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KEYNESIAN UNEMPLOYMENT, WICKSELLIAN INTEREST,  
AND NEOCLASSICAL GROWTH -- A SYNTHESIS

Pans Brems

#396

**College of Commerce and Business Administration**  
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KEYNESIAN UNEMPLOYMENT, WICKSELLIAN INTEREST, AND NEOCLASSICAL GROWTH  
———A SYNTHESIS

By Hans Brems  
University of Illinois at Urbana-Champaign 61801

100-word summary:

The paper tries to open the neoclassical growth model to Keynesian unemployment. First it uncovers the investment function inherent in neoclassical growth models. Second it uses the investment function uncovered to solve for a Wicksellian equilibrium rate of interest, i. e., a rate which equalizes saving and investment under steady-state growth. Third it shows that the rate thus found is the same for any nonzero level of physical output. To carry the economy to a steady-state full-employment growth path, monetary authorities may have to reduce the interest rate below its Wicksellian equilibrium, temporarily knocking the economy off its steady-state growth.



April 25, 1977

KEYNESIAN UNEMPLOYMENT, WICKSELLIAN  
INTEREST, AND NEOCLASSICAL GROWTH  
—A SYNTHESIS

By Hans Brems<sup>1</sup>  
University of Illinois at Urbana-Champaign 61801

1. PURPOSE

Keynesian short-run models and neoclassical growth models are different worlds. A Keynesian model knows unemployment, a neoclassical one doesn't. To Keynes, money mattered but mattered via investment and the rate of interest. No investment function is visible in neoclassical growth models——indeed it is widely believed to be absent.

In three steps the present paper will try to open the neoclassical growth model to Keynesian unemployment.

The first step is to uncover the investment function inherent in neoclassical growth models. That function will turn out to be general enough to encompass as special cases Wicksellian, Keynesian, post-Keynesian, and monetarist investment functions.

The second step is to use the investment function uncovered to solve for a Wicksellian equilibrium rate of interest, i. e., a rate of interest which equalizes saving and investment under steady-state



growth. In good accordance with what we have learned from Böhm-Bawerk, Wicksell, Fisher, and Schumpeter, that equilibrium rate will be found to be in direct proportion to the elasticity of output with respect to capital stock, in direct proportion to the rate of growth of output, and in inverse proportion to the propensity to save.

The third step is to show that the equilibrium rate of interest thus found does not guarantee full employment. On the contrary, to carry the economy to a steady-state full-employment growth path, monetary authorities may well have to reduce the rate of interest below its equilibrium level, temporarily knocking the system off its steady-state growth.

The neoclassical steady-state growth setting of the entire paper will be the usual one [11], [2], Ch. 5, [3] with only one modification: Labor employed will be the proportion  $\lambda$  of available labor force, where  $0 < \lambda \leq 1$ . At first  $\lambda$  is assumed not to be a function of time. Later  $\lambda$  is allowed to vary with time. With this modification, entrepreneurs produce the usual single good from labor and the usual immortal capital stock of that good. Capital stock is the



result of accumulated saving under an autonomously given propensity to save. Technology and available labor force are growing autonomously. The production function permits substitution between capital stock and labor.

Since economists possess no simple model of collective bargaining, we shall assume the money——but not the real——wage rate to be growing autonomously. The price of the single good is a variable adjusting, under profit maximization, to the autonomously growing money wage rate.

## 2. NOTATION

### Variables

$C$   $\equiv$  physical consumption

$g_v$   $\equiv$  proportionate rate of growth of variable  $v \equiv \kappa, P, S,$  and  $X$

$I$   $\equiv$  physical investment



$k$   $\equiv$  present gross worth of another physical unit of capital stock  
 $\kappa$   $\equiv$  physical marginal productivity of capital stock  
 $L$   $\equiv$  labor employed  
 $\lambda$   $\equiv$  the proportion of available labor force which is employed  
 $N$   $\equiv$  present net worth of entire physical capital stock  
 $n$   $\equiv$  present net worth of another physical unit of capital stock  
 $P$   $\equiv$  price of good  
 $r$   $\equiv$  nominal rate of interest  
 $\rho$   $\equiv$  real rate of interest  
 $S$   $\equiv$  physical capital stock  
 $X$   $\equiv$  physical output

Parameters

$\alpha, \beta$   $\equiv$  exponents of Cobb-Douglas production function  
 $c$   $\equiv$  propensity to consume  
 $F$   $\equiv$  available labor force  
 $g_p$   $\equiv$  proportionate rate of growth of parameter  $p \equiv F, M,$  and  $w$   
 $M$   $\equiv$  multiplicative factor of production function  
 $w$   $\equiv$  money wage rate



Parameters are stationary except  $F$ ,  $M$ , and  $w$  whose growth rates  $g_F$ ,  $g_M$ , and  $g_w$  are. All flow variables refer to the instantaneous rate of that variable measured on a per annum basis. The symbol  $e$  is Euler's number, the base of natural logarithms. Symbols  $t$  and  $\tau$  are time coordinates, to be used only in equations relating variables referring to different times.

### 3. THE SETTING: A NEOCLASSICAL MODEL OF STEADY-STATE GROWTH

Define the proportionate rate of growth

$$(1) \quad g_v \equiv \frac{dv}{dt} \frac{1}{v}$$



Define investment as the derivative of physical capital stock with respect to time:

$$(2) \quad I \equiv \frac{dS}{dt}$$

Let entrepreneurs apply a Cobb-Douglas production function

$$(3) \quad X = ML^\alpha S^\beta$$

where  $0 < \alpha < 1$ ;  $0 < \beta < 1$ ;  $\alpha + \beta = 1$ ; and  $M > 0$ . Let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:



$$(4) \quad \frac{w}{P} = \frac{\partial X}{\partial L} = \alpha \frac{X}{L}$$

Define physical marginal productivity of capital as the partial derivative of physical output with respect to physical capital stock:

$$(5) \quad \kappa \equiv \frac{\partial X}{\partial S} = \beta \frac{X}{S}$$

Let labor employed be the proportion  $\lambda$  of available labor force, where  $0 < \lambda \leq 1$  and  $\lambda$  is not a function of time:

$$(6) \quad L = \lambda F$$

Let consumption be a fixed proportion  $c$  of output:

$$(7) \quad C = cX$$



where  $0 < c < 1$ . Output equilibrium requires output to equal the sum of consumption and investment demand for it, or inventory would either accumulate or be depleted:

$$(8) \quad X = C + I$$

#### 4. STEADY-STATE GROWTH SOLUTIONS

By taking derivatives of all equations (2) through (8) with respect to time and then applying the definition (1), the reader may convince himself that the system is satisfied by the following stationary proportionate rates of growth:

$$(9) \quad g_C = g_X$$

$$(10) \quad g_I = g_X$$



- 10 -

$$(11) \quad g_K = 0$$

$$(12) \quad g_L = g_F$$

$$(13) \quad g_P = g_W - g_M/\alpha$$

$$(14) \quad g_S = g_X$$

$$(15) \quad g_X = g_M/\alpha + g_F$$

So there may be steady-state growth. Will there be? Assume that there is, then knock the system off its steady-state growth, and see if it will find its way back to such growth.

Use (1) and (2) to write  $I = g_S S$ , insert that and (7) into (8) and find:

$$g_S = (1 - c)X/S$$



Take the derivative of this with respect to time, use (1), and find the rate of acceleration of physical capital stock

$$(16) \quad z_{gS} = g_X - g_S$$

Into the production function (3) insert the employment function (6) with  $0 < \lambda \leq 1$  and  $\lambda$  not allowed to vary with time. Take the derivative of the outcome with respect to time, use (1), and find

$$g_X = g_M + \alpha g_F + \beta g_S$$

Insert this into (16) and find

$$(17) \quad z_{gS} = g_M + \alpha g_F - \alpha g_S$$

Knock  $g_S$  off its steady-state solutions (14) and (15). Differentiate (17) with respect to  $g_S$ , recalling that  $g_F$  and  $g_M$  are parameters:



$$(18) \quad \frac{dg_{gS}}{dg_S} = -\alpha$$

Since  $0 < \alpha < 1$ , a forced positive change of  $g_S$  will generate negative acceleration, and a forced negative change will generate positive acceleration. Once we stopped using force  $g_S$  would find its way back to its steady-state solutions (14) and (15). So the standard convergence proof applies even though in (6) labor employed was the proportion  $\lambda$  of the available labor force with  $0 < \lambda \leq 1$  and  $\lambda$  not allowed to vary with time. Not until Sec. 10 below shall we allow  $\lambda$  to vary with time.

## 5. PHYSICAL CAPITAL-STOCK EQUILIBRIUM

So far, no investment function is visible in our neoclassical growth



model. Is there one behind the scenes? Indeed there is, and we shall now derive it and find it general enough to encompass as special cases Wicksellian, Keynesian, post-Keynesian, and monetarist investment explanations. To derive it we must begin with physical capital-stock equilibrium. To entrepreneurs maximizing present net worth  $N$  of physical capital stock  $S$ , desired stock is the size of stock satisfying the first-order condition

$$(19) \quad \frac{\partial N}{\partial S} \equiv n = 0$$

where  $n$  is the present net worth of another physical unit of capital stock. Let us now find  $n$ .

Let entrepreneurs be purely competitive ones, hence price of output  $P$  is beyond their control. At time  $t$ , therefore, value



marginal productivity of another physical unit of capital stock is

$$(20) \quad \frac{\partial [P(t)X(t)]}{\partial S(t)} = \kappa(t)P(t)$$

As seen from time  $\tau$ , value marginal productivity at time  $t$  is  $\kappa(t)P(t)e^{-r(t - \tau)}$  where  $r$  is the stationary nominal rate of interest used as a discount rate. Define present gross worth of another physical unit of capital stock as the present worth of all its future value marginal productivities over its entire useful life

$$(21) \quad k(\tau) \equiv \int_{\tau}^{\infty} \kappa(t)P(t)e^{-r(t - \tau)} dt$$

Let entrepreneurs expect physical marginal productivity of capital stock to be growing at the stationary rate  $g_{\kappa}$ :

$$(22) \quad \kappa(t) = \kappa(\tau)e^{g_{\kappa}(t - \tau)}$$



and price to be growing at the stationary rate  $g_p$ :

$$(23) \quad P(t) = P(\tau)e^{g_p(t - \tau)}$$

Insert (22) and (23) into (21), define

$$(24) \quad \rho \equiv r - (g_k + g_p)$$

and write the integral (21) as

$$(25) \quad k(\tau) = \int_{\tau}^{\infty} \kappa(\tau)P(\tau)e^{-\rho(t - \tau)} dt$$

Neither  $\kappa(\tau)$  nor  $P(\tau)$  are functions of  $t$ , hence may be taken outside the integral sign. Our  $g_k$ ,  $g_p$ , and  $r$  were all said to be stationary, hence the coefficient  $-\rho$  of  $t$  is stationary, too. Assume  $\rho > 0$ . As a result find the integral (26) to be

$$(26) \quad k = \kappa P / \rho$$



Find present net worth of another physical unit of capital stock as its gross worth minus its price:

$$(27) \quad n \equiv k - P = (\kappa/\rho - 1)P$$

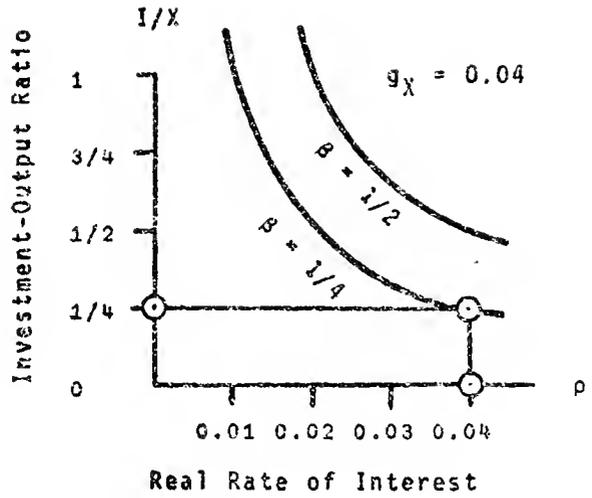
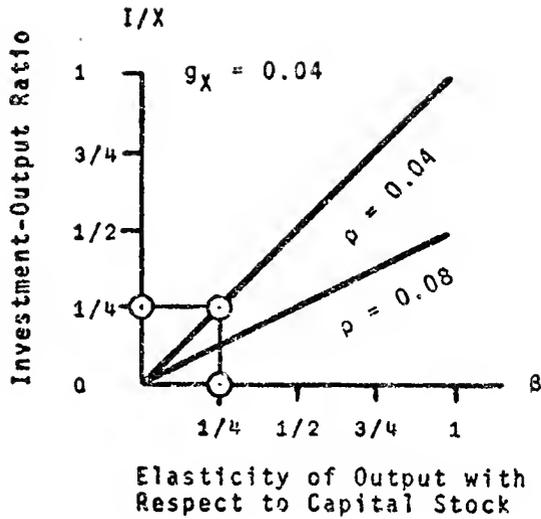
Applying the first-order condition (19) to (27), find equilibrium physical marginal productivity of capital stock

$$(28) \quad \kappa = \rho$$

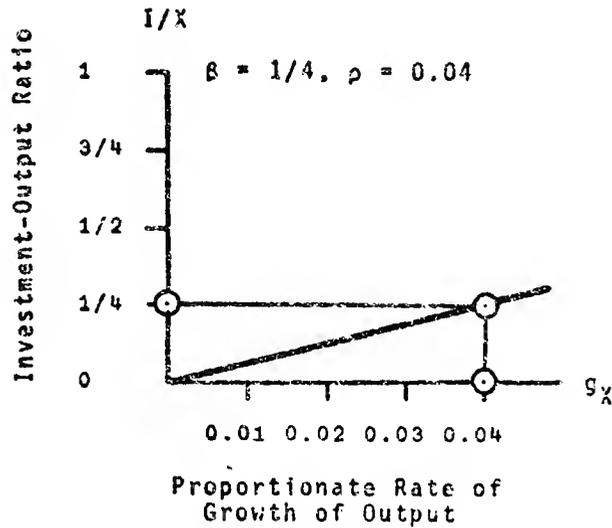
Finally take (5) and (28) together and find desired physical capital stock

$$(29) \quad S = \beta X/\rho$$





WICKSELL, KEYNES, AND THE MONETARISTS



POST-KEYNESIANS

FIGURE 1. THE INVESTMENT FUNCTION  $I = \beta X g_X / \rho$



## 6. THE INVESTMENT FUNCTION UNCOVERED

Applying the definition (2) to (29) find desired investment as the derivative of desired physical capital stock with respect to time. Use (1), recall that  $\rho$  is stationary, and find

$$(30) \quad I \equiv \frac{dS}{dt} = \beta X g_X / \rho$$

shown in Figure 1. So desired investment is in direct proportion to three things: First the elasticity  $\beta$  of output with respect to physical capital stock; second output  $X$  itself; and, third, the rate of growth  $g_X$  of output. Desired investment is in inverse proportion to  $\rho$ . What is  $\rho$ ? Insert the solution (11) into the definition (24) and find  $\rho = r - g_p$ , so  $\rho$  is simply the difference



between the nominal rate of interest  $r$  and the rate of inflation  $g_p$ . But that is exactly what monetarists call the real rate of interest. We have indeed found desired investment to be a function of the rate of interest—but of the real one, not the nominal one.

## 7. WICKSELL, KEYNES, MONETARISTS, AND POST-KEYNESIANS

Our investment function (30) is general enough to encompass as special cases Wicksellian, Keynesian, monetarist, and post-Keynesian investment explanations.

Wicksell [13], [14], and Keynes [7], [8], would have agreed<sup>2</sup> that, as in our model, investment  $I$  is the higher the higher the productivity  $\beta$  of capital and the lower a rate of interest—except that to Wicksell and Keynes that rate of interest was the nominal one  $r$ , to us it is the real one  $\rho$ .



Monetarists like Turgot [12], 49, Fisher [5], 8-9, and Mundell [10] distinguish between the nominal rate and the real rate of interest. The former equals the latter plus the rate of inflation and adjusts to the rate of inflation as easily and quickly as would the Keynesian marginal efficiency of capital. Consequently, investment would not be lower just because the nominal rate of interest were higher; only "an increase in the real interest rate lowers investment" [10], 16. As in our own model, then, investment  $I$  would be the lower the higher the real rate of interest  $\rho$ . Keynes [8], 142-143, knew Fisher's work but remained unconvinced by it.

Given an incremental capital coefficient, post-Keynesians like Domar [4] and Harrod [6], determine desired investment by the rate of growth of output: As in our own model, investment  $I$  is in direct proportion to the rate of growth  $g_X$  of output.

Our Appendix employs statistical proxies for the strategic variables used by Wicksell, Keynes, monetarists and post-Keynesians and offers a crude empirical test of the three investment functions on U. S. data 1947-74.



## 8. THE RATE OF INTEREST EQUALIZING SAVING AND INVESTMENT

Not until Sec. 5 above did we introduce the rate of interest—the nominal one called  $r$  and the real one called  $\rho$ . The rate of interest appeared neither in our system (1) through (8) nor in our solutions (9) through (15). Can we now find a solution for it? Now that we possess our investment function (30) we can. Insert the consumption function (7) into the equilibrium condition (8) and find

$$(31) \quad (1 - c)X = I$$

Output equilibrium, then, requires saving to equal investment, or inventory would either accumulate or be depleted. Then insert (30) into (31), assume a nonzero physical output  $X$ , divide on both sides by  $X$ , and write the result as



$$(32) \quad \rho = \beta g_X / (1 - c)$$

Our solution (15) expressed  $g_X$  solely in terms of the parameters  $\alpha$ ,  $g_F$ , and  $g_M$ . Our  $\beta$  and  $c$  are parameters. In (32) that leaves the real rate of interest  $\rho$  as the only variable. So (32) is a solution telling us what the real rate of interest  $\rho$  must be in order to equalize saving and investment.

Böhm-Bawerk, Wicksell, Fisher, and Schumpeter taught that highly productive capital, rapid growth of technology and labor force, and a low propensity to save would lead to a high equilibrium rate of interest. (32) is in complete accordance with such teachings, for its real rate of interest  $\rho$  is in direct proportion to the elasticity  $\beta$  of output with respect to capital stock, in direct proportion to the rate of growth  $g_X$  of output and in inverse proportion to the propensity to save  $1 - c$ .



The very fact that we could divide physical output  $X$  away brings to light an important property of (32): It holds for any nonzero level of physical output  $X$ , whether that level is a full-employment level ( $\lambda = 1$ ) or not ( $\lambda < 1$ ). If not, the latter level is an exact replica of the former, having exactly the same capital intensity  $S/L$ : Divide the production function (3) by physical capital stock  $S$ , divide desired physical capital stock (29) by physical output  $X$ , take the results together, and find desired capital intensity under steady-state growth.

$$(33) \quad S/L = (\beta M/\rho)^{1/\alpha}$$

The very fact that physical output  $X$  has disappeared from (33) shows that, like (32), (33) is the same for any nonzero level of physical output, whether that level is a full-employment level ( $\lambda = 1$ ) or not ( $\lambda < 1$ ).

Let us briefly relate our finding to Wicksell and Keynes.



## 9. WICKSELL AND KEYNES

Finding the solution (32) by dividing physical output  $X$  away brought to light an important property of that solution. The property is brought to light most glaringly if we look at our solution (32) first through Wicksellian, then through Keynesian glasses.

Wicksell [14], 193, 201, defined a rate of interest which would equalize saving and investment, and he called it the "normal" rate of interest<sup>3</sup>. Keynes in his Treatise [7] had also defined a rate of interest which would equalize saving and investment, and he called it the "natural" rate of interest. (It was when this agreement with Wicksell was first brought to his attention that Keynes made the disarming excuse [7], 199, that "in German I can only clearly understand what I know already.")

But in his General Theory [8], 243, Keynes corrected himself:



I had not then understood that, in certain conditions, the system could be in equilibrium with less than full employment... The 'natural' rate of interest...is merely the rate of interest which will preserve the status quo; and, in general, we have no predominant interest in the status quo as such.

We agree: Precisely because in our solution (32) we could divide physical output  $X$  away, that solution does hold for any nonzero level of physical output, hence does not guarantee full employment . But Keynes [8], 242, also said

I had...overlooked the fact that in any given society there is, on this definition, a different natural rate of interest for each hypothetical level of employment.

With that statement we must disagree as far as our steady-state growth model is concerned: Precisely because in (32) we could divide physical output  $X$  away, that solution is the same, not a different one, for any nonzero level of physical output.



### 10. HOW TO HANDLE UNEMPLOYMENT

If  $\lambda = 1$  well and good: By keeping the real rate of interest  $\rho$  at its equilibrium level (32) monetary authorities will keep the economy on its steady-state full-employment growth path. But what if  $\lambda < 1$ ? Until now  $\lambda$  has been assumed not to be a function of time. We must now drop that assumption. Into the production function (3) insert the employment function (6) with  $0 < \lambda < 1$  and  $\lambda$  now allowed to vary with time. Take the derivative of the outcome with respect to time, use (1), and find

$$g_X = g_M + \alpha(g_\lambda + g_F) + \beta g_S$$

Insert this into (16) and find



$$(34) \quad g_{gS} = g_M + \alpha(g_\lambda + g_F - g_S)$$

Sec. 4 failed to tell us how we managed to knock  $g_S$  off its steady-state solutions (14) and (15). But now that we possess (32) we know how: We reduce the real rate of interest  $\rho$  below its equilibrium level (32). According to our investment function (30) such a reduction will raise the investment fraction of output  $I/X$ , and represent a forced positive change of  $g_S$ . Differentiate (34) with respect to  $g_S$ , recalling that  $g_F$  and  $g_M$  are parameters:

$$(35) \quad \frac{dg_{gS}}{dg_S} = \alpha\left(\frac{dg_\lambda}{dg_S} - 1\right)$$

When  $\lambda < 1$   $g_\lambda$  may be positive and may be raised by raising  $g_S$ , hence  $dg_\lambda/dg_S$  may be positive, too. But once full employment  $\lambda = 1$  is reached  $g_\lambda$  must become zero, and so must  $dg_\lambda/dg_S$ : No forced raise in  $g_S$  can raise employment beyond full. At that time, then (35)



must become negative—and equal to (18). Consequently such a forced positive change of  $g_S$  would generate negative acceleration. Consequently, once we stopped using force and restored (32)  $g_S$  would find its way back to its steady-state solutions (14) and (15), and capital intensity  $S/L$  would find its way back to the value of (33) corresponding to the restored (32). But the temporary acceleration of  $g_S$  would have shifted the growth paths of the numerator (physical capital stock  $S$ ) as well as the denominator (employment  $L$ ) to permanently higher levels. With gentle nurturing, those levels could be full-employment levels.

## 11. CONCLUSION

Within the context of a neoclassical growth model we have derived an investment function (30) general enough to encompass as special cases Wicksellian, Keynesian, monetarist, and post-Keynesian



investment functions.

That investment function (30) was used to solve the system for its Wicksellian equilibrium ("normal") real rate of interest, i. e., the real rate which equalizes saving and investment under steady-state growth. The Wicksellian equilibrium ("normal") real rate of interest found (32) was in direct proportion to the elasticity of output with respect to capital stock, in direct proportion to the rate of growth of output, and in inverse proportion to the propensity to save.

But the solution (32) for the Wicksellian equilibrium ("normal") real rate of interest was found to hold for any nonzero level of physical output, hence did not guarantee full employment. To carry the economy to a steady-state full-employment growth path, monetary authorities might well have to reduce the real rate of interest below its Wicksellian equilibrium ("normal") level (32), temporarily knocking the system off its steady-state growth.



APPENDIX

THREE INVESTMENT FUNCTIONS CRUDELY TESTED, UNITED STATES 1947-74

It would be tempting to test empirically the Wicksellian, Keynesian, monetarist and post-Keynesian investment functions shown in Figure 1. The difficulty lies in finding suitable statistical proxies for the variables used. As a crude first approximation let us employ the following proxies where page numbers refer to Economic Report by the President, transmitted January, 1976.

$g_p$   $\equiv$  proportionate rate of growth of implicit price deflator of gross national product, 175.

$g_x$   $\equiv$  proportionate rate of growth of gross national product in 1972 dollars, 173.

$I$   $\equiv$  gross private domestic fixed investment in 1972 dollars, 172.



TABLE 1  
DATA FOR UNITED STATES INVESTMENT FUNCTION 1947-74

Year	I/X	$\kappa$	$\rho$	$\varepsilon_X$
1947	0.150	0.156	-0.102	-0.016
48	0.157	0.160	-0.041	0.041
49	0.143	0.116	0.037	0.006
50	0.156	0.154	0.006	0.087
51	0.139	0.121	-0.039	0.081
52	0.132	0.103	0.017	0.038
53	0.135	0.105	0.017	0.039
54	0.139	0.099	0.015	-0.013
55	0.147	0.126	0.009	0.067
56	0.145	0.123	0.003	0.021
57	0.141	0.109	0.005	0.018
58	0.132	0.086	0.022	-0.002
59	0.140	0.104	0.022	0.060
60	0.137	0.092	0.027	0.023
61	0.133	0.089	0.035	0.025
62	0.137	0.098	0.025	0.058
63	0.141	0.103	0.028	0.040
64	0.143	0.116	0.028	0.053
65	0.150	0.130	0.023	0.059
66	0.147	0.134	0.018	0.059
67	0.140	0.117	0.026	0.027
68	0.143	0.121	0.017	0.044
69	0.146	0.115	0.020	0.026
70	0.140	0.093	0.026	-0.003
71	0.145	0.097	0.023	0.030
72	0.153	0.106	0.031	0.057
73	0.155	0.128	0.015	0.053
74	0.142	0.149	-0.011	-0.018

---

Source: Economic Report by the President, transmitted January, 1976, 172, 173, 175, 238, and 261.



$\kappa$   $\equiv$  ratio of profits after tax to stockholders' equity, all manufacturing corporations, 261.

$r$   $\equiv$  nominal rate of interest  $\equiv$  yield of corporate Aaa bonds (Moody's), 238.

$\rho$   $\equiv$  real rate of interest  $\equiv$  nominal rate of interest  $r$  minus proportionate rate of growth of implicit price deflator  $g_p$ , 238 and 175.

$X$   $\equiv$  gross national product in 1972 dollars, 172.

Employing these proxies we have tried one multiple linear regression and three simple linear regressions. All regressions were unlagged. The four regressions were the following. First

$$(36) \quad \begin{array}{l} I/X = 0.108 + 0.290\kappa + 0.0508\rho + 0.0371g_p X \\ \text{Std. errors} \quad 0.0610 \quad 0.0435 \quad 0.0336 \\ \text{T-ratios} \quad 4.75 \quad 1.17 \quad 1.10 \end{array}$$

explains the investment-output ratio in terms of natural rate of



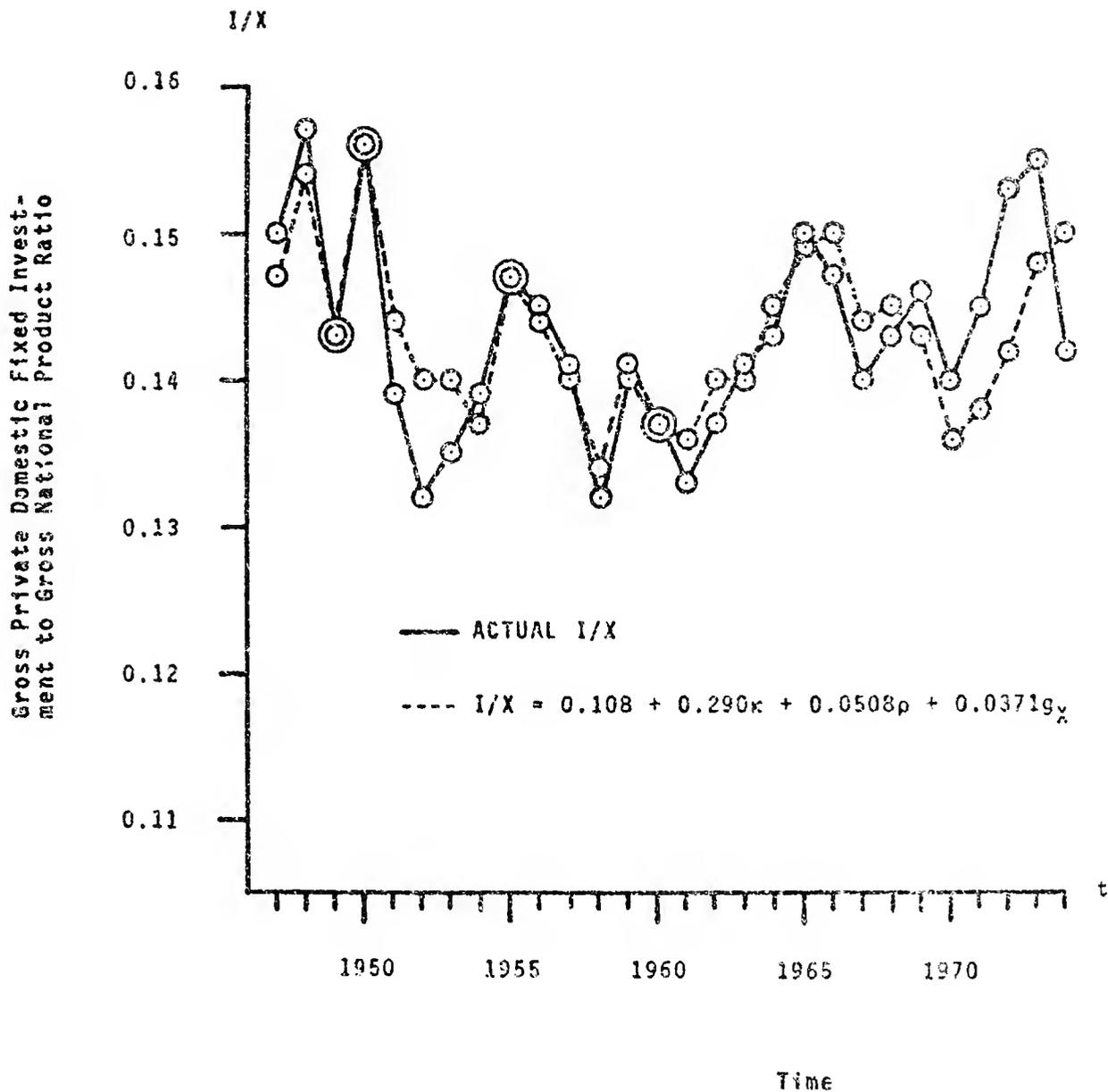


FIGURE 2  
INVESTMENT-OUTPUT RATIO, ACTUAL AND ESTIMATED FROM NATURAL RATE OF INTEREST  $\kappa$ ,  
REAL RATE OF INTEREST  $\rho$  AND RATE OF GROWTH OF OUTPUT  $g_\chi$ , UNITED STATES 1947-74



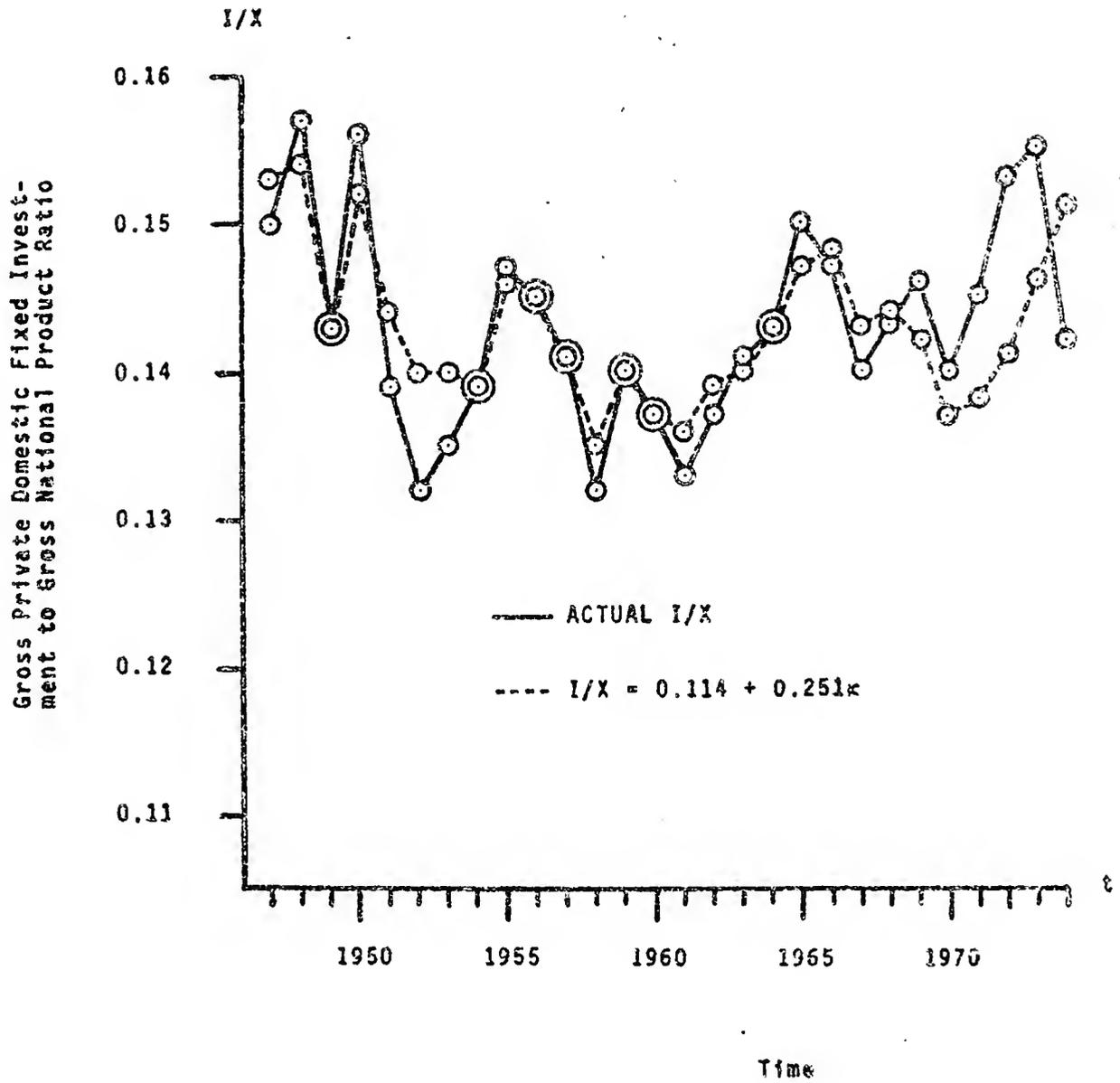


FIGURE 3  
INVESTMENT-OUTPUT RATIO, ACTUAL AND ESTIMATED FROM NATURAL RATE OF INTEREST  $\kappa$   
ALONE, UNITED STATES 1947-74



interest, real rate of interest, and rate of growth of output. Second,

$$(37) \quad \begin{array}{l} I/X = 0.114 + 0.251k \\ \text{Std. error} \quad 0.0444 \\ \text{T-ratio} \quad 5.64 \end{array}$$

explains the ratio in terms of natural rate of interest alone. Third,

$$(38) \quad \begin{array}{l} I/X = 0.144 - 0.0795\rho \\ \text{Std. error} \quad 0.0445 \\ \text{T-ratio} \quad -1.79 \end{array}$$

explains the ratio in terms of real rate of interest alone. Fourth,

$$(39) \quad \begin{array}{l} I/X = 0.140 + 0.0780g_X \\ \text{Std. error} \quad 0.0455 \\ \text{T-ratio} \quad 1.71 \end{array}$$

explains the ratio in terms of rate of growth of output alone.

It is quite clear that the proxies employed for the monetarist real rate of interest  $\rho$  and the post-Keynesian rate of growth of



output  $g_X$  add little to the explanation offered by the proxy employed for the Wickssellian natural rate of interest  $\kappa$ . We have 24 to 26 degrees of freedom, and the t-ratios of (36) and (39) as well as those of the last two coefficients of (36) are numerically well below 2. By contrast the t-ratios of the coefficients of  $\kappa$  in (36) and (37) are 4.75 and 5.64, respectively. Of (36) and (37), which is better?

The explanatory powers of (36) and (37) are shown in Figures 2 and 3, respectively. (36) adds little, if anything, to (37):

	Eq. (36)	Eq. (37)
Number of bull's eyes	4	7
Adjusted $R^2$	0.567	0.533
Durbin-Watson statistic	0.960	1.161

Both (36) and (37) are marred by some autocorrelation. But our main difficulty remains our use of proxies. Wicksell defined his natural rate of interest as the expected yield on newly created capital. Our proxy for that is the ratio of profits after tax to stockholders' equity in manufacturing corporations. For one thing, that is an average, not a marginal, rate. For another, it includes a leverage effect and excludes nonmanufacturing corporations and all noncorporations.



REFERENCES

- [1] Brems, H., "Wicksells tre Definitioner af Ligevægtsrenten," Nationaløkonomisk Tidsskrift 79, 1941, 199-228.
- [2] Brems, H., Labor, Capital, and Growth. Lexington, Mass., 1973.
- [3] Brems, H., "Reality and Neoclassical Theory," Journal of Economic Literature 15, March 1977, 72-83.
- [4] Domar, E. D., "Capital Expansion, Rate of Growth, and Employment," Econometrica 14, Apr. 1946, 137-147.



- [5] Fisher, I., "Appreciation and Interest," Publications of the American Economic Association 11, 4, Aug. 1896, 331-442.
- [6] Harrod, R. F., Towards a Dynamic Economics. London, 1948.
- [7] Keynes, J. M., A Treatise on Money, I. London, 1930.
- [8] Keynes, J. M., The General Theory of Employment, Interest, and Money. London, 1936.
- [9] Lindahl, E., Studies in the Theory of Money and Capital. London, 1939.
- [10] Mundell, R. A., Monetary Theory: Inflation, Interest, and Growth in the World Economy. Pacific Palisades, 1971.
- [11] Solow, R. M., "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics 70, Feb. 1956, 65-94.



- [12] Turgot, A. R. J., "Réflexions sur la formation et la distribution des richesses," Ephémérides du citoyen, Nov. 1769-Jan. 1770. English translation, Reflections on the Formation and the Distribution of Riches. New York, 1898.
- [13] Wicksell, K., Geldzins und Güterpreise. Jena, 1898. English translation, Interest and Prices. London, 1936.
- [14] Wicksell, K., Föreläsningar i nationalekonomi, II. Lund, 1906. English translation, Lectures on Political Economy, II. London, 1935.



F O O T N O T E S

<sup>1</sup>To Brian A. Montigney the author is indebted for checking all computer results.

<sup>2</sup>Wicksell's definition of the rate of interest on physical capital broadened from his early to his later work. In his early work [13], 102-103, Wicksell defined his "natural" rate of interest on physical capital as the in-kind rate prevailing in the absence of money. As Lindahl [9], 247, observed, such an in-kind rate is meaningful if "the productive process consists only in investing units of goods...of the same type as the final product"——as in our one-good Solow economy. In his later work [14], 193, Wicksell apparently found the in-kind rate too confined and defined his "natural rate of interest as "the expected yield on the newly created capital"——which is the same thing as Keynes' [8], 135, "marginal efficiency of capital".

Wicksell [13], 104, 134, defined his "money" rate of interest on loans as the lending rate prevailing under the use of money, Wicksell and Keynes agreed that, in the latter's language, "new investment will be pushed to the point at which the marginal efficiency of capital becomes equal to the [money] rate of interest" [8], 184.



<sup>3</sup>Wicksell implicitly defined three [1] different rates of interest. He used somewhat interchangeable labels but clearly distinguished three concepts:

First, his "natural" rate of interest was a rate of interest on physical capital and was defined [14], 193, as "the expected yield on the newly created capital"——the same thing as Keynes' "marginal efficiency of capital".

Second, Wicksell's "normal" rate of interest was a rate of interest on loans and was defined [14], 193, 201, as a rate of interest which would equalize saving and investment.

Third, Wicksell's "neutral" rate of interest was also a rate of interest on loans and was defined [14], 201——very implicitly ——as a rate of interest which would keep commodity prices stationary.

The present paper has had nothing to say about the third rate of interest.

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