J. A. Bursley
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INTERNATIONAL LIBRARY OF TECHNOLOGY

A SERIES OF TEXTBOOKS FOR PERSONS ENGAGED IN THE ENGINEERING PROFESSIONS AND TRADES OR FOR THOSE WHO DESIRE INFORMATION CONCERNING THEM. FULLY ILLUSTRATED AND CONTAINING NUMEROUS PRACTICAL EXAMPLES AND THEIR SOLUTIONS

ELECTRICITY AND MAGNETISM
ELECTRODYNAMICS
ELECTRICAL RESISTANCE AND CAPACITY
THE MAGNETIC CIRCUIT
ELECTROMAGNETIC INDUCTION
CHEMISTRY AND ELECTROCHEMISTRY
PRIMARY BATTERIES
ELECTRICAL MEASUREMENTS

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PREFACE

The International Library of Technology is the outgrowth of a large and increasing demand that has arisen for the Reference Libraries of the International Correspondence Schools on the part of those who are not students of the Schools. As the volumes composing this Library are all printed from the same plates used in printing the Reference Libraries above mentioned, a few words are necessary regarding the scope and purpose of the instruction imparted to the students of—and the class of students taught by—these Schools, in order to afford a clear understanding of their salient and unique features.

The only requirement for admission to any of the courses offered by the International Correspondence Schools, is that the applicant shall be able to read the English language and to write it sufficiently well to make his written answers to the questions asked him intelligible. Each course is complete in itself, and no textbooks are required other than those prepared by the Schools for the particular course selected. The students themselves are from every class, trade, and profession and from every country; they are, almost without exception, busily engaged in some vocation, and can spare but little time for study, and that usually outside of their regular working hours. The information desired is such as can be immediately applied in practice, so that the student may be enabled to exchange his present vocation for a more congenial one, or to rise to a higher level in the one he now pursues. Furthermore, he wishes to obtain a good working knowledge of the subjects treated in the shortest time and in the most direct manner possible.

In meeting these requirements, we have produced a set of books that in many respects, and particularly in the general plan followed, are absolutely unique. In the majority of subjects treated the knowledge of mathematics required is limited to the simplest principles of arithmetic and mensuration, and in no case is any greater knowledge of mathematics needed than the simplest elementary principles of algebra, geometry, and trigonometry, with a thorough, practical acquaintance with the use of the logarithmic table. To effect this result, derivations of rules and formulas are omitted, but thorough and complete instructions are given regarding how, when, and under what circumstances any particular rule, formula, or process should be applied; and whenever possible one or more examples, such as would be likely to arise in actual practice—together with their solutions—are given to illustrate and explain its application.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and to try and anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear is to give a picture or a diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives have been used, according to which will best produce the desired results. Half-tones have been used rather sparingly, except in those cases where the general effect is desired rather than the actual details.

It is obvious that books prepared along the lines mentioned must not only be clear and concise beyond anything heretofore attempted, but they must also possess unequaled value for reference purposes. They not only give the maximum of information in a minimum space, but this information is so ingeniously arranged and correlated, and the indexes are so full and complete, that it can at once be made available to the reader. The numerous examples and explanatory remarks, together with the absence of long demonstrations and abstruse mathematical calculations, are of great assistance in helping one to select the proper formula, method, or process and in teaching him how and when it should be used.

The first portion of this volume is devoted to the fundamental principles of electricity and magnetism, which form the foundation of electrical engineering. Every effort has been made to make the text clear and to cover the ground completely. Ohm's law, with its modifications and its applications, is treated at length. The methods of producing an electromotive force are fully described. The magnetic circuit and the interaction of currents and magnetic fields are fully explained. The subject of batteries has received special attention. One section is provided that treats of chemistry and electrochemistry, and furnishes the necessary instruction for a clear understanding of the action of batteries, which are described and illustrated in the section on Primary Batteries. The modern types of apparatus used in electrical measurements are well illustrated and their action explained. Numerous tests are given for making these measurements, as well as for determining the magnetic properties of iron.

The method of numbering the pages, cuts, articles, etc. is such that each subject or part, when the subject is divided into two or more parts, is complete in itself; hence, in order to make the index intelligible, it was necessary to give each subject or part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number it is preceded by the printer's section mark (§). Consequently, a reference such as § 16, page 26, will be readily found by looking along the inside edges of the headlines until § 16 is found, and then through § 16 until page 26 is found.

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ELECTRICITY AND MAGNETISM

INTRODUCTION

1. Electricity is the name given to the cause of all electrical phenomena. The word is derived from a Greek word meaning *amber*, that substance having been observed by the Greeks to possess peculiar properties that we now understand to be due to electricity.

Although electrical science has advanced sufficiently far to recognize the fact that the exact nature of electricity is unknown, yet recent research tends to demonstrate that all electrical phenomena are due to a peculiar strain or stress of a medium called *ether;* that when in this condition the ether possesses potential energy or capacity for doing work, as is manifested by attractions and repulsions, by chemical decomposition, and by luminous, heating, and various other effects. We do not know what electricity is; we only know the effects due to it and the laws that it follows.

2. Ether is a medium having the properties of matter of infinitely small density. All space, including the most complete vacuum, the space between the planets and the stars, as well as the interstices in the hardest crystal and the heaviest metal—in short, matter of every kind—is permeated by ether. It is comparable to an all-pervading jelly of almost perfect elasticity. An electrical disturbance produces a strain in this all-pervading jelly, or sets it in rapid vibration, thereby producing waves that travel through space and pass from one body to another with the velocity of light. When these waves in the ether reach a body capable of being excited by these particular waves, their motion is imparted to the molecules of that body, which in their turn begin to vibrate; that is, the body becomes electrically excited or electrified.

Light, heat, electricity, and magnetism are all supposed to be transmitted through space by some active condition of the ether, either in the form of longitudinal or of horizontal vibrations. If a bell is vibrating in a glass vessel, the sound can be heard from the outside; but if the vessel is put in communication with an air pump and exhausted, the sound grows fainter and fainter as the vacuum increases, showing that the sound needs the air for its transmission. A magnet enclosed in a glass vessel is just as active when the vessel is exhausted as when it is not. The filament of an incandescent lamp, although it glows in a vacuum, is visible from the outside of the globe, proving that air is not necessary for the transmission of light.

Furthermore, the sun's rays of heat and light are transmitted through space, a large portion of which must be devoid of air or other matter. It has also been noticed that an increase in the number of visible spots on the surface of the sun has a marked influence on magnetic needles, proving that the force of magnetism also travels through an apparently empty space. We must imagine the ether as all-pervading, as not alone surrounding material bodies, but penetrating through their interiors; that it, in fact, encircles the smallest particles, even the molecules and atoms.

Formerly, the phenomena of heat, light, magnetism, and electricity were all supposed to be actions of fluids; even today we speak of currents of electricity. This must not be taken too literally. It is more than likely that there is no flow existing, but simply a vibratory action. For instance, we do not speak of a current of heat; when we therefore speak of electricity as flowing, it is done more because of the facility it offers to explain the action of the various phenomena than because of a belief in the actual existence of a current.

3. In all probability, electricity is not a form of matter, because it does not possess most of the ordinary properties of matter. Electricity itself is not a form of energy, though energy may be necessary to move it under certain conditions, and electricity in motion is capable of performing work.

Electrical science is founded on the effects produced by the action of certain forces on matter, and all knowledge of the science is deduced from these effects. The study of the fundamental principles of electricity is an analysis of a series of experiments and the classification of the results in each particular case under general laws and rules. It is not necessary to keep in mind any hypothesis of the exact nature of electricity; its effects and the laws that govern them are quite similar to those of well-known mechanical and natural phenomena, and will be best understood by comparison.

4. Electricity may either appear to reside on the surface of bodies as a **charge,** or to flow through the substance or on the surface of a body as a **current.**

That branch of the science that treats of charges on the surface of bodies is termed *electrostatics*, and the charges are said to be *electrostatic*, or simply *static*, *charges* of electricity.

Electrodynamics is that branch which treats of the action of *electric currents*.

UNITS

FUNDAMENTAL UNITS

5. The electrical and magnetic units are based, as will be shown later, on the three fundamental units of *mass*, *length*, and *time*. They are, therefore, absolutely independent of all other considerations, and the system that they form is hence termed the **system of absolute units.**

These fundamental units are, respectively:

1. The *centimeter* as the unit of length.
2. The *gram* as the unit of mass.
3. The *second* as the unit of time.

This system is more often termed the **centimeter-gram-second system,** and is written **C. G. S. system.**

6. The **centimeter** was originally intended to represent exactly $\frac{1}{1000000000}$ of the distance from the pole to the equator along the surface of the earth. As a matter of fact, the centimeter is $\frac{1}{100}$ of a standard meter rod preserved in Paris and is equal to .3937 inch. Hence, 1 inch equals 2.54 centimeters, nearly.

7. The unit of mass, or quantity of matter, is the **gram.** It is the one-thousandth part of the mass of a standard weight preserved in Paris and was originally supposed to represent the quantity of matter contained in a cubic centimeter of pure water at the temperature of its maximum density, which is 3.9° C., or 39° F. As a matter of fact, the gram is equal in weight to 15.432 grains, or 453.6 grams is equal to 1 pound.

8. The unit of time is the **second,** and represents $\frac{1}{86400}$ part of a mean solar day.

SECONDARY UNITS

9. The **secondary units** derived from these fundamental units are defined as follows:

10. The unit of area is the **square centimeter,** and is the area contained in a square each of whose sides is 1 centimeter in length. 1 square centimeter equals .155 square inch; 1 square inch equals 6.452 square centimeters.

11. The unit of volume is the **cubic centimeter,** and is the volume contained in a cube each of whose edges is 1 centimeter in length. 1 cubic centimeter equals .06102 cubic inch; 1 cubic inch equals 16.387 cubic centimeters.

12. The unit of velocity, or the rate at which a body moves from one position to another, is defined as the velocity of a body moving through unit distance (1 centimeter) in unit time (1 second). The unit of velocity is, therefore, **1 centimeter per second.**

NOTE.—The word *per* in such expressions denotes that the quantity named before it is to be divided by the quantity named after it. Thus, to compute the velocity in centimeters per second, divide the number of centimeters by the number of seconds.

13. The unit of acceleration is that acceleration which imparts unit velocity to a body in unit time, or an acceleration of **1 centimeter-per-second per second.** The acceleration due to gravity imparts in 1 second a velocity considerably greater than this, for the velocity it imparts to falling bodies is about 981 centimeters (or about 32.2 feet) per second. The value differs slightly in different latitudes. At New York City the acceleration of gravity is $g = 980.26$; at the equator, $g = 978.1$.

14. The unit of force is the **dyne,** and is that force which, acting on a mass of 1 gram for 1 second, gives to it a velocity of 1 centimeter per second.

15. The unit of work is the **erg,** and is that amount of work performed when a force of 1 dyne is overcome through a distance of 1 centimeter; that is, the work done in pushing a body through a distance of 1 centimeter against a force of 1 dyne; the unit of work, the erg, therefore equals **1 dyne-centimeter.**

16. The unit of energy is also the **erg**; for the energy of a body is measured by the work it can do. The unit of energy, the erg, is therefore also **1 dyne-centimeter.**

17. The unit of power has no particular name in the C. G. S. system. It is defined as the **rate of doing work,** and is hence equal to **1 erg-per-second.**

18. The unit of heat, called the **gram-calory,** is the amount of heat required to raise the temperature of 1 gram of water 1° C. The unit of heat, called the **kilogram-calory,** is the amount of heat required to raise the temperature of 1 kilogram of water 1° C. Evidently, 1 kilogram-calory = 1,000 gram-calories.

ELECTRICAL UNITS

19. Unfortunately there are two distinct systems of electrical units, one called **electromagnetic units** and the other **electrostatic units.** The electromagnetic units are, moreover, subdivided into what are called **absolute,** or **C. G. S., units** and **practical units,** the practical units being merely some power of 10 times the C. G. S. unit. Thus, the practical unit of electromotive force is 10^8 times as large as the C. G. S. electromagnetic unit of electromotive force. Fortunately, however, the practical electrician seldom if ever needs to use any other than the practical electromagnetic and magnetic units. The electromagnetic system is derived from the force exerted between two magnet poles, and the electrostatic system from the force exerted between two electrostatic charges or quantities of electricity.

MAGNETIC UNITS

20. There are a number of magnetic units used in connection with the electromagnetic units. The magnetic units are derived from the force exerted between two magnet poles. These units are all C. G. S. units, and only a few of them have been given specific names, such as the maxwell and gauss. All electrical and magnetic units have been derived from the fundamental units of length, mass, and time.

PRACTICAL UNITS

21. Several of the absolute, or C. G. S., units would be inconveniently large and others inconveniently small for practical use. The **practical units** have therefore been adopted and named after distinguished men of science, such as **Ampere, Coulomb, Volta, Ohm, Joule,** and **Watt.** The various units will be explained as they naturally occur and will finally be collected into a table.

ELECTROSTATICS

STATIC CHARGES

22. When a glass rod or a piece of amber is rubbed with silk or fur, the parts rubbed have the property of attracting light particles of matter, such as pieces of silk, wool, feathers, gold leaf, pith, etc., which, after momentary contact, are repelled. These attractions and repulsions are caused by a static charge of electricity residing on the surface of those bodies. A body in this condition is said to be **electrified.**

23. A better experiment for demonstrating this action is to suspend a small pith ball by a silk thread from a support or bracket, as shown in Fig. 1. This apparatus is sometimes called an *electric pendulum.* If a static charge of electricity be developed on a glass rod by rubbing it with silk, and the rod be brought near the pith ball, the ball will be attracted to the rod, but, after momentary contact, will be repelled. By this contact, the ball receives a charge of the same nature as that on the glass rod, and as long as the two bodies retain their charges, mutual repulsion will take place whenever they are brought near each other. If a stick of *sealing wax*, electrified by being rubbed with fur, is brought near another pith ball, the same results will be produced, that is, the ball will fly toward the sealing wax and after contact will be repelled. But the charges, respectively, developed in these two cases are not of exactly the same character. For, if after the pith ball in the first case has been touched with the glass rod and repelled, the electrified sealing wax be brought in the vicinity, attraction takes place between the pith ball and the sealing wax. On the other hand, if the pith ball be

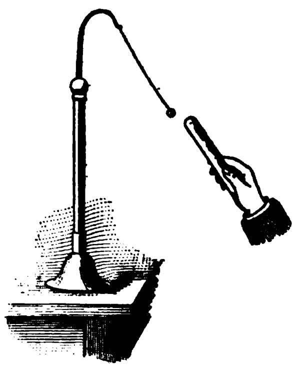

FIG. 1

charged with the electrified sealing wax, the pith ball will be repelled by the sealing wax and attracted by the glass rod.

24. We have, therefore, to distinguish between two kinds of electrification, and to make this distinction definite, the following designations have been adopted:

An electric charge developed on glass by rubbing it with silk has been termed a **positive charge** (+), and that developed on resinous bodies by friction with flannel or fur a **negative charge** (—).

25. Neither charge is produced alone, for there is always an equal quantity of both charges produced, one charge appearing on the body rubbed and an equal amount of the opposite charge on the rubber.

26. The intensity of the charge developed by rubbing the two substances together is independent of the actual amount of friction that takes place between the bodies. For, in order to obtain the highest possible degree of electrification from two dissimilar substances, it is only necessary to bring every portion of one surface into intimate contact with every particle, or every portion, of the other surface; when this is done, no extra amount of rubbing can develop any greater charge on either substance.

27. From these experiments are derived the following laws:

When two dissimilar substances are placed in contact, one of them always assumes the positive and the other the negative condition, although the amount may sometimes be so small as to render its detection very difficult.

Bodies electrified with similar charges are mutually repellent, while bodies electrified with dissimilar charges are mutually attractive.

28. Table I gives a list called the **electric series,** where the substances are arranged in such order that each receives a *positive* charge when rubbed with any of the bodies following, and a *negative* charge when rubbed with any of those that precede it.

TABLE I

THE ELECTRIC SERIES

1. Fur	6. Cotton	11. Sealing wax
2. Flannel	7. Silk	12. Resins
3. Ivory	8. The body	13. Sulphur
4. Crystals	9. Wood	14. Gutta percha
5. Glass	10. Metals	15. Guncotton

For example, glass when rubbed with fur receives a negative harge, but when rubbed with silk receives a positive charge.

ELECTROSTATIC INSTRUMENTS

29. Gold-Leaf Electroscope.—The **electroscope** is an instrument for detecting static charges of electricity and for determining their condition, whether positive or negative; but not for measuring the intensity of the charges. The pith ball suspended by a silk thread acts as a simple electroscope.

A more sensitive instrument, known as the **gold-leaf electroscope,** is shown in Fig. 2. It consists of two gold leaves *a* suspended in a glass jar *J*, which serves to protect them from drafts of air and to support them from contact with the earth. The gold leaves *a* are supported side by side in the jar by a brass rod or wire *x*, that passes through a glass tube *b*, which in turn passes through a cork in the mouth of the jar. The upper end of the brass rod is fastened to a flat metallic plate, or disk *c*.

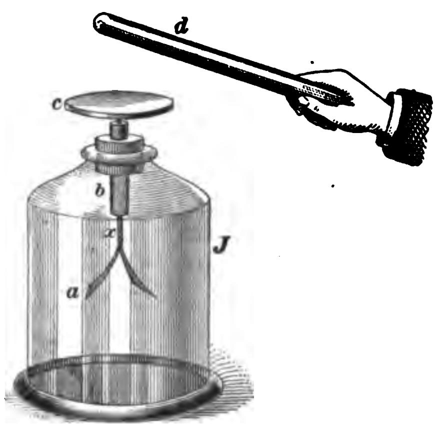

FIG. 2

This instrument can be made to show not only whether a body is electrified or not, but also to show the kind of electricity with which a body is charged.

Let us rub a glass rod *d* with silk and hold it in the neighborhood of the plate *c*, without making actual contact. The gold leaves will diverge. We know the glass rod is positively electrified, and, when it is brought near the plate *c*, a separation of the (previously neutralized) electric charges in the metallic portions of the electroscope takes place, the negative electricity being attracted to the plate *c* by the positive charge on the glass rod *d*, and the positive charge being repelled to the gold leaves. This charges the two gold leaves positively, and, as similarly charged bodies are mutually repellent, they are caused to diverge. If the rod is withdrawn without touching the plate *c*, the positive and negative charges on the electroscope reunite, thus allowing the gold leaves to come together. This action that causes a separation of the two electricities in a body by the proximity of a charged body, is called *electrostatic induction*. This will be treated farther on at greater length.

Again bring the rod near the electroscope, and this time touch the plate *c*. On removing the rod, the leaves remain diverged, because the negative charge that was attracted to the plate by the approaching positive charge on the rod is neutralized by an equal amount of the positive charge from the rod at the moment that the rod and plate touch, and when the rod is removed there is left only a positive charge on the gold leaves. The charge remaining on the gold leaves is, therefore, positive, and the gold leaves will remain diverged, although perhaps a trifle less after the rod is removed than just before the rod touched the plate, because the remaining positive charge is now free to distribute itself over the entire metal surface of the electroscope and not only over the gold leaves. The result is practically the same as if the rod had given up its positive charge to the electroscope. Knowing the character of this charge, we are able to compare other charges with it and find out whether they are of the same or of a different kind.

30. For instance, let the glass rod be rubbed with flannel. The question now is whether the glass rod is charged with positive or with negative electricity. On bringing it near the plate on the electroscope, the leaves show a tendency to close together. This is evidently because there is less repulsion between them; that is, their positive charge, previously received when the plate was touched with the glass rod that had been rubbed with silk, must have become less. Evidently, then, the rod must be negatively charged, because it attracts a part of the positive charge on the leaves toward it, leaving them less strongly charged. On the other hand, if the rod were positively charged, the gold leaves would spread wider apart, because the positive charge on them would be made stronger, due to a further separation of positive and negative charges, the negative being attracted to the plate, and the positive, repelled to the gold leaves, would increase the quantity of positive charge already there.

From these facts we may draw the conclusions that if the gold leaves diverge, the body being tested has a charge of the same kind as that on the electroscope; if the gold leaves approach each other, the charge is of the opposite kind.

31. The **torsion balance** is an instrument used to measure the force exerted between two electrified bodies. It consists of an arm or lever of some light, insulating material, such as a straw or piece of wood, provided at one end with a gilt pith ball *n*, Fig. 3, and suspended in a glass jar by a fine silver wire. The wire passes up through a glass tube and is fastened to a brass stopper *b*, called the **torsion head.** The torsion head is graduated in degrees, and is capable of being revolved around upon the glass tube. Another gilt pith ball *m* is fastened to the end of the vertical glass rod *a*, which is inserted through an opening in the top of the jar. A narrow strip of paper,

FIG. 3

also divided into degrees, encircles the glass jar at the level of the two pith balls.

32. To use the torsion balance, turn the torsion head around until the two pith balls *m* and *n* just touch each other. Remove the glass rod *a* and communicate the charge to be measured to the gilt ball *m*. Replace the glass rod in the jar. The two gilt balls will touch each other momentarily, and half of the charge will pass from *m* to *n*. As both balls possess similar charges, they will immediately repel each other; the ball *n*, being driven around, twists up the wire to a certain extent. The force of torsion in the wire will eventually balance the force of repulsion, and the ball *n* will come to rest when the balls are separated by a certain distance. In any wire, the force of torsion is proportional to the amount of twist, or, in this case, to the angle of torsion; hence, the force exerted between the two balls can be measured by the angle described by the ball *n*. If the arc through which the ball moves is not too large, the ball will practically move in a straight line, and it may therefore be said that the force of torsion is proportional to the arc, which may be read directly off the scale, instead of to the direct distance between the balls, which would have to be calculated from the arc.

33. Law Proved by Torsion Balance.—By means of the torsion balance it is possible to prove that *the force exerted between two small bodies statically charged with electricity varies inversely as the square of the distance between them.* Thus, if two electrified bodies 2 inches apart repel each other with a certain force, this force will be four times greater if the distance between them is decreased to 1 inch. This law holds good for both repulsion and attraction, and also when the charges on the two bodies are of unequal amounts.

At a given distance, the force of attraction or repulsion between two bodies will be *proportional to the product of the two quantities* of electricity with which they are charged. For instance, if one body is charged with

5 units, and another with 3 units of electricity, the force acting between them will be $5 \times 3 = 15$ times greater than it would be if each body had received but 1 unit.

Hence, if two charges, or quantities, of electricity Q and Q^1 are placed in air at a distance d apart they will attract or repel each other, depending on whether they are unlike or like charges, with a force F, such that

$$F = \frac{Q \times Q^1}{d^2} \qquad \textbf{(1)}$$

34. Electrostatic Unit Charge or Quantity of Electricity.—In formula **1**, if $F = 1$ dyne, $d = 1$ centimeter, and $Q = Q^1$, then Q and Q^1 must each be equal to 1. Hence, the electrostatic unit charge, or quantity, of electricity is that charge, or quantity, which, when placed in air at a distance of 1 centimeter from another equal and similar charge, or quantity, of electricity, will be repelled with a force of 1 dyne.

35. The Coulomb.—There is another unit of quantity based on what is called the electromagnetic system of units. This is called the **coulomb,** and its value is 3,000,000,000 times that of the electrostatic unit. That is, the coulomb is 3×10^9 electrostatic unit charges or quantities of electricity. The coulomb will be more fully explained later.

POTENTIAL

36. The term **potential,** as used in electrical science, is analogous with *pressure* in gases, *head* in liquids, and *temperature* in heat.

When an electrified body *positively* charged is connected to the earth by a conductor, electricity is said to *flow from* the body *to* the earth; and, conversely, when an electrified body *negatively* charged is connected to the earth, electricity is said to *flow from* the earth *to* that body. That which determines the *direction of flow* is the relative electrical *potential*, or *pressure*, of the two charges in regard to the earth.

37. It is impossible to say with certainty in which direction electricity really flows, or, in other words, to declare which of two points has the higher and which the lower electrical potential, or pressure. All that can be said with certainty is that, when there is a *difference* of *electrical potential*, or *pressure*, an electric current tends to flow *from* one point, which is said to be at the higher, *to* another point, which is said to be at the lower, potential, or pressure.

For convenience, it has been arbitrarily assumed and universally adopted that that electrical condition called *positive* is at a *higher*, *potential*, or *pressure*, than that called negative, and that an electric charge, or current, flows *from* a positively *to* a negatively electrified body.

38. The zero or normal level of water is taken as that of the surface of the sea, and the normal pressure of air as that of the atmosphere at the sea level; similarly, there is a zero pressure, or potential, of electricity in the earth itself. It may be regarded as a reservoir of electricity of infinite quantity, and its pressure, or potential, taken as zero. For this reason, all electric charges, or currents, have the tendency to reach this zero level, exactly as water on a mountain top tends to flow to the sea level. For this reason, it becomes necessary to insulate most electrical apparatus, otherwise the electric charge, or current, it generates or carries will leak away to the earth. The electrical condition that is called *positive* is assumed to be at a *higher* potential than the earth, and that called *negative* is assumed to be at a *lower* potential than the earth.

CONDUCTORS AND INSULATORS

39. Only that part of a dry glass rod that has been rubbed will be electrified; the charge does not pass to other parts of the rod. This is evident from the fact that the unrubbed parts produce neither attraction nor repulsion when brought near an electroscope. The same is true of a piece of sealing wax or resin. These bodies do not readily *conduct* electricity; that is, they *oppose*, or *resist*, the passage of electricity through them. Therefore, electricity can reside only as a charge on that part of their surfaces where it is developed. Experiments show that when a metal receives a charge at any point, the electricity immediately passes, or flows, through its substance to all parts. Metals, therefore, are said to be *good conductors* of electricity. Bodies have accordingly been divided into two classes; namely, **non-conductors,** or **insulators,** those bodies that offer a great, or high, resistance to the passage of electricity; and **conductors,** or those that offer a comparatively low resistance to its passage. This distinction is not absolute, for all bodies conduct electricity to some extent, while there is no known substance that does not offer some resistance to its flow.

40. Electrical resistance may be defined as a general property of matter, varying with different substances, by virtue of which matter *opposes,* or *resists,* the passage of electricity.

41. Conductance is the facility with which a body transmits electricity, and is the reciprocal, or opposite, of resistance. For instance, copper is of low resistance and high conductance; wood is of high resistance and low conductance.

Table II gives a list of conducting and non-conducting substances.

TABLE II

CONDUCTORS AND INSULATORS IN ORDER OF THEIR VALUE

Conductors	Insulators (Non-Conductors)	
Silver	Dry air	Glass
Copper	Shellac	Mica
Other metals	Paraffin	Ebonite
Charcoal	Amber	India rubber
Plumbago	Resin	Silk
Moist earth	Sulphur	Paper
Water	Wax	Oils

A general idea of these values may be obtained from the fact that water has about 6,754 million times greater resistance than copper.

42. In dividing the different substances into two classes, it should be understood that it is done only as a guide for the student. Between these classes are many substances that might be included in either, and no hard or fast line can be drawn. The list is arranged in order of the specific conductance or conductivity of the different substances, beginning with silver, which is the best conductor known.

ELECTROSTATIC INDUCTION

43. It has been seen that when electricity has been transferred from one body to another by actual contact, an attraction or repulsion will take place; and it was also seen, when considering the gold-leaf electroscope, how a charge could be present on the gold leaves when no actual contact had been made with charged bodies. In the latter case an electric charge is said to be *induced* in a conductor when that conductor is brought into the vicinity of an electrified body. This effect is termed **electrostatic induction,** and the range of space in which it can take place is called an **electrostatic field.**

44. If the conductor $A\ B$, Fig. 4, is supported from contact with the earth by an insulator, and is then brought into the electrostatic field of the conductor C, but not touching C, which is electrified with a positive charge, then:

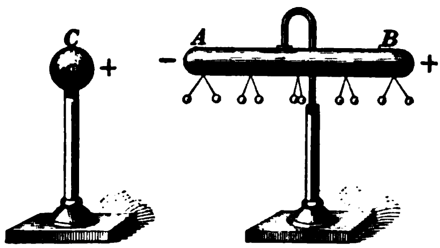

FIG. 4

1. A charge will be produced on $A\ B$, as is shown by the pith balls spreading apart.

2. This charge will be *negative* at the end A nearest C and *positive* at the end B farthest from C, as can be shown by an electroscope.

3. The charges at A and B are equal to each other; for, if the conductor $A\ B$ is removed from the vicinity of the

conductor *C* without having touched *C*, the opposite charges immediately neutralize each other; that is, no electrification will be indicated by the pith balls.

4. Again, as *C* is brought nearer and nearer *A*, the charges of opposite signs on the approaching surfaces attract each other more and more strongly until *C* is approached very near, and then the insulating capacity of the intervening substance (in this instance air) will break down, and the charges will rush across and reunite with such avidity that a spark will be produced between the two bodies. Two charges rushing together in this manner neutralize each other, leaving now free the induced positive charge, that was formerly repelled to the end *B* of the conductor; hence, positive electricity is distributed all over the surface of *A B*.

5. Or, if the conductor *A B* is momentarily touched by a conductor connected to the earth when it is under the influence of *C*, the positive charge will neutralize an equal quantity of negative electricity in the earth and the negative charge will remain when *A B* is removed from the field of *C*. The charge that passes to the earth from *A B* is called a **free charge,** while that charge which is held by the inductive influence of *C* is a **bound charge.** If the connection between *A B* and the ground is broken and *C* is removed, the induced negative charge *A B* is released; it is also free, and will now distribute itself over the whole surface of the conductor. Both free and bound charges can be negative or positive, depending on the sign of the charge on *C*.

45. When two conducting bodies, both electrified with equal dissimilar charges, are touched together momentarily, the two charges will neutralize each other, no trace of either remaining; but if they are unequal, the smaller charge will neutralize an equal amount from the larger and leave a charge that is equal to the difference between the two original charges, the sign of the remaining charge being the same as that of the larger one. Before the bodies can be separated, the remaining charge will divide equally between the two bodies. For example, two gilt balls *A* and *B* are charged respectively with +20 and −4 units of electricity. When the balls are placed in contact, the −4 charge on *B* will neutralize a +4 charge on *A* and leave a +16 charge, which immediately divides equally between the two balls; that is, a charge of +8 units remains on each ball when they are separated. If the two charges are similar but unequal in amount, they will, on being touched together, be distributed so that the intensity of the charge will be the same on each body.

ELECTROPHORUS

46. The **electrophorus,** Fig. 5, is an instrument devised for the purpose of obtaining an almost unlimited number of

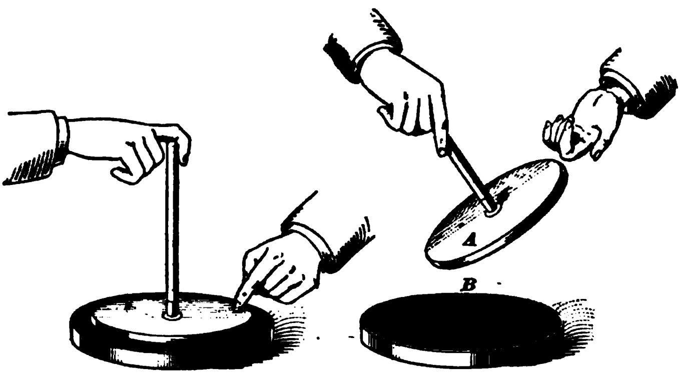

FIG. 5

static charges of electricity from one single charge, and is based on the principle of electrostatic induction. It consists of two main parts: a thin cake of resinous material cast in a round metal dish, or pan, *B*, about 1 foot in diameter, and a round disk *A*, of slightly smaller diameter, made of metal and provided with a glass handle. In modern instruments, *B* is usually made of ebonite.

47. When using the electrophorus, the resinous cake must first be beaten or rubbed with a warm piece of woolen cloth or fur. The disk, or cover, is then placed on the cake, the metal part is touched momentarily with the finger to liberate the *free* charge, and then removed by taking it up by the insulating handle. The metal part is now found to be powerfully electrified all over with a *positive* charge; so much so as to yield a considerable spark when the hand is brought near it. The spark is due to the positive charge escaping through the hand and body to the earth, where it is neutralized by an equal amount of negative electricity. The cover may be replaced, touched, and again removed, and will thus yield any number of sparks, the original charge on the resinous plate meanwhile remaining practically as strong as ever.

After the cake has been beaten with the fur, its condition is that of Fig. 6; it is charged with negative electricity. When

FIG. 6

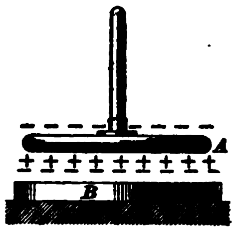

FIG. 7

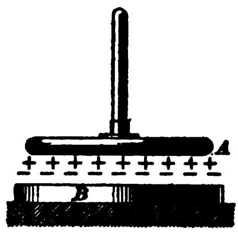

FIG. 8

the disk *A* is approaching the cake, the latter will act inductively on the disk, and attract a positive charge on its lower side and repel a negative charge to its upper side. These charges will increase in amount until they reach a maximum, when contact is made with the cake. This condition of cake and disk is represented in Fig. 7. Should the disk be now touched, the free negative charge will be neutralized by electricity flowing through the observer's body from the earth, while the positive electricity will remain as a bound charge, as shown in Fig. 8. If the disk is now lifted, the positive charge will be no longer bound, and will distribute itself all over the disk, as illustrated in Fig. 9.

The charges given to the disk will not diminish the original charge on the cake, as the action is purely inductive, and the recharging and discharging of the disk could go on forever if the cake were not subjected to a certain amount of leakage through the atmosphere, particularly when the air is damp. The charge must therefore be occasionally replenished. Evidently, the supply of energy represented by each charge must be drawn from some source, and it is of some interest to inquire into its origin. The fact is that when the disk is removed from the cake after being charged, it resists being removed more than when it was neutral. This supply of muscular energy is the real measure of the energy dissipated in each discharge of the disk.

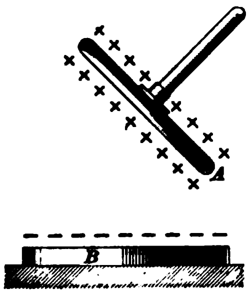

FIG. 9

48. A static charge of electricity is not usually distributed uniformly over the surface of conducting bodies. Experiments show that there is more electricity on the edges and corners than on their flatter parts.

The term **electric density** is used to signify the amount, or quantity, of electricity residing on a small unit of area of any part of a body, the distribution being supposed to be uniform over that small part of the surface.

The average electric density is the quotient arising from dividing the total charge of electricity in units of quantity residing on the surface of a body by the area of the surface in square inches. For example, a charge of 240 units of electricity is imparted to a sphere, the surface area of which is 40 square inches; then, the electric density over the surface of the sphere is $\frac{240}{40} = 6$ units of electricity per square inch.

THE CONDENSER

49. It has been shown that opposite charges attract and hold one another; that electricity cannot flow through glass and yet can act across it by induction. If a piece of tin foil is stuck on the middle of each face of a thin plate of glass, and one of the pieces is electrified with a positive charge and the other with a negative charge, the two charges will attract each other, or, in other words, they are held, or ***bound***, by each other. It will be found that these two pieces of tin foil may be charged much stronger in this manner than either of them could possibly be if they were stuck to the glass one at a time and then electrified. This property of retaining and accumulating a large quantity of static charges that two conductors possess when placed side by side and separated from each other by a non-conductor, is called **electrostatic capacity.**

50. A **condenser** is an apparatus for condensing or accumulating a large quantity of static electricity on a surface, and consists of two conductors separated by a layer of some non-conducting material called the **dielectric.** One of the conductors must be well insulated from the earth, but the other may or may not be connected to the earth by a conductor.

Let us take two plates *A* and *B*, Fig. 10, interpose a glass plate *C* between them, and see what will be the effect when

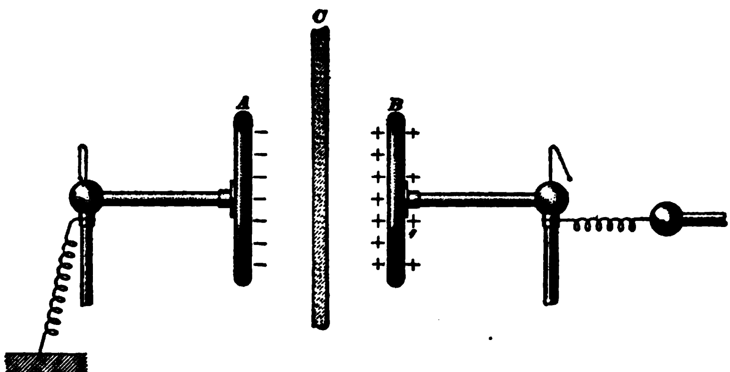

FIG. 10

B is charged with positive electricity from some generator of static electricity and *A* is connected with the ground. The positive charge on *B* will, through the glass, induce a negative charge on *A* and repel the positive electricity to the ground. The negative charge on *A* will collect on the face nearest *B* and react on the positive charge of the latter, attracting it nearer the glass; then the plate *B* will be capable of receiving more electricity from the generating source. This action through the glass will continue as long as the potential of the source is able to add new charges to *B*.

If the two plates are brought nearer to the glass, the attraction between the charges will increase and the inductive action will be greater; a larger quantity can therefore be accumulated on the plates. After the disks have been strongly charged, the wires may be removed and the disks placed farther from each other. The attraction between the charges will then be less; they will be less bound; and more of the charge will be free and able to spread over the entire surface. That this is so can be seen by watching the pith balls suspended from the conductors on each side. They will diverge, giving the impression that new charges have been added to *A* and *B*, while the fact is that the capacities of *A* and *B* have diminished, giving them the appearance of being more electrified than before, because there is a greater quantity of free charge.

The ground plate *A* has the effect of greatly increasing the capacity of an insulated conductor, the surface density on the side opposite the ground plate being very great.

It will be noticed that in Fig. 10 the pith-ball pendulums do not diverge through the same angle; this is a result of the method of charging the condenser. When *B* is connected to the generating source, the right side of *B* and its rod will have the same potential as the machine, while the left side of *A* and its rod will have zero potential. When *A* and *B* are disconnected from the ground and generator, respectively, *B* still retains the surplus of electricity residing on its right side, while the left side of *A* is still at zero potential; hence, the pendulums will remain as before.

51. One form of condenser, known as the **Leyden jar,** is shown in Fig. 11. It consists of a glass jar *J*, coated, up to a certain height, on the inside and outside with tin foil. A brass knob *a* is fixed on the end of a stout brass wire which passes downwards through a lid or stopper of dry, well-varnished wood, and connected by a small, loose brass chain with the inner coating of the jar.

FIG. 11

52. The jar may be charged by touching the knob *a* to the terminal *C* of an electrical machine, which will be considered later, or to the charged plate of an electrophorus, the jar being either held in the hand by the outer tin-foil coating or connected to the earth by a wire or chain. When a positive charge is thus imparted to the inner coating, it acts inductively on the outer coating and attracts a negative charge on the face of the outer coating nearest the glass and repels a positive charge to the outside of the outer coating. This outer charge passes through the hand or any conductor to the earth.

This form of Leyden jar has several weak points, the main one being the difficulty of keeping the outer and inner coatings well insulated from each other. It is known that moisture collects very readily on the surface of glass, and that this and the dust that is always liable to collect make it possible for the electric charge to leak from the brass wire across the stopper and the outside of the jar to the outside coating.

An improvement on this form of Leyden jar is one without any wooden lid or stopper, where the stout brass rod is fastened at the bottom to a plate that rests directly on the bottom of the jar, thus utilizing the whole length of the inside and outside glass surface, as well as the air space between them, as insulators.

53. Location of Charge.—Benjamin Franklin discovered that the seat of the charge in a Leyden jar is not on the tin foil, but on the glass. He proved this by so making the coatings of a jar that they could be separated from the jar after the latter had been charged. He then found that the coatings contained very little electricity. After having restored them to a neutral condition, the jar was put together again. It was now found to have a charge almost as large as before, proving that the coatings merely serve the purpose of distributing the charge over the surface of the dielectric.

54. Residual Charge.—It was also found that after a Leyden jar had been discharged, there remained a certain residue of charge on the glass, which after a while will emanate and collect on the surface and will be able to give a second spark. This can be repeated a number of times, each succeeding spark becoming feebler and feebler. It is known that the dielectric between two charged coatings is subject to a certain strain, which is so great that a Leyden jar when charged may even break under it.

55. Battery of Jars.—If the knobs and outer coatings of several Leyden jars are joined together (knobs to knobs and outer coatings to outer coatings), they will constitute what is called a **battery of Leyden jars.** The potential difference between the two coatings will be the same, but its capacity will increase in proportion to the number of jars. A battery of this kind must be handled with great care, as a shock from it may be very severe.

56. Conditions Governing Capacity.—That the area of the metal coatings influences the capacity of a condenser and is directly proportional to the same will hardly need a proof; because a large condenser may simply be supposed to be made up of several smaller ones, the area of one coating of the large condenser being equal to the sum of the areas of one set of coatings of the smaller condensers. Thus, if two Leyden jars have their coatings connected, inner with inner and outer with outer, they have the same electrostatic capacity as a single large jar with a coating of equal area. We have already seen that the nearer the two conductors are placed, the more intense is the inductive action between them. It can be proved experimentally that the capacity of a condenser with plain parallel plates is inversely proportional to the distance between the plates. It has also been found that the dielectric medium plays an active part in induction, and that the capacity of a condenser may be decreased or increased, depending on the substance used to separate the two coats or metal plates. It is found that the effect of electrostatic induction is greatly increased by placing some other substance than air, such as glass or paper, between the two bodies.

57. Inductivity.—The relative facility with which bodies allow electrostatic induction to act across them, is called their **inductivity.** The word *inductivity* is now coming into use in place of the inconveniently long term *specific inductive capacity* that was formerly used. Inductivity is analogous to the word conductivity, which is used in connection with the word conductors, and is much preferable to the older and longer term.

The inductivity varies with different substances, but almost all non-conductors are better than air, which is assumed to have an inductivity of 1. Inductivity of dielectrics will be considered in another section in connection with the capacity of condensers.

58. From what has been said, we conclude that the capacity of a condenser depends on (1) the size and form of the condenser plates, (2) the thinness of the dielectric medium between them, and (3) the inductivity of the dielectric medium.

59. Dielectric.—Any substance that allows electrostatic induction to act across it is termed a **dielectric.** All dielectrics are non-conductors. There is this distinction between a dielectric and an insulator: that the more resistance a substance offers to the flow of an electric charge or current, the better insulator it is; whereas, the less resistance a substance offers to an inductive influence across it, the better dielectric it is said to be. A good insulator may be a poor dielectric, but all dielectrics are insulators. Dry air, for instance, is a very good insulator but a poor dielectric.

60. Static Machines.—As the electric charges that are obtainable from an electrophorus are rather limited in quantity and of relatively low potential, machines were early devised for the production of large electrostatic charges at high potential. About the only practical use of electrostatic, or, as they are usually termed, static machines is for the treatment of certain diseases by physicians, for the operation of Roentgen-ray tubes, for college lectures, and for laboratories ; therefore, they are not of sufficient importance to warrant a detailed description in a Course of this character.

MAGNETISM

NATURAL MAGNETS

61. Near the town of Magnesia, in Asia Minor, the ancients found an ore that possessed a remarkably attractive power for iron. This attractive power they named **magnetism,** and a piece of ore having this power was termed a **magnet.** The ore itself has since been named **magnetite,** and has been found to be a chemical combination of about 72 parts of iron and 28 parts of oxygen, by weight.

62. A still more remarkable discovery was made concerning this ore. It was found that when a piece of the ore was hung from a thread, it invariably swung around to such a position that one of its ends pointed north and the other south. It was also observed that the same end always pointed north. Due to this fact, small pieces of the ore so suspended were used in navigation. Ships could be steered in any direction by its aid, because the direction of the north was always shown by one end of the stone. From this fact the name **lodestone** (meaning *leading stone*) was given to the natural ore.

ARTIFICIAL MAGNETS

63. When a bar or needle of hardened steel is rubbed with a piece of lodestone, it acquires magnetic properties similar to those of the lodestone without the latter losing any of its magnetism. Such bars are called **artificial magnets.**

Artificial magnets that retain their magnetism for a long time are called **permanent magnets.** The common form of artificial magnets is a bar of steel bent in the shape of a horseshoe and then hardened and magnetized. A piece of soft iron called an **armature,** or **keeper,** is placed across the two free ends, which helps to prevent the magnet from losing its magnetism.

64. If a bar magnet is dipped into iron filings, the filings are attracted toward the two ends and adhere there in tufts, while toward the center of the bar, half way between the

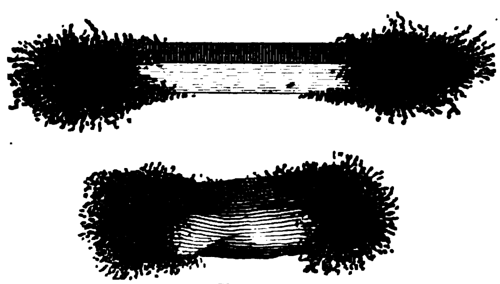

FIG. 12

ends, there is no such tendency (see Fig. 12). That part of the magnet where there is no apparent magnetic attraction is called the **neutral line,** and the parts around the ends where the attraction is greatest are called **poles.** An imaginary line drawn through the center of the magnet from end to end connecting the two poles together, is termed the **axis of magnetism.**

65. The **magnetic compass** consists of a magnetized steel needle, Fig. 13, resting on a fine point, so as to turn freely in a horizontal plane. When not in the vicinity of other magnets or magnetized iron, the needle will always come to rest with one end pointing toward the north and the other toward the south. The end pointing northwards is the north-seeking pole, and is invariably called the **north pole**; the opposite end is called the **south pole.** This polarity applies as well to all magnets.

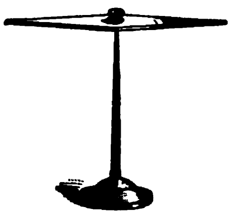

FIG. 13

66. If the north pole of one magnet is brought near the south pole of another magnet, *attraction* takes place; but if two north poles or two south poles are brought together, they *repel* each other. In general, *like magnetic poles repel one another; unlike poles attract.*

67. The earth is a great magnet whose magnetic poles coincide nearly, but not quite, with the true geographical north and south poles. By the laws of attraction and repulsion just given, it is seen why a freely suspended magnet, therefore, will always point in a north-south direction. The north-seeking pole of a magnetic needle, or other magnet, although invariably known as a north pole, is, of course, of opposite polarity to the north pole of the earth; nevertheless, the north-seeking pole is never called a south pole. If it were only customary to do so, it would be more correct to call the north-seeking pole a south pole, or to call the earth's north pole a south pole.

68. It is impossible to produce a magnet with only one pole. If a long bar magnet is broken into any number of parts, each part will still be a magnet and have two poles, a north and a south.

69. Magnetic substances are not necessarily magnets, that is, they do not always possess poles and neutral lines, but nevertheless they are capable of being attracted by a magnet. A piece of soft iron will attract either pole of a magnet, or will itself be attracted toward either pole of a magnet, but when not in the vicinity of a magnet it has no defined poles nor will it attract another piece of unmagnetized iron. In addition to iron and its alloys, the following metals are magnetic substances: nickel, cobalt, manganese, cerium, and chromium. These metals, however, possess magnetic properties in a very inferior degree compared with iron and its alloys. Most all other known substances are non-magnetic substances.

70. The space surrounding a magnet is called a **magnetic field**; or, in other words, a magnetic field is a region in which a magnetic pole is acted on by a force tending to pull it in some direction or other.

MAGNETIC LINES OF FORCE

71. Magnetic attractions and repulsions act in definite directions along imaginary lines that are called **lines of magnetic force**, or simply **lines of force.** Their position in any plane may be shown by placing a sheet of paper over a magnet, and sprinkling fine iron filings over the paper. In the case of a bar magnet lying on its side, the iron filings will arrange themselves in curved lines extending from the north to the south poles, as shown in Fig. 14. A view of the magnetic field looking toward either pole of a bar magnet

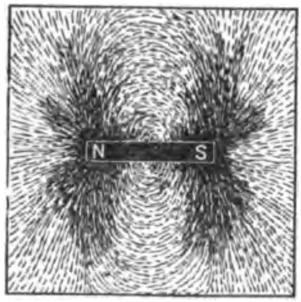

FIG. 14

would exhibit merely radial lines, as shown by the iron filings in Fig. 15.

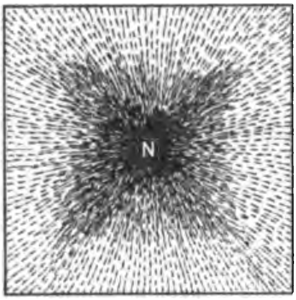

FIG. 15

72. Direction of Lines of Force.—*Every* line of force is assumed to pass *out from* the north pole, make a complete circuit through the surrounding medium, and return *into* the south pole; from thence through the magnet to the north pole again, as shown in Fig. 16.

This is called the **direction of the lines of force,** and the path that they take is called the **magnetic circuit.** Every line of force forms a complete magnetic circuit by itself.

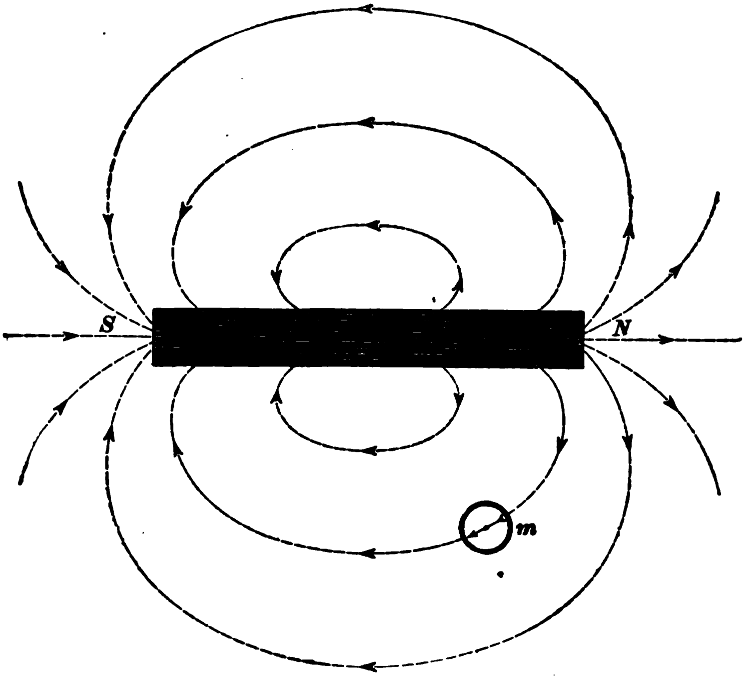

FIG. 16

The *direction of the lines of force* in any magnetic field can be traced by a small freely suspended magnetic needle, or a small compass such as is shown by *m* in Fig. 25. The north pole of the needle will always point in the direction of the lines of force, the axis of the needle lying parallel or tangent to the lines of force at that place. If the needle is moved bodily in the direction toward which its north pole points, its center or pivot will describe a path coinciding with the direction of the lines of force along that part of the magnetic field. In Fig. 16 the arrowheads indicate the direction of the lines of force. It will be noted that in Figs. 14, 15, 17 and 18, the magnetic lines are shown in one plane only, namely, in the plane of the paper. It should be borne in mind, however, that they extend from the magnet in every direction, above, below, and to both sides.

The direction of a line of force may also be defined as the direction in which an imaginary free north pole would move along the line of force if not acted upon by any force other than that due to the magnetic field.

73. Lines of force can never intersect each other. When two opposing magnetic fields are brought together,

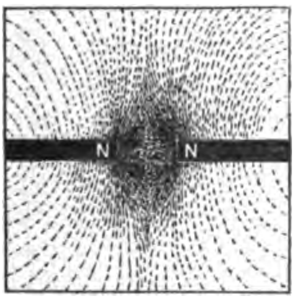

FIG. 17

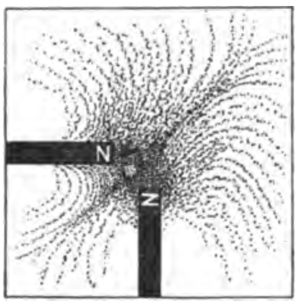

FIG. 18

the lines of force from each will be crowded and distorted from their original direction until they coincide in direction with those opposing and form a resultant field, in which the direction of the lines of force will depend on the relative strengths of the opposing fields. The action of the lines of force when opposing each other in direction is shown in Figs. 17 and 18, by the aid of iron filings.

74. Consequent Poles.—If two straight magnets are placed end to end with their like poles together, the compound bar will have three poles, one at each end of the same polarity and one in the middle of opposite polarity.

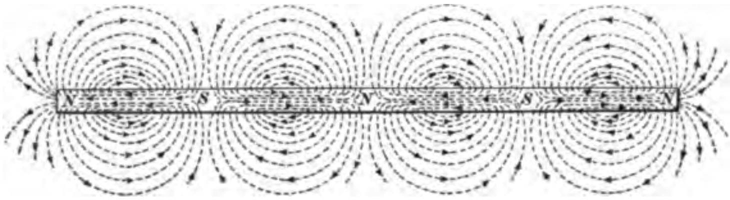

FIG. 19

The pole in the middle is really made up of two like or *consequent* poles. A single bar may be magnetized so as to have an odd number of poles, in addition to the two of like polarity at the ends; in this case the intermediate poles are called **consequent poles.** A bar so magnetized as to have consequent poles is shown in Fig. 19.

MAGNETIC FIELD PRODUCED BY A CURRENT OF ELECTRICITY

75. When a current of electricity flows through a conductor, such as a wire, a magnetic field is produced in the region surrounding the conductor. If a conductor through which there is flowing a current of electricity is threaded up through a piece of cardboard, and iron filings are sprinkled on the cardboard, they will arrange themselves in concentric circles around the wire, as shown in Fig. 20. This effect will be observed throughout the whole length of the conductor, and is

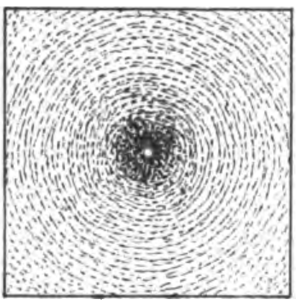

FIG. 20

caused entirely by the current. In fact, every conductor conveying a current of electricity can be imagined as completely surrounded by a sort of *magnetic whirl*, as shown in Fig. 21, the magnetic density decreasing as the distance from the conductor increases. If the electricity flows through the conductor in the direction of the vertical arrows, the direction of the lines of force around the conductor will be as shown; hence, the

FIG. 21

Rule.—*If the current in a conductor is flowing away from the observer, then the direction of the lines of force will be around the conductor in the direction of the hands of a watch.*

76. Screwdriver Rule.—If the axis of an ordinary screwdriver coincides with the axis of a conductor, then the direction in which the screwdriver is turned will be the direction of the lines of force that are set up around the conductor by a current flowing in the conductor in the direction in which the screwdriver as a whole moves in screwing or unscrewing an ordinary right-handed screw. That is, if the screwdriver is turned so as to screw an ordinary right-handed screw into a board, then the current will flow from the handle of the screwdriver toward the screw, and the direction of the lines of force will be around the axis of the screwdriver in the direction in which the screwdriver is turned.

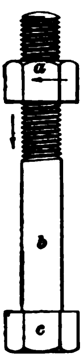

FIG. 22

77. Bolt-and-Nut Rule.—Another very simple method for remembering the connection between the lines of force surrounding a conductor and the direction of the current in the latter is the following: Imagine an ordinary nut *a*, Fig. 22, to represent the lines of force, and a bolt *b* to be the conductor, both nut and bolt having, as usual, a right-hand thread. If the bolt is placed with its head *c* downwards and the nut screwed on the bolt, it will turn toward the right, and will at the same time move downwards. The direction in which the nut revolves gives the direction of the lines of force, and the direction in which the nut proceeds indicates the direction of the current. It follows that, should the current be reversed, the lines of force will run around the bolt in an opposite direction.

MAGNETIC UNITS

78. Strength of Pole.—If two magnetic poles of strength m and m_1, respectively, are placed a distance d from each other, it has been determined that they act upon each other with a force F, such that

$$F = \frac{m \times m_1}{d^2} \qquad (2)$$

If F is 1 C. G. S. unit of force (a dyne), d 1 centimeter, and $m = m_1$, then m and m_1 are each C. G. S. unit poles. A **C. G. S. unit magnetic pole** may therefore be defined as a pole of such strength as to exert a force of 1 dyne on an equal pole when they are placed in air at a distance of 1 centimeter apart. No name has been adopted for the magnetic unit in which the strength of a magnetic pole is expressed. Hence, it is necessary, if $m = 5$, for instance, to say that the strength of this particular magnetic pole is 5 C. G. S. units.

79. Magnetic Moment.—A moment is the product of a force and a distance. Following this principle, the magnetic moment $\mathfrak{M}$ of a magnet in C. G. S. magnetic units is defined as the product of the strength m of one of its poles in electromagnetic C. G. S. units, and the distance l in centimeters between its poles; that is,

$$\mathfrak{M} = m\,l \qquad (3)$$

80. Intensity of a Magnetic Field.—If a magnetic pole of strength m is placed at a given point in a magnetic field, the force F that acts on the magnetic pole is such that

$$F = m \times \mathfrak{H} \qquad (4)$$

$\mathfrak{H}$ is called the strength or intensity of the magnetic field at the given point. In formula **4**, if F is 1 C. G. S. unit of force (a dyne), and m is 1 C. G. S. unit pole, then $\mathcal{H}$ is a field having an intensity, or strength, of 1 C. G. S. unit.

81. A C. G. S. unit field may be defined as a field of such intensity as to exert a force of 1 dyne on a C. G. S. unit pole. The word **gauss** has been adopted as the name for the C. G. S. unit of field density, strength, or intensity, as it is variously termed. It is equivalent to 1 line of force per square centimeter. For instance, if $\mathcal{H} = 5$, we may say that the field strength or intensity is 5 gausses, or that the field has 5 lines of force per square centimeter or 32.25 lines per square inch. The latter two designations are the ones most generally used at present. If a field has 5 lines of force *per square inch*, it *cannot* be said to have a strength of 5 gausses. In place of the expression a "line of force" the word **maxwell** has been adopted. Hence, instead of saying 5 lines of force, we may say 5 maxwells. Therefore a field having a strength of 5 lines of force per square inch may be said to have a strength of 5 maxwells per square inch, whereas, if it has 5 lines of force per square centimeter, it may be said to have a strength of simply 5 gausses.

82. Intensity of a Field at a Distance From a Pole. If m is the strength of a magnetic pole, it has been determined that the strength $\mathcal{H}$ of the magnetic field at a distance d from the pole is such that

$$\mathcal{H} = \frac{m}{d^2} \qquad (5)$$

in which $\mathcal{H}$ is expressed in C. G. S. (magnetic) units, m in C. G. S. (magnetic) units, and d in centimeters.

83. Uniform Field.—A magnetic field is uniform when it has the same intensity and direction at every point. In a limited space, such as a room, the earth's magnetic field is generally uniform, whereas the field in the air near a bar magnet is not generally uniform.

ELECTRODYNAMICS

POTENTIAL AND CURRENT

1. In connection with the subject of electrostatics, the terms potential and current have already been used and explained, but it will be well to now consider them a little further. It must be understood that electricity is a condition of matter and not matter itself, for it possesses neither weight nor extension. Consequently, the statement that electricity is flowing through a conductor must not be taken too literally; it must not be supposed that any material substance, such as a liquid, is actually passing through the conductor in the same sense as water flows through a pipe. The statement that electricity is flowing through a conductor is only another way of expressing the fact that the conductor and the space surrounding it are in different conditions than usual, and that they possess unusual properties. The action of electricity, however, is quite similar in many respects to the flow of liquids, and the study of electric currents is much simplified by the analogy.

2. In order to produce what is called an electric current, it is first necessary to cause a difference of electrical potential or pressure between two bodies or between two parts of the same body.

The term **electromotive force**, usually written **E. M. F.**, is employed to denote that which moves or tends to move electricity from one place to another. A difference of electrical potential has practically the same meaning as electromotive force.

3. An electromotive force may be produced or generated in a number of different ways, among which are the following:

(*a*) By friction and electrostatic induction.

(*b*) By dipping the ends of two strips of dissimilar materials into a liquid that has a greater tendency to chemically act on one material than on the other. The electromotive force is due to chemical action or affinity between the strips and the liquid.

(*c*) By moving a conductor across a magnetic field.

(*d*) By the contact of two dissimilar materials, especially when the junction *d*, Fig. 1, is at a different temperature than the two ends *a* and *b*, which are supposed to be at the same temperature. An electromotive force produced in this manner may be called a *thermoelectromotive force.*

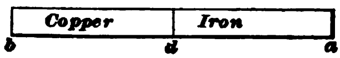

FIG. 1

ELECTROMOTIVE FORCE PRODUCED BY STATIC MACHINES

4. The production of a difference of potential and static charges, or currents, of electricity have already been considered under the subject of *Electrostatics.*

ELECTROMOTIVE FORCE DUE TO CHEMICAL ACTION

CHEMICAL ACTION IN A SIMPLE CELL

5. When a strip or plate of commercial zinc is dipped alone in a dilute solution of sulphuric acid, consisting of 1 part of sulphuric acid to about 20 times its volume of water, the zinc is attacked by the acid and a part of it is converted into a substance called sulphate of zinc, which dissolves in the solution. At the same time the liquid is decomposed and a gas called hydrogen is liberated from the immersed portion of the zinc, coming up from around the zinc in small bubbles. If the zinc is absolutely pure, the chemical actions take place more slowly; the bubbles of hydrogen do not immediately rise to the surface, but cover the surface of the zinc and seem to protect it from further action of the acid.

If, now, a strip of copper is placed in the same solution with the zinc, no change is observed as long as the two metals do not touch each other, but as soon as the two metals are allowed to touch each other or are connected by any conducting material, say a metal wire, as shown in Fig. 2, the chemical action becomes exceedingly vigorous, the zinc dissolves rapidly in the solution, and hydrogen gas is very freely given off from the immersed surface of the copper plate, and not from the zinc plate, as before. Thus, while the zinc is consumed, which may be determined by weighing it before and after the action has continued for, say, an hour, the only action apparent to the eye is the gas produced at the copper plate. Whenever the connection between the exposed ends is broken, all chemical actions cease and remain inactive until the two metals are again connected. The arrangement shown in Fig. 2 is called a *cell.*

6. By the use of a sufficiently delicate electroscope, it could be shown, when the plates are disconnected, that the exposed end of the copper was at a higher potential than the exposed end of the zinc, and hence when the plates are connected outside the liquid there is a continuous flow of positive electricity from the copper to the zinc through the external wire and a continuous flow of positive electricity from the zinc to the copper through the solution.

VOLTAIC CELL

7. Two Italian physicists, Volta and Galvani, were the first to construct the so-called **voltaic,** or **galvanic, cell,** as shown in Fig. 2. It is an apparatus for developing a continuous current of electricity, and consists, essentially, of a vessel A, containing saline or acidulated water, into which are submerged two plates of dissimilar metals C and Z, or one metal and a metalloid, as carbon.

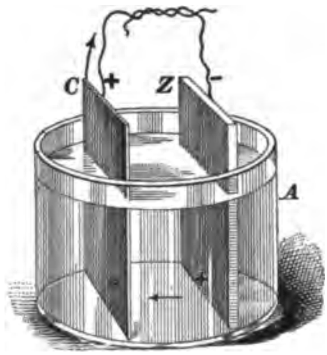

FIG. 2

8. The essential features of a voltaic cell are two dissimilar elements and an electrolyte into which the elements are dipped; and, furthermore, the electrolyte must act chemically more readily on one element than on the other in order to produce any difference of potential between the two elements. **Electrolyte** is the name given to the liquid that, as it transmits the current, is decomposed by it.

The two dissimilar metals, when spoken of separately, are called **voltaic elements**; when taken collectively, they are known as a **voltaic couple.**

In any voltaic cell the element that is the more vigorously acted on (chemically) by the electrolyte is known as the *positive element*, and the other as the *negative element*. While the submerged portion of the zinc plate Z in Fig. 2 is the positive element, as indicated by the plus sign (+), the exposed end, where the connecting wire is attached to it, is always known as the **negative terminal,** or **electrode,** as indicated by the minus sign (−). In a corresponding manner the submerged portion of the copper plate C is the negative element, and the exposed end, where the connecting wire is attached, the **positive terminal,** or **electrode,** as indicated by the minus (−) and plus (+) signs, respectively.

The element by which the current leaves any electrolytic cell is known as the **cathode,** and the element by which the current enters the electrolyte is known as the **anode.** Thus, in the cell already considered, the copper plate is the cathode and the zinc plate the anode.

9. Theory of the Action in a Simple Cell.—In the first place, sulphuric acid is a chemical compound, the chemical symbol for which is written H_2SO_4. As indicated by the symbol, sulphuric acid is composed of 2 atoms of hydrogen, 1 atom of sulphur, and 4 atoms of oxygen. The zinc and sulphuric acid in the solution act on each other, breaking up the sulphuric acid into two components, called *ions*. In this case it is broken up into an ion of hydrogen, written H_2, and an ion consisting of 1 atom of sulphur and 4 atoms of oxygen, written SO_4. The hydrogen ion is positively charged and the SO_4 ion is negatively charged.

Immediately on the breaking up of the sulphuric acid into these two ions, each of which has an equal but opposite charge, the SO_4 ion unites with some of the zinc, forming a chemical compound called zinc sulphate, the chemical formula for which is $ZnSO_4$. This zinc sulphate dissolves in the solution, and the only apparent action is the appearance of bubbles of hydrogen around the zinc plate. In forming the zinc sulphate, the SO_4 ions give up the negative charges, which they received when separated from the hydrogen ions, to the zinc plate; hence, the zinc plate is negatively charged and the hydrogen bubbles positively charged. There is probably a slight tendency for the same action between the copper and the sulphuric-acid solution, but it is less intense and there is no apparent chemical action on the copper, and, hence, there is very little, if any, negative charge produced on the copper plate.

Now, when the two plates are connected externally by a metallic wire, there is a rush of negative electricity through the external wire to the copper plate and the atoms of the hydrogen gas, which are positively charged, are transferred in some invisible manner from the zinc to the copper plate, where they now collect in bubbles and give up their positive charges, thereby neutralizing the negative charges coming through the external wire from the zinc. Thus, there is a tendency to equalize the potential of the copper and zinc plates, but the consumption of the zinc by the solution maintains a difference of potential of a trifle over 1 unit (or volt) between the zinc and copper plates.

The tendency to equalize the potentials of the two metal plates may be compared to the tendency of water flowing from one lake to another, to lower the level of the higher to that of the lower lake, but the springs and other natural agencies prevent it. In a similar manner, the chemical reactions that consume the zinc and break up the sulphuric acid prevent the equalizing of the potentials of the zinc and copper plates.

The flow of negative electricity from the zinc to the copper through the wire is equivalent to a flow of an equal amount of positive electricity in the opposite direction, that is, from the copper to the zinc through the wire. Hence, there is a constant flow of positive electricity from the copper, through the connecting wire, to the zinc, and from the zinc, through the solution, to the copper.

10. Continuous, or Direct, Current.—The equalizing flow that is constantly taking place from one plate to the other is known as a *continuous*, or *direct*, *current* of electricity. A **continuous current** always flows in the same direction and its strength does not vary in a pulsatory manner; whereas a **direct current** always flows in the same direction, but its strength may vary in a pulsatory manner. Since nearly all direct currents are also continuous, the two terms are used rather indiscriminately.

11. An **alternating current,** on the contrary, is one that is made to change the direction in which it flows in a conductor regularly a definite number of times per second. Alternating currents will not be considered here.

12. Direction of Current in a Cell.—The current produced in this cell will, therefore, flow from the exposed end of the copper through the conductor to the exposed end of the zinc, and from the submerged end of the zinc through the liquid to the submerged end of the copper.

It is found that the amount of electricity set free will depend on the quantity of zinc dissolved by the acid ; the latter quantity will also determine the quantity of hydrogen gas developed. It is clear that after a time the solution will have changed into one of sulphate of zinc, and will be unable to dissolve the zinc any further. Then the action of the cell will cease, and the acid will have to be renewed before more current will flow. Or, if there is sufficient acid, but all the zinc is consumed, then the zinc will have to be renewed before more current will flow.

13. Local Action.—Commercial zinc is not chemically pure, but contains impurities, such as small particles of iron, carbon, and other substances. When the zinc is immersed in any liquid that attacks the zinc more than the impurities, an electromotive force is set up, and since the two substances are connected through the metal, local currents are generated that eat away the zinc until the foreign substance is set free and falls away. This is called **local action.** When the zinc is amalgamated, that is, coated or alloyed with mercury, the mercury seems to cover up the impurities and to bring only the pure zinc to the surface. Moreover, the smooth surface seems to hold a film of hydrogen, when the cell is not at work, that protects the zinc from attack by the acid. Hence, amalgamation of the zinc prevents local action at all times.

14. **A voltaic battery** is a number of simple voltaic cells properly joined together.

Electrodes, or **poles,** of a cell or battery are the exposed ends of the plates or metallic *terminals* attached to the plates. The electrodes are used to connect the cell to any exterior conductor or to another cell. In electrical diagrams, cells are represented as drawn in Fig. 3. *M* and *N* each represent a single cell, *a* and *c* being the positive electrodes and *b* and *d* the negative electrodes. The two cells joined together in this manner constitute a battery,

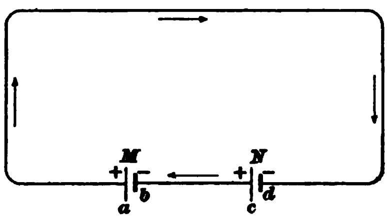

FIG. 3

a and *d* representing the terminals of the battery, as well as the positive electrode of *M* and the negative electrode of *N*, respectively. Where there is only one cell in a circuit it may be called a battery; in fact, a cell is often called a battery. The arrows indicate the direction in which the current flows. The various forms of cells and the actions that take place in them will be fully considered under the subject of *Batteries*.

15. In the case of two substances placed in a liquid, the resulting electromotive force may be said to be due to the **difference of potential** between the two substances. Just as in water pipes a difference of level produces a pressure and the pressure produces a flow, as soon as the water is turned on, so a difference of potential produces an electromotive force and the electromotive force sets up a *current*, as soon as the circuit is completed through which the electricity may flow.

The greater the intensity of the chemical action on one element than on the other, the greater will be the electromotive force of the cell. There is a large variety of metals and electrolytes that may be used to form a voltaic cell, and some combinations produce better results than others.

16. In Table I, various metals are arranged in a series such that any two of the substances form a voltaic couple and produce a difference of potential when submerged in a dilute solution of sulphuric acid; the one standing first on the list being the positive element or plate and the other the negative. For example, if iron and graphite are used, the iron will be the more vigorously acted on by the liquid, and will form the positive element; but if iron and zinc are used, the zinc will be the more vigorously acted on by the liquid, and will form the positive element.

TABLE I

THE ELECTROMOTIVE SERIES

1. + Sodium	5. Tin	9. Gold
2. Magnesium	6. Iron	10. Platinum
3. Zinc	7. Copper	11. – Graphite (carbon)
4. Lead	8. Silver	

The difference of potential will be greater in proportion to the distance between the positions of the two substances in the list. For example, the difference of potential developed between zinc and graphite is much greater than that developed between zinc and iron; in fact, the difference of potential developed between zinc and graphite is equal to the difference of potential developed between zinc and iron plus that developed between iron and graphite. The nature and cost of sodium and magnesium prohibit their use as electrodes in ordinary cells.

The maximum difference of potential developed by any voltaic couple placed in any electrolyte is about 2.25 volts; in the common forms of cells, the difference of potential developed averages from .75 to 2 volts.

17. Differences Between Current and Charge.—Electricity flowing as a continuous current differs usually from static charges in three important degrees; namely, its potential is much lower, its actual quantity is larger, and it is continuous.

A strong voltaic battery of several cells produces only a slight effect on a gold-leaf electroscope, and apparently none of its parts possesses the property of attracting light substances. The electromotive force produced by any well-known voltaic cell scarcely ever exceeds 2 volts, and it would require at least 4,000 cells (8,000 volts) to produce a spark between two balls separated by an air gap only $\frac{1}{10}$ inch in length; whereas a small electrostatic machine may produce sparks several inches in length. If, however, the actual quantity of electricity is measured by its effects in decomposing water, then the quantity produced by a simple voltaic cell as small as a thimble would give greater results than that from an electrostatic machine with plates 2 or 3 feet in diameter.

An electric current cannot be developed on the surfaces of non-conducting substances in the same manner as static charges, and it will never flow unless the conducting path is made entirely of conducting material or an extremely high potential is used.

18. A number of contacts of dissimilar metals can be so arranged as to add their electrical effects together; the difference of potential then developed will be greater in proportion to the number of contacts. Such an arrangement, as shown in Fig. 4, is called a **voltaic pile.** It is made by placing a pair of disks of zinc (chemical symbol *Zn*) and copper (chemical symbol *Cu*) in contact with each other, and then laying a piece of flannel or blotting paper, moistened with brine, on the copper disk. The pair of disks now form a voltaic couple. Several voltaic couples are placed together, and each pair separated by a moistened piece of flannel or blotting paper. One end of such a pile would then be terminated by a disk of copper and the other by a disk of zinc. The copper forms the positive electrode and the zinc the negative electrode. By joining these two electrodes together with a conductor, a current will flow from the positive to the negative *through* the conductor, and from the negative to the positive through the contacts.

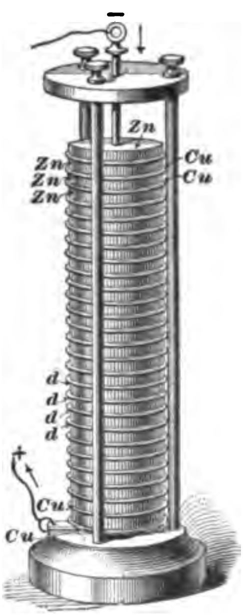

FIG. 4

CIRCUITS

19. A **circuit** is a path composed of a conductor, or of several conductors joined together, through which an electric current flows from a given point around the conducting path back again to its starting point.

A circuit is **broken** or **opened** when its conducting elements are disconnected in such manner as to prevent the current from flowing.

A circuit is **closed** or **completed** when its conducting elements are so connected as to allow the current to pass.

A circuit in which the conductors have come into contact with the ground, or with some electric conductor leading to the ground, is said to be a **grounded circuit,** or is called an **earth.**

The **external circuit** is that part of a circuit which is outside, or external, to the source of electricity.

The **internal circuit** is that part of a circuit which is included within the source of electricity.

In the case of the simple cell, the internal circuit consists of the two metallic plates, or elements, and the liquid, or electrolyte; an external circuit would be a wire or any conductor connecting together the free ends of the electrodes.

20. Conductors are said to be connected in **series** when they are joined so that the entire current must pass through each successively. For instance, in Fig. 5, all the current generated in the voltaic cell *B* must flow successively through the conductors r_1 and r_2. Hence, r_1 and r_2 are connected in series with each other. In this case, moreover, the battery *B* and conductors r_1 and r_2 are all connected in one series-circuit; *c*, *e*, and *d* represent connecting wires, and the arrows the direction in which the current is flowing.

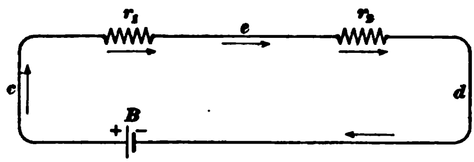

FIG. 5

21. A circuit divided into two or more branches, each branch transmitting part of the current, is a **divided circuit**; the conductors forming these branches are said to be connected in *parallel* or *multiple*. Each branch taken separately is called a **shunt.**

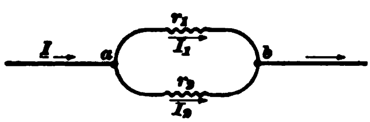

FIG. 6

Such a circuit is shown in Fig. 6. The conductors r_1 and r_2 are the two branches into which the portion *a b* of the circuit is divided. The conductors r_1 and r_2 are said to be connected in parallel or multiple, and r_1 is said to shunt r_2, or r_2 is said to shunt r_1. The total current I divides between the two branches, the portion I_1 flowing through r_1, and I_2 through r_2.

NOTE.—The letter I will be used in this section to represent a current, because it has been adopted for that purpose by an international convention of electrical engineers. However, the letter C has been used to represent current in the past, and is still sometimes so employed.

22. A battery of voltaic cells is said to be connected in series when the cells are arranged in one circuit by joining the positive electrode of one cell to the negative electrode of the adjacent one, so that the entire current passes successively through each, as shown in Fig. 7.

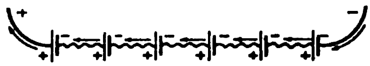

FIG. 7

23. A battery of voltaic cells is said to be connected in **multiple** or **parallel** when the positive electrodes of all the cells are connected to one main positive conductor, and all the negative electrodes are connected to one main negative conductor, as shown in Fig. 8.

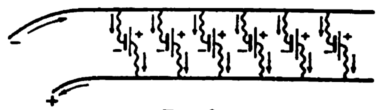

FIG. 8

24. When the series and multiple connections are combined, the battery is said to be connected in **multiple series** or **parallel series.** This is accomplished by joining several groups in **multiple** or **parallel,** the cells in each group being connected in series, as shown in Fig. 9.

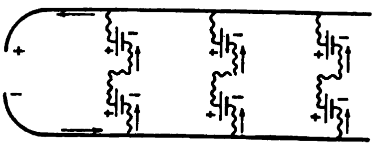

FIG. 9

ELECTROMOTIVE FORCE OF GROUPS OF CELLS

25. Cells Joined in Series.—When a number of cells are joined in series, as shown in Fig. 7, the total electromotive force of the group is equal to the sum of the electromotive forces of all the cells. When all the cells joined together are similar in kind and condition, as is usually the case, the total electromotive force of the group is equal to the electromotive force of one cell multiplied by the number of cells.

When joining together a number of cells in series, the positive pole of the first should be connected with the negative pole of the second cell, the positive of the second with the negative of the third, and so on throughout the whole series. It matters not which pole you commence with, provided you are careful not to connect like poles together. This must be as strictly observed in joining batteries hundreds of miles apart as if they stood side by side.

26. Cells Joined in Parallel.—When a number of cells, similar in kind and condition, are all joined in parallel, as shown in Fig. 8, the total electromotive force of the group is equal to the electromotive force of one cell.

This is due to the fact that all the positive electrodes have the same potential and all the negative electrodes the same potential; hence, joining all the positive electrodes together and all the negative electrodes does not alter the potential of either set, and hence the difference of potential between all the positive and all the negative electrodes is exactly the same as before they were connected together, that is, the same as that of one cell. A number of cells joined in parallel is really equivalent to one large cell, each immersed element of the latter being equal in area to the sum of the areas of the similar immersed elements of all the cells joined in parallel.

When the cells are connected in series, however, each positive electrode has a higher potential than the negative electrode to which it is joined; hence, the potential difference rises one step for each cell joined in the series.

27. Cells Joined in Parallel-Series.—When a number of cells similar in kind and condition are joined in a parallel-series set, as shown in Fig. 9, the total electromotive force of the group is equal to the electromotive force of one series-group; that is, to the electromotive force of one cell multiplied by the number of cells connected in series in one group.

EXAMPLE.—If the electromotive force of each cell is 1.8 volts, what is the total electromotive force of the groups of cells shown in Figs. 7, 8, and 9?

SOLUTION.—In Fig. 7, the cells are all connected in series; hence, the total electromotive force of the group is $6 \times 1.8 = 10.8$ volts. Ans.

In Fig. 8, the cells are all joined in parallel; hence, the total electromotive force of the group is 1.8 volts. Ans.

In Fig. 9 there are three parallel groups, each group consisting of two cells joined in series; hence, the total electromotive force of the group is equal to the electromotive force of one series-row, which is $2 \times 1.8 = 3.6$ volts. Ans.

ELECTROMOTIVE FORCE PRODUCED BY MOVING A CONDUCTOR ACROSS A MAGNETIC FIELD

28. Let the dots in Fig. 10 represent a uniform magnetic field of intensity $\mathcal{H}$, the lines of force being normal to the

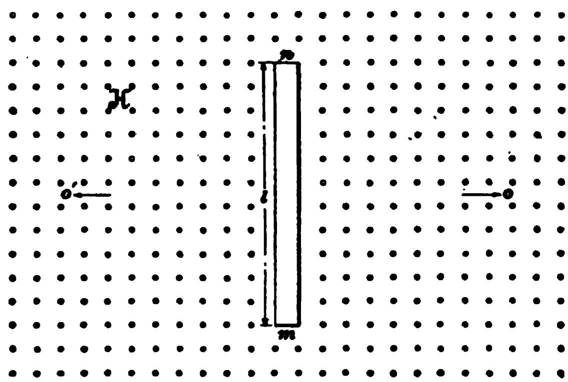

FIG. 10

plane of the paper, and $m\,n$ a conductor of length l lying in the plane of the paper. If the conductor is moved sidewise in the plane of the paper in the direction of the arrow o or o', with a velocity v, an electromotive force E will be developed, or induced, as it is called, in the moving conductor of such a value that

$$E = l\,\mathcal{H}\,v \qquad (1)$$

If l is measured in centimeters, v in centimeters per second, and $\mathcal{H}$ is the intensity of the magnetic field in

C. G. S. units, that is the number of lines of force per square centimeter, then E is the electromotive force in C. G. S. electromagnetic units.

29. The Volt.—The practical unit of electromotive force, or difference of potential, is the *volt*.

The volt is greater than the absolute, or C. G. S., electromagnetic unit of electromotive force.

1 **absolute, or C. G. S., unit** equals one one-hundred-millionths ($\frac{1}{100000000}$) volt.

1 **volt** equals one hundred million (100,000,000, or 10^8) absolute, or C. G. S., units.

Hence, $$E \text{ (in volts)} = \frac{l \mathcal{H} v}{10^8} \qquad (2)$$

A **kilovolt** is equal to 1,000 volts.

A **microvolt** is equal to $\frac{1}{1000000}$ volt.

1 **millivolt** is equal to $\frac{1}{1000}$ volt.

30. If the ends of the moving conductor $m\ n$, in Fig. 10, are joined by another conductor, so arranged as not to cut the lines of force at the same rate as $m\ n$, then the electromotive force induced in $m\ n$ will cause a current to flow through the closed circuit formed by the conductors.

In Fig. 11, let $a\ b\ c\ d$ represent conductors forming with $m\ n$ a closed circuit so arranged that the conductor $m\ n$ can slide sidewise on $a\ b$ and $d\ c$, as on a pair of rails. If the lines of force are directed upwards, that is, toward the observer, and the conductor $m\ n$ is slid along the rails $a\ b$ and $d\ c$ in the direction of the arrow o,

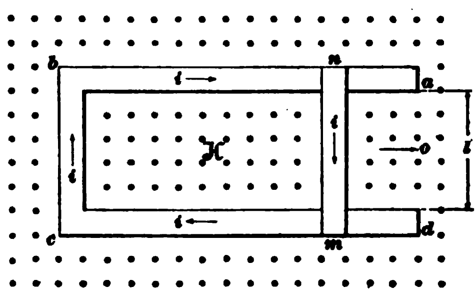

FIG. 11

then the electromotive force induced in $m\ n$ will be higher at m than at n, and hence cause a current to flow through the conductors in the direction of the arrows i.

31. The circuit *m c b n*, which is external to the moving wire, may be called the *external circuit*, and the moving wire *m n* may be called the *internal circuit*, since the electromotive force is generated in it. This is analogous to a system of water pipes, in some portion of which there is a pump that raises the water from the lowest to the highest level. The pressure at the top causes the water to flow by gravity through the system of pipes, which is analogous to the external electrical circuit, while the water is forced by the pump, which is analogous to the internal electrical circuit, from the lowest to the highest level through the pump itself.

Since the electric current flows in the external circuit in the direction *m c b n* when the field is upwards and the conductor is moved to the right, the point *m* is considered to be at a higher potential than *n*, or *m* is considered to be positive relative to *n*. The work done in moving the wire through the magnetic field produces a pressure from *n* to *m*, thereby making the potential higher at *m* than at *n*, the same as the pump forces the water from a lower to a higher level, where its potential energy is greater. Hence, the electric current flows in the external circuit from a point of higher to a point of lower potential or from a point of positive to a point of negative potential; whereas, in the moving wire, or internal circuit, the current flows from a point of lower to a point of higher potential or from a point of negative to a point of positive potential.

Reversing the direction of motion of *m n*, or the direction of the field, will reverse the direction of the electromotive force and the current. Reversing both the direction of the field and the motion of the conductor *m n* at the same time will not reverse the electromotive force and current.

When an electric current is thus produced in a system of conductors, all the mechanical energy, excepting that used to overcome the friction expended in moving the conductor, is converted into electrical energy. The generation of electrical energy in a dynamo depends primarily on the phenomenon described above.

32. When a current actually flows in the conductors *m n*, in Fig. 11, whirls of magnetic lines of force are set up around the conductors and the direction of these lines of force will be found (see Fig. 12) to be opposite to the field $\mathcal{H}$ on the one side (the left side of *m n*) and in the same direction as the field on the other side of each conductor (the right side of *m n*). Fig. 12 represents the imaginary condition of the field in a vertical plane at right angles to the conductor *m n*, and looking (see Fig. 11) from *m* to *n*, when *m n* is being forcibly moved in the direction of the arrow *o*. The field is so distorted by the opposing field set up by the induced current (flowing toward the observer, that is, from *n* to *m* in Fig. 11) that the density of the lines of force is increased on the right side of *m n* and decreased on the left side. The lines of force resist being thus crowded on the right and distorted or stretched, and hence mechanical work is required to overcome this resistance.

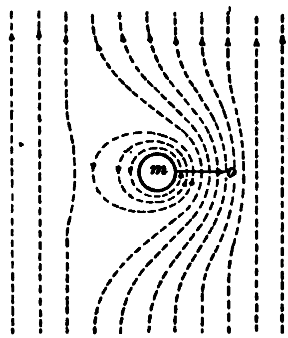

FIG. 12

33. Lenz's Law.—The facts just stated are expressed by **Lenz's law,** which is as follows:

When a conductor is moving in a magnetic field, a current is induced in the conductor in such a direction as to oppose, by its mechanical action, the motion to which the induced current is due. Another way of stating Lenz's law is as follows:

When an electric current is induced by the motion of a conductor through a magnetic field, the induced current has such a direction that the magnetic field set up by the induced current tends to oppose the motion.

In fact, whenever a current is induced in a conductor, no matter in what way, the current is always induced in such a direction as to oppose the inducing agent.

34. The various terms *electromotive force*, *pressure*, *difference of potential*, and *voltage* are, in general, used to signify the same thing; namely, that force which tends to move a current of electricity against the resistance of a conductor. The value of the E. M. F. in any circuit may be calculated, as will be shown later, when the resistance and current are known. Measuring instruments have been devised to indicate the E. M. F. directly.

35. Motion Produced by a Current.—If a current of electricity is caused to flow through the conductor $m\,n$, in Fig. 11, by a battery, or some other means that we need not consider now, then the reaction between the field $\mathcal{H}$ and the field, due to the current that encircles the wire, will tend to move the wire with a force F such that

$$F = I \times l \times \mathcal{H} \qquad (3)$$

If F is expressed in dynes, l in centimeters, $\mathcal{H}$ in lines of force per square centimeter, then I will be the strength of the electric current in C. G. S. units.

The direction in which the conductor will move can be determined by means of Lenz's law. That is, a current flowing in a conductor will cause the conductor to move across a magnetic field in such a direction that if the conductor had been forcibly moved in the same direction by an outside mechanical force, then the current so generated would flow in the opposite direction to the current that actually causes the motion in this case. The direction in which a current is induced by moving a conductor across a magnetic field has been stated in Arts. **30** and **33.** Further on this subject will be more fully treated.

36. A **C. G. S. unit current** may, consequently, be defined as a current of such strength that, flowing through a wire at right angles to a magnetic field of unit intensity, each centimeter of the wire is pushed sidewise with a force of 1 dyne. This definition follows from formula **3.**

37. The Ampere.—The practical unit of electric current is the **ampere.** The ampere is smaller than the absolute, or C. G. S. (electromagnetic), unit of current. One absolute, or C. G. S., unit of current equals 10 amperes;

1 ampere equals $\frac{1}{10}$ (or 10^{-1}) absolute, or C. G. S., unit of current.

A **milliampere** is equal to one one-thousandth ($\frac{1}{1000}$) ampere. If I is expressed in amperes, formula **3** must be written as follows:

$$F = \frac{I \times l \times \mathcal{H}}{10} \qquad (4)$$

THERMOELECTROMOTIVE FORCES AND THERMOELECTRIC CURRENTS

38. A difference of potential is developed by the mere contact of two dissimilar metals, and it varies, not only with the kind of metals and the physical condition of each, but also with their temperature. The greater difference of potential developed by heating a contact of two dissimilar metals can be shown in the following way: Solder or otherwise join together a copper and iron wire, as shown at d and b, Fig. 13, and include somewhere in the circuit an

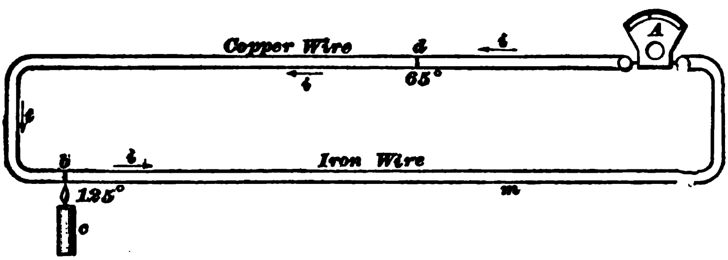

FIG. 13

instrument A that will detect or measure an electric current and also indicate the direction in which it flows If the junction b is heated to a temperature of 125° and the junction d is kept at the temperature of the room, say 65°, then the instrument A will show that a current of electricity flows from the copper wire across the hot junction b to the iron wire, through the iron wire and the instrument A to the junction d, then across this junction to the copper wire, as indicated by the arrows i.

If the junction b is cooled below the other parts of the circuit, a current will flow in the opposite direction, that is, from the iron through the contact b to the copper wire, etc., that is, in the opposite direction to the arrows i. In either case energy in the form of heat is converted into energy in the form of electricity. This phenomenon is known as the *Seebeck effect*, after the man who discovered it.

39. In general, the difference of potential is larger in proportion as the difference of temperature increases. The current produced in a given circuit will be proportional to the difference in temperature between the two junctions, provided the mean temperature of the two junctions has remained the same or nearly the same.

40. The thermoelectromotive force due to two junctions of dissimilar metals depends (1) on the metals employed, (2) on the difference of temperature between the junctions, and (3) on the mean or average temperature of the two junctions.

If one lead-iron junction is at a temperature of 149½° C., and another, in the same circuit, at 150½° C., it has been experimentally determined that the thermoelectromotive force produced is 18 microvolts, the direction of the current at the hotter junction being from the lead to the iron and at the colder junction from iron to lead. Hence, there is an electromotive force of 18 microvolts per degree C. at a mean temperature of 150° C. If the two junctions are at 49½° and 50½°, respectively, the electromotive force is 24 microvolts. Hence, there is an electromotive force of 24 microvolts per degree C. at a mean temperature of 50° C. It is evident, therefore, that this quantity, which is called the thermoelectric power of a metal, depends on the mean temperature of the junctions and the metals in contact with the lead. The thermoelectric power of a certain metal at some mean temperature may be defined as an experimentally determined quantity or coefficient by which to multiply the difference in temperature between two junctions formed by that metal and lead, the two temperatures of the two junctions being such as to give the mean temperature, in order to obtain the electromotive force of the two junctions in microvolts. The values 18 and 24 microvolts, just given, are the vertical distances between the lead and iron lines at 150° and 50°, respectively, in Fig. 14.

From the direction in which the current flows, it follows

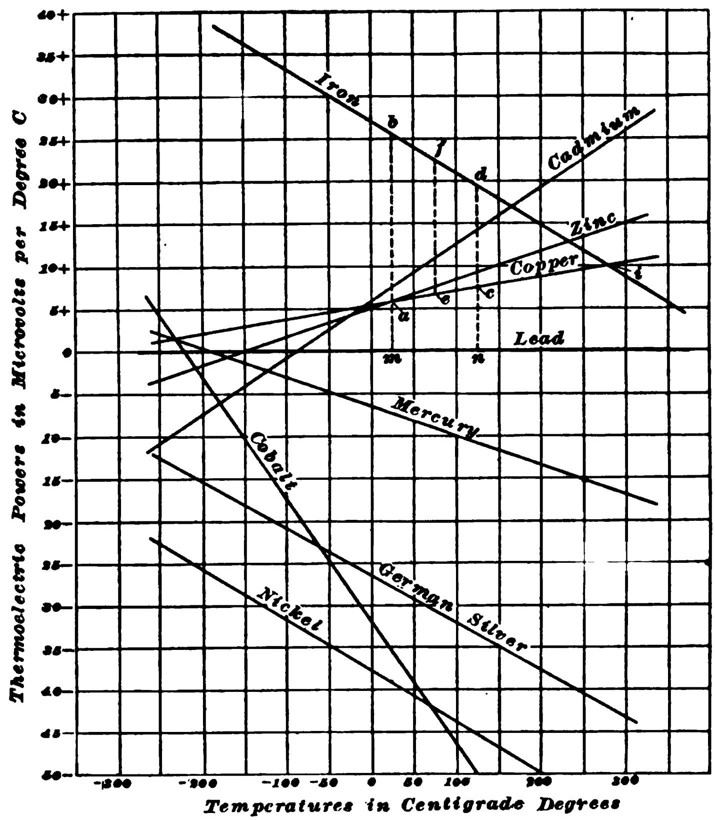

FIG. 14

that the difference of potential between iron and lead is greater at the colder junction than at the hotter junction, and that the entire piece of iron is at a higher or positive potential with reference to the lead. This will be evident if the iron and lead lines in Fig. 14 are carefully studied after reading the following article.

There appears to be no thermoelectric difference of potential between hot and cold lead when they are made to touch each other; in this respect lead differs from other metals. For this reason lead is taken as the standard with which to compare the thermoelectric power of other metals.

41. Thermoelectric Diagram.—The relation between thermoelectric powers of various metals is clearly shown by what is called the thermoelectric diagram given in Fig. 14. The lines representing the various materials in this diagram have been plotted from data experimentally determined. In this figure each division along the horizontal line represents 50° C., and each division along the vertical line represents 5 microvolts per degree C. The thermoelectromotive force due to two junctions between any of the metals there given can be readily determined from the diagram in the following manner:

Draw two vertical lines (perpendicular to the lead line) through points corresponding to the temperatures of the two junctions, and extend these two vertical lines sufficiently to cross the lines representing the thermoelectric power of the two substances forming the two contacts. Then the electromotive force in the circuit, due to the difference in temperature between the two junctions, is equal to the area enclosed by the two vertical lines and the two lines representing the thermoelectric power of the two substances. Thus, the area $abdc$ is equal to the electromotive force in a circuit containing two junctions of copper and iron when one junction is at 25° C. and the other at 125° C.

The area in any case may be obtained by multiplying the difference between the temperatures of the two junctions by the length of the vertical line erected at a point midway between the temperatures of the two junctions, extending from one to the other of the two lines representing the thermoelectric powers of the two substances.

In the case of copper and iron, with the two junctions at the temperatures 25° C. and 125° C., respectively, the thermoelectromotive force is equal to $125 - 25 = 100$ (represented by the horizontal distance mn), multiplied by the length of the vertical line ef. The line ef is really the thermoelectric power of the two metals copper and iron when the mean temperature of the two junctions is 75° C.

42. Thermoelectric Inversion. — Such a point as i, where the lines of two substances intersect, is called the *neutral temperature*, or the point of inversion of the two substances. When the junction of two substances is at their neutral temperature, there is no electromotive force produced at that junction. Furthermore, if one substance is thermoelectrically positive to another at a temperature below their neutral point, the former will be thermoelectrically negative to the other at a temperature above their neutral point.

In computing the thermoelectromotive force in case the neutral temperature lies between the temperatures of the two junctions, the smaller area on one side of the neutral point must be subtracted from the larger area on the opposite side of the neutral point.

43. If two dissimilar substances are joined at one point and the two free ends connected by a third substance, for instance, a long copper wire, the thermoelectromotive force developed will be exactly the same as if the two substances were connected directly together without the aid of the third substance, provided the two free ends that are joined to the copper are at the same temperature.

44. If a battery is connected in a circuit composed of two dissimilar wires, the current that passes across the junctions will heat one and cool the other, depending on the direction of the current. This is known as the *Peltier effect*, after its discoverer. It is the reverse of the Seebeck effect. Suppose the candle c, Fig. 13, is removed, the two junctions then being at the same temperature, and a voltaic cell connected in the circuit somewhere, as at m, so as to send a current through the circuit in the direction of the arrows i. The current produced by the voltaic cell is opposed by the contact difference of potential that exists between iron and copper at the junction b and is assisted by the contact difference of potential that exists at the junction *d*. Then the junction *b* will be cooled and the junction *d* will be heated, provided both junctions are at least below about 166° C., which is the point of inversion of copper and iron. At the junction cooled, heat is absorbed and converted into electrical energy, and at the junction heated, electrical energy is converted into heat.

45. Even the same metal in different physical conditions will develop a difference of potential if heated in a certain place. For instance, take an iron wire and heat it at some point. Then the hotter portion is at a lower potential than the colder portion, and there is a tendency for current to flow from the colder to the hotter portion. Copper would behave just the reverse, as is evident from the direction of the copper and iron lines in Fig. 14. Furthermore, if a current flows through a wire whose temperature is not uniform throughout, some portion of the wire is heated or cooled, depending on the direction of the current. This is known as the *Thomson effect*, after Sir William Thomson, or Lord Kelvin, who discovered it.

In some substances, for instance, copper, zinc, and cadmium, the electric current causes an absorption of heat, that is, produces a cooling effect, when the current flows in the direction in which the temperature is increasing, that is, from a cold to a hotter portion of the same metal, and vice versa. In some substances, for instance, iron, mercury, cobalt, German silver, and nickel, on the other hand, there is an absorption of heat when the current flows from a hot to a colder portion of the same substance. This effect is even more feeble than the Seebeck and Peltier effects, and all of them are usually very small, indeed, compared with the ordinary heating of a wire that is produced by an electric current.

46. Use of Thermoelectric Currents.—On account of the small difference of potential of thermoelectric currents, they have not been found of great practical value, except in determining high and very low temperatures, but they often become a source of great annoyance and error in accurate measurements with delicate instruments.

47. Thermoelectric Pyrometer.—An arrangement of electrical apparatus for measuring high temperatures by measuring the strength of current produced by the Seebeck effect is known as a **thermoelectric pyrometer.** The one known as *Le Chatelier's pyrometer* consists of a piece of platinum wire fused to a piece of platinum-rhodium wire; the two free ends, which are kept close together, and hence at the same temperature, usually that of the atmosphere, are connected by means of copper wire with some instrument capable of measuring small electric currents. The fused junction is usually protected by a fireclay tube closed at one end. The wires mentioned are used because their melting point is extremely high, and such a junction can be used for determining temperatures up to about the melting point of platinum, which is 3,227° F.

48. Thermoelectric Piles.—Although the electromotive force due to two junctions at two different temperatures may be quite small, nevertheless it is possible to arrange a large number of junctions in series in one circuit so as to measure extremely small differences in temperature. Such an arrangement of thermoelectric couples is called a **thermoelectric pile.** If the junctions are arranged in series in

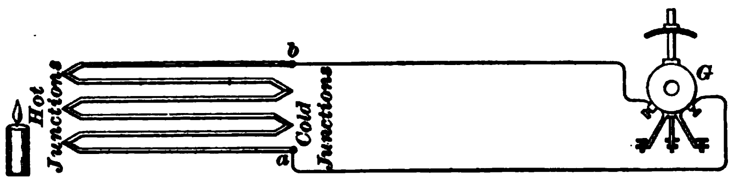

FIG. 15

one circuit, as shown in Fig. 15, whereby one set of alternate junctions is exposed to one temperature and the other set to another temperature, the electromotive forces of all pairs act in the same direction and the total electromotive force developed is proportional to the number of pairs of junctions used. By connecting the ends *a* and *b* to a galvanometer *G*, which is an instrument capable of measuring very small currents, it is possible to determine very small differences of temperature. For very small differences of temperature the currents produced are proportional to the differences of temperature between the hot and cold junctions.

A thermopile, made by Melloni, of a very large number of pairs of bismuth and antimony and arranged in the form shown at *D*, in Fig. 16, is said to have been sufficiently sensitive to detect the heat radiated by a fixed star or by the

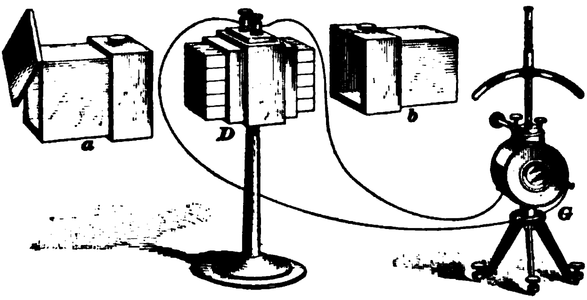

FIG. 16

hand. *G* is a galvanometer used for measuring the very small currents produced by the thermopile. For the detection of small differences of temperature, the resistance of the circuit, including the thermopile and the galvanometer, must be very low and the galvanometer must be very sensitive. The two tubes *a* and *b*, which are shown removed in Fig. 16, when in place screen the junctions of the thermopile from undesired radiations.

With a thermopile made of German silver and an alloy of zinc and antimony, with one set of junctions arranged around an inner circle and the other set around an outer circle, the inner set being heated by a gas flame and the outside set being kept cool by flowing water, it is possible to develop .04 volt per pair of elements. In this arrangement, originally due to Clamond, the source of energy is the gas flame. Similar thermopiles have been tested by a telephone company in this country with the intention of substituting them for primary batteries, but were not adopted. They are not efficient on account of the great heat losses, and hence have not been used commercially.

CURRENT AND QUANTITY OF ELECTRICITY

49. It will be well to consider here the relation between an electric current and quantity of electricity. The strength of an electric current can be described as a quantity of electricity flowing continuously past a given point every second, or, in other words, it is the rate of flow of electricity, just as the current expressed in gallons per minute is the rate of flow in liquids. When one practical unit quantity of electricity is flowing every second, continuously, then the rate of flow or the strength of the current is 1 ampere; if two unit quantities are flowing continuously every second, then the strength of the current is 2 amperes, and so on. It makes no difference in the number of amperes whether the current flows for a long period or for only a fraction of a second; if the quantity of electricity that would flow in 1 second is the same in both cases, then the strength of current in amperes is the same.

50. Electricity possesses neither weight nor extension, and, therefore, an electric current cannot be measured by the usual methods adopted for measuring liquids or gases. The quantity of a liquid that has passed a given point in a certain time may be measured by actually weighing the liquid. By dividing the weight obtained by the time elapsed, the rate of flow, that is, the quantity flowing per second, is obtained. The water flowing in a pipe is usually measured by means of a water meter that indicates the number of cubic feet of water that pass through it in a certain time. This quantity of water divided by the time in seconds would give the average rate at which water flowed through the meter, that is, the number of cubic feet per second. This rate at which the water flows corresponds to the strength of an electric current, and the total number of cubic feet of water to the quantity of current passing in a given time.

However, the strength of an electric current is easier to measure and generally more useful in practical work and in calculations than the quantity of electricity that passes in a given time.

QUANTITY OF ELECTRICITY

51. The Coulomb.—A **C. G. S.** (electromagnetic) **unit of quantity of an electric current** is that quantity which is conveyed by a C. G. S. unit current in 1 second.

The practical unit of quantity of an electric current is the **coulomb.**

The coulomb is smaller than the absolute electromagnetic unit of quantity of current. One absolute, or C. G. S., electromagnetic unit equals 10 coulombs; 1 coulomb equals $\frac{1}{10}$ (or 10^{-1}) absolute, or C. G. S., electromagnetic unit.

52. Relation of Ampere and Coulomb.—The relation of the ampere and the coulomb may be made clear by the water-flow analogy:

When a current of **water** *flows through a pipe, then, if the current has a certain fixed strength, a definite quantity of water will pass some point in a given time.*

When a current of **electricity** *flows through a conductor, then, if the current has a certain fixed strength in amperes, a definite number of coulombs of electricity will pass some point in a given time.*

53. The coulomb may be further defined as being such a quantity of electricity as would pass in 1 second through a circuit in which the strength of the current is 1 ampere.

One coulomb delivered per second therefore represents a current of 1 ampere.

One ampere flowing for 1 second will deliver 1 coulomb.

If Q = quantity of electricity in coulombs;
I = strength of current in amperes;
t = time in seconds;

then,

$$Q = It \qquad (5)$$

By transposition, $I = \frac{Q}{t}$ and $t = \frac{Q}{I}$

EXAMPLE.—Find the quantity of electricity in coulombs that flows around a circuit in $1\frac{1}{2}$ hours, when the strength of current is 12 amperes.

SOLUTION.—By formula **5**, the quantity of electricity

$Q = It = 12 \times 1.5 \times 3{,}600 = 64{,}800$ coulombs. Ans.

EXAMPLES FOR PRACTICE

1. Find the quantity of electricity in coulombs that passes in a circuit in which a current of 40 amperes flows for 55 seconds.
Ans. 2,200 coulombs

2. Find the quantity of electricity in coulombs that passes in a circuit in which a current of 13 amperes flows for 15 minutes.
Ans. 11,700 coulombs

3. In 1 hour, 36,000 coulombs of electricity pass through a closed circuit. If the flow is uniform during that time, what is the strength of the current? Ans. 10 amperes

4. How long will it take 72,000 coulombs of electricity to pass in a circuit in which the strength of current is 4 amperes? Ans. 5 hours

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CURRENT

54. The strength of an electric current is determined directly by the effect it produces, and the quantity of electricity that passes in a given time may be then calculated, if it is needed, by multiplying the strength of the current by the time.

The principal effects produced by an electric current are magnetic attractions and repulsions, chemical decomposition, and heating and luminous effects; of these, the three by means of which the strength of current is most generally measured are: (1) the action of a conductor or coil, through which the current flows, on a magnetic needle or on another coil through which a current is flowing, or the action of a magnet on a conductor or coil through which the current flows; (2) its chemical actions; and (3) the expansion of a conductor due to its being heated by the current that flows through it. These methods will be treated in the section on *Electrical Measurements.* However, an illustration of one of the methods used in measuring electric currents, and also a mode of determining the value of 1 ampere, will be given.

55. Electrolysis.—A current of electricity, when passing through water, decomposes it into two elements, hydrogen and oxygen. The separation of a chemical compound into its constituent parts or elements by an electric current is called **electrolysis.** The quantity of water decomposed is proportional to the strength of the current flowing, and also to the time during which it flows. For example, if a current of 2 amperes flowing for 1 second decomposes a certain quantity of water, then a current of 4 amperes flowing for 1 second will decompose twice that quantity, and if it flows for 2 seconds, it will decompose 4 times the original quantity. Consequently, a unit strength of current will decompose a certain quantity of water in a certain time.

56. It has been experimentally determined and universally accepted that 1 ampere is that strength of current which will decompose .00009349 gram, or .0014427 grain, of water in 1 second.

Let W = mass of water decomposed in grams;
w = mass of water decomposed in grains;
t = time in seconds required for decomposition;
I = current in amperes.

Then the strength of the current in amperes is given by the formulas

$$I = \frac{W}{t \times .00009349} \qquad (6)$$

$$I = \frac{w}{t \times .0014427} \qquad (7)$$

Rule.—*To find the strength of an electric current in amperes by the decomposition of water, divide the mass of* *water decomposed by the time in seconds required to decompose it, and then, if the mass of water is expressed in grams, divide the quotient by .00009349; but if expressed in grains, divide by .0014427.*

57. By transposing the preceding two formulas, we get

$$W = I \times t \times .00009349 \qquad (8)$$

and

$$w = I \times t \times .0014427 \qquad (9)$$

Rule.—*To find the mass of water that an electric current of a given strength can decompose in a given time, multiply the strength of the current in amperes by the time in seconds during which the current flows, and then, if the mass of water is to be expressed in grams, multiply the product by .00009349; but if in grains, multiply by .0014427.*

EXAMPLE 1.—The current from a voltaic cell decomposes water at the rate of 1.29492 grains per hour; what is the strength of current in amperes?

SOLUTION.— 1 hour = 3,600 seconds. By formula **7**, the strength of current

$$I = \frac{1.29492}{3,600 \times .0014427} = .25 \text{ ampere. Ans.}$$

EXAMPLE 2.—Find the number of grams of water decomposed in 3 hours by a current of 6 amperes.

SOLUTION.— 3 hours = 10,800 seconds. By formula **8**, the mass of water decomposed

$$W = .00009349 \times 6 \times 10,800 = 6.058 \text{ grams. Ans.}$$

EXAMPLE FOR PRACTICE

What strength of current will be required to decompose 25 grains of water in 4 hours? Ans. 1.203 amperes

58. If a current is made to flow from a silver electrode through a neutral solution of silver nitrate to a platinum electrode, then silver will be deposited on the platinum electrode. By weighing the silver deposited in a given time, the current flowing can be very accurately calculated, because the amount deposited by 1 ampere in 1 second (.001118 gram) has been very accurately determined experimentally. Hence, it has been universally accepted that the ampere is an unvarying current of such a strength that it will deposit silver (out of a neutral solution of silver nitrate, consisting of 15 parts, by weight, of silver nitrate and 85 of water) at the rate of .001118 gram per second. This ampere is called the *international ampere*, because this specification was adopted by an international convention of electrical engineers in 1893. This is the standard on which all accurate measurements of unvarying, or steady, currents are based.

RESISTANCE

HEATING EFFECT DUE TO RESISTANCE

59. If a thin piece of wire is connected across the electrodes of a voltaic cell, the wire may be so heated as to become red or white hot. The wire resists the passage of a current through it, and the electrical energy necessary to force the current through the wire against its **resistance** is converted into heat.

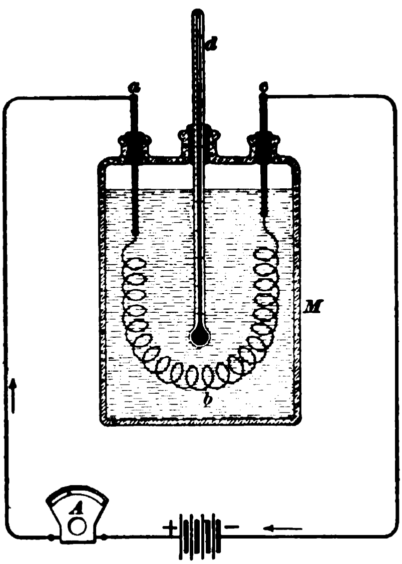

FIG. 17

The following experiment shows the principle of a method employed by Joule to determine the relation existing between the current I flowing through a conductor and the heat W produced. In the glass vessel M, Fig. 17, containing pure alcohol, place a thermometer d and a conductor $a\ b\ c$. By carefully measuring the current I (in C. G. S. units) that flows through the conductor by the instrument A, and the heat W (in ergs) produced in a certain time t (in seconds), it was determined that

$$W = I^2 R t \qquad (10)$$

That is, the heat produced in a conductor by a current is proportional to the time t during which the current flows, to the square of the current I, and to a property R of the conductor called its resistance. In other words, the resistance of a conductor $\left(R = \frac{W}{I^2 t}\right)$ is equal to the heat produced divided by the product of the square of the current I and the time t during which the current continues to flow. The amount of work, or heat, W was determined from the weight of water and its rise in temperature, and the application of well-known laws relating to heat.

60. C. G. S. Unit of Resistance.—If, in formula **10**, W is 1 erg, I 1 C. G. S. unit of current, and t 1 second, then R is 1 C. G. S. unit of resistance. Therefore, a conductor has 1 C. G. S. (electromagnetic) unit of resistance when 1 C. G. S. (electromagnetic) unit of current will develop heat in the wire at the rate of 1 erg per second.

61. The **ohm** is the practical unit of resistance. It is equal to 1,000,000,000 C. G. S. electromagnetic units of resistance; in other words, the ohm = 10^9 C. G. S. units of resistance. Hence, the ohm is larger than the C. G. S. unit of resistance.

One C. G. S. unit of resistance equals one-billionth $\left(\frac{1}{1000000000}\right.$, or $\left.10^{-9}\right)$ ohm.

62. The Joule.—If I is expressed in amperes, R in ohms, and t in seconds, then

$$J = I^2 R t \qquad (11)$$

and the product $I^2 R t$ is the work expended in joules, to designate which the letter J is used. The **joule** is the practical electromagnetic unit of electrical work. The joule is the amount of energy, or work, expended when

1 ampere continues to flow through a resistance of 1 ohm for 1 second. The unit of electrical work and its relation to mechanical units of work and to heat units will be fully discussed later.

EXAMPLE.—How much energy in joules is expended in a circuit during $1\frac{1}{4}$ hours when the strength of current is 14.2 amperes and the resistance of the circuit is 8 ohms?

SOLUTION.—First reduce the time to seconds. $1.25 \times 60 \times 60 =$ 4,500 seconds. By formula **11**, the electrical energy in joules $= J = I^2 R t = 14.2 \times 14.2 \times 8 \times 4{,}500 = 7{,}259{,}040$ joules. Ans.

TABLE II

VARIOUS VALUES OF THE OHM

Name	Height of Mercury Column	Cross-Section of Mercury Column	Use
British Association unit, written B. A. U. . .	104.8 cm.	1 sq. mm.	Out of use, because incorrect.
Legal ohm.	106.0 cm.	1 sq. mm.	Going out of use, because it is not as correct as the following.
International ohm (now commonly called ohm).	106.3 cm.	1 sq. mm.	Latest and most exact determination. Correct within $\frac{1}{5000}$ part. Now used in technical measurements and calculations.

63. The ohm is the only unit in electrical measurements for which a material standard can be adopted. The basis of any system of physical measurements is generally some material standard conventionally adopted as the unit; physical measurements in each system are made by comparison with the unit of that system.

The idea of utilizing a column of mercury of 1 square millimeter cross-section at 0° C. as the practical unit of resistance has been universally adopted, but it is a very difficult matter to accurately determine the exact height of this column. There are, therefore, various values of the unit often found quoted. Table II gives the most important of these various values in tabular form, with annotations denoting their use.

64. The relative values of these units, as accepted by United States Bureau of Standards, are as follows:

TABLE III

1 international ohm	=	1.01348 B. A. U.
1 international ohm	=	1.00283 legal ohms
1 legal ohm	=	.997178 international ohm
1 legal ohm	=	1.0106 B. A. U.
1 B. A. U.	=	.986699 international ohm
1 B. A. U.	=	.98949 legal ohm

65. The legal ohm has been extensively used, and many resistance coils still in use were calibrated in legal ohms. However, international ohms are now legalized and are rapidly coming into general use. Most all instruments containing resistance coils that were made since about 1893 were calibrated in international ohms. When the ohm is mentioned, we understand thereby the resistance of a column of mercury 106.3 cm. high, having a cross-section of 1 sq. mm., at 0° C. (or 32° F.).

66. Microhm.—It very often occurs in practical work that exceedingly small resistances are to be measured, for which the ohm as a unit causes unnecessary labor, because so very large. The absolute unit of resistance, on the other hand, is too small to do very well. Therefore, to facilitate calculations and measurements, a unit is used for such work having the value of one-millionth ($\frac{1}{1000000}$) ohm. This derived practical unit is called the **microhm.** Therefore, to express the resistance in microhms, multiply the resistance in ohms by 1,000,000; and, conversely, to express the resistance in ohms, divide the resistance in microhms by 1,000,000. For example, .0075 ohm = .0075 × 1,000,000 = 7,500 microhms, or 7,500 microhms = $\frac{7500}{1000000}$ = .0075 ohm.

67. Megohm.—Another similarly derived practical unit is the **megohm,** devised to facilitate calculations and measurements of exceedingly large resistances, and is equal to 1,000,000 ohms. Therefore, to express the resistance in megohms, divide the resistance in ohms by 1,000,000; and, conversely, to express the resistance in ohms, multiply the resistance in megohms by 1,000,000. For example, 8,500,000 ohms = $\frac{8500000}{1000000}$ = 8.5 megohms, or 8.5 megohms = 8.5 × 1,000,000 = 8,500,000 ohms.

The megohm is used mainly in the determination of the resistance of non-conductors and insulators.

EXAMPLES FOR PRACTICE

1. Give the equivalent resistance in microhms of .00425 ohm.
Ans. 4,250 microhms

2. Give the equivalent resistance in ohms of 375 microhms.
Ans. .000375 ohm

3. Give the equivalent resistance in megohms of 4,560,000 ohms.
Ans. 4.56 megohms

4. Give the equivalent resistance in ohms of 62.5 megohms.
Ans. 62,500,000 ohms

OHM'S LAW

68. In every electrical circuit there are particularly three factors, the true relation of which must be clearly understood. These three factors are:

1. The force tending to move the electricity.

2. The rate of flow of the electricity.

3. The resistance which the force must overcome to produce the flow of electricity.

These factors are respectively termed:

1. The **electromotive force** (written E. M. F. or E).
2. The **current** (written I).
3. The **resistance** (written R).

69. The relation of the three principal factors will be better understood by comparison with the flow of water through a pipe. The force that causes the water to flow through the pipe is due to the head or pressure; that which resists the flow is the friction of the water against the inside of the pipe, and varies with circumstances. The rate of flow, or the current, may be expressed in gallons per minute, and is a ratio between the head or pressure and the resistance caused by the friction of the water against the inside of the pipe. For, as the pressure or head increases, the rate of flow or current increases in proportion; as the resistance increases, the flow or current diminishes.

In the case of a continuous current of electricity flowing through a conductor, the electromotive force corresponds to the pressure or head of water, and the resistance that a conductor offers to the continuous current to the friction of the water in the pipe. The strength of a continuous current of electricity or the rate of flow of a continuous current of electricity is also a ratio—a ratio between the electromotive force and the resistance of the conductor through which the current is flowing. This ratio, as applied to electricity, was first discovered by Dr. G. S. Ohm, and has since been called **Ohm's law.**

70. Ohm's Law.—*The strength of a continuous current of electricity in a circuit is directly proportional to the electromotive force, and inversely proportional to the resistance of the circuit, and is equal to the quotient arising from dividing the electromotive force by the resistance.*

Ohm's law may be written thus:

$$\text{strength of current} = \frac{\text{electromotive force}}{\text{resistance}},$$

and is usually expressed algebraically as follows:

$$I = \frac{E}{R}.$$

71. Ohm's law expresses the relation between the three units of resistance, electrical pressure, and current. If any two of these values are known, the third is found by solving the simple equation of their relation.

The law may now be stated in practical units by the following rules and formulas:

72. Rule I.—*The strength in amperes of a continuous current* (I) *flowing in a closed circuit, when the electromotive force* (E) *and the total resistance* (R) *are known, is found by dividing the electromotive force in volts by the total resistance in ohms;* that is,

$$\text{current in amperes} = \frac{\text{electromotive force in volts}}{\text{resistance in ohms}},$$

or

$$I = \frac{E}{R} \qquad \textbf{(12)}$$

Rule II.—*The total resistance of a closed circuit* (R) *in ohms, when the electromotive force* (E) *and the continuous current* (I) *are known, is found by dividing the electromotive force in volts by the current in amperes;* that is,

$$\text{resistance in ohms} = \frac{\text{electromotive force in volts}}{\text{current in amperes}},$$

or

$$R = \frac{E}{I} \qquad \textbf{(13)}$$

Rule III.—*The total electromotive force* (E) *in volts developed in a closed circuit, when the continuous current* (I) *and the total resistance* (R) *are known, is found by multiplying the current in amperes by the total resistance in ohms;* that is,

$$\text{electromotive force in volts} = \text{current in amperes} \times \text{resistance in ohms},$$

or

$$E = I \times R \qquad \textbf{(14)}$$

73. The quantities I, E, and R in the foregoing formulas may also be expressed or measured in C. G. S. units as well as in amperes, volts, and ohms, but when using a formula all three quantities must be expressed in the same system of units. For example, I must not be expressed in amperes and R in C. G. S. units, nor can R be expressed in microhms or megohms.

As we have now learned the use of Ohm's law, it may be well to give some additional information regarding the relation of the units to one another.

74. Referring to formula **14**, we have volts = amperes × ohms. It follows from this that a volt would be that electromotive force, or difference of potential, that would force a current of 1 ampere through a resistance of 1 ohm.

A **C. G. S. unit of difference of potential** (or of electromotive force) would be that difference of potential (or electromotive force) that would force 1 C. G. S. unit of current through a conductor having 1 C. G. S. unit of resistance.

75. The ohm may be defined as the resistance that a conductor possesses when a difference of potential of 1 volt between its two ends causes a current of 1 ampere (that is, 1 coulomb per second) to flow through it.

A **C. G. S. unit of resistance** may be defined as the resistance that a conductor possesses when a C. G. S. unit of difference of potential between its two ends will cause 1 C. G. S. unit of current (that is, one C. G. S. unit of quantity per second) to flow through it.

76. An ampere may be defined as the current that would be produced in a conductor having a resistance of 1 ohm by a difference of potential of 1 volt.

A **C. G. S. unit of current** may be defined as the current that would be produced in a conductor having a resistance of 1 C. G. S. unit by a difference of potential of 1 C. G. S. unit.

77. The following examples show the application of Ohm's law:

Formula **12** determines the strength of current that will flow in a conductor of a given resistance, when the pressure in volts is known.

EXAMPLE 1.—A circuit has a resistance of 50 ohms and an available pressure of 100 volts; what is the strength of the current in amperes?

SOLUTION.—Applying formula **12**, amperes $= \frac{\text{volts}}{\text{ohms}}$; hence,

$$\text{amperes} = \frac{100}{50} = 2. \quad \text{Ans.}$$

EXAMPLE 2.—If the pressure in a conductor is 3 volts and the resistance is 15 ohms, how many amperes will flow?

SOLUTION.—Amperes $= \frac{\text{volts}}{\text{ohms}} = \frac{3}{15} = \frac{1}{5}$ ampere. Ans.

EXAMPLE 3.—What current can be made to flow through a circuit having a resistance of 10 ohms, if an E. M. F. of 100 volts is applied?

SOLUTION.— $E = 100$; $R = 10$; hence, by formula **12**, the required current

$$I = \frac{100}{10} = 10 \text{ amperes.} \quad \text{Ans.}$$

78. In case the electromotive force or difference of potential E is known, formula **13** must be used to calculate the resistance of the circuit that will allow a given current I to flow through it.

EXAMPLE 1.—The E. M. F. of a circuit is 500 volts; it is desirable to have a current of .5 ampere flowing in it; what should be the resistance of the circuit?

SOLUTION.—According to formula **13**, ohms $= \frac{\text{volts}}{\text{amperes}}$; hence,

$$R = \frac{500}{.5} = 1{,}000 \text{ ohms.} \quad \text{Ans.}$$

EXAMPLE 2.—Through what resistance can a current of 50 amperes flow, if the electromotive force is 500 volts?

SOLUTION.— $I = 50$; $E = 500$; hence, by formula **13**, the required resistance

$$R = \frac{500}{50} = 10 \text{ ohms.} \quad \text{Ans.}$$

79. To find how much pressure it will require to force a given current through a given resistance, it will be necessary to use formula **14.**

EXAMPLE 1.—How much pressure will it take to force a current of 18 amperes through a resistance of 5 ohms?

SOLUTION.—Formula **14** states that volts = amperes × ohms; hence,

$$18 \times 5 = 90 \text{ volts. Ans.}$$

EXAMPLE 2.—What voltage is required to send a current of 25 amperes through a resistance of 4 ohms?

SOLUTION.— $I = 25$; $R = 4$; hence, by formula **14,** the required voltage

$$E = 25 \times 4 = 100 \text{ volts. Ans.}$$

EXAMPLES FOR PRACTICE

1. The total resistance of a closed circuit is 49.3 ohms; if the current is 2.73 amperes, what is the total electromotive force in volts?
Ans. 134.589 volts

2. A difference of potential of 110 volts exists between the terminals of a conductor whose resistance is 20 ohms; find the current flowing through the conductor. Ans. 5.5 amperes

3. A circuit has an available pressure of 220 volts; what is its resistance if a current of 50 amperes can flow through it?
Ans. 4.4 ohms

APPLICATION OF OHM'S LAW

80. When applying Ohm's law, the following four facts should be carefully noted:

I. *The strength of a current (I) is the same in all parts of a closed circuit, except in the case of divided circuits.*

II. *In the case of a divided circuit, the sum of the currents in the separate branches is always equal to the current in the main or undivided circuit.*

III. *The total resistance of a circuit is the sum of the resistances of the internal circuit and of the external circuit, or its equivalent.*

IV. *The resultant electromotive force in a closed circuit is the algebraic sum of all the electromotive forces in that circuit.*

81. Fig. 18 represents a closed circuit having connected in series in it a battery B of 4 cells, an incandescent lamp L,

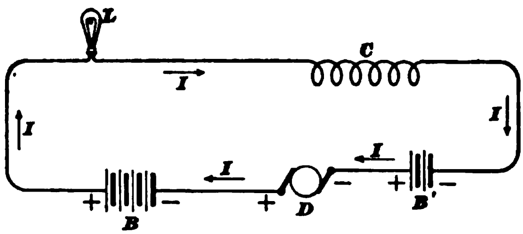

FIG. 18

a coil of wire C, a second battery B' of 2 cells, and a dynamo D.

NOTE.—A **dynamo** is a machine for converting the mechanical energy expended in driving it into electrical energy.

The current I has the same strength in all parts of that circuit; that is, the strength of the current flowing through B is exactly the same as that through L, C, B', and D; in other words, the same current flows through each.

82. If, as shown in Fig. 19, a main circuit a is split up in to two or more branches, as r_1, r_2, and r_3, that again reunite at b, then the current I in the main circuit subdivides through the various branches, and the current I in the main circuit is equal to the sum of the currents in all the branches; that is, $I = I_1 + I_2 + I_3$.

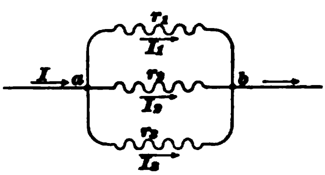

FIG. 19

EXAMPLE 1.—If the current in the three branches r_1, r_2, and r_3, of a divided circuit are 5, 7, and 9 amperes, respectively, what will be the current I in the main circuit?

SOLUTION.—The current I in the main circuit is equal to the sum of the currents in the three branches; hence, $I = 7 + 5 + 9 = 21$ amperes. Ans.

EXAMPLE 2.—If a main circuit in which a current of 18 amperes is flowing is subdivided into four parallel branches, the currents in three of the branches being 2, 4, and 5 amperes, respectively, what is the current in the remaining fourth branch?

SOLUTION.—The current in the main circuit is 18 amperes, and the sum of the currents in three branches $= 2 + 4 + 5 = 11$ amperes; hence, the current in the fourth branch $= 18 - 11 = 7$ amperes. Ans.

83. When Ohm's law is applied to the whole of a closed circuit, R must represent the entire resistance of the circuit, which includes the internal resistance of the battery or other source of current and all resistances connected in series in the external circuit.

In Fig. 20, suppose the total internal resistance of the battery is B ohms; the resistance of the lamp, r ohms; and the

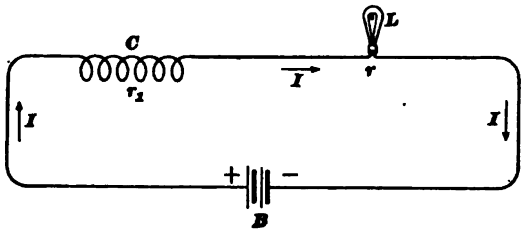

FIG. 20

resistance of the coil and all connecting wires, r_1 ohms; then the total resistance of the circuit will be $r + r_1 + B$ ohms.

The total resistance of the circuit shown in Fig. 18 is equal to the resistance of the lamp L + the resistance of the coil C + the internal resistance of the battery B' + the internal resistance of the dynamo D + the internal resistance of the battery B + the resistance of all the connecting wires. The resistance of connecting wires is very often, but not always, very small compared with the resistance of the apparatus and devices connected in the circuit, and in such cases the resistance of the connecting wires is not mentioned or considered in any way.

EXAMPLE 1.—The two electrodes of a simple voltaic cell are connected together by a copper wire, the resistance of which is 1 ohm. If the internal resistance of the cell is 4 ohms and the electromotive force developed is 2 volts, what is the strength of the current in all parts of the circuit?

SOLUTION.—Let r_i = the internal resistance and r_e = the external resistance; that is, the resistance of the copper wire. Then, the total resistance of the circuit,

$$R = r_i + r_e = 4 + 1 = 5$$

By formula **12**, the current

$$I = \frac{E}{R} = \frac{2}{5} = .4 \text{ ampere flowing through the circuit. Ans.}$$

EXAMPLE 2.—The total electromotive force developed in a closed circuit is 1.2 volts and the strength of the current flowing is .3 ampere; find the total resistance of the circuit.

SOLUTION.—By formula **13**,

$$R = \frac{1.2}{.3} = 4 \text{ ohms. Ans.}$$

EXAMPLE 3.—The internal resistance of a certain dynamo-electric machine is 10.9 ohms and the external resistance is 73 ohms; the electromotive force of the machine is 839 volts. Find the strength of the current flowing in the circuit.

SOLUTION.— $r_i = 10.9$; $r_e = 73$; $R = 10.9 + 73 = 83.9$. By formula **12**,

$$I = \frac{839}{83.9} = 10 \text{ amperes. Ans.}$$

84. In Fig. 21, cell B, having an electromotive force of 1.4 volts, is connected in a circuit with a dynamo D generating an electromotive force of 6 volts. Moreover, the

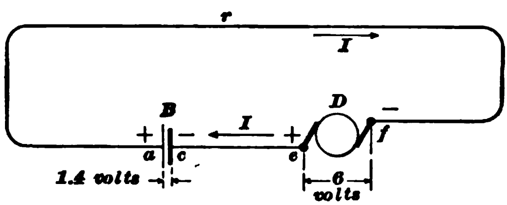

FIG. 21

electromotive forces, as indicated by the + and − signs, act in the same direction and hence tend to help each other in producing the current I that flows through the circuit. Consequently, the resultant electromotive force acting in the circuit is equal to $6 + 1.4 = 7.4$ volts.

If the cell B is reversed in the circuit, as shown in Fig. 22, then the electromotive forces of B and D oppose each other

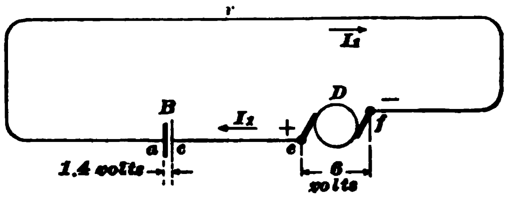

FIG. 22

and the resultant electromotive force acting in the circuit is 6 − 1.4 = 4.6 volts. This electromotive force acts in the direction of the greater of the two electromotive forces, and, hence, whatever current is produced flows in the direction of the arrows I_1. Storage cells, while charging, are connected with the dynamo in this manner.

The current flowing in either case is obtained by dividing the resultant electromotive force by the total resistance in the circuit.

EXAMPLE.—If the internal resistance of the battery B is 2 ohms, the internal resistance of the dynamo D, 1 ohm, and the resistance of the external circuit, 7 ohms, what will be the currents I and I_1 flowing in the circuits shown in Figs. 21 and 22?

SOLUTION.—Fig. 21: As already explained, the resultant electromotive force acting in the circuit in this case is 6 + 1.4 = 7.4 volts. The total resistance of the circuit = 2 + 1 + 7 = 10 ohms. Hence, the current $I = \frac{7.4}{10} = .74$ ampere. Ans.

Fig. 22: The resultant electromotive force = 6 − 1.4 = 4.6 volts, and the total resistance, as in preceding solution, is 10 ohms. Hence, the current $I_1 = \frac{4.6}{10} = .46$ ampere. Ans.

EXAMPLES FOR PRACTICE

1. The current from a simple voltaic cell decomposes water at the rate of 2.59686 grains per hour, and the total resistance of the circuit through which the current flows is 2 ohms. Find (*a*) the strength of the current, and (*b*) the total electromotive force developed by the cell.

Ans. { (*a*) .5 ampere; (*b*) 1 volt }

2. A battery of 10 cells connected in series generates an electromotive force of 1.2 volts per cell. If the total resistance of the 10 cells is 40 ohms and the resistance of the external circuit is 8 ohms, what is the strength of current flowing in the circuit? Ans. .25 ampere

3. Given internal resistance = 4 ohms; electromotive force = 1.5 volts; current = .2 ampere. Find the external resistance.

Ans. 3.5 ohms

4. Given electromotive force = 24 volts; current = .6 ampere. If the external resistance is 3 times the internal, what is the resistance of each?

Ans. { External, 30 ohms
Internal, 10 ohms

DROP OF POTENTIAL

85. Referring to the flow of water in pipes, we may tabulate the analogies as given in Table IV, a careful study of which will do much to assist the understanding of what is to follow.

86. The fourth analogy of the table states that the loss of pressure or E. M. F., due to the resistance of conductor, is termed **drop of potential.** This drop may be made clearer by the following:

Let Fig. 23 represent a tank *T* of water with a horizontal discharge pipe *EN*, which is provided with open vertical

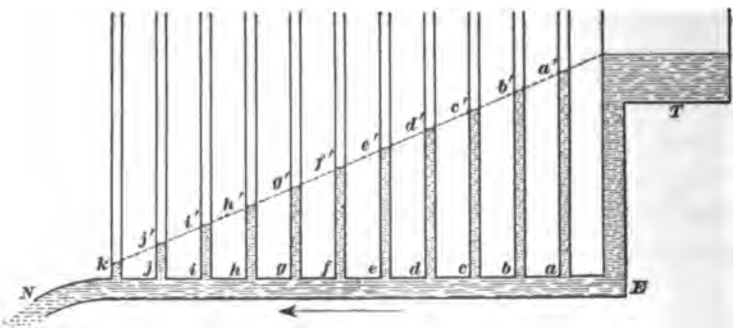

FIG. 23

tubes at *a*, *b*, *c*, etc. If the outlet at *N* is closed, the water in the vertical tubes will rise to the height of the water in the tank; but if the water is allowed to flow freely from the outlet at *N*, then the height of the water in the tubes will be

TABLE IV

ANALOGIES BETWEEN THE FLOW OF WATER AND ELECTRICITY

	Water in Pipes	Electricity in Conductors
I	Difference of level tends to make water fall from the upper level to the lower level.	Difference of potential tends to make electric current flow from point of high potential to point of lower potential.
II	Difference of level, hence acts as a pressure tending to cause a flow.	Difference of potential or E. M. F., hence acts as a pressure tending to cause a flow of current.
III	If not entirely obstructed, this pressure actually produces a flow of water.	If not entirely obstructed, this pressure or E. M. F. actually produces a flow of current.
IV	Some of this pressure is lost by friction of the water against inside walls of pipe.	Some of this pressure is lost by the electrical resistance of the conductor. The loss is called *drop of potential.*
V	This loss by friction is directly proportional to the length of the pipe, and inversely proportional to the diameter of the pipe.	This loss or drop due to resistance is directly proportional to the length of the conductor, and inversely proportional to its area of cross-section.
VI	No quantity of water can flow through a pipe without suffering some loss in this manner; in other words, there is no such thing as an absolutely frictionless pipe.	No quantity of electricity can flow through a conductor without suffering some loss in this manner; in other words, there is no such thing as an absolutely resistanceless conductor.

represented by the inclined line at *a'*, *b'*, *c'*, etc. The pressure, or head, of the water, which is measured by the height of the water in the tubes, decreases in the direction in which the water is flowing, so that the water that leaves the discharge outlet at *N* is under considerably less pressure than the water entering at *E*.

87. The same action takes place in a current of electricity flowing along a conductor, and can also be graphically shown. In Fig. 24, *B* represents a voltaic battery with the

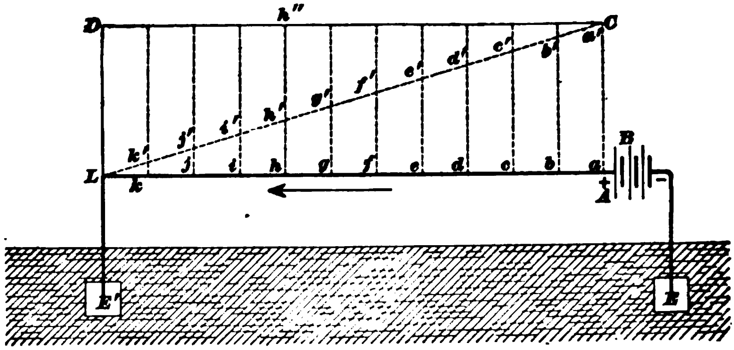

FIG. 24

negative electrode connected directly to the earth at *E*, and the positive electrode to a long conductor *A L*, which is also connected to the earth at *E'*. The battery may be regarded as a machine that raises the pressure or potential of electricity from zero (or that of the earth) to a height equal to the distance *a a'*; or, in other words, the distance *a a'* represents the total difference of potential across the terminals of the battery. If the circuit is opened or broken between *L* and *E'* so that no current flows, then the difference of potential between the conductor and the earth is the same at all points along the conductor, and is represented by the distances between the line *C D* and the conductor *A L*.

But when a current is allowed to flow along the conductor, the difference of potential between the conductor and the earth decreases in the direction in which the current is flowing. The vertical distances $b\,b'$, $c\,c'$, $d\,d'$, etc. represent this difference of potential at the points b, c, d, etc. along the conductor. The loss, or drop, of potential is represented by the vertical distances between the inclined line $C\,L$ and the horizontal line $C\,D$. This loss, or drop, also represents the difference of potential between the point a and any other point along the conductor. For example, at h the difference of potential between the conductor at that point and the earth is represented by the distance $h\,h'$; the loss or drop of potential is represented by the vertical distance $h'\,h''$, which distance also represents the difference of potential existing between the points a and h.

88. The graphical method of determining the difference of potential is seldom used. Ohm's law not only enables us to calculate the strength of the current in a closed circuit, but also the difference of potential in volts along that circuit. The difference of potential (E) in volts between any two points along a circuit is equal to the product of the strength of the current (I) in amperes and the resistance (R) in ohms of that part of the circuit included between those two points; or $E = I\,R$, which is an example of the use of Ohm's law. E, in this case, represents the loss or drop of potential in volts between the two points. If any two of these quantities are known, the third can be readily found, as already shown in connection with Ohm's law.

EXAMPLE.—Fig. 25 represents part of a circuit in which a current of 2.5 amperes is flowing. The resistance from a to b is 10 ohms; from b to c, 15 ohms; and from c to d, 20 ohms. Find the difference of potential between a and b, b and c, c and d, and a and d.

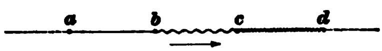

FIG. 25

SOLUTION.—Since, by Ohm's law, $E = I\,R$, then the difference of potential between

a and b is $2.5 \times 10 = 25$ volts;
b and c is $2.5 \times 15 = 37.5$ volts;
c and d is $2.5 \times 20 = 50$ volts;
a and d is $25 + 37.5 + 50 = 112.5$ volts;

or, in other words, the loss or drop in potential between a and d is 112.5 volts.

89. In a great many cases, it is desirable to have the current flow from the source a long distance to some electric receptive device, such as an incandescent lamp or an electric motor, and return without causing an excessive drop or loss of potential in the conductors leading to and from the two places. In such circuits, the greater part of the total generated electromotive force is expended in the receptive device itself, and only a small fraction of it is lost in the rest of the circuit. Under these conditions, it is customary to decide on a certain drop or loss of potential beforehand, and from that and the current calculate the resistance of the two conductors.

EXAMPLE.—It is desired to transmit a current of 10 amperes to an electrical device situated 1,000 feet from the source; the total generated E. M. F. is 110 volts, and only 5% of this potential is to be lost in the conductors leading to and from the two points. Find (*a*) the total resistance of the two conductors, and (*b*) the resistance per foot of the conductors, assuming each to be 1,000 feet long, and that the resistance of 1,000 feet is 1,000 times the resistance of 1 foot.

SOLUTION.—(*a*) 5% of 110 volts $= 110 \times .05 = 5.5$ volts, which represents the total drop or loss of potential on the two conductors. Let $E = 5.5$ volts; $I = 10$ amperes, and $R =$ the total resistance of the two conductors. Then, by Ohm's law, $R = \frac{E}{I} = \frac{5.5}{10} = .55$ ohm. Ans.

(*b*) The resistance of the conductor is directly proportional to its length, and hence the resistance of 1 foot $= \frac{1}{2000}$ of the resistance of 2,000 feet. But the resistance of 2,000 feet is .55 ohm; hence, the resistance per foot $= \frac{.55}{2000} = .000275$ ohm. Ans.

EXAMPLES FOR PRACTICE

1. In a part of a closed circuit the drop or loss of potential caused by the resistance of the conductor is 10 volts. If the current flowing is 4 amperes, what is the resistance of that part of the circuit?

Ans. 2.5 ohms

2. The total generated electromotive force in a circuit is 220 volts. A current of 10 amperes is transmitted to and from a receptive device situated some distance from the source, with a loss of potential of 10%. Find the total resistance of the two conductors leading to and from the two places. Ans. 2.2 ohms

TOTAL AND AVAILABLE E. M. F.

90. The difference of potential between the two electrodes of a simple voltaic cell when no current is flowing, that is, when the circuit is open, is always equal to the total electromotive force developed within the cell; but when a current is flowing, that is, when the circuit is closed, a certain amount of electromotive force is expended in forcing the current through and against the internal resistance of the cell itself. Consequently, the difference of potential between the two electrodes when the circuit is closed is always smaller than when the circuit is open. This difference of potential between the terminals of the cell when the circuit is closed is sometimes called the *available* or *external* electromotive force, to distinguish it from the *internal* or total generated electromotive force.

91. The available electromotive force is equal to the difference between the total generated electromotive force and that expended in forcing the current through the cell against the internal resistance when the circuit is closed. From Ohm's law, the loss or drop of potential in the cell itself is equal to the product of the internal resistance and the strength of current flowing.

Let E = total generated E. M. F.;
E' = available E. M. F.;
I = current flowing when the circuit is closed;
r_i = internal resistance of the cell;
r_e = an external resistance.

The drop or loss of potential in the cell $= I r_i$ and $E' = E - I r_i$.

For example, in a voltaic cell the total generated E. M. F. is 2 volts, and the internal resistance is 4 ohms. If the two electrodes are connected to an external resistance of 6 ohms, a current of .2 ampere will flow through the circuit, since $I = \frac{E}{r_i + r_e} = \frac{2}{4 + 6} = .2$ ampere. The loss or drop of potential in the cell $= I r_i = .2 \times 4 = .8$ volt. Then, $E' = E$

$- Ir_i = 2 - .8 = 1.2$ volts, which is the difference of potential available to force the current of .2 ampere through the external resistance of 6 ohms, since $Ir_e = .2 \times 6 = 1.2$ volts.

92. The total drop or fall of potential in a circuit containing a number of resistances connected in series is equal to the sum of the resistances multiplied by the current.

For instance, the total drop from *a* to *d* in Fig. 25, if the resistance of *a,b* = 10 ohms, *b c* = 15 ohms, *c d* = 20 ohms, and a current of 2.5 amperes is flowing through them, is equal to $(10 + 15 + 20) \times 2.5 = 45 \times 2.5 = 112.5$ volts. It will be noticed that this is exactly the same total drop as computed in a slightly different manner in the solution to the example in Art. **88.** In that solution the total drop was computed by adding together the difference of potential or drop across each resistance. Hence, the total drop may be calculated either way—whichever happens to be the most convenient.

In a closed circuit the current is computed by dividing the total or resultant electromotive force acting in the circuit by the total resistance of the circuit.

EXAMPLE.—In Fig. 18, suppose that the electromotive forces of the batteries *B* and *B'* and the dynamo *D* are 8, 4, and 13 volts, respectively, and their internal resistances are 12, 6, and 2 ohms, respectively, and that the resistances of the lamp *L* and coil *C*, including the connecting wires, are 50 and 30 ohms, respectively. What will be the strength of the current flowing in all parts of this circuit?

SOLUTION.—The total or resultant electromotive force $= 8 + 4 + 13 = 25$ volts, and the total resistance of the circuit $= 12 + 50 + 30 + 6 + 2 = 100$ ohms. Then, by Ohm's law, the current flowing through the entire circuit $= I = \frac{E}{R} = \frac{25}{100} = .25$ ampere. Ans.

FALL OF POTENTIAL THROUGHOUT A COMPLEX CIRCUIT

93. We will now analyze the fall of potential in a more complex circuit. Suppose that we have a circuit, as shown in Fig. 26, in which there is connected in series a dynamo generating an electromotive force E_1 of 38 volts, a conductor *b c* having a resistance of 3 ohms, a storage battery having an electromotive force E_3 of 12 volts and an internal resistance R_3 of 2 ohms, and finally a resistance coil R_4 of 7 ohms. These values are all indicated in the figure. The circuit is arranged so that the dynamo will charge the storage battery, and hence the electromotive force of the storage battery opposes that of the dynamo. The entire electromotive force of the dynamo, 38 volts, is represented by the line bf. In order to find the drop or fall of potential in each portion of the circuit, it will first be necessary to calculate

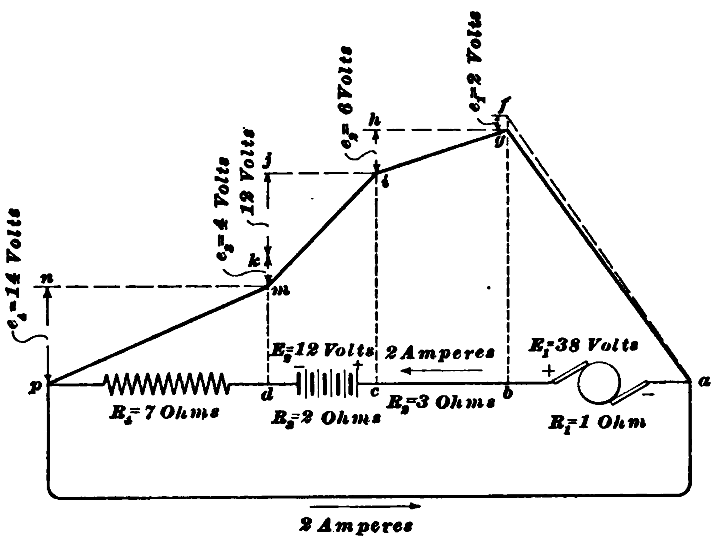

FIG. 26

the strength of the current. The resultant electromotive force in the circuit $= 38 - 12 = 26$ volts, since the electromotive force of the storage battery opposes that of the dynamo. The total resistance $R = R_1 + R_2 + R_3 + R_4 = 1 + 3 + 2 + 7 = 13$ ohms. Hence, by Ohm's law, the current $I = \frac{26}{13} = 2$ amperes. The line af shows the increasing pressure of the dynamo from the terminal a to the terminal b when the circuit is open. When the circuit is closed, and there is a current of 2 amperes flowing through it, there is a drop or fall of potential e_1 through the dynamo itself. This fall of potential is calculated by the formula $E = I \times R$, in which $I = 2$ amperes and $R = 1$ ohm. Hence, e_1, the fall of potential through the dynamo itself, $= I \times R_1 = 2 \times 1 = 2$ volts. Subtracting this fall of potential from the electromotive force bf, gives bg, equal to 36 volts, as the potential of the point b, assuming that the point a has zero potential, that is, the difference of potential between a and b, when 2 amperes is flowing in the circuit, is 36 volts. The fall of potential e_2 through the conductor $R_2 = 2 \times 3 = 6$ volts. Hence, the potential at the point c is represented by the line ci, which is equal to $bg - hi = 36 - 6 = 30$ volts. Hence, hi represents the fall of potential through the entire conductor R_2, or the difference of potential between b and c, and the potential along the conductor R_2 falls from g to i along the slanting line gi connecting these two points. From c to d there is, due to the storage battery, an electromotive force of 12 volts that opposes the electromotive force due to the dynamo, and hence there is a drop of 12 volts, represented by jk. From c to d there is a further drop due to the internal resistance of the storage battery. Since the internal resistance of the storage battery is 2 ohms, this drop will be equal to $2 \times 2 = 4$ volts. This fall of potential of 4 volts is represented by the line km. This leaves a potential $dm = ci - mj = 30 - 16 = 14$ volts at the point d. The difference of potential between c and d is evidently 16 volts. The potential is represented as falling from c to d along the line im. From d to p there is a resistance of 7 ohms, and hence a drop of $2 \times 7 = 14$ volts. This drop or fall of potential is represented by np, and the potential is represented as falling uniformly through the resistance R_4 along the line mp. The point p is supposed to be connected with the point a by a conductor having zero resistance, that is, p and a are practically the same point, but they could not be so represented in a figure of this nature; hence, we have assumed them to be joined together by a wire of negligible, or zero, resistance.

94. We will now add together the fall of potential due to resistance alone in each separate portion of the circuit:

$e_1 + e_2 + e_3 + e_4 = 2 + 6 + 4 + 14 = 26$ volts. This is equal to $E_1 - E_2 = 38 - 12 = 26$ volts. Hence, we see that the total electromotive force of the dynamo, less the 12 volts required to overcome the electromotive force of the storage battery, is consumed in forcing the current against the resistances of the various parts of the circuit. The fall of potential in the various portions of the circuit external to the dynamo is represented by the heavy line *g i m p*. The potential at any point in a circuit, and the fall of potential through any portion of a circuit in which the current and resistance are known, can be represented by a diagram of this nature.

ELECTRICAL RESISTANCE AND CAPACITY

RESISTANCE

1. The **resistance** that all substances offer to the passage of an electric current is one of the most important quantities in electrical measurements. It is that attribute of a conductor or of a circuit that determines the strength of the continuous electric current that can be sent through the conductor or the circuit by a given electromotive force, as shown by Ohm's law.

If a given conductor offers a resistance of 2 ohms to a current of 1 ampere, it offers the same amount, no more nor no less, to a current of 10 amperes. Hence,

The resistance of a given conductor is always constant at the same temperature, irrespective of the electromotive force or the strength of current flowing through it.

2. Resistance of Various Materials.—The resistance varies in different substances; that is, one substance offers a higher resistance to a current of electricity than another. In order to directly compare the resistances of different substances, however, the dimensions of the pieces to be measured must be equal. For, by changing its dimensions, a good conductor may be made to offer the same resistance as an inferior one. Under like conditions, annealed silver offers the least resistance of all known metals or conductors. Pure annealed copper comes next on the list, and then follow all other metals and conductors.

The resistance of a given conductor, however, is not always constant; it changes with the temperature, and also with the physical condition of the conductor. For instance, hard-drawn copper wire has a higher resistance than soft-drawn or annealed copper wire. The resistance of all metals increases as the temperature rises; the resistance of liquids, carbons, non-conductors, and a very few alloys decreases as the temperature rises. The amount of variation in the resistance caused by a change in temperature will be presently explained.

3. The **specific resistance,** or **resistivity,** of a substance is the resistance at some standard temperature, usually the freezing point of water, of a piece of the substance having unit length and unit sectional area. Specific resistance is usually defined as the resistance at 32° F. or 0° C. of a piece of the substance 1 centimeter long and 1 square centimeter in sectional area. This is sometimes designated the specific resistance per centimeter cube, in order to distinguish it from the resistance per inch cube. The term *resistivity* is gradually coming into use in place of the longer expression specific resistance; it is designated by the Greek letter ρ (spelled rho and pronounced rō).

The resistance R of a piece of any material may be expressed by the formula

$$R = \frac{\rho l}{a}, \qquad (1)$$

in which l = length of the piece;
a = sectional area; that is, the area at right angles to the direction of the current;
ρ = specific resistance of the material.

When ρ is the specific resistance per centimeter cube, l and a must be expressed in centimeters and square centimeters, respectively.

The specific resistance per centimeter cube of various substances is given in column 2 in Tables I and II. Occasionally ρ is defined as the resistance of an inch cube of the substance, that is, a piece 1 inch long and 1 square inch in

TABLE I

RESISTANCE OF VARIOUS METALS*

Metal	Specific Resistance† ρ	Resistance of 1 Mil-Foot in Ohms		Temperature Coefficient per Degree C. Between 0° and 100° C.	Temperature Coefficient per Degree F. Between 32° and 212° F.	Percentage Conductivity	Relative Resistance
		0° C. 32° F.	23.8° C. 75° F.				
1	2	3	4	5	6	7	8
Silver,[2] pure annealed	1.468	8.831	9.674	.004000	.002220	108.60	.925
Copper,[2] pure annealed	1.561	9.390		.004280	.002380	102.10	.980
Copper,[1] annealed	1.594	9.590	10.505	.004020	.002230	100.00	1.000
Silver, hard-drawn	1.629	9.799				97.80	1.022
Copper,[1] hard-drawn	1.631	9.810	10.745	.004020	.002230	97.80	1.022
Gold[2] (99.9% pure)	2.197	13.216		.003770	.002090	72.55	1.378
Aluminum (99.5% pure)	2.530	15.219				63.00	1.587
Aluminum[2] (commercial — 97.5% pure)	2.665	16.031		.004350	.002420	59.80	1.672
Magnesium[2]	4.355	26.197		.003810	.002120	36.60	2.732
Zinc[2] (very pure)	5.751	34.595		.004060	.002260	27.72	3.608
Iron,[2] approximately pure	9.065	54.529		.006250	.003470	17.50	5.714
Iron, "E. B. B." iron wire	9.759	58.702	65.190	.004630	.002570	16.20	6.173
Cadmium[2] (pure)	10.023	60.292		.004190	.002320	15.90	6.289
Palladium[2] (pure)	10.219	61.471		.003540	.001970	15.60	6.410
Platinum[2] (pure)	10.917	65.670		.003669	.002038	14.60	6.845
Iron, "B. B." iron wire	11.085	68.680	76.270	.004630	.002570	13.50	7.407
Nickel[2]	12.323	74.128		.006220	.003460	12.94	7.726
Tin[2] (pure)	13.048	78.489		.004400	.002450	12.22	8.184
Steel (wire)	13.495	81.179	90.150	.004630	.002570	11.60	8.621
Thallium[2] (pure)	17.633	106.070		.003980	.002210	9.04	11.060
Lead[2] (pure)	20.380	122.590	134.610	.004110	.002280	7.82	12.790
Antimony (pressed)	35.400	212.950		.004100		4.50	22.220
Mercury[2] (pure)	94.070	565.870	610.370	.000720	.000400	1.69	59.170
Bismuth (pressed)	130.800	786.810		.003540		1.22	81.970

* These resistances are given in international ohms.
† This is the resistance of a piece 1 centimeter long and 1 square centimeter in sectional area, at 0° C., in microhms.
[1] Determined by Matthiessen and taken as the standard.
[2] Determined by Fleming and Dewar.

TABLE II

RESISTANCES OF VARIOUS ALLOYS†

Substance	Specific Resistance‡ ρ	Resistance of 1 Mil-Foot in Ohms at 0° C. or 32° F.	Temperature Coefficient per Degree Centigrade	Temperature Coefficient per Degree Fahrenheit	Percentage Conductivity	Relative Resistance
1	2	3	5	6	7	8
Brass	7.200	43.310			22.15	4.515
Phosphor bronze, Commercial—*Cu*, *Sn*, *P*	8.479	51.005	.000640	.000356	18.80	5.319
Aluminum bronze	12.300	73.989	.001000	.000556	12.96	7.714
Platinum rhodium,[2] *Pt* 90, *Rh* 30	21.142	127.180	.001430*	.000795*	7.54	13.260
German silver,[3] *Cu* 50, *Zn* 35, *Ni* 15	21.250	127.800	.000400	.000220	7.50	17.300
Platinum silver,[3] *Pt* $66\frac{2}{3}$, Ag $33\frac{1}{3}$	24.900	149.800	.000310	.000170	6.40	15.600
German silver,[2] *Cu* 60, *Zn* 25, *Ni* 15	29.982	180.350	.000273*	.000152*	5.32	18.800
Platinum iridium,[2] *Pt* 80, *Ir* 20	30.896	185.850	.000822*	.000457*	5.16	19.380
Platinum silver,[2] *Pt* $33\frac{1}{3}$, *Ag* $66\frac{2}{3}$	31.582	189.980	.000243*	.000135*	5.05	19.800
Platinoid,[2] *Cu* 59, *Zn* 25.5, *Ni* 14, *W* 55	41.731	251.030	.000310*	.000172*	3.82	26.180
German silver,[3] *Cu* 55, *Zn* 20, *Ni* 35	45.540	271.100	.000330	.000180	3.50	28.600
Manganin,[2] *Cu* 84, *Ni* 4, *Mn* 12	46.678	280.790	.000000*		3.41	29.330
Constantan, *Cu* 58, *Ni* 41, *Mn* 1	{50 / 52}	{300.77 / 312.80}	±.000010	.000005	{3.19 / 3.07}	{31.35 / 32.57}
Reostene	76.468	459.990	.001100*	.000610*	2.08	48.080
Gray cast iron, *C* 3.46; graphite, 2.06; *Mn* .173; *S* .042; *Si* 2.04; *P* .151	114.000	684.000				
Carbon, arc light	{4400 / 8600}	{26500 / 51700}	.000520*	.000289*	{.0360 / .0186}	{2778 / 5376}

* These are the temperature coefficients at 15° C. or 59° F.; the others are mean temperature coefficients between the freezing and boiling temperatures of water. † Where the proportions are not given, the experimenters merely stated that they were made of the usual proportions. As this is not very definite, we cannot give the proportions. ‡ This is the resistance of a piece 1 centimeter long and 1 square centimeter in sectional area at 0° C. in microhms. [2] Determined by Fleming and Dewar. [3] Given by Jackson. *Pt* = platinum: *Ag* = silver; etc.

sectional area. In this case *l* and *a* must be expressed in inches and square inches, respectively, and *R* will then be the resistance of a piece whose dimensions are given in inches.

4. Relation Between Specific Resistance and Volume. The resistance of a conductor cannot be obtained by simply multiplying or dividing its specific resistance, that is, the resistance of a piece of unit length and unit sectional area, by the volume of the conductor. For suppose that a piece of a conductor has a sectional area of 1 square inch and a length of 10 inches; then there will be 10 cubic inches of metal in the conductor. Now, suppose that another conductor has a sectional area of ½ square inch and a length of 20 inches; there will also be 10 cubic inches of metal in this conductor. It can readily be seen that the conductor 20 inches long and having a sectional area of ½ square inch will have a much higher resistance than the conductor 10 inches long and having a sectional area of 1 square inch. Therefore, the resistance of a conductor cannot be obtained by simply multiplying or dividing the number of cubic inches by the resistance per cubic inch, unless the conductor happens to have a sectional area of exactly 1 square inch. The correct method for finding the resistance of a piece of a conductor when the specific resistance, length, and sectional area are known, is to use formula **1**; that is, multiply the specific resistance by the length and divide the product so obtained by the sectional area.

5. Resistance of Various Conductors.—A list of different substances is given in Tables I and II in the order of their relative resistances, beginning with annealed silver, which offers the least resistance. The second column gives the specific resistance, that is, the actual resistance in microhms of a piece of the substance 1 centimeter long and having a sectional area of 1 square centimeter at the freezing point of water; that is, at 32° Fahrenheit or 0° Centigrade. This is the specific resistance per centimeter cube. The resistance of a piece of any known dimensions of any substance given in the list can be determined by applying formula **1**, in which ρ is the corresponding specific resistance given in this table. Or formulas **3**, **4**, and **5** may be used in connection with the values given in Tables I and II.

EXAMPLE 1.—If the resistivity (specific resistance) of a certain metal is 2.53 (at 0° C.) microhms, what will be the resistance (at 0° C.) of a wire made of this metal, having a diameter of .080808 inch and a length of 1 mile?

SOLUTION.—According to formula **1**, $R = \frac{\rho l}{a}$, in which ρ (at 0° C.) $= .00000253$ ohm; $l = 5{,}280 \times 30.48$ cm. (5,280 being the number of feet in 1 mi. and 30.48 the number of centimeters in 1 ft.); and $a = (.080808)^2 \times .7854 \times 6.45$ (6.45 being the number of square centimeters in 1 sq. in.).

Hence, $$R = \frac{.00000253 \times 5{,}280 \times 30.48}{(.080808)^2 \times .7854 \times 6.45} = 12.309 \text{ ohms} \quad \text{Ans.}$$

EXAMPLE 2.—Find the resistance in ohms at 0° C. of a round column of mercury 180 centimeters high and .02 centimeter in diameter.

SOLUTION.—Use formula **1**, $R = \frac{\rho l}{a}$, in which $l = 180$; $a = .7854 \times (.02)^2$; and $\rho = 94.07$ microhms, or .00009407 ohm (from Table I) per centimeter cube. Hence,

$$R = \frac{.00009407 \times 180}{.7854 \times .0004} = 53.898 \text{ ohms} \quad \text{Ans.}$$

6. A **mil-foot** is a cylindrical piece of substance 1 foot long and 1 mil in diameter. A mil, which will be more fully considered later, is equal to .001 inch. Hence, the resistance of a mil-foot of a substance means the resistance of a wire, that is, a cylindrical piece of the substance 1 foot long and .001 inch in diameter. This term is frequently used in connection with ordinary conductors, especially copper wire. The resistance of a mil-foot of various substances is given in columns 3 and 4, Tables I and II.

The term **resistance per meter-millimeter**, which is occasionally used, means the resistance of a cylindrical wire 1 meter long and 1 millimeter in diameter.

7. Meter-Gram.—The specific resistance of a substance may be given in terms of length and mass instead of in length and sectional area. Thus the term **specific resistance per meter-gram** means the resistance of a piece of the substance 1 meter long and having a mass of 1 gram. That is, the sectional area, which is assumed to be approximately uniform at least, is such that a piece 1 meter long weighs exactly 1 gram. If k represents the length-mass specific resistance, then a wire of length l and mass m will have a resistance

$$R = \frac{k l^2}{m}; \qquad (2)$$

in which

k = resistance of a piece 1 meter in length and having a mass of 1 gram;
l = length in meters;
m = mass in grams of the piece.

A meter-gram of soft copper having a specific gravity of 8.89 (that is, k for soft copper) has a resistance of .14173 ohm.

NOTE.—Let k = resistance of a piece of a substance of unit length and unit mass. A wire of length l and mass m will have a mass per unit length of $\frac{m}{l}$, and its resistance per unit length is inversely proportional to $\frac{m}{l}$, because the greater the mass per unit length, the less will be the resistance. Hence, the resistance per unit length is equal to $\frac{k l}{m}$, when the mass per unit length $= \frac{m}{l}$. Furthermore, the resistance of l units of length will be l times as great; hence, $R = \frac{k l^2}{m}$, which proves the above formula. In order to reduce the resistance per unit length and mass of a substance to its corresponding resistance per unit length and sectional area, it is necessary to know not only its length and mass, but also its specific gravity. Mass equals the length times the sectional area times the specific gravity; from which the corresponding average sectional area can be calculated, when the mass, length, and specific gravity are known.

8. Conductance.—The word **conductance** is now coming into use in place of the word conductivity; it is the exact reciprocal of resistance. When the word conductance is used in this way, the word *resistivity* is generally used to signify specific resistance. Since conductance is the reciprocal of resistance, it naturally follows to call the reciprocal of the specific resistance, or resistivity, of a substance its specific conductance, or conductivity. Conductivity is, therefore, equivalent to specific conductance. However, the words conductivity and resistivity are not universally used in this way at present, but they are more logical and consistent than the older terms specific resistance and conductivity, which were not reciprocals of each other as formerly defined.

There is no established unit of conductance. Some writers use the word *mho*, which is ohm spelled backwards, as the name for the unit of conductance and designate it by G. A conductor having a resistance of ½ ohm will have a conductance of 2 mhos.

The **percentage conductivity** of a substance, given in column 7, Tables I and II, is the ratio the conductivity of that substance bears to that of the standard substance at the same temperature, usually Matthiessen's pure copper at 0° C.; the conductivity of the latter is taken as 100. The percentage conductivity is frequently used in specifications, it being a frequent requirement that the wire shall have a conductivity equal to 98 per cent. that of pure copper at the same temperature.

9. Multiples and Submultiples.—Expressed in ohms, the specific resistance of flint glass is 16,700,000,000,000,000,000 and that of annealed silver is .000001468. To prevent the constant repetition of zeros, prefixes have been adapted to express multiples and submultiples of a unit, as shown in Tables III and IV.

TABLE III

MULTIPLES

Prefix	Amount of Multiplication		
	Expressed in Words	Expressed in Figures	
deka	ten times	10	10
hecto	one hundred times	100	10^2
kilo	one thousand times	1,000	10^3
mega	one million times	1,000,000	10^6
bega	one billion times	1,000,000,000	10^9
trega	one trillion times	1,000,000,000,000	10^{12}
quega	one quadrillion times	1,000,000,000,000,000	10^{15}

TABLE IV

SUBMULTIPLES

Prefix	Amount of Division		
	Expressed in Words	Expressed in Figures	
deci	one-tenth	1 ÷ 10	10^{-1}
centi	one-hundredth	1 ÷ 100	10^{-2}
milli	one-thousandth	1 ÷ 1,000	10^{-3}
micro	one-millionth	1 ÷ 1,000,000	10^{-6}
bicro	one-billionth	1 ÷ 1,000,000,000	10^{-9}
tricro	one-trillionth	1 ÷ 1,000,000,000,000	10^{-12}

Using these prefixes, the specific resistance of flint glass would be said to be 16,700 quegohms (16,700 × 1,000,000,000,000,000 = 16,700,000,000,000,000,000 ohms) and that of annealed silver 1.468 microhms, since

$$\frac{1.468}{1,000,000} = .000001468 \text{ ohms}$$

TABLE V

Substance	Specific Resistance
Mica	84 tregohms
Gutta percha	449 tregohms
Hard rubber	28 quegohms
Paraffin (solid)	34 quegohms
Paraffin oil	8 tregohms
Porcelain	540 quegohms
Flint glass	16,700 quegohms
Olive oil	1 tregohm
Lard oil	350 begohms
Benzine	14 tregohms
Wood tar	1,670 tregohms
Ozocerite (crude)	450 tregohms

10. Specific Resistance of Insulating Materials.— The specific resistances of some of the substances commonly termed insulators are given in Table V.

Table VI gives the specific resistance of water and some of the more common solutions used as electrolytes.

TABLE VI

Solution	Specific Resistance per Centimeter Cube (International Ohms)
Sulphuric acid 5% acid, specific gravity 1.0330 at 18° C....	4.81
Sulphuric acid 10% acid, specific gravity 1.0700 at 18° C....	2.83
Sulphuric acid 20% acid, specific gravity 1.1414 at 18° C....	1.54
Sulphuric acid 25% acid, specific gravity 1.1700 at 18° C....	.99
Sulphuric acid 30% acid, specific gravity 1.2200 at 18° C....	1.36
Sulphuric acid 40% acid, specific gravity 1.3100 at 18° C....	1.48
Common-salt saturated solution at 18° C..................	5.09
Zinc sulphate (ordinary) saturated solution at 18° C.......	20.20
Copper sulphate (ordinary) saturated solution at 18° C....	29.90
Sal-ammoniac solution, specific gravity 1.07 at 18° C......	5.50
Water at 11° C..	3×10^5
Water at 76° C..	$1,196 \times 10^5$

NOTE.—The temperature coefficient of these liquids is about −.015 per 1° C.; that is, their resistance decreases about 1.5 per cent. for each degree centigrade rise in temperature.

EFFECT OF LENGTH AND SECTIONAL AREA ON RESISTANCE

11. Variation of Resistance With Length. — If the resistance and length of one conductor and the length of another conductor of the same sectional area and material are given, then the resistance of the second conductor may be obtained by the use of the following formula:

$$r_1 : r_2 = l_1 : l_2, \text{ or } r_2 = \frac{r_1 l_2}{l_1} \qquad (3)$$

In this formula,

r_1 = resistance of first conductor;
r_2 = required resistance of second conductor;
l_1 = length of first conductor;
l_2 = length of second conductor.

As in all examples of proportion, the two lengths must be reduced to the same unit.

Rule.—*The resistance of a given conductor increases as the length of the conductor increases; that is, the resistance of a conductor is directly proportional to its length.*

If the resistance and length of one conductor and the resistance of another conductor of the same sectional area and material, but of different length, are given, then the length of the second conductor may be obtained by solving formula **3** for l_2, which gives $l_2 = \frac{r_2 l_1}{r_1}$.

EXAMPLE 1.—Find the resistance of 1 mile of copper wire, if the resistance of 10 feet of the same wire is .013 ohm.

SOLUTION.— r_1 = .013 ohm; l_1 = 10 ft.; and l_2 = 1 mi. = 5,280 ft. Then, by formula **3**, the required resistance

$$r_2 = \frac{.013 \times 5{,}280}{10} = 6.864 \text{ ohms} \quad \text{Ans.}$$

EXAMPLE 2.—If the resistance of 11 inches of a German-silver wire is .022 ohm, what will be the length in feet of a piece of the same wire having a resistance of 2.4 ohms?

SOLUTION.— r_1 = .022 ohm; l_1 = 11 in.; r_2 = 2.4 ohms. By formula **3**, the required length

$$l_2 = \frac{r_2 l_1}{r_1} = \frac{2.4 \times 11}{.022} = 1{,}200 \text{ in.} = \frac{1200}{12} = 100 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the resistance per foot of a wire, if the resistance of 1 mile of the wire is 14.75 ohms. Ans. .002793 ohm

2. If the resistance of 18 inches of a certain piece of wire is .027 ohm, what is the resistance of 1,020 feet of the same wire? Ans. 18.36 ohms

12. Variation of Resistance With Sectional Area. If the sectional area of a conductor is increased, and other conditions remain unchanged, the resistance of the conductor will be decreased. For instance, if the sectional area be doubled, the resistance is halved; and, conversely, if the sectional area is halved, the resistance is doubled. The resistance of a conductor, therefore, increases with decreasing sectional area and diminishes with increasing sectional area. This may be expressed by the general rule:

Rule.—*The resistance of a conductor varies inversely as its sectional area.*

If the resistance and sectional area of one conductor and the sectional area of another conductor of the same length and material are given, then the resistance of the latter may be obtained.

Let r_1 = resistance of first conductor;
r_2 = required resistance of second conductor;
a_1 = sectional area of first conductor;
a_2 = sectional area of second conductor.

Then,

$$r_1 : r_2 = a_2 : a_1, \text{ or } r_2 = \frac{r_1 a_1}{a_2} \qquad (4)$$

If the resistance and sectional area of one conductor and the resistance of another conductor of the same length and material but different sectional area are given, then the sectional area of the second conductor may be obtained by solving formula **4** for a_2, which gives $a_2 = \frac{r_1 a_1}{r_2}$.

EXAMPLE 1.—The resistance of a conductor whose sectional area is .025 square inch is .32 ohm; what will be the resistance of a conductor whose sectional area is .125 square inch, other conditions remaining unchanged?

SOLUTION.— $r_1 = .32$ ohm; $a_1 = .025$ sq. in.; and $a_2 = .125$ sq. in. Then, by formula **4**, the required resistance

$$r_2 = \frac{.32 \times .025}{.125} = .064 \text{ ohm} \quad \text{Ans.}$$

EXAMPLE 2.—The sectional area of a conductor is .01 square inch and its resistance is 1 ohm; if its sectional area is decreased to .001 square inch, and other conditions remain unchanged, what will be its resistance?

SOLUTION.— $r_1 = 1$ ohm; $a_1 = .01$ sq. in.; and $a_2 = .001$ sq. in. By formula **4**, the required resistance

$$r_2 = \frac{1 \times .01}{.001} = 10 \text{ ohms} \quad \text{Ans.}$$

13. Variation of Resistance With Diameter.—The resistance of a conductor is independent of the shape of its cross-section. For example, the cross-section may be circular, square, rectangular, or irregular in shape; if the sectional area is the same in all cases, the resistances will be the same, other conditions being similar. When comparing the resistances of wires of circular cross-section, it is usually simpler to specify the diameter of the wire than its area. The sectional area of any wire of circular cross-section is, however, proportional to the square of the diameter; for the sectional area = diameter^2 × .7854.

Rule.—*The resistance of a conductor of circular cross-section is inversely proportional to the square of its diameter.*

For cylindrical wires, formula **4** may, therefore, be reduced to the following form:

$$r_1 : r_2 = d_2^2 : d_1^2, \text{ or } r_2 = \frac{r_1 d_1^2}{d_2^2} \qquad (5)$$

In this formula,

r_1 = resistance of first conductor;
r_2 = required resistance of second conductor;
d_1 = diameter of first conductor;
d_2 = diameter of second conductor.

If the resistance and diameter of one conductor and the resistance of another conductor of the same length and material but of different diameter are given, then the diameter of the second conductor may be obtained by solving formula **5** for d_2, which gives $d_2 = d_1\sqrt{\frac{r_1}{r_2}}$.

EXAMPLE 1.—The resistance of a round copper wire .12 inch in diameter is .64 ohm; find the resistance of a conductor whose diameter is .24 inch, the other conditions remaining unchanged.

SOLUTION.— $r_1 = .64$ ohm; $d_1 = .12$ in.; and $d_2 = .24$ in.
Then, by formula **5**, the required resistance

$$r_2 = \frac{.64 \times .12^2}{.24^2} = .16 \text{ ohm} \quad \text{Ans.}$$

EXAMPLE 2.—The diameter of a round wire is .1 inch and its resistance is 2 ohms; what would be its resistance if its diameter were decreased to .02 inch and the other conditions remained unchanged?

SOLUTION.— $r_1 = 2$ ohms; $d_1 = .1$ in.; and $d_2 = .02$ in.
By formula **5**, the required resistance

$$r_2 = \frac{2 \times .1^2}{.02^2} = \frac{2 \times .01}{.0004} = 50 \text{ ohms} \quad \text{Ans.}$$

14. By means of formulas **3**, **4**, and **5**, it is possible to readily solve problems like the following:

EXAMPLE 1.—If 1,000 feet of copper wire having a diameter of .05 inch has a resistance of 4 ohms, what will be the resistance of 2,500 feet of a rectangular ribbon of copper .06 inch wide and .02 inch thick?

SOLUTION.—First find the resistance of 2,500 ft. of a wire having a diameter of .05 in. For this purpose use formula **3**, in which $r_1 = 4$ ohms, $l_1 = 1{,}000$ ft., and $l_2 = 2{,}500$ ft. The resistance of 2,500 ft. of such a wire is

$$r_2 = \frac{4 \times 2{,}500}{1{,}000} = 10 \text{ ohms}$$

The sectional area of a copper wire having a diameter of .05 in. $= .7854 \times (.05)^2 = .0019635$ sq. in.

By formula **4**, in which $r_1 = 10$ ohms, $a_1 = .0019635$ sq. in., and $a_2 = .06 \times .02 = .0012$ sq. in., we now find the resistance of 2,500 ft. of a rectangular ribbon of copper having a sectional area of .0012 sq. in. to be

$$r_2 = \frac{10 \times .0019635}{.0012} = 16.36 \text{ ohms} \quad \text{Ans.}$$

EXAMPLE 2.—If the percentage conductivity of a certain substance is 60, what will be the resistance at 0° C. of 1,000 feet of a wire of this substance having a diameter of .080808 inch?

SOLUTION.—Matthiessen's standard copper, whose percentage conductivity is taken as 100, has a resistance per mil-foot at 0° C. of

9.59 ohms (see Table I). Hence, a wire having a percentage conductivity of 60 would have a resistance per mil-foot of $\frac{9.59}{.6}$ ohms. The resistance of 1,000 ft. of this wire, if it had the same diameter as that of the copper, that is, a diameter of 1 mil, or .001 in., would be $\frac{9.59 \times 1,000}{.6} = \frac{9,590}{.6}$ ohms (see Art. **11**). But if the wire has a diameter of .080808 in., instead of .001 in., then, according to formula **5**, it would have a resistance

$$R = \frac{9,590 \times (.001)^2}{.6 \times (.080808)^2} = 2.448 \text{ ohms} \quad \text{Ans.}$$

EXAMPLE 3.—Find the resistance in ohms at 32° F. of 1,000 feet of a wire .2 inch in diameter of German silver composed of 60 parts of copper, 25 parts of zinc, and 15 parts of nickel.

SOLUTION.—First use the proportion $r_1 : r_2 = l_1 : l_2$, that is, formula **3**, in which $r_1 = 180.35$ ohms = resistance of a mil-foot of this German-silver wire at 32° F. (see Table II), $l_1 = 1$ ft., and $l_2 = 1,000$ ft. Then,

$$r_2 = \frac{r_1 l_2}{l_1} = \frac{180.35 \times 1,000}{1} = 180,350 \text{ ohms}$$

Now use the proportion $r_2 : r_3 = (d_3)^2 : (d_2)^2$, that is, formula **5**, in which $r_2 = 180,350$ ohms, $d_3 = .2$, and $d_2 = .001$ in. (1 mil). Hence,

$$r_3 = \frac{180,350 \times (.001)^2}{(.2)^2} = 4.50875 \text{ ohms} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

The resistance of a piece of round copper wire .001 inch in diameter and 1 foot long is 10.8 ohms; use the same quality of copper and solve the following problems:

1. Find the resistance of 1,200 feet of round copper wire .102 inch in diameter. Ans. 1.2457 ohms

2. Find the resistance of 1 mile of round copper wire $\frac{1}{8}$ inch in diameter. Ans. 3.6495 ohms

3. Find the resistance of 1,500 feet of square copper wire .1 inch on a side. Ans. 1.2723 ohms

4. Find the resistance of 100 yards of copper wire .12 inch wide by .09 inch thick. Ans. .23562 ohm

NOTE.—The temperature of the copper in all the above problems is assumed to remain constant.

RESISTANCE OF CIRCUITS

SERIES-CIRCUITS

15. When a number of resistances are connected in series, the total resistance is equal to the sum of the separate resistances. For instance, in Fig. 1 the total resistance from *a* to *d* is equal to the sum of the resistances *a b*, *b c*, and *c d*. Hence, if the resistance of *a b* is 10 ohms, *b c* 15 ohms, and *c d* 20 ohms, the resistance from *a* to *d* = 10 + 15 + 20 = 45 ohms.

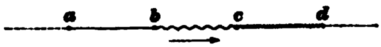

FIG. 1

PARALLEL, OR DIVIDED, CIRCUITS

16. A divided, or shunt, circuit is a branch or additional circuit provided at any part of a circuit through which the current branches, or divides, part flowing through the original circuit and part through the branch.

One branch of a divided circuit is said to be in multiple, or in parallel, with the other branch or branches into which the circuit is divided.

In the case of divided circuits, each of the branches acts as a shunt circuit to the others. A circuit or a portion of a circuit may be divided into any number of branches.

In treating divided or shunt circuits, only that part of the circuit will be considered which is divided into branches, each branch transmitting part of the current; the rest of the circuit is assumed to be closed through some electric source; as, for instance, a voltaic battery.

Before applying Ohm's law to divided circuits, it is necessary that the meaning of conductance be thoroughly understood. It has been explained that conductance is the inverse of resistance; or, in other words, it is the reciprocal of resistance. Therefore, conductance may be defined as being equal to $\frac{1}{R}$; from which it follows that the conductance is greater the less the resistance.

The conductance of any conductor is, therefore, one divided by the resistance of the conductor; and, conversely, the resistance of any conductor is one divided by the conductance of that conductor. For example, if the resistance of a circuit is 2 ohms, the conductance is represented by $\frac{1}{R} = \frac{1}{2}$; if the resistance is increased to 4 ohms, the conductance would be only one-half as much as in the first case, and would now be $\frac{1}{4}$.

17. Fig. 2 represents a divided circuit of two branches.

Let r_1 = resistance of one branch;
r_2 = resistance of the other branches;
I_1 and I_2 = currents in each branch, respectively;
I = current in the main circuit.

Then, $$I_1 + I_2 = I$$

When the current flows from a to b, if the resistances r_1 and r_2 are equal, the current will divide equally between the two branches. Thus, if a current of 4 amperes is flowing in the main circuit, 2 amperes will flow through each branch.

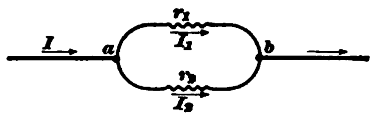

FIG. 2

When the resistances are unequal, the current will divide inversely as the respective resistances of the two branches; or, since the conductance is the reciprocal of the resistance, the current will divide in proportion to their respective conductances.

In Fig. 2 the conductances of the two branches are $\frac{1}{r_1}$ and $\frac{1}{r_2}$, respectively.

Therefore,

$$I_1 : I_2 = \frac{1}{r_1} : \frac{1}{r_2}; \text{ or, } I_2 = \frac{I_1 r_1}{r_2} \qquad (6)$$

EXAMPLE.—Given $I = 60$ amperes; $r_1 = 2$ ohms; $r_2 = 3$ ohms. Find I_1 and I_2.

SOLUTION.— $\frac{I_1}{I_2} = \frac{r_2}{r_1}$, or $\frac{I_1}{I_2} = \frac{3}{2}$, or $I_1 = \frac{3\,I_2}{2}$. But $I_1 + I_2 = 60$, or $I_1 = 60 - I_2$. Substituting for the value of I_1 gives $60 - I_2 = \frac{3\,I_2}{2}$. Simplifying gives $5\,I_2 = 120$, or $I_2 = 24$ amperes. Ans. $I_1 = 60 - 24 = 36$ amperes. Ans.

18. It is clear that two conductors in parallel will conduct an electric current more readily than one alone; that is, their joint conductance is greater than that of either taken alone. This being the case, their resistances must follow the inverse law, viz., the joint resistance of two conductors in parallel must be less than that of either taken alone.

Rule.—*If the resistances of two conductors are equal, their joint resistance when connected in parallel is one-half the resistance of either conductor.*

Suppose a conductor $A\,B$, Fig. 3, is split longitudinally into halves a and b. If $A\,B$ has a total resistance of 5 ohms between the points c and d, evidently the branches a and b must each have a resistance of 10 ohms, since they have only one-half of the sectional area of the whole conductor $A\,B$. Thus, their joint resistance does not amount to $10 + 10 = 20$ ohms, but to $\frac{10}{2} = 5$ ohms only.

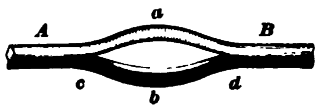

FIG. 3

19. When the resistances of two conductors are unequal, the determination of their joint resistance when connected in parallel involves some calculation.

In Fig. 2 the conductances of the branches are $\frac{1}{r_1}$ and $\frac{1}{r_2}$, respectively. Their joint conductance $= \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_2 + r_1}{r_1\,r_2}$; their joint resistance

$$R'' = 1 \div \frac{r_2 + r_1}{r_1\,r_2} = \frac{r_1\,r_2}{r_2 + r_1}. \qquad (7)$$

Rule.—*The joint resistance of two conductors in parallel is equal to the product of their resistances divided by the sum of their resistances.*

Example.—In Fig. 2, given $r_1 = 4$ ohms, $r_2 = 6$ ohms, and $I = 30$ amperes. Find the current I_1 and I_2 in each branch and the joint resistance of the branches from *a* to *b*.

Solution.— $\frac{I_1}{I_2} = \frac{6}{4}$, or $I_1 = \frac{6\,I_2}{4}$. But $I_1 + I_2 = 30$, or $I_1 = 30 - I_2$; substituting, $30 - I_2 = \frac{6\,I_2}{4}$. Reducing gives $10\,I_2 = 120$, or $I_2 = 12$ amperes. Ans. $I_1 = 30 - 12 = 18$ amperes. Ans.

By formula **7**, the joint resistance $R'' = \frac{r_1 r_2}{r_2 + r_1} = \frac{4 \times 6}{10} = 2.4$ ohms. Ans.

20. Fig. 4 represents a divided circuit of three branches. Let r_1, r_2, and r_3 = the resistances of the three branches, respectively; then, $\frac{1}{r_1}$, $\frac{1}{r_2}$, and $\frac{1}{r_3}$ represent the conductances of the three branches, respectively. Their joint conductance $= \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3}$. Since the joint resistance is the reciprocal of the joint conductance, then

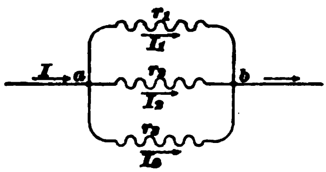

Fig. 4

$$R''' = 1 \div \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3} = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}, \tag{8}$$

which is the joint resistance of the three branches in parallel from *a* to *b*. We have, therefore, the following:

Rule.—*The joint resistance of three or more conductors in parallel is equal to the reciprocal of their joint conductance.*

When four or more resistances are connected in parallel, the following general formula for computing their joint resistance is usually the most convenient to employ:

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \text{etc.}} \tag{9}$$

Example.—In Fig. 4, given $r_1 = 5$ ohms, $r_2 = 10$ ohms, and $r_3 = 20$ ohms. Find their joint resistance from *a* to *b*.

Solution.—By formula **8**, the joint resistance

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{5 \times 10 \times 20}{(10 \times 20) + (5 \times 20) + (5 \times 10)} = \frac{1{,}000}{350}$$

$$= \frac{20}{7} = 2\tfrac{6}{7} \text{ ohms} \quad \text{Ans.}$$

21. In any divided circuit, the **difference of potential** between where the branches divide and where they unite is equal to the product of the sum of the currents in all the branches and their joint resistance in parallel, as will be apparent from a consideration of Ohm's law. For example, if the currents in the three branches, Fig. 4, are 16, 8, and 4 amperes, respectively, and the joint resistance from *a* to *b* is $2\frac{6}{7}$ ohms, then the difference of potential between *a* and *b* is $(16 + 8 + 4) \times 2\frac{6}{7} = 28 \times \frac{20}{7} = 80$ volts.

22. The **currents in the branches** of a divided circuit can be determined by finding the difference of potential between the points where the branches divide and where they unite, and dividing the result by the resistance of each branch. For example, in Fig. 4 assume that the difference of potenial between *a* and *b* is 80 volts, and that the separate resistances of the three branches are, respectively, 5, 10, and 20 ohms. Then the current in the first branch is $\frac{80}{5}$ = 16 amperes; in the second, $\frac{80}{10}$ = 8 amperes; and in the third, $\frac{80}{20}$ = 4 amperes.

23. The **resistances of the branches** of a divided circuit can be determined by finding the difference of potential between the points where the branches divide and where they unite, and dividing the result by the currents in each branch. For example, in Fig. 4 assume the difference of potential between *a* and *b* to be 80 volts, and the currents in the branches to be 16, 8, and 4 amperes, respectively; then, the resistance of the first branch is $\frac{80}{16}$ = 5 ohms; of the second, $\frac{80}{8}$ = 10 ohms; and of the third, $\frac{80}{4}$ = 20 ohms.

EXAMPLE 1.—The sum of the currents in the three branches r_1, r_2, and r_3 of a divided circuit is 52 amperes and the resistance of r_1 = 4 ohms, r_2 = 6 ohms, and r_3 = 8 ohms. How much current flows through each branch?

SOLUTION.—According to formula **8**, the joint resistance of the three branches $= \frac{4 \times 6 \times 8}{4 \times 6 + 4 \times 8 + 6 \times 8} = \frac{24}{13} = 1.846$ ohms. According to Ohm's law, the drop of potential between the two points where the branches are connected together $= 52 \times \frac{24}{13} = 96$ volts. Then, according to Ohm's law, the current

$$\left.\begin{array}{l} \text{in branch } r_1 = \frac{96}{4} = 24 \text{ amperes} \\ \text{in branch } r_2 = \frac{96}{6} = 16 \text{ amperes} \\ \text{in branch } r_3 = \frac{96}{8} = 12 \text{ amperes} \end{array}\right\} \text{Ans.}$$

EXAMPLE 2.—Four coils *A*, *B*, *C*, and *D* are connected in parallel; the resistance of *A* is 4 ohms; *B*, 5 ohms; *C*, 8 ohms; and *D*, 10 ohms. (*a*) What is the joint resistance of the four coils? (*b*) If the current flowing through *A* alone is 40 amperes, how much current flows through each of the other coils?

SOLUTION.—(*a*) The joint resistance of the four coils in parallel is equal to the reciprocal of the sum of the conductances of the four coils $= \frac{1}{4} + \frac{1}{5} + \frac{1}{8} + \frac{1}{10} = \frac{27}{40}$. Hence, the joint resistance of the four coils in parallel is equal to the reciprocal of $\frac{27}{40}$, that is, $\frac{40}{27} = 1.481$ ohms. Ans.

(*b*) The fall of potential through the coil $A = 40 \times 4 = 160$ volts. Since the four coils are joined in parallel, 160 volts must also be the fall of potential through *B*, *C*, and *D*; hence the current in $B = \frac{160}{5} = 32$ amperes, the current in $C = \frac{160}{8} = 20$ amperes, and the current in $D = \frac{160}{10} = 16$ amperes. Ans.

EXAMPLE 3.—As shown in Fig. 5, the two coils *A* and *B* are connected in parallel with each other, and in series with the coil *C* and the dynamo *D*. Suppose that the resistance of the coil *A* is 40 ohms; *B*, 60 ohms; *C*, 29 ohms; the internal resistance r_i, of the dynamo, 1 ohm; and the voltage across the terminals of the dynamo on open circuit 108 volts. (*a*) What will be the joint resistance of the two coils *A* and *B*? (*b*) What will be the total resistance of the circuit? (*c*) What will be the total current? (*d*) What will be the difference of potential across the terminals of the dynamo when the circuit is closed? (*e*) What will be the current in each coil?

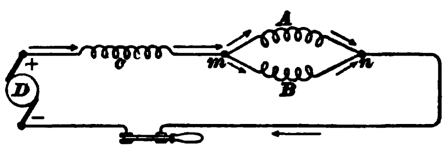

FIG. 5

SOLUTION.—(*a*) The joint resistance of the two coils *A* and *B* in parallel $= \frac{40 \times 60}{40 + 60} = 24$ ohms. Ans.

(*b*) The total resistance of the circuit is equal to the sum of the internal and external circuit, and hence the total resistance of the circuit $= 24 + 29 + 1 = 54$ ohms. Ans.

(*c*) According to Ohm's law, the total current

$$= \frac{\text{total electromotive force}}{\text{total resistance of the circuit}},$$

hence, $I = \frac{108}{54} = 2$ amperes. Ans.

(*d*) The difference of potential across the terminals of the dynamo when the circuit is closed is equal to the voltage, or electromotive force, on open circuit less the drop through the dynamo when the circuit is closed, that is, $E' = E - I r_i = 108 - 2 \times 1 = 106$ volts, which is the voltage across the dynamo terminals when 2 amperes are flowing through the circuit. Ans.

(*e*) The difference of potential across the two coils A and B, which are in parallel, $= I \times R$, in which R is the joint resistance of the two coils in parallel and I is the total current through both coils; hence $I \times R = 2 \times 24 = 48$ volts. Then the current in $A = \frac{48}{40} = 1.2$ amperes and the current in $B = \frac{48}{60} = .8$ ampere. The current in $C = 2$ amperes. Ans.

EXAMPLE 4.—Two branches of a divided circuit are both made of copper wire. One branch A is a round wire 1,000 feet long and .02 inch in diameter, the second branch B is a square wire .02 inch on a side and 1,000 feet long; if the sum of the currents in the two branches is 2.6 amperes, what is the current in each branch ?

SOLUTION.—The sectional area of A is $.02^2 \times .7854 = .00031416$ sq. in. The sectional area of B is $.02 \times .02 = .0004$ sq. in. The conductance of a conductor is proportional to its sectional area; therefore, the relative conductances of the two branches A and B are .00031416 and .0004, respectively. The current divides among the branches of a divided circuit in proportion to their conductances; therefore, if I_1 represents the current in branch A, and I_2 the current in branch B, then

$$I_1 : I_2 = .00031416 : .0004; \text{ or } I_1 : I_2 = .7854 : 1;$$

hence, $I_1 = .7854 \times I_2$. Now $I_1 + I_2 = 2.6$, or $I_1 = 2.6 - I_2$. Equating the values of I_1 gives $2.6 - I_2 = .7854 \times I_2$, or $I_2 = \frac{2.6}{1.7854}$ $= 1.4562$ amperes in B. Ans.

$$I_1 = 2.6 - 1.4562 = 1.1438 \text{ amperes in } A \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The resistances of two branches X and Y of a divided circuit are 13 and 29 ohms, respectively. Find their joint resistance in parallel. Ans. 8.9762 ohms

2. The sum of the currents in two branches X and Y of a divided circuit is 28 amperes. If the resistance of X alone is 7 ohms and the resistance of Y alone is 4 ohms, what is the current in each branch ?

Ans. { Current in branch X is 10.1818 amperes
Current in branch Y is 17.8182 amperes

3. The resistances of three branches of a divided circuit are, respectively, 36, 45, and 64 ohms. Find their joint resistance in parallel. Ans. 15.2381 ohms

4. The joint resistance of three conductors X, Y, and Z, connected in parallel, is 2.5 ohms. If the currents in the branches are, respectively, .6, .7, and .8 ampere, what is the resistance of each branch?

Ans. { Resistance of branch $X = 8.75$ ohms; Resistance of branch $Y = 7.5$ ohms; Resistance of branch $Z = 6.5625$ ohms }

5. The resistances of three branches X, Y, and Z of a divided circuit are, respectively, 2, 3, and 4 ohms. If the sum of the currents in the three branches is 26 amperes, what is the current in each branch?

Ans. { 12 amperes in branch X; 8 amperes in branch Y; 6 amperes in branch Z }

6. If the resistances of the two branches A and B, Fig. 5, are 79 and 98 ohms, respectively, and a current of 2 amperes is flowing in the main or undivided portion of the circuit, what is the difference of potential in volts between m and n? Ans. 87.48 volts

7. The joint resistance in parallel of two branches A and B of a divided circuit is 47.9 ohms; if the resistance of A is 2.7 times the resistance of B, what is the resistance of each branch?

Ans. { Resistance of branch $A = 177.23$ ohms; Resistance of branch $B = 65.6407$ ohms }

INTERNAL RESISTANCE OF BATTERIES

24. In a simple voltaic cell, the **internal resistance,** that is, the resistance of the two plates and the electrolyte, is of great importance, for it determines the maximum strength of current that can possibly be obtained from the cell. In the common forms of cells, the internal resistance may be excessively large, owing to the resistance of the electrolyte, the relative resistance of ordinary liquids used as electrolytes being from 1,000,000 to 20,000,000 times that of the common metals. In liquids, as in all conductors, the resistance increases as the length of the circuit increases, and diminishes as its sectional area increases. Consequently, the internal resistance of a simple voltaic cell is reduced by decreasing the distance between the two plates or elements and by increasing their active surfaces,

25. Cells in Series.—When a number of cells forming one battery are joined in series, as shown in Fig. 6, the total internal resistance of the battery is equal to the sum of the internal resistances of all the cells. When all the cells joined together in series are similar in kind, size, and condition, as is usually the case, the total internal resistance is equal to the resistance of one cell multiplied by the number of cells.

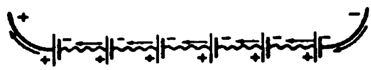

FIG. 6

26. Cells in Parallel.—When a number of cells are joined in parallel, as shown in Fig. 7, the total internal resistance of the battery is equal to the reciprocal of the sum of the conductances of all the cells. The sum of the conductances is the sum of the reciprocals of the resistance of each cell. When all the cells joined together in parallel are similar in kind, size, and condition, as is usually the case, the total internal resistance is equal to the resistance of one cell divided by the number of cells. A number of similar cells joined together in parallel is really equivalent to one larger cell, each immersed element of which has a surface equal in area to the sum of the areas of the immersed surfaces of the similar elements of all the cells joined in parallel.

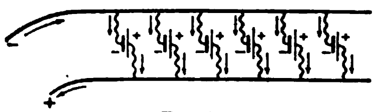

FIG. 7

27. Cells in Multiple Series.—When a number of cells of similar kind, size, and condition are joined in multiple series or parallel series, as shown in Fig. 8, the total internal resistance is obtained by multiplying the resistance of one cell by the number of cells in series in one group or row and dividing the product so obtained by the number of groups or rows joined in parallel.

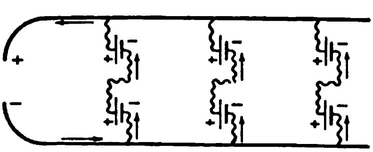

FIG. 8

28. The foregoing principles may be very nicely expressed by the following formula, in which b represents the internal resistance of one cell; s, the number of cells joined together in series in one row; p, the number of rows joined in multiple, or parallel; and B, the total resistance of the group of cells:

$$B = \frac{s\,b}{p} \qquad (10)$$

If all the cells are in series, there is only 1 row; that is, $p = 1$, hence $B = s\,b$. If all the cells are in parallel, then the number in series is one, that is, $s = 1$, hence $B = \frac{b}{p}$.

EXAMPLE.—If the internal resistance of each cell in Figs. 6, 7, and 8 is equal to 3 ohms, what is the total internal resistance of the whole group of cells in each figure?

SOLUTION.—In Fig. 6 there are 6 cells in series, hence the total internal resistance of the group is $6 \times 3 = 18$ ohms. Ans.

In Fig. 7 there are 6 cells in parallel, hence the total internal resistance of the group is $\frac{3}{6} = \frac{1}{2}$ ohm. Ans.

In Fig. 8 there are 3 groups of cells, each group consisting of 2 cells joined in series. Hence, in formula **10**, $b = 3$, $p = 3$, and $s = 2$; consequently, the total internal resistance $B = \frac{2 \times 3}{3} = 2$ ohms. Ans.

The internal resistance of the ordinary forms of cells varies from about .02 to 20 ohms.

29. Current in a Circuit Containing Voltaic Cells.—When a battery constitutes part of a circuit, the battery is not only acting as a source of E. M. F., but constitutes also a part of the total resistance of the circuit. The current that flows in a circuit containing a number of voltaic cells is equal to the total electromotive force of the cells divided by the total resistance of the circuit, the total resistance of the circuit being equal to the sum of the external and internal resistances. Hence, the current $I = \frac{E}{B + R}$.

The internal resistance of the battery is, under certain conditions, very effective, and in some cases determines the most suitable arrangement of the cells for the production of the proper current strength. The current that will flow in a circuit containing a number of voltaic cells and an external resistance may be computed by means of the following formula: Let I equal the current; e, the electromotive force of each cell; b, the internal resistance of each cell; s, the number of cells connected in series in one row; p, the number of rows of cells joined in parallel; and R, the total external resistance of the circuit; then,

$$I = \frac{s\,e}{\frac{s\,b}{p} + R} \qquad (11)$$

$s\,e = E =$ total electromotive force of the battery, and $\frac{s\,b}{p} = B =$ total internal resistance of the battery.

EXAMPLE 1.—The total electromotive force of a voltaic battery is 22 volts and its total internal resistance is 11 ohms; what is the resistance of the external circuit if .5 ampere is flowing through the circuit?

SOLUTION.—The current $I = \frac{E}{B + R}$ in which $I = .5$ ampere $E = 22$ volts, and $B = 11$ ohms. Solving for R, which is the unknown quantity, we have $R = \frac{E - IB}{I} = \frac{22 - .5 \times 11}{.5} = 33$ ohms. Ans.

EXAMPLE 2.—If, in Fig. 9, the electromotive force and internal resistance per cell be 1.5 volts and 3 ohms, respectively, the resistance of the line wire 20 ohms, and the resistance of the coils of each telegraph sounder 20 ohms, what will be the current flowing in the circuit?

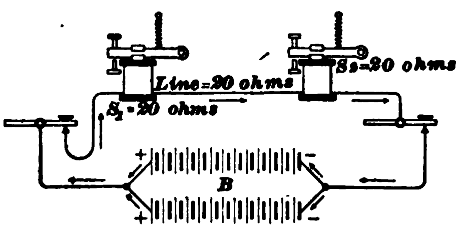

FIG. 9

SOLUTION.—In this figure, $s = 13$, $p = 2$, $R = 20 + 20 + 20 = 60$. Then, by formula **11**, we have

$$I = \frac{13 \times 1.5}{\frac{13 \times 3}{2} + 60} = .245 \text{ ampere} \quad \text{Ans.}$$

EXAMPLE FOR PRACTICE

A voltaic battery whose internal resistance is 36 ohms is connected to a copper wire having a resistance of 22 ohms; what is the total electromotive force in volts generated in the battery, if a current of 2 amperes flows under these conditions? Ans. 116 volts

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SIZES AND RESISTANCES OF WIRES

WIRE GAUGES

30. Sizes of Wire.—Unfortunately, various standards of wire gauges have been adopted by different manufacturers, with the result that there is a lack of uniformity in this direction, which frequently causes confusion. The standards by which the various sizes of wire are designated are usually termed **wire gauges.** In each gauge a particular number refers to a wire having a certain diameter. The size of wire generally decreases as the gauge number increases, but the law by which this decrease occurs is not the same in the different gauges.

31. Circular Measure.—The best method of designating the size of a wire is to express its diameter in *mils* and its sectional area in *circular mils.*

32. A **mil** is a unit of length used in measuring the diameter of wires, and is equal to $\frac{1}{1000}$ inch; that is, 1 mil = .001 inch.

33. A **circular mil** is a unit of area. If the diameter of a wire is given in mils, the square of this diameter gives its sectional area in circular mils. This method of expressing the area of cross-section of a wire is chosen in preference to expressing it in square inches, because a very simple relation exists between the circular mil and the diameter of a wire, so that either is more easily determined from the other than if the area were expressed in square inches.

The area of any circle in square measure is equal to $\pi r^2 = \frac{1}{4}\pi d^2$, where r is the radius and d the diameter of the circle.

If d is expressed in inches, the area $\frac{1}{4}\pi d^2$ will be in square inches. If d is in mils, the area $\frac{1}{4}\pi d^2$ will be in square mils. The area of a circle 1 mil in diameter is $\frac{1}{4}\pi \times 1^2 = \frac{1}{4}\pi$ square mil $= 1^2$ circular mil; and, hence, 1 square mil $= \frac{4}{\pi}$ circular mils. The area of any circle in circular mils will be equal to the area of that circle in square mils multiplied by the number of circular mils in 1 square mil; thus, the area of a circle d mils in diameter, expressed in circular mils C. M., is equal to $\frac{1}{4}\pi d^2 \times \frac{4}{\pi} = d^2$.

From this we see that the area of any circle expressed in circular mils is equal to the square of the diameter expressed in mils, that is, C. M. $= d^2$.

EXAMPLE 1.—What is the area in circular mils of a round wire having a diameter of .46 inch?

SOLUTION.— .46 inch = 460 mils. Since the area in mils is equal to the square of the diameter in mils, we have

$$460 \times 460 = 211{,}600 \text{ cir. mils} \quad \text{Ans.}$$

EXAMPLE 2.—Find the area of a round copper rod having a diameter of $\frac{3}{16}$ inch.

SOLUTION.— $\frac{3}{16}$ in. = 187.5 mils.

$$187.5 \times 187.5 = 35{,}156.25 \text{ cir. mils} \quad \text{Ans.}$$

If we know the area of a wire in circular mils, we may obtain the diameter in mils by extracting the square root of its area in circular mils.

EXAMPLE 3.—What is the diameter of a wire having a sectional area of 1,021.5 circular mils?

SOLUTION.— $\sqrt{1{,}021.5} = 31.961$ mils $= .031961$ in. Ans.

34. Brown & Sharpe, or American, Gauge.—In the United States, copper wire is usually designated by this gauge, which is generally termed B. & S. The rule by which the sizes of wire increase as the gauge numbers diminish is so simple in this gauge that it is surprising so few people understand it. If we take any gauge number as a basis of comparison, then, by adding 3 to the gauge number, we obtain the number of a wire having very nearly one-half the sectional area. To illustrate: One No. 7 wire will have the same sectional area as two No. 10's, as four No. 13's, as eight No. 16's, and so on. Similarly, by subtracting 3 from any gauge number, we obtain the number of a wire having very nearly twice the sectional area. Thus, one No. 1 has twice the area of a No. 4; one No. 10 has twice the area of a No. 13. A little study will show that the ratio between the area of each wire and the next smaller or larger is equal to the cube root of 2, or 1.26, for, in order to obtain the size of a wire of twice the area of a given wire, we must multiply the area of the given wire by the ratio three times, therefore the cube of the ratio must be equal to 2.

From the foregoing we may deduce the following rules, remembering that the resistance of a wire varies inversely as its sectional area:

Rule I.—*The ratio between the resistance of any wire in the B. & S. gauge and that of the next higher number is that of 1 to 1.26.*

Rule II.—*The ratio between the resistance of any wire in the B. & S. gauge and that of the next lower number is that of 1.26 to 1.*

A wire three sizes smaller than a given wire will have a resistance twice as great, and a wire three sizes larger will have a resistance one-half as great as that of the given wire.

EXAMPLE 1.—Find the resistance of 1,000 feet of No. 16 B. & S. gauge copper wire having given that the resistance of 1,000 feet of No. 10 wire is 1 ohm.

SOLUTION.—No. 16 is six sizes smaller than No. 10, and will therefore have $2 \times 2 = 4$ times the resistance. $4 \times 1 = 4$ ohms. Ans.

EXAMPLE 2.—The resistance of a No. 12 B. & S. gauge copper wire is 8.37 ohms per mile. What is the resistance (*a*) of a mile of No. 11? (*b*) of a mile of No. 18?

SOLUTION.—(*a*) $8.37 \div 1.26 = 6.64$ ohms. Ans.
(*b*) $8.37 \times 1.26 = 10.55$ ohms. Ans.

EXAMPLE 3.—The resistance of a No. 00 B. & S. gauge copper conductor is .411 ohm per mile. What is the resistance of a similar conductor of No. 3 gauge?

SOLUTION.—The third size smaller than No. 00 is No. 2. The resistance of No. 2 per mile is, therefore, $2 \times .411 = .822$ ohm. The resistance of No. 3 is 1.26 times that of No. 2, or $.822 \times 1.26 = 1.036$ ohms. Ans.

35. It is a very convenient fact to remember that the diameter of a No. 10 wire in the B. & S. gauge is very close to $\frac{1}{10}$ inch (.10189), and that the resistance of a No. 10 annealed copper wire per thousand feet is practically 1 ohm (.9972). For rough values, one can, by remembering these facts, compute the resistance or sectional area of any other size of copper wire in the B. & S. gauge without using the table. For accurate calculations, however, it is always better to consult a table giving the various properties of different sizes.

36. Other Wire Gauges.—Besides the Brown & Sharpe gauge, the following have been or are used to a considerable extent: Birmingham, or Stubs's, wire gauge, abbreviated to B. W. G.; Washburn & Moen Manufacturing Company's gauge; Trenton Iron Company's gauge; G. W. Prentiss's gauge; British Standard wire gauge, abbreviated S. W. G.; and the old English gauge. Table VII shows the diameters of the wires of the different gauge numbers according to each of these standards.

COPPER WIRE

37. Matthiessen's Standard.—Tables giving the resistance of the various sizes of copper wire are usually based on the grade of wire used by Matthiessen in determining the resistance of copper. The conductivity of the wire used by Matthiessen was at one time the highest known, but copper wire has since been produced having a somewhat higher conductivity. From Matthiessen's measurements, it has been determined that a piece of soft copper wire 1 foot long, and having a uniform diameter of .001 inch, has a resistance of 9.590 international ohms at a temperature of 0° C. Such a piece of wire is termed a *mil-foot*, meaning that its diameter is 1 mil and its length 1 foot. Inasmuch

TABLE VII

DIFFERENT STANDARDS FOR WIRE GAUGES

(Dimensions of Wires in Decimal Parts of an Inch)

Number of Wire Gauge	American, or Brown & Sharpe (B. & S.)	Birmingham, or Stubs (B. W. G.)	Washburn & Moen M'f'g Co., Worcester, Mass.	Trenton Iron Co., Trenton, N. J.	G. W. Prentiss, Holyoke, Mass.	Old English, From Brass Mfrs' List	British Standard (S. W. G.)	Number of Wire Gauge
000000			.4600					000000
00000			.4300	.4500				00000
0000	.460000	.454	.3930	.4000				0000
000	.409640	.425	.3620	.3600	.3586			000
00	.364800	.380	.3310	.3300	.3282			00
0	.324860	.340	.3070	.3050	.2994			0
1	.289300	.300	.2830	.2850	.2777			1
2	.257630	.284	.2630	.2650	.2591			2
3	.229420	.259	.2440	.2450	.2401			3
4	.204310	.238	.2250	.2250	.2230		.2320	4
5	.181940	.220	.2070	.2050	.2047		.2120	5
6	.162020	.203	.1920	.1900	.1885		.1920	6
7	.144280	.180	.1770	.1750	.1758		.1760	7
8	.128490	.165	.1620	.1600	.1605		.1600	8
9	.114430	.148	.1480	.1450	.1471		.1440	9
10	.101890	.134	.1350	.1300	.1351		.1280	10
11	.090742	.120	.1200	.1175	.1205		.1160	11
12	.080808	.109	.1050	.1050	.1065		.1040	12
13	.071961	.095	.0920	.0925	.0928		.0920	13
14	.064084	.083	.0800	.0800	.0816	.08300	.0800	14
15	.057068	.072	.0720	.0700	.0726	.07200	.0720	15
16	.050820	.065	.0630	.0610	.0627	.06500	.0640	16
17	.045257	.058	.0540	.0525	.0546	.05800	.0560	17
18	.040303	.049	.0470	.0450	.0478	.04900	.0480	18
19	.035890	.042	.0410	.0400	.0411	.04000	.0400	19
20	.031961	.035	.0350	.0350	.0351	.03500	.0360	20
21	.028462	.032	.0320	.0310	.0321	.03150	.0320	21
22	.025347	.028	.0280	.0280	.0290	.02950	.0280	22
23	.022571	.025	.0250	.0250	.0261	.02700	.0240	23
24	.020100	.022	.0230	.0225	.0231	.02500	.0220	24
25	.017900	.020	.0200	.0200	.0212	.02300	.0200	25
26	.015940	.018	.0180	.0180	.0194	.02050	.0180	26
27	.014195	.016	.0170	.0170	.0182	.01875	.0164	27
28	.012641	.014	.0160	.0160	.0170	.01650	.0148	28
29	.011257	.013	.0150	.0150	.0163	.01550	.0136	29
30	.010025	.012	.0140	.0140	.0156	.01375	.0124	30
31	.008928	.010	.0135	.0130	.0146	.01225	.0116	31
32	.007950	.009	.0130	.0120	.0136	.01125	.0108	32
33	.007080	.008	.0110	.0110	.0130	.01025	.0100	33
34	.006305	.007	.0100	.0100	.0118	.00950	.0092	34
35	.005615	.005	.0095	.0095	.0109	.00900	.0084	35
36	.005000	.004	.0090	.0090	.0100	.00750	.0076	36
37	.004453		.0085	.0085	.0095	.00650	.0068	37
38	.003965		.0080	.0080	.0090	.00575	.0060	38
39	.003531		.0075	.0075	.0083	.00500	.0052	39
40	.003145		.0070	.0070	.0078	.00450	.0048	40
41							.0044	41
42							.0040	42

as there are three different standard ohms, the British Association, or B. A., ohm, the legal ohm, and the international ohm, it is well to give the values of Matthiessen's standard in all of them. Table VIII is taken from the report of the Standard Wiring Table Committee of the American Institute of Electrical Engineers, and gives the resistances, at 0° C., not only of the mil-foot, but of the meter-gram, the meter-millimeter, and the cubic centimeter.

TABLE VIII

MATTHIESSEN'S STANDARD

Dimensions of Standard Specific Gravity = 8.89	Resistance at 0° C.		
	B. A. Ohms	Legal Ohms	International Ohms
Meter-gram soft copper	.14365	.14206	.14173
Meter-millimeter soft copper	.02057	.02035	.02030
Cubic centimeter soft copper	.000001616	.000001598	.000001594
Mil-foot soft copper	9.720	9.6120	9.590

The following values derived from Matthiessen's standard copper are frequently useful.

TABLE IX

Temperature 75° F., or 23.8° C.	Annealed Copper	Hard-Drawn Copper
Specific resistance (centimeter cube) in microhms	1.7464	1.7859
Resistance of a piece 1 inch long and 1 square inch sectional area in microhms	.68759	.70313

38. Pure annealed copper has a specific gravity of 8.89 at 60° F.; 1 cubic inch of it weighs .32 pound, and its melting point is about 2,100° F. As first manufactured, copper wire did not possess enough tensile strength to well adapt it for line wire, and for that reason and because of its greater expense, it was used but little for that purpose. The process of hard drawing copper wire has, however, greatly increased its tensile strength without seriously injuring its conductivity.

Tables X and XI give the resistances and weights for all sizes of copper wire, according to the B. & S. and the B. W. gauges, respectively. These tables are based on Matthiessen's standard.

TEMPERATURE COEFFICIENT

39. The resistance of any conducting body changes with variations in the temperature. The change in the resistance of a substance per ohm for unit change of temperature is known as the **temperature coefficient.** Thus a piece of copper wire which is known to have a resistance of 10 ohms at a temperature of 32° F. is found to have a resistance of 11 ohms at 77° F.

Variations in resistance due to variations of temperature often become quite important in practical work and it may be necessary to make an allowance for a change in temperature. If R_0 is the resistance of a piece of wire at 0° C. and a is the temperature coefficient of the substance, then if its temperature is raised from 0° to $t°$, its resistance has increased an amount $= R_0 \times a \times t$ ohms, and hence its resistance at $t°$ $= R_0 + R_0 \times a \times t$. Then we have

$$R = R_0(1 + a\,t) \qquad \textbf{(12)}$$

The resistance R_2 at $t_2°$ is similarly $= R_0\,(1 + a\,t_2)$. If R_0 is not known, but R_1, t_1, and t_2 are known, and it is desired to calculate the resistance R_2 at $t_2°$, then we have

$$R_2 = \frac{R_1(1 + a\,t_2)}{(1 + a\,t_1)}$$

40. A sufficiently approximate and more convenient formula for most purposes, and one which represents the effect of a change of temperature on the resistance of a substance, is as follows:

$$R_2 = R_1\,[1 + a\,(t_2 - t_1)], \qquad \textbf{(13)}$$

TABLE X

Gauge No.—B. & S.	Diameter in Mils, or 1/1000 Inch	Area in Circular Mils C. M. = d^2	Area in Square Inches $Area = \frac{d^2}{1,000,000} \times .7854$	Weights—Specific Gravity, 8.89				Resistance at 68° F., in International Ohms, Based Upon Matthiessen's Standard				
				Pounds per 1,000 Feet	Pounds per Mile	Feet per Pound	Ohms per Pound, Annealed	Ohms per 1,000 Feet		Ohms per Mile		Feet per Ohm, Annealed
	d	d^2						Pure Annealed	Hard Drawn	Pure Annealed	Hard Drawn	
0000	460.000	211,600.00	.1661900000	640.50000	3,381.400	1.561	.00007639	.04893	.050036	.25835	.26419	20,440.000
000	409.640	167,805.00	.1317900000	508.00000	2,682.200	1.969	.00012150	.06170	.063094	.32577	.33314	16,210.000
00	364.800	133,079.40	.1045200000	402.80000	2,126.800	2.482	.00019310	.07780	.079558	.41079	.42007	12,850.000
0	324.865	105,534.50	.0828870000	319.50000	1,686.900	3.130	.00030710	.09811	.100330	.51802	.52973	10,190.000
1	289.300	83,694.20	.0657320000	253.30000	1,337.200	3.947	.00048830	.12370	.126490	.65314	.66790	8,083.000
2	257.630	66,373.00	.0521280000	200.90000	1,060.600	4.977	.00077650	.15600	.159530	.82368	.84239	6,410.000
3	229.420	52,634.00	.0413390000	159.30000	841.090	6.276	.00123500	.19670	.201140	1.03860	1.06210	5,084.000
4	204.310	41,742.00	.0327840000	126.40000	667.390	7.914	.00196300	.24800	.253610	1.30940	1.33920	4,031.000
5	181.940	33,102.00	.0259990000	100.20000	529.060	9.980	.00312200	.31280	.319870	1.65160	1.68890	3,197.000
6	162.020	26,250.50	.0206180000	79.46000	419.550	12.580	.00496300	.39440	.403320	2.08250	2.12950	2,535.000
7	144.280	20,816.00	.0163510000	63.02000	332.750	15.870	.00789200	.49730	.508540	2.62580	2.68500	2,011.000
8	128.490	16,509.00	.0129670000	49.98000	263.890	20.010	.01255000	.62710	.641270	3.31110	3.38590	1,595.000
9	114.430	13,094.00	.0102830000	39.63000	209.240	25.230	.01995000	.79080	.808760	4.17530	4.27690	1,265.000
10	101.890	10,381.00	.0081548000	31.43000	165.950	31.820	.03173000	.99720	1.019900	5.26570	5.38480	1,003.000
11	90.742	8,234.00	.0064656000	24.93000	131.630	40.120	.05045000	1.25700	1.285400	6.63690	6.78690	795.300
12	80.808	6,529.90	.0051287000	19.77000	104.390	50.590	.08022000	1.58600	1.621800	8.37410	8.56330	630.700

13	71.961	5,178.40	.0040672000	15.68000	82.791	63.790	.12760000	1.99900	2.044300	10.55500	10.79400	500.100
14	64.084	4,106.80	.0032254000	12.43000	76.191	80.440	.20280000	2.52100	2.577900	13.31100	13.61200	396.600
15	57.068	3,256.70	.0025579000	9.85800	52.050	101.400	.32250000	3.17900	3.250800	16.78500	17.16500	314.500
16	50.820	2,582.90	.0020285000	7.81800	41.277	127.900	.51280000	4.00900	4.099600	21.16800	21.64600	249.400
17	45.257	2,048.20	.0016087000	6.20000	32.736	161.300	.81530000	5.05500	5.169200	26.69100	27.29400	197.800
18	40.303	1,624.30	.0012757000	4.91700	25.960	203.400	1.29600000	6.37400	6.518300	33.65500	34.41600	156.900
19	35.890	1,288.10	.0010117000	3.89900	20.595	256.500	2.06100000	8.03800	8.219600	42.44100	43.40000	124.400
20	31.961	1,021.50	.0008023100	3.09200	16.324	323.400	3.27800000	10.14000	10.372000	53.53900	54.74900	98.660
21	28.462	810.10	.0006362600	2.45200	12.946	407.800	5.21200000	12.78000		67.47900		78.240
22	25.347	642.40	.0005045700	1.94500	10.268	514.200	8.28700000	16.12000		85.11400		62.050
23	22.571	509.45	.0004001500	1.54200	8.142	648.400	13.18000000	20.32000		107.29000		49.210
24	20.100	404.01	.0003173300	1.22300	6.457	817.600	20.95000000	25.63000		135.53000		39.020
25	17.900	320.40	.0002516600	.96990	5.121	1,031.000	33.32000000	32.31000		170.59000		30.950
26	15.940	254.10	.0001995800	.76920	4.061	1,300.000	52.97000000	40.75000		215.16000		24.540
27	14.195	201.50	.0001582700	.61000	3.221	1,639.000	84.23000000	51.38000		271.29000		19.460
28	12.641	159.79	.0001255100	.48370	2.554	2,067.000	133.90000000	64.79000		242.09000		15.430
29	11.257	126.72	.0000995360	.38360	2.025	2,607.000	213.00000000	81.70000		431.37000		12.240
30	10.025	100.50	.0000789360	.30420	1.606	3,287.000	338.60000000	103.00000		543.84000		9.707
31	8.928	79.70	.0000625990	.24130	1.274	4,145.000	538.40000000	129.90000		685.87000		7.698
32	7.950	63.21	.0000496430	.19130	1.010	5,227.000	856.20000000	163.80000		864.87000		6.105
33	7.080	50.13	.0000393680	.15170	.801	6,591.000	1,361.00000000	206.60000		1,090.80000		4.841
34	6.305	39.75	.0000312210	.12030	.635	8,311.000	2,165.00000000	260.50000		1,375.50000		3.839
35	5.615	31.52	.0000247590	.09543	.504	10,480.000	3,441.00000000	328.40000		1,734.00000		3.045
36	5.000	25.00	.0000196350	.07568	.400	13,210.000	5,473.00000000	414.20000		2,187.00000		2.414
37	4.453	19.83	.0000155740	.06001	.317	16,660.000	8,702.00000000	522.20000		2,757.30000		1.915
38	3.965	15.72	.0000123450	.04759	.251	21,010.000	13,870.00000000	658.50000		3,476.80000		1.519
39	3.531	12.47	.0000097923	.03774	.199	26,500.000	22,000.00000000	830.40000		4,384.50000		1.204
40	3.145	9.89	.0000077634	.02993	.158	33,410.000	34,980.00000000	1,047.00000		5,528.20000		.955

TABLE XI

COPPER WIRE-BIRMINGHAM WIRE GAUGE

Gauge No. (B. W. G.)	Diameters in Mils, or 1/1000 Inch	Area in Circular mils. C. M. = d^2	Weights		Resistances in International Ohms, Based Upon Matthiessen's Standard at 68° F.	
			1,000 Feet	Mile	Ohms per 1,000 Feet	Ohms per Pound
0000	454	206,116	624.000	3,294.000	.05023	.00008051
000	425	180,625	547.000	2,887.000	.05732	.00010480
00	380	144,400	437.000	2,308.000	.07170	.00016400
0	340	115,600	350.000	1,847.000	.08957	.00025600
1	300	90,000	272.000	1,438.000	.11500	.00042230
2	284	80,656	244.000	1,289.000	.12840	.00052580
3	259	67,081	203.000	1,072.000	.15430	.00076010
4	238	56,644	171.000	905.000	.18280	.00106600
5	220	48,400	146.000	773.000	.21390	.00146000
6	203	41,209	125.000	659.000	.25130	.00201400
7	180	32,400	98.000	518.000	.31960	.00325800
8	165	27,225	82.000	435.000	.38030	.00461500
9	148	21,904	66.000	350.000	.47270	.00712900
10	134	17,956	54.000	287.000	.57660	.01061000
11	120	14,400	44.000	230 000	.71900.	.01650000
12	109	11,881	36.000	190.000	.87150	.02423000
13	95	9,025	27.300	144.000	1.14700	.04199000
14	83	6,889	20.800	110.000	1.50300	.07207000
15	72	5,184	15.700	83.000	1.99700	.12730000
16	65	4,225	12.800	68.000	2.45100	.19160000
17	58	3,364	10.200	54.000	3.07800	.30230000
18	49	2,401	7.300	38.400	4.31200	.59330000
19	42	1,764	5.300	28.200	5.87000	1.09900000
20	35	1,225	3.700	19.600	8.45200	2.27900000
21	32	1,024	3.100	16.400	10.11000	3.26200000
22	28	784	2.400	12.500	13.21000	5.56500000
23	25	625	1.900	10.000	16.57000	8.75600000
24	22	484	1.500	7.700	21.39000	14.60000000
25	20	400	1.200	6.400	25.88000	21.38000000
26	18	324	.980	5.200	31.96000	32.58000000
27	16	256	.770	4.100	40.45000	52.19000000
28	14	196	.590	3.100	52.83000	89.04000000
29	13	169	.510	2.700	61.27000	119.80000000
30	12	144	.440	2.300	71.90000	165.00000000
31	10	100	.300	1.600	103.50000	342.00000000
32	9	81	.250	1.300	127.80000	521.30000000
33	8	64	.190	1.020	161.80000	835.10000000
34	7	49	.150	.780	211.30000	1,425.00000000
35	5	25	.076	.400	414.20000	5,473.00000000
36	4	16	.048	.256	647.10000	13,360.00000000

in which R_1 = resistance at the lower temperature t_1; R_2 = resistance at the higher temperature t_2; t_1 = lower temperature; t_2 = higher temperature; a = temperature coefficient.

If t_1 and t_2 are given in centigrade (C.) degrees, then a is the temperature coefficient per degree C.; and if t_1 and t_2 are given in Fahrenheit (F.) degrees, then a is the temperature coefficient per degree F. The value of a for various metals and alloys, in both the centigrade and Fahrenheit scales, is given in columns 5 and 6, respectively, in Tables I and II. For pure, or good commercial, copper, Clark, Ford, and Taylor give $a = .004295$. If R_2 is the given resistance, then R_1 is obtained by dividing R_2 by $[1 + a(t_2 - t_1)]$; that is,

$$R_1 = \frac{R_2}{1 + a(t_2 - t_1)}$$

EXAMPLE.—An annealed copper conductor has a resistance of 15 ohms at a temperature of 20° C. What will be its resistance (*a*) at 50° C.? (*b*) at 8° C.?

SOLUTION.—(*a*) Let R_1 = the resistance at 20°, that is, $R_1 = 15$ ohms, $t_1 = 20°$, and $t_2 = 50°$. Then, by substituting in formula **13**, in which $a = .00402$ for annealed copper (see column 5, Table I), we get $R_2 = 15[1 + .00402(50 - 20)] = 15[1 + .00402 \times 30] = 16.809$ ohms. Ans.

(*b*) In this case, since 8° is less than 20°, R_1 = the resistance at 8°, $t_1 = 8°$, R_2 = the resistance at 20°, that is, $R_2 = 15$ ohms, and $t_2 = 20°$. It becomes necessary to solve for R_1 in formula **13**. Hence, we have

$$R_1 = \frac{R_2}{1 + a(t_2 - t_1)} = \frac{15}{1 + .00402(20 - 8)} = 14.31 \text{ ohms. Ans.}$$

41. As a matter of fact, the resistances of metals do not increase quite uniformly from 0° to 100° C. (32° to 212° F.). However, the average temperature coefficient for pure copper (.00223 for a change of 1° F.) is usually exact enough for a correction to 60° or 75° F., for all ranges of temperature that occur in the testing room; but for ranges below 50° F., or above 100° F., it is better to consult a table. The resistance of pure, or good commercial, copper when known for a given temperature may be reduced to its resistance at 75° F. by multiplying the known resistance by the factor given in Table XII corresponding to the given temperature. If the resistance is known at 75° F., then divide this value by the factor corresponding to the temperature to which it is desired to reduce it.

EXAMPLE.—The observed resistance of a copper wire is 12.74 ohms at 85° F. What is its resistance at 65° F.?

SOLUTION.—According to Table XII, the factor to reduce its resistance from 85° F. to 75° F. is .9787; hence its resistance at 75° F. = 12.74 × .9787 = 12.47 ohms. To reduce from 65° to 75°, the factor is 1.0222, hence its resistance at 65° = $\frac{12.47}{1.0222}$ = 12.20 ohms. Ans.

TABLE XII

TEMPERATURE COEFFICIENTS FOR COPPER WIRE

Temperature, Degrees F.	Factor	Temperature, Degrees F.	Factor	Temperature, Degrees F.	Factor	Temperature, Degrees F.	Factor
85.0	.9787	71.5	1.0077	58.0	1.0384	44.5	1.0708
84.5	.9797	71.0	1.0088	57.5	1.0395	44.0	1.0720
84.0	.9808	70.5	1.0099	57.0	1.0407	43.5	1.0733
83.5	.9818	70.0	1.0110	56.5	1.0419	43.0	1.0745
83.0	.9829	69.5	1.0121	56.0	1.0430	42.5	1.0757
82.5	.9839	69.0	1.0132	55.5	1.0442	42.0	1.0770
82.0	.9850	68.5	1.0144	55.0	1.0454	41.5	1.0783
81.5	.9861	68.0	1.0155	54.5	1.0466	41.0	1.0795
81.0	.9871	67.5	1.0166	54.0	1.0478	40.5	1.0808
80.5	.9882	67.0	1.0177	53.5	1.0490	40.0	1.0821
80.0	.9892	66.5	1.0188	53.0	1.0501	39.5	1.0833
79.5	.9903	66.0	1.0200	52.5	1.0513	39.0	1.0846
79.0	.9914	65.5	1.0211	52.0	1.0525	38.5	1.0858
78.5	.9924	65.0	1.0222	51.5	1.0537	38.0	1.0871
78.0	.9935	64.5	1.0233	51.0	1.0549	37.5	1.0884
77.5	.9946	64.0	1.0245	50.5	1.0561	37.0	1.0897
77.0	.9950	63.5	1.0257	50.0	1.0573	36.5	1.0910
76.5	.9967	63.0	1.0268	49.5	1.0585	36.0	1.0922
76.0	.9978	62.5	1.0279	49.0	1.0598	35.5	1.0935
75.5	.9989	62.0	1.0291	48.5	1.0610	35.0	1.0948
75.0	1.0000	61.5	1.0302	48.0	1.0622	34.5	1.0961
74.5	1.0011	61.0	1.0314	47.5	1.0634	34.0	1.0974
74.0	1.0022	60.5	1.0325	47.0	1.0646	33.5	1.0987
73.5	1.0033	60.0	1.0337	46.5	1.0659	33.0	1.1000
73.0	1.0044	59.5	1.0349	46.0	1.0671	32.5	1.1013
72.5	1.0055	59.0	1.0360	45.5	1.0683	32.0	1.1026
72.0	1.0066	58.5	1.0372	45.0	1.0695	31.5	1.1039

NOTE.—This table, which is given by Kempe in his "Handbook of Electrical Testing," is calculated from the exact formula $R_t = R_{32}[1 + .0023708(t - 32°) + .00000034548(t - 32°)^2]$, for pure, or good commercial, copper, as determined by Clark, Ford, and Taylor, in which t is expressed in Fahrenheit degrees.

The following table of centigrade and Fahrenheit degrees is given to facilitate the rapid conversion from one scale to the other.

TABLE XIII

TABLE OF CENTIGRADE AND FAHRENHEIT DEGREES

Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.
0	32.0	26	78.8	51	123.8	76	168.8
1	33.8	27	80.6	52	125.6	77	170.6
2	35.6	28	82.4	53	127.4	78	172.4
3	37.4	29	84.2	54	129.2	79	174.2
4	39.2	30	86.0	55	131.0	80	176.0
5	41.0	31	87.8	56	132.8	81	177.8
6	42.8	32	89.6	57	134.6	82	179.6
7	44.6	33	91.4	58	136.4	83	181.4
8	46.4	34	93.2	59	138.2	84	183.2
9	48.2	35	95.0	60	140.0	85	185.0
10	50.0	36	96.8	61	141.8	86	186.8
11	51.8	37	98.6	62	143.6	87	188.6
12	53.6	38	100.4	63	145.4	88	190.4
13	55.4	39	102.2	64	147.2	89	192.2
14	57.2	40	104.0	65	149.0	90	194.0
15	59.0	41	105.8	66	150.8	91	195.8
16	60.8	42	107.6	67	152.6	92	197.6
17	62.6	43	109.4	68	154.4	93	199.4
18	64.4	44	111.2	69	156.2	94	201.2
19	66.2	45	113.0	70	158.0	95	203.0
20	68.0	46	114.8	71	159.8	96	204.8
21	69.8	47	116.6	72	161.6	97	206.6
22	71.6	48	118.4	73	163.4	98	208.4
23	73.4	49	120.2	74	165.2	99	210.2
24	75.2	50	122.0	75	167.0	100	212.0
25	77.0						

42. As already stated, the resistances of metals do not increase quite uniformly from 0° to 100° C. (32° to 212° F.), but for most practical purposes formula **13** is sufficiently exact. For scientific work, however, it is usually necessary to employ the following more accurate formula:

$$R = R_o(1 + a\,t + b\,t^2), \qquad \textbf{(14)}$$

in which R = resistance at $t°$ centigrade, R_o = resistance at 0° centigrade, and t = temperature in centigrade degrees. For Matthiessen's pure copper $a = .004019$ and $b = .00000214$ when the temperature range is between 0° and 100° C., and $a = .003879$ and $b = .00000526$ when the temperature range is between 0° and 30° C. If the resistance R_o at 0° C. is not known or required, but R_1 is given and it is desired to calculate R_2, then

$$R_2 = \frac{R_1(1 + a\,t_2 + b\,t_2^2)}{(1 + a\,t_1 + b\,t_1^2)},$$

in which R_1 and R_2 are the resistances of the same piece of pure copper at the temperatures t_1 and t_2, respectively.

EXAMPLES FOR PRACTICE

1. Find the resistance in microhms at 75° F. of an inch cube of pure annealed silver. Ans. .6332 microhm

2. Find the resistance in ohms at 75° F. of 1 mile of square iron wire (E. B. B. quality) measuring .1 inch on each side. Ans. 27.0337 ohms

3. If a mile of a certain copper wire has a resistance of 13.6 ohms at 59.9° F. and its temperature coefficient is .00223, what will be its resistance at 75° F.? Ans. 14.0579 ohms

4. If a mile of a certain copper wire having a diameter of $\frac{1}{16}$ inch has a resistance of 14.06 ohms at 75° F., what will be the resistance between opposite faces of a cube measuring 1 inch on each side, that is, the resistance per inch cube? Ans. .6808 microhm

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ELECTROSTATIC CAPACITY

43. The **electrostatic capacity** of a conductor is measured by the quantity of electricity with which it must be charged in order to raise its electric potential from zero to unity. To make the meaning clearer, let us consider the capacity of a rubber bag when it is filled with water or gas. Its cubical contents is not limited to one definite quantity; on the contrary, it can vary between wide limits, depending on the pressure to which the water or gas is subjected. By pumping more gas into the bag, under an increasing pressure, the capacity of the bag will increase and also the pressure of the gas contained in it.

A charge of electricity will act in a similar manner. The number of coulombs of electricity residing on the surface of a conductor must not be considered as a fixed quantity, depending on the extent of its surface. It is, as seen in the case of the rubber bag, also dependent on the pressure: the higher the latter, the more compressed or dense the charge may be said to be. The smaller the above-mentioned bag is, the less quantity of gas will be required to raise the pressure; similarly with an electric conductor, the pressure will increase more rapidly if its capacity is small. A small conductor, such as an insulated sphere of the size of a pea, will not want so much as 1 unit of electricity to raise its potential from 0 to 1, and is therefore of small capacity; while a large sphere will require a large quantity to raise its potential to the same degree, and could therefore be said to be of large capacity. It is, then, necessary to know both the capacity of a conductor and the potential of the charge before any idea can be had of the quantity of electricity collected on the conductor.

44. A **condenser** is an appliance for storing up or holding electrostatic charges. It is usually made by taking a large number of sheets of tin foil and separating them by alternate sheets of waxed paper, mica, or other insulating material. The whole mass is pressed tightly together, one set of sheets constituting one terminal, and the alternate set the other, as shown in Fig. 10, where *p* represents the tin-foil sheets, *i* the insulating material between, and *T*, *T* the terminal posts. It should be noticed that there is no electrical connection between the two sets of plates connected to the two terminals *T*, *T*.

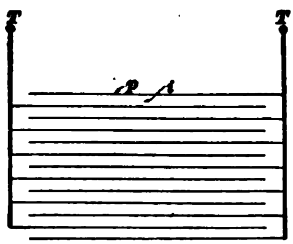

FIG. 10

PROPERTIES OF DIELECTRICS

INDUCTIVITY OF DIELECTRICS

45. If an uncharged metallic plate is covered with a sheet of mica and a charged metallic plate is set down on the mica, the upper plate will induce a charge in the lower plate that will be about 6 times greater than would be the case if the mica were replaced by dry air, the distance between the metal plates remaining exactly the same; hence, the mica has an inductive capacity 6 times greater than that of dry air. This quantity is known as the **inductivity of the dielectric.** It may be defined as the ratio of the capacity of a condenser when its plates are separated by that substance to the capacity of the same condenser when its plates are separated by the same thickness of air. Dry air at atmospheric pressure is considered the standard substance and its inductivity is taken as 1; hence, the inductivity of mica is 6.

The inductivity of a dielectric may be determined by ascertaining the thickness of a layer of the dielectric having the same inductive capacity as a given thickness of dry air; the thicker the layer, the higher is its inductivity. Thus, it is found that a thickness of resin of about 2.55 units has the same inductive capacity as 1 unit thickness of air; hence, resin is the better dielectric, and, air being taken as unity, has an inductivity of 2.55. Hence, if resin is interposed between two metal plates, they must be separated about $2\frac{1}{2}$ times the distance that would be necessary to obtain the same capacity with air.

46. The capacity of a condenser varies somewhat according to the method used in determining its capacity. On account of the so-called absorptive power of dielectrics, which will be explained later, a smaller capacity is obtained for a given condenser when it is measured by methods in which the condenser is charged and discharged a great number of times per second, than by methods in which the capacity is determined from one comparatively slow charge or discharge. Consequently, the values of the inductivity of dielectrics, as determined by different methods and by different authorities, varies greatly. Table XIV gives the inductivities of various substances, the inductivity of dry air at ordinary pressure being taken as unity. The values obtained by the so-called instantaneous methods are almost invariably lower than the values obtained by the slower charge and discharge methods.

DIELECTRIC STRENGTH

47. A property of insulating substances that is just as important as their specific resistance and inductivity is their **dielectric strength.** This property is the maximum difference of potential that an insulating substance can stand without being punctured. An insulating material may show a very high resistance when measured with a low voltage and a very sensitive galvanometer, while it may offer comparatively little resistance to the passage of a current when a very high voltage is used; whereas another material, which will not show so high a resistance with a low voltage, may better withstand disruption by a high voltage.

48. The dielectric strength of a substance is determined by placing it between two metal electrodes and increasing the difference of potential between the two electrodes by small steps until a spark passes through the dielectric. The dielectric is then said to be ruptured, or punctured, and the difference of potential between the electrodes at the step just preceding the rupture is the maximum strength of the dielectric in volts. If the maximum voltage so obtained is divided by the thickness of the substance in centimeters, we obtain the strength of the dielectric in volts per centimeter, which is a convenient unit for comparing the dielectric strength of various insulating materials. For such determinations very high voltages are required, too high to be

TABLE XIV

Material	Inductivity K
Air, vacuum at about .001 millimeter pressure	.9400
Air, vacuum at about 5 millimeters pressure	.9990
Hydrogen, at ordinary pressure	.9997 to 1.00026
Air, at ordinary pressure, standard	1.0000
Carbon dioxide, at ordinary pressure	1.00036 to 1.00095
Olefiant gas, at ordinary pressure	1.0007
Methane	1.0009
Sulphur dioxide, at ordinary pressure	1.0037
Manila paper	1.50
Carbon bisulphide	1.60 to 1.81
Paraffin, clear	1.68 to 2.32
Beeswax	1.86
Paraffin, solid	1.9936* to 2.32
Resin	1.77 to 2.55
Ozocerite	2.00
Petroleum	2.03 to 2.42
Ebonite	2.05* to 3.15
Turpentine	2.15 to 2.43
India rubber, pure	2.22 to 2.497
Sulphur	2.24 to 3.84
Gutta percha	2.46* to 4.20
Shellac	2.74* to 3.60
Olive and neatsfoot oils	3.00 to 3.16
Sperm oil	3.02 to 3.09
Glass	3.013* to 3.258*
Mica	4.00 to 8
Porcelain	4.38
Quartz	4.50
Flint glass, very light	6.57
Flint glass, light	6.85
Flint glass, very dense	7.40
Flint glass, double extra dense	10.10

* Results obtained by instantaneous methods.

measured by ordinary electrical instruments; however, it has been experimentally determined how many volts are required to produce a spark between needle points and between small spherical electrodes separated by air gaps of various lengths.

49. Dielectric Strength of Air.—At least from 300 to 400 volts is required to produce a spark across an air gap, no matter how short the gap is. Furthermore, the length of spark that can be produced through air depends on the pressure and temperature of the air, on the shape of the electrodes between which the spark is produced, and on whether a direct or alternating electromotive force is used. The dielectric strength, according to Thompson, increases "1 per cent. for a fall of 3 degrees of temperature or for a rise of 8 millimeters of pressure." Since 1 atmosphere will sustain a column of mercurcy 760 millimeters high, then a pressure of 8 millimeters is equivalent to $\frac{8}{760} = \frac{1}{95}$ of an atmosphere. If the difference of potential is nearly great enough to produce a spark, then a sudden reversal of the electromotive force will sometimes start a spark. The sparking distance is less with a very rapidly alternating electromotive force than with a steady, non-alternating electromotive force of the same potential. It requires a greater difference of potential to produce a spark in air between spheres or parallel disks than between needle points. To repeatedly obtain the same results, it is necessary to use each time new needle points or amalgamated, or newly polished, balls and disks.

The dielectric strength of a good vacuum and of very highly compressed air is very great. Such good vacuums have been produced as to render it impossible, with the electrostatic machines or induction coils heretofore used, to produce a spark across a vacuum even as short as 1 centimeter. Furthermore, the powerful induction coils heretofore used for such experiments have been unable to produce a spark across .05 centimeter of air when compressed to 40 or 50 atmospheres.

50. Table XV gives the difference of potential in volts necessary to produce a spark in air at ordinary atmospheric pressure between polished brass balls. The first two columns give the length in centimeters and inches, respectively, across which the volts in columns 3, 4, and 5 will produce a spark. The diameter of the balls used in each case is given at the head of the column.

TABLE XV

NUMBER OF VOLTS REQUIRED TO PRODUCE A SPARK BETWEEN BALLS IN AIR

Length of Spark Gap in		Diameter of the Balls		
Centimeters	Inches	1 Cm. = .3937 In.	2 Cm. = .787 In.	6 Cm. = 2.36 In.
		Volts	Volts	Volts
1	2	3	4	5
.02	.0079	1,560	1,530	
.04	.0157	2,460	2,430	
.06	.0236	3,300	3,240	
.08	.0315	4,050	3,990	
.10	.0394	4,800	4,800	4,500
.20	.0787	8,400	8,400	7,800
.30	.1181	11,400	11,400	10,800
.40	.1575	14,400	14,400	13,500
.50	.1969	17,100	17,100	16,500
.60	.2362	19,500	19,800	19,500
.70	.2756	21,600	22,500	22,500
.80	.3150	23,400	24,900	26,100
.90	.3543	24,600	27,300	29,000
1.00	.3937	25,500	29,100	32,700

The dielectric strength of air between the points of needles 2½ inches long, when an electromotive force that is reversed in direction 15,000 times a minute is used, is well shown by the curve in Fig. 11. In order to avoid the use of

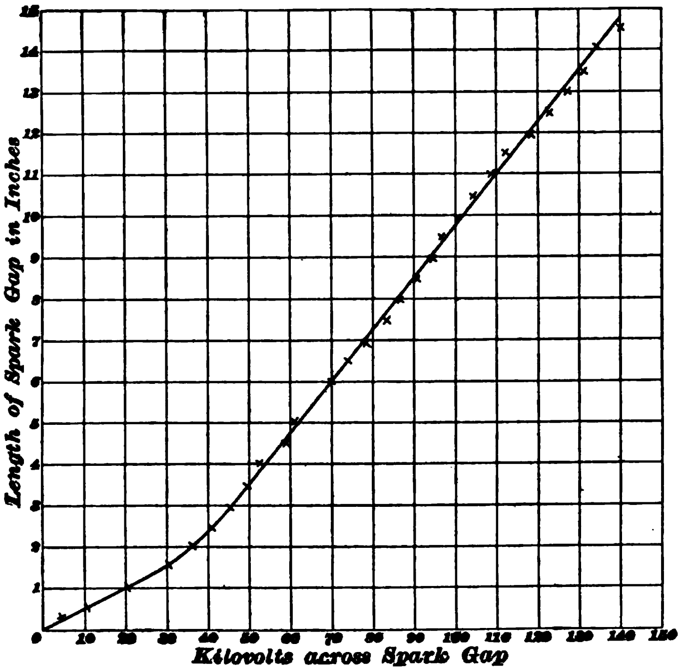

FIG. 11

large numbers, the divisions along the base line are marked kilovolts; 50 kilovolts, for instance, is equal to 50,000 volts.

51. Spark Gauge.—A spark gap in air, when especially constructed so that the distance between the sparking points can be very accurately determined, is called a **spark gauge.** Two pairs of electrodes are arranged in a parallel circuit and connected across a source capable of producing a very high difference of potential. One pair of electrodes forms the spark gauge and the space between the two electrodes of the other pair is filled with the insulating material to be tested. At first a spark is produced across a small air gap, then the length of the air gap is gradually increased until the voltage across the spark gap exceeds the dielectric strength of the insulating substance; the spark will then pass through the latter and rupture it instead of jumping across the air gap. As the voltage is increased, the insulating material is subjected to a continually increasing electric stress until it finally gives away and allows a flow of electricity in the form of a spark to pass through it. After once being punctured, the insulating quality of a solid material is practically destroyed because the material surrounding the puncture is carbonized and is therefore a fair conductor. Insulating liquids, such as an oil, are, however, nearly as good after being punctured as before, because the oil immediately fills up the hole produced by the spark.

TABLE XVI

DIELECTRIC STRENGTH OF VARIOUS SUBSTANCES

Substance	Strength in Volts per Centimeter	Substance	Strength in Volts per Centimeter
Oil of turpentine	94,000	Beeswaxed paper	540,000
Paraffin oil	87,000	Air (thickness 5 cm.)	23,800
Olive oil	82,000	CO_2 (thickness 5 cm.)	22,700
Paraffin (melted)	56,000	Oxygen (thickness 5 cm.)	22,200
Kerosene oil	50,000	Hydrogen (thickness 5 cm.)	15,100
Paraffin (solid)	130,000	Coal gas (thickness 5 cm.)	22,300

52. It may be assumed for some substances that are very homogeneous that the dielectric strength is directly proportional to the thickness of the material. That is, a piece 1 centimeter thick will stand twice the voltage of a piece of the same material ½ centimeter thick. While this is approximately true for some substances, other substances do not seem to follow this law, probably owing to the fact that the thicker the piece, the less homogeneous it becomes. There appears to be little uniformity in the results so far published of the dielectric strength of insulating materials. This is probably due to a lack of uniformity in the methods of testing, such as whether a direct or alternating current was used, and the frequency of the latter, also the length of time during which the maximum voltage was applied. The resistance of insulating materials, after they have been heated enough to expel all moisture, decreases very rapidly as the temperature increases.

Table XVI contains the results obtained by Macfarlane and Pierce.

In Table XVII are given the results published by Parshall and Hobart in "Electric Generators."

TABLE XVII

Substance	Thickness in Inches	Puncturing Voltage	Volts per $\frac{1}{1000}$ Inch
Composite sheets of mica and paper prepared so as to be moisture-proof	.005	3,600 to 5,860	
	.007	7,800 to 10,800	
	.009	8,800 to 11,400	
	.011	11,600 to 14,600	
Leatheroid	$\frac{1}{64}$ or .0156	5,000	320
	$\frac{1}{32}$ or .0313	8,000	256
	$\frac{3}{64}$ or .0469	12,000	256
	$\frac{1}{16}$ or .0625	15,000	240
	$\frac{1}{8}$ or .1250	15,000	120
	$\frac{3}{16}$ or .188	6,000	32
	$\frac{1}{4}$ or .250	6,000	24
Vulcanized fiber	$\frac{1}{8}$ or .125 to 1	about 10,000	
Hard rubber			500
Kiln-dried maple and other similar woods	1	10,000 to 20,000	
Vulcabeston	$\frac{1}{2}$ or .5	10,000	
Red pressboard	.030	10,000	
Red rope paper	.010	1,000	
Manila paper	.003	400	
Oiled cambric	.007	2,500 to 4,500	
Oiled cotton	.003	6,300 to 7,000	
Oiled paper	.004	3,400 to 4,800	
	.010	5,000	

ELECTRICAL CONDENSERS

CONDENSER CHARGE

53. If a battery be connected to the terminals of a condenser, as shown in Fig. 12, a current will flow into it and the plates will become charged. The flow of current will be a maximum the instant the electromotive force is applied, but will rapidly fall off, so that in a small fraction of a second the current will practically have ceased flowing and the condenser will be charged. This will be the state of affairs so long as the condenser remains connected to the battery; i. e., except for the instant when the battery is first connected, no current will flow, and the circuit will act simply as if it were broken. The condenser acts as if it acquired a counter electromotive force as it becomes charged, thus tending to keep the current out. This counter electromotive force increases as the condenser is charged, until it becomes equal and opposite to that of the battery. If the battery be disconnected and the terminals of the condenser connected together, the charge will flow out and will result in a current of short duration. This current will be a maximum when the terminals are first connected together, but it soon falls to zero.

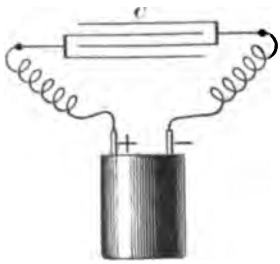
FIG. 12

UNITS OF CAPACITY

54. If the charge of a condenser in coulombs is Q, its capacity in farads C, and the difference of potential between the two plates E, then

$$Q = CE \qquad \textbf{(15)}$$

From formula **15**, it follows that

$$C = \frac{Q}{E}$$

and

$$E = \frac{Q}{C}$$

If Q and E are expressed in C. G. S. electromagnetic units, then C will be in C. G. S. electromagnetic units. The C. G. S. electromagnetic unit of capacity is not used in practical work. If Q and E are expressed in practical units, that is in coulombs and volts, respectively, then C will be in practical units, called *farads*. Hence, the **farad** is the practical unit of electrostatic capacity. The farad is equivalent to $\frac{1}{10^9}$ (or 10^{-9}) C. G. S. electromagnetic units of capacity. The farad is equivalent to 9.01×10^{11} electrostatic units of capacity. It is beyond the scope of this Course to derive formulas **15**, **17**, or **18**, or to prove the relation between electrostatic and electromagnetic units. We will state, however, that the ratio between the various C. G. S. electrostatic and C. G. S. electromagnetic units is equal to the velocity, or same power of that velocity, of light through air; that is, to some power of 3×10^{10}.

If a condenser be of such dimensions that a current of 1 ampere flowing into it for 1 second (that is 1 coulomb) causes the pressure across its terminals to rise 1 volt, its capacity is said to be 1 farad.

The flow of current into a condenser will always continue until the counter electromotive force of the condenser is equal and opposite to the electromotive force of the battery or generator to which the condenser is attached. A condenser sufficiently large to hold the quantity of electricity represented by 1 coulomb (1 ampere for 1 second), with a rise of potential of 1 volt, would have to be of enormous dimensions. The farad is, therefore, too large a unit for convenient use, and instead of it the microfarad is generally used.

55. One **microfarad** is equal to $\frac{1}{1000000}$ farad. Hence, it is necessary to divide the capacity as expressed in microfarads by 1,000,000 in order to express the same capacity in farads. It must be remembered that the microfarad is used only for convenience, and that in working out problems, capacities must generally be expressed in farads before substituting in formulas, because the farad is chosen with respect to the volt and ampere, and hence must be used in formulas along with these units. For example, a capacity of 10 microfarads as given in a problem would be substituted in formulas as .00001 farad. A microfarad $= \frac{1}{10^{15}}$ C. G. S. electromagnetic units of capacity.

56. From formula **15**, we see that the charge Q may be increased by increasing either the capacity C or the electromotive force E, or both. When condensers are used with static electrical machines, the potential is usually very high; while for galvanic batteries and other sources of current having low potential the capacity C must be increased in order to get a large charge Q.

We have seen in a previous section that two static charges Q and Q', when placed at a distance d apart in air, act upon each other with a force $F = \frac{Q\,Q'}{d^2}$. If the charges are placed in a dielectric whose inductivity is K (instead of in air whose inductivity is 1), then

$$F = \frac{K\,Q\,Q'}{d^2} \qquad \textbf{(16)}$$

If, in this formula, F is expressed in dynes and d in centimeters, then Q and Q' must be expressed in C. G. S. electrostatic units of quantity. This is not the same as coulombs; 3×10^9 of these units is equal to 1 coulomb.

CAPACITY OF CONDENSERS

57. The **capacity of a condenser** in electrostatic units is given by the formula

$$C = \frac{K\,a}{4\,\pi\,d}, \qquad \textbf{(17)}$$

in which K is the inductivity of the dielectric between the tinfoil or metal plates; a, the area in square centimeters of all the dielectric (insulating) sheets actually between and separating the condenser plates; and d is the average thickness in centimeters of the dielectric sheets between the metal plates. The electrostatic C. G. S. unit of capacity, in which C is given by this formula, has no name and is not used.

The capacity C of a condenser in microfarads is given by the formula

$$C = \frac{885\,K a}{10^{10}\,d}, \qquad (18)$$

in which K is the inductivity of the dielectric between the tin-foil or metal plates; a, the area in square centimeters of all the leaves or sheets of dielectric actually between and separating the condenser plates; and d, the thickness in centimeters of the dielectric leaves. When a and d are in square inches and inches, respectively, the formula becomes

$$C \text{ (microfarads)} = \frac{2{,}248\,K a}{10^{10}\,d} \qquad (19)$$

As is evident from the formula, the electrostatic capacity of a conductor, or system of conductors, is entirely independent of the quantity Q of electricity on them and of the difference of potential E between the two conductors separated by the dielectric. The capacity depends only on the quantities K, a, and d.

EXAMPLE 1.—A condenser is built up of two sheets of paraffined paper, the two measuring .006 inch thick, between sheets of tin foil measuring 8 × 8 inches. Find the total number of sheets of paper required to give a capacity of 1 microfarad. The inductivity of the paraffined paper may be taken as 1.8.

SOLUTION.—Solving formula **19** for the total area of the dielectric between the tin foil, we get $a = \frac{10^{10}\,d\,C}{2{,}248\,K}$. Substituting in this, we get $a = \frac{.006 \times 10^{10} \times 1}{2{,}248 \times 1.8} = 14{,}828$ sq. in. Now, each sheet is 8 × 8 = 64 sq. in.; hence, there must be $\frac{14{,}828}{64}$ separating leaves consisting of two sheets each, or $\frac{14{,}828 \times 2}{64} = 464$ sheets of paper, each somewhat larger than 8 × 8 in. Ans.

EXAMPLE 2.—A condenser is built up of leaves of mica, each .1 millimeter thick, between sheets of tin foil 7 × 10 centimeters. If there are 269 mica leaves, what is the capacity of the condenser in microfarads and in farads? (The inductivity of mica may be taken as 6.)

SOLUTION.—The formula, $C = \frac{885 K a}{10^{10} d}$, gives the capacity in microfarads when a and d are in square centimeters and centimeters, respectively. .1 mm. = .01 cm. By substituting in the above formula, we get $C = \frac{885 \times 6 \times 269 \times 7 \times 10}{10^{10} \times .01} = .999873$ microfarad, or practically 1 microfarad, or .000001 farad. Ans.

58. Condensers in Parallel.—Condensers may be connected in parallel, as shown in Fig. 13, or in series, as in Fig. 14. If two capacities are connected in parallel, the capacity of the two combined is equal to the sum of the two capacities. If C_1 and C_2 are the capacities of the two condensers shown in Fig. 13, the combined capacity C is equal to $C_1 + C_2$. The same holds true for any number of capacities connected in parallel. Hence, if a number of capacities are connected in parallel, the combined capacity is equal to the sum of all the capacities.

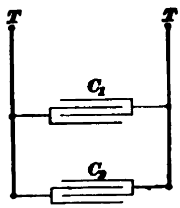

FIG. 13

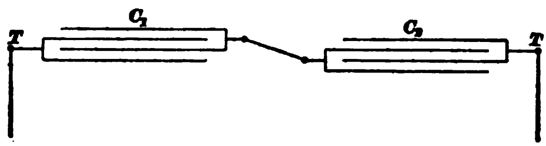

FIG. 14

59. Condensers in Series.—If two capacities are connected in series, the reciprocal of the combined capacity is equal to the sum of the reciprocals of the two capacities. If C_1 and C_2 are the capacities of the two condensers shown in Fig. 14, the combined capacity is obtained from the expression

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2};$$

or,

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}; \qquad (20)$$

This principle may be applied to any number of condensers connected in series; hence, we have the following formula for calculating the combined capacity of a number of condensers connected in series:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \text{etc.};$$

or,

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \text{etc.}} \qquad \textbf{(21)}$$

It is not convenient, as a rule, to further reduce the last formula for calculating the combined capacity when four or more condensers are connected in series.

By comparing the above with the laws governing the resistance of conductors, it will be seen that when combined in series and parallel condensers act just the opposite to resistances.

EXAMPLE.—What would be the total capacity, if three condensers of 2, 3, and 4 microfarads capacity were connected (*a*) in series and (*b*) in parallel?

SOLUTION.—(*a*) According to formula **21**, the capacity in series is equal to the reciprocal of the sum of their reciprocals. The sum of the reciprocals of the separate capacities $= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$, and the reciprocal of this sum $= \frac{12}{13} = .97$ microfarad. Ans.

(*b*) The capacity of all three in parallel is equal to the sum of the three capacities; hence, it is equal to $2 + 3 + 4 = 9$ microfarads. Ans.

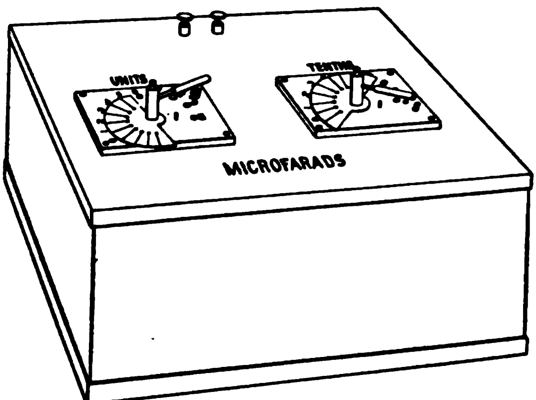

FIG. 15

60. Adjustable Condensers.—Condensers are often placed in a box and divided into sections, which may be cut in or out at will. Fig. 15 shows a condenser provided with switches *s*, *s* for cutting different sections in or out, and so varying the capacity. Condensers have been used to some extent commercially in connection with alternating-current motors and on telegraph and telephone circuits.

61. Fig. 16 gives a general view of a modern form of an adjustable condenser used in making electrical tests.

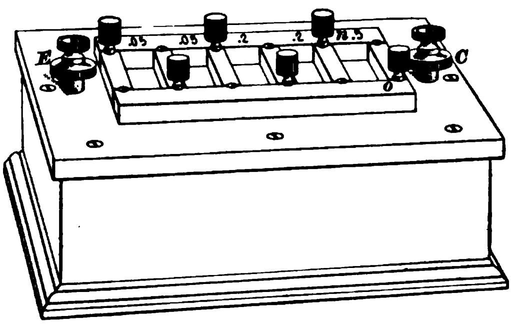

FIG. 16

The sections are connected between parallel brass blocks, as shown in Fig. 17, so that they may be joined either

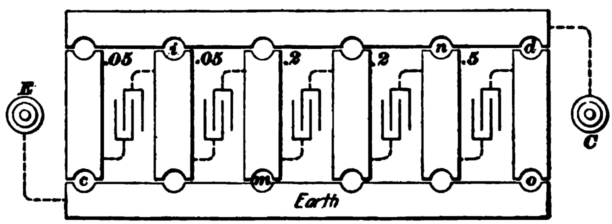

FIG. 17

in series or in multiple, or in combinations of series and multiple, this giving a much wider range of capacities for the same size and number of individual sections than can be secured by the ordinary pattern, shown in Fig. 18, with which only multiple combinations can be obtained. For instance, by placing plugs in the holes, as shown in

Fig. 16, all sections are connected in parallel, giving a capacity of 1 microfarad between the binding posts E and C. Withdrawing the plug o gives $.05 + .05 + .2 + .2 = .5$ microfarad. Withdrawing the plug n, or both n and o,

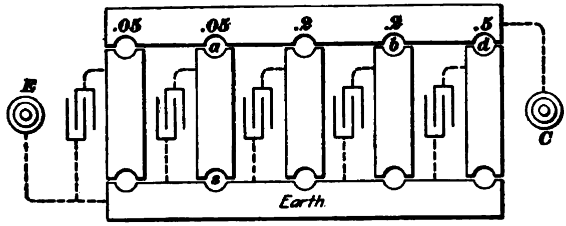

FIG. 18

would give $.05 + .05 + .2 = .3$ microfarad. If in Fig. 17 plugs are inserted only in the holes c and d, the five sections are all in series and the capacity, which is the smallest that can be obtained, may be calculated by formula **21** in the following manner:

$$\frac{1}{C} = \frac{1}{.05} + \frac{1}{.05} + \frac{1}{.2} + \frac{1}{.2} + \frac{1}{.5} = 52$$

Hence, $C = \frac{1}{52} = .0192$ microfarad.

EXAMPLE 1.—What would be the capacity between the binding posts C and E, Fig. 17, with plugs inserted only in holes c, i, m, n, and o?

SOLUTION.—When plugs are inserted in the holes named above, the condenser sections .05, .05, and .5 are connected in parallel across the two terminal bars. In addition, the two .2 sections, which are in this case connected in series with each other, are connected as one condenser, having, by formula **20**, a combined capacity of $\frac{.2 \times .2}{.2 + .2} = .1$ microfarad, in parallel with the other sections across the two terminal bars. Hence, the total capacity between the two terminals $= .05 + .05 + .5 + .1 = .7$ microfarad. Ans.

EXAMPLE 2.—What would be the capacity of the condenser shown in Fig. 17 when plugs are inserted only in holes c, i, m, and d?

SOLUTION.—The condenser sections .05 and .05 are connected in parallel across the two terminal bars. The three sections .2, .2, and .5

are connected in series with one another, having, therefore, by formula **21** a combined capacity $C = \frac{1}{\frac{1}{.2} + \frac{1}{.2} + \frac{1}{.5}} = \frac{1}{12} = .0833$ microfarad. This forms practically one condenser, which is in parallel with the two sections .05 and .05. Hence the total capacity of the condenser is now equal to .05 + .05 + .0833 = .1833 microfarad. Ans.

62. In Fig. 18 is shown a plan of the top of a condenser that has been and is still extensively used. The five condensers, or sections inside the box, are connected to the brass bars on top as shown by the dotted lines. By inserting plugs in the upper row of holes any number of sections may be connected in parallel and hence the condenser may have any of the following capacities: .05, .1, .2, .25, .3, .4, .45, .5, .55, .6, .7, .75, .8, .9, .95, and 1 microfarad. A plug inserted in any one of the lower holes will short-circuit that particular section; and if a plug is also inserted in the hole directly above, the entire condenser is practically short-circuited. For instance, if plugs are inserted in the holes *a* and *s*, the apparatus is useless as a condenser.

EXAMPLE.—When plugs are inserted only in the holes *a*, *b*, and *d* in Fig. 18, what is the capacity of the condenser between the terminals *C* and *E*?

SOLUTION.—In this case the sections marked .5, .2, and .05 are in parallel, giving a capacity of .5 + .2 + .05 = .75 microfarad. Ans.

63. Absorption.—When a condenser is charged, the difference of potential at its terminals does not rise instantly to its maximum value; that is, a certain time elapses before the condenser refuses to take in or absorb any more electricity. This apparent "soaking in" is due to an action on the surface of the dielectric. On discharging, a certain time also elapses before the previous charge is entirely removed; some has apparently soaked into the dielectric, giving rise to what is known as a **residual charge**, part of which can be obtained by again discharging. A condenser showing such a property is said to possess **residual absorption.** Furthermore, if the insulation between the plates is poor, the charge will leak away.

Hence, the actual capacity of a given condenser is not a very definite quantity, since it depends so much on its residual absorption and leakage. Furthermore, in measuring the capacity of a condenser, the result obtained will depend on the time during which it is charged or discharged. For these reasons it is difficult to accurately measure the capacity of a condenser and especially that of a poor condenser.

The capacity of most circuits met with in practice is quite small and its effect is not usually noticeable. However, long transmission lines have an appreciable capacity, the two wires constituting the plates of the condenser. The capacity of underground cables is often quite large and that of long submarine cables is very large, about $\frac{1}{3}$ microfarad per knot (2,029 yards). In the latter case the copper conductor constitutes one plate of the condenser and the sheath of the cable and the water the other plate.

64. Since certain values obtained from the best existing authorities were selected for this Section, the following values have been advocated by Mr. Carl Hering: 1 mil-foot (or preferably 1 circular mil-foot) of pure copper at 15° C., according to Professor Lindeck, is equal to 10.0275 ohms and according to Matthiessen's standard, it is equal to 10.1478 ohms; the resistivity of pure copper (per cubic centimeter) at 15° C., according to Professor Lindeck, is equal to 1.667 microhms, and according to Matthiessen's standard, it is equal to 1.687 microhms.

THE MAGNETIC CIRCUIT

ELECTRIC WORK AND POWER

1. Introductory.—Before taking up the subject of the magnetic circuit, which includes the magnetic properties of iron and steel, it will be well to first consider electric work and power, because the latter is used in connection with the property of a magnetic substance called hysteresis. Moreover, electric work and power is a very important subject, and one that the student may have occasion to use continually.

ELECTRIC WORK

2. When an electric current flows from a higher to a lower potential, electrical energy is expended and work is done. This energy is expended in heating the conductor constituting the circuit. When water flows through a pipe the friction of the water against the walls of the pipe produces heat, in an exactly similar manner as heat is produced, for instance, by rubbing sandpaper over a wooden surface. In the latter case, however, the friction is very great, and the heat produced is quickly felt by the hand, while in the case of water against metal pipes, the friction is comparatively very small, and the heat produced thereby is not perceptible to our sense of touch. Nevertheless, the heat is there, as the principle of the conservation of energy proves. This heat is dissipated into the surrounding atmosphere;

it is, therefore, not destroyed, but merely exists in another form, having gone to increase the temperature of the air.

Exactly so it is with the energy expended when an electric current flows through a conductor; the conductor resists the flow of the electric current, and hence a certain amount of electric energy is transformed into heat energy. This fact becomes very noticeable at times, for the conductor may become exceedingly hot—so hot, indeed, that unless due care is exercised, the wire carrying the current may be melted by the great heat produced.

3. Joule's Law.—It has been experimentally determined, as already explained, that when an electric current has passed through a substance, the development of heat was proportional (1) to the resistance of the conductor; (2) to the square of the current; (3) to the time during which the current flows.

The energy, or work, expended in a circuit may be very conveniently expressed by formulas for the various cases occurring in practical work. For use in the formulas that will be given,

let J = electric work in joules;
I = current in amperes;
t = time in seconds during which current flows;
E = electromotive force, or difference of potential, in volts;
R = resistance in ohms.

When the current, resistance, and time are known, then the work expended is given by the formula

$$J = I^2 R t \qquad (1)$$

This formula, which gives the work J in joules in terms of the current I in amperes, the resistance R in ohms, and the time t in seconds, is generally called **Joule's law.**

The **joule** may therefore be defined as that amount of energy which is expended during the time of 1 second by 1 ampere flowing through a resistance of 1 ohm.

The joule is the practical (electromagnetic) unit of energy, or work. It is greater than the C. G. S. unit of energy, or work, which is called the erg. One C. G. S. unit of work, the erg, equals one ten-millionth ($\frac{1}{10000000}$) part of a joule; or, 1 erg = 10^{-7} joules. One joule equals ten million (10,000,000) ergs; or, 1 joule = 10^7 ergs.

Example.—Find the amount of work done in joules when a current of 15 amperes flows for ½ hour against a resistance of 2 ohms.

Solution.— ½ hr. = 1,800 sec. By formula **1**, the electric work done

$$J = I^2 R t = 15 \times 15 \times 2 \times 1{,}800 = 810{,}000 \text{ joules} \quad \text{Ans.}$$

4. When the current, time, and electromotive force or difference of potential are given, the formula for calculating the work may be derived as follows :

Formula **1** may be written $J = I I R t$, and according to Ohm's law, $I R = E$; hence, substituting E for $I R$ in the above expression, we obtain

$$J = I E t \qquad (2)$$

This formula gives the work in joules in terms of the current in amperes, the electromotive force or difference of potential in volts, and the time in seconds.

Example.—Find the amount of work in joules done in 1 hour by a current of 25 amperes under an electromotive force of 20 volts.

Solution.— 1 hr. = 3,600 sec. By formula **2**, the electric work

$$J = I E t = 25 \times 20 \times 3{,}600 = 1{,}800{,}000 \text{ joules} \quad \text{Ans.}$$

5. When the resistance, time, and electromotive force, or difference of potential, are given, then according to Ohm's law, $I = \frac{E}{R}$, and inserting this value of I in the formula $J = I E t$, we obtain

$$J = \frac{E^2 t}{R} \qquad (3)$$

This formula gives the work in joules in terms of the time in seconds, the resistance in ohms, and the electromotive force, or difference of potential, in volts.

EXAMPLE.—What is the amount of work done in joules in 45 minutes in a circuit having 200 ohms resistance, the electromotive force being 110 volts?

SOLUTION.— 45 min. = 2,700 sec. By formula **3**, the electric work done

$$J = \frac{E^2 t}{R} = \frac{110 \times 110 \times 2{,}700}{200} = 163{,}350 \text{ joules} \quad \text{Ans.}$$

6. When the quantity of electricity and the electromotive force, or difference of potential, are given, a formula may be derived as follows: it has been explained that 1 ampere flowing for 1 second equals 1 coulomb; that is, $It = Q$. Substituting Q for It in the formula $J = IEt$, we obtain

$$J = QE \qquad (4)$$

This formula gives the work in joules in terms of the quantity of electricity in coulombs and the electromotive force or difference of potential in volts. One joule may now be defined as the amount of energy expended when 1 coulomb flows between two points in a conductor between which two points there is a difference of potential of 1 volt. One joule may therefore be said to be 1 **volt-coulomb,** just as in mechanics the work done by raising 1 pound through 1 foot is called 1 foot-pound.

The watt-hour and the kilowatt-hour are the two units of electric work that are extensively used in practice. However, they can be better understood after power, or the rate of doing work, has been considered.

ELECTRIC POWER

7. When there are several circuits, and the rates at which work is being done in the various circuits are to be compared, the element of time in which the work is done must be considered.

In practical mechanical work the unit of time is always 1 minute, and the unit that measures the work performed in a unit of time is the **foot-pound per minute.** This unit is called the **unit of mechanical power.** Power is, therefore, the rate of doing work, and hence the mechanical power exerted can always be determined by dividing the work done in foot-pounds by the time in minutes required to do it.

In electrical work the unit of time is the second, and the unit that measures the work performed in a unit of time is the **joule per second,** which unit has been named the **watt.** The *watt* is, therefore, the *practical* (electromagnetic) *unit of electric power.*

Hence, if in a certain electrical circuit, say 1,000 joules of work are done in 10 seconds, the power exerted is $1,000 \div 10 = 100$ joules per second, or 100 watts. If in another circuit the same work is done in 5 seconds, the power there exerted is $1,000 \div 5 = 200$ joules per second, or 200 watts—just twice as much. Hence, we say that the power exerted in the second circuit is twice that exerted in the first; and we understand thereby that if in both circuits work is done for the same length of time, twice as much work will be done in the second circuit as in the first.

8. The formula expressing the power exerted in an electrical circuit may be derived as follows: According to the last article, electric power is measured in watts, 1 watt being equal to 1 joule per second. According to the formula $J = IEt$, joules = amperes × volts × seconds. Dividing both sides of this equation by the time in seconds gives $\frac{\text{joules}}{\text{seconds}}$ = joules per second = amperes × volts. But joules per second = watts; hence, watts = amperes × volts.

If P = total watts expended in the circuit;
E = electromotive force in volts;
I = current in amperes,

then,

$$P = EI, \qquad (5)$$

which may be expressed by the following rule:

Rule.—*In an electrical circuit through which a direct current is flowing, the power in watts is equal to the product obtained by multiplying the current in amperes by the electromotive force in volts.*

EXAMPLE.—What is the power in watts in an electrical circuit in which .6 ampere flows under a pressure of 110 volts?

SOLUTION.— $I = .6$; $E = 110$; hence, by formula **5**,

$$P = EI = .6 \times 110 = 66 \text{ watts} \quad \text{Ans.}$$

Since 1 watt equals 1 joule per second, and 1 joule equals 10^7 ergs, then evidently 1 watt equals 10^7 ergs per second.

9. When the power is to be expressed by the current and resistance, the formula is obtained as follows: $J = I^2 R t$, but power $= \frac{\text{work}}{\text{time}}$ and watts $= \frac{\text{joules}}{\text{time in seconds}}$;

hence, $$P = \frac{J}{t} = \frac{I^2 R t}{t} = I^2 R;$$

that is, $$P = I^2 R, \qquad \textbf{(6)}$$

which may be expressed by the following

Rule.—*In an electrical circuit through which a direct current is flowing the power in watts is equal to the product obtained by multiplying the square of the current in amperes by the resistance of the circuit in ohms.*

EXAMPLE.—Determine the power expended in watts in an electrical circuit having a resistance of 183.3 ohms, through which a current of .6 ampere is flowing.

SOLUTION.—$I = .6$ ampere; $R = 183.3$ ohms; hence, by formula **6**,

$$P = I^2 R = .6 \times .6 \times 183.3 = 65.99 \text{ watts} \quad \text{Ans.}$$

NOTE.—It will be observed that this result is the same, within decimal limits, as that obtained from the example in Art. 8. It is, in fact, the same circuit.

10. When the power is to be expressed by the electromotive force and resistance, the formula is obtained as follows: According to formula **3**, we have $J = \frac{E^2 t}{R}$, but, as before, watts $= \frac{\text{joules}}{\text{time in seconds}}$; hence,

$$P = \frac{E^2}{R}, \qquad (7)$$

which may be expressed by the following

Rule.—*In an electrical circuit through which a direct current is flowing, the power in watts is equal to the quotient obtained by dividing the square of the electromotive force in volts by the resistance in ohms.*

EXAMPLE.—Determine the power in watts of an electrical circuit having a resistance of 183.3 ohms and an electromotive force of 110 volts.

SOLUTION.— $E = 110$ volts; $R = 183.3$ ohms; hence, by formula **7**,

$$P = \frac{E^2}{R} = \frac{110 \times 110}{183.3} = 66 \text{ watts} \quad \text{Ans.}$$

NOTE.—Observe that this is again exactly the same as the results obtained from the examples in Arts. **8** and **9**. It is, in fact, the same example in all three cases.

11. Kilowatt.—A unit of electrical power in extended use is the **kilowatt**, having the value of 1,000 watts. This unit is usually written K. W. One K. W. equals 1,000 watts. To reduce the power expressed in kilowatts to watts, it is necessary to multiply the number of kilowatts by 1,000. For use in formulas, kilowatts must generally be reduced to watts. For instance, 10 kilowatts divided by 50 amperes gives $\frac{10 \times 1,000}{50} = 200$ volts, and not $\frac{10}{50} = .2$ volt.

EXAMPLES FOR PRACTICE

1. Given electromotive force = 80 volts; resistance = 2 ohms. Find the power in watts and kilowatts.
Ans. 3,200 watts, or 3.2 kilowatts

2. Given resistance = 11.8 ohms; strength of current = 20 amperes. Find the power in watts. Ans. 4,720 watts

3. Given electromotive force = 112 volts; strength of current = 12 amperes. Find the power in watts. Ans. 1,344 watts

RELATIONS OF MECHANICAL, ELECTRICAL, AND HEAT ENERGIES

MECHANICAL EQUIVALENT OF HEAT

12. The distinguished English scientist, Joule, after whom the practical electric unit of energy is named, made elaborate experiments to determine exactly what relation existed between the units of work and heat. The experimental result that is now most generally accepted gives 778 foot-pounds of work as being exactly equivalent to the amount of heat required to raise the temperature of 1 pound of pure water 1° F., at or near 39° F., the temperature of its maximum density. This amount of heat is called **1 British thermal unit** (written B. T. U.). Therefore, we have the relation 778 foot-pounds = 1 British thermal unit and 1 foot-pound = .001285 British thermal unit. This relation is called the **mechanical equivalent of heat.**

From this relation follows the formula, in which F. P. represents foot-pounds.

$$\text{F. P.} = 778 \times \text{B. T. U.} \qquad (8)$$

Rule.—*To reduce the amount of heat developed in British thermal units to foot-pounds, multiply the number of British thermal units by 778.*

The last formula may be expressed by transferring the constant to the other side of the equation and taking its reciprocal, as follows:

$$\text{B. T. U.} = .001285 \times \text{F. P.}$$

From the experimental result showing the relation between foot-pounds and British thermal units, and the known relations existing between a pound and a gram, a foot and a centimeter, and a degree on the Fahrenheit and centigrade temperature scales, the relations between the various units of work and heat can be calculated.

RELATION BETWEEN JOULES AND FOOT-POUNDS

13. It has been stated that 1 joule is the work performed in an electrical circuit when a current of 1 ampere flows through a resistance of 1 ohm for 1 second. That is, $J = I^2 R t$. From the relations between foot-pounds and British thermal units, and between the English and absolute, or C. G. S., units, it can be shown that 1 joule equals .7373 foot-pound, or that 1 foot-pound equals 1.356 joules.* Therefore, when the work in joules is known, the work in foot-pounds is

$$\text{F. P.} = .7373\, J. \qquad \textbf{(9)}$$

which may be expressed by the

Rule.—*The equivalent work done in foot-pounds, when the work in joules is known, is obtained by multiplying the number of joules by .7373.*

The last formula may be expressed as follows:

$$J = 1.356 \text{ F. P.}$$

EXAMPLE 1.—Express the work accomplished in foot-pounds in a circuit where a current of 8 amperes flows for 2 hours, the electromotive force being 10 volts.

SOLUTION.— 2 hr. = 7,200 sec. = t. By the formula $J = I E t$, the electrical work done equals $J = 8 \times 10 \times 7{,}200 = 576{,}000$ joules. Expressed in foot-pounds this will be by the formula F. P. = .7373 J,

$$\text{F. P.} = .7373 \times 576{,}000 = 424{,}684.8 \text{ ft.-lb. Ans.}$$

* The relation between foot-pounds and joules may be determined in the following manner: Since 1 foot equals 30.479 centimeters, and 1 pound equals 453.59 grams, then 1 foot-pound = 30.479 × 453.59 centimeter-gram units of work. The next step is to reduce this work from centimeter-gram units to ergs. One erg is the work done in overcoming a force of 1 dyne through a distance of 1 centimeter. The force with which the earth attracts a mass of 1 gram varies at different points of the earth's surface, but this variation is usually neglected in practice and it is customary to take 981 as the acceleration of gravity in such reductions. Since $F = m a$, in which F equals the force in dynes, m the mass in grams, and a the acceleration in centimeters per second per second, then a force of 453.59 grams is equivalent to 453.59 × 981 dynes. Then, 1 foot-pound = 30.479 × 453.59 × 981 = 13,562,300 ergs; or 1 foot-pound = 1.35623×10^7 ergs. But 1 joule equals 10^7 ergs; hence, 1 foot-pound equals 1.35623 joules. From this it follows that 1 joule $= \frac{1}{1.35623} = .73734$ foot-pound.

EXAMPLE 2.—Find the amount of work done in foot-pounds by a current of 4 amperes flowing for 15 seconds against a resistance of 3 ohms.

SOLUTION.—By formula **1,** the electrical work done equals $J = 4 \times 4 \times 3 \times 15 = 720$ joules. The mechanical work done by the formula F. P. = .7373 J, is

$$\text{F. P.} = .7373 \times 720 = 530.856 \text{ ft.-lb. Ans.}$$

EXAMPLE 3.—Find the equivalent energy in foot-pounds of 600,000,000 ergs.

SOLUTION.—Since 1 joule equals 10,000,000 ergs (Art. **3**), then 600,000,000 ergs are equivalent to $\frac{600,000,000}{10,000,000} = 60$ joules. But 1 joule = .7373 ft.-lb.; then, 60 joules are equivalent to $60 \times .7373 = 44.238$ ft.-lb. Ans.

RELATION BETWEEN JOULES AND BRITISH THERMAL UNITS

14. It can be shown that for each joule of work expended in heating an electric conductor there is always developed .0009477 British thermal unit.* Hence, 1 joule equals .0009477 British thermal unit, and 1 British thermal unit equals 1,055 joules. If B. T. U. represents the heat developed in British thermal units, and J the work done in joules, then

$$\text{B. T. U.} = .0009477 J \qquad \textbf{(10)}$$

Rule.—*To reduce joules to British thermal units, multiply the number of joules by .0009477.*

The last formula may be expressed as follows:

$$J = 1,055 \times \text{B. T. U.}$$

---

* The relation between joules and British thermal units may be determined as follows: It has already been shown that 1 foot-pound equals 1.35623 joules, and it has been experimentally determined that 1 British thermal unit equals 778 foot-pounds, or 1 foot-pound equals $\frac{1}{778}$ British thermal unit; hence, by equating, we obtain 1.35623 joules equals $\frac{1}{778}$ British thermal unit. From this we get 1,055.14 joules equals 1 British thermal unit, or .00094774 British thermal unit equals 1 joule.

EXAMPLE 1.—Given an electrical circuit having a resistance of 3 ohms, through which a current of 5 amperes flows for 1 hour, determine (*a*) the work done in joules; (*b*) the number of foot-pounds this work is equivalent to; (*c*) the number of British thermal units developed.

SOLUTION.— $t = 3{,}600$ sec.; $I = 5$ amperes; $R = 3$ ohms; then,

(*a*) By the formula $J = I^2 R t$, the work equals $5 \times 5 \times 3 \times 3{,}600 = 270{,}000$ joules. Ans.

(*b*) According to the formula, F. P. $= .7373 J$, the work equals $270{,}000 \times .7373 = 199{,}071$ ft.-lb. Ans.

(*c*) According to Art. **12,** 1 ft.-lb. is equivalent to a heat development of .001285 B. T. U.; hence, $199{,}071 \times .001285 = 255.81$ B. T. U. Ans.

EXAMPLE 2.—Determine how many B. T. U. are developed in an electrical circuit having a resistance of 180 ohms, through which a current of 2 amperes flows for 1 minute.

SOLUTION.— $t = 60$ sec.; $I = 2$; $R = 180$; hence, by the formula $J = I^2 R t$, the number of joules $= 2 \times 2 \times 180 \times 60$, and the heat units developed are, according to the formula B. T. U. $= .0009477 J$,

B. T. U. $= .0009477 \times 2 \times 2 \times 180 \times 60 = 40.94$ B. T. U. Ans.

EXAMPLE 3.—Given an electrical circuit having a resistance of 3 ohms, in which a current of 2 amperes flows for 4 seconds, determine (*a*) the work in joules done in this circuit, and (*b*) the number of B. T. U. developed in the circuit.

SOLUTION.—$I = 2$; $t = 4$; $R = 3$. (*a*) By the formula $J = I^2 R t$, the work done $= 2 \times 2 \times 3 \times 4 = 48$ joules. Ans.

(*b*) By the formula B. T. U. $= .0009477 J$, we get $.0009477 \times 48 = .0454896$ B. T. U. Ans.

We therefore see that there is developed in the circuit of this example .04549 heat unit (B. T. U.) when 48 joules of work are done.

CALORIE

15. Besides the units of work and heat already mentioned, the unit of heat called the **calorie** is considerably used, especially in scientific investigations. There are two calories, called, respectively, the *large* and the *small calorie*, or the kilogram-calorie and the gram-calorie. The first is the quantity of heat required to raise the temperature of 1 kilogram of water 1° C. The second is the quantity of heat required to raise the temperature of 1 gram of water 1° C. When merely the one word calorie is used, the gram-calorie is generally, though not always, the one meant. The small calorie is often called the *gram-degree centigrade unit of heat*. Whenever the word calorie is used in this section, it means the gram, or small, calorie.

The following relations between joules and calories can be shown to be true.* One calorie equals 4.187, or 4.2 joules, and 1 joule equals .2388, or .24 calorie.

Then, $$H = .24\, I^2 R t = .24\, J, \qquad \textbf{(11)}$$

in which H is the number of calories produced in a wire having a resistance of R ohms by a current of I amperes in t seconds, and J is the work expended in joules. From this we have the

Rule.—*To reduce the work in joules expended in an electric conductor to the equivalent amount of heat developed in calories, multiply the number of joules by .24.*

Formula **11** can also be expressed as follows:

$$J = 4.2\, H$$

EXAMPLE.—If an insulated coil of wire having a resistance of 5 ohms is entirely immersed in water, how many calories will be expended in heating the water when a current of 10 amperes flows through the coil for 2 hours?

---

* The calorie is the heat required to raise the temperature of 1 gram of water 1° C.; that is, grams × degrees centigrade may be said to represent calories. Similarly, pounds × degrees Fahrenheit may be said to represent British thermal units. Since there are 453.59 grams in 1 pound, and $\frac{5}{9}$ centigrade degree in 1 Fahrenheit degree, then 1 B. T. U. = 453.59 × $\frac{5}{9}$ = 251.994 gram-degree centigrade units or calories. Hence, 1 B. T. U. = 251.994 calories. But it has been stated that 1 B. T. U. = 1,055.14 joules; hence, 251.994 calories = 1,055.14 joules; consequently, 1 calorie = $\frac{1{,}055.14}{251.994}$ = 4.1872 joules, and 1 joule = $\frac{1}{4.1872}$ = .23882 calorie.

SOLUTION.—The number of joules expended in the coil may be calculated by substituting in the formula $J = I^2 R t$, which gives $J = 10^2 \times 5 \times 2 \times 60 \times 60 = 3,600,000$ joules. Then, according to the formula $H = .24\ J$, we get $H = .24 \times 3,600,000 = 864,000$ calories. Ans.

It is shown in the foot-note in which is derived the relation between joules and calories, that 1 British thermal unit $= 251.994$, or 252, calories. Then 1 calorie $= \frac{1}{251.994}$ $= .0039683$ British thermal unit.

WATT-HOURS

16. The **watt-hour** is a unit of electrical energy or work. It is not a unit of power. A watt-hour is equivalent to the work done when 1 watt is expended for 1 hour. For instance, if 2 watts are expended in an electric circuit for 3 hours, then $3 \times 2 = 6$ watt-hours of work have been done; or if 1 watt is expended for 6 hours, then $1 \times 6 = 6$ watt-hours of work have been done. In both cases exactly the same amount of work has been done.

Since there are 3,600 seconds in 1 hour, then a watt-hour is 3,600 times greater than a joule, which is 1 watt for 1 second.

$$\begin{aligned} 1 \text{ watt-hour} &= 3,600 \text{ joules} \\ &= 859.8 \text{ calories} \\ &= 2,654 \text{ foot-pounds} \\ &= 3.412 \text{ British thermal units.} \end{aligned}$$

NOTE.—These relations are readily derived by multiplying the relation between joules and the various other units by 3,600. For instance, 1 watt-hour = 3,600 joules, but 1 joule = .23882 calorie; hence, 1 watt-hour $= 3,600 \times .23882 = 859.75$ calories. To be a little more exact, the following relation, taken from the note in Art. **15**, may be used: 1 joule $= \frac{251.994}{1055.14}$ calories. Hence, 1 watt-hour $= \frac{3600 \times 251.994}{1055.14}$ $= 859.77$ calories. This is the value used in Table I in expressing the relation between kilowatt-hours (1 kilowatt-hour = 1,000 watt-hours) and calories.

EXAMPLE.—Find the power in watts in a closed circuit in which an electrical energy equivalent to 785,584 foot-pounds is expended in 4 hours.

SOLUTION.—Since 1 watt-hour = 2,654 ft.-lb., then 785,584 ft.-lb. $= \frac{785{,}584}{2{,}654}$ watt-hours. But this work was done in 4 hours; hence, the rate at which it was done is $\frac{785{,}584}{2{,}654 \times 4} = 74$ watts. Ans.

17. The **kilowatt-hour** is a unit of electrical energy or work. A kilowatt-hour is equivalent to the work done when 1 kilowatt is expended continuously for 1 hour. Hence, if 3 kilowatts are expended in an electrical circuit for 4 hours, then $3 \times 4 = 12$ kilowatt-hours of work have been done. Since 1 kilowatt = 1,000 watts, then 1 kilowatt-hour = $3{,}600 \times 1{,}000 = 3{,}600{,}000$ times greater than a joule.

$$
\begin{aligned}
1 \text{ kilowatt-hour} &= 3{,}600{,}000 \text{ joules} \\
&= 859{,}800 \text{ calories} \\
&= 2{,}654{,}000 \text{ foot-pounds} \\
&= 3{,}412 \text{ British thermal units}
\end{aligned}
$$

The kilowatt-hour is generally used where electrical energy is charged for by meter.

EXAMPLE 1.—An electric-power station supplies a factory with 500 amperes. If the total drop of potential in the line wire between the power station and the factory is 50 volts, (*a*) how much power in kilowatts, (*b*) how much energy in kilowatt-hours, and (*c*) how much energy in joules is wasted in 10 hours in the line wire?

SOLUTION.—(*a*) The current I is 500 amperes and the difference of potential E is 50 volts; hence, the power wasted is $P = IE = 500 \times 50 = 25{,}000$ watts, or 25 kilowatts. Ans.

(*b*) The energy wasted is $25 \times 10 = 250$ kilowatt-hours. Ans.

(*c*) The time equals $60 \times 60 \times 10 = 36{,}000$ seconds. The energy wasted is $J = IEt = 500 \times 50 \times 36{,}000 = 900{,}000{,}000$ joules. Ans.

EXAMPLE 2.—Find the power in watts in a closed circuit in which an electrical energy equivalent to 125,341 foot-pounds is expended in 50 minutes.

SOLUTION.— 1 joule = .7373 ft.-lb.; therefore, $\frac{125{,}341}{.7373} = 170{,}000$ joules. Electric power equals $\frac{\text{joules}}{\text{time in seconds}} = \frac{170{,}000}{50 \times 60} = 56.67$ watts. Ans.

TABLE I

RELATIONS BETWEEN UNITS OF WORK

	Ergs	Joules	Kilowatt-Hours	Calories	Foot-Pounds	B. T. U.
1 erg (dyne-centimeter)..	1	10^{-7}	$2,778 \times 10^{-17}$	$.23882 \times 10^{-7}$	$.73734 \times 10^{-7}$	$.00094774 \times 10^{-7}$
1 joule (volt-coulomb)...	10^{7}	1	$2,778 \times 10^{-10}$	.23882	.73734	.00094774
1 kilowatt-hour.........	36×10^{12}	36×10^{5}	1	859,770	2,654,400	3,411.8
1 calorie (gram-degree C.)	4.1872×10^{7}	4.1872	$11,631 \times 10^{-10}$	1	3.0873	.0039683
1 foot-pound............	1.3562×10^{7}	1.3562	$37,673 \times 10^{-11}$	.32390	1	.001285
1 British thermal unit, B. T. U. (pound-degree F.).........	$1,055.1 \times 10^{7}$	1,055.1	$29,310 \times 10^{-8}$	252.00	778	1

NOTE.—Although five figures are given in most of the above values, it is rarely necessary to use more than three figures, and in very many cases two figures are sufficient. For instance, it is usually sufficient to use 1 calorie = 4.2 joules, or to be a little more exact, 1 calorie = 4.19 joules. This table was calculated on the basis of 1 B. T. U. being equal to 778 foot-pounds, and the acceleration of gravity (g) was taken as equal to 981 centimeters per second per second.

EXAMPLE 3.—The electrical energy expended in a closed circuit in 2 hours is equivalent to 4,246,848 foot-pounds; if the electromotive force is 200 volts, what is the strength of the current flowing in the circuit?

SOLUTION.— 1 joule = .7373 ft.-lb.; therefore, 4,246,848 ft.-lb. $= \frac{4,246,848}{.7373} = 576,000$ joules. Electrical energy in joules equals EIt by formula **2**. Therefore, $I = \frac{J}{Et} = \frac{576,000}{200 \times 60 \times 60 \times 2} = 4$ amperes. Ans.

18. We have now established relations between the various units in which mechanical work, electric work, and heat energy are measured, so that any one can now be mathematically expressed in terms of the others. Table I will be found very useful for all examples involving transformations of energy.

ELECTRICAL HORSEPOWER

19. In mechanical calculations the foot-pound per minute is found to be too small a unit for practical use; therefore a unit has been adopted having the value of 33,000 foot-pounds per minute, which is about equivalent to the power a strong horse can exert. This unit is, therefore, named the **horsepower.**

Similarly in electrical calculations the joule per second, that is, the watt, is very often too small a unit for practical use. In such cases either the kilowatt or the horsepower is used. The number of watts equivalent to 1 horsepower may be obtained in the following manner:

One mechanical horsepower = 33,000 foot-pounds per minute. But 33,000 foot-pounds per minute $= \frac{33,000}{60} = 550$ foot-pounds per second. Hence, 1 horsepower = 550 foot-pounds per second, or 1 foot-pound per second $= \frac{1 \text{ horsepower}}{550}$. And 1 joule = .7373 foot-pound; hence, 1 joule per second or 1 watt = .7373 foot-pound per second, and, hence, 1 foot-pound per second $= \frac{1 \text{ watt}}{.7373}$. We have, therefore, found the value of the foot-pound per second expressed both in horsepower and in watts; so that 1 foot-pound per second $= \frac{1 \text{ horsepower}}{550} = \frac{1 \text{ watt}}{.7373}$, from which we find the value of 1 mechanical horsepower $= \frac{550}{.7373}$ watts = 746 watts. This value, 746 watts, is sometimes termed **one electrical horsepower.**

20. The power exerted in any electrical circuit may now be expressed in horsepower units by the following

Rule.—*To express the rate of doing electrical work in horsepower units, find the number of watts and divide the result by 746.*

If H. P. = horsepower and P = watts, then

$$\text{H. P.} = \frac{P}{746} \qquad (12)$$

Since P has the various values given by formulas **5**, **6**, and **7**, the horsepower may also be expressed by the three following formulas:

$$\text{H. P.} = \frac{E\,I}{746} \qquad (13)$$

$$\text{H. P.} = \frac{I^2 R}{746} \qquad (14)$$

$$\text{H. P.} = \frac{E^2}{746\,R} \qquad (15)$$

21. A **kilowatt** is related to the horsepower by the following equations:

1 K. W. = 1,000 watts = 1.34 H. P.
1 H. P. = 746 watts = .746 K. W.

EXAMPLE.—The common incandescent electric light consists of a glass bulb containing a simple carbon conductor, the two free ends of which are connected to the source of the electric current. When the current flows through this conductor, it heats it to such a degree that it becomes white hot, or, as such a state is called, incandescent. If this conductor has a resistance of 189 ohms and the lamp is supplied with an electromotive force of 110 volts, determine the following points of interest: (*a*) What current does the lamp take? (*b*) How many watts does it consume? (*c*) How many B. T. U. are developed per second? (*d*) How many such lamps would 1 electrical horsepower keep burning? (*e*) What is the mechanical equivalent of the heat developed per second in the lamp? (*f*) For how many such lamps would 10 K. W. suffice?

NOTE.—Regard the lamp as a simple conductor of the stated resistance in solving all problems relating to it.

SOLUTION.—(*a*) $E = 110$; $R = 189$; hence, by Ohm's law,

$$I = \frac{E}{R} = \frac{110}{189} = .582 \text{ ampere} \quad \text{Ans.}$$

(*b*) By solution (*a*), $I = .582$; $E = 110$; hence, by formula **5**,

$$P = IE = .582 \times 110 = 64.02 \text{ watts} \quad \text{Ans.}$$

(*c*) By solution (*a*), $I = .582$; $R = 189$; $t = 1$ sec.; hence, by the formula $J = I^2 R t$, the number of joules $= .582 \times .582 \times 189 \times 1$. Then, by the formula B. T. U. $= .0009477 J$, we have B. T. U. $= .0009477 \times .582 \times .582 \times 189 \times 1 = .06067$, or .0607 B. T. U. Ans.

(*d*) By solution (*b*), the lamp consumes 64.02 watts. Since 1 H. P. = 746 watts, then 1 H. P. will supply $\frac{746}{64.02}$ = about 12 such lamps. Ans.

(*e*) By solution (*c*), the number of B. T. U. developed per sec. = .0607. Since 1 B. T. U. = 778 ft.-lb., then .0607 B. T. U. = $.0607 \times 778 = 47.22$ ft.-lb. per sec. Ans.

(*f*) Since 1 K. W. = 1,000 watts, then 10 K. W. = $10 \times 1{,}000 = 10{,}000$ watts. But by solution (*b*), 1 lamp requires 64.02 watts; hence, 10 K. W. will suffice for $\frac{10{,}000}{64.02}$ = about 156 such lamps. Ans.

EXAMPLES FOR PRACTICE

1. Find the rate of doing work in watts when a current of 40 amperes flows against a resistance of $2\frac{1}{2}$ ohms. Ans. 4,000 watts

2. Express the rate of doing work in horsepower units when a current of electricity produces a loss, or drop, of potential of 20 volts in passing through a resistance of 1 ohm. Ans. .5362 H. P.

3. How many watts in 4.5 horsepower? Ans. 3,357 watts

4. The power in an electric circuit is equivalent to 4 horsepower. If a current of 30 amperes is flowing, what is the electromotive force developed? Ans. 99.4667 volts

5. Given electromotive force = 500 volts; strength of current = 13 amperes. Find the power in horsepower units. Ans. 8.713 H. P.

MAGNETIC PROPERTIES OF IRON

MAGNETIC INDUCTION

22. When a magnetic substance, such as iron, is brought into a magnetic field, so that the magnetic lines of force reach it, the substance immediately becomes magnetic. The lines of force appear to crowd together and tend to pass through the magnetic substance. The substance so magnetized is usually, however, only a temporary magnet. When it is again removed from the magnetic field, its magnetism usually disappears. While under the influence of the magnetic field, however, it behaves as does any magnet, and has polarity, which is so distributed that its south pole is that pole where the magnetic lines enter it, while its north pole is in that portion of the substance where the magnetic lines leave it. The production of magnetism in a magnetic substance in this manner is called **magnetic induction.** The production of artificial magnetism in a hardened-steel needle or bar by contact with a lodestone is only a special case of magnetic induction.

MAGNETIC CIRCUITS

23. The length of a **magnetic circuit** represents the average lengths of all the lines of force measured from where they pass out from the north pole along their circuit through the surrounding medium to where they enter the south pole, plus their length in the magnet. In a short bar magnet, the length of the magnetic circuit may be exceedingly large and difficult to measure, because a great many of the lines of force travel a long distance through the surrounding medium before entering the south pole. In a longer bar, however, bent into the shape of a horseshoe, the lines of force pass out from the north pole and enter the south pole almost immediately, thus making the average length of the magnetic circuit comparatively short and easy to determine.

Every line of force must form a complete circuit. Although a line may apparently leave the end of a magnet and disappear in space it must eventually return to the opposite pole of the same magnet, however far it may go out into the surrounding space.

In every magnetic field there are certain stresses that tend to produce a tension along the lines of force and a pressure across them; that is, the magnetic lines tend to *shorten* themselves from end to end, and *repel* one another as they lie side by side.

FIG. 1

24. A simple magnetic circuit is one composed of some one magnetic substance having a uniform sectional area throughout its entire length, as shown in Fig. 1, which represents a simple ring.

25. A compound magnetic circuit is a circuit in which the lines of force pass consecutively through several different kinds of magnetic or non-magnetic substances, or one whose sectional area is not uniform in size. Fig. 2 represents a compound magnetic circuit in which the lines of force pass through two halves of an iron ring and across two air gaps.

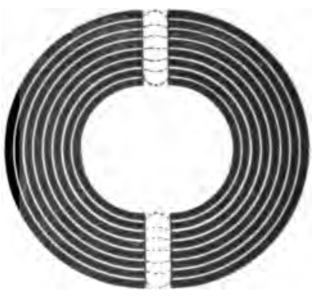

FIG. 2

26. A closed magnetic circuit is a circuit composed entirely of magnetic substances, and in which the lines of force do not pass across an air gap. A closed magnetic circuit may sometimes be a compound one, as would be the case, for instance, in Fig. 2 if the air gaps there shown were filled with some magnetic substance other than iron, or even with a different quality of iron than that of which the ring itself is composed.

27. The **sectional area** of a magnetic circuit at any point is the area of a plane through which the lines of force pass, the plane being taken perpendicularly to their direction at that point. In a rectangular bar magnet, the sectional area of the magnetic circuit at the neutral line will be the sectional area of the bar at that line, or the breadth of the magnet multiplied by its thickness.

The sectional area of the magnetic circuit outside the magnet would be an indeterminate quantity, because the lines of force spread apart and diverge in all directions before entering the south pole. But where the lines of force have only a small air gap to pass across, as in Fig. 2, the tendency to spread apart will be less, and the sectional area of the magnetic circuit may be taken as the area of the polar face.

For example, the sectional area of the magnetic circuit in a bar magnet .5 inch wide by .25 inch thick is $.5 \times .25 = .125$ square inch; that of a round bar magnet 1 inch in diameter is $1^2 \times .7854 = .7854$ square inch, since the area of a circle is equal to its diameter squared multiplied by .7854.

MAGNETIC UNITS

28. To properly define the strength of a magnet pole, a unit must be adopted by which this strength can be expressed. By universal agreement a magnet pole having unit strength is defined as a pole that meets the following conditions:

1. *It must, when placed at a distance of 1 centimeter from a similar pole having equal strength, repel this pole with a force of 1 dyne.*

2. *It must when placed in the center of a sphere having a radius of 1 centimeter send out such a number of lines of force that exactly 1 line of force passes through every square centimeter of the surface of the sphere.*

29. Number of Magnetic Lines per Unit Pole.—Directly from condition 2 of the preceding article the number of magnetic lines per unit pole may be calculated. It is there stated that a sphere of 1 centimeter radius receives 1 line of force per square centimeter of surface when a unit pole is situated at its center. This is equivalent to saying that a unit pole has as many magnetic lines as there are square centimeters on the surface of a sphere having a radius of 1 centimeter. If a sphere has a radius of 1 centimeter, its diameter equals 2 centimeters. The area of the surface of a sphere equals diameter squared times 3.1416; hence, area of the surface of this sphere equals $2^2 \times 3.1416 = 12.5664$, or approximately 12.57 square centimeters. But, as stated before, the number of square centimeters of surface equals the number of magnetic lines; hence, every magnet pole of unit strength emits 12.57 magnetic lines.

NOTE.—In this result, fractions of magnetic lines appear. Such fractions of magnetic lines are often obtained in calculations. They are treated in the same manner as are other fractions. Their significance may be made clear by the following consideration: Suppose we have a piece of cloth 1 inch wide and 1 inch long, that is, 1 inch square. Let us further suppose that, say, 13 pins were stuck vertically into this cloth. We could then say there are 13 pins per square inch. Assume now that one of these pins is removed, split lengthwise in half, and the one half again stuck into the cloth. Now we would say that there were only 12½, that is, 12.5 pins per square inch of cloth. Similarly, in the rule above, when we speak of 12.57 magnetic lines, we mean that a little over 12½ magnetic lines are sent out from every magnet pole of unit strength.

MAGNETIC FLUX

30. The **magnetic flux,** or **quantity of magnetism,** is expressed by the total number of magnetic lines of force passing along the magnetic circuit. In a bar magnet, for instance, the magnetic flux, or quantity of magnetism, would be that number of lines which pass through the metal from pole to pole, and which, if the magnet is imagined cut through at the neutral line, would pass through the surfaces thus produced.

The International Convention of Electrical Engineers at Paris, in 1900, adopted the name **maxwell** for the unit of magnetic flux. It was named after J. Clerk Maxwell, an

Englishman, who mathematically proved that electromagnetic disturbances, or waves, traveled with the same velocity through air as light waves. Hence, 1 maxwell is 1 line of force. A magnetic field through which 50,000 lines of force pass may be defined as a field having a strength of 50,000 maxwells. By universal custom the capital Greek letter phi (Φ), pronounced fi, is now being used to denote magnetic flux.

31. Magnetic Density.—If the sectional area of a bar magnet is divided into unit areas, for instance square centimeters, then the number of magnetic lines passing through each such unit area is termed the **magnetic density,** or the **flux density,** in the substance. Magnetic density is, therefore, the number of lines of force passing through a unit area measured perpendicularly to their direction. The word *induction* is used by some writers to signify magnetic density, but it is not so suggestive and hence not so desirable a term. The length of the magnetic circuit does not affect the magnetic density in that circuit as long as the total number of lines of force remains unchanged.

When every square inch of the sectional area of a magnetized substance has exactly the same number of lines of force passing through it, the magnetic density is said to be uniform. When this is not the case, the density is said to be non-uniform.

32. Unit of Magnetic Density.—At the Paris Convention already mentioned, the name **gauss** was adopted as the name for the unit of magnetic density, or flux density. Hence, 1 gauss is 1 line of force per square centimeter, and consequently 4,000 lines of force per square centimeter would be 4,000 gausses. These names for two of the C. G. S. magnetic units, in spite of the fact that they have been adopted by an International Convention of Electrical Engineers, are not yet extensively used. Lines of force and lines per unit sectional area are just about as convenient and more generally recognized than maxwells and gausses.

Since a unit magnet pole is supposed to send 1 line of force through every square centimeter of the surface of a sphere having a radius of 1 centimeter, then it follows that the unit of magnetic density is 1 line of force per square centimeter, or 1 gauss. Since 1 square inch equals 6.45 square centimeters, a density of 6.45 lines of force per square inch is equivalent to a density of 1 line of force per square centimeter. Unit magnetic, or flux, density may, therefore, be defined as a density of 6.45 lines of force per square inch. Unit flux density may also be defined as 6.45 maxwells per square inch. Since the gauss is the name for the C. G. S. unit of flux density, that is, 1 line of force per square centimeter, then a magnetic density of 35,000 lines per square inch would be equivalent to $\frac{35,000}{6.45}$ gausses, or C. G. S. units of magnetic density.

33. The magnetic density, or number of lines of force per unit area, is evidently equal to the total number of lines of force passing through the magnetic circuit divided by the sectional area of the magnetic circuit.

Hence we have the formula

$$\mathcal{B} = \frac{\Phi}{A'} \text{ or } \mathbf{B} = \frac{\Phi}{A}, \qquad \textbf{(16)}$$

in which Φ = total number of lines of force in the magnetic circuit in both cases;

$\mathcal{B}$ = magnetic density per square centimeter;

A' = sectional area of the magnetic circuit in square centimeters;

$\mathbf{B}$ = magnetic density per square inch;

A = sectional area of the magnetic circuit in square inches.

As far as practical, the French script letters, such as $\mathcal{B}$ and $\mathcal{H}$, and ordinary letters followed by a prime mark (′) will be used when the dimensions are given in centimeters, and the full-block letters, such as **B** and **H**, when the dimensions are given in inches.

When the sectional area of the magnetic circuit is expressed in square centimeters, the first expression in formula **16** is used. From this expression it follows that the magnetic density in lines of force per square centimeter is obtained by dividing the total number of lines of force by the sectional area of the magnetic circuit in square centimeters.

When the sectional area of the magnetic circuit is expressed in square inches, the second expression in formula **16** is used. From this expression it follows that the magnetic density in lines of force per square inch is obtained by dividing the total number of lines of force by the sectional area of the magnetic circuit in square inches.

For example, after measuring the magnetism in a straight bar magnet $\frac{1}{2}$ inch square and of any length, the total amount of magnetism at the neutral line is found to be 25,000 lines of force. The magnetic density in the bar is, therefore, by formula **16**, $\mathbf{B} = \frac{\Phi}{A} = \frac{25{,}000}{.5 \times .5} = 100{,}000$ lines of force per square inch. This is equivalent to saying that 100,000 lines of force would pass through the magnet if its sectional area were increased to 1 square inch and the lines of force were increased in the same proportion.

The total magnetism in a horseshoe magnet made of a bar of iron $1\frac{1}{2}$ centimeters square is 13,500 lines of force. The magnetic density in the bar is, therefore, by formula **16**, $\mathcal{B} = \frac{\Phi}{A'} = \frac{13{,}500}{1.5 \times 1.5} = 6{,}000$ lines of force per square centimeter. That is, 6,000 lines of force would pass through the magnet if its sectional area were reduced to 1 square centimeter, and the lines of force were reduced in the same proportion.

34. The total number of lines of force in a magnetic circuit, when the sectional area of the magnetic circuit and the magnetic density at that section are known, can be found by putting formula **16** in the following form:

$$\Phi = \mathcal{B} \times A'$$

or

$$\Phi = \mathbf{B} \times A$$

That is to say, the total number of lines of force in a magnetic circuit is obtained by multiplying the sectional area in square centimeters by the magnetic density per square centimeter, or by multiplying the sectional area in square inches by the magnetic density per square inch.

EXAMPLE 1.—In a certain part of a magnetic circuit the cross-section is .75 inch by .5 inch, and the magnetic density at that point is 50,000 lines of force per square inch; find the total number of lines of force in the magnetic circuit.

SOLUTION.—The sectional area of the magnetic circuit is $A = .75 \times .5 = .375$ sq. in. By formula **16**, the total number of lines of force $= \Phi = A\,\mathbf{B} = .375 \times 50{,}000 = 18{,}750$ lines of force. Ans.

EXAMPLE 2.—The cross-section of a magnetic circuit is a circle 1.5 centimeters in diameter, and the magnetic density is 3,000 lines of force per square centimeter; find the total number of lines of force passing through the circuit.

SOLUTION.—Sectional area $= A' = 1.5^2 \times .7854 = 1.76715$ sq. cm. By formula **16**, the total number of lines of force $= \Phi = 1.76715 \times 3{,}000 = 5{,}301$. Ans.

EXAMPLES FOR PRACTICE

1. Find the magnetic density in a round bar magnet $\frac{1}{2}$ inch in diameter when 3,927 lines of force pass through it.
Ans. 20,000 lines of force per sq. in., or 20,000 maxwells per sq. in.

2. Find the magnetic density in a bar magnet 2 centimeters wide by .75 centimeter thick, when 9,000 lines of force pass through it.
Ans. 6,000 lines of force per sq. cm., or 6,000 gausses

3. The magnetic density in a bar magnet .25 inch wide by .4 inch thick is 34,500 lines of force per square inch; find the total number of lines of force passing through the magnet.
Ans. 3,450 lines of force, or 3,450 maxwells

MAGNETIC PERMEABILITY

35. If there is a magnetic field in air, or other non-magnetic substance, produced by a solenoid, a permanent magnet, or otherwise, there will be a certain number of lines of force threading through each unit sectional area. The number of lines of force per square centimeter in air is usually denoted by the letter $\mathcal{H}$ and is called the *magnetizing force*, the *density in air*, or the *field density*. The name adopted for the unit magnetizing force is the gauss. Since 1 line of force per unit area is taken as the unit, the **magnetizing force** may be defined as the number of lines of force passing across a unit sectional area of the field, this sectional area always being normal to the lines of force. In a uniform field, in air, or other non-magnetic substance, the magnetizing force is the number of lines of force per unit area. In a non-uniform field the magnetizing force may be considered as the average number of lines of force per unit area.

Now, if a magnetic substance, such as soft iron, is placed in a magnetic field, it is a well-known fact that the magnetism in the iron is much more intense than was the magnetic field in the same space before the iron was introduced; that is, there are a great many more lines of force per unit area in the iron than in the same space before the iron was introduced. If a piece of soft iron is inserted in the magnetic circuit of a solenoid, the number of lines of force will usually be greatly increased and the iron will become highly magnetized. A magnetic substance is therefore a better conductor of magnetism than air or any other non-magnetic substance, and it is said to be more permeable than air.

The facility afforded by any substance to the passage through it of lines of force is called its **magnetic permeability,** or, simply, its **permeability.** The permeability of air is taken as 1, and that of soft iron may be 2,000, or even greater.

36. If we denote by $\mathcal{H}$ the density in air, that is, the number of the lines of force per unit area in the air space before iron is introduced, and by $\mathcal{B}$ the density in the iron after it is placed in the same space where the density in air was previously $\mathcal{H}$, then the ratio between $\mathcal{B}$ and $\mathcal{H}$, that is, $\frac{\mathcal{B}}{\mathcal{H}}$, is the magnetic permeability of the iron. Hence, if we denote the permeability by the Greek letter μ (pronounced *mu*), which is customarily used for this purpose, we have the formula

$$\mu = \frac{\mathcal{B}}{\mathcal{H}}, \text{ or } \mu = \frac{\mathbf{B}}{\mathbf{H}} \qquad (17)$$

μ will have exactly the same value calculated from either formula. By certain electrical measurements and calculations, which will be described in another section, the magnetizing force $\mathcal{H}$ or **H**, the resulting magnetic density $\mathcal{B}$ or **B** in iron and its permeability μ can be determined. A little further on we will explain how the magnetizing force may be calculated. There is no name for unit permeability, hence we can merely say that a certain specimen of iron has, for instance, a permeability of 1,520, which means that its permeability is 1,520 times greater than that of air.

EXAMPLE.—The permeability of a piece of steel is 850 when the magnetic density is 59,500 lines of force per square inch; find the field density required to produce that magnetic density.

SOLUTION.—From formula **17** we obtain $\mathbf{H} = \frac{\mathbf{B}}{\mu} = \frac{59,500}{850} = 70$ lines of force per sq. in. Ans.

EXAMPLE FOR PRACTICE

The magnetizing force acting on a piece of iron is 600, and the magnetic density produced is 54,300 lines of force per square inch; find the permeability at that stage of magnetization.

Ans. Permeability = 90.5

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37. The conductance of a conductor, if its temperature remains constant, does not depend on the strength of the current flowing through it, but the permeability μ of a magnetic substance does depend on the degree to which it is magnetized, as will be shown presently; that is, $\frac{\mathcal{B}}{\mathcal{H}}$ has not the same value for different degrees of magnetization even in the same piece of iron. By this we mean that if $\mathcal{H}$ is doubled, it does not follow that $\mathcal{B}$ is also doubled in value. In order to calculate μ, we must determine experimentally the value of the magnetic density $\mathcal{B}$ produced in the iron by each particular magnetizing force $\mathcal{H}$; because a given increase or decrease in the values of $\mathcal{H}$ will not always produce the same proportional increase or decrease in the value of $\mathcal{B}$.

For example, when a certain piece of iron was placed in a field where there previously existed 53 lines of force per square inch in air, it was found that there were 60,000 lines of force per square inch in the iron, giving a permeability of $\frac{\mathbf{B}}{\mathbf{H}} = \frac{60,000}{53} = 1,132$. When, however, the same piece of iron was placed in a field where there existed 106 lines of force in air (double the previous number), it was found that there were 82,500 lines of force per square inch in the iron, giving a permeability of $\frac{\mathbf{B}}{\mathbf{H}} = \frac{82,500}{106} = 778$. Hence, we see that the permeability of the same piece of iron was different for the two different values of the magnetizing force **H**, and furthermore, it did not change in proportion to the change in either **H** or **B**. In fact, the permeability has a different value for every value of **H**, increasing up to a certain point as the magnetizing force increases.

38. Magnetic Saturation.—In all kinds of magnetic substances, the permeability decreases when the magnetization is increased beyond the point just mentioned. As the magnetization increases beyond this point, the permeability continues to decrease and the substance approaches a certain limit of magnetization called magnetic saturation; that is, the substance becomes saturated with magnetism. A limit is never reached where perfect saturation takes place, but there is a limit beyond which it becomes impracticable to magnetize the substance.

The practical saturation point in wrought iron, soft annealed sheet iron, and cast steel is between 110,000 and 130,000 lines of force per square inch. Hence, in these metals, **B** may have any value from 0 to 130,000. In gray cast iron the practical saturation limit is from 60,000 to 70,000 lines of force per square inch.

The magnetizing force **H** is very seldom carried beyond 1,500, and, therefore, **H** may have any value between 0 and 1,500.

39. Before designing an electromagnet for any purpose, it is necessary to know the magnetic properties of the particular quality of iron to be used in the core, in order to be able to find its permeability at different stages of magnetization and its saturation limit. Tests are taken on small samples of the metal by electrical instruments, and the values of **B**, **H**, and μ are calculated from the readings of the instruments. As these tests require delicate instruments and a large number of careful measurements, it is customary to consult the results taken in some laboratory on an average quality of iron and its alloys. The results given in Table II have been found to agree very closely with the iron and steel ordinarily used in dynamos, motors, and other electrical apparatus.

Table II, which is very complete, may require a little explanation. It will be seen that the magnetic density is given both in square centimeters $\mathfrak{B}$ and in square inches **B**, and the corresponding values of the field density for the four magnetic materials are also given in square centimeters $\mathfrak{H}$ and in square inches **H**. There is also given for each material the permeability, which is the same, of course, whether the densities in square centimeters or in square inches are used in computing it, since it is merely a ratio between the field and magnetic densities, and, hence, $\mu = \frac{\mathfrak{B}}{\mathfrak{H}} = \frac{\mathbf{B}}{\mathbf{H}}$.

For instance, if cast steel, unannealed, with dimensions in centimeters, is to be used in designing an electromagnet, the corresponding values of $\mathfrak{B}$, $\mathfrak{H}$, and μ will be found in columns 1, 8, and 12. If the dimensions are given in square inches, then the proper values to use will be found in columns 2, 9, and 12.

If we desire to have a flux or magnetic density **B** of 30,000 lines of force per square inch (see column 2), we find that this would require for cast steel a magnetizing force, or

TABL

MAGNETIC QUALITY O

Magnetic Density Per		Sheet Iron, Annealed					Cast Steel, Unannealed			
		Magnetizing Force Per		Ampere-Turns Per		Permea-bility	Magnetizing Force Per		Ampere-Turns Per	
Square Centi-meter $\mathcal{B}$	Square Inch **B**	Square Centi-meter $\mathcal{H}$	Square Inch **H**	Centi-meter Length $\frac{IT}{l'}$	Inch Length $\frac{IT}{l''}$	μ	Square Centi-meter $\mathcal{H}$	Square Inch **H**	Centi-meter Length $\frac{IT}{l'}$	Inch Length $\frac{IT}{l}$
1,550	10,000	2.480	16	1.973	5.011	625.0	2.790	18	2.219	5.638
3,100	20,000	3.565	23	2.836	7.204	869.6	4.340	28	3.452	8.770
4,650	30,000	4.340	28	3.452	8.770	1,071.4	5.425	35	4.312	10.96
6,200	40,000	5.115	33	4.069	10.34	1,212.1	6.665	43	5.302	13.47
7,750	50,000	6.510	42	5.179	13.15	1,190.4	8.370	54	6.658	16.91
9,300	60,000	8.215	53	6.535	16.60	1,132.0	11.16	72	8.878	22.55
10,075	65,000									
10,850	70,000	10.54	68	8.384	21.30	1,029.4	15.35	99	10.85	31.01
12,400	80,000	14.57	94	11.59	29.44	851.0	22.63	146	18.00	45.73
13,950	90,000	21.39	138	17.02	43.22	652.2	34.88	225	27.74	70.47
15,500	100,000	33.17	214	26.39	67.02	467.3	58.13	375	46.24	117.45
16,275	105,000									
17,050	110,000	57.97	374	46.11	117.14	294.1	113.15	730	90.01	228.64
17,825	115,000						157.33	1,015	125.15	317.90
18,600	120,000	112.38	725	89.39	227.07	165.5				
19,375	125,000	166.63	1,075	132.55	346.69	116.3				
1	2	3	4	5	6	7	8	9	10	11

[ABLE II

.ITY OF IRON AND STEEL

...nealed		Wrought-Iron Forgings					Gray Cast Iron				
Turns	Permeability	Magnetizing Force Per		Ampere-Turns Per		Permeability	Magnetizing Force Per		Ampere-Turns Per		Permeability
Inch Length $\frac{IT}{l}$	μ	Square Centimeter $\mathcal{H}$	Square Inch H	Centimeter Length $\frac{IT}{l'}$	Inch Length $\frac{IT}{l}$	μ	Square Centimeter $\mathcal{H}$	Square Inch H	Centimeter Length $\frac{IT}{l'}$	Inch Length $\frac{IT}{l}$	μ
5.63	555.5	1.860	12	1.480	3.758	833.3	9.92	64	7.891	20.04	156.3
5.770	714.3	2.325	15	1.850	4.698	1,333.3	16.28	105	12.95	32.89	190.5
10.96	857.1	2.790	18	2.219	5.638	1,595.7	25.42	164	20.22	51.36	182.9
13.47	930.2	3.565	23	2.836	7.204	1,739.1	40.61	262	32.30	82.06	152.9
16.91	925.9	4.650	30	3.699	9.396	1,666.6	66.65	430	53.02	134.68	116.3
22.55	833.3	6.820	44	5.425	13.78	1,363.6	112.29	718	88.53	224.49	83.6
							159.65	1,030	127.0	322.60	63.1
31.01	707.1	10.08	65	8.015	20.36	1,076.9					
45.73	547.3	16.12	104	12.82	32.57	769.2					
70.47	400.0	31.00	200	24.66	62.64	450.0					
17.45	266.6	66.65	430	53.02	134.68	232.6					
		97.65	630	77.68	197.32	166.6					
28.64	150.7	160.43	1,035	127.62	324.16	106.3					
17.90	113.3										
11	12	13	14	15	16	17	18	19	20	21	22

field density, of 35 lines of force per square inch; the latter figure being given on the same line as the figure 30,000, but in column 9. If the dimensions are given in centimeters, and we desire to have the same density in the steel as above, we find that 30,000 lines per square inch **B** (column 2) is equivalent to 4,650 lines per square centimeter $\mathcal{B}$, which is given on the same line in column 1, and the corresponding required value of the field density per square centimeter $\mathcal{H}$ is given on the same line in column 8. If another one of the four materials given is to be used, the columns from which to obtain the proper values are determined in a similar manner. The columns headed ampere-turns will be considered later.

40. Since no two pieces of the same kind of iron or other magnetic substance, even from the same factory, are likely to have exactly the same magnetic qualities, it is impossible to give a table of values, or a curve, that will apply to every piece of wrought iron, or one that will apply to every piece of cast iron, or to every piece of steel; in fact, the magnetic qualities vary so much that each sample should be separately tested and its qualities determined for very exact work; but tables giving the values of the magnetic properties of an average piece of wrought iron, cast iron, sheet iron, and mild steel are generally exact enough for most calculations made in designing electromagnets and dynamo-electric machinery. The student must not expect any two tables or curves to agree exactly, since no two pieces of iron necessarily have exactly the same magnetic qualities. The magnetic qualities vary with the quality and kind of elements other than iron that are present, and depend, moreover, on whether the foreign elements are mechanically mixed or chemically combined with the iron. Hence, there is no end to the number of different tables of magnetic qualities that can be obtained for the various grades of iron and steel.

The effect of annealing magnetic materials is to increase their permeability at low stages of magnetization. In practice, however, it is found most economical to magnetize cast steel above 75,000 lines of force per square inch, and at such stages of magnetization annealing has practically no effect on its permeability.

The figures given in Table II for sheet iron, annealed, are the results of a test taken on pieces cut from sheets .014 inch thick, of soft-annealed charcoal iron of average quality. An average quality of wrought-iron forgings was used in the test from which the results for that material were determined.

The peculiarities of the various materials should be carefully noted. For example, it will be seen that at all stages of magnetization, cast iron, since it requires the largest magnetizing force for any given magnetic density, is vastly inferior to any one of the other three metals. To produce a density of 40,000 lines of force per square inch in cast iron requires that **H** = 262; whereas, in cast steel at the same density, **H** = 43, which indicates that at this density cast iron would require 262 ÷ 43, or about 6 times as much magnetizing force as would be required for cast steel. Therefore, other things being equal, it would be more economical to use cast steel rather than cast iron for magnetic purposes.

CURVES OF MAGNETIZATION

41. The most convenient way of representing the magnetic qualities of iron and other magnetic substances is to plot curves of magnetization and permeability on cross-section paper. On one sheet are plotted **magnetization curves**, which indicate the relation of the magnetizing force **H** to the magnetic density **B**; on the other sheet are plotted the resulting **permeability curves**, which indicate the relation of the permeability μ to the magnetic density **B**.

Suppose that we find by actual measurement that there are 10,000 lines of force **B** passing through each square inch of cross-section of an iron rod when the magnetizing force **H** is 16; that there are 20,000 lines of force per square inch **B** when **H** = 23, etc., as given in columns 2 and 4 in Table II. Then it is evident that **B** has increased from 10,000 to 20,000 when **H** increased from 16 to 23; hence, it is apparent that

B and **H** have not increased at the same rate, and it requires in such cases, where two related quantities as **B** and **H** do not vary proportionally, to plot a curve in order to show the manner in which the relative values change and also in order to determine intermediate values. For instance, we must plot a curve showing the relation between **B** and **H** in order to determine the value of **B** when **H** = 175, because we did not (and perhaps could not) measure the value of **B** exactly corresponding to **H** = 175. By means of a good curve, however, we can determine the value of **B** quite accurately. This curve is made by experimentally measuring, or determining in some way, the actual values of **B** corresponding to a series of values of **H**, such as are given in columns 2 and 4, Table II, for instance.

In order to avoid the tedious measurements and the erection of perpendicular lines for each value, cross-section paper, as shown in Fig. 3, is used. The lines are usually near enough together for all practical purposes. The cross-section paper should be divided into squares of about ½ inch, or about 1 centimeter, on a side, although it will be more accurate if these squares are still further divided into smaller ones $\frac{1}{10}$ inch or ⅕ centimeter on a side. The sheets should be at least 7½ inches wide by 9½ inches high. Although some cross-section paper is more desirable than others, nevertheless most any cross-section paper, if large enough, will answer the purpose.

The horizontal distances are called **abscissas**, and are indicated by numbers placed in the margin either above or below the chart. The abscissa of any point on a curve is its horizontal distance from the zero, or in this case, from the extreme left-hand vertical line. The vertical distances are called **ordinates**, and are represented by numbers placed in the margin either on the right-hand or left-hand side of the chart. The ordinate of any point on a curve is its vertical distance above or below the zero, or in this case, the vertical distance above the lowest horizontal line. The terms abscissa and ordinate, therefore, express clearly which set of divisions, the horizontal or vertical, is referred to.

42. On the sheet for the magnetization curves, Fig. 3 (reduced), the abscissas represent the different values of **H**, and each division represents 50 **H**. Starting with the extreme left-hand vertical line as zero, the remaining

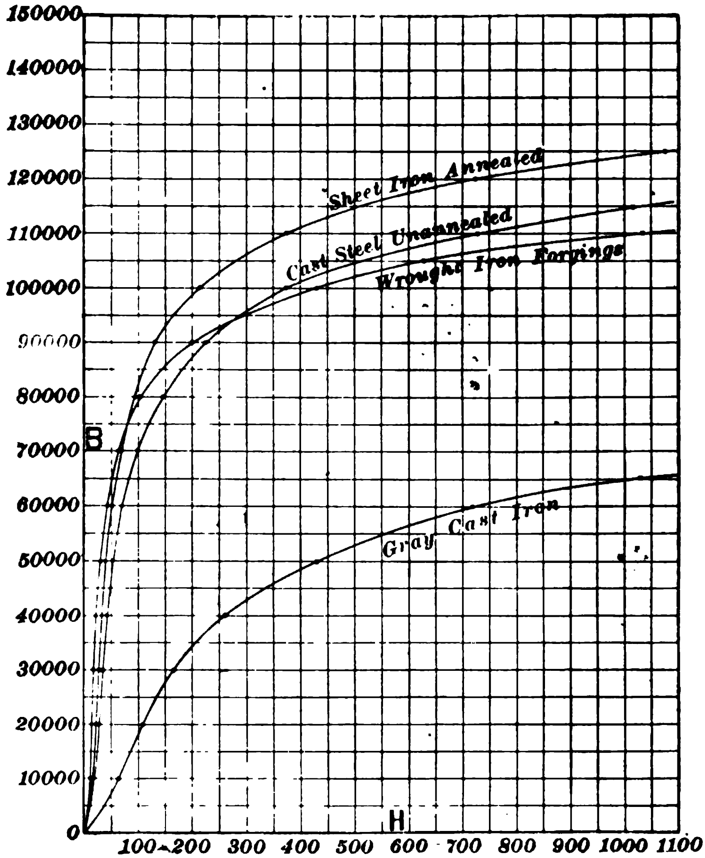

FIG. 3

vertical lines, which start from the lower horizontal line, are numbered consecutively in units of 50.

The ordinates represent the different values of the magnetic density **B**, and each division represents 5,000 **B**. Starting with the bottom line as zero, the remaining horizontal lines, that start from the extreme left-hand vertical line, are numbered consecutively in units of 5,000. In Fig. 4 are shown similar curves, plotted with field densities ℋ

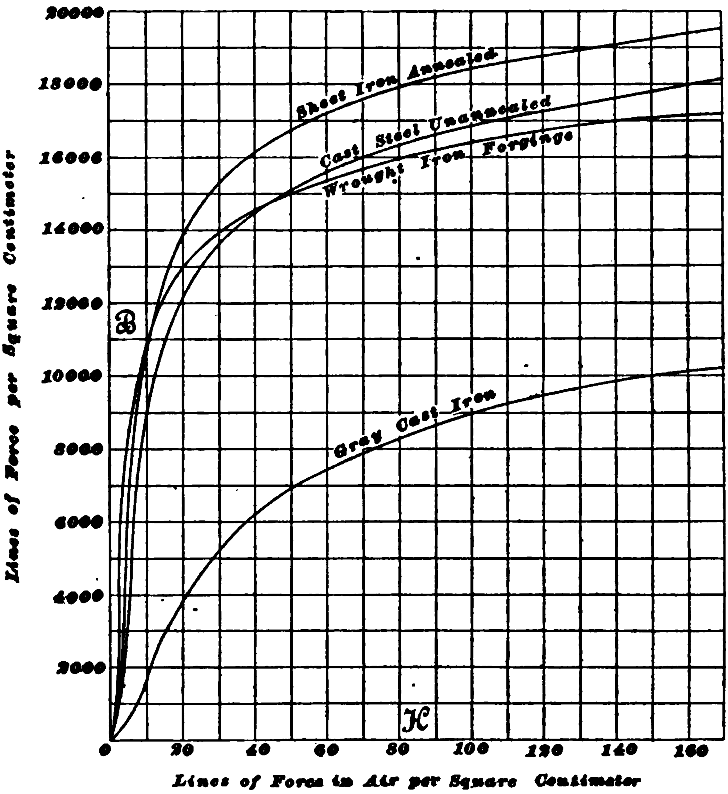

FIG. 4

and flux densities ℬ per square centimeter as abscissas and ordinates, respectively.

On the sheet for the permeability curves, Fig. 5 (reduced), the abscissas represent the different values of μ, each division representing 100 μ. Starting with the extreme left-hand line as zero, the divisions along the lower horizontal line are numbered consecutively in units of 100. The ordinates represent the different values of **B**, and are numbered as described for **B** on the sheet for magnetization curves.

43. Method of Plotting Curves.—In the first set of readings on cast iron, columns 2 and 19, in Table II, **H** = 64

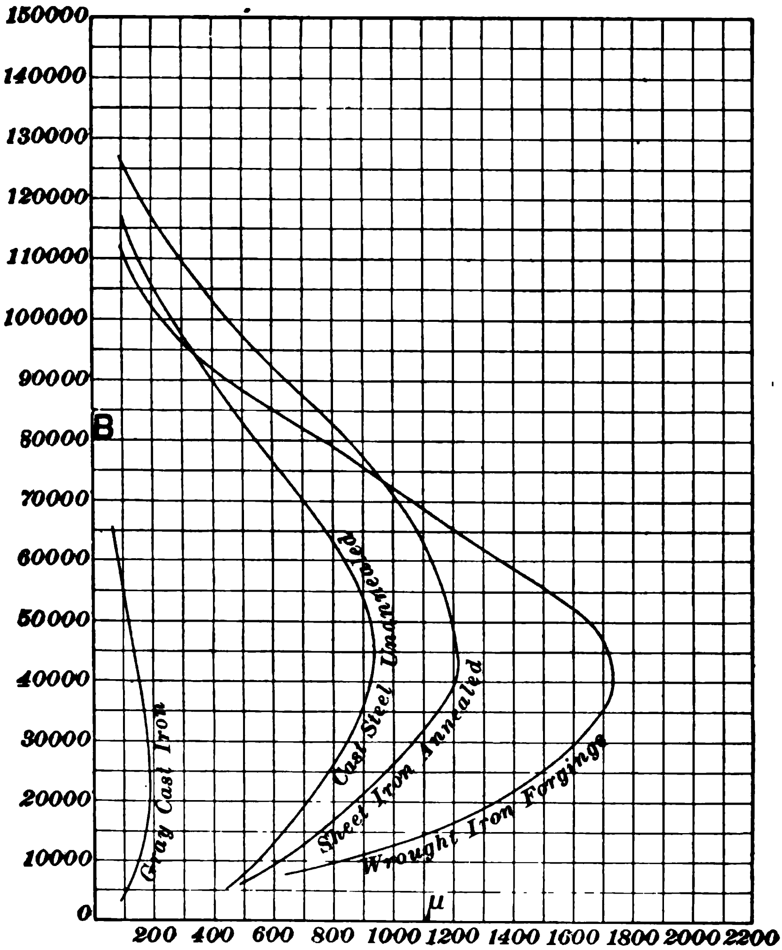

FIG. 5

and **B** = 10,000. To locate the first point, having an abscissa = 64 and an ordinate = 10,000, on the curve for cast iron, Fig. 3, follow up the extreme left-hand vertical line until the second division, which is marked 10,000, is reached, this being the value of **B** for **H** = 64. Now follow this third horizontal line, which represents **B** = 10,000, until a point is reached that represents 64 horizontal divisions. This point will be exactly $\frac{64}{100}$ of 2 divisions (2 horizontal divisions = 100 **H**) laid off along the horizontal line, and it will occur, as shown, a little to the right of the vertical line that represents 50 **H**. This distance to the right of the vertical line representing 50 **H** is between one-third and one-fourth of one horizontal division. For $\frac{64}{100} - \frac{50}{100} = \frac{14}{100}$ of 2 divisions or $\frac{14}{50}$ of 1 division, which is greater than one-fourth and less than one-third of 1 division. Hence, to locate the point, estimate a distance nearly one-third of a division from the second vertical line, which represents 50 **H**. Perhaps a better way, especially when many curves are to be plotted and considerably used, is to make all estimations in tenths and not in thirds, fourths, etc. For instance, the first point would be $\frac{14}{50} = .28$, or approximately three-tenths of 1 division, to the right of the second vertical line. After some practice, tenths of a division can be estimated about as accurately as thirds, fourths, and sixths.

A heavy dot (•), or a fine cross (×) placed at the point so located will represent the corresponding values of **B** and **H** for the first readings. The remaining readings in columns 2 and 19, Table II, are plotted in a similar manner, and afterwards all the dots are joined together by one long smooth curve. All the intermediate values of **H** and the corresponding values of **B** are now indicated by the curved line. This curve will enable one to determine the value of **B** corresponding to any value of **H** between zero and its highest value. For example, in the magnetization curve for cast iron, where **H** is 350, the corresponding value of **B** is about 46,000 lines of force per square inch.

In many cases, in fact in the majority of cases, the corresponding values of **B** and **H** are so very different that we cannot use the same scale for both; that is, we cannot let

1 division represent a flux density of 50 lines of force per square inch as well as a field density of 50 lines of force per square inch, because it would require such an inconvenient size and shape for the piece of paper on which the curve is plotted. Hence, we use any convenient unit for **H** and any other convenient unit for **B**. In this figure, 1 division in a horizontal direction represents a field density of 50 lines of force per square inch, whereas 1 division in a vertical direction represents a flux density of 5,000 lines of force per square inch. The same method is used for plotting the rest of the magnetization curves in Figs. 3 and 4 and the permeability curves in Fig. 5.

HYSTERESIS

44. When the magnetism of an electromagnet is rapidly reversed, that is, when the direction of the lines of force is suddenly changed several times in rapid succession by changing the direction of the magnetizing current, the iron or steel becomes heated, and a certain amount of energy will be expended. This effect is due to a kind of internal magnetic friction, by reason of which the rapid changes of magnetism cause the iron to grow hot. This effect is called **hysteresis.**

If we have a piece of iron that is perfectly neutral, that is, contains no residual magnetism whatever, and magnetize it by starting a current flowing in a coil surrounding the iron, the magnetism or magnetic density $\mathcal{B}$ will increase as we increase the magnetizing force $\mathcal{H}$ by increasing the current. The curve $O a$, Fig. 6, shows how $\mathcal{B}$ increases as $\mathcal{H}$ is increased, the iron being originally in a neutral magnetic condition. If, when the magnetizing force $\mathcal{H}$ reaches its maximum value $O m$, it is gradually decreased to zero, that is, the current is decreased to a very small value and then the circuit opened, the magnetic density $\mathcal{B}$ will decrease along the curve $a b$. Evidently the magnetism has not decreased along the same curve as when it increased. The magnetic substance resists any change in its magnetic condition, and the magnetism seems to lag behind the magnetizing force. **Magnetic hysteresis** is often defined as the tendency of a magnetic substance to persist in any magnetic state that it may have acquired. It should be noticed at the point *b* that although ℋ equals zero, ℬ has a value of about 9,000 lines per square centimeter. This represents the residual magnetism after the iron has been magnetized to a maximum density of about 11,200 lines per square centimeter and the magnetizing force then removed.

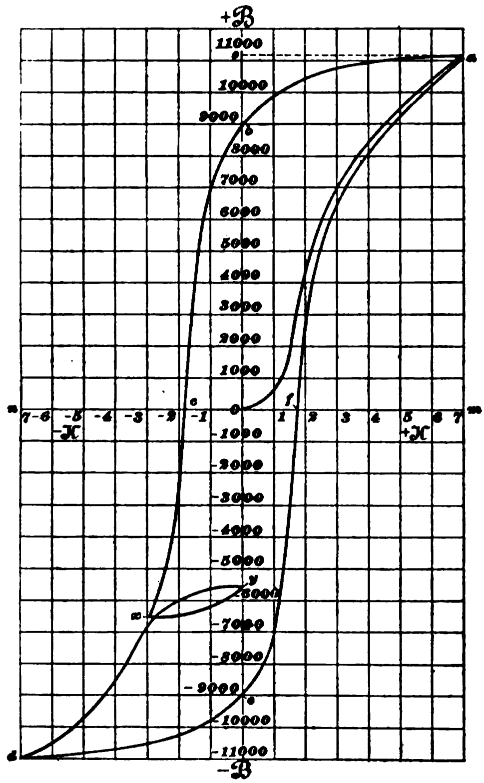

FIG. 6

45. If the current is started and increased gradually in the reverse direction, the magnetizing force being thereby also reversed in direction, the magnetism will first decrease from *b*, where it is about +9,000, to *c*, where it is zero. At the point *c*, ℋ has a negative value of about 1.8, and the iron possesses no magnetism, in spite of the fact that a magnetizing force of −1.8 is acting on it. This negative magnetizing force that is necessary to completely demagnetize a piece of iron is called its coercive force. Hence, the **coercive force** may be defined as the reverse magnetizing force necessary to completely remove the residual magnetism previously existing in a piece of magnetic material. The magnitude of the coercive force will depend not only on the magnetic quality of the iron, but also on the maximum flux density to which the material was magnetized the last time.

If the magnetizing force is gradually and continuously increased in the negative direction until its strength is equal to its previous maximum positive value, the flux density will increase in a negative direction along the curve *c d* until its maximum negative value is about the same as its previous maximum positive value. The small loop *x y* will be explained presently and need not now be considered as existing at all. If the magnetizing force is decreased from its value at *d* to zero, the magnetic density will decrease to a value *0 e*, and if the magnetizing force is then reversed in direction and increased to a value *0 f*, the magnetic density will be reduced to zero in spite of the fact that a magnetizing force of about 1.7 is acting on it. If the magnetizing force is further increased to its first maximum value, that is, to *0 m*, the magnetic density 𝔅 will increase in the positive direction along the line *f a* to about its first maximum value *m a*.

When a magnetic substance is magnetized so as to carry its magnetic flux through all the values represented by the curve *f–a–b–c–d–e–f*, it is said to have been carried through one complete cycle of magnetization. One cycle is made by two reversals of magnetism. For example, reversing the magnetism 40 times in 1 second will make 20 cycles in 1 second.

If at any point *x* of the cycle the value of ℋ, instead of being continuously increased, is decreased through a small series of values, and then increased gradually, the values of 𝔅 will be distributed over a small loop, as shown at *x y*. Otherwise there would be a smooth unbroken curve from *c* to *d*, the same as from *f* to *a*.

46. Now it requires the expenditure of a certain amount of work in the magnetizing coil to increase the magnetization of the iron from *f* to *a*. This work is proportional to the area enclosed by the lines *f–a*, *a–s*, *s–0*, and *0–f*. But when the magnetization decreases from *a* to *b* the iron restores to the magnetizing coil an amount of this work proportional to the area enclosed by *b–a*, *a–s*, and *s–b*. The difference, which is proportional to the area enclosed by the lines *f–a*, *a–b*, *b–0*, and *0–f*, is not restored to the coil, but is transformed into heat and is lost in heating the iron. This represents the energy lost by hysteresis in one-half cycle. Evidently the total energy lost in one complete cycle is proportional to the area enclosed by the complete hysteresis loop *d–e–f–a–b–c–d*. The heating of the iron is supposed to be due to a sort of friction between the molecules themselves. The energy so dissipated in heat cannot be entirely avoided; all magnetic materials seem to possess this property to a greater or less degree. In some qualities of iron and steel, the hysteresis is much less than in others, however.

If it were not beyond the scope of this Course, the statement that the area enclosed by the loop represents the energy lost in hysteresis could be mathematically proved, and moreover it could be proved that the hysteresis loss in ergs per cubic centimeter of iron for one complete cycle of magnetization is equal to the area in square centimeters enclosed by one hysteresis loop divided by 4π. Of course, the curves would have to be drawn accurately to a centimeter scale and measured with a planimeter in order to determine in this manner the energy so lost. Other more practical methods are employed to determine the loss due to hysteresis. These will be considered in another section.

This hysteresis loop is from a test on very soft iron. In other varieties, or specimens, of iron or other magnetic material, the loop may be wider, that is, the area enclosed may be larger; or it may be narrower, that is, the area enclosed may be smaller; but in every case there will be some loop, and the descending curve *a–b–c–d* will never be quite the same as that of the ascending curve *d–e–f–a* nor will either of the curves *f–a* or *a–b–c* exactly agree with the original magnetizing curve *0–a*. In designing electromagnets, the values for $\mathcal{H}$, $\mathcal{B}$, and μ are always taken or calculated from the curve *O a*, the loop being only employed to show or determine the hysteresis loss.

The magnetism in a piece of iron may be practically removed and the iron reduced to its original neutral magnetic condition by carrying it through a series of cycles, or hysteresis loops, of diminishing intensity. For this purpose a rapidly alternating current circulating in a coil surrounding the iron is very gradually reduced in strength to zero. Mechanical vibration of the iron assists in the above process of demagnetization and also in reducing the residual magnetism. Hence, permanent magnets should never be jarred or subjected to an alternating magnetic field if no change in their magnetic condition is desired.

47. The energy expended by hysteresis is furnished by the force that causes the change in the magnetism; in the case of an electromagnet, where the magnetism is reversed by the magnetizing force, the energy is supplied by the magnetizing current.

TABLE III

WATTS LOST PER CUBIC INCH PER CYCLE PER SECOND

Flux Density in Lines Per Square Inch B	Watts Per Cubic Inch, One Cycle Per Second *a*	Flux Density in Lines Per Square Inch B	Watts Per Cubic Inch, One Cycle Per Second *a*
30,000	.0042	90,000	.0244
40,000	.0067	95,000	.0267
50,000	.0095	100,000	.0289
60,000	.0128	105,000	.0312
65,000	.0145	110,000	.0337
70,000	.0164	115,000	.0362
75,000	.0183	120,000	.0387
80,000	.0202	125,000	.0414
85,000	.0223		

The loss of energy by hysteresis depends (1) on the quality of the magnetic substance; (2) on the volume of metal magnetized; (3) on the number of cycles per second; and (4) on the maximum density to which the substance is magnetized.

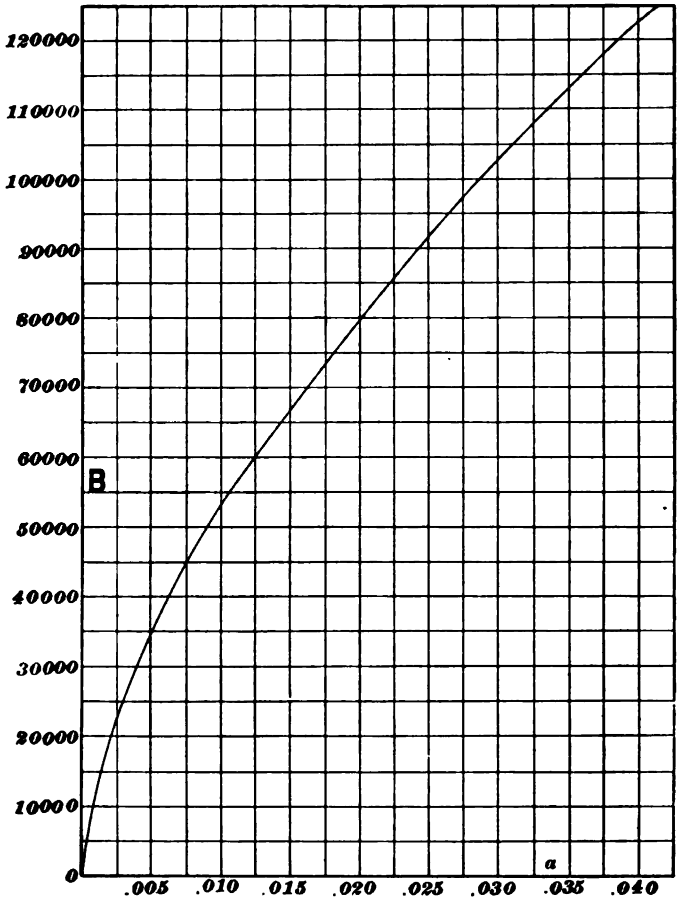

FIG. 7

Table III gives the power in watts expended by hysteresis in soft sheet iron when subjected to a rapid succession of cycles of magnetism at different maximum magnetic densities. The watts expended are directly proportional to the number of cycles per second and to the number of cubic inches of iron magnetized.

The readings given in Table III are plotted on a sheet of cross-section paper in Fig. 7, and the various points are connected by a curved line. The ordinates represent the different densities **B**, and the abscissas the corresponding number of watts expended in 1 cubic inch of iron for 1 cycle per second. By referring to the curve, all the intermediate values of **B** and the corresponding watts expended can be determined.

48. Let a = power in watts expended per cubic inch for one cycle per second;
V = volume of iron in cubic inches;
n = cycles per second;
P = total watts expended in hysteresis.

Then, the total loss due to hysteresis is

$$P = a V n \qquad (18)$$

Rule.—*To find the power expended by hysteresis in sheet iron at a given stage of magnetization, multiply the watts expended at that stage, as given in Table III, or Fig. 7, by the number of cubic inches of iron in the magnet and the number of cycles per second.*

EXAMPLE.—In an electromagnet, made with sheets of soft iron, there are 18 cubic inches of iron. Find the power in watts expended when the magnetizing current is reversed 70 times per second and the magnetism reaches a density of 90,000 lines of force per square inch.

SOLUTION.— 70 reversals are equivalent to 35 cycles = n. From Table III the watts expended per cubic inch for one cycle per second at a density of 90,000 are equal to .0244. Then, by formula **18**, the total power expended,

$$P = .0244 \times 18 \times 35 = 15.37 \text{ watts} \quad \text{Ans.}$$

NOTE.—The hysteresis loss in iron when subjected to magnetic reversals was experimentally determined, by Steinmetz, to vary as the 1.6 power of the magnetic density. He found that the watts lost in a mass of iron may be expressed as follows:

$$P = \frac{k V \mathcal{B}^{1.6} n}{10^7},$$

where P = watts lost on account of hysteresis;
k = a constant depending on the magnetic qualities of the iron under consideration.
V = volume of iron in cubic centimeters;
$\mathfrak{B}$ = maximum magnetic density (lines per square centimeter);
n = number of cycles per second.

The value of k will vary a great deal, depending on the quality of the iron. A fair value for k for annealed sheet iron and steel, such as used in dynamo and motor armatures, is .0035; for gray cast iron, .013; and for cast steel, .003.

The watts lost per cubic centimeter for one cycle per second are $\frac{k\,\mathfrak{B}^{1.6}}{10^7}$. Since 1 cubic inch = 16.38 cubic centimeters, then the watts lost per cubic inch for one cycle per second are $\frac{16.38\,k\,\mathfrak{B}^{1.6}}{10^7}$. If the density is expressed in lines per square inch **B**, then, since 1 square inch = 6.45 square centimeters, a density of **B** lines per square inch is equivalent to a density of $\frac{\mathbf{B}}{6.45} = \mathfrak{B}$ lines per square centimeter. Hence, the loss per cubic inch for one cycle per second, when the magnetic density is given in lines of force per square inch, is $\frac{16.38\,k\left(\frac{\mathbf{B}}{6.45}\right)^{1.6}}{10^7} = \frac{.8298\,k\,\mathbf{B}^{1.6}}{10^7}$. Then the total hysteresis loss in a given piece of iron is very nearly

$$P = \frac{.83\,k\,\mathbf{B}^{1.6}\,V n}{10^7}$$

In formula **18**, the constant $a = \frac{.83\,k\,\mathbf{B}^{1.6}}{10^7}$.

The mean of a large number of measurements by various persons seems to indicate that 1.5 is a more correct value than 1.6 for the exponent of $\mathfrak{B}$; however, 1.6 is much more generally used. To calculate the value of $\mathfrak{B}^{1.6}$ requires the use of logarithms. Although the formula is not very much used in practice, it has been given here to make the subject more complete. Since a knowledge of logarithms is not necessary in some of the Courses in which this section occurs, no questions requiring the use of this formula will be asked.

RESIDUAL MAGNETISM

49. Residual magnetism is the magnetism that a magnetic substance retains after being removed from a magnetic field or after the magnetizing force produced by a coil surrounding the magnetic substance has been reduced to zero. For instance, in Fig. 6, the residual magnetism, after the magnetizing force $O\,m$ has been applied and removed, is equal to $O\,b$ or about 9,000 lines per square centimeter. The larger the magnetic hysteresis in iron, the more persistently does it hold on to its residual magnetism. An iron that has a large hysteresis factor may retain but little magnetism, but it holds on to whatever amount it does have with a great deal of force, and it may require severe treatment to remove it. The tenacity with which it holds on to its residual magnetism is called its coercive force, which, as already defined, is the amount of negative magnetizing force that is required to reduce the residual magnetism to zero. In Fig. 6, $O\,c$ represents the coercive force necessary to remove the residual magnetism $O\,b$.

Soft irons, in which the hysteresis is very small, may, if very carefully handled, be made to retain considerable magnetism, but the slightest jar will generally remove it entirely, so that not even the slightest trace may be left. Its coercive force is very small. Soft iron and annealed steel usually retain only a small amount of residual magnetism, for there is usually some slight jar or disturbance, or perhaps a reverse induced current in the magnetizing coil that removes all or at least most of it.

A closed magnetic circuit of soft iron, that is, a magnetic circuit that consists of soft iron throughout its entire length, will exhibit a large amount of residual magnetism as long as the circuit remains unbroken. This tendency can be shown by a U-shaped electromagnet of soft iron, across the two ends of which is placed a well-fitted piece of iron called the *keeper*. If the circuit is magnetized by a current of electricity, the keeper will still adhere to the ends after the current is turned off, and may even require considerable force to detach it. But when once it is detached and the circuit broken, the keeper will not adhere again without the aid of the current.

Chilled iron and hardened steel retain residual magnetism in large quantities. Artificial or permanent magnets are made by placing a piece of hardened steel in a dense magnetic field or in contact with another magnet. A good permanent magnet is not necessarily one that retains a large amount of residual magnetism, although that is desirable, but it is preferably one that has a large coercive force, so that it will retain with fair treatment whatever residual magnetism it does have with as little loss as possible. Lodestone is the result of a natural residual magnetism.

INTENSITY OF MAGNETIZATION AND SUSCEPTIBILITY

50. If a long bar magnet has a pole strength m and a cross-section A', then $\frac{m}{A'}$ is called the **intensity of magnetization** of the bar and is denoted by the French script letter $\mathfrak{I}$, corresponding to the English letter I. Hence, $\mathfrak{I} = \frac{m}{A'}$. Suppose that we have a long solenoid of large area through the turns of which is circulating a current such that the field produced through the air inside the coil is $\mathcal{H}$ lines of force per square centimeter. Now introduce quite a slender iron rod of area A' inside the coil. Then $A'\mathcal{H}$ lines, due to the solenoid, will pass through the rod from end to end, producing poles of strength m at each end. Now each unit pole emits 4π lines of force; hence, the total number of lines passing through the rod from end to end due solely to the strength of the pole $= 4\pi m$. But there are $A'\mathcal{H}$ lines, due to the solenoid alone, also passing through the rod; hence, the total number of lines of force or flux passing through the rod from end to end is $\Phi = 4\pi m + \mathcal{H}A'$. Dividing all terms of this equation by A', we get $\frac{\Phi}{A'} = 4\pi\frac{m}{A'} + \mathcal{H}$. Now, $\frac{\Phi}{A'} = \mathcal{B}$ and $\frac{m}{A'} = \mathfrak{I}$; hence, $\mathcal{B} = 4\pi\mathfrak{I} + \mathcal{H}$. Dividing all terms by $\mathcal{H}$ gives $\mu = 4\pi\frac{\mathfrak{I}}{\mathcal{H}} + 1$. The ratio $\frac{\mathfrak{I}}{\mathcal{H}}$ is called the **magnetic susceptibility** of the iron, and is denoted by the Greek letter kappa κ. That is, $\kappa = \frac{\mathfrak{I}}{\mathcal{H}}$. Substituting κ for $\frac{\mathfrak{I}}{\mathcal{H}}$ in the last equation gives $\mu = 4\pi\kappa + 1$. The susceptibility is used very little in practical work.

51. Paramagnetism.—If a rod of iron is suspended in a magnetic field so that it is free to turn, it will place itself so as to lie in the direction of the lines of force. Nickel and cobalt, although less permeable than iron, will do the same. All substances that behave in this manner are called **paramagnetic** substances, and their permeability is greater than 1, that is, greater than the permeability of the air or of a vacuum in which the experiments have been tried. The following common substances are paramagnetic: iron, nickel, cobalt, manganese, chromium, and oxygen.

52. Diamagnetism.—There are some substances, such as bismuth and antimony, which, when suspended in a magnetic field in the form of a small bar and free to turn, will place themselves at right angles to the direction of the lines of force. All such substances are called **diamagnetic,** and their permeability is less than 1, that is, less than the permeability of air or vacuum. Experiments also show that paramagnetic substances are attracted and diamagnetic substances repelled by a magnet; at least, such is the case in a field that is not absolutely uniform in strength. No satisfactory explanation has been given for the behavior of diamagnetic substances. All diamagnetic substances are very weak, that is, their permeability is only a trifle less than 1. For instance, bismuth, which seems to be the strongest diamagnetic substance, and hence has the smallest permeability, has a permeability of .999969. The following common substances seem to be diamagnetic: bismuth, phosphorus, zinc, mercury, antimony, lead, silver, copper, gold, water, alcohol, and sulphur.

PRINCIPLES OF THE MAGNETIC CIRCUIT

53. The laws governing the production of magnetism, or lines of force, through a magnetic circuit are similar in some respects to the laws governing the production of a current in an electric circuit. In an electric circuit an electromotive force, which may be produced in various ways, is necessary before there is any tendency for a current to flow. In a magnetic circuit there is a force, called **magnetomotive force,** that corresponds to an electromotive force in an electric circuit. Magnetomotive force may be defined as that which produces magnetism, or as that which tends to drive lines of force along the magnetic circuit against the resistance offered by the magnetic circuit.

A magnetomotive force may be produced by a permanent magnet or by a wire (preferably, however, by several turns of wire) through which a current of electricity is flowing. Magnetism is said to be induced in a magnetic substance when it is placed in a magnetic field; that is, when it is acted on by a magnetomotive force.

54. Law of the Magnetic Circuit.—It has been shown that the strength of current that flows in an electric circuit is, by Ohm's law, equal to $\frac{E}{R}$, in which E = total or resultant electromotive force acting in the circuit, and R = the total resistance of the electric circuit. For the magnetic circuit we have a somewhat similar law, namely,

$$\text{magnetic flux} = \frac{\text{magnetomotive force}}{\text{reluctance}},$$

in which reluctance represents the magnetic resistance, and corresponds to electrical resistance in Ohm's law.

The magnetic flux is the quantity of magnetism passing through the magnetic circuit. This magnetic flux is usually expressed as so many lines of force, one line representing unit flux. Thus, one line of force, for which the name maxwell may be used, represents a unit quantity of magnetism, the same as an ampere represents the unit of current.

Magnetomotive force has already been defined. Unit magnetomotive force is that magnetomotive force which will produce unit flux, that is, one line of force, or one maxwell, through a magnetic circuit of unit reluctance. The word gilbert has been used by some writers as the name for the C. G. S. unit of magnetomotive force, but it is not very generally adopted. The French script letter $\mathfrak{F}$ is generally used to denote magnetomotive force, and $\mathfrak{R}$ to denote reluctance when the dimensions are given in centimeters.

The law given for the magnetic circuit may be expressed by the following formula:

$$\Phi = \frac{\mathfrak{F}}{\mathfrak{R}}, \qquad (19)$$

in which Φ = total number of lines of force threading through magnetic circuit;
$\mathfrak{F}$ = total magnetomotive force in magnetic circuit;
$\mathfrak{R}$ = total reluctance of magnetic circuit.

RELUCTANCE

55. Reluctance is the resistance that a magnetic circuit offers to the production of magnetic lines of force through it. We have seen that certain substances are good conductors of electricity; others are very poor conductors, and are termed non-conductors or insulators. For instance, pure copper is approximately 340,000,000,000,000,000,000 (or 34×10^{19}) times as good a conductor of electricity as porcelain. Furthermore, it has been experimentally determined that the best magnetic substance, soft iron, when magnetized to such a degree that it possesses its highest permeability is only about 2,500 times as good a conductor of magnetic lines of force as air, and that all non-magnetic substances, such as air, paper, wood, etc., seem to possess the same permeability as air. As we have seen, non-magnetic substances, air, for instance, will allow magnetic lines of force to pass through them. Although a non-magnetic substance may have some 2,500 times as great a reluctance as the same size piece of soft iron, nevertheless it is apparent that non-magnetic substances cannot rank as non-conductors of magnetism to the same degree that insulators rank as non-conductors of electricity. There is no known substance that can be called a very good magnetic insulator or non-conductor of magnetism.

56. The reluctance of the magnetic circuit depends on three quantities: (1) the length of the circuit, (2) the sectional area of the circuit, and (3) the permeability of the substances that form the circuit.

The reluctance increases as the length of the magnetic circuit increases; decreases as the sectional area increases; decreases as the permeability increases.

Let l' = length of a magnetic circuit in centimeters;
l = length of a magnetic circuit in inches;
A' = sectional area of the magnetic circuit in square centimeters;
A = sectional area of the magnetic circuit in square inches;
$\mathcal{R}$ = reluctance of the magnetic circuit in C. G. S. units, that is, when the dimensions are given in centimeters;
$\mathbf{R}$ = reluctance of the magnetic circuit in units for which there is no name, and when the dimensions are given in inches.

The reluctance of the magnetic circuit can then be expressed by either of the following formulas:

$$\mathcal{R} = \frac{l'}{A'\mu'}, \text{ or } \mathbf{R} = \frac{l}{A\mu} \qquad (20)$$

The C. G. S. unit of reluctance in which $\mathcal{R}$ in formula **20** is measured is sometimes called the **oersted,** after the man who first discovered the phenomenon of electromagnetism, in 1820, but this name is not very generally used. A centimeter cube of air has unit reluctance, that is, a reluctance of 1 oersted, because the permeability of air is taken as 1, and hence for a cube of air the reluctance $\frac{l'}{A'\mu} = 1$.

NOTE.—The reluctance $\mathbf{R}$ obtained when l and A are given in inches and square inches, respectively, must be multiplied by .3937, to give the reluctance $\mathcal{R}$ in C. G. S. units. That is, $\mathcal{R} = \frac{l\ .3937}{A\ \mu}$, when l and A are in inches and square inches, respectively. However, it will not usually be necessary to reduce the reluctance to C. G. S. units when the dimensions of the magnetic circuit are given in inches, as just explained, because formulas in which the reluctance occurs will be given in which such reductions are taken care of by the constant.

57. Reluctance of Non-Magnetic Substances.—All our magnetic units and formulas are based on the assumption that air has a permeability of 1. Since all non-magnetic substances have the same permeability as air, it follows that all non-magnetic substances have a permeability of 1 in our present system of units and formulas. There is no likelihood that this assumption will be changed for a good many years, even if it is ever changed. Hence, the reluctance of a magnetic circuit through air or other non-magnetic substance is equal to its length divided by its sectional area, that is, $\mathcal{R}$ for any non-magnetic substance $= \frac{l}{A}$.

58. For a simple magnetic circuit of uniform sectional area and material, and having no air gap in the path of the lines of force, the total reluctance of the magnetic circuit $= \frac{l'}{A'\ \mu}$. Substituting this for $\mathcal{R}$ in the formula $\Phi = \frac{\mathcal{F}}{\mathcal{R}}$, we get $\Phi = \frac{\mathcal{F}}{\frac{l'}{A'\ \mu}}$. Transposing this gives $\frac{\Phi\ l'}{A'\ \mu} = \mathcal{F}$, but $\frac{\Phi}{A'} = \mathcal{B}$ and $\frac{\mathcal{B}}{\mu} = \mathcal{H}$; consequently, we have $\mathcal{H}\ l' = \mathcal{F}$. Hence, the magnetomotive force $\mathcal{F}$ in a simple magnetic circuit is equal to the product of the magnetizing force $\mathcal{H}$ and the length of the magnetic circuit l'. $\mathcal{H}$ is the magnetizing force, or field density, but since $\mathcal{H} = \frac{\mathcal{F}}{l'}$, $\mathcal{H}$ is evidently equivalent to the magnetomotive force per unit length of the magnetic circuit, and the magnetomotive force $\mathcal{F}$ represents the work done on a unit magnetic pole in moving it, against the magnetizing force $\mathcal{H}$, once completely around the magnetic circuit whose length is l'.

In the case of a simple magnetic circuit, as in the ring shown in Fig. 8, l', which is the mean length of the magnetic circuit, is equal to $\pi (r_1 + r_2)$; r_1 and r_2 being the radii, respectively, of the inner and outer edges of the iron ring. The sectional area of the ring, since it has a circular cross-section, is $\frac{1}{4} \pi d^2$.

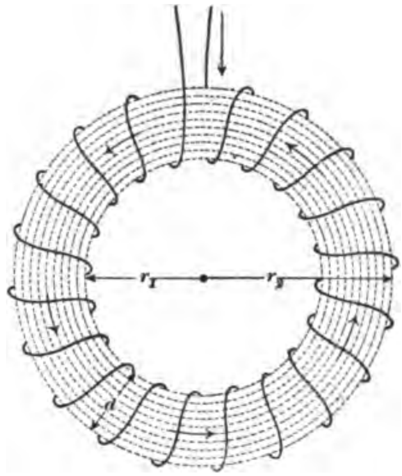

FIG. 8

59. Energy of a Magnetic Field. — We have seen that the permeability of a magnetic substance, which corresponds to the conductivity of a conductor, is not exactly analogous to the latter, because the permeability of a magnetic substance varies with its magnetic density, whereas the conductivity is independent of the current strength, provided the latter is not strong enough to appreciably heat the conductor. In another respect there is quite a difference between magnetic and electric circuits that has not yet been mentioned. We know that it requires the expenditure of energy, or work, to maintain an electric current; that the work done in maintaining a current I through a simple resistance R for t seconds $= I^2Rt$. Now, it requires no energy or work to maintain a steady magnetic field or flux after it is once established. However, work is required to establish or to increase the strength of a magnetic field, and the magnetic field, when allowed to fall to zero or to decrease in strength, tends to restore its energy to any electric circuit within its influence. Scientists have proved that the energy per cubic centimeter in a magnetic field is $W = \frac{\mu \mathcal{H}^2}{8\pi}$; in which μ is the permeability of the medium and $\mathcal{H}$ the intensity of the field. After a steady current is produced in a coil surrounding an iron core no expenditure of energy is necessary to maintain the magnetic field. Energy is required to maintain the current, but that energy is all consumed in heating the copper wire in the magnetizing coil, and can be calculated by the formula $J = I^2 R t$. This loss would be the same if the coil contained no magnetic substance. The fact that no energy is required to maintain a magnetic field must not be understood to contradict the fact that it requires work to move a magnetic pole around a magnetic circuit against the magnetizing force, and that energy is lost in heating the iron due to hysteresis, when the magnetizing current varies in strength or is an alternating one. The derivation of the formula for the energy in a magnetic field is beyond the scope of this Course.

MAGNETOMOTIVE FORCE

60. Strength of Field Around a Wire.—It has been proved by mathematical calculations, which are beyond the scope of this Course, that the magnetizing force $\mathcal{H}$ at a distance of r' centimeters from a long straight conductor carrying a current of I C. G. S. units is given by the following formula:

$$\mathcal{H} = \frac{2 I}{r'} \qquad (21)$$

The statement that the magnetizing force at a given point is $\mathcal{H}$ is equivalent to saying that the field density or the number of lines of force per square centimeter *in air* is $\mathcal{H}$. Furthermore, a unit magnetic pole is acted on by a force of 1 dyne by each line of force. Hence, the force acting on a unit pole when the field density is $\mathcal{H}$ is $\mathcal{H}$ dynes.

The paths of the lines of force produced by a current in a straight conductor are circumferences of circles about the axis of the wire as a center. Hence, the length l' of a line of force at a distance of r' centimeters from the axis of the conductor $= 2 \pi r'$ centimeters. Since the magnetizing force, or field strength, is $\mathcal{H}$ at all points at a distance of r' centimeters from the wire, it follows that the magnetomotive force, which represents the total work done in moving a unit pole completely around the magnetic circuit against the field, is

$$\mathcal{H}\, l' = \frac{2\, I}{r'} 2\, \pi\, r' = 4\, \pi\, I$$

It has already been shown that $\mathcal{H}\, l'$ represents the magnetomotive force acting in a simple magnetic circuit; hence, $4\, \pi\, I$ must be the magnetomotive force produced in the same circuit by one turn of a conductor carrying a current of I C. G. S. units.

It makes no difference whether the path of a line of force forms a circle or not; the work required to move a unit pole from any point in it around and back to its starting point is $4\, \pi\, I$. It requires no work to move a magnetic pole in a direction at right angles to a line of force, because a line of force does not oppose the motion of a pole in such a direction, and hence the work performed is proportional only to the distance the pole is moved along a line of force against the field. Furthermore, if the path of a magnetic pole is oblique to a line of force, the path may always be resolved into two components, one of which is along the line of force and the other at right angles to the line of force. Work is involved in moving the magnetic pole along the first-mentioned component only. Hence, the magnetomotive force always equals $4\, \pi\, I$, if the magnetic pole returns to its starting point, although the lines of force may not be circles.

61. If there are T turns in the coil, the intensity of the field produced by the same current will be T times :s great, and hence $\mathcal{H}\, l' = 4\, \pi\, I\, T = 12.57\, I\, T$, in which the current I is expressed in C. G. S. units. This is evident from the fact that it will require T times as much work to take a magnetic pole T times around the magnetic circuit as only once around.

If the current I is expressed in amperes, it is necessary to divide the last term ($12.57\ I\ T$) of the above expression by

10, since 10 amperes = 1 C. G. S. unit of current. Then, we have

$$\mathcal{H}\, l' = 1.257\, I\, T, \text{ or } \mathbf{H}\, l = 3.192\, I\, T,^* \qquad (22)$$

in which $\mathcal{H}$ = magnetizing force, or field density, per square centimeter;
l' = mean length of magnetic circuit in centimeters;
I = current in amperes;
T = total number of turns in magnetizing coil;
$\mathbf{H}$ = magnetizing force, or field density, per square inch;
l = mean length of magnetic circuit in inches.

Solving $\mathcal{H}\, l' = 1.257\, I\, T$ for $\mathcal{H}$, we obtain $\mathcal{H} = \frac{1.257\, I\, T}{l'}$.

Similarly, solving $\mathbf{H}\, l = 3.192\, I\, T$ for $\mathbf{H}$, we obtain $\mathbf{H} = \frac{3.192\, I\, T}{l}$. These two expressions enable us to calculate the field density produced inside a long solenoid and approximately inside any coil, l being the length of the coil.

62. It has been shown that

$$\mathcal{H}\, l' = \mathcal{F}, \text{ and}$$
$$\mathcal{H}\, l' = 1.257\, I\, T;$$

hence, the magnetomotive force

$$\mathcal{F} = 1.257\, I\, T$$

* The constant in this formula is derived as follows: $\mathcal{H}\, l' = 1.257\, I\, T$; $\mathcal{H}$ being the number of lines of force per square centimeter and l' the length of the magnetic circuit in centimeters. If $\mathbf{H}$ is the number of lines of force per square inch, then, since there are 6.45 square centimeters in 1 square inch, the equivalent density per square centimeter $\mathcal{H} = \frac{\mathbf{H}}{6.45}$. Furthermore, 1 inch = 2.54 centimeters, then l' in centimeters = 2.54 l in inches. Hence, we have $1.257\, I\, T = \frac{\mathbf{H}}{6.45} \times 2.54\, l$. Simplifying this gives $\mathbf{H}\, l = 3.192\, I\, T$, in which $\mathbf{H}$ is the magnetic density in lines of force per square inch and l is the length of the magnetic circuit in inches.

For dimensions in inches, we have the magnetomotive force $\mathbf{F} = 3.192\, I\, T$; in which, as before, I = current in amperes, and T = number of turns.

The principles relating to the magnetic circuit and explained in the preceding articles have been verified by experiment. For instance, it has been experimentally determined that the lines of force produced in a coil depend on the number of turns in the coil and on the current circulating therein. Hence, the current and the turns together act as a magnetizing force. This magnetizing force is, therefore, proportional to the product of current and turns. When the current strength is given in amperes, this product is called *ampere-turns*. It is found, furthermore, that the magnetizing force is independent of the size of the wire, and also that 20 amperes circulating around 5 turns exert precisely the same magnetizing force as 1 ampere circulating in 100 turns, or 50 amperes in 2 turns, all of which produce 100 ampere-turns.

63. Number of Lines of Force in a Solenoid.—The total number of lines of force passing through the air inside a solenoid may be calculated by multiplying the sectional area of the magnetic circuit by the field density inside the solenoid. For example, imagine a coiled conductor of 20 turns bent into a circular shape, as represented in Fig. 8, and inside of which there is no magnetic substance. Each line of force will form a complete ring inside the solenoid. Twenty amperes flowing through the conductor will give a magnetizing force of 400 ampere-turns. If the mean length of the magnetic circuit is 5 inches, then from formula **22** the magnetizing force $\mathbf{H} = 3.192 \times \frac{400}{5} = 255.36$, which means that a uniform magnetic field is produced in the air inside the solenoid in which the density is 255.36 lines of force per square inch of sectional area. Now, if the sectional area of the magnetic circuit is .5 square inch, there are $.5 \times 255.36 = 127.68$ lines of force produced in the coil. Or, if the sectional area is 1.5 square inches, there are $1.5 \times 255.36 = 383.04$ lines of force produced in the coil.

64. In the case of a long solenoid, the field inside is quite uniform and very dense compared to the field outside, as indicated in Fig. 9. The lines of force spread out into the space outside the coil. Theoretically, the flux density outside the coil is zero, since the area is infinitely great; because a definite quantity (the total magnetic flux) divided by an infinitely great quantity (the area of the space outside the solenoid) gives a zero quantity for the flux density. Since this is not quite true, all the magnetomotive force is not consumed in establishing a field of density $\mathcal{H}$ inside the solenoid; it is sufficiently exact, however, for practical purposes. If the distance $a\,b$, which is called the length of the solenoid, is l', then the total magnetomotive force due to a current of I amperes flowing through a solenoid of T turns produces a field of such an intensity $\mathcal{H}$ that the magnetomotive force $= \mathcal{H}\,l' = 1.257\,I\,T$. This is not quite true, because the lines of force spread out gradually as they approach the end of the solenoid, and the density does not reduce absolutely to zero immediately outside the ends of the solenoid. Moreover, this variation in flux density, which commences about three diameters inside each end of the solenoid, and the actual density outside the solenoid, cannot very well be determined, and hence it is practically impossible to make any exact allowance for these conditions in computing the magnetomotive force of a straight solenoid. The field density at the middle of a solenoid is given exactly by $\mathcal{H}$ or **H** in formula **22**, provided the solenoid is at least 6 times as long as its diameter. If a solenoid has a length of 20 or

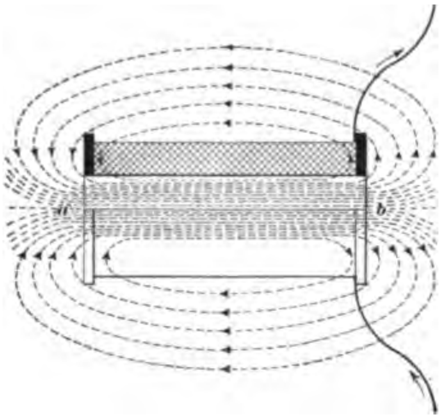

FIG. 9

more diameters, the error due to the variable density near the ends is small enough to be neglected. It has to be neglected in any case and l is taken as the length of the solenoid. When the solenoid is wound on a core of magnetic material that forms a closed, or very nearly closed, magnetic circuit, the end effects are practically negligible, no matter what may be the length of the solenoid, because then there is necessarily no variation in the density in the iron near the ends of the solenoid.

65. Ampere-Turns.—It has already been explained that $\mathcal{H} l' = 1.257\, I T$, or $\mathbf{H}\, l = 3.192\, I T$; in the first case the dimensions being in centimeters and in the second case in inches. From these we obtain, by transferring the constants to the other side of the equation and taking their reciprocals, the following formula:

$$I T = .796\, \mathcal{H} l', \text{ or } I T = .313\, \mathbf{H}\, l, \qquad \textbf{(23)}$$

in which $\mathcal{H}$ = magnetizing force, or field density, per square centimeter;
l' = mean length of magnetic circuit in centimeters;
$I T$ = ampere-turns in magnetizing coil;
$\mathbf{H}$ = magnetizing force, or field density, per square inch;
l = mean length of magnetic circuit in inches.

From these last two formulas can be readily calculated the ampere-turns required to produce a given magnetizing force in a simple magnetic circuit whose length is known.

66. It should be particularly noted that formula **22**, when solved for the field density, gives the number of lines of force per unit area for a coil in or near which there is no magnetic substance. When a magnetic substance is introduced into the core of a solenoid, the permeability is no longer 1, but becomes μ. For instance, if an iron ring having a sectional area of 6 square inches is wound with 100 turns of wire, and a current of 5 amperes is flowing through the wire, the mean length of the magnetic circuit (the mean circumference of the ring) being 10 inches, the magnetizing force **H** may be calculated by the formula $\mathbf{H} = \frac{3.192\, I\, T}{l}$, from which we obtain $\mathbf{H} = \frac{3.192 \times 5 \times 100}{10} = 159.6$ lines per square inch in air. The magnetic density produced in the iron ring by this magnetizing force depends on the permeability of the iron at that stage of magnetization. If the ring is made of cast iron, the curve for which is shown in Fig. 3, the magnetic density is found from the magnetization curve to be about 29,000 lines of force per square inch for a magnetizing force of 160 lines per square inch in air. Since the area of cross-section of the iron ring is 6 square inches, the total flux $\Phi = 29{,}000 \times 6 = 174{,}000$ lines of force, or maxwells.

COMPOUND MAGNETIC CIRCUIT

67. Reluctance of Compound Circuit.—The magnetic circuit is generally a compound one; that is, it is composed of two or more portions of different substances or of different sectional areas, or both. The total reluctance of the circuit would then be the sum of the reluctances of each substance. Let $\frac{l_1'}{A_1'\mu_1} = \mathcal{R}_1$ be the reluctance of the first substance, $\frac{l_2'}{A_2'\mu_2} = \mathcal{R}_2$, be the reluctance of the second, and so on. Then, the total reluctance of a compound magnetic circuit $= \mathcal{R}_1 + \mathcal{R}_2 +$, etc.

For instance, the field core and armature of a dynamo are usually of good soft iron, the yokes cast iron or steel, and the gaps between the field and armature cores are air. The reluctance of such a compound circuit must be computed by adding together the reluctances of each portion. If $\mathcal{R}_1 = \frac{l_1'}{A_1'\mu_1}$ is the reluctance of both field cores of an ordinary two-pole magnet, $\mathcal{R}_2 = \frac{l'_2}{A'_2\,\mu_2}$ the reluctance of the yoke, $\mathcal{R}_3 = \frac{l'_3}{A'_3\,\mu_3}$ the reluctance of the armature, and $\mathcal{R}_4 = \frac{l'_4}{A'_4}$ the reluctances of the two air gaps ($\mu = 1$ in this case), then, for the total reluctance of the magnetic circuit, we have $\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4$.

68. The flux through a compound magnetic circuit is equal to the magnetomotive force divided by the sum of the reluctances of all the parts. This is expressed by the formula

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \text{etc.}} \qquad \textbf{(24)}$$

Since $\mathcal{F} = 1.257\ I\,T$, and $\mathbf{F} = 3.192\ I\,T$, the above formula may be written in the following two forms:

$$.796\ \Phi\,(\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \text{etc.}) = I\,T \qquad \textbf{(25)}$$

or

$$.313\ \Phi\,(\mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3 + \text{etc.}) = I\,T$$

There will be as many terms in the denominator of formula **24** and in formulas derived from it as there are portions of one compound magnetic circuit in which the permeability, length, or sectional area varies.

It has been shown that $\mathcal{R} = \frac{l'}{A'\,\mu}$ and $\mathcal{F} = 1.257\ I\,T$ when centimeters are used, and that $\mathbf{R} = \frac{l}{A\,\mu}$ and $\mathbf{F} = 3.192\ I\,T$ when inches are used; hence, we have, by substituting in formula **24**, the following formula:

$$\Phi = \frac{1.257\ I\,T}{\frac{l'_1}{A'_1\,\mu_1} + \frac{l'_2}{A'_2\,\mu_2} + \frac{l'_3}{A'_3\,\mu_3} + \text{etc.}}, \qquad \textbf{(26)}$$

or

$$\Phi = \frac{3.192\ I\,T}{\frac{l_1}{A_1\,\mu_1} + \frac{l_2}{A_2\,\mu_2} + \frac{l_3}{A_3\,\mu_3} + \text{etc.}},$$

in which Φ = total flux, that is, the total number of lines of force, or maxwells;

l' and A' = lengths and sectional areas of the various portions in centimeters and square centimeters, respectively;

l and A = lengths and sectional areas in inches and square inches, respectively.

69. Formula **26** can be put in the following form:

$$1.257\, I\, T = \frac{\Phi\, l'_1}{A'_1 \mu_1} + \frac{\Phi\, l'_2}{A'_2 \mu_2} + \frac{\Phi\, l'_3}{A'_3 \mu_3} + \text{etc.}$$

But $\frac{\Phi}{A'} = \mathcal{B}$ and $\frac{\mathcal{B}}{\mu} = \mathcal{H}$; consequently, $\frac{\Phi\, l'}{A'\, \mu} = \mathcal{H}\, l'$.
Hence, we have

$$1.257\, I\, T = \mathcal{H}_1\, l'_1 + \mathcal{H}_2\, l'_2 + \mathcal{H}_3\, l'_3 + \text{etc.,} \qquad \textbf{(27)}$$

or

$$3.192\, I\, T = \mathbf{H}_1\, l_1 + \mathbf{H}_2\, l_2 + \mathbf{H}_3\, l_3 + \text{etc.}$$

in which $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ = field densities per square centimeters, that is, the number of lines of force per square centimeter in air;

l'_1, l'_2, l'_3 = lengths in centimeters of the various portions of the compound magnetic circuit;

$\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3$ = field densities per square inch, that is, the number of lines of force per square inch in air;

l_1, l_2, l_3 = lengths in inches of the various portions of the compound magnetic circuit.

It has been shown that 1.257 $I\,T$ and 3.192 $I\,T$ are magnetomotive forces; hence, we have the

Rule.—*The total magnetomotive force in a compound magnetic circuit is equal to the sum of the products of the magnetizing forces and lengths of the various portions of a compound magnetic circuit.*

70. In designing electromagnets and dynamos, it is usually more convenient to have the last formulas in the following form:

$$I\,T = .796\,\mathcal{H}_1 l'_1 + .796\,\mathcal{H}_2 l'_2 + .796\,\mathcal{H}_3 l'_3 + \text{etc.}, \qquad (28)$$

or $I\,T = .313\,\mathbf{H}_1 l_1 + .313\,\mathbf{H}_2 l_2 + .313\,\mathbf{H}_3 l_3 + \text{etc.}$

The separate members in the last formulas represent the ampere-turns necessary to drive the same flux through each separate portion of the magnetic circuit. The sum of the ampere-turns for each separate portion gives the total number of ampere-turns ($I\,T$) required to drive the same flux through the entire magnetic circuit.

If the dimensions and the flux through the magnetic circuit have been given, it is necessary to find the magnetizing forces $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, etc. before the ampere-turns can be calculated. The magnetizing force required will depend, as we have seen, not only on the kind and quality of the magnetic substance, but also on the magnetic density of the lines of force. The magnetic density is found by dividing the total number of lines of force that pass through a circuit by its sectional area. Consequently, the magnetic densities in the different substances that compose the magnetic circuit will be $\frac{\Phi}{A_1}$, $\frac{\Phi}{A_2}$, etc. Then, referring to the curves in Fig. 3, the field densities, or magnetizing forces, $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, etc. corresponding to $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, etc. can be readily found. Having given l_1, l_2, l_3, etc., and having determined $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, etc., the total number of ampere-turns $I\,T$ can be readily calculated by one of the last formulas. It is well to remember that the permeability of all non-magnetic substances is always 1, irrespective of the flux density, and hence $\mathbf{B} = \mathbf{H}$ in air, or in any other non-magnetic substance.

AMPERE-TURN CURVES

71. If, instead of plotting a curve with $\mathcal{B}$ as ordinates and $\mathcal{H}$ as abscissas, as in Fig. 4, a curve is plotted with $\mathcal{B}$ as ordinates and $.796\,\mathcal{H}$ (see formula **23**) as abscissas, then the ampere-turns for a simple magnetic circuit or for any portion of a compound magnetic circuit can be very readily calculated. The curves in Fig. 10 were plotted in this way,

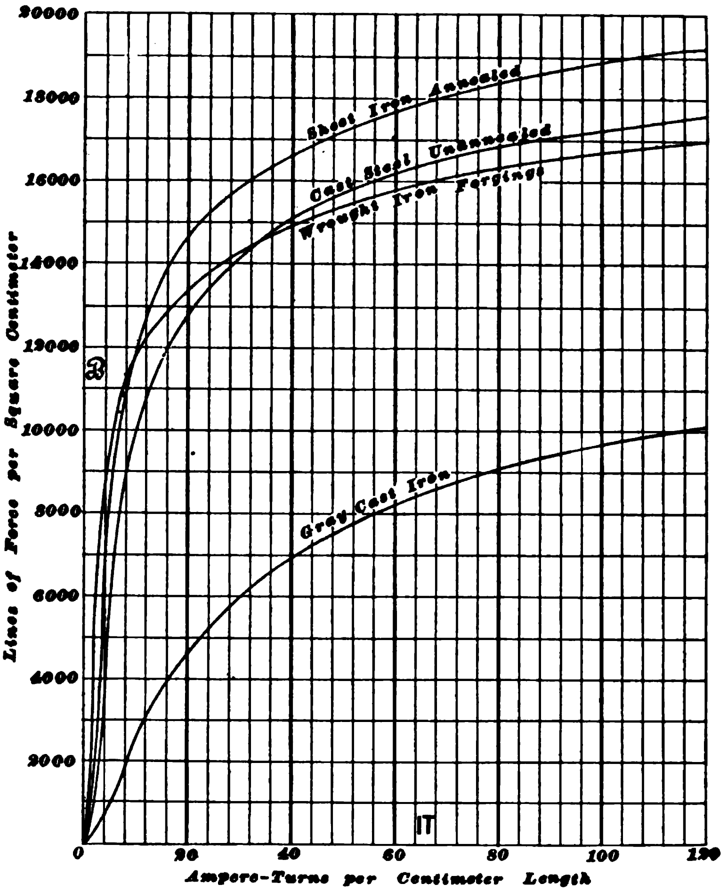

FIG. 10

that is, with $\mathcal{B}$ as ordinates and $.796\ \mathcal{H}$ as abscissas. Since $IT = .796\ \mathcal{H}\, l$, then $\frac{IT}{l} = .796\ \mathcal{H}$. Consequently, $.796\ \mathcal{H}$ is equal to the ampere-turns per centimeter of the length of the magnetic circuit. On a little investigation it will be evident that in Table II the numbers in the columns headed ampere-turns per centimeter length are .796 times as great as the corresponding numbers in the columns headed magnetizing force per square centimeter. Thus, 1.973 in column 5 = .796 × 2.48, the latter being the corresponding number in column 3. All the numbers in columns 5, 10, 15, and 20 were calculated in this manner. The numbers in columns 1 and 5 were used to plot the sheet-iron annealed curve in Fig. 10. The other curves in this figure were plotted in a similar manner.

Suppose, for example, that we have as a portion of a compound magnetic circuit a cast-steel core 9 centimeters long and having a sectional area of 5 square centimeters, and that we desire to determine the ampere-turns required to produce through it a flux of 80,000 lines of force. This would give a flux density of $\frac{80000}{5}$ = 16,000 lines per square centimeter. From the curve for cast steel in Fig. 10, we find that the ampere-turns per centimeter length $\frac{IT}{l'}$, that is, .796 $\mathcal{H}$, required to force 16,000 lines per square centimeter through cast steel is 56. Then the ampere-turns required to force the given flux through the core whose length is 9 centimeters = 56 × 9 = 504 ampere-turns.

72. The curves in Fig. 11 were plotted with the flux densities **B**, that is, lines of force per square inch as ordinates, and the ampere-turns per inch length, that is, .313 **H** (see formula **23**), as abscissas. Hence, these curves are used when the dimensions of the magnetic circuit are given in inches. In Table II the numbers in the columns headed ampere-turns per inch length are .313 times the corresponding numbers in the columns headed magnetizing force per square inch. All the numbers in columns 6, 11, 16, and 21 were calculated in this manner. The numbers in columns 2 and 6 were used to plot the sheet-iron annealed curve in Fig. 11. The other curves in this figure were plotted in a similar manner.

Sheet iron annealed is sometimes used when the densities are greater than 125,000 lines per square inch, and to avoid making the diagram unnecessarily large or reducing the

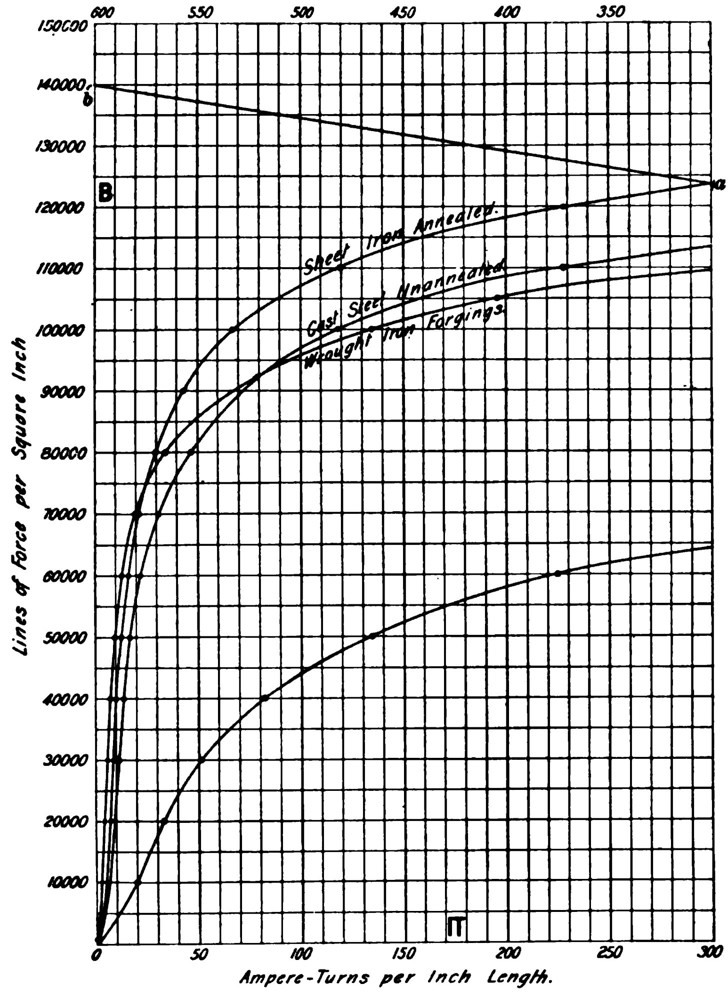

FIG. 11

scale, the curve beyond 300 ampere-turns per inch length has been plotted backwards and the abscissas belonging to this portion of this one curve are placed above the top line. This portion *a b* of the sheet-iron curve happens to be practically a straight line. As an example, suppose that we desired to find the ampere-turns required to produce a density of 135,000 lines per square inch in a piece of sheet iron 9 inches long. We follow the horizontal line representing 135,000 lines of force per square inch in Fig. 11, to the right until it intersects the line *a b*, which it happens to do at a point that has, according to the scale mentioned above, a value of about 510. Hence, the ampere-turns required to force this flux density through 9 inches of sheet iron = 510 × 9 = 4,590 ampere-turns.

Curves as given in Figs. 10 and 11 are used by designers of electrical apparatus and machinery in preference to such curves as given in Figs. 3 and 4.

EXAMPLE.—Find the ampere-turns required to drive an induction of 55,000 lines of force through the circuit of a horseshoe magnet made

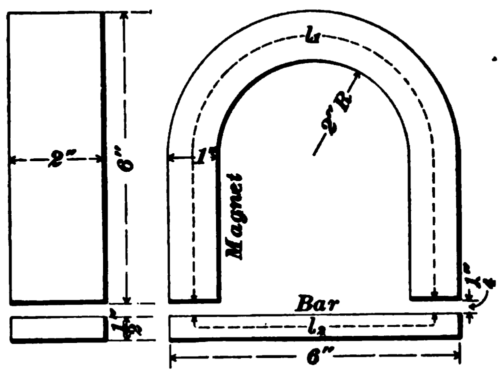

FIG. 12.

of cast iron, when a bar of wrought iron is placed across its two ends, but separated from them by an air gap of ¼ inch. The dimensions of the magnet and bar are shown in Fig. 12.

SOLUTION 1.—This magnetic circuit is a compound one, composed of three different substances: (1) the cast-iron magnet, (2) the wrought-iron bar, and (3) the two air gaps.

Let Φ = total induction;
l_1, l_2, and l_3 = the average lengths of the magnetic circuit in inches in magnet, bar, and total air gap, respectively;
A_1, A_2, and A_3 = the sectional areas, respectively;
$\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$ = the magnetic densities, respectively;
$\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ = the reluctances, respectively;
μ_1, μ_2, and μ_3 = the permeabilities, when the densities are $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$, respectively.

By formula **25**, the ampere-turns $IT = .313\,\Phi\,(\mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3)$. By formula **20**, the reluctance of the circuit in the cast-iron magnet is $\mathbf{R}_1 = \frac{l_1}{A_1 \mu_1}$. The length of the magnetic circuit in the cast-iron magnet is equal to the length of the dotted line in this portion of the figure. The length of this dotted line is equal to half the circumference of a circle plus the length of the two straight ends. Since the radius of the inner edge = 2 in., and the radius of the outer edge = 2 + 1 = 3 in., then the mean radius, that is, the radius of the dotted half-circle $= \frac{2+3}{2} = 2\frac{1}{2}$ in. The length of half a circumference $\left(\frac{2\pi r}{2} = \pi r\right)$ having a radius of $2\frac{1}{2}$ in. $= 2.5 \times 3.1416$ in. The length of one straight end of the cast-iron magnet = total height of magnet less the height of the curved portion. The radius of the outside edge is 3 in.; hence, the length of one straight end $= 6 - 3 = 3$ in.; consequently, the length of both ends $= 2 \times 3 = 6$ in. Therefore, the total length of the mean path of the lines of force through the cast-iron magnet $= l_1 = 2.5 \times 3.1416 + 6 = 13.854$ in. The sectional area of the magnet $= A_1 = 2 \times 1 = 2$ sq. in. By the formula $\mathbf{B}_1 = \frac{\Phi}{A_1}$, the density $= \frac{55{,}000}{2} = 27{,}500$ lines of force per square inch. From Fig. 5, μ is about 180 when $\mathbf{B} = 27{,}500$ in cast iron. Then the reluctance

$$\mathbf{R}_1 = \frac{l_1}{A_1 \mu_1} = \frac{13.854}{2 \times 180} = .03848$$

The reluctance of the circuit in the wrought-iron bar is $\mathbf{R}_2 = \frac{l_2}{A_2 \mu_2}$. The length of the magnetic circuit in the wrought-iron bar $= l_2 = 5 + .25 + .25 = 5.5$ in. The sectional area of the bar $= A_2 = 2 \times .5 = 1$ sq. in. $\mathbf{B}_2 = \frac{\Phi}{A_2} = \frac{55{,}000}{1} = 55{,}000$ lines of force per square inch. From Fig. 5, μ is about 1,520 when $\mathbf{B} = 55{,}000$ in wrought iron. Then the reluctance

$$\mathbf{R}_2 = \frac{l_2}{A_2 \mu_2} = \frac{5.5}{1 \times 1{,}520} = .00362$$

The two air gaps may be added together, and in the calculations a single air gap of double length, that is, $2 \times \frac{1}{4} = \frac{1}{2}$ in., considered. The reluctance of the circuit in the air gap is $\mathbf{R}_3 = \frac{l_3}{A_3 \mu_3}$. The length of the magnetic circuit in the two air gaps $= l_3 = .5$ in. The sectional area of the air gap $= A_3 = 2 \times 1 = 2$ sq. in. In the case of air, the permeability $\mu_3 = 1$. The reluctance is then

$$\frac{l_3}{A_3 \mu_3} = \frac{.5}{2 \times 1} = .25$$

By formula **25**, the necessary ampere-turns $= .313 \times 55{,}000 \times (.03848 + .00362 + .25) = .313 \times 55{,}000 \times .2921 = 5{,}028.50$, which means that a magnetizing force of 5,029 ampere-turns will have to circulate around the magnet arms to force 55,000 lines of force through the magnetic circuit. Ans.

This example was solved in the foregoing manner to show how to calculate and use the reluctances of the various portions of a compound magnetic circuit in connection with such curves as given in Fig. 5. The example could be solved more readily perhaps in the following manner, using the curves for ampere-turns as given in Fig. 11.

Solution 2.—Since the cross-section of the magnet = 2 sq. in., then the density in the magnet $= \frac{55000}{2} = 27{,}500$ lines of force per square inch. The ampere-turns per inch length required to produce such a density in cast-iron is found, from the cast-iron curve in Fig. 11, to be 47 ampere-turns per inch length. In the previous solution it has been shown that the length of the cast-iron magnet = 13.854 in.; hence, the ampere-turns required to produce a density of 27,500 lines of force per square inch throughout the whole length, 13.854 in., of the cast-iron magnet $= .313\,\mathbf{H}_1\, l_1 = 47 \times 13.854 = 651.14$ ampere-turns. The value of $.313\,\mathbf{H}_1$, which is 47, is obtained directly from the curve. In a similar manner the ampere-turns per inch length required to produce a density of $\frac{55000}{1} = 55{,}000$ lines per square inch in the wrought-iron bar is found from the wrought-iron curve in Fig. 11 to be 12 ampere-turns per inch length. Hence, the ampere-turns required to produce a density of 55,000 lines of force per square inch throughout the whole length, 5.5 in., of the wrought-iron bar $= .313\,\mathbf{H}_2\, l_2 = 12 \times 5.5 = 66$ ampere-turns. The density $\mathbf{B}_3$ in the air gap $= \frac{55000}{2} = 27{,}500$ lines per square inch. In the air gap, $\mu = 1$; hence, $\mathbf{H}_3 = \mathbf{B}_3 = 27{,}500$. Hence, the ampere-turns required to produce a density of 27,500 lines of force per square inch through the two air gaps $= .313 \times 27{,}500 \times .5 = 4{,}303.75$ ampere-turns. Then the total number of ampere-turns required to produce a magnetic flux of

55,000 lines of force in the given magnetic circuit = 651.14 + 66 + 4,303.75 = 5,020.89, or 5,021 ampere-turns. Ans.

By the method preceding this, 5,029 ampere-turns were obtained. $\dfrac{5029 - 5021}{\left(\dfrac{5029 + 5021}{2}\right)} = .0015$, or .15 per cent. Therefore the two results differ by less than .2 of 1 per cent. Either one is sufficiently correct. The difference is due to the fact that the curves cannot be plotted with sufficient accuracy to give closer results.

73. Reduction of Field Density in Lines per Square Centimeter to Ampere-Turns per Inch of Length. Magnetization curves will frequently be found plotted with $\mathcal{H}$, the lines of force per square centimeter in air, for abscissas and $\mathcal{B}$, the flux density in lines of force per square centimeter, for ordinates, as in Fig. 4, and it may be desirable to convert the values taken from such a curve into English measure; that is, $\mathcal{B}$ into flux density per square inch denoted by **B** and $\mathcal{H}$ into ampere-turns per inch of length. Now $\mathcal{H} = \dfrac{1.257\, I\, T}{l'}$, then ampere-turns per centimeter of length $\dfrac{I\, T}{l'} = \dfrac{\mathcal{H}}{1.257}$, and hence the ampere-turns per inch of length $= \dfrac{2.54 \times \mathcal{H}}{1.257} = 2.02\ \mathcal{H}$, 2.54 being the number of centimeters in 1 inch; therefore, the ampere-turns per inch of length are approximately equal to 2 $\mathcal{H}$; in which $\mathcal{H}$ is the magnetizing force in C. G. S. units; that is, the lines of force per square centimeter in air.

74. Reduction of Flux Density in Lines per Square Centimeter to Lines per Square Inch.—The flux density **B** in English measure, that is, the number of lines of force per square inch in any magnetic substance = 6.45 $\mathcal{B}$; in which $\mathcal{B}$ is the flux density in lines of force per square centimeter. Since there are 6.45 square centimeters in 1 square inch, then it is evident that there will be 6.45 times as many lines of force passing through 1 square inch as through 1 square centimeter when the density is the same; hence, $\mathbf{B} = 6.45\, \mathcal{B}$.

75. Design of Electromagnets.—Although it is beyond the scope of this section to take up the design of an electromagnet, it will be well to give here an outline of the method that may often be used. First, the total number of lines of force required and then the material to be used are decided on. Having selected the material for the various portions of the magnetic circuit, the magnetic densities are then selected, usually from experience, and the corresponding values of the magnetizing forces are obtained from the magnetization curves for the materials used.

The sectional areas of the various portions may then be determined by dividing the total flux by the flux densities. The length of each portion is estimated, the object being to make it no longer than necessary. The length of the cores will depend on the length of the magnetizing coils, which in turn depends on the number of turns, the size of wire, and the depth of the winding. The depth of the winding must be such that the surface of the coil will not have to radiate over from $\frac{1}{2}$ to 1 watt per square inch; that is, the area divided by $I^2 R$ must not exceed 1; I being the magnetizing current and R the resistance of the magnetizing coil. The exact amount of energy that can be safely radiated per square inch cannot be considered further here. This comes properly under the subject of dynamo design. Magnets used for telephone and telegraph apparatus do not usually depend on this heating limit, but rather on the resistance that will allow a given current to flow through it, at the same time producing the necessary number of ampere-turns.

If the lengths of the iron portions are first estimated, they may afterwards be corrected, if necessary. Since it is the air gap that usually requires the greatest number of ampere-turns, a slight variation in the lengths of the iron portions of the magnetic circuit will not appreciably affect the results. Having now the areas, lengths, densities, and magnetizing forces for the various portions, the ampere-turns required to produce the given flux in the various portions may be calculated. Or the ampere-turns for each portion of the compound magnetic circuit may be obtained directly from ampere-turn curves, such as those given in Figs. 10 and 11, after the flux densities are known. The sum of the ampere-turns for all portions gives the total number of ampere-turns required. To assist in designing the coils there is generally given the power in watts to be expended in the magnetizing coil, the electromotive force in volts, or the current in amperes. If both the watts and electromotive force are given, then the current is easily determined. If only the watts are given, then some standard electromotive force, such as 110, 220, or 500 volts, is usually taken and the current calculated. Having the current and the ampere-turns, the total number of turns may be obtained. Then it is necessary to choose a size of copper wire for the coils that will have the proper resistance so as to allow the desired current to flow through the coils, being careful that the wire used is large enough to carry the current without undue heating, otherwise the insulation may be charred and the coil will also be inefficient. On the other hand, unnecessarily large wire must not be used or it will make too large and bulky a coil, and hence costly and perhaps inefficient, because the outside turns are too far from the iron core, or the coil is unnecessarily long.

The size of wire may be calculated, if the voltage is known and the mean length of a turn can be determined, by the formula

$$A = \frac{12\, l\, I\, T}{E}, \qquad (29)$$

in which A is the sectional area of the wire in circular mils, l the mean length of one turn in feet, $I\,T$ the ampere-turns, and E the difference of potential in volts across the coil. The mean length of one turn need be calculated only approximately at first, and later, if necessary, a more correct value may be determined and the formula again solved for A.

ELECTROMAGNETIC INDUCTION

THE ELECTROMAGNET

FORMS OF ELECTROMAGNETS

1. A magnet produced by inserting a magnetic substance in the magnetic circuit of a solenoid is an **electromagnet**; the magnetic substance around which the current circulates is called the **core** (see Fig. 1). In the ordinary form, the magnetizing coil consists of a large number of turns of *insulated wire*; that is, wire covered with a layer or coating of some non-conducting or insulating material, usually silk or cotton; otherwise the current will not circulate around the magnet but will take a shorter and easier circuit from one coil to the adjacent one, or from the first to the last coil, by passing through the iron core.

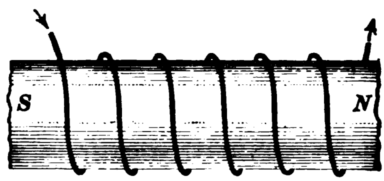

FIG. 1

The rule for determining the polarity of an electromagnet is the same as for a solenoid. It makes no difference whether the wire is wound in one layer or in any number of layers, whether it is wound toward one end and then back over this layer toward the other, or whether all layers are wound in the same direction; so long as the current circulates continually in the same direction around the core, the polarity of the magnet will remain unchanged.

2. As the practical designing of electromagnets for any purpose is beyond the scope of this section, only a few of the principal forms are illustrated here. The simplest form, shown in Fig. 2, consists of a straight bar of iron or steel *B* fitted inside a spool or bobbin *C* made of hard vulcanized rubber, fiber, wood, or some other inflexible insulating material. The magnetizing coil of fine insulated copper wire *w* is wound in layers in the bobbin.

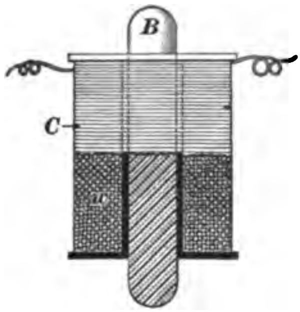

FIG. 2

3. A convenient form of electromagnet used for a variety of purposes is the **horseshoe,** or **U-shaped,** electromagnet, shown in Fig. 3. It usually consists of four parts, namely, two iron rods *M*, *M*, called *cores*, and over which are wound the *magnetizing coils c*; a straight bar of

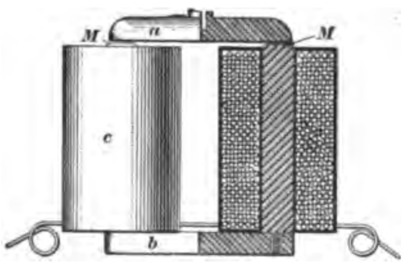

FIG. 3

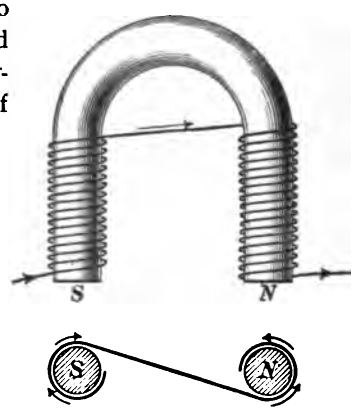

FIG. 4

iron *b*, called a *yoke*, joining the two cores together; and a straight bar of iron *a*, called the *armature*. When looking at the faces of the two cores, Fig. 4, the current should circulate around one core in an opposite direction to that around the other. If the current should circulate around both cores in the same direction when looking at their faces, the lines of force produced in the two cores would oppose one another and form two like poles at their free ends and a consequent pole in the yoke. The total number of useful lines of force produced by both coils would, under these conditions, be greatly diminished, and the magnet would exhibit very little or no magnetic attraction.

4. Another common form of electromagnet is known as the **iron-clad electromagnet.** In its simplest form, shown in Fig. 5, the central core, the yoke, and the outside shell *S* are made of one piece and completely surround and protect the coil on all sides except one end. The end *M* is usually covered by a disk of iron, thus forming practically a complete magnetic circuit. This form of electromagnet requires but one coil for its excitation. For some purposes the iron disk at the end *M* is pivoted so as to act as a movable armature. Relays used in telephone systems are often constructed on this principle, the armature being allowed to only move sufficiently to open and close a local circuit.

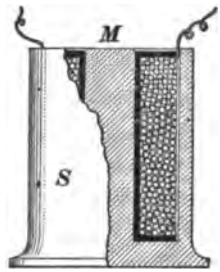

FIG. 5

5. An economical form of an iron-clad electromagnet for lifting weights is shown in Fig. 6. The magnet proper *M* is made in one casting, the coil *c*, after being wound on a suitable form, is thoroughly insulated by wrappings of cloth, mica, or tape, and then placed around the inside core of the magnet and held in position by a ring of brass or other non-magnetic metal *r* wedged between the core and the outside shell. The connections to the coil from an outside source are made to *leads*

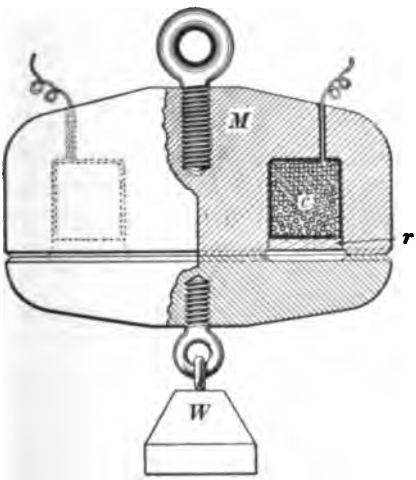

FIG. 6

(pronounced *leeds*) passing from the coil up through holes in the top of the magnet. By designing the magnet short in length and large in sectional area, the magnetic circuit can be made exceedingly short in proportion to its sectional area, thus realizing one of the conditions of an economical design.

6. Electromagnets may be divided into three general classes, according to their application, viz.:

1. Those for lifting weights and loads by adhesion.
2. Those for producing mechanical motion in an armature or a keeper; that is, for attracting an armature or a keeper through a distance.
3. Those for producing a magnetic field for dynamo-electric machines, and called field magnets.

PULL OF AN ELECTROMAGNET

7. The formula for the pull of a magnet upon its armature is $F = \frac{\mathcal{B}^2 A'}{8\pi}$,* in which F is the force, or pull, in dynes, $\mathcal{B}$ the magnetic density at the polar surface in lines of force per square centimeter, and A' the area of the polar surfaces in square centimeters. This fundamental expression for F may be easily transformed to one in which square inches and pounds are used as the units of measurement. A force of 1 gram is equivalent to 981 dynes, and there are 453.6 grams in a pound, so that we may write

$$F = \frac{\mathcal{B}^2 A'}{8 \times \pi \times 981 \times 453.6} = \frac{\mathcal{B}^2 A'}{11,183,600}, \text{ nearly} \qquad (1)$$

where F = pull in pounds;

$\mathcal{B}$ = magnetic density in lines per square centimeter;

A' = polar area in square centimeters.

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* The derivation of this expression requires the use of mathematics and a knowledge of the theories of electricity and magnetism that are beyond the scope of this Course.

One square inch is equal to 6.45 square centimeters, so that if we wish to express the magnetic density and polar area in terms of square-inch units instead of square centimeters, we will have, $F = \frac{\mathbf{B}^2 \times A \times 6.45}{11,183,600 \times 6.45^2}$, or

$$F = \frac{\mathbf{B}^2 A}{72,134,000}, \text{ nearly} \qquad (2)$$

where F = pull in pounds;
$\mathbf{B}$ = magnetic density in lines per square inch;
A = polar area in square inches.

That is, the tractive force of a magnet increases directly as the total area of the surface in contact with the armature, and as the square of the density of the lines of force in the magnetic circuit where it passes across that surface. Formulas **1** and **2** assume that the distribution of the lines of force is uniform throughout the entire contact surface. In actual practice it is impossible to obtain this result on account of magnetic leakage and other causes. The calculated load and the actual load lifted will generally differ—the actual being somewhat less than the calculated, due to the fact that some of the magnetic lines leak away from the attracting surfaces.

In nearly all electromagnets designed for traction there will be two contact surfaces, one at the north pole of the magnet and the other at the south pole; or, in other words, the total lines of force developed in the magnetic circuit are used twice in producing the traction of the magnet. If the two contact surfaces are symmetrical and equal in area, the total tractive force of the magnet will be twice the result obtained by considering one contact surface alone; but if the contact surfaces are unlike, the tractive force exerted by each surface should be calculated separately, and the two results thus obtained added together.

EXAMPLE.—What will be the approximate pull, in pounds, exerted on the armature of a horseshoe-shaped electromagnet, when the armature is in contact with the cores and the current used is $\frac{1}{4}$ ampere?

The length of the entire magnetic circuit is 6.75 inches, the polar area of one end of one core is .166 square inch, and the total number of turns of wire on the two coils is 940. The iron is extremely soft and permeable and the magnetization curve between **H** and **B** for annealed wrought iron given in a previous section may be assumed to apply to it. It may also be assumed that the flux density is constant throughout the magnetic circuit.

SOLUTION.—It is first necessary to determine the flux density **B** at the polar surfaces. By the formula $\mathbf{H} = \frac{3.192\, I\, T}{l}$, the magnetizing force

$$\mathbf{H} = \frac{3.192 \times .25 \times 940}{6.75} = 3.192 \times 34.8$$

$$= 111.08 \text{ or } 111 \text{ lines of force per square inch}$$

By consulting the curve for annealed wrought iron in the figure referred to above, we find that this magnetizing force gives a flux density **B** of about 94,400 lines of force per square inch.

By substituting in the formula $F = \frac{\mathbf{B}^2 A}{72{,}134{,}000}$, we get

$$F = \frac{(94{,}400)^2 \times 2 \times .166}{72{,}134{,}000} = 41.01 \text{ lb. Ans.}$$

MAGNETIC LEAKAGE

8. All the lines of force produced by a magnetomotive force cannot be confined along one path; some stray from the main circuit and take shorter paths; these constitute **magnetic leakage.** The amount of this leakage depends on the uniformity of the reluctance of the main circuit at all points—being least with the greatest uniformity. Its nature may be better understood by remembering that air is really a magnetic conductor, although its reluctance is much greater than that of iron or other magnetic substance. Consequently, when the reluctance of the main circuit becomes large at any point, some of the lines of force find a shorter and easier path for themselves through the surrounding air.

Fig. 7 represents a **U**-shaped electromagnet made of iron with a keeper of the same metal and sectional area. By placing the keeper tightly against the two ends, the reluctance becomes practically uniform throughout the entire

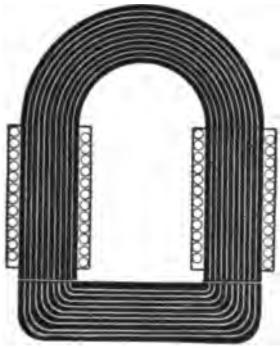

FIG. 7

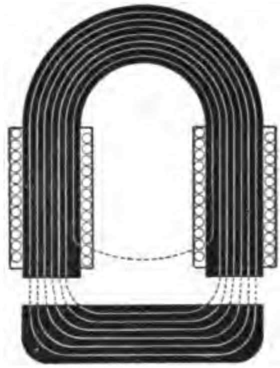

FIG. 8

magnetic circuit, and there is no perceptible leakage at any place. But if the reluctance of the circuit is changed by separating the keeper from the ends of the magnet by a small air gap, as in Fig. 8, the conditions are altered. In

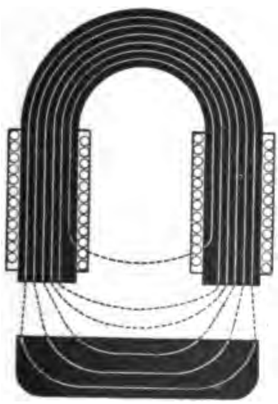

FIG. 9

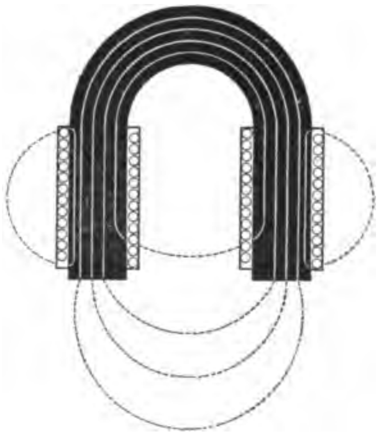

FIG. 10

the first place, the total number of lines of force will be reduced in all parts of the circuit, and in the second, some of the lines will leak across from end to end of the magnet without passing through the keeper. The larger the air gap between the keeper and the magnet, the greater will be the magnetic leakage. An approximate idea of the magnetic leakage is shown in Fig. 9, where the keeper is placed at a considerable distance from the ends of the magnet; Fig. 10 shows the state of the lines of force when the keeper is removed entirely.

9. Calculating the Magnetic Leakage. — Magnetic leakage may be defined as the difference between the total number of lines of force produced by the magnetomotive force and the number that are useful in attracting or lifting a given weight. As no definite laws govern magnetic leakage, it is almost impossible to calculate the number of stray lines of force in any compound magnetic circuit. After a magnet is built, the leakage can be determined with the proper instruments and under certain conditions. But in general, if the magnetic circuit is composed of magnetic substances whose permeabilities are high and no large air gaps are to be crossed, the magnetic leakage will be small.

If the total number of lines of force produced by the magnetizing coils and the useful number are known, the magnetic leakage can be expressed by a per cent. of the total number produced. Thus,

Let Φ = total number of lines of force;
Φ_u = number of useful lines of force;
Φ_s = number of stray lines of force;
p = per cent. of leakage.

Then

$$\Phi_s = \Phi - \Phi_u \qquad (3)$$

For example, assuming that 60,000 lines of force are produced by the magnetizing coils of an electromagnet, and that only 42,000 are useful in attracting an armature or lifting a weight, then, by this formula, the number of stray lines of force $\Phi_s = 60{,}000 - 42{,}000 = 18{,}000$.

The percentage of leakage is found from the formula

$$p = \frac{100\ \Phi_s}{\Phi} \qquad (4)$$

That is to say, the percentage of leakage is found by dividing the number of stray lines of force by the total number produced and multiplying the quotient by 100. In the above case

$$p = \frac{100 \times 18{,}000}{60{,}000} = 30\%\ \text{leakage}$$

10. To find the total number of lines of force when the percentage of leakage and the number of useful lines of force are known, use the following formula:

$$\Phi = \frac{100\ \Phi_u}{100 - p} \qquad (5)$$

Here we divide the useful lines of force by 100 minus the per cent. leakage and multiply the quotient by 100.

EXAMPLE.—Assuming that the magnetic leakage in an electromagnet is 25 per cent. and that there are 75,000 useful lines of force, how many lines of force are produced by the magnetizing coils?

SOLUTION.—By the last formula the total lines of force

$$\Phi = \frac{100 \times 75{,}000}{100 - 25} = \frac{7{,}500{,}000}{75} = 100{,}000$$

total lines of force produced by the magnetizing coils. Ans.

EXAMPLES FOR PRACTICE

1. 100,000 lines of force are produced by the magnetizing coils of an electromagnet and only 40,000 are useful. What is the percentage leakage? Ans. 60% leakage

2. In an electromagnet there are 27,000 stray lines of force and 63,000 useful; find the percentage of leakage. Ans. 30% leakage

3. The magnetic leakage in an electromagnet is 45 per cent. and there are 110,000 useful lines of force; find the total number of lines produced by the magnetizing coils. Ans. 200,000 lines of force

4. If the magnetic leakage in an electromagnet is 35 per cent. and there are 60,000 lines of force produced by the magnetizing coils, how many lines of force are useful? Ans. 39,000 useful lines of force

ELECTROMAGNETISM

REACTION BETWEEN CURRENTS AND MAGNETIC FIELDS

ACTION OF A CURRENT ON A MAGNET

11. If a conductor conveying a current of electricity be brought near a freely suspended magnetic needle, the needle will tend to place itself at right angles to the conductor, as indicated by the arrows in Fig. 11. Thus it is evident that an electric current and a magnet exert a mutual force

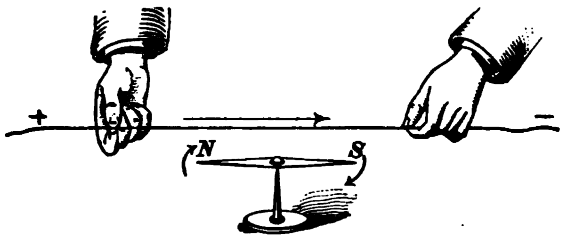

FIG. 11

on each other. Since a *magnetic field* is a region in which a magnetic needle is acted on by a force tending to turn it in some direction or other, it follows that the space surrounding a conductor, when an electric current is flowing through it, is a magnetic field.

If the experiment illustrated in Fig. 11 is tried under ordinary conditions, there is generally a disturbing influence due to the magnetism of the earth, and the compass needle can scarcely ever be made to set itself exactly at right angles to the wire in all positions.

12. If the current in a horizontal conductor is flowing toward the north and a compass is placed under the wire, the north pole of the needle will be deflected toward the west; by placing the compass over the wire, the north pole

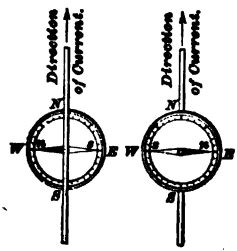

FIG. 12

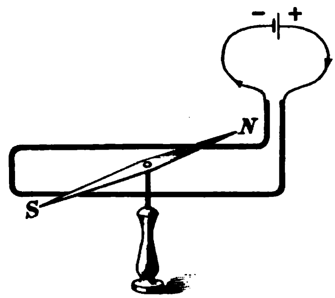

FIG. 13

of the needle will be deflected toward the east (see Fig. 12). Reverse the direction of the current in the conductor, and the needle will point in the opposite direction in each case, respectively.

If the conductor is placed over the needle and then bent back under it, forming a loop, as shown in Fig. 13, the tendency of the current in both top and bottom portions of the wire is to deflect the north pole of the needle in the same direction. From these experiments, knowing the direction of current in the conductor, the following rule is deduced for the direction of the lines of force around the conductor:

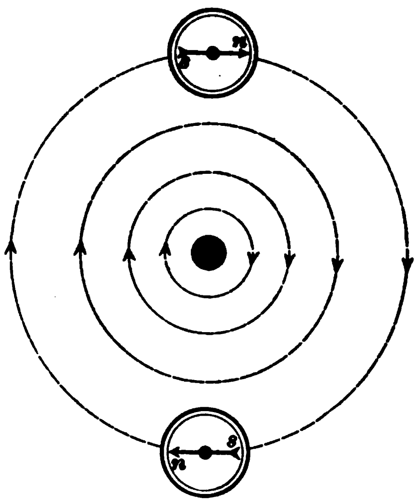

FIG. 14

Rule.—*If the current is flowing in the conductor away from the observer, then the direction of the lines of force will be around the conductor in the direction of the hands of a watch.*

The direction of the lines of force around a conductor is shown by the arrowheads and compass needles in Fig. 14, where the current is assumed to be flowing downwards, or away from the observer.

If the direction of the current is reversed, the direction of the lines of force around the conductor will also be reversed.

ACTION OF ONE CURRENT ON ANOTHER

13. Two parallel conductors, both transmitting currents of electricity, are either mutually attractive or repellent, depending on the relative direction of their currents. If the currents are flowing in the same direction in both conductors, as represented in Fig. 15, the lines of force tend to surround both conductors and contract, thus causing the conductors to attract each other. If, however, the currents are flowing in opposite directions, as represented in Fig. 16,

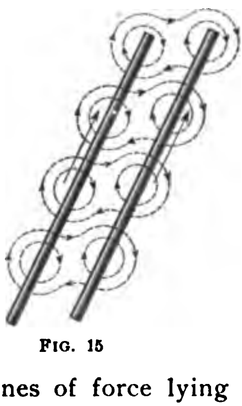

FIG. 15

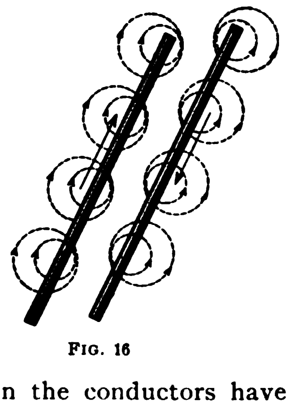

FIG. 16

the lines of force lying between the conductors have the same direction, and therefore the conductors repel each other.

MAGNETIC FIELD AROUND A SOLENOID

14. If the conductor carrying the current is bent into the form of a loop, as shown in Fig. 17, then all the lines of force around the conductor thread through the loop in the same direction. Any magnetic substance, therefore, such as *m*, when placed in front of the loop, will tend to place

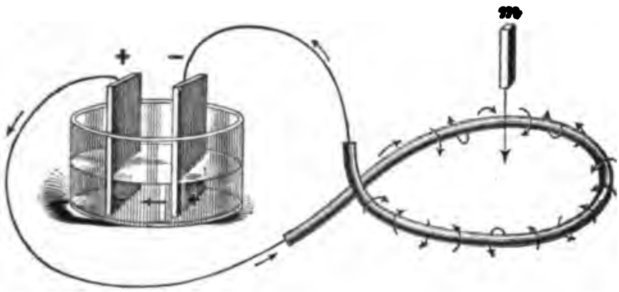

FIG. 17

itself with its longest axis projecting into the loop; that is, in the direction of the lines of force.

By bending the conductor into a long helix of several loops, the lines of force around and inside each loop will coincide in direction with those around and inside the adjacent loops, forming the equivalent of several long lines of force that thread through the entire helix, entering at one end and passing out through the other. The same conditions now exist in the helix as exist in a bar magnet; namely, the lines of force pass out from one end and enter the other. The imaginary appearance of a magnetic field produced around a solenoid through which a current is flowing is shown in Fig. 18. In fact, the helix possesses a north and south pole, a neutral line, and all the properties of attraction and repulsion of a magnet. If it is suspended in a horizontal position

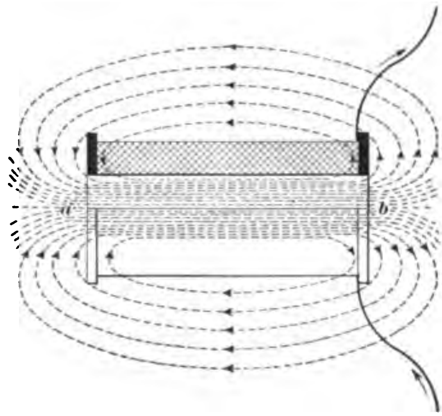

FIG. 18

and free to turn, it will come to rest pointing in a north-south direction.

A helix containing a number of turns of wire through which a current of electricity is flowing is called a **solenoid.** The polarity of a solenoid, or the direction of the lines of force that thread through it, depends on the direction in which the conductor is coiled and the direction of the current in the conductor.

By means of the following rule the polarity of a solenoid may be determined if the direction of the current is known:

Rule.—*In looking at the end of the helix, if the current flows around it in the direction of the hands of a watch, that end will be a south pole; if in the other direction, it will be a north pole.*

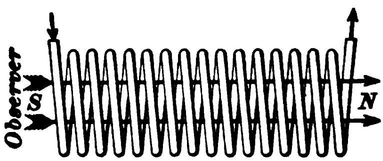

FIG. 19

Fig. 19 represents a conductor coiled in a right-handed helix. If the current flows into the coil from the end where the observer stands, that end will be a south pole, and the observer will be looking through the helix in the direction of the lines of force. The polarity of a solenoid can be reversed by reversing the direction of the current in the conductor.

INDUCED CURRENTS

15. Magnet Inducing a Current in a Coil.—The reaction between magnetic lines of force and a conductor forming a closed circuit can be shown by the following experiments, which any one can readily try. In Fig. 20, *a* is a solenoid whose terminal wires *c*, *d* are wound a number of times around an ordinary compass *e* in the same direction that the needle ordinarily points, that is, in a north-and-south direction. If a current passes through the wires around the compass, it will cause the compass needle to swing either to the right or left, depending on the direction of the current. If the bar magnet *b* is moved quickly into the interior of the solenoid, the compass needle will swing through an angle, proving that a current has been passing through the solenoid. As soon as the magnet stops its motion, the needle will swing back to its initial position and come to rest.

When the magnet is withdrawn from the solenoid, the needle will again swing through an angle, but in an opposite direction to that of its first deviation, proving that in this instance again a current was started, but in an opposite direction to the former current.

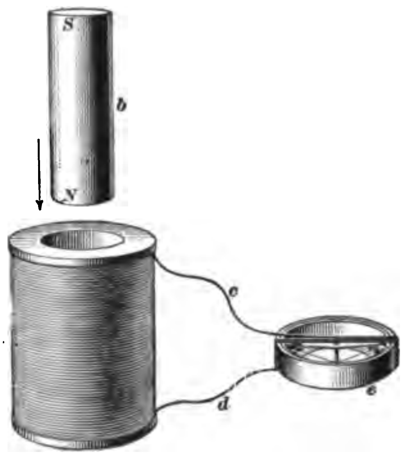

FIG. 20

It will also be found that no current flows as long as the magnet remains stationary, but only when a change takes place in the position of the magnet. The quicker these motions are made, the more will the needle deviate, and, consequently, the stronger must have been the current in the coil around the compass. Of course, the same effects will be produced if the bar magnet is stationary and the coil is moved.

16. A Solenoid Conveying a Current Acting as a Magnet.—If the magnet *b* is replaced by a solenoid *b* in which a current is flowing, as represented in Fig. 21, similar results may be produced. When the solenoid *b* is thrust into the solenoid *a*, a current will be induced in the solenoid *a*, in a direction opposite to the current that will be induced when the solenoid *b* is removed. It is further found that if the solenoid *b*, when conveying a current, is placed inside the solenoid *a*, and the circuit of the solenoid *b* is broken, a current will flow in the solenoid *a* in the same direction as if the solenoid *b* had been suddenly withdrawn while a current was flowing through it. Likewise it is found that on closing the circuit of *b* the effect on the solenoid *a* will be the same as if the solenoid *b*, while conveying a current, had been reinserted. In fact, any strengthening or weakening of the current in the

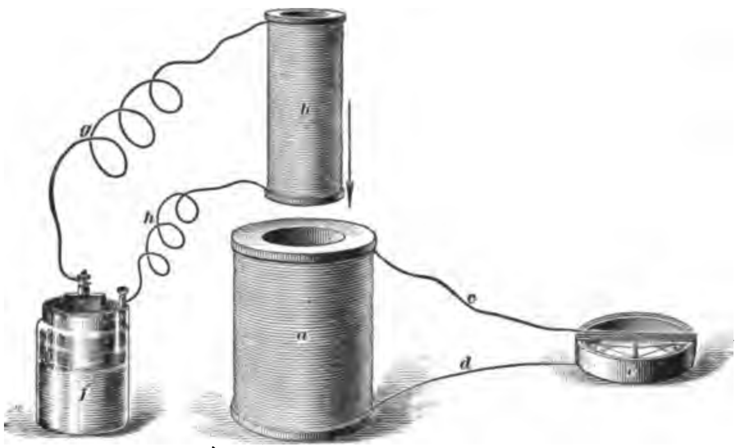

FIG. 21

solenoid *b* has the same effect as if the coil was approaching or receding from the solenoid *a*.

We see, then, that a current is flowing through the coil *a* only when one of the coils moves relatively to the other, or when the current in the coil *b* is changing in strength. As long as both coils remain stationary, or the current in the coil *b* does not vary in strength, no current will flow in the coil *a*.

That, in these experiments, the magnet *b* and the coil *b* both have the same effects on the coil *a* should not be surprising, as it has been shown that a solenoid, through which a current is flowing, produces lines of force that pass through and around it in the same manner as those of a permanent magnet.

In Fig. 22 the coil *a*, in which we wish to induce a current, is shown in cross-section, and the magnet *b* is surrounded by dotted lines b_1, indicating the position and direction of its magnetic lines of force. These lines of force pass through the surrounding copper wires without being diverted from their course, and as if the wires did not exist. If, now, the magnet is moved downwards, these lines of force will pass across the various turns of wire in the coils, and a certain interaction between the conductor and the lines of force will take place, whereby an electromotive force is created in the coil. There will then be a tendency to start a current, and if the circuit is closed, as in Figs. 20 and 21, a current will flow in the coil and show its presence by its action on the compass *e*.

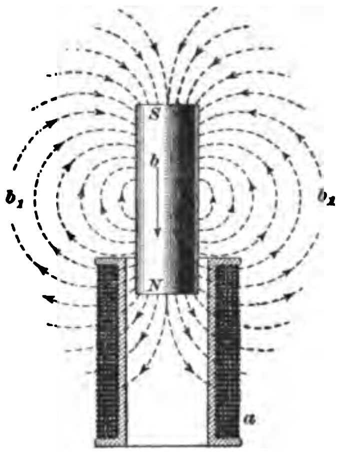

FIG. 22

MOTION OF A CONDUCTOR IN A MAGNETIC FIELD

17. It has been shown that a magnet and a conductor carrying a current of electricity exert a mutual force on each other; or, in other words, each tends to produce motion in the other. In general, when a conductor carrying a current of electricity is placed in a magnetic field, the conductor will tend to move in a definite direction and with a certain force, depending on the strength and direction of the current and on the direction and density of the lines of force. The direction of motion of a conductor carrying a current of electricity when placed in a magnetic field may be determined by the following rule:

Rule.—*Place thumb, forefinger, and middle finger of the left hand each at right angles to the other two, as shown in Fig. 23; if the forefinger shows the direction of the lines of force and the middle finger shows the direction of the current, then the thumb will show the direction of motion given to the conductor.*

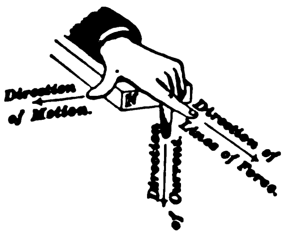

FIG. 23

By keeping the three fingers always at right angles to one another, this rule may be applied to determine the direction of motion of a conductor in any case in which the lines of force and the current are at right angles to each other; for the hand as a whole may be turned or placed in any position that may be necessary to make the middle and the forefinger point in the proper directions.

18. The direction of motion produced in the conductor can also be graphically shown. In Fig. 24, the dots represent an end view of the lines of force, and the heavy line a conductor conveying a current of electricity. If the direction of the lines of force is downwards, that is, away from the reader and perpendicular to the paper, and if the current flows in the direction indicated by the arrow heads on the conductor, then the conductor will be moved bodily to the right, as indicated by the two arrows.

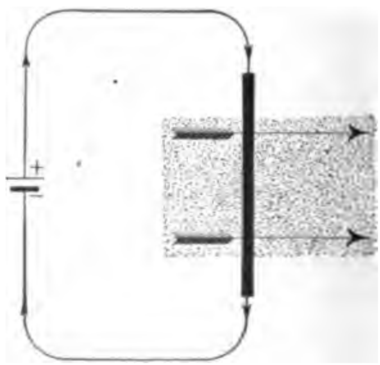

FIG. 24

This action is also true of an electric arc passing through a magnetic field, that is, a current of electricity passing or jumping in the form of a continuous spark between two electrodes across an air space traversed by lines of force, as indicated in Fig. 25. The arc or spark will be impelled to one side in the same direction as the conductor in the previous case. If the electrodes remain in a fixed position relative to the magnetic field, the arc may be blown out, that is, the spark may be extinguished, in which case the current will cease to flow in the circuit. In

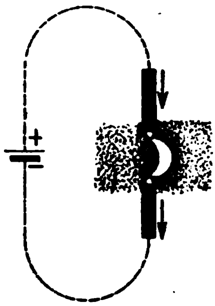

FIG. 25

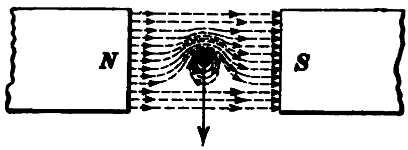

FIG. 26

both cases the motion is caused by the mutual action of the lines of force in the magnetic field and those produced by the current itself, as indicated in Fig. 26, where the current is assumed to be flowing downwards, that is, perpendicular to the paper and away from the reader. The lines of force in the magnetic field tend to coincide in direction with those around the current, and in doing so exert a crowding effect on the lines of force produced by the current, thereby tending to move the conductor or the arc, as the case may be.

19. The converse of this effect is also true, namely, when a conductor forming a closed circuit is moved across a magnetic field at right angles to the lines of force, a current is induced in the conductor.

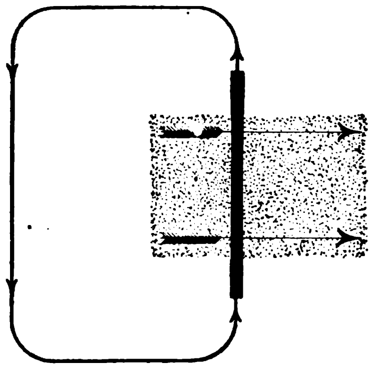

FIG. 27

This statement will be better understood by comparing the action in Fig. 24 with that in Fig. 27. In the former case, when a current is flowing in the direction indicated by the arrowheads, the conductor will move bodily to the right. In Fig. 27, however, when the conductor is moved to the right by some exterior means a current is induced in it which tends to flow in an opposite direction to the current that produces the same motion in the former case.

This generation of current is more correctly explained by saying that the motion of the conductor across the lines of force set up by the magnet produces an electromotive force in the conductor, which, when the circuit is completed, causes a current to flow. The direction of the current induced in the conductor will be at right angles to the lines of force and to the direction of motion of the conductor. The direction of induced currents may be easily determined by the following rule:

Rule.—*Place thumb, forefinger, and middle finger of the right hand each at right angles to the other two; if the forefinger shows the direction of the lines of force and the thumb shows the direction of motion of conductor, the middle finger will show the direction of the induced current* (see Fig. 28).

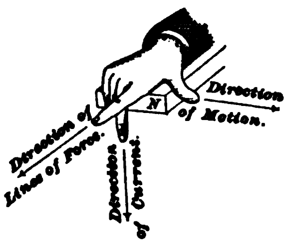

FIG. 28

By keeping the three fingers always at right angles to one another, this rule may be applied to determine the direction of an induced current in any case in which the lines of force and the direction of motion of a conductor are at right angles to each other; for the hand as a whole may be placed in any position that may be necessary to make the forefinger and the thumb point in the proper directions.

20. The **positive end** of a conductor in which an electromotive force is generated by moving it across a magnetic field is that end toward which the current tends to flow; the **negative end** is that from which the current tends to flow. Thus in Fig. 27, the upper end of the heavy moving conductor in which the electromotive force is generated is the positive end, and the lower end is the negative. Consequently the current flows from the negative to the positive terminals of the conductor in which the electromotive force is generated, and from the positive to the negative terminals through the external conductor.

It has been shown that an electric current will be induced in a coiled conductor when a pole of a magnet is suddenly inserted into the coil. The current will be continuous as long as there is a change in the same direction in the number of lines of force passing through the coil, but the current will cease to flow when the number of lines of force becomes constant, that is, when the lines of force inside the coil do not increase or diminish in number. The induced current will flow in the one direction if the number of lines of force are increasing, and in the opposite direction if the number of lines ot force are decreasing.

In reality, currents produced in a conductor cutting lines of force and currents induced in a coiled conductor by a change in the number of lines of force that pass through the coil are due to the same motion, for every conductor carrying a current of electricity forms a closed coil, and every line of force is a complete magnetic circuit by itself. Consequently. when any part of a closed coil is cutting lines of force, the lines of force are passing through the coil in a definite direction, and changing at the same rate as the cutting. In many calculations, however, it is more convenient to make a distinction between the two cases, and to consider that the current or, more strictly, the E. M. F., in the first case is generated by a conductor of a certain length cutting the lines of force at right angles; while, in the second case, the current in a closed coil is induced by a change in the number of lines of force passing through the coil.

21. The action of induced currents can be shown by a closed coil of any conducting material moving in a magnetic field. If it is moved in a uniform field along the lines of force, as in Fig. 29, so that only the same number of lines of force pass through it, no current will be generated.

Or, if the coil be moved across the lines of force in a uniform field, Fig. 30, as many lines of force are left behind as are gained in advancing, and there is, consequently, no change in the number of lines of force passing through the coil; hence there will be no current generated in it.

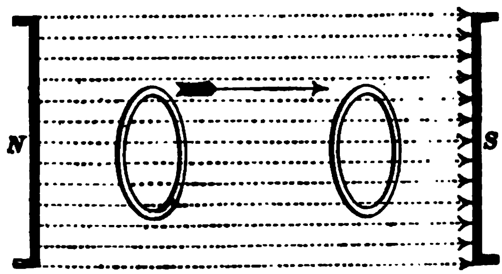

FIG. 29

Rotating the coil on a central axis perpendicular to the plane of the coil, like the rim of a pulley, will not generate a current, because there is no change in the number of lines of force passing through the loop. But if, as in Fig. 31, the

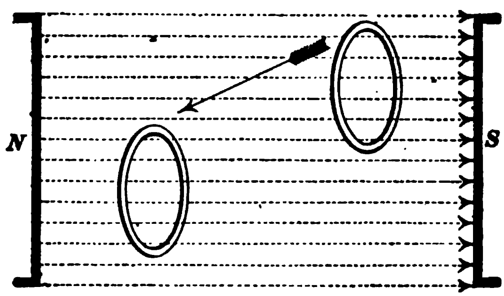

FIG. 30

coil be tilted as it is moved across the uniform field, or rotated around any axis in its own plane, then the number of lines of force that pass through it will be altered and a current will be generated. Where the magnetic field is not uniform, the removal of the coil bodily from a place where the lines of force are dense to where they are less dense, as from position *1* to position *2* in Fig. 32, will cause

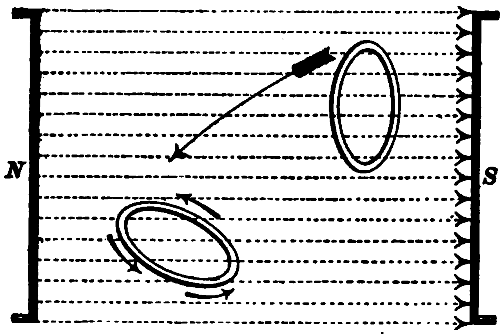

FIG. 31

the generation of a current in the coil; or if the coil is moved to a place where the direction of the lines of force is reversed, the effect will be the same.

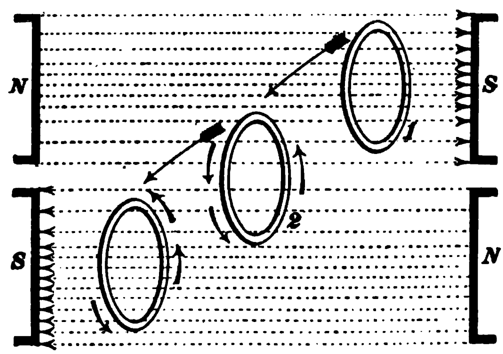

FIG. 32

22. The direction of induced currents in a closed coil may be determined by the following rule:

Rule.—*If the effect of the movement is to diminish the number of lines of force that pass through the coil, the current will flow in the conductor in the direction of the hands of a watch, as viewed by a person looking along the magnetic* *field in the direction of the lines of force; but if the effect is to increase the number of lines of force that pass through the coil, the current will flow in the opposite direction.*

In the explanations just given, it was stated that currents are generated by moving the conductor in a magnetic field. It must be remembered, however as shown in the beginning, that a current is merely the equalization of a difference of potential. Strictly speaking, therefore, it is not actually a current, but an electromotive force, that is developed by induction in the moving conductor; for, on opening the circuit, the electromotive force will still exist, but no current can flow. The word current is used merely to avoid complication.

DETERMINATION OF ELECTROMOTIVE FORCE

23. The electromotive force generated in a conductor cutting lines of force at right angles is proportional to the **rate of cutting,** which is found by dividing the number of lines cut by the time taken to cut them. One absolute unit of electromotive force is generated in a conductor when it is cutting lines of force at the rate of one line of force per second. By definition, 1 volt is equal to 100,000,000 (10^8) C. G. S. or absolute units; consequently, in order to generate an electromotive force of one volt, the rate of cutting must be 10^8 lines of force per second. This can also be expressed algebraically.

Let E = the electromotive force in volts;
Φ = the total number of lines of force cut by the conductor;
t = time in seconds taken to cut Φ lines of force.

Then, $$E = \frac{\Phi}{10^8 \times t} \qquad (6)$$

That is, the electromotive force in volts generated in a moving conductor is found by dividing the total number of lines of force cut by the conductor by the time taken and by 100,000,000.

If the total number of lines of force cut is the same, the electromotive force developed is the same, whether the lines of force proceed from a permanent magnet, electromagnet, or a coil carrying a current of electricity.

NOTE.—In a previous section of this Course the formula $E = \frac{l \mathcal{H} v}{10^8}$ was given. In this case $\frac{\Phi}{A}$ may be substituted for $\mathcal{H}$, in which Φ is the total number of lines of force cut, and A is the area, in square centimeters, swept over by the conductor. Then $E = \frac{l \Phi v}{10^8 A}$. $l \times v$ is the area swept over by a conductor l centimeters in length in 1 second, when moving with a velocity of v centimeters per second. Since $\frac{\Phi}{A}$ is the number of lines of force per unit area, then $l v \times \frac{\Phi}{A}$ is the total number of lines of force cut in 1 second, that is $\frac{l v \Phi}{A} = \frac{\Phi}{t}$, Φ being the total number of lines of force cut in t seconds, and hence $\frac{l v \mathcal{H}}{10^8} = \frac{l v \Phi}{10^8 A}$ $= \frac{\Phi}{10^8 t} = E$, as given by the last formula. E is the electromotive force, in volts, developed in a conductor l centimeters long when it is moving with a velocity of v centimeters per second across a magnetic field of such an intensity as to cut $\frac{\Phi}{t}$ lines of force per second.

24. According to Ohm's law, the current obtained from conductors cutting lines of force is equal to the quotient arising from dividing the total electromotive force generated by the total resistance of the circuit through which the current passes. In general, the total resistance is the resistance of the conductor cutting the lines of force, or the resistance of the internal circuit, plus the resistance of any conductor or conductors that complete the external circuit. If E represents the total electromotive force, in volts, R_i, R_e the resistance, in ohms, of the internal and external circuits, respectively; and I the current, in amperes; then $I = \frac{E}{R_i + R_e}$. It will be seen from this that a large or small induced current can be obtained from conductors cutting lines of force by simply changing the total resistance of the internal and external circuits. There is, however, a maximum limit to the amount of current obtained in this manner. The lines of force that are produced around the conductor by the current itself will always act in opposition to the lines of force producing the electromotive force, and will tend to distort or crowd them away from their original direction. The number of lines of force produced around the conductor by the current is directly proportional to the strength of the current; and, consequently, as the current becomes larger and larger, the lines of force cutting the conductor become more and more distorted and crowded away from their original direction, until the conductor no longer cuts all the lines of force, and, therefore, the electromotive force generated becomes smaller. To reduce this effect, the density of the magnetic field should be made large in proportion to the current to be generated.

PRODUCTION OF INDUCED ELECTROMOTIVE FORCE

ELECTROMAGNETIC INDUCTION

25. There are three ways of producing an electromotive force by induction in a coiled conductor, namely, by *electromagnetic induction*, by *self-induction*, and by *mutual induction*.

In **electromagnetic induction**, the change in the number of lines of force that pass through the coil is due to some relative motion between a coil and a magnetic field; as, for example, by thrusting a bar magnet into a coil, or by taking the magnet out of a coil, or by suddenly turning a coil in the magnetic field. Whenever there is such a relative movement between a conductor and the lines of force of a magnetic field as to cause the number of lines of force enclosed by the circuit to be increased or diminished, there will be set up in the conductor an electromotive force that tends to produce a current. The direction of this electromotive force will depend on the direction of the lines of force, and the direction of motion of the conductor and the value of the electromotive force will depend on the rate at which the number of lines of force enclosed by the circuit is increased or diminished by the motion of the conductor. This phenomenon is called electromagnetic induction; self-induction is one of the phenomena directly attributable to it.

SELF-INDUCTION

26. Mutual Action Between Turns of a Coil.—It is supposed that every conductor carrying a current is surrounded by what is called a magnetic field, or magnetic whirl. If each turn in a coil of wire carrying a current is surrounded by such a whirl, and if the various convolutions are close together, each will lie more or less within the field of the others. When the current flowing in one turn, for instance, suddenly increases, the lines of force set up around the wire forming this turn will expand, and in so doing will cut all the neighboring turns, thereby inducing in them an electromotive force that tends to produce a current in the opposite direction to the original current. On the other hand, when the current flowing in this particular turn diminishes, the lines of force will contract, and in so doing induce electromotive forces in each of the other turns that tend to produce currents in them in the same direction as the original current. Each turn of wire in the coil acts on all the others in the same manner as the particular turn that has been considered, thereby greatly magnifying the result. This phenomenon, that is, the action of one part of a circuit on the other parts of the same circuit, is termed **self-induction.** As an increase in the current flowing in one direction through a circuit always tends to induce a current in the opposite direction, while a decrease in the current tends to induce a current in the same direction, self-induction may be said to be that property of a circuit that tends to prevent any change in the strength of a current flowing in it. Self-induction has, therefore, been defined as the "inherent quality of an electric current that tends to impede the introduction, variation, or extinction of an electric current passing through an electric circuit." The circuit acts as if it possessed inertia that resists any change, and especially a sudden change, in the strength of the current flowing. Since self-induction tends to oppose any change being made in a current flowing in a circuit, the effect will be to make any change in current strength occur slightly later than it would be if the circuit possessed no self-induction.

27. Inductance, or the coefficient of self-induction, of a coil or circuit is defined as the total amount of cutting, or interlocking, of the lines of force and the turns of the coil, or circuit, that is produced when a current of 1 unit flows through it. Inductance is usually represented by L. Inductance may be possessed by a simple circuit as well as by a coil; that is, inductance is a property of any circuit and is not limited to a coil of wire of two or more turns. In the case of a simple circuit the number of turns is one. However, in deriving a general formula it is customary to consider a coil of any number of turns T.

If Φ is the total number of lines of force produced in a coii by I C. G. S. units of current flowing through it, 1 C. G. S. unit of current will produce $\frac{\Phi}{I}$ lines of force; since the total number of lines of force surrounding a conductor is directly proportional to the current, provided the permeability of the surrounding medium remains constant. If the coil has T turns, the total cutting or interlocking, when $\frac{\Phi}{I}$ lines of force are established or removed, will be $\frac{\Phi T}{I}$ because each line cuts through each of the T turns. But the inductance of a coil has been defined as the total amount of cutting of lines of force that is produced by unit current, hence $L = \frac{\Phi T}{I}$; in which L is the inductance in C. G. S. units, T the number of turns in the coil, and Φ the total number of lines of force set up in the coil by a current of I C. G. S. units strength. Inductance is evidently the ratio between the total induction $T \Phi$ through a circuit to the current I producing it.

28. A coil has 1 C. G. S. unit of inductance when 1 C. G. S. unit of current flowing through 1 turn produces 1 line of force. If the current I is expressed in amperes, then $L = \frac{10\,\Phi\,T}{I}$. The C. G. S. unit of inductance, in which L has so far been expressed, is too small for use as a practical unit, hence the *henry* has been adopted as the practical unit of inductance; it equals 1,000,000,000, or 10^9, C. G. S. units.

Hence, $$L = \frac{\Phi\,T}{10^8 I} \qquad (7)$$

in which L = the inductance of a coil or circuit, in henrys;
I = the current flowing through it in amperes;
T = the number of turns;
Φ = the total number of lines of force.

For a circuit to have an inductance of 1 henry, 100,000,000, or 10^8, lines of force must be produced in the magnetic circuit when 1 ampere is produced through the electric circuit.

The last formula is true for any coil in which Φ is the total number of lines when the current is I amperes. For a coil containing no magnetic material such as iron, L is entirely independent of the current I, because Φ is directly proportional to I; that is, Φ is doubled when I is doubled. When the coil contains an iron core, the lines of force do not always increase in direct proportion to the magnetizing force or to the ampere-turns in the coil. However, by representing the actual total number of lines of force set up by a current of I amperes by Φ, as we have done above, the formula given holds for coils with or without iron surrounding them.

EXAMPLE.—A coil of wire containing no iron has an inductance of .025 henry and 1,600 turns. What will be the total number of lines of force through a mean turn when 25 amperes is flowing through the coil?

SOLUTION.—By solving the formula $L = \frac{\Phi\,T}{10^8\,I}$ for Φ, we obtain $\Phi = \frac{L\,I\,10^8}{T} = \frac{.025 \times 25 \times 10^8}{1,600} = 39,060$ lines of force. Ans.

29. Inductance of a Solenoid.—The inductance of a coil in the neighborhood of which there is no magnetic substance has been given by the formula $L = \frac{\Phi T}{10^8 I}$, in which ΦT represents the flux turns; that is, the flux through the area enclosed by a mean turn multiplied by the number of turns in the coil. In a previous section it was shown that the number of lines of force per square centimeter produced inside a coil of T turns by a current of I amperes is $\mathcal{H} = \frac{4 \pi I T}{10 l'}$. If the area of the opening through the coil is A' square centimeters, then the total flux across this area is $A' \mathcal{H} = \frac{4 \pi I T A'}{10 l'}$ when there is no magnetic substance around or inside the coil. Substituting this value of $A' \mathcal{H}$ for the flux Φ in the formula $L = \frac{\Phi T}{10^8 I}$, we get

$$L = \frac{4 \pi T^2 A'}{10^9 l'} \qquad (8)$$

For a cylindrical coil the mean area is $\pi (r')^2$, hence the last formula reduces to $L = \frac{4 \pi^2 (r')^2 T^2}{10^9 l'}$; or,

$$L = \frac{.00000003948 (r')^2 T^2}{l'} \qquad (9)$$

in which L = the inductance, in henrys;
T = the total number of turns;
r' = the mean radius of the coil, in centimeters;
l' = the length of the coil, in centimeters.

If r is the mean radius of the coil, in inches, and l its length, in inches, then the inductance of the coil, in henrys $= \frac{4 \pi^2 (2.54 r)^2 T^2}{10^9 \times 2.54 \times l}$, or

$$L = \frac{.00000010028 r^2 T^2}{l} \qquad (10)$$

The last two formulas are strictly true only for a long coil or solenoid in which the length is 20 or more times the diameter and the total number of layers, compared to the radius, occupies very little space in depth or thickness. However, it may be used to determine approximately the inductance of any ordinary solenoid containing no magnetic material.

If the solenoid contains iron, the inductance will be μ times as great, because the flux produced will be μ times as great. Hence, for a coil having a magnetic core, the inductance $= \frac{4\pi T^2 A' \mu}{10^9 l'}$. But $\frac{A' \mu}{l'} = \frac{1}{\mathcal{R}}$, therefore the inductance, in henrys, is

$$L = \frac{4\pi T^2}{10^9 \mathcal{R}} \qquad (11)$$

in which $\mathcal{R}$ is the reluctance of the magnetic circuit. In order to determine either μ or $\mathcal{R}$, we must know $\mathcal{H}$ the magnetizing force, or $\mathcal{B}$ the magnetic density and also have a curve showing the magnetic quality of the iron; or else we must know both $\mathcal{H}$ and $\mathcal{B}$, because μ and $\mathcal{R}$ both depend on the kind of iron and the degree to which it is magnetized.

30. It will be evident from the formula $L = \frac{\Phi T}{10^8 I}$ that the inductance L may be increased by increasing T, the number of turns, or by increasing Φ, the total number of lines of force set up through the coil by a given current.

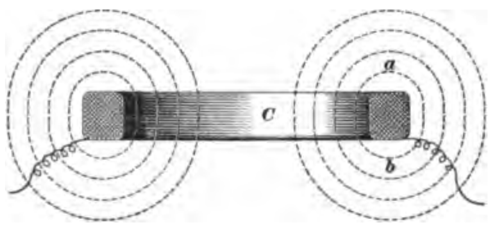

FIG. 33

The number of turns may be readily increased by winding more wire on a coil, in order to do which where a limited amount of space is available, it is usually necessary to wind with a smaller size of wire. The number of lines of force set up through a coil depends not only on the strength of the current but also on the character of the magnetic substance in and around the coil. A coil having no iron in or around it, as shown in Fig. 33, will have a very much lower inductance than a similar coil surrounded with iron, as shown in Fig. 34, for the reason that the number of lines of force set up by a given magnetizing force is much greater in iron than in air. We may say, therefore, that a large amount of iron in the core of a coil serves to greatly increase the inductance, and where the return path for the lines of force is also made of iron, so as to practically surround the coil with iron, as shown in Fig. 34, the inductance is still further increased, because the entire magnetic circuit is made of iron.

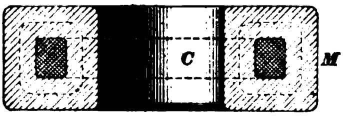

FIG. 34

The inductance L for a given coil is a constant quantity as long as the magnetic permeability of the material surrounding the coil does not change; this is the case where the coil is surrounded by air. Where iron is present, the inductance L is practically constant, provided the magnetism is not forced too high. In most cases arising in practice, the inductance L may be considered to be a constant quantity, just as the resistance R is usually considered constant although it varies with the temperature.

31. Electromotive Force of Self-Induction.—If a current be sent through such a coil as C, Fig. 33, lines of force will be set up as shown by the dotted lines. As long as the current remains steady, these lines will not change and the current will flow through the coil just as if it were an ordinary resistance; i. e., the current will follow Ohm's law; and if the voltage applied to the terminals is E volts and the resistance R ohms, the current will be determined by the relation $I = \frac{E}{R}$. If, however, the current is made to vary in any way, the number of lines of force threading the coil also varies, and hence an electromotive force is set up in the coil. This electromotive force, which is called the **electromotive force of self-induction,** tends to oppose any change in the current. Consequently, if a current varying in strength flows through a circuit that can set up lines of force so as to thread through the circuit, a counter electromotive force of self-induction is set up, and the current, while changing in strength, cannot be calculated by Ohm's law, since the effect of the self-induction is to apparently increase the resistance of the circuit. There are no self-induction effects present in circuits through which a steady current is flowing, and hence no induced electromotive forces are set up.

Circuits containing resistance can be made that have practically no self-induction; these are known as *non-inductive resistances.* Such circuits behave the same with regard to variable or alternating currents as to direct, i. e., the current flowing in them can be calculated by Ohm's law. Water resistances and incandescent lamps are practically non-inductive.

32. Extra Current.—When a circuit, especially if it contains a coil possessing inductance in addition to a battery or other source of continuous current, and a resistance is suddenly closed, the current does not instantly reach its steady final strength, which is equal to $\frac{E}{R}$, in fact an appreciable time elapses during which the strength of the current is increasing. Similarly, when a circuit through which a current equal to $\frac{E}{R}$ is flowing is opened, the current does not drop instantly to zero, but an appreciable time elapses during which the current is decreasing in strength. To be sure the time is usually very short, perhaps only a very small fraction of a second. During this time a variable current of very brief duration opposes or assists the main current. In the first case it tends to flow in the opposite direction, and in the second case in the same direction as the current from the battery. This brief current is called the **extra current**

due to self-induction. When the circuit is closed no extra current is, of course, perceptible, only the diminution in the strength of the current being apparent. When the circuit is broken the current is momentarily increased in strength, sometimes being greater even than the main current, and lasts longer than it would if the circuit possessed no inductance. This momentary increase in the strength of the current is the true extra current. If a current is changing in strength at a given instant at the rate of $\frac{i}{t}$ amperes per second in a circuit whose inductance is L henrys and resistance R ohms, the electromotive force of self-induction, in volts, is equal to $L \times \frac{i}{t}$ and the extra current is equal to $\frac{L}{R} \times \frac{i}{t}$ amperes; i being the change in the strength of the current during a short time t. For instance, if the current is changing at the rate of 500 amperes per second, the electromotive force of self-induction developed in a circuit whose inductance is .02 henry is equal to $.02 \times 500 = 10$ volts.

The ratio $\frac{L}{R}$ is known as the **time constant** of the circuit because it is a measure of the speed with which the current in that circuit increases or decreases in strength. In order for a current to change rapidly in strength, the ratio $\frac{L}{R}$ must be small. The time constant of a circuit may evidently be decreased by either decreasing its inductance or increasing its resistance, or by simultaneously decreasing its inductance and increasing its resistance. In other words, the time constant may be increased by increasing the value of the ratio $\frac{L}{R}$.

Electromagnets, such as telegraph relays, that should act quickly, should be designed so as to have a small time constant. On the other hand the tendency of a magnet, due to its self-induction, to spark when the circuit is broken is taken advantage of in the spark coils used for lighting gas.

Such a coil usually consists of a single winding connected directly in series in the circuit, the coil being so designed as to make the ratio $\frac{L}{R}$ very large. By suddenly breaking the circuit containing such a coil and about four primary cells, the spark produced is sufficiently intense to light gas.

EXAMPLE.—Find the inductance, in henrys, of a cylindrical coil 10 inches long, that has a mean diameter of 2 inches and contains 1,600 turns.

SOLUTION.—Since the diameter is 2 in. the radius will be 1 in., then by substituting in the formula $L = \frac{.00000010028\, r^2\, T^2}{l}$, we obtain

$$L = \frac{.00000010028 \times 1^2 \times 1,600^2}{10} = .02567 \text{ henry} \quad \text{Ans.}$$

EXAMPLE FOR PRACTICE

A solenoid 200 centimeters long consists of 2,500 turns, wound in a single layer, on a non-inductive core. The mean diameter of the coil is 3.6 centimeters. (*a*) What is the inductance, in henrys, of the solenoid? (*b*) If the core of this coil is made of magnetic material, what will be the inductance, in henrys, when 4 amperes flow through the coil, thereby causing the iron to have a permeability of 250?

Ans. { (*a*) .1599 henry
(*b*) 39.97 henrys }

MUTUAL INDUCTANCE

33. In mutual induction, two separate coiled conductors are placed near each other, so that the magnetic circuit produced by one, which carries a current of electricity, will be enclosed more or less by the other, as shown in Fig. 35. The coil *P*, around which the current is flowing, is called the *primary*, or *exciting*, *coil*; the other *S* is called the *secondary coil*. Any change in the strength of the current flowing around the primary coil will produce a change in

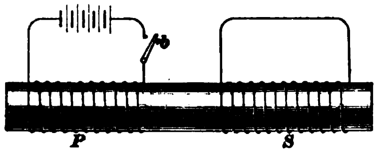

FIG. 35

the number of lines of force in the magnetic circuit, and, consequently, an electromotive force will be induced in the secondary coil. If the current in the primary coil is increasing, the electromotive force induced in the secondary coil will cause a current to flow around in the opposite direction to the current in the primary coil. If the current in the primary coil is decreasing, the induced electromotive force in the secondary coil will cause a current to flow around in the same direction as the current in the primary coil.

34. Mutual inductance, or the **coefficient of mutual induction**, as it is also called, may be defined as the product of the turns in one coil by the number of magnetic lines of force produced through it by unit current in another coil. It is, therefore, the total induction produced in one coil by unit current in another coil. It depends on the form, size, and relative position of the two circuits and on the magnetic permeability of the surrounding substances.

Suppose in Fig. 35 that the coil P has T turns, S has T_1 turns, and that the two coils are close together, or one over the other so that all the lines of force passing through P also pass through S; suppose further, that a current of I C. G. S. units in the coil P produces a flux through S of Φ lines of force. If there is no iron in the circuit $\frac{\Phi}{I}$ lines of force are produced per unit C. G. S. current. If the lines of force per unit current cut each of the T_1 turns in the coil S, then the whole coil S cuts $\frac{\Phi T_1}{I}$ lines of force and $M = \frac{\Phi T_1}{I}$ in which M is the mutual inductance in C. G. S. units.

The practical unit of mutual inductance is the same as the practical unit of self-inductance, namely the henry, which is equal to 1,000,000,000, or 10^9, C. G. S. units. In order that the last expression may give M in henrys, it is necessary to divide by 10^9, which gives

$$M = \frac{\Phi T_1}{10^8 I} \qquad \textbf{(12)}$$

where M = the mutual inductance in henrys;
I = the current in amperes in the primary coil;
Φ = the total flux through the secondary coil having T_1 turns.

The mutual inductance M, in henrys, may be defined as the number of times that 100,000,000 lines of force are cut by T_1 turns in the secondary coil when 1 ampere is turned on or off in the primary coil. The last formula is only approximately true, because it assumes that all the lines of force produced by the primary coil pass through the secondary coil; and this is rarely the case on account of magnetic leakage.

The induction produced in the secondary when a current of I amperes are turned on or off in the primary coil, that is the total cutting of lines of force by the secondary coil, is of course I times as great as for a current of 1 ampere. Furthermore, if the core, instead of being made of non-magnetic material is made of magnetic material whose permeability for a given magnetizing force is μ, then the total induction will be μ times as great, because Φ will be μ times greater with the magnetic than with the non-magnetic core.

INDUCTION COILS

35. An **induction coil** is an apparatus depending on the principle of mutual induction for producing pulsating currents of electricity of high electromotive force. Induction coils are sometimes called Ruhmkorff coils, from the name of a celebrated manufacturer of them, and consist, essentially, of two coils, primary and secondary, wound around a core consisting of a bundle of iron wires. In Fig. 36 the secondary coil S is composed of a large number of turns of fine insulated wire, while the primary coil P contains only a few turns of thick insulated wire. Both coils are wound on a spool O of insulating material fitting over the iron core C.

The primary circuit is automatically opened and closed at *D* in the following manner: *F* represents a spring that tends to keep the circuit closed between the platinum contact piece *D* attached to the spring *F* and the contact screw *K*. As soon, however, as the circuit is closed by the action of the spring, the current from the battery *B* begins to circulate through the primary coil *P* around the core *C*, thereby magnetizing the core and causing it to attract the

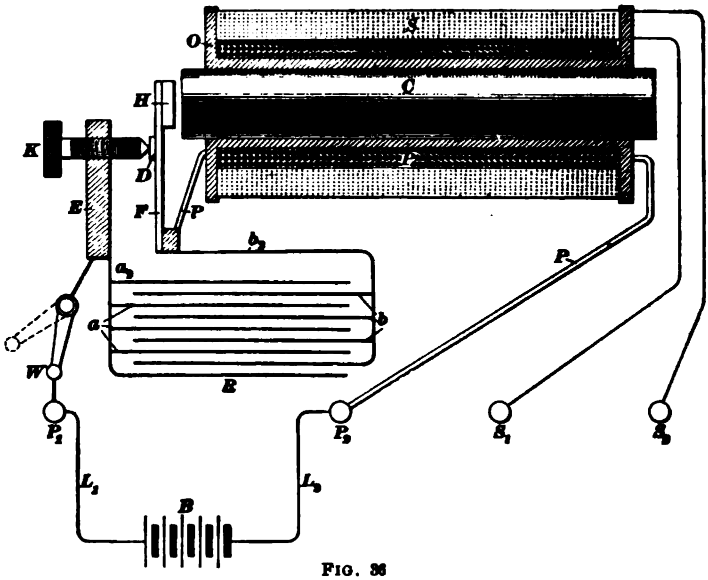

FIG. 36

iron armature *H*, thus breaking the primary circuit between *D* and the point of the adjustable screw *K*. On opening the circuit between *D* and *K*, the magnetism in the core begins to weaken, the spring once more closes the circuit, and the entire operation is again repeated. These actions take place in rapid succession, a large number of times a second, constantly producing a change in the number of lines of force passing through the core, and thereby inducing a current in the secondary coil. A switch *W* connects the post *E* with the terminal P_1, and the terminals P_1, P_2 are joined to the battery *B* by means of wires L_1, L_2. The terminals S_1, S_2 of the secondary coil are not, for the present, connected with each other; the secondary coil is therefore open.

A condenser R is connected across the break in order to overcome a difficulty encountered in the use of a simple make and break, as here shown at D, due to the fact that without a condenser the iron core does not lose much of its magnetism at break, and what it does lose it loses slowly. When the primary circuit is opened, its self-induction tends to keep the current flowing across the break; but when a condenser is used, this current flows into and charges the condenser instead of producing a spark across the break. After the current has been expended in charging the condenser, the latter immediately discharges through the circuit a_2–E–W–P_1–L_1–B–L_2–P_2–P primary coil P–P–b_2 and back to the condenser R. This current, being in the opposite direction to the current due to the battery B, demagnetizes the iron core with great rapidity and thus produces an enormous electromotive force in the secondary coil. The electromotive force of self-induction, to which the charging of the condenser is due, is sufficiently great to not only produce a discharge that overcomes the opposing electromotive force of the battery but it also overcomes the resistance of the battery and primary coil and produces therein an appreciable current. The spark at D is very much less with this condenser than it would be without it. By trial a condenser of proper capacity may be selected so that it is possible to practically interrupt the current and reduce the magnetism of the core to zero almost instantaneously, thereby producing the maximum induction in the secondary coil with a given current in the primary coil.

When the current is made in the primary coil its self-induction compels a rather gradual increase in the current strength in the primary coil, and consequently, the induced electromotive force in the secondary is comparatively small. When the current in the primary is broken, however, the current not only almost instantly decreases from its maximum value to zero, but it is quickly followed by the reverse current from the condenser; consequently there is produced a very intense electromotive force in the secondary winding. Hence, the tendency is to induce a very much greater current in the secondary winding in one direction than in the other. In most induction coils a spark gap in the circuit of the secondary winding gives this winding a very high resistance; consequently the electromotive force induced in the secondary coil, when the primary current is made, may be too weak to produce a spark, that is a current, across the air gap. Hence there may be no current in the secondary winding when the current in the primary is made. However, when the current in the primary is broken the electromotive force induced in the secondary is usually sufficient to force a current across the air gap. As a result a current may be produced practically in one direction only in the secondary winding. There is of course always a tendency to produce a current in both directions and doubtless there is a current in both directions in many cases.

For the operation of Roentgen ray tubes it is very desirable to prevent, as far as possible, a current flowing in both directions, because the so-called reverse current tends to disintegrate and blacken one of the electrodes of the Roentgen ray tube, which it is essential to preserve in first-class condition.

36. Reduction of Voltage in Secondary.—The voltage across the terminals of the secondary coil suffers a very great reduction when a current is allowed to circulate in this circuit. If this voltage is 300 when no current is flowing, the insertion of a comparatively low resistance across the secondary terminals may reduce it to 2 or 3 volts; the insertion of even 1,000 ohms in the secondary of a small coil may reduce the difference of potential across the secondary terminals to 10 or 15 volts. With large coils, these values will, of course, be greater. This reduction in voltage is caused partly by the reaction of the secondary current on the primary current, but principally by the self-induction in the secondary circuit itself. At the time when the induced electromotive force is at its maximum, and when a heavy current might flow, it is most effectually stopped by the powerful choking action of the coil, which does not permit a very sudden rise or fall of pressure. In addition to these inductive reactions, the difference of potential is reduced by the drop or loss of potential, through the resistance of the secondary coil, which is composed of a very long, thin wire. The secondary voltage, after it is subjected to all these influences, is not powerful enough to produce a very large current. If the resistance external to the secondary coil is very large, this resistance helps appreciably to limit the strength of the secondary current; whereas if it is small, the inductive reactions in the secondary coil limits the strength of the secondary current.

37. Relation Between Electromotive Forces in Primary and Secondary Coils.—Another point that may be difficult to understand is the relation between the pressures in the primary and secondary coils. If the coils *P* and *S*, Fig. 36, were made of wire of the same diameter and length, then the pressure and strength of the currents in both coils would practically be the same, not considering a certain loss depending on the efficiency of the combination. The more turns that the secondary coil contains, the more will it be exposed to the effects of the inductive influences of the primary coil. The electromotive force developed in the secondary coil would, up to a certain limit, increase in direct proportion to the increased number of turns that is wound in the same space on the bobbin. But as the total power in watts developed in the secondary coil cannot be increased without increasing the power supplied to the primary, it follows, as a consequence, that an increase in pressure can only be obtained at the expense of a decrease in current strength. For instance, a power of 50 watts may be developed in the secondary coil by a current of 5 amperes at 10 volts, .5 ampere at 100 volts, .05 ampere at 1,000 volts, or .005 ampere at 10,000 volts. The pressure in the secondary may be increased to from 200,000 to 300,000 volts, if there is a corresponding decrease in current.

Increasing the sectional area and decreasing the length of the wire in the secondary coil will decrease its voltage and increase its amperage, so that it is not only possible to increase the pressure of the current in the secondary coil over that in the primary, but also to decrease the pressure in the secondary coil with a corresponding increase, to a limited extent, in its current strength.

38. Comparisons Between Various Coils.—When coils are mentioned it is customary to simply state their sparking distance, say 8, 10, 12 inches, or whatever the same may be. This gives no idea whatever of the real power of the coil, any more than when speaking of a waterfall we would say that it is about 50 feet high. How much water per minute really flows is left to the imagination so that an estimate of its horsepower is impossible. The same argument applies to the rating of induction coils. It is necessary to know not only the pressure indicated by the sparking distance, but also the current volume. Two coils may be made to give exactly the same length of spark, but the sparks may be of very different nature. In one case it may be thick and intensely white, in the other thin and bluish. The former coil is the more powerful of the two and the more expensive to build. To send this increased current strength through the secondary winding, the primary and secondary coils must both be made of heavier wire, thereby increasing the expense both for copper and labor.

As the current in the secondary is not continuous, but interrupted, the current strength will depend on the volume of current sent through the spark gap at each interruption. Here, again, a deception is possible. If two coils send sparks of the same volume over the same air gap, they would be identical if this were done at similar intervals. But if one does this at double the number of interruptions per second, then evidently, double the quantity of current will flow across the air gap. Some coils can be made to show a high efficiency by slowing down the vibrator; but as the power of the coil, or the number of watts, depends on the product of amperes and volts, it is important to consider the relative strength of the current at each discharge, and the frequency, as well as the electromotive force.

Some small medical coils have a copper tube that slides over the iron core. This shields the iron core because the variable currents in the primary induce relatively large currents directly in this copper tube, which has a very low resistance. These induced currents oppose the primary currents; hence the resultant effect on the iron core is much reduced. Consequently the secondary cuts fewer lines of force. As this copper tube is gradually withdrawn, the iron core becomes more highly magnetized and the electromotive force induced in the secondary increases.

39. A straight iron core is always used in induction coils, for when the current in the primary is broken, the magnetic flux falls from its maximum value, not to zero, but to a value known as the residual magnetism. The residual magnetism in an open magnetic circuit is much less than in a closed magnetic circuit, so when the primary current is suddenly reduced to zero, the magnetism drops lower in an open magnetic circuit than in a closed one. As the electromotive force in the secondary is proportional to the reduction in the magnetic flux, it is greater with a straight core than with a complete circuit of iron.

An iron core is best made by filling an iron pipe of suitable length and diameter as full as possible with iron wire. A No. 24 B. & S. gauge iron wire is the most suitable, though No. 18 and No. 20 give satisfactory results and are more often used. Then fill the intervening space with clay and leave it in a coke or coal fire until the fire naturally goes out, in order to allow the wire to cool slowly, in order that it will be properly annealed. Each wire should then be sandpapered, to remove the excess oxide, dipped in boiling water, wiped dry, and coated with thin shellac while warm. The wires may then be bound together with paper and stout cord into a round bundle. If one end of the core is to be used to operate the circuit breaking device, it should be filed smooth, the other end being left rough.

Some of the above remarks concerning the iron core, as well as Table I, were given in the "Scientific American Supplement," November 15, 1902.

TABLE I

INDUCTION-COIL DATA

Length of Spark Gap Inches	Length of Core Inches	Diameter of Core Inches	Primary Wire B. & S. Gauge	Number of Layers in Primary Coil	Secondary Wire B. & S. Gauge	Secondary Wire Pounds	Condenser		Voltage of Battery
							Number of Sheets	Area of Each Sheet Square Inches	
1/8	3	1/4	23	2	36	1/8	25	2 × 1	2
1/4	3 3/4	3/8	23	2	36	1/4	40	2 × 1 1/8	2
3/8	4 3/4	1/2	22	2	36	3/4	45	4 × 2	4
1/2	6	1/2	19	2	36	5/8	50	4 × 2	4
3/4	7	3/4	16	2	36	3/4	60	4 × 4	6
1	8	1	16	2	34	1	100	7 × 5	12
2	10	1 1/8	14	2	34	3	100	9 × 7	12
3	11 3/4	1 1/4	14	2	33	5	150	9 × 7	12
6	14 3/4	1 1/2	12	2	33	10	200	9 × 9	16
12	19	1 5/8	10	3	33	12	60	12 × 8	10

It is beyond the scope of this course to discuss the design of induction coils; moreover, there are no exact rules or formulas by following which one may be made. They are constructed more from experience than from any predetermined calculations or designs.

A REPRESENTATIVE INDUCTION COIL

40. In Fig. 37 is shown an example of a large induction coil suitable for use in wireless telegraphy, Roentgen ray work, and electrotherapeutics. The iron core projects beyond each end of the core, because it has been found that this reduces the leakage of the lines of force and thereby increases the efficiency of the coil. This induction coil is wound so that it may be operated by from 3 to 9 storage-battery cells; that is, by from 6 to 18 volts. When it is desirable to operate the coil from a 110-volt circuit, a resistance, called a *rheostat*, must be inserted in series with the

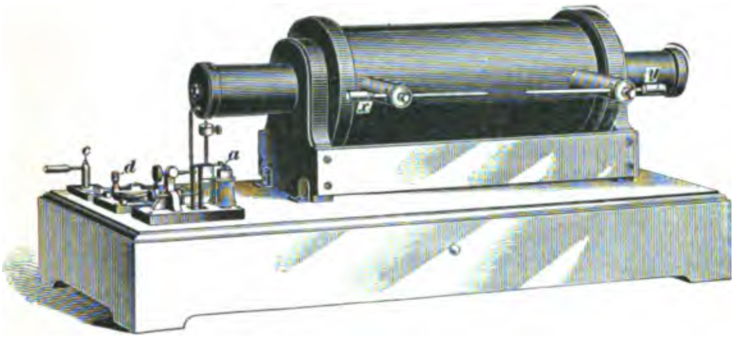

FIG. 37

primary winding in order to regulate the strength of the current in the primary coil. The two discharge rods *x* and *y* may be adjusted until the sparking points are the desired distance apart.

41. Independent Spring Interrupter.—A variation of the strength of the current in the primary of any induction coil will vary the rate of interruptions, if the interrupter is operated by the core of the induction coil. It is desirable that the rate of interruptions shall not only be independent of the current strength in the primary coil, but that the rate of interruptions should be easily regulated. Hence it is customary with the larger and better coils to operate the interrupter by connecting it in an independent circuit, either in parallel with the main primary coil or in circuit with a separate battery.

A separate enlarged view of the independent spring interrupter *a*, which sets upon the base of the large coil (see Fig. 37) is shown in Fig. 38. This interrupter consists of a small electromagnet *w* with two pole pieces *f*, *f* between which the armature *h*, which is fastened to the vertical spring *r*, can freely vibrate without coming against or even touching them. The flat springs *r*, *v* are fastened in the base; the spring *r* has a slight projection to hold the platinum contact piece to one side opposite the end of the screw *n* in order that the latter may clear the spring *v*.

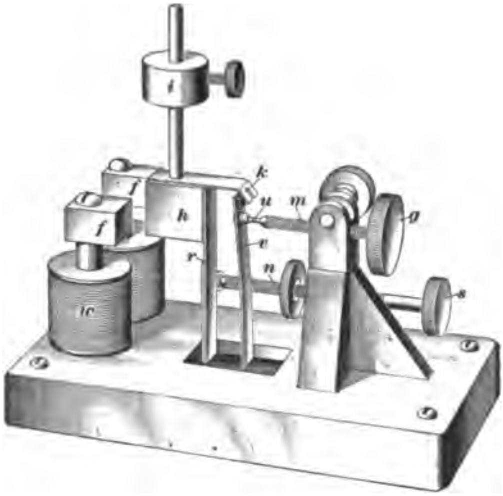

Fig. 38

42. Connections.— A diagram of connections of the induction coil, interrupter, and interlocking switch is shown in Fig. 39, which is lettered like Fig. 38 as far as practicable. The interlocking switch shown at *d*, Fig. 37, and the later one shown at the right of Fig. 39, make it impossible to send current through the primary winding of the induction coil before the interrupter is in operation. This is of importance, for if the coil is operated from a storage battery or electric-light circuit, it would be possible, if the interrupter were not first placed in operation, to send a larger current through the primary coil than it could safely stand and thus perhaps ruin it.

The metal blade *t*, which is pivoted on an extension of the plate *d*, makes contact with the metal piece *c* only when the handle *o* is raised sufficiently. After the blade *t* enters the jaw, or slot, in *c*, the handle, and with it the metal pieces *e*, *d* and the two switch blades *a*, *b* may be moved to either side. The blade *a* is mechanically fastened to, but electrically insulated from, the plate *d*, whereas the blade *b* is both mechanically fastened and electrically connected to the plate *d*.

The coils of the interrupter are supplied with current from the same source as the primary winding of the large coil. One terminal of the coils of the interrupter is connected (see Fig. 39) to the negative terminal of the battery *B*,

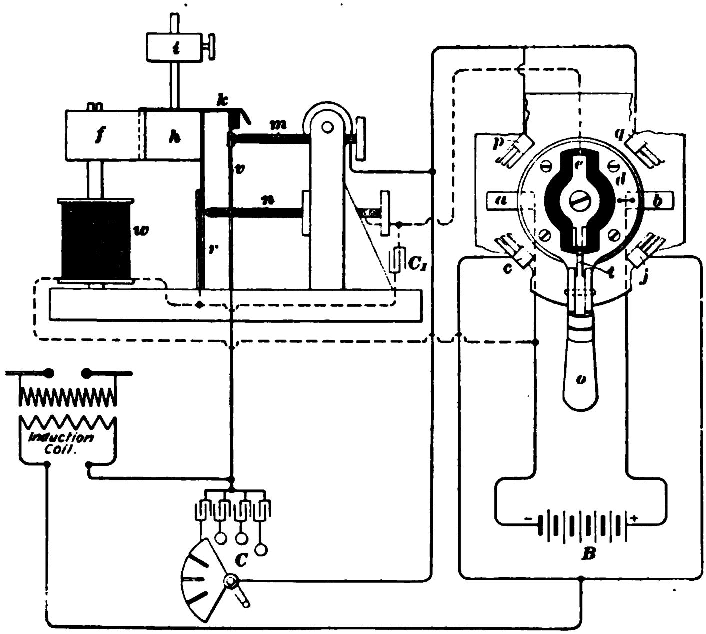

FIG. 39

the other terminal to the spring *r* and through the contact, the adjustable screw *n*, the metal piece *e*, and when the handle *o* is raised sufficiently, through the blade *t*, plate *d*, and switch blade *b* to the positive terminal of the battery *B*. When the interrupter, or vibrator, as it is also called, is at rest and the current is turned on, the armature is attracted and the circuit containing the coils *w* of the interrupter is broken between *r* and *n*; the spring *r* and armature *h* then spring back to their normal position. This action is repeated and produces a very uniform vibration of the spring and armature. To the vibrating spring *r* is secured a small vertical rod, on which slides a small weight *i*; the rate of vibration of the interrupter may be varied within suitable limits by raising or lowering the weight *i*, by removing it, or by increasing its weight. The frequency of a good spring vibrator varies from about 150 to 300 interruptions per second.

The current for the primary of the large induction coil is interrupted between platinum contact pieces, one of which is fastened to the spring *v* and the other to the end of the adjustable screw *m*. When the armature *h* is attracted by the pole pieces *f*, *f*, the hook *k* pulls the spring *v* so that *v* is separated from *m* a moment after *r* is separated from *n*. Thus, the currents through both circuits are interrupted through separate contacts. The contact surface between *v* and *m* must be larger than that between *r* and *n* since the former breaks a much larger current. The arm *k* has a piece of hard rubber, or other good insulating material, under the hooked end so that the springs *v*, *r* are always insulated from each other. The armature *h*, springs *r*, *v*, and screws *n*, *m* are so arranged and adjusted that *k* only comes in contact with *v* when, at full speed, the armature and arm *k* are drawn toward the magnet, and then the main current is only broken momentarily. The ratio between the make and break can be varied by screwing *m* farther in or out, since the farther it is screwed in, the longer it will be before the arm *k* reaches the spring and breaks the contact. It has been found that the longer the make is, or the period during which the spring *v* is in contact with the screw *m*, in comparison to the time they are separated, or the break, the greater will be the volume of the induced secondary current; that is, the volume of the secondary current is increased by increasing the ratio of the make to the break within a reasonable limit, and provided the sharpness of the break is not decreased

43. Interlocking Switch.— The connections of the interlocking switch are about as shown at the right side of Fig. 39. The brass contact blades *a*, *b* are fastened in a hard-rubber disk and are connected to the battery terminals through rubbing contacts under the disk. This could not very well be shown in this figure. By pulling up the handle *o* of the switch, the interrupter circuit is closed between *e* and *b* and the interrupter commences to vibrate. Then by turning the handle *o* to the right or left the knife blades *a*, *b* make contact with the jaws *p*, *j*, or with *c*, *q*, and the induction coil is put into operation. The switch is made to rotate in either direction so that the current through the primary of the induction coil may be readily reversed. A slot under and into which a piece projecting downwards from the handle enters prevents the turning of the switch and hence prevents the closing of the primary circuit of the induction coil before the interrupter circuit is closed between *e* and *t*; consequently, the primary circuit of the induction coil cannot be closed until the interrupter, which starts very quickly, is in full operation. For use with different sizes of coils this interrupter is made in slightly different forms and sizes, but the connections and operation are practically the same as shown here.

44. Condensers.— There are two condensers, one C across the break in the primary circuit of the induction coil and a smaller one C_1 across the break in the interrupter-coil circuit. The capacity of the condenser C can be readily varied within certain limits by the switch shown at *c*, Fig. 37, and C, Fig. 39. The condensers are placed in the base of the apparatus. It is not necessary that C_1 should be an adjustable condenser.

ELECTRIC WAVES

OSCILLATORY-CURRENT WAVES

45. The action of **oscillatory-current waves** in conductors may be explained by a mechanical analogy. Suppose that we have a pendulum suspended so that the bob is immersed in a heavy viscous liquid, like molasses. If the pendulum bob is pulled to one side and let go, it will slowly return to its normal central position, without vibrating or even once passing its central position. So it is with a current produced by discharging a charged condenser through a circuit possessing only a high resistance. The current rises to its maximum value almost instantly when the charged condenser terminals are joined by the high resistance, and then it gradually dies away.

Suppose that we now suspend the pendulum in air. If the pendulum bob is pulled to one side and let go, it will continue to vibrate for a long time. However, the friction at the point of suspension and between the pendulum and the air will eventually bring the pendulum to rest, each vibration being a little smaller than the preceding one. But as a matter of fact, the pendulum requires exactly the same time to make a small vibration as a large vibration, because its velocity decreases at such a rate that the time for each complete vibration remains the same, although the amplitude, or the distance the pendulum swings to one side of the center position, gradually diminishes. So it is with a current produced by discharging a charged condenser through a circuit containing sufficient inductance as well as resistance. As the current decreases in value an electromotive force of self-induction is produced that tends to keep the current flowing, hence the condenser is not only completely discharged, but it is also charged in the opposite direction, but to a lesser degree. A reverse discharge from the condenser then follows. As before, the condenser is discharged and then charged, the last charge being in the same direction as the original one. Thus the charges surge back and forth, gradually decreasing in strength, but the time for one complete cycle is the same throughout. The curve in Fig. 40 represents the

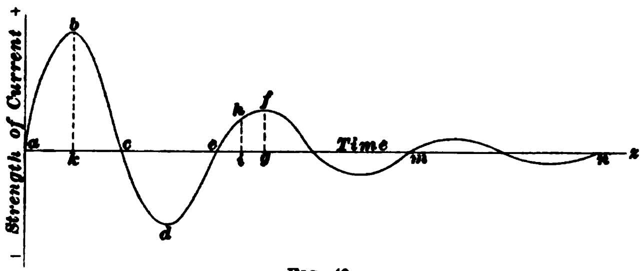

FIG. 40

manner in which the discharge current varies in strength. It first increases from *a* to *b*, then it dies away to zero at *c*, then it flows in the opposite direction, increasing in strength to *d*, and decreasing to zero at *e*. It has now passed (from *a* to *e*), through one complete cycle of changes. Distances along *a z* represent time and distances of points on the curve; above or below *a z* represent the relative strength of the current; values above *a z* represent the strength of the current in one direction, and values below represent the strength of the current in the opposite direction. If currents represented as above the line are called positive currents, then those below are negative currents, which merely means that the latter flow in an opposite direction to the former. Either direction of the current in the circuit may be considered as the positive direction. For instance, *i h* represents the strength of a positive current at the instant *i*, which is *a i* seconds later than *a*. The extreme vertical distances above or below *a z* are called the amplitudes of the various current discharges, or oscillations. For instance, *k b* is the amplitude of the curve after the time *a k* has elapsed, and *g f* is the amplitude of the curve after the time *a g* has elapsed since the current started at *a*. This particular amplitude corresponds to the maximum distance from the center position of the third swing of the pendulum. Since the time for each complete cycle is the same, the distance *a e* equals the distance *e m*, and *e m* equals *m n*; that is, *a e* is equal to the horizontal distance between any two similarly located points. The curve *a b c d e* represents one complete cycle.

If a condenser is made to produce a spark across an air gap in circuit with it, there are generally produced hundreds of millions of oscillations per second, but on account of this great rapidity they appear as a single spark. But their existence has been mathematically and experimentally proved, as well as the fact that each spark is an oscillatory current of about the form represented in Fig. 40.

WAVES THROUGH DIELECTRIC MEDIUMS

HERTZ'S EXPERIMENTS

46. A famous experiment of Hertz is illustrated by Fig. 41, where *I* represents an induction coil, *c* two small metallic balls separated by an air gap and connected to metal plates *a*, *b*, and to the secondary winding of the induction coil. The system *a c b* is called the *oscillator* and is given the proper electrostatic capacity by the plates *a*, *b*. *E* is a circular conductor about 28 inches in diameter, and placed as shown. The two balls at *d* are separated by an air gap whose length can be very minutely adjusted. If the capacity and inductance of the oscillator *a c b* and the receiving device *E* are carefully adjusted, sparks can be produced between the balls at *d* when sparks are produced at *c* by closing the key *k*. As there is no electrical connection whatever between the oscillator and the receiver, and the two are separated several feet, it has been universally accepted, as stated by Hertz, that the action is due to electromagnetic waves sent out by the currents that oscillated back and forth in *a c b* at the rate of about 100,000,000 cycles per second. The succession of waves sent out by Hertz's oscillator, about 12 complete cycles for each spark, induce in the receiver E electric oscillations of the same frequency as those in the oscillator. The effect of successive cycles on the receiver is cumulative, producing in it oscillatory currents that are in sympathy with those in the oscillator; just as one tuning fork may be made to vibrate in sympathy with another tuning fork of exactly the same pitch, when the latter is set vibrating by a blow and held in the proper

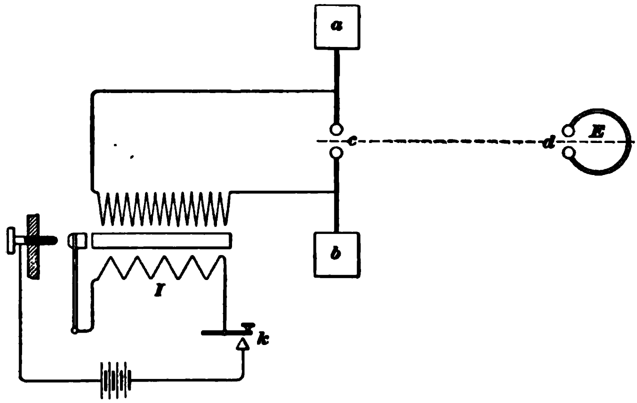

FIG. 41

position relative to the first fork. Each wave striking the resonator will tend to slightly increase the intensity of the electromotive force induced by the preceding wave, just as small pushes given to a pendulum at the proper times will make it swing violently. Hence the electromotive force in the resonator will increase until it is strong enough to produce a minute spark across d, the spark itself consisting of an oscillating current of exactly the same frequency as that in the oscillator, but of less intensity.

The receiver E is said to be in tune with the electromagnetic waves, or with the oscillator that emits the waves. The oscillator and receiver are said to be in tune, resonance, or syntony with each other. Hence the receiver, when in resonance with the oscillator, is often called an electric resonator.

ELECTROMAGNETIC WAVES

47. As already stated, a spark discharge is usually an oscillatory current, and hence produces an oscillatory magnetic field around the spark, and around the conductors connected on each side of the spark gap. This magnetic field increases in strength as the current increases, and decreases as the current decreases. Consequently, the magnetic field has the same frequency as the oscillating current, and is proportional to it in strength. Such rapid changes in the magnetic field surrounding the oscillating current produce disturbances in space that are supposed to travel as waves through space. These **electromagnetic waves,** which are also called **Hertzian waves,** after Hertz, of Germany, who first experimentally proved their existence, may be produced with such energy as to travel long distances. These are generally supposed to be the waves that produce wireless telegraph signals. It has also been proved that electromagnetic waves travel through space with the same velocity as light (although they have a different frequency of vibration), and that like light waves they may be reflected and refracted by certain substances, such as a prism of resin. The better conductor a substance is, the more opaque it seems to be to electromagnetic waves.

48. Just before a spark passes between two conductors separated by air or other dielectric, the dielectric is electrically strained; that is, an electric disturbance or displacement is produced in the surrounding region. Moreover, it is said that about the same magnetic field is set up by this disturbance as though an electric current actually flows. When the spark does pass, an oscillating current flows and an oscillating magnetic field is set up around the path of the current as an axis. This field restores part of its energy to the circuit as the current dies away, and part is doubtless radiated into space. When the potential difference is equalized by the sudden discharge, the electric tension in the dielectric is relieved, and displacement currents, or electric waves, are said to be sent out into space. As a result of the electric and electromagnetic disturbances, whether they are distinct or are one and the same phenomenon, disturbances, in the form of waves, are sent out into space in all directions; hence the energy due to these waves that is received at various distances decreases about as the cube of the distance from the originating point, or, according to a reliable authority, as the square of twice the distance. For this reason the difficulty in signaling through space increases very rapidly as the distance is increased.

If n is the frequency, that is, the number of complete cycles per second, l the wave length, and v the velocity of propagation, then

$$v = n\,l \qquad \textbf{(13)}$$

for any system of waves. Electromagnetic waves range from $2\frac{1}{2}$ inches to 18 miles in length and have a frequency from 480,000,000 to 10,000 periods per second, respectively; whereas light waves range from 165 millionths of an inch to 272 millionths of an inch in length and have a frequency of from 740 trillions to 434 trillions of periods per second, respectively. Oscillatory currents of any frequency may persist for a long or short time; that is, there may be a very large number of oscillations or only a very few. For instance, Hertz obtained with his apparatus sparks that consisted of about 12 oscillations, each oscillation lasting less than $\frac{1}{100000000}$ of a second, while others have succeeded in obtaining 20,000 oscillations before they died out.

UNITS

49. In the tables of magnetic and electrical units that will now be given, the quantity v, which occurs in the ratio between the practical and C. G. S. electrostatic units, equals the velocity of light in air and has a value of 3×10^{10} centimeters per second. The magnetic units that would correspond to the electrical units of the practical system are not used and have not been even named on account of their inconvenient magnitudes. The gilbert and oersted were adopted by the American Institute of Electrical Engineers in 1894, but are not very generally used. Not having been even sanctioned by any international convention these two units may have their names changed at some future time. All the practical electromagnetic units named, except the mho, have been adopted by some international convention and their use legalized by most of the important nations of the world. The C. G. S. electromagnetic and electrostatic units have not been given any names.

Dr. A. E. Kennelly has suggested that the prefixes *ab-* or *abs-* be applied to the names volt, ohm, etc., to designate the corresponding absolute electromagnetic units; thus *abvolt*, *absohm*, etc., mean the absolute, or C. G. S., electromagnetic units of E. M. F., resistance, etc. Similarly the prefix *abstat-* designates the corresponding C. G. S. electrostatic units. These terms are now being used by some writers.

TABLE II

MAGNETIC UNITS

Magnetic Quantities	Symbol	Defining Equation	Names of C. G. S. Units
Strength of pole	m	$m = \sqrt{F l^2}$	Has no name
Magnetic moment	$\mathcal{M}$	$\mathcal{M} = m l$	Has no name
Intensity of magnetization	$\mathcal{I}$	$\mathcal{I} = \frac{m}{A}$	Has no name
Magnetizing force or field density	$\mathcal{H}$	$\mathcal{H} = \frac{F}{m} = \frac{m}{l^2}$	Gauss, or 1 line of force per square centimeter
Susceptibility	κ	$\kappa = \frac{\mathcal{I}}{\mathcal{H}}$	Has no name
Magnetomotive force	$\mathcal{F}$	$\mathcal{F} = \mathcal{H} l$ or $\mathcal{F} = \frac{W}{m}$	Gilbert (Not internationally accepted)
Magnetic density or magnetic induction	$\mathcal{B}$	$\mathcal{B} = 4\pi\mathcal{I} + \mathcal{H}$	Gauss, or 1 line of force per square centimeter
Magnetic flux	Φ	$\Phi = \mathcal{B} A$	Maxwell, or 1 line of force
Permeability	μ	$\mu = \frac{\mathcal{B}}{\mathcal{H}}$	Has no name
Reluctance	$\mathcal{R}$	$\mathcal{R} = \frac{\mathcal{F}}{\Phi}$ or $\mathcal{R} = \frac{l}{A \mu}$	Oersted (Not internationally accepted)

NOTE.—In this and the following table l represents a length or distance, F a force, v a velocity, T the number of turns in a coil or circuit, t time, W work, and A an area.

TABLE III

ELECTRICAL UNITS

Electrical Quantities	Symbol	Defining Equation	Names of Practical Electromagnetic Units	Quantities by which to multiply Practical Electromagnetic Units to reduce to C. G. S. Electromagnetic Units	Quantities by which to multiply Practical Electromagnetic Units to reduce to C. G. S. Electrostatic Units $v = 3 \times 10^{10}$
Current	I or i	$I = \frac{F}{l\mathcal{H}}$	Ampere	10^{-1}	$v \times 10^{-1} = 3 \times 10^{9}$
Quantity of electricity	Q or q	$Q = It$ or $Q = \sqrt{Fl^2}$	Coulomb	10^{-1}	$v \times 10^{-1} = 3 \times 10^{9}$
Electromotive force	E or e	$E = \mathcal{H} l v = \frac{\Phi}{t}$ or $E = \frac{W}{Q}$	Volt	10^{8}	$10^{8} + v = \frac{1}{3} \times 10^{-2}$
Resistance	R	$R = \frac{E}{I}$ or $R = \frac{W}{I^2 t}$	Ohm	10^{9}	$10^{9} + v^2 = \frac{1}{9} \times 10^{-11}$
Resistivity	ρ	$\rho = \frac{RA}{l}$	Ohm		
Conductance	G	$G = \frac{1}{R}$	Mho Not internationally accepted		
Conductivity	γ	$\gamma = \frac{1}{\rho}$	Mho Not internationally accepted		
Work or energy	W or J	$J = EIt$	Joule	10^{7} ergs	
Power	P	$P = EI$	Watt	10^{7} ergs per second	
Capacity	C	$C = \frac{Q}{E}$	Farad	10^{-9}	$v^2 \times 10^{-9} = 9 \times 10^{11}$
Inductivity	K	$K = \frac{Q \text{ (air as dielectric)}}{Q' \text{ (other dielectric)}}$ or $K = \frac{4\pi Cl}{A}$	A number		
Inductance (self)	L	$L = \frac{\Phi T}{I}$	Henry	10^{9}	
Inductance (mutual)	M	$M = \frac{\Phi T}{I}$	Henry	10^{9}	

CHEMISTRY AND ELECTROCHEMISTRY

THE PRINCIPLES OF CHEMISTRY

INTRODUCTORY

1. In order to understand electric batteries and their working, it is necessary to understand the rudimentary principles of chemistry. Only those facts that bear directly on the internal actions of batteries are here given, and all other theories and facts not essential to the practical engineer or electrician are purposely omitted.

2. Matter.—Matter is anything that possesses weight, that is, is acted on by gravity. It presents itself to us in three physical states, called, respectively, the *solid*, the *liquid*, and the *gaseous*.

3. Division of Matter.—Science assumes three divisions of matter—*masses*, *molecules*, and *atoms*. A **mass** is any portion of matter appreciable by the senses. A **molecule** is the smallest particle of matter into which a body can be divided; it is the smallest particle that is capable of separate existence. An **atom** is the still smaller particle produced by the division of a molecule by chemical means, and is regarded by chemists as the unit quantity of chemical combination.

4. Physical and Chemical Properties.—**Physical properties** may be described as those properties that a body possesses as a result of its molecular condition, while **chemical properties** are those that a substance or body possesses as the result of the atomic composition of its molecule. Tenacity, which signifies the amount of cohesive attraction, and color, which is the result of the action of the molecules of a body on light, are properly termed physical properties, while such properties as combustibility, explosibility, affinity, etc. are chemical properties.

5. Difference Between Chemical and Physical Changes.—It is of great importance to distinguish between what are simply alterations in the physical properties of matter and what are chemical changes. A description of the following simple experiments will greatly facilitate this object:

1. If a piece of platinum wire is heated, we find that it soon becomes white hot, and if the wire be removed from the source of heat and allowed to cool it recovers its original brightness; the heat has not altered it in the least, not even tarnished it.

2. Next a piece of bright, new iron wire is heated in the same way and allowed to cool; the wire is tarnished but has otherwise not undergone any remarkable change.

3. If a piece of magnesium wire is placed in the flame, it will burn with a peculiar, dazzling, white light, depositing a white, easily powdered substance, composed of oxygen and magnesium, known as *magnesia*.

In the first of these experiments, the change is merely physical; the hot platinum wire possessed properties very different from those it had when cold, but as soon as it cooled again it regained its original character. With the iron wire, in the second experiment a slight, but permanent, change is produced on the surface, which is of a chemical character, but the main portion of the wire is unchanged. In the third experiment, there has been a very decided chemical action; the magnesium, as such, has entirely disappeared and a new substance has been produced and has taken its place. Chemical changes are changes that take place within the molecule; they alter the character of the molecule, and thus cause a change in the character of the substance itself.

6. Mechanical Mixtures and Chemical Compounds. It is important to further distinguish between a mere mechanical mixture and a chemical combination; this distinction is most easily made plain by the study of a few typical experiments.

If powdered sulphur and finely divided copper filings are mixed together, the characteristic color of the sulphur and of the copper disappear, and, to the unaided eye, the mixture will show a uniform greenish color. By the aid of a microscope, however, the particles of copper and sulphur may be distinctly seen lying side by side; the mixture can easily be separated again and the original colors restored by washing away the lighter particles of sulphur. Evidently no chemical combination has taken place, but an intimate mechanical mixture has been produced. If, however, this mixture is heated, a remarkable and different change takes place; the mixture will begin to glow, and on examining the mass it will be noticed that both the copper and the sulphur as such have disappeared. Neither can be distinguished even with the most powerful microscope, but in their place is a black substance with properties—such as form, appearance, specific gravity, etc.—that are essentially different from those possessed by either copper or sulphur. Here a chemical change has actually taken place; the copper and sulphur under the influence of heat have combined and a new substance with different properties has been obtained.

These two experiments show conclusively that a chemical combination is widely different from a mere mixture. In the first case, the resultant body is a mean in appearance, specific gravity, etc. between its constituents; each still has its individual properties. In the second case, the resultant body is more or less different in appearance and properties from those of which it is composed. Further, as will be presently demonstrated, when substances combine chemically they invariably do so in definite proportions. In a mixture, substances may evidently be present in any proportion whatever.

7. Indestructibility of Matter.—Although chemical action, such as combination, certainly can produce wonderful changes, there is one thing that cannot be accomplished: it can neither create nor destroy matter. By the most careful observation of all known cases of chemical action, it has been positively proved that a loss of matter never occurs, that matter is indestructible, and that even in such chemical actions as the burning and slow disappearance of a candle, there is simply a change of state but never an annihilation of matter. The truth of this fundamental principle of chemistry has been experimentally demonstrated by showing that the weights of substances acting chemically on one another are always the same after chemical changes and actions have taken place as they were before.

8. Division of Chemical Action.—Chemical actions may be divided into the following three classes:

1. *Synthetical reactions*, or actions of composition; that is, those actions in which a substance is formed by the union of two or more simpler ones.
2. *Analytical reactions*, or reactions of decomposition; that is, those reactions in which a substance is split up into two or more simpler ones.
3. *Metathetical reactions* are those that involve both decomposition and recomposition.

LAWS OF CHEMICAL COMBINATION

MOLECULES AND ATOMS

9. Chemical Definition of Molecule.—A molecule is a group of two or more atoms that are united by their affinity or mutual attraction. It is the smallest part of any substance that can be obtained and still exist in a free or uncombined state.

10. Classification of Molecules.—Molecules may be divided into two classes: (1) *Elemental* molecules, which are formed by like atoms; (2) *compound* molecules, which are formed by unlike atoms. Matter composed of molecules containing like atoms is termed *simple*, or *elementary*, *matter;* matter whose molecules are composed of dissimilar atoms is called *compound matter*.

11. The simplest way to distinguish elemental molecules from compound molecules is to cause a rearrangement of the atoms between two similar molecules. Elemental molecules do not yield any new kind of matter, whereas compound molecules produce elemental molecules. If, for instance, it is assumed that *a a* and *a a* are two molecules each composed of the two atoms *a* and *a*, it will be impossible to obtain, by any rearrangement, any other molecules than *a a* and *a a*. But should the molecules be *a b* and *a b*, that is, compound molecules, and each be composed of the dissimilar atoms *a* and *b*, a rearrangement of the atoms will give the elemental molecules *a a* and *b b*.

12. Number of Elemental Molecules.—Although the number of substances around us is, so to speak, unlimited, yet there are comparatively few elemental molecules; the number of those that has been positively ascertained is approximately seventy-six. As every elementary molecule is composed of atoms that are similar to the molecule, it is self-evident that the number of elemental atoms is also approximately seventy-six.

13. Nomenclature of Elemental Molecules and Atoms.—Elemental molecules and their atoms always possess the same name, which in some instances is the one by which these substances are known in every-day life, as, for instance, gold, silver, iron, copper, etc.; in other cases the name is chosen on account of striking properties. So, for instance, chlorine obtained its name from the Greek name of its color.

14. Avogadro's Law.—The Italian physicist Avogadro, in 1811, and, independently, Ampère, a French chemist, in 1814, as a result of various investigations and experiments, established the following law, which by right of priority is generally known as **Avogadro's law**.

Law.—*Equal volumes of all substances, in the gaseous state and at the same temperature and pressure, contain an equal number of molecules.*

From this law it obviously follows: (1) That the molecules of all bodies in a gaseous state must be of equal size, or at least occupy the same space. (2) That the weight of any molecule—compared with that of a molecule of hydrogen—is proportional to the weight of any given volume, also compared with an equal volume of hydrogen. If, for instance, 1 liter of chlorine weighs 35.5 times as much as 1 liter of hydrogen, one chlorine molecule must weigh 35.5 times as much as one molecule of hydrogen, if the above law is true.

15. If one volume of hydrogen and one volume of chlorine are mixed and exposed to the light, two volumes of hydrochloric acid are obtained, and assuming that one volume of hydrogen gas contains five hundred molecules of hydrogen and one volume of chlorine gas contains five hundred molecules of chlorine, we will have one thousand molecules of the compound. Submitting hydrochloric acid to an analysis, it is found that each of its molecules is composed of one atom of chlorine and one atom of hydrogen, and since the thousand molecules of hydrochloric acid were formed from five hundred molecules of hydrogen and five hundred molecules of chlorine, it is evident that each of these molecules must have furnished two atoms. From this fact we can state that a molecule of hydrogen is composed of two atoms.

If, then, we further assume that the weight of an atom of hydrogen is 1, so as to serve as a unit, the weight of a molecule of hydrogen, that is, its molecular weight, is 2.

16. Density.—By **density** of a body is meant its mass or quantity of matter, compared with the mass or quantity of matter of an equal volume of some standard body arbitrarily chosen. As hydrogen is chosen as this standard, the molecular weight of any substance may be obtained by multiplying its density by 2.

17. Classes of Elements.—The elements are divided into two general classes—the *metals* and the *non-metals*, or *metalloids;* but these classes are not so separated that all the elements on one side of a sharp boundary line can be said to be metals, and all on the other, non-metals. The elements can be arranged in a series that will pass gradually from strongly metallic elements on one hand to strongly non-metallic on the other, but there is a point in the series where an element resembles both a metal and non-metal. In the table of elements, those generally considered to be metals are printed in ordinary type and the non-metals in Italic.

18. Symbols.—To prevent constant repetition of the names of the elements, and to aid in expressing the composition of substances, abbreviations, or **symbols,** are used instead of names; these symbols consist of the initial letter, or the initial letter and another letter, of its name. Sometimes they are derived from the Latin names, which are often very different from the common ones. Such is the case with the elements sodium (*Na* from *natrium*), lead (*Pb* from *plumbum*), mercury (*Hg* from *hydrargyrum*), iron (*Fe* from *ferrum*), and copper (*Cu* from *cuprum*).

The names, symbols, and atomic weights of the most prominent elements are given in Table I. The numerals in the column headed Common Valence will be explained later.

19. Law of Definite Proportions.—It has been determined, as the result of many experiments, that when elements form a compound they always combine in definite proportions, which are always the same for the same compound. When oxygen and hydrogen combine to form water they always combine in the proportion of one part, by weight, of hydrogen to eight parts, by weight, of oxygen; or, expressed in percentage composition, 11.111 per cent., by weight, of hydrogen to 88.889 per cent., by weight, of oxygen. The same compound always contains the same elements combined in the same proportion by weight. This is known as the **law of definite proportions.**

20. Law of Multiple Proportions.—In many cases it is possible to get more than one compound from the same

TABLE I

Name of Element	Symbol	Atomic Weight	Common Valence	Chemical Equivalent
Aluminum . . .	*Al*	27.1	III	9.03
Antimony	*Sb*	120.2	III–V	40.067— 24.04
Arsenic	*As*	75.0	III–V	25. — 15.
Barium	*Ba*	137.4	II	68.7
Bismuth	*Bi*	208.5	III–V	69.5 — 41.7
Boron	*B*	11.0	III	3.67
Bromine	*Br*	79.96	I	79.96
Cadmium	*Cd*	112.4	II	56.2
Calcium	*Ca*	40.1	II	20.05
Carbon	*C*	12.0	IV	3.
Chlorine	*Cl*	35.45	I	35.45
Chromium . . .	*Cr*	52.1	II–VI	26.05 — 8.68
Cobalt	*Co*	59.0	II–III	29.5 — 19.67
Copper	*Cu*	63.6	I–II	63.6 — 31.8
Fluorine	*F*	19.0	I	19.
Gold	*Au*	197.2	III	65.73
Hydrogen	*H*	1.008	I	1.008
Iodine	*I*	126.85	I	126.85
Iron	*Fe*	55.9	II–III	27.95 — 18.63
Lead	*Pb*	206.9	II–IV	103.45 — 51.73
Lithium	*Li*	7.03	I	7.03
Magnesium . . .	*Mg*	24.36	II	12.18
Manganese . . .	*Mn*	55.0	II–VII	27.5 — 7.86
Mercury	*Hg*	200.0	I–II	200.0 —100.0
Nickel	*Ni*	58.7	II–III	29.35 — 19.57
Nitrogen	*N*	14.04	III–V	4.68 — 2.81
Oxygen	*O*	16.0	II	8.0
Palladium	*Pd*	106.5	IV	26.63
Phosphorus . . .	*P*	31.0	III–V	10.33 — 6.2
Platinum	*Pt*	194.8	IV	48.7
Potassium	*K*	39.15	I	39.15
Selenium	*Se*	79.2	II	39.6
Silicon	*Si*	28.4	IV	7.1
Silver	*Ag*	107.93	I	107.93
Sodium	*Na*	23.05	I	23.05
Strontium	*Sr*	87.6	II	43.8
Sulphur	*S*	32.06	II	16.03
Tellurium . . .	*Te*	127.6	II	63.8
Thallium	*Tl*	204.1	I–III	204.1 — 68.03
Thorium	*Th*	232.5	IV	58.13
Tin	*Sn*	119.0	II–IV	59.5 — 29.75
Tungsten	*W*	184.0	IV–VI	46.0 — 30.67
Uranium	*U*	239.5	IV–VI	59.88 — 39.92
Vanadium . . .	*V*	51.2	III–V	17.07 — 10.24
Zinc	*Zn*	65.4	II	32.7

NOTE.—The names of non-metallic elements are printed in Italic.

elements. It has been found that if two elements combine to form more than one compound, the different quantities of one that unite with a fixed quantity of the other, bear a simple ratio to each other. This is known as the **law of multiple proportions.** The following example will make the law more readily understood: Carbon and oxygen combine in two proportions. In carbon monoxide, three parts, by weight, of carbon combine with four parts, by weight, of oxygen; and in carbon dioxide, three parts of carbon combine with eight parts of oxygen. The ratio of oxygen in the two compounds, 4 to 8 or 1 to 2, is obviously a simple one.

21. Atomic Theory.—To account for the laws of definite and multiple proportions, we have the **atomic theory** of Dalton, which holds that there is a limit to the divisibility of matter and that these extremely small indivisible particles, called atoms, are of the same weight for the same element but that the weight of an atom of each element differs from the weight of an atom of all other elements. Chemical compounds result from the union of atoms of different elements; and since the atoms of any one element are all alike, it is easy to account for the laws of definite and multiple proportions by the atomic theory.

22. Atomic Weights.—Hydrogen combines with other elements in the smallest proportion, by weight, of any of the elements. The smallest proportion, by weight, of oxygen that has been known to enter into combination, is 15.88 times the corresponding weight of hydrogen. This weight for the element chlorine is 35.18 times the corresponding weight of hydrogen. These relative weights of the elements are called the **atomic weights.** If hydrogen is made the standard for comparison and the weight of its atom is called 1, the atomic weights of oxygen and chlorine, become, respectively, 15.88 and 35.18.

For some time it was thought that the atomic weight of oxygen was exactly 16 times the atomic weight of hydrogen; but by comparatively recent and more exact determinations of this atomic ratio the value has been found to be 15.88 : 1. It is, then, obvious that if we let the atomic weight of hydrogen equal 1, the atomic weight of oxygen will equal 15.88; or if we take oxygen as the standard for comparison and call its weight exactly 16, then the atomic weight of hydrogen must equal 1.008. The ratio 15.88 : 1, or the ratio between any two elements, is a definite quantity in each case, but the actual numbers used are arbitrary. Thus, we might call hydrogen 5; then the atomic weight of oxygen must be 5×15.88 or 79.4; or we might call oxygen exactly 100, when the atomic weight of hydrogen must be $\frac{100}{15.88} = 6.297$.

The two standards in present use are $H = 1$ and $O = 16$. If $H = 1$, then chlorine = 35.18, lead = 205.35, iron = 55.5, etc. If $O = 16$, then chlorine = 35.45, lead = 206.9, iron = 55.9, etc. The generally accepted atomic weights are based on the standard, $O = 16$. The atomic weights of the more important elements are given in the third column of Table I, and are based on the oxygen standard. In calculations where great accuracy is not required, it is customary to use round numbers for the atomic weight; thus, $H = 1$, instead of 1.008; $Cl = 35.5$, instead of 35.45; iron = 56, instead of 55.9; etc.

The atomic weights are, then, the relative weights of the atoms of the different elements. The actual weights of some of the atoms have been calculated, but these are extremely small and are unimportant here.

23. In writing the results of chemical reactions the symbols of the elements represent single atoms. Thus, H represents an atom of hydrogen; O, an atom of oxygen; Cl, an atom of chlorine; etc.

24. A compound is represented by a single molecule. Its formula is the representation of one of its molecules by the use of symbols and figures; thus, sodium chloride (common salt), whose molecules each consist of one atom of sodium, Na, and one atom of chlorine, Cl, is represented by the formula $NaCl$. Zinc sulphate, whose molecule consists of one atom of zinc, Zn, one atom of sulphur, *S*, and four atoms of oxygen, *O*, is represented by the formula $ZnSO_4$. The formula of a compound is represented by placing the symbols of the component elements side by side; where more than one atom of the same element occurs in a molecule, the figure expressing this number is written at the right and a little below the corresponding symbol.

25. The molecular weight of a compound is the sum of the weights of the atoms composing the compound. Thus a molecule of sulphuric acid has the formula H_2SO_4. The atomic weights of the elements are $H=1$, $S=32$, and $O=16$; then $(2 \times 1) + (1 \times 32) + (4 \times 16) = 98$, molecular weight of sulphuric acid. Nitric acid has the formula HNO_3. The atomic weight of nitrogen being 14, $(1 \times 1) + (1 \times 14) + (3 \times 16) = 63$, the molecular weight of nitric acid.

26. Affinity.—When different elements combine to form a compound, heat is generally produced; the development of heat, therefore, is the most usual indication that a chemical reaction is taking place. Certain elements have an attraction for, or a tendency to combine with, certain other elements; this attraction is expressed by saying that the elements have an **affinity** for each other. Different elements have different affinities for each other. Oxygen and hydrogen have a strong affinity for each other; oxygen and silver, a weak affinity; while oxygen and fluorine have no affinity and therefore do not combine. Elements that have a strong affinity form *stable compounds;* that is, the compounds are not readily decomposed. The weaker the affinity of the uniting elements the more unstable is the resulting compound. Lead oxide, *PbO*, is a stable compound, but lead peroxide, PbO_2, is unstable and is readily decomposed into oxygen and lead oxide, *PbO*. In compounds like lead peroxide the extra atom of oxygen is but loosely held or combined in the compound and is very readily given up.

Affinity is the attractive force of atoms and must not be confused with the attractive force of cohesion, which is the force holding particles of the same kind together in masses.

The difference between the two forces may be illustrated thus: Ice is a solid and the force of cohesion in this case acts strongly. By applying heat the ice melts to water; the particles cohere less strongly. By heating the water to 100° centigrade, or above, it is changed to steam, in which state there is no cohesion between the particles. The atoms of hydrogen and oxygen, which compose the ice, water, or steam, still have a strong affinity for each other and remain firmly combined.

27. Elemental Molecules.—In most cases atoms of the same element combine to form molecules of that element; thus, two hydrogen atoms may combine to form a molecule of hydrogen, H_2, and two oxygen atoms combine to form a molecule of oxygen, O_2. Atoms of the same element, however, have only a weak affinity for each other. At ordinary temperatures mercury atoms do not combine to form molecules.

28. Nascent State.—At the instant an element is set free from a compound, it is said to be in a **nascent state;** it is then more active, that is, it combines more readily with other elements. If the element does not combine with some other element at the moment it is set free, its atoms combine with each other, in which condition the element is less active.

29. Heat From Chemical Action.—When compounds are formed, there is usually a production of heat. The stronger the affinity of the uniting elements for each other, the more heat there is produced by the reaction; the amount of heat is, therefore, a measure of the force producing the combination. The heat given out by the reaction that produces a compound from its elements, is called the **heat of formation** of that compound, and when spoken of must refer to some definite amount of the substance; the quantity usually taken is the gram-molecular weight of the compound. This simply means that for a compound having a molecular weight of, say, 18, the heat of formation is the quantity of heat evolved when 18 grams of the compound are formed directly from its elements. The heat is generally measured in the lesser, or gram-degree, calorie, which is the quantity of heat required to raise 1 gram of water 1° centigrade. The heat of formation of sulphuric acid has been found to be 193,100 calories; this means that when 32 grams of solid sulphur, 64 grams of oxygen gas, and 2 grams of hydrogen gas combine to form 98 grams of sulphuric acid, there is evolved a quantity of heat sufficient to raise the temperature of 193,100 grams of water 1° centigrade.

In battery work, the heats of formation are not very useful, since most of the battery reactions do not involve the formation of compounds from their elements but rather from other compounds and elements together. A common case is where zinc combines with sulphuric acid; the first being an element and the second, a compound. The heat developed by such a reaction is called the *heat of combination, or reaction, of zinc and sulphuric acid;* the substances involved in the reaction always being named. This naming of the substances involved is necessary to distinguish from heat of formation, which always means the heat produced by the formation of a compound from its elements.

Table II gives the heats of formation of a few common substances. The term gram equivalent will be explained later.

30. The exact nature of chemical action is not known, any more than is the exact nature of electricity or heat; but it is similar to other physical phenomena in that it is a manifestation of energy. This energy is apparently stored in the atoms of the elements as potential energy, and causes them to have an affinity for, or a tendency to combine with, other atoms; the strength of this affinity depends on the relative amounts of potential energy stored in the combining atoms. Under proper conditions these affinities cause the atoms to combine, and their potential energy then appears as kinetic energy, usually in the form of heat, but under special conditions, in the form of an electric current. Thus, while chemical combination develops kinetic energy, to bring about a chemical decomposition, the same energy must be supplied.

TABLE II

HEATS OF FORMATION

Compound	Constituents	Calories per Gram Molecule	Calories per Gram Equivalent
Cadmium oxide	*Cd–O*	65,680	32,840
Carbon dioxide	*C–2O*	97,000	24,250
Copper oxide	*Cu–O*	37,200	18,600
Hydrochloric acid	*H–Cl*	22,000	22,000
Lead oxide	*Pb–O*	50,300	25,150
Iron oxide (ous)	*Fe–O*	68,200	34,100
Nickel oxide	*Ni–O*	60,840	30,420
Potassium chloride	*K–Cl*	105,600	105,600
Silver oxide	2*Ag–O*	6,000	3,000
Water	2*H–O*	68,000	34,000
Zinc oxide	*Zn–O*	85,400	42,700
Sulphuric acid (anhydrous)	$H_2–S–O_4$	193,000	
Sulphuric acid (with water)	$H_2–S–O_4–H_2O$	210,700	

31. Some elements do not produce heat when they combine with each other, but require heat, or energy in some form, to cause them to unite; compounds thus formed are called *endothermic compounds*, while those that produce heat in their formation are called *exothermic compounds*.

ACIDS, BASES, AND SALTS

32. An **acid** is a compound containing hydrogen, which hydrogen may be replaced by a metal when the acid is treated with a metal, a base, or an oxide of a metal. A **base** is a compound containing a metal combined with hydrogen and oxygen, the metal of which may be readily replaced by hydrogen when the base is treated with an acid. A **salt** is a compound formed by the action of an acid on a base, resulting in the replacement of the hydrogen of the acid by the metal of the base, and the formation of water.

Nitric acid has the formula HNO_3. The hydrogen may be replaced by the metal sodium by treating the acid with the base, sodium hydroxide, $NaOH$. When the two are brought together, the base exchanges its metal for the hydrogen of the acid and we have instead of the base, HOH or H_2O, which is water, and in place of the acid, $NaNO_3$, which is a salt called sodium nitrate. In general, combinations of the non-metallic elements with oxygen and hydrogen form acids, and combinations of the metals with oxygen and hydrogen form bases. H_2SO_4, H_3PO_4, and HNO_3 are acids, and $NaOH$, $Ba(OH)_2$, and $Zn(OH)_2$ are bases. Some acids do not contain oxygen; such are hydrochloric acid, HCl, and hydrobromic acid, HBr. Some metals form both an acid and a base; thus, with the metal chromium, $Cr(OH)_2$ is a base and H_2CrO_4 is an acid. Lead is another such metal; $Pb(OH)_2$ being a base and H_4PbO_4, a weak acid.

33. Chemical Equations.—Chemical reactions are expressed by means of equations in which the first member consists of all the substances that take part in the reaction and the second member consists of the compounds resulting from the reaction. Thus, the action of the base sodium hydroxide on nitric acid may be written $HNO_3 + NaOH = NaNO_3 + H_2O$. The action of sulphuric acid on the metal zinc may be written: $H_2SO_4 + Zn = ZnSO_4 + H_2$. In this case there is formed one molecule of the salt, zinc sulphate, and two atoms, or more properly, one molecule of hydrogen gas. The formation of water is represented thus: $2H + O = H_2O$.

34. The acids and bases that occur most frequently in battery work are given in Table III.

35. Valence.—By examining a number of compounds, such as HCl, H_2O, and SnO_2, it is found that one atom of hydrogen combines with one atom of chlorine; that two atoms of hydrogen combine with one atom of oxygen; and that two atoms of oxygen combine with one atom of tin. Thus it is evident that the atoms of different elements have different atom-affixing powers; that is, different powers for

TABLE III

COMMON ACIDS AND BASES

Name	Chemical Formula	Specific Gravity	
		Pure	Commercial (Average)
ACIDS			
*Hydrochloric	HCl	1.227	1.14 to 1.16
Nitric	HNO_3	1.530	1.33 to 1.41
Sulphuric	H_2SO_4	1.838	1.70 to 1.83
BASES			
†Potassium hydroxide . .	KOH		
†Sodium hydroxide	$NaOH$		
*Ammonium hydroxide . .	NH_4OH	.88	.9 to .96
†Calcium hydroxide (lime)	$Ca(OH)_2$		
†Barium hydroxide	$Ba(OH)_2$		

holding other atoms in combination. This power is called the **valence** or **valency** of the element. Hydrogen atoms stand among the lowest with respect to the power of affixing other atoms, and hydrogen is therefore assigned a valence of I, or we say that hydrogen is *monovalent*. (The prefixes mono-, di-, tri-, tetra-, penta-, etc., mean one, two, three, four, five, etc., respectively.) Since one atom of chlorine combines with one atom of hydrogen, chlorine is also monovalent. One atom of oxygen combines with two atoms of hydrogen; therefore, oxygen is divalent. Tin holds two atoms of the divalent element oxygen in combination; tin is, therefore, tetravalent. If it combined with hydrogen it would combine with four atoms. Tin, however, does not combine with hydrogen, but it does combine with the monovalent element chlorine to form the compound $SnCl_4$.

*These are gases (HCl and NH_3) that dissolve in water to form the respective acid and basic solutions.

†These are solids that are very soluble in water. Specific gravity of solution used in cells will depend on relative proportions of hydroxide and water.

Further, the atoms of some elements are capable of combining with different numbers of atoms of another element; thus, lead combines with oxygen in several proportions, for example, PbO, Pb_2O_3, and PbO_2. In PbO the lead is divalent; in Pb_2O_3, it is trivalent; and in PbO_2, it is tetravalent. The two most common valences of lead are two and four; many elements have more than one valence. The principal valences of each element are given in the fourth column of Table I. In compounds like Pb_2O_3 the valence of the metal is determined as follows: Oxygen is divalent; and since there are three oxygen atoms in the compound, there are six valences. There being two lead atoms in the molecule, each lead atom holds three of the oxygen valences. Therefore, lead is trivalent in the compound Pb_2O_3. Suppose that the valence of the manganese compound Mn_2O_7 were to be determined. In this case $O = 7$ atoms having $2 \times 7 = 14$ valences; $Mn = 2$ atoms; $14 \div 2 = 7$. Each atom of manganese holds seven valences of oxygen in this compound and manganese is, therefore, in this case a heptavalent atom (*hepta* is the Greek word for 7).

36. Perhaps a better idea of valence may be had by assuming the elements to have hands, links, or bonds, one for each valence — thus, the monovalent element hydrogen will have one hand ($H-$), divalent oxygen will have two hands ($O=$), tetravalent tin will have four hands ($=Sn=$), etc. When hydrogen and oxygen combine they take hold of hands thus, $\genfrac{}{}{0pt}{}{H}{H}\!\!>\!O$ or $H-O-H$, and when oxygen and tin combine they take hold of hands thus, $Sn\genfrac{}{}{0pt}{}{=O}{=O}$ or $O=Sn=O$. Tin is divalent in some compounds, as SnO, in which case it may be assumed that two of its hands are holding oxygen and the other two are either free or holding each other, thus, $=Sn=O$. It must not be understood that elements actually have hands or links or anything resembling these, but only that this assumption is a convenient way of representing the idea of valence. Of the actual method by which elements hold each other in combination, little is known. Sometimes the valence of an element is represented by placing accent marks at the right and a little above the symbol of the element, thus H', Cl', O'', Sn'', Sn'''', Fe'', Fe''', etc., or Roman numerals may be used for this purpose; thus, H^{I}, Sn^{IV}, etc.

37. Oxidation.—To oxidize a metal is meant, in the narrow sense of the term, to cause the metal to combine with oxygen; but in the broader sense, and the one generally used in chemistry, to oxidize a metal means to increase its valence by causing any other element to combine with it. Thus, by treating tin with hydrochloric acid, tin chloride, $SnCl_2$, is formed. The tin has been changed from the elemental state to a combined condition in which it is divalent, since it has combined with two atoms of the monovalent element chlorine. It may be said that inactive metallic tin has been changed to an active element with a valence of II. Under favorable conditions and by suitable treatment this compound, $SnCl_2$, can be changed to the compound $SnCl_4$; that is, the valence of the tin can be raised to such a degree that it will combine with four atoms of monovalent chlorine. These processes are known as *oxidation processes*, and the chemical compounds used to produce this change in valence are called *oxidizing agents*.

38. Reduction.—The reverse process is one of **reduction** and the reagents used are called *reducing agents*. For example, $SnCl_4$ may be reduced to $SnCl_2$ by treating the $SnCl_4$ with metallic tin. By treating $SnCl_2$ with metallic zinc (a metal having a stronger affinity for chlorine than tin) the compound is reduced to a metal and the zinc takes its place in combination with chlorine. These reactions may be represented by equations:

$$SnCl_4 + Sn = 2SnCl_2 \qquad (1)$$
$$SnCl_2 + Zn = ZnCl_2 + Sn \qquad (2)$$

A process of oxidation raises the valence, while one of reduction lowers the valence; both processes take place simultaneously when an oxidizing agent and a reducing agent are brought together. The former is reduced and the latter is oxidized.

Oxidizing agents are easily reduced themselves and so are capable of oxidizing other substances. They are used as depolarizers (a term that will be explained later) in batteries. The more important of the strong oxidizing agents used for this purpose are: potassium bichromate, $K_2Cr_2O_7$, chlorine gas, Cl_2, chromic acid, H_2CrO_4, nitric acid, HNO_3, lead peroxide, PbO_2, and manganese dioxide, MnO_2. The metals are the important reducing agents used in batteries.

39. Equivalent Weights.—An equivalent weight of an element (called chemical equivalent) is the weight of the element that combines with 1.008 parts, by weight, of hydrogen or 35.45 parts, by weight, of chlorine, or, that combines with the atomic weight of any monovalent element. In other words, the equivalent weight of an element is the atomic weight of that element reduced to the basis of monovalence. Therefore, if an element is divalent, the equivalent weight of that element is one-half of the atomic weight; if an element is trivalent, the equivalent weight is one-third of the atomic weight, etc. For example, the atomic weight of zinc is 65.4; since this metal is divalent, the equivalent weight is one-half of 65.4 or 32.7. The atomic weight of iron is 55.9; when the metal is divalent, the equivalent weight is 27.95, and when the metal is trivalent, the equivalent weight is 18.63. The atomic weights of all the monovalent elements are also the equivalent weights.

40. Basicity of Acids.—It was stated that an acid contains hydrogen and usually oxygen and that the hydrogen can be readily exchanged for the metal of a base. It is plain, therefore, that one hydrogen atom will be exchanged for one atom of a monovalent metal, or two hydrogen atoms will be exchanged for one atom of a divalent metal, etc.

Some acids have only one replaceable hydrogen atom, as nitric, HNO_3, and hydrochloric, HCl; these are called *monobasic acids*, since each molecule can hold in combination only one valence of a base-forming metal. For example, if the hydrogen atom of nitric acid is replaced by the metal sodium, we get the compound $NaNO_3$, while if replaced by the divalent metal zinc, we get the compound $Zn(NO_3)_2$. A trivalent metal, such as bismuth, will form the compound $Bi(NO_3)_3$. Sulphuric acid, H_2SO_4, has two replaceable hydrogen atoms and is, therefore, a diabasic acid. With the monovalent metal sodium two compounds may result, $NaHSO_4$ or Na_2SO_4, according to whether one hydrogen atom or both of them are replaced by the metal. Compounds like $NaHSO_4$, in which only a part of the hydrogen of an acid is replaced by a metal, are called *acid salts*. If all the replaceable hydrogen of an acid is exchanged for metal, a *neutral salt* results. One atom of a divalent metal can replace both hydrogen atoms of a dibasic acid; thus, zinc forms with sulphuric acid, zinc sulphate, $ZnSO_4$. Trivalent iron will form with sulphuric acid, the compound $Fe_2(SO_4)_3$. In writing the formulas of salts it must be remembered that the valences of the metal atoms entering the acid must be equal to the number of hydrogen atoms replaced.

EXAMPLE.—Phosphoric acid, H_3PO_4, is tribasic. What is the formula for the neutral salt zinc phosphate?

SOLUTION.—H_3PO_4 is tribasic, having three replaceable hydrogen atoms, and Zn is divalent, having two valences. The two figures are not equal, so such a multiple (the least common multiple) of each must be taken that the valences of the metal shall exactly equal the number of replaced hydrogen atoms. Two molecules of H_3PO_4 have six replaceable hydrogen atoms; three atoms of zinc have six valences. Therefore, the formula for zinc phosphate is $Zn_3(PO_4)_2$.

NOMENCLATURE OF CHEMICAL COMPOUNDS

41. Compounds are termed **binary, ternary, quaternary,** etc., according to whether they contain, respectively, two, three, four, etc. elements. Binary compounds of metallic elements combined with non-metallic elements are named by changing the last syllable of the name of the non-metallic element to *ide*; thus, compounds of oxygen, sulphur, chlorine, etc., with metals are, respectively, called oxides, sulphides, chlorides, etc. Examples are: zinc oxide, ZnO, sodium chloride, $NaCl$, potassium sulphide, K_2S In the case of binary compounds of one non-metallic element combined with another, the name of the more non-metallic element ends in *ide*; thus, sulphur dioxide, SO_2, and phosphorus pentachloride, PCl_5.

The prefixes mono-, di-, tri-, tetra-, penta-, hexa-, hepta-, etc. are frequently used to show the number of atoms of a non-metallic element present in the molecule of binary compounds, thus distinguishing between compounds having the same elements but combined in different proportions. For example, sulphur dioxide, SO_2, sulphur trioxide, SO_3, phosphorus trichloride, PCl_3, and phosphorus pentachloride, PCl_5. Other prefixes used for the same purpose are: sub-, proto-, sesqui-, and per-; for example, lead suboxide, Pb_2O, lead protoxide, PbO, lead sesquioxide, Pb_2O_3, and lead peroxide, PbO_2.

42. Acids.—Acids of chlorine that contain oxygen are named by using suffixes, or suffixes and prefixes, with "chlor." Similarly, acids of sulphur that contain oxygen are named by using suffixes, or suffixes and prefixes with "sulph." The most common of the chlorine acids, $HClO_3$, is called *chloric acid;* the acid containing one less atom of oxygen than chloric acid, is called *chlorous acid*, $HClO_2$; the acid containing a smaller portion of oxygen than chlorous acid, is called *hypochlorous acid*, $HClO$. (Hypo comes from the Greek and means under or less than.) The acid containing more oxygen than chloric acid is called *perchloric acid*, $HClO_4$. The common sulphur acid containing oxygen is called sulphuric acid, H_2SO_4, and the other acids of the series are named like the corresponding chlorine acids. The acid of chlorine that contains no oxygen is called *hydrochloric acid*, HCl; the sulphur acid that contains no oxygen may be called hydrosulphuric acid, H_2S, but is usually called *hydrogen sulphide*, and was formerly called sulphureted hydrogen.

43. Bases.—Bases are commonly called *hydroxides;* for example, sodium hydroxide, $NaOH$, and calcium hydroxide, $Ca(OH)_2$. The strongest bases are called *alkalies;* the common alkalies are sodium hydroxide, potassium hydroxide, and ammonium hydroxide. Metal oxides, such as sodium, potassium, and calcium oxides, readily combine with water to form hydroxides of the metals according to the equations, $Na_2O + H_2O = 2\,NaOH$; $CaO + H_2O = Ca(OH)_2$. From the fact that hydroxides may be obtained by the action of metals or metal oxides on water, they are called ***hydrates*** by some chemists.

44. Salts.—A salt derived from chloric acid by the replacement of the hydrogen atom by one of a metal, is called a *chlorate;* thus, if the substituting metal is potassium, the resulting salt is *potassium chlorate.* Salts derived from chlorous acid are called *chlorites;* those derived from hypochlorous acids are called *hypochlorites;* those derived from perchloric acid are called *perchlorates.* If the name of the acid ends in *ic*, that of the derived salt ends in *ate*; if in *ous*, that of the derived salt ends in *ite.* It might be expected that salts of hydrochloric acid would be called hydrochlorates, but since binary compounds of the metals with chlorine are called chlorides, the name hydrochlorate is unnecessary.

45. Since some of the metals have two common valences, there can be two neutral salts from the same metal and acid. In such cases the name of the metal (in some cases the Latin name is used and in others, the common name) has its last syllable changed to *ous* or *ic* to indicate which of the two salts is meant. When the metal has the lower valence, the name of the metal ends in *ous*; when the metal has the higher valence, the name of the metal ends in *ic.* Thus, $FeSO_4$ and $Fe_2(SO_4)_3$ are both iron sulphates, but to distinguish one from the other, the first is called ferrous sulphate, and the other ferric sulphate. The salt $SnCl_2$ is called stannous chloride and $SnCl_4$ is called stannic chloride. $HgNO_3$ is mercurous nitrate and $Hg(NO_3)_2$ is mercuric nitrate.

Sometimes bases and oxides are named in the same way; thus, $Fe(OH)_2$ is called ferrous hydroxide and $Fe_2(OH)_6$ is called ferric hydroxide; Hg_2O is called mercurous oxide and HgO is called mercuric oxide.

46. Many chemical compounds of commercial importance have commercial names that are different from the chemical names. Table IV gives the chemical name, the commercial name, and the formula for a number of compounds that are used in battery work.

TABLE IV

COMPOUNDS USED FREQUENTLY IN BATTERY WORK

Chemical Name	Commercial Name	Formula
Ammonium chloride	Sal ammoniac	NH_4Cl
Calcium hypochlorite	Chloride of lime	$Ca(ClO)_2$
Copper sulphate	Blue vitriol, or bluestone	$CuSO_4$
Ferrous sulphate	Green vitriol, or copperas	$FeSO_4$
Plumbo-plumbic oxide	Minium or red lead	Pb_3O_4
Lead oxide	Litharge	PbO
Manganese dioxide	Black oxide of manganese	MnO_2
Nitric acid	Aqua fortis	HNO_3
Potassium hydroxide	Caustic potash	KOH
Sodium chloride	Common salt	$NaCl$
Sodium hydroxide	Caustic soda	$NaOH$
Sulphuric acid	Oil of vitriol	H_2SO_4
Zinc sulphate	White vitriol	$ZnSO_4$
Copper oxide	Black oxide of copper	CuO

RADICALS

47. A **radical** may be considered as a chemical combination of two or more elements that, owing to its unsatisfied bonds, or links, is not able to exist in the free state, but is capable of acting similar to an element. For instance, in the compound H_2SO_4 (sulphuric acid) we find the group or the radical SO_4, and again in the salts of sulphuric acid, that is, in the sulphates, as for instance in $FeSO_4$, Na_2SO_4, $LnSO_4$, etc., this same group appears. Such a group having its origin in the acids is known as an *acid radical.* The group OH, which occurs in hydroxides, as for instance in $NaOH$, $Ba(OH)_2$, etc., is known as a *basic radical;* or, as it is more frequently called, the *hydroxide* or *hydroxyl radical.*

Where a radical occurs two or more times in a single compound it is usual to enclose it in parentheses and to indicate the number outside the parentheses, as for instance $Fe_2O_6H_6$ (ferric hydroxide) is usually written $Fe_2(OH)_6$; $Fe_2S_3O_{12}$ is generally written $Fe_2(SO_4)_3$.

48. Ammonia and the Ammonium Radical.—Ammonia, a compound formed by the combination of nitrogen and hydrogen and having the formula NH_3, is a gas that readily dissolves in water. When in solution, ammonia is supposed to be combined with a part of the water to form the base, ammonium hydroxide, NH_4OH, according to the equation, $NH_3 + H_2O = NH_4OH$. The compound NH_4OH has all the properties of a base, the radical NH_4 taking the place of a metal and, like a metal, replaceable by a hydrogen atom of an acid. The radical NH_4 is called **ammonium** from its similarity to metals in respect to chemical properties. With mercury, it forms an alloy with a metallic appearance; with acids, salts. Thus, $NH_4OH + HCl = NH_4Cl + H_2O$. NH_4Cl is ammonium chloride, commonly called *sal ammoniac*, and is a compound extensively used in certain batteries.

49. The valence of this radical is I, as may be seen from the following: Nitrogen has a valence of V, and since there are four atoms of monovalent hydrogen combined with the nitrogen in NH_4, $\left.\begin{matrix}H\\H\\H\\H\end{matrix}\right>N-$, there is left one free valence with which any other element may combine. Therefore, the radical considered as a whole, has a valence of I. When ammonium is in combination with a monobasic-acid radical, as NO_3, the formula of the compound is NH_4NO_3; when in combination with a dibasic-acid radical, as SO_4, the formula of the compound is $(NH_4)_2SO_4$.

50. Anhydrides.—Sulphur trioxide, SO_3, unites with water according to the equation, $SO_3 + H_2O = H_2SO_4$; sulphuric acid is the resulting compound. Sulphur trioxide is called the **anhydride** of sulphuric acid, because it is the acid with the elements of water taken out. Similarly, N_2O_5 is the anhydride of nitric acid, $N_2O_5 + H_2O = 2HNO_3$. The oxides of the non-metallic elements are generally anhydrides of acids; and the oxides of metallic elements, anhydrides of bases. There are, however, many exceptions to this rule.

ELECTROCHEMISTRY

ELECTROLYTIC ACTION AND THEORY

51. Electrochemical Decomposition.—A current of electricity passing through a conducting liquid of a compound nature, decomposes that liquid; for example, zinc chloride is decomposed, by an electric current, into chlorine gas and the metal zinc. Such decomposable bodies are called **electrolytes.** An arrangement for electrolytic decompósition consists of a vessel, or jar, that contains the conducting liquid, or *electrolyte*, and two conducting plates (usually of metal or carbon) that connect the source of the electric current with the electrolyte, thereby forming a closed circuit of which a section of the liquid is a part. The vessel, electrolyte, and plates constitute an electrolytic cell. Fig. 1 shows such an arrangement. The conducting plates are called *electrodes;* that electrode at which the current enters the electrolyte being called the **anode**, and the one at which the current leaves the electrolyte being called the **cathode**, it follows that the electrode that is connected with the positive pole of the dynamo, or other source of electric current, is the anode and that connected with the negative pole is the cathode.

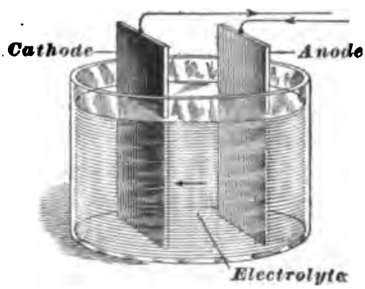

FIG. 1

52. All electrolytes are decomposable into two parts—one appearing at the anode and the other at the cathode. These products of electrolytic decomposition are called **ions.** Those ions that appear at the anode are called *anions* and those that appear at the cathode are called *cations.* Thus, when a solution of zinc chloride is the electrolyte, zinc will appear at the cathode and is therefore a cation; chlorine will appear at the anode and is an anion.

53. Kinds of Electrolytes.—The most common electrolytes are solutions of compounds in water, but most compounds that conduct electricity may be considered electrolytes, or conductors of the second class as they are frequently called to distinguish them from elementary conducting substances, which, of course, cannot be decomposed and which are often called conductors of the first class. Conductors of the first class are the metals and a few of the non-metals. A conductor of the second class may be a solution, a fused compound, or in some cases even a solid; many of the acids, bases, and salts form electrolytes when dissolved in water. Sulphuric acid, when pure, is a heavy liquid and a non-conductor of electricity; its water solution, however, is a good conductor and is an electrolyte. Zinc chloride, sodium chloride, and caustic soda are examples of compounds that become electrolytes when fused. Such compounds as silver sulphide and cop per sulphide are solid electrolytes.

54. Conductors of the first class have an increased electrical resistance with increase of temperature and are not decomposed by an electric current; those of the second class have a decreased resistance with increase of temperature and can be decomposed by an electric current. These are the distinguishing characteristics of the two classes of electric conductors.

55. Primary and Secondary Actions.—When a solution of zinc chloride is decomposed between electrodes that are not attacked by the products of the decomposition, the products are metallic zinc and chlorine; these are called *primary products* because they result directly from electrolytic decomposition. If a solution of sodium chloride, $NaCl$, is the electrolyte, the electric current will decompose the compound into metallic sodium and free chlorine. These are both primary products. Metallic sodium, however, has such a strong affinity for oxygen that it cannot remain uncombined in the presence of water, so as fast as it is liberated it is attacked by the water, forming hydrogen gas and sodium hydroxide, $Na + H_2O = NaOH + H$. The hydrogen gas escapes and the sodium hydroxide (caustic soda) remains in solution in the electrolyte. Hydrogen and sodium hydroxide are here *indirect*, or *secondary, products*. When an acid is electrolyzed (decomposed by an electric current), hydrogen is one of the ions and is in this case a primary product. Whenever the original products of electrolysis attack the electrodes, or combine with the electrolyte at the moment they are set free, secondary products result.

56. Theory of Electrolytic Dissociation.—When compounds dissolve in water with the formation of conducting solutions, it is believed that the molecules of such compounds are broken up into atoms or groups of atoms (radicals), each of which carries a static charge. Thus, when sodium chloride is dissolved in water it is supposed to be separated into charged atoms of chlorine and sodium. Ordinary sodium would decompose the water with the consequent evolution of hydrogen, and ordinary chlorine would color the water yellow. Since these phenomena do not attend the solution of common salt in water, it is assumed that the atoms have static charges of electricity that change their properties in some unknown manner, and a compound that has thus been resolved into ions is said to be dissociated; sulphuric acid may be dissociated into the ions H, H, and SO_4. The radical SO_4 ordinarily does not exist alone, but under the influence of the static charge it is enabled thus to exist and becomes an ion. When an electric current is flowing through an electrolyte, the ions are attracted to the electrodes and there give up their charges and appear as elements, or compounds as we commonly know them. When the ion SO_4 gives up its charge, it can no longer exist in an uncombined state; it either combines with some substance at the electrode or else breaks up into SO_3 and free oxygen, the SO_3 combining with the water of the electrolyte to form sulphuric acid and the oxygen escaping as a gas.

57. When a compound is dissociated there are always two kinds of ions (anions and cations) formed; the anions are attracted to the anode and the cations to the cathode. In general, the non-metallic elements and the radicals containing oxygen, are anions; while hydrogen, the ammonium radical, and the metals are cations. It is sometimes possible for the same element to act as an anion under one set of conditions and as a cation under another. Whether an element acts as an anion or a cation depends on whether the element is in combination with an element that is electropositive to it, or one that is electronegative to it. From a strong solution of phosphoric acid and with a strong current, phosphorus may be deposited on the cathode, under which conditions it is a cation; in the compound PH_3 (phosphoreted hydrogen), the phosphorus is electronegative to the hydrogen and therefore plays the part of an anion.

58. All the molecules of a compound are not dissociated into ions except in very dilute solutions in which the solvent has a strong ionizing power; that is, a power of resolving molecules into ions. Also, liquids vary as to ionizing power, water being one of the strongest. Many of the organic liquids (compounds of carbon) do not ionize molecules at all; and some compounds are not ionized in any solution. All conducting solutions contain ions; non-conducting solutions do not.

The electrolytic-dissociation theory accounts for many observed facts and is accepted by, perhaps, the majority of scientists in the fields of chemistry and physics; however, some experiments have recently been shown that seem to antagonize it. But whether or not this theory is accepted, the elements and radicals appearing at the electrodes may be called ions.

59. Amount of Substance Liberated by Electrochemical Action.—When an electrolyte is decomposed by an electric current, the amounts of the various substances liberated are proportional to the quantity of electricity passing through the electrolyte. The amount (weight) of any element that will be liberated by a given quantity of electricity is proportional to the chemical equivalent of that element. That is to say, a quantity of electricity that will liberate 1.008 grams of hydrogen will also liberate 8 grams of oxygen, 35.45 grams of chlorine, 107.93 grams of silver, etc.

60. Electrochemical Equivalent.—The amount, in grams, of any element that will be liberated by 1 coulomb (1 ampere for 1 second) of electricity is the **electrochemical equivalent** of that element. This quantity has been carefully determined by experiment for the element silver and is found to be .001118 gram. The method of obtaining this value is as follows: An electrolytic decomposition cell similar to that shown in Fig. 1 is used. Both electrodes are of silver and the electrolyte is a solution of a salt of silver, as silver nitrate, $AgNO_3$. The cathode is carefully cleaned and weighed before placing in the cell and after passing a current of known strength for a known time through the cell, the cathode is removed, carefully dried, and weighed. The increase in weight, in grams, divided by the number of coulombs passed through the cell, gives the weight, in grams, separated by 1 coulomb, which is the electrochemical equivalent of silver. The electrochemical equivalent being known, it is obvious that such an arrangement can be used to measure an electric current, and when used for this purpose the apparatus is known as a **voltameter.**

61. Calculation of Electrochemical Equivalents. Knowing that the quantity of an element liberated by electrochemical action is proportional to the chemical equivalent of that element, the electrochemical equivalent of any other element can be calculated. For example, the chemical equivalent of hydrogen is 1.008. The chemical equivalent of silver is 107.93 and is therefore 107.07 times that of hydrogen. The amount of hydrogen liberated per coulomb will be $\frac{1}{107.07} \times .001118$ or .000010442, which is the electrochemical equivalent of hydrogen. The methods of calculation may be expressed by a formula; thus:

Let c = chemical equivalent of any element whose electrochemical equivalent is known;

C = chemical equivalent of the element whose electrochemical equivalent is to be calculated;

z = electrochemical equivalent of the element whose chemical equivalent is c;

Z = electrochemical equivalent to be calculated.

Then,
$$Z = \frac{C}{c} \times z \qquad (1)$$

Applying this formula in the calculation of the electrochemical equivalent of oxygen and using silver as a basis for the calculation, $Z = \frac{8}{107.93} \times .001118 = .000082869$, the electrochemical equivalent of oxygen.

Using hydrogen for the basis for the calculation, the expression becomes $Z = \frac{8}{1.008} \times .00001044 = .000082857$, the electrochemical equivalent of oxygen, which agrees, sufficiently for all practical purposes, with that calculated from the values given for silver.

In Table V will be found the electrochemical equivalents of some of the common elements based on .001118 as the electrochemical equivalent of silver.

62. Coulombs per Gram Equivalent of a Substance.—If 1 coulomb of electricity will liberate .001118 gram of silver, to liberate 1 gram equivalent, or 107.93 grams of silver will require $\frac{107.93}{.001118} = 96{,}538$ coulombs of electricity. Similarly, to liberate 1 gram equivalent, or 8 grams of oxygen will require $\frac{8}{.000082869} = 96{,}538$

coulombs. It will require 96,538 coulombs to liberate a gram equivalent of any element or group of elements. This conforms to the law that the amount (weight) of any element that will be liberated by a given quantity of electricity is proportional to the chemical equivalent of that element.

63. Calculation of Amount of Liberated Substance.—Knowing the electrochemical equivalents of the elements, the amount of any element that will be liberated by an electric current of a given strength and in a given time, can be calculated.

EXAMPLE.—A current of 3 amperes is passed through a solution of silver nitrate for 1 hour and 20 minutes. How many grams of silver will be liberated?

SOLUTION.—The electrochemical equivalent of silver = .001118. 1 hr. and 20 min. = 80 min. = 4,800 sec. 4,800 × 3 = 14,400 ampere-seconds, or coulombs. If 1 coulomb will liberate .001118 gram of silver, then 14,400 coulombs will liberate 14,400 × .001118 = 16.099 grams of silver. Ans.

Groups of elements or radicals are dealt with the same as single elements. Using the figures in the above example, calculate the number of grams of nitric acid that will be liberated at the anode.

Since nitric acid is monobasic, the valence of its radical NO_3 is I. Therefore, its chemical equivalent is the sum of the atomic weights, $14 + 3 \times 16 = 62$. Applying this number in the formula $\frac{C}{c} \times z = Z$, $\frac{62}{107.93} \times .001118 = .000642$, the electrochemical equivalent of the radical NO_3. The number of coulombs from the preceding example is 14,400. Then, $14{,}400 \times .000642 = 9.245$ = number of grams of NO_3 liberated.

The NO_3 then breaks up, as shown by the equation, $2NO_3 = N_2O_5 + O$, and the N_2O_5 combines with water; thus, $N_2O_5 + H_2O = 2HNO_3$.

For each chemical equivalent of NO_3 (62) one chemical equivalent of nitric acid, HNO_3, is formed. Since 9.245 represents the amount of NO_3 in grams and since the chemical equivalents of NO_3 and HNO_3 are 62 and 63, respectively, then $\frac{63}{62} \times 9.245 = 9.394$ = the number of grams of nitric acid liberated. The same result can be obtained more directly by finding the electrochemical equivalent of HNO_3 (instead of NO_3) and multiplying this by 14,400. The longer method shows the steps by which the acid is formed.

EXAMPLES FOR PRACTICE

1. How many grams of zinc will be deposited from a solution of zinc sulphate by a current of 5 amperes flowing for 4 hours?
Ans. 24.39 grams

2. A certain current, flowing through a solution of silver nitrate, deposits 12 grams of silver in 2 hours. What was the average value of the current? Ans. 1.49 amperes

3. If 20 grams of zinc is to be deposited per hour in an electrolytic cell, what strength of current must be passed through the cell?
Ans. 16.4 amperes

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64. Polarization E. M. F.—When an electrolyte is decomposed by an electric current, the liberated elements have a tendency to recombine. Thus, when an electric current has been sent through a solution of zinc chloride and the salt has thereby been decomposed into zinc and chlorine, as soon as the electrolyzing current has been stopped, the zinc will commence to combine with the chlorine in contact with the other electrode, and if a galvanometer is connected in the circuit, a momentary current flowing in the opposite direction to the one that produced the decomposition, will be indicated. This tendency of the liberated elements to recombine acts as an opposing E. M. F., and is known as the **electromotive force of polarization.** With every electrolyte there is a minimum E. M. F. below which continuous decomposition cannot be effected. The combination of oxygen and hydrogen corresponds to an E. M. F. of 1.47 volts, so to decompose water continuously, an E. M. F. exceeding 1.47 volts will be required.

65. Polarization may be *permanent*, *transitory*, or there may be no polarization at all, according to the nature of the electrodes and the electrolyte. In the electrolysis of dilute

TABLE V

ELECTROCHEMICAL EQUIVALENTS

Name of Element	Valence	Usual Condition of Ion	Electrochemical Equivalent Grams per Coulomb
Bromine	I	Anion	.00082831
Chlorine	I	Anion	.00036723
Copper (ous)	I	Cation	.00065883
Copper (ic)	II	Cation	.00032942
Gold	III	Cation	.00068090
Hydrogen	I	Cation	.00001044
Iodine	I	Anion	.00131404
Iron (ous)	II	Cation	.00028953
Iron (ic)	III	Cation	.00019299
Lead	II	Cation	.00107164
Mercury (ous)	I	Cation	.00207180
Mercury (ic)	II	Cation	.00103590
Nickel	II	Cation	.00030404
Oxygen	II	Anion	.00008287
Potassium	I	Cation	.00040555
Silver	I	Cation	.001118
Sodium	I	Cation	.00023877
Tin (ous)	II	Cation	.00061636
Tin (ic)	IV	Cation	.00030818
Zinc	II	Cation	.00033874

sulphuric acid between electrodes that are not attacked by the products of the decomposition, the polarization is transitory; that is, it persists for a very brief time only. Oxygen and hydrogen are the products of the electrolysis and both being gases and insoluble in the electrolyte, they escape from the electrodes to the air almost as soon as they are liberated. When the electrolyzing current is stopped, a little of each gas remains clinging to, or in the electrolyte near, the electrode on which it was liberated and may cause a momentary current, but since a large proportion of the gases has escaped, what remains is quickly used up and the secondary current stops.

66. Permanent polarization is made use of in the so-called storage cells where the products of the decomposition of the electrolyte combine with the electrodes or are deposited on and become a part of the electrodes; moreover, they are conductors of electricity. When the electrolyzing, or charging, current is stopped, a current due to the E. M. F. of polarization may be obtained until, theoretically, all the products of the electrolysis have returned to their original condition and an equivalent amount of electrical energy has been given up.

67. In the case of the electrolysis of a solution of a salt between electrodes of the same metal as is contained in the salt, polarization does not occur. For example, take the electrolysis of a copper-sulphate solution between electrodes of copper. The electrolyte is split up into copper and the radical SO_4. The copper is deposited on the cathode and the radical attacks the copper of the anode forming copper sulphate. The result is that copper is simply transferred from the anode to the cathode. Both electrodes being always the same as to composition, it is obvious that one electrode will have the same affinity for the elements of the electrolyte as the other electrode, and therefore no polarization current will result. Different elements have different affinities for the same element and also different affinities for each other. Elements like oxygen, chlorine, etc. are said to be electropositive and in general the metallic elements are said to be electronegative. It is obvious that the elements can be arranged in a series, as shown in Table VI, such that each element is electropositive to those that follow in the list, and electronegative to those that precede it. The farther apart any two elements occur in the series, the greater is their affinity for each other, or the greater is the difference of their affinities for the same element. The same is true for the respective

E. M. F.'s of combination. The arrangement of the elements in such a manner is called the *electrochemical series*, or sometimes the *electromotive series*. The electric current generated by any two elements dipping in an electrolyte always passes from the more electropositive element through the electrolyte to the more electronegative element. The order of the elements in this series must not be taken as invariable, as a great deal depends on the temperature, concentration, and the chemical composition of the electrolyte.

TABLE VI

ELECTROCHEMICAL SERIES OF ELEMENTS

ELECTROPOSITIVE	Zinc	Mercury	Arsenic
Potassium	Iron	Silver	Selenium
Sodium	Cobalt	Antimony	Sulphur
Lithium	Nickel	Tellurium	Iodine
Barium	Thallium	Palladium	Bromine
Strontium	Lead	Gold	Chlorine
Calcium	Cadmium	Platinum	Oxygen
Magnesium	Tin	Silicon	Fluorine
Aluminum	Bismuth	Carbon	ELECTRONEGATIVE
Chromium	Copper	Boron	
Manganese	Hydrogen	Nitrogen	

PRIMARY AND STORAGE CELLS

68. It has been shown that an electric current passing through an electrolyte decomposes that electrolyte, the products appearing at the electrodes and, under the proper conditions, producing polarization. If these same substances are obtained from some source outside the cell, and are supplied to the electrodes in the proper form, their tendency to combine will set up an E. M. F. identical with the polarization E. M. F., and an electric current may be obtained. Such an arrangement is called a **primary, voltaic, or galvanic cell,** the two latter names being derived from the names of the two men, Volta and Galvani, who were the first great discoverers in this field of work. A simple primary cell is shown in Fig. 2, where Z is the zinc electrode and C the copper electrode; the arrows show the direction of the current furnished by the cell. A primary cell is, then, a device for the direct transformation of chemical potential energy into electrical energy. The action occurring in a primary cell may be considered as a case of reversed electrolysis, and is usually spoken of as voltaic action. Many primary cells are reversible; that is, after being discharged they may be restored to their original condition by sending an electric current through them in the proper direction. Such a cell may be used repeatedly without renewing any of the materials and is known as a **storage cell.**

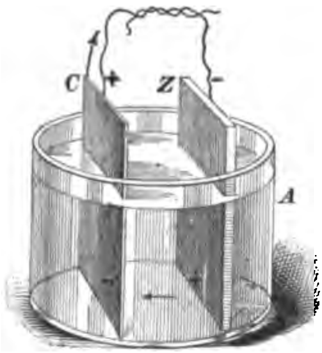

FIG. 2

69. Electrode Terminology.—The electrodes, or plates, of a primary or storage cell may be termed anodes or cathodes, as the case may be, but more frequently the terms *positive* and *negative* are used. Unfortunately, a great deal of confusion has arisen in the use of the latter terms. In primary batteries the zinc plate or anode is generally called the *positive electrode*, while the pole or point of connection between this plate and the external circuit above the electrolyte is called the *negative pole*, since it is the most negative point in the external circuit. The cathode (copper, carbon, platinum, etc.) is the *negative electrode* and its pole is *positive*, being the most positive point in the external circuit. These relations are shown in Fig. 2 by the use of + and − signs. In the case of the storage cell, it is customary in the United States to call the peroxide plate (cathode during discharge) the *positive electrode*, or *plate*, and its pole, the *positive pole*. The spongy metal plate (anode during discharge) is the *negative electrode*, or *plate*, and its pole is also *negative*.

The student should notice carefully the difference between the terms used in the two cases. The spongy lead and the peroxide of lead of storage-battery plates are called the *active materials*.

70. Nature of Voltaic Action.—A simple form of primary cell, Fig. 2, consists of an electrolyte of sulphuric acid with electrodes of zinc and copper. The radical SO_4 of the sulphuric acid has a greater affinity for zinc than for copper, so that when a complete circuit is established, it combines with the zinc and the hydrogen appears at the copper electrode. An electric current is produced and may be indicated by a galvanometer included in the circuit. Thus it is seen that the chemical action of a primary cell consists in the decomposition of the electrolyte, part of which attacks the anode and the other part appears at the cathode provided there is nothing at this electrode with which it can combine. This decomposition takes place throughout the space between the anode and the cathode; the hydrogen, however, does not appear throughout the electrolyte, but only at the surface of the cathode. As soon as a molecule of the electrolyte lying in contact with the zinc electrode is decomposed into its elements, the metal of the anode unites with those elements of the electrolyte with which it can combine (Zn with SO_4) and the free hydrogen atoms from the decomposed molecules that were in contact with the zinc plates combine with the SO_4 radicals of the next layer of molecules, thus displacing hydrogen atoms, which then combine with the SO_4 radicals of the third layer of molecules, and so on across the electrolyte until the last layer of molecules is reached, when the displaced hydrogen atoms from this layer of molecules, having nothing to combine with, remain free and appear as hydrogen gas at the copper plate or cathode. These decompositions and recompositions proceed across the electrolyte almost instantaneously, and since all but one of the molecules in a single line are reformed immediately after decomposition, the resulting energy is the same as if only one molecule took part in the reaction. This process is continued with extreme rapidity between all points of the opposed electrodes and furnishes a continuous electric current.

71. Voltaic Action Illustrated.—The accompanying diagrams, Fig. 3, represent the electrolytic action in the case of the zinc, *Zn*, sulphuric acid, H_2SO_4, copper cell. A single line of molecules of the electrolyte between the copper and the zinc electrodes is represented in (*a*). In this case the circuit is not completed or closed and the molecules are supposed to be arranged in an irregular order. For the sake of simplicity water is not represented as taking part in the voltaic action, though it probably does. Each small circle in the diagrams represents an atom or a radical and each large circle encloses a single molecule. When the circuit is completed, the molecules are supposed to instantly arrange themselves in lines with the SO_4 radical of each molecule, facing the zinc electrode, as shown in (*b*). The zinc then combines with the SO_4 radical of the nearest molecule of sulphuric acid. The two hydrogen atoms thus left free immediately combine with the SO_4 of the next molecule, thus displacing two hydrogen atoms, which immediately combine with the SO_4 of the third molecule, and so on across the liquid, as shown in (*c*). It is seen that the molecules, after a series of decompositions and recompositions, have their SO_4 radicals facing the cathode instead of the zinc anode, as at (*c*). They must therefore turn around before the next series of actions can take place. This explanation of voltaic action, while not accounting for all the observed phenomena, is simple and is therefore valuable to those beginning the study of electrochemistry and batteries.

72. Migration of Ions.—In the above explanation of electrolytic action in a cell, the liberated hydrogen came from the molecule nearest the cathode; it did not come across the electrolyte from the anode. There is, however, a movement of the ions. When the first hydrogen atoms are set free from the molecule nearest the anode, they combine with the SO_4 radical of the next molecule and so on across the liquid. In these recombinations the hydrogen atoms not only go to meet the SO_4 radical, but the SO_4

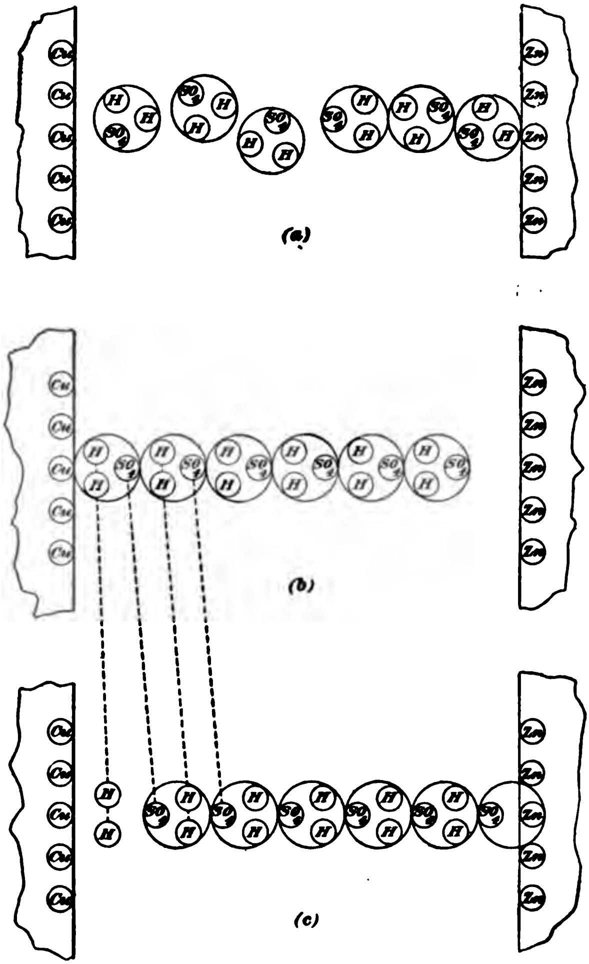

FIG. 3

radical goes to meet the hydrogen atoms. Hydrogen moves the faster and goes the greater part of the distance, so after a large number of decompositions and recompositions, the ions of a particular molecule have actually moved across the electrolyte; and since the hydrogen ions move more rapidly than the SO_4 ions, the electrolyte becomes more concentrated at the cathode (in the direction in which the hydrogen ion moves). Different kinds of ions move with different speeds and these speeds are called the **migration velocities of ions.** Table VII gives the approximate migration velocities for a few of the ions in water solution, at a temperature of 18° C., and with a difference of potential between the electrodes of 1 volt.

TABLE VII

MIGRATION VELOCITIES OF IONS

Cations	Centimeters per Hour	Anions	Centimeters per Hour
H	10.80	OH	5.60
K	2.05	Cl	2.12
NH_4	1.98	I	2.19
Na	1.26	NO_3	1.91
Ag	1.66		

73. Resistance of Electrolytes.—The resistance of the electrolytes is an important factor in battery work, since a large internal resistance in a cell reduces the amount of available current. A high internal resistance is not a serious defect where the cell is to furnish only a small current through a high external resistance. But where heavy currents are demanded the electrolyte must be as good a conductor as possible. Increasing the electrode surface decreases the internal resistance of a cell, but it is not always practical to increase this surface sufficiently to make up the difference between a high-resistance and a low-resistance electrolyte. For example, take two cells, one having an electrolyte of zinc sulphate and the other, sulphuric acid, both solutions being of such strength that their resistances have a minimum value; then in order for the cells to have the same internal resistance, the first must have an electrode surface about 20 times as great as the latter, the electrodes being the same distance apart in each case. In Table VIII the resistances of a few of the common electrolytes are given. The values given are the *specific resistances* (resistance between opposite parallel faces of a cube of the liquid 1 centimeter on a side). The first column gives the amount (weight) of acid or other substance dissolved in water. This is expressed as a percentage. The next column gives the specific resistance corresponding to the given percentage of substance in the solution, and the third column gives the temperature coefficient expressed as a percentage. For example, a solution of sulphuric acid and water composed of 5 parts by weight of acid and 95 parts by weight of water would have a specific resistance of 4.82 ohms. The resistance of all electrolytes decreases with increase in temperature and for each degree centigrade increase, this particular solution would decrease in resistance by 1.2 per cent.

With some electrolytes the resistance is least when the solution is strongest, that is, saturated; with others, the resistance decreases with increasing strength up to a certain point, from which point the resistance increases as the solution approaches saturation. Sodium chloride (common salt) and copper sulphate are representatives of the first class and zinc sulphate and sulphuric acid are representatives of the latter class. In Table VIII where a minimum resistance occurs for a given electrolyte, the resistance is printed in Italic. The resistance of the best conducting electrolytes is great compared with the resistance of the metals. The resistance of the best conducting sulphuric-acid solution is about 1,000,000 times that of copper.

The electrolyte of a primary cell is sometimes called the *exciting fluid*, or *excitant;* however, electrolyte is the best term to use.

74. Local Action.—When a piece of commercial zinc is used in a voltaic cell, the metal dissolves whether the external circuit is closed or not; this wasteful consumption of zinc is called **local action.** The hydrogen liberated at

TABLE VIII

RESISTANCE OF ELECTROLYTES

(*Kohlrausch, Wiedemann's Annalen*)

Composition Per Cent.	Nitric Acid HNO_3 Ohms	Nitric Acid Temp. Coefficient Per Cent.	Hydrochloric Acid HCl Ohms	Hydrochloric Acid Temp. Coefficient Per Cent.	Sulphuric Acid H_2SO_4 Ohms	Sulphuric Acid Temp. Coefficient Per Cent.	Silver Nitrate $AgNO_3$ Ohms	Silver Nitrate Temp. Coefficient Per Cent.	Caustic Potash KOH Ohms	Caustic Potash Temp. Coefficient Per Cent.	Zinc Sulphate $ZnSO_4$ Ohms	Zinc Sulphate Temp. Coefficient Per Cent.
5	3.90	1.5	2.55	1.6	4.82	1.2	39.3	2.2	5.84	1.9	52.3	2.2
10	2.18	1.4	1.59	1.6	2.57	1.3	21.4	2.2	3.19	1.9	31.4	2.3
15	1.64	1.4	1.35	1.6	1.85	1.4	14.7	2.2	2.36	1.9	24.1	2.3
20	1.41	1.4	*1.32*	1.5	1.54	1.5	11.6	2.1	2.01	2.0	21.9	2.4
25	1.31	1.4	1.39	1.5	1.40	1.5	9.50	2.1	1.86	2.1	*21.4*	2.6
30	*1.28*	1.4	1.52	1.5	*1.36*	1.6	8.11	2.1	*1.85*	2.3	22.9	3.0
35	1.31	1.4	1.70	1.5	1.39	1.7	7.18	2.1	1.97	2.4	28.5	4.0
40	1.37	1.5	1.95	1.5	1.48	1.8	6.44	2.1	2.23	2.7		
50	1.59	1.6			1.87	1.9	5.44	2.1				
60	1.96	1.6			2.70	2.1	4.80	2.1				
70	2.54	1.5			4.66	2.6						
80	3.76	1.3			9.13	3.5						

Composition Per Cent.	Copper Sulphate $CuSO_4$ Ohms	Copper Sulphate Temp. Coefficient Per Cent.	Magnesium Sulphate $MgSO_4$ Ohms	Magnesium Sulphate Temp. Coefficient Per Cent.	Sodium Sulphate Na_2SO_4 Ohms	Sodium Sulphate Temp. Coefficient Per Cent.	Alum $KAl(SO_4)_2$ Ohms	Alum Temp. Coefficient Per Cent.	Sodium Chloride $NaCl$ Ohms	Sodium Chloride Temp. Coefficient Per Cent.	Sal ammoniac NH_4Cl Ohms	Sal ammoniac Temp. Coefficient Per Cent.
5	52.3	2.2	39.3	2.3	24.8	2.4	39.3	2.0	14.9	2.2	10.9	2.0
10	31.4	2.3	24.1	2.4	14.7	2.5			8.33	2.1	5.67	1.0
15	24.1	2.3	*20.9*	2.5	11.3	2.6			6.15	2.1	3.89	1.7
20			20.9	2.7					5.14	2.2	2.98	1.6
25			24.1	2.9					4.70	2.3	2.50	1.5

the zinc represents the zinc dissolved without yielding useful energy to the external circuit, the energy being dissipated in the form of heat. The cause of local action may be seen by referring to Fig. 4, in which *a* is a piece of impure zinc suspended in the electrolyte *L*; *A* is a point where pure zinc is exposed to the electrolyte and *B* is a particle of some impurity exposed to the electrolyte. If the impurity is some metal or metalloid, such as iron or carbon, it is electronegative to the zinc and forms a closed circuit with the latter metal. The current will flow from *A*, through the electrolyte to the iron or carbon particle *B*, and the bar of zinc forms the external circuit of the couple. The arrow in the figure shows the direction of the current in the electrolyte. A stick of commercial zinc, in a sulphuric-acid electrolyte, is rapidly consumed by local action, on account of the large number of points of electronegative substances exposed, and the strongly electropositive character of the zinc. It is important that local action be prevented as much as possible so as to obtain the greatest amount of useful electric energy from a given weight of zinc. The purer the zinc, the less will be the local action on it; but chemically pure zinc is difficult to prepare and is therefore expensive, and the only practical way of lessening local action is by amalgamating the metal.

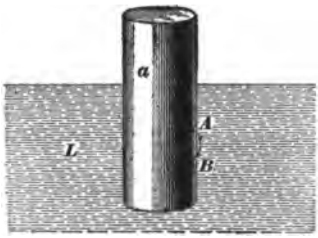

FIG. 4

75. Amalgamation.—The most common way of amalgamating zinc is to clean its surface with dilute sulphuric acid and then drop a little mercury on the cleaned metal, spreading the mercury around with a cloth until the entire surface of the zinc has a bright silvery appearance. The outer layer of zinc is thus alloyed with mercury, and as zinc is consumed from this alloy by voltaic action, the mercury strikes into fresh portions of the zinc, thus keeping the alloy rich in the latter metal.

When zinc is amalgamated it is in a large measure protected from local action. Zinc being more readily amalgamated than the impurities in it, passes to the surface and keeps the impurities covered up. In this way only pure zinc amalgam is exposed to the electrolyte. Assuming this explanation to be correct, we should expect that when the zinc becomes thin by dissolving away, the accumulated impurities would necessarily project through the amalgamated surface and cause local action to begin, and this is found to be the case. It is sometimes stated that the protection is due to a film of hydrogen gas collecting on the smooth surface of amalgam and preventing the electrolyte from coming into close contact with the metal. This explanation, however, is not very plausible.

76. Polarization and Depolarization.—If at or near the cathode there is some substance with which the free hydrogen left by the decomposition of the electrolyte may unite, the energy liberated by such formation will add to the E. M. F. of the cell. If this free hydrogen cannot unite with some substance at the cathode, it collects on the surface in bubbles as a gas. In addition to the reduction of the E. M. F. of the cell due to decomposition of the electrolyte, the formation of hydrogen also acts disadvantageously, as it forms in a layer on the surface of the cathode which enormously increases the internal resistance of the cell, thus diminishing the current that the E. M. F. of the cell can send through any given external resistance. The formation of hydrogen on the surface of the cathode is known as **polarization**, and its removal, by any means, mechanical or chemical, is called **depolarization;** the agent used is called the *depolarizer*.

77. If merely mechanical means of depolarization be used, the result is to prevent the increase of internal resistance of the cell; whereas, if the liberated hydrogen is caused to recombine at the cathode, by chemical means, not only is the internal resistance not increased, but the actual E. M. F. of the cell may be increased.

78. Mechanical and Physical Depolarization. Various mechanical and physical devices for depolarizing cells have been used; the cathode has been arranged to be agitated in the liquid, or to be entirely removed from the liquid at intervals; or the cathodes, and in some instances both electrodes, have been made in the form of disks, dipped for about half their diameter into the electrolyte. On rotating the disks, the hydrogen is prevented from remaining on the cathode by its motion. The power for performing these various movements has usually been derived from clockwork, and in some instances from the current given out by the battery. As examples of physical devices, increased cathode surface and roughened electrodes may be mentioned. It is obvious that with a large cathode surface the gas is distributed more than on a smaller surface, so polarization does not occur in so short a time. Roughened surfaces present numerous points from which the gas escapes more readily than from smooth surfaces. It is evident that such devices are commercially of little value, especially as chemical depolarizers may be easily used.

79. Chemical Depolarization.—The depolarization by chemical means may be accomplished by surrounding the negative element (cathode) with a solid or liquid substance, with which the free hydrogen may combine. This combination usually merely disposes of this element and prevents the bad effects of a deposit on the cathode. Under these circumstances the compound formed at the cathode is usually water, the depolarizer being a substance rich in oxygen, with which the hydrogen combines. This water has the effect of diluting the electrolyte, already weakened by the combination with the anode; but, by properly selecting the depolarizer with reference to the electrolyte, the chemical combination at the cathode may be such that it will, either directly or by further combination, replace the part of the electrolyte that has combined with the anode, thus keeping the electrolyte of the same composition and strength throughout the life of the anode or of the depolarizer.

Instances of both these classes of chemical depolarization will be noted in the description of the various cells. Chlorine gas is sometimes used as a depolarizer. It is somewhat soluble in water and so may be considered the same as a liquid depolarizer.

80. Rate of Depolarization.—The rate at which any depolarizer will act depends on many conditions; no depolarizer will keep the E. M. F. of a cell constant for all currents, for, after a certain limiting current has been reached, which depends on the sizes of the various parts of the cell, the formation of the free element of the electrolyte is more rapid than its absorption by, or combination with, the depolarizer, and the surplus will collect on the cathode. In the case of depolarizers that, by the formation of water, dilute the electrolyte, the E. M. F. will become less with continued use of the cell, even if the current output be small. These facts should be remembered in dealing with the various depolarizers.

81. Ampere-Hour Capacity of a Substance.—It has been shown that 96,538 coulombs of electricity will liberate 1 gram equivalent of any element or group of elements. It is also true that whenever 1 gram equivalent of any element combines with any other element, there is produced a quantity of electricity equal to 96,538 coulombs. It is then obvious that the greater the chemical equivalent of any element, the smaller will be the quantity of electricity that will be produced, for a given weight of the element, by its consumption in the voltaic cell. The electrical capacity of any metal, per unit of weight, depends on the chemical equivalent of that element and not on its specific gravity. For example, lead has a specific gravity of 11.3 and gold a specific gravity of 19.3, or gold is nearly twice as heavy as lead. The chemical equivalent of lead is 103.45 and that of gold is 65.73, or a little more than half that of lead and the dissolving of gold in the voltaic cell will produce nearly twice as many coulombs as the same weight of lead.

82. Calculation of Ampere-Hour Capacity of Metals. By a simple calculation the ampere-hour capacity per pound of substance consumed in a voltaic cell may be determined.

EXAMPLE.—How many ampere-hours should be given when 1 pound of zinc is consumed in a voltaic cell?

SOLUTION.— 1 gram equivalent of any element will give 96,538 coulombs. The chemical equivalent of zinc is 32.7; therefore, 32.7 grams of zinc will give 96,538 coulombs. 1 lb. (avoirdupois) = 453.6 grams. Therefore, 1 lb. of zinc will give $\frac{453.6}{32.7} \times 96{,}538 = 1{,}339{,}100$ coulombs. The number of coulombs, or ampere-seconds, divided by 3,600, gives the number of ampere-hours. Then, $\frac{1{,}339{,}100}{3{,}600} = 372.0 =$ number of ampere-hours due to the consumption of 1 lb. of zinc. Ans.

The value of 372 ampere-hours per pound is the theoretical capacity of zinc. The useful capacity, in practice, is less than the theoretical amount, owing to the impurities in commercial metal, losses in local action, etc. The ampere-hour capacity of any substance is the same whether it is consumed slowly or rapidly. Such conditions of the cell as voltage, and resistance of the electrolyte do not affect the ampere-hour capacity of the materials composing the cell. In other words, the capacity is a specific property of each substance and depends on its chemical equivalent.

83. General Formula for the Calculation of Ampere-Hour Capacity.—If we express 96,538 coulombs, or ampere-seconds, in ampere-hours we get $\frac{96{,}538}{3{,}600} = 26.82$. Then, letting W equal the chemical equivalent of the substance whose ampere-hour capacity is to be determined, and A equal the ampere-hour capacity per pound of substance,

$$A = \frac{453.6}{W} \times 26.82, \text{ or } A = \frac{12{,}165.6}{W} \qquad (2)$$

EXAMPLE.—If magnesium were consumed in a voltaic cell how many ampere-hours could be obtained, theoretically, per pound of metal consumed?

SOLUTION.—By referring to Table I it is seen that magnesium is a divalent element; its chemical equivalent, 12.18, is therefore half its atomic weight. In formula **2**, $W = 12.18$; hence, $A = \frac{12,165.6}{12.18} = 999$ ampere-hours per pound. Ans.

84. Efficiency of Depolarizer.—According to calculation, the active material of the positive plate of a storage battery should give 101.8 ampere-hours per pound. In practice, however, only from 30 to 40 per cent. of this output is obtained, owing to the fact that a part of the peroxide becomes surrounded by the insoluble sulphate formed during discharge, rendering it inaccessible to electrolytic action. In primary batteries, where the depolarizer is a solution, only a part of the depolarizer is available. As the discharge proceeds and the depolarizer is consumed, the solution becomes more and more dilute until, when a certain dilution has been reached, the depolarizing action is imperfect and the depolarizer must be renewed. The efficiency of the potassium-bichromate depolarizer of the plunge battery is about 45 per cent. of the theoretical. Nitric acid of commercial strength (specific gravity 1.4), containing about 66 per cent. of the pure acid, may be used for heavy currents until it contains only 39 per cent. of pure acid, or if only small currents are demanded, the acid solution may be used until it contains only 23 per cent. of pure acid. At this point the acid is too weak to depolarize and must be renewed. At the most, only 65 per cent. of the acid can be used before the liquid has become too weak to depolarize and more often only about 40 per cent. is used. Commercial nitric acid may be obtained with as low a specific gravity as 1.31. With such acid the percentage of pure acid available for depolarization is very low. In the case of copper-sulphate depolarizer, the product of reduction is metallic copper, which is deposited on the cathode. When the solution becomes too weak for efficient depolarization, it is not thrown away, because there are no products of reduction to saturate the solution and crystallize out, thus clogging up the cell. The depolarizing solution simply loses copper sulphate, and this is replaced from time to time by the addition of copper-sulphate crystals, thus keeping up the strength of the solution. When a copper-sulphate cell is in continued use, the depolarizer is almost completely used up. In the case of potassium-bichromate depolarizer, the products of reduction are potassium sulphate and chromium sulphate, which combine to form chrome alum. As the depolarizing solution becomes poorer in potassium bichromate it becomes more and more saturated with the reduction products. The solution must be thrown away from time to time to get rid of the reduction products, and each time a quantity of potassium bichromate must also be thrown away.

85. Battery Efficiency.—The efficiency of a battery, like that of any other device for producing power, is the ratio of the output to the input. The output of a cell may be expressed in ampere-hours or in watt-hours. The first does not take the voltage into consideration and hence is simply a measure of the total current output and not of the power output. The number of ampere-hours that should be theoretically obtained from the consumption of a given amount of material can be calculated, and the ratio of the ampere-hours actually obtained to the calculated number will give the **ampere-hour efficiency.** The ratio of the watt-hours delivered to the watt-hours represented by the total energy liberated by the chemical actions will give the **watt-hour efficiency** of a primary cell. This efficiency is less than the ampere-hour efficiency and is of most interest in connection with storage batteries, where the watt-hour efficiency is readily determined by taking the ratio of the watt-hours obtained on discharge to the watt-hours required to charge the cells. The ampere-hour efficiency of zinc in the primary battery can be made high (90 per cent. or above) by remelting and casting the scraps (pieces that have become too thin, parts that project above the electrolyte, etc.) and keeping the amount of local action as small as possible. Theoretically, 2 pounds of zinc and chlorine or 2 pounds of zinc and chromic acid should give about 1 horsepower-hour, and practically it ought not to require more than 3 pounds of either combination to give the same power. According to the figures given, we should expect a large amount of power from a small weight of battery, but since a large quantity of liquid is required to keep the materials in solution, and the weight of jars and accessories is considerable, the actual number of pounds of battery required to give a horsepower-hour is rather high.

86. Battery Equations.—Equations representing the chemical reactions that occur in cells are written the same as for any chemical reaction; the electrodes and the part of the electrolyte taking part in the discharge are written in the first member of the equation, and the products resulting from the discharge are written in the second member. The actions at the anode and those at the cathode may be expressed in separate equations or all the reactions may be expressed in one equation. For example, take the gravity cell in which the substances are copper, copper-sulphate solution, zinc-sulphate solution, and zinc (Cu, $CuSO_4$, $ZnSO_4$, Zn). The action at the anode is expressed, $Zn + H_2SO_4 = ZnSO_4 + H_2$; and that at the cathode is expressed, $CuSO_4 + H_2 = H_2SO_4 + Cu$. The sulphuric acid formed in this last reaction diffuses to the zinc, or anode, and furnishes the acid for the reaction expressed in the first equation. Both of these reactions may be expressed in one equation as follows: $Zn + H_2SO_4 + CuSO_4 = ZnSO_4 + H_2SO_4 + Cu$. Since in this cell sulphuric acid is both formed and decomposed, the result in the cell as a whole is expressed by the equation, $Zn + CuSO_4 = ZnSO_4 + Cu$. However, sulphuric acid is formed at one electrode (cathode) and decomposed at the other, and this affects the acid strength in the two parts of the electrolyte, until diffusion brings about a uniformity after the current is stopped. The total quantity of acid remains the same, but it is not always uniformly distributed; hence, the reactions in batteries may be viewed in two ways: (1) as total effects, and (2) as local effects occurring at the electrodes.

87. E. M. F. of a Cell.—In the electric cell the principal E. M. F.'s are set up at the junction with the electrolyte

(1) of the positive electrode, and (2) of the negative electrode. In cells having two liquids separated by gravity or by a porous cup, an E. M. F. may be set up at the junction of the two liquids, but it is generally unimportant.

If more than one set of actions can take place in a cell, the E. M. F. of each action must be added or subtracted to get the resulting E. M. F. according to the nature of the action. If the substance forming the anode has an affinity for one or more elements of the electrolyte, and the substance forming the cathode has an affinity for the other element or elements of the electrolyte, it is evident that the tendencies of these elements to combine with the anode and cathode, respectively, will assist each other, and the E. M. F.'s of each of the actions should be added together to give the resulting E. M. F. of the cell. When such depolarizers as nitric acid, lead peroxide, chromic acid, etc. are used in a cell, the result is an E. M. F. higher than the zinc alone could produce. Zinc has an attraction for the anion of the electrolyte, and the depolarizers have strong attractions for the cation (hydrogen) on account of the large amount of available oxygen they contain. If each of the substances forming the anode and cathode, respectively, has an affinity for the same element or elements of the electrolyte, it is evident that the tendency of these elements to combine with the anode will be partly balanced by their tendency to combine with the cathode; hence, the E. M. F. that would result from either action alone must be subtracted from the other to obtain the resulting E. M. F. In a copper-zinc cell having a sulphuric-acid electrolyte, both the anode and the cathode have an affinity for the same elements of the electrolyte, namely, the SO_4 radical. Zinc has the stronger attraction for this radical, and the difference between the two attractive forces determines the E. M. F. of the cell. If carbon had been used in place of the copper, the former having no affinity for the SO_4 radical, the full E. M. F. of the action on the zinc would appear.

It will be well to note at this point that zinc in contact with sulphuric acid has a greater E. M. F. than when the electrolyte contains zinc sulphate in solution in addition to the acid. The same is true for any metal and its salt. In a cell using zinc as the electropositive metal, the E. M. F. falls as the electrolyte around the zinc becomes more concentrated with zinc salt. A copper-zinc cell with a dilute sulphuric-acid electrolyte gives an E. M. F. of nearly .9 volt; the same cell having the copper electrode surrounded with a concentrated solution of copper sulphate gives an E. M. F. of nearly 1.1 volts. In the latter case, copper has a less E. M. F. opposing the E. M. F. at the zinc electrode, resulting in the increased E. M. F. of the cell.

88. The size of the electrodes does not affect the E. M. F. of a cell; a small cell having electrodes 1 millimeter square gives the same E. M. F. as a cell of the same materials having electrodes 1 meter square. The E. M. F. depends on the chemical properties of the materials used and not on the size of the electrodes nor on the distance between them.

89. Calculation of E. M. F. From Heat of Combination.—The combination of a gram-molecule of a substance with any other substance produces an amount of heat that is always the same (under the same conditions) for the same substance. Under the proper conditions this heat may appear in the form of electrical energy. The greater the affinity between the combining substances, the greater is the amount of heat or of electrical energy produced by their combination or required for their decomposition. The amount of energy is the same whether it appears as heat or electricity. The heat of any chemical reaction being known, the E. M. F. of combination can be calculated. It is known that $EQ = J$, where E is the E. M. F., in volts; Q is the quantity of electricity, in coulombs; and J is the energy, expressed in joules. Since 4.19 is the number of joules equivalent to 1 calorie, and if H is the number of calories evolved in the formation of 1 gram-equivalent of the substance whose E. M. F. of combination is to be calculated, $J = EQ = 4.19H$. Since H represents the heat evolved per gram-equivalent, we must let Q represent coulombs per gram-equivalent, and this we have shown to be 96,538 coulombs for any element or radical. The equation then becomes $E \times 96{,}538 = 4.19H$, or

$$E = \frac{4.19H}{96{,}538} = \frac{H}{23{,}040} \qquad (3)$$

Thus, dividing the number of calories of heat produced when 1 gram-equivalent of any substance is formed, by 23,040 gives the E. M. F. produced in the forming of that substance.

EXAMPLE.—Calculate the E. M. F. produced by the formation of water from its elements.

SOLUTION.—From Table II we find that the heat of formation of 1 gram-equivalent of water is 34,000 calories. Substituting the value for H in formula **3**, $E = \frac{34{,}000}{23{,}040} = 1.475$ volts. Ans.

NOTE.—In the case of binary compounds like water, the heat of formation and the heat of combination of one element with the other, amount to the same thing. With compounds like zinc sulphate, the heat of formation involves the combination of three elements—a reaction that does not take place at a single operation in a voltaic cell—however, the heat of combination of zinc with sulphuric acid involves a reaction merely between two substances and this reaction can take place in a voltaic cell.

It should be remarked that owing to the incomplete knowledge of thermochemical equivalents, and of the exact nature of the electrochemical actions in the cell, the E. M. F. of a cell can only in a very few instances be predetermined with accuracy. Secondary reactions often modify the result to such an extent that the E. M. F. actually obtained is quite different from that calculated from the energy liberated by the various reactions.

90. Choice of Anode Material.—From the preceding, it is seen that in order to give a high E. M. F., the metal chosen for the anode must be one whose salts have a comparatively high value for their heat of formation. Such metals are potassium, sodium, strontium, calcium, and magnesium; potassium salts have the highest heat of formation; the others, in the order given, have lower.

91. Suitability of Zinc for Anode.—Having a high heat of formation means, however, that a metal has a great affinity for the element necessary to form its salts or oxides;

this being the case, it is liable to combine with such elements whenever the opportunity presents itself, taking them from the air, from water, or from salts of other metals that have a lesser affinity for the salt-forming elements. Consequently, the metals in the list given could not be used in the presence of acids or solution of salts, or even of water, without decomposing the liquid and rapidly forming salts or oxides, nearly the whole of the energy of the action appearing as heat. In order, then, to have a practical cell, the metal should not be attacked by the electrolyte except when the cell is furnishing a current. This is the reason for the extensive adoption of zinc, it being a metal whose heat of formation is comparatively high, at the same time not high enough to cause its salts and oxides to be formed with any degree of rapidity when the cell is an open circuit. Besides, zinc is a cheap metal and, in proportion to the amount of chemical energy possessed, is cheaper than any other metal that can be used.

92. Carbon as Anode Material.—A great many investigators have attempted to use carbon in a primary cell in the place of zinc, but so far carbon-consuming cells have not been successful. Carbon cells would have the advantage of cheapness and lightness if the efficiency could be brought to the point reached by ordinary primary batteries. Theoretically, carbon will give 4,048.5 ampere-hours per pound; its E. M. F. of combination with oxygen, as calculated from its heat of combination with that element, is about 1 volt. Thus the calculated capacity of carbon is about 4,048.5 watt-hours per pound, while that of zinc combining with oxygen is about 670.0 watt-hours per pound. Notwithstanding the great advantage to be gained by the use of carbon, very little has been accomplished in this direction, and indeed, a number of authorities on this subject have pronounced it impossible to obtain electrical energy from carbon in this way.

93. Active Materials of Storage Batteries.—Up to the present time lead and its compounds have proved to be the materials best adapted for use in storage batteries; insolubility in the electrolyte used, and reversibility of the reactions, are the chief properties that give it the advantage. The high E. M. F. of the lead cell is also a strong point in its favor, a large part of the E. M. F. being due to the depolarizer PbO_2. Other metals, with their oxides, are now claiming attention in the storage-battery field; chief among these are iron and nickel (Edison cell) used in an electrolyte of potassium, or sodium, hydroxide. The E. M. F. of the new cell is not as high as that of the lead cell, but the ampere-hour capacity is much greater.

PRIMARY BATTERIES

VOLTAIC CELLS

INTRODUCTION

1. A **primary, voltaic,** or **galvanic, cell,** as it is variously called, is an apparatus for converting chemical energy directly into electric energy. The general conception of a primary cell includes the action of electrolysis; the cell consists of two conducting elements immersed in a solution that acts chemically on one element only or on one more than on the other. If the two elements or poles of the cell are joined by a continuous metallic wire or circuit, an electric current will flow in one direction through the metallic circuit as long as the circuit remains complete or closed, provided the chemical action is sufficient to maintain the electromotive force.

A **voltaic battery** is a combination of a number of separate voltaic cells properly joined together; however, the two terms, battery and cell, are used rather indiscriminately.

HISTORICAL

2. About the year 1786, Galvani, an Italian physiologist and physicist, discovered that frogs' legs suspended from an iron support by a copper wire, made violent movements when in contact with both the iron and the copper. Galvani supposed the movements due to a separation of positive and negative electricity at the junction of the nerves and the muscles.

Volta, another Italian physicist, showed that two dissimilar metals were necessary to the production of the phenomena. In 1799, he constructed a form of battery known as *Volta's pile*. Disks of zinc, wet cloth, and copper were piled up in the order given, thus producing a battery having a large number of couples joined in series and from which a considerable electromotive force was obtained. Later, Volta changed the form of the pile by substituting cups of salt water for the wet cloth, inserting copper and zinc strips in the liquid of the cups, and formed a battery of cells by joining the copper of one cup to the zinc of the next cup.

Fabroni, an Italian biographer, was perhaps the first to recognize that chemical reactions occur when dissimilar metals are immersed in water and brought into contact at some point, and stated (about 1791) that he believed the electric phenomena produced to be due to the slow combustion, or oxidation, of metal.

In 1801, Davy, an English chemist, experimented with acid electrolytes in primary cells. In 1828, Kemp, and later, Sturgeon, discovered the only practical method of reducing local action on zinc, that is, by amalgamation.

The important and well-known Daniell cell, Cu–$CuSO_4$–porous cup–$ZnSO_4$–Zn, was first described in 1836, and the Grove cell, Zn–H_2SO_4–porous cup–HNO_3–Pt, in 1839. The Smee cell, making use of a platinized silver negative electrode, appeared in 1840, and about the same time carbon was substituted for the platinum of the Grove cell. Bunsen, a German chemist of note, is generally credited with the latter improvement; he also developed the bichromate cell, $C\text{–}\begin{pmatrix} K_2Cr_2O_7 \\ H_2SO_4 \end{pmatrix}\text{–}Zn$. The Leclanché cell, using a solution of ammonium chloride as the electrolyte, was introduced about 1868. It has, perhaps, been modified more than any other primary cell; among these modifications is included the important class known as *dry cells*. The first successful dry cell was made by Gassner in 1888.

About the year 1865, cells using an alkaline electrolyte (potassium, or sodium, hydroxide solution) with a zinc anode and a copper oxide depolarizer in contact with an iron cathode, were investigated by Denys, Lalande, and Chaperon, and later by Edison and others. These investigations have resulted in the production of the Edison-Lalande and the Gordon cells—two cells that are in present use in the United States.

This brief account will give a general idea as to the length of time the primary battery has been in developing and also a few of the prominent names associated with this development.

CLASSIFICATION

3. Primary cells are frequently classed as *single-fluid* and *two-fluid cells;* but as such a classification has little reference to the principles of depolarization, it will not be used here. According to this classification bichromate cells would be treated in two separate classes. One class including those cells having the bichromate of potash depolarizer separated from the electrolyte by a porous partition or by gravity (one liquid being heavier than the other), and the other class treating of those cells having the depolarizer mixed with the electrolyte. It is readily seen that the two forms of the bichromate cell are the same in principle and should be treated under one head. The various cells will be here discussed in the four following classes:

1. *Cells Having No Chemical Depolarizer.*—This is the simplest form of primary cell, but on account of their rapid polarization, cells of this class are used only for intermittent work. They are commonly called *open-circuit cells.* This class includes those cells making use of mechanical and physical depolarization as well as those in which no method of depolarization is employed.

2. *Cells With a Liquid Depolarizer.*—In this class of cells the negative electrode, or cathode, is surrounded by a depolarizing liquid that may be separated from the electrolyte by

a porous partition or by gravity, or the depolarizing agent may be mixed with the electrolyte and is then in contact with both electrodes. The latter arrangement, however, is very apt to cause excessive local action.

3. *Cells With a Solid Depolarizer.*—This class is identical in action with the preceding one, the depolarizer, however, being a solid instead of a liquid. If the solid depolarizer is granular, or in the form of powder, it is often necessary to employ a porous partition between the cathode, which is surrounded by the depolarizer, and the electrolyte. This is merely to keep the depolarizer in place, and is dispensed with if the depolarizer is formed into a paste or solid body on the cathode. In fact, the depolarizer may itself form the cathode, if it be a solid conducting material, the office of the cathode being primarily to establish a connection between the electrolyte and the external circuit.

4. *Standard Cells.*—Since standard cells are designed for a special purpose (standards of electromotive force) and belong to more than one of the above classes, it is desirable to treat them together under a separate head.

Though a large number of different cells have been made, they are all included in one of these classes. Therefore, only a few typical cells of each class will be described, greater attention being paid to those cells that are commercially important.

4. Open- and Closed-Circuit Cells.—For practical purposes, primary cells may be roughly divided into two general classes: Those capable of furnishing, in a circuit of moderate resistance, a reasonably uniform current for quite a long time, and those capable of supplying a current only intermittently, and then only for a few seconds each time, but are able to stand for long intervals on open circuit without consumption of materials due to local action. The former are called **closed-circuit cells**, and the latter **open-circuit cells**. Some closed-circuit cells may be used to supply intermittent currents—that is, they may be used on circuits that are open the greater part of the time—

but open-circuit cells should never be used where a continuous current is required—that is, on circuits that are closed the greater part of the time. Gravity, Gordon, Edison-Lalande, and bichromate cells are samples of the closed-circuit type. Most closed-circuit cells deteriorate if left on open circuit too much of the time, and hence they are not usually suitable for intermittent work, where only small currents are required and the inactive periods are long. Leclanché and dry cells are the best examples of open-circuit cells. For intermittent work, for instance for electric bells and some types of telephones that are not in constant use, nor in use for long periods at any one time, good open-circuit cells are the most satisfactory. They are not, however, suitable where a continuous current is required, nor even for intermittent work unless the idle periods are sufficiently long and frequent to allow the cells time to recuperate.

5. Properties of a Good Primary Cell.—A good primary cell should have (1) a high and constant electromotive force; (2) a low internal resistance; (3) should give a constant current and therefore must be free from polarization; (4) should be free from local action; (5) the materials consumed should be cheap and efficient; (6) the electrolyte should be non-freezing; (7) the cell should require a minimum of attention for recharging and repairs; (8) and should not give off corrosive or poisonous fumes. No cell has yet been constructed that will fulfil all these requirements. In cases where the battery is kept in well-ventilated places, corrosive fumes may not be objectionable; in cases where the external resistance is great, as in telegraph work, a low internal resistance is not of much importance; in certain electroplating operations a high electromotive force is not needed; and when the work required of the battery is intermittent, as for ringing door bells, a constant current may not be necessary if the intervals of non-use are so distributed and of such duration that the electromotive force is kept near its normal value, and in such cases chemically depolarizing cells are not necessary.

From the description of batteries that follows, it will be seen that some cells are adapted to one purpose and some to another. Many of the cells of commercial importance will be treated in the several classes, and from the descriptions given it will be an easy matter for the student to determine the work for which a given cell is best adapted.

CELLS HAVING NO CHEMICAL DEPOLARIZER

6. Cells having no chemical depolarizer include those of the **Volta** type, which consist generally of an electrolyte of acid or saline solution, into which are placed two plates of metal, one of which (usually zinc) is acted on by the electrolyte. A simple form is illustrated in Fig. 1; its materials are zinc, dilute sulphuric acid, and copper, which give an electromotive force of about .9 volt. Hereafter electromotive force will be written E. M. F., an abbreviation very commonly used. Many modifications of the form of this type of cell have been suggested and used, such as making the elements in strips and rolling them around each other in a spiral form, with insulating material between, etc.; but all are open to the objection of rapid polarization.

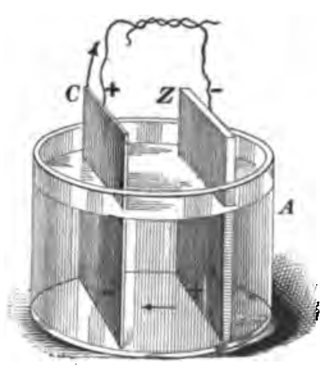

FIG. 1

In place of copper as a cathode many other elements have been used—for instance in the **Smee cell** a plate of platinum or platinized silver is used. A platinized silver plate is usually made of a corrugated plate of thin silver, the surface of which is covered with a very rough deposit of platinum. The escape of hydrogen from the cathode of the Smee cell is facilitated by the roughened surface of the deposited platinum, which presents a large number of points for the collection and escape of gas. In this case the depolarization is accomplished by physical means. The depolarization, however, is far from perfect.

Not long after the first use of the zinc–sulphuric acid–copper battery, it was found that the copper or other metallic cathode could be advantageously replaced with porous carbon, and many cells were so constructed. The E. M. F. of such a cell is about 1.35 volts ordinarily. To prevent the electrolyte from becoming exhausted too quickly, there is sometimes placed in the cell a porous earthenware pot or cup, filled with strong sulphuric acid. As the dilute acid outside the porous cup becomes weaker, the stronger acid diffuses through the sides of the porous cup and keeps up its strength. In some instances the carbon cathode has itself formed the porous cup. An objection to the use of porous cups in this type of cell is that its pores are liable to become clogged by deposits of zinc sulphate from the solution. The chemical reaction in this type of cell is expressed by the equation $Zn + H_2SO_4 = ZnSO_4 + H_2$.

Other acid electrolytes have been used in this type of cell. With either nitric or hydrochloric acid (diluted) the E. M. F. is not sensibly different from that with sulphuric acid as the electrolyte.

ZINC-AMMONIUM CHLORIDE-CARBON CELL

7. Of the saline electrolytes, the best exciting liquid is considered to be a solution of ammonium chloride (also called sal ammoniac). The E. M. F. of a zinc–ammonium chloride–carbon cell (without any depolarizer) varies from 1.3 to 1.4 volts, but when supplying a current of .2 ampere or more the E. M. F. soon drops lower and seldom regains its original value. The chemical reaction of this type of cell is expressed by the equation $Zn + 2H_4NCl = ZnCl_2 + 2H_3N + H_2$. The hydrogen passes off as a gas and the ammonia gas (H_3N), which is soluble in water, also passes off as a gas after the water becomes saturated with it.

To recharge an exhausted cell only a new zinc rod and a fresh solution of sal ammoniac are required. The carbon cylinder should be well soaked in water and exposed to the air and sun to remove the salts with which the pores become clogged.

8. There are a great variety of cells of this type in use for ringing bells, gas lighting, and doing other intermittent work. They are all alike in principle, but their mechanical construction differs somewhat.

In the *Law open-circuit cell*, shown in Fig. 2, the carbon electrode *c* is in the form of two hollow cylinders, one enclosing the other, as shown in section at (*b*). Each cylinder has a wide vertical slit in one side in which the zinc rod *z* hangs, being suspended from the cover *d*, which is made of

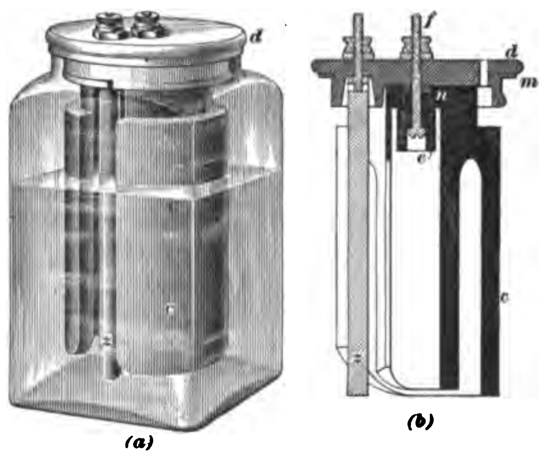

Fig. 2

compressed insulating material. A rubber band *m* under the cover produces a close fit between the cover and the jar. The terminal *f* is fastened by a nut on the top of the cover and makes contact through the carbon piece *e* with the cylinder *c*. This piece *e* is a separate piece of carbon and makes contact with the cylinders *c* only through a shoulder, the tightness of the joint depending on the nut on top of the cover. This arrangement does not seem very good. There is, however, a very soft rubber washer *n* that allows a tight joint to be made between *e* and *c*. The general appearance of the cell is shown at (*a*). The carbon element has a very large surface.

In the *Little Giant cell*, the hollow carbon cylinder is continuous, except for a hole in the side for the circulation of the electrolyte; the zinc, in the form of a rod, is suspended in the center of the carbon cylinder. The top of the cylinder is extended to form the cover of the cell, and the zinc is insulated from it by a porcelain bushing.

The *Laclede* is a similar cell, except that the carbon element is oval in shape, thus giving a larger amount of surface.

The *Hercules cell* employs a corrugated, solid, carbon cylinder, the zinc element being of sheet zinc bent into a cylinder surrounding the carbon; the two are prevented from touching by two thick rubber bands around the carbon cylinder.

Many other forms of carbon and zinc elements are used; the particular shape of the carbon has comparatively little to do with the satisfactory working of the cell, provided that the surface exposed to the liquids is very large; care and good design in the construction are more important. The element should be of such shape as not to be easily broken in transit, and, being usually molded into shape under pressure, should be of such proportions that it is cheap to make. The carbon should be made as porous as possible. Thus, the area of the internal circuit of the cell is made large, and at the same time advantage is taken of the slight depolarization occurring with a porous carbon of large surface, due to the oxygen that porous carbon absorbs from the air, and with which some of the evolved hydrogen combines. This depolarizing action takes place slowly, and, therefore, hydrogen forms on the cathode if a considerable current be taken from the cell, thus increasing the internal resistance. In intermittent work this is not objectionable, as the hydrogen is soon absorbed when the external circuit is opened. The connection between the binding post and the carbon in reliable cells is so made that there is little or no trouble from corrosion by capillary ascent of the solution. Corrosion may be caused by carelessly spilling the solution over the top and terminals.

Being very cheap and of common occurrence, sodium chloride is sometimes used. The E. M. F. of a zinc–sodium chloride–carbon cell is about 1.08 volts, which is somewhat lower than that obtained when ammonium chloride is used.

It has also been proposed to use sea-water as an electrolyte, by placing in the ocean immense plates of zinc and copper or carbon; this has never been commercially accomplished, for the consumption of zinc makes the cost of the electrical energy too great for this method to compete with others now in use.

Various other salts in solution have been used as electrolytes, such as ammonium nitrate, alum, potassium sulphate, zinc sulphate, zinc chloride, potassium hydrate (caustic potash), etc.

CELLS WITH A LIQUID DEPOLARIZER

9. Nitric acid, being rich in available oxygen, is often used as a depolarizing liquid. Its use is objectionable from the fact that, when deprived of a part of its oxygen, it gives off a gas, nitric oxide, which, on combining with the oxygen of air, becomes nitrogen peroxide, NO_2, a very disagreeable and even poisonous, corrosive gas; consequently, good ventilation is essential where cells with this depolarizer are used. The principal cells using this depolarizer are the *Grove* and *Bunsen*, and some of their derivatives.

GROVE CELL

10. The usual form of the **Grove cell** is shown in Fig. 3. The outer jar is rectangular in shape and contains a **U**-shaped positive element of zinc *z*. A narrow porous cup *d* is placed in the hollow formed by the zinc, and contains the strip of platinum *p* (negative element). The outer jar contains the dilute sulphuric-acid electrolyte and the porous cup is filled with the nitric-acid depolarizer. The E. M. F. of this cell varies from 1.9 to 2 volts. The chemical reactions are expressed by the equations $Zn + H_2SO_4 = ZnSO_4 + H_2$ and $H_2 + HNO_3 = HNO_2 + H_2O$.

Instead of the latter reaction the following is just as liable to take place: $3H_2 + 2HNO_3 = 4H_2O + 2NO$. The escaping nitric oxide, NO, combines with oxygen in the air, giving the disagreeable red, fuming, peroxide of nitrogen, NO_2.

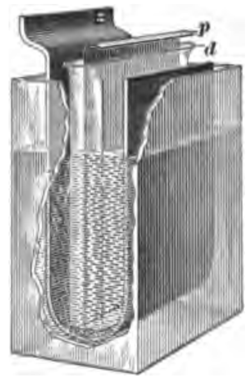

FIG. 3

The Grove cell is set up by filling the porous cup with strong nitric acid and the glass jar with diluted sulphuric acid, about twenty parts of water to one of acid. When not in use the plates should be removed from the solutions and washed and the nitric acid emptied out. Since the internal resistance of this cell is low and the depolarization good, comparatively large and constant currents may be obtained. The zinc must be kept well amalgamated or the local action becomes excessive.

BUNSEN CELL

11. The Grove cell is a very old type, and has been made in many forms, but the expensive platinum element has led to the adoption of the **Bunsen cell,** in which a carbon element is substituted for the platinum. The Bunsen cell consists of a carbon rod and nitric acid, as a depolarizer, placed in a round porous cup and the zinc and sulphuric acid in the jar. With commercial nitric acid, specific gravity about 1.33, the E. M. F. of the Bunsen cell is 1.89 volts ordinarily; if pure nitric acid, specific gravity 1.53, be used, the E. M. F. is increased to about 1.96 volts. About .35 volt is due to the action of the depolarizer. The chemical reactions are the same as for the Grove cell. Variations in the density of the nitric acid thus affect the E. M. F. of the cell only slightly, until the specific gravity of the solution falls to about 1.23; but at a density below this the acid has little or no effect as a depolarizer, although the liquid still contains about 30 per cent. of nitric acid.

As the commercial acid is most frequently used in the cell, only a small proportion of water is required to dilute it to a point where it cannot be used. In fact, where commercial acid is used, only about 13 per cent. of the actual amount of the pure acid in the solution can be utilized, if nitric acid alone be the depolarizer. In cases, however, where acid of 1.4 specific gravity is used a much greater percentage can be utilized. The water formed at the cathode by the process of depolarization, therefore, is disadvantageous on account of its dilution of the depolarizer.

Several investigators have mixed sulphuric acid with the nitric, in various proportions, with good results. Sulphuric acid has a strong affinity for water, and will combine with it in considerable quantity; consequently, the water formed at the cathode is absorbed by the sulphuric acid, leaving the nitric acid at its full strength.

Variations in the density of the exciting liquid also affect the E. M. F. of the cells to some extent, but not so much as a variation in the density of the depolarizer. The density ordinarily used is about 1.09 specific gravity (13 per cent. by weight of acid). At this point the E. M. F. due to the action of the exciting liquid on the zinc is about 1.53 volts. As the action of pure water alone on zinc will give an E. M. F. of about .9 volt, variations of the density of the exciting liquid from 13 per cent. (by weight) of acid down to pure water will reduce the E. M. F. about .6 volt. Increasing the density of the liquid to about 1.23 gives a maximum E. M. F. (due to the action of the acid on the zinc only) of about 1.6 volts; any further increase in the density does not increase the E. M. F. appreciably. To obtain the E. M. F. of the cell, to the above figures should be added the E. M. F. due to the action of the depolarizer, about .35 volt. It is somewhat difficult to maintain sulphuric acid, which has free access to the air, at a density much above about 1.10 on account of the absorption of water from the air by the acid; therefore, acid of about this density is ordinarily used.

The proportions of the two acids in the cells are about three of electrolyte to one of depolarizer, the depolarizer being of a specific gravity of about 1.33 to 1.4; with these proportions the cell will maintain its E. M. F. (within about 10 per cent.) for several days on a closed circuit. The average internal resistance of a quart size cell (as ordinarily constructed) is about 1 ohm. Such a cell will give about 1.25 amperes continuously until some material is exhausted.

12. Many modifications of the Grove and Bunsen cells have been made, some consisting merely in changes in the mechanical arrangement of the parts, others substituting various depolarizers, exciting liquids, or elements. For example, a carbon cup fitted with a tight cover has been used as cathode. When this is filled with nitric acid, the gas given off by the acid produces a pressure inside the cup that forces the acid through the pores of the carbon to the surface, where its depolarizing action takes place. This suppresses a part of the disagreeable fumes of the acid. To accomplish this same result, it has been proposed to cover the cell with an inverted vessel containing scrap tin, which will absorb the fumes. A layer of turpentine floating on the acid will prevent a large part of the fumes from being given off, as they combine with the turpentine. Where as large a current as this cell will give when nitric acid only is used as a depolarizer is not required, the following non-fuming solution may be used, all parts being taken by weight: Nitric acid (density 1.42), one part; chromic acid, three parts; sulphuric acid, six parts; water, five parts.

In the Bunsen cell it is difficult to maintain a good contact between the carbon and the terminal binding post.

ZINC–SULPHURIC AND NITRIC ACID–IRON CELL

13. When iron is placed in strong nitric acid it is not attacked, although this acid is a powerful oxidizing agent; iron in this condition is said to be in the passive state. When, however, the acid is diluted to a specific gravity of about 1.20, or lower, the iron is strongly attacked. Consequently, with a strong solution of nitric acid as a depolarizer, iron (usually cast iron) may replace the carbon element of the Bunsen cell, with good results, the E. M. F. being about 1.7 volts. Care must be taken, however, that the density of the depolarizer does not fall too low, or the negative element will be consumed. In fact, a cell of this class may be constructed with only iron and nitric acid as elements, in the following order: Iron (anode)–dilute nitric acid–porous cup–strong nitric acid–iron (cathode). If carbon is substituted for this iron cathode, we have the *Maiche cell*, whose E. M. F. is 1.5 volts; this cell gives a more constant current and less corrosive fumes than the Bunsen cell.

The E. M. F. of the zinc–sulphuric and nitric acid–iron cell is really generated in two parts: one at the surface of the anode, due to the action of the electrolyte (sulphuric acid) on the anode, and the other at or near the cathode, due to the action of the depolarizing liquid (nitric acid) on the hydrogen evolved. Varying the material of the anode or of the electrolyte will then affect that part of the E. M. F., and varying the depolarizer will affect the E. M. F. produced at the cathode.

BICHROMATE CELLS

BICHROMATE DEPOLARIZER

14. Another important depolarizer used in cells having a liquid depolarizer consists of a mixture of potassium bichromate and sulphuric acid. Sodium bichromate, or chromic acid, may be used in place of the potassium salt. Bichromate salts are derived from the oxide of chromium, having the formula CrO_3, technically known as *chromic acid*. This oxide when dissolved in water combines with a part of the water forming the true chromic acid, $H_2CrO_4 (CrO_3 + H_2O = H_2CrO_4)$. This acid forms a series of salts called *chromates* and a second series called *bichromates*. Potassium chromate has the formula K_2CrO_4 and potassium bichromate has the formula $K_2Cr_2O_7$. Bichromate salts have a large proportion of available oxygen and so are good depolarizers.

The chemical reactions in bichromate cells are rather complicated but are about as follows: At the anode, $Zn + H_2SO_4 = ZnSO_4 + H_2$; at the cathode, $K_2Cr_2O_7 + 4H_2SO_4 = K_2SO_4 + Cr_2(SO_4)_3 + 4H_2O + 3O$. The hydrogen of the first equation passes over to the cathode and there combines with the oxygen liberated in the second equation. It is obvious that three atoms of zinc must be dissolved to liberate enough hydrogen to combine with all the oxygen liberated by the reaction of one molecule of potassium bichromate. Representing both the anode and the cathode reactions in one equation, we have $3Zn + 7H_2SO_4 + K_2Cr_2O_7 = 3ZnSO_4 + Cr_2(SO_4)_3 + 7H_2O + K_2SO_4$.

The potassium sulphate and chromium sulphate combine to form a double salt called *chrome alum*, $K_2SO_4 + Cr_2(SO_4)_3 = 2KCr(SO_4)_2$. Chrome alum is not very soluble in water and so, as the reactions in the cell proceed, the double salt separates out in the form of crystals of a purplish color; these are apt to give trouble by adhering to the electrodes and to the porous cup (if one is used), thus causing an increase in the resistance of the cell. The crystals adhere firmly, and not being very soluble in water are difficult to remove.

When sodium bichromate is used the double salt formed has the formula $NaCr(SO_4)_2$; this salt is much more soluble in water than the corresponding potassium salt and does not give the trouble due to the formation of crystals, and hence is considered much preferable to potassium bichromate. If the sodium bichromate does not cost over 10 per cent. more than the potassium bichromate, it is the cheaper to use; because, for equal depolarizing capacity, about 11 per cent. (by weight) less of sodium bichromate is required.

If chromic acid is used in place of bichromate salts, no alum is formed. The chromium sulphate formed is very soluble in water. Both sodium bichromate and chromic acid give better results than potassium bichromate. It is claimed that the chromic acid is better than either of the bichromate salts. Its cost is a little greater, but its efficiency is enough greater to usually make up the difference in cost. Both chromic acid and sodium bichromate are readily soluble in water, while potassium bichromate requires heat to get a strong solution. This requires time and trouble and thus furnishes another reason for the preferred use of the acid or the sodium salt in bichromate cells.

GRENET CELL

15. A familiar type of bichromate cell is the **Grenet cell,** shown in Fig. 4, which consists of a bottle-shaped glass jar with a hard-rubber or porcelain cover. From this cover, two flat carbon plates *C, C* are suspended, parallel to and a short distance from each other; between them hangs a zinc plate *Z* supported by a sliding rod *R*, which may be drawn up, when the cell is not in use, until the zinc is entirely out of the liquid, so that local action may be prevented. It is held in any position by the thumbscrew *T*. On the top of the brass rod is a binding post B_1, the other terminal of the cell being the binding post *B*, which is connected to the two carbon plates.

FIG. 4

16. The **electrolyte for bichromate cells** may be made of three parts of potassium bichromate, dissolved in eighteen parts of water, to which is slowly added four parts of sulphuric acid, all parts by weight. The E. M. F. is 1.9 to 2.1 volts. At ordinary temperatures, variations in the proportion of bichromate in the solution, within moderate limits, do not change the E. M. F. or the internal resistance very much. Variations in temperature change the internal resistance, but not the E. M. F., the internal resistance decreasing as the temperature increases. With the above proportion of sulphuric acid and potassium bichromate in the solution, the sulphuric acid is first exhausted. Theoretically, for an equal life of both substances in the electrolyte, the correct proportions should be seven parts, by weight, of H_2SO_4, to three parts of $K_2Cr_2O_7$, which proportion is often used. In fact, however, it is more necessary to keep up the strength of the depolarizer, that is, the bichromate, so the first proportion will give better results.

If sodium bichromate is used in place of potassium bichromate, the percentage of salt and acid may remain about the same. The sodium salt is more soluble and hence a denser solution can be used, therefore replenishing is required less frequently. Crystals of the double sulphates of sodium and chromium do not form as with the bichromate solution; thus, the cells are more easily cleaned. Furthermore, sodium bichromate may be dissolved in cold water to form any density desired, which cannot be done with potassium bichromate. To make the solution for a battery, it is only necessary to place powdered sodium bichromate in the water and immediately add slowly the sulphuric acid. The mixture is ready for use as soon as it has cooled.

The battery solution, using chromic acid, is made by pouring 5½ pints of water upon 6 ounces of chromic acid and then adding slowly ½ pint of concentrated sulphuric acid, stirring constantly. Since about one-third less of chromic acid than of potassium bichromate is required, the cost of either as a depolarizer is about the same when the relative prices of chromic acid and potassium bichromate are 19 and 12 cents per pound, respectively.

17. Electropoion Fluid.—Bichromate mixtures are frequently sold by dealers in battery material under the name **electropoion fluid**; one of these mixtures is prepared as follows: Two parts, by weight, of sulphuric acid is mixed with four parts of water; in another vessel, one part of potassium bichromate is dissolved in three parts of boiling water; when cool, the two solutions are mixed together. This liquid, when diluted by not over five parts more of water, is suitable for use in most bichromate cells.

18. Plunge Batteries.—Cells of the bichromate type are often arranged to form what is called, in the United

States, a **plunge battery**, and in Europe, **Poggendorff's battery.** Such a battery does not differ in principle or material from the Grenet cell, but it usually consists of several cells connected in series so as to give an E. M. F. of 6 to 10 or more volts. All the elements being suspended from a wooden cross-bar, may be simultaneously raised out of, or lowered into, the liquid by a lever or windlass arrangement, as shown in Fig. 5, which represents a battery of five cells. The elements are of zinc and carbon, there being three plates of zinc *Z* and four of carbon *C* in each cell. The elements

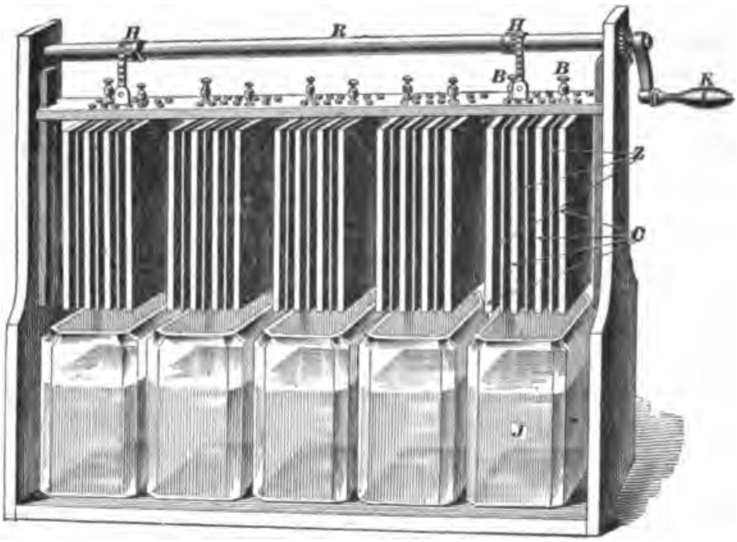

FIG. 5

should be raised from the liquid contained in the jars *J* when the cells are not in use. Each cell is provided with two binding posts *B*, *B*, one of which is connected to all the carbon and the other to all the zinc plates in the same cell. The various cells may then be used separately, or connected together in parallel or in series as desired.

By an ingenious arrangement of bichromate cells for cautery work, due to Chardin, the elements are normally held out of the liquid by a spring; by pressing a foot-lever they may be gradually lowered into the liquid. When just the ends of the elements are in the liquid, the internal resistance of the battery is considerable; but as the elements are lowered, and thus made to dip farther into the electrolyte, this resistance decreases rapidly. This arrangement provides a sensitive and easily managed method of control of the output of the battery.

19. Care of Bichromate Cells.—As a rule the electrolyte needs renewing when its color is a dark green. Should the color of the electrolyte be orange, and the cell yet show some weakening in its action, the addition of some sulphuric acid may improve it. If the bichromate cells are used daily, the electrolyte will need renewing in from 3 to 6 weeks. The elements should be removed and suspended in a jar with cold water in which about a tablespoonful of salt has been dissolved. Most of the impurities in the carbon will have been dissolved when the water has assumed a greenish hue. After a thorough soaking, the elements are rinsed off in cold water and thoroughly dried with a rag. They are then ready to be replaced in the battery. The importance of lifting the zincs out of the electrolyte when not in use should not be forgotten.

Trouble with bichromate plunge batteries is sometimes due to poor contact between the carbon and its metal terminal. In such cases the flat surfaces of the metal against which the carbon rests appear to make no connections. Good, clean, and firm contact should be made between the metal and the carbon, and to preserve this condition when once made, by excluding the battery solution from the joints, paint the whole, extending down the carbons about 1 inch, with a good insulating paint. Many carbons, when they have been in the solution awhile, have a tendency to crack or split at the connections and if not stopped the crack will extend, in the course of a little time, through the carbon. The insulating paint put on the joint as above directed seems to entirely prevent this cracking.

20. Amalgamation.—In many types of cells, including the bichromate, local action may be very much reduced and in some cases practically prevented by submitting the zinc to a process called **amalgamation.** By this means the iron and other impurities are separated from the zinc and made harmless. To amalgamate the zinc, it is first dipped in dilute sulphuric acid to clean the surface; then a little mercury is poured over it and rubbed into the surface with a rag or a piece of galvanized iron. When finished the surface should be as bright as silver. Another way to amalgamate zinc is to immerse it in an acid solution of mercuric nitrate. A small portion of the zinc is changed to zinc nitrate and liberates mercury, which combines with the remaining zinc; the result by either method, is that the whole of the zinc is covered with a pasty amalgam.

Where it is necessary to frequently amalgamate a large number of zincs the following solution may be used: Mix 1 pound of nitric acid with 2 pounds of hydrochloric acid, and add 8 ounces of mercury; when the mercury is dissolved, add 3 pounds more of hydrochloric acid. To amalgamate the zinc, immerse it in this solution, for 1 or 2 seconds, then remove it quickly to a dish of clean water, and rub it with a brush or cloth, when it will be found to be covered with a fine, even coat of mercury. This solution can be kept in a covered jar and used many times.

It is a peculiarity of the amalgam that it does not leave the zinc when the latter dissolves, but immediately attaches itself to fresh portions of the zinc plate. The surface will therefore appear bright and clean for a very long time. The theory of the protection afforded zinc by its amalgamation has been given in a previous Section. Another, and very effective, method of amalgamating zinc will be described in connection with the Harrison cell.

21. The bichromate cells so far described have the depolarizer mixed with the electrolyte, thus making them **single-fluid cells.** In these bichromate cells the local action on open circuit is apt to be excessive, hence the use of arrangements by which the zinc can be lifted from the electrolyte when the cell is not in use. Bichromate cells in which the depolarizer is separated from the electrolyte by a porous partition or by the difference in gravity between the two liquids, are the same in principle as those just described. Cells using liquid depolarizers separated from the electrolyte are sometimes called **two-fluid cells.**

FULLER CELL

22. The **Fuller bichromate cell,** one form of which is represented in Fig. 6, is a very excellent two-fluid cell, being economical in operation. It consists of a glass jar containing the depolarizer (electropoion fluid diluted about one-half), into which is hung the carbon cathode *C*. In the center of the jar is placed the porous cup *P*, into which is poured a little mercury, and the zinc, which is in the form of a rod or wire *W*, with a conical lump *Z* cast on the end, placed in position. The mercury serves to keep the zinc well amalgamated. The exciting liquid is either very dilute sulphuric acid, or, more commonly, pure water. The E. M. F. is 2.14 volts, and the internal resistance (of the type shown in Fig. 6) usually about 1 ohm, depending, however, on the thickness and character of the porous cup. There is very little local action in this cell when on open circuit and it does not polarize when in use, provided the current taken from it is not excessive. Although not really necessary, it is better to keep the porous cup, when not in use, in water in order to prevent disintegration of the cup due to the formation of crystals in its pores. To aid in preventing

FIG. 6

the diffusion of the two solutions, their surfaces should be on the same level in both the porous cup and glass jar.

This type of cell is largely used for telegraphic purposes in England, where it lasts, when not overworked, 1 year, the acid being renewed about ten times and the depolarizer five times. At the end of the year the cells are cleaned out and set up with new zincs and fresh solutions. It is not used in the United States on regular telegraph lines, but has been used quite extensively in telephone systems. For this purpose the cell has the disadvantage of being very unpleasant to handle, on account of the nature of the solution, and the fact that it produces very serious damage to whatever it happens to be spilled on. It has the advantage, however, of being able to produce a high and constant E. M. F. (2 volts), which it maintains for a considerable period when supplying as large a current as .6 ampere.

23. The cell used by The American Bell Telephone Company is termed the **standard Fuller cell,** and is the same as that shown in Fig. 6. In setting up this cell the following solution is generally used: Sodium bichromate, 6 ounces; sulphuric acid, 17 ounces; soft water, 56 ounces. If bichromate of sodium is not obtainable, bichromate of potassium may be substituted for it in equal quantities.

In mixing bichromate solutions great care should be taken to pour the sulphuric acid into the water very slowly. If the operation is reversed, the sudden formation of steam, due to the heat generated by the union between the acid and the water, is very likely to cause an explosion, throwing acid in all directions, and frequently doing much damage. It is well, also, to mix the solution in an earthenware jar, or, if it be mixed in the glass battery jar, the latter should be previously placed in a vessel containing cold water, in order to prevent the great heat produced from cracking the jar. After having mixed the solution, the jar should be a little less than half filled with it, and the porous cup put in place. In the bottom of the porous cup should be placed about a teaspoonful of mercury, after which the zinc electrode is put in place and the porous cup filled with water. A tablespoonful of common salt (3 ounces to a pint of water), or zinc sulphate (6 ounces to a pint of water), added to the water in the porous cup, will hasten the action and does not seem to have any bad effect whatever on the cell.

PARTZ CELL

24. Bichromate cells are often constructed in which the liquids employed have such a difference in their specific gravities that they may be placed one over the other in the cell, no porous partition being required to keep them from mixing The **Partz cell,** one form of which is illustrated in Fig 7. is an example. This cell is a bichromate cell, which uses a solution of sodium chloride, or of magnesium sulphate, as an electrolyte, surrounding the zinc *Z*, and a bichromate solution as a depolarizer, surrounding the carbon cathode *C*. The depolarizer, having a higher specific gravity than the electrolyte, remains at the bottom of the jar, and the two liquids are kept separate. As the depolarizer is weakened by use, it is from time to time strengthened by the introduction of crystals in the glass tube *T*, which is suspended in the cell, having a small opening below the normal level of the bichromate solution. The crystals used are what the manufacturers call *sulphochromic salt*, which is formed by the action of sulphuric acid on potassium bichromate, and when dissolved in water

FIG. 7

gives the same results as the electropoion fluid previously mentioned.

The cell shown, which employs a jar 6 inches by 8 inches, has an internal resistance of about 1 ohm with a solution of magnesium sulphate, and about .5 ohm with a solution of sodium chloride, the E. M. F. being the same, 1.9 to 2 volts, in either case. This cell is good for either open- or closed-circuit work, as the depolarization is very complete; at the same time, the local action on open circuit is almost imperceptible. The chrome-alum solution that forms, being heavier than the bichromate solution, descends to the lower part of the cell, so that the crystals form beneath the carbon plate, which is slightly raised from the bottom of the jar; consequently, the formation of these crystals does not appreciably increase the internal resistance of the cell.

25. Many of the nitrate and sulphate salts have been used as depolarizing liquids, and with a variety of electrolytes, generally acids; but the principal type of this class of cell, other than the Bunsen and the bichromate, is the type that employs as an electrolyte a salt of the metal of the anode, and as a depolarizer a salt of the metal of the cathode. The depolarizer is usually a salt formed by the same acid that formed the electrolyte salt; that is, if the electrolyte be a sulphate, the depolarizer is also a sulphate. In this case the action is about as follows: The SO_4 radicals attack the anode forming the sulphate of that metal; hydrogen decomposes the sulphate depolarizer, forming sulphuric acid and liberating the metal. The SO_4 of the sulphuric acid thus formed, combines with the anode; the hydrogen reduces the depolarizer, liberating the metal and forming more sulphuric acid, and so the action continues. The electrolyte, therefore, is continually added to, while the depolarizer is continually reduced.

Neglecting the intermediate reactions, which generally do not affect the E. M. F., it is evident that the E. M. F. of this type of cell is due to the energy given up by the formation of the salt of which the electrolyte is composed, less the energy required to decompose the salt of which the depolarizer is composed. Now, whatever may be the actual energy of the formation of the various salts, the difference between the energies of formation of the same salts of any two metals is the same, whatever the particular salt may be; for example, the difference between the heat formation of zinc sulphate and that of copper sulphate is the same as the difference between the heats of formation of zinc nitrate and copper nitrate.

For commercial use, the same considerations apply as to the other classes; that is, the materials used in the cell must be easily and cheaply obtained, even if they do not result in the highest possible E. M. F. The cells that best realize this condition are the Daniell cell and its derivatives.

DANIELL CELL

26. The **Daniell cell** uses for the anode, zinc; for the electrolyte, a solution (usually) of zinc sulphate, $ZnSO_4$; for the cathode, copper; and for the depolarizer, a solution of copper sulphate, $CuSO_4$. Sometimes in setting up the cell, dilute sulphuric acid is used instead of the zinc sulphate, but this soon forms a solution of zinc sulphate; hence, the result is the same as if the zinc sulphate were used originally. The E. M. F. of the Daniell cell is given several values by different investigators, ranging from 1.059 to 1.079 volts. The original form of the Daniell cell consisted of a glass jar, into which the zinc, in the form of a cylinder, was placed. Inside the zinc was a porous cup containing the cathode, a strip of sheet copper. The porous cup was filled with the $CuSO_4$ solution and the outer jar with the $ZnSO_4$ solution.

To prevent the gradual weakening of the depolarizer, it is usual to put a considerable amount of copper-sulphate crystals (commonly known as blue vitriol or bluestone) into the porous cup. As the liquid weakens, the crystals are gradually dissolved. Several modifications of the form of the original Daniell cell are in use, some of them designed to keep up the supply of copper sulphate as it is used.

The specific gravity, at ordinary temperature, of a saturated solution of $ZnSO_4$ is about 1.44, while that of a saturated solution of $CuSO_4$ is about 1.20; hence, if saturated solutions of these salts are used, the zinc-sulphate solution will be considerably heavier than the other; it has been found, however, that the best results are obtained from a saturated solution of copper sulphate, used with a solution of zinc sulphate diluted to have a specific gravity of about 1.10. The considerable difference in weight between the two solutions has led to their arrangement, one over the other, in the cell, the heavier copper sulphate being at the bottom. Daniell cells that depend on the difference of the specific gravities of the two liquids to keep them apart, are called gravity Daniell cells, or simply *gravity cells*. They are very extensively used for telegraph and fire-alarm work in the United States. As long as a current is flowing through the cell, the chemical action keeps the boundary line of the two liquids sharply defined; but when the current ceases to flow, the solutions gradually mix, and the copper sulphate, coming in contact with the zinc anode, sets up local actions, which cause a deposit of copper (appearing as a finely divided black coating) on the zinc, and a consumption of the zinc itself. To prevent this action, these cells should be used only on a circuit that is closed nearly all the time, which is the case on the telegraph and fire-alarm lines.

CROWFOOT CELL

27. The form of gravity Daniell cell most used in the United States is the familiar **crowfoot cell**, illustrated in Fig. 8, where Z is the zinc, from the shape of which the cell gets its name; C is the copper, which is connected to the external circuit by the wire W, which is insulated where it passes through the liquid. When the cell is set up the copper cathode is surrounded with copper-sulphate crystals. The standard form of this cell is of the following dimensions: The jar is 6 inches in diameter and 8 inches high. The copper element is made from three pieces of thin sheet copper 2 inches wide and 6 inches long riveted together in the middle; the outside pieces are then spread out, forming a six-pointed star. To the middle strip is riveted a piece of No. 16 insulated copper wire. The zinc usually has the shape shown in the illustration, and weighs 3 pounds. Many other forms of zincs are used more or less. The cell furnishes a working E. M. F. of 1 volt. For continuous working the most economical current output is about ¼ ampere. Its internal resistance varies considerably, depending on its condition, but 3 ohms may be taken as an average value.

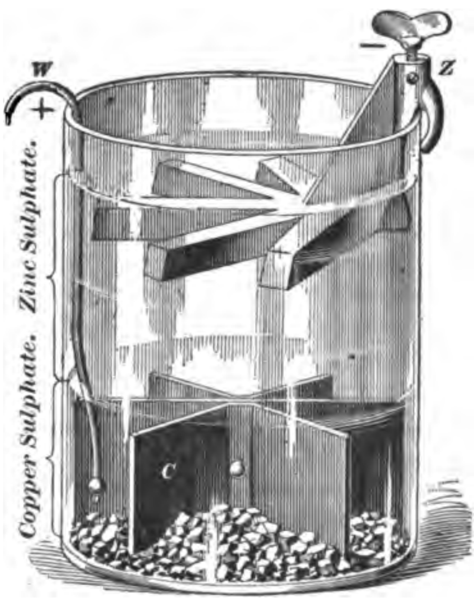

FIG. 8

Minotto's cell is a modification of the Daniell gravity cell. The copper plate is placed at the bottom of a glass jar, over it a layer of copper sulphate crystals, then a disk of cloth or canvas, then a layer of sawdust or sand, then another disk of cloth, on which rests the zinc element. The jar is filled with enough zinc-sulphate solution to cover the zinc. As it will require some time for the materials to become saturated, it is necessary to moisten the sawdust with the zinc-sulphate solution before placing it in the cell, if it is to be used immediately. A lead vessel is sometimes used in place of a glass jar. The sawdust acts as a sort of porous partition to assist in keeping the copper and zinc sulphates separate. The reactions in this cell are exactly the same as in the Daniell gravity cell. This cell has a very high internal resistance, usually from 11 to 19 ohms. However, it is simple in construction and requires no more attention than the ordinary crowfoot gravity cell. It is used in some countries for telegraph and signal circuits.

28. **Directions for setting up the crowfoot gravity cell** are as follows: Unfold the copper strip so as to form a star and place it in the bottom of the jar. The point where the copper connecting wire is riveted to the copper electrode should be near the bottom of the cell, and the insulated covering on the wire should come close to the riveted joint. Suspend the zinc about 4 inches above the copper by hooking the trip, or lug, on the side of the jar. The lug has a hole in it to receive a connecting wire. The method of suspending other forms of zincs will be evident from their construction. Pour sufficient clean water into the jar to cover the zinc and drop in the copper sulphate in small lumps. About 3 pounds is the proper amount to put in a cell to be used for heavy, continuous work, for instance, for the local-circuit batteries that run telegraph sounders; for the batteries in a main-line telegraph circuit, a smaller charge will be sufficient, and, in quadruplex telegraph circuits, the so-called "long" end of the battery will need less bluestone than the "short" end, because the former is not worked as continuously as the latter. The internal resistance may be reduced and the battery made immediately available by drawing about ½ pint of solution of sulphate of zinc from a battery already in use, and pouring it gently into the jar; or, when this cannot be done, by putting into the jar 4 or 5 ounces of pulverized sulphate of zinc previously dissolved in a cup of water. If there is no hurry for the cells, do not put in the zincs until the solutions have had time to settle to their normal conditions, which will require at least 24 hours. This prevents or reduces the formation of a black deposit on the zinc. When there is much of this black deposit, remove the zinc and brush or scrape it off. If no zinc sulphate is added in setting up the cell, it will be necessary to short-circuit the cell for some time (24 hours will not be too long) before it will be in good condition.

29. Caring for Cells.—Blue vitriol should be dropped into the jar as it is consumed, care being taken that it goes to the bottom and not on the zinc. The need of the blue vitriol is shown by the fading of the blue color, which should be kept at least as high as the top of the copper, but it should never reach the zinc. There should always be some bluestone crystals in the bottom of the jar.

After the battery has been started, no further attention is required, except to keep it supplied with bluestone and water, until the quantity of sulphate of zinc in solution has become too great. As long as the battery continues in action, there is an increase of the quantity of sulphate of zinc in solution in the upper part of the jar. When this becomes too dense (above 1.15 specific gravity), it will be necessary to draw out a portion of the top of the liquid with a battery syringe or a cup and replace it with clear water. A *hydrometer* is convenient for the purpose of testing the strength of this solution. A hydrometer usually consists of a small glass tube, the lower end of which is enlarged and partially filled with fine shot or mercury. The tube, when placed in a solution, floats in a vertical position. Some hydrometers are graduated by experiment with solutions of known specific gravity so that the scale indicates the specific gravity directly. When graduated according to the scale known as the *Beaumé*, the hydrometer is floated in water, and the point on the stem on a level with the surface of the water is marked 1°; then it is floated in strong undiluted sulphuric acid, and the corresponding point marked 65°. The intervening space is divided into 64 equal divisions, called *degrees*. Hydrometers will be more fully described and illustrated in connection with storage batteries.

When the specific gravity of the solution in the gravity cell is less than 15° on the Beaumé hydrometer scale, there is too little sulphate of zinc; when it is 30° or over, there is too much in solution, and it must be diluted. When the zincs become coated so as to interfere with the proper action of the battery, they must be taken out, scraped clean, and washed.

A gravity cell can be maintained in excellent condition by keeping it on a closed circuit about 60 per cent. of the time. If kept on open circuit too long a time the solutions mix and the cell is not ready for immediate use.

30. Cleaning Cells.—Cells that are used constantly should be cleaned out about once every 3 months. To do this, carefully remove the zinc, clean it by scraping with a knife, and wash it with plenty of water. Pour the clear liquid into a separate jar, leaving behind the oxide and dirt that may have gathered in the bottom of the jar. Now take out the copper, clean it and the jar, throwing away the sediment. Replace the copper, put around it some bluestone crystals, pour the clean liquid back into the jar, replace the zinc, and, without disturbing the liquid any more than is necessary, add enough water to cover the zinc. The battery will soon be ready for use, and short-circuiting the cell or battery should bring it into condition very rapidly. Some question the advisability of using any of the old solution over again, preferring to use only fresh solution, but this requires short-circuiting the battery for at least 24 hours, in order to bring it into working order, consuming both time and battery material. A fresh solution will, without doubt, give the best results where time and expense are not important.

31. The condition of a gravity cell may be judged from its appearance. When the cell is in good order, the solution is bright blue in color, the blue fading to a colorless solution before reaching the zinc. The batteries should not be allowed to freeze, for while frozen, the current is very much reduced or altogether stopped. Below 65° or 70° F., the internal resistance of a battery increases very rapidly. A battery works much more vigorously while warm, for heat is a promoter of chemical action. The connections should be kept free from dirt and corrosion in order to allow the current a low-resistance path through them.

32. Creeping of Salts.—If no precautions are taken to prevent it, the battery jar may after a time be found covered with crystals of salt adhering to the sides of the jar above the liquid and even extending over the top of the jar. These crystals, besides giving a dirty appearance to the cell, represent a considerable loss of salt from the cell and may also cause leakage of current. This **creeping of salt** is a phenomenon of capillarity. The solution, by capillarity, rises a little distance on the sides of the jar, and the water, evaporating, leaves crystals of the salt. More of the solution rises through the salt crystals and creeps a little higher and more crystals are deposited. This action is repeated and the salt gradually creeps over the sides of the jar. Strong solutions creep worse than dilute ones. The white zinc sulphate that creeps over the jars is a very fair conductor of electricity when moist, and if it extends from cell to cell, may cause considerable leakage and consequent waste of current and battery material. The creeping of salts in this and many other types of cell over the side of the glass jar is generally prevented by coating the upper part of the jar with paraffin. To do this, dip the inverted jar to a depth of about $\frac{1}{2}$ inch in a shallow dish of melted paraffin. Tip the jar sidewise, otherwise the air confined in the inverted jar will not allow the melted paraffin to rise to sufficiently coat the inside of the jar.

33. Oil on Gravity Cells.—Oil over the top of the solution will not only prevent the creeping of salts, provided it is poured on before the creeping commences, but it will also prevent evaporation of the solution. The oil makes it more difficult to clean the cell, but it saves the time that would otherwise be required for replenishing the cells with water. The oil may be removed with sand and a wet cloth. The advisability of using it is a disputed question and depends on local conditions. As common oil is very apt to rot the insulating covering of the wire running through it to the copper element, only a good quality of petroleum lubricating, or heavy paraffin, oil should be used.

CELLS WITH A SOLID DEPOLARIZER

34. The depolarizers that are used in this class of cell are generally substances containing a large proportion of oxygen, with which the free hydrogen unites, forming water; the remainder of the depolarizer is sometimes dissolved in this water, but more often remains at the cathode in a solid form, the water merely serving to dilute the electrolyte. In the first case, the solution formed usually acts to keep up the strength of the electrolyte. Some few of the elements that exist in the solid state, such as the metalloid tellurium, will unite directly with hydrogen, and might be used as depolarizing cathodes. Such elements are rare and are not used in commercial forms of cells.

Among the most widely used depolarizers are the oxides of manganese, copper, and lead, and the chlorides of some of the metals. The sulphates of mercury also have a large proportion of oxygen, or its equivalent in the form of the SO_4 radical, and are used for this purpose.

LECLANCHÉ CELL

35. The **Leclanché cell** is a well-known and widely used cell of this class. Its positive element (negative electrode) is zinc, usually in the form of a rod; the electrolyte is a solution of ammonium chloride, NH_4Cl (also called sal ammoniac); and the negative element is carbon, surrounded by manganese dioxide, MnO_2 (also called black oxide, or peroxide, of manganese), which is the depolarizer. This being in the form of a coarse powder, it is usually contained in a porous cup, which allows free access of the electrolyte to the depolarizer and negative element. Fragments of crushed coke (or carbon in other forms) are often mixed with the manganese dioxide to decrease the resistance of the contents of the porous cup.

Fig. 9 shows the usual form of this type of cell. The porous cup *P* contains the manganese dioxide and the carbon electrode, which projects from the top of the cup, and to which a binding post B is attached. The binding post is often placed on the side instead of the top of the carbon. The glass jar is circular, with a contracted top, in which a slight recess is usually formed to contain the zinc Z. The top of the zinc is provided with a binding screw B_1, which serves as the negative terminal of the cell, B being the positive. The top of the jar is coated with paraffin to prevent the creeping of salts over the top of the jar.

The cell illustrated in Fig. 9 is of the following dimensions: Jar, $4\frac{1}{2}$ inches in diameter, $6\frac{1}{2}$ inches high; zinc, $\frac{3}{8}$ inch in diameter, $6\frac{1}{2}$ inches high; porous cup, 3 inches in diameter, $5\frac{1}{2}$ inches high; carbon, 6 inches by $1\frac{1}{2}$ inches by $\frac{5}{16}$ inch high. The weight of the zinc rod is about 3 ounces, about two-thirds of which is below the level of the liquid. There are about 16 ounces of peroxide in the porous cup, and it requires 4 or 5 ounces of ammonium chloride to make sufficient solution for this size of cell. For each ounce of zinc consumed in the cell, 2 ounces of manganese dioxide and 2 ounces of ammonium chloride must also be consumed; so, from the amount of these materials contained in the cell, it follows that there is enough peroxide in the porous cup to last while four zincs are being consumed, while the ammonium chloride will not last longer than one zinc. As the zincs are usually replaced when eaten away to about $\frac{1}{8}$ inch or $\frac{1}{16}$ inch diameter, the solution need not be replaced until about two zincs have been consumed, and the contents of the porous cup will last

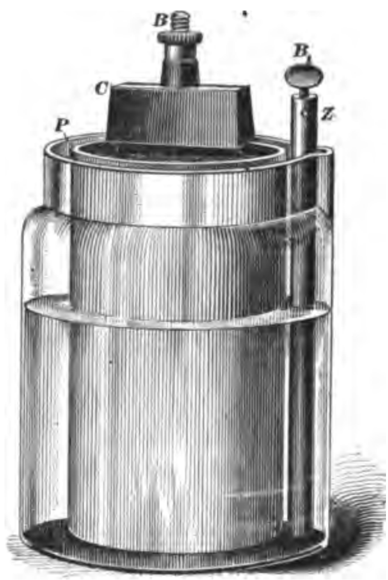

FIG. 9

as long as five or six zincs. The consumption of zinc in the Leclanché cell is about 23 ampere-hours per ounce of zinc, and as about 1¾ ounces of each zinc rod may be consumed, the life of each zinc is then about 40 ampere-hours. The E. M. F. of this type of cell varies from 1.4 to 1.7 volts and its internal resistance varies from .4 to 4 ohms.

It is usual to seal the carbon and depolarizer into the porous cup by some compound, such as sealing wax, leaving small tubes or holes, by which whatever gas is not absorbed by the depolarizer may escape. This sealing necessitates the entire renewal of the porous cup, with contents, when the depolarizer is exhausted. To obviate this expense, some makers use a carbon porous cup and place the zinc inside, at the center, the space between the zinc and carbon being filled with manganese dioxide.

36. Fastening Binding Posts to Carbon Electrode.—It is necessary to have the binding post substantially joined to the carbon and the corroding action of the electrolyte (due to its creeping up and through the porous carbon) on the binding post where the latter is joined to the carbon must be avoided. To do this was quite difficult for a time, but a good method is as follows: Drill a hole in the carbon the size of the binding-post screw, then enlarge the hole at the bottom, making it cone-shaped. This hole is then filled with a melted alloy consisting of two parts of bismuth and one part of tin, and the binding post is screwed in before it becomes too hard. On solidification the alloy expands, making a very tight and substantial joint. To prevent capillary attraction and the creeping of the electrolyte up the carbon, the top with the binding post in place should be thoroughly soaked, or at least coated over, with paraffin, wax, or insulating varnish or paint.

37. Hayden Cell.—Another form of the Leclanché cell is what is known as the **Hayden No. 2**, shown in Fig. 10. In this cell, the depolarizer, instead of being contained in a porous cup, is contained within a carbon cylinder that in itself forms the negative element of the battery. The zinc is cylindrical in form and surrounds the carbon cylinder; thus, by virtue of its large surface and the short distance between the two electrodes, producing a very low internal resistance. The carbon cylinder *C* is corrugated on its exterior surface so as to present as large a space as possible to the electrolyte, and contains the depolarizer *D*, composed of a mixture of manganese dioxide and crushed carbon in about equal portions, each being broken into particles somewhat smaller than peas. The carbon cylinder *C* engages the cover-plate *B*, also of carbon, by means of a screw thread, as shown. The positive terminal *T* of the cell is composed of the threaded stud *t*, the washer *t'*, and locking nut *t''*. The stud is secured in place by means of tin, which is melted and poured into the hole in the cover-plate, the plate itself being previously heated to a high temperature. After this, the entire cover-plate is boiled in paraffin, so as to prevent corrosion between the metallic terminal *T* and the carbon. Unless this or similar means is taken, this corrosion is sure to set in, due to the absorption of the chemicals in the solution by the porous carbon.

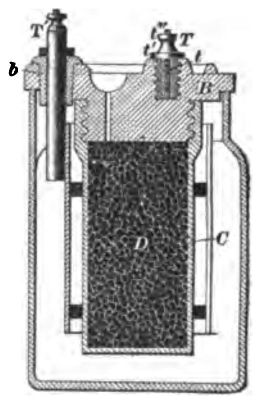

FIG. 10

Around the carbon cylinder are stretched two heavy rubber bands, the purpose of which is to maintain the zinc cylinder at a proper distance from the carbon. A zinc rod carrying the negative terminal *T'* of the cell passes through a porcelain bushing *b* in the cover-plate, it being soldered at its lower end to the zinc cylinder. Much trouble has been experienced in these cells, due to the rapid eating away of the zinc at the point where this rod joined it. This was undoubtedly due, in a large measure, to the presence of some foreign substance introduced by the solder, and also, to a less extent, to the fact that the action was more violent at that point. This trouble has, however, been entirely overcome by painting the plate and the rod in the vicinity of the joint with some material, such as a mixture of pitch and tallow, that adheres strongly to the surface, and prevents the action of the electrolyte at this place.

The proper place for the depolarizer is between the zinc and carbon, so that the oxygen is liberated on the carbon surface where the polarization occurs. The good results obtained with this form of Leclanché cell are probably due as much to the large carbon surface as to the depolarizing material.

38. Gonda Cell.—Another widely used form of Leclanché cell is the **Gonda-Leclanché**, which uses no porous cup whatever; the manganese dioxide is mixed with granulated carbon and some gummy substance, and compressed into cakes under great pressure. These cakes are attached to the sides of the carbon plate, and act in the same manner as the depolarizer in the regular form.

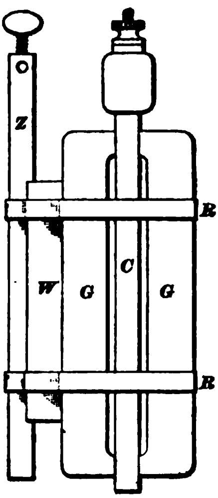

FIG. 11

Fig. 11 shows the construction of the elements of such a cell. The two cakes of depolarizer (called *gondas*) *G*, *G* are clamped one on each side of the carbon plate *C* by the soft-rubber bands *R*, *R*, which also serve to hold the zinc rod *Z* in place. The zinc lies in a groove in a block of wood or clay *W*, which serves to keep the zinc away from the gondas. This block is now generally omitted, both zinc and carbon being supported from a plate of insulating material, which also acts as a cover to the jar, and thick rubber bands are used to hold the gondas in place and to prevent the zinc touching them or the carbon. In other forms, the depolarizer is molded into a cylinder, in the center of which the zinc is supported. In this latter form, a second zinc electrode, consisting of a cylinder of sheet zinc encircling the cylindrical gonda, is sometimes used, a common terminal being connected to both zincs. The liquids and action of the gonda form are the same as in the regular Leclanché cell.

39. Commercial sal ammoniac often contains a considerable amount of impurities, in the shape of other salts, which materially reduce the life of the electrolyte; not sufficiently, however, to warrant the cost of using the chemically pure salt. However, it is very much better to use only a good quality of commercial sal ammoniac made especially for batteries. Ammonium chloride has been found to be the only salt that works well with manganese dioxide as a depolarizer, so the many other forms of cell that have been constructed, using this depolarizer, differ materially from the Leclanché type only in the mechanical arrangement of the parts. Cells using manganese dioxide for the depolarizer are used only for open-circuit work since the depolarizer acts slowly.

The principal chemical actions in the Leclanché cell are the formation of zinc chloride and ammonia, and the partial reduction of the manganese dioxide. Representing these reactions by equations, we have at the anode $Zn + 2NH_4Cl = ZnCl_2 + 2NH_4$. The NH_4 radical passes to the cathode and there breaks up into NH_3 and H $(2NH_4 = 2NH_3 + 2H)$. The hydrogen then reacts with the MnO_2 depolarizer as expressed in the equation $2MnO_2 + 2H = Mn_2O_3 + H_2O$. The zinc chloride of the first equation and the ammonia gas H_3N are dissolved in the water, unless the solution becomes saturated, when the H_3N escapes as a gas and a double salt of zinc and ammonium chloride appears as crystals on the zinc or settle to the bottom of the cell. Besides these, more complicated reactions may occur, but they do not affect the E. M. F. of the cell materially.

40. Care of Leclanché Cells.—The amount of sal ammoniac required to charge a Leclanché cell will depend on the amount of water required to fill the jar to the proper height; usually from 4 to 6 ounces is sufficient. It is best not to use too large a charge of sal ammoniac, and never put in more than will dissolve; 3 ounces of sal ammoniac to a pint of pure water is the best proportion. Too dense a solution allows the double salt of zinc and ammonium chloride that is gradually formed to settle to the bottom and crystallize around the zinc and carbon, thereby interfering with the proper action of the cell. Moreover, when the solution is extra dense at the bottom, the top portion of the zinc is eaten away more rapidly than the bottom, forming a cone-shaped zinc. This is due to local action between the two parts of the zinc that are in solutions of different specific gravities, the lower one of which also contains some zinc chloride. There should never be crystals or undissolved salts of any kind in the bottom or around the lower ends of either electrode. If such is the case it shows that either too much sal ammoniac was used in charging the cell, or that an excess of other salts have formed, probably due to impurities of some kind. In any case all the crystals should be removed by dissolving them in water or scraping them off.

The eating away of the zinc at the surface of the liquid is probably due to oxidation and can be avoided only by the use of a very closely adhering coating of insulating material around the zinc rod just where it passes through the surface of the solution. This is not usually considered necessary.

The jars usually have printed directions pasted upon them for setting up the cells and marks upon the jar to show how much water is required. The directions given should be carefully followed. Where porous cups are used the liquid should come to within about 1½ inches of the top of the porous cup. The cells with porous cups are not usually in a good condition for use until 10 or 12 hours after setting up, because time is required for the solution to soak through the porous cup. Where the porous cup contains one or more vent holes at the top the action may be hastened by pouring some of the sal-ammoniac solution into the porous cup through the vent holes, but the top of the cup should be wiped dry to prevent the corrosion and formation of salts around the binding post. If water only is poured in the vent holes, time will be required for the sal ammoniac to diffuse through the porous cup. If the solution evaporates pure water should be added from time to time to keep the level up to the proper height.

41. To recharge a Leclanché cell remove the various parts, throw away the old solution, wash off the various parts, and if necessary scrape the zinc and porous cup or carbon clean with a knife. Soaking the carbon and porous cup in water, where time will allow, will improve it. By baking the carbon only in an oven it could be improved; but, in most makes, this would also melt the coating over the top and should not be done. If the zinc seems to be rotten or mechanically weak when recharging the cell it should be no longer used. Recharge the cell as it was originally set up, using a new zinc, if the old one is practically consumed or unfit for further use. The terminals of the cell should be kept dry and water or solution should not be spilled all over them and the floor. If necessary, use a piece of sandpaper or a knife to brighten the wire, washers, or surfaces against which the wire is clamped when connecting the cells. If the coating around the top of the glass jar has come off, creeping of the salts will follow. Paraffin or ozokerite may be used to recoat the edge. A recharged Leclanché cell will not have as high an E. M. F. nor as long a life as a new cell, unless the depolarizer and carbon as well as the zinc are renewed, which practically makes a new cell of it.

42. Open-circuit cells, or batteries of such cells, should never be short-circuited. It is a common practice to touch a wire connected to one terminal to the other terminal of a battery to see if a spark and how much of a spark can be produced, and this is even done by those who should know better. This short-circuits the battery and the large momentary rush of current polarizes the cell and consumes the zinc and electrolyte. Probably more cells are injured in this way than in all other ways of misusing them combined. This practice cannot be too strongly condemned. There are no open-circuit cells and very few closed-circuit cells that will stand such treatment without suffering more or less injury.

43. Commercial Salts.—There are a number of salts on the market that are claimed to be superior to sal ammoniac for Leclanché cells, but it is doubtful if they are any better than good sal ammoniac prepared especially for use in batteries. The preparation and composition of these salts are trade secrets. One such salt has about the following composition: Zinc chloride, 20 parts; ammonium chloride, 76 parts; sodium sulphate, ½ part; water, 3½ parts.

DRY CELLS

44. The name **dry cells** is applied to cells in which the electrolyte is carried in the pores of some absorbent material, or combined with some gelatinous substance, so that the cell may be placed in any position without spilling the liquid. These cells are usually made with zinc and carbon elements, the zinc, usually forming the outside of the cell, being made into a sort of cylindrical can, in the center of which is the carbon, surrounded by its depolarizing compound. The space between them is filled with some absorbent material, such as mineral wool, asbestos, sawdust, blotting paper, etc., and the whole is then soaked in the exciting liquid; or, the exciting liquid is mixed with a hot solution of some gelatinous body, such as Irish moss, which mixture is poured into the cell; on cooling, it forms a soft jelly. The first method of preparation is most used. It is quite necessary for dry cells to have a depolarizer, as otherwise they would have to be made open to allow the hydrogen gas to pass off, which would also allow the small amount of water they contain to evaporate. To prevent this latter action, these cells are sealed with some resinous compound. The zinc can is covered with pasteboard to insulate it.

Owing to the presence of the absorbent material, the actual amount of liquid in these cells is comparatively small, and the sealing, if imperfect, allows the water to evaporate, in which case the cell ceases to act. A cell of this description may often be made to work when apparently exhausted by drilling a small hole in the seal and injecting a little water or sal-ammoniac solution, or by also drilling a small hole in the bottom and setting the cell in a glass jar containing water or sal-ammoniac solution.

The materials used in dry batteries are usually kept secret by their manufacturers; they all, however, answer to the above description as to construction, and the best types employ the same materials as the Leclanché battery; that is, a zinc anode, ammonium-chloride electrolyte, manganese-dioxide depolarizer, and carbon cathode. This form of cell is extremely convenient on account of its portability and the fact that it needs no attention until a new cell is required. Even if not used, the terminal potential gradually falls and the internal resistance rises, due to the drying up of the paste. Even the best dry cells will not usually remain in good condition longer than 3 years while the poorer ones depreciate considerably if kept 2 or even 1 year; no dry cell should be kept in stock over 1 year. The internal resistance and E. M. F. of fresh dry cells varies from .1 to .7 ohm and from 1.3 to 1.6 volts, respectively. Dry cells of ordinary size should not be used where a current exceeding about .15 ampere is required.

45. Gassner Cell.—This was one of the first successful dry cells. A cylindrical can of sheet zinc serves as the containing vessel and the positive element. The carbon element is in the center and occupies about one-half the space in the cell. The space between the carbon and the zinc is filled with the following mixture: Zinc oxide, one part; sal ammoniac, one part; plaster of Paris, three parts; zinc chloride, one part; water, two parts (all by weight). The zinc oxide in this mixture loosens and gives the mass greater porosity. The E. M. F. of this cell is about 1.3 volts.

46. Nungesser Dry Cell.—This cell is known as the *1900 Dry Battery* and is a very good open-circuit cell. It consists of a zinc outer case, or can, to which is attached a binding post forming the negative terminal of the cell. Just inside the zinc can and in close contact with it are several layers of insulating absorbent fiber that is thoroughly saturated with the electrolyte. A carbon pencil, or plate, occupies the center of the cell and the space between this and the fiber is filled with a solidly packed mass of granular carbon.

Great care is used in selecting only the purest and best-conducting carbon. A solid depolarizer is mixed with the carbon and the whole is saturated with the electrolyte. The top of the cell is sealed with a hard pitch. The E. M. F. is 1.6 volts and the average internal resistance of the 6″ × 2½″ size is about .1 ohm. There is little or no local action.

The life of this cell in terms of different kinds of work performed is stated to be as follows: In telephone service, when used with transmitters having a resistance of 30 to 35 ohms, the average life of the cell is not less than 15 months' continuous exchange service; with telephone transmitters of 65 to 75 ohms the average life is not less than 18 months and in many cases is considerably greater than this. For such work as door-bell and annunciator service the cell should last about 3 years.

FIG. 12

47. New Excelsior Dry Cell. This is an open-circuit dry cell made by the New Excelsior Dry Battery Manufactory, of New York. In this cell, shown in section in Fig. 12, the zinc can *a* is lined on the inside with absorbent paper *p*, which forms a porous partition and also prevents internal short circuits. The paper is thoroughly saturated with the electrolyte, which is a solution of zinc chloride and sal ammoniac in water. Inside the paper is a thin layer *d* of a white paste whose composition is a trade secret. A carbon pencil or plate *c* occupies the center of the cell and the space between is filled with a mixture of powdered carbon and manganese dioxide *m*, these also being saturated with the electrolyte; the manganese acts as the depolarizer. The E. M. F. is 1.5 volts. The regular size of this cell (6 inches by 2½ inches) is made in two qualities. The ordinary quality is rated to give 6 to 8 ampere-hours; and the extra quality, 10 to 12 ampere-hours. The ampere-hour rating of a cell is usually more desirable than a time rating, since the kind of work required of a cell varies so much. The New Excelsior dry cell is made in various sizes. A large size, known as the *Eclipse*, is much used for gas-engine ignition.

48. Other Dry Cells.—Many different dry cells have been made, but usually they do not differ much as to the materials used. Each manufacturer has given his cell a trade name by which it is generally known. Besides those mentioned there are the *Mesco*, *New Standard*, *Novak*, *O. K.*, *Mascot*, *Exeter*, *Phœnix Improved*, *Eastern*, *Ajax*, *Columbia*, and many others.

Meserole gives the following composition for a dry cell: Mineral carbon or graphite, one part; charcoal, three parts; white arsenic oxide, one part; peroxide of manganese, three parts; a mixture of dextrine or starch and glucose, one part; dry hydrate of lime, one part; all parts by weight. After mixing these well into a paste of suitable consistency, using for this purpose a solution of equal parts of ammonium chloride and common salt, add gradually one-tenth the volume of a solution of mercury bichloride and an equal volume of hydrochloric acid, mixing all together very thoroughly.

49. Some dry cells are superior to Leclanché cells and their output per unit weight is nearly always greater. About the only disadvantages of a good dry cell compared with a Leclanché cell is its liability to deteriorate even when standing idle and the fact that it cannot be charged with fresh materials. However, the Leclanché requires more attention and even when recharged is not the equivalent of a new cell. Good dry cells being so much more convenient, portable, occupying less space, and enough cheaper to offset their incapability of being recharged, are largely replacing wet Leclanché cells, especially for telephones, electric bells, and similar purposes. For private use, where the attention required is not an expense, the Leclanché cell may be more economical than a dry cell although more troublesome.

It is not practicable to charge exhausted dry cells of the ordinary type by passing a current of electricity through them in the proper direction. If, however, the cells are only badly polarized and not exhausted, a charging current of short duration will render them active again. There may be dry cells of unusual composition that can be charged by a current, but even then it does not usually pay to do it.

LALANDE-CHAPERON CELL

50. Another depolarizer that is used in important commercial cells is *cupric oxide*, CuO. The **Lalande-Chaperon cell** uses an iron or copper negative element surrounded with a layer of cupric oxide. The positive element is zinc, the electrolyte a solution of potassium (or sodium) hydrate (caustic potash). On closing the external circuit, the potassium-hydrate solution attacks the zinc, forming a compound of potassium and zinc oxides, known as *potassium zincate*, K_2ZnO_2, and liberating hydrogen, which combines with the oxygen of the cupric oxide forming water, and depositing metallic copper on the cathode.

If the surface of a solution of caustic potash is exposed to the air, it will gradually form, by the absorption of CO_2 gas from the air, potassium carbonate. To prevent this action, cells of this type have the surface of the liquid covered with a thin layer of heavy oil. The E. M. F. of this type of cell is about .7 volt, and its internal resistance is usually very low. It is capable of giving a greater output per unit weight than any other well-known cell, except the dry cell.

EDISON-LALANDE CELL

51. The **Edison-Lalande cell** is a modification of the Lalande-Chaperon. The cupric oxide is molded under pressure into plates of the requisite size, being first mixed with magnesium chloride, which, when the molded plates are heated, serves to bind the mass together. These plates are held in copper frames, which enclose the edges of the plates. The positive element in this cell is amalgamated zinc, and the electrolyte a solution of potassium hydrate, as in the Lalande-Chaperon cell. Two plates of zinc are used in most of the forms of this cell, one on each side of the cupric-oxide plate. There is practically no local action on open circuit.

One form of this cell is shown in Fig. 13, which represents a 150-ampere-hour cell. The cupric-oxide plate *C* is suspended in a copper frame *F*, *F* between the two zinc plates *Z*, *Z*, which are hung from each side of a lug on the porcelain cover of the jar. The sides of the copper frame of the oxide plate are carried through the cover supporting the plate and form terminals *B*, *B*, either of which may be used as the positive terminal of the cell. The copper frame is protected from the action of the liquid where it passes through, by tubes of insulated material *T*, *T*. A binding post B_1, on the bolt that supports the two zinc plates, serves as the negative terminal. A heavy paraffin oil is used in this cell to prevent the action of the air on the solution.

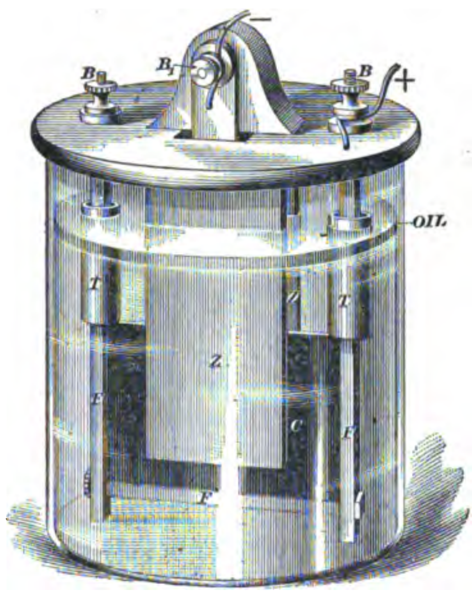

FIG. 13

The cell shown is $5\frac{1}{4}$ inches by $8\frac{1}{4}$ inches, outside dimensions, and will give a current of 3 amperes at a potential of about .7 volt for 50 hours, which is equivalent to about 100 watt-hours, with one charge of zinc, caustic potash, and copper oxide.

The internal resistance of the cell shown is about .07 ohm; the weight of the oxide plate is about $\frac{1}{4}$ pound. This type of cell is made in various sizes, ranging from a 15-ampere-hour cell for telephone and similar work, to 900-ampere-hour cells for running lamps, small motors, etc. The type R, 300-ampere-hour cell, which is 10 inches high by 6$\frac{3}{4}$ inches in diameter, has an internal resistance of about .03 ohm, and is able to give 14 amperes, although 4 amperes is its most efficient current output, and requires a 2-pound copper-oxide plate. The most efficient current output for the 50-ampere-hour cell is 2 amperes; for the 150-ampere-hour cell, 3 amperes; for the type S 300-ampere-hour cell, 6 amperes; and for the 600-ampere-hour cell, 7 amperes.

52. Directions.—For setting up, caring for, and renewing the type R cell, the following directions are given by the makers: Fill the jars with water to the brown line on the inside. Then open the can of granulated potash by cutting out the bottom (which is made of very thin tin) with a penknife. Add the potash gradually to the water, stirring the solution constantly until the potash is entirely dissolved, which will take about 3 minutes. When the solution cools it may be found necessary to add a little more water to bring it up to the brown line again. Put the elements in place, lift the cover slightly, and pour a layer of heavy paraffin oil (from the bottle furnished) on the solution in the jar, until it covers the blue line. As the potash will burn the skin and clothes, avoid splashing the liquid when stirring it.

Unless a short circuit should occur, the battery requires no attention until it is exhausted. A short circuit between the plates in the cell will exhaust the cell. A short circuit outside will exhaust the whole battery.

When the cell becomes exhausted, the solution and the remains of the zinc and oxide plates must be thrown away. All the remaining parts can be used again.

To take the cells apart, lift the lids, unscrew the bolts, and remove the zinc and oxide plates. Wash (with water) the copper frames, bolts, and rubber insulators, brightening up the metal, where corroded, with emery paper, especially the inside grooves of the copper frame sides. Pour away the solution carefully and set up cells with new potash, oxide plates, and zincs according to directions. All the parts that have been immersed in potash must be washed before they are handled.

To ascertain if the oxide plates are exhausted, pick into the body of the oxide plates with a sharp-pointed knife. If they are red throughout the entire mass they are completely exhausted and need renewing. If, on the contrary, there is a layer of black in the interior of the plate, there is still some life left, the amount being dependent entirely on the thickness of the layer of black oxide still remaining.

Too great stress cannot be laid on the necessity of observing (when setting up the cells) that the top of the oxide plate is fully 1 inch below the surface of the potash solution, and consequently about 1½ inches below the top of the oil. The difference of 1 inch in the height of the solution in the jars determines the success or failure of these batteries.

It is essential not to omit the layer of oil, for otherwise the life of the cell will not be over one-third what it should be.

W. R. Cooper, in "Primary Batteries," says: "It is not necessary to discard the copper-oxide plates when reduced by discharge. After soaking for a time in water to remove the excess of soda, they may be dried and reoxidized by heating to a red heat in air. A large gas burner may be used for this purpose; but in that case the plate should be protected from the direct action of the flame by a piece of thin sheet iron, otherwise reduction, instead of oxidation, is liable to take place in parts. A plate so treated is not likely to be as good as the original, and will give but a feeble current to start with, unless previously reduced on the surface, but will, nevertheless, have a serviceable life. The simplest way to reduce such a plate is to short-circuit the cell for a few minutes only. This short-circuiting should not be done with a new cell, for it, or its equivalent, has already been done by the makers."

GORDON CELL

53. The **Gordon cell,** which is a modification of the Lalande-Chaperon, is shown in Fig. 14. It is used to a considerable extent in this country for fire-, police-, and railway-signal systems and for many other purposes where a non-polarizing, closed-circuit battery is required. These cells are made with enameled steel, porcelain, or glass jars, and a cover of tin, porcelain, compressed fiber, or glass that fits the jar nearly water-tight. The negative element consists of a perforated tin cylinder (tinned-iron) *i* suspended by an iron rod from the center of the cover *l* and held in place by insulating washers and the brass connector. The perforated cylinder contains the black oxide of copper, which is the depolarizing agent. About $1\frac{1}{2}$ inches from the bottom of this cylinder are attached three porcelain lugs. Only two of these lugs *e, e* are shown in the figure. They sustain the weight of the zinc element *z* and at the same time insulate it from the negative element *i*. A copper wire *n*, insulated with rubber, connects with the zinc and passes through the cover of the jar and forms the negative terminal of the cell. The electrolyte is a solution of sodium hydroxide (caustic soda) in the proportion of $1\frac{1}{2}$ pounds of caustic soda to 6 pints of pure water. The water should especially be free from lime or carbonaceous materials. A layer of paraffin oil floats on the surface of the electrolyte; no kind of animal or vegetable oil must ever be used in Gordon or Edison-Lalande cells.

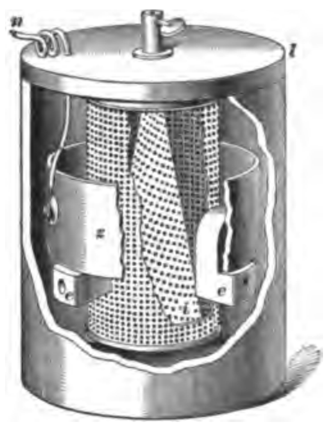

FIG. 14

A No. 1 Gordon cell, having a jar 6 inches in diameter and 8 inches high, holding 6 pints of electrolyte, and using $2\frac{3}{4}$ pounds of copper oxide for one charge, will give a steady current of 2 amperes at about .67 volt or of 5 amperes at about .5 volt, and is guaranteed by the makers to give 250 ampere-hours. It is claimed that the cell will give 25 per cent. greater capacity under ordinary conditions. At a discharge rate of .08 ampere, as required for some railway-signal systems, this size cell is warranted by the manufacturer to last 6 months without any attention whatever, and much longer where less current is required. The internal resistance may be as low as .04 ohm. This type of cell has been known to work without interruption at a temperature considerably below zero. The Gordon cell requires no attention until replenishment is necessary, for which purpose new cupric oxide, zinc, and caustic soda are required. Care must be taken not to splash or spill the solution for it is injurious to both the hands and the clothes.

To set up the cell, remove the cover by unscrewing the brass connector; place the copper oxide in the perforated cylinder; fill the No. 1, 6″ × 8″ jar, within 2 inches of the top and the No. 2, 4½″ × 6″ jar, within 1½ inches of the top with pure water; add the sodium hydroxide to the water slowly and with constant stirring (to prevent too great a production of heat); when all the caustic soda has been dissolved, place the elements (which are attached to the cover) in the jar and then lift the cover slightly and pour in the oil. The oil should be added last so that the elements will not come in contact with it, which would be the case if the elements were inserted after the oil had been put in. The liquid should then stand 1 inch, in the No. 1, and ¾ inch in the No. 2, from the top of the jar.

54. The reactions in these alkaline cells are represented by the equations: $Zn + 2KOH = K_2ZnO_2 + H_2$, at the anode; $CuO + H_2 = Cu + H_2O$, at the cathode. The salt K_2ZnO_2 is called *potassium zincate*. If sodium hydroxide is the electrolyte, the equations are the same except that Na is substituted for K.

55. Several oxides of lead have been used as depolarizers in single-liquid cells: plumbic oxide, PbO, known as *litharge*, which is in the form of a yellowish powder; peroxide of lead, PbO_2; and a combination of the oxide and the peroxide, Pb_3O_4, known as *minium*, or *red lead*, which is a brilliant red powder. As seen from its formula, the peroxide contains the most oxygen, and is the best depolarizer; for example, in the zinc–dilute sulphuric acid–carbon cell, replacing the carbon with lead peroxide increases the E. M. F. to 2.5 volts; the action of the sulphuric acid on the peroxide, however, forms lead sulphate, which is insoluble, and increases the internal resistance of the cell somewhat.

HARRISON CELL

56. A small size of the **Harrison cell** (No. 1) is shown in Fig 15. In this cell the element *n* consists of electrolytically prepared lead peroxide compressed around a conductor of hard lead. The electrolyte is a dilute solution of

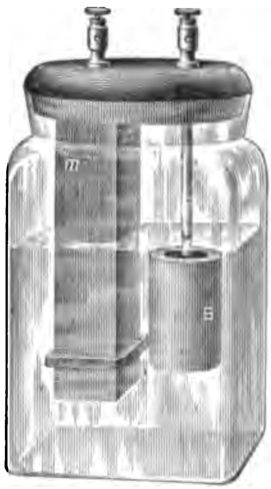

FIG. 15

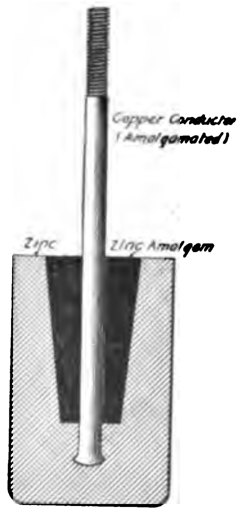

FIG. 16

sulphuric acid, which makes it necessary to keep the zinc element *z* well amalgamated. This is accomplished in the following manner: The zinc is cast in the form shown in Fig. 16, around an amalgamated copper-wire terminal.

The hollow space is then filled with melted zinc amalgam, which is of such a composition that it becomes quite solid when cold. When the zinc element is first placed in the electrolyte, there will be some local action on the amalgamated zinc, but the mercury soon spreads over the entire surface of the element and further local action is stopped. This method of amalgamation is more effectual than simple surface amalgamation. The E. M. F. of the Harrison cell is about 2.5 volts.

In place of the dilute-sulphuric-acid electrolyte, acid sulphate of potassium or sodium (bisulphate of potash or soda) may be used with good results. In any case the electrolyte must be pure or local action will take place in spite of good amalgamation. This may be due to the deposition of electronegative elements on the zinc from the impurities in the electrolyte. Such elements as iron, arsenic, and selenium, which do not amalgamate with mercury, are especially harmful in the electrolyte. If these impurities get into the cell, the trouble may be remedied, after they are all separated on the zinc, by removing this electrode and scrubbing off the impurities with clean water.

The equations expressing the reactions in this cell are: $Zn + H_2SO_4 = ZnSO_4 + H_2$, at the anode; $PbO_2 + H_2SO_4 + H_2 = PbSO_4 + 2H_2O$, at the cathode.

CHLORIDE DEPOLARIZERS

57. The principal chlorides used as depolarizing agents are the chlorides of mercury and silver. If the carbon of a zinc–ammonium chloride–carbon cell be placed in a porous cup and surrounded with a paste of mercurous chloride, the chemical action is as follows: The ammonium chloride attacks the zinc, forming zinc chloride and freeing ammonia and hydrogen, which attack the mercurous chloride and reform ammonium chloride, leaving free mercury at the negative pole. The ammonium-chloride solution is thus kept at its full strength until the mercurous chloride is entirely exhausted, and the hydrogen is recombined as fast as formed. Such a cell has an E. M. F. of 1.45 volts, which is maintained as long as the depolarizer lasts, if excessive currents are not used.

SILVER-CHLORIDE CELLS

58. In cells employing chloride of silver as a depolarizer a silver wire, or plate, coated with silver chloride is used for the negative element. The positive element is usually zinc, and the electrolyte a dilute solution of one of the chloride salts. With ammonium chloride, the E. M. F. is 1.03 volts; with zinc chloride, 1.02 volts; and with sodium chloride (common salt), .97 volt. The chemical reactions may usually be represented by the following equations when the electrolyte is ammonium chloride: $Zn + 2NH_4Cl = ZnCl_2 + 2NH_4$ and $2NH_4 + 2AgCl = 2NH_4Cl + 2Ag$. If the cell is worked hard, the reactions are more complicated and a gas is liberated.

While silver-chloride cells do not polarize much and recover promptly, they can be used to furnish only very small currents. They deteriorate in standing, are expensive on account of the silver required, and are apt to be troublesome and unreliable. They have been used by physicians and, in a compact form, for testing purposes in connection with portable testing sets.

OTHER DEPOLARIZERS

59. The various sulphates of mercury that are used as depolarizers are *mercuric sulphate*, *mercurous sulphate*, and a sulphate containing a still higher percentage of mercury, known as *turbith* (or *turpeth*) *mineral*. Either sulphate may be used in the zinc–dilute sulphuric acid–carbon cell without materially affecting the E. M. F., which, under these circumstances, is 1.3 to 1.5 volts. These sulphates, being slightly soluble, are usually employed in the form of a paste, made with water or the exciting liquid. In ordinary work the mercury sulphates are not extensively used, not only on account of the high cost of these salts, but because of their poisonous qualities. Still, these sulphates are excellent depolarizers, and are used in standard cells.

STANDARD CELLS

60. Standard cells are primary cells that have a definite and accurately known E. M. F. Of the large number of cells that have been made or suggested, only a few can meet the requirements for a standard E. M. F. The principal requirements are that they must be readily reproduced and preferably portable also, and always give the same E. M. F. under a given set of conditions; the change of E. M. F. with the change in the conditions (change in temperature, etc.) must be accurately known and the smaller the change the better. A portable standard cell is one so constructed that it may be shipped and even inverted for short periods without injury. Standard cells are usually made small in size and are otherwise designed for comparison of E. M. F. rather than to give a current. In fact, it is usual to have a high resistance in series with the cell so that only a minute current can be obtained from it, otherwise a slight polarization may result and appreciably change the E. M. F. In the so-called null methods of measurement this high resistance is not absolutely necessary because no current is taken from the cell, but it is always advisable to use a high resistance to prevent injury to the cell from accidental short circuits.

The use of such a cell is, briefly, as follows: A cell made according to directions will have a known E. M. F. A cell —or other source of current—of unknown E. M. F. may be compared with the standard cell, and the ratio between the two E. M. F.'s determined. The unknown E. M. F. can then be calculated. A number of standard cells have been devised but only three types are used to any extent; these are the Clark, Weston, and Daniell types.

CLARK CELL

61. In the **Latimer-Clark cell,** now known as the **Clark cell,** the positive and negative electrodes are of zinc and mercury, respectively; the electrolyte is a solution of zinc sulphate in distilled water, two parts, by weight, of zinc-sulphate crystals to one part of water; and the depolarizer is a paste of mercurous-sulphate and zinc-sulphate solution. The cell is usually made in the form shown in Fig. 17, when portability is desirable. The H-shaped vessel, like that shown in Fig. 20, is considered a better standard form but is not so portable. The E. M. F. of a carefully prepared cell varies with the temperature, and in order to obtain the correct value at any temperature other than 15°, the given value must be increased or decreased by a certain per cent. per degree variation in the temperature. The variation of the E. M. F. per degree centigrade is called the *temperature coefficient.* It varies for each type of cell and even for cells of the same kind, having different densities of electrolyte. The formula showing variation of E. M. F. with changes in temperature for the Clark cell is

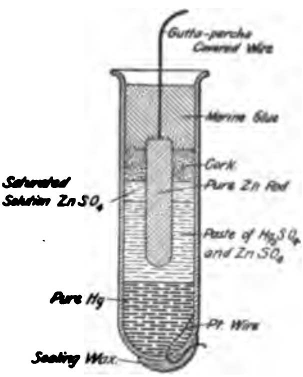

FIG. 17

$$E_t = E_{15} - .00119\,(t - 15°) - .000007\,(t - 15°)^2 \qquad (1)$$

E_t is the E. M. F. for any temperature t, and E_{15} is the E. M. F. of the cell at 15° C. The latest reliable determinations seem to show that the E. M. F. at 15° C. is 1.4333 volts, but 1.434 volts is legalized by many nations.

Take a specific case in which it is desired to know the E. M. F. of the Clark cell at 20° C. Substituting the known values in the above formula, we have $E_t = 1.433 - .00119\,(20 - 15)$ or $E_t = 1.427$ volts. In this calculation it was not necessary to consider the part of the equation, − .000007

$(t - 15°)^2$, because the quantity is too small to affect the third decimal in the result.

Desiring the E. M. F. at 0° C., we have, by substitution, $E_t = 1.4333 - .00119(0 - 15) - .000007(0 - 15)^2 = 1.450$ volts.

The greatest accuracy is demanded in the construction of this cell and in the determination of its temperature coefficient, because the cell is used as a standard in the measurement of unknown E. M. F.'s. It is used to supply only very minute currents; so it is made of conveniently small size. This cell is very valuable on account of its constancy, but its change with temperature makes it somewhat difficult to use with great precision, as thermometers are, as a rule, not very exact, their measurements depending largely on their physical condition. For a standard cell its temperature coefficient is rather large.

CARHART-CLARK STANDARD CELL

62. The **Carhart-Clark standard cell** is a modification of the Clark cell, the materials used being the same except that the zinc-sulphate solution is saturated at 0° C. instead of at 30°, as in the Clark cell. Since there is no crystallization or redissolving of crystals above 0° C., there is little or no change in the density of the zinc-sulphate solution at ordinary temperatures, and as a result the temperature coefficient of this cell is only about one-half that of the Clark cell. The change in E. M. F. of the Carhart-Clark cell with variation in temperature is expressed by the formula:

$$E_t = E_{15} - .00056(t - 15°) \qquad (2)$$

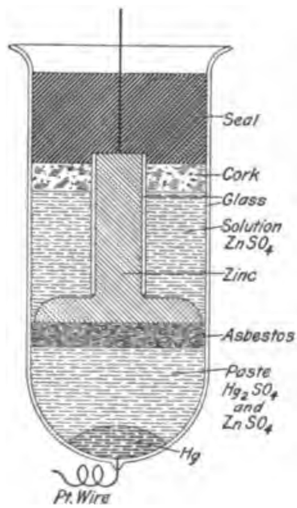

FIG. 18

The Carhart-Clark cell usually has an E. M. F. at 15° C. of 1.440 volts, but this value is always stated on the certificate that accompanies the cell.

A portable Carhart-Clark standard cell is shown in Fig. 18. A glass tube surrounding the stem of the pure cast-zinc electrode prevents local action between the top and bottom of the stem should there be even a slight difference in the density of the zinc-sulphate solution between the top and bottom of it. The asbestos, which is very carefully purified, together with the shape of the zinc makes the cell perfectly portable by preventing the possible mixing of the paste and the solution. Carhart says the cell is sealed with a compound of gutta percha and Burgundy pitch, with enough balsam of fir added to make the compound flow when hot, and that above this it is advantageous to put a mixture of powdered glass and sodium silicate. The Carhart-Clark cell is not injured by being temporarily inverted and will stand shipping. It has the advantages of being more portable, due to its construction, and of having a much lower temperature coefficient than the Clark cell.

Fig. 19

An exterior view of a case containing a portable standard cell is shown in Fig. 19. Usually a graphite resistance of about 10,000 ohms is permanently connected in series with the cell and placed in the same case, so the cell may not be injured by accidental short-circuiting. For convenience in taking the temperature of the cell, a small thermometer is fastened to the cover and projects into the cell. The thermometer is, when sold with the cell and case, bent so as to lie along the cover to lessen the liability of breaking.

It is quite common to put two standard cells in one case, with separate binding posts and carbon resistances for each cell. Then one cell can be used to check the other, and thus any relative change in the E. M. F.'s of the two cells may be readily detected.

WESTON CADMIUM CELL

63. A standard cell has been designed by Mr. Edward Weston that resembles the Clark cell, except that cadmium and cadmium sulphate are used instead of zinc and zinc sulphate. The chief advantage of the cadmium standard cell is its very low temperature coefficient, .00004 volt per degree centigrade. When there is an excess of cadmium-sulphate crystals, so that the solution is saturated at all temperatures between 0° and 30° C., the change of the E. M. F. with temperature is practically negligible and is usually so considered. The cell, as shown in Fig. 20, is similar in shape to one form of standard Clark cell, and consists of two short glass tubes *T*, *T*, connected together by a short tube *S*. In the bottom of the tubes are the elements *P*, *N*, to which connection is made by means of platinum wires *W*, *W*, which are sealed into the glass. It may be made in a portable form resembling Fig. 18. For the positive element, the metal cadmium in the form of an amalgam is used; and for the negative element, pure mercury in contact with a mixture of mercurous and cadmium sulphates. The electrolyte, which fills the vessel so as to connect the two limbs, is a saturated solution of cadmium sulphate, with an excess of cadmium-sulphate crystals added to insure its remaining saturated at all ordinary temperatures. The tops of the tubes are fitted with corks *C*, *C*, which are afterwards sealed in place, preferably with some resinous compound. The elements, being in a semiliquid condition, are each kept in place by a piece of cloth *F*, with a perforated cork *M* laid over it. When this is forced down the tube to the surface of the element, the

FIG. 20

cloth keeps the element in place, and the cork holds the cloth, the perforations allowing free access of the liquid to the elements. For this cell with an excess of crystals to preserve saturation the formula is as follows:

$$E_t = E_{20} - .000038\ (t - 20°) - .00000065\ (t - 20°)^2 \qquad (3)$$

The E. M. F. at 20° C. was formerly considered to be 1.019, but more recent determinations give 1.0187 volts. This cell is considered as about the best standard cell by Professor Carhart and many others.

STANDARD DANIELL CELL

64. The **standard Daniell cell** is easy to set up but has the disadvantage of not being portable and having to be made up fresh when used. It usually consists of a **U**-shaped glass tube, mounted on a wooden base; copper-sulphate solution is poured into one limb of the tube and zinc-sulphate solution into the other. The pouring must be carefully done so that the solutions will not mix. A copper-wire electrode is inserted in the copper-sulphate solution and an amalgamated rod of pure zinc in the zinc-sulphate solution. The copper wire may be of commercial copper but must have a fresh surface of copper electroplated on it before use.

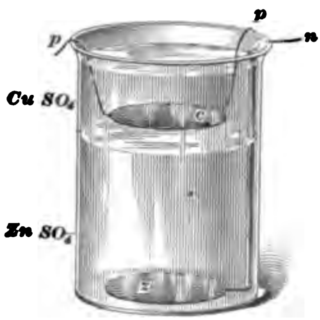

FIG. 21

The zinc-sulphate and the copper-sulphate solutions may be equidense, each having a specific gravity of 1.2 at 15° C., or a zinc-sulphate solution of specific gravity 1.4 may be used with a copper-sulphate solution of specific gravity 1.1 at 15° C. Other strengths might be used, but the ones just given are most frequently employed. The equidense solutions are prepared by dissolving 28.25 parts of pure crystallized copper sulphate in 71.75 parts of distilled water, and 32 parts of zinc-sulphate crystals in 68 parts of distilled water (all parts by weight). The zinc-sulphate solution of specific gravity 1.4 contains 55.5 parts of zinc-sulphate crystals dissolved in 44.5 parts of distilled water; and the copper-sulphate solution of specific gravity 1.1 contains 16.5 parts of the pure crystals dissolved in 83.5 parts of distilled water. Using the equidense solutions, the E. M. F. of the cell is 1.102 volts, and using the other set of solutions given, the E. M. F. of the cell becomes 1.072 volts. These values are for freshly made up cells. After the cells have stood for about an hour, the values increase by about .003 volt. The E. M. F. of this cell falls about .00015 volt per degree centigrade rise in temperature.

It is sometimes more convenient to make the cell in an ordinary glass vessel, as shown in Fig. 21. The electrodes are in the form of disks and the solutions used are generally the equidense solutions described above. The copper wire *n* running to the zinc disk *z* is insulated with glass tubing or rubber, where it passes through the solutions. The two solutions must be poured into the vessel so that they will not mix with each other. To accomplish this, half fill the vessel with zinc-sulphate solution; then place on this solution a disk of stiff note paper. By carefully pouring the copper-sulphate solution through a funnel, the tip of which nearly touches the note paper, the paper gradually rises and leaves the two solutions one above the other with a distinct boundary line between them. The copper plate *c* is supported by two or three copper wires *p*, *p*, one of which may be used for the positive terminal of the cell. These wires must not be fastened to the plate with solder, but preferably with copper rivets. For ordinary work it will be sufficiently accurate to call the E. M. F. 1.1 volts and neglect the temperature coefficient.

APPLICATION OF PRIMARY BATTERIES

65. Primary batteries as sources of electrical energy are used mainly in cases where a current is required very intermittently, such as in ringing bells, lighting gas, operating telephone transmitters, etc., or where a small but steady current is required for long periods of time, as in telegraphy, fire-alarms, and railway signals, or for laboratory and testing purposes. Their general use on a large scale, as sources of electrical energy for lighting or power purposes, is prohibited, at least at present, by the comparatively great cost of the material consumed and the expense of installation and maintenance. For example, the bichromate battery is about the cheapest in point of cost of materials consumed, and in this the materials alone would cost 28 cents per horsepower-hour on a large scale, while the total cost of electrical energy, using dynamos, is about 1 to 3 cents per horsepower-hour, ordinarily, and in many cases is much less. The cost of material in the silver-chloride battery is about $75 per horsepower-hour. In this cell, however, there is a valuable by-product (silver), which would reduce the cost considerably. This high cost of power does not, however, prevent batteries from being largely used for certain purposes. In such cases, the cost of materials consumed in producing the electrical energy is entirely offset by the little attention required and the constancy of the source of supply; and in many cases where current is used intermittently, the cost of the current from a battery in which the materials are consumed only as the current is used would actually be less than the cost of the power for driving an equivalent dynamo all the time. In large central offices, where the current required represents a considerable amount of energy, dynamos or storage batteries are replacing primary batteries to a large extent on account of the saving in space and in cost of maintenance. For telegraph and fire-alarm work, gravity batteries of the Daniell type are commonly used in the United States, as they possess the advantages of long life and little attention. For telephone work, the current supplied by the battery is small, and almost any good cell in which there is no local action and in which the depolarization is complete (at least for small currents) will give good results. The E. M. F. required is 1.5 to 3 volts; consequently, in some cases single cells may be employed; Leclanché, and dry cells are the most extensively used. Storage batteries at the central exchanges are, to a considerable extent, replacing primary batteries formerly located with the telephone instruments. In fire-alarm work a steady current of (usually) .04 ampere is used, the potential varying with the length of the circuit. Daniell gravity cells are used largely in this work, the zinc being made large and heavy to insure long life and, consequently, little attention. Gordon cells are also used to a considerable extent in this work, and in other signal systems as well. Several systems of block signaling on lines of railroads also employ electrical devices of such a character that Gordon, Edison-Lalande, and Daniell gravity cells are well suited and are quite extensively used. Another important application of batteries is in gas-engine ignition. For this work dry cells are often the most suitable. A cell of rather large size, such as the Eclipse made by the New Excelsior Dry Battery Manufactory, is required. Storage batteries are replacing primary batteries to some extent for ignition purposes.

Many devices require the application of a current intermittently; some, such as electric bells and other signals, electric gas-lighting apparatus, and the like, are used infrequently and irregularly, and the amount of electricity required is small, so that almost any voltaic cell will do, depolarizing or not, provided that there is no local action to cause waste when not in use. Therefore, cells with liquid depolarizers are not well adapted to this work, as in the long periods in which these cells are not called on to furnish current the two liquids will mix and usually cause local action. The cells most used for this work are the various zinc-carbon batteries with solid depolarizers; of the latter, some form of Leclanché, or dry cell usually gives the best results. In hotels and large buildings where the bell or signal service is practically continuous, depolarizing cells are required, such as large Leclanché cells, bichromates (with separate fluids), if of good modern construction, Edison-Lalande, Gordon, and the like.

66. Electric currents are much used in physicians' and surgeons' offices; currents of a few milliamperes in strength, but of from 75 to 100 volts E. M. F. are applied for curative purposes, while currents of 10 to 20 amperes in strength are used for heating cautery loops in surgical operations and for operating Roentgen ray induction coils, requiring an E. M. F. of from 4 to 16 volts. Miniature incandescent lamps, usually operated from the battery that furnishes current for the cautery, are also used to examine the interior of the body. The first appliance obviously requires a large number of cells of a small size. For occasional use, and when first cost is not such an object as compactness, a battery of small silver-chloride cells is sometimes, but not now often, used; while for more frequent use, requiring larger cells, some cheaper form of depolarizing cell is used. Obviously, if the cells selected have high E. M. F. (say 2 volts), a smaller number will be required than if the cells are of a low E. M. F.; however, the regulation of the current that is obtained, in some instances, by switching in or out of some of the cells, will be more uniform and gradual if the E. M. F. of each cell is low. Many physicians are now deriving all, or nearly all, their current from lighting and power circuits.

For furnishing the larger currents for cautery work, large cells should be selected, those that are so arranged as to have a minimum internal resistance being best. As the use of porous cups in a cell increases the internal resistance largely, cells that employ them are not well suited for this work. Bichromate cells are very convenient for this purpose, as their internal resistance is low and the E. M. F.

high and steady. It is usually convenient to use the form of bichromate cell in which the elements are raised from the liquid when the cell is not in use, as the purpose for which the current is used involves personal and immediate attention to all parts of the apparatus. Edison-Lalande and Gordon cells are also well adapted to work of this kind.

The most extensive application of cells of the Bunsen type is to electroplating and similar work, and cells of large size are made especially for this purpose. Such work being usually carried on in establishments especially fitted up for the purpose, the various unpleasant features of the Bunsen cell, which makes them objectionable for many purposes, may be readily provided for, and their high and constant E. M. F. utilized.

67. The minor applications of primary batteries are almost innumerable. A study of the requirements of such cases will usually determine the best type of cell to use, but attention should also be paid to the mechanical construction of the cells selected, as on this point often depends their life and suitability for the work they are called on to do.

The binding posts should be firmly and substantially fixed to the elements, and should be thoroughly protected from possible contact with the electrolyte, as the resulting action will so corrode the joint between the two as to destroy the contact, besides possibly eating away the connecting wires and breaking the circuit. Of the material of the positive element, as much as possible should be below the level of the liquid, as when that is consumed the remainder must be thrown away, and this may represent a considerable loss. Altogether, the cell should be substantial and compact, not liable to local action, and arranged so that its parts may be readily renewed with the least possible waste.

In general, it must be remembered that the consumption of material in a primary cell (assuming no local action) is proportional to the output in ampere-hours. The continuous output in watts depends not only on the amount of materials consumed, but on the E. M. F. of the cell and its internal resistance, so that, other things being equal, the higher the E. M. F. of a cell and the lower its internal resistance, the greater is its output, in watts, for a given cost of materials. As stated, the most economical metal to use for the positive element is zinc, and the amount of zinc consumed in a cell may be readily determined from the output in ampere-hours and the chemical equivalent of zinc (again assuming no local action); but to find the total cost of the energy, to this must be added the cost of the depolarizer consumed, if any, and the cost of labor in renewing the materials and caring for the cells. The substances resulting from the chemical actions that take place often have a market value; usually, however, the expense of collecting or preparing such substances for sale will be greater than the price they will bring, so that in ordinary cases this should not be taken into account.

It is evident that all the E. M. F. of a cell is not available for sending a current through the external circuit, but that a part is expended in overcoming the internal resistance. If the resistance of the external circuit is very great, this drop is of little importance; while if the external resistance is very small, the internal resistance practically determines the amount of current flowing.

The various methods of connecting the cells to form a battery, in parallel, series, or parallel series, have been given in another Section, but a little more on this subject will not be out of place here.

CONNECTING CELLS TOGETHER

68. Cells in Parallel.—Joining similar cells in parallel amounts to the same thing as using larger plates in a single cell, as can be seen by referring to Fig. 22. In Fig. 22 (*c*) *z*, *z* are positive elements or plates (negative terminals) of two similar cells and *c*, *c* the negative elements or plates (positive terminals) in the same cells. Since all the positive plates are joined together by conductors when in parallel, the plates might as well be joined together directly and placed in one vessel as shown in Fig. 22 (*b*) (without wires) as far as the result is concerned. If all the plates of one kind are joined together end to end, the result is one plate of twice the area of a plate of a single cell; when the plates are so joined, it is more convenient to place them in a single cell. Since the resistance of a cell varies inversely as the area of the plates, because the area of a plate is equivalent to the sectional area of the liquid across which the current

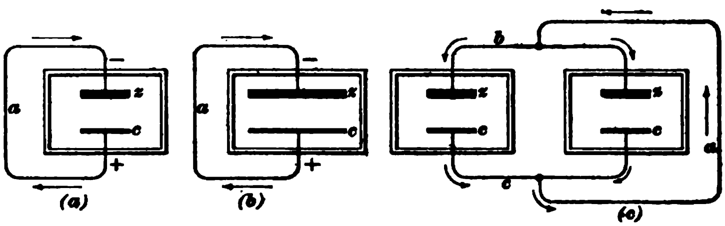

FIG. 22

flows; the resistance of the large cell, Fig. 22 (*b*), or that of the two small cells joined in parallel, Fig. 22 (*c*), will be one-half that of one of the small cells acting alone, Fig. 22 (*a*). Since the two cells joined in parallel are equivalent to a single cell having plates twice the size, the two cells in parallel will naturally have the same E. M. F. as the single large cell, which is the same as that of one of the small

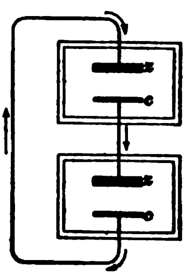

FIG. 23

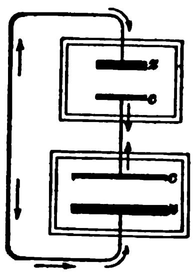

FIG. 24

ones. Hence, any number of similar cells joined in parallel will have the same E. M. F. as a single cell no matter what its size may be, but a resistance equal to that of one cell divided by the number of cells.

69. Cells in Series.—Fig. 23 shows two cells joined in series. Not only are the E. M. F.'s of the cells united, but their resistances also. Since the current has to go through both resistances one after the other, the length of the fluid column is doubled and consequently the resistance also. Hence, when similar cells are joined in series, the resistance must be that of one cell multiplied by the number of cells. Furthermore, the E. M. F. of each cell is superimposed on that of the others in the series, and therefore the total E. M. F. of the battery is equal to the sum of all the E. M. F.'s; or, in the case of similar cells, is the E. M. F. of one cell multiplied by the number of cells.

In Fig. 24, the cells of Fig. 22 (*a*) and (*b*) are joined in the manner indicated. There will be no current, because the E. M. F. of each cell is the same and acting in opposition to each other. That one cell is much larger than the other does not alter this fact. The arrows indicate the direction in which the E. M. F.'s tend to send a current.

EXAMPLE 1.—If the E. M. F. of each cell in Fig. 23 is 1.1 volts and the internal resistance of each cell is .6 ohm, what will be the E. M. F. and the internal resistance of the battery?

SOLUTION.—The E. M. F. of the battery will be $1.1 \times 2 = 2.2$ volts. Ans.

The internal resistance of the battery will be $.6 \times 2 = 1.2$ ohms. Ans.

EXAMPLE 2.—If the E. M. F. and internal resistance of each cell in Fig. 22 (*c*) is 1.1 volts and .6 ohm, respectively, what will be the E. M. F. and the internal resistance of the battery?

SOLUTION.—The E. M. F. of the battery, since similar cells are joined in parallel, will be equal to that of one cell, namely, 1.1 volts. Ans.

The internal resistance, since the total area of two similar plates joined together is twice that of one plate, will be $\frac{.6}{2} = .3$ ohm. Ans.

70. If several cells, all of the same size and kind, are connected in series, their total internal resistance will equal the resistance of one cell multiplied by the number of cells, and their total E. M. F. will equal the E. M. F. of one cell multiplied by the number of cells; if they are all connected in parallel, their total resistance will be equal to the resistance of one cell divided by the number of cells, while their total E. M. F. will be equal to that of a single cell. From this it follows that if the external resistance is very small, increasing the number of cells in series will not increase the current in the external circuit appreciably, as the resistance increases nearly as fast as the E. M. F.; while if the external resistance is great, increasing the number of cells in parallel will not appreciably increase the current flowing, as the total resistance is not much altered, while the E. M. F. remains the same.

71. Ampere-Hour Capacity.—It will now be seen that the ampere-hour capacity of a battery of several cells will be affected by the method of joining them. A number of similar cells taken separately each have a certain definite ampere-hour capacity. By joining the cells in parallel, the ampere-hour capacity of the battery will be that of a single cell multiplied by the number of cells in parallel, for it would be the same as constructing a single large cell containing an amount of material equivalent to that of all the smaller cells. If, however, the cells are joined in series, their internal resistance increases as fast as the E. M. F. increases and the current from the series (assuming no resistance in the external circuit) remains the same as for a single cell and the ampere-hour capacity is the same for the series as for a single cell.

Let us consider a case where there are ten cells, each having a capacity of 100 ampere-hours and an E. M. F. of 2 volts. When the cells are joined in parallel, the capacity of the battery will be 1,000 ampere-hours. When the cells are joined in series the capacity of the battery will be 100 ampere-hours. In the first case the E. M. F. is 2 volts, so that the battery will give 2,000 watt-hours; in the second case the E. M. F. is 20 volts, so that the battery will still give 2,000 watt-hours. The energy is the same in both cases, just as we should expect. Joining cells differently does not change the amount of energy that can be obtained from them but does change the values of the energy factor (quantity and E. M. F.); one increases as the other decreases.

72. Current in a Circuit.—According to Ohm's law the current in a circuit is equal to the total E. M. F. divided by the total resistance of the circuit. The total E. M. F. of a battery of similar cells connected in series is equal to the E. M. F. of one cell multiplied by the number of cells in series. Thus, if e is the E. M. F. of one cell and s the number of cells in series, then se is the total E. M. F. of the battery. The number of rows of cells in parallel does not affect this E. M. F. If there are s cells in series in one row and the internal resistance of each cell is b ohms, then, evidently, the total internal resistance of the one row of cells is sb ohms. If instead of one row of s cells in series there are p rows, each row having s cells in series, then the total internal resistance of the battery $= \frac{sb}{p}$. To get the total resistance of the circuit, the resistance of the external circuit, which we will call R ohms, must be added to the internal resistance of the battery. Doing this we get as the total resistance of the circuit $\frac{sb}{p} + R$. Then, according to Ohm's law, the current in the circuit is given by the formula:

$$I = \frac{se}{\frac{sb}{p} + R} \qquad (4)$$

Example.—Twenty-six cells, each having an E. M. F. of 1 volt and an internal resistance of 2 ohms, are connected so as to form 2 rows of 13 cells each in series. If the terminals of this battery are connected by an external resistance of 60 ohms: (a) what current will flow through the external circuit? (b) how much current will flow through each row of cells?

Solution.—For solving this example use the formula:

$$I = \frac{se}{\frac{sb}{p} + R},$$

in which the number of rows $p = 2$. Since there are 13 cells in series in each row, $s = 13$. Then, $I = \frac{13 \times 1}{\frac{13 \times 2}{2} + 60} = .18$ amperes, nearly. Ans.

73. Maximum Current.—It has been proved that a **maximum current** is obtained from a given number of cells through a given external resistance, when the grouping of the cells is such that the internal resistance of the battery is, as nearly as possible, equal to the external resistance. That is, arrange the cells so that $\frac{sb}{p}$ is, as nearly as possible, equal to R.

Let us assume that the total number of cells $s \times p = 12$, $e = 2$, $R = 3$, $b = 2$. Substituting in formula **4**, and taking the values of p given in each case, we get when

$$p = 1. \quad I = \frac{12 \times 2}{(\frac{12}{1} \times 2) + 3} = \frac{24}{24 + 3} = .9 \text{ ampere}$$

Total internal resistance $= 12 \times 2 \div 1 = 24$ ohms.

$$p = 2. \quad I = \frac{6 \times 2}{(\frac{6}{2} \times 2) + 3} = \frac{12}{6 + 3} = 1.3 \text{ amperes}$$

Total internal resistance $= 6 \times 2 \div 2 = 6$ ohms.

$$p = 3. \quad I = \frac{4 \times 2}{(\frac{4}{3} \times 2) + 3} = \frac{8}{2\frac{2}{3} + 3} = 1.4 \text{ amperes}$$

Total internal resistance $= 4 \times 2 \div 3 = 2\frac{2}{3}$ ohms.

$$p = 4. \quad I = \frac{3 \times 2}{(\frac{3}{4} \times 2) + 3} = \frac{6}{1\frac{1}{2} + 3} = 1.3 \text{ amperes}$$

Total internal resistance $= 3 \times 2 \div 4 = 1\frac{1}{2}$ ohms.

It is thus seen that the largest current is obtained when the internal resistance is $2\frac{2}{3}$ ohms, which approaches nearest to the value of the external resistance.

When the internal and external resistances are equal the efficiency is only 50 per cent. because half the energy is expended in the cell itself. The greater the ratio of external to internal resistance the greater will be the efficiency, because the greater will then be the proportion of the total energy expended in the external circuit. Very often it is not practical, nor is it always desirable to make the internal and external resistances equal. Furthermore, to make the efficiency very high would often require so many rows of cells in parallel as to be prohibitive on account of the first cost of the cells. No general rules can be given for the best arrangement or for the number of cells required, because both depend too much on local conditions, requirements, and cost of cells. In practice the arrangement that will give the desired current with the least number of cells is the one most generally used for primary cells. Ordinarily, in telephone, telegraph, and fire-alarm work the external resistance is high, while for ringing bells, gas-lighting, and similar work the resistance is low; batteries for these purposes should be grouped accordingly.

EXAMPLE.—It is desired to connect 32 cells, each having an E. M. F. of 2 volts and an internal resistance of 4 ohms, in a series-parallel group that will give the maximum current through an external resistance of 8 ohms.

SOLUTION.—Evidently the number of cells in series in each row multiplied by the number of rows will be the total number of cells; that is, 32. Hence $s \times p = 32$. Then $s = \frac{32}{p}$. For a maximum current $\frac{s\,b}{p} = R$, or $s = \frac{R\,p}{b}$, in which $b = 4$ and $R = 8$, hence $s = \frac{8\,p}{4} = 2\,p$. Since $s = \frac{32}{p}$ and $s = 2\,p$, then evidently $\frac{32}{p} = 2\,p$, from which we get $16 = p^2$ and hence $p = 4$. If $p = 4$ then $s = \frac{32}{4} = 8$. Hence the cells should be connected in 4 rows with 8 cells in series in each row. Ans.

This will give, by formula 4, a current $I = \frac{8 \times 2}{\frac{8 \times 4}{4} + 8} = 1$ ampere.

Any other arrangement of these 32 cells will give less than 1 ampere when the external resistance is 8 ohms.

TESTING CELLS

74. For a complete test of a cell the following quantities should be determined: (1) The E. M. F. of the cell when the external circuit is open; (2) the normal current it is capable of producing; (3) the internal resistance of the cell; (4) polarization, that is, the amount the E. M. F. is decreased on account of the polarization produced when the external circuit is closed; (5) efficiency.

75. Electromotive Force.—Other things being equal, the higher the E. M. F. for a given consumption of battery materials, the more efficient is the cell and the fewer cells are required to furnish a given E. M. F. The consumption of materials will usually depend only on the quantity of electricity developed and never on the E. M. F. of the cell. The instruments to be used, the formulas and the manner of making all the measurements necessary in testing cells are explained in *Electrical Measurements*.

76. Current.—The cell that will give the largest current is not necessarily the best cell by any means. The best cell to use will depend on the conditions to be met. One cell may give a very large current, but for only a short time, whereas another cell may give only a very small current, but will maintain such a current for a very long time. There is a best current output for each cell but its value is rather difficult to determine, but it may be said to be that current for which the cell gives the greatest chemical efficiency (to be explained presently), or for open-circuit cells, the current that the cell will maintain, without a great and sudden decrease in its value, for the longest time. This determines the life of the cell, also.

77. The *internal resistance* is a variable quantity and for that reason it is not necessary to attempt its measurement with great accuracy. It varies with the current the cell is generating. For any given current output it is determined by simultaneously measuring the difference of potential between the cell terminals and the current output and then opening the external circuit and immediately measuring the E. M. F. of the cell on open circuit. Then, if E is the E. M. F. of the cell on open circuit, V the difference of potential across the cell terminals when the external circuit is closed, and I the current, we have the internal resistance

$$b = \frac{E - V}{I} \qquad (5)$$

The lower the internal resistance, the larger is the proportion of the total energy that is utilized in the external circuit and therefore the greater is the electrical efficiency of the cell. Low internal resistance is desirable, but when the external resistance is comparatively high the internal resistance is of less importance. For large currents low internal resistance is necessary.

78. Polarization.—The greatest defect in most voltaic cells is the polarization, but the purpose for which the cell is to be used will determine how much polarization may be allowed. Thus the polarization in a Leclanché cell is very much greater than in the Edison-Lalande cell, but when small intermittent currents only are required the polarization of the Leclanché cell is not prohibitive by any means. Since the polarization varies with the current, it should be determined for the normal current that the cell is intended to generate. As polarization is a variable quantity, it may be determined with sufficient accuracy by first measuring the E. M. F. of the cell on open circuit, then close the circuit through a resistance that will give the desired current. When this current has been flowing the desired length of time, open the external circuit and immediately measure again the E. M. F. of the cell. The difference between the two measurements gives the amount of polarization for the desired current and time during which it flowed.

Successive measurements of E. M. F. after opening the external circuit will show the recovery of the cell due to depolarization.

79. Efficiency.—The *chemical efficiency* of a cell is the ratio of useful zinc consumption to total zinc consumption. The total zinc consumption may be determined by weighing the zinc before and after using the cell an observed length of time. The useful zinc consumption is the weight of zinc that would be electrolytically deposited by the same current delivered by the cell during the same length of time that the cell was in operation. This may be determined by calculation from the observed average current delivered by the cell, the time, and the electrochemical equivalent of zinc.

The ratio of the ampere-hours actually obtained from a cell to the ampere-hours that should be theoretically obtained from the consumption of a given amount of material is called the *ampere-hour efficiency*.

The *electrical efficiency* of the cell is the ratio of the energy expended in the external circuit to the total electrical energy developed, or to that expended in both the external and internal circuits. The ratio of the watt-hours expended in the external circuit to the total number of watt-hours developed by the cell is called the *watt-hour efficiency*. The energy expended in the internal circuit is I^2b. In the external circuit the energy expended is I^2R, when the entire fall of potential in the external circuit is expended in forcing the current through the resistance R, or it is $I \times V$, in which V is the difference of potential across the terminals of the cell when the circuit is closed. I is the current in amperes and b the internal resistance in ohms. In this case the electrical

$$\text{efficiency} = \frac{I^2R}{I^2(b+R)} = \frac{R}{(b+R)} \text{ or } \frac{V}{I(b+R)}.$$

Besides the tests indicated, account should be taken of mechanical defects, unequal corrosion of the zincs, facility with which the cell may be set up and subsequently cleansed, cost of the cell and of the material consumed, and kind of work for which the cell is suited and its probable life for such purposes.

TESTS ON OPEN-CIRCUIT CELLS

80. The tests applied to cells depends on the use for which the cell is intended and should approach the conditions under which the cell is to be used as closely as practical. It is not very often practical, for instance, to extend a test on a cell over a year so as to fulfil the conditions under which an ordinary dry cell is used. However, if a number of cells of a similar type are all tested in a uniform manner, the best one for the purpose can be selected by a comparison of the results. In testing open-circuit cells it is almost customary to use an external resistance of 5 ohms or 10 ohms and to plot curves showing the variation of current, internal resistance, terminal potential difference, E. M. F. (on open circuit) with the time and also the recovery of the E. M. F. with the time after the test.

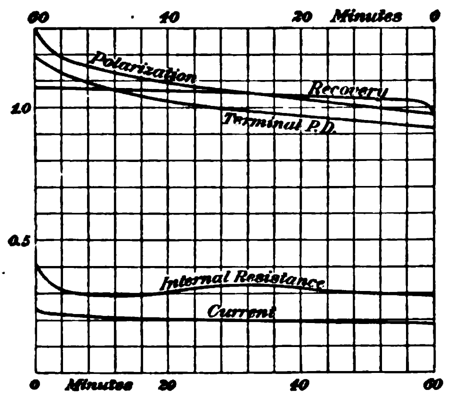

FIG. 25

In Fig. 25 is shown a set of curves, given by Carhart, who plotted them from actual tests made on a good dry cell. The same scale of ordinates applies to all the curves, although they do not represent the same electrical quantities. It will be seen from the polarization curve that the E. M. F. (on open circuit) falls, due to polarization, quite rapidly at first and then more gradually to the end of the test. It does not fall as rapidly, however, at the start as for most dry or Leclanché cells. For intermittent work it is desirable that there shall not be an excessively rapid fall in E. M. F. during the first minute because that is the very time the cell is most used. The recovery of the E. M. F. is shown for 60 minutes after the main test by the recovery curve, which is plotted backwards; the scale of minutes at the top applies only to this one curve. It starts, of course, where the polarization curve ends, rises very rapidly at first, then more and more gradually, but it does not again reach the initial E. M. F. of the cell. The internal resistance is low and the unusually strong current maintains its strength better than usual for a cell of this type.

Although a short test of this kind made on various cells shows the relative value of polarization, it gives no idea of the probable useful life of the cell, which is a very important property of cells used for intermittent work. All things considered, a good way to compare the life of Leclanché or similar open-circuit cells is as follows: Arrange the circuit so that the sum of the internal and external resistance shall be as near 10 ohms (or 5 ohms, if this is nearer the conditions under which the cell is to be used) as it is practical to make it and, without going to an unreasonable amount of trouble, try to keep the total resistance of the circuit about constant by adjusting the external resistance. Then allow the circuit to remain closed and note the time required for the current to decrease to one-half its theoretical maximum value, usually about $\frac{1}{2}\left(\frac{1.5}{10}\right) = .075$ ampere for zinc–ammonium chloride–carbon cells. The longer this time, the greater the life of the cell. This assumes that a current smaller than 75 milliamperes is not useful for the purpose for which such cells are generally used. An increase in internal resistance and hence a total resistance greater than 10 ohms will give a smaller average current but, other things being equal, a longer time will generally be required for the current to run down to 75 milliamperes for the polarization will be less. The life of various cells may be even better compared, by performing the test just indicated, but basing the comparison on the watt-hours given by the various cells. This will not require an adjustment of the external resistance after the run is started because the effect of the internal resistance is practically eliminated. The average current times the average potential difference at the terminals times the hours kept on closed circuit will give the useful watt-hours. Although local action in open-circuit cells is usually so small as to render chemical efficiency tests unnecessary, it is well, nevertheless, to watch for it, because local action on open circuit will render the cell unfit for intermittent work.

Sometimes tests are made extending over several days, automatic clockwork devices being arranged to close the circuit for any desired length of time at regular intervals, readings being taken as often as desired.

TESTS ON CLOSED-CIRCUIT CELLS

81. Closed-circuit cells should be tested with such a resistance in the internal circuit that the normal current generated will remain nearly constant, the polarization be slight, and the internal resistance nearly constant throughout the life of the cell. When the current, which has before

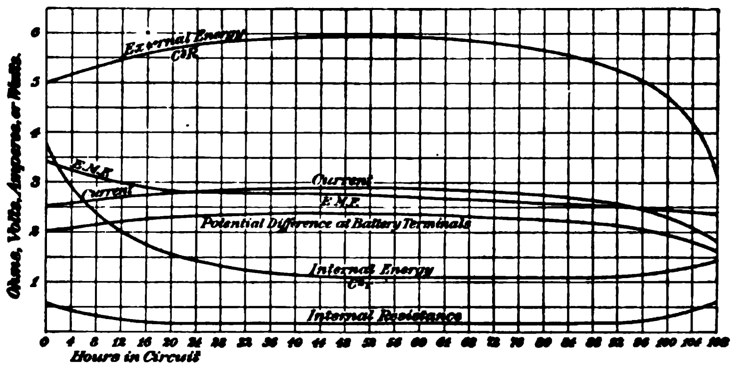

FIG. 26

remained fairly constant, begins to decrease rapidly, the useful life of the cell is assumed to be ended. The internal resistance of this type of cell must be low, otherwise the cell will be very inefficient and unsatisfactory on account of the resulting low-potential difference at its terminals on closed circuit. Local action in closed-circuit cells is usually of sufficient importance to warrant a chemical-efficiency test. If the cell is to remain a portion of the time on open circuit, the cell should remain on open circuit for the same percentage of time when the chemical-efficiency test is made. This becomes more important the greater the local action. Too much importance must not be attached to efficiency tests, because primary batteries are generally used only because they are the only convenient and suitable source of the current desired, their cost being a secondary consideration. As a source of power they cannot compare with dynamos in first cost, cost of maintenance, or space occupied.

In Fig. 26 is shown the results of a typical test made by Prof. A. E. Kennelly on four Edison-Lalande cells; the same scale is used for all curves. The four cells were joined in series with .8 ohm in the external circuit. The current, which is very steady, and the potential difference actually increase in value, due to the decrease in the internal resistance. The variation of the various quantities with the time is very nicely shown by the curves. To obtain the value for one cell of any ordinate on any curve except the current curve, the figures on the left must be divided by 4, since four cells were joined in series.

EXAMPLE.—The following data was obtained by a chemical-efficiency test made on a cell closed through an external resistance of .2 ohm: Weight of zinc before test, 2,508 grains; weight of zinc after 10.8 hours, 2,142 grains; mean current, 2.76 amperes; mean E. M. F., .7 volt. (*a*) What is the loss of zinc by local action, assuming that 1 coulomb deposits .0003387 gram of zinc? (*b*) What is the chemical efficiency? (*c*) What is the mean internal resistance if the mean terminal potential difference (on closed circuit) is .55 volt? (*d*) What is the mean electrical efficiency?

SOLUTION.—(*a*) The total consumption of zinc is 2,508 − 2,142 = 366 grams. 2.76 amperes flowing for 1 second will deposit .0003387 × 2.76 grams of zinc. Hence, in 108 hours, there will be deposited .0003387 × 2.76 × 108 × 60 × 60 = 363 grams. Hence, the loss of zinc due to local action is 366 − 363 = 3 grams, or less than 1 per cent. of the zinc consumed. Ans.

(*b*) The chemical efficiency is $\frac{363}{366} = .986$ or 98.6 per cent.

(*c*) Substituting in the formula $b = \frac{E - V}{I}$, we find the mean internal resistance to be $\frac{.7 - .55}{2.76} = .054$ ohm. Ans.

(*d*) The mean power expended in the external circuit $= (2.76)^2 \times .2 = 1.52$ watts. The mean power expended in the internal circuit $= (2.76)^2 \times .054 = .41$ watt. The electrical efficiency $= \frac{1.52}{1.52 + .41} = \frac{1.52}{1.93} = .787$, or nearly 79 per cent. Ans.

ELECTRICAL MEASUREMENTS

(PART 1)

ELECTROMAGNETIC MEASUREMENTS

1. A current of electricity is not a material substance, and, therefore, has no length, area, or weight by which it might be measured. It must, therefore, be measured by the effects that it produces.

These effects manifest themselves as follows: When a current of electricity is flowing in a conductor, the energy expended in overcoming the resistance of the conductor manifests itself as a **heat.** The amount of this energy is equal to the square of the current times the resistance; therefore, the heat generated in a circuit will be proportional to the square of the current if the resistance be constant, or to the resistance if the current be constant.

When a current of electricity flows through a conducting liquid, the liquid is decomposed. This decomposition is due to a chemical action of the current, known as **electrolysis,** and is distinct from the heating effect. The decomposition either liberates a certain amount of gas or deposits one or more of the elements of the liquid on one of the electrodes. The amount of liquid decomposed is directly proportional to the quantity (coulombs) of current; hence, the rate of decomposition, or the amount of liquid decomposed per unit of time, if the current is constant in strength, is proportional to the strength of the current in amperes.

When a current of electricity flows through a conductor, a magnetic field is set up around the conductor that tends to produce a relative motion in any other magnetic field in the vicinity; as, for instance, that emanating from a magnet pole. The force acting on such a pole will be directly proportional to the strength of the current, to the length of the conductor, to the strength of the magnet pole, and inversely proportional to the square of the distance between the conductor and the magnet pole when the conductor lies in the circumference of a large circle about the magnet pole as a center.

An instrument that measures a small current by its electromagnetic effect is called a *galvanometer*.

GALVANOMETERS

2. By the electromagnetic action the value of a current may be derived as follows: If a wire is bent into an arc of a circle with a radius of at least 15 centimeters, the strength of field $\mathcal{H}'$ produced at the center of the arc when a current I flows through the wire is proportional to the strength of the current and to the length of the arc, and inversely proportional to the square of the distance of the wire from the center; that is, inversely proportional to the square of the radius. If the wire forms a complete circumference of a circle, the length of the arc is $2\pi r$; hence, the strength of the field produced at the center of the circle is equal to $\frac{I \times 2\pi r}{r^2}$, or $\frac{2\pi I}{r}$ for one complete turn. If there are T turns in the coil, $\mathcal{H}' = \frac{2\pi I T}{r}$, provided that the radius of the coil is large compared to its length in a direction normal to the plane of the coil.

In this expression $\mathcal{H}'$ is the strength of the field, in C. G. S. units; I the current, in C. G. S. units; T the total number of complete turns in the coil; and r the mean radius of the coil in centimeters. The field $\mathcal{H}'$ is uniform within a small area at the center of the coil and has a direction normal to the plane passing through the coil and its center. If the current I is expressed in amperes, since 10 amperes = 1 C. G. S. unit, the strength of the field at the center of the coil is

$$\mathcal{H}' = \frac{\pi I T}{5 r} \qquad (1)$$

A rigid derivation of this formula requires the use of mathematics beyond the scope of this Course. It is evident that if $\mathcal{H}'$, r, and T were known that the current I could be calculated from formula **1.** But it is not often convenient to determine the value of $\mathcal{H}'$; hence, I is never calculated directly from this formula.

3. If a magnet pole can be influenced by a known constant force in one direction, and if by exerting on it a second force, due to a current circulating in a coil but acting in a different direction, the resultant of the two forces can be accurately determined, then the value of the second force may be determined and the strength of the current producing it may be calculated. This known constant force is furnished by the earth itself, which is a magnet of such size that for short distances the direction of its lines of force may be considered as perfectly parallel. The actual direction of the earth's field is not horizontal but at an angle to the horizontal so that the actual field may be said to be made up of two components—a horizontal and a vertical. The horizontal component is the one most frequently used in measurements. A small bar magnet placed in a horizontal position across the earth's field of force will have equal and opposite forces acting on its poles or ends, since the lines of force act in a parallel direction; this results in turning the magnet about its center, if the magnet is free to move about a vertical axis, until the forces act in a direct line with the center. It then points in the direction of the horizontal component of the earth's magnetism. The magnet is so supported that it can move only in a horizontal plane.

This is illustrated by the magnet in the common compass. The force of the earth's field tends to keep the magnet parallel to the lines of force of the earth's field, and, consequently, the magnet points north and south.

Fig. 1 illustrates this action. The direction of the earth's field of force is represented by the line *a b*. A bar magnet, *NS*, placed across this line at an angle with it, will have equal and opposite forces acting on the poles *N* and *S*, as shown by the arrows. These forces may be considered as parallel to the line *a b*; so, if the magnet be free to turn about its center, these forces will bring it to a state of rest in such a position that the magnet coincides with the line *a b*.

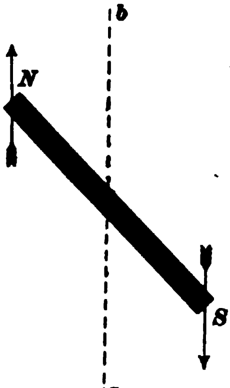

FIG. 1

If the magnet *N S* be acted on by another force at an angle with *a b*, the magnet will come to rest in a position where the two forces balance. Furthermore, if a coil, through which a current is flowing, is placed so that the plane of the coil is not only vertical but also coincides in direction with the earth's horizontal component of magnetism, then the magnetic field produced by the coil at and near its center will be at right angles to the earth's horizontal component. This component tends to make a magnet suspended on a vertical axis through its center lie in the plane of the coil, whereas the field due to the coil tends to make the magnet lie normal to the plane of the coil. In Fig. 2, the magnet *N S* is acted on by the earth's field along the line *a b*, the direction of the force on the *N* pole of the magnet being along the line *d N*, and that on the *S* pole along the line *c S*, as indicated by the arrowheads. In addition, another force due to the field set up by the current in the coil is acting along the line *x y*, at right angles

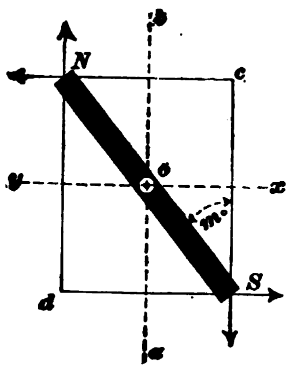

FIG. 2

to $a\,b$, the direction of the force on the N pole being along the line $c\,N$, and on the S pole being along the line $d\,S$, as indicated by the arrows. Under the influence of these two forces the magnet is deflected into the position shown, where it remains at rest, making the angle $m°$ with the line $a\,b$.

Calling the strength of the horizontal component of the earth's field $\mathcal{H}$, the strength of the field acting along the line $x\,y\,\mathcal{H}'$, and the strength of each pole of the magnet p, then the forces acting on the N pole of the magnet are equal to $\mathcal{H} \times p$ in the direction $d\,N$, and $\mathcal{H}' \times p$ in the direction $c\,N$; the forces acting on the S pole are equal to $\mathcal{H} \times p$ in the direction $c\,S$, and $\mathcal{H}' \times p$ in the direction $d\,S$. The two equal forces $\mathcal{H}\,p$ acting in opposite directions, $d\,N$ and $c\,S$, on the two poles of the magnet form a couple tending to rotate the magnet about its center o. The moment of this couple is equal to one force multiplied by the perpendicular distance between the lines of action. The moment of the couple produced by the force $\mathcal{H}\,p$ is, therefore, equal to $\mathcal{H}\,p \times c\,N$, tending to rotate the magnet in the same direction in which the hands of a watch move. Similarly, the force $\mathcal{H}'p$ produces a couple equal to $\mathcal{H}'p \times d\,N$ tending to rotate the magnet in a direction opposite to that in which the hands of a watch move. When the magnet is in equilibrium, that is, at rest, these two moments are equal; hence, $\mathcal{H}'p \times d\,N = \mathcal{H}\,p \times c\,N$, or $\mathcal{H}' \times d\,N = \mathcal{H} \times c\,N$. Since this last equation does not contain p, it follows that the deflection of the magnet is independent of the strength of the magnet. From the last equation we obtain $\frac{c\,N}{d\,N} = \frac{\mathcal{H}'}{\mathcal{H}}$. The tangent of an angle is equal to the side opposite divided by the side adjacent. In Fig. 2, $c\,N$ is the side opposite the angle $m°$, and $S\,c$, which is equal to $d\,N$, is the side adjacent; therefore, $\frac{c\,N}{S\,c}$ or $\frac{c\,N}{d\,N}$ is the tangent of the angle $m°$, and we obtain $\mathcal{H}' = \mathcal{H} \times \tan m°$. But it has also been shown that $\mathcal{H}' = \frac{\pi\,I\,T}{5\,r}$. Equating these two values for $\mathcal{H}'$ gives $\frac{\pi\,I\,T}{5\,r} = \mathcal{H} \tan m°$, from which we obtain

$$I = \frac{5\,\mathcal{H}\,r}{\pi\,T} \tan m^\circ \qquad (2)$$

This formula enables us to determine the current I in amperes when we know the value of the earth's horizontal component of magnetism $\mathcal{H}$ in C. G. S. units (that is, in dynes, or in lines of force per square centimeter), the mean radius r of the coil in centimeters, the total number of turns T in the coil, and the angle of deflection m° produced by the current of I amperes passing through the coil.

4. Tangent-Galvanometer Constant.—For any given coil of a given tangent galvanometer the factors $\frac{5\,r}{\pi\,T}$ remain constant and its value may be computed once for all. Furthermore, if the galvanometer is set up where $\mathcal{H}$ does not change then $\frac{5\,r\,\mathcal{H}}{\pi\,T}$ may be computed once for all, and if $\frac{5\,r\,\mathcal{H}}{\pi\,T} = K$, then

$$I = K \tan m^\circ \qquad (3)$$

K is called the **constant** of the galvanometer for a given magnetic field $\mathcal{H}$. If K is known, a current may be measured by causing it to flow through the galvanometer coil, noting the steady deflection produced, and then multiplying the tangent of the angle of this deflection by the constant K. An instrument depending on this principle for measuring a current is called a *tangent galvanometer.*

5. The **horizontal component** $\mathcal{H}$ of the earth's field has been accurately measured at various places and times. But it is constantly changing. At some places it changes much more than at others, and as the yearly variation is not very definitely known for many places it is almost impossible to keep the values revised up to date. For this reason such instruments as the tangent galvanometer, whose deflection depends on the value of $\mathcal{H}$, are rapidly being displaced by instruments whose deflections are practically independent of $\mathcal{H}$.

Table I gives the value of the earth's horizontal component $\mathcal{H}$ for some well-known cities for the year 1903, except where otherwise stated. These values were determined by the United States Coast and Geodetic Survey.

TABLE I

HORIZONTAL COMPONENT OF THE EARTH'S MAGNETISM FOR THE YEAR 1903

Place	Horizontal Component $\mathcal{H}$ in Lines of Force per Square Centimeter, or in Dynes	Place	Horizontal Component $\mathcal{H}$ in Lines of Force per Square Centimeter, or in Dynes
Albany	.1709	Omaha	.1980
Baltimore	.1960	Philadelphia	.1950
Boston	.1755	Salt Lake City	.2268
Buffalo	.1706	San Francisco	.2495
Chicago	.1858	Scranton, Pa. (1902)	.1825
Cincinnati	.2056	Seattle	.1903
Cleveland	.1850	St. Louis	.2126
Denver	.2248	Washington	.2035
El Paso, Tex.	.2770	Halifax	.1653
Indianapolis	.2070	Montreal	.1516
Milwaukee	.1775	Quebec (1900)	.1481
Minneapolis (1900)	.1681	Toronto (1900)	.1691
New Orleans	.2788	Victoria, V.I. (1900)	.1878
New York	.1895		

6. It is necessary that the lines of force that influence the magnet be practically parallel within the range covered by the swing of the magnet. With the earth's field this is the case, but with a coiled conductor, whose radius is large compared to its length at right angles to the plane of the coil, this only holds true for a very small space relative to the diameter of the coil at the center of the coil. The magnet of a tangent galvanometer must, therefore, be short, compared with the diameter of the coil. A magnet $\frac{3}{4}$ inch long can be used with a coil 8 inches in diameter with accurate results. As the deflections of such a magnet can scarcely be read directly, a very light glass or aluminum pointer attached to it, usually at right angles, extends over a scale on which the deflections may be read.

Fig. 3 gives a top view of a simple **tangent galvanometer** in which *NS* is the coil of wire and *P* the pointer

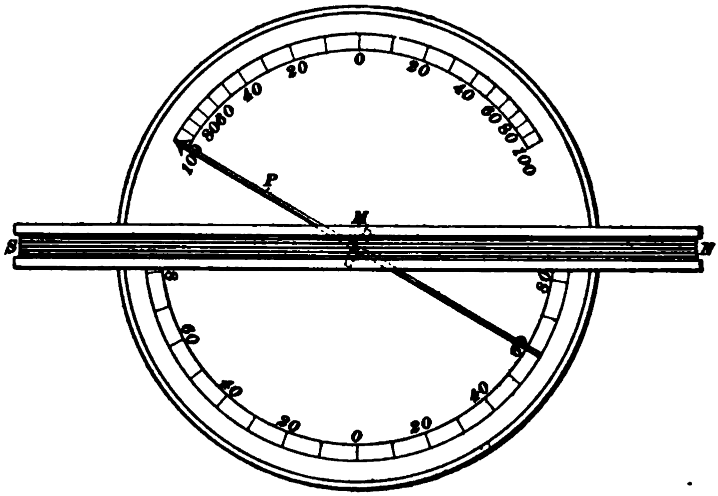

FIG. 3

attached to the small permanent magnet *M*. Two scales are shown, one on each side of the coil. One is divided into degrees, the other into divisions proportional (but not equal) to the tangents of the angles represented by the divisions on the degree scale. While the tangent scale is the most convenient to use, nevertheless degree scales are more often used because they are generally more accurate. If the angle of deflection is read in degrees, the corresponding tangent of the angle may be readily found in a table of Natural Tangents.

7. Controlling Magnet.—In order that a variety of current strengths may be measured with the same instrument, it is customary to wind the coil in two or more parts, varying in the number of turns and size of wire. The terminals of these parts of the coil are led out to binding posts *b*, *b*, *b*, *b*, Fig. 4, on the base of the instrument, so that either one or all the parts of the coil may be used. Even this method of winding does not give much range to the instrument. Another way of regulating its indications is to vary the effective earth's field by placing a permanent bar magnet, called a *controlling magnet*, usually in the plane of the coil and parallel to it, but often in any position that may be convenient and still give the desired result.

Fig. 4

Fig. 4 shows a tangent galvanometer, with an adjustable controlling magnet *m*. If this controlling magnet be so placed that its *S* pole corresponds in direction with the *N* pole of the magnet of the instrument, its field will be added to the earth's field, so that a given current will give a smaller deflection than if the controlling magnet were removed. Hence, a larger current may be measured with a controlling magnet in this position. If the polarity of the controlling magnet be reversed, the opposite effect will result, and the instrument will give a deflection with a very small current. Hence, smaller currents may be measured with a controlling magnet when its north pole points toward the north, provided the controlling magnet is not so strong or so near the galvanometer needle as to not only neutralize the earth's field, but to produce in the opposite direction a stronger field than that originally due to the earth alone. Since the field produced near a permanent magnet may be very strong, the latter condition may be readily produced.

8. Drift.—Controlling magnets are used on many forms of galvanometers. There is a difficulty, known as **drift,** which is increased, when a controlling magnet is used to make the galvanometer more sensitive. This difficulty is due to the fact that the direction of the earth's field is continually changing slightly, and its effect is to make the zero point of the instrument vary from time to time. This effect may be shown by the diagram in Fig. 5. In (*a*), *n o* represents the direction and magnitude of the force due to the earth's field, and *n m* the direction and magnitude of the force due to the controlling magnet; the resultant *n s* is the direction that the magnet of the instrument will assume. If the direction of the earth's field change through a slight angle to the position shown in (*b*), the resultant is the line $n_1\ s_1$, and its direction is at an angle of nearly 180° to the resultant *n s*. If the controlling magnet had not been used, there would have been a slight drift, but the use of the controlling magnet to lessen the effective field very much magnifies the effect of any change in the direction of the earth's field.

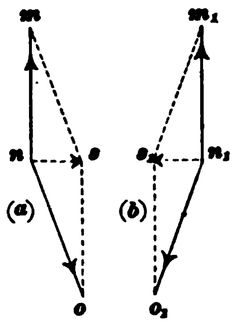

FIG. 5

9. Calibration of a Tangent Galvanometer.—When a controlling magnet is used, it is necessary to find the deflection that a certain known current will produce, as the actual value of the controlling field is no longer known and K in the formula $I = K \tan m°$ cannot very well be computed. But the galvanometer may be used to measure currents in the following manner: Send a known current through the galvanometer coil and note the deflection. Then, putting the formula $I = K \tan m°$ in the form $K = \frac{I}{\tan m°}$ and knowing I and $m°$, the value of K may be computed. Any other current may then be measured by multiplying the tangent of the angle of deflection that the unknown current produces by this value of K. This method of determining the constant of a galvanometer or other measuring instrument is called **calibration.**

This may seem a roundabout way of measuring a current and the question may arise, Why not measure the current by the method in which the calibrating current was measured? In reply we would say that while preferable, it may be very inconvenient or even impossible. For instance, the value of the standardizing current may be determined by a very accurate but slow method, requiring at least 30 minutes to determine one value, which would be entirely impracticable for most purposes. Or, the standardizing current may be measured by a very accurate instrument whose use it is possible to obtain for only a short time, whereas the instrument calibrated is to be used constantly, day after day. Most practical instruments must be calibrated in some way from time to time to detect any change or inaccuracy.

10. The **range** of any current-measuring instrument is the extent of variation of current that the instrument is capable of measuring. For instance, one galvanometer may be capable of measuring any current between 0 and .001 ampere, another may be capable of measuring any current between 0 and .1 ampere. The last instrument has a range one hundred times greater than the first, but it will probably not measure currents less than .001 ampere as accurately. Instruments constructed to measure large currents will very seldom measure small currents accurately, because the scale divisions are too close together. For instance, there is a voltmeter, which is an instrument for measuring difference of potential in volts, that will accurately measure as high as 600 volts with a scale divided into divisions of 4 volts each; hence, its range is from 0 to 600 volts in steps of 4 volts. To be sure, the deflections of the pointer may be read closer than 4 volts by estimating its location when it lies between two divisions; thus, a reading of 385 volts may be made with fair accuracy. Such a reading would indicate that the needle was one-fourth of 1 division (since one-fourth of 4 = 1) beyond the 384 mark. On the other hand, there is a voltmeter that only reads from 0 to .02 volt, but it can be read directly from the scale to .0002 volt, there being 100 divisions, each representing .0002 volt.

11. Setting Up a Tangent Galvanometer.—The plane of the galvanometer coil must be, at least, approximately in the magnetic meridian and vertical; the center of the scale should coincide with the center of the needle, and both should be in the plane of the coil. The suspension, if a fiber, should be free from torsion; if a pivoted bearing, it should be as free from friction as possible. If the coil is not in the magnetic meridian a given current will produce a larger deflection when it flows in one direction through the coil of the galvanometer, than when it flows in the opposite direction. It is therefore best when using a tangent galvanometer to read the deflections at both ends of the pointer, then reverse the direction of the current through the galvanometer, which reverses the deflection of the needle, and again read the deflections at both ends of the pointer. The average of these four deflections should be taken as the correct deflection of the galvanometer. This process eliminates most of the errors due to imperfect adjustment of the galvanometer. This cannot very well be done when the scale on one side is marked in degrees and on the other side in tangents, or numbers proportional to the tangents of the corresponding angles.

12. The following example will illustrate the application of the formulas for the tangent galvanometer:

EXAMPLE.—Galvanometer No. 1 has a coil of only one turn and a mean diameter of $7\frac{7}{8}$ inches. Galvanometer No. 2 has a coil of three turns and a mean diameter of $7\frac{7}{8}$ inches; furthermore, a controlling magnet increases its range. The two galvanometers are connected in series so that the same current must flow through them and are set up where the earth's horizontal component is .194 line of force per square centimeter. On sending a current through the two instruments, the deflection of No. 1 is 52°, while the deflection of No. 2 is but 38°. (*a*) What current is passing through the galvanometers? (*b*) What is the value of the galvanometer constant of No. 2? (*c*) What is the strength of field at the needle of galvanometer No. 2?

SOLUTION.—(*a*) The diameter of the coils in both instruments is $7\frac{7}{8}$ inches, or 20 centimeters; hence, the radius r = 10 centimeters. Consider now only galvanometer No. 1. For use in the formula $I = \left(\frac{5\,\mathcal{H}\,r}{\pi\,T}\right)$ tan $m°$, we have $\mathcal{H}$ = .194, r = 10, and T = 1. The deflection $m°$ = 52°. From a table of Natural Tangents it is found that the tan 52° = 1.28, nearly. Substituting these values in the formula, $I = \left(\frac{5 \times .194 \times 10}{3.1416 \times 1}\right) 1.28 = 3.952$ amperes. Ans.

(*b*) In No. 2, the current is 3.952 amperes and the angle of deflection produced in this galvanometer is 38°. From a table of Natural Tangents it is found that tan 38° = .7813, nearly; then, substituting in the formula $I = K$ tan $m°$, we have $3.952 = K \times .7813$, $K = \frac{3.952}{.7813} = 5.058$. Ans.

(*c*) From this value of K and the dimensions of the galvanometer coil, the strength of field $\mathcal{H}$ at the galvanometer needle may be readily calculated as follows: It has been shown that $K = \frac{5\,\mathcal{H}\,r}{\pi\,T}$, from which we obtain $\mathcal{H} = \frac{K\,\pi\,T}{5\,r}$. Hence, $\mathcal{H} = \frac{5.058 \times 3.1416 \times 3}{5 \times 10} = .9534$ dyne or lines of force per sq. cm. Ans.

This value of $\mathcal{H}$ represents the combined value of the field due to the earth and to the controlling magnet. As will be seen, the intensity of this field is nearly five times that of the earth alone; so galvanometer No. 2 may be used to measure currents of about five times the strength that No. 1 will measure under the same conditions.

EXAMPLE FOR PRACTICE

A galvanometer has a coil of 12 turns and a mean diameter of 12 inches. When set up where the earth's horizontal component is .147, a certain current passing through it produces a deflection of 42°. What is the strength of this current, in amperes? Ans. .268 ampere

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REFLECTING GALVANOMETER

13. If the needle of a tangent galvanometer be suspended by a fiber of raw silk or other similar material without twist, and if a beam of light reflected from a small mirror attached to the needle be used instead of a pointer, accurate measurements of very small deflections can be obtained. Such an instrument is known as a **reflecting**

galvanometer. Two arrangements for the optical observation of deflections are used.

14. In the **telescope-and-scale** arrangement, a small plane mirror is attached to the deflecting system of the galvanometer and a telescope and stand carrying a horizontal straight scale, suitably divided, is set up directly in front of the galvanometer in such a manner that portions of the horizontal scale will be reflected by the mirror of the galvanometer into the tube of the telescope. Such a telescope and scale is shown in Fig. 6. Special scales are provided for this purpose, the numbers on which are reversed, so that when a reflection in the mirror is viewed through the telescope they will appear normal. This method of reading the galvanometer is more desirable than that using a scale and lamp, which will be presently described, because the readings may be made with greater accuracy by means of a telescope containing cross-hairs and further, because the presence of a lighted lamp is not necessary. However, a lamp placed to illuminate the scale (not the galvanometer mirror) will often render the reading of the scale much easier.

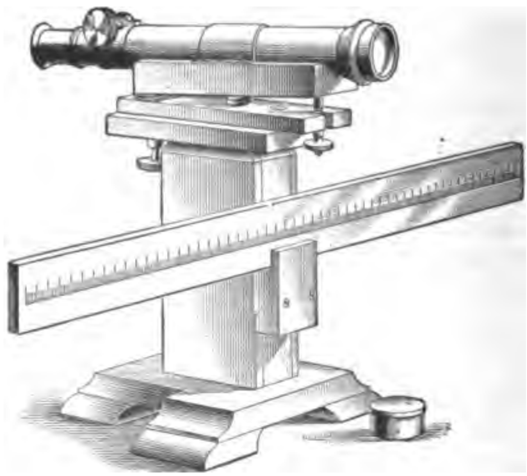

FIG. 6

15. The **lamp-and-scale** arrangement, Fig. 7, consists of a suitable support for a scale that is placed parallel to the plane of the galvanometer mirror and has a small slit below its middle. A lamp placed close behind this scale throws, through the slit, a beam of light that the mirror reflects, producing on the scale a bright spot that moves in accordance with the movements of the mirror.

This arrangement requires a reasonable dark room or the scale must be covered enough to make it dark. For the illuminated slit, a vertical wire stretched across a suitable

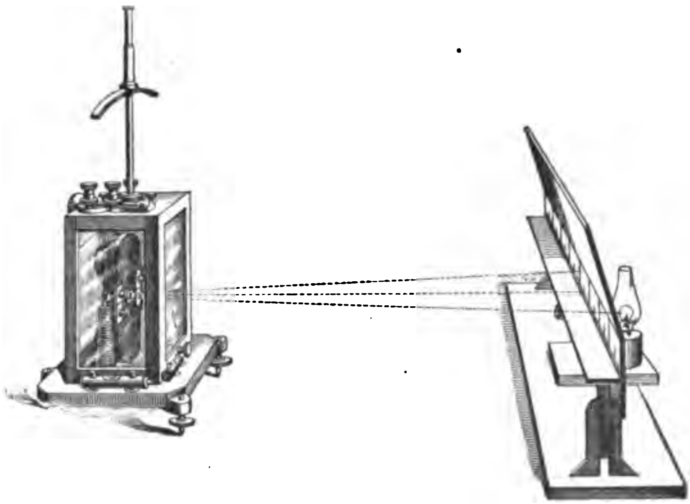

Fig. 7

convex lens may be substituted. There will then appear on the scale a vertical black line across a bright spot of light.

A somewhat different arrangement is shown in Fig. 8. The metal case *l* contains a gas, oil, or, preferably, an incandescent lamp. The light coming out of a small door, falls on the lens *h*, which concentrates it on a plane mirror *r*, which merely reflects it to a mirror fastened to the movable system of the galvanometer; the galvanometer mirror reflects the light back on the scale *s*. Either a plane or concave mirror with its focus on the scale may be used on the galvanometer. In case a plane mirror is used, a vertical wire should be stretched across a circular door in the case *l* or across the converging lens *h*, and the latter adjusted along its support until there appears on the scale a bright spot of light with a sharply focused vertical line across it. The scale is usually made, as shown in Fig. 8, on a piece of ground glass through which the bright spot illuminates the scale to allow the observer to read the deflection by standing in front of the scale; that is, with the scale between the observer and the galvanometer.

Although the zero of the scale may be either at the center or at one end, it is better to have it at the extreme end; then

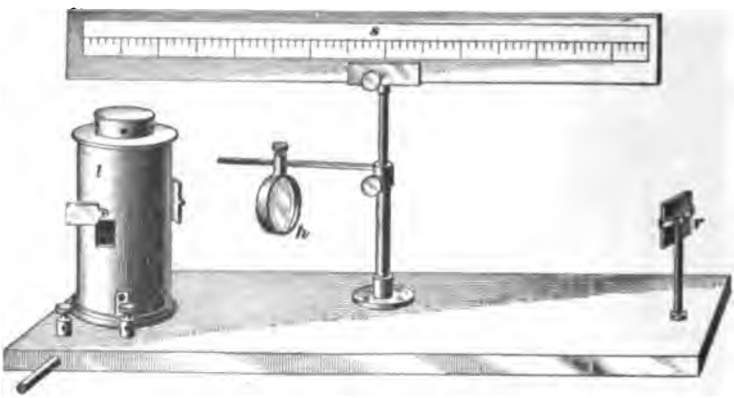

FIG. 8

it will not be necessary to note the direction of the deflection in addition to its amount, for the reading itself will show whether it is to the left or right of the middle position of the spot of light, which should be noted down any way. The actual deflection in this case is the difference between the at rest and deflection readings; in any case it is the actual distance the spot of light moves across the scale.

16. The angle between the original beam and the reflected beam will be equal to twice the angle of deflection of the mirror. Allowance for this fact must be made when it is necessary tc determine the angle, or the tangent of the angle of deflection of the deflecting galvanometer. This may be explained as follows: Let *o n*, Fig. 9, be the normal, undeflected position of the mirror and suppose it is parallel to the scale *b f*. A ray of light along *a c* is then reflected back along *c a*. If the mirror is deflected, through any angle $m°$, to the position $n' o'$ the ray *a c* will be reflected along *c b*, the angle *a c e*, called the *angle of incidence*, being equal to the angle *e c b*, called the *angle of reflection*. The line *c e* is drawn normal or perpendicular to the mirror $o' n'$. Since *a c* is perpendicular to *n o* and *e c* perpendicular to $n' o'$, then perpendicular line *e c* must have moved through the same angle as the mirror $o' c$; hence, the angle *a c e* = angle $o c o'$ = angle $m°$. But the angle *a c e* = angle *e c b*, because it is a well-known fact that the angle of incidence equals the angle of reflection, consequently, the angle $a c b = 2 \times$ angle $m°$. The distance *a b* is usually called the *deflection of the galvanometer*. The tangent of the angle *a c b* is, according to trigonometry, equal to $\frac{a b}{a c}$, that is, $\tan a c b = \frac{a b}{a c}$. But the angle *a b c* is twice the angle through which the mirror is deflected, that is, $\frac{a b}{a c} = \tan 2 m°$.

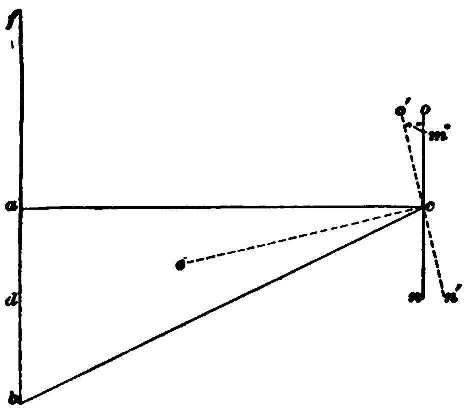

FIG. 9

Furthermore, it is not correct to say that $\tan \frac{1}{2} (a c b)$ or $\tan m° = \frac{a b}{2 a c}$, because $\frac{1}{2} \tan 2 m°$ is not equal to $\tan m°$. Consequently, it is not strictly correct to assume that scale deflections are proportional to the tangent of the angle of deflection of the mirror or needle. However, the error is less than 1 per cent. for deflections not exceeding 200 scale divisions on a scale distant 1,000 scale divisions from the mirror. For very small angles, such as are usually obtained with the ordinary arrangement, having a scale about 50 centimeters long set about 1 meter from the galvanometer, it is generally correct enough, except for the most accurate measurements, to assume that the tangent of the angle of deflection of the mirror equals the deflection on the scale divided by twice the distance of the scale from the mirror. When using a scale, care should be taken that it is parallel to the plane of the galvanometer mirror in its undeflected position, otherwise equal angular deflections of the mirror on each side of its normal zero position will not give equal deflections. Moreover, it is usually preferable to have the spot of light rest at about the center of the scale when the galvanometer system is in its normal at rest position.

17. Astatic Magnets.—The magnet of a reflecting galvanometer sometimes consists of a number of small magnets, made from bits of steel needles or pieces of watch spring, one-half of the magnets arranged with their poles opposing the remainder, which makes the magnet **astatic**; that is, the earth's field has almost no directive force on the magnetic system of the instrument. By using a strong controlling magnet, the instrument is made almost independent of the earth's field, and thus errors or drift due to variations in the horizontal component of the earth's magnetism are rendered of little effect.

18. Damping.—When the magnetic system with its mirror is suspended by a long fiber, considerable difficulty in reading may be met with, owing to the length of time required for the needle to come to rest after being deflected. This is corrected by **damping** the moving parts of the instrument, which may be effected by suspending from the needle a small fan, or vane, of very light construction, which, by reason of the friction of the air on the blades of the fan as it rotates causes the needle to swing more slowly and come to rest more quickly. The damping effect is increased by placing the vane in a small and almost air-tight chamber.

This damping effect is an important feature of most measuring instruments. Other methods are used, one of which is to enclose the moving magnetic needle in a cavity in a block of copper; the movement of the needle then sets up little eddy currents in the copper block, which retard the movement of the needle, giving the desired damping effect without affecting the final deflection.

THOMSON GALVANOMETERS

19. It is often desirable to use an instrument for indicating the presence of very small currents without necessarily measuring their value. For this purpose a tangent galvanometer must be considerably modified. By inspecting the formula $I = \frac{5\,\mathcal{H}\,r}{\pi\,T}\tan m^\circ$ for the tangent galvanometer, we see that to produce a given deflection m° the current I will be a minimum when the controlling field, as $\mathcal{H}$ is called, and the mean radius r of the coil have minimum values and there are a maximum number of turns T in the coil. The controlling effect of the field is reduced by using an astatic system of needles, because the controlling force is then due only to the difference of two very nearly equal reactions; that is, the two reactions between $\mathcal{H}$ and the two reversed sets of needles. If one set is exactly equal to the other in every respect, the system will point indifferently in any direction when suspended in the earth's field. If one set of needles is slightly stronger than the other, which is invariably the case in practice, the directing force due to the earth's field will even then be very small.

Ordinarily $\mathcal{H}$ in the formula for the tangent galvanometer is understood to be the field due to the earth's horizontal component, but it is really the field acting on the needles from whatever source it may be due, except that produced by the current in the coil. Then $\mathcal{H}$ in the formula may be made much weaker than the earth's field by using a permanent bar magnet so placed as to neutralize nearly all or to very slightly more than neutralize the strength of the earth's field. There is then left an exceedingly weak controlling field at the needles. Furthermore, the radius of the coil may be reduced until there is merely room at the center of the coil for the needles to rotate freely. There is a limit to the thickness of the coils and to the number of turns because in a short, or flat, coil, as used in galvanometers, the effect of a turn on the needle with a given current diminishes as the radius of the turn increases, as will be evident from a consideration of the formula $\mathcal{H} = \frac{\pi I T}{5 r}$, in which $\mathcal{H}$ is the intensity of the field at the center of a very short coil due to a current of I amperes. Furthermore, an increase in the number of turns beyond a certain limit increases the size of the coil and consequently increases the length of the outside turns so much that the increase in the resistance of the coil more than offsets the advantage gained by the increased tendency of these outside turns to rotate the needle.

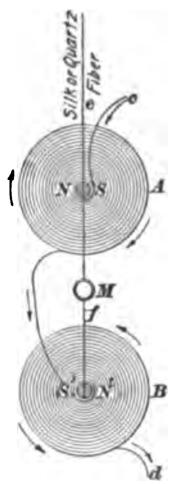

FIG. 10

20. The magnets are arranged to form an astatic system; the little magnets of one set, all of which point in one direction, being hung a little below the other set, all the magnets of which point in the opposite direction. Generally, two or four coils are used. When two coils are used, one is placed close to and directly behind the other and one set of needles is arranged to rotate in a little cavity between them; the other set of needles rotates immediately below or above the coils. The two sets are fastened rigidly to a very fine glass or quartz fiber, to which a very small light mirror is also fastened. When four coils are used, each set of needles is arranged to rotate between two coils. This arrangement is indicated in Fig. 10, in which A represents two coils, one behind the other, and B two coils, one behind the other. The very small needles $N S$ and $N' S'$, which are usually made of magnetized, glass-hard steel, piano wire, are glued to small pieces of very thin mica, which are in turn glued to a very fine glass, or preferably a quartz, fiber *ef*. The needles rotate about the fiber as an axis in a small recess between and in the center of the coils. To the fiber is also fastened a very small light mirror *M* by means of which the deflection may be observed. The rotating, or needle, system should be very light. The needle systems are usually suspended by means of very fine fibers of unspun silk or quartz. No matter how many coils there may be or whether they are connected in series or in multiple, the current must circulate in each coil in such a direction that they all tend to rotate the needle system in the same direction; otherwise, they will oppose instead of assist one another, as they should do.

In very sensitive galvanometers there is very little clearance between the coils, the needles are very short, the whole suspended system weighs only a few milligrams, and the suspending quartz or silk fibers are so fine that they can scarcely be seen with the naked eye. Galvanometers of this type may be made to distinctly detect currents that are as small as .000000000001 or 10^{-12} ampere. Although the deflections of very few sensitive reflecting galvanometers follow the law of the tangent galvanometer, nevertheless the angle of the deflections obtained is generally so small that the deflections themselves are proportional, to within about 1 per cent., to the currents producing them. Where the ratio of two deflections is used, especially if the deflections are nearly equal in value, the ratio may be nearer correct than either deflection. Hence, such galvanometers may be used for comparative measurements, as well as for the detection, of very small currents.

This form of instrument is known as the **Thomson, or Kelvin, galvanometer.** The one shown in Fig. 7 is a four-coil instrument with a controlling magnet on a rod above the case, which can be moved up or down or rotated about a vertical axis. It has a lamp and scale for observing the deflections.

THE D'ARSONVAL GALVANOMETER

21. An electromagnetic measuring instrument that is quite extensively used is the **D'Arsonval galvanometer,** which derives its name from its inventor, a French physicist. For the detection of very minute currents the Thomson galvanometer is best suited, but its use is attended with many difficulties which render it unfit for many forms of practical work. As a laboratory instrument, however, where it can be properly shielded from the magnetic fields set up by neighboring electrical machinery or by trolley or lighting currents, it is unexcelled. The D'Arsonval galvanometer, however, is sensitive enough for nearly all practical work, and possesses the advantage of being practically free from the effects of external fields. It is now made in portable form, so that it can be unpacked and set up in a few moments.

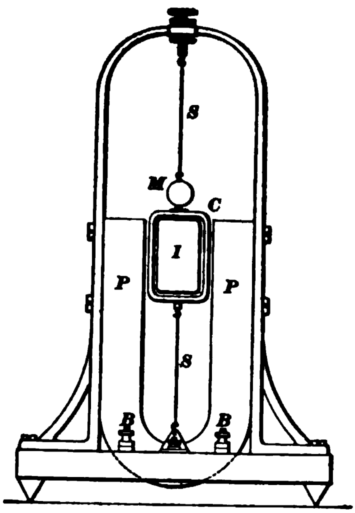

FIG. 11

The principle of the D'Arsonval galvanometer differs slightly from that of the galvanometers so far described. It consists, as shown in Fig. 11, of a large permanent horseshoe magnet *PP*, between the poles of which is suspended a coil of wire *C*. Current which is led to the coil by means of the suspension causes the coil to rotate about its axis, the tendency of the coil being to place itself at right angles to the lines of force. This tendency is opposed by the suspension, which may be a spring or fine wire. A pointer may be attached to the coil to indicate its deflection, though usually a mirror *M* is used, from which a reflected beam of light forms the pointer, as in any reflecting galvanometer. In many forms of this instrument a soft-iron core *I* supported between the poles of the magnet from the rear (a space, in which the coil swings, being left between the core and the magnet) serves to increase the strength of the field in which the coil moves.

By suitably shaping the poles of the magnet, the intensity of the magnetic field in various parts may be so varied that the movement of the beam of light will be directly proportional to the current in the coil. Fig. 11 represents an early form of the D'Arsonval galvanometer. Connection from the binding posts *B*, *B* to the coil *C* is made through fine platinum, or phosphor-bronze, wires *S*, *S*.

The damping of the moving coil, which is often very desirable, may be affected by winding the coil on a bobbin of thin copper or other non-magnetic metal. The movement of this bobbin with the coil through the field generates eddy currents in the bobbin, which produce the required damping effect.

One of the chief advantages of this instrument is the fact that external fields, such as the earth's magnetism, have little effect on it, so that it requires no controlling magnet or correction for the earth's field, and may be used near dynamos and large masses of iron without being affected. They are not as sensitive as Thomson galvanometers; hence, they are not as suitable in a laboratory as the latter for the mere detection of very minute currents. Many of the commercial forms of portable instruments are built on the principle of the D'Arsonval galvanometer.

22. One form of D'Arsonval galvanometer quite extensively used is shown in Fig. 12, in which *P* are the permanent magnets by which the field of force in which the coil is suspended is maintained. The needle and suspension are placed within the tube *T*, which is shown in the left-hand portion of the figure. This tube may be removed from the frame of the instrument when it is desired to change coils. In some instruments the tube may also be completely removed from the inner rib sustaining the coil system, thus making the latter readily accessible when it is necessary to make repairs of the working parts within. The coil system, as the coil *N* and its supporting parts are termed, is shown in detail at the right of the figure. *R* is a rib supporting

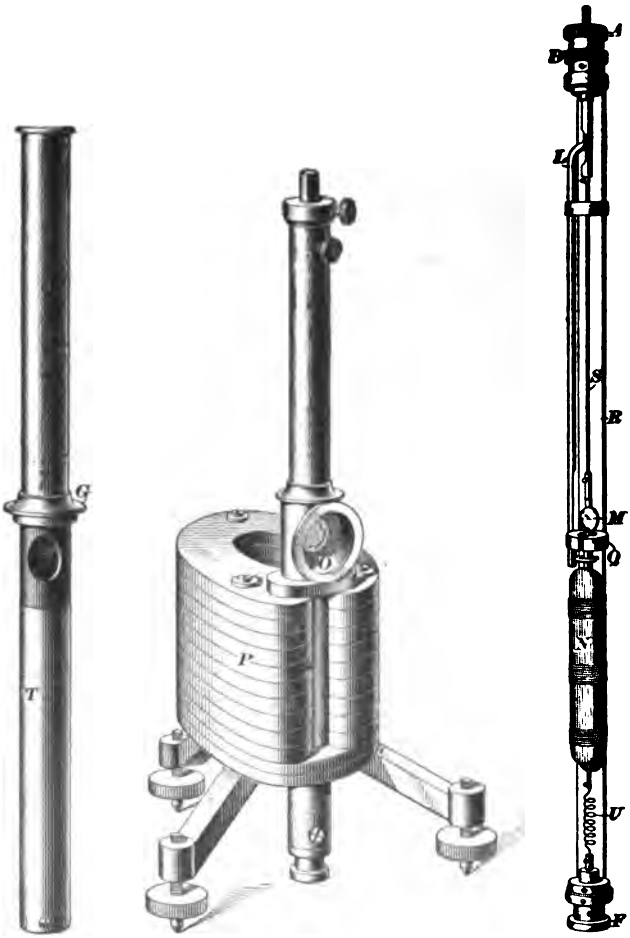

FIG. 12

at the top and the bottom the torsion heads *B*, *F*. Secured above the rectangular coil *N*, which consists of many turns of fine wire, is a mirror *M* that reflects a ray of light through the window *O*. The coil is suspended by a straight, elastic fiber *S* of some conducting material, such as phosphor bronze, while its lower part is connected by a coil of wire *U*, usually of the same material, with the lower torsion head *F*. Current is led through the coil *N* by means of the suspension fiber *S* and the coil *U*. The torsion of the suspending fiber tends to hold the needle in a certain normal position, which may be regulated by turning the torsion heads *B*, *F*, usually by turning *B* alone. When it is desired to move the instrument, the thumbscrew *A* at the top of the system may be tightened, thus drawing up the rod *L* and causing the fork carried by its lower end to engage a disk *Q*, which raises the needle just enough to remove its weight from the suspending fiber *S*.

Owing to the shape of the pole pieces, the deflections are practically proportional to the deflecting currents. Interchangeable coils and suspending systems are made so that the instrument may be used either as a dead beat or ballistic galvanometer; *dead beat* means that the coil, when deflected, does not swing back and forth before it comes to rest like the needle of an undamped galvanometer will do, but it immediately swings to and remains at its proper point of deflection. This is accomplished by making the weight and lateral dimensions of the coil as small as possible in order to give it the smallest possible moment of inertia; furthermore, it may be damped by winding the coil on a non-magnetic metal bobbin or by fastening a mica vane moving in a confined air chamber that is no larger than absolutely necessary to allow it to rotate without touching the walls of the confining chamber. Or, both means may be used to increase the damping effect.

ROWLAND D'ARSONVAL GALVANOMETER

23. The **Rowland** form of the D'Arsonval galvanometer, shown in Figs. 13 and 14, is used not only in laboratories and testing rooms but also with portable testing sets. It may be screwed against a wall and the telescope and scale attachment thrown up and locked out of the way when not in use. Or, the galvanometer backboard may be placed in a metal tripod so that the whole instrument may be set up and used on a table. Some instruments are constructed so that the backboard may also serve as a table tripod. The upper suspension is a straight phosphor-bronze strip, 6 inches long and rolled from a wire $1\frac{1}{2}$ mils in diameter. The lower conductor is a helical spring made from a similar strip of the same length. The suspension system has all the necessary adjustments so that the coil may be suspended free from twist in the suspension conductors and raised or lowered.

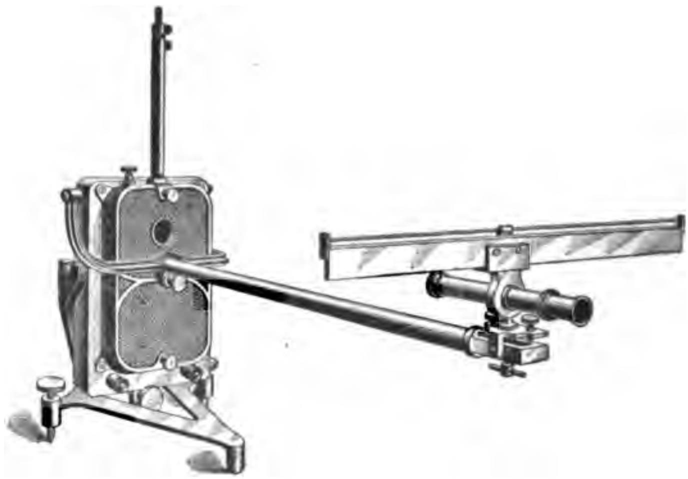

FIG. 13

In one type of this instrument a light aluminum vane may be readily attached to the coil system, thus rapidly converting an undamped coil into a damped coil that is very dead beat and more suitable for most measurements. The vane *d* is slid in a groove behind the mirror *c*, and swings within an almost air-tight box. Fixed to the mirror support is a light cross-wire *e* which, when the vane is removed, prevents the spinning around of the suspension system by an excessive current. By means of the milled screw-head *f* in Fig. 14, to which is attached the stiff wire *i*, the coil may be lifted off the suspending conductor and firmly clamped. The mirror *c* is tilted, as shown enlarged at (*b*), to prevent the reflection from the mirror and that from the surface of the glass window being thrown in the same direction, thus interfering with each other. Those who have used galvanometers giving trouble from this cause will appreciate this simple but very desirable improvement. Whether fastened to a wall or placed on a table, the instrument may be leveled by screws provided for that purpose. The shape of the pole pieces is such as to give deflections almost perfectly proportional to the current through the coil. The working parts of the instrument are protected by a front cover that may be quickly and readily removed. The telescope and scale have all the adjustments necessary. The scale, which is ½ meter long, divided into millimeters, is set exactly ½ meter from the mirror.

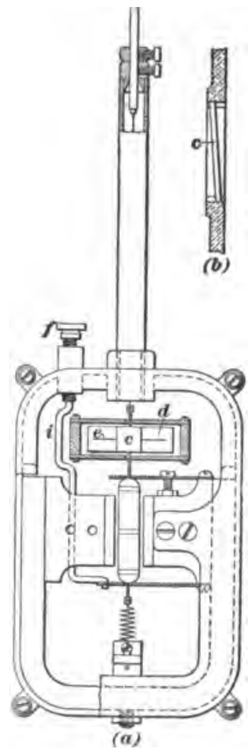

FIG. 14

BALLISTIC GALVANOMETER

24. A **ballistic galvanometer** is one used to measure instantaneous currents; that is, currents that last but a very brief interval of time. For this purpose, it is essential that the movable system shall be damped as little as possible and its weight need not be made small; in fact, it is sometimes weighted. Some form of reflecting galvanometer is generally used for this purpose. If a momentary current passes through the coils of a ballistic galvanometer, and especially one having a heavy suspended system, the impulse given to the suspended system does not cause it to move appreciably until after the current has ceased, owing to the inertia of the heavy moving parts, which results in a slow, undamped swing of the system after the impulse has ceased. The maximum angle of swing must be read by watching the spot of light, reflected from the mirror attached to the suspended system, move across a suitably divided scale, and noting the point at which the spot of light ceases to move and begins to swing back. The quantity of electricity (the number of coulombs) that passes through the coils of the instrument is proportional to the sine of one-half this angle of deflection of the needle.

$$Q = K' \sin \frac{m^\circ}{2} \qquad (4)$$

The deflection being usually small, the quantity of electricity may be regarded as directly proportional to the angle of deflection. As the use of the mirror and ray of light instead of a pointer merely doubles the angle of deflection, then $\sin \frac{m^\circ}{2}$ is approximately equal to $\frac{d}{4s}$, s being the distance between the scale and the mirror, measured, of course, in the same units as d. With the galvanometer at a fixed distance from the scale, $4s$ remains constant; hence, it will introduce no serious error to consider the quantity of electricity proportional to the swing of the spot of light across the scale, and the last formula may be modified to read

$$Q = Kd \qquad (5)$$

where d is deflection in scale divisions. It will be seen that K in formula 5 is equal to $\frac{K'}{4s}$, in which K' is the constant in formula 4.

For accurate results, a ballistic galvanometer should have as little damping as possible. It will depend on the method used and the accuracy desired as to whether it is necessary to correct for damping.*

*The throw of a ballistic galvanometer may be corrected for damping in a simple manner, provided the damping is small and the throw small. If the throw is so small that the angle may be taken for the sine of the angle, then $d = d_1 + \frac{1}{4}(d_1 - d_2)$, in which d_1 is the first throw and d_2 the following one in the same direction, that is, on the same side of the scale, and d is the throw corrected for the damping. The theory and complete formulas for damping are rather complicated and are seldom if ever used in practical work; hence, they are not given here.

The ballistic galvanometer is used to determine the magnetic qualities of iron and the electrostatic capacity of condensers, lines, and cables.

25. Fig. 15 shows one form of ballistic galvanometer in which C, C_1 are two coils, either of which may be swung back, as shown, to examine or remove the magnetic system. Each coil is supported from a brass strip, both of which are clamped in place by the nut N. Connections to the coils are made from the terminals P, P_1, while the coils are connected together by a flexible conductor F, which allows either coil to be swung aside without disturbing the connec-

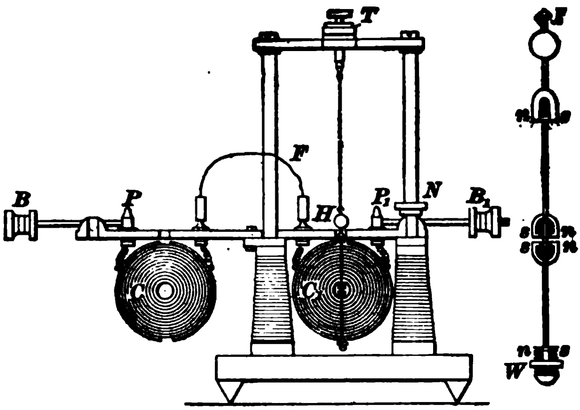

Fig. 15

tions. When in use, the instrument is surrounded by a case (not shown) through insulating bushings, or small air spaces, in which the binding posts B, B_1 project.

The magnetic system, an enlarged section of which is shown at the right, is suspended by a fine quartz fiber from the torsion head T, the magnets and mirror being hooked on to the lower end of the suspension by the hook I.

The magnets are thimble-shaped and are sometimes filled with lead to give extra weight. The system is rendered astatic by the arrangement of polarities as shown, the upper and lower magnets being the stronger, and therefore directing the system. An external controlling magnet may be used with this system, but it is not very often necessary. The sensibility of the system is varied by screwing the small soft-iron ring *W* up or down on the lower magnet. If the ring is screwed up, it short-circuits some of the lines from that magnet, thus weakening its effect on the system.

26. For many purposes a properly constructed D'Arsonval galvanometer may be used as a ballistic galvanometer. When it is desirable to make one galvanometer and one movable system answer for measuring transient as well as currents of longer duration, a D'Arsonval galvanometer is sometimes used with merely the damping vane removed when transient currents are to be measured or compared.

When a D'Arsonval galvanometer is used for ballistic tests, the movable system, being damped as little as possible, may be brought to rest at or near its zero position very quickly by closing a key that short-circuits the galvanometer coil. When the coil is thus short-circuited through a low external resistance, the E. M. F. generated in the coil when it swings and cuts lines of force, produces such a relatively large current that its reaction on the field very soon brings the coil to rest. Thus, considerable time is saved that would otherwise be wasted while waiting for the coil to come to rest. The two terminals of the key should be connected directly to the terminals of the galvanometer. The key may be closed as soon as the observation has been obtained. A D'Arsonval, when used as a ballistic galvanometer, should require about 15 seconds to make one complete oscillation, thus allowing the transient current that it is intended to measure time enough to have passed through the galvanometer before the system has moved much, if at all.

27. The galvanometers described comprise the principal forms of galvanometers in use. The selection of any one instrument for a test depends on its particular fitness for that work. All galvanometers, however, are merely current measurers, or, in some cases, current indicators only, and certain features of their use and certain apparatus used with them are common to all.

CALIBRATING A BALLISTIC GALVANOMETER

28. The constant K for a ballistic galvanometer may be determined by sending a known quantity of electricity Q through the galvanometer and noting the deflection d it produces. Then from the formula $Q = K\,d$, we get $K = \frac{Q}{d}$. Knowing K, any other quantity Q', that produces an observed deflection d', may be calculated, for then $Q' = \left(\frac{Q}{d}\right) d'$. Two ways of calibrating a ballistic galvanometer are given; the one to be used will depend on the measurement to be made and, sometimes, on which may happen to be the more convenient.

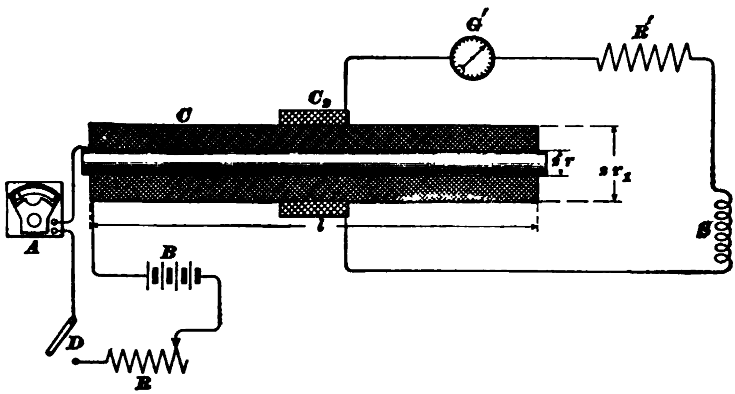

FIG. 16

29. Long-Coil Method.—The long-coil, or Thomson, method of calibrating a ballistic galvanometer, consists in connecting the galvanometer in series with a coil of a known number of turns that is wound over a long coil whose area, length, and number of turns are known, this coil being wound on a long non-metallic core. If a known current is made or broken in the latter coil, the number of lines of force cut by the first, or secondary, coil can be accurately calculated. In Fig. 16, let C represent a coil, called *primary coil*, wound on a non-magnetic core, usually a wooden rod or a glass tube, the length of the coil C being great compared to its outside diameter.

Let the length of the coil be l centimeters, the number of turns T, and its mean sectional area A square centimeters. The mean area of a coil is one-half the sum of the inside and outside areas. If the coil is cylindrical, as is usually the case, then the mean area $A = \frac{\pi}{2}(r^2 + r_1^2)$, r and r_1 being the inside and outside radii of the coil, in centimeters. Over the middle of the primary coil is wound a so-called secondary coil C_2, which is comparatively short, is usually of much finer wire and has a large number of turns T_2. The field density produced inside of the secondary when a current of I amperes flows through the primary coil is given by the formula $\mathcal{H} = \frac{1.257\,I\,T}{l}$, which has been explained in a previous Section. The sectional area of the primary is A; hence, the total number of lines of force surrounded by the secondary is $\frac{1.257\,A\,I\,T}{l}$. All these lines are cut by each turn in the secondary coil whenever the primary circuit is closed or opened; hence, the entire secondary cuts $\frac{1.257\,A\,I\,T\,T_2}{l}$ lines of force. Since the E. M. F. developed in any circuit is given by the formula $E = \frac{\Phi}{10^8\,t}$, in which t is the time, in seconds, during which there is a total change of Φ lines of force, then the E. M. F. developed is $\frac{1.257\,A\,I\,T\,T_2}{10^8\,l\,t}$. Now $Q = It$ and $I = \frac{E}{R}$, hence $Q = \frac{Et}{R}$, in which R is the total resistance of the galvanometer circuit. In the last expression substituting for E its value given above, we get for the quantity of electricity, in coulombs, that will be produced, $Q = \frac{1.257\,A\,I\,T\,T_2}{10^8\,l\,R}$. The time t has been eliminated from the expression for Q. Hence, this expression for Q is independent of the rate of change; for, assuming that the number of lines of force changes uniformly for 1 second, and that the turns of the secondary coil are such that 1 volt is generated in that coil, then, if the resistance of the entire secondary circuit is 1 ohm, 1 ampere will flow for 1 second, or as long as that E. M. F. is being generated; that is, the quantity of electricity will be 1 coulomb. If the number of lines of force be changed by the same amount, but in 2 seconds, only ½ volt will be generated in the secondary coil, and only ½ ampere will flow in the secondary circuit, but it will flow for 2 seconds, and the quantity of electricity will be the same as before. The same holds true if the rate of change in the number of lines is not uniform, which is usually the case.

If this quantity Q produces a deflection d of the galvanometer, then $Q = \frac{1.257\,A\,I\,T\,T_1}{10^8\,l\,R} = Kd$, from which $K = \frac{1.257\,A\,I\,T\,T_1}{10^8\,l\,R\,d}$. Then, if an unknown quantity Q' coulombs produces a deflection d' of the same galvanometer, we have

$$Q' = \frac{1.257\,A\,I\,T\,T_1\,d'}{10^8\,l\,R\,d} \text{ or } Q' = Kd' \qquad (6)$$

in which A is given in square centimeters and l in centimeters. If the sectional area is A square inches and the length l inches, then

$$Q = \frac{3.192\,A\,I\,T\,T_1\,d'}{10^8\,l\,R\,d} \qquad (7)$$

In many measurements R in the last two formulas may be eliminated and, therefore, need not be known, provided that it is not changed after the galvanometer is once calibrated.

30. Standard Condenser Method.—Another way to calibrate a ballistic galvanometer is to charge a standard condenser of known capacity with a known E. M. F. and then to immediately discharge the condenser through the galvanometer. It is known that the deflection of a ballistic galvanometer, provided the damping is not excessive, is proportional to the quantity of electricity flowing through the galvanometer; that is, $Q : Q' = d : d'$, Q being a quantity that produces a throw d, and Q' another quantity that produces a throw d'. Hence, $Q' = \frac{Q\,d'}{d}$. But Q may be calculated, if a condenser of known capacity C is charged with a known E. M. F. E, by the formula $Q = E\,C$. Hence, substituting this value for Q in the last expression, we get

$$Q' = \left(\frac{E\,C}{d}\right)d' \qquad (8)$$

Hence, if an unknown quantity of electricity Q' produces a throw d', the value of Q' may be calculated by the last formula, if the galvanometer is calibrated by observing the throw d produced when a condenser of known capacity C charged to a known potential E is discharged through the same galvanometer. $\frac{E\,C}{d}$ may be called the constant of the ballistic galvanometer as determined by this method.

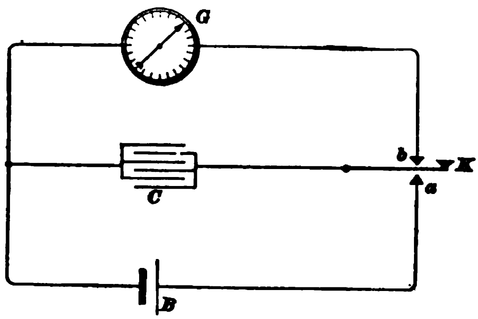

FIG. 17

The most convenient connections for the calibration of a ballistic galvanometer by this method are shown in Fig. 17. When the battery circuit is closed at the end a of the double contact key K the condenser C of known capacity is charged; when the circuit is closed at the end b by releasing the key, the condenser is discharged through G. The throw due to discharge should be observed as quickly as possible after charging the condenser, and the mean of several observations should be used for d in the last formula. E is the E. M. F. of a standard cell B.

SENSITIVENESS OF GALVANOMETERS

31. There are in use two ways of specifying the *sensitiveness*, *sensibility*, or *figure of merit* of a galvanometer, as it is variously termed. It is defined either, (*a*) as the current, in amperes, required to produce unit deflection, or (*b*) as the total resistance of the galvanometer circuit through which an E. M. F. of 1 volt will produce exactly unit deflection. When the figure of merit of reflecting galvanometers is given it is understood, unless specifically stated to the contrary, to imply that the scale was placed at the recognized standard distance of 1 meter from the galvanometer mirror and, furthermore, in the case of galvanometers with which a controlling magnet may be used, the controlling magnet is not supposed to have been used unless it is so stated and its position noted. The first definition is the one usually preferred by scientific investigators, but the latter one is most frequently used commercially.

In the case of galvanometers with the scale permanently secured to the galvanometer frame and at a fixed distance from the mirror, the figure of merit given usually applies to the permanent fixed distance, although this distance may not be even mentioned. The distance should be mentioned by all means, however, and it would be well to also state what its figure of merit would be if the scale were placed 1 meter from the mirror. This would readily enable the direct comparison of the sensitiveness of various galvanometers.

The sensibility of the form of D'Arsonval galvanometer illustrated in Fig. 12 may be as high as 1,500 megohms with a coil having a resistance of 3,500 ohms. A 20-ohm coil may have a sensibility of 80 megohms; a 200-ohm coil, a sensibility of 200 megohms; and a 1,000-ohm coil, a sensibility of 800 megohms.

The Rowland form of D'Arsonval may be made to have any sensibility up to about 500 megohms; but over 300 megohms is not recommended for most purposes, because a sensibility greater than this tends to greater accuracy than is generally required or is usually possessed by the accessory apparatus employed, while the delicacy of manipulation necessitated is correspondingly and uselessly increased.

Galvanometers are invariably designated by their resistance, because that is most convenient. However, it is well to remember that it is the number of turns and not the resistance of a galvanometer that determines its sensibility. High-resistance galvanometers are usually the more sensitive in circuits of equally high resistance, merely because, by the use of fine wire, it is possible to get more turns in the same space and consequently more ampere-turns for a given current.

GALVANOMETER SHUNTS

32. If a resistance be connected in parallel with a galvanometer, the current will divide between the two branches of the circuit, as shown in Fig. 18, and the galvanometer is said to be *shunted* by the resistance.

The drop in volts in each branch will be the same; that is, $I_s S = I_g G$, where

I_s = current in the shunt;
I_g = current in the galvanometer;
S = resistance of the shunt;
G = resistance of the galvanometer.

The total current divides through the galvanometer and shunt inversely as their resistances, that is, $I_g : I_s = S : G$, or $I_s = \frac{I_g G}{S}$. But the total current $I = I_s + I_g$, or $I_s = I - I_g$. Equating these two values of I_s and solving for I, we get $\frac{I_g G}{S} = I - I_g$; solving for I, gives

$$I = I_g\left(\frac{G + S}{S}\right) \qquad (9)$$

That is, the current in the main circuit, or the total current, is obtained by multiplying the current in the galvanometer branch by the quantity $\frac{G + S}{S}$, which is known as the *multiplying power* of the shunt. It is the amount by which any given shunt will multiply the range of a particular galvanometer. Thus, by inserting a known resistance in parallel with a galvanometer of known resistance, the total current flowing may be calculated from the current flowing in and measured by the galvanometer. A resistance arranged for such use with a galvanometer is known as a **galvanometer shunt.**

This affords a convenient means of increasing the range of a galvanometer, as by inserting the proper shunts, currents of any reasonable multiple of the normal range of the galvanometer may be measured.

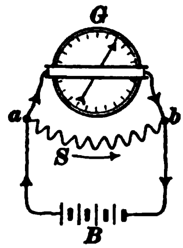

Fig. 18

33. To find the shunt resistance required to make the multiplying power of the shunt any desired amount, divide the resistance of the galvanometer by the multiplying power of the shunt desired less 1. Let the desired multiplying power of the shunt be m, then, since the multiplying power of a shunt of resistance S is $\frac{G+S}{S}$, we may write $\frac{G+S}{S} = m$, from which we obtain,

$$S = \frac{G}{m-1} \qquad (10)$$

Example.—If a galvanometer has a resistance of 500 ohms, what must be the resistance of a shunt in order to give a multiplying power of 100?

Solution.—The resistance of the shunt must be such as to satisfy the formula $S = \frac{G}{m-1}$, in which $G = 500$ ohms and $m = 100$. Substituting these values we obtain

$$S = \frac{500}{100-1} = \frac{500}{99} = 5.0505 \text{ ohms. Ans.}$$

34. Compensating Resistance.—It is evident that introducing the shunt into the circuit in parallel with the galvanometer reduces the resistance of that part of the circuit (between a and b, Fig. 18). In some delicate measurements it is desirable that this resistance be not altered, so that galvanometer shunts are sometimes mounted in connection with a second resistance, known as a **compensating resistance**, which is introduced into the circuit in series with the galvanometer and its shunt. This resistance is given such a value that its resistance, plus the combined resistance of the galvanometer and its shunt connected in parallel, is equal to the resistance of the galvanometer alone. Its value for any particular case may be calculated from the formulas for divided circuits. Such an arrangement is only used where it is necessary to maintain a very constant current in the main circuit.

EXAMPLE 1.—If a galvanometer whose resistance is 21 ohms gives a deflection of 40 with a current of 2 amperes and no shunt, what will be the resistance of the shunt that must be used to cause 16 amperes in the main circuit to give the same deflection?

SOLUTION.—The multiplying power of this shunt is evidently 8; because $\frac{16}{2} = 8$; therefore, $8 = \frac{G+S}{S} = \frac{G}{S} + 1$, then $\frac{G}{S} = 7$. But $G = 21$, hence $\frac{21}{S} = 7$, or $S = \frac{21}{7} = 3$ ohms. Ans.

EXAMPLE 2.—What must be the value of a compensating resistance if used with the galvanometer and its shunt in the above example?

SOLUTION.—Let G equal resistance of the galvanometer = 21 ohms and S equal the resistance of the shunt = 3 ohms. The joint resistance R of the galvanometer and shunt, by the formula for the joint resistance of two resistances connected in parallel, $= \frac{GS}{G+S}$. Then $R = \frac{21 \times 3}{21 + 3} = \frac{63}{24} = 2.625$ ohms. The compensating resistance introduced must be such as to make the total resistance 21 ohms, that is, the same as the resistance of the galvanometer circuit before any shunt was used; hence, the compensating resistance R_c plus the joint resistance of the galvanometer and shunt must be equal to the galvanometer resistance, or $R_c + R = G$; substituting the values gives $R_c + 2.625 = 21$, or $R_c = 21 - 2.625 = 18.375$ ohms. Ans.

EXAMPLE FOR PRACTICE

What is the resistance of a galvanometer if a shunt of 10 ohms resistance has a multiplying power of 8? Ans. 70 ohms

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35. With their galvanometers the makers are usually prepared to furnish shunts of $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ of the resistance of the instrument, which increase the range of the instrument 10, 100, or 1,000 times. Applying the formula $I = I_g\left(\frac{G+S}{S}\right)$, we obtain for the three shunts the following currents:

$$I = I_g\left(\frac{G+S}{S}\right) = I_g\left(\frac{1+\frac{1}{9}}{\frac{1}{9}}\right) = 10\ I_g$$

$$I = I_g\left(\frac{1+\frac{1}{99}}{\frac{1}{99}}\right) = 100\ I_g$$

$$I = I_g\left(\frac{1+\frac{1}{999}}{\frac{1}{999}}\right) = 1{,}000\ I_g$$

In the foregoing cases the multiplying powers of the shunts are obviously 10, 100, and 1,000, respectively.

There are two kinds of galvanometer shunts in use; the *ordinary shunt*, and the *Ayrton*, or *universal*, *shunt.*

36. **The ordinary galvanometer shunt,** which has been extensively used, is arranged as shown in Fig. 19, so that any one of three resistances *a*, *b*, or *c* may be connected across the galvanometer terminals *D* and *F*, thereby reducing the sensitiveness of the galvanometer to $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ of what it would be with no shunt. When a plug is inserted at *4* the galvanometer is short-circuited; but when one plug is inserted at *1*, *2*, or *3*, the galvanometer is shunted by the resistance *a*, *b*, or *c*, respectively. The blocks *A*, *B*, and *C* are marked either with the figures $\frac{1}{999}$, $\frac{1}{99}$, and $\frac{1}{9}$, indicating the ratio between the resistances of the coils terminating at each block and that of the galvanometer, or with $\frac{1}{1000}$, $\frac{1}{100}$, or $\frac{1}{10}$, thereby indicating the fraction of the total current that passes through the galvanometer. For instance, when

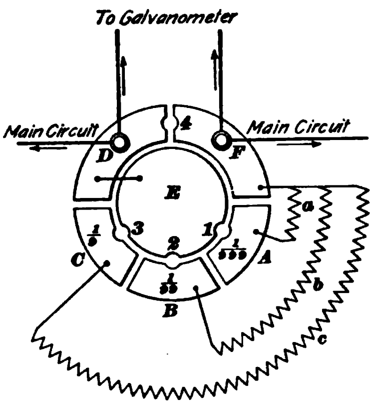

Fig. 19

a plug is inserted in the hole *2*, only $\frac{1}{100}$ of the current in the main circuit will pass through the galvanometer, and hence this particular shunt coil is said to have a multiplying power of 100, whereas its resistance is $\frac{1}{99}$ that of the galvanometer.

An ordinary shunt can be used only with the galvanometer for which it was made, and its use produces more variation in the joint resistance of the galvanometer and shunt than the Ayrton shunt. Moreover, when used with ballistic galvanometers for the measurement of transient currents, it causes an amount of damping for which no allowance can be made.

37. The **Ayrton, or universal, shunt**, which was devised by Ayrton and Mather, has met with steadily increasing favor since its first appearance about 1894. In this shunt the coils are so arranged that their relative multi-

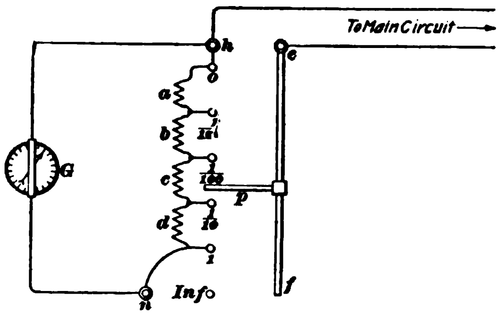

Fig. 20

plying powers, whatever may be the actual resistance of the galvanometer, are always the same. The resistance is connected, as shown in Fig. 20, directly across the galvanometer terminals, and one of the main circuit leads may be connected, through the bar ef and sliding arm p, to various points along the shunt resistance hn. If the resistance of $a + b + c + d$, that is from h to n, is 10,000 ohms, then the resistance of a is made equal to 10 ohms, $a + b = 100$, and $a + b + c = 1{,}000$ ohms.

When p is placed on contact *1*, the entire 10,000 ohms is connected across the main circuit as well as across the galvanometer terminals. This is the most sensitive arrangement when the shunt is used, and hence its multiplying power is considered to be 1. When the arm rests on the contact marked *0* the galvanometer and the resistances *a*, *b*, *c*, *d*, which are in series with it, are short-circuited; when the arm rests on the $\frac{1}{1000}$ contact, the coil *a* shunts the galvanometer and the resistances *b*, *c*, *d*, the latter still being directly in series with the galvanometer. In this position of the arm $\frac{1}{1000}$ as much current will flow through the galvanometer as will flow through it if the arm rested on the contact *1*, assuming that in each case the same total current flows in the main circuit. That is, if I_g represents the current in the galvanometer when the arm rests on *1*, then $\frac{1}{1000} \times I_g$ represents the current in the galvanometer when the arm rests on the point marked $\frac{1}{1000}$.

Similarly, $\frac{1}{100}$ and $\frac{1}{10}$ as much current will flow through the galvanometer when the arm rests on the points marked $\frac{1}{100}$ and $\frac{1}{10}$, respectively, as will flow through it if the arm rests on *1*. This means that when the arm rests on the contact marked, for instance, $\frac{1}{1000}$, the multiplying power of the shunt is 1,000 times as great as it is when the arm rests on contact *1*. If in any test, depending on two or more galvanometer readings, one reading is obtained with the arm resting on the point $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, or 1, then no subsequent reading can be used, without making some inconveniently long calculations, with the shunt circuit broken or disconnected, because the multiplying power will not then be that marked on the various contacts. When *p* is placed on the infinity contact (*Inf.*), the circuit is open; that is, an infinite resistance is connected across the main circuit.

The Ayrton shunt is preferred to the ordinary kind, because the same shunt can be used with galvanometers of various resistances, since its relative multiplying power does not depend on the resistance of the galvanometer. This is a particularly good feature in connection with D'Arsonval galvanometers, since the replacing of broken suspending wires is very apt to alter the resistance of the galvanometer, which will produce with the ordinary shunt inaccurate results by reason of the change in the ratio of the galvanometer and shunt resistances.

With this shunt the galvanometer is always shunted by the total resistance of the shunt. The initial sensitiveness of the galvanometer with a 1:1 ratio is less, therefore, than would be that of the galvanometer alone. This decrease of initial sensitiveness is, however, very small, provided the total shunt resistance is relatively large compared with that of the galvanometer. Therefore, the Ayrton shunt should have quite a high resistance compared with any galvanometer with which it is used. For this reason, combined with the fact that a

FIG. 21

resistance needlessly high is unnecessarily expensive, these shunts are made for commercial use in three sizes: One of 100,000 ohms for galvanometers having a resistance of 20,000 ohms or more; one of 10,000 ohms for galvanometers of from 2,500 to 5,000 ohms resistance; and one of about 5,000 ohms for galvanometers of 2,500 ohms or less resistance. On account of the high resistance in the shunt, slight variations in contact resistance between the arm and the studs produce no appreciable error, and hence a sliding contact, which is more convenient than plugs, can be used. The general appearance of an Ayrton shunt box is shown in Fig. 21.

One great advantage of the universal shunt is that it is accurate when used in ballistic galvanometer tests, which is not true of the other two forms. It is no more expensive than the old-style shunt and is much preferable.

REVERSING SWITCH

38. Reversing switches are used to reverse the direction in which a current, from a battery or other source of direct current, flows in some portion of a circuit. In Fig. 22 is shown a circuit containing an easily made reversing switch *D*, which consists of four holes *a*, *b*, *c*, *d* nearly full of mercury and four binding posts *e*, *f*, *g*, *h*, each connected to the mercury in the hole nearest it. If the mercury in hole *a* is connected to the mercury in hole *b* and that in *c* to that in *d* by two short pieces of bare wire, then current from *B* will flow in the circuit in the direction shown by the arrows *w*, *v*. If the connections of *D* are changed so that *a* is connected to *c* and *b* to *d*, current will flow as shown by the arrows *x*, *u*. Thus, the current flowing through the galvanometer has been reversed in direction, although the current in the battery circuit has flowed in the same direction in each case.

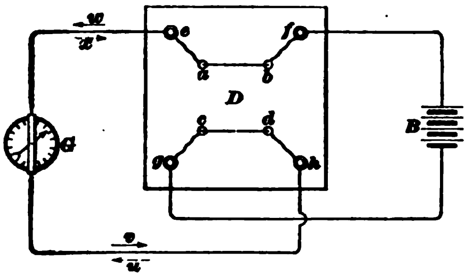

FIG. 22

39. The **Pohl commutator** is a simple switch that may be used for rapidly reversing the direction of a current in a circuit and as a convenient switch for rapidly changing connections in various tests. It consists, as shown in Fig. 23, of a block of hard, dry wood or other good insulating material, in which are the six mercury cups *a*, *b*, *c*, *d*, *e*, and *f*, each of which is connected to the adjacent binding post. When it is to be used as a reversing switch, the cups *b*, *e* are connected by means of the loose wires *m*, *n*, one of which is bent up so as not to touch the other where they cross. The wires *j*, *i* are connected together, and rigidly fastened in an insulating handle *h*, which insulates them from two similar wires *k* and *l* that are also rigidly fastened together and in the same handle. These wires then form a rocking switch, the wires *i*, *k* being somewhat longer than the pieces at right angles to them. Thus, in one position the rocking-arm connects the mercury cup *a* to *b* and *c* to *d*; in the other position,

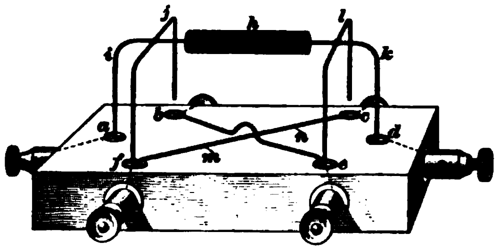

FIG. 23

as shown in the figure, *a* to *f* and *d* to *e*. The arm may be slowly or very rapidly rocked back and forth.

When used as a reversing switch one circuit terminates at the binding posts connected to *a* and *d*, and the other at binding posts connected to either *b* and *c* or *e* and *f*. When the connecting wires *m*, *n* are removed, the switch may be used to simply connect a circuit that is joined to *a* and *d* to either *b* and *c* or to *e* and *f*. This makes a simple double-throw, double-pole switch out of it.

ELECTROCHEMICAL MEASUREMENTS

40. The decomposition of liquids by the electric current affords a means of measuring the current that gives very accurate results; but it is too slow a method for most purposes. It is chiefly used for determining galvanometer and ammeter constants, as it is not well suited for the measurement of commercial currents; that is, currents used for lighting, power, etc.

Electrochemical measurements require a voltameter, which may be defined as an apparatus for determining the quantity of electricity that passes through it by measuring the amount of decomposition of a liquid that it produces. There are so-called copper, silver, and other voltameters.

COPPER VOLTAMETER METHOD

41. If a current of electricity is sent through a solution of copper sulphate (blue vitriol), the decomposition of the liquid by the current will cause a deposit of copper on the negative plate. The weight of copper deposited in a given time is proportional to the current flowing—1 ampere

TABLE II

Square Centimeters of Cathode per Ampere	12° C. or 54° F.	23° C. or 73° F.	28° C. or 82° F.
50	.0003288	.0003286	.0003286
100	.0003288	.0003283	.0003281
150	.0003287	.0003280	.0003278
200	.0003285	.0003277	.0003274
250	.0003283	.0003275	.0003268
300	.0003282	.0003272	.0003262

will deposit .0003286 gram of copper in 1 second. Moderate variations in the proportions of copper sulphate in the solution do not affect the result appreciably. The variation of the electrochemical equivalent of copper with temperature and current density is shown in Table II.

The copper electrodes should be of such size that there should be from 8 to 15 square inches (50 to 100 square centimeters) of surface to be deposited upon for each ampere of current.

When copper is deposited from copper-sulphate solution, sulphuric acid is set free, which dissolves a portion of the positive plate, forming copper sulphate, thus keeping the amount of copper sulphate in solution practically constant. The positive plate does not lose in weight exactly in proportion to the current passing, so in measurements of this description the gain in weight of the negative plate only is measured.

VOLTAMETERS

42. An excellent voltameter is shown in Fig. 24. The vessel *t* should be of glass or other insulating material, of sufficient size to allow the square part of the three plates to hang entirely below the surface of the liquid. The stiff spring clips for holding the plates are fastened to an insulating strip *m*, the two outside clips being connected to the binding post *d* and the middle clip to *a*; the two binding posts are thoroughly insulated from each other. By means of the screw *n*, all the plates can be removed together from the

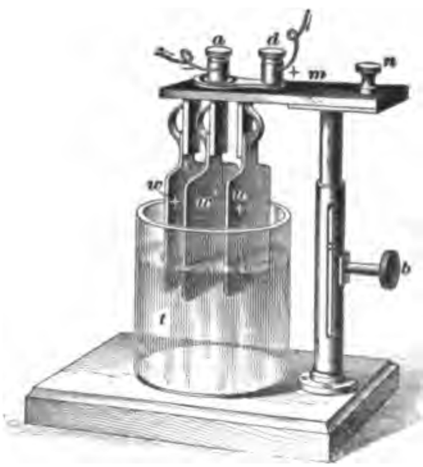

FIG. 24

solution and they may be held at any height by means of the rack and pinion *b*.

43. Plates.—Three plates should be used—one negative or *gain plate*, suspended between two positive or *loss plates*, which should be of the same shape and material as the gain plate, but somewhat smaller and thicker. The gain plate should be of very thin copper, so that its gain in weight will be enough to make considerable difference between its weights before and after the test. The plates should be cut approximately square, the corners clipped and rounded, and only a narrow neck left where they pass through the surface of the electrolyte.

For measuring small currents, when the stand shown in Fig. 24 cannot be made or obtained, the following arrangement may be resorted to. Make the narrow neck that projects from the middle of one end of the plate, long enough to bend into a hook by which the plate may be hung in the liquid. The electrodes may then be suspended from pieces of heavy copper wire, or rod, that rest on the edges of the trough a short distance from one another. The necessary connections may be made between the rods and the battery.

44. Voltameter for Large Currents.—A form suitable for large plates suspended in rectangular glass jars, is shown in Fig. 25. For measuring a large current a number of large plates may be used, each gain plate being placed between two loss plates, there being, therefore, one more loss than gain plates. The gain plates are all connected together, so as to be in parallel, and the loss plates are all connected together for the same reason.

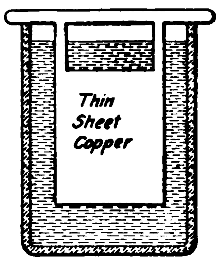

FIG. 25

45. Ryan Spiral-Coil Voltameter.—Prof. H. J. Ryan advocates the use of a voltameter in which the electrodes are made of wires of suitable diameter wound in spiral form, one spiral about twice the diameter of the other.

These are suspended, as shown in Fig. 26, in the electrolyte, one within the other, the smaller inner coil being connected to the negative side of the battery circuit and the larger outer coil to the positive side. The inner side is thus made the cathode, or gain electrode, and hence is the one to be weighed.

Professor Ryan claims the following advantages for this form: Freedom of the electrodes from angles and sharp corners, hence greater firmness of deposit; easy removal and facility of cleaning the cathode, it being only necessary to clamp one end of the spiral in a vise, pull it straight, and sandpaper down the wire; greater uniformity of deposit, the cathode being surrounded by the anode and the whole being perfectly symmetrical and concentric. The cathode must be washed the same as any gain electrode. This form is convenient, for by raising or lowering the electrodes the resistance of the voltameter may be varied and the current kept constant in spite of slight variations in the E. M. F. of the battery. However, plates are more generally used.

FIG. 26

46. Copper-Sulphate Solution.—This liquid is made by dissolving one part (by weight) of copper-sulphate crystals in five parts (by weight) of water, and adding 1 per cent. of strong sulphuric acid (1 per cent. is about three teaspoonfuls to the quart). The acid serves to dissolve such impurities as may exist in the copper sulphate. The density of the copper-sulphate solution should be from 1.15 to 1.18. It is best to make a fresh solution for each run. Commercial sulphuric acid and copper sulphate are sufficiently pure.

47. Preparation of Plates.—The gain electrodes must be prepared with great care. The plate should be rubbed smooth with emery cloth or sandpaper, washed thoroughly with water (preferably hot), to which about 5 per cent. of strong sulphuric acid has been added, then rinsed thoroughly with cool water and allowed to dry. It must not be handled with the fingers but with a piece of clean cloth or paper.

The gain plate should be weighed to the ten-thousandth part of a gram by means of an analytical or chemical balance. The plate may be weighed by some druggist, in which case it will be necessary, if his weights are given in ounces, drams, and scruples, to reduce them to grains and use formula **12.** This preparation of the gain plate should

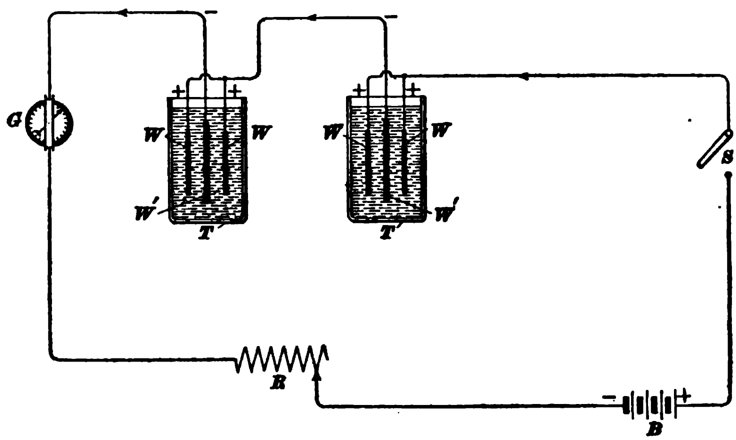

Fig. 27

not be made until all the rest of the apparatus is ready and a preliminary run has been made to adjust the apparatus and the current to the strength desired, because long exposure of the clean surface to the air will oxidize it.

When the deposition of copper is finished, the gain plate should be at once placed in cool water to each quart of which ten drops of strong sulphuric acid have been added, then washed very carefully in cool water and finally weighed again as soon as it is dry.

48. Connections may be made as shown in Fig. 27, in which R is an adjustable resistance used to assist in keeping the current constant; *G*, an ammeter or galvanometer to be calibrated; *T*, *T*, the copper voltameters; *W*, the anode or loss plates; and *W'*, the cathode, or gain plates. Two voltameters should always be used if possible, one as a check against the other. The gain plates should be weighed separately and their average used in calculating the result. The plates should be ½ to ¾ inch apart. For the battery *B* some kind of cells, such as Edison-Lalande, gravity, or storage battery, that will maintain an almost constant current, should be used.

When all is ready, the switch *S* is closed and the exact time noted. The deflection of the instrument to be calibrated should be noted from time to time, and any change in the deflection corrected by changing the resistance *R*. After sufficient time, at least 30 minutes, has elapsed, the switch is opened and the exact time again noted.

As soon as possible, the negative plates are taken out, washed and dried carefully, and accurately weighed. Then, the current that has been flowing may be calculated by the following formulas:

Let w_1 = original weight of gain plate;
w_2 = weight after the current has passed;
t = time in seconds during which the current flows;
I = strength of current in amperes.

Then, if the weights are in grams,

$$I = \frac{w_2 - w_1}{.0003286\, t} \qquad (11)$$

If the weights are in grains,

$$I = \frac{w_2 - w_1}{.005068\, t} \qquad (12)$$

As shown in Table II, the electrochemical equivalent will vary somewhat with the temperature and the current density.

After finding the current that has been passing, the constant of the instrument can be determined by the formula $K = \frac{I}{\tan m^\circ}$ for a tangent galvanometer, or $K = \frac{I}{d}$ for a reflecting galvanometer or ammeter:

EXAMPLE.—The negative plate is a sheet of copper about $2\frac{1}{2}$ inches square and about $\frac{1}{16}$ inch thick. After cleaning, it weighs 29.62 grams. The current being allowed to pass for 75 minutes, the plate weighs 31.33 grams. A tangent galvanometer in circuit gives a deflection of 42°. (*a*) How many amperes were passing, and (*b*) what was the galvanometer constant?

SOLUTION.—(*a*) In this example, $w_1 = 29.62$ grams; $w_2 = 31.33$ grams; $t = 75 \times 60 = 4{,}500$ sec. Then, by formula $I = \frac{w_2 - w_1}{.0003286\, t}$, the current

$$I = \frac{31.33 - 29.62}{.0003286 \times 4{,}500} = 1.156 \text{ amperes. Ans.}$$

(*b*) Use the formula $I = K \tan m°$; then, $\tan 42° = .9004$, and $K = \frac{I}{\tan m°} = \frac{1.156}{.9004} = 1.284$. Ans.

NOTE.—The weight of copper deposited per ampere per second may be taken in grains (troy) instead of grams, and the result worked out in the same way. 1 gram = 15.432 grains (troy).

SILVER VOLTAMETER

49. The most accurate method of measuring an electric current is by the use of a **silver voltameter.** An international convention of electrical engineers defined the international ampere as a current that, when passed through a solution of pure silver nitrate in accordance with the following specifications, will deposit .001118 gram of silver per second. For currents as large as 1 ampere, the cathode on which the silver is deposited should take the form of a platinum bowl not less than 10 centimeters in diameter and from 4 to 5 centimeters in depth. It also serves as a containing vessel for the electrolyte, which should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of silver nitrate to 85 parts by weight of water. The anode should be a plate of pure silver about 30 square centimeters in area and 2 or 3 millimeters thick. This plate is supported by platinum wires in a horizontal position near the top of the solution in the platinum bowl. To prevent detached articles from falling from the anode to the bottom of the bowl the anode should be wrapped with clean filter paper.

Before being used, the platinum bowl should be washed with nitric acid, then with distilled water, dried at about

160° C., left to cool in a dry place, and finally weighed. It should then be placed upon a clean, brightened sheet of copper that is connected through the ammeter or galvanometer to be calibrated to the negative terminal of the battery. The anode is then suspended in position and is connected through a metal-wire resistance and a simple switch to the positive terminal of the battery. The resistance of the circuit, exclusive of the silver voltameters and battery, should be at least 10 ohms in order that variations in the resistance of the cell may not produce large fluctuations in the strength of the current. The platinum bowl is nearly filled with the electrolyte, the silver plate being entirely covered with the solution just before the run is commenced and the external resistance adjusted from time to time, if necessary, to keep the current constant in strength. After the run, the duration of which must be carefully noted, the solution is removed from the bowl, the deposit thoroughly washed with distilled water, the bowl dried at about 160° C., cooled and weighed. The current, in amperes, may then be calculated from the formula

$$I = \frac{w_2 - w_1}{.001118\,t} \qquad (13)$$

in which w_1 = original weight of the platinum bowl;

w_2 = weight after the current has flowed for t seconds;

t = time, in seconds, during which the current flows;

I = average strength of the current in amperes.

50. The calibration of ammeters or other instruments is determined by means of the silver voltameter in exactly the same way as with the copper voltameter, so that no further directions regarding this seems to be necessary here. In place of the expensive platinum bowl, three silver plates and the silver solution may be substituted for the copper plates and the copper solution used in the voltameter shown in Fig. 24. The silver voltameter gives more accurate results than the copper voltameter because 1 coulomb produces a heavier deposit of silver than of copper.

Solutions of salts of other metals besides those of copper and silver may be used as the electrolyte, with corresponding metals as electrodes. For reliable results, however, silver or copper is always used.

MEASUREMENT OF POTENTIAL

51. The **international standard volt** is an E. M. F. that will cause a current of 1 international standard ampere to flow through a resistance of 1 international standard ohm. The international standard volt is represented with sufficient accuracy for all ordinary purposes by $\frac{1000}{1434}$ of the E. M. F. of the standard Clark cell at a temperature of 15° C. The E. M. F. and temperature coefficients of the various standard cells are given in *Primary Batteries*.

52. If two points between which a difference of potential exists are connected by a conductor, a current will flow from one to the other, its value depending on the resistance of the conductor and the difference of potential between the two points. If this conductor be the coil of a galvanometer, it is obvious that the divisions on the scale may be marked to read volts directly.

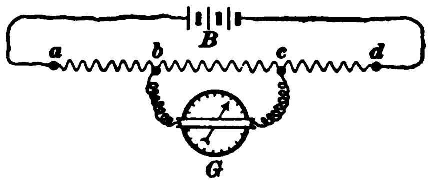

FIG. 28

In Fig. 28, a current flows from the battery *B* through the resistance *a b c d*; there will, therefore, be a certain fall of potential along *a b c d*. It may be desired to measure the difference of potential between *b* and *c*.

If a galvanometer whose resistance is approximately that of a part of the circuit *b c* is connected to the points *b* and *c*, the current flowing from *a* to *b* will divide at *b*, and a part flow through the galvanometer *G*. The whole current will again flow from *c* to *d*. If the resistance of the galvanometer is known, the current flowing through it, as measured by the deflection of the needle, is also a measure of the difference of potential between *b* and *c*, but this difference of potential is not the same as it was before the galvanometer was connected. The galvanometer being placed in parallel with a part of the circuit reduces the total resistance of the circuit, and as the distribution of resistance between *a* and *d* is changed, the distribution of the fall of potential will also be changed. In order, therefore, to measure the difference of potential between *b* and *c*, the instrument used should be so constructed that it will not measurably alter the conditions of the circuit. If the galvanometer has a very high resistance as compared with *b c*, so that the current passing through it will be a very small percentage of the total current in the circuit, the conditions will not be altered sufficiently to introduce any serious error.

53. When a difference of potential exists between two points between which no current is flowing, as a battery with no external circuit made, it is usually the case that any considerable current flowing will reduce this difference of potential, owing to the internal resistance of the battery or other generator of the E. M. F. To measure this difference of potential requires a galvanometer of such resistance that a very small current will flow through it, in order that the conditions of the circuit will not be sensibly changed; so that commercial measuring instruments that are constructed on the galvanometer principle are divided into two classes:

1. Instruments of low resistance, so arranged that a considerable current is required to give readable deflections, usually with the scales so marked that the deflection of the needle will give the proper value, in amperes, of the current passing through the instrument. These are called **amperemeters,** or more briefly **ammeters.** Ammeters constructed to measure small currents, for instance from 0 to 1.5 amperes or less, in steps, say, of .001 ampere, or 1 milliampere, are usually called **milliammeters.** The instrument here mentioned has a range from 0 to 1,500 milliamperes.

2. Instruments of high resistance, so arranged that very small currents will give readable deflections, and with the scales usually so marked that the deflections of the needle will give the proper value, in volts, of the difference in potential between the points to which the instrument is connected. Such instruments are called **voltmeters.** Voltmeters constructed to measure small differences of potential, for instance from 0 to .2 volt or less, in steps, say, of .002 volt, or 2 millivolts, are usually called **millivoltmeters.**

Difference of potential is most easily measured by voltmeters. Measurement of potential by voltmeters and in other ways will be considered more fully later.

MEASUREMENT OF RESISTANCE

OHM'S LAW METHOD

54. The resistance of a conducting body may be measured in a number of ways. One of the most common is called the **Ohm's law** or the **voltmeter-and-ammeter method.** It consists in passing a current through the unknown resistance and measuring the amperes flowing and the drop in volts through the resistance; the resistance then being calculated from Ohm's law. Fig. 29 shows the arrangement of the apparatus. *a b c d* is a resistance of which it is desired to know the resistance of the part *b c*. A current from the battery *B* flows through the ammeter *A M* and the resistance. The drop in volts from *b* to *c* is measured by the voltmeter *V M*.

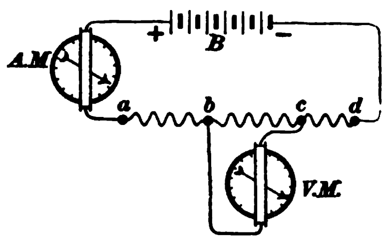

Fig. 29

EXAMPLE.—If the current flowing from *a* to *d* be 2.2 amperes, and the drop from *b* to *c* be 6.25 volts, what is the resistance of the part of the circuit *b c*?

SOLUTION.—By the formula for Ohm's law, $R = \frac{E}{I}$. $E = 6.25$ and $I = 2.2$, then $R = \frac{6.25}{2.2} = 2.841$ ohms. Ans.

EXAMPLE FOR PRACTICE

If the current be found to be 21.25 amperes, and the drop in potential 4.6 volts, what is the resistance? Ans. .2165 ohm

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POTENTIAL-DIFFERENCE METHOD

55. The voltmeter-and-ammeter method of measuring resistance is often not convenient, and is many times impossible to use. Another method, variously known as the **potential-difference, fall-of-potential,** or the **comparison-of-potential, method,** is to compare the unknown resistance with one or more known resistances. It may be done by connecting a known and the unknown resistance in series, and, on sending a current through the two, measuring the drop, in volts, across each. The resistances will be directly proportional to the fall of the potential, and the current need not be measured.

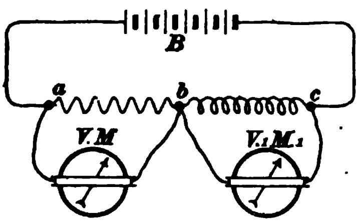

FIG. 30

In Fig. 30, current from the battery B flows through the known resistance ab and the unknown resistance bc. Voltmeters VM and V_1M_1 measure the fall of potential across each. If E is the fall of potential from a to b and E_1 the fall from b to c, then $ab : bc = E : E_1$.

The same voltmeter might readily be used for both readings, the connections being arranged as shown in Fig. 31. In this figure, U represents a double-throw switch, for which a Pohl commutator may be conveniently used. G may be a high-resistance galvanometer or a voltmeter of suitable range. By means of the double-throw switch U, the galvanometer circuit may be very quickly transferred from a–b to c–d or vice versa. In one position one conductor connects *i* with *e*, the other conductor connects *o* with *n*; in the other position of the switch *i* and *m* are connected together and *o* and *v*. *W* and *S* are reversing switches so that readings of galvanometer deflections may be taken on both sides of the scale with current flowing first in one and then in the opposite direction through *a–b–c–d*. The mean of four, eight, etc., deflections eliminates errors due to thermo-currents and to false zero readings of the galvanometer. *R* is a high resistance that may be used to prevent the flow of too large a current in the galvanometer circuit. In final measurements it may be retained or cut out as seems most suitable. A complete set of readings should be obtained as quickly as possible, so that the condition of the battery may

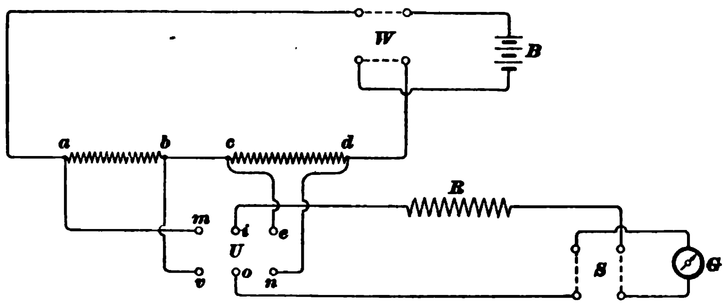

FIG. 31

not change in the meantime. If a voltmeter is used in place of *G* the resistance *R* will probably not be required. This method is used principally for measuring low resistances, that is, resistances not exceeding .1 ohm. Other methods are more suitable for measuring higher resistances and also for resistances much less than .001.

56. Precautions.—To avoid the increase of resistance that would be caused by a rise in temperature of the resistances if a current flows through them continuously, the battery reversing switch should be open except during the time actually required to take the readings. Evidently the more sensitive the galvanometer the smaller need be the current and hence the less will be the increase in resistance. An unsteady battery is another source of error. For this test a steady, uniform current must be obtained if good results are desired.

EXAMPLE.—If the resistance ab is known to be 2 ohms, and the drops as measured by $V'M$ and V'_1M_1 are 4.25 volts in ab and 6.12 volts in bc, what is the resistance of bc?

SOLUTION.—As the resistances are directly proportional to the drops of potential, $4.25 : 6.12 = 2 : x$;

or, $$x = \frac{2 \times 6.12}{4.25} = \frac{12.24}{4.25} = 2.88 \text{ ohms. Ans.}$$

EXAMPLE FOR PRACTICE

If the drop through the known resistance is 6.28 volts and through the unknown 2.25, what is the unknown resistance if the known is 3.5 ohms? Ans. 1.254 ohms

WHEATSTONE BRIDGE

57. Measurements of resistance are usually made by means of the **Wheatstone bridge**, which is very accurate for all resistances except those very large or very small, and possesses the additional desirable features of great simplicity and portability.

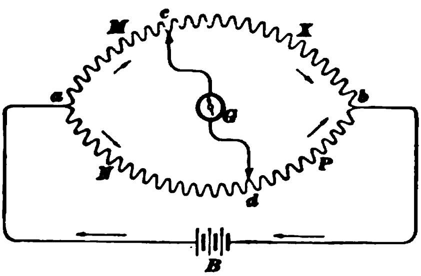

FIG. 32

In Fig. 32, acb and adb represent any two resistances joined in parallel. There is a difference of potential between the points a and b which is due to the current that flows from the battery B. The difference of potential between a and some point c in acb must be less than the difference of potential between a and b. Similarly, the difference of potential between a and some point d in adb must be less than between a and b. Therefore, if any point in acb, as c, be selected, there must be in adb

some point, as *d*, so located that the difference of potential between *a* and *d* is exactly the same as the difference of potential between *a* and *c*. If *d* is this point, the fall of potential from *a* to *c* equals the fall of potential from *a* to *d*. But *a* is common to both paths; hence, *c* and *d* must have exactly the same potential. If a galvanometer is connected from *c* to *d*, no current will flow through it because the points *c* and *d* were so selected that there is no difference of potential between them; for no current will flow between two points, even if connected together by a wire of little or no resistance, when there is no difference of potential between them. Consequently, if *c* and *d* have no difference of potential, exactly the same current must flow in *a c* as flows in *c b*; for otherwise some current (the difference between the two currents) would have to flow in *c d* and the galvanometer would show this by a deflection which would be contrary to the supposition that there is no difference of potential between *c* and *d*. Similarly, exactly the same current flows in *a d* as in *d b*. But notice that this does not imply, by any means, that the current in *a c b* is equal to that in *a d b*; in fact, it is very seldom that this is so. Furthermore, the resistance of *a c b* is not necessarily, and in fact is very seldom, equal to the resistance of *a d b*.

Let I_c be the current in *a c b*; I_d, the current in *a d b*; M, the resistance of *a c*; N, the resistance of *a d*; X, the resistance of *c b*; and P, the resistance of *d b*, when there is no difference of potential between the points *c* and *d*. Then, by Ohm's law, the fall of potential from *a* to *c* $= M \times I_c$ and the fall of potential from *a* to *d* $= N \times I_d$. But there is no difference of potential between the points *c* and *d*; hence, $M I_c = N I_d$. Similarly, the fall of potential from *c* to *b* must equal that from *d* to *b*; hence, $X I_c = P I_d$. Dividing one of these equations by the other we get $\frac{M I_c}{X I_c} = \frac{N I_d}{P I_d}$, then $\frac{M}{X} = \frac{N}{P}$ or $\frac{M}{N} = \frac{X}{P}$. That is, $M : N = X : P$. That is, the resistance of *a c* is to the resistance of *a d* as the resistance of *c b* is to the resistance of *d b*.

From this proportion, it is evident that if *a c*, *a d*, and *d b* are known, the resistance of *cb* may be readily calculated. This affords a means for measuring resistance that, as will be shown, is very flexible and is used more than any other method.

58. In Fig. 33, *M*, *N*, and *P* represent three known resistances, which may be varied by known amounts. An unknown resistance *X* is connected from *c* to *b*. A current flows from the battery *B* through the circuit. Any one, or all, of the resistances *M*, *N*, and *P* may be adjusted until the galvanometer *G* gives no deflection, thereby indicating that the points *c* and *d* are at the same potential; then, we have the proportion $M : N = X : P$.

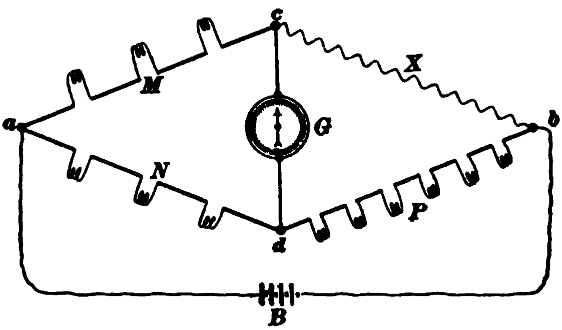

FIG. 33

It is obvious that if *M* be equal to *N*, *X* will be equal to *P*, while if *X* be a very high or very low resistance it may be measured by changing the ratio of *M* to *N*. In any case, in which no current flows in the galvanometer, we have

$$X = \frac{M}{N} \times P \qquad (14)$$

This method of measuring resistance is known as the Wheatstone bridge method, and the instrument used is called a Wheatstone bridge, or, more commonly, a bridge. As this principle is extensively used in electrical measurements it should be thoroughly understood.

59. In practice, the arms *M*, *N*, which are called the ***ratio***, ***bridge***, or ***balance arms***, and *P*, which is called the ***rheostat arm***, are made up of a number of carefully prepared resistance coils, accurately adjusted to different resistances, fixed in a box, on the top of which are arranged blocks of brass, which form the terminals of the coils. The brass blocks are usually so situated that by inserting a metallic plug between any two of them the corresponding resistance coil is cut out, or short-circuited; that is, the current passes from block to block through the plug instead of going through the coil, as this path offers practically no resistance to the current. In this way the resistance of the arms of the bridge is changed.

Fig. 34 illustrates a section of a box of coils showing the brass blocks and the method of cutting out the coils. *a*, *b*, *c*, *d*, *e*, *f*, and *g* are the brass blocks, to which are connected the coils *1*, *2*, *3*, *4*, *5*, and *6*. The brass plug *P* is made to fit tightly between the blocks.

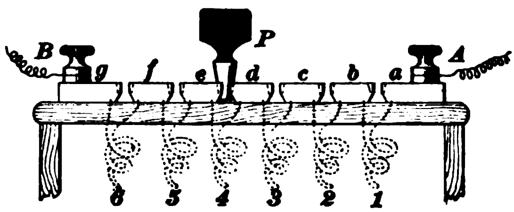

FIG. 34

Between adjacent brass blocks there are reamed slightly conical holes, or sockets, in which the conical plugs *P* should fit very exactly. If the fit is not good and the plugs not clean, the resistance at the contacts will not be negligible, especially with coils of low resistance. To properly insert a plug, press it down firmly and at the same time slightly turn it; great care must be taken not to force a plug too firmly into place, otherwise their insertion and removal wears the conical surfaces unnecessarily and is liable to loosen the brass blocks. The plugs should be kept free from dust, oxide, and grease. They may be cleaned by rubbing with a cloth moistened with a very weak solution of oxalic acid, alcohol, or benzol and dipped into prepared chalk or whiting. Alcohol will attack lacquer and hence must be kept off lacquered parts. The sockets may be cleaned by using a plug of wood. The contact resistance of a well-fitting and clean plug and socket should not average over .00005 ohm, but a slight looseness, poor fit, or a small amount of dirt between the contact surfaces will increase the resistance to .001 ohm or more, and careless treatment will make it very much greater. Thus, plug contact resistance may be a source of error in very exact work.

The current from the battery should never be allowed to flow continuously through the resistance coils, as it may introduce errors owing to the heating effect of the current; hence, the battery circuit and galvanometer circuit are each provided with a key. On pressing the battery circuit key, the current passes through the bridge, and on then pressing the galvanometer key, it is seen, from the motion or lack of motion of the needle, whether the resistance in the arms is properly adjusted. It is usual to make the arms M and N of comparatively few coils, with ratios of 10; for example, 1, 10, 100, and 1,000 ohms in each arm. By cutting out, for instance, all but the 1-ohm coil in one arm, and leaving all the coils in the other, the ratio of M to N is 1,111 to 1, or 1 to 1,111, as the case may be; so that an instrument thus arranged will measure resistances varying from 1,111 times the largest value of P to $\frac{1}{1111}$ of the smallest value of P. It is always customary, however, to use such resistance coils in M and N as to give an even multiple of 10 for the value of $\frac{M}{N}$; that is, $\frac{M}{N}$ is almost always made equal to 1, 10, 100, 1,000, $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$. This makes the arithmetical reduction a very simple matter.

For bridge measurements requiring considerable accuracy, it is best to use a sensitive reflecting galvanometer, which will detect a very slight difference in potential between c and d in Fig. 33.

60. In Fig. 35 is shown an arrangement of a bridge in which $H\ G$ corresponds to arm M in Fig. 33, $E\ F$ to arm

N, $A\ B\ C\ D$ to arm P, and x to the unknown resistance. K, K' are the keys for closing the battery and galvanometer circuits, respectively. The number opposite each coil represents the resistance of that coil. It will be seen that with the resistance coils in $A\ B\ C\ D$, this arm may be made to have any resistance, from 1 to 2,110 ohms inclusive, in steps of 1 ohm, by the cutting in or cutting out of coils by means of the plugs, as at *b*, *c*, *d*, etc.

EXAMPLE.—If in Fig. 35 with the plugs in the holes shown, the galvanometer gives no deflection on pressing the keys K, K', what is the resistance of x?

SOLUTION.—In arm M ($H\ G$) the 10-ohm coil is in circuit, the others are short-circuited by the plugs *r*, *s*, and *u*. In arm N ($E\ F$) the 1,000-ohm coil is in circuit, the rest being short-circuited by the plugs

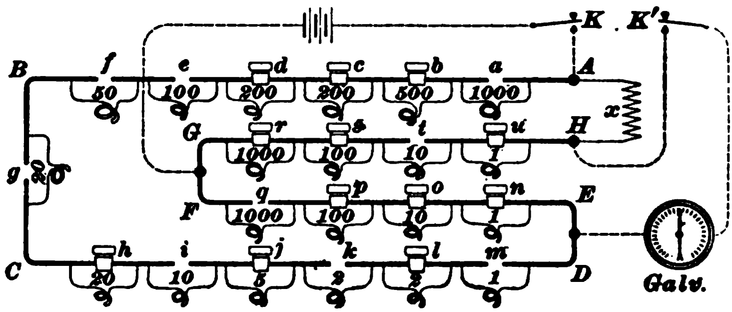

FIG. 35

n, *o*, and *p*. In arm P ($A\ B\ C\ D$) the 1,000-, 100-, 50-, one 20-, 10-, one 2-, and 1-ohm coils are in circuit, the rest being short-circuited by the plugs *b*, *c*, *d*, *h*, *j*, and *l*. The resistances are, therefore,

$$M = 10 \text{ ohms};$$
$$N = 1{,}000 \text{ ohms};$$
$$P = 1{,}000 + 100 + 50 + 20 + 10 + 2 + 1 = 1{,}183 \text{ ohms}.$$

By substituting in the formula $X = \frac{M}{N} \times P$, we get $\frac{10}{1{,}000} \times 1{,}183 = 11.83$ ohms. Ans.

EXAMPLE FOR PRACTICE

What plugs would have to be inserted in P to measure a resistance of 21.7 ohms in x, if the 1-ohm coil only be used in M and the 10-ohm coil only be used in N?

Ans. { *a*, *b*, *c*, *e*, *f*, *g*, *h*, *l*, *m*; or, *a*, *b*, *d*, *e*, *f*, *g*, *h*, *l*, *m*; or, *a*, *b*, *c*, *e*, *f*, *g*, *h*, *k*, *m*; or, *a*, *b*, *d*, *e*, *f*, *g*, *h*, *k*, *m*.

61. The resistance of the coils is usually stamped on the top of the box, that for each individual coil being marked beside the space between the brass blocks to which the coil is attached, so that, after having made the necessary adjustments, it is easy to read off the resistance in either arm of the bridge by adding the figures opposite the spaces unfilled by plugs.

The coils themselves are wound on spools of insulating material, and in reliable instruments are carefully standardized. In order that the current flowing through a resistance coil of a considerable number of turns should not create a magnetic field or produce a brief current due to its self-induction, either of which might affect the galvanometer, the coils are wound *non-inductively;* that is, for each turn around the spool in one direction the wire is wound a turn in the opposite direction, so that the magnetic effects are neutralized. The usual method of winding the spool is to measure off the length of wire required and fold it in the middle; then, starting at this fold, the two parts of the wire are wound on as one wire. A current circulating in a spool so wound will pass through one half the wire in one direction and the other half in the reverse; so the magnetic effects, as well as the self-induction, are rendered practically zero.

In making resistance measurements with a Wheatstone bridge, it is not necessary to know either the current flowing or the E. M. F. of the source of current; so almost any source of direct current of low E. M. F. is suitable for bridge work. It is customary to use two or three primary cells, except for measuring high resistances, when more cells may be necessary.

62. Modified Anthony Bridge.—A much superior but more expensive form of Wheatstone bridge, known as a **modified Anthony bridge,** is shown in Fig. 36. The coils are arranged so that more than one plug need not be used in each of the units, tens, hundreds, and thousands rows of the rheostat and one plug in each ratio arm. Thus, plug resistance is reduced to a minimum. The ratio arms are interchangeable by a simple plug device. With plugs in holes u and o we have $\frac{M}{N} = \frac{X}{P}$ and the bridge will read, with various plugs in the positions shown, 25,030 ohms. With plugs in s and v, the ratio arms will be reversed and the bridge will read 250.3 ohms.

63. Best Arrangement of Arms.—That arrangement of resistances in a Wheatstone bridge that will give the greatest sensitiveness—that is, that will correspond to a maxi-

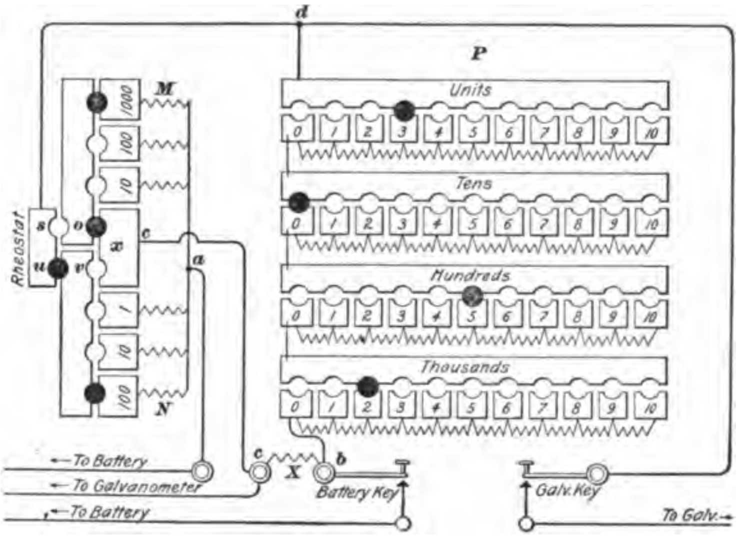

FIG. 36

mum deflection of the galvanometer for a given change in the resistance of the rheostat arm—is when $M = N = X = P = G$, where G is the galvanometer resistance. The best value for the resistance of the galvanometer is the parallel resistance of the arms of the bridge connecting the galvanometer terminals; that is, if connected as in Fig. 33, then make $G = \frac{(M+N)(X+P)}{M+N+X+P}$. These conditions, of course, can rarely be attained in practice and are to be regarded, therefore, chiefly as the ideal condition, that should be approached as closely as possible or practical. Fortunately quite sufficient sensitiveness for ordinary purposes can be had even when the ideal condition is widely departed from.

When, as is usual, the galvanometer resistance is larger than the internal resistance of the battery, the galvanometer should be connected from the junction of the two arms having the highest resistances to the junction of the remaining two arms having the lowest resistances. For instance, in Fig. 33, the galvanometer should be connected from *c* to *d* and the battery from *a* to *b*, when *M* and *X* have the greater resistances and *N* and *P* the smaller resistances. The bridge will work all right with the galvanometer connected from *a* to *b* and the battery from *c* to *d*, but it will not be as sensitive an arrangement as the other; unless the internal resistance of the battery is greater than the resistance of the galvanometer, in which case it will be the more sensitive.

Not over .2 to .3 ampere should ever be allowed to flow through any arm of an ordinary bridge, as an excessive current not only heats and, therefore, increases the resistance of the coils, but it may permanently injure them.

64. The form of bridge best adapted for general testing purposes has a rheostat capable of being adjusted to any resistance from 1 to about 11,000 ohms. The ratio arms should be capable of having values of 10, 100, and 1,000 ohms, thus being able to obtain multipliers from $\frac{1}{100}$ to 100.

Some form of D'Arsonval galvanometer is most convenient for use with ordinary Wheatstone bridges. These galvanometers have the advantage of not being affected by the proximity of other magnetic fields, and are, moreover, sufficiently sensitive for ordinary testing. Of course, for the most accurate tests, some form of reflecting Thomson galvanometer may be necessary.

In portable bridges, there is generally included in one case a compact and portable form of D'Arsonval galvanometer, two keys for opening and closing the galvanometer and battery circuits, and from 1 to 4 cells of dry battery, in addition to the resistances forming the three arms of the bridge. Such a portable set adds greatly to the ease with which rapid tests may be made, inasmuch as it is not necessary to carry extra batteries and a separate galvanometer and to connect them up every time a test is to be made.

65. Interpolation Method.—Low resistances that cannot be measured very well by the ordinary method of balancing a Wheatstone bridge may often be measured with sufficient accuracy by interpolation of deflections. Suppose that a resistance of about .01 ohm is to be measured. Make the bridge arms 1,000 and 1, respectively; unplug 10 ohms from rheostat, and suppose that the needle of the galvanometer swings to + or right side. Try 5 ohms, and suppose that it reverses, now swinging to the − or left side. Another trial demonstrates that the correct value lies between 7 and 8; that is, between .007 and .008 ohm. To determine the result more accurately, note the values of the two reverse deflections when 7 and 8 ohms, respectively, are out. In the former case suppose that the deflection is −1.4 divisions; in the latter case, +1.1 divisions. The 8 comes more nearly balancing, or in other words, the true value is more nearly 8 than 7. Now divide the larger deflection by the sum of the two deflections, and annex the quotient to the smaller value removed from the rheostat. Thus, $\frac{1.4}{1.4+1.1} = \frac{14}{25}$ and $.007 + \frac{14}{25}$ of $.001 = .00756$ ohm and $.007 + \frac{14}{25}$ of $.001 = .00756$ ohm is the resistance desired. To accurately measure low resistances in this manner requires a galvanometer combining low resistance with high sensitiveness and a cell having high E. M. F. and low internal resistance.

66. High Resistance by Wheatstone Bridge.—It is sometimes desirable to determine by means of a Wheatstone bridge a resistance that is too high to be measured by it in the ordinary direct manner. Provided there can be conveniently obtained a resistance, such as a number of incandescent lamps connected in series or a high-resistance field rheostat whose resistance can be directly measured by the Wheatstone bridge, the value of the unknown resistance can be determined in the following manner: First measure the lower resistance and let it be y ohms. Then connect this resistance y in parallel with the high unknown resistance and measure the joint resistance of the two connected in parallel and let this be z ohms. Then, if x is the unknown high resistance, we have $z = \frac{xy}{x+y}$, from which we get

$$x = \frac{yz}{y-z} \qquad (15)$$

Where x is above the top limit of the bridge y should be as high, as can be accurately measured on the bridge, or as

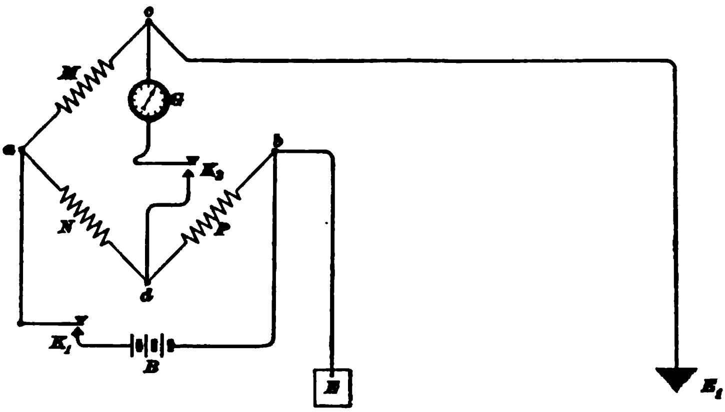

FIG. 37

high as can be obtained, say, at least several thousand ohms. When y is accurately known or measured and x is not too high, this is a very good method. This method may be used to check up resistances that have been measured separately.

An example will serve to illustrate this way of measuring a high resistance. Let the resistance y, measured by itself with the bridge in the ordinary way, be 60,201 ohms and let the result z obtained by measuring y and the unknown high resistance in parallel be 57,010 ohms; then the unknown high resistance $x = \frac{60{,}201 \times 57{,}010}{60{,}201 - 57{,}010} = 1{,}075{,}600$ ohms.

67. By grounding the distant end of a line, the resistance of the conductor that makes up the line may be measured by the Wheatstone bridge. Grounding a circuit consists in connecting it electrically with the earth, usually by means of a metal plate buried in moist earth, or to the pipes of a water or gas system. Grounding is conventionally represented as at E or E_1, in Fig. 37. The resistance of the earth varies from a fraction of an ohm to 50 or more ohms, depending on the nature of the soil and the kind and size of the ground plates. It is usual to consider the resistance of an earth circuit of at least several miles as about 10 ohms. The resistance of the earth, if the grounding is well done, may usually be neglected in measurements in which it enters, provided the resistance measured is large compared with about 10 ohms. A very common way of representing a Wheatstone bridge and its connections is shown in Fig. 37.

PRECISION IN MEASUREMENTS

68. Mathematical results can be obtained with absolute accuracy with proper attention, but any measurements that can be made are liable to error; that is, it cannot be determined that the measurement is absolutely correct. For example, an absolutely rectangular portion of the top of a table 37.5 inches long and 20 inches wide has a surface area of absolutely 750 square inches, no more and no less, but it would be impossible to lay out a surface on a table or anywhere else that would be known to have a surface area of exactly 750 square inches.

Results from a series of measurements cannot be expected to have a greater degree of accuracy than the instruments with which such measurements are made; and, conversely, it is unnecessary labor to use very accurate and sensitive instruments to obtain results that it is only necessary to know approximately.

Each of a series of measurements should be made with a degree of precision corresponding to the effect it will have on the final result. For example, if it be desired to find the cubic inches of iron in a bar about 20 feet long and about $\frac{3}{4}$ inch square by measuring its length and width and thickness, it would be absurd to carefully measure the length to eighths of an inch with a graduated scale, and then to estimate the width and thickness by using the end joint of the thumb as an inch and estimating by the eye the fraction of that distance that would equal the width or thickness of the bar.

In making delicate tests that require a high degree of accuracy, the subject should be carefully studied, and precautions taken to move as far as possible any source of error; the reading should be repeated several times, and, if possible, repeated with different methods and apparatus. Even then the best that can be said is that the results are as nearly accurate as the apparatus will allow, to the best of one's judgment.

So, in making measurements, electrical or otherwise, care should be taken to make the apparatus, methods of using it, and the necessary calculations as accurate as the required degree of precision of the final result requires. At the same time unnecessary labor in making one part of the work precise beyond a point where the unavoidable errors in another part would neutralize such precision should be avoided.

69. This leads to the consideration of how many significant figures to retain in the readings, calculations, and results to obtain results within the accuracy of the instruments used. By **significant figures** is meant the number of digits, not counting the zeros that are used merely to indicate the position of the decimal point.

For example, 20467, 28.321, 45.670, and .00010569 would each have five significant figures. If it were necessary to use but four significant figures, these values would be written

20470, 28.32, 45.67, or .0001057; that is, if the figure dropped be 5 or greater, the next figure to the left is increased 1; if less than 5, the figure to the left is unchanged. Zeros are sometimes significant figures, as in 45.670 and in the example, 25 × 4 = 100. There are three significant figures in the answer, 100, as the example has been carried out far enough to show that the value of units and tens is 0 in each case. In the number 20,470 where four significant figures are required, the last zero indicates that the actual value of the number is known to be within 5 units either way from 0, because if it had been 20,475 or greater it would have been written 20,480, and if 20,464 or less it would have been written 20,460. Whereas, if five significant figures were required, 20,470 would indicate that the last number was known to be within .5 unit either way from 0, for a reason similar to that just given.

The requirements of the calculations and results of observations in this respect are as follows: (*a*) If any one of the measurements cannot be determined within 1 per cent., four significant figures retained in any reading, calculation, or result will give an answer correct within the limits of precision of the measurements. (*b*) If any one of the measurements cannot be determined within less than .1 per cent., but can be within 1 per cent., five significant figures are required; and (*c*) if not within .01 per cent., but within .1 per cent., six significant figures are required.

The degree of precision of the various instruments used in making electrical measurements can be obtained either from careful calibration or from the maker's guarantee, and results obtained from such instruments may be calculated with the allowable degree of accuracy by observing the requirements given.

ELECTRICAL MEASUREMENTS

(PART 2)

MEASUREMENT OF RESISTANCE

LOW RESISTANCE

SLIDE-WIRE BRIDGE

1. For measuring low resistances, a modification of the Wheatstone bridge, known as the **slide-wire, or meter, bridge**, is used; a diagram of it is shown in Fig. 1. A wire $a\,b$ of uniform cross-section is stretched between the heavy copper blocks c, d; R is a known and X an unknown resistance, both of which are connected at one end to the heavy copper block e, and at the other to the blocks c, d, respectively. The galvanometer is connected between the block e and a contact piece n sliding on the wire $a\,b$. This is a form of the Wheatstone bridge where the arms M, N are replaced by $R, a\,n$, and the adjustable resistance by $n\,b$. From the consideration of the principles of the Wheatstone bridge, $R : X = a\,n : n\,b$; or, $X = R \times \frac{n\,b}{a\,n}$.

The copper blocks c, e, and d are made heavy, so that they will introduce no appreciable resistance into either arm of the bridge. As the wire $a\,b$ is of uniform cross-section, its absolute resistance need not be known; as the resistance of the two parts $a\,n$ and $n\,b$ will be directly proportional to their lengths, the formula

$$X = R \times \frac{n\,b}{a\,n} \qquad (1)$$

will hold good if *a n* and *n b* represent length instead of resistance. It is not even necessary to know the actual lengths of *a n* and *n b*; their ratio is sufficient. It is customary, however, to make the length of the wire *a b* one meter in this form of slide-wire bridge; whence the name *meter bridge*. The slider *n* is usually arranged so that one end

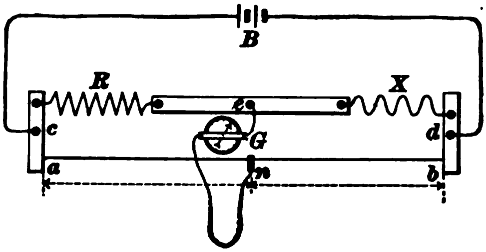

FIG. 1

slides along a scale parallel to or underneath the wire, the scale being divided into any convenient number of divisions—in the case of a meter bridge, into millimeters; so that the lengths *a n* and *n b* may be read directly from the scale. The known resistance *R* is not generally made adjustable; instead standard coils are used, the general sizes being .1, 1, and 10 ohms, the particular coil used being selected according to the resistance *X*. This makes the construction of the bridge much cheaper than the ordinary form, and as standard resistance coils of great accuracy may be purchased, the bridge may be cheaply and easily constructed.

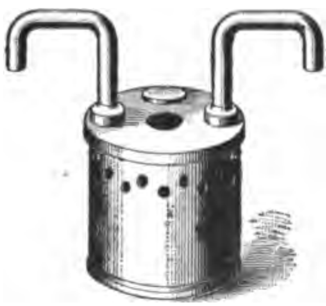

FIG. 2

2. These standard resistance coils are usually of the form shown in Fig. 2. The resistance coil itself is enclosed in a brass shell, and the whole filled with paraffin. The two projecting wires are of heavy copper, and serve as terminals. In order to insure good contact, when great accuracy of measurement is required, the terminals of the copper bars *c*, *e*, and *d* (Fig. 1), where the resistances *R* and *X* are attached, are usually made in the form of mercury cups, instead of binding posts, so that in connecting the standard resistance coil it is only necessary to hang the ends of the terminals in the mercury cups.

It is not at all necessary that the wire of the slide-wire bridge be stretched out straight, as shown in Fig. 1. This is a very convenient way to make such a bridge, but they are often built with the wire wrapped around an insulating cylinder, or stretched around the edge of a support, which may be circular or square, or of other shape, the main point being to support and insulate a long piece of bare wire so that the ratio of the distance from any point on the wire to both ends of the wire may be determined.

The slide-wire bridge is more especially suited, as stated, to the measurement of low resistances, such as determining the specific resistance of metals, electrolytes, etc.

RESISTANCE OF ELECTROLYTES

3. The resistance of a solution, or electrolyte, cannot be accurately measured by the ordinary Wheatstone bridge with a direct current on account of polarization. However, with an alternating current in place of a direct current and a telephone

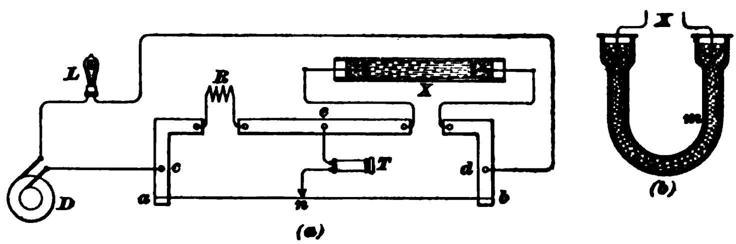

FIG. 3

receiver in place of a galvanometer, the bridge method may still be used. For most purposes, a slide-wire bridge with rather a high-resistance (small) slide wire is most suitable. The connections may be arranged as shown in Fig. 3, in which D represents an alternating-current dynamo or a device, such as an induction coil, that reverses the direct current supplied by a battery. If an alternating current generated at about 110 volts is used, a 110-volt, 16-candle-power incandescent lamp L should usually be connected in series with the source of current so that not more than $\frac{1}{2}$ ampere can be sent through the bridge. An alternating current at a pressure of 5 to 12 volts is very convenient. R is a known and X the electrolytic resistance to be measured. In this case the electrolyte is held in a glass tube between proper electrodes and corks at the ends. The solution should be free from bubbles of air or gas. The pointer is touched at different points along the bridge wire $a\,b$ until a balance point n is found that causes no sound, or at least a minimum sound, in the telephone receiver T.

The plates dipping in the electrolyte should be made of similar material and preferably of such material that the electrolyte will not act chemically on them. For this reason platinum and gold are by far the best because they may be used with practically any electrolyte; they are expensive, however, and other metals may often be used. The telephone receiver is very sensitive, in fact it is sometimes difficult or impossible to find a point where no sound is produced. This is probably due to a lack of balance in the inductance or capacity of the various arms of the bridge, and the point at which a minimum sound is produced must be accepted as the balancing point.

4. Resistivity of Electrolytes.—The resistance of an electrolytic cell depends not only on the temperature and composition of the electrolyte, but also on the size and shape of the electrodes and the vessel. Moreover, it is the resistivity or specific resistance of the electrolyte that is generally desired. When this is the case, the resistance of an electrolytic cell having fixed electrodes and filled with a solution whose specific resistance ρ (Greek letter rho) is known should be first determined. Let this resistance be R. Then refill the same cell with the given electrolyte and measure the resistance of the cell, which will be called R_1. In this case, the form of cell shown at Fig. 3 (*b*) is very suitable. The platinum electrodes *r*, *s* are slightly curved so that no bubbles can be retained under them and they are permanently suspended from hard-rubber, or fiber, covers so that when the covers are in position the two electrodes are always in the same relative position. The glass vessel *m* should always be filled to the same height.

Let the temperature of the standard electrolyte, when its resistance R is measured, be $t°$ C., the temperature of the given electrolyte when its resistance R_1 is measured be $t_1°$ C., and the specific resistance of the standard electrolyte at $t°$ C. be ρ, then the specific resistance of the given electrolyte ρ_1 at $t_1°$ C. may be obtained from the proportion $\rho_1 : R_1 = \rho : R$, or

$$\rho_1 = \frac{R_1 \rho}{R} \qquad (2)$$

When the specific resistance of a low- or high-resistance electrolyte is to be determined, the vessel should preferably be standardized by the use of a standard low- or high-resistance electrolyte, respectively. Any one of the following solutions may be used as a standard: Sulphuric-acid solution, consisting of 30.4 per cent. pure sulphuric acid has a specific gravity of 1.224; its specific resistance at the temperature $t°$ C. may be calculated by the following formula:

$$\rho = 1.36[1 - .0163\ (t - 18°)] \qquad (3)$$

Sodium-chloride (common salt) solution, consisting of 26.4 per cent. of sodium chloride, has a specific gravity of 1.201 and at the temperature $t°$ C. a specific resistance of

$$\rho = 4.66[1 - .022\ (t - 18°)] \qquad (4)$$

Magnesium-sulphate solution, consisting of 17.3 per cent. of magnesium sulphate, has a specific gravity of 1.187 and at the temperature $t°$ C. a specific resistance of

$$\rho = 20.45[1 - .026\ (t - 18°)] \qquad (5)$$

These solutions may be made of the required strength by diluting a too strong solution until a hydrometer shows that the proper specific gravity has been obtained. All specific resistances given above refer to the resistance of a centimeter cube in international ohms. By means of these formulas the value of ρ to be used in the formula $\rho_1 = \frac{R_1 \rho}{R}$, may be computed for the temperature $t°$ C. at which R was measured by the slide-wire bridge.

5. Conversion of Temperature Scales.—Temperatures may be converted from the Fahrenheit to the centigrade scale, or vice versa, by the following rules:

Rule I.—*To convert Fahrenheit to centigrade, subtract 32, multiply by 5, and divide by 9.*

For example, $50° \text{ F.} = \frac{(50 - 32)\,5}{9} = 10° \text{ C.}$

Rule II.—*To convert centigrade to Fahrenheit, multiply by 9, divide by 5, and add 32.*

For example, $100° \text{ C.} = \frac{9 \times 100}{5} + 32 = 212° \text{ F.}$

CONDUCTIVITY BRIDGE

6. To determine whether the wires used for electric light, power, telephone, or telegraph circuits come up to the conductivity specified, some convenient method is desirable. The ordinary Wheatstone or box bridge is not suitable, because reliable results cannot be obtained with it unless several ohms, at least, of the wire to be tested can be measured; as this requires such a long piece of No. 14 or larger sizes of copper wire it is decidedly an inconvenient method. Some sort of a slide-wire bridge is much better.

In Fig. 4 is shown a diagram of connections of a very convenient and good form of conductivity bridge. The resistances P, Q are exactly equal, while A is somewhat larger; their exact value must be known. The arrangement of the resistances forms a slide-wire bridge, the sample wire *a b*, a portion of which is to be measured, forming the slide wire. The coils *A*, *P* are now in parallel and their joint resistance is equal to $\frac{A\,P}{A+P}$.

Let r_1, r_2, r_3, and r_4 represent, as indicated, the resistances of the side connections, including the sheet-copper strips, joints, and ends of the sample wire under test, to the points where the scale commences and ends. The bridge is balanced by closing the battery circuit and adjusting the position of *v* until no deflection of the galvanometer is produced when its circuit is closed. Let this point be at a

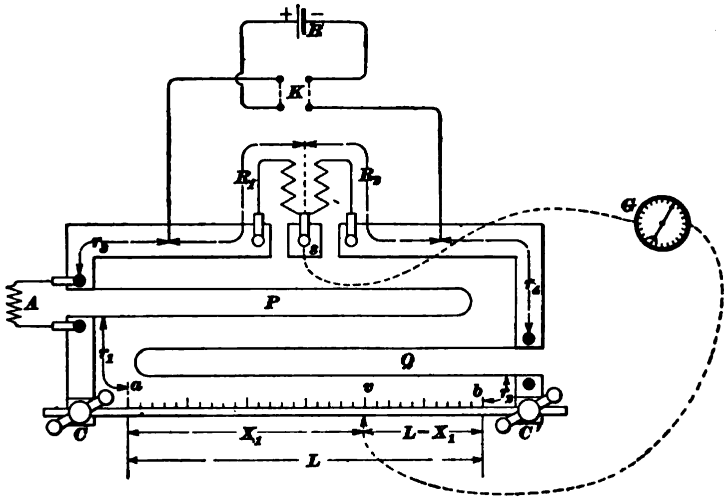

FIG. 4

distance X_1 from *a* and hence $L - X_1$ from *b*, in which *L* is the total length of the scale. Further, let *Z* be the resistance of one unit length of the sample wire *a b*. The following proportion then holds true between the four arms of the bridge:

$$\frac{R_1}{R_2} = \frac{r_3 + \frac{A\,P}{A+P} + r_1 + Z X_1}{r_4 + Q + r_2 + Z(L - X_1)}$$

The battery should be reversed by means of the switch *K* and another balance, which may or may not be exactly the same as the first one obtained. In the equation just given, X_1 should be the average value of the two balanced positions obtained with the battery direct and reversed. This procedure eliminates errors due to thermoelectric currents.

Now transpose the coil A, putting it on the right side in parallel with the coil Q, and obtain a new balance by shifting v until no deflection of the galvanometer is produced. Reverse the battery, as before in getting X_1, and obtain another balance. Suppose that the average of these two readings is X_2 inches from the left end of the scale and hence $L - X_2$ from the right end. Then we may write the proportion

$$\frac{R_1}{R_2} = \frac{r_2 + P + r_1 + Z X_2}{r_4 + \frac{A Q}{A + Q} + r_3 + Z(L - X_2)}$$

By solving these two equations, assuming that P and Q are equal, we get

$$Z = \frac{P^2}{(P + A)(X_1 - X_2)} \qquad (6)$$

Hence, if we know the resistances of P and A, and also Q, since it must be exactly equal to P, and if we have obtained, by balancing the bridge, the quantities X_1, X_2 we can evidently calculate the value of Z, which is the resistance of a unit length (generally 1 inch, 1 foot, or 1 centimeter) of the wire under test.

If the resistances (including the metal connections) are all made of copper, adjusted to be correct at some one temperature, say 70° F., then measurements made on copper samples will be entirely independent of temperature, and hence, at whatever temperature the tests are made, provided all parts of the bridge and sample are at the same temperature, the measurements are correct for 70° F.

7. Conductivity bridges, as they are called, depending on this principle and requiring samples only 30 inches in length, are now made in convenient form by several manufacturers of electrical instruments. The samples need only be stretched along the scale and held in position by clamps provided for that purpose.

The resistances P, Q are made of No. 22 B. & S. copper wire rolled into strips and adjusted to have a resistance of $\frac{1}{10}$ ohm each; the coils R_1, R_2 have a resistance of 1 ohm each. For measuring copper wires Nos. 4 and 5 B. & S., the shunt, as the coil A is called, has a resistance of about 28.5 ohms; for Nos. 6 and 7, the shunt is about 13.2 ohms; for Nos. 8, 9, and 10, the shunt is about 4.9 ohms; and for Nos. 11 and 12, the shunt is about 3.23 ohms. The shunt coils have marked on them the value of $\frac{P^2}{P+A}$ (see formula 6) and not the actual resistance of the coil A. The entire bridge is enclosed in a case to prevent changes in temperature of any part of the bridge by air-currents or radiation from any warm body. Each foot of the scale is divided into 200 parts and by means of a vernier one-fifth of a division can be read; hence, adjustments and readings can be made to $\frac{1}{1000}$ foot. A D'Arsonval galvanometer of about 4 to 20 ohms resistance and a light system, is suitable to use with this conductivity bridge.

8. Precautions.—This method assumes a uniform temperature for all the resistances, and hence great care must be taken that temperature disturbances do not occur. The sample and shunt resistances should be handled as little as possible, sudden changes in the temperature of the room should be avoided, the mercury contacts of the shunt resistances should be kept well amalgamated and scrupulously clean, and the current passed through the bridge should not exceed that furnished by one or two dry cells, nor should the battery circuit be closed any longer than is absolutely necessary to obtain a balance. Since the resistance of the bridge is very low, it is generally a good plan to insert 5 or 10 ohms directly in series with the battery to keep down the strength of the current. To determine if the various resistances are at the same temperature obtain a balance on a clean sample or standard wire, and after waiting a time obtain another balance. Any difference between the two balances indicates that some resistances are changing in temperature more than others.

The advantages of the conductivity bridge are: First, all errors due to uncertain contact resistances are eliminated; second, if all coils are made of copper the measurements on copper wires do not have to be corrected for temperature; third, measurements can be made very quickly, thus enabling a large number of samples to be tested in a day.

INTERNAL RESISTANCE OF BATTERIES

9. The **internal resistance of a battery** or cell is a very variable quantity, depending on the kind of cell, size of electrodes, distance between electrodes, condition of the solution and the electrodes, and the current flowing through the cell. It is somewhat difficult to measure and very exact results can hardly be expected. There are a number of methods for determining it, but for practical purposes the **voltmeter-and-ammeter method,** which is described in connection with measurements with these instruments, is the most satisfactory. Several methods that have proved satisfactory and do not require both a voltmeter and an ammeter will be given here.

HALF-TANGENT METHOD

10. For the **half-tangent method** of determining the internal resistance of cells an adjustable resistance box and a tangent galvanometer are generally used; however, it may be made with an ordinary Wheatstone bridge set, the bridge being used merely as an adjustable resistance. A much more convenient way is to use an ammeter or a milliammeter in place of the tangent galvanometer.

In series with the cell or battery whose internal resistance is desired, connect an adjustable known resistance and the tangent galvanometer. Use such a coil of the galvanometer or arrange the adjustable resistance so that a deflection between 60° and 80° is obtained. It is best in this case to use as little of the adjustable resistance as possible. With cells, like the gravity, having an appreciable internal resistance, no resistance external to the cell is usually necessary. If the deflection is too large it is preferable to use a galvanometer coil having fewer turns. Suppose that there is a small resistance a external to the cell; this must include the galvanometer resistance unless the latter is small enough to be neglected. Note the galvanometer deflection, in degrees, and from a table of Natural Tangents obtain the tangent corresponding to this angular deflection. Divide this tangent in half and from the same table obtain the degrees corresponding to this half tangent. Then increase the known adjustable resistance until the deflection is reduced to the degree obtained from the table. Let the known total external resistance now be c ohms. Since the tangent of the second deflection is half the tangent of the first deflection, it is evident that the current has been reduced one-half and the total resistance must, therefore, have been increased to double its first value. Hence, if b is the internal resistance of the cell or battery, then $2(b + a) = b + c$. From which we get

$$b = c - 2a \qquad (7)$$

Evidently, if no external resistance a is used when the first deflection is obtained, b will be equal to c.

11. Half-Deflection Method Using an Ammeter. An ammeter or milliammeter of suitable range can be used in place of the tangent galvanometer, and it is much more convenient. All that is necessary is to adjust the resistances, so that one reading on the scale, that is one current, is just one-half the other. The internal resistance is worked out in the same manner, using formula **7**.

EXAMPLE.—A gravity cell connected in series with a tangent galvanometer and a resistance of .3 ohm, gave a deflection of 65°. The resistance of the galvanometer and all connecting wires was .1 ohm. The tangent of 65° was found in a table of Natural Tangents to be 2.14451. The angle, having a tangent of $\frac{1}{2}$ (2.14451) = 1.07225, was found in the same table to be very nearly 47°. It was found necessary to add an extra resistance of 3.05 ohms in series with the cell and galvanometer in order to reduce the deflection from 65° to 47°. What was the internal resistance of the cell?

SOLUTION.—By formula **7**, internal resistance $b = c - 2a$, in which $c = 3.05 + .3 + .1 = 3.45$ and $a = .3 + .1 = .4$. Hence, $b = 3.45 - .8 = 2.65$ ohms. Ans.

CONDENSER METHOD

12. Connect the cell B whose internal resistance is to be measured as shown in Fig. 5. G must be a ballistic, or slow-moving, galvanometer, in order that its first swing may be proportional to the quantity of electricity passing through it. With the switch S open, depress the key K so as to charge the condenser C, release the key quickly so as to discharge the condenser through the galvanometer. Let E be the E. M. F. of the cell on open circuit (that is, with S open) and d the observed first swing of the galvanometer. With a suitable resistance at R, close S and repeat the above operation of charging and discharging C, d' being the observed first swing of the galvanometer and E' the difference of potential across the battery terminals when the battery circuit is closed through R and S. Since the first or extreme swing of the galvanometer is proportional to the quantity of electricity passing through it, and the quantity of electricity that a given condenser will receive is proportional to the E. M. F. applied to its terminals, then it follows that $d : d' = E : E'$. But the total E. M. F. $E = I(R + b)$, I being the current and b the internal resistance, whereas $E' = IR$. Hence, $d : d' = I(R + b) : IR$, or $d : d' = R + b : R$. Solving this for b, we get

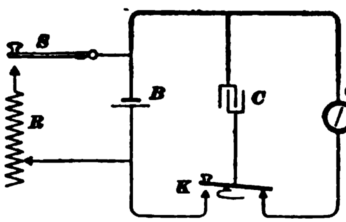

FIG. 5

$$b = R\left(\frac{d - d'}{d'}\right) \qquad (8)$$

13. E. M. F. of Cell.—These same connections may be utilized to determine the E. M. F. of a cell. Keep S open, or better, remove S and R altogether. First connect a standard cell, whose E. M. F. E is known, at B. Charge the condenser and observe the first swing d when the condenser is discharged through the galvanometer. Repeat this operation with the cell, whose E. M. F. is desired, connected at B in place of the standard cell. Let its E. M. F. be E' and the observed first swing be d', then $d : d' = E : E'$, or

$$E' = \frac{E\,d'}{d} \qquad (9)$$

MEASUREMENT OF HIGH AND INSULATION RESISTANCE

14. Insulation.—In order to transmit electricity from one point to another, that is, to make the electric current follow the conductor, it is necessary that the conductor should be separated from all points between which and the conductor there is a difference of potential by substances whose resistance is so high that that difference of potential can establish no appreciable current.

If two conductors supplying current to a lamp, for example, were laid directly on the ground, more or less of the current would flow directly from one conductor to the other through the earth, the earth being a fairly good conductor. If the wires were surrounded by glass tubes, the resistance offered to the passage of the current from wire to wire through the glass and the earth would be so great that the current would be infinitesimal, and the full strength of the current could be utilized in the lamp. Or, if the wires were suspended in perfectly dry air upon clean glass knobs attached to poles, the resistance between conductors, or from the conductors to the earth, would be comparatively enormous. The joint resistance through all such insulators that are in parallel is known as the **insulation resistance** of the circuit, and it is obvious that it should be as great as possible. The insulation resistance of a circuit may, therefore, be defined as the joint resistance of all paths, through substances that insulate the circuit more or less perfectly, to the earth and other conducting bodies.

In almost all electrical appliances, insulating materials are as necessary as conducting materials, and the measurement of the insulation resistance of such apparatus is often very important. For telegraph, telephone, and high-potential power-transmission lines, bare wires supported on glass or porcelain insulators are used. From the high specific resistance of glass it would be reasonable to suppose that this would insulate the wires very thoroughly from the earth, which would be the case were it not that the surface of the glass insulators is generally covered with a film of dust and moisture, which is of much less resistance than the glass. Glass insulators are, therefore, made so as to give a considerable length of surface between the point of attaching the wire and the point of support of the glass. Fig. 6 shows such an insulator, which is supported by a wooden pin with a thread cut on the end, which screws into the thread molded in the glass B. The wire being fastened in the groove C, any leakage of current from the wire must pass over the surface of the glass from C to the supporting pin. The length of this surface is materially increased by the groove A. This form of insulator is known as a *petticoat insulator*.

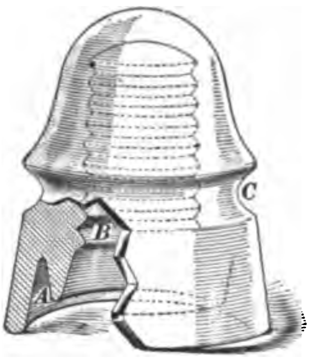

FIG. 6

The insulation resistance of one of these insulators is, of course, very high, even if considerable moisture is present, but as in a long line strung on these insulators the insulation resistances are all in multiple, the total insulation resistance of the line may be low.

TANGENT-GALVANOMETER METHOD

15. Fig. 7 shows the principle of a method of measuring the approximate insulation resistance of a line L, in which G is a galvanometer, B a battery, and R a known resistance, which should be high, say 10,000 ohms. K is a key or switch, which, when contact is made with terminal *b*, connects the resistance *R* through the galvanometer to the battery *B*; when contact is made to the terminal *a*, the battery is connected to the earth by the earth plate *E*, which may be a metal plate buried in moist earth, or the wire may be attached to a water or gas pipe, which, being buried in the earth, makes an excellent earth connection.

By connecting the battery to the resistance *R* by means of the switch *K*, the needle of the galvanometer will be deflected a certain amount, which should be noted. Then, on connecting the battery to the earth plate *E*, the circuit will be completed through the insulation resistance between the line *L* and the earth. The current flowing from the battery to the various insulators, over the insulators and

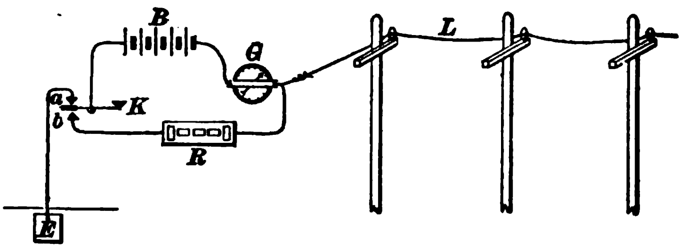

FIG. 7

poles to the ground will again produce a deflection of the galvanometer needle. The currents that flow through the known resistance *R* and the insulation resistance of the line *L* will be inversely proportional to those resistances; so, knowing the ratio of the currents and the resistance *R*, the insulation resistance of the line may be calculated.

If a tangent galvanometer be used, the resistances will be inversely proportional to the tangents of the angles of deflection; that is,

$$R : \tan d_x^\circ = X : \tan d^\circ$$

where R = known resistance;
X = insulation resistance;
d° = angle of deflection when R is in circuit;
d_x° = angle of deflection when X is in circuit.

From the foregoing proportion,

$$X = \frac{R \tan d^\circ}{\tan d_x{}^\circ} \qquad (10)$$

EXAMPLE 1.—The known resistance is 10,000 ohms; the deflection of the galvanometer when R was in circuit was 60°; the deflection of the galvanometer when the line was in circuit was 33°; what was the insulation resistance of the line in ohms?

SOLUTION.—Tan $d^\circ = 1.732$; tan $d_x{}^\circ = .649$; $R = 10{,}000$; therefore,

$$X = \frac{10{,}000 \times 1.732}{.649} = 26{,}700 \text{ ohms, nearly. Ans.}$$

As the number of paths for the current through the insulation increases with the length of the line, the insulation resistance of the line decreases as the length of the line increases; so the total insulation resistance multiplied by the length of the line gives the insulation resistance per unit of length. The usual unit of length for overhead telegraph and telephone lines is 1 mile.

EXAMPLE 2.—What is the insulation resistance per mile in the above example if the line is: (*a*) 7.5 miles long? (*b*) ½ mile long?

SOLUTION.—(*a*) 26,700 × 7.5 = 200,250 ohms, or .2 megohm, practically. (*b*) 26,700 × ½ = 13,350 ohms. Ans.

For ordinary telegraph and telephone work 200,000 megohms per mile is about the insulation resistance required.

This method of testing requires a sensitive galvanometer of fairly low resistance, and gives approximately precise results for resistances not exceeding about 30,000 ohms.

WHEATSTONE BRIDGE METHOD

16. Another method of measuring insulation resistance is to make this resistance one arm of a Wheatstone bridge, as represented in Fig. 8. By making the resistance of M great in proportion to N, resistances as high as 2,000,000 ohms may be measured with a bridge as ordinarily arranged.

The method described under High Resistances by Wheatstone Bridge may sometimes be used to measure the insulation resistance when it is too high to be measured in this ordinary way. The known, or previously measured, high resistance is connected from *c* to *b* (see Fig. 8) and one end of the line wire is joined to *c* when measuring the joint resistance in parallel of the insulation and the high resistance.

17. Elimination of Earth Currents.—Earth currents will often render measurements of resistances, where the ground is used as a part of the circuit, as in the last measurement, very unreliable. They may oppose or aid the testing current. When the earth currents are fairly steady, their effect may be eliminated by making a measurement, then reversing the battery and making another measurement. In this case the battery and galvanometer keys are closed in the order named. This is the ordinary procedure and is known as **balancing to a true, or scale, zero.** The

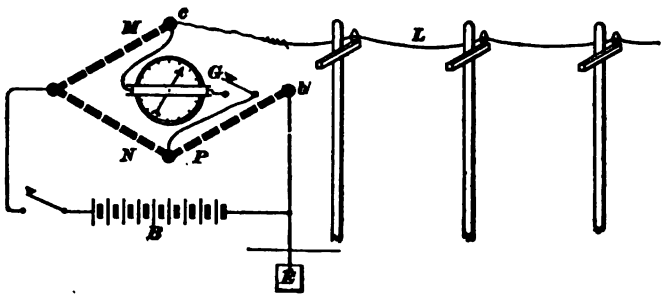

FIG. 8

average of the two measurements may be taken as the value of the resistance measured, but the geometric mean seems to give a more accurate result. That is, if R_1 is the result of one balance and R_2 the result of another for which the battery has been reversed, then the most correct value is $R = \sqrt{R_1 \times R_2}$, provided R_1 and R_2 do not differ very much. For good results, the earth current should not only be steady but it should also be small compared with the testing current.

Another way, known as **balancing to a false zero,** is to close the galvanometer key first. If earth or other currents are present, a deflection will be obtained; this deflection is known as the *false zero*. Then balance the bridge to this false zero by adjusting the rheostat arm with the galvanometer key closed, until no change is produced in the galvanometer deflection when the battery key is closed. This procedure is not very practical if there is sufficient inductance or capacity in any of the four arms of the bridge to produce a confusing momentary kick of the galvanometer when the battery key is closed. Similar kicks are sometimes due to the direct action of a magnetic field produced by a coil whose resistance is being measured because the coil has been placed too near the galvanometer.

18. The wire used for electric-light and power circuits is, except in special cases, covered with insulation instead of being bare. This insulation must not only have a high specific resistance, but it must be able to meet various other requirements. For overhead electric-light and power circuits, for example, the insulation on the wire must stand the abrasion of tree branches, etc., be reasonably fireproof and waterproof, able to withstand the action of the weather, and flexible enough to allow the wire to be reeled or strung in place without injury to the insulation.

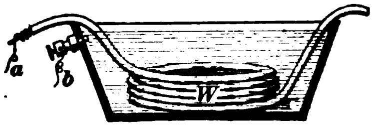

FIG. 9

It is obvious that many substances of high specific resistance, such as glass or porcelain, will not fill some of the above conditions. In fact, there is scarcely any one substance that will answer. The best grades of insulated wire are usually made with a layer of rubber, or some compound composed largely of rubber, surrounding the wire, and protected by an additional covering of braided cotton or similar material, soaked in some reasonably fireproof and weather-proof compound. Cables are now extensively used in all kinds of electrical work. They consist of one or more conductors, insulated and then enclosed in a protecting sheath of lead or by an armor of iron or steel wire or ribbon.

In order to thoroughly test the insulation resistance, contact should be made with the whole outer surface of the insulation. This is best done by immersing the wire in a metallic tank of water, slightly salted to make it conducting, as shown in Fig. 9. For testing insulated wires, a copper or copper-lined tank is preferable, but for lead-covered cables a lead-lined or iron tank is generally used. The insulation resistance is measured between the water surrounding the wire and the wire itself; that is, between *a b*. Connection with the water is made by a binding post attached to the metal tank; if the tank be glass, wood, or china, a metal plate dipping in the water is used. A long piece of wire prepared for test in this way will have a large area of insulating material between two conducting bodies; i. e., between the wire and the water. Consequently, a long cable or insulated wire possesses electrostatic capacity as well as conductor and insulation resistance. Both the resistance of the conductor and the capacity increase as the length of the cable or wire increases, but the insulation resistance decreases as the length increases.

In testing the insulation resistance of long pieces of wire in water, its capacity may interfere materially with readings, especially if the Wheatstone bridge method is used. With some other methods of testing, however, it is usually sufficient to wait, after closing the circuit until the current has become reasonably steady before taking readings. Where the surface area of the insulation is small, as when only a short length is being tested, the electrification is hardly perceptible and ordinarily will have no effect on the readings, even if a bridge be used.

DIRECT-DEFLECTION METHOD

19. In making insulation tests the methods already outlined may be followed in some cases, but the method known as the **direct-deflection method** is the one most generally used in practical work. It is suitable for measuring resistances from about 1 megohm up to about 90,000 megohms.

For tests of extreme accuracy, the Thomson galvanometer is best suited, but the use of this instrument is attended with many difficulties that render it unfit for many forms of practical work. The D'Arsonval galvanometer is sensitive enough for nearly all practical work. The Ayrton, or universal, shunt is the most desirable form of galvanometer shunt, although the ordinary form is still considerably used. The shunt, in any case, should preferably have multiplying values of 10, 100, and 1,000.

It is customary, in using the direct-deflection method of measuring insulation resistance, to first obtain the deflection through a known resistance, using a suitable known shunt around the galvanometer, with the given battery, and from it to calculate the deflection, in scale divisions, that would be produced were the entire current of the same battery to pass through the galvanometer and a resistance of 1 megohm; this latter quantity is called the *constant* of the galvanometer. After the constant is obtained, the deflection is taken with the insulation resistance of the line or cable substituted for the known resistance. In taking the galvanometer constant, it is usually necessary to use the shunt having a multiplying power of 1,000 (sometimes called the $\frac{1}{999}$ shunt), for otherwise the deflection will usually be so large as to go off the scale.

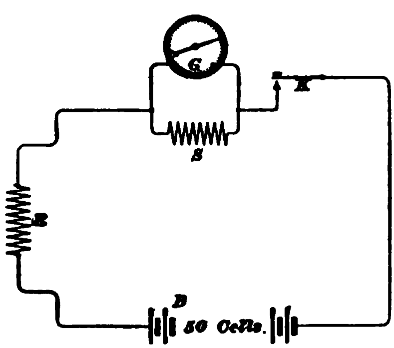

FIG. 10

20. Taking the Constant.—The circuit for determining the galvanometer constant is shown in Fig. 10, where G is the galvanometer; S, the shunt; B, a battery of 50 or 100 cells; and R, a standard resistance—for instance, 100,000 ohms; i. e., $\frac{1}{10}$ megohm. On closing the key K, a certain deflection d will be noted in the galvanometer. If the shunt used has a multiplying power of 1,000, it is evident that without the shunt the deflection would have been 1,000 times as large, could it have been measured. Further, if a resistance of 1 megohm had been used instead of $\frac{1}{10}$ megohm, the deflection would have been only one-tenth of d. Therefore, we may say that the deflection K produced by the current from the battery, passing through 1 megohm and through the galvanometer, without the shunt, would have been $K = \frac{1}{10} \times 1{,}000 \times d$.

If m represents the multiplying power of the shunt, d the deflection, and R the resistance expressed in megohms, then the constant K may be expressed by the formula

$$K = R m d \qquad (11)$$

The following general rule, therefore, may be given for calculating the constant:

Rule.—*Multiply the deflection by the multiplying power of the shunt and by the resistance in the standard resistance box, expressed in megohms or a fraction thereof.*

EXAMPLE.—In taking a constant, a $\frac{1}{10}$-megohm box was used and a deflection obtained of 247 scale divisions, the multiplying power of the shunt being 1,000. What was the constant?

SOLUTION.— $\frac{1}{10} \times 1{,}000 \times 247 = 24{,}700$. Ans.

As the standard high resistance possesses practically no electrostatic capacity or absorption, the deflection becomes steady very quickly.

21. Deflection Through Insulation.—After taking the constant, the material whose insulation resistance is to be determined, say a cable or line, is substituted for the standard resistance, the connections being then substantially those shown in Fig. 11. In the case of telephone and telegraph cables all the wires of the cable, except the one being measured, should be bunched together and connected with the sheath, the sheath itself being grounded. At the start use, as a precaution, a small shunt, the one whose multiplying

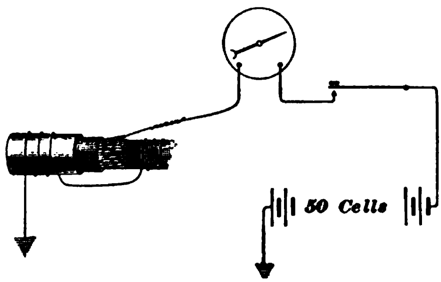

FIG. 11

power is 1,000 or 100, and increase the resistance of this shunt until a suitable deflection is obtained. If the insulation resistance is rather high, usually the highest shunt in an Ayrton shunt or no shunt (that is, a shunt of infinite resistance) in the ordinary shunt will be required. On closing the key, a certain deflection of the galvanometer will be obtained at once, but this deflection, instead of remaining constant as it did with the circuits shown in Fig. 10, will be seen to slowly diminish on account of the *absorption* or *electrification* of the cable, as it is sometimes called. This would seem to indicate that the insulation resistance of the cable was increasing. On account of the first rush of current, when the battery circuit is closed, it is necessary in insulation tests by the direct-deflection method to use a key that normally short-circuits the galvanometer, the key being only opened when the current has settled down to a steady value. (Such a key is shown properly connected at K in Fig. 13.) If the insulation resistance of a cable is good the deflection will be found to steadily decrease, tending eventually to a constant value. After about ½ minute, on a short length of ordinary telephone cable, the electrification will practically have ceased; but in determining the insulation resistance of nearly all, except submarine, cables it is customary to allow 1 minute for electrification, after which the reading is taken. This should be so stated in the results of the test, thus: "Insulation resistance per mile after 1 minute's electrification, 400 megohms."

22. Calculation of Insulation Resistance.—When making insulation tests at the factory, the whole cable, excepting the two ends, of course, is submerged in a tank of salt water. The manufacturers generally use as high as 200 volts in making this test. This requires a known resistance of 500,000 ohms (½ megohm) and a galvanometer shunt with a multiplying power of about 1,000 in order to obtain the constant K.

In taking the deflection through the insulation resistance, a certain deflection at the end of 1 minute was observed, which will be called d', the galvanometer shunt this time being one whose multiplying power will be called m'. Without this shunt it is evident that the deflection would have been m' as large, i. e., $m' \times d'$, could it have been measured directly. It is also evident that the deflections, if no shunts or the same shunts are used, will vary inversely as the resistance in circuit with the galvanometer, and, therefore, where X is the required insulation resistance, the following proportion will hold: $X : 1 = K : d' \times m'$. Solving for X, we have

$$X = \frac{K}{d'\,m'} \qquad (12)$$

or,

$$X = \frac{R\,m\,d^*}{m'\,d'} \qquad (13)$$

*This is the formula for computing the insulation resistance that is generally used, but it is not strictly accurate because the combined resistance of the galvanometer shunt and the internal resistance of the battery should be added to that of the known resistance and to that of the insulation resistance, in making the computations. However, the error introduced by neglecting this is usually so small that it is generally neglected in making ordinary insulation tests.

The exact formula for calculating the insulation resistance X is as follows:

$$X = \frac{d\left(\frac{G+S}{S}\right)\left[R + \left(\frac{G \times S}{G+S}\right) + B\right]}{d'\left(\frac{G+S'}{S'}\right)} - \left(\frac{G \times S'}{G+S'} + B\right)$$

In this formula d is the deflection obtained through the known resistance R, and S the resistance of the galvanometer shunt used in obtaining this deflection; d' is the deflection obtained through the unknown insulation resistance X, and S' the resistance of the shunt used in obtaining this deflection; G is the resistance of the galvanometer; B the internal resistance of the whole battery; $\frac{G+S}{S}$ and $\frac{G+S'}{S'}$ are the multiplying powers of the two shunts used. With the ordinary and Ayrton universal shunt boxes the multiplying powers are usually 1, 10, 100, and 1,000, and are marked on the shunt boxes. $\frac{G \times S}{G+S}$ and $\frac{G \times S'}{G+S'}$ are the joint resistances of the galvanometer and shunt in parallel when d and d', respectively, are obtained. When the Ayrton shunt box is used G includes not only the galvanometer resistance but also that portion of the shunt resistance that is directly in series with the galvanometer, and S' is the remainder of the shunt resistance, that is, the portion across the main circuit. With Ayrton shunts $\frac{G \times S}{G+S}$ has more nearly a constant value for all values of S than with the ordinary shunts.

Rule.—*The insulation resistance is equal to the constant of the galvanometer divided by the product of the multiplying power of the second shunt used and the deflection obtained through the insulation.*

When the shunt resistance has an infinite value, that is, when an ordinary shunt is entirely cut out of the circuit and no shunt is, therefore, used, the value of m' is 1. In such a case, X would be simply the constant K divided by the deflection d'. In order to determine the insulation resistance per mile, multiply the measured insulation resistance of the cable by its length expressed in miles or a fraction thereof.

EXAMPLE 1.—In taking the constant of a galvanometer for an insulation test, a deflection of 184 scale divisions was obtained with a $\frac{1}{10}$-megohm box, and a shunt whose multiplying power was 1,000. The deflections, taken through the insulation resistance of two wires, one at a time, in a cable 10,123 feet long, with a shunt whose multiplying power was 10, were as follows: 19 and 25 scale divisions, respectively. What was the insulation resistance of each of the wires?

SOLUTION.—The constant of the galvanometer is equal to $184 \times 1{,}000 \times \frac{1}{10} = 18{,}400$.

Insulation resistance, first wire, $\frac{18{,}400}{10 \times 19} = 96.84$ megohms. Ans.

Insulation resistance, second wire, $\frac{18{,}400}{10 \times 25} = 73.6$ megohms. Ans.

EXAMPLE 2.—What is the insulation resistance per mile of each of the wires in the preceding example?

SOLUTION.—Length of cable $= \frac{10{,}123}{5{,}280} = 1.917$ miles.

Insulation resistance per mile, first wire, $96.84 \times 1.917 = 185.62$ megohms. Ans.

Insulation resistance per mile, second wire, $73.6 \times 1.917 = 141.1$ megohms. Ans.

CAPACITY, ABSORPTION, AND LEAKAGE

23. When one terminal of a battery is connected to the conductor of a cable or insulated wire and the other pole to the sheath of the cable, or to the water in a tank in which the cable or insulated wire is immersed, quite a large momentary rush or charge of electricity will spread itself over the inner and outer surfaces of the insulation, due to the *electrostatic capacity* of the cable or insulated wire. (In order to avoid constant repetition, the word cable must hereafter be understood to apply also to a long piece of insulated wire or anything that possesses, like a cable, some electrostatic capacity, absorption, and leakage.) Moreover, the cable has a property called *absorption;* that is, the insulating material apparently absorbs some of the electricity. The quantity absorbed by the insulation flows with considerable strength at first, but it gradually decreases the longer the E. M. F. is applied to the cable; theoretically, it never becomes absolutely zero. The exact nature of this phenomenon is not known, but it has been held by eminent authority to be due to polarization of the insulation. The quantity that actually *leaks* through the insulation into the surrounding lead sheath or water will then form a steady current (assuming that the insulation is good and that there is no electrolysis). Hence, the current that first rushes into the cable consists of three parts—that which goes to charge the cable and very quickly becomes zero, that which is absorbed by the insulating material and diminishes gradually toward zero, and that which leaks through the insulation because it is not an absolutely perfect insulator and would continue, for good insulating material, to have the same strength for a very long time.

When the source of E. M. F. is removed and the cable conductor grounded, the current formerly used in charging the cable as a condenser and the current absorbed by the insulation now flow out in the reverse direction, but otherwise in the same manner that they flowed in; that is, the current that charged the cable as a condenser quickly rushes out and that absorbed by the insulation flows out with considerable strength at first, but gradually decreases. There now being no outside E. M. F., there will be no true leakage current. There may, however, be a current due in a long cable, to a difference of potential between two portions of the earth's surface, or in any defective cable to electrolysis.

24. Charge-and-Discharge Curve.—The process of soaking in and out proceeds at a continually decreasing rate, commencing rapidly and gradually dying away along a curve like that shown in Fig. 12. The ordinates of this curve represent the current flowing through one end of a cable at any instant during an insulation test, and the horizontal axis represents the time. The left-hand half of the diagram represents the current flowing for 5 minutes after connecting the battery to one end of a cable conductor, and the right-hand half represents the current flowing for 5 minutes after disconnecting the battery and grounding the same end of the cable. The left-hand curve indicates how the current due to absorption is superimposed on that due to leakage and how the former tends continually to diminish with time, and finally, to leave only the leakage current, the value of which

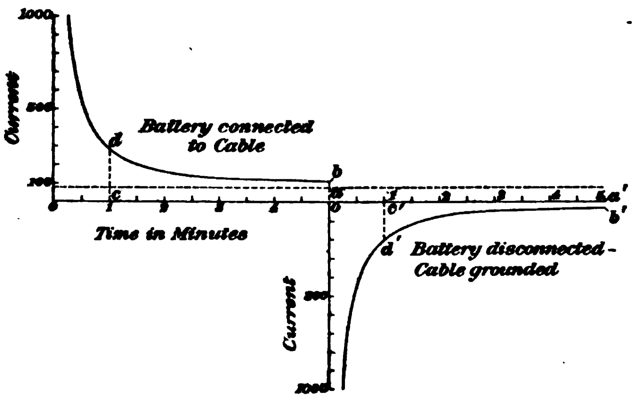

FIG. 12

is the measure of the true dielectric resistance of the cable. The right-hand half of the diagram shows that the absorbed charge soaks out in the same manner as it soaked in. The difference between any two corresponding ordinates represents the true leakage current. For instance, $ob - a'b' = oa$; hence, oa represents the true leakage current from which the insulation resistance may be computed.

In order to get, especially on a long submarine cable, an absolutely true value of the leakage current oa it would be necessary to wait an indefinitely long time before taking a reading. Evidently this is impractical, but results that are satisfactory in practice can be obtained from the difference between two readings observed at the end of exactly the same time intervals after connecting one end of the cable to the battery and again after grounding the same end. Hence, to obtain the value of the true leakage current the deflections are observed at exactly each minute, for, say, 5 minutes, after connecting the battery to one end of the cable conductor, and again at exactly each minute for 5 minutes after disconnecting the battery and grounding the same end of the cable. For instance, the difference between the deflections observed after 1 minute of charge and after 1 minute of discharge, should be very nearly equal to the deflection due to the leakage current only, and hence $c\,d - c'\,d' = o\,a$. If there is no earth current and the progress of absorption has been steady, the difference between any two corresponding readings should be approximately constant. Very smooth and symmetrical absorption is exhibited by only a practically perfect cable and such a cable would probably have a resistance too high to be measured by the direct-deflection method. The variation of the earth current would also interfere with a smooth absorption curve in submarine cables.

Though the actual insulation resistance of the cable may change from day to day, the constant relation between the charge and discharge pairs has been found to always exist if the cable is in good condition, while in imperfect cables the relation is very jerky and irregular. A simple taking of the deflections and the comparisons of their values is, hence, just as good and better in the case of cables that are tested daily, than the working out of the actual value of the insulation resistance, for a comparison of charge and discharge deflections themselves shows whether the absorption has been uniform or regular. Where there is any appreciable absorption the insulation resistance should be calculated from the last charge deflection, or from the difference between charge and discharge readings for corresponding intervals of time.

25. Preparation of the Ends of Insulated Conductors.—More leakage is apt to occur from the ends of the core of a cable by conduction over the surface of the insulating material from the sheath to the core, especially if this surface is damp and dirty, than by conduction throughout miles of the dielectric. To reduce this leakage, the dielectric should be pared down with a clean knife, should be kept dry, and should not be touched during or after trimming. In case the distant end is not easy of access and it is liable to be disturbed, a very useful plan with cables insulated with gutta percha, rubber, and compounds is to seal the end; that is, heat and draw the insulating material completely over it, adding a coating of compound. This cannot be done if paper or fiber constitutes part of the insulating material, but in that case the end may be dipped several times in melted paraffin wax or compound. Some recommend that the free ends of rubber-insulated and all submarine cables should be bared of the outer insulation down to the rubber for a space of about 2½ inches, and the rubber itself tapered with a clean, sharp knife for a length of about 1½ inches from the end. A piece of wire for connecting purposes should then be soldered or otherwise firmly fastened to one or both ends of the conductor, and the ends finally coated three or four times with clean, melted paraffin for about 3½ inches from the end. The paraffin should not be heated above the temperature of the boiling point of water, which can be done by placing the can of paraffin inside one of boiling water. For if the wax is melted over a flame it may burn and its insulation properties be partially destroyed. The coating of the ends in this manner is said to prevent end leakage and may be used in both insulation and capacity tests.

Cables containing two or more separate conductors, such as telephone and telegraph cables, should have all the conductors at the distant end spread out so that each conductor is separated and well insulated from every other conductor.

The ends of insulated wire may be prepared by carefully trimming with a sharp, clean knife, much as a pencil is sharpened, but with a much longer taper. Avoid touching it and keep it dry and clean. No coating of any kind is necessary. The two ends of the conductor should project at least 2 feet out of the water in which the coil of wire is placed for testing, and should be connected together. The water should be slightly salted to reduce its resistance and if possible its temperature should be nearly 75° F., which is the standard testing temperature.

26. Price's Guard Ring.—Errors due to surface leakage over the ends of a cable to the conductor may be eliminated almost completely when making insulation tests by the use of **Price's guard ring.** Price's method consists, as shown in Fig. 13, in wrapping a thin, copper, guard wire *V* two or three times around the long, clean tapered surface of the insulation at the ends over which the current is apt to

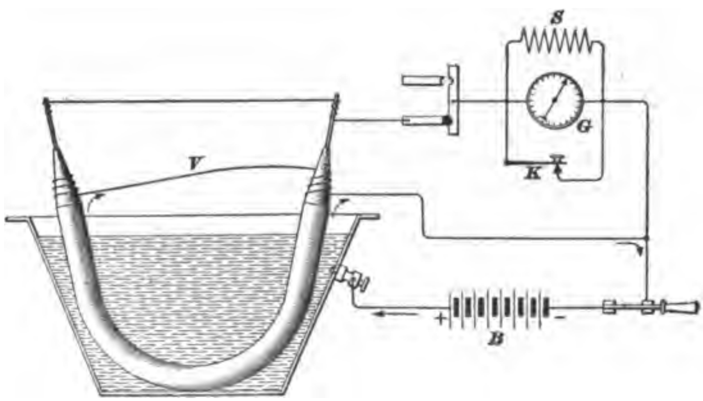

FIG. 13

leak. This guard wire is connected to some point between the galvanometer and battery. From inspection of the figure it will be seen that the current leaking over the tapered-end surface of the cable is shunted around the galvanometer; hence, the deflection of the latter is not increased by the leakage over the ends of the cable.

27. Construction of a High Resistance.—If a high resistance is not at hand, one may be readily made for temporary use by marking with a soft pencil on a strip of ground glass. Clamp, or otherwise securely fasten, pieces of tin-foil, to which are soldered connecting wires, to the ends of the glass strip so as to make contact with the pencil marks; then connect the pencil marks by means of the tin-foil and connecting wires to the proper binding posts and measure its resistance, adding or removing a small amount of graphite until about the desired value is secured. Then place another piece of glass of equal size on top of the first and firmly bind and seal the two together, in order to keep out air and moisture and to prevent any relative movement of the tin-foil terminals and glass strips.

PRACTICAL CONNECTIONS FOR DIRECT-DEFLECTION METHOD

28. A complete and practical diagram of connections to be used in the direct-deflection method of measuring insulation resistance is shown in Fig. 14. *G* represents a sensitive reflecting galvanometer, preferably a good D'Arsonval. *K*, which is a short-circuiting, or normally closed, key, is connected across the galvanometer terminals to protect the galvanometer from a sudden rush of current when charging

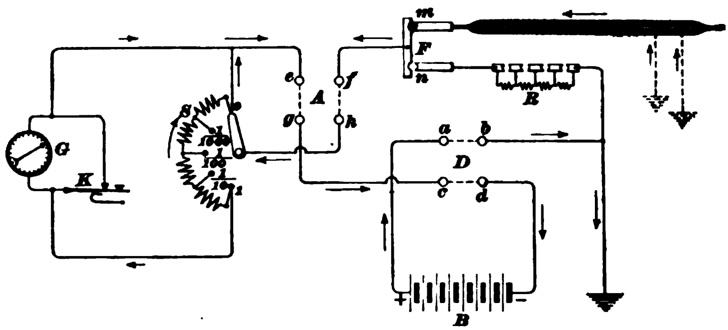

FIG. 14

or discharging or when a low resistance is unexpectedly encountered. *S* is a galvanometer shunt, preferably an Ayrton shunt, which has already been explained. One reversing switch *D* is used in the battery circuit and another *A* in the galvanometer circuit—the former for reversing the direction of the current in the whole circuit and the latter for reversing the direction of the current through the galvanometer only.

The short-circuiting key and galvanometer-reversing switch used in this test should have their various metal parts highly insulated. Sometimes a short-circuiting key, a discharge key for use in capacity tests, and a reversing switch are combined in one device, called a *combination cable-testing key*. This makes a very convenient but rather a complicated and expensive key. R is a $\frac{1}{10}$-, $\frac{1}{2}$-, or 1-megohm resistance box, preferably an adjustable 1-megohm box, and F a convenient arrangement whereby either the cable or the high resistance R may be connected to the testing circuit. If used, F must be very highly insulated and is usually made of brass pieces mounted on good hard rubber or fiber, sometimes on pillars of this material.

A preliminary test, to determine if the insulation resistance is high enough to measure by the direct-deflection method, may be made by placing the shunt switch on the $\frac{1}{1000}$ point, and a plug in hole m, closing the reversing switch D, so as to connect the zinc or negative terminal of the battery to the conductor, and also closing the switch A. The zinc, or negative, terminal of the battery is connected to the cable conductor, so that current will flow from the water to the conductor in order to secure maximum leakage. According to Prof. S. W. Holman, this maximum leakage is probably due to the deposition of "metallic copper on the conductor at the break, thus gradually making a fault bare and reducing its resistance to a minimum, after which free hydrogen may accumulate and increase the apparent resistance. When the current flows in the opposite direction, a coating of chlorides and oxides is probably formed at the fault and their comparatively high resistance tends to partly seal up the fault. Much polarization causes irregularities in the flow of current and indicates a very faulty insulation."

Having made the proper connections, cautiously depress the key K. If the deflection remains off the scale after waiting a reasonable time, which will depend on what is being tested, the insulation resistance is quite low, probably too low to test in this manner. However, by reducing the number of cells in the battery or waiting until the flow of current due to absorption is over it may be possible to keep the deflection on the scale. If there is no deflection when *K* is depressed, move the shunt switch, step by step, toward the point *1* to see if a deflection can be obtained. If there is a deflection put the shunt switch in such a position as to give a good, readable deflection. If this preliminary test is made the cable must be thoroughly discharged by opening *D* and putting plugs in both holes *m* and *n* and all the plugs in *R* for at least a little longer time than the battery was connected to the conductor.

29. To make the test it is customary to first obtain the constant of the apparatus. To do this remove the plug at *m*, allowing the plug at *n* to remain, and close *A* and *D* in such positions as to get the deflection on the same side of the zero as when the insulation to be measured is in circuit. Cautiously depress the key *K* and note the deflection. If this deflection is not somewhere near that obtained in the preliminary test, adjust, if possible, either *R* or *S*, or both, until it is. After the deflection becomes steady, write it down. Then reverse *D* and *A* so as to keep the deflection on the same side of the zero position and obtain a second deflection. The average of these two readings, both of which should be recorded, will be *d*, the galvanometer deflection obtained through the known resistance *R*. Write down also the resistance, or position, of the shunt switch or plugs and the value of the known resistance *R*. Now open *D* and put plugs in *m* and *n* in order to thoroughly discharge the cable. Then remove the plug at *n*, connect *c* to *b* (in order not to include the battery), close *A*, and cautiously open *K*; if the galvanometer deflection fails to decrease to zero within 5 or 10 minutes, except perhaps for a long submarine cable which may require more time, or if the deflection is large and irregular, the trouble is probably due to imperfections—such as minute perforations through which the water penetrates. In such a case the insulation is very poor and a further test is scarcely necessary. This preliminary discharge test should never be omitted, as the existence of a charge may introduce a serious error in later observations. It may require several hours for the deflections to become constant in the case of very long submarine cables.

To measure the leakage current, place a plug in hole *m* only, close *A*, and with the second hand of a watch exactly at the minute, close the switch *D*, so as to connect the negative terminal of the battery to the conductor of the cable and note the time. After about 10 or 15 seconds, open *K* and endeavor to get the deflection steady, adjusting the shunt if necessary to obtain a suitable deflection, and read the deflection exactly at the minute. Thus, 1 minute has elapsed, during which time the cable conductor has had time to become charged and the deflection more nearly measures the leaking current. After depressing the key *K* the deflection will diminish, the rate at which it diminishes depending on the amount and quality of the insulating material. One minute is the time generally allowed for the charge to soak into ordinary land cables, insulated wires, and overhead lines, although it is well to record readings at each minute for 5 or more minutes. If the deflection still continues to decrease the 1-minute reading is usually taken, nevertheless, and used in calculating the insulation resistance. [Discharge readings may be obtained in the following manner: Exactly on the minute open *D*, reverse *A* (so as to get deflections on the same side of the scale), and connect *c* to *b*. At the end of each minute, for 5 minutes or more, note the discharge deflections.]

The cable should be discharged by connecting it directly to the ground by inserting plugs at *m* and *n* and all plugs in *R* for several minutes. A deflection should now be obtained at the end of 1 minute (or preferably at the end of each minute for 5 or more minutes) with the opposite (positive) pole of the battery connected to the conductor. To do this and to keep the galvanometer deflection on the same side of the zero position as before, which is rather preferable, close *D* so as to connect *a* to *c* and *b* to *d* and close *A* so as to connect *e* to *f* and *g* to *h*. (Discharge readings may be obtained, if desired, as before by opening D, reversing A, and connecting c to b.)

The average insulation resistance of the cable is usually considered to be the average result obtained with both positive and negative poles of the battery connected to the conductor and is computed from the average deflection after a charge of 1 minute. Some prefer to use the mean of four deflections for both d and d' in the formula $X = \frac{R\,m\,d}{m'\,d'}$. The four deflections in each case are obtained by taking two readings, one on each side of the scale, with D in one position, which is done by merely reversing A for the second reading; and similarly two more readings with D reversed in position. Thus, one deflection is obtained on each side of the scale for each direction of the current through the insulation and also through the known resistance.

30. Trolley currents frequently cause trouble in this test. If the trolley current is fairly steady the error that would otherwise produce inaccurate results is eliminated by reversing the battery as directed. In one position of the battery reversing switch D, the trolley current augments the battery current; whereas, in the other position of D, the trolley current diminishes the battery current, and the deflections are therefore diminished in one direction as much as they are increased in the opposite direction by the trolley current. If the trolley current is so unsteady as to cause jerky or irregular deflections, then only approximately correct results can be obtained, and there is no remedy except possibly to make the test at night, when no cars are running.

31. Earth Currents.—The presence of an earth, or electrolytic, current may be detected by connecting the cable conductor directly through a galvanometer to the ground. If the cable has previously been thoroughly discharged, a small, steady deflection will indicate an earth current. In order to obtain a deflection due to the earth current that can be directly used in correcting the deflection due to the combined leakage and earth current, the same galvanometer shunt should be used in both cases, or else the earth-current deflection, if obtained without any galvanometer shunt, must be divided by the multiplying power of the shunt used in observing the combined leakage and earth-current deflection. The deflection due to the earth current should be subtracted or added to the combined leakage and earth-current deflection, according as to whether the deflection due to the earth current is in the same or reverse direction, respectively, to the deflection due to the combined leakage and earth current.

When, as sometimes occurs on long submarine cables, earth currents prevent the deflections from becoming steady and regular, it is well to take a reading 15 seconds before and after the minute as well as at the minute, the average of these three readings being considered as the minute reading. On land cables there is not apt to be an appreciable earth current unless it is from a street-railway circuit.

When a test is being made on 1,000 or 2,000 miles of submarine cable or on any long cable or insulated wire for which a complete and thorough study of the behavior of the insulation is required, deflections should be observed at the end of the second minute, and so on until the rate of absorption becomes so slow that a minute causes, practically, no further decrease in the deflection. Five minutes is a good average duration for tests of long cables and long insulated wires, but for special tests observations may be taken for 15 to 30 minutes.

32. As many insulations deteriorate after having been under water some time, tests should be made of the insulation resistance at intervals during a considerable time to observe this deterioration, if there be any. For example, readings taken after the wire had been immersed 15 minutes, 1 hour, 3 hours, 10 hours, 24 hours, would show any serious effect that wetting would have.

If a break occur in the insulation under the water, the water will come in contact with the metal wire, and the ensuing electrolysis will liberate bubbles of gas, which will alternately collect and pass off at the break. This will so vary the resistance that the current will not be steady enough to allow its value to be read. The galvanometer needle will irregularly swing back and forth, and it will be useless to attempt to measure the insulation resistance, especially as the action will indicate defective insulation.

It is not always necessary to immerse the wire in water, although this is very convenient, as the water makes contact with the entire outer surface of the insulation, as the wire does with the entire inner surface, besides testing the waterproof qualities of the insulation. For some tests the wire may be closely wrapped around a smooth, bright, metal bar—a section of shafting for example—and the resistance between this bar and the wire measured. Or, two pieces of the wire may be twisted together, and the resistance between the two wires measured.

It is often desirable to test the insulating qualities of sheets of paper, mica fiber, or similar substances. A convenient way to prepare them for such a test is to make two smooth brass plates smaller than the pieces of insulation to be tested, which should be placed between them. The insulation resistance may then be measured between the two brass plates, and from the area and length (thickness) of the piece of insulation between the plates its specific resistance may be calculated. Many other methods of preparing insulation for tests will suggest themselves as occasion requires.

33. Although a storage battery, a lighting or a power circuit may be used, and is for some reasons preferable, for insulation or capacity tests, if carefully and properly fused, nevertheless there is so much danger of injuring the testing instruments that a primary battery of 50 or 100 cells is generally employed. In a testing room where the battery may be permanently set up, Leclanché cells are about the most satisfactory. Dry cells are cheaper, however, and are often used, but their internal resistance is apt to increase considerably, especially when they are kept in a dry, heated room. Where the battery must be carried around, a case of

50 or more chloride-of-silver cells or other compact battery will have to be used. Some makers advertise a small semi-dry cell for testing purposes.

INSULATION RESISTANCE BY LEAKAGE METHOD

34. The direct-deflection method is not suitable for measuring resistances over 100,000 megohms with E. M. F.'s less than several hundred volts. There are, however, the variously called loss of charge, fall of charge, or leakage methods for measuring higher insulation resistances, such as that of short lengths of cable and insulated wire, and joints in cables. These methods require, in the insulation to be tested, some capacity as well as a very high resistance. At least as accurate results for very high resistances can be obtained by the leakage method to be given here as by any other method. Moreover, the calculations required are less complicated than for other leakage methods.

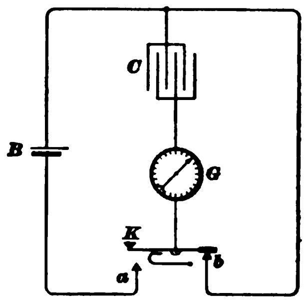

FIG. 15

The method consists in charging the cable, or other object whose insulation resistance is to be measured, as a condenser, then allowing it, while insulated, to leak for an observed number of seconds, and finally again charging it to the same full potential through the galvanometer. The quantity of electricity Q passing through the galvanometer replaces the charge that has leaked away during the observed time t; hence, the E. M. F. divided by $\frac{Q}{t}$ gives the insulation resistance. For calibrating the ballistic galvanometer required any of the methods given may be used. If the condenser method is used, connect the apparatus as shown in Fig. 15, in which G is the ballistic galvanometer, C a condenser of known capacity, B a standard Clark or other similar cell of known E. M. F., and K a charge-and-discharge key. All apparatus and connections must be very highly insulated throughout this test, for which reason connecting wires should be made air lines as much as possible. The condenser must first be thoroughly discharged. Then by depressing the key so that it touches *a*, the condenser is charged, producing a throw of the galvanometer; and on letting up the key so that it touches *b*, the condenser discharges, producing another deflection. Let d_1 be the mean of the two deflections, E_1 the E. M. F. of the cell, and C_1 the capacity of the condenser; then the quantity of electricity per unit deflection, that is, the constant of the ballistic galvanometer, has already been shown to be $K = \frac{C_1 E_1}{d_1}$.

The cable to be tested is then connected as shown in Fig. 16. If the cable is on a reel, it should be immersed in

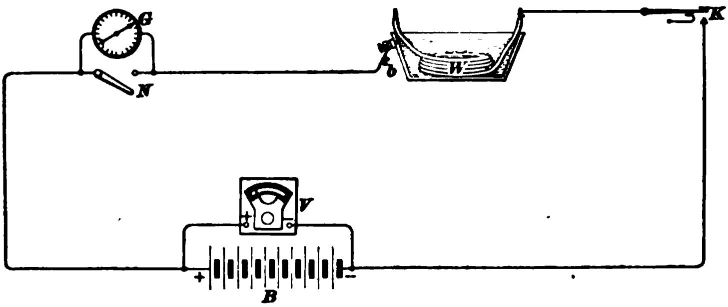

FIG. 16

a tank of water. Particular care should be taken to insulate the ends of the cable, so as to avoid surface leakage. The guard-ring method of eliminating surface leakage may also be applied. *N* is a switch or plug by which the galvanometer may be short-circuited. With the switch *N* and key *K* closed, charge the cable. For a preliminary test a charge lasting 1 minute is sufficient, as poor insulation may render a longer charge useless. Then open the circuit at *K* for a carefully observed number of seconds, say 30, in the meantime opening the switch *N*, and noting the zero reading of the galvanometer. At the end of the 30 seconds, close the key *K* and note the throw d_2 of the galvanometer. This throw corrected for the zero reading, indicates the quantity of electricity passing through the galvanometer, and hence the quantity required to replace the part of the charge lost by leakage or absorption during the observed time that the cable was disconnected and a portion of its charge allowed to leak away. In order to obtain a series of values that will show the condition of the insulation and the amount of absorption, repeat the above observation after equal periods of charge of at least 1 minute, and for equal periods of discharge, say 30 seconds.

If Q_2 is the quantity of electricity corresponding to the throw d_2, t the number of seconds during which the cable was disconnected and the charge equivalent to Q_2 escaped, and E_2 the E. M. F. of the battery that may be most conveniently measured by a voltmeter V, then $Q_2 = \frac{E_2}{R} t = K d_2$.
But it has already been shown that $K = \frac{C_1 E_1}{d_1}$. Substituting this value for K in the preceding equation, we obtain

$$R = \frac{E_2}{E_1} \times \frac{d_1}{d_2} \times \frac{t}{C_1} \qquad (14)$$

If C_1 is expressed in farads R will be in ohms, but it is usually more convenient to express C_1 in microfarads in which case R will be in megohms; t must be expressed in seconds. This formula is true provided the leakage is not great enough to appreciably lower the potential of the cable during the time that the charge is allowed to leak away. Constant results will not be obtained unless the insulation is very high, and even then not unless the cable is charged until absorption ceases, in many cases at least for ½ hour. If the deflections decrease as the time the cable is allowed to remain on open circuit is lengthened, it indicates absorption; the greater the decrease the greater is the absorption. The higher the insulation resistance, the easier is the application of the method, for it merely requires that the observation be taken after longer discharge or leakage intervals in order to get sufficiently large deflections. With any one cable the discharge intervals, or the E. M. F.'s, or both, should be regulated to give desirable deflections. The insulation resistance calculated from successive deflections obtained by this or any other leakage method, will increase until absorption, which may continue for hours, ceases.

THE TESTING OF INSULATORS

35. Although no method of testing insulators, or any insulating material for that matter, can equal a practical trial under conditions of actual service, nevertheless, whatever tests are made should duplicate as nearly as possible the electrical and mechanical strains to which the insulation will be exposed under the most severe conditions that will ever be met with in practice. All insulating materials are most apt to break down on long-applied electric stress. The prepared cloth wrappings used on the windings of electrical machinery will stand instantaneously two or three times the potential that they will carry continuously. Glass and porcelain are not affected by time to the same extent as organic materials, but both are punctured by long-continued applications of lower pressures than they have withstood when tested. But it is impractical to test each insulator with a given number of volts continuously as it would be in service, especially on high-voltage transmission systems, and the ordinary insulation resistance tests alone are not sufficient.

Insulators should first be inspected to see that they are free from cracks, bubbles, or pits that will impair their strength or in which moisture can lodge. If of porcelain, the glaze should cover all the outer surfaces. The glaze is of no insulating value in itself, but dirt sticks to unglazed surfaces. Experience has shown that porcelain insulators that are not absolutely non-absorbent are worthless. The best porcelain shows a polished fracture like glass. If there is any doubt about the quality of the porcelain in this respect, it should be broken into small pieces, kept in a hot, dry place for some time, weighed, and immersed in water for a day. When taken out of the water and all visible drops and moisture wiped off, the weight should be the same as at first.

It is also well (if the insulator is of a type that seems to require it) to try samples for mechanical strength. When mounted on pins the insulator should stand a side strain of at least ten times the pressure exerted by the air on the conductor with a wind velocity of, say, 100 miles an hour. With built-up insulators (two or more parts) it would also be well to test them in tension along the axis of the pin. In transmission lines crossing depressions, such an upward pull is not infrequently exerted on the insulator.

PUNCTURE TEST

36. A puncture test should be made by setting the insulator upside down, as shown at *W*, Fig. 17, in a vessel containing salt water, filling the pinhole also with salt water. The connections for this test are also shown in this figure. *D* represents an alternating-current dynamo, and *T* an alternating-current transformer, which works on the same principle as an induction coil. It is so constructed that an ordinary

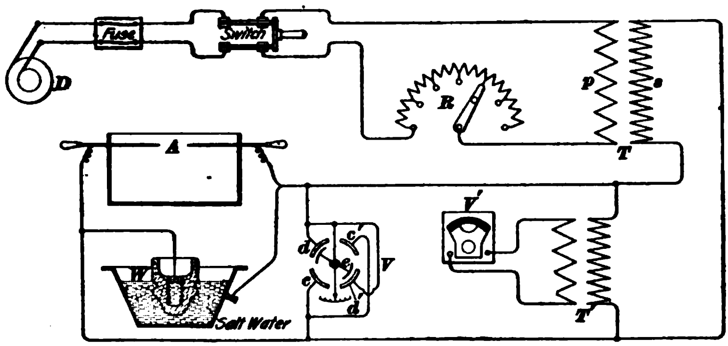

FIG. 17

alternating potential, say 125 volts, acting on the primary winding *p* may produce a potential as high even as 100,000 volts in the secondary winding *s*. A spark gap *A* and the insulator under test are connected in parallel. The spark gap prevents the application of a greater potential to the insulator than will produce a spark across its points at the particular distance to which they may have been separated. The points of the spark gap are set such a distance apart that a spark will jump across them when the voltage has been raised to the highest value at which it is desired to test the insulators. This distance may be determined by the table or curve given in *Electrical Resistance and Capacity*. If a spark merely jumps across the gap A when the points are set at the proper distance apart, so that the desired voltage is obtained, and does not puncture the insulator or creep over its surface from the water inside to the water outside, the insulator has withstood this *breakdown test*, as it is also called. New needles should be used at the gap after each discharge across it, otherwise the potential necessary to produce a spark across the gap may not follow the table or curve referred to above.

Either a high-voltage electrostatic voltmeter, or an ordinary low-voltage (up to 300 volts) alternating-current voltmeter may be used in addition to, or if necessary, even in place of the spark gap; in the figure, V represents an electrostatic voltmeter. Briefly stated, it consists of four insulated quadrants d, d', c, and c' and two movable vanes fastened together, a spiral spring at the center tending to prevent the movable vanes and pointer from rotating about the center from their zero position. The quadrants c, c' being connected to one side of the circuit are always oppositely charged to those that are connected to the other side of the circuit d, d' and the movable vanes e. Hence, c, c' tend to attract and d, d' to repel the movable vanes. The scale is calibrated to read, in volts, the difference of potential between the two sets of quadrants. This voltmeter may be used for high voltages on either alternating- or direct-current circuits.

The high voltage in the testing circuit may be reduced, or stepped down, as it is called, by the small transformer T', which is made purposely so that an ordinary low-voltage, alternating-current voltmeter V' may be used. The reading of the voltmeter V' must be multiplied by the ratio of transformation of voltage due to T' or else V' must have its scale calibrated so as to read directly the voltage in the high-potential circuit. Either voltmeter may be used.

37. Care should be taken that all high-potential connecting wires form air lines and are separated a distance at least somewhat greater than the distance between the sparking points at *A*. Start the test with all the resistance in *R*, then gradually reduce *R*, leaving it remain in each position 1 minute. Thus slowly increase the potential between the inside and outside of the insulator *W* until the insulator either punctures or arcs over the surface or until the desired test potential is reached, thereby causing a spark to pass across the air gap *A*.

The puncture test as here explained may be used for similar tests on almost any insulating material.

If an insulator is built up of several parts, each part should be able to withstand a pressure greater than it will have to sustain when the complete insulator is tested. If it is to be tested for 100,000 volts and is made in two parts, each part might, for instance, be tested with 70,000 volts. The object of this is to have the weak parts rejected before they are assembled. A fair puncture test for an insulator is twice the potential for which it is to be employed, applied between the head and the interior for 1 minute. For example, the insulators for a 50,000-volt line should each stand 100,000 volts.

A 1-minute test is not so severe as a continuous application of an equal potential, but insulators that have passed this test stand up well in service. New types of insulators for high-potential circuits should be tested both wet and dry, to determine the potentials that will arc over them. The dry test is of little value, as the potential at which the arc jumps from the head to the pin can be predetermined by measuring the shortest distance between them and referring to a curve of sparking distances in air. In a wet arcing test, a stream of water from a sprinkler nozzle under a pressure of at least 50 pounds to the square inch should be played on the insulator at an angle of, say, 30° from the horizontal. This will be similar to the condition that exists in a rain or wind storm. The insulator should not arc over from the wire to the pin at less than the potential that will exist in service between any two conductors.

ELECTROSTATIC CAPACITY

38. The **electrostatic capacity** of well-insulated condensers may be measured by any one of a number of methods, and the method for which the apparatus at hand is most suitable, would be the one generally selected. But the various methods fail to a greater or less extent when applied to leaky condensers, submarine and underground cables, and overhead lines. Where there is much leakage, the leakage current interferes with the instantaneous charge or discharge currents and moreover the discharge from cables cannot be completed instantaneously, both because induction may delay the discharge and because the dielectric gives up only slowly the portions of the charge that it has absorbed. The *direct-deflection method* of measuring electrostatic capacity is the simplest and probably the most generally used, except perhaps for alternating-current apparatus, for which the *alternating-current method* would usually be preferred. The results obtained by the direct-deflection method will hardly be correct, even under favorable conditions, to within 1 per cent. and it is not as exact, perhaps, as Gott's method, but it is quicker and more simple.

DIRECT-DEFLECTION METHOD FOR MEASURING CAPACITY

39. In the **direct-deflection method,** the capacity is measured by comparing the extreme swing of a galvanometer produced by discharging the cable through the galvanometer, with that produced by discharging through the same galvanometer a condenser of known capacity charged to the same potential.

To observe the extreme throw of the ballistic galvanometer when the standard condenser is discharged through it, the apparatus may be connected as shown in Figs. 15 or 18. *G* is a ballistic galvanometer, *C* the standard condenser, *B* a battery of from 1 to 15 cells, and *K* a discharge key resting normally against the contact *b*, but capable of being pressed against the contact *a*. The capacity of the condenser *C* should, preferably, be adjustable from about $\frac{1}{10}$ to 1 microfarad. The best results are obtained when the capacity of the standard, or known, condenser is very nearly equal to that of the unknown, and the deflections therefore nearly the same. However, a moderate divergence from this condition of equality does not materially reduce the precision. In case an adjustable condenser is not available, one having

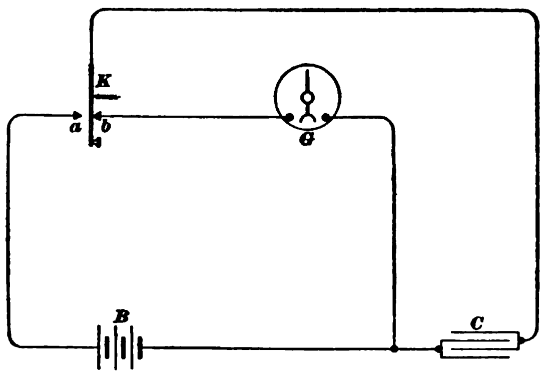

FIG. 18

a capacity of about $\frac{1}{10}$ microfarad will be found most suitable for use with telephone and telegraph cables.

When the key is pressed against *a*, a current from the battery charges the condenser. The charging should be allowed to continue for about 15 seconds, in order to give the charge a chance to soak in. The key should then be suddenly released, which will establish such connections as to allow the condenser to discharge through the galvanometer. A certain throw, or kick, of the galvanometer needle will take place, and this extreme reading should be noted down. Several readings should be taken to avoid error and to obtain an average.

The cable or line is then substituted for the condenser, as shown in Fig. 19, where the wire *n* leading from the galvanometer and battery is represented as connected to the one wire of a telephone cable to be measured, all the other wires being insulated from it and connected with the sheath of the cable. The key *K* should be connected to the sheath, as shown. If the line whose capacity is being measured is of bare wire, the wire *n* from the battery and galvanometer should be connected to it, while the wire from the lever of the key should be grounded. Several readings are then taken on the galvanometer after charging for, say, 15 seconds

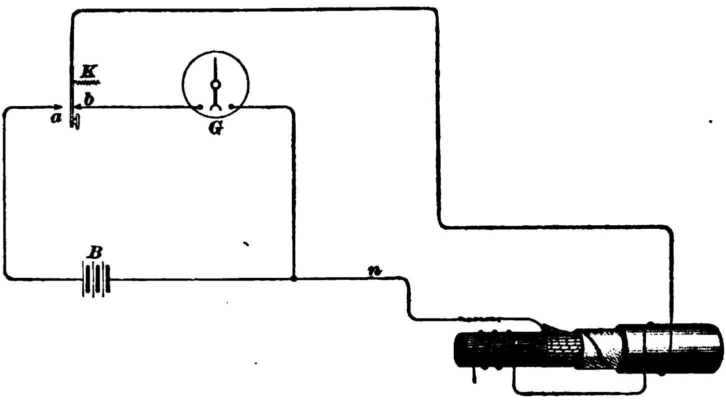

Fig. 19

and the average of them obtained as before. If the deflection is found to increase with the duration of the charge, due to absorption, some duration should be found by trial so great that the deflection is not further increased by longer charging. For a condenser showing no signs of absorption, 1 or 2 seconds is enough. Between readings, the condenser or line, as the case may be, should be fully discharged by holding the key in the discharging position for at least as long a time as the cable was charged.

If no shunt is used on the galvanometer, or if the same shunt is used in each case, the two capacities will vary in proportion to the respective readings of the galvanometer; thus, calling *d* the deflection obtained with the standard condenser, d' that with the cable, C the capacity of the standard condenser, and C' the capacity of the cable, we have $C' : C = d' : d$; or,

$$C' = \frac{Cd'}{d} \qquad (15)$$

The capacity of a cable per mile is found by dividing its total capacity by the length of the cable in miles.

40. Use of Shunts in Measuring Transient Currents.—Since the relative opposition offered by a galvanometer and by an ordinary shunt to transient (brief and variable) currents does not vary the same as their relative resistances to steady currents, it follows that the multiplying power of an ordinary galvanometer shunt is not the same for transient as for steady currents. Hence, if very accurate results are desired, an ordinary shunt should not be used in connection with the measurement of transient currents, as in this or any similar method, unless exactly the same shunt is used when obtaining two comparative deflections (d and d' in this case) that enter into the computations. However, an Ayrton shunt may be used, because its multiplying power is the same for transient as for steady currents. If the same shunt is used in both cases no account need be taken of the shunt, but if it is necessary to use different shunts in obtaining the deflections d and d', then each deflection must be multiplied by the multiplying power of the shunt used when observing it.

41. A complete and practical diagram of connections for determining the capacity of a cable by the direct-deflection method is shown in Fig. 20; A and D represent reversing switches. It will be noticed that the galvanometer is so connected in the circuit that the charge as well as the discharge flows through it, as in Fig. 15. If it is desired to have only the discharge pass through the galvanometer, as in Figs. 18 and 19, then connect d to i and disconnect d from h and e from i, and connect b to e and h to E and disconnect b from E. In Fig. 20 a universal shunt is shown connected to the galvanometer. The cable must first be thoroughly emptied of any stray charge by placing a plug in m, closing A, and placing *p* upon *0*, or by connecting *e* to *h*. *K* should be a key that can be clamped down in contact with *a*, but when released will immediately spring against *b*. With *p* upon a suitable point, a plug in *m*, the reversing switches *A* and *D* closed, and the galvanometer system perfectly at rest—its position of rest being noted—depress the switch *K* and note the throw of the galvanometer. Reverse the switch *A*, so as to obtain the next deflection on the same side of the scale, and when the galvanometer comes to rest, let up the key *K* and observe the discharge deflection. If possible, adjust the

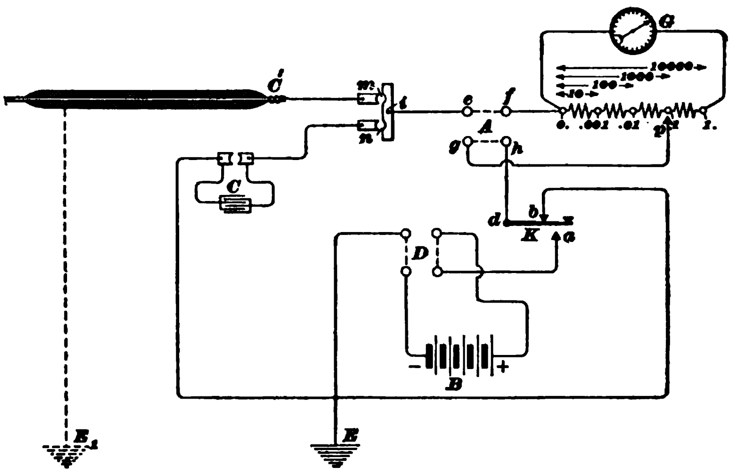

FIG. 20

galvanometer shunt so as to obtain as large a deflection as can be accurately read.

Practically all the charge of the condenser is yielded instantaneously when it is discharged, but the cable discharges at first only a large percentage of the charge it received, the remaining portion of the charge is given up so slowly that the swing of the galvanometer system is not affected by it; moreover, some of the charge may have escaped due to poor insulation. Consequently, the capacity computed from the discharge deflections is too small. It would be too large, if computed from the charge deflections because the leakage as well as the absorption would tend to increase the swing. Probably the mean of both charge and discharge deflections will give the best result, especially when the insulation resistance and capacity are low. When the insulation resistance is high and the absorption rather large then experience seems to show that the discharge deflection gives the most accurate result. In order to reduce to a minimum the error due to leakage the test on overhead bare wires should be made in dry weather when the insulation resistance is a maximum.

To determine the deflection from the known condenser *C*, shift the plug from *m* to *n* and proceed in the same manner. A mean of a number of observations should be taken on both the condenser and cable. Between each pair of observations the cable or condenser must be thoroughly discharged.

42. Proper Galvanometer.—In order that the throw shall be a correct measure of the charge (or discharge), the whole charge should pass through the galvanometer before the system has had time to move far from its position of rest; and, furthermore, the system should swing with but little air resistance, so that the magnitudes of successive throws do not fall off rapidly. These conditions are satisfied by a sensitive, reflecting, ballistic galvanometer. With a galvanometer of the Thomson type, the position of the controlling magnet of the instrument may be arranged to sufficiently decrease the strength of the earth's field in which the moving system rotates; and the mass of the moving system may in some cases be sufficiently increased so that their combined effect will cause the system to require from 10 to 15 seconds to make half a complete vibration, that is, one swing in one direction and back to zero. Thus, a charge of several seconds duration may act on the needle before it has moved far from its position of rest. A D'Arsonval galvanometer may also be used, if its time for making half a complete vibration is as long as that mentioned above. No correction for damping is required because the correction factor is proportional to the throw and thus the throws from both the cable and condenser are reduced by the same fraction, and hence the ratio of the uncorrected is the same as that of the corrected throws.

EXAMPLE.—A test was made to determine the capacity of a cable 2,000 feet long. The capacity of the standard condenser was 1 microfarad. The deflection, or throw, from the standard condenser was 37 divisions, and that from the cable was 42. What was the capacity of the cable, per mile?

SOLUTION.—The capacity of the cable $C' = \frac{42 \times 1}{37} = 1.135$, and the capacity of a mile of such a cable $= 1.135 \times \frac{5,280}{2,000} = 2.996$, or practically 3 microfarads. Ans.

GOTT'S METHOD FOR MEASURING CAPACITY

43. Gott's method for measuring electrostatic capacity is quite as convenient and more accurate than the direct-deflection method, and moreover any kind of a sufficiently sensitive reflecting galvanometer may be used. It does not, therefore, require a ballistic galvanometer, which many companies do not possess. From the diagram for

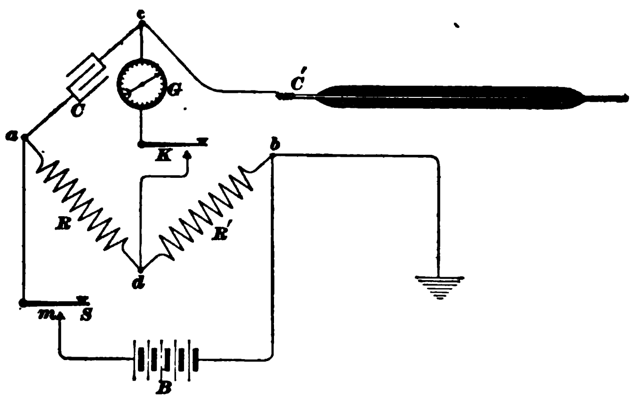

FIG. 21

Gott's method, shown in Fig. 21, it will be seen that the apparatus is arranged like a Wheatstone bridge, the known capacity of a standard condenser C and the unknown capacity of a condenser or cable C', forming two arms and the adjustable resistances R and R' forming the other two arms. To carry out the determination, give R and R' any values, but make

$R + R'$ as large as possible or convenient, close the switch S upon m, and after 5 or 10 seconds close the key K and note the deflection of the galvanometer G. The battery circuit must remain closed until after the galvanometer deflection is observed. Then open S and thoroughly discharge both C and C' by holding K closed at least several times longer than the battery circuit previously remained closed. After readjusting R or R' (always keeping $R + R'$ large), the above operation is repeated till on closing K, with S resting on m, no deflection of the galvanometer is produced. Now, the charge Q on a condenser is such that $Q = E\,C$, in which E is the potential difference between the two terminals of the condenser and C its capacity. Similarly, for the condenser C', $Q' = E'\,C'$. Then E represents the difference of potential from a to c and E' the difference of potential from c through C' to b when S is closed. Since the two condensers were charged in series, it follows that the quantities of electricity Q, Q' on the two condensers must be exactly equal; hence, $E\,C = E'\,C'$, or $\frac{E}{E'} = \frac{C'}{C}$. Moreover, R and R' were adjusted so that when K was closed no deflection of the galvanometer was produced; hence, the points c, d must have exactly the same potential. But the difference of potential E between a and c equals the difference of potential between a and d, and similarly the difference of potential E' between c and b equals that between d and b. But the difference of potential E between a and d equals $I \times R$ and the difference of potential between d and b equals $I \times R'$, because the same current I flows through both R and R'. Hence, $\frac{E}{E'} = \frac{R}{R'}$. But it has been shown that $\frac{E}{E'} = \frac{C'}{C}$; hence, $\frac{C'}{C} = \frac{R}{R'}$, or

$$C' = C\frac{R}{R'} \qquad \textbf{(16)}$$

If the insulation of the cable is less than several megohms to the microfarad the capacity obtained by Gott's method would be appreciably greater than the actual capacity. Both leakage and absorption tend to increase the apparent capacity of the cable as obtained by this method. The best conditions are to have the known and unknown capacities about equal; the total resistance $R + R'$ should be high and the battery should supply as large a current as R and R' will safely carry. The three arms of a Wheatstone bridge may be used for $R + R'$, the large resistances in two ratio arms being used for one and the rheostat arm for the other. Care must be taken not to cut out too much resistance from one of the two resistances without also inserting a corresponding amount in the other one, otherwise $R + R'$ may be made so small as to allow the battery to send too large a current through it. When a plug is inserted in one resistance another plug can usually be withdrawn from the other resistance. To protect the resistance, the battery circuit should not be closed by the switch S any longer than is really necessary. Where C' is a condenser, it is not necessary to ground b, merely connect the condenser between c and b. In this method it is not essential that the battery or switch S should be very highly insulated from the ground, but all other connections and instruments should be very highly insulated from the ground and from one another.

On account of absorption and leakage of cables, even with well-insulated condensers, the ratio $\frac{R}{R'}$ will change with a change in the duration of charging; hence, in the final adjustments of R and R' the duration of charge should always be the same. With long submarine cables the duration of charge should be at least 15 seconds, or even 30 seconds in order to obtain the full capacity. On cables having much less absorption, such as electric-light, telephone, and telegraph cables, about 5 seconds is sufficient. Gott's method has the disadvantage that the error due to leakage increases with the duration of charge, but it is a zero or null deflection method and hence either a Thomson or D'Arsonval galvanometer is suitable; the galvanometer should be sensitive enough to detect the smallest change it is possible to make in R or R'.

ABSOLUTE MEASUREMENT OF CAPACITY

44. The capacity of a condenser, line, or cable may be determined without the use of a standard condenser. To obtain results that do not need to be corrected for damping a ballistic galvanometer should be used that occupies at least 10 to 15 seconds in making half a complete oscillation. First connect the galvanometer, a high resistance, and a battery in a series-circuit and observe the steady deflection d_1 produced. The deflection should be the mean of an equal number of readings on both sides of the scale obtained by the use of reversing switches in the battery and galvanometer circuits. Then connect the same galvanometer and the same battery as shown in Fig. 15, so that the instantaneous swings may be determined when the condenser is charged and discharged. Call the mean of an equal number of charge and discharge deflections on each side of the scale d_2. Also observe the time t, in seconds, required by the galvanometer system to make half a complete oscillation; that is, one vibration from one to the other side of the scale, or from the center to one side and back to the center. This is best determined by noting the total time for a number of vibrations and dividing this time by the number of vibrations. Then, the unknown capacity is given in farads by the formula

$$C = \frac{.3183\, t\, d_2}{R\, d_1} \qquad (17)$$

If R is expressed in megohms, C will be in microfarads. R is the total resistance of the circuit; hence, $R = R_1 + B + G$, in which G is the galvanometer resistance, B the internal resistance of the battery, and R_1 all other resistance in the circuit. If a shunt of resistance S is used with the galvanometer when determining the deflection d_1, then in the above formula $R\, d_1 = \left(R_1 + B + \frac{G S}{G + S}\right) d_1 \left(\frac{G + S}{S}\right)$. The internal resistance B of the battery can usually be neglected. An Ayrton shunt may be, but preferably no shunt should be, used when determining the deflection d_2. The derivation of the last and the next three formulas belong properly to a treatise on the subject of alternating currents, and are beyond the scope of this Section.

MEASUREMENT OF CAPACITY BY ALTERNATING CURRENTS

45. Method No. 1.—Connect the condenser C whose capacity is to be measured in series with an alternating-current ammeter A, a generator of alternating currents D, and an alternating-current voltmeter V across the terminals of the condenser, as shown in Fig. 22. If the resistance and inductance of the whole circuit is negligible, compared to $\frac{1}{2\pi n C}$, then the capacity C is given by the formula

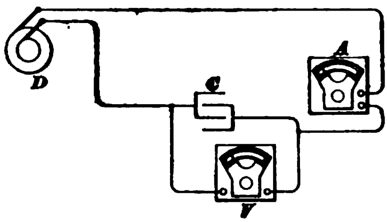

Fig. 22

$$C = \frac{I}{2\pi n E} \qquad (18)$$

in which I = current, measured by A;
E = difference of potential measured by V;
n = frequency or number of cycles per second made by the alternating current.

Read A and V as nearly simultaneously as possible. If the generator D has p pairs of poles and makes s revolutions per second, $n = ps$. If I is expressed in amperes and E in volts, C will be in farads.

If a non-inductive resistance R is included in series with the condenser, and the voltmeter is connected so as to measure the drop of potential through both R and C, then, the inductance being negligible, we have

$$C = \frac{I}{2\pi n \sqrt{E^2 - I^2 R^2}} \qquad (19)$$

R should be measured by a Wheatstone bridge or with direct current if a voltmeter and ammeter are used.

46. Method No. 2.—The capacity may be measured in the following manner: Connect a non-inductive resistance (an incandescent lamp or graphite resistance) in series with the condenser, measure, as nearly simultaneously as possible, the difference of potential E' across the non-inductive resistance and the difference of potential E across the condenser terminals; then,

$$C = \frac{E'}{ER2\pi n} \qquad (20)$$

A known resistance R but no ammeter is required in this method.

MEASUREMENT OF INDUCTANCE

47. The inductance,* or self-inductance, of a coil is a quantity that is strictly constant only when no magnetic material, masses of metal, or closed coils are near it. For if there is magnetic material near the coil its inductance will depend on the strength of the current passing through the coil; and if there is any mass of metal or closed coils near it currents may be induced in the metal or closed coils, thereby tending, as a rule, to reduce the apparent inductance of the coils under measurement. Moreover, the hysteresis in any iron near the coil would produce the same result in methods using alternating or reversed currents. Hence, constant values cannot be expected from measurements of the inductance of coils unless care is taken to remove all iron, metal, or closed coils from the neighborhood of the coil while its inductance is being measured. When the

*Some writers make the following distinction between inductance, self-inductance, and mutual inductance: *Inductance* is the number of interlinkages (flux × turns) of an electric circuit with the lines of force produced by unit current in the circuit. *Self-inductance* is the number of interlinkages of an electric circuit with the lines of force produced by unit current in this circuit and not linked with a second circuit. *Mutual inductance* of one circuit upon a second circuit is the number of interlinkages of the second electric circuit with the lines of force produced by unit current in the first electric circuit.

inductance is itself variable, there is no use in striving for great accuracy of measurement.

If the coil has an iron core or surrounding shell the strength of the current through the coil when the measurement is made should be determined and stated. The inductance of a coil when it contains iron should be preferably determined with exactly the same current flowing through it as when in use. This condition cannot always be readily fulfilled, however. As the derivations of the formulas that will be given for the methods of measuring inductance are rather complex and difficult it will not be practical to derive them here.

MAXWELL-RIMINGTON METHOD

48. Rimington's modification of the Maxwell method of measuring the inductance of a coil, is a null method and avoids the successive adjustments necessary in the method, as originally devised by Maxwell. A diagram of connections is shown in Fig. 23; *M*, *N*, and *P* represent three non-inductive resistances for which the three arms of a Wheatstone bridge may be used, while *R* is the coil whose inductance *L* is to be determined. *N* represents the total resistance of the arm *a d*, while *n* represents only the resistance of a portion *d e*; consequently, some form of adjustable resistance must be used for the arm *a d* so that the position of *e* can be adjusted without in any way altering the total resistance from *a* to *d* after the proper total resistance for *a d* has once been determined.

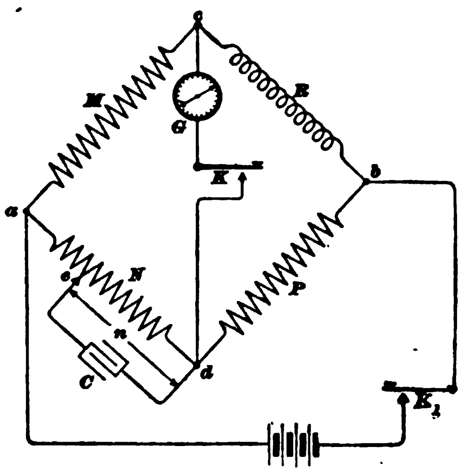

FIG. 23.

First balance the bridge in the usual manner by closing K_1 for 1 or 2 seconds and then K, adjusting M, N, and P until the galvanometer G gives no deflection; from the values of M, N, and P so obtained R can be computed if not already known. It is well to add enough non-inductive resistance to the arm $c\,b$ in series with the coil R, if its resistance is low, so that M may be made equal to N; in any case R in the formula to be given will be the total resistance of the arm $c\,b$. Move e along $a\,d$ until a point is found where no kick, or deflection, is produced after first closing K and then K_1. Let n be the resistance from d to e after this balance is obtained. The induction of the coil, or arm $c\,b$, may then be calculated by the formula

$$L = \frac{Cn^2R}{N} \qquad (21)$$

L will be given in henrys when the capacity C is expressed in farads and the resistances n, R, and N in ohms.

It is preferable to have an adjustable condenser for C so that it may be varied, if necessary, to assist in obtaining a balance and also, in order to obtain several values of L for various values of C, from which a mean value of L may be computed. In most cases G may be an ordinary Thomson or D'Arsonval galvanometer, but if the rate of change of current in the condenser C varies so much from the rate of change of the current in the coil L as to make G deflect in spite of C and L being otherwise balanced, then the time of a complete vibration of the system of the galvanometer must be increased and should not be less than 4 or 5 seconds, in order that the variable currents may settle down before G has time to move. Although a ballistic galvanometer would be suitable in the latter case, its use requires more time and it is not apt to be as sensitive as a more dead-beat instrument; hence, the latter is to be preferred whenever it can be used.

ALTERNATING-CURRENT METHOD

49. To determine the inductance of a coil with alternating currents connect the coil *c d*, as shown in Fig. 24, in series with an alternating-current ammeter *A* and an alternating-current dynamo *D*, using if necessary an adjustable resistance *S* to regulate the strength of the current. Let *L* be the inductance and *R* the resistance of the coil *c d*. Across the terminals of the coil connect an alternating-current voltmeter *V*, preferably an electrostatic instrument. Read *A* and *V* as nearly simultaneously as possible. Then the inductance, in henrys, may be calculated from the formula

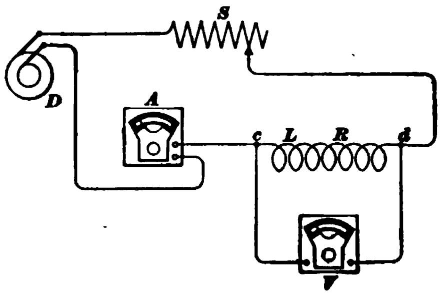

FIG. 24

$$L = \frac{\sqrt{E^2 - I^2 R^2}}{2 \pi n I} \qquad (22)$$

in which E = difference of potential from c to d;
I = current in the coil $c\,d$;
R = resistance of the coil;
n = frequency or number of cycles per second made by the alternating current.

If E is expressed in volts, I in amperes, and R in ohms, then L will be in henrys. R and n must, of course, be known or else determined. R can be measured by a Wheatstone bridge or with direct current and a voltmeter and ammeter as here connected; but it cannot be calculated from the voltmeter and ammeter readings as obtained in this measurement with alternating currents. The frequency $n = p \times s$, in which p is the number of pairs of field magnet poles on the dynamo D and s the number of revolutions per second made by the armature of the dynamo. To use this method there must be no appreciable electrostatic capacity between the points c and d.

MISCELLANEOUS MEASUREMENTS AND APPARATUS

POTENTIOMETER

50. The **potentiometer** is coming to be recognized in America, as it has been in England and Europe, as one of the most accurate and satisfactory arrangements of apparatus for the measurement of E. M. F. and current. It is also suitable for comparing resistances that are too low to be accurately measured by the ordinary Wheatstone bridge. There are about as many different arrangements of resistance and switches constituting a potentiometer as there are makers, but the fundamental principles, which we shall explain, are practically the same in all.

The principles of the potentiometer can be explained by the aid of Fig. 25, in which R is an adjustable resistance whose value need not be known, and D a steady source of E. M. F., for which a storage battery of one or more cells answers admirably. The E. M. F. of D must be at least a trifle greater than that of B. Like poles of D and B must be joined together at the same end of a very uniform bare wire $a\ b$, stretched over an accurately divided scale. The distance $a\ b$ is usually divided into 1,000 or 1,500 equal parts. The slide wire $a\ b$ should be of convenient length and of such a size that the current from D will never heat it appreciably. It is not convenient to make $a\ b$ very large in cross-section; hence, it is always advisable to keep all circuits

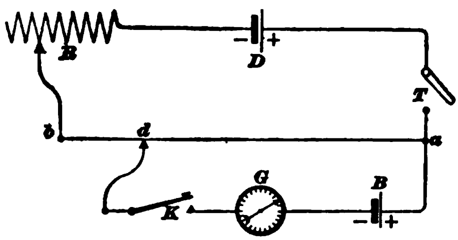

FIG. 25

open as much as possible in order to avoid a change in the resistance of *a b* due to heating. *G* may be almost any sufficiently sensitive galvanometer, preferably 1,000 ohms or more in resistance.

At *B* is first placed a standard cell whose E. M. F. is known, and the slider *d* is set at the division on the scale corresponding to this E. M. F. Suppose that *a b* is divided into 1,500 equal divisions and that the E. M. F. of the standard cell *B* is 1.431 volts at 17.7° C., its temperature when the test is being made. Then set *d* at a point 1,431 divisions from *a* and adjust *R* until the galvanometer gives no deflection when both circuits are closed, first at *T* and then at *K*. It will be seen that when *G* gives no deflection the E. M. F. of *B* must just balance the fall of potential from *a* to *d* due to the current supplied by *D*. The battery *B* is not producing any current when *d* is at the proper point on *a b*, and hence the difference of potential between *a* and *d* is exactly equal to the E. M. F. of *B*. However, if *d* is too near *a* the potential of *B*, being greater than the fall of potential from *a* to *d*, will cause current to flow from *B* through *a b* and *G*, thereby producing a deflection of the galvanometer in a certain direction. If, on the other hand, *d* is too near *b*, the fall of potential from *a* to *d*, due to the current supplied by *D*, will be greater than the E. M. F. of *B*; hence, current will then flow from *a* through *B* and *G* to *d*, producing a deflection of the galvanometer in the opposite direction to that produced when *d* was too near *a*. When a balance has been obtained at exactly 1,431 divisions from *a*, *a d* represents 1.431 volts, and hence each division represents $\frac{1.431}{1,431} = \frac{1}{1000}$, or .001 volt.

Any E. M. F. not exceeding 1.5 volts may now be measured by substituting it for the standard cell, at *B*, being careful to connect the positive terminal, or point of higher potential, to *a*, where the positive terminal of *D* is connected. Now, without disturbing *R* or any part of the circuit containing *D*, and with *T* closed, adjust *d* along the slide wire *a b* until no deflection of the galvanometer is produced on closing *K*. Suppose that the scale reads 1,324 at the point *d* where a balance is obtained. Then the E. M. F. is 1.324 volts. If the galvanometer is sufficiently sensitive *d* may be adjusted and the scale read to one-tenth of a single division, in which case the E. M. F. can be measured, by estimating tenths of a division, to $\frac{1}{10000}$, or .0001 volt.

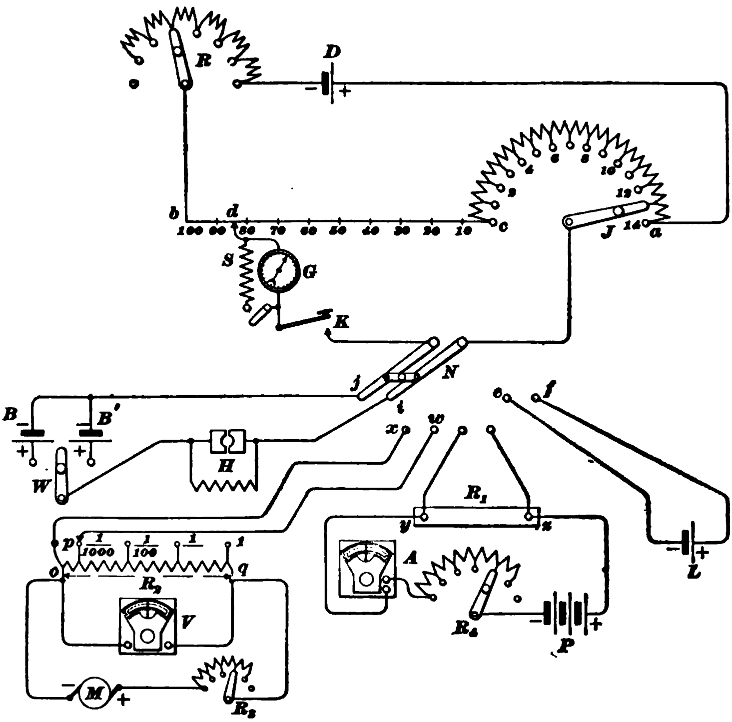

FIG. 26

51. Instead of having all the resistance in one slide wire *ab*, as represented in Fig. 25, it is more customary to use a number of equal resistance coils for parts or even for all of *ab*. One arrangement is shown in Fig. 26. The slide wire *cb*, which is 25 inches long, has a resistance of 2 ohms and is divided into 100 equal parts. The resistance *ca* consists of fourteen coils, each one equal to the resistance of *cb*.

Since each coil of *c a* is equal to 2 ohms and hence equivalent to 100 scale divisions, then the fourteen coils represent 1,400 scale divisions and *c b* 100 scale divisions, or 1,500 divisions in all from *a* to *b*. Then each scale division represents $\frac{1}{1500}$ of the total resistance from *a* to *b*. With one storage cell at *D*, *R* may be readily adjusted to give exactly 1.5 volts from *a* to *b*, in which case each division along *c b* represents $\frac{1}{1000}$ or .001 volt. It will be noticed that the resistance *c a* is numbered from *c* and *c b* also from the same point *c*. Since tenths of one scale division can be estimated and the apparatus is sufficiently sensitive to detect a lack of balance of one-tenth of a scale division, readings may be made with fair accuracy to $\frac{1}{10000}$ volt. This arrangement avoids the use of so long a slide wire, or a slide wire of larger cross-section may be used. Some makers use a large number of coils of suitable resistances, with switches making contacts with any coil desired in place of the graduated wire and sliding contact. In this way a higher resistance can be obtained for *a b* in a more convenient sized case, the wear on a slide wire is avoided and fully as good if not better results can be obtained with it. In order to obtain balances without difficulty, and also accurate results, it is essential that the battery *D* shall be very constant, that is, not polarize nor rapidly run down even with constant use, and contacts in this battery circuit must be good, for variable contact resistances in series with *D* are just as annoying and as much a source of error as a variable E. M. F. at *D*. The E. M. F. of a standard cell may be relied on as correct within a few hundredths of 1 per cent. of its given value, but in order to secure reliable results and to prevent permanent injury to the cell it is very necessary to avoid drawing any appreciable current from it. The potentiometer method, being a zero method, requires no current from the standard cell after a balance has been secured. To prevent the flow of any appreciable current from the standard cell, a high resistance *H* should be included in the standard-cell circuit until a balance has been nearly secured; that is, during the preliminary adjustments. *H* should have a resistance of 10,000 ohms or more, but its value need not be definitely known. *S* is any suitable resistance that may be used to shunt the galvanometer during preliminary adjustments. The procedure is to first obtain a balance with the standard cell, then with the unknown E. M. F. to be measured, and again with the standard cell. The latter balance is taken as a check on the first balance. The switch *N* is almost necessary in order to rapidly shift the galvanometer circuit from the standard cell to other sources of E. M. F.

52. To use the potentiometer, set *J* and *d* to correspond to the reading of the standard cell. Suppose that the standard cell, at its temperature at the time of making the test, has an E. M. F. of 1.381 volts. Then set *J* upon coil *13*, representing 1.3, and *d* at a point on the scale marked *81*, thus *J* to *d* is equal to 1,381 divisions, or 1.381 volts. To balance the bridge adjust *R* until no deflection of the galvanometer *G* is obtained when *K* is closed. For the final adjustment, the circuit through *S* should be opened and *H* short-circuited. It is quite customary to supply two similar standard cells for a potentiometer, so that one may be used to check the other. A balance should be obtained first with *B* and this balance checked with *B'*. If they do not agree something is wrong with one or both standard cells, perhaps their temperatures have changed, or the E. M. F. of *D* has changed, or the potentiometer resistance somewhere in the circuit *D–R–b–c–a* has changed for some reason, probably from a change in temperature or on account of a poor or variable contact. To measure the E. M. F. of *L*, which must not exceed 1.5 volts in this case, turn *N* so as to connect points *e f* to the potentiometer. Terminals of *D* and *L* of the same polarity must be connected toward the same end of the resistance *a b*. Without disturbing the circuit *D–R–b–a* (for the accuracy of the measurement depends on maintaining a constant current in this circuit) adjust the positions of *J* and *d* until the galvanometer gives no deflection, *S* being cut out as before, for the final adjustment. Then the reading between the positions of *J* and *d* will give the E. M. F. of *L*.

53. Measurement of Higher E. M. F's.—Whenever it is necessary to measure potential differences greater than that at the terminals of the potentiometer resistance *a b*, say over 1.5 volts, the potential difference to be measured is applied to the terminals of a resistance (R_s in Fig. 26) and the sliding potentiometer contacts are connected by means of switch N and movable contact p to other points along the same resistance, including between them a resistance that is an even part, say $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ of the whole resistance of R_s. If the fall of potential from p to o is measured then that from q to o = 1,000 times that from p to o.

The measurement of currents, as in calibrating ammeters, by the potentiometer is done by measuring the difference of potential at two points on a standard resistance (R_1 in

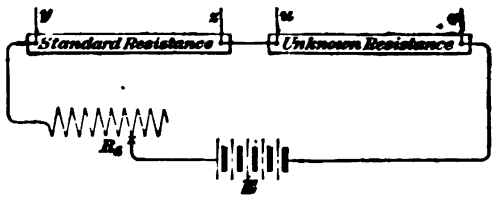

FIG. 27

Fig. 26), which, in order to avoid inconvenient calculations, should be exactly 1, $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ ohm. As these low resistances have to carry large currents, they must be made so that they will not heat to a sufficient extent to introduce errors. Knowing R_1 and measuring the fall of potential from z to y, the current in R_1 can be calculated by Ohm's law.

54. Measurement of Resistance by Potentiometer. The potentiometer is suitable for the measurement, or comparison, of low resistances that will carry sufficient current to give a good reading on the potentiometer. Two low resistances, a standard and the one whose value is to be measured, must be connected, as shown in Fig. 27, in series with an adjustable resistance R_s and a source of very constant current E, preferably a storage battery. R_s is adjusted and the number of cells at E varied until the fall of potential from y to z and from u to v will give suitable values to be measured by the potentiometer. First obtain a balance with yz connected to the potentiometer and then with uv. The current from E must be the same when balances are obtained for yz and for uv in order to get accurate results. Then the unknown resistance is to the standard known resistance as the potentiometer reading for uv is to the potentiometer reading for yz. If all quantities in this proportion, except one resistance, are known, the value of the latter can be computed.

CALIBRATION OF VOLTMETERS

55. A voltmeter may be calibrated by connecting it in parallel with a voltmeter of similar range whose readings are known to be correct. Adjust the potential difference in any convenient manner to a suitable value and read both voltmeters as nearly simultaneously as possible. Readings may thus be taken at as many different points on either scale as desirable.

Another way is to connect the voltmeter across a suitable known resistance that is in series with a standard ammeter, or other instrument, by means of which the current can be accurately measured. Adjust the current to give desired readings on the voltmeter scale and read both instruments simultaneously. The known resistance multiplied by the current flowing through it, as measured by the ammeter, gives the number of volts that the voltmeter should indicate.

A third method, which is very satisfactory, requires the use of a potentiometer and a suitable resistance, usually rather high, whose value is accurately known.

CALIBRATION OF VOLTMETER BY POTENTIOMETER

56. A very convenient arrangement for calibrating a voltmeter by the potentiometer is shown in Fig. 26. In addition to the potentiometer a resistance R_1, sometimes called a *volt box*, is used. The total resistance of R_1 should be 10,000 or 15,000 ohms, divided as indicated, so that even fractions of its total resistance may be connected to the potentiometer through contacts $x\,w$ and switch N. Thus, the range of the potentiometer may be considerably extended and an E. M. F. much higher than that of D may be measured. The potentiometer is first balanced with the standard cell B in the usual manner, placing the switch N upon contacts j and i. Then turn N to $x\,w$, vary R_s until M—a dynamo, storage battery, or electric-light or power mains—furnishes a current that will cause the voltmeter V, which is to be calibrated, to give the reading desired. Then connect w to such a point along R_s, say to p, that the fall of potential between p and o will be less than the total fall of potential from a to b, which is usually 1.5 volts, and balance the potentiometer by adjusting the position of d and J until there is no deflection. Note the readings of the voltmeter and potentiometer when a balance is obtained. The reading on the potentiometer scale from d to J multiplied by the ratio $\frac{o\,q}{o\,p}$ gives the fall of potential from o to q that the voltmeter should indicate.

For example, suppose that it is desired to determine how near the 150-volt reading of a voltmeter is correct. Balance the potentiometer with the standard cell B, correcting the E. M. F. of B for its temperature, and suppose that each division of $a\,b$ now corresponds to .001 volt and $a\,b$ to 1.5 volts. Then turn switch N to $x\,w$, connect w to p, the $\frac{1}{100}$ part of $o\,q$, adjust R_s until V reads exactly 150, and then balance the potentiometer by adjusting the position of d. Suppose a balance is obtained with d at 1,487, on the scale of $a\,b$. Then the fall of potential from o to p is 1.487 volts and the fall of potential from o to q is $1.487 \times 100 = 148.7$ volts. Hence, the voltmeter reads $150 - 148.7 = 1.3$ volts too high when it points to 150 on its scale.

A little different procedure may be pursued as follows: After balancing the potentiometer with the standard cell B, turn switch N to $x\,w$, place d on the point of the scale corresponding to the reading desired on the voltmeter and also connect w to a suitable point along $o\,q$, then vary R_s until the potentiometer is balanced.

An Ayrton galvanometer shunt may be conveniently used for the resistance *o q*. It is often convenient to have more than three subdivisions of *o q*, and hence the Ayrton shunt that is made with six subdivisions ($\frac{1}{1000}$, $\frac{5}{1000}$, $\frac{10}{1000}$, $\frac{50}{1000}$, $\frac{100}{1000}$, and $\frac{500}{1000}$) is to be preferred. More cells may be used at *D* and a number of standard cells in series at *B* to give the potentiometer a higher range, but this can be applied only to a limited extent with a given potentiometer, because too large a current would overheat the resistance *a b*.

If the range of the voltmeter is less than the range of the potentiometer then no resistance *o q* will be required; the points *o* and *q* will then be connected directly to *x* and *w*, respectively. The desired reading on the voltmeter may be obtained without the rheostat *R*, if a number of cells at *M* can be suitably varied.

CALIBRATION OF A VOLTMETER BY FRANKLIN'S METHOD

57. If a good potentiometer is not available, standard cells may be used to calibrate a voltmeter. Connect, as shown in Fig. 28, an adjustable resistance *R*; a steady source of E. M. F. *D*, preferably a storage battery; the voltmeter to be calibrated *V* and a high resistance (10,000 or more ohms) *H*; and adjust *R* until no deflection of the galvanometer *G* is observed when *K* is closed. *B* represents any desirable number of standard cells. Then the E. M. F. of the standard cells, corrected for temperature, gives the E. M. F. at the terminals of the voltmeter. By varying the number of standard cells at *B*, as many points, as desirable, on the voltmeter scale may be calibrated. The high resistance *H* may be short-circuited and *S* cut out for the final adjustment. The E. M. F. of *D* must exceed that of *B*.

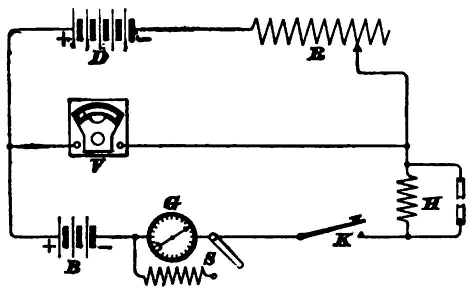

FIG. 28

58. Carhart's Method.—A slightly different method is as follows: Connect as shown in Fig. 29, in which D is a storage battery of a sufficient number of cells to give the desired reading on the voltmeter V. R and R' are adjustable and known resistances. R' should be preferably, for a high-reading voltmeter, at least, as high as 100,000 ohms, while the range of R will depend on the number of standard cells used at B and the reading desired on the voltmeter. Adjust the resistances R, R' until no deflection of the galvanometer is produced when the key K is closed, the high resistance H being short-circuited and S open for the final balance. Read the voltmeter, the resistances at R, R', and the temperature of the standard cell, or cells, at B. Then the reading of the voltmeter should be

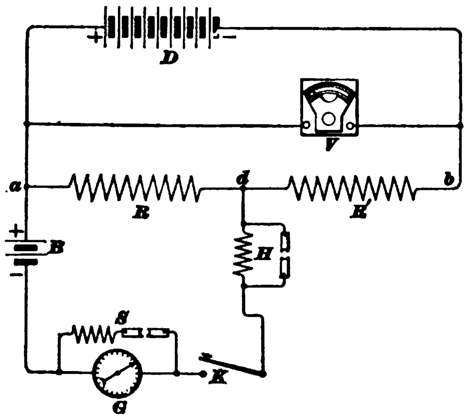

FIG. 29

$$V = sE\frac{R + R'}{R} \qquad (23)$$

in which E = E. M. F. of one standard cell corrected for temperature;

s = number of standard cells used at B.

The voltage may be changed by varying the number of cells at D and another balance obtained.

For a very low range voltmeter a slide wire can be used instead of the resistance boxes R, R' and the point d moved along the slide wire until no deflection is obtained. Then, since $\frac{R + R'}{R}$ is merely a ratio, the distance $a\,d$ can be used in place of R and $a\,b$ in place of $R + R'$, which assumes that the resistance per scale division is uniform all along *a b*. This assumption is usually made in all slide-wire bridges and potentiometers. The slide wire is suitable for calibrating voltmeters that do not read over 3 or 4 volts.

CALIBRATION OF AMMETERS

59. An ammeter may be calibrated by connecting it in series with an ammeter of similar range whose readings are known to be correct, a source of current, and an adjustable resistance. Vary the resistance until a desirable reading is obtained on the standard ammeter and read both ammeters as nearly simultaneously as possible. The readings of the ammeter may thus be compared at any number of points on their scales.

Another way is to connect the ammeter in series with a copper, or silver, voltameter, as explained in connection with voltameters, and the current determined by the voltameter for one reading of the ammeter. This is one of the most accurate methods, but it is very slow, as the calibration of a single scale reading on the ammeter requires a run of at least 30 minutes, and at least double that time to clean and weigh the plates and make the necessary calculations.

A third method, which is very satisfactory, requires the use of a potentiometer and a low resistance whose value is accurately known.

CALIBRATION OF AMMETER BY POTENTIOMETER

60. The connections for calibrating an ammeter by a potentiometer are shown in Fig. 26. R_1 is a standard low resistance of suitable value. The switch N is first placed in connection with a standard cell B whose E. M. F. is corrected for its temperature, and the potentiometer is balanced in the usual manner. The rheostat R_4 is adjusted to give the lowest reading desired on the ammeter A and the switch N placed so as to connect the potentiometer to the terminals of R_1. Then J and d are adjusted until no deflection of the galvanometer G is obtained and the readings of the ammeter and potentiometer are simultaneously noted. From the potentiometer reading is obtained the drop of potential, in volts, from z to y. This drop divided by the resistance of R_1 gives the current flowing through the ammeter A. In a similar manner other points on the ammeter scale may be calibrated.

For example, if the E. M. F. of the standard cell is 1.431 volts at its temperature 17.7° C., the contacts J and d will be set to read 1,431 and R regulated to give no deflection of G. Then with $y z$ connected by the switch N to the potentiometer, suppose a balance is obtained with d at 1,334 and that the resistance of R_1 is $\frac{1}{10}$ ohm; then the current flowing through the ammeter A will be $1.334 \div \frac{1}{10} = 13.34$ amperes.

Ammeters for measuring large currents usually consist of a standard low resistance, called the *ammeter shunt*, and a sufficiently sensitive instrument (really a very low reading voltmeter) connected to the terminals of the shunt. In calibrating such an instrument, the ammeter shunt, if its resistance is known, can be connected in the place of R_1 in this figure. Although the instrument will be in parallel with its shunt, nevertheless its scale is calibrated so as to read directly the amperes flowing through the main circuit in which its shunt is connected. In this case the instrument readings are directly proportional to the difference of potential between z and y, but the resistance of R_1 is constant; hence, the readings are also proportional to the current and the scale can, therefore, be marked to indicate directly the current in $y z$. The resistance of $y z$ will be negligible compared to that of the instrument, and hence the current passing through the instrument will be insignificant compared with that through R_1.

CALIBRATION OF AN AMMETER BY FRANKLIN'S METHOD

61. A good potentiometer furnishes the best means for calibrating ammeters and voltmeters. When a potentiometer is not available, standard cells and a standard resistance may be used to calibrate an ammeter by connecting it as shown in Fig. 30, in which D is a steady source of E. M. F., preferably a storage battery, A the ammeter to be calibrated, R a standard known resistance, and G a high-resistance galvanometer. With a suitable number of standard cells connected in series at B, adjust the resistance P until the galvanometer gives no deflection when K is closed; then the E. M. F. between the terminals of the standard resistance R is equal to the E. M. F. of the battery B consisting of standard cells. The E. M. F. of the standard cells should be reduced to the observed

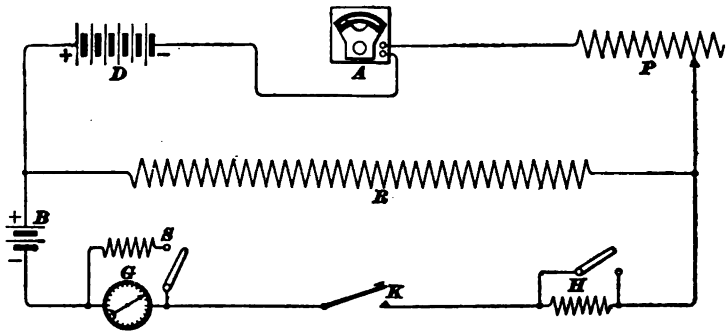

FIG. 30

temperature. This E. M. F. divided by R will give the current through the ammeter A. The high resistance H, equivalent to about 10,000 or more ohms per cell at B, should be short-circuited and the shunt S open-circuited when making the final adjustment. By the use of a different number of standard cells, various points on the ammeter scale may be calibrated. The E. M. F. of D must exceed that of B.

PORTABLE TESTING SETS

62. Portable testing sets usually consist of compact collections of instruments and apparatus necessary for making the measurements for which they are intended. Thus, there are portable Wheatstone bridge sets, cable-testing sets, and ohmmeters. They are suitable for use in the office as well as outdoors, while some are constructed so that the ordinary workman, knowing little or nothing about electrical measurements, may use them. Since full directions for the use of portable testing sets, of which there are many kinds and modifications, usually accompany them, it is not advisable to attempt to give them all here. However, as a sample, it will be well to fully explain the use of one portable set that is quite extensively used for measuring resistances of various magnitudes and for several other purposes. It will not be necessary to explain the principles of the various tests, as they have already been given.

ACME PORTABLE SETS

63. The **Acme portable testing set,** which is shown in Fig. 31, is claimed by Queen & Co., its makers, to be suitable for measuring resistances over a very wide range, and with simple accessories, to be equally adapted to measure insulation resistance, compare differences of potential, measure internal resistance of batteries, check ammeters and voltmeters, and locate grounds and crosses on line circuits. The set, which consists of a Wheatstone bridge, a portable form of D'Arsonval galvanometer, the necessary keys, and a battery of four special dry cells is contained in one portable case. The coils of the Wheatstone bridge, as in all reliable bridges, are wound of platinoid wire, having a low and uniform temperature coefficient. The combined resistance of the rheostat coils amounts to 11,110 ohms and in each bridge arm there are three coils of 1, 10, and 100 ohms and 10, 100, and 1,000 ohms, respectively. A special commutator allows the use of ratios ranging from $\frac{1}{1000}$ to $\frac{1000}{1}$. The theoretical range of the bridge is, therefore, from .001 ohm to 11,110,000 ohms, though for resistances above 1 megohm additional battery is required, because the galvanometer is not then sufficiently sensitive. For measuring insulation resistances that require more than four cells, cases containing from 12 to 100 small and compact silver-chloride cells may be used.

Ba is a single-contact key and *Ga* a galvanometer and short-circuit key. When the latter key is depressed it closes the galvanometer circuit, and when released it first opens the galvanometer circuit and then touches a top contact *g*, which short-circuits the galvanometer, thereby bringing it to rest almost instantly. This enables tests to be made quickly, as no time is lost waiting for a swinging needle to come to rest. When necessary, external standard high and low resistances may be used in connection with the set.

By comparing the plan view of the circuits of the Acme set and the simplified diagram of it as shown in Figs. 31 and 32, respectively, with the theoretical diagram of the Wheatstone

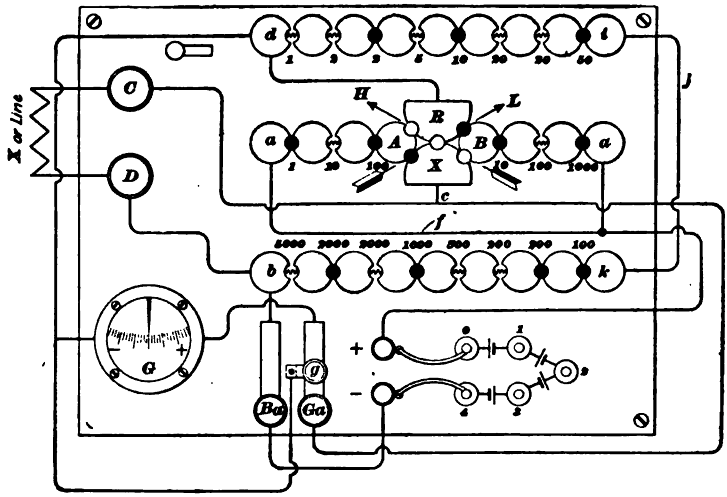

Fig. 31

bridge, shown in the preceding Section, the fact that the three are identical in principle will be readily understood.

The top row of blocks, Fig. 31, is connected, inside the case, to the bottom row by a heavy wire or bar *j*. These two rows, that is *d i k b*, constitute the rheostat *R*, in Fig. 32, in which any resistance from 1 to 11,110 ohms may be obtained by removing the proper plugs. Each half of the middle row *a*, *a* constitutes a bridge arm, designated *A* and *B*, respectively, as may be seen by comparing Figs. 31 and 32. The two blocks, *a*, *a*, Fig. 31, that are common to the two bridge arms, are joined by a heavy wire *f* and are also connected to the battery terminal marked +. All the connections constituting part of any arm of the bridge are made sufficiently heavy so as to add no appreciable resistance to the arms. One galvanometer terminal is connected to the blocks *d* and *R* and also to the top contact *g* of the key *Ga*, while the other connects through the lever of the key *Ga* and, when depressed, through its lower front contact to the block *X* and binding post *C*. The resistances contained in this set form three arms of a Wheatstone bridge; the fourth arm, which is the unknown resistance, is joined to the *X* or line posts *D*, *C*. This is shown diagrammatically in Fig. 32, where the corresponding parts are lettered similar to Fig. 31.

64. The commutator consists of the blocks *A*, *B*, *R*, and *X*, and two plugs. When the plugs are in the position shown in Fig. 32 (*a*), the bridge arm *A* is connected to the rheostat, and the bridge arm *B* to the line or unknown resistance *X*. But if the plugs have the position shown in Fig. 32 (*b*), the connections of the bridge arms are interchanged, the one formerly connected to the rheostat now being connected to the line, and the one formerly connected to the line now being joined to the rheostat; therefore, whatever ratio existed formerly between the bridge arms is reversed. Or, to put it more conveniently, when the commutator plugs have the position shown in Fig. 32 (*a*), $\frac{A}{B} = \frac{R}{X}$; when they have the position shown in Fig. 32 (*b*), $\frac{A}{B} = \frac{X}{R}$.

65. To measure resistance, connect the terminals of the resistance to be measured to the line posts *C*, *D*, as shown in Fig. 31, and place the battery connectors first on the two tips *0*, *1*. This throws one cell of the battery into circuit, which is sufficient until balance is roughly attained. Now unplug the 100-ohm coil in each bridge arm, and place the commutator plugs as in Fig. 32 (*a*) or (*b*). Remove plugs from the rheostat until the aggregate resistance unplugged is, as nearly as may be guessed, equal in value to that of the unknown resistance. Then press the battery key, and, holding that down, momentarily press the galvanometer key. If the galvanometer needle swings toward the + mark, let us suppose that the resistance unplugged in the rheostat is too high and should be reduced; if the deflection is toward the − mark, the resistance is then too low and should be increased. By altering the resistance a value will soon be found wherein a slight change either way will reverse the deflection of the galvanometer needle. The rest of the battery may now be put in circuit by moving one battery connector from tip *1* to tip *4*. If the keys be again pressed—first the battery key, then the galvanometer key—a greater deflection will be obtained than before for the same variation in the rheostat, and therefore the adjustment can be made more accurately. With bridge arms of equal value this is the best result that can be obtained, but by selecting more suitable values for the two bridge arms a considerably higher degree of accuracy may be secured. The arrows and their letters on the top of the set (see Fig. 31) facilitate the setting of the commutator plugs. If measuring a high resistance, set the plugs in the direction indicated by arrow *H*; if measuring a low resistance, set the plugs in the direction indicated by arrow *L*.

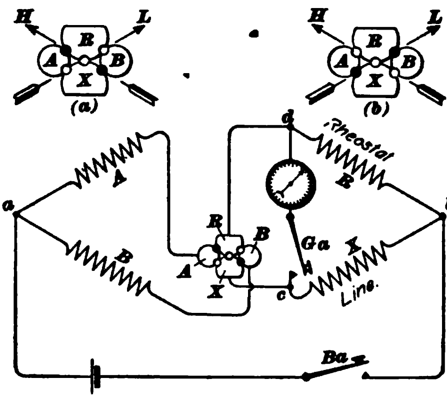

FIG. 32

66. The **bridge ratio** used in any measurement is important. Experiment will show the fact that certain values for the two bridge arms give more accurate measurements than others. This increased accuracy of measurement is in some cases so marked as to warrant care in selecting the proper values. Table I shows the values of A and B, respectively, to be chosen when measuring any resistance within the range of the set. This is applicable to about any Wheatstone bridge having the same resistances in the rheostat and balance arms.

TABLE I

			Place Plugs in Commutator as Shown in
Below 1.5 ohms, make	A = 1,	B = 100	Fig. 32 (*b*)
Between 1.5 and 11 ohms, make	A = 1,	B = 100	Fig. 32 (*b*)
Between 11 and 78 ohms, make	A = 10,	B = 100	Fig. 32 (*b*)
Between 78 and 1,100 ohms, make	A = 100,	B = 1,000	Fig. 32 (*b*)
Between 1,100 and 6,100 ohms, make	A = 100,	B = 100	Fig. 32 (*a*) or (*b*)
Between 6,100 and 110,000 ohms, make	B = 1,000,	A = 100	Fig. 32 (*a*)
Between 110,000 and 1,110,000 ohms, make	B = 100,	A = 10	Fig. 32 (*a*)
Between 1,110,000 and 11,110,000 ohms, make . .	B = 1,000,	A = 1	Fig. 32 (*a*)

67. In measuring very high resistances, it is advisable, where considerable accuracy is desired, to utilize (with the proper care) an outside battery of higher E. M. F., which may be done provided great care is exercised to balance carefully with its own battery first and to put not over a 1-ampere fuse in the circuit. It is impracticable to measure above 10,000,000 ohms with a small Wheatstone bridge, partly because of leakage across the bridge arms, which changes their ratio, and partly because of leakage across the line posts, which acts as a shunt to the resistance being measured.

Low resistances may be measured by arranging the commutator plugs in the direction of the arrow L and using, for very low resistances, the interpolation method explained in connection with the Wheatstone bridge.

68. To compare E. M. F.'s the Acme set may be arranged so as to constitute, as shown in Fig. 33, a potentiometer. Connect all the cells in the set in series in the usual way, taking care, however, not to reverse them by crossing the battery cords. Plug the commutator as shown in Fig. 33 (*a*), and remove 1,000 ohms from bridge arm *B*; from the rheostat unplug, say, 5,000 ohms. Connect one of the cells whose E. M. F.'s are to be compared at *S*, with its positive terminal to the + battery post and its negative terminal to the line post *C*. A simple diagram of the connections so formed is

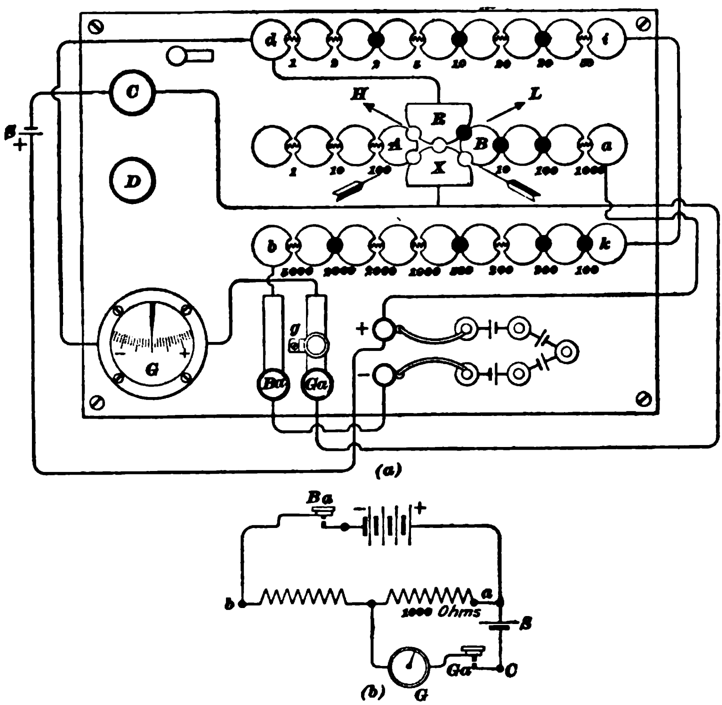

FIG. 33

shown at Fig. 33 (*b*). On pressing first the battery key *Ba* and then the galvanometer key *Ga*, the needle swings one way or the other. A value may quickly be found wherein a variation in the rheostat arm *d i k b* of 1 ohm either way reverses the deflection. To the resistance unplugged in the rheostat add that unplugged in the arm *B* and divide this sum by the resistance unplugged in the arm *B*; this gives the ratio between the potentials of the battery of the set and the cell tested, respectively. This operation repeated with any number of cells gives their values in terms of the E. M. F. of the battery in the set from which their relative values may be obtained. If desired, standard cells may replace the battery in the set, in which case the first measurement gives at once the value of the E. M. F. of the cell tested in terms of the standard cells. This method is identical in principle with Carhart's method for the calibration of a voltmeter, which has already been given. If the E. M. F. of the cell or battery being tested exceeds that of the battery in the set, it is only necessary to reverse the positions of the two batteries.

This set may be used as a potentiometer to calibrate or check voltmeters and ammeters and to determine the E. M. F. and internal resistance of cells. To determine the latter, first measure the E. M. F. of the cell on open circuit and then when shunted with a known resistance. This is practically the volt-and-ammeter method for determining the E. M. F. and the internal resistance of cells, which is described elsewhere.

Insulation resistance may be measured by the aid of an external known high resistance, the direct-deflection method being used. The manner of connecting the bridge and performing the various measurements are fully described and illustrated by examples in pamphlets that accompany the Acme and similar sets.

OHMMETERS

69. Ohmmeters are instruments from whose scale may be directly read the value, in ohms, of a resistance that is being measured. The principle of the *slide-wire ohmmeter*, which is a specially calibrated slide-wire Wheatstone bridge, may be explained by means of Fig. 34. When a plug is inserted at *1*, the coil *h* constitutes the arm *M* and two long slide wires *a F* and *F b*, joined by a bar *F* of negligible resistance, constitute two adjustable arms *N*, *P* of a Wheatstone bridge. The telephone receiver *T* is held to the ear, the key *K* closed, and the slide wire *aFb* touched at various points with the contact piece *d* until a point is found where gently tapping the contact piece against the slide wire produces no sound, or at least a minimum sound in the telephone receiver *T*. The reading of the scale under the point so found gives the resistance being measured directly in ohms. With one fixed value for the arm *M*, the slide wire may be calibrated to read directly in ohms by inserting known resistances at *X*, beginning with about 1 ohm and increasing in value until the whole slide wire has been calibrated. The divisions per ohm are not, of course, equal in length but gradually decrease in length as the resistance at *X* increases. Having once calibrated the bridge wire with known resistances at *X*, any unknown resistance at *X*—that would bring the balance point *d* somewhere on the bridge wire—may be measured and its resistance read directly from the point of balance on the scale.

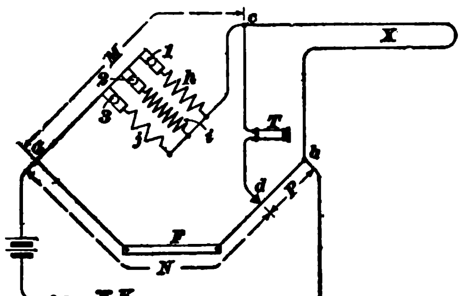

FIG. 34

If a second coil *i* having exactly ten times the resistance of *h* is used instead of *h*, then the resistance being measured will be ten times the scale reading obtained by balancing the bridge. Similarly, if a third coil *j* having a resistance of one-tenth of *h* is used, the resistance measured will be one-tenth the scale reading obtained by balancing the bridge. Thus, by means of three fixed coils resistances from about .01 to 200,000 ohms may be measured directly. The lower and higher resistances will not be measured as accurately as resistances of intermediate values. On account of the difficulty of obtaining a slide wire of uniform size and resistance and the wear upon it, such ohmmeters cannot be expected to give as accurate results as a regular Wheatstone bridge. They are convenient instruments and measurements can be made rapidly with them. With the addition of two small dry cells, which are usually fastened in the case, all the apparatus required is contained in one portable case.

70. I. E. S. Ohmmeter.—A diagram of connections for one form of slide-wire ohmmeter made by the Illinois

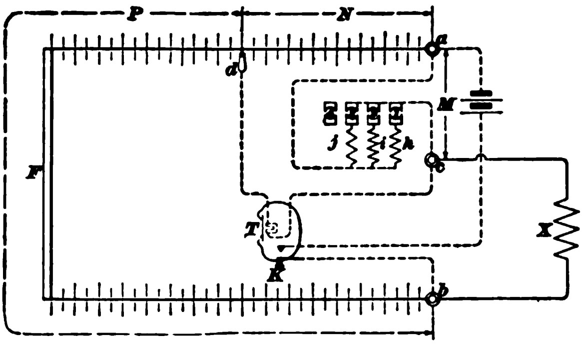

FIG. 35

Electric Specialty Company, of Chicago, is shown in Fig 35. The general appearance of the instrument is shown in Fig. 36. A scale on one side of the slide wire is made with black ink. From this scale the value of an unknown resistance may be read directly in ohms when a plug is in hole *1*. When the plug is in hole *2* or *3* the scale reading is to be multiplied or divided, respectively, by 10. A scale on the other side of the slide wire is made with red ink.

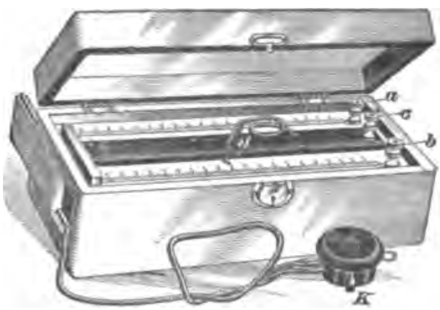

FIG. 36

The divisions on this scale are equally spaced, the total length of the wire being divided into exactly one hundred equal parts. This scale is very convenient for certain measurements, such as the mere comparison of two unknown resistances, a principle that is extensively used for locating grounds and crosses on overhead and cable lines and for other purposes. When the instrument is to be used for this purpose, the slide wire on one side of the point of balance is one arm, the slide wire on the other side a second arm, and the external resistance the remaining two arms of the bridge. For such tests none of the coils *h*, *i*, *j* (see Fig. 35) are needed; hence, the plug is placed in an idle hole *4*.

71. Two inductances may be compared, provided the resistance of each coil is small compared to its inductance, by connecting the coils *X* and *Y* as shown in Fig. 37. *I* represents an induction coil, or other source of alternating current, which should preferably be used instead of a battery. Obtain a balance by first closing *K* and adjusting *d* until a point giving a minimum sound is found. In case a battery

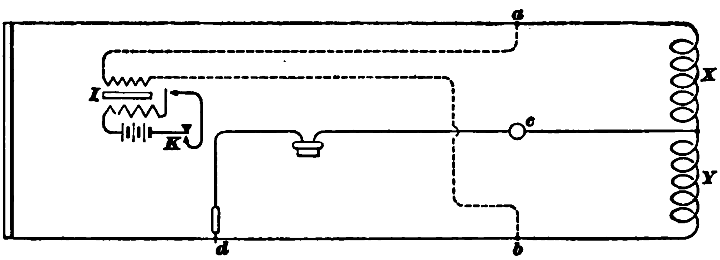

FIG. 37

is used across *a b* instead of the secondary circuit of an induction coil, the circuit must be closed at *d* before the battery circuit is closed. After a balance has been obtained, the inductance of *X* is to the inductance of *Y* as the length *a d* is to the length *d b*, the lengths *a d* and *b d* being read off the slide wire from the red or equally divided scale.

Two capacities may be compared in a similar manner, but in this case the capacities are inversely proportional to the resistances; that is, the capacity at *X* is to the capacity at *Y* as the length *d b* is to the length *a d*. In either case, if one inductance or one capacity is known the other can, of course, be calculated.

The distance along a conductor to an accidental ground may be determined by connecting as shown in Fig. 38, provided a good wire *a f*, of the same material and size is available and can be joined to the faulty wire *b f* at some point *f* beyond the fault *e*. Then, if *d* is the point of balance, the distance $be = \frac{\text{length } db \times \text{distance } afb}{\text{length } adb}$.

The resistance of electrolytes may be measured, as with any bridge, by connecting the electrolytic resistance in the unknown arm of the bridge and using in the place of the bridge battery an alternating current from an ordinary induction coil or other source.

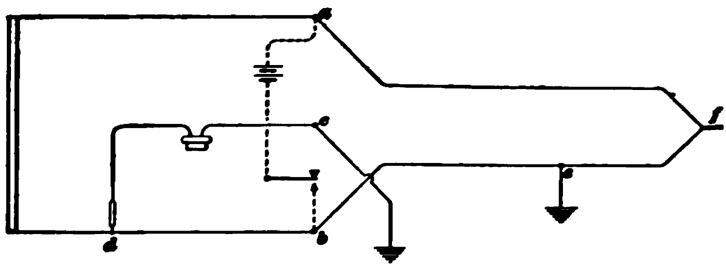

Fig. 38

72. In the I. E. S. ohmmeter, an external view of which is shown in Fig. 36, the battery key *K* is included in the receiver case, so the key can be operated by the hand that holds the receiver to the ear, leaving the other hand free to manipulate the contact piece *d*. The Illinois Electric Specialty Company makes another instrument, in which the receiver is replaced by a sufficiently sensitive and portable D'Arsonval galvanometer. It is placed in the base of the ohmmeter, there being a round glass through which the needle may be seen. The key in the battery circuit is a small push button mounted in the hard rubber top.

ELECTRICAL MEASUREMENTS

(PART 3)

COMMERCIAL INSTRUMENTS

VOLTMETERS AND AMMETERS

1. Such accurate and reliable portable measuring instruments can now be obtained that many measurements may be made with as great a degree of precision and much greater facility than with the various galvanometers and other apparatus so far described. The latter are more suitable for use in the laboratory than in the shop or station. Some of the best portable instruments made are the Weston, the general form of which is shown in Fig. 1. They are made on the principle of the D'Arsonval galvanometer, as shown in Fig. 2. Fig. 3 shows the magnetic circuit. The permanent magnet *A A* has soft-iron pole pieces *P*, *P* fastened to it by the screws *S*, *S*, and bored out to make a cylindrical opening. In the center of this opening a stationary soft-iron cylinder *C* is supported by a screw *M* passing through a lug on the brass plate *B*. This cylinder being of less diameter than the opening through the pole pieces, a narrow gap is left between the pole pieces and the iron core,

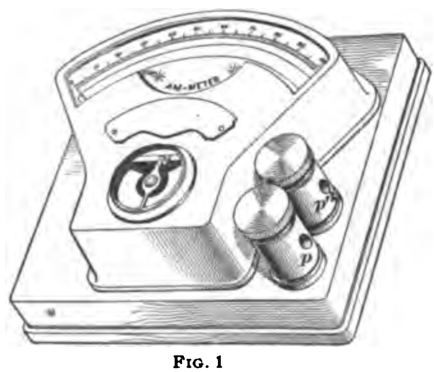

FIG. 1

as shown. The lines of force from the permanent magnet pass across this gap, making a strong and uniform magnetic field.

The movable part of the instrument is shown in Fig. 4. It consists of a rectangular coil *C* of fine wire wound on an aluminum or thin copper bobbin, which is suspended vertically between two delicate jeweled bearings. Two flat horizontal spiral springs *S*, *S* oppose the tendency of the coil to rotate, and at the same time conduct the current to

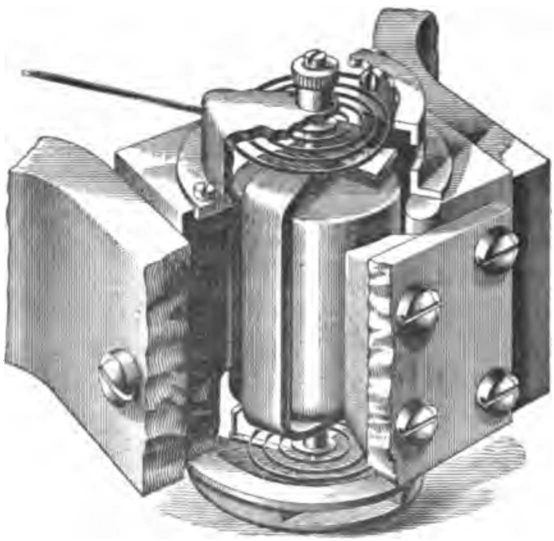

FIG. 2

and from the suspended coil. A thin aluminum pointer *P*, attached to the coil, moves over a scale and indicates the deflection of the coil from its normal position, which is shown in Fig. 2. When a current is sent through the movable coil, there is a tendency for the coil to rotate through the magnetic field, which it will do until the torsion of the spiral springs equals the force with which the coil tends to move, then the coil will come to rest, and the pointer will indicate the angle of deflection of the coil. The magnetic field being practically uniform, the angle of deflection is closely proportional to the current in the coil, so that the scale divisions are very uniform, as shown by Fig. 5, which is a scale about three-fourths size.

2. Dead-Beat Instruments.—The copper or aluminum bobbin on which the coil is wound, in moving through the magnetic field, has an E. M. F. set up in it that causes a current to circulate around the bobbin as long as the bobbin moves. This current circulates in the opposite direction to the current in the coil; hence, it tends to oppose the motion of the coil. As this tendency exists only when the bobbin is moving, it has the effect of preventing the needle from swinging too far over the scale, thus bringing it quickly to rest at the proper point. This *damping effect* is due almost entirely to the currents generated in the bobbin. The friction is so slight that it has practically no effect on the position the needle will take. This is shown by the fact that the needle having been deflected by a current will respond to very minute variations in that current; that is, the instruments are very *sensitive*. An instrument possessing the feature of coming to rest quickly at the proper point is known as a **dead-beat** instrument; this is a very important

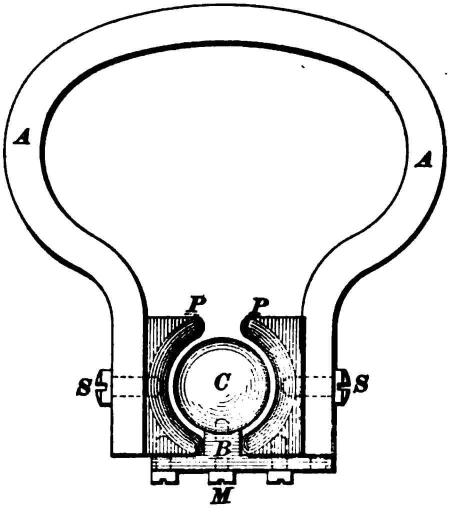

FIG. 3

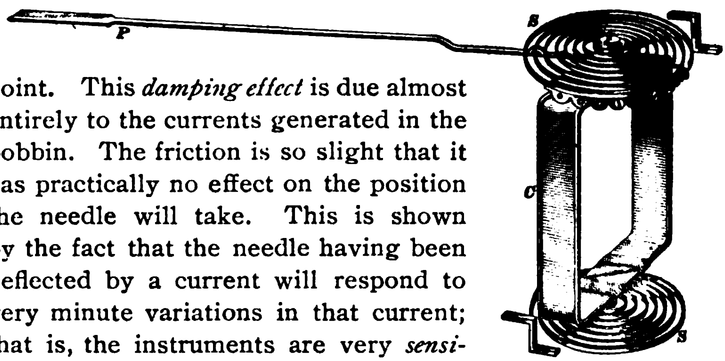

FIG. 4

feature, for it assists the rapidity of taking measurements very materially.

3. The moving system is practically the same for all direct-current Weston ammeters and voltmeters. If the instrument is designed for a voltmeter, a high resistance, located in the back of the case, is connected in series with the movable coil; if for an ammeter, for measuring all except small currents, the movable coil is connected in parallel with a short, thick piece of copper or some alloy, called an *ammeter shunt*, so that only a small part of the current passes through the movable coil, and the resistance of the instrument, that is, the joint resistance of the shunt and the movable coil, is extremely low. By reason of this extremely low resistance of the ammeters and the high resistance of the voltmeters, they consume very little energy and may be left continuously in circuit without undue heating.

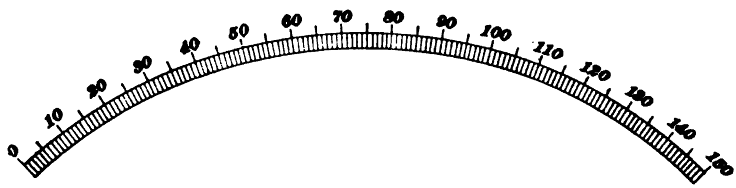

FIG. 5

For example, a 15-ampere Weston ammeter has an internal resistance of .0022 ohm; when measuring a 10-ampere current, the drop (IR) is .022 volt, and the watts expended (IE) = .22, or about $\frac{1}{3400}$ horsepower. The resistance of a 150-volt voltmeter is about 18,000 ohms. Measuring 110 volts, the instrument would take $\frac{110}{18{,}000}$ = .0061 ampere, nearly, with a consumption of energy of .671 watt, nearly, or about $\frac{1}{1100}$ horsepower.

The conducting parts of the instrument are made of an alloy having a very low temperature coefficient, so that moderate changes in the temperature of the instrument do not affect its readings appreciably. Beneath the needle just inside the scale is a mirror. On looking down on the needle, by getting the needle directly over its image in the mirror, errors due to not getting the needle in line with the scale (known as *parallax*) are eliminated. These several features make these instruments very reliable and convenient for making all sorts of electrical measurements, and as they may be obtained in a great variety of ranges, their use is very general.

AMMETERS

4. The interior of a **Weston direct-current ammeter** is shown in Fig. 6. The current entering the instrument at binding post *A* passes directly to a copper plate *a*, and thence through a number of parallel circuits to the plate *b*, from which it passes to the binding post *B*. The parallel circuits between the two plates consist of wire wrapped non-inductively around the permanent magnet *e*; they are shown by the dotted lines in the figure. The magnet serves merely as a convenient support for the wires. The coil *c* is also connected across the two plates. The joint resistance of the non-inductively wound wires is so proportioned as to allow the necessary amount of current to flow through the coil *c* to cause the proper deflection of the coil and pointer *p*.

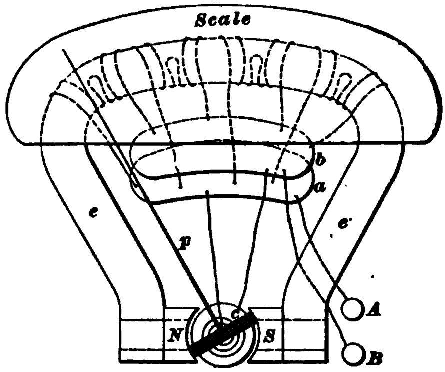

FIG. 6

Except for measuring small currents it is not customary to send all the current through the swinging coil; in fact, it has already been shown that in the ordinary Weston ammeter part of the current passes through non-inductively wound coils that shunt the swinging coil. For measuring large currents it is not practical to even include this shunt resistance in the same case with the rest of the instrument. An ammeter shunt is shown in Fig. 7. It has a very low resistance and a large surface so that it may keep cool by being able to radiate the heat generated in it by the current. The ammeter shunt is always connected in series in the circuit in which the current is to be measured. For example, suppose that we wish to measure the current supplied by dynamo *A*, Fig. 8, to whatever circuits may be connected across the so-called bus-bars *B C*. The ammeter shunt *S* will be connected in series with the dynamo, as shown, and two small wires *1*, *2* join the terminals of the shunt to the ammeter *M*. As a result only a small portion of the current passes through the instrument *M*. But as the resistances of the shunt and ammeter are fixed with regard to each other, the current through the ammeter will always be a fixed proportion of that in the main circuit, and the scale may be marked so as to read the main current and not the current actually flowing through the instrument. The instrument and shunt may be placed at convenient points and connected together by the light flexible conductors, which are usually sent out with the shunt and should never be altered in length. If they are too long, they should be coiled up out of the way; if too short, another shunt with long enough leads and its corresponding instrument should be obtained. The object, of course, is to use leads of exactly the same resistance with a given shunt and its instrument,

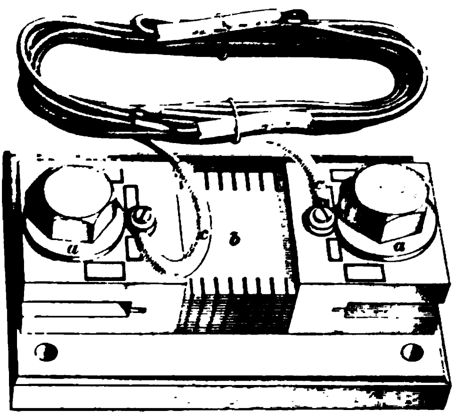

FIG. 7

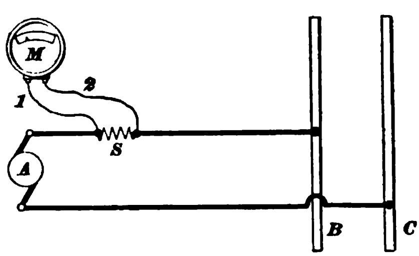

FIG. 8

Fig. 7 shows the ordinary type of ammeter shunt. It consists of the two terminals *a*, *a* connected together by the flat strips *b*, which are made of an alloy that has practically a constant resistance regardless of ordinary temperature changes; *c*, *c* are the small flexible conductors, or cables, that run to the instrument. The shunts and instruments are always numbered to correspond, and care should be taken to see that these numbers agree before connecting the instruments. Many makes of ammeters other than the Weston are used in connection with shunts.

5. The **range** of an ammeter may be increased by connecting a shunt across its terminals. Let R be the resistance of an ammeter, S the resistance of a shunt connected around the ammeter terminals, I the highest reading, that is, the present range of the ammeter, and I', the range desired. To produce the same reading, the current and fall of potential through the ammeter itself must be the same with as without the shunt, the drop through the shunt will be exactly the same as that through the ammeter, and the current I' in the main circuit minus the current I in the ammeter, will be equal to the current $(I' - I)$ in the shunt.

To produce the same reading I with and without a shunt it is necessary that $S\,(I' - I) = R\,I$; hence,

$$S = \frac{R\,I}{I' - I} \qquad (1)$$

Hence, to increase the range of an ammeter, having a resistance of R ohms, from I to I' amperes, a shunt S whose resistance may be calculated by the formula just given must be connected across the ammeter terminals. When shunted, the indicated reading must be multiplied by $\frac{I'}{I}$ to give the total current flowing in the main circuit.

EXAMPLE.—(*a*) What resistance shunt must be used with an ammeter whose resistance is .0022 ohm and whose maximum reading is 15 amperes, in order that currents up to 135 amperes may be measured? (*b*) By what factor, or constant, must the scale readings be multiplied to give the current in the main circuit when the shunt is used?

SOLUTION.—(*a*) By substituting in the formula $S = \frac{R I}{I' - I}$, we get

$$S = \frac{.0022 \times 15}{135 - 15} = .000275 \text{ ohm. Ans.}$$

(*b*) The scale readings must be multiplied by $\frac{135}{15} = 9$. Ans.

VOLTMETERS

6. The **Weston voltmeter,** Fig. 9, is based on the same principles as the Weston ammeter, and in appearance is quite similar to it. Its internal resistance, as in all voltmeters, is exceedingly large; for indicating up to 150 volts it is about 18,000 ohms, while the resistance of a Weston ammeter, measuring currents up to 15 amperes, is only .0022 ohm. It will be seen that, owing to the great resistance, the current passing through a voltmeter is exceedingly small. For example, when indicating 150 volts, the current, by Ohm's law, is only 150 ÷ 18,000 = .0083 ampere. All voltmeters are provided with at least two terminals, or binding posts p, p', Fig. 9. Conductors, called *voltmeter leads*, connect them to the two points between which the difference of potential, or the E. M. F., is to be measured.

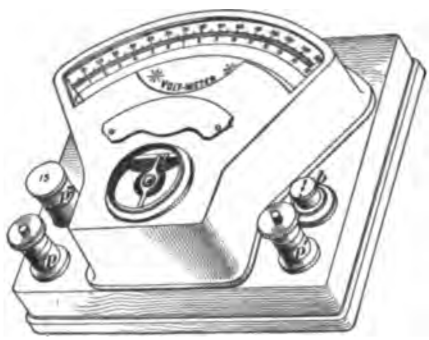

FIG. 9

Weston voltmeters usually have a third binding post p'', which when used with p' give deflections that must be read from a graduated scale situated directly under the upper scale. The upper scale is usually some even multiple of the lower scale. For instance, a voltmeter with a range of 150 volts may have, when using the third binding post, a range of 15 volts, and hence the divisions on the upper scale will have ten times the value of those on the lower scale.

The majority of voltmeters are also provided with a contact button b, which when pressed closes the circuit and allows the index needle to be deflected by the current. When the pressure on the button is relaxed, the circuit is opened and the index needle returns to the zero mark. With some voltmeters the circuit may be kept closed by depressing the button and then giving it a little turn to one side.

In Fig. 10 is shown the working parts of a single-scale, Weston, direct-current voltmeter. The resistance coil a is wound non-inductively, one terminal being connected to the binding post a' and the other to the coil c that is free to turn about a vertical axis. From the other end of this coil the circuit is completed through the key to the binding post d.

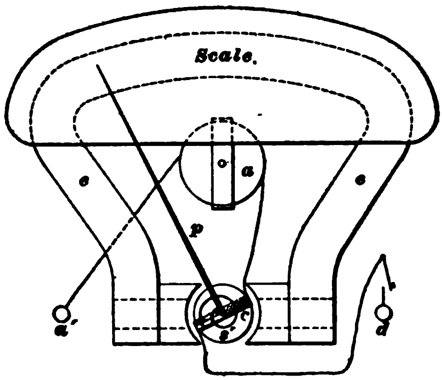

FIG. 10

7. The range of a voltmeter may be increased by connecting a resistance in series with the voltmeter. Let R be the resistance of the voltmeter, R' the resistance connected in series with it, V the highest reading of the voltmeter, and V' the highest reading desired. To produce the same reading in each case the current must be the same; hence, $\frac{V}{R} = \frac{V'}{R'+R}$, $\frac{R'+R}{R} = \frac{V'}{V}$, $\frac{R'}{R} = \frac{V'}{V} - 1$, or

$$R' = R\left(\frac{V' - V}{V}\right) \qquad (2)$$

When the resistance R' is connected in series with the voltmeter, the scale readings must be multiplied by $\frac{V'}{V}$ to give the difference of potential across both the added resistance and the voltmeter, that is, across the circuit whose difference of potential is being measured.

Example.—(*a*) What resistance must be connected in series with a voltmeter whose highest reading is 150 volts and whose resistance is 18,000 ohms in order to use it to measure up to 600 volts? (*b*) By what constant must its scale readings be multiplied to give the potential difference across both the voltmeter and resistance?

Solution.—(*a*) By substituting in the formula $R' = R\left(\frac{V' - V}{V}\right)$, we get

$$R' = 18{,}000\left(\frac{600 - 150}{150}\right) = 54{,}000 \text{ ohms. Ans.}$$

(*b*) The scale readings must be multiplied by $\frac{600}{150} = 4$. Ans.

8. Double, or two-scale, voltmeters are usually connected internally as shown in Fig. 11. The resistance of the non-inductive coil *a b* together with that of the movable coil (that is, from *a* to *p'*) may be 15,000 ohms for the 15-volt scale and the total resistance from *p* to *p'* 150,000 ohms. Since from *p* to *p'* is ten times the resistance from *p''* to *p'*, it follows that ten times the potential difference will be required between *p* and *p'* as between *p''* and *p'* to give the same current through the movable coil and the same deflection of the pointer. When using a double-scale voltmeter, care must be taken not to apply too high a voltage to the terminals of the lower resistance coil that is associated with the lower reading scale. For instance, if 125 volts is connected to the 15-volt terminals, the instrument may be ruined by the excessive current that will flow through it. If in doubt about the value of the potential to be measured, use the higher reading scale first; then, if the reading is less than the highest reading on the lower reading scale, use the other terminal and the lower reading scale.

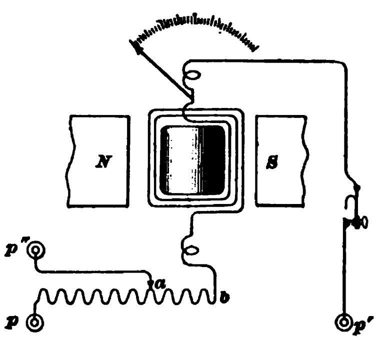

Fig. 11

9. The **Cardew voltmeter** is an instrument that measures a current or difference of potential by the heating effect of the current. In it a long wire, usually made of some platinum alloy, has one end fastened to the dial end of a long tube; the wire then passes over a pulley at the opposite end of the tube and back to the dial end. A spring attached to the middle of the wire keeps it stretched taut. When a current is sent through the wire, the heat caused by its passage expands the wire; the increase in length is taken up by the spring, and the motion of the middle of the wire that is attached to the spring is transmitted to a pointer by suitable multiplying gear, so that the motion of the pointer over the dial is a measure of the amount of expansion of the wire.

The wire is usually of small diameter and considerable specific resistance, so that it has resistance enough to allow the instrument to be used as a voltmeter for potentials less than about 100 volts without external resistance. This voltmeter may be used either for alternating or direct currents, is remarkably dead beat, and simple in construction. Its internal resistance is low for a voltmeter, and, in consequence, it takes considerable current, enough in many instances to seriously affect some conditions of an experiment. This instrument has no particular law of deflections by which the scale is divided; the principal divisions are determined by comparing the instrument with a standard, and the intermediate divisions interpolated.

There are several instruments made on this principle, commonly known as *hot-wire instruments;* the Cardew and Stanley are the most widely known in the United States.

MEASUREMENTS WITH VOLTMETERS AND AMMETERS

10. Many of the measurements described as being made with some form of galvanometer can be made with good commercial instruments. In the Weston instruments, the terminals of the ammeter are both on the same (right) side of the instrument (see Fig. 1), and are made large and heavy, while in the voltmeters the terminals are on opposite sides (see Fig. 9), are made small, and are usually covered with rubber, in order that they may be handled without danger from shocks.

Measurements of current strength or difference of potential are very simple. To measure the number of amperes flowing in a circuit, it is only necessary to connect an ammeter of proper capacity in series with the rest of the circuit and read the amperes directly from the position of the pointer on the scale. The difference of potential between two points in a circuit, or the E. M. F. of a battery, or other source of E. M. F., may be readily measured by connecting the terminals of a voltmeter of suitable range to the proper points of the circuit and reading the voltage direct.

11. Measurement of Current by Ammeter.—The methods of connecting voltmeters and ammeters for measuring difference of potential and currents in various circuits should be thoroughly understood. Suppose, for example, that it is desired to determine the strength of current flowing from the battery *B* in Fig. 12, through the circuit and the

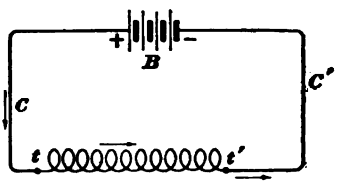

FIG. 12

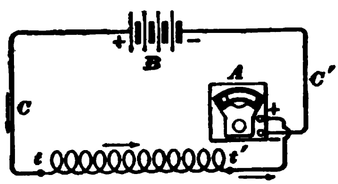

FIG. 13

difference of potential produced between the ends *t*, *t'* of an unknown resistance by means of an ammeter and voltmeter. *C*, *C'* represent conductors of negligible resistance for making the necessary connections. The first step is to determine the strength of the current flowing in the circuit by the use of an ammeter. Assuming that the battery is constant, that is, that the E. M. F. developed in it does not vary, appreciably, then as long as the resistance of the circuit is not altered, the strength of the current will remain unchanged and will be the same in all parts of the circuit. Hence, if an ammeter is inserted in any part of the circuit, as shown in Fig. 13, it will measure the strength of current flowing through the entire circuit. As has been stated, the internal resistance of the ammeter should be so small that its insertion makes no appreciable change in the total resistance of the circuit, and therefore does not to any extent affect the current flowing. For convenience, assume that the strength of the current flowing in the circuit is found to be 1.2 amperes. The next operation is to find the E. M. F. required to drive a current of 1.2 amperes through the resistance t–t'; or, in other words, to find the difference of potential between the terminals t, t' when a current of 1.2 amperes is flowing in the circuit. This is accomplished by connecting the terminals t, t', Fig. 14, of the unknown resistance R, to the two binding posts p, p' of the voltmeter $V M$ by two voltmeter leads l, l'. Any small wires of reasonable length can be used for voltmeter leads, as the current they transmit is exceedingly small, owing to the extremely high resistance of the voltmeter. After pressing the contact button, assume that the needle indicates a potential of 6 volts; this, then, is the E. M. F. required to force a current of 1.2 amperes through the unknown resistance R; or, in other words, the difference of potential between the terminals t and t' is 6 volts.

12. Measurement of Resistance by Voltmeter and Ammeter.—From the readings of the current and voltage, as obtained in the last article and by the application of Ohm's law, the resistance R of the circuit between t and t' can be determined. It has been shown that a current of 1.2 amperes flowing through the resistance R produces a difference of potential at the ends of R of 6 volts; hence, from Ohm's law, $R = \frac{E}{I} = \frac{6}{1.2} = 5$ ohms.

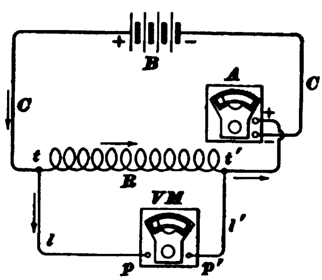

FIG. 14

When the voltmeter is connected as shown in Fig. 14, the ammeter measures the current through the voltmeter in addition to the current through the resistance R. For very accurate results the current through the voltmeter should be calculated and then subtracted from the ammeter reading. The current through the voltmeter is equal to its own reading divided by its own resistance. Hence, if I is the current flowing through the ammeter, E the reading of the voltmeter, and R_v its resistance, the current through the resistance R is equal to $I - \frac{E}{R_v}$. Then $R = \frac{E}{I - \frac{E}{R_v}}$. The current $\frac{E}{R_v}$ is generally so small compared to I that the correction is not necessary in ordinary work.

Instead of connecting the voltmeter from t to t', as shown in Fig. 14, it may be connected as shown in Fig. 15. The voltmeter then measures the fall of potential through the ammeter in addition to the fall of potential through the resistance R. For very accurate results the fall of potential through the ammeter, when the voltmeter is connected so as to include the ammeter, should be subtracted from the voltmeter reading. If E is the voltmeter reading when connected from t to C', I the ammeter reading, and R_a the resistance of the ammeter, IR_a is the fall of potential through the ammeter, hence the fall of potential through the resistance R only would be $E - IR_a$. Then $R = \frac{E - IR_a}{I}$. The drop through the ammeter is generally so small compared to E that this correction is not necessary in ordinary work.

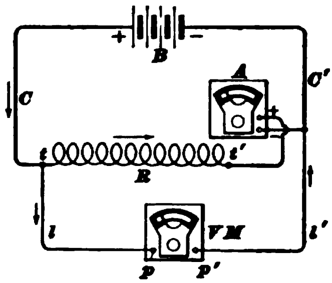

Fig. 15

The voltmeter and ammeter are generally connected as shown in Fig. 14, although more correct results may usually be obtained by connecting the voltmeter so as to include the ammeter with the resistance, as shown in Fig. 15; because the resistance of the ammeter is so small that neglecting the drop through it usually introduces less of an error than neglecting the current through the voltmeter in spite of the high resistance of the latter. In any particular case, when the resistances of the two instruments are known, or can be measured, a little calculation will show which method will introduce the least error. For very accurate results it may be necessary to apply the corrections indicated above, whichever method of connecting is employed. Electrostatic voltmeters consume no current. With such a voltmeter connected as shown in Fig. 14, no correction would be required and the correct resistance of R would be obtained by dividing the voltmeter reading by the ammeter reading. Such a voltmeter should never be connected, as shown in Fig. 15, so as to include the drop through the ammeter with that through the resistance to be measured.

13. By using instruments of the proper range, very low or very high resistances may be measured. Fig. 16 shows a way of measuring a very low resistance—in this case a section of copper rod. Here a current from the battery B, measured by the ammeter A, flows through the section of copper rod R; the drop between the points C and D is measured by the voltmeter V. As the drop through a short section of copper rod will be very slight, except with an enormous current, a voltmeter capable of measuring very small differences of potential must be used. They may be had to measure from 0 to .05 volt, such an instrument being known as a *millivoltmeter*.

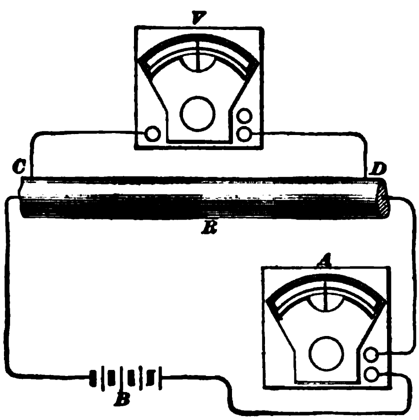

FIG. 16

EXAMPLE.—If, in Fig. 16, the reading of the ammeter be 34.5 amperes, and that of the millivoltmeter be .00875 volt, what is the resistance of the copper rod between C and D?

SOLUTION.— $R = \frac{E}{I} = \frac{.00875}{35.4} = .000247$ ohm. Ans.

This fall-of-potential method may be used to measure the resistance of dynamo or motor armatures. The current that is used in making the measurement, and which may be supplied by a storage battery or from lighting- or power-circuit mains, should preferably be made equal to the current that is generated by the dynamo or supplied to the motor when in regular use. The circuit is connected through a suitable ammeter to two diametrically opposite bars of the commutator, to which the voltmeter is also attached, as shown in Fig. 17. It is advisable not to connect the voltmeter wires to the brushes, but to attach them by the screws, or by mere pressure, to the ends of the armature wires,

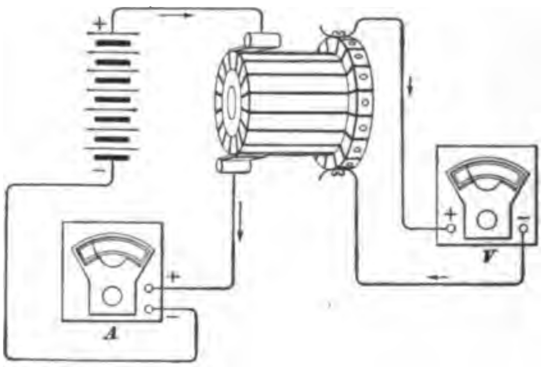

FIG. 17

as shown, in order not to include the contact resistance at the brushes. The resistance of each coil in the armature may also be determined by connecting the voltmeter wires successively with adjacent commutator bars. As to what portion of the whole current passes through any one coil will depend on the way in which the armature has been wound.

The same method may also be used to measure the resistance of station or switchboard ammeters or their shunts. The millivoltmeter is connected to the binding posts of the station ammeter and a standard ammeter is connected in series with the station ammeter. Another application, for which a millivoltmeter is especially well adapted, is the location of bad contacts occurring in switches, cut-outs, safety devices, joints, etc., on switchboard wiring and leads. In such cases the fault is shown by an excess of the deflection of the needle over the amount to be expected under ordinary circumstances.

High resistances may be measured in a similar manner by using a low-reading ammeter (milammeter) and a high-reading voltmeter.

14. Resistance Measurement With Voltmeter and Known Resistance.—A resistance R may be measured by connecting it in series with a battery or dynamo B and with a known resistance R', as shown in Fig. 18. This is practically the fall-of-potential method, using a voltmeter in place of a galvanometer. With the voltmeter first connected across the ends of R', as shown in the figure, let the reading be E' volts; with the voltmeter connected across the ends of the unknown resistance R, let the reading be E volts; then $R : R' = E : E'$, or

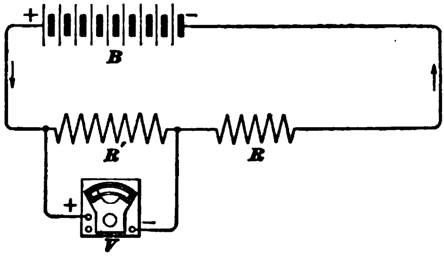

FIG. 18

$$R = \frac{R' E}{E'} \qquad (3)$$

For instance, if $R' = .26$ ohm, $E' = 2.6$ volt, and $E = 3.69$ volts, then $R = \frac{.26 \times 3.69}{2.6} = .369$ ohm.

If R' is very different from R, for instance very small compared with R, it is sometimes more convenient, if one has a double-scale voltmeter, to measure E on the higher reading scale and E' on the lower.

15. High-Resistance Measurement With Voltmeter.—High-reading voltmeters may be used to measure very high resistances, such as insulation resistances. The method of connecting up for such a test is shown in Fig. 19, in which R is the insulation, or high resistance, to be measured; $B\,C$ a battery or other source of E. M. F., which should be as high as convenient, as long as it is within the range of the voltmeter VM, and K, a switch for short-circuiting the resistance R. As the resistance of the switch K is practically nothing, it is evident that when it is closed the voltmeter is connected directly to the terminals of the battery and will measure its E. M. F., and when the switch K is open the resistance R is in series with the voltmeter. The formula for finding the value of R in ohms is

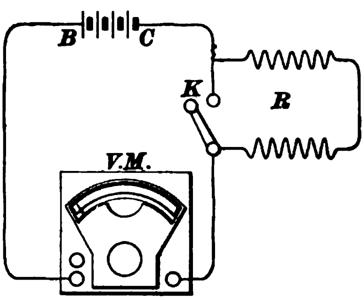

FIG. 19

$$R = r\left(\frac{d}{d_1} - 1\right) \qquad (4)$$

where d = deflection of voltmeter with the resistance R not in circuit;

d_1 = deflection of voltmeter with resistance R in circuit;

r = resistance of voltmeter.

NOTE.—This formula is derived as follows: The difference of potential across the terminals of the battery must remain practically constant during the test. Then the drop through the voltmeter, when the switch K is closed, is equal to the drop through both the voltmeter and resistance R in series, when the switch K is open. Let I be the current through the voltmeter and d the reading when the switch K is closed, and let I_1 be the current and d_1 the reading when the switch K is open. Then $I\,r = I_1 R + I_1 r$. As the deflection of the voltmeter needle is proportional to the current, this may be written $d\,r = d_1 R + d_1 r$; or, $d\,r - d_1 r = d_1 R$; or, $\frac{d\,r}{d_1} - \frac{d_1 r}{d_1} = R$; or, $\frac{d\,r}{d_1} - r = R$; hence, $r\left(\frac{d}{d_1} - 1\right) = R$, which is the formula given.

EXAMPLE.—If the E. M. F. of the battery, as measured by the voltmeter, is 100 volts, and the deflection, when the resistance to be measured is in circuit, is 40 volts, what is the value of that resistance in ohms if the resistance of the voltmeter is 18,000 ohms?

SOLUTION.—In this case $d = 100$, $d_1 = 40$, $r = 18,000$. Then, by the formula $R = r\left(\frac{d}{d_1} - 1\right)$,

$$R = 18,000\left(\tfrac{100}{40} - 1\right) = 18,000 \times 1.5 = 27,000 \text{ ohms. Ans.}$$

16. Insulation-Resistance Measurement With a Voltmeter.—One of the most convenient methods for measuring the insulation resistance of telephone, telegraph, electric-light, power circuits, and dynamos and motors is the voltmeter method, for the measurement may be made while the system is in operation, no complete or partial shutting down of the system being necessary. The only instrument required is a reliable commèrcial voltmeter. Suppose that we have a lighting circuit, as shown in Fig. 20 (*a*), where

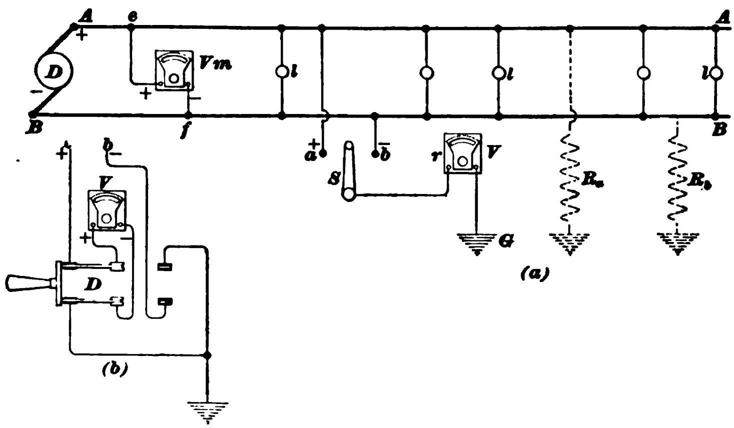

FIG. 20

D is a dynamo, *A A* and *B B* the mains, and *l* the lamps. If the main *A A* or *B B* becomes grounded by accidentally rubbing against wet trees, posts, metal pipes, or grounded telegraph or telephone wires, the insulation between the wire *A A* or *B B* and the ground will be poor; if both sides of the circuit happen to be poorly insulated, waste current will flow from one side to the ground and then to the other side. This current is not only a loss but it may cause fire or other damage along its path to earth, and it should be stopped if appreciable.

Let R_a represent the joint resistance of all leakage paths between the main $A A$ and the ground, and R_b the same between $B B$ and the ground. First connect a suitable voltmeter between the mains, as at $e f$, and let the reading be V_m volts. Then connect the same voltmeter between G and the main $A A$ by closing switch S on contact a, and let this reading be V_a volts. Finally turn S to b, thereby connecting the voltmeter V between G and $B B$, and let the reading be V_b volts. To obtain the last reading, it will be necessary, with most voltmeters, to reverse the connections at the voltmeter in order to make the needle deflect in the right direction. If r is the resistance of the voltmeter and R the joint resistance of all possible paths between both mains and the ground, the insulation resistance of the whole system will be

$$R = r\left[\frac{V_m}{V_a + V_b} - 1\right]^* \qquad (5)$$

If the insulation resistance of one side is extremely high and the other side is poorly insulated or partially grounded,

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*This formula may be derived as follows: The joint resistance of all paths between the two mains and the ground is $R = \frac{R_a \times R_b}{R_a + R_b}$, since the resistance R_a between $A A$ and ground is in parallel with the resistance R_b between $B B$ and ground. When the voltmeter is connected between $A A$ and G, r and R_a are in parallel with each other and in series with R_b, and since the fall of potential along a circuit varies directly as the resistance, then $V_a : V_m = \frac{r \times R_a}{r + R_a} : \frac{r \times R_a}{r + R_a} + R_b$ or $V_a = V_m\left[\frac{\frac{r \times R_a}{r + R_a}}{\frac{r \times R_a}{r + R_a} + R_b}\right]$. Similarly, when the voltmeter is connected between $B B$ and G, we have $V_b = V_m\left[\frac{\frac{r \times R_b}{r + R_b}}{\frac{r \times R_b}{r + R_b} + R_a}\right]$. Adding the two equations and simplifying, we get $V_a + V_b = V_m\left[\frac{r(R_a + R_b)}{r(R_a + R_b) + R_a \times R_b}\right]$. Then, taking the reciprocal of each member gives $\frac{1}{V_a + V_b} = \frac{1}{V_m}\left[1 + \frac{R_a \times R_b}{r(R_a + R_b)}\right] = \frac{1}{V_m}\left(1 + \frac{R}{r}\right)$, since $\frac{R_a \times R_b}{R_a + R_b} = R$. Solving for R, we get $R = r\left[\frac{V_m}{V_a + V_b} - 1\right]$.

the voltmeter reading between the partially grounded side and the ground will be practically zero, because there is no path for any current back to the well-insulated main. For instance, if *A A* is poorly insulated but the insulation resistance of *B B* is very high, the reading V_a obtained with the voltmeter connected between *A A* (the faulty side) and the ground will be practically zero and the above formula reduces to $R = r\left[\frac{V_m}{V_b} - 1\right]$ (which is the same as formula 4). Hence, to measure the insulation resistance of one side of the system only, it will be necessary to take two readings only, one between the two mains and the other between the good side and the ground. If the reading between each side and the ground is zero, the circuit is free from grounds and leaks and the insulation resistance is too high to be measured by the voltmeter used. It is usually best to make the test for both sides. If the line *A A* is dead-grounded at some point, V_b will be found equal to V_m and consequently the insulation resistance R will be practically zero. If the ratio $\frac{V_m}{V_b}$ is very much larger than 1, the formula reduces approximately to $R = r\left(\frac{V_m}{V_b}\right)$, a form that is often used for rough or approximate work.

The resistance r of the voltmeter is generally marked on the voltmeter box, or case. If the voltmeter has two scales and the reading V_a or V_b is less than the largest reading on the lower reading scale, it is evident that a more accurate reading may be obtained by using the lower reading scale. In the formula, r will be the resistance of the coil used in obtaining the reading V_a or V_b. The resistance of the coil used in obtaining the reading V_m will not enter into the result. Care must, of course, be used to avoid connecting the smaller coil across the mains, for if this is done the smaller coil will probably be burned out.

When this test is to be made repeatedly a very convenient arrangement is shown at (*b*). The double-throw switch *D* is so connected as to make the voltmeter deflect in the proper direction whether connected to *a*, by throwing the switch to the left, or to *b* by throwing the switch to the right. All switches and connections should be very much better insulated than the system to be tested and the voltmeter readings should be taken one after the other as quickly as possible, as the formula assumes that all readings are observed simultaneously. Slight variations in the E. M. F. of the source of supply do not affect the result very materially, and when an approximately constant E. M. F. is available from an electric-lighting circuit or other source, insulation tests may be made with great facility by merely connecting the voltmeter in series with the E. M. F. and the insulation resistance. On the assumption that the E. M. F. has a constant (known) value, a table may be prepared showing the insulation resistance corresponding to various deflections of the voltmeter.

17. The insulation resistance of a dynamo may be measured in the same way as that of a line circuit. Suppose the pressure to be 150 volts. One binding post of the voltmeter is connected to the proper terminal of the dynamo (while running) and the other one to earth as shown in Fig. 21. If on closing the key, or switch, *K* momentarily, the needle of a 150 voltmeter is not deflected appreciably from its zero position, it is certain that the insulation of the machine is quite high. The same test should be made on the other terminal of the machine. If there is a deflection, the insulation resistance of the machine may be calculated in the same manner as the insulation of a line circuit. The line circuit should be disconnected from the machine while taking the readings, otherwise the combined insulation resistance of the machine and line will be obtained.

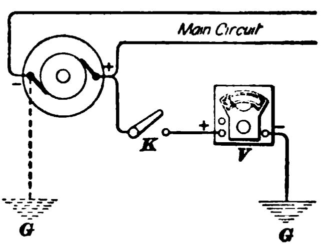


Fig 21

E. M. F. OF CELLS

18. Wheatstone's Method Using a Voltmeter.—The connections for comparing the E. M. F.'s of voltaic cells by Wheatstone's method, using a voltmeter in place of a galvanometer, are shown in Fig. 22. Suppose that the E. M. F.'s of two cells to be compared are E_1 and E_2. First connect E_1 so as to be in series with the voltmeter V and the resistance R, and note the reading d when the key K is closed. Then increase the external resistance, by opening the key K, by r_1 ohms, and note the deflection d'. Repeat these two observations with the cell E_2 connected in place of E_1. First make the resistance R of such a value as to give, with the key K closed, exactly the same deflection d as with the cell E_1, then open the switch K and make the additional resistance r_2 of such a value as to give exactly the same deflection d' as under similar conditions with the cell E_1. Then, $E_1 : E_2 = r_1 : r_2$.

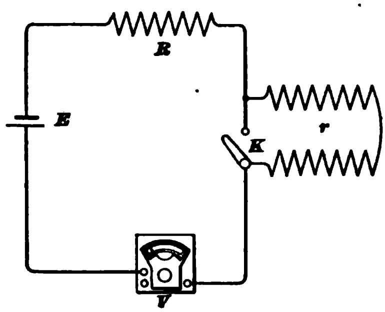


Fig. 22

If E_1 is smaller than E_2, the resistance of the voltmeter itself may be taken for R_1. It is preferable to make r_1 about twice as large as the combined resistance of E_1 and R_1. It is not necessary that the internal resistance of the cells should be small compared with the resistance of the voltmeter. This method is correct to about 1 per cent. with a suitable voltmeter.

Example.—With the switch K closed, let the total resistance in circuit with a cell E_1 be 600 ohms and the reading d 1.45 volts. With K open and a resistance of 1,200 ohms at r, let the reading d' be .48 volt. Then another cell E_2 is inserted in the circuit in place of E_1. With the key K closed, the resistance in circuit with the cell had to be increased to 636 ohms to give a deflection d equal to 1.45 volts; and with the key K open, the resistance at r had to be increased to 1,275 ohms to give the deflection d' equal to .48 volt. What is the relative value of the E. M. F.'s of the two cells?

SOLUTION.—We have $E_1 : E_2 = r_1 : r_2$ or $E_1 : E_2 = 1{,}200 : 1{,}275$. Hence, $E_2 = 1.0625\ E_1$. If E_1 is known, the value of E_2 can be calculated.

19. Volt-and-Ammeter Method.—By this method both the internal resistance and the E. M. F. of the cell may be determined from the same observations, and, moreover, the measurements may be made when the cell or battery is generating current at its normal or desired rate. It, therefore, may be made to give the internal resistance under normal working conditions. The connections for this method are shown in Fig. 23, in which A is an ammeter and V a voltmeter of suitable ranges, R a resistance of such a value that the battery B to be tested will furnish its normal amount of current, and K a switch or key. Let B be the internal resistance of the battery, I the current measured by the ammeter A, E the E. M. F. of the cell measured by the voltmeter V when the key K is open, and V the difference of potential, also measured by the voltmeter V, at the terminals of the cell when the key K is closed and I amperes are flowing through R. The resistance of the voltmeter should be at least a thousand times that of the battery, or, better, several thousand times. The ammeter must be of so small a resistance as to allow the battery to work at its normal rate of output, and it is better that the ammeter resistance should be very low.

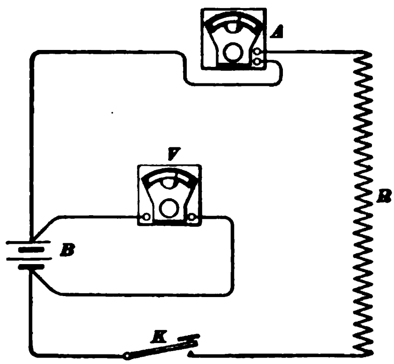


FIG. 23

With the key K open, read the voltmeter. This will be E, the E. M. F. of the battery when practically no current is flowing; that is, when the battery is practically on open circuit. Then close the key K and immediately read simultaneously, or as nearly so as possible, both the ammeter and the voltmeter. These two readings give the current I

and the difference of potential V at the battery terminals when I amperes are flowing through the circuit. Then, $E - V$ is the drop or fall of potential necessary to drive the current I through the battery against the internal resistance B. But this fall of potential, by Ohm's law, is $B \times I$; hence,

$$B = \frac{E - V}{I} \qquad (6)$$

This is the internal resistance at an output of I amperes. At another rate it may be different. If the total resistance R external to the battery is known the ammeter will not be necessary, for the current I is equal to $\frac{V}{R}$ and can, therefore, be calculated.

It may be mentioned here that the internal resistance as well as the difference of potential between the terminals of a cell, when it is closed through an external resistance, are not constant quantities, but depend on the strength of the current passing through the cell. This current, which is equal to $\frac{V}{R}$, may be regulated as required by adjusting R. The key K is generally kept closed, and is opened only as long as is necessary to observe the value of E.

EXAMPLE.—With the switch K open, the voltmeter read 1.5 volts. With the switch K closed, so that the current from the cell B could flow through a resistance R of 4 ohms, no ammeter being used, the voltmeter read 1.35 volts. What are the E. M. F. and the internal resistance of the cell?

SOLUTION.—The E. M F. E of the cell, which is given directly by the voltmeter when the switch K is open, is 1.5 volts. The internal resistance of the cell may be computed by the formula $B = \frac{E - V}{I}$. In this example $I = \frac{V}{R}$. Substituting this for I and then substituting the values of the various quantities in the resulting formula, we get $B = R\left(\frac{E - V}{V}\right) = 4\left(\frac{1.5 - 1.35}{1.35}\right) = .44$ ohm. Ans.

MEASUREMENT OF POWER

20. The power expended in a direct-current circuit may be determined by measuring the current and the difference of potential, the product of these two measurements giving the power expended. Such measurements can be readily made with a voltmeter and an ammeter. One way to connect a voltmeter and an ammeter for determining the power, in watts, expended in a circuit is shown in Fig. 24. The ammeter A and voltmeter V are connected in the circuit so as to measure the power consumed by the group of lamps L. The ammeter A is connected in series in the circuit in such a position as to measure the total current supplied to the lamps L. Notice that it does not measure the current supplied to the motor M, nor to the lamps L_1, nor even to the voltmeter V. The voltmeter is connected so as to measure

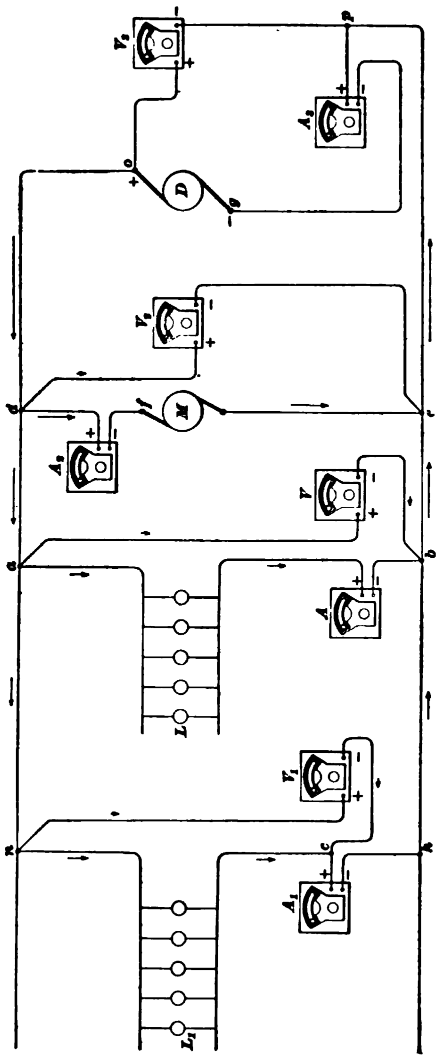


Fig. 24

the difference of potential across the circuit *a–b* to which the group of lamps is connected. The power, in watts, expended in the lamp circuit *L* would be considered as the product of the readings of the voltmeter and ammeter, the two being read as nearly simultaneously as possible. With the voltmeter connected as shown in this figure, it measures the drop of potential through the ammeter in addition to the drop of potential through the lamps, and hence the power determined in this manner is not absolutely correct, being a little too large. If R_a is the resistance of the ammeter, I the ammeter reading, and E the voltmeter reading, the true drop of potential through the lamps $= E - IR_a$ and the watts consumed by the lamps only $= I(E - IR_a)$.

If the voltmeter is connected across *n c*, the ammeter A_1 measures the current through the voltmeter V_1 in addition to the current through the lamps L_1, and consequently the product of the two meter readings will not give the exact power consumed by the lamps, being a little too large. If R_v is the resistance of the voltmeter, E the reading of the voltmeter, and I the reading of the ammeter, the current flowing through the lamps $= I - \frac{E}{R_v}$ and the watts consumed by the lamps only $= E\left(I - \frac{E}{R_v}\right)$.

Although the correct results can be calculated, as explained, by allowing for the drop through the ammeter A_1 or for the current through the voltmeter V_1, as the case may be, it is seldom necessary to do so in practical work and it is customary to consider the product of the voltmeter and ammeter readings as correct enough. More accurate results may usually be obtained with the instruments connected as shown at A and V than as shown at A_1 and V_1. When an electrostatic voltmeter, which requires no current whatever, is used, the voltmeter should be connected across *n c*, for the product of the two readings then gives the correct number of watts consumed by the lamps only.

21. If the power consumed by the motor *M* only is to be determined, the ammeter A_2 should be connected in series with the motor only, for instance, somewhere between *df*, and the voltmeter V_2 across *de*. If the total power expended in all the lamps and the motor is to be determined, the ammeter A_3 should be connected in series with the dynamo *D*, so as to measure the current for both the motor and all the lamps; for instance, it could be inserted in the main circuit somewhere between the points *e* and *g*, and the voltmeter V_3 from *o* to *p*, so as to measure the total drop through the entire external circuit. The power expended, in watts, will be considered as the product of the ammeter and voltmeter readings. The power developed by the dynamo is equal to the power expended in the motor and lamp circuits and in the main leads *n o* and *h p*.

Voltmeters and ammeters cannot be used in this manner to determine directly the power consumed in alternating-current circuits.

The resistance of the lamp circuit *L* may be calculated from the readings of the voltmeter *V* and ammeter *A*, but the resistance of the motor circuit, when the motor is running, cannot be calculated in a similar manner because all the power consumed by the motor is not used in overcoming simple electrical resistance. Moreover, it would not be safe to hold the motor still and apply the potential used in running it unless considerable extra resistance is connected in series with the motor armature. A direct-current-motor armature usually has a very low resistance, much less than 1 ohm even, and the potential ordinarily used to run it will soon burn out the armature if it cannot revolve at nearly its normal speed. It follows that the total resistance of this circuit, or any circuit to which running motors are connected, cannot be calculated from the readings of a voltmeter and ammeter when the motor is running.

SIEMENS DYNAMOMETER

22. An instrument that may be used for measuring currents, E. M. F.'s, and power on both direct- and alternating-current circuits is the **Siemens dynamometer.** This instrument is constructed on the principle that a coil of wire carrying a current will tend to rotate, if suspended in a magnetic field in which the direction of the lines of force are not parallel with the plane of the coil. The working parts of this instrument, one form of which is shown in Fig. 25, consists of two rectangular coils of wire, one of which *A* is fixed and the other *B* movable. The normal position of the movable coil is with its plane at right angles to the plane of the fixed coil. It is suspended in this position by jewel bearings or sometimes by a fiber. Connection is made to the coil *B*, which is free to swing a limited amount, by means of two mercury cups *c*, *c'* into which the ends of the coil dip. To the top of this coil is attached a light helical spring *d*, the other end of which is attached to a milled nut *e*, called the *torsion head.* A pointer attached to this head moves over the circular scale *f*. A second pointer *g* attached to the swinging coil is opposite a zero mark on the scale when this coil is at right angles to the

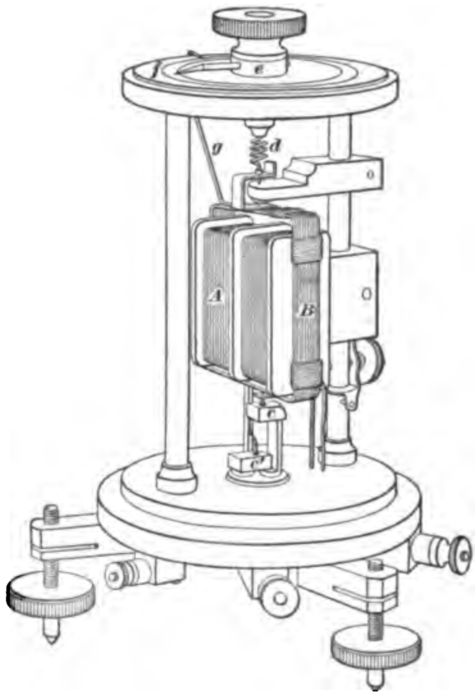


Fig. 25

fixed coil. When no current flows through the instrument, both pointers stand at the zero point of the scale; but, on the passage of a current through both coils, the swinging coil is slightly deflected, its motion being limited by stops not shown in the figure. The fixed coil is usually wound in two parts of unequal number of turns and size of wire; either coil may be used, thus varying the range of the instrument.

USE OF DYNAMOMETER

23. Measuring a Current.—When a current flows through the two coils, the mutual action of the coils tends to move them into parallel planes. The effect is to rotate the movable coil about its vertical axis; but by turning the milled nut, a tension may be put on the spring that will return the coil to its original position. The force exerted by the spring on the coil is proportional to the angle through which the milled head attached to the spring is turned; so, the pointer attached to the milled head *e* indicates the force required to pull the coil back to its central position. This force is proportional to the product of the two currents in the two coils. If the coils are connected in series so that the same current flows through both coils, the rotating force is proportional to the square of the current. That is, if the two coils are connected in series, doubling the current in one coil doubles it in the other, so the mutual force of both coils is doubled, and the force acting on the movable coil is quadrupled; hence, the torsion of the spring necessary to bring the pointer on the swinging coil back to zero, is proportional to the square of the current. That is, $I^2 : I_1^2 = d : d_1$; in which I is the current producing the deflection d, and I_1 the current producing the deflection d_1.

The Siemens dynamometer is usually calibrated to read amperes by connecting both the fixed and movable coils in series with a standard direct-current ammeter. The current, in amperes, is measured by the ammeter and the torsion required to keep the movable coils in its zero position is measured by the scale on the dynamometer, the current being reversed and the mean of two readings being used. The current for any one reading may, of course, be determined by any reliable method; for instance, by the copper- or silver-voltameter method, then any other reading of the dynamometer will vary as the square of the current producing it. Since $I_1^2 : I^2 = d_1 : d$, then $I = \frac{I_1}{\sqrt{d_1}} \sqrt{d}$, or

$$I = k\sqrt{d} \qquad (7)$$

in which k may be called the constant of the instrument and is experimentally determined. In this case $k = \frac{I_1}{\sqrt{d_1}}$; that is, k is equal to a known current divided by the square root of the reading that it produces. Hence, if a known current I_1 produces a reading d_1 and an unknown current I produces a reading d, the value of the unknown current I can be readily calculated.

24. Measuring the Difference of Potential.—When the dynamometer is used to measure difference of potential the fixed and movable coils and a sufficiently high non-inductive resistance are connected in series across the two points in a circuit between which the difference of potential is to be measured. Then the deflections are proportional to the square of the currents as before, but the currents, since the resistance remains constant, are proportional to the potential differences; hence, the deflections are proportional to the squares of the potential differences. The total resistance of the dynamometer circuit must be high so as not to appreciably alter the total amount of current, or its distribution, when the dynamometer is connected to the circuit. Its function is then exactly the same as an ordinary voltmeter.

25. Measuring Power.—If the swinging-coil circuit has a constant and high resistance and is connected between two points having a difference of potential, the current through the swinging coil will be proportional to the difference of potential between those two points; and if the fixed coil is connected in series with a circuit joining the same two points, the whole current in the main circuit will pass through the fixed coil. Consequently, the torsion will be proportional to the product of potential difference and current; that is, to the power being expended in the circuit in series with which the current, or fixed coil is connected, and across which the potential, or movable coil is connected. The use and connections of a Siemens dynamometer when used to measure power, that is, as a wattmeter, will be more fully considered a little further on.

26. Siemens dynamometers are seldom made direct reading, but are furnished with a table, or curve, that gives the current strength in amperes, the difference of potential in volts, or the power in watts corresponding to the various deflections. Intermediate values may be interpolated or calculated.

If the earth's horizontal field is in the plane of the movable coil, it tends to rotate that coil. Consequently, it will produce an error in the resulting measurements, unless it is so small compared to the field set up by the fixed coil, as not to appreciably affect the torsion produced by the mutual action of the two coils. If the earth's field is at an angle with the plane of the movable coil, it may be resolved into two components, one of which is in the plane of the fixed and the other in the plane of the movable coil. The component in the plane of the movable coil may still be strong enough to produce an error. Hence, a dynamometer should be set up so that the plane of the movable coil is approximately at right angles to the earth's horizontal field. However, no error will be produced, even if the earth's horizontal field is in the plane of the movable coil, provided the mean of two readings be taken, for one of which the current through both coils is reversed in direction. In such cases the earth's field increases the torsion for one reading as much as it decreases it for the other.

The Siemens dynamometer is even more suitable for measuring alternating currents, E. M. F.'s, and power, for then the earth's field has no effect on it. When alternating current from the same source flows through both the fixed and movable coils, the field set up by the fixed coil changes in unison with the field set up by the movable coil, thus always tending to produce a deflection in one direction.

The Siemens dynamometer is a slow-reading instrument; that is, it requires time to adjust the torsion head in order to obtain each reading, and this is an objection to its use. It is not a direct-reading instrument and its construction is not such as to make it readily portable. For these reasons it is not used to any great extent in practical measurements, but it is used largely as a standard laboratory instrument for checking commercial instruments on account of its reliability and accuracy.

Instruments, depending on the Siemens dynamometer principle, are now made in portable commercial forms. The long spiral spring is replaced by flat spiral springs, like those used in Weston ammeters and voltmeters, and the scales may be calibrated to read amperes, volts, or watts, directly. Some makes require the application of torsion by means of a milled screw head to bring the swinging coil back to its normal position, while other makes do not even require this adjustment. Sometimes two fixed coils are wound with a different number of turns and different scales for use with the two coils.

The maximum voltage and current that can be used with each coil of such instruments are stated in the directions sent with them. So-called multipliers, that is, non-inductive resistances, are also made for use with such instruments. The multipliers increase the capacity of the instrument usually in volts only, the maximum-current capacity remaining unchanged.

WATTMETERS

27. The power expended in a direct-current circuit, being the product of the current and E. M. F., may be determined by measuring the factors separately and multiplying them together. Instruments have been designed, however, that automatically perform this multiplication, thus measuring directly the power expended in watts; such an instrument is called a **wattmeter.**

The Siemens dynamometer may be used as a wattmeter in both direct- and alternating-current circuits. When thus used it measures the product of the current in one coil and the difference of potential between the ends of the circuit containing the other coil. The stationary coil is connected in series with the main circuit, consequently the total current flows through it. The swinging coil is usually connected in series with a resistance great enough to prevent the full difference of potential between the mains sending more than a small amount of current through the movable coil. This coil and the resistance are then connected in parallel with the rest of the circuit, as shown in Fig. 26, where F is the fixed coil of the instrument; M, the movable coil; R, the resistance that is connected in series with M; D, the source of electricity; and C, the external circuit, the energy expended in which it is desired to measure.

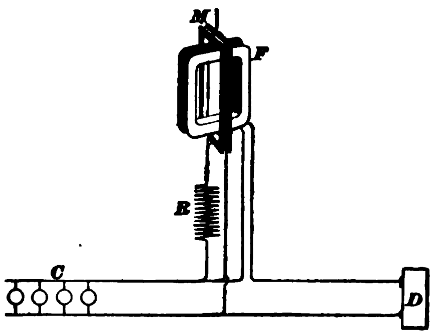


FIG. 26

Since the resistance of the swinging-coil circuit is constant and comparatively large, the current flowing through it will at all instants be proportional to the difference of potential acting on its terminals; and the current in the fixed coil will always be equal to the current in the circuit; hence, the torque action between the two coils will at all instants be proportional to the product of current and difference of potential. Consequently, the force acting on the movable coil and the torsion of a spring required to bring it to zero position, varies directly as the power or watts expended in the circuit. When variations occur in both current and difference of potential simultaneously, the force required to bring the coil to the zero position is still proportional to the watts expended.

The resistance R, Fig. 26, used with the movable coil, is usually made a separate piece of apparatus. When this resistance is made non-inductive, as it generally is, so that the total inductance of the swinging-coil circuit is so small as to have no appreciable effect on a variable current, the wattmeter may be used for measuring the energy expended in alternating-current as well as in direct-current circuits. A wattmeter for use on both direct- and alternating-current circuits must average up all the instantaneous values of the product of current and difference of potential; consequently, its indications must be affected by both. When the Siemens dynamometer is used in an alternating-current circuit the field set up by the fixed coil changes in unison with the field set up by the swinging coil, and hence the torque action between the two coils is in the same direction for each direction of the current.

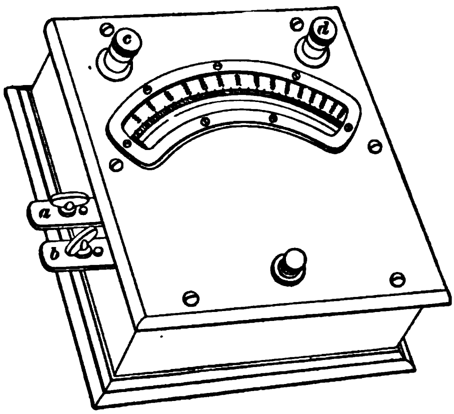


FIG. 27

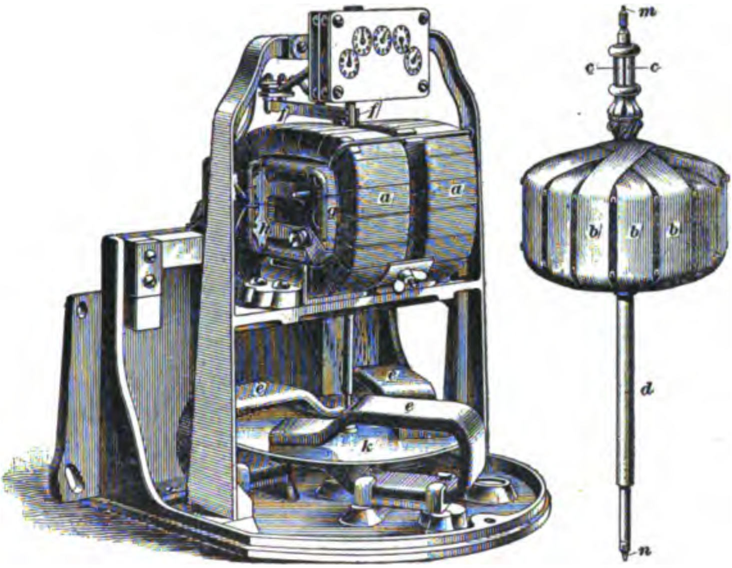
28. Portable Direct-Reading Wattmeters.—In addition to the Siemens wattmeter, there have been introduced several forms of **portable direct-reading wattmeters,** which are giving good satisfaction.

The Siemens wattmeter is not direct reading, and is, therefore, not as convenient for commercial work as the portable direct-reading types. Fig. 27 shows a Weston portable wattmeter, which is constructed about the same as the Weston voltmeter, except that fixed coils, composed of a few turns of heavy copper conductor, which carry the main currrent replace the permanent magnets. The heavy binding posts a, b at the side of the case are the terminals of the current coil and the small binding posts *c*, *d* on the top connect with the swinging coil. In using wattmeters, care should be taken not to get the connections mixed, because if the current coil should, by mistake, be connected across the circuit, the instrument will, in all probability, be burnt out, as the resistance of this coil is very low and the resulting current will be enormous. In order that the readings of a wattmeter for use on alternating-current circuits may be reliable, the self-induction of the swinging-coil circuit should be very small.

29. Weston Compensated Wattmeter.—In measuring power with a voltmeter and ammeter the product of the two readings includes not only the power consumed in the lamps or other device, but also the power consumed in one of the instruments. The same error occurs in the results obtained by the use of an ordinary, or uncompensated, wattmeter or Siemens dynamometer. In such instruments the error is usually very small, however, on account of the very high resistance of the potential coil. This error is eliminated in the Weston compensated wattmeter by winding the wire leading to the potential coil alongside of each turn in the current coil, the current circulating in the two in opposite directions. The result is the same as if the current in the potential coil were subtracted from the current in the current coil as far as the magnetic action of the current coil is concerned.

RECORDING WATTMETERS

30. The Siemens wattmeter gives the instantaneous value of the watts expended in the circuit. Instruments that show the value of the watts at any instant are frequently called **indicating wattmeters** to distinguish them from **recording wattmeters**, which measure the total work done during a given time. A recording wattmeter indicates the product of the watts and time; i. e., the watt-hours. Strictly speaking, these recording instruments are not wattmeters at all, because they do not record watts. The watts represent the rate at which work is done, whereas, a so-called recording wattmeter records the watt-hours or total work done during a given time; they are, therefore, joule meters or watt-hour meters. Large numbers of recording wattmeters are used for measuring the electrical energy supplied to customers on electric-light and power circuits. They measure the total energy supplied during a given period, say a month, and the number of watt-hours, or kilowatt-hours, is read off a dial similar to that on a gas meter.



(a) (b)

FIG. 28

The Thomson recording wattmeter, shown in Fig. 28 (*a*), with the cover removed, is one of the most prominent examples of this class of measuring instruments. Its construction is simple, the principle being, broadly, that of the Siemens dynamometer. The movable coil neither surrounds the fixed coil nor is it held to zero position, but it revolves between two fixed coils *a*, *a*. The movable coil is shown in Fig. 28 (*b*); it consists of a number of coils *b*, *b*, *b* wound across a suitable support. The ends of these coils are connected to a number of silver strips *c c* and the whole moving element is mounted on a shaft *d* with a hardened-steel pivot at the lower end and a worm-gear *m* at the upper. This revolving armature is essentially the same in its construction as those used in dynamos and motors, with the important difference that no iron is used in its construction. Current is carried into the revolving part by two small spring contacts, or brushes, *f*, *f*, Fig. 28 (*a*), that make contact with the silver strips in such a manner that the effective plane of the movable coil, or armature, always takes a position at right angles to the plane of the fixed coils and a continuous rotation is thus maintained instead of a simple deflection, as in the indicating wattmeter.

The connections of this instrument are made on the same principle as those of the Siemens wattmeter. The fixed coils are connected in series with the main circuit while the movable coil in series with a resistance and a so-called shunt coil *g*, thereby forming a circuit of high resistance which is connected across the main circuit. This shunt coil consists of a number of turns of fine wire and is mounted on an adjustable brass frame *h* so that the coil can be moved in or out; that is, to or from the armature. The object of this coil is to compensate for friction on light loads; it provides a magnetic field almost sufficient to move the armature when there is no current in the series-coils; hence, when a small load is placed on the meter, it starts up.

The amount of energy expended in the circuit to which the meter is connected is measured by the rotation of the movable coil. The worm on the upper end of the shaft engages with a set of gears that operate a dial similar to a gas-meter dial so that the energy expended in a certain time can be read directly from the dial, in watt-hours, by noting the readings at the beginning and end of the interval of time and taking the difference between the two. In the earlier types of Thomson meter, the reading as taken from the dials had to be multiplied by a constant in order to obtain the true reading; this constant was marked on the dial. The later types are direct reading except in the large sizes, where the dial reading has to be multiplied by 10 or 100 in order to give the true reading.

The friction of the apparatus is exceedingly small and it is necessary to provide a retarding force that will oppose the motion of the movable coil and make its speed proportional to the load on the circuit. An aluminum disk *k*, Fig. 28 (*a*), is mounted on the lower part of the shaft and revolves between the poles of permanent magnets *e*, *e*, *e*; in the earlier types of meter this disk was made of copper. As the disk revolves it cuts across the lines of force passing between the poles of the magnets and the E. M. F.'s set up thereby cause currents to eddy around in the disk. These tend to retard the motion of the disk and the higher the speed the greater the retardation. As in the Siemens wattmeter, the force acting to rotate the movable coil increases directly as the watts; therefore, the number of revolutions of the moving system of the meter will be directly proportional to the watts expended in the circuit. The Thomson meter will operate on either direct or alternating current and will give accurate results if the commutator, pivot, and jewel are kept in good condition.

The Thomson meter is really a small electric motor so constructed that its speed will be accurately proportional to the watts. There are a number of other meters of the motor type in common use, some of which are adapted for alternating current only.

EDISON CHEMICAL METER

31. Most of the measuring instruments in commercial use depend on the electromagnetic effect of a current for their action; perhaps the only electrochemical current meter that has been in commercial use is the **Edison chemical meter.** At one time it was extensively used by the Edison illuminating companies, but it has been superseded by recording wattmeters. In the Edison chemical meter a fixed proportion of the current passing through the meter is shunted through an electrolytic bath consisting of two zinc plates dipping in a solution of sulphate of zinc. The plates, solution, and connectors are mounted in little glass jars, and two jars are set up in each meter, one to act as a check on the other. At the end of a certain fixed time (usually 30 days) the jars and their contents are replaced by others, and the ampere-hours of current that have been used by the customer calculated from the gain in weight of the negative plate. By various ingenious devices in the several parts of the meter, the effects of various sources of error are almost entirely eliminated. It is necessary, however, to use great care in removing the jars, and in cleaning and weighing the zinc plates.

SPEED INDICATORS

32. For many electrical measurements, it is necessary to know the rate of revolution of certain moving parts of machinery. The number of revolutions made by the machinery in 1 minute or other length of time does not necessarily give its rate of revolution at any one instant,

FIG. 29

so that for accurate work the ordinary revolution counter is scarcely suitable. Instruments, called **tachometers**, are made that indicate by the position of a needle on a dial the rate of revolution at each instant of the apparatus to which they are connected. The principle of these instruments is similar to that of a centrifugal engine governor; two weights are thrown from their center of rotation by centrifugal force, and their tendency to move is overcome by a spring. By suitable gearing, the motion of the weights is made to actuate a pointer that moves over a suitably divided dial, thus indicating at each instant the rate of rotation of the weights.

Fig. 29 shows a form of tachometer that, being belted to a pulley of suitable diameter by a light belt, will give the rate of revolution of that pulley.

The form shown in Fig. 30 is to be held in the hands. A three-sided point on one of the spindles of the instrument is intended to be thrust into the center mark of a revolving shaft, when the rate of revolution of that shaft is indicated on the dial. It is usual to make three little ridges in the sloping sides of the center mark of the shaft with a three-sided punch (supplied with the instrument), to insure that the point on the tachometer shaft will not slip when the instrument is applied.

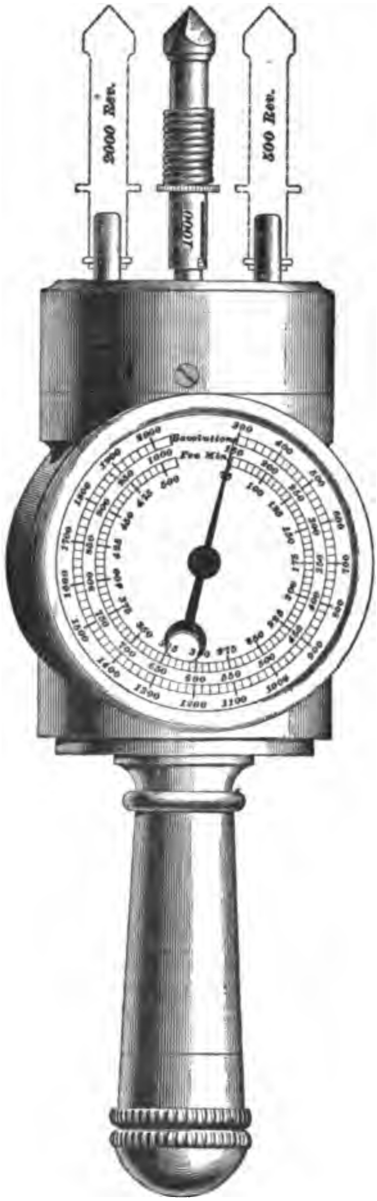


FIG. 30

DETERMINATION OF MAGNETIC PROPERTIES OF IRON

BALLISTIC-GALVANOMETER METHOD

33. A method for determining the magnetic properties of iron is shown in Fig. 31, where the apparatus and connections are indicated. A sample of the iron to be tested, preferably in the form of a ring, is wound throughout its length with insulated wire, so that if a steady current is sent through the wire the ring will be magnetized. If a second coil of wire be wound for a short distance over the first coil,

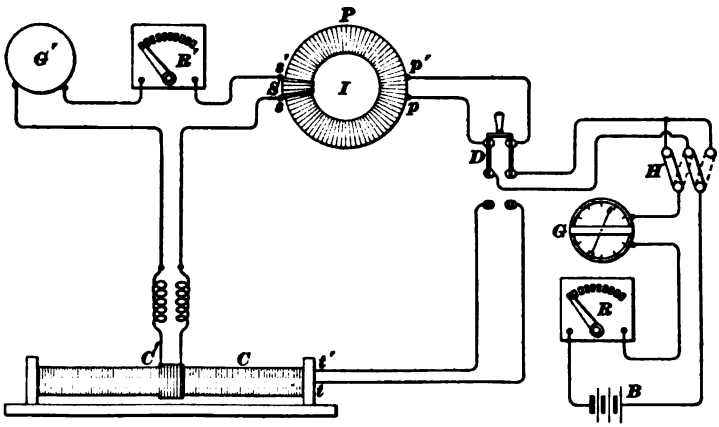


FIG. 31

any change in the number of lines of force in the ring will induce an E. M. F. in the second coil. A ring so wound is represented in Fig. 32, where R is the iron ring, PP_1 the primary or magnetizing coil, SS the secondary coil.

If known currents are sent through the primary coil PP_1, the magnetizing force **H** may be calculated. Any change in the magnetizing current will produce a change in the number of lines of force in the iron ring, which will be indicated by a swing of the galvanometer needle, and the amount of this swing will indicate the relative amount of change in the number of lines of force passing through the secondary coil.

In Fig. 31, *I* is a flat ring of the iron to be tested. Upon it is evenly wound a known number of turns of insulated wire, the terminals being *p*, *p'*; this is called the *primary* or magnetizing coil. A known number of turns of insulated wire are wound on a wooden or other non-magnetic core, with the terminals at *t* and *t'*; the coil *C* is called the *calibrating* coil. On the iron ring *I* and also at the center of the length on the coil *C* are wound secondary coils, consisting of a known number of turns of insulated wire. *H* is a reversing switch.

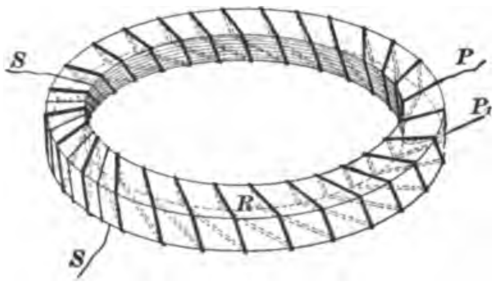


FIG. 32

As the dimensions of the coil *C*, the number of turns, and the current passing through the coil are measurable, the exact number of lines of force per square inch, designated by **H**, in air, wood, or other non-magnetic medium, may readily be calculated as follows: The setting up of a certain number of lines of force in the core of *C* will cause the needle of the ballistic galvanometer *G'* to give a certain kick. The strength of the current sent through the coil *C* from the battery *B*, the switch *D* being closed in the lower position instead of as shown in this figure, is adjusted by means of the rheostat *R* and the resistance of the galvanometer circuit is adjusted by means of *R'* until the sudden opening or closing of the switch *D* in the lower position produces a convenient deflection, or kick, of the galvanometer *G'*. The resistance *R'*, in fact the entire resistance of the galvanometer circuit, should preferably not be altered afterwards unless the galvanometer is recalibrated. Thus, the galvanometer G' may be calibrated by observing the kick d produced when the coil C' cuts a certain number of lines of force that can be calculated. Then, if the kick d' of the galvanometer G' produced by cutting an unknown number of lines of force by the coil S is observed, the total number of lines of force and the flux density in the iron specimen can be calculated.

The switch D being closed in the upper position, a succession of currents of different values, measured by an ammeter, or galvanometer, G, may be sent through the primary coil P of the iron ring I, thus producing therein a certain number of lines of force, which number is proportional to the observed kick of the needle of the ballistic galvanometer. These results may be tabulated, or else plotted so as to give a magnetization curve.

34. Principles Involved.—Let us suppose that the coil C has T turns, a length of l inches, a mean sectional area of A square inches, and that making a current of I amperes through the coil C will produce a deflection d of the galvanometer G' when the total resistance of the galvanometer circuit is R ohms. Then the total number of lines of force set up inside the coil C' is given by the formula $A\,\mathbf{H} = \frac{3.192\,I\,T\,A}{l}$. If there are T_1 turns in the secondary coil C', the total cutting of lines of force by this coil is $\frac{3.192\,I\,T\,A\,T_1}{l}$ and the quantity of electricity produced is $\frac{3.192\,I\,T\,T_1\,A}{l\,R}$. Suppose that this produces a deflection d. Let A_1 be the sectional area, in square inches, of the iron ring, or other specimen to be tested, and T_2 the number of turns in the secondary coil S. If a flux density $\mathbf{B}$ is set up in the iron by sending a current I_1 through the primary coil P, then the total flux inside the secondary coil S will be $A_1\,\mathbf{B}$. Supposing that the cutting of these lines of force causes the galvanometer G' to give a deflection d; the quantity of electricity Q' produced in the galvanometer circuit by the induced E. M. F. set up in the secondary coil S is such that $\frac{A_s \mathbf{B} T_s}{R_s} = Q'$, R_s, as before, being the total resistance of the galvanometer circuit. This produces a deflection d'. Considering the galvanometer deflections as proportional to the quantities of electricity that produce the deflections we may write the proportion, $\frac{3.192\, I T T_s A}{l R} : d = \frac{A_s \mathbf{B} T_s}{R_s} : d'$. Solving this for $\mathbf{B}$, the desired and unknown quantity, we get

$$\mathbf{B} = \frac{3.192\, I T T_s A R_s d'}{l T_s A_s R d} \qquad (8)$$

This is a general formula that will apply to almost any case. Generally, for any one test at least, the value of $\frac{3.192\, T T_s A}{l T_s A_s}$ will remain constant and may be called the constant of the apparatus. Then this value may be calculated once for the whole set of observations. For a series of values it is better to determine a new calibrating deflection d every time the resistance of the galvanometer circuit is altered, because it is easier and much more accurate to do this than to measure R and R_s. Hence, for any such series of deflections for which $R = R_s$, these quantities cancel each other and we get $\mathbf{B} = \left(\frac{3.192\, I T T_s A}{l T_s A_s d}\right) d'$. The value of the parenthesis, which may be called the *calibration* constant, need be computed only once for each set of observations during which the resistance of the galvanometer circuit remains constant and d was the deflection obtained when the circuit was closed so that I amperes flowed through the calibrating coil. $\mathbf{B}$ is calculated by simply multiplying each subsequent deflection d' by this constant quantity.

If a current I_4 through the primary coil P produced the flux density $\mathbf{B}$, then the corresponding magnetizing force $\mathbf{H} = \frac{3.192\, I_4 T_4}{l_4}$, in which T_4 is the number of turns in the primary coil P and l_4 is the mean length of the magnetic circuit in the iron ring or specimen. This is the value of **H** that produces the flux density **B**.

As the diagram now stands, the calibrating coil is out of the circuit, and the primary coil *P* of the iron ring is being energized by current from the battery *B*. The energizing current is regulated by the adjustable resistance *R*, and may be calculated from the E. M. F. of the battery and the resistance of the primary circuit, but is in practice always measured by an ammeter *G*. The reversing switch *H* is used to start, stop, or reverse the current in either the primary coil *P* or the calibrating coil *C*, depending on the position of the switch *D*. The adjustable resistance *R'* is used to adjust the range of the ballistic galvanometer *G'*, as the currents induced in *S* and *C* may vary widely.

35. The test of the iron may be made in a variety of ways. The two most used are the step-by-step and the reversal methods.

The *step-by-step* method consists of suddenly increasing or decreasing the magnetizing current in the primary coil *P* by quickly moving the handle of the rheostat *R* a step at a time. The swing of the galvanometer *G'* at each step indicates the change in the number of lines of force corresponding to a change in the magnetizing force. The total number of lines at any point may be determined by adding together the previous changes.

The *reversal* method is to reverse the current in the primary by throwing the switch *H*. The lines of force will then change from a certain number in one direction down to zero, and then to about the same number in the opposite direction. This change will cause a swing of the galvanometer *G'*, and one-half this swing is taken to represent the number of lines of force in the circuit due to the magnetizing force that has been reversed. By increasing or decreasing this magnetizing force by successive steps, and reversing each time, the *curve of magnetization* may be obtained.

One objection to the step-by-step method is that an error in one of the early observations will be included in the whole series, as they are all added together; but with care in taking the readings, however, this need not occur. With the method of reversals, however, the residual magnetism, more especially in hard iron and steel, introduces an error; as, when the magnetizing force is reversed, there will not be quite as many lines of force in the circuit after the reversal as before, with the same magnetizing force.

Although a ring is the preferable form, almost any shape of specimen forming a closed magnetic circuit may be used and even a straight piece, if its length is 200 or more times its diameter, will give satisfactory results.

EXAMPLE.—From the following data and information, work out the magnetizing force **H** in the primary coil of the iron ring, and the resulting density of lines of force **B**, using the step-by-step method; from these results plot a magnetization curve on cross-section paper, showing the magnetic qualities of the iron. The switch D was first closed in the lower position, so that the current from the battery B passed through the calibrating coil C, the primary coil P being out of circuit. The elements of the present primary circuit have resistances as follows:

Resistance of primary calibrating coil C = 3 ohms.

Internal resistance of the battery B = 1.2 ohms.

Resistance of rheostat R, ten steps of 4 ohms each = 40 ohms.

Resistance of balance of primary circuit, including connections = 1.1 ohms.

The elements of the secondary circuit have the following resistances:

Resistance of rheostat R', ten steps of 200 ohms each = 2,000 ohms.

Resistance of ballistic galvanometer G' = 500 ohms.

Resistance of balance of secondary circuit, including both secondary coils = 10 ohms.

The battery B has 6 cells, each furnishing a constant E. M. F. of 1.9 volts.

The secondary coil S consists of 120 turns of No. 22 insulated wire.

The calibrating coil C is wound on a wooden rod 30 inches long and 2 inches in diameter, and consists of 1,200 turns of No. 18 insulated wire, wound evenly in two layers.

The secondary coil C', wound at the center of the length of the primary calibrating coil C, has 260 turns of No. 22 wire.

The ballistic galvanometer G' is of the type already described, having a scale about 4 feet long, and reads from zero at the center to 225 at each end. In the first part of the measurement, when the galvanometer G' was being calibrated, all the resistance in the rheostat R' was cut out of the circuit. Then, with all the resistance in the rheostat R cut out of the primary circuit, the galvanometer G' gave an extreme swing of 48 scale divisions when the primary circuit containing coil C was suddenly closed or opened by means of the switch D. Then the circuit connections were arranged by closing D in the upper position exactly as shown in Fig. 31, the primary calibrating coil C having been thus replaced in the primary circuit by the primary coil P of the flat-iron ring I, and all the resistance, 2,000 ohms, in the rheostat R' was cut into the circuit and the switch H was open; that is, in a midway position. The dimensions of the iron ring are: 5 inches inside diameter, 6¼ inches outside diameter, and 1 inch thick. The primary coil P is wound evenly over the entire ring I, and consists of 800 turns of No. 18 insulated wire, having a resistance of .8 ohm.

The manner of performing the experiment is to turn in the whole resistance of the rheostat R, 40 ohms, and then suddenly close the switch H and note the first swing of the galvanometer. After the spot of light has settled down to zero, the hand of the rheostat R is suddenly moved to the second contact. This cuts out 4 ohms resistance, which allows an additional amount of current to flow through the circuit. The addition of this quantity of current sets up additional lines of force, and the additional lines of force set up a current through the ballistic galvanometer G' and the first swing must be carefully noted. In this manner the rheostat is moved around the successive steps, and the readings noted as follows:

Resistances of R in Ohms	Deflection of Galvanometer G' in Divisions	Resistances of R in Ohms	Deflection of Galvanometer G' in Divisions
40	220.6	16	16.3
36	11.1	12	19.6
32	12.9	8	26.0
28	7.7	4	30.7
24	14.8	0	40.4
20	13.2		

SOLUTION.—The calculations should be made in tabular form, for the sake of clearness. The following calculations will have to be made, and a column may properly be assigned for the result of each calculation:

1. The resistance of the primary circuit.
2. The current in the primary circuit.
3. The magnetizing force **H** of the primary coil.
4. The deflection of the ballistic galvanometer G' in scale divisions.
5. The corresponding change in the number of lines of force per square inch in the iron ring I.
6. The flux density **B** per square inch in the iron.

Column 1 is found by adding the resistances of the elements of the primary circuit through the coil P, the several values of the adjustable rheostat R having been given in the example; for illustration, the first quantity equals $.8 + 1.2 + 1.1 + 40 = 43.1$ ohms. The second quantity $= 43.1 - 4 = 39.1$. The rest are found in the same manner.

1 Resistance of Primary Circuit	2 Current in Primary Circuit	3 H Magnetizing Force in Primary Coil, per Square Inch	4 Deflection of Galvanometer Divisions	5 Change in Number of Lines of Force, per Square Inch	6 B Lines of Force in Iron, per Square Inch
43.1	.2645	37.40	220.6	56,560	56,560
39.1	.2916	41.20	11.1	2,850	59,410
35.1	.3248	45.90	12.9	3,310	62,720
31.1	.3666	51.84	7.7	1,970	64,690
27.1	.4207	59.50	14.8	3,800	68,490
23.1	.4935	69.78	13.2	3,390	71,880
19.1	.5968	84.40	16.3	4,180	76,060
15.1	.7550	106.80	19.6	5,020	81,080
11.1	1.0270	145.20	26.0	6,670	87,750
7.1	1.6060	227.10	30.7	7,870	95,620
3.1	3.6770	520.00	40.4	10,360	105,980

The values in column 2 are found by dividing the total E. M. F. of the cells, 11.4 volts, by the respective resistances given in column 1. The values of the magnetizing force in the primary coil P for column 3 can be calculated from the formula $\mathbf{H} = \frac{3.192\,I\,T}{l}$, where the number of turns $T = 800$, and the current $I = .2645$, .2916, .3248 ampere, etc.; the length of the magnetic circuit l is determined from the dimensions of the ring, as follows: The mean diameter of the ring $= \frac{5 + 6\frac{1}{2}}{2} = 5\frac{3}{4}$ in. Length $l = 5\frac{3}{4} \times 3.1416 = 18.06$ in. The first value of $\mathbf{H}$ for insertion in the table $= \frac{3.192 \times .2645 \times 800}{18.06} = 37.40$ lines of force per sq. in. The deflections of the galvanometer, column 4, were given in the example.

The change in the number of lines of force per square inch, that is, the additional number of lines of force per square inch due to the increases of primary current, noted in column 2, when the resistance of the rheostat R' equals 2,000 ohms, is found in the following manner: It has already been explained that $\mathbf{B} = \left(\frac{3.192\,I\,T\,T_1\,A\,R'}{l\,T_2\,A_2\,R\,d}\right) d'$.

First calculate the value of the calibration constant enclosed in the parenthesis. The current I in the primary calibrating coil C, according to Ohm's law, is

$$I = \frac{E}{R} = \frac{6 \times 1.9}{3 + 1.2 + 1.1} = \frac{11.4}{5.3} = 2.15 \text{ amperes}$$

T, the number of turns in the primary calibrating coil $C = 1{,}200$, and T_2, the number of turns in the secondary C' of the calibrating

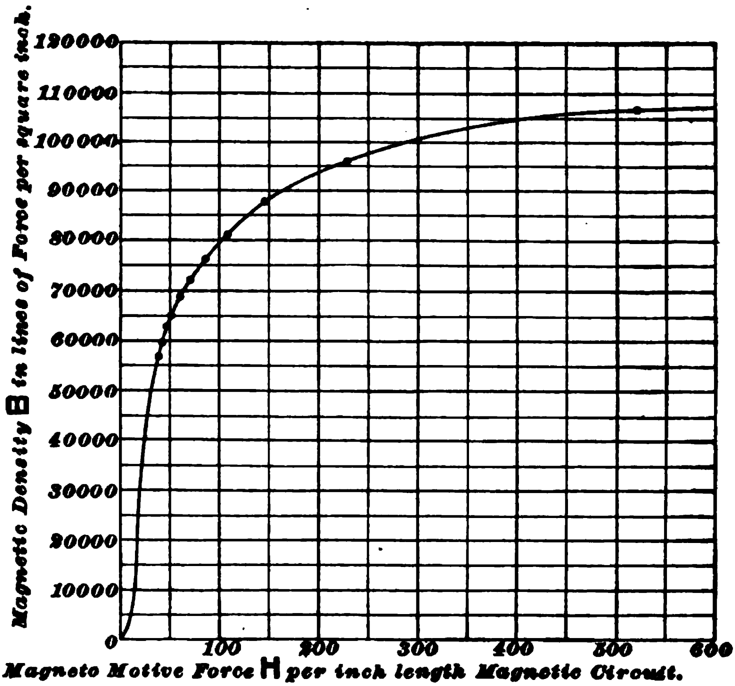


FIG. 33

coil = 261. The sectional area A of the calibrating coil $C = 3.1416 \times \left(\frac{2}{2}\right)^2 = 3.1416$ sq. in. To be strictly correct the mean area should be used, but the depth of the winding, a trifle less than twice the diameter of the insulated No. 18 wire, may be neglected since it is small compared to the inside diameter, which is 2 in. R', the resistance of the galvanometer circuit, when the deflections due to the currents induced in the secondary S were observed = 2,000 + 500 + 10 = 2,510 ohms. The length l of the primary calibrating coil C = 30 in. T_3 the number of turns in the secondary S on the iron ring = 120. The sectional area A_s of the iron ring $= \frac{3}{4} \times 1 = .75$ sq. in., and R, the resistance of the galvanometer circuit, when the deflection d of 48 scale divisions was observed $= 500 + 10 = 510$ ohms. Substituting these values in the formula for **B** gives

$$\mathbf{B} = \left(\frac{3.192 \times 2.15 \times 1{,}200 \times 261 \times 3.1416 \times 2{,}510}{30 \times 120 \times .75 \times 510 \times 48}\right) d' = 256.4\, d'$$

For the increase in magnetic density from zero to the first value of **B** we get $\mathbf{B} = 256.4 \times 220.6 = 56{,}561$, or 56,560, which is exact enough and fully as close as can be used in plotting a curve. The deflection 11.1 corresponds to an increase in the current from .2645 to .2916 ampere. Using this deflection in the same formula, we get for the corresponding change in the number of lines of force per square inch, $\mathbf{B} = 256.4 \times 11.1 = 2{,}846$, or 2,850 as the increase in the number of lines of force per square inch. In this manner all the values in column 5 are obtained. The total number of lines of force per square inch, column 6, corresponding to the respective magnetizing forces tabulated in column 3, are obtained by adding each number in column 5 to the sum of all the preceding numbers in the same column, that is, by adding each number in column 5 to the number last set down in column 6. The first number in column 6 would of course be 56,560, the same as the first number in column 5. The second number in column 6 $= 56{,}560 + 2{,}850 = 59{,}410$ lines of force per sq. in.

The values of the magnetizing force **H** and the corresponding density **B** in the iron are now plotted on a sheet of cross-section paper, and the points so obtained connected by a line which then forms the magnetization curve of the piece of iron under test. This curve is shown in Fig. 33.

NOTE.—The student is advised to perform the computations enumerated for at least one complete horizontal row in the table, to better comprehend the rules and principles involved.

THE PERMEAMETER

36. The **permeameter** is an apparatus for determining quickly and with a fair degree of accuracy the magnetic qualities of iron. The method was proposed by S. P. Thompson and is suitable for use in shops. One practical arrangement of apparatus is shown in Fig. 34. The sample rod of iron to be tested slips through a hole in the top of a large wrought-iron yoke w and through a magnetizing coil having a known number of turns. The lower end of the specimen is very accurately faced and rests on a part of the yoke that is scraped to a truly plane surface, so that the two will form as good a joint as possible. One end of a stout cord *i*, which passes over a pulley *m* that is firmly secured to a support *n*, is fastened to the spring balance and the other end to a lever *l* that is pivoted at *g*. When a current flows through the magnetizing coil the force required to separate the specimen from the surface of the yoke is measured by means of the spring balance and the current is measured by the ammeter *A*. To obtain the pull, the lever *l* is forced down and the reading of the spring balance is observed just before or as the specimen separates from the yoke. The mean of three or four readings should be taken while the current is maintained constant. The spring balance should be carefully calibrated from time to time over its whole range. If there is a correction, a calibration curve for correcting the readings will be found very convenient. From the current, the number of turns in the magnetizing coil and the mean length of the magnetic circuit, the magnetizing force $\mathcal{H}$ can be calculated.

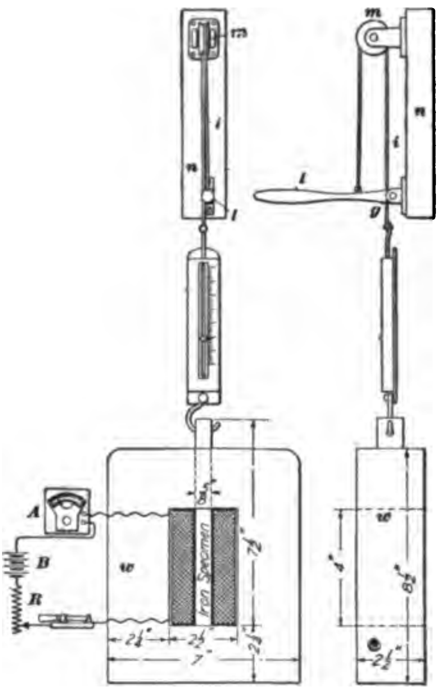


FIG. 34

In *Electromagnetic Induction* it was stated that $F = \frac{\mathcal{B}^2 A'}{11,183,600}$, in which F is the pull in pounds necessary to detach an armature from a core when the density is $\mathcal{B}$ lines per square centimeter and the polar area A' square centimeters. In this case, since the core moves inside a stationary magnetizing coil, the $\mathcal{H}$ lines of force are not broken and hence it is more accurate to consider the pull as proportional to $(\mathcal{B} - \mathcal{H})^2$ rather than to $\mathcal{B}^2$. Hence, we have $F = \frac{(\mathcal{B} - \mathcal{H})^2 A'}{11{,}183{,}600}$, from which is obtained the formula

$$\mathcal{B} = 1{,}317\sqrt{\frac{F}{A}} + \mathcal{H} \qquad (9)$$

in which F = pull, in pounds;
A = polar area, in square inches.

Hence, $\mathcal{H}$ and the corresponding values of $\mathcal{B}$ can both be determined for each magnetizing current and a curve can be made showing the relation between a series of values of $\mathcal{B}$ and $\mathcal{H}$.

PERMEAMETER USED BY WESTINGHOUSE COMPANY

37. The permeameter used by the Westinghouse Electric and Manufacturing Company has a different method of applying the pull, as shown in Fig. 35, but is the same in principle. (For this illustration and description credit is due the Electrical Engineers' Pocketbook, by H. A. Foster.) This apparatus is fastened securely to the table and the rigid arrangement of the brass frame *d* is such that the balance is suspended exactly over the center of the rod and yoke so that all side pull is avoided. As the handle *e* of a pinion *j* is turned, the rack *i* is raised and the pull is applied to the specimen through a spring balance *f*. A spring buffer *o* allows the sample to rise perfectly free but only for a distance of $\frac{1}{8}$ inch, when it takes up the jar resulting from the sudden release of the specimen. Two spring balances with long scales are used; one measures up to 30 pounds and the other to 100 pounds. While the spring balances were originally made self-registering, this feature is no longer used as measurements can be made quicker and with sufficient accuracy without it. For a sample $\frac{5}{8}$ inch in diameter, the maximum pull for cast iron will be about 25 pounds, and for mild cast steel about 70 pounds. The dimensions of the bar, yoke, and coil used by this company are the same as shown in Fig. 34. The sample is finished very accurately to $\frac{5}{8}$ inch in diameter in order that it may fit nicely in the hole through the yoke. The surfaces of both rod and hole must be very smoothly finished to reduce the friction as much as possible. The magnetizing coil has 223 turns and the mean length l' of the magnetic circuit is estimated as 11.74 centimeters. By means of the adjustable resistance R, the current can be varied by steps of .01 ampere from zero to 12.5 amperes, the latter giving a maximum magnetizing force $\mathcal{H}$ of 300 lines per square centimeter. This is about as high a value as is ever required in practice. The magnetizing force is calculated from the formula $\mathcal{H} = \frac{1.257\,I\,T}{l'}$, and in this case, since $T = 223$ and $l' = 11.74$, therefore, $\mathcal{H} = 23.8\ I$. With a rod $\frac{5}{8}$ inch in diameter, the formula $\mathcal{B} = 1{,}317\sqrt{\frac{F}{A}} + \mathcal{H}$ reduces to $\mathcal{B} = 2{,}380\sqrt{F} + \mathcal{H}$. By having a table of square roots for each possible value of F given by the spring balance used, it will be seen that $\mathcal{H}$ and $\mathcal{B}$ can both be very quickly calculated from corresponding observed readings of the ammeter and the spring balance.

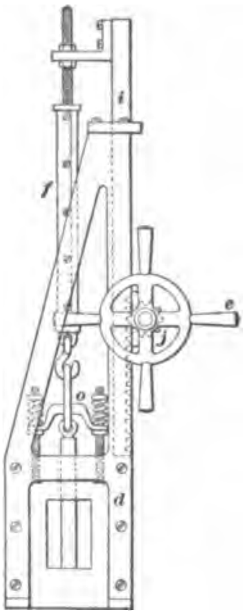


FIG. 85

The specimen to be tested is first demagnetized by placing it in the field of an electromagnet, through the coil of which an alternating current flows while the specimen is gradually removed from the field. The specimen is then placed in the yoke, care being taken to see that it can move without friction and that it rests perfectly flat on the yoke, from which all dust and dirt have been carefully removed. A series of observations with different currents are taken, beginning with the smallest current, and gradually increasing it to its greatest value. No reading should ever be taken if the current decreases with the specimen in place. If this happens the sample must be first demagnetized and the current brought up to the strength desired without allowing it to decrease at all. The most convenient way is to adjust the current with the specimen removed, then insert the specimen, and give it a turn to insure good contact between the yoke and specimen.

Measurements made with the permeameter are subject to several sources of error that should be eliminated as far as possible. First, there is an error due to the unavoidable air gap between the specimen and yoke at the upper and lower contact surfaces; these air gaps increase the reluctance and no allowance can be made for the error thus introduced. But by careful work this can be reduced to a certain point and there maintained constant. Second, the magnetic lines that leak from the specimen at the lower end are not cut by all the turns on the coil and the error due to this increases as the magnetic density increases. Third, there are errors in the calibration of the spring balance and errors in reading the same. The spring balance can be read to within less than 1 per cent. and as the square root of the pull is used, the error on this account is quite small, especially with large pulls. For this reason very small values of $\mathcal{B}$ cannot be determined with much accuracy.

With the permeameter, measurements may be made very quickly, and on a large number of specimens, which is very desirable for ordinary shop use. While it is a good workshop method and gives values that are comparatively correct enough, it does not give very correct absolute values.

38. A permeameter may also be fitted with a small coil for testing the specimen ballistically. A small coil, surrounding the specimen below the main magnetizing coil, is attached to one or two springs so arranged as to make the coil fly out suddenly when the rod is pulled up. A ballistic galvanometer connected in series with this small coil will give deflections proportional to the lines of force cut by the coil. In this way the ballistic and permeameter methods can be used to check one another.

PERMEABILITY BRIDGES

39. A **permeability bridge** is an apparatus for determining in a simple way suitable for workshop use the magnetic densities in iron corresponding to given magnetizing forces. The **Ewing permeability bridge** is shown in Fig. 36. It enables the curve of magnetic induction and magnetizing force (the $\mathcal{B}$–$\mathcal{H}$ curve) to be determined with ease, in a way that much resembles the measurement of resistance by a Wheatstone bridge.

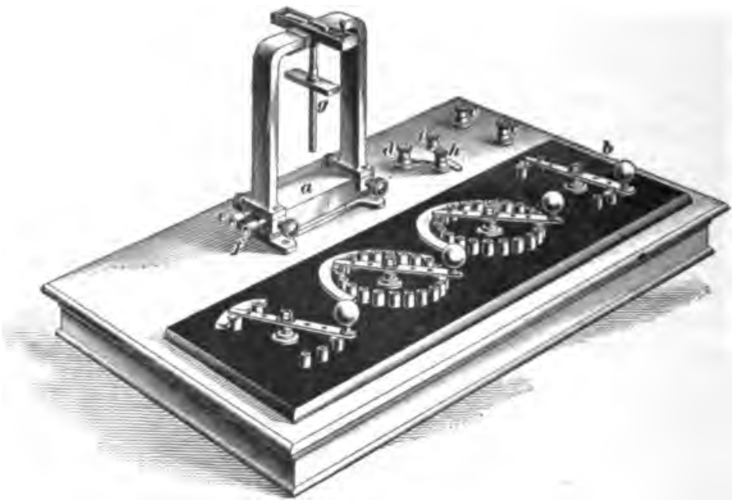


FIG. 36

The sample *f* under test is in the form of a short bar, which is turned to the same diameter as a standard bar *e* supplied with the instrument. The $\mathcal{B}$–$\mathcal{H}$ curve of the standard bar having been determined beforehand, particulars of its quality are furnished by the makers along with the instrument. The test of any sample is made by comparing it with the standard, in order to see what magnetizing force will produce the same induction in the sample as a given magnetizing force produces in the standard. By repeating this comparison with several values of the magnetizing force, a series of readings are obtained that enable the $\mathfrak{B}$–$\mathfrak{H}$ curve for the test piece to be drawn.

The two bars, namely, the standard *e* whose $\mathfrak{B}$–$\mathfrak{H}$ curve is known and the bar *f* to be tested, are slipped into parallel magnetizing coils, which are hidden by the cover *a*, and their ends joined by two short yokes of soft iron. In the figure, the yokes appear at the ends of the brass cover *a*, which contains the two coils. From the yokes two long iron horns project upwards until they nearly meet; in the gap between them a box containing a compass needle is placed. This corresponds to the galvanometer of a Wheatstone bridge; it serves as a detector to show when the two yokes are at the same magnetic potential, or, in other words, to show when there is no magnetic flux across from yoke to yoke through the two iron horns. Such a condition will be produced only when the magnetic flux in the two bars is the same. In that case, the magnetic circuit consisting of the two bars and the yokes will be complete in itself. All the lines that go from right to left along one bar will return from left to right along the other; the yokes will be at the same magnetic potential, the horns will remain unmagnetized, and the detector will not deflect. If both bars have the same permeability, the state of balance will be produced by having the same number of ampere-turns act on each; but if they differ in quality, the condition of balance can still be produced by altering the relative number of ampere-turns. To do this, the number of turns in the magnetizing coil of the sample bar is altered by means of the dial switches, while the same current passes through both coils.

To get rid of all effects of hysteresis in the bars, and in the yokes and horns, a reversing key *b* is worked while the adjustment proceeds, and the balance is perfect when each reversal produces no permanent displacement of the compass needle between the horns. A transient kick will even then in general be observed, owing to the difference in the time rate with which the two bars take up their magnetism. In practice, the adjustment, by means of the dial switches, is very readily made and takes no longer than the corresponding process in measuring resistances. To prevent the current from altering while the adjustment is going on, the switches are furnished with a second set of contacts, which throw in compensating resistances as the number of turns in the magnetizing coil is reduced, with the effect that the total resistance of the circuit remains unchanged. The clear length of each bar is 12.56 centimeters (4π), and the number of turns in the magnetizing coil of the standard bar is 100. Hence, the magnetizing force due to its coil is 10 C. G. S. units for each ampere of current. This allows any required magnetizing force to be easily applied, with the aid of an ammeter and an adjustable resistance outside the instrument. Further, since the relation of $\mathcal{B}$ to $\mathcal{H}$ is known for the standard bar, a knowledge of the current is enough to show at what value of the flux density $\mathcal{B}$ the comparison is being made, and $\mathcal{B}$ is, of course, the same for both bars when the condition of balance is produced. The number of turns on the other bar then gives, by its ratio to 100, the ratio of the magnetizing force required for that bar to the known force applied to the standard. For instance, if the number of turns on the sample is 126, which is given on the dial, the sample requires $\frac{126}{100}$ of the magnetizing force that the standard requires to produce the same flux density. When the current is 1 ampere, $\mathcal{H}$ for the standard is 10 and the magnetizing force acting on the sample is $\frac{10 \times 126}{100}$, or 12.6 The value of $\mathcal{B}$ is found by reference to the table accompanying the standard bar. Hence, a point in the $\mathcal{B}$-$\mathcal{H}$ curve of the bar under test is determined, and by changing the current as many points as are wished may be found.

The dial switches give the means of increasing the number of turns on the test piece up to 210. In cases where the test piece is magnetically much worse than the standard, a ratio of rather more than 2 to 1 may be insufficient; and to provide for that there is a two-way arrangement *d* by which the number of turns on the standard bar is readily reduced to 50, with the result that $\mathcal{H}$ becomes 5 instead of 10 per ampere, and the sample under examination may then have applied to it more than four times as much magnetizing force as the standard bar. The sensitiveness of the compass needle is adjusted by raising or lowering the directing magnet *g*, and the latter is manipulated so as to bring the compass needle into a central position before each reversal of the key.

When it is desired to test the permeability of sheet metal, the samples are formed by piling up a number of straight strips, giving a total cross-section equal to that of the standard, and a different form of yokes is required. In the arrangement used with bars the latter are slipped through holes in which they are a loose fit, and then pressed against the middle portion of the yokes by a pair of setscrews outside.

This permeability bridge may be applied to determine the $\mathcal{B}$–$\mathcal{H}$ curve for low as well as for high magnetizing forces, using currents ranging down to .2 ampere or even less. Complete directions for the use of the Ewing permeability bridge are given in a pamphlet that the makers send with the bridge and hence it is not necessary to give them here.

40. The **Holden permeability bridge,** designed for the General Electric Company, depends on practically the same principles, but instead of varying the number of turns and keeping the current through both coils the same, as in the Ewing bridge, the number of turns is kept the same and the current in each coil is varied until a compass, placed directly over the middle of the two bars, gives no deflection.

DETERMINATION OF HYSTERESIS

BY USE OF WATTMETER

41. When the iron is to be used in alternating-current apparatus, it is usually desirable to determine the loss due to hysteresis by means of an alternating current and as nearly as possible under working conditions. This can be done by using an alternating current of the proper frequency, a wattmeter, and an alternating-current ammeter. In Fig. 37,

D represents a generator of alternating current of the proper frequency n, F the current coil, E the potential coil, and r the non-inductive resistance of an ordinary wattmeter, A an alternating-current ammeter, S the sample of iron to be tested, and M an adjustable resistance. The sample must consist of a pile of disks, or long and relatively narrow strips, of thin sheet iron or steel, and all disks and strips must be well shellaced and dried before they are placed together, as the method cannot be used with solid or electrically connected rings or rods on account of the loss due to the currents induced in the iron by the alternating current in the coil, and which in this method is inseparable from that due to hysteresis. The insulating of the disks from one another

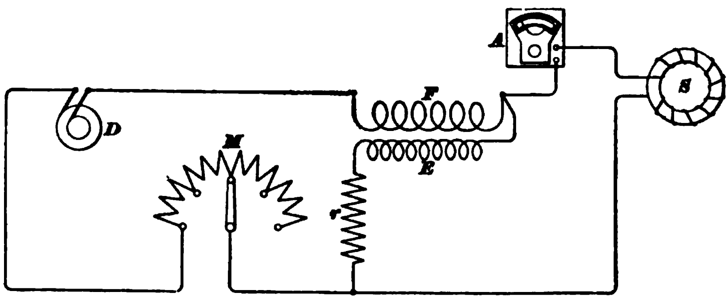


Fig. 37

by shellac or tissue paper prevents the generation of induced currents of sufficient magnitude to impair the accuracy of the results obtained.

Adjust the resistance M or the voltage of the generator D, to give the current desired. Let P' be the reading of the wattmeter, in watts; I the reading of the ammeter, in amperes; n the frequency, R the resistance, in ohms, of the coil of wire wound on the sample S and preferably including also the resistance of the ammeter A; and V the volume of the iron sample. The volume of the iron sample is equal to the product of the thickness of one sheet, its width, mean length, and the number of sheets. Since $I^2 R$ represents the watts lost in the copper winding and preferably also in the ammeter A, then, if V is in cubic centimeters, the watts lost in hysteresis per cubic centimeter for 1 per cycle per second (or the joules per cubic centimeter per cycle) is given by the formula

$$P = \frac{P' - I^2 R}{V n} \qquad (10)$$

The resistance R can be measured by any suitable method, but it cannot be correctly determined by Ohm's law from the readings of an alternating-current voltmeter and ammeter obtained with an alternating current. It may be thus determined, however, if a direct current is used.

EWING'S HYSTERESIS METER

42. A **hysteresis meter** is an instrument for measuring the hysteresis in sheet iron and steel. **Ewing's hysteresis meter** does this in a simple way suitable for workshop as well as laboratory use. The operation of this instrument, which is shown in Fig. 38, is entirely mechanical, and requires no knowledge of electrical testing, and it has the further advantage of using easily prepared samples. A few strips of the iron to be tested are cut, or stamped, $\frac{5}{8}$ inch wide and 3 inches long. They are filed to the exact length when clamped in a gauge, which is provided with the instrument, and are then inserted in the carrier *a*, which is made to revolve by turning a handle. The carrier turns between the poles of a permanent magnet *b*, which is suspended on a knife edge at *c*. In consequence of the hysteresis of the specimen, which causes it to resist any change in the magnetic state that it may have acquired, the magnet is deflected, and the amount of its deflection, which increases with the hysteresis, is observed by means of a pointer and scale. From the observed deflection the hysteresis of the specimen may be determined. The deflections are practically the same, with even a great variation in the thickness of the pile of test pieces, so that no correction has to be made for such variation. The magnetic density is practically the same in all specimens, notwithstanding differences in the permeability of the iron, because the permanent magnet remains constant in strength and the reluctance of the air gaps remains constant. Moreover, the reluctance of the air gaps plus that of the permanent magnet is so large in comparison with the reluctance of any specimen of iron likely to be tested, that a change in the reluctance of the iron specimen has no appreciable effect on the total reluctance of the circuit.

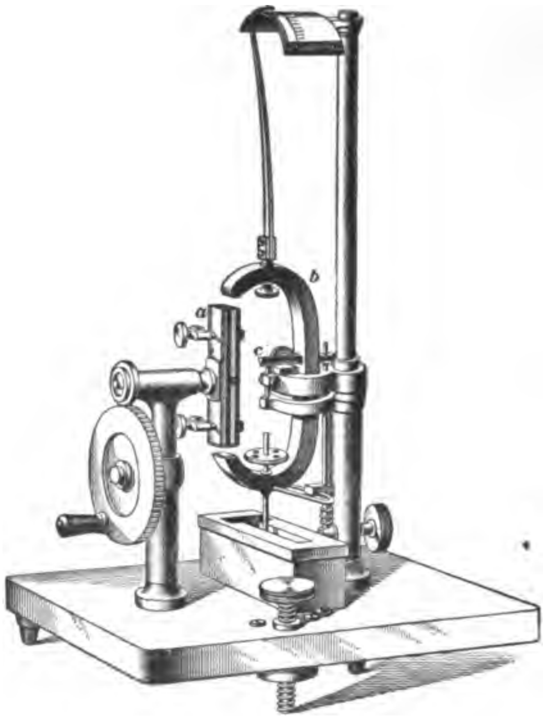


FIG. 38

Hence, the magnetomotive force and reluctance are practically constant quantities and consequently the flux is constant for any specimen. The hysteresis of a specimen varies about as the 1.5 or 1.6 power of the flux density. If the flux density is constant then the hysteresis of any one specimen remains constant.

Two standard samples (with Professor Ewing's certificate) are provided with the instrument, having stated amounts of hysteresis. The hysteresis of any other specimen is determined simply by comparing the deflection produced by it with the deflections produced by the standard samples. As the standard samples may change somewhat, Professor Ewing recommends that they be recalibrated from time to time. That is, their hysteresis loss should be carefully determined by the ballistic-galvanometer method either by the owner of the instrument or by the maker, to whom the standard samples may be returned for recalibration. This also applies to the standards used in permeability bridges. Complete directions for the use of Ewing's hysteresis meter are given in a pamphlet that the makers send with the instrument and hence it is not necessary to give them here.

43. The most important electrical measurements and their general applications have been described, but for particular cases these methods must often be combined and many modifications made in details.

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