A

KEY

TO THE

ELEMENTARY TREATISE

ON

ALGEBRA.

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KEY.

[In the first edition of the Algebra, the Articles are numbered erroneously from the commencement of Section XXVI. To make that edition correspond with the Key, increase by 2 the number of each Article, commencing with the Section specified.]

SECTION I.

ART. 4.

1. Performed.
2. Horse $100. Saddle $10.
3. 6 bushels of each.
4. 11 of each.
5. 5 melons; 10 pears; 30 peaches.
6. 15 days. Man $20; boy $10.
7. 105, 210, and 315.
8. 25, 35, and 40.
9. 10 hours. One goes 80 miles, the other 70 miles.
10. 1st 8, 2d 16, 3d 24 guineas.
11. Wife $36000; elder son $27000; younger son $18000.
12. 120 cavalry; 360 artillerymen; 720 infantry.
13. A 8 days, 72 rods; B 16 days, 112 rods; C 80 days, 400 rods.
14. 8 yds, 16 yds, and 24 yds respectively.
15. Sheep $2; calf $4; cow $20.
16. Greater 14 galls; less 7 galls.
17. Beef 10 lbs; mutton 20 lbs.
18. 32 of each.
19. 80 guineas; 240 crowns.
20. 10, 20, 30, 60 and 30 miles, on the respective days.

21. 8 bushels of each.
22. Cows $25, oxen $50 each.

SECTION II.

ART. 7.

1. Performed.
2. 1st $23000; 2d $33000.
3. $73\frac{1}{3}$ and $156\frac{2}{3}$ miles.
4. A's 15, B's 35, C's 50 years.
5. 1st 8 lbs; 2d 22 lbs; 3d 30 lbs; 4th 60 lbs.
6. 6, 8, 10, 12, and 14 years respectively.
7. Black $30; green $32; blue $35.
8. A's 30, B's 20 years.
9. Each daughter $1000; younger son $2000; elder son $5000.
10. Performed.

ART. 9.

1. 493 and 400 votes respectively.
2. 5 men; 20 boys; 180 girls.
3. 150 cavalry; 450 riflemen; 1700 infantry.
4. A $2750; B $8250; C $22000; D $32000.
5. A $377; B $7; C $107.
6. 10 years in France; 60 in England; 10 in America.
7. 1500 English; 900 Irish; 500 French.
8. 50 eagles; 90 half-eagles; 250 dollars.
9. Broadcloth 50; cassimere 144; silk 288 yards.
10. $2000.
11. A $10000; B $10600; C $20800; D $20300; E $30800
12. 80 oxen; 215 cows; 215 calves; 395 sheep.
13. 20 students; 55 merchants; 65 officers.

SECTION III.

ART. **11**.

1. Performed.
2. Finer $6; coarser $4 per yard.
3. A's 50; B's 40 years.
4. 20 each.
5. Sum distributed 153d. Eldest 60d.; 2d 42d.; 3d 30d.; 4th 21d.
6. 1st 10s.; 2d 7s.; 3d 8s.; 4th 5s.; 5th 3s.; 6th 2s.
7. Smaller 40 galls.; larger 80 galls.
8. Wheat 8s.; rye 6s. per bushel.
9. 45 scholars in all. He wished to place 7 in a row.
10. 10 lbs; 95 cents.
11. 10 shillings.
12. $60.
13. 22 years.
14. 12 years ago.
15. 150 sheep.
16. 8 days; 720 miles from the starting place.
17. Horse $200; chaise $275.
18. Wheat 8s.; oats 3s.
19. $1000.
20. 125 miles.

SECTION IV.

ART. **12**.

1st, 2d, 3d, performed.

4. 60 years old. 20 years in Germany; 15 in France; 24 in England.

5. Whole estate $5040. He owed his creditors respectively $630, $720, and $672.

6. Whole mixture 32 oz. $12\frac{2}{3}$ oz. gold; 16 oz. silver; $3\frac{1}{3}$ oz. copper.

7. Whole estate $22500. Wife's portion $6500; eldest son's $6500; 2d son's $5750; youngest son's $3750.

8. 80 years.

9. A's 40; B's 30 years.

10 30 inches.

11. 72.

12. $150.

13. $75.

14. 1st 30 cents; 2d and 3d 15 cents each.

15. Whole number 30. The 1st had 10; 2d 6; 3d 4; 4th 10

16. 1st horse $100; 2d horse $150; chaise $300.

17. Income $1200; he spends $650.

18. 45 sheep.

19. 1st £56; 2d £35; 3d £7.

20. 280 miles in all. 40 miles 1st day; 56 2d day; 70 3d day.

SECTION V.

ART. **13**.

1st, 2d, 3d, performed.

4. A $300; B $200.

5. Man 40; woman 30 years.

6. Man's age 50; woman's 45 years

7. £50.

8. From A to B 40 miles, from B to C 60 miles; from A to C 100 miles.

9. 1st 105 yds; 2d 120 yds; 3d 135 yds.

10. Each has £150.

11. 100 yards.

12. Each has $140.

13. A $350; B $400; C $475.

14. 30 cows; 60 sheep.

15. $40.
16. $1000.
17. Cider $4; beer $6 per barrel.
18. 54 and 60.
19. Shorter 20 rods; longer 40 rods.
20. Chaise $233\frac{1}{3}$; horse $166\frac{2}{3}$.
21. $100.
22. 1st car 18 miles; 2d car 20 miles per hour.
23. $150.
24. $110.
25. 139.

SECTION VI.

ART. **15.**

1. Horse $150; chaise $250.
2. 80 gallons.
3. Greater 35; less 25.
4. Performed.
5. 20 tons.
6. $72.
7. Greater 75; less 72.
8. 100 gallons each.
9. 90 eggs.
10. Time past 40 years.
11. Lower, middle and upper parts respectively, 12, 16 and 20 feet. Whole pole, 48 feet.
12. 20.

SECTION VII.

ART. **17.**

1. $14abcm$.
2. $845c$.
3. $21abxy$.
4. $24apqr$.

5. $51\,a\,g\,h\,x$
6. $156\,a\,x\,y$.
7. $4\,m\,n\,p\,q$.
8. $20\,a\,m\,p$.
9. $21\,m\,q\,s$.
10. $90\,a\,q\,x$.
11. Performed.

ART. **19**.

1. Performed.
2. $54\,a\,b^2\,c$.
3. $301\,a^4\,b^6$.
4. $46\,a^7\,m^7\,x$
5. $399\,a^2\,m\,x$.
6. $11\,x^6\,y^8$.
7. $72\,a^3\,b^3\,c^3\,d^3$.
8. $146\,m^9\,n^5\,x^4$.
9. $125\,a^8\,b^7\,c^6\,d^2$.
10. $162\,p^4\,q^2\,x^7\,y^9$.
11. $12\,a^6\,h^6\,n^8\,x^7$
12. $567\,a^3\,r^2\,t^7\,y$.
13. $132\,p^3\,x^6\,y^3$.
14. $36\,a^3\,m^6\,n^2\,p^5$
15. $39\,x^3\,y^3\,z^4$.
16. $170\,a^5\,b^2\,c\,x$.
17. $210\,a^5\,b^3\,c$.
18. $54\,c^2\,p\,x^4\,y^3$.
19. $28\,a\,p\,x^6\,y^7$.
20. $3\,a^7\,m^5\,x^4\,y^7$.

SECTION VIII.

ART. **22**.

1. $4\,a^2\,b - a\,b^2$.
2. $3\,a\,b\,c^2$.
3. $8\,a\,b\,c^3 + a\,b^3\,c - 6\,a\,b^3 - 4\,m\,n$.
4. $-\,c\,p^3\,r + 7\,m^2\,n^2 + 7\,g\,h$.
5. $2\,m\,n\,r - 8\,p^3\,q + 10\,b\,c$.
6. $27\,x^2 - 8\,y^3 + 5\,m$.
7. $7\,a^3\,p\,m - 3\,a\,p\,m^3 + a\,m\,p^2 + 6$.
8. $m\,n^2\,p^3\,r^4 + 12\,m^2\,n - m^2\,n^2\,p^3\,r^4$.

SECTION IX.

ART. **25**.

1. $20\,a + 2\,b + 16\,c + 4\,d + 6$.
2. $12\,a\,b + 4\,c\,d + 4\,m^2\,n + 3\,m$.

3 $18ab+5cd+5m^2+4cm$.

4. $13a^2b^2+8m^2-18mn+3b+4xy$.

5. $16bc-3ab+5ac+12cd+4mn+10an$.

6. $8a-8b+c-17d+25$.

7. $-8a+6c-109d+31e-10f+g$.

8. $17a^2b-13ab^2-10ab+18c^3d-10d^4-12m^2n+11mn^2+mn$.

9. $8a^3+14b^3-7bx^2-8cx^2-x^2+13x-9hy^2-2ky^5-5ky-9hy+7py^2$.

SECTION X.

ART. **27**.

1st, 2d, performed.

3. $3ab-2c^2+3bc$.

4. $-6ac-6ab+9bc+7cd-12am$..

5. $8m-6x-f-3d-8$.

ART. **28**.

1. $18a^2x-6bc+10x^2-ax+b-6$.

2. $10ax^3-36a^2x^2+49a^3x-6a^4$.

3. $26ab-18bc^2+26b^2+c^2-m^2x+47$.

4. $7a^2xy-8bx^2y+30cxy^2-29y^5$.

5. $18x^2y^2+48xy+4b^2c^2+cd^3+c^3d+m$.

ART. **29**.

1. $am-(bc-dm)$.

2. $abc-d^2-(3my-xy)$.

3. $a+b-(c+d)$.

4. $a^2+b^2+2ab-(a+b)$.

5 $4m^2+12mn+9n^3-(2m+3n)$.

6. $4ab+7am-(x^2-y^2)$.

7. $7mn+x^2-(pq-y^3)$.

SECTION XI

ART. **31.**

1st, 2d, 3d, performed.

4. $2a^2+5a-12$.

5. a^4+4b^4.

6. $37a^2bc^2-12a^2c^4+22ac^4d^2-21a^2b^2-26abc^2d^3-8c^4d^4$.

7. $126a^6c^2+30a^3bc^2-36b^2c^2+108a^3cxy^2-28a^3cx^2y+33a^3c^3+72bcxy^2+12bcx^2y-24bc^3-24x^3y^3+2c^2x^2y+36c^2xy^2-3c^4$.

8. $3a^6b-10a^5b^2+17a^4b^3-17a^3b^4+8a^2b^5-ab^6$

9. $42b^5-217b^3c^2-11b^2c^3+8bc^4-2c^5$.

ART. **32.**

1. $a^3+a^2b-2ab^2-ac^2+bc^2$.

2. $a^4-4a^4b^2+8a^2b^3-4b^4$.

3. m^8-n^8.

4. $2a^2b+2b^2c+2c^3$.

5. $7x^3+x^2y-12xy^2-6y^3$.

6. $10acx+2a^2c+5abc+2x+6a+8b$.

7. $4abc+2a^2d+2b^2d$.

8. $4ax^2-4axy+ac^2+4bx^2-4bxy+bc^2-a^3c^2m-ac^2xy+a^3d^2m+ad^2xy-a^2c^2mx-c^2x^2y+a^2d^2mx+d^2x^2y$.

9. $13b^3c^2x-8b^2c^3x^2+18c^5x^2$.

10. $112b-4bx+a^2b+448-16x+14a^2+6a^2bc-7a^2c^2$.

SECTION XII.

ART. **40.**

1. Performed.
2. $4a^2bcd$.
3. $5a^2b^3cd$.
4. $12b^6h^3$
5. $25m^5y^6$.
6. $31a^2b^2m^2$.

7. $3\,m^3\,a^6\,b^4\,y$.
8. $a^2\,h^2\,n^2$.
9. 15.
10. $9\,a^4\,m^4\,x^3\,y$.
11. $81\,x\,y^3$.
12. $2\,a^4\,b^2$.
13. $3\,m\,x^4\,y^3\,z$.
14. $126\,a^4\,b^4\,d^2$.
15. $37\,m\,n\,p^7$.
16. $23\,n^2\,r^5\,s$.
17. $3\,x^2\,y^{10}$.
18. $7\,b^{14}$.
19. $79\,m^6\,n^5$.
20. $103\,x$.

SECTION XIII.

ART. **42**.

1. $a\,b+m$.
2. $4\,y-2\,x+y^2$.
3. $a\,m^2+2\,m^3\,x-3$.
4. $6\,d-4\,b\,c-2\,b^2$.
5. $3\,a^3\,b^3-1$.
6. $3\,x^2\,y+1-6\,x$.
7. $-2\,c+1+6\,a^2\,b^4\,c^2$

ART. **44**.

1. $a^2+2\,a\,m+m^2$.
2. $a^3+3\,a^2\,x+3\,a\,x^2+x^3$.
3. $m^3-3\,m^2\,x+3\,m\,x^2-x^3$.
4. $2\,x^3-4\,x^2+8\,x-16$.
5. $3\,a^2-a\,b-2\,b^2$.
6. $4\,a^2-5\,a\,b+2\,b^2$.
7. $2\,x^2-3\,a\,x+a^2$.
8. $a^4+a^3\,z+a\,z^3+z^4$.
9. $a^4+4\,a^3\,x+12\,a^2\,x^2+16\,a\,x^3+16\,x^4$.
10. $x^4-x^3+x^2-x+1$.
11. $-5\,a^2+4\,a\,b+3\,b^2$.
12. $m^6+m^5\,x+m^4\,x^2+m^3\,x^3+m^2\,x^4+m\,x^5+x^6$.
13. $5\,a^3\,b^2\,c^3-2\,a^2\,b^2\,c^4-3\,a\,b^2\,c^5-7\,b\,c^6$.

ART. **46**.

1. $x^8+x^7\,y+x^6\,y^2+x^5\,y^3+x^4\,y^4+x^3\,y^5+x^2\,y^6+x\,y^7+y^8$.
2. $81\,a^4+108\,a^3\,b+144\,a^2\,b^2+192\,a\,b^3+256\,b^4$.

3. $x^9 - x^8 y + x^7 y^2 - x^6 y^3 + x^5 y^4 - x^4 y^5 + x^3 y^6 - x^2 y^7 + x y^8 - y^9$.

4. $1 - m + m^2 - m^3 + m^4 - m^5 + m^6 - m^7$.

5. $m^4 - m^3 n + m^2 n^2 - m n^3 + n^4$.

6. $m^8 - m^7 + m^6 - m^5 + m^4 - m^3 + m^2 - m + 1$.

7. $1 + m^2 + m^4 + m^6 + m^8 + m^{10} + \frac{m^{12}}{1 - m^2}$.

8. $a - a x y + a x^2 y^2 - a x^3 y^3 + a x^4 y^4 - a x^5 y^5 + \frac{a x^6 y^6}{1 + x y}$.

ART. 47.

1. a^2.

2. $(m + 1)(a + b)$.

3. $2(x^2 + x + 1)(a + m)$.

4. $(a + b)(x^2 - x y + y^2)$.

5. $9(m^2 - n^2)(x^4 - x^3 y + x^2 y^2 - x y^3 + y^4)$.

6. $3 m (x^3 - y^3)(m^2 + n^2)$.

7. $5(m^2 - 1)(x^4 - x^3 + x^2 - x + 1)$.

SECTION XIV.

ART. 48.

1. $\frac{a b c x}{m}$.

2. $\frac{b m x + c m x}{a}$.

3. $\frac{a m + b m + a n + b n}{c}$.

4. $\frac{4 c x y + 3 x^2 y}{a - x}$.

5. $\frac{m^2 + 2 m n + n^2}{b - c}$.

6. $\frac{12 a^4 + 25 a^3 b}{b^2 + 2 a c}$.

7. $\frac{a^2 x - b^2 x + a^2 y - b^2 y}{b + c}$.

8. $\frac{4 a^2 c^2 - 4 b^2 c^2}{m^2 + 2 m n + n^2}$.

9. $\frac{50 a x^3 + 26 a x^2 y + 125 a x^4 + 65 a x^3 y}{17 b + 3 c}$.

10th and 11th performed.

ART. **49.**

1. $\frac{4a+3xy}{mx}$.
2. $\frac{3ab+4ay}{12xy-19x^2y^2}$.
3. $\frac{m^2-n^2+ab}{a-b}$.
4. $\frac{14ac+25b^2+43}{3a+2a^2b-6xy}$.
5. $\frac{4bm^2+11ax^2-93}{x-y}$.
6. $\frac{3bc+4xy}{a+b}$.
7. $\frac{3ab}{5(x^2-y^2)}$.
8. $\frac{7mx+3by}{2(a-b)}$.
9. $\frac{ax+by}{7(m^5-m^4y+m^3y^2-m^2y^3+my^4-y^5)}$.
10. $\frac{15xy}{2(a+b)(x^4-x^3y+x^2y^2-xy^3+y^4)}$.
11. $\frac{a+bcm}{3(c+d)}$.
12. $\frac{3ac+xy}{(m+n)(x^4-x^3y+x^2y^2-xy^3+y^4)}$.

13th and 14th performed.

15. $a+b$.
16. $m^2+2mc+c^2$.
17. $3a^2bc+4xy$.
18. $4bcm-12x^2y$

ART. **50.**

1. $\frac{a^2cm}{xy}$.
2. $\frac{a^2m}{d}$.
3. $\frac{x^4y^4}{ac^2}$.
4. $\frac{7a^3p^2q^5}{xy^3}$.
5. $\frac{3ax+3bx}{n}$
6. $\frac{a^4x^3y^2}{2m^2}$.
7. $\frac{bc^2y-bcdy}{x-m}$.
8. $\frac{5a^2c+4abc}{7(x-y)}$.
9. $\frac{a^2+2ax+x^2}{2(a+m)}$.
10. $\frac{abcm^2+a^2x+b^2cm^2+abx}{ab(m^2+1)}$

SECTION XV

ART. **51.**

1. $\frac{2a}{7m}$.
2. $\frac{3y^2}{11bc}$.
3. $\frac{2b^2y}{4m^2-n^2}$.
4. $\frac{17}{bc+4xy}$.
5. $\frac{a+b}{m}$.
6. $\frac{am+3m^2+4c}{116+cd}$.
7. $\frac{a+b}{x+y}$.
8. $\frac{a^2+ab+b^2}{3bc-xy}$.
9. $\frac{3x^2+2x}{ab+7b^2}$.
10. $\frac{6x-7}{11m+4n^2}$.
11. Performed.

ART. **52.**

1. $\frac{3a}{bcm}$.
2. $\frac{x+y}{b^2-c^2}$.
3. $\frac{m+n}{4x^3+4x^2y}$.
4. $\frac{3ab+4b}{9x^2-16m^2}$.
5. $\frac{x+y}{m+n}$.
6. $\frac{x+y}{a+b}$.
7. $\frac{3+7x}{4p+4q+bcp+bcq}$
8. $\frac{a^2-b^2}{x^3-y^3}$.
9. $\frac{2m^2+3x}{b^2+c^2}$.
10. $\frac{x^2+xy+y^2}{a^2}$.
11. $\frac{m^3-m^2n+mn^2-n^3}{x+y}$.

ART. **53.**

1. $\frac{2a^2}{3m^4}$.
2. $\frac{3xy^2}{7ab}$.
3. $\frac{3m^2xy}{5a^3b^3}$
4. $\frac{1}{7ay^2}$.

5. $\frac{7m^2n}{x^2+xy}$.

6. $\frac{m}{4x+4y}$

7. $\frac{x-y}{21mn}$.

8. $\frac{2(x-y)(a+b)}{3am^2}$.

9. $\frac{2(m^2+mn+n^2)}{5axy}$.

SECTION XVI.

ART. **54**.

1. Performed.
2. $a(b+c-2m)$.
3. $2x(2a+y+6)$.
4. $5x^2(5+6y-3m)$.
5. $3^3abc(3+ab+2a^2)$.
6. $3^2m^2(11xy+12pq+2r)$.
7. $2^2.3ab(1+2c-3x)$.

ART. **57**.

1. $1, a, a^2, b, b^2, ab, a^2b, ab^2, a^2b^2$.

2. $1, a, a^2, b, ab, a^2b, b^2, ab^2, a^2b^2, c, ac, a^2c, bc, abc, a^2bc, b^2c, ab^2c, a^2b^2c, c^2, ac^2, a^2c^2, bc^2, abc^2, a^2bc^2, b^2c^2, ab^2c^2, a^2b^2c^2, c^3, ac^3, a^2c^3, bc^3, abc^3, a^2bc^3, b^2c^3, ab^2c^3, a^2b^2c^3$.

3. $1, 3, 9, x, 3x, 9x, x^2, 3x^2, 9x^2, y, 3y, 9y, xy, 3xy, 9xy, x^2y, 3x^2y, 9x^2y, y^2, 3y^2, 9y^2, xy^2, 3xy^2, 9xy^2, x^2y^2, 3x^2y^2, 9x^2y^2, y^3, 3y^3, 9y^3, xy^3, 3xy^3, 9xy^3, x^2y^3, 3x^2y^3, 9x^2y^3$.

4. $1, 2, 5, 10, 25, 50, m. 2m, 5m, 10m, 25m, 50m, m^2, 2m^2, 5m^2, 10m^2, 25m^2, 50m^2$.

5. 1, 2, 3, 6, 5, 10, 15, 30, 25, 50, 75, 150.

6. 1, 2, 4, 3, 6, 12, 5, 10, 20, 15, 30, 60, 13, 26, 52, 39, 78, 156, 65, 130, 269, 195, 390, 789.

7. $1, 2, 4, a, 2a, 4a, b, 2b, 4b, ab, 2ab, 4ab, a+4, 2(a+4), 4(a+4), a(a+4), 2a(a+4), 4a(a+4), b(a+4)$ $2b(a+4), 4b(a+4), ab(a+4), 2ab(a+4), 4ab(a+4)$.

8. 1, 5, a, $5a$, $2b+5m$, $5(2b+5m)$, $a(2b+5m)$, $5a(2b+5m)$.

SECTION XVII.

ART. **59.**

1. $4a^2bc$.
2. $5a$.
3. $15a$.
4. m^2+1.
5. $x+1$.
6. $x+2$.
7. x^3-8x-3.
8. $2a+3x$.
9. $a-b$.

SECTION XVIII

ART. **61.**

1. Performed.
2. $36a^3m^4y^3$.
3. $150a^3m^3x^3$.
4. $36ax^2y^5$.
5. $12ab(2b+c)$, or $24ab^2+12abc$.
6. $27m^2x^4(1+2mx)$, or $27m^2x^4+54m^3x^5$.
7. $(a+b)^2(a-b)$, or $a^3+a^2b-ab^2-b^3$.
8. $(a+b)(a-b)(a^2+ab+b^2)$, or $a^4+a^3b-ab^3-b^4$.

SECTION XIX.

ART. **63.**

1. Performed.
2. $\frac{3x}{5y}$.
3. $\frac{3a^3c}{2m^2}$.
4. $\frac{31m^3}{15xy^2}$.
5. $\frac{1}{12mxy^2}$.
6. $\frac{1}{7a^3m}$.
7. $\frac{am+35x}{bcx}$.
8. $\frac{xy+2}{3x^2y^2}$.
9. $\frac{25a^2bc^2}{3-5abc}$.
10. $\frac{2b+13m^2}{144a}$.
11. $\frac{1}{3ax-2a^2}$.
12. $\frac{3am^2}{a^2-2m^2}$.
13. Performed.
14. $\frac{xy}{x+y}$.
15. $\frac{a+b}{a-b}$.
16. $\frac{5(a+b)}{8a}$, or $\frac{5a+5b}{8a}$
17. $\frac{a^2+ax+x^2}{a-x}$.
18. $\frac{3(x-y)}{4(x^2-xy+y^2)}$.
19. $\frac{9ab^2}{5(x^3-x^2y+xy^2-y^3)}$.

ART **64.**

1. Performed.
2. $\frac{xy}{2abc}$.
3. $\frac{1}{2xy}$.
4. $\frac{c}{4x^2}$.
5. $\frac{abc}{m+n}$.
6. $\frac{23x}{2xy+3y^2}$.
7. $\frac{1}{2(x-y)}$.
8. $\frac{1}{3}$.
9. $\frac{1}{3(x^2+xy+y^2)}$.

SECTION XX.

ART. **65.**

1. $\frac{4\,a\,b^4}{21\,c}$

2. $\frac{48\,x^3\,y^3}{35\,b\,c\,m^2}$.

3. $\frac{12\,x^3+9\,b^2\,x^2}{76+9\,m^2}$.

4. $\frac{8\,x^2+2\,x}{7}$.

5. $\frac{3\,a^3-3\,a^2\,b}{56}$.

6. $\frac{3\,x-4}{4\,a\,x^2-6\,a}$.

7. $\frac{9\,x}{2}$.

8. $\frac{5\,c\,x^2\,y}{27\,b\,m^4}$.

9. $\frac{3}{y}$.

10. $\frac{21\,a\,b\,c\,x-21\,b\,c\,x^2}{a}$.

11. $\frac{4\,a\,b\,c\,x+4\,b\,c\,x^2}{3\,a^2\,m^2-3\,a\,m^2\,x}$.

12. $\frac{3\,a^2\,m\,x^3+3\,a^2\,m\,y^3}{2}$.

SECTION XXI.

ART. **68.**

1. $\frac{h\,n\,p+h\,m\,q+g\,n\,q}{h\,n\,q}$.

2. $\frac{29\,a\,y+35\,x}{35\,y}$.

3. $\frac{24\,a\,b^2+9\,b^3\,x+22\,a^2\,y}{42\,a^2\,b^3}$

4. $\frac{27\,q^2\,x^2\,y+6\,a\,b\,p^2+2\,p\,x\,y^2}{12\,p^3\,q^2}$.

5. $\frac{5\,a\,x\,y^2+5\,b\,x\,y^2+33\,x^2\,y+117\,a}{225\,x^3\,y^2}$.

6. $\frac{17\,a-4\,b}{162}$.

7. $\frac{63\,m\,n^4\,x\,y + 70\,b\,c\,n^3 + 60\,h\,x}{525\,m^3\,n^4}$.

8. $\frac{150\,m\,x^2\,y^2 + 70\,b\,c\,x\,y^2 + 35\,c^2\,y^2 + 56\,a\,x^2}{350\,x^3\,y^2}$

9. $\frac{105\,a^3 + 364\,a^2 + 390\,b}{910\,a}$.

10. $\frac{2 + 2\,x^4}{1 - x^4}$.

11. $\frac{2}{1 - x^2}$.

12. $\frac{3 + 7\,a + 7\,b}{a^2 - b^2}$.

13. $\frac{3\,a^2\,b + 6\,a^2 + 5\,b^2\,m}{4\,a\,b^3 + 8\,a\,b^2}$.

14. $\frac{14\,b - 9\,a\,m}{12\,m^2}$.

15. $\frac{3\,a^2 + 11\,b^2}{196}$.

16. $\frac{2\,a\,c\,x}{b^2 - c^2}$.

17. $\frac{4\,x^2 + 3}{3\,x}$.

18. $\frac{17\,a^2 - 2\,b}{252}$.

19. $\frac{32\,x - 3\,a\,x + 7\,a}{56\,a^2}$

20. $\frac{4\,a^2\,x - 8\,a\,x^2 - 6\,a\,x}{21\,a - 42\,x}$

21. $\frac{4\,x^2}{1 - x^4}$.

22. $\frac{b\,c + m\,p\,x + m\,p\,y}{x + y}$.

23. $\frac{5a+3b}{4}$.

24. $\frac{11m^3+7mn^2-2m^2n+n^3}{7m-2n}$

25. $\frac{4a^2-2b^2}{a^2+b^2}$.

26. $\frac{12m^2xy-3a+7b}{4xy}$.

27. $\frac{5h^3n^3x+h^3n^3}{x+1}$.

28. $\frac{7ab+4c^2-2m^3}{m^3}$

29. $\frac{9x+4bc-a^2-ab-ax-bx}{a+x}$.

30. $\frac{x^2+y^2-mx+my+x-y}{x-y}$.

SECTION XXII.

ART. 69.

1. $\frac{3m}{2}$.

2. $\frac{7ac^3}{3}$.

3. $\frac{4n(x^2+y^2)}{m^2}$.

4. $\frac{5c(a+b)}{2}$.

5. $\frac{5m}{3n}$.

6. $\frac{a^3x^5}{3b^2c^3}$.

7. $\frac{21ay}{bmx}$.

8. $\frac{4bx}{81a^3mp^2}$.

9. $\frac{2b^2(x+y)}{3}$

10. $\frac{10x}{3}$.

11. $\frac{4(x-3)}{5}$.

12. $\frac{9x-3}{x}$.

13. $\frac{4m}{ab}$.

14. $\frac{2a^2x^3}{7my}$.

15. $\frac{2m}{3x^2(x-y)}$.

16. $\frac{9mx^2y^2(1+3b)}{4(2bc+6)}$.

17. $\frac{31a(22m+p)}{429bp^2}$.

18. $\frac{16bc}{11x^3(m-p)}$.

19. $\frac{h^2n^2}{5x(m-1)}$.

20. $\frac{5bc^2}{4(x+y)}$.

SECTION XXIII.

ART. **70.**

1. Performed.

2. $\frac{a^2+b^2c}{a+bc}$.

3. $\frac{3bcd-ad+bc}{b+bdm}$.

4. $\frac{bcm+c^2m-3b^2+6bc}{8c-4b+bd+cd}$.

5. $\frac{2ab+3bc}{5m^2+3a+12c}$.

6. $\frac{ac^2-abc+bcd-c^2d}{a^2-b^2+bc}$.

7. $\frac{d^2}{c}$.

8. $\frac{7b^2c+150d-33b+6abd}{8ad+7bc-33}$

9. $\frac{61b-30a}{18b-9a}$.

SECTION XXIV.

ART. **74.**

1. 1st 8; 2d 3.
2. 1st \$12; 2d \$16.
3. Cow \$25; ox \$35.
4. A \$12; B \$20.
5. Potatoes 2s.; corn \$1 per bushel.
6. Wheat 10 bushels; rye 8 bushels. Sold 5 and 3.
7. Father's \$1; son's 2s.
8. $\frac{5}{7}$.
9. $\frac{9}{10}$.
10. He bought 200 lbs. of coffee and 300 lbs. of tea; he sold 160 lbs. of coffee and 200 lbs. of tea.
11. 200 eggs at 2 for a cent; 100 at 5 for 3 cents
12. Smaller 75, larger 100 gallons.
13. Wheat 9s., oats 3s. per bushel.
14. Larger 160, smaller 80 gallons.
15. Upper part 15 feet; lower part 25 feet.
16. A \$500; B \$700.

SECTION XXV.

ART. **75.**

1st, 2d and 3d performed.

4. Flour \$7, rice \$8 per barrel; sugar \$10 per box.
5. 1st 2; 2d 3; 3d 5.
6. A \$2; B \$0·75; C \$0·50.
7. 1st \$500; 2d \$600; 3d \$800.
8. A's 60, B's 80, C's 90 years.
9. 1st 5s.; 2d 6s.; 3d 7s.; 4th 8s.
10. A \$0·30, B \$0·40, C \$0·50, D \$0·60 per lb.
11. Wheat \$2, rye \$1, barley \$1$\frac{1}{2}$, oats \$$\frac{1}{2}$, potatoes \$$\frac{1}{4}$ per bushel.

SECTION XXVI.

ART. 76.

1st and 2d performed.
3. 2100.
4. 10500.
5. 12600.
6. 4500.
7. 35000.
8. $\frac{3}{7}$.
9. 21.
10. 140.
11. $\frac{8}{43}$.
12. $\frac{1}{12}$.
13. 18.
14. 37.
15. 47.
16. 100.
17. 220.
18. 425.
19. 90.
20. 48.
21. 880.
22. 1011
23. 4.
24. —6.
25. 112.
26. 180.
27. 115.
28. 510.
29. 187.
30 33.
31. —33.
32. 3.
33. 187.
34. 1639.
35. 561.
36. 561.
37. 121.
38. 8470.
39. 4.
40. 2.
41. 24.
42. 0.
43. 18.
44. 1.
45. 1.
46. 2.
47. $\frac{3}{2}$.
48. —1.
49. $\frac{1}{3}$.

The remaining answers in this article being indefinite, are not given.

SECTION XXVII.

ART. 77.

1st and 2d performed
3. 60 and 90.
4. 90 and 140.
5. 540 and 660.
6. 11 and 16.
7. 16 and 19.
8. $33\frac{1}{2}$ and $36\frac{1}{2}$.
9. $17\frac{1}{4}$ and $30\frac{1}{4}$.
10. $32\frac{5}{6}$ and $66\frac{5}{6}$
11th, 12th and 13th performed.
14. 9 oxen, 45 cows, 315 sheep.
15. Performed.
16. $2\frac{8}{11}$ days.
17. $3\frac{3}{10}$ hours.
18. Performed.
19. A $200; B $150; C $100
20. A $54; B $27; C $18.
21. $200; $150; $100; $50.

22. A \$128; B \$80; C \$64
23. Performed.
24. \$69·1875.
25. $p = \frac{i}{tr}$.
26. \$75.
27. $r = \frac{i}{tp}$.
28. ·055 or $5\frac{1}{2}$ per cent.
29. $t = \frac{i}{rp}$.
30. 7 years, 1 month.
31. Performed.
32. \$862·0625.
33. $p = \frac{a}{1+tr}$.
34. \$250.
35. $r = \frac{a-p}{tp}$.
36. ·065 or $6\frac{1}{2}$ per cent.
37. $t = \frac{a-p}{rp}$.
38. $3\frac{1}{2}$ years.
39. $\frac{art}{1+tr}$.
40. £1 4s. $8\frac{1}{4}$d.
41. $t = \frac{1}{r}$, doubled; $t = \frac{2}{r}$, tripled.
42. $16\frac{2}{3}$ years, doubled; $33\frac{1}{3}$ years, tripled.
43. 20 years, doubled; 40 years, tripled.
44. $\frac{a}{1+n}$; $\frac{an}{1+n}$.
45. 1st $\frac{an}{m+n}$; 2d $\frac{am}{m+n}$.
46. $\frac{a}{1+m+n}$; $\frac{am}{1+m+n}$; $\frac{an}{1+m+n}$

47. $\frac{bcd-ab}{c-b}$; $\frac{ac-bcd}{c-b}$.

48. $\frac{am+bmn}{m+n}$; $\frac{an-bmn}{m+n}$

49. $\frac{mnp}{n-m}$.

50. $\frac{amn}{mn-m-n}$ guineas.

51. $\frac{anq}{nq-mq-np}$ dollars.

52. $\frac{ab}{b-a}$ days.

53. $\frac{bc}{c-a}$.

54. $\frac{5a}{b-1}$; $\frac{a(4+b)}{b-1}$; $\frac{a(3+2b)}{b-1}$; $\frac{a(2+3b)}{b-1}$; $\frac{a(1+4b)}{b-1}$; $\frac{5ab}{b-1}$.

55. $\frac{15a}{16}$ shillings.

56. Wrought $\frac{c+bn}{a+b}$ days; idle $\frac{an-c}{a+b}$ days.

57. $\frac{b+d}{c-a}$ beggars.

58 A's $\frac{a+b}{2}$ years; B's $\frac{a-b}{2}$ years.

59. Boots $\frac{ad-b}{ac-1}$ dollars; shoes $\frac{bc-d}{ac-1}$ dollars.

60. 1st $\frac{ac(b-1)}{bc-1}$; 2d $\frac{ab(c-1)}{bc-1}$.

61. Numerator $\frac{ap(m+n)}{mq-np}$; denominator $\frac{an(p+q)}{mq-np}$.

62. Numerator $\frac{ap(m+n)}{np-mq}$; denominator $\frac{an(p+q)}{np-mq}$.

63. Particular answer to 61st $\frac{4}{9}$; particular answer to 62d $\frac{11}{15}$.

SECTION XXVIII.

ART. 84.

From the 1st to the 5th inclusive, performed.

6. This question as enunciated, if x represents the number, gives

$$\frac{7x}{10} = \frac{5x}{7} + 5.$$

From this equation we find

$$x = -350.$$

Changing the sign of x in the original equation, we have

$-\frac{7x}{10} = -\frac{5x}{7} + 5$; or, by transposition,

$$\frac{5x}{7} = \frac{7x}{10} + 5.$$

To rectify the question, therefore, we enunciate it thus:

What number is that $\frac{5}{7}$ of which exceeds $\frac{7}{10}$ of it by 5?

7. This question gives the number of years $= -40$; and the equation being modified accordingly, we find that the question should have been:

How many years after marriage was his age to hers as 7 to 6?

8. If $\frac{x}{y}$ represent the fraction, the question gives

$$\frac{x}{y-2} = \frac{1}{5}; \text{ and } \frac{x-2}{y} = \frac{5}{13}.$$

From these equations we find

$$x = -3, \text{ and } y = -13.$$

Changing the original equations, in a manner similar to that pursued in the solution of the 5th question, we have

$$\frac{x}{y+2} = \frac{1}{5}; \text{ and } \frac{x+2}{y} = \frac{5}{13}; \text{ from which the modification to be made in the enunciation is manifest.}$$

9. Let x = the greater, and y = the less; then,

$$x - y = 20, \text{ and } 6x - 3y = 96.$$

These equations give $x = 12$, and $y = -8$.

Changing the sign of y in the original equations, we have

$$x + y = 20, \text{ and } 6x + 3y = 96;$$ and the question should be changed accordingly.

10. Let x and y represent the number of gallons, which flow through the cocks A and B, respectively, in a minute. Then,

$$5x + 3y = 24, \text{ and } 7x + 5y = 32.$$

These equations give $x = 6$ gallons, and $y = -2$ gallons.

Changing the sign of y in the original equations, we have

$$5x - 3y = 24, \text{ and } 7x - 5y = 32.$$

Hence it appears that water flows *out*, instead of flowing *in*, through the cock B.

11. A's \$5000; B's \$3000; C's — \$2000.

Hence, either C is in debt \$2000; or he possesses \$2000, and a certain number of times his estate is *subtracted*, in each case, instead of being *added*.

SECTION XXIX.

ART. 85.

1. Let x represent his money in cents. Then

$$\frac{x}{3} + \frac{5x}{12} + 40 = \frac{3x}{4} + 49.$$

This equation gives

$3x - 3x = 9$; that is, $(3-3)x$ or $0 \cdot x = 9$; therefore,

$x = \frac{9}{0}$.

Hence, the question is *impossible*, or the value of x is *infinite*.

2. Let x = A's age; then, $x + 10$ = B's, $x + 20$ = C's, and $x - 34$ = D's. We have, therefore,

$$\frac{x+10}{2} + \frac{2x+40}{3} + \frac{5x-170}{6} = 2x - 10.$$

This equation gives

$12x - 12x$ or $0 \cdot x = 60 - 60$; and

$x = \frac{0}{0}$.

Hence, the value of x is *indeterminate;* consequently, their ages cannot be ascertained from the conditions.

3. If x represent the number killed to-day, and y the number killed yesterday, we have

$$\frac{x}{2} = \frac{y}{3} + 5, \text{ and } y = \frac{3x}{2} - 5.$$

These equations become, by multiplication and transposition,

$3x - 2y = 30$, and $3x - 2y = 10$, in which the first members are identical, while the second members are different. Hence, the two conditions are incompatible with each other.

By subtracting and canceling the terms containing y, we have

$3x - 3x = 20$, or $0 \cdot x = 20$, and $x = \frac{20}{0}$.

Or, by subtracting and canceling the terms containing x, we have

$2y - 2y = 20$, or $0 \cdot y = 20$, and $y = \frac{20}{0}$.

The problem, therefore, is *impossible;* or the values of x and y are *infinite.*

Other forms for the answers may be obtained by different methods of elimination; but they are all essentially the same, because they are all infinite.

SECTION XXX.

ART. 86.

1. 28.
2. 53.
3. 78.
4. 29.
5. 36.
6. 93.

7th and 8th performed.

ART. 87.

1. 37.
2. 49
3. 19.
4. 351.

5. ·709.
6. 109.
7. 190.
8. 1049
9. 1197.
10. 12325.
11. 30021.
12. 50309.

SECTION XXXI

Art. 89.

1. $\frac{4}{5}$.
2. $\frac{7}{8}$.
3. $\frac{9}{11}$.
4. $\frac{3}{17}$.
5. $\frac{25}{33}$.
6. $\frac{79}{112}$.

Art. 90.

1. $\frac{8}{9}$ —
2. $\frac{8}{12}$ —.
3. $\frac{6}{11}$ —.
4. $\frac{8}{22}$ +.
5. $\frac{10}{15}$ +.
6. $\frac{23}{25}$ —.

Art. 92.

1. 1·4142 +.
2. 5·1962 —.
3. 5·8095 —.
4. 12·1370 +.
5. 5·8676 —.
6. 18·0537 +.
7. 0·8528 +.
8. 0·8292 —.
9. 0·8660 +.
10. 0·03937 +.
11. 0·03915 +.
12. 2·01008 +.

SECTION XXXII.

Art. 93.

1. A's 21, B's 27 years.
2. A 10, B 12 miles per hour. A went 1)0, B 120 miles.
3. 5 yards and 6 yards.
4. 10 rows; 12 trees in a row; 960 bushels.
5. 30 miles.
6. 50·662 + inches.

7. 12·649 + rods.
8. 43·818 — rods.
9. 16.
10. Greater 60; less 15.
11. Length 30, breadth 25, height 20 feet.
12. 37 yards in each; prices 42s. and 32s. per yard.
13. 30 persons; $150 each.
14. Greater 57; less 45.
15. $150.

SECTION XXXIII.

ART. **95.**

1. By the 2d statement, 9 yards.

ART. **97.**

1st and 2d performed.

3. 15 rods.
4. 9 rods; $3 per rod.
5. Greater 90; less 85.
6. Length 100, breadth 60 rods.
7. 10 sheep; 13 calves.
8. 40 hours; 4 miles per hour.
9. $100.
10. 80 feet.
11. Coffee $0·10, tea $0·50 per pound.
12. 4 rods.
13. 225.
14. 5 sons.
15. 12 fowls.
16. A's stock $100.
17. 2 rods.
8. 10 gallons.

SECTION XXXIV.

ART. **100**.

1. Performed.
2. 67.
3. 39.
4. 91.
5. 77.
6. 335.
7. 364.
8. 909.
9. 3009

SECTION XXXV.

ART. **101**.

1. $\frac{7}{9}$.
2. $\frac{6}{13}$.
3. $\frac{8}{11}$.
4. $\frac{23}{27}$.
5. $\frac{35}{8}$ or $4\frac{3}{8}$.

ART. **102**.

1. $\frac{41}{63}+$.
2. $\frac{105}{195}-$.
3. $\frac{84}{187}+$.

ART. **103**.

1. 1·2599 + or 1·260 —.
2. 1·9129 + or 1·913 —.
3. 4·8629 + or 4·863 —.
4. 1·1447 + or 1·145 —.
5. 2·951 +.
6. 0·292 +.
7. 2·3129 + or 2·313 —.
8. 10·790 +.
9. 0·956 +.
10. 0·7688 + or 0·769 —
11. 1·957 +.
12. 2·3339 + or 2·334 —.
13. 1·542 +.
14. 0·183 +.
15. 2·044 +.
16. 2·823 +.

SECTION XXXVI

ART. **104**.

1. 4, 6 and 10.
2. Length 9, breadth 5, depth 7 feet.

3. 5 and 10.
4. Length 30, depth 8, breadth 22 feet.
5. 16 feet.
6. 18 feet.
7. 6 and 12.
8. 22·071 + inches.
9. Side of the base 31·072 + feet; altitude 124·288 + feet.
10. Radius 2 feet; length 7 feet.
11. 1·8837 + feet.

SECTION XXXVII.

ART. **105.**

1. $49\,a^2\,m^4$.
2. $64\,b^4\,c^2\,x^8$.
3. $225\,a^8\,m^6\,p^{14}$.
4. $128\,x^{14}\,y^{21}$.
5. $b^{26}\,c^{78}\,d^{91}$.
6. $1024\,b^{30}\,c^{20}\,d^{90}$.
7. Performed.
8. $p^{mn}\,q^{mt}$.
9. Performed.
10. $3^m\,p^{mx}\,q^{my}$.
11. $81\,p^8\,q^8$.
12. $-32\,x^{15}\,y^{35}$.
13. $-343\,a^6\,b^6\,c^3\,d^3$.
14. $64\,a^6\,m^{18}\,n^{12}\,p^{30}\,q^{42}\,x^6\,y^6$
15. Performed.
16. $\dfrac{9\,b^2\,c^2}{m^4}$.
17. $\dfrac{125\,m^6\,n^3}{216\,x^9\,y^6}$.
18. $\dfrac{1}{81\,p^8\,x^8}$.

SECTION XXXVIII.

ART. **106.**

1. $(6\,m - n + p)^4$
2. $(b + c + d)^{18}$.
3. $(4\,a\,b + 4\,x\,y)^{14}$.
4. $(x - y)^{26}$
5. $(a + b)^2\,(a - b)^6$.

6. $243\,(x-y)^{15}\,(p-q)^{25}$.
7. $8\,(a+b+c)^{3m}$.
8. $a^4\,m^4\,(c-d)^{4m}\,(x+y)^{4n}$.
9. $(a+b+c)^{3n}$.
10. $(a+b)^{2n}\,(c-d)^{3n}$.
11. $(x+2\,y)^{mn}$.
12. $(a+x)^{mp}\,(n-x)^{mq}$.

13th and 14th performed.

15. $\dfrac{(b+c)^4}{(x+y)^4}$.
16. $\dfrac{(m+n)^6}{(p+q)^9}$.
17. $\dfrac{16\,(a+b)^4\,(x-y)^8}{81\,(c+d)^{32}}$.
18. $\dfrac{a^7\,(m+n)^{21}\,(x-y)^{35}}{b^7\,(r+s)^{14}\,(a+b+c)^{21}}$.

ART. **107**.

1. $m^2-2\,m\,x+x^2$.
2. $a^3\,c+3\,a^2\,b\,c+3\,a\,b^2\,c+b^3\,c$.
3. $a^2\,m+2\,a\,b\,m+b^2\,m+2\,a\,c\,m+2\,b\,c\,m+c^2\,m+a^2\,n+2\,a\,b\,n+b^2\,n+2\,a\,c\,n+2\,b\,c\,n+c^2\,n$.
4. $a^3\,x^2+2\,a^3\,x\,y+a^3\,y^2$.
5. $4\,c^2+12\,c\,d+9\,d^2$.
6. $\dfrac{a^2+2\,a\,b+b^2}{c^3+3\,c^2\,d+3\,c\,d^2+d^3}$.

SECTION XXXIX.

ART. **113**.

1. $a^7+7\,a^6\,b+21\,a^5\,b^2+35\,a^4\,b^3+35\,a^3\,b^4+21\,a^2\,b^5+7\,a\,b^6+b^7$.
2. $x^{10}+10\,x^9\,y+45\,x^8\,y^2+120\,x^7\,y^3+210\,x^6\,y^4+252\,x^5\,y^5+210\,x^4\,y^6+120\,x^3\,y^7+45\,x^2\,y^8+10\,x\,y^9+y^{10}$.
3. $m^5-5\,m^4\,n+10\,m^3\,n^2-10\,m^2\,n^3+5\,m\,n^4-n^5$.

4. $b^{11} + 11\,b^{10}\,c + 55\,b^9\,c^2 + 165\,b^8\,c^3 + 330\,b^7\,c^4 + 462\,b^6\,c^5 + 462\,b^5\,c^6 + 330\,b^4\,c^7 + 165\,b^3\,c^8 + 55\,b^2\,c^9 + 11\,b\,c^{10} + c^{11}$.

5. $x^{13} - 13\,x^{12}\,y + 78\,x^{11}\,y^2 - 286\,x^{10}\,y^3 + 715\,x^9\,y^4 - 1287\,x^8\,y^5 + 1716\,x^7\,y^6 - 1716\,x^6\,y^7 + 1287\,x^5\,y^8 - 715\,x^4\,y^9 + 286\,x^3\,y^{10} - 78\,x^2\,y^{11} + 13\,x\,y^{12} - y^{13}$.

6. Performed.

7. $x^5 + 10\,x^4\,y + 40\,x^3\,y^2 + 80\,x^2\,y^3 + 80\,x\,y^4 + 32\,y^5$.

8. $216\,a^3 + 540\,a^2\,x + 450\,a\,x^2 + 125\,x^3$.

9. Performed.

10. $32\,a^5 + 240\,a^4\,x + 720\,a^3\,x^2 + 1080\,a^2\,x^3 + 810\,a\,x^4 + 243\,x^5$.

11. $64\,x^3 - 144\,x^2\,y + 108\,x\,y^2 - 27\,y^3 + 48\,a\,x^2 - 72\,a\,x\,y + 27\,a\,y^2 + 12\,a^2\,x - 9\,a^2\,y + a^3$.

12. $a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3 + 3\,a^2\,c + 6\,a\,b\,c + 3\,b^2\,c + 3\,a^2\,d + 6\,a\,b\,d + 3\,b^2\,d + 3\,a\,c^2 + 6\,a\,c\,d + 3\,a\,d^2 + 3\,b\,c^2 + 6\,b\,c\,d + 3\,b\,d^2 + c^3 + 3\,c^2\,d + 3\,c\,d^2 + d^3$.

13. $a^6 + 12\,a^5\,b - 6\,a^5\,c + 60\,a^4\,b^2 - 60\,a^4\,b\,c + 15\,a^4\,c^2 + 160\,a^3\,b^3 - 240\,a^3\,b^2\,c + 120\,a^3\,b\,c^2 - 20\,a^3\,c^3 + 240\,a^2\,b^4 - 480\,a^2\,b^3\,c + 360\,a^2\,b^2\,c^2 - 120\,a^2\,b\,c^3 + 15\,a^2\,c^4 + 192\,a\,b^5 - 480\,a\,b^4\,c + 480\,a\,b^3\,c^2 - 240\,a\,b^2\,c^3 + 60\,a\,b\,c^4 - 6\,a\,c^5 + 64\,b^6 - 192\,b^5\,c + 240\,b^4\,c^2 - 160\,b^3\,c^3 + 60\,b^2\,c^4 - 12\,b\,c^5 + c^6$.

14. $a^5 + 5\,a^4\,b + 10\,a^3\,b^2 + 10\,a^2\,b^3 + 5\,a\,b^4 + b^5 - 10\,a^4\,c - 40\,a^3\,b\,c - 60\,a^2\,b^2\,c - 40\,a\,b^3\,c - 15\,a^4\,d - 60\,a^3\,b\,d - 90\,a^2\,b^2\,d - 60\,a\,b^3\,d - 15\,b^4\,d + 40\,a^3\,c^2 + 120\,a^2\,b\,c^2 + 120\,a\,b^2\,c^2 + 40\,b^3\,c^2 + 120\,a^3\,c\,d + 360\,a^2\,b\,c\,d + 360\,a\,b^2\,c\,d + 120\,b^3\,c\,d + 90\,a^3\,d^2 + 270\,a^2\,b\,d^2 + 270\,a\,b^2\,d^2 + 90\,b^3\,d^2 - 80\,a^2\,c^3 - 360\,a^2\,c^2\,d - 540\,a^2\,c\,d^2 - 270\,a^2\,d^3 - 160\,a\,b\,c^3 - 720\,a\,b\,c^2\,d - 1080\,a\,b\,c\,d^2 - 540\,a\,b\,d^3 - 80\,b^2\,c^3 - 360\,b^2\,c^2\,d - 540\,b^2\,c\,d^2 - 270\,b^2\,d^3 + 80\,a\,c^4 + 480\,a\,c^3\,d + 1080\,a\,c^2\,d^2 + 1080\,a\,c\,d^3 + 405\,a\,d^4 + 80\,b\,c^4 + 480\,b\,c^3\,d + 1080\,b\,c^2\,d^2 + 1080\,b\,c\,d^3 + 405\,b\,d^4 - 32\,c^5 - 240\,c^4\,d - 720\,c^3\,d^2 - 1080\,c^2\,d^3 - 810\,c\,d^4 - 243\,d^5$.

Erratum. In 2d line of 14th, insert $- 10\,b^4\,c$.

SECTION XL

ART. **115.**

1. 5.
2. 12.
3. 73.
4. 73.
5. 2·11.
6. 13.

SECTION XLI.

ART. **117.**

1. $\pm a^2 m^3 x$.
2. $\pm 8 x y$.
3. $7 a^3 p q^2$.
4. $-9 a^2 b c^4$.
5. $\pm 2 a^2 b^4 c^3$.
6. $\pm \frac{2a}{5x^2}$.
7. $\frac{3x^3 y}{4m^4}$.
8. $-\frac{2a^2 b^3}{5 x y^2}$
9. $\frac{3 a^2 b^3 c}{4 x^4 y^5}$.

ART. **119.**

In this article the answers are indefinite, and, therefore, the, are not given.

SECTION XLII.

ART. **120.**

1. $3x^2 + 2x + 5$.
2. $a^2 + 6ab + 9b^2$.
3. $3x^3 - 2x^2 + x - 1$
4. $3a^2 - 2ab + 5b^2$.
5. $2x^2 + 2ax + 4b^2$.
6. $x^3 + 2x^2 - x + 2$.
7. $2x^2 + \frac{3x}{2} + 5$.

ART. **121.**

1. $3a + 3x$.
2. $2x^3 + 5a^2$.
3. $5c^2 - 2yz$.
4. $2a^2 - b^3$.
5. $3x + 2y$.

SECTION XLIII.

ART. **122.**

1. $5\,a\,b^2\sqrt{5\,a\,b}$.
2. $4\,c^2\,(5\,a\,b)^{\frac{1}{2}}$
3. $3\,a^3\,b^2\sqrt[3]{4\,c^2}$.
4. $3\,b\,c\sqrt{5\,a}$.
5. Performed.
6. $2\,a\sqrt[3]{3\,a-b}$.
7. $a\,(2\,b^2+a^2\,b\,c)^{\frac{1}{3}}$
8. $a\sqrt[3]{1+b^3}$.
9. $4\,a\sqrt[4]{3\,a^2-1}$.
10. $24\sqrt{6\,a^3\,b-3}$.
11. $\frac{8}{9\,b}\sqrt{3\,a\,b}$.
12. $\frac{5\,a^2}{3\,c\,d}\,(5\,b\,d)^{\frac{1}{2}}$.
13. $\frac{1}{2}\,c\sqrt[3]{2\,a^2\,b^2}$.
14. $\frac{1}{2}\,b\,(6\,a\,b^2)^{\frac{1}{3}}$.
15. $\frac{a}{4\,c}\sqrt{3\,b\,c}$.
16. $\frac{2\,a}{3\,c}\sqrt{\frac{3\,a\,b-2\,b^2}{3\,c^2-d}}$.
17. $\frac{7\,a}{5\,m}\sqrt{\frac{28\,a^2\,b-7\,a\,b}{20\,m^2-10\,m\,n}}$.
18. $\frac{4\,a}{3\,c}\sqrt[3]{\frac{5\,b+10\,a^2}{c\,m+2\,m^2}}$.

ART. **123.**

1. $\sqrt{9\,a^2\,b\,x^2}$.
2. $\sqrt{\frac{20\,b\,c}{9}}$.
3. $\sqrt[3]{\frac{8\,(a-b)}{27\,(c+d)}}$.
4. $\sqrt{\frac{c\,(a+b)^2}{3\,d}}$.
5. $(12\,a^2\,b)^{\frac{1}{2}}$.
6. $(125\,x\,y)^{\frac{1}{3}}$.
7. $(16\,a^4\,b^4\,x+32\,a^4\,b^4\,y)^{\frac{1}{4}}$.
8. $\left(\frac{27\,a^3\,m^2}{64\,b^3}\right)^{\frac{1}{3}}$.

SECTION XLIV

ART. **125.**

1. $4\,a^2\,(3\,x)^{\frac{1}{2}}$.
2. $14\,(2)^{\frac{1}{2}}$.
3. $5\,(5)^{\frac{1}{3}}$.
4. $23\,(3)^{\frac{1}{2}}$.

5. $8\,(2)^{\frac{1}{2}}$.

6. $8\,(2)^{\frac{1}{3}}$.

7. $5\,a\,b^{\frac{1}{2}} + 8\,a\,b^{\frac{1}{3}}$

8. $\frac{17}{15}\,(15)^{\frac{1}{2}}$.

9. $\frac{4}{5}\,a\,(10)^{\frac{1}{2}}$.

10. $\frac{1}{2}\,(13 + 2\,c)\,(a\,b)^{\frac{1}{2}} + (x\,y)^{\frac{1}{2}}$.

11. $(2)^{\frac{1}{2}}$.

12. $2\,x\,(3\,a)^{\frac{1}{2}}$.

13. $4\,a\,(2\,b)^{\frac{1}{3}}$.

14. $2\,a\,(3\,a\,b^2)^{\frac{1}{3}}$.

15. $\frac{11}{60}\,(6)^{\frac{1}{2}}$.

16. $(5)^{\frac{1}{2}}$.

17. $(a\,b)^{\frac{1}{2}} + (10\,c - 7)\,(m)^{\frac{1}{3}}$.

18. $\frac{5}{72}\,(3\,a\,b)^{\frac{1}{2}} + \frac{1}{60}\,(25\,m)^{\frac{1}{3}}$.

19. $21\,a\,b^2$.

20. $10\,a^{\frac{3}{2}}\,b^{\frac{3}{2}}\,c^{\frac{3}{2}}$, or $10\,a\,b\,c\,(a\,b\,c)^{\frac{1}{2}}$

21. $3\,m^2\,a^{\frac{39}{40}}\,c^{\frac{13}{20}}$.

22. $75\,x^{\frac{5}{6}}\,y^{\frac{7}{6}}$, or $75\,y\,(x^5\,y)^{\frac{1}{6}}$

23. Performed.

24. $30\,(10)^{\frac{1}{2}}$.

25. Performed.

26. $8\,(216)^{\frac{1}{6}}$.

27. 140.

28. $a^{\frac{m+n}{m\,n}}$

29. $15\,a^{\frac{m\,q+n\,p}{n\,q}}$.

30. $(648000)^{\frac{1}{12}}$.

31. $(a+b)^{\frac{13}{12}}$.

32. $12\,a\,(c-d)^{\frac{57}{56}}$

33. $12\,a^2\,b\,(a+b)^{\frac{1}{3}}\,(x-y)^{\frac{41}{30}}$

34. $35\,(m+n)^{\frac{5}{6}}\,(c-d)^{\frac{8}{15}}$.

35. 4.

36. $3-17\,(6)^{\frac{1}{2}}$

37. 41.

38. a^2.

39. $a^{\frac{2}{3}}\,b^{\frac{1}{3}}\,c^{\frac{4}{5}}$.

40. $2\,a^{\frac{1}{10}}\,b^{\frac{3}{2}}\,c$.

41. $\frac{3}{4}\,a^{\frac{1}{6}}\,b^{\frac{1}{3}}\,c^{\frac{5}{3}}$

42. 6.

43. 15.

44. $8\,(2)^{\frac{1}{2}}$.

45. $a^{\frac{n-m}{mn}}$.

46. $\frac{3}{4}\,a^{\frac{p-m}{mp}}\,b^{\frac{q-n}{nq}}$

47. $\frac{55}{42}\,(2)^{\frac{2}{3}}$

48. $(a-b)^{\frac{1}{8}}$.

49. $\dfrac{3\,(x-y)^{\frac{4}{6}}}{a}$.

50. $\dfrac{(a+b)^{\frac{1}{8}}\,(c-d)^{\frac{2}{15}}}{3\,m}$

51. $4\,a^{\frac{2}{3}}\,b^{\frac{1}{2}}$.

52. $125\,a^6\,b^{\frac{3}{5}}\,c$.

53. $\dfrac{8\,a^{\frac{3}{2}}\,b^2}{27\,x^2\,y^3}$.

54. $\frac{1}{16}\,(3)^{\frac{4}{3}}$.

55. $\frac{288\, a^{\frac{5}{4}}\, b^5\, c^{10}}{m}$.

56. $(a+b)^{\frac{3}{5}}$.

57. $27\,(x-y)^{\frac{7}{3}}$.

58. $32\,(x^2-y^2)^{\frac{35}{8}}\,(b-c)^{\frac{15}{4}}$.

59. $a^{\frac{m}{6}}\, b^{\frac{3m}{7}}\, c^m$.

60. $a^{\frac{m}{n}}\, b^{\frac{m}{p}}\, c^m\, d^m$.

61. $\frac{8\,(a+b)^{\frac{3}{5}}}{27\,(c-d)^{\frac{2}{5}}}$.

62. $\frac{4\,(x+y)^{\frac{2}{5}}\,(c-d)^{\frac{1}{3}}}{25\,(m+n)^{\frac{1}{2}}\,(x-y)^{\frac{3}{2}}}$.

63. $a^{\frac{1}{7}}\, b^{\frac{2}{5}}$.

64. $3\, a^{\frac{1}{5}}\, b^{\frac{2}{7}}$.

65. $2^{\frac{1}{3}}\, a^{\frac{1}{6}}\, b^{\frac{1}{3}}$.

66. $3^{\frac{1}{4}}\, a^{\frac{3}{28}}\, b^{\frac{1}{2}}\, c^{\frac{1}{4}}$.

67. $10^{\frac{1}{m}} a^{\frac{3}{4m}}\, x^{\frac{1}{m}}\, y^{\frac{1}{3m}}$

68. $6^{\frac{1}{m}}\, a^{\frac{p}{q\,m}}\, b^{\frac{r}{s\,m}}$

69. $(a+b)^{\frac{1}{6}}$

70. $4\,(a-b)^{\frac{2}{5}}\,(c-d)^{\frac{1}{6}}$.

71. $2\,a\,(m-n)^{\frac{2}{3}}\,(c-d)^{\frac{1}{6}}$.

72. $\frac{3^{\frac{1}{3}}\,(x-y)^{\frac{2}{21}}\,(a^2+d)^{\frac{2}{3}}}{5^{\frac{1}{3}}\, b^{\frac{1}{3}}\,(m+n)^{1\frac{2}{15}}}$.

SECTION XLV

ART. **132.**

1. $7\sqrt{2}$.
2. $6\sqrt{b}$.
3. $(6a+5)\sqrt{y}$.
4. $8\sqrt[3]{4}$.
5. $43\sqrt{3}$.
6. $\frac{4}{5}\sqrt{10}$.
7. $191\sqrt{3}$.
8. $\frac{21}{4}\sqrt[3]{2}$.
9. $\frac{1}{6}(3a+4x^2)\sqrt{b}$.
10. $-13\sqrt{3}$.
11. $8\sqrt[3]{2}$.
12. $8\sqrt{7}-\sqrt[3]{3}$.
13. $ab(3a+5)\sqrt{2ab}$.
14. $(a-c)\sqrt{5c}$.
15. $\left(\frac{a(ade+bce-bd^2)}{b^2de}\right)\sqrt{bc}$.
16. $3\sqrt{2}$.
17. $4\sqrt{7}$.
18. $2\sqrt[3]{3}$.
19. $\sqrt{5}$.
20. $2\sqrt[3]{5}$.
21. $\frac{4}{45}\sqrt{15}$.
22. $\sqrt{2}$.

23. $\sqrt[3]{9}$.
24. $2a(2a-x)\sqrt{5x}$.
25. $2a(4-a)\sqrt[3]{b}$.
26. $4(\sqrt[3]{4}-\sqrt{2})$.
27. $\frac{2}{15}\sqrt[3]{75}$.
28. $8\sqrt[3]{2}$.
29. $\frac{1}{6}\sqrt{3}+\frac{1}{12}\sqrt{6}$
30. 12.
31. 4.
32. 2.
33. $\sqrt{ab}$.
34. $6a\sqrt{bc}$.
35. $5a^2c\sqrt[6]{ac}$.
36. $30\sqrt{10}$.
37. $6\sqrt[6]{432}$.
38. $4a^2-9b$.
39. $21+18\sqrt{3}+28\sqrt[3]{2}+24\sqrt[6]{108}$.
40. $1-\sqrt{5}$.
41. $1+\sqrt{6}$.
42. $bc\sqrt[15]{a^{13}}$.
43. $\frac{1}{2}$.
44. $\frac{9}{40}\sqrt{15}$.
45. 12.
46. $72\sqrt[12]{23328}$
47. $24\sqrt[30]{a^{21}b^{21}m^{10}}$.
48. $\frac{1}{6}\sqrt[42]{(\frac{1}{2})^{21}(7)^{14}b^6}$.
49. 2.

50. $4\sqrt{b}$.

51. $\frac{3}{5xy}\sqrt{xy}$.

52. $12\sqrt{2}$.

53. $\frac{3}{4}\sqrt{10}$.

54. $\sqrt[6]{7}$.

55. $\frac{3}{2}\sqrt[15]{a^2b^2}$.

56. $\frac{a}{2bc}\sqrt[6]{4c^4x^3y^3}$.

57. $\frac{a^2+2a\sqrt{b}+b}{a^2-b}$.

58. $\frac{1}{7}(10-\sqrt{2})$.

59. $\frac{1}{2}(\sqrt{3}-1)$.

60. $\sqrt[3]{a^2}$.

61. $25\sqrt[5]{b^4}$.

62. $9\sqrt{ab}$.

63 $64\sqrt{ax}$.

64. $a^4bc\sqrt{b}$.

65. $a^3b^3m^{10}\sqrt[3]{ab}$.

66. $x^my^m\sqrt{a^mb^m}$

67. $\sqrt[n]{xy}$.

68. $\frac{3}{8}\sqrt{3}$.

69. $\frac{1}{36}a^2$.

70. 3.

71. $\sqrt{(a+b)^{10}}$ or $(a+b)^5$.

72. $2\sqrt[3]{ab}$.

73. $3\sqrt[18]{ab}$

74. $4\sqrt[9]{a\,m}$.
75. $2\sqrt[5]{a\,b\,c^2}$.
76. $\frac{1}{2}\sqrt{a\,b}$.
77. $\sqrt[10]{a\,m}$.
78. $\sqrt[3m]{(a+b)^2}$.
79. $\sqrt[3]{(a-b)^2}$.
80. $\sqrt[mn]{x+y}$.
81. $\sqrt[6]{9\,(a+b)}$

SECTION XLVI

ART. **134.**

1. $7\,m^7\,n^{-5}\,x^2$.
2. $12\,a^{-6}\,b^{-5}\,c^5$.
3. $30\,a^{\frac{1}{6}}\,b^{\frac{2}{5}}\,c^{\frac{4}{5}}$.
4. $100\,a^{\frac{6}{5}}\,b^3\,c^{\frac{4}{3}}$.
5. 1.
6. $\frac{9\,a^2\,c\,m^3}{4}$.
7. $14\,a^3\,m^{-1}\,r^{-1}\,x\,y$.
8. $\frac{49\,a^{-3}\,b^{-1}\,c^{-3}\,m^{-3}\,x^2\,y}{5}$.
9. $\frac{4\,(c-d)\,(a+b)^{-3}}{3}$.
10. $\frac{49\,a^3\,b\,(x+y)^2\,(m-n)\,(m+n)^{-1}\,(c-d)^{-1}}{5}$
11. $\frac{a^{-2}\,m^{-1}}{2}$.
12. $\frac{5\,a^3\,x^{-9}\,y^{-2}}{3}$.
13. $\frac{a^5\,b^{-1}\,c^{-1}\,m\,x^7}{4}$
14. $3\,a^3\,b^3\,c^2\,m^{-1}\,x^4$.
15. $\frac{7\,a^{-1}\,b^{-2}\,c^{-2}\,x^{-1}}{3}$.

16. $\frac{21\,(a+b)\,(x+y)\,(c-d)^{-1}}{2}$.

17. $\frac{22\,a^{-1}\,(2\,c-3\,d)^{-3}\,(m+n)^{-1}\,(x-y)^{-1}}{5}$.

18. $9\,a^{-2}\,b^{-4}\,c^{6}$.

19. $64\,a^{-6}\,b^{6}\,c^{-9}$.

20. $16\,a^{2}\,b^{-\frac{4}{3}}\,c^{-\frac{8}{7}}$.

21. $1000\,m^{-\frac{1}{2}}\,x^{-1}\,y^{6}$.

22. $4\,a^{-1}\,b^{-2}\,c^{3}$.

23. $2\,a^{-3}\,b\,c^{-2}$.

24. $3\,a^{\frac{1}{4}}\,b^{-\frac{1}{4}}\,c^{-\frac{1}{2}}\,d^{-1}$.

25. $3\,a^{\frac{1}{6}}\,b^{-\frac{1}{9}}\,c^{-\frac{1}{12}}$.

26. $3^{\frac{1}{5}}\,a^{-\frac{1}{10}}\,x^{\frac{1}{5}}\,y^{-\frac{1}{7}}\,m^{-\frac{1}{15}}$.

SECTION XLVII.

ART. **136.**

1. Performed
2. 20.
3. 51.

SECTION XLVIII.

ART. **137.**

1. 16.
2. $5\frac{1}{2}$.
3. 5 and 15.
4. $12\frac{3}{4}$.
5. 6.
6. 1st term 10; 3d 18.

SECTION L.

ART. **143.**

1. Performed.

2. $n=\dfrac{l+d-a}{d}$; $S=\dfrac{(l+a)(l+d-a)}{2d}$.

3. $l=\dfrac{2S-an}{n}$; $d=\dfrac{2(S-an)}{n(n-1)}$

4. $n=\dfrac{2S}{a+l}$; $d=\dfrac{(l+a)(l-a)}{2S-(l+a)}$.

5. Performed.

6. $l=\dfrac{-d\pm\sqrt{(2a-d)^2+8dS}}{2}$; $n=\dfrac{d-2a\pm\sqrt{(2a-d)^2+8dS}}{2d}$

7. $a=\dfrac{2S-dn(n-1)}{2n}$; $l=\dfrac{2S+dn(n-1)}{2n}$.

8. $a=l-d(n-1)$; $S=\dfrac{2nl-dn(n-1)}{2}$.

9. $a=\dfrac{2S-nl}{n}$; $d=\dfrac{2(nl-S)}{n(n-1)}$.

10. $a=\dfrac{d\mp\sqrt{(d+2l)^2-8dS}}{2}$; $n=\dfrac{d+2l\pm\sqrt{(d+2l)^2-8dS}}{2d}$

Let the learner verify the preceding formulæ, by making $a=1$, $d=2$, $n=4$, $l=7$, and $S=16$.

SECTION LI.

ART. **145.**

1. 78 strokes.
2. $l=65$; $S=425$
3. $l=70$; $S=1360$.
4. $l=48\frac{1}{4}$; $S=903\frac{3}{4}$.
5. $l=26\frac{5}{8}$; $S=414\frac{3}{4}$.
6. The progression is 20, 25, 30, 35, 40, 45, 50, 55.

7. The progression is 6, $6\frac{2}{3}$, $7\frac{1}{3}$, 8, $8\frac{2}{3}$, $9\frac{1}{3}$, 10.

8. The progression is 5, 7, 9, 11, &c., all the odd numbers to 41 inclusive.

9. 300 strokes.

10. 950 trees.

11. 31700 feet.

12. $1608\frac{1}{3}$.

13. $S = an^2$.

14. 8 days.

15. 5 days and 10 days; 55 miles and 80 miles.

16. 12, 17, 22.

17. 4, 6 and 8.

18. 14, 12, 10, 8 and 6.

19. 531.

SECTION LII.

ART. **151.**

1. Performed.

2. $q = \sqrt[n-1]{\frac{l}{a}}$; $S = \frac{\sqrt[n-1]{l^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{l} - \sqrt[n-1]{a}}$.

3. $a = \frac{l}{q^{n-1}}$; $S = \frac{l(q^n-1)}{q^{n-1}(q-1)}$.

4. $a = \frac{S(q-1)}{q^n-1}$; $l = \frac{Sq^{n-1}(q-1)}{q^n-1}$.

Let the learner verify the preceding formulæ, by making $a=2$, $q=3$, $n=3$, $l=18$, and $S=26$.

SECTION LIII

ART. **152.**

1. $l=640$; $S=1275$.

2. $l=\frac{1}{81}$; $S=1\frac{40}{81}$

3. 2, 10 and 50.
4. $4095.
5. Performed.
6. 3, 15, 75, 375 and 1875.
7. 7, 35 and 175.
8. 8, 24 and 72.
9. Any progression in which the ratio is 2.
10. 3, 5 and 7.
11. 6.
12. 10.
13. 7·937 +.
14. 178·535 +.
15. 2.
16. 6.
17. 21.
18. $91\frac{1}{3}$.
19. Progression 2, 6, 18, 54, 162.
20. Progression 5, 30, 180, 1080.
21. Progression 3, 15, 75, 375, 1875, 9375, 46875.
22. Progression 1, 2, 4, 8, 16, 32, 64, 128, 256, 512.

SECTION LIV.

ART. **153.**

1st to 5th inclusive, performed.
6. 626.
7. 10, or −2.
8. 4.
9. $\frac{9}{4}$.
10. 9.
11. 81.

12th to 15th inclusive, performed.
16. 2, or −3.
17. $\pm\sqrt{2}$, or $\pm\sqrt{-6}$.
18. 3, or $\sqrt[3]{-19}$.
19. 25, or 49.
20. ± 8, or $\pm\sqrt{-1331}$
21. 729, or $(-\frac{1}{4}^3)^6$.
22. 3, or $\sqrt[3]{-41}$.
23. 2, or −3.
24. 25, or 81.
25. $(\frac{7}{8} \pm \sqrt{-\frac{47}{64}})^3$.

26. 27, or $(-\frac{6}{10})^{\frac{3}{2}}$.

27. $\left(\frac{2 \pm \sqrt{724}}{3}\right)^2$.

28. 81, or $(-\frac{6}{3})^4$.

29. 4, or $\frac{64}{9}$.

30. 4, or 1.

31. $\left(\frac{\pm\sqrt{b^2 \mp 4ac} - b}{2a}\right)^{\frac{4}{3}}$.

32d and 33d performed.

34. 9, or -12.

35. 10, or -2.

36. 1, or $1 \pm \sqrt{60}$.

37. 6, or -9.

38. 9, or $5 + \sqrt[3]{25}$.

39. 5, or -6, or $\frac{\pm\sqrt{377} - 1}{2}$

40. 3, or $-\frac{1}{2}$, or $\frac{5 \pm \sqrt{1329}}{4}$.

SECTION LV.

ART. **154.**

1. Performed.

2. $x = 2\frac{1}{2}$, $y = 4$; or $x = 2$, $y = 5$.

3. $x = 18$, $y = 3$; or $x = 12\frac{1}{2}$, $y = -2\frac{1}{2}$.

4. $x = 2$, $y = 3$; or $x = -46$, $y = 15$.

5. $x = 4$, $y = 2$; or $x = -4$, $y = -2$.

6th to 11th inclusive, performed.

12. $x = 6$, $y = 3$; or $x = -6$, $y = -3$.

13. $x = 7$, $y = 3$; or $x = 3$, $y = 7$.

14. $x = 3$, $y = 5$; $x = -3$, $y = -5$; $x = 5$, $y = 3$; or $x = -5$, $y = -3$.

15. $x=9, y=3$; or $x=-9, y=-3$.

16. $x=49, y=25$.

17. $x=625, y=16$.

18. $x=5, y=4$; or $x=-5, y=-4$.

19. $x=7, y=3$; or $x=-3, y=-7$.

20. $x=1, y=2$; $x=-1, y=-2$; $x=2, y=1$; or $x=-2, y=-1$.

21. $x=2, y=4$; $x=4, y=2$; $x=3+\sqrt{21}, y=3-\sqrt{21}$; or $x=3-\sqrt{21}, y=3+\sqrt{21}$.

22. $x=1, y=2$; or $x=2, y=1$.

23. $x=4, y=9$; or $x=9, y=4$.

24. $x=8, y=2$; or $x=2, y=8$.

25th and 26th performed.

27. $x=20, y=16$.

28. $x=5, y=3$.

29. $x=8, y=6$; $x=-8, y=-6$; $x=6, y=8$; or $x=-6, y=-8$.

30. $x=14, y=12$.

31. $x=27, y=148\frac{1}{2}$; or $x=6, y=33$.

32. $x=5, y=\pm 3$.

33. Performed.

34. $x=3, y=1$; $x=-3, y=-1$; $x=8\sqrt{\frac{1}{6}}, y=\sqrt{\frac{1}{6}}$; or $x=-8\sqrt{\frac{1}{6}}, y=-\sqrt{\frac{1}{6}}$.

35. $x=4, y=3$; $x=-4, y=-3$; $x=11\sqrt{\frac{1}{43}}, y=-12\sqrt{\frac{1}{43}}$; or $x=-11\sqrt{\frac{1}{43}}, y=12\sqrt{\frac{1}{43}}$.

36. $x=1, y=2$; $x=-1, y=-2$; $x=\frac{2}{5}, y=-\frac{11}{5}$; or $x=-\frac{2}{5}, y=\frac{11}{5}$.

37. $x=4, y=2$; $x=-4, y=-2$; $x=-\sqrt{7}, y=\sqrt{7}$; or $x=\sqrt{7}, y=-\sqrt{7}$.

SECTION LVI.

ART. **155.**

1. 2048.
2. 65536.
3. 1048576.
4. 4194304.
5. 131072.
6. 4194304.
7. 32.
8. 32.
9. 16.
10. 1024.
11. 16.
12. 64.
13. 32768.
14. 16384.
15. 65536.
16. 1048576.
17. 1048576.
18. 1048576.
19. 32.
20. 8.
21. 8.
22. 16.
23. 8.
24. 8.

SECTION LVII.

ART. **162.**

1. $3{\cdot}1048284$.
2. $4{\cdot}7581015$.
3. $\bar{2}{\cdot}1048284$.
4. $\bar{3}{\cdot}4456042$.
5. $5{\cdot}5152060$.
6. $4{\cdot}1041671$.
7. $0{\cdot}4392806$.
8. $2{\cdot}5228783$.
9. $2{\cdot}6389892$.
10. $2{\cdot}0467815$

ART. **163.**

1. 30.
2. 2000.
3. 31·55868.
4. 1309·534.
5. 221754·6.
6. 1·63472.
7. 0·3048.
8. 0·0593562.
9. 0·0016891.
10. 0·0002155.

ART. **167.**

1. $\bar{1}{\cdot}8593785$, or $9{\cdot}8593785$.
2. $\bar{3}{\cdot}7604225$, or $7{\cdot}7604225$.
3. $\bar{4}{\cdot}5789370$, or $6{\cdot}5789370$
4. $\bar{1}{\cdot}8239087$, or $9{\cdot}8239087$

5. $\bar{1}$·2179083, or 9·2179083.
6. 0·7262335.
7. 0·5682017.
8. 2·6596926.
9. 2·1628630.
10. $\bar{1}$·3322956, or 9·3322956

ART. **168**.

1. 0·273603.
2. 0·0536312.
3. 0·0002.
4. 0·0000000471435.
5. 0·00003.
6. 0·772481.
7. 0·0676142.
8. 0·00274074.

SECTION LVIII.

ART. **169**.

1st to the 13th inclusive, performed.

14. 151·807.
15. 77190.
16. 324·324.
17. 0·0035991.
18. 7·71707.
19. 3·01182.
20. 3581·242.
21. 4577·626.
22. 49·9336.
23. 5932·52.
24. 4770·1902.
25. 0·187295.
26. 9·33333.
27. 7·1197.
28. 55·54569.
29. 768·575.
30. 0·0000000243.
31. 165·3817.
32. 2·236068.
33. 3·48428.
34. 0·422838.
35. 0·77992.
36. 0·99285.
37. 0·867362.
38. 0·71254.
39. 0·80818.
40. 1·11204.
41. 0·59905.
42. 0·4809.
43. 172·002.
44. Performed.
45. 2·3774.
46. 1·1291.
47. 1·9699.
48. Performed.
49. 7·05.
50. 30·787.
51. Progression 7, 8·6394, 10·662, 13·16, 16·242, 20

ART. **170.**

1. Performed.

2. $q = \frac{S-a}{S-l}$; $n = \frac{\log. l - \log. a}{\log. (S-a) - \log. (S-l)} + 1.$

3. $l = \frac{a+S(q-1)}{q}$; $n = \frac{\log. (a+Sq-S) - \log. a}{\log. q}$.

4. $a = lq - S(q-1)$; $n = \frac{\log. l - \log. (S-Sq+lq)}{\log. q} + 1$

5. $S = 12285$; $n = 12$.

6. $q = 3$; $n = 8$.

7. $l = 177147$; $n = 10$.

8. $a = 2$; $n = 8$.

SECTION LIX.

ART. **171.**

1st to 5th inclusive, performed.

6. \$569·801.

7. \$1948·67.

8. \$333·24.

9. \$1829·71.

10. \$250.

11. 14·137 years, or 14 years, 1 month, 29 days

12. 0·06085, or $6\frac{1}{10}$ per cent. nearly.

13. 11·8956 years, or 11 years, 10 months, 22 days

14. 18·854 years, or 18 years, 10 months, 7 days.

15. 14·2067 years, or 14 years, 2 months, 14 days

16. \$644·781.

17. 83310.

18. 0·03205.

19. 26·91 years, or 26 years, 10 months, 28 days.

ART. **172.**

1. Performed. 2. \$3558·094

SECTION LX.

ART. **173**.

1st 2d and 3d performed.
4. $7686·22.
5. $7857·98.
6. $1169·84.
7. 19·387 years, or 19 years, 4 months, 19 days.

MISCELLANEOUS QUESTIONS

1. A $948, B $1896, C $2844, D $4740.
2. 3 yards at 9s.; 5 yards at 7s.
3. 8 years.
4. $871\frac{1}{3}$.
5. 60 years.
6. 55.
7. 4 days.
8. 20.
9. Breadth 30, length 40 rods.
10. 10 eagles; $30.
11. A $60; B $48.
12. $1000.
13. $50.
14. £400, £500, £700.
15. 800 men.
16. $20.
17. £280.
18. 14 beggars; 32s.
19. $6\frac{3}{4}$.
20. 40, 20 and 12.
21 Wheat 10s., rye 6s. per bushel
22. $\frac{4}{21}$.
23. 23.
24 21 and 16.

25. 17 and 8.
26. 24s.
27. 53.
28. A $20; B $50.
29. Greater 79; less 35.
30. A £300; B £350.
31. Length 20, breadth 18 feet
32. A $1; B $1½; C $2.
33. 15 and 12.
34. 10 and 8.
35. 12 and 19.
36. 8 and 15.
37. 40 sheep.
38. A £10; B £20; C £30; D £40.
39. A 6, B 5 miles per hour.
40. 93.
41. 8 hours.
42. 6 hours, or 31 hours.
43. 3 and 243.
44. A 204, B 200 acres.
45. 5 and 4.
46. 36 and 24.
47 Breadth 100, length 150 feet.
48. 12 and 10.
49. Breadth 40, length 50 rods.
50. 16 and 9.
51. 6 and 4.
52. 18 and 6.
53. 3 and 5.
54. 5 and 9.
55. 81 scholars.
56. 23.
57. Depth of 1st 4 feet, that of 2d 5 feet.
58. 8.
59. 25.
60 Court 30 by 19 feet; height of fence 4 feet.

61. 12 yards.
62. 150 miles.
63. 25.
64. A \$200; B \$300.
65. 9 and 15.
66. 741.
67. A \$81; B \$41; C \$21; D \$11; E \$6.

Explanation.

Let v, w, x, y, and z represent the money of A, B, C, D, and E, respectively, when they began to play; and let S represent the known amount of their money, which is 5 . 32 or \$160.

In the first game A loses the whole amount minus his own share, or $S - v$; he will therefore have left $v - (S - v)$, or $2v - S$. After this his money is doubled four times; he will then have $32v - 16S$; this must be equal to \$32 or $\frac{1}{5}S$. Hence,

$$32v - 16S = \tfrac{1}{5}S, \text{ from which we have}$$
$$v = \frac{81S}{160} = \frac{81 \,.\, 160}{160} = \$81.$$

B, in the first game, doubles his money; he will then have $2w$; in the second game he loses $S - 2w$, and will have left $4w - S$. Afterwards his money is doubled three times; he will then have $32w - 8S$. Hence,

$$32w - 8S = \tfrac{1}{5}S, \text{ and } w = \frac{41S}{160} = \$41.$$

In like manner, for C's money we find

$$32x - 4S = \tfrac{1}{5}S, \text{ and } x = \frac{21S}{160} = \$21;$$

for D's money,

$$32y - 2S = \tfrac{1}{5}S, \text{ and } y = \frac{11S}{160} = \$11;$$

and for E's money,

$$32z - S = \tfrac{1}{5}S, \text{ and } z = \frac{6S}{160} = \$6.$$

68. A 40, B 60 eggs.
69. 10 and 14.
70. 4 and 2.
71. 4, 6, and 11.
72. Depths 4 feet and 5 feet.
73. 5 and 3.
74. 6.
75. 16.
76. $50.
77. 7 and 3.
78. 7, 5 and 3.
79. 1, 3, 5, and 7.
80. 2, 6, and 18.
81. 10, 20, 40, and 80.
82. £94 18s. 0d. 1 qr.
83. $491·585.