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## K E Y

EXAMPLES FOR PRACTICE -

IN

WRITTEN ARITHMETIC.

FOR THE USE OF TEACHERS.

BY FREDERIC A. ADAMS, PRINCIPAL OFDUMMERACADEMX.

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## TO TEACHERS.

It is essential to the pupil's progress that he be required to master thoroughly each principle in the First Part, before he attempts to solve the questions that come under it in the Second. In this way alone can he proceed intelligently and pleasantly in his work.

In no instance in the Key is the process of the solution given, but only the result.

The few suggestions that are made, in connection with some of the more difficult examples, are designed only as hints, to stimulate and encourage the student; not to supply the place of study on his part.

No question is given, the solution of which is not provided for in the principles and rules which the Arithmetic contains.

> Dummer Academy, September $16,1846$.

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## KEY TO WRITTEN ARITHMETIC.

## SECTION I.

1. 1314. 
1. 1442. 
1. 2068. 
1. 2134. 
1. 937. 
1. 2278. 
1. 2063. 
1. 2145. 
1. 1846. 
1. 2364. 
1. 2443. 
1. 2594. 
1. 1582. 
1. 21941. 
1. 17102. 
1. 6495. 
1. 6262. 
1. 6522. 
1. 6650. 
1. 23434. 
1. 15708. 22. 16060 . 23. 18033. 24. 11957. 25. 18739. 26. 2234842. 27. 6313374. 28. 4284811. 29. 4055313. 30. 16888340.
1. Performed. 32. 12478. 33. 23928.
2. 215140. 
1. 569069. 
1. 761973. 
1. 371897. 
1. 1274915. 
1. 62784. 
1. 358915 .

SECTION II.

1. 383. 
1. 555. 
1. 195. 
1. 20. 
1. 117. 
1. 750. 
1. 417. 
1. 71. 
1. 4082. 
1. 1894. 
1. 1751. 
1. 688. 
1. 110. 
1. 177. 
1. 826. 
1. 1464. 
1. 58403. 
1. 214514. 
1. 623217. 
1. 86201 .
2. 5642. 
1. 128 years.
2. 156 years.
3. 44 years.
4. 57 years.
5. 101620. 
1. 102018. 
1. 51447. 
1. 86921. 
1. 127291. 
1. 412863. 
1. 545796. 
1. 510313. 
1. 350674. 
1. 356469 .
2. 581564. 
1. Performed.
2. Performed.
3. 165. 
1. 276. 
1. 1388. 
1. 1542. 

## SECTION III.

1. Performed.
2. 11764 .
3. 47478. 
1. 37088 .
2. 75166. 
1. 45496. 
1. 83835 .
2. 90972 .
3. 216335. 
1. 559322. 
1. 711648. 
1. 174262. 
1. 761940. 
1. 389408. 
1. 328287. 
1. 3378219. 
1. 8330688. 
1. 2764557. 
1. 246694. 
1. 357838. 
1. 31044114. 
1. 253296936. 
1. 639107315. 
1. 677282208. 
1. 11744226. 
1. 2788998514.
2. 622233540. 
1. 286616918. 
1. 53072684. 
1. 2331512816.
2. 836626937432.
3. 59933126820 .
4. 764183400522.
5. 122162496384.
6. 406478903992.
7. 464003894669.
8. 7500. 
1. 45375. 
1. 40140. 
1. 36015. 
1. 9125. 
1. 755232. 
1. 88960. 
1. Performed.
2. 17521555. 
1. 2272392510 .
2. 469580064. 
1. 321354. 
1. 18653328. 

## SECTION IV.

1. Performed.
2. 346. 
1. 579. 
1. 976. 
1. $825+41$.
2. 376. 
1. 345. 
1. 798. 
1. 5279. 
1. $52672+79$.
2. $14+63$.
3. $20605+31$.
4. $4292+17$.
5. $2074+66$.
6. $120899+18$.
7. $104683+10$.
8. $14793419+16$.
9. $2224+12$.
10. $22947+24$.
11. 7157 dollars.
12. 71 barrels.
13. $16+21793$.
14. $98+2699$.
15. $37+39467$.
16. Performed.
17. 192877. 
1. $49459+5$.
2. $23853+4$.
3. $85527+3$.
4. $2410069+3$.
5. $24451677+1$.
6. $103978+4$.
7. $163684+9$.
8. $1172845+2$.
9. $2231+2$.
10. $3129+8$.
11. $5108+15$.
12. $4365+9$.
13. $30629+7$.

## Miscellaneous Examples.

1. 5109 dollars.
2. 186 dollars.
3. 1674 dollars.
4. 647 dollars.
5. Whole amount, $\$ 3297$; each barrel, $\$ 5 \frac{82}{643}$.
6. 994 dollars.
7. 216 dollars.
8. 192 dollars.
9. Lost $\$ 34.00$.
10. $\$ 180.00$ more than his salary.

## SECTION V.

1. 9 feet.
2. 126 feet.
3. 48 inches.
4. 204 inches.
5. 1584 inches.
6. 576 inches.
7. 756 inches.
8. 1351 inches.
9. 1984 inches.
10. 213 inches.
11. 396 feet.
12. $1435 \frac{1}{2}$ feet.
13. 112266 inches.
14. $119 \frac{1}{2}$ feet.
15. 6168 inches.
16. 2620 shillings.
17. 15620 shillings.
18. 15160 shillings.
19. 228 pence.
20. 1812 pence.
21. 34036 pence.
22. 4188 farthings.
23. 232 ounces.
24. 1501 ounces.
25. 5031 pounds.
26. 30520 pounds.
27. 857 pennyweights.
28. 3174 pennyweights.
29. 2304 grains.
30. 38140 .
31. 216 pints.
32. 1818 quarts.
33. 2016 gills.
34. 5568 quarts.
35. 4672 pints.
36. 7392 quarts.
37. 1872 square inches.
38. 22869. 
1. 5096 อั2.
2. $37298 \frac{1}{4}$ square feet.
3. 14751 square feet.
4. 29376 solid inches.
5. 910656 solid inches.
6. 32760 solid inches.
7. 781 solid feet.
8. 3484 solid feet.
9. 5773 solid feet.
10. 375 quarters.
11. 315 quarters.
12. 3108 nails.
13. $427 \frac{1}{2}$ inches.
14. 4176 nails.
15. $24634 \frac{1}{2}$ feet.
16. 1128600 inches.
17. 5247 pence.
18. 960 sixpences.
19. 3806 sixpences.
20. 3620 threepences.
21. 5040 fourpences.
22. 28 times.
23. 112 parcels.
24. $51 \frac{3}{16}$ times.
25. 19200 acres.
26. 480000 acres.

## SECTION VI.

1. $17 £ 8$ s.
2. $250 £$.
3. $684 £$.
4. $46 £$.
5. $47 £ 1 \mathrm{~s}$.
6. $84 £ 5 \mathrm{~s}$.
7. $5 £ 17 \mathrm{~s} .1 \mathrm{~d}$.
8. $16 £ 19 \mathrm{~s}$.
9. $6 £ 17 \mathrm{~s} .3 \mathrm{~d}$.
10. $243 \frac{1}{2} \mathrm{~s}$.
11. 74 days.
12. 576 d .17 h .
13. 1364 days, 50 m .
14. 6236 h .56 m .29 sec .
15. 30 gals. 1 pt.
16. 2646 gals.
17. $215 \frac{4}{6} \frac{3}{3}$ bls.
18. 433 pks. 5 qts.
19. 1506 bu. 2 pks. 7 qts. 1 pt.
20. 120 bu. 6 qts.
21. 141 fur. 34 r.
22. 121 m .6 fur. 1 r .
23. 4629 r. $2 \frac{1}{2} \mathrm{ft}$.
24. 1 fur. 6 ft . 4 in .
25. 112 m .3 fur. 33 r .10 ft .
26. 2112 steps.
27. 57600. 
1. 1 cwt. 2 qrs. 11 lbs.
2. 3 cwt. 2 qrs. 21 lbs.
3. 2 qrs. 9 lbs. 8 oz.
4. 17 lbs.
5. 3 cwt. 1 qr. 19 lbs .2 oz.
6. 5 lbs. 4 oz.
7. 6 lbs. 3 oz. 11 dwt.
8. 3 lbs. 1 oz. 2 dwt. 4 gr.
9. 5 sq . ft.
10. 114 sq. yds. 3 sq. ft.
11. 1 sq. yd. 6 sq. ft. 43 sq. in.
12. 12 square rods.
13. 20 square rods, 186 feet.
14. 3 solid yds. 5 s . ft.
15. 111 solid feet, 126 in.
16. 1 solid ft. 605 in.
17. 32 solid yds. 12 ft .
18. 1 solid yd. 565 in .
19. 18 E. E. 2 qrs.
20. 90 qrs. 1 n.
21. 29 yds. 3 n.
22. 83 E. E. 6 in.
23. 373 E. E. 3 qrs.
24. 138 lots, 1 rod over.
25. 459 times, $1 \frac{1}{2}$ feet over.
26. 31 loads, 18 cwt. over.
27. 177 G. 15 s .
28. 60 E. E.
29. 93 balls, 8 lbs. over.
30. 440 times.
58.107 kegs, 1 gal. over.

## SECTION VII.

1. Performed.
2. $22 £ 19 \mathrm{~s}$. 8 d .
3. $79 £ 12 \mathrm{~s}$. 4 d .3 qrs.
4. $230 £ 18 \mathrm{~s}$.
5. $2 £ 6 \mathrm{~s} .11 \mathrm{~d}$.
6. 12 lb .2 oz. 0 dwt. 7 gr .
7. 23 lb .3 oz .10 dwt .22 gr .
8. 21 lb .1 oz .12 dwt .22 gr .
9. 25 lb .8 oz .14 dwt .4 gr .
10. 16 cwt .0 qr .3 lb .10 oz .
11. 5 T. 12 cwt. 1 qr. 10 lb .23 .1 A. 29 p. 63 ft .78 in.
12. 4 T. 2 cwt. 1 qr. 12 lb. 24. 1 R. 9 p. 112 ft. 112 in.

3 oz.
13. 2 T. 10 cwt. 19 lb.
14. 5 m .1 fur. 14 r .1 ft .6 in. 15. 105 m .7 fur. 31 r .
16. 1 fur. 32 r. 2 ft .6 in . 17. 1 fur. 18 r. 12 ft .3 in. 18. 1 m .34 r .2 ft .8 in .
19. 1 m .6 fur. 10 r .16 ft . 20. 32 m .21 r.
21. 18 А. 23 p. 22. 35 A. 23 p.
25.8 s. yds. 13 ft. 190 in. 26. 26 s. yds. 21 ft .74 in . 27. 101 gals. 2 qts. 28. 90 bu. 3 pks. 1 qt. 29. 114 bu. 2 pks. 30. 36 yds. 2 qrs. 31. 21 d .6 h .14 m . 32. 4 Y. 25 w. 6 d. 33. 26 deg. $35^{\prime} 4^{\prime \prime}$. 34. 35 deg. $25^{\prime} 30^{\prime \prime}$. 35. 95 deg. $27^{\prime} 22^{\prime \prime}$.

## SECTION VIII.

1. Performed.
2. $4 £ 3 \mathrm{~s} .11 \mathrm{~d}$.
3. 3£ 8s. 8d.
4. 2 bu. 7 qts.
5. 23 bu. 2 pks. 5 qts.
6. 1 Y. 10 mo .26 d .
7. 8 Y. 9 mo. 3 d .
8. 8 gals. 2 qts.
9. 13 gals. 2 qts. 1 pt.
10. 6 fur. 36 r .
11. 10 miles, 38 rods.
12. 1 cwt .1 qr .22 lb .
13. 2 cwt. 2 qrs. 26 lb .
14. 1 Y. 2 mo. 10 d .
15. 1 Y. 14 d.
16. 9 mo .17 d .
17. 3 Y. 11 mo .22 d .
18. 1 Y. 9 mo. 5 d .
19. 1 Y. 9 mo .22 d .
20. 5 mo .19 d .
21. 1 Y. 10 mo. 24 d .
22. 3 Y. 6 mo .1 d .
23. 2 Y. 9 mo. 7 d.
24. 2 deg. $57^{\prime}$.
25. 13 deg. $23^{\prime}$.
26. 10 deg. $26^{\prime}$.
27. 9 deg. $16^{\prime}$.

## SECTION IX.

1. Performed.
2. $8 £ 11 \mathrm{~s}$.
3. 18£ 12s. 9 d .3 qrs.
4. 1 d .2 h .53 m .
5. 2 d. 7 h. 10 m .6 sec .
6. 117 d .18 h .24 sec.
7. 140 bu. 2 pks.
8. 187 bu. 1 pk. 2 qts.
9. 96 bu. 2 pks. 2 qts.
10. 3 m .2 fur. $27 \mathrm{r} .7 \frac{1}{2} \mathrm{ft}$.
11. 39 m .6 fur. 13 r .2 ft. 15.213 yds. 3 qrs. 3 in.
12. 1 T. 1 qr.
13. 2 T. 19 cwt. 2 qrs.
14. 2 T. 3 qrs. 8 lb .
15. 48 lb .6 oz .15 dwt .
16. 87 lb .7 oz. 5 dwt. 12 gr . 18. 68 A. 1 R. 33 r.
17. 216 A. 1 R. 36 r.

## SECTION X.

1. Performed.
2. 1£ 15s. 9 d .
3. $6 £ 4 \mathrm{~s}$. 3 d .
4. 6 h. 43 m .15 sec .
5. 9 h .11 m .41 sec .
6. 14 d. 8 h .15 m .3 sec .
7. 8 bu .3 pks. 1 qt.
8. 15 bu. 2 pks. 3 qts. 1 pt.
9. $9 £ 13 \mathrm{~s}$. 4 d .
10. $38 £ 6 \mathrm{~s} .7 \mathrm{~d}$.
11. 3 lb .4 oz .14 dwt .6 gr . 12. 2 T. 10 cwt. 2 qrs. $9 \frac{1}{3} \mathrm{lb}$. 13. 1 bu. 2 pks. $1 \frac{13}{\mathrm{z}} \mathrm{qt}$. 14. 8 A. 3 R. $33 \frac{1}{2}$ r.

## SECTION XI.

1. 1 T. 3 cwt. 2 qrs. 14 lb .
2. $142 £ 8 \mathrm{~s} .9 \mathrm{~d}$.
3. 1 T. 17 cwt .
4. 49 m .52 sec . past 12 .
5.59 m .12 sec. past 1.
5. 9 m .20 sec . past 12 .
6. 15 minutes 44 seconds past 7, A. м.
7. 3 m .56 sec . past 7, A. м.
9.6 m .28 sec . past 11, A. M.

## SECTION XII.

1. $29,23$.
2. 2, 7, 29.
3. Prime; 19,$19 ; 2,7,53$;

Prime ; 2, 2, 79.
4. Prime; 2, 2, 7, 29; 7, 107 ; 2, 2, 3, 3, 31; $3,2,41 ; 2,2,2,1013$.
5. 2, 7, 19 ; 2, 2, 13, 17 ; 3, $3,3,71$; $2,2,2,47$.

## SECTIONXIII.

1. Performed.
2. $\frac{9}{10}$.
3. $\frac{24}{2}$.
4. $\frac{5}{9}$.
5. $1 \frac{1}{2}$.
6. Performed.
7. 437. 
1. 1, or Prime to each other.
2. 1 ,
3. 53. -11 . 32. - 12. 79.
1. 2.         - 14. 2. -15 . 2.
1. $\frac{1}{125}$.
2. $\frac{13}{1}, \frac{1}{2} \frac{1}{4}, \frac{1}{3} \frac{3}{4}$.
3. $\frac{1}{1} \frac{1}{3}, \frac{4}{5}, \frac{7}{8}$.
4. $\frac{23}{2}, \frac{3}{10}, \frac{6}{11}$.
5. Performed.
6. Performed.
7. $4 \frac{7}{8}, 2 \frac{1}{7}, 3 \frac{6}{11}$.
8. $7 \frac{3}{8}, 6 \frac{7}{12}, 2 \frac{28}{5}$.
9. $2 \frac{5}{6}, 111 \frac{3}{7}, 21_{16}^{5}, 18$.

## SECTION XIV.

1. Performed.
2. Performed.
3. $\frac{21}{3}$.
4. $1 \frac{3}{7}$.
5. $1 \frac{10}{8}$.
6. $\frac{63}{3}$.
7. $\frac{126}{18}$.
8. 164. 
1. 7 halves.
2. $\frac{21}{5}$.
3. $\frac{5}{3}$.
4. 145. 
1. $1 \frac{6}{9} \frac{3}{3}, 7 \frac{85}{7}, 1 \frac{273}{4}$.
2. $\frac{6}{8}$.
3. $+\frac{2}{5}$.
4. $\frac{2}{2} \frac{1}{4}, \frac{24}{80}, \frac{88}{9}$.
5. $\frac{14}{26}, \frac{36}{56}, \frac{52}{60}$.
6. Performed.
7. $\frac{445}{8}, \frac{7 \frac{7}{8}}{9}, \frac{119}{7}$.
8. $\frac{15}{4}, \frac{2 \frac{1}{7}}{5}, \frac{24}{6}$.
9. $\frac{20 \frac{5}{7}}{5}, \frac{93 \frac{1}{2}}{11}, \frac{29 \frac{1}{3}}{4}$.
10. $\frac{89 \frac{1}{3}}{4}, \frac{129 \frac{1}{2}}{7}, \frac{157 \frac{1}{2}}{5}$
11. $\frac{25 \frac{1}{2}}{3}, \frac{34}{4}, \frac{42 \frac{1}{2}}{5}, \frac{76 \frac{1}{2}}{9}$.
12. $\frac{97 \frac{1}{2}}{5}, \frac{78}{4}, \frac{136 \frac{1}{2}}{7}$.
13. $\frac{27 \frac{3}{4}}{3}, \frac{46 \frac{1}{4}}{5}, \frac{74}{8}$.
14. $\frac{185 \frac{1}{2}}{14}, \frac{198 \frac{3}{4}}{15}$.
15. $\frac{144 \frac{1}{2}}{17}, \frac{110 \frac{1}{2}}{13}$.
16. $\frac{141 \frac{3}{4}}{7}, \frac{162}{8}$.
17. $\frac{65 \frac{1}{3}}{4}, \frac{81 \frac{2}{3}}{5}$.
18. $\frac{425 \frac{1}{2}}{37}, \frac{218 \frac{1}{2}}{19}$.

## SECTION XV.

1. $\$ 45$.
2. 404. 
1. $\$ 52.12 \frac{1}{2}$.
2. $255 \frac{3}{4}, 105 \frac{5}{6}$.
3. $466 \frac{2}{3}$ lbs., $3266 \frac{2}{3} \mathrm{lbs}$.
4. $68 \frac{1}{1}^{\frac{7}{6}}$ bush.
5. $\frac{5}{14}$ cwt. ; $130 \frac{5}{14}$ cwt.
6. \$18咟.
7. $37 \frac{9}{20}, 13 \frac{2}{2} \mathrm{~T}$.
8. $32 \frac{7}{1}, 48 \frac{1}{2}, 52 \frac{1}{2}$.
9. $\frac{13}{20} \mathrm{cwt}$.
10. $2 \frac{3}{16}, 2 \frac{2}{3}, 2 \frac{1}{63}$.
11. $14 \frac{1}{2} 7,115 \frac{3}{4}, 42 \frac{1}{4}$ f.

## SECTION XVI.

1. $409 \frac{1}{2}$ bottles.
2. $31 \frac{1}{2}$ loaves.
3. $2 \frac{1}{6} \frac{2}{4}, 4 \frac{5}{28}, \frac{24}{3}$ ?
4. \$876.90.
5. $11{ }_{1}^{7}{ }^{7}$ coats.
6. $35 \frac{1}{3}$ blocks.
7. $3_{1 \frac{3}{14}} \mathrm{lbs}$., $1173_{14} \frac{3}{4} \mathrm{lbs}$.
8. $47 \frac{1}{4}, 1 \frac{1}{2} \frac{5}{4}, 9 \frac{5}{57}$.
9. $414 \frac{3}{8}, 56 \frac{4}{7}, 617 \frac{1}{2}$. 10. $25 \frac{1}{3}$.
10. $1_{35}^{2}, 7 \frac{17}{24}$.
11. $2 \frac{1}{4}$ í, $12 \frac{22}{2}$.
12. $7_{2}^{2}$, $1_{1}^{4} 7$.
13. $356 \frac{1}{8}, 49 \frac{1}{2}$.
14. $\$ 145.12 \frac{1}{2}$.
15. $\$ 196.87 \frac{1}{2}$.
16. \$96.00.
17. $\$ 42.87 \frac{1}{2}$.
18. $\$ 7911.62 \frac{1}{2}$.

## SECTION XVII.

1. $2_{\mathrm{I}^{7}}^{7}, 1_{1 \frac{1}{1}}$.
2. $\frac{1}{4}, \frac{1}{3} 3, \frac{1}{3}$.

3. $2 \frac{1}{36}, 1 \frac{75}{85}$.
4. $5 \frac{3}{1}, 7 \frac{5}{7} \frac{3}{7}, \frac{3}{6} 8$.
5. $106 \frac{4}{8} 7$.
6. $80 \frac{1}{4}$.
7. $\$ 1.03$ 等.
8. $44 \frac{3}{4}$ bushels.
9. $23 \frac{9}{16}$ lbs.
10. $5 \frac{3}{8} \frac{3}{8}, 112 \frac{1}{21}$.
11. $22 \frac{3}{20}, 91974$.
12. $41 \frac{5}{84}, 8_{\frac{69}{2} 38}$.
13. $5 \frac{1}{3} \frac{1}{3} \frac{3}{6}, 8 \frac{25}{52} 29$.
14. $18 \frac{48}{5}$ 8, $21+\frac{25}{259}$.
15. $13{ }_{19}^{951}{ }^{580}$.

## SECTION XVIII.

1. $\frac{1}{64}$.
2. $1 \frac{1}{9} \overline{2}$.
3. $\frac{5}{2} 24$.
4. $\frac{1}{72}$.
5. $\frac{1}{48}$.
6. $6 \frac{1}{4} 0$.
7. $T^{\frac{1}{8} 0}$.
8. $\frac{13}{3} \frac{39}{0}$.
9. $5 \frac{1}{2} 80,5 \frac{2}{8} \sigma^{\circ}$
10. $\frac{1}{3} \frac{09}{20}$.
11. $\frac{205}{224}$.
12. $16 \frac{6}{8} 9$.
13. $\frac{25}{256}$.
14. $\frac{73}{512}$.
15. $1 \frac{1}{6} 0$.
16. $\frac{1}{6}$.
17. $\frac{3}{1} 2$.
18. $\frac{29}{672}$.
19. $\frac{1}{15}$.
20. Performed.
21. $8 \mathrm{~s} .10 \frac{2}{3} \mathrm{~d}$.
22. 18 h .40 m .
23. 4200 m .
24. $16 \mathrm{~m} .21 \frac{9}{1 \mathrm{~T}} \mathrm{sec}$.
25. 2 qts. $1 \frac{1}{3}$ pts.; 9 gals.
$1 \mathrm{qt} .1 \frac{3}{5} \mathrm{pts}$.
26. 10 oz. 16 dwt.
27. 2 oz .2 dwt. $8_{17}^{8}$ grs.
28. $68 \frac{4}{7}$ sq. rods.
29. 1 sq. ft. 6012 in .

## SECTION XIX.

1. $1^{\frac{1}{3}}$.
2. $5 \frac{1}{2} \frac{7}{8} 0$.
3. $4^{2} \frac{2}{2} 4$.
4. $4 \frac{51}{4} \frac{1}{8} \sigma$.
5. $\frac{4}{8929} 8$.
6. $\frac{187}{4480}$.
7. $\frac{11}{12} 0$.
8. $\frac{55}{2} \frac{5}{7} \frac{3}{8} 0^{-}$
9. 123263 .
10. $\frac{143}{5} \frac{3}{4}$.
11. $4 \frac{3}{8}$.
12. $\frac{43}{336}$.
13. $\frac{2033}{48}$.
14. $\frac{1}{48}$.
15. $\frac{9}{640}$.
16. $\frac{21997}{89280}$.
17. $\frac{11}{9}$.
18. $\frac{13}{336}$.
19. $\frac{1}{24}$.
20. 1 考音 $\sqrt{2}$.
21. $1 \frac{1}{4} \frac{27}{8} 0$.

## SECTION XX.

1. $\$ 7.87 \frac{1}{2}$.
2. $\$ 1.96 \frac{7}{8}$.
3. $\$ 55.50$.
4. $\$ 27.12 \frac{1}{2}$.
5. $\$ 24.06 \frac{1}{4}$.
6. $\$ 27.18 \frac{3}{4}$.
7. 78 bu.
8. \$29.25.
9. $\frac{3}{4} \mathrm{cwt}$.
10. $21 \frac{3}{5} \mathrm{cwt}$.
11. $67 \frac{1}{5}$ cubic feet.

* 12. $13 \frac{1}{32}$ cords.

13. $\frac{20}{297}$.
14. $12^{2}$ of a sq. rod.
15. $\mathrm{T}^{20} 9 \mathrm{~g}$ of a sq. rod.
16. $\$ 1.256+$.
17. $26 \frac{1}{2} \frac{7}{6}$ gals.

18. $\$ 11.10 \frac{5}{27} ; \$ 16.14 \frac{2}{27}$.
19. $\$ 32.00$.
20. \$1209.60.
21. $\$ 2.95_{15}^{5}$; $\$ 3.544_{1}{ }^{6}$.
22. $\$ 45.61 \frac{107}{107}$; $\$ 30.88_{\frac{8}{189}}$.

## SECTION XXI.

1. 201.025 .
2. 9683.322.
3. 568.1496 .
4. 336.325 .
5. 3921.065 .
6. 159.09 ; 778.73 .
7. 15.685 ; 13.143.
8. 132.385 ; 173.41 .
9. 16.593; 84.16.
10. 648.079 ; 255.076.
11. 3.776 ; . 4933.
12. . 684 ; . 5584.
13. 10814.1 ; 7816.581.
14. 582.75 ; 3.925.
15. 35.25; 1177.6.
16. 2.226 ; 15.6 .
17. . 1244 ; . 5583 .
18. 262.81 ; 801.325.
19. 22.2882 ; . 89. 20. 44.375 ; 27.5625.
20. 7000 ; $6433 \frac{1}{3}$. 22. . 11 ; $248 \frac{14}{7}$. 23. $1708 \frac{1}{3}$; $7727{ }_{1}{ }^{3}$. 24. $1000 ; 10000$. 25. . 0005 ; . 0032.

## SECTION XXII.

1. . 375.
2. . 625.
3. . 125 ; 875 .
4. . 0625 ; . 1875 ; . 3125 .
5. . 5625 ; . 6875 ; . 9375 .
6. . $59375 ; .71875 ; .84375$.
7. $\left\{\begin{array}{l}.148+; .2903+; \\ .1470+.\end{array}\right.$
8. $\left\{\begin{array}{l}.6111+; .1428+; \\ .0172+.\end{array}\right.$
9. $\left\{\begin{array}{l}.0487+; .2658+; \\ .9393+.\end{array}\right.$
10. 

$\left\{\begin{array}{c}.0012+; ~ \\ .0079+\end{array}\right.$
11. Performed.
12. $.1_{1}^{9}$; $.18{ }_{\mathrm{T}_{\mathrm{T}}^{2}}$; $.181_{\mathrm{T}^{9} \mathrm{~T}}$.
13. . $2105263 \frac{3}{19}$.
14. . $538461538{ }_{\frac{6}{13}}$.
15. . $3809523809 \frac{1}{2}+$.

## SECTIONXXIII.

1. $.2958+$.
2. $0160416 \frac{2}{3}$.
3. . $0198+$.
4. . 09375.
5. . 3125.
6. . 2070 .
7. $4646+$ +
8. . $389+$.
9. . $0743+$ +
10. .2208+.
11. . 1255 +.
12. . 078125.

## SECTION XXIV.

1. Performed.
2. 4 feet 1.14 inches.
3. 15 feet 6.12 inches.
4. 4 feet 3.48 inches.
5. 13 shillings.
6. 16 shillings.
7. $8 \frac{2}{5}$ pence.
8. $1 \frac{23}{25}$ pence.
9. 3s. 9 d . 2.4 qrs.
10. 2 pks. 7 qts. 1.36 pts.
11. 3 pks. 4 qts. 1.6 pts.
12. 112 rods.
13. 144 rods.
14. 7 minutes 12 seconds.
15. 9 minutes.
16. 2 d. 9 h .7 m .12 sec .
17. 1 d .3 h .12 m .57 .6 sec .
18. 50.4 seconds.
19. 97.408 cubic feet.
20. 25 solid feet 1036.8 in .
21. $94 \frac{1}{2} \frac{8}{8}$ feet.
22. $16 \frac{1}{2} \frac{6}{6}$ feet.

## SECTION XXV.

1. $\$ 7.125$.
2. \$41.205.
3. $\$ 49.625$.
4. $\$ 9.125$.
5. $\$ 50.52$.
6. $\$ 2.145$ loss.
7. \$8.875.
8. $\$ 7.45$.
9. $\$ 5.78$.
10. \$4.905.
11. $\$ 1.05$.
12. $\$ 1.575$.
13. $\$ 0.625$.
14. $\$ 0.6975$.
15. \$0.984375.
16. \$0.584375.
17. $\$ 71.25$.
18. $\$ 91.875$.
19. \$22.68.
20. \$121.50.
21. \$29.75.
22. \$5.52.
23. $\$ 0.0$ ō.
24. 350 lbs .
25. 12.5 times.
26. 15.5 lbs .
27. 2625. 
1. 1375. 
1. 15 barrels.
2. 17.5 barrels.
3. 3 tons.
4. 13.5.
5. 17 tons.
6. 16 times.
7. \$3.29.

## SECTIONXXVI.

1. $\$ 8.66 \frac{1}{2} \frac{53}{24}$ gain.
2. $\$ 67.08$ gain.
3. $\$ 23.36 \frac{4}{5} \frac{1}{6}$ gain.
4. $\$ 53.02$ gain.
5. \$16.333 gain.
6. $\$ 8.04 \frac{3}{4}$ gain.
7. $\$ 1.68$ gain.
8. $\$ 9.88$ gain.
9. $\$ 5.06$ gain.
10. \$3.23 gain.
11. $\$ 15.75$ gain.
12. $\$ 7.05$ gain.
13. $\$ 76.01$ loss.
14. $\$ 1.99$ loss.
15. \$14.84 gain.
16. $\$ 7.70$ gain.
17. $\$ 23.37 \frac{1}{2}$ gain.
18. $\$ 10.73$ gain.
19. $\$ 9.25$ gain.
20. $\$ 24.45$ gain.
21. $\$ 2.24$ gain.
22. $\$ 40.35$ gain.
23. $\$ 3.00$ gain.
24. $\$ 0.75$ loss.
25. \$6.86 gain.

## SECTIONXXVII.

1. Performed.
2. $\$ 386.66 \frac{2}{3}$.
3. $\$ 462.22 \frac{2}{9}$.
4. $\$ 284.444$.
5. $\$ 742.22$.
6. $\$ 2311.11 \frac{1}{9}$.
7. $\$ 374.666$.
8. $\$ 554.00$.
9. $\$ 162.111$ 十.
10. Performed.
11. $16 £ 14 \mathrm{~s} .1 \frac{1}{5} \mathrm{~d}$.
12. $41 £ 18 \mathrm{~s} .9 \frac{3}{5} \mathrm{~d}$.
13. $126 £ 19 \mathrm{~s} .6_{19}^{9} \mathrm{~d}$.
14. $144 £ 0 \mathrm{~s} .8{ }_{10}{ }^{1} \mathrm{~d}$ d.
15. Performed.
16. $\$ 506.40$.
17. $210 £ 7 \mathrm{~s} .6 \mathrm{~d}$.
18. Performed.
19. $\$ 402.083+$.
20. Performed.
21. $\$ 136.31 \frac{1}{4}$.
22. $\$ 43.40$.
23. $\$ 19.83 \frac{1}{9}$.

## SECTION XXVIII.

1. Performed.
2. $\$ 14.0866$.
3. \$4.0477 1 .
4. $\$ 26.8024 \frac{7}{12}$.
5. $\$ 4.2599 \frac{1}{3}$.
6. $\$ 3.604+$.
7. $\$ 5.496+$.
8. $\$ 50.912+$.
9. \$68.142.
10. $\$ 36.10$.
11. $\$ 43.00$.
12. $\$ 0.626+$.
13. $\$ 0.445+$.
14. $\$ 25.666+$.
15. $\$ 0.327+$.
16. $\$ 0.47$.
17. $\$ 2.521+$.
18. \$1.896.
19. $\$ 0.698+$.
20. $\$ 0.535+$.
21. $\$ 0.962+$.
22. $\$ 2.703+$.
23. $\$ 18.084+$.
24. \$19.78.
25. \$39.56.
26. $\$ 26.373+$.
27. $\$ 23.076+$.
28. $\$ 16.483+$.
29. $\$ 75.00$.
30. $\$ 7.16+$.
31. $\$ 57.318+$.
32. Performed.
33. $\$ 2.903+$.
34. $\$ 2.047$-.
35. $\$ 0.607+$.
36. $\$ 4.488+$.
37. $\$ 0.210+$.
38. \$5584.041.
39. $\$ 1039.461+$.
40. \$501.40.
41. \$137.37.
42. $\$ 74.829$.
43. $\$ 567.50$.
44. $\$ 753.876$. 45. $\$ 859.575$ +.
45. \$42.201.

## SECTION XXIX.

1. $\$ 143.32$.
2. $\$ 273.00$.
3. $\$ 308.887$.
4. $\$ 565.55$.
5. $\$ 210.439+$.
6. \$323.34.
7. \$528.011. •
8. $\$ 112.36$.
9. $\$ 119.101$ +.
10. $\$ 126.247+$.
11. $\$ 3.821+$.
12. $\$ 310.00$.
13. $\$ 265.00$.
14. \$280.90.
15. $\$ 297.754$.
16. $\$ 315.619+$.
17. $\$ 5.619+$.

## SECTION XXX.

1. $\$ 468.367$.
2. \$337.30.
3. $\$ 921.445$.
4. $\$ 153.80$.
5. $\$ 310.776$.
6. $\$ 496.584$.
7. $\$ 805.845$.
8. $\$ 46.146$ +.
9. $\$ 121.339$.
10. $\$ 352.096$.

## SECTION XXXI.

1. $\$ 98.95$.
2. \$443.025.
3. $\$ 247.375$.
4. \$509.34.
5. \$593.70.
6. $\$ 146.925$.
7. $\$ 74.587+$.
8. $\$ 989.50$.
9. $\$ 548.52$.
10. \$149.175.
11. $\$ 100.00$.
12. $\$ 520.00$.
13. $\$ 600.00$.
14. $\$ 10055.30+$ -

## SECTION XXXII.

1. 10 per cent.
2. $13 \frac{1}{8} \frac{8}{9}$ per cent.
3. $11 \frac{1}{2} \frac{0}{3} 9$ per cent.
4. $\$ 4.816$.
5. $2 \frac{1}{2}$ per cent.
6. $\$ 0.316+$.
7. $\$ 644.061$.
8. $6 \frac{5}{6} \frac{2}{3}$ per cent.
9. \$5.32.
10. $19 \frac{1}{5} \frac{1}{1}$ per cent.
11. $43_{\frac{2}{1} \frac{28}{9} \frac{3}{9} \frac{1}{8} \div 3}$ per cent.
12. $39 \frac{2}{3} \frac{59}{4} \frac{98}{3} \frac{8}{2} \frac{3}{3}$ per cent.
13. $26 \frac{28869999}{7} \frac{9}{6} \frac{1}{51}$ per cent.
14. $151+$ per cent.
15. $61+$ per cent.
16. $62+$ per cent.
17. $10+$ per cent.
18. $16+$ per cent.
19. $20+$ per cent.
20. $\$ 300$.
21. \$88.
22. $\$ 20.236$ 十
23. \$15.63.
24. \$522.50.
25. \$658.
26. $\$ 1036.80$.
27. $\$ 260.25$.
28. $\$ 622.08$.
29. \$37.38.
30. $9 \frac{8}{13}$ per cent.
31. $\$ 0.51 \frac{1}{2}+$.
32. \$806.25.
33. $\$ 7735$.
34. \$4.75.
35. \$11958.04+.
36. $\$ 12000$.
37. $\$ 781.50$.
38. Whole gain, \$22.32; gain per cent. $34+$.
39. Whole gain, $\$ 135.02$; gain per cent. 73+.
40. Whole gain, $\$ 17.375$; gain per cent. $41+$.
41. Whole gain, $\$ 393.50$;
gain per cent. 20+.
42. Whole gain, $\$ 306$;
gain per cent. $21+$.

## SECTION XXXIII.

1. $\$ 0.649$.
2. $\$ 0.565$.
3. $\$ 0.1208$.
4. Performed.
5. 3 bu. oats, 1 bu. rye.
6. 1 bu o oats, 1 bu . wheat.
7. 1 bu . oats, 3 bu. wheat.
8. 2 bu. oats, 1 bu. wheat.
9. 35 barley, 7 corn, 14 rye.
10. 13 at 25 cents, 5 at 40 cents, 5 at 33 cents.
11. 15 at 20 cents, 18 at 45 cents, 9 at 54 cents.
12. 1 lb . at 6 cents, 3 lbs . at 10 cts., 2 lbs. at 8 cts. and 1 lb . at 11 cts .
13. 1 gal. at 60 cts., 2 gals. at 75 cts., 1 gal. water, 7 gals. at 80 cts.
14. 5 bu. at 25 cts., 10 bu . at $30 \mathrm{cts} ., 10 \mathrm{bu}$. at 33 cts., 15 bu. at 45 cents, 17 bu. at 50 cents.
15. 24 gals. at $\$ 1,80$ gals. at 74 cents.
16. 50 lbs. at 8 cts., 100 lbs . at 11 cts ., 100 lbs . at 7 cts., 150 lbs. at 12 cents.
17. Performed.


## SECTION XXXV.

1. 225 square rods.
2. $240 \frac{1}{4}$ square rods.
3. 289 square rods.
4. $306 \frac{1}{4}$ square rods.
5. $462 \frac{1}{4}$.square rods.
6. 800 square rods.
7. $1874 \frac{1}{4}$ square rods.
8. 7191 square rods.
9. 32 rods.
10. 71 acres, 58 rods.
11. $295_{\text {去 }}$ rods.
12. $132 \frac{12}{13}$ inches.
13. $3 \frac{3}{4}$ feet.
14. $16 \frac{7}{8}$ feet.
15. $4 \frac{4}{7}$ yards.
16. 342 yards.
17. 3420 feet.
18. \$39.33.
19. 1040 rods.
20. 182 dollars.
21. 4 acres.
22. 6 acres 132 rods.
23. 5 acres 33 rods.
24. 20 rods.
25. $36 \frac{2}{5}$ rods.

## SECTION XXXVI.

1. Performed.
2. Performed.
3. Performed.
4. $27 \mathrm{ft} .6^{\prime}$.
5. $45 \mathrm{ft}. 1^{\prime}$.
6. $137 \mathrm{ft} .9^{\prime} 2^{\prime \prime}$.
7. $19 \mathrm{ft}. 10^{\prime}$.
8. $15 \mathrm{ft} .7^{\prime} 5^{\prime \prime}$.
9. $127 \mathrm{ft} .1^{\prime \prime}$.
10. $1 \frac{3}{8}$ cords 14 feet.
11. 5 cords $1^{\prime}$.
12. $1 \frac{1}{8}$ cord 1 ft .
13. 106 ft . $5^{\prime} 9^{\prime \prime} 4^{\prime \prime \prime}$.
14. 1 cord 10 ft . 15. $14 \frac{1}{8}$ cords 3 ft .
15. $1 \frac{2}{8}$ cords 7 ft .

## SECTION XXXVII.

1. Performed.
2. 15. 
1. 17. 
1. 19 .
2. 25. 
1. 28. 
1. 29. 
1. 31. 
1. 125. 
1. 246 ; 324; 462.
2. 1.6.
3. 1.5 .
4. 12.5.
5. 3.7.
6. 100. 
1. 1.3.
2. .9.
3. .1.
4. $\frac{2}{3}$.
5. $\frac{3}{4}$.
6. $1 \frac{4}{5}$.
7. $1 \frac{6}{7}$.
8. $\frac{1}{3}$.
9. $\frac{4}{9}$.
10. .5.
11. 8.366

27
28. 7.745 +.
29. 14.142+.
30. 12.64+.
31. 3600 square rods.
32. 9 rods.
33. $39.001+$.
34. 17 miles.
35. 33.03 + feet.
36. 21.26 - feet.
37. 26.83 + inches.
38. 23.49 feet.
39. 17.32 feet.
40. 10.39 feet.
41. 6.92 inches.
42. 6.92 inches.

## SECTION XXXVIII.

1. 5. 
1. 6. 
1. 8. 
1. 9. 
1. Performed.
2. 14 .
3. 15. 
1. 17. 
1. 21. 
1. 24. 
1. 36. 
1. 125. 
1. 134. 
1. 156. 
1. 2.5.
2. 3.3.
3. $2.15+$.
4. 2.71 +.
5. $4.64+$
6. $29.5+$ inches.
7. $41.7+$ inches.
8. 5.03 feet.

SECTION XXXIX.

1. $\$ 8.68$.
2. $\$ 23.33 \frac{1}{3}$.
3. $\$ 0.95$.
4. $69 \frac{2}{3}$ miles.
5. $5 \frac{5}{7}$ dollars.
6. $13_{\mathrm{I}^{7}}^{7}$ cords.
7. 64 months.
8. $10 \frac{5}{4}$ per cent.
9. $6 \frac{3}{8} \frac{8}{7}$ per cent.
10. A, $\$ 112.50 ; \mathrm{B}, \$ 67.50$.
11. C, $\$ 150$; D, $\$ 90$.
12. $\$ 18717$; $\$ 1522_{1}^{2}$.
13. $\$ 10 \frac{30}{6} 7$; $\$ 9 \frac{3}{6} 7$.
14. $\$ 24 \frac{1}{3} ; \$ 12 \frac{1}{6}$.
15. 19.79 rods.
16. 22.5 acres.
17. 50.84 acres.
18. $\left\{\begin{array}{l}\text { Length, } 84.84 \text { rods. } \\ \text { Width, } 48.08 \text { rods. }\end{array}\right.$
19. $2 \mathrm{ft} .9 .94+$ inches.
20. 116.41 rods.
21. $7.19+$ feet.
22. $24.19+\mathrm{lbs}$.
23. $38.41+\mathrm{lbs}$.
24. 2030.56 lbs .
25. $8.23+$ cords.
26. 7.65 + tons.
27. $8315.77+\mathrm{lbs}$.
28. $108232 \frac{1}{21}$.
29. 27 hours 8 minutes.
30. $35 \frac{2}{3} \frac{2}{5}$ days.
31. $2 \frac{4}{5}$ weeks.
32. 472 2. days.
33. 524 atays. of TMI
34. Performed
35. Performed.
36. 1921 hours.
37. \$29.60.
38. $23_{13}^{13}$ oz.
39. $13_{\text {1T }}$ days.
40. $8 \frac{24}{5}$ men.
41. $\$ 300$.
42. $\$ 280$; $\$ 320$.
43. $\$ 299 \frac{2}{2} 7 ; \$ 7 \frac{1}{2} \frac{1}{7} ; \$ 105 \frac{1}{2} \frac{5}{7}$.
44. 
45. $\$ 150$; $\$ 450$.
46. $\$ 160 ; \$ 330$; $\$ 350$.


## SECTION XL.

1. 75. 
1. 105; 181; 177; 213. 14. 5000 .
2. 190; 223; 241.
3. $157 ; 265 ; 317$.
4. $90 ; 200 ; 400$.
5. 1; 67; 571 .
6. 17 ; 197.
7. $45 ; 169$.
8. Performed.
9. 2. 
1. 4. 
1. 156. 
1. 590. 
1. 124800 feet.
2. 168; 14196.
3. 6. 
1. 32.2 feet.
2. 257.6 feet.
3. 402.5 feet.
4. 579.6 feet.
5. 7 sec. ; height, 778.9 ft .
6. 8 sec. ; height, 1030.4 ft .
7. 144.9 feet.

## SECTION XLI.

1. 4094. 
1. 1536. 
1. \$327.68.
2. 320000,000 .
3. $\$ 1111111.111$.

## SECTION XLII.

1. 113.0976 feet.
2. 3 miles 7 furlongs 16 rods 10 feet 6 inches十.
3. 76 feet 11 inches.
4. 24899 miles 1 furlong 31 rods 7 inches + .
5. 73 r. $8 \mathrm{ft} .8 \frac{1}{2} \mathrm{in}$.
6. $907 \mathrm{r} .15 \mathrm{ft} .2 \frac{1}{2} \mathrm{in}$.
7. $452 \mathrm{ft} .4 \frac{1}{2} \mathrm{in}$.
8. $35 \mathrm{r} .7 \mathrm{ft} .3 \frac{1}{2} \mathrm{in}$.
9. $35 \mathrm{r} .13 \mathrm{ft} .11_{4} \frac{18}{2} \frac{8}{5} \mathrm{in}$.
10. The circle is greater by
1.115 rods.

## SECTION XLIII.

1. $4.714+$ cubic feet.
2. 21.2058 cubic feet.
3. 22400 cubic feet.
4. 973.3724 cubic feet.
5. 75.3984 cubic feet.
6. 197343519.161326 m .
7. 1423.672 cubic inches.
8. 904 feet 1347 inches.

## SECTION XLIV.

1. 86 and 54.
2. 333 and 239.
3. $115 \frac{1}{2}$ and $71 \frac{1}{2}$.
4. $123 \frac{1}{2}$ and $66 \frac{1}{2}$.
5. 47 and 33.
6. 59 and 41.
7. 85 rods long, 35 rods broad.
8. 156 r. long, 84 r . wide.
9. $53 \frac{8}{9}$ feet; $36 \frac{1}{3}$ feet.
10. $77 \frac{1}{15}$ feet; $42 \frac{1}{5}$ feet.
11. Performed.
12. $6396 ; 1551 ; 8096$.
13. 1 acre $19 \frac{1}{5}$ rods.
14. $4 \mathrm{ft} .10 \frac{7}{8} \mathrm{in}$.
15. 13.1 rods; 5.69 rods.
16. $\frac{13331}{80}$.
17. Height, $47.62+\mathrm{ft}$; $\frac{1}{8 \frac{1}{6}}$ of the whole remained to be built.
18. As $2: 1$; as sq. r. of $2: 1$; that is, as $1.414+$ to 1 .
19. 10 miles; 10 miles; $14.422+$ miles.
20. $4.97+$ inches. The an- 39. $9.09+$ inches. swer to the 18th ques- $40.3 .66+$ inches. tion furnishes the hint $41.23+\frac{1}{2} \mathrm{lbs}$. for the solution of this. $42.60 \frac{1}{3} \frac{\mathrm{l}}{\mathrm{O}} \mathrm{lbs}$.
21. $8 \mathrm{~m} .4_{154}^{49} \mathrm{sec}$.

22. $14.97+\mathrm{ft}$. Regard the stick as the frustrum of a pyramid. See 17th question.
23. 25.525 - acres; or, 25 acres 2 roods 4 rods.
24. $486 \frac{1}{4}$ lbs.
25. $31.82+$ lbs.
26. $5.46+$ inches.
27. $6.88+$ inches.
28. $254.6+\mathrm{lbs}$.
29. $14.47+\mathrm{lbs}$.
30. $2049 \frac{1}{3} \mathrm{lbs}$.
31. 33 ft .4 .256 inches.
32. $18.8+$ lbs.
33. $182 \frac{2}{9}$ lbs.
34. $906 \frac{2}{3}$ lbs.
35. 2688 lbs.
36. 448 lbs.
37. $5_{1} \frac{73}{87}$ inches.
38. $72 \frac{2}{9}$ lbs.
39. $4_{1 \frac{1}{2}}$ inches.
40. 33 feet.
41. $9.69+$ lbs.
42. 82274.73 lbs .
43. 7.25 + feet.
44. $2.52+$ inches.
45. $94_{1^{2} 3} \mathrm{cwt}$.
46. $60_{\mathrm{T}^{3} \mathrm{~T}} \mathrm{cwt}$.
47. $75.6617+\mathrm{cwt}$.
48. $20.57+$ cwt.
49. 3918 cwt .
50. $122_{7}^{\frac{6}{77}} \mathrm{cwt}$.
51. $84 \frac{1}{2} \frac{2}{5} \mathrm{cwt}$.
52. 48 永 2 cwt .
53. $59+\frac{1}{5} \mathrm{cwt}$.
54. $20 \frac{17}{80} \mathrm{cwt}$.
55. $77 \frac{25}{25} \frac{3}{8} \mathrm{cwt}$.
56. $42 \frac{2}{3} \mathrm{cwt}$.
57. $17 \frac{7}{9} \mathrm{cwt}$.
58. As 1 to 12.
59. 1728 lbs .

## A PPENDIX.

1. $\$ 351.872+$.
2. $\$ 265.476+$.
3. $\$ 397.584$.
4. Dec. 22, 1845.
5. \$588.
6. $\$ 176.50$.
7. For himself and oxen, $\$ 1.729+$ each ; for his son, $\$ 0.864+$.
8. A gained $\$ 72.244$; B received $\$ 4.078$ per day; oxen, $\$ 2.719$ per day.
9. $\left\{\begin{array}{l}\text { A, } 6337 \text { lbs. } 14 \mathrm{oz.} \text {; } \\ \text { B, } 8222 \text { lbs. } 2 \mathrm{oz} .\end{array}\right.$
10. $\$ 79.703$ +.
11. $\$ 104.50$.
12. $\$ 29.589$.
13. $2.1212+$; $1.4142+\mathrm{ft}$.
14. $50 \frac{3}{5}+$ cubic feet.
15. $34.57+\mathrm{in}$. $; 21.81+\mathrm{in}$.
16. 40 cubic ft. ; 630 in .
17. 1st, $12 ; \cdot 2 \mathrm{~d}, 8 ; 3 \mathrm{~d}, 30 \mathrm{yds}$.
18. $\$ 40$.
19. 16 years old.
20. A's age, 24; B's age, 54 . 21. $31+\frac{1}{9}$ gallons.
21. .416666+.
22. . 3015873015.
23. .22916666+.
24. 95855760 .
25. 1013 ; 1019.
26. 2 and 1163.
27. $\left\{\begin{array}{c}\frac{820743}{1078231} ; \frac{859342}{1078231} \text {; } \\ 10485244 \\ 1078231\end{array}\right.$
28. 1s. 7.13566 d .
29. $\$ 60.21+$.
30. $\$ 114.118+$.
31. \$630.892.
32. $\$ 1244.581$ +.
33. $\$ 460.804$ +.
34. $\$ 137.181+$.
35. 3 mo .11 d .
36. $\$ 787.60$.
37. \$335.447.
38. $\$ 684.611$ t.
39. $\$ 222.283+$.
40. 607662 sq. in.
41. $\$ 25.85$.
42. 1472 sq. ft.
43. $\$ 174.37 \frac{1}{2}$.
44. 98.706+
45. $132.02+$.
46. 376.85 +.
47. $108.526+$ +
48. $\left\{\begin{array}{l}\text { Width, } 10.733 \text { in. } \\ \text { Thickness, } 5.366 \text { in. }\end{array}\right.$
49. $\{$ Width, 11.384 in. Thickness, 3.794 in .
50. 10 in. long, 10 in . wide,
51. $8.769+$ - 5 in. thick.
52. $12.01+$ +
53. $3.141+$ -
54. $\{$ One gains $\$ 179.287$. \{ The other, \$159.712.
55. $30 \frac{2 \pi}{306}$.
56. $281+\frac{13}{43}$.
57. $\$ 2218.978$
58. 470.016 gallons.
59. $2 \mathrm{ft} .11 .87+\mathrm{in}$.
60. 15 ft .6 .393 in .
61. $12+$ per cent.
62. $1_{13} \frac{3}{13}$.
63. $10_{14}^{4}$, first term.
64. 3460. 
1. $\frac{6561}{32768}$.
2. $1 \frac{5}{2} 7$, fifth term.
3. $\$ 120.38 \frac{1}{4}$.
4. $\$ 121.659+$.
5. $532 \frac{1}{2} \frac{17}{2} 93$.
6. $\frac{14850}{5940} ; \frac{34587}{5940} ; \frac{14}{5} \frac{1}{5} \frac{8}{40}$.
7. 150 bu. 2 pks. 7.154 qts.
8. $347 \frac{1}{12}$ cubic feet.
9. The first $\frac{17}{4}$, second $\frac{1}{45}$, third $\frac{13}{4}$ of whole pay.
10. $193 \frac{1}{7} \frac{7}{7}$ lbs.
11. $738 \frac{38}{6} \frac{8}{9}$ lbs.
12. $27.36+\mathrm{lbs}$.
13. $21.89+\mathrm{lbs}$.
14. 34.2 lbs.
15. 3.3 inches.
16. 18 and 60.
17. 16 and 66.
18. \$937.11.
19. $478 \frac{174}{23}$.
20. 200; 440 ; 880 .
21. $30 \frac{19}{40}$.
22. 54.77 + rods.
23. 32.76 rods.
24. .586.
25. . 846.
26. $15 \frac{3}{3} \frac{0}{0} 01$ times.
27. Afternoon :

Liverpool, 4h 32 m 20 s . Greenw'h, 4h 44m 16s. Havre, 4 h 45 m 20 s . Paris, $\quad 4 \mathrm{~h} 53 \mathrm{~m} 36 \mathrm{~s}$. 93. Before noon:
N. Y. 4 m 16 s past 7. Wash. 52 m 12s past 6. Cin. 22 m 32 s past 6.
94. 56 rods.
95. $\frac{1+\frac{91}{5}}{5} \frac{23 \frac{3}{5} 5}{15}$
96. $\$ 8.4713+$.
97. 5.241.
98. $185.31+\mathrm{lbs}$.
99. 갛훈.
100. $382 \frac{1}{2}$ square feet.
101. 1075.572 cubic feet.
102. \$79.998.
103. 24.
104. 5. 81+.
105. $7.9422+$.
106. $161 \frac{1}{2}$.
107. $4417 \frac{1}{2}$.
108. 264 lbs.
109. 19445159 oz.
110. $31.82+$ lbs.
111. $11.69+\mathrm{lbs}$.
112. \$68.623.
113. 389.225 + times.
114. $1015988{ }^{5}$ inches.
115. $\{$ Weight, 400 lbs . Power, 80 lb .
116. $\frac{33721 .}{127600}$.
117. $.36328+\mathrm{bu}$.
118. 9s. 1.488 d .
119. 21892 men.
120. 399.
121. 975 13
122. 98010 tiles.
123. $27.852+$ feet.
124. $2917 \frac{7}{8}$ feet.
125. $31.46+$ miles.
126. .823.
127. $\$ 682.0613$ +.
128. $\$ 1338.225$.
129. $4.4722+$ lbs.
130. 6 ; $60 ; 120$.
131. 9 and 78.
132. $\$ 1807.81$ द
133. 121.38+.
134. \$364.86.
135. $\frac{11994323}{1677216}$.
136. $3065^{\frac{8}{324} 9}{ }^{2}$.
137. 14.23+ lbs.
138. $233.26+$ cubic in.
139. $182.34+$ lbs.
140. 334.9 + inches.
141. $198.25+\mathrm{imp}$. gals.
142. \$18.928.
143. \$2.07.
144. 238.075 + miles.
145. $\$ 70.982$.
146. $18+$ cents.
147. 20 m .32 sec. past 7 P.м.
148. $3045 \frac{1}{4}$ square feet.
149. \$4.445.
150. 9929.22 lbs .
151. \$347.04.
152. $\$ 2961.504+$.
153. \$5555.053.
154. $\$ 27.106+$.
155. \$213.55.
156. \$16.05.
157. $4_{1}^{5{ }^{59}{ }^{59}}{ }^{2}$ barrels.
158. $27 \frac{1}{1}$.
159. $31_{\frac{737}{737}}$.
160. . 140625 of a $£$.
161. 31622400.
162. $22 \frac{1458}{58} 8^{5}$ sec. past 12.
163. 23 m .49 sec . before 1.
164. $\left\{\begin{array}{l}\mathrm{A}, \$ 184.658 . \\ \mathrm{B}, \$ 527.252 . \\ \mathrm{C}, \$ 334.088 .\end{array}\right.$
165. \$24.031+.
166. $\$ 1083.75$.
167. 19.1+ per cent gain.
168. $7.90+$ inches.
169. $207 \frac{9}{25}$ pipes.
170. $161.29+$ sq. inches.
171. The ratio of 72900 to 35693.
172. 30 sq. r. $3.5+\mathrm{sq}$. ft.
173. 3.6685 cubic inches.
174. $142 \frac{1}{29} 4$ yards.
175. 49.
176. 36.
177. $4900+1120+64$.
178. 8.
179. 1.
180. 27.
181. 40.89 +.
182. $98.4+$.
183. 1288.252+ cubic in.
184. 5.77 + inches.
185. $\left\{\begin{array}{l}\text { Thickness, } 7.427 \text { + in. } \\ \text { Width, } 9.902 \text { + in. }\end{array}\right.$ Length, $14.854+$ in.
186. 6112 cubic feet.
187. 2, 1873 ; 2, 37, 7, 19.
188. 361962656.
189. 2.
190. 26890.756 times.
191. $224.089+$ times.
192. $\$ 2219.444$ +.
193. $\$ 98.90+$.
194. Gained, $\$ 113.138$.
195. Lose, \$13.72.
196. $8 \frac{4}{96}$ days.
197. 9.013 + inches.
198. $322_{5}^{1}$ horses.
199. \$23.10.
200. 101.07十 rods.
201. 78.57 + rods.
202. 18.09+ rods.
203. $1520.534+$ sq. feet.
204. $210.487+$ inches.
205. 286.47 + sq. rods.
206. $323.066+\mathrm{sq}$. rods. 207. 5.3 inches.
208. 35.076 times.
209. $322.2286+\mathrm{lbs}$.
210. 35.3 + inches.
211. 907.
212. 9973.
213. 1; the numbers are prime to one another.
214. $3,3,89$; 2, 3, 3, 13, 13 ; 2, 293 ; 2, 2, 227.
215. 7 bushels of corn, 7 of rye, and 66 of oats.
216. $1 \frac{3}{4} \frac{0}{5} \frac{13}{2}$.
217. . 057614238 of a $£$.
218. \$575.24.
219. \$2.325.
220. \$236.28.
221. $\$ 2.33 \frac{1}{3}$.
222. $2.4217+$
223. 12 cwt. 3 qrs. 26 lb .8 oz.
224. 1 m .1 fur. 17 r .12 ft . $\frac{3}{4} \mathrm{in}$.
225. 1 Y. 235 d. 13 h .47 m . 45 sec .
226. 1006 A. 2 R. 24 r.
227. 3 T. 6 cwt. 1 qr. 9 lbs. $9 \mathrm{oz} .9 \frac{3}{5}$ drs.
228. $\$ 12.328$ +.
229. $14 \frac{2}{3}$.
230. 19.
231. 1264.
232. $7323 \frac{3}{7}$.
233. 1258.75.
234. $2.885+$ feet.
235. 15.82 rods.
236. 405.72 rods.
237. $68.87+$ feet.
238. 54.75 inches.

## REMARKS

ON THE

## INVESTIGATION AND DISCLSSSION OF ARITHMETICAL QUESTIONS.

No Arithmetic can be so minute and full in its explanations, and so regularly progressive in its examples, as to supersede the necessity of the teacher's aid. If such a thing were possible, that circumstance alone would render the work worthless as a school-book. It would substitute the lifeless printed page for the living soul of the instructor, animating his pupils by his eye, his tone, and his whole manner, and adapting himself each moment to the present exigency. The Arithmetic should be the teacher's aid, and not his substitute. The amount of aid required of the teacher in the use of a particular book, will depend on the natural ability of the scholars, and on the thoroughness with which they may have studied the preceding parts of the book.

It is important that the more backward members of a class be not pressed forward more rapidly than they can go and accomplish their work well. To secure this point, the more forward scholars should have some collateral study, in which they may engage, and thus be saved from feeling that they are losing their time while waiting for their classmates.

Another way of effecting the same, would be to allow the more backward pupils to omit, sometimes, a recitation in some other branch, in which the connection is less rigorous than it is in Arithmetic ; and, consequently, the injury from occasional interruption not so great.

The manner in which the teacher shall give the requisite aid to his pupils in Arithmetic, is a point of great importance, and cannot, perhaps, be too thoroughly considered by those engaged in this branch of instruction. How much aid shall the teacher give - and how should he give it?

To the first part of this question the answer is, the teacher should give aid enough, till the pupil is satisfied, and has accomplished all that the question required. How great this amount of aid should be, the teacher will readily determine
after well considering the second part of the question, -how he shall give it.

To render the suggestions that will be offered on this point more intelligible, we will suppose a pupil brings to the instructor the 9th example in Section XLIV.

Pupil. Will you show me how to find the answer to this question?

Teacher. What is the particular difficulty in the question?
P. I do not know how to find the answer.
T. How far can you go in the question?
P. I do n't know how to go to work at all in it.
T. Will you read the question?
$P$. "There is a triangle, the hypotenuse and one leg of which measure together 90 feet; the other side measures 40 feet; what are the lengths of the two first-named sides respectively?"
T. What kind of a figure does this question relate to ?
$P$. A triangle.
T. Can you draw such a figure on your slate?

The pupil in drawing the figure may be so heedless as not to draw a right-angled triangle. If so, the teacher may ask, What kind of a triangle is it to which the terms hypotenuse and leg apply?
$P$. A right-angled triangle.
T. Draw your figure, then, accordingly.

After the pupil has drawn the figure, the teacher will notice if he has made a good approximation to the relative lengths of the lines, as stated in the question. This, though not essential to the reasoning, is yet an important point, in order to secure close attention to every thing in the question. After the figure is drawn correctly, the investigation may proceed.
T. You may set down the length of the lines as far as they are given in the question.
$P$. The base, $a b$, measures 40 feet; the
 hypotenuse and the other leg, that is, $a c$ and $a b$, added together, measure 90 feet.
T. Very well; now this is a right-angled triangle. Tell me what you know of a rightangled triangle.
$P$. The square of the hypotenuse is equal to the sum of the squares of the other two sides.
T. What more do you know about it?

Here the pupil will, perhaps, pause. If necessary, let him go back and review the statements made respecting the rightangled triangle. When he is prepared, he will answer.
$P$. The square of one leg is equal to the difference between the square of the hypotenuse and the square of the other leg.
T. Now apply these statements to the triangle you have in hand.

Here, among other statements, the pupil will say, "The square of $a b$, or 1600 , is equal to the difference between the square of $a c$ and the square of $c b$."
T. Well; now, by the principle laid down on the page above this example, what is the difference between the squares of two numbers equal to?

Here another pause may be necessary; but let the pupil have no aid till he has overcome the difficulty, and is ready to proceed.
$P$. The difference of the squares of two numbers is equal to the product of their sum and difference.
T. Apply this to the triangle in hand, using the number given instead of the general term $a b$ square.
P. 1600 is equal to the product of the sum of the hypotenuse and base, multiplied by the difference between them.

This answer will bring the pupil to the last step of the solution, which he will probably take without difficulty. Having gone through the work, let him review it without aid, and then be required to solve another question proposed by the teacher, involving the same principles, but varying the quantities.

After the class has thoroughly mastered the principles involved in a question like this, it will be useful occasionally to enter on the full discussion of it, by varying the conditions. This may be done in the above example in the following manner.

Question. Fix your eyes on the triangle $a c b$, as drawn on the board, and answer this question: If the vertex $c$ descends perpendicularly towards $\bar{b}$, what quantities in the triangle will be altered?

Answer. The line $c b$ will become shorter.
Q. What other change ?
A. The line $a c$ will become shorter.
Q. What other change?
A. The sum of the two lines $a c$ and $c b$ will become smaller.
Q. Answer this question: Will the difference between those two lines be altered?
A. (After some pause.) The difference will become greater; for the line $c b$ diminishes faster than $a c$.
Q. Is there any quantity in the triangle that has not changed all this while?
$A$. The base $a b$ has not altered.
Q. In working this question, what quantity was the dividend?
A. The square of $a b$, or 1600 .
Q. What was the divisor, - and what was the quotient?
A. The sum of the two other lines was the divisor, and their difference was the quotient.
Q. In moving the vertex $c$ down, as above stated, what term in the operation becomes smaller?
A. The divisor.
Q. Now, if the divisor becomes smaller without altering the dividend, what will be the effect on the quotient?
A. It will become greater.
Q. Now, is this just what you found true of the difference of the lines, on moving $c b$ down towards the base?

By this series of questions, they will be led to the knowledge that the equation remains true through various changes in its conditions; and also, that a change in one factor is compensated by an opposite change in the other, whenever the equation requires that the product remain the same. By carrying the change above indicated in the line $b c$ still further, till the line vanishes at $b$, they will see that the proposition is still true; the sum of the two lines is then equal to the base,-for one of them disappears, and the other coincides with the base. And the difference is also equal to $a b$; for one is $a b$, and the other is zero. The proposition then becomes this: $(a b)^{2}=(a b+0) \times(a b-0)$ which is identical.

The point $c$ may be moved upward in the line $b c$ continued, and thus the discussion carried still further.

Another change in conditions is, to let the point $a$ move towards $b$ in the line of the base. Here the pupils may be led to see that both sides of the equation are made smaller by the change; inasmuch as the base is shortened, and both the
sum and the difference of the two other lines are made smaller.

These are but indications of the changes that may be made in the conditions of a question. Where a class is well prepared for an exercise of this kind, the introduction of it occasionally will greatly enlarge their powers of apprehension and invention. It would open the door for them to new and wider views, and lead them to look for much that is interesting in every proposition they study. It may be said that such a course would render the progress of scholars very slow. In reply, I would entreat instructors not to forget that the growth of mind is not to be measured by the extent of ground nominally gone over. A single new principle, well mastered and made the stimulus to reasoning and invention, contributes more to the pupil's mental growth, than whole weeks of the most expert practice in an old and beaten track.

It may not be amiss to add another example, illustrating the manner of investigating a question with a class.

Section XLIV., Example 20. In repairing a meetinghouse, it was thought desirable to alter the form of the posts, which were one foot square. It was proposed to cut away the corners, so as to make them regular eight-sided prisms. How wide must each face be, so as to have all the eight faces of exactly the same width?

Supposing the class to have been directed each to draw a section of the post on his slate, and do every thing he can towards finding the answer; they may afterwards meet the teacher for the investigation of the question.

Let a section of the post be drawn on the board.


Question. What can you now state respecting the figure on the board?

Answer. It is 12 inches on a side.
Q. What more can you state about it?
A. The sides of the octagon are all equal; $e b$ to $b c, b c$ to $c d$, and so of the others. The line $a e$ is equal to $a b$, and $c d$ to $d h$.
Q. What can you state respecting the triangle e a $b$ ?
$A$. The square of $e b$ is equal to the sum of the squares of $e a$ and $a b$.
Q. What more can you tell?

Here the class will pause. Let them dwell on the question till some one of them sees the next step and answers.
$A$. The square $e b$ is equal to twice the square of $a b$.
Q. Is that true? What do the rest of the class think of it?

Here let the class have time to apprehend the new step they have taken.
Q. Now look again, and tell me if there is any other line whose square is equal to twice the square of $a b$ ?
$A$. The square of $b c$.
Q. What is the ratio of the square of $b c$ to the square of $a b$ ?
A. The ratio of 2 to 1 .
Q. What is the ratio of the line $b c$ to the line $a b$ ?
A. The ratio of the square root of 2 to the sq. root of 1 .
Q. If, then, we call the line $a b 1$, what will be the proportional length of $b c$ ?
$A$. It will be the square root of 2 .
Q. How may we express the proportional length of the whole line, made up of the parts $a b, b c$, and $c d$ ?
$A$. The part $a b$ will be $1 ; b c$ will be the square root of 2 ; and $c d$ will be 1 , for it is equal to $a b$. The whole line, therefore, will be $2+$ sq. root of 2 .
Q. Very well. Now the sq. root of 2 you can find; you will then have the proportional value of the whole line, and the proportional value of each part. What is the thing which you seek?
A. The length of the part $b c$.
$Q$. The proportional length, or the actual length?
A. The actual length.

Teacher. You must now form a proportion with the data you have, so as to give you the part you seek.

Here let the class, without any aid, but by repeated trials, form the required proportion in general terms.

As the proportional length of the whole line is to the proportional length of the part b c , so is the actual length of the whole line to the actual length of the part b c.

Before closing the recitation, it will be well to go over the investigation once more. The class may then be dismissed to perform the work. When they have obtained the answer, they should, before reading it, go through the investigation once more; and then let the instructor originate a question
involving the same principles, to be answered at the next recitation.

It will often happen that a smaller amount of aid will be needed than is indicated in the investigations given above. Much will depend on the age and maturity of mind in the class, and on their previous training. The object with the teacher should be to give just that amount of aid that will in the highest degree both stimulate and reward the student's efforts; the less the better, provided the student does not become discouraged.

## Investigation of Question 23. Section XLIV.

Two men purchase, in equal shares, a stick of hewn timber, 40 ft . long, 2 ft . square at the larger end, and 1 ft . square at the smaller end; how far from the larger end shall they cut it in two, so that each may have exactly one half?

Question. What step can you take towards solving this question?

Answer. It is a part of a pyramid. We may complete the pyramid, and then the whole pyramid will be 80 feet long.
Q. If that is done, what can you state further?
A. We can find the solid contents of the whole pyramid; and also the solid contents of the added part.
Q. If you can find those, what more can you ascertain?
$A$. We can find the solid contents of the stick, by subtracting the added part from the whole pyramid; and then we can find the solid contents of one half of the stick.
Q. Now consider what you have found, and see if you can take one step more?

This important step the class should not be aided in. Let them dwell on it till some one is ready to proceed.
A. We can find the solid contents of a pyramid made up of the added part of half of the stick.
Q. Compare now the added part with the whole pyramid; what is the ratio of their solid contents?
A. As the cubes of their corresponding dimensions; the solid contents of the added part is to the solid contents of the whole, as the cube of 40 is to the cube of 80 ; or as 1 is to the cube of 2 .
Q. If, then, you can find the solid contents of the added part 1, what will you call the solid contents of the whole?
A. 8.
Q. What will be the proportional solid contents of the stick? A. 7.
Q. What number will express the contents of the pyramid embracing the added part and half of the stick?
A. The added part is 1 ; the stick is 7 ; half of it is $3 \frac{1}{2}$; the proportional contents of the pyramid then will be $4 \frac{1}{2}$.
$Q$. What proportion now will you form between the solids and the cubes of their dimensions that will give you the distance from the vertex to the dividing line for the 4th term?

Here let the class have no direct assistance, but find by trial the required proportion, in general terms.

The contents of the added part is to the contents of the pyramid embracing the added part and half of the stick, as the cube of the length of the former is to the cube of the length of the latter. Or, using numbers, $1: 7 \frac{1}{2}:: 40^{3}: \square$.

The statement of this proposition may very well be made a separate study for the class at their desks. In this case they may be instructed to write out all the propositions that they make trial of in their study, and in the recitation state what proportions were either incorrect or inappropriate, and the reason why they were so.
yondo.
$\frac{1}{4}=0$


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