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KEY TO THE UNIVERSE

RESISTANCE

PROPULSION



NEW THEORY OF ITS MECHANISM

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KEY TO THE UNIVERSE.



Planetary and Stellar Worlds “roll upon their wings in their glory, in the midst of the power of God.”



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KEY TO THE UNIVERSE,

OR A

NEW THEORY OF ITS MECHANISM.

FOUNDED UPON A

- I. CONTINUOUS ORBITAL PROPULSION, ARISING FROM THE VELOCITY OF GRAVITY AND ITS CONSEQUENT ABERRATIONS;
- II. RESISTING ETHEREAL MEDIUM OF VARIABLE DENSITY.

WITH

MATHEMATICAL DEMONSTRATIONS AND TABLES.

BY

ORSON PRATT, SEN.

193846

SECOND EDITION,

FROM THE FIRST EUROPEAN EDITION.

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P R E F A C E .

To the first edition of this work, published in England in the early part of the present year, the following was prefixed:

“A new theory of the mechanism of the universe is propounded by the author, not with a design of subverting, or, in the least degree, interfering with the great law of universal gravitation. On the contrary; it is intended to greatly extend the universality of that law. Modern astronomers and scientists have excluded the immense ocean of ethereal substance from the catalogue of gravitating matter: thus limiting the grand discovery of Newton, intended to be universal, to such gross materials only, as they may consider endowed with the gravitating power. The aim of the author, therefore, is to vindicate the UNIVERSALITY of the law: to rescue it from the environed limits sought to be thrown around it: and to give it that unlimited freedom of action, which the distinguished name, “UNIVERSAL,” so appropriately and definitely imports.

A new theory of celestial mechanism is, at first, startling to those who have not given the subject their special attention. They have unhappily formed an idea, that all the varied phenomena of the universe can be

accounted for, by the grand discoveries already made. But this erroneous notion is not general. Mathematicians and the able expounders of the Newtonian system, have clearly pointed out numerous out-standing and very important movements, among celestial bodies, which cannot be explained without the aid of some new theory. Hence, La Place and others introduced the "Nebular Hypothesis," which seemed, for a time, to render a sufficient cause for certain movements observed. But, as astronomy advanced, the hypothesis weakened: and recent observations prove its total inefficiency, as a cause, to expound the phenomena alluded to. Hence, the astronomical world are again at sea, without "rudder or compass" to guide them, in respect to the causes which have hitherto so perplexed their most skillful navigators.

Astronomical science, in its present advanced condition, needs a theory which will answer, as far as possible, the following reasonable questions:—

FIRST,—Why are the orbital movements of planets, asteroids and satellites, in the solar system, in one general direction, namely, from west to east, instead of moving indiscriminately in all directions?

SECOND,—Why do planetary bodies rotate upon their respective axes? Why do they rotate from west to east, instead of the contrary direction? Is there any law governing their diurnal periods?

THIRD,—Why do the eccentricities of planetary orbits differ? Why do the orbits so closely approximate circular forms? Will they eventually become circular? Were they once greatly elongated, like those of comets?

FOURTH,—Why are the planes of planetary orbits confined within the narrow limits of the Zodiac? Why are they slightly inclined to each other? Will they ever

become co-incident? Did their inclinations ever have a far greater range than they now have? Did any of the planets ever revolve in retrograde orbits?

FIFTH,—Why do not the planes of diurnal rotations co-incide with the respective orbital planes? Did these planes ever co-incide? If so; are there any causes which will compel them into a future co-incidence?

SIXTH,—If the satellites of Uranus were originally detached from the primary by rotation and condensation, according to the prevailing hypothesis, why have they a retrograde motion in their orbits, contrary to that of all others in the system?

SEVENTH,—If the “Nebular Hypothesis” be true, how is it possible for the inner satellite of Mars to revolve around its primary three times while the planet rotates but once?

EIGHTH,—Observation seems to indicate, that there is an intra-Mercurial planet, so near the Sun, as to have a period less than the solar rotation; how can this be possible, if the theory of La Place be true?

NINTH,—Will cometary orbits ever be converted into those of a planetary form?

Until a theory is propounded sufficiently comprehensive to include, at least, partial answers to the above questions, we may consider the great science of astronomy but imperfectly developed. Indeed, a new theory is imperatively called for. As none have recently volunteered their services in this great enterprise, the author, unaided and alone, has launched his humble barque upon this great unexplored ocean, with a compass of his own invention. How far he may succeed, in shunning the mists and fogs which others have, unfortunately, encountered, will be known after the experiment.

Mathematical demonstrations of the fundamental laws of the new theory have been given. But, in all cases, the author has endeavored to simplify these analytical investigations, by strictly avoiding the use of the higher Calculus, and confining the demonstrations to the simple algebraical rules of ratio and proportion.

The Tables, in the Appendix, have required a vast amount of labor, in preparing them in accordance with the new theory. But if the mathematical theorems and laws, developed, shall be of any general utility in advancing a true knowledge of the mechanism of the universe, the author will consider himself amply rewarded for all his wearisome toil."

The second edition has not been enlarged or changed from the first; hence, it is deemed unnecessary to extend the Preface.

O. PRATT, SEN.

HISTORIAN'S OFFICE,
SALT LAKE CITY,
UTAH TERRITORY,
JULY, 1879.

CONTENTS.

CHAPTER I.

ETHEREAL MEDIUM.

1. Two Theories. 3. Universal Ether. 7. Immense Quantity of Ethereal Matter. 8. Resistance of Ether. 9. Elasticity of Ether. 11. Power of Ethereal Waves. 12. Ethereal Waves Electric. 13. Ethereal Waves Magnetic. 14. Their Chemical Properties. 15. Attraction and Repulsion of Ether. 16. The Ethereal Medium of Variable Density. 17. Does Density affect the Wave-Velocity of Ether? 18. How to determine the Question—its importance. Pages 13—20.

CHAPTER II.

ROTATING ETHEREAL ENVELOPES.

19. Ethereal Surface of Rotation; Definition. 20, 21. Law of Ethereal Semi-diameters—Examples. 22. Remarkable Forms of the Ethereal Envelopes. 23. Ethereal Medium External to the Rotating Envelopes. 24. Law of Planetary Velocity. 25. Circulating Currents External to the Envelopes—their Periodic Times increase with the latitude, when the Axial Distances are the same; Examples. 26. Planes of Motion, Perpendicular to the Axes. 27, 28. Tendency to Equilibrium—its effects upon the Rotation of Bodies. 29, 30. The Author's Theory Confirmed by Recent Observations—Sun-Spot-Periods Increase with the Solar Latitude. 31. Sun's Rotation less than the Sun-Spot-Periods. 32. Rotation Diminished by Ethereal Resistance, unless Balanced by a Constant Acceleration. Pages 21—29.

CHAPTER III.

GRAVITATION.

33. Gravitation is not an Instantaneous Force. 34. Gravitating Force transmitted with the Velocity of Light. 35. Aberration of the Gravitating Force. 36, 39. Aberration illustrated by a right angled triangle. 40. Aberration of Force in Elliptic Orbits the same as that of Light. 41. Law of Aberrating Velocity, expressed in terms of Planetary Distances. 42. Orbital Accelerations. 43. Intensity of the Earth's Gravity toward the Sun. 44. Intensity of the Earth's Orbital Acceleration, expressed in terms of Earth's Gravity. 45. Or-

bital Acceleration, expressed in Pounds Weight. 46. Earth's Fall towards the Sun in one second. 47, 48. Excess of Orbital Space, gained in one second, by the Aberrating Force—Space gained in one year—Increase of Periodic Time. 49. Aberrating Intensities vary with those of Gravity. 50. Law of Aberrating Forces. 51. Definition. 52. Law of Aberrating Forces, expressed in terms of Planetary Distances. 53. The law applicable to the Satellites. Pages 30—39.

CHAPTER IV.

COMPOUND ORBIT OF THE SUN.

54. Solar Orbit. 55. Aberrating Intensities vary as the Masses. 56. Apparent Places of Planets as seen from the Sun. 57. Law of Aberrating Forces, as exerted by Planets on the Sun. 58. How to find the resultant action of all the Planetary Aberrating Forces. 59. Acceleration of the Sun in his Orbit, arising from the Aberrating Force of the Earth. 60. Pounds pressure, exerted by the Earth, in propelling the Sun in his Orbit. 61. Excess of Orbital Space, gained by the Sun in one year—Enlargement of his Orbit, and the Increase of his Periodic time. 62. Algebraic expression of the Law, given in Par. 57. 63. Accelerations of Rotation. 64, 65. Law of the Aberrating Force of Rotation. 66. Law, relating to the Rotating Particles of the Envelope. Pages 40—44.

CHAPTER V.

ABERRATIONS IN ELLIPTIC ORBITS.

67. Elliptic Orbits. 68. Law of Velocity in an Elliptic Orbit. 69. Law of Angular Velocity. 70, 71. Aberrating Velocity—its Law in Elliptic Orbits. 72. Law of Aberrating Forces in Elliptic Orbits. 73. Angular and Aberrating Velocity at the mean Distance. 74. A Particle at rest must describe an Orbit around a body in Motion. 75. How to estimate the joint Aberrating Forces of two bodies in Motion. 76. When the Aberrating Forces of two bodies become neutralized or Zero. 77. General Theorem. Pages 45—51.

CHAPTER VI.

RESISTING MEDIUM.

78. Ethereal Resistance unlike that of Gross Matter. 79. Transfused Resistance. 80. Mass Resistance. 81. Velocity Resistance of one particle. 82. Resistance of a given number of particles. 83. In a medium of Uniform Density, Resistance is as the Square of the Velocity. 84. Density Resistance. 85. General Law of Resistance. 86. Law of Density of the Ethereal Medium, in terms of the Sun's Distances. 87. Law of Resistance, expressed in terms of the Sun's Distance. 88. Resistances of the Planets vary as their Orbital Accelerations—Under what conditions the two Antagonistic Forces balance each other. Pages 52—56.

CHAPTER VII.

RESISTANCES IN ELLIPTIC ORBITS.

89. General Law. 90. To find the Point in an Ellipse where the two Antagonistic Forces Balance—Balancing Point when the Ellipse becomes a Circle, or a

straight line—Discussion of the Formula for Ellipses of any Eccentricity—Elliptic Perturbations—Minute Excess of Resistance—Decrease of Eccentricity and of the Periodic Times—Instability of Hyperbolic and Parabolic Orbits. 91. Example 1. Balancing Point in the Earth's Orbit—Example 2. Difference of the Two Forces at the Perihelion—Example 3. Difference at Aphelion—Example 4 and 5. Difference at Two Intermediate Points—Example 6. Balancing Point in the orbit of Venus. 92. How to find the Radius Vector for any given time. 93. Variation of the two Forces in an Ellipse. 94. Sums of the two Forces in an Ellipse. Pages 57—65.

CHAPTER VIII.

INVARIABLE ELLIPTIC ORBITS.

95. Intensity of the two forces not necessarily equal. 96. How to find their ratio at mean distance. Two Examples. 97. Method of calculation General. 98. Minute apparent discrepancy; probable cause. Pages 66—68.

CHAPTER IX.

ETHEREAL CURRENTS.

99. Solar Rotation the Origin of Ethereal Currents. 100. The Currents must be Circular and their planes perpendicular to the Axis of the Solar Envelope. 101. Thickness of the two Polar Wings at 1000 millions of miles from the Sun's Center. 102. Immense Extension of the Currents in Planes Perpendicular to the prolonged Solar Axis. 103. Is there a law of Velocity for these Ethereal Currents, depending on Solar Distances? Example. 104. Two important data required. 105. Resulting Consequences, if the Ethereal Currents had a Planetary Velocity. How the two Forces must be proportioned in Ethereal Currents of any given Velocity. Pages 69—72.

CHAPTER X.

REVOLUTION FROM WEST TO EAST.

106. "Nebular Hypothesis." Its Insufficiency as a Cause to account for observed phenomena. 107. Upon what a New Theory should be founded. 108. Direction of Planetary Orbits dependent on Ethereal Currents. 109. An extreme case of Retrograde Motion considered. 110. Universality of the New Theory; its great Contrast, when Compared with the limited Hypothesis of La Place. Pages 73—76.

CHAPTER XI.

DIURNAL OR AXIAL ROTATION OF PLANETS.

111. Is Rotation the Result of a single Impulse, acting in the form of Projection? Or, is it the Result of Continuous Forces still operating? 112. Rotations of Machinery. 113. How Celestial Rotations are Generated. Initial Rotations. Direction of Rotation. 114. Causes for the Inclination of the Planes of Rotation to the Planes of the respective Orbits. 115. How Retrograde Rotations are converted into Direct ones. 116. Retrograde Satellites of Uranus. Their orbits must be converted into the Direct Form, long before their axial Rota-

tions obtain the same Form. 117. Necessary Data for Calculating Diurnal Periods. Pages 77—80.

CHAPTER XII.

DIURNAL PERIODS OR PERIODS OF ROTATION.

118. Approximate Law for Diurnal Periods, as Published by the Author about a quarter of a century ago. 119. A Rotation upon an Axis is a Stable Equilibrium. Any derangement in the Period, cannot be Permanent. The two Forces will work their own Adjustments. 120. Example. Periods of Rotation the result of a state of Equilibrium between the two Forces. 121. Effects of Terrestrial Aberration upon the Diurnal Period of the Earth. 122. Further Development of the Theory left to others who have time and means at their command. 123. Remarks on the Causes of Solar Rotation. 124. How the Aberrating Force is increased. The Solar Rotation the result of the Equilibrium of the two opposing Forces. Pages 81—93.

CHAPTER XIII.

REDUCTION OF COMETARY ORBITS TO A PLANETARY FORM.

125. The Data necessary for Reducing Orbits may be obtained by careful Observation of *one* Orbit. 126. Hyperbolic Orbits require a long Period, before they are conquered to the Elliptic Form. 127. Parabolic Orbits easily subdued to the Elliptic Form. Causes of great Eccentricity. Worlds in Embryo. Their future Destiny among Planets. The Great Creator works by Law. 128. The work of Creation has its immense Periods. 129. The great Problem of Reducing a Cometary Orbit to a Planetary Form. 130. Other Systems of the Universe. The Extension of the New Theory to them. Pages 94—104.

MISCELLANEOUS EXAMPLES, ILLUSTRATING THE NEW THEORY.

APPENDIX.

TABLE I.—Relative to the Aberrating and Resisting Forces in the Earth's Orbit, calculated for each Day. Pages 107—112.

TABLE II.—Relative to the Aberrating and Resisting Forces in the Orbit of Venus, calculated for each Day. Pages 113—116.

TABLE III.—Relative to the Aberrating and Resisting Forces in the Orbit of Mercury, calculated for each Day. Pages 117, 118.

KEY TO THE UNIVERSE.

CHAPTER I.

ETHEREAL MEDIUM.

1. *Two Theories.* 3. *Universal Ether.* 7. *Immense Quantity of Ethereal Matter.* 8. *Resistance of Ether.* 9. *Elasticity of Ether.* 11. *Power of Ethereal Waves.* 12. *Ethereal Waves Electric.* 13. *Ethereal Waves Magnetic.* 14. *Their Chemical Properties.* 15. *Attraction and Repulsion of Ether.* 16. *The Ethereal Medium of Variable Density.* 17. *Does Density affect the Wave-Velocity of Ether?* 18. *How to determine the Question—its importance.*

1. CORPUSCULAR AND UNDULATORY THEORIES.—Since the days of Newton, until very recently, scientists have been divided, in regard to the manner in which light is transmitted through space. Two theories were propounded; one called the *Corpuscular*; the other, the *Undulatory*. The former assumes that light consists of extremely minute particles or corpuscles, projected from luminous bodies, with immense velocity; that such particles impinge upon the optic nerve and produce the sensation of seeing. The Undulatory theory claims that all space is occupied with a substance hypothetically called Ether, extremely rarefied and elastic in its nature; that the molecules of luminous bodies, being themselves in a constant state of tremulous agitation, impart the same to the adjoining molecules of ether; that this jar or vibration is transmitted from molecule to molecule, forming a wave; that the displacements of the

molecules in a wave are not that of extension from and compression toward the point of their origin, but in circular forms, whose planes are transverse or perpendicular to the line of motion of the wave; that the wave thus formed, travels through the ethereal space with the immense velocity of 185420 miles per second, which is over 660 thousand times swifter than the tidal wave of the ocean, and over 820 thousand times the velocity of longitudinal sound-waves. It is still further assumed that a white luminous body forms a continuous succession of mixed waves, varying in length, from 37640 to 59750 waves in one inch; that the longer waves are formed more slowly than the shorter ones, but travel with the same velocity; that only 468 millions of millions of the longer waves are successively originated in one second; while 727 millions of millions of the shorter ones are formed in the same time. It is also assumed that the variations of color are merely variations of wave-lengths, and the rapidity with which they are formed.

2. The Corpuscular theory is now generally discarded: it is found to be insufficient to account for many phenomena exhibited by light.

The Undulatory theory, when examined by the aid of mathematical analyses, renders a sufficient cause, for nearly all the curious and complicated exhibitions, so wonderfully and universally manifested by the immense ocean of ethereal substance. Those outstanding phenomena not yet brought within the domain of analytical investigation, will undoubtedly yield, one by one, as research, in the exact sciences, shall be extended.

3. In my future investigations, I shall adopt the theory of a universal ether, with two great and important modifications, namely:—

I. That the ethereal matter, like all other matter, is subject to *Gravity*.

II. That the ethereal matter, like all other matter, possesses the quality of *Resisting and being Resisted*.

4. It has been almost universally assumed, without any substantial evidence, that the ethereal substance has no gravitation; that it is uniformly distributed throughout space; and that it manifests no resistance to moving bodies, revolving in

or passing through it. In support of these assumptions, it is urged that, if the ethereal medium were possessed of gravity, it would collect in greater density, around all stellar and planetary bodies, leaving the intermediate spaces, in a comparatively rarefied condition; that it would necessarily resist all moving bodies in their orbits, bending them into curves of an inward spiral form, approaching nearer and nearer to the great central masses, around which they respectively revolve; and that it would finally plunge the whole universe into irretrievable ruin. Such, undoubtedly, would be the consequences, if there were no antagonistic propelling force, to counteract or balance the resisting force.

5. A non-resisting substance is philosophically inconceivable. That which is non-resisting, when uniformly distributed, cannot possibly manifest resistance, when its particles are condensed to any assignable degree. If all the ethereal substance of nature were collected into one cubic mile, all other substance, by this vague hypothesis, would pass through it, without the least resistance, as if the space were an absolute vacuum. Such an assumption, however popular, is unworthy a place in the annals of philosophy.

6. The observed decrease in the periodic times of Encke's comet, has, in some small degree, dissipated the idea that the ethereal medium is a non-resisting substance. Scientists are beginning to abandon, though reluctantly, this most unphilosophical and inconceivable notion. But it is difficult to free the human mind, at once, from popular traditional errors. It requires time to discipline and accustom the intellect to new fields of research. Consequently, there are some, even now, who cling, with great tenacity, to the absurd conjectures of former times, as though age had sanctified them, and made them true. Science, however, in its accelerated and triumphal march, cannot afford to wait for those timid, sluggish followers. "What is truth," is the inspiring watch-word of the age; it animates the bosom and is on the tongue of every lover of intellectual progress.

7. IMMENSE QUANTITY OF ETHEREAL MATTER. — Every cubic inch of space must contain, at least, fifty-three millions

of millions of luminous waves. It has been demonstrated by numerous and skillful experiments, that one inch in length of the extreme red rays of light, contains 37640 waves. And it can easily be demonstrated, and is generally admitted, that the depth of a luminous wave is almost infinitesimally smaller than its length. But if the depth of each wave, measured transversely to its line of motion, be assumed equal to its length, then the number of waves in a cubic inch must be

$= (37640)^3 = 53,327,207,744,000$. The extreme violet gives 59750 waves in one linear inch; hence, a cubic inch is

$= (59750)^3 = 213,311,234,375,000$ waves.

As each wave undoubtedly consists of numerous molecules, how extremely minute must be the pores between these molecules! the spaces unoccupied must be infinitesimally small, and yet all the stellar and planetary bodies of the universe perform all their evolutions in the midst of this immensity of substance. Resisted they must be; and without a compensating accelerating force, a universal ruin must speedily ensue.

8. WAVES OF ETHER, LIKE OTHER MATTER, ARE CAPABLE OF RESISTING AND BEING RESISTED.—The luminiferous waves are originated by the action of gravitating matter; when reflected, their course is changed by other matter; they are retarded and their course altered, when passing through water, glass, crystals, and other transparent materials; they are refracted into curvilinear paths, in passing through the atmosphere obliquely to the earth's surface: they are absorbed, and their momentum destroyed by opaque materials: they act upon the optic nerve, and impart to it a tremulous motion. That which can exhibit all these complicated phenomena, must, therefore, have the power of resisting and being resisted.

9. THE ETHEREAL MEDIUM IS INTENSELY ELASTIC.—When the equilibrium of the ether is, in the least, destroyed, as in the formation of luminiferous waves, the particles almost instantaneously return to their former state of repose. So rapidly are these disturbances and restorations to repose, performed, that over seven hundred millions of millions of waves of violet light are successively formed and destroyed, in one second of time.

And as the return of each wave to its equilibrium, occupies the same time as the disturbance, it follows, that over seven hundred millions of millions of restorations successively transpire, in one-half of a second. All these restorations are produced by the elastic force; therefore, the ethereal medium is intensely elastic.

10. THE ETHEREAL WAVES CAN BE CONCENTRATED OR DISPERSED.—Both the luminiferous and heat waves can be either refracted or reflected to a focus, and *vice versa*, they can be either refracted or reflected from a focus into a variety of paths.

11. THE ETHEREAL WAVES CAN CHANGE THE FORM OF ALL MATTER.—When these waves enter solids in the form of heat, the solids are dilated or expanded, and sometimes converted into liquids, or even into gases. By these ethereal waves, chemical compounds are torn asunder, and their elements are made to appear; by them, steamers of great weight plow the ocean against both wind and current; by them, lengthy trains, of heavily laden cars, rush with terrible speed from ocean to ocean, till distance itself seems almost annihilated; by these almost infinitesimal undulations lofty mountains are upheaved, and their smoking summits tell of the fiery billows which rage, in awful grandeur, far beneath; by these vibrations of the subtle ether, the earth itself rocks to and fro, and its very foundations tremble as if about to divide asunder. Solids, liquids, gases, compounds, elements, and all terrestrial phenomena, bow in humble reverence, and submit themselves to the powerful control of this most potent substance—the ethereal medium.

12. THE ETHEREAL WAVES ARE ELECTRIC.—The electricity imparted to the earth, by the solar rays, is experimentally known, and almost universally admitted. Our earth is a great reservoir of electric waves, receiving a constant supply from the great solar fountain, and radiating the same into the surrounding spaces. The ethereal substance intervening between the sun and earth, when electrically agitated manifests itself in the form of electricity.

13. THE ETHEREAL WAVES ARE MAGNETIC.—The experi-

ments upon steel sewing needles, placed in the violet rays of the sun, show most conclusively that those rays are magnetic. All our experiments, in forming magnets, prove that the presence of a magnetized body is necessary to impart its power to an unmagnetized body. If the violet solar rays were destitute of this property, or, in other words, if the peculiar waves of the ethereal medium, called violet, were not magnetic, no other substance, in their presence would be magnetically affected.

14. THE ETHEREAL WAVES POSSESS CHEMICAL PROPERTIES.—There are certain waves of ether, not far from the violet end of the solar spectrum, that are especially denominated *Chemical*. This peculiar chemical action is no where more beautifully illustrated, than in the department of photography. But in the great laboratory of nature, the ethereal substance, in the form of heat and electric waves, manifests its most wonderful and powerful chemical operations.

15. THE ETHEREAL WAVES BOTH ATTRACT AND REPEL.—Steel needles, when magnetized, exhibit polarity: unlike poles attract; like poles repel. When parallel waves of ether proceed in the same direction, they attract each other: when they proceed in opposite directions, they repel each other. This is demonstrated by innumerable experiments on galvanic currents. In one form and direction of the wave-current, the particles of bodies cohere, with great tenacity. The cohesion is modified and changed as the wave-current varies. If the variation becomes sufficiently great, cohesion is destroyed, and the molecules exist in the liquid form; a still greater variation in the magnitude, intensity and direction of these waves, separates liquid substances into a gaseous form, in which state, their molecules become mutually repulsive.

Is it reasonable to assume, that the ethereal substance, possessing so many active qualities, is independent of the great law of universal gravitation? If light, heat, electricity, magnetism, chemical affinity, and cohesive attraction, are interwoven so closely with the wave-motions of ether, would not analogy strongly urge the claims of this, the most active of all substances, to a high and most powerful position, among the universal list of all other materials? How can we, with any

degree of assurance, exclude it from that universal power with which all other matter is endowed?

16. THE ETHEREAL MEDIUM IS NOT OF UNIFORM DENSITY.—The ethereal medium, by virtue of its gravitation toward all other matter, must collect around all worlds, and nebulous forms of other substances, in the form of atmospheres or envelopes. These ethereal coverings or envelopes must increase in density, as their respective centers of gravitation are approached.

As gross matter is not impervious to ethereal matter, the latter must infuse itself through all the interior of worlds, its density increasing, until the centers of gravity are reached. Outward from the surfaces, the densities will decrease, as far as the ethereal envelopes of rotating bodies can extend. Beyond the surfaces of these rotating envelopes, the densities will still continue to decrease, until reaching the limits of equal gravitation, between world and world. Such must be the condition of the ethereal medium, under the force of gravity.

17. *Is it probable that the Velocity of the ethereal waves is Variable, according to the Density of the Ether through which they are propagated?*

The average velocity of light, in passing diametrically across the earth's orbit, is 185,420 miles per second. But as some portion of its path lies near the sun, in close proximity to the densest strata of the ethereal medium, may not the luminiferous waves be continually retarded for 493.096 seconds, during which they describe the first half of their path? And in the second half of their journey, may they not be continually accelerated, during an equal interval of time? If so, the sum of their retardations will be exactly equal to the sum of their accelerations, and the average velocity will be as stated above.

18. To those who may be anxious to test, by observation, the variable velocity of light, (if such exist,) arising from the gravitating power, and consequent variable density of the ethereal medium, the writer would suggest the following:—

Let the observer, with good instruments, carefully determine, from the eclipses of one of Jupiter's satellites, the exact time of the passage of light, through different chords of the

earth's orbit. The exact lengths of these chords are easily calculated, it is evident, that the shorter the chords, the greater will be the distance from the sun, and the less will be the density of the medium, and the greater *may* be the velocity of light, and the less *may* be the time in passing over equal distances. If such variations be found to exist, the exact determination of these data, will be of immense importance in the future development of astronomy: for the data, thus obtained, will determine.

First, the relative densities of the different strata of ether, intervening between the earth's orbit and the sun:

Second, the relative elastic forces of these strata, as compared with their respective forces of gravitation toward the sun.

These discoveries would, very probably, develop some law of density, depending on the distance from the sun: if so, such law could be extended into the vast distances beyond the present boundaries of our system, till we reach the sphere of equal gravitation, between our system and others.

Such a law would also develop the law of planetary resistance, depending on their respective masses, velocities, distances, and the relative densities of the medium in which they move.



CHAPTER II.

ROTATING ETHEREAL ENVELOPES.

19. *Ethereal Surface of Rotation; Definition.* 20, 21. *Law of Ethereal Semi-diameters—Examples.* 22. *Remarkable Forms of the Ethereal Envelopes.* 23. *Ethereal Medium External to the Rotating Envelopes.* 24. *Law of Planetary Velocity.* 25. *Circulating Currents External to the Envelopes—their Periodic Times Increase with the latitude, when the Axial Distances are the same; Examples.* 26. *Planes of Motion, Perpendicular to the Axis.* 27, 28. *Tendency to Equilibrium—its effects upon the rotation of Bodies.* 29, 30. *The Author's Theory Confirmed by Recent Observations—Sun-spot-Periods Increase with the Solar Latitude.* 31. *Sun's Rotation less than the Sun-spot-Periods.* 32. *Rotation Diminished by Ethereal Resistance, unless Balanced by a Constant Acceleration.*

19. ETHEREAL SURFACE OF ROTATION.—The ethereal medium, not only collects around worlds in greater density, but it also partakes of the rotatory motion of the body around which it gathers. This rotation necessarily produces an envelope of definite form and dimension, depending on the balanced condition, between the centripetal and centrifugal forces. The rotating envelope can only extend to the distance, where the rotating velocity becomes sufficiently great to balance the gravity of the surface particles. The vast ethereal ocean beyond these limits, cannot rotate in the same time as the world within. But if it revolves at all, each particle must have a velocity slower than the surface rotation.

Definition.—The distances from the center of a rotating globe to the limiting surface of the ethereal envelope, or to that surface which can possibly rotate, in the same time, as the central globe of grosser matter, without being thrown off by the centrifugal force, will be called, *Ethereal semi-diameters.*

20. Let D, D' represent the perpendicular axial distances of any two points in a limiting ethereal surface of rotation; let d, d' represent the respective distances of the two points from the center of gravity of the rotating globe, which distances, by definition, will be the ethereal semi-diameters; let v, v' represent the respective rotative velocities of the two points.

The rotating velocities of any two ethereal surface points vary as their perpendicular axial distances; thus

$$v : v' :: D : D' \dots \dots \dots (1)$$

The rotating velocity and the planetary velocity, of any ethereal surface point, must be the same; otherwise, the point would lose its characteristic as a surface point.

According to a well known law, the planetary velocity of an ethereal surface point varies inversely as the square root of its distance from the center of gravity; that is, inversely as its ethereal semi-diameter; hence, we have

$$v : v' :: \frac{1}{\sqrt{d}} : \frac{1}{\sqrt{d'}} \dots \dots \dots (2)$$

By (1) and (2) we obtain

$$\frac{1}{\sqrt{d}} : \frac{1}{\sqrt{d'}} :: D : D';$$

hence

$$d : d' :: \frac{1}{D^2} : \frac{1}{D'^2} \dots \dots \dots (3)$$

Thus we have the following

21. LAW.—In a rotating globe, whose axis is infinitely prolonged, the ethereal semi-diameters of its envelope vary as the inverse squares of the perpendicular axial distances of the points of their intersection with the limiting surface.

In illustration of this law, the following examples are given.

Example 1.—Find the equatorial ethereal semi-diameter of the sun, the period of his equatorial rotation being 25 days, and his distance from the earth being 91430000 miles.

Let t = the earth's orbital period;

$t' = 25$ days = the solar-equatorial-ethereal-surface-rotation;

Let d = the sun's mean distance from the earth;

d' = the required equatorial ethereal semi-diameter.

By Kepler's law, we have

$$t^2 : t'^2 :: d^3 : d'^3;$$

hence, when reduced to figures, we have

$$d' = 15298548 \text{ miles from the sun's center.}$$

Example 2.—If the equatorial ethereal semi-diameter of the sun, be called unity or 1, what will be the ethereal semi-diameter, at any ethereal surface point, where the perpendicular axial distance is equal to $\frac{1}{2}$? And what the solar latitude of such point?

By formula (3), we have

$$d' = \frac{1}{\left(\frac{1}{2}\right)^2} = 4;$$

hence, when the axial distance is $\frac{1}{2}$ the equatorial, the ethereal semi-diameter will be four times that of the equatorial.

The solar latitude of the point is found by the common method; that is, by finding the angle included between the ethereal semi-diameter and the axial distance, or the hypotenuse and the base.

$$\text{Latitude} = 82^\circ 49' 10''.$$

Example 3.—If the perpendicular axial distance of a sur-

face point, on the sun's ethereal envelope be $\cdot 7937$, what will be its ethereal semi-diameter? and what the solar latitude?

$$\text{Answer} \left\{ \begin{array}{l} 1 \cdot 587401 \\ \text{Latitude} = 60^\circ. \end{array} \right.$$

Example 4.—If the axial distance be $\cdot 1$, what will be the length of the ethereal semi-diameter?

Answer, 100 times the ethereal semi-diameter at the equator.

Example 5.—If the axial distance of a surface point be $\cdot 8908981$, what will be the ethereal semi-diameter? and what the solar latitude?

$$\text{Answer} \left\{ \begin{array}{l} 1 \cdot 259920 \\ \text{Latitude} = 45^\circ. \end{array} \right.$$

Example 6.—If the axial distance is $\cdot 9531843$, what the semi-diameter, and the solar latitude?

$$\text{Answer} \left\{ \begin{array}{l} 1 \cdot 1006424 \\ \text{Latitude} = 30^\circ. \end{array} \right.$$

22. By the solution of these few problems, and the mathematical law which determines the form and dimensions of these ethereal envelopes, we can readily perceive that every sun, planet, asteroid, satellite, and comet which has a rotation on an axis, must have an ethereal envelope of a certain figure, and must be governed by the same law; and that the dimension of each depends upon the mass of the body and period of its rotation.

What appears very remarkable is the sameness and peculiarity of this figure, having no resemblance to any spheroid, ellipsoid, paraboloid, hyperboloid, or other common known solid. Although it is a figure generated by the mechanical laws of central force and axial rotation, yet its properties do not seem to have been noticed, so far as the author is aware, by any of the investigators of celestial mechanism.

23. THE ETHEREAL MEDIUM, OUTWARD FROM THE ETHEREAL ROTATING ENVELOPES.—Outward from these ethereal surfaces of rotation, there must be circulating currents of ether,

revolving in the same planes, and in the same directions, as the interior rotating bodies, and their respective envelopes. For a rotating surface imparts its motion to adjoining strata, and these again to others, and so on, to an indefinite distance. The velocities of these successive currents, if not retarded would necessarily be the same as those of planetary bodies: and, therefore, from the ethereal surface, outward, the velocities must continually become slower and slower, while the periodic times become greater.

24. The law of mean velocities, depending on planetary distances may be derived from Kepler's law: thus

Let t, t' represent the periodic times of any two planets in their orbital revolutions: let d, d' be their respective mean distances from the sun; v, v' their respective velocities; then, by Kepler's law, we have

and $t^2 : t'^2 :: d^3 : d'^3;$
 but $t : t' :: \sqrt{d^3} : \sqrt{d'^3}; \dots \dots \dots (1)$

$t : t' :: \frac{d}{v} : \frac{d'}{v'};$

hence $\frac{d}{v} : \frac{d'}{v'} :: \sqrt{d^3} : \sqrt{d'^3};$

and $\frac{1}{v} : \frac{1}{v'} :: \sqrt{d} : \sqrt{d'}; \dots \dots \dots (2)$

hence $v : v' :: \frac{1}{\sqrt{d}} : \frac{1}{\sqrt{d'}} \dots \dots \dots (3)$

Therefore, we have the

LAW.—*Planetary mean velocities vary as the inverse square roots of the planets' mean distances from the sun.*

25. If tangent lines, touching the equatorial ethereal surface of the sun, be drawn at right angles to the equatorial plane, they will be parallel to the axis; and hence, all particles

of ether, situated in those lines, will have equal axial distances; but their central distances will vary, as the secants of the solar latitudes, and their planetary velocities will vary as the inverse square roots of these secants; (Par. 24;) and therefore, the periodic times, around the solar axis, in equal circumferences, must vary directly as the square roots of the secants; that is, at 4 times the distance the velocity will be $\frac{1}{2}$, while the periodic time around the solar axis will be 2. At 9 times the distance from the solar center, the velocity will be $\frac{1}{3}$, and the periodic time 3; and so on. And, therefore, the periodic times of the circulating strata, at equal distances from the axis, will increase as you recede from the equatorial plane. The following examples will still further illustrate.

If the ethereal equatorial surface of the sun rotates in 25 days, at the same axial distance, in solar latitude $37^{\circ} 8' 13''$, the planetary velocity of the ether is such that it would require 28 days to perform one complete revolution. In solar latitude 45° , the axial distance being the same, the periodic time would be *29d 17h 28m 52s*. The axial distance being the same, if we take a distance from the equatorial plane equal to the sine of the latitude 45° , the periodic time of the revolutions of the ether around the solar axis will be = *27d 16h*.

26. It should be borne in mind, that the planetary revolutions of the ethereal strata, must not be calculated for orbits around the center of gravity of the sun; for these (which would be their natural orbits if left free to move) they are prevented from describing, by the intervening strata; but they move in circles, parallel to the equatorial plane, and at right angles to the solar axis.

If lines be drawn parallel to the solar axis, and exterior to the equatorial tangents, which are also parallel to the axis, the revolving particles, at different distances from the equatorial plane, along these exterior lines, will follow the same law, as expressed in Par. 25; that is, for the same solar latitude, the velocities will be less, and the periodic times greater, because of the increased distance of these lines from the axis; and these periodic times will still further increase as the solar latitude increases.

27. *Constant Tendency of the Ethereal Medium to a state of Equilibrium.*

The intensely elastic properties of the ethereal medium (Par. 9.) must necessarily operate to preserve an equilibrium; and when such equilibrium is disturbed, it will operate with great force or rapidity to restore the medium to its former state of repose. When a train of cars passes rapidly through the atmosphere at rest, the air is thrown into a violent motion; but as soon as the cause ceases to act, the elastic power of the medium soon restores it to its former condition. The same is true, when the disturbance arises from rotating bodies. As long as the rotation of the body continues, there will be a constant rotation of the air in the immediate vicinity, and also a constant effort of the air to counteract the rotation, and to preserve itself in a state of equilibrium; and when the rotation ceases, the circulating currents do not long survive, being overpowered by the friction and elasticity of the surrounding medium.

The ethereal strata, immediately exterior to the rotating ethereal envelopes, when seeking to acquire a planetary velocity, if successful, for a moment, cannot preserve such velocity; for the still more distant strata, having a slower motion, must, by friction, retard the interior strata, diminishing their centrifugal force, until regaining a small fraction of weight they again slightly press upon the rotating surface, when they begin again to re-acquire the velocity lost by friction. These alternate processes will be repeated so rapidly as to become continuous while rotation continues.

28. *The Effects of the Equilibrium Tendency on the Rotation of Worlds.*

Besides the ethereal envelopes surrounding worlds, the most of them have also atmospheres of less dimensions, composed of grosser substances, which only extend a comparatively short distance above the solid or liquid portions. These gross atmospheres have a constant tendency to rotate in the same time as the denser mass beneath. But this they are prevented from doing, by the constantly retarding action of the ocean of ether, exterior to these rotating atmospheres and ethereal

envelopes. This retarding action does not exhibit itself with equal force in different latitudes of the rotating body. At the equator it is a *minimum*; in the neighborhood of the poles, it is a *maximum*. The periodic times of these rotating atmospheric and ethereal strata, will increase with the latitude. The true cause of this, has been explained in Paragraphs 25 and 26. It necessarily arises from the excess of gravitating force, acting on the ether, not being so fully counteracted by the centrifugal force, as in the equatorial regions. This greater pressure, with greatly retarded velocities of the ethereal strata exterior to the rotating surface must retard the atmosphere with increased force as you recede from the equator.

29. *The Author's Theory is Confirmed by Recent Observations.*

Recently observers have carefully noted the periodic times of rotation of sun-spots in different solar latitudes. At the solar equator they rotate in about 25 days, as the distance from the solar equator increases, the periodic time of rotation increases. About 40° of latitude, or nearly half-way from the equator to the poles, the periodic time of these spots is a little less than 28 days. Now it is a fact, worthy of note, that in latitude 45° , at a distance above the ethereal surface, only amounting to a little less than one-tenth of the equatorial ethereal semi-diameter, the periodic time of the circulating currents having a planetary velocity is 28 days.

30. These sun-spots are probably dark clouds, floating in the denser or grosser atmosphere, and rotate in the same time as the respective strata which they occupy. These sun-spots, as supposed by some, may possibly be the dark body of the sun, far beneath its luminous atmosphere, and seen through openings. If such be the case, these openings must travel around with the rotating current, exhibiting successive portions of the dark body beneath. As you recede outward from these sun-spots through the chromosphere and coronal regions into the invisible ethereal envelope, the ethereal substance must be more and more retarded, until the region of planetary velocity is reached, when the retardations will become much more rapid, until the velocities will finally become greatly reduced.

31. *The Real Period of the Sun's Rotation must be Less than 25 days.*

For the equatorial sun-spots have a period of 25 days; and these are retarded by the causes already named. And though this retardation is reduced at the solar equator to a minimum, yet it must be appreciable, and may vary some hours from the real rotation of the great interior mass.

32. *Will the Real Rotation of the Sun be Diminished by its Ethereal Resistance?*

If there is no accelerating force equal to the resistance, the velocity of rotation must be diminished, until it is altogether overcome. And the same is true in regard to all rotating worlds. It is also true in regard to all orbital revolutions. An end must come to all axial and orbital motions, and the universe be reduced to chaos, unless there is a compensating accelerating force, sufficiently powerful to counteract the resisting force of the ethereal medium. Such a force is imperatively called for, and such a force does exist, as we shall in the next chapter, proceed to show.



CHAPTER III.

GRAVITATION.

33. *Gravitation is not an Instantaneous Force.* 34. *Gravitating Force transmitted with the Velocity of Light.* 35. *Aberration of the Gravitating Force.* 36, 39. *Aberration illustrated by a right-angled triangle.* 40. *Aberration of Force in Elliptic Orbits the same as that of Light.* 41. *Law of Aberrating Velocity, expressed in terms of Planetary Distances.* 42. *Orbital Accelerations.* 43. *Intensity of the Earth's Gravity toward the Sun.* 44. *Intensity of the Earth's Orbital Acceleration, expressed in terms of Earth's Gravity.* 45. *Orbital Acceleration, expressed in Pounds Weight.* 46. *Earth's Fall toward the Sun in one second.* 47, 48. *Excess of Orbital Space, gained in one second, by the Aberrating Force—Space gained in one year—Increase of Periodic Time.* 49. *Aberrating Intensities vary with those of Gravity.* 50. *Law of Aberrating Forces.* 51. *Definition.* 52. *Law of Aberrating Forces, expressed in terms of Planetary Distances.* 53. *The law applicable to the Satellites.*

33. GRAVITATION IS NOT AN INSTANTANEOUS FORCE.—It has been assumed, since the days of Newton, that the force called gravitation is transmitted from world to world instantaneously. This assumption has remained without proof until now. It was undoubtedly made to correspond with another equally erroneous assumption that there was no resisting medium; and when compelled to admit that all space was filled with an ethereal substance, there was still further a necessity to assume that this ocean of ether had no gravitating nor resisting properties. And thus one absurd assumption has been heaped upon another, in order to maintain an assumed hypothesis, namely, that there is no resistance, and therefore, no need of an accelerative force to maintain the rotative and orbital motions of the planets. For

to admit that gravitation, like light, needs time for its transmission through space would involve, as we shall presently prove, the necessity of also admitting an accelerating orbital force, which would strike a fatal blow to the other assumptions.

34. *The Gravitating Force is Transmitted with the Velocity of Light.*

The velocity of light has been very accurately determined. And it is very firmly believed that the heating, magnetic, electric, and chemical rays, as well as the rays of different colors, are transmitted with equal velocity. In this respect, the solar radiations are believed to follow the same law as the atmospheric radiations of sound. Sounds of every tone are conveyed through the same medium, with the same velocity. Why should the radiations of gravity depart from this law? Why should Neptune receive this solar force as soon as Mercury? How can force be transmitted, or pass through space, without occupying time? Time and space are essential characteristics of all motion; take away either of these, and we can form no conception of motion. An instantaneous motion is inconceivable. In assuming, therefore, that the gravitating force is transmitted with velocity, involving time, we only assume that which is analogous to all the motions of nature. In further assuming that it has the same velocity as light and other solar radiations, we are supported by the analogous phenomena of the transmission of sounds.

35. *Aberration of the Gravitating Force.*

It is evident, that a traveling force, having the same velocity as light, will, when combined with a planet's orbital velocity, produce an aberration or deviation from the real center of gravity. The amount of this aberration will be the same as the aberration of light, which, in the case of the earth, is equal to the mean deviation of about $20'' \cdot 25$ of an arc. This is called the mean or constant angle of aberration; that is, the earth, if its orbit be considered circular, moves over the arc of $20'' \cdot 25$, during the 8 minutes, and $13 \cdot 096$ seconds in which the force of gravitation is transmitted from the sun to the earth.

36. The angle of the aberration of light has been calculated on the supposition, that gravitation was transmitted from

the sun to the earth instantaneously,—that when the earth is in perihelion or aphelion, it moves in a right angle to this line of instantaneous force—that the line connecting the real and apparent centers of the sun, being parallel to the line of the earth's motion, must also be at right angles to the same line of force,—and that a line, drawn from the earth to the apparent place of the sun, is the hypotenuse of this right-angled triangle. This process of reasoning and the conclusion are correct, so far as the aberration of light is concerned.

37. But if gravitation moves with the velocity of light, the line of force and the line of light will be identical: both will proceed, not from the real place of the sun at the instant of their reaching us, but from the apparent or aberrating position of that luminary. Hence, when the earth is in aphelion, it does in reality move at right angles to the supposed line of instantaneous force, and consequently must move with an acute angle to the line of the traveling or aberrating force; and therefore, the line joining the centers of the real and apparent sun, which is parallel to the line of the earth's motion, must also make an acute angle with the same line of traveling force. These three lines form a right-angled triangle; the right angle being at the sun's real center; the apparent sun's distance is the hypotenuse; the line joining the real and aberrating centers is the perpendicular; and the line connecting the earth with the real center is the base. The base, if the earth's orbit be considered circular, is less than one-half a mile shorter than the hypotenuse, there being only 2326 feet difference. The acute angle, included between the base and hypotenuse is the angle of aberration, the mean of which is as above stated: the mean of the other acute angle is $89^{\circ} 59' 39'' \cdot 75$

38. Still considering the earth's orbit circular, and its radius equal to 91430000 miles, an arc of $20'' \cdot 25$ would be nearly equal to its sine or equal to 8976.1192599286 miles, which would be equal to the perpendicular of the triangle we have just described.

39. If the sides of this right-angled triangle were considered as consisting of some rigid material, as, for instance, iron-wire; and if the triangle were made to revolve around its right-

angle, the greater acute angle would describe the circle of aberration, and the other acute angle would describe a circular orbit of the earth, with the base as radius. The momentary direction of the earth's path would be constantly perpendicular to the base.

40. The effects of aberration in an elliptic orbit, calculated with reference to the traveling force of gravity, will be the same as the phenomena observed in connection with the aberrations of light.

41. *The orbits being considered circular, the Aberrating Velocities at the Different Planets, vary as the Inverse Square Roots of their Distances from the Sun.*

The aberrating velocities in circular orbits vary directly as the orbital velocities; and these velocities vary as the inverse square roots of their distances; (Par. 24;) and therefore, the aberrating velocities of the force of gravitation must vary as the inverse square roots of these distances.

Hence, the points toward which the planets gravitate, cannot be the real center of the sun, but they are movable points, at a distance from such center equal to the angle of aberration. As one faces the sun, this point for each planet is continually on the right hand of the sun's center, and circulates around it, in the same orbital period of the planet, and from west to east in the same direction as the orbital motion.

42. *The Aberrating Force of Gravity Accelerates the Orbital Motion of a Planet.*

The points of gravitating aberration, being on that side of the sun's center, towards which the planets are moving, must necessarily accelerate their orbital motions.

The perpendicular of the right-angled triangle, described in Paragraph 37, represents both in quantity and direction the aberrating force of gravity. Complete the parallelogram, by drawing lines parallel to the base and perpendicular; the hypotenuse will be the diagonal of this parallelogram. The force which this diagonal represents may be resolved into two simple forces, represented by the base and perpendicular; but the latter is parallel and equal to the line which the planet describes, during the transmission of the gravitating force from the sun. Of these two simple forces, the base, being equal

to the real sun's distance, represents the central force; and the earth's path, the accelerating orbital force. These forces are in the following proportion. (See Par. 38.)

Central force : *Orbital force* :: 91430000 : 8976·11926.

If the Orbital force be called unity or 1, then

8976·11926 : 91430000 :: 1 : 10185·91635788;

or
$$\frac{\textit{Orbital force}}{\textit{Central force}} = \frac{1}{10185 \cdot 91635788}.$$

Therefore, the earth is accelerated in its orbit, by the accelerating force of gravity, equal to the $\frac{1}{10185 \cdot 91635788}$ -th part of its central or gravitating force towards the sun.

All the other planets have a similar accelerating orbital force.

43. *To find the Intensity of the Earth's Gravity toward the Sun.*

The earth's mean radius is equal to 3955·94943182 miles. Let the earth's force of gravity toward her center, at the distance of her mean radius, and when freed from the counteracting effect of the centrifugal force of rotation, be equal to unity or 1; let the sun's mean distance from the earth, (91430000 miles,) be expressed in terms of the earth's mean radius, being equal to

$$\frac{91430000}{3955 \cdot 94943182} = 23364 \cdot 80826992676;$$

let the mass of the sun be expressed in terms of the earth's mass, being equal to 314760. Then by Newton's law of Universal Gravitation, we shall have

$$\begin{aligned} \frac{\textit{Sun's mass}}{(\textit{distance})^2} &= \frac{314760}{(23364 \cdot 80826992676)^2} \\ &= G' = \cdot 000576574051819522, \end{aligned}$$

which is equal to the intensity of the earth's gravitation toward the sun, compared with unity or the intensity of gravity toward the earth's center at the distance of her mean radius.

44. *To find the Intensity of the Earth's Accelerating Orbital Force, in Terms of the Intensity of the Earth's Gravity.*

The intensity of the earth's central force towards the sun, is determined in terms of terrestrial gravity, in the last paragraph. Hence, (Par. 42,) we have

$$10185 \cdot 91635788 : 1 :: \cdot 000576574051819522 \\ : \cdot 000000056605025170217 = G,"$$

which is the intensity of the earth's accelerating orbital force, in terms of the intensity of gravity toward the earth's center, at the distance of her mean radius.

45. *To Find the Intensity of the Earth's Accelerating Orbital Force, expressed in Pounds Weight.*

A cubic inch of distilled water weighs 252·458 grains avoirdupois; and 7000 grains make one pound; hence 1 cubic mile of distilled water will weigh 9183125415170 $\frac{1}{2}$ lbs and 4432 grs. The density of the earth is 5·6604 times heavier than water; hence, 1 cubic mile of the earth must be equal to (9183125415170 $\frac{1}{2}$ lbs 4432grs.) \times 5·6604, which is equal to 51980163100031·8518 $\frac{1}{2}$ lbs. The volume of the earth is 2597560-14917 cubic miles. These last two numbers, multiplied together give a product of 13,502,160,021,599,966,659,335,933 $\frac{1}{2}$ lbs, as the weight of the whole earth. This multiplied by the intensity of the earth's accelerating orbital force, as determined in Paragraph 44, will give a product equal to

$$764,290,107,874,962,825\frac{1}{2}\text{lbs} = p.$$

This is the pressure in pounds weight, in the direction of the tangent of the earth's orbit, when considered circular. This is an accelerating force, constantly acting to increase the velocity of the orbital motion.

46. *To Find the Earth's Fall toward the Sun in one Second.*

Bodies at the surface of the earth, at the distance of her mean radius from the center, in latitude 45°, fall in one second

16·08538 feet; and were it not for the centrifugal force of rotation, they would fall 16·1131467 feet in the first second.

Let G = the intensity of gravity at the earth's surface;

G' = the intensity of gravity of the earth toward the sun.

Let f = fall in one second toward the earth's center;

f' = fall of the earth in one second toward the sun.

then we shall have

$$G : G' :: f : f';$$

or

$$1 : G' :: 16·1131467 : f';$$

substitute the value of G' , (Par. 43,) and we shall find

$$f' = \cdot 009290422280381360 \text{ feet.}$$

The velocity gained at the end of one second is = $2f'$.

$$v = 2f' = \cdot 018580844560762720 \text{ feet per second.}$$

47. *To Find the Increased Orbital Space over which the Earth must Move in one second by the Action of the Accelerating Aberrating Force of Gravity.*

Let f'' = the increased space required; then

$$\text{Central force} : \text{Orbital force} :: f' : f'';$$

or (Par. 44)

$$10185·91635788 : 1 :: f' : f'';$$

therefore, as f' is known, (Par. 46,) we have

$$f'' = \cdot 000000912085074525 \text{ feet.}$$

This is the excess of orbital space, moved over in one second, at the end of which, the velocity gained will be $2f''$.

$$v = 2f'' \cdot 000001824170149050 \text{ feet per second.}$$

48. *To Find the Increased Orbital Space over which the Earth must Move in one Sidereal Year, (= 31558149·6 seconds,) by the Action of the Accelerating Aberrating Force of Gravity.*

Let s = the required increased space, then, by the law of acceleration, we have

$$s = f'' \times (31558149 \cdot 6)^2;$$

therefore, by multiplying, and reducing to miles, we have

$$s = 172038 \cdot 040602598266 \text{ miles.}$$

Thus, it will be perceived, that the aberrating force of gravity will lengthen the earth's orbital path, over 172000 miles in one sidereal year, during which the earth will continually recede from the sun in a spiral path. If s be added to the circumference of the earth's orbit, and the same be considered circular, the semi-diameter will be increased from 91430000 miles to 91457380·704562 miles making a difference of over 27380 miles. And by Kelper's law, the periodic time would be increased from 1 sidereal year, to 1 sidereal year, 3 hours, 56 minutes, 17·2 seconds.

In like manner, if there is no resisting medium, all the planets must be accelerated in their orbits, by the aberrating force of gravity, and recede in spiral paths from their common center into the far distant regions of space.

49. *The Intensities of the Aberrating Force, acting under the same angle, vary directly as the Intensities of Gravity.*

Let i, i' represent the aberrating intensities, corresponding to any change of mass or intensity toward which a body may be gravitating; let g, g' represent the gravitating intensities.

Because the angle of aberration is supposed to remain the same, the parallelogram, representing these forces, (Par. 42,) will also be constant; and however much the intensity, represented by the diagonal, may change, the same proportional intensity of change must characterize the two simple forces, into which the compound force has been decomposed, and which are represented by two of the sides of the parallelogram; therefore,

$$i : i' :: g : g'.$$

50. *The Aberrating Tendencies vary directly as the Intensity of the Force, multiplied into the Aberrating Velocity.*

It is seen by the last Paragraph, that the intensity of the aberrating force varies as the intensity of gravity; and it is also evident, that the same intensity produces greater or less results in the exact proportion to the velocity of aberration.

Let a, a' be the velocities of aberration for any two circular orbits; let i, i' be the corresponding intensities of the aberrating force; and let F, F' be the aberrating tendencies, resulting from the joint action of the intensities and velocities. For the reasons above mentioned, we shall have

$$F : F' :: ia : i'a'.$$

51. *Definition.*—Though a and a' are not properly forces, yet they, in their combined state with i, i' , represent forces; therefore, $ia, i'a'$, or their representatives F, F' , will be called, *the aberrating forces of gravity.*

52. *In the Planetary System, the Orbits being considered Circular, the Aberrating Forces of Gravity vary Directly as the Masses and Inversely as the Fifth Powers of the Square Roots of the Distances from the Gravitating Center.*

Let d, d' represent the respective distances of any two bodies from the sun.

By Par. 49 we have

$$i : i' :: g : g';$$

Newton's law gives

$$g : g' :: \frac{1}{d^2} : \frac{1}{d'^2};$$

hence

$$i : i' :: \frac{1}{d^2} : \frac{1}{d'^2};$$

by Par. 41

$$a : a' :: \frac{1}{\sqrt{d}} : \frac{1}{\sqrt{d'}};$$

hence

$$ia : i'a' :: \frac{1}{\sqrt{d^5}} : \frac{1}{\sqrt{d'^5}};$$

but (Par. 50)

$$F : F' :: ia : i'a';$$

therefore

$$F : F' :: \frac{1}{\sqrt{d^5}} : \frac{1}{\sqrt{d'^5}}.$$

53. *In the Secondary Systems, the Satellites are governed by the same Laws in relation to their Primaries, as the Planets are in relation to the Sun.*

For First,—The circular orbital velocities of the satellites of any one system, around their common center of gravity, are as the inverse square roots of their respective distances from such center. (Par. 24.)

Second,—The aberrating velocities, being as the orbital velocities, are also as the inverse square roots of the same distances. (Par. 41.)

Third,—Newton's law of gravity is the same for the secondary systems as for the planetary, and consequently, the intensities of the orbital accelerating forces of the satellites must vary as the intensities of gravity; (Par. 49;) and therefore, the aberrating forces of gravity, represented by F, F' , (Pars. 51, 52,) must vary as the fifth powers of the square roots of the distances of the satellites from their common center.



CHAPTER IV.

COMPOUND ORBIT OF THE SUN.

54. *Solar Orbit.* 55. *Aberrating Intensities vary as the Masses.* 56. *Apparent Places of Planets as seen from the Sun.* 57. *Law of Aberrating Forces, as exerted by Planets on the Sun.* 58. *How to find the resultant action of all the Planetary Aberrating Forces.* 59. *Acceleration of the Sun in his Orbit, arising from the Aberrating Force of the Earth.* 60. *Pounds pressure, exerted by the Earth, in propelling the Sun in his Orbit.* 61. *Excess of Orbital Space, gained by the Sun in one year—Enlargement of his Orbit, and the Increase of his Periodic Time.* 62. *Algebraic expression of the Law, given in Par. 57.* 63. *Accelerations of Rotation.* 64, 65. *Law of the Aberrating Force of Rotation.* 66. *Law, relating to the Rotating Particles of the Envelope.*

54. SOLAR ORBIT.—It is known that the sun revolves from west to east, around the common center of gravity of the system, in a very irregular orbit. If the system consisted of the sun and only one planet, each would revolve around the common center of gravity, in the same plane, and in balancing orbits precisely similar, the motions being parallel, but in opposite directions. The dimensions of these two balancing elliptic orbits, would be inversely as the masses, describing them, while the periodic times would be equal, and therefore, their mean velocities would be inversely as the masses. If a second planet, be introduced into the system, there will be three orbits—two planetary and one solar; but the solar will be compounded of its two simple balancing orbits. In like manner, if there be n planets, there will be $n + 1$ orbits,—the solar orbit being compounded of n simple orbits. The sun will describe this irregular compound balancing orbit, in a period equal to that of the most distant planet of the system.

55. *The Aberrating Intensities, mutually existing between the Earth and Sun, are directly as their Masses.*

It has been proved, (Par. 49,) that the aberrating intensities vary as the force of gravity; but gravity, when the distance is the same, varies directly as the mass; therefore, the aberrating intensities of the two bodies must vary as their masses.

The same is true in regard to the mutual aberrating intensities, existing between the sun and any other planet.

56. Planetary bodies, as seen from the sun, will not appear in their true position, but in the position that they were in at the instant the light left them. During the interval in which light performs its journey from each planet to the sun, each will describe an arc equal to its angle of aberration.

The same is also true, in regard to the angles of the aberration of forces as well as light, as the former are being transmitted from the respective planets to the sun.

57. *The Aberrating Force, exerted by any Planet to Accelerate the Sun in his little Balancing Orbit, is directly as the Mass of the Planet, and inversely as the Fifth Power of the Square Root of its Distance.*

This general proposition is demonstrated by a similar process of reasoning, to that in Paragraphs 52, 55 and 56.

58. To find the resultant intensity and direction of all the combined aberrating forces of the planets, exerted upon the sun in any given moment, it is necessary to introduce into the problem, the relative positions of the planets and sun in regard to the common center of gravity of the system; and from these data, connected with the law, expressed in the preceding Paragraph, the problem, though tedious, can be solved.

59. *To find the Intensity of the Earth's Action in Accelerating the Orbital Motion of the Sun, in terms of the Earth's Gravity.*

Let G'' = the aberrating intensity of the solar action, in accelerating the orbital motion of the earth; let I = the aberrating intensity of the earth's action in accelerating the orbital motion of the sun. Then (Par. 55) we shall have

or	<i>sun's mass</i>	:	<i>earth's mass</i>	::	G''	:	I ;
	314760	:	1	::	G''	:	I ;

as G'' is known, (Par. 44,) we have

$$I. = .00000000000017983551.$$

This is the force, exerted by the mean aberrating gravity of the earth, in accelerating the sun in his balancing orbit, expressed in terms of the earth's gravity at the distance of her mean radius, the earth's gravity being called unity or 1.

60. *To Find the number of Pounds pressure, exerted by the Aberrating Force of the Earth's Gravity, in Accelerating the Sun in his Balancing Orbit.*

Let p' = the required number of pounds.

By Par. 55 we have

$$\begin{array}{l} \text{sun's mass} \quad : \quad \text{earth's mass} \quad :: \quad p \quad : \quad p'; \\ \text{or} \quad 314760 \quad : \quad 1 \quad :: \quad p \quad : \quad p'; \end{array}$$

as p is known, (Par. 45,) we have

$$p' = 2,428,167,835,414\frac{1}{2}\text{lbs.}$$

61. *To Find the Increased Space, over which the Sun must Move in his Balancing Orbit, in one Sidereal Year, by the Action of the Accelerating Aberrating Force of the Earth's Gravity.*

Let s' = the increased space required.

By Par. 55, we have

$$\text{sun's mass} \quad : \quad \text{earth's mass} \quad :: \quad s \quad : \quad s';$$

as s is known, (Par. 48,) we have

$$s' = .54656894333 \text{ of a mile.}$$

Thus it will be perceived, that by the action of the earth's aberrating force, the solar balancing orbit will, in one sidereal year, be increased over one-half a mile, which would increase his periodic time, in his increased balancing orbit, from one sidereal year to $1y \ 3h \ 56m \ 17.2s$. (See Par. 48.)

It will also be perceived, that the sun's balancing orbit with that of the earth, will no longer be an ellipse, but a spiral

of the same form as the longer spiral orbit of the earth, each receding from the common center of gravity between the two bodies. Such must necessarily be the phenomena, under the mutual action of aberrating forces, unless counteracted by a resisting medium whose opposing forces shall exactly balance the aberrating forces.

62. To find, in terms of the earth's gravity, the action of the aberrating force of any other planet, in accelerating the sun, in the balancing orbit existing between him and such planet, the law, expressed in Par. 57, must be used.

To express this law algebraically,

let m = the earth's mass ;

m' = planet's mass, expressed in terms of the earth's mass ;

d = the earth's distance from the sun ;

d' = the planet's distance from the sun ;

F'' = the aberrating force, exerted by the earth on the sun ;

F''' = the required aberrating force, exerted by the planet on

the sun ;

then

$$\frac{m}{\sqrt{d^5}} : \frac{m'}{\sqrt{d'^5}} :: F'' : F''' ;$$

or

$$F''' = \sqrt{\left\{ \frac{d}{d'} \right\}^5} \cdot \frac{m' F''}{m} .$$

If m and d be each made equal to unity or 1, then the formula becomes

$$F''' = \frac{m'}{\sqrt{d'^5}} \cdot F'' .$$

63. ACCELERATIONS OF ROTATION.—In a rotating world of a globular form every particle without the axis will be accelerated in the direction of the tangent to its circular path. These accelerations are caused by the aberrating forces of gravity. The line of motion of every particle in a rotating body makes an angle with the central lines of traveling force; and the velocity of force must necessarily have a certain ratio to the velocity of each particle, giving rise to the phenomena of force aberration.

64. When the density, throughout a rotating globe, is the same, we have the following

LAW.—The aberrating Force of Gravity upon any Particle, situated either on the surface or in the Interior, will be jointly as its Central Distance multiplied into its perpendicular Axial Distance.

For in such a globe, the intensity of the force of gravity of any particle toward the center is directly as the distance,, and its rotating velocity is as its axial distance: but the aberrating force (Par. 51) of gravity upon any particle is as its gravitating intensity multiplied into its rotating velocity.

Let i, i' , represent the gravitating intensities of any two particles toward the center; d, d' , their respective central distances; v, v' , their rotating velocities; D, D' , their axial distances; A, A' , their aberrating forces.

Then

$$\begin{array}{l} \text{and} \\ \text{hence} \\ \text{but} \\ \text{therefore} \end{array} \quad \begin{array}{l} i : i' :: d : d'; \\ v : v' :: D : D'; \\ iv : i'v' :: dD : d'D'; \\ A : A' :: iv : i'v'; \\ A : A' :: dD : d'D'; \end{array}$$

or

$$A' = \frac{d'D'}{dD} . A.$$

65. It should be remembered, that the expressions $iv, i'v'$, in the last Paragraph, are not true representatives of the aberrating forces, only when the momentary spaces, described with the velocities v, v' , are at right angles to the lines of gravitating force, or those radiating from the center of gravity, which is always the case in rotating bodies.

66. The mass of the ethereal envelope, enclosing a rotating globe, is extremely small, compared with the interior mass of gross matter; therefore, the aberrating or accelerating force of rotation, exerted by such interior mass, upon any ethereal particle of the envelope, will be directly as its perpendicular axial distance, and inversely as the square of its distance from the center.

CHAPTER V.

ABERRATIONS IN ELLIPTIC ORBITS.

67. *Elliptic Orbits.* 68. *Law of Velocity in an Elliptic Orbit.* 69. *Law of Angular Velocity.* 70, 71. *Aberrating Velocity—its Law in Elliptic Orbits.* 72. *Law of Aberrating Forces in Elliptic Orbits.* 73. *Angular and Aberrating Velocity at the mean distance.* 74. *A particle at rest must describe an Orbit around a body in Motion.* 75. *How to estimate the joint Aberrating Forces of two bodies in Motion.* 76. *When the Aberrating Forces of two bodies become neutralized or Zero.* 77. *General Theorem.*

67. ELLIPTIC ORBITS.—Thus far our investigations have been confined to circular motions. But the planets, and the most of comets, revolve in elliptic orbits, in each of which, the angle of aberration, the velocity of the body, and the intensity of the focal force, are constantly changing. At the extremity of the minor-axis, as the body approaches the perihelion, the angle, between the radius vector and the line of motion, is acute, being then at its *minimum* value: from this point the angle increases, becoming a right angle at the perihelion and still opening out into an obtuse angle, it attains its *maximum* value at the other extremity of the minor axis: it now begins to decrease, passing through a right angle at the aphelion, still decreasing until reaching again the *minimum* point.

The velocity of the body, and the intensity of the focal force, increase from the aphelion where they are the least, to the perihelion where they are the greatest, and decrease from the latter to the former.

68. The velocity of a planetary body, moving in an elliptic orbit around a focal force is expressed by the following

LAW.—*If a body move in an elliptic orbit, under the influence of a focal force, varying inversely as the squares of its*

distances from the focus, the squares of its velocities in any two points of its orbit, will vary directly as its distances from the upper focus, and inversely as its distances from the lower focus.

Let d, d' , be any two distances of the body from the lower focus; let k, k' , be the distances from the upper focus; h, h' , the distances of the apsidal points from the lower focus; p, p' , the perpendiculars let fall from the lower focus on the two tangents drawn from the points in the orbit at the distances d and d' .

From the First Math. Tract of Dr. Matthew Stewart, Prop.

21, Cor., we have

$$p^2 : hh' :: d : k;$$

hence

$$p^2 = \frac{hh'd}{k};$$

also

$$p'^2 : hh' :: d' : k';$$

$$p'^2 = \frac{hh'd'}{k'};$$

therefore

$$p^2 : p'^2 :: \frac{hh'd}{k} : \frac{hh'd'}{k'} :: \frac{d}{k} : \frac{d'}{k'};$$

hence

$$\frac{1}{p^2} : \frac{1}{p'^2} :: \frac{k}{d} : \frac{k'}{d'};$$

but (See James Adam's *Ellipse, Centrip. Forces*, Cor. 1, Prop. 1.)

$$v^2 : v'^2 :: \frac{1}{p^2} : \frac{1}{p'^2} :: \frac{k}{d} : \frac{k'}{d'}.$$

69. *In an Elliptic Orbit, the Angular Velocity of a body around the focus varies inversely as the square of its distance from the focus.* (See Dr. Stewart's First Math. Tract, Prop. VI.)

70. The real velocity in an Ellipse may be resolved into two velocities; one in the direction of the radius vector; the other in a direction perpendicular to the radius vector: the former has no aberrating effect; the latter gives rise to the aberrating velocity of gravity.

71. *In an Elliptic Orbit, the Aberrating Velocity, when the Mass of the central body remains the same, varies inversely as the distance of the moving body from the focal force.*

Let d and d' be the distances from the lower focal force of any two points in an ellipse; let V and V' be the angular velocities of a body at those two points; and let a and a' be the aberrating velocities, or those parts of the velocities which are at right-angles to the radius vector.

The actual velocities at right-angles to the radius vector, must be as the angular velocities multiplied into the respective distances: hence

$$\begin{aligned} \text{but (69)} \quad a & : a' & :: & Vd & : & V'd'; \\ & & & \frac{1}{d^2} & : & \frac{1}{d'^2}; \end{aligned}$$

hence

$$Vd & : & V'd' & :: & \frac{1}{d} & : & \frac{1}{d'};$$

therefore

$$a & : & a' & :: & \frac{1}{d} & : & \frac{1}{d'}.$$

72. *The Aberrating Forces in any two points in an Elliptic Orbit vary inversely as the cubes of their distances from the focal force.*

Since we have (71)

$$a : a' :: \frac{1}{d} : \frac{1}{d'};$$

also (50 and 51)

$$i : i' :: \frac{1}{d^2} : \frac{1}{d'^2};$$

hence

$$ia : i'a' :: \frac{1}{d^3} : \frac{1}{d'^3}; \dots\dots\dots (1)$$

but

$$F : F' :: ia : i'a';$$

therefore

$$F : F' :: \frac{1}{d^3} : \frac{1}{d'^3}.$$

Cor. If the mean distance in an Ellipse be represented by d , and the intensity of gravity at such distance be represented by i , and if both d and i be each taken as unity or 1, and if $F=a$, then

$$F' = \frac{a}{d'^3}.$$

For (1)

$$a : i'a' :: 1 : \frac{1}{d'^3};$$

therefore

$$i'a' = F' = \frac{a}{d'^3}.$$

73. To find the Angular Velocity in an Ellipse, at the mean distance from the focal force.

Let d be the mean distance; let c represent the semi-conjugate axis; let v be the angular velocity in a circle at the mean distance; and V , the required angular velocity in the ellipse.

The velocity in an ellipse at the mean distance from the focus, is the same as the velocity in a circle at the same distance. Also the mean distance is to the conjugate semi-axis, as the angular velocity in the circle to the angular velocity in the ellipse. (See Centripetal Forces, by James Adams, Prop. 8, Cor. 2.)

hence

$$d : c :: v : V;$$

therefore

$$V = \frac{cv}{d}.$$

Cor. 1.—*The Angular Velocity, at the mean distance in an Elliptic Orbit, is equal to the Aberrating Velocity.*

For the momentary arc described, at right angles to the radius vector, is the same as the momentary angular arc: therefore

$$V = \frac{cv}{d} = a.$$

Cor. 2. *When $c=d$, the ellipse becomes a circle; and we have*

$$V = v = a.$$

When $c=0$, V and a each equal zero, and the ellipse is resolved into a straight line.

Cor. 3. *If v and d be each assumed as unity or 1, we have*

$$V = c = a.$$

74. A body, moving at an angle with a line connecting it with a particle at rest, exerts upon the latter an aberrating force, causing it to revolve in a spiral curve around the moving body. This spiral orbit will continually decrease in its eccentricity.

For the apparent place of the moving body, as viewed from the particle at rest, will always be behind its true place. The amount of displacement, the first moment, will be equal to the aberrating velocity of the body, during the interval in which its.

force is transmitted to the particle. Both the apparent and true places continue moving. The particle at rest will, for the first moment, commence falling, not toward the real place of the moving body, but toward its apparent place. The next moment the apparent place is changed, but the course of the falling particle is not changed in an equal degree; its centrifugal force, gained the first moment, causes it to fall toward a point, behind the apparent place. The third moment, the apparent position of the moving body is still further changed, and the velocity and centrifugal force of the particle are much greater, and the tangential line of motion makes a still greater angle with the line to the apparent center. This angle, centrifugal force, and velocity continue, from moment to moment, to increase, urging the particle still more and more away from the apparent central line. And when the particle has fallen to a point whose distance from the apparent center is equal to nearly one-half its original distance, it will have acquired a velocity sufficiently great to maintain it in a circular or any elliptic orbit at such mean distance. Let us now suppose the aberrating force to cease, and the moving body to become stationary, it is evident that the particle would necessarily revolve in an elliptic orbit, whose mean distance and eccentricity would depend upon the mass and angular velocity of the moving body, and the original distance of the particle when at rest.

75. In estimating the aberrating forces of two bodies in motion, if their aberrating velocities are opposite their sum must be taken: but if they are in the same direction, or nearly so, their difference will be the amount. In the falling of the particle which we have been considering, the sum of the relative aberrating velocities must be used, so long as the aberrating angle increases: but as the particle recedes from the perihelion, the angle which its path makes with the radius vector continues increasing, but the aberrating angle does not increase in the same proportion, because both motions are now inclined in the same direction; and when their relative velocities are equal the aberrating angle ceases; but as the particle now has the swiftest velocity, and its direction being momentarily changed, its aberration will soon gain the ascendancy and recede towards

the aphelion, under the retarding influence of gravity. But this retardation is not so great as it would be if gravity alone acted. The angle of aberration begins to increase diminishing more and more the retardation, until the particle arrives at its aphelion. In consequence of the retardation being lessened, the particle will reach its aphelion at a greater distance than it originally had, when at rest. Thus the major-axis is lengthened, and the dimensions of the spiral curve increased. At the aphelion, the spiral path makes an angle with the radius vector of 90° .

The next revolution, under the influence of still greater aberrating angles, the eccentricity will be diminished, and the orbit enlarged: this will continue each succeeding revolution, until the spiral curve becomes nearly or quite circular, unless the process is sooner arrested by a resisting medium.

76. When two bodies gravitate directly towards, or recede directly from each other, they exert no aberrating forces: also when they are moving with equal velocities in the same direction, parallel to each other, or when their relative aberrating velocities in, or nearly in, the same direction, are equal, the aberrating forces are neutralized.

77. From the foregoing considerations, and from the numerous mechanical laws known to exist, we are warranted in adding the following

GENERAL THEOREM.

Every Particle of Matter in the Universe transmits its Force to every other Particle with the Velocity of Light, by Virtue of which every Moving Particle exerts an Aberrating Force upon every other Particle at right angles to the connecting line, directly as its Mass, multiplied into its Velocity of Aberration, and Inversely as the square of its Distance from each.

CHAPTER VI.

RESISTING MEDIUM.

78. *Ethereal Resistance unlike that of Gross Matter.* 79. *Transfused Resistance.* 80. *Mass Resistance.* 81. *Velocity Resistance of one Particle.* 82. *Resistance of a given number of Particles.* 83. *In a Medium of Uniform Density, Resistance is as the Square of the Velocity.* 84. *Density Resistance.* 85. *General Law of Resistance.* 86. *Law of Density of the Ethereal Medium, in terms of the Sun's Distances.* 87. *Law of Resistance, expressed in terms of the Sun's Distance.* 88. *Resistances of the Planets vary as their Orbital Accelerations—Under what conditions the two Antagonistic Forces Balance each other.*

78. ETHEREAL RESISTANCE UNLIKE THAT OF GROSS MATTER.—In Chapter I. we have dwelt upon the existence, and some of the properties of the ethereal medium: but in the present Chapter, we propose to investigate more fully the single property of its resistance to moving planetary matter. All gaseous substances of a gross nature offer a resistance to the passage of all other gross substances through them. These resistances are not altogether proportional to the quantity of matter resisted, but depend also upon the form and magnitude of the surfaces passing through the gas. This is principally owing to the impervious nature of these substances: the gas cannot penetrate freely the surfaces and interior of bodies, and therefore, the resistance is almost wholly confined to the surface. If the transfusion were perfect, the moving body would have no resistance, whatever might be its form or magnitude. But perfect transfusion of one substance through another, without resistance, is not known in nature.

79. *Transfused Resistance.*—Some of the waves of light are resisted and reflected; some are transmitted through transparent substances, with a slight degree of resistance in velocity;

others are absorbed and destroyed. The waves of heat are not resisted as much as those of light, but they slowly penetrate bodies where light cannot follow. The electric and magnetic waves are more generally diffused, traveling by the aid of good conductors, with great velocities. It may be said that such bodies offer but slight resistance to their transfusion. The ether, itself, which exhibits all these varieties of waves, is no doubt more transfusable than any of its waves or tremulous agitations. It exists in all space; it is transfused through all worlds; it enters largely into the composition of all substance. Its resistance is a transfused resistance, not affected in the least by the form or magnitude of surfaces of gross matter, but only by the quantity of matter which moving bodies contain.

80. *When the Velocities of Bodies, moving in Ether of the same Density, are the same, the Resistance is as the Quantity of Matter in the Bodies Resisted.*

As ethereal resistance does not depend upon form nor surfaces, (Par. 79,) but upon masses, it is quite evident, that each atom of every substance will be equally resisted; and hence, the whole resistance, offered to a body, will be the sum of the resistances of its several particles; and consequently, the resistance, offered to different bodies, will be as the number of atoms which each contains; and therefore, as their respective quantities.

81. *The Resistance, offered by an Ethereal Particle, is as the Velocity with which it is struck.*

If the velocity be doubled or tribled, the resistance will be doubled or tribled. If the ethereal particle be struck with n times the velocity, its resistance will be n times greater.

82. *When the Velocity of contact is equal, the Resistance will be as the Number of Ethereal Particles struck.*

It is evident that if two particles of ether be struck by a moving body, the resistance will be doubled; if n particles be impinged upon, the resistance will be n times increased.

83. *The Resistance of an Ethereal Medium of Uniform Density is as the Square of the Velocity of a body moving therein.*

If a body move with n times the velocity, it will go, in the

same time, n times as far, and will meet with n times as many ethereal particles, and will impinge upon each particle with n times greater velocity; and therefore, the resistance will be equal to n times the particles multiplied into n times the velocity, which is equal to either the square of the number of particles, or the square of the velocity.

84. *When the Velocity of a body is the same, the Resistance of the Ethereal Medium is as its Density.*

If the ethereal medium contains twice, or thrice, or n times the number of particles in a given volume, its density will be twice, or thrice, or n times greater. And as the resistance varies as the number of particles, (Par. 82,) it, therefore, must vary as the density of the medium.

85. *The Resistance of the Ethereal Medium is as its Density multiplied into the Square of the Velocity of a body moving therein.*

It was proved, (Par. 84,) that with equal velocity, the resistance varies as the density. It was also proved, (Par. 83,) that the resistance varies as the square of the velocity; therefore, the resistance of the ethereal medium varies directly and jointly as its density, multiplied into the square of the velocity of a body moving therein.

86. *The Density of the Ethereal Medium, in the Solar System, varies Inversely as the Cube of the Square Root of the Distance from the Sun.*

The law of density, when circular orbits are permanent, must be such, that the resistance to planetary orbital circular motion will be exactly equal to, and balance the planetary orbital circular accelerations, arising from the aberrating force of gravity. In the next two paragraphs, it will be proved, that the law of density, just expressed, does fulfill these requirements, and maintain the stability of the system, so far as circular orbits are concerned.

87. *In an Ethereal Medium, whose Density varies Inversely as the Cube of the Square Roots of the respective Distances of the Planets from the Sun, the Resistances will vary Inversely as the Fifth Powers of the Square roots of such Distances.*

Let d, d' , be the distances of two planets from the sun ;
 v, v' , their orbital velocities ;

D, D' , the densities of the medium at the respective distances ;

r, r' , the resistances at the respective distances.

Then (Par. 86,) we have

$$D : D' :: \frac{1}{\sqrt{d^3}} : \frac{1}{\sqrt{d'^3}}; \dots \dots \dots (1)$$

and (Par. 85)

$$r : r' :: Dv^2 : Dv'^2; \dots \dots \dots (2)$$

combining (1) and (2)

$$r : r' :: \frac{v^2}{\sqrt{d^3}} : \frac{v'^2}{\sqrt{d'^3}}; \dots \dots \dots (3)$$

by Par. 24

$$v^2 : v'^2 :: \frac{1}{d} : \frac{1}{d'}; \dots \dots \dots (4)$$

combining (3) and (4)

$$r : r' :: \frac{1}{\sqrt{d^5}} : \frac{1}{\sqrt{d'^5}}.$$

This law of resistance is similar to the law of acceleration, imparted by the aberrating force of gravitation. (Sec Par. 52.)

88. *The Resistance of the Ethereal Medium to the Orbital Motion of the Planets, considered as circular, varies as their Orbital Accelerations, under the influence of the Aberrating Force of Gravity.*

For (Par. 52) we have

$$F : F' :: \frac{1}{\sqrt{d^5}} : \frac{1}{\sqrt{d'^5}};$$

and (Par. 87)

$$r : r' :: \frac{1}{\sqrt{d^5}} : \frac{1}{\sqrt{d'^5}};$$

therefore

$$F : F' :: r : r'.$$

Cor. If $F = r$, then $F' = r'$; therefore, under these last conditions, the two forces will exactly balance each other, and the circular orbits and periodic times will remain unchanged. Any other law of density of the ethereal medium, than that expressed in Par. 86, would destroy the stability of the system.



CHAPTER VII.

RESISTANCES IN ELLIPTIC ORBITS.

89. *General Law.* 90. *To find the Point in an Ellipse where the two Antagonistic Forces Balance—Balancing Point when the Ellipse becomes a Circle, or a Straight Line—Discussion of the Formula for Ellipses of any Eccentricity—Elliptic Perturbations—Minute Excess of Resistance—Decrease of Eccentricity and of the Periodic Times—Instability of Hyperbolic and Parabolic Orbits.* 91. *Example 1. Balancing Point in the Earth's Orbit—Example 2. Difference of the Two Forces at the Perihelion—Example 3. Difference at Aphelion—Examples 4 and 5. Difference at two intermediate points—Example 6. Balancing Point in the Orbit of Venus.* 92. *How to find the Radius Vector for any given time.* 93. *Variation of the Two Forces in an Ellipse.* 94. *Sums of the Two Forces in an Ellipse.*
89. **GENERAL LAW.**—*If a body move in an Elliptic Orbit, under the influence of a focal force, varying inversely as the squares of the distances from the focus, and if it be resisted by the ethereal medium, whose density varies inversely as the cube of the square root of the distances from the lower focus, the resistance will vary directly as the distance to the upper focus, and inversely as the fifth power of the square root of the distance of the lower focus.*

LET d, d' , be the distances of the body at any two points of its orbit, from the lower focus; let k, k' , be the corresponding distances to the upper focus; let v, v' , be the corresponding velocities; D, D' , the densities of the resisting medium; R, R' , the corresponding resistances.

Then, by (68) we have

$$v^2 : v'^2 :: \frac{k}{d} : \frac{k'}{d'}$$

we also have (by hypothesis)

$$D : D' :: \frac{1}{\sqrt{d^3}} : \frac{1}{\sqrt{d'^3}};$$

hence

$$v^2 D : v'^2 D' :: \frac{k}{\sqrt{d^5}} : \frac{k'}{\sqrt{d'^5}};$$

but (85)

$$R : R' :: v^2 D : v'^2 D';$$

therefore

$$R : R' :: \frac{k}{\sqrt{d^5}} : \frac{k'}{\sqrt{d'^5}}.$$

90. *To find the distance from the lower focus, or the radius vector, in any given ellipse, where the resisting and aberrating forces balance each other, the density of the resisting medium being the same as in Par. 89., and d, i, k and R , being, at the mean distance, each equal to unity.*

Let the angular velocity in a circle (whose radius is equal to the mean distance in the ellipse) = 1; let the angular velocity in the ellipse at the mean distance = a . Then we shall have (see James Adams Ellipse, Centripetal Forces, Prop. 8, Cor 2,)

$$d : \text{conjugate semi-axis} :: \text{Ang. vel. in a circle} : a;$$

or

$$1 : \text{conju. semi-axis} :: 1 : a;$$

therefore

$$\text{conjugate semi-axis} = a.$$

At the mean distance, the angular velocity in the ellipse is equal to the aberrating velocity. (Par. 73, Cor. 1.) Therefore, by Par. 72, Cor., we have

$$a : a' i' :: 1 : \frac{1}{d'^3};$$

hence

$$a' i' = \frac{a}{d'^3}.$$

By Par. 89 we have,

$$1 : R' :: 1 : \frac{k'}{\sqrt{d'^5}};$$

hence

$$R' = \frac{k'}{\sqrt{d'^5}}.$$

Because (by hypothesis) the resisting and aberrating forces are required to balance each other, we have

$$\frac{a}{d'^3} = \frac{k'}{\sqrt{d'^5}}.$$

In this equation, let $d' = x$; then $k' = 2 - x =$ the distance to the upper focus, and we shall have

$$2 - x = a \frac{\sqrt{x^5}}{x^3} = \frac{a}{\sqrt{x}};$$

squaring both sides and reducing, we have

$$x^3 - 4x^2 + 4x - a^2 = 0 \dots \dots \dots (1)$$

In this equation, when $a^2 = 1$, one of the values of x will equal 1, the ellipse resolving itself into a circle. When $a^2 = 0$, one value of $x = 0$; the other two roots will each = 2; hence, the lower focus will be on the ellipse, at the lower apsis; and the upper focus will be on the ellipse, at the upper apsis; hence, the ellipse will be resolved into a straight line whose length will equal 2; that is, twice the mean distance. As a^2 cannot be greater than 1, nor less than nothing, it follows, that one of the values of x , expressing the required distance, or, in other words, the radius vector, to the point where the two forces balance, can never be less than one, nor greater than two. Therefore, by finding the required root from equation (1), we shall have the distance of the point, where the resisting and aberrating forces balance each other.

Cor. 1. The less the eccentricity of an ellipse, the nearer the point of the balancing forces approaches the mean distance. And when becoming a circle, the balancing forces remain equal throughout the entire orbit.

Cor. 2. When $a^2 = 0$, $2 - x = k' = 0$; hence, $R' = a'i' = 0$; but as i' is greater than nothing, therefore, $a' = 0$; therefore, the resisting and aberrating forces each equal nothing.

SCHOLIUM. By solving numerically equation (1) for all possible values of a^2 , between its limits of one and nothing, it will be perceived that two of the values of x never touch the elliptic orbit, except when the ellipse becomes a straight line; one value being at some point within the curve; the other, at some point without the curve; the former always being less than the perihelion distance, and the latter always greater than 2, or the aphelion distance; while the third value of x will always be between the limits of 1 and 2, and will be the true distance sought for, where the resisting and aberrating forces balance. It is evident that there will be a point, on each side, of the major-axis, at equal distances from the lower focus, where the two forces will balance. Between these two points, by the way of the perihelion, the sum of the resistances exceeds the sum of the aberrating forces: but between these two points, by the way of the aphelion, the sum of the aberrating forces exceeds the sum of the resistances. When the resistances preponderate, there will result a slight perturbation from the elliptic curve in an inward elliptic spiral: on the other hand, when the aberrating forces preponderate, the result will be a minute perturbation in an outward elliptic spiral. These two forces, when their action in the whole orbit is considered, become partially restorative to each other, leaving a minute balance in favor of resistance.

This residual resistance will be greater, where the eccentricity is greater: but its tendency is to decrease the eccentricity, and reduce elliptic to circular orbits. When an elliptic orbit, under the influence of the residual force, has closely approximated the circular form, as is the case with all our planetary orbits, the excess of resistance becomes inappreciably small, requiring immense periods to attain to a perfect equilibrium in

a circular orbit, where the resistance and aberrating forces, will, throughout the whole orbit, be exactly equal.

As another consequence, attending the propelling and resisting forces, all cometary bodies which may assume, for a moment, parabolic or hyperbolic orbits, cannot maintain themselves in such orbits; for the residual resisting force will compel them into orbits of the elliptic form, and afterwards still further reduce them to the planetary form of small eccentricity, from which, after the lapse of ages, they will reach their final destiny in circular orbits of different dimensions.

Another tendency of these two antagonistic forces is to continually correct, in a measure, any derangements which may happen in a system of bodies. If from some extraneous cause, the eccentricity of a cometary orbit should suddenly be increased, as has been the case, in some rare instances, the residual force begins slowly but surely to work a restoration, so far as the gradual diminution of the eccentricity is concerned.

When the changes upon the whole orbit are taken into the calculation, the residual force, except in circular orbits, is always in favor of resistance; therefore the result will always be an inward elliptic spiral, which will, not only diminish the eccentricity, but shorten the transverse axis, and diminish the mean distance, and consequently lessen the periodic time. These minute perturbations of cometary orbits may, by close observation, be detected, in cases, where the eccentricity is very great. It is in the diminution of the periodic time, that the phenomena, alluded to, will more readily and satisfactorily develop themselves.

91. The foregoing propositions will now be more fully illustrated, by introducing a few examples.

Example 1.—Let the mean distance of the earth from the sun = 1; let its semi-minor axis = $\cdot 99985578 = a$; let the density of the resisting medium be the same as in Par. 89; let i , k , and R , be, at the mean distance, each = 1; let a' , i' , k' , d' , R' , represent the same quantities, as in the former propositions: at what distance from the sun will the earth, in its elliptic orbit, be equally acted upon by the propelling and resisting forces?

By Par. 90, the angular velocity $a =$ semi-minor axis; hence, formula (1) of the same Paragraph becomes

$$x^3 - 4x^2 + 4x - (.99985578)^2 = 0.$$

In this equation find the value of x between 1 and 2, which will be the distance required.

$$x = d' = 1.00028834$$

From this example, it will be seen, that the radius vector, x , is only a small fraction greater than the mean distance; therefore, the two points, where the antagonistic forces balance each other, are situated near the extremities of the minor axis, on the aphelion side of the same.

Example 2.—Find the values of the resisting and aberrating forces at the perihelion point of the earth's orbit, the data being the same as in Ex. 1.

$$R' = \frac{k'}{\sqrt{d'^5}} = \frac{\text{dis. of upper focus}}{\sqrt{(\text{dis. of Perihelion})^5}} = \frac{1.01678880}{\sqrt{(.98321120)^5}};$$

therefore $R' = 1.06075164$

$$a'i' = \frac{a}{d'^3} = \frac{.99985578}{(.98321120)^3} = 1.05195439$$

$R' - a'i' = .00879725 =$ excess of Resistance at the perihelion $= B.$

Example 3.—Find the values of the aberrating and resisting forces, at the aphelion point of the earth's orbit, the data being the same as in Example 1.

$$a'i' = \frac{a}{d'^3} = \frac{.99985578}{(1.01678880)^3} = .95114144$$

$$R' = \frac{k'}{\sqrt{d'^5}} = \frac{\text{dis. of upper focus}}{\sqrt{(\text{dis. of Aphelion})^5}} = \frac{.98321120}{\sqrt{(1.01678880)^5}} \\ = .94312647$$

$d'i - R' = .00801497 =$ excess of aberrating force, at the Aphelion $= A$.

Therefore

$B - A = .00879725 - .00801497 = .00078228 =$ residual force of resistance at the perihelion, above the excess of aberrating force at the aphelion.

Example 4.—What is the difference between the resisting and aberrating forces, where the latus-rectum of the earth's orbit cuts the ellipse?

$$\frac{1}{2} \text{latus-rectum} = \frac{(\text{semi-minor axis})^2}{\text{semi-transverse axis}} = .99971158$$

$$R' = \frac{k'}{\sqrt{d'^5}} = \frac{1.00028842}{\sqrt{(.99971158)^5}} = 1.00101004$$

$$d'i = \frac{a}{d'^3} = \frac{.99985578}{(.99971158)^3} = 1.00072141$$

$$\text{Required difference} = .00028863$$

Example 5.—What is the difference between the aberrating and resisting forces, where the ordinate, passing through the upper focus of the earth's orbit, perpendicularly to the transverse axis cuts the ellipse?

The ordinate, in this case, is equal to the latus-rectum; therefore

$$d'i' = \frac{a}{d'^3} = \frac{.99985578}{(1.00028842)^3} = .99899114$$

$$R' = \frac{k'}{\sqrt{d'^5}} = \frac{.99971158}{\sqrt{(1.00028842)^5}} = .99899110$$

$$\text{Required difference} = .00000004$$

It will be seen, by this example, that the two antagonistic forces very nearly balance. The balance would have been complete, if the radius vector, d' , had been about seven miles less. (See Example 1.)

Example 6.—If the elements of the orbit of Venus be represented by the same symbols as those of the Earth, and those at the mean distance of Venus from the sun, be considered as unity, a , representing the angular velocity = the semi-conjugate axis, at what distance from the sun, will the two antagonistic forces balance each other?

For Venus, equation (1), Par. 90, becomes

$$x^3 - 4x^2 + 4x - \cdot 999952661705 = 0,$$

the last term being equal to a^2 ; hence

$$x = 1 \cdot 000047336054$$

When $a^2 = \frac{1}{2}$ lat. rec., we have

$$2 - a^2 = 1 \cdot 000047338295$$

The value of $2 - a^2$ does not differ from x to eight places of decimals. The cause of this very small difference is the near approximation of the orbit of Venus to a circle. When a circular orbit is reached, $2 - a^2 = x = 1$; but in all other conditions $2 - a^2 > x$; that is, $2 - a^2$ is the distance from the lower focus to that point in the ellipse from which an ordinate let fall perpendicularly to the transverse axis, would pass through the upper focus.

92. The sum of the aberrating forces throughout the whole of an elliptic orbit, deducted from the sum of the resisting forces, will, as stated in the preceding paragraphs, give a minute remainder, which alone becomes affective in gradually, (and almost imperceptibly in an orbit of small eccentricity,) changing its elements. To calculate approximately the sum of each of these opposing forces, it is first necessary to determine the length of the radius vector, for equal intervals of time, between the aphelion and perihelion points. The necessary data, which enter into this calculation, are derived from Kepler's law of the equable description of areas in equal times, by which, first, the true anomaly of the sun is deduced from the mean anomaly, for any given time from the aphelion; and second, the length of the radius vector is derived from the true anomaly. (See Robi-

son's Mechanical Philosophy, p. 191; also Fig. 37, Plate 9, p. 236.)

But as the logarithm of the radius vector, for each of the minor planets, is given for mean noon of each day in the year, in the English Nautical Almanac, it is a matter of easy calculation, to determine the amount of the two opposing forces, for each of these intervals of time. In the Appendix, Tables I, II, III, I have given the results of these calculations, for each day at noon, in the year 1854, for the Earth and Venus, and in 1852, for Mercury. It is not necessary to extend the Tables, only from the aphelion to the perihelion of each orbit.

93. By an inspection of these Tables, it will be perceived, that each of these forces has its *minimum* value at the aphelion, and its *maximum* at the perihelion; and that the resisting force has a wider range, between these limits, than the aberrating force; that is, the resisting force at the aphelion is less than the aberrating force; but at the perihelion, the resisting force is greater than the aberrating force: consequently there must be two intermediate points, in the whole orbit, where these forces are equal. (Par. 90.)

94. The *sums* of the two forces are calculated for the Earth, Venus and Mercury, in the three Tables referred to. These sums only extend from the aphelion to the perihelion of each orbit. And the calculation is based upon the assumption that the two forces in circular orbits are exactly equal; and consequently, that the present orbits are variable, and slowly approximating to a permanent, invariable, circular form; and that the major-axes and periodic times are slowly diminishing. I shall, in the next chapter, examine the intensity or ratio of the two forces, which will insure the stability or permanency of elliptic orbits.

CHAPTER VIII.

INVARIABLE ELLIPTIC ORBITS.

95. *Intensity of the two Forces not necessarily equal.* 96. *How to find their ratio at mean distance. 2 Examples.*
 97. *Method of calculation General.* 98. *Minute apparent discrepancy; probable cause.*

95. In the preceding investigations, it has been assumed that the two forces in circular orbits were equal in intensity, having a tendency to reduce all cometary and planetary orbits to a circular form. But it is quite evident, that such an assumption is not necessary: indeed, it cannot be true, if any orbits of an elliptic form have already become permanent in their nature, (planetary perturbations excepted,) unless we admit a slow rotation of the ether around the Sun.

96. To find the ratio of the two forces at the mean distance, in any invariable elliptic orbit.

First, calculate each of these forces for equal intervals of time, say for every day from the aphelion to the perihelion of the given ellipse. (Assuming the two forces to be equal in a circular orbit whose radii are equal to the mean distance of the given ellipse.)

Second, find the sums of the two forces from the aphelion to the perihelion.

Let s , be equal to the sum of the aberrating forces;

S , be equal to the sum of the resisting forces;

r , be equal to the ratio of the two sums.

Then we shall have

$$r = \frac{s}{S} \dots\dots\dots (a)$$

Example 1. What must be the intensity of the resisting force, (say for the year 1854,) at the mean distance in the

earth's elliptic orbit, to maintain its invariability, or, in other words, to render the elements of the orbit invariable. (Extraneous perturbations excepted.)

In Table I., Appendix, the sum of each of the two forces is given, the resisting force at mean distance being assumed equal to unity. Therefore, by the last formula, we have

$$r = \frac{s}{S} = \frac{183 \cdot 94357449}{183 \cdot 96105127} = \cdot 999904973656$$

That is, the intensity of the resisting force necessary to render the earth's orbit a permanent circle, is to the intensity of the resisting force necessary to sustain it permanently in its present elliptic form, as 1 to $\cdot 999904973656$. With this value of r , the ellipticity, major-axis, orbital period, and all other elements of the earth's orbit must remain invariable. Indeed, were the earth forced by some extraneous action, into a circular orbit, or even into a cometary orbit, this value of r would restore it to its present form.

Example 2. What must be the intensity of the resisting force, (say in 1854,) at the mean distance, in the orbit of Venus, to render its elements invariable?

In Table II. Appendix, the sums of the two forces are calculated, from the aphelion to the perihelion of the orbit of Venus. These calculations are founded upon the supposition, that the two forces, at the mean distance in a circular orbit, are exactly equal; and consequently, that their ratio is unity. According to the law of resistance, these forces are inversely as the fifth powers of the square roots of the distances; hence, for Venus each equals $2 \cdot 2473179933$. (Those at the Earth's distance being unity.)

By formula (a) we have

$$r = \frac{s}{S} = \frac{256 \cdot 18253159}{256 \cdot 18681393} = \cdot 999983284308;$$

this ratio is a constant quantity, to be multiplied into each of the resisting forces in Table II., which will render the sum of

the resistances equal to the sum of the aberrating forces.

Therefore

$$R = 2 \cdot 2473179933 \times \cdot 999983284308 = 2 \cdot 24728042782 =$$

the resistance, at the mean distance in the orbit of Venus, if its present orbit is invariable. The difference between the intensity of the resisting force, necessary to make the orbit circular, and that necessary to maintain it in its present elliptic form, is

very small, being less than the $\frac{1}{50000}$ -th part of the former force.

97. In the same manner, the value of the ratio, r , may be calculated for every other orbit in the solar system; and if necessary, the resistances in each can be expressed in pounds weight. (Par. 45.)

98. If we assume that the orbits of the Earth and Venus have already attained their invariable form, then there is a very minute discrepancy between the calculated resisting forces, and those which should exist, according to the law of density. For, in the case of the Earth's orbit, we have (see Ex. 1, Par. 96.)

$r = \cdot 999904973656$; and by the law of resistance we have

$$1 : \frac{1}{\sqrt{d^3}} :: r : \text{the resistance at the mean distance of Venus.}$$

or expressed in numbers

$$1 : 2 \cdot 2473179933 :: \cdot 999904973656 : 2 \cdot 2471044389$$

The fourth term, in this proportion, should, according to the law of density, be the resistance at the mean distance of Venus: but it is somewhat less than R , as expressed in Ex. 2. Par. 96. What is the cause of the apparent discrepancy? Is the law not exact? Or is there some other data which should be taken into the calculation? May not R be too great in consequence of ethereal currents circulating from west to east around the Sun? In the next chapter we shall examine the effects of circulating currents of ether, in diminishing resistance.

CHAPTER IX.

ETHEREAL CURRENTS.

99. *Solar Rotation the Origin of Ethereal Currents.* 100. *The Currents must be Circular and their Planes Perpendicular to the Axis of the Solar Envelope.* 101. *Thickness of the two Polar Wings at 1000 millions of miles from the Sun's Center.* 102. *Immense Extension of the Currents in Planes Perpendicular to the Prolonged Solar Axis.* 103. *Is there a Law of Velocity for these Ethereal Currents, depending on Solar Distance? Example.* 104. *Two Important Data Required.* 105. *Resulting Consequences, if the Ethereal Currents had a Planetary Velocity. How the two Forces must be Proportioned in Ethereal Currents of any given Velocity.*

99. IN Chapter II., I have demonstrated that an ethereal medium, having, like other matter, the property of gravity, must necessarily accumulate, in a condensed state, around all worlds, and must rotate with them. I have also demonstrated the form of these rotating ethereal envelopes, and have shown that when not counteracted by the external ether, their poles extend to infinity. I have also determined the equatorial limit of the solar envelope, and demonstrated that it extends over fifteen millions of miles from the sun's center. I have also demonstrated, (Par. 21. Ex. 2.) that the two great polar wings of the solar envelope, at a distance of over sixty millions of miles, from the sun's center, have a thickness or diameter perpendicular to the prolonged solar axis, of over fifteen millions of miles. Can this immense elongated solar envelope swiftly rotate without generating ethereal currents exterior to its surface? Analytical mechanics, common sense, reason, all answer, No!

100. What must be the nature of these ethereal currents? Are they circular or elliptic? Do they circulate in planes per-

pendicular to the axis of the envelope? Or are their planes of motion inclined to the plane of the solar equator? That these currents cannot be elliptic is evident from the circulation of currents in our terrestrial atmosphere. Exterior currents are buoyed up by the density and elasticity of the interior strata; hence, they must generally be circular. The planes of motion of these circular currents cannot be inclined to the solar equator, because of the elasticity of the circulating ether already intervening between them and such equator. (See Paragraphs 23, 26.) Therefore, these exterior currents of the revolving ether are circular, and the planes of rotation are perpendicular to the axis of the ethereal envelope.

101. The polar envelope wings of the sun, at the distance of one thousand millions of miles, from his center, must have a thickness of over three million and seven-hundred thousand miles, provided that there is no exterior resistance to diminish or contract them. If we extend the solar envelope no farther, north and south,—or suppose the wings to be contracted or limited by resistance to these limits, we still have a polar diameter of two thousand millions of miles, around which the ethereal strata rotate from west to east in planes perpendicular to the axis. Of course, the greatest velocity of these currents will be in the solar equator: in which position the retardation will be at its *minimum* value. (Par. 28.)

102. How far does the great ocean of rotating currents in the solar equatorial regions extend? Do they merely reach to the orbit of Mercury? Why may not their potent influence be sensibly felt at Neptune? Can any one assign any reason, why they should not be extended to those immense boundaries of equal gravitation, between our system and others? Are ten millions of millions of miles too far for ethereal motion to be propagated? Is motion lost by distance? The answers to all these questions are plain. Motion, if not counteracted, must extend to affinity. The grand rotation of the Sun, with its immense polar wings, must exert a rotatory influence, for millions of millions of miles. All bodies entering his wide dominions, must partake, not only of his controlling power of gravitation, but also of the great controlling power of rotation

which, in connection with resistance, must determine their orbital paths, and prescribe their annual periods, and point out their axial rotations.

103. There is, undoubtedly, a law, depending on the distance from the Sun, determining the velocity of these ethereal currents. If such law could be discovered, it would be a simple problem to determine the exact amount, by which the resistance is diminished in each planetary orbit: and by this means we could also calculate the exact eccentricity of each invariable orbit.

Example.—If the two forces in the orbit of Venus, when considered circular, were exactly equal, in a stationary resisting medium, what must be the velocity of the ethereal current necessary to reduce the orbit to an invariable elliptic form such as now exists?

Let v = the velocity of Venus at the mean distance = 1.1757985108 , that of the Earth being unity;

let v' = v - velocity of the ether;

let r = the resistance when the two forces are equal = 2.2473179933 .

let R = the diminished resistance, as calculated from Table II. = 2.2472804278 . (See Par. 96. Ex. 2.)

Then we shall have

$$r : R :: v^2 : v'^2 ;$$

or in numbers

$$2.2473179933 : 2.2472804278 :: (1.1757985108)^2 : (1.1757886836)^2$$

or $1.1757985108 - 1.1757886836 = .0000098272 =$ the required velocity of the ethereal current = $11\frac{1}{3}$ inches per second. Such a velocity of the rotating ethereal current would render the elements of the orbit of Venus invariable under the conditions specified in the problem.

104. In a similar manner, and under the same conditions, we can calculate the velocity of the ethereal current, at any given mean distance of a planet, providing that we know the

planet's eccentricity, and are assured that the orbit is invariable. But two great and important data remain yet undiscovered, namely, first, the intensity of the resistance compared with the known intensity of aberration; and second, the law of the velocity of the ethereal medium, depending on its perpendicular distance from the axis of the solar envelope.

105. If the ether had a planetary velocity, varying inversely as the square root of the Sun's distance, the planets would be wafted along, if their orbits were circular, without meeting with any resistance. But the aberrating force would soon send them adrift in the wilds of space. On the other hand, if the velocity of the ether were $\frac{1}{2}$, $\frac{1}{4}$, or any other fraction of planetary velocity, the intensity of resistance must be increased sufficiently to balance the sum of the aberrating forces in all invariable orbits.



CHAPTER X.

REVOLUTION FROM WEST TO EAST.

106. "*Nebular Hypothesis.*" *Its Insufficiency as a Cause to account for observed phenomena.* 107. *Upon what a New Theory should be founded.* 108. *Direction of Planetary Orbits dependent on Ethereal Currents.* 109. *An extreme case of Retrograde Motion considered.* 110. *Universality of the New Theory; its great Contrast, when Compared with the limited Hypothesis of La Place.*

106. It has been known for centuries, that the planets of our system revolve in their orbits from west to east. The discovery of a great number of Asteroids, during the present century, all revolving in the same direction, shows most conclusively that there must be some general cause for this great phenomenon. La Place, in the last century, propounded a very ingenious theory which has been very generally received, under the name of the "*Nebular Hypothesis.*" The fundamental characteristics of which, are that the Sun and all the planets once existed in a nebulous state, and that by the combined laws of condensation and rotation nebulous patches were detached in the form of rings, which afterwards were broken up and formed nebulous planets which by the same process detached satellites. In this manner he endeavors to account for the revolution of planets from west to east, and for their diurnal rotations in the same direction: This hypothesis, however fertile in its consequences, is beginning to wane. The satellites of Uranus move in retrograde orbits: many of the asteroids have orbits greatly inclined to the solar equator: great numbers of comets have orbits from east to west: the inner satellite of Mars revolves around its primary over three times while Mars rotates once, which proves, beyond all controversy, that it could not have been detached, according to the nebular hypothesis. It is also believed that there is an intra-Mercurial planet, having a less period than

the Sun's rotation: if so, it could not have been detached by such rotation.

107. If, then, there are so many evidences, adverse to the hypothesis of La Place, so many out-standing phenomena irreconcilable with it, is it not necessary that some new field of research should be explored, more in accordance with advanced discoveries? The mechanism of the heavens should be founded upon exact laws,—should embrace within its domain the workings of every part of the grand machinery. A theory, which does not take within its scope the universe as a whole,—that does not point out the mechanism of its parts,—that does not exhibit the adjustments, as the necessary results of the infallible laws of force,—should be received with a measure of distrust. Advanced research, oftentimes, calls for an alteration of theories, and sometimes for an entire renunciation of them. A theory shown to be insufficient or untrue in many respects, should not be cherished and upheld, because of its antiquity, or its general popularity, or because there is no other known theory more in accordance with existing phenomena. First, free the mind from error, and it will be better prepared for new fields of research, and to decide as to the truth or falsity of any new theories which may be propounded.

108. If we discard the "nebular hypothesis," is there any hopes of accounting for the general movements of the bodies of our system in one direction—from west to east? Why do they not move indiscriminately in all directions? The theory advanced, in these pages, gives an easy solution of these questions. We have shown, in Chapter IX., that the great ethereal ocean, surrounding the Sun, must partake of his rotation,—must rotate from west to east, in planes perpendicular to the axis of the ethereal envelope,—and that these currents must extend north and south for thousands of millions of miles, and that their influence is undoubtedly felt far beyond the known boundaries of our system. Can these rotatory currents exist, and bodies, falling into them from the distant regions of space, not be influenced by them? As well might we say, our ships will not be influenced by the trade-winds which they may encounter.

109. Let us suppose, for instance, an extreme case of ret-

rograde motion. We will say a comet enters the boundaries of our system, directly in the plane of the solar equator, with a retrograde motion from east to west, or, in other words, in direct opposition to the ethereal currents. This would be a case of unstable equilibrium. The least deviation from that exact plane could not be recovered. Any slight cause, such as a planetary perturbation, would destroy the equilibrium, and the comet would deviate either north or south of the equatorial plane: and the action of the ethereal current would continually augment the inclination, until the plane of its orbit became perpendicular to the solar equator. From this position, under the influence of these same currents, the angle of inclination would rapidly decrease, until nearing the equatorial plane, when the decrease would become diminished more and more, till finally the two planes would coincide, which would be a state of stable equilibrium. Before the orbit could be converted from a retrograde to a direct condition, or from an unstable form to one of stability, immense ages might intervene, and many thousands of cometary revolutions might be performed, till the eccentricity becomes reduced from a cometary to a planetary form, and its invariable orbit is attained.

110. Thus the new theory accounts for all the phenomena, so far as orbital revolutions are concerned, that are attempted to be explained by the "nebular hypothesis." And in addition, embraces within its domain those out-standing facts, so fatal to the common theory. By the new theory a nebulous mass may come from vast distances,—enter our system, be swayed by the attraction of Mars, revolve around him in a cometary orbit of great eccentricity, in a retrograde direction, and in the course of ages swing around into a direct orbit, have its eccentricity diminished, its major-axis and periodic time shortened, and finally attain to an invariable orbit, so near the surface of Mars as to pass around him three times while the primary revolves but once. All this is possible, as the necessary consequences of the two great antagonistic forces developed by the new theory. Can the theory of La Place trace the history of nebulous matter through all these adverse stages to its final consummation as the inner satellite of Mars? Can the combined theories

of all ages do this? But we have not yet shown, the full and extended capacities of the new theory. There are other great phenomena, constantly before our eyes, which no theory has hitherto sufficiently explained. I refer to the diurnal or axial rotations of the planets in one general direction from west to east; and the inclinations of their axes to their respective orbits. These subjects will be investigated, in a brief and general manner, in the next chapter.



CHAPTER XI.

DIURNAL OR AXIAL ROTATION OF PLANETS.

111. *Is Rotation the Result of a single Impulse, acting in the form of Projection? Or, is it the Result of Continuous Forces still operating?* 112. *Rotations of Machinery.* 113. *How Celestial Rotations are Generated. Initial Rotations. Direction of Rotations.* 114. *Causes for the Inclination of the Planes of Rotation to the Planes of the respective Orbits.* 115. *How Retrograde Rotations are converted into Direct ones.* 116. *Retrograde Satellites of Uranus. The Orbits must be converted into the Direct Form, long before their axial Rotations obtain the same Form.* 117. *Necessary Data for Calculating Diurnal Periods.*

111. WHY do planetary bodies rotate upon axes? Who do they rotate in one general direction from west to east? Why are the planes of rotation inclined at various angles, to the planes of the respective orbits? Are there no laws which govern these things? Was the immense machinery set in motion by a single impulse, exerted for a moment, and then left to itself for all future ages? Or are the phenomena the result of continuous powers, still operating? These are questions well worthy of the consideration of all scientists: they are questions far out of the reach of any former theory yet propounded.

112. I propose to show how the *initial* rotatory movements first commenced. But first, let me refer to the rotation of bodies here on the Earth. A steamer, for instance, with side wheels may float with the current of a river without the least rotation of the wheels. But let such steamer hoist her sails to the winds, instead of using steam, so as to propel the vessel in any direction, and the wheels will immediately begin to rotate; the upper halves revolving in the same direction as the vessel. This arises from the difference of resistance upon the wheels; the

lower halves being resisted by the water; the upper halves being resisted by a less dense medium—the air. A wind-mill constructed with shafts like the side wheels of a steamer, when carried through stationary air of equal density, will not rotate, however swift its conveyance. But were it possible to construct such a wind-mill one mile in diameter, having its upper and lower wings each one-half a mile in length from the axis, it would, if conveyed in any direction, in a stationary atmosphere commence rotating in the direction of its conveyance: this would arise from the difference of density in the atmospheric medium; the upper wings would not experience the same resistance as the lower ones. The more rapid the conveyance, the greater the velocity of rotation. This would also be the case, if the atmosphere were moving in any given direction or velocity, providing that the flight of the wind-mill exceeded the atmospheric current. By these few simple illustrations, we cannot but see, how rotation is generated, and how its direction and velocity are modified.

113. Let us now apply these same mechanical laws on a more grand and magnificent scale: let our wind-mills become planets or moving worlds: let the great ocean of ethereal matter be the theatre of their movements. I have already demonstrated that this ethereal matter, if subject to gravity, must vary in density as the inverse cube of the square root of its distance from the Sun. Can worlds be conveyed in this variable medium, without generating rotation? They can, if they only float with the ethereal currents: but if they have orbital velocities far greater than these currents, the unequal density of the medium must generate rotation. If the planet revolves in its orbit from west to east the direction of the rotation must be from west to east. If the orbit be from north to south the rotation upon the axis must be from north to south. Whatever may be the direction of its orbital path, the planet is compelled, in its *initial* rotations, in the same direction.

114. If there were no swinging around of orbits, there would be no inclination of equatorial planes to these respective orbits. For, in all instances, the orbital and equatorial planes would coincide. If we suppose a nebulous cometary mass to

come from the immensity of space, and to enter our system in an orbital plane exactly perpendicular to the plane of the solar equator: this mass would begin to acquire an initial rotation upon an axis: the direction would be from north to south: its equator and orbital plane would coincide. But after a few revolutions, the orbital plane, by the action of the ethereal currents, would swing around, say 30° , making an inclination with the solar equator of 60° . Is it possible for the axis of rotation also to swing around 30° in the same period? All will see, at once, the impossibility of such an occurrence. For the orbit is acted upon, with greater mechanical advantages, in altering its inclination, than in altering the position of rotation. Hence, there must arise an angle of inclination, between the rotative plane and the orbital plane. The swinging of the latter must, so to speak, out-run the swinging of the former. The plane of the planet's orbit will have accomplished its destined journey and coincide with the solar equator, long before the plane of rotation will have arrived at the same destination.

115. We shall now speak of orbits and rotations of a retrograde character. It is evident, from what has been stated, that if a cometary mass were to enter the boundaries of our system, having a retrograde motion, either in the plane of the solar equator, or at any given angle with that plane, the rotation generated will be in the same direction as the orbit, and consequently retrograde. As the orbit, under the action of the ethereal current, begins to increase its inclination, the equatorial plane will begin to change in the same direction, but in a much slower manner: after the elapse of ages, the orbit will have arrived at its *maximum* inclination or 90° , while the plane of rotation will be lingering behind, unable to keep up with the orbital change. As the ethereal currents continue operating, the orbit is next changed from a retrograde character to one of direct motion: thus we may have, for a while, the curious phenomenon of a direct orbit, with a retrograde rotation upon an axis. But such a condition of things cannot forever continue: the same mechanical cause which has converted a retrograde orbit into a direct one, must necessarily convert the

retrograde rotation into a direct one: they are both the effects of the same cause: the one being accomplished more speedily than the other, giving rise to that hitherto unaccountable phenomenon, called the inclination of the axes to the respective orbital planes.

116. In our solar system, we have many instances of retrograde comets; but none of retrograde planets. In the system of Uranus we have the phenomenon of retrograde satellites, whose orbits are said to be almost perpendicular to the plane of the orbit of the primary. If we had instruments sufficiently powerful to detect the planes of rotation of these satellites, we would, undoubtedly, find such rotations retrograde in their character. Such must be their condition, according to the mechanical laws by which they are governed. In future ages, these satellite paths will be converted into direct orbits; and these secondary bodies will gradually take up their line of march, in the same direction and order, as the other satellite groups of the solar system.

117. If we can determine the amount of change in the inclination of any given orbit, in any given period, it is evident that we shall be in possession of data, which will enable us to calculate, at least, approximately the intensity of the force acting upon such orbit: and, in like manner, we can determine approximately the intensity of force, necessary to produce the observed deviations of axial inclinations. When we have made such calculations for several orbits and axial rotations, we shall have the requisite data, for determining the law of intensity of these mechanical forces, as depending on distances from the Sun. It is to be hoped, that some of our great mathematicians, who have means and leisure, will attack these formidable problems, and discover the law so immensely important to the future progress of Astronomy. It is in this direction also, that we may hope to discover the law, governing the periods of diurnal rotation, and thus be able to calculate such periods, with all the accuracy which so harmoniously and beautifully attends the calculation of orbital periods. But this highly interesting subject will be briefly examined in the next Chapter.

CHAPTER XII.

DIURNAL PERIODS OR PERIODS OF ROTATION.

118. *Approximate Law for Diurnal Periods, as Published by the Author about a quarter of a century ago.* 119. *A Rotation upon an Axis is a Stable Equilibrium. Any derangement in the Period, cannot be Permanent. The Two Forces will work their own adjustments.* 120. *Example. Periods of Rotation the Result of a state of Equilibrium between the Two Forces.* 121. *Effects of Terrestrial Aberration upon the Diurnal Period of the Earth.* 122. *Further Development of the Theory left to others who have Time and Means at their Command.* 123. *Remarks on the Causes of Solar Rotation.* 124. *How the Aberrating Force is increased. The Solar Rotation the Result of the Equilibrium of the Two Opposing Forces.*

118. MANY have been the investigations of astronomers, since the days of Kepler, to discover the law, governing the periods of diurnal rotation. But all such attempts have been apparently fruitless. The author, about one quarter of a century ago, published some of his own researches upon this subject: the law of planetary rotation was expressed as follows:—

“THE PERIODS OF PLANETARY ROTATIONS UPON THEIR AXES VARY AS THE SQUARE OF THE CUBE ROOTS OF THEIR DENSITIES.”

This law, as was demonstrated at the time, seemed to be a very close approximation to the observed periods. But as he could not, at that time, show any cause for such a law, it was partially abandoned. But lately, since propounding the theory set forth in these pages, he has often been reminded of the law, formerly published, in several periodicals, as follows:—

“TO THE EDITOR OF &c.,

Dear Sir,—Permit me to announce to the world, through

your valuable paper, an astronomical discovery, made by me on the eleventh day of November, 1854. I allude to a law governing planetary rotation. Telescopic observation reveals to us the fact that many of the bodies of the solar system, not only have a progressive velocity in their orbits, but a rotative motion upon their axes. Heretofore, the only means known to astronomers, by which to determine the exact period of a planet's rotation, has been the careful observation of the movements of spots upon its surface. For instance, spots are discerned on the eastern limb of a planet's disc, which, instead of remaining stationary, gradually move across the disc in a westerly direction, disappear for a few hours at the western limb, and again re-appear in their former position on the eastern limb. These spots are believed to be portions of the surface of a planet darker than the adjacent parts, and carried around by a rotation of the planet on its own axis from west to east.

By such observation, the period of the rotation of the planet Mars has been determined to be *24h. 37m. 23s.* mean solar time; Jupiter's period, *9h. 54m. 12s.*; and Saturn's period, *10h. 29m. 17s.* Mercury and Venus are situated so near the sun that it is extremely difficult to distinctly discern spots upon their surfaces. It is believed, however, by some astronomers that their observations are sufficiently exact to assign to Mercury a rotative period of about *24h. 5m.*, and to Venus a period of *23h. 21m. 21s.* It is still maintained by some eminent observers that there is a small degree of uncertainty remaining as to the rotative periods of Mercury and Venus. Uranus is supposed by some observers to rotate on its axis in *9h. 30m.*; but this planet is so far distant from us that it is generally believed that the present powers of the telescope are incapable of revealing any spots upon its surface sufficiently distinct to determine whether it has a rotation or not. Whether the asteroids and the planet Neptune rotate, the astronomical instruments of the present day are utterly inadequate to determine.

Do all the planets of the solar system rotate? Observation has, as yet, been entirely unable to answer this question. It is supposed from analogy that rotation is as extended in its operations as the progressive motions of planets in their orbits—that

every planet turns upon its axis, producing the agreeable alternations of light and darkness, day and night. But how to demonstrate this analogical supposition—how to ascertain the periods of rotation of such planets as are beyond the reach of observation, has been a problem unsolved by the astronomical world.

Many eminent and distinguished astronomers have eagerly sought after some law, connecting the rotative periods of the planets with some known data of the solar system, such as their distances from the sun—their orbital velocities—their masses, &c.; but all their laborious researches to develop such law have not been crowned with success—the law of planetary rotation has eluded their grasp.

Firmly believing, from my early youth, that the diurnal periods of the planets were the results of some hidden law, I have endeavored, at different times, to discover the same, so as to determine the periods of rotation by calculation instead of observation. After many fruitless researches in regard to the original causes of planetary motion, I was led by the indications of certain hypothesis to seek for the law of rotation connected with the masses and diameters of the planets, or, in other words, with their densities. These investigations resulted in the development of the following beautiful law:—

THE CUBE ROOTS OF THE DENSITIES OF THE PLANETS ARE AS THE SQUARE ROOTS OF THEIR PERIODS OF ROTATION.

Or, which amounts to the same thing—THE SQUARES OF THE CUBE ROOTS OF THE DENSITIES OF THE PLANETS ARE AS THEIR PERIODS OF ROTATION.

But as the densities of globes are proportional to their masses or quantities of matter, divided by their volumes or by the cubes of their diameters, it follows that the rotation of the planets, considered as spheres, is proportional to their masses and diameters. The law, therefore, may be expressed in terms of the masses and diameters, as follows:—

THE SQUARES OF THE CUBE ROOTS OF THE MASSES OF THE PLANETS DIVIDED BY THE SQUARES OF THEIR DIAMETERS ARE AS THEIR PERIODS OF ROTATION.

To illustrate the correctness of this law, I will give the following examples:—

Example 1.—Given the mass of the earth equal to 1; its equatorial diameter, 7925·5 miles; its period of rotation, 23h. 56m. 4·090475s mean solar time, which is equal to one absolute sidereal day: also the mass of the planet Mercury equal to 0·0627694 and its diameter 3140 miles; it is required to find the period of Mercury's rotation.

$$\frac{(1)^{\frac{2}{3}}}{(7925 \cdot 5)^2} : \frac{(0 \cdot 0627694)^{\frac{2}{3}}}{(3140)^2} :: 23h. 56m. 4 \cdot 090475s. : 24h. 5m.$$

Example 2.—Given the mass of Venus equal to 0·9043346; its diameter 7800 miles; and the mass, diameter, and rotative period of the earth, as in the first example. Required the period of the rotation of Venus.

$$\frac{(1)^{\frac{2}{3}}}{(7925 \cdot 5)^2} : \frac{(0 \cdot 9043346)^{\frac{2}{3}}}{(7800)^2} :: 23h. 56m. 4 \cdot 090475s. : 23h. 21m. 21s.$$

Example 3.—Given the mass of Mars equal to 0·1453337; its diameter, 4108·26 miles; and the mass, diameter, and rotation of the earth, as in the first example. Required the period of the rotation of Mars.

$$\frac{(1)^{\frac{2}{3}}}{(7925 \cdot 5)^2} : \frac{(0 \cdot 1453337)^{\frac{2}{3}}}{(4108 \cdot 26)^2} :: 23h. 56m. 4 \cdot 090475s. : 24h. 37m. 23s.$$

Example 4.—Given the mass of Jupiter equal to 371·7547; its diameter 88592·7 miles; those of the earth as in the former examples. Required the period of the rotation of Jupiter.

$$\frac{(1)^{\frac{2}{3}}}{(7925 \cdot 5)^2} : \frac{(371 \cdot 7547)^{\frac{2}{3}}}{(88592 \cdot 7)^2} :: 23h. 56m. 4 \cdot 090475s. : 9h. 54m. 12s.$$

Example 5.—Given the mass of Saturn and its rings equal to 289·0281; its diameter, 79160 miles; those of the earth as in the former examples. Required the period of the rotation of Saturn.

$$\frac{(1)^{\frac{2}{3}}}{(7925 \cdot 5)^2} : \frac{(289 \cdot 0281)^{\frac{2}{3}}}{(79160)^2} :: 23h. 56m. 4 \cdot 090475s.$$

$$: 10h. 29m. 17s.$$

Example 6.—Given the mass of Uranus equal to 20·625488; its diameter 34500 miles; those of the earth as in the former examples. Required the period of the rotation of Uranus.

$$\frac{(1)^{\frac{2}{3}}}{(7925 \cdot 5)^2} : \frac{(20 \cdot 625488)^{\frac{2}{3}}}{(34500)^2} :: 23h. 56m. 4 \cdot 090475s.$$

$$: 9h. 30m.$$

Example 7.—Given the mass of Neptune equal to 26·87671; its diameter, 41500 miles; those of the earth as in the former examples. Required the period of the rotation of Neptune.

$$\frac{(1)^{\frac{2}{3}}}{(7925 \cdot 5)^2} : \frac{(26 \cdot 87671)^{\frac{2}{3}}}{(41500)^2} :: 23h. 56m. 4 \cdot 090475s.$$

$$: 7h. 49m. 57s.$$

Example 8.—Given the mass of Neptune equal to 26·87671; its diameter, 37500 miles; those of the earth remaining as in the former examples. Required the period of the rotation of Neptune.

$$\frac{(1)^{\frac{2}{3}}}{(7925 \cdot 5)^2} : \frac{(26 \cdot 87671)^{\frac{2}{3}}}{(37500)^2} :: 23h. 56m. 4 \cdot 090475s.$$

$$: 9h. 35m. 32s.$$

If in any of these examples the mass be divided by the cube of the diameter, the quotient will be the density of the planet. And if the density of the earth be taken as unity or 1,

the densities of the other planets, deduced from the data given in the foregoing examples will be as in the second column of the following table:—

Name of Planet.	Densities (the earth's being 1.)	Masses (the earth's being 1.)	Diameters in miles.	Diameters (the earth's being 1.)	Periods of rotation in absolute sidereal days (the earth's being 1.)
Mercury.....	1.00934	0.9637694	3140	0.3961895	1.00622
Venus.....	0.96395629	0.9043346	7800	0.984165	0.975824
Earth.....	1.000000	1.0000000	7925.5	1.000000	1.000000
Mars.....	1.04346	0.145337	4108.26	0.51836	1.02877
Jupiter.....	0.26616	371.7547	88592.7	11.17818	0.41377
Saturn and rings	0.29007	289.0281	79160	9.988013	0.43820
Uranus.....	0.25005	20.625488	34500	4.353038	0.39692
Neptune.....	0.1872026	26.87671	41500	5.2362627	0.327246
Neptune.....	0.253715	26.87671	37500	4.7315627	0.400771

If in the examples given above, the densities in the second column of the table be substituted for the masses and diameters, and the absolute sidereal period of the earth's rotation be considered as unity or 1, the calculations will be greatly shortened, for then the squares of the cube roots of the densities would be equal to the periods of rotation, expressed in absolute sidereal days, as in the last column of the above table.

Example.—If the density of Mars be 1.04346, what will be its period of rotation?

$$(1.04346)^{\frac{2}{3}} = 1.02877 \text{ sidereal days, which is the answer.}$$

The ratio of one mean solar day to one absolute sidereal day is as 1.00273791 to 1. Therefore, by dividing the sidereal days in the last column of the table by this ratio, the quotient will be solar days, which may be easily reduced to hours, minutes, and seconds, mean solar time.

To express the law of planetary rotation in general algebraical formula, applicable to the periods of the rotation of all the primary planets, let M , D , P , represent respectively the mass, diameter, and rotative period of the earth; and let m , d ,

p , represent the mass, diameter, and rotative period of any planet, then we will have

$$\frac{P. D^2. m^{\frac{2}{3}}}{d^2. M^{\frac{2}{3}}} = p; \text{ or in terms of the densities and periods, thus—}$$

$$\frac{(Planet's \text{ den.})^{\frac{2}{3}} P}{(Earth's \text{ den.})^{\frac{2}{3}}} = p; \text{ or, if the earth's rotative period and}$$

density be each taken as unity or 1, then,

$$(Planet's \text{ density})^{\frac{2}{3}} = \text{rotative period.}$$

As the rotative periods depend upon the masses and diameters of the planets, any errors entering into these elements by the imperfections of observation will necessarily affect the calculated periods of rotation in a proportionate degree. This will be more manifest by referring to the 7th and 8th examples in regard to the mass and diameter of Neptune. The mass which I have adopted in these two examples is that calculated by Professor Struve, from his own observations of the satellite attending the primary. The diameter, 41500 miles, is that given by Sir John Herschel in his "Outlines of Astronomy." The assumed diameter, 37500 miles, in conjunction with the same mass, gives a difference of 1*h.* 45*m.* 35*s.* in the calculated periods of rotation. Many astronomers have adopted 35000 miles for the diameter.

The diameter of Neptune is probably not yet known within several thousand miles, for an error of observation of three-tenths of a second of an arc, would at that great distance produce an error of over 4000 miles in the calculated diameter of the planet. The same statement is equally applicable to the determination of the mass. An error of observation on the dimensions of the orbit of Neptune's satellite, though it might be only a small fraction of a second of an arc, yet it would

produce a great difference in the calculated mass. We can, therefore, in our calculations, only approximate the true periods of rotation in proportion as observation approximates the true ratio of the masses and diameters of the planets.

There is a great difference in the observations of astronomers from which the mass and diameter of Saturn are computed. Some have adopted 67000 miles as the diameter; others, 73000; and others again 79160. Some have adopted the mass much less than we have assigned in the foregoing table. If we take the diameter at 67000 miles, and the mass at $175 \cdot 245657$, the ratio and the period of rotation will remain the same as in the table.

The renowned French mathematician—Le Verrier, to whose calculations the world are indebted for the discovery of Neptune, has revised the tables containing the elements of the planet Mercury, and has computed its mass much less than formerly received: hence, I have adopted Mercury's mass less than stated in the old tables.

It will be seen by the formula which I have given, that if the rotation is known by observation, the ratio of the mass and diameter can be calculated; and that if any two of the elements are known, the third can be calculated.

The periods of the rotations of the ultra-zodiacal planets or asteroids cannot be calculated until observation shall determine their masses and diameters.

Whether the law of planetary rotation can be extended to the rotative periods of the satellites, attending the four exterior planets of the solar system, is not known. It is supposed by some, from observation, that the periods of the rotation of the satellites are equal to their periods of revolution around their primaries: but this needs confirmation by further observation of greater perfection and accuracy than the present powers of the telescope seem capable of affording.

From the masses and diameters of the four satellites of Jupiter, as given in Herschel's "Outlines of Astronomy," I find by the application of the law of rotation, the following relative or proportional periods. (Assuming the period of the

rotation of the 1st satellite, nearest to the planet, to be equal to unity or 1.

Jupiter's satellites.	Proportional periods of rotation, as calculated.
1st.	1.00000000
2nd.	1.788478493
3rd.	1.635488852
4th.	1.375555069

If it be true, that the rotative periods of these satellites are equal to the periods of their revolutions around their primaries, then the law does not apparently hold good for these secondary systems, unless the diameters and masses are affected with considerable errors; for instance, if the apparent angular diameter of the second satellite be reduced the one-twentieth of a second of an arc, it would reduce the real diameter 116 miles, which would give a calculated period of rotation, such as should exist in order to correspond precisely with the ratio of the periods of revolution in their orbits. In the cases of the third and fourth satellites, there would have to be a greater correction in order to make the rotative and orbital periods of the same length. A mistake of a small fraction of a second of an arc, might easily be made in the apparent angular diameters. Likewise, as the masses are deduced from observations of the minute perturbations which the satellites exercise upon each other, it is evident that a minute error in such observations, would give a much greater error in the calculated masses. Therefore, masses and diameters might be assumed, within the limits of unavoidable errors, which would give calculated periods of rotation for the four satellites of the same length as their orbital periods.

This remarkable law, connecting the periods of rotation with the masses and diameters of the planets, appears to point to some more original law of a higher order of generalization. Such was the case in regard to Kepler's law, connecting the orbital periods of the heavenly bodies with their distances from their respective centers of motion. Newton demonstrated Kepler's law to be a necessary result of the more general law of universal gravitation.

Providence may raise up a Newton in our day who shall disclose to us the reason why the cube roots of the densities of

the planets are as the square roots of their periods of rotation.

I intend, in some future communication, to present an hypothesis which will, if I am not mistaken, account for this curious law obtaining in the solar system.

With the most sincere desire for the development and diffusion of useful knowledge, I subscribe myself your most humble servant,

ORSON PRATT, SENIOR."

August 4th, 1855."

119. It is evident, according to the new theory, that a rotation upon an axis, is a state of stable equilibrium, wherein the two opposite forces are equal. The aberrating force is a known force, and its intensity can be calculated, for any orbital velocity, or for any rotative velocity, and, if necessary, can be expressed in pounds weight. When the period of rotation becomes invariable, it is certain that the resisting force becomes a known quantity, and can also be expressed in pounds weight, the one force balancing the other. If the diurnal period of the Earth were to be lengthened out, say to forty-eight hours, the aberrating force would preponderate, and would necessarily restore the rotation to its present period. On the other hand, if the rotative period were shortened, say to twelve hours, the resisting force, being in the ascendancy, would diminish the velocity of rotation to its present period. Thus the period of rotation is one of stability, working its own adjustments, and preventing by its laws, any permanent derangements, from extraneous causes.

120. *Example.*—If the sum of the solar aberrating forces were concentrated at one point, and acted perpendicularly to that equatorial radius of the Earth directly opposite the Sun, and if the propulsion were parallel to the tangent of the orbit in a direction from west to east, and at a distance from the Earth's center equal to 25·1643635 miles, what must be the sum of the resisting forces, expressed in pounds weight, concentrated at the same point, to produce the present annual and diurnal periods?

In Paragraph 45, we have the *continuous* orbital force,

equal to 764290107874962825 lbs. It is also proved, in works on Celestial Dynamics, that if a force be applied at the distance of 25·1643635 miles from the Earth's center, of sufficient intensity to project it from a state of rest, with a velocity necessary to its present orbit, the same projection will also produce the present diurnal period. If, therefore, instead of projection or an instantaneous impulse, we apply a *continuous* propelling force, whose intensity, when the orbital velocity is attained, shall be exactly balanced by a *continuous* resisting force, the same results must follow in regard to the annual and diurnal periods. But the *continuous* aberrating force is susceptible of calculation, and is demonstrated to be the number of pounds above expressed. Therefore, as the two forces must be in equilibrium when the periods are constant, the resisting force must also be equal to the same number of pounds.

121. But the *resultant* solar aberrating force, and the point of its application to the Earth, have not yet been calculated, neither that of the resisting forces nor the point of its application. But besides these necessary data: there is another one of great importance, to be incorporated in the same problem, namely, *terrestrial* aberration, or that arising from the velocity of each particle in the diurnal rotation. It is quite evident that each particle in a rotating globe, has a definite velocity, compared with the velocity of the gravity of every other particle in the same globe. The velocity of gravity being known, and the velocity of every particle in a rotating globe being also known, it becomes an easy problem to determine the exact aberrating force, exerted upon any given particle in the direction of the tangent of the rotation. (See Paragraphs 64 and 65.)

122. I have not sufficient leisure, at my command, to calculate the sum of all these terrestrial aberrating forces, and add them to the sum of the solar; and thus point out the amount of force due to each of these causes, in producing terrestrial rotation. I leave this work for others, under more favored circumstances. It is to be hoped, that the great mathematicians of the age, will thoroughly develop this new theory, in all its consequences: it is a theory, which, in its very beginning, seems to be exceedingly fruitful, in accounting for out-standing

phenomena which have baffled all former theories. We have seen, as one instance, comets, traversing our system, in hyperbolic and parabolic orbits. We have had it announced in our ears, that such have taken an everlasting farewell of our system, which is necessarily true, if the old theory be true. But, by the new theory, these two classes of orbits, are unstable. They must yield to the two forces in operation: and it is only a matter of time for them to be converted into ellipses of great eccentricity. These cometary ellipses, must, in their turn, become orbits of a planetary nature, and finally take their invariable position, with orbits of small eccentricity. Such, and such by necessity, are the mechanical consequences of the new theory.

123. The great solar center of our system rolls upon his two great polar wings in the midst of the powers of eternity. Whence arises his rotative forces? From a higher and more extended source than that developed in his own system. Our Sun with all the grand retinue of worlds, attending him, is but a wheel in the vast machinery of the universe. The ethereal ocean, in which he performs his evolutions, stretches forth to infinity in all directions. Each system has its revolutions, its rotations, its currents. No matter can move in ethereal space without encountering ethereal currents, whose densities are ever variable, and ever varying; whence arise rotations of bodies, which, in their turn, generate aberrating forces, and these again increase resisting forces, and these adjusted in equal quantities produce equilibrium; and out of all these actions and re-actions, propulsions and resistances, come forth harmony and beauty, as manifested in regular invariable periodic times. Such is the manner, in which the Almighty power of the great Creator is so wonderfully manifest! such the exhibitions of his infinite wisdom!

124. A body, once commencing to rotate, from any cause whatsoever, must necessarily generate an aberrating force among its own parties, which will increase the velocity of rotation; and the accelerations will continue, until made uniform by continued resistance, There is no reason to doubt, but such must be the condition of our great solar center. The sum

of the aberrating forces which arise from his own rotation, is equal to the sum of the resisting forces which the rotation encounters. The aberrating forces can be calculated; hence, the sum of the resisting forces to his rotation will become known, and may be expressed in pounds weight.



CHAPTER XIII.

REDUCTION OF COMETARY ORBITS TO A PLANETARY FORM.

125. *The Data necessary for Reducing Orbits may be obtained by careful Observation of one Orbit.* 126. *Hyperbolic Orbits require a long Period before they are conquered to the Elliptic Form.* 127. *Parabolic Orbits easily subdued to the Elliptic Form. Causes of Great Eccentricity. Worlds in Embryo. Their future Destiny among Planets. The Great Creator works by Law.* 128. *The work of Creation has its Immense Periods.* 129. *The Great Problem of Reducing a Cometary Orbit to a Planetary Form.* 130. *Other Systems of the Universe. The Extension of the New Theory to them.*

125. THERE are no definite laws by which we can, at once, deduce from the changeable elements of a cometary orbit, its final destiny in an invariable orbit of a planetary form. In each revolution all of its elements are changed. The exceedingly great preponderance of the resisting force, will diminish the dimensions of the orbit, the major-axis, the eccentricity, and the periodic time. When we have learned, by observation, the amount of this diminution in one revolution, of any given comet, we shall be in possession of sufficient data, to calculate, at least, approximately, the ultimate dimensions of its future invariable orbit. If its orbit be retrograde, and we learn the amount of increase in its inclination to the solar equator for any given time, we shall also be able to calculate the time which must elapse, before the orbit shall be converted into one of direct motion, and also how long before it shall be confined like the planets, within the limits of the zodiac. When we have made these calculations for one cometary orbit, we shall be in a condition to declare approximately the absolute intensity of the resisting force at different distances from the Sun; and as the law of variation of density is already known, these

intensities will be known for other regions of cometary space : and thus we shall be in possession of facts which will enable us more readily to solve the great problem of the future destiny of all other cometary orbits, without being obliged to wait for, or learn by the slow process of observation, the data which are necessary to enter into the solution.

126. In some orbits, of a hyperbolic character, comets may proceed to vast distances, before the resisting force conquers them, and sways them from their infinite paths, into curves of an elliptic form. But however obstinate in their course, conquered they must be, unless they pass beyond the bounds of equal gravitation between our system and others, in which case, they can never return : but their direction and velocity may become so modified, as to insure their captivity in an elliptic orbit around the next great center which they may approach.

127. Comets, revolving in parabolic orbits, are not so intractable as those named in the last paragraph. It requires but comparatively a small resistance, to bend their curves into elliptic forms. If they were compelled to assume the elliptic form, when they had exhausted nearly all their velocities in receding from the Sun, then they would be very near the aphe-
lion of their assumed orbits : and the eccentricities would be very great. But if they were compelled, at an earlier part of their journey into elliptic curves, the eccentricities would be less. When a cometary orbit has, by the resisting force, been converted from the hyperbolic into the parabolic form, its escape from our system is next to impossible. It may then be said, that the elements of a new world are secured ; for it only requires time, to condense these elements and properly organize them. By the unchangeable laws of force, they must take their position among those great bodies of our system which have preceded them, and attained to their invariable orbits. Thus the process of organization or the creation of a new world, is accomplished by the word of God, or which is the same thing, by his unchangeable laws. For by law worlds are formed and properly organized ; by law, the materials move from distant space in regular cometary orbits ; by law, the orbits swing around to their destined position ; by law, they are compelled

into invariable orbits; by law, their periodic times become permanent; by law, higher adjustments are brought about, until the new creation becomes perfected, and fully prepared for higher and more God-like purposes, than merely to traverse the wilds of space. By law, it is studded with vegetation, and filled with animal life, and peopled with moral and intelligent beings, who also by law may attain to immortality and endless life. All these laws came from the Great Supreme Architect of the universe—the Governor of all worlds, who speaks, and eternity is filled with his voice, who ordains, and the immensity of creation obeys.

128. Time is an essential element in the workings of all law. Creation, or more definitely, an organization of the eternal elements, requires time. The great Creator may design a new world to be added to our solar system. The materials may not be found in sufficient quantity in the vicinity of our system, having been exhausted in the formation of the worlds already within its bounds. But he beholds an infinity of matter, existing in the inter-stellar spaces, in the far off depths of eternity. He beholds these nebulous patches in all of their evolutions: he perceives that the vast revolution of some particular part will eventually bring it within the sphere of the attraction of our system. The orbit in which it is caught may be of a hyperbolic nature; it wheels its course with an accelerated speed towards the perihelion of its orbit: it passes this point with inconceivable velocity: it departs for depths unknown: in its wild flight, it passes far beyond the uttermost planet of our system: but the firm grasp of the resisting force has not been loosened: its prey, as if wearied with unusual exertion, yields by degrees to the victorious captor; wheels itself into an elliptic orbit, with its future destination fixed and definite. All this has required time: ages innumerable may have intervened, in the grand revolutions of nebulous matter, before the detachment of this cometary portion, under the potent influence of the solar energy. Long ages may intervene, before this captured nebulous matter, shall be fully subdued to an orbit of an invariable character. Thus the grand work of creation has its definite periods. It has a period for collecting the eternal

elements in a nebulous condition : it has a period of revolution, until, in fortunate circumstances, it is detached in a cometary form ; it has a period of cometary revolution, until compelled to assume a planetary orbit, and take its position among inhabited worlds ; it has a period adapted to vegetable and animal life ; and, finally, it will have a period in its highest, most perfect and glorified condition, as the eternal abode of immortal, glorified beings.

129. The greatest and most important problem in Celestial Mechanics is to determine with mathematical certainty, the dimension and position of the invariable orbit, which must be assumed by each cometary body : or, in mathematical language, Given the elements of any hyperbolic, parabolic, or elliptic orbit, to find the dimension and position of its future invariable orbit. When this problem is properly solved, we shall be in a condition to say, whether any given comet will become a primary planet or a satellite ; if a planet, whether its orbit will be interior or exterior to that of the earth ; if a satellite, which planet will be favored with this new neighbor, to cheer up and gladden its nights.

130. In our investigations thus far, we have said but little about other systems. It has been our intention to exhibit the results of the new theory in our own system ; if it works well at home, we may venture to extend its operations to other domains. Indeed, from the very nature of the ethereal medium we are obliged to admit that it is governed by the same laws in other systems as in this. Therefore, it must gravitate toward the centers of all worlds ; the velocity of the gravitating force is undoubtedly the same as in our system ; hence, the aberrating forces must be the same ; the density of the medium must follow the same law ; the resisting properties must produce the same results : therefore, when we have learned the wonderful workings of this part of the universal machinery, we shall have gained a knowledge of the great fundamental principles which characterize the whole. Hence, it is unnecessary to launch forth into unfathomable depths, in search of information which may be more easily learned at home. Study our own system, which, in most respects, is a beautiful sample of many others. Study

its mechanism,—its laws,—its tremendous forces,—its skillful adjustments,—its wonderful revolutions and axial rotations. Study out of this great book of nature whose pages are open and directly before us; and we shall be better prepared to mount upwards into the lofty heights of eternity, and gaze upon the untold wonders of infinity.

MISCELLANEOUS EXAMPLES, ILLUSTRATING THE NEW THEORY.

131. *Example 1.*—When the velocity of the wind at the sea level is one mile per hour, or $\cdot 0002\frac{1}{2}$ of a mile per second, it imparts a perpendicular pressure of $\cdot 00492\frac{1}{5}$ avoirdupois on one square foot. With what velocity must the earth move in its orbit, in such a medium, to meet with the same resistance which it encounters at its mean distance, by the resistance of the ethereal medium, the velocity of the earth in ether, being $18\cdot 2\cdot 03590512$ miles per second, and its ethereal resistance being equal to the aberrating force in a circular orbit? Also when the velocities are the same, what will be the exact ratio of the resistance of the air to that of ether? (For the resistance of the ether, when equal to the aberrating force, see Paragraph 45.)

Example 2.—If the specific gravity of gases vary as the resistances which they offer to bodies passing through them, and one cubic foot of air weighs $\cdot 07670839\frac{1}{5}$ avoirdupois, what will be the specific gravity of the ethereal medium, compared with that of the air at sea level, at the mean distance of the earth in its orbit? Or, in other words, what will one cubic foot of ether weigh?

Example 3.—If the distance of the earth from the Sun be 91430000 miles; and the solar semi-diameter be 430000 miles, and the sun's mass = 314760 times that of the earth; and if the density of the ethereal medium varies as the inverse cube of the square root of the Sun's distance, what will a cubic foot of the ethereal medium weigh at the Sun's surface?

Example 4. = What must be the height of a stratum of the ethereal medium, next to the surface of the Sun, to exert the

same pressure upon the solar surface, as our atmosphere does upon the earth's surface, namely, 2120.04lbs. per square foot?

132. The four preceding examples, are propounded on the supposition, that the ethereal medium exerts its resistance, like other gaseous media, on the surface of bodies passing through it: but, according to the new theory, such is not the fact: its resistance is in proportion to the masses of bodies, whatever may be the form of their surfaces: therefore it is impossible to deduce the specific gravity of the ethereal medium, by a comparison of mass-resistance with resistances depending on surface only. (See Paragraphs 78, 79 and 80.) For instance, a cubic foot of the earth, at its mean density, passing through air, having the same density as at sea-level, with a velocity equal to that of the earth at its mean distance from the Sun, — will generate a resistance equal to 21129256.7024lbs. Whatever may be the amount of matter added the resistance remains the same, provided that only one square foot of the surface is exposed to the resistance. The same cubic foot of the earth when made to pass through the ethereal medium with the same velocity meets with a resistance equal to .000020245444lb. If the density of the cubic foot were doubled or tribled, its resistance would be doubled or tribled: if its form were altered into any shape whatsoever, it would have no influence in diminishing or increasing the resistance. Therefore, it is impossible to deduce the specific gravity of the ether, as supposed in the four examples of the preceding Paragraph.

133. Notwithstanding the discordance existing between mass-resistance and surface resistance, when the quantity varies, yet there is much importance attached to a careful comparison, when the masses are the same; for by such means we are able to point out the intensity of the forces acting, whatever may be their mode of action: this is clearly illustrated in the instance given in the last Paragraph. It is instructive to compare the intensity of the ethereal resistance with that of the air, and thus learn something of the nature of the forces, entering so extensively into the mechanical movements of the universe.

134. *Example 1.*—To find a general formula, or cubic

equation with known co-efficients, one of the roots of which shall be the value of the radius vector in any given elliptic orbit, where the resisting and aberrating forces are equal.

Let the mean distance of the earth from the Sun,—the orbital velocity,—the intensity of gravity,—and the resistance, be each equal to unity or one.

Let a = the semi-major axis of the given elliptic orbit ;

c = the semi-conjugate axis ;

v = the velocity of a planet or comet, at the mean distance in the given ellipse ;

V = the aberrating velocity at the mean distance in the ellipse ;

r = the resistance at the distance a ;

R' = the resistance at any point in the elliptic orbit ;

F = the aberrating force at the mean distance ;

F' = the aberrating force at any point in the given ellipse ;

g = the intensity of gravity at mean distance in the ellipse ;

x = the required radius vector ;

$2a - x$ = the distance of the required point from the upper focus of the ellipse.

To find F' .

we have (Par. 24)

$$1 : \frac{1}{\sqrt{a}} :: 1 : v; \text{ or } v = \frac{1}{\sqrt{a}};$$

by the law of gravity

$$1 : \frac{1}{a^2} :: 1 : g; \text{ or } g = \frac{1}{a^2};$$

by Par. 73, we have

$$a : c :: v : V :: \frac{1}{\sqrt{a}} : V; \text{ or } V = \frac{c}{\sqrt{a^3}};$$

by Par. 77, General Theorem

$$Vg = \frac{c}{\sqrt{a^7}} = F;$$

but (Par. 72) we have

$$F : F' :: \frac{1}{a^3} : \frac{1}{x^3}; \text{ or } \frac{c}{\sqrt{a^7}} : F' :: \frac{1}{a^3} : \frac{1}{x^3}$$

or

$$F' = \frac{c}{\sqrt{a}} \frac{1}{x^3} \dots\dots\dots(1)$$

To find R' .

by Par. 87, we have

$$1 : r :: 1 : \frac{1}{\sqrt{a^5}}; \text{ hence, } r = \frac{1}{\sqrt{a^5}}$$

by Par. 89, we have

$$r : R' :: \frac{2a - a}{\sqrt{a^5}} : \frac{2a - x}{\sqrt{x^5}};$$

hence

$$R' = r \sqrt{a^3} \cdot \frac{2a - x}{\sqrt{x^5}} \dots\dots\dots(2)$$

Substituting the value of r , and reducing, we have

$$R' = \frac{1}{a} \cdot \frac{2a - x}{\sqrt{x^5}} \dots\dots\dots(3)$$

Equating (1) and (3) we have

$$\frac{c}{\sqrt{a}} \cdot \frac{1}{x^3} = \frac{1}{a} \cdot \frac{2a - x}{\sqrt{x^5}}$$

Reducing we obtain

$$x^3 - 4ax^2 + 4a^2x - ac^2 = 0 \dots\dots\dots(4)$$

As a and c are both known quantities, the numerical value of x can easily be found. As a particular case for this general theorem, let $a = 1$, as assumed for the semi-major axis of the earth's orbit, and we obtain the radius vector x , as found in Par. 91, Ex. 1. Also Par. 90.

Example 2.—What is the length of the radius vector in the orbit of Encke's comet where the two forces balance each other?

- Let a = semi-major axis = 209500000 miles;
- let c = semi-conjugate axis = 111282523 miles;
- let x = the radius vector required.

Then we shall have $x^3 - 4ax^2 + 4a^2x - ac^2 = 0$; one of the values of x in this equation is the required distance.

$$x = 330384629 \text{ miles.}$$

Lines drawn from the Sun, representing the other two values of x , do not terminate on the elliptic curve, one being less than the perihelion distance, and the other greater than the aphelion distance.

135. *Example 1.*—The earth's sidereal rotation being accomplished in $23h\ 56m\ 4.09s$, what will be the equatorial semi-diameter of its ethereal envelope?

Answer, 26254 miles.

Example 2.—How far from the center of the earth, measured in the direction of her axis prolonged, will the two polar wings of her envelope have one half the thickness of the equatorial ethereal diameter; the thickness being measured in a line perpendicular to the prolonged axis?

Answer, 104192½ miles.

The solutions of the last two problems depend upon the supposition, that the envelope form is not changed, or in the

least degree altered, by the retarding pressure of exterior ethereal currents.

Example 3.—What is the velocity of rotation per second, (if not retarded by exterior currents,) of the equatorial surface of the solar envelope?

Answer, 44·50167 miles.

Example 4.—If the two opposite forces in Venus' orbit become equal in intensity, only when the orbit becomes circular, (excluding the action of ethereal currents,) what will be the diminution of her semi-major axis in one revolution? And what will be the diminution of her year in one thousand revolutions?

Answer, $\left\{ \begin{array}{l} \text{Dimin. semi-axis in one rev.} = 9\cdot0288 \text{ feet.} \\ \text{Dimin. yr. in 1000 rev.} = \frac{3}{4} \text{ of one second.} \end{array} \right.$

Example 5.—As Venus proceeds from the aphelion to the perihelion of her orbit, what time intervenes, between her arrival at the point where the aberrating and resisting forces balance, and at the point of her mean distance from the Sun?

Answer, 5h 51m 38s.

Example 6.—If there are no ethereal currents around the Sun from west to east, and if the invariable orbit of Mercury must be circular, how much will the semi-major axis be diminished in one revolution? and consequently, how much would be the diminution of the period?

Answer, $\left\{ \begin{array}{l} \text{Dimin. Semi-axis in one rev.} = 98\cdot86 \text{ miles.} \\ \text{Diminished Period in one rev.} = 31\cdot85s. \end{array} \right.$

Example 7.—What will be the value of each of the two forces in Mercury's orbit, when they are equal? and what the length of the radius vector?

Answer, $\left\{ \begin{array}{l} F = R = 9\cdot313127509. \\ \text{Rad. vec.} = \cdot40285102867405. \end{array} \right.$

Example 8.—What must be the velocity of the rotation of the ethereal current around the Sun, at Mercury's mean distance, to render the present elements of the orbit invariable?

Answer, 2254 feet and 7 inches per second, which is nearly twice the velocity of sound in the air.

Example 9.—What must be the velocity of the rotation of the ethereal current around the Sun, at the earth's mean distance, to render the present elements of her orbit invariable?

Answer, 4 feet and $6\frac{2}{3}$ inches per second.

PROBLEM NOT YET SOLVED.

Example 10.—Given the elements of a comet, in a hyperbolic orbit, to determine the dimension and position of its future invariable orbit, provided that the two forces become equal in circular orbits only, and that the ethereal currents vary in velocity as the inverse square roots of the distance from the Sun, and that the velocity of the current, at the mean distance of the earth's orbit, is 4 feet 7 inches per second.



APPENDIX.

TABLE I.—EARTH.

1854.	(Par. 134, Ex. 1.)	(Par. 134, Ex. 1.)
JULY.	Formula (1), $a = 1.$	Formula (3), $a = 1.$
Noon.	F'	F'
1	·9 5 1 2 1 5 6 5	·9 4 3 2 0 8 7 0
2	·9 5 1 2 2 4 2 2	·9 4 3 2 1 8 8 0
3	·9 5 1 2 4 6 5 2	·9 4 3 2 4 5 3 3
4	·9 5 1 2 8 1 9 6	·9 4 3 2 8 6 6 3
5	·9 5 1 3 3 0 0 0	·9 4 3 3 4 2 3 9
6	·9 5 1 3 8 9 1 3	·9 4 3 4 0 3 0 4
7	·9 5 1 4 5 8 7 0	·9 4 3 4 9 3 0 4
8	·9 5 1 5 3 9 7 8	·9 4 3 5 8 7 2 8
9	·9 5 1 6 2 8 4 4	·9 4 3 6 9 1 0 9
10	·9 5 1 7 2 6 3 0	·9 4 3 8 0 5 3 3
11	·9 5 1 8 3 4 7 8	·9 4 3 9 3 2 1 7
12	·9 5 1 9 5 2 6 1	·9 4 4 0 6 9 8 9
13	·9 5 2 0 8 0 8 7	·9 4 4 2 1 9 5 7
14	·9 5 2 2 1 8 9 1	·9 4 4 3 8 0 8 7
15	·9 5 2 3 6 7 6 1	·9 4 4 5 5 4 3 5
16	·9 5 2 5 2 7 3 9	·9 4 4 7 4 1 2 0
17	·9 5 2 6 9 9 7 8	·9 4 4 9 4 5 1 1
18	·9 5 2 8 8 5 4 3	·9 4 5 1 5 9 6 7
19	·9 5 3 0 8 4 2 2	·9 4 5 3 9 1 8 5
20	·9 5 3 2 9 7 6 1	·9 4 5 6 4 0 9 8
21	·9 5 3 5 2 6 7 4	·9 4 5 9 0 8 5 9
22	·9 5 3 7 7 1 1 1	·9 4 6 1 9 4 0 0
23	·9 5 4 0 3 1 5 2	·9 4 6 5 0 1 2 0
24	·9 5 4 3 0 8 2 6	·9 4 6 8 2 1 4 1
25	·9 5 4 6 0 1 7 8	·9 4 7 1 4 2 1 7
26	·9 5 4 9 1 1 0 9	·9 4 7 5 2 5 5 4
27	·9 5 5 2 3 8 2 2	·9 4 7 9 0 7 7 2
28	·9 5 5 5 8 1 3 3	·9 4 8 3 0 8 5 6
29	·9 5 5 9 4 0 0 0	·9 4 8 7 2 7 1 1
30	·9 5 6 3 1 2 4 4	·9 4 9 1 6 2 3 9
31	·9 5 6 6 9 9 5 7	·9 4 9 6 1 4 5 7
	29·5 4 3 9 1 1 9 7	29·3 0 5 1 3 0 5 5

TABLE I.—EARTH.

1854.	(Par. 134, Ex. 1.	(Par. 134, Ex. 1.
Aug.	Formula (1), $a = 1.$)	Formula (3), $a = 1.$)
Noon.	F'	R'
	59·5 4 3 9 1 1 9 7	29·3 0 5 1 3 0 5 5
1	·9 5 7 0 9 9 5 6	·9 5 0 0 8 1 6 7
2	·9 5 7 5 1 1 5 2	·9 5 0 5 6 3 0 4
3	·9 5 7 9 3 4 8 9	·9 5 1 0 5 7 3 9
4	·9 5 8 3 6 9 1 1	·9 5 1 5 6 4 4 4
5	·2 5 8 8 1 2 6 7	·9 5 2 0 0 2 6 1
6	·9 5 9 2 6 5 2 2	·9 5 2 6 1 0 9 8
7	·9 5 9 7 2 6 5 2	·9 5 3 1 4 9 6 7
8	·9 6 0 1 9 4 0 0	·9 5 3 6 9 5 8 7
9	·9 6 0 6 6 8 9 1	·9 5 4 2 5 0 4 4
10	·9 6 1 1 5 2 2 2	·9 5 4 8 1 4 8 9
11	·9 6 1 6 4 3 1 1	·9 5 5 3 8 8 1 5
12	·9 6 2 1 4 1 3 3	·9 5 5 9 7 0 1 1
13	·9 6 2 6 4 6 4 4	·9 5 6 5 6 2 3 9
14	·9 6 3 1 6 3 5 6	·9 5 7 1 6 3 4 8
15	·9 6 3 6 8 8 6 7	·9 5 7 7 7 6 6 3
16	·9 6 4 2 2 3 3 3	·9 5 8 4 0 2 2 2
17	·9 6 4 7 6 8 8 9	·9 5 9 0 3 8 3 3
18	·9 6 5 3 2 6 2 2	·9 5 9 6 8 9 2 4
19	·9 6 5 8 9 5 1 1	·9 6 0 3 5 3 4 8
20	·9 6 6 4 7 7 7 8	·9 6 1 0 3 3 8 0
21	·9 6 7 0 7 3 5 6	·9 6 1 7 2 9 6 7
22	·9 6 7 6 8 2 2 2	·9 6 2 4 4 0 4 4
23	·9 6 8 3 0 4 2 2	·9 6 3 1 6 6 8 9
24	·9 6 8 9 3 9 5 6	·9 6 3 9 0 8 8 9
25	·9 6 9 5 8 8 6 7	·9 6 4 6 6 6 5 6
26	·9 7 0 2 5 1 8 2	·9 6 5 4 4 1 0 0
27	·9 7 0 9 2 7 1 1	·9 6 6 2 2 9 5 6
28	·9 7 1 6 1 3 4 1	·9 6 7 0 3 0 5 6
29	·9 7 2 3 1 0 9 1	·9 6 7 8 4 5 1 1
30	·9 7 3 0 1 8 4 4	·9 6 8 6 7 1 2 2
31	·9 7 3 7 3 4 5 5	·9 6 9 5 0 7 3 3
	59·4 4 8 0 6 5 5 0	59·0 3 1 0 1 6 6 1

TABLE I. — EARTH.

1854.	(Par. 134, Ex. 1.)	(Par. 134, Ex. 1.)
SEPT.	Formula (1), $a = 1.$)	Formula (3), $a = 1.$)
Noon.	F'	I'
	59·4 4 8 0 6 5 5 0	59·0 3 1 0 1 6 6 1
1	·9 7 4 4 5 8 6 4	·9 7 0 3 5 2 8 9
2	·9 7 5 1 8 9 1 1	·9 7 1 2 0 5 5 7
3	·9 7 5 9 2 5 7 8	·9 7 2 0 6 5 4 5
4	·9 7 6 6 6 8 4 1	·9 7 2 9 3 2 3 3
5	·9 7 7 4 1 4 7 7	·9 7 3 8 0 3 9 8
6	·9 7 8 1 6 5 2 3	·9 7 4 6 7 9 5 6
7	·9 7 8 9 1 7 5 0	·9 7 5 5 5 8 3 3
8	·9 7 9 6 7 2 5 0	·9 7 6 4 3 9 4 4
9	·9 8 0 4 3 1 3 6	·9 7 7 3 2 5 2 3
10	·9 8 1 1 9 2 9 5	·9 7 8 2 1 4 5 5
11	·9 8 1 9 5 8 4 1	·9 7 9 1 0 8 0 7
12	·9 8 2 7 2 7 2 7	·9 8 0 0 0 5 4 5
13	·9 8 3 4 9 9 3 3	·9 8 0 9 0 7 0 0
14	·9 8 4 2 7 7 0 4	·9 8 1 8 1 4 2 0
15	·9 8 5 0 5 9 0 9	·9 8 2 7 2 7 3 9
16	·9 8 5 8 4 8 1 8	·9 8 3 6 4 8 1 8
17	·9 8 6 6 4 3 1 8	·9 8 4 5 7 6 2 5
18	·9 8 7 4 4 5 0 0	·9 8 5 5 1 2 1 6
19	·9 8 8 2 5 5 6 8	·9 8 6 4 5 8 3 0
20	·9 8 9 0 7 3 8 6	·9 8 7 4 1 3 3 0
21	·9 8 9 9 0 0 2 3	·9 8 8 3 7 7 6 7
22	·9 9 0 7 3 4 0 9	·9 8 9 3 5 0 8 1
23	·9 9 1 5 7 6 1 4	·9 9 0 3 3 3 7 5
24	·9 9 2 4 2 5 2 3	·9 9 1 3 2 4 8 8
25	·9 9 3 2 8 3 1 8	·9 9 2 3 2 6 0 2
26	·9 9 4 1 4 6 9 8	·9 9 3 3 3 3 6 0
27	·9 9 5 0 1 5 2 3	·9 9 4 3 4 7 1 6
28	·9 9 5 8 8 8 4 1	·9 9 5 3 6 6 4 0
29	·9 9 6 7 6 4 5 5	·9 9 6 3 8 8 6 4
30	·9 9 7 6 4 3 4 9	·9 9 7 4 1 4 3 2
	89·0 1 8 2 6 6 3 2	88·5 3 4 3 2 7 4 9

TABLE I. — EARTH.

1854.	(Par. 134, Ex. 1.	(Par. 134, Ex. 1.
Oct.	Formula (1), $a=1$.)	Formula (3), $a = 1$.)
Noon.	F'	R'
	89·0 1 8 2 6 6 3 2	88·5 3 4 3 2 7 4 9
1	·9 9 8 5 2 2 5 6	·9 9 8 4 3 9 8 9
2	·9 9 9 4 0 1 8 6	·9 9 9 4 6 6 0 2
	Mean distance October 2d 12h 30m 16s.	Mean distance October 2d 12h 30m 16s.
3	1·0 0 0 2 8 0 1 8	1·0 0 0 4 9 0 9 0
4	1·0 0 1 1 5 7 5 1	1·0 0 1 5 1 4 3 2
5	1·0 0 2 0 3 2 0 3	1·0 0 2 5 3 4 6 4
6	1·0 0 2 9 0 3 2 3	1·0 0 3 5 5 1 0 4
7	1·0 0 3 7 6 9 5 1	1·0 0 4 5 6 1 8 9
8	1·0 0 4 6 3 2 4 1	1·0 0 5 5 6 8 7 5
9	1·0 0 5 4 9 1 9 0	1·0 0 6 5 7 1 6 9
10	1·0 0 6 3 4 7 3 3	1·0 0 7 5 6 9 7 2
11	1·0 0 7 1 9 9 3 0	1·0 0 8 5 6 3 5 7
12	1·0 0 8 0 4 7 8 0	1·0 0 9 5 5 3 3 7
13	1·0 0 8 8 9 2 7 9	1·0 1 0 5 3 9 0 7
14	1·0 0 9 7 3 5 5 8	1·0 1 1 5 2 2 2 6
15	1·0 1 0 5 7 7 2 1	1·0 1 2 5 0 4 0 8
16	1·0 1 1 4 1 7 4 4	1·0 1 3 4 8 4 1 5
17	1·0 1 2 2 5 7 5 8	1·0 1 4 4 6 4 1 0
18	1·0 1 3 0 9 6 9 7	1·0 1 5 4 4 3 3 3
19	1·0 1 3 9 3 8 4 6	1·0 1 6 4 2 4 8 2
20	1·0 1 4 7 8 0 6 1	1·0 1 7 4 0 7 4 9
21	1·0 1 5 6 2 3 6 0	1·0 1 8 3 9 0 6 3
22	1·0 1 6 4 6 6 5 1	1·0 1 9 3 7 3 8 3
23	1·0 1 7 3 0 9 6 0	1·0 2 0 3 5 7 0 6
24	1·0 1 8 1 5 2 4 6	1·0 2 1 3 3 9 9 1
25	1·0 1 8 9 9 5 3 1	1·0 2 2 3 2 3 1 1
26	1·0 1 9 8 3 6 1 5	1·0 2 3 3 0 3 7 7
27	1·0 2 0 6 7 3 4 1	1·0 2 4 2 7 9 9 5
28	1·0 2 1 5 0 7 0 6	1·0 2 5 2 5 2 3 6
29	1·0 2 2 3 3 5 8 5	1·0 2 6 2 1 8 9 1
30	1·0 2 3 1 5 8 2 5	1·0 2 7 1 7 8 0 1
31	1·0 2 3 9 7 2 7 1	1·0 2 8 1 2 7 7 3
	120·3 7 0 7 7 9 4 9	119·9 5 0 6 4 7 8 6

TABLE I.—EARTH.

1854.	(Par. 134, Ex. 1.	(Par. 134, Ex. 1.
Nov.	Formula (1), $a = 1.$)	Formula (3), $a = 1.$)
Noon.	F'	R'
	120·3 7 0 7 7 9 4 9	119·9 5 0 6 4 7 8 6
1	1·0 2 4 7 7 8 7 7	1·0 2 9 0 6 7 6 5
2	1·0 2 5 5 7 6 1 2	1·0 2 9 9 9 7 5 1
3	1·0 2 6 3 6 2 8 8	1·0 3 0 9 1 5 0 5
4	1·0 2 7 1 3 9 8 4	1·0 3 1 8 2 0 4 3
5	1·0 2 7 9 0 4 5 0	1·0 3 2 7 1 2 6 2
6	1·0 2 8 6 5 6 8 7	1·0 3 3 5 8 9 8 8
7	1·0 2 9 3 9 6 6 8	1·0 3 4 4 5 2 6 2
8	1·0 3 0 1 2 5 8 9	1·0 3 5 3 0 2 7 4
9	1·0 3 0 8 4 2 7 5	1·0 3 6 1 3 8 4 2
10	1·0 3 1 5 4 8 6 9	1·0 3 6 9 6 1 4 6
11	1·0 3 2 2 4 3 5 7	1·0 3 7 7 7 1 8 4
12	1·0 3 2 9 2 7 6 2	1·0 3 8 5 6 9 2 6
13	1·0 3 3 6 0 2 1 4	1·0 3 9 3 5 5 9 8
14	1·0 3 4 2 6 7 1 4	1·0 4 0 1 3 1 2 9
15	1·0 3 4 9 2 3 8 7	1·0 4 0 8 9 6 6 4
16	1·0 3 5 5 7 2 5 5	1·0 4 1 6 5 2 8 8
17	1·0 3 6 2 1 3 6 0	1·0 4 2 4 0 0 6 0
18	1·0 3 6 8 4 2 2 4	1·0 4 3 1 4 0 7 7
19	1·0 3 7 4 7 7 0 9	1·0 4 3 8 7 2 9 6
20	1·0 3 8 0 9 8 5 6	1·0 4 4 5 9 7 8 4
21	1·0 3 8 7 1 2 6 8	1·0 4 5 3 1 3 2 5
22	1·0 3 9 3 1 8 4 2	1·0 4 6 0 1 9 5 2
23	1·0 3 9 9 1 6 5 5	1·0 4 6 7 1 6 9 9
24	1·0 4 0 5 0 7 1 9	1·0 4 7 4 0 5 1 9
25	1·0 4 1 0 8 8 2 5	1·0 4 8 0 8 2 4 9
26	1·0 4 1 6 5 8 5 1	1·0 4 8 7 4 7 5 8
27	1·0 4 2 2 1 7 0 7	1·0 4 9 3 9 8 5 5
28	1·0 4 2 7 6 2 8 3	1·0 5 0 0 3 4 7 8
29	1·0 4 3 2 9 5 4 3	1·0 5 0 6 5 5 5 7
30	1·0 4 3 8 1 1 5 4	1·0 5 1 2 5 6 7 8
	151·4 1 8 5 6 9 3 3	151·1 7 7 6 2 7 0 0

TABLE I.—EARTH.

1854.	(Par. 134, Ex. 1.)	(Par. 134, Ex. 1.)
DEC.	Formula (1), $a = 1.$	Formula (3), $a = 1.$
Noon.	F'	R'
	151·41856933	151·17762700
1	1·04431200	1·05184007
2	1·04479615	1·05240484
3	1·04526322	1·05294903
4	1·04571253	1·05347269
5	1·04614386	1·05397549
6	1·04655518	1·05445449
7	1·04694843	1·05491323
8	1·047332458	1·05535109
9	1·047668213	1·05576808
10	1·04802295	1·05616553
11	1·04834734	1·05654355
12	1·04865507	1·05690231
13	1·04894783	1·05724329
14	1·04922609	1·05756768
15	1·04949080	1·05787610
16	1·04974155	1·05816837
17	1·04998019	1·05844659
18	1·05020797	1·05871183
19	1·05042415	1·05896387
20	1·05062881	1·05920232
21	1·05082107	1·05942634
22	1·05100121	1·05963634
23	1·05116812	1·05983098
24	1·05132349	1·06001174
25	1·05146441	1·06017585
26	1·05159080	1·06032323
27	1·05170121	1·06045195
28	1·05179492	1·06056125
29	1·05187119	1·06065000
	Sums of the two forces equal 29d 0h 16m 3ss. 4	Sums of the two forces equal 29d 0h 16m 3ss. 4
30	1·05192791	1·06071614
	Perihelion 30d 14h 2m.	Perihelion 30d 14h 2m.
	1·05193000	1·06072000
	183·94357449	183·96105127

TABLE II.—VENUS.

1854. Greenwich Time. Noon.	<i>Aberrating Force.</i> (that in a circular orbit at the Earth's mean distance = 1.) $F' = \frac{c}{\sqrt{a}} \frac{1}{x^3}.$ For the values of $a, c, x,$ see Par. 134, Ex. 1.	<i>Resisting Force.</i> (that at the Earth's mean distance = 1.) $R' = r \sqrt{a^3} \frac{2a-x}{\sqrt{x^5}}.$ For the values of $a, c, x,$ see Par. 134, Ex. 1.
June 1d 14h 6m Aphelion.	2.20185533	2.19432551
2	2.20186599	2.19433435
3	2.20189797	2.19437522
4	2.20196497	2.19445302
5	2.20206548	2.19457019
6	2.20220051	2.19472857
7	2.20236954	2.19492530
8	2.20257462	2.19516312
9	2.20281066	2.19544015
10	2.20308173	2.19575655
11	2.20338782	2.19611356
12	2.20372538	2.19650832
13	2.20409695	2.19694114
14	2.20450203	2.19741476
15	2.20493909	2.19792498
16	2.20540812	2.19847224
17	2.20590964	2.19905730
18	2.20644315	2.19967958
19	2.20700711	2.20033831
20	2.20760305	2.20103433
21	2.20822843	2.20176360
22	2.20888426	2.20252946
23	2.20957107	2.20333114
24	2.21028571	2.20416496
25	2.21102944	2.20503273
26	2.21180051	2.20593347
27	2.21260153	2.20686749
28	2.21342857	2.20783328
	61.76153866	61.56901263

TABLE II.—VENUS.

1854. Greenwich Time. Noon.	<i>Aberrating Force.</i> (that in a circular orbit at the Earth's mean distance = 1.) $F' = \frac{c}{\sqrt{a}} \cdot \frac{1}{x^3}.$ For the values of $a, c, x,$ see Par. 134, Ex. 1.	<i>Resisting Force.</i> (that at the Earth's mean distance = 1.) $R' = r \sqrt{a^3} \frac{2a-x}{\sqrt{x^5}}.$ For the values of $a, r, x,$ see Par. 134, Ex. 1.)
June 29	6 1.7 6 1 5 3 8 6 6	6 1.5 6 9 0 1 2 6 3
“ 30	2.2 1 4 2 8 1 7 3	2.2 0 8 8 2 9 1 7
July 1	2.2 1 5 1 6 0 2 0	2.2 0 9 8 5 3 8 0
2	2.2 1 6 0 6 4 2 8	2.2 1 0 9 0 9 4 3
3	2.2 1 6 9 9 3 8 8	2.2 1 1 9 9 4 6 8
4	2.2 1 7 9 4 6 6 7	2.2 1 3 1 0 5 8 2
5	2.2 1 8 9 2 2 9 6	2.2 1 4 2 4 5 5 2
6	2.2 1 9 9 2 0 9 2	2.2 1 5 4 0 9 9 3
7	2.2 2 0 9 4 0 8 2	2.2 1 6 6 0 0 5 0
8	2.2 2 1 9 8 2 6 5	2.2 1 7 8 1 6 0 6
9	2.2 2 3 0 4 3 5 9	2.2 1 9 0 5 4 0 7
10	2.2 2 4 1 2 5 1 3	2.2 2 0 3 1 6 3 8
11	2.2 2 5 2 2 5 1 3	2.2 2 1 5 9 9 9 8
12	2.2 2 6 3 4 1 5 4	2.2 2 2 9 0 4 8 2
13	2.2 2 7 4 7 8 4 6	2.2 2 4 2 2 9 8 1
14	2.2 2 8 6 2 9 7 4	2.2 2 5 5 7 0 5 0
15	2.2 2 9 7 9 6 9 2	2.2 2 6 9 3 5 1 0
16	2.2 3 0 9 7 8 4 1	2.2 2 8 3 1 3 8 9
17	2.2 3 2 1 7 3 3 3	2.2 2 9 7 0 8 3 1
18	2.2 3 3 3 8 0 9 3	2.2 3 1 1 1 7 4 1
19	2.2 3 4 6 0 1 5 5	2.2 3 2 5 4 1 4 8
20	2.2 3 5 8 3 1 9 6	2.2 3 3 9 7 7 4 1
21	2.2 3 7 0 7 2 6 8	2.2 3 5 4 2 4 8 4
22	2.2 3 8 3 2 3 2 0	2.2 3 6 8 8 3 9 9
23	2.2 3 9 5 8 1 9 6	2.2 3 8 3 5 3 1 3
24	2.2 4 0 8 4 7 9 4	2.2 3 9 8 3 0 3 9
25	2.2 4 2 1 1 9 1 7	2.2 4 1 3 1 3 1 1
26	2.2 4 3 3 9 7 4 1	2.3 4 2 8 0 4 6 1
	2.2 4 4 6 1 7 7 2	2.2 4 4 2 9 8 4 2
	124.1 6 1 3 1 9 5 4	123.8 8 2 9 5 5 1 9

TABLE II. — VENUS.

1854. Greenwich Time. Noon.	<i>Aberrating Force.</i>	<i>Resisting Force.</i>
	(that in a circular orbit at the Earth's mean distance = 1.)	(that at the Earth's mean distance = 1.)
	$F' = \frac{c}{\sqrt{a}} \cdot \frac{1}{x^3}$	$R' = r \sqrt{a^3} \frac{2a-x}{\sqrt{x^5}}$
	For the values of $a, c, x,$ see Par. 134, Ex. 1	For the values of $a, r, x,$ see Par. 134, Ex. 1.
July 27	124·1 6 1 3 1 9 5 4	123·8 8 2 9 5 5 1 9
$F' = R'$	2·2 4 5 9 6 1 8 6	2·2 4 5 7 9 6 3 6
	2·2 4 6 9 5 1 3 5 8	2·2 4 6 9 5 1 3 5 8
28	2·2 4 7 2 4 8 4 5 Mean distance 2 ^{sd} 0h 19m 6s.	2·2 4 7 2 9 7 6 6 Mean distance 2 ^{sd} 0h 19m 6s.
29	2·2 4 8 5 3 5 7 5	2·2 4 8 7 9 9 4 6
30	2·2 4 9 8 2 2 2 8	2·2 5 0 3 0 0 4 9
31	2·2 5 1 1 0 9 3 3	2·2 5 1 8 0 2 0 6
Aug. 1	2·2 5 2 3 9 4 3 0	2·2 5 3 3 0 1 0 2
2	2·2 5 3 6 7 6 6 8	2·2 5 4 7 9 7 3 6
3	2·2 5 4 9 5 5 2 1	2·2 5 6 2 8 8 7 8
4	2·2 5 6 2 2 8 1 2	2·2 5 7 7 7 3 5 4
5	2·2 5 7 4 9 5 3 4	2·2 5 9 2 5 1 8 0
6	2·2 5 8 7 5 7 2 9	2·2 6 0 7 2 4 4 6
7	2·2 6 0 0 1 0 8 8	2·2 6 2 1 8 6 3 9
8	2·2 6 1 2 5 5 2 1	2·2 6 3 6 3 8 0 0
9	2·2 6 2 4 9 1 1 5	2·2 6 5 0 7 9 9 2
10	2·2 6 3 7 1 5 1 0	2·2 6 6 5 0 7 7 7
11	2·2 6 4 9 2 7 6 1	2·2 6 7 9 2 1 4 8
12	2·2 6 6 1 2 6 1 8	2·2 6 9 3 2 0 0 4
13	2·2 6 7 3 1 1 5 2	2·2 7 0 7 0 2 5 8
14	2·2 6 8 4 8 3 3 3	2·2 7 2 0 6 9 6 1
15	2·2 6 9 6 3 6 9 8	2·2 7 3 4 1 4 6 5
16	2·2 7 0 7 7 5 3 9	2·2 7 4 7 4 2 9 8
17	2·2 7 1 8 9 5 8 1	2·2 7 6 0 4 9 4 6
18	2·2 7 2 9 9 7 9 1	2·2 7 7 3 3 5 0 6
19	2·2 7 4 0 7 9 5 8	2·2 7 8 5 9 7 1 0
20	2·2 7 5 1 4 1 8 8	2·2 7 9 8 3 6 3 8
21	2·2 7 6 1 8 2 7 2	2·2 8 1 0 4 9 7 1
22	2·2 7 7 2 0 0 0 0	2·2 8 2 2 3 6 4 3
23	2·2 7 8 1 9 4 7 4	2·2 8 3 3 9 6 2 9
	187·5 1 7 9 3 0 1 4	187·3 1 3 1 7 2 0 3

TABLE II. — VENUS.

1854. Greenwich Time. Noon.	<i>Aberrating Force.</i> <small>(that in a circular orbit at the Earth's mean distance = 1.)</small> $F' = \frac{c}{\sqrt{a}} - \frac{1}{x^3}.$ For the values of $a, c, x,$ see Par. 134, Ex. 1.	<i>Resisting Force.</i> <small>(that at the Earth's mean distance = 1.)</small> $R' = r \sqrt{a^3} \frac{2a-x}{\sqrt{x^5}}.$ For the values of $a, r, x,$ see Par. 134, Ex. 1.
	187·5 1 7 9 3 0 1 4	187·3 1 3 1 7 2 0 3
Aug. 2 4	2·2 7 9 1 6 5 7 9	2·2 8 4 5 2 8 6 5
2 5	2·2 8 0 1 1 2 1 1	2·2 8 5 6 3 2 9 1
2 6	2·2 8 1 0 3 2 1 1	2·2 8 6 7 0 5 2 5
2 7	2·2 8 1 9 2·5 7 9	2·2 8 7 7 4 7 6 2
2 8	2·2 8 2 7 9 2 6 7	2·2 8 8 7 5 8 9 6
2 9	2·2 8 3 6 3 1 9 4	2·2 8 9 7 3 7 6 1
3 0	2·2 8 4 4 4 2 6 3	2·2 9 0 6 8 3 8 5
3 1	2·2 8 5 2 2 4 2 1	2·2 9 1 5 9 4 6 3
Sept. 1	2·2 8 5 9 7 5 7 9	2·2 9 2 4 7 1 4 1
2	2·2 8 6 6 9 5 7 9	2·2 9 3 3 1 0 8 0
3	2·2 8 7 3 8 6 3 2	2·2 9 4 1 1 6 4 4
4	2·2 8 8 0 4 3 6 8	2·2 9 4 8 8 2 3 6
5	2·2 8 8 6 6 9 4 7	2·2 9 5 6 1 2 4 2
6	2·2 8 9 2 6 2 6 3	2·2 9 6 3 0 4 2 0
7	2·2 8 9 8 2 0 6 3	2·2 9 6 9 5 5 6 5
8	2·2 9 0 3 4 6 0 3	2·2 9 7 5 6 7 9 7
9	2·2 9 0 8 3 6 5 1	2·2 9 8 1 3 9 9 2
1 0	2·2 9 1 2 9 3 6 8	2·2 9 8 6 7 2 9 8
1 1	2·2 9 1 7 1 3 2 3	2·2 9 9 1 6 2 1 6
1 2	2·2 9 2 0 9 9 4 7	2·2 9 9 6 1 2 4 1
1 3	2·2 9 2 4 4 8 1 5	2·3 0 0 0 1 8 9 1
1 4	2·2 9 2 7 6 1 5 8	2·3 0 0 3 8 4 3 8
1 5	2·2 9 3 0 3 7 0 4	2·3 0 0 7 0 6 0 6
1 6	2·2 9 3 2 7 6 3 2	2·3 0 0 9 8 4 9 4
1 7	2·2 9 3 4 7 9 3 7	2·3 0 1 2 2 1 2 6
1 8	2·2 9 3 6 4 3 9 2	2·3 0 1 4 1 3 6 7
1 9	2·2 9 3 7 7 2 4 9	2·3 0 1 5 6 3 2 4
2 0	2·2 9 3 8 6 2 6 3	2·3 0 1 6 6 8 7 7
2 1	2·2 9 3 9 1 6 4 0	2·3 0 1 7 3 1 6 8
21d 22h Pe- rihelion.	Sums of the two forces equal 21d 9h 35m 37·2s 2·2 9 3 9 3 3 0 7	Sums of the two forces equal 21d 9h 35m 37·2s 2·3 0 1 7 5 0 7 9
	256·1 8 2 5 3 1 5 9	256·1 8 6 8 1 3 9 3

TABLE III.—MERCURY.

1852. Greenwich Time. Noon.	<i>Aberrating Force.</i> (that in a circular orbit at the Earth's mean distance = 1.) $F' = \frac{c}{\sqrt{a}} \cdot \frac{1}{x^3}$ For the values of a, c, x , see Par. 134, Ex. 1.	<i>Resisting Force.</i> (that at the Earth's mean distance = 1.) $R' = r \sqrt{a^3} \frac{2a-x}{\sqrt{x^5}}$ For the values of a, r, x , see Par. 134, Ex. 1.
Aphelion Nov. 3d 23h 55m	5·9 8 9 9 6 1 4 3	5·3 3 8 6 9 9 1 9
Nov. 5	5·9 9 5 4 0 0 3 2	5·3 4 5 1 9 1 5 8
6	6·0 1 1 6 1 9 7 6	5·3 6 4 5 5 3 8 2
7	6·0 3 8 7 3 9 7 6	5·3 9 6 9 2 7 5 8
8	6·0 7 6 9 1 1 5 9	5·4 4 2 5 0 2 3 8
9	6·1 2 6 3 8 1 7 5	5·5 0 1 5 7 3 5 5
10	6·1 8 7 4 2 6 0 4	5·5 7 4 5 0 4 7 5
11	6·2 6 0 4 6 9 8 9	5·6 6 1 7 3 4 8 3
12	6·3 4 5 9 1 3 3 7	5·7 6 3 8 2 5 3 3
13	6·4 4 4 3 1 7 9 8	5·8 8 1 4 3 0 3 0
14	6·5 5 6 2 5 1 8 6	6·0 1 5 2 3 8 2 4
15	6·6 8 2 4 5 7 2 7	6·1 6 6 1 4 2 0 3
16	6·8 2 3 6 8 6 0 8	6·3 3 5 0 5 0 1 4
17	6·9 8 0 8 3 9 0 8	6·5 2 3 0 4 2 3 1
18	7·1 5 4 8 5 4 4 7	6·7 3 1 2 4 7 1 5
19	7·3 4 6 8 7 3 2 7	6·9 6 1 0 2 3 5 8
20	7·5 5 8 0 1 7 9 4	7·2 1 3 7 1 1 4 2
21	7·7 8 9 6 0 0 4 1	7·4 9 0 8 6 6 0 4
22	8·0 4 3 0 1 7 0 9	7·7 9 4 1 3 8 5 4
23	8·3 1 9 6 9 8 5 4	8·1 2 5 2 0 5 3 9
24	8·6 2 1 2 2 5 9 4	8·4 8 5 9 1 5 7 3
25	8·9 4 9 2 2 0 8 7	8·8 7 8 1 5 5 4 6
26	9·3 0 5 3 7 2 2 3	9·3 0 3 8 5 8 4 7
$F' = R'$	9·3 1 3 1 2 7 5 0 9	9·3 1 3 1 2 7 5 0 9
27	9·6 9 1 3 0 2 7 3	9·7 6 4 8 7 6 7 8
28	10·1 0 8 6 4 4 7 1	10·2 6 3 0 4 5 2 6
Mean dis.	10·4 9 7 0 1 1 9 0	10·7 2 6 2 3 2 7 2
29	10·5 5 8 8 2 7 8 0	10·7 9 9 9 2 0 0 6
30	11·0 4 3 0 4 1 2 9	11·3 7 6 7 6 2 6 3
	203·0 1 0 0 7 3 4 7	193·4 9 9 1 4 2 5 4

TABLE III.—MERCURY.

1852. Greenwich Time. Noon.	<p><i>Aberrating Force.</i></p> <p>(that in a circular orbit at the Earth's mean distance = 1.)</p> $F' = \frac{c}{\sqrt{a}} \frac{1}{x^3}.$ <p>For the values of $a, c, x,$ see Par. 134, Ex. 1.</p>	<p><i>Resisting Force.</i></p> <p>(that at the Earth's mean distance = 1.)</p> $R' = r\sqrt{a^3} \frac{2a-x}{\sqrt{x^5}}.$ <p>For the values of $a, r, x,$ see Par. 134, Ex. J.</p>
Dec. 1 2 3 4 5 6 7 8 9	<p>203·0 1 0 0 7 3 4 7</p> <p>11·5 6 2 2 3 2 6 1</p> <p>12·1 1 6 7 5 2 0 3</p> <p>12·7 0 6 2 0 0 0 8</p> <p>13·3 2 9 2 8 6 8 5</p> <p>13·9 8 3 5 4 5 8 9</p> <p>14·6 6 4 8 8 2 5 2</p> <p>15·3 6 7 6 2 9 7 8</p> <p>16·0 8 3 9 4 1 5 8</p> <p>16·8 0 3 7 0 3 5 8</p>	<p>193·4 9 9 1 4 2 5 4</p> <p>11·9 9 4 5 0 6 7 3</p> <p>12·6 5 3 3 6 5 2 0</p> <p>13·3 5 2 6 2 8 7 2</p> <p>14·0 9 0 5 2 3 2 9</p> <p>14·8 6 3 8 9 0 5 6</p> <p>15·6 6 7 6 6 3 7 9</p> <p>16·4 9 4 9 6 3 2 1</p> <p>17·3 3 6 4 0 4 8 9</p> <p>18·1 8 0 0 2 9 3 5</p>
10 11 12 13 14 15 16 17	<p>Sums of the forces equal 9d 23h 58m 21s.</p> <p>17·5 1 4 6 7 4 3 0</p> <p>18·2 0 2 1 1 8 1 1</p> <p>18·8 4 9 6 6 7 6 5</p> <p>19·4 3 9 5 0 5 4 8</p> <p>19·9 5 3 5 3 3 1 5</p> <p>20·3 7 4 2 2 1 7 2</p> <p>20·6 8 6 3 5 8 2 3</p> <p>20·8 7 7 7 7 5 1 3</p> <p>20·9 4 0 9 8 4 6 7</p>	<p>Sums of the forces equal 9d 23h 58m 21s.</p> <p>19·0 1 1 5 2 0 2 8</p> <p>19·8 1 3 7 7 4 1 1</p> <p>20·5 6 7 9 3 2 9 4</p> <p>21·2 5 3 5 8 3 0 0</p> <p>21·8 5 0 1 0 4 9 0</p> <p>22·3 3 7 6 1 9 3 6</p> <p>22·6 9 8 9 4 3 2 7</p> <p>22·9 2 0 3 5 6 6 5</p> <p>22·9 9 3 4 4 5 1 0</p>
Perihelion 17d, 23h, 36m.	506·4 6 7 0 8 6 8 3	521·5 8 0 3 9 7 8 9





