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INVESTIGATION OF COST VARIANCES: A CASE OF
MULTIPLE-STEP INVESTIGATIONS

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Introduction

The investigation of cost variances has been the subject of several papers[1] in recent years. These papers develop models that lead to a decision as to whether or not to investigate a process after a cost variance is observed. The investigation is usually conceived as a single-stage investigation process with a constant cost; in other words, the decision does not allow for a choice among alternative investigative procedures.[2]

Yet such a choice is apt to be necessary in practice. Accounting variances arise from a multiplicity of causes. When a variance is observed, it is reasonable to suppose that the structure of any insuing investigation will be influenced by available information about the likelihood of the various possible causes of the variance and about the costs and savings associated with discovery and correction[3] of each variance-causing condition.

The purpose of this paper is to consider a multiple-stage investigation process in which the decision involves a choice among several investigative techniques and a determination of the order of their application.[4] Although the complexity of the decision rule increases with the complexity of the investigative process, the decision rule may be operated by a simple graphical technique.

The Single-Step Investigation

A process is in one of two states; it is either <u>in control</u> or <u>out of control</u>. If the process is in control, then corrective action cannot improve its operation. If it is out of control, then corrective action will secure certain benefits. Unfortunately, we cannot know with certainty which state actually exists unless we investigate and incur the attendant costs. Clearly we do not wish to investigate a process when the cost of doing so exceeds the benefits obtained thereby. Since the state of the process is uncertain before investigation, we formulate a decision model that leads to a choice between investigating and not investigating so as to maximize the net expected benefits.

Prior to the decision, we formulate probabilities about the process being in or out of control and about the probability of observing variances of different sizes given that the process is in control and given that it is out of control. $P_o(IC)$ designates the prior probability that the process is in control and $P_o(OC) = 1 - P_o(IC)$ is the probability that the process is out of control. $P(X;IC)$ and $P(X;OC)$ designate the conditional probabilities of observing a variance X given that the process is in control and given that the process is out of control, respectively.

When a variance, X, occurs, we revise our prior probabilities according to Bayes Theorem as follows:

$$P_n(IC) = P_n(IC; X) = \frac{P(X; IC) \, P_o(IC)}{P(X;IC) \cdot P_o(IC) + P(X;OC) \cdot P_o(OC)}$$

$$P_n(OC) = 1 - P_n(IC)$$

where $P_n(IC)$ is the revised probability that the process is in control given an observed variance X. Notice that three probabilities are required to reach the revised estimates—the prior probability that the process is in control, $P_o(IC)$; the conditional probability of a variance X when the process

is in control, $P(X;IC)$; and the conditional probability of a variance X when the process is out of control, $P(X;OC)$.

Having revised the probability that the process is in control, we proceed to calculate the expected value of the actions open to us. In the single-stage investigation problem only two actions are available-- investigate or do not investigate. The loss matrix associated with this simple decision is given below:

Action	States	
	In Control (IC)	Out of Control (OC)
Investigate (I)	C	C
Do not Investigate (NI)	0	L

The table shows the outlay costs associated with each combination of action and state. Investigation incurs a known constant cost, C, and an out-of-control process occasions a constant loss, L, which is avoided if an investigation is undertaken. The decision maker wishes to choose between investigating and not investigating so that he minimizes the expected outlay cost. The expected cost of investigation, $E(I)$, is the sum of two products: (1) the cost of investigation multiplied by the probability that the process is in control and (2) the cost of investigation multiplied by the probability that the process is out of control, that is,

$$E(I) = C \cdot P(IC) + C \cdot P(OC)$$

The expected cost of not investigating, $E(NI)$, is the product of (1) the loss and the probability that the process is out of control, that is,

$$E(NI) = L \cdot P(OC)$$

The decision maker calculates E(I) and E(NI) and chooses the action with
the smaller expected cost. Alternatively, the decision rule may be formulated
as a test of the revised probability that the process is in control. The
expected cost of investigating will be less than the expected cost of not
investigating only if the revised probability that the process is in control
is greater than the ratio (C - L) / L. This is the familiar result obtained
by Bierman, Fouraker and Jaedicke and by Dyckman.

Multiple-Step Investigations

The single-step investigation model reviewed above contemplates a fixed
set of investigative procedures with a constant cost. In practice, however,
it is unlikely that the search for causes of an observed deviation of variance
exhibits such simplicity. It is probable that different variance-causing
conditions call for different investigative procedure investigation and that
the decision is not merely whether or not to investigate, but also how to
investigate.

The development of a multiple-step investigation model requires the
partitioning of investigative procedures into groups or steps containing
only inseparable elements--procedures that must be performed together.
As in the single-step model, if a step is undertaken the investigative
activity is completely specified, and exactly the same procedures are
applied regardless of the occasion on which the step is undertaken. It is
therefore, reasonable to view the cost of each step as more or less constant.

The multiple-step model considered here constrains the relationship
between time and conditions that dislodge a process from control. No more
than one such condition may be operative in any given period. Such a
constraint simplifies the investigation because once the condition is

discovered, we need to look no further. On the other hand, if several

conditions may combine to effect an out-of-control state, then the in-

vestigation must determine the extent of the variance attributable to a

particular condition in addition to the presence or absence of the condition.

Moreover, the presence of a condition alone may not bring loss of control

although its presence in combination with other conditions will bring loss

of control. Cases of this complexity are beyond the scope of this paper.

Each stage of investigation is directed toward the discovery of a

well-defined condition whose presence creates an out-of-control situation

and causes a known loss. The outcome of each stage is binary in the sense

that the condition is found to be present or absent. Moreover, the in-

vestigative techniques are assumed to be error-free, which precludes an

erroneous conclusion as to the presence or absence of the condition under

investigation. Finally, the corrective procedures which follow discovery

of such conditions are assumed to be effective in avoiding the known loss.

The revision of probabilities, after the observation of a variance

X, in the presence of several out-of-control states, requires an application

of Bayes theorem. In general if the process is in one of J states, S_i, we

revise each of the J prior probabilities, $P_o(S_i)$, according to Bayes Theorem

as follows:

$$P_n(S_i) = P_n(S_i;X) = \frac{P(X;S_i) \cdot P_o(S_i)}{\sum\limits_{j=1}^{J} P(X;S_j) \cdot P_o(S_j)} \quad , \quad i = 1,2,\ldots,J$$

In the cases examined here, three states are possible: (1) an in-control

state, (IC); (2) an out-of-control state owing to the presence of condition A,

(OCA); and (3) a second out-of-control state owing to condition B, (OCB).

After observing a variance X, the three prior probabilities are revised as

follows:

$$P_n(IC) = \frac{P(X;IC) \cdot P_o(IC)}{P(X;IC)P_o(IC) + P(X;OCA)P_o(OCA) + P(X;OCB)P_o(OCB)}$$

$$P_n(OCA) = \frac{P(X;OCA) \cdot P_o(OCA)}{P(X;IC)P_o(IC) + P(X;OCA)P_o(OCA) + P(X:OCB)P_o(OCB)}$$

$$P_n(OCB) = 1 - P_n(IC) - P_n(OCA)$$

The revised probabilities are conditional on the observation of a variance X and require prior knowledge of the conditional density functions of X given each of the three possible states as well as prior estimates of the probability that each state occurs.

The Case of Two Investigative Steps Unconstrained as to Combination or Order of Application

The allowable combinations of investigative steps and the order in which they are applied may be constrained in many different ways. For example, Dyckman[5] considers a case in which two investigative procedures---an experatory investigation and a full investigation--are alternative to one another and may not be applied together in any order. The case considered here imposes no constraints as to the allowable combinations of procedures or as to the order of their application.

When the two investigative steps may be undertaken in any order, the decision entails a choice among five alternative actions: (1) do not investigate, (NI); (2) investigate for condition A only, (IA); (3) investigate for condition B only, (IB); (4) investigate first for condition A, then for condition B, (IAB); and (5) investigate first for condition B, then for condition A, (IBA). The loss associated with each action under the three possible states of the process is given in Table 1.

If the objective is to minimize the expected loss, then the decision requires a determination of the expected loss of each action (as given in the far right-hand column of Table 1) and a selection of the action with the smallest expected loss.

The selection of the minimum-loss action requires at most ten pairwise comparisons of the expected values of the five alternative actions. Each of the ten comparisons may be represented by an inequality that specifies the choice of one action over another. For example, the action NI (no investigation) is preferred to the action IA (investigate for A only) when E(NI) is less than E(IA), that is, when

$$L_a P_n(A) + L_b P_n(B) < C_a + L_b P_n(B)$$

or, equivalently,

$$L_a \cdot P_n(A) - C_a < 0.$$

If the inequality is satisfied, then NI is preferred to IA and IA (and the comparisons involving IA) may be eliminated from further consideration. If $L_a \cdot P_n(A) - C_a = 0$, then we are indifferent between IA and NI. On the other hand, if $L_a \cdot P_n(A) - C_a > 0$, then IA is preferred to NI and NI is eliminated from further considerations. We then proceed to compare the remaining (non-eliminated) pairs of expected values in a similar way. Eventually a preferred action will be indicated. The ten pairwise comparisons of expected values are given in Table 2. Although this procedure is somewhat tedious it may be simplified by a simple graphical method.

TABLE 1.—Loss Matrix for Case of Three Investigative Steps
where the Order of Their Application is Unconstrainted

Actions	States			Expected Value
	IC	OCA	OCB	
NI	0	L_a	L_b	$E(NI) = L_a P_n(A) + L_b P_n(B)$
IA	C_a	C_a	$C_a + L_b$	$E(IA) = C_a + L_b \ P(B)$
IB	C_b	$C_b + L_a$	C_b	$E(IB) = C_b + L_a \ P(A)$
IAB	$C_a + C_b$	C_a	$C_a + C_b$	$E(IAB) = C_a + C_b [1 - P_n(A)]$
IBA	$C_a + C_b$	$C_a + C_b$	C_b	$E(IBA) = C_b + C_a [1 - P_n(B)]$
Probability	$P_n(IC)$	$P_n(A)$	$P_n(B)$	

TABLE 2.—Inequalities Leading to a Minimum Expected Cost Action

Preference	Inequality Condition
NI over IA	$L_a P_n(A) - C_a < 0$
NI over IB	$L_b P_n(B) - C_b < 0$
NI over IAB	$(L_a+C_b) P_n(A)+L_b P_n(B)-C_a-C_b < 0$
NI over IBA	$L_a P_n(A)+(L_b+C_a)P_n(B)-C_b-C_a < 0$
IA over IB	$L_b P_n(B)-L_a P_n(A)+C_a-C_b < 0$
IA over IAB	$L_b P_n(B)+C_b P_n(A)-C_b < 0$
IA over IBA	$(L_b+C_a) P_n(B) -C_b < 0$
IB over IAB	$(L_a+C_b) P_n(A) -C_a < 0$
IB over IBA	$L_a P(A) + C_a P(B) -C_a < 0$
IAB over IBA	$C_a P(B) -C_b P(A) < 0$

Illustration of Graphical Procedure

Since the costs and avoidable losses associated with investigation are known constants, each pair-wise comparison of expected values represents a partition of the probability space which gives all possible combinations of $P_n(A)$ and $P_n(B)$. If the space is partitioned for each of the ten comparisons, each resulting region of the space identifies pairs of $P_n(A)$ and $P_n(B)$ that signal a unique least-cost action. Once the graph is determined, the choice among the five alternative actions requires only the calculation of $P_n(A)$ and $P_n(B)$ and the location of the region on the graph with the pair of probabilities is associated.

A numerical illustration will clarify the application of the model and the graphical procedure. Suppose that condition A leads to a loss of $100 and investigation for condition A costs $50; condition B leads to a loss of $300 and investigation for B costs $100. The graph in Figure 1 is derived from this data by a simple procedure. First the data is substituted into each of the ten inequalities given in Table 2 and the corresponding boundary lines are drawn on the graph. Each inequality in Table 2 gives rise to a line of the graph. For example, the line specifying the preference relation between the actions IA and IB is given by the equation $300P(B) - 100P(A) - 50 = 0$ or, equivalently, $6P(B) - 2P(A) - 1 = 0$. Probability pairs above this line indicate a preference for action IB over IA while pairs below indicate a preference for IA over IB. The other lines are obtained from the remaining nine inequalities in a similar way.

Next, each cell in the resulting partition of the probability space is identified with the least-cost action. The performance of this second step is straight-forward, but its rationale requires some explanation.
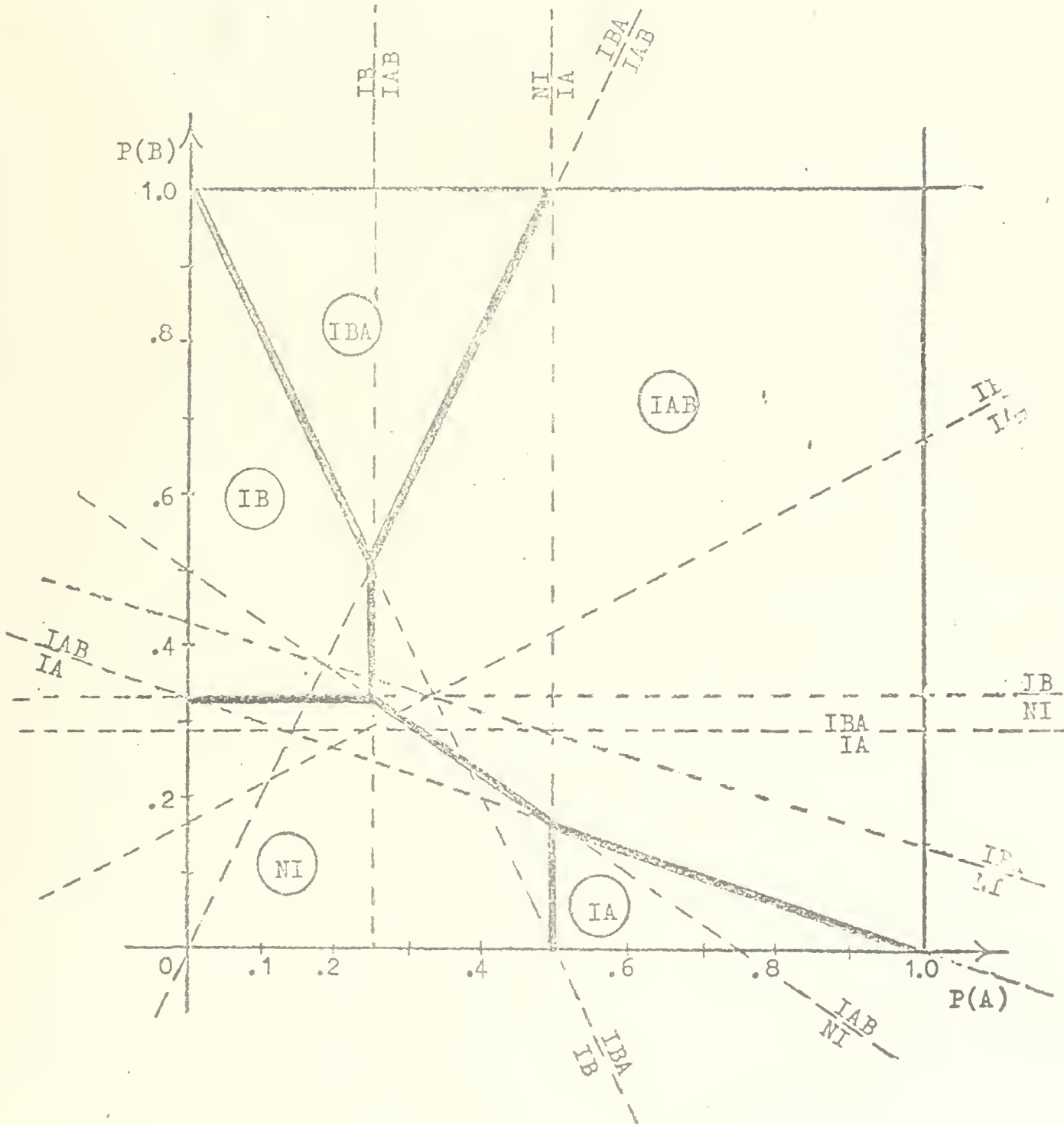
FIGURE 1

In general, we will not investigate if $P_n(A)$ and $P_n(B)$ are sufficiently small. Clearly, we will not investigate if $P_n(A)$ and $P_n(B)$ are both equal to zero which corresponds to the origin. Nor will we investigate in those cells whose boundaries include the origin. As we move to other regions whose boundaries do not adjoin the origin, we still do not investigate unless a boundary is crossed that signals an investigation of some kind. When such a boundary is crossed, all points within the region entered signal the kind of investigation designated as we cross the boundary. In general, moving from any region in which the decision is known, to an adjacent region, leaves the decision unchanged unless the shared boundary involves the decision of the first region; if it does, the decision is changed to the other action involved.

In other words, the second step in generating the graph begins by locating the cells that adjoin the origin of the probability space and by designating the probability pairs contained therein as signaling no investigation. We proceed from these "no investigation" regions to adjacent regions and, then, to other adjacent regions until every region of the probability space has been identified with a least-cost action according to the following rule: as one moves from any region in which the least-cost action is known, to an adjoining region, determine whether or not the pair of actions associated with the boundary crossed includes the least-cost action of the region from which one moves. If the pair does not include the action signaled in the first region, then the least-cost action of the second region is the same as that of the first. If the pair does include the action signaled by the first region, then the least-cost action of the second region is the other member of the pair. In this way every cell of the partitioned probability space is identified with a least-cost action.

After construction of the graph, the decision rule may be implemented by merely calculating $P_n(A)$ and $P_n(B)$, plotting the corresponding point on the corresponding point on the graph, and taking the least-cost action identified with the region within which the point falls. For example, if the revised probabilities, $P_n(A)$ and $P_n(B)$, are found to be 0.6 and 0.4, respectively, then the action IAB is signaled.

Summary

The complexity of the investigation process has an important impact on the decision rule in models for the investigation of cost variances. In general, the more complex the representation of the investigation process, the more complicated the decision rule. In some instances, complexity is an impediment to the application of a model despite the fact that it provides a better fit to reality. In the case examined here, however, a simple graphical procedure permits a straight-forward application of the model despite the complication introduced in the decision rule.

FOOTNOTES

[1]See, for example, H. Bierman, L. E. Fouraker, and R. K. Jaedicke, "A Use of Probability and Statistics in Performance Evaluation," The Accounting Review, XXXVI (July, 1961), 409-417; and T. R. Dyckman, "The Investigation of Cost Variances," Journal of Accounting Research, VII (Autumn, 1969), 215-244.

[2]Dyckman (1969) considers an exception in the form of a three-action model for the purpose of choosing between an exploratory investigation, a full investigation and no investigation. The full investigation is assured of correcting an out-of-control state, if one exists, but the exploratory investigation may fail to do so with known probability. This analysis differs from the analysis in this paper in that the two investigative procedures are strictly alternative to another and cannot be undertaken together. See pp. 228-230.

[3]The analysis presented here assumes that the cost of correcting an out-of-control condition, over and above the cost of the investigation leading to its discovery, is negligible. A cost of correction can be incorporated into the analysis without great difficulty. See Harold Bierman, Jr. and Thomas R. Dyckman, Managerial Cost Accounting. New York: The Macmillan Company, 1971, pp. 33-53.

[4]Bierman and Dyckman (1971) suggest this among other possible extensions of the analysis. See p. 52.

[5]Dyckman (1969), pp. 228-230.