LIPPINCOTT'S

PRACTICAL ARITHMETIC

EMBRACING

THE SCIENCE AND PRACTICAL APPLICATIONS OF NUMBERS

BY

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PREFACE.

THAT Arithmetic is both a science and an art is not only generally conceded, but emphatically affirmed. One, therefore, investigating the methods of instruction adopted in school-rooms would logically expect to find both aspects of the subject distinctly put in evidence. What is universally acknowledged and proclaimed as essential and vital to any true system of arithmetical education the investigator, however, would fail to find distinctly characterizing either the instructions given by the average teacher or the work required of the average pupil. What we mean is, that if he saw anything notable, he would see Art conspicuous in the foreground, while Science, if visible at all, sat mute far back. There might be seen, to be sure, remarkable skill displayed in many instances, and results brought forth with surprising facility; but, in it all, Science, that alone imparts life to action and informs the mind, would have little or no part.

The great error is that arithmetical exercises and problems are too frequently—nay, almost invariably—set up like so many ten-pins, to be knocked down by mechanical action, without any inquiry as to the underlying and fundamental principles upon which action is based. In a word, the schools, with little exception, are not making the best use of

Arithmetic as an educational force by ignoring the fact that it has principles to be explained, induction and analysis to explain them, and a philosophical reason for every step necessary to be taken.

The text-books used may sometimes be seriously at fault, want of time may be an impediment, and other hindrances may be numerous; but the live and intelligent teacher will find in the least scientific treatise food for quickening thought and moulding the mental state. It must be admitted, however, that the teacher, in the conscientious performance of his duty, has a difficult environment, and needs all the help that text-books can furnish him.

The book that we now introduce to the public we have aimed to make what it assumes to be,—a *Practical Arithmetic;* —practical, not so much by devising short processes and labor-saving schemes as by laying a scientific foundation to be studied and mastered as the essential preliminary to the intelligent and skilful use of any device of mere art; practical, therefore, as a teacher's true assistant, bringing to his hand a full supply of definitions, inductive steps, illustrations, principles, analyses, syntheses, processes, rules, and suggestions, needful to him in his high vocation,—a vocation that is highest when most devoted to "bright-eyed Science," and lowest when it rests content with the pretentious and empty forms of mere "mechanic art."

The text-book, even in its best estate, replete with science and art-full, can have little philosophical efficiency except when intelligently used as a means to an end. In the school-room, where a book is expected to promote the high aims of education, the intelligent use of it must begin, if it begins at all, with the teacher; for it is he alone whose very office it is, through voice and action, to stir into quickening force the words of the text, that otherwise may fall as good seed upon sterile ground. Every teacher ought to know—what every pupil soon learns—that "to hear illustrations and explanations from living lips is a different thing from struggling through them on the printed page." Every page is to be learned, however,—mastered,—and made emphatically the pupil's own; and, as a suggestion pertinent here, we quote the philosophic words of John Locke: "The great art to learn much is to undertake a little at a time."

The author would gladly express his thanks to all who in any way made contributions of help. To one friend, whose devotion to the work never faltered, he acknowledges lasting obligation.

J. M. R.

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GENERAL SUGGESTIONS.

1. There is no royal road to a knowledge of Arithmetic, and in this book no attempt has been made to preclude the necessity for laborious effort without which, it has been wisely said, life gives nothing to mortals.

2. Self-reliance is the basis of action, and "self-activity is the law of growth." To render the pupil self-reliant, self-helpful, and self-acting in the face of difficulties, is the object the teacher should keep steadily in view.

3. Two ideas are fundamental: I. Knowledge cannot be successfully built except on knowledge already acquired. The lesson to be learned to-morrow must start its growth in the lesson learned to-day. II. Lessons assigned a class must not be made too easy for some, nor too difficult for others. Careful judgment is, therefore, required that oral explanations and illustrations be neither too ample nor too meagre. The least active mind must be made to understand, and, at the same time, the most active brain must be required to labor.

4. In each division of the subject, as treated, will be found inductive steps, definitions, principles, processes, explanations, rules, exercises, and problems,—all to be thoroughly learned and intelligently recited. Mastery of the exercises will give facility in performing the operations required by the problems.

5. The solution of a problem requires three distinct steps:

I. The indication in arithmetical language of the operations to be performed.

II. The mechanical performance of the operations indicated.

III. The statement of the reasoning by which the operations as indicated were obtained, and also the elucidation of any merely mechanical step that has been taken to reach the final result.

If in each subject the introduction, including principles, processes, and explanations, be systematically and thoroughly acquired, the exercises and problems that follow will seem not forbidding obstacles, but, as it were, beckoning friends.

6. Too much importance cannot be attached to the method of dealing with a problem, as pointed out above. The frequent suggestions made throughout the book attest the author's belief in the excellence of the system proposed. One advantage is that the first step—the indication in arithmetical language of the work to be done,—*really solves the problem*, and that here, in many cases, the pupil's work may be considered as satisfactorily closed. Every teacher must determine for himself, and every intelligent teacher will successfully determine, how far his pupils need to work out and recite the details of a solution. He must go far enough to be convinced that they have got within them a conception of the truth, and are able to declare it. But how is this possible, unless he recognizes the great fact that every pupil is an individual, has a distinct individuality, and is, as far as possible,

to be individually approached and trained, "not for school, but for life"?

To summarize:

1. Do not go too fast; hasten slowly.

2. Assign lessons with care, keeping in mind that "too much is not good."

3. Repeat constantly; "repetition is the mother of all learning."

4. Require hard work; "the harder a pupil has worked for what he knows and can do, the better for him."

5. Be methodical, enthusiastic, persistent, and patient.

6. Remember the ancient maxim, that "to the boy is due the highest reverence."

PRACTICAL ARITHMETIC

PART I.

DEFINITIONS.

1. A **Unit** is a *single thing* or *one.*

2. A **Number** is a *unit* or a *collection of units.*

3. **Arithmetic** is both a *Science* and an *Art:* as a science, it investigates the *principles* of numbers; as an art, it applies those principles to *practical purposes.*

4. A **Principle** is a *fundamental truth* or *ground of action.*

5. A number is **Concrete** or **Denominate** when its unit is *named,* as in *one man, two books, three ships.*

6. A number is **Abstract** when its unit is *not named,* as *one, two, three.*

When named, the unit of a number is one of the things expressed by the number, as *one tree, one man.*

When not named, the unit of a number is simply *one.*

7. A **Simple Denominate** number has a single unit, as in *five feet.* A **Compound Denominate** number has two or more related units, as in *three yards two feet six inches.*

What is the unit of the concrete number *three ships?* Of the abstract number *three?*

Tell which of the following numbers are concrete and which abstract, and what is the unit of each:

1. Ten men.	7. One apple.	13. Four horses.
2. Three.	8. Seven.	14. Six wagons.
3. Nine boys.	9. Five pounds.	15. Sixty.
4. Eleven girls.	10. Fifty-five.	16. Sixty-seven.
5. Twenty-one.	11. Twenty-nine.	17. Twenty ships.
6. Seventeen.	12. Twelve.	18. Twenty-nine.

8. **Analysis** (Greek, *taking apart*) examines the separate parts of a subject, or proposition, and their connection with each other; it solves problems by a comparison of their elements; it reasons from the given number to *one*, and then *from one* to the required number; it reasons, also, from particular instances to general principles.

9. **Synthesis** (Greek, *putting together*) unites separated parts, in accordance with their obvious relations.

10. A **Rule** is founded on some *principle*, and is a *precise direction* for solving a problem.

11. A **Problem** is a practical question *requiring a solution.*

12. A **Solution** consists of a *process* and an *explanation* made by the application of a rule or by *analysis* and *synthesis.*

NOTATION AND NUMERATION.

1. **Notation** is the art of *writing numbers.*
2. **Numeration** is the art of *reading numbers.*
3. There are *three* methods of notation in common use:
    1. The word method.
    2. The Arabic or figure method.
    3. The Roman or letter method.
4. The Arabic method employs the Arabic figures: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

5. The word method names these figures and expresses their values as follows:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.
One, two, three, four, five, six, seven, eight, nine, naught (cipher, zero).

The script forms are as follows:

1 2 3 4 5 6 7 8 9 0

These figures are frequently called digits (Latin, *digitus, finger*); those preceding 0 are called significant figures.

ARABIC NOTATION.

1. Each of the first nine numbers, you perceive, is expressed by a single digit; higher numbers are expressed by *combinations of the digits.*

One prefixed to naught (10) is ten.

2. Our system of notation is "due to the fact that we have ten fingers," and the basis of it is the first ten numbers formed into a model group.

10 is one ten, or simply *ten* (Latin, "*decem*").
11 is *eleven* (Gothic, "*ain, one; lif, ten*"), one and ten.
12 is *twelve* (Gothic, "*tva, two; lif, ten*"), two and ten.
13 is *thirteen,* three and ten.
14 is *fourteen,* four and ten.
15 is *fifteen,* five and ten.
16 is *sixteen,* six and ten.
17 is *seventeen,* seven and ten.
18 is *eighteen,* eight and ten.
19 is *nineteen,* nine and ten.
20 is *twenty* (*teen* becomes *ty*).

3. The numeral names that precede "*teen*" follow "*ty*" and a hyphen (-), as follows: 21 is twenty-one, that is, twice ten and one; 30 is thirty; 31 is thirty-one; 40 is forty; 42 is forty-two.

4. Any significant figure, located as four in 40, has its value increased ten-fold and denotes *tens.*

Locate 5 thus, and name the number; also 6, 7, 8, 9.

The value of a significant figure in units' place is called its **Simple Value.**

The value of a significant figure otherwise placed is called its **Local Value.**

5. 44 is four tens and four units, or forty-four.

Which four has the increased or *Local Value?* Which has only its *Simple Value?*

99 is ninety-nine, and is the largest number that can be expressed with two figures.

6. 100 is ten tens, or *one hundred.*
200 is twenty tens, or *two hundreds.*
300 is thirty tens, or *three hundreds.*

Any significant figure thus located expresses hundreds.

444 is four hundreds, four tens, four units, or four hundred forty-four.

404 is thus read: "four hundred four," not "four hundred *and* four." The naught (0) indicates the absence of tens.

7. Again, 404, or any three digits thus written together, constitute a *period* with units in the *first place,* tens in the *second place,* and hundreds in the *third place.* The left hand period may contain but one or two digits.

8. Ten units grouped make a single ten-group. Ten ten-groups make a single hundred-group. Ten one-hundred-groups make one thousand, written 1000.

9. Any single thing is a unit; a single ten-group may, therefore, be considered a unit; so also, a single hundred-group.

10. On this principle a digit in the first place denotes units of the *first order;* in the second place, units of the *second order;* in the third place, units of the *third order*, etc.

PRINCIPLES.

1. The first nine numbers are expressed by the nine digits (1, 2, 3, etc.), taken singly.

2. Numbers above nine are expressed by combining the digits and giving them local values.

3. Naught (0) has no value, but is used to fill a vacant place and to fix the values of the significant figures.

4. Local value increases from right to left, ten units of any order making one unit of the next higher order.

EXERCISES.

NOTE.—Pupils should be carefully drilled in giving the digits their correct forms. Ill-formed figures often lead to erroneous results.

1. Write the Arabic numerals.

2. Write their names.

3. Write the significant figures.

4. Write figures enough to make a period.

5. Write a period and indicate the absence of tens and units.

6. Write two thousand three hundred seventy-five.

7. How many places have you written? How many orders? How many periods?

8. How many units of any order make one unit of the next higher order?

9. Our system of notation puts how many units in a group?

10. What is a unit? When may ten or a hundred be considered a unit?

11. Write units of the fourth order, and show the absence of units of the first, second, and third orders.

12. Express 1898 in words, remembering what was said about "and."

NUMERATION TABLE.

1. Places, orders, and periods may be carried on indefinitely from right to left.

2. The whole subject may now be concisely presented in tabular form. The first six periods are as follows:

NAMES OF PERIODS.	Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
NAMES OF PLACES.	Hundred-quadrillions,	Ten-quadrillions,	Quadrillions,	Hundred-trillions.	Ten-trillions,	Trillions,	Hundred-billions,	Ten-billions,	Billions,	Hundred-millions,	Ten-millions,	Millions,	Hundred-thousands,	Ten-thousands,	Thousands,	Hundreds,	Tens,	Units,
PLACES AND ORDERS.	8	8	8,	8	8	8,	8	8	8,	8	8	8,	8	8	8,	8	8	8
PERIODS.	6TH.			5TH.			4TH.			3D.			2D.			1ST.		

3. The periods are separated from each other by commas.

4. The periods from the first to the twenty-second are named as follows:

1. Units.
2. Thousands.
3. Millions.
4. Billions.
5. Trillions.
6. Quadrillions.
7. Quintillions.
8. Sextillions.
9. Septillions.
10. Octillions.
11. Nonillions.
12. Decillions.
13. Undecillions.
14. Duodecillions.
15. Tredecillions.
16. Quatuordecillions.
17. Quindecillions.
18. Sexdecillions.
19. Septendecillions.
20. Octodecillions.
21. Novendecillions.
22. Vigintillions.

EXERCISES.

1. Write five, fifty-five, five hundred fifty-five, and state what each five expresses.

2. Write a number consisting of four digits, and point off the first period with a comma; also, read the number.

3. Write a number consisting of two full periods; name the periods, and read the number.

4. In 876, of what order and place is each figure?

5. In writing nine hundred seven, how will you express the tens? Write the number.

6. Write a number consisting of three periods; name the periods, and read the number.

7. Write numbers consisting respectively of four periods, five periods, six periods, and read each of the numbers.

8. Write a number with a significant figure located in the seventh place. What will occupy the other six places?

9. If a number to be written omits a period, or an order, or a place, what must in all cases supply the vacancy?

10. Write a number with 7 in the sixth place, 5 in the fourth place, and 2 in the first place.

11. Write a number with five full periods and one partial period.

12. Point off into periods and read 405651320.

13. In the preceding number, how many units of the first order? How many of the eighth order? How many ten-thousands? How many ten-millions?

RULE FOR NUMERATION.

1. Begin at the right and mark off the number into periods.

2. Begin at the left, read each period separately, naming each period except that of units.

EXERCISES.

Copy and read:

1.	89.	13.	571320179.
2.	134.	14.	35627003.
3.	946.	15.	1000000000.
4.	1664.	16.	4321078654.
5.	5790.	17.	4141414441.
6.	83405.	18.	62340007313.
7.	624151.	19.	141662223143.
8.	731052.	20.	700706831455.
9.	8000000.	21.	31671240630231.
10.	763303454.	22.	19870006345965216129\12.
11.	900058798.	23.	1234567891011121314151.
12.	100100001.	24.	6171819202122232425262.

1. Write the number *five hundred twenty-three.*

Process. **Explanation.**

523

ANALYSIS.—Five hundred twenty-three means five hundreds, two tens, three units, represented by the digits 5, 2, and 3.

SYNTHESIS.—We, therefore, write 3 in the first place, which is the place of simple units, 2 in the second place, which is the place of tens, and 5 in the third place, which is the place of hundreds.

2. Write *six hundred thirty-seven thousand one hundred six.*

Process. **Explanation.**

637,106

ANALYSIS.—Since the given number contains 637 thousands and 106 units, there are in it two full periods.

SYNTHESIS.—We, therefore, write the digits of the thousands, 637, as the second period, and the digits of the units, 106, as the first period.

3. Express in figures:
   1. Five hundred sixty-four.
   2. Seven hundred fifty-nine.
   3. Four thousand eighty-one.
   4. One thousand two hundred.
   5. Twenty-five thousand seven.
   6. Forty-one thousand nineteen.
   7. Six thousand six hundred six.
   8. One hundred thirty-one thousand.
   9. Sixty-five thousand four hundred seventy-nine.
   10. One million five hundred three thousand five hundred ninety-three.
   11. Ninety-one million three hundred forty-five thousand.
   12. Twelve thousand nine hundred seventy-eight.
   13. Thirty-one billion three hundred thirteen million six hundred seventy-two thousand four hundred eleven.
   14. One hundred sixty-four million eighteen thousand.
   15. One hundred fifteen quadrillion four hundred forty-four trillion five hundred three billion four million two hundred fifty thousand one.

DECIMAL PARTS OF A UNIT.

1. By placing a mark (.) called the decimal point after units of the first order, the numeration and notation table is extended to express parts of a unit, on the decimal scale:

Units.		Tenths.	Hundredths.	Thousandths.
5	.	5	5	5

The above number is thus read: "five and five hundred fifty-five thousandths."

2. The decimal point (.) is always read "and."
   6.7 is read "six *and* seven tenths."
   7.89 is read "seven *and* eighty-nine hundredths."
   .005 is read "five thousandths." The naughts are only recognized as giving local value to 5.
   1.234 is read "one *and* two hundred thirty-four *thousandths.*
3. The decimal point never acts as a *period.*

EXERCISES.

1. Read: .08, .75, .006, 3.079.
2. Write: Twenty-seven hundredths.
   Eleven thousandths.
   One *and* four hundred six thousandths.
   Thirteen *and* twenty-five thousandths.
3. Read: 17.6, 1.76, .196, .144.
4. Write: Five hundred six thousandths.
   Five hundred *and* six thousandths.
5. Read: 325.72, 325, 32.5, .072.
6. Write: Five hundred four thousandths.
   Five hundred *and* four hundredths.
7. Read: 6.050, 7.200, 872.003, .409.
8. Write: Seven tenths four thousandths.
   Nine *and* seventy thousandths.
9. Express "and" by a sign.
10. What word interprets the decimal point.
11. What is the difference between a decimal point and a period?
12. Form a number by writing the digits *six, seven, eight, nine,* and *zero,* in their natural order; then place the decimal point in all the different positions you can; finally read, in succession, the different numbers you have thus formed.

13. State the effect of moving the point one place to the right; one place to the left.

14. How many fold does a removal one place increase or diminish the value expressed?

UNITED STATES MONEY.

1. The currency of the United States has the decimal system.

Table.

10 mills	make	1	cent.
10 cents	make	1	dime.
10 dimes	make	1	dollar.
10 dollars	make	1	eagle.

2. $ is the dollar sign, and, prefixed to an abstract number, renders it concrete: 10 becomes $10, read "ten dollars."

3. *The dollar is the Unit,* and the decimal point is invariably placed between the dollars and dimes of any sum of money: $5.60 is read "five dollars *and* sixty cents," or "five dollars *and* six dimes."

EXERCISES.

1. Express in figures nine dollars *and* twenty-five cents six mills.

Process. **Explanation.**

$9.256

ANALYSIS.—Given: nine dollars, twenty-five cents, six mills. Twenty-five cents are two dimes and five cents. *The dollar is the unit.*

SYNTHESIS.—Write the dollar sign, 9, and a point; and after the point 2 dimes, 5 cents, and 6 mills, in their natural order.

2. Express in figures thirty-one dollars *and* nine cents five mills.

Process.	**Explanation.**
$31.095	ANALYSIS.—Given: thirty-one dollars, *no dimes*, nine cents, five mills. PRINCIPLE.—*0 supplies a vacant place.*

SYNTHESIS.—Write dollar sign, 31, and a point; 0 in dimes' place; and 9 cents and 5 mills in their order.

3. Write six dollars and eighty-five cents.
4. Read $2.235, $202.025, $112.25.
5. Write five hundred dollars and nine cents five mills.
6. Write two thousand dollars.
7. Write forty dollars and four cents.
8. Read $313112.13, $20000.32.
9. Write twelve dollars and five cents six mills.
10. Write six thousand one dollars and one mill.
11. Write seven million dollars and seventy-seven cents.
12. Read $.05, $.03, $.62, $.70.
13. Copy and read the following:

1.	$8.53.	5.	$236.06.	9.	$796.844.
2.	$13.75.	6.	$20000.	10.	$.16.
3.	$39.05.	7.	$2104.083.	11.	$.057.
4.	$49.34.	8.	$6001.102.	12.	$12.500.

14. Write the following:
    1. Eight dollars and fifty cents.
    2. Two hundred two dollars and two cents five mills.
    3. Five dollars.
    4. Five hundred dollars.
    5. One hundred twelve dollars and twenty-five cents.
    6. Four dollars and eighty-seven cents.
    7. Ninety-seven cents eight mills.
    8. Six hundred twenty dollars and nine cents.
    9. Twelve million seven hundred thousand dollars.

10. Three thousand ten dollars and fifty cents.
11. Seventy dollars and ten cents.
12. Six million dollars and eighty cents.
13. Four cents. Ten cents. Nine mills.

ROMAN NOTATION.

1. This system of notation employs *seven capital letters.*

Table.

I. denotes one,	1.	C. denotes one hundred,	100.
V. denotes five,	5.	D. denotes five hundred,	500.
X. denotes ten,	10.	M. denotes one thousand,	1,000.
L. denotes fifty,	50.	$\overline{\mathrm{M}}$. denotes one million,	1,000,000.

2. All other numbers are expressed by combining or repeating these letters:

I.	1.	XIV.	14.	C.	100.
II.	2.	XV.	15.	CCCC., or CD.	400.
III.	3.	XVI.	16.	D.	500.
IV.	4.	XVII.	17.	DCCCC., or CM.	900.
V.	5.	XVIII.	18.	M.	1000.
VI.	6.	XIX.	19.	MD.	1500.
VII.	7.	XX.	20.	MDCLXV.	1665.
VIII.	8.	XXI.	21.	MDCCXLIX.	1749.
IX.	9.	XXX.	30.	MDCCCLXXIX.	1879.
X.	10.	XL.	40.	$\overline{\mathrm{V}}$.	5,000.
XI.	11.	L.	50.	$\overline{\mathrm{L}}$.	50,000.
XII.	12.	LX.	60.	$\overline{\mathrm{C}}$.	100,000.
XIII.	13.	XC.	90.	$\overline{\mathrm{M}}$.	1,000,000.

From the repetitions and combinations observable above, we derive the following

PRINCIPLES.

1. Repeating I., X., C., or M. repeats its value.

XX. denotes 20; CC. denotes 200. V., L., and D. cannot be thus repeated.

2. Prefixing I., X., or C. to a letter of greater value diminishes that value by I., X., or C.

3. Affixing I., V., X., L., C., or D. to a letter of greater value increases that value by I., V., X., L., C., or D.

4. Inserting I., X., or C. between two letters,* each of greater value, diminishes the united value of the two by I., X., or C.

* The first of the two must not be of less value than the second. XIV., not VIX., denotes 14; XIX. denotes 19.

5. A bar placed over a letter, except I., increases its value a thousand-fold.

$\overline{\text{C}}$. denotes 100,000.

6. IIII. is sometimes used instead of IV., as on the dials of clocks and watches. 400 may be expressed by CCCC. or by CD.

EXERCISES.

1. Read the following combinations:

1. XV.	10. XLV.	19. DCCXC.
2. IV.	11. XCIX.	20. MXXIX.
3. XIV.	12. LXV.	21. $\overline{\text{V}}$DLV.
4. XXIV.	13. CIX.	22. $\overline{\text{DL}}$DC.
5. XIX.	14. CXI.	23. $\overline{\text{CCX}}$DVI.
6. XXXIX.	15. XCI.	24. $\overline{\text{VIII}}$.
7. XXXIII.	16. DCXC.	25. $\overline{\text{C}}$CXC.
8. XXIX.	17. CCCXXXIX.	26. CXLIX.
9. XLIX.	18. DCCXXXIV.	27. MMD.

28. $\overline{\text{LXX}}$DCCCXCIX. 29. $\overline{\text{MDXCV}}$DCCCLXIV.

2. Write in Roman characters the following:

1.	15.	11.	18.	21.	27.
2.	36.	12.	42.	22.	81.
3.	87.	13.	66.	23.	95.
4.	56.	14.	86.	24.	40.
5.	49.	15.	63.	25.	45.
6.	99.	16.	100.	26.	534.
7.	1050.	17.	3600.	27.	5000.
8.	5010.	18.	587.	28.	436.
9.	789.	19.	207.	29.	999.
10.	1898.	20.	8004.	30.	76,959.

3. Which of these are correct expressions and which incorrect?

VV.	DD.	VXX.
LV.	CCC.	VLC.
XIL.	XCC.	VDC.
LXIX.	MMXL.	CIXVIIX.
XLX.	XLIX.	LXXXVIII.
XCIX.	CIXXVII.	DMCC.

REVIEW.

1. Define the following terms:

1. Unit.	11. Solution.
2. Number.	12. Notation.
3. Arithmetic.	13. Numeration.
4. Principle.	14. Word method.
5. Concrete number.	15. Arabic method.
6. Abstract number.	16. Roman method.
7. Analysis.	17. Simple value.
8. Synthesis.	18. Local value.
9. Rule.	19. Zero.
10. Problem.	20. Period.

21. Decimal point.
22. United States Money.
23. Mill.
24. Cent.
25. Dime.
26. Dollar.
27. Eagle.
28. Significant figures.

2. Repeat the four principles of notation.
3. Name the periods from the 1st to the 22d.
4. Repeat the five principles of the Roman notation.
5. Repeat the rule for numeration.

ADDITION.

INDUCTIVE STEPS.

1. How many units are 5 units and 3 units? 2 tens and 7 tens? 4 thousands and 6 thousands?

2. A certain field has 7 acres, and an adjoining field 8 acres. How many acres in both fields?

Process.

7 acres
8 acres
———
15 acres

Explanation.

Since one field contains 7 acres and the other 8 acres, the two fields contain 7 acres and 8 acres, which are 15 acres.

3. If on one shelf there are 9 books and on another shelf 5 books, how many books are on both shelves?

Write and explain the process.

4. How many pounds are 8 pounds and 6 pounds?

Write and explain.

5. The process of thus uniting quantities in a single quantity is called *adding*.

6. What is the unit of 8 pounds? Of 6 pounds? Of 14 pounds?

7. Can quantities having *like* units be added?

8. Add 6 pounds and 5 dollars. Can you show a process? If you can, is your result 11 pounds or 11 dollars?

9. What, then, does addition require as to the units to be added?

10. What does addition require as to the unit of the result or sum?

11. Numbers having like units are called **Like Numbers.**

DEFINITIONS.

1. **Addition** is the process of finding the *sum* of two or more like numbers. The sum is, therefore, the result of addition.

2. A **Sign** indicates some process or condition. The sign of addition is an upright cross, $+$. It is read "plus."

3. The **Sign of Equality** is two short horizontal lines, $=$. It is read "equals," or "is equal to." $3 + 2 = 5$, is read "3 plus 2 equals 5." $3 + 2 = 5$, being an expression of *equality*, is called an **Equation.**

PRINCIPLES.

1. **Only like numbers and orders can be added.**
2. **The numbers added and their sum are like numbers.**

EXERCISES.

1. Find the sum of 120, 331, and 478.

Process. **Explanation.**

$$\begin{array}{r} 120 \\ 331 \\ 478 \\ \hline 929 \end{array}$$

ANALYSIS.—There are three numbers to be added, each containing units, tens, and hundreds.

PRINCIPLE.—*Only like orders can be added.*

SYNTHESIS.—Hence we write the numbers with the units' figures (0, 1, 8) in the first column on the right, the tens' figures (2, 3, 7) in the second column, and the hundreds' figures (1, 3, 4) in the third column.

The sum of the first column is 9 units; the sum of the second column is 12 tens $=$ 1 hundred $+$ 2 tens. We write the 2 tens, and add the 1 hundred to the hundreds' column, making 9 hundreds. Hence the sum required is 929.

2. Find the sum of 25, 206, and 9837.

Process.

$$\begin{array}{r} 25 \\ 206 \\ 9,837 \\ \hline 10,068 \end{array}$$

Explanation.

ANALYSIS.—

25 =	2 tens + 5 units.
206 =	2 hundreds + 0 tens + 6 units.
9,837 =	9 thousands + 8 hundreds + 3 tens + 7 units.

PRINCIPLE.—*Only like orders can be added.*

SYNTHESIS.—We, therefore, write the units' figures (5, 6, 7) in the first column; the tens' figures (2, 0, 3) in the second column; the hundreds' figures (2 and 8) in the third column; and the 9 thousands alone in the fourth place. Adding the first column we have 18 units = 1 ten and 8 units. We add the 1 ten to the tens' column and have 6 tens. Adding the third column we have 10 hundreds = 1 thousand and 0 hundreds. We say finally 1 thousand + 9 thousand = 10 thousand.

RULE FOR ADDITION.

1. See that the numbers to be added are like numbers.

2. Write units of the same order in the same column.

3. Begin at units' column, and find the sum of each column separately.

4. Write the units of a sum, but add the tens with the next column.

5. Write the entire sum of the last column.

EXERCISES.

1. Find the sum of:

(1.)	(2.)	(3.)	(4.)	(5.)
234	134	712	473	535
365	542	314	321	213

(6.)	(7.)	(8.)	(9.)
\$2.14	\$6.10	\$31.12	\$231.25
\$3.31	\$2.11	\$41.23	\$542.30
\$1.50	\$1.34	\$20.44	\$210.44

(10.)	(11.)	(12.)	(13.)
4134	2460	3782	469
8104	3782	1856	7206
3910	3673	1916	39
45	418	3061	6

2. What is the sum of 2213, 1123, 3201, 2112?
3. What is the sum of:
   1. 3210 + 2136 + 3752 + 2331?
   2. 3561 + 5103 + 6385 + 5632?
   3. 73,250 + 3102 + 16,287 + 1210 + 7542?
   4. 50,673 + 520 + 16,302 + 2531 + 7204?
   5. 154,632 + 54,231 + 16,302 + 2120 + 8023?
4. Find the sum of:

	(1.)		(2.)		(3.)		(4.)		(5.)		(6.)		
1.	888	+	777	+	666	+	555	+	543	+	735	=	?
2.	444	+	333	+	222	+	111	+	210	+	141	=	?
3.	000	+	999	+	234	+	423	+	924	+	287	=	?
4.	578	+	287	+	342	+	760	+	553	+	765	=	?
5.	504	+	167	+	359	+	578	+	751	+	432	=	?
6.	105	+	483	+	142	+	263	+	351	+	109	=	?

Add the foregoing both vertically and horizontally.

5. What is the sum of 37 + 375 + 3754 + 37,546 + 64 + 645 + 4573 + 57,373?

6. Add $317.50, $610.10, $514.085, $6.16.

7. What is the sum of four hundred sixty-two, three thousand two hundred fourteen, seventy-nine thousand six hundred fifty-nine, two hundred eighty-four?

8. What is the sum of eighteen dollars and five cents, fifty-one dollars, fifty-one cents, ten dollars and ten cents, eighteen dollars and twenty-four cents, thirty-five dollars?

9. Write the following numbers with Arabic numerals and find their sum: DCCCCXXXVI., MDXVI., MMMMCCIV., CLIV., XCVII., CLXIX.

10. $4682 + 19{,}783 + 100 + 6402 + 178 + 19 = ?$

PROBLEMS.

Let the pupil first indicate the solution of each problem by using the signs, + and =.

1. A. owns 345 sheep, B. owns 295, C. owns 436, and D. owns 524. How many sheep do all own?

Process Indicated.

$345 + 295 + 436 + 524 =$ number of sheep required.

Process.

$$\begin{array}{r} 345 \\ 295 \\ 436 \\ 524 \\ \hline 1600 \end{array}$$

Explanation.

ANALYSIS.—There are four flocks of sheep: A.'s = 345.
B.'s = 295.
C.'s = 436.
D.'s = 524.

In each number the unit is 1 sheep; hence the numbers are *like* and may be added.

SYNTHESIS.—$345 + 295 + 436 + 524 = 1600$. Hence all own 1600 sheep.

2. How many acres are in three fields, containing respectively 23 acres, 34 acres, and 38 acres?

3. A man bought a horse for \$250, a carriage for \$175, a harness for \$74.50, a whip for \$1.25, a carriage blanket for \$3.45. What did he pay for all?

4. A. bought 7590 pounds of pea coal, 3765 pounds of nut coal, 6834 pounds of stove coal, and 2505 pounds of bituminous coal. How much coal did he purchase?

5. In a primary school there are 386 children in first grade, 258 in second grade, 237 in third grade, and 184 in fourth grade. How many pupils in the four grades?

6. Spain has an area of 195,773 square miles; France, 204,091; Switzerland, 15,922; Italy, 112,622. How great is the area of the four countries?

7. The battle-ship "Oregon" sailed from San Francisco to Callao, 4,012 miles; from Callao to Sandy Point, 2,666 miles; from Sandy Point to Rio, 2,228 miles; from Rio to Bahia, 745 miles; from Bahia to Barbadoes, 2550 miles; from Barbadoes to St. Thomas, 346 miles; from St. Thomas to Key West, 1040 miles. Find the total number of miles she sailed.

8. The monthly pay of a major-general in the United States army is $625; of a brigadier-general, $458.33; of a colonel, $291.67; of a lieutenant-colonel, $250; of a major, $208.33; of a captain, mounted, $166.67; of a captain, not mounted, $150; of a chaplain, $125. Find total monthly pay of the eight officers.

9. In 1897 the organized military strength of the State of New York was 13,894 men; of Pennsylvania, 8521; of Illinois, 6260; of Ohio, 6004; of Massachusetts, 5154; of New Jersey, 4297; of California, 3909; of Georgia, 4450; of South Carolina, 3127; of Texas, 3023. What was the entire military strength of the ten States in 1897?

10. In 1890 the population of Cincinnati was 216,239; of Cleveland, 92,829; of Toledo, 31,584; of Columbus, 31,274; of Dayton, 30,473. How many inhabitants had these cities altogether in 1890?

11. A merchant received money for goods as follows: On Monday, $357.15; on Tuesday, $463.87; on Wednesday, $279.19; on Thursday, $318.67; on Friday, $687.27; on Saturday, $348.48. Find the total receipts.

12. A builder bought a lot for $650, built upon it a house costing $5845, a barn and carriage-house costing $1075.50; he paid for fencing $215.75, for grading $87.50. For what must he sell the property to gain $640?

13. The provinces of Cuba, with the population of each, are as follows:

Province.	White.	Colored.
Havana	344,417	107,511
Pinar del Rio	167,160	58,731
Matanzas	153,169	116,401
Santa Clara	249,345	109,777
Puerto Principe . . .	54,232	13,557
Santiago de Cuba . .	157,980	114,339

Find the total population of Cuba.

14. The land forces of Japan are as follows: infantry, fifty-six thousand thirty-seven; cavalry, $\overline{\text{V}}$DCCLX; artillery, seven thousand 818; engineers and train, $\overline{\text{IV}}$CCCXXVI. What is the total land force?

15. January has 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, December 31. How many days in a year?

16. If a school session closes on the 29th of June and opens again on the 10th of September, how many days' vacation will there be?

17. What are the expenses of a factory for a year, if the manager receives $1850, the engineer $850, the fireman $650, the bookkeeper $800, the fuel costs $1600, the raw material $111,110, and the pay-roll of the other employees amounts to $55,000?

18. Add 2, 6, 8, 7, 1, 2, 8, 5, 3, 2, 8, 9.

19. Add IV., VII., II., V., V., II., IX., VI., II., VIII., VII., V.

20. A capitalist made the following deposits in a bank: August 4, 1897, $484.50; August 7, $985.25; August 10, $436.75. In a second bank as follows: August 14, $2657.76;

August 18, $1386.25; August 22, $2096.65. How much did he deposit in each bank? How much in both banks?

	(1.)	(2.)	(3.)	(4.)
21.	7651	5005	10,475	$453.48
	8923	4567	72,482	4,938.78
	4554	2299	46,552	85,473.89
	5421	9900	62,651	3,457.96
	6432	8877	62,272	835.47
	9888	7788	67,286	53.49
	8797	6655	40,025	9.87
	5032	5566	82,827	82.75
	8060	4433	40,050	875.39
	2134	3344	24,165	48.34

(5.)	(6.)	(7.)	(8.)
8410	6546	4828	595
9836	3210	3424	579
984	785	293	8574
543	156	788	3250
9758	5634	2763	386
8574	7654	5612	984
451	696	942	5849
876	321	397	6546
7864	3288	5945	429
5849	2188	8020	451
762	785	694	8765
321	564	432	5634
3250	7688	6131	543
8765	5861	9876	762
688	978	750	3210
642	643	976	3288

(9.)	(10.)	(11.)
.85	$901.09	.3789
463.27	91.85	.7398
39.99	387.24	4.217
1.58	19,877.46	3.95
6,598.86	19.90	45.007
9,005.79	104.99	4.256
95,783.04	3,972.87	3.520
2,469.98	79,841.24	23.3
956.83	18.72	29.317
14,816.00	3,120.50	343.28
3,947.25	14.12	1899.

REVIEW.

1. Define the following terms:
   1. Like numbers.
   2. Unlike numbers.
   3. Addition.
   4. Sum.
   5. Sign of Addition.
   6. Equation.
   7. Indicated process.
   8. Process.
2. Repeat the principles of Addition.
3. Repeat the rule for Addition.
4. Invent five problems in Addition and indicate their solution.

SUBTRACTION.

INDUCTIVE STEPS.

1. How many are 6 units less 3 units? 7 tens less 5 tens? 8 millions less 4 millions?

2. If you have $9 and spend $5, how many dollars do you retain?

Process.	Solution.
\$9 \$5 \$4	If I have \$9 and spend \$5, I retain the difference between \$9 and \$5, which is \$4.

Is that explanation analytical or synthetical?

3. \$5 + \$4 = how many dollars?

4. Was it analysis or synthesis that gave you the \$9?

5. Does the synthesis, then, prove the correctness of the analysis?

6. Robert is 10 years of age and Richard is 8. What is the difference of their ages?

Write and explain the process. Prove the correctness of the result.

7. There were 7 bunches of ripe grapes on a vine; a fox took 2 bunches. How many bunches remained?

8. Have you been finding the difference between like numbers?

9. Finding the difference between two numbers is called *Subtracting.*

10. What is the difference between 6 horses and 3 sheep?

11. Subtraction of numbers makes what requirement as to their units?

DEFINITIONS.

1. **Subtraction** is the process of finding the *difference* between two *like* numbers.

2. The greater number is called the **Minuend**; the less number is called the **Subtrahend**; the result is called the **Difference** or **Remainder.**

3. The **Sign of Subtraction** is a short horizontal line, —, called *minus* (less), and is always placed *after* the minuend and *before* the subtrahend.

7 — 5 = 2 is read "7 minus 5 equals 2." The form, 7 — 5 = 2, is called what?

PRINCIPLES.

1. Only like numbers and orders can be subtracted.
2. Subtrahend + Remainder = Minuend.

1. From 54 subtract 33.

Process.

$$\begin{array}{r} 54 \\ 33 \\ \hline 21 \end{array}$$

Explanation.

The minuend, 54 = 5 tens + 4 units;
the subtrahend, 33 = 3 tens + 3 units.

2 tens + 1 unit = 21.

PRINCIPLE.—*Only like orders can be subtracted.*

We therefore write the 3 units under the 4 units and the 3 tens under the 5 tens. We then say "4 units − 3 units = 1 unit; 5 tens − 3 tens = 2 tens. Hence the remainder is 21."

Proof.

PRINCIPLE.—*The subtrahend + the remainder = the minuend.*

33 + 21 = 54.

2. From 469 subtract 327.

Process.

$$\begin{array}{r} 469 \\ 327 \\ \hline 142 \end{array}$$

Explanation.

469 = 4 hundreds + 6 tens + 9 units.
327 = 3 hundreds + 2 tens + 7 units.

1 hundred + 4 tens + 2 units = 142.

PRINCIPLE.—*Only like orders can be subtracted.*

We therefore write the 4, 6, and 9 of the minuend, and under them the 3, 2, and 7 of the subtrahend, with units under units, tens under tens, and hundreds under hundreds. We now say "9 units − 7 units = 2 units; 6 tens − 2 tens = 4 tens; 4 hundreds − 3 hundreds = 1 hundred. Hence the difference is 142." *Show proof.*

EXERCISES.

Copy, subtract, explain, prove:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
824	569	997	965	896	8953
413	245	743	752	544	3420

(7.)	(8.)	(9.)	(10.)
\$59.86	\$75.39	\$56.89	\$52.90
\$34.24	\$40.30	\$45.76	\$31.50

(11.)	(12.)	(13.)	(14.)
62,979	98,316	945.791	\$798.945
30,825	71,004	523.150	\$653.620

PROBLEMS.

NOTE.—Let the pupil first indicate the solution of each problem by using the minus sign, —.

1. An army went into battle with 6878 men, and came out with only 4345 men. How many men were missing?

Process Indicated.

6878 men — 4345 men = the number missing.

Process.	Explanation.
6878 4345 2533	1. Since the army went into battle with 6878 men and came out with only 4345, the number missing was 6878 *minus* 4345. 2. Since the unit of both the numbers is *one man*, the numbers are *like* and can be subtracted, the less from the greater; units from units, tens from tens, etc. Therefore we say "8 units — 5 units = 3 units, 7 tens — 4 tens = 3 tens, 8 hundreds — 3 hundreds = 5 hundreds, 6 thousands — 4 thousands = 2 thousands. Hence the number of men missing was 2533."

Proof.

PRINCIPLE.—*Subtrahend + Remainder = Minuend.*

4345 + 2533 = 6878.

2. A grain dealer, having 7890 bushels of wheat, sold 6370 bushels. How many bushels had he remaining?

Process Indicated.

7890 bushels — 6370 bushels = the bushels remaining.

3. Watches were invented at Nuremburg in 1477. How many years ago?

4. If I borrow $6798, and afterwards pay $3534, how much do I still owe?

5. Under a call for volunteers, California's quota was 3237 men; Arkansas's quota, 2025 men. Find the difference?

6. The population of Spain in 1820 was about 11,000,000; at present (1899) it is 17,550,216. Find the increase.

7. The exports of the United States from the Philippine Islands last year amounted to $4,982,857; their imports, to $162,446. Find the excess of the exports over the imports.

8. The population of Havana is 198,720, of Santiago, 71,300. Find the difference.

9. The telescope was invented in 1610. How many years between that date and 1899?

10. Harvey discovered the circulation of the blood in 1619. How many years after the invention of the telescope?

CHIEF DIFFICULTY OF SUBTRACTION.

1. From 594 take 368.

Process.

$$\begin{array}{r} 594 \\ 368 \\ \hline 226 \end{array}$$

Proof.

$$\text{Add} \begin{cases} 368 \\ 226 \end{cases} \quad \overline{594}$$

Explanation.

ANALYTIC AND SYNTHETIC.

594 = 5 hundreds + 9 tens + 4 units.
368 = 3 hundreds + 6 tens + 8 units.

The difficulty is that 8 units cannot be taken from 4 units. But one of the 9 tens = 10 units; 10 units + 4 units = 14 units. Hence we write:

$$\left.\begin{array}{l} 594 = 5 \text{ hundreds} + 8 \text{ tens} + 14 \text{ units} \\ 368 = 3 \text{ hundreds} + 6 \text{ tens} + 8 \text{ units} \end{array}\right\} \text{Subtracting}$$

we have 2 hundreds + 2 tens + 6 units = 226.

2. What is the first principle of subtraction?

3. On what principle does the proof depend?

4. From 703 take 549.

Process.

$$\begin{array}{r} {}^{6\ 9\ 13} \\ 703 \\ 549 \\ \hline 154 \end{array}$$

Explanation.

703 = 7 hundreds + 0 tens + 3 units
549 = 5 hundreds + 4 tens + 9 units

The difficulty is that we cannot take 9 units from 3 units, nor 4 tens from 0 tens. But one of the 7 hundreds = 10 tens; one of the 10 tens = 10 units; 10 units + 3 units = 13 units. Hence we may write

$$\left.\begin{array}{l} 703 = 6 \text{ hundreds} + 9 \text{ tens} + 13 \text{ units} \\ 549 = 5 \text{ hundreds} + 4 \text{ tens} + 9 \text{ units} \end{array}\right\} \text{Subtracting we}$$

have 1 hundred + 5 tens + 4 units = 154.

5. Give the principles of subtraction and prove the work.

6. From 367.280 take 298.356.

Process.

$$\begin{array}{r} 367.280 \\ 298.356 \\ \hline 68.924 \end{array}$$

Explanation.

$$\left.\begin{array}{l} 367.280 = 367 \text{ units} + 280 \text{ thousandths} \\ 298\ 356 = 298 \text{ units} + 356 \text{ thousandths} \end{array}\right\} \text{Sub-}$$

tracting we have 68 units + 924 thousandths = 68.924.

7. Where must the point always be placed in the remainder?

RULE FOR SUBTRACTION.

1. See that the numbers to be subtracted are like numbers.

2. Write the subtrahend under the minuend, units under units, etc.

3. Beginning at the right, subtract each lower figure from the one above it.

4. When necessary, increase the upper figure by 10 and diminish by 1 the next upper figure on the left.

EXERCISES.

1. Copy, subtract, explain, prove:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
864	1095	937	865	2537	954
559	867	645	593	658	893

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
2957	2794	3908	4002	8923	9114
1038	2406	2609	3962	2095	6983

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
$35.56	$40.19	$74.36	$82.99	$53.44	$6.12
$32.49	$38.02	$58.19	$58.03	$19.78	$5.125

(19.)	(20.)	(21.)	(22.)
618.724	9,651,782	69,503.48	8888.88
529.728	8,241,509	38,298.75	7890.10

2. What is the value of:

1. 81,214 — 53,467?	5. 862,493 — 729,603?
2. 104,321 — 58,461?	6. 998,765 — 567,890?
3. 831,408 — 337,529?	7. 9,327,325 — 3,586,143?
4. 740,037 — 357,320?	8. 4,986,384 — 2,998,796?

PROBLEMS.

1. If a man owes $97.66 and pays $70.89, how much does he then owe?

Process Indicated.

$97.66 — $77.89 = how much he then owes.

Process.	Explanation.
$97.66 70.89 $26.77	Since he owes $97.66, and pays $70.89, he still owes the difference between $97.66 and $70.89. Since the numbers have the same unit, *one dollar*, they are like numbers, and their difference can be found. It is $26.77.

NOTE.—In each problem let the process be indicated first, and then performed and explained.

2. A man bought some land for $8765, and sold it for $10,890. What was his gain?

3. The first line of telegraph was established in the United States in 1844. How long ago?

4. In 1890 the population of the United States was 62,622,250, and in 1840 it was 17,063,353. How much did it increase in the 50 years?

5. A man was born in 1785: what was his age in 1830?

6. How old was George Washington at the time of his death? He was born in 1732, and died in 1799.

7. 29,400 feet is the greatest depth of water measured. 37,000 feet is the greatest height reached by a balloon. Find by how much the greatest height reached exceeds the greatest depth reached.

8. The displacement of the battle-ship "Alabama" is 11,525; of the cruiser "Charleston," 3730. How much does the displacement of the "Alabama" exceed that of the "Charleston"?

9. The estimated population of the United States in 1800 was 5,308,483; in 1898 it was 74,500,000. Find the growth in population in the 98 years.

10. How many dollars must be added to $4872 to make $8021?

11. How many dollars increased by $74,015 make a million dollars?

12. What number must be taken from \$6412 to leave \$5366?

13. Find the value of \$8.052 — \$3.687.

14. John has \$20.19 and James has \$40. How much more money has James than John?

15. A bankrupt has \$6456 assets, and owes \$33,860. How much more does he owe than he can pay?

16. Mt. Everest is 29,062 feet high; Mt. Whitney is 14,900 feet high. How much higher is the former than the latter?

17. The height of Mt. Cenis, an Alpine peak, is 11,792 feet; the height of the pass over it is 6884 feet. How much higher is the mountain than the pass?

18. At an election 3245 persons voted, and the candidate elected received 1808 votes. How many did the defeated candidate receive?

19. Benjamin Franklin died in 1790, and was 84 years old at his death. When was he born?

20. Subtract MMMIX. from $\overline{\mathrm{LXVII}}\mathrm{XI}$.

ADDITION AND SUBTRACTION IN COMBINATION.

EXERCISES.

1. What is the value of 20,324 + 4756 — 13,186?

Process.

$$\begin{array}{r} 20{,}324 \\ 4{,}756 \\ \hline 25{,}080 \\ 13{,}186 \\ \hline 11{,}894 \end{array}$$

Explanation.

ANALYTIC AND SYNTHETIC.

The sign + signifies that I must add 20,324 and 4756. Adding, the sum is 25,080.

The sign — signifies that I must subtract from that sum 13,186. Subtracting, the remainder is 11,894.

2. What is the value of:

1. 23,732 — 9478 + 9273?
2. 25,657 + 10,898 — 2597?

3. $20{,}201 - 9022 + 2002$?
4. $132{,}571 - 90{,}798 + 78{,}318$?
5. $\$238.70 - \$53.36 + \$22.27$?

A **Parenthesis**, ( ), or **Vinculum**, ———, indicates that all the quantities it incloses are to be considered as a single quantity; as, $(2 + 5 + 10 + 13)$, or $\overline{2 + 5 + 10 + 13}$.

2. What is an **Equation**?

3. Prove the following equations to be correct:

First perform the operations indicated within the parentheses.

1. $40 - (2 + 5 + 10 + 13) = 10.$
2. $(355 + 637 + 403) - 977 = 418.$
3. $2543 - \overline{504 + 600 + 725} = 714.$
4. $10{,}000 - (275 + 220 + 35 + 3675) = 5795.$
5. $(300 + 100 + 95 + 60 + 125) - \overline{125 + 25 + 40} = 490.$

4. Complete the following partial equations:

1. $(350{,}000 + 225{,}100 + 4000 + 96{,}000) - 450{,}120 = ?$
2. $23{,}191{,}876 - \overline{3{,}204{,}313 + 434{,}495} = ?$
3. $(367 + 875 + 1012) - \overline{423 + 912} = ?$
4. $(36 + 200 + 150) - 331 = ?$

PROBLEMS.

Note.—The indicating of a solution often facilitates the completing of it. The pupil should be faithfully drilled in the use of signs to indicate the actual solution to be made.

1. Mr. A. gave his note for \$6000. He paid at one time \$3586 and at another time \$2000. How much remained to be paid?

Process Indicated.

$\$6000 - (\$3586 + \$2000) =$ debt remaining.

Process.

Paid	$3586
Paid	2000
Total paid .	5586
Note	$6000
Paid	5586
Balance due,	$414

Explanation.

Since he paid $3586 at one time and $2000 at another time, he paid at both times $3586 + $2000, or $5586.

Since his note, or debt, was $6000, he still owes the difference between $6000 and $5586, or $414.

2. A man finished a journey of 972 miles in 3 days; the first day he travelled 398 miles; the second day, 409 miles. How many miles did he travel the third day?

Process Indicated.

972 miles — (398 miles + 409 miles) = miles travelled third day.

3. A produce dealer had in bank $6032, and checked out on one day $2360, and on the next day, $2307. How much had he left in bank?

4. A cattle dealer had 982 cattle, bought 621 more, lost by disease 32, sold 416. How many remained?

5. How much does the sum of 3694 and 5005 exceed the difference of 10,532 and 3903?

6. In 1880 there were 16,120 Indians and 75,025 Chinese in California. How many were there of both, and how many more Chinese than Indians?

7. A. sells a house to B. for $3486; B. sells it to C. at a gain of $360; C. sells it to D. at a loss of $285. What does D. pay for the house?

8. I have a yearly income of $10,000. I pay $450 for rent, $230 for fuel, $50 for medical attendance, and $4786 for all my other expenses. What have I saved at the end of the year?

9. A tract of land containing 2753 acres was divided among four persons, A., B., C., D. A.'s share was 679 acres,

B.'s was 47 acres more, C. had 75 acres less than B., and D. had the remainder. What were the shares of B., C., and D.?

10. A man has an income of $1845; he spends $645 for board, $456 for clothing, and $297 for other expenses. What has he saved at the end of the year?

11. A man deposits in bank $2374. At one time he draws out $897, at another, $543, and at a third time, $689. How much has he remaining in bank?

12. I bought 24 shares of bank stock for $2863, and paid a broker $22 for purchasing the same; afterwards sold it for $3000. What was my profit?

13. A farmer invests $18,975 as follows: in land, $11,893; in horses, $1575; in mules, $4297; in stock, $937; the remainder in tools. How much did he expend for tools?

14. A man bought three houses; for the first he gave $3585; for the second, $5260; for the third, as much as for the other two. He sold them all for $15,280. Did he gain or lose, and how much?

15. From the sum of 874 and 398 subtract their difference.

16. I have a bin that holds 936 bushels. I put into it 383 bushels, and again 457 bushels. How much more will the bin hold?

17. Some excursionists made a journey that cost them $492.97. Railroad fares cost $203.26; hack hire cost $48.36; steamboat fare cost $72.46. The remainder was expended for food. What did their food cost?

18. A merchant bought silk for $486, muslin for $286, linen for $346, and sold the whole for $1200. How much did he gain?

19. An estate worth $23,460 was bequeathed to a wife and two children. The widow received $7820; the son received $3400 less; and the daughter, the balance. Find the daughter's share.

REVIEW.

1. Define the following terms:

1. Minuend.	6. Analysis.
2. Subtrahend.	7. Synthesis.
3. Difference	8. Solution.
4. Remainder.	9. Parenthesis.
5. Minus.	10. Vinculum.

2. Repeat the principles of Subtraction.
3. Repeat the rule for Subtraction.
4. Invent five problems involving both Addition and Subtraction, and indicate the process of solution.

MULTIPLICATION.

INDUCTIVE STEPS.

1. How many are $3 + 3$?
2. How many are $3 + 3 + 3$?
3. How many are two times 3?
4. How many are three times 3?
5. $3 + 3 + 3 = 9$, and 3 times $3 = 9$. The first operation is addition; the second is multiplication. Which is the shorter?
6. Is Multiplication a short kind of addition?
7. What will 4 apples cost at 2 cents apiece?

Process.

ADDITION.	MULTIPLICATION.
2 cents	
2 cents	
2 cents	2 cents
2 cents	4
8 cents	8 cents

Explanation.

Since 1 apple costs 2 cents, 4 apples will cost 4 times 2 cents, which are 8 cents.

In the preceding process, is 2 cents concrete? Is 8 cents concrete? Is the 4 concrete or abstract?

8. What is the cost of 5 yards of ribbon at 10 cents a yard?

Write and explain the process.

In the preceding process, did you write 5 yards or simply 5? What kind of number is 5?

You took 10 cents *five* times; hence 5 is called the **Multiplier**, and the process is called **Multiplication.**

DEFINITIONS.

1. **Multiplication** is a *short process* of finding the sum of two or more equal numbers; or of taking a number as many times as there are *units in another number.*

2. The **Multiplicand** is the number to be taken or repeated.

3. The **Multiplier** is the number which shows how many times the multiplicand is to be taken or repeated.

4. The **Product** is the result of the multiplication.

5. The Multiplicand and Multiplier are called **Factors** of the product.

6. The **Sign of Multiplication** is an oblique cross, $\times$.

$5 \times 4 = 20$, is read "5 times 4 are 20"; or, "5 multiplied by 4 equals 20."

7. What are the factors of 20?

Is $4 \times 5 = 5 \times 4$ a correct equation?

Proof.

* * * * *
* * * * *
* * * * *
* * * * *

The 20 stars as arranged equal 5 stars in a line taken 4 times, or 4 stars in a column taken 5 times.

Hence *the product is the same in whatever order the factors are taken.*

8. *What is a parenthesis and what does it signify? A vinculum?*

134 — (9 + 6) × 3 signifies that you must take 9 + 6 three times and subtract the result from 134.

Process.

$(9 + 6) \times 3 = 45$; $134 - 45 = 89$.

PRINCIPLES.

1. The multiplier must be considered an abstract number.
2. The product and multiplicand are like numbers.
3. Either factor may be taken as the multiplier.

EXERCISES.

For Analytic and Synthetic Explanation.

1. What is the product of 347 multiplied by 3?

1st Process.	2d Process.	3d Process.
	347	
	3	
347	21	
347	12	347
347	9	3
1041	1041	1041

Explanation.

1st Process.—We say, "A product is the result of multiplication. Since multiplication is a short process of adding equal numbers, we can find the product by addition; adding, we have 1041."

2d Process.—We say, "Since 347 must be taken 3 times, each order of units must be taken 3 times. 3 times 7 units = 21 units; 3 times 4 tens = 12 tens; 3 times 3 hundreds = 9 hundreds; adding, we have 1041."

3d Process.—The shortest process is generally the best in practice. We say, "3 times 7 = 21; we reserve the 2; 3 times 4 = 12, 12 and 2 reserved are 14; we reserve the 1; 3 times 3 = 9, 9 and 1 reserved = 10. Hence the product is 1041."

Proof.

The first and second processes are proof of the accuracy of the third.

2. Complete, explain, and prove the following:

(1.)	(2.)	(3.)	(4.)	(5.)
365	674	756	327	408
2	4	3	6	5

(6)	(7.)	(8.)	(9.)	(10.)
\$5.09	\$7.95	\$12.75	\$55.06	\$43.60
5	8	7	9	12

3. How many places in the product must be pointed off for cents?

4. Multiply:

1. 8692 by 8.	6. 13,896 by 3.
2. 5328 by 7.	7. 52,209 by 5.
3. 10,318 by 5.	8. 68,387 by 7.
4. 7289 by 9.	9. 79,588 by 4.
5. 17,345 by 4.	10. 91,983 by 9.

5. Find the value of:

1. 420×9.	5. $40{,}527 \times 4$.
2. 9059×2.	6. $305{,}238 \times 5$.
3. $78{,}059 \times 3$.	7. $40{,}597 \times 6$.
4. $1{,}790{,}478 \times 7$.	8. $910{,}362 \times 8$.

The parts of an equation, right and left of the sign of equality, are called its *members*.

6. Find a second member for each of the following:

1. $127 + (2 + 8) \times 9 + 85 =$
2. $209 - (27 + 4) \times 5 =$
3. $3300 + 86 \times \overline{6 + 4} =$
4. $3246 - \overline{329 + 524} \times 3 =$
5. $9203 - 6 \times (350 - 239) =$
6. $(275 + 262) \times 3 - 2 \times (68 - 39) =$
7. $1935 - 195 + 186 \times 4 =$

PROBLEMS.

Let the pupil indicate the solution of each problem by using the sign $\times$.

1. If sound moves 1092 feet in a second, how far does it move in 5 seconds?

Process Indicated.

$1092 \times 5 =$ how far it moves.

Process.

$$\begin{array}{r} 1092 \text{ feet} \\ 5 \\ \hline 5460 \text{ feet} \end{array}$$

Explanation.

Since sound moves 1092 feet in 1 second, in 5 seconds it moves 5 times 1092 feet; 1092 feet $\times$ 5 = 5460 feet.

What principles are involved?

2. When wheat is worth $1.25 per bushel, what is the value of 9 bushels?

3. What will 7 horses cost at $175.35 each?

4. Since in 1 mile there are 1760 yards, how many yards are there in 9 miles?

5. Find the cost of 4886 sheep at $6 a head.

6. Each workman in an iron-foundry is paid $605 a year: what do 11 men receive at that rate?

7. A bushel of corn weighs 56 pounds: find the weight of 12 bushels.

8. The distance to the moon is 240,000 miles: what is 10 times that distance?

9. If the distance of the earth from the sun is about 91,430,000 miles, how many miles is 9 times that distance?

10. If an army major receives monthly $208.33, what is the monthly pay of 12 majors?

11. Mr. White owns 3 houses, and the first house is worth $3872; the second, 3 times as much; and the third, 7 times as much. Find the cost of the 3 houses.

Process Indicated.

$3872 + (3872 \times 3) + (3872 \times 7) =$ cost required.

12. A farmer's wife took to a store 3 pounds of butter worth 33 cents a pound, and bought 12 yards of calico at \$.08 a yard. Find the balance due her.

13. Which are worth more, 7 cows at \$35 apiece or 3 horses at \$75 apiece?

14. A lady bought a bicycle for \$100; she rented it to a friend for 5 months at \$3 a month, and finally sold it for \$75. Did she gain or lose, and how much?

15. A man sold three houses; for the first he received 3575 dollars; for the second, \$950 less; for the third, three times the difference between the price of the first and second. What did he receive for the three?

CHIEF DIFFICULTY OF MULTIPLICATION.

1. Multiply 438 by 234.

1st Process.	2d Process.
438	438
234	234
1752	1752
13140	1314
87600	876
102,492	102,492

Proof.

234
438
1872
702
936
102,492

Explanation.

1st Process.—

$$234 = \begin{cases} 4 \text{ units} = 4 \text{ units.} \\ 3 \text{ tens} = 30 \text{ units.} \\ 2 \text{ hundreds} = 200 \text{ units.} \end{cases}$$

Therefore, we are to multiply 438 firstly by 4 units, secondly by 30 units, thirdly by 200 units, and then find the sum of the three partial products.

Multiplying by 4 we have 1752 units; multiplying by 30 we have 13140 units; multiplying by 200 we have 87600 units. The sum of these products is 102,492 units.

2d Process.—Since 13,140 units = 1314 tens, and since 87,600 units = 876 hundreds, we omit the ciphers, and, writing 1314 as tens, and 876 as hundreds, the significant figures keep their relative positions, and the result of the addition is the same as before.

Proof.—State the principle on which the proof depends.

2. Find the product, explain the process, and show proof of accuracy:

(1.)	(2.)	(3.)	(4.)	(5.)
317	483	847	657	343
15	16	23	26	37

(6.)	(7.)	(8.)	(9.)	(10.)
\$13.35	\$17.85	\$30.23	\$42.93	\$55.66
33	43	72	81	29

Put a decimal point in each product.

3. Multiply:

1. 382 by 19.
2. 384 by 35.
3. 405 by 53.
4. 534 by 37.
5. 645 by 73.
6. 843 by 54.
7. 917 by 46.
8. 903 by 74.
9. 593 by 91.
10. 578 by 83.
11. 619 by 96.
12. 777 by 86.
13. 910 by 52.
14. 732 by 62.
15. 839 by 93.
16. 327 by 67.
17. 931 by 95.
18. 733 by 123.
19. 981 by 234.
20. 891 by 345.

4. How many are 837×38?

5. How many are \$57.30 $\times$ 665?

6. $2537 \times 47 =$ how many?

7. $5386 \times 65 =$ how many?

8. $4860 \times 9574 = ?$

9. \$357.63 $\times 47 = ?$

10. Find the second member of the following unfinished expressions:

1. \$9.672 $\times 635 =$
2. $5732 \times 891 =$
3. $51{,}526 \times 527 =$
4. $97{,}601 \times 987 =$

11. Multiply five thousand nine hundred sixty-five by six thousand nine.

12. Multiply four hundred sixty-two thousand six hundred nine by itself.

13. Multiply eight hundred forty-nine million six hundred seven thousand three hundred six by nine hundred thousand two hundred four.

14. Multiply 704 million 130 thousand 496 by three thousand three hundred one.

15. Multiply one hundred twenty-three *and* 45 hundredths by 804.

16. Multiply 415 *and* 5 hundredths by 367.

17. Multiply 113 dollars *and* 41 cents by 613.

18. Multiply XLVIII. by XIX.

19. Multiply CDLXIV. by CDIV.

20. Form an equation of 220,056,121, and 26,626,776.

21. Find the value of $(3467 \times 7004) - (3467 \times 704)$, and form an equation.

PROBLEMS.

1. Find the cost of a farm of 202 acres at $102 per acre.

Process Indicated.

$\$102 \times 202 =$ cost.

2. A farmer had 105 rows of trees, each row containing 105 trees. How many trees had he?

3. If horses are worth $117 each, and oxen $85.50 a pair, what must I pay for 18 horses and 5 pairs of oxen?

4. If 786 yards of cloth can be made in one day, how many yards can be made in 1252 days?

5. A grocer's sales average $19 a day for the month of March; leaving out 5 days for Sundays, how much money did he receive during the month?

6. John takes 1434 steps in going to school; if he goes and returns twice a day, how many steps will he take in 24 days?

7. There are 5280 feet in a mile. How many feet are there in 18 miles?

8. If James sells 57 papers a day and Thomas 65 papers, how many more does Thomas sell than James in 54 days?

Indicate the process by using the signs, $-$, $\times$, $=$.

9. If I buy 17 tons of iron at \$38.75 a ton, and 26 tons at \$40.25 a ton, how much shall I gain by selling the whole at \$42.50 a ton?

10. $(? + ?) \times ? - (? \times ? + ? \times ?) = ?$ Substitute a number for each of the interrogation marks in the first member, solve, and state your problem.

SHORT PROCESSES.

When there are ciphers on the right of multiplicand, or of multiplier, or of both.

1. Multiply 2 by 30.

Process. **Explanation.**

$$\begin{array}{r|l} 2 & \\ 3 & 0 \\ \hline \multicolumn{2}{c}{60} \end{array}$$

The factors of 30 are 3 and 10. We say "$2 \times 3 = 6$; and, by annexing a cipher to 6, we multiply it by 10, and have 60."

2. Multiply 30 by 4.

$$\begin{array}{r|l} 3 & 0 \\ 4 & \\ \hline \multicolumn{2}{c}{120} \end{array}$$

The factors of 30 are 3 and 10. We say "$3 \times 4 = 12$; annexing a cipher to 12 multiplies it by 10, and we have 120."

Does the order in which factors are used in multiplying affect the result?

3. Multiply 40 by 500.

$$\begin{array}{r|l} 4 & 0 \\ 5 & 00 \\ \hline \multicolumn{2}{c}{20,000} \end{array}$$

$$40 = 4 \times 10.$$
$$500 = 5 \times 100.$$
$$4 \times 5 \times 10 \times 100 = 20,000.$$

We say "$4 \times 5 = 20$; and, annexing one cipher, we multiply by 10 and have 200; annexing two ciphers to that result, we thus multiply it by 100, and have 20,000."

RULE.

Cut off and reserve the ciphers on the right; then multiply, and to the product obtained annex the ciphers reserved.

EXERCISES AND PROBLEMS.

1. Multiply 486 by 10. By 100. By 400.
2. Multiply 9560 by 40. By 80. By 1000.
3. Multiply 2870 by 600. By 800. By 900.
4. Multiply 2490 by 300. By 3000. By 4400.
5. Multiply 59,700 by 360. By 4300. By 7600.
6. Multiply 42,300 by 320. By 3700. By 57,000.
7. Multiply 4,871,000 by 270,000. By 304,000.
8. Multiply \$7849.93 $\times$ 400. By 5000.
9. Multiply 600,700 $\times$ 6000. By 4,004,000.
10. Multiply CDXL. by $\overline{\text{M}}$. By $\overline{\text{LIX}}$.

Process.

$$\begin{array}{r|l} \$6 & 000 \\ 2 & 0 \\ \hline \multicolumn{2}{r}{\$120{,}000} \end{array}$$

11. If the yearly pay of a rear-admiral is \$6000, how much will he have received in 20 years?

12. One mile contains 5280 feet. How many feet in 600 miles?

13. There are 350 rows of trees in an orchard, 120 trees in a row, and 3000 apples on each tree. How many apples in the orchard?

14. One acre contains 160 square rods. How many square rods in 300 acres?

15. One pound avoirdupois contains 7000 troy grains. How many grains in 230 pounds?

16. A short ton = 2000 pounds. How many pounds in 570 tons?

17. The circumference of the earth = about 25,000 miles. One mile = 1760 yards. How many yards around the earth?

18. If one bushel of corn costs $.65, what will 1000 bushels cost?

19. At $160 an acre, what will 500 acres cost?

20. One hour = 60 minutes; one minute = 60 seconds. How many seconds in 24 hours?

MULTIPLICATION BY FACTORS.

1. You have learned that the multiplicand and multiplier are called the factors of the product.

What factors will produce 4? 6? 8? 10? 12? 15? 16? 18?

2. All numbers that can be thus factored are called **Composite numbers.**

PRINCIPLE.

Multiplication may be performed by using the factors of the multiplier.

EXERCISES.

1. Multiply 5 by 6, using the factors of 6.

Process.

$6 = 3 \times 2.$

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \\ 2 \\ \hline 30 \end{array}$$

Explanation.

$6 = 3 \times 2$; therefore, we say "6 times 5 = 2 *times* 3 *times* 5; 3 times 5 = 15, and 2 times 15 = 30."

Proof.

$$\left.\begin{array}{l} \left.\begin{array}{ccccc} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{array}\right\} = 15 \\ \\ \left.\begin{array}{ccccc} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{array}\right\} = 15 \end{array}\right\} = 30$$

PROOF.—Six rows of 5 stars each = $5 \times 6 = 30$, the whole number. 3 rows of 5 stars each = $5 \times 3 = 15$. 2 groups of 15 each = $15 \times 2 = 30$, the whole number.

2. Multiply 438 by 15.

Process.

$15 = 5 \times 3.$

$$\begin{array}{r} 438 \\ 5 \\ \hline 2190 \\ 3 \\ \hline 6570 \end{array}$$

Explanation.

Since the factors of 15 are 5 and 3, we first, for convenience, multiply by the larger factor, 5, and that result by 3, and obtain 6570.

Proof.

$438 \times 15 = 6570.$

3. Multiply, using factors:

1. 6809 by 49.
2. 435,261 by 63.
3. 310,204 by 48.
4. 97,387 by 45.
5. 91,849 by 36.
6. 4953 by 81.
7. 14,953 by 144.
8. 2348 by 21.

PROBLEMS COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION.

First indicate the process.

1. I have 10 bags of coffee, each containing 50 pounds. How many pounds of coffee have I?

2. If hay is worth $14.50 per ton, and oats $.56 a bushel, what will be the cost of 27 tons of hay and 200 bushels of oats?

3. A drover bought 43 cows at $22 each, 64 sheep at $13 each, and 16 horses at $135 each, and sold them all for $4010. How much did he gain?

4. A freight train consists of 28 cars, and each car contains 136 casks of lime, weighing 240 pounds each. How many pounds of lime in the whole cargo?

5. Sound travels at the rate of 1092 feet in a second. If between the flash of lightning and the clap of thunder there were 9 seconds, how far distant was the cloud that produced the flash?

6. If 250 pounds of charcoal are used in making a ton of gunpowder, how many pounds will be used for 1280 tons of gunpowder?

7. What sum of money will be required to pay a regiment of 987 men for a year's services, at $18 a month for each man?

8. A merchant sold 324 barrels of apples at $4.75 a barrel, and gained $162 on the transaction. What did his apples cost him?

9. A cattle train is made up of 17 cars, and each car contains 53 sheep. The average weight of the sheep is 115 pounds. How much do they all weigh?

10. A farmer bought a farm containing 10 fields; 3 of the fields contained 9 acres each; 3 other of the fields, 12 acres each; the remaining 4 fields, each 15 acres. How many acres in the farm? What was the cost of the farm at $21 an acre?

11. If it requires 1716 pickets to fence one side of a square lot, how many pickets will be required to fence 13 lots of the same size and shape?

12. Mrs. Brown bought 12 yards of oilcloth at 65 cents a yard, and 32 yards of ingrain carpet at 75 cents a yard. What did she pay for all?

13. One day = 86,400 seconds; one year = 365 days. If light moves at the rate of 186,000 miles in a second, how far distant is a star whose light is one year in reaching the earth?

14. An army lost in battle 315 killed, 417 wounded; the enemy lost in killed and wounded 17 times as many. How many soldiers were killed and wounded in this battle?

15. If 1327 barrels of flour will feed the inhabitants of a city for one day, how many barrels will supply them for two years?

16. The Erie Railroad is about 425 miles long, and cost sixty-five thousand dollars a mile. When 9,645,635 dollars were paid, what was the balance due?

17. Two vessels are 4500 miles apart, and travel toward each other; one at the rate of 91 miles a day, and the other at the rate of 85 miles a day. How far apart are they at the end of 24 days?

18. Two vessels start from New York for Liverpool; one sails at the rate of 138 miles a day; the other, at the rate of 215 miles a day. How far will they be apart at the end of nine days?

19. What is the product of three hundred eleven million two hundred twenty-one thousand multiplied by two hundred three thousand one hundred five?

MISCELLANEOUS EXERCISES.

1. Complete these equations: $(16 - 11 + 2) \times 5 = ?$ $(4 + 15) \times (15 - 4) \times 6 = ?$

2. Use the signs, ( ), +, —, ×, =, in forming an equation of your own.

3. An unfinished equation is: 63,915 + (?) = one million. Find the required part.

4. (?) + 4872 = 8021. Complete the equation.

5. 5301 — (?) = 4255. Complete the equation.

6. Multiply three million three by one hundred thousand one.

7. Write the immediately preceding numbers in Roman numerals.

8. Multiply 3008 by 132, using the two factors of 132 whose difference is 1.

9. Find the value of $\overline{6145 - 3408 + 1931} \times 3400 - (33{,}600 \times 105)$.

10. A drover had 690 sheep; he sold 340 to one man, 324 to another, and then bought enough to make his number 700. How many did he buy?

REVIEW.

1. Define the following terms:

1. Multiplication.	7. Parenthesis.
2. Multiplicand.	8. Vinculum.
3. Multiplier.	9. Proof.
4. Product.	10. Members.
5. Factors.	11. Composite number.
6. Sign of Multiplication.	12. Equation.

2. Repeat the principles of Multiplication.

3. Repeat the rule for Multiplication when the factors are composed of significant figures with ciphers on their right.

4. What is the principle respecting factors of the multiplier?

5. Illustrate the principle.

6. Invent five problems that will involve Addition, Subtraction, and Multiplication. Indicate the solution by the signs, $+$, $-$, $\times$.

DIVISION.

INDUCTIVE STEPS.

1. Since $2 \times 3 = 6$, the 2 and 3 are called what?

2. Then if 2 is one factor of 6, what is the other?

3. Why must 3 be the other?

4. Because 2 times 3 is 6, we say that 2 is contained in 6 *three* times.

5. How many times is 4 contained in 8?

Process.

$$\begin{array}{l} 4\,)\,8\,(\,2 \\ \quad\; \underline{8} \\ \quad\; 0 \end{array}$$

Explanation.

We say "4 is contained in 8 *two* times, because 2 times $4 = 8$."

6. How many times is 5 contained in 10?

Write the process and explain.

7. The process of finding how many times one number is contained in another is called **Dividing**.

8. Divide 16 by 8, and show that division is a short method of subtraction.

1st Process.

$$\left.\begin{array}{r}16\\ 8\end{array}\right\}\text{Subtract.}$$
$$\left.\begin{array}{r}8\\ 8\end{array}\right\}\text{Subtract.}$$
$$0$$

2d Process.

$$\begin{array}{r}8\,)\,16\,(\,2\\ \underline{16}\\ 0\end{array}$$

Explanation.

1st Process.—We say "Subtracting 8 *once*, we have 8 remaining; subtracting 8 a second time, we have 0 remaining; therefore, 8 is contained in 16 *two* times."

2d Process.—Since 8 times 2 = 16, 8 is contained in 16 two times.

9. One factor of 24 is 3, what is the other factor?

10. Dividing by 7 separates a number into how many equal parts?

One of seven equal parts is called one-seventh, written $\frac{1}{7}$.

$\frac{1}{7}$ of 28 equals what?

11. If a farmer pays $28 for 7 sheep, how much is that apiece?

Process.

$$\begin{array}{r}7\,)\,28\,(\,4\\ \underline{28}\\ 0\end{array}$$

Explanation.

Since he pays $28 for 7 sheep, he pays for each one-seventh of $28, or $4.

Did you divide by 7 sheep, or simply by 7? Is 7, then, an abstract or a concrete number?

Is your answer 4 or $4?

12. How many sheep at $4 apiece can a farmer buy for $28?

If $4 is one factor of $28, is $7 the other factor? Is 7 sheep? Is 7?

What, however, does the 7 indicate?

DEFINITIONS.

1. Division is the process of finding *how many times* one number is contained in another, or of finding *one of the equal parts* of a number.

Note.—This latter operation is called *Partition*.

2. The **Dividend** is the number to be divided.

3. The **Divisor** is the number by which we divide.

4. The **Quotient** is the result obtained.

5. The number which is sometimes left after dividing is called the **Remainder**.

When the remainder is 0, the division is said to be **exact.**

6. The **Sign of Division** is ÷.

21 ÷ 7 = 3, is read "21 divided by 7 equals 3."

Take notice that the dividend is written *before* the sign; the divisor *after* the sign.

In practice it is found convenient to indicate division thus:

$$7\,\underline{)\,21}\atop{\;\;\;3}\;;\text{ or thus: }\begin{array}{l}7\,)\,21\,(\,3\\ \quad\;\;\underline{21}\end{array}\;;\text{ or thus: }\frac{21}{7}=3.$$

PRINCIPLES.

1. **Dividing a number by one of its factors gives the other factor for the quotient.**

2. **When the divisor is an abstract number, the dividend and quotient are like numbers.**

3. **When the dividend and divisor are like numbers, the quotient is an abstract number.**

4. **The divisor multiplied by the quotient reproduces the dividend.**

EXERCISES

For Analytic and Synthetic Explanation.

The Divisor not exceeding 12.

1. If 6 is one factor of 24, what is the other?

Process.

Divisor. Dividend. Quotient.

$$\begin{array}{lll}6\,) & 24 & (\,4\\ & \underline{24} & \\ & \;\,0 & \end{array}$$

Explanation.

We say "Since 6 is one factor of 24, and since 6 times 4 = 24, 4 is the quotient, or other factor."

State the principle.

2. 474 has a factor, 2; find the other factor.

Process.

$$\begin{array}{l} 2)474(237 \\ \underline{4} \\ \;7 \\ \;\underline{6} \\ \;14 \\ \;\underline{14} \\ \;\;0 \end{array}$$

Explanation.

We say "474 = 4 hundreds, 7 tens, 4 units. 4 hundreds ÷ 2 = 2 hundreds. Bring down 7 tens; 7 tens ÷ 2 = 3 tens and 1 ten remaining. 1 ten and 4 units = 14 units; 14 units ÷ 2 = 7 units. The quotient is 2 hundreds, 3 tens, 7 units, or 237.

Proof.

$$237 \times 2 = 474.$$

State the principle.

3. 5 is one factor of 35; find the other.
4. 8 is one factor of 48; find the other.
5. 7 is one factor of 49; find the other.
6. 9 is one factor of 72; find the other.
7. 12 is one factor of 108; find the other.
8. 11 is one factor of 132; find the other.
9. 12 is one factor of 144; find the other.
10. Divide 144 by 8.

We may express the division in four different ways:

(1.)

$$\begin{array}{l} 8)144(18 \\ \;\;\;\underline{8} \\ \;\;\;64 \\ \;\;\;\underline{64} \\ \;\;\;\;0 \end{array}$$

(2.)

$$\begin{array}{l} 8\,\underline{)144} \\ \;\;\;\;18 \end{array}$$

(3.)

$$\frac{144}{8} = 18$$

(4.)

$$144 \div 8 = 18$$

The first is called **Long Division**; the second, third, and fourth, **Short Division.**

Explanation.

144 = 1 hundred 4 tens 4 units. 1 is not divisible by 8, hence we say "1 hundred + 4 tens = 14 tens; 14 tens ÷ 8 = 1 ten, with 6 tens remaining; 6 tens + 4 units = 64 units; 64 units ÷ 8 = 8 units, with 0 units remaining. Therefore 18 is the exact quotient."

Proof.

$$18 \times 8 = 144$$

11. Solve by Long Division, explain, and prove the following:

(1.)	(2.)	(3.)	(4.)
3) 849 (	5) 940 (	7) 497 (	8) 992 (

Process.

```
5 ) 7506 ( 1501
    5
    25
    25
     06
      5
      1 rem.
```

(5.)	(6.)	(7.)
9) 7506 (	10) 41,690 (	11) 103,961 (
(8.)	(9.)	(10.)
12) 113,820 (	11) 57,893 (	12) 74,856 (
(11.)	(12.)	(13.)
8) 38,496 (	9) 43,281 (	12) 2964 (

(14.)	(15.)	(16.)
3) 12,414 (	5) 32,795 (	4) 374,864 (
(17.)	(18.)	(19.)
3) 629,274 (	5) 947,860 (	6) $1589.10 (
(20.)	(21.)	(22.)
7) $6472.69 (	8) $1025.68 (	9) $1999.98 (

Place a decimal point in the quotient.

12. Solve by Short Division the following:

(1.)	(2.)	(3.)	(4.)
6) 1698	10) 1980	7) 994	8) 1984
(5.)	(6.)	(7.)	(8.)
9) 15,012	10) 41,690	11) 103,961	12) 113,820

(9.)	(10.)	(11.)	(12.)
11) 57,893	12) 74,856	8) 38,496	9) 43,281

(13.)	(14.)	(15.)	(16.)
7) 193,760	12) 29,640	3) 24,828	5) 32,795

(17.)	(18.)	(19.)	(20.)
4) 374,864	3) 629,274	5) 947,860	6) 158,910

(21.) $\frac{\$48.56}{8} = ?$ (22.) $\frac{\$6.44}{7} = ?$ (23.) $\frac{\$976.50}{9} = ?$

Place a decimal point in the quotient.

(24.) Pounds. $\frac{360,723}{11} = ?$ (25.) Bushels. $94,000 \div 8 = ?$ (26.) Rods. $760,344 \div 12 = ?$

In (24), (25), (26), are your quotients abstract or concrete? Give the principle.

The Divisor exceeding 12, but less than 100.

1. Divide 34,028 by 13.

Process.

```
     th. h.t.u.  th.h.t.u.
13 ) 34,028 ( 2617
     26
     ---
      80 h.
      78
      ---
       22 t.
       13
       ---
        98 u.
        91
        ---
         7
```

Explanation.

34,028 = 34 thousands 0 hundreds 2 tens 8 units. 34 thousands ÷ 13 = 2 thousands, with 8 thousands remaining; 8 thousands + 0 hundreds = 80 hundreds; 80 hundreds ÷ 13 = 6 hundreds, with 2 hundreds remaining; 2 hundreds + 2 tens = 22 tens; 22 tens ÷ 13 = 1 ten, with 9 tens remaining; 9 tens + 8 units = 98 units; 98 units ÷ 13 = 7 units, with 7 units remaining.

2. Divide 5684 by 14.
3. Divide 6480 by 15.

4. Divide 2672 by 16.	13. $1504 \div 47 = ?$
5. Divide 2928 by 17.	14. $5289 \div 43 = ?$
6. Divide 4147 by 18.	15. $9464 \div 52 = ?$
7. Divide 8797 by 19.	16. $5612 \div 61 = ?$
8. Divide 6872 by 24.	17. $6336 \div 72 = ?$
9. Divide 8519 by 27.	18. $4557 \div 21 = ?$
10. Divide 9672 by 31.	19. $3264 \div 24 = ?$
11. Divide 28,490 by 15.	20. $2664 \div 37 = ?$
12. Divide 18,476 by 42.	21. $3465 \div 99 = ?$

PROBLEMS.

1. 3 feet = 1 yard. How many yards in 927 feet?

Process.

LONG DIVISION.

feet. feet.

```
3 ) 927 ( 309
    9
    --
     27
     27
     --
      0
```

Explanation.

Since 3 feet = 1 yard, 927 feet equal as many yards as 3 is contained times in 927. $927 \div 3 = 309$. Therefore 927 feet equal 309 yards.

How is the 0 in the quotient obtained?

In the process, is 309 an abstract or a concrete number? State the principle.

Is the process long or short division?

Write the process in short division and explain the steps.

2. I paid 285 cents for a railroad ticket at 3 cents a mile. How many miles did I ride?

3. If you buy 12 pounds of soap for 96 cents, how much do you pay for a pound?

4. If the circumference of a wheel is 12 inches, how many times will it revolve in moving 1728 inches?

5. If it takes 5 bushels of wheat to make a barrel of flour, how many barrels can be made from 65,890 bushels?

6. A merchant has 1620 yards of calico, which he wishes to cut into 15-yard patterns. How many patterns will he have?

7. How many times must you take 7 dollars to make 567 dollars?

8. A boat sails 1872 miles, going at the rate of 18 miles an hour. How many hours does it sail?

9. How many sacks, each containing 55 pounds, can be filled from 2035 pounds of flour?

10. If I divided \$576 equally among some men, giving to each man \$8.00, how many men were there?

11. If 4 weeks make a month, how many months are there in 264 weeks?

12. Into how many lots of 39 acres each can a tract of land containing 6318 acres be divided?

13. The circumference of a wheel of a bicycle is 7 feet. How many revolutions will it make in going 18,480 feet?

14. How many sheep at \$9 a head can be bought for \$1377?

15. A man bought land at \$87 an acre, paying \$31,755 for it. How many acres did he buy?

16. How many feet are there in a mile, if 42 miles contain 221,760 feet?

17. Divide four hundred eighteen thousand six hundred forty-eight by twenty-four.

18. If the post-office sends 13,125 pounds of mail-matter in bags, each holding 75 pounds, how many bags will it require?

19. An estate worth 2943 dollars is to be divided equally among a father, mother, 3 daughters, and 4 sons. What will be the portion of each?

20. Solve the following equation:

$$(5280 - 1760 + 144) \times \frac{1728}{72} \div 12 = ?$$

21. I bought 15 horses at \$75 a head; at how much per head must I sell them to gain \$210?

EXERCISES.

The Divisor exceeding 100.

1. Divide 145,260 by 108.

Process.

```
108 ) 145,260 ( 1345
      108
      ---
       372
       324
       ---
        486
        432
        ---
         540
         540
         ---
```

Explanation.

145 thousand $\div$ 108 $=$ 1 thousand, with 37 thousand remaining; 37 thousand $+$ 2 hundred $=$ 372 hundred; 372 hundred $\div$ 108 $=$ 3 hundred, with 48 hundred remaining; 48 hundred $+$ 6 tens $=$ 486 tens; 486 tens $\div$ 108 $=$ 4 tens, with 54 tens remaining; 54 tens $+$ 0 units $=$ 540 units; 540 units $\div$ 108 $=$ 5, with 0 remaining.

2. Divide:

1. 1,874,774 by 162.
2. 1,206,528 by 192.
3. 815,898 by 421.
4. 199,864 by 301.
5. 315,008 by 428.
6. 503,652 by 564.
7. 705,776 by 728.
8. 892,696 by 839.
9. 902,260 by 916.
10. 683,537 by 987.

3. Find the value of:

1. 395,630 $\div$ 750.
2. 683,537 $\div$ 987.
3. 900,503 $\div$ 173.
4. 456,007 $\div$ 560.
5. 881,881 $\div$ 700.
6. 341,517 $\div$ 529.
7. 237,607 $\div$ 837.
8. 1,056,566 $\div$ 326.
9. 10,365,051 $\div$ 3021.
10. 2,159,450 $\div$ 2465.
11. 496,839,715 $\div$ 1047.
12. 9,325,814 $\div$ 2042.
13. 27,227,704 $\div$ 6472.
14. 47,254,149 $\div$ 4674.

4. Find the value of:

1. 352,107,193,214 $\div$ 210,472.
2. 558,001,172,606,176,724 $\div$ 2,708,630,425.
3. 123,456,789,102,345,678 $\div$ 1,234,567,890.
4. 987,654,321,000,000,000 $\div$ 9,876,543,210.
5. 2,016,722,783,975,663,729 $\div$ 41,927,081.

PROBLEMS.

1. A man has 12,000 dollars to invest in land. How many acres can he buy at 125 dollars an acre?

2. There are 47,520 yards in 27 miles. How many yards are there in one mile?

3. There are 640 acres in a square mile. How many square miles are there in the District of Columbia, which contains 38,400 acres?

4. If the earth in its revolution round the sun moves 1,641,600 miles a day, how far does it move in one second, a day containing 86,400 seconds?

5. If 4671 building lots are worth 1,985,175 dollars, how much is one building lot worth?

6. What number of dollars must be multiplied by 124 to produce 40,796 dollars?

7. There are 31,173 verses in the Bible. How many verses must be read each day, that it may be read through in a common year?

8. Pennsylvania contains 45,125 square miles, and Delaware contains 2050 square miles. How many states the size of Delaware could be made from Pennsylvania?

9. How long can 125 men subsist on an amount of food that will last one man 4500 days?

10. If 1988 hogsheads of molasses cost 115,304 dollars, what will one hogshead cost?

11. A balloon is said to have ascended 37,000 feet. How many miles? (One mile = 5280 feet.)

12. If one of two factors of 4,312,695 is 1205, what is the other factor?

13. A man has 8000 dollars; he buys two houses for 4500 dollars, and invests the remainder in land at 140 dollars an acre. How many acres of land can he buy?

14. If the distance from the earth to the sun is 91,430,000 miles, how long will it take light from the sun to reach us, if it moves 186,000 miles a second?

15. How many years will it take a man to save $5475, if his savings average one dollar per day, reckoning 365 days to the year?

16. A railroad that cost $4,076,500 was divided into 8153 equal shares. What was the cost of each share?

17. There are 231 cubic inches in a gallon. How many gallons in a tank that contains 139,755 cubic inches?

18. The salary of the President of the United States is $50,000 a year. How much does he receive each day?

19. If a pound of cotton can be spun into a thread 70 miles long, how many pounds of it must be spun to reach around the world, a distance of 25,000 miles?

20. Two trains on the same railway are 689 miles apart. If they start at the same time and run toward each other, one averaging 27 miles per hour, and the other 26 miles, in how many hours will they meet?

21. Find the value of $15 \times 37{,}153 - 73{,}474 - 67{,}152 \div 4 + 40{,}734 \times 2$.

Suggestion: Use $\times$ and $\div$ first.

22. Find the value of $(7854 - 4913) \times 3 - (20{,}352 - 5194) \div 53 - 6 + (395{,}456 - 2364) \div 556$.

23. Find the value of $(12 + 7 - 9) \times 5 \div 10$.

24. Find the value of $(5 + 7 - 3) \times 3 + (3 + 5 - 4) \div 4$.

25. Find the value of $(828 - 475 - 325) + (982 - 620 - 82)$.

26. Find the value of $849 \times 4 \div 3 - 714 \times 4 \div 3 - 135 \times 4 \div 3$.

27. Find the value of (LI. — III. + I.) ÷ VII. + (III. × V. — IX.) ÷ III.

28. Find the value of (XXVII. + XXII. — XIX) × VI.

29. Find the value of (CCCLXI. — CCI.) × ($\overline{\text{CCCXX.}}$ — $\overline{\text{CCCXII.}}$).

30. Find a second factor of 4807, taking 11 as the first factor.

MISCELLANEOUS EXERCISES.

1. The minuend is 900,000 and the subtrahend is 323,456. What is the difference?

2. The minuend is 300,400 and the difference 197,325. Find the subtrahend.

3. The subtrahend is 204,054 and the difference is 9735. What is the minuend?

4. The product of two numbers is 567,204, and one of the numbers is 141,801. Find the other number.

5. The multiplier is 3007 and the multiplicand is 3007. What is the product?

6. The product is 24,483 and the multiplier is 3. What is the multiplicand?

7. The product is 24,402 and the multiplier is 21. Find the multiplicand?

8. The product is 20,692 and the multiplicand is 739. Find the multiplier.

9. The divisor is 437, the quotient is 730, and the remainder is 89. What is the dividend?

10. The divisor is 954, the quotient is 840, the remainder 227. Find the dividend.

11. What number divided by 573 will give a quotient of 205 and a remainder of 89?

12. Of what number is 623 both the divisor and the quotient?

13. The sum of two numbers is 21,000,000; one of the numbers is 12,113,141. Find the other number.

14. Divide 18,490,700 by 73,000.

15. Multiply 5690 by 3008. Prove by division.

16. Show that $(26 \times 26 - 15 \times 15) \div (26 + 15) = 26 - 15$.

17. How many times in succession can 3589 be subtracted from 241,462? What will be the remainder?

18. A certain number is contained 41 times in 1043, with 18 as a remainder. What is the number?

19. What number is that which, divided by 12, the quotient multiplied by 8, and 580 added to the product, equals 740?

20. Divide 9,999,999 by 33,300.

MISCELLANEOUS PROBLEMS.

1. If a ship sails 10 miles an hour, in how many days will it cross the Atlantic Ocean, 2880 miles?

Process Indicated.

$$\frac{2880 \div 10}{24} = \text{the number of days required.}$$

Process.

$$\frac{2880}{10} = 288 \text{ hours.}$$

$$\begin{array}{r} 24\,)\,288\,(\,12 \text{ days.} \\ \underline{24} \\ 48 \\ \underline{48} \end{array}$$

Explanation.

1. Since the ship sails 10 miles an hour, it will sail 2880 miles in $2880 \div 10 = 288$ hours.

2. Since 24 hours $= 1$ day, 288 hours $= \frac{288}{24}$ days $= 12$ days.

NOTE.—Carefully indicate each solution.

2. How many barrels of apples, at \$2.75 a barrel, must be given for 6 barrels of cranberries, at \$8.25 a barrel?

3. How many pounds of coffee, worth \$.12 a pound, must be given for 368 pounds of sugar, worth \$.09 a pound?

4. A young farmer earns \$60 a month and spends \$25. In what time can he save enough to pay for a farm of 50 acres, at \$28 an acre?

5. A grocer bought 250 pounds of coffee for $82.50, and sold it at $.37 a pound. What did he gain?

6. $(309 - 76) + (4426 - 309) + (6375 - 4426) + 76 = 9375$ is a defective equation to what extent?

7. There were 24,012 public schools in Pennsylvania in 1893, with 994,407 pupils. How many pupils, on an average, in each school?

8. Multiply the sum of 276 and 347 by three times their difference?

9. A park is 48 rods long and 32 rods wide. How many times must a boy go around it on his bicycle to travel 45 miles, there being 320 rods in a mile? How many times must he go around the park to travel one mile?

10. A man dying, left three tracts of land to be divided equally among his six children. The first tract contained 1118 acres; the second, three times as much lacking 193 acres; the third, twice as much as the other two lacking 105 acres. What was each one's share?

11. A.'s house cost $7825, which was $4218 less than the cost of the farm. What was the cost of both?

12. The diameter of the earth at the poles is 41,707,620 feet, and at the equator, 41,847,426 feet. How much does the equatorial diameter exceed the polar diameter?

13. What will 53,000 bricks cost at $7.25 per M.?

14. Mr. Gill, a drover, purchased 36 head of cattle, at $64 a head, and 88 sheep, at $5.00 a head. He sold the cattle for $40.00 a head, and the sheep for $4.00 apiece. Did he lose, and how much?

15. Of two boys, one was lazy and did not rise till nine o'clock, while the other was active and rose every morning at six. Allowing 365 days to the year, how many hours did the lazy boy lose in five years?

16. There are two numbers, the greater of which is 25

times 670, and their difference 55 times 81. Find the less number.

17. I bought 87 acres of land at \$50 an acre, and paid \$3150 in cash, and the balance in labor at \$240 a year. How many years of labor did it take?

18. A farmer has 1000 head of cattle in five fields. In the first he has 315 head; in the second, 175 head; in the third, 300 head; and in the fourth, the same number as in the fifth. How many has he in the fifth?

19. If a man sells 19 bushels of potatoes at \$.55 a bushel, 23 bushels of oats at \$.53 a bushel, and with the proceeds buys 8 yards of broadcloth, how much does he pay a yard for the broadcloth?

20. If a newsboy buys papers at \$.08 a dozen, and sells them at \$.01 apiece, how much can he clear in March, if he averages 120 papers a day?

Suggestion: $(.01 \times 12 - .08) \times \frac{120}{12} \times 31 = ?$

ANALYSIS AND REVIEW.

"Analysis reasons from the given number to one, and from one to the required number."

1. A man bought 13 horses for \$2405. What would he pay for 37 horses at the same rate?

Process Indicated.

$$\frac{\$2405 \times 37}{13} = \text{sum received.}$$

Process.

13 horses cost \$2405.
1 horse costs \$185.
37 horses cost \$6845.

Explanation.

1. Since 13 horses cost \$2405, 1 horse will cost \$2405 ÷ 13, or \$185.

2. Since 1 horse costs \$185, 37 horses will cost \$185 × 37 or \$6845.

Let the pupil indicate the solution by using the appropriate signs.

2. If 25 pounds of sugar cost \$2.50, what will 36 pounds cost?

3. If I exchanged 40 barrels of flour for 61 yards of cloth at \$4 a yard, how much did I get per barrel for the flour?

Indicate the process and explain.

4. A carriage maker sold 15 carriages for \$1875. How much would he receive for 25 carriages, selling them at the same rate?

5. 190 bushels of corn cost \$100.70. At what rate must it be sold to gain 13 cents a bushel?

6. If 93 oranges cost \$5.58, what will 75 oranges cost?

7. If 12 yards of cloth cost \$48.00, what will 7 yards cost?

8. If 16 horses cost \$1952, what will 22 horses cost at \$6 less a head?

9. A. paid \$27,144 for a farm, at the rate of 15 acres for \$3510. How many acres did he buy?

10. If 46 acres of land produce 2484 bushels of corn, how many bushels will 120 acres produce?

INDICATED SOLUTIONS.

As we have already attempted to show, the solution of any problem should first be indicated by means of signs, and afterwards carried to completion as the signs direct.

In completing a solution indicated, a parenthesis or a vinculum must be removed first. The other signs, whether within a vinculum or not, may be safely used in the following order: $\times$, $\div$, $-$, $+$.

$(12 \div 3) \times 2 = 4 \times 2 = 8$; but $12 \div 3 \times 2 = 12 \div 6 = 2$.

1. Perform the operations indicated in $(48 \times 2 - 84 \div 6 \times 2) + 7 - 3$.

   1. $(48 \times 2 - 84 \div 6 \times 2) + 7 - 3$.
   2. By removing sign $\times$, $(96 - 84 \div 12) + 7 - 3$.
   3. By removing sign $\div$, $(96 - 7) + 7 - 3$.
   4. By removing sign $-$, $89 + 4$.
   5. By removing sign $+$, 93.

2. Find the value of $48 \times 2 - 84 \div 6 \times 2 + 7 - 3$.

3. If 6 men can do a piece of work in 10 days, how long will it take 5 men to do the work?

We may let x stand for the required number of days, and write an equation thus:

Process.

$$x = \frac{10 \times 6}{5} = 12$$

$$x = 12 \text{ days.}$$

Explanation.

ANALYSIS.—Since 6 men require 10 days, 1 man will require 6×10 days. Hence, 5 men will require $\frac{6 \times 10}{5}$ days. Performing the operations indicated, we have $\frac{10 \times 6}{5} = \frac{60}{5} = 12$ days.

4. If 12 men can build a school-house in 25 days, how long will it take 25 men to build it?

5. How many pounds of butter, at \$.23 a pound, must be given for 5 pounds of raisins at \$.11 a pound, 2 pounds of tea, at \$.63 a pound, and a barrel of sugar, at \$9?

Suggestion: $\frac{5 \times \$.11 + 2 \times \$.63 + 9}{\$.23}$

6. Find the value of:

   1. $28 \times 6 \div 14 + 9 \times 8 \div 12 + 42 \div 7 \times 3$.
   2. $99 \times (8 + 51) \times 10 - (7 \times 104 + 26)$.
   3. $(105 \div 21 + 80 \div 5) \times (81 + 36 \div 9)$.
   4. $\frac{16 \times 3125 - 127 \times 0 + \overline{(380 \div (100 \div 50)}}{239}$.
   5. $\frac{\overline{(125 \times 30) \div (25 \times 25)} \times \overline{(32 - 21)} - 55 \div 5}{11}$.

7. A lady paid a store bill of \$784, giving 30 twenty-dollar bills, 4 one-dollar bills, and the remainder in five-dollar bills. How many five-dollar bills did she use?

$$\frac{\$784 - (30 \times \$20 + 4 \times \$1)}{\$5}$$

8. Two trains leave New York for Chicago, 900 miles, at the same hour, one averaging 30 miles an hour, the other 45 miles an hour. How long will the second train be in Chicago before the first arrives?

9. How many men will it take to do a piece of work in 26 days that 39 men can do in 76 days?

10. How long can 125 men subsist on an amount of food that will last 3 men 4500 days?

11. A quantity of provisions lasts an army of 2500 men 72 days. How long would it last 18,000 men?

12. I bought a carriage for \$140, a horse for \$125, and a set of harness for \$18; kept them a month at an expense of \$17.25, and then sold the team for \$300. Did I gain or lose, and how much?

13. A pedler sells beets, six in a bunch, at 10 cents a bunch, and gains one cent on each bunch. Find the cost per C.

$$\text{Suggestion: } \frac{10 - 1}{6} \times 100 = ?$$

14. I paid \$86.40 for 1440 blocks of granite. What was the price per M?

$$\text{Suggestion: } \frac{\$86.40}{1440} \times 1000 = ?$$

15. If 8 acres of land cost \$656, what will 35 acres cost at \$4 more per acre?

16. From $\overline{126 + (16 + 4) \times 2}$ take $\overline{(48 \div 2) + 34 \times 6 \div (17 - 15)}$.

GENERAL PRINCIPLES OF DIVISION,

If 24 is the dividend (*D.*), 4 the divisor (*d.*), and 6 the quotient (*Q.*), we have

$$\frac{24,\ D.}{4,\ d.} = 6,\ Q.$$

We will now notice the effect upon *Q*, if we multiply and divide *D.* and *d.* by 2, as follows:

1.	$\frac{24 \times 2}{4} = \frac{48}{4} = 12$	*Q.* is multiplied by 2.
2.	$\frac{24 \div 2}{4} = \frac{12}{4} = 3$	*Q* is divided by 2.
3.	$\frac{24}{4 \times 2} = \frac{24}{8} = 3$	*Q.* is divided by 2.
4.	$\frac{24}{4 \div 2} = \frac{24}{2} = 12$	*Q.* is multiplied by 2.
5.	$\frac{24 \times 2}{4 \times 2} = \frac{48}{8} = 6$	*Q.* is unchanged.
6.	$\frac{24 \div 2}{4 \div 2} = \frac{12}{2} = 6$	*Q.* is unchanged.

"Analysis reasons from particular instances to general principles."

Reasoning from the particular instances above, we derive the following

PRINCIPLES.

1. **Multiplying *D.* multiplies *Q*.**
2. **Multiplying *d.* divides *Q*.**
3. **Dividing *D.* divides *Q*.**
4. **Dividing *d.* multiplies *Q*.**
5. **Multiplying both *D.* and *d.* does not change *Q*.**
6. **Dividing both *D.* and *d.* does not change *Q*.**

Let *D.* = 1728 and *d.* = 144. Find *Q.*, and illustrate each of the above six principles.

SHORT PROCESSES IN DIVISION.

When there are ciphers at the right of the divisor, the process of division is readily simplified.

The Divisor 1 with Ciphers annexed.

1. Divide 539 by 10.

Process.

$$1|0\,)\,\underline{53|9}$$
$$53|9$$

Quo. Rem.

Explanation.

Cutting off the digit 9 from the dividend, and the 0 from the divisor, we have 53 tens $\div$ 1 ten $=$ 53, with 9 remaining. [Principle 6.]

RULE.

For each cipher in the divisor cut off a digit from the right of the dividend.

2. Divide:

By 10.	By 100.	By 1000.
1. 6327.	6. 3267.	11. 6173.
2. 5327.	7. 5327.	12. 5432.
3. 9732.	8. 9273.	13. 8650.
4. 9267.	9. 5533.	14. 3000.
5. 2567.	10. 1234.	15. 5678.

The Divisor any Significant Figure with Ciphers annexed.

1. Divide 7436 by 3000.

Process.

$$3|000\,)\,\underline{7|436}$$
$$2\ 1436$$

Quo. Rem.

Explanation.

Cutting off 436 from *D.* and 000 from *d.*, we have 7 thousands $\div$ 3 thousands $=$ 2, with one thousand remaining. 1 thousand $+$ 436 $=$ 1436. Hence *Q.* $=$ 2, and *R.* $=$ 1436.

Repeat the principle involved.

2. Divide:

1. 673 by 20.	5. 1074 by 80.
2. 957 by 30.	6. 1096 by 90.
3. 686 by 40.	7. 5736 by 200.
4. 790 by 50.	8. 7300 by 300.

3. Complete the following:

1. $873 \div 600 =$	7. $10{,}432 \div 4000 =$
2. $1052 \div 700 =$	8. $10{,}037 \div 5000 =$
3. $1095 \div 800 =$	9. $9396 \div 6000 =$
4. $1073 \div 900 =$	10. $9116 \div 7000 =$
5. $5327 \div 2000 =$	11. $10{,}370 \div 8000 =$
6. $8645 \div 3000 =$	12. $10{,}573 \div 9000 =$

The Divisor any Number with Ciphers annexed.

1. Divide 5658 by 3200.

Process.

```
32|00 ) 56|58 ( 1 Quo.
        32
        ----
        2458 Rem.
```

Explanation.

Cutting off 58 from *D.* and 00 from *d.*, we have 56 hundreds, quotient, 58 units remaining; 56 hundreds ÷ 32 hundreds = 1, quotient, with 24 hundreds remaining; 24 hundreds + 58 units = 2458, entire remainder.

2. Find the value of:

1. $97{,}658 \div 3300 =$	7. $500{,}896 \div 11{,}000 =$
2. $59{,}625 \div 4600 =$	8. $485{,}432 \div 23{,}400 =$
3. $78{,}695 \div 5300 =$	9. $306{,}959 \div 30{,}500 =$
4. $89{,}765 \div 4400 =$	10. $940{,}938 \div 24{,}500 =$
5. $68{,}543 \div 6400 =$	11. $768{,}448 \div 32{,}300 =$
6. $954{,}000 \div 350 =$	12. $533{,}337 \div 38{,}000 =$

REVIEW.

1. Define the following terms:

1. Division.	7. Long Division.
2. Divisor.	8. Short Division.
3. Dividend.	9. Analysis.
4. Quotient.	10. Solution.
5. Remainder.	11. Principle.
6. 0 as remainder.	12. Parenthesis.

2. What are the principles of division?

3. In a solution indicated by the signs you have learned to use, in what order is it always safe to use these signs?

4. Invent five problems whose solution may be indicated by five different signs.

PROPERTIES OF NUMBERS.

DEFINITIONS AND INDUCTIVE STEPS.

1. A **Factor** (Latin, "maker") of a number is one of the numbers which, multiplied together, produce the number, as in $2 \times 3 \times 4 = 24$.

2. Write two factors that will produce 24. Write *four* factors $= 24$.

3. Form an equation, putting *five* factors $= 300$.

4. An **Exact Divisor** of a number is *one of its factors.* *What are the exact divisors of 6?*

5. Since $2 \times 3 \times 5 = 30$, are 2, 3, and 5 factors of 30, or exact divisors of 30?

6. If you have the equation $2 \times ? = 6$, how can you obtain the required factor?

7. Since $2 \times 3 \times ? = 30$, how can you obtain the required factor?

8. Then if a number and all its factors are given except one, how do we find that one?

9. Has 2, or 3, or 5, any factors except itself and 1?

10. A number that has no factors or exact divisors except itself and one is a **Prime number**, as 2, 3, 5, 7, 11, etc.

11. A number that has factors or exact divisors other than itself and one is a **Composite number**, as 4, 6, 8, 9, etc.

12. The **Prime numbers** between 1 and 100 are as follows: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

13. All the other numbers between 1 and 100 are called what? [See 11.]

14. Why are they so named? *Ans.: Because they are composed of factors.*

15. $12 = 3 \times 4$ is a correct equation. What kind of factor is 3? What kind is 4?

What are the two equal prime factors of 4? Re-write the equation with *three* prime factors in the second member.

16. An **Even number** is exactly divisible by 2.

17. An **Odd number** is not exactly divisible by 2.

18. Is 3 an exact divisor of 12? Of twice 12? Of three times 12? Of any number of times 12?

19. 3 is an exact divisor of 12 and 21. Is it an exact divisor of their sum? *Illustrate.*

Is it an exact divisor of their difference? *Illustrate.*

20. Since numbers are either prime or composite, *factors* are either prime or composite.

PRINCIPLES.

1. Every composite number is the product of its prime factors.

2. Every prime or composite factor of a number exactly divides that number.

3. Every exact divisor of a number is one of its prime factors or the product of two or more of its prime factors.

4. Every exact divisor of a dividend exactly divides any number of times that dividend.

5. A common divisor of two numbers or dividends exactly divides their sum.

6. A common divisor of two numbers or dividends exactly divides their difference.

7. Any factor of a number becomes a quotient when the number itself becomes a dividend, and its other factor, or the product of its other factors, becomes a divisor.

Suggestion.—Pupils should be required to illustrate each of the foregoing principles.

EXERCISES.

1. Write:
   1. Three prime numbers exceeding 100.
   2. The composite numbers between 75 and 100.
   3. An equation, using three composite numbers as factors.
   4. An equation, using three prime numbers as factors.

   What kind of number is the second member? Mention the exact divisors of it.
2. Write:
   1. Three even numbers.
   2. Three odd numbers.
   3. The prime numbers between 1 and 50.
   4. The prime numbers between 50 and 100.
   5. The single *even* prime number.
   6. Three *odd* numbers that are not prime.
3. Finish the following equations:
   1. $1 \times 2 \times 89 \times 97 = ?$
   2. $3 \times 5 \times 7 \times 31 = ?$
   3. $47 \times ? \times 53 = 12{,}455.$
   4. $? \times 1 \times 67 \times 89 = 59{,}630.$
   5. $41 \times 43 \times 47 \times 0 = ?$
4. Why are not 49, 51, and 63 prime numbers?

FACTORING.

Factoring is the process of obtaining the factors or exact divisors of a number. The number factored is, therefore, a *dividend.*

The most important problem in this connection is to find the *prime factors* of a number, as a sure means of obtaining certain required divisors or dividends.

EXERCISES

For Analytic and Synthetic Explanation.

1. What are the prime factors of 108?

Process.

$$\begin{array}{r|r} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$108 = 2 \times 2 \times 3 \times 3 \times 3.$

Explanation.

Since the prime factors of a number are exact divisors of the number, we find all the prime numbers that exactly divide 108. 108 being an *even* number, is divisible by 2; 54 being even, is divisible by 2. Dividing by 3, and again by 3, the quotient is 3, a prime number. Hence the prime factors of 108 are 2, 2, 3, 3, 3.

Note.—In finding the prime factors of a number, use the prime numbers as divisors in order of their values, beginning with the lowest one that will divide the given number.

2. What are the prime factors:

1. Of 72?
2. Of 35?
3. Of 64?
4. Of 46?
5. Of 336?
6. Of 111?
7. Of 385?
8. Of 429?
9. Of 925?
10. Of 492?
11. Of 1320?
12. Of 8424?
13. Of 7698?
14. Of 743?
15. Of 3675?
16. Of 4536?
17. Of 168?
18. Of 231?
19. Of 178?
20. Of 180?
21. Of 144?
22. Of 315?
23. Of 420?
24. Of 660?
25. Of 740?
26. Of 945?
27. Of 1728?
28. Of 4284?
29. Of 1682?
30. Of 997?
31. Of 4620?
32. Of 5250?
33. Of 798?
34. Of 484?
35. Of 1280?
36. Of 1898?
37. Of 5460?
38. Of 3420?
39. Of 1470?
40. Of 1492?
41. Of 2310?
42. Of 2772?
43. Of 1600?
44. Of 8364?
45. Of 2585?
46. Of 1997?
47. Of 4851?
48. Of 7623?

3. Find the composite factors of 40.

Process.

$$\text{Prime factors.}\left\{\begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}\right. \qquad \begin{array}{c}\text{Prime}\\ \text{factors}\\ \text{combined.}\end{array}\left\{\begin{array}{lr} 2 \times 2 & = 4 \\ 2 \times 2 \times 2 & = 8 \\ 2 \times 5 & = 10 \\ 2 \times 2 \times 5 & = 20 \end{array}\right\}\text{Composite factors.}$$

1. Of 102, 105, 108, 221, 715, 845.
2. Of 84, 250, 735, 9800, 11,165.
3. Of 231, 78, 415, 852, 452, 1227.

REVIEW OF PRINCIPLES.

[See page 72.]

1. Factor 54, and illustrate Principle 1.
2. Factor 36, and illustrate Principle 2.
3. Factor 108, and illustrate Principle 3.
4. Factor 144, and illustrate Principle 4.
5. Factor 231 and 154, and illustrate Principle 5.
6. Factor 360 and 320, and illustrate Principle 6.
7. Factor 1728, and illustrate Principle 7.

MULTIPLICATION BY FACTORS.

1. What will 24 carriages cost at $257 each?

Process.

$24 = 4 \times 6$; $\$257 \times 6 = 1542$; $1542 \times 4 = \$6168$.

2. In like manner find the cost of:
    1. 35 cows at $53 each.
    2. 22 violins at $10.35 each.
    3. 72 cords of wood at $4.65 a cord.

4. 99 books at $2.18 apiece.
5. 123 hats at $5.65 apiece.
6. 51 acres of land at $125 an acre.
7. 49 barrels at $1.25 apiece.
8. 63 bags of salt at $1.875 a bag.
9. 21 shot-guns at $55.50 apiece.
10. 121 paper-weights at $.555 apiece.
11. 132 horses at $132 a head.
12. 144 rifles at $87.50 apiece.
13. 34 yards of silk at $2.56 a yard.
14. 81 bushels of wheat at $.95 per bushel.

DIVISION BY FACTORS.

When the divisor is a composite number, division may sometimes be readily performed by using factors of the divisor.

1. Divide 3598 by 14.

Process.

$$\begin{array}{r|l} 2 & 3598 \\ \hline 7 & 1799 \\ \hline & \ \ 257 \end{array}$$

Explanation.

The factors of 14 are 2 and 7. Dividing 3598 into two equal parts, and each of those 2 equal parts into 7 equal parts, we thus obtain 7 times 2 or 14 equal parts, each equal to 257.

2. Divide, using factors:

1. $8445 \div 15$.
2. $7776 \div 24$.
3. $23,296 \div 32$.
4. $1152 \div 64$.
5. $1855 \div 35$.
6. $16,340 \div 38$.
7. $9345 \div 105$.
8. $1152 \div 72$.
9. $3648 \div 96$.
10. $42,336 \div 49$.
11. $37,464 \div 42$.
12. $153,160 \div 56$.

The chief difficulty in dividing by factors is to find the true remainder. Notice the following explanation:

3. Divide 4753 by 140, using factors.

Process.		Explanation.
		$140 = 4 \times 5 \times 7.$
4	4753	
5	1188	. . 1 unit of 4753 remaining.
7	237	. . 3 units of 1188 remaining $= 3 \times 4 = 12$ units of 4753.
	33	. . 6 units of 237 remaining $= 6 \times 5 \times 4 = 120$ units of 4753
		$1 + 12 + 120 = 133$, true remainder.

The true remainder must be a part of 4753.

Why must partial remainder 3 be multiplied by 4?

Why must partial remainder 6 be multiplied by 5×4?

4. Divide, using factors:

1. 7304 by 24.
2. 4104 by 45.
3. 3276 by 27.
4. 3275 by 56.
5. 2184 by 49.
6. 3824 by 32.
7. 3548 by 72.
8. 1299 by 56.

CANCELLATION.

Cancellation abridges the process of division by *striking out a common factor* from dividend and divisor.

Striking out a common factor is in effect dividing both dividend and divisor by the same number. [State the principle, page 68.]

1. Divide $(96 \times 9 \times 8)$ by (12×16).

Process.

$$\frac{\overset{\overset{4}{\cancel{8}}}{\cancel{96}} \times 9 \times \cancel{8}}{\cancel{12} \times \underset{\cancel{2}}{\cancel{16}}} = 4 \times 9 = 36.$$

Explanation.

Cancelling 12 and 96, we have 8 as the result in the dividend; cancelling 8 and 16, we have 2 as the result in the divisor; cancelling 2 and 8, we have 4 as the result in the dividend. The entire divisor having been cancelled, the quotient is the product of the uncancelled factors 4 and 9, which is 36.

2. Divide $4 \times 2 \times 8 \times 21$ by $36 \times 8 \times 2$.

Process.

$$\frac{\cancel{4} \times \cancel{2} \times \cancel{8} \times \overset{7}{\cancel{21}}}{\underset{\underset{3}{\cancel{9}}}{\cancel{36}} \times \cancel{8} \times \cancel{2}} = \frac{7}{3}$$

Explanation.

We first cancel 2 and 8 in both dividend and divisor; next, 4 and 36, obtaining 9 in the divisor; finally, we cancel 9 and 21, rejecting from each the factor 3, and obtain 3 in the divisor and 7 in the dividend.

3. Divide, using cancellation:

1. $18 \times 24 \times 35$ by $6 \times 8 \times 7$.
2. $30 \times 10 \times 9 \times 4$ by $8 \times 5 \times 6$.
3. $6 \times 7 \times 9 \times 11$ by $2 \times 3 \times 7 \times 3 \times 21$.
4. $10 \times 6 \times 84 \times 42$ by $12 \times 5 \times 24 \times 7$.
5. $144 \times 75 \times 15 \times 32 \times 23$ by $432 \times 25 \times 8 \times 30$.
6. $400 \times 18 \times 30 \times 42$ by $270 \times 20 \times 30 \times 14$.
7. $28 \times 56 \times 400$ by 112×280.

PROBLEMS.

1. A farmer exchanged 15 barrels of apples, each containing 3 bushels, at $.80 a bushel, for 8 pieces of cloth, each containing 30 yards. Find the price of the cloth per yard.

Process.

$$\frac{\cancel{15} \times 3 \times \overset{.10}{\cancel{.80}}}{\cancel{8} \times \underset{2}{\cancel{30}}} = \frac{3 \times .10}{2} = \frac{.30}{2} = .15.$$

Explanation.

The farmer had 15 times 3 bushels of apples worth 15 times 3 times $.80. He got therefor 8 times 30 yards of cloth, which cost him as much per yard as 8×30 is contained times in $15 \times 3 \times .80$. By cancellation we obtain 15 cents as the cost per yard.

2. A miller bought 12 loads of wheat, each containing 130 bushels, at $1.25, and gave in exchange 8 loads of flour at $6.25 a barrel. How many barrels were there in a load?

3. I exchanged apples at $1.50 per bushel for 25 days' labor at $1.20 per day. How many bushels of apples did it take?

4. Three pieces of cloth containing 20 yards each, worth $5 a yard, were exchanged for 5 pieces of cloth containing 40 yards each. What was the second kind of cloth per yard?

5. How many pounds of coffee at 24 cents per pound are required to pay for 3 hogsheads of sugar, each weighing 1464 pounds, and worth 15 cents per pound?

6. Four farms containing 80 acres each, worth $65 per acre, were exchanged for 5 farms containing 95 acres each. What was the value per acre of the farms received in exchange?

7. How many firkins of butter, each containing 50 pounds, at 18 cents a pound, must be given for 3 barrels of sugar, each containing 200 pounds, at 9 cents a pound?

8. If 25 Jersey cows each give 8 quarts of milk a day, at 5 cents a quart, how many pieces of matting of 40 yards each, at 50 cents a yard, will pay for the milk of 12 days?

9. A tailor bought 5 pieces of cloth, each piece containing 24 yards, at 3 dollars a yard. How many suits of clothes, at 18 dollars a suit, must be made from the cloth to pay for it?

10. A grocer bought 7 chests of Souchong tea, containing 24 pounds each, at $1.05 per pound. How many firkins of butter, at $.35 a pound, will be required to pay for the tea, each firkin containing 56 pounds?

11. I bought 24 barrels of apples, each containing 2 bushels, at the rate of 75 cents a bushel. Find the number of cheeses, each weighing 30 pounds, at 15 cents a pound, that will pay for the apples.

12. How many days' work, at $1.80 a day, will pay for 84 bushels of corn, at $.45 a bushel?

13. If 52 men can dig a trench in 15 days, working 10 hours a day, in how many days will 25 men dig a similar trench, working 12 hours a day?

COMMON DIVISORS.

INDUCTIVE STEPS.

1. What number is a divisor of both 6 and 8?

What one of both 9 and 12?

What one of both 20 and 24?

2. *Two*, *three*, and *four* are in this case called **Common factors** or **Common divisors.**

3. Numbers $\left\{\begin{array}{l} 6 = 2 \times 3 \\ 8 = 2 \times 2 \times 2 \\ 20 = 2 \times 2 \times 5 \end{array}\right\}$ prime factors.

What single prime factor is common to all the numbers?

What, then, is the common divisor of 6, 8, and 20?

What prime factor is common to 8 and 20 only?

Is it the same 2 that is common to all the numbers, or is it a different 2?

Is there a 2 that is not common to any two of the numbers?

What two other factors are not common to any two of the numbers?

4. Numbers $\left\{\begin{array}{l} 15 = 3 \times 5 \\ 30 = 2 \times 3 \times 5 \\ 45 = 3 \times 3 \times 5 \end{array}\right\}$ prime factors.

What two prime factors are common to all the numbers?

The numbers have what two common divisors?

Is the product of 3 and 5 a common divisor?

State the principle.

Is 15 the *greatest* common divisor of 15, 30 and 45? Why?

DEFINITIONS.

1. A **Common Divisor** of two or more numbers is a number that *exactly divides each of them.*

2. The Greatest Common Divisor (G. C. D.) of two or more numbers is the *greatest* number that exactly divides each of them.

3. Numbers that have *no* Common Divisor are said to be *prime to each other.*

PRINCIPLE.

The G. C. D. of two or more numbers is the product of all their common factors.

EXERCISES.

1. What is the G. C. D. of 42, 56 and 70?

Process.

$42 = 2 \times 3 \times 7$

$56 = 2 \times 2 \times 2 \times 7$

$70 = 2 \times 5 \times 7$

$2 \times 7 = 14$, G. C. D.

Explanation.

Resolving the given numbers into their prime factors, we have $42 = 2 \times 3 \times 7$, $56 = 2 \times 2 \times 2 \times 7$, $70 = 2 \times 5 \times 7$. By inspecting these prime factors, we find that 2 and 7 are the only prime factors common to all the numbers. Hence their product, 14, is the G. C. D. of 42, 56 and 70.

2. Find the G. C. D. of:

1. 21, 35, 56.	5. 6, 12, 30.
2. 12, 18, 24.	6. 15, 25, 30.
3. 14, 35, 63.	7. 12, 18, 72.
4. 9, 27, 36.	8. 105, 35, 70.

An abridgment of the above method is as follows:

Process.

$$\begin{array}{r|l} 5 & 105,\ 35,\ 70 \\ \hline 7 & \ \ 21,\ \ 7,\ 14 \\ \hline & \ \ \ \ 3,\ \ 1,\ \ 2 \end{array}$$

G. C. D. $= 5 \times 7 = 35$.

Explanation.

Dividing by the common prime factors 5 and 7, the quotients 3, 1, 2 are seen to be *prime* to one another. Hence 5 and 7 are all the factors common to all the numbers, and 5×7 or 35 is the G. C. D.

What is meant by "prime to one another"?

3. Find the G. C. D. of the following:

1. 28, 42, 70.	5. 16, 48, 80.
2. 84, 126, 210.	6. 84, 126, 210.
3. 45, 105, 135.	7. 120, 240, 600.
4. 60, 100, 200.	8. 44, 154, 110.

9. 51, 105, 243.
10. 36, 84, 132.
11. 36, 81, 135.
12. 42, 54, 60.
13. 75, 300, 450.
14. 144, 576, 720.
15. 13, 91, 143.
16. 14, 98, 112.
17. 180, 300, 900.
18. 360, 288, 720, 648.
19. 290, 435, 232.
20. 17, 27, 36.
21. 30, 42, 63.
22. 296, 407.
23. 2121, 1313.
24. 1326, 3044, 4520.

Nos. 22, 23 and 24 may be reserved and factored by the next process.

When the numbers are not readily factored, a method founded on principle 6, page 72, is adopted.

1. What is the G. C. D. of 169 and 195?

Process.

```
          169)195(1
              169
              ---
               26)169(6
                  156
                  ---
G. C. D. = 13)26(2
              26
              --
               0
```

Explanation.

By Principle 6, a common divisor of 169 and 195, divides the remainder, 26, and consequently 156 and the remainder 13. Since 13 exactly divides itself and 26, it is a common divisor of 169 and 195.

The G. C. D must also divide 26 and 13, and since it must divide 13 it cannot exceed 13. Therefore 13 is the G. C. D. of 169 and 195.

2. Find the G. C. D. of:

1. 187 and 209.
2. 322 and 391.
3. 186 and 217.
4. 205 and 246.
5. 329 and 423.
6. 424 and 583.
7. 488 and 671.
8. 296 and 407.
9. 849 and 1132.
10. 426 and 784.
11. 1215 and 1878.
12. 1071 and 1870.
13. 3696 and 1440.
14. 6237 and 3520.
15. 333 and 592.
16. 423 and 752.
17. 697 and 820.
18. 901 and 1060.
19. 3471 and 1869.
20. 1584 and 2772.

When it is required to find the G. C. D. of more than two numbers, first find the G. C. D. of two of them, then of that G. C. D., and one of the remaining numbers, and so on for all the numbers. The last G. C. D. will be the G. C. D. of all the numbers.

3. Find the G. C. D. of:

1. 492, 744, 1044.
2. 944, 1488, 2088.
3. 216, 408, 740.
4. 945, 1560, 22,680.
5. 43, 473, 215, 344.
6. 121, 181, 221, 241.
7. 561, 6732, 1728.
8. 630, 1134, 1386.
9. 462, 1764, 2562.
10. 7955, 8769, 6401.

PROBLEMS.

1. What is the length of the longest chain that will measure exactly the length and the width of a field 484 rods long and 420 rods wide?

2. Three fields containing 24 acres, 18 acres and 42 acres are to be cut each into the least number of smaller fields of equal size. Find the size of the fields.

3. Two vats contain respectively 7875 and 16,128 gallons. Find the cask of greatest capacity that will exactly measure both vats.

4. What is the length of the longest pole with which you can measure the three lengths, 132, 156, and 168?

5. In a village some of the walks are 56 inches wide, some 70 inches, and others 84 inches. What is the width of the widest flagging that will suit all the walks?

6. What is the greatest length of board that can be used without cutting in fencing a triangular field whose sides are 80, 112 and 144 feet?

7. The Erie Railroad has 3 side-tracks of the following lengths: 3013, 2231, and 2047 feet. Find the length of the longest rail that will exactly lay each side-track.

8. A grain dealer has 2722 bushels of wheat, 1822 bushels of corn and 1226 bushels of beans which he wishes to "ship" in the smallest number of bags of equal size. Find the size of the bags.

9. Find the size of the largest equal packages that will contain without mixing 60 pounds of one kind of tea, 75 pounds of a second kind, and 100 pounds of a third kind.

10. There is a triangular field whose sides are 288, 450, and 390 feet. What is the least number of rails that will enclose it, with a fence 5 rails high?

COMMON DIVIDENDS.

INDUCTIVE STEPS.

1. In the expression, $30 \div 5 = 6$, which number is the dividend? Which the divisor?

2. In the expression, $30 \div 6 = 5$, which is the divisor?

3. Since 30 is divisible by both 5 and 6, it is called a *common dividend* of 5 and 6.

4. Can you find a number less than 30 that is a common dividend of 5 and 6?

5. Since such a number cannot be found, 30 is called the **Least Common Dividend (L. C. Dd.)** of 5 and 6.

6. What are the prime factors of 5 and 6? 2, 3, 5. What are the factors of 30? 2, 3, 5.

7. Hence we see that the L. C. Dd. of two or more numbers is composed only of the factors of those numbers.

8. The prime factors of 42 are 2, 3, 7. Is 42 the L. C. Dd. of 6 and 7? Why? Of 3 and 14? Why? Of 2 and 21? Why?

9. Does 42 contain any other factors than those of 6 and 7, 3 and 14, or 2 and 21?

10. Numbers $\left\{\begin{array}{l} 6 = 2 \times 3 \\ 8 = 2 \times 2 \times 2 \\ 20 = 2 \times 2 \times 5 \end{array}\right\}$ prime factors.

What prime factor is common to all the numbers?

What prime factor is common to 8 and 20 only?

Is this 2 a different 2 from the other?

What three prime factors are not common to any two of the numbers?

How many different prime factors are common?

How many are not common?

How many different factors in all?

Name them.

Which one is common to all the numbers?

Which one is common to two of the numbers?

Which three are not common?

11. Three classes of different prime factors are to be recognized: (1) Factors that are common to all the numbers; (2) Factors that are common to some of the numbers; (3) Factors that are not common to some of the numbers.

12. Can you form a L. C. Dd. of two or more numbers without using all the different prime factors of those numbers? Why not?

A factor common to all the numbers will be taken how often as a factor of the L. C. Dd.?

A factor common to some of the numbers will be taken how often as a factor of the L. C. Dd.?

Will a factor not common to some of the numbers be used as a factor of the L. C. Dd.?

DEFINITIONS.

1. A **Dividend** of a number exactly contains that number.

NOTE.—The word *Multiple* has commonly been used instead of Dividend.

2. A Common dividend of two or more numbers exactly contains each of them.

3. The Least common dividend (L. C. Dd.) of two or more numbers is the *least* number that exactly contains each of them.

PRINCIPLE.

The L. C. Dd. of two or more numbers equals the product of all the different prime factors of the numbers, and no other factors.

First Method.

What is the L. C. Dd. of 20, 30, 70?

Process.

$20 = 2 \times 2 \times 5$
$30 = 2 \times 3 \times 5$
$70 = 2 \times 5 \times 7$

L. C. Dd. $= 2 \times 5 \times 7 \times 3 \times 2$.

Explanation.

We first resolve the numbers into their prime factors. The L. C. Dd. equals the product of all their different prime factors. The factors common to all the numbers are 2 and 5. The factors not common to some of the numbers are 2, 3, and 7. Hence the factors of the L. C. Dd are 2, 5, 2, 3, and 7, and the L. C. Dd. $= 2 \times 2 \times 3 \times 5 \times 7 = 420$.

RULE.

Resolve the given numbers into their prime factors.

Select all the different factors, common and not common, and find their product.

EXERCISES.

Find the L. C. Dd. of:

1. 28, 21, 14.
2. 60, 48, 36.
3. 28, 32, 64.
4. 16, 24, 36.
5. 15, 45, 60.
6. 45, 30, 72.
7. 20, 24, 27.
8. 35, 40, 42.
9. 80, 60, 200.
10. 30, 32, 36.
11. 56, 72, 96.
12. 20, 24, 36.
13. 22, 33, 55.
14. 36, 40, 48.
15. 30, 50, 80.
16. 25, 45, 75.
17. 36, 48, 64.
18. 100, 450, 900.

Since a common factor enters but once into the L. C. Dd., an abridged method is usually adopted.

Second Method.

Process.

$$\begin{array}{r|l} 2 & 8,\ 6,\ 20 \\ \hline 2 & 4,\ 3,\ 10 \\ \hline & 2,\ 3,\ \ 5 \end{array}$$

L. C. Dd. $= 2 \times 2 \times 2 \times 3 \times 5$.

Explanation.

2, a common factor of 8, 6, and 20, is a factor of the L. C. Dd. 2, a common factor of 4 and 10, is a factor of the L. C. Dd. 2, 3, and 5, in the last line, are the factors that are not common. Hence L. C. Dd. $= 2 \times 2 \times 2 \times 3 \times 5 = 120$.

Find the L. C. Dd. of the following:

1. 15, 60, 75.
2. 8, 12, 40.
3. 14, 35, 56.
4. 9, 18, 27, 54.
5. 27, 36, 45, 90.
6. 18, 21, 24, 27.
7. \$240, \$270, \$180, \$150.
8. 9, 10, 14, 15, 18.
9. 12, 26, 52.
10. 18, 24, 36.
11. 12, 18, 24.
12. 18, 36, 72, 108.
13. 12, 48, 36, 70.
14. 17, 51, 34, 85.
15. 21, 24, 26, 28, 30.
16. 45, 50, 60, 63, 84.

Third Method.

The two foregoing methods of finding the L. C. Dd. show that two kinds of factors come into play,—*factors common* and *factors not common.*

Since the G. C. D. of two numbers is the product of their *common factors,* the quotients of the numbers divided by the G. C. D. are either the prime factors not common or the products of those factors. Hence, to find the L. C. Dd. of two numbers not readily factored:

1. **Find the G. C. D. of the numbers.**
2. **Divide the numbers by the G. C. D.**
3. **Find the product of the G. C. D. and the quotients.**

1. Find the L. C. Dd. of 849 and 1132.

Process.

(1.)

849) 1132 (1
   849
G. C. D. = 283) 849 (3
   849

(2.)

$283\,\underline{)\,849,\ 1132}$
    3,   4

(3.)

$$\text{L. C. Dd.} = 283 \times 3 \times 4 = 3396.$$

2. *How many kinds of factors does the L. C. Dd. of* 849 *and* 1132 *contain?*

3. *Why did we first find the G. C. D. of* 849 *and* 1132*?*

4. *Why did we divide* 849 *and* 1132 *by* 283*?*

NOTE.—When the L. C. Dd. of more than two numbers is required, first find the L. C. Dd. of two of them, and then the L. C. Dd. of the result and a third number, and so on.

5. Find the L. C. Dd. of:

1. 261 and 319.
2. 731 and 817.
3. 527 and 589.
4. 91 and 117.
5. 135 and 144.
6. 169 and 221.
7. 357 and 612.
8. 1417 and 1469.
9. 17,640 and 18,375.
10. 1110 and 777.
11. 4087 and 4757.
12. 1728 and 1898.
13. 321 and 314,259.
14. 7854 and 86,394.
15. 9797 and 10,403.
16. 9523 and 11,663.
17. 2479 and 3589.
18. 3045 and 6195.
19. 568 and 712.
20. 11,023 and 6493.
21. 1485 and 2160.
22. 30,072 and 133,784.
23. 9,144,407 and 10,347,059.
24. 1177, 1391, 1819.
25. 2943, 2616, 4578.
26. 31, 124, 217, 310.
27. 113, 452, 1000, 1492.
28. 135, 144, 356, 612.

PROBLEMS.

1. What is the least number of oranges that can be divided equally among 21, 24 or 30 boys?

2. Find the least number of acres in a farm that can be divided exactly into lots of 12, 15 or 18 acres.

3. What is the smallest sum of money I can consume in paying workmen 12, 14, 16, 18 or 20 dollars a week?

4. A man desires to purchase a piece of cloth that can be cut without waste into parts 5, 6, or 8 yards long. How many yards must the piece contain?

5. How many bushels does the smallest bin contain that can be emptied by taking out 7 bushels, 10 bushels, or 30 bushels at a time?

6. If 5 boys start together and run around a square in 12, 15, 16, 18 and 20 minutes respectively, in how many minutes will they all meet at the starting-point if they continue their course around the square?

7. How many quarts does the smallest vessel hold that can be filled by using a 3-quart measure, a 4-quart measure, a 5-quart measure or a 6-quart measure?

8. What is the smallest sum of money that can be wholly expended in buying horses at $75, cows at $50, or sheep at $9?

9. What is the shortest piece of cord that can be cut into pieces 10, 12, 15, 16 or 18 feet long?

10. A heap of pebbles can be made up into groups of 25; but when made up into groups of 18, 27 or 32 there is in each case a remainder of 11. Find the least number of pebbles such heap can contain.

11. What is the smallest number that can be divided by 360, 460, 636, and 748, respectively, and leave a remainder of 260?

REVIEW.

1. Divide $5 \times 15 \times 80 \times 56 \times 91$ by $10 \times 5 \times 16 \times 78$.

2. Divide $18 \times 15 \times 90$ by $12 \times 27 \times 25$.

3. Find the greatest number that will divide each of the two numbers 849 and 1132, and explain the process.

4. What is the L. C. Dd. of 4, 9, and 29?

5. How many barrels of sugar, 240 pounds each, at 5 cents a pound, can be exchanged for 8 pieces of sheeting, of 45 yards each, at 10 cents a yard?

6. How much does the L. C. Dd. of 1751 and 2369 exceed their G. C. D.?

7. Find the difference between the G. C. D. of 1, 3, 5, 7, 9, and the L. C. Dd. of 2, 4, 6, 8, 10.

8. Find the least number of oranges that, arranged in groups of 6, 7, 8, or 9, have just 5 over in each case.

9. I have just money enough to buy a whole number of dozens of oranges at \$.40 a dozen, or a whole number of baskets of peaches at \$1.25. How much money have I?

10. Four boys start together to run around a square; the first can run around in 12 minutes, the second in 15 minutes, the third in 16 minutes, and the fourth in 18 minutes. How long will it be before they all meet at the starting point?

11. Define the following terms:

1. Integer.	9. Factoring.
2. Fraction.	10. Cancellation.
3. Factor.	11. Common divisor.
4. Exact divisor.	12. Greatest common divisor.
5. Prime number.	13. Dividend.
6. Composite number.	14. Common dividend.
7. Even number.	15. Least common dividend.
8. Odd number.	16. Prime to one another.

12. Repeat the principles pertaining to Prime and Composite numbers; to the G. C. D.; to the L. C. Dd.

13. Repeat the rules for finding the L. C. Dd.

14. Invent 5 problems that will involve the use of the L. C. Dd.

FRACTIONS.

INDUCTIVE STEPS.

1. Divide 5 apples into 2 equal parts.

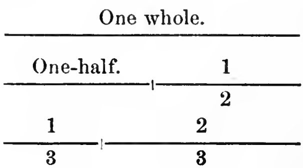
$5 \div 2 = 2$, with one apple remaining undivided. To indicate that one apple remains to be divided into 2 parts, we write it thus, $\frac{1}{2}$, and call the expression one-half. The exact quotient, therefore, of $5 \div 2$ is $2\frac{1}{2}$, read "two and one-half." $\frac{5}{3} = 1\frac{2}{3}$, read "one and two-thirds." All the remainders, resulting from division, are commonly written over their divisors, and thus form numerical expressions called **Fractions**, because they denote *parts of a unit.*

2. It is the divisor that names the fraction. If the divisor is 2, one part ($\frac{1}{2}$) is called *one-half;* two parts ($\frac{2}{2}$) *two halves.* If the divisor is 3, one part ($\frac{1}{3}$) is called *one-third;* two parts ($\frac{2}{3}$) *two-thirds.*

3. In like manner we have *fourths, fifths, sixths, sevenths, eighths, ninths, tenths,* etc.

$\frac{5}{5}$ is read "five-fifths," and is equal to one. Any quantity divided by itself gives one for quotient.

4. $\frac{1}{2}$ may denote one-half of a whole line. $\frac{2}{3}$ may denote two-thirds of a line.



In this way we might illustrate *fourths, fifths, sixths,* etc.

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., are called *fractional units.*

5. $\frac{3}{2}$ denotes how many fractional units?

6. What is the value of $\frac{3}{2}$?

7. Read the following fractions: $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$, $\frac{21}{30}$, $\frac{19}{19}$. Which has a decimal divisor? Which is equal to one?

8. Write two-thirds, four-ninths, seven-twelfths, ten-seventeenths. Write a fraction whose value is *one.*

DEFINITIONS.

1. An **Integral unit** is a *whole* or undivided unit.

2. An **Integer** is a whole unit or a collection of whole units.

3. A **Fractional unit** is one of the equal parts into which the unit is divided.

4. A **Fraction** is a fractional unit, or a collection of fractional units.

As we have seen, a fraction is the expression of a division that cannot always be performed, and is written with the number to be divided (dividend) above a horizontal line, and the divisor below that line.

5. The number below the line is called **Denominator,** because, as we have seen, it names the fractional unit.

6. The number above the line is called **Numerator,** because it numbers the fractional units.

7. The Numerator and Denominator are called the **Terms of the Fraction.**

8. A **Common fraction** is expressed by writing both numerator and denominator, as in $\frac{7}{8}$, $\frac{21}{43}$, $\frac{16}{51}$.

9. A **Decimal fraction** is usually expressed by simply writing a point before the numerator, as in .5, .37, .25.

10. A **Proper fraction** has the numerator less than the denominator, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{9}{10}$, etc.

11. An **Improper fraction** has the numerator equal to or greater than the denominator, as $\frac{7}{7}$, $\frac{11}{5}$, $\frac{13}{8}$, etc.

12. A Mixed Number consists of an integer and a fraction, as $2\frac{1}{3}$, $7\frac{3}{5}$, etc., read, "two and one-third, seven and three-fifths."

EXERCISES.

Analysis examines the separate parts of a subject and their connection with one another.

1. Analyze $\frac{7}{9}$.

9 is the denominator, is a divisor, makes and names the fractional unit.

7 is the numerator, is a dividend, and numbers the fractional units.

$\frac{7}{9}$ is a proper fraction, its terms are 7 and 9, and its value is less than one.

2. Analyze: $\frac{5}{6}$, $\frac{7}{8}$, $\frac{6}{7}$, $\frac{25}{7}$, $\frac{5}{4}$, $\frac{9}{15}$, $\frac{11}{5}$, $\frac{8}{8}$, $\frac{5}{2}$, $\frac{5}{9}$, $\frac{13}{30}$, $\frac{184}{196}$, $\frac{9}{13}$, $\frac{24}{31}$, $\frac{81}{66}$, $\frac{45}{139}$, $\frac{19}{134}$.

3. Name the proper and improper fractions and mixed numbers among these: $\frac{9}{5}$, $3\frac{1}{7}$, $\frac{107}{100}$, $4\frac{2}{15}$, $\frac{8}{8}$, $\frac{3}{17}$, $\frac{9}{23}$, $11\frac{2}{9}$, $3\frac{15}{17}$, $\frac{19}{100}$, $\frac{19}{3}$, $\frac{13}{13}$, $18\frac{13}{27}$, $\frac{50}{41}$, $27\frac{11}{100}$, $\frac{23}{80}$, $\frac{11}{7}$, $\frac{19}{20}$, $3\frac{4}{5}$.

4. Write with figures:

1. Five-ninths.	11. Sixty-five hundredths.
2. Ten-elevenths.	12. 110 ninetieths.
3. Seven-twenty-firsts.	13. 211 eighths.
4. Six and two-thirds.	14. Thirty and seven-eighths.
5. Seventy-eightieths.	15. Five twenty-fifths.
6. Ninety-one ninetieths.	16. Fifteen-sixteenths.
7. 314-tenths.	17. Nine thirtieths.
8. 1898-millionths.	18. Four and two-fifths.
9. Ten and five-sixths.	19. Three-thirds.
10. Ten-tenths.	20. Two and twelve-twentieths.

5. Write:

1. A common fraction.	3. A proper fraction.
2. A decimal fraction.	4. An improper fraction.

5. An improper fraction equal to one.

Point out the *terms* of the fractions you have written.

REDUCTION OF FRACTIONS.

Reduction changes the *terms* of a fraction without changing its value. The change is to **Higher Terms**, to **Lower Terms**, or to **Lowest Terms**.

Reduction of Fractions to Higher Terms.

Is not $\$\frac{1}{2} = \$\frac{2}{4}$?

May we not multiply the terms of $\frac{1}{2}$ by 2 and thus obtain $\frac{2}{4}$?

PRINCIPLE.

Multiplying both terms of a fraction by the same number does not change the value of the fraction. (See p. 68.)

NOTE.—The pupil should perceive that fractions are, by their very nature, subject to the principles of division.

EXERCISES.

1. Change $\frac{3}{4}$ to twentieths.

Process.

$20 \div 4 = 5$

$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

Analysis.

$1 = \frac{20}{20}$

$\frac{1}{4} = \frac{5}{20}$

$\frac{3}{4} = \frac{15}{20}$

Explanation.

The division shows that the terms of the fraction must be multiplied by 5 to change fourths to twentieths. Multiplying both 3 and 4 by 5 we have $\frac{15}{20}$.

RULE.

Divide the required denominator by the given denominator, and multiply both terms of the fraction by the quotient.

2. Reduce:

1. $\frac{14}{30}$ to 60ths.
2. $\frac{13}{40}$ to 80ths.
3. $\frac{11}{20}$ to 40ths.
4. $\frac{9}{100}$ to 1000ths.
5. $\frac{31}{90}$ to 270ths.
6. $\frac{42}{75}$ to 150ths.

7. $\frac{9}{15}$ to 90ths.
8. $\frac{7}{11}$ to 110ths.
9. $\frac{5}{9}$ to 99ths.
10. $\frac{3}{7}$ to 49ths.
11. $\frac{1}{4}$ to 60ths.
12. $\frac{3}{8}$ to 24ths.
13. $\frac{7}{10}$ to 70ths.
14. $\frac{19}{37}$ to 74ths.
15. $\frac{11}{21}$ to 42nds.
16. $\frac{18}{19}$ to 38ths.
17. $\frac{5}{6}$ to 30ths.
18. $\frac{41}{50}$ to 100ths.
19. $\frac{2}{2}$ to 10ths.
20. $\frac{18}{1}$ to 12ths.

Reduction of Fractions to Lower Terms.

Is not $\$\frac{2}{4} = \$\frac{1}{2}$?
May we not divide both terms of $\frac{2}{4}$ by 2 and obtain $\frac{1}{2}$?

PRINCIPLE.

Dividing both terms of a fraction by the same number does not change the value of a fraction. (See page 68.)

EXERCISES.

1. Reduce $\frac{12}{16}$ to eighths.

Process.

$$16 \div 8 = 2$$
$$\frac{12 \div 2}{16 \div 2} = \frac{6}{8}$$

Explanation.

The division of 16 by 8 shows that both terms of the fraction must be divided by 2 to change sixteenths to eighths. Dividing both 12 and 16 by 2, we have $\frac{6}{8}$.

RULE.

Divide the given denominator by the required denominator, and divide both terms of the fraction by the quotient.

2. Reduce:

1. $\frac{24}{30}$ to 15ths.
2. $\frac{16}{36}$ to 9ths.
3. $\frac{64}{80}$ to 10ths.
4. $\frac{120}{144}$ to 12ths.
5. $\frac{21}{28}$ to 4ths.
6. $\frac{180}{320}$ to 16ths.
7. $\frac{276}{300}$ to 100ths.
8. $\frac{144}{1728}$ to 12ths.
9. $\frac{490}{630}$ to 9ths.
10. $\frac{1776}{1898}$ to 949ths.

Reduction of Fractions to Lowest Terms.

Reduction to lowest terms requires the terms of the fraction to be divided by their greatest common factor (G. C. D.).

EXERCISES.

1. Reduce $\frac{1760}{5280}$ to lowest terms.

Process.

$$1760\,)\,5280\,(\,3$$
$$\underline{5280}$$
$$0$$

$$\frac{1760)1760}{1760)5280} = \frac{1}{3}$$

Explanation.

The G. C. D. of 1760 and 5280 is 1760. Dividing both terms of the fraction by 1760 we obtain $\frac{1}{3}$, the lowest terms. (See Principle, p. 95.)

RULE.

Divide both terms of the given fraction by their G. C. D.

Continued division by a common factor will secure lower or lowest terms.

$$\frac{4)1760}{4)5280} = \frac{4)\;440}{4)1320} = \frac{11)110}{11)330} = \frac{10)10}{10)30} = \frac{1}{3}.$$

The terms are the lowest when they are prime to each other.

2. Reduce to lowest terms :

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
$\frac{8}{22}$	$\frac{7}{28}$	$\frac{8}{56}$	$\frac{9}{63}$	$\frac{6}{22}$	$\frac{7}{8}$
(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
$\frac{18}{24}$	$\frac{16}{20}$	$\frac{232}{376}$	$\frac{225}{270}$	$\frac{280}{380}$	$\frac{289}{340}$
(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
$\frac{42}{63}$	$\frac{36}{45}$	$\frac{84}{105}$	$\frac{150}{180}$	$\frac{210}{252}$	$\frac{462}{504}$
(19.)	(20.)	(21.)	(22.)	(23.)	(24.)
$\frac{240}{288}$	$\frac{324}{356}$	$\frac{336}{378}$	$\frac{525}{735}$	$\frac{192}{216}$	$\frac{792}{864}$
(25.)	(26)	(27.)	(28.)	(29.)	(30.)
$\frac{650}{780}$	$\frac{1769}{1920}$	$\frac{288}{864}$	$\frac{648}{720}$	$\frac{288}{1024}$	$\frac{864}{1296}$

(31.)	(32.)	(33.)	(34.)	(35.)	(36.)
$\frac{726}{792}$	$\frac{1680}{4312}$	$\frac{1694}{1848}$	$\frac{9660}{11040}$	$\frac{3003}{3696}$	$\frac{4770}{6678}$

(37.)	(38.)	(39.)	(40.)	(41.)
$\frac{10605}{11445}$	$\frac{6161}{7171}$	$\frac{1710}{14364}$	$\frac{5040}{17160}$	$\frac{4692}{161976}$

Reduction of Integers and Mixed Numbers.

We have learned that $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, or $\frac{5}{5}$, etc., equal one.
How many halves in one whole thing?
How many thirds? Sixths? Tenths?
How many thirds in two? In $2\frac{2}{3}$?

EXERCISES.

1. Reduce $8\frac{3}{4}$ to fourths.

Process.

$8 = \frac{32}{4}$

$8\frac{3}{4} = \frac{32}{4} + \frac{3}{4} = \frac{35}{4}$

Explanation.

Since $1 = \frac{4}{4}$, $8 = \frac{32}{4}$; and $8 + \frac{3}{4} = \frac{32}{4} + \frac{3}{4} = \frac{35}{4}$.

RULE.

Multiply the integer by the denominator, to the product add the numerator, and write the sum over the denominator.

2. Reduce :

1. $6\frac{1}{4}$ to fourths.
2. $2\frac{1}{3}$ to thirds.
3. $12\frac{1}{2}$ to halves.
4. $9\frac{6}{7}$ to sevenths.
5. $16\frac{1}{4}$ to fourths.
6. $13\frac{5}{8}$ to eighths.
7. $314\frac{7}{21}$ to twenty-firsts.
8. $673\frac{8}{12}$ to twelfths.
9. $702\frac{10}{11}$ to elevenths.
10. $122\frac{1}{15}$ to fifteenths.
11. 15 to fifths.
12. $13\frac{1}{6}$ to sixths.
13. $18\frac{3}{11}$ to elevenths.
14. $5\frac{1}{9}$ to ninths.
15. $5\frac{1}{9}$ to eighteenths.
16. $272\frac{5}{11}$ to elevenths.
17. $278\frac{4}{9}$ to ninths.
18. $946\frac{3}{13}$ to thirteenths.
19. $615\frac{3}{5}$ to fifths.
20. $241\frac{4}{21}$ to twenty-firsts.

Have your results been proper or improper fractions?

3. Reduce to improper fractions the following:

1. $9\frac{3}{4}$.	6. $223\frac{3}{16}$.	11. $210\frac{4}{7}$.	16. $15\frac{1}{42}$.
2. $17\frac{1}{8}$.	7. $13\frac{4}{9}$.	12. $16\frac{8}{25}$.	17. $108\frac{6}{25}$.
3. $28\frac{4}{11}$.	8. $504\frac{4}{5}$.	13. $62\frac{1}{31}$.	18. $51\frac{3}{17}$.
4. $27\frac{3}{5}$.	9. $114\frac{7}{10}$.	14. $159\frac{1}{14}$.	19. $40\frac{16}{19}$.
5. $49\frac{2}{7}$.	10. $312\frac{11}{12}$.	15. $67\frac{4}{15}$.	20. $864\frac{13}{97}$.

Reduction of Improper Fractions.

How many dollars in $\$\frac{6}{2}$? In $\$\frac{18}{3}$?
How many units in $\frac{16}{4}$? In $\frac{13}{4}$? In $\frac{37}{4}$? In $\frac{48}{5}$? In $\frac{51}{3}$?
What kind of numbers are your results?

EXERCISES.

1. Reduce $\frac{504}{7}$ to an integer and $\frac{505}{7}$ a mixed number.

Process.

$\frac{504}{7} = 72$
$\frac{505}{7} = 72\frac{1}{7}$

Explanation.

Since $\frac{504}{7}$ indicates the division of 504 by 7, we divide and obtain the integer 72.

RULE.

Perform the division indicated.

2. Reduce the following improper fractions:

1. $\frac{68}{13}$.	10. $\frac{40}{19}$.	19. $\frac{396}{25}$.	28. $\frac{1956}{56}$.
2. $\frac{57}{20}$.	11. $\frac{93}{31}$.	20. $\frac{351}{17}$.	29. $\frac{1047}{13}$.
3. $\frac{721}{50}$.	12. $\frac{117}{20}$.	21. $\frac{827}{20}$.	30. $\frac{22625}{225}$.
4. $\frac{242}{69}$.	13. $\frac{87}{28}$.	22. $\frac{923}{36}$.	31. $\frac{9047}{5063}$.
5. $\frac{77}{20}$.	14. $\frac{81}{27}$.	23. $\frac{384}{37}$.	32. $\frac{24840}{16200}$.
6. $\frac{93}{25}$.	15. $\frac{655}{28}$.	24. $\frac{600}{32}$.	33. $\frac{29400}{17640}$.
7. $\frac{157}{35}$.	16. $\frac{543}{32}$.	25. $\frac{6820}{172}$.	34. $\frac{39700}{23820}$.
8. $\frac{65}{57}$.	17. $\frac{144}{36}$.	26. $\frac{16315}{104}$.	35. $\frac{1180410}{184800}$.
9. $\frac{75}{15}$.	18. $\frac{1728}{12}$.	27. $\frac{3802}{15}$.	36. $\frac{1976832}{138240}$.

REDUCTION OF UNLIKE FRACTIONS.

1. Have $\frac{1}{2}$ and $\frac{2}{3}$ like or unlike fractional units?

2. By reduction to higher terms, $\frac{1}{2}$ equals how many sixths? $\frac{2}{3}$ equals how many sixths?

3. Are $\frac{3}{6}$ and $\frac{4}{6}$ like fractions? Why?

4. What, then, is the difference between Like and Unlike fractions?

5. Have $\frac{3}{6}$ and $\frac{4}{6}$ a common denominator?

6. What is the least common dividend of the denominators 2 and 3?

DEFINITIONS.

1. **Like fractions** have the same fractional unit.

2. **Unlike fractions** have not the same fractional unit.

3. Like fractions have a **Common denominator.**

4. Like fractions may have a **Least common denominator.** (L. C. D.)

PRINCIPLES.

1. A common denominator of two or more fractions is a common dividend of their denominators.

2. The least common denominator of two or more fractions is the least common dividend of their denominators.

EXERCISES.

1. Reduce $\frac{5}{3}$ and $\frac{7}{8}$ to like fractions.

Process.

$3 \times 8 = 24$

$\frac{5}{3} = \frac{5 \times 8}{3 \times 8} = \frac{40}{24}$

$\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$

Explanation.

A common dividend of 3 and 8 is 24; therefore 24 is a common denominator of $\frac{5}{3}$ and $\frac{7}{8}$. To reduce $\frac{5}{3}$ to twenty-fourths we multiply both terms by 8; to reduce $\frac{7}{8}$ to twenty-fourths we multiply both terms by 3.

2. Reduce $\frac{5}{6}$, $\frac{7}{9}$, and $\frac{11}{12}$ to fractions having their *least common denominator.*

Process.

L. C. Dd. of 6, 9, 12 is 36.

$$\frac{5}{6} = \frac{5 \times 6}{6 \times 6} = \frac{30}{36}$$
$$\frac{7}{9} = \frac{7 \times 4}{9 \times 4} = \frac{28}{36}$$
$$\frac{11}{12} = \frac{11 \times 3}{12 \times 3} = \frac{33}{36}$$

Explanation.

The least common denominator of the fractions is the least common dividend of their denominators. The L. C. Dd. of 6, 9, 12 is 36. We therefore multiply the terms of $\frac{5}{6}$ by 6, the terms of $\frac{7}{9}$ by 4, the terms of $\frac{11}{12}$ by 3.

The same results may be obtained by reasoning thus: Since $1 = \frac{36}{36}$, $\frac{1}{6} = \frac{6}{36}$, and $\frac{5}{6} = \frac{30}{36}$. Reduce the two other fractions in like manner.

RULE.

Find the L. C. Dd. of the denominators, divide it by each denominator, multiply both terms of each fraction by the quotient obtained by its denominator.

State the principle involved. (See page 68.)

Brief directions are:

1. **Find the L. C. Dd.**
2. **Divide by the denominators.**
3. **Multiply the numerators by the quotients.**
4. **Place the products over the L. C. Dd.**

Before applying the rule reduce mixed numbers to improper fractions and fractions to their lowest terms.

3. Reduce $\frac{3}{7}$, $\frac{5}{8}$, $\frac{6}{12}$ to like fractions having their L. C. D.

Process.

Introductory, $\frac{6}{12} = \frac{1}{2}$.
1. L. C. Dd. of 7, 8, 2 is 56.
2. $\frac{56}{7}, \frac{56}{8}, \frac{56}{2} = 8, 7, 28$.
3. $3 \times 8, 7 \times 5, 1 \times 28 = 24, 35, 28$.
4. $\frac{24}{56}, \frac{35}{56}, \frac{28}{56}$.

4. Reduce in similar manner the following:

1. $\frac{1}{3}, \frac{5}{6}, \frac{5}{8}$.
2. $\frac{5}{6}, \frac{4}{9}, \frac{7}{12}$.
3. $\frac{5}{7}, \frac{3}{28}, \frac{15}{35}$.
4. $\frac{3}{4}, \frac{3}{7}, \frac{7}{8}$.

5. $\frac{2}{5}, \frac{5}{9}, \frac{12}{16}$.
6. $\frac{5}{6}, \frac{7}{8}, \frac{12}{14}$.
7. $\frac{3}{6}, \frac{9}{15}, \frac{18}{24}$.
8. $\frac{3}{4}, \frac{4}{9}, \frac{8}{13}$.
9. $\frac{8}{12}, \frac{10}{16}, \frac{8}{15}$.
10. $\frac{7}{8}, \frac{3}{4}, 2\frac{1}{2}$.
11. $8\frac{1}{9}, 2\frac{1}{6}, 4\frac{1}{4}$.
12. $\frac{3}{5}, \frac{7}{20}, \frac{13}{30}$.
13. $\frac{16}{36}, \frac{9}{12}, \frac{12}{16}$.
14. $3\frac{4}{7}, \frac{1}{2}, 7, 1\frac{1}{4}$.
15. $9\frac{1}{4}, \frac{4}{5}, \frac{9}{10}, \frac{5}{8}$.
16. $2\frac{1}{5}, 4\frac{4}{9}, 4, \frac{7}{6}$.
17. $8, 7\frac{1}{3}, \frac{5}{6}, \frac{8}{9}$.
18. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$.
19. $\frac{5}{14}, \frac{27}{35}, \frac{3}{68}, \frac{99}{112}, \frac{100}{119}, \frac{63}{170}$.
20. $\frac{7}{20}, 6\frac{1}{4}, \frac{9}{10}, 7, \frac{3}{5}, 1\frac{1}{2}$.
21. $\frac{18}{55}, \frac{19}{60}, \frac{20}{77}, \frac{21}{132}, \frac{22}{165}, \frac{23}{288}$.
22. $\frac{1}{3}, \frac{8}{13}, \frac{9}{12}, \frac{3}{28}, 7\frac{1}{3}, \frac{21}{28}$.

ADDITION OF FRACTIONS.

INDUCTIVE STEPS.

1. What is the sum of 2 books and 3 books?
2. What is the sum of $\frac{2}{5}$ and $\frac{3}{5}$?
3. Of $\frac{4}{9}$ and $\frac{2}{9}$? Of $\frac{3}{11}$ and $\frac{5}{11}$?
4. What is the fractional unit of $\frac{3}{11}$? Of $\frac{5}{11}$? Of $\frac{8}{11}$? Are these, then, like or unlike fractions?
5. What kind of fractions can be added?
6. Can you directly add $\frac{5}{12}$ and $\frac{6}{13}$?
7. If these fractions had a like or common denominator, could you add them?
8. How do you reduce unlike fractions to like fractions?

PRINCIPLES.

1. Only like fractions can be added.

2. Unlike fractions can be reduced to like fractions and then added.

EXERCISES.

1. Find the sum of $\frac{2}{15}, \frac{7}{15}$ and $\frac{9}{15}$.

Process.

$$\frac{2}{15} + \frac{7}{15} + \frac{9}{15} = \frac{18}{15} = 1\frac{3}{15} = 1\,\frac{1}{5}.$$

State the principle involved.

2. Find the sum of $\frac{3}{8}$, $\frac{5}{6}$, $\frac{4}{9}$.

Process.

1. L. C. Dd. of 8, 6, 9 $= 72$.
2. $\frac{72}{8}, \frac{72}{6}, \frac{72}{9} = 9, 12, 8$.
3. $3 \times 9, 5 \times 12, 4 \times 8 = 27, 60, 32$
4. $\frac{27+60+32}{72} = \frac{119}{72} = 1\frac{47}{72}$.

Explanation.

Since the fractions are unlike, we render them like by reducing them to fractions having the L C. D. 72.

$$\frac{3}{8} + \frac{5}{6} + \frac{4}{9} = \frac{27}{72} + \frac{60}{72} + \frac{32}{72} = \frac{119}{72} = 1\frac{47}{72}.$$

3. Find the sum of $4\frac{1}{5}$, $3\frac{1}{4}$, $4\frac{1}{2}$ and $5\frac{2}{10}$.

Process.

Introductory, $\frac{2}{10} = \frac{1}{5}$

$$\begin{aligned}
4\tfrac{1}{5} &= 4 + \tfrac{4}{20} \\
3\tfrac{1}{4} &= 3 + \tfrac{5}{20} \\
4\tfrac{1}{2} &= 4 + \tfrac{10}{20} \\
5\tfrac{1}{5} &= 5 + \tfrac{4}{20} \\
\hline
& \;16 + \tfrac{23}{20} =
\end{aligned}$$

$16 + 1\frac{3}{20} = 17\frac{3}{20}$

Explanation.

The numbers to be added are composed of integers and fractions. We therefore add the integers and fractions separately, and then unite their sums. $4 + 3 + 4 + 5 = 16$. After reduction to twentieths the sum of the fractions is $\frac{23}{20}$, or $1\frac{3}{20}$. $16 + 1\frac{3}{20} = 17\frac{3}{20}$, the sum total.

RULE.

1. Reduce the fractions, giving them a common denominator.

2. Add the integers and the fractions separately, and unite their sums.

4. Find the sum of the following:

1. $\frac{5}{6}, \frac{7}{9}, \frac{9}{12}$.
2. $4\frac{1}{5}, 3\frac{1}{4}, 4\frac{1}{2}, 5\frac{2}{10}$.
3. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$.
4. $\frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{11}{12}$.
5. $\frac{4}{25}, \frac{6}{30}, \frac{8}{12}, \frac{11}{15}$.
6. $8\frac{1}{2}, 10\frac{5}{6}, 14\frac{3}{4}$.
7. $3\frac{3}{5}, 4\frac{2}{3}, 1\frac{5}{6}, 2$.
8. $\frac{11}{35}, \frac{7}{40}, 10, \frac{23}{60}$.
9. $\frac{5}{14}, \frac{6}{11}, 9\frac{1}{2}$.
10. $\frac{11}{38}, \frac{14}{57}, \frac{17}{76}$.
11. $4\frac{3}{4}, 2\frac{1}{8}, 3\frac{3}{8}, 7\frac{1}{6}, 8\frac{15}{16}$.
12. $\frac{7}{8}, \frac{3}{4}, \frac{7}{12}, \frac{1}{2}$.

13. $\frac{42}{140}$, $\frac{9}{70}$, $\frac{7}{28}$, $\frac{1}{14}$.
14. $\frac{9}{17}$, $\frac{3}{17}$, $\frac{14}{17}$, $\frac{11}{17}$.
15. $8\frac{9}{17}$, $6\frac{3}{17}$, $5\frac{14}{17}$, $\frac{11}{17}$.
16. $\frac{5}{6}$, $\frac{11}{12}$, $\frac{8}{15}$, $\frac{7}{20}$, $\frac{13}{30}$.
17. $6\frac{1}{2}$, 4, $\frac{7}{8}$, 8.
18. $15\frac{1}{3}$, $17\frac{6}{7}$, $\frac{5}{9}$.
19. $900\frac{1}{10}$, $450\frac{4}{5}$, $75\frac{19}{25}$.
20. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$.

5. What is the value of:

1. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$?
2. $\frac{4}{7} + \frac{3}{4} + \frac{11}{14}$?
3. $4\frac{1}{2} + 3\frac{3}{4} + \frac{11}{12}$?
4. $\frac{4}{5} + \frac{5}{6} + \frac{7}{8} + \frac{15}{16}$?
5. $\frac{1}{11} + \frac{1}{7} + \frac{2}{3} + \frac{1}{21}$?
6. $5\frac{4}{5} + 18\frac{7}{15} + 25\frac{9}{30}$?
7. $187\frac{1}{3} + 197\frac{2}{5} + 746\frac{1}{9}$?
8. $1764\frac{3}{4} + 7867\frac{2}{3} + \frac{3}{27}$?
9. $21\frac{1}{9} + 33\frac{1}{8} + 62\frac{1}{10} + 75\frac{1}{12}$?
10. $\frac{6}{7} + \frac{7}{8} + \frac{8}{9} + \frac{9}{10} + \frac{10}{11} + \frac{11}{12}$?

6. Answer the following inquiries:

1. $\frac{1}{11} + \frac{1}{7} + \frac{2}{3} + \frac{1}{21} = ?$
2. $\frac{7}{18} + \frac{7}{13} + \frac{3}{24} = ?$
3. $\frac{3}{90} + \frac{1}{15} + \frac{3}{7} = ?$
4. $187\frac{1}{3} + 1976\frac{2}{5} + 746\frac{1}{9} = ?$
5. $8\frac{7}{8} + 9\frac{8}{9} + 12\frac{11}{12} = ?$
6. $7\frac{16}{55} + 8\frac{8}{15} + 9\frac{19}{33} = ?$
7. $11\frac{7}{8} + 10\frac{1}{12} + 7\frac{1}{16} = ?$
8. $8\frac{3}{4} + 6\frac{2}{3} + 2\frac{9}{10} = ?$
9. $5\frac{1}{2} + 6\frac{2}{3} + 7\frac{11}{24} + 9\frac{17}{48} + 3\frac{11}{16} + 2\frac{1}{6} = ?$
10. $9\frac{3}{4} + 10\frac{6}{7} + 11\frac{2}{3} + 5\frac{17}{42} + 7\frac{8}{21} + 18\frac{5}{6} = ?$

PROBLEMS.

1. I bought 3 pieces of cloth containing $125\frac{7}{8}$, $96\frac{3}{4}$, and $48\frac{2}{3}$ yards. How many yards in the three pieces?

2. A merchant sold a customer $22\frac{1}{2}$ yards silk, $3\frac{1}{4}$ yards paper muslin, $1\frac{1}{8}$ yards silesia, $5\frac{3}{4}$ yards cambric, and $5\frac{1}{3}$ yards ruffling. How many yards were sold?

3. A farmer divides his farm into 5 fields. The first contains $26\frac{7}{12}$ acres, the second $40\frac{16}{21}$ acres, the third $51\frac{6}{7}$ acres, the fourth $59\frac{3}{4}$ acres, and the fifth $62\frac{2}{3}$ acres. How many acres in the farm?

4. A bicycler rode $27\frac{3}{4}$ miles on Monday, $33\frac{1}{3}$ miles on Tuesday, $37\frac{3}{4}$ miles on Wednesday, and $42\frac{2}{5}$ miles on Thursday. How far did he ride in the four days?

5. A dry-goods merchant sold a lady $18\frac{3}{4}$ yards of flannel, $21\frac{7}{8}$ yards of silk, and as many yards of calico as of both the other goods. How many yards in all did he sell?

SUBTRACTION OF FRACTIONS.

INDUCTIVE STEPS.

1. From $\frac{5}{9}$ subtract $\frac{3}{9}$.
2. $\frac{8}{13} - \frac{6}{13} =$ what?
3. If you have $\$\frac{7}{8}$ (of a dollar) and spend $\$\frac{3}{8}$, how much have you left?
4. If you have $\$\frac{7}{8}$ and spend $\$\frac{1}{4}$, how do you find the remainder?
5. What kind of fractions can be subtracted without reduction.
6. What kind require reduction? Reduction to what?
7. Give four brief directions for such reduction.
8. What introductory step is sometimes necessary?

PRINCIPLES.

1. **Only like fractions can be subtracted.**
2. **Unlike fractions can be reduced to like fractions and then subtracted.**

EXERCISES.

1. Find the difference between $\frac{8}{11}$ and $\frac{5}{11}$.

Process.

$$\frac{8}{11} - \frac{5}{11} = \frac{3}{11}$$

Explanation.

Since $\frac{8}{11}$ and $\frac{5}{11}$ are like fractions, having a common denominator, 11, their difference is 8 elevenths — 5 elevenths, or 3 elevenths.

2. What is the difference between $\frac{9}{16}$ and $\frac{2}{5}$?

Process.

$$\frac{9}{16} - \frac{2}{5} = \frac{45}{80} - \frac{32}{80} = \frac{13}{80}$$

Explanation.

Since $\frac{9}{16}$ and $\frac{2}{5}$ are unlike fractions, we reduce them to eightieths, making them like fractions. $\frac{9}{16} - \frac{2}{5} = \frac{45}{80} - \frac{32}{80} = \frac{13}{80}$.

3. Subtract $7\frac{5}{8}$ from $11\frac{3}{8}$.

Process.

$$\begin{aligned} 11\tfrac{3}{8} &= 10\tfrac{11}{8} \\ 7\tfrac{5}{8} &= \underline{\ 7\tfrac{5}{8}\ } \\ &\ \ \ 3\tfrac{6}{8} = 3\tfrac{3}{4} \end{aligned}$$

Explanation.

We subtract integers and fractions separately. $\frac{5}{8}$ cannot be taken from $\frac{3}{8}$; but 1, taken from 11, equals $\frac{8}{8}$; $\frac{8}{8} + \frac{3}{8} = \frac{11}{8}$; $\frac{11}{8} - \frac{5}{8} = \frac{6}{8}$ or $\frac{3}{4}$. $10 - 7 = 3$. Uniting the two results, we have $3\frac{3}{4}$, the remainder.

RULE.

1. Reduce unlike fractions to a common denominator.

2. Write the difference of the numerators over the common denominator.

3. Subtract integers and fractions separately, and unite the results.

4. From $\frac{5}{9}$ take $\frac{2}{9}$. From $\frac{3}{5}$ take $\frac{1}{5}$.
5. From $\frac{3}{5}$ take $\frac{1}{3}$. From $\frac{5}{6}$ take $\frac{4}{5}$.
6. From $\frac{7}{8}$ take $\frac{3}{11}$. From $\frac{6}{13}$ take $\frac{5}{17}$.
7. From $\frac{13}{20}$ take $\frac{4}{15}$. From $\frac{14}{25}$ take $\frac{7}{32}$.
8. From $\frac{5}{8}$ take $\frac{3}{15}$. From $\frac{17}{30}$ take $\frac{7}{24}$.
9. From $10\frac{14}{36}$ take $\frac{13}{99}$. From 112 take $75\frac{1}{2}$.
10. From $606\frac{3}{4}$ take $70\frac{1}{2}$. From $506\frac{2}{3}$ take $418\frac{1}{2}$.
11. What is the value of:

1. $\frac{58}{85} - \frac{39}{85}$.	7. $1198\frac{3}{5} - 149\frac{2}{3}$.
2. $\frac{237}{900} - \frac{4}{30}$.	8. $589\frac{2}{3} - 67\frac{3}{4}$.
3. $\frac{38}{95} - \frac{7}{105}$.	9. $75\frac{11}{15} - 63\frac{11}{20}$.
4. $\frac{13}{70} - \frac{1}{15}$.	10. $72\frac{11}{16} - 58\frac{19}{24}$.
5. $\frac{77}{336} - \frac{6}{300}$.	11. $49\frac{1}{12} - 36\frac{3}{10}$.
6. $\frac{97}{262} - \frac{7}{131}$.	12. $42 - \frac{29}{100}$.

12. Find the value of:

1. $51 - 20\frac{6}{11}$.	8. $48\frac{3}{16} - 22\frac{2}{6}$.	15. $76\frac{30}{101} - 68\frac{5}{26}$.
2. $66 - 36\frac{4}{13}$.	9. $35\frac{8}{9} - 29\frac{6}{7}$.	16. $24\frac{11}{19} - 9\frac{6}{25}$.
3. $64 - 59\frac{3}{8}$.	10. $44\frac{2}{3} - 27\frac{2}{3}$.	17. $43\frac{10}{11} - 28\frac{30}{33}$.
4. $38 - 37\frac{8}{11}$.	11. $48\frac{2}{3} - 9$.	18. $10\frac{18}{31} - 9\frac{20}{30}$.
5. $59 - 32\frac{14}{21}$.	12. $73\frac{1}{2} - 27\frac{2}{3}$.	19. $28\frac{34}{47} - 16\frac{35}{47}$.
6. $111 - 31\frac{16}{17}$.	13. $22\frac{2}{9} - 7\frac{1}{2}$.	20. $56\frac{9}{11} - 29\frac{3}{7}$.
7. $36\frac{6}{9} - 27\frac{2}{3}$.	14. $88\frac{4}{11} - 53\frac{10}{27}$.	21. $65\frac{11}{13} - 30\frac{1}{6}$.

13. Answer the following inquiries:

1. $\frac{5}{9} + \frac{3}{4} - \frac{1}{12} = ?$
2. $\frac{8}{9} + \frac{16}{18} - 1\frac{1}{3} = ?$
3. $2\frac{12}{13} - 2\frac{1}{26} + 3\frac{4}{39} = ?$
4. $5\frac{3}{10} - 4\frac{2}{15} + 3\frac{4}{5} = ?$
5. $13\frac{2}{11} + 19\frac{3}{4} - 26\frac{1}{2} = ?$
6. $9\frac{5}{8} - 3\frac{17}{24} + 4\frac{6}{16} = ?$
7. $\frac{49}{200} - \frac{19}{150} + \frac{23}{50} = ?$
8. $\frac{17}{30} - \frac{53}{200} + 2\frac{1}{5} = ?$
9. $3\frac{4}{7} + 9\frac{1}{14} + 6\frac{2}{3} = ?$
10. $36 - 2\frac{1}{9} - 4\frac{3}{8} - 6\frac{1}{3} = ?$
11. $20 - 8\frac{1}{9} - 6\frac{1}{10} - \frac{5}{6} = ?$
12. $200 - 30\frac{4}{5} - 17\frac{1}{12} - 26\frac{19}{20} = ?$
13. $39 - \frac{9}{100} - \frac{13}{20} - \frac{11}{60} = ?$
14. $53 - \frac{8}{15} - 2\frac{3}{5} - 9\frac{8}{10} = ?$
15. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{7} - \frac{1}{9} - \frac{1}{10} = ?$

NOTE.—The teacher will suggest the shortest method of answering the above inquiries.

PROBLEMS.

1. $3\frac{1}{4}$ yards, $4\frac{5}{8}$ yards, and $12\frac{1}{2}$ yards were cut off from a piece of silk containing $30\frac{1}{4}$ yards. How many yards remained?

2. A man spent $\frac{1}{5}$ of his income for rent, $\frac{1}{4}$ for food, and $\frac{1}{8}$ for other expenses. What part of his income remained?

3. A farmer sold $\frac{1}{3}$ of his corn to one man, $\frac{3}{6}$ to another, and had 50 bushels remaining. How much corn had he at first?

4. Show that the fraction $\frac{11}{13}$ is greater than $\frac{5}{6}$ and less than $\frac{6}{7}$.

5. If I pay my grocer $\$18\frac{3}{4}$, my coal dealer $\$27\frac{1}{2}$, and my tailor $\$22\frac{1}{2}$, how much will I have left out of four 20-dollar bills?

6. $\frac{4}{13}$ of a pole is in the mud, $\frac{5}{21}$ of it is in the water, and the rest of it is in the air. What part of it is in the air?

7. Show that $13\frac{4}{9} - 2\frac{5}{18} - 6\frac{9}{16} + 3 - 1\frac{5}{12} + 8\frac{7}{8} - \frac{23}{72} - 10\frac{19}{48} = 4\frac{25}{72}$ is a correct equation.

8. Find the second members of these:

1. $\overline{450 + (12 \times 5)} - 86\frac{9}{40} - 50\frac{1}{8} + \frac{1}{4} = ?$
2. $8\frac{5}{6} + 3\frac{2}{3} - 6\frac{5}{11} = ?$
3. $59 - (27\frac{4}{7} - 4\frac{1}{2}) = ?$
4. $52\frac{1}{3} + \overline{67 - 35\frac{7}{30}} - 5\frac{11}{15} = ?$
5. $231 - 62\frac{2}{9} + 101\frac{10}{27} - \frac{2}{3} = ?$
6. $453 - (32\frac{3}{15} + \frac{5}{3} - 10) = ?$
7. LXXVII. $- \frac{7}{17} +$ CLXIX. $- 11\frac{7}{51} = ?$

MULTIPLICATION OF FRACTIONS.

INDUCTIVE STEPS.

1. How much is 2 times 3 dollars?
2. How much is 2 times 3 sevenths?
3. How much is 2 times $\frac{4}{7}$?
4. How much is 5 times $\frac{5}{9}$?
5. Multiply $\frac{6}{11}$ by 10. $\frac{9}{13}$ by 3.
6. Have you been multiplying numerators or denominators?
7. Then what effect has multiplying the numerator?
8. What is 3 times $\frac{2}{9}$? What are the lowest terms of $\frac{6}{9}$?

9. Then 3 times $\frac{2}{9} = \frac{2}{3}$. How could you have obtained $\frac{2}{3}$ more directly than by multiplying the numerator?

10. What effect, then, has dividing the denominator?

11. Is that effect in agreement with principle 4, page 68? Why?

PRINCIPLE.

Multiplying the numerator or dividing the denominator multiplies the fraction.

EXERCISES.

1. Multiply $\frac{7}{16}$ by 4.

Process.

$\frac{7}{16} \times 4 = \frac{7}{4}$

Explanation.

According to the principle we may multiply the numerator or divide the denominator. Since the denominator, 16, is divisible by 4, we divide and obtain the result, $\frac{7}{4}$.

2. Multiply 4 by $\frac{7}{17}$.

Process.

$4 \times \frac{7}{17} = \frac{28}{17} = 1\frac{11}{17}$

Explanation.

Since the denominator, 17, is not exactly divisible by 4, we multiply the numerator by 4 and obtain the result, $\frac{28}{17} = 1\frac{11}{17}$.

RULE.

To find the product of an integer and a fraction divide the denominator or multiply the numerator by the integer.

3. Multiply:

1. $\frac{9}{14}$ by 7.	8. $\frac{3}{20}$ by 4.	15. $\frac{18}{37}$ by 18.
2. $\frac{7}{25}$ by 5.	9. $\frac{17}{22}$ by 11.	16. $\frac{17}{18}$ by 6.
3. $\frac{18}{21}$ by 7.	10. $\frac{15}{28}$ by 14.	17. $\frac{105}{108}$ by 18.
4. $\frac{7}{48}$ by 6.	11. $\frac{3}{11}$ by 5.	18. $\frac{15}{250}$ by 10.
5. $\frac{9}{56}$ by 8.	12. $\frac{6}{26}$ by 3.	19. $\frac{5}{59}$ by 28.
6. $\frac{17}{24}$ by 3.	13. $\frac{12}{23}$ by 9.	20. $\frac{8}{37}$ by 19.
7. $\frac{16}{39}$ by 13.	14. $\frac{15}{39}$ by 14.	21. $\frac{144}{1728}$ by 12.

4. Multiply $8\frac{5}{8}$ by 4.

Process.

$$\begin{array}{r} 8\frac{5}{8} \\ 4 \\ \hline 2\frac{1}{2} \\ 32 \\ \hline 34\frac{1}{2} \end{array}$$

Explanation.

4 times $\frac{5}{8} = \frac{5}{2} = 2\frac{1}{2}$.
4 times $8 = 32$.
$32 + 2\frac{1}{2} = 34\frac{1}{2}$.

5. Find the value of:

1. $9\frac{2}{3} \times 6$.	8. $28\frac{15}{26} \times 10$.	15. $45\frac{13}{18} \times 60$.
2. $7\frac{2}{3} \times 9$.	9. $\frac{121}{128} \times 48$.	16. $19\frac{18}{21} \times 14$.
3. $8\frac{9}{10} \times 5$.	10. $\frac{229}{1728} \times 144$.	17. $25\frac{19}{25} \times 15$.
4. $18\frac{7}{8} \times 8$.	11. $18\frac{3}{4} \times 10$.	18. $46\frac{15}{26} \times 13$.
5. $21\frac{2}{3} \times 4$.	12. $9\frac{5}{6} \times 21$.	19. $54\frac{25}{42} \times 35$.
6. $6\frac{1}{2} \times 13$.	13. $8\frac{8}{9} \times 24$.	20. $65\frac{19}{51} \times 68$.
7. $17\frac{1}{5} \times 9$.	14. $63\frac{4}{7} \times 56$.	21. $77\frac{11}{121} \times 77$.

6. Multiply:

1. 9 by $\frac{7}{18}$.	8. 100 by $\frac{3}{25}$.	15. \$406 by $\frac{3}{14}$.
2. 57 by $\frac{10}{12}$.	9. 144 by $\frac{11}{16}$.	16. \$718 by $\frac{13}{17}$.
3. 88 by $\frac{3}{4}$.	10. 51 by $\frac{5}{17}$.	17. \$825 by $\frac{23}{45}$.
4. 17 by $\frac{8}{51}$.	11. 75 by $\frac{19}{30}$.	18. $\$\frac{369}{147}$ by 49.
5. 12 by $\frac{19}{48}$.	12. 90 by $\frac{9}{20}$.	19. $\$\frac{175}{273}$ by 26.
6. 124 by $\frac{7}{16}$.	13. 91 by $\frac{11}{26}$.	20. \$400 by $\frac{3}{5}$.
7. 153 by $\frac{9}{17}$.	14. \$318 by $\frac{6}{25}$.	21. $\$\frac{231}{144}$ by $\frac{9}{7}$.

STEPS TO GENERAL RULE.

1. Both expressions, 3×4 and 4×3, $=$?
2. What principle do you find established on page 37?
3. How much is $\frac{1}{2} \times 6$? $\frac{1}{2} \times 18$?
4. How much is $6 \times \frac{1}{2}$? $18 \times \frac{1}{2}$?
5. How much is $\frac{1}{2}$ of 6? $\frac{1}{2}$ of 18?
6. "Of" between a fraction and a following number is equivalent to what sign?
7. Express $27 \times \frac{2}{3}$ by using "of."

8. In how many ways can you indicate the product of 16 and $\frac{7}{8}$?

PRINCIPLES.

1. Fractions, as factors, may be used in any convenient order.

2. A fractional multiplier may be used as expressing the part of the multiplicand to be taken.

EXERCISES.

1. Multiply $\frac{5}{8}$ by $\frac{2}{3}$.

Process.

$$\frac{\cancel{2}}{3} \text{ of } \frac{5}{\underset{4}{\cancel{8}}} = \frac{5}{12}$$

Explanation.

$\frac{5}{8} \times \frac{2}{3} = \frac{2}{3}$ of $\frac{5}{8}$, Principle 2. $\frac{1}{3}$ of $\frac{5}{8} = \frac{5}{8 \times 3}$. $\frac{2}{3}$ of $\frac{5}{8} = \frac{5 \times \cancel{2}}{\underset{4}{\cancel{8}} \times 3} = \frac{5}{12}$. See Principle, page 68.

GENERAL RULE.

Write the integers and mixed numbers in fractional form; cancel common factors, and find the product of the remaining factors of the numerators for a new numerator, and of the denominators for a new denominator.

2. Find the value of:

1. $\frac{8}{9}$ of $\frac{12}{20}$.
2. $\frac{11}{25}$ of $\frac{35}{44}$.
3. $\frac{15}{32}$ of $\frac{55}{56}$.
4. $\frac{20}{81}$ of $\frac{63}{50}$.
5. $\frac{41}{48}$ of $\frac{80}{99}$.
6. $\frac{4}{5}$ of $\frac{7}{12} \times \frac{8}{35}$ of $\frac{25}{32}$.
7. $\frac{24}{35}$ of $\frac{25}{16} \times \frac{16}{50}$ of $\frac{49}{75}$.
8. $\frac{45}{51}$ of $\frac{17}{40}$ of $\frac{9}{25}$ of $\frac{10}{11}$.
9. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7} \times \frac{8}{9}$ of $\frac{10}{71}$.
10. $\frac{4}{5} \times \frac{9}{15}$ of $\frac{11}{12}$ of $\frac{2}{3}$.

3. Reduce:

1. $\frac{4}{5} \times 3\frac{1}{3}$.
2. $\frac{6}{12} \times 12\frac{1}{3}$.
3. $9\frac{1}{2} \times \frac{5}{7}$.
4. $2\frac{9}{10} \times 1\frac{1}{2}$.
5. $\frac{25}{40} \times \frac{4}{5}$.
6. $\frac{17}{18} \times \frac{9}{54}$.
7. $\frac{4}{12} \times \frac{2}{3}$.
8. $\frac{7}{8} \times 2\frac{2}{3}$.
9. $\frac{72}{31} \times 3\frac{3}{5}$.
10. $1\frac{9}{11} \times 1\frac{37}{40}$.
11. $\frac{12}{15} \times \frac{9}{10}$.
12. $\frac{6}{7} \times \frac{10}{12} \times 6\frac{3}{5}$.

4. Reduce:

1. $\frac{3}{7}$ of $1\frac{2}{5} \times \frac{5}{6}$ of $\frac{9}{12}$ of 4.
2. $\frac{15}{27} \times \frac{9}{20} \times \frac{4}{21} \times \frac{5}{7} \times \frac{45}{51}$.
3. $\frac{17}{40} \times \frac{9}{25} \times \frac{3}{11} \times \frac{24}{51} \times \frac{17}{48}$.
4. $\frac{12}{15} \times \frac{7}{16} \times \frac{31}{45} \times \frac{15}{62} \times \frac{5}{7}$.
5. $\frac{9}{10} \times \frac{17}{20} \times \frac{10}{11} \times \frac{3}{5} \times \frac{7}{12}$.
6. $\frac{24}{35} \times \frac{7}{15} \times \frac{15}{18} \times \frac{9}{13} \times \frac{25}{94}$.
7. $\frac{47}{36} \times \frac{32}{75} \times \frac{29}{120} \times \frac{60}{93} \times \frac{31}{114}$.
8. $\frac{3}{4} \times \frac{2}{3} \times \frac{6}{7} \times \frac{7}{8} \times \frac{3}{5} \times \frac{5}{7}$.
9. $\frac{6}{11} \times \frac{5}{18} \times \frac{5}{9} \times \frac{7}{8} \times \frac{5}{3} \times \frac{16}{21}$.
10. $\frac{9}{11} \times \frac{10}{13} \times \frac{26}{40} \times \frac{11}{27} \times \frac{7}{12} \times \frac{5}{6}$.

5. What is the value of:

1. $\frac{5}{9}$ of $\frac{3}{7}$ of $5 \times \frac{3}{11}$ of $\frac{8}{3}$ of $3\frac{1}{2}$?
2. $\frac{3}{5}$ of $\frac{7}{12}$ of $8 \times \frac{5}{7}$ of $\frac{9}{10}$ of 15?
3. $3\frac{1}{2} \times \frac{3}{7} \times 4 \times \frac{3}{4}$ of 7?
4. $5\frac{1}{4}$ times $\frac{7}{9} \times 18 \times \frac{5}{6}$ of 3 times $\frac{1}{5}$ of 4?
5. $\frac{8}{25}$ of $15 \times \frac{3}{5}$ of $\frac{9}{11}$ of $\frac{10}{21}$ of 6?
6. $\frac{8}{17}$ of $3\frac{4}{9}$ of $\frac{34}{49} \times \frac{7}{7}$ of 49?
7. $\frac{16}{23}$ of $\frac{18}{24} \times \frac{28}{45}$ of $\frac{115}{116}$?
8. $\frac{19}{27}$ of $\frac{102}{95} \times \frac{144}{189}$ of $\frac{273}{1728}$ of $\frac{1898}{1898}$?

PROBLEMS.

Required the cost of:

1. 45 pairs of shoes at $\$1\frac{1}{4}$ per pair.
2. $\frac{9}{10}$ of a yard of cloth at $\$\frac{5}{6}$ a yard.
3. 120 yards of ribbon at $16\frac{3}{4}$ cents a yard.
4. $4\frac{4}{5}$ tons of hay at $\$16\frac{7}{8}$ per ton.
5. 465 Rochester lamps at $\$7\frac{3}{5}$ apiece.
6. 250 tons of coal at $\$6\frac{3}{8}$ a ton.
7. $12\frac{7}{8}$ cords of wood at $\$5\frac{3}{4}$ a cord.
8. If $16\frac{1}{2}$ feet make a rod, how many feet are there in $5\frac{1}{2}$ rods?

9. There are $24\frac{3}{4}$ cubic feet in a perch of stone. How many cubic feet in $5\frac{3}{11}$ perches?

10. Mr. Lipmann bought a lot of crockery, of which the retail price was $\$576\frac{3}{8}$, but he got a reduction of $\frac{1}{3}$ for wholesale and $\frac{3}{50}$ for cash. What amount did he pay?

DIVISION OF FRACTIONS.

INDUCTIVE STEPS.

1. 1 divided by 1 equals what?
   4 divided by 1 equals what?
   $\frac{1}{2}$ divided by 1 equals what?
   $\frac{1}{8}$ divided by 1 equals what?
2. If $\frac{1}{8} \div 1 = \frac{1}{8}$, $\frac{1}{8} \div \frac{1}{3}$ equals how many times $\frac{1}{8}$?
3. If $\frac{1}{8} \div \frac{1}{3} = 3$ times $\frac{1}{8}$, $\frac{1}{8} \div \frac{2}{3} = \frac{3}{2}$ times $\frac{1}{8}$. Hence, $\frac{1}{8} \div \frac{2}{3} = \frac{1}{8} \times \frac{3}{2}$ or $\frac{3}{2}$ of $\frac{1}{8}$.
4. Divide in like manner $\frac{3}{4}$ by $\frac{5}{6}$ and $\frac{7}{8}$ by $\frac{9}{10}$.

What principles on page 68 did you apply?

What change in the form of the divisor do you observe?

EXERCISES.

1. Divide $\frac{5}{7}$ by $\frac{2}{3}$.

Process.

$$\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2} = \frac{15}{14} = 1\frac{1}{14}$$

Explanation.

$\frac{5}{7} \div 1 = \frac{5}{7}$. $\frac{5}{7} \div \frac{1}{3} = \frac{5 \times 3}{7}$. $\frac{5}{7} \div \frac{2}{3} = \frac{5 \times 3}{7 \times 2} = \frac{5}{7} \times \frac{3}{2}$. $\div$ has become $\times$, and $\frac{2}{3}$ has become $\frac{3}{2}$, *i.e.*, has become inverted.

2. Divide 4 by $\frac{7}{8}$.

Process.

$$4 \div \frac{7}{8} = \frac{4}{1} \times \frac{8}{7} = \frac{32}{7} = 4\frac{4}{7}$$

Explanation.

$4 = \frac{4}{1}$. Inverting $\frac{7}{8}$ and writing sign $\times$, we have $\frac{4}{1} \times \frac{8}{7} = \frac{32}{7} = 4\frac{4}{7}$.

3. Divide $\frac{9}{10}$ by 3.

Process.

$$\frac{9}{10} \div 3 = \frac{\overset{3}{\cancel{9}}}{10} \times \frac{1}{\cancel{3}} = \frac{3}{10}$$

Explanation.

$\frac{9}{10} \div 3 = \frac{1}{3}$ of $\frac{9}{10} = \frac{\overset{3}{\cancel{9}}}{10} \times \frac{1}{\cancel{3}}$, which by cancellation gives $\frac{3}{10}$.

RULE.

1. Give integers and mixed numbers fractional form.

2. Invert all divisors.

3. Cancel factors common to numerators and denominators.

4. Find the product of the remaining factors.

4. Divide:

1. 15 by $\frac{5}{7}$.

Process.

$$\frac{\overset{3}{\cancel{15}}}{1} \times \frac{7}{\cancel{5}} = 21$$

2. 18 by $\frac{3}{7}$.
3. 63 by $\frac{7}{9}$.
4. 25 by $\frac{8}{3}$.
5. 49 by $\frac{7}{6}$.
6. 75 by $\frac{15}{16}$.
7. 32 by $\frac{8}{9}$.
8. 45 by $\frac{45}{70}$.
9. 64 by $\frac{16}{17}$.

5. Divide:

1. $\frac{12}{13}$ by 6.

Process.

$$\frac{\overset{2}{\cancel{12}}}{13} \times \frac{1}{\cancel{6}} = \frac{2}{13}$$

2. $\frac{115}{127}$ by 5.
3. $\frac{31}{16}$ by 8.
4. $\frac{111}{10}$ by 6.
5. $\frac{66}{8}$ by 60.
6. $\frac{47}{4}$ by 12.
7. $\frac{3}{45}$ by 15.
8. $\frac{125}{131}$ by 5.
9. $\frac{162}{231}$ by 9.

6. What is the value of:

1. $\frac{20}{49} \div \frac{4}{7}$?
2. $\frac{35}{36} \div \frac{5}{6}$?
3. $\frac{27}{24} \div \frac{3}{4}$?
4. $\frac{60}{130} \div \frac{65}{9}$?
5. $\frac{13}{24} \div \frac{7}{8}$?
6. $\frac{24}{29} \div \frac{8}{9}$?
7. $\frac{32}{49} \div \frac{4}{7}$?
8. $\frac{13}{14} \div \frac{13}{15}$?
9. $\frac{81}{90} \div \frac{27}{45}$?

7. Find the value of:

1. $\frac{3}{7}$ of $\frac{5}{8}$ of $16 \div \frac{3}{5}$ of $\frac{6}{7}$ of $5\frac{1}{3}$.

Process.

$$\frac{\cancel{3}}{\cancel{7}} \times \frac{5}{8} \times \frac{\cancel{16}}{1} \times \frac{5}{\cancel{3}} \times \frac{\cancel{7}}{\underset{2}{\cancel{6}}} \times \frac{\cancel{3}}{\cancel{16}} = \frac{25}{16} = 1\frac{9}{16}.$$

2. $\frac{9}{5}$ of $\frac{15}{13}$ of $5\frac{1}{3} \div 4\frac{1}{3}$ times $\frac{1}{6}$ of 17.
3. $\frac{2}{5}$ of $2\frac{1}{2} \div 5\frac{1}{3}$.
4. $\frac{1}{6}$ of $\frac{5}{3}$ of $\frac{9}{11}$ by 7 times $\frac{3}{8}$ of $\frac{6}{7}$.
5. $(\frac{3}{7} \div \frac{3}{10}) \times \frac{5}{14}$.
6. $\frac{2}{3}$ of $4\frac{1}{2} \div \frac{7}{9}$ of $3\frac{3}{5}$.
7. $\frac{3}{5}$ of $\frac{3}{7}$ of $15 \div \frac{2}{5}$ of $\frac{5}{9}$ of 6.
8. $2\frac{1}{5}$ of $2\frac{1}{2} \div \frac{3}{14}$ of $3\frac{2}{3}$.
9. $\frac{5}{8}$ of $\frac{6}{11}$ of $22 \div \frac{3}{10}$ of $\frac{5}{7}$ of 16.
10. $\frac{3}{8}$ of $\frac{5}{7}$ of $\frac{14}{15} \div 6$.
11. $\frac{3}{14}$ of $3\frac{2}{3}$ of $6 \div \frac{7}{9}$ of 6 times $1\frac{2}{7}$.
12. $\frac{5}{9}$ of $3\frac{1}{3}$ of $\frac{6}{25} \div 5\frac{1}{2}$.
13. $8\frac{1}{4}$ times $\frac{1}{3}$ of $7 \div \frac{3}{4}$ of $\frac{2}{3}$ of 5.
14. $\frac{3}{10} \div \frac{2}{5}$ of $2\frac{1}{4}$ of $1\frac{5}{7}$.
15. $\frac{2}{9}$ of $35\frac{1}{2} \div \frac{5}{7}$ of $8\frac{2}{5}$.
16. $\frac{6}{13}$ of $\frac{14}{15} \div \frac{5}{9}$ of $\frac{7}{8}$.
17. $\frac{5}{11}$ of $\frac{17}{21} \div \frac{2}{21}$ of $\frac{22}{29}$.
18. $\frac{16}{19}$ of $\frac{18}{23} \div \frac{7}{20}$ of $3\frac{1}{57}$.
19. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8} \div \frac{5}{6}$ of $\frac{14}{15}$ of $\frac{20}{21}$.
20. $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{8}{9} \div \frac{7}{8}$ of $\frac{9}{10}$ of $\frac{11}{14}$ of $\frac{16}{27}$.

PROBLEMS.

1. If $7\frac{3}{5}$ yards of cloth cost $\$47\frac{1}{2}$, what is the price per yard?

2. If a man spends $\$\frac{2}{3}$ per day for cigars, in how many days will he spend $\$17\frac{1}{2}$?

3. If $\frac{5}{6}$ of a ton of hay costs \$15, what is the cost of one ton?

4. A man has $229\frac{1}{2}$ pounds of honey, which he wishes to pack in boxes containing $8\frac{1}{2}$ pounds each. How many boxes will he require?

5. A man owning $\frac{15}{16}$ of a ship, sold $\frac{3}{5}$ of his share, and divided the remainder equally among his three sons. What part of the ship did each son own?

6. The product of two numbers is $\frac{13}{14}$, and one of the numbers is $1\frac{1}{14}$. What is the other number?

7. What number multiplied by $1\frac{3}{8}$ will produce $14\frac{3}{4}$?

8. How many yards of cloth at $\$3\frac{2}{5}$ per yard can be bought for $\$317\frac{2}{3}$?

9. When wheat is selling at $\$1\frac{7}{8}$ per bushel, how many bushels can be bought for $3168?

10. For $\$8\frac{7}{10}$ how many thousand feet of gas at $\$1\frac{1}{4}$ per thousand can be bought?

COMPLEX FRACTIONS.

A Complex Fraction has a fraction in one or both of its terms; as, $\frac{\frac{2}{3}}{4}$, $\frac{\frac{2}{3}}{\frac{4}{5}}$, $\frac{4\frac{1}{2}}{5\frac{3}{4}}$, $\frac{\frac{3}{5} \text{ of } \frac{6}{8}}{\frac{4}{9} \times \frac{5}{3}}$.

1. Simplify $\frac{5\frac{1}{3}}{6\frac{1}{4}}$.

Process.

$5\frac{1}{3} \div 6\frac{1}{4} = \frac{16}{3} \times \frac{4}{25} = \frac{64}{75}$

Explanation.

Since $\frac{5\frac{1}{3}}{6\frac{1}{4}}$ signifies that $5\frac{1}{3}$ is to be divided by $6\frac{1}{4}$, we proceed according to the rule for division, and obtain $\frac{64}{75}$.

2. Simplify:

1. $\frac{2\frac{3}{11}}{3\frac{3}{4}}$.

2. $\frac{\frac{5}{9}}{8\frac{1}{3}}$.

3. $\frac{5\frac{1}{9}}{8\frac{4}{11}}$.

4. $\frac{1\frac{4}{5} \text{ of } 3\frac{1}{7}}{4\frac{1}{8} \text{ of } \frac{9}{10}}$.

5. $\frac{\frac{3}{7} \text{ of } 2\frac{1}{17}}{1\frac{2}{3} \div 2\frac{3}{7}}$.

6. $\frac{\frac{2}{3} \text{ of } \frac{5}{6}}{\frac{2}{3} \text{ of } 9}$.

7. $\frac{\frac{2}{5} \text{ of } 9}{7}$.

8. $\frac{3}{\frac{1}{5} \text{ of } \frac{7}{3}}$.

9. $\frac{\frac{3}{5} \text{ of } \frac{5}{6}}{4\frac{1}{3} \times 3}$.

10. $\frac{2\frac{3}{5} \times \frac{9}{11}}{3\frac{5}{7} \div 4\frac{1}{8}}$.

11. $\dfrac{\frac{23}{26}}{\frac{9}{13}}$; also, $(\frac{3}{7} \text{ of } 2\frac{1}{17}) \div \dfrac{1\frac{2}{3}}{2\frac{3}{7}}$.

12. $\dfrac{6\frac{1}{7}}{5\frac{1}{3}}$; also, $4 \div \dfrac{1}{4 - \frac{1}{4}}$.

13. $\dfrac{1\frac{4}{5} \text{ of } 3\frac{1}{7}}{4\frac{1}{8} \text{ of } \frac{9}{11}} \times \dfrac{2}{3 - \frac{4}{5}}$.

14. $\dfrac{3\frac{3}{7}}{11\frac{1}{4}} \text{ of } \dfrac{3\frac{3}{8}}{2\frac{2}{5}} \div \dfrac{3}{4 - \frac{5}{6}}$.

15. $\dfrac{3\frac{5}{11}}{1\frac{2}{5} \times 9\frac{1}{11}}$.

16. $\dfrac{\frac{1}{7}}{8} \times \dfrac{\frac{8}{7}}{9\frac{1}{2}} \times \dfrac{7\frac{1}{9}}{\frac{8}{9}}$.

17. $\dfrac{2\frac{3}{4} \times 7\frac{7}{11}}{\frac{1}{2} \times \frac{3}{4} \times 18\frac{2}{3}}$.

18. $\frac{1898}{1898} \text{ of } \dfrac{9\frac{1}{10}}{2\frac{10}{33}}$.

FRACTIONAL RELATIONS.

1. In the equation, $\frac{1}{2}$ of $4 = 2$, the $\frac{1}{2}$ expresses the relation of 2 to 4. If the question is asked, "What is the fractional relation of 2 to 4?" the answer simply reverses the equation, —"$2 = \frac{1}{2}$ of 4."

This equation may be derived analytically, thus: Since $1 = \frac{1}{4}$ of 4, 2, being twice 1, $= \frac{2}{4}$ or $\frac{1}{2}$ of 4.

2. In like manner show the fractional relation of 3 to 9. Of 5 to 8.

What part of 8 is 5? Does the answer show the relation of 5 to 8?

3. What part of \$5 is \$$\frac{1}{2}$?

Since $\$1 = \frac{1}{5}$ of \$5, \$$\frac{1}{2}$, being $\frac{1}{2}$ of \$1, $= \frac{1}{2}$ of $\frac{1}{5}$ of five dollars, or $\frac{1}{10}$ of five dollars.

4. In like manner find the fractional relation of \$$\frac{3}{8}$ to \$6.

5. What part of 7 acres is $\frac{5}{6}$ of an acre?

6. Is \$5 any part of 10 acres?

7. What is the fractional relation of 7 men to 9 trees?

PRINCIPLE.

Only like numbers can have fractional relation to each other.

EXERCISES.

To find the Fractional Relation between Two Numbers.

1. Form an equation to show the fractional relation of:

1. 8 to 24.
2. 13 to 26.
3. 12 to 18.
4. 10 to 15.
5. 9 to 27.
6. 35 to 40.
7. 16 to 24.
8. 15 to 35.
9. 19 to 95.
10. 20 to 110.
11. $\frac{2}{5}$ to 5.
12. $\frac{3}{7}$ to 10.
13. $\frac{5}{8}$ to 7.
14. $\frac{4}{9}$ to 9.
15. $\frac{3}{4}$ to 16.
16. $\frac{5}{8}$ to 26.
17. $\frac{3}{9}$ to 7.
18. $\frac{4}{5}$ to 15.
19. $\frac{7}{8}$ to 16.
20. $\frac{9}{10}$ to 3.
21. $6\frac{2}{3}$ to 40.
22. $6\frac{1}{4}$ to 425.
23. $2\frac{4}{5}$ to 42.
24. $6\frac{2}{5}$ to 128.
25. $6\frac{3}{4}$ to 75.
26. $12\frac{1}{2}$ to 180.
27. $\frac{1}{2}$ of $3\frac{3}{4}$ to 84.
28. $\frac{3}{4}$ of $\frac{5}{6}$ to 75.
29. $8\frac{1}{3}$ to $\frac{1}{2}$ of 90.
30. $\frac{3}{4}$ of $2\frac{1}{3}$ to $\frac{5}{6}$ of 18.

2. Find the fractional relation of:

1. $\frac{2}{3}$ to $\frac{3}{5}$.

Suggestion: $\frac{2}{3} = \frac{10}{15}$ and $\frac{3}{5} = \frac{9}{15}$; $10 = \frac{10}{9}$ of 9.

2. $\$\frac{1}{4}$ to $\$\frac{3}{4}$.
3. $\$\frac{3}{8}$ to $\$\frac{5}{4}$.
4. $\$\frac{1}{10}$ to $\$\frac{1}{2}$.
5. $\$\frac{7}{8}$ to \$1.
6. $\$\frac{5}{7}$ to $\$\frac{6}{11}$.
7. $\$\frac{5}{8}$ to $\$\frac{2}{3}$.
8. $\$\frac{3}{2}$ to $\$\frac{6}{5}$.
9. $\$\frac{3}{8}$ to $\$\frac{6}{7}$.
10. $\$\frac{6}{7}$ to $\$\frac{4}{5}$.
11. $\$\frac{3}{10}$ to $\$\frac{5}{9}$.
12. $2\frac{1}{5}$ to $\frac{3}{5}$.
13. $1\frac{1}{4}$ to $2\frac{3}{4}$.
14. $2\frac{1}{3}$ to $7\frac{1}{8}$.
15. $7\frac{1}{3}$ to $2\frac{1}{8}$.
16. $3\frac{1}{3}$ to $8\frac{1}{7}$.
17. \$6 to \$100.
18. $\$8\frac{4}{9}$ to $\$100\frac{5}{9}$.
19. $\frac{2}{3}$ of $1\frac{2}{3}$ to $3\frac{1}{3}$.
20. $9\frac{10}{3}$ to $12\frac{9}{26}$.
21. $1\frac{3}{5}$ to $3\frac{1}{2} \times \frac{8}{9}$ of $\frac{4}{7}$.

To find a Number from its Fractional Relation to Another Number.

1. In the equation, $3 = \frac{1}{3}$ of 9, the $\frac{1}{3}$ expresses the relation of 3 to 9.

2. If the question arise, 3 is $\frac{1}{3}$ of what number? what is the answer?

3. The analytical answer is what?

Suggestion: $\frac{1}{3}$ of the number $= 3$; $\frac{3}{3}$, the whole of the number, $=$ what?

4. Answer the following inquiries:

1. 12 is $\frac{1}{4}$ of what number?
2. 24 is $\frac{4}{5}$ of what number?

   Suggestion: $\frac{1}{5}$ of the number $= \frac{1}{4}$ of 24.

3. 24 is $\frac{3}{8}$ of what number?
4. 28 is $\frac{5}{7}$ of what number?
5. 48 is $\frac{7}{3}$ of what number?
6. $\frac{15}{17}$ is $\frac{5}{8}$ of what number?

   Suggestion: $\frac{1}{8}$ of the number $= \frac{1}{5}$ of $\frac{15}{17}$.

7. $\frac{28}{32}$ is $\frac{7}{11}$ of what number?
8. $\frac{24}{19}$ is $\frac{12}{7}$ of what number?
9. $\frac{52}{67}$ is $\frac{144}{144}$ of what number?
10. $\frac{1}{2}$ of $3\frac{1}{5}$ is $\frac{5}{7}$ of what number?

REVIEW.

1. The sum of two fractions is $\frac{8}{9}$, and their difference is $\frac{1}{7}$. Required the fractions.

Suggestion: Were the difference 0, the fractions would be $\frac{4}{9}$ and $\frac{4}{9}$. Hence the greater fraction $= \frac{4}{9} + \frac{1}{2}$ of $\frac{1}{7}$; the less $= \frac{4}{9} - \frac{1}{2}$ of $\frac{1}{7}$.

2. If a man can cut in one day $\frac{1}{2}$ of a field containing 7 acres of wheat, how many acres can he cut in $\frac{5}{7}$ of a day?

3. Reduce $\frac{1}{2}$, $\frac{5}{6}$, $\frac{3}{8}$ and $\frac{1}{4}$ to equivalent fractions whose denominators shall be 24.

4. Add $\frac{9}{14}$, $\frac{14}{18}$, $4\frac{5}{8}$, $15\frac{2}{3}$, and explain fully.

5. Find the value of $\dfrac{\frac{1}{2} \text{ of } \frac{5}{7} \text{ of } 7\frac{3}{8}}{21\frac{1}{14}}$.

6. The product of three numbers is $\frac{6}{7}$; two of the numbers are $2\frac{1}{2}$ and $\frac{7}{9}$; what is the third?

7. A housekeeper bought 6 mahogany chairs at $3\frac{4}{5}$ dollars each, and gave for them 2 ten-dollar bills and one five-dollar bill. What change ought she to receive?

8. Find the sum of $1\frac{7}{12} + 3\frac{11}{15} + 4\frac{19}{20}$.

9. A box contains 345 eggs. What is their value at \$.16$\frac{2}{3}$ a dozen?

10. If $\frac{5}{8}$ of an acre of land cost 101 dollars, what will $\frac{3}{4}$ of an acre cost?

11. Reduce $816\frac{5}{11}$ to an improper fraction.

12. Subtract $\frac{1}{3}$ of $\frac{9}{10}$ from $\dfrac{8\frac{2}{3} + 2\frac{1}{4}}{4\frac{1}{5}}$.

13. Simplify $\dfrac{16}{5\frac{1}{3}}$; also, $\dfrac{1}{3\frac{1}{5}} - \dfrac{2\frac{1}{4}}{9}$.

14. If $\frac{3}{10}$ of an acre of land is worth \$79$\frac{1}{4}$, what is 1 acre worth?

15. Reduce $\dfrac{\frac{4}{5} \text{ of } \frac{1}{3}}{8\frac{1}{4}}$ to a simple fraction.

16. From what must $6\frac{3}{5}$ be subtracted to leave $\frac{1}{2}$ of $3\frac{1}{9}$?

17. At $\frac{7}{8}$ of a dollar per bushel, what will be the cost of $\frac{4}{9}$ of a bushel of potatoes?

18. $\frac{8}{9}$ of 27 is $\frac{6}{7}$ of what number?

19. From $\frac{3}{4}$ of $\frac{4}{5}$ take $\frac{1}{2}$ of $\frac{2}{3}$.

20. Simplify $\dfrac{42 \times 18 \times 75 \times 6}{25 \times 17 \times 14 \times 9}$.

21. A merchant paid $85\frac{1}{4}$ dollars for $15\frac{1}{2}$ tons of coal. How much did the coal cost him per ton?

22. Simplify $8\frac{7}{24} - 2\frac{7}{8} - 3\frac{1}{4} + 6\frac{3}{16} - 5\frac{2}{3}$.

23. 150 is $\frac{10}{7}$ of what number?

24. \$150 was paid for a horse and saddle. If the cost of the saddle was $\frac{3}{7}$ of the cost of the horse, what was the cost of each?

25. $700 - \frac{7}{100} = ?$

26. Find the quotient of $20\frac{4}{7} \div \frac{2}{11}$ of $8\frac{8}{35}$.

27. How many lemons, at $\frac{6}{25}$ of a dollar a dozen, will pay for 80 oranges at $2\frac{1}{8}$ cents each?

28. Four loads of hay weigh respectively $1723\frac{5}{8}$, $2317\frac{3}{4}$, $1547\frac{3}{8}$, and $1357\frac{1}{2}$ pounds. What is the total weight of the hay?

29. Reduce $\frac{1232}{1540}$ to its lowest terms.

30. A farmer had $\frac{1}{5}$ of his sheep in one pasture, $\frac{1}{4}$ in another, and the remainder, which were 77, in a third pasture. How many sheep had he?

31. If I give A. $\frac{1}{3}$ of my money, B. $\frac{1}{4}$ of it, and C. $\frac{1}{6}$ of it, what part of my money have I left?

32. The divisor is $\frac{88}{315}$, and quotient $\frac{252}{143}$. What is the dividend?

33. A man bought land for \$5130, and sold it so as to gain $\frac{1}{19}$ of the cost, the gain being \$3 per acre. How many acres did he buy?

34. After buying a suit of clothes for \$60 I found I had $\frac{3}{7}$ of my money left. How much had I at first?

35. What number, diminished by the difference between $\frac{3}{4}$ and $\frac{3}{5}$ of itself, leaves a remainder of 34?

36. Divide $\frac{3}{4}$ of $3\frac{1}{3}$ by $\frac{5}{9}$ of $\frac{15}{16}$.

37. If $3\frac{3}{4}$ bushels of oats will sow an acre, how many bushels will it take to sow $7\frac{1}{3}$ acres?

38. Reduce $\dfrac{\frac{7}{9} \text{ of } 2\frac{1}{7}}{2\frac{1}{9}} \times \dfrac{\frac{2}{3}}{\frac{1}{2} \text{ of } \frac{1}{6}}$.

39. Find the value of $5 + 6\frac{4}{7} - 7\frac{7}{10} + \frac{23}{30}$.

40. The circumference of a bicycle wheel is $7\frac{1}{2}$ feet; the circumference of another bicycle wheel is $7\frac{6}{7}$ feet. How many more times will the smaller wheel turn than the larger in going 5280 feet?

41. Divide $(\frac{5}{2} - \frac{2}{5})$ by $(\frac{4}{3} - \frac{3}{4})$.

42. Reduce $\frac{15}{6}$ to 786ths.

43. If $\frac{6}{7}$ of a cord of wood cost $\$6\frac{4}{5}$, what will 10 cords cost?

44. Find the sum and product of $\frac{7}{5}$, $\frac{1}{3}$ and $\frac{5}{8}$.

45. What is the value of $(\frac{2}{3}$ of $\frac{5}{6}$ of $3\frac{3}{4} + 8\frac{2}{3}) \div (10\frac{1}{2} - 7\frac{5}{12})$?

46. A shepherd, being asked how many sheep he had, answered that $\frac{5}{7}$ of $\frac{3}{4}$ of the whole number was 45. How many had he?

47. Find the G. C. D. and the L. C. Dd. of 833, 1127, 1421, 343.

48. What part of $\frac{2}{3}$ is $\frac{1}{2}$?

49. If $3\frac{1}{9}$ yards of cloth cost 84 cents, how much is that per yard?

50. The reciprocal of $\frac{7}{8}$ is $1 \div \frac{7}{8}$. What are the sum, the difference, and the product of $\frac{7}{8}$ and its reciprocal?

51. Reduce to a common denominator and add $\frac{3}{8} \times \frac{4}{5} \times \frac{5}{9}$, $\frac{7}{15}$, $\frac{3}{4}$, and $\frac{9}{10}$.

52. A lot which cost $400 was sold for $480. What part of the cost was gained?

53. How much is the sum of $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ greater or less than $\frac{3}{4}$ of the sum of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{9}$?

54. If bricks cost $8.50 a thousand, what is the cost of one brick?

55. Simplify $\dfrac{\dfrac{6\frac{1}{4}}{4\frac{1}{4}}}{12}$. Give the principles involved.

56. A regiment lost in battle 250 men, which was $\frac{2}{9}$ of the regiment. What was the number of men before the battle?

57. Divide 98 by $11\frac{11}{13}$, and multiply the quotient by $\frac{3}{7}$ of $8\frac{3}{4}$.

58. Reduce $\frac{7}{10}$, $\frac{10}{12}$, $\frac{14}{15}$ to their L. C. D.

59. A barrel of beef, which holds 200 pounds, was $\frac{2}{3}$ full. How many pounds would there be left in it after $53\frac{3}{5}$ pounds were taken out?

60. A man having $\$5\frac{1}{4}$ bought a knife, and then had left $\$4\frac{9}{10}$. How much did the knife cost?

61. Mr. Gould sold a cow for $30, which was $\frac{3}{5}$ of what she cost him. How much did he lose?

62. At $9\frac{1}{2}$ dollars a barrel, how many pounds of flour can be bought for $\$3\frac{1}{6}$? [One barrel = 196 pounds.]

63. When hay is worth $\$9\frac{3}{4}$ a ton, what will $\frac{2}{3}$ of $3\frac{1}{5}$ tons cost?

64. A. and B. kill an ox. A. takes $\frac{5}{8}$ and B. the remainder. If B.'s share weighs $361\frac{1}{2}$ pounds, what is the weight of the ox?

65. What fraction of $18\frac{3}{4}$ is $6\frac{2}{3}$?

66. If $2\frac{3}{4}$ acres of land cost $220, what will be the cost of $17\frac{7}{8}$ acres? Indicate the work and cancel.

67. If 15 men do a piece of work in $10\frac{2}{3}$ days, how long would it take 5 men to do the same work?

68. If 5 be added to both terms of the fraction $\frac{8}{7}$, will its value be increased or diminished?

69. If A. can do a piece of work in 5 days and B. in 8 days, how long will it take both to do it?

70. There are two numbers whose sum is 140, one of which is $\frac{3}{4}$ the other. What are the numbers?

71. The product of two numbers is 6, and one of them is 1846. What is the other?

72. If 3 dozen lemons cost $\$1\frac{1}{8}$, what will be the cost of 56 lemons?

73. If $7\frac{1}{2}$ pounds of rice cost $.90, how many pounds can be bought for $1.10?

74. A clerk earns $\$1\frac{3}{5}$ a day, and spends $\$\frac{2}{3}$ a day. How much does he save in a year?

75. Multiply $\frac{5\frac{1}{3}}{18\frac{2}{5}}$ by $\frac{11\frac{11}{12}}{12\frac{1}{10}}$.

76. $(\frac{39}{72} \times \frac{45}{56}) \div (\frac{44}{54} \times \frac{42}{32})$.

77. Margaret, in attempting to divide a fraction by $\frac{18}{19}$, inverted the dividend instead of the divisor, and obtained a quotient of $\frac{41}{38}$. What was the given fraction?

78. If a man's brain is $\frac{1}{50}$ of his weight, and weighs $3\frac{3}{32}$ pounds, what is his weight?

79. Which is the greater, $\frac{40}{103}$ or $\frac{5}{13}$?

80. When land is worth 100 dollars per acre, what part of an acre will be worth $26\frac{2}{3}$ dollars?

81. A cistern can be filled by one pipe in 15 hours, and by another in 20 hours. In what time can the two pipes fill it flowing together?

82. What part of $\frac{12\frac{1}{2}}{\frac{4}{7}}$ is $\frac{\frac{2}{3} \times \frac{3}{4}}{\frac{1}{2}}$?

83. What is the quotient of $1\frac{1}{2}$ divided by its reciprocal?

NOTE.—The reciprocal of a quantity is 1 divided by that quantity.

84. Change $\frac{5}{7}$ to a fraction whose denominator shall be 35.

85. Find the least number of apples that, arranged in groups of 8, 9, 10, or 12, will have just 6 over in each case.

86. Three times a number plus $\frac{2}{3}$ of it, plus $4\frac{6}{7}$ times the number plus $\frac{1}{9}$ of it, are how many times the number?

87. If $\frac{3}{5}$ of a steeple casts a shadow $83\frac{2}{3}$ feet long, how long is the shadow cast by $\frac{5}{9}$ of it?

88. A man has $4\frac{6}{7}$ bushels of potatoes, which is $\frac{5}{9}$ of the quantity that he planted. How many did he plant?

89. A man who received $\frac{7}{8}$ of his father's property gives to his own son $\frac{3}{4}$ of what he received. Who then has $\frac{7}{32}$ of the whole?

90. Two men require $8\frac{1}{2}$ days to take account of a stock of goods. Six men would need what time?

91. What fraction is the quotient of $\frac{990}{2184} \div \frac{96}{336}$?

92. In sowing a field, one kind of seed is used at the rate of $12\frac{1}{4}$ bushels to 5 acres. What will be required to sow $22\frac{2}{7}$ acres, using $\frac{5}{7}$ as much to the acre as before?

93. When oysters yield $1\frac{1}{4}$ gallons to the bushel, a 25-gallon barrel can be filled from how many bushels in the shell?

94. $\frac{1}{4}$ of a bushel of berries is picked; $\frac{1}{2}$ of them are sold to one man, $\frac{1}{4}$ of the remainder to another. What fractional part remains unsold?

95. Oranges are bought at 3 for \$.05 and sold at 4 for \$.09. What is gained on a box of 9 dozen, 1 in 12 of which are worthless.

96. $21 - \frac{8}{9} - \frac{8}{9} - \frac{8}{9} - \frac{8}{9} = ?$

97. Find the G. C. D. and the L. C. Dd. of 45, 90, 100, and 200.

98. A., B., and C. can do a piece of work in 10 days. A. can do it in 25 days, and B. in 30 days. In what time can C. do it?

99. A. and B. together had \$5700. $\frac{2}{3}$ of A.'s money was equal to $\frac{3}{5}$ of B.'s. How much had each?

100. Define:

1. Fraction.
2. Decimal fraction.
3. Common fraction.
4. Fractional unit.
5. Denominator.
6. Numerator.
7. Proper fractions.
8. Improper fractions.
9. Mixed number.
10. Reduction.
11. Higher terms.
12. Lower terms.
13. Lowest terms.
14. Like fractions.
15. Unlike fractions.
16. Common denominator.
17. Least common denominator.
18. Fractional relation.

101. Repeat:

1. The principles of Addition of Fractions.
2. The rule for Addition of Fractions.
3. The brief directions for finding L. C. D. of Fractions.
4. The principles of Subtraction of Fractions.
5. The rule for Subtraction of Fractions.
6. The principles of Multiplication of Fractions.
7. The rules for Multiplication of Fractions.
8. The principles of Division of Fractions.
9. The rules for Division of Fractions.
10. The principle of Fractional Relation.

102. Invent and solve:

1. Five problems in Reduction of Fractions.
2. Five problems in Addition of Fractions.
3. Five problems in Subtraction of Fractions.
4. Five problems in Multiplication of Fractions.
5. Five problems in Division of Fractions.
6. Five problems in Relation of Fractions.
7. Five miscellaneous problems in Fractions.

DECIMAL FRACTIONS.

DEFINITIONS.

1. A **Decimal Fraction** denotes one or more of the decimal divisions of a unit.

2. **Decimal Fractions** are usually called **decimals** (Latin, decem, "ten").

3. A **Pure Decimal** consists of decimal figures only, as .234.

4. **A Mixed Decimal** consists of an integer and a decimal, as 23.005.

5. **A Complex Decimal** has a common fraction on the right of the decimal, as $.06\frac{2}{3}$.

NOTATION AND NUMERATION.

1. By placing a mark (.), called the decimal point, after units of the first order, the numeration and notation table is extended to express parts of a unit on the decimal scale.

2. The relation of decimals and integers to each other is clearly shown by the following

Numeration Table.

Hundred-millions.	Ten millions.	*Millions.*	Hundred-thousands.	Ten-thousands.	*Thousands.*	Hundreds.	Tens.	UNITS.	Tenths	Hundredths.	*Thousandths.*	Ten-thousandths.	Hundred-thousandths.	*Millionths.*	Ten-millionths.	Hundred-millionths.
9	8	7	6	5	4	3	2	1 .	2	3	4	5	6	7	8	9
INTEGERS.									DECIMALS.							

By examining this table we see that:—

Tenths are expressed by one figure.
Hundredths are expressed by two figures.
Thousandths are expressed by three figures.
Ten thousandths are expressed by four figures.
Hundred thousandths are expressed by five figures.
Millionths are expressed by six figures.

3. The decimal point is a separatrix, not a period; it is read "and."

Remember that the name of the 6th decimal order is *Millionths,* and give orally the names of the following orders: 6th order, 5th order, 4th order, 3d order, 2d order, 1st order, 3d order, 5th order, 4th order, 6th order, 1st order, 5th order, 2d order, 4th order, 3d order, 6th order, 5th order, 4th order, 3d order, 2d order, 1st order, 6th order.

In what decimal place do you find: Millionths? Thousandths? Tenths? Hundredths? Ten-thousandths? Hundred-thousandths? Ten-millionths? Hundredths? Millionths? Thousandths?

4. Read the following: 1.2, 1.03, 1.004, 1.0005, 1.00006, 1.000007, 2.008, 3.09, 4.0001, 5.000002, 6.00003, 7.0004, 8.9, 9.10.

PRINCIPLES.

1. Decimals and integers are subject to the same law of local value.

2. Each cipher inserted between the decimal point and the first figure of a decimal diminishes the value of the decimal ten-fold.

3. Annexing ciphers to a decimal does not alter its value.

.05 = .050, for 0 thousandths add nothing to 5 hundredths.

4. The denominator of a decimal, when expressed, is 1 with as many ciphers annexed as there are orders, or places, in the decimal.

Read 7.039.

ANALYSIS.

7 is an integer representing 7 units, and is read "seven." The decimal point is read "and" 0 denotes the absence of tenths, and is not read. 3 hundredths + 9 thousandths is read "39 thousandths." Hence 7.039 is read "7 and 39 thousandths"

RULE.

Read the decimal as an integral number, and add the decimal name of the right-hand figure.

EXERCISES.

1. Read the following:

1. .7.	18. 29.15625.	35. 6.839.
2. .36.	19. 341.63456.	36. $.24\frac{2}{3}$.
3. .625.	20. 1001.000089.	37. $3.70\frac{2}{3}$.
4. .025.	21. .6305.	38. 7.039.
5. .0005.	22. $.446\frac{1}{2}$.	39. 8.1367.
6. .12345.	23. .00371.	40. $7.0308\frac{3}{4}$.
7. .789123.	24. .0506.	41. $9.1007\frac{5}{6}$.
8. .405607.	25. .087345.	42. 146.0302056.
9. .890123.	26. 6.00056.	43. 376.932474.
10. .456789.	27. 11.04735.	44. 2.234006.
11. 8.54.	28. 63.04048.	45. 487.000081035.
12. 85.4.	29. 100.000001.	46. 586.0004003256.
13. 9.213.	30. 734.819181.	47. .5.
14. 7.389.	31. 341.63456.	48. 5.078.
15. 12.3601.	32. .684.	49. 8.008.
16. 19.0032.	33. .084.	50. 6.2040.
17. 25.00081.	34. .004.	51. 37.40253.

2. Write decimally 13 thousandths.

ANALYSIS.

13 thousandths = one hundredth + 3 thousandths. 0 tenths are given. As the number is a pure decimal, the expression of it must begin with the decimal point. Hence 13 thousandths expressed decimally is .013.

RULE.

Write the number as an integer, and give the right-hand figure the place indicated by the decimal name of the number.

3. Express decimally:
   1. Seven tenths. Twelve hundredths.
   2. Nine tenths. Seventeen hundredths.

3. Four hundredths. 42 hundredths.
4. 125 thousandths. 22 thousandths.
5. 20 hundredths. Eight thousandths.
6. 30 thousandths. 206 thousandths.
7. 3027 ten-thousandths.
8. Three hundred ten-thousandths.
9. Forty-two ten-thousandths.
10. 145 hundred-thousandths.
11. Fifty-one hundred-thousandths.
12. One hundred seven millionths.
13. 306 ten-millionths.
14. 3259 hundred-thousandths.
15. 429 ten-millionths.
16. 4268 hundred millionths.
17. 13,760 millionths.
18. Three hundred forty-two millionths.
19. One hundred forty-five hundred thousandths.
20. 703,205 millionths.

4. Express as mixed decimals the following:

1. $5\frac{4}{10}$.	8. $24\frac{208}{1000}$.	15. $73\frac{318}{100000}$.
2. $7\frac{9}{10}$.	9. $27\frac{49}{1000}$.	16. $4\frac{13}{1000}$.
3. $8\frac{25}{100}$.	10. $54\frac{175}{10000}$.	17. $64\frac{5}{10}$.
4. $9\frac{7}{100}$.	11. $74\frac{9}{10000}$.	18. $33\frac{418}{100000}$.
5. $12\frac{5}{1000}$.	12. $86\frac{3216}{10000}$.	19. $\frac{9}{1000000}$.
6. $15\frac{29}{1000}$.	13. $48\frac{21875}{100000}$.	20. $\frac{4000}{10000}$.
7. $15\frac{406}{1000}$.	14. $56\frac{4279}{100000}$.	21. $95\frac{375}{100000}$.

UNITED STATES MONEY.

1. Read $12.925 as a mixed decimal, and as dollars, cents, and mills.

It is read "12 *and 925-thousandths dollars*," or "12 *dollars, ninety-two cents, five mills.*"

2. Read in like manner the following:

1. \$89.06.	5. \$59.375.	9. \$1.375.
2. \$94.254.	6. \$86.047.	10. \$0.876.
3. \$69.045.	7. \$344.002.	11. \$0.093.
4. \$195.005.	8. \$20.25.	12. \$0.001.

3. Express decimally $\$\frac{48}{100}$, $\$20\frac{1}{4}$, $\$35\frac{1}{2}$, $\$\frac{13}{1000}$, $\$4\frac{9}{10}$, $\$\frac{135}{1000}$, $\$64\frac{5}{10}$, $\$33\frac{41}{100}$, $\$\frac{684}{1000}$, $\$\frac{84}{1000}$, $\$\frac{4}{1000}$, $\$6\frac{839}{1000}$, $\$68\frac{39}{100}$, five cents, five dimes, five mills, five dollars five cents five mills.

REDUCTION.

To Like Decimals.

$\$\frac{6}{100} = \$\frac{60}{1000}$. Therefore \$.06 = \$.060.

PRINCIPLE.

Annexing ciphers to a decimal does not alter its value.

EXERCISES.

1. Reduce .7, .05, and .304 to like fractions.

Process.

.7 = .700
.05 = .050
.304 = .304

Explanation.

Thousandths is the lowest order given, hence all the fractions must be reduced to thousandths. Since annexing ciphers to a decimal does not alter its value, we annex two ciphers to .7, thus rendering it 700 thousandths; one cipher to .05, thus rendering it 50 thousandths.

RULE.

By annexing ciphers give each decimal the same number of decimal places.

2. Reduce to like decimals the following:
   1. .25, .025, .37.
   2. .523, 4.36, 5.0315.
   3. .4036, .5, .375.

4. .06, .008, .4267, .026.
5. .409, 3.61, .75, .10055, 19.6.
6. 7.07, 5.0909, 1.9090, 19.099.
7. .12, .8, 306.973, .004, 48.56.
8. .0436, .04506, .82.
9. .8104, .0008, 8000.4.
10. 8.1, .43, .68, 3.96.

To a Common Fraction.

1. What is the denominator of .125?
2. What is its numerator?
3. Write .125 as a common fraction.
4. What part of the expression .125 did you omit?

EXERCISES.

1. Reduce .375 to a common fraction.

Process.

$$.375 = \frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}.$$

RULE.

Write the decimal, omitting the decimal point; supply the decimal denominator, and reduce the fraction to its lowest terms.

2. Reduce the following decimals according to the rule:

1. .45.	8. 4.0125.	15. 23.075.
2. .027.	9. .4355.	16. .354.
3. .72.	10. 10.25.	17. .00625.
4. 1.39.	11. .0005.	18. .05375.
5. .375.	12. .5000.	19. 15.064.
6. .625.	13. 10.25.	20. .005396.
7. 4.75.	14. 15.725.	21. .0007890.

COMPLEX DECIMALS.

EXERCISES.

1. Reduce $.9\frac{1}{3}$ to a common fraction.

Process.

$.9\frac{1}{3} = \frac{9\frac{1}{3}}{10} = \frac{28}{30} = \frac{14}{15}$

Explanation.

Multiplying both terms of $\frac{9\frac{1}{3}}{10}$ by 3, we obtain $\frac{28}{30}$.

2. Reduce in like manner:

1. $.16\frac{2}{3}$.	8. $.04\frac{2}{5}$.	15. $\$66.66\frac{2}{3}$.
2. $.3\frac{3}{4}$.	9. $.037\frac{1}{2}$.	16. $\$25.14\frac{2}{7}$.
3. $.56\frac{1}{4}$.	10. $.562\frac{1}{2}$.	17. $\$50.06\frac{1}{4}$.
4. $.33\frac{1}{3}$.	11. $\$5.9\frac{2}{7}$.	18. $\$100.87\frac{1}{2}$.
5. $.12\frac{1}{2}$.	12. $\$12.18\frac{3}{4}$.	19. $\$700.37\frac{1}{2}$.
6. $.16\frac{2}{3}$.	13. $\$33.03\frac{1}{3}$.	20. $\$1000.11\frac{1}{9}$.
7. $.87\frac{1}{2}$.	14. $\$55.83\frac{1}{3}$.	21. $\$33.62\frac{1}{2}$.

COMMON FRACTIONS.

1. What is the denominator of a common fraction that may be directly expressed as a decimal?

2. If $\frac{1}{5}$ be reduced to a decimal, what is the smallest denominator it can have?

3. $\frac{2}{5}$ = how many 10ths?

4. How is $\frac{4}{10}$ written decimally? How $\frac{5}{8}$?

$$\frac{2 \text{ units}}{5} = \frac{20 \text{ tenths}}{5} = .4$$

$$\frac{5 \text{ units}}{8} = \frac{5000 \text{ thousandths}}{8} = .625$$

5. How does the number of places in the quotients agree with the number of ciphers annexed?

EXERCISES.

1. Reduce $\frac{3}{8}$ to a decimal.

Process.

$\frac{3.000}{8} = .375$

Explanation.

We find by trial that three ciphers must be annexed to 3 to secure a complete quotient. The three ciphers annexed require the pointing off of three decimal places in the quotient.

RULE.

Annex ciphers to the numerator and divide by the denominator. Point off in the quotient as many decimal places as there are ciphers annexed.

2. Reduce the following to decimals:

1. $\frac{1}{4}$.	9. $\frac{113}{256}$.	17. $\frac{14}{40}$.	25. $\frac{372}{1250}$.
2. $\frac{3}{4}$.	10. $\frac{3}{5}$.	18. $\frac{23}{80}$.	26. $\frac{11}{1600}$.
3. $\frac{5}{8}$.	11. $\frac{3}{8}$.	19. $\frac{15}{32}$.	27. $\frac{15}{1280}$.
4. $\frac{7}{8}$.	12. $\frac{4}{5}$.	20. $\frac{21}{32}$.	28. $\frac{347}{2560}$.
5. $\frac{5}{16}$.	13. $\frac{1}{16}$.	21. $\frac{51}{64}$.	29. $\frac{1}{10000}$.
6. $\frac{7}{16}$.	14. $\frac{3}{20}$.	22. $\frac{201}{256}$.	30. $\frac{3476}{15625}$.
7. $\frac{15}{16}$.	15. $\frac{17}{20}$.	23. $\frac{129}{1032}$.	31. $\frac{1899}{32768}$.
8. $\frac{17}{32}$.	16. $\frac{13}{25}$.	24. $\frac{7}{800}$.	32. $\frac{3856}{81920}$.

NOTE.—It is not possible in every case to render the division exact by annexing ciphers. Frequently a remainder occurs, which may be used as the numerator of a fraction; or it may be disregarded, and the sign + employed to denote the incompleteness.

3. Reduce $\frac{3}{7}$ to a decimal.

Process.

$$\frac{3.0000}{7} = .4285\tfrac{5}{7}, \text{ or } .4285+.$$

4. Reduce to decimals the following:

1. $\frac{1}{3}$.	6. $\frac{5}{11}$.	11. $\frac{29}{39}$.	16. $\frac{1}{15}$.
2. $\frac{5}{9}$.	7. $\frac{6}{19}$.	12. $\frac{8}{6}$.	17. $\frac{1}{120}$.
3. $\frac{5}{13}$.	8. $\frac{5}{21}$.	13. $\frac{11}{29}$.	18. $\frac{1}{150}$.
4. $\frac{7}{30}$.	9. $\frac{15}{17}$.	14. $\frac{41}{47}$.	19. $\frac{307}{1728}$.
5. $\frac{3}{7}$.	10. $\frac{3}{35}$.	15. $\frac{74}{97}$.	20. $\frac{1898}{3}$.

5. Give decimal form to the fractions in these:

1. $16\frac{6}{9}$.	6. $76\frac{1}{3}$.	11. $31.0\frac{4}{5}$.
2. $35\frac{4}{5}$.	7. $98\frac{1}{2}$.	12. $.000\frac{6}{19}$.
3. $.93\frac{1}{4}$.	8. $.54\frac{7}{8}$.	13. $3.00\frac{1}{160}$.
4. $4.5\frac{1}{5}$.	9. $5.32\frac{4}{9}$.	14. $4627\frac{9}{14}$.
5. $.34\frac{3}{4}$.	10. $48.6\frac{2}{5}$.	15. $1899\frac{15}{31}$.

ADDITION.

INDUCTIVE STEPS.

1. $\frac{7}{10} + \frac{2}{10} =$ what? $.7 + .2 =$ what? $.7 + .02 =$ what? $.7 + .32 =$ what?

2. If $.7 + .02 = .72$, and if $.7 + .32 = 1.02$, does the addition of decimals differ from the addition of integers?

Obviously, the addition of decimals and the addition of integers are governed by the same principles.

EXERCISES.

1. What is the sum of 4.18, .005, and 5.6057?

Process.

$$\begin{array}{rcr} 4.18 & = & 4.1800 \\ .005 & = & .0055 \\ 5.6057 & = & \underline{5.6057} \\ & & 9.7912 \end{array}$$

Explanation.

By writing the decimal points in a vertical column the digits of like orders fall in the same column, and we proceed as in addition of integers. To mark the place of tenths in the sum we put a decimal point therein under the column of points.

RULE.

Write the decimals so that the decimal points will fall in a vertical column; add as in addition of integers, and place a point in the sum under the column of points.

2. Find the value of:

1. $87.79 + 47.05 + 245.406$.
2. $.027 + 1.39 + 48.6 + 72.978$.
3. 4 dollars + 16 cents + 87 cents + 95 mills.
4. \$.047 + \$6.210 + \$.47 + 3 mills.
5. $12.34 + 432.015 + 302.23 + .00025$.
6. $106 + .106 + 1.06 + 10.6$.
7. \$5.18 + \$3.09 + \$46 + \$54.185.
8. $25.002 + 206.31 + 505.05 + 1.015 + 8.33$.
9. $.718 + 643.5 + 29.21 + 114$.
10. $72.5 + 7.29 + 4.009 + 3.275 + .4$.
11. $.9374 + 13.21 + 45.135 + 1.0006 + 82.16$.
12. 9 tenths + 18 hundredths + 125 thousandths + 125.
13. $.61692 + 243.734 + 901 + 68.45213 + 8.386$.
14. $2.352 + .0008 + 5.0856 + 9.6823$.
15. $\$63.75 + \$9.60 + 7\frac{1}{2}$ cents $+ \$.80 + \$\frac{5}{8} + \$.375$.
16. $\$364.12\frac{1}{2} + \$1.18\frac{3}{4} + \$100.25 + \$63.50$.
17. $\$48\frac{15}{100} + \$.97 + (\$\frac{4}{5} + \$.62\frac{1}{2} + \$\frac{5}{8})$.
18. $.3 + 2.03 + \frac{75}{60} + \frac{605}{100} + \frac{116}{100} + 6.4837$.
19. $.1 + .01 + .001 + .0001 + .00001 + 11.111$.
20. $\$9.90 + \$99.99 + \$999.999 + \$\frac{1}{9}$.

PROBLEMS.

1. A wagon cost \$25.37, a horse \$180.90, a set of harness \$10.05, a mowing machine \$85.37, a cow \$28.08, and a pair of mules \$225.75. What was the entire cost?

2. If I purchase a penknife for $87\frac{1}{2}$ cents, a quire of paper for 25 cents, a bottle of ink for $18\frac{3}{4}$ cents, and a box of pens for $62\frac{1}{2}$ cents, how much money will pay the bill?

3. Add ten thousand *and* one millionth, four hundred-thousandths, ninety-six hundredths, forty-seven million sixty thousand *and* eight billionths.

4. Five pieces of silver weigh as follows: .33 pounds, 1.275 pounds, .5 pounds, 2.375, and 1.324 pounds. How much do the five pieces weigh?

5. Find the sum of 2 decimal units of the 2d order, $2\frac{1}{2}$ of the 3d order, $4\frac{1}{2}$ of the 4th order, $3\frac{1}{8}$ of the 5th order, $5\frac{1}{16}$ of the 6th order, and $9\frac{3}{8}$ of the 7th order.

SUBTRACTION.

1. What is the difference between $\frac{9}{10}$ and $\frac{7}{10}$? Between .9 and .7?

2. What is the value of .94 — .02? Of .94 — .06?

Obviously, the subtraction of decimals and the subtraction of integers are governed by the same principles.

EXERCISES.

1. From 86.71 take 27.004.

Process.

$$\begin{array}{r} 86.710 \\ \underline{27.004} \\ 59.706 \end{array}$$

Explanation.

The numbers being written with the decimal points in a vertical column, the digits of like orders fall in the same column, and the subtraction proceeds as in the case of integers. The 7 tenths must be separated from the 9 units by the decimal point.

RULE.

Write the decimals so that the decimal points will fall in a vertical column; subtract as in subtraction of integers, and place a point in the difference under the column of points.

2. Subtract:

(1.)	(2.)	(3.)
82.19	274.684	8468.3684
14.21	217.423	1764.8342

(4.)	(5.)	(6.)
8.4062	4256.85	\$18.86
.6434	.00564	13.685

3. Subtract 284.7654 from 321.07659.
4. Subtract 17.2398 from 27.06.
5. Subtract 29.9189 from 240.775.
6. Subtract 70.2574 from 365.71.
7. Subtract .006 from .00609.
8. Subtract 204.01 from 889.009.
9. Subtract 89.009 from 204.01.
10. Subtract 8.0999 from 1000.
11. Subtract 24.869 from 36.
12. Subtract 84 ten-millionths from 84 millionths.
13. Subtract 9 dollars 37 cents 5 mills from 27 dollars 8 cents.
14. Subtract 406.375 from 2370.001.
15. Subtract .074532 from 1.0003246.
16. Subtract $\frac{991}{100}$ from $10.333\frac{2}{5}$.
17. Find the value of:

1. \$125.75 — \$41.095.	4. \$107.003 — \$.479.
2. \$1.375 — \$.88.	5. 5 — 5 hundred-millionths.
3. \$2349 — \$29.33.	6. 3.625 — 1.5625.

PROBLEMS.

1. A man's income is \$3000 a year; he spends \$487.50. How much does he lay up?

2. I bought a farm for \$2560; paid at one time \$1046, and at another time \$807.87. What remains unpaid?

3. A gentleman's income was \$12,384.16, and his expenses the same year were \$9864.18. How much of his income was left?

4. A man did .37 of a piece of work the first day and .33 of it the second day. What part of the work was left for him to do the third day?

5. John walks 3.475 miles and James 3.8005 miles. Which walks the farther, and how much?

6. Simplify:

1. $8.763 - 4.12 + 78.326 - 68.0816$.
2. $(\$.87\frac{1}{2} + \$14\frac{3}{5}) - (\$5.10 + \$.75)$.
3. $(155.006 - .32) + (80.0032 + 55.1)$.
4. $12.07 - 11.432765$.
5. $\$48\frac{15}{100} + \$.97 - (\$\frac{4}{5} + \$.62\frac{1}{2} + \$\frac{5}{8})$.

MULTIPLICATION.

1. $\frac{3}{10} \times \frac{7}{100} =$ what? $.3 \times .07 =$ what?
2. How many decimal places in both factors?
3. How many decimal places in their product?

PRINCIPLE.

The product of decimal factors has as many decimal places as the factors.

EXERCISES.

Process.

1. Multiply 9.06 by .045.

$$\begin{array}{r} 9.06 \\ .045 \\ \hline 4530 \\ 3624 \\ \hline .40770 \end{array}$$

Explanation.

$9.06 = \frac{906}{100}$; $.045 = \frac{45}{1000}$; $\frac{906}{100} \times \frac{45}{1000} = \frac{40770}{100000}$. Or, since the factors have $2 + 3$ or five decimal places, the product must have five decimal places (Principle).

RULE.

Multiply without regarding the decimal point, but in the product point off from the right as many places for decimals as there are decimal places in the factors.

NOTE.—Should the result of a multiplication not contain as many figures as the factors contain decimal places, we must supply the deficiency by prefixing ciphers, as in $.02 \times .003 = .00006$.

2. What is the value of:

1. 13.2×2.475.
2. $.132 \times 2.475$.
3. $.236 \times 12.13$.
4. $9.06 \times .045$.
5. $.008 \times 751.1$.
6. 70×387.45.
7. 70.07×387.45.
8. $4.2 \times .065$.
9. $2000 \times .075$.
10. $.436 \times .46$.
11. $.579 \times .035$.
12. 3.94×3.84.
13. $5384 \times .0064$.
14. $.014 \times 6.2 \times .007$.
15. $200 \times 3\frac{2}{5} \times .006$.
16. $947.36 \times .00423$.
17. $6\frac{2}{5} \times 7\frac{3}{8} \times .8\frac{1}{2}$.
18. $.305 \times .00046$.
19. 10000×8.6213.
20. 8.47×9.432.
21. $.84 \times 9.60$.
22. 3.468×2.008.
23. $8\frac{1}{3} \times 5.076$.
24. $28.8 \times 4\frac{1}{2}$.
25. $8.375 \times 6\frac{1}{4}$.
26. $\frac{5}{8} \times 2.5$.
27. $156\frac{1}{4} \times .625$.
28. $1.776 \times .24$.
29. 1.603×2.564.
30. $.0069 \times 95.6$.
31. $2000 \times .075$.
32. $8000 \times .0755$.
33. $.785 \times .0191$.
34. $.00432 \times .00037$.
35. $8\frac{1}{2} \times .07\frac{1}{4} \times 10$.
36. $.37 \times 10000$.
37. $16\frac{1}{8} \times 14.55$.
38. $277\frac{3}{8} \times 12.004$.
39. 3 hundredths $\times$ 3 thousandths.
40. Four hundred thousand two hundred sixty-eight ten-millionths by two hundred sixty and two hundred seventy-five thousandths.

THE DECIMAL POINT AS A MULTIPLIER.

1. $.0004 \times 10 =$ what? $0.004 \times 10 =$ what? $00.04 \times 10 =$ what? $000.4 \times 10 =$ what?

2. Since $.0004 \times 10 = 0.004$ and $0.004 \times 10 = 00.04$, how does the decimal point become a multiplier?

3. To become a multiplier, does it move toward the right or the left?

4. Its removal one place to the right multiplies the number by what? Its removal two places multiplies by what? Three places?

PRINCIPLE.

Every removal of the point one place toward the right multiplies the number by ten.

RULE.

To multiply by a number consisting of 1 with ciphers annexed, remove the decimal point as many places towards the right as there are ciphers in the multiplier.

EXERCISES.

1. Multiply .394 by 100.

Process. **Explanation.**

39.4

Since the multiplier is one with two ciphers annexed, we remove the decimal point two places towards the right, and have 39 and 4 tenths as product.

2. Multiply:

1. 8.7 by 10.	7. 9.2 by 10.
2. .0069 by 10.	8. 7.49 by 100.
3. 95.6 by 100.	9. .036 by 100.
4. .0453 by 100.	10. 854.3 by 1000.
5. 4.069 by 1000.	11. 1.00182 by 10,000.
6. .000094 by 10,000.	12. 76.541 by 1,000,000.

PROBLEMS.

1. Find the value of:
   1. 57 horses, at $86.375 each.
   2. 200 barrels of flour, at $8.53½ each.
   3. 25⅛ yards of cloth, at $5½ a yard.
   4. 236 bushels of oats, at $.515 a bushel.
   5. 36¾ bushels of clover seed, at $4.52 a bushel.
   6. 1000 pounds of wool, at $.375 per pound.
   7. 280 barrels of apples, at $3⅜ a barrel.

8. 100 cords of wood, at \$5.47 a cord.

9. $305\frac{9}{10}$ acres of land, at $\$82\frac{3}{4}$ an acre.

2. A lady made the following purchases: 47 yards of sheeting, at $\$.14\frac{1}{2}$ per yard; 9 yards of ribbon, at $\$.45\frac{1}{2}$ per yard; 38 yards of silk, at \$3.46 per yard. What did her purchases cost her?

3. Multiply six hundred twenty-five ten-millionths by three *and* eight thousandths.

DIVISION.

1. $.6 \times .9 =$ what?

Since one factor of .54 is .6, what is the other factor?

Since $.54 \div .6 = .9$, how does the number of decimal places in the dividend compare with the number in the divisor and quotient?

2. $.054 = .09 \times .6$.

Assuming .054 to be a dividend and .09 to be a divisor, what is the quotient?

Since the dividend has 3 decimal places and the divisor 2, how can you operate with 3 and 2 to find the number of places in the quotient?

PRINCIPLE.

The number of the decimal places in the quotient equals the number of places in the dividend minus the number in the divisor.

EXERCISES.

1. Divide 82.32 by 2.1.

Process.

```
2.1 ) 82.32 ( 39.2
      63
      ---
      193
      189
      ---
        42
        42
        --
```

Explanation.

$2.1 = \frac{21}{10}$, divisor; $82.32 = \frac{8232}{100}$; $\frac{8232}{100} \times \frac{10}{21} = \frac{392}{10} = 39.2$. Or, dividing without regard to the decimal point, we have 392 as quotient. Since the dividend has two decimal places and the divisor one, the quotient has *one;* hence the quotient sought is 39.2.

RULE.

Divide without regard to the decimal point, but finally point off from the right of the quotient as many figures for decimals as the number of decimal places in the dividend exceeds the number of those in the divisor.

NOTES.—1. When the quotient does not contain as many figures for decimals as the rule requires, supply the deficiency by prefixing ciphers.

2. Before beginning to divide, it is best to make the number of decimal places in the dividend at least equal to the number of decimal places in the divisor.

3. When the process of division has used only as many decimal places of the dividend as equal the number of decimal places of the divisor, the quotient will be an integer.

2. Divide:

1. 21.6 by .006 (Apply Notes 2 and 3).
2. .4913 by 1.7.
3. 2.1952 by .028.
4. .5964 by 35 (Note 1).
5. 26.01 by 51.
6. .456 by .06.
7. 4375 by .25 (Note 2).
8. 89.756 by 8.
9. 36.792 by 4.2.
10. 44.98 by 1.3.
11. .0002 by .02.
12. 325.72 by 34.
13. 10,864.2 by 5432.1.
14. 3.2572 by 3.4.
15. 467.37 by 100.
16. .003125 by .125.
17. .03759 by .01253.
18. .13 by .026 (2 and 3).
19. .75 by .025.
20. 7 by .007.
21. .4 by .008.
22. .005 by .0015.
23. .0003 by 3.75.
24. .018 by 3600.
25. 2.0064 by 2.09.

3. Divide:

1. 1235.434256 by 20.074.
2. 195.388698 by 6.0708.
3. 273.2879688 by 6.0708.
4. 3.859243392 by 3.5702.
5. .00020596611 by .03507.
6. 625 ten-thousandths by 25 millionths.

THE DECIMAL POINT AS A DIVISOR.

1. $4000 \div 10 =$ what? $400.0 \div 10 =$ what? $40.00 \div 10 =$ what? $4.000 \div 10 =$ what?

2. Since $4000 \div 10 = 400.0$ and $400.0 \div 10 = 40.00$, how does the decimal point become a divisor?

3. To become a divisor, does the decimal point move towards the right or the left?

4. Its removal one place to the left divides the number by what? Its removal two places divides the number by what? Three places?

PRINCIPLE.

Every removal of the point one place toward the left divides the number by ten.

RULE.

To divide by a number consisting of 1 with ciphers annexed, remove the decimal point as many places toward the left as there are ciphers in the divisor.

EXERCISES.

1. Divide 48.26 by 100.

Process. **Explanation.**

.4826

Since the divisor is 1 with two ciphers annexed, we remove the decimal point two places toward the left, and have .4826 as quotient.

2. Divide:

1. 534.79 by 100.	6. 4956.74 by 10,000.
2. 492.568 by 1000.	7. .038649 by 100,000.
3. 24.9653 by 1000.	8. 82.253 by 1,000,000.
4. 5.908 by 100.	9. $9.391 by 10.
5. .07156 by 1000.	10. 785.437 by 10,000.

PROBLEMS.

1. Find the value of a single one if:

1. 144 eggs cost \$2.88.
2. 20 francs = \$3.86.
3. 20 shillings = \$4.8665.
4. 25 dress patterns = 102.50 yards.
5. 125 bushels of oats cost \$36.50.
6. 100 acres of land cost \$3156$\frac{1}{2}$.
7. 72.50 C. cigars cost \$84.10.
8. 1.440 M. bricks cost \$10.44.
9. 22 days' work = \$29.70.
10. .62 of a ton of hay cost \$11.47.
11. 7$\frac{1}{2}$ acres of land cost \$70.125.
12. .7$\frac{3}{4}$ yards of cloth cost \$.73625.
13. 5 weeks' provisions cost \$47.31$\frac{1}{4}$.

2. Find the cost of 8.25 tons of hay when 2.2 tons cost \$31$\frac{1}{3}$.

3. Find the value of $(6.25 \div 3\frac{1}{4}) \div (3\frac{7}{8} - .275)$.

SHORT PROCESSES.

When the Multiplier approximates 100, 1000, etc.

1. How much less than 10 times a number is 9 times that number?

2. How much less than 100 times a number is 98 times that number?

3. Multiply 4965 by 99.

Process.	Explanation.
496,500	496,500 = 100 times 4965.
4965	4965 = 1 time 4965.
491,535	491,535 = 99 times 4965.

4. Multiply:

1. 4993 by 99.	6. 597,076 by 995.
2. 4967 by 98.	7. 575,854 by 98.
3. 59,678 by 999.	8. 954,367 by 96.
4. 98,849 by 98.	9. 697,547 by 996.
5. 457,836 by 997.	10. 5,064,367 by 97.

5. What cost 496 bushels of wheat at $.98 per bushel?

6. Find the cost of 240 acres of land at $96 per acre?

7. What is the value of $.0755 \times 997$?

8. 2484 pounds of tea at $.96 = what?

9. Find the value of $\frac{67,898 \times 999}{.012345}$.

When One Part of the Multiplier is an Exact Divisor of Another Part.

1. 6 is how many times 3?

2. When you have taken a number 3 times, how many times that product will 6 times the number be?

3. Multiply 1728 by 63.

Process.

$$\begin{array}{r} 1728 \\ 63 \\ \hline 5184 \\ 10368 \\ \hline 108,864 \end{array}$$

Explanation.

6 tens = 2 tens × 3. 1728 × 3 = 5184 units. 5184 × 2 tens = 10,368 tens. Adding the two products we have 108,864.

NOTE.—Be careful in placing the first figure of each partial product.

4. Multiply:

1. 4795 by 124 [12 = 4 × 3].	9. 7495 by 735.
2. 4936 by 93 [9 = 3 × 3].	10. 5349 by 927.
3. 7935 by 123.	11. 23,894 by 756.
4. 25,384 by 142.	12. 47,523 by 918.
5. 39,764 by 246.	13. 47,596 by 2505.
6. 79,546 by 328.	14. 39,864 by 8024.
7. 57,324 by 217.	15. 49,975 by 9045.
8. 4793 by 945 [45 = 5 × 9].	16. 35,656 by 642.

To Multiply or Divide when the Multiplier or Divisor is an Aliquot Part of 10, 100, or 1000.

Process.

$$4\overline{)189800}$$
$$47450$$

1. Multiply 1898 by 25.

Explanation.

Since $25 = \frac{100}{4}$, we multiply by 100 and divide the product by 4.

Process.

$$\begin{array}{r} 1900 \\ 4 \\ \hline 76.00 \end{array}$$

2. Divide 1900 by 25.

Explanation.

$\frac{100}{4}$ inverted $= \frac{4}{100}$; hence we multiply by 4 and point off two places to the right

Process.

$$3\overline{)189900}$$
$$63300$$

3. Multiply 1899 by $33\frac{1}{3}$.

Explanation.

Since $33\frac{1}{3} = \frac{100}{3}$, we multiply by 100 and divide the product by 3.

Process.

$$\begin{array}{r} 1728 \\ 3 \\ \hline 51.84 \end{array}$$

4. Divide 1728 by $33\frac{1}{3}$.

Explanation.

$\frac{100}{3}$ inverted $= \frac{3}{100}$; hence we multiply by 3 and point off two places to the right.

Process.

$$8\overline{)569600}$$
$$71200$$

5. Multiply 5696 by $12\frac{1}{2}$.

Explanation.

Since $12\frac{1}{2} = \frac{100}{8}$, we annex two ciphers and divide the product by 8.

Process.

$$\begin{array}{r} 3.14156 \\ 8 \\ \hline .2513248 \end{array}$$

6. Divide 3.14156 by $12\frac{1}{2}$.

Explanation.

$\frac{100}{8}$ inverted $= \frac{8}{100}$; hence we multiply by 8 and point off five plus two places to the right.

Process.

$$6\overline{)78.54}$$
$$13.09$$

7. Multiply .7854 by $16\frac{2}{3}$.

Explanation.

Since $16\frac{2}{3} = \frac{100}{6}$, we multiply by 100 and divide the product by 6.

8. Divide 1492 by $16\frac{2}{3}$

Process.

$$\begin{array}{r} 1492 \\ \underline{6} \\ 89.52 \end{array}$$

Explanation.

$\frac{100}{6}$ inverted $= \frac{6}{100}$; hence we multiply by 6 and point off two places to the right.

25, $33\frac{1}{3}$, $12\frac{1}{2}$, and $16\frac{2}{3}$ are *aliquot* parts of 100, *i.e.*, they are such parts as exactly divide 100.

Other aliquot parts of 100 may be dealt with similarly; also aliquot parts of 10 and 1000, etc.

EXERCISES.

1. Multiply:

1. 2556 by 25.	6. 40002 by $16\frac{2}{3}$.
2. 7.36 by 50.	7. 205.59 by $166\frac{2}{3}$.
3. 72.06 by $33\frac{1}{3}$.	8. 380.087 by $12\frac{1}{2}$.
4. 207.27 by $3\frac{1}{3}$.	9. 5908 by $14\frac{2}{7}$.
5. 9.087 by $333\frac{1}{3}$.	10. 390.8 by $2\frac{1}{2}$.

2. Divide:

1. 404 by 25.	6. 399099 by $333\frac{1}{3}$.
2. 5005.09 by $2\frac{1}{2}$.	7. 7906.73 by $16\frac{2}{3}$.
3. 407.709 by 50.	8. 970008 by $166\frac{2}{3}$.
4. 33659 by $33\frac{1}{3}$.	9. 5227.38 by $12\frac{1}{2}$.
5. 9304.75 by $3\frac{1}{3}$.	10. 470058 by $14\frac{2}{7}$.

PROBLEMS.

Finding the Cost of Articles sold by the 100, 1000, and Ton.

1. If 100 articles cost a certain price, how many times that price will 500 articles cost?

2. If 1000 articles cost a certain price, how many times that price will 6000 articles cost?

3. How many articles does C. represent? M., how many?

Process.

$$\begin{array}{r} 26.73 \\ \underline{2.25} \\ 13365 \\ 5346 \\ \underline{5346} \\ \$60.1425 \end{array}$$

4. What will 2673 feet of timber cost at $2.25 per C.?

Explanation.

2673 feet = 26.73 C. feet. Since C. feet cost $2.25, 26.73 C. feet cost $2.25 × 26.73 = $60.1425.

5. What is the value of 1262 fence pickets at $12½ per M.?

Suggestion: 1262 pickets = 1.262 M. pickets.

6. I sold 6000 cigars at $4.20 per C. Find the amount received therefor.

7. I paid $10.44 for 1440. What was the price per M.? (1440 = 1.440 M.)

8. If the price of gas be $1.75 per M., find the amount of a man's bill when 12,240 cubic feet have been consumed.

9. What cost 23½ M. feet of pine at $55?

10. What cost 4⅜ M. brick at $8?

11. A contractor furnished the following materials for a house: 34,600 bricks at $9.60 per M., 7960 feet of lumber at $16 per M., 9050 feet of flooring at $22.50 per M., 7600 shingles at 40 cents per C. Find the total cost of the materials.

12. If 2000 pounds cost $10.00, what will 1000 pounds cost?

13. At $5.00 per M., what will 3200 pounds cost?

14. 2000 pounds = one ton. If one ton of iron costs $30, what will 1000 pounds cost? What will 4000 pounds cost?

15. What must I pay for 7850 pounds of stone at $2.20 a ton?

Process.

$$\begin{array}{r} 2\,\underline{)\,\$2.20} \\ 1.10 \\ \underline{7.850} \\ \$8.63500 \end{array}$$

Explanation.

2000 lbs. cost $2.20.
1000 lbs. cost $1.10.
7850 lbs. = 7.850 M.

Since M. cost $1.10, 7.850 M. cost $1.10 × 7.850 = $8.63½.

16. What will 3426 pounds of plaster cost at \$3.48 per ton?

17. How much must be paid for 6745 pounds of clay at \$15.25 a ton?

18. For 7890 pounds of hay at \$16.60 per ton?

19. For 27,936 pounds of fertilizer at \$18.50 per ton?

20. For 7330 pounds of wool at \$5.50 per ton?

21. For 9041 pounds of iron at \$125 a ton?

REVIEW.

Indicate the processes.

1. What is the quotient when 3 is divided by 3 thousandths?

2. If the divisor is 207, dividend 4776, quotient 23, find the remainder.

3. The product of two numbers is $\frac{4}{9}$, and one of them is $\frac{5}{24}$ of 2. What is the other?

4. Reduce $.094\frac{2}{3}$ to a common fraction.

5. Reduce $\frac{128}{512}$ to a decimal.

6. Add 3.5 tons, 2.25 tons, 5 tons, 5.486 tons, 2.986 tons, 3.6 tons, 2.336 tons, and 2.376 tons.

7. Add .0273 and $\frac{306}{2700}$.

8. Subtract .00976 from 2.03.

9. Find the value of .21 of $\frac{2}{3} \times 50 \times .011\frac{3}{7}$.

10. When pork is selling at \$6.25 per hundred-weight, how much can be bought for \$725?

11. Divide the sum of six thousandths and six millionths by their difference. Find six decimal places.

12. Multiply 732.89 by $33\frac{1}{3}$. [Use short process.]

13. Multiply 92.5674 by $333\frac{1}{3}$.

14. Divide 96.325 by $12\frac{1}{2}$.

15. How many pounds of coffee can be bought for \$16.25 if $5\frac{1}{2}$ pounds can be bought for $\$1.78\frac{3}{4}$?

16. $(2.04 \div 17 + .235 \times 5000) - \frac{7}{8} = ?$

17. Divide 250,000 by .00005.

18. I paid .33 of a sum of money for a slate, .17 for a book, and .375 for a pair of skates. What fractional part of the sum was left?

19. Simplify $\dfrac{4.5 + \frac{3}{5}}{7.375 + \frac{3}{4} - \frac{1}{8}}$, expressing the result as a decimal.

20. What is the value of 95,150 bricks at \$7.25 per M.?

21. Subtract .0507009 from .08.

22. Reduce $.15\frac{2}{9}$ to a common fraction.

23. Find the cost of 560 pineapples at \$13.35 per C.

24. Multiply 39,864 by 3609. [$36 = 4 \times 9$.]

25. Add $\frac{2}{11}$, $\frac{9}{5}$, and $\frac{1}{8}$; subtract $\frac{7}{35}$ from $\frac{4}{13}$ of $\frac{65}{56}$; subtract the second result from the first, and take $\frac{1}{8}$ of the difference.

26. If 15 tons of hay cost \$125.25, what will 35 tons cost?

27. A long ton $= \frac{2240}{2000}$ of a short ton. Reduce to a mixed decimal.

28. Write as decimals and as simple common fractions: $\frac{1\frac{1}{4}}{100}$, $\frac{1\frac{1}{4}}{10}$, $\frac{2\frac{1}{2}}{1000}$.

29. Find the result of $1.76 \times 49.647 \div .0088$.

30. Add $\frac{14}{21}$, $\frac{12}{16}$, $\frac{25}{30}$. Express the result as a decimal.

31. What cost 164,960 pounds of coal at \$6.00 per short ton?

32. Bought 100 sheep at \$3.375 a head, and sold them at \$3.875. What did I gain on each, and on the whole number?

33. What would $7\frac{4}{5}$ bales of cotton cost, each bale weighing 537.5 pounds, at $\$.11\frac{3}{4}$ a pound?

34. A horse and bridle are worth \$178.50; but the horse is worth 20 times the bridle. Find the value of each.

35. Reduce $.7708\frac{1}{3}$ to a common fraction.

36. Multiply 793.295 by .0001.

37. How much iron in 89,276 pounds of ore if .72 of it is pure iron?

38. An agent charged \$5.85 for collecting a bill of \$260. What was his charge per dollar?

39. If .35 of a share in a mining company is worth \$31.15, what is the value of 15 shares?

40. A grocer sold 8970 pounds of sugar at \$4.75 a hundred pounds. How much did he receive?

41. A farmer exchanged 9 tons of hay worth \$16.87$\frac{1}{2}$ a ton for oats at 31$\frac{1}{4}$ cents a bushel. How many bushels did he receive?

42. If 49 yards of broadcloth cost \$251.12$\frac{1}{2}$, what would be the price per yard?

43. If one acre of land costs \$38.75, how much can be bought for \$3560?

44. How many days' work at \$1.25 a day must be given for 6 cords of wood worth \$4.12$\frac{1}{2}$ a cord?

45. Bought a roll of carpet containing 82 yards for \$45, and sold it for 75 cents a yard. Find the amount of profit?

46. Find the value of $60\frac{4}{25} + 49\frac{3}{50} + 18\frac{7}{20} + 6\frac{3}{4} + 90\frac{1}{2}$.

47. Find the cost of 3700 cedar rails at \$5.75 per C.

48. If a man earns \$12$\frac{1}{2}$ a week and spends \$7$\frac{5}{8}$ per week, in how many weeks can he save \$500?

49. What is the value of 86,260 bricks at \$7.50 per M.

50. Express as a decimal $\dfrac{(\frac{3}{4} - \frac{1}{11}) \times (4 + \frac{3}{5})}{(2\frac{1}{2} + \frac{6}{7}) + (4 - 2\frac{2}{3}) \times 6}$.

51. What cost 49.76 pounds of raisins at 12$\frac{1}{2}$ cents a pound?

52. What cost 65 yards of muslin at 16$\frac{2}{3}$ cents a yard?

53. Find the cost of 85 bushels of apples at 33$\frac{1}{3}$ cents a bushel.

54. What must you pay for 50 pairs of gloves at 125 cents a pair?

ACCOUNTS AND BILLS.

DEFINITIONS.

1. A **Debt** is that which one person owes to another, whether money, goods, or services.

2. A **Credit** is that which is due from one person to another; or, that which is paid towards cancelling a debt.

3. A **Debtor** is the person who owes.

4. A **Creditor** is the person to whom a debt is due.

5. An **Account** is a record of debts and credits.

6. The **Balance of an account** is the difference between the sums of the debts and credits.

7. A **Bill** describes the goods sold by giving quantity and price.

8. The **Footing of a bill** is the total cost.

9. A **Receipt** acknowledges the payment of a bill at its foot, thus: "Received payment,

"JAMES JOHNSON."

Common Abbreviations.

@,	at.	Cwt.,	hundred weight.	Mdse.,	merchandise.
%,	account.	Do.,	the same.	No.,	number.
Acc't,	account.	Doz.,	dozen.	Pay't,	payment.
Bal.,	balance.	Dr.,	debtor.	Pd.,	paid.
Bbl.,	barrel.	Fr't,	freight.	Per,	by
Bo't,	bought.	Hhd.,	hogshead.	Rec'd,	received.
Bu.,	bushel.	Inst.,	this month.	Ult.,	last month.
Co.,	company.	Int.,	interest.	Yd.,	yard.
Cr.,	creditor.	Lb.,	pound.	Yr.,	year.

Bills are usually written in the following form:

LANCASTER, PA., July 31, 1898.

MR. R. B. RISK,

Bought of BAIR & STEINMAN.

2500 ft. Boards,	@ $27.50 per M.	$68	75
1875 ft. do.	" 25.00 " "	46	88
1650 Laths,	" .32 " C.	5	28
1520 Pickets,	" 15.00 " M.	22	80
7500 Shingles,	" 6.50 " "	48	75
		$192	46

Find the footings of the following bills:

(1.)

CHICAGO, ILL., Aug. 3, 1898.

MR. JOHN TODD,

Bought of RIGGS & CARTER.

25,000 ft. Pine Boards,	@ $15.00 per M.	$	
8,500 " Plank,	" 9.50 " "		
11,850 " Scantling,	" 7.00 " "		
4,970 " Timber,	" 3.25 " "		
6,398 " do.	" 4.00 " "		
		$	

Received payment,

RIGGS & CARTER.

NOTE.—Finding the value of the different items of a bill is called "making the extensions."

(2.)

TRENTON, N. J., Aug. 4, 1898.

DR. J. C. GOOD,

Bought of STEPHEN SMITH.

35 lbs. Coffee	@ $.30	$	
5 lbs Tea	" .50		
30 lbs. Mackerel	" .15		
5 gals. Molasses	" .60		
20 lbs. Sugar	" .05½		
3 doz. Eggs	" .18		
2 lbs. Cheese	" .10		
3 lbs. Butter	" .20		
		$	

(3.)

ATLANTA, GA., Aug. 5, 1898.

MR. SHERMAN ROGERS,

To PAUL R. JONES, *Dr.*

To 48 bbl. Pork	@ $12.50	$	
" 138 bbl. Flour	" 7.15		
" 4 bbl. Molasses, 169 gal.	" .40		
" 30 firkins Butter, 2200 lb.	" .17		
" 4 boxes Raisins	" 4.60		
" 4 bbl. Kerosene, 164 gal.	" .19		
" 30 doz. cans Fruit	" 2.50		
" 2 bundles Tobacco	" .40		
" 12 doz. Spices	" 1.12½		
		$	

Set in bill form the following purchases, find the footings, and assume that the bills were paid:

4. Mrs. T. N. Butcher bought of Hervey Martin, 15 yd. of carpet @ \$1.00; 50 yd. of muslin @ $12\frac{1}{2}$ cts.; 18 yd. of calico @ $9\frac{1}{4}$ cts.; 5 pairs of hose @ 75 cts.; 15 yd. of gingham @ $11\frac{1}{2}$ cts.; and 25 yd. of Canton flannel @ $10\frac{1}{4}$ cts.

5. Mr. D. F. Lovett bought of S. Q. Lowrey, 8679 ft. of hemlock @ \$13.85 per M.; 9640 ft. of flooring @ \$24.75 per M.; 6709 ft. of pine @ \$50.00 per M.; 4926 ft. of oak @ \$35.00 per M.; 8457 ft. of ash flooring @ \$40.00 per M.

6. Mr. H. K. Landman bought of B. A. Gross, 47 bu. of wheat @ \$.87; 60 bu. of corn @ \$.60; 50 bu. of oats @ \$.33; 30 cwt. of flour @ \$3.50; 160 bu. of bran @ \$.18; 83 lb. of corn meal @ \$.05.

7. Mr. John Rodgers bought of William H. Cartwright, 100 lb. of breakfast bacon @ \$.10; 55 lb. of lard @ \$.08; $37\frac{1}{2}$ lb. of picnic hams @ \$.06; 45 lb. of tallow @ \$.05$\frac{1}{2}$; 25 lb. of creamery butter @ \$.17; 10 doz. Western eggs @ \$.12$\frac{1}{2}$; 16 lb. of fowls (hens) @ \$.15; 5 lb. of cheese @ \$.10$\frac{1}{2}$; $12\frac{1}{2}$ lb. of Rio coffee @ \$.17$\frac{1}{2}$.

DENOMINATE NUMBERS.

Denominate Numbers are *Simple* or *Compound*.

A Compound Denominate Number is composed of units of two or more denominations that have among them a certain natural relation; as, 4 feet 6 inches, or 3 bushels 2 pecks 1 quart.

Compound Denominate Numbers have their origin in the existence of the various **Measures** in common use.

The **Measure of Value** is **Money,** which is also called **Currency.**

1. **United States Money** consists of *Coin* and *Paper Money.* Coin is called *Specie.* The Coins are:

GOLD.		SILVER.	
The Double Eagle	= $20.00.	The Dollar.	= $1.00.
Eagle	= 10.00.	Half-Dollar	= .50.
Half-eagle	= 5.00.	Quarter-Dollar	= .25.
Quarter eagle	= 2.50.	Dime	= .10.

The Nickel Coin = $.05.
The Bronze Coin = .01.

Other United States coins found in circulation are not now coined.

Paper money is issued in the form of bills whose face value is one dollar and upward.

The Unit of United States Money is the Dollar.

Table.

10 Mills (m.)	=	1 Cent (ct.).
10 Cents	=	1 Dime (d.).
10 Dimes	=	1 Dollar ($).
10 Dollars	=	1 Eagle (E.).

$ *d.* *cts.* *m.*
1 = 10 = 100 = 1000.
Scale: 10, 10, 10 (Decimal).

2. **Canadian Money** has the denominations of the United States money, except the gold coins, which are the *Sovereign* and *Half-Sovereign.*

3. **French Money** has the following denominations: *Centime, Decime,* and *Franc.*

The Unit is the Franc.

Table.

10 Centimes (ct.) (*son-teems*)	=	1 Decime (dc.).
10 Decimes (*des-seems*)	=	1 Franc (fr.).

Fr. *dc.* *ct.*
1 = 10 = 100. = $0.193.
Scale: 10, 10 (Decimal).

4. **English or Sterling Money** is the currency of Great Britain. The coins are:

Gold.		Silver.	
The Sovereign	= 20 shillings.	Crown	= 5 shillings.
Half-Sovereign	= 10 "	Florin	= 2 "
Guinea	= 21 "	Shilling.	
		Six-penny piece.	
		Three-penny piece.	

Copper: Penny, Half-penny, Farthing (four things).

The Unit is the Pound or Sovereign.

Table.

4 Farthings (far.) = 1 Penny (d.).
12 Pence = 1 Shilling (s.).
20 Shillings = 1 Pound (£).

£.	s.	d.	far.	
1 =	20 =	240 =	960 =	\$4.8665.

Ascending Scale: 4, 12, 20.

REDUCTION DESCENDING.

INDUCTIVE STEPS.

1. Since 4 farthings = 1 penny, how many farthings = 2 pence? 3 pence? 4 pence? 5 pence?

2. Since 12 pence = 1 shilling, how many pence = 6 shillings? 10 shillings? 12 shillings?

3. How many shillings in £2? In £2 6 shillings?

4. How many pence in £2 6s. 5d.?

5. £$\frac{1}{2}$ = how many shillings?

6. $\frac{1}{4}$s. = how many pence?

7. £$\frac{1}{4}$ = how many shillings?

Reduction descending changes a denominate number from a higher to a lower denomination without changing its value.

EXERCISES.

1. Reduce £4 12s. 8d. to farthings.

Process.

$$\begin{array}{r} £4\ 12s.\ 8d. \\ \underline{20} \\ 80s. \\ \underline{12s.} \\ 92s. \\ \underline{12} \\ 1104d. \\ \underline{8d.} \\ 1112d. \\ \underline{4} \\ 4448 \text{ far.} \end{array}$$

Explanation.

Since £1 = 20s., £4 = 80s. 80s. + 12s. = 92s.

Since 1s. = 12d., 92s. = 92 × 12d. = 1104d. 1104d. + 8d. = 1112d.

Since 1d. = 4 far., 1112d. = 1112 × 4 far. = 4448 far.

Hence £4 12s. 8d. = 4448 farthings.

2. Reduce £$\frac{5}{9}$ to pence.

Process.

£$\frac{5}{9}$ = $\frac{5}{9}$ of 20s. = $\frac{100}{9}$s. $\frac{100}{9}$s. = $\frac{100}{9}$ of 12d. = $\frac{1200}{9}$d. = $133\frac{1}{3}$d. Or, £$\frac{5}{9} \times \frac{20}{1} \times \frac{12}{1}$ = $\frac{400}{3}$d. = $133\frac{1}{3}$d.

3. Reduce £$\frac{4}{7}$ to integers of the lower denominations.

Process.

£$\frac{4}{7}$ = $\frac{4}{7}$ of 20s. = $\frac{80}{7}$s. = $11\frac{3}{7}$s.
$\frac{3}{7}$s. = $\frac{3}{7}$ of 12d. = $\frac{36}{7}$d. = $5\frac{1}{7}$d.
$\frac{1}{7}$d. = $\frac{1}{7}$ of 4 far. = $\frac{4}{7}$ far.
Hence, £$\frac{4}{7}$ = 11s. 5d. $0\frac{4}{7}$ far.

RULE.

Multiply by the numbers of the scale in reverse order, beginning with that number that makes one of the highest given denomination; and, as you proceed, add to the products the given numbers of lower denominations.

4. Reduce:

1. £24 6s. to shillings.
2. £40 9s. 6d. to farthings.
3. £35 6s. 8d. to pence.
4. 7s. 6d. 2 far. to farthings.
5. £14 18s. 11d. to pence.
6. £92 15s. 8d. 2 far. to farthings.

7. £56 4s. $10\frac{4}{5}$d. to farthings.
8. £$\frac{3}{4}$ + $\frac{3}{4}$s. + $\frac{3}{4}$d. to pence.
9. £3. 5s. $7\frac{3}{4}$d. to farthings.
10. $\frac{3}{4}$s. to farthing.
11. £$\frac{3}{8}$ to pence.
12. £$\frac{3}{7}$ to farthings.
13. £$\frac{8}{9}$ to integers of lower denominations.
14. £$\frac{3}{7}$ to integers of lower denominations.
15. £$\frac{2}{7}$ to integers of lower denominations.
16. \$150 to mills.
17. \$17.28 to cents.
18. 10 eagles to mills.
19. 19 francs to decimes.
20. 19 francs 8 decimes to centimes.

REDUCTION ASCENDING.

INDUCTIVE STEPS.

1. How many pence in 8 farthings? In 12 farthings? In 24 far.?
2. How many shillings in 12d.? In 36d.? In 108d.?
3. 40 shillings equal how many pounds? 60 shillings?
4. If you take £2 out of 45s., how many shillings remain?
5. If you reduce 45s. to pounds, what is your result?

Reduction ascending changes a denominate number from a lower to a higher denomination without changing its value.

EXERCISES.

1. How many pounds are there in 8365 pence?

Process.

12) 8365d.
20) 697s. + 1d.
£34 + 17s.

8365d. = £34 17s. 1d.

Explanation.

Since 12d. = 1s., $\frac{1}{12}$ of the number pence = the number of shillings. Since 20s. = £1, $\frac{1}{20}$ of the number of shillings = the number of pounds. Hence, 8365d. = £34 17s. 1d.

2. Reduce $\frac{5}{9}$d. to the fraction of a pound.

Process.

$$\tfrac{5}{9}\text{d.} = \tfrac{5}{9} \text{ of } \tfrac{1}{12}\text{s.} = \tfrac{5}{108}\text{s.}$$
$$\tfrac{5}{108}\text{s.} = \tfrac{5}{108} \text{ of } £\tfrac{1}{20} = £\tfrac{5}{2160} = £\tfrac{1}{432}.$$

Or,

$$\frac{\cancel{5}}{9}\text{d.} \times \frac{1}{12} \times \frac{1}{\underset{4}{\cancel{20}}} = £\frac{1}{432}.$$

RULE.

Divide by the numbers of the scale, beginning with the number of the given denomination that makes one of the next higher, and continuing until the required denomination is reached.

3. Reduce $\frac{4}{5}$s. to a fraction of a pound.
4. Reduce $\frac{4}{7}$ far. to a fraction of a shilling.
5. Change 495d. to units of higher denominations.
6. Change 4257 shillings to pounds.
7. Reduce 4697d. to pounds.
8. Reduce 5967s. to pounds.
9. Reduce 6969 far. to pounds.
10. Reduce 4995d. to pounds.
11. Reduce 5796 far. to shillings.
12. Reduce 59,678 far. to pounds.
13. Reduce £4 15s. 6d. to far.
14. Reduce 59,607 far. to pounds.
15. Reduce 25,392 far. to £, s., d.
16. Reduce £26 to dollars.
17. Reduce $567 to pounds.
18. Reduce $394.45 to pounds.
19. Reduce $48.60 to pounds.
20. Reduce £36 to dollars.
21. Reduce 25,488 far. to £, s. and d.
22. Reduce $973.30 to pounds.

23. Reduce \$1,216,625 to pounds.
24. Reduce \$1084.40 to cents.
25. Reduce 3596 cents to dollars.
26. Reduce 48,567 mills to dollars.
27. Reduce 1930 francs to dollars.
28. Reduce 3846 francs to dollars.
29. Reduce 4856 dollars to francs.
30. Reduce 5968 centimes to francs.
31. Reduce $\frac{4}{5}$ centime to the fraction of a franc.
32. Reduce £$\frac{11}{12}$ to integers of lower denominations.

MEASURES.

1. Extension has three dimensions,—*length, breadth,* and *thickness.*

2. These dimensions are the origin of three different kinds of Measures,—*Linear Measures, Surface Measures,* and *Measures of Volume* or *Capacity.*

LINEAR MEASURES.

Linear Measures are used in determining *lengths* and *distances.*

Common Linear Measure.

12 Inches (in.)	= 1 Foot (ft.).
3 Feet	= 1 Yard (yd.).
$5\frac{1}{2}$ Yards / $16\frac{1}{2}$ Feet	= 1 Rod (rd.).
320 Rods	= 1 Mile (mi.).
40 Rods	= 1 Furlong (fur.).
8 Furlongs	= 1 Mile.

mi.	*rd.*	*yd.*	*ft.*	*in.*
1 =	320 =	1760 =	5280 =	63,360.

Scale: 12, 3, $5\frac{1}{2}$, 320.

Surveyors' Linear Measure.

7.92 Inches	= 1 Link (l.).
25 Links	= 1 Rod (rd.).
4 Rods, 100 Links	= 1 Chain (ch.).
80 Chains	= 1 Mile (m.).

mi.	*ch.*	*rd.*	*l.*	*in.*
1 =	80 =	320 =	8000 =	63,360.

Scale: 7.92, 25, 4, 80.

The following have special uses:

3 Barleycorns	= 1 Inch.	Shoe-length measure.
4 Inches	= 1 Hand.	Horse-height measure.
6 Feet	= 1 Fathom.	Sea-depth measure.
3 Feet	= 1 Pace.	Pacing measure.
5 Paces	= 1 Rod.	Pacing measure.
1.15 Statute Miles	= 1 Geographical, or Nautical Mile.	
3 Geographical Miles	= 1 League.	
60 Geographical Miles, 69.16 Statute Miles	= 1 Degree	of Latitude, or of Longitude at the Equator

EXERCISES.

1. In 12 rd. 3 yd. 2 ft. how many feet?
2. In 5 mi. 18 rd. 4 yd. how many yards?
3. Change 7 rd. 5 ft. 6 in. to inches.
4. Reduce 5 miles to inches.
5. In 5 mi. 2 fur. 4 yd. how many feet?
6. In $5\frac{1}{2}$ yd. of ribbon how many inches?
7. How many inches in one mile?
8. Reduce 3 mi. 4 yd. 2 ft. to feet.
9. Reduce 13,769 ft. to mi., fur., rd., etc.
10. In 15,347 in. how many yd., ft., etc.?
11. Change 250,497 ft. to miles, etc.
12. Reduce 77,565 in. to miles, etc.
13. Reduce 5 miles to links.
14. Reduce 9 miles 54 ch. to links.

15. Reduce 11 mi. 68 fathoms to fathoms.
16. Reduce 43,000 l. to miles.
17. Reduce 79,400 l. to miles.
18. Reduce 9968 fathoms to miles.
19. In 481,401,716 in. how many degrees?
20. If a horse is $15\frac{1}{2}$ hands high, find his height in feet.
21. Change 29,763 l. to higher denominations.
22. How many inches in 2 degrees at the equator?
23. Reduce 677,653 in. to higher denominations.
24. Reduce 7912 mi. (the diameter of the earth) to inches and to paces.
25. A ship was sailing in $12\frac{1}{2}$ fathoms of water. How deep was the water?

SURFACE MEASURES.

1. An **angle** is formed by two straight lines drawn from the same point.

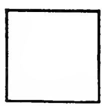
Angle.

Right Angles.

2. When a line is drawn from a point between the ends of another line, making the two angles *equal*, the angles are called **right angles**.

3. A **Rectangle** is a figure having four sides and four right angles.

Rectangle.



Square.

4. A **Square** is a rectangle with four equal sides.

A

1	2	3
4	5	6
7	8	9
10	11	12

B

5. The **Area** of a surface is the number of *square units* it contains.

The area of the rectangle A. B. is 12 squares; they may be square inches, square feet, or square rods, etc.

Since there are 4 rows of 3 squares each, the area, 12 squares, is denoted by the product of 4 by 3; *i.e.*, by the product of the length by the breadth.

If by 4, four squares are meant, is 3 to be considered an abstract or a denominate number?

FORMULA (Indicated Process).

Area of rectangle = Length × Breadth.

NOTE.—The rule implies that the length and breadth are expressed in the same denomination.

Common Square Measure.

Table.

144	Square Inches (sq. in.)	=	1 Square Foot (sq. ft.).
9	Square Feet	=	1 Square Yard (sq. yd.).
$30\frac{1}{4}$	Square Yards	=	1 Square Rod (sq. rd.). 1 Perch.
160	Square Rods	=	1 Acre (A.).
640	Acres	=	1 Square Mile (sq. mi.). 1 Section of Land.

sq. mi.	*A.*	*sq. rd.*	*sq. ft.*	*sq. in.*
1 =	640 =	102,400 =	27,878,400 =	4,014,489,600.

Scale: 144, 9, $30\frac{1}{4}$, 160, 640.

NOTE.—40 Perches = 1 Rood (R.); 4 Roods = 1 Acre.

Units.

The *unit* for land is the *acre;* for plastering, ceiling, etc., it is the *sq. yd.;* for paving, glazing, and stone-cutting, it is the *sq. ft.;* for roofing, flooring, and slating, it is a *square 10 ft. by 10 ft., or 100 sq. ft.*

Surveyors' Square Measure.

Table.

625 Square Links (sq. l.)	=	1 Sq. Rd., or Perch (P.).
16 Square Rods	=	1 Sq. Chain.
10 Square Chains	=	1 Acre.
640 Acres	=	1 Sq. Mile.
36 Sq. Miles	=	1 Township (Tp.).

EXERCISES.

Reduce:

1. 54 A. 10 P. to perches (sq. rd.).
2. 624 P. to sq. yds.
3. 5 A. 145 P. to sq. ft.
4. 250 P. 17 sq. yd. to sq. inches.
5. 2 A. 70 P. 30 sq. yd. to sq. inches.
6. 10 sq. yd. 4 sq. ft. 15 sq. in. to sq. inches.
7. 3,737,796 sq. ft. to acres, etc.
8. 3 A. 37 sq. rd. 5 sq. yd. 7 sq. ft. to sq. inches.
9. 295,376 sq. in. to units of higher denominations.
10. 31 sq. mi. 48 A. to sq. ch.
11. 2,000,000 sq. l. to acres.
12. 10,800 sq. ch. to sq. miles.
13. $\frac{7}{9}$ of A. to sq. inches.

Process.

$$30\tfrac{1}{4} = \frac{121}{4}. \quad \frac{7}{\cancel{9}} \times \frac{160}{1} \times \frac{121}{4} \times \frac{\cancel{9}}{1} \times \frac{144}{1} = 4{,}878{,}720 \text{ sq. in.}$$

14. $\frac{7}{9}$ of A. to units of lower denominations.

Process.

$\frac{7}{9}$ A. $= \frac{7}{9}$ of 160 sq. rd. $= 124\frac{4}{9}$ sq. rd.

$\frac{4}{9}$ sq. rd. $= \frac{4}{9}$ of $\frac{121}{4}$ sq. yd. $= 13\frac{4}{9}$ sq. yd.

$\frac{4}{9}$ sq. yd. $= \frac{4}{9}$ of 9 sq. ft. $= 4$ sq. ft.

Hence $\frac{7}{9}$ A. $= 124$ sq. rd. 13 sq. yd. 4 sq. ft.

15. $\frac{5}{7}$ A. to units of lower denominations.
16. $\frac{3}{8}$ sq. rd. to lower denominations.
17. $\frac{2}{5}$ A. to sq. chains.
18. $\frac{1}{4}$ section of land to sq. l.
19. 25,373,896 sq. l. to higher denominations.
20. 8 chains to sq. rods.
21. 1 sq. mi. 4 sq. ch. 53 sq. l. to links.
22. 47,916 sq. ft. to higher denominations.
23. 871,200 sq. ft. to acres.
24. 89,794,172 sq. in. to acres, etc.
25. 20,000 sq. ft. to sq. rods.

PROBLEMS.

Indicate the process first, and abridge the work by cancellation.

1. A rectangular piece of land is 40 rd. long and 12 rd. wide. Find the acres in it.

2. A floor is 8 ft. by 16 ft. How many sq. ft. in it?

3. A ceiling is 17 ft. by 20 ft. How many sq. yds. in it?

4. The side of a square is 6 feet. Find its area.

5. A table is 6 ft. by 2 ft. Find its area in sq. in.

6. A garden is 656 ft. by 93 ft. Find the sq. rd. in it.

7. A fence surrounding a mile race-course is 6 ft. high. How many sq. yd. in it? Find the cost at 10 cents a sq. yd.

8. A room is 20 by 30 feet. How much will it cost to carpet the room with carpet 1 yd. wide at $1.00 per yd.?

9. A field contains 12 acres and is 24 rds. wide. Find its length.

10. I bought 10 acres of land at $200 an acre and sold it at 8 cts. a sq. ft. Find the gain.

11. Find the cost of a piece of oil-cloth 25 feet long and 16 feet 9 inches wide, at 95 cents a square yard.

12. What will it cost to carpet a room 18 feet long and $25\frac{3}{4}$ feet wide at $1.25 a yard, the carpet being $\frac{3}{4}$ yd. wide?

13. A school-room measures as follows: Length, 72 ft.; width, $22\frac{1}{2}$ ft.; height, 16 ft. Deduct 245 sq. ft. for doors and windows, and find the cost of plastering at $16\frac{1}{2}$ cts. per sq. yd.

14. Show the difference between 6 sq. ft. and 6 feet square.

15. A room is to be plastered and painted; its length is 20 ft., its width 18 ft., its height 12 feet; the rate will be $33\frac{1}{3}$ cts. per sq. yd. Find the cost of the work.

16. Find the value of a field 180 rd. long and $94\frac{1}{2}$ rd. wide at $18 an acre.

17. How many yds. of carpeting, 3 ft. 6 in. wide, will it take to cover a floor 21 ft. wide and 36 ft. long, the carpet running lengthwise?

18. A room measures 18 ft. × 15 ft. × 10 ft. Find the cost of papering it with paper 24 in. wide at $.85 a roll, 8 yd. in a roll, making a deduction of 20 sq. yd. for openings.

19. A sidewalk is 10 ft. wide, exclusive of the curb, and is 100 ft. long. How many 4 × 8 bricks in it?

20. A 15 by 18 ft. room is to be carpeted. Which will be the cheaper way to run yard-wide strips, lengthwise or breadthwise?

21. A room is 18 ft. wide and 9 ft. high. After deducting from the area of one end two windows 6 ft. × $4\frac{1}{2}$ ft., find the number of sq. yd. remaining to be plastered.

22. A rectangular piece of land 1320 yds. long and 2 rods wide was taken for public use. How much was due the owner at $160 an acre?

23. A barn is roofed with shingles put 6 in. to the weather. Find the cost at $12 per M. if the roof is 60 feet long, each side being 32 feet, the first course along the eaves being doubled. [1000 shingles to 110 sq. feet.]

24. Find the cost of slating a roof 64 ft. 9 in. long and 45 ft. wide at $$15.37\frac{1}{2}$ per square.

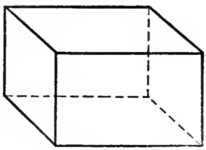
25. How many bricks will pave a sidewalk 25 ft. by 10 ft., a brick measuring 8 in. $\times$ 4 in. $\times$ 2 in.?

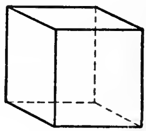
26. How many bricks, set on end, will pave a sidewalk containing one-half the last area?

MEASURES OF VOLUME.

1. A *Solid* has *length*, *breadth*, and *thickness*.

2. A **Rectangular Solid** is bounded by *six* rectangular faces.

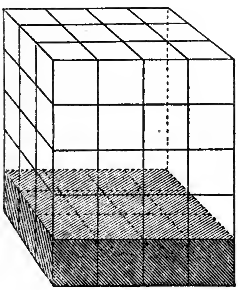




3. A **Cube** has six *equal* faces and twelve *equal* edges.

4. **Volume,** or **Solid Contents,** of a body is the number of *cubic units* it contains.

5. If on a rectangle of 12 sq. ft., as a base, we erect a rectangular solid 5 feet high, the structure will contain 3 times 4, or 12, cubic feet for each of the 5 feet of height. Hence, the *volume* of the solid will be 5 times 12, or 60, cubic feet.



RULE.

The volume of a rectangular solid is the number of cubic units denoted by the product of its length, breadth, and thickness.

NOTE.—The rule implies that the three dimensions are all of the same denomination.

Cubic Measure.

Table.

1728 Cubic Inches (cu. in.)	=	1 Cubic Foot (cu. ft.).
27 Cubic Feet	=	1 Cubic Yard (cu. yd.).
16 Cubic Feet	=	1 Cord Foot (cd. ft.).
8 Cord Feet, or 128 Cubic Feet	=	1 Cord of Wood (cd.).

Scale: 1728, 16, 8, and 1728, 27.

EXERCISES.

Reduce:

1. 15 cu. yd. 18 cu. ft. 16 cu. in. to cu. inches.
2. 730,960 cu. in. to cu. yd., etc.
3. 32 cu. ft. 114 cu. in. to cu. inches.
4. 174,964 cd. ft. to cords.
5. 7680 cu. ft. to cords.
6. 2160 cu. ft. to cu. yd.
7. 62,950 cu. in. to cu. ft.
8. 3 cd. ft. 8 cu. ft. to cu. ft.
9. 78,976 cd. ft. to cords.
10. 8797 cu. ft. to cords.
11. 466 cd. 124 cu. ft. to cu. ft.
12. 1216 cu. ft. to cords.
13. 19,528 cd. ft. to cords.
14. 988 cu. ft. to cu. yd.
15. $\frac{7}{9}$ cd. to units of lower denominations.

FORMULÆ.

1.	Volume	= length × breadth × height (thickness).
2.	Height, Thickness	= volume ÷ length × breadth.
3.	Breadth	= volume ÷ length × height.
4.	Length	= volume ÷ breadth × height.

UNITS.

The *cubic foot* for bricklaying, masonry, and hewn timber.

The *cubic yard* for embankments, excavations, and masonry.

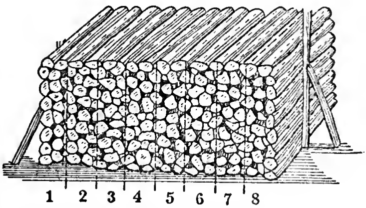
A *cubic yard* of common earth is sometimes called a *load.*

The *perch* of stone, $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, equal to $24\frac{3}{4}$ cu. ft. It is customary, however, to call 25 cu. ft. a perch.

Brick.—The size of a *common* brick is $8 \times 4 \times 2$ in., and for ordinary calculation it is sufficiently accurate to reckon 27 bricks to the cubic foot, laid dry, or $22\frac{1}{2}$ laid in mortar.

Brickwork is generally estimated by the *thousand bricks.* In estimating *material,* allowance is made for openings in walls, as doors, windows, etc.

In estimating *labor*, the length of each wall is measured on the *outside*, and thus each corner is measured twice.



Sometimes, by *special contract*, an allowance is made for *one-half* the openings and corners.

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is a *cord.*

A *cord foot* is one foot in length of such a pile; that is, 1 ft. long, 4 ft. wide, and 4 ft. high.

PROBLEMS.

1. Find the volume of:
   1. A block of marble 9 ft. long, 5 ft. wide, $3\frac{1}{2}$ ft. thick.
   2. A cube whose edge measures 2 ft. 9 in.
   3. A pile of wood 36 ft. long, 6 ft. high, 4 ft. wide.

      Suggestion: $\frac{36 \times 6 \times 4}{128}$. Cancel.
   4. The contents of a box measuring 18 in. by 16 in. by 14 in.
   5. A cube 1 in. long, 1 in. wide, 1 in. high.
   6. The earth in a cellar 36 ft. $\times$ 24 ft. $\times$ $5\frac{1}{2}$ ft.
   7. Excavation for a reservoir 80 ft. by 60 ft. by 10 ft.

8. Excavation for a cistern measuring 15 ft. by 14 ft. by 13 ft.
9. A cube whose sides are each 6 in. square.
10. A brick measuring $8 \times 4 \times 2$ in.

2. Why are there 16 cu. ft. in a cord-foot of wood?

3. Find the perches of masonry in a wall 38 ft. by 4 ft. by $1\frac{1}{2}$ ft.

Suggestion: $\frac{38 \times 4 \times 1\frac{1}{2}}{24\frac{3}{4}}$. Cancel.

4. Reduce 16 p. $14\frac{1}{2}$ cu. ft. to cu. ft.

5. A cube whose edge is 9 yds. equals in volume how many cubes whose edge is one foot?

6. How many blocks of one cubic inch can be sawed from a cube of 7 ft., if there is no waste in sawing?

7. How many cords in a pile of wood measuring 72 ft. by 13 ft. by 9 ft.?

8. Find the cost of digging a cellar 50 ft. long, 40 ft. wide, $5\frac{1}{2}$ ft. deep, at $\$1\frac{1}{2}$ per cubic yard?

9. How many cubic feet in two-thirds of a cubic yard?

10. How many cd. ft. in 5 cords of wood?

11. How many cu. yd. of air in a room 24 ft. by 18 ft. by 12 ft.?

12. Find the number of loads of ashes in a rectangular pit measuring 12 ft. by 4 ft. by 2 ft. 6 in.?

13. Find the cost of building a stone wall 55 rds. long, $4\frac{1}{2}$ ft. high, and 1 yd. thick, at \$7.25 a perch?

14. How many bricks would make the bulk of 9 loads?

15. How many perches of masonry in a wall 6 ft. high, 2 ft. thick, inclosing a plot of ground 40 rds. square?

16. How many bricks, laid in mortar, will build a house 57 ft. long, 45 ft. wide, and 30 ft. high, the wall to be $1\frac{1}{2}$ ft. thick; deducting for 15 windows, $8\frac{1}{2}$ by $3\frac{1}{2}$ ft., and 8 doors, $7\frac{2}{3}$ by $3\frac{1}{4}$ ft. wide?

17. What will it cost to dig a cellar 35 ft. long, 22 ft. wide, and 6 ft. deep, at $.25 for the removal of a cubic foot of earth?

18. A pile of wood containing 19$\frac{7}{8}$ cd. is 66 ft. long and 6 ft. wide. Find its height?

19. Prescribing 18 in. for the height and 6 ft. for the length of a wagon body, how wide must it be made to hold a load?

20. Find the length of a wall, 1 ft. thick and 4 ft. high, that 1000 bricks, laid in mortar, will build.

21. A river 30 ft. deep and 20 yd. wide flows 4 mi. an hour. How many cu. ft. of water pass a given point in a minute?

Board Measure.

1. Boards whose thickness is *one inch or less* are measured by the square foot. A board 12 ft. long, 1 ft. wide, 1 in. thick = 12 sq. ft. *board measure.*

2. All hewn or squared lumber is estimated and sold by board measure, thickness beyond one inch becoming a factor.

EXERCISE.

How many feet, board measure, are there in a timber 40 ft. long, 9 in. wide, and 6 in. thick?

Process.

$$\frac{\overset{20}{\cancel{40}}}{1} \times \frac{9}{\underset{2}{\cancel{12}}} \times \frac{\cancel{6}}{1} = 180 \text{ ft. board measure.}$$

Explanation.

9 in. = $\frac{9}{12}$ ft. Since $\frac{40}{1} \times \frac{9}{12}$ = the number of board feet when the thickness is one inch, 6 times that, or $\frac{40}{1} \times \frac{9}{12} \times \frac{6}{1}$ = the number of board feet when the thickness is 6 in. By cancelling we have 180 ft.

FORMULA.

Number of feet board measure = length (feet) × width (feet) × thickness (inches).

NOTE.—When the board tapers uniformly, use the *mean width, i.e.,* half the sum of the two end widths.

PROBLEMS.

1. Find the number of board feet in the following pieces of lumber:

1. 16 ft. by 9 in.	6. 15 ft. by 18 in. by 4 in.
2. 15 ft. by 10 in.	7. 35 ft. by 15 in. by 12 in.
3. 12 ft. by 11 in.	8. 12 ft. by 5 in. by 4 in.
4. 11 ft. by 12 in.	9. 40 ft. by 9 in. by 6 in.
5. 14 ft. by 16 in.	10. 16 ft. by 8 in. by $\frac{3}{4}$ in.

2. Find the number of feet in a joist 20 ft. long, 8 in. wide, and 4 in. thick.

3. A man has 70 planks measuring 16 ft. by 8 in. and $1\frac{3}{4}$ in. thick. How many feet, board measure, has he?

4. Find the width of a 2-in. plank 16 ft. long that contains 50 ft., board measure.

5. Find the contents of a board 18 ft. long, 1 ft. 8 in. wide at one end and 14 in. at the other.

6. Find the cost of 38 boards 16 ft. long, $12\frac{1}{2}$ in. wide, at $\$2\frac{3}{4}$ per C.

7. Find the cost of the following, at $20 per M.:

160 boards 16 ft. by 11 in.
170 boards 15 ft. by 10 in.
70 plank 14 ft. by 10×3 in.
70 scantling 12 ft. by 4×2 in.
50 rafters 25 ft. by 5×3 in.

8. How many feet, board measure, are there in a plank 17 ft. long, 22 in. wide at one end, 13 in. wide at the other, and 3 in. thick?

9. Find the cost of 72 boards, each 11 ft. long, 16 in. wide, and $\frac{3}{4}$ in. thick, at $16.50 per M.

10. In a tapering board, 11 ft. long, 18 in. wide at one end, 11 inches wide at the other, and $\frac{7}{8}$ in. thick, how many feet?

11. A room is 20 by 25 ft. What will be the cost of floor-

ing with $1\frac{1}{2}$ in. flooring, at $24 per M., allowing one-eighth for matching?

12. A field 160 yds. long by 120 yds. wide is to be enclosed with a fence 4 boards high, each board 6 in. wide. Find the cost of the boards at $18 per M.

MEASURES OF CAPACITY.

Liquid Measure.

Liquid Measure is used in measuring all kinds of liquids.

Table.

4 Gills (gi.) = 1 Pint (pt.).
2 Pints = 1 Quart (qt.).
4 Quarts = 1 Gallon (gal.).

gal.	*qt.*	*pt.*	*gi.*
1 =	4 =	8 =	32.

Scale: 4, 2, 4.

1. The **capacity of cisterns**, reservoirs, etc., is commonly expressed in *gallons* or in *barrels.*

2. The *standard liquid gallon* of the United States contains 231 cubic inches.

3. The *barrel* (bbl.), as a measure of capacity, is estimated at $31\frac{1}{2}$ gallons; the *hogshead* (hhd.) at 63 gallons; neither, as a commercial vessel, holds a fixed quantity.

4. The *beer gallon*, of 282 cubic inches, is no longer in use.

EXERCISES.

1. Reduce the following:
   1. 7 pt. to gills.
   2. 8 qt. to gills.
   3. 37 gal. to pints.
   4. 795 pt. to gallons.
   5. 4957 gi. to gallons.
   6. 4 gal. 5 qt. 1 pt. 4 gi. to gills.
   7. 5 bbl. 7 gal. to gills.
   8. 9560 gi. to barrels.

9. $\frac{3}{4}$ gal. to the fraction of a barrel.
10. 1 bbl. $\frac{1}{4}$ gal. 1 pt. to pints.

2. How many cu. in. in 8 gal.?
3. How many gal. in 4956 cu. in.?

PROBLEMS.

1. A cistern is 16 ft. long, 12 ft. wide, and 9 ft. deep. How many bbl. of water does it hold?

2. How many hhd. will a cistern contain that is 12 ft. long, 7 ft. wide, and 2 ft. 10 in. deep?

3. Find the cu. in. in the space that will hold 63 bbl.

4. If a vat contains 54,762 cu. in., how many bbl. of water will it hold?

5. If a cu. ft. of water weighs 1000 ounces, what is the weight of 10 hhd. of water?

6. How many cu. ft. in a cistern that contains 100 hhd.?

7. A cistern 8 ft. long by 6 ft. wide contains 5 ft. of water. How many gal. of water are therein?

8. How deep must a cistern be to contain 40 hhd. if it is 14 ft. long by 6 ft. wide?

Apothecaries' Liquid Measure.

Apothecaries' Liquid Measure is used for measuring liquids required by medical prescriptions.

Table.

60 Drops (gtt) or minims (♏)	= 1 Fluid drachm (f ʒ).
8 Fluid drachms	= 1 Fluid ounce (f ℥).
16 Fluid ounces	= 1 Pint (O.).
8 Pints	= 1 Gallon (Cong.).

Gtt. is from the Latin *gutta*, a drop.
Minim is from the Latin *minimus*, the least.
O. is from the Latin *octarius*, one-eighth.
Cong. is from the Latin *congius*, gallon.

EXERCISES.

Reduce :

1. 12 pints to fluid drachms.
2. 48 fluid ounces to pints.
3. 12 gal. 3 pt. to fluid ounces.
4. 51 pt. to gallons.
5. 34 Cong. 3 O. 1 f℥ 3 fʒ to ♏.
6. 1860 ♏ to gallons.
7. 16 Cong. 6 O. 7 f℥ to fʒ.
8. 27,408 fʒ to Cong.
9. 4 Cong. 2 O. 15 f℥ 7 fʒ to ♏.
10. 8,472,347 ♏ to Cong., etc.

Dry Measure.

Dry Measure is used in measuring dry substances, such as grain, roots, fruit, etc.

Table.

2 Pints (pt.)	=	1 quart (qt.).
8 Quarts	=	1 Peck (pk.).
4 Pecks	=	1 Bushel (bu.).

bu. *pk.* *qt.* *pt.*
1 = 4 = 32 = 64.

Scale : 2, 8, 4.

A standard bushel contains 2150.42 cubic inches.
7 cubic feet of corn in the ear equals 3 bushels of shelled corn.

Liquid and Dry Measures Compared.

1. Since 1 liquid gallon = 231 cu. in.,
   1 liquid quart = how many cu. in.?
   1 liquid pint = how many cu. in.?
   1 liquid gill = how many cu. in.?
2. Since 256 dry gills = 1 bu. = 2150.42 cu. in.,
   1 dry gill = how many cu. in.?
   1 dry pint = how many cu. in.?
   1 dry quart = how many cu. in.?
   1 dry gallon = how many cu. in.?

	1 gal.	*1 qt.*	*1 pt.*	*1 gi.*
3. Liquid Measure:	231,	$57\frac{3}{4}$,	$28\frac{7}{8}$,	$7\frac{7}{32}$ cu. in.
4. Dry Measure:	$268\frac{4}{5}$,	$67\frac{1}{5}$,	$33\frac{3}{5}$,	$8\frac{2}{5}$ cu. in.

EXERCISES.

Reduce:

1. 12 pt. to quarts.
2. 5 bu. to pecks.
3. 32 qt. to bushels.
4. 16 qt. 1 pt. to pints.
5. 899 qt. to bushels.
6. 25 bu. 3 pk. 7 qt. to quarts.
7. 38 bu. 5 qt. 1 pt. to pints.
8. 42 pt. to quarts.
9. 402 pt. to pecks.
10. 19 pk. 7 qt. 1 pt. to pints.

PROBLEMS.

1. How many cu. in. in 8 bu.? 21 bu.?

2. How many bu. in 27,692 cu. in.?

3. How many cu. in. in a bin 9 ft. long, 8 ft. wide, and 6 ft. high?

4. How many bu. will a bin hold that is 10 ft. long, 7 ft. wide, and 7 ft. high?

5. What must be the depth of a bin to contain 250 bu. of grain, its length being 12 ft. and its width 6 ft.?

6. What must be the length of a bin whose width is 6 ft. and depth $4\frac{1}{2}$ ft. to contain 400 bu. of rye?

7. A bin 8 ft. long, 7 ft. wide, and 5 ft. deep is $\frac{3}{4}$ full of oats. What is the value of the oats at $.25 a bushel?

8. If a vessel holds 700 gal. of water, how many bu. of grain will it contain?

9. How many bu. of grain will a bin hold that is 9 ft. 5 in. long by 3 ft. 6 in. wide by 7 ft. deep?

10. If rain falls to the depth of $1\frac{1}{4}$ in., how many cu. in. fall on one acre?

11. A rectangular box exactly contains a bu. The length of the box is 16 in., its width 15.5 in. Find its depth.

12. How many bu. of corn will a bin hold that is 8.5 ft. long, 4.25 ft. wide, and $7\frac{1}{2}$ ft. deep?

13. The capacity of a bin is 1583.2 bu.; its length is 9 ft. and depth 9 ft. Find its width.

14. How many cu. ft. in 600 bu. of wheat?

15. How many cu. ft. in 500 bu. of potatoes, heaped measure?

1 bu. heaped meas. $= \frac{5}{4}$ bu. stricken meas.

16. How many cu. ft. in a bushel?

17. How many gal. in a cubic foot?

18. A bin 18 ft. long, $4\frac{4}{9}$ ft. wide contains 40 cu. yd. Find its depth.

19. A bin measures 6 ft. 5 in. by 3 ft. 9 in. by 4 ft. 6 in. How many bushels of wheat will it hold? How many bushels of corn in the ear?

MEASURES OF WEIGHT.

Weight is the measure of the force that attracts bodies to the earth.

Avoirdupois Weight.

Avoirdupois Weight is used in weighing all materials except *gold* and *silver*.

Table.

16 Ounces (oz.) = 1 Pound (lb.).
100 Pounds = 1 Hundred-weight (cwt.).
20 Hundred-weight = 1 Ton (T.).

T.		*cwt.*		*lb.*		*oz.*
1	=	20	=	2000	=	32,000.

Scale: 16, 100, 20.

The pound = 7000 grains.
The ounce = 16 drachms.
The long ton = 2240 lbs.

Measures Much Used.

1 Firkin (of butter)	= 56 lb.
1 Cental (of grain, flour)	= 100 lb.
1 Quintal (of dried fish)	= 100 lb.
1 Keg (of nails)	= 100 lb.
1 Barrel (of flour)	= 196 lb
1 Barrel (of pork or beef)	= 200 lb.
1 Barrel (of salt at N. Y. Works)	= 280 lb.
1 Cask (of lime)	= 240 lb.

Pounds in a Bushel.

Wheat,	60 lb.	Beans,	60 lb.	Wheat Bran,	20 lb.
Rye,	56 lb.	Buckwheat,	42 lb.	Salt,	56 lb.
Corn,	56 lb.	Flax Seed,	56 lb.	Corn Meal,	50 lb.
Barley,	48 lb.	Hemp Seed,	44 lb.	Corn in Ear,	68 lb.
Oats,	32 lb.	Potatoes,	60 lb.	Clover Seed,	60 lb.
Peas,	60 lb.	Onions,	60 lb.	Timothy Seed,	45 lb.

Note.—Slight variations from the above exist among the States.

EXERCISES.

Reduce:

1. 3 T. to pounds.
2. 5 T. 4 cwt. 6 lb. to pounds.
3. 10 T. 327 lb. to pounds.
4. 95,000 oz. to tons.
5. 2 T. to ounces.
6. 10,406 lb. to tons.

PROBLEMS.

1. At $.05$\frac{1}{2}$ a pound, what will 3 cwt. of sugar cost?
2. What will 4$\frac{1}{4}$ lb. of confections cost, at $.04$\frac{1}{4}$ per ounce?
3. At $.08 per lb., what are 8 cwt. of beef worth?
4. How many grains in 9 lb.?
5. What will 400 lb. of coal cost at $6.50 per T.?
6. How many bushels of corn meal in 2 T.?
7. How many pounds will 1000 bu. of oats weigh?
8. How many T. will 500 bu. of potatoes weigh?

9. Find how much more 160 bu. of wheat weigh than 200 bu. of barley.

10. Find the cost of 3024 lb. of rye, at $.66 a bu.

11. Find the value of 5 firkins of butter, at $.17 per lb.

12. In 30,000 lb. of pork how many barrels are there?

13. What is the weight of 105 bu. $3\frac{1}{2}$ pk. of potatoes?

14. If a dealer has 27 long tons of coal and sells 7780 lb., how many short tons remain?

15. At $10 a barrel, what will a bag of flour cost, weighing 59 lb.?

Troy Weight.

Troy Weight is used in weighing gold, silver, and precious stones.

Table.

24 Grains	= 1 Pennyweight (pwt.).
20 Pennyweights	= 1 Ounce (oz.).
12 Ounces	= 1 Pound (lb.).

lb.	*oz.*	*pwt.*	*gr.*
1 =	12 =	240 =	5760.

Scale: 24, 20, 12.

The *carat*, equal to 4 grains, is commonly used in weighing precious stones. *Carat*, as the unit of fineness for gold, means $\frac{1}{24}$. Gold 14 carats fine is $\frac{14}{24}$ gold and $\frac{10}{24}$ alloy.

EXERCISES.

1. Reduce:
   1. 9 pwt. 12 gr. to grains.
   2. 204 gr. to pennyweights.
   3. 70 lb. 2 oz. 19 pwt. 16 gr. to grains.
   4. 4438 gr. to ounces.
   5. 5 lb. 11 oz. 15 pwt. 10 gr. to grains.
   6. 8356 gr. to ounces.
   7. 150 pwt. to oz.
   8. 1 lb. to grains.

2. How many grains in a pound avoirdupois?

3. How many more grains in a pound avoirdupois than in a pound troy?

4. Which is heavier, and how much, a pound of lead or a pound of silver?

5. How many pounds avoirdupois are there in 185 troy pounds?

6. Find the value in troy weight of 9 lb. 10 oz. avoirdupois.

7. If an ounce of gold is worth $25.75, what is the value of a pennyweight?

8. If 1898 sovereigns weigh 40.6172 lb. troy, how many grains does one sovereign weigh?

Apothecaries' Weight.

Apothecaries' Weight is used in prescribing and mixing dry medicines.

Medicines are bought and sold by avoirdupois weight.

Table.

20 Grains (gr.)	=	1 Scruple (℈).
3 Scruples	=	1 Drachm (ʒ).
8 Drams	=	1 Ounce (℥).
12 Ounces	=	1 Pound (lb.).

lb.		*oz.*		*dr.*		*sc.*		*gr.*
1	=	12	=	96	=	288	=	5760.

Scale: 20, 3, 8, 12.

Scruple: Latin, *scrupulus*, a little stone.
Drachm: Greek, *drachma*, a piece of money.
Ounce: Latin, *uncia*, one-twelfth.

1. *1 oz. Avoirdupois equals how many grains?*
2. *1 oz. Troy equals how many grains?*
3. *1 oz. Apothecaries' equals how many grains?*

EXERCISES.

Reduce:

1. 96℈ to drachms.
2. 144℥ to pounds.
3. 8 lb. 6℥ to drachms.
4. 816ʒ to pounds.
5. 18 lb. 5℥ 4ʒ 1℈ to scruples.
6. 952℥ to pounds.
7. 17 lb. 5ʒ 2℈ 17 gr. to grains.
8. 105,840 gr. to pounds.

PROBLEMS.

1. Express 4567 grains apoth. in higher units.

2. How many 4-grain pills can be made from 6ʒ 2℈?

3. How many ounces of quinine will be required to make 1440 2-grain pills?

4. How many 5-grain pills can be made from an avoirdupois pound of quinine?

5. Change 1℥ 5ʒ 1℈ 16 gr. to the fraction of a pound.

6. Reduce 8 lb. avoirdupois to apothecaries' weight.

7. How much does a druggist gain who buys 15 lb. (avoir.) of drugs at $2.75 per pound and sells the same at $.20 per drachm, apoth. weight?

8. An apothecary bought 50 lb. 8 oz. of opium at 45 cents an ounce, and sold it at 3 cents a scruple. How much did he gain?

MEASURES OF TIME.

Time is measured by *centuries, years, months, weeks, days, hours, minutes,* and *seconds.*

The *unit* is the *day,* which is determined by the *revolution of the earth on its axis.* The year is determined by the *revolution of the earth around the sun.*

Table.

60 Seconds (sec.)	=	1 Minute (min.).
60 Minutes	=	1 Hour (hr.).
24 Hours	=	1 Day (da.).
7 Days	=	1 Week (wk.).
365 Days	=	1 Year (yr.).
366 Days	=	1 Leap year.
100 Years	=	1 Century (cen.).

yr.		*mo.*		*da.*		*hr.*		*min.*		*sec.*
1	=	12	=	365	=	8760	=	525,600	=	31,536,000.

Scale: 60, 60, 24, 365, 100.

1. A year is called **Leap Year** when the number denoting it is *divisible by* 4 *and not by* 100, *or is divisible by* 400.

2. A year is called a **Common Year** when the number denoting it cannot be thus divided.

3. The **Calendar** divides the year into weeks and months.

Table.

1. January,	31 days, Jan.		7. July,	31 days, July.	
2. February,	28 or 29 days, Feb.		8. August,	31 days, Aug.	
3. March,	31 days, Mar.		9. September,	30 days, Sept.	
4. April,	30 days, Apr.		10. October,	31 days, Oct.	
5. May,	31 days, May.		11. November,	30 days, Nov.	
6. June,	30 days, June.		12. December,	31 days, Dec.	

A Useful Stanza.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone;
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

EXERCISES.

Reduce:

1. One day to seconds.
2. 86,400 seconds to days.

3. One week to minutes.
4. 20,160 minutes to weeks.
5. 4 da. 6 h. 56 min. to min.
6. 5776 min. to days.
7. 32 hr. 32 min. 41 sec. to seconds.
8. 87,990 sec. to hours.

PROBLEMS.

1. Express 49,796 sec. in higher denominations.
2. Express 49,598 sec. in units of higher orders.
3. How many days from April 10th to September 12th?
4. How many days from March 22d to July 17th?
5. Giving three months to each, which is the longer, summer or winter?
6. Which of these are leap years: 1600, 1660, 1666, 1700, 1776, 1790, 1794, 1800, 1898, 1900?
7. How many seconds are there in 7 hr. 38 min. 49 sec.?
8. How many days and hours in $\frac{2}{3}$ week?
9. Find the number of hours in a week.
10. What part of a week is 1 day 18 hours?
11. A man borrows some money July 17 and promises to repay it in 60 days. On what day is it due?
12. How many leap years in the nineteenth century?
13. How many hours in the month of August?
14. If you read French 30 min. each day for 5 yr., how much time do you thus spend?
15. On what day will $\frac{2}{5}$ of a common year end? $\frac{2}{3}$ of a leap year?

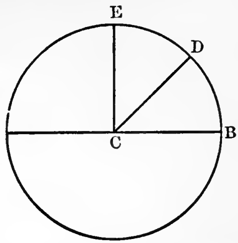
CIRCULAR MEASURE.

Circular Measure is used to measure *angles*.

A **Circle** is a figure made by a *bounding* line which is everywhere equally distant from a *centre-point.*

The **Circumference** of the circle is the bounding line.

An **Arc** is any part of the circumference, as BD.



A **Quadrant** is an arc equal to $\frac{1}{4}$ of the circumference, as BE.

A **Degree** is $\frac{1}{360}$ of the circumference of a circle.

An angle whose sides meet at the centre is measured by the arc included between its sides. The angle BCD is measured by the arc BD. A right angle is measured by a quadrant, or 90°.

Table.

60 seconds (″)	= 1 Minute (′).
60 Minutes	= 1 Degree (°).
30 Degrees	= 1 Sign (S.).
12 Signs, or 360°	= 1 Circumference (C.).

The Sign is used in astronomical calculations.

EXERCISES.

1. Reduce:

   1. 56′ 25″ to seconds.
   2. 3830″ to minutes.
   3. 23° 36′ to minutes.
   4. 856′ to degrees.
   5. 35° 48′ 59″ to seconds.
   6. 99,800″ to degrees.
   7. 4 S. 29° 26′ 33″ to seconds.
   8. 490,833″ to signs.

2. How many minutes in 2 quadrants?

3. How many seconds in a right angle?

4. $2\frac{1}{2}$ quadrants are what part of a circumference?

5. In 811,480″ how many signs?

6. In what time does a fixed point in the earth's surface pass through 15° 15′ 15″?

7. Where is a degree of latitude equal to 60 geographical miles?

8. Sixty-nine and 16 hundredths statute miles equal one degree on what surface?

MISCELLANEOUS TABLES.

Counting.

12 Units = 1 Dozen.	12 Gross = 1 Great gross.
12 Dozen = 1 Gross.	20 Units = 1 Score.

Paper.

24 Sheets = 1 Quire.	2 Reams = 1 Bundle.
20 Quires = 1 Ream.	5 Bundles = 1 Bale.

Books.

A book composed of sheets folded in:

2 leaves is a folio.	12 leaves is a duodecimo.
4 leaves is a quarto.	16 leaves is a 16mo.
8 leaves is an octavo.	18 leaves is an 18mo.

EXERCISES.

Reduce:

1. 45 gross of crayons to units.
2. 222 dozen bottles of ink to gross.
3. 12 great gross to units.
4. 3 reams of paper to quires.
5. 5 bundles of paper to sheets.

PROBLEMS.

1. What would 9600 sheets of foolscap cost at $.25 per quire?

2. Find the cost of 4 dozen brushes @ $.55 each.

3. A dealer bought paper at $8 per ream and sold it at $.30 a quire. Did he gain or lose, and how much?

4. In an octavo book of 960 pages how many sheets?

5. How many years are 3 score and 10?

6. How many sheets in 2 bundles 1 ream 15 quires 10 sheets?

7. How many units in 8 gross 9 dozen?

8. If 12 dozen of buttons are worth \$1.08, what are 11 buttons worth?

9. The use of 48 screws per day implies the use of how many gross in 6 weeks?

10. 500,000 copies of a daily newspaper are sold on an average per diem. Reckoning 3 sheets for each copy, how many reams of paper are used in a month?

REDUCTION OF DENOMINATE FRACTIONS.

SPECIAL EXERCISES.

Reduction Descending.

1. Reduce $\frac{6}{7}$ of a rod to units of lower denominations.

Process.

$\frac{6}{7}$ of $\frac{11}{2}$ yd. $= \frac{66}{14}$ yd. $= 4\frac{5}{7}$ yd.
$\frac{5}{7}$ of 3 ft. $= \frac{15}{7}$ ft. $= 2\frac{1}{7}$ ft.
$\frac{1}{7}$ of 12 in. $= \frac{12}{7}$ in. $= 1\frac{5}{7}$ in.
$\frac{6}{7}$ rd. $=$ 4 yd. 2 ft. $1\frac{5}{7}$ in.

Explanation.

$5\frac{1}{2}$ yd. $= \frac{11}{2}$ yd. Since 1 rd. $= \frac{11}{2}$ yd., $\frac{6}{7}$ rd. $= \frac{6}{7}$ of $\frac{11}{2}$ yd. $= 4\frac{5}{7}$ yd. Since 1 yd. $=$ 3 ft., $\frac{5}{7}$ yd. $= \frac{5}{7}$ of 3 ft. $= \frac{15}{7}$ ft. $= 2\frac{1}{7}$ ft. Since 1 ft. $=$ 12 in., $\frac{1}{7}$ of a ft. $= \frac{1}{7}$ of 12 in. $= \frac{12}{7}$ in. $= 1\frac{5}{7}$ in. Hence $\frac{6}{7}$ rd. $=$ 4 yd. 2 ft. $1\frac{5}{7}$ in.

2. Reduce .795 lb. Troy to units of lower denominations.

Process.

$$\begin{array}{r} .795 \text{ lb.} \\ \underline{12} \\ 9.540 \\ \underline{20} \\ 190.800 \end{array}$$

Explanation.

Since 12 oz. $=$ 1 lb., 12 times the number of pounds $=$ the number of ounces; 12 times .795 $=$ 9.540 oz. Since 20 pwt. $=$ 1 oz., 20 times the number of ounces $=$ the number of pwt. $.540 \times 12 = 10.8$ pwt. Hence .795 lb. $=$ 9 oz. 10.8 pwt.

3. Reduce $\frac{5}{288}$ gal. to gills.

Process.

$$\frac{5}{\cancel{288}\;\cancel{72}\;\cancel{36}\;9} \times \cancel{4} \times \cancel{2} \times \cancel{4} = \frac{5}{9} \text{ gi.}$$

Explanation.

Since 1 gal. $=$ 4 qt., 1 qt. $=$ 2 pt., and 1 pt. $=$ 4 gi., we multiply by the numbers of the scale, 4, 2, 4, and obtain by cancellation $\frac{5}{9}$ gills.

EXERCISES.

1. Reduce:

   1. $\frac{21}{868}$ of a bushel to the fraction of a pint.
   2. $\frac{1}{640}$ da. to minutes.
   3. $\frac{2}{3}$ rd. to yards, feet, and inches.
   4. .065 of a gallon to integers of lower denominations.
   5. $\frac{3}{7}$ of a ton to lower denominations.
   6. $\frac{3}{11}$ of an acre to lower denominations.
   7. .007 of a bushel as a decimal of a pint.
   8. .796 of a lb. troy to lower denominations.
   9. £.686 to lower denominations.
   10. .436 of a ream to integers.
   11. .875 of a leap year to integers.
   12. .795 of a league to integers.
   13. $\frac{9}{20}$ cu. yd. to lower denominations.
   14. .115625 lb. troy to integers.

2. Reduce:

   1. $\frac{3}{160}$ bu. to pints.
   2. £$\frac{1}{1728}$ to pence.
   3. $\frac{5}{9}$ lb. troy to integers.
   4. $\frac{2}{3}$ mi. to integers.
   5. $\frac{3}{7}$ bu. to integers.
   6. $\frac{5}{9}$ A. to integers.
   7. .1845 gal. to integers.
   8. .15625 bu. to integers.
   9. $\frac{1}{12000}$ T. to ounces.
   10. $\frac{7}{18720}$ mi. to inches.
   11. $\frac{5}{9}$ da. to integers.
   12. $\frac{44}{51}$ of a rod to integers.
   13. .03125 T. to integers.
   14. $\frac{7}{9}$ yd. to integers.
   15. $\frac{3}{32}$ mi. to yards.
   16. $\frac{4}{15}$ mi. to rod.

Reduction Ascending.

1. Reduce $\frac{8}{9}$ gi. to the fraction of a gal.

Process.

$$\frac{\overset{2}{\cancel{8}}}{9} \times \frac{1}{\cancel{4}} \times \frac{1}{\cancel{2}} \times \frac{1}{4} = \frac{1}{36}$$

Explanation.

4 gi. = 1 pt., therefore $\frac{1}{4}$ the number of gi. = the number of pt.; 2 pt. = 1 qt., therefore $\frac{1}{2}$ the number of pt. = the number of qt.; 4 qt. = 1 gal., therefore $\frac{1}{4}$ the number of qt. = the number of gal.; hence $\frac{8}{9}$ gi. = $\frac{8}{9} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{36}$ gal.

2. Reduce .375 wk. to the fraction of a year.

Process.

$.375$ wk. $= \frac{3}{8}$ wk.

$\frac{3}{8} \times 7 = \frac{21}{8}$ da.

$\frac{21}{8} \times \frac{1}{365} = \frac{21}{2920}$ yr.

Explanation.

$.375 = \frac{375}{1000} = \frac{3}{8}$. $\frac{3}{8}$ wk. $= \frac{3}{8}$ of 7 da. $= \frac{21}{8}$ da. 365 days $= 1$ yr., therefore $\frac{21}{8}$ da. $= \frac{21}{8} \times \frac{1}{365} = \frac{21}{2920}$ yr.

EXERCISES.

Reduce:

1. $\frac{7}{9}$ gi. to gal.
2. $\frac{13}{16}$ min. to da.
3. $\frac{13}{15}$ ft. to mi.
4. $5\frac{3}{7}$ oz. to T.
5. $\frac{4}{7}$ in. to rd.
6. $2\frac{2}{5}$ pt. to bu.
7. $\frac{6}{11}$ sec. to deg.
8. $3\frac{1}{5}$ min. to da.
9. .45 lb. to T.
10. $\frac{4}{7}$ cu. in. to cu. ft.
11. $\frac{4}{5}$ sq. yd. to A.
12. $\frac{7}{9}$ pt. to bbl.
13. $\frac{4}{9}$ gi. to gal.
14. $4\frac{1}{17}$ sq. in. to sq. rd.
15. $9\frac{1}{4}$ in. to mi.
16. $\frac{2}{3}$ of $\frac{3}{8}$ ♏ to cong.
17. $5\frac{1}{3} \times 7\frac{1}{4}$ cu. in. to cd.
18. 1 li. to mi.
19. $\frac{75}{76}$ lb. to T.
20. .89725 oz. to cwt.

THE FRACTIONAL RELATION OF ONE DENOMINATE NUMBER TO ANOTHER.

EXERCISES.

1. What part of 4 ft. 7 in. is 3 ft. 4 in.?

Process.

4 ft. 7 in. $=$ 55 in.

3 ft. 4 in. $=$ 40 in.

$\frac{40}{55} = \frac{8}{11}$

Explanation.

1 in. $= \frac{1}{55}$ of 55 in.; therefore, 40 in. $= \frac{40}{55}$ of 55 in.

$\frac{40}{55}$ reduced $= \frac{8}{11}$.

2. What decimal part of £1 is 16s. 8d.

1st Process.

£1 $=$ 240d.

16s. 8d. $=$ 200d.

$£\frac{200}{240} = £\frac{5}{6} = £.8333+$.

Explanation.

Both quantities must be reduced to the same denomination.

£1 $=$ 240d. 16s. 8d. $=$ 200d. 200d. $= \frac{200}{240}$ or $\frac{5}{6}$ of 240d.

$\frac{5}{6}$ reduced to a decimal $= £.8333+$.

3. What fraction of:
   1. 1 yd. is 2 ft. 9 in.?
   2. 1 mi. is 4 rd. $2\frac{1}{2}$ yd.?
   3. 1 A. is 24 sq. rd. 33 sq. yd.?
   4. $4\frac{3}{4}$ lb. is $5\frac{1}{2}$ oz.?
   5. 3 mi. is 5 rd. 2 yd. 2 ft. 2 in.?
   6. 3 bbl. are 13 gal. 3 qt. 3 pt. 3 gi.?
   7. 3 bu. is 1 bu. 3 pk. 4 qt.
   8. 5 lb. troy is 6 oz. 6 pwt. 6 gr.?
   9. 4 lb. avoirdupois is 4 lb. troy?
   10. A 6-in. cube is 6 cu. in.?
   11. 65 ch. is 1430 ft.?
   12. 365 da. is 4 wk. 4 da. 4 hr.?
   13. 360° is 40° 40′?
   14. £1 is 18s. $5\frac{3}{4}$d.?
   15. 1 cwt. is 16 lb. 11 oz.?

4. What decimal fraction of 1 bu. is 3 pk. 6 qt. 1 pt.?

2d Process.

$$\begin{array}{r|l}
2 & 1. \\ \hline
8 & 6.5 \\ \hline
4 & 3.8125 \\ \hline
 & .953125
\end{array}$$

Explanation.

2 pt. = 1 qt.; therefore $\frac{1}{2}$ the number of pt. = the number of qt. $\frac{1}{2}$ of 1 = .5 qt., which added to 6 qt. = 6.5 qt. 8 qt. = 1 pk.; therefore $\frac{1}{8}$ of the number of qt. = the number of pk. $\frac{1}{8}$ of 6.5 = .8125 pk., which added to 3 pk. = 3.8125 pk. 4 pk. = 1 bu.; therefore $\frac{1}{4}$ the number of pk. = the number of bu. $\frac{1}{4}$ of 3.8125 = .953125 bu.

5. What decimal fraction of:
   1. 1 S. is 6° 25′ 36″?
   2. 1 mi. is 5 rd. 3 ft. 10 in.?
   3. 1 yd. is 31 in.?
   4. 1 T. is 3 cwt. 48 lb. 9 oz.?
   5. 1 A. is 1 R. 39 P.?
   6. 1 T. is 6 cwt. 75 lb.?
   7. 1 da. is 11 hr. 55 min. 41.7 sec.?

8. 1 A. is 4276 sq. ft.?
9. 1 lb. is 14 oz.?
10. 82 mi. 70 rd. is 10 mi. 10 rd.?
11. 228 bu. 3 pk. is 8 bu. 2 pk. 6 qt.?
12. 1 lb. is 4 oz. 8 pwt. 12 gr.?
13. 1 mi. is 765 yd. 9 in.?
14. 1 cd. is 4 cd. ft. 8 cu. ft.?

REVIEW.

1. What will be the cost of:

1. 1 T. 15 cwt. 36 lb. of sugar @ 3 cts. a pound?
2. 3 lb. 9 oz. 13 pwt. of gold dust @ $.75 a pwt.?
3. 8 tons of coal @ $.26$\frac{1}{4}$ a cwt.?
4. 9 barrels of flour at $.03 a pound?
5. 16 lb. 9 oz. butter at $.30 a pound?
6. 4 pk. 5 qt. cherries at 10 cts. a quart?
7. 40 rd. 8 ft. 9 in. fence at $.80 per ft.?
8. 25 bu. of seed at 8 cts. a pint?
9. 7 bu. 3 pk. 2 qt. blackberries at 7 cts. a qt.?
10. 14 hhd. of molasses at 12 cts. a qt.?

2. How many bu. of wheat in 1260 lb.?

3. How many min. in the yr. 1898?

4. How many cords of wood in a pile 4 ft. wide, 7 ft. high, 70 ft. long?

5. How many days of 12 hrs. each will it require to make a million figures if one figure is made each second?

6. How many bu. of carrots will a 10-acre field produce if each sq. rd. produces 5 bu.?

7. How many sec. are there in 365 da. 5 hr. 48 min. 49 sec.?

8. How many bu. of oats in 2000 lb.?

9. How many sec. from 7 A.M., Aug. 15th, to Dec. 7th, 7 P.M.?

10. How many kegs, each holding 7 gal. 3 qt. 1 pt., can be filled from 11 hhd. of wine?

11. How many degrees in a quadrant measured on a meridian of the earth's surface? How many miles?

12. If 1 ton of phosphorus is used in making 10,000,000 matches, how many gr. of phosphorus on each match?

13. If a cistern holds 4890 gal. of water, how many bbl. does it hold?

14. If hay at $15 per T. is exchanged for flour at $5.85 per bbl., how many bbl. will a ton of hay buy?

15. If a druggist put 8℥ 4ʒ 5℈ of a medicinal substance in 2-gr. pills, how many pills did he make?

16. If a man constructed a cistern 12 ft. long and 8 ft. wide to hold 150 bbl., how high did he make it?

17. If 10 bales of goods weigh 22 cwt. 86 lb., what will 155 bales of like size weigh?

18. If a silver dollar weighs $412\frac{1}{2}$ gr., what will 1,000,000 dollars weigh?

19. If a bbl. of flour costs £1 4s. 9d., how many bbl. can be bought for £275 10s. 3d.? (Reduce before dividing.)

20. If a man travels 24 mi. 7 fur. 30 rd. in a day, how long will it take him to travel 300 mi. 6 fur. 20 rd.?

Suggestion: 40 rd = 1 fur.

21. If a cu. ft. of ice weighs 58.1 lb., how many tons will an ice-house hold that is 45 ft. long, 32 ft. wide, and 20 ft. high?

22. Find the cost of 1 qt. of olive oil when 1 doz. pt. cost $3.50.

23. Find the number of gal. in a cistern $5\frac{1}{2}$ ft. square and 7 ft. deep.

24. Find the cost of covering the floor of a hall $46\frac{1}{2}$ ft. long and 14 ft. 9 in. wide with matting $1\frac{1}{4}$ yd. wide at $.25 a yard.

25. If a glacier moves uniformly 100 ft. a year, how far will it go in 181 days?

26. If a man earns $3 per day and pays $6 a week for board, etc., how much can he save in 7 mo.?

27. A square lot, having 32 chains on a side, contains how many acres?

28. How many times will the wheel of a carriage 17.5 ft. in circumference revolve in going 1 mi. 5 ft.?

29. How many board ft. in 3 planks 12 ft. long, 9 in. wide, and $3\frac{1}{2}$ in. thick?

30. What will it cost to carpet a room 18 ft. by 24 ft. with carpet $\frac{3}{4}$ yd. wide at $1.25 per yd., the breadths to run lengthwise?

31. What decimal part of a yr. has passed with August 15th?

ADDITION OF DENOMINATE NUMBERS.

In the addition of simple numbers we have a *uniform decimal scale;* in the addition of compound numbers we have a *varying scale;* apart from this there is no difference in the process of adding.

EXERCISES.

1. What is the sum of 12 lb. 5 oz. 13 pwt., 21 lb. 8 oz. 15 pwt., 13 lb. 7 oz. 10 pwt., 51 lb. 3 oz. 17 pwt.?

Process.

	12	20
lb.	oz.	pwt.
12	5	13
21	8	15
13	7	10
51	3	17
99	1	15

Explanation.

Units of the same denomination must stand in the same column.

The scale is 24, 20, 12. We use 20 and 12.

The sum of the pwt. is 55. 55 pwt. = 2 oz 15 pwt. We write the 15 under the column of pwt. and add the 2 oz. with the column of ounces. The sum of the oz. is 25, which equals 2 lb. and 1 oz. We write the 1 oz. under the column of oz. and add the 2 lb. with the column of lb., making 99 lb.

2. What is the sum of 37 A. 159 P. 25 sq. rd. 8 sq. ft. 126 sq. in., 20 A. 110 P. 30 sq. rd. 8 sq. ft. 131 sq. in., 345 A. 111 P. 16 sq. rd. 7 sq. ft. 99 sq. in?

Process.

	160	$30\frac{1}{4}$	9	144
A.	P.	sq. rd.	sq. ft.	sq. in.
37	159	25	8	126
20	110	30	8	131
345	111	16	7	99
404	62	$12(\frac{1}{2}) =$	7	68
		$\frac{1}{2} =$	$4(\frac{1}{2}) =$	72
404	62	13	2	140

Explanation.

$\frac{1}{2}$ sq. yd. $= 4\frac{1}{2}$ sq. ft.
$\frac{1}{2}$ sq. ft. $= 72$ sq. in.

Adding 4 sq. ft. and 72 sq. in., we have a result free from fractions.

3. Find the sum of the following:

(1.)

rd.	yd.	ft.	in.
140	5	2	7
225	0	3	9
402	4	0	10

Suggestion: Reduce the $\frac{1}{2}$ yd. occurring in the result to feet and inches.

(2.)

mi.	rd.	yd.	ft.	in.
5	251	4	2	9
5	184	4	0	6
8	256	5	1	7
7	159	4	0	8

(3.)

A.	P.	sq. yd.	sq. ft.	sq. in.
112	80	21	5	0
108	75	16	4	0
93	57	12	0	0
115	18	28	0	0

(4.)

lb.	oz.	pwt.	gr.
15	9	17	11
14	8	16	23
15	6	3	18
12	10	0	19
24	3	16	5
13	0	14	0

(5.)

T.	cwt.	lb.	oz.
4	6	38	9
9	12	49	12
14	4	44	11
	9	20	10
21	5	12	8
7	9	65	6

6. 6 mi. 80 rd. 3 yd. 2 ft. 1 in., 4 mi. 75 rd. 1 yd. 2 ft. 7 in., 5 mi. 170 rd. 2 yd. 1 ft. 8 in.

4. Find the value of $\frac{5}{6}$ mi. + $13\frac{1}{3}$ rd.

Process.

$$\begin{array}{lrl} \frac{5}{6}\text{ mi.} & = & 266\text{ rd. }3\text{ yd. }2\text{ ft.} \\ 13\frac{1}{3}\text{ rd.} & = & \underline{13\text{ rd. }1\text{ yd. }2\text{ ft. }6\text{ in.}} \\ \text{Sum} & = & 279\text{ rd. }5\text{ yd. }1\text{ ft. }6\text{ in.} \end{array}$$

Explanation.

5 yd. 1 ft. 6 in. = $16\frac{1}{2}$ ft. = 1 rd. 279 rd. + 1 rd. = 280 rd.

5. Find the value of:
   1. $\frac{3}{8}$ mi. + .46 rd. + $3\frac{5}{8}$ rd.
   2. $.00\frac{7}{8}$ sq. yd. + $.04\frac{3}{4}$ sq. ft. + .0008 sq. in.
   3. £$\frac{5}{8}$ + 3.75s. + .975d.
   4. .2965 T. + .8725 cwt. + .3725 cwt. + .1625 lb.
   5. $\frac{2}{5}$ lb. + $3\frac{2}{9}$ oz. + $5\frac{2}{3}$ pwt.
   6. $\frac{3}{73}$ yr. + $\frac{9}{56}$ wk. + $\frac{7}{12}$ hr.
   7. $\frac{3}{16}$ mi. + $\frac{2}{3}$ rd. + $\frac{3}{8}$ yd.
   8. $\frac{14}{21}^{\circ} + \frac{12}{16}' + \frac{25}{30}''$.
   9. £$\frac{5}{8}$ + $\frac{1}{7}$ of $5\frac{5}{9}$s.
   10. $\frac{3}{8}$ wk. + $\frac{5}{6}$ hr. + $\frac{7}{12}$ min.
   11. $\frac{3}{5}$ A. + $\frac{4}{9}$ sq. rd. + $\frac{2}{3}$ sq. yd.
   12. $27\frac{4}{7}$ cwt. + $26\frac{7}{8}$ lb. + 14 oz. [112 lb. = 1 cwt.]
   13. $1\frac{2}{3}$ hhd. + 36 gal. 3 qt. $1\frac{1}{4}$ pt. + $\frac{7}{8}$ gal. + 2 qt. $\frac{3}{4}$ pt. + 1.75 pt.
   14. $\frac{3}{7}$ of £13 + $\frac{1}{3}$ of $\frac{1}{2\frac{4}{5}}$ of $\frac{3}{5}$ of £2 12s. + $\frac{5}{7}$ of 9d.

6. Add $\frac{1}{7}$ of $\frac{2}{6}$ of a guinea to .4 of .375 of £1, and express the sum as the decimal of a crown (5s.).

7. Express .05735 mi. + 46.25 yd. as the decimal of 7 fur.

8. What is the value of 1.1375 fathoms + .875 yd. + 2.965 ft. + 9.75 in. in feet.

SUBTRACTION OF DENOMINATE NUMBERS.

EXERCISES.

1. From 2 mi. 116 rd. 4 yd. 0 ft. 4 in. take 1 mi. 120 rd. 2 yd. 1 ft. 8 in.

Process.

mi.	rd.	yd.	ft.	in.
2	116	4	0	4
1	120	2	1	8
	316	1	1	8

Explanation.

Units of the same denomination must stand in the same column. Since we cannot subtract 8 in. from 4 in., we add to the 0 ft. one of the 4 yds.; 1 yd. = 3 ft.; now having 3 ft., instead of 0 ft., we add to the 4 in. one of the 3 ft.; 1 ft. = 12 in.; 12 in. + 4 in. = 16 in.; 16 in. — 8 in. = 8 in. Proceeding to the feet, we say, "1 ft. from 2 ft. leaves 1 ft." Proceeding to the yards, we say, "2 yd. from 3 yd. leaves 1 yd." One of the 2 mi. added to 116 rd. gives us 320 + 116 = 436; 436 rd. — 120 rd. = 316 rd.

(2.)

	£	s.	d.
From	37	17	9
take	29	18	10

(3.)

	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
From	18	40	25	6	100
take	9	50	13	7	140

(4.)

	T.	cwt.	lb.	oz.
From	5	13	21	13
take	3	19	2	14

(5.)

	lb.	oz.	pwt.	gr.
From	284	0	0	0
take	100	9	17	21

(6.)

	yr.	wk.	da.	hr.	min.	sec.
From	99	36	5	31	46	49
take	81	46	6	32	47	50

(7.)

	S.	°	′	″
From	12	25	20	$43\frac{2}{3}$
take	10	28	49	$57\frac{5}{8}$

8. From 1 hhd. 38 gal. 3 qt. 2 pt. take 60 gal. 2 qt. 1 gi.
9. From 8 lb. take 1 lb. 1℥ 2ʒ 2℈.
10. From 5 T. take 10 lb. 8 oz.
11. From $\frac{3}{8}$ oz. take $\frac{7}{8}$ pwt.

Process.

$\frac{3}{8}$ oz. $=$ 7 pwt. 12 gr.
$\frac{7}{8}$ pwt. $=$ 21 gr.
6 pwt. 15 gr.

Explanation.

Reduce the fractions to lower denominations and then subtract.

12. From $2\frac{1}{5}$ oz. take $\frac{7}{8}$ pwt.
13. From $\frac{1}{2}$ da. take $\frac{2}{3}$ min.
14. From $\frac{4}{15}$ hhd. take $\frac{5}{7}$ qt.
15. From $\frac{1}{4}$ wk. take .9 da.
16. From $\frac{3}{4}$ pk. take .0625 bu.
17. From .625 Troy lb. take 4.25 Troy oz.
18. From $\frac{2}{11}$ sq. rd. take $\frac{3}{4}$ sq. yd.
19. From 45 sq. yd. take 45 sq. in.
20. From 360° take $\frac{4}{11}$ of a quadrant.
21. Find the lapse of time between July 4, 1890, and August 15, 1898.

Process.

1898	8	15
1890	7	4
8	1	11

Explanation.

July is the 7th month and Aug. the 8th month of the calendar.

22. Between Jan. 9, 1842, and Mar. 4, 1898.

Process.

1898	3	4
1842	1	9
56	1	25

Explanation.

January is the 1st month and March the 3d month of the calendar. 1 month = 30 days in most computations.

23. Between Mar. 2, 1857, and July 4, 1866.
24. Between Jan. 5, 1844, and Mar. 16, 1862.
25. Between May 3, 1804, and Dec. 16, 1871.
26. The Spanish-American war began April 21, 1898, and ended Aug. 12, 1898. Find the difference of the dates.
27. The American civil war began April 11, 1861, and ended April 9, 1865. How long did it continue?

28. The Revolution commenced April 19, 1775, and closed Jan. 20, 1783. How long did the war last?

29. Columbus discovered America Oct. 11, 1492. How long ago did that event occur?

30. A note dated Aug. 10., 1882, was paid Nov. 11, 1887. How long did it run unpaid?

MULTIPLICATION OF DENOMINATE NUMBERS.

EXERCISES.

1. Multiply 5 gal. 3 qt. 1 pt. 3 gi. by 9.

Process.

gal.	qt.	pt.	gi.
5	3	1	3
			9
53	2	1	3

Explanation.

9 times 3 gi. = 27 gi. = 6 pt. 3 gi. We reserve the 6 pt. to add to the next product. 9 times 1 pt. = 9 pt.; 9 pt. + 6 pt. reserved = 15 pt. = 7 qt. and 1 pt. We reserve the 7 qt. 9 times 3 qt. = 27 qt. 27 qt. + 7 qt. reserved = 34 qt. = 8 gal. and 2 qt. We reserve the 8 gal. 9 times 5 gal. = 45 gal.; 45 gal. + 8 gal. reserved = 53 gal.

2. Multiply 18 lb. 9 oz. 4 pwt. 16 gr. by 11.
3. Multiply 2 T. 2 cwt. 46 lb. 7 oz. by 8.
4. Multiply 9 mi. 3 fur. 20 rd. 3 yd. 2 ft. by 6.
5. Multiply 6 yr. 5 mo. 15 da. 18 hr. by 12.
6. Multiply 26 cd. 3 cd. ft. 12 cu. in. by 18.
7. Multiply £9 17s. 6d. 1 far. by 28.
8. Multiply 5 T. 8 cwt. 64 lb. 8 oz. by 37.
9. Multiply 7 mi. 4 fur. 15 rd. 3 yd. 2 ft. 8 in. by 48.
10. Multiply 5 lb. 7 oz. 15 pwt. 19 gr. by 75.
11. Multiply 21 lb. 9℥ 2ʒ 1℈ 16 gr. by 25.
12. Multiply 9 A. 3 R. 22 P. 6 sq. yd. 5 sq. ft. by 10.
13. Multiply $25° 37' 51''$ by 16.
14. Multiply £10 18s. 7d. 2 far. by 29.
15. Multiply 8 mi. 120 rd. 4 yd. by 26.

DIVISION OF DENOMINATE NUMBERS.

EXERCISES.

1. Divide 76 lb. 10 oz. 14 pwt. 12 gr. by 6.

Process.

	lb.	oz.	pwt.	gr.
6)	76	10	14	12
	12	9	15	18

Explanation.

To divide a quantity by 6 is to take $\frac{1}{6}$ of it. $\frac{1}{6}$ of 76 lb. = 12 lb. and 4 lb. remaining; 4 lb. = 48 oz.; 48 oz. + 10 oz. = 58 oz. $\frac{1}{6}$ of 58 oz. = 9 oz. and 4 oz. remaining; 4 oz. = 80 pwt.; 80 pwt. + 14 pwt. = 94 pwt.; $\frac{1}{6}$ of 94 pwt. = 15 pwt. and 4 pwt. remaining; 4 pwt. = 96 gr.; 96 gr. + 12 gr. = 108 gr.; $\frac{1}{6}$ of 108 gr. = 18. Hence the quotient is 12 lb. 9 oz. 15 pwt. 18 gr.

2. Divide:

1. 112 T. 16 cwt. 66 lb. by 7.
2. 17 bu. 3 pk. 4 qt. by 8.
3. 29 lb. 5℥ 3ʒ 1℈ by 9.
4. 125 S. 24° 12′ by 10.
5. 427 A. 131 sq. rd. by 11.
6. 342 gal. 2 qt. 1 pt. 2 gi. by 5.
7. 16 T. 1300 lb. by 12.
8. 120 mi. 313 rd. 3 yd. 2 ft. by 12.
9. £31 5s. 8d. by 4.
10. 196 cd. 4 cd. ft. 12 cu. ft. by 36.
11. £275 10s. 6d. by £1 4s. 9d.

Process.

£275 10s. 6d. = 66,126d.
£1 4s. 9d. = 297d.
66,126d. ÷ 297d. = $222\frac{64}{99}$.

What rule is derivable from the process?

12. 48 T. 9 cwt. 23 lb. 8 oz. by 6 T. 1 cwt. 15 lb. 7 oz.
13. 200 mi. 6 fur. 18 rd. by 24 mi. 7 fur. 22 rd.
14. 31 cwt. 18 lb. by 3 lb. 8 oz.

15. 13 lb. 7 oz. 15 pwt. by 2 oz. 10 pwt.
16. $5\frac{5}{6}$ mi. by 7 ft. 4 in.
17. 118 bu. 2 pk. by 7 bu. 1 pk. 5 qt.
18. 35 wk. 3 da. 15 hr. 25 min. by 17 wk. 6 da. 22 hr. 39 min.
19. 61 ft. 3 in. by 8 ft. 7 in.
20. A quadrant by 27° 14′ 45″.

LONGITUDE AND TIME.

INDUCTIVE STEPS.

1. Does the earth revolve on its axis from west to east or from east to west?

2. It revolves *once* in how many hours?

3. Does the sun actually revolve, or only *appear* to revolve around the earth?

4. If the earth revolves from west to east, do Eastern or Western people behold the sun first?

5. Has a place 30° east of Philadelphia later or earlier time?

6. When it is noon in New York, is it afternoon or forenoon in Chicago?

7. Through how many degrees does the sun appear to travel in 24 hrs.?

8. Then how many degrees of longitude and how many hours are compassed in a day?

Time.		*Longitude.*
24 hrs.	=	360°.

Dividing the equation by 24, we have:

1 hr.	=	15°.

Dividing by 60, we have:

1 min.	=	15′.

Dividing again by 60, we have:

1 sec.	=	15″.

RULE.

To reduce time to longitude, multiply by 15; to reduce longitude to time, divide by 15.

PROBLEMS.

1. The difference of time between two places is 1 hr. 15 min. 30 sec. Find the difference of longitude.

Process.

hr.	min.	sec.
1	15	30
		15
18°	52′	30″

Explanation.

Since we are required to reduce time to longitude, we multiply the given hours, minutes, and seconds by 15, and obtain 18° 52′ 30″.

2. The difference of longitude between New York and Baltimore is 2° 36′. Find the difference of time.

Process.

$$15\,\underline{)\,2^\circ\ 36'}$$
10 min. 24 sec.

Explanation.

Since we are required to reduce longitude to time, we divide the given number of degrees and minutes by 15, and obtain 10 min. 24 sec. as the difference of time.

The **Meridian** of a place is an imaginary line running from North Pole to South Pole through that place.

A meridian divides longitude into *east longitude* and *west longitude*, making 180° of each.

The meridian of Greenwich, near London, or of Washington, D. C., is commonly reckoned from.

3. The longitude of Washington (from Greenwich) is 77° 2′ 48″ W., and of San Francisco 122° 24′ 15″ W. Find the difference of longitude and the difference of time.

4. If the difference of time between San Francisco and Philadelphia is 3 hr. 9 min. 7 sec., what is the longitude of Philadelphia?

5. The difference in time between Berlin and New York is 5 hr. 49 min. 35 sec. Find the difference in longitude.

6. If Berlin is 13° 23′ 43″ E., find how much of the preceding difference is west longitude.

7. In travelling west my watch seemed to gain 20 min. How many degrees did I travel?

8. Constantinople is 28° 59′ E. When it is noon in Greenwich, what is the time in Constantinople?

9. When one place is in west longitude and the other in east longitude, do you add or subtract to find the difference of longitude?

10. New York is 74° 3′ west; Paris, France, is 2° 20′ east. Find the difference of time.

11. When it is noon at Boston (71° 3′ 30″ west), what is the time at Paris (2° 20′ 22″ east)?

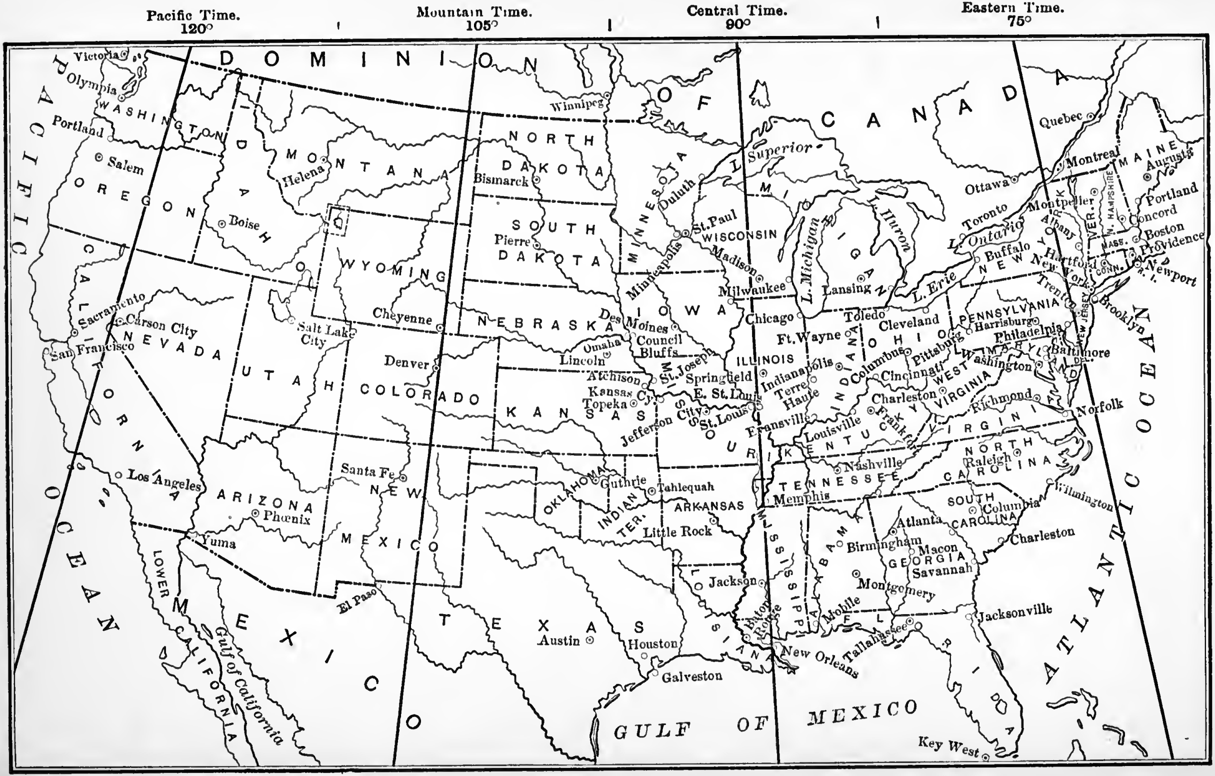
12. Canton in China is 113° 14′ 30″ east longitude and Washington is 77° west longitude. When it is midnight on July 4th, at Washington, what time will it be at Canton?

Standard Time.

Nov. 18, 1883, the United States was divided into four time-belts, each 15° wide, named respectively, *Eastern*, *Central*, *Mountain*, and *Pacific*. The local time of the middle meridian of each time-belt was adopted as the standard time of the whole belt.

1. Eastern time is that of the 75th meridian.
2. Central time is that of the 90th meridian.
3. Mountain time is that of the 105th meridian.
4. Pacific time is that of the 120th meridian.

All places lying within 7° 30′ of the middle meridian have the time of that meridian.



Pacific Time.
120°
Mountain Time.
105°
Central Time.
90°
Eastern Time.
75°
DOMINION OF CANADA
PACIFIC OCEAN
ATLANTIC OCEAN
GULF OF MEXICO
MEXICO
LOWER CALIFORNIA
Gulf of California
Victoria
Olympia
WASHINGTON
Portland
Salem
OREGON
IDAHO
Boise
MONTANA
Helena
NORTH DAKOTA
Bismarck
SOUTH DAKOTA
Pierre
WYOMING
Cheyenne
CALIFORNIA
Sacramento
San Francisco
Los Angeles
NEVADA
Carson City
UTAH
Salt Lake City
COLORADO
Denver
ARIZONA
Phœnix
Yuma
NEW MEXICO
Santa Fe
El Paso
TEXAS
Austin
Houston
Galveston
NEBRASKA
Lincoln
Omaha
KANSAS
Topeka
Atchison
Kansas Cy.
OKLAHOMA
Guthrie
INDIAN TER.
Tahlequah
ARKANSAS
Little Rock
LOUISIANA
Baton Rouge
New Orleans
MISSISSIPPI
Jackson
MINNESOTA
Minneapolis
St.Paul
Duluth
Winnipeg
IOWA
Des Moines
Council Bluffs
MISSOURI
St.Joseph
Jefferson City
St.Louis
WISCONSIN
Madison
Milwaukee
L. Superior
L. Michigan
L. Huron
L. Erie
L. Ontario
MICHIGAN
Lansing
ILLINOIS
Chicago
Springfield
E. St.Louis
INDIANA
Indianapolis
Ft.Wayne
Terre Haute
Evansville
OHIO
Toledo
Cleveland
Columbus
Cincinnati
KENTUCKY
Louisville
Frankfort
TENNESSEE
Nashville
Memphis
ALABAMA
Birmingham
Montgomery
Mobile
GEORGIA
Atlanta
Macon
Savannah
FLORIDA
Tallahassee
Jacksonville
Key West
SOUTH CAROLINA
Columbia
Charleston
NORTH CAROLINA
Raleigh
Wilmington
VIRGINIA
Richmond
Norfolk
WEST VIRGINIA
Charleston
Pittsburg
PENNSYLVANIA
Harrisburg
Philadelphia
MARYLAND
Baltimore
Washington
DEL.
NEW JERSEY
Trenton
NEW YORK
Buffalo
Albany
New York
Brooklyn
Toronto
Ottawa
Montreal
Quebec
Montpelier
VER.
N. HAMPSHIRE
Concord
MAINE
Augusta
Portland
MASS.
Boston
CONN.
Hartford
R. I.
Providence
Newport

PROBLEMS.

1. St. Paul is in longitude 93° 5′ W. Find the difference between the local and the standard time of St. Paul.

Process.

$$
\begin{array}{r}
93^\circ\ 5' \\
\underline{90^\circ\quad} \\
15\,\underline{)\ 3^\circ\ 5'} \\
12 \text{ min. } 20 \text{ sec.}
\end{array}
$$

Explanation.

St. Paul is within 7° 30′ of 90°, and is within the central time-belt. Its distance from the 90th meridian is 3° 5′, which, divided by 15, gives 12 min. 20 sec., the difference of time required.

2. Boston is in longitude 71° 3′ 30″. Find the difference between the local and the standard time of Boston.

3. Pittsburg is within 7° 30′ from Philadelphia, which lies close to the middle meridian of the eastern belt. When it is noon at Philadelphia, what is the standard time at Pittsburg?

4. Galveston is in longitude 94° 50′ W. When it is noon there by local time, what hour is it by standard time?

5. St. Louis is in longitude 90° 15′ 15″ W. When it is noon there by standard time, what is the local time?

MISCELLANEOUS PROBLEMS.

1. If one doz. pints of oil cost $4.00, what is the cost of one qt.?

2. A gentleman in travelling found at a certain railroad station that his watch was 1 hr. and 25 min. slow. What direction was he travelling? How far had he travelled?

3. A note dated June 12, 1896, was paid Jan. 5, 1897. How long did the note run?

4. How many steps $\frac{2}{3}$ yd. long will a man take in walking 1 mi. and 580 yd.?

5. If 10 grain bins contain 254 bu. 3 pk. 7 qt. 1 pt., what does 1 bin contain?

6. Since noon the sun has seemed to pass through 10° 43′ 35″. What is the time of day?

7. If a cu. ft. of water weighs 1000 oz., how many lb. avoirdupois does a cu. yd. of water weigh?

8. When it is 1 hr. 37 min. 12 sec. P.M. at Bangor (68° 47′ W.), what is the time at St. Paul (93° 5′ W.)?

9. A crib measuring 16 ft. × 6 ft. 9 in. × 7 ft. is full of corn in the ear. How many bu. of shelled corn will there be?

10. In 556,688 ft. how many miles?

11. How many gal. of air in a room 16 ft. long, 11 ft. wide, and 10 ft. high?

12. How many bu. of shelled corn will fill a vat that holds 6000 gal. of water?

13. A block of marble 4 ft. long and $2\frac{1}{2}$ ft. wide contains $12\frac{1}{2}$ cu. ft. How thick is the block?

14. How many bu. in 6 tons of oats?

15. How much is gained on 65 doz. eggs bought at \$.15 a doz. and sold at the rate of $1\frac{1}{3}$ doz. for \$.25?

16. What is the cost of 4 tons and 468 pounds of hay at \$12 a ton?

17. A firkin of butter weighed 61 lb. 12 oz. How much did the vessel itself weigh?

18. If a man can do a piece of work in 22 hr. 30 min. 25 sec., what part of it can he do in 13 hr. 11 min. 15 sec.?

19. Divide 3 gal. 2 qt. 2.03 pt. by 18, and reduce the result to the decimal of a barrel.

20. What decimal of a lb. avoirdupois is a lb. troy?

21. How many bu. of potatoes in 2240 lb?

22. The longitude of New York is 74° 0′ 3″ W.; of London, 0° 5′ 48″ W. Find the difference of time between the two cities. Which has the earlier time?

23. If a bicycle wheel 7 ft. 4 in. in circumference makes 3 revolutions in a second, at what rate per hour is the rider going?

24. How many francs equal \$1.00?

25. Reduce £3 8s. 4d. to dollars, U. S. currency.

26. The annual cost of Spanish royalty is 9,500,000 pesetas. Reduce to U. S. money. (Peseta = $.193.)

27. Latitude is distance north or south from the Equator. If a man travels due north from the Equator 2500 mi., what latitude does he reach? ($1° = 69\frac{1}{6}$ mile.)

28. 3780 gal. of water will fill how many barrels?

29. If hyoscine hydrobromate is worth $12.50 a grain, what will be the cost of 12 tablets of the drug, each containing .01 of a grain?

30. A owns $\frac{5}{11}$ of a farm, and B owns the remainder. $\frac{2}{3}$ of the difference between their shares is 16 A. 80 sq. rd. Find the share of each in acres.

REVIEW.

1. Define:

1. Denominate Number.
2. Compound Denominate Number.
3. Money.
4. U. S. Money.
5. Sterling Money.
6. Reduction.
7. Reduction Descending.
8. Reduction Ascending.
9. Extension.
10. Linear Measures.
11. Surface Measures.
12. Measures of Volume.
13. Measures of Capacity.
14. Angle.
15. Rectangle.
16. Square.
17. Area.
18. Solid.
19. Rectangular Solid.
20. Cube.
21. Volume.
22. Solid Contents.
23. Board Measure.
24. Weight.
25. Troy Weight.
26. Apothecaries' Weight.
27. Circular Measure.
28. Circle.
29. Circumference.
30. Arc.
31. Quadrant.
32. Degree.
33. Fractional Relation.
34. Uniform Scale.
35. Varying Scale.
36. Longitude.
37. Standard Time.

2. Repeat the table of:
   1. U. S. Money.
   2. English Money.
   3. French Money.
   4. Linear Measure.
   5. Surveyors' Linear Measure.
   6. Surveyors' Square Measure.
   7. Liquid Measure.
   8. Apothecaries' Liquid Measure.
   9. Dry Measure.
   10. Avoirdupois Weight.
   11. Troy Weight.
   12. Apothecaries' Weight.
   13. Time.
   14. Months (Stanza).
   15. Circular Measure.
   16. Counting.
   17. Paper.
   18. Books.

3. Name the:
   1. U. S. Coins.
   2. English Coins.

4. Repeat the rule for:
   1. Reduction Descending.
   2. Reduction Ascending.
   3. Area of Rectangle.
   4. Volume.
   5. Time to Longitude.
   6. Longitude to Time.
   7. Board Measure.
   8. Fractional Relation.

5. What is the unit of:
   1. U. S. Money?
   2. Canadian Money?
   3. French Money?
   4. English Money?

6. What is the unit for:
   1. Land?
   2. Plastering?
   3. Paving?
   4. Roofing, etc.?
   5. Bricklaying, etc.?
   6. Excavations, etc.?
   7. Brickwork?
   8. Grain?

PART II.

PERCENTAGE.

INDUCTIVE STEPS.

1. A man earned $5 and spent $1.00. What fractional part of the $5 did he spend? What part of $10 would he have spent? What part of $50? What part of 100?

2. $20 out of $100 means 20 *per hundred*, or 20 *per cent.*

3. What is the meaning of 10 per cent? Of 25 per cent.? Of 50 per cent.? Of 75 per cent.? Of 100 per cent.?

4. Having taken 100 per cent. of a sum of money, how much is left?

5. How much is 1 per cent. of $100? Of $200? Of $1000?

6. What is 5 per cent. of $200? Of 100 acres? Of 500 men?

7. What is 6 per cent. of $600? Of 900 sheep? Of 1200 yards?

DEFINITIONS.

1. **Percentage** means computation *by the hundred,* and has 100 for its unit. One per cent. of any number is $\frac{1}{100}$ of it; 5 per cent. is $\frac{5}{100}$ of it.

Per cent. is a contraction of the Latin *per centum*, by the hundred.

2. The result of computation is also called Percentage.

$\frac{5}{100}$ of $1000 = $50, the percentage.

3. The **Symbol** for per cent. is %. Per cent., however, may be expressed in *five* different ways: 6 per cent. = 6% = .06 = $\frac{6}{100}$ = $\frac{3}{50}$. The best way in any given case is the one that affords the shortest solution.

4. The **Rate** per cent. is the *number* of hundredths; $\frac{5}{100}$ indicates that the rate is 5 per cent.

5. The number on which the *percentage is computed* is the **Base**. Attention, therefore, must be given to *Base, Rate,* and *Percentage.*

6. **Amount** is the Base plus the Percentage.

7. **Difference** is the Base minus the Percentage.

EXERCISES.

1. Use 10, 100, $16\frac{2}{3}$, 125, $\frac{1}{2}$, .00625, in five different ways to express rate per cent.

Process.

$$10 \text{ per cent.} = 10\% = .10 = \frac{10}{100} = \frac{1}{10}.$$
$$100 \text{ per cent.} = 100\% = 1.00 = \frac{100}{100} = 1.$$
$$16\tfrac{2}{3} \text{ per cent.} = 16\tfrac{2}{3}\% = .16\tfrac{2}{3} = \frac{16\frac{2}{3}}{100} = \left(\frac{50}{300}\right) = \frac{1}{6}.$$
$$125 \text{ per cent.} = 125\% = 1.25 = \frac{125}{100} = \frac{5}{4}.$$
$$\tfrac{1}{2} \text{ per cent.} = \tfrac{1}{2}\% = .005 = \frac{\frac{1}{2}}{100} = \frac{1}{200}.$$
$$.00625 \text{ per cent.} = .00625\% = \frac{.00625}{10000000} = \frac{1}{16000}.$$

2. In like manner express as rate per cent. the following numbers: 15, 20, 25, 40, 50, 55, 65, 75, 96, 45, 85, $12\frac{1}{2}$, $33\frac{1}{3}$, $8\frac{1}{3}$, $6\frac{1}{4}$, $66\frac{2}{3}$, $87\frac{1}{2}$, $37\frac{1}{2}$, $11\frac{1}{9}$, $18\frac{3}{4}$, $62\frac{1}{2}$, 250, 375, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{7}{25}$, $\frac{11}{50}$, $\frac{3}{10}$, $\frac{1}{6}$, .3, .06, $.012\frac{1}{2}$, .001.

3. Change $\frac{1}{10}$, 1, $\frac{1}{6}$, $\frac{5}{4}$, $\frac{1}{200}$, $\frac{1}{16000}$ into the symbol form.

Process.

$$\frac{1}{10} = \frac{10}{100} = 10\%.$$
$$1 = \frac{100}{100} = 100\%.$$
$$\frac{1}{6} = \frac{100}{600} = \frac{16\frac{2}{3}}{100} = 16\tfrac{2}{3}\%.$$
$$\frac{5}{4} = \frac{500}{400} = \frac{125}{100} = 125\%.$$
$$\frac{1}{16000} = \frac{1 \div 160}{100} = \frac{.00625}{100} = .00625\%.$$

4. Change the following fractions into the symbol form: $\frac{1}{16}$, $\frac{1}{12}$, $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{3}{10}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{7}{10}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$.

5. Give the following symbol forms their simplest fractional form: 90%, $87\frac{1}{2}$%, 80%, 75%, 70%, $66\frac{2}{3}$%, $62\frac{1}{2}$%, 60%, 50%, 40%, $37\frac{1}{2}$%, $33\frac{1}{3}$%, 30%, 25%, 20%, $16\frac{2}{3}$%, $12\frac{1}{2}$%, 10%, $8\frac{1}{3}$%, $6\frac{1}{4}$%.

To Find the Percentage.

EXERCISES.

1. What is 25% of \$24.00?

Process.

$25\% = \frac{1}{4}$. $\frac{1}{4}$ of \$24.00 = \$6.00.

Or, $\$24.00 \times .25 = \$6.00$.

Explanation.

The base is \$24.00; the rate is 25%; the percentage is required. Since 25% = $\frac{1}{4}$, 25% of \$24.00 = $\frac{1}{4}$ of \$24.00 = \$6.00, the percentage.

Hence the formula:

Percentage = Base × Rate.

NOTE.—As an important preparation, let the pupil write in a table the following rates, both as common and decimal fractions, and make solution with both forms.

2. What is:

1. 70% of 30 sheep?
2. $33\frac{1}{3}$% of 9 books?
3. $28\frac{1}{7}$% of 35 ft?
4. $12\frac{1}{2}$% of 48 A.?
5. 50% of 600 yd.?
6. $62\frac{1}{2}$% of 64 da.?
7. 4% of 200 gal.?
8. 80% of 60 horses?
9. 25% of 124 yd.?
10. 5% of 700 men?
11. 4% of 1000 horses?
12. 40% of 800 lb.?
13. $37\frac{1}{2}$% of 160 oxen?
14. $33\frac{1}{3}$% of \$900?
15. 15% of \$500?
16. 20% of 400 bu.?
17. 70% of 500 yr.?
18. 90% of 9000 sec.?
19. $87\frac{1}{2}$% of 1600 books?
20. 80% of 1000 horses?
21. 75% of 1000 oz.?
22. $66\frac{2}{3}$% of 1500 gal.?
23. $62\frac{1}{2}$% of \$2400?
24. 30% of 10,000 fr.?
25. $16\frac{2}{3}$% of $\frac{7}{8\frac{1}{6}}$ of a bbl.?
26. $12\frac{1}{2}$% of $\frac{4}{7}$ of a yr.?
27. 10% of a million?
28. $8\frac{1}{3}$% of 1728 cu. in.?
29. $6\frac{1}{4}$% of 144 sq. in.?
30. $\frac{3}{4}$% of \$2856.00?

PROBLEMS.

1. Of 600 trees, $33\frac{1}{3}\%$ are peach trees. Find the number of peach trees?

2. A bicycle marked \$90 was sold at a reduction of $12\frac{1}{2}\%$. Find the reduction and the selling price.

3. If a man owes \$3564 and pays 30% of it, how many dollars does he pay?

4. Mr. A. deposited in bank \$963, and afterwards drew out 5% of it. How many dollars did he draw out?

5. A farmer who had a flock of 540 sheep, sold $33\frac{1}{3}\%$ of them. How many did he sell, and how many had he left?

6. A merchant bought goods for \$630, and sold them at a gain of 23%. How much did he gain?

7. How much is made by selling at 20% profit a house which cost \$10,500?

8. How much is lost by selling at 8% below cost 163 tons of coal which cost \$6.00 per ton.

9. I sold a horse which cost me \$250 at a loss of 35%. What did I get for him?

Is the difference asked for?

10. A merchant paid \$.80 a yard for silk. For how much must he sell it to gain $12\frac{1}{2}\%$?

Is the amount to be sought?

11. I bought a bill of goods amounting to \$986.60, from which was deducted 5%. Find the percentage allowed and the amount paid.

12. A certain mine yields 60% of metal, and of the metal $\frac{3}{4}\%$ is silver. Find how much silver and how much other metal are obtained from 1300 tons of ore.

13. In a school of 80 children $17\frac{1}{2}\%$ are girls. Find the number of boys.

14. Assuming that gunpowder contains 75% of saltpetre, 10% of sulphur, 15% of charcoal, find how many pounds of each there are in a ton of powder.

15. Express as a rate per cent. $.33333\frac{1}{3}$ and apply it to 99,999 as a base. Change $.33\frac{1}{3}\%$ to a common fraction in its lowest terms, and apply it to 99,900 as a base.

To Find the Rate.

Since Percentage is the product of Base and Rate, obviously

$$\text{Rate} = \frac{\text{Percentage}}{\text{Base}}.$$

EXERCISES.

1. What rate per cent. of \$276 is \$82.80?

Process.

$$\text{R.} = \frac{\text{P.}}{\text{B.}} = \frac{82.80}{276} = .30 = 30\%.$$

By Analysis.

$$276 = 100\% \text{ of B.}$$
$$1 = \frac{100}{276}\% \text{ of B.}$$
$$82.80 = \frac{8280}{276} = 30\% \text{ of B.}$$

Explanation.

The base is \$276; the percentage is \$82.80; the rate is required. Since the rate is required, we use the formula $\text{R.} = \frac{\text{P.}}{\text{B.}}$, and obtain R. = 30%.

2. What per cent. of:

1. \$450 is \$90?
2. \$12 is 15 cents?
3. 15 lb. is 5 lb. 10 oz.?
4. 250 head of cattle is 4 head?
5. $\frac{3}{5}$ of 80 is $\frac{1}{2}$ of 120?
6. 380 pages is 120 pages?
7. \$465 is \$130.20?
8. \$832 is \$807.04?
9. \$1041.66$\frac{2}{3}$ is \$62.50?
10. 93 yd. is 6.51 yd.?
11. 812 T. is 203 T.?
12. \$5600 is \$1600?
13. 64% is $5\frac{1}{3}\%$?
14. 4.5% is $3\frac{3}{8}\%$?
15. $\frac{8}{9}$ is $\frac{2}{5}$?
16. 1 is .35?
17. 8 is .375?
18. 1 T. is 75 lb.?
19. 6 A. is 5 sq. rd.?
20. £11 is £1 2s.?

PROBLEMS.

1. A farmer raised 5390 bu. of grain and sold 1078 bu. What per cent. of it did he sell?

2. A merchant having 375 yd. of cloth, sold 150 yd. What per cent. did he sell?

3. As agent, I sold a house for $5000, and received as remuneration $50. What rate per cent. did I receive?

4. A lady having invested funds to the amount of $4750, on withdrawing the money received $4987.50. What per cent. did she gain?

5. If 8 lb. of an article loses 4 oz. in weight by drying, find what per cent. of water escaped.

6. A baseball team won 15 games and lost 9 games. What per cent. of its games did it win?

7. $1\frac{1}{2}$ times a number is what per cent of it?

8. If $56.70 is paid for the use of $1260, what is the rate per cent.?

9. A boy misspells 55 words out of 660. What per cent. does he misspell?

10. .875 is what per cent. of .125?

11. In one month clover seed advances from $6.50 to $7.00 per 100 lb. What was the rate per cent. of increase?

12. In one year mixed hay advanced from $9 to $11.75 per ton. Find the rate per cent. of increase.

13. When in one year the production of wool in the United States increased from 298 million lb. to 309 million lb., what was the rate per cent. of increase?

14. When tallow fell from 5 cents to $4\frac{1}{8}$ cents per lb., what was the rate per cent. of the fall?

15. If carpet which should be 1 yd. wide is only $34\frac{1}{2}$ in. wide., what per cent. should be deducted from the price?

16. What per cent. of 1 rd. 3 yd. 2 ft. 5 in. is 7 ft.?

17. What per cent. of MMMDLXX. is 25 per cent. of MMDCCCLVI?

18. Gas is reduced from $1.50 to $1.00 per M. What per cent. of the original cost is saved?

19. If $\frac{1}{3}$ of a ton of coal is sold for what 1000 lb. cost, what is the gain per cent.?

20. The cost was $3486, the selling price was $4161. Find the gain and the gain per cent. Also point out the base, the amount, the percentage, and the difference.

To Find the Base.

Since Percentage is the product of Base and Rate, obviously

$$\text{Base} = \frac{\text{Percentage}}{\text{Rate}}.$$

EXERCISES.

1. $82 is $12\frac{1}{2}$ per cent. of what base?

Process.

$12\frac{1}{2}\% = .12\frac{1}{2} = .125$

$\text{B.} = \frac{\text{P.}}{\text{R.}} = \frac{82}{.125} = \$656.$

By Analysis.

$12\frac{1}{2}\%$ of B. $= \$82.$

1% of B. $= \frac{82}{12\frac{1}{2}} = \frac{164}{25}.$

100% of B. $= \frac{16400}{25} = \$656.$

Explanation.

The percentage is $82 00; the rate is $12\frac{1}{2}\%$; the base is required.

Since the base is required, we use the formula, $\text{B.} = \frac{\text{P.}}{\text{R.}}$, and obtain B. = $656.

2. Find of what number:

1. 385 is $12\frac{1}{2}\%$.
2. 396 is 11%.
3. 250 is 15%.
4. 8.25 is $33\frac{1}{3}\%$.
5. $64.36 is 10%.
6. 38.6 bu. is 13%.
7. 168 men is 8%.
8. 462 oxen is 7%.
9. 12 is $16\frac{2}{3}\%$.
10. 70 is $66\frac{2}{3}\%$.
11. 300 is $33\frac{1}{3}\%$.
12. 100 is $62\frac{1}{2}\%$.

13. 72 is $44\frac{4}{9}\%$.
14. 48 is $37\frac{1}{2}\%$.
15. 84 is $87\frac{1}{2}\%$.
16. 126 is 90%.
17. 24 is $8\frac{1}{3}\%$.
18. 10 is $6\frac{1}{4}\%$.
19. $1\frac{3}{4}$ is 14%.
20. 5 cwt. is 40%.
21. 75 yd. is $18\frac{3}{4}\%$.
22. \$.50 is $31\frac{1}{4}\%$.
23. 160 is $106\frac{2}{3}\%$.
24. 75 lb. is $6\frac{1}{4}\%$.
25. 837 gal. is 6%.
26. 9006 is .06%.

3. \$281.25 is $37\frac{1}{2}\%$ of what number?

4. If 28% of a number = \$71.68, what is the number?

5. If 25% of a number = \$324, what is 40% of that number?

PROBLEMS.

1. A man sold a horse at a gain of \$15, which was 15% of the cost. Find the cost and selling price.

2. A farmer sold 384 barrels of apples, which was 96% of all he had. How many had he?

3. A farm was sold for \$536 less than cost, which was at a loss of 20%. What was the cost of the farm?

4. The immigrants of a population number 143,000 persons, or 11% of the whole. Find the total population.

5. A farmer sold 110A. 43 sq. rd. of land, which was 20% of his land. How much land had he at first?

6. A man pays \$500 rent a year; 85% of this sum is $33\frac{1}{3}$ per cent. of $\frac{3}{4}$ of his income. Find his income.

7. On the sale of a patent \$1600 was lost. What was the value, if the rate of loss was 16%?

8. A sale resulted in a loss of \$38.46, which was $\frac{1}{2}\%$ of the cost. Find the cost.

9. By selling an article for $66\frac{2}{7}\%$ of its cost, \$23.25 was realized. What was the cost?

10. At a gain of $\frac{4}{7}\%$, a profit of \$36 was realized. What was the cost?

11. 30% of B.'s money is in a bank, and 50% in a farm; the remainder, \$2000, is in P. R. R. stock. How much money does B. own?

To Find the Base when the Rate and the Amount or Difference are Given.

EXERCISES.

1. What number increased by $6\frac{1}{4}\%$ of itself equals 510?

Process.

$6\frac{1}{4}\% = \frac{1}{16}$. $\frac{16}{16} + \frac{1}{16} = \frac{17}{16} = 510$. $\frac{1}{16} = 30$, $\frac{16}{16} = 480$.

Or, $1 + .06\frac{1}{4} = 1.06\frac{1}{4} = 510$. $510 \div 1.0625 = 480$.

Explanation.

$\frac{16}{16}$, the number, $+ \frac{1}{16}$ of the number equals $\frac{17}{16}$ of the number $= 510$; and since $\frac{17}{16}$ of the number $= 510$, $\frac{1}{16}$ of the number $= \frac{1}{17}$ of $510 = 30$; and since $\frac{1}{16} = 30$, $\frac{16}{16} = 16$ times 30 or 480.

Or, since once the number and $.06\frac{1}{4}$ times the number $= 510$, that number must equal $510 \div 1.06\frac{1}{4}$, or 480.

Hence we have the formula:

$$\text{Base} = \frac{\text{Amount}}{1 + \text{Rate}}; \text{ or, } B. = \frac{A.}{1 + R.}.$$

2. What number diminished by $\frac{3}{4}\%$ of itself equals 794?

Process.

$\frac{3}{4}\% = \frac{3}{400}$.

$\frac{400}{400} - \frac{3}{400} = \frac{397}{400} = 794$.

$\frac{1}{400} = 2$.

$\frac{400}{400} = 800$. Or,

$1 - .0075 = .9925$

$794 \div .9925 = 800$.

Explanation.

$\frac{400}{400}$, the number, $- \frac{3}{400}$ of the num. $= \frac{397}{400}$ of the num. $= 794$; and since $\frac{397}{400}$ of the num. $= 794$, $\frac{1}{400} = \frac{1}{397}$ of $794 = 2$; and since $\frac{1}{400} = 2$, $\frac{400}{400} = 400$ times 2 or 800. Or, since once the num. $- .0075$ times the num. $= .9925$ times the num., and since $.9925$ times the num. $= 794$, that num. must equal $794 \div .9925$, or 800.

Hence we have the formula:

$$\text{Base} = \frac{\text{Difference}}{1 - \text{Rate}}; \text{ or, } B. = \frac{D.}{1 - R.}$$

3. What number increased by:

1. 15% of itself = 4830?
2. 27% of itself = 508?
3. $33\frac{1}{3}$% of itself = 984?
4. $16\frac{2}{3}$% of itself = 658?
5. $62\frac{1}{2}$% of itself = 1820?
6. 10% of itself = 15,400?
7. 15% of itself = 690?
8. $8\frac{3}{4}$% of itself = 4140.15?
9. 2% of itself = 516?
10. $16\frac{2}{3}$% of itself = $\frac{11}{12}$?
11. $16\frac{2}{3}$% of itself = 1400?
12. $6\frac{1}{4}$% of itself = 901?
13. 16% of itself = 261?
14. 3% of itself = $2\frac{5}{8}$?
15. 140% of itself = 630?
16. $\frac{3}{4}$% of itself = 1897?

4. What number diminished by:

1. 25% of itself = 1200?
2. 55% of itself = 1240?
3. $33\frac{1}{3}$% of itself = 1260?
4. 11% of itself = 4539?
5. 3% of itself = 2667.50?
6. 12% of itself = \$616?
7. $14\frac{2}{7}$% of itself = 96?
8. 36% of itself = 336?
9. 28% of itself = $\frac{1}{2}$?
10. $6\frac{1}{4}$% of itself = 675?

PROBLEMS.

1. A man owes \$14,300, which is 8% more than his property is worth. Find the value of his property?

2. In 5 years the population of a town has increased 25%. The population is now 7675. What was it 5 yr. ago?

3. A regiment lost $6\frac{1}{4}$% of its men, and then had left 750. Find the original number.

4. Having sold 30% of my land, I had then 34 acres remaining. How many acres had I at first?

5. Two farms contain 630 acres, and one farm is 25% larger than the other. Find the size of each farm?

6. What was the cost of hats sold at \$1.00 apiece, which was at a loss of $16\frac{2}{3}$%?

7. A horse was sold for \$112, and the loss was $33\frac{1}{3}$%. What did he cost?

8. Silk sold at \$5.50 a yd. brings in a profit of 10%. What did it cost?

9. A wheat speculator lost at one time 10% of his money, and at another time $12\frac{1}{2}\%$ of the remainder, and had then \$50,400 left. How much had he previous to those losses?

10. A farmer sold two cows for \$50 apiece. On the one he gained 25%; on the other he lost 25%. Did he gain or lose by the sales, and how much?

11. An army lost 25% of its men in battle, and 25 per cent. of the remainder were discharged. 2223 men still remained. Find the original number of men in the army.

12. A farmer having sold 110 A. 43 sq. rd. of land had 80% left. How much had he at first?

13. A patent was sold for \$8000. The seller lost 84% of the original value. Find the original value?

14. A flock of sheep was increased by 250% of itself, and then numbered 1400. Find the original number.

15. How many lb. of tallow must be mixed with $9\frac{1}{2}$ lb. of rosin that the mixture may contain 24% of tallow?

16. By selling goods at 60 cents a pound, 8% is lost. What advance must be made in the price to gain 15%?

REVIEW EXERCISES.

1. $P. = B. \times R.$ 2. $R. = \frac{P.}{B.}$ 3. $B. = \frac{P.}{R.}$

4. $B. = \frac{A.}{1 + R.}$ $\therefore A. = B. \times (1 + R.).$

5. $B. = \frac{D.}{1 - R.}$ $\therefore D. = B. \times (1 - R.).$

1. Show how formulæ 2 and 3 are derived from formula 1.
2. Show how formulæ 4 and 5 are derived.
3. Invent five problems to which above formulæ may respectively apply.

Find the value of (?):

	Rate %.	Base.	Percentage.	Amount.	Difference.
1	$16\frac{2}{3}$	\$10.35	?	?	?
2	15	2500 cd.	?	. . . . . .	. . .
3	$12\frac{1}{2}$	?	246 tons.	?	. . .
4	$83\frac{1}{3}$	?	7550 T.	. . . . . .	?
5	?	365 da.	$29\frac{1}{5}$ da.	. . . . . .	. . .
6	$\frac{9}{10}$	\$16,200	?	?	. . .
7	?	\$10,800	\$19,200	. . . . . .	. . .
8	?	?	144 cu. ft.	1728 cu. ft.	. . .
9	?	5760 gr.	?	7000 gr.	. . .
10	$87\frac{1}{2}$	196 lb.	?	. . . . . .	. . .
11	$9\frac{1}{2}$	?	$328\frac{8}{25}$	?	?
12	23	?	?	. . . . . .	10,318
13	30	?	?	. . . . . .	\$29.24
14	?	\$10,000	\$120	?	. . .
15	9	?	?	872	. . .
16	$\frac{3}{4}$	3	?	?	?
17	$37\frac{1}{2}$	2	?	. . . . . .	$1\frac{1}{4}$
18	?	$\frac{8}{9}$	$\frac{2}{5}$	. . . . . .	. . .
19	?	1 bu.	8 qt.	. . . . . .	. . .
20	?	?	24	144	21
21	?	?	300	. . . . . .	645

REVIEW PROBLEMS.

1. If a merchant sells $\frac{3}{4}$ of an article for what $\frac{7}{8}$ of it cost, what is his gain per cent.?

Suggestion: Find the value of P. and use Formula 2.

2. If I buy a farm for \$5950, for what must I sell it to gain 5%?

Suggestion: Use Formula 4.

3. I received for my horse \$164, and thereby lost 18% of his value? What was his value?

4. If I buy boots at \$3.25 a pair and sell them at \$3.87$\frac{1}{2}$ a pair, what per cent. do I gain?

5. In a school, 77 pupils are present, which is 87$\frac{1}{2}$% of the number enrolled. Find the number enrolled.

6. $\frac{5}{6}$ is $\frac{7}{8}$% of what number?

7. A man sold flour at \$11 per bbl., by which he gained 37$\frac{1}{2}$% of the cost. He raised the price to \$13.50. What does he gain per cent. by this advance?

8. If the bread made from a barrel of flour weighs 33$\frac{1}{3}$% more than the flour, what is the weight of the bread?

9. A regiment lost in a campaign 400 men out of 965. What was the rate per cent. of loss?

10. I shall be obliged to use to-day 53$\frac{7}{8}$% of my money to pay a note for \$8620. How much money have I?

11. In a certain school, during 1895, the attendance of pupils was 824, which was 3% more than in 1894. What was the attendance in 1894?

12. In 1897 I gained in business \$9207, which was 7% less than I gained in 1896. How much did I gain in 1896?

13. In a mixture of copper and zinc, the copper is 1$\frac{1}{2}$ times the zinc. Find the percentage of each ingredient in the mixture.

14. What per cent. of a pound troy is a pound avoirdupois?

15. A piece of land is bought for \$3600, and a man who owns $\frac{2}{5}$ of it sells $\frac{1}{2}$ of his share for \$800. What rate per cent. does he gain?

COMMERCIAL DISCOUNT.

1. **Commercial Discount** means *deduction from the price of merchandise or from the amount of a bill,* and is computed at *some rate per cent.*

2. When two or more discounts are allowed, the first is counted off, and the second from the remainder, and so on.

3. (1.) The Price or Bill is the Base. (2.) The Rate per cent. is the Rate. (3.) The Discount is the Percentage. (4.) The Price or Bill — the Discount = the Net Price or Net Amount = the Difference.

Hence formulæ 1 and 5 on page 218 become:

1. **Discount = Price or Bill × Rate.**
2. **Net Price or Net Amount = Price or Bill × (1 — R.).**

PROBLEMS.

1. What is the *net amount* of a bill of \$500 discounted at 20%?

By Formulæ.

Net Amt. = Bill × (1 — R.) = \$500 × (1 — .20) = \$500 × .80 = \$400.

By Analysis.	Brief Process.
$20\% = \frac{1}{5}$.	$5 \mid 500$
$\frac{1}{5}$ of \$500 = \$100.	100
\$500 — \$100 = \$400.	400

2. What is the *net amount* of a bill of \$500 with $33\frac{1}{3}$ and $12\frac{1}{2}$ off?

Analysis.	Brief Process.
$33\frac{1}{3}\% = \frac{1}{3}$; $12\frac{1}{2}\% = \frac{1}{8}$;	$3 \mid 500.00$
$\frac{1}{3}$ of \$500 = \$166.66$\frac{2}{3}$;	$166.66\frac{2}{3}$
\$500 — \$166.66$\frac{2}{3}$ = \$333.33$\frac{1}{3}$;	$8 \mid 333.33\frac{1}{3}$
$\frac{1}{8}$ of \$333.33$\frac{1}{3}$ = \$41.66$\frac{2}{3}$;	$41.66\frac{2}{3}$
\$333.33$\frac{1}{3}$ — \$41.66$\frac{2}{3}$ = \$288.66$\frac{2}{3}$.	$288.66\frac{2}{3}$

3. A bicycle marked \$90 was sold at a discount of $12\frac{1}{8}\%$. Find the discount and selling price.

4. An agent sold a book, price \$5.50, at 20% discount. What did he get for it?

5. A piano was marked \$650, but was sold at a discount of 20 and 8. Find the selling price.

6. Is 50 off \$2000 the same as 25 and 25 off? Find the difference, if any.

7. A merchant bought \$850 worth of goods and received 25 and 10 off. What was the net amount of his bill?

8. What is the cash value of a bill of goods amounting to \$2157.25 at 15 and 3 off?

9. Valentines marked \$18 were sold at 20, 15, and 5 off. What was the selling price?

10. A bill of goods amounted to \$3268.36. What was the net value if 15 and 3 were counted off?

11. A bill of envelopes amounted to \$86.40. On a credit of 30 da. the reduction was $\frac{1}{6}$ and 5 off; for cash $2\frac{1}{2}$ off. With all rates off, what was paid?

12. Find the net amount of a bill for \$762 subject to the following discounts: 40, 5, and 10.

13. What is the discount on a bill of goods, if 20%, 15%, and 5% are successively made?

14. Find the net cost of 20,000 bags at \$4.50 per M., with 60, 10, and 5 off.

15. Which is the better discount for the buyer, 40 and 10 off, or 30 and 20 off?

16. After getting 25% off, a book cost me \$4.50. What was the mark price of the book?

17. What single discount is equal to the following double discounts?

1. 30 and 10%.
2. 40 and 15%.
3. 50, 5, and 5%.
4. 15, 10, and 5%.

18. What must a merchant ask for an article which cost \$40, so that he may deduct 20% and still gain 10%?

19. A bill of \$670 is subject to the following discounts: 20, 15, 10, 5. Find the net amount of the bill.

20. What must I ask for an article that cost me \$3.60, so that I may deduct $12\frac{1}{2}\%$ and still make a profit of $16\frac{2}{3}\%$?

GAIN AND LOSS.

1. **Gain** or **Loss** in business is computed at some *rate* per cent. of the cost.

2. (1.) The Cost is the Base. (2.) The Rate per cent. is the Rate. (3.) The Gain or Loss is the Percentage. (4.) The Selling Price (Cost + Gain) is the Amount. (5.) The Selling Price (Cost — Loss) is the Difference.

3. Hence the formulæ on page 218 become:

1. $\text{G. or L.} = \text{C.} \times \text{R.}$

2. $\text{R.} = \dfrac{\text{G. or L.}}{\text{C.}}$. 4. $\text{C.} = \dfrac{\text{S. P.}}{1 + \text{R.}}$ (gain).

3. $\text{C.} = \dfrac{\text{G. or L.}}{\text{R.}}$. 5. $\text{C.} = \dfrac{\text{S. P.}}{1 - \text{R.}}$ (loss).

NOTE.—Let the pupil derive formulæ for S. P. from formulæ 4 and 5.

EXERCISES.

1. I bought corn at 40 cents a bushel, and sold it at a gain of $12\frac{1}{2}\%$. How much did I get for it?

By Formula.

$12\frac{1}{2}\% = \frac{1}{8}$. G. = C. × R.
G. = $40 \times \frac{1}{8} = 5$.
$40 + 5 = 45$, selling price.

By Analysis.

$12\frac{1}{2}\% = \frac{1}{8}$. $\frac{8}{8}$, cost, + $\frac{1}{8}$, gain, = $\frac{9}{8}$, selling price.
$\frac{8}{8} = 40$; $\frac{1}{8} = 5$; $\frac{9}{8} = 45$, selling price.

2. I bought sugar at 5 cts. per pound, and sold it at $5\frac{1}{2}$ cts. per pound. What was my gain per cent.?

By Formula.

$5\frac{1}{2} - 5 = \frac{1}{2}$, gain.

$R. = \frac{G.}{C.} = \frac{\frac{1}{2}}{5} = \frac{1}{10}.$

$\therefore R. = 10\%.$

By Analysis.

$5\frac{1}{2} - 5 = \frac{1}{2}$, gain. Cost, 5, $= 100\%$; $1 = 20\%$. Gain, $\frac{1}{2} = 10\%$.

3. I sold a horse and lost \$50, which was 20% of the cost. What was the cost?

By Formula.

$C. = \frac{L.}{R.}$. $C. = \frac{\$50}{.20} = \$250.$

By Analysis.

Loss, $20\% = \$50$. $1\% = \frac{50}{20} = \frac{5}{2}$. Cost, $100\% = \frac{500}{2} = \$250$.

4. I sold wheat at \$1.00 per bu., and gained $12\frac{1}{2}\%$ of the cost. What was the cost per bu.?

By Formula.

$C. = \frac{S.\ P.}{1 + R.} = \frac{\$1.00}{1 + .12\frac{1}{2}} = \frac{2.00}{2.25} = \$.88\frac{8}{9}.$

By Analysis.

Gain, $12\frac{1}{2}\% = \frac{1}{8}$. $\frac{8}{8}$, cost, $+ \frac{1}{8}$, gain, $= \frac{9}{8}$, selling price. $\frac{9}{8} = \$1.00$; $\frac{1}{8} = \frac{1.00}{9}$; $\frac{8}{8} = \frac{8.00}{9} = \$.88\frac{8}{9}$, cost.

5. I sold hay at \$10 per ton and lost 10% of the cost. What was the cost of the hay?

By Formula.

$C. = \frac{S.\ P.}{1 - R.} = \frac{10}{1 - .10} = \frac{10}{.90} = \$11.11 +.$

By Analysis.

Loss, $10\% = \frac{1}{10}$. Cost, $\frac{10}{10}$ — loss, $\frac{1}{10} = \frac{9}{10}$, s. p. $\frac{9}{10} = \$10$. $\frac{1}{10} = \frac{10}{9}$. $\frac{10}{10} = \frac{100}{9} = \$11.11 +$.

6. Cost, \$10,500; rate, 20%. Gain? Selling price?
7. Cost, \$6; rate, 8%. Loss? Selling price?
8. Cost, \$8560; selling price, \$10,700. Gain? Rate?
9. Gain, \$6.30; rate, 14%. Cost? Selling price?
10. Cost, \$700; rate, 15%. Gain? Selling price?
11. Selling price, \$19; rate, 5%. Loss? Cost?

12. Selling price, \$175; rate, 30%. Loss? Cost?
13. Cost 12 cts.; selling price, 10 cts. Loss? Rate?
14. Cost, $9\frac{1}{2}$ cts.; rate, $12\frac{1}{2}$%. Gain? Selling price?
15. Cost, \$.75; selling price, \$1.00. Gain? Rate?
16. Cost, \$1.00; selling price, \$1.25. Gain? Rate?
17. Cost, \$250; rate, 35%. Loss? Selling price?
18. Cost, \$1.75; selling price, \$1.25. Loss? Rate?
19. Cost, \$.06; selling price, \$.05. Loss? Rate?
20. Gain, \$.12; rate, 8%. Cost? Selling price?
21. Gain, 10 cts.; rate, 10%. Cost? Selling price?
22. Selling price, \$180; rate, 20%. Cost? Gain?
23. Selling price, \$230; rate, 8%. Gain? Cost?
24. Selling price, \$4.56$\frac{1}{2}$; rate, 17%. Loss? Cost?
25. Selling price, \$49.95; loss, \$4.05. Loss, %? Cost?
26. Gain, \$47.25; rate, $7\frac{1}{2}$%. Selling price? Cost?
27. Loss, \$38.46; rate, $\frac{1}{2}$%. Cost? Selling price?
28. Cost, \$75.52; rate, $3\frac{1}{8}$%. Gain? Selling price?
29. Selling price, \$24.975; loss, \$2.025. Rate? Cost?
30. Cost, \$1939.50; rate, $\frac{5}{9}$%. Loss? Selling price?

PROBLEMS.

1. A man sold his house at a profit of 15%. If he paid \$3000 for it, how much did he get for it?

2. What per cent. is lost by selling tea at \$.75 that cost \$1.00?

3. A man sold a horse at an advance of \$75, which was a gain of 25%. What was the cost of the horse?

4. A boot and shoe dealer lost 9% by selling boots at \$3.75 a pair. What was the cost of the boots?

5. What per cent. is gained on goods sold at double their cost?

6. I bought a horse for \$500 and sold it for \$300. What per cent. did I lose?

7. The selling price was \$30, the gain 25%. What was the cost?

8. A dry-goods merchant sells goods at $12\frac{1}{2}$ cts. above their cost and makes a gain of 8%. Find the cost.

9. By selling a house for \$3500 I lose \$500. What is my loss per cent.?

10. How shall I mark goods that cost me \$1.00 a yd., then deduct 15% from that mark, and still realize 2%?

11. I bought 480 barrels of flour at \$4.50 a barrel and sold it for \$2880. Find the gain per cent.

12. A cargo of flour was bought for \$690. For what must it be sold to gain $66\frac{2}{3}$%?

13. I sold tea for 114% of its cost and made a profit of \$.07 a lb. What was the selling price?

14. I paid \$30 for a vase. I desire to gain 30% on it, after dropping 40% from the asking price. What price shall I ask?

15. When 4% is lost on cheese sold at 12 cts. a lb., what was the cost?

16. I sold a lot of goods for \$200 and thereby gained 15%. Had I sold them for \$220, what per cent. would I have gained?

17. If a wagon was purchased at 20% less than \$50, and afterwards sold at 25% more than cost, at what price was it sold?

18. A merchant selling goods at a certain price loses 5%; but if he had sold them for \$20 more he would have gained 3%. What did the goods cost him?

19. If a merchant sells goods for $\frac{3}{4}$ of their cost, what per cent. does he lose?

20. I sold a quantity of potatoes for \$850 which cost me \$970. What per cent. did I lose?

21. An agent gets a discount of 40% from the retail price

of articles and sells them at the retail price. What is his gain per cent.?

22. When coal was sold at $4.56\frac{1}{2}$ per ton there was a loss of 17%. What was the cost?

23. A druggist gained 300% by retailing quinine at $3.00 per ounce. How much did it cost him per ounce?

24. A grain dealer sold 1380 bu. of wheat at $1.00 per bu. and lost 8%. What per cent. would he have gained had he sold at $1.20 per bu.?

25. A drover bought 100 cows at $20 a head. If 20 were killed by accident, for how much must he sell the remainder per head to gain 25% on the cost of the whole number bought?

COMMISSION.

1. **Commission** is the percentage allowed an agent for buying or selling goods or transacting other business.

2. Commission is computed on money *collected* by him or on money *paid out* by him.

3. The *Collection* or the *Payment* is the **Base.**

4. The *Rate* per cent. is the **Rate.**

5. The *Commission* is the **Percentage.**

6. The *Payment* + the *Commission* is the **Amount.**

7. The *Collection* — the *Commission* is the **Difference.**

Hence the formulæ on page 215 become:

$$1.\ \text{Com.} = \text{Coll. or Payt.} \times \text{R.}$$

$$2.\ \text{R.} = \frac{\text{Com.}}{\text{Coll. or Payt.}}.$$

$$3.\ \text{Payt.} = \frac{\text{Amount}}{1 + \text{R.}}.$$

$$4.\ \text{Coll.} = \frac{\text{Difference}}{1 - \text{R.}}.$$

PROBLEMS.

1. An attorney collects a debt of $500 on a commission of 3%. What is his commission?

By Formula.

$$\text{Com.} = \text{Coll.} \times \text{R.}$$
$$500 \times .03 = \$15.00.$$

By Analysis.

$$100\% = \$500.$$
$$1\% = \$5.$$
$$3\% = \$15.$$

2. A tax collector receives $180 for collecting taxes on a 3% commission. What is the amount collected?

By Formula.

$$\text{Com.} = \text{Coll.} \times \text{R.} \qquad \therefore \text{Coll.} = \frac{\text{Com.}}{\text{R.}}.$$
$$\text{Coll.} = \frac{\$180}{.03} = \$6000.$$

By Analysis.

$$3\% = \$180.$$
$$1\% = \$60.$$
$$100\% = \$6000.$$

3. Find the value of the goods that can be purchased for $420, if the agent's commission is 5%.

By Formula.

$420 = the Payment + the Commission = the Amount.

$$\text{Payt.} = \frac{\text{Amt.}}{1 + \text{R.}} = \frac{420}{1.05} = \$400, \text{ value of goods.}$$

By Analysis.

$$100\% = \text{Payt.}$$
$$5\% = \text{Com.}$$
$$105\% = \$420.$$
$$1\% = \$4.$$
$$100\% = \$400.$$

4. An agent sells goods at $2\frac{1}{2}\%$ commission. After deducting his commission, he remits his employer $3763.50. How much money did he collect for the goods sold?

By Formula.

$3763.50 = the Collection — the Commission = the Difference.

$$\text{Coll.} = \frac{\text{Difference}}{1 - \text{R.}} = \frac{3763.50}{1 - .02\frac{1}{2}}$$
$$= \frac{3763.50}{.975} = \$3860.$$

By Analysis.

$$100\% = \text{Coll.}$$
$$2\tfrac{1}{2}\% = \text{Com.}$$
$$97\tfrac{1}{2}\% = \$3763.50.$$
$$1\% = \frac{3763.50}{97\frac{1}{2}}.$$
$$100\% = \frac{376350}{97\frac{1}{2}} = \$3860.$$

5. An agent sold $2275 worth of goods at 2% commission. What was his commission?

6. A commission merchant received $318.25 for selling $12,730 worth of bankrupt goods. What was his rate of commission?

7. A merchant sent his agent in Cincinnati $7000 to invest in pork, after deducting his commission at $2\frac{1}{2}$%. What was his commission, and how much did he invest?

8. An agent received a certain sum of money to invest in goods after deducting his commission of 3%. He invested $6250. What sum did he receive?

9. If I send my agent $4050 to invest in goods after deducting 3% commission, what sum will he invest?

10. What is the commission at 3% for selling 125 bbl. of potatoes at 2.37\frac{1}{2}$ per bbl.?

11. A commission merchant receives $2\frac{1}{2}$ commission for buying grain for a customer. The cost of the grain and his commission = $4223. How much does the grain cost?

12. Find the amount of an agent's sales when his commission at 5% = $37.65.

13. A real estate agent collects the annual rent of a house and retains $13.25 as his commission at $2\frac{1}{2}$%. What is the rental of the house?

14. My attorney collected a bill for me at a commission of $12\frac{1}{2}$% and paid me a net sum of $56. How much money did he collect?

15. An agent collected 20% of an account of $860, charging 4% commission. Find commission and sum paid over.

16. My agent collected 90% of a debt of $5600 and charged $7\frac{1}{2}$% commission. How much should I receive from him?

17. A sale of real estate returned, as net proceeds, $2396.49, after paying $324.18 charges and a commission of 2%. For how much did it sell?

18. Had sold for me 500 bbl. of apples at \$4.50 per bbl., paying $2\frac{1}{2}\%$ commission; had bought for me with the proceeds wheat at 70 cts. a bu., paying 3% commission. How many bu. of wheat did I obtain?

19. How much commission must be paid to a collector for collecting an account of \$928.75 at $3\frac{3}{4}\%$?

20. An agent's commission for selling grain was \$76.80, at 4%. How much did he get for the grain?

21. A real estate agent sold a house for \$7500 and charged $\frac{3}{4}\%$ commission. Find the net proceeds of the sale.

22. An agent sold goods to the amount of \$8725. What was his commission at $2\frac{1}{2}\%$?

23. A consignee sells \$6742 worth of woollen goods, charging $2\frac{1}{2}\%$ commission and $1\frac{1}{4}\%$ for insuring payment. What sum will he pay over to the consignor?

24. I send \$10,000 to my correspondent in New Orleans to invest in cotton. His commission is $\frac{1}{2}\%$ for buying. What sum does he invest and what is his commission?

25. A man receives \$1500 commission on his yearly sales. What is the amount of his sales if he is allowed $\frac{1}{4}\%$ commission?

26. To be invested in cotton at 15 cents a lb., \$21,630.00; commission allowed, $2\frac{1}{2}\%$; marine insurance paid, $1\frac{1}{4}\%$; cartage and freight paid, $1\frac{1}{4}\%$. Find the sum invested in cotton and the number of lb. of cotton bought.

REVIEW.

Formulæ to be used.

1. I sold a horse, which cost me \$250, at a loss of 35%. What did I get for him?

2. What is an agent's commission on the purchase of an estate for \$30,000, at $1\frac{1}{2}\%$?

3. By selling a watch for \$19, the seller loses 5% on his outlay. What would have been his loss or gain per cent. if he had sold the watch for \$23.75?

4. A merchant's prices are 25% above cost price; if he allows a customer 12% on his bill, what profit does he make?

5. If my broker buys for me goods worth \$13,000, and his commission is $1\frac{1}{2}$%, how much must I pay him?

6. A speculator sells at a profit of 75%, but his purchasers fail, and only pay 25 cents on a dollar. How much does the speculator gain or lose by this venture?

7. A man gains 15% in buying an article, and again 15% in selling it. Find the whole of his gain per cent.

8. If goods marked at 45% above cost are sold at 40% off, what is the gain or loss per cent.?

9. If 8% be lost by selling an article for \$25.50, what per cent. is gained or lost if it be sold at \$38.00?

10. A carriage is sold for \$175, which is 30% less than cost. What was the cost?

11. An army lost 18% of its men by disease and desertion, and then lost 14% of the remainder in battle; the number then remaining was 84,624. Of how many men did the army consist at first?

12. If I sell a piano, which cost \$275, for \$315, what is the rate per cent. of gain?

13. If I buy coffee at 16 cents, and sell it at 20 cents a pound, what per cent. do I make?

14. A cargo of wheat was sold for \$12,500, by which a gain of 25% was made. What was the amount of net gain, after paying \$150 freight and \$75 for other charges?

15. If the commission is $1\frac{1}{2}$ per cent., what bill will \$3950 buy?

16. An auctioneer sells for me a carriage for \$140, a table

for $15, 50 yd. of carpet at 60 cts. a yd. His commission is $2\frac{1}{2}\%$. What will be due me for the goods?

17. A commission merchant, receiving $2\frac{1}{2}\%$ commission, had 410 bu. of potatoes sent him, with orders to sell at 96 cents per bu. He held them until he received $492 above his commission. What per cent. was made by holding them?

18. At what price must I sell goods that cost $\$\frac{3}{8}$ to gain 20%?

19. What per cent. of $90 is $33\frac{1}{3}\%$ of $67.50?

20. Having purchased a farm for $9000, and spent $2500 in improvements, I sold it for $13,800. What per cent. did I make on my investment?

21. A man sold a set of harness for $15, and lost $16\frac{2}{3}\%$. If he had sold it at a profit of 20%, what would he have received?

22. What per cent. is gained by selling 15 ounces of tea for what a pound costs?

23. A speculator sold 2760 bu. of wheat at $1.00 per bu., and lost 8%. How much per cent. would he have gained had he sold at $1.20 a bu.?

24. Which is the better, a discount of 25% and 10% off the remainder, or a discount of $33\frac{1}{3}\%$ off?

25. A broker sells 4000 bu. of wheat, and, after deducting his commission of 2%, remits by check $4900. At what price per bu. did he sell the wheat?

26. I sent a commission agent 500 bbl. of potatoes, which he sold at $2.50 per bbl. His charges were: commission, $2\frac{1}{4}\%$; storage, $1\frac{1}{2}\%$; cartage, $9.00. How much was due me?

27. A drummer earns $3000 annually. $1500 is guaranteed; the remainder is his commission, at 5%. What are his annual sales?

28. I bought 1000 gross of screws at 27 cents, at a discount of 15, 10, and 5. I sold the lot at cost plus 30%. What was my gain?

29. Offered cattle for sale at 25% above cost, but was obliged to sell them for 14% less than that mark, and gained thereby $170. What did the cattle cost? What did I ask for them? How much did I sell them for?

30. Received $2020 to buy with; commission, 1% Find the cost.

31. Collection, $14,000; commission, $420. Find the rate.

STOCKS AND BONDS.

DEFINITIONS.

1. Stock is *invested capital,* and is represented by *certificates* which attest the ownership of a certain number of *shares.*

2. Bonds are written *obligations,* in which an agreement is made to *pay a specified amount on or before a specified date,* with interest.

3. The **Face-Value** is the *sum mentioned* in certificates and bonds. When stocks and bonds sell for their *face-value,* they are said to be at *par.* When they sell for *more* than their face-value, they are said to be at a *premium.* When they sell *below* their face-value, they are said to be at a *discount.*

4. Coupons certify to *interest due,* and are *cut off* and *surrendered* when the interest is *paid.*

5. Bonds are issued by *corporations* organized under law, and take their name from the name of the corporation that has issued them. "Buffalo Railway—4—Q——84," means stock issued by the Buffalo Railway Company, *rate* 4%, interest *payable quarterly,* now selling at $84 per share,—*i.e.,* at a discount.

6. Stock Brokers buy and sell stocks and bonds; their commission is called *brokerage.* Brokerage is reckoned at $\frac{1}{4}\%$ or $\frac{1}{8}\%$ on the par value.

7. A Quotation is a *published statement* of the current *selling price* of a stock.

Quotations.

Government Bonds.	Stocks.
1. U. S. 4s, registered, 1906, $110\frac{3}{4}$.	6. Adams Express 4s, 105.
2. U. S. 5s, coupon, 1904, $112\frac{3}{4}$.	7. Penna. 4 p. c., 110.
3. Currency 6s, 1899, $102\frac{1}{4}$.	8. Schuylkill R. R. 5s, $105\frac{1}{2}$.
4. Cherokee 4s, 1899, 101.	9. N. C. Railway $4\frac{1}{2}$s, $104\frac{1}{2}$.
5. U. S. small bonds, $105\frac{3}{4}$.	10. L. V. R. R. Coal 5s, 94.

8. The *Par Value* is the *Base.*

9. The *Rate of Premium,* or *Discount,* is the *Rate.*

10. The *Premium,* or the *Discount,* is the *Percentage.*

11. The *Quotation Value* (ab. par) is the *Amount.*

12. The *Quotation Value* (bel. par) is the *Difference.*

Hence the formulæ on page 218 become:

1. $\text{Prem. or Disc.} = \text{P. V.} \times \text{R.}$

2. $\text{R.} = \dfrac{\text{Prem. or Disc.}}{\text{P. V.}}$.

3. $\text{P. V.} = \dfrac{\text{Prem. or Disc.}}{\text{R.}}$.

4. $\text{P. V.} = \dfrac{\text{Q. V. (ab. par)}}{1 + \text{R.}}$ or $\dfrac{\text{Q. V. (bel. par)}}{1 - \text{R.}}$.

5. $\text{Q. V.} = \text{P. V.} \begin{cases} + \text{Prem.} \\ - \text{Disc.} \end{cases}$

Income is computed on the face of a bond, at the face-rate. 5 shares ($500) U. S. 4s will yield $4 per share, or $20.00 income.

MODEL SOLUTIONS.

1. Find the premium on 25 shares U. S. 4s quoted at $110\frac{3}{4}$; also the annual income derivable therefrom.

$110\frac{3}{4} - 100 = 10\frac{3}{4}$. $\therefore$ R. $= 10\frac{3}{4}\%$. Par value of 25 shares = $2500. *Formula:* Prem. = P. V. × R. $2500 × .1075 = $268.75. Income = $2500 × .04 = $100.

2. The par val. of U. S. bonds = \$25,000, and the premium = \$500. Find the rate and the quotation.

Formula: R. $= \frac{\text{Prem.}}{\text{P. V.}}$. $\frac{500}{25000} = \frac{1}{50} = 2\%$. $100\% + 2\% = 102\%$, the quotation.

If these bonds were U. S. 5s, what income would they yield?

3. I bought railroad stock quoted at 96, and the discount I obtained was in all \$24. How many shares of stock did I buy?

$100 - 96 = 4$. $\therefore$ R. $= .04 = 4\%$.

Formula: P. V. $= \frac{\text{Disc.}}{\text{R.}}$. $\frac{24}{.04} = \$600 = 6$ shares.

If this is 6% stock, what is my yearly income therefrom?

4. I bought railway stock quoted at $102\frac{1}{2}$, investing \$10,250. Find the number of shares I bought, and my income therefrom at 5%.

Formula: P. V. $= \frac{\text{Q. V.}}{1 + \text{R.}}$. $\frac{10250}{1.025} = \$10,000 = 100$ shares.

5% of \$10,000 = \$500, income.

5. If you buy railway stock quoted at 84, and invest \$3360, how many shares will you buy, and what will be your income therefrom if the stock pays 4%?

Formula: P. V. $= \frac{\text{Q. V.}}{1 - \text{R.}}$. $\frac{3360}{.84} = \$4000 = 40$ shares.

What will be your income at 4%?

6. How much must I invest in 6% stock at $102\frac{1}{2}$ to secure me an annual income of \$300, brokerage $\frac{1}{4}$?

Suggestions: What will be the income from 1 share? How many shares will yield \$300? What will the shares cost you at $102\frac{1}{2}$? What will the brokerage be at $\$\frac{1}{8}$ per share? What will the total cost of the investment be?

7. What per cent. does an investment in Coney Island Brooklyn 6s offer me if the stock is quoted at 140?

Suggestions: \$140 invested yields what income? That income is what per cent. of \$140?

8. If I wish to obtain 7% on my investment, wnat must I pay for a 5% stock?

Suggestion: If $\frac{7}{100}$ of a sum = \$5.00, what must that sum be? Or, if 5 is the percentage and 7 is the rate, what is the base?

PROBLEMS.

NOTE.—The numbers in the following problems refer to the Bond and Stock Quotations found on a preceding page. Brokerage is not considered unless mentioned.

1. Find the market value of 150 shares of No. 1; also the income and the rate the investment pays.

2. Find the cost of investment in No. 10 to secure an income of \$250.

3. Mr. A. owns 90 shares of No. 2. What is his income?

4. What rate do Nos. 6, 7, 8, 9, 10 severally pay on the investments?

5. Mr. B. has \$41,309 to invest. Which will secure him the larger income, No. 3 or No. 4?

6. Which will give him the higher rate on his investment?

7. How many shares of No. 3 can he buy, paying $\frac{1}{8}$% brokerage?

Suggestion: Cost of a share = $102\frac{1}{4} + \frac{1}{8}$.

8. If I wish to obtain 7% on my investment, what must I pay for a 6% stock?

9. How much will 55 shares C. C. C. and I. R. R. stock cost at $28\frac{3}{4}$, brokerage $\frac{1}{4}$%?

10. How many shares of railroad stock at 3% discount can be bought for \$2139.50, brokerage $\frac{1}{4}$%?

11. Which is more profitable, and how much, to invest \$6000 in 6% stock purchased at 75%, or 5% stock purchased at 60%?

12. What sum must I invest in Louisiana 7s at $107\frac{1}{4}$ to secure an annual income of \$1750?

13. Which affords the greater per cent. of income, bonds bought at 125, which pay 8%, or bonds which pay 6%, bought at a discount of 10%?

14. At what price must I purchase 15% stock that it may yield the same rate of interest as 6% stock purchased at 90?

15. What is the cost of 125 U. S. 6s at 104, brokerage $\frac{1}{8}$%?

16. How many shares must a broker sell to realize $10.50, commission at $\frac{1}{8}$%?

17. B paid $10,989 for U. S. 6s at 110$\frac{7}{8}$, brokerage $\frac{1}{8}$%. What was his income?

18. What sum must be invested in U. S. 5s at 116$\frac{1}{2}$, brokerage $\frac{1}{8}$%, to secure an annual income of $160?

19. What is the annual income from investing $4446 in 5$\frac{1}{2}$% stock at 92$\frac{1}{2}$, brokerage $\frac{1}{8}$%?

20. What will be the cost of 17 shares of canal stock at 93$\frac{7}{8}$ and 143 shares gas stock, par value $10, at 102$\frac{3}{4}$?

21. A man invested $9562.50 in the stock of a city bank at 127$\frac{1}{2}$. If a dividend of 3$\frac{1}{2}$% is declared, what amount of dividend would he get?

22. 6% bonds were sold at 118; the proceeds were invested in 4$\frac{1}{2}$% bonds. If the former and the latter incomes were the same, at what quotation were the 4$\frac{1}{2}$% bonds bought?

23. 200 shares of stock, par value $25, and sold at 102$\frac{1}{2}$, $\frac{1}{8}$% being retained for brokerage, how much is paid over?

24. Which is the better investment, U. S. 5s at 98$\frac{1}{4}$%, or U. S. 6s at 108$\frac{3}{4}$%, brokerage $\frac{1}{4}$% in each case?

25. Which is the more costly, and how much more, 15 shares of N. Y., N. H. & H., at 85, or 13 shares N. Y. & N. E., at 102, if the brokerage in each case is $\frac{1}{4}$%?

26. How much money must be invested in U. S. 4$\frac{1}{2}$s to yield a quarterly income of $225, bonds selling at 105$\frac{1}{4}$?

27. A man invested some money in bonds, at par, bearing 6% interest, and received \$300 semi-annually. What was the sum invested?

28. A 5% stock is quoted at $85\frac{1}{2}$. A purchaser pays brokerage at $\frac{1}{4}$%. What rate per cent. does he receive on his investment?

29. A lady would secure by investment an annual income of \$650. How much 5% stock must she buy at par for the purpose?

30. How much stock can be bought for \$14,178 when the quoted price is $208\frac{1}{2}$?

31. Find the quoted price of railroad stock when the cost of 250 shares, including brokerage at $\frac{1}{8}$%, is \$30,312.50.

32. What income will \$10,120 yield if invested in 4% stock bought at 115?

33. If a 6% stock is quoted at 120, what rate per cent. will an investor receive on his money?

34. If I invest \$1500 in 3% stock at 75, what is my income and what rate per cent. do I get on my investment?

35. If I exchange 48 shares of a 9% stock at 176 for U. S. 4s at $116\frac{1}{2}$, how much must I add to my investment to secure the same income?

36. What sum of money must be invested in Louisville & Nashville Railroad certified gold 4% bonds at $84\frac{1}{2}$ to produce an annual income of \$320, brokerage $\frac{1}{4}$?

37. If \$8000 5% stocks are sold at 90 and the proceeds invested in $3\frac{1}{2}$% stocks at 60, find the increase or decrease in income?

38. Find the price of a $3\frac{1}{2}$% bond that will be as profitable an investment as a 6% bond at par.

39. Mr. A. bought U. S. 6s for 108, kept them a year, and then sold them at $118\frac{2}{5}$. What rate of interest did the investment pay him for that year?

40. What sum must be invested in 6% stocks, worth 95, to yield an income of $4500?

41. Which would yield the larger income, $11,400 invested in 7% stock, at 95, or the same amount invested in 5% stock, quoted at 57?

42. At what rate must a 5% stock be sold to produce 8% on the investment?

43. If I buy 6% stock at 15% discount, what is the rate of interest on the investment?

44. If I give a house and lot worth $2000 for 175 shares ($10) N. Y. Gas Co.'s stock, what is the rate of premium?

45. How many shares of bank stock, selling at 5% discount, can be bought for 250 shares of insurance stock, selling at 14% premium?

46. How many shares of stock, par 25, can be bought for $2730, when quoted at 105?

47. A capitalist bought stock at 65, and after receiving a dividend of $5\frac{1}{2}$%, sold it at 82, and made $1125. How much stock had he, and what per cent. did he realize?

48. If stock is bought at $3\frac{1}{2}$% discount, and sold at a premium of $2\frac{1}{4}$%, and the gain is $258.75, what is the par value of the stock?

49. I bought bank stock at $96\frac{1}{4}$, and sold it at $112\frac{1}{8}$, thereby gaining $3556. How many shares were there?

50. What must I pay for 6% bonds to realize $5\frac{1}{2}$% on my investment, brokerage $\frac{1}{8}$%?

51. In order to realize 6% annually on an investment, what must I give for bonds that pay a semi-annual interest of 3%, if I immediately reinvest the semi-annual interest at 6%?

52. If 5% bonds are bought at 90, what is the rate of income on the investment?

53. A lady desiring to invest money, considered 5s at 108, 6s at 124, and 7s at 129. Which was preferable?

54. A broker charges $25 at $\frac{1}{4}\%$ for buying Pennsylvania R. R. ($50). How many shares did he buy?

INSURANCE.

1. **Insurance** is security guaranteed for loss by fire or other specified causes.

2. **Property Insurance** includes:

1. Fire Insurance. 2. Marine Insurance. 3. Live Stock Insurance.	Premium computed as percentage.

3. **Personal Insurance** includes:

1. Life Insurance. 2. Accident Insurance. 3. Health Insurance.	Premium computed at a certain sum per $1000.

4. The written agreement is called the **Policy**; the sum named in the policy is called the **Face**; the sum paid annually, semi-annually, or quarterly is called the **Premium**.

1. The *Face* (the amount insured) is the *Base.*
2. The *Rate of Premium* is the *Rate.*
3. The *Premium* is the *Percentage.*

Hence we have the following formulæ:

1. Face × Rate = Premium.
2. Premium ÷ Face = Rate.
3. Premium ÷ Rate = Face.

MODEL SOLUTIONS.

1. How much will it cost to insure a house worth $3000 at $1\frac{1}{4}\%$?

$1\frac{1}{4}\% = .01\frac{1}{4} = \frac{1\frac{1}{4}}{100} = \frac{5}{400} = \frac{1}{80}$.
Formula: Premium = Face × Rate = $\$3000 \times \frac{1}{80} = \$37.50$.

2. A merchant insures his store, valued at $4850, for $\frac{4}{5}$ of its value at $\frac{7}{8}\%$. What is the premium?

$\frac{4}{5}$ of $4850 = \$3880$. $\frac{7}{8}\% = \frac{7}{800} = .00875$.
Formula: Face × Rate = $\$3880 \times .00875 = \$33.95$.

3. The insurance on a barn at $\frac{3}{4}\%$ costs \$18. What is the face of the policy?

$\frac{3}{4}\% = \frac{\frac{3}{4}}{100} = \frac{3}{400} = .0075.$

Formula: Face = Premium ÷ Rate = 18 ÷ .0075 = \$2400.

Or, $\frac{3}{4}\%$ = \$18. $\frac{1}{4}\%$ = \$6. 1% = \$24. 100% = \$2400.

If I pay \$30 insurance on a \$3000 house, what is the rate?

Formula: Rate $= \frac{\text{Premium}}{\text{Face}} = \frac{30}{3000} = \frac{1}{100} = .01 = 1\%.$

Or, \$3000 = 100%. \$1 = $\frac{100}{3000}\% = \frac{1}{30}\%$. \$30 = $\frac{30}{30}\% = 1\%$.

PROBLEMS.

1. If a man pays \$30 insurance at $1\frac{1}{4}\%$, what amount of insurance does he get?

2. A vessel and cargo valued at \$2840 are insured at $3\frac{1}{2}\%$. What is the premium?

3. A man has a house worth \$5600. He insures it at $1\frac{1}{5}\%$ on $\frac{6}{7}$ of its value. Find the cost of insurance.

4. What is the total premium on a house worth \$4500 insured for 5 years at $1\frac{1}{2}\%$?

5. How much is the premium for insuring a stock of goods for \$15,000 at $1\frac{1}{4}\%$?

6. Mr. Jacobs paid \$652.50 for insuring property valued at \$43,500. What was the rate?

7. A vessel and cargo were insured for $\frac{2}{3}$ of their value at $1\frac{1}{4}\%$. The premium was \$2475. At what price were the vessel and cargo valued?

8. \$3.75 was the premium on $\frac{3}{4}$ the value of some furniture at 1% a year. What was its insurance valuation?

9. One company offers to take a \$12,000 risk at $1\frac{1}{2}\%$ for five years, and another at $\frac{1}{4}\%$ a year. Which is the cheaper?

10. An insurance company loses \$3528 by the wreck of a carload of flour which it had insured for \$3600. What was the rate of insurance?

11. A merchant imports a cargo from Liverpool, England,

worth £1500 and insures it at $\frac{4}{5}\%$. Find the premium in U. S. money.

12. For what sum must a policy be made out to cover the insurance on a property of \$2100 at $\frac{4}{5}\%$?

13. If it cost \$93.50 to insure a store for one-half of its value, at $1\frac{3}{8}\%$, what is the store worth?

14. A person insured his house for $\frac{3}{4}$ of its value at 40 cents per \$100, paying a premium of \$73.50. What was the value of the house?

15. At $\frac{4}{5}\%$, how much insurance can be effected upon a store for \$108?

16. For what sum should a cargo worth \$74,496 be insured at 3% so that, in case of loss, the owner may recover both the value of the cargo and the premium paid?

17. A man has a house worth \$5600. He insures it at $1\frac{1}{5}\%$ on $\frac{5}{7}$ of its value. Find the cost of insurance.

18. If a tax of \$12 is paid on a house and lot valued at \$1200, what is the rate per cent. of tax?

19. A vessel worth \$28,000 was insured at $1\frac{3}{4}\%$, and the cargo, worth \$15,000, at $2\frac{1}{5}\%$. Both were totally lost. What was the loss to the insurer?

20. A man 25 years of age has his life insured for \$6000 at \$19.85 on \$1000 annually. What annual premium does he pay?

21. If a man 35 years of age takes out a life policy for \$8500 at \$22.70 on \$1000 annually, and dies at the age of 60, how much does the amount insured exceed the sum of the premiums?

22. If Mr. B. takes out a life policy for \$8000, what is his yearly premium at the rate of \$26.50 on \$1000?

23. At the age of 28 years I took out an endowment policy for \$10,000. What is my yearly premium at the rate of \$45.15 on \$1000?

24. I insure my life for \$8000, paying \$19.80 per \$1000 per year. What do I pay the company if I live 20 years after insurance?

25. If a person who is insured for \$5000, at an annual premium of \$28.90 per \$1000, dies after 9 payments, how much more will his heirs get than has been paid in premiums?

26. A lady insures her life for \$8000, at an annual payment of \$29.30 per \$1000. If she lives 15 years, what amount will she have paid in premiums?

DIRECT TAXES.

1. A **Tax** is a sum of money levied on persons in behalf of the public welfare.

2. A **Poll Tax** is levied on the person. A **Property Tax** is levied on property.

3. **Assessors** determine the value of property.

4. A **Tax-Collector** collects the taxes; his salary is commonly a percentage of the sum collected.

5. **Property Tax** is reckoned at some rate per cent. on the value of the property assessed.

MODEL SOLUTION.

A tax of \$15,600 is to be raised in a town in which the taxable property is \$3,200,000; there are 1000 persons who pay a poll-tax of \$2.00 each. What is the rate of taxation? What is A.'s tax, whose property is valued at \$6000, and who pays a single poll-tax?

1. The poll-tax = \$2.00 × 1000 = \$2000.
2. Total tax, \$15,600 — \$2000 = \$13,600, tax to be raised on property.
3. \$13,600 ÷ 3,200,000 = .004¼. Rate = 4¼ mills on a dollar.
4. A.'s tax = \$6000 × .004¼ = \$25.50, on property.
5. \$25.50 + \$2.00 = \$27.50, A.'s entire tax.

Hence the formulæ:

1. **Rate of Taxation = [Total Tax − Poll Tax] ÷ Total Valuation.**

2. **Each Citizen's Tax = His Valuation × Rate + His Poll Tax.**

PROBLEMS.

1. A certain town wishes to raise \$1644 by taxation. The property of the town is assessed at \$224,000. There are 400 polls, assessed at \$0.75 each. What is the tax on \$1?

2. At the above rate, what would be A.'s tax if he pays for real estate valued at \$3655, for personal property valued at \$980, and for 2 polls?

3. If a tax-collector receives \$54 for collecting \$1800, what is his rate of commission?

4. A tax-collector receives \$180 for collecting taxes on a 3% commission. What is the amount collected?

5. How many dollars on \$1000 must be levied on \$597,600 to raise \$5976 tax?

6. How much is a man taxed who was assessed for one poll \$0.75, and on property valued at \$5390, the rate being $\frac{1}{2}$%?

7. In a town whose taxable property is valued at \$5,463,000 a tax of \$9560.25 is raised. What is the rate of taxation?

8. My property, which cost me \$7800, is taxed at $\frac{2}{3}$ of its value. If my tax is \$15.60, what is the rate of taxation?

9. What sum must be assessed to raise \$3750, besides paying 2% for collection? What would be the taxable valuation of property to raise that sum if the rate were .003275?

10. A tax of \$14,250 is to be assessed on a town; the real estate is valued at \$1,200,000 and the personal property at \$750,000; there are 400 polls, each of which is taxed \$1.50. What is the rate of taxation?

11. What is the assessed value of property taxed \$87.50 at the rate of 5 mills on a dollar?

12. A tax of \$28.50 is to be raised on a town, and sufficient besides to pay for collecting at 5%. If the rate is $\frac{1}{2}$ cent on a dollar, what is the property worth?

13. In a certain district a school-house is to be built at a cost of \$18,527. What amount must be assessed to cover this and the collector's fees at 3%?

14. Find the entire tax that must be assessed in order that a town may receive \$12,134 after the collector deducts his commission of $2\frac{1}{2}$%.

After the tax rate has been determined, the computation of a tax list is facilitated by the use of a table.

Table.

Rate, \$0.015.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
\$1 . . .	.015	\$4 . . .	.06	\$7 . . .	.105
2 . . .	.03	5 . . .	.075	8 . . .	.12
3 . . .	.045	6 . . .	.09	9 . . .	.135

15. By using the table, find the tax on \$8450.

Process.

$$\begin{array}{lrll} \text{Tax on} & \$8000 = & \$120.00, & 1000 \text{ times } .12. \\ \text{Tax on} & 400 = & 6.00, & 100 \text{ times } .06. \\ \text{Tax on} & 50 = & \underline{\quad .75}, & 10 \text{ times } .075. \\ & & \$126.75. & \end{array}$$

16. In like manner find the tax of:

1. C. H. Anheier, on \$910.
2. R. B. Bates, on \$2356.
3. G. B. Caldwell, on \$3600.

4. M. F. Dooley, on $9855.
5. Z. S. Eldridge, on $10,864, paying 2 polls, at $1.50.
6. J. S. Escott, on $20,200, paying 1 poll, at $1.50.
7. S. R. Flynn, on $31,750, paying 3 polls, at $1.50.
8. E. J. Graham, on $111,368, paying 2 polls, at $1.50.
9. C. P. Hatch, on $200,500, paying 5 polls, at $1.50.
10. E. J. Johnson, on $567,005, paying 2 polls, at $1.50.

INDIRECT TAXES.

1. **Indirect Taxes** are levied upon merchandise. They consist of **Duties**, levied on imported goods, and of **Internal Revenue**, levied on domestic goods.

2. Duties are of two classes,—**Specific** and **Ad Valorem.**

3. **Specific Duties** are levied on each yard, pound, etc., of the article. **Ad Valorem Duties** are levied at a rate per cent. of the cost of the article in the country in which it was bought.

4. *Specific Duties* are computed on the *net measure* or *weight*, *Tare* being allowed for the weight of box or wrappings, and for *Breakage*, *Leakage*, etc.

PROBLEMS.

NOTE.—In the examples that follow, the present tariff rates (1899) are used.

1. What is the duty on 1250 lbs. of desiccated apples imported at the rate of 2 cents per pound?

Process.

$$1250 \times .02 = \$25.00, \text{ duty.}$$

2. A merchant imported 1000 yd. of Brussels carpet costing in Europe 3 shillings per yd. What was the duty at 40%?

Process.

£1 = \$4.8665; 1s. = \$.243325; 3s. = .729975. ∴ The cost of 1000 yd. = 1000 × .729975 = \$729.975.

40% of \$729.975 = \$291.99, duty.

3. A merchant imported \$1250 worth of silk beaded goods. What was the duty at 60%?

4. What is the duty, at 3 cts. per lb., on 175 bags of coffee, each containing 115 lb., valued at 20 cts. per lb.

5. Find the duty on 100 boxes of Castile soap, containing each 110 lb., costing 20 liras per cwt., at $1\frac{1}{4}$ cts. per lb., tare allowed, 5%.

6. What is the duty on 400 boxes of cigars, each box containing 500 cigars, gross weight 400 lb., costing 80 cts. per lb. in Havana, at the rate of \$4.50 per lb. and 25% ad valorem, together with the *internal revenue tax* of \$3.00 per 1000 cigars?

7. If an imported piano cost in Europe \$200 and was subject to a duty in New York of 45%, at what price must it be sold to gain 25%?

8. What is the duty, at $2\frac{1}{2}$ cts. a pound, on 3750 lb. of coffee, allowing 5% for tare?

9. What is the duty on 500 lb. of raisins, in boxes, valued at 10 cts. a pound, allowing 15% for tare, when the duty is $2\frac{1}{2}$ cts. a pound?

10. What is the duty, at $1\frac{7}{8}$ cts. per pound, on 7 T. of steel anvils, of 2240 lb. each, invoiced at 20 cts. a pound?

11. An importer paid duties amounting to \$386.75. If the duty was 25% of the value of the goods, what was their value?

12. What will be the duty, at 55 cents per sq. yd., on 6 pieces of cloth, each containing 54 yd., 32 in. wide?

13. A merchant imported from Havana 25 hhd. of W. I. molasses, which was invoiced at 40 cents per gal. Allowing $\frac{1}{2}\%$ for leakage, what was the duty at 6 cents a gallon?

14. What is the duty, at 44 cents per lb., and 55% ad valorem, on 700 yd. of cloth, invoiced at \$1.60 per yd., one yd. weighing $1\frac{1}{2}$ lb.?

15. The duty on certain cotton goods is $5\frac{1}{2}$ cents per sq. yd., and 20% ad valorem. Find the duty on 267 pieces, 30 in. wide, each piece containing 37 yd., and costing 7 cents per yd.

16. I imported 100 tons of iron, costing $1\frac{1}{2}$d. per lb., on which I paid a duty of \$4.00 per ton. The freight was 6s. per ton. What was the entire cost in U. S. currency?

17. An importer bought 1000 pieces of certain goods at \$40 per piece; the duty thereon was 50%; the freight, etc., was \$1200. How must the goods be sold to gain 25%?

18. A quantity of bookbinders' calf-skins cost \$630, including \$15 for freight and \$102.50 for duty. What was the rate ad valorem?

19. If the importation of $83\frac{1}{3}$ doz. of gloves doubled their cost, which was 50 fr. per dozen, what was gained on each pair, and on the entire lot by selling them at \$2.00 per pair?

20. Find the total cost of glassware on which \$311.85 for duty at 45% ad valorem was paid, and 16% for breakage was allowed.

21. If the gross cost is \$2630, the freight \$100, the duty \$330, what is the rate?

22. 100 pieces of French goods were invoiced at \$40 per piece; the duty paid was 50%; the freight, etc., amounted to \$1500. How must they be sold to gain 20%?

23. The invoice price of goods is \$1.00 per yard; the ad

valorem duty is 20%; the specific duty is \$0.20. Find the gross value of a single yard.

24. The specific duty is \$0.44 per lb.; the ad valorem duty is 60%; the gross cost is \$244. Find the invoice price of 100 lb.

25. The ad valorem duty is 50%; the invoice price is \$500; the selling price at a profit of 25% is \$1000 on 100 lb. Find the gross value and the specific duty.

INTEREST.

1. **Interest** is money paid for the *use* of money, and depends both upon a certain rate per cent. and the length of time the money is in use.

2. The money used is the **Principal (Base)**.

3. The interest for one year is the **Percentage**.

4. The interest for a longer or a shorter time than one year is the product of the percentage and the time expressed in years or in the fraction of a year.

5. The time, when expressed in months, must be divided by 12; when expressed in days, by 360.

6. Since percentage equals the product of principal (base) and rate, we have the following formulæ:

$$1.\ \text{Interest} = \text{Principal} \times \text{Rate} \times \text{Years};$$
$$\text{or, } (\text{Int.} = \text{Pr.} \times \text{R.} \times \text{Y.}).$$

$$2.\ \text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Months}}{12};$$
$$\text{or, } \left(\text{Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{mo.}}{12}\right).$$

$$3.\ \text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Days}}{360};$$
$$\text{or, } \left(\text{Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{da.}}{360}\right).$$

MODEL SOLUTIONS.

1. What is the interest of \$550 at 5% for 4 yr.?

Process.

$$\text{Int.} = \text{Pr.} \times \text{R.} \times \text{Y.} = \$550 \times .05 \times 4 = \$110.$$

Explanation.

Since the int. is required for exactly 4 yr., we use the formula, Int. = Pr. × R. × Y. = \$550 × .05 × 4 = \$110.

Or, we may explain thus:

Since the rate per cent. is 5, the int. for 1 yr. = .05 of \$550 = \$27.50. Since the int. for 1 yr. = \$27.50, the int. for 4 yr. = 4 times \$27.50 = \$110.

2. What is the interest of \$500 for 5 yr. and 2 mo. at 6%?

Process.

$$5 \text{ yr. } 2 \text{ mo.} = 62 \text{ mo.}$$

$$\text{Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{mo.}}{12} = \frac{5\cancel{00} \times .\cancel{06} \times \overset{31}{\cancel{62}}}{\underset{\cancel{200}}{\cancel{12}}} = 5 \times 31 = \$155.$$

Explanation.

Since the int. is required for 62 mo., we use the formula, Int. = $\frac{\text{Pr.} \times \text{R.} \times \text{mo.}}{12} = \frac{500 \times .06 \times 62}{12}$. By cancellation we have 5 × 31 = \$155.

Or,

The int. of \$500 for 1 yr. at 1% = \$5.00; at 6%, therefore, it = \$5 × 6, for 1 yr.; for 1 mo. it $= \frac{\$5 \times 6}{12}$; for 62 mo. it $= \frac{\$5 \times \cancel{6} \times \overset{31}{\cancel{62}}}{\underset{\cancel{2}}{\cancel{12}}} = \$155.$

3. What is the interest of \$222.50 for 10 yr. 8 mo. 21 da. at 3%?

Process.

$$\left.\begin{array}{rl} 10 \text{ yr.} &= 120 \text{ mo.} \\ 8 \text{ mo.} &= \quad 8 \text{ mo.} \\ 21 \text{ da.} &= 0.7 \text{ mo.} \end{array}\right\} = 128.7 \text{ mo.} \quad \text{Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{mo.}}{12}.$$

$$= \frac{\overset{.55625}{\cancel{222.50}} \times \cancel{.03} \times 128.7}{\underset{400}{\cancel{12}}} = .55625 \times 128.7 = \$71.59.$$

Or,

$$\left.\begin{array}{rl} 10 \text{ yr.} &= 3600 \text{ da.} \\ 8 \text{ mo.} &= 240 \text{ da.} \\ 21 \text{ da.} &= 21 \text{ da.} \end{array}\right\} = 3861 \text{ da.} \quad \text{Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{da.}}{360} =$$

$$\frac{222.50 \times .03 \times 3861}{360} \text{ by cancellation } = \frac{.89 \times 1287}{16} = \$71.59.$$

NOTE.—The work may be simplified by removing decimals before cancelling.

EXERCISES.

$$\text{Int.} = \text{Pr.} \times \text{R.} \times \text{Y.}$$

1. Find the interest of:

1. \$100 for 1 yr. at 8%.
2. \$200 for 2 yr. at 6%.
3. \$300 for 3 yr. at 10%.
4. \$400 for 4 yr. at 7%.
5. \$500 for 5 yr. at 6%.
6. \$600 for 6 yr. at 10%.
7. \$700 for 7 yr. at 7%.
8. \$800 for 8 yr. at 6%.
9. \$900 for 9 yr. at 6%.
10. \$1000 for 10 yr. at 8%.

$$\text{Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{mo.}}{12}.$$

2. Find the interest of:

1. \$590 for 3 yr. 7 mo. at 7%.
2. \$600 for 4 yr. 11 mo. at 10%.
3. \$830 for 5 yr. 10 mo. at 6%.
4. \$950 for 5 yr. 9 mo. at 6%.
5. \$1070 for 6 yr. 11 mo. at 6%.
6. \$470 for 3 yr. 8 mo. at 10%.
7. \$2359 for 4 yr. 7 mo. at 12%.

8. \$3597 for 6 yr. 9 mo. at 8%.
9. \$2300 for 4 yr. 6 mo. at 6%.
10. \$7000 for 1 yr. 7 mo. at 7%.

3. Find the interest of:
    1. \$10 for 1 yr. 1 mo. 3 da at 5%.
    2. \$121 for 2 yr. 2 mo. 6 da. at 6%.
    3. \$25.16 for 3 yr. 3 mo. 9 da. at 6%.
    4. \$36.24 for 4 yr. 4 mo. 12 da. at 7%.
    5. \$48.20 for 5 yr. 5 mo. 15 da. at 7%.
    6. \$2000 for 6 yr. 6 mo. 18 da. at 6%.
    7. \$590.50 for 7 yr. 7 mo. 21 da. at 6%.
    8. \$640.82 for 8 yr. 8 mo. 24 da. at 10%.
    9. \$725.83 for 9 yr. 9 mo. 27 da. at 6%.
    10. \$618.24 for 10 yr. 10 mo. 3 da. at 6%.

$$\text{Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{da.}}{360}.$$

4. Find the interest of:
    1. \$7000 for 1 yr. 6 mo. 7 da. at 3%.
    2. \$8300 for 2 yr. 5 mo. 5 da. at 4%.
    3. \$670 for 4 yr. 8 mo. 8 da. at $4\frac{1}{2}$%.
    4. \$950 for 6 yr. 6 mo. 10 da. at 5%.
    5. \$500 for 8 yr. 7 mo. 11 da. at $5\frac{1}{2}$%.
    6. \$700 for 8 yr. 6 mo. 13 da. at 6%.
    7. \$3000 for 5 yr. 8 mo. 14 da. at 7%.
    8. \$600 for 4 yr. 9 mo. 16 da. at 8%.
    9. \$300 for 4 yr. 8 mo. 17 da. at 9%.
    10. \$536 for 3 yr. 10 mo. 19 da. at 10%.

The Amount equals the Principal plus the Interest.

1. Find the amount of:
    1. \$1 for 3 yr. 3 mo. 3 da. at 6%.
    2. \$125 for 4 yr. 4 mo. 4 da. at 6%.

3. \$24.50 for 5 yr. 5 mo. 5 da. at 7%.
4. \$1000 for 6 yr. 6 mo. 6 da. at 10%.
5. \$280.75 for 7 yr. 7 mo. 7 da. at 6%.

2. Find the interest of:
   1. \$2000 for 5 mo at $3\frac{1}{2}$%.
   2. \$6030 for 15 da. at $4\frac{1}{2}$%.
   3. \$700 for 6 mo. 20 da. at $5\frac{1}{2}$%.
   4. \$60.70 for 11 mo. 27 da. at $6\frac{1}{4}$%.
   5. \$400 for 30 da. at 6%.
   6. \$1670 from April 1 to Dec. 25 at 7%.
   7. \$4440 from Feb. 4 to June 8 at 5%.
   8. \$1060 from April 13, 1897, to Dec. 21, 1898, at $4\frac{1}{2}$%.

Six Per Cent. Method.

MODEL SOLUTION.

At 6% the interest of \$1.00 for one year = \$.06; for one month = $\frac{1}{12}$ of \$.06 = \$.00$\frac{1}{2}$; for one day = $\frac{1}{30}$ of \$.00$\frac{1}{2}$ = \$.000$\frac{1}{6}$.

Hence, writing 6 cents for every year, $\frac{1}{2}$ a cent for every month, and $\frac{1}{6}$ of a mill for every day, we have the formula:

$$\text{Int.} = \text{Pr.} \times \left\{ \begin{array}{ll} \$.06 & \times \text{ yr.} \\ .00\frac{1}{2} & \times \text{ mo.} \\ .000\frac{1}{6} & \times \text{ da.} \end{array} \right\} \text{add.}$$

What is the interest of \$236 for 3 yr. 4 mo. 18 da., at 6%?

Process.

$$\text{Int.} = \$236 \times \left\{ \begin{array}{l} .18 \\ .02 \\ .003 \end{array} \right\} = 236 \times .203 = \$47.91.$$

Explanation.

For 3 yr. we write \$.18; for 4 mo., \$.02; for 18 da., \$.003; the sum of these three is \$.203. Since the int. of \$1 is \$.203, the int. of \$236 is 236 times \$.203, or \$47.91.

EXERCISES.

1. Find the interest of:
    1. $560 for 3 yr.
    2. $636 for 5 mo.
    3. $700 for 60 da.
    4. $236 for 1 yr. 10 mo. 18 da.
    5. $35.60 for 4 yr. 9 mo. 24 da.
    6. $2000 for 4 mo. 15 da.
    7. $390.86 for 6 yr. 24 da.
    8. $3000 for 5 yr. 11 mo. 25 da.
    9. $6030 for 6 yr. 6 mo. 7 da.
    10. $700 for 4 yr. 8 mo. 9 da.
2. Find the amount of:
    1. $45.70 for 5 yr. 10 mo.
    2. $443.76 for 9 mo. 12 da.
    3. $1085.93 for 3 yr. 6 mo. 18 da.
    4. $627.92 for 5 yr. 8 mo. 19 da.
    5. $113.96 for 9 yr. 9 mo. 9 da.
    6. $5090 for 10 yr. 10 mo. 10 da.
    7. $3500 for 11 yr. 11 mo. 11 da.
3. Find the interest of $700 for 3 yr. 10 mo. 13 da., at 5%.

Process.

$$
\begin{array}{lll}
3 \text{ yr.} & = .18 & \\
10 \text{ mo.} & = .05 & \\
13 \text{ da.} & = \underline{.002\frac{1}{6}} & \\
 & \ .232\frac{1}{6} \times 700 = 162.517, \text{ int. at } 6\%. & \\
 & \qquad\qquad\qquad 6\,\underline{)\,162.517} & \\
 & \qquad\qquad\qquad\quad 27.086, \text{ int. at } 1\%. & \\
 & \qquad\qquad\qquad\quad \underline{\qquad\quad 5} & \\
 & \qquad\qquad\qquad \$135.430, \text{ int. at } 5\%. &
\end{array}
$$

NOTE.—Observe how the six per cent. method is applicable when other rates are given.

4. Find the interest of:
   1. \$760 for 3 yr. 11 mo. 12 da., at 5%.
   2. \$4030 for 5 yr. 3 mo. 7 da., at 7%.
   3. \$26.74 for 4 yr. 2 mo. 6 da., at $5\frac{1}{2}$%.
   4. \$3000 for 6 yr. 6 mo. 6 da., at $4\frac{1}{2}$%.
   5. \$2736 from July 12, 1897, to Sept. 15, 1898, at 5%.
   6. \$526 from Nov. 10, 1898, to June 16, 1900, at 7%.
   7. \$600 from May 15, 1890, to July 11, 1898, at $6\frac{1}{2}$%.

EXACT INTEREST.

Exactness requires that in reckoning interest for less than one year 365 days should be considered one year, and not 360 days. Hence

$$\text{Exact Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{exact No. of days}}{365}.$$

NOTE 1.—To find the exact number of days between two dates reckon each year as 365 days, and give to each month the number of days assigned it in the calendar. To be more exact, 366 days should be reckoned for each leap year and 29 days to February of every leap year, but in the following problems leap years are not considered.

NOTE 2.—Since 5 days $= \frac{1}{73}$ of 365 days, common interest diminished by $\frac{1}{73}$ of itself will give exact interest for any number of days less than 365. CAUTION.—*This does not apply to interest reckoned for one or more entire years.*

Find the exact interest of \$840, at 6%, from Mar. 3, 1894, to Aug. 24, 1897.

MODEL SOLUTION.

From Mar. 3, 1894, to Mar. 3, 1897 = 3 yrs. = 365 da. × 3 = 1095 days

$$\text{From Mar. 3, 1897, to Aug. 24, 1897} = \overset{\text{Mar.}}{28} + \overset{\text{Apr.}}{30} + \overset{\text{May}}{31} + \overset{\text{June}}{30} + \overset{\text{July}}{31} + \overset{\text{Aug.}}{24} = \underline{174 \text{ days}}$$

Exact No. of days = 1269 days

$$\text{Exact Int.} = \frac{\text{Pr.} \times \text{R.} \times \text{Exact No. of days}}{365} = \frac{\overset{168}{\cancel{840}} \times .06 \times 1269}{\underset{73}{\cancel{365}}} = \$175.23$$

EXERCISES.

1. Find the exact interest of:
   1. \$960 from Feb. 5, 1898, to Dec. 26, 1898, at 6%.
   2. \$2370 from Apr. 10, 1887, to Aug. 15, 1890, at 5%.
   3. \$3500 from Jan. 1, 1891, to Nov. 20, 1895, at 7%.
   4. \$2670 from May 29, 1890, to Mar. 4, 1891, at 6%.
   5. \$4440 from Feb. 5, 1898, to Dec. 25, 1898, at 5%.
2. Find the exact amount of:
   1. \$747.37 from March 22, 1896, to Aug. 5, 1896, at 6%.
   2. \$837.46 from April 3, 1896, to Dec. 21, 1897, at 9%.
   3. \$1094.94 from Sept. 2, 1896, to Sept. 2, 1898, at 10%.
   4. \$231.03 from Sept. 1, 1897, to Feb. 28, 1899, at 6%.
   5. \$556.44 from Jan. 1, 1893, to April 21, 1900, at $7\frac{1}{2}$%.

To Find the Principal, the Rate, and the Time.

FORMULÆ.

Since Interest = Pr. × R. × yr., obviously

$$1.\ \text{Pr.} = \frac{\text{Int.}}{\text{R.} \times \text{yr.}}. \qquad 2.\ \text{R.} = \frac{\text{Int.}}{\text{Pr.} \times \text{yr.}}. \qquad 3.\ \text{Yr.} = \frac{\text{Int.}}{\text{Pr.} \times \text{R.}}.$$

Again, since Pr. × R. × yr. + Pr. = Amount, and since Pr. × R. × yr. + Pr. = Pr. × (R. × yr. + 1), we have, Pr. × (R. × yr. + 1) = Amount, and

$$4.\ \text{Pr.} = \frac{\text{Amt.}}{\text{R.} \times \text{yr.} + 1}.$$

MODEL SOLUTIONS.

1. What principal will in 3 yr. 8 mo. 12 da. yield \$1117.48 interest at 5%?

Process.

$$\left.\begin{array}{lll}3\text{ yr.} &= 36 & \text{mo.}\\ 8\text{ mo.} &= 8 & \text{mo.}\\ 12\text{ da.} &= 0.4 & \text{mo.}\end{array}\right\} = 44.4\text{ mo.} = 3.7\text{ yr.}\quad \text{Pr.} = \frac{\text{Int.}}{\text{R.} \times \text{yr.}}$$

$$= \frac{1117.48}{.05 \times 3.7} = \frac{1117.48}{.185} = \$6040.43 +.$$

2. What principal will in 4 yr. 7 mo. 6 da. amount to \$859.52 at 4%?

Process.

$$\left.\begin{array}{lll}4\text{ yr.} &= 48 & \text{mo.}\\ 7\text{ mo.} &= 7 & \text{mo.}\\ 6\text{ da.} &= 0.2 & \text{mo.}\end{array}\right\} = 55.2\text{ mo.} = 4.6\text{ yr.}\quad \text{Pr.} = \frac{\text{Amt.}}{\text{R.} \times \text{yr.} + 1}$$

$$= \frac{859.52}{.04 \times 4.6 + 1} = \$725.946.$$

Or,

\$1.00 in the given time will amount to $.01 \times 4 \times 4.6 + 1.00 = \$1.184$. Since \$1.00 amounts to \$1.184, and since some number of dollars multiplied by 1.184 yields \$859.52, that number must be $\$859.52 \div 1.184$, or \$725.946.

3. At what rate per cent. will \$4220 produce \$503.235 interest in 2 yr. 7 mo. 24 da.?

Process.

$$\left.\begin{array}{lll}2\text{ yr.} &= 24 & \text{mo.}\\ 7\text{ mo.} &= 7 & \text{mo.}\\ 24\text{ da.} &= 0.8 & \text{mo.}\end{array}\right\} = 31.8\text{ mo.} = 2.65\text{ yr.}\quad \text{R.} = \frac{\text{Int.}}{\text{Pr.} \times \text{yr.}}$$

$$= \frac{503.235}{4220 \times 2.65} = \frac{503.235}{11183} = .045 = 4\tfrac{1}{2}\%.$$

4. In what time will the interest on \$4220 at $4\frac{1}{2}\%$ amount to \$503.235?

Process.

$$\text{Yr.} = \frac{\text{Int.}}{\text{Pr.} \times \text{R.}} = \frac{503.235}{42.20 \times 4.5} = 2.65 = 2 \text{ yr. } 7 \text{ mo. } 24 \text{ da.}$$

EXERCISES.

1. Find the principal that will:
   1. Produce \$180 int. in 6 yr. at 4%.
   2. Produce \$126 int. in 6 yr. at $6\frac{1}{4}$%.
   3. Produce \$200 int. in 16 yr. 6 mo. at 5%.
   4. Produce \$823.30 int. in 1 yr. 11 mo. at 6%.
   5. Produce \$6 int. in 14 mo. at 5%.
   6. Produce \$25 int. in 144 da. at $4\frac{1}{2}$%.
   7. Produce \$669.64 int. in 1 yr. 7 mo. 12 da. at 6%.
   8. Produce \$2624.65 int. in 2 yr. 6 mo. at 5%.
   9. Produce \$1680 in 6 yr. at 4%.
   10. Produce \$840 in 3 yr. at $4\frac{1}{2}$%.
   11. Amount to \$45,056.92 in 2 yr. 6 mo. at 5%.
   12. Amount to \$3000 in 42 da. at $5\frac{1}{2}$%.
   13. Amount to \$595.20 in 16 mo. at 6%.
   14. Amount to \$3189.375 in 2 yr. 2 mo. at 5%.
   15. Amount to \$10,523.475 in 1 yr. 11 mo. 21 da. at $4\frac{1}{2}$%.
   16. Amount to \$360.18 in 4 yr. 6 mo. 18 da. at 5%.
   17. Amount to \$770.50 in 2 yr. 7 mo. 15 da. at 6%.
   18. Amount to \$47,187.58 in 3 yr. 8 mo. 25 da. at $4\frac{1}{2}$%.
   19. Amount to \$5133.30 in 4 yr. 6 mo. 27 da. at 6%.
   20. Amount to \$950 in 3 yr. 3 mo. 3 da. at 7%.
2. At what rate will:
   1. \$1800 gain \$396 in 3 yr. 8 mo.?
   2. \$852 gain \$106.50 in 2 yr. 6 mo.?
   3. \$660 gain \$192.50 in 5 yr. 10 mo.?
   4. \$840 gain \$107.80 in 2 yr. 4 mo.?

5. \$144 gain \$128.52 in 12 yr. 9 mo.?
6. \$220 gain \$82.36 in 3 yr. 8 mo.?
7. \$420 gain \$42.30 in 2 yr. 9 mo. 24 da.?
8. \$9.10 gain \$5.115 in 9 yr. 9 mo. 9 da.?
9. \$100 double itself in 3 yr.? 5 yr.? 6 yr.?
10. Any principal treble itself in 7 yr.? 8 yr.? 20 yr.?

3. Find the time in which:
   1. \$500 will produce \$60 interest at 6%.
   2. \$1200 will produce \$48 interest at 8%.
   3. \$230 will produce \$27.60 interest at 6%.
   4. \$25.20 will produce \$8.30 interest at 7%.
   5. \$70.50 will produce \$26.50 interest at 7%.
   6. \$50 will produce \$50 interest at 6%.
   7. \$300 will double itself at 8%.
   8. \$200 will double itself at 5%. 6%. 7%.
   9. Any principal will double itself at $4\frac{1}{2}$%.
   10. Any principal will treble itself at 6%. 7%. 8%.

PROBLEMS.

1. Find the exact interest of \$680.20, at $7\frac{1}{2}$%, for 73 days.

2. What sum, bearing interest at $4\frac{1}{2}$%, will yield an annual income of \$1500?

3. Find the amount of \$1040 for 2 mo. 3 da., at 6%.

4. How long must \$1952.46 be on interest, at 6%, to amount to \$2284.38?

5. At what rate per cent. will \$6000 produce \$500 interest in 1 yr. 10 mo. 7 da.?

COMPOUND INTEREST.

Compound Interest is interest computed, at certain intervals, on both the principal and unpaid interest. Such intervals are commonly 1 yr., 6 mo., or 3 mo.

MODEL SOLUTIONS.

1. Find the amount of $70, at compound interest for 3 yr., at 6%; also the compound interest.

Process.

Int. for 1st yr. = Pr. × R. × yr. = 70 × .06 = $4.20. Amt. = $74.20.

Int. for 2d yr. = 74.20 × .06 = $4.45. Amt. = $78.65.

Int. for 3d yr. = 78.65 × .06 = $4.72. Amt. = $83.37.

(Amt.) $83.37 — (Pr.) $70.00 = $13.27, compound interest.

Hence the formula:

Compound Int. = Final Amount — Principal.

2. Find the compound interest of $630 for 2 yr. 6 mo., at 5%.

Process.

Amt. for 1st yr. = $661.50.
Amt. for 2d yr. = 694.58.
Amt. for 6 mo. = 729.31.
$729.31 — $630 = $99.31.

Explanation.

2 yr. = two full intervals; 6 mo. = ½ an interval. We therefore find the amount of $694.58 for the half interval, 6 mo.

PROBLEMS.

1. Find the compound interest of $200, at 7%, for 3 yr. 6 mo.

2. What is the amount of $458.50 for 2 yr., interest compounded semi-annually, at 6%?

Suggestion: Compute for four intervals at 3%.

3. Compute the compound interest of $580 for 1 yr. 3mo., interest compounded quarterly, at 8%.

Five intervals, 2%.

4. Find the compound interest, at 6%, on $2000 for 1 yr. 10 mo., interest payable semi-annually.

5. What is the compound interest of $525.75 for 3 yr. 4 mo., at 6%?

6. Find the compound interest on $1050 for 1 yr. 6 mo., at 5%, interest being compounded quarterly.

7. Compute the compound interest of $600 for 2 yr. 3 mo., at 4%, interest being compounded semi-annually.

8. Find the compound interest of $20,000 for 6 mo., at 6%, interest being compounded monthly.

ANNUAL INTEREST.

Annual Interest is interest on the *principal and each year's interest* from the time each interest is due until settlement. Annual interest is computed when the words "with interest payable annually" are in the contract.

MODEL SOLUTION.

Find the interest of $300 for 3 yr. 6 mo. 20 days at 4%, payable annually.

Process.

3 yr. 6 mo. 20 da. = 1280 da.

$$\text{Int.} = \frac{\overset{10}{\cancel{300}} \times \overset{.01}{\cancel{.04}} \times 1280}{\underset{\underset{3}{\cancel{90}}}{\cancel{360}}} = \frac{128}{3} = \$42.67, \text{ for the whole time.}$$

Int. for each of the 3 yr. = $12.00. The $12 will be on interest:

Firstly, for 2 yr. 6 mo. 20 da.
Secondly, for 1 yr. 6 mo. 20 da.
Thirdly, for 6 mo. 20 da.
4 yr. 8 mo. = total time = 56 mo.

$$\text{Int.} = \frac{\cancel{12} \times .04 \times 56}{\cancel{12}} = 56 \times .04 = \$2.24.$$ Total Int. = $42.67 + $2.24 = $44.91.

Hence the following brief directions:

1. **Find int. of Pr. for whole time.**
2. **Find int. of Pr. for one yr.**
3. **Find the sum of the time intervals.**
4. **Find int. on the one year's int., for the sum of the time intervals.**
5. **Find the sum of int. first found and int. last found.**

EXERCISES.

1. Find the annual interest of:
    1. $360 for 4 yr. 5 mo. 16 da. at 6%.
    2. $250 for 3 yr. 9 mo. 12 da. at 7%.
    3. $3500 for 4 yr. 6 mo. at 6%.
    4. $1247.75 for 3 yr. 5 mo. 10 da. at 6%.
    5. $987.25 for 4 yr. 9 mo. 6 da. at 4%.
    6. $1098.36 for 5 yr. 10 mo. 7 da., at 5%.
2. Find the amount, at annual interest, of:
    1. $360 for 4 yr. 5 mo. 16 da. at 5%.
    2. $250 for 3 yr. 9 mo. 12 da. at 7%.
    3. $600 for 3 yr. 4 mo. 12 da. at 6%.
    4. $840 for 4 yr. 8 mo. 18 da. at $5\frac{1}{2}$%.
    5. $2180 for 6 yr. 11 mo. 27 da., at $4\frac{1}{2}$%.
    6. $1070 for 5 yr. 10 mo. 24 da. at 4%, the interest of the first two years having been paid.

PROMISSORY NOTES.

1. A **Promissory Note** is a promise, made in writing, to pay a sum of money *on demand* or at a *specified time*.

2. The **Face** of a note is the sum of money named in it.

3. The **Maker** of a note *signs* it. The **Payee** receives payment for it. The **Holder** has rightful possession of it.

4. The **Endorser** of a note writes his name on the back of it, and thus becomes responsible for payment of it.

5. A **Negotiable Note** is one that is transferable.

6. Notes are said to be negotiable or transferable when they contain the words "or bearer," or "or order," but no transfer of the latter can be made without the endorsement of the payee.

To insure the negotiability of a note, in Pennsylvania the words "without defalcation" should be added. In New Jersey the words "without defalcation or discount" should be added; in Missouri, "negotiable and payable without defalcation or discount."

7. The words "with interest" render the note interest-bearing from its date.

8. A note not containing the words "with interest" begins to bear interest at maturity if not paid.

9. The words "value received" are proof that the note represents actual value.

10. The day of *maturity* of a note is the day when it becomes due.

11. In any case, when the rate per cent. is not specified the legal rate is always understood.

12. Interest computed at a higher rate than the law allows is called *usury*.

13. In many States the time of payment is postponed *three days*, called "Days of Grace."

14. A *Protest* is a notice sent to the endorsers that the maker of the note has failed to pay it. The protest, to be valid, must not be sent later than the last day of grace.

15. A note signed by two or more persons, who thus become jointly and severally responsible for its payment, is called a Joint or Several Note.

Forms of Notes.

(1.)

486\frac{25}{100}$

CHICAGO, ILL., Sept. 1, 1898.

Three months after date, *I* promise to pay *Edward L. Baker*——————————————— *Four hundred eighty-six*——————————— $\frac{25}{100}$ *Dollars, with interest at 5%,* for value received.

ROBERT H. KING.

(2.)

486\frac{25}{100}$

WASHINGTON, D. C., Sept. 1, 1898.

Four months after date, *I* promise to pay *Edward L. Baker*——————————————or bearer, *Four hundred eighty-six*——————————— $\frac{25}{100}$ *Dollars, with interest at 7%,* for value received.

ROBERT H. KING.

(3.)

486\frac{25}{100}$

PHILADELPHIA, PA., Sept. 1, 1898.

Six months after date, *I* promise to pay *Edward L. Baker*——————————————or order, *Four hundred eighty-six*——————————— $\frac{25}{100}$ *Dollars,* without defalcation, for value received.

ROBERT H. KING.

(4.)

486\frac{25}{100}$

TRENTON, N. J., Sept. 1, 1898.

On demand, I promise to pay *Edward L. Baker* *Four hundred eighty-six*——————————— $\frac{25}{100}$ *Dollars, with interest at 6%,* without defalcation or discount.

ROBERT H. KING.

(5.)

486\frac{25}{100}$

ST. LOUIS, MO., Sept. 1, 1898.

Four months after date, we jointly and severally promise to pay *Edward L. Baker*——————or order, *Four hundred eighty-six*——————$\frac{25}{100}$ *Dollars, with interest at 3%,* for value received, negotiable and payable without defalcation or discount.

ROBERT H. KING.
JOHN C. TAYLOR.

(6.)

486\frac{25}{100}$

ATLANTA, GA., Sept. 1, 1898.

Sixty days after date, *I* promise to pay *Edward L. Baker*——————or order, *at the Atlanta National Bank*—————— *Four hundred eighty-six*——————$\frac{25}{100}$ *Dollars, with interest,* value received.

ROBERT H. KING.

QUESTIONS AND EXERCISES.

1. Find when the above notes will severally mature.
2. Compute the amount due on each at maturity.
3. Point out which are negotiable and which non-negotiable.
4. Point out which are interest-bearing from date and which from maturity.
5. When may a "demand" note be collected?
6. When is a time note collectible?
7. When no rate per cent. is specified in a note, what rate is understood?
8. What is the legal rate in your State?
9. Can a note be protested after its maturity?

10. If the maker of a note fails to pay it, who is held responsible for payment of it?

11. Write a negotiable note, in favor of George Hudson, for $500.50, using your own name as that of maker.

12. Write a note from the following data: Face, $347.56; negotiable; time, 60 days; payee, George Jones; maker, Hiram Smith; rate, 6%; place, Reading, Pa.

13. Write a non-negotiable note.

14. Write a note that will bear interest from date.

15. Write a note payable at a bank.

16. Write a note with an endorsement.

NOTE.—Latin, *dorsum*, the back.

17. Write a note payable with *annual* interest.

18. Assume a date for settlement, and compute the amt. due on said note.

19. Find the day of maturity and amount due, having given:

1. $631.36, Feb. 13, 1898, 63 da., 6%.
2. $796.56, Apr. 23, 1898, 90 da., 5%.
3. $397.86, Sept. 6, 1898, 5 mo., 4%.
4. $1055.51, Nov. 21, 1898, 4 mo., 6%.
5. $631.36, Nov. 6, 1898, 33 da., 7%.
6. $937.72, Jan. 17, 1898, 6 mo., 8%.
7. $2632.98, Apr. 30, 1898, 1 mo., 10%.
8. $2849.65, June 23, 1898, 15 da., 6%.
9. $984.05, Aug. 11, 1898, 3 yr., 4½%.
10. $1968.10, Sept. 3, 1898, 3 mo., 5½%.

PARTIAL PAYMENTS.

1. The payment of part of a note or other obligation is called a **Partial Payment**.

2. Notes on which partial payments have been endorsed are computed chiefly by two rules: **The Merchants' Rule** and **The United States Rule.**

The Merchants' Rule.

The *Merchants' Rule* applies to notes settled *within a year*. The method is as follows:

MODEL SOLUTION.

600\frac{00}{100}$.

PITTSBURG, PA., Feb. 25, 1898

For value received, I promise to pay John Wayland, or order, Six Hundred Dollars, on demand, with interest from date.

JAMES BROWN.

On this note were made the following payments: May 25, 1898, $156.00; Aug. 25, 1898, $200.00; Nov. 25, 1898, $185.00. What was due on Feb. 20, 1899?

Process.

Date of settlement,	1899, 2, 20.
Date of note,	1898, 2, 25.
Interval,	11 mo. 25 da.

Principal (note)	$600.00.
Interest for 11 mo. 25 da.	35.50.
Amount	$635.50.

1899, 2, 20.	1899, 2, 20.	1899, 2, 20.
1898, 5, 25.	1898, 8, 25.	1898, 11, 25.
8 mo. 25 da.	5 mo. 25 da.	2 mo. 25 da.

1st payment and int. for 8 mo. 25 da.	$162.89.	
2d payment and int. for 5 mo. 25 da.	205.84.	
3d payment and int. for 2 mo. 25 da.	187.62.	556.35.
Balance due at settlement		$79.15.

Hence the formula:

Balance = Amount of Face due at time of settlement — the sum of the Payment-Amounts due at time of settlement.

PROBLEMS.

1. A note for $960, on demand, with interest at 7%, dated Feb. 1, 1897, was endorsed as follows: May 11, 1897, $300; Oct. 16, 1897, $366. How much was due Dec. 16, 1897?

2. What amount is due Nov. 27, 1898, on a note for $800, dated Jan. 15, 1898, with interest at 6%, on which are the following endorsements: May 3, 1898, $300; July 9, 1898, $400?

3. A man holds a note of $460, dated Jan. 20, 1898, on which the following payments are endorsed: $140, Mar. 26, 1898; $100, June 16, 1898; $160, Oct. 14, 1898. Settlement is made Dec. 22, 1898. Find the balance due, interest at 5%.

4. What is due Dec. 20, 1898, on a note for $1300, dated Feb. 10, 1898, with interest at $6\frac{1}{2}$%, on which is the following endorsement: June 7, 1898, $900?

5. A note of $1100, dated April 1, 1898, payable on demand, with interest at 6%, bears the following endorsements: June 6, $300; Aug. 5, 236.48; Nov. 19, $333. What is due Jan. 1, 1899?

(6.)

696\frac{36}{100}$

BIRMINGHAM, ALA., April 4, 1898.

Nine months after date, for value received, I promise to pay to the order of Paul Stakeman, Six Hundred Ninety-six and $\frac{36}{100}$ Dollars, with interest at 8%.

ROBERT S. CAMPBELL.

This note was endorsed as follows: July 9, 1898, $436; Sept. 4, 1898, $95.40; Oct. 3, 1898, $100. What was due on the note at maturity?

7. A note of $946.36, dated Aug. 1, 1898, payable on demand, with interest at $5\frac{1}{2}$%, bears the following endorse-

ments: Sept. 21, \$268.60; Oct. 22, \$280.36; Nov. 6, \$300; Dec. 2, \$90. What remained due after Dec. 2?

8. A note for \$4300, dated Feb. 8, 1898, has the following endorsements on it: Mar. 20, 1898, \$900; April 20, 1898, \$700; July 25, 1898, \$600; Aug. 17, 1898, \$500; Nov. 25, 1898, \$400. What is due Jan. 1, 1899, at 6%?

(9.)

$\$9600\frac{00}{100}$

HARRISBURG, PA., July 20, 1898.

Thirty * days after date, for value received, I promise to pay Charles Davenport, or order, Nine Thousand Six Hundred Dollars, without defalcation.

JAMES H. BOYD.

Endorsements: Aug. 30, \$200; Oct. 12, \$400; Nov. 10, \$600; Dec. 20, \$800. What is due July 15, 1899?

10. A note for \$7000, at 90 days, dated Sept. 25, 1898, has the following endorsements: Dec. 31, \$300; Jan. 18, 1899, \$500; May 20, 1899, \$600; Aug. 10, \$100. What was due Sept. 1, 1899?

11. On a note for \$2898, dated Jan. 4, 1897, and bearing interest at $5\frac{1}{2}\%$, the following payments were made: Jan. 20, 1897, \$600; Feb. 25, 1897, \$700; June 10, 1897, \$400; Sept. 30, 1897, \$360. How much is due Jan. 1, 1898.

The United States Rule.

This rule applies to notes settled beyond the limit of a year, and *forbids* the deducting of a payment unless the payment *equals* or *exceeds* the interest due. The compounding of interest is thus prevented.

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* Interest must not be computed for these 30 days.

MODEL SOLUTION.

$\$475\frac{50}{100}$

NEW YORK, May 1, 1895.

I promise to pay George Jenkins, or order, Four Hundred Seventy-five$\frac{50}{100}$ Dollars, on demand, with interest at 7%, for value received.

THOMAS WRIGHT.

Endorsements: Dec. 25, 1895, $50.00; July 10, 1896, $15.75; Sept. 1, 1897, $25.50; June 14, 1898, $104.00. How much is due April 15, 1899?

Process.

Face of note		$475.50.
Date of 1st pay't, 1895, 12, 25.		
Date of note, 1895, 5, 1.		
Interval 7 mo. 24 da.		
Int. of face for interval		21.64.
Amount		$497.14.
1st payment (exceeding interest)		50.00.
1st balance		$447.14.

As the next two payments will not equal or exceed the interest due, we compute the interval to June 14, 1898.

Date of 4th pay't, 1898, 6, 14.		
Date of 1st pay't, 1895, 12, 25.		
Interval, 2 yr. 5 mo. 19 da.		
Int. of 1st balance for interval		77.29.
Amount		$524.43.
2d payment	$15.75	
3d payment	25.50	
Sum less than int. due	$41.25	
4th payment	104.00	145.25.
2d balance		$379.18.
Date of settlement, 1899, 4, 15.		
Date of 4th pay't, 1898, 6, 14.		
Interval, 10 mo. 1 da.		
Int. of 2d bal. for interval		22.19.
Balance due April 15, 1899		$401.37.

RULE.

1. Find the amount of the principal to the given date at which the payment, or sum of the payments, is equal to or greater than the interest.

2. From this amount deduct the payment, or the sum of the payments.

3. Consider the remainder as a new principal and proceed as before.

PROBLEMS.

(1.)

3000\frac{00}{100}$

PHILADELPHIA, PA., Feb. 26, 1895.

On demand, I promise to pay George Palmer Three Thousand Dollars, with 6% interest.

JOHN JAY.

Payments were made on this note as follows: Sept. 10, 1895, $25.00; Jan. 1, 1896, $500.00; Oct. 25, 1896, $75.00; April 4, 1897, $1500.00. How much was due Feb. 20, 1898?

(2.)

750\frac{00}{100}$

LYNN, MASS., July 15, 1893.

Three months after date, I promise to pay George Mason, or order, Seven Hundred and Fifty Dollars, for value received.

GEORGE PALMER.

Payments were made as follows: Aug. 3, 1896, $75.00; May 1, 1897, $560.00. What was due Feb. 20, 1898?

3. A note was given Jan. 1, 1890, for $700. The following payments were endorsed upon it: May 6, 1890, $85; July 1, 1891, $40; Aug. 20, 1891, $100; Jan. 10, 1893, $350. How much was due Sept. 30, 1894, interest at 6%?

4. What is due Aug. 18, 1894, on a note for $400, dated April 1, 1892, with interest at 6%, on which are the following

endorsements: Jan. 13, 1893, $50; Sept. 22, 1893, $10; April 25, 1894, $125.

(5.)

750\frac{00}{100}$

PHILADELPHIA, PA., May 10, 1895.

Three years after date, for value received, I promise to pay Thomas Newbury, or order, Seven Hundred and Fifty Dollars, with interest, without defalcation.

SAMUEL TOWNSEND.

Endorsements: Jan. 15, 1896, $124.75; Sept. 12, 1896, $20; Dec. 16, 1897, $216.80. How much remained due May 10, 1898?

6. A note for $5600 was given May 1, 1895, and was endorsed as follows: Oct. 17, 1895, $350; Feb. 18, 1896, $455; July 10, 1896, $318.50. What was due May 1, 1897, interest at 7%?

7. What is due Dec. 31, 1898, on a note for $2800, dated Oct. 10, 1894, with interest at 7%, on which are the following endorsements: April 1, 1895, $66.60; July 21, 1896, $300; June 15, 1898, $300?

(8.)

4600\frac{00}{100}$

CHICAGO, ILL, April 6, 1896.

On demand, I promise to pay George K. Brown, or order, Four Thousand Six Hundred Dollars, for value received, with interest at 5%.

CHARLES MOREHEAD.

Endorsements: July 10, 1897, $1360; Oct. 4, 1897, $500; Jan. 16, 1898, $660; June 21, 1898, $700. How much was due Jan. 21, 1899?

9. A note for $8580 was given July 12, 1894. Endorsed: Jan. 8, 1895, $300; April 26, 1896, $500; July 16, 1897,

$335; Oct. 8, 1897, $250. What was due at settlement, Jan. 1, 1898?

10. A note was drawn in Michigan for $2774.65, payable in 2 years, with interest, and dated March 15, 1896. Payments were made as follows: July 30, 1896, $100; Dec. 8, 1896, $200; Jan. 5, 1897, $250; May 17, 1897, $600; Jan. 1, 1898, $600. How much remained unpaid, April 1, 1898?

BANK DISCOUNT.

1. A **Bank** is an institution established for the purpose of *receiving, loaning,* and *issuing* money.

NOTE.—All banks do not issue money.

2. For cashing notes in advance of their maturity, banks make a deduction from their face value. This deduction is called **Bank Discount.**

3. Bank discount depends upon *Face, Rate,* and *Time,* and is computed precisely like simple interest.

4. The *Time of Discount* of a note is the interval between the *day of its presentation* and the *day of its maturity.* This interval is commonly called *time to run.*

NOTE.—In some States the time to run is increased by 3 days, called "Days of Grace."

5. The *Proceeds* of a note equal its *Face less the Discount.*

MODEL SOLUTION.

2360\frac{00}{100}$

PHILADELPHIA, PA., Feb. 26, 1897.

Three months from date, I promise to pay to the order of George Gross, at the West Philadelphia Bank, Twenty-three Hundred and Sixty Dollars, for value received.

JAMES JENKINS.

This note was presented at bank for discount April 1, 1897. Find: 1. The day of maturity. 2. The time to run. 3. The discount. 4. The proceeds.

Process.

Feb. 26, 1897 + 3 mo. = May 26, 1897, the day of *maturity*.

Day of maturity,	1897	5	26
Day of presentation,	1897	4	1
		1 mo.	25 da., *time to run*.

$$\text{Discount for 55 da.} = \frac{2360 \times .06 \times 55}{360} = \$21.64.$$

Face of note	=	$2360.00
Discount	=	21.64
Proceeds	=	$2338.36

That is, the bank took the note and paid in cash for it $2338.36.

Hence the brief directions are:

1. **Find the day of maturity.**
2. **Find the time to run.**
3. **Find the discount (simple interest).**
4. **Find the proceeds (subtract discount from face).**

EXERCISES.

Find the discount and proceeds of:

1. $350 for 30 da. at 5%.
2. $400 for 90 da. at 6%.
3. $540 for 60 da. at 7%.
4. $600 for 60 da. at 8%.
5. $2000 for 3 mo. at 10%.
6. $80.60 for 90 da. at $5\frac{1}{2}$%.
7. $5000 for 18 da. at $6\frac{1}{2}$%.
8. $780 for 40 da. at $7\frac{1}{2}$%, with grace.
9. $600 for 2 mo. 12 da. at $8\frac{1}{2}$%, with grace.
10. $1000 for 90 da. at 10%, with grace.

PROBLEMS.

Apply the brief directions to the following notes:

(1.)

500\frac{00}{100}$

SAN FRANCISCO, CAL., Feb. 20, 1898.

Sixty days after date, I promise to pay James Warner, or order, Five Hundred Dollars, value received.

JOHN GORDON.

Discounted Mar. 15, 1898, at 7%.

(2.)

800\frac{00}{100}$

BALTIMORE, MD., Feb. 1, 1898.

Ninety days after date, I promise to pay to the order of Peter Welsh Eight Hundred Dollars, for value received.

HENRY BRYCE.

Discounted April 1, 1898, at 6%.

(3.)

400\frac{00}{100}$

PHILADELPHIA, PA., Jan. 5, 1898.

Ninety days after date, I promise to pay Charles Garrett, or order, Four Hundred Dollars at the Girard Bank, for value received, without defalcation.

JOHN WATERMAN.

Discounted Jan. 10 at 6%.

(4.)

465\frac{75}{100}$

WASHINGTON, D. C., April 20, 1898.

Six months after date, for value received, I promise to pay Alfred Rickert, or order, Four Hundred Sixty-five$\frac{75}{100}$ Dollars, at the First National Bank.

WESLEY EVANS.

Discounted June 23 at 7%.

5. Find the bank discount of a note for \$3600, dated March 6, 1898, and payable 3 mo. after date, with interest at 5%, if discounted May 13, 1898, at 6%.

6. Find the proceeds of a note for \$2400, dated Aug. 26, 1898, payable 90 days after date, with interest, at $5\frac{1}{2}$%, and discounted Oct. 1, 1898, at 6%.

To find the Face of a Note.

It is sometimes necessary to determine what face to give a note in order to secure a certain sum as proceeds.

Find the face of a note that, discounted for 60 days at 6%, will yield \$500 as proceeds.

Process.

Discount of \$1.00 = .01.
Proceeds of \$1.00 = 1.00 — .01 = .99.
\$500 ÷ .99 = \$505.05.

Explanation.

Since \$1.00, as face, discounted for 60 da. at 6%, yields \$.99, the question is, How many dollars as face will yield \$500 as proceeds? Obviously, \$500 ÷ .99 = \$505.05.

Hence the formula:

Face = Given Proceeds ÷ Proceeds of \$1.00.
Face = Given Discount ÷ Discount of \$1.00.

EXERCISES.

Find the face in each of the following instances:

1. Proceeds, \$800; time 60 da.; rate, 6%.
2. Proceeds, \$989.50; time, 2 mo.; rate, 6%.
3. Proceeds, \$3000; time, 90 da.; rate, 6%.
4. Proceeds, \$15,000; time, 2 mo.; rate, 7%. Grace.
5. Proceeds, \$240; time, 3 mo.; rate, 5%. Grace.
6. Proceeds, \$975; time, 2 mo.; rate, 7%. Grace.
7. Discount, \$40; time, 90 da.; rate, 6%.
8. Discount, \$4.18; time, 60 da.; rate, 6%.

9. Discount, \$8.48; time, 60 da.; rate, 5%.
10. Discount \$17.50; time, 2 mo. 12 da.; rate, 7%.

PROBLEMS.

1. I wish to borrow \$400 at a bank. For what sum must I draw my note, payable in 60 da., so that when discounted at 6% I shall receive the desired sum?

2. What is the face of a note at 60 days which yields \$780 when discounted at a bank? Rate, 5%.

3. Suppose you buy goods in Philadelphia to the amount of \$1248.50, and give your note in payment drawn at 6 mo. What must be the face of the note?

4. For how large a sum must a note be drawn, payable in 3 mo., that the net proceeds may be \$7500 after deducting the bank discount at 8%?

5. A Chicago merchant sold goods, and received in payment for them a 6-mo. note, which he had immediately discounted at 7%. If he received \$1898 in cash for the note, for what sum had he sold the goods?

6. For what amount must a note be made payable in 3 mo., so that when discounted in Baltimore at the legal rate (6%), the proceeds may be \$1420.

7. For what sum must a note be drawn, payable in 3 mo., so that when discounted in Montana at the legal rate (10%), the proceeds may be \$1000?

8. In Oregon I suffered a discount of \$6.18 on a 6-mo. note at the legal rate (8%). Find the face of my note?

TRUE DISCOUNT.

1. The **Present Worth** of a debt is a sum which, put at interest, *amounts to the debt* when due.

2. **True Discount** is the *difference between the present worth and the debt.* Finding the present worth is the same as finding

what principal will in a given time, and at a given rate, amount to a given sum.

Hence we have, from page 256, Pr. $= \frac{\text{Amt.}}{\text{R.} \times \text{yr.} + 1}$, which becomes:

$$\textbf{P. W.} = \frac{\textbf{Amt. or Debt}}{\textbf{R.} \times \textbf{yr.} + \textbf{1}}.$$

MODEL SOLUTION.

What present worth, or principal, will amount to $1000 in 8 mo. at 6%? Also, find the true discount.

Process.

$$\text{P. W.} = \frac{\text{Amt.}}{\text{R.} \times \text{yr.} + 1} = \frac{\$1000}{.06 \times \frac{2}{3} + 1} = \frac{\$1000}{1.04} = \$961.54.$$

$1000 — $961.54 = $38.46, true discount.

Explanation.

Since the P. W. stands to the Amt. in the relation of principal, we use the formula, P. W. $= \frac{\text{Amt}}{\text{R.} \times \text{yr.} + 1}$, and obtain $961.54. $1000 — $961.54 = $38.46, true discount.

Or, we may say: $1.00 amounts to $1.04; therefore, it will require the quotient of $1000 ÷ 1.04 to amount to $1000. Hence the P. W. = $961.54.

EXERCISES.

Find the present worth and true discount of:

1. $400, due 1 yr. hence, at 6%.
2. $200, due $1\frac{1}{2}$ yr. hence, at 6%.
3. $180, due 1 yr. 5 mo. hence, at 5%.
4. $600, due 2 yr. 3 mo. hence, at 8%.
5. $350, due 2 yr. 6 mo. 9 da. hence, at 7%.
6. $1500, due 2 mo. 21 da. hence, at 5%.
7. $2000, due 2 yr. 3 mo. 6 da. hence, at 6%.
8. $487.75, due 3 yr. hence, at 7%.
9. $422.00, due $2\frac{1}{4}$ yr. hence, at 6%.
10. 479.37\frac{1}{2}$, due 3 yr. hence, at 5%.

PROBLEMS.

1. Find the present worth and true discount of \$200, due in 3 yr. 8 mo. 16 da., at $5\frac{1}{2}\%$.

2. Find the *bank* discount on \$1000, due in 9 mo., without grace, money being worth 6%.

3. Find the difference between the bank discount and the true discount of \$1000, due in 9 mo., rate 5%.

4. Find the true discount on \$980, due in 6 mo., money being worth $4\frac{1}{2}\%$.

5. Money being worth 6%, find the difference between the true discount and the bank discount of a note for \$525, due in 10 mo., without interest.

6. If I buy goods for \$3000 on 3 mo. credit, what discount should I receive if I pay cash, money being worth $5\frac{1}{2}\%$?

7. If I pay a debt of \$9450 2 yr. 6 mo. 15 da. before it is due, what discount should I receive, money being worth 8%?

8. What is the aggregate present worth of two notes, each for \$800, due at the end of one and three years respectively, the rate of bank discount being 7%?

9. I wish to place at 6% interest a sum that will amount to \$832.50, from Jan. 9, 1897, to Nov. 9, 1898. What is the sum?

10. If you owe \$500, to be paid in 1 yr., without interest, what ought you in equity to pay *now* in order to cancel the debt, if money is worth 7%?

REVIEW.

1. On property worth \$15,000, fire caused a loss of \$3840. Find the rate per cent. of loss.

2. An agent makes 20% by selling a book for \$2.88. Had he sold it for \$4.00, what per cent. would he have made?

3. Find the interest on $960 for 7 yr. 6 mo. 27 da., at $4\frac{1}{2}\%$. Also, find the interest at 9%.

4. Find the rate per cent. when $1758 amounts to $1869.34 in 8 mo.

5. Find the time when the principal, at 7%, is doubled.

6. What principal will amount to $2222.22 in 2 yr. 2 mo. 2 da., at 5%?

7. The face of a note is $1975; its date, Sept. 12, 1898; its time, 3 mo.; its day of discount, Sept. 26, 1898; its rate of discount, $5\frac{1}{2}\%$. Find its day of maturity, etc.

8. Find the compound interest of $4000 for 2 yr. 6 mo., at 5% per annum.

9. Find the annual interest of $1600 for 4 yr. 8 mo., at 6%. Also, find the annual interest at 4%.

10. Find the present worth of $6450, due in 6 mo., without grace, money being worth 6%.

11. Find the proceeds of a note for $2500, payable in 90 da., without grace, discount, $5\frac{1}{2}\%$.

12. How much greater is the interest on $25,000 for 3 yr. 6 mo., at 6%, at compound interest, than at annual interest?

13. What was due Jan. 1, 1898, on a note for $1150, dated Sept. 1, 1894, at 7%?

14. The interest on a note from Aug. 3 to Dec. 27, at 10% per annum, was $33.00. What was the face of the note?

(15.)

$\$850\frac{00}{100}$.

PROVIDENCE, R. I., April 29, 1890.

For value received, we promise to pay to Webster, Arnold & Co., Eight Hundred and Fifty Dollars, ninety days after date, with interest at 6%.

CHARLES HATHAWAY.
JOHN TODD.

Endorsements: Oct. 13, 1890, \$40; Jan. 9, 1891, \$32; Aug. 21, 1891, \$125; Dec. 1, 1891, \$10; March 16, 1892, \$80. What was due Nov. 11, 1892?

16. What is the difference between the simple and the compound interest of \$2362.75 for 2 yr. 2 mo. 2 da. at 10%?

17. Having sold 15% of my stock one month, 10% of it the next month, and 25% the third month, I had remaining \$2650 worth of goods. What stock had I before I began to sell?

18. Discounted a note of \$309.59 for 90 days, at 10%, and invested the proceeds in flour at \$10 per barrel. How many barrels did I purchase?

19. If at $7\frac{3}{10}$% discount \$75.15 is received on a 60-day note 20 days after its date, what is the face?

20. What principal at $6\frac{3}{4}$% interest will gain \$85.60 from May 4, 1897, to Jan. 6, 1898?

21. If $\frac{7}{18}$ of the price received for an article equals the loss, what is the loss per cent.?

22. Find the exact interest of \$1200 at 5% from Aug. 19, 1896, to March 4, 1898.

23. Write a note that will be negotiable either in Pennsylvania or New Jersey.

EXCHANGE.

1. A **Bank Draft**, or **Bill of Exchange**, is a written order directing one person to pay a specified sum to another.

2. A **Sight Draft** directs payment to be made *at sight* or on presentation.

3. A **Time Draft** directs payment to be made at a certain time *after sight or date.*

4. An **Acceptance** is a draft with the word "accepted" written across its face, together with the name of the *acceptor,* who thus makes *himself* responsible for payment.

5. The method of paying by draft money due at a distance is called **Exchange.**

6. Drafts are bought and sold, and are described as *at par, i.e.*, as having their face value; as *at a premium, i.e.*, as having more than their face value; and as *at a discount, i.e.*, as having less than their face value.

General Form of a Draft.

No. 275. *Philadelphia, Pa., May 10, 1899.*

At sight [or . . . . days after sight]

Pay to the order of Alexander Richardson

Two hundred $\frac{00}{100}$ *Dollars,*

and charge to account of

\$ 200$\frac{00}{100}$. *Simon Osgood.*

To Brown, Jones & Co.

Demand Draft.

\$ 274$\frac{17}{100}$ **AMERICAN EXCHANGE BANK.**

St. Louis, Mo., Sept. 7, 1898.

Pay to the order of

Jonathan Wills

Two hundred seventy-four $\frac{17}{100}$ *Dollars,*

value received, and charge the same to account of

To Mechanics' National Bank, Philadelphia, Pa. *S. A. Battaile, Cashier.*

No. 9.

The method of using the above draft is as follows: Assume that Jonathan Wills, of St. Louis, Mo., owing Wm. E. Smith, of Philadelphia, $274.17, and purposing to pay the debt, enters the American Exchange

Bank in St. Louis, and, by depositing the requisite sum of money, obtains the draft. He then writes on the back thereof, "Pay to the order of Wm. E. Smith," and signs his own name. He finally forwards the draft to Smith in Philadelphia, who, taking it to the Mechanics' National Bank and writing his name on the back, receives the money.

Collection Draft.

$200x	Chicago, Ill., *March 15*, 18*99*.
	At ten days' sight, Pay to the order of
John Hill	
Two hundred	$\frac{00}{100}$ Dollars,
value received, and charge to account of	
To *Henry Smith*, *New York, N. Y.*	*John Hill.*

The method of using the above draft is as follows: Assume that Henry Smith, of New York, owes John Hill, of Chicago, $200, and that Hill, desiring to collect the money, prepares the above draft, and having endorsed it thus: "Pay to George Jones, Cashier, or order, for collection," sends it to the bank in New York of which Jones is cashier. Jones, receiving the draft, presents it to Smith, who writes upon its face, "Accepted," signs his name, and the draft is said to be *honored*. Smith is now under obligation to pay to the bank the $200 at the end of the ten days. Should Smith fail to pay, the draft is said to be *dishonored*, and is, in consequence, protested unless marked "without protest."

DOMESTIC EXCHANGE.

PROBLEMS.

1. What will be the cost of a sight draft on New York for $6000 at $\frac{3}{8}\%$ premium?

Process.

$$\$6000 \times .00\tfrac{3}{8} = \$22.50, \text{ premium.}$$
$$\$6000 + \$22.50 = \$6022.50, \text{ cost of draft.}$$

2. What is the cost of a draft on Chicago for \$4200 at $\frac{3}{4}\%$ discount?

Process.

$$\$4200 \times .00\tfrac{3}{4} = \$31.50, \text{ discount.}$$
$$\$4200 - \$31.50 = \$4168.50, \text{ cost.}$$

3. Find the cost of a draft for \$1000, payable in 60 days after sight, when exchange is $\frac{1}{4}\%$ premium, and interest 6%.

Process.

$$\begin{array}{rr} \text{Draft} = & \$1000.00 \\ \text{Discount for 60 da., at } 6\% = & \underline{10.00} \\ & \$990.00 \\ \text{Premium at } \frac{1}{4}\% = & \underline{2.50} \\ \text{Cost of draft} = & \$992.50 \end{array}$$

4. The draft was for \$580; the time 30 da. after sight; the exchange is at a premium of 3%. Find the cost.

5. What will be the cost in Philadelphia of a draft on Boston for \$1800, payable 60 days after sight, exchange being at a premium of 2%?

6. What must be paid in Detroit for a draft of \$3000 on Boston at 30 days, exchange being $\frac{1}{4}\%$ premium?

7. If exchange on Chicago is $1\frac{1}{4}\%$ premium, what will be the cost in Savannah, Ga., of a sight draft for \$3000?

8. What will be the cost of a sight draft on New York for \$6400, at $1\frac{1}{4}\%$ premium?

9. Find the cost of a draft on Omaha for \$1400, payable in 60 days, when exchange is $\frac{1}{2}\%$ premium, and interest 5%?

10. Find the cost of a draft on Baltimore for \$1237.50, payable in 30 da. after sight, exchange being $\frac{1}{8}\%$ discount, and interest 5%?

11. A San Francisco merchant bought goods in New York valued at \$5284. What will be the cost of a 3 mo. draft for the amount on New York at $\frac{3}{4}\%$ premium?

12. What must be paid for a draft of $900 on New Orleans at 90 da., exchange at $\frac{3}{4}\%$ discount, and interest 5%?

13. If a Boston firm owes a bill in Chicago of $8750, what must they pay for a draft on Chicago, exchange at $\frac{3}{8}\%$ premium?

14. What must be the face of a draft to pay $500, exchange being at $1\frac{1}{2}\%$ premium?

Process.

$1.00 + .015 = 1.015, cost of a draft for $1.00.

Hence $500 ÷ $1.015 = $492.61 = the draft that $500 will buy.

15. Find the face of a draft, drawn at 30 da., that will pay $369.72, exchange being at $3\frac{1}{4}\%$ discount.

Analysis.

Discount of $1.00 for 30 da. at $6\% = .005$. $1.00 — .005 = .995. .995 — .0325 = .9625, face that will pay $1.00. $369.72 ÷ .9625 = $384.125, face that will pay $369.72.

16. How large a sight draft can be purchased on Chicago for $6836 when the rate of exchange is $\frac{1}{4}\%$ premium?

17. What is the face of a 90-day draft on Philadelphia bought for $4600 at 6%, exchange $1\frac{1}{2}\%$ premium?

18. What is the face of a draft on St. Paul for 60 days which may be bought for $2000, exchange being $\frac{7}{8}\%$ discount and interest 7%?

19. How large a draft on sight on San Francisco can be purchased for $3500 if exchange is at $\frac{1}{8}\%$ premium?

20. If I pay $325.05 for a draft payable 60 days after sight, what is the face of the draft if exchange is 1% discount and interest 6%?

21. A draft payable 90 days after sight was bought for

$2756 when exchange was $\frac{3}{4}\%$ discount and interest 6%. What was its face?

22. If exchange is $\frac{1}{8}\%$ premium, how large a draft will $1201.50 buy?

23. If exchange is at $1\frac{3}{4}\%$ premium, what bill of exchange can be bought for $762, in current funds, supposing a discount of $\frac{1}{2}\%$ is charged on the funds?

RATIO AND PROPORTION.

1. Ratio is the relation which one quantity has to another of the same kind, and is expressed by a common fraction, as $\frac{2}{3}$ and $\frac{12}{7}$. These fractions express the ratio of 2 to 3 and 12 to 7. The same ratios may be expressed thus: 2 : 3 and 12 : 7. 2 and 12 are called *antecedents*. 3 and 7 are called *consequents.*

2. Since ratios have a fractional form, they are governed by the principles that govern fractions. $\frac{2}{3} = \frac{6}{9}$; therefore 2 : 3 and 6 : 9 express the same ratio.

3. Ratio cannot exist between two quantities of different kinds. If it be required to find the ratio between 5 pounds and 25 ounces, the pounds must first be reduced to ounces, or the ounces to pounds. 5 lbs. = 80 oz. The ratio of 80 oz. to 25 oz. $= \frac{80}{25} = \frac{16}{5}$.

4. A proportion is an equation composed of two ratios. Since $\frac{8}{4} = \frac{18}{9}$, the *expression* is called a proportion, and the terms 8, 4, 18, and 9 are called *proportionals.* The proportion may also be written thus: 8 : 4 = 18 : 9; or, 8 : 4 :: 18 : 9, and is thus read: "The ratio of 8 to 4 equals the ratio of 18 to 9"; or, "8 is to 4 as 18 is to 9."

5. The first and last terms of a proportion are called *extremes;* the second and third terms are called *means.*

6. The proportion $8 : 4 = 18 : 9$ is also $\frac{8}{4} = \frac{18}{9}$. Reducing these fractions to a common denominator, we have $\frac{8 \times 9}{4 \times 9} = \frac{18 \times 4}{9 \times 4}$. Since the fractions are equal and the denominators are equal, the numerators are equal; that is, $8 \times 9 = 18 \times 4$. But 8 and 9 are the extremes, and 18 and 4 are the means. Hence we have the following fundamental principle:

The product of the extremes equals the product of the means.

7. No four terms are proportional unless, when arranged as means and extremes, they *conform* to the foregoing principle. 2, 5, 6, and 12 are not proportionals, since they cannot be arranged to make the product of the extremes *equal* the product of the means.

8. If any one of the terms of a proportion is wanting, it may readily be found. For convenience we will call the unknown term x, and find what number x equals.

(*a.*) If 6, 8, and 9 are the first, second, and third terms of a proportion, what is the fourth term? By using x we have $6 : 8 = 9 : x$. The product of the extremes being equal to the product of the means, we have the *equation*, 6 times $x = 8$ times 9; or, $6x = 72$. Since 6 times $x = 72$, once $x = \frac{72}{6} = 12$. Hence 12 is the fourth term, and the completed proportion is $6 : 8 = 9 : 12$.

(*b.*) Let the second term be wanting, as in $6 : x = 3 : 7$. Applying the principle we have 3 times $x = 6$ times 7; or, $3x = 42$. $x = \frac{42}{3} = 14$, the second term. The proportion completed is $6 : 14 = 3 : 7$.

Hence the formulæ:

$$\text{(A.) Required Extreme} = \frac{\text{Product of Means}}{\text{Given Extreme}}.$$

$$\text{(B.) Required Mean} = \frac{\text{Product of Extremes}}{\text{Given Mean}}.$$

To find any required term the other three terms must be given; hence arose the old name, **Rule of Three.**

EXERCISES.

Find the value of x in:

1. $8 : 12 = 16 : x$.	9. $20 : 6 = 12 : x$.
2. $18 : 10 = x : 30$.	10. $280 : 16 = 140 : x$.
3. $16 : x = 12 : 24$.	11. $\frac{1}{2} : 5 = x : 6$.
4. $x : 5 = 8 : 20$.	12. $\frac{2}{3} : x = \frac{1}{2} : 10$.
5. $x : 16 = 20 : 80$.	13. $6 : \frac{2}{9} = 10 : x$.
6. $18 : x = 14 : 42$.	14. $.20 : .05 = 11 : x$.
7. $12 : 10 = x : 22$.	15. $1.7 : 1.9 = 1.5 : x$.
8. $16 : 8 = 24 : x$.	16. $2.8 : 3.9 = .07 : x$.

The Principles of Proportion Applied to Practical Problems.

1. Most practical problems under this rule involve the use of concrete (denominate) numbers, and in each example there are only two different kinds of quantities.

2. Every problem furnishes *two like quantities* and a third quantity of *like denomination with the required answer.*

ILLUSTRATIONS.

1. If 8 lb. of sugar cost 40 cents, what will 20 lb. cost?

Process.

(*a.*) The ratio of the pounds equals the ratio of the costs.

(*b.*) The ratio of the pounds is $8 : 20$; hence the ratio of costs is $40 : x$, since 20 lb. cost more than 8 lb. Therefore we have:

$8 : 20 = 40 : x$. Or, $40 : x = 8 : 20$.

$8x = 800$.

$x = 100$, the cost of 20.

Analysis.

If 8 lb. = 40 cts.,
1 lb. = 5 cts.;
20 lb. = 100 cts.

Note.—The pupil will observe that because x is to be greater than its antecedent, the second term must be greater than *its* antecedent. It is only in this way that the equality of ratios can be preserved.

2. If 6 men can do a piece of work in 5 days, in how many days can 10 men do the work?

Process.

We find that the answer, or x, will be less than the third quantity, for 10 men will not require so long a time as 6 men. We have, therefore:

$10 : 6 = 5 : x.$ Or, $6 : 10 = x : 5.$

$10x = 30.$

$x = 3$ da.

Analysis.

If 6 men = 5 days,
1 man = 30 days;
10 men = 3 days.

THE RULE OF THREE.

1. Let x represent the required term or answer.

2. With x and the quantity of like denomination form a ratio.

3. Compare the two terms of the ratio, and determine from the conditions of the problem whether x is greater or less than the other term.

4. With the two given like quantities form a ratio equal to the first.

5. Express the equality of the ratios, and apply formula A or B, as the case may require.

PROBLEMS.

NOTE.—Solve the following problems both by analysis and proportion.

Suggestion: Let x represent the fourth term.

1. If 31 yd. of cloth cost \$62, what will 21 yd. cost?

2. How long will it take 24 men to do a piece of work that 8 men can do in 12 days?

3. How far can a certain load be carried for \$34, if \$64 will carry it 100 miles?

4. If 231 men have provisions for 8 mo., how long will the same provisions last 308 men?

5. If 95 cents will buy one bushel of wheat, how many bushels will \$11.75 buy?

6. A man owes \$2500, and can pay only \$1000. How much does he pay on a dollar?

7. If 28 yd. of oil-cloth, .875 yd. wide, cover a certain floor, how many yards 1.25 yd. wide will cover the same floor?

8. If 22 bu. 3 pk. of corn be produced on one acre, how many acres will produce 546 bu.?

9. If two men earn \$72 in 6 da., how much will 30 men earn in the same time?

10. If $12\frac{1}{2}$ tons of hay cost \$180.25, what will $81\frac{1}{2}$ tons cost?

Suggestion: Let x represent the third term in the following problems.

11. A regiment of 960 men has provisions for 40 days. How long will it last if the regiment is reinforced by 240 men?

12. A field can be mowed in 4 days of 11 hours each; how many days of 9 hours each will it take?

13. At the time when a man 5 ft. 9 in. in height casts a shadow 4 ft. 6 in. long, what is the height of a tree that casts a shadow 52 ft. 6 in. long?

14. If a locomotive runs $96\frac{3}{4}$ miles in $3\frac{1}{2}$ hours, how many miles will it run in $5\frac{7}{8}$ hours?

15. A wheel makes 75 revolutions in 5 min. How many does it make in an hour?

16. A. can do a piece of work in 6 days, B. can do it in 7 days. If B.'s wages are \$2.10 per day, how much should A. receive per day?

17. If a 5-cent loaf of bread weighs 8 ounces when flour is worth \$5, what should such a loaf weigh when flour is at \$6?

18. $\frac{7}{9}$ yd. cost $\$\frac{5}{6}$. Find the cost of $\frac{3}{4}$ yd.

19. If a cistern containing 3000 gal. leak 1 gal. 2 qt. a min., how long will it take to empty it?

20. If 42 yd. of carpet 2 ft. 3 in. wide are required for a room, how many yd. of carpet 2 ft. 4 in. wide will be required?

Suggestion: Let x represent the second term in the following problems.

21. If a train, at the rate of $\frac{5}{13}$ of a mile per min., requires $3\frac{1}{4}$ hr. to make a certain distance, how long will it require at the rate of $\frac{7}{15}$ of a mile a min.?

22. If a train travels $\frac{1}{4}$ of a mile in 18 sec., how many miles an hour does it travel?

23. A. gains 4 yd. on B. in running 30 yd. How many yd. will he gain while B. is running $97\frac{1}{2}$ yd.?

24. If a man spends \$276 in the three summer months, how much will he spend in a year at the same rate per day?

25. If 28 men mow a field of grass in 12 days, how many men will be required to mow it in 8 days?

26. If 17 men can mow a field in 9 days, how long would it take to mow half of it if 5 men refuse to labor?

27. If $14\frac{1}{2}$ yd. of cloth cost \$$19\frac{1}{2}$, how much will $19\frac{7}{8}$ yd. cost?

28. If $\frac{3}{16}$ of a ship costs £273 2s. 6d., what will $\frac{5}{32}$ of her cost?

29. If $2\frac{1}{2}$ gal. of molasses cost 65 cents, what will $3\frac{1}{2}$ hhd. cost?

30. If a steeple 150 ft. high casts a shadow 210 ft., what is the length of the shadow cast, at the same time, by a staff 5 ft. high?

Suggestion: Let x represent the first term in the following problems.

31. If the interest of \$600 for 6 mo. is \$15, what principal will gain \$64 in the same time?

32. If $15\frac{1}{2}$ yd. of silk that is $\frac{3}{4}$ yd. wide will make a dress, how many yards of muslin that is $1\frac{1}{8}$ yd. wide will be required to line it?

33. If I borrow \$500 and keep it 1 yr. 4 mo., for how long a time should I lend \$240 as an equivalent for the favor?

34. A butcher in selling meat sells $14\frac{11}{16}$ oz. for a pound. How much does he cheat a customer who buys of him to the amount of $30?

35. In what time can a man pump 64 hhd. of water if he can pump 12 hhd. in 2 hr. 15 min.?

36. How many men can do in 24 days a piece of work which would employ 40 men 6 days?

37. If $\frac{1}{2}$ of $\frac{3}{4}$ of $6\frac{1}{2}$ bbl. of beef cost $78, how much will $\frac{7}{8}$ of $\frac{5}{6}$ of $3\frac{1}{5}$ bbl. cost?

38. If 450 tiles, each 12 in. square, will pave a cellar, how many tiles that are 9 in. by 8 in. will pave the same?

39. If a distance of 48 miles is represented on a map by $1\frac{3}{4}$ in., what distance is represented on the same map by $7\frac{7}{8}$ in.?

40. Twenty-four men in 30 days can finish a piece of work. After 16 days 11 men quit work. In how many days can the rest finish the work?

COMPOUND PROPORTION.

A **Compound ratio** indicates the product of *two* or *more simple ratios;* for instance, $\frac{3}{7} \times \frac{5}{3}$ is a compound ratio, being the product of the simple ratios $\frac{3}{7}$ and $\frac{5}{3}$, written $\begin{matrix}3:7\\5:3\end{matrix}$.

A **Compound proportion** has one of its ratios compound.

ILLUSTRATION.

If 5 men build a wall 6 ft. high in 7 days of 8 hr., in how many days of 9 hr. can 10 men build a wall 11 ft. high?

Process.

The second ratio is simply 7 da. : x da.

The first ratio is compound, and we construct it as follows:

We write 10 : 5, for 10 men require less time than 5 men.

We write 6 : 11, for 11 ft. require more time than 6 ft.

We write 9 : 8, for 9 hr. per day require fewer days than 8 hr.

Hence the proportion is: $\left.\begin{cases} 10 : \ 5 \\ 6 : 11 \\ 9 : \ 8 \end{cases}\right\} : : 7 : x.$

By formula A, $x = \dfrac{\overset{\overset{2}{\cancel{4}}}{\cancel{5}} \times 11 \times \cancel{8} \times 7}{\underset{2}{\cancel{10}} \times \underset{3}{\cancel{6}} \times 9} = \frac{154}{27} = 5\frac{19}{27}$ days.

By Analysis: $\dfrac{7 \times 5 \times 11 \times 8}{1 \times 10 \times 6 \times 9} = 5\frac{19}{27}$ da.

Since 5 men require 7 da., 10 men require a shorter time, *i. e.*, $\frac{5}{10}$ of 7 da.; since 6 ft., etc.

PROBLEMS.

1. If 6 men can mow 24 acres of grass in 2 days, by working 10 hours per day, how many days will it take 7 men to mow 56 acres by working 12 hrs. per day?

2. If 4 men mow 15 A. in 5 da. of 14 hr., in how many da. of 13 hr. can 7 men mow $19\frac{1}{2}$ A.?

3. If 810 bricks, 8 in. long and 4 in. wide, are required for a walk 36 ft. long and 5 ft. wide, how many bricks will be required for a walk 66 ft. long and 4 ft. wide?

4. If the interest on \$640 for 4 yr. 6 mo. is \$172.8, what is the interest on \$820 for 2 yr. 8 mo. at the same rate?

5. If it requires 275 yd. of cloth $\frac{3}{4}$ yd. wide to make 75 garments, how many yards of cloth $1\frac{1}{4}$ yd. wide will it require to make 215 such garments?

6. If it costs \$2.40 to carry 20 cwt. 50 miles, what will it cost to carry 40 cwt. 40 miles at the same rate?

7. If 12 candles, 8 weighing a pound, last from 5 o'clock to 11, how many candles, 6 weighing a pound, will last from 7 o'clock to 11?

8. A farmer owning 25 horses traded them for sheep. If 3 horses are worth 12 cows, 6 cows are worth 42 pigs, and 25

pigs are worth 30 sheep, how many sheep did he get for his horses?

9. If 200 men in 12 days of 8 hr. each can dig a trench 160 yd. long, 6 yd. wide, and 4 yd. deep, in how many days of 10 hr. will 90 men dig a trench 450 yd. long, 4 yd. wide, and 3 yd. deep?

10. 5 compositors, in 16 da. of 14 hr. each, can compose 20 sheets of 24 pages in each sheet, 50 lines in a page, 40 letters in a line. In how many days of 7 hr. each will 10 compositors compose a volume containing 40 sheets, 16 pages in a sheet, 60 lines in a page, 50 letters in a line?

11. At 6%, what principal will gain $27 in 9 months?

12. If 12 horses eat 10 bu. of oats in 8 da., how many bushels will 30 horses eat in 40 days?

13. If it takes 22 reams of paper to make 1000 copies of a book of 11 sheets, how many reams will be required to make 4500 copies of a book of 7 sheets?

14. If a field 60 rods long and 20 rods wide cost $500, what will a field 15 chains long and 8 chains wide cost?

15. If a piece of iron 7 ft. long, 4 in. wide, and 6 in. thick weighs 600 lb., how much will a piece of iron weigh that is 16 ft. long, 8 in. wide, and 4 in. thick?

16. If a 6-cent loaf weighs 8 ounces when wheat is $1.25 per bu., how much bread may be bought for 50 cents when wheat is $1.00 per bushel?

17. A ship's crew of 32 men, at a daily allowance of 3 lb. to each man, have provisions enough for 45 days. If they now rescue a crew of 16 men, what can be allowed each man daily to make the provisions hold out 40 days?

18. 4000 copies of a book, containing 420 pages, were printed from 650 reams of paper; how many reams of paper would have been required to print 7000 copies, containing 528 pages, of the same size?

19. If \$500 will gain \$16.50 in 4 mo. 12 da., at 9%, how much will \$750 gain in 2 yr. 9 mo. 8 da., at 6%?

20. If 3280 42-lb. shot cost \$3000, how many 32-lb. shot can be bought for \$4200?

21. How many hours a day must 5 men work to mow a field in 8 days, that 7 men can mow in 6 days of 10 hours?

22. If 25 horses can consume a bin of grain in 40 days, in what time will a bin of twice the size be consumed, if 7 horses are added when the grain is $\frac{2}{3}$ eaten?

23. \$600 gains \$72 in 2 years. In how many years at the same rate will \$92 gain \$54?

CAUSE AND EFFECT.

Since like causes produce like effects, we have the following general formula:

1st cause : 2d cause : : 1st effect : 2d effect.

ILLUSTRATIONS.

1. If 4 men earn \$144, how much will 6 men earn in the same time and at the same rate?

Process.

Let x be the required effect, representing what 6 men will earn, and we have:

$$\begin{array}{ccccccc} \text{1st c.} & & \text{2d c.} & & \text{1st ef.} & & \text{2d ef.} \\ 4 & : & 6 & = & 144 & : & x; \end{array}$$

$$4x = 864;$$

$$x = 216, \text{ ans.}$$

2. If 4 men earn \$144 in 12 days, how much will 6 men earn in 10 days at the same rate?

Note.—Here there are compound causes, consisting of men and days.

Process.

Let x dollars be the required effect of 6 men and 10 days, and we have:

$$\left.\begin{matrix} 4 : 6 \\ 12 : 10 \end{matrix}\right\} :: 144 : x. \qquad x = \frac{\cancel{6} \times \overset{5}{\cancel{10}} \times \overset{36}{\cancel{144}}}{\cancel{4} \times \underset{2}{\cancel{12}}} = \$180.$$

PROBLEMS.

1. If 3 workmen can board 4 weeks for $54, how many can board 13 weeks for $585?

$$\text{Suggestion: } \left\{\begin{matrix} \text{c.} & & \text{c.} \\ 3 & : & x \\ 4 & : & 13 \end{matrix}\right\} :: \begin{matrix} \text{ef.} & & \text{ef.} \\ 54 & : & 585. \end{matrix}$$

2. If 36 men earn $1296 in 13 days, how much will 42 men earn in 87 days?

3. If 12 horses consume 40 bu. of oats in 8 days, how long will 140 bu. of oats last 16 horses?

Suggestion: Let x days be a cause.

4. If it cost $15 to carry 20 tons $1\frac{1}{2}$ miles, what will it cost to carry 400 tons $\frac{1}{2}$ of a mile?

5. If A. can do $\frac{2}{3}$ of a piece of work in 5 da., working 8 hr. a da., how long will it take him to do the whole piece, working 10 hr. a day?

6. If 12 horses in 5 da. draw 44 loads of stone, how many horses will draw 132 loads the same distance in 18 da.?

Note.—If additional practice is needed in applying the principle of cause and effect, any of the previous problems in proportion may be used.

PROPORTIONAL PARTS.

A number may be divided into parts which are proportional to two or more given numbers.

ILLUSTRATIONS.

1. Divide the number 180 into three parts that shall be to one another as 3, 4, and 5.

Process.

$$\begin{aligned}
\text{Let } 180 &= 3 + 4 + 5 = 12 \text{ parts.}\\
\text{If } 12 \text{ parts} &= 180.\\
3 \text{ parts} &= \tfrac{3}{12} \text{ of } 180 = 45.\\
4 \text{ parts} &= \tfrac{4}{12} \text{ of } 180 = 60.\\
5 \text{ parts} &= \tfrac{5}{12} \text{ of } 180 = \underline{75.}\\
&\qquad\qquad\qquad\quad 180.
\end{aligned}$$

2. Divide 940 in the proportion of $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{4}$.

Process.

The L. C. D. of the fractions is 60. $\frac{1}{5} = \frac{12}{60}$; $\frac{1}{3} = \frac{20}{60}$; $\frac{1}{4} = \frac{15}{60}$.

$$\begin{aligned}
940 &= 12 + 20 + 15 = 47 \text{ parts.}\\
\text{If } 47 \text{ parts} &= 940.\\
12 \text{ parts} &= \tfrac{12}{47} \text{ of } 940 = 240.\\
20 \text{ parts} &= \tfrac{20}{47} \text{ of } 940 = 400.\\
15 \text{ parts} &= \tfrac{15}{47} \text{ of } 940 = \underline{300.}\\
&\qquad\qquad\qquad\quad 940.
\end{aligned}$$

PROBLEMS.

1. Divide 60 into two parts that are to each other as 5 and 7.

2. Divide 1200 into parts proportional to 11, 12, 13, 14.

3. Divide 780 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

4. Three men caught 120 fish. How many did each catch, their proportions being as 2, $1\frac{1}{2}$, and $\frac{1}{2}$?

5. Divide a profit of $13,384 among three partners, the first owning $\frac{8}{34}$, the second $\frac{12}{34}$, the third $\frac{14}{34}$.

6. Divide $9 into parts that are to each other as .05, .10, .25, and .50.

7. A house, a farm, and a store cost $18,000. The farm cost twice as much as the house, and the store three times as much as the house. How much did each cost?

8. Three men agree to pay \$60 rent for a pasture lot; the first pastures 3 cows, the second 5 cows, and the third 4 cows. How much should each pay?

9. If gunpowder contains nitre, charcoal, and sulphur in the proportion of 15, 3, and 2, and if in a quantity of gunpowder there is 20 cwt. of charcoal, find the weight of nitre and sulphur therein.

PARTNERSHIP.

1. **Partnership** is of two kinds,—*Simple* and *Compound*.

2. It is simple when the capital of the partners continues in the business for the *same* time.

3. It is compound when the capital of the partners continues in the business for *different lengths of time.* Time, therefore, has to be considered in the proportional division of gains or losses.

ILLUSTRATIONS.

1. Two men, A. and B., enter into partnership, and gain in 1 yr. \$500. What part of the gain did each own if A.'s capital was \$3000 and B.'s \$2000?

Process.

A.'s capital is to B.'s as 3 to 2.
$\$500 = 3 + 2 = 5$ equal parts.
If 5 parts $= \$500$,
3 parts $= \frac{3}{5}$ of $\$500 = \$300$, A.'s share;
2 parts $= \frac{2}{5}$ of $\$500 = \$200$, B.'s share.

2. But suppose A. had \$3000 in business for 3 yr., and B. had \$2000 in business for 2 yr.; how would \$1300 gain be divided proportionally?

Difference of time must be considered as well as difference of capital.

Process.

$$\$3000 \text{ for } 3 \text{ yr.} = \$9000 \text{ for } 1 \text{ yr.}$$
$$\$2000 \text{ for } 2 \text{ yr.} = \$4000 \text{ for } 1 \text{ yr.}$$
$$\$9000 : \$4000 = 9 : 4.$$
$$\$1300 = 9 + 4 = 13 \text{ equal parts.}$$
$$\text{If } 13 \text{ equal parts} = \$1300,$$
$$9 \text{ parts} = \tfrac{9}{13} \text{ of } \$1300 = \$900, \text{ A.'s.}$$
$$4 \text{ parts} = \tfrac{4}{13} \text{ of } \$1300 = \$400, \text{ B.'s.}$$

PROBLEMS.

1. A. and B. engage in trade. A. furnishes $300, and B. $400 of the capital. They gain $182. What is each one's share of the gain?

2. Two persons form a partnership. A. puts in $450 for 7 mo. and B. $300 for 9 mo. They lose $156. How much is each man's share of the loss?

3. A., B., and C. paid $220.50 for a pasture. A. put in 9 cows for $2\frac{1}{2}$ mo., B. 12 cows for 2 mo., C. 18 cows for $1\frac{1}{2}$ mo. How much of the rent ought each to pay?

4. C., D., and E. formed a partnership for carrying on business. C. furnished $800 for 6 mo., D. $1800 for 8 mo., and E. $1500 for 4 mo. They gained $940. How should the gain be divided?

5. A., B., C., and D. traded in company. A. put in $7500, B. $7000, C. $9500, and D. $8000. What was each partner's share of a profit amounting to $9280?

6. Three men buy a house for $1200. A. furnishes $600, B. $400, C. $200. They sell the house for $1500. How much money should each receive?

7. M. and N. entered into partnership. M. put $200 into the business for 5 mo. and N. $300 for 4 mo. They gained $132. Find the share of each?

8. Two men hire a pasture for $42. One puts in twice as many head of cattle as the other. What should each pay?

9. A., B., and C. buy a house for $15,000. A. supplies $4000, B. $5000, C. the remainder. The yearly rental being $1000, to what part of it is each entitled?

10. E., F., and G. bought a block of stores for $46,000. E. furnished $\frac{3}{8}$ of the money, F. $11,500, and G. the rest. The property was sold for $48,300. What was the gain of each?

11. A., B., and C. engaged in manufacturing. A. invested $4500 for 6 mo., B. $5000 for 8 mo., and C. $6500 for 7 mo. They gained $4500. Find each partner's gain.

12. The profits were $4800. Patterson's share was $3000. How many eighths of the capital did he own?

13. X. and Y. hire a bicycle for $4.50 a week. If X. uses it on Tuesday and Friday, and Y. the rest of the week, except Sunday, what does each pay?

14. Smith and Jones united in a partnership. Smith contributed $240 for 8 mo., and B. $560 for 5 mo. They lost $118. How much did each man lose?

15. A man dies owing three creditors, $8050, $2970, and $7170, respectively. If his assets, after deducting expenses, are $13,646, how much will each creditor receive?

16. A. and B. enter into partnership with capitals of $3500 and $8700. A. is to have 12% of the profits for his services as manager. Divide a gross profit of $1906.25.

17. S. and T. engaged in speculation. S. employed $1260 for 8 mo., and T. $980 for 6 mo. They lost $957.60. Apportion this loss.

18. A man failing in business, paid 50 cents on the dollar. With assets of $40,000, how much would X., Y., and Z. receive, whose claims against him were respectively $2000, $3000, and $4000?

19. Apportion a loss of $5600 among three partners whose capital was invested in the proportion of 1, 4, and 5.

20. A. commenced business January 1st, with a capital of $3400. April 1st he took B. into partnership with a capital of $2600; at the expiration of the year, they had gained $750. What is each one's share of the gain?

21. Our standard gold coin consists of 900 parts of gold, 90 parts silver, 10 parts copper. What is the quantity of each metal in 50 pounds of coin?

22. A. and B., contractors, received $857.50 for grading a roadway. A. furnished 5 men 20 days, and 6 others for 15 days; B. furnished 10 men for 12 days, and 9 others for 20 days. What was each contractor's share?

23. C. and D. form a partnership. C. invests $5000; D. $10,000. During the year C. draws $1500 of the profits and D. draws $1000. At the end of the year the business is disposed of for $20,000. What amount should each receive?

24. P., Q., and R. buy a lot for $600. After selling it, P. receives $220 as his share of the proceeds, Q. receives $280 and R. $300. How much did each invest originally?

25. A partnership is formed between A., with a capital of $1500, and B., with a capital of $4000. Six months thereafter they take in C., with a capital of $4000. How should a profit of $3500 be divided at the end of the year?

26. V. and W. rented a field for a year for $200. V. put in 6 horses for the whole time, W. put in 5 horses for 11 months and 3 horses for 5 months. How much of the rent had each to pay?

27. A., B., and C. entered into partnership for one year. A. put in $5000, B. $6000, and C. $4000. At the end of six months A. withdrew $2000, and C. put in $8000 more. The profits at the end of the year were $6000. What was each man's share?

28. B. and C., trading together, find their stock to be worth \$3500, of which C. owns \$2100. They have gained 40% on their first capital. What did each put in?

AVERAGES.

An **average** is expressed by the *ratio* of the *sum* of two or more quantities to the *number* of the quantities.

ILLUSTRATION.

A Fahrenheit thermometer registered 30° at 8 A.M., 56° at M., and 40° at 6 P.M. What was the average temperature of the day between 8 and 6 o'clock?

Process.

1. The sum of the quantities is $30 + 56 + 40 = 126$.
2. The number of them is 3.
3. Their average is $\frac{126}{3} = 42$.

Therefore the average temperature is 42°.

PROBLEMS.

1. A tax-collector received on Monday, \$430.74; on Tuesday, \$380.88; on Wednesday, \$448.60; on Thursday, \$420.79; on Friday, \$367.44; on Saturday, \$508.73. What did he receive daily on an average?

2. In a school the largest attendance present for each of six months was as follows: 1st mo., 125; 2d mo., 130; 3d mo., 128; 4th mo., 125; 5th mo., 132; 6th mo., 122. Find the average attendance for the 6 months.

3. A goldsmith combined 2 oz. of gold 16 carats fine, 2 oz. 18 carats fine, and 6 oz. 22 carats fine. What is the fineness of the composition?

4. If one dozen eggs weigh 1 lb. 2 oz., what is their average weight?

5. If a man owes a debt, due in 3 mo., and a like debt, due in 5 mo., when may he pay both debts at once?

AVERAGING OR EQUATING OF PAYMENTS.

The **averaging** or **equating** of payments due at different times consists in finding an equitable time for including all payments in one.

ILLUSTRATION.

A. owes B. $250, due in 3 mo., and $350, due in 5 mo. Find the average term of credit?

NOTE.—If the payments were equal, the average term of credit would be $\frac{5+3}{2} = 4$ mo. Since they are not equal, we must consider both time and payments.

Short Process.

$$\begin{array}{lr} 250 \times 3 = & 750 \\ \underline{350} \times 5 = & \underline{1750} \\ 6|00 &)\,25|00 \\ & 4\frac{1}{6}\text{ mo.} \end{array}$$

Explanatory Process.

$$\left.\begin{array}{l} \$250 \text{ for } 3 \text{ mo.} = \\ \$350 \text{ for } 5 \text{ mo.} = \end{array}\right\} \$1 \text{ for} \left\{\begin{array}{r} 750 \text{ mo.} \\ 1750 \text{ mo.} \end{array}\right.$$

Total, $1 for 2500 mo.

If the term of credit on $1 be 2500 mo., the term of credit on $600 is $\frac{1}{600}$ of 2500 mo. $= \frac{2500}{600} = \frac{25}{6} = 4\frac{1}{6}$ mo.

Hence the rule, briefly stated, is:

1. Multiply the debts by the terms of credit.

2. Divide the sum of the products by the sum of the debts.

PROBLEMS.

1. Equate the time for payment of $400, due in 3 mo.; $600, due in 7 mo.; and $300, due in 10 mo.

2. Find the average time of payment of $3500, due in 5 mo.; of $1600, due in 8 mo.; of $1500, due in 10 mo.; and of $600, due in 9 mo.

3. Mr. Jenkins has bought \$1200 worth of goods on 6 months' credit and \$600 worth on 3 months' credit. For what time should he give a note for the whole amount, \$1800?

4. \$1680 is to be paid in four equal instalments, in 1, 2, 3, and 4 mo. respectively. Equate the time.

5. \$500 is due in 8 mo., \$900 in 6 mo., \$1000 in 3 mo., \$1200 in cash [$1200 \times 0 = 0$]. Find the term of credit for a single payment of the whole indebtedness.

6. Equate the time for the payment of \$5000, due Feb. 1; of \$4000, due June 1; of \$3000, due Aug. 1, and of \$3000, due Oct. 1.

Suggestion: Count time from Feb. 1.

7. A person owes a certain sum, of which $\frac{1}{4}$ is payable in 8 mo., $\frac{1}{3}$ in 9 mo., and the balance in 12 mo. Equate the time of payment.

8. Johnson & Co. sold a bill of lumber on the following terms: \$1500 cash, \$3000 payable in 30 days, and \$2000 payable in 90 days. When may the whole debt be cancelled by one payment?

9. If a person lends me \$250 for 8 mo., for how long ought I to lend him \$480 as an equivalent?

10. I bought on July 5th goods to the amount of \$2400. \$630 was to be paid at once, \$820 in 8 mo., and \$950 in 9 mo. What is the equated time for the payment of the whole?

11. A man owes \$600, of which $\frac{1}{3}$ is to be paid in 1 yr., and the remainder in 2 yr. Equate the time, and find the present value, money being worth 6%.

12. I bought bills of goods as follows: June 1, \$250, on 3 mo. credit; July 5, \$300, on 3 mo. credit; Aug. 6, \$150, on 3 mo. credit; Oct. 2., \$400, on 2 mo. credit. Find the equated time of payment.

Process.

$$\begin{array}{rcr} 250 \times 0 & = & 00000 \\ 300 \times 34 & = & 10200 \\ 150 \times 66 & = & 9900 \\ \underline{400} \times 92 & = & \underline{36800} \\ 1100 & & 56900 \end{array}$$

$56900 \div 1100 = 51\frac{8}{11}$

Sept. 1 $+ 51\frac{8}{11} =$ Oct. 23

Explanation.

1. Add the terms of credit to their respective dates.

2. Find the interval between the earliest resulting date and each of the other dates.

3. Multiply the debts by their respective intervals, and proceed as before.

13. Mr. B. bought goods as follows: April 15, \$150 on 2 mo. credit; May 10, \$200 on 3 mo. credit; June 5, \$250 on 4 mo. credit. Find the equated date of payment.

14. What is the average time at which the following bills become due: Feb. 1, 1898, \$200 on 1 mo. credit; March 10, 1898, \$500 on 3 mo. credit; April 12, 1898, \$275 on 2 mo. credit; and May 1, 1898, \$400 on 4 mo. credit?

15. I owe Mr. Wilson \$100, to be paid on the 15th of July; \$200 on the 15th of August, and \$300 on the 9th of September. What is the mean time of payment?

16. Find the equated time for the payment of \$112.30 due July 6, \$115.25 due July 30, \$232.15 due Sept. 4, and \$102.36 due Oct. 1.

17. A merchant bought goods as follows: Mar. 19th, \$350 on 4 mo.; Apr. 1st, \$430 on 130 da.; May 16th, \$540 on 95 da.; June 10th, \$730 on 3 mo.; what is the average time for the payment of the whole?

18. \$1200 worth of mdse., bought Nov. 5, and \$1000 worth bought on the following Jan. 9, have a credit of 2 mo. When may both be paid at once?

19. A man bought the following bills of goods: Jan. 15, \$600 on 2 mo. credit; Feb. 1, \$300 on 3 mo. credit; March 25, \$550 on 30 da. credit; and April 8, \$400 on 60 da. credit. Find the equated time of payment.

20. Find the equated time of payment for the following obligations:

1. $400, due June 15; $375, due July 11; $195, due Sept. 4.
2. $1394.50, due Dec. 1, 1898; $129.80, due Dec. 10, 1898; $960, due Feb. 1, 1899.

21. A. owes $600, due in 8 mo. If he pays $160 in 3 mo. and $120 in 6 mo., when should he pay the balance?

$$8 \text{ mo.} - 3 \text{ mo.} = 5 \text{ mo.}$$
$$8 \text{ mo.} - 6 \text{ mo.} = 2 \text{ mo.}$$

Therefore A. has to his credit:

$$\left.\begin{array}{l}\$160 \text{ for } 5 \text{ mo.} = \$1 \text{ for } 800 \text{ mo.}\\ \$120 \text{ for } 2 \text{ mo.} = \$1 \text{ for } 240 \text{ mo.}\end{array}\right\} = \$1 \text{ for } 1040 \text{ mo.}$$

But A. still owes $600 — 280 = $320.

$$\$1 \text{ for } 1040 \text{ mo.} = \$320 \text{ for } \tfrac{1040}{320} = 3\tfrac{1}{4} \text{ mo. (after 8 mo.).}$$

22. B. owes $1600, due in 5 mo.; $2400 due in 7 mo. If at the end of 5 mo. he pays $2800, when should the balance be paid?

23. A man owes $2000, due in 8 mo. He pays $500 in 2 mo. and $800 in 3 mo. When in equity should he pay the balance?

24. A. owed B. $2000, payable in 4 mo., but at the end of 1 mo. he paid him $500, at the end of 2 mo. $500, and at the end of 3 mo. $500. In how many months is the balance due him?

25. A. owes $800, due in 5 mo.; $1200, due in 7 mo. If at the end of 5 mo. he pays $1400, when should the balance be paid?

26. A merchant owes $5400, due in 9 mo. If he pays $2300 in 4 mo., $2000 in 5 mo., and $600 in 7 mo., when should he pay the balance?

27. A modiste bought goods to the amount of $425 on a credit of 20 da. and $380 on a credit of 30 da. At the end

of 15 da. she paid $450, and at the end of 20 da. she paid $150. When can the remainder be equitably paid?

28. What is the average date of payment for the following three notes: March 10, 1898, $240; April 12, 1898, $260; May 14, 1898, $320?

29. I bought goods to the amount of $1200 on the following terms: $\frac{1}{4}$ payable in cash, $\frac{1}{4}$ payable in 2 mo., the balance in 6 mo. When may the whole in equity be paid at once?

30. I owe $600, due in 5 mo.; $1000, due in 10 mo., and $1200, due in $7\frac{5}{6}$ mo. What is the average term of credit?

INVOLUTION.

INDUCTIVE STEPS.

1. In the equation $3 \times 3 = 9$, there are how many equal factors? What is the product of those factors?

2. In the equation $5 \times 5 \times 5 = 125$, how many times is 5 taken as a factor? What is 125 called?

3. The product of equal factors is also called the *power*.

4. Find the product or power of 6 taken twice as a factor.

5. Find the power of six taken 3 times as a factor?

6. When a number is taken twice as a factor, the product is called the *second* power of the number.

7. When a number is taken 3 times as a factor, the product is called the *third* power of the number; when taken four times, the *fourth* power, and so on.

8. Write the second power of 2; of 3; of 4; of 5; of 7; of 8; of 9.

9. Write the third power of 2; of 3; of 4; of 5; of 7; of 8; of 9.

10. What is the product of $\frac{3}{8}$ by $\frac{3}{8}$? Of $\frac{3}{8}$ by $\frac{3}{8}$ by $\frac{3}{8}$?

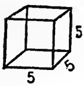
11. What, then, is the second power of $\frac{3}{8}$? Third power?

12. What is the second power of $\frac{4}{9}$? Third power?

13. The equation $2 \times 2 \times 2 = 8$, expressing the third power of 2, is commonly written thus: $2^3 = 8$. The ^{3}, indicating the number of times 2 is taken as a factor, is called the *Exponent.*

14. Write an equation showing by an exponent the third power of 4; the fourth power of 5; the fifth power of 6.

15. For the reason that the product of two equal factors equals the area of a square, and the product of three equal factors denotes the volume of a cube, the second power of a number is also called the *Square*, and the third power of a number the *Cube.*



16. **Involution** is the process of finding the power of a number.

EXERCISES.

The *first* power of a number is the number itself.

1. Write the first power of the numbers represented by the digits.

2. Write an equation to denote the square of each of the following numbers: 1, 3, 5, 7, 9, 10, 15, 25.

3. Write an equation to denote the cube of the following numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

4. Find the value of x in each of the following equations: $x = 0^2$, $x = 1^2$, $x = 2^2$, $x = 3^2$, $x = 4^2$, $x = 5^2$, $x = 6^2$, $x = 3^3$, $x = 4^3$, $x = 4^4$, $x = 5^2$, $x = 5^3$, $x = 6^3$.

5. In like manner show the square of 20, 30, 40, 50, 60, 70, 80, 90, 100.

6. Also, the cube of 10, 20, 30, 40, 50, 60, 70, 80.

7. In this manner, $(\frac{1}{2})^2 = \frac{1}{4}$, write the square of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$; of $\frac{2}{7}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{5}{10}$.

8. In like manner write the third power of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

9. Write the second power of .1, .2, .3, .4, .5, .6, .7, .8, .9, in this manner: $.1 \times .1 = .1^2 = .01$.

10. Find the value of $.1^3$, $.2^3$, $.3^3$, $.4^3$, $.5^3$, $.6^3$, $.7^3$, $.8^3$, $.9^3$.

11. Find the value of $(\frac{3}{4})^2$, $(.4)^2$, $(\frac{2}{9})^3$, $(\frac{1}{6})^3$, $(.1)^4$, $(.02)^3$.

In the equation $x = 4^2$, the value of x is the square of 4.

Process.

$$x = 4^2 = 4 \times 4 = 16.$$

In like manner find the value of x in the following:

1. $x = 25^2$.	8. $x = 10^3$.	15. $x = .09^3$.
2. $x = 35^2$.	9. $x = 21^3$.	16. $x = .05^4$.
3. $x = 88^2$.	10. $x = (\frac{2}{3})^3$.	17. $x = .005^3$.
4. $x = 101^2$.	11. $x = .001^3$.	18. $x = 2.05^2$.
5. $x = 13^2$.	12. $x = .15^3$.	19. $x = (25\frac{1}{3})^2$.
6. $x = 9^3$.	13. $x = .04^3$.	20. $x = (4.500\frac{1}{7})^2$.
7. $x = 17^3$.	14. $x = .125^3$.	21. $x = (21.65\frac{3}{8})^2$.

INVOLUTION BY ANALYSIS.

NOTE.—If this subject be considered too difficult, it may be omitted.

$32^2 = (30 + 2)^2 = (\text{tens} + \text{units})^2 = (t + u)^2$.

We will now square 32 as tens and units.

$$\begin{array}{rclcr}
32 = 30 + 2 & = & & & t + u \\
32 = \underline{30 + 2} & = & & & \underline{t + u} \\
2^2 & = & 4 = & & u^2 \\
\left.\begin{array}{r}30 \times 2\\ 30 \times 2\end{array}\right\} & = & \left\{\begin{array}{l}60 = t \times u\\ 60 = t \times u\end{array}\right\} & = & 2t \times u \\
30^2 & = & \underline{900} = & & \underline{t^2} \\
 & & 1024 = & & t^2 + 2t \times u + u^2.
\end{array}$$

Hence we have a very important principle:

The square of any number consisting of tens and units = the tens² + 2 times the tens × the units + the units².

To illustrate, find the square of 35 in accordance with this principle.

$$35^2 = (\text{tens} + \text{units})^2 = \left\{ \begin{array}{r l} 900 & = t^2 \\ 300 & = 2t \times u \\ 25 & = u^2 \\ \hline 1225 & = t^2 + 2t \times u + u^2. \end{array} \right.$$

In like manner find the square of 45, 56, 97, 21, 38, 63, 75, 88, 19, 24.

We will now proceed to find the cube of 32.

$32^3 = 32^2 \times 32^1.$

$$32^2 = \left. \begin{array}{r} 4 \\ 60 \\ 60 \\ 900 \end{array} \right\} \times 32 = \begin{array}{r l} 8 & = u^3 \\ \left. \begin{array}{r} 120 \\ 120 \\ 120 \end{array} \right\} & = 3t \times u^2 \\ \left. \begin{array}{r} 1800 \\ 1800 \\ 1800 \end{array} \right\} & = 3t^2 \times u \\ 27000 & = t^3 \\ \hline 32{,}768 & = t^3 + 3t^2 \times u + 3t \times u^2 + u^3. \end{array}$$

Hence the principle:

The cube of any number consisting of tens and units = the tens³ + 3 times the tens² × the units + 3 times the tens × the units² + the units³.

For example: $35^3 = (30 + 5)^3 = 30^3 + 3 \times 30^2 \times 5 + 3 \times 30 \times 5^2 + 5^3 = 27{,}000 + 13{,}500 + 2250 + 125 = 42{,}875.$

In like manner find the cube of 35, 22, 19, 53, 38, 27, 47, 48, 45, 63, 73, 77.

Involution, as we have seen, develops the power from the root. The reverse process, that extracts the root from the power, is called *Evolution*.

EVOLUTION.

INDUCTIVE STEPS.

1. The numbers represented by the digits and their squares are:

Numbers,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Squares,	1,	4,	9,	16,	25,	36,	49,	64,	81.

NOTE.—Pupils should memorize this table of squares and roots.

2. The numbers are the square roots of their squares: 1 is the square root of 1; 2 is the square root of 4; 3 is the square root of 9, and so on.

3. A square number is the product of *two equal factors*, either of which is the *square root* of that square number. $64 = 8 \times 8$; 64 is a square number, and 8 is its square root.

4. Since $27 = 3 \times 3 \times 3$, 27 is a cube number, and 3 is its cube root; and since $16 = 2 \times 2 \times 2 \times 2$, 16 is a fourth power, and has 2 for its fourth root.

5. Evolution is the process of finding the *roots* of numbers. A number having an exact root is a *perfect* power.

6. The *radical* or *root sign* is $\sqrt{}$. The square root of 64 may be thus expressed: $\sqrt{64}$, or thus: $64^{\frac{1}{2}}$.

SQUARE ROOT.

By Factoring.

The prime factors of 16 are $\overline{2 \times 2} \times \overline{2 \times 2} = 4 \times 4$; therefore $\sqrt{16} = 4$. $36 = \overline{2 \times 2} \times \overline{3 \times 3} = 6 \times 6$; therefore $\sqrt{36} = 6$.

In like manner find the square root of:

1. 144.	5. 576.	9. 1764.	13. 3969.
2. 196.	6. 676.	10. 1936.	14. 5184.
3. 256.	7. 1296.	11. 2601.	15. 6400.
4. 324.	8. 1225.	12. 2916.	16. 8281.

Periods and Roots Compared.

Separating the following squares into two-figure periods as far as possible, we have:

One period.	Two periods.	Three periods.
$\sqrt{1'} = 1.$	$\sqrt{1'00} = 10.$	$\sqrt{1'00'00} = 100.$
$\sqrt{81'} = 9.$	$\sqrt{98'01} = 99.$	$\sqrt{99'80'01} = 999.$

Obviously, one period in the square gives but one figure in the root; two periods in the square, two figures in the root; three periods in the square, three figures in the root. Hence the principle:

The number of figures in the square root equals the number of two-figure periods into which the square can be pointed off, beginning at units.

NOTE.—The period on the extreme left may contain but a single figure.

EXERCISES.

1. In accordance with the foregoing principle, state how many *periods* the squares of the following roots contain: 4, 9, 32, 99, 317, 999, 3163, 9999, 21, 115, 4156, 19, 316, 6184, 35, 584, 8196.

2. Show that the squares of the numbers from 4 to 9, inclusive, give full periods.

Suggestion: $4^2 = 16$; $9^2 = 81$.

3. Show that the squares of the numbers from 32 to 99, inclusive, give two full periods.

4. Show that the squares of the numbers from 317 to 999, inclusive, give three full periods.

5. Show that the squares of the numbers from 3163 to 9999, inclusive, give four full periods.

6. State the exact number of figures contained in the squares of the following numbers: 3, 5, 21, 29, 41, 66, 97, 125, 200.

7. Point off the following numbers into periods, and tell the number of figures in the root of each: 196, 1296, 2809, 5625, 400,689, 516,961, 182,329, 23,804,641, 9,991,921.

Extraction of Square Root.

General Method.

Find the square root of 190,969.

	Square.	Root.
1. The number pointed off is	19′09′69	(437
2. The greatest square in 19 is	16	
3 The square root of 16 is 4 (the first figure of the root)		
4. The remainder, with the second period, is . . .	309	
5. Twice the root-figure 4 is 8, the trial divisor.		
6. 30 ÷ 8 gives 3 for the second root-figure.		
7. 3 annexed to 8 gives 83, the complete divisor.		
8. 83 × 3	249	
9. The remainder, with the third period annexed, is	6069	
10. Twice 43 or 86 is the second trial divisor.		
11. 606 ÷ 86 gives 7 for the third root-figure.		
12. 7 annexed to 86 gives 867, the second complete divisor.		
13. 867 × 7	6069	
Therefore the exact root of 190,969 is 437.	0	

In a treatise on arithmetic, a scientific explanation of the square root is scarcely admissible, as methods essentially algebraic or geometrical have to be adopted.

The algebraic discussion, in brief, is as follows:

Let t represent the *tens* of a root, and u the *units*. As we have already seen (page 310), a square number equals $t^2 + 2t \times u + u^2$. We will now proceed to find from this expression its root, $t + u$.

1. The square is . . . . $t^2 + 2t \times u + u^2$.
2. $\sqrt{t^2} = t$, the tens of the root.
3. Subtracting . . . . t^2

we have remaining . . . . $2t \times u + u^2$.

4. Dividing $2t \times u$ by $2t$ (twice the tens), we obtain u, the units of the root.
5. Adding u to the divisor, $2t$, we have $2t + u$.
6. Multiplying $2t + u$ by u, we have . . . . . . $2t \times u + u^2$.
7. Subtracting, we have remaining 0

Therefore the root of $t^2 + 2t \times u + u^2$ is $t + u$.

Brief directions are:

1. Point off the number into two-figure periods.

2. Find in the first period the greatest square and its root.

3. Subtract and annex the next period for a remainder.

4. Divide the remainder by twice this root to find the second figure of the root.

5. Annex the quotient to both root and divisor.

6. Multiply by the units.

7. Apply (3), (4), (5), and (6) again, if necessary.

To apply the rule:

Find the square root of 53,361.

Process.

1st step . .	5′33′61 (231	. . . $t^2 + 2t \times u + u^2 \,(t + u$
2d step . .	4	. . . t^2
3d step . 4th step . 5th step .	43) 133	. . $2t + u\,)\, 2t \times u + u^2$ (+ 4)
6th step . .	129	. . . . . $2t \times u + u^2$
7th step . .	461) 461	
	461	
	0	

The above formula furnishes two figures of the root. Calling 23 the tens of the root, $2t = 46$.

EXERCISES.

Find the square root of:

1. 100.	11. 2809.	21. 674,041.
2. 10,000.	12. 3969.	22. 784,996.
3. 625.	13. 4489.	23. 944,784.
4. 961.	14. 7056.	24. 998,001.
5. 2704.	15. 9216.	25. 5,875,776.
6. 6889.	16. 16,129.	26. 6,270,016.
7. 15,625.	17. 70,756.	27. 12,574,116.
8. 141,376.	18. 118,336.	28. 30,858,025.
9. 160,801.	19. 262,144.	29. 40,005,625.
10. 100,000,000.	20. 368,449.	30. 29,735,209.

SQUARE ROOT OF COMMON AND DECIMAL FRACTIONS.

INDUCTIVE STEPS.

1. What is the square root of $\frac{1}{4}$?

$$\sqrt{\tfrac{1}{4}} = \tfrac{1}{2}; \text{ for } \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4}.$$

2. What is the square root of $\frac{4}{9}$?

$\sqrt{\frac{4}{9}} = \frac{2}{3}$; for $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$. Hence, $\sqrt{\text{Fraction}} = \dfrac{\sqrt{\text{Numerator}}}{\sqrt{\text{Denominator}}}$.

3. Find the square root of .25.

$$\sqrt{.25} = .5; \text{ for } .5 \times .5 = .25.$$

4. The $\sqrt{.4}$ = what?

Not .2, for $.2 \times .2 = .04$.
But $\sqrt{.4} = \sqrt{.40} = \sqrt{.4000}$.

By rule: .40′00 (.632

$$\begin{array}{r|l} & .40'00\,(\,.632 \\ & 36 \\ 123 & 400 \\ & 369 \\ 1262 & 3100 \\ & 2524 \end{array}$$

Hence, $\sqrt{.4} = .632+$.

In such cases annex ciphers and make periods *from the point toward the right.*

PROBLEMS.

1. Find the square root of:

1. $\frac{1}{9}$.	5. $\frac{81}{169}$.	9. $\frac{2304}{5184}$.	13. $\frac{898704}{935089}$.
2. $\frac{4}{16}$.	6. $\frac{121}{441}$.	10. $\frac{2704}{4225}$.	14. $\frac{1000000}{1000000000}$.
3. $\frac{9}{100}$.	7. $\frac{64}{225}$.	11. $\frac{55225}{784996}$.	15. $\frac{1002001}{1234321}$.
4. $\frac{100}{10000}$.	8. $\frac{225}{729}$.	12. $\frac{781456}{978121}$.	16. $\frac{4937284}{11108889}$.

2. Find the square root of:

1. .09.	6. .12345.	11. .003969.
2. .9.	7. .763876.	12. 1.679616.
3. .0144.	8. .30858025.	13. 204.7761.
4. .144.	9. .093636.	14. .00009801.
5. .0100.	10. .099225.	15. .00010201.

3. Find the square root of:

1. $\frac{7}{8}$.	6. $\frac{11}{16}$.	11. $\frac{3}{8}$.	16. $\frac{2}{3}$.
2. $\frac{8}{9}$.	7. $\frac{12}{25}$.	12. $\frac{30}{32}$.	17. $\frac{5}{21}$.
3. $\frac{144}{1728}$.	8. $\frac{1}{2}$.	13. $\frac{3}{4}+\frac{5}{6}+\frac{6}{9}$.	18. $65\frac{16}{125}$.
4. $\frac{1225}{4489}$.	9. $\frac{3}{4}$.	14. $\frac{32}{125}$.	19. $102\frac{1}{100}$.
5. $\frac{289}{2704}$.	10. $\frac{5}{8}$.	15. $1\frac{13}{16}$.	20. $\frac{349}{800}$.

Suggestion: $\frac{7}{8} = .875$; $\sqrt{.87'50} =$ what?

SQUARES.

Since the area of a square lot whose side is 12 rods equals 12×12 or 144 square rods, a side of the lot $= \sqrt{144}$.

Hence the formula:

$$\textbf{Side of Square} = \sqrt{\textbf{Area}}.$$

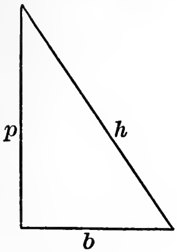
PROBLEMS.

1. What is the side of a square whose area is 1225 sq. ft.?
2. What is the side of a square whose area is 2025 sq. rd.?
3. What is the side of a square farm containing 40 A.?
4. A square plot of ground contains 320 A. How many feet long is each side?
5. A circular pond has an area of 529 sq. rd. What is the side of a square of equal area?
6. If an acre of land be laid out in a square farm, what will be the length of each side in rods?
7. To arrange 7225 men in the form of a square, how many men must be put in each line?
8. What would it cost to fence a square lot containing 640 A. at $4.00 per rod?
9. If it cost $312 to enclose a field 216 rd. long and 24 rd. wide, what will it cost to enclose a square field of equal area with a like kind of fence?
10. The attempt to form a square of 10,200 men excluded 200 of the men. How many men stood in each line of the square?
11. If the faces of a cubical box measure 23,064 sq. in., how many linear inches in one of its edges?
12. Which will cost the more to fence, a field measuring 40 by 80 rd. or a field of the same area in the form of a square? How much more at $1.33\frac{1}{3}$ per rod?

TRIANGLES.

A **Triangle** is a figure bounded by three straight lines.

A **Right Triangle** has one right angle.



Right Triangle.

h denotes the hypotenuse, the side opposite the right angle; *p*, the perpendicular; *b*, the base.

These three lines are so related that $h^2 = b^2 + p^2$.

Hence it follows that

$$\left.\begin{array}{l} 1.\ h = \sqrt{b^2 + p^2} \\ 2.\ b = \sqrt{h^2 - p^2} \\ 3.\ p = \sqrt{h^2 - b^2} \end{array}\right\} \text{Formulæ.}$$

1. The base of a right triangle is 10 feet, its perpendicular 15 feet. Find its hypotenuse.

$h. = \sqrt{b^2 + p^2} = \sqrt{100 + 225} = \sqrt{325} = 18$ very closely.

Nos.	*h*	*p*	*b*
1	*x*	3	2
2	6	*x*	5
3	8	7	*x*
4	10	9	*x*
5	12	*x*	11
6	*x*	13	14

2. Find the sides indicated by *x* in the table, using formulæ 1, 2, and 3.

No. 3. $b = \sqrt{h^2 - p^2} = \sqrt{64 - 49} = \sqrt{15} = 3.87 +$.

3. The perpendicular of a right triangle is 30 ft. and the hypotenuse is 50 ft. What is the base?

4. A square floor contains 400 sq. ft. Find the length of the longest straight line that can be drawn thereon.

5. A tree 150 ft. high stood on the bank of a stream. A part broken off 125 ft. from the top exactly measured the distance to the opposite bank. How wide was the stream?

6. How far from a tower 40 ft. high must the foot of a ladder 50 ft. long be placed that it may exactly reach the top of the tower?

7. The inner dimensions of a box are 36, 24, and 12. Find the length of the longest straight rod that can be put therein.

8. A ladder 40 ft. long is so placed in a street that, without being moved at the foot, it will reach a window on one side 33 ft. and on the other side 21 ft. from the ground. What is the breadth of the street?

9. $x = \sqrt{h^2 - b^2}$. Draw a figure for this equation, and write x upon the line to be found.

10. Make a ten-foot pole the hypotenuse, and find exact lengths for the base and perpendicular.

Three Sides Given to Find the Area.

1. If the three sides of a triangle are 2, 5, and 6, what is its area?

Process.

(*a*.) $\frac{2+5+6}{2} = \frac{13}{2} = 6.5.$

(*b*.) $6.5 - 2 = 4.5$; $6.5 - 5 = 1.5$; $6.5 - 6 = .5$.

(*c*.) Area $= \sqrt{6.5 \times 4.5 \times 1.5 \times .5} = \sqrt{21.9375} = 4.68.$

Brief directions are:

1. Find half the sum of the sides.

2. From the half sum subtract each side separately.

3. Find the square root of the product of the half sum and the three remainders.

2. What is the area of a triangle whose sides are respectively 4 in., 5 in., and 6 in.?

3. Find the area of a triangular lot whose sides are respectively 20, 25, and 28 rods.

4. Find the area of a triangular farm whose sides are 400 yd., 500 yd., and 600 yd.

5. What is the area of a triangle whose sides are 6, 8, and 12 ft.?

CUBE ROOT.

1. The **Cube Root** of a number is one of its three equal factors. $216 = 6 \times 6 \times 6$; 216 is therefore a cube, and 6 is its *cube root*.

2. The numbers represented by the digits and their cubes are:

Numbers,	0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Cubes,	0,	1,	8,	27,	64,	125,	216,	343,	512,	729.

NOTE.—This table should be memorized.

3. The numbers are the cube roots of their cubes. 1 is the cube root of 1; 2 is the cube root of 8; 3 is the cube root of 27, and so on.

4. The cube root of 216 may be thus expressed: $\sqrt[3]{216}$ or $216^{\frac{1}{3}}$.

Cube Root Found by Factoring.

The prime factors of 64 are $\overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} = 4 \times 4 \times 4$; therefore $\sqrt[3]{64} = 4$.

$216 = \overline{2 \times 2 \times 2} \times \overline{3 \times 3 \times 3} = 6 \times 6 \times 6$; therefore $\sqrt[3]{216} = 6$.

In like manner find the cube root of 27, 125, 343, 512, 729, 4096, 42,875, 166,375, 185,193.

Periods and Roots Compared.

1. Separating the following numbers into three-figure periods as far as possible, we have:

One Period.	Two Periods.	Three Periods.
$\sqrt[3]{1'} = 1.$	$\sqrt[3]{1'000} = 10.$	$\sqrt[3]{1'000'000} = 100.$
$\sqrt[3]{729'} = 9.$	$\sqrt[3]{980'001} = 99.$	$\sqrt[3]{998'000'001} = 999.$

2. Obviously one period in the cube gives but one figure

in the root; two periods in the cube, two figures in the root; three periods in the cube, three figures in the root.

Hence the principle:

The number of figures in the cube root equals the number of three-figure periods into which the number can be pointed off, beginning at units.

NOTE—The period on the extreme left may contain only one or two figures.

Extraction of the Cube Root.

General Method.

Find the cube root of 74,088.

	Cube. Root.
1. The number, pointed off into periods, is	74′088 (42
2. The greatest cube in 74 is	64
3. The cube root of 64 is 4, the first figure of the root.	
4. Subtracting and annexing the second period, we have	10088
5. 300 times the root-figure $4^2 = 4800$, the trial divisor.	
6. $10088 \div 4800$ gives 2 for the second figure of the root.	
7. The complete divisor consists of:	
(*a.*) The trial divisor, 4800	
(*b.*) 30 times 4×2, or 240	
(*c*) $2 \times 2 = 2^2$, or 4	
Sum = 5044	
8. Multiplying the sum, 5044, by 2, we have	10088
9. Subtracting, we have	0

Therefore the cube root of 74,088 is 42.

The process, freed from explanation, stands thus:

	74′088 (42
	64
4800	10088
240	
4	
5044	10088
	0

The algebraic discussion is as follows:

We have seen that the cube of any number consisting of tens and units = the tens^3 + 3 times the tens^2 × the units + 3 times the tens × the units^2 + the units^3. For example, $35^3 = (30 + 5)^3 = 30^3 + 3 \times 30^2 \times 5 + 3 \times 30 \times 5^2 + 5^3$. That is, the cube of a two-digit number consists of four parts, which may be presented thus:

$$
\begin{array}{lll}
a. & 30^3 & = t^3. \\
b. & \overline{3 \times 30^2} \times 5 & = \overline{3 \times t^2} \times u. \\
c. & \overline{3 \times 30} \times 5^2 & = \overline{3 \times t} \times u^2. \\
d. & 5^3 & = u^3.
\end{array}
$$

By regarding these four parts we may readily see how the cube root of a number may be obtained.

1. What is the cube root of 42,875?

Process.

$$
\begin{array}{rr}
 & 42'875\,(\,35 \\
2700 & 27 \\ \cline{2-2}
450 & 15\,875 \\
25 & 15\,875 \\ \hline
3175 & 0
\end{array}
$$

Pointing off, we have:	42′875
Finding Part a, t^3, we have:	27
$\sqrt[3]{27} = 3$, the *tens* of the root.	
Subtracting 27, we have remaining parts b, c, d =	15 875
Assume $\overline{3 \times t^2} \times u = 15875$.	

Dividing by the factor $\overline{3 \times t^2}$, we shall obtain the other factor, the units.

$\overline{3 \times t^2} = 2700$; $15{,}875 \div 2700 = 5$, the *units* of the root.

Having thus found by trial the units, we must now form the parts b, c, d, and subtract their sum.

$$
\left.\begin{array}{l}
b.\ 3 \times 30^2 \\
c.\ 3 \times 30 \times 5 \\
d.\ 5 \times 5
\end{array}\right\} \times 5 = \left\{\begin{array}{r}
2700 \\
450 \\
25
\end{array}\right\} \times 5 = 3175 \times 5 = \begin{array}{r} 15\,875 \\ \hline 0 \end{array}
$$

Hence, the cube root of 42,875 is 3 tens + 5 units = 35.

The rule, briefly stated, is:

1. **Point off the number into three-figure periods.**
2. **Find in the first period the greatest cube and its root.**

3. Subtract and annex the second period.

4. To find the second figure of the root, divide the remainder by 300 times the square of the first root-figure.

5. To this divisor add 30 times the product of the two root-figures; also, the square of the second figure.

6. Multiply the sum by the second root-figure.

7. Then apply again 3, 4, 5, 6, and 7, if necessary.

To apply the rule:

2. Find the cube root of 79,507.

Process.

	(1.) (2.)(4.)
(4.)	79′507 (4 3
4800 (4.)	(2.) 64
360 (5.)	(3.) 15 507
9 (5.)	(6.) 15 507
5169 (6.)	0

3. Find the cube root of 2,048,383.

Process.

	2′048′383 (127, Ans.
300 (4.)	1
60 (5.)	1048 (3.)
4 (5.)	728 (6.)
364 (6.)	320383 (7.)

Now call the root 12 *tens*, and proceed to find the units by (4).

144	320383, Rem.
300	320383
43200 (4.)	0 (6.)
2520 (5.)	
49 (5.)	
45769 (6.)	

EXERCISES.

Find the cube root of:

1. 614,125.	8. 2,000,376.	15. 592,704.
2. 74,088.	9. 153,990,656.	16. 1,860,867.
3. 15,625.	10. 41,063,625.	17. 34,328,125.
4. 32,768.	11. 12,167.	18. 145,531,576.
5. 103,823.	12. 32,768.	19. 264,609,288.
6. 1,953,125.	13. 79,507.	20. 1,879,080,904.
7. 5,545,233.	14. 59,319.	21. 12,895,213,625.

CUBE ROOT OF COMMON AND DECIMAL FRACTIONS.

1. What is the cube root of $\frac{1}{27}$?

$$\sqrt[3]{\tfrac{1}{27}} = \tfrac{1}{3}; \text{ for } \tfrac{1}{3} \times \tfrac{1}{3} \times \tfrac{1}{3} = (\tfrac{1}{3})^3 = \tfrac{1}{27}.$$

FORMULA.

$$\sqrt[3]{\text{Fraction}} = \frac{\sqrt[3]{\text{Numerator}}}{\sqrt[3]{\text{Denominator}}}.$$

2. What is the cube root of .8?

Not .2, for $.2 \times .2 \times .2 = .008$.

$$\sqrt[3]{.8} = \sqrt[3]{.800000}.$$

By rule: .800′000 (.92 +

```
                 .800'000 ( .92 +
                  729
 24300            71000
   540            49688
     4            21312, Rem.
 -----
 24844
```

Hence $\sqrt[3]{.8} = .92 +$.

In such cases annex ciphers, and make periods *from the point toward the right.*

3. Find the cube root of:

1. $\frac{8}{27}$.	4. $\frac{64}{512}$.	7. $\frac{19683}{166375}$.	10. $\frac{68921}{148877}$.
2. $\frac{1}{216}$.	5. $\frac{125}{729}$.	8. $\frac{12167}{32768}$.	11. $\frac{274625}{438976}$.
3. $\frac{27}{343}$.	6. $\frac{1728}{46656}$.	9. $\frac{39304}{46656}$.	12. $\frac{551368}{614125}$.

4. Find the cube root of:

1. .008.	6. 2.197.	11. 7.
2. .08.	7. 9.261.	12. 34.965783.
3. .8.	8. 185.193.	13. 41.063625.
4. .125.	9. .1.	14. .000001.
5. .25.	10. 6.	15. .0000001.

Suggestion: $\sqrt[3]{.08} = \sqrt[3]{.080} = .4$, etc. Find two more places.

5. Find the cube root of:

1. $\frac{3}{4}$.	3. $\frac{3}{9}$.	5. $\frac{2}{3}$.	7. $\frac{1728}{1898}$.	9. $\frac{5760}{7000}$.
2. $\frac{9}{13}$.	4. $\frac{5}{4}$.	6. $\frac{5}{6}$.	8. $\frac{1492}{1776}$.	10. $\frac{8000}{63360}$.

Suggestion: $\frac{3}{4} = .75$; $\sqrt[3]{.750} =$ what?

VOLUME.

Volume of a cube = side^3. Therefore:

$$\sqrt[3]{\textbf{volume}} = \textbf{side of cube.}$$

PROBLEMS.

1. A cubical cistern contains 1331 solid feet. What is the length of one side of the cistern?

Volume = 1331. $\sqrt[3]{1331} = 11$, length of one side.

2. A cubical pedestal contains 373,248 cu. in. What is the length of one of its sides?

3. A cubical box contains 474,552 cu. in. What is the area of one of the surfaces of the box?

4. How much paper will cover the six surfaces of a cubical box whose volume is $\frac{27}{64}$ cu. ft.?

5. What is the depth of a cubical box that will hold a bushel?

6. A wagon-box holds 100 bu. The length is twice the width and the width and depth are equal. Find the dimensions.

7. Find the cost, at 83 cts. per square yard, of lining the inside of a cubical box holding 900 gal. of water.

8. Find the height of a cubical pile of wood containing 179 cords.

SIMILAR FIGURES.

1. **Similar figures** have the same shape, but differ in size.

2. Figures are either *surfaces* or *solids.*

3. A surface has *dimensions* and *area.*

4. A solid has *dimensions* and *volume.*

5. The relation of similar figures is in accordance with the following general principles:

Similar Surfaces.

1. The areas of similar surfaces are to each other as the squares of their like dimensions.

2. The like dimensions of similar surfaces are to each other as the square roots of their areas.

PROBLEMS.

1. Two surfaces having the same shape are to each other as 144 to 36. What is the ratio of their lengths?

Process.

$$
\begin{aligned}
L. : l. &= \sqrt{144} : \sqrt{36}. \\
L. : l. &= 12 : 6. \\
L. : l. &= 2 : 1.
\end{aligned}
$$

Hence the ratio of their lengths is 2 : 1.

2. The radius of a certain circle is 5 ft. What is the radius of another circle containing twice the area of the first?

Suggestion: $5 : R. = \sqrt{1} : \sqrt{2}$.

3. The surfaces of two bodies having the same shape are as 100 : 25. What is the ratio of their widths?

4. If the area of a circle, whose diameter is 2 ft., is 6.2832 sq. ft., what is the diameter of a circle whose area is 25.1328 sq. ft.?

Suggestion: 25.1328 = 4 times 6.2832.

5. A farmer has a field 50 rd. wide by 80 rd. long, which contains 25 A. Find the dimensions of a similar field containing 16.81 A.

6. If a horse tied to a stake by a rope 8.79 rd. long can graze upon $1\frac{1}{2}$ A. of land, how long must the rope be that he may graze upon 6 A.?

7. If a pipe whose diameter is 1.5 in. fills a cistern in 5 hours, in what time will a pipe whose diameter is 3 in. fill the same cistern?

8. A half-inch pipe discharges a barrel of water in a certain time. How much will a 2-in. pipe discharge in the same time?

9. If a 1-in. pipe discharges 1 gal. in 45 seconds, how much will a 2-in. pipe discharge in 60 seconds?

10. A rectangular piece of land has a width of 160 ft. and is valued at $1200. What is the value of a similar piece of land having twice the length and breadth?

Similar Solids.

1. The volumes of similar solids are to each other as the cubes of their like dimensions.

2. The like dimensions of similar solids are to each other as the cube roots of their volumes.

PROBLEMS.

1. Of two spheres, one is 1000 times the size of the other. If the diameter of the smaller is 6 inches, how many feet are in the diameter of the larger?

Process.

$$\sqrt[3]{1} : \sqrt[3]{1000} = 6 \text{ in.} : x.$$
$$1 : 10 = 6 \text{ in.} : x.$$
$$x = 10 \times 6 = 60 \text{ in.} = 5 \text{ feet.}$$

2. The diameter of a ball weighing 32 lb. is 6 in. What is the diameter of a ball weighing 4 lb.?

3. The diameters of two spheres are respectively 4 and 12 in. The larger sphere is how many times the smaller?

4. If a 2-in. globe of gold is worth $500, what is the value of a 6-in. globe of gold?

5. If the diameter of the sun is 112 times as long as that of the earth, how much greater is the mass of the sun than that of the earth?

6. If the diameter of the moon is 2000 mi. and that of the earth is 8000 mi., what is the ratio of their volumes?

7. The weights of two cylinders of the same shape are as 27 to 64. What is the ratio of their lengths?

Process.

$$\text{l.} : \text{L.} = \sqrt[3]{27} : \sqrt[3]{64}.$$
$$\text{l.} : \text{L.} = 3 : 4.$$

8. If a log $1\frac{1}{2}$ ft. in diameter contains 35 cu. ft., what is the diameter of a log of the same length that contains 105 cubic feet?

9. If a pyramid of hay 12 ft. high contains 8 tons, how high is a similar pyramid that contains 60 tons?

MENSURATION.

Mensuration treats of the measurement of *lines, surfaces,* and *volumes.*

Important Suggestion.—Experience has shown that much, if not all, of the difficulty in mensuration results from the pupil's failure fully to understand the terms used in describing surfaces and solids, and from the consequent failure to get a clear conception of the objects themselves. Therefore it is suggested that pupils be required to learn all definitions. This can best be done by a careful study of the figures in connection with the definitions. Concrete illustration should be used whenever possible, and pupils should be permitted to handle objects. In the absence of geometrical forms, pupils should draw correct and neat figures to represent the conditions of each problem. Time thus spent will produce good results.

DEFINITIONS.

1. A **Line** has *length,* but no width.

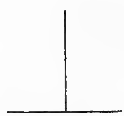
2. A **Straight Line** is one which has the same direction throughout its whole length. It is the shortest distance between two points.

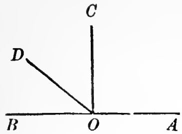
3. A **Curved Line** is one which changes its direction at every point in its length.

4. **Parallel Lines** are equidistant throughout their whole length.

5. A **Horizontal Line** is a line parallel to the horizon. The line $A\,B$ is horizontal.

6. When two straight lines meet or intersect in such manner as to form right angles, they are said to be **Perpendicular,** the one to the other.





7. A **Vertical Line** is one that is perpendicular to the horizon. $C\ O$ is a vertical line.

8. An **Angle** is the amount of divergence of two lines which meet at a point. The point is called the **Vertex.** In the angle $A\ O\ C$, O is the vertex.

The size of an angle is not dependent upon the length of the lines which form the angle.

9. There are three kinds of angles:

1. **Right Angle.**
2. **Acute Angle,** less than a right angle.
3. **Obtuse Angle,** greater than a right angle.

Draw an angle of each kind.

10. A **Diagonal** is a straight line joining opposite angles.

11. The **Perimeter** measures the bounding line of a surface.

12. An **Inscribed Figure** is the largest figure of a given kind that can be drawn within another. (See page 343.)

13. A **Circumscribed Figure** is the smallest figure of a given kind that can be drawn about another. (See page 343.)

14. **Concentric Circles** are those having the same centre. The space between two concentric circles is called a **Ring.**

Draw two concentric circles.

SURFACES.

1. **Surface** is the outside of anything. Every surface has two dimensions,—*length* and *breadth.*

2. **Area** is the extent of a surface, and is estimated in square units; as, square inches, square feet, square yards, etc.

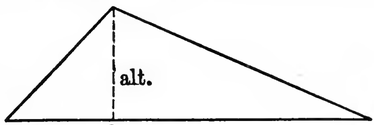
3. A **Plane Surface** is flat, like the walls and the floor of the school-room. Name some plane surfaces.

4. A **Curved Surface** is like that of a ball. Name some curved surfaces.

5. Surfaces are bounded by straight or curved lines; hence the terms *rectilinear* and *curvilinear* as applied to surfaces.

TRIANGLES.

1. A **Triangle** is a plane surface having three angles and three sides. Every triangle has two dimensions, *altitude* and *base.*



2. Triangles, classified according to their angles, are of three kinds:

1. **Right Triangle,** having *one right angle.*
2. **Obtuse-Angled Triangle,** having *one obtuse angle.*
3. **Acute-Angled Triangle,** having *three acute angles.*

Draw a triangle of each kind.

3. Triangles classified according to their sides are of three kinds:

1. **Equilateral Triangle,**—*all sides equal.*
2. **Isosceles Triangle,**—*two sides equal.*
3. **Scalene Triangle,**—*no two sides equal.*

Draw a triangle of each kind.

4. We have learned (page 164) that the area of a rectangle is the product of the length and breadth (base and altitude). Every triangle is regarded as one-half of a rectangle having the same base and altitude; hence the formula for the area of a triangle is:

$$\textbf{Area of triangle} = \frac{\textbf{base} \times \textbf{altitude}}{\textbf{2}}.$$

By drawing figures and by cutting paper let pupils prove the foregoing.

PROBLEMS.

1. The base of a triangle is 150 yd. and its altitude is 75 yd. What is its area?

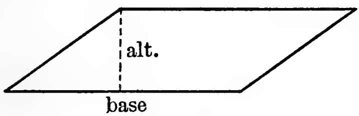
2. Required the area of a triangle whose base is 40 rd. and altitude 30 rd.

3. What is the area of an equilateral triangle whose sides are each 10 chains?

4. A board 5 ft. long has the shape of an isosceles triangle and measures at its base 15 inches. Find the number of square feet it contains.

5. Find the area of a right triangle, base 23.1 ft., altitude 32.1 ft.

PARALLELOGRAM.



Rhomboid.

1. A **Parallelogram** is a plane surface whose opposite sides are parallel.

2. There are four parallelograms:

1. **Square**—Sides parallel and equal; four right angles.
2. **Oblong**—Sides parallel; opposite sides equal, adjacent sides unequal; four right angles.
3. **Rhombus**—Sides parallel and equal; two angles obtuse and two acute.
4. **Rhomboid**—Sides parallel; opposite sides equal; two angles obtuse and two acute.

3. The altitude of the Rhombus and the Rhomboid is the perpendicular distance between the parallel sides.

4. Make correct forms of the parallelograms. Draw the diagonal and mark the altitude.

5. The formula for the area of a parallelogram is:

$$\text{Area} = \text{base} \times \text{altitude}.$$

PROBLEMS.

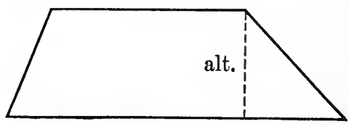
1. A field in the form of a square is 64 rd. long. Find its area in acres.

2. How many square feet in an oblong board 90 in. long and 14 in. wide?

3. A pane has the form of a rhombus, measures 16 in. on each side, and the perpendicular distance between its sides is one-half the length of a side. Find its area.

4. Find the area in acres of a rhomboidal field which measures 10 ch. in length and 8 ch. in breadth.

TRAPEZOID.



1. A Trapezoid is a four-sided plane figure having two sides parallel.

2. The *altitude* of a trapezoid is the perpendicular distance between the parallel sides.

3. The formula for the area of a trapezoid is:

$$\text{Area} = \frac{\text{sum of parallel sides}}{2} \times \text{altitude}.$$

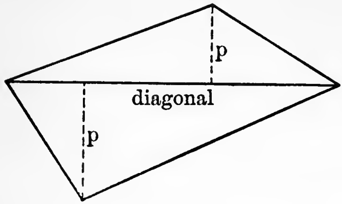
What do you get when you divide the sum of the parallel sides by 2?

PROBLEMS.

1. Find the area of a trapezoid with parallel sides of 50 rd. and 78 rd., and with a distance between them of $39\frac{1}{2}$ rd.

2. A trapezoidal field contains $12\frac{1}{2}$ A. Its parallel sides are 220 rd. and 180 rd. How far apart are the parallel sides?

THE TRAPEZIUM.



1. A **Trapezium** is a four-sided plane surface having no two of its sides parallel.

2. The *diagonal* of a trapezium is a straight line connecting opposite angles. The diagonal divides the trapezium into two triangles.

3. The altitude of each triangle is the perpendicular distance between the diagonal and the opposite angle.

4. To find the area of a trapezium, the diagonal and the altitude of each triangle being given, first find the area of each triangle, then add the areas.

5. The following is the formula:

$$\text{Area} = \text{diagonal} \times \frac{\text{sum of altitudes}}{2}.$$

PROBLEMS.

1. A field has the form of a trapezium with a diagonal length of 1000 ft., and with perpendicular distances of 450 and 350 ft. Find the area.

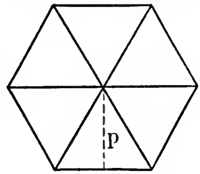
2. Require the area of a trapezium whose diagonal measures 145 ft. and the altitudes of the two triangles are 34 and 44 ft. respectively.

THE REGULAR POLYGON.

1. Every plane surface bounded by straight lines has as many angles as it has sides.

Naming plane figures according to the number of angles each contains, we have the following:

Triangle, *three angles;* **Quadrangle**, *four angles;* **Pentangle**, or **Pentagon**, *five angles;* **Hexagon**, *six angles;* **Heptagon**, *seven angles;* **Octagon**, *eight angles;* **Nonagon**, *nine angles;* **Decagon**, *ten angles*, etc.



Hexagon.

2. **Polygon** is a general term, and is applicable to any figure having three or more angles.

3. A **Regular Polygon** is one having all its angles and sides equal.

4. Any regular polygon may be divided into as many equal triangles as the polygon has sides. If the base and the altitude of the triangles be known, the area of the polygon may be found by multiplying the area of one triangle by the number of triangles.

5. The formula for the area of a regular polygon is:

$$\text{Area} = \text{perimeter} \times \frac{\text{perpendicular}}{2}.$$

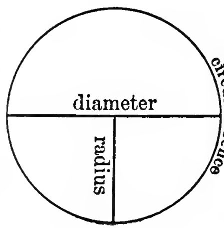
NOTE.—The word "perpendicular" is here used to denote the altitude of one triangle.

PROBLEMS.

1. Find the area of a hexagon whose sides are each 12 in. and the perpendicular distance from the centre to a side is 8 in.

2. What is the area of a regular pentagon whose side is 15 ft. and the altitude of the triangles into which it may be divided is 8.602 ft.?

THE CIRCLE.



1. A **Circle** is a plane surface bounded by a curved line every point of which is equally distant from a point within the circle called the centre. The point where the straight lines in the figure meet is the **Centre** of the circle.

2. The **Circumference** of a circle is the bounding line.

3. The **Diameter** is the distance across the circle measured through the centre.

4. The **Radius** is one-half of the diameter.

By a geometrical process, it has been found that if the diameter of a circle is 1, the circumference is 3.1416. Hence, if we know the diameter of a circle, we may find the circumference by multiplying the diameter by 3.1416; and knowing the circumference, we may find the diameter by dividing the circumference by 3.1416.

The number 3.1416 is called the ratio of a circumference to its diameter. Pupils should remember this number, as it is of much use in measuring circular surfaces, etc.

5. The following formulæ apply to the circle:

1. $\text{Circumference} = \text{diameter} \times 3.1416.$
2. $\text{Diameter} = \dfrac{\text{circumference}}{3.1416}.$
3. $\text{Area} = \text{circumference} \times \dfrac{\text{radius}}{2}.$
4. $\text{Area} = \text{Radius}^2 \times 3.1416.$
5. $\text{Area} = \text{diameter}^2 \times .7854.$

NOTE.—Observe that .7854 is one-fourth of 3.1416.

MISCELLANEOUS PROBLEMS.

1. What is the circumference of a circle having a diameter of 21 ft.?

2. What is the diameter of a circle 33 yd. in circumference?

3. What is the circumference of a circle whose radius is 16 yd.?

4. What is the area of a circle whose circumference is 18 in.?

5. Find the perimeter of a triangle whose sides are respectively $3\frac{1}{2}$ ft., $4\frac{3}{4}$ ft., and $5\frac{7}{8}$ ft.

6. A horse is tied by a rope 7 rd. long, and can reach 2 ft. beyond the end of the rope. How much surface can he graze over?

7. Find the circumference of a circle whose diameter is 14 ft.

8. Find the diameter of a circle whose circumference is 1 ft.

9. Find the area of a circle whose radius is 7 yd.

10. The radius of a grass plot is 42 ft. Find the area of a walk 4 ft. wide running around the grass plot.

11. Find the area of a triangle whose base is 10 ft. and altitude $2\frac{2}{5}$ ft.

12. What is the area of a trapezium the diagonal of which is 110 ft., and the perpendiculars to the diagonal are 40 ft. and 60 ft. respectively?

13. If a horse is tethered by a rope 20 rd. long, over how much surface can he graze?

14. The base of a triangle is 300 yd. and its altitude is 150 yd. Find the area.

15. Two opposite sides of a quadrangular field are parallel, and are 140 yd. and 170 yd. long. The shortest measure across the field is 90 yd. What is the area?

16. A rectangular tank is 12 ft. long, 4 ft. wide, and 3 ft. high. How many square feet of sheet lead will be required to line it?

17. A diagonal of a field in the form of a trapezium is 17 chains 56 links; the perpendiculars to that diagonal from the opposite angles are 8 chains 82 links, and 7 chains 73 links. What is the area?

18. Find the diameter of a circle whose circumference is 316 ft.

19. What is the circumference of a circular pond whose diameter is 45 rods?

20. What is the area in acres of a circular island whose circumference is 2 miles?

21. A farm in the form of a trapezoid has its parallel sides 72 ch. and 84 ch. in length, and the perpendicular distance between them is 40 ch. How large is the farm?

22. How many rods of fence will be needed to go round a circular park containing 120 A.?

Suggestion: Draw figures to illustrate the following problems.

23. A circular yard 200 feet in diameter has a walk 6 feet wide bordering on the circumference and extending entirely around the yard. What is the area of the walk?

24. If within a circle 10 feet in diameter a circle 6 feet in diameter be drawn so that the two circles shall meet at one point, what will be the area of the crescent thus formed?

25. The side of the largest regular hexagon that can be inscribed within a circle 6 ft. in diameter is equal to the radius of the circle. How much waste will there be in cutting such hexagon from the circle?

26. After making the hexagon in problem 25, suppose you should decide to make from the hexagon as large a circle as possible, what would be the diameter of the circle?

VOLUMES.

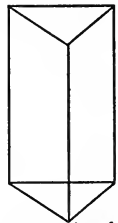
1. A **Solid** has three dimensions, *length*, *breadth*, and *thickness*.

2. The **Volume** of a solid is the number of cubic units which it contains; it may be cubic inches, cubic feet, etc.

3. The **Lateral Surface** of a solid is the area of its sides or faces. This is also called **Convex Surface.**

4. To find the volume of a solid three dimensions or their equivalent must be given; and to find any one of the dimensions of a solid the volume and two dimensions or their equivalent must be given.

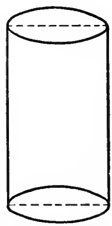
THE PRISM AND CYLINDER.



Base a triangle

1. **Prism** is a solid whose ends are equal parallel polygons, and whose sides are rectangles. The ends are called *bases* and the sides are called *lateral faces.*

2. The form of the base gives a prism its distinguishing name. If the base be a triangle, the prism is called a *triangular prism;* if the base be a square, the prism is called a *square prism;* if the base be a pentagon, the prism is called a *pentangular prism,* etc.



Base a circle

3. A **Cylinder** is a solid with circular ends and uniform diameter. The ends are called the *bases,* and the curved surface is called the *lateral surface,* or *convex surface.*

4. The following formulæ apply to prisms and cylinders:

1. **L. S. = Perimeter of Base × Altitude.**
2. **Vol. = Area of Base × Altitude.**

PROBLEMS.

1. Find the lateral surface of a pentangular prism, the side of the base being 8 in. and the height 35 in.

2. Find the lateral surface of a cylinder whose height is 25 in. and diameter of the base 15 in.

3. Find the lateral surface of a triangular prism 24 ft. high, the sides of the base being 3 ft., 4 ft., and 5 ft.

4. Find the entire surface of a cylinder 9 ft. high and 3 ft. in diameter.

5. Find the entire surface of a prism 18 in. square and 7 ft. high.

6. Find the entire surface of a prism 18 in. high, the base being a triangle whose sides are 3 in., 4 in., and $4\frac{1}{2}$ in.

7. Estimate the volumes of the solids described in problems 2, 3, 4, 5, and 6.

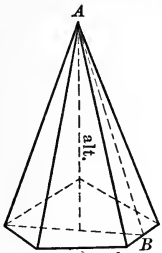
8. What must be the diameter of a cylindrical tank 10 ft. deep to contain 8460.288 gal.?

9. A rectangular bin 5 ft 4 in. long and 3 ft. 2 in. wide contains 64 bu. What is the depth?

10. If you cut a cylinder as large as can be made from a prism 6 in. square and 18 in. long, how much of the prism will be wasted?

THE PYRAMID AND CONE.

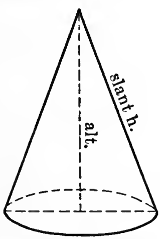
1. A **Pyramid** is a solid having a regular polygon for a base and ending in a point at the top.



Base a reg. polygon

Draw a triangular pyramid. A square pyramid.

2. A **Cone** is a solid having a circular base and tapering to a point.



Base a circle

3. The point of a pyramid and of a cone is called the **Vertex**.

4. The **Altitude** of a cone and of a pyramid is a straight line drawn from the vertex perpendicular to the base.

5. The **Slant Height** of a pyramid is a straight line drawn from the vertex perpendicular to one side of the base, as *A B*.

6. The **Slant Height** of a cone is a straight line drawn from the vertex to any point on the circumference of the base.

1. L. S. $=$ Perimeter of Base $\times \frac{\text{Slant Height}}{2}$.

2. Vol. $=$ Area of Base $\times \frac{\text{Altitude}}{3}$.

PROBLEMS.

1. Find the lateral surface of a hexagonal pyramid whose slant height is 20 ft. and each side of the base 5 ft.

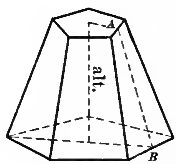
2. What is the extent of the lateral surface of a cone the base of which is 27 in. in diameter and the slant height 5 ft.?

3. If wheat be piled in a corner of a rectangular room in such manner as to form a portion of a cone, how many bushels are in the pile if the top of the pile is 8 ft. from the floor and the outer edge 5 ft. from the angle formed by the walls?

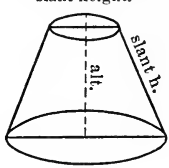
4. A conical glass is 7 in. deep and $5\frac{1}{2}$ in. in diameter. What part of a gallon will it hold?

5. A quadrangular pyramid is 16 in. square at the base and 3 ft. high. In making from this pyramid the largest possible cone, how much must be cut off?

FRUSTUM OF PYRAMID AND OF CONE.



The line A B is the slant height.



1. A **Frustum of a Pyramid** is that part of a pyramid which remains when the top is cut off by a plane parallel to the base.

2. A **Frustum of a Cone** is that part of a cone which remains when the top is cut off by a plane parallel to the base.

3. The **Altitude** and the **Slant Height** of frustums are found in the same manner as in the case of the pyramid and the cone.

4. The following formulæ are applicable to pyramids and cones:

$$1.\ \text{L. S} = \frac{\text{Sum of Perimeters of the 2 Bases}}{2} \times \text{Slant Height.}$$

$$2.\ \text{Vol.} = [\text{Sum of Bases} + \sqrt{\text{Product of Bases}}] \times \frac{\text{Altitude}}{3}.$$

PROBLEMS.

1. Find the lateral surface of a frustum of a pentangular pyramid if the side of the lower and upper bases be 3 ft. and 2 ft., respectively, and the slant height 9 ft.

2. What is the entire surface of a frustum of a cone, the bases being 16 in. and 10 in. in diameter and the altitude 30 in.

Suggestion: First find the slant height.

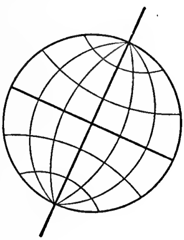
3. What is the volume of the frustum described in the second problem?

4. Find the volume of a frustum of a pyramid $4\frac{1}{2}$ ft. square at the lower base, $2\frac{1}{2}$ ft square at upper base, and $6\frac{1}{2}$ ft. high.

5. At $1.25 a square foot what will be the cost of lining with copper a vat in the shape of an inverted frustum of a cone if the upper diameter is 7 ft., the lower diameter 5 ft., and the depth 6 ft.?

THE SPHERE.

1. A **Sphere** is a solid bounded by a curved surface of which every point is equally distant from a point called the centre.



2. The following formulæ are for the surface and volume of a sphere:

1. Sur. = Diameter × Circumference.
2. Sur. = Diameter² × 3.1416.
3. Vol. = Sur. × $\frac{R.}{3}$.
4. Vol. = Diameter³ × .5236.

NOTE.—.5236 is one-sixth of 3.1416.

PROBLEMS.

1. What is the surface of a sphere 18 in. in diameter? Its volume?

2. The diameter of a sphere is 12 in., the circumference is 37.6992 in. What is the surface?

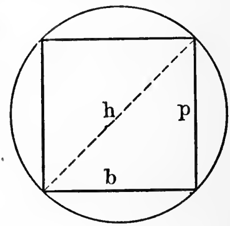
3. What is the volume of a sphere the surface of which is 78.54 sq. in. and the radius is 2.5 in.?

4. If the diameter of a cannon-ball is 15 in., what is the volume? What is the surface?

5. A hemispherical bowl 12 in. in diameter is filled with water. An iron ball put into the water is just large enough to extend from the bottom of the bowl to the surface of the water. Find the amount of water that remains in the bowl after the sinking of the ball.

CIRCLE AND LARGEST SQUARE.

h is obviously both *diameter* of the circle and *hypotenuse* of a right triangle; b and p are *base* and *perpendicular*, and also *sides* of the *square*. Since $b = p$, $h^2 = 2b^2$. Let $h = 10$; then $2b^2 = 100$, and $b^2 = \frac{100}{2}$. Taking the square root, we have $b = \sqrt{\frac{100}{2}}$. Hence the formula:



$$\text{Side of square} = \sqrt{\frac{\text{diameter}^2}{2}}.$$

When the diameter $= 1$, the side of the square $= \sqrt{\frac{1}{2}}$ or $.5 = .7071 +$, and the formula becomes:

$$\text{Side of square} = \text{diameter} \times .7071.$$

PROBLEMS.

1. If b and p each equal 1 (see figure), what is the length of h?

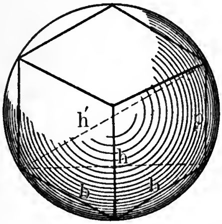
2. If p or b equal 1, what is the length of the circumference?

3. When the diameter of a circle equals 5, what is the side of the inscribed square?

4. Find the area of the inscribed square and of the circumscribed circle, when the diameter equals 5.

SPHERE AND LARGEST CUBE.

h' is obviously both diameter of the sphere and hypotenuse of the erect right triangle, h', h, p; h is the hypotenuse of the horizontal triangle, h, b, b. $h^2 = 2b^2$. $(h')^2 = h^2 + p^2$. Hence $(h')^2 = 2b^2 + p^2$. But $b = p$; therefore $(h')^2 = 3b^2$. Let $h' = 10$; then $3b^2 = 100$, and $b^2 = \frac{100}{3}$. Taking the square root, we have $b = \sqrt{\frac{100}{3}}$. Hence the formula:



$$\text{Side of Cube} = \sqrt{\frac{\text{diameter}^2}{3}}.$$

When the diameter $= 1$, the side of the cube $= \sqrt{\frac{1}{3} \text{ or } .3333, \text{ etc.}} = .57735 +$, and the formula becomes:

$$\text{Side of cube} = \text{diameter} \times .57735.$$

PROBLEMS.

1. What is the volume of a pyramid whose base is a rectangle 13 by 14 feet, and whose height is 18 feet?

2. What is the volume of a cylinder 108 in. in diameter and 10 ft. long?

3. What is the lateral surface of a cone whose base is 10 ft. in diameter and slant height 20 ft.? Find also the entire surface.

4. Find the surface of a sphere whose radius is 12 inches.

5. How many gallons will a hollow globe contain whose inside diameter is 20 inches?

6. What is the lateral surface of a triangular prism whose sides are each 6 feet and whose altitude is 8 feet?

7. What is the lateral surface of a quadrangular pyramid whose base is 15 feet square and the slant height 18 feet?

8. What is the lateral surface of a cone whose base is 10 ft. in diameter and whose slant height is 10 ft.?

9. Find the volumes in problems No. 6, 7, and 8.

10. Required the surface of the frustum of a cone whose slant height is 12 feet, diameter of lower base 10 ft. and upper base 6 feet. What is the volume?

11. Find the entire surface of the frustum of a triangular pyramid whose slant height is 40 in., and the sides of the upper base 4 in. and the lower base 10 in.

12. Required the contents of a cannon ball whose diameter is 9 inches. What is the surface?

13. At 45 cents a square foot, how much will it cost to gild a ball 25 inches in diameter?

14. Find how many cubic inches of iron there are in a hollow sphere, the diameter being 15 inches long and the shell 3 inches thick?

15. A cylindrical can is 6 inches deep and 4 inches in diameter. If a cone of the same height and diameter be placed in the can, how much water will be required to fill the remaining space?

16. In the above problem, what is the ratio of the volume of the cone and cylinder? Does this show why 3 is used in the formula for the volume of a cone?

17. Find the side of the greatest square that can be inscribed in a circle whose diameter is 10 feet?

18. Find the edge of the greatest cube that can be cut from a wooden ball whose diameter is 5.5 inches.

19. I have a cubical box whose faces each contain 64 square inches. Find the diameter of the sphere that will exactly contain the box.

20. I have a circular garden whose circumference is 31.416 rods. I wish to reduce it within the circumference to the largest possible square form. Find the area of the square.

GENERAL REVIEW.

The following problems have been selected from the examination papers of the University of the State of New York.

They are introduced here for the purpose of affording a complete review of the principles and methods set forth in the previous pages of the book.

It is suggested that the best efforts of both teacher and pupil be applied to these problems, and that the science and art of arithmetic, as already illustrated, be faithfully recalled, studied afresh, and securely fixed in mind.

Let every solution, therefore, proceed systematically, and every principle involved be distinctly stated.

1. Define *sum*, and illustrate your definition by a practical example.

2. A man deposits in bank \$986.46. At different times he has drawn the following amounts: \$314.18, \$49.25, \$57.62, \$39.84, \$25.13. Find the amount remaining in the bank.

3. Find the least number of bushels of grain that can be exactly measured either by a 3-quart, a peck, a 20-quart, or a bushel measure.

4. Reduce $\frac{20265}{22967}$ to its lowest terms.

5. Simplify $\frac{\frac{3}{5} \times 1.25}{5\frac{3}{4} - 4.25}$ and express the result both as a common and as a decimal fraction.

6. Define *composite number* and give an example.

7. Make a receipted bill for the following: Harold Kirby bought of Pliny Hall, 10 lb. sugar at 5 cts., $\frac{1}{2}$ lb. tea at 60 cts., 3 lb. coffee at 40 cts., 1 sack flour at \$1.50.

8. If the shadow of a post 6 ft. high is 4 ft. 6 in. long, what is the height of a tree whose shadow at the same time is 125 ft. long? (Solve by analysis.)

9. What would it cost to dig a cellar 80 ft. × 35 ft. × 8 ft. at $.84 per cubic yard?

10. A railway train runs $\frac{2}{3}$ of a mile in $\frac{4}{5}$ of a minute. Find its velocity per hour? (Solve by analysis.)

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11. Define *quotient,* and give an illustration.

12. Find the prime factors of 1001 and 1309, and from these factors form the G. C. D., and the L. C. Dd. (least common multiple) of the two numbers.

13. A field 10 chains 50 links long and 8 chains 40 links wide produces 40 bushels of oats per acre; what is the value of the crop at 35 cents a bushel?

14. Find the sum of $9\frac{3}{5}$, $8\frac{1}{4}$, $5\frac{2}{3}$, and $\frac{5}{12}$. Express the result both as a fraction in lowest terms and as a decimal.

15. What part of an ounce (apothecaries' weight) is 5 drachms and 2 scruples?

16. Find the cost of a stick of timber 40 ft. long, 12 in. wide, 9 in. thick, at $12.50 per M., board measure.

17. A roll of wall paper 8 yd. long and 18 in. wide costs 25 cts. What will be the cost of paper for the four walls of a room 30 ft. × 27 ft. × 9 ft., no allowance being made for openings?

18. I bought 240 barrels of apples at $1.75 a barrel; lost 40 barrels through frost; at what price a barrel must I sell the remainder to gain 25% on the money invested?

19. If 2 men plough 15 acres in 5 days, working 10 hours a day, how many acres will 3 men plough in 4 days, working 8 hours a day?

20. Define *greatest common divisor* and *least common dividend* (multiple). Illustrate.

21. What is meant by *cancellation?*

22. Simplify $\frac{1}{3}$ of $\frac{3\frac{1}{7}}{4}$ of $2\frac{1}{3} \times 14$.

23. What part of a bushel is contained in a rectangular box 3 in. deep and 4 in. square? [A bushel = 2150.42 cu. in.]

24. From sixty subtract forty-seven and sixteen ten-millionths and express the decimal as a common fraction.

25. Find the cost of carpeting a room 18 ft. long, 15 ft. wide, with carpet 27 in. wide, at 75 cts. a yard.

26. Define *divisor, root, proportion, fraction.*

27. I retail oranges at 3 cts. each, gaining 150% on the purchase price. What did the oranges cost a dozen?

28. I sell an article at an advance of 25% on the cost and then discount the bill 5% for cash payment. My net gain is $63.75. Find the cost.

29. A cubic foot of water weighs $62\frac{1}{2}$ lb. Find the weight of a barrel of water.

30. On a bill of goods amounting to $485.50 I receive commercial discounts of 15%, 10%, and 5%. Find the net cost of the goods.

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31. What principal loaned for 1 yr. and 3 mo. at 6% simple interest will amount to $1000?

32. A 30-day note discounted at a New York bank yields $358.02. What was the face of the note?

33. A note for $500 at 90 days, with interest at 6%, is discounted at a bank 30 days after it is dated. Find the proceeds.

34. A certain stock pays annual dividends of 4%. At what rate must it be bought to pay 5% on the investment?

35. Find the square root of 4,004,231 to *two* places of decimals.

36. If I buy 10 shares of railway stock at 80 and sell them at 90, how many dollars do I gain and what is the rate per cent. of profit?

37. Find the smallest number that will exactly contain 15, 18, 21, 24, and 30.

38. Two men hire a pasture for \$30. A. puts in 8 horses for 10 weeks and B. 6 horses for 12 weeks. How much should each pay?

39. A house valued at \$6000 is insured for $\frac{3}{4}$ of its value at the rate of $\frac{1}{2}$ of 1% a year. How much is the annual premium?

40. Find the prime factors of 1226, 1938, and 2346. Indicate which of these factors must be combined to produce (*a*) the greatest common divisor, (*b*) the least common dividend.

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41. Make a receipted bill of the following: Sold this day to Anson White, 3 bbl. flour, at \$3.75; 75 lb. sugar, at 5 cts.; 10 lb. coffee, at 35 cts.; 2 lb. tea, at 60 cts.

42. Find the amount at simple interest of \$865.35 for 1 yr. 5 mo. 17 da. at $4\frac{1}{2}$%.

43. In a certain school district the assessed valuation of property is \$136,395, and the amount to be raised by local tax is \$785.72. Find the amount of A.'s tax, whose property is assessed at \$8500.

44. A bar of iron in the form of a cylinder, 6 feet long and 2 inches in diameter, is forged into a square bar whose cross-section is $2\frac{1}{4}$ square inches. Find the length of the new bar.

45. A man plants corn on $\frac{1}{5}$ of his land, potatoes on $2\frac{1}{2}$ times as much, and sows the remainder with wheat. He sells the wheat at 60 cts. a bushel, and receives for it \$180. If the yield of wheat was 20 bushels an acre, how much land had he?

46. Simplify the following: $\frac{3\frac{1}{5} \times 2\frac{2}{3}}{\frac{1}{2} \times \frac{4}{9} \times \frac{1}{5}}$.

47. A note for \$624 is dated August 26, 1893; July 15, 1894, there was paid on it \$62.50. Find the amount now due.

48. Find the amount of \$685 at $4\frac{1}{2}\%$ simple interest from July 1, 1894, to the present time.

49. Define and illustrate *dividend, power, ratio, factor.*

50. I buy hats at \$18 a dozen and sell them at \$2.50 apiece. Find the gain per cent.

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51. I sell goods at a discount of 10% from the marked price and still make a profit of 8%. How many per cent. above cost was the marked price?

52. What single discount is equal to a commercial discount of 10%, 10%, and 5%?

53. Find the square root of 1,080,234 to *two* decimal places.

54. Find the least possible cost of carpeting a room 15 feet long, 12 feet wide, with carpet $\frac{3}{4}$ yd. wide, at 75 cts. a running yard.

55. Write the table of avoirdupois weight. For what is this weight used?

56. Two men start from the same point on a level plain and travel, one due north at the rate of 3 miles an hour, the other due east at the rate of 5 miles an hour. How far apart will they be at the end of 10 hours?

57. Divide one millionth by eight ten-thousandths, and express the result in words.

58. Find the prime factors of 2964, and all the different composite factors into which the prime factors may be combined.

59. Define *minuend, multiplication, prime factor, common divisor, ratio.*

60. Find the amount at simple interest, at 5%, of $860 from Sept. 1, 1894, to the present time.

---

61. Show that if four quantities are in proportion the product of the means equals the product of the extremes.

62. How much is due Aug. 15, 1893, on an interest-bearing promissory note for $250, dated Buffalo, June 1, 1886, on which $50 was paid Dec. 24, 1886, and $10 Jan. 5, 1888?

63. Find the cost, at $7 per 100 sq. ft., of slating a trapezoid of which the parallel sides are 64 ft. and 32 ft., and the perpendicular distance between them is 20 ft.

64. Simplify and express decimally $\frac{18\frac{1}{2} \times 7\frac{2}{5}}{8\frac{3}{4} - 6\frac{1}{5}}$.

65. Find the square root of 8.5849.

66. Find the cost of shingles required to cover a roof 40 ft. long, 20 ft. wide at $5.00 a thousand, if it requires 36 shingles to cover 5 sq. ft.

67. Find the amount due this day on a note given in New York May 10, 1890, for $500, with interest, a payment of $35 having been made July 5, 1891.

68. Reduce to its lowest terms $\frac{(1\frac{2}{3} + 1\frac{3}{4}) \times 3}{\frac{7}{8} \times 2\frac{1}{2} - \frac{4}{5} \div \frac{3}{7}}$.

69. Define *least common dividend, factor, numerator, divisor, root, proportion, fraction.*

70. A cistern is 6 ft. square. How deep must it be to hold 30 bbl. of water?

---

71. Find the least common dividend (multiple) and the greatest common divisor of 45, 70, and 105.

72. How many times will a wheel 4 ft. in diameter revolve in going one mile?

73. Find the diagonal of a rectangle whose sides are 15 ft. and 20 ft.

74. I invest $6000 in 6% bonds at 125. What rate per cent. do I receive on the investment and what is the income from it?

75. A field is 42 rd. long and 35 rd. wide. Find its value at $37.50 an acre.

76. A man 6 ft. high casts a shadow 42 in. long. Find the height of a flagstaff which at the same time casts a shadow 28 ft. long.

77. Multiply 2 thousand 9 ten-millionths by 30 thousand 2 and 7 tenths, and divide the product by 3 ten-thousandths.

78. An agent remits to me $247.38, after retaining a commission of 5% for collection. What sum did he collect? What was the amount of his commission?

79. Three men engage in partnership. A. puts in $1200, B. $1550, C. $1900. They gain $350. What is each man's share of the profits?

80. The owner of $\frac{3}{11}$ of a mine sold $\frac{9}{10}$ of his share for $40,500. What should he who owns $\frac{3}{5}$ of the mine get for $\frac{5}{9}$ of his share?

---

81. If 18 men can dig 128 yards of ditch in 32 days, how many yards can 12 men dig in 64 days?

82. If a square field contains 10 acres, what is the length of the diagonal?

83. At what price must 6% bonds be bought to yield 4% on the investment?

84. If 8 men reap 36 acres of grain in 9 days, working 9 hours a day, how many men will reap 48 acres in 12 days, working 12 hours a day?

85. Find the cost, at 35 cts. per cubic yard, of excavating a trench 6 rods long, $1\frac{1}{2}$ yards wide, 1 foot 6 inches deep.

86. A note for \$560, payable in 90 days, is discounted at a bank 30 days after it is dated. Find the proceeds.

87. Find the amount of \$945.15 from December 15, 1891, to November 22, 1892, at $4\frac{1}{2}\%$ simple interest.

88. Divide \$720 among A., B., and C., so that the number of dollars they receive shall be as the numbers 5, 6, and 7.

89. A merchant marks an article \$2.80, but in selling it takes off 5% for cash. If the rate of his profit is 33%, what was the cost of the article?

90. What part of an ounce is 5ʒ 2℈?

---

91. Find the amount of \$375 for 11 mo. 17 da., at $4\frac{1}{2}\%$ simple interest.

92. Find the cost, at 25 cts. a rod, of building a fence round a square 10-acre field.

93. How many gold rings, each weighing 5 pwt. 18 gr., can be made from 2 oz. 6 pwt. of gold?

94. Find the face of a 60-day note which, when discounted at a New York bank, will yield \$250.

95. If it costs \$80 to plough a field 40 rods by 80 rods when we pay \$5 a day for man and team, how much will it cost to plough a field 30 rods by 60 rods if we pay \$4 a day?

Suggestion: Solve by proportion and by analysis.

96. What number divided by the sum of $\frac{4}{5}$ and $2\frac{1}{3}$ will give a quotient of $2\frac{7}{20}$?

97. If rain-drops are falling directly downward, how much more ground surface would be protected from the rain by a board 20 feet long and 18 inches wide when in a horizontal position than when one end of it is elevated 9 feet higher than the other?

98. A certain town raised a tax of $4607.50. The real estate was valued at $420,000, the personal property at $189,000, and 1250 persons paid a poll-tax of $1.25 each. Find the tax on $1.00 of the property.

99. How high must be a pile of wood 10 feet long and $2\frac{1}{2}$ feet wide to contain one cord?

100. How much should be paid for 40 shares of railroad stock at $3\frac{1}{2}\%$ discount and $\frac{1}{8}\%$ brokerage?

---

101. Find the diagonal of a cubical block each of whose edges is 20 inches.

102. How many dollars would a man gain in buying 240 shares of railroad stock at $3\frac{3}{8}\%$ discount and selling them at $1\frac{7}{8}\%$ premium?

103. A note for $350, dated October 17, 1865, was paid April 11, 1868, with interest at 7%. Find the amount paid.

104. What would be the cost of 50 boards, each 12 feet long, 8 inches wide, and $1\frac{1}{2}$ inches thick, at $4\frac{1}{2}$ cts. a foot, board measure?

105. At 30 cts. a sq. yd., how much will it cost to plaster the four walls and ceiling of a room 15 ft. × 18 ft. and 9 ft. high, no allowance being made for openings?

106. For what sum must I make a bank note at 90 days, that the proceeds may be $150?

107. Simplify $\dfrac{3\frac{5}{6} + 1\frac{1}{12} - 4\frac{1}{4}}{\frac{14}{15} \text{ of } \frac{5}{28}}$.

108. Find the true discount and the present worth of $412, due in 6 mo., without interest.

109. Find the cost of 1478 lb. of coal at $4.60 per ton.

110. How much lumber will be required to ceil the four walls of a room 16 ft. × 18 ft., and 10 ft. high, and how much will the lumber cost at $16 per M.?

111. If 5 men can dig a trench 10 rods long, 2 ft. wide, and 5 ft. deep in 4 days, how many men will it take to dig a trench 40 rods long, 2 ft. wide, and 4 ft. deep in 8 days?

Suggestion: Solve by proportion.

112. I buy stocks at 80 and sell them at par. Find the per cent. profit.

113. A. owns $\frac{3}{11}$ of a farm worth $15,422, and sells $\frac{2}{3}$ of his share. Find the value of what he has left.

114. A pension of $140 per year is four years in arrears. Find the amount now due at 5% compound interest.

115. How many bushels will a bin contain that is 9 ft. long, 4 ft. wide, 6 ft. deep? How many bushels, heaped measure?

116. Constantinople is in longitude 28° 59′ E. and Philadelphia 75° 10′ W. When it is 4 A.M. in Philadelphia, what time is it at Constantinople?

117. Find the diagonal of a right parallelopiped whose edges are 6 ft., 8 ft., and 4 ft.

118. Find in inches to two places of decimals the diagonal of a cube whose volume is 9 cu. ft.

119. The diameter of the base of a cone is double that of the base of a cylinder of the same volume. Find the ratio of their altitudes.

120. A locomotive runs $\frac{3}{4}$ of a mile in $\frac{4}{5}$ of a minute. How many feet does it run in a second?

---

121. The base of a certain triangle is 40 ft., its altitude is 30 ft. Find the area of a similar triangle whose base is 25 ft.

122. I buy an article by avoirdupois weight and sell it at the same price per pound by Troy weight. Do I gain or lose, and how many per cent.?

123. Find the cost of a draft on Chicago for \$1000 at 60 days' sight, money being worth 5% and exchange at $1\frac{1}{2}\%$ premium.

124. When it is noon in Philadelphia, what is the time in Paris, 2° 20′ E. long.? In San Francisco, 122° 25′ 40.76″ W. long.?

125. A. is in longitude 18° E. and B. 23° W. Find the difference of time between A. and B., and give the reason for each step in the process.

126. Find the depth of a cylindric cistern whose bases are 8 ft. in diameter and whose capacity is 100 barrels.

127. Find the amount of \$436 at $4\frac{1}{2}\%$ simple interest from January 1, 1893, to the present time.

128. The wheels of a sulky are $4\frac{1}{2}$ ft. apart. In driving around a circular track, the inner wheel traverses 1 mile. How far does the outer wheel go?

129. Write a full analysis of the following: $\frac{2}{3}$ is $\frac{4}{5}$ per cent. of how many times $\frac{5}{8}$?

130. Find the face of a note at 60 days, without interest, which will yield \$750 proceeds when discounted at a New York bank.

---

131. Find the exact interest of \$590 from Sept. 18, 1893, to March 1, 1894, at $4\frac{1}{2}\%$.

132. Find the side of a square which is equal in area to a right triangle whose base is 24 ft. and hypotenuse 40 ft.

133. Find the cube root of 1796.63 to two places of decimals.

134. Find the surface and volume of a sphere whose diameter is 4 ft.

135. Find the volume and the entire surface of a square pyramid the side of whose base is 2 ft., and whose slant height is 6 ft.

136. Find the smallest number that will exactly contain 15, 18, 21, 24, 30, and 91.

137. Find the rate per cent. of interest on an investment in government 3% bonds bought at 115.

138. Find the value of $43{,}562 \times 21{,}894 \div 986$.

139. Show your knowledge of the use of signs by indicating the solution of the following: A man earns $37.50 a month for 6 months and $50 a month for 9 months. He invests his earnings in railway stock at 75. The stock pays a dividend of 4%, and the money thus received is divided among his children, each receiving as many dollars as there are children. How many children are there?

140. If it requires 40 min. for a pipe 4 in. in diameter to fill a tank 20 ft. by 10 ft. by 6 ft., how long will it take a pipe 3 in. in diameter to fill a tank 30 ft. by 12 ft. by 8 ft.

---

141. If a ball $2\frac{1}{2}$ ft. in diameter weighs 400 lb., what is the diameter of a similar ball that weighs 1 T. 1200 lb.?

142. A box made of 2-inch plank is 3 feet 4 inches long, 2 feet 8 inches wide, and 1 foot 6 inches high; it has no lid. How much will it cost to cover the box completely inside and outside with gold leaf, at $2 per square foot?

143. A bushel measure and a peck measure have been made of the same shape. Find the ratio of their heights.

144. How many feet of lumber are there in three $1\frac{1}{4}$-inch 16-foot boards whose breadths are respectively 12, 14, and 15 inches?

145. A railway one chain wide runs across a section of land parallel with one side. Find the price of the land in the railway at $25 per acre?

146. If the ox that Milo carried was $6\frac{1}{2}$ ft. in girth when it weighed 1000 lb., what was the girth of the ox when it weighed 2000 lbs.?

147. If I pay $.62\frac{1}{2}$ a cord for sawing wood 4 feet long into 3 pieces, how much more should I pay for sawing wood 8 feet long into pieces of the same length?

148. A room is 18 ft. long, 15 ft. wide, and 10 ft. high. What is the distance from an upper corner to the opposite lower corner?

149. If a 5-in. cube of granite weighs 12 lb., what will a cubic foot weigh?

150. I have a trapezium of land, measuring 30, 40, 60, 70 rd., with a diagonal of 50 rd. Find the area of the field.

151. Given the frustum of a square pyramid: height, 20 ft.; side of upper base, 8 in.; side of lower base, 20 in. Find its volume.

152. Find the surface of a cube that contains 5268.024 cubic inches.

153. A spherical balloon contains 28974.25 cubic feet. Find the number of square yards of silk required to make it.

154. What must be the market price of 3% stock, that it may give $3\frac{1}{3}$% interest after deducting 35 cts. from every $12 of the income?

155. David Palmer borrows this day of Samuel Hill $350, and gives his note for this amount for 4 months at 6%. Make out the promissory note in proper form.

156. When it is 3 P.M. at Rome, longitude 12° 27′ east, it is 8.20 A.M. at Chicago; find the longitude of Chicago.

157. A and B run a mile in opposite directions: A's running is to B's as $6\frac{1}{2} : 5\frac{1}{2}$; B gets 4 seconds start, during which time he runs $12\frac{1}{10}$ yards. Find when he will pass A.

ANSWERS.

Page 14.

1\. 1. Fifteen.
2. Four.
3. Fourteen.
4. Twenty-four.
5. Nineteen.
6. Thirty-nine.
7. Thirty-three.
8. Twenty-nine.
9. Forty-nine.
10. Forty-five.
11. Ninety-nine.
12. Sixty-five.
13. One hundred nine.
14. One hundred eleven.
15. Ninety-one.
16. Six hundred ninety.
17. Three hundred 39.
18. Seven hundred 34.
19. 790.
20. 1029.
21. 5555.
22. 550600.
23. 210506.
24. 8000.
25. 200090.
26. 149.
27. 2500.
28. 70899.
29. 1595864.

Page 15.

2\. 1. XV.
2. XXXVI.
3. LXXXVII.
4. LVI.
5. XLIX.
6. XCIX.
7. ML.
8. MMMMMX = $\overline{\mathrm{V}}$X.
9. DCCLXXXIX.
10. MDCCCXCVIII.
11. XVIII.
12. XLII.
13. LXVI.
14. LXXXVI.
15. LXIII.
16. C.
17. $\overline{\mathrm{III}}$DC. or MMMDC.
18. DLXXXVII.
19. CCVII.
20. $\overline{\mathrm{VIII}}$IV.
21. XXVII.
22. LXXXI.
23. XCV.
24. XL.
25. XLV.
26. DXXXIV.
27. $\overline{\mathrm{V}}$.
28. CDXXXVI.
29. CMXCIX.
30. $\overline{\mathrm{LXXVI}}$CMLIX.

ADDITION.

Page 18.

1\. 1. 599.
2\. 676.
3\. 1026.
4\. 794.
5\. 748.
6\. $6.95.
7\. $9.55.
8\. $92.79.
9\. $983.99.

Page 19.

10\. 16193.
11\. 10333.
12\. 10615.
13\. 7720.

2\. 8649.

3\. 1. 11429.
2\. 20681.
3\. 101391.
4\. 77230.
5\. 235308.

4\. 1. 4164.
2\. 1461.
3\. 2867.
4\. 3285.
5\. 2791.
6\. 1453.
1\. 2519.
2\. 3046.
3\. 1965.
4\. 2690.
5\. 3332.
6\. 2469.

5\. 104367.
6\. $1447.845.
7\. 83619.
8\. $132.90.

Page 20.

9\. 7076.
10\. 31164.

2\. 95 acres.
3\. $504.20.
4\. 20694.
5\. 1065.

Page 21.

6\. 528408.
7\. 13587.
8\. $2275.00.
9\. 58639.
10\. 402399.
11\. $2454.63.
12\. $8513.75.

Page 22.

13\. 1646619.
14\. 73941.
15\. 365.
16\. 72 days.
17\. $171860.
18\. 61.
19\. LXII.
20\. $1906.50, $6140.66, $8047.16.

Page 23.

21\. 1. 66892.
2\. 58434.
3\. 508785.
4\. $96229.42.
5\. 67573.
6\. 46997.
7\. 51871.
8\. 49845.

Page 24.

9\. 134083.44.
10\. 108349.98.
11\. 2356.9657.

SUBTRACTION.

Page 26.

1\. 411.
2\. 324.
3\. 254.
4\. 213.
5\. 352.
6\. 5533

Page 27.

7\. $25.62.
8\. $35.09.
9\. $11.13.
10\. $21.40.
11\. 32,154.
12\. 27,312.
13\. 422.641.
14\. 145.325.

2\. 1520.

Page 28.

4\. $3264.
5\. 1212.
6\. 6,550,216.
7\. $4,820,411.
8\. 127,420.
9\. 289.
10\. 9 yrs.

Page 30.

1\. 1. 305.
2\. 228.
3\. 292.
4\. 272.
5\. 1879.
6\. 61.
7\. 1919.
8\. 388.
9\. 1299.
10\. 40.
11\. 6828.
12\. 2131.
13\. $3.07.
14\. $2.17.
15\. $16.17.
16\. $24.96.
17\. $33.66.
18\. $.995.
19\. $88.996.
20\. 1,410,273.
21\. 31204.73.
22\. 998.78.

2.	1. 27,747.	5. 132,890.
	2. 45,860.	6. 430,875.
	3. 493,879.	7. 5,741,182.
	4. 382,717.	8. 1,987,588.

Page 31.

2. $2125.	8. 7795.
4. 45,558,897.	9. 69,191,517.
5. 45 yrs.	10. $3149.
6. 67 yrs.	11. $925,985.
7. 7600 ft.	

Page 32.

12. $1046.	17. 4908 ft.
13. $4.365.	18. 1437.
14. $19.81.	19. 1706.
15. $27,404.	20. $\overline{\mathrm{LXIV}}$II.
16. 14,162 ft.	

1. 23,527.	2. 33,958.

Page 33.

3. 13,181.	5. $207.61.
4. 120,091.	

1. 224,980.	3. 919.
2. 19,553,068.	4. 55.

Page 34.

2. 165.	6. 91,145; 58,905.
3. $1365.	7. 3561.
4. 1155.	8. $4484.
5. 2070.	9. D. 697.

Page 35.

10. 447.	15. 796.
11. 245.	16. 96.
12. 115.	17. 168.89.
13. 273.	18. 82.
14. 2410.	19. 11,220.

MULTIPLICATION.

Page 39.

2.	1. 730.	6. $25.45.
	2. 2696.	7. $63.60.
	3. 2268.	8. $89.25.
	4. 1962.	9. $495.54.
	5. 2040.	10. $523.20.
4.	1. 69536.	6. 41688.
	2. 37296.	7. 261045.
	3. 51590.	8. 478709.
	4. 65601.	9. 318352.
	5. 69380.	10. 827847.
5.	1. 3780.	5. 162108.
	2. 18118.	6. 1526190.
	3. 234177.	7. 243582.
	4. 12533346.	8. 7282896

6.	1. 302.	4. 687.	7. 2484.
	2. 54.	5. 8537.	
	3. 4160.	6. 1553.	

Page 40.

2. $11.25.	7. 672.
3. 1227.45.	8. 2,400,000.
4. 15840.	9. 822870000.
5. $29316.	10. $2499.96.
6. $6655.	11. $42,592.

Page 41.

12. .03.	14. Lost $10.
13. Cows, 20.	15. 9050.

Page 42.

2.	1. 4755.	6. $440.55.
	2. 7728.	7. $767.55.
	3. 19481.	8. $2176.56.
	4. 17082.	9. $3477.33.
	5. 12691.	10. $1614.14.

3. 1. 7258.
2. 13440.
3. 21465.
4. 19758.
5. 47085.
6. 45522.
7. 42182.
8. 66822.
9. 53963.
10. 47974.
11. 59424.
12. 66822.
13. 47320.
14. 45384.
15. 78027.
16. 21909.
17. 88445.
18. 90159.
19. 229554.
20. 307395.

4. 31806.
5. $38104.50.
6. 119239.
7. 350090.
8. 46529640.
9. $16808.61.
10. 1. $6141.720. 2. 5107212. 3. 27154202. 4. 96332187.

Page 43.

11. 35843685.
12. 214007086881.
13. 764,819,895,290,424.
14. 2,324,334,767,296.
15. 99253.80.
16. 152323.35.
17. $69520.33.
18. CMXII.
19. $\overline{\text{CLXXXVII}}$CDLVI.
20. 5859385041295896.
21. 21,842,100.

1. $20604.
2. 11025.
3 $2533 50.
4. 984072.
5. $494.
6. 137664.

Page 44.

7. 95040.
8. (65 — 57) × 54 = 432.
9. (17 + 26) × $42.50 — (17 × 38.75) + (26 × 40.25) = $122.25.

Page 45.

1. 4860, 48600, 194400.
2. 382400, 764800, 9560000.
3. 1722000, 2296000, 2583000.
4. 747000, 7470000, 10956000.
5. 21492000,256710000,453720000.
6. 13536000, 156510000, 2411100000.
7. 1315170000000, 1480784000000.
8. $3139972.00, $39249650.00.
9. 3604200000, 2405202800000.
10. 440000000, 25,960,000.
12. 3168000.
13. 126000000.
14. 48000.
15. 1610000.
16. 1140000.
17. 44000000.

Page 46.

18. $650.
19. $80,000.
20. 86,400.

Page 47.

1. 333641.
2. 27421443.
3. 14889792.
4. 4382415.
5. 3306564.
6. 401193.
7. 2153232.
8. 49308.

1. 500.
2. $503.50.
3. $72.
4. 913,920.
5. 9328.

Page 48.

6. 320,000.
7. 213,192.
8. $1,377.
9. 103,615
10. $2,583.
11. 89,232
12. $31.80.

13. 5,865,696,000,000.
14. 13,176.
15. 968710.
16. $17,979,365.

Page 49.

17. 276.
18. 693.
19. 63,210,541,205,000.
1. $(16 - 11 + 2) \times 5 = 35$.
2. $(4 + 15) \times (15 - 4) \times 6 = 1254$.
3. $63915 + 936085 = 1000000$.
4. $3149 + 4872 = 8021$.
5. $5301 - 1046 = 4255$.
6. 300,003, 300,003.
7. $\overline{\mathrm{MMM}}\mathrm{III}$, $\overline{\mathrm{C}}\mathrm{I}$.
8. 397,056.
9. 12,343,200.
10. 674.

DIVISION.

Page 54.

1. 283.	9. 5263.	17. 209758.
2. 188.	10. 6238.	18. 189572.
3. 71.	11. 4812.	19. $264.85.
4. 124.	12. 4809.	20. $924.67
5. 834.	13. 247.	21. $128.21.
6. 4169.	14. 4138.	22. $222.22.
7. 9451.	15. 6559.	
8. 9485.	16. 93716.	

1. 283.	4. 248.	7. 9451
2. 198.	5. 1668.	8. 9485.
3. 142.	6. 4169.	

Page 55.

9. 5263.	13. 27680.
10. 6238.	14. 2470.
11. 4812.	15. 8276.
12. 4809.	16. 6559.
17. 93716.	22. $.92.
18. 209758.	23. $108.50.
19. 189572.	24. 32793.
20. 26485.	25. 11750.
21. $6.07.	26. 63362.
2. 406.	3. 432.

Page 56.

4. 167.	13. 32.
5. 172, with 4 rem.	14. 123.
6. 230, with 7 rem.	15. 182.
7. 463.	16. 92.
8. 286, with 8 rem.	17. 88.
9. 315, with 14 rem.	18. 217.
10. 312.	19. 136.
11. 1899, with 5 rem.	20. 72.
12. 439, with 38. rem.	21. 35.

2. 95.	4. 144.	6. 108.
3. 8.	5. 13178.	

Page 57.

7. 81.	15. 365.
8. 104.	16. 5280.
9. 37.	17. 17443, with 16 rem.
10. 72.	18. 175.
11. 66.	19. 327.
12. 162.	20. 7328.
13. 2640.	21. $89.
14. 153.	

Page 58.

1. 1. 11,572, with 110 rem.
   2. 6284.
   3. 1938.
   4. 664.
   5. 736.
   6. 893.
   7. 969, with 344 rem.
   8. 1064.
   9. 985.
   10. 692, with 533 rem.

3. 1. 527, with 380 rem.
2. 692, with 533 rem.
3. 5205, with 38 rem.
4. 814, with 167 rem.
5. 1259, with 581 rem.
6. 645, with 312 rem.
7. 283, with 736 rem.
8. 3241.
9. 3431.
10. 876, with 110 rem.
11. 474,536, with 523 rem.
12. 4567.
13. 4207.
14. 10,110, with 9 rem.

4. 1. 1,672,940, with 165,534 rem.
2. 206,008,604, with 24 rem.
3. 100,000,000, with 102,345,678 rem.
4. 100,000,000.
5. 48,100,720,009.

Page 59.

1. 96.	6. 329.	10. 58.
2. 1760.	7. 85 +.	11. 7 +.
3. 60.	8. 22 +.	12. 3579.
4. 19.	9. 36.	13. 25.
5. 425.		

Page 60.

14. 491 sec.	21. 548,501.
15. 15.	22. 9238.
16. 500.	23. 5.
17. 605.	24. 28.
18. $137 nearly.	25. 308.
19. 357 +.	26. 0.
20. 13.	27. IX.

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28. CLXXX.
29. $\overline{\text{MCCLXXX}}$
30. 437.

1. 576,544.	9. 319,099.
2. 103,075.	10. 801,587.
3. 213,789.	11. 117,554.
4. 4.	12. 388,129.
5. 9,042,049.	13. 8,886,859.
6. 8161.	14. 253, with 21,700 rem.
7. 1162.	
8. 28.	

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15. 17,115,520.
16. ——.
17. 67, with 999 rem.
18. 25.
19. 240.
20. 300, with 9999 rem.

2. 18. 3. 276. 4. 40.

Page 63.

5. $10.00.
6. 3000 too much in 2d member.
7. 41.
8. 132 699.
9. 90.
10. 2122.
11. 19,868.
12. 139,806.
13. $384.25.
14. Lost $952.
15. 5475 hr.
16. 12,295.

Page 64.

17. 5 yr.	19. 2.83.
18. 105.	20. $12.40.

Page 65.

2. $3.60.	5. $.66.	8. $2552
3. $6.10.	6. $4.50.	9. 116.
4. $3125.	7. $28.00.	10. 6480.

Page 66.

2. 93. 4. 12 5. 47.

1. 20. 3. 1785. 5. 5.
2. 57,656. 4. 210.

Page 67.

7. 36. 11. 10. 14. $60.
8. 10 hr. 12. $.25. 15. $3010.
9 114. 13. $1.50. 16. 40.
10. 108.

Page 69.

		Q.	Rem.		Q.	Rem.
2.	1.	632,	7.	9.	55,	33.
	2.	532,	7.	10.	12,	34.
	3.	973,	2.	11.	6,	173.
	4.	926,	7.	12.	5,	432.
	5.	256,	7.	13.	8,	650.
	6.	32,	67.	14.	3,	000.
	7.	53,	27.	15.	5,	678
	8.	92,	73.			

		Q.	Rem.		Q.	Rem.
2.	1	33,	13.	5.	13,	34.
	2.	31,	27.	6.	12,	16.
	3.	17,	6.	7.	28,	136.
	4.	15,	40.	8.	24,	100.

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		Q.	Rem.		Q.	Rem.
3.	1.	1,	273	7.	2,	2432.
	2.	1,	352.	8.	2,	37.
	3.	1,	295	9.	1,	3396.
	4.	1,	173.	10.	1,	2116.
	5.	2,	1327.	11.	1,	2370.
	6	2,	2645.	12.	1,	1573.

		Q.	Rem.		Q.	Rem.
2.	1.	29,	1958.	7.	45,	5896.
	2.	12,	4425.	8.	20,	17432.
	3.	14,	4495.	9.	10,	1959.
	4.	20,	1765.	10.	38,	9938.
	5.	10,	4543	11.	23,	25548.
	6.	2725,	250.	12.	14,	1337.

PROPERTIES OF NUMBERS.

Page 74.

1. 2, 2, 2, 3, 3.
2. 5, 7.
3. 2, 2, 2, 2, 2, 2.
4. 2, 23.
5. 2, 2, 2, 2, 3, 7.
6. 3, 37.
7. 5, 7, 11.
8. 3, 11, 13.
9. 5, 5, 37.
10. 2, 2, 3, 41.
11. 2, 2, 2, 3, 5, 11.
12. 2, 2, 2, 3, 3, 3, 3, 13.
13. 2, 3, 1283.
14 743, prime number.
15. 3, 5, 5, 7, 7.
16. 2, 2, 2, 3, 3, 3, 3, 7.
17. 2, 2, 2, 3, 7.
18. 3, 7, 11.
19. 2, 89.
20. 2, 2, 3, 3, 5.
21. 2, 2, 2, 2, 3, 3.
22. 3, 3, 5, 7.
23. 2, 2, 3, 5, 7.
24. 2, 2, 3, 5, 11.
25. 2, 2, 5, 37.
26. 3, 3, 3, 5, 7.
27. 2, 2, 2, 2, 2, 2, 3, 3, 3.
28. 2, 2, 3, 3, 7, 17.
29. 2, 29, 29.
30. 997, prime number.
31. 2, 2, 3, 5, 7, 11.
32. 2, 3, 5, 5, 5, 7.
33. 2, 3, 7, 19.
34. 2, 2, 11, 11.
35. 2, 2, 2, 2, 2, 2, 2, 2, 5.
36. 2, 13, 73.
37. 2, 2, 3, 5, 7, 13.
38. 2, 2, 3, 3, 5, 19.

39. 2, 3, 5, 7, 7.
40. 2, 2, 373.
41. 2, 3, 5, 7, 11.
42. 2, 2, 3, 3, 7, 11.
43. 2, 2, 2, 2, 2, 2, 5, 5.
44. 2, 2, 3, 17, 41.
45. 5, 11, 47.
46. 1997, prime number.
47. 3, 3, 7, 7, 11.
48. 3, 3, 7, 11, 11.

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3. 102. $\begin{cases} 2 \times 3 = 6. \\ 2 \times 17 = 34. \\ 3 \times 17 = 51. \end{cases}$

105. $\begin{cases} 3 \times 5 = 15. \\ 3 \times 7 = 21. \\ 5 \times 7 = 35. \end{cases}$

108. $\begin{cases} 2 \times 2 = 4. \\ 2 \times 3 = 6. \\ 2 \times 3 \times 3 = 18. \\ 2 \times 3 \times 3 \times 3 = 54. \\ 2 \times 2 \times 3 = 12. \\ 2 \times 2 \times 3 \times 3 = 36. \end{cases}$

221 has prime factors only.

715. $\begin{cases} 5 \times 11 = 55. \\ 5 \times 13 = 65. \\ 11 \times 13 = 143. \end{cases}$

845. $\begin{cases} 5 \times 13 = 65. \\ 13 \times 13 = 169. \end{cases}$

The answers to the remaining eleven examples are omitted.

2. 1. \$1855. 3. \$334.80.
2. \$227.70.

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4. \$215.82. 10. \$67.155.
5. 694.95. 11. \$17424.
6. \$6375. 12. \$12600.
7. \$61.25. 13. \$87.04.
8. \$118.125. 14. \$76.95
9. \$1165.50.

2. 1. 563. 7. 89.
2. 324. 8. 16.
3. 728. 9. 38.
4. 18. 10. 864.
5. 53. 11. 892.
6. 430. 12. 2735.

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	Q.	R.		Q.	R.
1.	304,	8.	5.	44,	28.
2.	91,	9.	6.	119,	16.
3.	121,	9.	7.	49,	20.
4.	58,	27.	8.	23,	11.

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3. 1. 45. 3. $\frac{11}{7}$. 5. 46. 7. 20.
2. 45. 4. 21. 6. 4.

2. 39.

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3. 20. 9. 20.
4. 1.50. 10. 9.
5. 2745. 11. 8.
6. 43.80 very nearly. 12. 21.
7. 6. 13. 26.
8. 6.

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2. 1. 7. 3. 7. 5. 6. 7. 6.
2. 6. 4. 9. 6. 5. 8. 35.

3. 1. 14. 3. 15. 5. 16. 7. 120.
2. 42. 4. 20. 6. 42. 8. 22.

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1. 9. 3. 15. 13. 21. 3.
10. 12. 16. 14. 22. 37.
11. 9. 17. 60. 23. 101.
12. 6. 18. 72. 24. 2.
13. 75. 19. 29.
14. 144. 20. 1.

2\. 1. 11.
2\. 23.
3\. 31.
4\. 41.
5\. 47.
6\. 53.
7\. 61.
8\. 37.
9\. 283.
10\. 2.
11\. 3.
12\. 17.
13\. 48.
14\. 11.
15\. 37.
16\. 47.
17\. 41.
18\. 53.
19\. 267.
20\. 396.

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3\. 1. 12.
2\. 8.
3\. 4.
4\. 15.
5\. 43.
6\. 1.
7\. 3.
8\. 126.
9\. 42.
10\. 37.

1\. 4.
2\. 6.
3\. 63.
4\. 12.
5\. 14.
6\. 16.
7\. 23.

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8\. 2.
9\. 5.
10\. 940.

Page 86.

1\. 84.
2\. 720.
3\. 448.
4\. 144.
5\. 180.
6\. 360.
7\. 1080.
8\. 840.
9\. 1200.
10\. 1440.
11\. 2016.
12\. 360.
13\. 330.
14\. 720.
15\. 1200.
16\. 225.
17\. 576.
18\. 900.

Page 87.

1\. 300.
2\. 120.
3\. 280.
4\. 54.
5\. 540.
6\. 1512.
7\. $10,800.
8\. 630.
9\. 156.
10\. 72.
11\. 72.
12\. 216.
13\. 5040.
14\. 510.
15\. 10,920.
16\. 6300.

Page 88.

1\. 2871.
2\. 13,889.
3\. 10013.
4\. 819.
5\. 2160.
6\. 2873.
7\. 4284.
8\. 160,121.
9\. 441,000.
10\. 7770.
11\. 290,177.
12\. 1,639,872.
13\. 314,259.
14\. 86,394.
15\. 1,009,091.
16\. 1,038,007.
17\. 240,463.
18\. 179,655.
19\. 50,552.
20\. 473,989.
21\. 23,760.
22\. 71,842,008.
23\. 31,154,994,649.
24\. 260,117.
25\. 329,616.
26\. 4340.
27\. 42,149,000.
28\. 3,268,080.

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1\. 840.
2\. 180.
3\. 5040.
4\. 120.
5\. 210.
6\. 720.
7\. 60.
8\. 450.
9\. 720.
10\. 875.
11\. 82,063,340.

Page 90.

1\. 490.
2\. 3.
3\. 283.
4\. 1044.
5\. 3.
6\. 40,170.
7\. 119.
8\. 509.
9\. $10.
10\. 720.

FRACTIONS.

Page 94.

2\. 1. $\frac{28}{60}$.
2\. $\frac{26}{80}$.
3\. $\frac{22}{40}$.
4\. $\frac{90}{1000}$.
5\. $\frac{93}{270}$.
6\. $\frac{84}{150}$.

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7. $\frac{54}{90}$. 8. $\frac{70}{110}$. 9. $\frac{55}{99}$. 10. $\frac{21}{49}$. 11. $\frac{15}{60}$. 12. $\frac{9}{24}$. 13. $\frac{49}{70}$. 14. $\frac{38}{74}$. 15. $\frac{22}{42}$. 16. $\frac{36}{38}$. 17. $\frac{25}{30}$. 18. $\frac{82}{100}$. 19. $\frac{10}{10}$. 20. $\frac{216}{12}$.

2. 1. $\frac{12}{15}$. 2. $\frac{4}{9}$. 3. $\frac{8}{10}$. 4. $\frac{10}{12}$. 5. $\frac{3}{4}$. 6. $\frac{9}{16}$. 7. $\frac{92}{100}$. 8. $\frac{1}{12}$. 9. $\frac{7}{9}$. 10. $\frac{888}{949}$.

Page 96.

2. 1. $\frac{4}{11}$. 2 $\frac{1}{4}$. 3. $\frac{1}{7}$. 4. $\frac{1}{7}$. 5. $\frac{3}{11}$. 6. $\frac{7}{8}$. 7. $\frac{3}{4}$. 8. $\frac{4}{5}$. 9. $\frac{29}{47}$. 10. $\frac{5}{6}$. 11. $\frac{14}{19}$. 12. $\frac{17}{20}$. 13. $\frac{2}{3}$. 14. $\frac{4}{5}$. 15. $\frac{4}{5}$. 16. $\frac{5}{6}$. 17. $\frac{5}{6}$. 18 $\frac{11}{12}$. 19. $\frac{5}{6}$. 20. $\frac{81}{89}$. 21. $\frac{8}{9}$. 22. $\frac{5}{7}$. 23. $\frac{8}{9}$. 24. $\frac{11}{12}$. 25. $\frac{5}{6}$. 26. $\frac{1769}{1920}$. 27. $\frac{1}{3}$. 28. $\frac{9}{10}$. 29. $\frac{9}{32}$. 30. $\frac{2}{3}$.

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31. $\frac{11}{12}$. 32. $\frac{30}{77}$. 33. $\frac{11}{12}$. 34. $\frac{7}{8}$. 35. $\frac{13}{16}$. 36. $\frac{5}{7}$. 37. $\frac{101}{109}$. 38. $\frac{61}{71}$. 39. $\frac{5}{42}$. 40. $\frac{42}{143}$. 41. $\frac{23}{794}$.

2. 1. $\frac{25}{4}$. 2. $\frac{7}{3}$. 3. $\frac{25}{2}$. 4. $\frac{69}{7}$. 5. $\frac{65}{4}$ 6. $\frac{109}{8}$. 7. $\frac{6601}{21}$. 8. $\frac{8084}{12}$. 9. $\frac{7732}{11}$. 10. $\frac{1831}{15}$. 11. $\frac{75}{5}$. 12. $\frac{79}{6}$. 13. $\frac{201}{11}$. 14. $\frac{46}{9}$. 15. $\frac{92}{18}$. 16. $\frac{2997}{11}$. 17. $\frac{2506}{9}$. 18. $\frac{12301}{13}$. 19. $\frac{3078}{5}$. 20. $\frac{5065}{21}$.

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3. 1. $\frac{39}{4}$. 2. $\frac{137}{8}$. 3. $\frac{312}{11}$. 4. $\frac{138}{5}$. 5. $\frac{345}{7}$. 6. $\frac{3571}{16}$. 7. $\frac{121}{9}$. 8. $\frac{2524}{5}$. 9. $\frac{1147}{10}$. 10. $\frac{3755}{12}$. 11. $\frac{1474}{7}$. 12. $\frac{408}{25}$. 13. $\frac{1923}{31}$. 14. $\frac{2227}{14}$. 15. $\frac{1009}{15}$. 16. $\frac{631}{42}$. 17. $\frac{2706}{25}$. 18. $\frac{870}{17}$. 19. $\frac{776}{19}$. 20. $\frac{83821}{97}$.

2. 1. $5\frac{3}{13}$. 2. $2\frac{17}{20}$. 3. $14\frac{21}{50}$. 4. $3\frac{35}{69}$. 5. $3\frac{17}{20}$. 6 $3\frac{18}{25}$. 7. $4\frac{17}{35}$. 8. $1\frac{8}{57}$. 9. 5. 10. $2\frac{2}{19}$. 11. 3. 12. $5\frac{17}{20}$. 13. $3\frac{3}{28}$. 14. 3. 15. $23\frac{11}{28}$. 16. $16\frac{31}{32}$. 17. 4. 18. 144. 19. $15\frac{21}{25}$. 20. $20\frac{11}{17}$. 21. $41\frac{7}{20}$. 22. $25\frac{23}{36}$. 23. $10\frac{14}{37}$. 24. $18\frac{3}{4}$. 25. $39\frac{28}{43}$. 26. $156\frac{7}{8}$. 27. $253\frac{7}{15}$. 28. $34\frac{13}{14}$. 29. $80\frac{7}{13}$. 30. $100\frac{5}{9}$. 31. $1\frac{48}{61}$. 32. $1\frac{8}{15}$. 33. $1\frac{2}{3}$. 34. $1\frac{2}{3}$. 35 $6\frac{31}{80}$. 36. $14\frac{3}{10}$.

UNLIKE FRACTIONS.

Page 100.

1. $\frac{8}{24}, \frac{20}{24}, \frac{15}{24}$.
2. $\frac{30}{36}, \frac{16}{36}, \frac{21}{36}$.
3. $\frac{20}{28}, \frac{3}{28}, \frac{12}{28}$.
4. $\frac{42}{56}, \frac{24}{56}, \frac{49}{56}$.

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5. $\frac{72}{180}, \frac{100}{180}, \frac{135}{180}$.
6. $\frac{140}{168}, \frac{147}{168}, \frac{144}{168}$.
7. $\frac{10}{20}, \frac{12}{20}, \frac{15}{20}$.
8. $\frac{351}{468}, \frac{208}{468}, \frac{288}{468}$.
9. $\frac{80}{120}, \frac{75}{120}, \frac{64}{120}$.
10 $\frac{7}{8}, \frac{6}{8}, \frac{20}{8}$.
11. $\frac{292}{36}, \frac{78}{36}, \frac{153}{36}$.
12. $\frac{36}{60}, \frac{21}{60}, \frac{26}{60}$.
13. $\frac{16}{36}, \frac{27}{36}, \frac{27}{36}$.
14. $\frac{100}{28}, \frac{14}{28}, \frac{196}{28}, \frac{35}{28}$.
15. $\frac{370}{40}, \frac{32}{40}, \frac{36}{40}, \frac{25}{40}$.
16. $\frac{198}{90}, \frac{400}{90}, \frac{360}{90}, \frac{105}{90}$.

17. $\frac{144}{18}, \frac{132}{18}, \frac{15}{18}, \frac{16}{18}$.

18. $\frac{30}{60}, \frac{20}{60}, \frac{15}{60}, \frac{12}{60}, \frac{10}{60}$.

19. $\frac{3400}{9520}, \frac{7344}{9520}, \frac{420}{9520}, \frac{8415}{9520}, \frac{8000}{9520}, \frac{3528}{9520}$.

20. $\frac{7}{20}, \frac{125}{20}, \frac{18}{20}, \frac{140}{20}, \frac{12}{20}, \frac{30}{20}$.

21. $\frac{36288}{110880}, \frac{35112}{110880}, \frac{28800}{110880}, \frac{17640}{110880}, \frac{14784}{110880}, \frac{8855}{110880}$.

22. $\frac{364}{1092}, \frac{672}{1092}, \frac{819}{1092}, \frac{117}{1092}, \frac{8008}{1092}, \frac{819}{1092}$.

ADDITION.

Page 102.

1. $2\frac{13}{36}$. 5. $1\frac{19}{25}$. 9. $10\frac{31}{77}$.
2. $17\frac{3}{20}$. 6. $34\frac{1}{12}$. 10. $\frac{173}{228}$.
3. $3\frac{1}{8}$. 7. $12\frac{1}{10}$. 11. $26\frac{17}{48}$.
4. $4\frac{187}{504}$. 8. $10\frac{733}{840}$. 12. $2\frac{17}{24}$.

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13. $\frac{3}{4}$. 16. $3\frac{1}{15}$. 19. $1426\frac{33}{50}$.
14. $2\frac{3}{17}$. 17. $19\frac{3}{8}$. 20. $1\frac{201}{280}$.
15. $21\frac{3}{17}$. 18. $33\frac{47}{63}$.

1. $2\frac{1}{4}$. 5. $\frac{73}{77}$. 9. $191\frac{151}{360}$.
2. $2\frac{3}{28}$. 6. $49\frac{17}{30}$. 10. $5\frac{9713}{27720}$.
3. $9\frac{1}{6}$. 7. $1130\frac{38}{45}$.
4. $3\frac{107}{240}$. 8. $9632\frac{19}{36}$.

1. $\frac{73}{77}$. 5. $31\frac{49}{72}$. 9. $34\frac{5}{6}$.
2. $1\frac{49}{936}$. 6. $25\frac{2}{5}$. 10. $63\frac{25}{28}$.
3. $\frac{37}{70}$. 7. $29\frac{1}{48}$.
4. $2909\frac{38}{45}$. 8. $18\frac{19}{60}$.

1. $271\frac{7}{24}$. 2. $37\frac{23}{24}$. 3. $241\frac{13}{21}$.

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4. $141\frac{7}{30}$. 5. $81\frac{1}{4}$.

SUBTRACTION.

Page 105.

4. $\frac{1}{3}, \frac{2}{5}$. 8. $\frac{17}{40}, \frac{11}{40}$.
5. $\frac{4}{15}, \frac{1}{30}$. 9. $10\frac{17}{66}, 36\frac{1}{2}$.
6. $\frac{53}{88}, \frac{37}{221}$. 10. $536\frac{1}{4}, 88\frac{1}{6}$.
7. $\frac{23}{60}, \frac{273}{800}$.

11. 1. $\frac{19}{85}$. 5. $\frac{251}{1200}$. 9. $12\frac{11}{60}$.
2. $\frac{13}{100}$. 6. $\frac{83}{262}$. 10. $13\frac{43}{48}$.
3. $\frac{1}{3}$. 7. $1048\frac{14}{15}$. 11. $12\frac{47}{60}$.
4. $\frac{5}{42}$. 8. $521\frac{11}{12}$. 12. $41\frac{71}{100}$.

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12. 1. $30\frac{5}{11}$. 8. $25\frac{41}{48}$. 15. $8\frac{275}{2626}$.
2. $29\frac{9}{13}$. 9. $6\frac{2}{63}$. 16. $15\frac{161}{475}$.
3. $4\frac{5}{8}$. 10. 17. 17. 15.
4. $0\frac{3}{11}$. 11. $39\frac{2}{3}$. 18. $\frac{85}{93}$.
5. $26\frac{1}{3}$. 12. $45\frac{5}{6}$. 19. $11\frac{46}{47}$.
6. $79\frac{1}{17}$. 13. $14\frac{13}{18}$. 20. $27\frac{30}{77}$.
7. 9. 14. $34\frac{295}{297}$. 21. $35\frac{53}{78}$.

13. 1. $1\frac{2}{9}$. 6. $10\frac{7}{24}$. 11. $4\frac{43}{45}$.
2. $\frac{4}{9}$. 7. $\frac{347}{600}$. 12. $125\frac{1}{6}$.
3. $3\frac{77}{78}$. 8. $2\frac{301}{600}$. 13. $38\frac{23}{300}$.
4. $4\frac{29}{30}$. 9. $19\frac{13}{42}$. 14. $40\frac{1}{15}$.
5. $6\frac{19}{44}$. 10. $23\frac{13}{72}$. 15. $\frac{541}{1260}$.

1. $9\frac{7}{8}$. 2. $\frac{17}{40}$.

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3. 300.
4. $\frac{11}{13} = \frac{462}{546}, \frac{5}{6} = \frac{455}{546}, \frac{6}{7} = \frac{468}{546}$.
5. $11\frac{1}{4}$.
6. $\frac{124}{273}$.
8. 1. $373\frac{9}{10}$. 4. $78\frac{11}{30}$. 6. $429\frac{2}{15}$.
2. $6\frac{1}{22}$. 5. $269\frac{13}{27}$. 7. $234\frac{23}{51}$.
3. $35\frac{13}{14}$.

MULTIPLICATION.

Page 108.

3. 1. $\frac{9}{2}$. 12. $\frac{9}{13}$.
2. $\frac{7}{5}$. 13. $4\frac{16}{23}$.
3. $\frac{18}{3} = 6$. 14. $5\frac{5}{13}$.
4. $\frac{7}{8}$. 15. $8\frac{28}{37}$.
5. $\frac{9}{7} = 1\frac{2}{7}$. 16. $5\frac{2}{3}$.
6. $2\frac{1}{8}$. 17. $17\frac{1}{2}$.
7. $5\frac{1}{3}$. 18. $\frac{3}{5}$.
8. $\frac{3}{5}$. 19. $2\frac{22}{59}$.
9. $\frac{17}{2} = 8\frac{1}{2}$. 20. $4\frac{4}{37}$.
10. $7\frac{1}{2}$. 21. 1.
11. $\frac{15}{11} = 1\frac{4}{11}$.

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5. 1. 58.	8. $285\frac{10}{13}$.	15. $2743\frac{1}{3}$.
2. 69.	9. $45\frac{3}{8}$.	16. 278.
3. $44\frac{1}{2}$.	10. $19\frac{1}{12}$.	17. $386\frac{2}{5}$.
4. 151.	11. $187\frac{1}{2}$.	18. $605\frac{1}{2}$.
5. $86\frac{2}{3}$.	12. $206\frac{1}{2}$.	19. $1910\frac{5}{6}$.
6. $84\frac{1}{2}$.	13. $213\frac{1}{3}$.	20. $4445\frac{1}{3}$.
7. $154\frac{4}{5}$.	14. 3560.	21. 5936.
6. 1. $3\frac{1}{2}$.	8. 12.	15. 87.
2. $47\frac{1}{2}$.	9. 99.	16. $549\frac{1}{17}$.
3. 66.	10. 15.	17. $421\frac{2}{3}$.
4. $2\frac{2}{3}$.	11. $47\frac{1}{2}$.	18. 123.
5. $4\frac{3}{4}$.	12. $40\frac{1}{2}$.	19. $16\frac{2}{3}$.
6. $54\frac{1}{4}$.	13. $38\frac{1}{2}$.	20. 240.
7. 81.	14. $76\frac{8}{25}$.	21. $2\frac{1}{16}$.

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2. 1. $\frac{8}{15}$.	5. $\frac{205}{297}$.	8. $\frac{27}{220}$.
2. $\frac{7}{20}$.	6. $\frac{1}{12}$.	9. $\frac{256}{4473}$.
3. $\frac{825}{1792}$.	7. $\frac{28}{125}$.	10. $\frac{22}{75}$.
4. $\frac{14}{45}$.		
3. 1. $2\frac{2}{3}$.	5. $\frac{1}{2}$.	9. $\frac{1296}{155}$.
2. $6\frac{1}{6}$.	6. $\frac{17}{108}$.	10. $3\frac{1}{2}$.
3. $6\frac{11}{14}$.	7. $\frac{2}{9}$.	11. $\frac{18}{25}$.
4. $4\frac{7}{20}$.	8. $2\frac{1}{3}$.	12. $4\frac{5}{7}$.

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4. 1. $1\frac{1}{2}$.	5. $\frac{1071}{4400}$.	8. $\frac{9}{56}$.
2 $\frac{25}{833}$.	6. $\frac{30}{611}$.	9. $\frac{250}{2673}$.
3. $\frac{153}{22000}$.	7. $\frac{2726}{115425}$.	10. $\frac{35}{432}$.
4. $\frac{1}{24}$.		
5. 1. $3\frac{1}{33}$.	4. 147.	7. $\frac{28}{87}$.
2 27.	5. $\frac{2592}{385}$	8. $\frac{221}{2430}$.
3. $31\frac{1}{2}$.	6. $55\frac{1}{9}$.	
1. $56\frac{1}{4}$.	4. 81.	7. $74\frac{1}{32}$.
2. $\frac{3}{4}$.	5. 3534.	8. $90\frac{3}{4}$.
3. $20.10.	6. $1593\frac{3}{4}$.	

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9. $130\frac{1}{2}$.	10. $361\frac{1}{5}$ nearly.

DIVISION.

Page 113.

4. 2. 42.	5. 42.	8. 70.
3. 81.	6. 80.	9. 68.
4. $9\frac{3}{8}$.	7. 36.	
5. 2. $\frac{23}{127}$.	5. $\frac{11}{80}$.	8. $\frac{25}{131}$.
3. $\frac{31}{128}$.	6. $\frac{47}{48}$.	9. $\frac{6}{77}$.
4. $1\frac{17}{20}$.	7. $\frac{1}{225}$.	
6. 1. $\frac{5}{7}$.	4. $\frac{54}{845}$.	7. $1\frac{1}{7}$.
2. $1\frac{1}{6}$.	5. $\frac{13}{21}$.	8. $1\frac{1}{14}$.
3. $1\frac{1}{2}$.	6. $\frac{27}{29}$.	9. $1\frac{1}{2}$.

Page 114.

7. 2. $\frac{2592}{2873}$.	9. $2\frac{3}{16}$.	16. $\frac{288}{325}$.
3. $\frac{3}{16}$.	10. $\frac{1}{24}$.	17. $5\frac{45}{484}$.
4. $\frac{10}{99}$.	11. $\frac{11}{14}$.	18. $\frac{4320}{6923}$.
5. $\frac{25}{49}$.	12. $\frac{8}{99}$.	19. $\frac{63}{100}$.
6. $1\frac{1}{14}$.	13. $7\frac{7}{10}$.	20. $1\frac{43}{77}$.
7. $2\frac{25}{28}$.	14. $\frac{7}{36}$.	
8. 7.	15. $1\frac{17}{54}$.	
1. $6\frac{1}{4}$.	3. 18.	5. $\frac{1}{8}$.
2. $26\frac{1}{4}$.	4. 27.	

Page 115.

6. $\frac{13}{15}$.	8. $93\frac{22}{51}$.	10. $6\frac{24}{25}$.
7. $10\frac{8}{11}$.	9. $1689\frac{3}{5}$.	
2. 1. $\frac{20}{33}$.	5. $1\frac{2}{7}$.	9. $\frac{1}{26}$.
2. $\frac{1}{15}$.	6. $\frac{5}{54}$.	10. $2\frac{29}{80}$.
3. $\frac{11}{18}$.	7. $\frac{18}{35}$.	
4. $1\frac{11}{21}$.	8. $6\frac{3}{7}$.	

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11. $1\frac{5}{18}$; $1\frac{2}{7}$.	15. $\frac{19}{70}$.
12. $1\frac{17}{112}$; 15.	16. $\frac{16}{931}$.
13. $1\frac{11}{21}$.	17. 3.
14. $\frac{19}{42}$.	18. $3\frac{723}{760}$.

Page 117.

1\. 1. $\frac{1}{3}$. 2. $\frac{1}{2}$. 3. $\frac{2}{3}$. 4. $\frac{2}{3}$. 5. $\frac{1}{3}$. 6. $\frac{7}{8}$. 7. $\frac{2}{3}$. 8. $\frac{3}{7}$. 9. $\frac{1}{5}$. 10. $\frac{2}{11}$. 11. $\frac{2}{25}$. 12. $\frac{3}{70}$. 13. $\frac{5}{56}$. 14. $\frac{4}{81}$. 15. $\frac{3}{64}$. 16. $\frac{5}{208}$. 17. $\frac{1}{21}$. 18. $\frac{4}{75}$. 19. $\frac{7}{128}$. 20. $\frac{3}{10}$. 21. $\frac{1}{6}$. 22. $\frac{1}{68}$. 23. $\frac{1}{15}$. 24. $\frac{1}{20}$. 25. $\frac{9}{100}$. 26. $\frac{5}{72}$. 27. $\frac{5}{224}$. 28. $\frac{1}{120}$. 29. $\frac{5}{27}$. 30. $\frac{7}{60}$.

2\. 2. $\frac{1}{3}$. 3. $\frac{3}{10}$. 4. $\frac{1}{5}$. 5. $\frac{7}{8}$. 6. $\frac{55}{42}$. 7. $\frac{15}{16}$. 8. $\frac{5}{4}$. 9. $\frac{7}{16}$. 10. $\frac{15}{14}$. 11. $\frac{27}{50}$. 12. $\frac{11}{3}$. 13. $\frac{5}{11}$. 14. $\frac{56}{171}$. 15. $\frac{176}{51}$. 16. $\frac{70}{171}$. 17. $\frac{3}{50}$. 18. $\frac{76}{905}$. 19. $\frac{1}{3}$. 20. $\frac{962}{963}$. 21. $\frac{9}{10}$.

Page 118.

1. $\frac{65}{126}$, $\frac{47}{126}$.
2. $2\frac{1}{2}$.
3. $\frac{12}{24}$, $\frac{20}{24}$, $\frac{9}{24}$, $\frac{6}{24}$.
4. $21\frac{359}{504}$.
5. $\frac{1}{8}$.

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6. $\frac{108}{245}$.
7. $2\frac{1}{5}$.
8. $10\frac{4}{15}$.
9. \$4.79$\frac{1}{6}$.
10. $121\frac{1}{5}$.
11. $\frac{8981}{11}$.
12. $2\frac{377}{1260}$.
13. 3; $\frac{1}{16}$.
14. $264\frac{1}{6}$.
15. $\frac{16}{495}$.
16. $8\frac{7}{45}$.
17. $\frac{7}{18}$.
18. 28.
19. $\frac{4}{15}$.
20. $6\frac{6}{17}$.
21. $5\frac{1}{2}$.
22. $2\frac{11}{16}$.
23. 105.
24. 105, 45.

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25. $\frac{69993}{100}$.
26. $13\frac{3}{4}$
27. 85.
28. $6946\frac{1}{4}$.
29. $\frac{4}{5}$.
30. 140.
31. $\frac{1}{4}$.
32. $\frac{32}{65}$.
33. 90.
34. 105.
35. 40.
36. $4\frac{4}{5}$.
37. $27\frac{1}{2}$.
38. $6\frac{6}{19}$.
39. $4\frac{67}{105}$.
40. 32.
41. $3\frac{3}{5}$.

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42. $\frac{1965}{786}$.
43. $79\frac{1}{3}$.
44. $2\frac{43}{120}$, $\frac{7}{24}$.
45. $3\frac{18}{37}$.
46. 84.
47. 49, 3,889,277.
48. $\frac{3}{4}$.
49. 27.
50. $2\frac{1}{56}$, $\frac{15}{56}$, 1.
51. $2\frac{17}{60}$.
52. $\frac{1}{5}$.
53. Greater by $\frac{7}{120}$.
54. $\frac{17}{20}$ cents.
55. $17\frac{11}{17}$.
56. 1125.
57. $31\frac{1}{44}$.
58. $\frac{21}{30}$, $\frac{25}{30}$, $\frac{28}{30}$.

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59. $79\frac{11}{15}$.
60. $\frac{7}{20}$.
61. 20.
62. $65\frac{1}{3}$.
63. $20\frac{4}{5}$.
64. 964.
65. $\frac{16}{45}$
66. 1430.
67. 32.
68. Diminished.
69. $3\frac{1}{13}$.
70. 60, 80.
71. $\frac{3}{923}$.
72. $1\frac{3}{4}$.
73. $9\frac{1}{6}$.
74. $340\frac{2}{3}$.

Page 123.

75. $\frac{650}{2277}$.
76. $\frac{1755}{4312}$.
77. $\frac{36}{41}$.
78. $154\frac{11}{16}$.
79. $\frac{40}{103}$.
80. $\frac{4}{15}$.
81. $8\frac{4}{7}$.
82. $\frac{8}{175}$.
83. $2\frac{1}{4}$.
84. $\frac{25}{35}$.
85. 366.
86. $8\frac{40}{63}$.
87. $77\frac{38}{81}$.
88. $8\frac{26}{35}$.
89. The man

Page 124.

90. $2\frac{5}{6}$.
91. $1\frac{61}{104}$.
92. 39.
93. 20.
94. $\frac{3}{32}$.
95. $42\frac{3}{4}$.
96. $17\frac{4}{9}$.
97. 5, 1800.
98. $37\frac{1}{2}$.
99. 2700, 3000.
100. 150.

DECIMALS.

Page 131.

2\. 1. $\frac{9}{20}$.
2. $\frac{27}{1000}$.
3. $\frac{18}{25}$.
4. $1\frac{39}{100}$.
5. $\frac{3}{8}$.
6. $\frac{5}{8}$.
7. $4\frac{3}{4}$.
8. $4\frac{1}{80}$.
9. $\frac{871}{2000}$.
10. $10\frac{5}{20}$.
11. $\frac{1}{2000}$.
12. $\frac{1}{2}$.
13. $10\frac{1}{4}$.
14. $15\frac{29}{40}$.
15. $23\frac{3}{40}$.
16. $\frac{177}{500}$.
17. $\frac{1}{160}$.
18. $\frac{43}{800}$.
19. $15\frac{8}{125}$.
20. $\frac{1349}{250000}$.
21. $\frac{789}{1000000}$.

Page 132.

2\. 1. $\frac{1}{6}$.
2. $\frac{3}{8}$.
3. $\frac{9}{16}$.
4. $\frac{1}{3}$.
5. $\frac{1}{8}$.
6. $\frac{1}{6}$.
7. $\frac{7}{8}$.
8. $\frac{11}{250}$.
9. $\frac{3}{80}$.
10. $\frac{9}{16}$.
11. $5\frac{13}{14}$.
12. $12\frac{3}{16}$.
13. $33\frac{1}{30}$.
14. $55\frac{5}{6}$.
15. $66\frac{2}{3}$.
16. $25\frac{1}{7}$.
17. $50\frac{1}{16}$.
18. $100\frac{7}{8}$.
19. $700\frac{3}{8}$.
20. $1000\frac{1}{9}$.
21. $33\frac{5}{8}$.

Page 133.

2\. 1. .25.
2. .75.
3. .625.
4. .875.
5. .3125.
6. .4375.
7. .9375.
8. .53125.
9. .44140625.
10. .6.
11. .375.
12. .8.
13. .0625.
14. .15.
15. .85.
16. .52.
17. .35.
18. .2875.
19. .46875.
20. .65625.
21. .796875.
22. .78515625.
23. .125.
24. .00875.
25. .2976.
26. .006875.
27. .01171875.
28. .135546875.
29. .0001.
30. .222464.
31. .05795 +.
32. .04707 +.

4\. 1. $.3333\frac{1}{3}$.
2. $.5555\frac{5}{9}$.
3. $.3846\frac{2}{13}$.
4. $.2333\frac{1}{3}$.
5. $.4285\frac{5}{7}$.
6. $.4545\frac{5}{11}$.
7. $.3157\frac{17}{19}$.
8. $.2380\frac{20}{21}$.
9. $.8823\frac{9}{17}$.
10. $.0857\frac{1}{7}$.
11. $.7435\frac{35}{39}$.
12. $1.33\frac{1}{3}$.
13. $.3793\frac{3}{29}$.
14. $.8723\frac{19}{47}$.
15. $.7628\frac{84}{97}$.
16. $.0666\frac{2}{3}$.
17. $.0083\frac{1}{3}$.
18. $.0066\frac{2}{3}$.
19. .17766 +.
20. $632.66\frac{2}{3}$.

Page 134.

1. $16.66\frac{2}{3}$.
2. 35.8.
3. .9325.
4. 4.52.
5. .3475.
6. $76.33\frac{1}{3}$.
7. 98.5.
8. .54875.
9. $5.3244\frac{4}{9}$.
10. 48.64.
11. 31.08.
12. .0003158—.
13. 3.00625.
14. 4627.6428 +.
15. 1899.4839.

ADDITION.

Page 135.

1. 380.246.
2. 122.995.
3. 5.125.
4. 6.730.
5. 746.58525.
6. 117.766.
7. 108.455.
8. 745.707.
9. 787.428.
10. 87.474.
11. 142.4430.
12. 126.205.
13. 1222.18905.
14. 17.1207.
15. 75.225.
16. 529.0625.
17. 51.170.
18. 17.2737.
19. 11.22211.
20. $1110.0001\frac{1}{5}$.

1. 555.52.
2. 1.9375.
3. 47070000.960041008.

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4. 5.804.
5. .022987875.

SUBTRACTION.

1. 67.98.
2. 57.261.
3. 6703.5342.

Page 137.

4. 7.7628.
5. 4256.84436.
6. 5.175.

3. 36.31119.
4. 9.8202.
5. 210.8561.
6. 295.4526.
7. .00009.
8. 684.999.
9. 115.001.
10. 991.9001.
11. 11.131.
12. .0000756.
13. 17.705.
14. 1963.626.
15. .9257926.
16. .4234.

17. 1. 84.655.
 2. .495.
 3. 2319.67.
 4. 106.524.
 5. 4.99999995.
 6. 2.0625.

1. 2512.50.
2. 706.13.
3. 2519.98.

Page 138.

4. .3.
5. James, .3255.

1. 14.8874.
2. 9.625.
3. 289.7892.
4. .637235.
5. 47.07.

MULTIPLICATION.

Page 139.

2. 1. 32.67.
 2. .3267.
 3. 2.86268.
 4. .4077.
 5. 6.0088.
 6. 27121.5.
 7. 27148.6215.
 8. .273.
 9. 150.
 10. .20056.
 11. .020265.
 12. 15.1296.
 13. 34.4576.
 14. .0006076.
 15. 4.08.
 16. 4.0073328.
 17. 40.12.
 18. .0001403.
 19. 86213.
 20. 79.88904.
 21. 8.064.
 22. 6.963744.
 23. 42.3.
 24. 129.6.
 25. 52.34375.
 26. 1.5625.
 27. 97.65625.
 28. .42624.
 29. 4.110092.
 30. .65964.
 31. 150.
 32. 604.
 33. .0149935.
 34. .0000015984.
 35. 6.1625.
 36. 3700.
 37. 234.61875.
 38. 3329.6095.
 39. .00009.
 40. 10.41797537.

Page 140.

2. 1. 87.
 2. .069.
 3. 9560.
 4. 4.53.
 5. 4069.
 6. .94.
 7. 92.
 8. 749.
 9. 3.6.
 10. 854300.
 11. 10018.2.
 12. 76541000.

1. 1. 4923.375.
 2. 1707.
 3. 138.1875.
 4. 121.54.
 5. 166.11.
 6. 375.
 7. 945.

Page 141.

8. 547.
9. 25313.225.

2. 142.39.
3. .000188.

DIVISION.

Page 142.

1. 1. 3600.
 2. .289.
 3. 78.4.
 4. .01704.
 5. .51.
 6. 7.6.
 7. 17500.
 8. 11.2195.
 9. 8.76.
 10. 34.6.
 11. .01.
 12 9.58.
 13. 2.
 14. .958.
 15. 4.6737.
 16. .025.
 17. 3.
 18. 5.
 19. 30.
 20. 1000.
 21. 50.
 22. $3.33\frac{1}{3}$.
 23. .00008.
 24. .000005.
 25. .96.

3. 1. 61.544.	4. 1.08096.
2. 32.185.	5. .005873.
3. 45.0167966 +.	6. 2500.

Page 143.

2. 1. 5.3479.	6. .495674.
2. .492568.	7. .00000038649.
3. .0249653.	8. .000082253.
4. .05908.	9. .9391.
5. .00007156.	10. .0785437.

Page 144.

1. 1. .02.	8. .00725.
2. .193.	9. 1.35.
3. .243325.	10. 18.5.
4. 4.1.	11. 9.35.
5. .292.	12. .95.
6. 31.565.	13. 9.4625.
7. .0116.	

2. 117.50.	3. .534188.

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4. 1. 494307.	6. 594090620.
2. 486766.	7. 56433692.
3. 59618322.	8. 91619232.
4. 9687202.	9. 694756812.
5. 456462492.	10. 491243599.

5. $486.08.	7. 75.2735.
6. $23040.	8. 2384.64.

9. 5494540461.725 +.

4. 1. 594580.	9. 5508825.
2. 459048.	10. 4958523.
3. 976005.	11. 18063864.
4. 3604528.	12. 43626114.
5. 9781944.	13. 119227980.
6. 26091088.	14. 319868736.
7. 12439308.	15. 452023875.
8. 4529385.	16. 22,891,152.

Page 147.

1. 1. 63900.	6. 666700.
2. 368.	7. 34265.
3. 2402.	8. 4751.0875.
4. 690.9.	9. 84400.
5. 3029.	10. 977.

2. 1. 16.16.	6. 1197.297.
2. 2002.036.	7. 474.4038.
3. 8.15418.	8. 5820.048.
4. 1009.77.	9. 418.1904.
5. 2791.425.	10. 32904.06.

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5. $15.775.	10. $35.
6. $252.	11. $693.545.
7. $7.25.	12. $5.00.
8. $21.42.	13. $16.
9. $1292.50.	14. $60.

Page 149.

16. 5.96.	19. 258.408.
17. 51.43.	20. 20.1575.
18. 65.487.	21. 565.0625.

1. 1000.	9. .08.
2. 15.	10. 116.
3. $1.06\frac{2}{3}$.	11. 1.002002 +.
4. $\frac{71}{750}$.	12. $24429\frac{2}{3}$.
5. .25.	13. 30855.8.
6. 27.534.	14. 7.706.
7. $.1406\frac{1}{3}$.	15. 50.
8. 2.02024.	

Page 150.

16. 1174.245.
17. 5000000000.
18. .125.
19. .6375.
20. $689.8375.
21. .0292991.
22. $\frac{137}{900}$.

23. 74.76.
24. 143869176.
25. $\frac{1201}{4928}$.
26. 292.25.
27. 1.12.
28. .0125, .125, .0025, $\frac{1}{80}$, $\frac{1}{8}$, $\frac{1}{400}$.
29. 9929.4.
30. 2.25.
31. 494.88.
32. $0.50, $50.
33. 492.61875.
34. $170.
35. $\frac{37}{48}$.
36. .0793295.

Page 151.

37. 64278.72.
38. .0225.
39. 1335.
40. 426.075.
41. 486.
42. 5.125.
43. $91\frac{37}{31}$.
44. 19.8.
45. 16.50.
46. $224\frac{41}{50}$.
47. $212.75.
48. $102\frac{22}{39}$.
49. $646.95.
50. $\frac{4669}{17490}$.
51. $6.22.
52. 10.83\frac{1}{3}$.
53. 28.33\frac{1}{3}$.
54. $62 50.

BILLS.

Page 153.

1. 580 44.

Page 154.

2. 22.94.
3. 2167.16.

Page 155.

4. 30.95.
5. 1204.93.
6. 231.34.
7. 29.74.

COMPOUND NUMBERS.

Page 158.

4. 1. 486.
2. 38856.
3. 8480.
4. 362.
5. 3587.
6. 89074.

Page 159.

7. $539995\frac{1}{5}$.
8. $189\frac{3}{4}$.
9. 3151.
10. 36.
11. 90.
12. $411\frac{3}{7}$.
13. 17, 9, $1\frac{1}{3}$.
14. 8, 6, $3\frac{3}{7}$.
15. 5, 8, $2\frac{2}{7}$.
16. 150000.
17. 1728.
18. 100000.
19. 190.
20. 1980.

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3. $\frac{1}{25}$.
4. $\frac{1}{84}$.
5. 2, 1, 3.
6. 212.85.
7. $19.57\frac{1}{12}$.
8. $298\frac{7}{20}$.
9. $7\frac{83}{320}$.
10. $20\frac{39}{48}$.
11. $120\frac{3}{4}$.
12. $62\frac{79}{480}$.
13. 4584.
14. $62\frac{29}{320}$.
15. £26. 9s.
16. $126.529.
17. £116 10s. 2d. 2 far.
18. £81.0541 +.
19 £9.9866 +.
20. $175.194.
21. £26 11s.
22. £200.

Page 161.

23. £250000.
24. 108440.
25. $35.96.
26. $48.567.
27. $372.49.
28. $742.278.
29. 25160.62.
30. 59.68.
31. $\frac{1}{125}$.
32. 18s. 4d.

Page 162.

1. 209.
2. 8903.
3. 1452.

4. 316800.
5. 27732.
6. 198.
7. 63360.
8. 15854.
9. 2 m. 4 fur. 34 rd. 2 yd. 2 ft.
10. 426, 0, 11.
11. 47, 141, 3, 1 ft. 6 in.
12. 1 m. 1 fur. 31 rd. 12 ft. 3 in.
13. 40000.
14. 77400.

Page 163.

15. 9748.
16. $5\frac{3}{8}$.
17. $9\frac{37}{40}$.
18. $11\frac{18}{55}$.
19. 126.631°.
20. 5 ft. 2 in.
21. 3 mi. 57 ch. 63 links.
22. 7603200 or 8763955.2.
23. 10, 222, 2, 2, 1.
24. 501304320; 13925120.
25. 75 ft.

Page 165.

1. 8650.
2. 18876.
3. $257276\frac{1}{4}$.
4. 9823032.
5. 15328440.
6. 13551.
7. 85, 129, 8, 3, 108.
8. 20275956.
9. $7.15\frac{1}{4}$, 8.32.
10. 198880.
11. 20.
12. $1\frac{11}{16}$.

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15. 114, 8, 5, $113\frac{1}{7}$.
16. 11, 3, $13\frac{1}{2}$.
17. 4.
18. 16000000.
19. 253, 7, 6, 146.
20. 128.
21. 64040053 sq. l.
22. 1.1 A.
23. 20.
24. 14, 50, $12\frac{1}{2}$, 5, 92; or, 14, 50, 13, 1, 20.
25. $73\frac{503}{1089}$.

1. 3 A.
2. 128.
3. $37\frac{7}{9}$.
4. 36.
5. 1728.
6. $224\frac{32}{363}$.
7. 3520, $352.
8. $66\frac{2}{3}$.
9. 80.
10. $32848.
11. 44.20.
12. $\$85.83\frac{1}{3}$.

Page 167.

13. $80.65.
14. 30.
15. $47.11.
16. $\$1913\frac{5}{8}$.
17. 72.
18. $8.50 or $13.28.
19. 4500.
20. Neither.
21. 12.
22. $480.
23. $425.45.
24. $447.99.

Page 168.

25. 1125.
26. 2250.

VOLUME.

Page 169.

1. 730960.
2. 15, 18, 16.
3. 55410.
4. $21870\frac{1}{2}$.
5. 60.
6. 80.
7. $36\frac{371}{864}$.
8. 56.
9. 9872.
10. $68\frac{93}{128}$.
11. 59772.
12. $9\frac{1}{2}$.
13. 2441.
14. $36\frac{16}{27}$.
15. 6, 3, 960.

Page 170.

1. 1. $157\frac{1}{2}$ cu. ft.
   2. $20\frac{51}{64}$ cu. ft.
   3. $6\frac{3}{4}$.
   4. $2\frac{1}{3}$.
   5. 1 cu. in.
   6. 176 cu. yd.
   7. $1777\frac{7}{9}$. cu. yd.

Page 171.

8. $101\frac{1}{9}$ cu. yd.
9. 216 cu. in.
10. 64 cu. in.

2. Vol. of a cd. ft. $= 1 \times 4 \times 4$.
3. $9\frac{7}{33}$.
4. $410\frac{1}{2}$.
5. 19683.
6. 592704.
7. $65\frac{13}{16}$.
8. $611.11.
9. 18.
10. 40.
11. 192.
12. $4\frac{4}{9}$.
13. $3588.75.
14. 6561.
15. 1280. No allowance for corners.
16. 178,687.

Page 172.

17. $1155.
18. $6\frac{14}{33}$.
19. 3.
20 $4 \times 11\frac{1}{9}$.
21. 633600.

Page 173.

1. 1. 12.
   2. $12\frac{1}{2}$.
   3. 11.
   4. 11.
   5. $18\frac{2}{3}$.
   6. 90.
   7. 525.
   8. 20.
   9. 180.
   10. $10\frac{2}{3}$

2. $53\frac{1}{3}$.
3. $1306\frac{2}{3}$.
4. $1\frac{9}{16}$.
5. $25\frac{1}{2}$.
6. $\$17\frac{5}{12}$.
7. $180.88.
8. $74\frac{3}{8}$.
9. $17.424.
10. $13\frac{7}{24}$.
11. $20.57.

Page 174.

12. $60.48.

MEASURES OF CAPACITY.

1. 1. 28.
   2. 64.
   3. 296.
   4. $99\frac{3}{8}$.
   5. $154\frac{29}{32}$.
   6. 176.
   7. 5264.
   8. 9.484.

Page 175.

9. $\frac{1}{42}$.
10. 255.

2. 1848.
3. 21.5 +.

1. 410.36.
2. 28.26.
3. $458419\frac{1}{2}$.
4. 7.525.
5. $5263\frac{43}{64}$.
6. $842\frac{3}{16}$.
7. $1795\frac{25}{77}$.
8. $4\frac{1}{96}$.

APOTHECARIES' MEASURE.

Page 176.

1. 1536.
2. 3.
3. 1584.
4. $6\frac{3}{8}$.
5. 2,112,660.
6. .030273.
7. 17,208.
8. $214\frac{1}{8}$.
9. 268,740.
10. 137, 7, 2, 5, 47.

DRY MEASURE.

Page 177.

1. 6.
2. 20.
3. 1.
4. 33.
5. $28\frac{3}{32}$.
6. 831.
7. 2443.
8. 21.
9. $25\frac{1}{8}$.
10. 319.

CAPACITIES.

1. 17203.36, 45158.82.
2. 12.877.
3. 746,496.
4. $393\frac{3}{4}$.

5. 4.32.
6. 18.43.
7. 42.19.
8. 75.2.
9. 185.4.
10. 7,840,800.
11. 8.671.

Page 178.

12. 217.718.	16. 1.24 +.
13. 24.323.	17. 7.5 nearly.
14. $746\frac{2}{3}$.	18. $13\frac{1}{2}$.
15. $497\frac{7}{9}$.	19. 87 +, 70 nearly.

MEASURES OF WEIGHT.

Page 179.

1. 6000.	4. 2.97.
2. 10,406.	5. 64,000.
3. 20,327.	6. 5.203 T.

WEIGHTS AND VALUES.

1. 16.50.	5. 1.30.
2. 2.89.	6. 80.
3. 64.00.	7. 32,000.
4. 63,000.	8. 15.

Page 180.

9. The same.	13. $6352\frac{1}{2}$.
10. 35.64.	14. $26\frac{7}{20}$.
11. 47.60.	15. $3.01.
12. 150.	

1. 1. 228.	5. 34,450.
2. $8\frac{1}{2}$.	6. $17\frac{49}{120}$.
3. 404,632.	7. $7\frac{1}{2}$.
4. $9\frac{59}{240}$.	8. 5760.

Page 181.

2. 7000.	6. $11\frac{803}{1152}$.
3. 1240.	7. $1.28\frac{3}{4}$.
4. Lead, 1240.	8. 123.274.
5. $152\frac{8}{35}$.	

1. $437\frac{1}{2}$. 2. 480. 3. 480.

Page 182.

1. 32 lb.	5. 5317℈.
2. 12 lb.	6. $79\frac{1}{3}$ lb.
3. 816ʒ.	7. 98,277 gr.
4. $8\frac{1}{2}$ lb.	8. $18\frac{3}{8}$ lb.

1. 9℥ 4ʒ 0℈ 7 gr.	5. $\frac{17}{120}$.
2. 100.	6. $9\frac{13}{18}$.
3. 6℥.	7. 308.75.
4. 1400.	8 164.16.

MEASURES OF TIME.

Page 183.

1. 86,400. 2. 1 da.

Page 184.

3. 10,080.	5. 6176.	7. 117,161.
4. 2.	6. $4\frac{1}{90}$.	8. $24\frac{53}{120}$.

1. 13. 49. 56.
2. 13. 46. 38.
3. 155.
4. 117.
5. Summer.
6. 1600, 1660, 1776.
7. 27;529.
8. 4 da. 16 hr.
9. 168.
10. $\frac{1}{4}$.
11. Sept. 15.
12. 24.
13. 744.
14. 38 da.
15. May 26, Aug. 31.

CIRCULAR MEASURE.

Page 185.

1. 1. 3385.	5. 128,939.
2. $63\frac{5}{6}$.	6. 27.7139.
3. 1416.	7. 537,993.
4. $14\frac{4}{15}$.	8. 4.545.

2. 10,800.
3. 324,000.
4. $\frac{5}{8}$.
5. $7\frac{1387}{2700}$.
6. 1 hr. 1 min. 1 sec.

COUNTING.

Page 186.

1. 6480.	4. 60.
2. $18\frac{1}{2}$.	5. 4800.
3. 20,736.	
1. 100.	5. 70.
2. 26.40.	6. 2770.
3. Lost $2.00.	7. 1260.
4. 60.	

Page 187.

8. $.08\frac{1}{4}$. 9. 12. 10. 93,750.

Page 188.

1. 1. $1\frac{17}{31}$ pt.	8. 9, 11, .96.
2. $2\frac{1}{4}$ min.	9. 13, 8, 2.56.
3. 3 yd. 2 ft.	10. 8, 17.28.
4. 2.08 gills.	11. 320, 6.
5. 8, 57, $2\frac{2}{7}$.	12. 2, 3, 3.2.
6. 43, 19, 2, 36.	13. 12, 259.2.
7. .448.	14. 1, 7, 18.
2. 1. $1\frac{1}{5}$.	9. $2\frac{2}{3}$.
2. $\frac{5}{36}$.	10. $23\frac{9}{13}$.
3. 6, 13, 8.	11. 13, 20.
4. 5, 13, 5, 6.	12. 4, 2, $2\frac{14}{17}$.
5. 1, 5, $1\frac{3}{7}$.	13. 62, 8.
6. 88, 26, 8.	14. 2, 4.
7. 1.476.	15. 165.
8. 5 qt.	16. $85\frac{1}{3}$ rd.

REDUCTION ASCENDING.

Page 189.

2. 1. $\frac{7}{9} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{4} = \frac{7}{288}$.
2. $\frac{13}{23040}$.
3. $\frac{13}{79200}$.
4. $\frac{19}{112000}$.
5. $\frac{2}{6[illegible]}$.
6. $\frac{3}{80}$.
7. $\frac{1}{6600}$.
8. $\frac{1}{450}$.
9. .000225.
10. $\frac{1}{3024}$.
11. $\frac{1}{6050}$.
12. $\frac{1}{324}$.
13. $\frac{1}{72}$.
14. $\frac{23}{222156}$.
15. $\frac{37}{253440}$.
16. $\frac{1}{245760}$.
17. $\frac{29}{165888}$.
18. $\frac{1}{8000}$.
19. $\frac{3}{6080}$.
20. .00056078125.

FRACTIONAL RELATIONS.

Page 190.

3. 1. $\frac{11}{12}$.	9. $\frac{144}{175}$.
2. $\frac{49}{3520}$.	10. $\frac{1}{36}$.
3. $\frac{759}{4840}$.	11. $\frac{1}{3}$.
4. $\frac{11}{152}$.	12. $\frac{193}{3190}$.
5. $\frac{17}{2970}$.	13. $\frac{61}{540}$.
6. $\frac{65}{432}$.	14. $\frac{887}{960}$.
7. $\frac{5}{8}$.	15. $\frac{267}{1600}$.
8. $\frac{101}{960}$.	

DECIMAL RESULTS.

Page 190.

5. 1. .21423 +.	5. .49375.
2. .01635.	6. .3375.
3. .861 +.	7. .497 +.
4. .174.	

Page 191.

8. .09816 +.	12. .36875.
9. .875.	13. .434
10. .122 +.	14. .5625.
11. .038 —.	

REVIEW.

1. 1. 106.08.
   2. 684.75.
   3. 42.
   4. 52.92.
   5. 4.97.
   6. 3.[illegible]
   7. 535.
   8. 128.
   9. 17.50
   10. 423.36.

2. 21.
3. 525600.
4. 15.3125.
5. $23\frac{4}{27}$.
6. 8000.
7. 31556929.
8. $62\frac{1}{2}$.
9. 9892800.

Page 192.

10. 88.
11. 90°.
12. 1.4.
13. $155\frac{5}{21}$.
14. $2\frac{22}{39}$.
15. 2090.
16. 6.57.
17. 17, 14, 33.
18. $58928\frac{4}{7}$ lb.
19. $222\frac{21}{33}$.
20. $12\frac{38}{799}$.
21. 836.64.
22. $.58\frac{1}{3}$.
23. 1584.
24. $15.50.

Page 193.

25. $49\frac{43}{73}$.
26. 336.
27. $102\frac{2}{5}$.
28. 302.
29. $94\frac{1}{2}$.
30. 80.
31. .6219 +.

ADDITION.

Page 194.

1. 2, 129, 0, 1, 2.
2. 27, 213, 2, 1, 0.
3. 429, 72, 17, 4, 72.
4. 96, 3, 9, 4.
5. 57, 7, 31, 8.

Page 195.

6. 16. 6. 2. 1. 10.

5. 1. 124 rd., 1 ft. 4.83 in
   2. 18.1808 sq. in.
   3. 16s. 3.975d.
   4. 7 cwt. 17 lb. 10.6 oz.
   5. 8 oz. 6 pwt. $2\frac{2}{3}$ gr.
   6. $16\frac{43}{288}$ da.
   7. $60\frac{97}{132}$ rd.
   8. $40\frac{55}{72}'$.
   9. 13s. $3\frac{11}{21}$d.
   10. 2 da. 15 hr. 50 min. 35 sec.
   11. 96 rd. 14 yd. 1 sq. ft.
   12. 27 cwt. 91 lb. 12 oz.
   13. 2 hhd. 17 gal. 2 qt. 0 pt. 3 gi.
   14. £5 15s. $8\frac{1}{7}$d.

6. .8 crown.
7. .095575.
8. 13 ft. 2.73 in.

Page 196.

1. 2. £7 18s. 11d.
   3. 8 A. 150 sq. rd. 11 sq. yd. 7 sq. ft. 104 sq. in.
   4. 1 T. 14 cwt. 18 lb. 15 oz.
   5. 183 lb. 2 oz. 2 pwt. 3 gr.
   6. 17 yr. 41 wk. 5 da. 23 hr. 58 min. 59 sec.
   7. 1s. 26° 30′ $46\frac{1}{24}''$.
   8. 41 gal. 1 qt. 1 pt. 3 gi.
   9. 6 lb. 10℥ 5ʒ 1℈.
   10. 4 T. 1989 lb. 8 oz.
   11. 6 pwt. 15 gr.

Page 197.

12. 2 oz. 3 pwt. 3 gr.
13. 11 hr. 59 min. 20 sec.
14. 16 gal. 2 qt. 0 pt. $3\frac{31}{35}$ gi.
15. 21 hr. 24 min.
16. 4 qt.
17. 3 oz. 5 pwt.
18. 4 sq. yd. 6 sq. ft. 108 sq. in.

19. 44 sq. yd. 8 sq. ft. 99 sq. in.
20. 327° 16′ $21\frac{9}{11}$″.
23. 9. 4. 2.
24. 18. 2. 11.
25. 67. 7. 13.
26. 3 mo. 21 da.
27. 3. 11. 28.

Page 198.

28. 7. 9. 1.
30. 5. 3. 1.
2. 206, 5, 11, 8.
3. 16, 19, 71, 8.
4. 56, 5, 4, 0.
5. 77, 6, 9, 0.
6. 474, 6, 0, 216.
7. 276, 10, 7, 0.
8. 200, 19, 86, 8.
9. 361, 3, 13, 5, 0, 6.
10. 423, 8, 4, 9.
11. 544, 4, 7, 2, 16.
12. 98, 3, 22, 5, 0, 72.
13. 13, 20, 5, 36.
14. 317, 0, 1, 2.
15. 234, 18, 5.

DIVISION.

Page 199.

1. 16, 2, 38.
2. 2, 0, $7\frac{1}{2}$.
3. 3, 3, 2, 0, $8\frac{8}{9}$.
4. 12, 17, 25, 12.
5. 38, 142, 24, 6, 108.
6. 68, 2, 0, $1\frac{1}{5}$.
7. 1, 775.
8. 10, 26, 0, 2, $1\frac{1}{2}$.
9. 7, 16, 5.
10. 5, 3, 11.
12. 8.
13. $8\frac{201}{3991}$.
14. $890\frac{6}{7}$.

Page 200.

15. $65\frac{1}{2}$.
16. 4200.
17. 16.
18. $1\frac{176686}{181359}$.
19. $7\frac{14}{103}$.
20. $3\frac{29745}{98085}$.

LONGITUDE AND TIME.

Page 201.

3. 45° 21′ 27″; 3 hr. 1 min. $25\frac{4}{5}$ sec.
4. 47° 16′ 45″.

Page 202.

5. 87° 23′ 45″.
6. 74° 0′ 2″.
7. 5°.
8. 55 min. 56 sec. past 1 P.M.
10. 5 hrs. 5 min. 32 sec.
11. 53 min. $35\frac{7}{15}$ sec. past 4 P.M.
12. 40 min. 58 sec. past noon, July 5.

STANDARD TIME.

Page 204.

2. 15 min. 46 sec.
3. Noon.
4. 19 min. 20 sec. past noon.
5. 1 min. 1 sec. before 12.

MISCELLANEOUS.

1. $66\frac{2}{3}$ ct.
2. Eastward; 21° 15′.
3. 6 mo. 23 da.
4. 3510.
5. 25, 1, 7, $1\frac{9}{10}$.
6. 42 min. $54\frac{1}{3}$ sec. past noon.

Page 205.

7. $1687\frac{1}{2}$.
8. Noon.
9. 324 bu.
10. $105\frac{143}{330}$.

11. $131655\frac{5}{7}$.
12. $644\frac{56476}{107521}$.
13. $1\frac{1}{4}$ ft.
14. 375.
15. $2.43\frac{3}{4}$.
16. 50.808.
17. 5 lb. 12 oz.
18. $\frac{1899}{3241}$.
19. .00662 bbl.
20. .82285 +.
21. $37\frac{1}{3}$.
22. 4, 55, 37. London.
23. 15.
24. $5\frac{35}{193}$.
25. $16.6272.

Page 206.

26. $1,833,500.
27. $36° 8' 40\frac{40}{83}''$.
28. 120.
29. 1.50.
30. 123 A., 120 sq. yd.; B, 148 A., 80 sq. rds.

PERCENTAGE.

Page 210.

2. 1. 21.
2. 3.
3. $9\frac{17}{20}$.
4. 6.
5. 300.
6. 40.
7. 8.
8. 48.
9. 31.
10. 35.
11. 40.
12. 320.
13. 60.
14. 300.
15. .75.
16. 80.
17. 350.
18. 8100.
19. 1400.
20. 800.
21. 750.
22. 1000.
23. 1500.
24. 3000.
25. $\frac{1}{7}$.
26. $\frac{1}{14}$.
27. 100,000.
28. 144.
29. 9.
30. $21.42.

Page 211.

1. 200.
2. 11.25, 78.75.
3. 1069.20.
4. 48.15.
5. 180, 360.
6. 144.90.
7. 2100.
8. $78.24.
9. 162.50.
10. $.90.
11. 49.33, 937.27.
12. $5\frac{17}{20}$, $774\frac{3}{20}$.
13. 66.

Page 212.

14. 1500, 200, 300.
15. $33\frac{1}{3}\%$; 33333. $\frac{1}{300}$; 333.

THE RATE.

2. 1. 20%.
2. $1\frac{1}{4}\%$.
3. $37\frac{1}{2}\%$.
4. $1\frac{3}{5}\%$.
5. 125%.
6. $31\frac{11}{19}\%$.
7. 28%.
8. 97%.
9. 6%.
10. 7%.
11. 25%.
12. $28\frac{4}{7}\%$.
13. $8\frac{1}{3}\%$.
14. 75%.
15. 45%.
16. 35%.
17. $4\frac{11}{16}\%$.
18. $3\frac{3}{4}\%$.
19. $\frac{25}{48}\%$.
20. 10%.

Page 213.

1. 20%.
2. 40%.
3. 1%.
4. 5%.
5. $3\frac{1}{8}\%$.
6. $62\frac{1}{2}\%$.
7. 150%.
8. $4\frac{1}{2}\%$.
9. $8\frac{1}{3}\%$.
10. 700%.
11. $7\frac{9}{13}\%$.
12. $30\frac{5}{9}\%$.
13. $3\frac{103}{149}\%$.
14. $17\frac{1}{2}\%$.
15. $4\frac{1}{6}\%$.
16. $25\frac{5}{67}\%$.

Page 214.

17. 20%.
18. $33\frac{1}{3}\%$.
19. 50%.
20. $675, $19\frac{211}{581}\%$.

THE BASE.

2. 1. 3080.	7. 2100.	
2. 3600.	8. 6600.	
3. $1666\frac{2}{3}$.	9. 72.	
4. 24.75.	10. 105.	
5. $643.60.	11. 900.	
6. $296\frac{12}{13}$.	12. 160.	

Page 215.

13. 162.	20. $12\frac{1}{2}$.
14. 128.	21. 400.
15. 96.	22. $1.60.
16. 140.	23. 150.
17. 288.	24. 1200.
18. 160.	25. 13950.
19. $12\frac{1}{2}$.	26. 15,010,000.

3. 750. 4. 256. 5. 518.40.

1. 100; 115.	6. 1700.
2. 400.	7. 10000.
3. 2680.	8. 7692.
4. 1300000.	9. $35.
5. 551 A. 55 sq. rd.	10. 6300.

Page 216.

11. 10000.

Page 217.

3. 1. 4200.	9. $505\frac{15}{17}$.
2. 400.	10. $\frac{11}{14}$.
3. 738.	11. 1200.
4. 564.	12. 848.
5. 1120.	13. 225.
6. 14000.	14. $2\frac{113}{206}$.
7. 600.	15. $262\frac{1}{2}$.
8. $3807\frac{1}{29}$.	16. $1882\frac{354}{403}$.
4. 1. 1600.	6. $700.
2. $2755\frac{5}{9}$.	7. 112.
3. 1890.	8. 525.
4. 5100.	9. $\frac{25}{36}$.
5. 2750.	10. 720.

1. $13240.74\frac{2}{27}$.	5. 280; 350.
2. 6140.	6. $1.20.
3. 800.	7. 168.
4. $48\frac{4}{7}$.	8. 5.00.

Page 218.

9. 64000.	13. 50000.
10. $6\frac{2}{3}$ loss.	14. 400.
11. 3952.	15. 3 lb.
12. 551 A. 55 sq. rd.	16. 15 cts.

Page 219.

1. $1.72½; $12.07½; 8.62½.
2. 375.
3. 1968; 2214.
4. 9060; 1510.
5. 8%.
6. 145.80; 16345.80.
7. $177\frac{7}{9}$%.
8. $9\frac{1}{11}$%; 1584.
9. $21\frac{19}{36}$%; 1240.
10. 171.5.
11. 3456; $3784\frac{8}{25}$; $3127\frac{17}{25}$.
12. 13400; 3082.
13. $41.771; $12.531.
14. $1\frac{1}{5}$%; 10120.
15. 800; 72.
16. $\frac{9}{400}$; $3\frac{9}{400}$; $2\frac{391}{400}$.
17. $\frac{3}{4}$; $2\frac{3}{4}$.
18. 45%.
19. 25%.
20. 20%; 120.
21. $31\frac{47}{63}$%; 945.

1. $16\frac{2}{3}$%.

Page 220.

2. $6247.50.	6. $95\frac{5}{21}$.
3. $200.	7. $68\frac{3}{4}$%.
4. $19\frac{3}{13}$%.	8. $261\frac{1}{3}$.
5. 88.	9. $41\frac{87}{193}$%.

10. $16000.
11. 800.
12. 9900.
13. 40%; 60%.
14. $121\frac{19}{36}$%.
15. $11\frac{1}{9}$%.

COMMERCIAL DISCOUNT.

Page 222.

3. 10.91\frac{1}{4}$; 79.08\frac{3}{4}$.
4. $4.40.
5. $478.40.
6. 50 off = $1000 off; 25 and 25 off = $875 off.
7. $573.75.
8. $1778.65.
9. $11.63.
10. $2694.76 +.
11. $66.69.
12. $390.91.
13. 35.4%.
14. $30.78.
15. The former.
16. $6.00.

1. 37%.
2. 49%.
3. $54\frac{7}{8}$%.
4. $27\frac{13}{40}$%.

Page 223.

18. $55.
19. $389.54.
20. $4.80.

GAIN AND LOSS.

Page 224.

6. 2100; 12600.
7. 48; 5.52.
8. 2140; 25%.
9. 45; 51.30.
10. 105; 805.
11. 1; 20.

Page 225.

12. 75; 250.
13. $16\frac{2}{3}$%; 2 cts.
14. 0.01\frac{3}{16}$; 0.106\frac{7}{8}$.
15. 25; $33\frac{1}{3}$%.
16. 25; 25%.
17. $87.50; 162.50.
18. .50; $28\frac{4}{7}$%.
19. 1; $16\frac{2}{3}$%.
20. 1.50; 1.62.
21. 100; 110.
22. 150; 30.
23. $212.96\frac{8}{27}$; $17.03\frac{19}{27}$.
24. $.935; $5.50.
25. $7\frac{1}{2}$%; $54.
26. 630; 677.25.
27. $7692; 7653.54.
28. $2.36; 77.88.
29. $7\frac{1}{2}$%; $27.
30. 10.775; 1928.725.

1. $3450.
2. 25%.
3. $300.
4. 4.12\frac{8}{91}$.
5. 100%.
6. 40%.

Page 226.

7. $24.
8. 1.56\frac{1}{4}$.
9. $12\frac{1}{2}$%.
10. $1.20.
11. $33\frac{1}{3}$%.
12. $1150.
13. 57 cts.
14. $65.
15. $12\frac{1}{2}$ cts.
16. $26\frac{1}{2}$%.
17. $50.
18. $250.
19. 25%.
20. $12\frac{36}{97}$.
21. $66\frac{2}{3}$.

Page 227.

22. $5.50.
23. 75 cts.
24. $10\frac{2}{5}$%.
25. $31.25.

COMMISSION.

Page 229.

5. \$45.50.
6. $2\frac{1}{2}\%$.
7. \$170.73; \$6829.27.
8. \$6437.50.
9. \$3932.04.
10. \$8.91.
11. \$4120.
12. \$753.
13. \$530.
14. \$64.
15. \$6.88; \$165.12.
16. \$4662.
17. \$2776.193.

Page 230.

18. 3042.65 bu.
19. \$34.83.
20. \$1920.
21. \$7443.75.
22. \$218.125.
23. \$6489.175; \$252.825.
24. \$9950.25; \$49.75.
25. \$600,000.
26. \$20600; $137333\frac{1}{3}$ lb.

1. \$162.50. 2. \$450.

Page 231.

3. Gain, $18\frac{3}{4}\%$.
4. 10%.
5. \$195.00.
6. $56\frac{1}{4}\%$.
7. $35\frac{5}{17}\%$.
8. 13%.
9. Gain, 37+%.
10. \$250.
11. 120000.
12. $14\frac{6}{11}$.
13. 25%.
14. \$2275.
15. \$3891.625.
16. \$180.375.

Page 232.

17. $28\frac{8}{39}\%$.
18. $\$\frac{9}{20}$.
19. 25%.
20. 20%.
21. \$21.60.
22. $6\frac{2}{3}\%$.
23. $10\frac{2}{5}\%$.
24. The latter.
25. \$1.25.
26. \$1194.125.
27. \$30000.
28. \$58.87.

Page 233.

29. $\$2266.66\frac{2}{3}$; $2833.33\frac{1}{3}$; $2436.66\frac{2}{3}$.
30. \$2000.
31. 3%.

STOCKS.

Page 235.

6. \$5137.50. 7. $4\frac{2}{7}$.

Page 236.

8. $71\frac{3}{7}$.

1. \$16612.50; \$600.
2. \$4700.
3. \$450.
4. $3\frac{17}{21}$; $3\frac{7}{11}\%$; $4\frac{156}{211}\%$; $4\frac{64}{209}\%$, $5\frac{15}{47}\%$.
5. No. 3, \$2424; No. 4, \$1636.
6. No. 3, $5\frac{355}{409}\%$; No. 4, $3\frac{97}{101}\%$.
7. 403 shares +.
8. $85\frac{5}{7}$.
9. \$1595.
10. 22 shares.
11. 5% at 60 by \$20.00.
12. 26812.50.

Page 237.

13. 6% at 90.
14. 225%.
15. $13015\frac{5}{8}$.
16. 84.

17. \$594.
18. 3732.
19. 264.
20. 3065.20.
21. 262.50.
22. $88\frac{1}{2}$.
23. 5118.75.
24. The latter.
25 The latter, \$50.50.
26. 21,050.

Page 238.

27. 10,000.
28. $5\frac{285}{343}$.
29. \$13,000.
30. \$6780.
31. $121\frac{1}{8}$.
32. 352.
33. 5%.
34. \$60; 4%.
35. 4134.
36. 6780.
37. Increase, \$20.
38. $58\frac{1}{3}$.
39. $15\frac{5}{27}$%.

Page 239.

40. 71,250.
41. 5%.
42. $62\frac{1}{2}$
43. $.07\frac{1}{17}$.
44. $14\frac{2}{7}$.
45. 300.
46. 104.
47. 5000; $22\frac{1}{2}$%.
48. \$4500.
49. 224 shares.
50. $109\frac{19}{88}$.
51. $101\frac{1}{2}$.
52. $5\frac{5}{9}$%.
53. 7s.

Page 240.

54. 200 shares.

INSURANCE.

Page 241.

1. \$2400.
2. \$99 40.
3. \$57.60.
4. \$337.50.
5. \$187.50.
6. $1\frac{1}{2}$%.
7. \$297,000.
8. 375.
9. The 2d
10. 2%.
11. \$58.39.

Page 242.

12. 2116.80.
13. 13,600.
14. 24,500.
15. \$13,500.
16. \$76,800.
17. \$48.
18. 1%.
19. \$42,180.
20. \$119.10.
21. \$3676.25.
22. 212.
23. 451.50.

Page 243.

24. \$3168.
25. \$3699.50.
26. \$3516.

TAXES.

Page 244.

1. 6 mills.
2. \$29.31.
3. 3%.
4. \$6000.
5. \$10.
6. \$27.70.
7. $.001\frac{3}{4}$.
8. .003.
9. 3826.53.
10. .007.

Page 245.

11. 17,500.
12. 6000.
13. \$19,100.
14. \$12,445.13.

TAX FROM TABLE.

Page 246.

16. 1. \$13.65.
2. 35.34.
3. 54.
4. 147.83.
5. 165.96.
6. 304.50.
7. 480.75.
8. 1673.52.
9. 3015.
10. 8508.08.

INDIRECT TAXES.

Page 247.

3. \$750.
4. \$603.75.
5. \$130.625.
6. \$2480.
7. \$362.50.
8. $\$89.06\frac{1}{4}$.
9. $\$10.62\frac{1}{2}$.
10. \$294.
11. \$1547.

Page 248.

12. $158.40.
13. $94.03.
14. $1078.
15. $591.09.
16. $6629.12.
17. $76500.
18. 20%.
19. $.39 +; $391.66.
20. $825.
21. 15%.
22. $9000.
23. $1.40.

Page 249.

24. $125. 25. $800; $.50.

INTEREST.

Page 251.

1\. 1. $8. 2. $24. 3. $90. 4. $112. 5. $150. 6. $360. 7. $343. 8. $384. 9. $486. 10. $800.

2\. 1. $147.99. 2. $295.00. 3. $290.50. 4. $327.75. 5. $444.05. 6. $172.33⅓. 7. $1297.45.

Page 252.

8\. $1942.38. 9. $621.00. 10. $775.83⅓.

3\. 1. $.546. 2. $15.85. 3. $4.94. 4. $11.08. 5. $18.42. 6. $786. 7. $270.74. 8. $559.65. 9. $427.88. 10. $402.17.

4\. 1. $319.08. 2. $806.94. 3. $141.37. 4. $310.07. 5. $236.88. 6. $358.52. 7. $1198.16⅔. 8. $230.13. 9. $127.28. 10. $208.30.

1\. 1. $1.20. 2. $157.58.

Page 253.

3\. $33.81. 4. $1651.67. 5. $408.82.

2\. 1. $29.17. 2. $11.31. 3. $2.14. 4. $3.76. 5. $2.00. 6. $87.03. 7. $76.47. 8. $80.56.

Page 254.

1\. 1. $100.80. 2. $15.90. 3. $7.00. 4. $26.67. 5. $10.29. 6. $45. 7. $142.27. 8. $1077.50. 9. $2358.73. 10. $197.05.

2\. 1. $61.70. 2. $464.62. 3. $1317.23. 4. $843.40. 5. $180.80. 6. $8406.98. 7. $6008.92.

Page 255.

4\. 1. $150.10. 2. $1486.51. 3. $6.15. 4. $879.75. 5. 160.74. 6. 58.912. 7. $318.07.

Page 256.

1\. 1. $51.13. 2. $396.73. 3. $1196.81. 4. $122.45. 5. $196.45.

2\. 1. $764.07. 2. $966.93. 3. $1313.93. 4. $251.73. 5. $861.15.

Page 258.

1\. 1. \$750.
2\. \$336.
3\. \$242.42.
4\. \$7159.13.
5\. \$102.86.
6\. \$1388.89.
7\. \$6903.51.
8\. \$2099.72.
9\. \$7000.00.
10\. \$6222.22.
11\. \$40050.59.
12\. \$2980.87.
13\. \$551.11.
14\. \$2877.63.
15\. \$9664.54.
16\. \$293.43.
17\. \$665.66.
18\. \$40396.
19\. \$4027.69.
20\. \$773.56.

2\. 1. 6%.
2\. 5%.
3\. 5%.
4\. $5\frac{1}{2}$.

Page 259.

5\. 7%.
6\. $10\frac{127}{605}\%$.
7\. $3\frac{681}{1183}\%$.
8\. $5\frac{3}{4}\%$.
9\. $33\frac{1}{3}\%$; 20%; $16\frac{2}{3}\%$.
10\. $28\frac{4}{7}\%$; 25%; 10%.

3\. 1. 2 yr.
2\. $\frac{1}{2}$ yr.
3\. 2 yr.
4\. 4 yr. 8 mo. 14 da.
5\. 5 yr. 4 mo. 13 da.
6\. $16\frac{2}{3}$ yr.
7\. $12\frac{1}{2}$ yr.
8\. 20 yr.; $16\frac{2}{3}$ yr.; $14\frac{2}{7}$ yr.
9\. $22\frac{2}{9}$ yr.
10\. $33\frac{1}{3}$ yr.; $28\frac{4}{7}$ yr.; 25 yr.

1\. \$10 20.
2\. \$33,333$\frac{1}{3}$.
3\. \$1050.92.
4\. 2 yr. 10 mo.
5\. $4\frac{1}{2}\%$.

COMPOUND INTEREST.

Page 260.

1\. \$53.58.
2\. \$516.05.
3\. \$60.366.
4\. \$229.17.

Page 261.

5\. \$110.95.
6\. \$81.25.
7\. \$55.94.
8\. \$607.55.

ANNUAL INTEREST.

Page 262.

1\. 1. \$106.52.
2\. \$72.77.
3\. \$1045.80.
4\. \$277.33.
5\. \$202.56.
6\. \$360.58.

2\. 1. \$447.36.
2\. \$322.778.
3\. \$730.056.
4\. \$1080.44.
5\. \$2958.37.
6\. \$1246.68.

Page 266.

19\. 1. April 17, \$637.99.
2\. July 22, \$806.517.
3\. Feb. 6, \$404.49.
4\. March 21, \$1076.62.
5\. Dec. 9, \$635.41.
6\. July 17, \$975.228.
7\. May 30, \$2654.92.
8\. July 8, \$2856.77.
9\. Aug. 11, 1901, \$1116.90.
10\. Dec. 3, \$1995.16.

Page 268.

1\. \$335.99.
2\. \$122.20.
3\. \$71.95.
4\. \$441.40.
5\. \$261.68.
6\. \$85.23.
7\. \$19.08.

Page 269.

8\. \$1330.88.
9\. \$8039.47.
10\. \$5747.13.
11\. \$914.80.

Page 271.

1\. \$1316.76.
2\. \$287.72.
3\. 257.21.
4\. \$267.14.

Page 272.

5. $506.49.
6. $5204.67.
7. $2958.49.
8. 1860.32.
9. $8972.43.

Page 273.

10. $1295.71.

BANK DISCOUNT.

Page 274.

1. $1.46; $348.54.
2. $6; $394.00.
3. $6.30; $533.70.
4. $8.00; $592.00.
5. $50; $1950.00.
6. $1.11; $79.49.
7. $16.25; $4983.75.
8. $6.99; $773.01.
9. $10.63; $589.37.
10. $25.83; $974.17.

Page 275.

1. Apr. 21, 1898; 37 da.; $3.60; $496.40.
2. May 2, 1898; 31 da.; $4.13; $795.87.
3. Apr. 5, 1898; 85 da.; $5.67; $394.33.
4. Oct. 20, 1898; 119 da.; $10.78; $454.97.

Page 276.

5. June 6, 1898; 24 da.; $14.58; $3630.42.
6. Nov. 24, 1898; 54 da.; $21.897; $2411.10.

1. $808.08.
2. $999.495.
3. $3045.69.
4. $15,186.03.
5. $243.14.
6. $987.09.
7. 2666.66\frac{2}{3}$.
8. $418.00.

Page 277.

9. $1017.60.
10. $1250.00.

1. $404.04.
2. $786.55.
3. $1287.11 +.
4. $7653.06 +.
5. $1966.84.
6. $1441.62 +.
7. $1025.64 +.
8. $154 50.

TRUE DISCOUNT.

Page 278.

1. $377.36; $22.64.
2. $183.48; $16 52.
3. $168.09; $11.91.
4. $508.47; $91.53.
5. $297.43; $52.57.
6. $1483.31; $16.69.
7. $1760.56; 239.44.
8. $403.099; 84.65.
9. $371.806; 50.194.
10. $416.848; 62.527.

Page 279.

1. $166.09; 33.91.
2. $45.
3. $1.355.
4. $21.56.
5. $1.25.
6. $40.69.
7. $1596.815.
8. $1408.82.
9. $750.
10. $467.29.

1. 25$\frac{3}{5}$%.
2. 66$\frac{2}{3}$%.

Page 280.

3. $327.24; $654.48.
4. 9$\frac{1}{2}$%.
5. 14$\frac{2}{7}$ yr.
6. $2004.51.
7. Dec. 12; 2 mo. 16 da.; $22.93; $1952.07.
8. $520.25.
9. $497.92; $320.86.

10. \$6262.136.
11. \$2465.625.
12. \$13.66.
13. \$1418.33.
14. \$813.69.
15. \$678.37.

Page 281.

16. \$32.17.
17. \$5300.
18. $30\frac{37}{200}$ bbls.
19. \$75.76.
20. \$1886.50.
21. 28%.
22. \$92.38.

EXCHANGE.

Page 284.

4. \$594.50.
5. \$1818.
6. \$2992.50.
7. \$3037.50.
8. \$6480.
9. \$1395.33.
10. \$1230.79.
11. \$5244.37.

Page 285.

12. \$882.
13. \$8782.81.
16. \$6818.95.
17. \$4600.
18. \$2041.65.
19. \$3495.62.
20. \$325.05.
21. \$2819.44.

Page 286.

22. \$1200.00.
23. \$745.15.

RULE OF THREE.

Page 288.

1. 24.
2. 54.
3. 32.
4. 2.
5. 4.
6. 54.
7. 26.4.
8. 12.
9. 3.6.
10. 8.
11. $\frac{3}{5}$.
12. $13\frac{1}{3}$.
13. $\frac{10}{27}$.
14. 2.75.
15. $1.6\frac{13}{17}$.
16. .0975.

Page 289.

1. 42.
2. 4.
3. $53\frac{1}{8}$.
4. 6.
5. $12\frac{7}{19}$.

Page 290.

6. .40.
7. 19.6.
8. 24.
9. 1080.
10. 1175.23.
11. 32.
12. $4\frac{8}{9}$.
13. $67\frac{1}{12}$.
14. $162\frac{45}{112}$.
15. 900.
16. 2.45.
17. $6\frac{2}{3}$.
18. .80 +.
19. 33 hr. 20 min.
20. $40\frac{1}{2}$.

Page 291.

21. $2\frac{19}{28}$.
22. 50.
23. 13.
24. 1095.
25. 42.
26. $6\frac{3}{8}$.
27. $26\frac{169}{232}$.
28. £227 12s. 1d.
29. \$57.33.
30. 7.
31. 2560.
32. $10\frac{1}{3}$.
33. 2 yr. 9 mo. 10 da.

Page 292.

34. \$2.46 +.
35. 12.
36. 10.
37. $74.66\frac{2}{3}$.
38. 900.
39. 216.
40. $25\frac{11}{13}$.

COMPOUND PROPORTION.

Page 293.

1. $3\frac{1}{3}$.
2. 4.
3. 1188.
4. \$131.20.
5. 473.
6. \$3.84.
7. 6.
8. 840.

Page 294.

9. 30.
10. 32.
11. \$600.
12. 125.
13. 63.
14. \$800.
15. $1828\frac{4}{7}$.
16. $8\frac{1}{3}$ ten-ounce loaves.
17. $2\frac{1}{4}$.
18. 1430.

Page 295.

19. \$124.75.
20. 6027.
21. $10\frac{1}{2}$ hr.
22. $74\frac{1}{6}$ da.
23. $9\frac{18}{23}$ yr.

CAUSE AND EFFECT.

Page 296.

1. 10.
2. \$10118.77.
3. 21.
4. \$100.
5. 6.
6. 10.

PROPORTIONAL PARTS.

Page 297.

1. 25, 35.
2. 264, 288, 312, 336.
3. 360, 240, 180.
4. 60, 45, 15.
5. $3149.17\frac{11}{17}$, $4723.76\frac{8}{17}$, $5511.05\frac{15}{17}$.
6. .50, 1.00, 2.50, 5.00.
7. 3000, 6000, 9000.

Page 298.

8. 15, 25, 20.
9. 100, 20, $13\frac{1}{3}$.

PARTNERSHIP.

Page 299.

1. \$78, \$104.
2. \$84; \$72.
3. \$67.50, \$72, \$81.
4. \$179.048, \$537.142, \$223.810.
5. \$2175, \$2030, \$2755, \$2320.
6. \$750, \$500, \$250.
7. \$60, \$72.

Page 300.

8. \$14, \$28.
9. $\$266\frac{2}{3}$, $\$333\frac{1}{3}$, \$400.
10. \$862.50, \$575.00, \$862.50.
11. \$1080, \$1600, \$1820.
12. $\frac{5}{8}$.
13. \$1.50, \$3.00.
14. \$48, \$70.
15. $\$6039.04\frac{1624}{1819}$; $\$2228.00\frac{1300}{1819}$.
16. \$481.25; \$1196.25.
17. \$710, \$352.80.
18. \$1000, \$1500, \$2000.

Page 301.

19. \$560, \$2240, \$2800.
20. \$273 365 +, \$476.635 +.
21. 45 lb., $4\frac{1}{2}$ lb., $\frac{1}{2}$ lb.
22. \$332.50, \$525.
23. \$6000, \$14000.
24. \$165, \$210, \$225.
25. 700, $1866\frac{2}{3}$, $933\frac{1}{3}$.
26. \$101.41, \$98.59.
27. $\$1333.33\frac{1}{3}$, 2000, $2666.66\frac{2}{3}$.

Page 302.

28. \$1000, \$1500.

AVERAGES.

1. $426.19\frac{2}{3}$.
2. 127.
3. 20.

Page 303.

4. $1\frac{1}{2}$ oz.
5. 4 mo.

AVERAGE OF PAYMENTS.

1. $6\frac{6}{13}$.
2. $7\frac{1}{24}$.

Page 304.

3. 5 mo.
4. $2\frac{1}{2}$ mo.
5. $3\frac{4}{9}$.
6. May 26.
7. 10 mo.

8. 1 mo. 12 da.
9. $4\frac{1}{6}$ mo.
10. $6\frac{71}{240}$ mo.
11. \$545.45; 1 yr. 8 mo.
12. Oct. 23d.

Page 305.

13. Aug. 20.
14. June 21.
15. Aug. 22.
16. Aug. 21.
17. Aug. 19.
18. Feb. 3.
19. Apr. 22.

Page 306.

20. July 11; Dec. 26.
21. $3\frac{1}{4}$ mo.
22. 9 mo.
23. 18 mo.
24. 10 mo.
25. 9 mo.
26. $50\frac{2}{5}$.
27. 50 da.

Page 307.

28. April 15.
29. $3\frac{1}{2}$ mo.
30. 8 mo.

INVOLUTION.

Page 308.

1. 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.
2. $1^2 = 1$, $3^2 = 9$, $5^2 = 25$, $7^2 = 49$, $9^2 = 81$, $10^2 = 100$, $15^2 = 225$, $25^2 = 625$.
3. $1^3 = 1$, $2^3 = 8$, $3^2 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$, $0^3 = 0$.
4. $x = 1$, $x = 4$, $x = 9$, $x = 16$, $x = 25$, $x = 36$, $x = 27$, $x = 64$, $x = 256$, $x = 25$, $x = 125$, $x = 216$.
5. $x = 400$, $x = 900$, $x = 1600$, $x = 2500$, $x = 3600$, $x = 4900$, $x = 6400$, $x = 8100$, $x = 10{,}000$.
6. $x = 1000$, $x = 8000$, $x = 27{,}000$, $x = 64{,}000$, $x = 125{,}000$, $x = 216{,}000$, $x = 343{,}000$, $x = 512{,}000$.
7. $(\frac{1}{3})^2 = \frac{1}{9}$, $(\frac{1}{4})^2 = \frac{1}{16}$, $(\frac{1}{5})^2 = \frac{1}{25}$, $(\frac{1}{6})^2 = \frac{1}{36}$, $(\frac{2}{7})^2 = \frac{4}{49}$, $(\frac{3}{8})^2 = \frac{9}{64}$, $(\frac{4}{9})^2 = \frac{16}{81}$, $(\frac{5}{10}) = \frac{25}{100}$.

Page 309.

8. $(\frac{1}{2})^3 = \frac{1}{8}$, $(\frac{2}{3})^3 = \frac{8}{27}$, $(\frac{3}{4})^3 = \frac{27}{64}$, $(\frac{5}{6})^3 = \frac{125}{216}$.
9. .01, .04, .09, .16, .25, .36, .49, .64, .81.
10. .001, .008, .027, etc.
11. $(\frac{3}{4})^2 = \frac{9}{16}$, $(.4)^2 = .16$, $(\frac{2}{9})^3 = \frac{8}{729}$, $(\frac{1}{6})^3 = \frac{1}{216}$, $(.1)^4 = .0001$, $(.02)^3 = .000008$.

1. $x = 625$.
2. $x = 1225$.
3. $x = 7744$.
4. $x = 10{,}201$.
5. $x = 169$.
6. $x = 729$.
7. $x = 4913$.
8. $x = 1000$.
9. $x = 9261$.
10. $x = \frac{8}{27}$.
11. $(.001)^3 = .000000001$.
12. $15^3 = .003375$.
13. $.04^3 = .000064$.
14. .001953125.
15. .000729.
16. .00000625
17. $.005 = .000000125$.
18. 4.2025.
19. $(25\frac{1}{3})^2 = 641\frac{7}{9}$.

20. $(4.500\frac{1}{7})^2 = 20.251285\frac{36}{49}$.
21. 468.884869 +.

Page 312.

1. 12.	7. 36.	13. 63.
2. 14.	8. 35.	14. 72.
3. 16.	9. 42.	15. 80.
4. 18.	10. 44.	16. 91.
5. 24.	11. 51.	
6. 26.	12. 54.	

Page 315.

1. 10.	11. 53.	21. 821.
2. 100.	12. 63.	22. 886.
3. 25.	13. 67.	23. 972.
4. 37.	14. 84.	24. 999.
5. 52.	15. 96.	25. 2424.
6. 83.	16. 127.	26. 2504.
7. 125.	17. 266.	27. 3546.
8. 376.	18. 344.	28. 5555.
9. 401.	19. 512.	29. 6325.
10. 10,000.	20. 607.	30. 6453.

SQUARE ROOT.

Page 316.

1. 1. $\frac{1}{3}$.	9. $\frac{48}{72}$.
2. $\frac{2}{4}$.	10. $\frac{52}{65}$.
3. $\frac{3}{10}$.	11. $\frac{235}{886}$.
4. $\frac{10}{100}$.	12. $\frac{884}{989}$.
5. $\frac{9}{13}$.	13. $\frac{948}{967}$.
6. $\frac{11}{21}$.	14. $\frac{1000}{10000}$.
7. $\frac{8}{15}$.	15. $\frac{1001}{1111}$.
8. $\frac{15}{27}$.	16. $\frac{2222}{3333}$.
2. 1. .3.	9. .306.
2. .9486 +.	10. .315.
3. .12.	11. .063.
4. .3794.	12. 1.296.
5. .1.	13. 14.31.
6. .3513 +.	14. .0099.
7. .874.	15. .0101.
8. .5555.	

3. 1. .9354 +.	11. .6123 +.
2. .9428 +.	12. .9684 +.
3. .2886 +.	13. 1.5.
4. $\frac{35}{67}$.	14. .5059 +.
5. $\frac{17}{52}$.	15. 1.3462 +.
6. .8291 +.	16. .8164 +.
7. .6928 +.	17. .488 nearly.
8. .70716 +.	18. 8.0702 nearly
9. .86602 +.	19. 10.1.
10. .7905 +.	20. .66 +.

SQUARES.

Page 317.

1. 35.	8. \$5120.
2. 45.	9. \$187.20.
3. 80 sq. rd.	10. 100.
4. 3733.523 ft.	11. 62 in.
5. 23 rd.	12. The rectangular field.
6. 12.649 + rd.	\$18.34⅔.
7. 85.	

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2. h. = 3.605 +.
p. = 3.316 +.
b. = 4.358 +.
p. = 4.8 nearly.
h. = 19.104.

3. 40.	5. 122.4 +.
4. 28.284 +.	6. 30 ft.

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7. 44.9 very nearly.
8. 56.69 +.
10. b = 6, p = 8.

2. 9.921 +.	4. 99215.67 +.
3. 242.63 +.	5. 21.33 +.

CUBE ROOT.

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$\sqrt[3]{27} = 3$, for $3 \times 3 \times 3 = 27$.

$\sqrt[3]{125} = 5$, for $5 \times 5 \times 5 = 125$.

$\sqrt[3]{343} = 7$, for $7 \times 7 \times 7 = 343$.

$\sqrt[3]{512} = 8$, for $8 \times 8 \times 8 = 512$.

$\sqrt[3]{729} = 9$, for $9 \times 9 \times 9 = 729$.

$\sqrt[3]{4096} = 16$, for $2^3 \times 2^3 \times 2^3 = 4096$.

$\sqrt[3]{42875} = 35$, for $5^3 \times 7^3 = 42875$.

$\sqrt[3]{166375} = 55$, for $5^3 \times 11^3 = 166375$.

$\sqrt[3]{185193} = 57$, for $3^3 \times 19^3 = 185193$.

Page 324.

1. 1. 85.
2. 42.
3. 25.
4. 32.
5. 47.
6. 125.
7. 177.
8. 126.
9. 536.
10. 345.
11. 23.
12. 32.
13. 43
14. 39.
15. 84.
16. 123.
17. 325.
18. 526.
19. 642.
20. 1234.
21. 2345.

Page 325.

1. $\frac{2}{3}$. 2. $\frac{1}{6}$. 3. $\frac{3}{7}$. 4. $\frac{4}{8}$. 5. $\frac{5}{9}$. 6. $\frac{12}{36}$. 7. $\frac{27}{55}$. 8. $\frac{23}{32}$. 9. $\frac{34}{36}$. 10. $\frac{41}{53}$. 11. $\frac{65}{76}$. 12. $\frac{82}{85}$.

1. .2.
2. .43 +.
3. .928 +.
4. .5.
5. .627 +.
6. 1.3.
7. 2.1.
8. 5.7.
9. .464 +.
10. 1.816 +.
11. 1.91 +.
12. 3.27.
13. 3.45.
14. .01.
15. .00464 +.

1. .908 +. 2. .88 +. 3. .61 +. 4. 1.07 +. 5. .87 +. 6. .94 +. 7. .96 +. 8. .94 +. 9. .93 +. 10. .501 +.

VOLUME.

2. 72.
3. 6084 sq. in.
4. $3\frac{3}{8}$ sq. ft.

Page 326.

5. 12.9 + in.
6. Length, 95.11 in.; width and depth, 47.55 in.
7. $13.48.
8. 28.4 + ft.

SIMILAR FIGURES.

Page 327.

2. R. $= \sqrt{50}$.
3. 2 : 1.
4. D. = 4.
5. L. = 65.6; W. = 41.
6. R. = 17.58.
7. $1\frac{1}{4}$ hr.
8. 16 bbl.
9. $5\frac{1}{3}$ gal.
10. $4800.

Page 328.

2. 3.
3. 27 times.
4. $13500.
5. 1404928.
6. 1 : 64.
8. 2.16 ft.
9. 23.49 ft.

Page 332.

1. 5625 sq. yd.
2. 600 sq. rd.
3. 43.30 sq. ch.
4. $3\frac{1}{8}$ sq. ft.
5. 370.75 sq. ft.

Page 333.

1. $25\frac{3}{5}$ A.
2. $8\frac{3}{4}$ sq. ft.
3. $\frac{8}{9}$ sq. ft.
4. 8 A.

1. 2528 sq. rd. 2. 10 rd.

Page 334.

1. $1469\frac{259}{1089}$ sq. rd. 2. $20\frac{280}{363}$ sq. rd.

Page 335.

1. 288 sq. in. 2. 322.575 sq. ft.

Page 336.

1. 65.97 ft.
2. 10.5 + yd.
3. 100.5312 yd.
4. 25.78 sq. in.
5. $14\frac{1}{8}$ ft.

Page 337.

6. 159.315 sq. rd.
7. 43.9824 ft.
8. .3183 ft.
9. 153.9384 sq. yd.
10. 1105.84 + sq. ft.
11. 12 sq. ft.
12. 5500 sq. ft.
13. 7.854 A.
14. 22,500 sq. yd.
15. 13,950 sq. yd.
16. 144 sq. ft.
17. 145.309 sq. ch., or 14.5309 A.
18. 100.58 ft.
19. 141.372 rd.

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20. 203.716 A.
21. 312 A.
22. 491.189 rd.
23. 3883.0176 sq. ft.
24. 50.2656 sq. ft.
25. 4.8744 sq. ft.
26. 5.2 very nearly.

Page 339.

1. 1400 sq. in.
2. 1178.1 sq. in.
3. 288 sq. ft.
4. 98.9604 sq. ft.
5. 46.50 sq. ft.

Page 340.

6. 218.77 sq. in.
7. 4417.875 cu. in.; 144 cu. ft.
   63,6174 cu. ft.; 15.75 cu. ft.;
   105.84 cu. in.
8. 144 in. = 12 ft.
9. $4.71\frac{7}{12}$ ft.
10. 139.0608 cu. in.

Page 341.

1. 300 sq. ft.
2. 17.6715 sq. ft.
3. 42.07 bu.
4. .24 gal.
5. 659.2512 cu. in.

Page 342.

1. $112\frac{1}{2}$ sq. ft.
2. 1510.95 sq. in.
3. 4052.664 cu. in.
4. 81.8 cu. ft.
5. $167.80.

1. 1017.8784 sq. in.; 3053.6352 cu. in.
2. 452,3904 sq. in.

Page 343.

3. 65.45.
4. 1767.15 cu. in.; 706.86 sq. in.
5. 339.2928 cu. in.

1. $\sqrt{2}$.
2. $3.1416 \times \sqrt{2}$.
3. $5 \times .7071$.
4. 12.50; 19.64.

Page 344.

1. 1092 cu. ft.
2. 636.174 cu. ft.
3. 314.16 sq. ft.; 392.70 sq. ft.
4. 1809.5616 sq. in.
5. 18.13 gal.
6. 144 sq. ft.
7. 540 sq. ft.

Page 345.

8. 157.08 sq. ft.
9. 124.704 cu. ft.; 1226.25 cu. ft.; 227.766 cu. ft.
10. 408.408 sq. ft.; 607.133 + cu. ft.
11. 890.22 sq. in.
12. 381.7044 cu. in.
13. $6.14.
14. 1385.4456 cu. in.
15. 50.2656 cu. in.
16. 1 to 3; yes.
17. 7.071 ft.
18. 3.175425 in.
19. 13.85 in.
20. 50 sq. rd.

GENERAL REVIEW.

Page 346.

2. $500.44.
3. 15.
4. $\frac{15}{17}$.
5. $\frac{1}{2}$; .5.
7. $3.50.
8. $166\frac{2}{3}$ ft.

Page 347.

9. $696.89.
10. 50 mi.
12. 77; 17017.
13. $123.48.
14. $23\frac{14}{15}$; $23.93\frac{1}{3}$.
15. $\frac{17}{24}$.
16. $4.50.
17. $7.25.
18. $2.625.
19. $14\frac{2}{5}$ A.

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22. $8\frac{5}{9}$.
23. $\frac{5}{224}$ nearly.
24. 12.9999984.
25. $30.00, or $31.50.
27. $14\frac{2}{5}$ cts.
28. $340.
29. $263.18\frac{23}{64}$.
30. $352.84.
31. $930.23.
32. $359.82.
33. $502.42.
34. 80.
35. 2001.05.

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36. $100; $12\frac{1}{2}\%$.
37. 2520.
38. $$15\frac{15}{19}$; $14\frac{4}{19}$.
39. $22.50.
40. G. C. D. = 2; L. C. Dd. = $2 \times 3 \times 17 \times 23 \times 19 \times 613$.
41. $19.70.
42. $922.35.
43. $48.96.
44. 8.3776 ft.
45. 50 A.

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46. 192.
50. $66\frac{2}{3}\%$.
51. 20%.
52. $23\frac{1}{20}\%$.
53. 1039.34.
54. $21.00.
56. 58.308.
57. One hundred twenty-five hundred-thousandths.
58. 2, 2, 3, 13, 19; 4, 6, 26, 38, 39, 57, 247, 12, 52, 76, 78, 114, 741, 494, 988, 1482.

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62. $281.53.
63. $67.20.
64. 53.6862 +.
65. 2.93.
66. $28.80.
68. $31\frac{73}{77}$.
70. 3.509 ft.
71. 630; 5.
72. 420.17 times.

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73. 25.
74. $4\frac{4}{5}\%$; $288.
75. $344.53.
76. 48 ft.
77. 20091.8081.
78. $260.40; $13.02.
79. $90.32; $116.67; $143.01.
80. $55000.
81. $170\frac{2}{3}$ yd.
82. 56.56 + rd.
83. 150.
84. 6 men.
85. $8.6625.

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86. $554.40.
87. $984.96.
88. $200; $240; $280.
89. $2.00.
90. $\frac{17}{24}$.
91. $391.27.
92. $40.00.
93. 8.
94. $252.53.
95. $36.
96. $7\frac{109}{300}$.
97. 3.21 sq. ft.

Page 354.

98. $.005 on $1.
99. 5.12 ft.
100. $3865.
101. 34.64 in.
102. $1260.
103. $410.84.
104. $27.00.
105. $28.80.
106. $152.29.
107. 4.
108. $400; $12.00.
109. $3.40.
110. 680 sq. ft.; $10.88.

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111. 8 men.
112. 25%.
113. $1402.
114. $633.588.
115. 173.57 bu.; 138.85 bu.
116. 56 min. 36 sec. past 10 A.M.
117. 10.7 ft.
118. 43.23 in.
119. 4 : 3.
120. $82\frac{1}{2}$ ft.
121. $234\frac{3}{8}$.
122. $21\frac{19}{36}\%$.

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123. $1006.54.
124. 10 min. past 5 P.M.; 50 min. 57.28 sec. past 8 A.M.
125. 2 hr. 44 min.
126. 8.2 ft., very nearly.
128. 1 mi. 9 yd. 1 ft. 3.23 in.
129. $133\frac{1}{3}$ times.
130. $757.59.
131. $11.93.
132. 19.6 ft., very nearly.
133. 12.15.
134. 50.2656; 33.5104.
135. 24 sq. ft.; 7.88 cu. ft.

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136. 32760.
137. $2\frac{14}{23}\%$.
139. 6 children.
140. $170\frac{2}{3}$ min.
141. 5 ft.
142. $100.
143. $\sqrt[3]{4} : 1$.
144. $68\frac{1}{3}$ ft.
145. $200.

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146. 8.2 ft., nearly.
147. \$0.15$\frac{5}{8}$.
148. 25.47 ft.
149. 165$\frac{111}{125}$ lb.
150. 2069.69 sq. rd.
151. 28.8 cu. ft.
152. 12 sq. ft. 88.56 sq. in.
153. 507 sq. yd.
154. 87$\frac{3}{8}$.
156. 87° 33′ W.
157. 268$\frac{5}{6}$ sec. after B

APPENDIX.

The following subjects are presented in an appendix, not because they are unimportant, but for the reason that thus placed they may more distinctly constitute a supplementary course which the pupil may elect to study or not, as circumstances may incline him.

1. Duodecimals.
2. Metric System.
3. Specific Gravity.
4. Foreign Exchange.
5. Arithmetical Progression.
6. Geometrical Progression.
7. Compound Interest (Table).
8. Annuities.
9. Circulating Decimals.
10. G. C. D. and L. C. Dd. of Fractions.
11. Thermometer.
12. The Clock.
13. Work.
14. Averaging of Accounts.
15. Miscellaneous Exercises and Problems.

DUODECIMALS.

Duodecimals (Latin, *duodecim*, *twelve*) are fractions of which 12 of any order equal one of the next higher order.

The unit is the *foot*, which is divided into 12 equal parts called *primes* ($'$), each prime ($'$) being divided into 12 equal parts called *seconds* ($''$), each second ($''$) in the same manner into 12 *thirds* ($'''$), and each third ($'''$) into 12 *fourths* ($''''$).

Table.

12 fourths ($''''$) = 1 third ($'''$).
12 thirds ($'''$) = 1 second ($''$).
12 seconds ($''$) = 1 prime ($'$).
12 primes ($'$) = 1 ft.

Hence $1' = \frac{1}{12}$ of a ft.

$1'' = \frac{1}{12}$ of $1' = \frac{1}{12}$ of $\frac{1}{12}$ ft. $= \frac{1}{144}$ of a ft.

$1''' = \frac{1}{12}$ of $1'' = \frac{1}{12}$ of $\frac{1}{144}$ of 1 ft. $= \frac{1}{1728}$ of a ft.

$1'''' = \frac{1}{12}$ of $1''' = \frac{1}{12}$ of $\frac{1}{1728}$ of 1 ft. $= \frac{1}{20736}$ of a ft.

Duodecimals are employed principally by artisans in the measurement of lines, surfaces, and solids.

The adding and subtracting of duodecimals differ in no respect from the adding and subtracting of other compound numbers.

EXERCISES.

1. What is the sum of 12 ft. 7′ 10″, 17 ft. 8′ 9″, and 35 ft. 5′ 8″?
2. Add 7 ft. 1′ 3″ 6‴, 1 ft. 3″ 6‴ 1⁗, 7 ft. 8′ 7″ 9⁗, 8 ft. 10′ 6″.
3. Add 123 ft. 5′ 6″ 8‴, 217 ft. 9′ 10″ 8‴, and 352 ft. 7′ 9″ 4‴.
4. Add 6 ft. 4′ 2″, 15 ft. 4″ 3‴, 8′ 9″ 4‴ 3⁗, and 7⁗.
5. What is the sum of 186 ft. 5′ 9″ 4‴ 5⁗, 218 ft. 7′ 10″ 10‴ 8⁗, and 235 ft. 6′ 9″ 7‴ 10⁗.
6. From 9 ft. 1′ 3″ take 2 ft. 6′ 1″ 3‴.
7. From 18 ft. 3″ 9⁗ take 10 ft. 2′ 2″ 6‴.
8. From 275 ft. 5′ 6″ 8‴ take 127 ft. 8′ 4″ 5‴.
9. From 225 ft. 0′ 2″ 5‴ 7⁗ subtract 117 ft. 5′ 9″ 8‴ 5⁗.
10. From a board 15 ft. 7′ 6″ in length, 3 ft. 8′ 11″ were sawed off. What was the length of the piece left?

MULTIPLICATION.

What is the product of 8 ft. 5′ 4″ and 5′ 3″?

Process.

8 ft.	5′	4″	
	5 ft. 3′		
2 ft.	1′	4″	0‴
42 ft.	2′	8″	
44 ft.	4′	0″	0‴

Explanation.

$5' = \frac{5}{12}$, $4'' = \frac{4}{144}$, $3' = \frac{3}{12}$.

$4'' \times 3' = \frac{4}{144} \times \frac{3}{12} = \frac{12}{1728} = 12''' = 1''\ 0'''$; $5' \times 3' = \frac{5}{12} \times \frac{3}{12} = \frac{15}{144} = 15''$; $15'' + 1'' = 16'' = 1'\ 4''$.

$8 \times 3' = 8 \times \frac{3}{12} = \frac{24}{12} = 24'$; $24' + 1' = 25' = 2$ ft. $1'$.

$4'' \times 5 = \frac{4}{144} \times 5 = \frac{20}{144} = 20'' = 1'\ 8''$.

$5' \times 5 = \frac{5}{12} \times 5 = \frac{25}{12} = 25'$; $25' + 1' = 26' = 2$ ft. $2'$.

$8 \times 5 = 40$; $40 + 2 = 42$ ft.

Adding the two partial products, we have 44 ft. 4′ 0″ 0‴.

NOTE.—′ and ″, etc., are called indices. Since a product has as many indices as both its factors, by the use of the indices the multiplication may be performed, in practice, without the fractions.

PROBLEMS.

1. A board is 7 ft. 5′ 8″ in length and 2 ft. 4′ 7″ in breadth. What is its area?

2. How many cubic feet in a wall 80 ft. 9′ long, 3 ft. 4′ high, and 1 ft. 8 in. wide?

3. Find the surface to be plastered in a room 18 ft. 3′ long, 15 ft. 2′ wide, and 10 ft. 3′ high, allowing 8′ for the width of the baseboard.

4. A pile of wood is 255 ft. long, 4 ft. 6′ high, and 8 ft. 4′ wide. Find the number of cords in it.

5. What are the solid contents of a block of stone 3 ft. 2′ long, 2 ft. 3′ 6″ wide, and 4 ft. 2′ high?

6. What would it cost to plaster a wall 32 ft. 8′ long and 9 ft. high at 17 cents per square yard?

7. How many loads of earth must be taken out in digging a cellar that is to be 45 ft. 6 in. long, 25 ft. wide, and 10 ft. 9 in. deep?

DIVISION.

Divide 14 ft. 9′ 8″ by 3 ft. 5′.

Process.

3 ft. 5′) 14 ft. 9′ 8″ (4 ft. 4′
13 ft. 8′
1 ft. 1′ 8″
1 ft. 1′ 8″

Explanation.

14 ÷ 3 = 4 + ; (3 ft. 5′) × 4 = 13 ft. 8′; subtracting we have 1 ft. 1′ 8″.

1 ft. 1′ = 13′; 13′ ÷ 3 = 4′ + ; (3 ft. 5′) × 4′ = 1 ft. 1′ 8″; subtracting we have 0. Hence 4 ft. 4′ is the exact quotient.

PROBLEMS.

1. A floor contains 216 sq. ft. 5′ 10″ 6‴, and is 10 ft. 6′ wide. How long is it?

2. The square contents of a quilt are 14 ft. 6 in.; it is to be lined with stuff 2 ft. 7 in. wide. Find the length of the lining.

3. A stick of timber is 3 ft. 2 in. wide, and 2 ft. 9 in. thick; it contains 176 cu. ft. 4′ 1″ 6‴. Find its length.

4. Find the cost of carpeting a room 15 ft. long, 12 ft. wide with carpet 27′ wide, at 75 cts. a yard.

5. A plank is 5′ thick, 20 ft. 2′ long, and contains 14 cu. ft. 8′′′. How wide is it?

THE METRIC SYSTEM.

The **Metric System** is a decimal system of weights and measures; the fundamental unit is the *metre*. The **Standard Metre** is a bar of very hard metal, whose length, as determined by French scientists, is $\frac{1}{10000000}$ of a quadrant of a meridian, and is equal to about 39.37 inches.

This system was first adopted in France in 1795, and is now used also in Germany, Spain, Portugal, Belgium, and Greece, as well as in Mexico, Brazil, and most of the other States of South America. Its use is allowed by law in Great Britain and in the United States, but as yet it finds small favor, except among scientists.

The principal units of the Metric System are:

1. The **Metre** (^{m}), for lengths.
2. The **Are** (^{a}), or square dekametre (^{qdkm}), for surfaces.
3. The **Stere** (st), or cubic metre (^{cbm}), for volumes.
4. The **Litre** (^{l}), or cubic decimetre* (^{cbdm}), for small volumes.
5. The **Gramme** (^{g}), for weights; equal to the weight of one cubic centimetre* of water at 4° C. = 39.2° F.

The units are all divided and multiplied decimally. Subdivisions are indicated by Latin prefixes; multiples, by Greek prefixes.

Latin.
- The prefix *milli* means $\frac{1}{1000} = .001$.
- The prefix *centi* means $\frac{1}{100} = .01$.
- The prefix *deci* means $\frac{1}{10} = .1$.

Greek.
- The prefix *deka* means 10.
- The prefix *hekto* means 100.
- The prefix *kilo* means 1000.
- The prefix *myria* means 10,000.

Metric numbers are written decimally, with the point placed immediately after the *unit*, as 10.15^{m}, which may be read "10 and $\frac{15}{100}$ metres," or, "10 metres, 1 decimetre, 5 centimetres."

---

* 1 dekametre = 10 metres; 1 decimetre = $\frac{1}{10}$ metre; 1 centimetre = $\frac{1}{100}$ metre.

LINEAR MEASURE.

Table.

10 millimetres (^{mm})	$=$ 1 centimetre (^{cm}).
10 centimetres	$=$ 1 decimetre (^{dm}).
10 decimetres	$=$ 1 metre (^{m}).
10 metres	$=$ 1 dekametre (^{dkm}).
10 dekametres	$=$ 1 hektometre (^{hm}).
10 hektometres	$=$ 1 kilometre (^{km}).
10 kilometres	$=$ 1 myriametre (^{Mm}).

The *metre* is very little more than 39.37 in. The *kilometre* is a little *less* than $\frac{5}{8}$ of a mile.

Reduction from one denomination of the table to another is made by *simply moving the decimal point to the right or left:* to the *right* for *lower* denominations; to the *left* for *higher* denominations. Thus it will be seen that operations with metric numbers are similar to those with decimals.

Illustrations.

$3568^{m} = 35.68^{hm} = 3.568^{km}$; $3.568^{km} = 356.8^{dkm} = 35{,}680^{dm} = 356{,}800^{cm}$.

SURFACE MEASURE.

Table.

100 sq. millimetres (^{qmm})	$=$ 1 sq. centimetre (^{qcm}).
100 sq. centimetres	$=$ 1 sq. decimetre (^{qdm}).
100 sq. decimetres	$=$ 1 sq. metre (^{qm}).
100 sq. metres	$=$ 1 sq. dekametre (^{qdkm}).
100 sq. dekametres	$=$ 1 sq. hectometre (^{qhm}).
100 sq. hectometres	$=$ 1 sq. kilometre (^{qkm}).

In the measurement of *land surface*, the

Sq. metre	$=$ 1 centare (^{ca}).
Sq. dekametre (^{qdkm})	$=$ 1 are (^{a}).
Sq. hectometre (^{qhm})	$=$ 1 hectare (^{ha}).

The *are* equals about $10\frac{3}{4}$ sq. ft. The *hectare* equals about $2\frac{1}{2}$ acres.

Illustrations.

1. Change 5^{qhm}, 3^{qdkm}, 9^{qm}, to sq. metres.

$$\begin{array}{lr} 5^{qhm} = & 50{,}000^{qm} \\ 3^{qdkm} = & 300^{qm} \\ 9^{qm} = & \underline{9^{qm}} \\ & 50{,}309^{qm} \end{array}$$

2. How many ares in 158^{a} and 3561^{ca}?

$$\begin{array}{lr} 158^{a} = & 158^{a} \\ 3561^{ca} = & \underline{35.61^{a}} \\ & 193.61^{a} \end{array}$$

MEASURE OF VOLUME.

Table.

1000 cu millimetres (^{cmm}) = 1 cu. centimetre (^{ccm}).
1000 cu. centimetres = 1 cu. decimetre (^{cdm}).
1000 cu. decimetres = 1 cu. metre (^{cbm}) = 1 stere.

The *stere* is used in measuring wood, etc.

10 decisteres (^{ds}) = 1 stere (st).
10 steres = 1 dekastere (^{dkst}).

Illustrations.

1. Reduce $57{,}000^{ccm}$ to cu. decimetres.

From cu. centimetres to cu. decimetres there is but one step; hence $57{,}000^{ccm} = 57\ ^{cdm}$.

2. How many steres of wood in a pile 8^{m} long, 1.6^{m} high, and 1^{m} wide?

$$\text{Volume} = 1.6^{m} \times 8^{m} \times 1^{m} = 12.8 \text{ steres.}$$

CAPACITY.

Table.

10 millilitres (^{ml}) = 1 centilitre (^{cl}).
10 centilitres = 1 decilitre (^{dl}).
10 decilitres = 1 litre (^{l}).
10 litres = 1 dekalitre (^{dkl}).
10 dekalitres = 1 hektolitre (^{hl}).
10 hektolitres = 1 kilolitre (^{kl}) = 1 cu. metre.

The *litre* is used in measuring liquids, grain, etc.
The *hektolitre* is used in measuring large quantities.
One *millilitre* = 1 cu. centimetre.

Illustrations.

1. How many hektolitres of air in a room 6^m long, 5^m wide, and 3^m high?

$$\text{Volume} = 6^m \times 5^m \times 3^m = 90^{cbm} = 90^{kl} = 900^{hl}.$$

2. If the value of a hektolitre of grain is \$1.80, what is the value of a dekalitre?

$1^{hl} = 10^{dkl}$; therefore, the value of a dekalitre $= \frac{1}{10}$ of \$1.80 = \$0.18.

WEIGHT.

Table.

10 milligrammes (^{mg})	= 1 centigramme (^{cg}).
10 centigrammes	= 1 decigramme (^{dg}).
10 decigrammes	= 1 gramme (^{g}) = wt. of 1^{ccm} of water.
10 grammes	= 1 dekagramme (^{dkg}).
10 dekagrammes	= 1 hektogramme (^{hg}).
10 hektogrammes	= 1 kilogramme (^{kg}) = 1^{cdm} of water.
10 kilogrammes	= 1 myriagramme.
10 myriagrammes	= 1 quintal.
10 quintals = 1000^{kg}	= 1 tonneau, or ton (^{t}) = 1^{cbm} of water.

1 cu. decimetre = 1 litre; 1 litre of water = 1 kilogramme in wt.

Illustration.

Find the cost of 12^{hg}, 6^{dkg}, and 3^{dg} of sugar, at the rate of 15 cts. per kilogramme.

$$\begin{aligned} 12^{hg} &= 1200^g \\ 6^{dkg} &= 60^g \\ 3^{dg} &= \underline{0.3^g} \\ &\quad 1260.3^g = 1.2603^{kg};\ 1.2603 \times .15 = 18.9 \text{ cts.} \end{aligned}$$

SPECIFIC GRAVITY.

The **Specific Gravity** of a substance is the *ratio* of the weight of a given volume of it to the weight of an *equal volume* of water.

Table.

Wt. of 1 cu. centimetre of water	= 1 gramme.
Wt. of 1 cu. decimetre of water	= 1 kilogramme.
Wt. of 1 cu. metre of water	= 1 ton.

Therefore, the specific gravity of a substance is the number representing the *grammes* in a *cu. centimetre* of the substance, the *kilogrammes* in a *cu. decimetre* of the substance, and the *tons* in a *cu. metre of the substance.*

FORMULÆ.

$$\text{Specific Gravity} = \frac{\text{Wt. in }^{\text{g}}}{\text{ccm}} = \frac{\text{kg}}{\text{cdm}} = \frac{\text{Tons}}{\text{cbm}}.$$

$$\text{Volume in }^{\text{ccm}} = \frac{\text{Weight in }^{\text{g}}}{\text{Specific Gravity}}.$$

Illustrations.

1. What is the specific gravity of a substance of which 8.4^{ccm} weighs 33.6^{g}?

$$\text{Sp. gr.} = \frac{\text{g}}{\text{ccm}} = \frac{33.6}{8.4} = 4.$$

2. What is the volume of a body whose specific gravity is 2.5 and whose weight is 5 tons?

$$\text{Vol.} = \frac{\text{Wt.}}{\text{Sp. Gr.}} = \frac{5}{2.5} = 2^{\text{cbm}}.$$

APPROXIMATE EQUIVALENTS.

Table.

Metre	=	1.1 yds.	Yard	=	$.9^{\text{m}}$
Kilometre	=	$\frac{5}{8}$ mi.	Mile	=	1.6^{km}
Sq. metre	=	$1\frac{1}{5}$ sq. yd.	Sq. yard	=	$\frac{5}{6}^{\text{qm}}$
Hektare	=	$2\frac{1}{2}$ acres.	Acre	=	$\frac{2}{5}^{\text{ha}}$
Cu. centimetre	=	$\frac{1}{16}$ cu. in.	Cu. inch	=	16^{ccm}
Cu. metre	=	1.3 cu. yd.	Cu. yard	=	$\frac{10}{13}^{\text{cbm}}$
Stere	=	$\frac{3}{11}$ cord.	Cord	=	$3\frac{2}{3}^{\text{st}}$
Litre	=	$1\frac{1}{16}$ liq. qt.	Liq. quart	=	$\frac{16}{17}^{\text{litre}}$
		$\frac{9}{10}$ dry qt.	Dry quart	=	$1\frac{1}{9}^{\text{litres}}$
Hektolitre	=	$2\frac{5}{6}$ bu.	Bushel	=	$\frac{6}{17}^{\text{hl}}$
Gramme	=	$15\frac{1}{2}$ grs.	Pound av.	=	$\frac{5}{11}^{\text{kg}}$
Kilogramme	=	$2\frac{1}{5}$ lbs. av.	Pound troy	=	$\frac{5}{13}^{\text{kg}}$

Illustration.

At 20 cts. a litre, what will be the cost of 150 qt. of olive oil?

$$1 \text{ liq. qt.} = \tfrac{16}{17}\text{ l.};\ 150 \text{ qt.} = \tfrac{150}{1} \times \tfrac{16}{17}\text{ l.};\ \tfrac{2400}{17} \times \tfrac{20}{1} = \$28.24.$$

PROBLEMS.

1. Reduce $57,654^{m}$ to millimetres; to dekametres; to centimetres; to hektometres; to decimetres; to kilometres.

2. Find the sum, in metres, of 243^{m}, 265^{cm}, 4264^{mm}, $.012^{km}$.

3. Find the difference, in metres, between $.628^{km}$ and 3158^{dm}.

4. Reduce $360,000^{qdm}$ to sq. metres; to sq. hektometres; to sq. kilometres.

5. Find the cost of 2.8^{ha} of land at $1.05 an are.

6. Find the ares in 258^{a} and 4672^{ca}.

7. Reduce $57,000^{ccm}$ to cu. decimetres; to cu. metres.

8. How many cu. decimetres in a bin measuring $12^{m} \times 6.48^{m} \times 4.13^{m}$?

9. How many steres of wood in a pile measuring $9^{m} \times 2.6^{m} \times 1^{m}$?

10. Find the hectolitres of air in a room measuring $5^{m} \times 4^{m} \times 3^{m}$?

11. How many hektolitres of corn in a crib measuring $9^{m} \times 3^{m} \times 2^{m}$?

12. How many litres in a cistern whose dimensions are $2^{m} \times 1^{m} \times .5^{m}$?

13. Reduce 3^{t} to grammes; to milligrammes; to dekagrammes; to decigrammes.

14. How many kilogrammes of water will be held in a cistern that is cylindrical in shape and is 1.5^{m} in diameter and 4^{m} deep?

15. From 18^{km} take 18^{mm}.

16. How many metres of muslin, at $0.25 per metre, must be given in exchange for 300^{hl} of oats, at $1.20 per hektolitre?

17. What is the area (in ares) of a floor 3.25^{m} long and 2.5^{m} wide?

18. A grocer buys butter at $0.28 per lb., and sells it at $0.60 per kilogramme. What per cent. does he gain or lose?

19. At 39 cts. a metre, what would it cost to cut a ditch $9^{dkm}\ 10^{m}\ 7^{dm}$ in length?

20. How many metres in one-half a mile?

21. If the diameter of a ball is 63^{cm}, find the surface and volume of the ball in inches.

22. The specific gravity of sea-water is 1.026, and that of milk 1.032; find the weight of a hektolitre of each in pounds and in kilogrammes.

23. Find the volume of 63^g of platinum, if its specific gravity is 21. Find the volume also in cu. inches.

24. If a pedestrian goes 125^m in a minute, what is his rate in miles per hour?

25. Find the hektares in a lot that is 130^m square.

26. What is the depth of a bin 12^m long and 8^m wide, to hold 2000^{hl} of wheat?

27. At 25 cts. a quart, what is the cost of $10^l\ 5^{dl}$ of oil?

28. How many metres in 100 mi. 20 rd. 6 yd. 3 ft. 2 in.?

29. Reduce 9 myrialitres $5^{hl}\ 6^l\ 10^{dl}$ to litres.

30. A vessel full of alcohol, specific gravity .916, weighs 7.4^{kg}. When empty it weighs 500^g. How many litres will the vessel hold?

FOREIGN EXCHANGE.

Domestic Exchange, of which we have treated, takes place between different parts of the same country. **Foreign Exchange** takes place between *different countries.*

The methods of computation are the same in both, except that the latter requires the reduction of the currency of one country to that of another.

In foreign exchange the practice is to send *three separate bills* in different ways; each bill being so conditioned that the payment of one of them cancels the other two, and the non-payment of any one keeps all valid.

Form of Draft.

£500 New York, *April 1,* 18*98.*

At sight of this First of Exchange (Second and Third of the same tenor and date unpaid) pay to the order of

———*John Jefferson*———

Five hundred pounds———

Value received, and charge to account of

To *Baring Brothers,* *Bliss & Morton.*
London.

VALUE OF FOREIGN COINS.

Proclaimed by Law, July 1, 1897.

Argentine Republic	Peso	$ .965
Austria	Crown	.203
Belgium	Franc	.193
Bolivia	Boliviano	.443
Brazil	Milreis	.546
British Poss., N. A.	Dollar	1.000
Central American States	Peso	.443
Chili	Peso	.365
China, Chefoo	Tael	.686
Haikwan	Tael	.730
Shanghai	Tael	.655
Tientsin	Tael	.695
Colombia	Peso	.443
Cuba	Peso	.926
Denmark	Crown	.268
Ecuador	Sucre	.443
Egypt	Pound	4.943
France	Franc	.193
Finland	Mark	.193
German Empire	Mark	.238
Great Britain	Pound	4.866½
Greece	Drachma	.193
Hayti	Gourde	.965
India	Rupee	.211
Italy	Lira	.193
Japan	Yen { Gold	.997
	Yen { Silver	.478
Liberia	Dollar	1.000
Mexico	Dollar	.482
Netherlands	Florin	.402
Newfoundland	Dollar	1.014
Norway	Crown	.268
Peru	Sol	.443
Portugal	Milreis	1.080
Russia	Rouble (Gold)	.772
Spain	Peseta	.193
Sweden	Crown	.268
Switzerland	Franc	.193
Turkey	Piaster	.044
Venezuela	Bolivar	.193

Quotations.

March 28, 1899.

"Sterling Exchange, $4.85\frac{3}{4}$ @ 4.86; $4.83\frac{1}{4}$ @ 4.84.

Paris Exchange, $5.18\frac{3}{4}$ less $\frac{1}{16}$ @ $5.18\frac{3}{4}$; $5.21\frac{1}{4}$ less $\frac{1}{16}$ @ $5.21\frac{1}{4}$."

That is, in London the value of a pound sterling in United States money varied from $\$4.85\frac{3}{4}$ to \$4.86 for sight bills, and from $\$4.83\frac{1}{4}$ to \$4.84 for 60-day bills; in Paris the value of a United States dollar varied from $5.18\frac{3}{4}$ less $\frac{1}{16}$ fr. to $5.18\frac{3}{4}$ for sight bills, and from $5.21\frac{1}{4}$ less $\frac{1}{16}$ ct. to $\$5.21\frac{1}{4}$ for 60-day bills.

Drafts are at a *premium* or at a *discount*, in accordance with the relative condition of trade between two countries.

For instance, when New York owes London more than London owes New York, bills on London are *above par*, or at a premium; the case being reversed, bills on London are at a discount.

Illustrations.

1. What was the cost of a sight draft on Paris for 1000 francs, March 28, 1899?

$$5.18\tfrac{3}{4} \text{ francs} = \$1.00.$$

$$\text{Hence } 1000 \text{ francs} = 1000 \div 5.18\tfrac{3}{4} = \$192.77, \text{ cost.}$$

2. What was the cost of a bill for £300 on London at the same date?

$$\text{Since } £1 = \$4.86,\ £300 = \$4.86 \times 300 = \$1458.00.$$

PROBLEMS.

1. Find the cost of a draft on Liverpool for £1000, premium at $4\frac{1}{2}\%$. [See table above for value of £1.]

2. On Jan. 1, 1898, the interest debt of Spain was 528,185,659 pesetas. Find the value of this sum in United States money?

3. On the same date Spain's income was estimated at \$152,970,000. Find the value in pesetas and show the amount left, after paying the interest debt, to carry on civil and military transactions.

4. What is the cost of a bill on Berlin for 2000 marks?

5. What is the cost of a draft on Glasgow for £500, exchange being at par?

6. A Chicago merchant bought a bill of exchange on London for £795 15s., sterling exchange, as quoted above for 60-day bill. Find the cost of the bill.

7. A merchant paid $530 for a draft on Paris, exchange at 5.18¼. Find the *face of the draft.*

8. Find the cost of a draft on Hamburg for 13,700 marks.

9. If a Philadelphia merchant pays $3058.50 for a bill on Manchester, Eng., for £533 15s., what is the rate of exchange?

10. What must a merchant in St. Louis pay for a bill on Havre for 18,875 francs, exchange being quoted at 5.20⅝?

11. Find the cost of a draft on London for £500 12s. 6d.

12. Find the cost of a draft on Paris for 2800 francs.

13. Find the cost of a draft on Frankfort for 6500 marks.

14. How many dollars must be paid in Cairo (Egypt) for a draft having a face value of £250, exchange being at 1½% premium?

15. A New York merchant owing 15,000 francs in Paris remits by exchange on London. Find the cost of his draft in U. S. dollars, 25.22 francs being equal to £1.

ARITHMETICAL PROGRESSION.

1. An "Arithmetical Progression is a series of numbers that increase or decrease by a *common difference*, as 7, 10, 13, 16, 19, 22; or, 12, 10½, 9, 7½, 6." The common difference in the first series is 3; in the second it is 1½. The numbers constituting a series are called its *terms.*

2. Two principal cases arise: (1.) *To find any term of a series;* (2.) *To find the sum of the terms of a series.*

1. To Find Any Term.

Let it be required to find the sixth term, or any term, of the series 7, 10, 13, 16, 19, etc.

Examining the construction of the series, we find the common difference to be 3, and the

1st term $= 7$	4th term $= 7 + (3 \times 3)$
2d term $= 7 + (3 \times 1)$	5th term $= 7 + (3 \times 4)$
3d term $= 7 + (3 \times 2)$	

From which we see that, to form the terms following the first, the common difference was multiplied in succession by *one, two, three,* and *four.* Hence the sixth term $= 7 + (3 \times 5)$, and *any term* $= 7 + (3 \times$ the number of the term *less one*).

Were the series a *decreasing* one, the products above would require to be subtracted instead of being added, and *any term* would equal first term — (3 × the number of the term *less one*).

FORMULA.

Any term = 1st term ± (Com. dif.) × (No. of the term — 1).

NOTE.—The sign ± is read "plus or minus."

To apply the formula, let it be required to find the fortieth term of the series $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, etc.

The com. dif. $= \frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$; 40th term $= \frac{1}{2} + \frac{1}{4}(40 - 1) = \frac{1}{2} + \frac{39}{4} = \frac{41}{4} = 10\frac{1}{4}$.

2. To Find the Sum of the Terms.

Let it be required to find the sum of five terms of the series 1, 3, 5, 7, 9.

The series, as given, is	1,	3,	5,	7,	9
The order reversed, is	9,	7,	5,	3,	1
Twice the sum is	10 +	10 +	10 +	10 +	10

or 10 × 5.

Once the sum is $\frac{10 \times 5}{2} = 25$.

10 is the sum of 1 and 9, the first and last terms of the given series; 5 is the number of terms.

Hence the formula:

$$\textbf{Sum of terms} = \frac{\textbf{1st term + Last term}}{\textbf{2}} \times \textbf{Number of terms.}$$

To apply the formula, let it be required to find the sum of 8 terms of the series 12, $10\frac{1}{2}$, 9, etc.

Since we do not know the eighth term, we proceed to find it by the formula for any term.

$$\text{8th term} = 12 - (1\tfrac{1}{2} \times 7) = 12 - 10\tfrac{1}{2} = 1\tfrac{1}{2}.$$

$$\text{Sum of 8 terms} = \frac{12 + 1\frac{1}{2}}{2} \times 8 = \frac{108}{2} = 54.$$

GEOMETRICAL PROGRESSION.

1. A Geometrical Progression is a series of terms in which any term is equal to the product of the preceding term and a *factor, which is constant throughout the series*, as 4, 8, 16, etc., whose constant factor is 2. The constant factor of a series is called its *common ratio.*

2. Two principal cases arise: (1.) *To find any term of a series;* (2.) *To find the sum of the terms of a series.*

1. To Find Any Term.

Illustration.

Let it be required to find the sixth term or any higher term of the series 2, 4, 8, 16, etc.

The common ratio is 2, and

$$\begin{aligned} \text{The 1st term} &= 2, \\ \text{2d term} &= 2 \times 2^1, \\ \text{3d term} &= 2 \times 2^2, \\ \text{4th term} &= 2 \times 2^3, \\ \text{5th term} &= 2 \times 2^4. \end{aligned}$$

From which we see that, to form the terms following the first, the common ratio was raised successively to the first, second, third, and fourth powers. Hence, the sixth term $= 2 \times 2^5$, and *any term* $= 2 \times 2$ *raised to a power* denoted by the *number of the term less one.*

Were the series a *decreasing one*, the common ratio would be $\frac{1}{2}$, the *reciprocal* of 2.

FORMULA.

Any term = 1st term × Com. ratio raised to a power whose index is one less than the number of the term.

To apply the formula, let it be required to find the sixth term of the series, 5, 10, 20, etc.

The com. ratio is 2; the 6th term $= 5 \times 2^5 = 5 \times 32 = 160$.

2. To Find the Sum of the Terms.

Illustration.

Let it be required to find the sum of five terms of the series 2, 6, 18, etc.

The ratio is 3, and the sum of five terms is

$$2 + 6 + 18 + 54 + 162.$$

Multiplying this sum by 3, the common ratio, we have

$$6 + 18 + 54 + 162 + 486.$$

Subtracting the upper sum from the lower, we have, *twice* the sum $= 486 - 2$; therefore, *once* the sum $= \frac{486 - 2}{2}$.

But, 486 = last term × the ratio, while — 2 means "subtract the first term," and 2, divisor, = the ratio — 1.

Hence the formula:

$$\text{Sum of terms} = \frac{\text{Last term} \times \text{Ratio} - \text{1st term}}{\text{Ratio} - 1}.$$

When the ratio is *less than* 1, the series is a *decreasing* one, and the formula becomes:

$$\text{Sum of terms} = \frac{\text{1st term} - \text{Last term} \times \text{Ratio}}{1 - \text{Ratio}}.$$

To apply the formula, let it be required to find the sum of 3, 12, 48, etc., to 6 terms. The ratio is 4.

The 6th term $= 3 \times 4^5 = 3 \times 1024 = 3072$. The sum $= \frac{3072 \times 4 - 3}{4 - 1} = \frac{12288 - 3}{3} = \frac{12285}{3} = 4095$.

3. Compound interest problems may be solved by the use of the formula for finding any term, since the principal and the yearly amounts form a geometrical series.

Illustrations.

1. Find the amount of \$60, at compound interest for 3 yr., at 5%.

The required amount will be 60 times the amount of \$1.00 for the same time.

The series is

$$\$1, \$1 \times 1.05, \$1 \times (1.05)^2, \$1 \times (1.05)^3.$$

The ratio is 1.05, and last term $= \$1 \times (1.05)^3 = \$1.157625$.

$$\$1.157625 \times 60 = \$69.46, \text{ Amt.}$$

Hence the formulæ:

Amount = (1 + Rate, raised to a power denoted by the number of years) × Principal.

$$\textbf{Principal} = \frac{\textbf{Amount}}{\textbf{(1 + Rate, raised to a power denoted by the number of years.}}$$

2. Find what principal will in 3 yr., compound interest at 6%, amount to \$1898.04.

$$\text{Principal} = \frac{1898.04}{1 \times (1.06)^3} = \frac{1898.04}{1.191016} = \$1593.72.$$

4. Since such computations as the foregoing cannot be readily made, *tables* showing the amount of \$1 at compound interest for various rates and lengths of time are in common use.

Table I.

Showing the Amount of $1 at Compound Interest for:

Yr.	2%	2½%	3%	3½%	4%	4½%	5%	5½%	6%	7%
1	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000	1.05500	1.06000	1.07000
2	1.04040	1.05063	1.06090	1.07123	1.08160	1.09203	1.10250	1.11303	1.12360	1.14490
3	1.06121	1.07689	1.09273	1.10872	1.12486	1.14117	1.15763	1.17424	1.19102	1.22504
4	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551	1.23882	1.26248	1.31080
5	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628	1.30696	1.33823	1.40255
6	1.12616	1.15969	1.19405	1.22926	1.26532	1.30226	1.34010	1.37884	1.41852	1.50073
7	1.14869	1.18869	1.22987	1.27228	1.31593	1.36086	1.40710	1.45468	1.50363	1.60578
8	1.17166	1.21840	1.26677	1.31681	1.36857	1.42210	1.47746	1.53469	1.59385	1.71819
9	1.19509	1.24886	1.30477	1.36290	1.42331	1.48610	1.55133	1.61909	1.68948	1.83846
10	1.21899	1.28009	1.34392	1.41060	1.48024	1.55297	1.62889	1.70814	1.79085	1.96715
11	1.24337	1.31209	1.38423	1.45997	1.53945	1.62285	1.71034	1.80209	1.89830	2.10485
12	1.26824	1.34489	1.42576	1.51107	1.60103	1.69588	1.79586	1.90121	2.01220	2.25219
13	1.29361	1.37851	1.46853	1.56396	1.66507	1.77220	1.88565	2.00577	2.13293	2.40985
14	1.31948	1.41297	1.51259	1.61870	1.73168	1.85194	1.97993	2.11609	2.26090	2.57853
15	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893	2.23248	2.39656	2.75903
16	1.37279	1.48451	1.60471	1.73399	1.87298	2.02237	2.18287	2.35526	2.54035	2.95216
17	1.40024	1.52162	1.65285	1.79468	1.94790	2.11338	2.29202	2.48480	2.69277	3.15882
18	1.42825	1.55966	1.70243	1.85749	2.02582	2.20848	2.40662	2.62147	2.85434	3.37993
19	1.45681	1.59865	1.75351	1.92250	2.10685	2.30786	2.52695	2.76565	3.02560	3.61653
20	1.48595	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330	2.91776	3.20714	3.86968

Formulæ for the application of the above table:

$$\text{Required Amt.} = \text{Amt. in Table} \times \text{Principal.}$$

$$\text{Principal} = \frac{\text{Amt.}}{\text{Amt. in Table}}.$$

$$\text{Time or Rate} = \frac{\text{Amt.}}{\text{Principal}}.$$

The quotient, as found in the table, will indicate the Time or the Rate, as the case may be.

Illustration.

In what time will 1700 dollars amount to $2551.25, at 7%?

$$\text{Time} = \frac{\text{Amt.}}{\text{Pr.}} = \frac{2551.25}{1700} = 1.50074.$$

Referring to the 7% column in the table, we find that 1.50074 represents 6 years.

ANNUITIES.

1. An **Annuity** is a sum of money, generally of a uniform amount, payable at regular intervals of time.
2. A **Perpetual Annuity** continues forever.
3. A **Certain Annuity** continues for a fixed term of years.
4. A **Contingent Annuity** depends upon the "continuance of some status, such as the life of a person."
5. An **Annuity in Reversion** begins at some future date.
6. An **Annuity in Arrears** remains unpaid.
7. The **Final Value** of an annuity is the *final* amount of an annuity at compound interest.
8. The **Present Value** of an annuity will amount to the final value, at compound interest.
9. To facilitate computation, tables are customarily used.

FORMULÆ.

$$\text{Present Value} = \text{Pres. Val. of \$1 in Table} \times \text{Annuity.}$$

$$\text{Amount of An.} = \frac{\text{Pres. Val.}}{\text{Pres. Val. of \$1 in Table}}.$$

Table II.

SHOWING THE PRESENT VALUE OF AN ANNUITY OF $1 PER ANNUM, AT COMPOUND INTEREST FROM 1 YR. TO 40 YRS., AT $3\frac{1}{2}\%$ AND AT 4%.

Yr.	$3\frac{1}{2}\%$	4%	Yr.	$3\frac{1}{2}\%$	4%
1	0.96618	0.96154	21	14.69797	14.02916
2	1.89969	1.88610	22	15.16713	14.45112
3	2.80164	2.77509	23	15.62041	14.85684
4	3.63708	3.62990	24	16.05837	15.24696
5	4.51505	4.45182	25	16.48152	15.62208
6	5.32855	5.24214	26	16.89035	15.98277
7	6.11454	6.00206	27	17.28537	16.32959
8	6.87396	6.73275	28	17.66702	16.66306
9	7.60769	7.43533	29	18.03577	16.98372
10	8.31661	8.11090	30	18.39205	17.29203
11	9.00155	8.76048	31	18.73628	17.58849
12	9.66333	9.38507	32	19.06887	17.87355
13	10.30274	9.98565	33	19.39021	18.14765
14	10.92052	10.56312	34	19.70068	18.41120
15	11.51741	11.11839	35	20.00066	18.66461
16	12.09412	11.65230	36	20.29049	18.90828
17	12.65132	12.16570	37	20.57053	19.14258
18	13.18968	12.65930	38	20.84109	19.36786
19	13.70984	13.13394	39	21.10250	19.58449
20	14.21240	13.59033	40	21.35507	19.79277

Illustrations.

1. What is the present value of an annuity for $600 for 6 yrs. at 4%?

Pres. val. = Pres. val. (table) × An. = $5.242124 × 600 = $3145.27.

2. A man 49 yrs. of age pays $8686.64 for a life annuity. Reckoning interest at $3\frac{1}{2}\%$, find the amount of the annuity.

Here we must first find the man's expectancy of life. Referring to the Carlisle Table, we find it to be somewhat less than 22 years.

$$\text{Amt. of An.} = \frac{\text{Pres. val. of \$1}}{\text{Pres. val.}} = \frac{\$8686.64}{15.16713} = \$572.73.$$

Carlisle Table of the Expectancy of Life.

Age.	Expectancy.	Age.	Expectancy.	Age.	Expectancy.	Age.	Expectancy.	Age.	Expectancy.
0	38.72	20	41.46	40	27.61	60	14.34	80	5.51
1	44.68	21	40.75	41	26.97	61	13.82	81	5.21
2	47.55	22	40.04	42	26.34	62	13.31	82	4.93
3	49.82	23	39.31	43	25.71	63	12.81	83	4.65
4	50.76	24	38.59	44	25.09	64	12.30	84	4.39
5	51.25	25	37.86	45	24.46	65	11.79	85	4.12
6	51.17	26	37.14	46	23.82	66	11.27	86	3.90
7	50.80	27	36.41	47	23.17	67	10.75	87	3.71
8	50.24	28	35.69	48	22.50	68	10.23	88	3.59
9	49.57	29	35.00	49	21.81	69	9.70	89	3.47
10	48.82	30	34.34	50	21.11	70	9.18	90	3.28
11	48.04	31	33.68	51	20.39	71	8.65	91	3.26
12	47.27	32	33.03	52	19.68	72	8.16	92	3.37
13	46.51	33	32.36	53	18.97	73	7.72	93	3.48
14	45.75	34	31.68	54	18.28	74	7.33	94	3.53
15	45.00	35	31.00	55	17.58	75	7.01	95	3.53
16	44.27	36	30.32	56	16.89	76	6.69	96	3.46
17	43.57	37	29.64	57	16.21	77	6.40	97	3.28
18	42.87	38	28.96	58	15.55	78	6.12	98	3.07
19	42.17	39	28.28	59	14.92	79	5.80	99	2.77

EXERCISES AND PROBLEMS.

1. Find the twelfth term of 3, 6, 9, etc.

2. Find the twentieth term of 1, 8, 15, 22, etc.

3. Find the seventh term of 99, 92, 85, etc.

4. Find the twenty-fifth term of 100, 96, 92, etc.

5. Find the number of strokes made in a day by a clock striking the hours only.

6. The first term of an arithmetical progression is 7, the last term 79, and the number of terms 15. What is the sum of the series?

7. The series is $\frac{1}{2}$, $\frac{3}{4}$, 1, etc. Find the one-hundredth term and the sum of the series.

8. A body falling for 12 sec. passes through $16\frac{1}{12}$ ft. the first second, and increases its speed $32\frac{1}{6}$ ft. each succeeding second. What is the whole extent of its fall?

9. 100 apples were placed in a row 2^m apart, and a basket was placed 2^m from the first apple. A boy, starting at the basket, brought to the basket the first apple, next the second apple, and

so on until all were brought. How far did the boy walk in performing the labor?

10. A debt can be discharged in a year by paying \$1 the first week, \$3 the second week, \$5 the third week, and so on. Find the last payment and the amount of the debt.

11. Find the sixth term of the series 2, 6, 18, etc.

12. Find the sixth term of the series 8, 4, 2, etc.

13. Find the seventh term of the series $\frac{3}{2}$, $\frac{1}{2}$, $\frac{1}{6}$, etc.

14. Find the nineteenth term of the series 4, 7, 10, etc.

15. In going a 9-days' journey a man travelled 30^{km} the first day, and constantly thereafter increased his daily distance 10^{km}. How far did he travel the last day, and how many *miles* altogether?

16. Starting with a man's immediate parents and running back 10 generations, compute the number of his ancestors.

17. What is the amount of \$1 at compound interest for 6 yrs. at 7%? Solve in three ways.

18. What is the amount of \$2000 for 9 yrs. at 6%? Solve by the table.

19. What principal, at compound interest for 5 yrs. at 6%, will amount to \$267,646?

20. In what time will the same principal, at 5%, amount to the same sum?

21. A man 23 yrs. old has a life annuity of \$700. Find its present value at $3\frac{1}{2}$%.

22. A woman 36 yrs. old has a life annuity of \$1200. Find its present value at 4%.

23. A person 76 yrs. old has a life annuity of \$3000. Find its present value at $3\frac{1}{2}$%.

24. A dowager 50 yrs. old has a jointure of \$4500. Find its present value, interest at 4%.

25. A widow 29 yrs. old has a dower of \$1800. Find its present value, interest at $3\frac{1}{2}$%.

26. A man 35 yrs. old pays \$9368.10 for a life annuity. Find the amount of the annuity, interest at $3\frac{1}{2}$%.

27. A life annuity costs a person 44 yrs. old \$5933.35. Find the amount of the annuity, interest at $3\frac{1}{2}$%.

28. Find the amount of \$365 at compound interest for 20 yrs. at 5%.

29. Find the amount of \$1728 at compound interest for 25 yrs. at 6%.

CIRCULATING DECIMALS.

$\frac{2}{5} = \frac{2.0}{5} = .4$, quotient exact. $\frac{3}{7} = \frac{3.0}{7} = .4285 +$, quotient inexact.

To reduce a common fraction to a decimal, we annex ciphers to the numerator, perform the operation indicated, and point off in the quotient as many places for decimals as there are ciphers annexed. Annexing a cipher to the numerator multiplies it by the factors 2 and 5; hence, when the denominator contains no other factors than 2 and 5, the quotient will be exact; when it contains other factors than 2 and 5, the quotient will be inexact.

$\frac{5}{13}$ reduced to a decimal becomes .384615, and so on without end; but in this instance a very noticeable peculiarity is that the 384,615 will be constantly repeated, however far the reduction be carried.

Such a decimal fraction is called a *circulating* or *repeating* decimal. To show the fact of repetition we place a point over the first and the last digit, thus: $\dot{3}8461\dot{5}$. $\frac{3}{11} = .272727$, etc. $= .\dot{2}\dot{7}$.

The constantly repeating figures are called a *repetend.* A *mixed repetend* begins with one or more non-repeating figures, as $.5\dot{2}4\dot{3}$.

The practical advantage of recognizing circulates will appear in the following illustrations:

1. Reduce $.\dot{7}\dot{2}$ to a common fraction.

$$\begin{array}{rl} 100 \text{ times } .\dot{7}\dot{2} &= 72.7272 \ldots\ldots \\ 1 \text{ time } .\dot{7}\dot{2} &= .7272 \ldots\ldots \\ \hline 99 \text{ times } .\dot{7}\dot{2} &= 72 \\ 1 \text{ time } .\dot{7}\dot{2} &= \frac{72}{99} = \frac{8}{11}. \end{array}$$

2. Reduce $.11\dot{7}\dot{2}$ to a common fraction.

$$.11\dot{7}\dot{2} = .11\frac{72}{99} = .11\frac{8}{11} = \frac{11\frac{8}{11}}{100} = \frac{129}{1100}.$$

Hence the formula:

$$\frac{\text{Digits of repetend}}{\text{As many 9's as digits}} = \text{value of repetend.}$$

EXAMPLES.

Reduce the following repetends to common fractions:

1. $.\dot{3}$.	7. $.\dot{7}5\dot{3}$.	13. $.\dot{8}5\dot{2}$.	19. $.\dot{0}00\dot{3}$.	25. $.2\dot{2}9\dot{7}$.
2. $.\dot{4}$.	8. $.\dot{2}1\dot{6}$.	14. $.\dot{1}4\dot{4}$.	20. $.\dot{2}4678\dot{9}$.	26. $2.1\dot{8}7\dot{3}$.
3. $.\dot{6}$.	9. $.\dot{5}3\dot{1}$.	15. $.\dot{5}2\dot{7}$.	21. $.25\dot{6}\dot{4}$.	27. $.4\dot{3}0\dot{6}$.
4. $.\dot{3}\dot{6}$.	10. $.\dot{0}23\dot{4}$.	16. $.\dot{0}00\dot{9}$.	22. $.8\dot{7}1\dot{6}$.	28. $5.04\dot{1}\dot{5}$.
5. $.\dot{2}\dot{1}$.	11. $.\dot{8}23\dot{2}$.	17. $.\dot{0}4\dot{8}$.	23. $.35\dot{1}3\dot{5}$.	29. $.84\dot{2}3\dot{4}$.
6. $.\dot{0}1\dot{8}$.	12. $.\dot{8}\dot{1}$.	18 $.\dot{0}819\dot{9}$.	24. $3.04\dot{1}\dot{2}$.	30. $4.2\dot{6}7\dot{4}$.

THE GREATEST COMMON DIVISOR OF FRACTIONS.

Find the G. C. D. of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$.

Each of the given fractions divided by the G. C. D. must give an integer for quotient. In dividing by a fraction, we invert the divisor and then multiply. Hence the G. C. D. sought must have for its numerator the G. C. D. of the given numerators and the L. C. Dd. of the given denominators.

The G. C. D. of 2, 4, and 6 = 2.
The L. C. Dd. of 3, 5, and 7 = 105.
Hence the G. C. D. of $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{6}{7} = \frac{2}{105}$.

FORMULA.

$$\text{G. C. D.} = \frac{\text{G. C. D. of Numerators}}{\text{L. C. Dd. of Denominators}}.$$

THE LEAST COMMON DIVIDEND OF FRACTIONS.

Find the L. C. Dd. of $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{8}{15}$.

The L. C. Dd. sought, when divided by each of the given fractions, must give an integral quotient. In dividing by a fraction, we invert the divisor and then multiply. Hence the L. C. Dd.

required must have for its numerator the L. C. Dd. of the given numerators and the G. C. D. of the denominators.

The L. C. Dd. of 2, 4, and 8 = 8.
The G. C. D. of 3, 9, and 15 = 3.

Hence the L. C. Dd. of $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{8}{15} = \frac{8}{3}$.

FORMULA.

$$\text{L. C. Dd.} = \frac{\text{L. C. Dd. of Numerators}}{\text{G. C. D. of Denominators}}.$$

EXERCISES.

1. Find the G. C. D. of:

1. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.
2. $\frac{7}{12}$, $\frac{9}{16}$, $\frac{3}{20}$, $\frac{5}{24}$.
3. $12\frac{1}{2}$, $3\frac{1}{3}$, $17\frac{1}{7}$.
4. 10, $2\frac{1}{2}$, $\frac{3}{4}$.
5. $1\frac{8}{13}$, $1\frac{7}{15}$, $1\frac{3}{20}$.
6. $\frac{22}{27}$, $\frac{11}{45}$, $\frac{77}{36}$.

2. Find the L. C. Dd. of:

1. $\frac{8}{9}$, $\frac{7}{12}$, $\frac{3}{4}$.
2. $\frac{5}{6}$, $\frac{10}{18}$, $\frac{12}{15}$.
3. $6\frac{3}{7}$, $3\frac{6}{7}$, $2\frac{1}{7}$.
4. $\frac{22}{18}$, $\frac{7}{82}$, $\frac{9}{24}$.
5. $\frac{1}{13}$, $\frac{1}{11}$, $2\frac{1}{2}$, 5, $6\frac{1}{3}$.
6. $\frac{11}{45}$, $\frac{22}{27}$, $\frac{77}{36}$.

THE THERMOMETER.

The **Thermometer** is an instrument for measuring change of temperature by means of the expansion of liquid substances. Mercury is the pre-eminently suitable substance.

Thermometers are of three principal kinds: The *Fahrenheit*, the *Centigrade*, and the *Reaumur*. The Centigrade is used largely for scientific purposes.

It is sometimes necessary to transform readings from one scale to another.

	Freezing Point.	Boiling Point.	Interspace.
Fahrenheit	32°	212°	180°
Centigrade	0°	100°	100°
Reaumur	0°	80°	80°

Hence the number of degrees F. — 32° = $\frac{180}{100}$ or $\frac{9}{5}$ C. = $\frac{180}{80}$ or $\frac{9}{4}$ R.

From which we deduce the formulæ:

1. $(F. - 32°) \times \frac{5}{9} = C.$
2. $(F. - 32°) \times \frac{4}{9} = R.$
3. $C. \times \frac{9}{5} = F. - 32°.$
4. $R. \times \frac{9}{4} = F. - 32°.$

1. Change 40° F. to C.

$$40^\circ - 32^\circ = 8^\circ. \quad 8^\circ \times \tfrac{5}{9} = \tfrac{40^\circ}{9} = 4\tfrac{4}{9}^\circ \text{ C.}$$

2. Change 40° C. to F.

$$40^\circ \times \tfrac{9}{5} = \tfrac{360}{5} = 72^\circ. \quad 72^\circ + 32^\circ = 104^\circ \text{ F.}$$

The minus sign (—) prefixed to a reading signifies *below zero.*

3. Change —10° C. to F.

$$-10^\circ \times \tfrac{9}{5} = \tfrac{-90^\circ}{5} = -18^\circ$$

We now have —18° + 32°. Difference of sign implies *subtraction.* —18° + 32° = 14° F. above zero.

4. Change —30° C. to F.

$$-30^\circ \times \tfrac{9}{5} = \tfrac{-270^\circ}{5} = -54^\circ.$$

$$-54^\circ + 32^\circ = -22^\circ \text{ F.; that is, } 22^\circ \text{ below zero.}$$

5. Change 32° F. to C. and R.
6. Change 50° C. to F. and R.
7. Change —20° C. to F. and R.

THE CLOCK.

The hour and minute hands are together at 12 o'clock, and the minute hand may be regarded as setting out at that point to rejoin the hour hand. In 60 minutes the minute hand will have returned to 12, but the hour hand will have passed on to 1. In 60 minutes, therefore, the minute hand has gained on the hour hand 11 spaces. Hence, a single space was gained in $\frac{1}{11}$ of 60 minutes = $5\frac{5}{11}$ minutes.

1. At what time between 4 and 5 o'clock are the hands of a clock together?

At 4 o'clock the hands are 4 spaces apart. Since the minute hand gains 1 space in $5\frac{5}{11}$ minutes, it will gain 4 spaces in 4 times $5\frac{5}{11}$ minutes = $21\frac{9}{11}$ minutes. Hence, the hands will be together at $21\frac{9}{11}$ minutes past 4.

2. When will the hands of a clock be together between:

1. 6 and 7 o'clock?
2. 8 and 9 o'clock?
3. 1 and 2 o'clock?
4. 3 and 4 o'clock?
5. 9 and 10 o'clock.
6. 5 and 6 o'clock.

3. When will the hands of a clock be opposite each other between :

1. 12 and 1 o'clock?
2. 3 and 4 o'clock?
3. 9 and 10 o'clock?
4. 11 and 12 o'clock?

4. When will the hands of a clock be at right angles between :

1. 2 and 3 o'clock?
2. 4 and 5 o'clock?
3. 9 and 10 o'clock?
4. 11 and 12 o'clock?

5. When between those hours will they make with each other an angle of 30°? Of 60°? Of 120°? Of 150°?

WORK.

1. A. can do a certain piece of work in 6 days, B. in 8 days, and C. in 9 days. How long will it take them to do it together?

A. can do $\frac{1}{6}$ of the work in 1 day.
B. can do $\frac{1}{8}$ of the work in 1 day.
C. can do $\frac{1}{9}$ of the work in 1 day.
All can do $\frac{1}{6} + \frac{1}{8} + \frac{1}{9} = \frac{29}{72}$ in 1 day.
All can do $\frac{1}{72}$ of the work in $\frac{1}{29}$ of 1 day.
All can do $\frac{72}{72}$ of the work in $\frac{72}{29} = 2\frac{14}{29}$ days.

2. A. and B. together can do a piece of work in $2\frac{1}{2}$ days, A. and C. in $3\frac{1}{3}$ days, B. and C. in $3\frac{3}{4}$ days. How long will it take the three working together to do the work, and how long will it take each alone?

A. and B. in one day can do $\frac{1}{2\frac{1}{2}}$ of the work.

A. and C. in one day can do $\frac{1}{3\frac{1}{3}}$ of the work.

B. and C. in one day can do $\frac{1}{3\frac{3}{4}}$ of the work.

2A. + 2B. + 2C. in one day can do $\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{3}} + \frac{1}{3\frac{3}{4}}$ of the work.

A. + B. + C. in one day can do $\frac{1}{5} + \frac{1}{6\frac{2}{3}} + \frac{1}{7\frac{1}{2}}$ of the work $= \frac{1}{5} + \frac{3}{20} + \frac{2}{15} = \frac{12+9+8}{60} = \frac{29}{60}$ of the work.

In one day A. can do $\frac{29}{60} - \frac{1}{3\frac{3}{4}} = \frac{29}{60} - \frac{16}{60} = \frac{13}{60}$ of the work.

Hence A. can do $\frac{60}{60}$ in $\frac{60}{13}$ or $4\frac{8}{13}$ days.
Find time required by B. and C.

3. If it takes A., working alone, 4 days, B. 3 days, and C. $4\frac{1}{2}$ days to do a piece of work, how long will it take them to do the work if all three work together?

4. A. can do a piece of work in 10 days, A. and C. can do it in 7 days, and A. and B. can do it in 6 days. How long will it take them all to do it?

5. One pipe can fill a cistern half full in $\frac{3}{4}$ of an hour, and another can fill it three-quarters full in $\frac{1}{2}$ an hour. How long will it take both pipes together to fill the cistern?

6. Pipes A. and B. can fill a cistern in 3 minutes and 5 minutes respectively, and C. can empty it in $7\frac{1}{2}$ min. In what time will the cistern be filled when A., B., and C. are all open?

7. A., B., and C. together can do a piece of work in 10 days, A. and B. together in 12 days, B. and C. together in 20 days. How long will it take each alone to do the work?

8. A. does $\frac{4}{11}$ of a piece of work in 6 days, when B. comes along and helps him, and they finish it in 5 days. How long would it take B. alone to do the work?

9. A. and B. can do a piece of work in 4 days, A. and C. in 6 days, and A., B., and C. in 3 days. In how many days can each do the work alone?

10. A reservoir has two sluices, one of which alone would drain it in 7 hours and the other in 13 hours. How soon would it be emptied if both were opened together?

AVERAGING OF ACCOUNTS.

1. Find the average term of credit of the following account:

DR. JACOB HART. CR.

1898				1898			
Jan. 1	To Mdse	448	00	Jan. 20	Amt. br. forwd	560	00
Feb. 4	" Cash	364	00	Feb. 11	By 1 Carriage	264	00
" 20	" "	232	00	" 25	" Cash	900	00

Process.

DR.				CR.			
Due.	*Da.*	*Items.*	*Prod.*	*Due.*	*Da.*	*Items.*	*Prod.*
Jan. 1,	00	448	00000	Jan. 20,	19	560	10640
Feb. 4,	34	364	12376	Feb. 16,	46	264	12144
" 20,	50	232	11600	" 25,	55	900	49500
		1044	23976			1724	72284
						1044	23974
				Balances,		680	48308

$48308 \div 680 = 71$ da. Jan. $1 + 71$ da. $=$ March 13.

Explanation.

Jan. 1, the earliest date, was assumed to be the starting point or focal date. From Jan. 1 to Jan. 1 there are 0 days; from Jan. 1 to Feb. 4 there are 34 days; from Jan. 1 to Feb. 20 there are 50 days. On the credit side we proceed in a similar way, saying from Jan. 1 to Jan. 20 are 19 days, and so on. The balance of the products divided by the balance of the items gives the average term of credit, 71 da., which, added to Jan. 1, gives us March 13 as the day of payment.

2. When should interest begin on the following account:

DR.	JACOB JOHNSON.						CR.
1898				1898			
Jan. 1	To Mdse, 3 mo.	145	86	May 11	By Cash	11	00
" 12	" " 5 "	37	48	July 12	" "	15	00
June 3	" " 3 "	12	25	Oct. 12	" "	82	00
Aug. 4	" " 2 "	66	48				

Process.

DR.				CR.			
Due.	*Da.*	*Items.*	*Prod.*	*Due.*	*Da.*	*Items.*	*Prod.*
April 1,	00	145.86	0000.00	May 11,	40	11.00	440.00
June 12,	72	37.48	2698.56	July 12,	102	15.00	1530.00
Sept. 3,	155	12.25	1898.75	Oct. 12,	194	82.00	15908.00
Oct. 4,	186	66.48	12365.28				
		262.07	16962.59			108.00	17878.00
		108.00					16962.55
Bal. of Items,		154.07		Bal. of Prod.,			915.41

$915 \div 154.07 = 6$ da. April 1 — 6 da. $=$ March 26, 1898.

$$\left.\begin{array}{l}\text{Jan. } 1 + 3 \text{ mo.} = \text{April } 1\\ \text{Jan. } 12 + 5 \text{ mo.} = \text{June } 12\\ \text{June } 3 + 3 \text{ mo.} = \text{Sept. } 3\\ \text{Aug. } 4 + 2 \text{ mo.} = \text{Oct. } 4\end{array}\right\}$$ which shows April 1 to be the focal date. We now proceed as before.

It must be observed that when the balances are both on the same side of the account, the term of credit must be added to the focal date; otherwise subtracted.

3. Find the average term of credit of the following account:

DR. RICHARD STEVENS, *in Acct. with* HENRY BECK. CR.

1898				1898			
Apr. 10	To Mdse	150	00	Apr. 30	By Cash	250	00
" 30	" "	400	00	May 1	" "	200	00
May 16	" "	100	00	Jun. 27	" "	400	00
Jun. 24	" "	500	00				

4. What will be the cash balance of the following account Jan. 1, 1899, interest at 6%?

DR. ENOCH HOBSON. CR.

1898				1898			
July 10	To Mdse, 2 mo.	500	00	July 20	By Cash	400	00
Aug. 1	" " 3 "	700	00	Aug. 20	" "	1000	00
Sept. 9	" " 1 "	800	00				
" 20	" " 2 "	600	00				

REVIEW.

Simplify the following:

1. $\dfrac{3\frac{1}{5} + 2\frac{2}{3}}{\frac{1}{2} \times \frac{4}{9} \times \frac{1}{5}}$.

2. $\dfrac{3\frac{1}{4} - 2\frac{1}{2}}{1\frac{1}{2} \times \frac{1}{8}} \div \frac{9}{11}$.

3. $1\frac{3}{5} + 2\frac{3}{8} + \frac{3}{4} \times \frac{2}{5} \times \dfrac{1\frac{2}{9} - \frac{5}{7}}{2\frac{1}{2}}$.

4. $\dfrac{4\frac{2}{3} + 5\frac{1}{2}}{9\frac{3}{4} - 4\frac{2}{3}} \times \dfrac{\frac{3}{5} \times 2\frac{1}{6}}{130}$.

5. $\dfrac{26.7 - 11.80 + 6.45}{\frac{5}{8} \times 3\frac{11}{15} \times .72}$.

6. $\dfrac{3\frac{3}{4} - \frac{2}{7} \times 4.2}{\frac{5}{18} \div \frac{20}{27}}$.

7. $\dfrac{4\frac{3}{8} \times \frac{9}{14} + 1.8}{8\frac{4}{7} \div \frac{21}{46}}$.

8. $\dfrac{\frac{11}{27} \div (\frac{13}{9} + 2\frac{5}{6})}{.25 \times \frac{4}{7}}$.

9. $\dfrac{2\frac{1}{3} + 4\frac{1}{4}}{4\frac{1}{6}}$.

10. $1\frac{1}{8} + \frac{2}{5} \times 1\frac{1}{2} + 3 + \frac{7}{16}$.

11. $\dfrac{4\frac{3}{8} - 1\frac{3}{4}}{4\frac{2}{7} \times 5\frac{3}{5}}$.

12. $\dfrac{3\frac{1}{2} \times 2\frac{1}{3}}{4} - \dfrac{1\frac{2}{3}}{2\frac{1}{8} + 1\frac{1}{4}}$.

13. $\dfrac{\frac{3}{5} \times 1.25}{5\frac{3}{4} - 4.25}$.

14. $\dfrac{3\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}$.

15. $\dfrac{7\frac{7}{12} \times \frac{2}{13}}{1\frac{1}{2} - \frac{1}{6}}$.

16. $4\frac{1}{4}$ of $1\frac{1}{68}$ of $\frac{2}{23} + 3\frac{1}{2} + 2\frac{1}{4}$ of $1\frac{1}{3}$.

17. $\dfrac{3\frac{2}{3} \text{ of } 1\frac{7}{22}}{\frac{2}{5} \text{ of } 6\frac{2}{3}}$.

18. $\dfrac{1\frac{1}{12} \times \frac{11}{15}}{1\frac{1}{2} - 1\frac{1}{6}}$.

19. $\dfrac{2\frac{3}{4} \text{ of } 1\frac{1}{11} + \frac{8}{17} \text{ of } 2\frac{5}{6}}{2\frac{3}{4} \text{ of } 1\frac{1}{11} - \frac{8}{17} \text{ of } 2\frac{5}{6}}$.

20. $\dfrac{\frac{3}{5} + 5\frac{5}{6} \text{ of } \frac{2}{7}}{\frac{3}{5} \text{ of } 5\frac{5}{6} + \frac{2}{7}}$.

21. $\frac{.4}{5} + \frac{2\frac{5}{6}}{5\frac{2}{3}} + \frac{21}{48} + \frac{2\frac{4}{9}}{1\frac{7}{18}}$.

22. $\frac{4}{5\frac{1}{3}} \times 14\frac{1}{7} \times \frac{2\frac{3}{4}}{4} \times \frac{5}{7\frac{1}{3}} \times \frac{1\frac{1}{5}}{2\frac{3}{4}} \times 6$.

23. $\frac{\frac{4}{7} \text{ of } \frac{3}{8}}{5\frac{1}{14} - 4\frac{51}{56}} - \frac{\frac{5}{11} \text{ of } \frac{13}{15}}{7\frac{15}{22} - 6\frac{1}{2}}$.

24. $\frac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}}$.

25. $3\frac{1}{21} + 2\frac{3}{7} - 3\frac{3}{4} + 6\frac{1}{2} + 10\frac{2}{3}$.

26. $\frac{.0\dot{6}5\dot{4}}{1.2}$.

27. $\frac{7\frac{2}{5}}{2\frac{2}{5}} - \frac{2\frac{1}{3} + \frac{1}{33}}{1\frac{3}{11}}$ of $1\frac{5}{16}$.

28. $(2\frac{1}{9} + 1\frac{7}{8}) \div (2\frac{1}{9} - 1\frac{7}{8})$.

29. $\frac{2\frac{1}{2} + 3\frac{1}{4} - 4\frac{1}{7}}{1\frac{3}{7} \text{ of } 2\frac{1}{4}}$.

30. $\frac{1\frac{1}{8} + \frac{1}{4}}{1\frac{1}{8} - \frac{1}{4}}$.

31. $\frac{3\frac{1}{3} - 4\frac{1}{4} + 5\frac{1}{5}}{2\frac{2}{7} \text{ of } 3\frac{1}{5}}$.

32. $\frac{4\frac{1}{2} + 2\frac{2}{3}}{6\frac{2}{7} - 4\frac{1}{8} \text{ of } 1\frac{7}{33}} \div 4\frac{7}{9}$.

33. $\frac{3\frac{1}{4} - 2\frac{1}{3}}{3\frac{1}{2} + 3\frac{1}{6}} + 7\frac{1}{4}$.

34. $\frac{3 - 4\frac{1}{7} + 2\frac{1}{3}}{3 \times 2\frac{1}{3} - 4\frac{1}{7}}$.

35. $\frac{6\frac{1}{4} \text{ of } 4\frac{4}{5}}{11\frac{1}{5} - 6\frac{7}{16}}$.

36. $\frac{.04478256 \div 5.48}{.036 \times 2.043}$.

37. $1\frac{6}{7}$ of $1\frac{3}{4} + \frac{6\frac{2}{7} - 5\frac{1}{2}}{6\frac{2}{7} + 5\frac{1}{2}} - 2\frac{2}{3}$.

38. $6\frac{1}{4}$ of $1\frac{3}{5} + \frac{7}{10} + 2\frac{1}{2}$ of $\frac{3\frac{2}{5}}{1\frac{2}{15}} + 2\frac{1}{9}$.

39. $\frac{2\frac{1}{2} - \frac{5}{6}}{2\frac{1}{2} + \frac{5}{6}} - \frac{1}{4}$ of $29\frac{1}{3} + \frac{2\frac{5}{18}}{\frac{1}{3}}$.

40. $3\frac{1}{4} + 2\frac{1}{3}$ of $5\frac{1}{4}$ of $1\frac{2}{49} - 2\frac{1}{10} \times \frac{3}{7}$.

41. $3.01 - 5.\dot{3}1\dot{4} + 2.4$.

42. $\frac{5}{4} + 2\frac{23}{24} + \frac{13}{56} - \frac{1}{16} + ? = 5$.

43. $\frac{15\frac{3}{4} - 4\frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 23\frac{1}{3} + 2\frac{12}{36}}$.

44. $\frac{.02048}{.00003125}$.

45. $\frac{17\frac{1}{2}}{1\frac{1}{4}} \div 12\frac{5}{6} + \frac{4}{55}$ of $9\frac{3}{8}$.

46. $\frac{5\frac{1}{3} + 4\frac{3}{4}}{5\frac{1}{3} - 4\frac{3}{4}} \div 1\frac{4}{7}$ of $1\frac{4}{7}$.

47. $\frac{2\frac{4}{5} - 1\frac{1}{2} + 9\frac{1}{11}}{4\frac{1}{5} - 2\frac{1}{4} + 13\frac{7}{11}}$.

48. $.11\frac{1}{9} + .6666\frac{2}{3} + .222222\frac{2}{9}$.

49. $\frac{2\frac{1}{9} + 4\frac{5}{12}}{1\frac{3}{4} \times 3\frac{1}{3}} \div \frac{1}{6}$ of $.6\frac{5}{7}$.

50. $\frac{1}{4 + \frac{1}{4 + \frac{1}{4}}}$.

51. $\frac{1\frac{1}{2} \times 2\frac{1}{4} + 2\frac{1}{3} \times 5\frac{4}{9}}{2\frac{1}{4} - 1\frac{1}{2} \text{ of } 2\frac{1}{3} + 5\frac{4}{9}}$.

52. $\frac{2\frac{1}{2}}{2\frac{1}{6}}$ of $\frac{2}{3} + \frac{1\frac{1}{4} - \frac{11}{12}}{\frac{1}{6} + \frac{11}{12}} - \frac{2\frac{11}{15}}{2\frac{3}{5}}$.

53. $\frac{3\frac{1}{3} \text{ of } 4\frac{1}{3}}{(2\frac{1}{2} - \frac{1}{3}) \text{ of } (3\frac{1}{2} - \frac{1}{4})}$.

54. $\frac{.321 \times .321 - .179 \times .179}{.321 - .179}$ of \$5.

55. $\frac{.562 \times .562 - .188 \times .188}{.562 - .188}$ of \$75

56. $\frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{2} + \frac{1}{3}}$ of $\frac{\frac{1}{4} - \frac{1}{5}}{\frac{1}{4} + \frac{1}{5}}$ of $\frac{\frac{1}{6} - \frac{1}{7}}{\frac{1}{6} + \frac{1}{7}}$ of 585.

57. $\frac{(.00056542)^2}{(12.534)^2}$.

58. $5\frac{1}{3}$ of $\frac{1}{1\frac{1}{3} + \frac{1}{2\frac{1}{4}}} \div \frac{4\frac{1}{5} + 5\frac{1}{4}}{4\frac{1}{4} + 3\frac{2}{5}}$.

59. $\frac{2\frac{1}{3} - 3\frac{2}{3} + 2\frac{1}{24}}{\frac{7}{9} - \frac{8}{81} + \frac{23}{324} + \frac{1}{3} \text{ of } \frac{8}{9}}$.

60. $\dfrac{3\frac{3}{8} - \frac{27}{125}}{\frac{7}{9} \text{ of } 9\frac{1}{2} \text{ of } 8\frac{1}{3}} \div \dfrac{2\frac{1}{2} - 1\frac{3}{5}}{3\frac{1}{2} \text{ of } 4\frac{2}{9} \text{ of } 6\frac{1}{4}}.$

61. $\dfrac{7\frac{1}{2} + 1\frac{17}{22}}{8\frac{2}{3} + 3\frac{23}{33}} - \dfrac{2\frac{1}{9} - 1\frac{2}{3}}{1\frac{2}{5} \text{ of } 1\frac{3}{7}}.$

62. $\dfrac{2\frac{1}{3} \text{ of } 4\frac{1}{10} - (2\frac{1}{3} + 4\frac{1}{19})}{17\frac{1}{2} \text{ of } \frac{20}{57}}.$

63. $\dfrac{\frac{4}{13} \text{ of } 1.82 + \frac{3}{5} \text{ of } .35}{7} + 1.$

64. $\dfrac{1\frac{1}{15} \text{ of } 2\frac{5}{8}}{5\frac{1}{5} - 4\frac{1}{2}} \div \dfrac{2\frac{1}{2} + 1\frac{2}{3}}{81\frac{2}{3} - 62\frac{1}{2}}.$

65. $\dfrac{3.125}{6\frac{1}{4}}$ of 2 yd. 2 ft. $11\frac{1}{4}$ in.

66. $(\frac{2}{5} \text{ of } \frac{3}{7} + \frac{3}{7} \text{ of } 2\frac{1}{3}) \times \dfrac{\frac{2}{3} \text{ of } \frac{5}{6}}{\frac{5}{4} + \frac{4}{5}}.$

67. $3\frac{1}{6} \times \frac{8}{57} + \dfrac{6\frac{1}{6} - 5\frac{4}{5}}{6\frac{1}{6} + 5\frac{1}{5}} - 2\frac{1}{9} \div 5\frac{3}{7}.$

68. $\dfrac{1}{2 + \dfrac{1}{3 + \frac{1}{5}}}.$

69. $\dfrac{7\frac{1}{5} \times 5\frac{1}{7}}{7\frac{1}{5} - 5\frac{1}{7}} \div \dfrac{5\frac{1}{3} \times 3\frac{1}{5}}{5\frac{1}{3} + 3\frac{1}{5}}.$

70. $\dfrac{\frac{1}{2} + \frac{1}{3} - \frac{1}{6}}{\frac{1}{2} \text{ of } \frac{1}{3} \text{ of } \frac{1}{6}}.$

71. $\dfrac{2}{3 + \dfrac{4}{5 - \frac{6}{7}}}.$

72. $\dfrac{3\frac{7}{8} - 4\frac{1}{3} + 2\frac{1}{12}}{9\frac{1}{36} - 7\frac{19}{27}}.$

73. $\sqrt{\dfrac{5.29}{7.29} \times 20\dfrac{341}{841}}.$

74. $\dfrac{2\frac{5}{7}}{5} - 10\frac{5}{6} \div 1\frac{12}{13} + \frac{3}{7} \text{ of } 3\frac{3}{4} \text{ of } 3\frac{4}{5}.$

75. $\dfrac{7.5 + 3.75 - 6.375}{3.75 + 2.3 - 4.25}.$

76. $78\frac{1}{7} + 23\frac{4}{21} - (3\frac{5}{7} - 2\frac{26}{35}) \times \dfrac{\frac{5}{11}}{\frac{3}{14}}.$

77. $\dfrac{11\frac{1}{3} + 5\frac{1}{6} - 15\frac{1}{2}}{4\frac{1}{21} + 7\frac{19}{91} + \frac{1}{13} - 10}.$

78. $2\frac{2}{3} - \frac{6}{8} + \frac{4}{5} - \frac{20}{24} - \frac{2}{5} - \frac{1}{2}.$

79. $\dfrac{7\frac{6}{11}}{8\frac{7}{22}} \text{ of } \dfrac{5\frac{1}{12}}{81\frac{4}{11}} \div \dfrac{2\frac{5}{6}}{7\frac{1}{2}}.$

80. $\dfrac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} + 2\frac{1}{7}} \div \dfrac{\frac{3}{7} + \frac{1}{2}}{\frac{5}{7} - \frac{1}{2}}.$

81. $1\frac{2}{3} \text{ of } \dfrac{.53}{.6\dot{1}} \text{ of } \dfrac{.825}{.416}.$

82. $\dfrac{\frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}}{\frac{1}{2} + \frac{3}{4} + \frac{6}{7} + \frac{13}{14} + \frac{27}{28}}.$

83. $\dfrac{3\frac{1}{15} - 4\frac{3}{20} + 2\frac{1}{12}}{3\frac{1}{25} - 2\frac{1}{10}} \times \dfrac{3\frac{2}{15}}{\frac{2}{3} + \frac{3}{8} - \frac{4}{48}}.$

84. $\dfrac{5}{6 + \dfrac{7}{8 + \frac{10}{11}}}.$

85. $\dfrac{.0038425 - .00183}{.035}.$

86. $\dfrac{\frac{1}{3} \text{ of } 2.179 - \frac{5}{6} \text{ of } .8684}{\dfrac{2\frac{1}{3}}{5} - \dfrac{2}{2\frac{5}{8}} + \dfrac{1\frac{5}{6}}{4\frac{5}{7}}}.$

87. $\dfrac{6.757}{2.17\dot{4}\dot{2}} \times \dfrac{.\dot{2}5\dot{9}}{2.78}$ of 12s. $9\frac{3}{4}$d.

88. $\dfrac{.203 \times .0003 \times 16}{.008 \times .0029}.$

89. $\dfrac{\frac{629}{17} \text{ of } \sqrt{9\frac{43}{49}}}{\dfrac{215}{5\frac{3}{8}}} \div \sqrt[3]{67419143}.$

90. $\sqrt[3]{\dfrac{3\frac{7}{9} \times 1\frac{1}{7} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}}.$

91. $\sqrt[3]{\dfrac{\dfrac{1\frac{8}{9}}{11\frac{1}{3}} - 2\frac{5}{6} \text{ of } 1\frac{11}{34} + 4\frac{1}{4}}{\frac{2}{7} + 1\frac{8}{9} \div \frac{17}{18} \div \frac{1}{28}}}.$

92. $\dfrac{.5 \times .006}{\frac{9}{5} \times \frac{4}{5} \times (\frac{1}{4})^2} + \dfrac{\frac{1}{5} \text{ of } \frac{15}{16} \text{ of } (\frac{2}{3})^3}{1.6 + .625}.$

93. $\frac{1}{3} \times \frac{1 - \frac{1}{3}}{2} \times \frac{2 - \frac{1}{3}}{3} \times \frac{3 - \frac{1}{3}}{5} \times \frac{4 - \frac{1}{3}}{4}.$

94. $.01\dot{6} + 4.0808 - .0\dot{0}0\dot{8} + 50.1 - .1966.$

95. $\frac{\frac{3}{11}\left(\frac{2}{19} \text{ of } 2\frac{5}{7} + \frac{1}{3} \text{ of } 1\frac{2}{7}\right)}{\frac{2}{13} \times 1\frac{5}{8} \times 1\frac{1}{7} - \frac{1}{11}} \div \frac{\frac{1}{3}}{\frac{2}{9} \text{ of } 3 - \frac{1}{17} \times 5\frac{2}{3}}.$

96. $\frac{7}{11} \text{ of } \frac{\frac{19}{81} - \frac{9}{49} - \frac{1}{27}}{\frac{1}{2} + \frac{1}{14} - \frac{5}{9}} \text{ of } (3\frac{2}{5} + \frac{1}{2} - 2\frac{1}{10}).$

97. $\frac{1}{2} \text{ of } (\frac{1}{3} + \frac{1}{4} + \frac{1}{5}) + 7 \times (\frac{1}{35} + \frac{1}{36}) - \frac{1}{12} \div \left(\frac{11\frac{1}{2}}{15\frac{1}{3}} - \frac{1}{4}\right).$

98. $1\frac{4}{7} \text{ of } 5\frac{4}{11} - 1\frac{4}{23} \text{ of } 5\frac{4}{9} + 1\frac{45}{19} \text{ of } 2\frac{4}{17}.$

99. $\frac{3}{5}(\frac{3}{2} \times \frac{1}{6} + \frac{3}{8} \div \frac{2}{3}) - \frac{1\frac{2}{3}}{6\frac{2}{3}} \text{ of } \frac{19}{20} + \frac{3}{7} \text{ of } \frac{6\frac{5}{12}}{3\frac{2}{3}}.$

100. $\sqrt{7.4538 - 6.8 \div 8.5 - 2.03 \times 1.17}.$

101. $\frac{2}{3} \text{ of } \frac{\frac{1}{7} - \frac{1}{2} \text{ of } \frac{1}{11}}{\frac{1}{9} - \frac{1}{5} \text{ of } \frac{1}{7}} \text{ of } \frac{1}{2} \text{ of } \frac{\frac{7}{8} - \frac{3}{5} \text{ of } \frac{17}{18}}{\frac{3}{8} \text{ of } \frac{3}{7} - \frac{1}{14}}.$

MISCELLANEOUS PROBLEMS.

1. Prove the product will be the same in whatever order the factors be taken.

2. Prove that in division of fractions, multiplying the dividend by the divisor *inverted* will produce the quotient.

3. Show how you determine whether a given common fraction can be exactly expressed as a decimal, and give reasons.

4. What factors of two or more numbers must be combined to produce their greatest common divisor, and what ones to produce the least common dividend.

5. State a method of multiplying a fraction by a fraction, and demonstrate the correctness of the method.

6. On what theory was the length of the metre originally determined?

7. Explain the process of finding the greatest common divisor by division.

8. Prove that any common divisor of two numbers is a divisor of their sum and of their difference.

9. Explain a method of finding the greatest common divisor of two fractions.

10. Given interest, principal, and time, how may the rate be found?

11. Explain and illustrate a method of finding the least common dividend of fractions.

12. Indicate the following operations by signs in one connected expression: The sum of 3 and 4 multiplied by the difference between 9 and 5, and the product divided by 2 times 7. Perform the operations indicated.

13. Write a complex fraction. State the reasons for regarding it as complex. Reduce it to a simple fraction, and this result to a decimal.

14. Distinguish between a compound and a denominate number; also, interest and discount. Illustrate by examples.

15. Write a number which shall be at the same time simple, composite, abstract, and even. State why it fills each of these requirements.

16. Name the principal unit of length, of surface, of capacity, and of weight in the metric system, and show the relation among these units.

17. The sum of two numbers is 260 and their difference is 12; find the numbers and demonstrate the principle involved.

18. A. and B. can do a piece of work in three days; B. and C. can do it in four days; A. and C. can do it in six days; if all work together for the same length of time, what part of the sum paid to all should each receive?

19. Find the fourth term of the following proportion, and demonstrate the principle on which it is based: $8 : 12 = 10 : x$.

20. A cylindric vessel is 8 ft. in diameter; how deep must it be to contain 75 bbl. of water?

21. Find the square root of 104976, and give a reason for each step in the process.

22. Deduce a rule for finding the sum of an arithmetic series, and illustrate its use by finding the sum of ten terms of the series whose first term is 2 and whose common difference is 4.

23. Find the sixth root of 191102976, and show why you believe your method to be correct.

24. Indicate the following by signs: The difference of 9 and 5 is multiplied by 8, this product is divided by 10 and the quotient increased by 1, the sum is squared, increased by 2, and the cube root of the result taken.

25. A franc is worth 9.5d.; a mark is worth 11.7d.; a pound sterling is worth $4.86. Find the value of $100 in each of the three other currencies.

26. Explain and illustrate a method of finding the least common denominator of fractions.

27. Insert four geometric means between 3 and 96. Insert two arithmetic means between 3 and 96.

28. Assuming that iron is 7.8 times as heavy as water, find the weight in kilogrammes of a round bar of iron .60 centimetres in diameter and 3 metres long.

29. Find the cost in United States money of a bill of exchange on London for £12 15s. 9d., exchange being at $4.86.

30. A room 5^{m} long, 4^{m} wide, and 3^{m} high has opening from it one door 2^{m} high, $1\frac{1}{2}^{m}$ wide, and two windows, each $2\frac{1}{2}^{m}$ high, 1^{m} wide. Find the cost of plastering the walls and ceiling at 15 cts. a square metre, deducting half the openings.

31. In an arithmetic progression of 8 terms, the first term is 3 and the last is 31; find the remaining terms.

32. Show the exact value of the decimal .666 . . . . to infinity.

33. The length of a tank which holds 100 bbl. of water is twice its height, and its height is twice its width; find its dimensions to the nearest inch.

34. Loaned $6000 to be paid, with interest at 6%, in six equal annual instalments; what is the amount of each payment?

35. Explain a method of finding difference of longitude from difference of time, and show its application in finding the longitude of a place.

36. A bar of aluminum 2^{cm} thick and 2^{cm} wide weighs $1\frac{1}{2}^{kg}$; find its length, assuming that aluminum is $2\frac{1}{2}$ times as heavy as water.

37. State the process of finding the cost in U. S. money of a time draft on a foreign country, giving the reasons for each step.

38. Reduce the repetend $.16\dot{2}1\dot{3}$ to a common fraction in its lowest terms.

39. The diameter of a cylindric vessel is 42^{cm} and its depth is $6\frac{1}{2}^{dm}$; how many litres of water will it hold and how many kilos will this water weigh?

40. What factors of two or more numbers must be combined to produce their greatest common divisor, and what ones to produce the least common dividend? Give reasons.

41. Find the G. C. D. and the L. C. Dd. of $\frac{4}{5}$, $\frac{5}{9}$, $\frac{3}{4}$. Explain the process fully.

42. When it is Monday, 7 A.M., at San Francisco, longitude 122° 24′ 15″ W., what day and time of day is it at Berlin, longitude 13° 23′ 55″ E.?

43. A gallon contains 231 cu. in.; a cubic foot of water weighs 62.5 lb.; mercury is 13.5 times as heavy as water. How many gallons of mercury will weigh a ton?

44. Find the sum of $23.\dot{3}$, $42.\dot{6}\dot{1}$, $78.3\dot{4}5\dot{2}$.

45. Find the value of $\sqrt{2-\sqrt{2+\sqrt[3]{.8}}}$.

46. At how many minutes after 3 o'clock will the hour and minute hands of a watch be opposite each other?

47. A general formed his army into a solid square, and had 200 men left over; he then received a reinforcement of 1000 men, and, increasing each side of the square by 5 men, lacked 25 men to complete the square; how many men were there in the original army?

48. Find the cubic inches in a pail 12 in. deep, 16 in. wide at top and 12 in. at bottom.

49. A life annuity costs a person 44 yrs. old \$5933.35. Find the amount of the annuity, interest at $3\frac{1}{2}\%$.

50. A person investing in a 4% stock receives $4\frac{3}{8}\%$ for his money. What is the price of the stock?

51. A piece of work is to be completed in 30 days, and 15 men are employed upon it; at the end of 24 days the work is only half done. How many more men must be employed to fulfil the contract?

52. A man buys eggs at a certain price per score, and sells them at half that price per dozen. What is his gain or loss per cent.?

53. A.'s present age is to B.'s as 9 is to 5; three years ago the proportion was 10 to 3. Find the present age of each.

54. Simplify $1\frac{2}{3}$ of $\frac{.53}{.61}$ of $\frac{.825}{.416}$.

55. Simplify $\dfrac{5\frac{5}{6}}{6\frac{6}{7}+\dfrac{7\frac{1}{2}}{8\frac{1}{3}+3\frac{1}{4}}}$.

Some Commercial Laws Tabulated.

States and Territories.	Grace Allowed.	Legal Rate of Interest.	May Contract for	Penalty for Usury.	Arrest for Debt.
Alabama	Yes	8%		Forfeit interest	No
Arizona	Yes	7%	Any rate	None	No*
Arkansas	Yes	6%	10%	Forfeit claim	No*
California	No ‖	7%	Any rate	None	No*
Colorado	No	8%	Any rate	None	No*
Connecticut	No ‖	6%	Any rate	None	No*
Delaware	Yes	6%	Any rate	Forfeit principal	No*
District of Columbia	No	6%	10%	Forfeit interest	No
Florida	No	8%	10%	Forfeit interest	No
Georgia	Yes	7%	8%	Forfeit excess of int.	No
Idaho	No ‖	7%	12%	Forfeit int. & 10% of pr.	No*
Illinois	No ‖	5%	7%	Forfeit interest	No*
Indiana	Yes	6%	8%	Forfeit excess of int.	No*
Indian Territory	Yes	6%	10%	Void as to prin. and int.	
Iowa	Yes	6%	8%	Void as to interest	No*
Kansas	Yes	6%	10%	Forfeit interest over 10%	No*
Kentucky	Yes	6%		Forfeit interest	No*
Louisiana	Yes	5%	8%	Forfeit interest	No†
Maine	No	6%	Any rate	None	Yes
Maryland	No	6%		Forfeit excess of int.	No
Massachusetts	No	6%	Any rate	None	Yes
Michigan	Yes	6%	8%	Forfeit excess of int.	No‡
Minnesota	Yes	7%	10%	None	No
Mississippi	Yes	6%	10%	Forfeit interest	No
Missouri	Yes	6%	8%	Forfeit interest	No
Montana	No ‖	8%	Any rate	None	No*
Nebraska	Yes	5%	10%	Forfeit interest	No*
Nevada	Yes	7%	Any rate	None	Yes
New Hampshire	No	6%		Forft 3 times exc. & costs	Yes
New Jersey	No ‖	6%		Forfeit interest	No*§
New Mexico	Yes	6%	12%	Forfeit interest	Yes
New York	No ‖	6%		Forfeit claim	No*
North Carolina	Yes	6%		Forfeit interest	Yes
North Dakota	No ‖	7%	12%	Forfeit interest	No*
Ohio	No	6%	8%	Forfeit excess of 8%	No*
Oklahoma	Yes	7%	12%		
Oregon	No ‖	8%	10%	Forfeit claim	No*
Pennsylvania	No ‖	6%		Forfeit excess of int.	No*
Rhode Island	Yes	6%	Any rate	None	Yes§
South Carolina	Yes	7%	8%	Forfeit interest	Yes
South Dakota	Yes	7%	12%	Forfeit interest	No*
Tennessee	Yes	6%		Forfeit excess of int.	No
Texas	Yes	6%	10%	Forfeit interest	No
Utah	No ‖	8%	Any rate	None	Yes
Vermont	No ‖	6%		Forfeit excess of int.	Yes§
Virginia	Yes	6%		Forfeit interest	Yes
Washington	Yes	7%	12%	None	Yes
West Virginia	Yes	6%		Forfeit excess of int.	Yes
Wisconsin	No ‖	6%	10%	Forfeit interest	Yes
Wyoming	Yes	8%	12%	None	Yes§

* Except in cases of fraud. ‡ Except in cases of fraud and breaches of trust.

† Except to secure person of debtor to answer suit. § Except females.

‖ Forbidden absolutely.

ANSWERS TO APPENDIX.

DUODECIMALS.

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1. 65 ft. 11′ 3″.
2. 24 ft. 8′ 8″ 10⁗.
3. 693 ft. 11′ 2″ 8‴.
4. 22 ft. 1′ 3″ 7‴ 10⁗.
5. 640 ft. 8′ 5″ 10‴ 11⁗.
6. 6 ft. 7′ 1″ 9‴.
7. 7 ft. 10′ 0″ 6‴ 9⁗.
8. 147 ft. 9′ 2″ 3‴.
9. 107 ft. 6′ 4″ 9‴ 2⁗.
10. 11 ft. 10′ 7″.

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1. 17 ft. 9′ 6″ 11‴ 8⁗.
2. 448 cu. ft. 7′ 4″.
3. 927 sq. ft. 3′ 4″.
4. 74.7 cords.
5. 30 ft. 2′ 10″ 2‴.
6. $5.55\frac{1}{3}$.
7. $452\frac{193}{216}$ loads.

1. 20 ft. 7′ 5″.
2. 5 ft. 7′ 4″ 3‴, etc.
3. 20 ft. 3′.
4. $21.00.

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5. 1 ft. 8′.

METRIC SYSTEM.

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1. 57654000^{mm}; 5765.4^{dkm}; 5765400^{cm}; 576.54^{hm}; 576540^{dm}; 57.654^{km}.
2. 261.914^{m}.
3. 312.2^{m}.
4. 3600^{qm}; $.36^{qhm}$; $.0036^{km}$.
5. $294.
6. 304.72^{a}.
7. 57^{cdm}.
8. 321148.8.
9. 23.4^{st}.
10. 600^{hl}.
11. 540^{hl}.
12. 10^{hl} or 1000^{l}.
13. 3000000^{g}; $3,000,000,000^{mg}$; 300000^{dkg}; 30000000^{dg}.
14. 7068.6^{kg}.
15. 17999.982^{m}.
16. 1440.
17. $.08125^{a}$.
18. He loses $2\frac{46}{77}\%$.
19. $39.273.
20. $.800^{m}$.
21. 1932.69 sq. in.; 7989.46 cu. in.
22. Sea water 102.6^{kg}; milk 103.2^{kg}; sea-water 225.72 lb.; milk 227.04 lb.

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23. 3^{ccm}; $\frac{3}{16}$ cu. in.
24. 8250 yd. $= 4\frac{11}{16}$ mi.
25. 1.69^{ha}.
26. $2\frac{1}{12}^{m}$.
27. $2.79.
28. 16104.176^{m}.
29. 90507^{l}.
30. 7.5325^{l}.

FOREIGN EXCHANGE.

Page 410.

1. 5085.49\frac{1}{4}$.
2. $101939832.187.
3. 792590673.575 pesetas; $51030167.813.
4. $476.
5. 2433.25.
6. $3851.43.

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7. 2746.725 francs.
8. $3260.60.
9. $5.73.
10. $3625.45.
11. $2436.29.
12. $540.40.
13. $1547.00.
14. $1254.286.
15. $2894.43.

ARITHMETICAL PROGRESSION.

Page 418.

1. 36.	6. 645.
2. 134.	7. $25\frac{1}{4}$; $1287\frac{1}{2}$.
3. 57.	8. 2316.
4. 4.	9. 20200.
5. 156.	

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10. 103; 2704.	20. 5 yrs.
11. 486.	21. $14771.75.
12. $\frac{1}{4}$.	22. $20715.436.
13. $\frac{1}{486}$.	23. $18343.62.
14. 58.	24. $63131.22.
15. 110; 630.	25. $36001.188.
16. 2048.	26. $500.
17. $1.50073.	27. $360.
18. $3378.96.	28. $968.45.
19. $209707.90	29. $7416.39.

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1. $\frac{1}{3}$.	11. $\frac{2744}{3333}$.	21. $\frac{2539}{9900}$.
2. $\frac{4}{9}$.	12. $\frac{9}{11}$.	22. $\frac{4354}{4995}$.
3. $\frac{2}{3}$.	13. $\frac{284}{333}$.	23. $\frac{13}{37}$.
4. $\frac{4}{11}$.	14. $\frac{16}{111}$.	24. $3\frac{34}{825}$.
5. $\frac{7}{33}$.	15. $\frac{527}{999}$.	25. $\frac{17}{74}$.
6. $\frac{2}{111}$.	16. $\frac{1}{1111}$.	26. $2\frac{104}{555}$.
7. $\frac{251}{333}$.	17. $\frac{16}{3333}$.	27. $\frac{239}{555}$.
8. $\frac{8}{37}$.	18. $\frac{911}{11111}$.	28. $5\frac{137}{3300}$.
9. $\frac{59}{111}$.	19. $\frac{1}{3333}$.	29. $\frac{187}{222}$.
10. $\frac{26}{1111}$.	20. $\frac{27421}{111111}$.	30. $4\frac{1336}{4995}$.

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1.	1. $\frac{1}{60}$.	3. $\frac{5}{42}$.	5. $\frac{1}{780}$.
	2. $\frac{1}{240}$.	4. $\frac{1}{4}$.	6. $\frac{11}{540}$.
2.	1. 168.	3. $\frac{135}{7}$.	5. 95.
	2. $\frac{60}{3}$.	4. 693.	6. $\frac{154}{9}$.

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5. C., 0°; R., 0°.
6. F., 122°; R., 40°.
7. F., —4°; R., —16°.

2. 1. $32\frac{8}{11}$ min. past 6.
   2. $43\frac{7}{11}$ min. past 8.
   3. $5\frac{5}{11}$ min. past 1.
   4. $16\frac{4}{11}$ min. past 3.
   5. $49\frac{1}{11}$ min. past 9.
   6. $27\frac{3}{11}$ min. past 5.

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3. 1. $32\frac{8}{11}$ min. past 12.
   2. $49\frac{1}{11}$ min. past 3.
   3. $16\frac{4}{11}$ min. past 9.
   4. $27\frac{3}{11}$ min. past 11.

4. 1. $27\frac{3}{11}$ min. past 2.
   2. $5\frac{5}{11}$ min. past 4.
   3. $32\frac{8}{11}$ min. past 9.
   4. $43\frac{7}{11}$ min.; $11\frac{1}{11}$ min. past 11.

5. $5\frac{5}{11}$ min. past 2.

3. $1\frac{7}{29}$ days.

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4. $4\frac{17}{22}$ days.
5. $\frac{6}{13}$ of an hour.
6. $2\frac{1}{2}$ min.
7. A., 20 da.; B., 30 da.; C., 60 da.
8. B., 15 da.
9. B., 6 da.; A. and C., each 12 da.
10. 4 hr. 33 min.

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3. 29 da. fr. Apr. 10. 4. $1198.60.

1. 132.
2. $4\frac{8}{9}$.
3. $4\frac{151}{4200}$.
4. $\frac{1}{50}$.
5. 12.71.
6. $6\frac{4}{5}$.
7. $\frac{18081}{73600}$.
8. $\frac{2}{3}$.
9. $1\frac{29}{50}$.
10. $5\frac{13}{80}$.
11. $\frac{7}{64}$.
12. $1\frac{355}{648}$.
13. $\frac{1}{2}$.
14. $\frac{7}{8}$.
15. $\frac{7}{8}$.
16. $6\frac{7}{8}$.
17. $1\frac{13}{16}$.
18. $2\frac{23}{60}$.
19. $2\frac{3}{5}$.
20. $\frac{476}{795}$.

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21. $2\frac{311}{400}$.
22. $13\frac{1}{56}$.
23. 1.
24. $\frac{53}{122}$.
25. $18\frac{25}{28}$.
26. $.0\dot{5}4\dot{5}$.
27. $\frac{31}{48}$.
28. $16\frac{15}{17}$.
29. $\frac{1}{2}$.
30. $1.\dot{5}7142\dot{8}$.
31. $\frac{1799}{3072}$.
32. $1\frac{1}{6}$.
33. $7\frac{31}{80}$.
34. $\frac{5}{12}$.
35. $6\frac{88}{127}$.
36. $.\dot{1}$.
37. $\frac{13}{20}$.
38. $20\frac{14}{45}$.
39. 0.
40. $15\frac{1}{10}$.
41. $.09\dot{5}6\dot{8}$.
42. $\frac{209}{336}$.
43. $1\frac{1}{36}$.
44. 655.36.
45. $1\frac{17}{22}$.
46. 7.
47. $\frac{2}{3}$.
48. 1.
49. 10.
50. $\frac{17}{72}$.
51. $3\frac{5}{6}$.
52. $\frac{1}{39}$.
53. $2\frac{2}{39}$.
54. $2.50.
55. $56.25.
56. 1.
57. .000000002035.
58. $2\frac{3}{7}$.
59. $\frac{17}{21}$.

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60. $5\frac{53}{200}$.
61. $\frac{19}{36}$.
62. $\frac{259}{500}$.
63. 1.11.
64. $\frac{92}{5}$.
65. 1 yd. 1 ft. $5\frac{5}{8}$ in.
66. $\frac{20}{63}$.
67. $\frac{49}{558}$.
68. $\frac{16}{37}$.
69. 9.
70. 24.
71. $\frac{58}{115}$.
72. $1\frac{5}{22}$.
73. 3.848.
74. $1\frac{1}{60}$.
75. $2.708\frac{1}{3}$.
76. $99\frac{3}{11}$.
77. $\frac{3}{4}$.
78. $\frac{59}{60}$.
79. $\frac{913}{6086}$.
80. $\frac{159}{3146}$.
81. 2.8665.
82. $\frac{1}{4}$.
83. $3\frac{11}{23}$.
84. $\frac{14}{19}$.
85. .0575.
86. .02847 +.
87. 3s. 8d. $\frac{935}{1668}$.
88. 42.
89. $\frac{1}{140}$.
90. .98 —.
91. .238 +.
92. .058302.

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93. $\frac{22}{729}$.
94. $54.0000\dot{6}5\dot{8}$.
95. 1.
96. 1.
97. $\frac{223}{360}$.
98. $9\frac{1551}{2737}$.
99. 1.
100. 2.068501.
101. $1\frac{205}{572}$.