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A Little Bit of Evidence on the Intertemporal Dependence in the Volatility of Stock Prices

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In this note, I present a little bit of evidence on the temporal dependence in stock returns by focusing on second moments. Specifically, I find that the variance or volatility changes over time with some degree of persistence. When the volatility is high, it tends to remain high before returning to a normal level; and when volatility is low, it tends to remain low. This kind of behavior suggests that stock returns are not independent over time even though the series seem to be serially uncorrelated (zero autocorrelation coefficients.)[1]

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In the empirical literature on stock return distributions, there is much evidence supporting models in which the variance parameter changes randomly over time (see the papers by Blattberg and Gonedes (1974), Clark (1973), Epps and Epps (1976), and Kon (1984)). These studies and others have treated stock returns over discrete time intervals as subordinated processes: the stock return or the log of one plus the stock return is normally distributed with a directing process determining the variance each period. Blattberg and Gonedes note that if we take Brownian motion and randomize the variance of the process with an inverted gamma--2 process, the resulting distribution is a student t, which they apply to stock returns. Another approach is to use the mixture-of-normals model in which we first randomly draw mean and variance parameters from a set of possible parameter values and then generate stock returns using the normal distribution with the randomly drawn parameter values. In these applications, stock returns are independent over time: the variance parameter drawn this period is independent of the draw in any other period. In Feller's (1971, pp. 346-47) terminology, the directing process has "stationary independent increments."

If we were to compute monthly standard deviations for stock returns using the daily data, we would expect the monthly estimates to be distributed randomly around the unconditional variance if the underlying stock returns are independent over time.[2] If we look at these monthly standard deviations over time, what we see is a persistent pattern. In Figure 1, I have plotted the monthly standard deviations for the NYSE-AMEX value-weighted return series taken from the CRSP daily file. The sample period is July 1962 to December 1983 and the following calculation has been made for each month:

$$\hat{\sigma}_i^2 = \frac{1}{(T_i - 1)} \sum_{t=1}^{T_i} (\ln (1+R_{it}) - \hat{\mu}_i),$$

where $\hat{\mu}_i$ is the sample mean of $\ln (1+R)$ for month i. Alternatively, one could use daily highs and lows to compute more efficient extreme value estimators for the variances and standard deviations. I then treat the 258 estimates of the monthly standard deviations as a time series and compute the first order autocorrelation coefficient. The estimate for the NYSE-AMEX data is .5872. Whether we compute the non-Neumann ratio or a t-statistic using a standard error of $n^{-1/2}$, we shall reject the null hypothesis of serial independence at extremely low significance levels. Similar calculations have been made with the daily S&P 500 data for the same period and the autocorrelation coefficient for the monthly standard deviations is .6263.

For a second approach to test this phenomena in the data, I apply the autoregressive conditional heteroskedasticity (ARCH) model of Engle (1982) to monthly data on ex post excess returns on the market. For the market return, I use the value-weighted returns on the NYSE taken from the monthly CRSP file. For the risk-free return, I use the returns on one-month Treasury bills in Ibbotson and Sinquefield (1982) and I update the data by using prices quoted in the Wall Street Journal. The excess return is computed as follows:

$$RP_t = \ell n(1+R_{mt}) - \ell n(1+R_{ft}).$$

The sample period is 1947 to 1983. In this model we treat the expected excess return, the market risk premium, as a constant and use the sample mean as the estimator of this constant. I then apply Engle's LaGrange multiplier test to check for the ARCH disturbance. The following regression is run on the residuals, $u_t = RP_t - \hat{\mu}$:

$$u_t^2 = \underset{(.00017)}{.001780} + \underset{(.0476)}{.1248}\, u_{t-1}^2 + \underset{(.0478)}{.1019}\, u_{t-2}^2 + \underset{(.0476)}{.1008}\, u_{t-3}^2 + e_t$$

$F(3,436) = 7.09$

$R^2 = .0465$ $\qquad\qquad$ $TR^2 = 20.46$

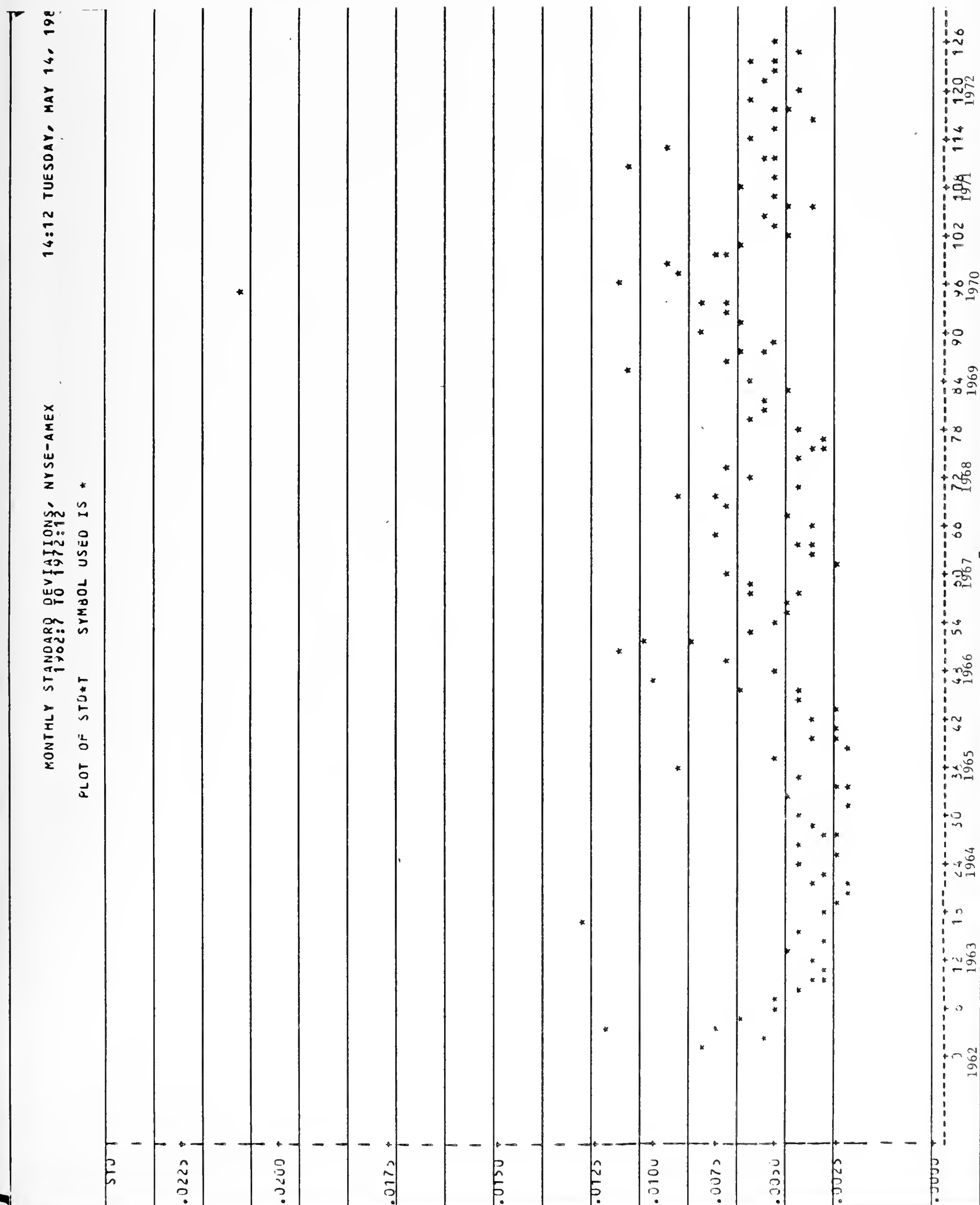
TR^2 is Engle's LaGrange multiplier test statistic, and in this application it is distributed as Chi-squared with three degrees of freedom under the null hypothesis of no conditional heteroscedasticity in the error term. The test statistic is significant at the 0.5% level, but the low R^2 for the regression suggests that these results are not as dramatic as those of the previous test.

The two tests outlined here indicate strong evidence of intertemporal dependence in the volatility of stock prices. This phenomena cannot be explained by models in which stock returns are distributed independently over time, which is the case with the class of subordinated processes which have been frequently applied to stock returns. One possible explanation is a diffusion process of the following form:

$$dP = \alpha P_t \, dt + \sigma_t P_t \, dz,$$

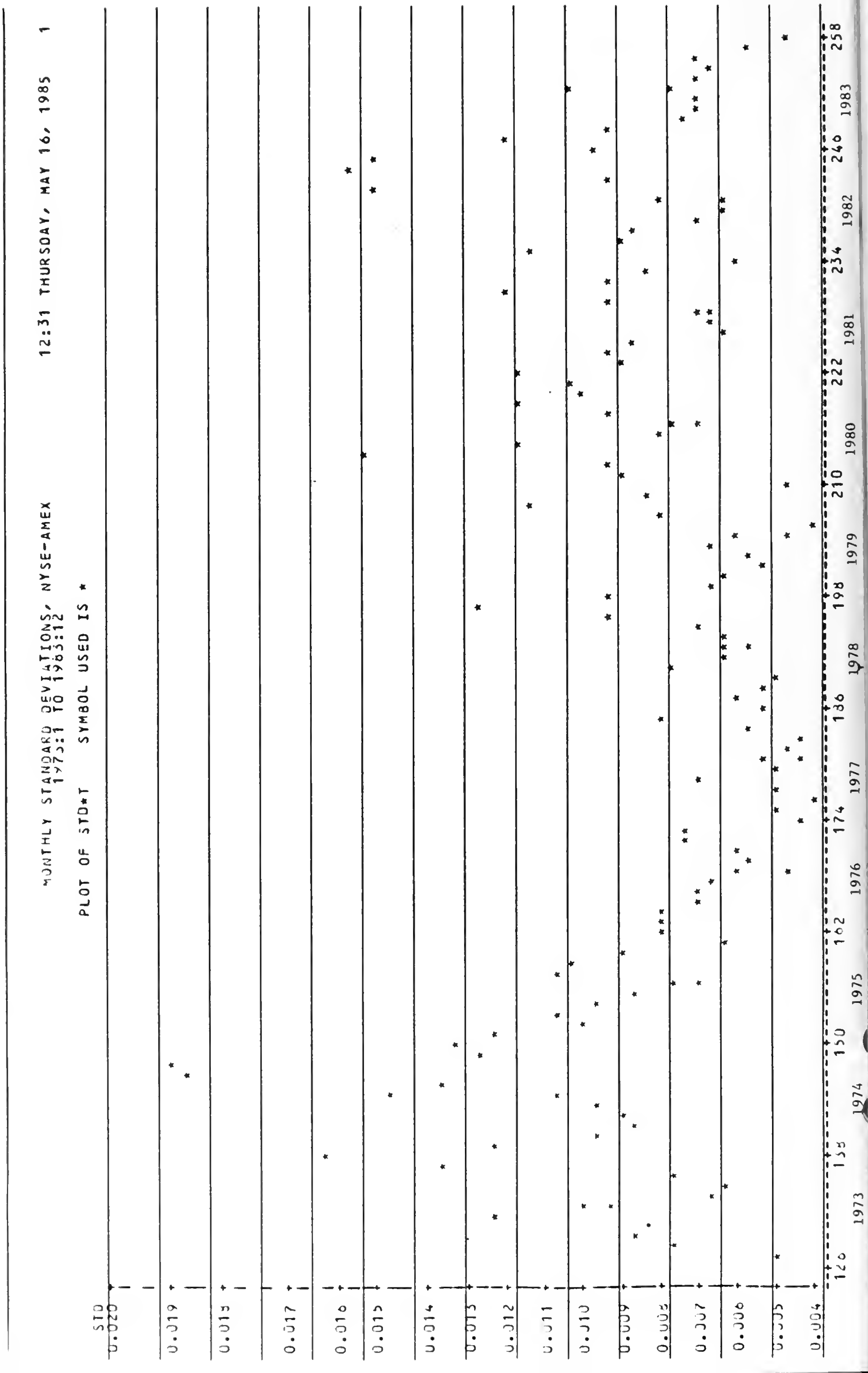
where σ_t is itself a diffusion process driven by a separate Brownian motion process, dq. In addition, one can easily incorporate a mean-reverting tendency in the standard deviation process. This kind of process for stock returns could be very important for valuing and understanding options, because the variance rate plays an important role in option pricing.

FIGURE 1



MONTHLY STANDARD DEVIATIONS, NYSE-AMEX
1962:7 TO 1972:12
PLOT OF STD*T SYMBOL USED IS *
14:12 TUESDAY, MAY 14, 19
STD
0.0225
0.0200
0.0175
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972

FIGURE 1 cont.



FOOTNOTES

1. Fama (1970) has documented empirical evidence on the lack of autocorrelation in stock return data.

2. Of course, we probably need to assume that the unconditional variances exist. So done.

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