AN INTRODUCTION TO THE DESIGN OF

BEAMS, GIRDERS AND COLUMNS

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THE DESIGN OF BEAMS

GIRDERS AND COLUMNS

IN

MACHINES AND STRUCTURES

WITH

EXAMPLES IN GRAPHIC STATICS

BY

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PREFACE.

The design of beams, in relation to strength, stiffness and convenience of construction, is a study that appeals to all classes of engineers and architects. For in all machines and structures beams appear in one form or another; and little progress can be made in scientific designing without a proper understanding of the principles or fundamental facts underlying their construction. Hence great prominence is rightly given to this subject in all courses of applied mechanics, machine and building construction, and naval architecture.

The following chapters, based on articles originally contributed to *The Mechanical World*, deal in an elementary manner with the main principles and considerations involved in designing beams and columns of such forms and materials as commonly occur in machines and structures.

Although points of mathematical intricacy have been intentionally avoided, my aim has been to make the treatment thorough, within the limits prescribed, and the mode of presentation such as can be easily understood. On questions of fundamental importance I have aimed at going to the root of the matter, giving full proofs of the leading formulæ: while on points of less significance I have touched but lightly. The diagrams interspersed throughout the text are very numerous, and include examples illustrative of the graphical method of estimating the stresses in the several members of braced girders.

A long experience, not only in the lecture-room but also in the designing and estimating offices of several large engineering works, has taught me the great value of numerical examples in imparting clearness of view and facility in applying principles to practice. Accordingly I have introduced a liberal number of fully worked out examples, of a kind that will be found helpful to young engineers and draughtsmen, whether engaged in the mechanical or the structural branches of engineering. For in the drawing office the problems presented to the estimator and the draughtsman are always special or particular, never general. In other words, each problem refers to a particular case, and must be solved *numerically*, in response either to a definite inquiry or a definite order. Hence a draughtsman always prefers numerical examples to general investigations, though the latter are more important in college work.

As denoted by its title, this book is strictly *introductory* in its aim and scope. Consequently no attempt has been made to exhaust the subject, which may be pursued much further in several larger books, such as Anglin's "Design of Structures" and Fidler's "Bridge Construction." For a brief summary of beam formulæ and data, reference may be made to that convenient little drawing-office companion, "The Mechanical World Pocket Diary," which still contains, in a slightly abridged form, the section on "Beams and Girders" contributed by me in the year 1897. Lastly, an account of some recent laboratory experiments on the strength of columns will be found in the paper on that subject by Prof. W. E. Lilly, read before the Institution of Mechanical Engineers in February 1905.

W. H. ATHERTON.

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THE DESIGN OF BEAMS.

CHAPTER I.

INTRODUCTORY.

Definition of Beam.—In everyday English the term "beam" has many widely different meanings. Thus we speak of a beam of sunlight, of the beam of a ship, and of a weaver's beam, as well as of any large and straight piece of timber or iron serving to support some part of a building.

In the technology of engineering, however, the term "beam" has a very definite significance, and refers to any detail whatever of a machine or structure that is subjected to bending action, by being loaded transversely or obliquely at some distance from its support or supports. The characteristic feature of every beam is that the forces acting on it tend to bend or deflect it, and finally to break it crosswise.

A **Girder** is a large or main beam of iron or steel supported at both ends, and is usually a composite structure, built up of several parts riveted or otherwise secured together. The term "truss" is also used to denote a large pin-jointed girder or roof principal.

The distinction between a beam and a girder is not very rigidly observed in practice, however; many engineers regarding the two terms as synonymous. At the same time the construction of riveted girders, as a branch of engineering, is always referred to as girder-work, and never as beam-work.

Engine Beams.—A good example of the kind of beam that occurs in machinery is the main beam (Fig. 1) of a large compound pumping-engine of the rotative type. This is a single heavy iron casting, 28ft. long by 5ft. 9in. deep at the centre, of I section; consisting of broad top and bottom flanges connected by a thin web and by cross ribs at the bosses. The outline of the engine beam is tapered, not for the sake of appearance merely, but chiefly to secure uniformity of strength, most metal being put where the bending action is greatest.

In the year 1876 an exceptionally large wrought-iron riveted beam was constructed for a Cornish pumping-engine at the Hull Corporation Waterworks, having a steam cylinder of 96 inches diameter and a stroke of 12 feet. This beam was 40 feet long from centre to centre, 8 feet deep, and weighed 40 tons. It was found to be quite satisfactory in every respect, though many engineers at that time objected to the use of built-up engine beams. Probably the best material now available for engine beams of moderate dimensions is Siemens' cast steel, while those of very large size are best built up of steel plates and angles.

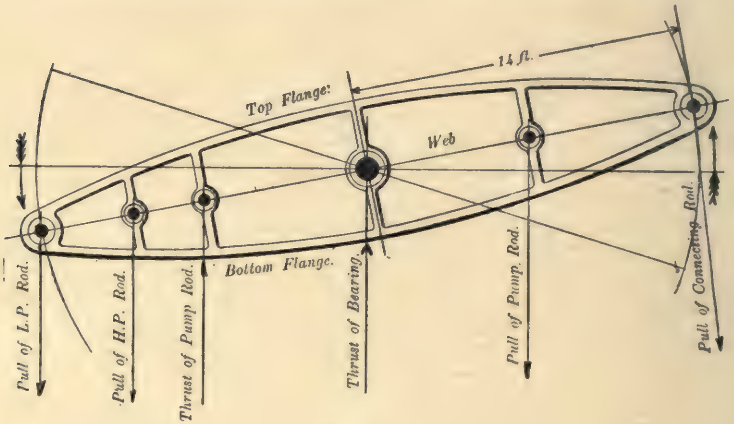


Fig. 1.

Gun Beams.—Another instructive example of the mechanical engineer's beam is the so-called "slide beam" shown in Fig. 2; which is one of a pair of cast steel slides, 30 inches deep, forming the recoil ways of a 68-ton gun, as fitted on many first-class battleships. The two beams are spaced 4 feet apart, and between them is situated the hydraulic recoil buffer, which ties them securely together. The whole system of gun, carriage, slide beams and gear swings about 9-inch pins fastened to the turntable structure, the movement being controlled by a powerful hydraulic elevating ram.

A still more interesting illustration of the varied forms and uses of beams is seen in Fig. 3, showing the mode of supporting and handling a ponderous 68-ton gun, by slinging it between a

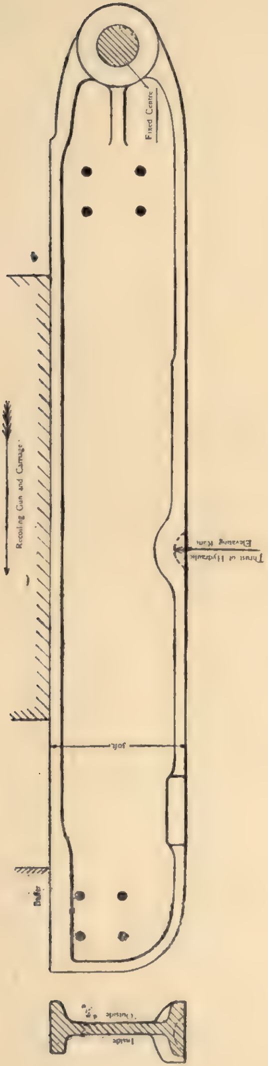

Fixed Centre
Recoiling Gun and Carriage
Thrust of Hydraulic Elevating Ram
10ft.
Buffer
Outside
Inside

Fig. 2.

pair of levers or beams technically known as *elevators*. This is the method of mounting adopted in all hydraulic and hydro-pneumatic disappearing gun-carriages, ranging in size from the 6-inch 5-ton mounting up to that for the 13·5-inch gun illustrated. The gun

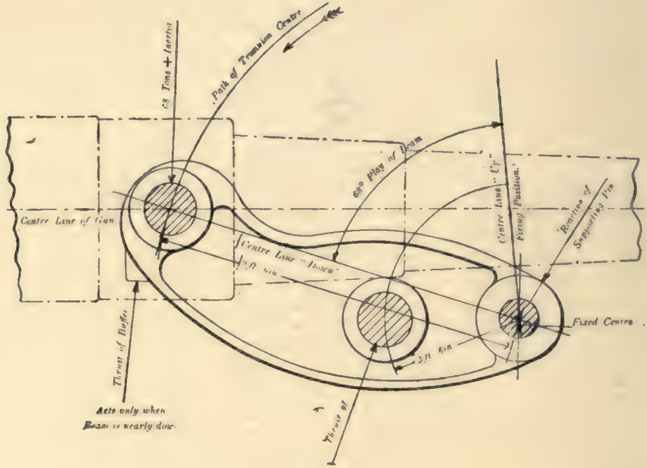


Fig. 3.

is embraced by a trunnion-hoop, whose two trunnions are carried by the pair of heavy cast steel elevators of I section. In the position here represented, the gun is supposed to have just fired a shot at the enemy, to have then immediately disappeared from view behind a parapet, and now to be on the point of coming to rest in the position for reloading.

Cranes.—The 20-ton foundry crane jib outlined in Fig. 4 is an example of the special type of beam known as the *cantilever*, which is the generic name for all beams secured at one end and free at the other. This curved form of jib is designed to give as much headway as possible round the crane, and is built up of steel plates and angle bars. The side frames are double, and support a traversing carriage or "monkey," from which the load is hung; the hoisting being done by a pair of cylinders, 6 in. diameter by 12 in. stroke, bolted to the pillar.

The girders of travelling cranes, as commonly used in engineering workshops, are also familiar examples of beams. They are usually either of I or of box section, as shown in Fig. 5, and are "fish-bellied" in elevation; that is, deepest at the centre of the

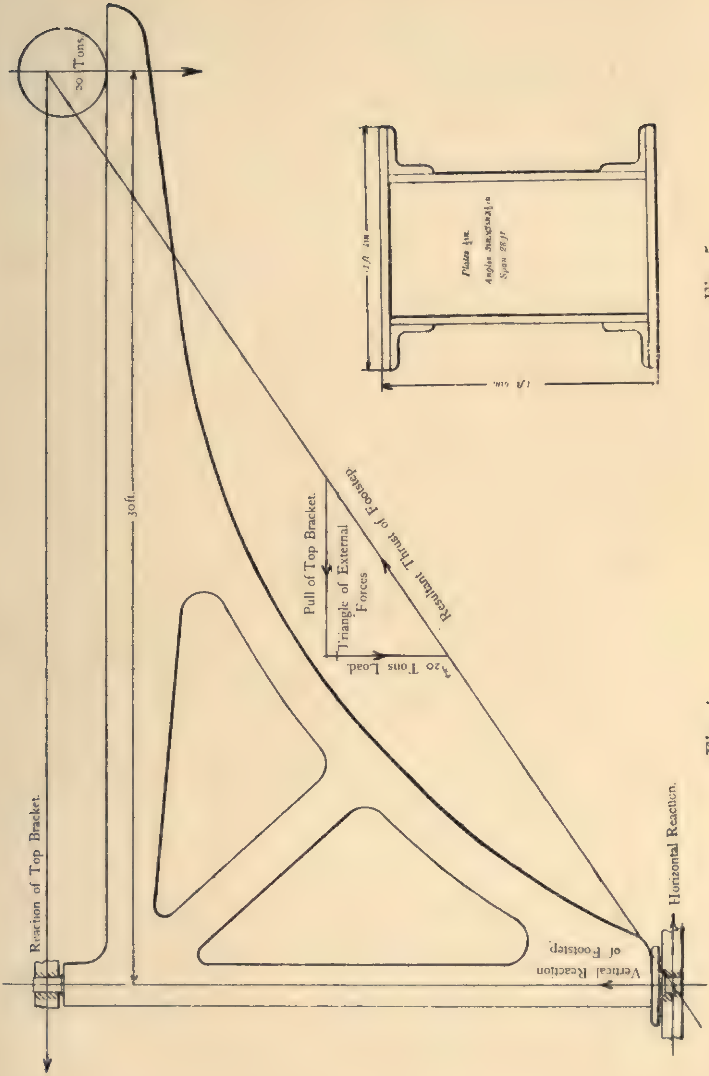

20 Tons.
30 ft.
Reaction of Top Bracket.
Pull of Top Bracket.
Triangle of External Forces
Resultant Thrust of Footstep.
20 Tons Load.
Vertical Reaction of Footstep.
Horizontal Reaction.
Span 28 ft

Fig. 4.

Fig. 5.

span, where the bending action is greatest. Their ends are supported on "end-carriages," which run on the elevated "roadway" or "runway" down the shop. A steel box girder of the given section, and of 28 foot span, will carry safely a live load of 13 tons at the centre, allowing for its own weight. Any desired increase of strength can be got by increasing the depth and adding extra plates to the top and bottom flanges.

Sundry Examples.—Iron and steel beams are of extensive and increasing application in all structural work, such as bridges, roofs, large buildings, railway stations, pit-head frames, and ships. Many recent buildings, displaying handsome stonework exteriors, are essentially steel-work structures, the stone-work being little more than a thin shell covering the steel skeleton.

Many parts of machines are really beams in a more or less disguised form ; such as the crank-shaft of a pair of coupled engines carrying a heavy fly-wheel and driving by teeth, ropes or belt (Fig. 6). Here are six transverse forces tending to bend the

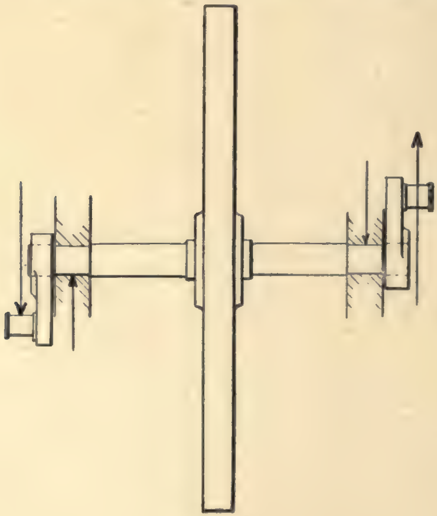


Fig. 6.

shaft, namely : the weight of the fly-wheel, the pull or thrust on two crank-pins, the reactions of two bearings, and, finally, the pull of the ropes or other drive. But this is a complicated example, because several of the forces are continually changing in direction and amount, besides which the shaft is subjected to torsion as well as to bending.

Amongst other examples of beams we may mention :—The cross-slide and uprights of a metal-planing machine ; a lathe, planing or other machine bed supported on feet ; most machine frames, as those of hydraulic riveters, punching and shearing machines ; all cranks and levers, as those of engines and testing machines ; overhanging crank-pins, brackets and hangers ; the arms of fly-wheels, pulleys, and gearing ; the teeth of wheels ; plate springs ; the girder stays of boilers ; bicycle and motor-car frames ; the entire hull of a ship, and a vast variety of other details and structures.

General Remarks.—From a consideration of these examples it is manifest that beams constitute a class of engineering details so numerous and important as to make it well worth our while looking carefully into the elementary theory of their strength and stiffness, so as to become quite familiar with the fundamental principles on which they are designed. For it is not possible to design beams properly and with confidence by simply looking at existing examples, or even by copying them, though this is certainly a useful exercise. It is necessary to *reason* about them, starting with first principles, and to work through many numerical examples. In this way only is it possible to get to the root of the matter, and to acquire facility in the application of scientific principles to practical designing.

In the drawing-office the problem before us is either to design a beam to stand a certain load, or to examine some proposed or existing beam to see if it is strong enough for its work, by calculating the stress likely to come on the material. We shall not concern ourselves here with workshop processes, except as influencing design ; nor with the problem of the erection of the completed beam in position, which is not always a simple one.

CHAPTER II.

APPLIED FORCES AND REACTIONS.

Force.—As we shall very often have occasion to use the term "force," a few words about it will be in season. Force may be regarded from either the statical or the dynamical point of view. For the statical problems with which we shall chiefly have to do it is sufficient to regard a force as any push, pull, pressure, thrust, or resistance—terms whose meaning is brought home to us in our daily experience, and needing no formal definition. The modern dynamical definition of force is "time rate of change of momentum," but for our present purpose this is quite needlessly mystifying.

So far as we are concerned, force is measured in terms of some unit of weight, as that of a ton of iron. There are only two units of force that will be of much use to us—namely, a *pound* for light work and a *ton* for heavy work. We could do very well without hundredweights and quarters, and, in fact, it is rather a pity that such units exist at all. We have far too many units. Unfortunately, also, there are three different tons to bear in mind. The English ton contains 2240 lbs., the American ton 2000 lbs., and the French ton of 1000 kilos., about 2205 lbs. Bridge engineers occasionally find it convenient to measure forces in hundredweights. Continental engineers usually express forces in kilogrammes (kgs.).

It is commonly said that we know all about a force when we know (1) its amount or magnitude in tons or other unit; (2) its point of application, or where it acts; (3) how it acts—*i.e.*, its direction up or down, to right or left, or otherwise. But practically there is at least one other thing that must be known before the force is completely specified; namely, whether it is "live" or "dead"—an important distinction which we now proceed to consider.

Relative Effect of Live and Dead Loads.—From a consideration of the various examples of beams already presented, it will have now become clear that we have to distinguish between two kinds of applied forces, differing widely in their nature and effect. These are usually styled *live* and *dead* loads respectively. A "live" load is a suddenly applied force or *shock*, as that due to the recoil of a gun or the pressure of a gust of wind on an exposed structure. Also forces suddenly reversed in direction, as the alternating force on the crosshead of a double-acting engine, are live loads, and require special consideration. A "dead" load, on the other hand, is a gradually-applied force or steady pressure, never changing in any respect, like the weight of a building or bridge.

The destructive effect of a live load, it is important to observe, is twice that of a dead load of the same nominal amount. This statement admits of exact proof, and as the proof is easy and convincing, we now give it, so as to clear up any uncertainty that may exist on the matter. A little thoughtful experimenting with a spring balance or a piece of elastic is instructive in this connection.

Case 1.—Consider the effect of a load of 1lb. applied *gradually* to a helical spring S (Fig. 7). As the support is carefully removed from the pound weight the tension on the spring increases from nothing up to 1 lb., and therefore the opposing elastic force or elasticity of the spring will also increase from nothing up to 1lb.

But the pull of a spring is proportional to its extension, so that S will stretch a certain distance AB, and remain extended as long as the load is applied. Draw BF to represent to scale the maximum pull of S thus set up—viz., 1lb.—and join AF. The series of horizontals parallel to BF show the gradual increase of spring tension, and the area of the triangle ABF represents the work done by gravity in extending the spring. For, work done = mean pull × extension = $\frac{1}{2}$BF × AB = area ABF.

Case 2.—Next suppose the load to be applied *suddenly*. Then gravity at once acts with the full force of 1lb., whereas the tension of the spring starts at nothing and increases gradually as W falls lower and lower. Set off AC to represent the constant force of gravity, and complete the triangle ABF and the rectangle ABFC.

Consider the state of affairs when W has fallen through the former total distance AB. The work done by gravity equals the constant force AC into the distance AB, and is thus represented by the area of the rectangle ABFC. Similarly, the work done by the varying elasticity of the spring is shown by the area ABF. And since the work done by gravity, up to the point B, exceeds that done by elasticity, the body W will continue to fall until its store of energy has been absorbed by the spring. Completing the diagram of work, we see that this will be the case when the work done by gravity equals the work done by elasticity, and, therefore, when area ADGC = area ADEA, or triangle GEF = triangle CAF, or AB = FG, or AD = 2AB, or DE = 2BF, by proportion. This result conclusively proves that the greatest pull which comes on the spring due to the live load, as shown by DE, is double the greatest pull induced by the dead load, as shown by BF.

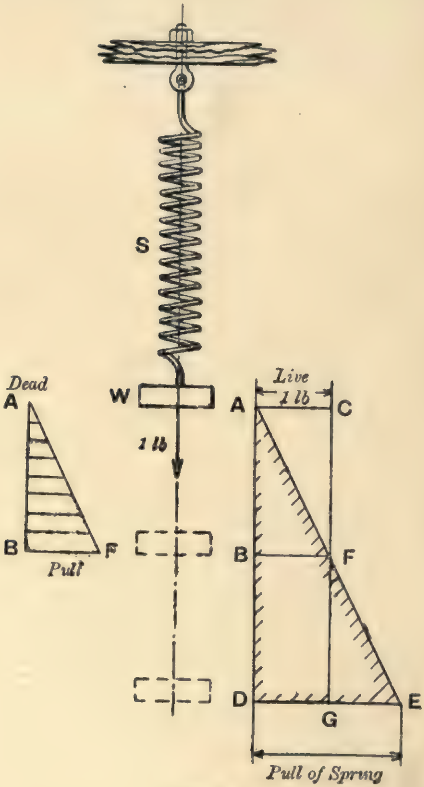


Fig. 7.

In Case 2 the spring will oscillate for a time, finally coming to rest when the tension on it is 1lb.

This proof, although for clearness we have only spoken of a spring, is evidently true of *any* elastic body whose resistance increases proportionally to the deformation. But all metals and alloys used in engineering may be regarded as perfectly elastic within the range of stress comprised between their limits of elasticity, experiments showing that specimens of them return approximately to their old positions after being pulled out or compressed slightly. Hence this investigation applies to all engineering details and structures, and fully explains why a live load is so much more destructive than a dead load. It also teaches us that if both live and dead loads act simultaneously on a structure, in order to find their combined effect we must add twice the live loads to the dead loads. In designing machinery we have in nearly all cases to consider the forces as live, and use a correspondingly high factor of safety.

Determination of External Forces.—In the preceding figures of engine beam, gun beams, and others, it will have been noticed that the external forces acting on the beam are indicated. Now, in designing a beam scientifically, the first thing to do is to find these forces, both as regards magnitude and direction. Occasions, unfortunately, arise when it is not possible to calculate the external forces with any approach to exactness, by reason of want of exact information, or the uncertainty and variability of the conditions of operation. The framing of machine tools forms an example to the point. Then the beam cannot be designed scientifically at all, but must be proportioned according to individual judgment, or from experience or precedent, or by the sure but expensive method of trial and error. But in many important cases, fortunately, there is no great difficulty in estimating the loads with tolerable accuracy; and it is with these cases alone that we shall concern ourselves.

Consider a few instances. In crane work the greatest load to be lifted is always known, and a rough estimate can be made of the weight of the structure. This information, taken along with a general arrangement of the machine, suffices to determine the external forces completely. In designing a bridge, again, the test load forms one of the items of the specification, and the weight of the structure itself admits of estimation; while the maximum wind pressure to which it is likely to be exposed is fairly well known from experimental observation. In the case of engine beams, and in steam and hydraulic work generally, the required load can be readily calculated from a knowledge of the areas of pistons or rams and the intensity of the steam or water pressure. Lastly, in estimating the forces which act on details that are rapidly set in motion and stopped, such as engine crossheads and gun trunnions, the extremely important influence of the inertia of matter must not be lost sight of. Knowing the mass of the moving parts and their acceleration at any instant, corrections for the inertia reaction can be made by applying Newton's Second Law of Motion, viz , that Force = Mass × Acceleration.

We shall find it convenient to divide external forces into two kinds—namely, *applied* forces and *reactions* or dead resistances. An example will best show the distinction. The weight of a train passing over a bridge is an applied force; while the upward thrusts of the two abutments, which oppose and balance the weight of the bridge and train, are reactions. Reactions are always mutual, never occurring as solitary forces. We have seen that in designing machinery the applied forces cannot in all cases be estimated; but if known, the corresponding reactions can be found by invariable rules. Assuming, then, that the applied forces have already been ascertained by a preliminary calculation or otherwise, we now proceed to consider the question of how to find the balancing forces or reactions.

Examples.

Case of Parallel Forces.—To take a simple example, suppose we have a beam or lever with equal arms (Fig. 8), weighing 2 tons, having a force of 10 tons acting at each end: What is the reaction

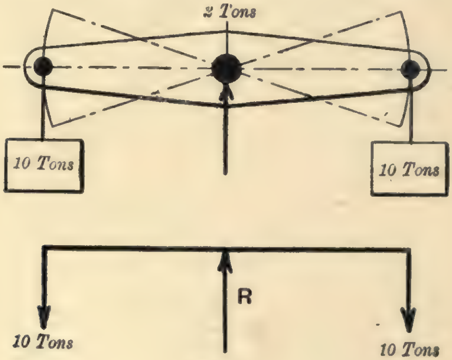


Fig. 8.

or back-thrust of the bearings supporting it? The lever may be regarded as the scale beam of a large balance or weighing-machine.

The universal rule for vertical forces is that the sum of the upward forces is equal to the sum of the downward forces. Here there is only one upward force—namely, the required reaction R; and there are three known downward forces.

$$\therefore R = 10 + 10 + 2 = 22 \text{ tons.}$$

Note that the *length* of the beam has nothing to do with the matter, except as influencing its own weight.

To modify this problem a little, so as to show the relation between two modes of loading and supporting, take the case of a beam of 2 tons weight, resting on supports at the ends, and carrying a load of 20 tons at the middle, as in Fig. 9. What are the pressures on the supports?

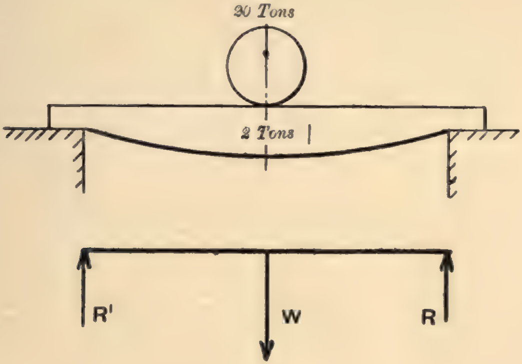


Fig. 9.

The downward pressures on the supports are equal and opposite to the reactions or upthrusts of the supports, say R and R′. Hence

$$R + R' = 20 + 2 = 22 \text{ tons.}$$

But since the load is in the middle, and the beam itself is supposed to be symmetrical, R must equal R′.

$$\therefore 2R = 22, \text{ or } R = 11 \text{ tons.}$$

Thus the pressure on each support is 11 tons.

When the arms are unequal (Fig. 10), as in the case of an air-

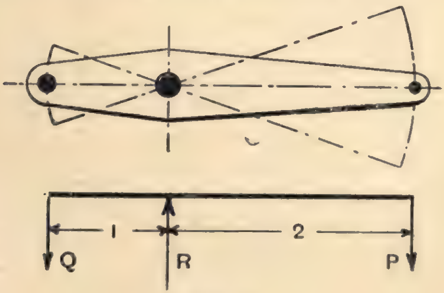


Fig. 10.

pump lever, where the stroke of the engine piston is generally twice that of the pump bucket, we still have

$$R = P + Q,$$

disregarding the weight of the lever, as we can always safely do in light work. With the given ratio of arms Q must equal twice P in order to balance.

In the corresponding case of a beam supported at both ends (Fig. 11), we find the reactions R and R′ by applying the principle of the lever—that is, by taking turning-moments about either end, say A, and forming an equation thus—

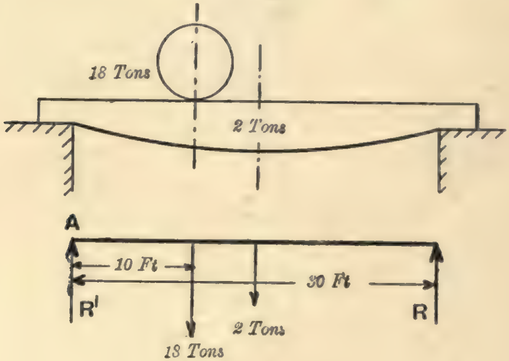


Fig. 11.

$$\left.\begin{array}{l}\text{Moments tending to turn}\\ \quad\text{the beam one way round}\end{array}\right\} = \left\{\begin{array}{l}\text{Moments tending to turn}\\ \quad\text{the beam the opposite way}\\ \quad\text{round.}\end{array}\right.$$

This principle is plainly true, seeing that the beam neither turns in one direction nor the other, but remains at rest under the action of the forces. Hence, in the present case,

$$R \times 30\text{ft.} = 18 \text{ tons} \times 10\text{ft.} = 180 \text{ foot-tons.}$$
$$\therefore R = 6 \text{ tons; and } R' = 18 - 6 = 12 \text{ tons.}$$

Thus, disregarding the beam's own weight, the reactions are inversely proportional to their distances from the load. The weight of the beam is taken into account by adding 1 ton to each reaction, the beam being symmetrical.

In the same way, taking another instance, if a 40-ton locomotive is standing on a bridge of 400ft. span, at a distance of 100ft. from one end, we find that the part of the engine's weight borne by the near abutment is 30 tons, while the other takes 10 tons only.

These are simple but most important examples, because the same principle applies in every case, no matter how many forces may act on the beam together. An interesting application of the principle of moments is seen in the 100-ton lever-testing machine (Fig. 12). The test bar A is placed between

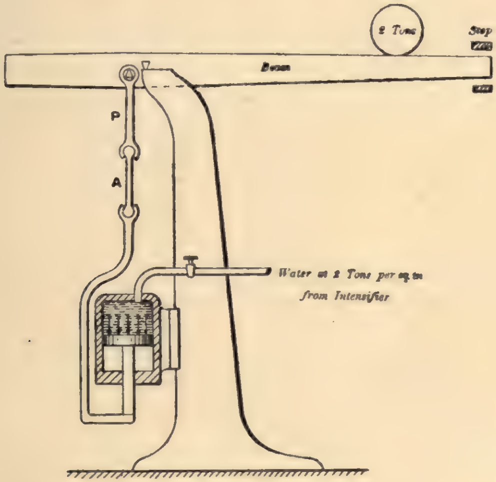


Fig. 12.

an hydraulic press and a lever acting as a steelyard, along which can be rolled a counterpoise or jockey weight of 2 tons. The load is applied to the specimen by the hydraulic press, and measured by the steelyard, its amount being deduced from the position of the weight. In order to gain sufficient sensitiveness, the fulcrum of the lever and the supports of the shackles are hard steel knife-edges, bearing on hard steel planes. The figure, of course, is purely diagrammatic. Actually there are crossheads above and below the cylinder.

Taking the jockey weight at 160 in. from the fulcrum, and the leverage of the hydraulic press as 4 in., let us find the pull on the specimen and the reaction of the fulcrum, neglecting the weight of the lever.

The equation of moments is:

$$P \times 4\text{in.} = 2 \text{ tons} \times 160 \text{ in.};$$
$$\therefore P = 80 \text{ tons, and } R = 82 \text{ tons.}$$

Examples of levers "centred" at one end and loaded at two other points are very common in practice, as in the case of lever safety valves, lever handles, and connections generally. An illustration on a large scale is seen in the 68-ton gun beam (Fig. 3). In the typical example represented in Fig. 13 the reaction of the pin R

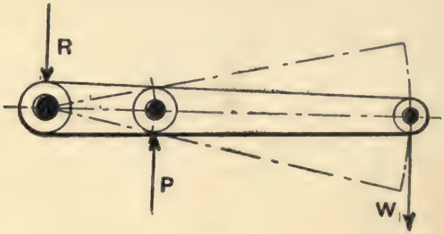


Fig. 13.

is the difference between P and W. The nearer P is to W the smaller is R, and when the two are directly opposite, the reaction vanishes entirely. There is no need, therefore, to fit such a large fulcrum pin as when P and W act on opposite sides of the fulcrum, if we have strength alone in view. For this reason the fulcrum pin in Fig. 3 has been made comparatively small.

Culmann's Graphical Method.—The above is usually the readiest and most accurate mode of proceeding, using a slide-rule for the arithmetical work; but if drawing instruments are at hand an interesting graphical method is also available, due to Professor Culmann of Zurich. It is especially useful when the forces are not parallel. To apply it, letter the spaces between the external forces, as in Fig. 14, according to Mr. Bow's excellent system. Set off $ab = 4$ tons, and $bc = 3$ tons, to any convenient scale. Select any point O, called the "pole," and join Oa, Ob, Oc. Produce the vertical lines of action of the forces, and, beginning on any of these lines, draw OA, OB, OC in the link polygon parallel to the corresponding lines in the polar diagram. Draw the closing side of the polygon OD, and the dotted polar line Od parallel to it. Then the required reactions are cd and da to the force scale. Also the intersection of OA and OC, in the link polygon, fixes the line of action of the resultant of the loads. A feature to be noted is the relationship of the three diagrams as regards lettering.

We shall see later that the shaded link polygon is a bending moment diagram, to a certain scale, which depends on the length of Ob chosen. Remembering this, it is generally advisable to make the polar distance Ob equal to ten linear units on the same scale as the beam. Thus, if the beam is drawn to a scale of an inch to a foot, then Ob should be 10 in. But sometimes it is more convenient to make $Ob = 20$ linear units, as here, in order to economise height on the paper. Apart, however, from ease of drawing and multiplying, the polar distance may be anything.

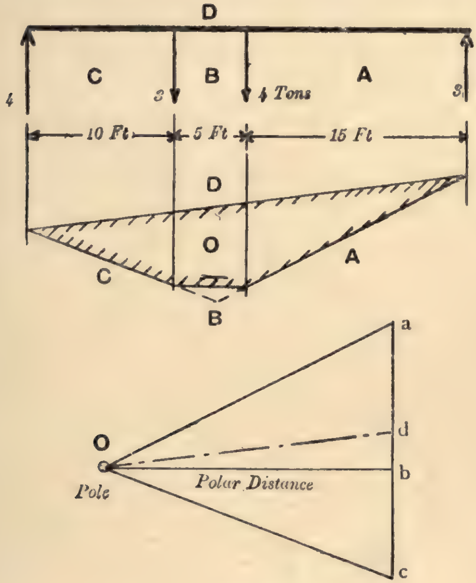


Fig. 14.

Intersecting Forces.—In continuing the question of how to find the external forces acting on a beam, we have next to consider how to proceed when the lines of action of the forces are not parallel, but meet at a point, as in the case of a jib crane (Fig. 4) or the bell-crank lever (Fig. 15) which we may assume drives a pump.

Suppose we have already estimated that the load on the pump rod will be 2 tons; and now, as a necessary preliminary to designing the L lever, pins, and supporting bracket, we want to find the pull P required to drive the pump, and the reaction R of the bearings.

Since the arms are in the ratio 2 to 1, the force P must be half of W, and therefore, disregarding friction, equal to 1 ton. If we took into account the friction of the bearings, we should have to increase P by, say, 10 per cent. For any other ratio of arms we should have:

$$P \times P\text{'s arm} = W \times W\text{'s arm},$$

just as for a straight lever.

To find the reaction R, produce P and W to meet at D, and join CD. Measure off, to some force scale, $Dw = 2$ tons, and complete the rectangle $Dwrp$. Then rD represents the reaction of

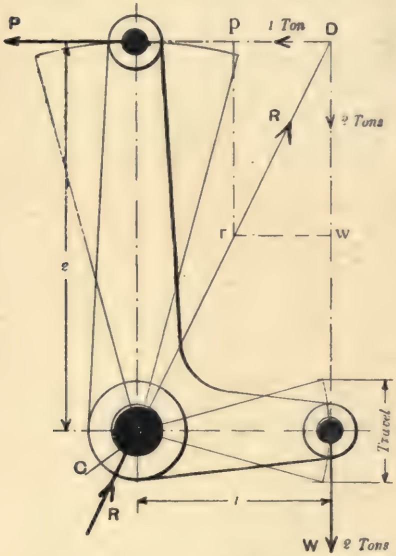


Fig. 15.

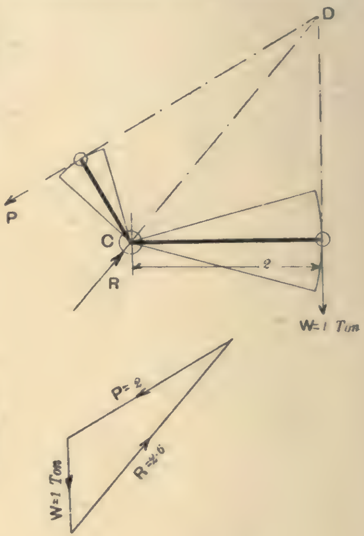


Fig. 16.

the bearing in direction and magnitude, by the well-known proposition of the parallelogram of forces. Also Dp represents P, which is thus found by another method.

A second way of finding the magnitude of R is to use arithmetic and Euclid I. 47, by which we have:

$$R^2 = P^2 + W^2 = 1 + 4$$
$$\therefore R = \sqrt{5} = 2{\cdot}24 \text{ tons.}$$

Incidentally we learn from this result that the brasses supporting the rocking-shaft or gudgeon should be divided, *not* horizontally, but at right angles to the line CD; because it is a fundamental principle of machine designing that, as far as possible, all bearings should be adjustable in the direction in which they wear, which is the line of resultant thrust. In the present example, however, this line is not quite inclined at a constant angle, owing to the motion of the lever. Further, the effect of the weight of the lever and connections would be to make the actual line of thrust more nearly vertical.

Occasionally we have to deal with forces meeting at an angle greater or less than 90°, as in Fig. 16. In this case, given the load W, say 1 ton, and the direction of P, our object is to find the balancing forces completely. Produce the lines of action of P and W to meet at D, and join CD. We have now 3 forces meeting at a point and keeping the lever in equilibrium. Hence these forces can be represented in magnitude and direction by the sides of a triangle taken in order. Draw this triangle by setting off W equal to 1 ton and drawing parallels to the other forces. This fixes P and R in both magnitude and direction.

The final case is when we have to find the reactions due to any number of intersecting forces, some of which may be parallel. This can readily be done by means of Culmann's graphic method already described for the special case of parallel forces.

Pressure on Guide Bar.—The following is an interesting example of the mode of finding the load which comes on a beam: A large rolling mill engine has a pair of cylinders $60\frac{1}{2}$ in. diameter and 6 ft. stroke. The connecting-rods are 15 ft. centres in length. The pressure due to the obliquity of the connecting-rod will come on either top or bottom guide bars, according to the direction of rotation of the crankshaft. The bottom bars are probably supported throughout their entire length, but the top bars may be taken as supported only at the ends. Assuming that the steam pressure at half-stroke is 80lb. per square inch effective, what will be the transverse pressure or thrust of one pair of slide-blocks on the top bars? (See Fig. 17.)

Load on piston = area × steam pressure,
= 2875 sq. in. × 80lbs. per sq. in.
= 230,000lbs. = 102 tons.

The thrust on the guide bar will be greatest when the connecting-rod is most inclined to it—that is, when the crank is vertical. This is not quite at half-stroke, but the difference is negligible for our present purpose.

Taking the coefficient of friction as 0·1, the angle of friction will be that whose tangent is 0·1, or say 6°; which is therefore the inclination of the guide-bar reaction.

On drawing to scale a triangle (Fig. 17) whose sides represent the forces meeting at G, we find that the reaction of the top guide bar, and consequently the equal and opposite thrust, is 20·4 tons, which is the required total force. The weight, however, of the

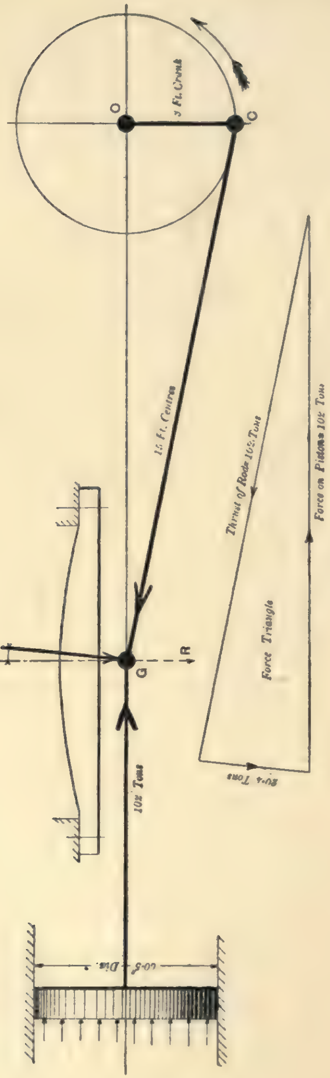


Fig. 17.

crosshead, motion-block, and rod would reduce this by a few tons. As there are two top bars to each cylinder, the load is equally divided between them. On the return stroke of the piston, the thrust will slope 6° on the other side of the vertical, because friction always *opposes* the motion. Hence, in designing the guide bars, one should take the thrust as vertical; but the end pressure due to the slide friction must be provided for in the supports. In the force triangle as drawn, the thrust of the connecting-rod happens to scale the same as the force on the piston.

If we choose to neglect friction, it is quicker to find the pressure on the guide bars by arithmetic than by drawing a triangle of forces. Thus, taking moments about the crankpin C, we have:

$$\text{Reaction of guide} \times \text{its arm} = \text{piston load} \times \text{its arm};$$

$$\text{or, } R \times GO = 102 \text{ tons} \times 3 \text{ ft.}$$

$$\text{But } GO = \sqrt{15^2 - 3^2} = \sqrt{216} = 14{\cdot}7 \text{ ft.}$$

$$\text{Hence, } R = \frac{306}{14{\cdot}7} = 20{\cdot}8 \text{ tons.}$$

Forces on a Bracket.—As another instructive exercise, let us find the external forces acting on a bracket required to balance a pull P of 1000 lbs. inclined at 30° to the vertical (Fig. 18), arising

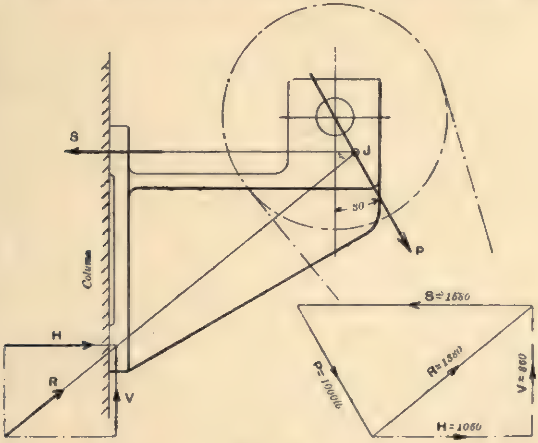


Fig. 18.

from the tension of a belt. Looking at the figure we see that to balance P there must be a horizontal pull S on the wall or column at the top row of bolts; also a horizontal thrust H at the bottom, and lastly a vertical force V coming on either the bolts or a lip on the column carrying the bracket. Now H and V have a diagonal resultant R, which must pass through the intersection J of the directions of P and S. We then have three balancing forces at J, and therefore easily find R and S by drawing to scale a triangle of forces. Resolving R horizontally and vertically, as shown in the force polygon, we finally get H and V, as shown.

A large part of all books on "Statics" consists of investigations and examples relating to the determination of forces required to balance certain given forces; which examples, however, are for the most part of a remarkably academic or unpractical character, useful only as mental drill. The present part of our subject, therefore, might easily be extended to great length; but sufficient practical examples have been now worked out to show how to proceed in the most commonly occurring cases.

Laws of Equilibrium.—Before leaving this matter, however, it will be useful to summarise the so-called general conditions or laws of equilibrium of a rigid structure or detail acted on by any number of forces in one plane. These are as under:—

(1) The geometrical or vector sum of all the external forces must be *nothing*.

(2) The algebraic sum of all the turning moments, taken about any centre of rotation whatever in the plane of the forces, must also be *nothing;* opposite directions being considered of unlike sign.

Under these circumstances the beam or other body will remain at rest or move without acceleration.

Condition (1) is satisfied if a *closed* polygon can be drawn to represent the forces, or if the algebraic sums of the vertical and horizontal components of the forces are severally *nought*, according as we choose to work graphically or analytically.

The commonest particular case of all is when a body is kept in equilibrium by *three* forces only. Then the force lines, if not parallel, are bound to meet at a common point (a useful fact to note), and the force polygon degenerates to a simple triangle of forces.

We have already made some use of these important principles to find the reactions induced by given loads, and numerous other applications will appear as we proceed.

CHAPTER III.

COUPLES. THE EQUILIBRIUM OF A BEAM. EXAMPLES.

Couples.—A pair of forces may act on a lever or other detail in such a way as to produce only a *turning* effect. Suppose, for instance, we have a pulley round which is coiled a rope kept taut by equal weights, as in Fig. 19. It needs little reflection to perceive that the two equal pulls, acting on opposite sides of the pulley, tend simply to rotate the latter, without causing any lateral pressure on the shaft. The effect is similar to that produced by two equally strong men

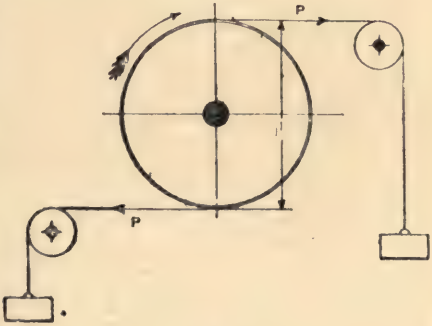


Fig. 19.

pushing at the ends of opposite capstan bars on board a ship. Such a pair of equal forces, acting on a body in opposite directions along parallel lines any distance l apart, forms what is called a "statical couple" or a "pure torque," the perpendicular distance between the lines of action of the forces being known as the "leverage" or "arm" of the couple. If, however, we cut away one half of the rope, as in Fig. 20, and make fast the cut end of the other to the pulley, we still have a turning effect; but the lob-sided or unsymmetrical action of the single applied force causes a pressure equal to it on the shaft bearings. The pull of the rope

Fig. 20.

and the reaction of the bearings now constitute a new couple of half the former valne, which tends to rotate the pulley about a new axis.

In considering the strength of beams, couples are of paramount importance; so that it is essential to secure clear ideas about them before going further. A couple is measured by the product of either force P and the arm l, the value Pl being styled the "moment of the couple." The word *moment* is here used in the sense of *importance*, as in the sentence, "It is a matter of no moment." This product is the same as the moment of either force about any point in the line of action of its mate.

A peculiarity of a couple is that the algebraic sum of the moments of its two forces, taken about any chosen point in their plane of action, is always the same, and equal to the moment Pl of the couple. From this it appears that a couple may be shifted anywhere in its plane without altering its rotational effect on a rigid body. Thus in Fig. 21 no difference is made in the turning effect by shifting the couple from position Fig. 20 to position Fig. 21. A further fact worth remembering is that any number of unbalanced forces acting in one plane are either equivalent to a single *force* or to a *couple*, this result being arrived at by successively combining the forces.

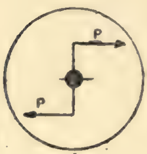


Fig. 21.

As regards the *sign* of couples, it is customary amongst physicists to consider a couple *negative* when it tends to rotate clockhand-wise the body on which it acts, and *positive* when it has the opposite effect. But Rankine, in his "Civil Engineering," p. 139, says: "The turning of a body is said to be right-handed when it appears to a spectator to take place in the same direction with that of the hands of a watch, and left-handed when in the opposite direction; and couples are designated as right-handed or left-handed according to the direction of the turning which they tend to produce." So that Rankine's right-handed couple is the physicist's negative couple. (See Thomson and Tait's "Elements of Natural Philosophy," p. 204.) Thus the matter is as yet in an unsettled state, no uniformity of practice prevailing.

Couples on a Crane.—A single couple cannot keep anything in equilibrium, because its two forces, though equal, are not directly opposed. *Two* couples, however, may. A very good example of the balance of two couples is presented by the jib crane shown in Fig. 4. Here the applied couple, tending to *overturn* the crane, consists of the 20-ton load at the end of the jib and the equal vertical reaction of the footstep, disregarding the weight of the crane itself. The arm of this couple is 30 ft., and therefore its moment is 20 tons × 30 ft. = 600 foot-tons, or ton-feet, as some modern writers prefer to say.

The *righting* couple, of opposite sign, is made up of the horizontal reaction of the footstep and the equal reaction of the top bracket. To preserve equilibrium, its moment must be 600 foot-tons also. If we take the arm of the righting couple as 20 ft., the magnitude of each horizontal reaction will be 600 ÷ 20 = 30 tons; the same result as we got by drawing a triangle of external forces.

Graphic Representation of Couples.—A statical couple can be represented graphically in exactly the same way as a force can; for, like the latter, it is a directed quantity or vector. This is done by drawing a line at right angles to the plane of its two forces, through any chosen point of reference, as in Fig. 22. Make the length of this line represent the moment of the couple, to some scale, and make its direction indicate the sign of the

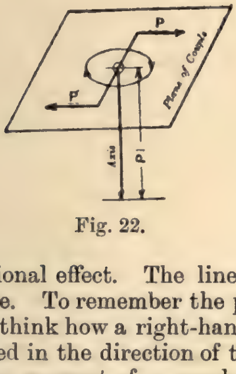


Fig. 22.

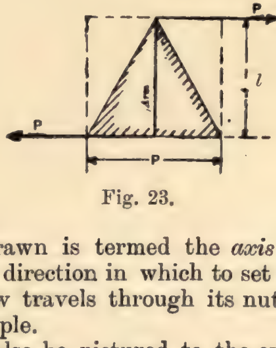


Fig. 23.

rotational effect. The line so drawn is termed the *axis* of the copule. To remember the proper direction in which to set off the axis, think how a right-hand screw travels through its nut when rotated in the direction of the couple.

The moment of a couple can also be pictured to the eye very well by twice the area of a triangle (Fig. 23), whose base is drawn to represent one of the forces P, and whose height is the arm of the couple. That is, the area of a rectangle of base P and height l represents the moment.

A final fact concerning couples, having an important bearing on graphic methods, is that any number of unbalanced couples acting on the same body can be reduced to a single resultant couple, just as forces can; the axes of the couples being taken instead of the lines used to represent forces.

Bending Moment and Resisting Moment.—The equilibrium of a loaded beam resembles a strife between two equal antagonists. The name of the assailant is "Bending Moment," and his opponent is known as "Resisting Moment." As these two antagonists are of exactly equal strength, neither prevails over the other; the result being a deadlock. The harder B.M. presses, the harder R.M. resists, though he is never the aggressor. Should, however, Bending Moment happen to be stronger than his opponent, the inevitable result will be yielding on the part of Resisting Moment, and finally total collapse or *failure.*

The "resisting moment" at any section of a beam is the true measure of the *strength* of the beam at that section, since it represents the beam's ability to resist its foe, the "bending moment." It might, then, be equally well styled the "strength moment"; but it is commonly referred to as the "moment of resistance of the section to bending."

In scientific language the bending moment on a beam at a given transverse section is defined as the algebraic sum of the

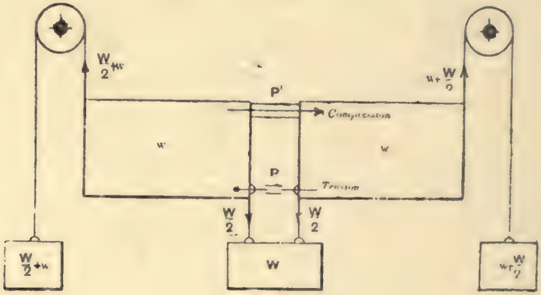


Fig. 24.

moments of all the external forces acting on one side of that section. It represents the influence tending to bend or deflect, and ultimately to rupture the beam. This bending moment is wholly *external* to the beam, its magnitude being governed only by the nature of the loads and their position along the beam. It has no reference to the cross-section or to the material of the beam.

On the other hand, the moment of resistance with respect to bending of a stated *section* of a beam, being the measure of the capacity of that section to resist the bending moment, is wholly *internal.* Its value depends on the size and shape of the cross-section, and on the kind of material composing the beam. It has nothing to do either with the length of the beam or the external forces, except indirectly. Thus the bending moment at a section measures the tendency of the beam to *break* there, while the resisting moment measures its tendency to *stand.*

Beam Model.—The forces acting on a beam can be very well illustrated and made more tangible by a simple model, as indicated in Fig. 24; the pulleys being fixed to uprights or to a wall. Here, small w denotes the weight of each half of the beam, and large W the central load. The width of the gap is much exaggerated. Working with such a model, it is found that a chain or string at the lower part of the section suffices to maintain equilibrium; while at the upper part a stiff block or packing piece is necessary, a chain being quite useless to prevent collapse. It is thus experimentally proved that the lower internal force P connecting the fibres must be a pull, and the upper internal force P^1 must be a thrust. Hence the lower part of a beam loaded at the middle and supported at the ends is in tension, and the upper part is in compression.

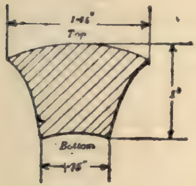


Fig. 25.

This fact is also clearly brought out by bending a substantial square bar of mild steel on a testing machine. When the originally straight bar has assumed a U shape, it is seen that the upper part of the section is distinctly bulged out, as in Fig. 25, while the lower part is thinned. The figure was sketched from an actual mild-steel bar 1 in. square originally, which had been so bent double. A similar effect is easily observed by bending a square piece of india-rubber.

Ideal Example. — In order to form a proper conception of the relation between bending moment and resisting moment it will be advisable, in the first instance, to consider a very simple hypothetical case of the strength of a loaded beam. Imagine, then, a girder (Fig. 26) 10ft. deep and 40ft. span, with equal top and bottom flanges composed of mild-steel plates 2 ft. wide and 1 in. thick, connected by a web so thin that its strength may be left entirely out of account. The problem before us is to ascertain what load, say W lb., this girder will safely carry at the middle

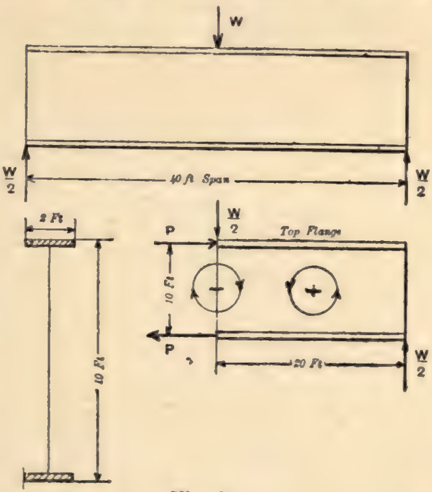


Fig. 26.

of its span; allowing a safe stress of 10,000lbs. per square inch on the material both for tension and compression, and regarding the weight of the girder itself as negligible.

As one-half of the load is necessarily transferred to each abutment, by the principle of symmetry, we isolate in imagination the right-hand half of the girder, as shown, and consider what forces keep it in equilibrium. These forces form two pairs. The first pair consists of $\frac{1}{2}$W, acting downwards at the centre, and its comrade the equal upward reaction of the abutment. These two forces together constitute an *overturning* couple whose moment is $\frac{1}{2}W \times 20$ft. = 10Wft.-lbs. This is the moment of the bending couple, or, shortly, the bending moment at the centre of the girder. The second pair of forces consists of a horizontal thrust of Plb. on the top flange, and an equal horizontal pull on the bottom flange. These two constitute a *righting* couple whose moment is very nearly $P \times 10$ft.; which is the moment of the resisting couple, or the moment of resistance.

Now, in order to preserve equilibrium, the righting couple must be numerically equal to the overturning couple, and of opposite sign; so that the required relation between P and W is given by the equation $P \times 10 = 10W$, from which $P = W$; a first important result that could not readily have been foretold.

Further, we know that the horizontal force P must be equal to the cross-section in square inches of either flange, multiplied by the stress per square inch allowed on the material; consequently—

W or P = width × thickness × stress.
= 24in. × 1in. × 10,000lbs. per square inch.
= 240,000lbs.
= 120 American tons, or 107 English tons.

This is the safe central load sought. It is plain that the moment of resistance might be greatly increased by deepening the girder, without using any more material in the flanges.

This mode of calculation gives a close enough approximation when applied to deep girders which have nearly all the material concentrated in the flanges, such as lattice girders, the arm of the resisting couple being taken as the distance between the centres of gravity, or *centroids*, of the flanges, and the stress per square inch regarded as uniform all over the flange section.

Practical Examples.—(1) A road bridge over the Danube at Vienna has four spans, each of about 260ft. The main girders are of the lattice type, 24ft. deep, each flange consisting mainly of four wrought-iron plates $\frac{3}{4}$in. thick and 44in. wide. We propose to get a rough idea of what bending moment each girder will safely withstand, taking the safe maximum stress at 8000lbs. per square inch.

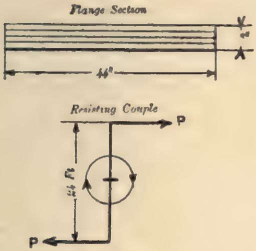


Fig. 27.

The area of each flange (Fig. 27) is 44 × 3 = 132 sq. in. This section of metal will safely stand a pull of 132 × 8000lbs. = 470 tons, say. The arm of the resisting couple is, say, 24ft., though really 3in. less. Hence the moment of resistance is 470 tons × 24ft. = 11,280 foot-tons. In smaller work it is usually advisable to reduce such moments to inch-tons rather than to leave them in foot-tons.

The value of the resisting moment just found must evidently also be that of the bending moment which the section can safely withstand, the one being always equal to the other. Observe that the *span* of the girder has not entered into the question at all.

(2) As another instructive example, we shall next calculate the approximate moment of resistance to bending of a beam having the section shown in Fig. 28, the angle irons being $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$in. A stress on the metal of 3 tons per square inch is allowed.

We shall neglect the unshaded part of the web altogether, and regard the thrust on the top flange and the pull on the bottom one as concentrated at the respective centres of figure or centroids of the flange sections.

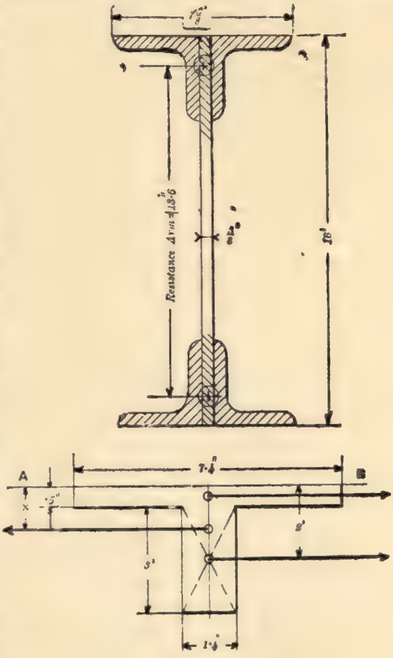


Fig. 28.

To find the position of the centroid of either flange section, we reduce the *actual* shape to the more simple geometrical form shown in the lower diagram, and take area moments about the axis AB, thus :—

$$x \times \text{whole area} = (\tfrac{1}{4}\text{in.} \times \text{area of upper rectangle}) + (2\text{in.} \times \text{area of lower rectangle})$$

Now, upper area $= 7{\cdot}4 \times 0{\cdot}5 = 3{\cdot}7$ sq. in.
And lower area $= 1{\cdot}4 \times 3{\cdot}0 = 4{\cdot}2$ sq. in.

So that whole area of each flange $= 7{\cdot}9$ sq. in.

It is hardly worth while using a planimeter to find these areas in a simple case like the present.

Putting these values in the above equation gives

$$x \times 7{\cdot}9 = (0{\cdot}25 \times 3{\cdot}7 + 2 \times 4{\cdot}2) = 9{\cdot}32,$$

$$\therefore x = \frac{9{\cdot}32}{7{\cdot}9} = 1{\cdot}18 \text{ inches.}$$

To get the length of the resistance arm we subtract twice this distance from the depth of the beam, thus:

Arm of couple $= 16 - 2{\cdot}36\text{in.} = 13{\cdot}64\text{in.}$

If now we *assume* that the stress on the metal is uniform over the shaded areas, then the total thrust on the top flange and the pull on the lower flange will each be equal to: Area $\times$ stress $= 7{\cdot}9 \times 3$ tons $= 23{\cdot}7$ tons. And the approximate moment of resistance will be

$$\text{Horizontal force} \times \text{arm} = 23{\cdot}7 \text{ tons} \times 13{\cdot}6\text{in.} \doteq 322 \text{ inch-tons.}$$

The accurate value, calculated by a more elaborate method, is 288 inch-tons; so that this is only to be regarded as a very rough approximation.

Taking the value of the resisting moment as 320 inch-tons, let us next find roughly what load this beam would safely carry at the centre of a 24ft. span.

The moment of the bending couple, or the bending moment at the centre, is half the length of the beam multiplied by either reaction. As this product must be equal to the resisting moment just found, the equation of moments is $\tfrac{1}{2}W \times 12 \times 12\text{in.} = 320$ inch-tons.

$$\therefore W = \frac{320}{72} = 4{\cdot}4 \text{ tons, say.}$$

This handy method is near enough for some purposes; but before leaving it, a word of caution is necessary. It should only be adopted in cases where the flanges are *thin* relatively to the total depth of the beam. When applied to solid beams, the method gives results which are altogether erroneous; because in their case the assumption that the stress is uniform all over the section is far from being true.

CHAPTER IV.

STRESS, STRAIN AND ELASTICITY.

Stress and Strain.—There are still a good many engineers who fail to discriminate between the technical meanings of the oft-recurring words stress and strain; the result being great ambiguity and often confusion of ideas. In popular language, no doubt, these terms are used indifferently the one for the other; but to the mind of a scientific man they convey entirely different ideas, the essential distinction between which it will be wise to point out and emphasise before attempting to deduce a formula for the strength of solid beams of rectangular or any other section.

As a scientific term the word *stress* is regularly used, unfortunately, in at least two distinct senses, as determined by the context. Having in mind the first of these, an engineer, in speaking of the stress on the tie-rod of a jib crane, for instance, refers to *the whole internal action* on the tie, regarded in its complete dual aspect of a pull inclining upwards towards the jib head, and an equal and opposite pull inclining downwards towards the attachment of the tie to the pillar. Much the same is meant in speaking of a stress diagram for a loaded bridge girder; such a diagram being a geometrical figure made up of straight lines whose lengths represent to scale the total pulls and thrusts on the several members of the girder. The stress diagram of a particular structure might be more fully called "the diagram of the external and internal balanced forces acting upon and within that structure." Sometimes this figure is referred to simply as the *force* diagram, but the name *stress* diagram is far more common.

As used in this primary sense, a formal definition of the term under consideration may be framed as follows:—"*Stress* is the mutual action between two bodies or portions of matter, taking into account the whole action between them." It is variously described as attraction, repulsion, tension, pressure, and shearing, according to the mode of action. This is the most comprehensive meaning of the term. It includes what appears to be (but probably is not) action at a distance, as gravitation and magnetic attraction.

The definition of the term given by Unwin * in relation to the strength of materials, reads:—"Stresses are the molecular actions within the material which are called into existence by external forces or loads, and which resist deformation."

Both the above definitions are rather academic, as exact definitions usually are. For practical purposes it will be sufficient, so far, to regard the stress on a bar as the whole *resistance* of the bar to external forces. If there are no external forces, then there is no stress.

The second sense in which engineers employ the word stress is "resistance per unit area of cross-section." Thus, in saying that the safe stress for steel bars in tension is about 10,000 pounds per square inch, one uses the term in this sense. To distinguish the present from the first named, we may speak of "intensity of stress" or "stress intensity," but as a matter of fact the word intensity is usually omitted.

A similar double usage exists with regard to the term "pressure," which may mean either the total pressure on a piston, say, or only the intensity of pressure, measured in pounds per square inch.

The collateral term "strain" is also scientifically used in two entirely distinct senses. The first of these is "any alteration of size or shape of a body subjected to a stress," or in one word, "any deformation." The second sense is "the change of length per unit length." Both of these meanings are correct, though the former is much the more comprehensive of the two. Some writers, however, refer to the latter as the more *exact* definition of longitudinal strain; but it is preferable to regard it merely as the more *restricted* usage. It is one much in favour among physicists.

Professor Rankine introduced the modern distinction between stress and strain. In an essay on the strength and qualities of wood and metals, written in 1868, that illustrious investigator says: "*Stress* means at once the intensity of the *load* tending to alter the shape of a solid body, and the intensity of the equal and opposite *resistance* which the body opposes to that load. The word *strain* is commonly used, sometimes in the same sense with the word *stress*, and sometimes to denote the *measure of the alteration of shape* corresponding to a given stress. In precise language it is necessary that each word should have but one meaning; and therefore in the present essay the word *strain* will be used to denote *alteration of shape*."

It appears then that stress and strain stand to one another in

* "Machine Design," p. 25.

the relation of cause and effect, longitudinal strain being the alteration of length *caused* by a longitudinal or direct stress. In the language of a useful colour convention, one may say that stress belongs to the *red* order of ideas, and strain to the *blue*.

Elastic Limit.—To illustrate this matter, consider the so-called stress-strain diagram for a mild-steel bar pulled asunder in a testing machine, as drawn by an autographic test-recorder. The ordinary form of stress-strain diagram for *tension* specimens is shown in Fig. 29. Autographic diagrams for *compression* tests are seldom drawn. In studying the elastic properties of a material,

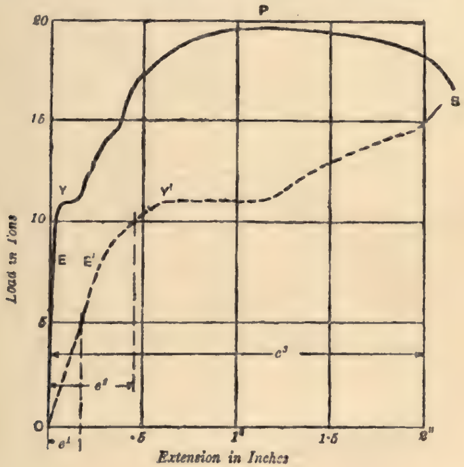


Fig. 29.

however, it is convenient to consider an ideal combined diagram, as in Fig. 30, showing both tension and compression features.

The first effect of the pull on the bar is to cause a state of stress, tending to separate the particles; which is accompanied by a definite small increase of length or stretch, easily seen by experimenting on a piece of india-rubber, though invisible in the case of a steel bar. For clearness, therefore, part of the curve is redrawn in dotted lines to a greatly exaggerated horizontal scale. As the line OE, in Figs. 29 and 30, is practically straight, it follows that the pulls and stretches are proportional within a certain range of load whose upper boundary is the ordinate of the point E, which marks the "elastic limit" with regard to tension of the steel bar being tested. An alternative name for this notable stress is the "limit of elasticity." The lower elastic limit point *e* in Fig. 30 refers to compression. Thus there are two limits of elasticity for each kind of material. The fact here graphically shown, that stress varies directly as strain within the limits marked by E and *e*, is known as Hooke's law, after an investigator of that name, contemporaneous with Newton. Numerically stated, this law asserts that if 1 ton stretches the test-bar $\frac{1}{100}$in., then 2 tons will stretch it twice that amount, and so on.

Thus the elastic limit of any material, as regards either tension or compression, is the greatest stress per square inch that it will stand without permanent deformation or "set." So long as the

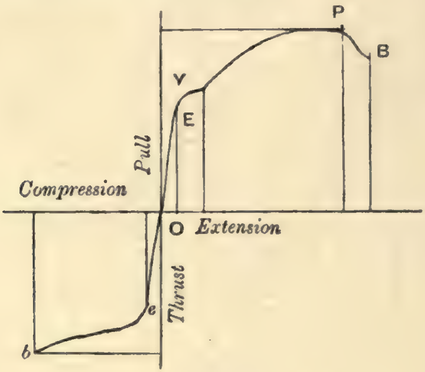


Fig. 30.

stress does not exceed this limit, the bar springs back to its original length as soon as the load is removed. It is a curious experimental fact, however, that the tensile elastic limit, though fairly constant for a particular material under normal conditions, can yet be artificially raised to almost any extent short of the breaking stress by subjecting the material for several hours to a stress exceeding the normal or "primitive" elastic limit preparatory to a fresh application of the load.* But of course in ordinary commercial testing this is never done, the whole test only occupying a few minutes. One should remember that the normal tensile elastic limit for mild steel in tension is about 15 tons per square inch, and for untempered tool steel it is about 38 tons on the inch.

* This peculiarity is sometimes referred to as the phenomenon of elastic hysteresis, being somewhat analogous to that of magnetic hysteresis or "lag."

Referring again to Figs. 29 and 30, it will be noticed that the line *curves* slightly between E and Y. This indicates a deviation from the proportionality of stress to strain. There appears a marked jump or singularity in the curve at Y, this being styled the "yield point." But in diagrams actually drawn by the testing-machine, the yield point Y is much more clearly defined than the elastic point E, and differs in position very little from it. Hence, in commercial testing, Y is commonly regarded as the elastic limit point. The elastic limit of the material is then defined as the load represented by the ordinate of Y, divided by the original cross-section of the bar.

After E is passed, the material is said to be stressed beyond its elastic limit, the elongation being partly plastic and partly elastic —that is to say, on releasing the bar from its load, it only partly springs back. The precise extension for any increase of load now depends to a great extent upon the length of *time* during which the load is allowed to act. The more rapid the loading, the steeper the curve.

The point P, where the maximum load is reached, marks the *plastic* limit. About this point begins the final *local contraction*, which immediately precedes the fracture of the bar, as shown in Fig. 31. Then the bar loudly breaks, even though the load is relieved by running back the jockey weight, as indicated by the drooping end of the curve. This instructive diagram vividly teaches the important lesson that stress is decidedly *dis*-proportional to strain, outside of the elastic limits of the material.

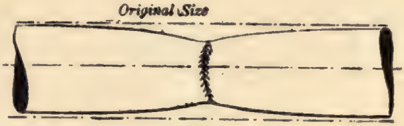


Fig. 31.

Stress and Strain (Resumed).—Now, in calling Fig. 29 a stress-strain diagram, what do we signify by the terms stress and strain? Evidently by stress we mean either the total pull on the specimen or the entire resistance at any instant of the latter; and by strain we mean the total extension in inches of the test-piece up to that instant. Thus the terms are used in their broader senses, or, as some would perhaps prefer to say, in their *cruder* senses.

There yet remain for more detailed consideration the other significations already mentioned which this rather ill-used pair of technical terms may properly bear. These meanings are more subtle, and need some nicety of discrimination. The *stress* on a bar at any instant, in the sense of resistance per unit area, is found by dividing the whole pull in tons or pounds by the area in square inches of the cross-section. It is thus easy to deduce a new scale of stresses in any stress diagram to accord with the new definition.

Strain is rather less easy to realise and to measure. It is the ratio of the extension of the bar under the pull to the original length of the bar, and is therefore always a small fraction. Put another way, strain is the fraction of its length by which a bar lengthens under a given tensile stress. So defined, strain is *not* measured in inches, nor in any other linear unit, because the ratio of one length to another is simply a pure fraction. But strain can be expressed as so much per cent., like any other fraction, such as efficiencies.

It finally appears, then, that to speak of a strain of a hundredth of an inch is to talk nonsense, according to the latest definition of strain; and there are not a few scientific men who would ignore any other meaning, labelling as ignoramuses all those who measure strains in inches. Nevertheless, engineers and shipbuilders find the word strain a very handy one to express any sort of deformation, and many of them ignore the more restricted academic usage of the term altogether; which, however, is more convenient for calculation. In the midst of this chaos it is therefore necessary, as yet, always to scan thoughtfully the context or setting of the word before one can decide what a man exactly means when he chooses to use this much-abused term, *strain*.

By looking at the exaggerated stress-strain diagram dotted in Fig. 29 it is readily seen that the value of the strain, due to a fixed increase of load, changes continuously after the elastic limit is passed, and that the extension increases enormously faster than the load.

Thus, for the first 5 tons of load put on to the test-bar,

$$\text{the strain} = \frac{\text{increase of length}}{\text{initial length}} = \frac{e_1}{L};$$

and for the next 5 tons put on,

$$\text{the strain} = \frac{\text{stretch due to second increment}}{\text{previous length}} = \frac{e_2}{L+e_1};$$

and similarly for the third 5 tons,

$$\text{the strain} = \frac{e_3}{L+e_1+e_2}; \text{ and so on.}$$

We thus get the values of the *average* strain for increments of 5 tons. These values, being very small fractions, are not plotted to scale in diagrams.

Ultimate Strength.—The conventional ultimate strength of a piece of material is found by dividing the load at which the test-bar breaks by the original section of the bar; but in finding the *actual* ultimate strength, the divisor is the final or reduced section at the point of fracture. In the case of ductile materials like wrought iron and mild steel, the difference between the true and the conventional strengths is very great, the local contraction of area before fracture, as shown in Fig. 31, being as much as 50 to 60 per cent. of the original area. There is no visible contraction in the case of cast-iron.

Factor of Safety.—Practical experience shows that the ordinary *working* load on any detail must be a good deal less than the *breaking* load. Now it is easy to find the latter, but often impossible to fix the former with much attempt at precision. Partly because of this uncertainty, it is necessary to divide the actual breaking stress by some number to get the *safe* working stress. This number is the factor of safety.

A variety of definitions of the factor of safety, however, may be given; according as the ultimate or only the elastic strength of the material is considered, and also the mean or the maximum working load. Primarily the factor of safety is the divisor found necessary to provide a sufficient margin of strength for unknown contingencies and uncertainties, arising from imperfect materials and workmanship, deterioration, and unforeseen or accidental forces. The *actual* factor is thus the ratio of the ultimate strength of the piece to the *greatest* load on it. But in designing machinery it is often very troublesome to find the greatest load that comes on a detail—as, say, the piston rod of an engine suddenly started. It is far easier to calculate the statical load, or the load when running steadily. Hence an *apparent* factor of safety is generally used, which is the ratio of the breaking strength to the steady working load.

The use of a too high factor of safety means waste of material, increase of first cost, and excessive weight. Too low a factor means breakdowns. We have, therefore, to steer between two evils. The proper factor to adopt in designing details of a particular class can only be found by the method of trial and error. But the present generation of engineers has to thank preceding generations for having already found out the right factors to use in all ordinary cases. Using the term factor of safety in the sense of the statical breaking stress divided by the ordinary working stress, Professor Unwin gives the annexed values of this factor as suitable for the different materials and classes of loads named :—

Material.	Dead Loads.	Live Loads.		Shocks.
		Case 1.	Case 2.	
Cast iron	4	6	10	15
Wrought iron and steel .	3	5	8	12
Timber	7	10	15	20

As regards the live loads, Case 1 applies to stress of one kind only, either tension or compression; while Case 2 applies to equal *alternate* stresses of different kinds. The piston rod of a double-acting steam-engine is an example of a detail subject to such alternating stresses. The shocks referred to in the last column are probably such as those that come on the teeth of wheels of rolling-mill trains, approaching the nature of blows. The usual factor of safety for machine details is 6.

Modulus of Elasticity.—This is another technical term often used, the meaning of which had better be explained here once for all. The name co-efficient of elasticity is sometimes employed instead. The word modulus literally means "measure;" and the whole expression means, in general terms, the measure of the force with which a sample of material tends to spring back to its original position after having been stretched or otherwise deformed within the elastic limits. If the tendency to spring back is very great, the modulus is high, and if small the modulus is relatively low.

The orthodox definition of modulus of elasticity is *the ratio of the increase of stress to the resulting increase of strain, within the elastic limits.* Hence, knowing the values of the stress and the strain, the elastic modulus of a stated material is got by simple division. Its value is nearly constant for the same sort of material.

The modulus of elasticity of a substance is sometimes said to be the stress required to double the length of a bar of that substance, on the supposition that stress remains proportional to strain until the extension reaches the extraordinary value named. But as that hypothesis is absurd, this definition had better be avoided. It is no use in finding the actual value of the modulus.

To distinguish the ordinary modulus from other elastic moduli with which we are not now concerned, it is referred to as the modulus of *direct* elasticity, or simply as Young's modulus, after its first investigator, Dr. Thomas Young.* The use of this

* A distinguished natural philosopher who died in 1829.

modulus is to enable us to calculate beforehand the strain that will be produced by a certain stress, and so to design beam springs

Material.	Modulus of Direct Elasticity.	
	Pounds per square inch.	Tons per square inch.
Tempered steel	36,000,000	16,000
Mild steel	30,000,000	13,400
Wrought iron	29,000,000	13,000
Cast iron	17,000,000	7,600
Fir and oak	1,500,000	670

with some exactness, and also to estimate the deflection of beams in general.

The figures above given may be taken as average values of Young's modulus for some common materials; but different specimens of nominally the same sort of stuff differ in elastic value by quite 5 per cent.

It is possible to find the modulus of elasticity of a material from a so-called stress-strain diagram, if its scales are known. For this purpose we need only the elastic or straight-line part of the diagram. Let A square inches be the section of the bar, L inches its initial length, and e inches the elongation for a pull of P tons, e and P being found from the diagram; then

Young's modulus = stress ÷ strain,

$$\text{or E tons per square inch} = \frac{P}{A} \div \frac{e}{L}.$$

Two methods of finding Young's modulus experimentally are in common use. The first of these is applicable only to long, slender wires made of the material whose modulus is sought. It consists in measuring the extension of the wire under the influence of a known load by means of the arrangement shown in Fig. 32. A and B are wires of iron, steel, copper, or other material, from 10 to 20 ft. long, hung side by side from the same secure support. A is kept taut by a constant weight W, while B carries a tray for the reception of the weights required to stretch it. A has clamped to it an engraved plate,

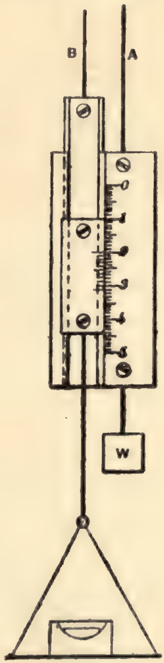


Fig. 32.

and B is fitted with a brass slide on which is engraved a vernier, by which readings can be taken to one-tenth of a scale division. The value of the modulus is calculated in the manner above described, the data being the diameter of the wire, its length above the slide, and the observed stretch of the wire due to a definite increase of load beyond that needed to keep the wire B taut.

In chap. iii. we have clearly shown that on bending a beam there is a shortening of the top fibres and an extension of the bottom ones. On this fact is based the second method of determining Young's modulus of elasticity. It consists in observing the deflection of a light beam, made of the desired material, due to a definite load placed at its centre. From this deflection and the dimensions of the beam, the value of E is found by aid of the formula :

$$\text{Deflection} = \frac{WL^3}{48EI},$$

which will be fully explained in due course. (See chap. xiv.)

Thus, the long wire affords a direct, and the elastic beam an indirect, method of evaluating the modulus E. Both methods should give the same result ; provided that the material be not in either case stressed beyond its elastic limits.

Examples.—In order to get exact ideas on the relation between stress, strain, and elasticity, it will be advisable at this stage to work through a few simple numerical examples.

(1) A vertical wire 100in. long is found to stretch 1in. under a certain load. Find the *strain* due to that load.

$$\text{Strain} = \frac{\text{increase of length}}{\text{original length}}$$

$$= \frac{1\text{in.}}{100\text{in.}} = \frac{1}{100} = 0{\cdot}01 = 1 \text{ per cent.}$$

(2) A bar of copper 10in. long is found to crush up 0·1in. under a heavy load. Calculate the *strain* due to that load.

$$\text{Strain} = \frac{0{\cdot}1\text{in.}}{10\text{in.}} = 0{\cdot}01 = 1 \text{ per cent.}$$

(3) Calculate what load is required to stretch 100ft. of wrought-iron wire, $\frac{1}{4}$in. diameter, to the extent of 1in.

$$\text{Strain} = \frac{1\text{in.}}{100 \times 12\text{in.}} = \frac{1}{1200}.$$

$$\text{Stress} = \text{strain} \times \text{modulus of elasticity}$$

$$= \frac{1}{1200} \times 29{,}000{,}000 = 24{,}167\text{lbs. per square inch.}$$

(This is about the tensile elastic limit of wrought-iron.) Hence the load sought

= stress × area of section
= 24,167lbs. per square inch × 0·049in.
= 1184 pounds.

(4) A wrought-iron tie, 20ft. long, is loaded to 17,000lbs. per square inch of section by a *dead* load. How much will it stretch under this load?

Stress = 17,000lbs. per square inch.

$$\text{Strain} = \frac{\text{stretch}}{\text{length}} = \frac{e}{240\text{in.}}.$$

$$\text{But } \frac{\text{stress}}{\text{strain}} = \text{modulus} = 29{,}000{,}000,$$

$$\therefore 17{,}000 \div \frac{e}{240} = 29{,}000{,}000;$$

$$\therefore e = \frac{17 \times 24}{2900} = 0{\cdot}14\text{in.},$$

which is the required extension.

(5) The front pillars of a vertical engine are of wrought-iron, 3in. diameter. The back pillars are of cast-iron, with a minimum section of 18 sq. in. All are 8ft. long. Each pillar has to sustain a working *live* load of 25,000lbs. Calculate the elongation of each pillar under this load.

Note.—As the load is *live* we must double the nominal load:
(1) For the *front* pillars:—

$$\text{Stress} = \frac{\text{load}}{\text{area}} = \frac{25{,}000\text{lbs.} \times 2}{7{\cdot}07 \text{ sq. in.}}$$

$$= 7070\text{lbs. per square inch};$$

$$\text{and strain} = \frac{\text{stretch}}{\text{length}} = \frac{e}{96\text{in.}}.$$

Now, strain = stress ÷ modulus;

$$\therefore \frac{e}{96} = \frac{7070}{29{,}000{,}000};$$

$$\therefore \text{Stretch } e = 0{\cdot}0234\text{in.}$$

(2) For the *back* pillars:—

$$\text{Stress} = \frac{25{,}000\text{lbs.} \times 2}{18 \text{ sq. in.}} = 2780\text{lbs. per square inch};$$

$$\text{also strain} = \frac{e'}{96\text{in.}}.$$

$$\text{Hence, } \frac{e'}{96} = \frac{2780}{17{,}000{,}000};$$

$$\therefore \text{ Stretch } e' = 0{\cdot}0157\text{in.}$$

The full consideration of the relation between stress and strain is a difficult subject, and for our present purpose unnecessary. Our attention has therefore been confined to direct longitudinal stress, strain, and elasticity. We have next to show the bearing of this preliminary matter on the question of the strength and stiffness of beams.

CHAPTER V.

STRENGTH OF RECTANGULAR BEAMS.

In chap. iii. it was shown how to calculate the resistance to bending offered by a deep girder composed of *thin* flanges connected by a web of negligible strength, as in Fig. 33. It was there explained that the moment of resistance of any section of such a girder is got by first finding the safe resultant *pull* (P) on the fibres composing the lower flange (which, from the nature of a couple, must also equal the resultant *thrust* on the top flange),

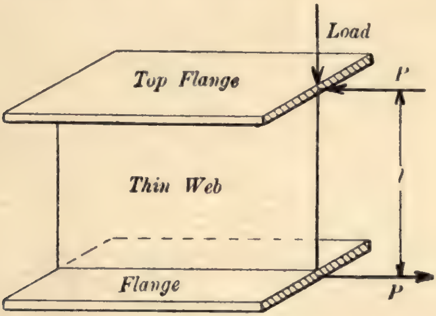


Fig. 33.

and then multiplying this force by the length of the resistance arm (l)—namely, the distance betwen the two forces just named, acting at the centres of area of the respective flanges. This distance (l) is sometimes called the *effective* depth of the girder.

There is no advantage in designing one flange stronger than the other, since a girder is sure to fail at the weaker flange before any demand can be made on the surplus strength of the other. But it does not follow that the two flanges should have exactly equal areas, even if the material be equally strong in tension and compression, because the compression flange must be stiff enough to resist buckling. The aim should be to make the top flange as capable of sustaining a *thrust* as the bottom flange is able to resist a *pull* of equal magnitude.

Preliminary Assumptions.—So far the matter looks simple enough. We next pass on to investigate the resisting capacity of solid beams of rectangular cross-section. This is more difficult. The question at once arises, How are we to find the magnitude and positions of the resultant thrust and pull when the girder has very *thick* flanges or no web at all? In this, as in all physical investigations of some difficulty, the problem must be simplified and brought within the range of practical mathematics by making certain assumptions and reasoning from them. Then, if our premises are quite correct, our conclusions will be absolutely true; but if our hypotheses are only roughly true, then will our inferences be in error to at least an equal extent. Consequently, it is of the greatest importance to realise the exact nature of the assumptions made to simplify the theory of bending, and to see how far they are justifiable.

In the first place, we assume or make the proviso that the material composing the beam shall not be stressed beyond its elastic limits, so ensuring that the stress may be always proportional to the strain. Experiments on test-pieces show that this proviso is easily satisfied by allowing a suitable factor of safety, say not less than 3, for a *steady* load.

Another fundamental assumption is that the modulus of elasticity of the material with regard to *compression* has the same value as the modulus with respect to *tension*—or, in other words, that the ratio of stress to strain is constant throughout the entire elastic range of the material.

Further, in the theory of bending it is taken for granted that the section of the beam is symmetrical about a vertical line through its centre of gravity, as in Fig. 34, and not as in Fig. 34A. So long as this condition is observed, the section of the beam may be of any shape whatever.

In the absence of any statements to the contrary, the *vertical* plane is taken to be the plane of bending of a beam, as in the case of ordinary girders.

Neutral Surface and Axis.—Since the upper fibres of a beam, as ordinarily supported and loaded, are undoubtedly crushed up a little, and the lower fibres pulled out a trifle, it does not seem a very unlikely supposition that somewhere between the top and bottom fibres there is a certain layer which is not strained at all. For it stands to reason that the self-same fibres cannot possibly be both shortened and pulled out at the same time; and so, between the compressed and extended fibres, there must be some sort of line of demarcation, either sharp or ill-defined: a region of no longitudinal *strain*, and therefore of no longitudinal *stress*. The vertical or shearing stress we leave out of account for the present, reserving it for future consideration in chap. viii.

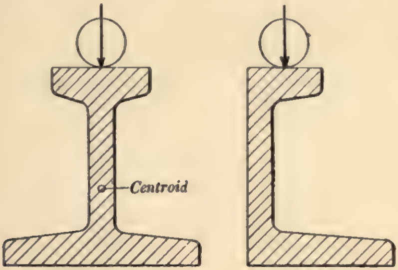


Fig. 34. Fig. 34A.

Obviously true as this argument appears to ordinary mortals, there are yet certain individuals who deny it. But, whether true or not, it is the custom of engineers to assume the existence of such a layer of unstressed fibres. It is called indifferently the "neutral *layer*" or the "neutral *surface*" of the beam.

The neutral layer of a straight beam is usually regarded as a *plane* surface, the separation between the shortened and lengthened

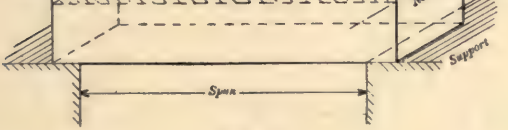


Fig. 35.

fibres being quite sharply defined. Then the transverse section of this unique surface is a straight line, which is styled the "neutral *axis*" of the section, *not* of the beam. (See Fig. 35.) The line of intersection of the neutral surface with the neutral axis of the section is referred to by some writers as the "neutral axis of the *beam*," and by others as its "geometrical axis."

The idea of a sharp surface of division is adopted rather as a mathematical convenience than as a physical fact. But even granting the clear separation of compressed and extended fibres, it must not be forgotten that in a loaded beam the neutral surface is not quite a plane, nor is the neutral axis exactly a straight line, though the deviation is so slight, for ordinary amounts of deflection, that it may be safely neglected in questions of strength.

Location of the Neutral Axis.—We have next to locate or find the position of the imagined neutral axis, the problem being; Given a certain plane transverse section of a beam, symmetrical about a vertical axis, to draw the neutral axis. There is a practical rule that enables us to do this easily—namely, find the "centre of gravity" of the section (otherwise known as the "centre of area," the "centre of figure," and the "centroid,") and through it draw a straight line perpendicular to the direction of loading. The line so drawn is the conventional neutral axis of the section.

This rule is much too often stated without the slightest explanation or qualification. But we require a reason for it, as the justice of the rule is not obvious on the face of it. Why should the neutral axis pass through the centroid of the section of the beam? The reason assigned by one writer is, "Because there is then just as much metal above the neutral axis as below it." He adds that "this can only hold when the material is as strong in tension as in compression"; thus implying that, for beams made of such a material as cast-iron—which is far stronger in compression than in tension—the neutral axis does *not* pass through the centre of area of the section.

As the assertion that the neutral axis *does* pass through the centroid of every section lies at the very root of the regular strength formulæ for beams, and as so many engineers seem to have no idea why it does, but are content to take the rule on trust, without troubling themselves about the rationale of the matter, it will surely be worth while looking carefully into the grounds of this assumption. Mere rules, divorced from principles, are to be distrusted, especially if they seem to be opposed to common sense.

If the assumption be allowed, then the position of the neutral axis must depend only upon the geometrical form of the section, and be entirely independent of the character of the material composing the beam. It cannot therefore matter in the least whether the section refers to a steel, a cast-iron, a lead, a wood, or an india-rubber beam. Bearing in mind the diverse elastic qualities of these several materials, one is led to ask, Is this reasonable? Hardly so; indeed, it seems most unlikely. The statement can scarcely be universally true; it surely needs some qualification. But still, the error will not be rectified by saying that the location of the neutral axis depends simply upon the ratio of the *ultimate* tensile and compressive strengths of the material; because in practice we have only to deal with material unstressed beyond its elastic limits, and therefore far removed from its breaking-point.

The exact statement of the case is this: The neutral axis of a stated plane transverse section of a loaded beam will pass through the centroid of that section, provided that the tensile modulus of elasticity of the material composing the beam is exactly equal to its compressive modulus of elasticity, but not otherwise. In other words, the material must be such that

$$\frac{\text{pull per unit area}}{\text{extension per unit length}} = \frac{\text{thrust per unit area}}{\text{compression per unit length}}.$$

In the case of materials for which the tension modulus is less than the compression modulus, the neutral axis should be placed rather nearer to the compressed face of the beam than the centre of area of the section. But, as a matter of fact, the shifting of the neutral axis due to this inequality is so small that it is commonly neglected altogether in practice. The whole question of the proper location of the neutral axis is one of considerable difficulty, and has given rise to much contention. (See, for example, a discussion in *The Engineer* extending over March, April, and May of the year 1897.)

In a small work on *Solid Beams and Girders*, published in 1872, Mr. W. Donaldson combats the ordinary theory of beams, and undertakes to prove that "the neutral axis does not necessarily in all materials ever pass through the centre of gravity, and does not maintain an invariable position, but that it is continuously changing its position with every change in the magnitude of the stress." He further shows mathematically that the true relation existing between the distance (h_c) of the neutral axis from the top of the beam and its distance (h_t) from the bottom, the stress (f_c) that actually comes on the top fibres and the stress (f_t) on the bottom fibres, the compressive modulus of elasticity (E_c) of the material and the tensile modulus (E_t), is as follows:

$$\frac{h_c}{h_t} = \frac{f_c}{f_t} \times \frac{E_t}{E_c},$$

which holds good both for rectangular and flanged sections. For rectangular sections alone it simplifies to

$$\frac{h_c}{h_t} = \sqrt{\frac{E_t}{E_c}}.$$

Reliable experimental data as to the relative values of the tensile and compressive moduli of elasticity for the ordinary materials of construction appear to be wanting. Authorities like Rankine, Unwin, and Lord Kelvin give only one value of the modulus of direct elasticity for each material. They do not discriminate between tensile and compressive values. Donaldson, however, says that for *wrought* iron $E_t = 28$ millions, and $E_c = 22$ millions of pounds per square inch. Concerning *cast* iron he says, in one place, that $E_t = E_c$, and in another place that the maximum value of E_t is about 14 millions, and of E_c about 13 millions of pounds per square inch.

Further, in Lanza's *Applied Mechanics* there is a table of experiments on *cast* iron, showing that for stresses up to two tons per square inch the tensile modulus is *greater* than the compressive modulus, but that for a stress of about three tons per square inch the two moduli are *equal* in value; and finally, that for a stress of seven tons to the inch, the values of E_t and E_c are about $9\frac{1}{2}$ and 12 millions of pounds per square inch respectively, thus showing a curious reversal as the stress is increased.

As regards wrought iron and steel, it is generally understood that the values of E_t and E_c are practically equal. Still, the need for further experiment and research is evident. The truth is that it is so much easier and cheaper to determine Young's modulus by stretching a long wire than by compressing a thick bar that experiments of the latter kind appear to have been sadly neglected.

Example.—To find the position of the neutral axis in the case of a wrought-iron beam of rectangular section 10 in. deep, assuming $E_t = 28$ millions and $E_c = 22$ millions of pounds per square inch (Fig. 36):

From the last formula we have

$$\frac{h_c}{h_t} = \sqrt{\frac{28{,}000{,}000}{22{,}000{,}000}} = \sqrt{1 \cdot 27} = 1 \cdot 128 \quad . \quad (1)$$

Also depth of section $= h_c + h_t = 10\text{in.}$. (2)

Substituting from (1) in (2) gives

$$\begin{aligned} 1 \cdot 128\, h_t + h &= 10 \\ \therefore 2 \cdot 128\, h_t &= 10 \\ \therefore \quad h_t &= 4 \cdot 7 \text{ in.,} \end{aligned}$$

which locates the neutral axis. This example is given only for the sake of illustrating the use of the formula, and so impressing an important theoretical point. Henceforth, according to custom, we shall suppose that the tensile modulus of elasticity has the same value as the other, and consequently that the neutral axis passes through the centroid of the section. The proof that it then does so needs careful consideration, but the following demonstration will perhaps make the matter intelligible.

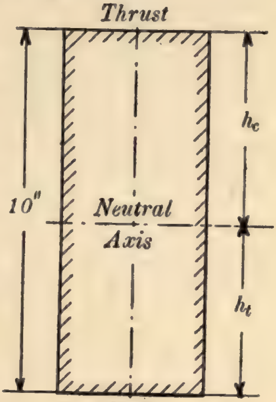


Fig. 36.

Proof that the Neutral Axis of a Section Passes through the Centre of Area.—(1) Take a small rectangular bar of india-rubber (Fig. 37), and bend it *slightly* by placing the forefingers F, F at the ends, and the thumbs T, T a little distance from the ends (Fig. 38). The equal pressures of the right-hand finger and thumb now form a couple, and those of the left hand form an equal and contrary couple, the forces being supposed to remain practically parallel. The deflection is greatly exaggerated in the figure. Under these circumstances the bending moment at any section between T and T is uniform, and equal to the moment of either couple. Consequently the curvature is uniform also; or, in other words, TT is an arc of a circle. Further, there is no shearing stress.

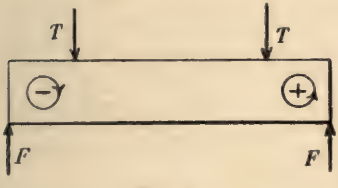


Fig. 37.

(2) Let us regard the little india-rubber bar as a small fraction, say the 100th part, of a long bar, which is imagined to be bent round by the external forces so as to form a complete ring of the same large radius as before.* This long bar, after the imaginary bending, is shown in elevation and section by Fig. 39. In looking at this rather misleading figure, however, one has to bear in mind that it is not drawn to scale, the thickness of the beam being magnified, let us say, 1000 times. Another difficulty in admitting this step arises from the fact that it does not seem possible to bend round a bar, however long, in this way by the

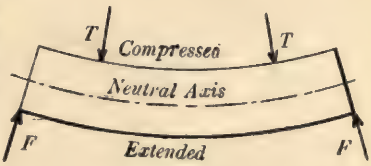


Fig. 38.

* This imaginary process is performed by Unwin in his " Machine Design," vol. i. p. 50.

action of a pair of couples, as theory requires. Hence it seems preferable to regard the ring either as merely the *geometrical* continuation of the outline of the material between T and T, or else as made up of a large number of small beams bent in exactly the same way.

The magnitudes and nature of the stresses in the bar will not be in the least altered by the supposed formation of the entire ring, so that we can safely reason on the ring as a whole.

(3) Now dismiss the introductory idea of an india-rubber bar, and substitute the metal or wood beam which it has served to exemplify. Let r be the radius of the layer of fibres unaltered in length by bending the beam—that is, the neutral layer. Before the long beam was bent every fibre was the same length as this neutral layer—namely, the circumference of a circle r inches radius. Hence,

$$\text{Original length} = \text{diameter} \times \frac{\text{circumference}}{\text{diameter}}$$

$$= 2r \times 3{\cdot}1416 = 2\pi r \text{ inches.}$$

(4) Consider the alteration in the length of a layer of fibres situated y inches *beyond* the neutral surface—that is, at a radius of $(r + y)$ inches. After bending, the new length of these fibres will be 2π times the new radius—that is, $2\pi\ (r + y)$ inches.

The amount of *stretching*, being the *new* length minus the *old* length, is therefore $2\pi\ (r + y) - 2\pi r = 2\pi y$ inches.

Similarly, the amount of *shortening* of the fibres situated at the same distance, y inches, *within* the neutral surface, being the *old* length minus the *new* length, is

$$2\pi r - 2\pi\ (r - y) = 2\pi y \text{ inches.}$$

(5) From the definition of the tensile modulus of elasticity (E_t), the relation between the tensile stress and strain is

$$E_t = \frac{\text{stress on fibres}}{\text{strain of fibres}} = f_t \div \frac{\text{alteration of length}}{\text{original length}}$$

$$= f_t \div \frac{2\pi y}{2\pi r}, \text{ from steps (3) and (4),}$$

$$= f_t \times \frac{r}{y} \text{ pounds per square inch.}$$

Similarly, the expression for the compressive modulus of elasticity is $E_c = f \times \frac{r}{y}$ pounds per square inch.

(6) If we assume that $E_c = E_t$, then from step (5) the compressive stress (f_c) will equal the tensile stress (f_t) in magnitude.

Granting this, we can drop subscripts and say that the common modulus of direct elasticity

$$E = f \times \frac{r}{y}, \text{ or that } f = E\frac{y}{r} \text{ pounds per square inch.}$$

This equation shows that the stress on any layer is proportional to its distance, y inches, from the neutral surface ; whose position, however, is not yet fixed. Any value may be given to y compatible with the size of the beam. It appears, then, that to assume the equality of the moduli is equivalent to assuming that the stress varies as the distance from the neutral axis.

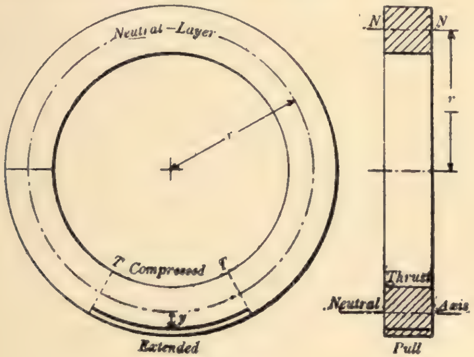


Fig. 39.

(7) Looking now at the transverse section of the beam (Fig. 39), and calling a square inches the area of a strip y inches from the neutral axis, we have total pull on strip = stress × area = $f \times a$, which, by the last result,

$$= E\frac{y}{r} \times a \text{ pounds.}$$

(8) Regarding compressive stresses as positive, and tensile stresses as negative, the total combined stress over the entire section of the beam is the algebraic sum of the stresses on every strip. This is conveniently expressed as

$$\Sigma\left(E\frac{y}{r} \times a\right) \text{ pounds.}$$

But the resultant thrust and pull together constitute a couple which balances the bending couple. Also, the algebraic sum of the two equal and oppositely directed forces composing any couple is zero. Consequently, the last expression = o.

(9) Now E and r are the same for all the strips of the section, and so may be placed outside the brackets thus:—

$$\frac{E}{r}\,\Sigma ay = o.$$

And as E and r are neither of them nothing, it must follow that the other term $\Sigma ay = o$.

(10) This final step is the crux of the whole question. The sole condition that a line may pass through the centre of area of a plane figure is that the turning moment of the figure about that line shall be nothing. This only amounts to saying that the sum of all the products, got by multiplying each little part by the distance of that part from the line in question, must be zero. But this is precisely what the last equation states to be the case here. It therefore shows that the neutral axis must pass through the centre of area of the transverse section of the beam, which is what we set out to prove.

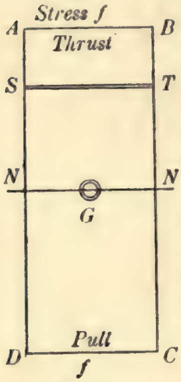


Fig. 40.

But remember that this result is only true if $E_c = E_t$. On this point Professor Cotterill remarks that "in perfectly elastic material the value of E is the same for compression as for tension;" and he goes on to regard all materials of construction as perfectly elastic within their limits of elasticity.

Resistance Areas.—It is a matter of some interest to see how the effective resistance of a transverse section of a loaded beam can be graphically represented. Let us confine our attention for the present to the simple rectangular section ABCD (Fig. 40). Through the centroid G of the figure draw the neutral axis NN. Let us agree that there is no stress along the central layer of fibres, also that the stress is a safe maximum of f tons per square inch at the top edge, and is equal in value at the bottom edge. The question now arises, How does the stress vary between AB and DC; or, in other words, is it possible to predict the stress on any intermediate layer?

This question has already been answered algebraically in the preceding demonstration; but it will also be advisable to obtain the same result geometrically, which may be clearer to some. Fig. 41 shows, in an exaggerated fashion, how the two sections at *ab* and *cd* of the beam, though quite parallel (and, say, 1in. apart) *before* the beam is bent, are afterwards inclined to one another at a perceptible angle; owing to the material crushing up slightly along *ac* and stretching along the edge *bd*.

This deformation of the beam can be beautifully seen by bending a piece of india-rubber of square section, after marking it at *ab* and *cd* with a penknife. Or of course a large india-rubber bar, 3 or 4in. square and 2 or 3ft. long, if available, will exhibit the features of interest even better. Experimenting thus, we observe with our own eyes that the lines *ab* and *cd* do *not* remain straight when the bar is bent nearly double, but that for a slight amount of bending, the deviation from straightness is not visible. We therefore conclude that the cross-sections whose elevations are *ab* and *cd* remain plane or flat for such small deflections as occur in actual beams.

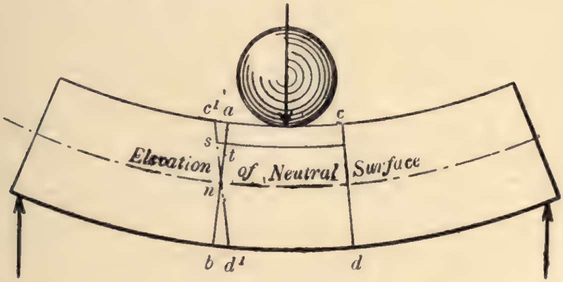


Fig. 41.

From these observations, together with the assumptions already stated, we can, by the aid of a little reasoning, deduce the stress on any layer whatever between the extremes *ac* and *bd*. Through the point *n* (Fig. 41) draw the line c^1d^1 parallel to *cd*. Next, for clearness, isolate the pair of triangles anc^1, bnd^1, and draw them in Fig. 42, alongside of cross-section ABCD. Then ac^1 represents the shortening of the top fibres of the unit block *abdc*, and bd^1 represents the extension of the bottom fibres per unit length of the beam. The strain of any other layer of fibres, such as ST in section, is represented by the length of the horizontal *st* opposite to it in the strain triangle (Fig. 42). So we have now ascertained how the *strain* varies between AB and CD, the strain of any layer being directly proportional to the distance of the layer above or below the neutral axis.

But we are less concerned with the variation of the strain than with the distribution of the *stress*. This information is given by one more step in the reasoning. As we have agreed not to trespass beyond the elastic limits of the material, we can safely say that the stress is proportional to the strain. Applying this fact, we finally conclude that the *stress* on any strip like ST is exactly proportional to the distance of that strip from the neutral axis. This result is clearly set forth in the stress triangles of Fig. 43; the compressive stresses, or pressures per square inch, being set off on the right, and the tensile stresses, or pulls per square inch, on the left of the vertical line. From this pair of stress triangles we can readily construct what is called the "resistance area of the section;" in fact, these triangles actually do form the resistance area of the section of a beam 1 in. broad.

The "safe resistance area of a section of a beam" may be defined as a geometrical figure which is drawn to represent, in magnitude

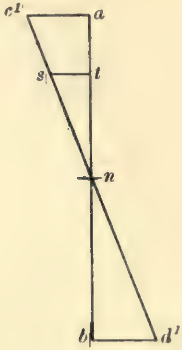


Fig. 42.

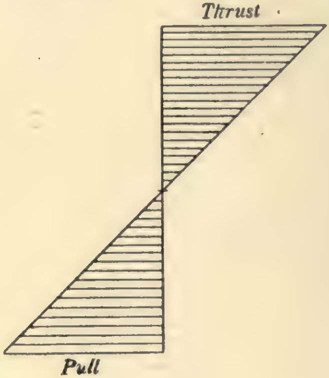


Fig. 43.

and sign, the resistance exerted by every layer of the beam at the section in question, when the extreme fibres on *one* side of the neutral axis are stressed to the greatest safe extent. It may be briefly styled the stress diagram of the section.

This geometrical figure is divided into two equal parts at the neutral axis. The upper area represents the resultant *thrust* on the section on the beam, and the lower area the resultant *pull*. Hence

$$\tfrac{1}{2}\,(\text{resist. area}) \times \text{resist. arm} = \text{mom. of resist. of section.}$$

It would be more consistent with the usual geometrical meanings of the terms *area* and *figure* to call this diagram the "*figure* of resistance" of the section. Then we could conveniently define the resistance *area* as "half the area of the figure of resistance," and we should have the simple relation, Resist. area × resist. arm = mom. of resistance.

It is usual to draw the resistance area or diagram, not exactly as in Fig. 43, but symmetrical about a vertical line, as in subsequent figures. This is merely a matter of convenience however. The method of drawing such a figure for any given breadth and depth of beam can be best explained by reference to a particular example.

Example.—To draw from first principles the resistance diagram of a section 12in. deep and 4in. wide of a steel beam in which the maximum stress allowed on the top and bottom fibres is 6 tons per square inch; also to deduce the moment of resistance of the section.

As an aid to clear thinking, let us, in imagination, isolate the right-hand half of the beam, and consider what forces keep it at

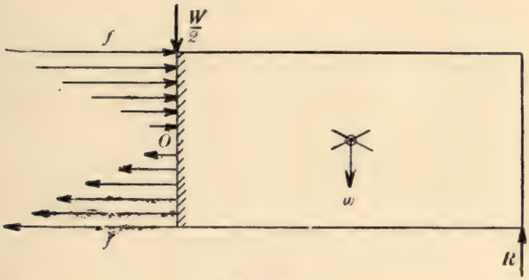


Fig. 44.

rest (see Fig. 44). These are (1) half the central load, (2) half the weight of the beam, (3) the reaction of the abutment, and (4) the resistance of the left-hand half of the beam. We are at present concerned only with the last of these.

The forces constituting the resistance of the beam are innumerable. Every fibre above the neutral axis of the imaginary central section is *shoving*, to a greater or less extent, to prevent bending; and below that axis every fibre is *pulling*, with the same aim. But those fibres near to the neutral axis are much less stressed than those remote from it, and are far from doing all they are capable of. How is it possible, then, to take account of all these pushes and pulls, giving due credit to each?

First Method.—Several methods are available. Perhaps the easiest of these to understand, and also a very instructive one, is the following:—Draw the section to a large scale (Fig. 45), find its centre, and through it draw the neutral axis. Divide up the whole section into a number of rectangles (the more the better for accuracy), and find the resisting moment of each pair separately, in the manner already described for deep flanged girders. Adding all these moments together gives the moment of resistance of the entire section. In this method the stress is assumed to be uniform over each pair of rectangles symmetrically situated with respect to the neutral axis.

In applying this method to find the resisting moment of a section 12in. deep, *six* is a convenient number of rectangles to use for the purpose of illustration. Remembering that the stress on any strip varies as the distance of the strip from the neutral axis, we then get, from Fig. 45 and simple multiplication, the values tabulated below:—

Rectangles.	Area.	Stress.	Force.	Arm.	Moment.
Pair—	Sq. in.	Tons per sq. in.	Tons.	Inches.	Inch-tons.
Outer . .	8	5	40	10	400
Middle .	8	3	24	6	144
Inner . .	8	1	8	2	16
					560

Thus the resisting moment of the whole section works out to be 560 inch-tons. But this is not a very close approximation. The regular formula gives the figure 576. If, however, we take 12 rectangles, instead of only 6, we get the very close result 572 inch-tons.

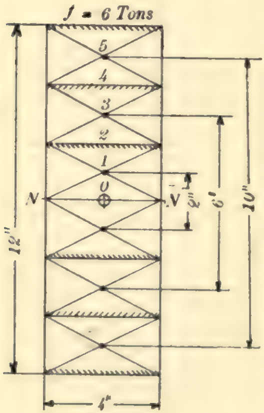


Fig. 45.

This method is applicable to any section whatever, even the most irregular; but it is rather laborious, because a large number of rectangles must be taken. It has the great merit, though, of being perfectly obvious in principle, even to non-mathematical men, and is extremely easy to keep in mind.

As a further test of the accuracy of the method here proposed, it was applied to find the moment of resistance of a circular section 12in. diameter, the section being drawn half-size and divided up into six pairs of rectangles. Without special care the result came out to be 1026 inch-tons, allowing a stress of 6 tons per square inch on the extreme fibres. The figure 1018 was obtained by the use of the exact formula. Comparing these, the error is seen to be only about $\frac{3}{4}$ per cent.; which is of no account in practical work, as the strength of different samples of mild steel varies at least 5 per cent. Nevertheless, the moments of resistance of *regular* sections can be calculated much quicker by aid of the standard formulæ and a slide-rule.

Second Method.—Returning now to the example in hand, a second mode of ascertaining the moment of resistance of the given section consists in finding the resistances of several separate portions of the section, and multiplying their sum by the *average* resistance arm, as described in detail below.

Having found the neutral axis NN (Fig. 46), divide the section into strips 1in. thick, say. To the right of the section draw a vertical centre line and the indefinite horizontals *ab*, *cd*, *fe*, &c. Now the resistance offered by the *top* strip ABEF = its area × average stress over it = (4×1) inch^2 × $\frac{1}{2}(6+5)$ tons per square inch = 22 tons. This is also the resistance of the bottom strip.

Similarly, the resistance offered by each of the second pair of strips is $4 \times 4{\cdot}5 = 18$ tons; and so on for the rest.

Represent these values by the areas of the shaded rectangles in Fig. 47, assuming a convenient scale. Each half of the stepped resistance area so drawn represents a resistance of

$$22 + 18 + 14 + 10 + 6 + 2 = 72 \text{ tons.}$$

Now if the strips had been taken only $\frac{1}{10}$in. thick instead of 1in., the projecting steps in the resistance diagram would have been only one-tenth of their present width; and finally, on taking the strips extremely thin, the figure would become a pair of triangles, as shown in Fig. 48. The area of each triangle represents one of the equal forces composing the resistance couple.

These triangles are the true resistance areas, showing how the resistances of adjacent horizontal layers of the beam change, by imperceptible stages, from a maximum at the top and bottom, to nothing at the central layer. The comparative uselessness of the material in the region of the neutral axis of the section is thus displayed in a striking manner; at least, as regards its share in resisting the bending moment. This central material, however, has subordinate duties to perform, as will appear hereafter.

It may not yet appear how the actual width of the resistance triangles is arrived at. This is best ascertained by reasoning from Figs 46 and 47. Consider a very thin outermost strip, say $\frac{1}{100}$in. thick. Its resistance = its area × mean stress on it = 4in. × $\frac{1}{100}$in. × 6 tons/inch^2 $\doteq \frac{6}{25}$ ton.

This force must be represented by a portion of the resistance

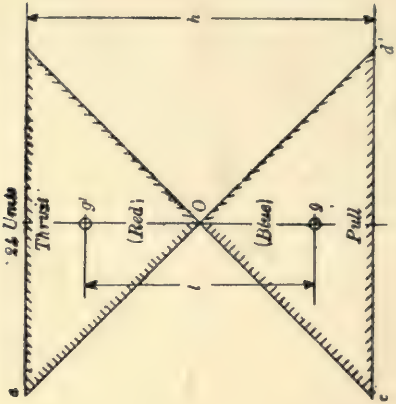


Fig. 48.

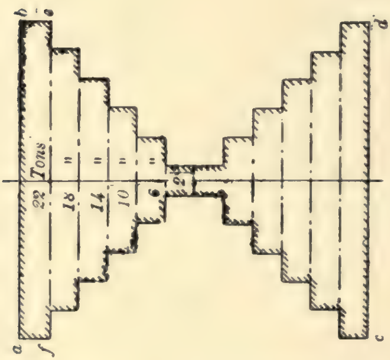


Fig. 47.

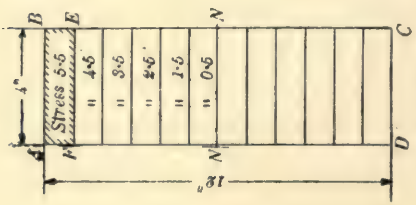


Fig. 46.

area x inches wide, and therefore of area $= x$ inch $\times \frac{1}{100}$in. $= \frac{x}{100}$ sq. in. So that we must have an area of $\frac{x}{100}$ sq. in. representing a resistance of $\frac{6}{25}$ ton, and consequently $x = 100 \times \frac{6}{25} = 24$. Thus the extreme width of the present figure of resistance will be 24in., to the same scale as that to which the beam section is drawn.

More generally, the extreme width of the resistance figure for a rectangular section will be numerically equal to the breadth of the section multiplied by the stress allowed on the extreme fibres. If drawn, for convenience, either narrower or wider than this, as here, then the number of units of area in it will *not* be equal to the number of units of resistance of the section, but only proportional thereto.

In spite of what is elsewhere stated, it is not wholly correct to say that the extreme width of the resistance area is the product of the breadth of the section *into* the stress on the extreme layers of fibres, and that it represents so many units of *resistance*. For, using dimensional equations, breadth $\times$ stress $=$ inches $\times \frac{\text{tons}}{\text{inch}^2}$ $= \frac{\text{tons}}{\text{inch}}$, so that *width* really represents tons per *linear* inch.

But *areas*, on the contrary, really do represent forces, both numerically and dimensionally, for in Fig. 47 area $abef$ = width $ab \times$ height $af = \frac{\text{tons}}{\text{inch}} \times$ inches $=$ tons of force.

The moment of resistance of the section is the whole thrust (P tons) multiplied by the length (l inches) of the resistance arm. The thrust has been found to be seventy-two tons, but the *arm* yet remains to be discovered. To find it, we inquire, "At what point of the upper triangle must a single force act in order to have the same effect as the whole lot of forces acting at the several parts of the beam section?" A little reflection will make it clear that this point must be none other than the centre of gravity (or centroid) of the triangle abO (Fig. 48). So also for the lower triangle.

Now, the centroid of any triangle is known to be situated on a straight line joining the vertex to the middle point of the base, and at a distance of one-third the length of this line from the base. This fact fixes g and g', the points of application of the resultant pulls and pushes of the fibres. From Fig. 48 the required resistance arm is then seen to be gg', whose length is—

$$h - 2\left(\tfrac{1}{3} \times \tfrac{h}{2}\right) = \tfrac{2}{3}h \text{ inches.}$$

In the present case this is $\frac{2}{3} \times 12$ in. $= 8$ in. Hence the moment of resistance of the section $= 72$ tons $\times$ 8 in. $= 576$ inch-tons.

Case of Unequal Limiting Stresses.—Consider next a beam (Fig. 49) of cast-iron, which may be stressed to 6 tons per square inch in compression, but only to 1·5 ton in tension.

In Fig. 50 the areas of the upper and lower triangles represent the resistances which the corresponding halves of the section are *capable* of sustaining. But clearly, on applying an increasing load to the beam, the stress on the bottom layer would reach its limiting safe value of 1·5 ton per square inch long before the stress on the top layer could reach the limiting safe value of 6 tons per square inch.

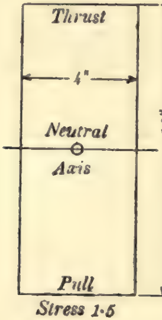


Fig. 49.

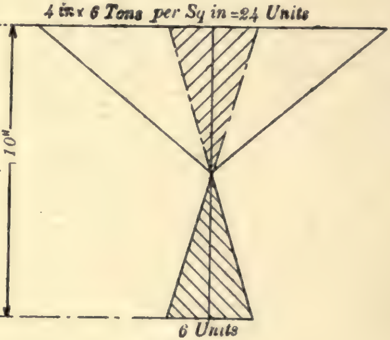


Fig. 50.

Also, if the load were increased to such an extent as to induce a stress of 6 tons per square inch on the top layer of fibres, then the bottom layer would be stressed to the same intensity: assuming that the stress varies as the distance from the neutral axis. But this value, being dangerously high, is not allowable. Instead, therefore, of utilising to the full the superior compressive strength of cast-iron, the stress is unavoidably kept down to the same low value as the tensile stress. Hence the actual safe resistance area can only consist of the two equal shaded triangles, these representing the two equal resultant forces forming the resisting couple, whose moment is 100 inch-tons.

Apparently, then, the surplus strength of the top half is of no benefit whatever, the beam being not a bit stronger than if made of a material only as strong in withstanding thrust as pull. Of course, in making this statement, we disregard for the moment all lateral and shearing forces, as well as all considerations of stiffness. A solid rectangular section is thus shown to be even less economical for a cast-iron beam than for one of wrought-iron, and is, therefore, quite unsuitable for heavy work.

Equivalent Areas.—Take a section ABCD (Fig. 51) of a rectangular beam, and draw the diagonals crossing at G; then the shaded area between them is known as the "equivalent area" of the section, or sometimes as the *effective* area. It is so called because it represents a section (Fig. 52) of an imaginary beam (Fig. 53) having all its layers stressed to the same extent as the extreme top or bottom layer, and equivalent in value to the actual

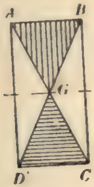


Fig. 51.

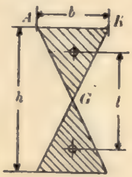


Fig. 52.

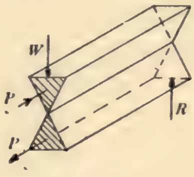


Fig. 53.

beam so far as regards its resistance to pure bending. In this equivalent beam all the apparently useless material is cut away; on the principle that if a chain is only as strong as its weakest link, it is useless to make the middle links stronger than the end ones.

The equivalent area of a section must not be confused with the *resistance area* previously discussed. By the sole use of *equivalent* areas it is only possible to compare beams of the *same* material equally stressed; but by means of *resistance* areas we can compare beams of *different* materials unequally stressed.

The equivalent area of any section of a beam is that area which, when taken along with a hypothetical uniform stress, has the same moment of resistance as the actual section when taken along with the actual varying stress. The equivalent area is drawn to the same scale as the actual section, and is such that half its area multiplied by the length of the resistance arm, and also by the maximum safe stress on the material, gives the moment of resistance of the section to bending.

Thus, referring to *any* beam, the resisting moment of a section may be indifferently expressed either as

$$\tfrac{1}{2} \text{ equivalent area} \times \text{resistance arm} \times \text{stress},$$

or as

$$\text{Effective flange area} \times \text{effective depth of beam} \times \text{stress},$$

expressions which have exactly the same meaning.

Strength Formula.—To deduce from this statement a convenient formula for the moment of resistance of a rectangular section of any given material, we put out of sight the actual section of the beam, and consider only the equivalent section (Fig. 52). The stress on it being uniform all over, the total thrust on the upper half (which equals the pull on the lower half) is:—

Area ABG × the uniform stress allowed.

That is,

$$\text{Force P} = \tfrac{1}{2}b \times \tfrac{1}{2}h \times f$$
$$= \tfrac{1}{4}bh \times f \quad . \quad . \quad . \quad . \quad (1)$$

(In the case of a general beam this may be called the effective flange resistance.) Also, from the rule for the position of the centre of area of a triangle:—

$$\text{Resistance arm } l = \tfrac{2}{3}h \quad . \quad . \quad . \quad (2)$$

But

Moment of resistance = force × arm of couple.

$$\therefore \text{M} = \text{P} \times l.$$

Inserting (1) and (2) in this, we have

$$\text{M} = (\tfrac{1}{4}bh \times f) \times \tfrac{2}{3}h$$
$$\therefore \text{M} = \tfrac{1}{6}bh^2 \times f \quad . \quad . \quad . \quad . \quad (3)$$

which is the formula sought.

Modulus of a Section.—This extremely useful formula for the resisting moment of a rectangular section may be split up into two parts. The part f refers to the safe stress per square inch allowed on the material, and has nothing to do with the shape of the section. The other part takes account of the shape and size of the section, and is conveniently referred to as the strength modulus of the section with respect to bending, a term introduced by Professor Unwin. This quantity is symbolised by the letter Z. Hence, for *any* section of a beam,

Moment of resistance = modulus of section × stress;

or, $\text{M} = \text{Z} \times f$.

Each shape of section has its own particular strength modulus, according to the disposition of the material with respect to the neutral axis. But it is rather troublesome to deduce from first principles the moduli of sections other than those of solid rectangular beams and deep-flanged girders with very thin webs. As regards the latter important case, we know that the moment of resistance is:—

$$\text{P} \times l = (f \times \text{area of flange}) \times l.$$

But $\text{M} = f \times \text{Z}$.

Hence $\text{Z} = \text{area of flange} \times l$.

Here l may be taken as the total depth of the girder, without material error, though it really refers to the distance between the centres of area of the flanges—*i.e.*, the *effective* depth of the beam (see Fig. 33).

It thus appears that the ability of a beam to resist a bending moment at a stated section depends only on two things—(1) the bending modulus of that section as found by calculation from a drawing; and (2) the greatest safe stress which the material will bear, as determined by experiments with a testing-machine, combined with the use of a factor of safety.

Much confusion prevails as regards the proper units in which strength moduli should be measured. The unit of measurement may be found in two ways. We know, from previous work, that stress is measured in either tons per square inch or pounds per square inch, and consequently the moment of resistance in either inch-tons or inch-pounds. Hence to find the dimensions of the third quantity we write the equation

$$\text{M (inch} \times \text{ton)} = \text{Z} \times f\frac{\text{ton}}{\text{inch}^2},$$

which transposes to

$$\text{Z} = (\text{M inch} \times \text{ton}) \div \left(f\frac{\text{ton}}{\text{inch}^2}\right)$$

$$= \frac{\text{M}}{f}\text{ inch}^3.$$

We therefore conclude that the strength modulus of a section is a geometrical quantity of the nature of length raised to the third power, like volume. Some people object to this, but anything more rational has yet to be proposed. The same method is regularly applied to ascertain the physical dimensions of much more intangible electrical quantities.

Another way is to reason directly from the formula,

$$\text{Z} = \tfrac{1}{6}bh^2,$$

from which, since b and h both refer to linear magnitudes, it at once follows that Z has the dimensions $(\text{length})^3$.

General Conclusions.—From the preceding result and the fundamental relation that the bending moment equals the working resisting moment, we get, for the particular case of a *rectangular* beam supported at both ends and loaded at the centre, the important equation

$$\tfrac{1}{4}\text{WL} = \tfrac{1}{6}bh^2 \times f;$$

which concisely sums up the whole question. It may also be written as—

$$W = \frac{2}{3}\frac{bh^2}{L} \times f,$$

where

W tons = greatest safe central load,
b inches = breadth of beam,
h inches = depth of beam,
L inches = span of beam,
$f \frac{\text{tons}}{\text{inch}^2}$ = safe stress on extreme fibres.

The general conclusions to be inferred from this formula are the following :—

(1) The resisting capacity or strength of a rectangular beam is directly proportional to its *breadth ;* so that doubling the breadth of a beam also doubles its strength, all other things being unaltered.

(2) The strength of a beam is directly proportional to the square of its *depth ;* so that doubling the depth quadruples the strength. Increase of strength is therefore most economically gained by increase of depth. (The question of *stiffness* will be considered hereafter.)

(3) The load which a beam will safely carry is inversely proportional to its *span ;* so that doubling the span of a beam halves the load it will carry.

(4) The resistance of a beam is directly proportional to the stress put on the outside fibres, so long as this stress does not exceed the elastic limits of the material.

The solid rectangular form is not an economical section; but it is much used for wood beams and for such forged details of machinery as cranks, levers, wheel teeth, and plate springs. Ribbed and box sections are in every way preferable for large work, such as engine columns and bedplates.

CHAPTER VI.

APPLICATIONS.

Example 1.—A beam 15ft. long by 10in. deep by 10in. broad sustains a certain central load. It is proposed to substitute for it another beam of like material, the length of the new beam being 18ft. and the breadth 8in. What should be its depth to enable it to carry the same load as the first beam, with the same factor of safety?

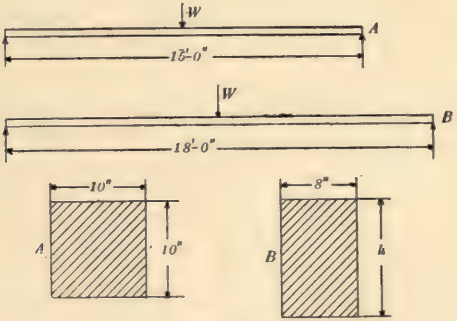


Fig. 54.

Considering the *first* beam A (Fig. 54):

Resisting moment $= \frac{1}{6}bh^2 \times f = \frac{1}{6} \times 10 \times 100 \times f.$

Bending moment $= \frac{W}{2} \times \frac{L}{2} = \frac{W}{4} \times 15 \times 12.$

Hence

$$\tfrac{1}{6} \times 1000 f = 45W \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly, for the *second* beam B we have

$$\tfrac{1}{6} \times 8h^2 \times f = 54W \quad . \quad . \quad . \quad . \quad . \quad (2)$$

With the same factor of safety, the stress f is the same in each case. Dividing (2) by (1) gives

$$\frac{8h^2}{1000} = \frac{54}{45} = \frac{6}{5};$$

$$\therefore h^2 = \frac{6}{5} \times 125 = 150$$

$$\therefore h = 12{\cdot}25.$$

Thus the required depth is $12\frac{1}{4}$ inches.

Example 2.—Determine the necessary depth of the rectangular section of the guide bar of an engine (Fig. 55), in order that the stress on the extreme fibres may not exceed 5 tons per square inch. The length of the connecting-rod is twice the stroke of the piston, and the width of the guide bar is 6 in. The greatest obliquity of the connecting-rod may be taken to occur, without much error, when the guide block is at the centre of the 4 ft. span. It has already been estimated that the total steam pressure on the piston will amount to 25 tons, and will cause a normal reaction of the guide bar of 6·5 tons.

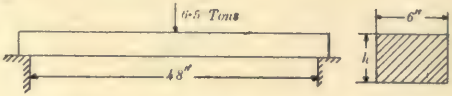


Fig. 55.

Greatest bending moment $= \frac{1}{4}$ WL

$= \frac{1}{4} \times 6{\cdot}5 \times 48 = 78$ inch-tons.

Equating this to the moment of resistance gives

$$\tfrac{1}{6} \times 6\text{in.} \times h^2 \times 5 \text{ tons} = 78 \text{ inch-tons}:$$

$$\therefore h^2 = 15{\cdot}6, \text{ and } h = 3{\cdot}95.$$

Hence the requisite depth is, say, *4 inches*.

Example 3.—A cast-iron bar $1\frac{1}{8}$ in. diameter, when tested in direct tension, broke under a load of 9·85 tons. Another bar, 4in. square and 40in. span, made of the same kind of iron, is loaded as a beam. What safe central load will it carry, allowing a factor of safety of 6?

$$\text{Tensile stress on test bar} = \frac{\text{load}}{\text{area}};$$

$$\therefore \text{ breaking stress} = \frac{9{\cdot}85 \text{ tons}}{0{\cdot}994 \text{ sq. in.}} = 9{\cdot}91 \text{ tons per sq. in.}$$

Working stress (f) allowed is $\frac{9{\cdot}91}{6} = 1{\cdot}65$ tons per sq. in.

Resisting moment $= \frac{1}{6}bh^2 \times f$

$= \frac{1}{6} \times 4 \times 16 \times 1{\cdot}65 = 17{\cdot}6$ inch-tons.

But

$$\text{Bending moment} = \text{resisting moment};$$
$$\therefore \tfrac{1}{4}\text{W} \times 40\text{in.} = 17{\cdot}6 \text{ inch-tons};$$
$$\therefore \text{W} = 1{\cdot}76 \text{ tons.}$$

Example 4.—Estimate what *distributed* load a uniform rectangular beam, 10in. by 5in. section (Fig. 56), would safely carry over a span of 100in., both when set on the flat and when placed

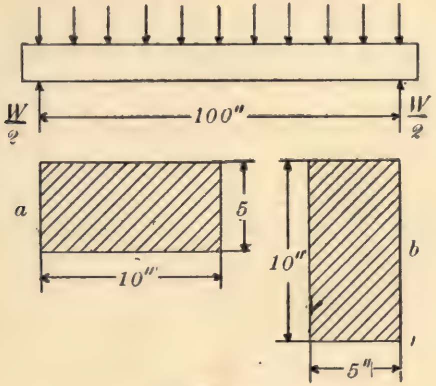


Fig. 56.

on edge. The material and the factor of safety are the same as in Example 3; also the weight of the beam is to be allowed for.

(1) To estimate the *weight* of the beam :—

$$\text{Volume of metal} = \text{section} \times \text{length.}$$
$$= 10 \times 5 \times 100 \text{ cub. in.}$$

Now a cubic inch of cast-iron weighs 0·26 of a pound.

$$\therefore \text{weight of beam} = 5000 \times 0{\cdot}26 = 1300\text{lb.}$$

In the present case the beam's own weight is of little importance, but in the case of very long span beams it is all-important.

(2) To estimate the resisting moment of the section when the beam is on the flat (Fig. 56, *a*) :—

$$\text{Resisting moment} = \tfrac{1}{6} \times \text{breadth} \times \text{depth}^2 \times \text{stress}$$
$$= \tfrac{1}{6} \times 10 \times 5^2 \times (\tfrac{1}{6} \times 9{\cdot}9);$$

or, $$\text{M} = 41{\cdot}6\text{in.}^3 \times 1{\cdot}65 \text{ tons/in.}^2$$
$$= 68{\cdot}7 \text{ inch-tons.}$$

(3) When set on edge (Fig. 56, *b*) :—

$$\text{M} = \tfrac{1}{6} \times 5 \times 10^2 \times 1{\cdot}65 = 137{\cdot}5 \text{ inch-tons,}$$

which is double the former value.

(4) In each case, from Fig. 56 :—

$$\text{Bending moment} = \left(\frac{W}{2} \times \frac{L}{2}\right) - \left(\frac{W}{2} \times \frac{L}{4}\right)$$

$$= \tfrac{1}{8}WL = \frac{100}{8}W \text{ inch-tons.}$$

(5) Hence, in the first case,

$$12 \cdot 5W = 68 \cdot 7 ; \therefore W = 5 \cdot 5 \text{ tons} ;$$

and in the second case $W = 11$ tons.

But each of these values has to be diminished by 1300lbs., on account of the weight of the beam itself.

Example 5.—A cast-iron *cantilever* (Fig. 57) 10 ft. long, is loaded with a uniform load of 1 ton per foot run. Its width is

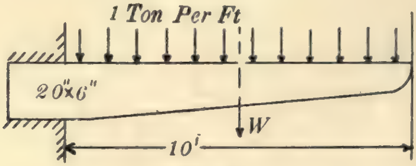


Fig. 57.

6in. Estimate a suitable depth at the wall, allowing a stress of $1\frac{1}{2}$ tons per square inch.

Total load = 10 × 1 ton = 10 tons.

Now a load uniformly distributed over the whole length of the beam produces the same bending moment at the wall as a single load of the same amount concentrated at the centre. (That this is so can be convincingly shown by integrating, between the limits zero and L, the several bending moments due to each element of length.)

Hence,

B.M. at wall = total load × $\frac{1}{2}$ length
= 10 tons × 60in. = 600 inch-tons.

Also,

Moment of resistance = $\frac{1}{6}bh^2 \times f$
= $\frac{1}{6}$ × 6in. × h^2 × 1·5 ton per square inch.

Equating these results gives

$$1 \cdot 5h^2 = 600 ; \therefore h^2 = 400.$$

Hence the required depth is 20 inches.

Such a beam would doubtless carry the load safely, but the form of section is uneconomical.

Example 6.—It is estimated that the total pressure on the teeth of the pinion designed to rotate the turret of a battleship carrying two 67-ton guns will amount to 20,000 lbs. under the most severe conditions likely to occur. The driving pinion and the rack embracing the turret are of steel, and have cut teeth. Allowing a safe stress of 5000lbs. per square inch, estimate a suitable pitch and breadth of face, assuming that the teeth bed well across the whole width of the gearing.

The pitch of the teeth is governed partly by the diameter of pinion that can be got in the available space, the teeth being kept as few in number as will ensure satisfactory working. The breadth of face is less restricted. In this case it is not safe to assume that more than one pair of teeth is simultaneously in gear. The wear on these teeth is but trifling, as they are only in occasional use. Consequently it is needless to make any allowance for wear.

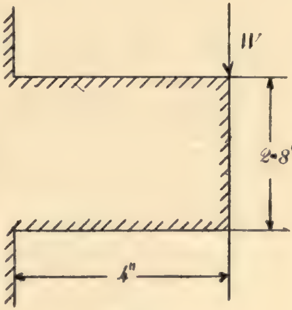


Fig. 58.

Here, as in all designing, one has to work tentatively: making certain assumptions, seeing to what results they lead, and then, if necesary, going back and altering.

From previous experience, assume a pitch of 6in. The thickness of the tooth of the pinion at the root will then be 2·8in., say. The length of the tooth will be $0{\cdot}65 \times 6$in. = 3·9in., or, say, 4in.

Treating the tooth as a rectangular cantilever (Fig. 58) loaded at the end, as is usual,

$$\begin{aligned}\text{Bending moment} &= \text{force} \times \text{arm.}\\ &= 20{,}000\text{lbs.} \times 4\text{in.}\\ &= 80{,}000 \text{ inch-pounds.}\end{aligned}$$

Also,

$$\begin{aligned}\text{Resisting moment} &= \tfrac{1}{6}bh^2 \times f\\ &= \tfrac{1^2}{6} \times b \times (2{\cdot}8)^2 \times 5000.\end{aligned}$$

Hence,

$$\tfrac{1}{6}b \times 7{\cdot}82 \times 5000 = 80{,}000;$$
$$\therefore b = 12{\cdot}3 \text{ in., nearly.}$$

As this is a reasonable width, the pitch need not be altered. The pinion may have from 12 to 15 teeth, and therefore a diameter of from 23 to 28·6in. The hydraulic training engines and the gearing are in duplicate, as a safeguard against the breakdown or disablement of one set.

The most important factor in the strength of a wheel tooth is its thickness at the root. For this reason the slope-backed form of tooth shown in Fig. 59 is occasionally adopted for heavy gearing whose direction of rotation is never reversed. Such teeth, while 50 per cent. stronger than those of ordinary form of the same pitch, are no more expensive to make, after once a properly shaped wheel-block or segment pattern has been prepared for the foundry.

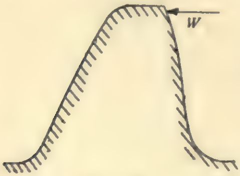


Fig. 59.

Example 7.—Fig. 60 represents the largest section of the arm of a spur wheel, its length from boss to rim being 20in. The wheel has six arms (Fig. 61). Calculate what driving pressure the wheel will stand at the rim, assuming that each arm takes one-sixth of the whole load.

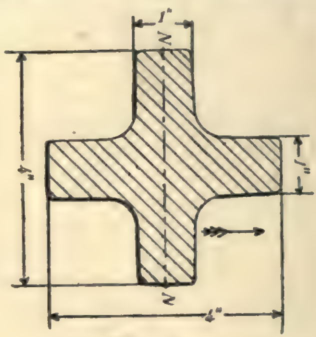


Fig. 60.

The corners are well-rounded or filleted, in order to increase the strength and improve the appearance. The feathers are also tapered slightly, to facilitate moulding. But these features may safely be disregarded for purposes of calculation. It is also usual to neglect the influence of the lateral feathers or stiffeners, which impart but little additional strength in the plane of bending (indicated by the arrow), and are often cast thinner than the rest of the wheel.

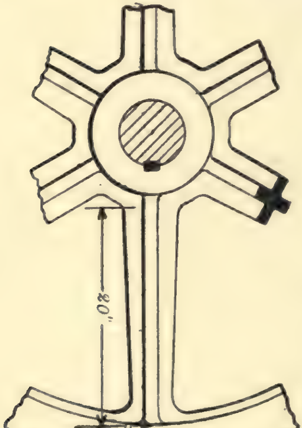


Fig. 61.

Regarding each arm (Fig. 61) as a cantilever fixed at the boss and free at the rim, the bending moment on each is—

$$\frac{\text{Driving pressure} \times \text{length of arm}}{\text{Number of arms}} = \tfrac{1}{6}P \times 20\text{in.}$$

The modulus of the section, omitting feathers, is—

$$\tfrac{1}{6} \times 1\text{in.} \times 4^2 = 2{\cdot}67\text{in.}^3,$$

and including feathers is $2{\cdot}79\text{in.}^3$, so that their influence is very slight. This arises from their proximity to the neutral axis.

Allowing a working stress of 3000lbs. per square inch, the moment of resistance of the section of the arm is—

The strength modulus × the stress
$= 2{\cdot}7 \times 3000 = 8100$ inch-pounds.

Equating the bending and resisting moments gives

$$\tfrac{1}{6}P \times 20 = 8100$$
$$\therefore P = 2430 \text{ pounds},$$

which is the driving pressure sought. Conversely, using the same equations, we can design the arms of a wheel to stand any desired driving pressure.

Example 8. — Two wooden beams are required to carry a water tank 10ft. square and 3ft. deep over a 10ft. span (Fig. 62). Estimate a suitable section.

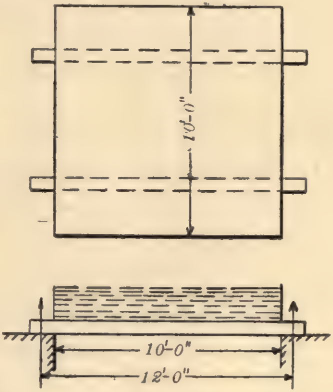


Fig. 62.

(1) To find the weight of the *tank* itself, which is assumed to be built up of wrought-iron plates ½in. thick.

Area of plates = area of bottom + sides.
$= 10^2 + 4\,(10 \times 3).$
$= 100 + 120 = 220$ sq. ft.

Now, ½in. plate weighs 20lbs. per square foot. Hence the weight is 220 × 20lbs. = 4400lbs.

(2) To find the weight of the greatest volume of *water* the tank can hold.

Capacity = area of base × depth.
$= 10^2 \times 3 = 300$ cub. ft.

Now, 1 cub. ft. of fresh water weighs 62·5lbs. Hence the weight of water is 300 × 62·5lbs. = 18,750lbs.

(3) Neglecting the weight of the beams, the *total* uniformly distributed load is, therefore—

$$4400 + 18{,}750 = 23{,}150\text{lbs.}$$

Allowing the odd 850lbs. for rivet heads, overlapping of plates and weight of beams, we may call this 24,000lbs.—that is, 12,000lbs. per beam.

(4) Taking the effective span as 12ft., the bending moment at the centre of each beam is

$$\tfrac{1}{8}\mathrm{WL} = \tfrac{1}{8} \times 12{,}000 \times 144\text{in.}$$
$$= 216{,}000 \text{ inch-pounds.}$$

Any beam having this value for its central moment of resistance will answer the purpose.

(5) Take a trial breadth of 10in. Then to find the depth h we say—

Resisting moment $= \frac{1}{6}bh^2 \times f =$ bending moment.

Allowing a stress of 800lbs. per square inch, this becomes—

$$\tfrac{1}{6} \times 10 \times h^2 \times 800 = 216{,}000.$$
$$\therefore h^2 = 162.$$
$$\therefore h = 13\text{in., say.}$$

Thus a strong enough section for each beam is 10in. × 13in. Whether this would be the best section to use under the circumstances depends on what scantlings of timber are available.

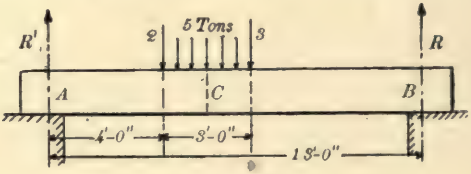


Fig. 63.

(6) Taking the density of the timber (fir) as 40lbs. per cubic foot, the weight of each beam is

$$\frac{10\text{in.} \times 13\text{in.}}{144} \times 12\text{ft.} \times 40\text{lbs.} = 433\text{lbs.}$$

Example 9.—A fir beam, of 13ft. effective span, is loaded in the manner shown by Fig. 63, a distributed load of 5 tons being applied between the two concentrated loads. Calculate the bending moment at C, midway between the points of application of the two local loads, and estimate a suitable section.

(1) To find the reactions. Taking moments about A, we have

$$\mathrm{R} \times 13\text{ft.} = (3 \text{ tons} \times 7\text{ft.}) + (5 \text{ tons} \times 5{\cdot}5\text{ft.}) + (2 \text{ tons} \times 4\text{ft.}).$$
$$= 21 + 27{\cdot}5 + 8 = 56{\cdot}5\text{ft.-tons.}$$
$$\therefore \mathrm{R} = 56{\cdot}5 \div 13 = 4{\cdot}35 \text{ tons.}$$

The other reaction is the total load *minus* R, or

$$R^1 = (2 + 5 + 3) - 4{\cdot}35 = 5{\cdot}65 \text{ tons.}$$

(2) To find the bending moment at C. Taking the moments of the forces acting to the left of C, we get—

$$M = (5{\cdot}65 \text{ tons} \times 5{\cdot}5\text{ft.}) - (2 \text{ tons} \times 1{\cdot}5\text{ft.}) - (2{\cdot}5 \text{ tons} \times 0{\cdot}75\text{ft.}).$$
$$= 31{\cdot}1 - 3 - 1{\cdot}87 = 26{\cdot}2 \text{ foot-tons.}$$

(3) To choose a suitable stress. We find from Molesworth that the *ultimate* tensile strength of spruce fir is 10,100lbs. per square inch, and the crushing strength 6500lbs. per square inch. The tensile strength need not further concern us; because, if the beam breaks at all, it will break in the weakest part—viz., by the crushing of the extreme top fibres. Allowing a factor of safety of 8, the safe compressive strength will be—

$$\tfrac{1}{8} \times 6500\text{lbs.} = 800\text{lbs. per square inch, say.}$$

(4) To find a suitable section. Assume tentatively a breadth of 12in. Then, as the resisting moment must equal the bending moment of 26·2ft.-tons, we have—

$$\tfrac{1}{6} \times 12\text{in.} \times h^2 \times 800\text{lbs.} = 26{\cdot}2 \times 12 \times 2240 \text{ inch-pounds.}$$
$$\therefore h^2 = 441, \text{ and } h = 21\text{in.}$$

Thus the ratio of depth to breadth is 21 in. ÷ 12 in. = 1·75.

If for any reason this section should be thought unsuitable, or, if we have any difficulty in obtaining such a beam, then a different value of b may be chosen and h calculated afresh. Here a better breadth would be 14 in. If a likely piece of timber happens to be available, its moment of resistance should be calculated, in order to see whether the same can be worked in.

Ratio of Depth to Breadth in Rectangular Beams.—It is worth noticing here that the *strongest* beam which can be cut from a round log (Fig. 64) is one whose depth is $\sqrt{2}$ or 1·41 times its breadth, and that the *stiffest* beam has a depth of $\sqrt{3}$ or 1·73 times its breadth. Geometrically put, the perpendiculars from the corners cut off from $\frac{1}{3}$ and $\frac{1}{4}$ the diameter in the respective cases. Hence, when a single rectangular beam has to be cut out of a cylindrical log, the depth should be from $1\frac{1}{2}$ to $1\frac{3}{4}$ times the breadth.

As timber beams, however, are not usually cut singly out of round logs, the above ratios are not of great value. The most scientific mode of designing a beam is to fix the ratio of depth to span with a view to stiffness, and then compute the breadth with a view to strength. But the need for lateral stiffness and stability, as well as the limitations imposed by the market sizes of timber obtainable, must not be lost sight of.

Notes on Timber.—Though mechanical engineers have little to do with timber beams, the following brief notes, condensed from Rankine and other sources, are not without interest:

Pine timber of the best sort is the produce of the red pine, or Scottish fir, grown in Norway, Sweden, Russia, and Poland. The best is exported from Riga, the next from Memel and Dantzic.

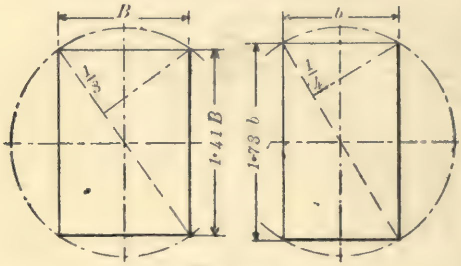


Fig. 64.

The same species of tree grows also in Britain, but is inferior in strength. This Baltic fir is the best of all timber for straight beams, ties, and straight pieces in framework generally.

The length of Baltic fir logs varies from 25 to 45 ft. The section is square, and of the following sizes:

Name.	Size.	Characteristics.
	Inches.	
Stettin (Prussian) . .	18 to 20	——
Dantzic ,, . .	13 to 16	Longest logs; coarse, but strong, large knots.
Memel ,, . .	12 to 14	Uniform and durable.
Riga (Russian) . .	10 to 12	Best and most uniform; small hard knots.
Swedish . . .	10 to 12	——
Norwegian . . .	8 to 9	——

Pine timber for use as beams and framework is also obtained from various other species, chiefly North American, of which the best are the yellow pine and white pine. It is softer and less durable than the red pine of Northern Europe, but lighter, and can be had in larger logs.

White fir, or deal timber of the best kind, is the produce of the spruce fir, grown in Norway, Sweden, and Russia. The best is that known as Christiana deal. Much of this timber is sawn up for sales into pieces of various thicknesses suited for planking. Boards 7in. wide are known as *battens*, those 9in. wide as *deals*, and those 11 or 12in. wide as *planks*. They are to be had of various lengths, but the most usual length is about 12ft. This is an excellent kind of timber for planking, light framing, and joiners' work.

Example 10.—The top of the combustion chamber of a marine boiler is supported by a number of similar girder stays (Fig. 65) 28in. span, 8in. deep, and $8\frac{1}{2}$in. apart; each carrying three

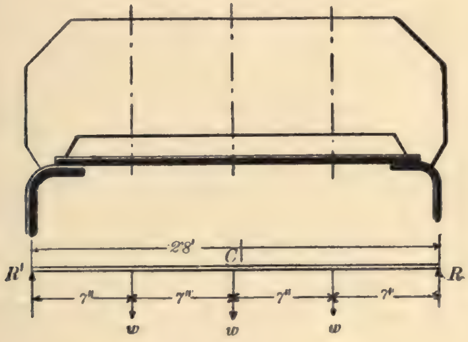


Fig. 65.

$1\frac{1}{8}$in. bolts, spaced equally. The steam pressure is 180lbs. per square inch, and the stress permissible on the extreme fibres of the stays is 9000lbs. per square inch. Estimate the breadth of each stay.

The function of the girder stays is to prevent sagging of the combustion chamber top or crown, under the steam pressure. The ends of the flat crown plate are supported directly by the front and back plates of the chamber, and the central portion by means of the suspension bolts.

(1) To find the pull on each bolt. A little reflection will show that each bolt has to support an area of $8\frac{1}{2}$ by 7in. of flat plate. Hence the pull (w) = $8{\cdot}5 \times 7 \times 180$lbs. = 10,710lbs.

(2) To find the equal reactions of the front and back plates—

$$R = \tfrac{1}{2} \times 3w = \tfrac{3}{2} \times 10{,}710 = 16{,}065\text{lbs.}$$

(3) To find the greatest bending moment, which is at centre

of span, take moments about C of the forces on the right-hand half of the stay, thus—

$$M = (16{,}065\text{lbs.} \times 14\text{in.}) - (10{,}710\text{lbs.} \times 7\text{in.}).$$
$$= 150{,}000 \text{ inch-pounds, say.}$$

(4) To find the breadth of a girder, equate the resisting and bending moments, thus—

$$\tfrac{1}{6}b \times 8^2 \times 9000 = 150{,}000.$$
$$\therefore b = 1{\cdot}56\text{in.} = 1\tfrac{9}{16}\text{in.}$$

Each stay may be a solid bar, with drilled bosses, or it may consist of two plates riveted together, and distance pieces. The girder stays of mercantile marine boilers are in practice necessarily designed in accordance with the hard-and-fast rules of one or other of the registration societies—Lloyd's or the Board of Trade. Not so in naval and locomotive work.

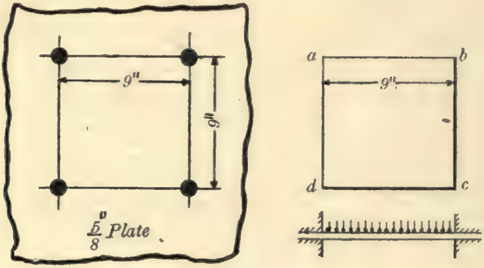


Fig 66.

Example 11.—The screwed stays supporting the back and sides of the combustion chamber of a marine boiler are spaced as shown in Fig. 66. The plates are $\frac{5}{8}$in. thick, and the steam pressure is 150lbs. per square inch. Determine the maximum stress on the flat plates induced by this pressure.

Each square piece of plate like *abcd* may be mentally isolated and regarded, with fair accuracy, as a beam *fixed* all round and loaded uniformly. The bending moment caused by a uniformly-distributed load acting on a beam fixed at both ends is commonly taken as—

$$\tfrac{1}{12}(\text{total load} \times \text{span}).$$

In the case of a beam fixed on *four* sides it seems reasonable to assume that the bending moment will be only one-half of this amount. Making this assumption, the equation of moments is—

$$\tfrac{1}{6} \times \text{breadth} \times (\text{thickness})^2 \times \text{stress} = \tfrac{1}{24} \times \text{load} \times \text{span}.$$
$$\therefore \tfrac{1}{6} \times 9 \times (\tfrac{5}{8})^2 \times \text{stress} = \tfrac{1}{24} \times 9^2 \times 150 \times 9.$$

Whence the stress = 7776lbs. per square inch.

Working with symbols, this method gives the formula—

$$\text{Stress} = \tfrac{1}{4} \times \frac{L^2}{t^2} \times p;$$

where L is the pitch of the stays, t the plate thickness, and p the steam pressure. Unwin's rule (p. 93, "Machine Design") is the same in form as this, but the constant is there taken as 0·222 instead of 0·25.

The necessary diameter of the stays at the bottom of the threads may be readily estimated from the consideration that each sustains a direct pull of $9^2 \times 150$lbs.

Example 12.—A lever safety valve A (Fig. 67) is $2\frac{1}{2}$in. diameter. The lever is estimated to weigh 10lbs., its centre of gravity being about 19in. from the fulcrum (knife-edge). The valve weighs 2lbs. Estimate what load W is required for the steam to blow

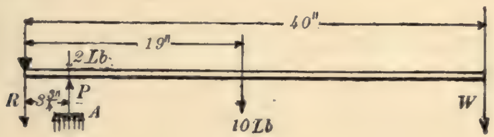


Fig. 67.

off at a pressure of 90lbs. per square inch, and also what should be the greatest depth of the lever, if the thickness is $\frac{1}{2}$in. The stress on the metal must not exceed 5000lbs. per square inch.

(1) Total pressure P = area × pressure.

$$= 4{\cdot}91 \times 90\text{lbs.} = 442\text{lbs.}$$

(2) Taking moments about the fulcrum—

$$(W \times 40\text{in.}) \div (10\text{lbs.} \times 19\text{in.}) = (442 - 2)\ 3\tfrac{3}{4}\text{in.}$$
$$\therefore 40W + 190 = 1650,$$
$$\therefore W = 1460 \div 40 = 36{\cdot}5\text{lbs.}$$

(3) The bending moment is greatest directly over the valve, and is—

$$W \times (40\text{in.} - 3\tfrac{3}{4}\text{in.}) + 10\text{lbs.} \times (19\text{in.} - 3\tfrac{3}{4}\text{in.})$$
$$= (36{\cdot}5\text{lbs.} \times 36{\cdot}25\text{in.}) + (10\text{lbs.} \times 15{\cdot}25\text{in.})$$
$$= 1323 + 153 = 1476 \text{ inch-pounds.}$$

(4) Equating the resisting and bending moments gives—

$$5000 \times \tfrac{1}{6} \times 0{\cdot}5 \times h^2 = 1476$$
$$\therefore h^2 = 3{\cdot}55, \text{ and } h = 1{\cdot}9 \text{ in.}$$

The lever should therefore have a maximum depth of, say 2 inches.

A Caution.—In working the preceding practical examples, liberal use has been made of the fact that the moment of resistance of a rectangular section is given by the formula

$$M = \frac{1}{6}bh^2 \times f.$$

Now it has to be particularly observed that the stress f here referred to is *not* the breaking stress, but some chosen safe stress well within the elastic range of the material composing the beam. If we were innocently to substitute for f the tensile breaking stress of the material, and multiply out, the value of M so found would *not* be the *ultimate* moment of resistance of the beam at the section under consideration; or, in other words, it would not be equal to the bending moment that would break the beam, but a good deal less. How much less cannot be stated very exactly; but Sir B. Baker, in his paper on "The Practical Strength of Beams" (*Proc. I.C.E.*, vol. lxii., part iv.), says that the actual ultimate moment of resistance of the rectangular section of a beam (presumably of

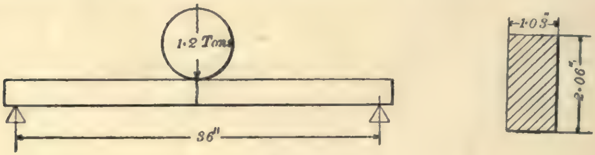


Fig. 68.

mild steel) is found by experiment to be about 70 per cent. greater than the so-called theoretical value.

This apparent discrepancy between experiment and reasoning does not, however, prove that the ordinary formula for the strength of a rectangular beam is quite wrong, and therefore useless, as some have stated. It merely shows that if we ignore or lose sight of the assumptions on which it is founded, and use the formula rashly, it will of course yield totally unexpected and seemingly erroneous results. To every formula built on an experimental basis the same remark applies.

The following example will serve to exemplify a common error and to point a moral:—

A cast-iron bar (Fig. 68) which calipered 1·03in. wide by 2·06in. deep was tested for transverse strength in a testing-machine. The span was 36in. The bar broke suddenly with a load of 1·2 tons applied at the centre. Calculate from these data the greatest *tensile* stress induced in the material:—

The bending moment at the centre is

$$\frac{1}{4}WL = \frac{1}{4} \times 1{\cdot}2 \text{ tons} \times 36\text{in.} = 10{\cdot}8 \text{ inch-tons.}$$

The resisting moment is $\frac{1}{6}bh^2 \times f$,

$$= \frac{1}{6} \times 1{\cdot}03 \times (2{\cdot}06)^2 \times f,$$
$$= 0{\cdot}73\ f \text{ inch-tons.}$$

Equating these moments, we have

$$0{\cdot}73f = 10{\cdot}8;$$
$$\therefore f = 14{\cdot}8 \text{ tons per square inch.}$$

Now as cast-iron is weaker in tension than in compression, a uniform beam is sure to give way, if at all, primarily by failure of the stretched side. Hence many people would conclude that 14·8 tons per square inch was the greatest tensile stress induced in the sample of cast-iron tested. This conclusion seems all right, but as a matter of fact it is all wrong. The real maximum stress is *less* than 14·8 tons per square inch.

The mistake lies in the tacit assumption that stress varies as strain right up to the breaking-point, which is not true. For a non-plastic material like cast-iron the result may not be *much* wrong; but in the case of a metal that draws out visibly before fracture, like wrought-iron or mild steel, the result thus found would certainly be wide of the mark.

The true use of the above formula has already been shown in previous examples. By its aid we can rapidly calculate what load a beam will safely carry when the top and bottom layers of material are subjected to a chosen stress that is well within the elastic limits. And this is all we want in practice; because engineers design beams, not to break, but to *stand* with a liberal margin of safety. The exact breaking load of a solid beam does not admit of calculation from first principles, or at any rate not of simple intelligible calculation. Moreover, its value is rather a matter of curiosity or of scientific interest than of practical service in designing machines and structures.

Plastic Bending.—It is instructive to consider *why* the actual breaking load of a beam is greater than the hypothetical breaking load, as calculated by the elastic formula. It is because the plastic yielding of the material more remote from the neutral axis throws a higher stress on the material nearer to that axis than the ordinary theory takes account of, and so tends to equalise the stress over the section, as shown in Fig. 69. Instead, therefore, of the pair of dotted triangles, showing that the extension or compression of any layer of fibres is proportional to the distance of that layer from the neutral layer, we have a pair of figures more nearly resembling parabolas (Fig. 69). If we only knew the exact form of these curves, we could calculate readily enough the true moment of resistance of the section just before the rupture of the beam; but it is not easy to see how we are to obtain that information.

This simple explanation seems sufficient; but a further reason why the elastic formula fails to give the correct breaking loads is often put forward. It is that the simple theory of bending takes no account of the adhesion, or horizontal and vertical attach-

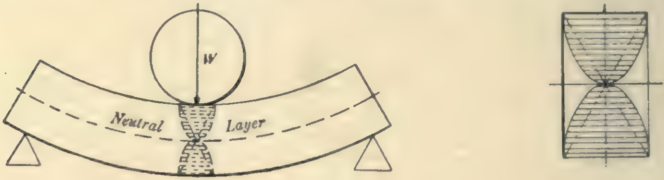


Fig. 69.

ment to one another, of the successive layers of material composing the beam, as shown in section by Fig. 70. By mathematical reasoning (as in Cotterill's "Applied Mechanics," p. 402) it can be shown that when the beam is bent *within the elastic limits*, the lateral connection of the parts can have but a very trifling influence on its resistance to bending, unless the ratio of breadth to

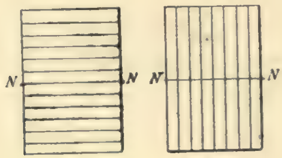


Fig. 70.

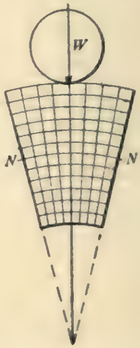


Fig. 71.

depth be great, as in the case of a wide thin plate. But when the breaking stress is approached, similar reasoning shows that the connection of the layers may greatly increase the resisting moment. Fig. 71 is intended to show, in an exaggerated manner, the nature of the deformation of the horizontal and vertical layers due to the *elastic* bending of the beam.

Assumptions.—It will now be advisable to summarise and note carefully the various assumptions made in framing the ordinary theory of bending. These are as follow :—

(1) The beam is of uniform section from end to end. But many actual beams are not uniform.

(2) The forces are applied in such a way as to cause only a bending moment, and no other kind of straining action. But all actual beams have to withstand shearing forces, in addition to bending moments.

(3) The material of the beam is perfectly uniform in composition throughout; or, in a word, is homogeneous. But actual materials of construction are only nearly so.

(4) The material is perfectly elastic; or, in other words, stress is proportional to strain.

(5) Sections of the beam that are plane or flat before bending remain so after bending. This is true only for a limited range of stress.

(6) The very thin layers into which the whole beam is imagined to be split up in establishing the formulæ are unattached to one another.

These are six large assumptions, no doubt; but without making them it is not possible to bring the subject within the range of practical mathematics, so great is the simplification they bring about. Moreover, sufficient evidence that the simplified theory of bending is not so very far wrong is afforded by the fact that the modulus of elasticity of a material, as ascertained by experiments on the deflection of a beam, comes out practically the same as the tensile modulus determined by the direct stretching of a bar of like material, provided always that the stress be kept well within the elastic range of the material experimented upon.

CHAPTER VII.

MOMENT OF RESISTANCE OF I, CHANNEL, AND CIRCULAR SECTIONS.

Moment of Resistance of I Sections.—Leaving rectangular beams, we next pass on to consider the strength of the much more economical form of section shown in Fig. 72. Hitherto, in finding the resisting moment of flanged beams at a given section, we have been content to neglect the influence of the web entirely. This procedure is usual, and is quite legitimate as regards deep girders; but it will not do for comparatively shallow beams with thick webs, except as a first approximation. The method to be then adopted, or rather choice of methods, will be made clear by the aid of an example.

Fig. 72 is the section of a steel locomotive coupling rod. The greatest safe stress on the material is six tons per square inch. It is required to ascertain its moment of resistance as a beam, disregarding entirely the rounding of the corners.

The transverse load is here the centrifugal force arising from the rapid rotation of the rod, and acts alternately up and down. This is increased slightly during half a revolution by the weight of the rod. There is also a longitudinal load, but this does not immediately concern us.

Method I.—Several ways of determining the moment of resistance of the rod are available, and it will be instructive to compare them. The easiest to grasp is the "bit-by-bit method," which consists in dividing the whole section into symmetrical slices (Fig. 73), then finding the resistance of each pair separately, multiplying these by their respective resistance arms, and finally adding all the results together, as detailed on p. 84; the product of the first two columns (area × stress) is a force in each case.

Thus the moment of resistance of the entire section is 32·5 inch-tons. This result is quite accurate enough; but a practical objection to the method is its slowness, owing to the necessity of

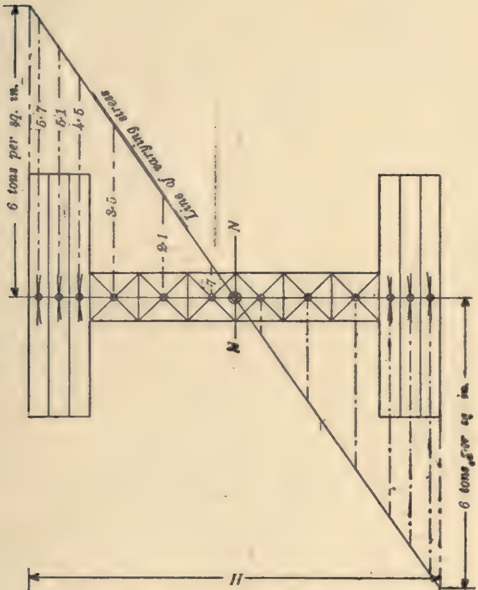


Fig. 73.

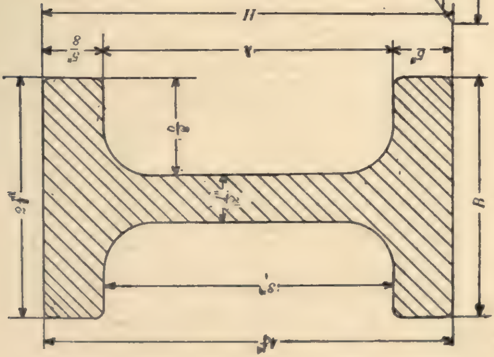


Fig. 72.

drawing the section carefully to scale and of making a good many measurements. Nevertheless, it is a good general method to be able to fall back upon.

Area.	Stress.	Arm.	∴ Moment.
	Tons per sq. in.	Inches.	Inch-tons.
2·5in. × 0·21in.	5·7	4·05	12·12
2·5in. × 0 21in.	5·1	3·63	9·73
2·5in. × 0·21in.	4·5	3·21	7·58
0·5in. × 0·5in.	3·5	2·5	2·19
0·5in. × 0·5in.	2·1	1·5	0·79
0·5in. × 0·5in.	0·7	0·5	0·09
			32·50

Method II.—By means of a *resistance area*, the several operations being described in complete detail below :—

(1) The centre of area or centroid of this symmetrical section is at once placed at the centre of its depth, and the neutral axis drawn through it.

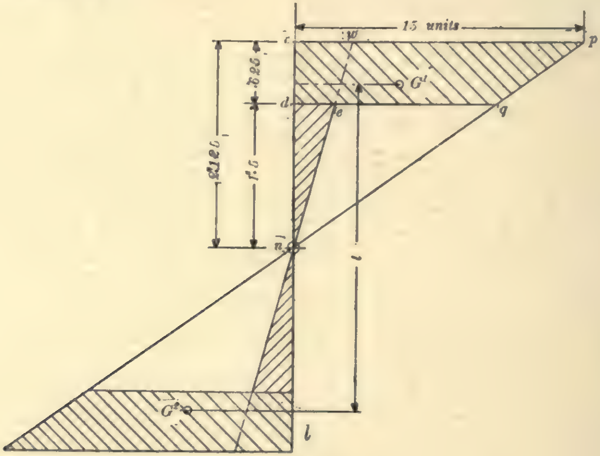


Fig. 74.

(2) Draw the centre line *cl* of the figure of resistance (Fig. 74) at right angles to the neutral axis, and set off *cp* equal

to the breadth of the flange multiplied by the stress allowed at the top edge—that is,

$$2{\cdot}5\text{in.} \times 6 \text{ tons per square inch} = 15 \text{ units},$$

any convenient scale of representation being chosen. Join *pn*, and project *dq* from the lower edge of the top flange. Then *cdqp* is the resistance area of the top flange. We can find its numerical value either by the use of a planimeter (being careful as to the scales) or preferably by arithmetic, as follows:—

Since the stress varies as the distance from the neutral axis, we can find what *dq* represents by the proportion

$$dq : cp = nd : nc,$$

which, on substituting the known values, becomes

$$dq : 15 = 1{\cdot}5 : 2{\cdot}125;$$

whence

$$dq = 10{\cdot}58 \text{ units}.$$

Now, as the area of a trapezoid is half the sum of the parallel sides multiplied by the height, the area *cdqp* represents

$$\tfrac{1}{2}(15 + 10{\cdot}58) \times 0{\cdot}625 = 8 \text{ tons}.$$

This is the resistance of the top flange. It is sometimes called the total stress on the top flange. By working in this way it is not absolutely necessary, so far, to draw an accurate figure of the section; though it is advisable to do so, as a check on the arithmetical work.

(3) To draw the resistance area for the *web*, make *cw* equal to the thickness of the web multiplied by the stress allowed at the top edge of the section—that is,

$$\tfrac{1}{2} \text{ in.} \times 6 \text{ tons per square inch} = 3 \text{ units}.$$

Join *wn*, so finding *e*. Then *de* is a measure of the resistance of the top layer of fibres of the web, and *den* is the resistance area of the top half of the web.

To find how much thrust the area *den* represents, we write the proportion

$$de : dq = \text{thickness of web} : \text{width of flange};$$

which, on inserting the known values, becomes

$$de : 10{\cdot}58 = 0{\cdot}5 : 2{\cdot}5;$$

consequently

$$de = \frac{10{\cdot}58 \times 0{\cdot}5}{2{\cdot}5} = 2{\cdot}116 \text{ units}.$$

An alternative way is to measure *de* directly from a large figure drawn strictly to scale.

Now, as the area of any triangle is half the base into the height, the triangle *den* represents

$$\tfrac{1}{2}\, dn \times de = \tfrac{1}{2} \text{ of } 1{\cdot}5 \times 2{\cdot}116 = 1{\cdot}59 \text{ tons.}$$

(4) By adding together the last two results, we find that the entire *thrust*, say, of the part of the section above the neutral axis is $8 + 1{\cdot}59 = 9{\cdot}59$ tons, as represented by the shaded area. The resistance area of the lower half is similar, and represents the total *pull* over the section.

(5) It is now necessary to find the centroid G^1 of the upper shaded area, which is the most troublesome part of the whole process. For this step we need an accurate figure, and to avoid confusion the construction is given separately in Figs. 75 and 76.

Bisect *cp* at *m*, and *dq* at m^1 (Fig. 75). Join mm^1. Produce *cp*, making $pt = dq$. Also produce *qd*, making $db = cp$. Join *bt*, cutting mm^1 at *g*. This point is the centroid of the trapezoid *cdqp*.

To find the centroid of the triangle *den*, bisect *de* at *a*, and *en* at *f*. Join *an*, *df*. Their intersection g^1 is the point sought.

Lastly, to find G^1 we must divide the line g^1g into segments inversely proportional to the areas of the trapezoid and triangle, thus:—

$$gG^1 : g^1G^1 = \text{area } den : \text{area } cdqp.$$

From steps (2) and (3) we know that

$$\text{Area of } den : \text{area } cdqp = 1{\cdot}59 : 8\,;$$

so that

$$\begin{aligned} gG^1 : g^1G^1 &= 1{\cdot}59 : 8 \\ &= 1 : 5{\cdot}03. \end{aligned}$$

To divide the line g^1g in this proportion, along any line *gk* drawn from *g* at any angle to gg^1, as in Fig. 76, set off $gh = 1$in. or other convenient unit, and $hk = 5{\cdot}03$ of such units. Join kg^1 and draw hG^1 parallel to it. Thus the centroid G^1 of the upper resistance area is determined. The centroid G^2 of the lower resistance area (Fig. 74) is similarly situated, and is at once located by direct measurement.

(6) The next step is to measure the length of the resistance arm. This is the vertical distance between G^1 and G^2. It measures 3·38in. From this and step (4), the moment of resistance to bending of the section is

$$\begin{aligned} &\text{Resistance of half of section} \times \text{resist. arm} \\ &= 9{\cdot}59 \text{ tons} \times 3{\cdot}38\text{in.} = 32{\cdot}43 \text{ inch-tons,} \end{aligned}$$

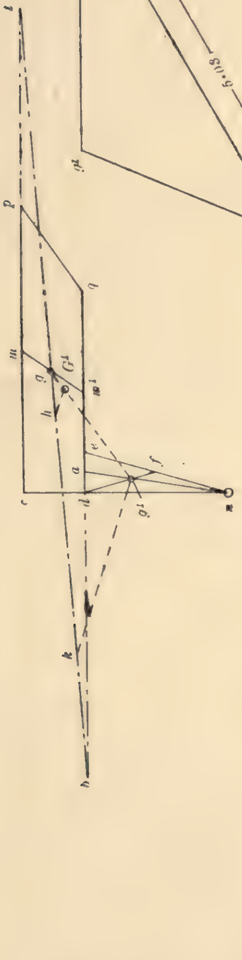


Fig. 75.

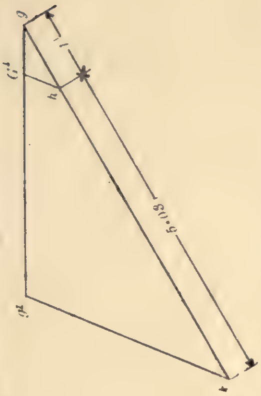


Fig. 76.

a result which agrees remarkably well with the 32·5 inch-tons arrived at by the first method.

This second method is both more difficult and occupies more time than the first.

Method III.—By means of an *equivalent area* (Fig. 77):—

(1) Locate the centroid G of the entire section. Join AG, BG, CG, and DG, thus finding the points E, F, H, J. To take account of the web, join also K, L, M, N, to G. The shaded figure thus determined is the section of an imaginary beam of equivalent value to the actual beam in resisting a bending moment, having every layer stressed to the same extent as the outer layers of the actual beam.

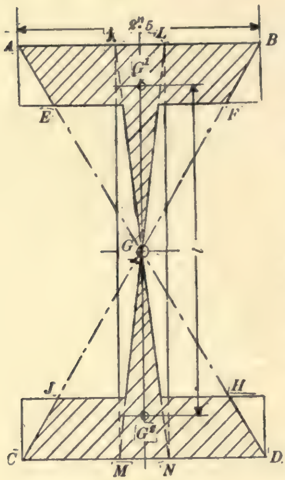


Fig. 77.

(2) The centroids G^1 and G^2 of the upper and lower parts are located by the geometrical process already described under Method II. Another way, mechanical, is to cut out a stiff paper or sheet-metal template, and find the point about which it will balance. This can be done either by trial and error, or by suspending the template from two corners in succession and using a plumb-line to locate G^1.

(3) The area of each shaded part is also found, in the manner already described, to be 1·598sq. in. Hence the moment of resistance of the section is

$$\tfrac{1}{2} \text{ shaded area} \times \text{lever arm} \times \text{stress}$$
$$= 1{\cdot}598 \times 3{\cdot}38 \times 6 = 32{\cdot}42 \text{ inch-tons.}$$

This method is slightly less cumbrous than that of resistance areas.

Method IV.—By the use of a *formula.*—In previous chapters it has been stated that the moment of resistance of any section of a beam is got by multiplying the modulus of that section by the stress allowed on the extreme layers of the material. Now for the present we simply assert that the modulus of an **I** section, like Fig. 72, is given by the formula

$$Z = \frac{BH^3 - bh^3}{6H},$$

which, for the case in hand, becomes

$$Z = \frac{2 \cdot 5 \times (4 \cdot 25)^3 - 2 \times 3^3}{6 \times 4 \cdot 25}$$

$$= \frac{(2 \cdot 5 \times 76 \cdot 77) - (2 \times 27)}{25 \cdot 5} = 5 \cdot 42 \text{inch}^3.$$

Hence the moment of resistance is

$$M = 6 \frac{\text{tons}}{\text{inch}^2} \times 5 \cdot 42 \text{inch}^3 = 32 \cdot 5 \text{ inch-tons.}$$

This last is by far the shortest and least laborious way of proceeding, a slide-rule being used, if at hand. It is also the most accurate, and does not need the section to be drawn to scale. But one must remember that, as we have not yet proved the formula made use of, the result has been found rather by faith than by the more intellectual and satisfying process of reasoning from first principles. Still, the close agreement of the results obtained by processes so different is in itself strong evidence of the truth of each method employed.

BEAMS OF CHANNEL SECTION.

Strength of an Unsymmetrical Section.—Fig. 78 is a section of a beam unsymmetrical about the neutral axis, and therefore different from any hitherto considered in the preceding pages. It is required to estimate its moment of resistance to bending. The following method of proceeding is easy to understand, and has the great advantage of being applicable to any section whatever. It is based on the now familiar fact that the stress on any part of a section varies directly as the distance of that part from the neutral axis of the section.

(1) After drawing the section to scale, reduce it to the simple equivalent section (Fig. 79), putting on all needful dimensions. For simplicity the rivet holes will be disregarded.

(2) Find the areas of the three rectangles, also their moments about the top edge of the section. Arrange these values in tabular form, add them up, and by division locate the centre of area G of the figure. Thus :—

Area.		Arm.	Moment.
	Inch².	Inch.	Inch³.
10in. × 0·5in. = . .	5	0·25	1·25
3·5in. × 2in. = . .	7	2·25	15·75
1in. × 10in. = . .	10	9·0	90
	22		107

Hence $x = 107 \div 22 = 4{\cdot}87$in. Set off this distance, and through G draw the neutral axis.

(3) At that edge of the section which is the farther from the neutral axis—viz., the bottom edge (Fig. 80)—set off *ab* to represent the limiting stress on the extreme fibres, say six tons per square inch. Join *b*G, and produce it to cut the top edge. Ordinates to this

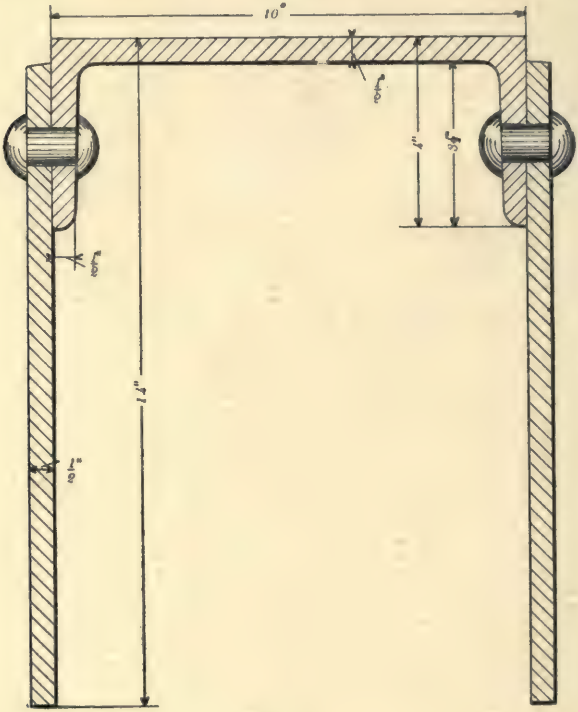


Fig. 78.

diagonal line represent the varying stress over the section due to bending.

(4) Divide the section into a number of strips—the more the better for accuracy, the fewer the better for speed. The lengths of the ordinates at the centres of area of the several strips represent the average stresses over them. Measure all these ordinates with a decimal scale, and set them down as shown in Fig. 80, also their distances from the neutral axis.

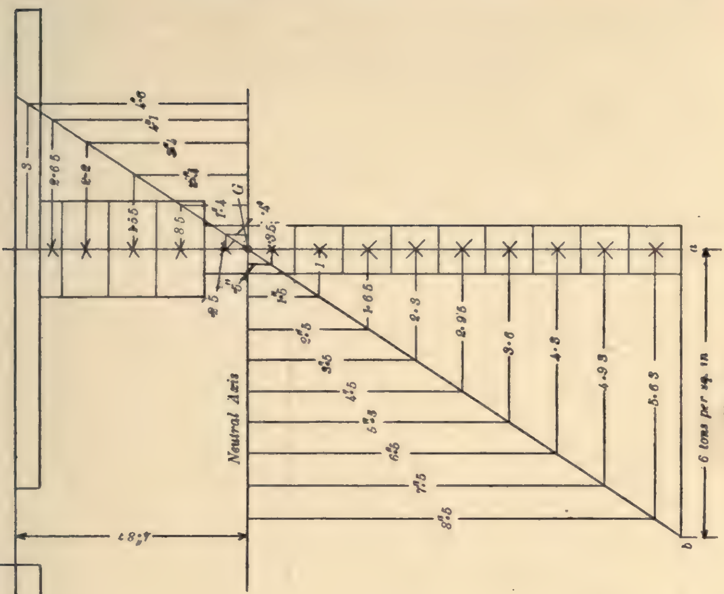


Fig. 80.

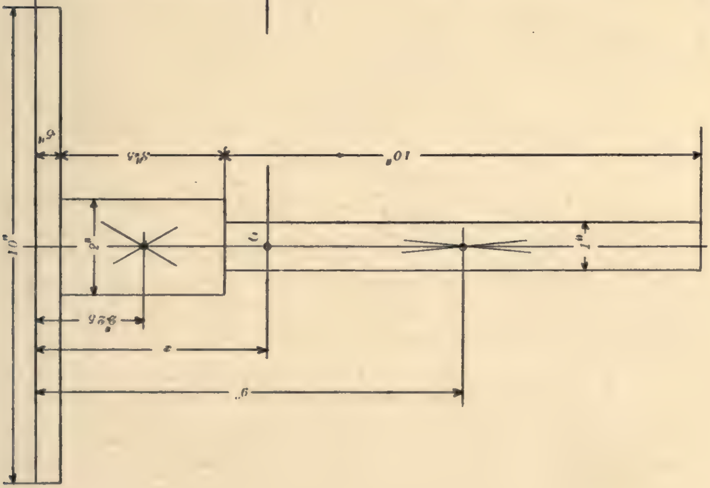


Fig. 79.

(5) Arrange the areas, stresses, and arms in tabular form; multiply each row of figures together; add the resulting moments, and so get the total resistance which the upper and lower parts are separately capable of exerting without anywhere exceeding the stress allowed. Thus :—

Area.	Stress.	Arm.	Moment.
Inch^2.	Ton/inch^2.	Inch.	Inch-tons.
Upper part—			
10in. × 0·5in. = 5	3·00	4·6	69·0
2in. × 0·5in. = 1	2·65	4·1	10·9
2in. × 1in. = 2	2·20	3·4	15·0
,, 2	1·55	2·4	7·45
,, 2	0·85	1·4	2·38
1in. × 0·87in. = 0·87	0·25	0·4	0·09
			104·8
Lower part—			
1in. × 1in. = 1	0·35	0·5	0·18
,, 1	1·00	1·5	1·50
,, 1	1·65	2·5	4·13
,, 1	2·3	3·5	8·05
,, 1	2·95	4·5	13·29
,, 1	3·6	5·5	19·80
,, 1	4·3	6·5	27·95
,, 1	4·93	7·5	37·00
1in. × 1·1in. = 1·1	5·63	8·5	52·60
			164·5

Hence the required total amount of resistance is $104{\cdot}8 + 164{\cdot}5 = 269{\cdot}3$, or, say, 270 inch-tons.

An alternative mode of proceeding, styled the "moment of inertia method," will be discussed later. It gives the same result as the above process, and is shorter in application; but it demands considerably more mathematical knowledge for its proper understanding.

BEAMS OF CIRCULAR SECTION.

Example 1.—To find the moment of resistance of a beam 12in. diameter, when the stress on the extreme fibres is six tons per square inch.

First Method.—Divide the section (Fig. 81) into twelve strips 1in. broad, and reduce them by the eye or the planimeter to rectangles of equal area. Then, beginning with the outermost pair of strips, multiply each area by the average stress over the strip and by the distance between the centres of rectangles equally distant from the neutral axis; thus finding the moment of the resisting couple for each pair of strips. Tabulate the products and add them, as below :—

Area.	Stress.	Arm.	Moment.
Inch².	Ton/inch².	Inch.	Inch-tons.
4·5	5·5	11	273
8·0	4·5	9	324
9·6	3·5	7	235
10·8	2·5	5	135
11·6	1·5	3	52
11·9	0·5	1	6
			1025

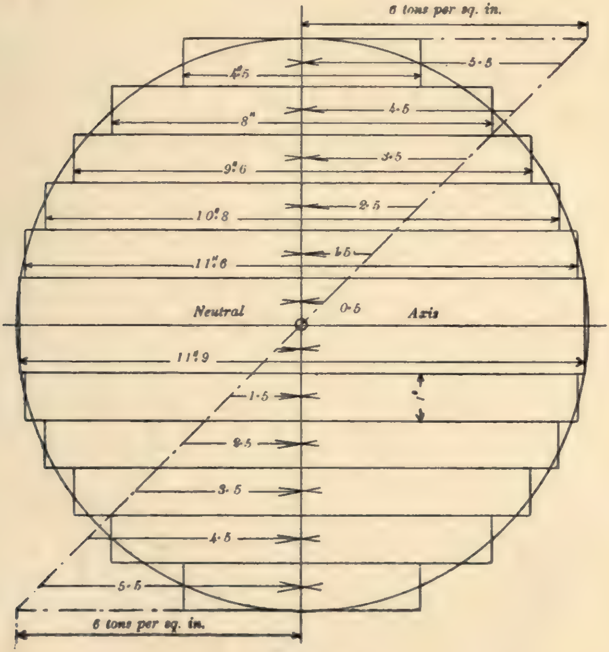


Fig. 81.

Second Method.—It is instructive to compare this result with that given by the following very useful formula, which can be directly deduced only by the aid of the integral calculus, viz. :—

$$M = \frac{\pi}{32} \times D^3 \times f = 0{\cdot}0982\ D^3 f,$$

the symbols having the meanings given below :—

M inch-tons = moment of resistance to bending of a circular section.
D inches = diameter of section.
f ton/inch² = stress on extreme fibres.
π or 3·1416 = circumference ÷ diameter of section.

Of course, if f be expressed in pounds per square inch, M will be expressed in inch-pounds. For a 12in. section and a stress of 6 tons per square inch, this rule gives—

$$M = 0{\cdot}0982 \times 1728 \times 6 = 1020 \text{ inch-tons.}$$

Thus the two methods agree very well, and the agreement could be made even closer by taking more strips; but so slight a difference is of no practical importance. The formula can be further verified by taking a large number of circles of different sizes, treating them like Fig. 81, and comparing the results with those obtained by the use of the formula. The latter can be slightly simplified for practical use, thus :—

$$M = \tfrac{1}{10} D^3 f.$$

This is amply accurate enough, because 0·1 is so near to 0·0982 that it may always be used in place of the latter, without material error. It must be remembered that the other factor f has very far from an absolutely fixed value, different engineers allowing widely different working stresses.

Crank-pins.—A suitable stress to allow for steel crank-pins and shafts subjected to fluctuating forces is 5000lbs. per square inch, which should not be exceeded, unless lightness is specially desirable, as in the engines of warships. An ample margin of safety is thereby secured, and the formula for the moment of resistance of the cross-section of a crank-pin becomes—

$$M = 500 D^3 \text{ inch-pounds.}$$

In the case of crank-pins and like details, where the stress on the metal is continually changing in value between wide limits, an unusually high factor of safety is essential for longevity. Crank-pins are not designed simply to be strong enough, but rather with a view to durability and freedom from heating during long runs. Hence ample *bearing surface* is necessary. In high-speed engines it is best to make the crank-pins long, and not excessively large in diameter; but if the length be unduly restricted, then sufficient bearing area must be given by increasing the diameter, as in locomotives. Lightness is best attained by the use of *hollow* pins; the holes in which also increase the radiating surface, and therefore promote cool running.

Example 2.—It needs little reflection to see that a *solid* circular section cannot be an economical one as regards weight; because most of the material is situated near to the neutral axis, where it has very little leverage in resisting the bending moment. Weight

Fig. 82.

for weight, a *hollow* beam is much stronger; because the material is placed farther away from the neutral or unstressed layer of fibres. This is the secret of the lightness of modern bicycle frames, and, in fact, of all tubular girders.

If we were to make a hollow beam, like Fig. 82 in section, of the same weight as a 12in. solid beam, but of metal only 1in. thick, we should find its moment of resistance to be surprisingly greater than in the former case. The following calculation shows the exact gain in strength :—

Let d inches be the diameter of the hole. Then $(d + 2)$ inches is the outside diameter of the beam, say D. Now, as the *weight* is the same as that of a 12in. solid beam, we must have—

Area of hollow section = area of solid section, or, $$\frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4} \times 12^2.$$

Cancelling out and substituting for D in terms of d, we get—

$$(d + 2)^2 - d^2 = 144,$$
$$\therefore d^2 + 4d + 4 - d^2 = 144,$$

and finally, $$d = 35.$$

Hence the internal diameter is 35in., and the external size 35 + 2 = 37in.

Now the moment of resistance of a tubular or hollow circular section is given by the important formula—

$$M = \frac{\pi}{32} \times \frac{D^4 - d^4}{D} \times f,$$

as can be proved by the aid of the calculus.

Applying this formula to the case in hand, we find that

$$M = 0\cdot098 \times \frac{37^4 - 35^4}{37} \times 6 = 5936 \text{ inch-tons.}$$

Comparing this result with that of Example 1, preceding, we see that the strength of the hollow beam is to the strength of the solid beam as 5936 is to 1020, or as 5·8 to 1. The gain of *strength* is thus very great; and it can be shown that the gain in *stiffness* by this more rational distribution of material is still greater.

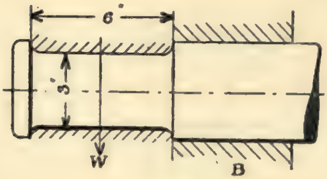


Fig. 83.

Example 3.—The journal of an axle (Fig. 83) is 3in. diameter and 6in. long, the wheel boss B coming close up to it. Estimate what distributed load W the journal will bear safely, allowing a maximum stress of 8000lbs. per square inch.

Since the maximum bending moment equals the resisting moment, we have—

$$W \times 3\text{in.} = \tfrac{1}{10} \times 3^3 \times 8000, \text{ say,}$$
$$\therefore W = 9 \times 800 = 7200\text{lbs.}$$

$$\text{Bearing pressure} = \frac{7200}{6 \times 3} = 400\text{lbs. per sq. in.}$$

Example 4.—A travelling crane axle (Fig. 84) has to be made strong enough to carry a load of 12 tons. Calculate its diameter for a safe stress of 4 tons per square inch.

The wheel or "runner" is supposed to be keyed on the *middle* of the axle, but when this is not the case the reactions of the bearings will be unequal. The maximum bending moment is usually calculated as if the load or reaction of the rail were concentrated at the centre of the wheel, though actually distributed over a considerable length of the axle. On this assumption the equation of moments is—

$$\tfrac{1}{10}d^3 \times 4 \text{ tons} = 6 \text{ tons} \times 5\text{in.}$$
$$\therefore d^3 = 75, \text{ and } d = 4\tfrac{1}{4}\text{in.}$$

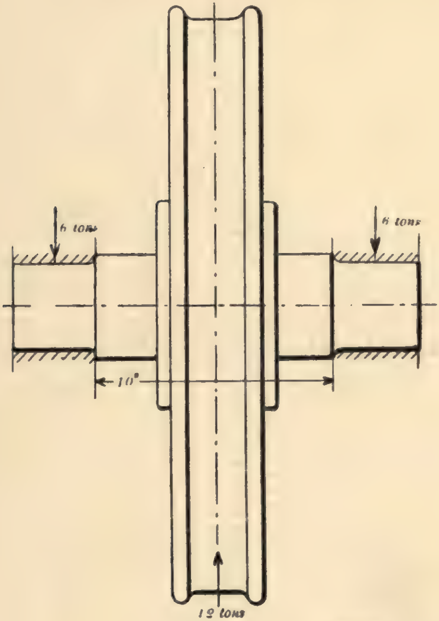


Fig. 84.

This is the necessary diameter at the centre of the span. Beyond the wheel seating the axle may be tapered without loss of strength; but such tapering is not usual, as it would add to the cost of turning the axle in the lathe. The journals are made large enough to resist the shearing force, and to provide sufficient bearing area.

Example 5.—A solid wrought-iron shaft, 9 in. diameter, is supported at each end in a horizontal position (Fig. 85). Calculate for what span the material would be stressed to the elastic limit of 30,000lbs. per square inch merely by its own weight.

Weight of shaft = volume × weight of 1 cub. in.

$$\therefore W = \frac{\pi}{4}d^2 \times L \times 0{\cdot}28 \quad . \quad . \quad . \quad . \quad (1)$$

Treating the weight as a distributed load, the maximum or central bending moment is

$$M = \tfrac{1}{8}WL \quad . \quad . \quad . \quad . \quad (2)$$

Inserting in (2) the value of W given by (1), we get

$$M = \tfrac{1}{8}\left(\frac{\pi}{4}d^2L \times 0{\cdot}28\right)L$$

$$= \frac{\pi}{32}d^2L^2 \times 0{\cdot}28 \quad . \quad . \quad . \quad . \quad (3)$$

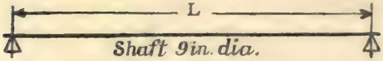


Fig. 85.

Equating this bending moment to the resisting moment of a circular section, we have the general equation

$$\frac{\pi}{32}d^2L^2 \times 0{\cdot}28 = \frac{\pi}{32}d^3 \times f.$$

Removing the common factors, there results

$$L^2 = \frac{f \times d}{0{\cdot}28}$$

Substituting the highest elastic value of the stress f and also the given diameter, 9in., this becomes

$$L^2 = \frac{30{,}000 \times 9}{0{\cdot}28} = 964{,}300.$$

Extracting roots, the required length is 982in., or nearly 82ft. Beyond this length the shaft would take a permanent set under the influence of the earth's attraction or gravity.

CHAPTER VIII.

SHEARING ACTION IN BEAMS.

HITHERTO we have strictly confined our attention to the *bending* action caused by loads acting transversely on a beam, and to the mode in which this action is resisted in certain important cases. But before we can proceed to design beams scientifically, the shearing action which always accompanies bending must likewise receive consideration.

Probably the simplest example of shearing action is that presented by the pins of a flat-link chain (Fig. 86) subjected to a pull

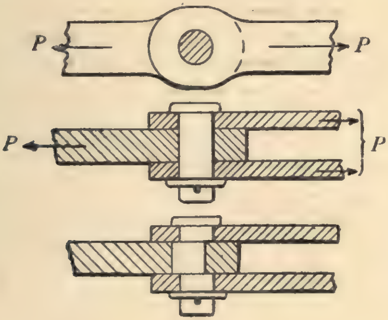


Fig. 86.

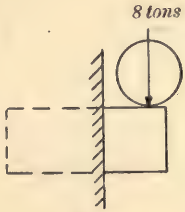


Fig. 87.

of P tons. Here the tendency of the two opposing forces is to produce the effect shown in the lowest view—that is, to shear the pin across in two planes parallel to the plane containing P. Of course, this effect will not actually occur unless the pin be made disproportionately weak; but the *tendency* to shear exists, nevertheless.

The nature of the shearing action in a *beam* is not so easily realised as in the above simple case. It is best understood by considering, in the first instance, a very short cantilever (Fig. 87).

Here, as the leverage is so short, there is clearly very little bending action, the main tendency of the load being to shear the beam bodily across. This tendency is referred to as the "vertical shearing force." The equal and opposite resistance which the material offers to this force is often called the "shearing stress." The word "stress," however, is frequently used so ambiguously that it seems better to speak of the "shearing *resistance*" as an

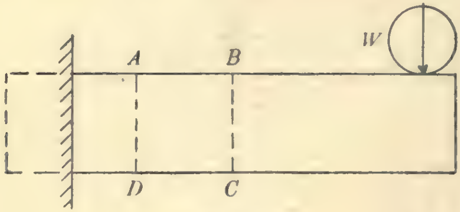


Fig. 88.

alternative to the "total shearing stress." The term "stress" is then available for expressing the resistance of the material per unit area of cross-section. Adopting this nomenclature, the relation between shearing force and shearing resistance is analogous to that between bending moment and moment of resistance.

Thus, if the transverse load is 8 tons (Fig. 87), and the section of the bar 2in. by 4in., then the shearing *force* is –8 tons, the

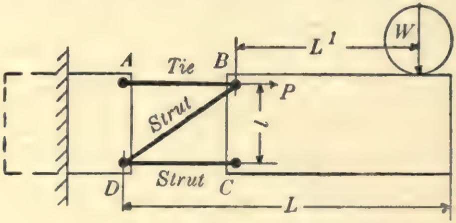


Fig. 89.

negative sign showing that the force acts downwards. The shearing *resistance* is + 8 tons, because it acts upwards. Lastly, the shearing *stress* is the shearing resistance ÷ the area of cross section = 8 tons ÷ (2in. × 4in.) = 1 ton per square inch.

Equilibrium of a Cantilever.—Passing from the case of a very short cantilever to that of a long one, the true relation between the several forces acting on a loaded cantilever (Fig. 88) may be brought out very clearly by supposing a piece to be cut out and a system of three sets of pin-jointed bars substituted for it, as in Fig. 89. To preserve lateral stability there must be at least two bars in each set. The load is then just as well supported as by the original beam, provided that the bars are properly proportioned. Let us examine the several effects caused by removing the bars one set at a time, and by that means ascertain their individual functions.

In the first place, the disastrous effect of taking away the *upper*

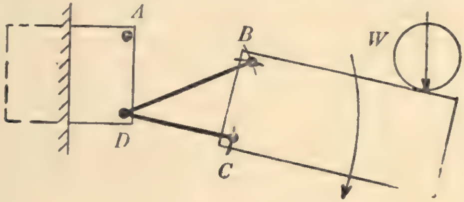


Fig. 90.

links is graphically shown in Fig. 90, where the free part of the beam is now rotating about the pin D as a pivot. The function of the set A B, is therefore to prevent turning about D, by maintaining a constant distance between A and B. Since this distance tends to increase, the upper set of links must be in tension. The measure of the tendency of the free or right-hand part of the beam to turn round D (that is, the turning moment about D) is W times the arm WA (Fig. 89).

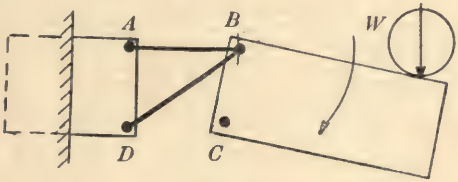


Fig. 91.

This product is what we are already quite familiar with as the bending moment at the section AD. Further, so long as the ties prevent the rotation about D, the opposing turning moment, which maintains stability, must be the pull P on all the ties multiplied by the leverage l. This product also we have become familiar with as the moment of resistance of the section AD.

Secondly, the no less calamitous effect of removing the *lower* set of bars is shown in Fig. 91, the free part of the beam now rotating about the pin B. Hence the precise duty of the bars C D is to prevent angular motion about B. Since the distance CD tends to diminish, the lower bars must be in compression, or, in a word, they are *struts*. The initial turning-moment about B is W × L[1]

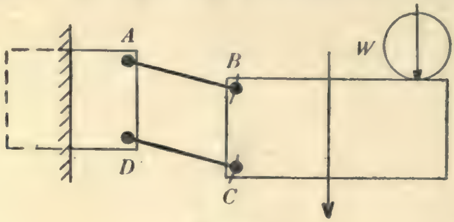


Fig. 92.

(Fig. 89), and this is numerically equal to the total thrust on the bars D C multiplied by the leverage l, from which relation the thrust is readily found. Thus the united function of the upper and lower sets of bars is to resist the *rotative* tendency of the load.

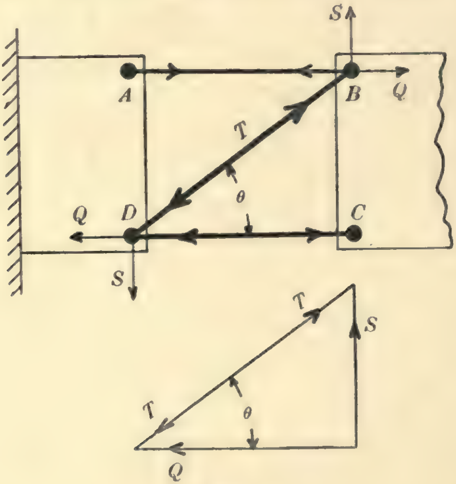


Fig. 93.

Lastly, Fig. 92 shows the peculiar effect of removing the diagonal bars—namely, a vertical displacement or bodily drop of the free part, in parallel ruler fashion. Accordingly the function of the set of diagonals B D is to prevent one part of the beam *sliding* bodily relatively to the other, simply by keeping the distance between the pins B and D constant. And since this distance tends to lessen under the influence of the load, it follows that the bars B D must be *struts*.

It should be observed that the equilibrium could be equally well preserved by substituting for the struts B D a set of diagonal ties connecting A to C, which would effectually prevent the distance AC from increasing.

In a *solid* beam these diagonal shearing-stress actions still exist, though their mode of action is less clearly defined. In the case of a rectangular beam the whole body of metal is available for resisting them ; in a plate girder, the web performs this office ; and lastly, in the familiar lattice girder the diagonal bars resist the shearing forces.

Vertical and Horizontal Shearing Forces.—In Fig. 93 the total thrust T on the assumed single diagonal strut is shown resolved into its horizontal and vertical components Q and S. These may be found either by graphic construction or by calculation, knowing that

$$Q = T\cos\theta, \text{ and } S = T\sin\theta.$$

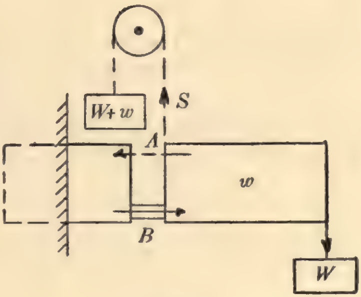


Fig. 94.

The vertical component S alone resists the sliding action and keeps the right-hand part of the beam in position. The effect of the horizontal component Q is, at the point B, to increase the tension on the *tie* AB ; and at the point D to diminish the thrust on the *strut* CD to an equal extent.

The tendency of the right-hand part of the beam to slide downwards is the vertical shearing force at the section BC. The holding-up force, or resistance to shearing, is numerically equal to this, though of opposite sign.

The vertical shearing force at a given section of a beam loaded in any manner is more precisely defined as the resultant of all the forces acting on one (either) side of the section resolved vertically, upward forces being regarded as positive, and downward forces as negative. To preserve consistency of sign, however, the right-hand side of the section should always be chosen. For instance, in the simple case represented by Fig. 94, the load W is negative, and so is the shearing force W + w at the section AB. But the shearing resistance S is positive. Unless this convention of signs be carefully observed, confusion is sure to arise in dealing with difficult cases.

The effect that the vertical shearing force tends to produce in a solid beam can be illustrated by reference to a model composed of

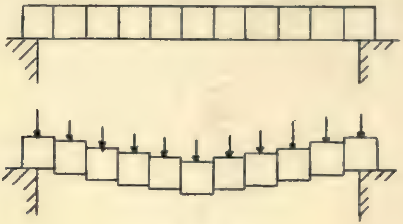


Fig. 95.

a large number of segments or short blocks strung tightly together, as in Fig. 95. When such a beam is loaded sufficiently these blocks slip past one another, as shown in the lower view. In an actual solid beam, this *tendency* of adjacent segments to slide past one another still exists, though it cannot take place owing to the cohesion of the particles.

Fig. 96.

Accompanying the *vertical* shearing force in a solid beam there is invariably a *horizontal* shearing force of equal amount. Fig. 96 shows a beam cut into a number of thin planks, say four. When a load is applied, these planks slide over each other slightly, as in the lower view. The cause of this sliding is the *horizontal* shearing force.

To prove that at any point in a beam the horizontal shearing force is numerically equal to the vertical shearing force, imagine a very small cube of material situated at the neutral layer of an unloaded rectangular beam. The elevation of this cube is a square, shown greatly magnified in Fig. 97. On loading the beam, the cubic block is distorted slightly, and its elevation becomes a rhombus, as shown greatly exaggerated in the figure.

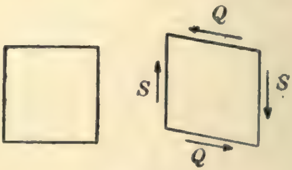


Fig. 97.

To what *forces* is this distortion due? Since there is no direct thrust or pull at the neutral layer of a beam, the distortion must be solely due to the vertical shearing couple SS acting on the very small block. This couple is opposed by the originally horizontal couple QQ. And since there is equilibrium, and the arms of the couple are horizontal, it necessarily follows that Q equals S; that is, the horizontal shearing force numerically equals the vertical shearing force at the place considered.

CHAPTER IX.

CONSTRUCTION OF SHEARING FORCE AND BENDING MOMENT DIAGRAMS.

Diagrams of Shearing Force and Bending Moment show very clearly how the shearing force and the bending moment vary from point to point in the length of a beam. In turning our attention to these, it will be best, in the first instance, to consider numerically a few simple but oft-recurring cases, and to regard the weight of the beam itself as negligible in comparison with the magnitude of the load applied.

Example 1.—The simplest case of all is that of a cantilever loaded at one end with a single load, as shown in Fig. 98. To construct the shearing force (or S.F.) diagram, draw a base or zero line A_1B_1 and measure downwards from it at a distance A_1S representing the load of 10 tons, to any convenient scale. Complete the rectangle A_1F. The equal ordinates to the line SF indicate that the vertical shearing force at any section between A and B is uniformly equal to 10 tons in magnitude, and acts downwards, or in the *negative* direction. There is no shearing force on the short length of beam to the right of the load.

Before starting to draw the bending moment diagram, we first decide that moments which tend to rotate the beam about the root A, in the same direction as that travelled by the hands of a clock, shall be regarded as *negative*, and be measured *downwards* from the base line A_2B_2. Though this is in accordance with the best mathematical usage, some writers prefer to adopt a different convention of signs, regarding *upward* forces as negative, and watch-hand rotation as positive—that is, precisely contrary to the practice we shall here adopt.

As regards the *magnitude* of the bending moment, we have to reflect that the value will increase uniformly from the point of application B of the load to the point of support A, in exact proportion as the leverage or *arm* of the force increases.

To proceed with the diagram, therefore, set off a distance A_2M to represent in magnitude and sign the greatest bending moment of $-$ 10 tons $\times$ 100in. $=$ $-$ 1000 inch-tons, to any convenient scale of moments; and join MB_2. Then any ordinate to this sloping line represents the value of the bending moment at the section of the beam directly over that ordinate.

From this simple B.M. diagram it is evident that a cantilever, loaded at the end, should not be made parallel or of uniform depth, if economy of material is desired, but rather of tapering

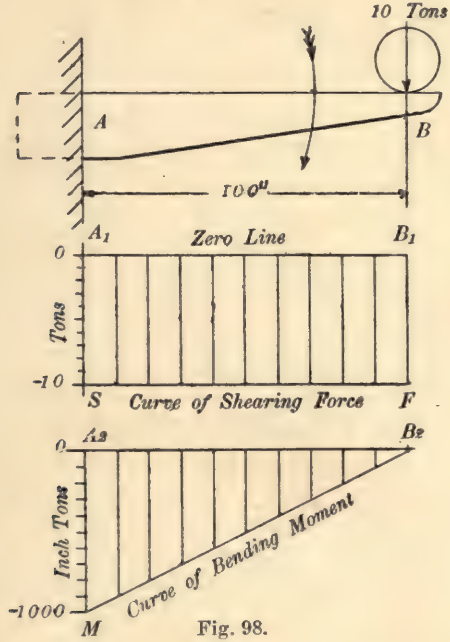


Fig. 98.

profile; the thinnest part being made of sufficient section to withstand the shearing force, and the thickest part strong enough to resist the greatest bending moment.

Example 2.—Consider next the case of a beam (Fig. 99) loaded at the ends A, B, and supported on a fulcrum at some intermediate point C; like the beam of a lever testing-machine or of a pumping-engine. Such a beam may be regarded as two cantilevers united at C.

The total reaction or upthrust of the bearings is the sum of the downward loads—that is, 6 tons. The shearing force between A and C is the algebraic sum of the forces to the right of A—

namely, + 6 − 2 = 4 tons, which is set off at A_1, above the base line A_1B_1. The S.F. between C and B is − 2 tons, which is set off downwards at C_1. Complete the shearing force rectangles as shown.

The bending moment at C is − 2 tons × 100in. = − 200 inch-tons, which is the moment tending to turn the beam clockwise. The equal opposing moment is − 4 tons × − 50in. = 200 inch-tons, distances to the left of C being regarded as *negative.*

Further, the B.M. at A is (6 tons × 50in.) − (2 tons × 150in.) = 300 inch-tons − 300 inch-tons = 0, which is also the B.M. at B. Similarly, the moment at D is (6 × 25) − (2 × 125) =

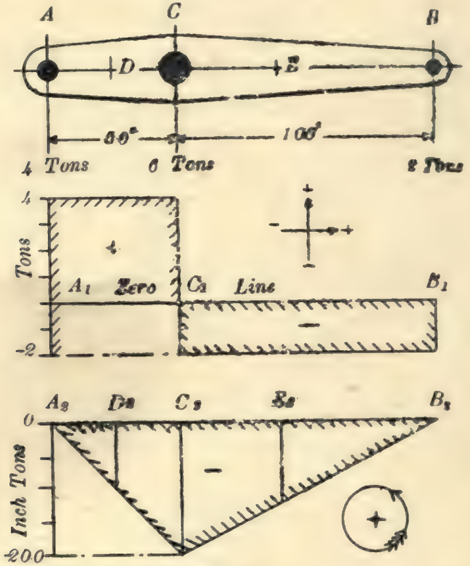


Fig. 99.

150 − 250 = − 100 inch-tons, and at E is − 2 tons × 60in. = − 120 inch-tons. All these results are fully represented in the bending moment triangle of Fig. 99.

It should be borne in mind that, in designing actual beams, the construction of shearing force and bending moment diagrams is quite a preliminary process, useful only as a means to an end. The designer's ultimate aim is to determine the proper shape and size of the beam, both as a whole and in detail. But this he is unable to do with absolute certainty without first of all finding out what each section of the beam really has to stand—information which is most clearly expressed by means of S.F. and B.M. diagrams.

Example 3.—Fig. 100 shows the shearing force and bending moment diagrams for an ordinary cantilever loaded with *two* forces. After drawing the upper part of the S.F. diagram as before, produce the line of action of the 8-ton force, and set off a

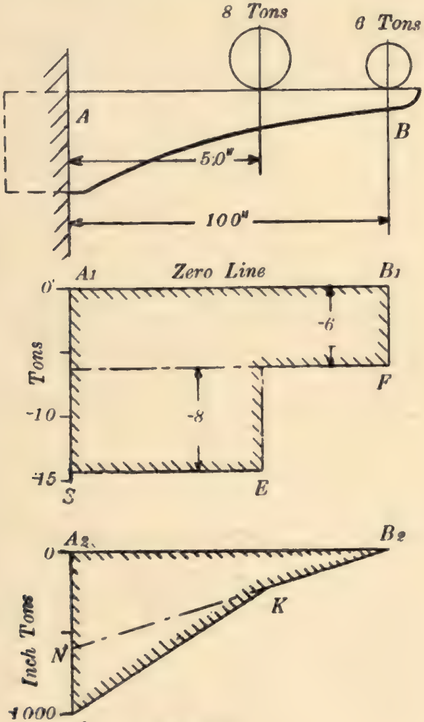


Fig. 100.

further distance downwards of 8 units. Then SE, drawn parallel to the zero line, completes the figure, which is merely the sum of the two rectangles arising from the separate forces. Similarly, if n loads act on any beam, then the S.F. diagram is the sum of all the n rectangles due to the several loads.

To construct the bending moment diagram, set off A_2N to represent a moment of $-$ 6 tons $\times$ 100in. $=$ $-$ 600 inch-tons, and join NB_2. This disposes of the outer load. Then make

NM represent − 8 tons × 50in. = − 400 units, and join MK. The triangle NMK thus formed is the addition due to the 8-ton load. Thus the B.M. diagram is simply the sum of the triangles due to the two separate loads. In the same way, to generalise, if there are n loads, the bending moment diagram is the sum of all the n triangles arising from the separate action of those loads,

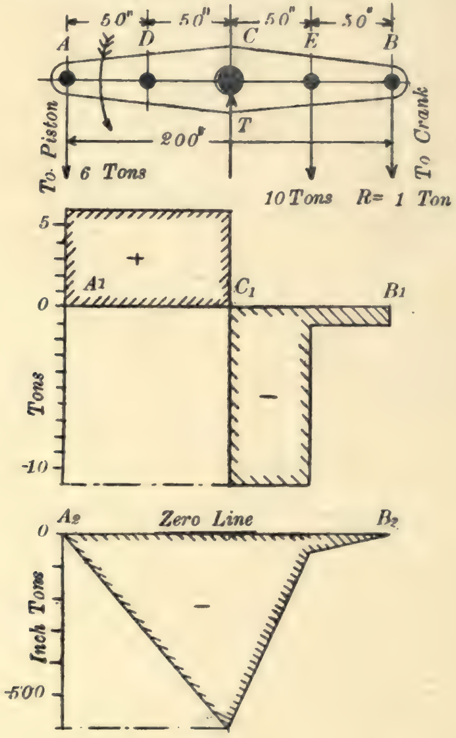


Fig. 101.

and may be drawn by the method of superposition. One lesson taught by the B.M. diagram of Fig. 100 is that, for economy of material, the cantilever should be well splayed out at the root.

When a beam is so designed that at each section the greatest safe moment of resistance is precisely proportional to the bending moment, the beam is rather curiously said to be of *uniform strength*, because it is equally likely to give way at each section.

A beam of uniform strength is *not* one whose moment of resistance is the same at every cross-section, in conventional language.

Example 4.—Fig. 101 indicates a pumping-engine beam 200in. long. A double-acting pump, having a resistance of 10 tons, has to be worked from it. It is undecided whether to drive the pump from the pin D or from E. Show, by constructing S.F. and B.M. diagrams, which is the better position.

First: Let the pump be driven from E. For simplicity, we may neglect the obliquity of the connecting-rod, the pull R along which is got by taking moments about C, thus—

$$(R \times 100\text{in.}) + (10 \text{ tons} \times 50\text{in.}) = 6 \text{ tons} \times 100\text{in.}$$
$$\therefore 100\, R + 500 = 600 \text{ inch-tons.}$$
$$\therefore R = 1 \text{ ton.}$$

This is the pull on the rod at the instant considered, when part of the driving effort is being expended in accelerating the fly-wheel of the engine. But the *pull* changes to a *thrust* at a later period of the stroke, when a demand is made on the fly-wheel's store of energy for the maintenance of motion.

The upthrust T of the beam gudgeon bearings at C is the sum of all the downward forces acting on the beam, and equals $6+10+1=17$ tons. The shearing force diagram is now easily drawn, as indicated, the greatest S.F. being -11 tons.

As to the bending moment, at B it is plainly nothing. At E it is -1 ton $\times$ 50in. $= -50$ inch-tons. At C it is ($-$ 1 ton $\times$ 100in.) $-$ (10 tons $\times$ 50in.) $= -600$ inch-tons. At D, again, the bending moment is (-1 ton $\times$ 150 in.) $-$ (10 tons $\times$ 100in.) $+$ (17 tons $\times$ 50in.), which items add up to -300 inch-tons. Lastly, at A the B.M. vanishes. By drawing straight lines through the points thus found, the B.M. diagram is determined.

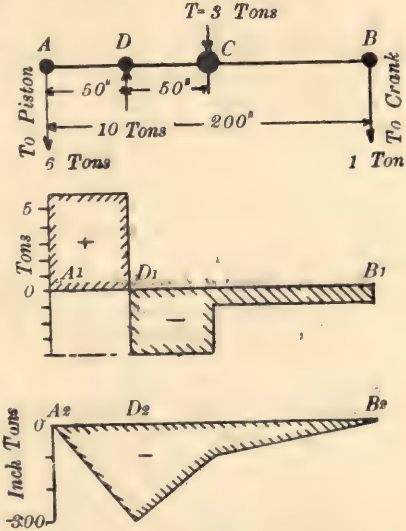


Fig. 102.

Secondly: Let the pump be driven from D, as in Fig. 102. Then the vertical reaction T of the bearings at C is such that

$$T - 6 + 10 - 1 = 0;$$

and therefore T = − 3 tons, showing that the pressure now comes on the *caps* of the bearings, which react downwards. Compared with the former case, the magnitude of the reaction is reduced to the extent of 17 − 3 = 14 tons, a very considerable improvement.

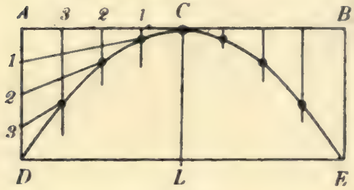


Fig. 103.

The shearing force diagram is readily drawn, in the manner already described. The greatest S.F. is now only 6 tons instead of − 11 tons.

The bending moment at C is − 1 ton × 100in. = − 100 inch-tons, and at D it is (− 1 × 150) − (3 × 50) = − 300 inch-tons. Hence the greatest bending moment has changed from − 600 inch-tons to − 300 inch-tons. It finally appears, then, that the second arrangement is decidedly the better of the two; for not only is the load on the gudgeons greatly relieved, but the beam scantlings may also be materially reduced. Further, owing to the diminished friction, the mechanical efficiency of the engine would undoubtedly be higher in the second case than in the first.

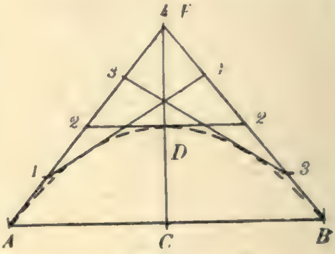


Fig. 104.

How to Draw a Parabola.—A simple method of drawing a parabolic curve of a given height on a given base will now be described, as we shall require it immediately. After drawing the enclosing rectangle ABED (Fig. 103), draw the centre line CL, then divide AC into any number of equal parts—say four—and AD into the same number. Figure the points as shown, and draw radial lines to C. The intersections of similarly-named radial and vertical lines are points on the parabola sought, and through these the curve is drawn freehand, or by the aid of a suitable template.

Another useful method of drawing a parabola through three points, A, B, and D, is that of tangents, as shown in Fig. 104, where CD is the given height of the curve. Produce CD to F, making DF equal to CD. Join AF and BF. Divide these lines into a similar number of parts, number the points as shown, and join points numbered alike. The series of tangents thus drawn determine the shape of the curve very accurately.

Example 5.—The important case of a cantilever loaded uniformly (Fig. 105) next demands attention. A uniformly-distributed load, however great, may always be regarded as a large number of small equal loads acting at a short distance apart. Thus an 8-ton load may, with fair accuracy, be split up into eight 1-ton loads, and the boundaries of the shearing force and bending-moment diagrams then drawn, in the manner already described, as a series of stepped straight lines.

But happily a much shorter method is available. The greater the extent to which the subdivision of the distributed load is carried, the nearer will the S.F. curve approach a single straight line, and in the limit the steps disappear entirely, and we get the dotted straight line.

Reasoning in the same way, we find that the shape taken by the bending-moment curve, when the subdivision of the load is carried on indefinitely, is a parabola with axis vertical and vertex at the free end of the beam. It requires a slight knowledge of co-ordinate geometry to see that this necessarily follows from the form of the equation of the B.M. curve, namely,

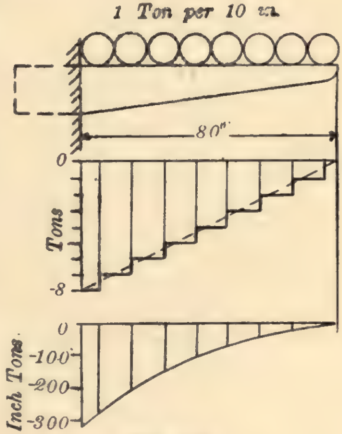


Fig. 105.

$$\text{Moment} = \tfrac{1}{2}wx^2,$$

where w pounds per inch run is the load, and x inches the distance of any section from the free end of the beam. The *greatest* bending moment is $\frac{1}{2}wL^2$ inch-pounds, which is got by multiplying the total load wL by the mean leverage $\frac{1}{2}L$.

Example 6.—Fig. 106 illustrates the instructive case of a cantilever uniformly loaded with four tons over half its length only. The shearing force is uniform over the unloaded half, and dwindles down to nothing at the free end.

The greatest bending moment is the same as that due to a load of four tons, concentrated at the mean leverage 75in. from the wall, and therefore equals – 4 tons × 75in. = – 300 inch-tons. Also the B.M. at the *centre* of the beam is – 4 tons × 25in. = – 100 inch-tons. The curve of moments is completed by a parabolic arc.

Example 7.—Passing from cantilevers to beams supported at *both* ends, or girders, the simplest case is that shown in Fig. 107, where the entire load is supposed to be concentrated at the very centre, though in reality it is spread over an appreciable area.

To get a right view of the magnitude and sign of the shearing

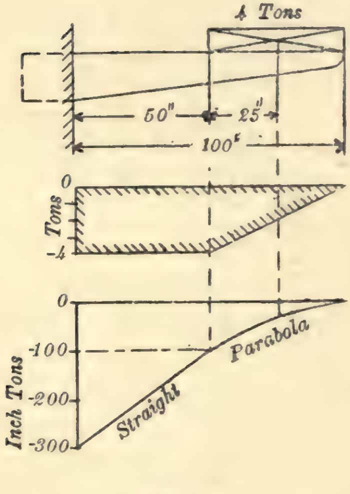


Fig. 106.

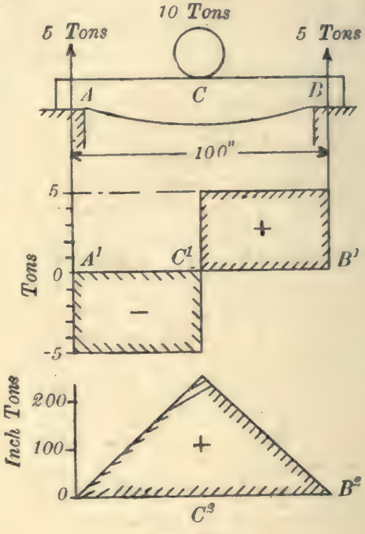


Fig. 107.

force at any section, we find the resultant of the vertical forces acting on the *right-hand* side of that section. Thus the S.F. anywhere between A and C is – 10 tons + 5 tons = – 5 tons, and anywhere between C and B is + 5 tons. The S.F. therefore changes sign at C; or, in other words, there is no shearing force at the very centre of the beam.

By taking moments about A, the bending moment at that point is found to be

$$(-\ 10 \text{ tons} \times 50\text{in.}) + (5 \text{ tons} \times 100\text{in.}) = 0;$$

and, by taking moments about C, the greatest B.M. is seen to be 5 tons × 50in. = 250 inch-tons. Lastly, the B.M. at D, half-way between A and C, is

$$(-10 \text{ tons} \times DC) + (5 \text{ tons} \times DB)$$
$$= (-10 \text{ tons} \times 25\text{in.}) + (5 \text{ tons} \times 75\text{in.})$$
$$= -250 + 375 = 125 \text{ inch-tons.}$$

Or it may be found by dealing with the forces to the *left* of D, thus: 5 tons × − 25 in. = − 125 inch-tons. This differs in *sign* only, as is always the case when opposite sides of the section are considered.

Example 8.—In the case of a single local load, not at the centre, as in Fig. 108, the vertical shearing forces, on the two parts of the beam right and left of the load, are of opposite sign, and in

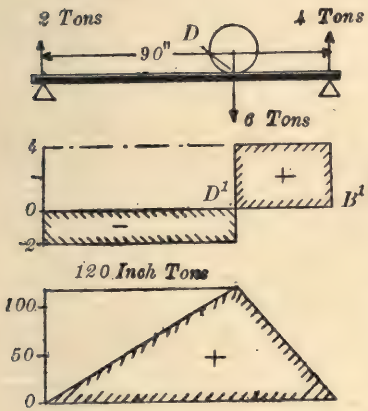


Fig. 108.

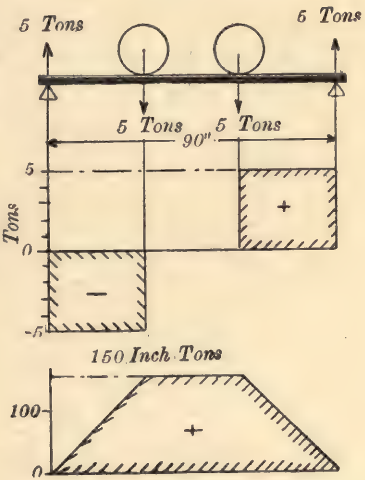


Fig. 109.

magnitude equal to the respective reactions. This is evident from the definition of vertical shearing force. The bending moment at D is 4 tons × 30in. = 120 inch-tons. The common case of a lever centred at D, and loaded unequally at A and B, can be treated similarly.

Example 9.—A rather peculiar case, illustrated by Fig. 109, is that where two *equal* loads, say of 5 tons, are situated so as to divide the span into three equal parts. The S.F. and B.M. curves can be got by adding together the curves due to each load, due regard being paid to sign. There is no shearing force between the two loads—the zone of uniform bending moment—as the shearing effect of one load is counteracted by that due to the other load. This is an illustration of the general rule that where the B.M. curve is parallel to the zero line, the S.F. vanishes.

Example 10.—In Fig. 110 is represented the case of two *unequal* loads. The reactions are first found by taking moments about A. The shearing forces and bending moments at the various points are then calculated, ordinates representing them set up, and the diagrams drawn through the summits of these. The details will now present no difficulty.

Example 11.—The common case of a beam loaded uniformly is dealt with in Fig. 111. The shearing force at C is the upward

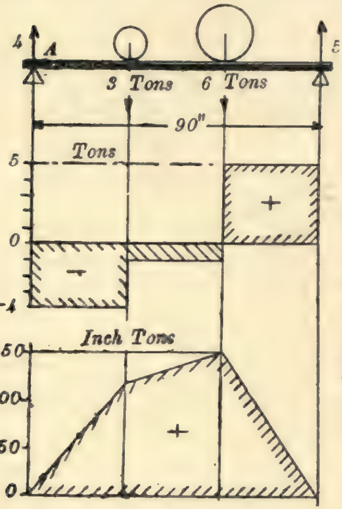


Fig. 110.

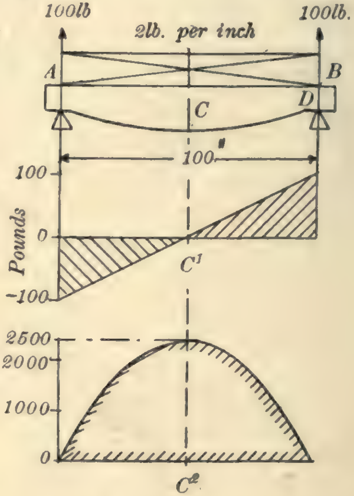


Fig. 111.

reaction at B, less the downward load between C and B, and equals + 100lbs. − 100lbs. = 0. The S.F. at A is the sum of all the forces to the right of A, and equals + 100lbs. − 200lbs. = − 100lbs. The S.F. at B is + 100lbs.; as may be seen by considering the forces acting to the right of a section D, situated a very short distance from B.

The bending moment at C is the sum of the moments of all the forces to the right of C, and equals

$$(100\text{lbs.} \times 50\text{in.}) - (100\text{lbs.} \times 25\text{in.})$$
$$= 5000 \text{ inch-lbs.} - 2500 \text{ inch-lbs.} = 2500 \text{ inch-pounds.}$$

The bending moment curve is a parabola, drawn in the manner already fully described. (See Fig. 103.)

Example 12.—Reserving for future consideration the peculiar case of a beam *fixed* at both ends, instead of merely supported, we shall now consider, as a crowning example of the value of shearing-force and bending-moment diagrams in imparting clearness of view, the rather complex case of a beam or girder supported and loaded in the way shown by Fig. 112. If the preceding examples have been carefully followed, this will be easily understood; but otherwise it will appear mysterious.

(1) Make a skeleton sketch of the beam, inserting all the loads and distances, as in Fig. 112. This is essential.

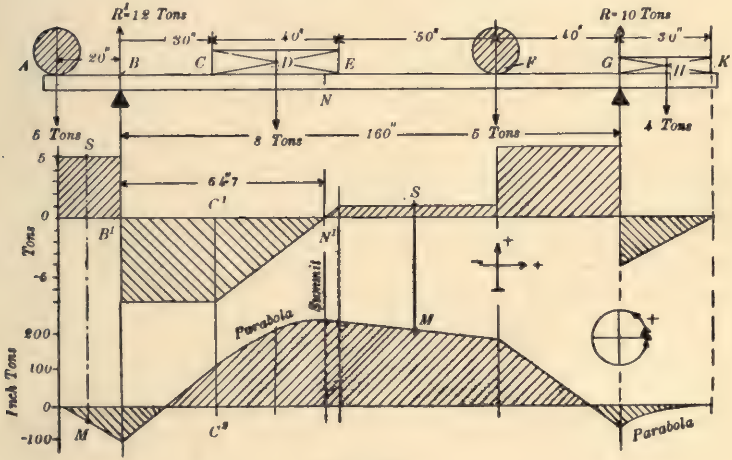


Fig. 112.

(2) To find the *reactions* of the supports, take moments about B. Since the beam is in equilibrium the algebraic sum of the turning moments must be nothing, thus:

$$(R \times 160\text{in.}) - (4 \text{ tons} \times 175\text{in.}) - (5 \text{ tons} \times 120\text{in.})$$
$$- (8 \text{ tons} \times 50\text{in.}) + (5 \text{ tons} \times 20\text{in.}) = 0.$$

Multiplying out, this equation becomes

$$160R - 700 - 600 - 400 + 100 = 0;$$

from which we get

$$160R = 1600, \text{ or } R = 10 \text{ tons}.$$

This is the reaction of the right-hand support. Again,

Sum of ↓ reactions = sum of ↑ loads.

Therefore

$$R^1 + 10 \text{ tons} = 22 \text{ tons};$$

and so

$$R^1 = 22 - 10 = 12 \text{ tons}.$$

Show these values in the sketch.

(3) To calculate the *shearing force* at any vertical section, it is best to isolate the part of the beam on the *right-hand* side of that section, and to consider only the external forces acting on this part. Choosing the sections marked K, H . . . B, A in succession and proceeding thus, we get the following results :—

Just to Right of the Section Lettered.	The Shearing Force is	Tons.
K	0	= 0
H	− 2	= − 2
G	− 4	= − 4
F	− 4 + 10	= + 6
E	− 4 + 10 − 5	= + 1
D	− 4 + 10 − 5 − 4	= − 3
C	− 4 + 10 − 5 − 8	= − 7
B	The same	= − 7
A	− 4 + 10 − 5 − 8 + 12	= + 5

Set up or down these values from a base line to any convenient scale and sign convention. Join the points thus found by straight lines parallel to the base line, between the lines of action of the concentrated loads, and by sloping straight lines beneath the parts uniformly loaded. Note that at three points the shearing force changes sign, and therefore passes through zero value.

(4) To calculate the *bending moment* at any vertical section, regard only the turning moments acting on the part of the beam to the right of that section. In this way we arrive at the results in the table on p. 119.

Set up or down these values from a zero line, in accordance with any chosen scale and sign convention. The points thus located connect by sloping straight lines between the lines of action of concentrated loads, and by parabolic arcs beneath the parts uniformly loaded. Observe that the varying ordinate to the bending-moment curve passes through one maximum value and two minima values, these occurring at the points of zero shearing force.

Just to Right of the Section Lettered	The Bending Moment is
	Inch-tons.
K	—— 0
H	− 2 tons × 7·5in. . . . = − 15
G	− 4 tons × 15in. . . . . = − 60
F	−(4 tons × 55in.) + (10 tons × 40in.) = 180
E	−(4 tons × 105in.) + (10 tons × 90in.) −(5 tons × 50in.) = 230
D	−(4 tons × 125in.) + (10 tons × 110in.) −(5 tons × 70in.) −(4 tons × 10in.) = 210
C	Similarly, or working on the left and changing sign . . . . = 110
B	Similarly, or working on the left and changing sign . . . . = − 100
A	—— = 0

Relations between S.F. and B.M. Curves.—A careful comparison of the foregoing curves will make it clear that in every beam the trigonometrical tangent of the angle of slope to the zero line of the bending-moment curve is numerically proportional to the shearing force, and opposite in sign: a connection which enables us to derive one curve from the other.

This extremely important fact is perhaps best brought home to one's mind by imagining a point, such as a pencil point, to move along each curve. Referring to Fig. 112, let M be the point which travels along the bending-moment curve, and S the associated point which travels along the shearing-force curve, S being always directly above M. Out of the infinite number of possible positions of the pair of moving points, two only are shown in the figure. Then so long as the point M preserves its straight course along the B.M. curve, the point S moves along the S.F. curve parallel to the zero line. But as soon as M changes its direction of motion and inclines upwards, S also alters its path, instantly passing to the other side of the zero line and then continuing to move as before. Hence, when the shearing force changes sign, the bending moment is a maximum or a minimum, as shown in Fig. 112.

When the point M travelling along the B.M. curve, reaches the parabolic hill, the point S immediately answers by rising along a uniform slope until that hill is left behind. While M is on the parabola, the direction of its motion is continually changing; the tangent of the angle of slope being proportional to the horizontal distance from the summit of the hill.

It would be tedious to continue this comparison further; but however far pursued, it will be found that if θ be the angle of slope of the bending moment curve, and C some number depending on the scales of the diagrams, the relation

$$\text{Shearing force} = C \times \tan\theta$$

always holds good, at least as regards magnitude. Readers who are familiar with the differential calculus will at once recognise the analytical expression of this fact, but it need not be introduced here.

A further important connection between the curves of S.F. and B.M. should be noted. The difference between the *areas* of the positive and negative parts of the shearing-force diagram lying directly below any length of the beam represents the change of bending moment in that length.

For instance, working from the shearing-force diagram in Fig. **112**, the area of that diagram below the part AC represents to some scale the bending moment at C; thus:—

$$(5 \text{ tons} \times 20\text{in.}) \sim (7 \text{ tons} \times 30\text{in.}) = 100 \sim 210 = 110 \text{ inch-tons};$$

this being also represented by the ordinate of the bending-moment curve.

Again, to find the *greatest* bending moment on the beam we can measure the distance $B_1 N_1$ from the diagram drawn to scale, and then say that the area of the triangle on the base $C_1 N_1$ measures the increase of moment between C and N—namely,

$$\tfrac{1}{2}(7 \text{ tons} \times 34{\cdot}7\text{in.}) = 121{\cdot}5 \text{ inch-tons}.$$

Hence the greatest bending moment is—

$$\begin{aligned} &\text{B.M. at C} + \text{increase between C and N} \\ &= 110 + 121{\cdot}5 = 231 \text{ inch-tons}. \end{aligned}$$

This interesting principle therefore affords a second way of deriving the bending-moment curve from the curve of shearing force.

Tabular Mode of Calculating Bending Moments.—In connection with Fig. 112 it will be convenient to describe a method of proceeding that has the advantage of materially reducing the labour of calculation, as well as the number of figures employed. It fails, however, when *distributed* loads have to be dealt with, and therefore all the loads will be regarded as concentrated ones. The entire calculation is included in the following table, the method of obtaining it being described below.

(1) Write down the external forces acting on the beam, beginning at either end (say A), and paying due attention to the signs of the forces (up or down).

(2) In column (2) insert the uniform shearing forces between each two external forces. These are got by continued addition from column (1). If we had started at the other end of the beam, we should have got the opposite signs, as in Fig. 112.

(3) In the next column place the distances between the forces tabulated in column (1).

(4) Multiply together each row in columns (2) and (3), and so get column (4), which consists of the increments of bending moment between each two external forces.

(5) The bending moments in the last column are obtained from column (4) by repeated addition.

Nothing could be neater than this handy method.

(1) External Force.	(2) Shearing Force.	(3) Distance Between Forces.	(4) Product of (2) and (3).	(5) Bending Moment.
Tons.	Tons.	Inches.	Inch-tons.	Inch-tons.
At A, − 5				0
	− 5	20	− 100	
At B, + 12				− 100
	+ 7	50	+ 350	
At D, − 8				+ 250
	− 1	70	− 70	
At F, − 5				+ 180
	− 6	40	− 240	
At G, + 10				− 60
	+ 4	15	+ 60	
At H, − 4				0

CHAPTER X.

STRENGTH OF ROLLED JOISTS.

Rolled Joists.—A beam of uniform I section, having both its flanges and its web formed in the rolling mill out of a single steel ingot, as shown in Fig. 113, is technically known as a "rolled joist." The flanges, it will be noticed, are placed well away from the neutral surface (or zone of minimum usefulness); the bulk of the material is thereby much more advantageously situated for resisting a bending moment than is the case either in a rectangular beam or in a beam of circular section. The flanges are rolled at an angle of about 98° to the web, to which they are united by generous fillets. The taper and rounds render the joist easier to roll, stronger, and of neater appearance.

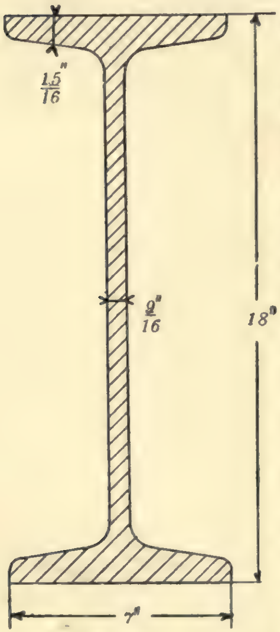


Fig. 113.

Rolled joists may be obtained up to 60 ft. in length and 24 by 7½ in. in section. They were formerly made of wrought iron, but are now usually of mild steel; because that material is stronger, more uniform in quality, and no dearer than iron. Of recent years their use has greatly extended, and they are now applied to many purposes for which wood beams were formerly employed. This extended application of rolled steel joists is due to their cheapness, convenience, and relatively fireproof nature. Unlike wood, such joists will not burn; and, unlike cast-iron, they will not crack; but they *will* warp and bend under intense heat.

Still, with all their advantages, rolled joists are not suitable for beams of long span; because from their mode of manufacture they are necessarily of uniform section from end to end, and therefore a good deal heavier at the ends than is required. In other words, a *rolled* girder is not of uniform strength in the same sense that a *built-up* girder can be made; and this drawback leads to considerable waste of material.

The depth of a rolled joist should not, as a rule, be less than $\frac{1}{20}$th of the span to be bridged over, lest the deflection or sag should be excessive. It is true that the depth of the joists used for supporting the floors of buildings is occasionally less than this; but still it is best, wherever feasible, to select a joist whose depth is about $\frac{1}{12}$th or $\frac{1}{15}$th of the span.

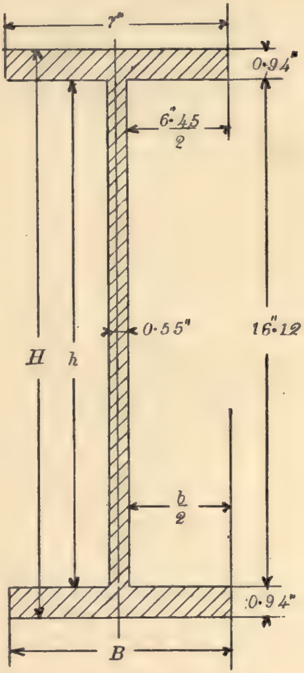


Fig. 114.

Some of the varied purposes to which rolled steel joists are applied are the following:—(1) The main girders of foot bridges and the cross girders of somewhat larger bridges. (2) The girders of light overhead travelling cranes and the roadways (or runways) of heavier cranes, say up to 20 tons lifting capacity. (3) The jibs of cheap jib cranes; but it must be confessed that the appearance of these is not good. (4) The framework or skeleton of large public buildings, warehouses, hotels, and offices; such as the twelve to thirty-storey "sky-scrapers" which are now so prevalent in the large cities of the United States and in other places where ground is very valuable. (5) Lastly, rolled joists find considerable application in the construction of large workshops, piers, and towers.

The Strength of Joists.—In order to gain definite ideas on the strength of joists, let us calculate, by several rival methods, what uniformly distributed dead load can safely be put on a rolled steel joist of the section shown in Fig. 113, and weighing 75lbs. per lineal foot.

I. First of all reduce the given section to the equivalent but simpler geometrical form shown in Fig. 114, and express the dimensions in decimal notation. Great refinement is not only quite unnecessary here, but positively absurd, as steelmakers cannot guarantee to roll sections without an allowance of $2\frac{1}{2}$ per cent. over or under standard dimensions. Moreover, the working stress permitted by different engineers varies at least 25 per cent. even on similar work. Still, for the sake of comparing different modes of calculation, the dimensions in Fig. 114 have been given to two decimal places, even though a rather less degree of accuracy would suffice in practice.

In the next place, the modulus of the section (which is sometimes strangely confounded with the moment of resistance of the section) is most accurately calculated by the aid of the formula

$$Z = \frac{BH^3 - bh^3}{6H},$$

the symbols being explained in Fig. 114. On substituting the given dimensions in this formula, it becomes

$$Z = \frac{(7 \times 18^3) - (6{\cdot}45 \times 16{\cdot}12^3)}{6 \times 18} \text{ in.}^3,$$

which works out to 127·7 inch units as the modulus of the section with respect to the bending.

So far we have regarded only the shape and size of the transverse section; but we have now to consider the strength of the material composing the joist. The tensile breaking stress or ultimate strength of mild steel is 28 to 30 tons per square inch. so that, allowing a factor of safety of 5, we may take 6 tons per square inch as a working stress, and still be well on the safe side, as the load is steady. Then the safe moment of resistance to bending of the joist section is

$$M = f \times Z = 6 \frac{\text{ton}}{\text{in.}^2} \times 127{\cdot}7 \text{ in.}^3 = 766 \text{ inch-tons.}$$

It is assumed that the joist is so well stayed sideways that the compression flange will not fail by crippling or buckling. This action, which consists in a lateral bending and wrinkling of the fibres, has to be carefully guarded against in long girders and columns.

To proceed with the example—the bending moment on a uniformly loaded beam of 28ft. span is

$$\tfrac{1}{8}WL = \tfrac{1}{8} \times W \times 28 \times 12 = 42W \text{ inch-tons.}$$

Hence, equating the bending and resisting moments, we have

$$42W = 766; \quad \therefore W = 18{\cdot}2 \text{ tons,}$$

so that a steel joist 18 × 7 in. will safely carry 18 tons under the stated conditions. But it must not be forgotten that this includes the weight of the beam itself, and therefore a correction has to be applied before the residual or effective load can be stated.

As the joist weighs 75lbs. per foot run, its weight between the supports is $28 \times 75 = 2100$lbs., or, say, a ton. Deducting this, it finally appears that the useful distributed load that can be safely borne by a joist 18×7in. is 17 tons. This weight-correction becomes of much greater importance in the case of a long-span girder, the weight of which may exceed the useful load. This factor, therefore, puts a practical limit to the span of a bridge. For instance, it would not be feasible to construct a bridge with a clear span of five miles; because it would be incapable of supporting its own weight.

II. **Safe Load on a Joist in terms of its Weight.**—The above-described exact method of calculating what load a given joist will safely carry is rather laborious, especially in the absence of a slide-rule or tables; for which reason certain formulæ of a less refined or rough-and-ready nature are much used in practice, and are well worth knowing.

One of these is a neat empirical formula for estimating the safe load that may be put on a rolled steel joist when given the principal dimensions and the weight per foot. Thus,

$$W = 0{\cdot}7\,(w - 0{\cdot}3BH)\frac{H}{L},$$

in which

W tons = safe distributed dead load.
w lbs. = weight of joist per lineal foot.
B inch = breadth of flange.
H inch = total depth of joist.
L feet = clear span.

Applying this formula to the case of a joist 18 by 7in. by 75lbs. per foot, and of 28ft. span, we get

$$W = 0{\cdot}7\,(75 - 0{\cdot}3 \times 7 \times 18)\frac{18}{28},$$

which works out to 16·74 tons.

Now by the usual laborious method we found that 18 tons could be borne, including the weight of the joist, and 17 tons exclusive of that weight; so that the result is about right, even without correcting for the weight of the beam itself. This is also the case for all reasonable spans, but not for spans much greater than twenty times the depth of the joist. To emphasise this point let us see how the two methods will agree for a joist of the same section as that last considered (Fig. 113), but having the extreme span of 100ft. We disregard, for the present, the question of *stiffness*.

The bending moment on a uniformly-loaded beam of 100ft. span is

$$\tfrac{1}{8}\mathrm{WL} = \tfrac{1}{8}\mathrm{W} \times 100 \times 12 = 150\mathrm{W} \text{ inch-tons.}$$

Also the moment of resistance of the section shown in Fig. 113 has been already found to be 766 inch-tons. Equating the movements gives

$$150\mathrm{W} = 766\,; \quad \therefore \mathrm{W} = 5{\cdot}11 \text{ tons.}$$

It remains to correct for the weight of the beam. This is 75lbs. per foot, or 7500lbs. altogether—that is, 3·35 tons. Hence the safe useful load is only

$$5{\cdot}11 - 3{\cdot}35 = 1{\cdot}76 \text{ tons.}$$

Next, trying the short practical rule, we get

$$\mathrm{W} = 0{\cdot}7\,(75\text{lbs.} - 0{\cdot}3 \times 7\text{in.} \times 18\text{in.})\,\frac{18\text{in.}}{100\text{ft.}}$$

which amounts to 4·68 tons. This is the *total* distributed load. Deducting 3·35 tons for the weight of the joist itself, there remains only 1·33 tons as the greatest useful or *net* load it would be advisable to put on the beam. Evidently, then, the weight correction is of essential importance in the case of beams of abnormal span.

III. **Second Approximate Formula for Joists.**—Another very handy method of dealing with the strength of a rolled joist is to neglect the web entirely, and confine one's attention to the flanges; knowing that, if these alone are able to withstand the bending moment, then the whole beam will certainly be strong enough in that respect. The theory of this method has already been fully gone into in former pages, where it was shown that the moment of resistance is

$$\left(\begin{array}{c}\text{Area of one flange,}\\ \text{in square inches}\end{array}\right) \times \left(\begin{array}{c}\text{mean stress over flange,}\\ \text{in tons per square inch}\end{array}\right) \times \left(\begin{array}{c}\text{effective depth of beam,}\\ \text{in inches}\end{array}\right).$$

Instead of the *effective* depth of the joist, which is the distance between the centres of area of the flanges, we may, in this type of beam, substitute the *total* depth, without introducing much error. Also, in place of the *mean* flange stress we may safely take the stress on the *extreme* fibres. Then, on making these approximations, the formula for the moment of resistance of an I section becomes

$$\mathrm{M} = f \times \mathrm{A} \times \mathrm{H},$$

the letters being explained in Fig. 115. Assuming uniform loading, the equation of moments will then be

$$\tfrac{1}{8}W \times L = f \times A \times H,$$

and therefore

$$W = \frac{8f \times A \times H}{L},$$

the span L and depth H being in inches. By the aid of this simple formula it is easy to calculate, with fair accuracy, what uniformly-distributed load a joist of given dimensions will carry safely. If the beam is loaded at the centre, instead of all over, then only half the load found by the above formula can be carried.

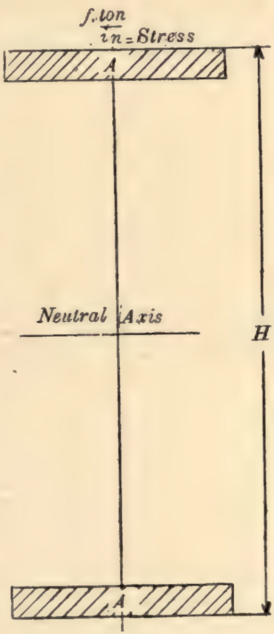


Fig. 115.

The last-named formula will now be applied to calculate the uniform load that a steel joist 7 × 18in. (Fig. 113) is capable of carrying with safety. The stress (f) allowed is six tons per square inch, the flange area (A) is about 7 × 0·94in. = 6·58 sq.in., the depth (H) of the joist is 18in., and the span (L) is 28 × 12 = 336in. Inserting these values in the formula, it becomes

$$W = 8 \times 6 \frac{\text{ton}}{\text{in.}^2} \times 6{\cdot}58\text{in.}^2 \times 18\text{in.} \div 336\text{in.},$$

which works out to very nearly 17 tons. The accurate value of the safe distributed load is 18·2 tons, as we have already found by the usual scientific method, so that the error is not great, and moreover it is on the right side. The approximate formula may therefore be used with every confidence. But of course it is still necessary to make a correction for the weight of the joist itself, especially if of long span. In the present case, however, this weight is less than one ton, and therefore is not of much consequence.

IV. Third Approximate Formula for the Strength of I Beams.—The last-mentioned method of finding the safe load ignores the web entirely, and therefore gives results somewhat too low. The method to be now described takes into account the web, but gives it rather undue credit, so that the results obtained are a trifle too high.

Referring to the half-section shown in Fig. 116, the respective resisting powers of the flange and the half-web are proportional to the areas ABCD and *ab*G, as fully explained in ch. vii. Now the area *ab*G is very nearly one-fourth the area of the web,

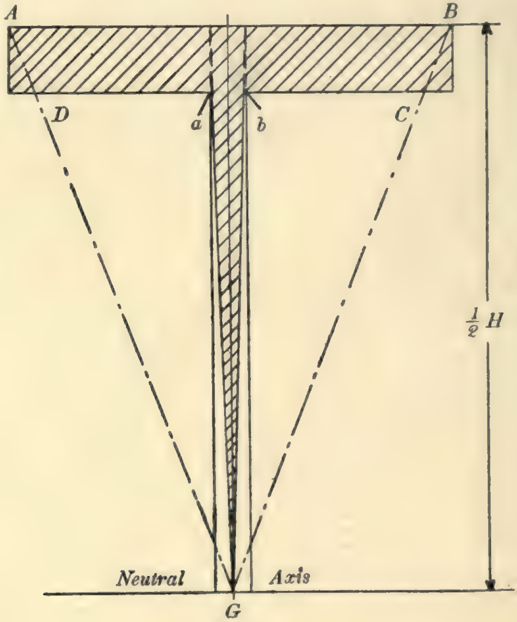


Fig. 116.

and may conveniently be taken as exactly so when the web is thin. Also the distance of the centroid of this triangle from the neutral axis is two-thirds of the depth of the half-web, and it may be regarded with no great error as one-third of the total depth of the beam H. Hence, the amount of resistance of the half-web alone is approximately

$$\left(\begin{matrix}\text{stress on}\\ \text{extreme fibres}\end{matrix}\right) \times \frac{\text{area of web}}{4} \times \frac{\text{depth of beam}}{3},$$

or in symbols

$$f \times \frac{a}{4} \times \frac{\text{H}}{3};$$

and therefore the moment of resistance of the *whole* web is twice this—namely

$$\tfrac{1}{6}f \times a \times \mathrm{H}.$$

This is simply the web correction. And it has already been shown that the moment of resistance of the flanges is approximately

$$f \times \mathrm{A} \times \mathrm{H}.$$

Hence the approximate moment of resistance of the whole section is the resisting moment of the flanges *plus* the resisting moment of the web, or

$$\mathrm{M} = f\mathrm{AH} + \tfrac{1}{6}fa\mathrm{H}$$

$$= f\mathrm{H}\left(\mathrm{A} + \frac{a}{6}\right).$$

Now, for a joist uniformly loaded, the bending moment is $\frac{1}{8}$WL, and therefore

$$\tfrac{1}{8}\mathrm{WL} = f\mathrm{H}\left(\mathrm{A} + \frac{a}{6}\right),$$

whence

$$\mathrm{W} = \frac{8f\mathrm{H}}{\mathrm{L}}\left(\mathrm{A} \times \frac{a}{6}\right).$$

This formula is highly esteemed and much used by some engineers, and not at all by others. Applying it to the case of a joist 18 × 7in., having a flange area of 6·58 sq. in., a web area of 8·87 sq. in., and a span of 28ft., we get, by substituting the given values, the expression—

$$\mathrm{W} = \frac{8 \times 6 \times 18}{28 \times 12}\left(6\ 58 + \frac{8{\cdot}87}{6}\right),$$

which simplifies to 20·7 tons, including the weight of the joist itself. As this result is too high, and the second approximate result too low, the mean of them should be almost exact—viz.,

$$\tfrac{1}{2}\,(17 \text{ tons} + 20{\cdot}7 \text{ tons}) = 18{\cdot}85 \text{ tons}.$$

The true value of the safe load, as calculated in the standard way, was found to be 18·2 tons, inclusive of the joist's own weight.

On considering the comparative merits of the two last-described formulæ, we on the whole prefer the simpler one; because its error is on the safe side and also of less amount than the error of the more complex formula.

Ultimate Strength of I Joists.—In treating of rectangular beams, it was stated that the breaking strength of such beams could not be calculated in any rational manner. The same is true of rolled joists, though in their case much less error is introduced by forgetting that rational strength formulæ only hold good so long as the elastic limits of the material are not exceeded. Nevertheless, it will be interesting to calculate what result will be obtained by the use of a rule which Sir Benjamin Baker gave many years ago for calculating the *breaking* strength of beams. This rule was tested and verified by numerous experiments of his, in which a close agreement between the calculated and the experimental results was exhibited.

In Baker's original paper on "The Practical Strength of Beams," read before the Institution of Civil Engineers in the year 1880, the following passages occur, and will no doubt be read with interest:—

"The theory of transverse stress has engaged the attention of mathematicians for many years, and certain hypotheses have been, and still are, generally accepted, although every practical engineer knows that in the majority of cases the calculated results based upon these hypotheses are widely at variance with those obtained by experiment. Engineers, however, cannot afford to wait until a rational theory of transverse stress is agreed upon, and no doubt many engineers besides the author have framed certain rules for their own guidance, which have given results agreeing with experiment, and otherwise answered their purpose as well as if an unassailable theory had been arrived at. A comparison of these practical rules can hardly fail to be useful, both to the scientific experimentalist who has leisure to make special tests to elucidate a theory, and to the engineer whose first object is to make sure that his structure possesses the required strength. The author, therefore, proposes to illustrate the method of calculation which he has found during the past fifteen years to give satisfactory results in the instance of many thousands of tons of beams of every variety of cross-section. . . ."

"The average results of a very large number of experiments show that, as regards deflection under transverse stress, a rail as a beam behaves exactly in accordance with the ordinarily-accepted theory, with this important distinction: that the maximum deflection within the elastic limit is greater than theory would indicate by an amount ranging from 5 to 50 per cent., according to the cross-section of the rail. Experiments by Mr. W. H. Barlow, F.R.S., President Inst. C.E., on other descriptions of beams would have indicated such a conclusion, and that the increase in the elastic deflection, as in the elastic and ultimate strength, must necessarily be included within the limits of 0 and 70 per cent., because the increase is nil in the instance of a steel-plate girder with a thin web, and averages 70 per cent. in a solid bar of rectangular cross-section. In estimating the probable increase in the case of a beam, such as a rail, having a cross-section between these two extremes of girder and bar, the first impulse naturally would be to assume that it would approach the limit of 70 per cent. in the same proportion as the section of the rail approached the solid rectangular bar; that is to say, that the increase would be 70 per cent. multiplied by the sectional area of the rail, and divided by the area of the enclosing rectangle. This simple assumption the author has found to be sufficiently near the truth for all practical purposes."

These words, coming from a man who has attained eminence in his profession, and become famous as the joint engineer of the stupendous Forth Bridge, are deserving of thoughtful consideration. Let us, therefore, apply Baker's method to the case of a joist 18 by 7in. by 28ft. span, weighing 75lbs. per foot.

Let M inch-tons be the moment of resistance calculated in the ordinary way, but using the *ultimate* tensile stress, say 30 tons per square inch, in place of the safe working stress. Thus M is the *ultimate* moment of resistance, as ordinarily understood. Now, in preceding pages it has been shown that the modulus of the section in question is 127·7in.^{3}, so that the ultimate moment $M = 127{\cdot}7 \times 30 = 3831$ inch-tons.

Next, let a sq. in. be the area of the entire section of the beam—namely, 22 sq. in.,—and A sq. in the area of the enclosing or circumscribing rectangle—namely, $18 \times 7\text{in.} = 126$ sq. in. Then, according to Baker, the *true ultimate* moment of resistance will not be M simply, but

$$M + 70 \text{ per cent. of } M \times \frac{a}{A};$$

that is,

$$M\left(1 + 0{\cdot}7\,\frac{a}{A}\right).$$

On substituting the known values in this formula we get

$$3831\left(1 + 0{\cdot}7 \times \frac{22}{126}\right) = 3831 \times 1{\cdot}122 = 4300 \text{ inch-tons, say,}$$

and as the bending moment under a distributed breaking load W is $\frac{1}{8}$WL, we must have

$$\tfrac{1}{8} \times W \times 28 \times 12 = 4300,$$

from which

$$W = \frac{4300 \times 8}{28 \times 12} = 102 \cdot 3 \text{ tons.}$$

This is the breaking load according to Baker's rule. But some other engineers would make use of the approximate formula already deduced namely,

$$W = \frac{8f \times A \times H}{L},$$

to determine with sufficient accuracy the probable uniformly distributed breaking load, A being here the flange area. This rule gives, for the same joist,

$$W = \frac{8 \times 30 \times 6 \cdot 58 \times 18}{28 \times 12} = 84 \cdot 6 \text{ tons.}$$

However, the precise load that will *break* a given beam is not of great interest in practice; the important question rather is, What will the beam safely carry? This question has been fully answered.

Trade Catalogues.—Before leaving this important matter of the strength of single joists, it should be mentioned that most steel makers issue lists of the sections rolled by them, which are gladly furnished to customers and prospective buyers as being likely to facilitate business. These lists contain tables of safe loads for various sections and spans of joists, which have been calculated for stresses of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of the breaking stress of the steel employed in their manufacture. When such tables are available there is no need to enter into strength calculations at all, save as a check; the nearest section usually kept in stock being simply picked out from the list of standard sections. Some men, however, prefer entirely to ignore tables of this kind and to make their own calculations. We do not advise this altogether, as it is possible to use tables intelligently: but it may be necessary to warn the inexperienced that unless they know what factor of safety has been allowed in calculating standard tables, and unless the nature of the load be taken into account in the selection of a section, very serious errors are likely to arise. At any rate, there is no doubt that every engineer and architect should be readily able to ascertain for himself the size of beam required for a definite purpose without any assistance whatever from standard tables.

The table of British standard beams on pp. 134–135 is based on the list of Messrs. Dorman, Long and Co., Ltd., and that issued by the Engineering Standards Committee: but the arrangement has been considerably modified, useless decimals* omitted, and minor features passed over—all with a view to securing greater clearness and suitability for drawing-office use. The loads have also been re-calculated. The standard angle between the flange and the web of **I** beams is 98°.

* It is absurd to write such a value as 2654·769 inch units as the moment of inertia of a beam section. It should be remembered that it is not commercially practicable to roll steel joists to a closer degree of accuracy than 2½ per cent. of the nominal weight per foot. Hence the absurdity of too many significant figures.

BRITISH

Ref. No.	Size.	Weight per foot.	Area of Section.	Mean Thickness. Web.	Mean Thickness. Flange.	Centres of Flange Holes.	Modulus of Section for Bending.	Moment of Inertia about N.A.
BSB.	Inches.	Lbs.	Sq. in.	Inch.	Inch.	Inch.	Inch.^{3}	Inch^4.
30	24×7½	100	29·4	·6	1·1	4·5	221	2655
29	20×7½	89	26·2	·6	1	4·5	167	1670
28	18×7	75	22·1	·55	·93	4	128	1150
27	16×6	62	18·2	·55	·85	3·5	91	726
26	15×6	59	17·4	·5	·88	3·5	84	629
25	15×5	42	12·4	·42	·65	2·75	57	428
24	14×6	57	16·8	·5	·87	3·5	76	533
23	14×6	46	13·5	·4	·7	3·5	63	441
22	12×6	54	15·9	·5	·88	3·5	63	376
21	12×6	44	12·9	·4	·72	3·5	53	315
20	12×5	32	9·4	·35	·55	2·75	37	220
19	10×8	70	20·6	·6	·97	4·75	69	345
18	10×6	42	12·4	·4	·74	3·5	42	212
17	10×5	30	8·82	·36	·55	2·75	29	146
16	9×7	58	17·1	·55	·92	4	51	230
15	9×4	21	6·18	·3	·46	2·25	18	81
14	8×6	35	10·3	·44	·6	3·5	28	111
13	8×5	28	8·24	·35	·58	2·75	22	89
12	8×4	18	5·3	·28	·4	2·25	14	56
11	7×4	16	4·7	·25	·39	2·25	11·2	39·2
10	6×5	25	7·35	·41	·52	2·75	14·5	43·6
9	6×4½	20	5·88	·37	·43	2·5	11·6	34·7
8	6×3	12	3·53	·26	·35	1·5	6·74	20·2
7	5×4½	18	5·3	·29	·45	2·5	9·08	22·7
6	5×3	11	3·24	·22	·38	1·5	5·44	13 6
5	4¾×1¾	6·5	1·91	·18	·33		2·85	6·77
4	4×3	9·5	2·8	·22	·34	1·5	3·76	7·53
3	4×1¾	5	1·47	·17	·24		1·84	3·67
2	3×3	8·5	2·5	·2	·33	1·5	2·52	3·79
1	3×1½	4	1·18	·16	·25		1·11	1·66
	H B	w	A	t	T	C	Z	I
1	2	3	4	5	6	7	8	9

STANDARD BEAMS.

Safe Distributed Dead Load, for various spans, with an extreme fibre stress of 7½ tons per sq. in.

4ft.	6ft.	8ft.	10ft.	12ft.	14ft.	16ft.	18ft.	20ft.	24ft.	28ft.	32ft.
Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.
			110	92	79	69	61	55	46	39	34
		104	84	70	60	52	46	42	35	30	26
		80	64	53	45	40	35	32	26	22	20
		56	45	38	32	28	25	22	19	16	
		52	42	35	30	26	23	21	17		
		36	28	23	20	18	16	14	12		
	64	48	38	32	27	24	21	19	16		
	52	38	31	26	22	19	17	15	13		
	52	38	31	26	22	19	17	15			
	44	32	26	22	19	16	14	13			
	30	22	18	15	13	11	10	9			
	56	42	34	28	24	21					
	36	26	21	18	15	13					
	24	18	14	12	10	9					
	42	32	25	21	18						
22	15	11	9	7·5	6·4						
34	24	17	14	12							
28	18	14	17	9							
17·5	11	8·7	7	5·8							
14	9·4	7	5·6	4·7							
18	12	9	7·2								
14·5	9·6	7·2	5·8								
8·4	5·6	4·2	3·4								
11·3	7·6	5·7									
6·8	4·5	3·4									
3·6	2·4	1·8									
4·7	3·1										
2·3	1·5										
3·2											
1·4											
W											
10	11	12	13	14	15	16	17	18	19	20	21

CHAPTER XI.

MOMENT OF INERTIA.

Moment of Inertia is a term, much used by writers on the theory of beams, which may now be advantageously explained. It has already been made clear, in former pages, that the modulus of a cross-section of a beam—also spoken of as "the strength modulus" and "the section modulus"—is a geometrical quantity whose value depends solely on the shape and size of the section, and that, when multiplied by the safe stress on the extreme fibres, it gives the moment of resistance of the section. This is symbolically expressed thus—

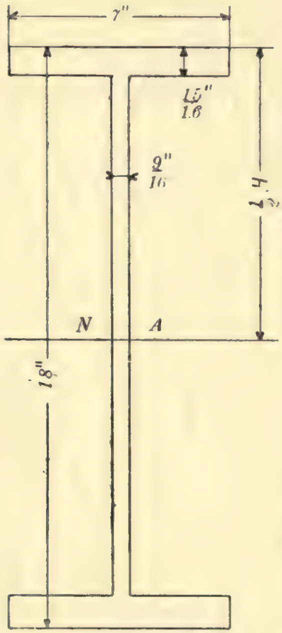


Fig. 117.

$$Z \times f = M,$$

the corresponding dimensional equation being—

$$\text{Inch}^3 \times \frac{\text{pound}}{\text{inch}^2} = \text{inch-pound.}$$

Now, the moment of inertia of a section is a geometrical quantity also, since it has no reference to mass or time; but it is of one dimension higher in length than the section modulus, to which it is related in the following way:—The moment of inertia of a section, relative to the neutral axis, is a quantity which, when divided by the distance of the extreme fibres from that axis, gives the strength modulus of the section in question.

Thus, in a symmetrical section, as Fig. 117,

$$I \div \tfrac{1}{2}H = Z,$$

the actual numerical values here being—

$$1152\text{in.}^4 \div 9\text{in.} = 128\text{in.}^3.$$

In the case of a section unsymmetrical about the neutral axis

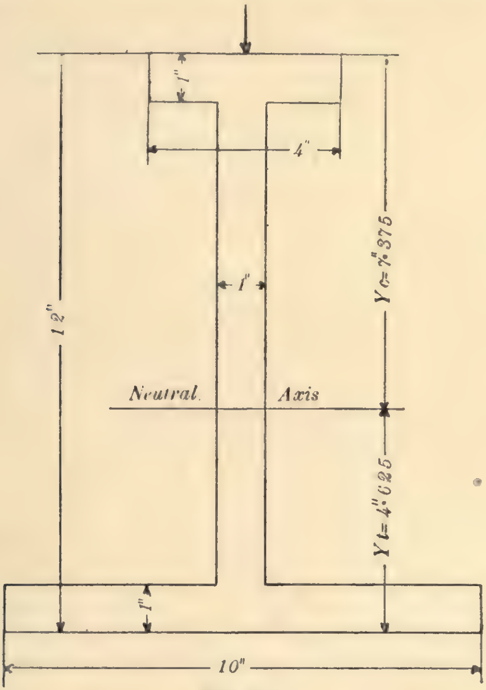


Fig. 118.

as Fig. 118, there are *two* strength moduli, one compressive and the other tensile. Then we have the two relations—

$$I \div y_c = Z_c = \text{compressive modulus},$$

and

$$I \div y_t = Z_t = \text{tensile modulus}.$$

In the section figured, for instance, these become—

$$462\text{in.}^4 \div 7{\cdot}375\text{in.} = 62{\cdot}6\text{in.}^3,$$

and

$$462\text{in.}^4 \div 4{\cdot}625\text{in.} = 100\text{in.}^3;$$

the lesser of these values being taken as the effective modulus of the section.

So far the moment of inertia of a given section has been merely connected with the strength modulus of that section. In actual calculations, however, the moment of inertia is not usually derived from the modulus, but the latter from the former. It is necessary, therefore, to be able to find the moment of inertia of a figure without reference to the section modulus; and this can always be done by the aid of the following fundamental rule, which will also serve to *define* the quantity in question:—

Rule.—To find the *moment of inertia* of a given plane figure, with respect to an assigned axis, divide the figure into narrow strips parallel to that axis, multiply the area of one strip by the *square* of its mean distance from the axis, and repeat this operation time after time until the entire figure has been dealt with piecemeal. The sum of all the resulting products is the moment of inertia of the section: approximately if a limited number of strips have been taken, accurately only when an infinite number of strips have been taken. In the latter case, the integral calculus is required. The axis must always be specified.

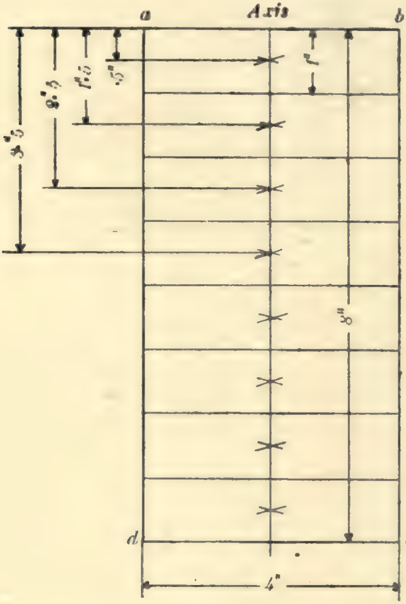


Fig. 119.

A few numerical examples will make clear the application of this important rule.

Example 1.—To find the moment of inertia of a rectangle 4in. × 8in. (Fig. 119), relatively to one short edge *ab*. Divide the figure into, say, eight equal strips, measure the distances of their centres of area from *ab*, square them, multiply across, and add the products, arranging the work as follows:—

Area of strip.	Distance from *ab*.	Square of Distance.	Product.
In.^{2}	In.	In.^{2}	In.^{4}
1×4	0·5	0·25	1
4	1·5	2·25	9
4	2·5	6·25	25
4	3·5	12·25	49
4	4·5	20·25	81
4	5·5	30·25	121
4	6·5	42·25	169
4	7·5	56·25	225
32			680

Hence the *approximate* moment of inertia of the given rectangle about the axis *ab*, is 680in.^{4}

On taking an infinite number of strips, instead of only eight, and performing the summation by the aid of the integral calculus, the general result obtained for the moment of inertia of a rectangle about one edge is—

$$I = \tfrac{1}{3}BH^3,$$

which in the present case becomes—

$$\tfrac{1}{3} \times 4 \times 8^3 = 682{\cdot}7\text{in.}^4,$$

so that our approximate result is a trifle too low.

Closer agreement, if desired, can be attained by more minute subdivision; but it is much quicker to use the above simple formula rather than the fundamental rule.

Similarly, the moment of inertia of the same rectangle, about the long edge *bc*, is—

$$\tfrac{1}{3} \times 8 \times 4^3 = 170{\cdot}7\text{in.}^4.$$

Example 2.—To find the moment of inertia of a rectangular section, 4in. by 8in., about the *neutral* axis, proceed as indicated in Fig. 120 and the following table :—

Area of Strip.	Distance from Axis.	Distance.^{2}	Product.
In.^{2}	In.	In.^{2}	In.^{4}
4	3·5	12·25	49
4	2·5	6·25	25
4	1·5	2·25	9
4	0·5	0·25	1
			84

Hence the inertia-moment of the part of the section above the neutral axis is about 84in. units. Also, that of the part below the axis is the same in magnitude and sign, since the square of a negative quantity is itself positive. The moment of inertia of the entire section is therefore about 168 inch units.

We can test this result by means of a general rule or formula, easily found from the consideration that the moment of inertia of

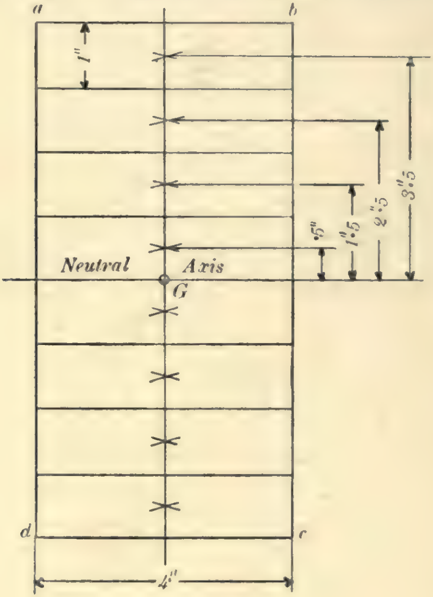


Fig. 120.

the whole figure, relatively to the neutral axis, is twice that of half the figure about the same axis, thus—

$$I_{NA} = 2\left[\tfrac{1}{3} \times B \times \left(\frac{H}{2}\right)^3\right] = \tfrac{1}{12}BH^3,$$

which is a very useful formula. Hence, in the present example, the accurate value is—

$$\tfrac{1}{12} \times 4 \times 8^3 = 170{\cdot}7\text{in.}^4.$$

This, again, differs but slightly from the above approximate result. Similarly, the moment of inertia of the same rectangle, with respect to the *vertical* axis through G, Fig. 120, is—

$$\tfrac{1}{12} \times 8 \times 4^3 = 42{\cdot}7\text{in.}^4.$$

Geometrical and Dynamical Moments of Inertia Compared.—Before proceeding to deal with less simple sections, it appears advisable to say something about the use of the term "moment of inertia" in this connection. One may very reasonably ask, "How can a mere geometrical figure have a moment of *inertia* at all, since it has no mass, and therefore no inertia? Is not inertia a property of tangible bodies only?" The objection is a just one. Strictly speaking, it is undoubtedly absurd to speak of the moment of inertia of a figure necessarily devoid of inertia; and the practice of so doing may seem to suggest a strange forgetfulness of the fundamental meaning of the term inertia.

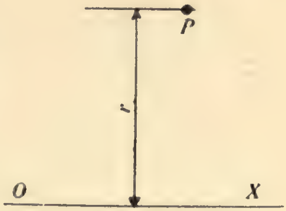


Fig. 121.

A similar usage, however, exists in regard to the term "centre of gravity," which is commonly employed—instead of the less familiar "centroid"—in reference to mere weightless figures; though well known to be strictly applicable only to *bodies*. And in precisely the same way, by analogy with the dynamical term, we have come to speak conventionally of the moment of inertia of a plane figure, because no better name has been agreed on. Some people, it is true, prefer to speak always of the "*geometrical* moment of inertia," but that name is rather cumbrous; while others justify the conventional term by regarding a plane figure as a very thin sheet of heavy material.

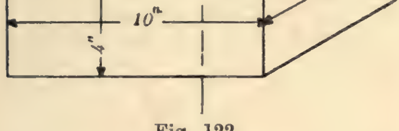


Fig. 122.

For the sake of comparison, a definition of the "dynamical" or "mass moment of inertia" will now be given and exemplified.

The moment of inertia of an indefinitely small body or *particle* P (Fig. 121), relatively to a given axis or straight line OX, is the product of the mass m of the body by the square of its perpendicular distance r from the axis. Thus, if P weighs 1lb. and r measures 10in., then the moment of inertia of P is—

$$1 \text{ pound} \times (10 \text{ inches})^2 = 100 \text{ pound-inch}^2.$$

Further, the moment of inertia of an extended *body* (Fig. 122) about a given axis is the sum of the moments of inertia of all its parts; and is approximately found by dividing the body into a good many small parts, multiplying the mass of each of these by the square of its mean distance from the axis, and finally adding the products together. Accurately, however, the moment of inertia is the limiting value of the sum of the products when the small parts are made infinitely numerous. The calculus method of finding the moment of inertia, based on the latter definition, is applicable only to *regular* bodies, while the approximate method can be used for any body whatever, and is therefore more generally useful.

For instance, if a rectangular block of cast-iron, Fig. 122, measuring 10 × 6 × 4in., weighs $62\frac{1}{2}$lbs., its moment of inertia about a vertical axis passing through its centre of gravity is—

$$I = \frac{M}{12}(l^2 + b^2)$$

$$= \frac{62 \cdot 5}{12}(10^2 + 6^2) = 708 \text{ pound-inch}^2.$$

The formula is arrived at by the process of integration.

The units here made use of have been selected merely for convenience of illustration. But in solving technical problems involving the use of the dynamical moment of inertia, such as fly-wheel calculations, engineers commonly use 32·2lbs. as the unit of mass, and 1ft. as the unit of length, the force unit being 1 pound.

$$\text{Unit moment of inertia is then } \frac{1\text{lb.}}{32 \cdot 2} \times 1\text{ft.}^2.$$

The dynamical or true moment of inertia will not further concern us in these pages however. In future, then, whenever the moment of inertia is spoken of without qualification, the *geometrical* quantity will be meant, the unit being an inch raised to the fourth power.

CHAPTER XII.

NUMERICAL APPLICATIONS.

Any one who can find the moment of inertia of any plane figure whatever has made an important step towards being able to investigate the transverse strength and the deflection of any beam that admits of mathematical treatment. Hence it will be advisable to dwell sufficiently long over this important question to render absolutely clear the method of procedure in the case of every type of beam section usually met with in practice. Most of the sections in general use are made up of rectangles, and can be rapidly dealt with by the successive application of the formula already given for the simple rectangular section. The method of designing a beam, it should be remembered, is largely *tentative*—that is to say, we *assume* a likely section and then examine into its sufficiency, in the manner illustrated by the following examples:—

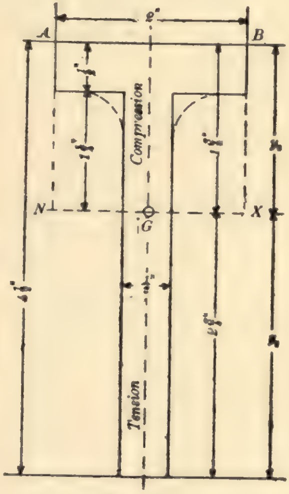


Fig. 123.

Example 1. — A wrought-iron beam of the given T-section (Fig. 123) is loaded uniformly over the upper flange and supported at both ends. Find the moment of inertia about the neutral axis of the section, also the strength moduli. What dead load will the beam safely carry over an 8ft. span?

(1) Locate the neutral axis NX (Fig. 123) by taking area moments about the top edge AB of the section, thus:—

Moment of flange = area × mean distance from AB

$= 2\text{in.} \times \frac{1}{2}\text{in.} \times \frac{1}{4}\text{in.} = 0{\cdot}25\text{in.}^3$

Also moment of web $= \frac{1}{2}\text{in} \times 4\text{in.} \times 2\frac{1}{2}\text{in.} = 5\text{in.}^3$

Now the entire area of the section is 3 sq. in., and the moment of the whole figure about AB is the sum of the moments of the parts; hence—

$$3y_c = 5{\cdot}25, \ \therefore y_c = 1{\cdot}75\text{in.},$$

which fixes the position of the centre of area G and the neutral axis.

(2) The moment of inertia of the part of the section above NX about that axis is the same as the moment of inertia of the large rectangle ABXN about one edge NX, lessened by the collective moment of inertia of the two side rectangles outside of the section. We have already seen that the moment of inertia of a rectangle about one edge is—

$$\tfrac{1}{3} \times \text{base} \times \text{height.}^3$$

Applying this rule to the case in hand gives—

$$I_1 = \tfrac{1}{3} \times 2\text{in.} \times (1{\cdot}75\text{in.})^3 = 3{\cdot}572\text{in.}^4$$
$$I_2 = \tfrac{1}{3} \times 1{\cdot}5\text{in.} \times (1{\cdot}25\text{in.})^3 = 0{\cdot}976$$

$$\text{Difference} = 2{\cdot}596$$

This is the inertia moment of the *upper* part of the section; that of the *lower* part, taken about NX is—

$$I_3 = \tfrac{1}{3} \times 0{\cdot}5\text{in.} \times (2{\cdot}75\text{in.})^3 = 3{\cdot}466\text{in.}^4$$

Adding together these two results, the moment of inertia of the entire section is $6{\cdot}062\text{in.}^4$

(3) The modulus of the section as regards compression is—

$$Z_c = I \div y_c = 6{\cdot}06\text{in.}^4 \div 1{\cdot}75\text{in.} = 3{\cdot}46\text{in.}^3,$$

and with respect to tension is—

$$Z_t = I \div y_t = 6{\cdot}06\text{in.}^4 \div 2{\cdot}75\text{in.} = 2{\cdot}2\text{in.}^3$$

In the case of a *cantilever* these values would be interchanged, but in both cases the moment of resistance of the section is—

$$Z_c \times f_c = Z_t \times f_t,$$

where f_c and f_t are the compressive and tensile stresses actually induced in the extreme layers of the material. If f_t must not exceed 5 tons per square inch, then—

$$Z_t\ f_t = 2{\cdot}2\text{in.}^3 \times 5\,\frac{\text{ton}}{\text{in.}^2} = 11 \text{ inch-tons},$$

and f_c will be—

$$\frac{11}{Z_c} = \frac{11}{3{\cdot}46} = 3{\cdot}18 \text{ tons per square inch.}$$

As this compressive stress is considerably lower than the tensile stress, it is certain that such a beam, if loaded to destruction, would fail by the tearing asunder of the part in tension. The flange not only gives an excess of direct compressive strength, but also affords ample security against *failure by buckling*, to which the compressed part of every beam is liable. For this reason the material is not so uneconomically disposed as might at first sight appear.

(4) The greatest bending moment induced by a distributed load—namely, $\frac{1}{8}$WL—is now to be put equal to the above moment of resistance, thus:—

$$\tfrac{1}{8}W \times 96\text{in.} = 11 \text{ inch-tons,}$$

from which—

$$W = 0{\cdot}92 \text{ ton.}$$

Thus the required uniform load may be taken as 1 ton.

Example 2.—To find the moment of inertia of the given unsymmetrical section (Fig. 124) with regard to the neutral axis. This style of section, when provided with generous fillets and feathers, is that adopted for beams of cast-iron; a material which is from four to six times stronger in compression than in tension.

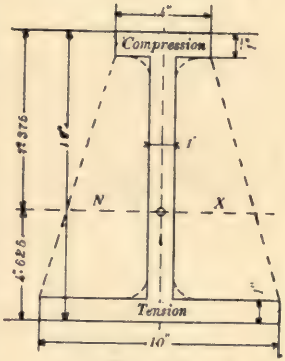


Fig. 124.

The tensile breaking strength of average cast-iron is given by Unwin as 17,500lbs. per square inch, and the compressive as 95,000lbs. per square inch; so that, from the point of view of ultimate strength, the tensile flange should have about 5·4 times the area of the compressive flange. But as beams are not loaded to their breaking-point, this proportion is not the best.

The *elastic limit* of cast-iron in tension (if it can properly be said to have any, which Unwin doubts) is about 10,500lbs. and in pressure is about 21,000lbs. per square inch. Hence, from the point of view of elastic strength, the tension flange should apparently have twice the area of the compression flange. On the whole, however, a ratio of 4 to 1 seems best to satisfy practical requirements, and is that usually adopted.

(1) To proceed with the example in hand, first locate the neutral axis NX (Fig. 124) by taking moments about the top edge of the section, in the manner already described.

(2) The moment of inertia of the upper part of the section about the line NX is—

$$\tfrac{1}{3} \times 4\text{in.} \times (7{\cdot}375\text{in.})^3 - \tfrac{1}{3} \times 3\text{in.} \times (6{\cdot}375\text{in.})^3$$
$$= \tfrac{1}{3}(1600 - 777) = 274 \text{ inch.}^4$$

(3) The moment of inertia of the lower part about NX is—

$$(\tfrac{1}{3} \times 10 \times 4{\cdot}625^3) - (\tfrac{1}{3} \times 9 \times 3{\cdot}625^3)$$
$$= \tfrac{1}{3}(990 - 427) = 188 \text{ inch.}^4$$

(4) Adding these two results, the moment of inertia of the whole section is 274 + 188 = 462 quartic inches.

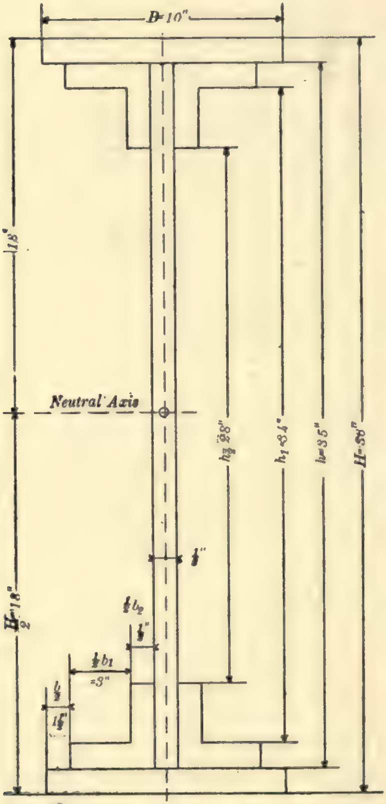


Fig. 125.

Example 3.—Fig. 125 shows the section of a plate girder with a single ½in. web and 3 × 3 × ½in. angle irons. Find its moment of inertia about the neutral axis, and also the safe load that the girder will support. The span is 28ft., and the stress allowed is 5 tons per square inch. The rivet holes and the rounds in the angle irons may here be neglected.

(1) The moment of inertia of a symmetrical section like the present may be found at one operation by the formula—

$$I = \tfrac{1}{12}(BH^3 - bh^3 - b_1h_1^3 - b_2h_2^3),$$

which, on substituting the values marked on the section, becomes—

$$I = \tfrac{1}{12}(10 \times 36^3 - 3 \times 35^3 - 6 \times 34^3 - 1 \times 28^3) = \tfrac{1}{12} \text{ of } 80{,}160 = 6680\text{in.}^4$$

(2) The strength moduli are equal in value, viz. :—

$$Z \quad Z_c = I \div \frac{H}{2} = \frac{6680}{18} = 371\text{in.}^3$$

(3) For a uniform load the maximum bending moment is $\frac{1}{8}$WL. Equating this to the moment of resistance gives—

$$\tfrac{1}{8}W \times 28 \times 12 = 5 \times 371,$$

the stress being 5 tons per square inch.

$$\therefore W = \frac{1855}{42} = 44 \text{ tons},$$

or the load per lineal foot is $\frac{44}{28} = 1{\cdot}57$ tons.

Example 4.—A wrought-iron box girder of 30ft. span has the uniform section shown in Fig. 126. Required the moment of

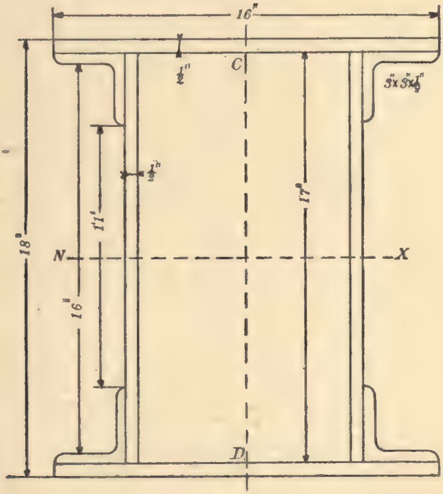


Fig. 126.

inertia of the section and the distributed live load the girder will safely carry, neglecting rivets.

(1) Draw the equivalent solid section (Fig. 127), by simply closing up the web plates and the angles. This section will have the same moment of inertia about the neutral axis NX as the original box section, but a much less moment about the vertical axis CD. In other words, an I beam has less lateral stability, and is more liable to buckle under a heavy load than a box beam of equal section.

This method of reducing the actual section to a simpler equivalent section is a very valuable artifice; its utility being specially marked in dealing with complex sections, such as the midship section of a ship.

(2) The moment of inertia of the equivalent section can now be accurately found as described in Example 3 above, or approximately as follows; the latter method, though not quite correct in principle, having the advantage of clearly bringing out the relative importance of the different parts of the section.

Consider in succession the approximate moments of inertia of the four rectangles composing the upper half of the section about the neutral axis NX.

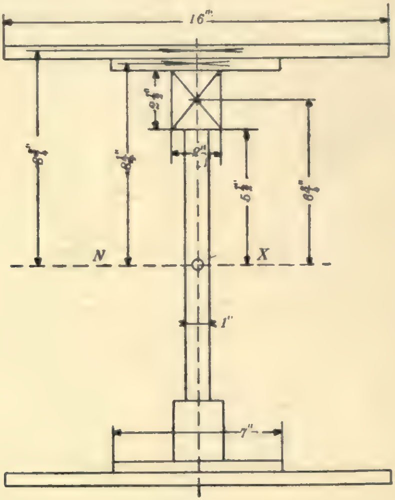


Fig. 127.

$$\text{I of 1st rectangle} = \text{area} \times (\text{mean distance})^2$$
$$= 16 \times \tfrac{1}{2} \times (8\tfrac{3}{4})^2 = 612\text{in.}^4$$
$$\text{I of 2nd rectangle} = 7 \times \tfrac{1}{2} \times (8\tfrac{1}{4})^2 = 238$$
$$\text{I of 3rd rectangle} = 2 \times 2\tfrac{1}{2} \times (6\tfrac{3}{4})^2 = 228$$
$$\text{I of 4th rectangle} = \tfrac{1}{3} \times 1 \times (5\tfrac{1}{2})^3 = 55$$

Hence the moment of inertia of the upper half . . . . . $= 1133\text{in.}^4$

The moment of inertia of the lower half of the section about NX is the same, and, therefore, the total moment is about 2266in.^4 The result obtained by using the formula given in Example 3, p. 146, is 2273in.^4, the difference being quite insignificant.

(3) The strength modulus is the moment of inertia ÷ the half-depth; that is—

$$2270\text{in.}^4 \div 9\text{in.} = 252\text{in.}^3$$

Hence the moment of resistance for a safe compressive stress of 6000lbs. per square inch is—

$$252\text{in.}^3 \times 6000\text{lb./in.}^2$$

Also, the bending moment at the centre of a beam uniformly loaded is—

$\frac{1}{8}$ of load per inch × length^2,

or—

$\frac{1}{8}w \times (30 \times 12)^2$ inch-pounds.

On equating the two moments, we get—

$\frac{1}{8}w \times 900 \times 144 = 252 \times 6000$,

from which we find that the load per *inch*, or w, is 93·3 lbs. The safe load per *foot* is, therefore,

$12 \times 93{\cdot}3 = 1120$lbs.

(4) Lastly, the girder itself is estimated to weigh about 150lbs. per lineal foot, so that the *net* safe load which may be put on the beam will be 1120 − 150 = 970lbs. per foot of length.

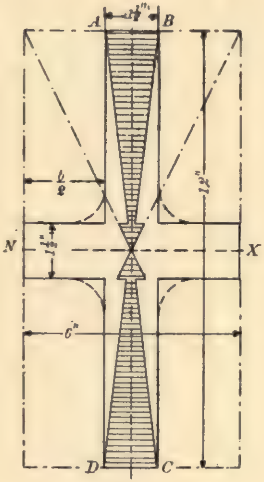


Fig. 128.

Example 5.—A beam of the given cruciform section (Fig. 128) is 15 ft. long, and is supported at both ends, the material being cast-iron. State the inertia moment and the resisting moment of the section. Also estimate the greatest safe distributed load that the beam will sustain.

(1) The section is most conveniently divided up into three rectangles, and treated as follows:—

Moment of inertia of main rectangle ABCD about NX is

$$\tfrac{1}{12} \times 1{\cdot}5 \times 12^3 = 216\text{in.}^4$$

The collective moment of inertia of the two remaining rectangles is—

$$\tfrac{1}{12} \times 4{\cdot}5 \times 1{\cdot}5^3 = 1{\cdot}26\text{in.}^4$$

Hence the moment of inertia of the entire section is 217in.^{4} This calculation shows that the side feathers are not much use in resisting the bending moment, though of service in imparting lateral stiffness. Fig. 128 graphically illustrates this fact, the shaded area being the hypothetical section of an equivalent beam, all the fibres of which are stressed to the same extent.

(2) The resisting moment of the section is the safe stress multiplied by the section modulus, thus:—

$$M = f \times \frac{I}{6\text{in.}} = 1{\cdot}5 \frac{\text{ton}}{\text{in.}^2} \times \frac{217\text{in.}^4}{6\text{in.}} = 54{\cdot}2 \text{ inch-tons.}$$

(3) Equating the bending and resisting moments gives—

$$\tfrac{1}{8}W \times 15 \times 12 = 54{\cdot}2.$$
$$\therefore W = 2{\cdot}4 \text{ tons.}$$

Example 6.—Compare the *weights* and the *strengths* of two round steel beams, one of them being 12in. diameter solid, and the other 12in. diameter outside and 1in. thick, as in Fig. 129.

(1) Weight of A per inch = volume of slice × density of

$$\text{material} = \frac{\pi}{4} \times 144 \times 0{\cdot}29\text{lb.}$$

$$\text{Weight of B per inch} = \frac{\pi}{4}(144 - 100) \times 0{\cdot}29\text{lb.}$$

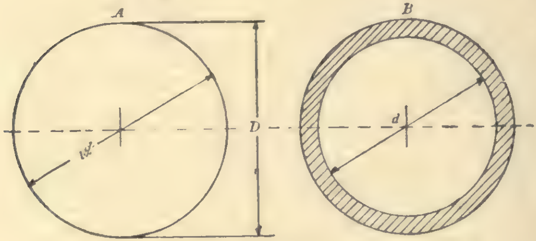


Fig. 129.

Hence the *weights* are as 144 to 44, or as 3·3 to 1, say.

(2) The moment of inertia of a circular section about the neutral axis is easiest found by aid of the formula—

$$\frac{\pi}{64} \times \text{diameter}^4 = {\cdot}0491\, D^4.$$

The strength modulus of such a section is therefore

$$\frac{\pi}{64} D^4 \div \tfrac{1}{2} D = \frac{\pi}{32} D^3.$$

(3) The moment of inertia of a tubular section like B is given by the formula—

$$\frac{\pi}{64}(D^4 - d^4).$$

Dividing this by half the diameter gives the strength modulus of the hollow section thus—

$$\frac{\pi}{32}\frac{D^4 - d^4}{D}$$

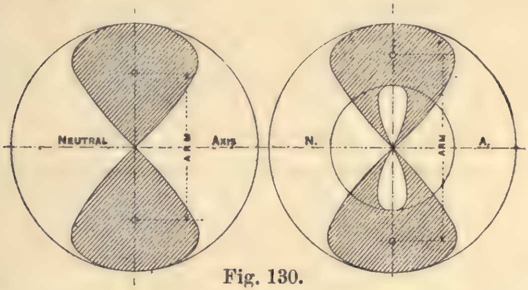


Fig. 130.

(4) The external diameter of the two sections being equal, their relative *strengths* are in the ratio—

$$\frac{D^4}{D^4 - d^4} = \frac{12^4}{12^4 - 10^4} = 1{\cdot}93 \text{ to } 1.$$

Comparing this result with the relative weights, we see that the hollow beam is much more economical of material than the solid beam.

In Fig. 130 the shaded parts are the equivalent areas for solid and hollow circular sections. These diagrams show very clearly the small reduction of strength caused by a considerable saving of material, but are otherwise valueless in practice.

Example 7.—Fig. 131 represents the section of a cast-iron tube 12in. diameter, having a 9in. hole cored out of it. During the process of casting, however, the core has become displaced, so that the metal, instead of being of uniform thickness, is only $\frac{3}{4}$in. thick on the top side, and $2\frac{1}{4}$in. thick on the bottom. Notwithstanding this peculiarity the tube is erected as a beam and carries a load of six tons at the centre of a 25ft. span. It is required to ascertain the moment of inertia of this eccentric section, and also the greatest stress to which the metal is exposed, leaving out of account the weight of the beam itself.

(1) Let A denote the area of the larger circle, with centre C, and a the area of the smaller circle; also let x be the distance of C from the neutral axis, the unit being 1in. The position of the neutral axis is readily found by applying the principle of moments, thus:—

$$A \times \text{its arm} = a \times \text{its longer arm},$$

the point G being regarded as a fulcrum, and the two areas as of opposite sign. Now, as the areas of circles vary as the squares of their diameters, we may write

$$12^2 \times x = 9^2 (x + \tfrac{3}{4}).$$

Dividing by 9 gives—

$$16x = 9x + \frac{27}{4}.$$

$$\therefore x = \frac{1}{7} \times \frac{27}{4} = 0{\cdot}964\text{in.}$$

Hence $\quad x + \tfrac{3}{4} = 0{\cdot}964 + 0{\cdot}75 = 1{\cdot}714\text{in.},$

which locates the centroid G, and therefore the neutral axis.

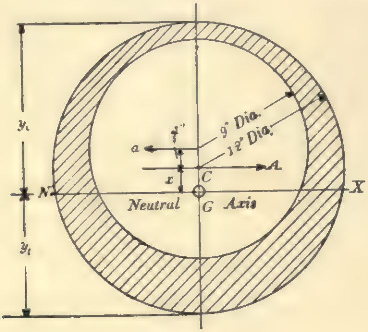


Fig. 131.

(2) To find the moment of inertia of the section we make use of the important fact that the moment of inertia of a given area about any axis (or straight line) is equal to its moment of inertia about a parallel axis through its centroid, *plus* the product of the area by the square of the distance between the two axes. Thus, for the 12in. circle, we have—

$$\begin{aligned} I_{NX} &= I_c + Ax^2 \\ &= \left(\frac{\pi}{64} \times 12^4\right) + \left(\frac{\pi}{4} \times 12^2 x^2\right) \\ &= \left(\frac{\pi}{4} \times 12^2\right)\left(\frac{144}{16} + 0{\cdot}964^2\right) \\ &= 113{\cdot}1\,(9 + 0{\cdot}93) = 1123\text{in.}^4 \end{aligned}$$

Similarly, for the 9in. hole, we have—

$$\begin{aligned} I_{NX} &= \frac{\pi}{4} 9^2 \left(\frac{81}{16} + 1{\cdot}714^2\right) \\ &= 63{\cdot}6\,(5{\cdot}06 + 2{\cdot}94) = 509\text{in.}^4. \end{aligned}$$

The difference between these results—viz., 614in.^4—is the moment of inertia of the whole section about the neutral axis.

(3) Accordingly, the strength modulus of the section with respect to compression is, from Fig. 131—

$$Z_c = \frac{I}{y_c} = \frac{614}{6{\cdot}964} = 88{\cdot}1\text{in.}^3,$$

and with regard to tension is—

$$Z_t = \frac{I}{y_t} = \frac{614}{5{\cdot}036} = 122\text{in.}^3$$

(4) Lastly, as the bending moment at the centre of the beam is 450 inch-tons, the greatest *stress* on the material is—

$$\frac{M}{Z_c} = \frac{450}{88{\cdot}1} = 5{\cdot}11 \text{ tons per square inch in compression.}$$

and—

$$\frac{M}{Z_t} = \frac{450}{122} = 3{\cdot}68 \text{ tons per square inch in tension.}$$

Example 8.—A pair of cast-iron beams are required to carry an estimated central load of 40 tons across a span of 15ft. The load consists of a heavy fly-wheel, gearing, pump work, and a water column. It is proposed to use two beams of the section shown in Fig. 132, the ends being bolted down. Are they of sufficient strength and stiffness?

As the load is not altogether statical, but of varying intensity, and therefore causing vibration, it should be considered as a *live* load, and equivalent to a statical or dead load of 80 tons. Each beam, therefore, should be capable of safely carrying at its centre a dead load of 40 tons.

(1) Reduce the proposed section to the equivalent form, Fig. 133, and locate the neutral axis by taking moments about the top edge, thus:—

Area.	Arm.	Moment.
Sq. in.	Inch.	Inch^3.
12in. × 1in. = 12	0·5	6
2in. × 21in. = 42	11·5	484
16in. × 2in. = 32	23	736
86		1226

Hence, referring to Fig. 133, and using c to denote the distance of the centroid of the section from the top edge, we have—

$$c \times 86 = 1226, \quad \therefore c = 14{\cdot}2\text{in.}$$

Consequently,

$$t = 24\text{in.} - 14{\cdot}2\text{in.} = 9{\cdot}8\text{in.}$$

(2) The moment of inertia of the *upper* part of the section about NX is—

$$\tfrac{1}{3}(\text{BH}^3 - bh^3)$$
$$= \tfrac{1}{3}(12 \times 14{\cdot}2^3 - 10 \times 13{\cdot}2^3)$$
$$= \tfrac{1}{3}(34{,}400 - 23{,}000) = 3800\text{in.}^4.$$

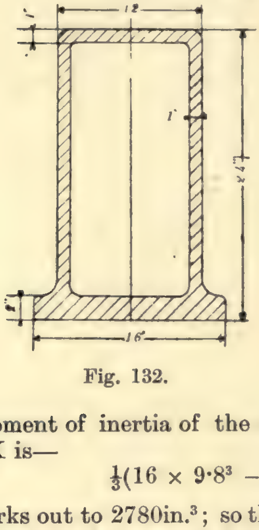


Fig. 132.

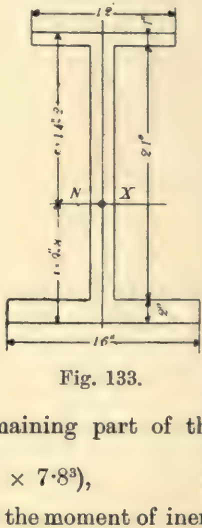


Fig. 133.

The moment of inertia of the remaining part of the section about NX is—

$$\tfrac{1}{3}(16 \times 9{\cdot}8^3 - 14 \times 7{\cdot}8^3),$$

which works out to 2780in.^{3}; so that the moment of inertia of the entire section is 6580 quartic inches.

(3) The modulus of the section as regards compression is—

$$Z_c = 6580 \div 14{\cdot}2 = 467\text{in.}^3;$$

and in respect to tension is—

$$Z_t = 6580 \div 9{\cdot}8 = 673\text{in.}^3.$$

(4) The greatest bending moment is—

$$\tfrac{1}{4}\text{WL} = \tfrac{1}{4} \times 40 \text{ tons} \times 180\text{in.} = 1800 \text{ inch-tons.}$$

(5) Since resisting moment = bending moment,

$$f_c \times 467 = 1800, \quad \therefore f_c = 3{\cdot}86 \text{ tons/in.}^2$$

and

$$f_t \times 673 = 1800, \quad \therefore f_t = 2{\cdot}68 \text{ tons/in.}^2.$$

A tensile stress of 2·68 tons per square inch is too high for cast-iron, so that it will be advisable to increase the *depth* of the beam to, say, 30in., and the *width* of the tension flange to 18in. The result will be a great gain in stiffness, as the stiffness of a beam varies as the *cube* of its depth.

If economy of metal is desired, the beam should be made deepest at the middle, where the bending moment is greatest; but the ends must be of sufficient section to withstand the shearing force. It is left to the reader, as an instructive exercise, to calculate the maximum stresses of tension, pressure, and shearing, in the case of a strengthened beam, allowing for its own weight.

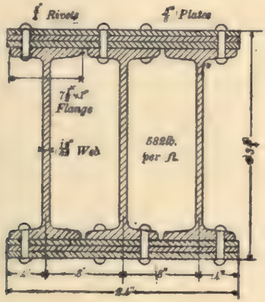


Fig. 134.

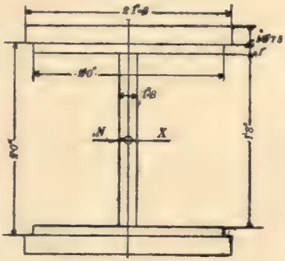


Fig. 135.

Example 9.—Fig. 134 represents the section of a *compound joist* steel girder, weighing 582lbs. per foot of length, this being one of Messrs. Dorman, Long and Co.'s heaviest stock sections. The clear span is 30ft., and the effective span may be taken, for safety, as 32ft. Calculate the magnitude of the uniformly distributed statical load that the girder will carry when the stress on the steel farthest from the neutral axis is 6·4 tons per square inch. Allowance must be made for the loss of strength due to the rivet-holes, as well as the increase of stress due to the girder's own weight.

The three $\frac{3}{4}$in. rivet-holes in the plane of section of the *bottom* flange constitute a direct loss of valuable sectional area, which must evidently be deducted from the gross section. The rivet-holes in the *upper* or compression flange, however, cause no appreciable loss of strength, provided that the rivets fill them completely; so that no deduction need apparently be made for them. But as there is always the possibility of imperfect workmanship, and as the calculation is also simplified by keeping the section symmetrical about the neutral axis, it is better to allow for the rivet-holes in *both* flanges. The several steps of the calculation are then as follows:—

(1) Draw the equivalent solid section, Fig. 135. The effective width of the flange plates is $24\text{in.} - 3 \times 0{\cdot}8\text{in.} = 21{\cdot}6\text{in.}$, and that of the joists is $3\,(7{\cdot}5\text{in.} - 0{\cdot}8\text{in.}) = 20{\cdot}1\text{in.}$ Also the combined web thickness is $3 \times 0{\cdot}6\text{in.} = 1{\cdot}8\text{in.}$ The neutral axis of a symmetrical section passes through the centre of its depth, and is therefore located without calculation.

(2) The next step is to find the moment of inertia of the whole section about the neutral axis NX. That of the *web* is—

$$\tfrac{1}{12} \times 1{\cdot}8\text{in.} \times (18\text{in.})^3 = \quad 875\text{in.}^4,$$

and of the joist *flanges* is—

$$\tfrac{1}{12} \times 20\text{in.} \times (20^3 - 18^3)\text{in.}^3 = \quad 3617\text{in.}^4,$$

and of the *plates* is—

$$\tfrac{1}{12} \times 21{\cdot}6\text{in.} \times (23{\cdot}75^3 - 20^3)\text{in.}^3 = \quad 9740\text{in.}^4.$$

$$\text{Total} \quad 14{,}232\text{in.}^4.$$

(3) The bending modulus of the section with respect to either tension or pressure is—

$$Z = \frac{\text{moment of inertia}}{\text{half depth}} = \frac{14{,}232\text{in.}^4}{11{\cdot}875\text{in.}} = 1200\text{in.}^3.$$

(4) Hence the resisting moment is—

$$M = f \times Z = 6{\cdot}4\,\frac{\text{ton}}{\text{in.}^2} \times 1200\text{in.}^3 = 7680 \text{ inch-tons.}$$

(5) Equating the bending and the resisting moments gives—

$$\tfrac{1}{8}W \times 32 \times 12 = 7680,$$

from which the *gross* load W that can be borne is found to be 160 tons.

(6) The weight of such a girder, 32ft. long, is—

$$32 \times 582\text{lbs.} = 18{,}600\text{lbs.} = 8{\cdot}3 \text{ tons.}$$

Making this deduction, the safe uniformly distributed *net* load is 152 tons.

Example 10.—A proposed beam is loaded and supported as shown in Fig. 112, p. 117, where the shearing force and bending-moment diagrams were fully explained. From these it appears that the greatest shearing force is 7 tons, and the greatest bending moment about 240 inch-tons. Let us in the first instance design a cast-iron beam of *uniform* section, capable of withstanding this moment, and then afterwards see how material can be saved by reducing the section where less strength is called for.

(1) Taking the loads as statical, or steady, a *tensile* stress of $1\frac{1}{2}$ tons per square inch may be allowed on cast-iron, and a shearing stress of two tons per square inch.

The *cross-section* may be either of the *box* or the *ribbed* form, the latter being chosen in the present case, as the expense of a core-box will be saved. A beam of box section, however, looks heavier and more substantial than an equally strong beam of flanged section; which is sometimes an advantage.

(2) The proper depth to be given to the beam is an important consideration. The strength of a beam varies as the *square* of its depth, and the deflection inversely as the *cube* of the depth. Hence great depth is highly advantageous where height is available. Choose tentatively, or as a trial value, a depth of one-tenth the span—viz., 16in.

(3) A suitable mean *breadth* of flange for a cast-iron girder is about half the depth of beam —that is, 8in.; and the compression flange may be taken as about one-third of the width of the tension flange. This proportion makes the top flange 4in. and the bottom flange 12in. wide. Insert these dimensions on the sketch, Fig. 136.

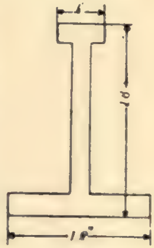


Fig. 136.

(4) We have next to determine the thickness of the flanges and the web. In parallel beams the *web* should be designed strong enough to bear the whole shearing force without aid from the flanges. In all cases the shearing stress over the section is greatest at the region of the neutral axis, and vanishes at the flanges. The minimum section of the web should be equal to the shearing force divided by the shearing stress, that is—

$$7 \text{ tons} \div 2 \text{ tons per square inch} = 3{\cdot}5 \text{ sq. in.}$$

Now the depth of the web is about 14in., so that the least thickness necessary for strength is—

$$3{\cdot}5 \text{ sq. in.} \div 14\text{in.} = 0{\cdot}25\text{in.}$$

But there are other considerations besides strength to be kept in view. The web must be of sufficient thickness to ensure a sound casting and also to allow ample margin for *corrosion*. We have therefore to fall back on practical experience. Engineers know that in the case of a cast-iron girder 16in. deep, $\frac{1}{4}$in. thickness of metal is not enough to satisfy foundry requirements; and so they employ some empirical rule based on former successful practice. A fair proportion for the web thickness of a cast-iron girder is about three-quarters of the thickness of the bottom flange; but this rule is of no avail when the latter is unknown, as here.

(5) We proceed, then, to find the necessary thickness of the bottom flange from a consideration of the bending moment to be resisted. This process is *tentative*, especially when the web is taken into account, but is much simplified by neglecting the web and the fillets. Let a be the area of the top flange, $4a$ that of the bottom flange, and l the effective depth of the section, the unit being the inch (see Fig. 137). Then the resisting moment of the section is $P \times l$, which equals either—

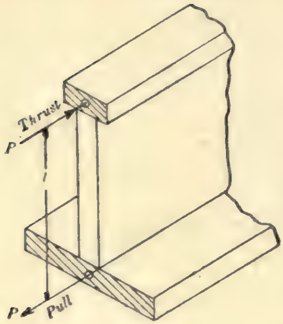


Fig. 137.

$$a \times f_c \times l, \text{ or } 4a \times f_t \times l,$$

each of which must equal the bending moment. Now l is really rather less than 16in.; but for convenience of calculation, and also to allow some credit to the web, the full depth of the beam should be taken. Hence we have the equation—

$$4a \times 1{\cdot}5 \text{ ton/in.}^2 \times 16\text{in.} = 240 \text{ inch-tons},$$

from which the area a of the top flange is found to be 2·5 sq. in., and of the bottom flange 10 sq. in. Hence the required thickness of the top flange is 2·5 sq. in. ÷ 4in. = 0·625in., and of the bottom flange 10 sq. in. ÷ 12in. = 0·833in. Practical experience, however, tells us that these thicknesses are too little for a safe casting; and so we had better reduce our depth and width and increase our thicknesses.

(6) Assume Fig. 138 as a likely section, though the flanges are still rather thin, and let us find the actual stresses by the moment of inertia method. To locate the centre of area, take moments about the top edge, thus:—

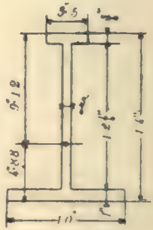


Fig. 138.

Area.	Arm.	Moment.
Inch².	Inch.	Inch³.
3·5in. × 0·75in. = 2·63	0·375	0·987
0·75in. × 12·25in. = 9·19	6·875	63·2
10in. × 1in. = 10·00	13·500	135
21·82		199·187

Let c be the distance of the centroid from the top edge of the section; then—

$$21{\cdot}82c = 199{\cdot}19, \quad \therefore c = 9{\cdot}12\text{in.}$$

(7) The moment of inertia about NX of the part of the section in compression is—

$$\tfrac{1}{3}(3{\cdot}5 \times 9{\cdot}12^3 - 2{\cdot}75 \times 8{\cdot}37^3),$$

which reduces to 343in.^4. Also the moment of inertia about NX of the remaining part of the section is—

$$\tfrac{1}{3}(10 \times 4{\cdot}88^3 - 9{\cdot}25 \times 3{\cdot}88^3),$$

which reduces to 420in.^4. Hence the total moment of inertia is $343 + 420 = 763$ quartic inches.

(8) The compressive modulus of the section is—

$$Z_c = \frac{763}{9{\cdot}12} = 83{\cdot}6\text{in.}^3,$$

and the tensile modulus is—

$$Z_t = \frac{763}{4{\cdot}88} = 156\text{in.}^3.$$

Finally, the maximum bending moment being 240 inch-tons, we have for the required stresses—

$$83{\cdot}6 \times f_c = 240, \quad \therefore f_c = 2{\cdot}87 \text{ tons/in.}^2,$$

and

$$156 \times f_t = 240, \quad \therefore f_t = 1{\cdot}54 \text{ tons/in.}^2.$$

(9) To keep within the limit of $1\frac{1}{2}$ tons per square inch, the bottom flange should be made $1\frac{1}{8}$in. thick. The web may also with advantage be tapered from $\frac{3}{4}$in. thick at the top to 1in. at the bottom, but this tapering causes more trouble in pattern-making. The top flange is of ample compressive strength. The corners should be well rounded, as in the final section, Fig. 139, to avoid weak planes of crystallisation, and the flanges slightly tapered, both for the sake of appearance and ease of moulding. Also "feathers" or "stiffeners," $\frac{3}{4}$in. thick, should be arranged at intervals of 40in., in order to tie the flanges together better and to prevent buckling of the web.

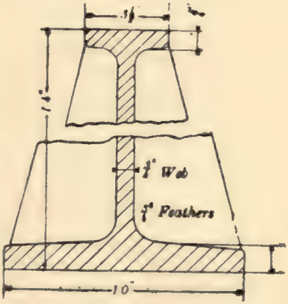


Fig. 139.

In Fig. 140 is given an elevation of this parallel or uniform cast-iron beam, suitable for carrying the loads indicated. The

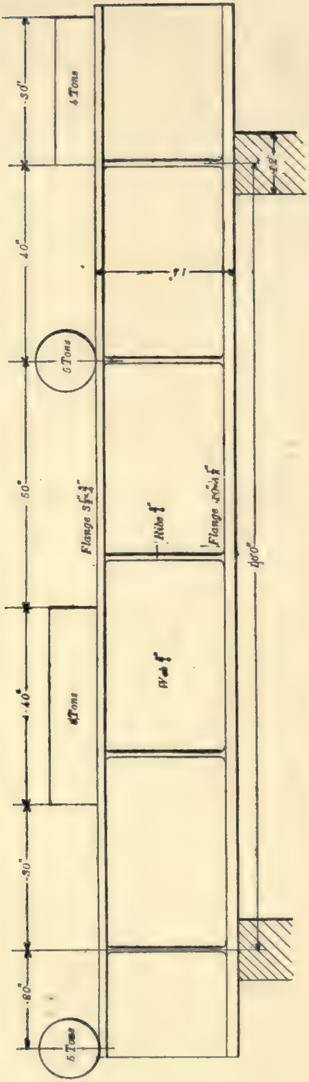


Fig. 140.

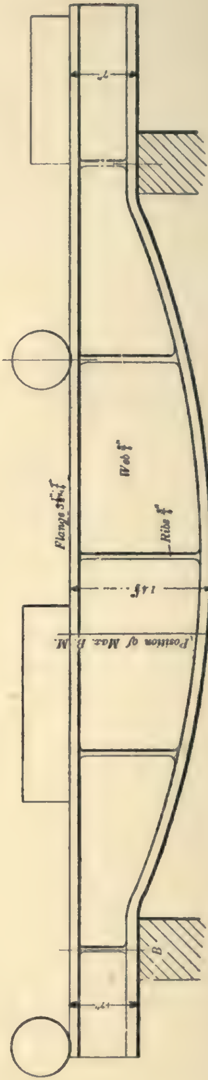


Fig 141

vertical scale, it should be noted, is twice the horizontal. Such a beam is simple to make, but the material is not used to the best advantage, because there is an excess of strength at the ends. We now proceed to show how the design may be so modified as to effect a saving of metal.

Alternative Design.—A beam is conventionally said to be of uniform strength when at every section the resisting moment is proportional to the bending moment. But of course the ends, where the bending moment is usually nothing, must still be made of sufficient section to withstand the shearing force, and the general appearance of the beam cannot be entirely disregarded. In the present case a sufficient approach to uniform strength will be secured by retaining the section already designed (Fig. 139) at the place of greatest bending moment, making the ends of the beam equal in depth and parallel (for the sake of symmetry), and completing the elevation by a fair curve, as in Fig. 141. Each flange will then be of uniform section throughout, the web alone varying.

To find the depth of the beam at the left-hand support B, where the bending moment is 100 inch-tons, we write

$$\left(\begin{array}{c}\text{Area of}\\ \text{bottom flange}\end{array}\right) \times \left(\begin{array}{c}\text{tensile stress}\\ \text{allowed}\end{array}\right) \times \left(\begin{array}{c}\text{length of}\\ \text{resistance arm}\end{array}\right)$$

$$= \text{bending moment},$$

$$\therefore (10\text{in.} \times 1\tfrac{1}{8}\text{ in.}) \times 1{\cdot}5 \text{ ton/in.}^2 \times l = 100.$$

$$\therefore l = \frac{10}{1{\cdot}125 \times 1{\cdot}5} = 5{\cdot}9\text{in.}$$

This is the *effective* depth. The total depth is about 1in. more, and may be taken as 7in. at both supports. Then, if we regard the shearing force as taken solely by the web, the shearing stress at B will be 7 tons ÷ (0·75in. × 5in.) = 1·86 ton per square inch, which is not too great.

For the sake of appearance and ease of manufacture the depth of the beam in the present case is best made greatest at the centre of the span, instead of at the section of maximum bending moment. By drawing a circular arc to pass through the point P, and allowing a sufficient seating at each end of the beam, we get the outline shown in Fig. 141, the greatest depth being rather less than 14½in. The vertical scale of the figure, it should be observed, is again twice the horizontal scale. The vertical stiffeners or feathers are placed symmetrically 40in. apart, and of the same thickness as the web.

The length of the *bearing surface*, assumed as 12in. at each end, depends on the material of the support, the magnitude of the reaction, and the bearing pressure allowed, thus:

$$\text{Bearing area} = \frac{\text{reaction of support}}{\text{bearing pressure}}.$$

If the beam were intended to rest on brickwork it would be necessary to increase the bearing area by casting feet on the bottom flange. The following are safe values for the *bearing pressures* on various materials:—

Cast-iron	200	tons per square foot.	
Wrought iron and mild steel . .	80	,,	,,
Granite	15	,,	,,
Sandstone	12	,,	,,
Brick set in cement and capped with a stone template	6	,,	,,
The same set in mortar . . .	4	,,	,,
Plain brickwork	2	,,	,,

The ends of girders resting on masonry are bedded on either sheet lead or roofing felt.

Example 11.—Fig. 142 shows the section of a steel *flange rail*, weighing 70lbs. per yard. It is required to find its moment of inertia both by calculation and graphically; also the moment of resistance of the section corresponding to a stress on the extreme fibres of five tons per square inch.

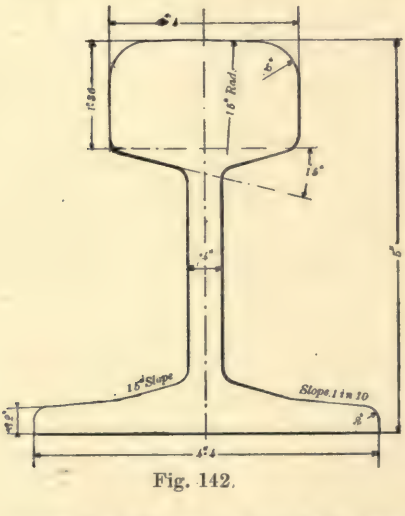


Fig. 142.

Sir Benjamin Baker, in his paper already alluded to on "The Practical Strength of Beams," remarks that: "Of all classes of iron and steel beams, *rails* hold the most important position, for not only do they outnumber all other descriptions of beams by hundreds of millions, but at least a thousand pieces of rails are tested to destruction, purposely and in actual work, for every single specimen of rolled joist or riveted girder." Rails, therefore, are justly entitled to some consideration.

It is difficult, if not impossible, to calculate the actual stress on a rail caused by the passage of a train over it. As Sir Benjamin Baker remarks: "The stress occurring upon a rail in actual work is a matter outside the limits of theoretical investigation. . . . On paper, the problem presented by a cross-sleeper road appears to be identical with that of a continuous girder bridge of seven or eight spans; and the late Mr. Heppel and many others have so treated it. As a matter of fact this method is entirely wrong, both on theoretical and practical grounds. Theoretically so, because the rail rests upon *elastic* supports in the form of compressible wooden sleepers; and practically so, because of the uncertainty as regards packing of ballast and state of decay of the timber. The experiments of Baron von Weber, M. Inst. C.E., have shown that an average wooden sleeper compresses about one-fifth of an inch under a pressure equivalent to the weight on a heavy driving-wheel; and as an ordinary rail would deflect only that amount if the sleeper were entirely removed and the rail supported by the adjoining ones, it will be seen at once how utterly misleading must be any conclusions based upon the hypothesis of *rigid* supports." Notwithstanding the difficulty stated, it is still desirable for an engineer to be able to ascertain quickly, and without the aid of expensive testing apparatus, the transverse strength of a rail of given section, and to compare the merits of one section with those of another. Hence we will now proceed to solve the example proposed.

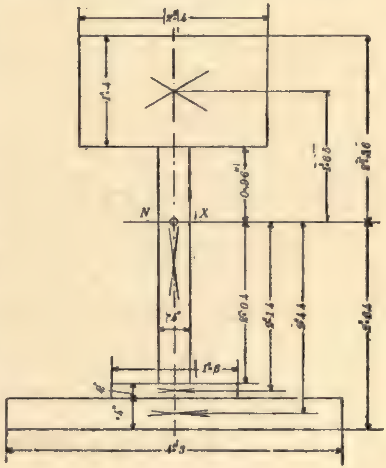


Fig. 143.

(1) Reduce the section to the approximately equivalent simple shape shown in Fig. 143. In this process a piece of tracing paper laid over the section is a valuable aid.

(2) Find the centre of area of the figure by taking moments about either the top or the bottom edge—say the latter in the present example, thus:—

Area.	Arm.	Moment.
Sq. In.	In.	$In.^3$
2·4in. × 1·4in. = 3·36	4·3	14·44
0·4in. × 3in. = 1·20	2·1	2·52
1·6in. × 0·2in. = 0·32	0·5	0·16
4·3in. × 0·4in. = 1·72	0·2	0·34
6·60		17·46

Consequently, the distance of the centre of area of the section from its lower edge is $17{\cdot}46\text{in.}^3 \div 6{\cdot}6\text{in.}^2 = 2{\cdot}64\text{in.}$ This fixes the neutral axis NX of the section.

(3) The moment of inertia about NX of the top rectangle is equal to its moment of inertia about a parallel axis through its centre of area, *plus* its area multiplied by the square of the distance between the two axes, or in figures

$$\tfrac{1}{12} \times 2{\cdot}4\text{in.} \times (1{\cdot}4\text{in.})^3 + 3{\cdot}36\text{in.}^2 \times (1{\cdot}65\text{in.})^2 = 0{\cdot}55\text{in.}^4 + 9{\cdot}15\text{in.}^4 = 9{\cdot}7\text{in.}^4.$$

Moment of inertia about NX of *upper* part of web =

$$\tfrac{1}{3} \times 0{\cdot}4\text{in.} \times (0{\cdot}96\text{in.})^3 = 1{\cdot}18\text{in.}^4.$$

The moment of inertia about NX of the *lower* part of the web is

$$\tfrac{1}{3} \times 0{\cdot}4\text{in.} \times (2{\cdot}04\text{in.})^3 = 1{\cdot}13\text{in.}^4;$$

and that of the smallest rectangle is

$$\tfrac{1}{12} \times 1{\cdot}6\text{in.} \times (0{\cdot}2\text{in.})^3 + 0{\cdot}32\text{in.}^2 \times (2{\cdot}14\text{in.})^2 = 0{\cdot}0011\text{in.}^4 + 1{\cdot}466\text{in.}^4 = 1{\cdot}467\text{in.}^4;$$

and that of the bottom rectangle similarly is

$$\tfrac{1}{12} \times 4{\cdot}3 \times 0{\cdot}4^3 + 1{\cdot}72 \times 2{\cdot}44^2 = 10{\cdot}27\text{in.}^4.$$

Totalling all these items, the moment of inertia about NX of the entire section is 23·75 quartic inches.

(4) Modulus of section for part *above* NX . . $\left.\right\} = \dfrac{23{\cdot}75\text{in.}^4}{2{\cdot}36\text{in.}} = 10{\cdot}05\text{in.}^3.$

Modulus of section for part *below* NX . . $\left.\right\} = \dfrac{23{\cdot}75\text{in.}^4}{2{\cdot}64\text{in.}} = 9\text{in.}^3.$

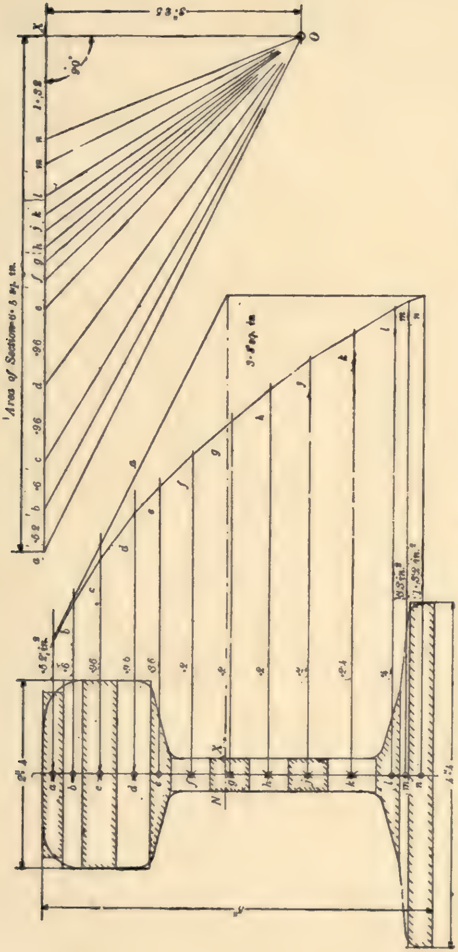


Fig. 144A.

Fig. 144B.

Fig. 144.

Taking the smaller of these values, the moment of resistance of the entire section is—

$$9 \text{ in.}^3 \times 5 \text{ ton/in.}^2 = 45 \text{ inch-tons.}$$

The bending moment cannot safely exceed this amount.

The *graphic method* of finding the moment of inertia of the same rail section next claims attention.

(1) Draw the section full size (Fig. 144), and divide it as shown into a convenient number of parts or layers—the more the better for accuracy. Find their areas and add them. The total area thus found is 6·5 sq. in.

(2) Draw the horizontal line aX (Fig. 144A) to represent the total area of the section, and along it measure the lengths *ab*, *bc*, etc., proportional to the areas of the several layers. Any convenient scale may be used, such as 1in. to 1 sq. in. Thus *ab* represents the area of the *top* layer—viz., 2·1in. × $\frac{1}{4}$in. = 0·52 sq. in., and the last division nX represents the area of the *bottom* layer—viz., 4·4 in. × 0·3 in. = 1·32 sq. in.

(3) Draw the perpendicular XO (Fig. 144A) equal to $\frac{1}{2}a$X. Join O to the points a, b, c, etc., thus forming a polar diagram.

(4) In Fig. 144, draw a series of indefinite horizontals, passing through the centre of area of every layer. Then, starting at any point in the first horizontal, draw a sloping line parallel to bO in the polar diagram, stopping it at the horizontal through b. Continue the construction of Fig. 144B with a sloping line drawn parallel to cO, then with another parallel to dO, and so on until the last horizontal is reached. Complete the polygon by drawing a vertical line from the bottom horizontal, and a sloping line parallel to aO from the top horizontal. The point of intersection of these two closing lines fixes the height of the centre of area of the rail section, and a horizontal through it is the neutral axis of the section.

(5) By means of a planimeter, or otherwise, measure the area of the polygon (Fig. 144B). It is 3·3 sq. in.

(6) Then the moment of inertia of the section is—

$$\text{Area of section} \times \text{area of polygon}$$
$$= 6{\cdot}5\text{in.}^2 \times 3{\cdot}3\text{in.}^2 = 21{\cdot}5\text{in.}^4$$

This graphic process is inferior to calculation on several grounds. It occupies far too much time for practical use, and needs too much apparatus. The section must be accurately drawn, and a planimeter is almost a necessity. In the rival method it is not absolutely necessary, though desirable, to draw the section to scale. Further, owing to an accumulation of small inaccuracies to construction, the final result is not likely to be very accurate.

On the other hand, not much arithmetic is needed in applying the graphic method, and in the absence of a slide-rule this is a consideration. The process is also very easy to remember, no complex formulæ being used. On the whole, however, the graphic method described, though deemed worthy of a place in "Molesworth," is of more academic interest than technical value.

CHAPTER XIII.

COMPARATIVE STRENGTHS OF CIRCULAR, RECTANGULAR, AND ELLIPTICAL TUBES OF EQUAL WEIGHT.

Some important experiments on the transverse strength of wrought-iron *welded* tubes were made fifty years ago for Mr. Robert Stephenson by Mr. John Hosking, particulars of which are not without interest and profit even to this generation of engineers.

The tubes experimented on were identical in everything except shape, and in each case $\frac{3}{16}$ in. thick, the other dimensions being shown in Fig. 145. The tubes B and C were made from round

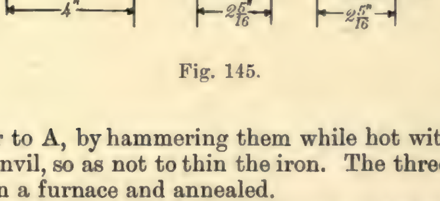


Fig. 145.

tubes similar to A, by hammering them while hot with mallets on a prepared anvil, so as not to thin the iron. The three tubes were finally put in a furnace and annealed.

In each experiment a *saddle* 6in. wide was placed at the middle of the tube, and from this the load was hung. The length between the supports was 6ft. The weights were laid on quietly, a little at a time, and the deflection thereby produced at the centre of the span carefully measured. The results are given in Mr. Edwin Clark's voluminous work on "The Britannia and Conway Tubular Bridges," from which the following table has been prepared :—

Load in Tons.	Deflection in Inches.		
	Circular Tube.	Rectangular Tube.	Elliptical Tube.
0·35	0·057	0·065	0·065
0·7	0·14	0·12	0·122
1·05	0·215	0·177	0·187
1·4	0·285	0·232	0·245
1·5	0·31	0·255	0·262
1·6	0·335	0·272	0·277
1·7	0·367	0 29	0·295
1·8	0·442	0·31	0·317
1·9	0·845	0·33	0·34
2·0	Not measured.	0·36	0·37
2·1	—	0·405	0·392
2·2	—	0·485	0·435
2·3	—	0·625	0·482
2·4	—	0·9	0·58
2·5	—	1·45	0·73
2·6	Failed suddenly.	—	1·035
2·65	—	2·2	1·23
3·15	—	Tube failed.	—
3·46	—	—	Tube failed.

The general trend of the experiments is best seen by plotting the loads and deflections to scale on squared paper, the curves in Fig. 146 being thus obtained. It will be noticed that the deflection of the circular tube increases much more rapidly than that of the others, owing to its smaller depth. The sudden bend marks the elastic limit of the material.

All the tubes gave way by the top or compression part becoming first distorted. In the case of tubes A and C the sides were forced *outwards*, and the tubes became flattened at the place of application of the load. In the case of the rectangular tube, one side buckled *inwards*, the tube yielding sideways and becoming much twisted. No injury to the tubes could be detected near the bearing ends.

As the elliptical tube stood the heaviest load—viz., 3·46 tons—its section would appear to be the best. But the merits of the three *shapes* of section cannot be fairly compared until they have been reduced to the same depth; since, though the area of each section is the same, the *leverage* of the resistance increases with increase of depth, quite apart from the geometrical form of the section. The correction for unequal depth was made by deducing the several values of a coefficient C from the formula—

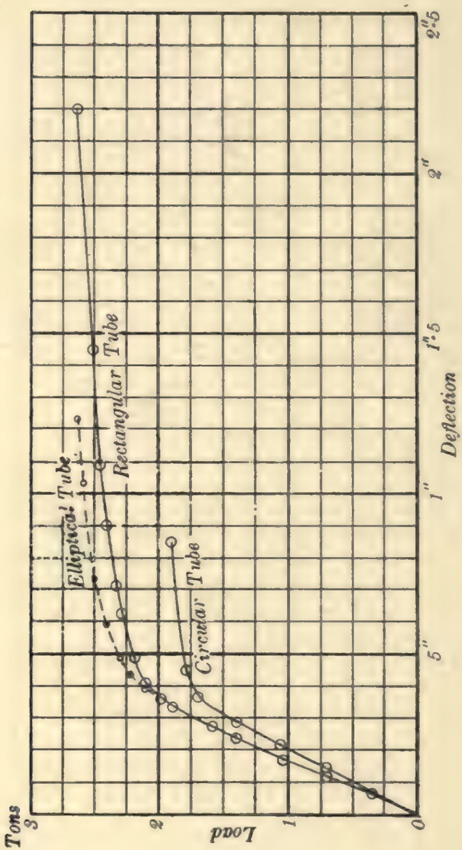

Elliptical Tube
Rectangular Tube
Circular Tube
Tons
Load
3
2
1
0
Deflection
5"
1"
1"5
2"
2"5

Fig. 146.

$$C = \frac{\text{load} \times \text{length}}{\text{area} \times \text{depth}}.$$

Thus, in the case of the *round* tube,

$$C = \frac{2{\cdot}6 \times 72}{2{\cdot}246 \times 4} = 20.9.$$

Similarly, for the so-called *oval* tube, $C = 22{\cdot}25$; and for the *box* section, $C = 23{\cdot}53$; where C gives the relative value of the form, independent of the depth. It therefore appears that the rectangular tube is the strongest form.

Tubes of elliptical section find some application in cycle construction, but they are very little used in large structures. It was once intended, however, that the Britannia Bridge should be built of two such tubes; but the oval section was finally abandoned in favour of the box section, owing to the greater ease of construction and superior strength of the latter with a given depth and the same weight of iron.

Experiments on Box Girders.—Prior to the construction of the Britannia and Conway tubular bridges, some highly instructive preliminary experiments were carried out with a view to determining the best form of section for the tubular girders proposed to be used in their construction. In the light of the lessons thus learnt, more elaborate experiments were afterwards made on a large model one-sixth the size of the final girders of the Britannia Bridge. A brief account of one set of the preliminary experiments will serve to confirm and emphasise the general principles of strength enunciated in preceding chapters. For the particulars of these experiments we are indebted to Mr. Edwin Clark, the resident engineer of the Britannia Bridge, whose own words have been made use of to some extent.

The preliminary experiments on the transverse strength of wrought-iron girders were devised by Mr. Stephenson without any other object than to test generally the properties of such structures—to discover in what manner they might be expected to fail, and to ascertain practically their applicability to purposes of construction. They furnished valuable practical hints on the best methods of construction, pointed out the road for future investigation, and supplied some data for deductive reasoning. Moreover, they confirmed Mr. Stephenson's view, that "a wrought-iron tube is the most efficient as well as the most economical description of structure that can be devised for a railway bridge of 450ft. span across the Menai Straits." This opinion, it should be remembered, was expressed over fifty years ago by the most eminent railway engineer of the day. Since then the art of bridge building has advanced greatly.

The rectangular tubes experimented on varied in length from $17\frac{1}{2}$ to 24ft., and in depth from $\frac{1}{15}$ to $\frac{1}{20}$ of the length, or from 8 to $18\frac{1}{2}$ in. The thickest plates used were $\frac{1}{4}$in. thick. Weights were hung either from a hole in the bottom of the tube under test, or from a bar passing through the sides and resting on a cushion on the bottom plates, the part pierced being strengthened by a plate riveted around the hole. As shown in Fig. 147, the web plates were flanged (or bent over) and united to the flange plates by rivets, no angle irons being used. This mode of connection, it may be remarked, is much employed in modern ship-building, and also in boiler work, on account of the saving of weight and labour effected; the bending-over of the plates being rapidly and cheaply performed by special flanging machines. Strangely, however, the method of flanging seems never to be used in ordinary girder work.

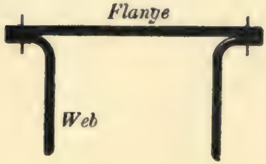


Fig. 147.

The model tubes were first of all tested with the thickest plates at the *bottom*, and loaded until failure took place. Subsequently the same girders were repaired, inverted, and re-tested with the thickest plates at the *top;* when much better results were got than before, thick plates being far less liable to buckle than thin ones. The following table gives the results of some of the experiments:

Clear Span.	Depth.	Width.	Thickness of Plate.		Ultimate Deflection.	Breaking Load.
			Top.	Bottom.		
Ft.	In.	In.	In.	In.	In.	Lb.
17·5	9·6	9·6	0·075	0·075	1·10	3,738
17·5	9·6	9·6	0·272	0·075	1·13	8,273
17·5	9·6	9·6	0·075	0·142	0·94	3,788
17·5	9·6	9·6	0·142	0·075	1·88	7,148
17·5	18·25	9·25	0·059	0·149	0·95	6,812
17·5	18·25	9·25	0·149	0·059	1·73	12,188
18	13·25	7·5	0·142	0·142	1·71	13,680
19	15·4	7·75	0·23	0·18	1·59	22,469

It is highly interesting to learn what Robert Stephenson had to say about these results. In his official report to the directors of the Chester and Holyhead Railway, dated 1846, we read as follows: "In the first series of experiments, this remarkable and unexpected fact was brought to light—viz., that in such tubes the power of wrought iron to resist compression was much less than its power to resist tension, being exactly the reverse of that which holds with cast-iron. In *cast-iron* beams for sustaining weight the proper form is to dispose of the greater portion of the material at the *bottom* side of the beam; whereas with *wrought iron* these experiments demonstrate beyond any doubt that the greater portion of the material should be distributed on the *upper* side of the beam. We have arrived, therefore, at a fact having a most important bearing upon the construction of the tube—viz., that rigidity and strength are best obtained by throwing the greatest thickness of material into the upper side."

Examining the figures in detail, we notice that in the first experiment on a tube 9·6in. square, with top and bottom plates of equal thickness, the breaking load was 3738lbs.; but on riveting a thicker plate to the top side, the load necessary to break the beam advanced to 8273lbs., the strength being more than double the original.

The next experiment was made on a tube in which the bottom plate was about twice the thickness of the top plate. The gain in strength due to simply turning the beam upside down is very noticeable. The same is true of the deeper tube having the section 18¼ by 9¼ in., in regard to which Sir William Fairbairn, in his report to the directors, remarks: "Loading this tube with 6812lbs. (the *thin* plate being uppermost), it becomes wrinkled, with a hummock rising on the top side, so as to render it no longer safe to sustain the load. Take, however, the same tube, and reverse it with the *thick* plate upwards, and you not only straighten the part previously injured, but you increase the resisting powers from 6812 to 12,188lbs."

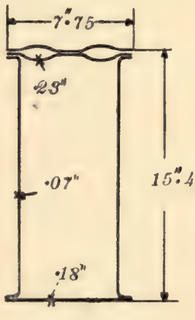


Fig. 148.

"Let us now examine," continues Fairbairn, "the tube in the last experiment, where the top is composed of corrugated iron"—as Fig. 148—"forming two tubular cavities. This presents the best form for resisting the 'puckering' or crushing force. Having loaded the tube with increasing weights, it ultimately gave way by tearing the sides from the top and bottom plates, at nearly the instant after the last weight was laid on. The greatly increased strength indicated by this form of tube is highly satisfactory. The results here obtained are so essential to this inquiry, and to our knowledge of the strength of materials in general, that I have deemed it essential to direct attention to facts of immense value in the proper and judicious application of the material of the proposed structure. Strength and lightness are *desiderata* of great importance, and the circumstances above stated are well worthy the attention of the mathematician and engineer."

After these comments of the leading authorities of their day on bridge building, it is fitting to introduce here some critical remarks of the distinguished engineer, the late Sir William Anderson, taken from his excellent lecture on "The Interdependence of Abstract Science and Engineering," delivered before the Institution of Civil Engineers in 1893. These remarks reveal the vast advance of scientific bridge designing during the last half-century. Alluding to the history of the development of iron and steel bridge building, which necessarily followed the introduction of railways, Anderson says:

"The principles which underlie the determination of stresses in braced structures, such as roofs and frameworks of various kinds, as well as those in solid bars subjected to the action of transverse forces (*i.e.*, beams), have long been known, and early in this century Navier made them the subjects of lectures at the Ecole des Ponts et Chaussées; yet engineers in this country seem to have been but dimly aware of them, or, at any rate, to have made but little use of the knowledge at their disposal. It is difficult, from the published histories of such enterprises as the Conway and Britannia bridges, to arrive at any conclusion as to the extent of knowledge, or rather ignorance, which existed among engineers before these works were commenced. It is probable that some, of a specially scientific turn of mind, but who were not in conspicuous practice, had a deeper insight into principles than the men whose great natural genius and knowledge of affairs placed them in prominent positions in the great railway enterprises of the day. It is sufficiently evident, however, from the long series of purely tentative experiments by which the proportions of the Conway and Britannia bridges were determined, as well as from the singular vagaries to be noticed in the smaller bridges of that day, that only the haziest ideas of the disposition of stresses and of the functions of the component members of girders existed. This naturally led to timidity as to the capacity of girders to carry, unaided, the loads it was sought to impose, and induced a preference for masonry or for suspension bridges, with respect to which much wider experience was at command.

"In the experimental investigations of the time, the function of the web or vertical member of a girder was completely ignored, for it was looked upon merely as the means of keeping the top and bottom flanges in their relative positions, while the essential difference in effect of a uniformly distributed load, or of a rolling load, as compared with a load concentrated at the centre, on the vertical member of a girder, and even on the flanges, appears to have been overlooked till made evident by the results of experiment; and the grave doubts which arose as to whether the girders of the two great tubular bridges could be made self-supporting are apparent to this day in the preparations made in the piers and abutments for the introduction of auxiliary chains. Yet the principle that a force cannot change its direction unless combined with another force acting in a direction inclined to it, was perfectly well known, and should have led to the discovery that it is only by diagonal stresses in the vertical members that the load resting on a beam can be transmitted to the abutments, or be made to produce effects at right angles to its own direction in the flanges, and that the stresses due to loads concentrated at the centre were very different to those arising, both in the vertical web and in the flanges, from the action due to a load distributed in a given manner along the top or the bottom flanges, and that a rolling load would produce effects peculiar to itself.

"Since 1848 the supremacy of theory over rule-of-thumb has gradually but surely asserted itself, though at times the want of common sense and experience in the application of abstract principles, as well, perhaps, as ill-judged efforts to produce cheap structures, has led to disasters quite as serious as those which arose from want of theoretical knowledge; and in this respect the competent and successful engineer will still show himself as the man who in his work is careful to make theory and practice walk side-by-side, the one ever aiding and guiding the other, neither asserting undue supremacy. This course, in its highest development, we may, I think, assert is that adopted by our leading engineers, with the result that this country may claim the honour of such a structure as the Forth Bridge, for the design and construction of which no tentative experiments were needed, though the form and mode of construction were very special, if not absolutely new, and the dimensions, both in span and height, so gigantic that the authors of the design could have derived but little aid from previous experience."

CHAPTER XIV.

THE DEFLECTION OF BEAMS.

LEAVING, for the present, the consideration of the *strength* of beams, we now pass on to the question of their *stiffness*, which, in many cases, is equally important.

When an originally straight beam (Fig. 149) is loaded by a force of sufficient magnitude, the beam visibly *bends* or assumes a curved form, and the load falls through a certain height. This vertical movement is styled the *deflection* of the beam. The amount of deflection varies from point to point, but its *greatest* value is alone important, and this is what is ordinarily meant by the deflection of the beam.

The general question of the deflection of beams of any section and profile, loaded in any manner whatever, is one of considerable

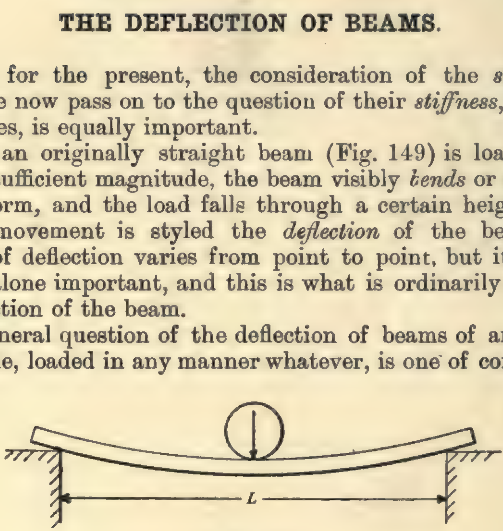


Fig. 149.

mathematical difficulty, so that only the more common cases will be here considered.

The general formula for calculating the maximum deflection of any beam of uniform section, when loaded in any simple or standard manner, is the following :—

$$\text{Deflection} = C \times \frac{WL^3}{EI}.$$

The deflection is expressed in inches, the usual symbols being Δ (delta) and d; but there is no universal agreement. Rankine uses v. Sometimes y is used. L is the length of the beam unsupported, in inches. W is the load or force applied, either in tons or pounds. The formula ceases to be true when W exceeds the elastic limit load, the stress due to a given load being calculated by the equation—

Bending moment = stress × modulus of section.

I is the geometrical moment of inertia of the section with respect to its neutral axis, expressed in quartic inches ($inch^4$). E is the modulus of elasticity of the material, or ratio of stress to strain within the elastic limits. This is a measure of the *stiffness* of the material. Approximate values of E for various materials of construction are tabulated below :—

Material.	Modulus of Elasticity.		Tensile Elastic Limit.
	Lbs. per sq. in.	Tons per sq. in.	Lbs. per sq. in.
Cast-iron (average)	17,000,000	7,600	10,500
Riveted wrought-iron girder	18,000,000	8,000	—
Wrought iron	29,000,000	13,000	30,000
Mild steel	30,000,000	13,400	40,000
Tool steel	30,000,000	13,400	80,000
Tempered steel	36,000,000	16,000	190,000
Oak	1,500,000	670	5,000(?)
Pine	1,600,000	715	4,000(?)

C is a numerical coefficient whose value depends on the mode of supporting and loading the beam, as under :—

Case.	How Supported.	Position of Load.	C.
1	Fixed at one end.	At free end.	$\frac{1}{3}$
2	ditto.	Uniformly distributed.	$\frac{1}{8}$
3	Supported at both ends.	At centre.	$\frac{1}{48}$
4	ditto.	Uniformly distributed.	$\frac{5}{8} \times \frac{1}{48}$
5	Fixed at both ends.	At centre.	$\frac{1}{4} \times \frac{1}{48}$
6	ditto.	Uniformly distributed.	$\frac{1}{8} \times \frac{1}{48}$

Deflection of Rectangular Beams.—Since the moment of inertia of a rectangular section about the neutral axis is—

$$\frac{1}{12}BH^3,$$

for a beam of this shape supported at both ends the general deflection formula becomes—

$$= \frac{W}{4E} \times \left(\frac{L}{H}\right)^3 \times \frac{1}{B}.$$

$$d = \tfrac{1}{48} \times \frac{WL^3}{E} \times \frac{12}{BH^3}$$

This important formula shows that the deflection of a beam under a given load varies inversely as the breadth of the beam and directly as the cube of the ratio of the span to the depth. Hence to secure sufficient stiffness long beams require to be made very deep.

It is highly instructive to compare the *stiffness* formula with the *strength* formula for uniform beams of rectangular section. Regarding the strength of a beam as measured by the load it will support, we may say that—

$$\text{Strength varies as } f \times \frac{H^2}{L} \times B.$$

Again, taking the numerical measure of stiffness as the inverse or reciprocal of deflection, we may write—

$$\text{Stiffness varies as } E \times \left(\frac{H}{L}\right)^3 \times B.$$

From these two expressions it is clear that the effect of doubling the *breadth* of a beam is simply to double both its strength and its stiffness, whereas the effect of doubling the *depth* alone is to quadruple the strength and to increase the stiffness no less than eight-fold, the span, of course, being kept constant. Thus, depth is a much more potent factor than breadth.

If we have two beams of the same material, one being 4in. wide, 6in. deep, and 10ft. span, and the other 4in. wide, 12in. deep, and 20ft. span, they will be of equal stiffness—*i.e.*, they will deflect equally under the same load, although the longer beam will carry twice as heavy a load as the other, and therefore will be twice as strong.

A common allowance for the deflection of a beam that is intended to be fairly rigid is from $\frac{1}{1000}$ to $\frac{1}{500}$ of the span.

Beams designed with the express object of yielding considerably under working conditions—*i.e.*, plate *springs*—are much used as cushioning devices for the prevention of shock, as well as in certain instruments for measuring forces.

Before giving the mathematical proof of the general deflection formula, a few numerical examples of its use will be introduced.

Example 1.—A steel spring (Fig. 150) of 10in. free length is loaded with 100lb. at the end. The uniform section is 2in. by $\frac{1}{4}$in. Calculate the greatest stress and the deflection.

$$\text{Bending moment} = 100\text{lbs.} \times 10\text{in.} = 1000\text{in.-lb.}$$
$$\text{Strength modulus} = \tfrac{1}{6} \times 2\text{in.} \times (\tfrac{1}{4}\text{in.})^2 = \tfrac{1}{48}\text{in.}^3$$

But bending moment = stress × modulus,

$$\therefore \text{stress} = 1000 \div \tfrac{1}{48} = 48{,}000 \text{ lb./in.}^2$$

As this is well within the elastic limit of tempered steel, the spring is safe.

Moment of inertia of section about neutral axis is—

$$\tfrac{1}{12} \times 2\text{in.} \times (\tfrac{1}{4}\text{in.})^3 = \tfrac{1}{384}\text{in.}^4$$

According to Unwin, the modulus of elasticity of tempered steel is 36,000,000lbs. per square inch, though of course it varies

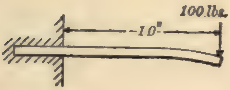


Fig. 150.

Fig. 151.

more or less with every sample of steel. Substituting the known values in the formula—

$$d = \frac{1}{3}\frac{WL^3}{EI},$$

we get

$$d = \frac{1}{3} \times \frac{100 \times 1000}{36{,}000{,}000} \times 384,$$

or deflection = 0·356 inch.

The question of the superposition of spring plates is one of considerable interest and importance. If we had *two* perfectly smooth and identical plates (Fig. 151), the deflection under a given load would be *halved*. But actual plates never are very smooth, and so they rub on each other with considerable friction. Hence the actual deflection for the case of two plates is somewhat less than half that for one plate. How much less is not calculable, but must be found by experiment. It is customary, however, in spring calculations, to take the deflection as inversely proportional to the number of plates.

Experiment.—The author once made an experiment on the deflection of a small pitch-pine beam, measuring 0·96in. wide by

1·18in. deep, the span being 36in. Its total volume was 49 cub. in., and its weight 1·2lbs.; so that the density of this particular sample of pitch-pine was 42·3lbs. per cubic foot. The deflection of the centre of the beam was measured by a dial arrangement, the finger being actuated by a small drum, round which was coiled a cord, and one end of the latter was fastened to the beam. The following readings were taken, the uniform increment of load being 28 pounds:—

Load.	Deflection.	Increment.
Lb.	In.	In.
28	0·075	
		0·100
56	0·175	
		0·115
84	0·290	
		0·122
112	0·412	
		0·118
140	0·530	
		0·130
168	0·660	
		0·120
196	0·780	
		0·140
224	0·920	
		0·130
252	1·05	
		0·180
280	1 23	
		0·210
308	1·44	
		—
336	Broke.	

These observations have been plotted in Fig. 152. It will be seen that the deflection remains fairly proportional to the load until the breaking-point is approached.

Example 2.—Calculate the modulus of elasticity of the sample of pitch-pine used in the above experiment, the deflection caused by an addition to the load of 28lbs. being taken as 0·12in.—

$$\text{Since } d = \frac{W}{4E} \times \left(\frac{L}{H}\right)^3 \times \frac{1}{B},$$

$$\therefore\ 0{\cdot}12 = \frac{28}{4E} \times \left(\frac{36}{1{\cdot}18}\right)^3 \times \frac{1}{0{\cdot}96}$$

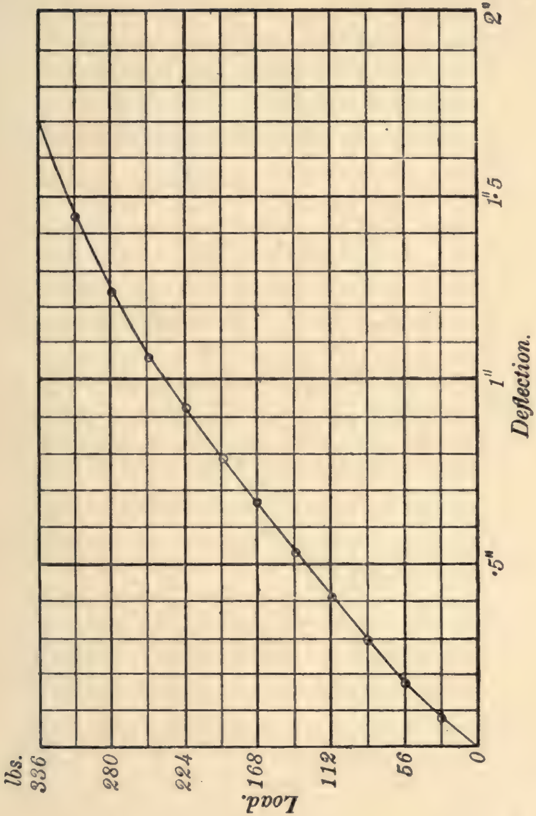


Fig. 152.

from which we find—

$$E = \frac{7 \times 30{\cdot}5^3}{0{\cdot}12 \times 0{\cdot}96} = 1{,}720{,}000\text{lbs. per square inch.}$$

Example 3.—A wrought-iron girder of the section given on page 147, Fig. 126, and of 30ft. span, weighs 2 tons, and carries a uniform load of 13 tons. Calculate the probable deflection at the centre.

If there were no joints in this girder, we should take 13,000 tons per square inch as the modulus of elasticity of wrought iron; but for a riveted girder like the present, the usual value taken is 8000 tons, to allow for the inevitable yielding of the joints.

The moment of inertia of the section about the neutral axis is 2273 inch^4 units. Inserting the proper values in the deflection formula—

$$d = \frac{5}{8} \times \frac{1}{48}\frac{WL^3}{EI},$$

we get

$$d = \frac{5}{384} \times \frac{15}{8000} \times \frac{360^3}{2273} = 0{\cdot}5\text{in.}$$

This is also the amount of initial *camber* that should be given to the beam during construction.

Example 4.—A beam of uniform section is supported at both ends and loaded in the middle. Find the ratio of depth to span, in order that the deflection may not exceed $\frac{1}{1000}$ of the span, when the stress is 8000lbs. per square inch, and the modulus of elasticity is 28,000,000lbs. per square inch.

Step 1.—By the conditions of the question,

$$\frac{WL^3}{48\,EI} = \frac{L}{1000}.$$

Dividing by L, and substituting the given value of E, we get—

$$\frac{WL^2}{48 \times 28{,}000{,}000\,I} = \frac{1}{1000};$$

$$\therefore \frac{WL^2}{48 \times 28{,}000} = I \quad . \quad . \quad . \quad . \quad (1)$$

Step 2.—Again, for a beam of any symmetrical section, we know that—

$$\frac{\text{Moment of inertia of section}}{\text{Modulus of section}} = \frac{\text{depth}}{2},$$

or

$$\frac{I}{Z} = \frac{H}{2}.$$

Now Z = moment of resistance ÷ stress,

$$= M \div f.$$

Hence, substituting for Z,

$$I = \frac{M}{f} \times \frac{H}{2}.$$

But for a beam loaded at the centre,

$$M = \tfrac{1}{4} WL.$$

$$\therefore I = \frac{WL}{4f} \times \frac{H}{2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Step 3.—Combining results (1) and (2), we have

$$\frac{WL^2}{48 \times 28{,}000} = \frac{WLH}{8f}.$$

$$\therefore \frac{L}{6 \times 28{,}000} = \frac{H}{f}.$$

The given stress being 8000lbs. per square inch, this becomes—

$$\frac{L}{H} = \frac{6 \times 28{,}000}{8000} = \frac{21}{1}.$$

Hence the depth of a uniform wrought-iron beam must not be less than $\frac{1}{21}$ of the span, if the deflection has not to exceed $\frac{1}{1000}$ of the span.

Proof of the Deflection Formula.—Sufficient examples having been given of the use of the formula for calculating the deflection of beams, it remains to show on what basis it is framed. Under one aspect it may be regarded simply as a convenient general statement of the results of numerous experiments on the stiffness of beams of various lengths, sections, and materials. The formula has been experimentally verified over and over again, and is generally taken to be approximately true, so long as the material is not stressed beyond its elastic limits.

But as a matter of fact the deflection formula has really been deduced by mathematical reasoning from first principles; and as the process of deduction is both instructive and interesting, we propose to consider its leading steps, taking care, however, to dwell more on the physical ideas involved than on the symbolical expression of those ideas. The mathematical reader will have no difficulty in supplying the complete symbolism.

(1) Consider a cantilever of rectangular section (Fig. 153) bent by a heavy load W, but not so heavy as to cause a permanent set. The thickness of the beam and the deflection are both greatly exaggerated in the figure, for the sake of clearness. Choose two transverse sections of the beam AB, CD, very near to each other,

so as to isolate in imagination a thin slice of material. Before the beam is bent, the faces of this slice are parallel to each other, but after bending they radiate to a common centre and include a very small angle, say i. In the figure the thickness of the slice is enormously exaggerated, and also the magnitude of the angle.

(2) Because of the bending, the top fibres of the slice under consideration pull out by a very trifling amount, which we may

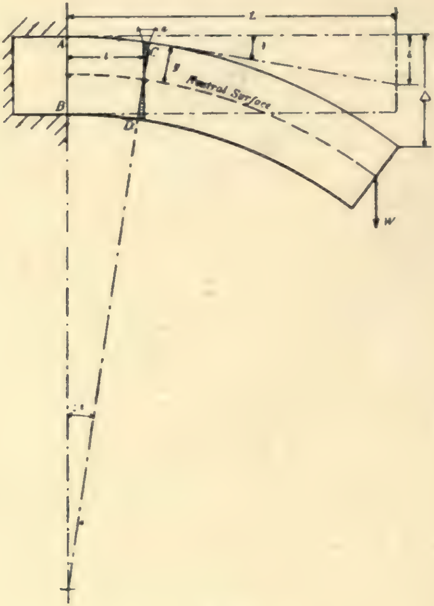


Fig. 153.

call e. The bottom fibres are crushed up to the same extent, if the neutral axis is at the middle of the depth of the beam.

(3) We find an expression for the strain at the top edge by dividing the extension e by the original thickness of the slice t.

(4) Another expression for the same strain is found by applying Hooke's law, that stress is proportional to strain. Calling the ratio of stress to strain, or modulus of elasticity of the material, E, and the stress on the top fibres f, it follows that the strain is the stress f divided by the modulus E. We make the assumption that the compressive modulus of elasticity has the same value as the tensile modulus.

(5) Having thus got two statements of the same thing from different points of view, we equate them, and so get an expression for the unknown very small extension e in terms of three better-known quantities, the stress f, the modulus E, and the thickness of the slice t, thus—

$$e = \frac{f}{E} t.$$

(6) Looking at the small upper shaded triangles (Fig. 153), and remembering that the short arc e is found by multiplying the radius y by the circular or radian measure of the angle i, we introduce into our work the known or calculable distance of the top of the beam from the neutral axis, and also the unknown factor i.

(7) In order to get rid of the objectionable angle i, we draw tangents to the top surface of the beam at the points A and C; which tangents then include the angle i. Our aim is now to express this angle in terms of two other quantities—viz., the distance L of the thin slice AC from the free end of the beam, and the small deflection or drop δ of the end due to the bending of this slice alone.

This we do by again applying the useful relation that arc = radius × angle.

We are compelled, however, to make the assumptions that the tangents are of equal length, and that the end of the cantilever moves in an arc so short as to be practically a vertical line. Since the deflection is always extremely small compared with the length of the beam, and the slice is also very thin, these assumptions are perfectly valid.

(8) We have now found two expressions for the same quantity i—namely,

$$i = \frac{e}{y}, \text{ and } i = \frac{\delta}{L},$$

by equating which we eliminate i and get a formula for the small deduction δ in terms of the length of the beam L, the distance y, and the small extension of the top edge e, thus—

$$\delta = \frac{e}{y} L.$$

Of these three quantities the first two may remain, being easily measurable; but the third item e must be got rid of without delay, as being much too intangible. Hence, in place of the infinitesimal extension e, we substitute the value of e in terms of the stress f, the modulus E, and the short length t, as already found in step (5), thus—

$$\delta = \frac{f}{E} t \times \frac{L}{y}.$$

We then have a formula for the small deflection in terms of five quantities, all of which are measurable save t.

(9) The next move is to get rid of the distance y, and bring in the moment of inertia I of the section, so as to make the formula apply to any shape of section whatever. We do this by making use of the proportion that

$$\frac{\text{Moment of resistance of section}}{\text{Moment of inertia of section}}$$

$$= \frac{\text{Stress on material at top edge}}{\text{Distance of neutral axis from top edge}},$$

or

$$\frac{M}{I} = \frac{f}{y}.$$

On writing $\frac{M}{I}$ in place of $\frac{f}{y}$ in the last formula, and then inserting the bending moment WL in place of M, we get

$$\delta = \frac{WL}{E} t \times \frac{L}{I}$$

$$= \frac{W}{EI} L^2 t.$$

This formula gives the deflection for which the slice nearest the wall is responsible; expressed in terms of the load W on the end of the beam, the modulus of elasticity E of the material of the beam, the moment of inertia I of the uniform section of the beam, the total length L, and finally the infinitesimal length t.

Of course if we had been considering a slice at the *middle* of the beam, instead of close up to the wall, then our L would have had only *half* its value in this formula.

(10) So far we have reasoned on the distortion of only a single slice of the beam, of thickness t. The next question is: Knowing the deflection at the end due to this microscopic length, how are we to find the *total* deflection due to the bending of the entire beam?

Two methods are available, one approximate and the other exact. In the first, we divide up the beam into short lengths, calculate the deflection at the end for each piece separately by the above formula, and then add the results together. Unfortunately this method, though instructive, is very laborious, and is therefore never used.

By the very ingenious mathematical process of the addition of infinitesimals known as integration, the summation can be rapidly done at one operation, and once for all. The rule is very simple: add *one* to the index of the varying quantity, and divide by the new index, the constants remaining unchanged. So, applying to the case in hand this handy method of adding together all the small effects, we finally get the total deflection—

$$\Delta = \frac{1}{3}\frac{WL^3}{EI}.$$

The last bit of work is probably the only part that will not be quite intelligible to the majority of readers, after writing down all the algebraic steps and doing a little thinking. The full working of this step is given below.*

It remains to adapt the deflection formula for a *cantilever* to the more common case of a beam supported at the ends and loaded centrally (Fig. 154). Such a beam may be regarded as loaded at both ends by the reactions and fixed in the middle; or, in other words, as two inverted cantilevers. Hence, writing $\frac{W}{2}$ in place of W, and $\frac{L}{2}$ instead of L, in the last result, we get—

Fig. 154.

$$\text{Deflection} = \frac{\frac{W}{2} \times \left(\frac{L}{2}\right)^3}{3\,EI},$$

or

$$\Delta = \frac{WL^3}{48\,EI},$$

which is the required expression.

---

* Using standard notation, let dx be the thickness of any vertical slice of the beam, and x its distance from the free end, then

$$\text{Deflection} = \frac{W}{EI}\int_{x=o}^{x=L} x^2 dx,$$

or $$\Delta = \frac{W}{EI}\left[\frac{x^3}{3}\right]_o^L = \frac{1}{3}\frac{WL^3}{EI},$$

CHAPTER XV.

TYPES OF GIRDERS.

CAST-IRON, as a material for girders, is cheap, and has the advantage of being easily moulded to any desired shape, whether plain or ornamental. But it has several serious drawbacks. On account of the low tensile strength of cast-iron, girders of this material are very heavy, and therefore quite unsuited for long spans. They are also liable to contain dangerous flaws which cannot be perceived, and they give no warning before fracture. This unreliability is especially marked when, after being heated, cast-iron girders are suddenly cooled by water, as might happen in the case of a building on fire. Further, the expense of a wood pattern is a considerable item, when only a single girder is required. For these reasons cast-iron girders are very little used now in structural work, having been displaced by rolled joists and built-up steel girders. Malleable cast-iron is suitable for light levers.

Moulded beams of *cast-steel* find some application in heavy machinery, as in the case of the slide-beams carrying the heavy guns of battleships and forming the recoil path (Fig. 2). Cast-steel is never used for bridge girders, however; as very large steel castings are difficult to cast, and are also less reliable than mild-steel built-up girders. The latter are moreover much more convenient to transport and erect.

The largest market size of rolled I beam has a depth of 24in. and a width of $7\frac{1}{2}$in. Single joists, however, are often strengthened by riveting to each flange a broad plate, which need not extend the full length of the beam. When a beam is required to have a stronger section than a rolled joist, we may use either a compound joist girder, such as that shown in Fig. 134, or else a compound channel girder, as shown in Fig. 155. These can be rapidly built up, from rolled sections and flat bars, with a minimum amount of riveting.

A *plate girder* is very commonly used when the span exceeds 20ft. Such a girder (Fig. 156) consists of a thin continuous vertical member or web, connected at the top and bottom by angle bars and rivets to two or more thicker plates forming the flanges or booms. The bulk of the material being thus situated at the greatest distance from the neutral axis is well placed for withstanding the horizontal thrust and pull.

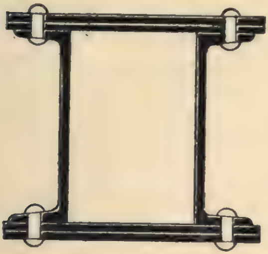


Fig. 155.

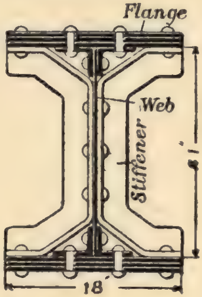


Fig. 156.

While the flanges of a plate girder oppose the bending moment, the web connecting them resists the shearing force; and as this is greatest at the ends of the girder, the web plates near the abutments are sometimes made thicker than at the centre. To prevent the web buckling or crumpling up, it is stiffened at intervals of about 4ft. by means of vertical T or L bars spaced closer near the supports. These bars also serve to cover the joints in the web plates. The deeper the girder the greater the tendency of the web to buckle. Hence a *deep* girder should have either thicker plates or more vertical stiffeners than a *shallow* girder.

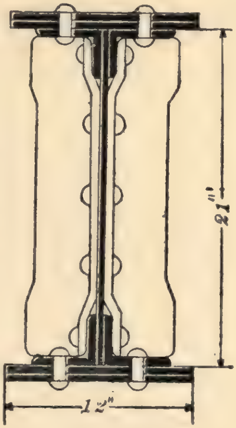


Fig. 157.

The *web stiffeners* are usually either bent outwards to clear the angle bars, as shown in Fig. 156, or else cranked and fitted close to the angles, as in Fig. 157, the former method being preferable when the width of the flanges admits of it. A third method is to use *straight* stiffeners, kept clear of the angles by means of packing pieces; which, however, add to the weight of the girder without increasing its strength.

The *joints* in the flanges of plate girders are generally made by cover plates, as shown in the perspective sketch Fig. 158, an equal number of rivets being symmetrically placed on each side of the joint. In designing such a joint the aim is to keep its strength as nearly as possible equal to that of the original plate. When there are several layers of plates it is convenient to group the joints, and cover them by one pair of plates, as shown in Fig. 159.

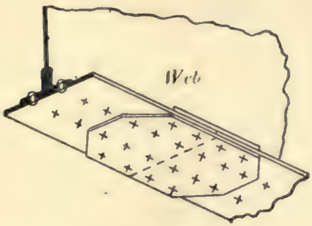


Fig. 158.

A plate-girder bridge is considered to be the best form of road or railway bridge for spans not exceeding 60ft.; while above that span lattice and braced girders are preferable. Plate girders are also largely used in the construction of modern workshops and city buildings, as well as for the cross-girders of overhead travelling cranes, and for the roadways (or runways) supporting such cranes, when the pitch of the columns exceeds about 15ft.

A plate-girder with *two* vertical webs (Fig. 126) is termed a box plate girder, or simply a box girder. Such a girder is stiffer laterally than a single-web girder, and is used for the larger spans. One practical objection to a box girder is that it cannot be painted inside, after erection, unless made so large as to admit of the passage of a man. Further, for a given weight of metal in the web,

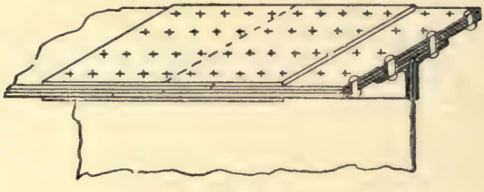


Fig. 159.

the surface exposed to corrosion is greater than in the case of a single-web girder.

The structural weight of a small girder is unimportant compared with the load it can carry; so that for small spans cast-iron girders are practicable. But as the span increases the structural weight becomes relatively more and more important, and it is necessary to use plate girders; then lattice, bow-string, and other braced girders. Finally, when we reach such enormous spans as those of the Brooklyn (1595ft.) and Forth (1710ft.) bridges, engineers are compelled to resort to either suspension chains or double cantilevers to enable them to bridge over such distances at all.

Since the bending moment on a uniformly-loaded girder is greater at the centre than elsewhere, the cross-section should not be uniform from end to end, if it is desired to economise material. Most girders are therefore constructed either of *uniform* depth, with the flanges of variable section, or else of *variable* depth and uniform flange section. The former are broadly classed as *parallel* girders, and the latter as *parabolic* girders; because the outline of the curved flange is approximately a parabola.

In properly-designed *parallel* girders, with flanges of variable thickness, the total flange stress is everywhere nearly proportional to the bending moment; so that the bending moment diagram also represents, to a certain scale, a diagram of flange stress.

In *parabolic* girders the depth at any section is made roughly proportional to the bending moment induced by a uniform load; and when this is the case the total horizontal stress in each flange is nearly uniform throughout the span, as can be seen from the well-known relation—

$$\begin{pmatrix}\text{Resistance}\\ \text{of flange}\end{pmatrix} \times \begin{pmatrix}\text{effective depth}\\ \text{of section}\end{pmatrix} = \begin{pmatrix}\text{moment of resist-}\\ \text{ance of section}\end{pmatrix};$$

or, in symbols,

$$A \times f \times l = M.$$

Such girders as are used for the overhead travelling cranes of workshops, with straight top flanges and curved bottom flanges, are often referred to as *fish-bellied* girders; and those of the inverted form are known as *hog-back* or *saddle-back* girders. In the latter type the depth at the ends is commonly *half* the depth at the centre; and in the former it is half the central depth *plus* 3in.

For a given span, girders with curved flanges are both lighter and more elegant than those with parallel flanges, but they are more costly to construct, and where the head-room is very limited, parallel girders must often of necessity be employed.

CHAPTER XVI.

BRACED GIRDERS AND STRESS DIAGRAMS.

In addition to ordinary plate girders, which are characterised by *continuous* webs, there are in general use many types of parallel girders with *open* webs, some of which will now be briefly described, and their merits compared.

The Zig-zag Truss, or the Warren Girder, as it is more commonly styled, after the name of its originator, Captain Warren, is perhaps the best-known type of parallel braced girder. It was first introduced about the year 1850. Though regarded with disfavour by some eminent engineers of that period, as Sir William Fairbairn, who preferred the plate girder, it has since been much used in bridge-work; owing to its being more economical of material than a plate girder, when the span is considerable.

Fig. 160 shows the usual form of Warren girder, having in this case eight sections, bays or panels. Instead of a continuous web we have a number of bracing bars or diagonals, inclined at 60°.

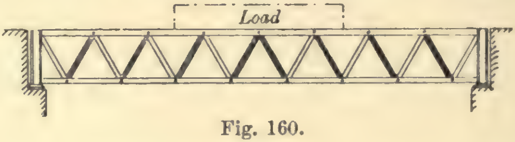


Fig. 160.

Fig. 161.

These diagonals are connected to the upper and lower flanges or booms either by pins or rivets, thus forming a single system of equilateral triangles. The thick bars are in thrust, and the others in tension.

In Fig. 161 is shown a modified style of Warren girder, suitable for a "through" bridge, its diagonals being inclined at an angle of 45°. Here the upper joints are loaded by vertical ties or suspenders, which divide the girder into ten panels. The vertical posts merely serve to support and stiffen the upper flange of the girder. But if the load were to be carried on the upper flange, instead of on the lower, then the posts referred to would act as struts, and become more actively employed.

A third type of Warren girder, illustrated in Fig. 162, is suitable for a so-called "deck" bridge—that is, one in which the load is

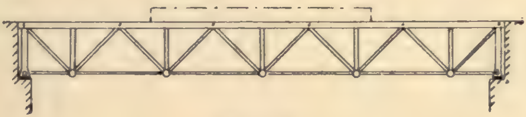


Fig. 162.

carried by the top flanges of the girders. It is derived from Fig. 161 by inverting the latter and removing one set of verticals. In this case the verticals act as struts instead of as ties, and the stresses in the diagonals are also reversed.

If we were to leave out altogether the verticals from Figs. 161 and 162, the omission would make no difference to the stresses, provided the load were equally divided between all the joints. But, as a rule, the division is *not* equal, the roadway and the live load being carried by the joints of one flange, while the other flange is loaded with only half the weight of the main girder. Hence, as a matter of fact, the stresses would be somewhat modified by omitting the verticals.

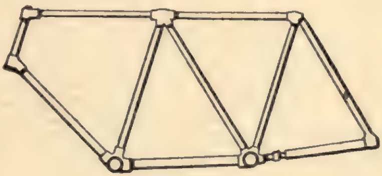


Fig. 163.

As seen from Fig. 163, makers of modern cycles have taken a hint from the Warren girder in designing multicycle frames, thus securing great rigidity, combined with lightness.

Example of a Warren Girder Bridge.—Many fine examples of Warren girder bridges exist, such as the elegant Crumlin Viaduct, which carries the Taff Vale Railway at a level of about 200 ft. above the valley of the Ebbw, in South Wales. The spans of this bridge are 150 ft. The top booms of the main girders are rectangular cells of the section shown in Fig. 164, the bottom booms consisting of chains of wrought-iron plates, set on edge and riveted together. The diagonal ties are flat links. The struts are built up of wrought-iron plates and angle-irons to form the cruciform section shown in Fig. 165. Pins, $3\frac{1}{4}$ in. diameter, connect the bracing bars to the booms, assisted by riveted gusset plates in the case of the bottom boom. In order to increase the lateral stability, it has been found desirable to connect the four booms of the girders by a deck or platform of iron plates.

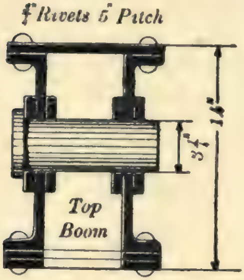


Fig. 164.

Some Warren girders have been made with the top boom of cast-iron, circular in section, the diagonal struts being also of cast-iron. But this construction is now obsolete. Cast-iron is certainly strong in compression, but its range of elasticity is so small as to render it ill-adapted for resisting *sudden* forces. Hence, when cast-iron is used for railway bridges a very high factor of safety is necessary.

To Find the Stresses on a Warren Girder Carrying a Dead Load.—The graphical method of finding the total stresses on the several members of a Warren girder, when carrying only a dead load, is very simple, being an application of the well-known theorems of the "triangle" and "polygon of forces." It should be borne in mind that each member of a *perfect* or ideal braced girder, other than fastenings, is subject only to either direct pull or to direct thrust, there being no transverse force, except at the joints. Hence the pull or thrust, in tons, across every transverse section of any chosen member is the same; also the stress, in tons per square inch, is uniform all over any such section. In the case of *actual* braced girders it is assumed, for convenience, that there are frictionless pin joints at the meeting-points of the axes of the bars, which are supposed to meet on the centre lines of the pins. Although the practical conditions are rather different, the results are approximately correct.

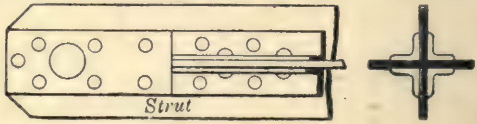


Fig. 165.

Fig. 166 is the frame diagram of a Warren girder of 50 feet span, made up of five bays each of 10 ft., and forming one of a pair of girders carrying a double line of rail. The weight of one girder and half of the platform is taken as one ton per lineal foot of span. This is the *dead* load. There is also a *travelling* load of

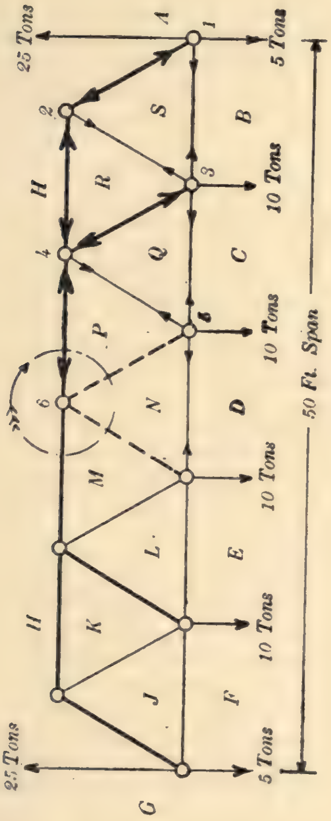

25 Tons
A
1
5 Tons
2
S
B
H
R
3
10 Tons
4
Q
C
P
5
10 Tons
50 Ft. Span
6
N
D
M
10 Tons
L
E
H
K
10 Tons
J
F
25 Tons
5 Tons
G

Fig. 166.

$1\frac{1}{2}$ tons per foot run; but to simplify matters this is neglected for the present. The joints of the lower boom, at each of which there is a cross girder, carry the entire load.

After transferring the distributed load to the joints, letter the frame diagram as shown in Fig. 166—Bow's system of lettering or notation being used—placing a capital letter between the line of action of each two external forces, and also in each triangular space. Then, in this system, the bar joining the points marked 1 and 2 is called the bar SH or HS; and so for the rest.

To construct the stress or force diagram (Fig. 167) choose a convenient scale, and set off the line of loads *abcdefg*. The upward reactions are then represented by the lines *gh* and *ha*, the sum of the upward forces being equal to the sum of the downward forces. Starting at a joint where only *three* bars meet, as that marked 1, draw a line through *b* parallel to BS, in Fig. 166, and another through *h* parallel to SH, so finding the *s* in Fig. 167. Then *bs* represents the total stress or force on the first bay of the lower boom, and *sh* that on the first diagonal bracing bar. The arrows show that BS is a tie, since it *pulls* from the joint, and SH a strut, since it *pushes* on the pin. The auxiliary diagram marked (1) clearly shows the first step of the construction.

Passing next to the joint marked 2, draw the horizontal *hr* and the diagonal *sr* to intersect it at *r*. In placing arrow-heads on the bars, take care to go round the joint in the watch-hand direction, and round the *triangle of forces* in the direction *srh*. We thus find that RH is a strut and SR is a tie.

Turning our attention now to the joint marked 3, we place opposing arrows on the bars already dealt with, and then complete the *polygon of forces bcqrs*, representing, in magnitude and direction, all the forces acting at the point in question. For the sake of clearness, detached force diagrams have been drawn for three of the joints, though unneceesary in practice. In the same way we might deal with every remaining joint; but as the loading is symmetrical, it is quite sufficient to draw one-half of the complete diagram of forces, the other being similar. The construction of the diagram for an unsymmetrical load has been left as an instructive exercise to the reader.

The magnitudes of the forces and the nature of the stresses on the several members are best given in the form of a table, as that on p. 198. This table is a valuable guide in proportioning the various members of the girders. Observe that the thrust on the upper boom and the pull on the lower boom increase from the ends of the girder to the middle, while the stress on the *end* diagonals is the greatest. In fact, a uniform statical load causes

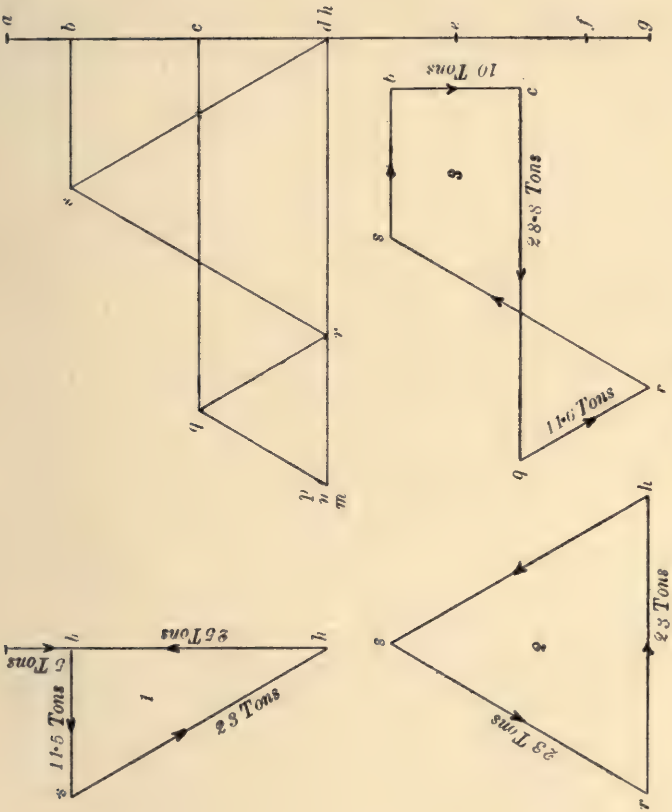

10 Tons
28·8 Tons
11·6 Tons
25 Tons
5 Tons
11·5 Tons
23 Tons
23 Tons
23 Tons

Fig. 167

no stress at all on the two middle diagonals, though a *travelling* load would.

It is worth noticing that the stress in any segment of either boom is equal to the sum of the horizontal components of the stresses in all the diagonals between that segment and the nearest abutment.

This important fact is evident from the force diagram, since, for example, cq is equal to the projection on it of $hs + sr + rq$. Or, we may put it this way:—

$$\begin{aligned} cq &= hs \cos.60^\circ + sr \cos.60^\circ + rq \cos.60^\circ. \\ &= 23 \times \tfrac{1}{2} + 23 \times \tfrac{1}{2} + 11{\cdot}6 \times \tfrac{1}{2}. \\ &= \tfrac{1}{2} \times 57{\cdot}6 = 28{\cdot}8, \end{aligned}$$

which verifies the graphical work.

Most writers denote a compressive stress by a plus (+) sign, and a tensile stress by a negative (−) sign. To remember this convention, we reflect that a *cross*-section is a good form for a strut, and that a flat bar makes a good tie.

——	Bar.	Force.	Stress.
		Tons.	
Upper boom . .	RH	23	Compressive
	PH	34·6	"
Lower boom . .	BS	11·5	Tensile
	CQ	28·8	"
	DN	34·7	"
Diagonals . . .	HS	23	Compressive
	SR	23	Tensile
	RQ	11·6	Compressive
	QP	11·6	Tensile
	PN	0	Neutral

In designing a braced girder for a bridge of small or moderate span, from few data, we first of all find the stresses in the several members, neglecting the structural weight entirely. The required dimensions of the parts are next approximately calculated and the weights estimated. The revised stresses, allowing for the girder's own weight, can then be found, and the scantlings increased as may be necessary.

This method is too slow, however, for general application. Bridge designers have available numerous practical rules and tables giving the approximate weights of all ordinary types of bridges, which much facilitate the work of designing and save a great deal of time. In the case of long-span bridges the weight of the structure itself forms so large a proportion of the total load to be borne that rules for determining the theoretical weight of the main girders are indispensable. In all cases such rules are very convenient when estimating the probable cost of a proposed bridge and comparing the economic merits of different designs. The interesting subject of the theoretical weight of bridges, however, is too large to be intelligibly treated here; and the reader who desires information upon it is referred to Fidler's extensive "Treatise on Bridge Construction," or to Baker's "Long-span Railway Bridges."

Lattice Girders.—The lattice girder is an important type of parallel braced girder. Its web consists of diagonal bars inclined at an angle of about 45°, and forming two or more systems of triangles. The girder diagrammed in Fig. 168 has four systems; one set of diagonals being thickened, to show this arrangement more clearly.

Fig. 168.

The perspective view Fig. 169 will give a good idea of the general appearance and construction of a light riveted lattice girder road bridge of rather longer span (200ft.) than usual, especially for so small a width as 14ft. The bridge illustrated was constructed by the Berlin Iron Bridge Company, Connecticut. A careful examination of the diagonals of one girder will show that there are *four* systems of triangles in the web. The manner of bracing together the two girders overhead should be noticed.

Fig. 170 is a very clear view of a riveted lattice girder bridge of 143ft. span, with a roadway 16ft. wide. This is an example of the so-called "single-intersection" type of web, in which there are only *two* systems of triangles. For medium spans this type of girder is very economical, and it is well adapted for a light highway bridge.

The distinction between a *lattice* and a *trellis* girder does not appear to be very clearly defined. The majority of engineers regard the two as identical. Others hold that a trellis girder differs from a lattice girder of the same span in having more numerous and lighter bracing bars, so as to form a comparatively close network.

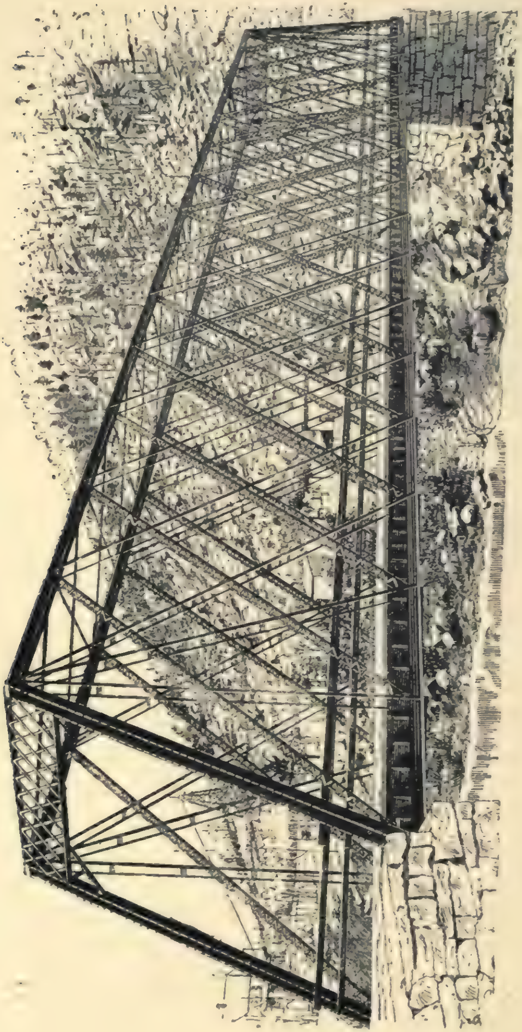


Fig. 169.

Fig. 170.

A convenient distinction is to say that a trellis girder has more than two systems of triangles.

The designers of the earlier lattice bridges considered it an advantage to use a great many bracing bars, riveted together at every intersection; this mode of construction enabling stock sections of iron and steel—such as channels and tees—to be used for the struts of quite large girders. This was the case in the *Blackfriars Railway Bridge* across the Thames, which was finished in 1864, and cost £28 per ton of ironwork. The superstructure of this bridge is arranged for four lines of rails. The extreme width over the girders is $53\frac{1}{2}$ft. There are *three* main girders, the centre one being double the strength of the others, and each girder has *three* sets of diagonals. The platform and tracks are carried by cross girders secured to the bottom flanges of the main girders. The central span is $202\frac{1}{2}$ft., the height of the main girders $15\frac{1}{2}$ft., and their width $4\frac{1}{2}$ft. The top and bottom flanges or booms are of trough section, with side plates 21in. deep and $\frac{3}{4}$in. thick, riveted by 5 by 5 by $\frac{3}{4}$in. angle-irons to the top and bottom tables, in which there are from one to four plates in the outer girders, and from one to five in the middle girder. The *ties* of the lattice work connecting the flanges are made of flat bar, decreasing from 10 by 1in. at the ends to 5 by $\frac{3}{8}$in. at the centre of the span. The *struts* are made of double channel irons, set some distance apart, and braced together by light zigzag bracing and pipe stays. They are riveted to the *inside* of the vertical trough plates, while the ties are riveted to the *outside*.

According to Professor Unwin,* the riveting of the struts to the ties of a lattice girder at every intersection certainly stiffens the bars against bending in the vertical plane, but at the same time the multiplication of the number of diagonals and the reduction of their section weaken them as regards transverse bending. Further, the smaller the section of the individual struts and ties, the greater is the proportionate waste of material near the centre of the girder, where there is no shearing-force; because the diagonals cannot in practice be reduced below certain fixed dimensions, partly owing to the necessity of providing a margin of thickness to allow for corrosion. For these reasons in the best practice the number of systems of triangles in the bracing is reduced as much as possible, the limit being the practical conditions that fix the largest size of a single strut.

The well-known *Charing Cross Bridge* across the River Thames is still a notable example of lattice girder work. It was designed by Sir John Hawkshaw, and completed in 1863; but since then it

* "Wrought-iron Bridges and Roofs," p. 72.

has been widened. The principal spans are 154ft. clear. Originally, over each span there were two main girders of the single-intersection type, each weighing 190 tons. The extreme depth of the girders is 14ft., and the effective depth $\frac{1}{12}$ of the clear span. Below the main girders are fixed cross girders, forming a platform carrying four lines of railway. Fig. 171 shows in section the top and bottom booms, which consist of five thicknesses of $\frac{5}{8}$in. plates, and four vertical ribs, which are strengthened round the pin holes by 1in. cover plates. The 1in. rivet holes were drilled. The diagonal *ties* have the form shown in Fig. 172. They vary in section from 12 by $2\frac{1}{2}$in. over the piers to 6 by 2in. at the middle of the span, where the shearing-force is least. The *struts* (Fig. 173) are built-up of forged bars, fastened together by bolts passing through

Fig. 171.

Fig. 172.

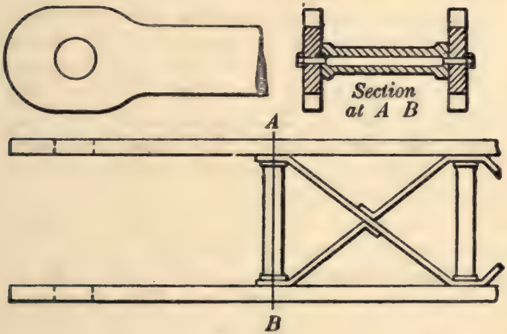


Fig. 173.

cast-iron distance-pieces, and by light diagonal bracing. The steel pins connecting the main booms to the diagonals vary in diameter from 5in. at the centre of the span to 7in. over the piers, where the shearing force is greatest. In addition to the diagonal struts and ties a series of vertical suspension bars connect the joints of the upper and lower booms, dividing each girder into fourteen panels. These transfer part of the load from the lower to the upper joint, but precisely what part is not easy to determine. The *end pillars* over the abutments are plate-work boxes, the sides being stiffened with T-irons. At the abutments, or shore supports, the girders rest on roller bedplates; over the piers they are carried by granite blocks. The greatest deflection at the middle of the girder permitted by the specification was $\frac{1}{500}$ of the span.

Plate Girders *versus* **Lattice Girders.**—It is interesting to compare the respective advantages and drawbacks of two types of girders much used in the construction of railway bridges—namely, the plate girder with one or two webs, and the lattice girder; a subject which in the past has been keenly discussed by eminent engineers.

Both Sir William Fairbairn and Robert Stephenson were distinctly in favour of plate girders, arguing that—

1. The plate girder is more rigid than the braced girder.

2. The continuous web assists the booms in resisting horizontal forces, and therefore adds to the strength of the girder.

3. The booms being of the same weight, whatever the type of girder, the only advantage of the braced girder over the plate girder must be a small saving in the weight of the web only.

In answer to these arguments, Professor Unwin * remarks: "If it were true that the economy of braced girders is confined to a fractional saving in the weight of the web, many engineers, admitting the more simple and homogeneous construction of the plate web, would consider its superiority established. But a saving of weight in any part implies that the dead load to be carried is lessened, and therefore involves a cumulative saving in the weight of every part of the girder. . . . The braced girder is not necessarily less rigid than the plate girder. The most important source of economy in the braced girder, however, is the greater *depth* which may be given to it. The depth of plate web girders is practically limited to from $\frac{1}{12}$ to $\frac{1}{15}$ of the span; but in braced girders a greater depth is constantly and successfully adopted. . . . In America a proportion of depth of $\frac{1}{8}$ the span in large bridges, and $\frac{1}{7}$ in small bridges, is a usual allowance. The stress on the booms, and consequently their sectional area, is in the inverse ratio of the depth; and the saving of weight in the booms much more than compensates for the increased weight of the web."

Again, according to Sir Benjamin Baker, the amount of metal in the shape of stiffeners required to prevent the *buckling* of the

* "Wrought-iron Bridges and Roofs," p. 82.

deep thin plates of a plate girder of large span is sufficient to form the struts of a lattice girder: so that the effective duty of a plate web is little more than the resistance offered to *tensile* forces. But these forces are more economically resisted by lattice bars than a solid plate; because the section of a bar can always be made proportional to the pull on it, whilst in the case of the plate web a certain minimum thickness of plate must be provided at all points.

On account of corrosion this limiting thickness of web plates is never less than ¼in.; and in places not easily accessible for painting, it should be fixed at ⅜in. Material is thus wasted throughout a great part of the length of the girder; and, as the influence of the web on the weight of a large plate girder is considerable, the economical depth of such a girder is less than that of a lattice girder designed to carry the same load across the same span. Hence the sectional area of the flanges of the plate girder needs to be made greater than the area of the flanges of the deeper lattice girder.

It therefore appears that the practical advantage of the lattice girder over the other for long spans is due to the greater *depth* that may be economically adopted in the case of the former, and not to the smaller quantity of metal needed to construct girders of *equal* depth, but different types. The lattice construction of web undoubtedly enables the section of the metal to be more closely proportioned to the varying stress than does a continuous plate web. And since this reduction of weight in the web somewhat lessens the total load on the girder, the weight of metal in the flanges must be to some extent affected by the design of web. But this saving is important only in the case of fairly long spans; for the weight of the girder itself, when the span is short, is but a small fraction of the gross load.

Stress Diagram for a Lattice Girder.—A skeleton diagram of a lattice girder of the single-intersection type is given in Fig. 174. Assuming the girder to be loaded with five unequal loads, acting at the joints shown, and the weight of the girder itself to be neglected, it is required to determine, by graphic construction, the total stresses in the several members.

After lettering the frame diagram as shown, find the reactions by taking moments about one end, thus:

$$\text{R} \times 40\text{ft.} = (2 \text{ tons} \times 40\text{ft.}) + (8 \text{ tons} \times 30\text{ft.}) + (6 \text{ tons} \times 20\text{ft.}) + (4 \text{ tons} \times 10\text{ft.}) + (2 \text{ tons} \times 0)$$

$$= (80 + 240 + 120 + 40) \text{ foot-tons.}$$

$$\therefore \text{R} = \frac{480\text{ft.-tons}}{40\text{ft.}} = 12 \text{ tons.}$$

Hence, $\text{R}_1 = 22 \text{ tons} - 12 \text{ tons} = 10 \text{ tons.}$

These reactions may also be found graphically by drawing a polar diagram and a polygon of bending moments, but the method is slower than that adopted.

Consider, now, the joint marked 1. Of the four forces acting

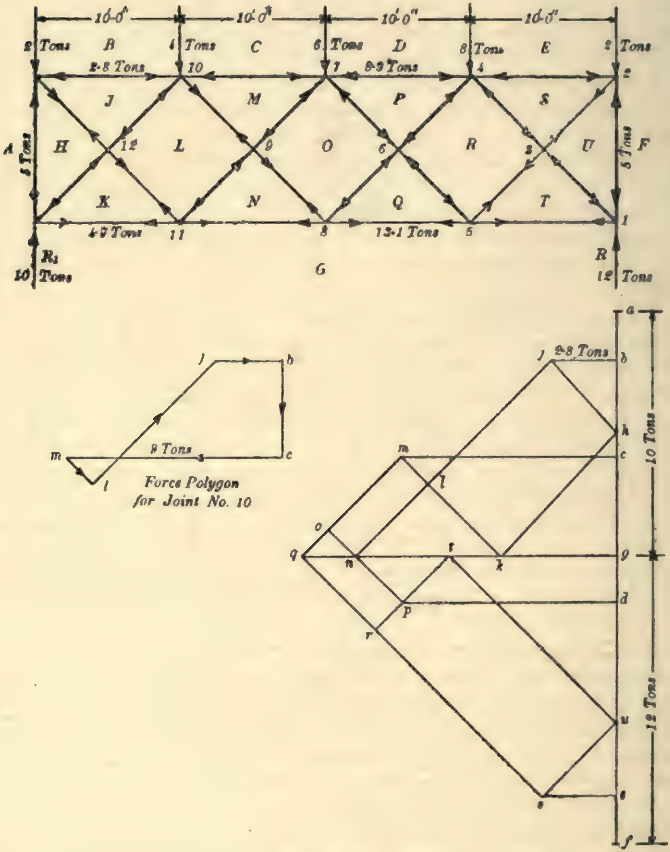


Fig. 174.

there, only one is known, viz., R; and as it is impossible to draw a polygon of forces when but a single force is known completely, and the directions of three others, we must try if some other joint is not easier to attack. On looking at all the other joints where a known force acts, however, the same difficulty is encountered. Some special artifice is therefore necessary to overcome the difficulty. We must turn our attention again to the joint marked 1, and as a preliminary step ascertain by general reasoning the thrust on the end pillar UF, before it will be possible to make any progress with the stress diagram.

Consider, then, the load BC of 4 tons. The assumption is often made that half of this force will be transmitted down each diagonal. But this assumption is hardly justifiable; for it by no means follows that because two jointed bars are equally inclined, therefore the stresses on the bars are equal. A much more reasonable procedure is the following:—

We know that the structure is rigid, because it cannot possibly deform without straining some of the bars. And since it is rigid, the part of any vertical load that is transmitted to either abutment is inversely proportional to the distance of the load from the abutment considered, just as if the beam were solid. Hence, of the 4 tons applied at joint No. 10, a force of 3 tons passes down the bar JL to the left-hand support, and 1 ton is transmitted down LM, up OQ, and finally down RS to the right-hand support. The stress is bound to be transmitted in this roundabout way, because a *vertical* force cannot be transmitted by *horizontal* bars, arranged as in Fig. 174. Thus we see that the vertical component of the stress in JL due to the 4-ton load is 3 tons, and in the bars LM, OQ, and RS, it is 1 ton.

Reasoning in exactly the same way, we find that the pressure on the right-hand support due to the 8-ton load is 6 tons, and on the left-hand support is 2 tons; also that the vertical component of the stress in RS due to this load is 6 tons, and in the bars QR, MO, and KL is 2 tons.

As regards the effect of the central load of 6 tons, it is plain that the vertical component of the stress due to it will be 3 tons in the diagonals OP and RT. Summing up, then, the left-hand end pillar UF takes no part of the force AB, no part of BC, 3 tons of CD, no part of DE, and 2 tons of EF, or a total of 5 tons. Similarly, AH takes 5 tons in all. This equality, however, is only a coincidence.

After the stress in *one* of the end pillars has been found in this way, the stress diagram can be completed in the ordinary manner; taking the joints in the order figured, and drawing parallels to the bars meeting at each joint. Then scale off the stresses and tabulate them, distinguishing between ties and struts. On some of the bars the stresses have been marked. A detached force polygon for one joint has also been drawn, to make the method of proceeding quite clear.

It is remarkable that the total stresses on the several members of a braced girder would not in the least depend on the actual lengths of the bars, if we could neglect their weights; but would depend only on the loads and the inclination of the bars, for a given general arrangement. In other words, the *scale* of the girder would not influence the magnitude of the forces; so that the same stresses would be found whether the span were taken to be 40 or 400ft. The explanation of this apparent paradox is that the depth of the girder, and therefore the length of the resistance arm (or the arm of the resisting couple) increases proportionally to the effort arm (or the arm of the bending couple), as the span is increased. But of course every one knows that, as a matter of fact, the stresses on the parts depend a very great deal on the span; because the parts have weight, and therefore greatly add to the load, at least in the case of long-span girders.

The Linville Girder.—We have now to deal with another type of parallel braced girder, diagrammed in Fig. 175, which has been largely adopted in bridge work. This type of girder has received a good many names, being variously known as the N girder, the Linville girder, the Whipple-Murphy girder, and the Pratt truss. At any rate, if there is any difference at all between the forms of girder so named, it is in comparatively insignificant details.

As shown in Fig. 175, the N girder consists mainly of a stiff upper boom adapted to resist a thrust, a bottom boom or chord which resists a pull, a number of vertical posts or struts, and two sets of oppositely-inclined diagonal ties, the inclination being about 45°. The special feature of this style of girder is that the struts are the shortest possible for a given depth of girder, and are therefore very economical. It is most suitable for medium spans.

When the Linville truss is intended to support a *rolling* load, the more central panels must be counterbraced, as indicated by the dotted lines. The extra bracing bars are introduced to avoid putting some of the diagonals in compression in certain positions of the rolling load, due to the reversal of the shearing force. Whether a given panel needs to be counterbraced or not depends upon the relative magnitude of the dead and rolling loads. When the dead load forms a large proportion of the total load, only a few of the central panels or bays need be counterbraced; but when the dead load is small compared with the rolling load, all the bays must be counterbraced save the two end ones.

The Linville truss or girder is suitable for either a "deck" bridge or a "through" bridge. It is also well adapted for use in a bridge of the "half-deck" type, an excellent example of which is represented in Fig. 176 (p. 210). This is an American single-track

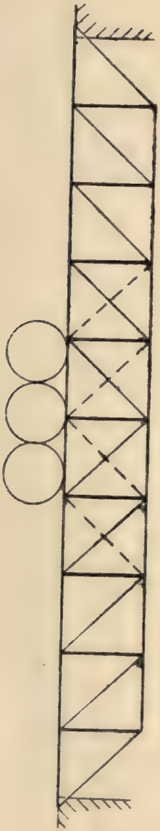


Fig. 175.

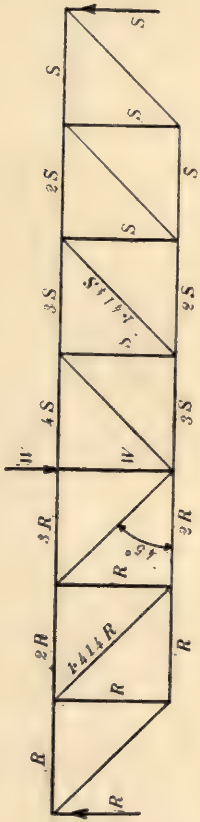


Fig. 179.

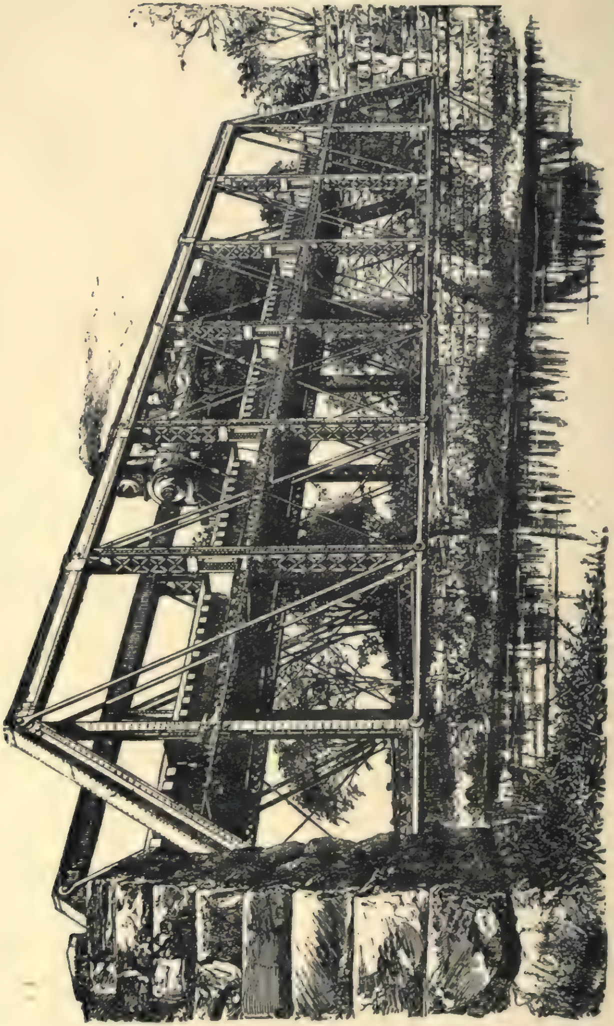


Fig. 176.

railway bridge, by the Berlin Iron Bridge Company. Notice the counterbracing of the four bays remote from the abutments, and the lightness of the linked tension chord.

Example.—A Linville girder of 60ft. span and 10ft. depth has

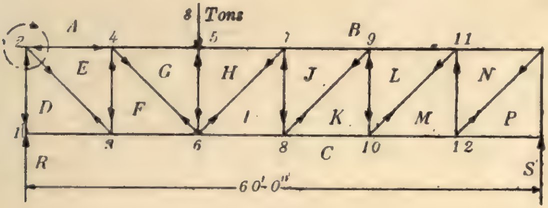


Fig. 177.

six equal spans, and is loaded with 8 tons, applied as shown in Fig. 177. It is required to draw the stress diagram.

(1) Calculate the reactions R and S thus:

$$R \times 60\text{ft.} = 8 \text{ tons} \times 40\text{ft.}$$
$$\therefore R = 5\tfrac{1}{3} \text{ tons,}$$
$$\text{and } S = 8 - 5\tfrac{1}{3} = 2\tfrac{2}{3} \text{ tons.}$$

A little thought will now make it plain that the thrust on the vertical post GH directly under the load must be 8 tons; also

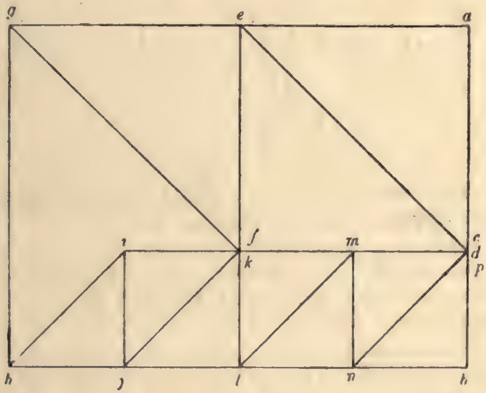


Fig. 178.

that the thrust on the post to the *left* of GH is $5\frac{1}{3}$ tons, and the thrust on the posts to the *right* of GH is $2\frac{2}{3}$ tons.

(2) Draw *ab* (Fig. 178) to represent 8 tons to a large scale, and

set off *bc* to represent $2\frac{2}{3}$ tons to the same scale. Measuring off on this load line a distance *ad* equal to $5\frac{1}{3}$ tons, *d* coincides with *c;* which informs us that there is *no stress* in DC, the first member of the tension chord. The same applies to the final member PC. Thus the girder is theoretically complete without these bars and the two end posts. But, for convenience of support, they are retained in the case of the girders of *through* bridges, though omitted from the girders of *deck* bridges, as in Fig. 175.

(3) Passing on now to the joint marked 2, draw *ae* parallel to AE, and *de* to DE, thus fixing *e*. Then *ae* represents the thrust in the first section of the boom, and *de* the tension in the first diagonal. Place the significant arrow heads on both ends of each bar, and then turn to the joint 3.

It would be tedious to follow all the joints through in detail, and it is quite unnecessary; as, when once fairly started, the process of constructing the stress diagram is very easy and quite mechanical. Fig. 178 shows the complete diagram, from which the stresses can be scaled off and tabulated, as under:

Top Boom.		Lower Chord.		Verticals.		Diagonals.	
Bar.	Tons.	Bar.	Tons.	Bar.	Tons.	Bar.	Tons.
AE	5·33	CD	0	AD	5·33	DE	7·54
AG	10·66	CF	5·33	EF	5·33	FG	7·54
BH	10·66	CI	8	GH	8	HI	3·77
BJ	8	CK	5·33	IJ	2·66	JK	3·77
BL	5·33	CM	2·66	KL	2·66	LM	3·77
BN	2·66	CP	0	MN	2·66	NP	3·77
				PB	2·66		

It is desirable to know how to find the stresses on the members of a Linville girder by calculation, as well as graphically. Take the simple case of a girder with a single load W placed unsymmetrically, as in Fig. 179 (p. 209). Let θ be the angle between the diagonals and the horizontal, R and S the reactions. Then the stress on each vertical post to the left of W is R, and on those to the right of W it is S. The stress on the diagonals to the left of W is R sec θ, and on those to the right is S sec θ. The stress on the first member of each boom is R tan θ, and on the last is S tan θ. On the second member of each boom the stress is 2R tan θ, and on the last but one 2S tan θ; and so on until the post below the load is reached. When θ is 45°, tan θ is 1 and sec θ is 1·414, the stresses becoming those indicated in Fig. 179.

Linville Girder with Duplex Bracing.—The skeleton diagram Fig. 180 shows in half-elevation a form of Linville truss with inclined terminal struts and duplex bracing, which is largely used in through bridges of long span. The counterbraces are shown dotted. Compared with the simple form of Linville girder of equal span, the number of diagonal and vertical bars is doubled, and the stress on each reduced to one-half. With the duplex bracing, a vertical section taken anywhere, as at AB, intersects one or more of the diagonals, and the tensile stress in the lower

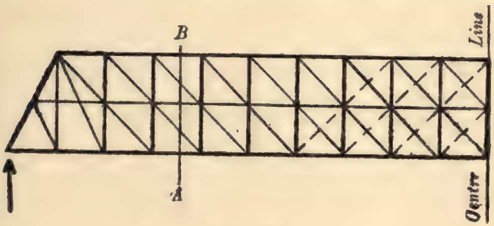


Fig. 180.

chord is less than the compressive stress in the upper boom by the sum of the horizontal components of the stresses in the diagonals cut through.

Ohio River Bridge.—Fig. 181 is an elevation in outline of a single-track railway bridge over the river Ohio, at Beaver, Pennsylvania; which will give a good idea of the application of the Linville girder or Pratt truss, and also serve to emphasise the distinction between a through bridge and a deck bridge. The length of the bridge proper is nearly 1378ft., and of the iron approach viaduct 1080ft. Beginning at the left-hand abutment B, there is first of all a short span of nearly 31ft., for which plate girders are used. The next span, of about 181ft., is bridged by a pair of Pratt deck trusses Y, fully counterbraced. The fifth and sixth spans Y are similar. The 440ft. between the piers CC is spanned by two double-intersection Pratt through trusses Z, partially counterbraced. This type of girder is also utilised for the fourth span Z. The approach viaduct consists of 36 spans, each of 30ft., for which plate girders are employed, supported by ironwork trestles.

An elevation of the longest Pratt truss, to a larger scale, is given in Fig. 182. It has twenty-one panels of 21ft. 1in. each, making a total length of 442ft. 9in. between the centres of the end pins. The depth of the girder or truss between the centres of the chords or booms is 42ft. 2in. The distance between the

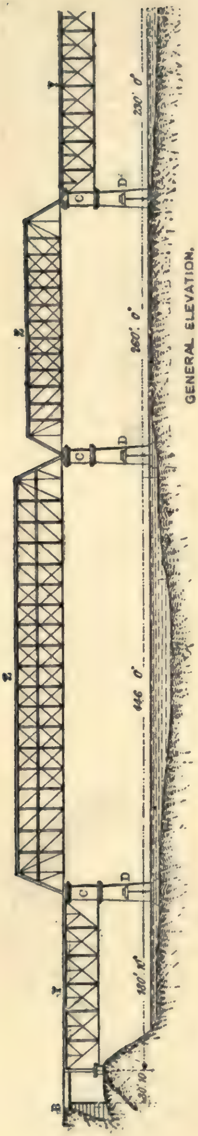


Fig. 181.

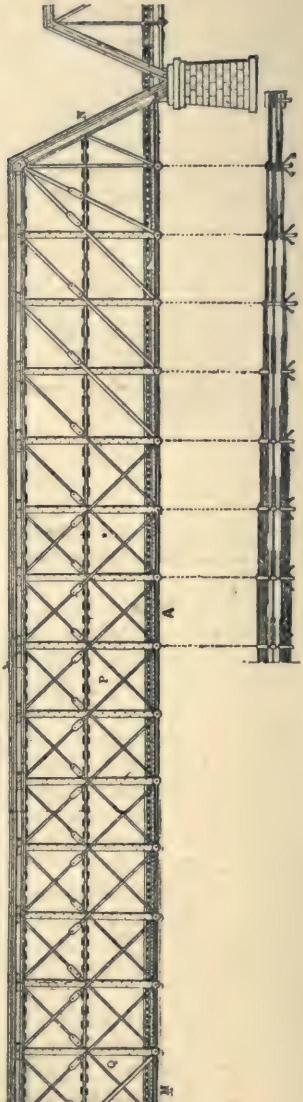


Fig. 182.

centres of the trusses is only 18ft., so that the bridge is quite narrow.

The *upper boom* of each main girder is built up of a top plate 42in. wide, with three lines of web plates 24in. deep, joined together and stiffened by eight lines of 4in. angle-irons. The *lower chord* is formed of 8in. links, with forged eyes for the reception of the pins.

Each *end-post* is inclined, and similar in construction to the upper boom. The intermediate posts or verticals are double, each half being made up of an 8in. beam, with plates varying from 12 to 9in. wide, riveted to each flange. All these posts are stiffened longitudinally at their centres by a strut which extends the full length of the girder from one end-post to the other. It consists of two 6in. channels, stiffened with straps.

The main tie-bars and the counterbracing-bars are in two lengths, each line of ties being joined together by means of two splice-plates and pins. The links composing the main ties vary from 6in. wide at the ends of the truss to 3in. wide at the centre of the span, where the shearing force is least. The counterbracing ties are square bars. All the pins in the lower chord are 6in. diameter; those in the upper boom vary in diameter from 6 to 4in., the centre ones being the smallest. The pins used for coupling the ties have each a diameter equal to at least three-fourths of the width of the bars coupled.

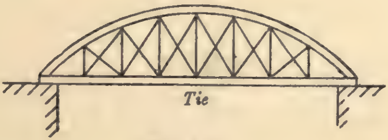


Fig. 183.

To allow for expansion the free end of the main span rests on eleven turned rollers 3in. diameter, set in a frame having a planed base $1\frac{1}{8}$in. thick. At one end of the truss the roller frame, and at the other end the baseplate of the post, rests on a wrought-iron deadplate 5ft. wide and $1\frac{1}{2}$in. thick, which extends the full width of the pier in order to receive the ends of the adjoining spans.

Bowstring Girders.—The essential features of an ordinary bowstring girder (Fig. 183) are:—A stiff parabolic arch, springing from two shoes, tied together by a horizontal tie which is slung from the arch by vertical bars. The resemblance to the archer's bow is obvious.

The *inverted* bowstring girder has the general form shown in

Fig. 184, the *lower* boom being arched, and the verticals struts. In both forms of girder the arched boom relieves the web of part of the shearing force. In fact, when the curve is a parabola, and the load uniformly distributed over the span, the entire shearing force at any section is balanced by the vertical part of the stress on the boom. In that case no diagonal bracing is necessary.

Fig. 184.

But when a bowstring girder has to carry a *rolling* load, the shearing force is no longer entirely transmitted through the arched boom, and one of three methods of bracing is adopted to give the necessary stiffness—viz., (1) vertical struts and diagonal ties (Fig. 183); (2) vertical struts and gusset stays; and (3) lattice bracing, as in Fig. 185.

Rankine, in his "Civil Engineering," page 563, says:—"The proper figure for the centre line or neutral curve of the bow is a parabola; but a circular segment is often used in practice. The cross-section of the bow, like that of the upper boom of a

Fig. 185.

lattice girder, must be of a form suited for resisting thrust. A cylindrical tube is the strongest form; an inverted trough shape, either cast or built of plates and angle-bars, is convenient for the attachment of the suspending pieces. These have usually an I-shaped section, with the greatest breadth transverse, to give them lateral stability; and for the same purpose they widen towards the bottom, where they are riveted to the ends of the plate or box beams that form the cross joists. The main tie is best made of parallel flat bars on edge, and is made fast to the shoes at each end by gibs and cotters. The diagonal braces are round or flat rods."

Apart from its more elegant appearance, one great advantage of a bowstring over a parallel girder is that the stress on its members is nearly uniform from the centre to the abutments. In a parallel girder it is difficult to distribute the material to the greatest advantage, because the flange stress varies so much. But in the bowstring construction, the stress being nearly uniform, there is little need to vary the section of the parts.

According to Unwin, all the bars and plates of a bowstring girder loaded uniformly might be made uniform in section from end to end of the span, without sensible waste of material; because the stress on the arched boom increases from the centre to the ends by only 8 per cent. when the ratio of span to depth is 10, and by 12 per cent. when that ratio is 8; while the tension on the horizontal boom and on the vertical ties is quite uniform. A bowstring girder costs more per ton than a parallel girder, however; a practical drawback that limits its adoption.

The stresses on the members of a bowstring girder loaded uniformly are readily calculated by the following simple formulæ:—

Let L feet be the effective span, H feet the effective depth, n the number of bays, and w tons the load per foot run. Then the thrust on the arched rib at the *centre* of the span, and also the uniform pull on the straight boom, is—

$$P = \frac{wL^2}{8H} \text{ tons.}$$

The thrust in tons at any point of the curved boom, at a horizontal distance of x feet from the centre, on a section normal to the curve, is—

$$\sqrt{P^2 + w^2x^2}.$$

The tension in tons on each of the suspending rods is—

$$\frac{wL}{n}$$

The Bowstring Suspension Girder is diagrammed in Fig. 186. The tie hangs in a catenary curve, and helps the bow to support the verticals. The light bracing is needed to transmit the load

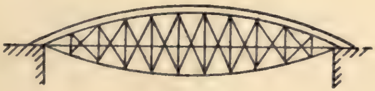


Fig. 186.

to the booms and to resist the distortion due to a rolling load. In Brunel's bowstring suspension bridge, over the Tamar at Saltash, with spans of 445ft., the bow is a wrought-iron oval tube, stiffened by transverse diaphragms, and the tie consists of a pair of chains. The depth of the girders is one-eighth of the span.

In a girder of this form it is possible to secure uniformity of stress on any normal section of the booms throughout the entire span, by adopting a suitable curvature. The horizontal thrust due to the weight of the bow or arch should balance the horizontal pull due to the weight of the chain.

The Bollman Truss.—Fig. 187 is the frame diagram of a Bollman bridge truss of three bays. This is an American form of girder, seldom seen in England. The span is divided into an equal number of bays by two, three, or more vertical struts, each of which is independently stayed by its own pair of tie-rods. Thus each load is supported by a separate triangular truss,

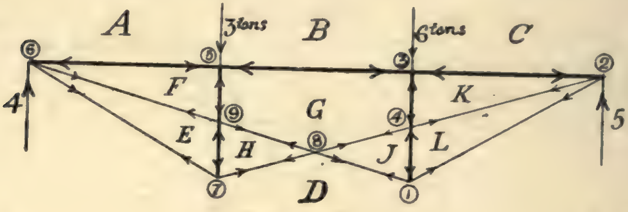


Fig. 187.

extending the whole length of the span. The top or compression boom is common to all the triangular elements, and the stress on it is the sum of the horizontal stresses induced by each loaded triangular frame. Usually the load is not concentrated at the joints, but distributed over part of the beam; and before the stresses can be found the case must be reduced to that of a truss loaded at the joints only. The loads on the struts may be found by supposing the beam to be pin-jointed at the points of junction with the struts.

One method of finding the stresses on the several members of a Bollman truss is to draw a separate stress diagram for each triangular frame, and then to add the results in order to get the total thrust on the top boom; which thrust, it should be observed, is uniform throughout the span. This is termed the method of *superposition*.

Another way is to construct a complete stress diagram, as in Fig. 188. To do this, letter the frame diagram as in Fig. 187, and calculate the reactions or supporting forces by the method of moments. The next move is peculiar. Neither of the supporting points can, for the moment, be successfully attacked; because four forces meet at each, and one force only is known. Start, therefore, at the lower joint marked 1, where only *three* forces act, of which the vertical one—namely, the six tons transmitted by the strut—is known. Draw *jl* to represent this force, and from its extremities draw the lines marked 2 and 3, their intersection

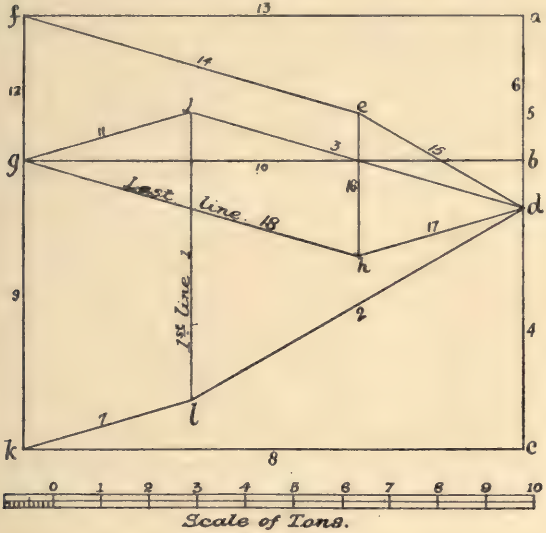


Fig. 188.

fixing *d*. Next set off on the vertical through *d* the known reactions, thus fixing *a* and *c*. The order of the remaining operations necessary to complete the stress diagram is clearly indicated by consecutive numbers.

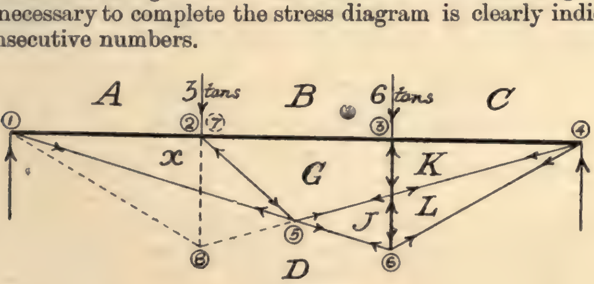


Fig. 189.

An alternative method of drawing the complete stress diagram for a loaded Bollman truss is to employ a special artifice known as a *substituted frame ;* a simpler arrangement of bars being substituted for the actual arrangement, with the object of facilitating the initial steps. The method is shown in Figs. 189 and 190. The dotted strut and ties are temporarily disregarded, and a

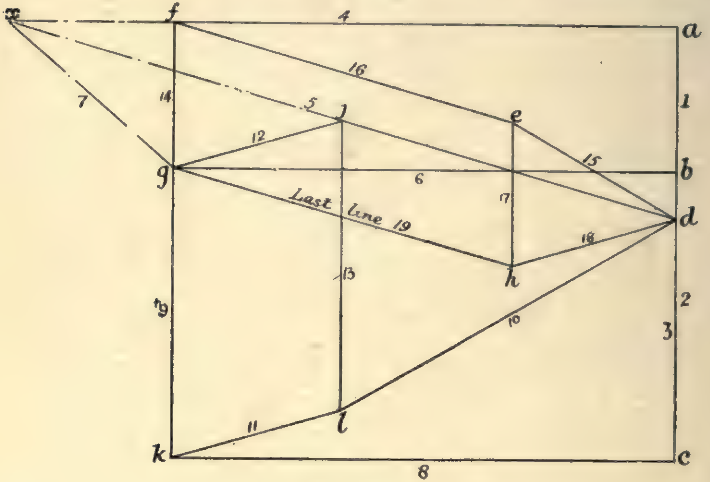


Fig. 190.

single inclined strut substituted. Then we can begin the stress diagram in the usual way by drawing the load-line and setting off the reactions. The point x is next found, and then g. The substituted frame is now dispensed with, having served its purpose. The stress diagram can be completed with ease, the steps being all numbered in the figures. A small freehand sketch will suffice

Fig. 191.

to represent the substituted frame, and the exact inclination of the extra bar can be taken from the original frame diagram.

The Fink Truss.—The object of combining several triangular trusses, as in the Bollman girder, is to support the roadway of a bridge at a sufficient number of points to prevent undue deflection of any part. The same end may be attained by the use of secondary trusses, as shown in Fig. 191, the whole forming a braced girder known as the Fink truss. In the United States, girders of this type have been constructed exceeding 200ft. in length. The usual number of divisions or bays is either eight or sixteen. Both the thrust on the central strut and the pulls on the long tie-rods are the same as if the secondary trusses were absent. The Fink truss hardly finds a place amongst modern engineering structures, and is practically obsolete. Indeed, on this side of the Atlantic examples are decidedly scarce. A good drawing of a long-span Fink truss, however, may be found in the article on "Bridges" in Spon's "Dictionary of Engineering."

Cost of Girders.—The cost of steel girders varies a good deal from time to time, according to the price of material and the current rate of wages. Thus in 1894 steel plates cost £5 5*s*. per ton, whereas in January 1900 the cost was £8 per ton. Still, a few rough figures may be useful.

Medium sections and lengths of rolled steel joists cost about £9 a ton, and heavy sections and long lengths about £1 a ton extra. Plain girder-work can be had at as low as £10 a ton, which is about the bottom limit. This includes the cost of erection. Lattice girders cost more per ton than plate girders, and bow-string girders still more. But girder-work seldom costs more than £16 a ton, including erection; unless the site is unusually inaccessible, and skilled labour very costly, as in the centre of India and Africa.

CHAPTER XVII.

THE STRENGTH OF COLUMNS.

THE *vertical* compression members of structures are variously styled columns, pillars, posts, struts, and stanchions; the term strut being also applicable to *inclined* members subjected to thrust. Crane jibs and sheer poles are examples of inclined struts. *Piers* are columns of great size and structural complexity, as a rule, used for supporting the superstructures of bridges. In architecture, every column consists of a *base* or foot; a *shaft*, which forms the main body; and a *capital* or head. The capital is often highly ornamented, and is distinctly characteristic of the style of architecture to which the building belongs. A column is distinguished from a pier by the shaft being either cylindrical or polygonal, and in but few pieces; a pier often consisting of a group of shafts substituted for a column. A *pilaster* is a square column, usually attached to a wall.

The least transverse dimension of a column is termed the *diameter* (*i.e.*, "measure across"), regardless of the shape. *Short* columns, or those whose length differs little from 8 to 12 diameters, according to the nature of the material, fail chiefly by crushing, but partly by bending. *Long* columns, or those whose length exceeds about 30 diameters, fail by bending; one side being crushed or "crippled," and the other torn asunder, as in the case of a beam.

The resistance of a long column depends on the material, on the ratio of its length to its diameter, on the shape and size of the cross-section, on the form and mode of supporting the ends, and on the direction in which the load acts with reference to the axis of the column.

Cast-iron and mild steel are the materials commonly used by engineers for columns. Timber struts are occasionally useful, as in the case of crane jibs, and for temporary purposes. Stone columns are less used than formerly in engineering structures.

Cast-iron pillars are most commonly of the hollow cylindrical form, where appearance is considered; but many have an H or -|-

section, these being cheaper to make. Rolled steel joists are often used for light columns where appearance is no object, as in boiler houses. The following table gives the calculated *breaking* loads in tons of rolled steel joists, when used as columns with the ends *fixed*. The *working* load should not exceed one-fourth the tabulated value for stationary loads, and one-sixth for moving loads :

Section.	Weight per Foot.	Length of Column.				
		8ft.	10ft.	12ft.	15ft.	20ft.
In.	Lbs.	Tons.	Tons.	Tons.	Tons.	Tons.
20 × 7½	89	607	563	514	435	309
18 × 7	75	508	467	423	352	245
16 × 6	62	394	346	301	230	155
15 × 6	59	377	333	292	226	152
14 × 6	57	356	321	283	225	150
14 × 6	46	293	259	224	171	115
12 × 6	54	350	321	283	222	153
12 × 6	44	286	260	230	182	124
12 × 5	32	188	160	131	99	63
12 × 5	39	226	189	155	115	73
10 × 6	45	292	267	236	185	127
10 × 5	35	203	172	143	105	68
10 × 5	29	173	149	124	90	60
9 × 7	58	397	370	338	283	201
8 × 6	35	228	208	183		
8 × 4	19	93	72	58		
7 × 3¾	16	73	55	44		
6 × 5	25	151	133	112		
6 × 3	13	44	32	24		
5 × 5	24	150	133	115		
5 × 3	11	43	32	24		
4 × 3	9½	37	27	20		
3 × 3	10	42	31	24		

Two joists, braced together by light diagonal lattice bars, make an excellent column. Figs. 192 to 199 show the usual forms of sections of riveted steel columns manufactured by Messrs. Dorman, Long and Co., Limited.

In Fig. 200 is shown an exceptionally heavy steel column, 32ft. 9in. long, to carry 400 tons. The section is worthy of note.

Gordon's Formula.—No entirely satisfactory formula has yet been proposed for calculating the greatest load that a given column will support. The best for practical use is probably that


Fig, 192.

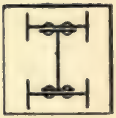


Fig. 193.

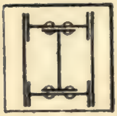


Fig. 194.

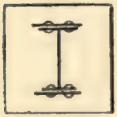


Fig. 195.

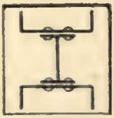


Fig. 196.

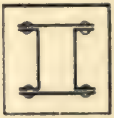


Fig. 197.

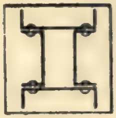


Fig. 198.

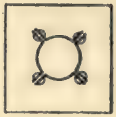
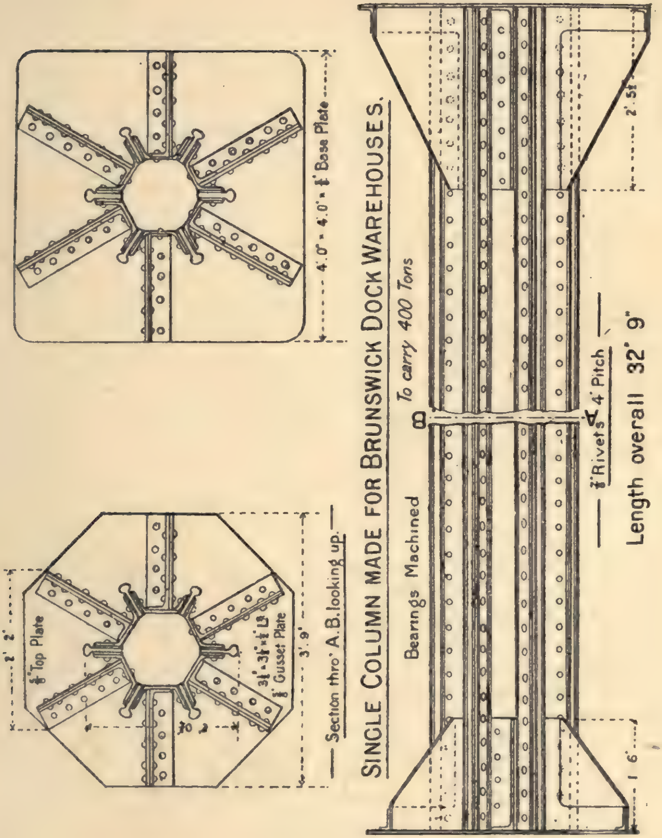


Fig. 199.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4' Pitch
Rivets
Section thro' A.B. looking up.
Top Plate
Gusset Plate
Base Plate
3' 9"
2' 2"
2' 5"
1' 6"

Fig. 200.

known as Gordon's, which was originally proposed by Tredgold. The values of the constants were deduced, however, by Gordon, from an analysis of Hodgkinson's experiments on columns. The formula in question, for columns with *flat* or *fixed* ends, is :—

$$P = \frac{fA}{1 + cr^2}.$$

Where

P = the axial breaking load (lbs. or tons).
f = the corresponding maximum stress.
A = the area of cross-section (square inches).
r = the ratio of length to diameter.
c = a constant, its value depending on the material and shape of section.

Gordon gave the following values :—

Material.	Section.	f Breaking Stress.	c.
		Lbs. per sq. in.	
Wrought-iron .		36,000	$\frac{1}{3000}$
Cast-iron . .		80,000	$\frac{1}{800}$
Cast-iron . .		80,000	$\frac{1}{400}$

For columns *rounded* or jointed at *both* ends, as connecting-rods, write $4c$ instead of c in the above formula.

For columns rounded at *one* end and fixed at the other, write $2c$ instead of c. Piston rods come under this head.

In designing a column, a probable value of the length-diameter ratio r must be provisionally assumed, and afterwards altered if found necessary.

The factor of safety should be about 6. In practice, however, it is better to deal with *working* loads and stresses than with *ultimate* or *breaking* loads, as in the convenient table on p. 227.

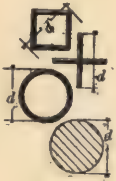
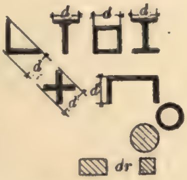
Example.—A hollow cylindrical cast-iron column, with well-spread flat ends, is 20ft. high and 12in. diameter, the metal being 1in. thick. What is the greatest live load it will safely carry?

Here f = 16,000lbs. per square inch.
$A = \pi \times 11\text{in.} \times 1\text{in.} = 34{\cdot}5$ sq. in.
$c = \frac{1}{800}$.
$r = 20\text{ft.} \div 1\text{ft.} = 20$.

$$\text{Hence } P = \frac{16{,}000 \times 34\cdot5}{1 + (\frac{1}{800} \times 20^2)}$$
$$= 368{,}000\text{lbs.} = 164 \text{ tons.}$$

If the length were increased to 40 ft., the column ought by Gordon's rule to stand 80 tons safely. But having regard to the possibility of inferior metal and flaws in casting, it would be advisable to keep well within that load.

VALUES OF *f* AND *c* FOR COLUMNS FLAT AT BOTH ENDS.

Material.	Shape of Section.	Safe Stress *f* for Live Load.	*c*.
Cast-iron.		Lbs. per. sq. inch. 16,000	$\frac{1}{530}$ $\frac{1}{800}$ $\frac{1}{800}$ $\frac{1}{400}$
Wrought Iron.		*f*. 7,000 6,000 6,000 6,000	*c*. $\frac{1}{900}$ $\frac{1}{3500}$ $\frac{1}{2250}$ $\frac{1}{3000}$
Mild Steel.		10,000	$\frac{1}{1400}$ $\frac{1}{2400}$

N.B.—Gordon's formula is strictly applicable only to columns of uniform section from top to bottom.

Hodgkinson's Rules.—From his own experiments on *cast-iron* columns or pillars, Hodgkinson deduced the following formulæ, applicable to *circular* sections only :—

Case I.—When the length is at least thirty times the diameter.

(*a*) For *solid* cylindrical pillars—

$$P = c \,.\, D^{3·6} \div L^{1·7}.$$

(*b*) For *hollow* cylindrical pillars—

$$P = c(D^{3·6} - d^{3·6}) \div L^{1·7}.$$

Where

P tons = breaking load, statical.
D in. = diameter of pillar, external.
d in. = diameter of pillar, internal.
L ft. = length of pillar.
c = a coefficient, whose values are as under:

Type of Pillar.	*c*.
Solid, with flat ends . . .	44·2 tons.
Hollow, with flat ends . . .	44·2 ,,
Solid, with rounded ends . .	15 ,,
Hollow, with rounded ends . .	13 ,,

Case II.—When the length is less than thirty times the diameter, the crushing load of the pillar is

$$P = \frac{b \times c}{b + \frac{3}{4}c}$$

where b tons is the breaking load of the pillar, calculated by the appropriate formula in Case I., and c is the crushing load of a short block of the same sectional area—viz.,

$$c = A \times 49 \text{ tons.}$$

Hodgkinson's formulæ are adapted for logarithmic computation only, on account of the fractional indices. A 3-figure log. table or a slide-rule will suffice.

Example.—To calculate the breaking load on a hollow, flat-ended, cylindrical column 12 in. diameter, 1 in. thick, and 40 ft. high, the material being cast-iron.

Here log. $D^{3·6} = 3·6 \log. 12 = 3·6 \times 1·079 = 3·88$;
and log. $d^{3·6} = 3·6 \log. 10 = 3·6 \times 1 = 3·6$;
and log. $L^{1·7} = 1·7 \log. 40 = 1·7 \times 1·602 = 2·724$.

Therefore, $D^{3·6} = 7600$; $d^{3·6} = 4000$, say; and $L^{1·7} = 530$. Hence, by Hodgkinson's formula,

$$P = \frac{44·2(7600 - 4000)}{530} = 300 \text{ tons.}$$

This is the "dead" or statical *breaking* load, allowing a factor of safety of 4, the *safe* dead load $= \frac{300}{4} = 75$ tons. By Gordon's formula the *breaking* load of this column is

$$P = \frac{80,000 \times 34\cdot5}{1 + \frac{1}{800} \times 40^2} \text{lbs.} = 410 \text{ tons.}$$

Thus, the two formulæ made use of do not, in this case, agree very well.

Timber Struts.—For the ultimate strength of oak and red-pine struts or posts, fixed at both ends, Rankine gives the rule—

$$P = 3,000,000 \left(\frac{h}{3}\right)^2 A.$$

Where

A sq. in. = sectional area.
h in. = least transverse dimension.
L in. = length.

Using a factor of safety of 10, the formula for *square* timber struts becomes

$$P = 300,000 \frac{h^4}{L^2}.$$

In the case of struts freely jointed at both ends, the strength is reduced to one-fourth. The resisting to crushing of *green* timber is only about half that of well-seasoned timber.

The breaking strength of timber struts may also be computed by Gordon's formula, f being taken as 7200lbs. per square inch, and c as $\frac{1}{250}$.

Euler's Formula.—The above formulæ are semi-empirical. Euler, however, investigated the strength of long columns mathematically, assuming perfectly elastic material, and deduced the following *rational* formula for the breaking load of a column *fixed* at both ends:—

$$P = 4\pi^2 \frac{EI}{L^2},$$

in which

P = breaking load (pounds).
π = 3·1416.
E = modulus of elasticity of the material (pounds per square inch).

I = least moment of inertia of the section about an axis through its centre of area (quartic inches).

L = length of column (inches).

For a column *rounded* at both ends take P = $\frac{1}{4}$ the above value. The formula is to be applied only when the ratio $\frac{L}{h}$ or $\frac{L}{d}$ exceeds the values given below, h being the least side of a rectangular section and d the diameter of a circular section :—

Case.	Material.	$L \div h$.	$L \div d$.
Both ends rounded	Wrought iron	28	24
	Cast-iron	11·5	10
	Wood	13·5	11·5
Both ends fixed	Wrought iron	56	48
	Cast-iron	23	20
	Wood	27	23

Experiments on the strength of cast-steel columns are wanted.

Example.—A mild steel column, fixed at both ends, is 50ft. high, 12in. diameter, and $\frac{1}{2}$in. thick. Find the statical breaking load.

Here E = 30,000,000lbs. per square inch.

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(12^4 - 11^4) = 300\text{in.}^4$$

$$L = 50 \times 12 = 600\text{in.}$$

Hence, by Euler's formula,

$$P = \frac{4 \times 9{\cdot}85 \times 30{,}000{,}000 \times 300}{600 \times 600}\text{lbs.} = 440 \text{ tons.}$$

The *safe* load will be about $\frac{440}{5}$ = 88 tons.

Conclusion.—For further information on the strength and design of columns and struts, the reader should consult Professor T. Claxton Fidler's "Treatise on Bridge Construction," chaps. x. and xi.: where it is shown that the strength of columns cannot be defined by any hard-and-fast line, even when the average elasticity of the column and the ultimate strength of the material are accurately known; but that the strength may have any value less than that of the ideal column within certain limits. Hence the strength of columns must be represented by an area within which the results of individual experiments will fall at haphazard. In Fig. 201, for example, the upper limit is the ideal or Euler

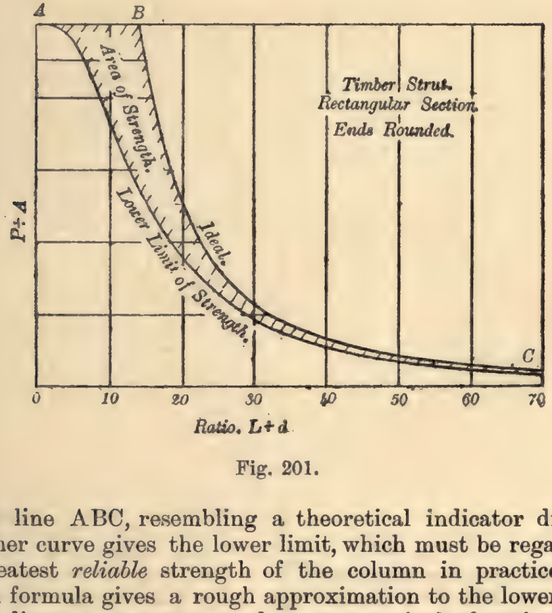


Fig. 201.

column line ABC, resembling a theoretical indicator diagram. The other curve gives the lower limit, which must be regarded as the greatest *reliable* strength of the column in practice. The Gordon formula gives a rough approximation to the lower curve, when ordinates represent pounds per square inch of sectional area, and abscissæ represent the ratio of length to diameter.

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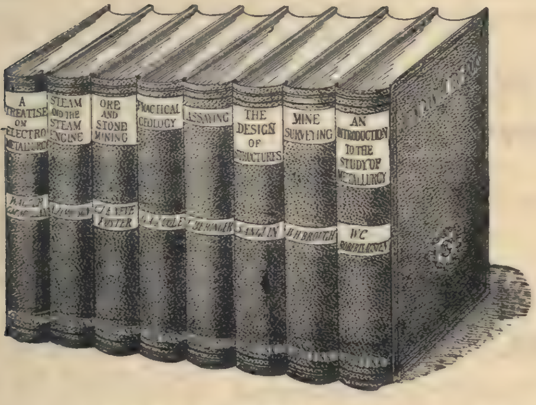

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