MACHINE SHOP MATHEMATICS

Florian Cajori.

WENTWORTH-SMITH MATHEMATICAL SERIES

MACHINE-SHOP MATHEMATICS

BY

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PREFACE

Purpose of the Work. This work has been prepared to meet the needs of students who expect to become machinists, either in the special line of automobile construction or in the more general lines of the machine shop. It is therefore strictly limited in scope to the needs of those who are entering upon this kind of work, and it treats only of such topics as experience has shown are demanded by the practical machinist who is determined to advance in his vocation.

Work Presupposed. The student is supposed to have covered the work laid down in the authors' "Fundamentals of Practical Mathematics," or its equivalent, and therefore to be familiar with the use of whole numbers, common fractions, decimals, per cents, proportion, and the common tables of measure as applied to practical problems. While it is not an absolute essential that the student should have mastered the slide rule or should be thoroughly acquainted with the elements of trigonometry and with the metric system, it is desirable that he should have at least a fair working knowledge of these subjects.

Topics Considered. A glance at the Contents will show the topics considered, the relative amount of attention given to each, and the sequence in which they are taken up. In general it may be said that the choice of topics and the time allotted to each are conditioned by the actual needs of the student, while the sequence is based chiefly upon the question of relative difficulty, although due attention has been given to the dependence of one topic upon another. The first thing that is needed is a knowledge of the measuring instruments actually used in the machine shop, since without this knowledge the rest of the work is meaningless; the second topic relates to speeds and feeds, this being the first thing that the student meets in the use of a machine; the third topic, tapers and taper turning, follows naturally, and so on throughout the book.

The authors believe that they have succeeded here, as in their earlier book in this field, in eliminating nonessentials, in emphasizing the great principles, and in presenting the matter in a new but perfectly natural form, with definite and valuable applications which initiate the student into the actual work of the shop. They hope that their efforts will meet with the approval of teachers and students alike.

Acknowledgment. The authors wish to express their thanks to the following manufacturers who have given permission to use the illustrations shown on the pages mentioned: South Bend Lathe Works, South Bend, Ind., p. 8; R. K. LeBlond Machine Tool Co., Cincinnati, Ohio, p. 62; Cincinnati Milling Machine Co., Cincinnati, Ohio, p. 63; Brown and Sharpe Manufacturing Co., Providence, R.I., pp. 70, 81, 91, 99, 105, and 115; Meisel Press Manufacturing Co., Boston, Mass., pp. 97, 103, and 112; Niles-Bement-Pond Co., New York, N.Y., p. 110; Hoeffer Manufacturing Co., Freeport, Ill., p. 118; Ohio Machine Tool Co., Kenton, Ohio, p. 124; Lynd-Farquhar Co., Boston, Mass., p. 126; Garvin Machine Co., New York, N.Y., p. 128; and the Putnam Machine Works, Fitchburg, Mass., p. 132. Other well-known manufacturers supply similar machines of a high degree of precision, and they would, no doubt, have been equally willing to give permission to use their illustrations had it been requested.

CONTENTS

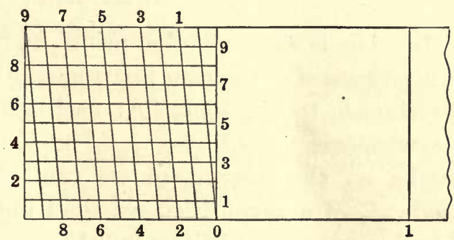
MACHINE-SHOP MATHEMATICS

CHAPTER I

MEASURING INSTRUMENTS

Measuring Lengths. In measuring short distances we take each distance with a pair of dividers, transfer it to a steel ruler, and then read off the length. Since, however, rulers are seldom graduated beyond $\frac{1}{64}''$, this method is not accurate enough for fine work.

For finer work a *diagonal scale*, which applies the principle of parallel lines to measuring lengths, is sometimes used by draftsmen. In the scale here shown the distance from the vertical line 1 to 0 is $1''$. From the vertical line 1 to the point where diagonal line 0 cuts horizontal line 8, the distance is $1.08''$; to the point where diagonal line 5 cuts horizontal line 6, the distance is $1.56''$; and so on, so that by this method we can readily measure to $0.01''$.

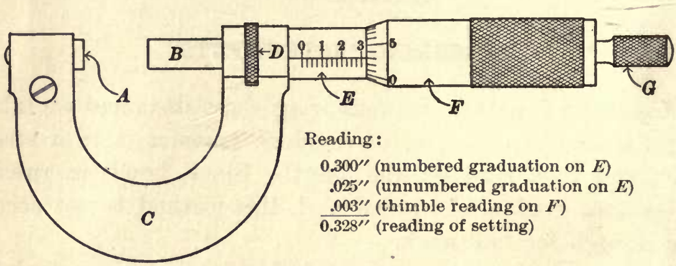


PART OF A DIAGONAL SCALE

For the work required of a skilled machinist, however, a higher degree of accuracy than is obtainable by either of the two methods given above is necessary. A method of obtaining this greater precision by the application of the principle of the screw thread is explained on page 2.

Caliper. A caliper is an instrument for measuring thickness or diameters. One of the most common forms is a pair of compasses with legs curved inward for measuring outside diameters and the thickness of plates. The legs may also curve outward for measuring inside diameters.

Micrometer Caliper. A caliper of the type shown in this figure is called a *micrometer caliper*, or simply a *micrometer.*



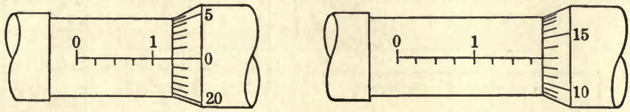
MICROMETER CALIPER

A, anvil; *B*, screw; *C*, frame; *D*, lock nut; *E*, barrel; *F*, thimble; *G*, ratchet stop

In this instrument the principle, which is also used in other types of measuring instruments, is that of a screw with 40 threads to the inch. At each revolution, therefore, the screw moves lengthwise $\frac{1}{40}''$, or $0.025''$. If by means of marks on the instrument we can tell when the screw has made $\frac{1}{25}$ of a revolution, we shall know when it has moved lengthwise $\frac{1}{25}$ of $\frac{1}{40}''$, or $0.001''$.

In the caliper shown above the thimble is attached to the screw and moves along the barrel. When the screw is against the anvil, the 0 on the thimble coincides with the 0 on the barrel. The barrel is graduated to $\frac{1}{40}''$, the numbered lines being $0.1''$ apart. Each revolution of the screw opens or closes the caliper $\frac{1}{40}''$, or $0.025''$, and therefore moves the thimble one graduation on the barrel. Each of the 25 graduations on the thimble indicates $\frac{1}{25}$ of $0.025''$, or $0.001''$.

Directions for Reading a Micrometer. In reading a micrometer caliper of the type shown on page 2 proceed as follows: *Count each numbered graduation on the barrel as 0.100″, each unnumbered graduation as 0.025″, and each graduation on the thimble as 0.001″. The total is the reading of the setting.*



MICROMETER READINGS

The reading of the figure at the left is 0.125″; that of the one at the right is 0.188″

The student should carefully examine the above illustration and the one given on page 2 and should verify the readings. It is only by actually using the instrument, however, that he will acquire the necessary facility in reading it.

Exercises. Reading the Micrometer Caliper

1. If the thimble of a micrometer caliper is given five revolutions, how far has the caliper opened or closed?

2. What is the diameter of a piece of stock when the reading of the numbered graduations on the barrel is 7 and the thimble reading is 22?

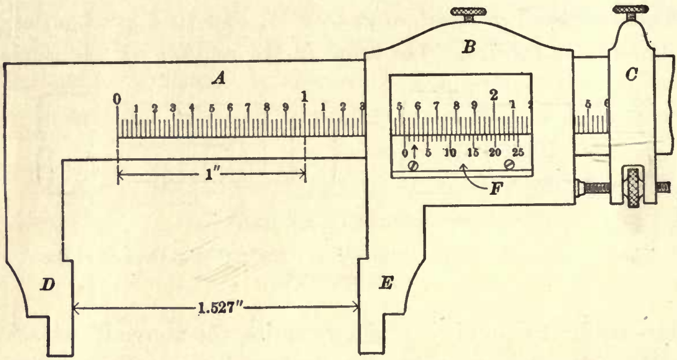
3. Reduce $\frac{7}{32}''$ to a decimal to the nearest 0.001″ and tell how you would set the caliper to have this reading.

4. How many turns must you give the thimble to measure a thickness of $\frac{5}{8}''$?

5. If, when the screw is against the anvil, the thimble is given $\frac{1}{5}$ of a turn, what is the reading of the micrometer?

6. How many turns must you give the thimble to have a reading of 0.375″? of 0.261″? of $\frac{3}{4}''$? of 0.955″?

Vernier Caliper. Calipers are also made with a device known as a *vernier*, by the aid of which we can read measurements to 0.001″. Such a caliper is illustrated in the figure below.



VERNIER CALIPER

A, beam; *B*, sliding head; *C*, clamp; *D*, solid jaw; *E*, sliding jaw; *F*, vernier

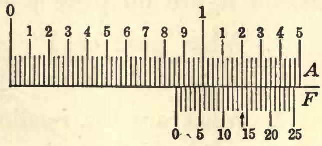
The vernier is based upon the principle that although the eye cannot easily decide upon a fractional part of a small space, it can readily tell when two lines appear to coincide.

On beam *A* each unnumbered division is $\frac{1}{40}$″, or 0.025″, while on the vernier *F* each unnumbered division is 0.024″. Hence 25 divisions on *F* are together equal to 24 divisions on *A*, and each division on *F* is 0.001″ shorter than each division on *A*. If, therefore, 0 on *F* is slightly to the right of a mark on *A*, but 1 on *F* coincides with a mark on *A*, then we see that 0 is 0.001″ to the right of the mark at its left.

In the above illustration, line 2 (indicated by the arrow) on *F* coincides with a line on *A*. The reading of the scale on *A* is 1.000″ + 0.500″ + 0.025″, or 1.525″. Adding the 0.002″ from *F*, we have 1.527″ as the reading of this setting.

For the clear understanding of this work the student should have a vernier caliper in his hands, setting it for various widths of objects to be measured and reading off the measurements.

Reading the Vernier. The detail of part of the scale of a vernier caliper is shown below and illustrates more clearly than the figure on page 4 the method of reading the vernier caliper. Since the smaller of the numbered divisions on *A* represent 0.1″, we first see that we have 0.8″ plus two unnumbered divisions of 0.025″ each, or 0.850″ in all.

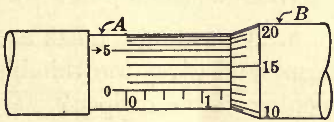


PART OF THE SCALE OF A VERNIER CALIPER

A, scale on beam; *F*, vernier scale

We also see that line 14 (indicated by the arrow) on the vernier scale coincides with a line on the scale on the beam. We therefore add 0.014″ to 0.850″, the result being 0.864″, the reading of this setting.

Micrometer Caliper with a Vernier. By combining the vernier and the micrometer we can obtain readings to 0.0001″. In the figure here shown a vernier of ten divisions is marked on the barrel *A* and occupies the same space as nine divisions on the thimble *B*. The micrometer reading is 0.137″, according to the explanation given on page 3.



THE SCALE OF A MICROMETER CALIPER WITH A VERNIER

A, barrel; *B*, thimble

Since line 5 (indicated by the straight arrow at the left on the barrel *A*) of the vernier coincides with a line on the thimble, we add 0.0005″ to 0.137″ and the reading of this setting is 0.1375″.

Thus, by the aid of a relatively simple device we can find the diameter of a rod or the thickness of a sheet of metal to a higher degree of accuracy than would seem possible if one did not know the ingenious mathematical principle involved in the vernier.

Exercises. Verniers and Micrometers

1. How would you set the vernier micrometer in the second figure on page 5 to read 0.4843″?

2. What part of 1″ is indicated by the sixteenth line from the zero mark on the beam of a vernier caliper?

3. What are the readings on the beam scale and on the vernier scale of a vernier caliper for a thickness of $\frac{7}{8}$″?

4. How would you set a vernier micrometer to read 0.5781″? to read 0.8704″?

5. What are the readings on the barrel, the thimble, and the vernier of a ten-thousandths micrometer for a setting of 0.1093″? for a setting of 0.2789″?

6. How far does the screw of a micrometer move at each revolution of the thimble?

7. How would you set a vernier micrometer to read $\frac{9}{16}$″?

8. A machinist takes a micrometer caliper which is fully closed and gives the thimble three revolutions. What is the opening of the caliper?

9. How would you set a ten-thousandths micrometer to read 0.1878″? to read 0.6593″?

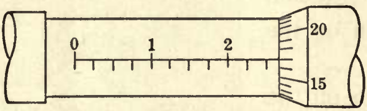
10. Through what part of a revolution must the thimble be turned to move the screw of a micrometer caliper 0.004″? to move the screw 0.017″?

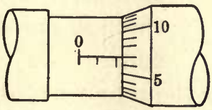
11. Which, if either, gives the more accurate measurement, a vernier caliper or a micrometer caliper? State fully the reason for your answer.

12. Find the distance between the anvil and the screw of a vernier micrometer when the reading of the numbered graduations on the barrel is 4, the thimble reading is 6, and the vernier reading is 3.

13. Find the distance as in Ex. 12 when the reading of the numbered graduations on the barrel is 7, that of the unnumbered graduations is 3, the thimble reading is 4, and the vernier reading is 6.

14. Find the reading of a micrometer caliper for each of the settings which are represented in the following figures:





15. In the following table let A represent the reading of the numbered graduations on the barrel of a vernier micrometer, B the reading of the unnumbered graduations on the barrel, C the thimble reading, and D the vernier reading. Copy the table and insert the number representing the reading of each setting in the blank spaces in the column marked E:

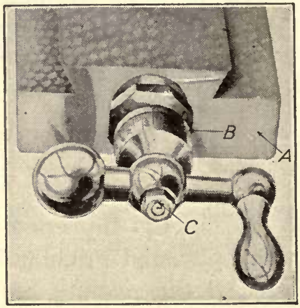
A	B	C	D	E
6	2	19	0	
1	0	6	2	
9	3	1	9	
7	1	20	4	
5	1	0	7	
0	0	21	1	
8	3	18	6	
6	0	0	3	
3	3	3	3	

Gaging the Depth of a Cut. The principle of the micrometer screw is used on various machines in gaging the depth of cut. The picture below shows a graduated collar which is attached to the cross-feed screw of a lathe. For convenience let

x = the number of inches in the endwise movement of the feed screw for each rotation through one division on the collar;

N = the number of divisions on the collar;

L = the number of inches in the *lead* of the screw; that is, the distance that the screw advances in a single turn.



GRADUATED COLLAR ON CROSS FEED

A, cross slide; *B*, graduated collar; *C*, cross-feed screw

We then have the formula

$$x = \frac{L}{N}.$$

Exercises. Graduated Feed Screws

1. Find the distance that a cross-slide feed screw with a lead of 0.2″ moves when the collar, which has 100 divisions, is turned through five divisions.

2. The table of a milling machine is moved by a double-thread feed screw the lead of which is $\frac{1}{4}$″. Into how many divisions should the collar of the feed screw be divided in order that the table shall advance 0.001″ when the collar is turned through one division?

3. The screw that raises the knee of a milling machine is a single-thread screw which moves the knee 0.001″ when the collar, which has 100 divisions, is turned through one division. Find the lead of the screw.

CHAPTER II

SPEEDS AND FEEDS

Cutting Speed. The *cutting speed* of a machine is always given in feet per minute (F.P.M.), but the expression has somewhat different meanings for different machines.

In turning work on a lathe it means the number of linear feet, measured on the surface of the work, which passes the edge of the cutting tool in one minute.

On a planer it means the rate in F.P.M. at which the work passes the tool.

On a shaper it means the rate in F.P.M. at which the tool passes the work.

On a milling machine it means the *surface speed* of the cutter; that is, the speed of a point on the rim of the cutter.

The cutting speed is not always the same, even on the same machine, but depends upon the following conditions:

1. The kind and quality of the material.
2. The kind and quality of the cutting tool.
3. The depth of the cut.
4. The *feed* of the tool, that is, the distance traveled sideways by the tool in one revolution of the work on a lathe, or a similar distance in other types of machines.
5. The lubrication.

Thus, on a milling machine the cutting speed may be 30 F.P.M. for steel, 50 F.P.M. for cast iron, and 90 F.P.M. for brass. On such a machine a heavy flow of oil is necessary on the cutter when milling steel, and a failure to provide for it not only changes the speed but materially affects the life of the cutter.

Cutting Speeds of Lathes. The following table gives the cutting speed in F.P.M. for roughing each of the materials specified with lathe tools of carbon steel or of high-speed steel:

MATERIAL	CARBON STEEL	HIGH-SPEED STEEL
Annealed tool steel .	20	45
Machine steel . . .	30	65
Wrought iron . . .	30	65
Cast iron	40	90
Brass	90	190

On finishing cuts the speed is 50% greater than it is in roughing.

Letting C be the cutting speed in F.P.M., R the number of R.P.M. of the work, D the diameter of the work in inches, and taking $3\frac{1}{7}$ as the value of π, we have these formulas:

$$C = \frac{\pi RD}{12} \qquad R = \frac{12\,C}{\pi D}$$

The factor 12 is necessary for reducing inches to feet.

Exercises. Cutting Speeds of Lathes

1. If a tool of high-speed steel is used, at how many R.P.M. should a cast-iron pulley 18″ in diameter be roughed?

Use the formula for R, cancel, and give the result to the nearest unit.

2. Find the speed of the finishing cut in boring the hub of a cast-iron pulley to a diameter of $2\frac{5}{16}$″ with a carbon-steel tool; with a high-speed steel tool.

3. Find the speed at which a lathe should run to rough a brass bushing $\frac{15}{16}$″ in diameter with a high-speed steel tool.

4. Some machine-steel axles are roughed with a carbon-steel tool in an axle lathe which has a fixed speed of 75 R.P.M. What is the diameter of each axle?

Cutting Speeds of Planers. Planers are usually so belted as to give only one cutting speed. This speed is usually about 25 F.P.M. and is used for brass, cast iron, and steel. Only the direct stroke is a cutting stroke, the return stroke being more rapid, usually two or three times as rapid as the cutting stroke. It is customary to say that the return stroke is then 2 to 1 or 3 to 1; and to speak of the planer as a 2-to-1, or a 3-to-1, planer.

In setting up the work on a planer a distance of 1″ is always allowed at each end of the work for the tool to clear.

Exercises. Cutting Speeds of Planers

1. How long will it take to make one cut the length of a cast-iron block 14′ 10″ long, the cutting speed of the planer being 25 F.P.M. and the return stroke being 3 to 1?

Allowing 1″ at each end of the work for the tool to clear, the total forward movement is 15′. Hence the forward movement will take $\frac{15}{25}$, or $\frac{3}{5}$, of a minute. How long will it take to make the cut?

2. A planer to be used for planing cast iron is belted to give a cutting speed of 45 F.P.M. If the return speed is 90 F.P.M., how many strokes per minute will the planer make when the length of the stroke is 2′ 6″? when it is 6′? when it is 8′ 9″? when it is 12′ 6″?

3. If a 2-to-1 planer makes six strokes per minute and the length of each stroke is 3′ 4″, what is the cutting speed?

4. How long will it take to make 36 cuts on an iron casting 43″ long, the cutting speed of the planer being 45 F.P.M. and the return stroke being 2 to 1?

5. On a casting 1′ 7″ long a 3-to-1 planer is making 12 strokes per minute. What is the cutting speed?

6. In Ex. 5 how many strokes per minute would be required to have the same cutting speed on a 2-to-1 planer?

Cutting Speeds of Milling Machines. The table below gives the cutting speeds in F.P.M. for milling each of the following materials on a milling machine with carbon-steel cutters:

Annealed tool steel	30	Cast iron	45
Machine steel	40	Brass	90

For high-speed steel cutters the speeds are 100% higher.

In finding the cutting speeds of milling cutters we use the formulas given on page 10, letting D be the diameter of the cutter in inches instead of the diameter of the work.

Exercises. Cutting Speeds of Milling Machines

1. Find the cutting speed of a side-facing cutter 6″ in diameter running at 30 R.P.M.

Use the formula for C given on page 10, and give the result to the nearest unit. Use cancellation whenever possible.

2. Find the cutting speed of a milling cutter $1\frac{7}{8}''$ in diameter running at 85 R.P.M.

3. Find the number of R.P.M. for milling a brass casting with a high-speed steel cutter 4″ in diameter.

4. Find the cutting speed of a milling cutter $2\frac{3}{4}''$ in diameter running at 55 R.P.M.

5. How many R.P.M. should a carbon-steel cutter $3\frac{1}{4}''$ in diameter make in milling a square on a machine-steel shaft?

6. Babbitt metal can be milled at a cutting speed of 120 F.P.M. Find the number of R.P.M. for a $4\frac{1}{2}$-inch cutter when milling a piece of this metal.

7. Using a carbon-steel cutter $5\frac{1}{2}''$ in diameter, find the number of R.P.M. for milling a cast-iron lathe bed.

Speeds of Drills. In the case of drills the number of R.P.M. for the drill is found by dividing a certain number called a *constant*, because it is always the same for the given material, by the diameter D of the drill. For example:

For machine steel, $\text{R.P.M.} = \frac{100}{D}$.

For cast iron, $\text{R.P.M.} = \frac{125}{D}$.

For brass, $\text{R.P.M.} = \frac{225}{D}$.

In these cases the constants are 100, 125, and 225 respectively.

The depth of cut for small drills is usually from 0.002″ to 0.005″ for each revolution, while for large drills it is from 0.005″ to 0.020″.

Exercises. Speeds of Drills

1. Find the number of R.P.M. for a $\frac{7}{8}$-inch drill when drilling a cast-iron lathe bed.

2. How fast should you run a drill $\frac{1}{16}''$ in diameter when drilling brass bearings?

3. When centering a machine-steel shaft with a $\frac{1}{4}$-inch drill, how fast should the drill rotate?

4. If a lathe has a fixed speed of 250 R.P.M., what is the largest diameter of drill that should be used on cast iron?

5. Find the number of R.P.M. for a #30 drill when drilling machine steel.

A #30 drill has a diameter of 0.1285″.

6. Find the number of R.P.M. for a $1\frac{3}{16}$-inch drill when drilling cast-iron surface plates.

7. Find to the nearest $\frac{1}{64}''$ the largest diameter of drill that should be used in drilling brass at 350 R.P.M.

Exercises. Review

1. Find the number of R.P.M. of a lathe for roughing the outside of the brass cylinder, which is shown in the blueprint on page 15, with a tool of carbon steel.

2. Find the speed of the finishing cut in boring the hole in the same cylinder with a high-speed tool.

3. Find the number of R.P.M. for drilling the center hole in the spur-gear blank.

4. Using a carbon-steel tool, find the number of R.P.M. for roughing the face *R* of the spur-gear blank.

5. If the spur-gear blank in Ex. 4 were made of machine steel instead of cast iron and if the tool were of high-speed steel, should the speed of the lathe be increased or should it be decreased when making the cut? How much should be the increase or decrease?

6. What should be the number of R.P.M. for drilling the hole in the machine-steel wheel shown in the blueprint?

7. Find the number of R.P.M. for a finishing cut over the outside of the steel wheel with a tool of carbon steel.

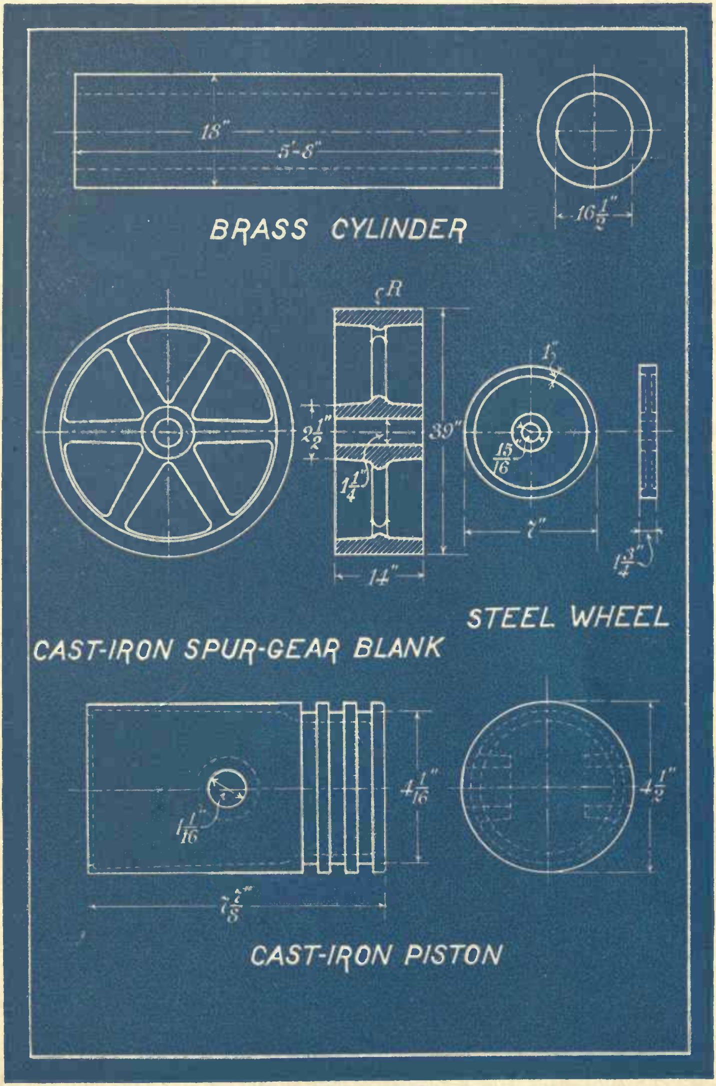
8. Find the number of R.P.M. for drilling the $1\frac{1}{16}$-inch hole in the cast-iron piston.

9. Using a high-speed tool, find the number of R.P.M. for roughing the outside of the piston.

10. Using a carbon-steel tool, find the number of R.P.M. for finishing the piston-ring grooves shown in the blueprint.

11. Find the number of R.P.M. for milling a brass casting with a high-speed cutter $7\frac{1}{2}''$ in diameter.

12. If a drill press is belted to make 365 R.P.M., find to the nearest $\frac{1}{64}''$ the largest diameter of drill that can be used when drilling cast iron.


18"
5'-8"
16 1/2"
BRASS CYLINDER
R
2 1/2"
39"
1 1/4"
14"
1"
15/16"
7"
1 3/4"
STEEL WHEEL
CAST-IRON SPUR-GEAR BLANK
4 1/16"
1 1/16"
4 1/2"
7 7/8"
CAST-IRON PISTON

Cutting Speed of Shapers. There are two leading kinds of shapers, namely, geared shapers and crank shapers.

The cutting speed of a geared shaper is found in the same way as that of a planer, as explained on page 11.

In the case of a crank shaper, however, the number of strokes per minute depends upon the speed of the cone pulley which drives the shaper. In this kind of shaper the return stroke has about twice the speed of the forward stroke; that is, it is said to be about 2 to 1.

For example, if a crank shaper is making 50 R.P.M., if the length of the stroke is 1′, and if the return stroke is twice as rapid as the forward stroke, what is the cutting speed of the shaper?

Since the return stroke is twice as fast as the forward one, it takes half as long to make it. Hence the forward stroke takes $\frac{2}{3}$ and the return stroke takes $\frac{1}{3}$ of the time required for one revolution.

In 1 min. there are 50 forward strokes of 1′ each and 50 return strokes of the same length, the 50 forward strokes taking $\frac{2}{3}$ of a minute and the 50 return strokes taking $\frac{1}{3}$ of a minute.

Hence the cutting speed is 50′ in $\frac{2}{3}$ min., or $\frac{3}{2}$ of 50′ in 1 min.

That is, the cutting speed is 75 F.P.M.

Exercises. Cutting Speeds of Shapers

1. The ram of a crank shaper makes 85 strokes per minute, and the length of the cut is 2″. Find the cutting speed.

In problems dealing with shapers take the ratio of speeds as 2 to 1, as given above, unless otherwise stated.

2. A crank shaper is cutting at the rate of 40 F.P.M. on a 14-inch cut. Find the number of R.P.M.

3. If a crank shaper makes 18 R.P.M. and the length of the cut is 9″, what is the cutting speed?

4. If a crank shaper is cutting a 4-inch casting at the rate of 50 F.P.M., what is the number of R.P.M.?

5. Solve Ex. 4 for a cutting speed of 40 F.P.M.

Expression of Feeds. Feeds can be expressed in several different ways, the following being the most common:

1. By the number of revolutions which the work makes while the tool is advancing 1″.

That is, we may speak of a feed as 75 revolutions to the inch.

2. By the number of inches that the tool advances along the work at each revolution or stroke.

In the above case we may express the feed as $\frac{1}{75}$″ per revolution.

Standard Feeds. The following are certain standard feeds in common use, each being expressed in inches per revolution:

MATERIAL	LATHE	PLANER	MILLING MACHINE	DRILLS
Annealed tool steel	$\frac{1}{75}$ to $\frac{1}{16}$	$\frac{1}{64}$ to $\frac{1}{8}$	0.002 to 0.015	0.002 to 0.015
Machine steel or wrought iron	$\frac{1}{60}$ to $\frac{1}{8}$	$\frac{1}{64}$ to $\frac{1}{4}$	0.005 to 0.025	0.002 to 0.015
Cast iron	$\frac{1}{50}$ to $\frac{1}{16}$	$\frac{1}{16}$ to $\frac{1}{2}$	0.005 to 0.075	0.005 to 0.020
Brass	$\frac{1}{40}$ to $\frac{1}{16}$	$\frac{1}{32}$ to $\frac{1}{8}$	0.002 to 0.125	0.005 to 0.020

Exercises. Feeds

1. A milling cutter revolves 85 times while the table is moving 1″. What is the feed in inches per revolution?

2. In 1 min. a lathe tool moves $1\frac{7}{8}$″ while the work makes 250 revolutions. What is the feed in revolutions per inch?

3. A shaper makes 15 strokes while the table moves $1\frac{1}{4}$″. What is the feed in inches per stroke?

4. A drill making 244 R.P.M. has a feed of 0.008″. How long will it take it to drill through a plate $2\frac{1}{2}$″ thick?

Exercises. Review

1. The bench block shown in the blueprint on page 19 is made of cast iron and $\frac{3}{16}''$ was left on the surfaces marked *f* for finishing to the dimensions given. If a roughing cut is taken with a $\frac{1}{8}$-inch feed and a finishing cut with a $\frac{1}{4}$-inch feed, how long will it take to plane the top of the casting on a 3-to-1 planer with a cutting speed of 25 F.P.M.?

In such problems find the results to minutes, calling any fraction a whole minute, unless otherwise directed.

2. The die-shoe dimensions given in the blueprint are for the rough machine-steel casting. Taking one cut with a 0.015-inch feed and using a carbon-steel side-milling cutter having a 3-inch face and a diameter of 4″, how long will it take to machine the entire surface of the shoe?

It will require six cuts on the top of the shoe, five on the bottom, one at each end, and one on each long side.

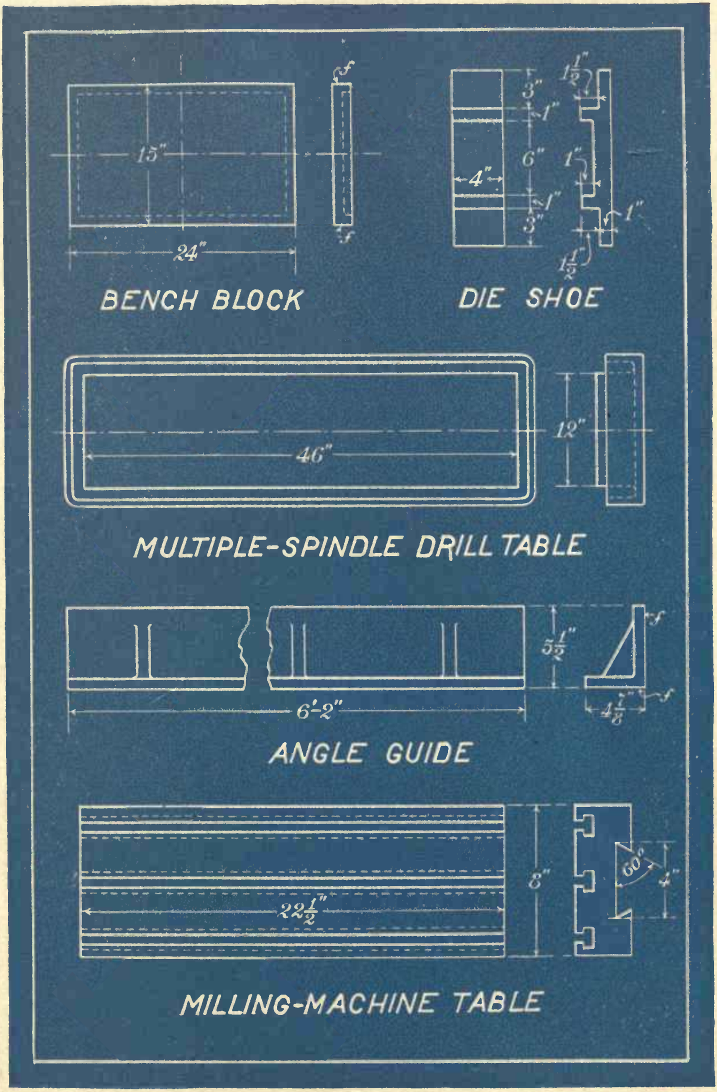
3. The multiple-spindle drill table is made of cast iron. Find the time required to plane the top on a 2-to-1 planer with a cutting speed of 25 F.P.M., giving it one roughing cut with a $\frac{3}{16}$-inch feed and one finishing cut with a $\frac{5}{16}$-inch feed.

4. Using a 2-to-1 planer with a cutting speed of 25 F.P.M. and a $\frac{1}{16}$-inch feed, how long will it take to make one cut over the two surfaces marked *f* on the machine-steel angle guide?

5. How many R.P.M. should be made by a carbon-steel T-slot cutter $1\frac{1}{8}''$ in diameter when milling the T-slots in the cast-iron milling-machine table?

6. What should be the speed of a 4-inch high-speed cutter when milling the 4-inch groove in the milling-machine table?

7. Calculate the time required to take a single cut over the top of the milling-machine table, using a $\frac{1}{32}$-inch feed on a 3-to-1 planer with a cutting speed of 25 F.P.M.


15"
24"
f
f
BENCH BLOCK
3"
1"
6"
1"
3"
4"
$1\frac{1}{2}$"
1"
1"
$1\frac{1}{2}$"
DIE SHOE
46"
12"
MULTIPLE-SPINDLE DRILL TABLE
6'-2"
$5\frac{1}{2}$"
f
$4\frac{1}{8}$"
f
ANGLE GUIDE
$22\frac{1}{2}$"
8"
60°
4"
MILLING-MACHINE TABLE

8. Find the time required to drill through a cast-iron plate $1\frac{9}{16}''$ thick with a $\frac{7}{8}$-inch drill and a feed of $0.008''$.

In this case give the result to the nearest second.

9. A machine-steel cylinder $4'\ 9''$ long is to be turned in a lathe to a diameter of $6\frac{1}{2}''$. Using a $\frac{3}{32}$-inch feed and a high-speed tool, how long will it take to make the roughing cut?

10. In Ex. 9, using a $\frac{1}{4}$-inch feed, how long will it take to make the finishing cut?

11. Using a carbon-steel cutter, how many R.P.M. should a $\frac{9}{16}$-inch end mill make while cutting the slots in a cast-iron milling-machine table?

12. Using a $\frac{1}{32}$-inch feed and a high-speed tool, how long will it take for a roughing cut over the face of a cast-iron pulley which has a diameter of $28''$ and a face $15''$ wide?

13. If a 2-to-1 planer makes eight strokes of $5'\ 6''$ per minute, what is the cutting speed of the planer?

14. Some machine-steel axles $9'\ 2''$ long are turned down to $3\frac{3}{8}''$ in diameter. If two cuts are made with a high-speed tool, the roughing cut with a $\frac{1}{20}$-inch feed and the finishing cut with a $\frac{1}{40}$-inch feed, how long should it take to finish 75 axles?

First find the time to the next full minute for each axle.

15. How many R.P.M. must a 2-to-1 crank shaper make to attain a cutting speed of 35 F.P.M. on a $7\frac{1}{2}$-inch casting?

16. How much time is needed to make a single cut with a $\frac{1}{32}$-inch feed over one side of an iron casting $22''$ wide and $8'\ 4''$ long on a planer whose forward speed is 35 F.P.M. and whose return speed is 60 F.P.M.?

17. Find the number of R.P.M. for drilling a cast-iron block $\frac{3}{4}''$ thick with a drill $\frac{1}{2}''$ in diameter. Find the feed necessary to drill through the block in 25 sec.

CHAPTER III

TAPERS AND TAPER TURNING

Taper. The difference in diameter of a piece for a unit of length is called the *taper* of the piece.

The piece may be round like a lathe center or flat like a taper gib.

The taper is usually stated by giving the difference in diameter in inches for a foot of length, as, for example, $\frac{1}{2}''$ per foot.

Common Tapers. The tapers most frequently used are:

Morse: approximately 0.625″ per foot (all sizes)
Brown and Sharpe: 0.500″ per foot (all except #10)
Jarno: 0.600″ per foot (all sizes)

Symbols. The following symbols or abbreviations are used in connection with tapers:

T. P. I. = taper per inch	D = larger diameter
T.P.F. = taper per foot	d = smaller diameter
T.P.L. = taper in any length	l = length of the taper

All the measurements are always stated in inches.

Formulas. The following formulas may be referred to in solving the exercises on tapers:

$$\text{T.P.I.} = \frac{\text{T.P.F.}}{12} \qquad D = d + \left(\frac{l \times \text{T.P.F.}}{12}\right)$$

$$\text{T.P.F.} = \frac{12(D - d)}{l} \qquad d = D - \left(\frac{l \times \text{T.P.F.}}{12}\right)$$

$$\text{T.P.L.} = \frac{l \times \text{T.P.F.}}{12} \qquad l = \frac{12(D - d)}{\text{T.P.F.}}$$

Exercises. Tapers

1. Find the T.P.F. of the taper collar shown in the blueprint on page 23.

2. If the taper of the end mill is 0.625″ per foot, what is the length of the tapered part?

3. If the T.P.F. of the milling-machine arbor is 0.500″, find the diameter of the larger end of the taper.

4. The T.P.F. of the lathe center is $\frac{9}{16}''$. Find the length A.

5. Find the T.P.F. of the flat drill shown in the blueprint.

6. Find the T.P.F. of a boring bar which is tapered 16″ of its length to diameters of $1\frac{3}{4}''$ and $\frac{7}{8}''$ respectively.

7. The shank of a drill is $3\frac{1}{2}''$ long, $D = 0.421''$, and $d = 0.375''$. Find the T.P.F.

8. A drill collet has a T.P.F. of 0.602″ and $d = \frac{5}{8}''$. Find the diameter at a point $3\frac{1}{2}''$ from the smaller end.

9. The following formulas, in which N is the number of the taper in the table, D is the diameter of the larger end in inches, d is the diameter of the smaller end in inches, and l is the length of the taper in inches, are used in a Jarno taper:

$$D = \frac{1}{8}N \qquad d = \frac{1}{10}N \qquad l = \frac{1}{2}N$$

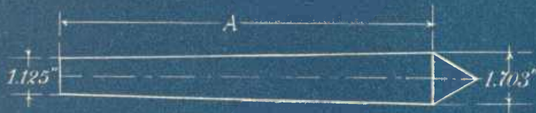
Copy the following table and fill the columns, using decimals:

N	D	d	l	N	D	d	l	N	D	d	l
1				7				13			
2				8				14			
3				9				15			
4				10				16			
5				11				17			
6				12				18			

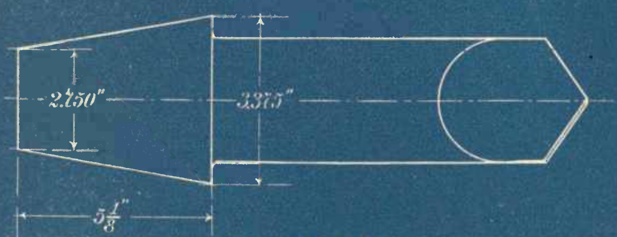
TAPER COLLAR

END MILL

MILLING-MACHINE ARBOR



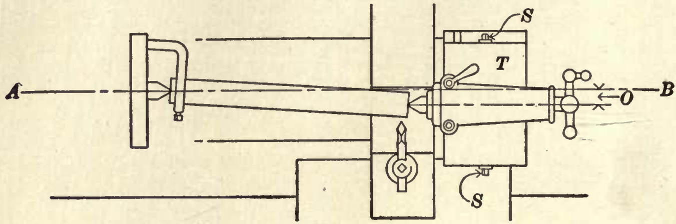
LATHE CENTER



FLAT DRILL

Taper Turning. There are three ways of turning tapers:

1. By offsetting the tailstock, that is, by moving the two centers out of alignment by means of the screws *S*, as shown.



OFFSET TAILSTOCK

A–B, center line of the lathe; *S*, *S*, screws for moving tailstock; *T*, tailstock; *O*, offset of tailstock center

To find *O*, the amount of offset, these formulas are used:

a. When the taper runs the entire length of a bar,

$$O = \frac{1}{2}(D - d).$$

For example, if a bar is to be turned taper to diameters of $1\frac{1}{4}''$ and $1''$,

$$O = \tfrac{1}{2}(D - d) = \tfrac{1}{2}(1\tfrac{1}{4}'' - 1'') = \tfrac{1}{2} \times \tfrac{1}{4}'' = \tfrac{1}{8}''.$$

b. When the taper runs only part of the length of a bar,

$$O = \frac{(D - d)L}{2l},$$

where *L* is the total length of the bar in inches.

For example, if a taper 6″ long with diameters of $2\frac{1}{2}''$ and 2″ respectively is to be turned on a bar 18″ long,

$$O = \frac{(D - d)L}{2l} = \frac{(2\frac{1}{2} - 2) \times 18''}{2 \times 6} = \frac{18''}{2 \times 2 \times 6} = \frac{3''}{4}.$$

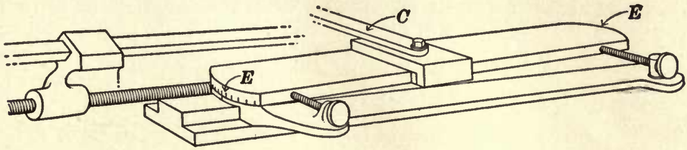
c. When part of a bar is to be tapered to a given T.P.F.,

$$O = \frac{1}{24}(\text{T.P.F.} \times L).$$

For example, if a T.P.F. of 0.6″ is to be turned on a bar 12″ long,

$$O = \tfrac{1}{24}(\text{T.P.F.} \times L) = \tfrac{1}{24} \times 0.6 \times 12'' = 0.3''.$$

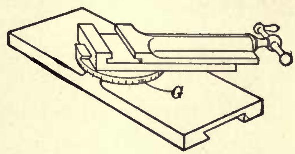
2. By using the taper attachment shown below, which causes the tool to feed transversely at the same time that it



TAPER ATTACHMENT

The graduations on the ends *E*, *E* are in T.P.F., and *C* is attached to the cross slide of the lathe. The graduations at one end are usually in tenths and at the other end in eighths

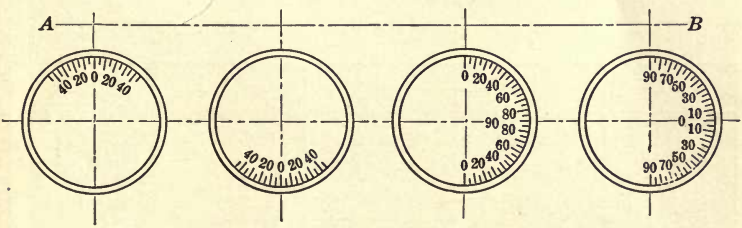
feeds longitudinally, thus turning a taper. This attachment can be used for turning either external or internal tapers. In using a taper attachment neither the tailstock offset nor the distance between centers need be considered.



COMPOUND REST

The base *G* is graduated in degrees

3. By using the compound rest, one form of which is here shown. Since the tool can be set at any desired angle, very steep tapers can be cut with the compound



METHODS OF GRADUATING THE BASE OF A COMPOUND REST

The line *A–B* represents the center line of the lathe

rest. The base of the rest is usually graduated in degrees in one of the four ways shown in the figure above.

The method of setting the compound rest is described on page 28.

Exercises. Offset Method

1. Find the T.P.F. of the drill-holder handle shown in the blueprint on page 27.

2. In Ex. 1 how far should the tailstock center be offset in order to turn the handle to the required taper?

3. The crowned pulley is to be turned on a 10-inch arbor. What distance should the tailstock center be offset?

4. Find the T.P.F. of the crowned pulley.

5. If the thimble is to be turned on a $6\frac{1}{2}$-inch mandrel, find the T.P.F. and the distance which the tailstock center should be offset in turning the taper.

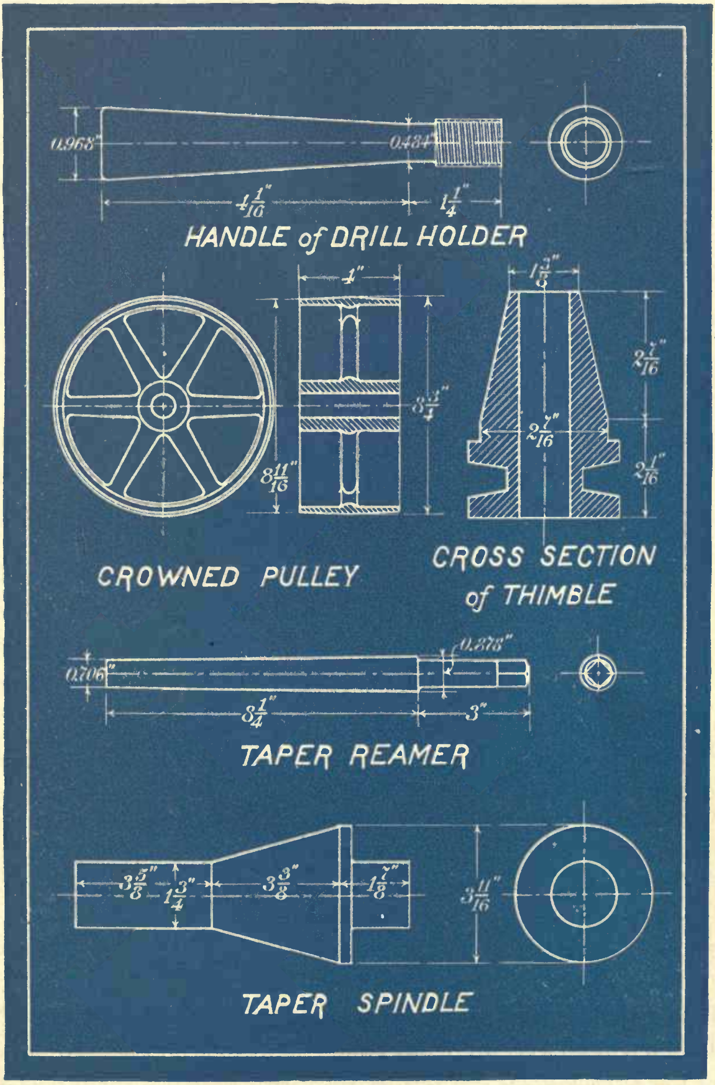
6. In turning the taper reamer, what should be the offset of the tailstock center? What is the T.P.F.?

7. In turning the taper spindle, what should be the offset of the tailstock center?

8. A taper 15″ long with diameters of 1.234″ and 0.984″ respectively is to be turned on a shaft 17″ long. Find the offset for the tailstock center.

9. Copy the following table of measurements of Morse standard taper reamers and supply the missing numbers:

No.	*D*	*d*	*l*	T.P.F.
0	0.369″	0.252″	$2\frac{1}{4}$″	
1		0.369	$2\frac{13}{16}$	0.600″
2	0.741		$3\frac{3}{8}$	0.602
3	0.979	0.778		0.602
4	1.280	1.020		0.623
5	1.790	1.475	6	
6	2.559	2.116	$8\frac{1}{2}$	
7		2.750	12	0.625


0.968"
0.484"
$4\frac{1}{16}$"
$1\frac{1}{4}$"
HANDLE of DRILL HOLDER
4"
$1\frac{3}{8}$"
$8\frac{3}{4}$"
$8\frac{11}{16}$"
$2\frac{7}{16}$"
$2\frac{7}{16}$"
$2\frac{1}{16}$"
CROWNED PULLEY
CROSS SECTION of THIMBLE
0.878"
0.706"
$8\frac{1}{4}$"
3"
TAPER REAMER
$3\frac{5}{8}$"
$1\frac{3}{4}$"
$3\frac{3}{8}$"
$1\frac{7}{8}$"
$3\frac{11}{16}$"
TAPER SPINDLE

M

Angles. A steep taper is usually referred to as an *angle*, and in designating the angle we generally give either the *included angle* or the *angle with the center axis.* In the figure below the angle *DAC* is the included angle and the angle *a* is the angle with the center axis. The following formula is used in finding *a*, the angle with the center axis:

$$\tan a = \frac{D - d}{2\,l}$$

The included angle is found by multiplying *a* by 2.

For example, if $D = 0.875''$, $d = 0.250''$, and $l = 1\frac{1}{4}''$, we have

$$\tan a = \frac{D - d}{2\,l} = \frac{\frac{7}{8} - \frac{1}{4}}{2 \times 1\frac{1}{4}} = \frac{1}{2} \times \frac{4}{5} \times \frac{5}{8} = \frac{1}{4} = 0.2500.$$

From the table on page 146 we find that, to the nearest minute,

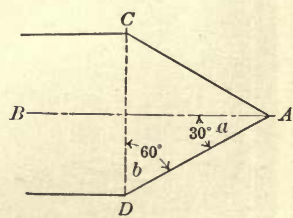
$$0.2500 = \tan 14^\circ\, 2'.$$

Therefore *a*, the angle with the center axis, is 14° 2′, and 2 *a*, the included angle, is 28° 4′.

If the taper per foot is given, the above formula becomes

$$\tan a = \frac{\text{T.P.F}}{24}.$$

Setting the Compound Rest. To turn a given angle the compound rest shown on page 25 is swiveled from its zero position, which is perpendicular to the center line of the lathe, so that the axis of the feed screw of the rest shall be parallel to the side of the angle. Thus, to turn angle *a* in this figure the rest is swiveled so that the graduated base is set at the complement of *a*; that is, at 90° − 30°, or 60°. To turn angle *b*, which is measured from the perpendicular to the center axis, the rest is set direct; that is, at 60°.



Exercises. Use of the Compound Rest

Find the included angles for the following tapers per foot:

1. $0.500''$. **2.** $0.602''$. **3.** $\frac{9}{16}''$. **4.** $0.750''$.

5. A drill collet is to be bored taper by the aid of a compound rest. If it is required that $D = 1\frac{1}{16}''$, $d = \frac{13}{16}''$, and $l = 4\frac{1}{8}''$, find the angle made with the center axis.

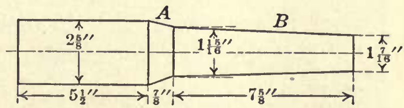
Find the angles made with the center axis in these tapers:

6. $D = 3\frac{1}{2}''$, $d = 1\frac{1}{4}''$, $l = 1''$. **8.** $D = 2\frac{1}{2}''$, $d = 1\frac{1}{2}''$, $l = 3''$.

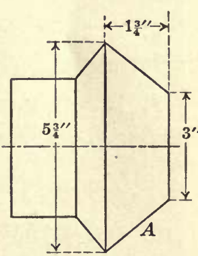
7. $D = 3''$, $d = 1''$, $l = 2\frac{1}{2}''$. **9.** $D = 4.7''$, $d = 3''$, $l = 3.2''$.

10. In turning the tapers *A* and *B* on the taper bearing shown in this figure, at what angles should the compound rest be set?

First find the angle which each taper makes with the center axis of the bearing.



11. Find the T.P.F. of taper *A* on the bevel-gear blank here shown. At what angle should the compound rest be set to cut the taper?



12. If in a taper socket $D = 0.435''$, $d = 0.300''$, and $l = 2\frac{3}{4}''$, find the angle made with the axis.

13. If in the taper shank of a drill $D = 0.475''$, $d = 0.388''$, and $l = 1\frac{3}{4}''$, find the included angle.

14. A counter bore has a taper shank in which $D = 0.416''$, $d = 0.312''$, and $l = 1\frac{5}{8}''$. Find the included angle.

15. A bevel-gear blank is to be turned to an included angle of 134°. Find the T.P.F. of the blank and the angle at which to set the compound rest.

Exercises. Review

1. The center punch shown in the blueprint on page 31 is to be turned in a chuck with the aid of a compound rest. Find the angle which each taper makes with the axis.

2. The rotary oiler shown in the blueprint is used to oil the ways of a planer. Find the included angle.

3. In the drill socket shown in the blueprint find the T.P.F. and the angle made with the axis.

4. In the speed-lathe center shown in the blueprint find the T.P.F. and the included angle.

5. A milling-machine arbor 18″ long is to be tapered $4\frac{1}{2}''$ on one end to 0.500″ per foot. Find the offset of the tailstock center for turning the required taper.

6. A reamer has a T.P.F. of 0.625″ and its diameters are 0.500″ and 0.750″ respectively. Find l.

7. If the maximum offset of the tailstock center of a lathe is $1\frac{1}{4}''$, what is the greatest T.P.F. that can be cut on a shaft 6″ long? on a shaft 2′ 3″ long? on a shaft 3′ 6″ long?

8. A taper bushing has diameters of $2\frac{3}{8}''$ and $1\frac{7}{8}''$, respectively, and a T.P.F. of 0.500″. Find l.

9. In the taper shank of a cutter which has a T.P.F. of 0.600″, $d = \frac{15}{16}''$ and $l = 3\frac{5}{8}''$. Find D.

In each of the following find the angle made with the axis:

10. $D = 0.354''$, $d = 0.279''$, and $l = 3\frac{5}{8}''$.

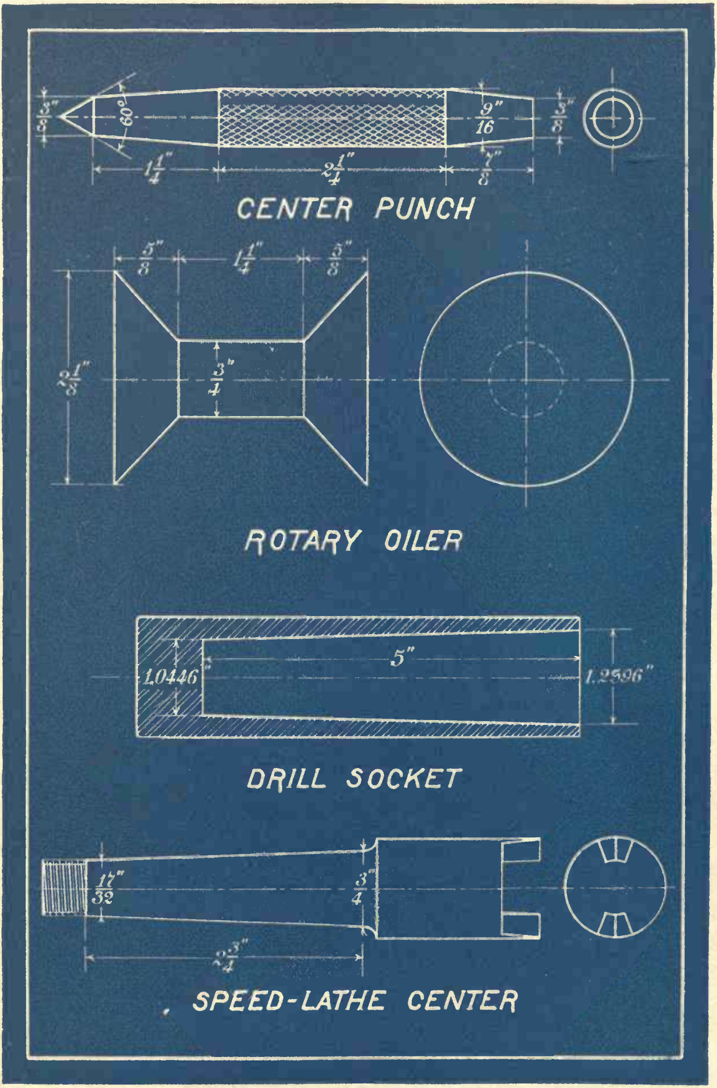
11. $D = 1.125''$, $d = 0.900''$, and $l = 4\frac{1}{2}''$.

12. $D = 1.231''$, $d = 1.020''$, and $l = 4\frac{1}{16}''$.

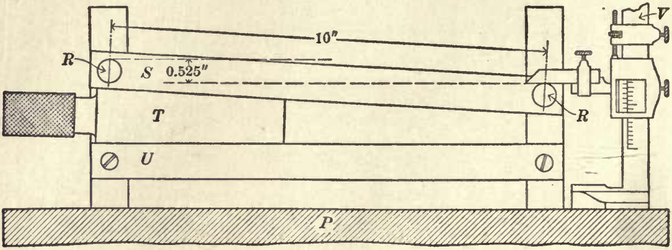
13. $D = 2.494''$, $d = 2.116''$, and $l = 7\frac{1}{4}''$.

14. $D = 1.2888''$, $d = 1.0446''$, and $l = 5\frac{1}{16}''$.

15. $D = 3.976''$, $d = 2.165''$, and $l = 3\frac{1}{8}''$.


60°
3/8"
9/16"
3/8"
1 1/4"
2 1/4"
7/8"
CENTER PUNCH
5/8"
1 1/4"
5/8"
2 1/8"
3/4"
ROTARY OILER
5"
1.0446"
1.2596"
DRILL SOCKET
17/32"
3/4"
2 3/4"
SPEED-LATHE CENTER

Measuring Tapers with a Sine Bar. An instrument known as a *sine bar* is often used to measure the angle of a taper.



MEASURING TAPERS WITH A SINE BAR

P, scraped surface plate; *R*, *R*, plugs; *S*, hardened-steel sine bar; *T*, taper plug gage; *U*, straight edge; *V*, vernier height gage

The taper to be measured is placed on the straight edge *U*, which is parallel to the surface plate *P*, and the sine bar *S*, which has two plugs *R*, *R* set 10″ apart, is clamped along the taper. Then *r*, the difference in height in inches between the plugs, is found by means of the height gage *V*. Letting *A* be the included angle, we have the following formulas:

$$\sin A = \frac{r}{10} \qquad r = 10 \sin A$$

For example, in the above figure $r = 0.525''$, and we have

$$\sin A = \frac{r}{10} = \frac{0.525}{10} = 0.0525\text{; whence } A = 3^\circ\,1'.$$

Therefore the included angle of the taper plug gage is $3^\circ\,1'$.

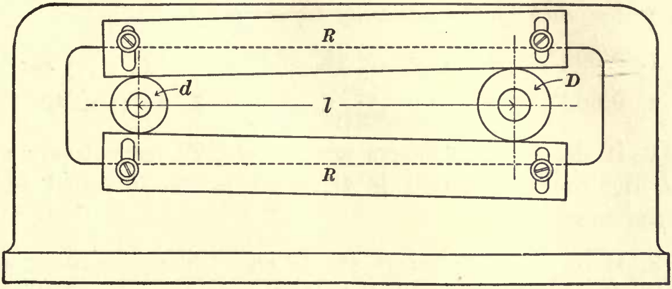
Testing Tapers. To test a taper for a given angle, the difference in height *r* of the plugs is found from the second formula, and bar *S* is set to this distance by means of the height gage. The taper is then tested between bars *S* and *U*.

For example, what should be the difference in height of the plugs for testing a taper which is to have an included angle of $26^\circ\,30'$?

We have $$r = 10 \sin A = 10 \times 0.4462 = 4.462.$$

Hence the difference in height of the plugs should be 4.462″.

Measuring Tapers with Disks. The angle of a taper may also be measured by means of two disks of unequal diameters.



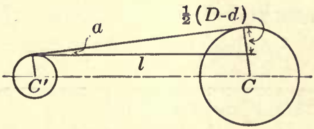
MEASURING TAPERS WITH DISKS

R, *R*, hardened-steel edges; *D*, *d*, disks of different diameters; *l*, distance between centers of disks

The disks are placed as shown above, and the straight edges *R*, *R*, which are made of hardened steel and carefully ground, are adjusted so that the tangent lines form the taper.

Taking *a* as the angle with the center axis, *D* as the larger diameter, *d* as the smaller diameter, and *l* as the distance between the centers, as shown in the figure below, we have

$$\sin a = \frac{\frac{1}{2}(D-d)}{l};$$



whence $$\sin a = \frac{D-d}{2l}.$$

Angle *a* can then be found from a table of sines, and from it we can find $2a$, the included angle of the taper.

Furthermore, from the formula for sin *a* we have

$$l = \frac{D-d}{2 \sin a},$$

so that, given *D*, *d*, and the angle with the axis, we can find *l*.

Exercises. Measuring Tapers

Find the difference in height of the plugs for setting a sine bar to test each of the following tapers per foot:

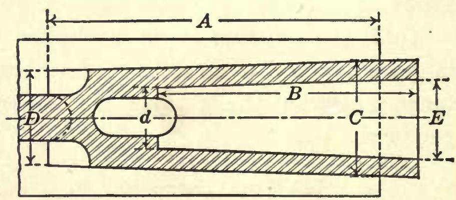
1. 0.600″.	**3.** 0.623″.	**5.** $\frac{3}{4}''$.	**7.** $1\frac{5}{16}''$.	**9.** $2\frac{1}{4}''$.
2. 0.602″.	**4.** 0.630″.	**6.** $\frac{7}{16}''$.	**8.** $1\frac{3}{4}''$.	**10.** $2\frac{3}{4}''$.

11. If the disk diameters are 1″ and 2″ respectively and if l, the center distance, is 4″, what is the included angle of the taper?

12. If the disk diameters are 1″ and 1.5″ respectively and if $l = 4''$, what is the T.P.F.? the included angle? the angle formed with the axis?

13. If the taper is 0.5″ per foot and the disk diameters are 0.75″ and 1″ respectively, what is l?

14. The figure and table below show certain dimensions of Morse standard tapers, all the measurements being given in inches. Copy the table and fill in the blank columns marked *D*, *d*, and "Angles with Axis."



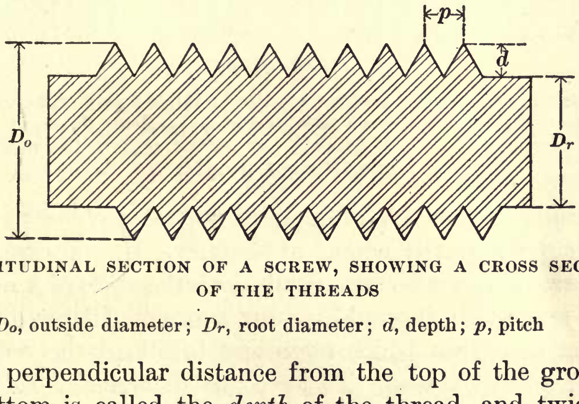
T.P.F. of Outer Hole	T.P.F. of Inner Hole	*A*	*B*	*C*	*E*	*D*	*d*	Angles with Axis
0.602	0.600	$3\frac{1}{4}$	$2\frac{3}{16}$	0.938	0.475			
.623	.602	$4\frac{1}{8}$	$2\frac{5}{8}$	1.231	0.700			
.630	.602	$5\frac{1}{4}$	$3\frac{1}{4}$	1.748	0.938			
.626	.623	$7\frac{3}{8}$	$4\frac{1}{8}$	2.494	1.231			
.626	.630	$7\frac{3}{8}$	$5\frac{1}{4}$	2.494	1.748			

CHAPTER IV

SCREW THREADS

Thread. When a uniform spiral groove is cut around a cylindric bar, the material left by the cutting tool forms a projection which is called a *thread*, or a *screw thread.*

The diameter of the cylindric bar on which the screw thread is cut is called the *outside diameter* of the thread.



LONGITUDINAL SECTION OF A SCREW, SHOWING A CROSS SECTION OF THE THREADS

D_o, outside diameter; D_r, root diameter; d, depth; p, pitch

The perpendicular distance from the top of the groove to the bottom is called the *depth* of the thread, and twice this depth is called the *double depth* of the thread.

The bottom of the groove is called the *root* of the thread, and the diameter of the screw measured at the bottom of the groove is called the *root diameter.*

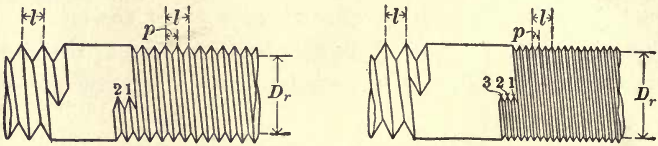
The distance from the center of one thread to the center of the next thread is called the *pitch* of the screw thread.

The advance that the screw makes in one revolution is called the *lead* of the screw.

Multiple Thread. A thread formed by cutting two or more uniform spiral grooves around a bar is called a *multiple thread.*

A multiple thread is said to be a double thread, a triple thread, a quadruple thread, and so on, according as the number of grooves thus cut is two, three, four, and so on.

The pitch and the lead of a single thread are equal, but the lead of a double thread is twice the pitch. Similarly, in a triple thread the lead is three times the pitch, in a quadruple thread the lead is four times the pitch, and so on.



MULTIPLE THREADS

The left-hand figure shows a double thread and a single thread of the same lead, and the right-hand figure shows a triple thread and a single thread of the same lead. In each figure l shows the lead of each thread, p shows the pitch, and D_r shows the root diameter

Multiple threads are often used where machine parts are to be moved by the action of a screw. If, for example, a feed screw is desired which shall move the table of a machine $\frac{1}{4}''$ per revolution, it would require a screw of large diameter in order to have a $\frac{1}{4}$-inch pitch and fulfill all the requisites for strength of thread. To keep the diameter of the screw to an appropriate size a double thread of $\frac{1}{8}$-inch pitch might be used, and the desired lead of $\frac{1}{4}''$ would be thus obtained.

Hand of a Screw. A screw that advances when turned clockwise is called a *right-hand screw.*

A screw that advances when turned counterclockwise is called a *left-hand screw.*

The right-hand screw is the common form, and, unless otherwise stated, a right-hand thread is always understood in speaking of a screw.

Exercises. Threads

1. A screw has 18 single threads to the inch. Find the pitch and the lead of the screw.

2. In the single-thread screw shown on page 35 suppose that the outside diameter is $1\frac{1}{16}''$ and that the depth of the thread is $\frac{3}{16}''$. Find the root diameter.

It is evident from the figure that the root diameter is equal to the outside diameter minus the double depth of the thread.

3. In the double-thread screw shown on page 36 suppose that there are 16 threads to the inch; that is, 8 double threads. Find the pitch and the lead of the screw.

4. If the root diameter of a double-thread screw is $1\frac{5}{8}''$ and the depth of the thread is $\frac{3}{32}''$, find the outside diameter of the screw.

5. In the triple-thread screw shown on page 36 suppose that the pitch is $\frac{1}{16}''$. What is the lead of the screw?

6. The cross slide on a lathe is moved by a screw that has 16 threads per inch, and one revolution of the screw moves the slide $\frac{1}{4}''$. How would you designate the thread?

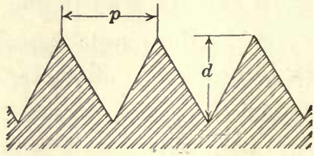
7. A lead screw has a triple thread with 18 threads to the inch. Find the pitch and the lead.

8. When the feed screw on a milling machine makes one revolution it moves the table $\frac{1}{4}''$. If the screw has 8 threads to the inch, how would you designate the thread?

9. How many revolutions must be made by a triple-thread feed screw of $\frac{1}{6}$-inch pitch in order to move the table which it controls a distance of $1\frac{1}{2}''$?

10. When the screw that raises the knee on a milling machine makes one revolution it moves the knee $0.200''$. If the screw has 5 threads to the inch, how should the thread be designated?

Sharp V-Thread. The thread of which the cross section is an equilateral triangle is called a *sharp V-thread.* Since the cross section of this thread is an equilateral triangle, all the angles are 60°. The depth of the thread is the altitude of the triangle, and the pitch is equal to the base, as is evident from the figure at the right.



SHARP V-THREAD

p, pitch; *d*, depth

In a sharp V-thread the pitch and the depth of the thread are found by the following formulas:

$$\text{Pitch} = \frac{1''}{\text{number of threads to } 1''}$$

$$\text{Depth} = \frac{0.8660''}{\text{number of threads to } 1''} = 0.8660 \times \text{pitch}$$

The table below shows the standard number of V-threads (T) per inch for screws of the following diameters (D_o):

D_o	T	D_o	T	D_o	T	D_o	T
$\frac{1}{4}''$	20	$\frac{13}{16}''$	10	$1\frac{3}{4}''$	5	$2\frac{7}{8}''$	4
$\frac{5}{16}$	18	$\frac{7}{8}$	9	$1\frac{7}{8}$	$4\frac{1}{2}$	3	$3\frac{1}{2}$
$\frac{3}{8}$	16	$\frac{15}{16}$	9	2	$4\frac{1}{2}$	$3\frac{1}{8}$	$3\frac{1}{2}$
$\frac{7}{16}$	14	1	8	$2\frac{1}{8}$	$4\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{2}$
$\frac{1}{2}$	12	$1\frac{1}{8}$	7	$2\frac{1}{4}$	$4\frac{1}{2}$	$3\frac{3}{8}$	$3\frac{1}{4}$
$\frac{9}{16}$	12	$1\frac{1}{4}$	7	$2\frac{3}{8}$	$4\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$
$\frac{5}{8}$	11	$1\frac{3}{8}$	6	$2\frac{1}{2}$	4	$3\frac{5}{8}$	$3\frac{1}{4}$
$\frac{11}{16}$	11	$1\frac{1}{2}$	6	$2\frac{5}{8}$	4	$3\frac{3}{4}$	3
$\frac{3}{4}$	10	$1\frac{5}{8}$	5	$2\frac{3}{4}$	4	4	3

Unless otherwise stated the diameter of a screw, as D_o in the table above, means the outside diameter.

Since the sharp V-thread offers no advantage over the United States Standard thread described on page 40 and the sharp edges are easily injured, its manufacture is being discontinued.

Exercises. Sharp V-Threads

1. Find the depth of a sharp V-thread with a pitch of $\frac{1}{4}''$.

In the following problems use the table and the formulas on page 38. Carry all computations involving decimals to four decimal places, but in the final result discard the fourth place, giving the dimension correct to the nearest 0.001″.

2. Find the double depth of a sharp V-thread with 9 threads to the inch; with 11 threads to the inch.

3. Find the depth of a sharp V-thread with $3\frac{1}{2}$ threads to the inch; with $4\frac{1}{2}$ threads to the inch.

4. Find the tap-drill size of a sharp V-thread tap which has an outside diameter of $\frac{1}{4}''$.

A tap is a tool for cutting internal threads, and the tap-drill size is the size of the drill for boring the hole to be tapped. In actual practice, to avoid breaking taps, the tap-drill size for ordinary work is generally taken a little larger than the root diameter of the thread on the tap, but in all problems in this book the tap-drill size is to be taken as equal to the root diameter.

5. A screw having 5 sharp V-threads to the inch has a root diameter of 1.4036″. Find the outside diameter.

6. Find the tap-drill size for a sharp V-thread tap which has an outside diameter of $1\frac{1}{8}''$.

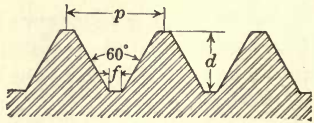
7. If the root diameter of a sharp V-thread screw is 4.3072″ and the outside diameter is 5″, what is the depth of the thread and the number of threads per inch?

Find the double depth of a sharp V-thread, given that the pitch of each thread is as follows:

8. $\frac{1}{14}''$. **9.** $\frac{1}{24}''$. **10.** $\frac{1}{3}''$. **11.** $\frac{1}{16}''$. **12.** $\frac{1}{8}''$.

13. Find the tap-drill size for a tap $1\frac{3}{8}''$ in diameter with 6 double sharp V-threads to the inch.

United States Standard Thread. A sharp V-thread flatted an equal amount at the top and bottom, as here shown, is known as the *United States Standard thread* and is commonly designated as the U. S. S. thread.



UNITED STATES STANDARD THREAD

d, depth; *f*, flat; *p*, pitch

S. A. E. Thread. A thread of the same shape as the U. S. S. thread, but which differs only in the number of threads per inch, bears the name of the Society of Automobile Engineers and is known as the *S. A. E. thread.*

In each of these threads the following formulas are used:

$$\text{Pitch} = \frac{1''}{\text{number of threads to } 1''}$$

$$\text{Depth} = \frac{0.6495''}{\text{number of threads to } 1''} = 0.6495 \times \text{pitch}$$

$$\text{Flat (top and bottom)} = \frac{1}{8} \times \text{pitch}$$

The table below shows standard U.S.S. and S.A.E. threads:

U.S.S. THREAD						S.A.E. THREAD			
D_o	T	D_o	T	D_o	T	D_o	T	D_o	T
$\frac{1}{4}''$	20	$1''$	8	$2\frac{1}{4}''$	$4\frac{1}{2}$	$\frac{1}{4}''$	28	$\frac{7}{8}''$	14
$\frac{5}{16}$	18	$1\frac{1}{8}$	7	$2\frac{1}{2}$	4	$\frac{5}{16}$	24	1	14
$\frac{3}{8}$	16	$1\frac{1}{4}$	7	$2\frac{3}{4}$	4	$\frac{3}{8}$	24	$1\frac{1}{8}$	12
$\frac{7}{16}$	14	$1\frac{3}{8}$	6	3	$3\frac{1}{2}$	$\frac{7}{16}$	20	$1\frac{1}{4}$	12
$\frac{1}{2}$	13	$1\frac{1}{2}$	6	$3\frac{1}{4}$	$3\frac{1}{2}$	$\frac{1}{2}$	20	$1\frac{3}{8}$	12
$\frac{9}{16}$	12	$1\frac{5}{8}$	$5\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$\frac{9}{16}$	18	$1\frac{1}{2}$	12
$\frac{5}{8}$	11	$1\frac{3}{4}$	5	$3\frac{3}{4}$	3	$\frac{5}{8}$	18	. .	. .
$\frac{3}{4}$	10	$1\frac{7}{8}$	5	4	3	$\frac{11}{16}$	16	. .	. .
$\frac{7}{8}$	9	2	$4\frac{1}{2}$	$4\frac{1}{4}$	$2\frac{7}{8}$	$\frac{3}{4}$	16	. .	. .

In this table D_o stands for the diameter of the screw and T for the number of threads per inch.

Exercises. U.S.S. and S.A.E. Threads

1. Find from the formula on page 40 the depth of a U.S.S. thread with 8 threads to the inch.

The U.S.S. thread was devised by William Sellers in 1869. It is also known as the Sellers thread and as the Franklin Institute thread.

2. Find the tap-drill size for a $1\frac{3}{8}$-inch S.A.E. thread.

3. Find the double depth of a U.S.S. thread of $\frac{1}{11}$-inch pitch; of $\frac{1}{12}$-inch pitch.

4. If the root diameter of a U.S.S. thread is 4.2551″ and there are $2\frac{5}{8}$ threads to the inch, what is the outside diameter of the screw?

5. What is the root diameter of an S.A.E. thread $\frac{7}{16}$″ in diameter? $\frac{3}{4}$″ in diameter?

6. Find the tap-drill size of a $3\frac{1}{2}$-inch U.S.S. thread.

7. If the root diameter of an S.A.E. thread of $\frac{1}{14}$-inch pitch is 0.9072″, what is the outside diameter?

Find the width of the point of a thread tool for cutting each of the following numbers of U.S.S. threads to the inch:

8. 3. **9.** $4\frac{1}{2}$. **10.** 7. **11.** 13. **12.** 18. **13.** 20.

The width of the point of the cutting tool is the same as the width of the flat at the bottom of the groove.

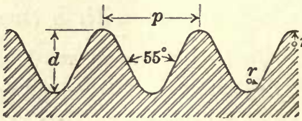
14. If the root diameter of a U.S.S. thread is 1.7113″ and the outside diameter is 2″, what is the depth of the thread?

15. Find to the nearest $\frac{1}{64}$″ the proper size of drill for boring a hole which is to be tapped with a full $1\frac{1}{8}$-inch U.S.S. thread; with a full $\frac{3}{8}$-inch S.A.E. thread.

A full thread is a thread cut to the depth given by the formula. For ordinary work a tap drill which will give about 75% of a full thread is generally used, and tables of tap drills often list the drill sizes in sixty-fourths of an inch which will give such a thread.

Whitworth Standard Thread. The thread of which the cross section is of the form here illustrated is called the *Whitworth Standard thread.*

This thread was devised by Sir Joseph Whitworth in 1841 and, with slight modifications, is still the British standard.



WHITWORTH STANDARD THREAD

d, depth; p, pitch; r, radius of curvature

In such a thread the pitch, the depth, and the radius are found by the following formulas:

$$\text{Pitch} = \frac{1''}{\text{number of threads to } 1''}$$

$$\text{Depth} = \frac{0.6403''}{\text{number of threads to } 1''} = 0.6403 \times \text{pitch}$$

$$\text{Radius} = \frac{0.1373''}{\text{number of threads to } 1''} = 0.1373 \times \text{pitch}$$

The following table shows the standard number of threads per inch for Whitworth threads of the following diameters:

D_o	T	D_o	T	D_o	T	D_o	T
$\frac{1}{16}''$	60	$\frac{9}{16}''$	12	$1\frac{1}{4}''$	7	$1\frac{15}{16}''$	$4\frac{1}{2}$
$\frac{3}{32}$	48	$\frac{5}{8}$	11	$1\frac{5}{16}$	7	2	$4\frac{1}{2}$
$\frac{1}{8}$	40	$\frac{11}{16}$	11	$1\frac{3}{8}$	6	$2\frac{1}{8}$	$4\frac{1}{2}$
$\frac{5}{32}$	32	$\frac{3}{4}$	10	$1\frac{7}{16}$	6	$2\frac{1}{4}$	4
$\frac{3}{16}$	24	$\frac{13}{16}$	10	$1\frac{1}{2}$	6	$2\frac{1}{2}$	4
$\frac{7}{32}$	24	$\frac{7}{8}$	9	$1\frac{9}{16}$	6	$2\frac{3}{4}$	$3\frac{1}{2}$
$\frac{1}{4}$	20	$\frac{15}{16}$	9	$1\frac{5}{8}$	5	3	$3\frac{1}{2}$
$\frac{5}{16}$	18	1	8	$1\frac{11}{16}$	5	$3\frac{1}{4}$	$3\frac{1}{4}$
$\frac{3}{8}$	16	$1\frac{1}{16}$	8	$1\frac{3}{4}$	5	$3\frac{1}{2}$	$3\frac{1}{4}$
$\frac{7}{16}$	14	$1\frac{1}{8}$	7	$1\frac{13}{16}$	5	$3\frac{3}{4}$	3
$\frac{1}{2}$	12	$1\frac{3}{16}$	7	$1\frac{7}{8}$	$4\frac{1}{2}$	4	3

In this table D_o stands for the diameter of the screw and T for the number of threads per inch.

Exercises. Whitworth Threads

1. Find the depth of a Whitworth thread of $\frac{1}{48}$-inch pitch.

2. Find the double depth of a Whitworth thread with 7 threads to the inch.

Find the radius for the point of the tool for cutting a Whitworth thread of which the pitch in each case is as follows:

3. $0.4''$. **4.** $\frac{1}{3}''$. **5.** $\frac{1}{5}''$. **6.** $\frac{1}{9}''$. **7.** $\frac{1}{24}''$. **8.** $\frac{1}{60}''$.

9. Find the tap-drill size for a Whitworth thread of which the outside diameter is $\frac{5}{8}''$.

10. If the root diameter of a Whitworth thread is $3.3231''$ and there are 3 threads to the inch, what is the outside diameter of the screw?

11. If the root diameter of a Whitworth thread is $5.2377''$ and there are $2\frac{1}{2}$ threads to the inch, what is the outside diameter of the screw?

12. Find the root diameter of a Whitworth thread with an outside diameter of $\frac{1}{16}''$.

13. If the outside diameter of a Whitworth thread is $4\frac{1}{2}''$ and the root diameter is $4.0546''$, what is the pitch?

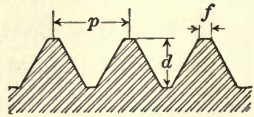
14. Find the root diameter of a Whitworth thread with an outside diameter of $3\frac{1}{2}''$.

15. If the diameter of a Whitworth screw is $\frac{1}{4}''$, how many threads are there to the inch? What is the pitch? the depth of thread? the root diameter?

16. If the root diameter of a Whitworth thread with 8 threads to the inch is $0.9024''$, what is the outside diameter?

17. If the diameter of a Whitworth screw is $3''$, how many threads are there to the inch? What is the pitch? the depth of thread? the root diameter?

Metric Threads. A *metric thread* is similar in shape to the U. S. S. thread shown on page 40, but the measurements are in millimeters (1 mm. = 0.0394″).



METRIC THREAD

d, depth; *f*, flat; *p*, pitch

Of the two standard metric threads in general use, the first is the International Standard thread, adopted at Zurich in 1898, and the second is the French Standard. These threads differ but little, as will be seen by comparing the measurements of the threads in the table below.

The following formulas are used with these two threads:

$$\textbf{Depth} = 0.6495 \times \text{pitch}$$

$$\textbf{Flat (top and bottom)} = \tfrac{1}{8} \times \text{pitch}$$

The following table shows standard pitches for both International Standard and French Standard metric threads:

International Standard			French Standard		
Diameter of screw		Pitch	Diameter of screw		Pitch
mm.	in.	mm.	mm.	in.	mm.
3	0.1181	0.55	3	0.1181	0.50
4	.1575	0.70	4	.1575	0.75
5	.1969	0.85	5	.1969	0.75
6	.2362	1.00	6	.2362	1.00
7	.2756	1.00	7	.2756	1.00
8	.3150	1.25	8	.3150	1.00
9	.3543	1.25	9	.3543	1.00
10	.3937	1.50	10	.3937	1.50
11	.4331	1.50	11	.4331	1.50
12	.4724	1.75	12	.4724	1.50
14	.5512	2.00	14	.5512	2.00
16	.6299	2.00	16	.6299	2.00
18	.7087	2.50	18	.7087	2.50
20	.7874	2.50	20	.7874	2.50
22	.8661	2.50	22	.8661	2.50

Exercises. Metric Threads

1. Find from the formula on page 44 the depth of a French Standard thread of 1-millimeter pitch.

The introduction of French and Italian automobiles into this country and our recent expansion of foreign trade render computation in metric units necessary as well as desirable.

In problems dealing with metric threads give the final results to the nearest 0.1 mm., unless otherwise specified.

2. Find the double depth of a French Standard thread of 3-millimeter pitch.

3. Find the root diameter of a 10-millimeter International Standard thread.

4. Find the pitch of an International Standard thread of which the outside diameter is 27 mm. and the root diameter is 23.10 mm.

5. Find the outside diameter of a French Standard thread of which the root diameter is 48.86 mm. and the pitch is 5.50 mm.

6. For particularly accurate work the formula for the depth of an International Standard thread is as follows: $depth = 0.64952 \times pitch$. If the pitch is 4.500 mm., find the depth to the nearest 0.01 mm., using this formula.

7. In Ex. 6 if the depth is 0.974 mm., what is the pitch?

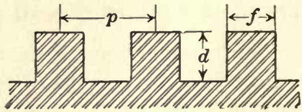
8. Find the width of the flat of an International Standard thread with an outside diameter of 11 mm.

9. If the root diameter of a French Standard thread is 23.10 mm. and the pitch is 3 mm., what is the outside diameter of the screw?

10. If the outside diameter of a French Standard thread is 36 mm. and the root diameter is 30.80 mm., what is the pitch of the thread?

Square Thread. The thread of which the cross section is a square, as shown in the figure, is called a *square thread.*

In theory the cross sections of a square thread and of the space between two successive threads are both squares. In practice the groove is slightly wider and deeper than the width of the thread. For our present purposes we shall take the theoretical measurements.



SQUARE THREAD

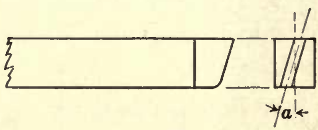
d, depth; *f*, flat; *p*, pitch

In such a thread the pitch, the depth, and the flat are found from the following formulas:

$$\textbf{Pitch} = \frac{1''}{\textbf{number of threads to } 1''}$$

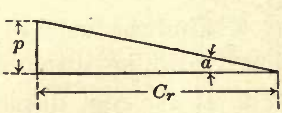
$$\textbf{Depth} = \textbf{flat} = \tfrac{1}{2} \times \textbf{pitch}$$

Square-Thread Tool. The tool used in cutting a square thread is shaped like a parting tool, or cutting-off tool, except that it has a side clearance, or rake, which varies for the pitch and diameter of the thread. The distance that the tool travels around the work is the root circumference of the thread; the distance that it travels sideways is the pitch. Letting the pitch p and the root circumference C_r be two sides of a right triangle, as in this figure, a is the angle for grinding the side clearance of the tool and is found by the use of tangents.



SQUARE-THREAD TOOL

a, angle for grinding the tool



For example, find the angle of clearance of the tool for cutting a square thread of $\frac{1}{4}$-inch pitch on a shaft 2″ in diameter.

$$\tan a = \frac{p}{C_r} = \frac{0.25}{3.1416 \times (2 - 0.25)} = \frac{0.25}{5.4978} = 0.0455.$$

Therefore $a = 2° 36'.$

Exercises. Square Threads

1. From the formulas on page 46 find the pitch, the depth, the double depth, and the flat of a square thread with 8 threads to the inch.

2. If the outside diameter of a screw with square threads is $2\frac{3}{4}''$, and there are 8 threads to the inch, what is the root diameter of the screw?

3. At what angle should the tool be ground for cutting the thread referred to in Ex. 2?

4. If a double square thread has a diameter of $1\frac{1}{8}''$ and a lead of $\frac{1}{8}''$, what is the root diameter?

5. In cutting a square thread with 2 threads to the inch on a shaft $4\frac{1}{2}''$ in diameter, what angle of side clearance should be allowed for the tool?

6. If the double depth of a square thread is 0.0417″, what is the number of threads to the inch?

7. Find the angle for grinding the side clearance of the tool used in cutting a square thread having an outside diameter of $1\frac{3}{4}''$ and a pitch of $\frac{5}{16}''$.

8. A triple square-thread screw of outside diameter $1\frac{5}{8}''$ has a lead of $\frac{1}{4}''$. Find the root diameter of the thread.

9. Find the angle for grinding the side clearance of the tool for cutting the thread in Ex. 8.

10. Find the outside diameter of a square thread which has a root diameter of $3\frac{5}{16}''$ and a pitch of $\frac{7}{8}''$.

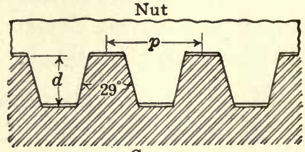
11. Find the angle for grinding the tool used in cutting the thread described in Ex. 10. What would be the angle if the thread were a double thread of the same lead?

12. If the root circumference of a square thread is 2.3562″ and the pitch is $\frac{1}{4}''$, what is the outside diameter?

Acme Thread. The thread illustrated in the figure below is called an *Acme thread.* The nature of this thread will be understood from the illustration.

Owing to the ease of cutting and to the greater strength secured, the Acme thread has to a large extent replaced the square thread.

From the formulas given below it will be seen that the depth of the Acme screw thread differs by 0.0100″ from that of the square thread.



ACME THREAD

d, depth; *p*, pitch

The following formulas are used in computing measurements of Acme screw threads and tap threads:

$$\text{Pitch} = \frac{1''}{\text{number of threads to } 1''}$$

$$\text{Depth for screws} = \tfrac{1}{2} \times \text{pitch} + 0.0100''$$

$$\text{Depth for taps} = \tfrac{1}{2} \times \text{pitch} + 0.0200''$$

$$\text{Width of flat at root for screws} = 0.3707 \times \text{pitch} - 0.0052''$$

$$\text{Width of flat at root for taps} = 0.3707 \times \text{pitch} - 0.0052''$$

$$\text{Width of flat at top for screws} = 0.3707 \times \text{pitch}$$

$$\text{Width of flat at top for taps} = 0.3707 \times \text{pitch} - 0.0052''$$

$$\text{Width of space at top for screws} = 0.6293 \times \text{pitch}$$

$$\text{Width of space at top for taps} = 0.6293 \times \text{pitch} + 0.0052''$$

$$\text{Diameter of tap} = \text{diameter of screw} + 0.0200''$$

$$\text{Root diameter for screws and taps} = D_o - (\text{pitch} + 0.0200'')$$

The symbol D_o in the last formula stands for "outside diameter."

It is evident from the above formulas that the clearance at the bottom of an Acme thread between the nut and the screw is obtained by making the outside diameter of the nut thread 0.0200″ larger than that of the screw thread.

Exercises. Acme Threads

1. Find from the formula on page 48 the width of the flat at the root of an Acme screw thread having $2\frac{1}{2}$ threads to the inch.

2. Find the depth of an Acme tap thread of $\frac{1}{8}$-inch pitch.

3. Find the width of the flat at the top of an Acme screw thread having 12 threads to the inch.

4. Find the root diameter of an Acme tap thread having 4 threads to the inch and an outside diameter of $2\frac{1}{4}''$.

5. Find the outside diameter of an Acme screw thread having a root diameter of 3.4443″ and $3\frac{1}{2}$ threads to the inch.

6. A boring bar with a diameter of $3\frac{5}{8}''$ is threaded with an Acme screw thread which has a double depth of 0.2422″. Find the pitch of the thread.

7. In Ex. 6 find the width of the flat at the root.

8. Find the width of the flat at the top and also at the root of an Acme screw thread of $\frac{5}{8}$-inch pitch.

9. Find the root diameter of an Acme tap thread which has an outside diameter of $1\frac{9}{16}''$ and 7 threads to the inch.

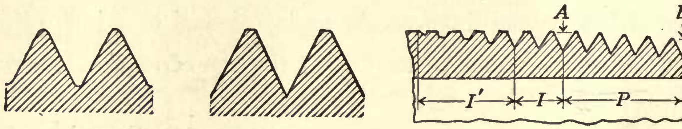
10. In an Acme screw thread of $\frac{3}{16}$-inch pitch find the width of the flat at the top, the width of the flat at the root, and the width of the space at the top.

11. If the outside diameter of an Acme screw thread having 3 threads to the inch is $\frac{15}{16}''$, what is the root diameter? the outside diameter of the corresponding tap thread?

12. If the root diameter of an Acme screw thread is 3.2982″ and the outside diameter is $4\frac{1}{2}''$, what is the pitch of the thread?

13. Find the outside diameter of an Acme screw thread with 12 threads to the inch and a root diameter of 1.1467″.

Briggs Pipe Threads. A *Briggs pipe thread* differs from the threads so far considered in that for a certain distance, A–B in figure III below, the outside of the pipe is tapered. The threads P, which are on the taper, are perfect threads, the



I II III

BRIGGS PIPE THREADS

Figure I shows a cross section of a perfect standard thread as devised by Robert Briggs, a British engineer, in 1882; figure II shows a similar cross section of the modified Briggs thread; figure III shows a longitudinal section of the complete thread, the letters in this figure being explained in the text

next two threads I, not being on the taper, are imperfect on top, and the next four threads I' are imperfect at the bottom as well, owing to the chamfer of the threading die.

The nominal inside diameter (N.I.D.) is the size under which the pipe is listed and differs considerably from the actual inside diameter (A.I.D.), which is the exact measurement, as is seen in the table on page 51. The actual outside diameter (A.O.D.) is measured at the large end of the taper, which has a T.P.F. of $\frac{3}{4}''$ between points A and B.

The following formulas are used with Briggs threads and show the differences between the standard and modified forms:

$$\text{Pitch} = \frac{1''}{\text{number of threads to } 1''}$$

$$\text{Depth (standard)} = 0.8 \times \text{pitch}$$

$$\text{Depth (modified)} = 0.833 \times \text{pitch}$$

$$\text{Flat (modified)} = \frac{1}{26} \times \text{pitch}$$

The modified form of Briggs thread is more easily cut and hence has come into more general use.

Table of Briggs Threads. The table below shows the standard measurements and the number of threads (T) per inch:

N.I.D.	A.I.D.	A.O.D.	T	N.I.D.	A.I.D.	A.O.D.	T
$\frac{1}{8}''$	0.270″	0.405″	27	2″	2.067″	2.375″	$11\frac{1}{2}$
$\frac{1}{4}$	.364	.540	18	$2\frac{1}{2}$	2.468	2.875	8
$\frac{3}{8}$	.494	.675	18	3	3.067	3.500	8
$\frac{1}{2}$	.623	.840	14	$3\frac{1}{2}$	3.548	4.000	8
$\frac{3}{4}$	.824	1.050	14	4	4.026	4.500	8
1	1.048	1.315	$11\frac{1}{2}$	$4\frac{1}{2}$	4.508	5.000	8
$1\frac{1}{4}$	1.380	1.660	$11\frac{1}{2}$	5	5.045	5.563	8
$1\frac{1}{2}$	1.610	1.900	$11\frac{1}{2}$	6	6.065	6.625	8

Exercises. Briggs Pipe Threads

1. Find the depth of a modified Briggs pipe thread having 27 threads to the inch.

2. Find the double depth of a modified Briggs pipe thread having $11\frac{1}{2}$ threads to the inch.

3. Find the width of the flat of a modified Briggs pipe thread with 8 threads to the inch.

Find the root diameter at the large end of the taper of a Briggs Standard thread for each of the following sizes of pipes (N.I.D.):

4. $\frac{1}{8}''$. **5.** $\frac{1}{4}''$. **6.** $\frac{1}{2}''$. **7.** $1\frac{1}{4}''$. **8.** $3''$. **9.** $4''$.

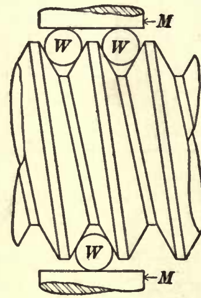
10. In the Briggs Standard thread the length of the perfect thread, P in figure III on page 50, is $\frac{0.8\,D + 4.8}{N}$, where D is the A.O.D. and N is the number of threads per inch. Find the length of the perfect thread for a $\frac{3}{8}$-inch pipe.

As in Ex. 10, find the length of the perfect thread in each case for pipes of the following sizes:

11. $\frac{3}{4}''$. **12.** $1''$. **13.** $1\frac{1}{4}''$. **14.** $1\frac{1}{2}''$. **15.** $2''$. **16.** $3\frac{1}{2}''$.

Measuring Threads. In addition to the outside diameter and the root diameter of a screw, there is also the *pitch diameter*, or the *angle diameter*, which is used in measuring threads. The pitch diameter is theoretically the outside diameter minus the single depth of the thread; that is, it includes half the depth of the thread on opposite sides of the screw.

The best means of measuring the pitch diameter of a thread is by the use of a special thread micrometer. In such an instrument the anvil is V-shaped so as to fit over the thread, and the screw, pointed to 60°, is slightly rounded so as to enter the groove.



TESTING THE PITCH DIAMETER OF A U.S.S. THREAD

M, *M*, micrometer jaws; *W*, *W*, *W*, wires

In practice, however, the pitch diameter is not actually measured, but is tested by the use of the ordinary micrometer and of three wires of the same diameter placed as here shown. The wires used are hardened, tempered, accurately ground, and lapped to size. One wire is placed in the angle of the thread, and the other two wires are placed in adjacent angles on the opposite side. The micrometer is placed over the wires, and the reading is taken.

By trigonometry it can be shown that if M is the measurement over the wires, D_o the outside diameter of the screw, p the pitch, C a certain constant, and D_w the diameter of the wires, the following formula is true:

$$M = D_o - pC + 3\,D_w$$

The pitch diameter of the screw is correct (that is, it is equal to the theoretical pitch diameter) when the micrometer reading over the wires is equal to the result from the formula.

Illustrative Problems. 1. Find the correct measurements over wires of a $\frac{3}{4}$-inch U. S. S. thread, the diameter of the measuring wires being 0.05773″ and the constant, which is used for all U. S. S. threads, being 1.5155.

We first refer to the table of U. S. S. threads on page 40, and find that when the outside diameter of the thread is $\frac{3}{4}''$ there are 10 threads to the inch. The pitch, therefore, is $\frac{1}{10}''$.

Using the formula given on page 52, we find M, the measurement over the wires, as follows:

$$M = D_o - pC + 3\,D_w = 0.75 - \frac{1.5155}{10} + 3 \times 0.05773 = 0.77164$$

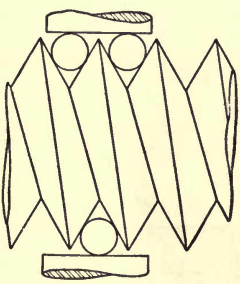
Since a vernier micrometer gives readings only to 0.0001″ we give the above result as 0.7716″. If the reading of the caliper agrees with this result, the pitch diameter is correct.

2. Find the correct micrometer reading over wires for a $\frac{5}{16}$-inch sharp V-thread, the diameter of the wires being 0.035″ and the constant for sharp V-threads being 1.732.

For sharp V-threads we find on page 38 that when $D = \frac{5}{16}''$ the number of threads is 18, and hence the pitch is $\frac{1}{18}''$. We therefore have

$$\begin{aligned} M &= D_o - pC + 3\,D_w \\ &= 0.3125 - \frac{1.732}{18} + 3 \times 0.035 \\ &= 0.3213. \end{aligned}$$

The correct reading for a $\frac{5}{16}$-inch sharp V-thread is therefore 0.3213″.



TESTING THE PITCH DIAMETER OF A SHARP V-THREAD

Use of the Three-Wire Method. If, in cutting the sharp V-thread described above, it is desired to test the depth of cut, the wires are placed as shown in the figure and the micrometer reading is taken over them. If, for example, this reading is 0.01″ more than 0.3213″, the thread must be cut 0.01″ deeper to be correct; if the reading is less than 0.3213″, the thread has been cut too deep.

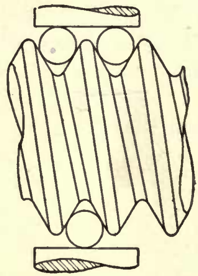
Exercises. Measuring Threads

1. Find the correct reading over wires for a $2\frac{1}{8}$-inch Whitworth thread, the wires being 0.150″ in diameter and the constant for this thread being 1.6008.

The formula for the correct reading of a Whitworth thread differs from the one given on page 52. It is as follows:

$$M = D_o - pC + 3.1657\ D_w$$

For the pitch see the table on page 42.



TESTING THE PITCH DIAMETER OF A WHITWORTH THREAD

2. Find the correct reading over wires for a $1\frac{1}{8}$-inch U.S.S. thread, using wires 0.090″ in diameter.

3. Find the correct reading over wires for a sharp V-thread $1\frac{5}{8}$″ in diameter, using wires 0.150″ in diameter.

4. Find the correct reading over wires for a U.S.S. thread 3″ in diameter, using wires 0.200″ in diameter.

5. Copy the following table and in the proper space insert the correct measurement over wires for each thread:

U. S. S. Thread			Whitworth Thread		
Outside diameter	Diameter of wires	Measurement over wires	Outside diameter	Diameter of wires	Measurement over wires
$\frac{1}{2}$″	0.050″		$\frac{7}{16}$″	0.04026″	
$\frac{7}{8}$	0.06415		$1\frac{3}{8}$	0.09394	
$1\frac{5}{8}$	0.10497		2	0.12526	
$2\frac{1}{4}$	0.12830		$3\frac{1}{4}$	0.17344	
$3\frac{1}{3}$	0.17623		$3\frac{3}{4}$	0.18789	

Exercises. Review

1. Find the lead of a double-thread screw which has 12 threads to the inch; which has 20 threads to the inch.

2. Find the root diameter of an S.A.E. thread that is $\frac{11}{16}''$ in diameter.

3. Find the double depth of a Briggs Standard pipe thread for a $4\frac{1}{2}$-inch pipe.

4. In an Acme screw thread with a pitch of $\frac{7}{16}''$ find the width of the flat at the top, the width of the flat at the root, and the width of the space at the top.

5. What is the pitch of a square thread when the double depth is $0.2222''$? How many threads are there to $1''$?

6. If the root diameter of an International Standard thread is 5.70 mm. and the pitch is 1 mm., what is the outside diameter? the width of the flat?

7. At what angle should the thread tool be ground to cut a square thread with 6 threads to the inch and an outside diameter of $1\frac{1}{4}''$?

8. Find the tap-drill size of a $1\frac{5}{8}$-inch U.S.S. tap.

9. Find the root diameter of an Acme tap thread $2\frac{1}{8}''$ in outside diameter with 5 threads to the inch.

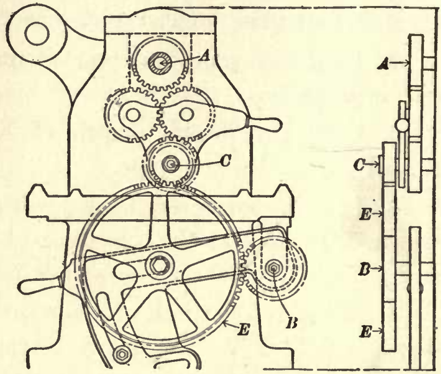
10. Find the correct reading over wires for a sharp V-thread $\frac{11}{16}''$ in diameter, the diameter of the wires being $0.070''$.

11. The root diameter of a U.S.S. thread with $3\frac{1}{2}$ threads to the inch is $2.6288''$. Find the outside diameter.

12. The root diameter of a Whitworth Standard thread is $4.5343''$ and the outside diameter is $5''$. Find the pitch of the thread.

13. If the outside diameter of an Acme screw thread having $1\frac{1}{3}$ threads to the inch is $3\frac{1}{4}''$, what is the root diameter?

Screw-Thread Cutting. The cutting of a thread in a lathe is a mechanical operation in which the selection of the proper change gears is a matter of greatest importance.



LATHE HEADSTOCK SHOWING SIMPLE GEARING FOR CUTTING THREADS

A, lathe spindle; *B*, lead screw; *C*, stud shaft; *E*, intermediate gear

In order to cut the required number of threads per inch, the number of revolutions of the work must be exactly equal to the number of threads to be cut during the time that the carriage carrying the tool rest moves exactly 1″ along the bed of the lathe. Change gears are used so as to give the proper number of revolutions to the *lead screw*, which moves the carriage, gears of different sizes being placed on the stud shaft and on the lead screw according to the size of thread to be cut. For fine threads the carriage moves slowly, but for coarse threads it moves rapidly along the work.

A simple-geared lathe has only one change of speed between the stud shaft and the lead screw.

Since the gear on the lathe spindle has the same number of teeth as the driven gear on the stud spindle, the two spindles have the same speed. The intermediate gear connecting the driving gear on the stud and the gear on the lead screw may be of any size since it has no effect on the rate of turning of the lead screw.

The lead of the lathe can be found by counting the threads per inch on the lead screw, or by placing gears of the same number of teeth on the stud shaft and on the lead screw and then counting the number of revolutions of the lead screw while the carriage moves 1″.

Illustrative Problem. If the lead screw of a lathe has 6 threads per inch and it is desired to cut 12 threads per inch on a screw, what change gears are required?

In this case, since the lead screw will revolve 6 times while the carriage advances 1″, and the work must revolve 12 times during the same period, change gears with a ratio of 6 to 12, or of 1 to 2, should be placed on the stud shaft and the lead screw. That is, expressed as a general proportion, which may be used in any problem requiring the use of simple gearing, we have the following:

$$\frac{\text{Number of teeth on outside stud gear}}{\text{Number of teeth on lead-screw gear}} = \frac{\text{number of threads per inch on lead screw}}{\text{number of threads per inch on work}}$$

Each term of the ratio of the gears is multiplied by a number such that the result is a ratio whose terms correspond to the numbers of teeth on two of the gears with which the lathe is equipped.

In this case we have $\frac{4 \times 6}{4 \times 12} = \frac{24}{48}$.

That is, a gear with 24 teeth should be placed on the end of the stud shaft, and a gear with 48 teeth on the lead screw. An intermediate gear of the proper size is used to connect the two gears.

Variation of Lathes. The number of threads per inch on the lead screw varies on different makes of lathes, and this makes it necessary to have different sets of change gears. In our problems we shall use the following sets of gears:

For a lead screw with 5 threads per inch: 25, 25, 30, 35, 40, 45, 50, 55, 60, 65, 69, 70, 80, 90, 100, 110, 120.

For a lead screw with 6 threads per inch: 24, 24, 32, 40, 44, 48, 52, 56, 60, 64, 72, 110.

For a lead screw with 8 threads per inch: 24, 24, 28, 32, 36, 40, 44, 48, 52, 56, 64, 69, 72, 80, 96.

In the exercises use only gears which are shown above as belonging to the set for the lead screw specified in each problem.

Exercises. Screw-Thread Cutting

1. What change gears are required on a lathe with a lead screw of $\frac{1}{5}$-inch pitch in cutting a screw thread of $\frac{1}{16}$-inch pitch? in cutting a thread with a pitch of 0.125″?

Find the change gears required on a lathe with a lead screw of $\frac{1}{5}$-inch pitch for cutting each of the following numbers of threads per inch:

2. 12. **3.** 7. **4.** 13. **5.** 18. **6.** $11\frac{1}{2}$.

Find the change gears required on a lathe having a lead screw with 8 threads per inch for cutting each of the following numbers of threads per inch:

7. 4. **9.** 9. **11.** 12. **13.** 18. **15.** 10.

8. 5. **10.** 16. **12.** 7. **14.** 6. **16.** 20.

17. What is the pitch of the thread that is cut on a lathe with a lead screw of $\frac{1}{5}$-inch pitch when there is a 25-tooth gear on the stud and a 40-tooth gear on the lead screw?

Find the change gears required on a lathe having a lead screw with 6 threads per inch for cutting each of the following numbers of threads per inch:

18. 6. **19.** 10. **20.** 13. **21.** 18. **22.** $27\frac{1}{2}$.

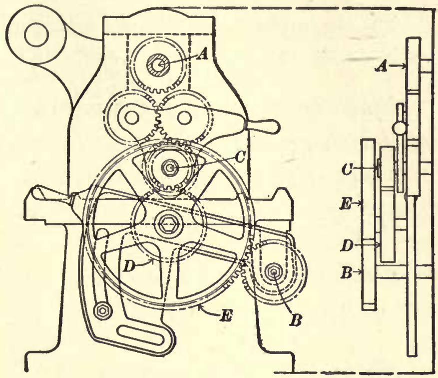
23. Find the largest number of threads per inch that can be cut with simple gearing on a lathe which has a lead screw of $\frac{1}{5}$-inch pitch; of $\frac{1}{6}$-inch pitch; of $\frac{1}{8}$-inch pitch.

24. Find the smallest number of threads per inch that can be cut with simple gearing on a lathe which has a lead screw of $\frac{1}{5}$-inch pitch; of $\frac{1}{6}$-inch pitch; of $\frac{1}{8}$-inch pitch.

25. Find the change gears necessary to cut a thread with a lead of $\frac{3}{8}$″ if the lead screw of the lathe has 5 threads per inch; if the lead screw has 8 threads per inch.

Compound Gearing. A *compound gearing* consists of two gears keyed together to revolve at the same rate and held in position by a bracket. The gears are introduced between the gear on the stud and that on the lead screw and are usually in the ratio of 2 to 1, but the general principle of operation is the same with gears of any other ratio.

Since the cutting of a very small or a very large number of threads per inch with simple gearing would require the use of gears with a very large number of teeth, it is necessary to use compound gearing on a lathe.



LATHE HEADSTOCK SHOWING COMPOUND GEARING FOR CUTTING THREADS

A, lathe spindle; *B*, lead screw; *C*, stud shaft; *D*, first gear on compound, driven; *E*, second gear on compound, driving

Illustrative Problem. Find the change gears required on a lathe with a lead screw of $\frac{1}{5}$-inch pitch in cutting a screw thread with 30 threads per inch.

Solving as for simple gearing, the ratio of the gears is found to be $\frac{5}{30}$, and since a 25-T gear is the smallest one that goes with a lead screw of $\frac{1}{5}$-inch pitch, we shall need a gear with 150 teeth if we use only simple gearing. To avoid this we split the ratio $\frac{5}{30}$ into factors, thus:

$$\frac{5}{30}=\frac{1}{6}=\frac{1\text{ (gear on stud)}}{3\text{ (gear on lead screw)}}\times\frac{1\text{ (driving gear on compound)}}{2\text{ (driven gear on compound)}}$$

We therefore see that the gears on the stud and the lead screw must have the ratio of 1 to 3, such as 30 T to 90 T, while the gears on the compound must have the ratio of 1 to 2, such as 25 T to 50 T, the smaller gear being the driving gear and the larger gear being the driven gear.

M

Exercises. Screw-Thread Cutting

1. Find the change gears required on a lathe with a lead screw of $\frac{1}{6}$-inch pitch in cutting 25 threads to the inch.

This example shows that the gears on the compound are not necessarily in the ratio of 2 to 1, the principle being the same for any ratio.

Find the change gears required on a lathe with a lead screw of $\frac{1}{6}$-inch pitch in cutting a thread with each of the following numbers of threads to the inch:

2. 28. **3.** $7\frac{1}{2}$. **4.** $3\frac{1}{2}$. **5.** 32. **6.** 42.

Similarly, for a lead screw of $\frac{1}{8}$-inch pitch, the numbers being:

7. 1. **8.** $1\frac{1}{2}$. **9.** 22. **10.** 36. **11.** 28.

Similarly, for a lead screw of $\frac{1}{5}$-inch pitch, the numbers being:

12. 2. **13.** 23. **14.** $\frac{5}{16}$. **15.** $\frac{3}{4}$. **16.** $1\frac{1}{8}$.

Find the change gears required on a lathe having a lead screw with 5 threads to the inch in cutting a thread with each of the following leads:

17. $\frac{4}{13}''$. **18.** $\frac{3}{5}''$. **19.** $\frac{6}{7}''$. **20.** $\frac{8}{9}''$. **21.** $\frac{2}{11}''$.

22. Find the change gears required on a lathe having a lead screw with 6 threads to the inch in cutting a metric screw with a lead of 2 mm.

First find the number of threads to the inch ($1'' = 25.4$ mm.) corresponding to the given lead in millimeters. It will be found that a gear with 127 teeth is required to cut a metric thread on a lathe which has an English lead screw. In Exs. 22 and 23 assume that the lathe is equipped with such a gear in addition to those listed on page 57.

23. On a lathe having a lead screw with 6 threads to the inch, find the change gears required for cutting a metric thread with a lead of 3 mm.; with a lead of 3.5 mm.

CHAPTER V

INDEXING AND SPIRAL CUTTING

Indexing. The process of dividing a circumference into equal parts is called *indexing.*

Indexing is done by the aid of a milling-machine fixture known as the *dividing head*, *spiral head*, or *index head*, and which is described on page 63. The dividing head is ordinarily employed in obtaining equal divisions on a circumference, but may be used to obtain equal divisions of various kinds on other types of work.

There are various kinds of indexing, — direct, simple, angular, compound, and differential. These will be considered and explained in the following pages.

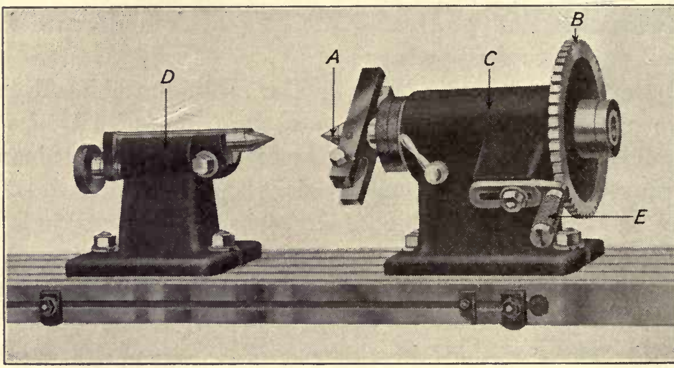
Among the cases in which the machinist will make use of indexing are the following: in cutting spur, bevel, and spiral gears; in cutting worm wheels; in cutting plate and cylindric cams; and in making milling-machine cutters, counter bores, reamers, and twist drills.

The dividing-head spindle may be set at any angle from about 5° below the horizontal to about 10° beyond the vertical. The swiveling block is usually graduated about its circumference in fourths of a degree, but some machines have a vernier that reads to $\frac{1}{12}°$.

Work is held in a dividing head in a manner similar to that used on an engine lathe, the three following general methods being used: (1) by mounting it on centers, (2) by holding it on an arbor, and (3) by securing it in a chuck.

When the work is held on an arbor or mounted on centers it is necessary to use the dividing-head tailstock, which has an adjustable center capable of being raised or lowered.

Index Center. The index center shown below is used to obtain exact and rapid spacing on work that requires only a small number of divisions upon the periphery, as is the case when milling bolt heads and fluting reamers and taps. It is used only on machines that are not fitted with a dividing head, since the dividing head will do all that an index center can perform, and has many additional advantages.



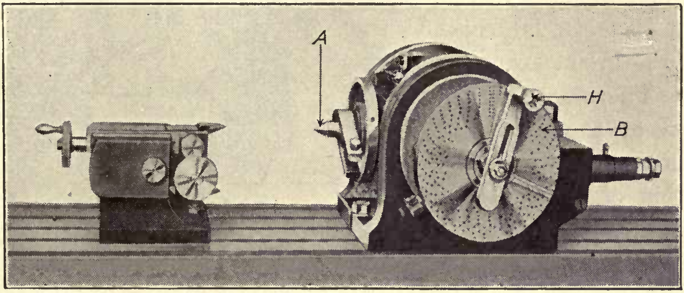
INDEX CENTER

A, spindle; *B*, index plate; *C*, headstock; *D*, footstock; *E*, stop pin

The index center consists of two main parts, the headstock *C* and the footstock *D*. The spindle *A* is moved freely by the index plate *B*, which is fastened to it. The spindle is locked at the desired division by means of the stop pin *E*, which is attached to the headstock. There is an even number of grooves, usually 24, 36, or 48, on the circumference of the index plate, and the number of divisions which can be made on the work is limited to the number of grooves and to factors of this number.

Direct Indexing. It is seen from the above description that the index plate gives direct motion to the spindle, and the use of the instrument in this way is called *direct indexing.*

Dividing Head. Instead of connecting the index plate *B* directly to the spindle *A*, as in the index center, the index crank *F* of the dividing head here shown is connected to

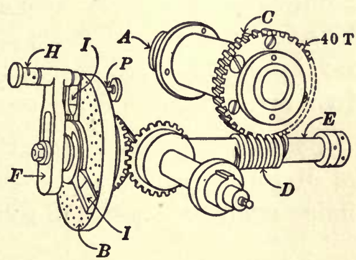


DIVIDING HEAD

A, spindle; *B*, index plate; *H*, index pin

shaft *E*, on which is cut a single-thread worm *D*. The worm meshes with the 40-tooth worm wheel *C* on the spindle, thus transmitting the motion from the crank.

The index plate *B* is not fastened to shaft *E* and can be held stationary by the stop pin *P*. The index pin *H* is adjustable and, by the tension of a spring, is held in any one of the holes in the index plate. The sector arms *I*, *I* can be set to include any desired number of holes, thus rendering it unnecessary to count the holes every time that the index crank is turned.

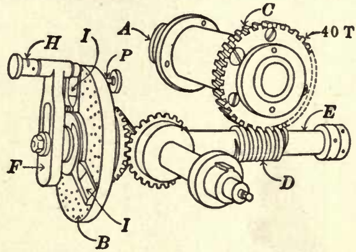


MECHANISM OF DIVIDING HEAD

A, spindle; *B*, index plate; *C*, worm wheel; *D*, single-thread worm; *E*, shaft; *F*, index crank; *H*, index pin; *I*, *I*, sector arms; *P*, stop pin

In direct indexing the worm is thrown out of mesh, and the spindle is locked in the desired position by a special stop pin.

Simple Indexing. For simple indexing we make use of the dividing head already shown on page 63 and here represented again in detail.



MECHANISM OF DIVIDING HEAD

A, spindle; *B*, index plate; *C*, worm wheel; *D*, single-thread worm; *E*, shaft; *F*, index crank; *H*, index pin; *I*, *I*, sector arms; *P*, stop pin

The worm wheel *C*, which has 40 teeth, is driven by the single-thread worm *D*, on shaft *E*. Therefore every revolution of the worm *D*, which corresponds to a single revolution of the index crank *F*, moves the worm wheel *C* exactly one tooth and moves the spindle *A*, which is attached to it, $\frac{1}{40}$ of a revolution.

Therefore 40 revolutions of the index crank *F* cause the spindle *A*, which carries the work, to make one revolution. Hence if 40 divisions are required on the work, one revolution of *F* must be made between each pair of cuts; if 20 divisions are required, two revolutions of *F* are necessary; if 10 divisions are required, four revolutions of *F* are necessary; and so on.

Formula for Simple Indexing. Letting *D* be the number of divisions required and *C* the number of revolutions of the index crank, we have the following simple formula:

$$C = \frac{40}{D}$$

For example, using the above instrument, find the indexing required to obtain 80 divisions.

We have $$C = \frac{40}{D} = \frac{40}{80} = \frac{1}{2}.$$

Therefore any circle of holes in the index plate may be used provided the number of holes is divisible by 2. In this case we might take a 16-hole circle and move the index crank 8 holes for each division.

Index Plates. Different makes of dividing heads have different sets of index plates. In our exercises we shall use plates which have circles with the number of holes as follows:

BROWN AND SHARPE

Plate 1: 15, 16, 17, 18, 19, 20
Plate 2: 21, 23, 27, 29, 31, 33
Plate 3: 37, 39, 41, 43, 47, 49

CINCINNATI MILLING MACHINE CO.

One plate drilled on both sides, as follows:

First side: 24, 25, 28, 30, 34, 37, 38, 39, 41, 42, 43
Second side: 46, 47, 49, 51, 53, 54, 57, 58, 59, 62, 66

Exercises. Simple Indexing

1. If a spur gear is to have 36 teeth, find the indexing required on a Cincinnati dividing head.

Since $C = \frac{40}{36} = 1\frac{1}{9}$, we must have a circle with the number of holes divisible by 9, in this case 54. The indexing required is therefore one revolution of the index crank plus 6 holes on the 54-hole circle.

2. Find the indexing required on a B. & S. (Brown and Sharpe) dividing head for 7 divisions.

Find the indexing on a B. & S. dividing head for each of the following divisions:

3. 3.	**5.** 45.	**7.** 98.	**9.** 145.	**11.** 205.
4. 26.	**6.** 68.	**8.** 116.	**10.** 164.	**12.** 312.

Find the indexing on a Cincinnati dividing head for each of the following divisions:

13. 5.	**15.** 17.	**17.** 44.	**19.** 70.	**21.** 92.
14. 9.	**16.** 23.	**18.** 55.	**20.** 85.	**22.** 110.

Angular Indexing. If the dividing head requires 40 turns of the index crank for one revolution of the spindle, that is, to turn the work through 360°, one turn of the crank will produce a turning of $\frac{1}{40}$ of 360°, or 9°. The process of indexing work by angles is known as *angular indexing.*

Therefore, if one revolution of the index crank gives an angle of 9°, two holes on the 18-hole circle or three holes on the 27-hole circle of a B. & S. dividing head will correspond to 1°. Halves and thirds of a degree are indexed by taking one hole on the 18-hole circle or one hole on the 27-hole circle respectively. On a Cincinnati dividing head the 54-hole circle is used, six holes corresponding to 1°, and so on.

For example, using a B. & S. dividing head, find the indexing for an angle of 25°.

Since one turn of the index crank gives an angle of 9°, the number of turns of the crank will be $\frac{25}{9}$, or $2\frac{7}{9}$. The indexing required is, therefore, two revolutions of the crank plus 14 holes on the 18-hole circle.

Exercises. Angular Indexing

1. Find the indexing on a B. & S. head for an angle of 11°.

2. Find the indexing on a Cincinnati dividing head for an angle of $17\frac{1}{2}$°.

Using a B. & S. dividing head, find the indexing for each of the following angles:

3. 1°.	**5.** $4\frac{1}{2}$°.	**7.** 13°.	**9.** 33°.	**11.** $44\frac{2}{3}$°.
4. $2\frac{1}{3}$°.	**6.** $8\frac{2}{3}$°.	**8.** $17\frac{1}{3}$°.	**10.** $40\frac{1}{2}$°.	**12.** $45\frac{1}{3}$°.

Using a Cincinnati dividing head, find the indexing for each of the following angles:

13. 3°.	**15.** 11°.	**17.** 23°.	**19.** $56\frac{1}{2}$°.	**21.** $79\frac{2}{3}$°.
14. $5\frac{1}{2}$°.	**16.** $19\frac{2}{3}$°.	**18.** $31\frac{1}{3}$°.	**20.** 75°.	**22.** $82\frac{1}{2}$°.

Compound Indexing. To obtain divisions which are beyond the range of simple indexing a device known as *compound indexing* is sometimes employed.

In this method the index crank is first turned a definite amount in the regular way, and then the stop pin holding the index plate is disengaged and the index plate is turned either in the same direction or in the opposite direction. The indexing is thus compounded of two separate movements which are, in reality, two simple-indexing operations.

Compound indexing should be used only when there is no other convenient way of obtaining the division. This is because of the chances of error due to the fact that the holes must be counted when moving the index plate, as will be seen in the example below. The method has been largely superseded by the differential method explained on page 70.

If, for example, the index crank of a B. & S. dividing head is turned 40 holes on the 18-hole circle, that is, two revolutions plus four holes, and then, the stop pin being disengaged, the index plate is turned 40 holes on the 19-hole circle, that is, two revolutions plus two holes, in the same direction, the two movements will cause the spindle holding the work to make $\frac{1}{18} + \frac{1}{19}$, or $\frac{37}{342}$, of a revolution.

If, however, the index plate is turned in the opposite direction from the index crank, the spindle will make $\frac{1}{18} - \frac{1}{19}$, or $\frac{1}{342}$, of a revolution. Since the two revolutions of the index crank and the index plate are in opposite directions, they offset each other, and the same result is obtained by moving the index crank four holes on the 18-hole circle in one direction and the index plate two holes on the 19-hole circle in the opposite direction.

Thus, by combining the two movements we can obtain 342 divisions with the index plates listed on page 65. With simple indexing we should have required a plate with a circle of 171 holes.

Illustration of Compound Indexing. For example, to find the indexing required on a B. & S. dividing head to obtain 147 divisions, we proceed as follows:

1. Find two factors of 147.

That is, $147 = 3 \times 49.$

2. Try to find two circles on the same plate, one with 49 holes and the other with perhaps ten holes less. In this case let us try the 49-hole and 39-hole circles.

3. Factor the difference between the numbers of holes in the two circles.

That is, $49 - 39 = 10 = 2 \times 5.$

4. Take the indicated product of these two sets of factors as the numerator of a fraction.

That is, the numerator is $(3 \times 49) \times (2 \times 5)$.

5. Take for the denominator of this fraction (1) the factors of 40, that is, of the number of turns of the index crank to one turn of the dividing-head spindle; (2) the factors of the larger circle; (3) the factors of the smaller circle.

That is,

$$(1) \quad 40 = 2 \times 2 \times 2 \times 5,$$
$$(2) \quad 49 = 7 \times 7,$$
$$(3) \quad 39 = 3 \times 13,$$

and hence the denominator is $(2 \times 2 \times 2 \times 5) \times (7 \times 7) \times (3 \times 13)$.

6. Now cancel, and if all the factors in the numerator cancel out, the correct circles have been chosen.

In this case we have
$$\frac{(\overset{\cancel{7}}{\cancel{49}} \times \cancel{3}) \times (\cancel{2} \times \cancel{5})}{(2 \times 2 \times \cancel{2} \times \cancel{5}) \times (\cancel{7} \times \cancel{7}) \times (\cancel{3} \times 13)},$$

and therefore the correct circles have been chosen.

If all the factors in the numerator do not cancel out, other numbers representing the numbers of holes in the circles of the index plates must be chosen until two circles are found that will permit the cancellation of all these factors.

7. Take the product of the uncanceled factors in the denominator as the numerator of each of two fractions, called the *indexing fractions*, in which the numerator is the number of holes to be turned, and the denominator is the number of holes in the circle on which the number turned is counted.

In this case the numerator of the indexing fractions is $2 \times 2 \times 13$, or 52, and the fractions which give the desired divisions are $+\frac{52}{39}$ and $-\frac{52}{49}$, the signs + and − being used to show that the two movements are to be made in opposite directions.

That is, by turning 52 holes on the 39-hole circle in one direction and 52 holes on the 49-hole circle in the other direction, we shall have the proper movement of the spindle.

This is seen from the fact that

$$\frac{52}{39} - \frac{52}{49} = \frac{40}{147}.$$

If any whole number is subtracted from each of two fractions, the difference of the fractions is not affected. In this case, therefore, we may subtract 1 from each fraction, and we have

$$\frac{52}{39} - 1 = \frac{13}{39}, \text{ and } \frac{52}{49} - 1 = \frac{3}{49}.$$

Hence we give the indexing fractions required to obtain 147 divisions by compound indexing as $+\frac{13}{39}$ and $-\frac{3}{49}$.

Exercises. Compound Indexing

Using compound indexing on a B. & S. dividing head, find the indexing fractions for each of the following divisions:

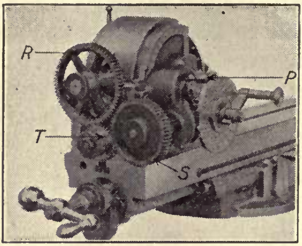
1. 69. **2.** 51. **3.** 99. **4.** 217. **5.** 294.

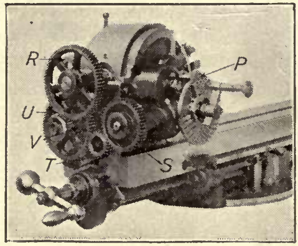
6. By the compound method, find the indexing fractions for cutting a spur gear with 77 teeth on a milling machine which is equipped with a B. & S. dividing head.

7. Find the fractions in Ex. 6 for a gear with 87 teeth.

8. Find the indexing fractions for cutting 129 teeth on the rim of a ratchet wheel, using compound indexing on a B. & S. dividing head.

Differential Indexing. The method of indexing known as *differential indexing* is the same in general principle as compound indexing. It differs, however, from the latter in that the index plate is revolved by a suitable gearing interposed between it and the dividing-head spindle, the stop pin holding the index plate being disengaged altogether. The movement





DIVIDING HEADS GEARED FOR DIFFERENTIAL INDEXING

The index plate of the dividing head shown at the left is connected to the spindle by simple gearing; that of the dividing head shown at the right, by compound gearing. The letters show the following parts: *P*, stop pin; *R*, gear on spindle (driving); *S*, gear on bevel-gear shaft, or worm, (driven); *T*, intermediate gear; *U*, first gear on stud (driving); *V*, second gear on stud (driven)

of the index plate takes place when the crank is turned, the index plate moving either in the same direction as the crank or in the opposite direction.

Therefore, in using differential indexing to obtain a given number of divisions we have to find two factors, (1) the indexing fraction and (2) the change gears which will turn the index plate the required amount in the proper direction during the movements of the index crank.

Change Gears. The change gears furnished with a B. & S. dividing head for differential indexing and spiral milling have the following numbers of teeth:

24 24 28 32 40 44 48 56 64 72 86 100

In solving the exercises on differential indexing use only these gears.

Illustration of Differential Indexing. For example, let us consider the differential indexing on a B. & S. dividing head for 59 divisions.

In simple indexing for 60 divisions the movement of the index crank is $\frac{40}{60}$, or $\frac{2}{3}$, of a turn for each cut.

If the crank is given $\frac{2}{3}$ of a turn 59 times, it makes $39\frac{1}{3}$ turns, or $\frac{2}{3}$ of a turn less than the 40 turns required for one revolution of the work. Hence the index plate must move in the same direction as the crank $\frac{2}{3}$ of a revolution while the work revolves once.

We therefore have

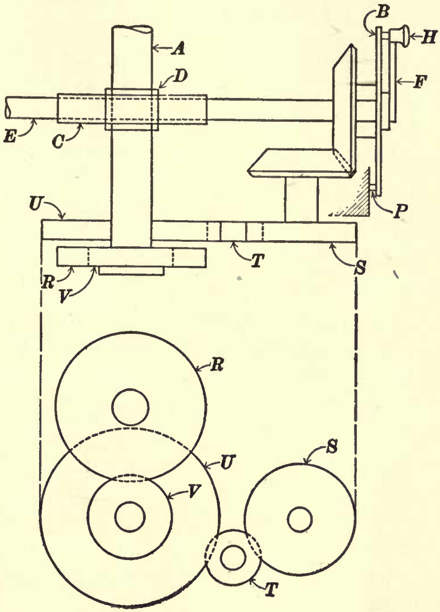
$$\frac{40}{60} = \frac{2}{3},$$

$$59 \times \frac{2}{3} = 39\frac{1}{3},$$

and $40 - 39\frac{1}{3} = \frac{2}{3}$.

Hence the ratio of the gears is $\frac{2}{3}$ to 1, or 2 : 3,

and $$\frac{16 \times 2}{16 \times 3} = \frac{32}{48}.$$



MECHANISM OF DIVIDING HEAD GEARED FOR DIFFERENTIAL INDEXING

A, spindle; *B*, index plate; *C*, worm wheel; *D*, worm; *E*, shaft; *H*, index pin; *P*, stop pin; *R*, gear on spindle (driving); *S*, gear on bevel-gear shaft, or worm, (driven); *T*, intermediate gear; *U*, first gear on stud (driving); *V*, second gear on stud (driven)

A 32-T gear (driving) is placed on the special differential-indexing center in the spindle of the dividing head, and a 48-T gear (driven) is placed on the bevel-gear shaft which turns the index plate. An intermediate gear is used to make the index plate move in the same direction as the index crank, which is given $\frac{2}{3}$ of a turn for each cut; that is, the indexing fraction may be given as $+\frac{2}{3}$.

Exercises. Review

1. Find the differential indexing on a B. & S. dividing head for 175 divisions.

For 180 divisions, $\frac{40}{180} = \frac{2}{9}$.

Then $175 \times \frac{2}{9} = 38\frac{8}{9}$, and $40 - 38\frac{8}{9} = 1\frac{1}{9}$.

Hence the required gearing must be in the ratio of $1\frac{1}{9}$ to 1, or 10 : 9. Since no two gears will give this ratio, we have

$$\frac{10}{9} = \frac{40}{36} = \frac{8}{9} \times \frac{5}{4}.$$

Gears in the ratio of 8 (driving) to 9 (driven) are used on the spindle and bevel-gear shaft, and gears in the ratio of 5 (driving) to 4 (driven) are used on the stud. We therefore have

$$\frac{8 \times 8}{8 \times 9} = \frac{64}{72}, \quad \text{and} \quad \frac{8 \times 5}{8 \times 4} = \frac{40}{32}.$$

The arrangement of the gears is as follows: spindle, 64 T; bevel-gear shaft, 72 T; first gear on stud, 40 T; second gear on stud, 32 T. With compound gearing an intermediate gear is not needed to make the index plate turn in the same direction as the index crank. The indexing fraction may be given as $\frac{4}{18}$, which is equivalent to $\frac{2}{9}$ of a turn.

Find the differential indexing on a B. & S. dividing head for each of the following divisions:

2. 53. **4.** 107. **6.** 161. **8.** 199. **10.** 271.

3. 67. **5.** 112. **7.** 173. **9.** 214. **11.** 321.

Using a Cincinnati dividing head, find the simple indexing movement for each of the following divisions:

12. 116. **13.** 145. **14.** 205. **15.** 245. **16.** 392.

Using a B. & S. dividing head, find the indexing movement for each of the following angles:

17. 12°. **18.** $21\frac{1}{2}$°. **19.** $29\frac{1}{3}$°. **20.** $53\frac{2}{3}$°. **21.** $66\frac{1}{2}$°.

22. Using compound indexing on a B. &. S. head, find the indexing fractions for milling 273 slots on a feed disk.

Spirals. A winding cut made by a tool moving around a cone and at the same time advancing along its slant line at a uniform rate is called a *spiral cut.*

Since the method of feeding the work is the same in the case of a straight reamer as in that of a taper reamer, it is customary to give the name "spiral cut" to the helical flutes of straight drills and reamers and also to the spiral teeth in mills, gears, and cutters.

The dividing head is used not only for indexing but also for the milling of spirals. When a spiral is being milled the work is turned slowly by the dividing head, while the table of the milling machine feeds lengthwise.

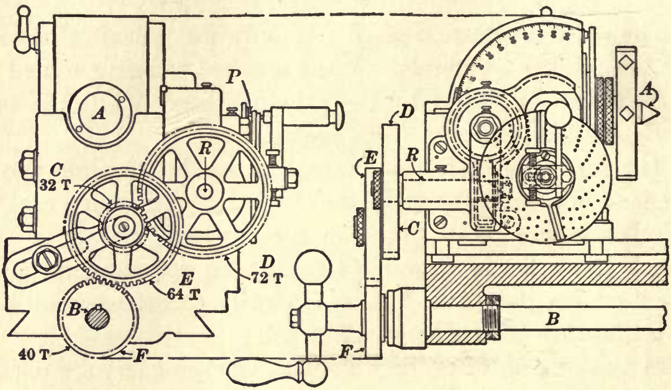
The *lead* of a spiral is figured in much the same way as the lead of a screw thread; that is, it is the distance that the spiral advances in making one turn around the work. The spiral, however, is spoken of as having so many inches to one turn instead of as having so many threads to one inch, as is the case with a screw thread.

In the process of milling a spiral, change gears are used to connect the dividing head with the feed screw on the table of the milling machine, so that the spindle of the dividing head rotates a definite amount in proportion to the distance that the table travels. The action of this gearing is similar to that used in cutting a thread in a lathe.

Spirals are always cut on a universal milling machine, the table of which can be set at any desired angle to the spindle holding the milling cutter. The table is set at such an angle as will bring the milling cutter in line with the cut to be made on the work, as is explained on page 78. The angle at which the table is set has no effect on the lead of the spiral, which is determined by the change gears used. If the table were left in its normal position at right angles to the cutter spindle, a spiral of the same lead would be obtained, but the shape of the cut would be changed.

Spiral Milling. The figure below shows both the side view and the end view of a dividing head mounted on the table of a milling machine and arranged for spiral milling.

The rotary movement of the dividing-head spindle *A*, which drives the work, is caused by the turning of the feed screw *B*, which also causes the table to move longitudinally.



DIVIDING HEAD CONNECTED BY COMPOUND GEARING WITH FEED SCREW OF MILLING MACHINE

A, spindle; *B*, feed screw; *C*, first gear on stud (driving); *D*, gear on bevel-gear shaft, or worm, (driven); *E*, second gear on stud (driven); *F*, gear on feed screw (driving); *P*, stop pin; *R*, bevel-gear shaft; *T*, teeth, as in 32 T

The motion of the feed screw *B* is transmitted through the compound train of change gears *F*, *E*, *C*, *D* to the bevel-gear shaft *R*, which has a set of bevel gears inside the dividing head connecting with the shaft on which the worm which drives the spindle is cut, as will be seen in the figures on pages 63 and 71. The gear on the feed screw is customarily spoken of as the "gear on screw," while that on shaft *R* is called the "gear on worm." When the dividing head is used for spiral milling, the stop pin *P*, which holds the index plate, is disengaged altogether.

Lead of a Milling Machine. If the change gears are in the ratio of 1 to 1 (that is, if gear D revolves once for each revolution of F), the lead of the milling machine is the distance that the table travels while the spindle revolves once.

This distance is a constant used in figuring change gears and varies with different makes of machines.

If R is the number of revolutions of the feed screw for one revolution of the spindle with 1-to-1 gearing and F is the lead of the feed screw, M, the lead of the milling machine, is found as follows:

$$M = RF$$

For example, if with 1-to-1 gearing $R = 40$ on a milling machine which has $F = \frac{1}{4}''$, we have

$$M = RF = 40 \times \tfrac{1}{4}'' = 10''.$$

Change Gears. Letting L be the lead of the spiral to be cut and M the lead of the milling machine, the ratio of driven to driving gears is found from the following formula:

$$\frac{\text{Gears driven}}{\text{Gears driving}} = \frac{L}{M}$$

Letting T_w represent the gear on worm (driven), T_s the gear on screw (driving), S_1 the first gear on stud (driving), and S_2 the second gear on stud (driven), the above ratio is split into two factors as follows:

$$\frac{\text{Gears driven}}{\text{Gears driving}} = \frac{T_w}{T_s} \times \frac{S_2}{S_1}$$

The terms of each of the ratios $T_w : T_s$ and $S_2 : S_1$ are multiplied by a number which will give numbers in the list of gears on page 70.

To find the lead of a spiral which will be cut with a given combination of change gears we have the following formula:

$$L = \frac{M \times \text{product of driven gears}}{\text{product of driving gears}}$$

Illustrative Problems. 1. Find the change gears required on a milling machine, which has a lead of 10″, for cutting a spiral with a lead of 36″.

Using the formulas given on page 75, we have

$$\frac{\text{Gears driven}}{\text{Gears driving}} = \frac{L}{M} = \frac{36}{10} = \frac{T_w}{T_s} \times \frac{S_2}{S_1} = \frac{9}{5} \times \frac{4}{2};$$

whence $\frac{T_w}{T_s} = \frac{9}{5} = \frac{8 \times 9}{8 \times 5} = \frac{72}{40}$, and $\frac{S_2}{S_1} = \frac{4}{2} = \frac{16 \times 4}{16 \times 2} = \frac{64}{32}$.

We therefore have the following arrangement of gears: gear on worm (driven), 72 T; gear on screw (driving), 40 T; first gear on stud (driving), 32 T; and second gear on stud (driven), 64 T.

2. Find the change gears required on a milling machine with a lead of 10″ for cutting a spiral with a lead of 12.80″.

$$\frac{\text{Gears driven}}{\text{Gears driving}} = \frac{L}{M} = \frac{12.80}{10} = \frac{128}{100} = \frac{T_w}{T_s} \times \frac{S_2}{S_1} = \frac{64}{100} \times \frac{2}{1}.$$

Then $\frac{T_w}{T_s} = \frac{64}{100}$, and $\frac{S_2}{S_1} = \frac{2}{1} = \frac{28 \times 2}{28 \times 1} = \frac{56}{28}$.

We therefore have the following arrangement of gears: gear on worm (driven), 64 T; gear on screw (driving), 100 T; first gear on stud (driving), 28 T; and second gear on stud (driven), 56 T.

3. What is the lead of the spiral that will be cut on a milling machine which has a lead of 10″ when the following arrangement of change gears is used: gear on worm, 86 T; first gear on stud, 40 T; second gear on stud, 48 T; and gear on screw, 44 T?

Using the formula given at the foot of page 75, we have

$$L = \frac{M \times \text{product of driven gears}}{\text{product of driving gears}} = \frac{\cancel{10} \times 86 \times \overset{\overset{3}{\cancel{12}}}{\cancel{48}}}{\underset{4}{\cancel{40}} \times \underset{11}{\cancel{44}}} = \frac{258}{11} = 23.4545.$$

Hence the lead of the spiral that will be cut is 23.45″.

Exercises. Cutting Spirals

1. If a milling machine has a lead of 10″, what change gears are needed to cut a spiral with a lead of 16″?

2. If a milling machine has a lead of 10″, what change gears are needed to cut a spiral with a lead of 22.50″?

3. For milling a spiral the change gears on a milling machine which has a lead of 10″ were placed as follows: a 64-T gear as the gear on worm, a 40-T gear as the gear on screw, a 32-T gear as the first gear on stud, and a 56-T gear as the second gear on stud. What is the lead of the spiral which was cut with this arrangement of gears?

Find the change gears required on a milling machine with a lead of 10″ for milling each of the following spiral leads:

4. 0.80″.	**7.** 4.125″.	**10.** 18.75″.	**13.** 31.50″.
5. 2.20″.	**8.** 12.60″.	**11.** 21.00″.	**14.** 37.50″.
6. 3.50″.	**9.** 17.20″.	**12.** 25.80″.	**15.** 52.50″.

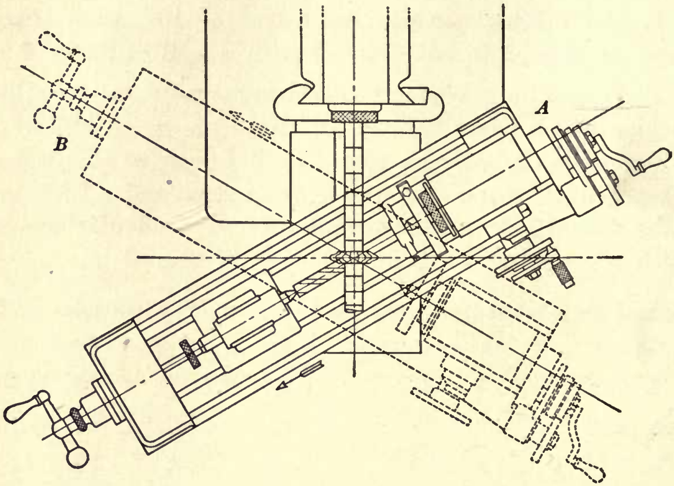
16. Find the change gears required on a milling machine with a lead of 10″ in cutting a spiral of one turn in $13\frac{1}{3}$″.

17. In milling a spiral the following arrangement of change gears was used: gear on worm, 86 T; gear on screw, 56 T; first gear on stud, 28 T; second gear on stud, 48 T. If the lead of the milling machine was 20″, what was the lead of the spiral which was milled?

18. What is the lead of the spiral which will be cut on a milling machine with a lead of 10″ when there is a 64-T gear on the bevel-gear shaft, or worm, a 32-T gear as the first gear on the stud, a 48-T gear as the second gear on the stud, and a 56-T gear on the feed screw?

19. What would be the lead of the spiral in Ex. 18 if the gears on the stud were reversed?

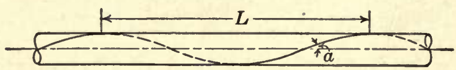
Setting the Table. When cutting a spiral the table of the milling machine is set at the angle which the spiral cut is to make with the axis of the work, as explained on page 73.



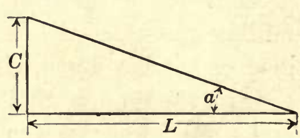
MILLING-MACHINE TABLE SET TO ANGLE FOR SPIRAL CUTTING

The position of the table shown by *A* is for cutting a right-hand spiral, and that shown by *B* is for cutting a left-hand spiral

As shown in the figures below, there is a fixed relation connecting the spiral angle *a*, the lead *L*, and the circumference of the work *C*.



If we let the base of a right triangle be equal to the lead *L*, as shown in the second figure, and let the altitude be equal to the circumference *C*, the angle *a* formed by the hypotenuse and the base is the required angle at which to set the table of the milling machine, and is found by the use of tangents.



Illustrative Problem. Find the angle at which to set the table of a milling machine when cutting a spiral with a lead of 28.00″ on a $3\frac{1}{4}$-inch blank.

Referring to the lower figures on page 78, in which C represents the circumference of the work and L the lead of the spiral, we see that

$$\tan a = \frac{C}{L}.$$

Then $$\tan a = \frac{C}{L} = \frac{3.1416 \times 3\frac{1}{4}}{28} = 0.3647,$$

and hence $$a = 20^\circ\, 2'.$$

Since the scale by which angle a is set is usually graduated only to fourths of a degree, we give the above result as 20°.

Exercises. Setting the Table

Find the angle at which to set the table of a milling machine to cut each of the following:

1. A spiral with a lead of 13.12″ on a spiral-gear blank which is $2\frac{1}{4}$″ in diameter.

In all problems dealing with setting the table of a milling machine give the results to the nearest $\frac{1}{4}$°.

2. A spiral of 8.95-inch lead on a 1-inch twist drill.

3. A spiral with a lead of 15.75″ on a milling cutter which is to be $\frac{5}{8}$″ in diameter.

4. A spiral with a lead of 0.67″ on a $\frac{1}{8}$-inch spindle.

5. A spiral with a lead of 67.45″ on a spiral-gear blank which has a diameter of $5\frac{5}{8}$″.

6. If the angle of the spiral on a twist drill $\frac{11}{16}$″ in diameter is $19\frac{1}{2}$°, what is the lead?

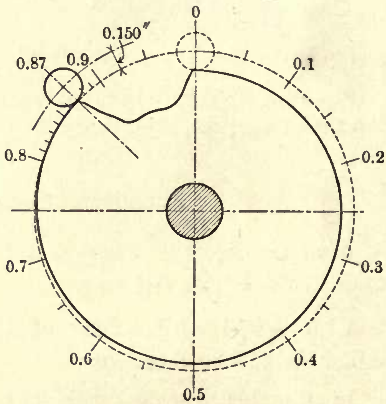
7. If the table of a milling machine is set at an angle of $10\frac{1}{4}$° for cutting a spiral with a lead of 68.57″, what is the diameter of the work?

Cams. A rotating part of a machine that gives a reciprocating or oscillating motion to another part is called a *cam.*

The piece to which the motion is thus given is called the *follower,* and the contact between the two is called the *line of contact.*

A cam may also have a reciprocating or oscillating motion.

The distance that the follower is moved by the action of the cam is called the *rise,* or *lead,* of the cam. Thus, the cam shown in this figure has a constant rise in 0.87 of its circumference, and in one revolution imparts to the follower a movement of 0.150″.



A CAM OF ONE LOBE

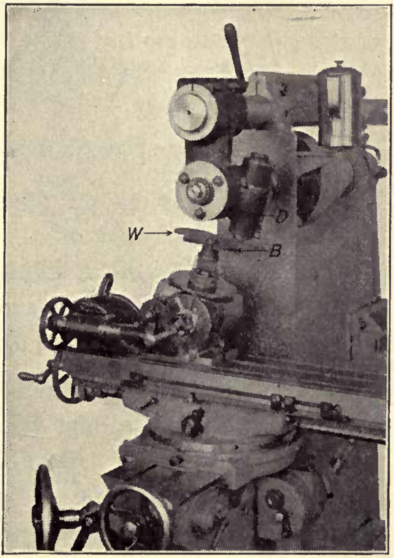
The lobe extends through 0.87 of the circumference, or approximately 313°, and the cam has a lead of 0.150″

Cams are made in many different shapes, and often have more than one lobe, but we shall consider only what are known as *plate cams,* or *peripheral cams.*

Plate cams having a constant rise, such as are used on automatic screw machines, can be cut on a universal milling machine. Since these cams usually have shorter leads than can be obtained on a milling machine by any practical combination of change gears, it is necessary when cutting them to use the vertical-spindle attachment of the milling machine to hold the milling cutter.

Such cams are required in great variety, each differing from the others by only a few thousandths of an inch. Since there are practically no duplications, it is impracticable to cut them by means of master cams.

Milling Cams. When the dividing head is set with its spindle at right angles to the table of the milling machine, the lead, which will be cut on a cam if milled for a complete revolution, is the same as the lead of the spiral cut with the same change gears. If, however, the dividing head and the vertical-spindle attachment are both set at an angle, any required lead can be obtained, provided it is less than the spiral lead for which the milling machine is geared. Therefore change gears, which will give a longer lead than that to be cut on the cam, are first selected from a table of spiral leads.



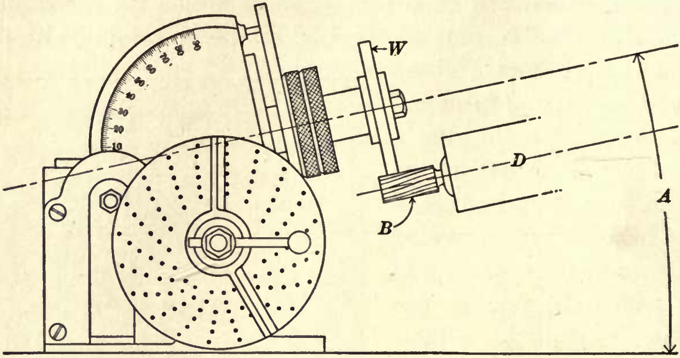
MILLING A SCREW-MACHINE CAM

B, milling cutter; *D*, vertical-spindle attachment; *W*, cam being cut

The dividing head is then elevated from the horizontal a certain angle, which is found by the formula given on page 82, and the vertical-spindle attachment, which drives the milling cutter, is swiveled around from its vertical position an amount equal to the complement of the angle at which the dividing head is set. This places the milling cutter in line with the spindle of the dividing head so that the edges of the cam will be cut parallel with its axis.

Since the graduations for elevating the dividing head are usually marked only in fourths of a degree, in all problems dealing with cams the angles are to be figured to the nearest $\frac{1}{4}°$.

Finding the Angle. If A is the angle for setting the dividing head, c the lead of the cam, or its rise if continued at the



DIVIDING HEAD SET TO ANGLE FOR MILLING A CAM

A, angle of dividing head; B, cutter; D, vertical-spindle attachment; W, cam

given rate for a complete revolution, and s the lead of the spiral for which the milling machine is geared, we have

$$\sin A = \frac{c}{s}.$$

If the rise of the cam is given only for n, a definite number of hundredths of the circumference, the formula becomes

$$\sin A = \frac{c}{ns}.$$

Illustrative Problems. 1. Find the angles at which to set the dividing head and the vertical-spindle attachment in milling a cam with a lead of 0.602″, the milling machine being geared to cut a spiral with a lead of 0.67″.

We have $$\sin A = \frac{c}{s} = \frac{0.602}{0.67} = 0.8985,$$

and hence $$A = 63° \, 58'.$$

Calculating to the nearest $\frac{1}{4}°$, the dividing head is elevated 64°, and the vertical-spindle attachment is set at 90° − 64°, or 26°, from the vertical.

2. In Ex. 1 find the angles if the cam were to have a lead of 0.150″ in 300°, or in 0.83 of the circumference.

We have $$\sin A = \frac{c}{ns} = \frac{0.150}{0.83 \times 0.67} = 0.2697,$$

and hence $$A = 15° \, 39'.$$

The dividing head is elevated $15\frac{3}{4}°$ and the vertical-spindle attachment is swiveled around $90° - 15\frac{3}{4}°$, or $74\frac{1}{4}°$, from the vertical.

Exercises. Cutting Plate Cams

1. Find the angle at which to elevate the dividing head when cutting a cam with a lead of 1.249″, the milling machine being geared to cut a spiral with a lead of 1.333″.

2. A milling machine is geared to cut a spiral with a lead of 1.860″. At what angles should the dividing head and the vertical-spindle attachment be set for milling a cam with a lead of 1.801″?

3. A cam having a rise of 0.634″ in 0.75 of its circumference is to be milled on a milling machine geared to cut a spiral with a lead of 0.930″. Find the angle at which to elevate the dividing head.

4. In Ex. 3 find the angle at which to set the vertical-spindle attachment if the given rise were to be obtained in 0.83 of the circumference of the cam.

A milling machine being geared to cut a spiral with a lead of 0.916″, find the angle at which to elevate the dividing head for milling a cam with each of the following leads:

5. 0.669″.	**8.** 0.789″.	**11.** 0.733″.	**14.** 0.898″.
6. 0.691″.	**9.** 0.712″.	**12.** 0.859″.	**15.** 0.771″.
7. 0.701″.	**10.** 0.853″.	**13.** 0.751″.	**16.** 0.904″.

Exercises. Review

1. If the feed screw on the table of a milling machine has a lead of 0.4″ and it requires 40 revolutions of the feed screw with 1-to-1 gearing to turn the dividing-head spindle once, what is the lead of the machine?

2. Find the change gears required on a milling machine with a lead of 10″ for milling a cutter with spiral teeth of 9.60-inch lead.

3. Find the angle at which to elevate the dividing head when milling a cam with a rise of 0.150″ in 0.75 of the circumference, the machine being geared to cut a spiral with a lead of 0.67″.

Find the angle at which to set the table to cut a spiral with a lead of 13.760″, given the diameter of each piece as follows:

4. $\frac{1}{8}''$.	**6.** $\frac{1}{4}''$.	**8.** $\frac{3}{8}''$.	**10.** $\frac{1}{2}''$.	**12.** $\frac{3}{4}''$.
5. $1\frac{5}{8}''$.	**7.** $1\frac{3}{4}''$.	**9.** $2\frac{1}{2}''$.	**11.** $1\frac{7}{8}''$.	**13.** $3\frac{15}{16}''$.

14. Find the lead of the spiral which is cut on a milling machine with a lead of 10″ when there is a 100-T gear on the worm, a 24-T gear as the first and a 64-T gear as the second gear on the stud, and a 28-T gear on the screw.

15. Find the angles at which to set the dividing head and the vertical-spindle attachment when milling a cam with a rise of 0.136″ in 0.85 of the circumference, the milling machine being geared to cut a spiral with a lead of 0.67″.

Find the change gears required on a milling machine with a lead of 10″ for milling each of the following spiral leads:

16. 0.90″.	**19.** 3.75″.	**22.** 12.375″.	**25.** 27.50″.
17. 1.44″.	**20.** 6.60″.	**23.** 13.125″.	**26.** 31.25″.
18. 2.45″.	**21.** 8.25″.	**24.** 16.875″.	**27.** 42.00″.

CHAPTER VI

GEARS

Nature of Gears. If two wheels are mounted on parallel axes, and if the rims of the wheels are placed in close contact, when one is turned the other will also turn if there is sufficient friction. The surfaces thus placed in contact are called *friction surfaces.*

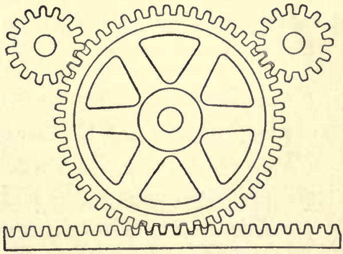
The circumferences of these wheels are called *pitch circles*, and the diameters of these circles are called *pitch diameters.*

To prevent slipping the friction surfaces are usually grooved so as to form teeth which mesh together. Wheels thus provided with teeth are known as *gear wheels*, or *gears*.

A toothed wheel in which the teeth are cut parallel to the axis and at right angles to the sides is called a *spur gear.*

In the spur gearing here shown the smaller gears are called *pinions* and the straight bar is called a *rack.*

Besides spur gears there are gears on conical surfaces which transmit power to shafts at an angle to each other. Such gears are known as *bevel gears.* Gears having teeth on the inside of the rim are called *internal gears.*



SPUR GEARING

Teeth. It is evident that the teeth may have various shapes, but there are only two systems in general use,—the involute, or single-curve form of tooth, and the cycloidal, or double-curve form. The cycloidal form is being rapidly replaced by the involute form.

Terms Used. The figure on page 87 illustrates the definitions of the following terms, which we shall use in connection with the work on spur gearing:

Addendum. The height (a) of a tooth above the pitch circle.

Center distance. The distance (D_c) between the centers of the shafts to which the gears are fastened.

Chordal pitch. The distance from the center of one tooth to the center of the next tooth, measured on a straight line; that is, on the chord of the pitch circle.

Circular pitch. The distance (P_c) between the centers of two adjacent teeth, measured on the pitch circle.

It includes the equivalent of one space and one tooth.

Clearance. The distance (c) from the working-depth circle to the whole-depth circle to allow freer action of the gears.

Dedendum. The depth (d) of a tooth between the pitch circle and the working-depth circle.

Diametral pitch. The number of teeth per unit of the pitch diameter of the gear.

Thus, if there are 48 teeth on a gear with a pitch diameter of 12″, the diametral pitch of the gear is $\frac{48}{12}$, or 4.

Unless otherwise stated, the "pitch" of a gear means the "diametral pitch."

Face of a tooth. The working surface (Fa) outside the pitch circle and extending the width of the tooth.

Flank of a tooth. The working surface (Fl) inside the pitch circle and extending the width of the tooth.

Outside circle. The circumference (Y) of the gear blank.

Outside diameter. The diameter (D_o) of the outside circle.

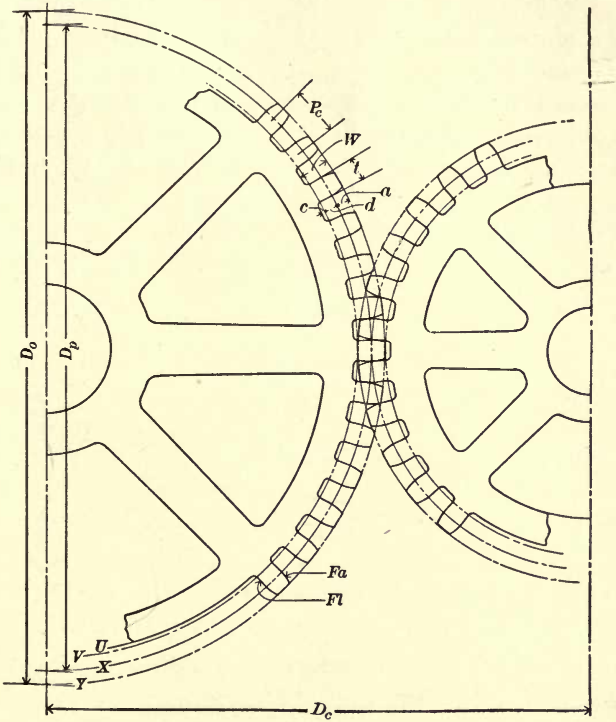
Pitch circle. The circumference of the friction cylinder.

Pitch diameter. The diameter (D_p) of the pitch circle.

Pitch line. The circumference line (X) of the pitch circle.

Root. The total depth of a tooth inside the pitch circle, equivalent to the dedendum plus the clearance.

Thickness of tooth. The thickness (t) of the tooth, measured on the pitch line in the same manner as the circular pitch.



TERMS USED IN SPUR GEARING

a, addendum; c, clearance; d, dedendum; D_c, center distance; D_o, outside diameter; D_p, pitch diameter; Fa, face; Fl, flank; P_c, circular pitch; t, thickness of tooth on pitch line; U, whole-depth circle; V, working-depth circle; W, whole depth; X, pitch circle; Y, outside circle

Whole depth. The distance (W) from the outside circle to the circle (U) formed by the bottom of the grooves.

Working depth. The distance from the outside circle to the circle (V) formed by the top of the teeth of the mating gear.

Symbols. We shall use the following symbols in the formulas and computations for spur gearing:

a	= addendum	P_c	= circular pitch
c	= clearance	P_d	= diametral pitch
d	= dedendum	r	= root
D_o	= outside diameter	t	= thickness of tooth
D_p	= pitch diameter	W	= whole depth
N	= number of teeth	W'	= working depth

Spur-Gearing Formulas. The following formulas used in practical work may be referred to in solving the exercises:

$$a=\frac{P_c}{\pi}=\frac{1}{P_d}$$

$$c=\frac{t}{10}$$

$$d=\frac{P_c}{\pi}=\frac{1}{P_d}$$

$$D_o=\frac{(N+2)P_c}{\pi}=D_p+\frac{2P_c}{\pi}=(N+2)a=\frac{N+2}{P_d}=D_p+\frac{2}{P_d}=\frac{D_p(N+2)}{N}$$

$$D_p=\frac{NP_c}{\pi}=\frac{ND_o}{N+2}=D_o-\frac{2P_c}{\pi}=\frac{N}{P_d}=D_o-\frac{2}{P_d}$$

$$N=\frac{\pi D_p}{P_c}=D_pP_d=D_oP_d-2$$

$$P_c=\frac{\pi}{P_d}=\frac{\pi D_p}{N}=\frac{\pi D_o}{N+2}$$

$$P_d=\frac{N}{D_p}=\frac{\pi}{P_c}=\frac{N+2}{D_o}$$

$$r=\frac{1}{P_d}+c$$

$$t=\frac{P_c}{2}=\frac{\pi}{2P_d}$$

$$W'=\frac{2}{P_d}$$

$$W=2a+c=\frac{2}{P_d}+c$$

$$\pi=3.1416$$

$$\frac{1}{\pi}=0.3183$$

Exercises. Spur Gears

1. A gear has a circular pitch of 0.7854″. Find the pitch.

Remember that the word "pitch" when used alone always refers to the diametral pitch, and use the second formula for P_d on page 88. The difference in the use of the term "pitch" as applied to gears and to screw threads should be noted. Thus, we speak of a screw with 4 threads to the inch as a screw of $\frac{1}{4}$-inch pitch, while a gear which has 4 teeth to each inch of the pitch diameter is a 4-pitch gear.

2. What is the circular pitch of a gear which has a pitch of 26?

Carry all computations for dimensions of gears to four decimal places, but in the final result discard the fourth place, giving the dimensions correct to the nearest 0.001″.

3. Find the outside diameter of a gear having 70 teeth and a pitch of 8.

4. Find the thickness of tooth on the pitch line of a gear with a pitch of 16.

5. A gear has a pitch of 12 and an outside diameter of 5″. Find the number of teeth.

6. If $P_c = 2.5133''$, what is the value of P_d? Write a statement of the meaning of the problem, not using the symbols.

The value of P_d should be given as $1\frac{1}{4}$.

7. If $P_d = 9$, what is the value of P_c? Write a statement as in Ex. 6.

8. If the outside diameter of a gear is 18″ and the pitch is 6, what is the number of teeth?

9. If a gear has 24 teeth and a pitch of 18, what is the outside diameter?

10. What should be the diameter of the gear blank for a gear which is to have 46 teeth and the circular pitch of which is to be 0.1309″?

11. Find the whole depth of a tooth on a gear having a pitch of 10.

It will be seen from the formulas on page 88 that to find the whole depth W we must use the formula $W = \frac{2}{P_d} + c$, in which we find c from the formula $c = \frac{t}{10}$ and t from the formula $t = \frac{\pi}{2\,P_d}$. Since $\pi = 3.1416$, we have $\frac{\pi}{2} = 1.5708$, and hence $t = \frac{1.5708}{P_d}$. If we substitute this value for t in the formula $c = \frac{t}{10}$, we shall then have $c = \frac{1.5708}{10\,P_d}$, or $c = \frac{0.1571}{P_d}$. Using this value for c, we have

$$W = \frac{2}{P_d} + c = \frac{2}{P_d} + \frac{0.1571}{P_d} = \frac{2.1571}{P_d}.$$

Such a formula when once developed may be used in finding the whole depth of tooth for any given pitch.

12. How many teeth should there be on a gear having a pitch of 12 and an outside diameter of $3\frac{3}{4}''$?

13. If $P_d = 1\frac{3}{4}$, what is the value of P_c? Write a statement of the meaning of the problem, not using the symbols.

14. If the outside diameter of a gear is $4''$ and the number of teeth is 46, what is the pitch?

15. Find the number of teeth for a gear having a pitch of 14 and an outside diameter of $7\frac{2}{7}''$.

16. Find the thickness of a tooth on an 8-pitch gear.

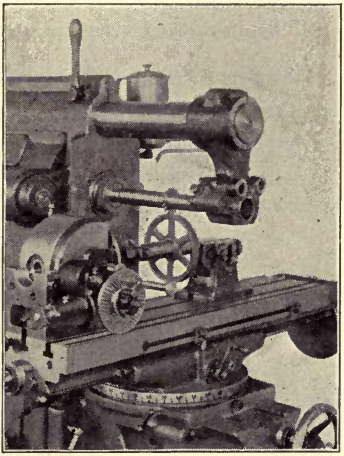
17. Find the whole depth of a tooth on an 11-pitch gear.

18. Find the pitch of a 75-tooth gear having an outside diameter of $19\frac{1}{4}''$.

19. The pitch diameter of a gear is $1.8182''$ and the pitch is 22. Find the number of teeth and the outside diameter.

20. The circular pitch of a gear is $1\frac{3}{4}''$. Find the working depth of a tooth.

Cutting Spur Gears. In cutting spur gears on a milling machine a cutter of the proper pitch is placed on the arbor of the machine, the table of which is set at zero. The center line on the cutter teeth must be central with the axis of the gear blank, which is held on an arbor between the dividing-head and tailstock centers. The gear is indexed by one of the methods described in Chapter V.



CUTTING A SPUR GEAR ON A MILLING MACHINE

Center Distance. In speaking of a pair of gears the smaller gear, if one is smaller than the other, is called the *pinion*, the other being called the *gear*. In finding the center distance D_c of two spur gears in mesh, we use the formula

$$D_c = \frac{N_g + N_p}{2 P_d},$$

in which N_g and N_p represent the number of teeth on the gear and the number of teeth on the pinion respectively, and P_d is the diametral pitch.

There is also a convenient rule used in finding the pitch diameters of two spur gears in mesh, given the center distance and the ratio $v_g : v_p$ of the velocities of the gear and the pinion in lowest terms. This rule expressed by formulas is as follows, D_{pg} and D_{pp} being the pitch diameters of the gear and the pinion respectively:

$$D_{pg} = 2\, v_p \times \frac{D_c}{v_g + v_p}$$

$$D_{pp} = 2\, v_g \times \frac{D_c}{v_g + v_p}$$

Illustrative Problems. **1.** Two gears in mesh have 90 teeth and 70 teeth respectively. If the pitch of the gears is 10, what is the center distance?

$$D_c = \frac{N_g + N_p}{2 P_d} = \frac{90 + 70}{2 \times 10} = \frac{160}{20} = 8.$$

Therefore the center distance of the pair of gears is 8″.

2. Find the pitch diameters of two gears that are 12″ between centers and have velocities in the ratio of 1 to 2.

The smaller gear, or pinion, will obviously have the larger velocity, so that in the ratio of velocities 2 represents the velocity of the pinion and 1 represents that of the gear.

$$D_{pg} = 2\, v_p \times \frac{D_c}{v_g + v_p} = 2 \times 2 \times \frac{12}{2+1} = 2 \times 2 \times 4 = 16.$$

$$D_{pp} = 2\, v_g \times \frac{D_c}{v_g + v_p} = 2 \times 1 \times \frac{12}{2+1} = 2 \times 1 \times 4 = 8.$$

Therefore the pitch diameters of the gear and the pinion are 16″ and 8″ respectively.

Exercises. Spur Gears

1. Two gears in mesh have 36 teeth and 24 teeth respectively. If their pitch is 8, what is the center distance?

2. Two gears in mesh have 128 teeth and 76 teeth respectively. If their pitch is 18, what is the center distance?

3. If the ratio of the velocities of two gears in mesh is 4 to 5 and the center distance of the gears is 27″, what are the pitch diameters?

4. If the center distance of two gears in mesh is 16″ and the ratio of the velocities is 2 to 3, what are the pitch diameters of the gears?

5. The ratio of the velocities of two gears in mesh is 3 to 4, and the center distance is 21″. If the gears have a pitch of 10, what is the outside diameter of each gear?

6. In Ex. 5 find the number of teeth on each of the gears.

7. If two 6-pitch gears in mesh have 132 teeth and 66 teeth respectively, what is the center distance?

8. If the velocity ratio of two gears in mesh is 5 to 6 and the distance between the centers is 33″, what are the pitch diameters of the gears?

9. If the pitch in Ex. 8 is 6, find the outside diameters.

10. Find the number of teeth on each of the gears in Ex. 8.

11. If two 3-pitch gears in mesh have 182 teeth and 96 teeth respectively, what is the center distance?

12. If the velocity ratio of two gears in mesh is 3 to 1 and the distance between the centers of the gears is 48″, what are the pitch diameters?

13. If the pitch in Ex. 12 is 4, find the outside diameters.

14. Find the number of teeth on each of the gears in Ex. 12.

15. The center distance of two gears in mesh is 6″. One gear has 48 teeth and its pitch is 16. Find the number of teeth and the outside diameter of the other gear.

16. If the center distance of two gears in mesh is 12″, and the gears have a velocity ratio of 4 to 6, find the pitch diameters. If the pitch of the gears is 10, find the number of teeth on each gear and also find the outside diameters.

17. If the gears in Ex. 16 are cut on a milling machine which is equipped with a Cincinnati dividing head, find the indexing required in cutting each gear.

18. If the pitch diameters of two gears in mesh are $4\frac{1}{2}''$ and $13\frac{1}{2}''$ respectively, find the velocity ratio and the distance between the centers of the gears.

19. Two gears mesh together, one having 76 teeth and an outside diameter of 5″, and the other 49 teeth and an outside diameter of 3″. Find the center distance of the gears.

Comparative Sizes of Gear Teeth. In order that the student may have an idea of the comparative sizes of the gear teeth that he is using, several figures are shown on page 95. In some of these cases both P_d and P_c are given, but either may be found from the other by the formulas on page 88.

Exercises. Gear Teeth

Find the circular pitch of a gear when the diametral pitch in each case is as follows:

1. $\frac{3}{4}$. **3.** 14. **5.** 18. **7.** 22. **9.** 28. **11.** 38.

2. $1\frac{1}{2}$. **4.** 16. **6.** 20. **8.** 24. **10.** 32. **12.** 46.

Find the diametral pitch of a gear when the circular pitch in each case is as follows:

13. 0.0628″. **15.** 0.0873″. **17.** 0.1047″. **19.** 0.2856″.

14. 0.0714″. **16.** 0.0924″. **18.** 0.1208″. **20.** 0.8976″.

21. Find the thickness on the pitch line of a $\frac{1}{2}$-P_d gear tooth; of a $1\frac{3}{4}$-P_d gear tooth.

A $\frac{1}{2}$-P_d gear tooth is a tooth on a gear which has a pitch of $\frac{1}{2}$.

22. Find the addendum of a 4-inch-P_c gear tooth.

A 4-inch-P_c gear tooth is a tooth on a gear in which $P_c = 4''$.

23. Find the whole depth of a $2\frac{1}{2}$-P_d gear tooth.

24. Find the thickness on the pitch line of a $\frac{15}{16}$-inch-P_c gear tooth; of a $1\frac{3}{16}$-inch-P_c gear tooth.

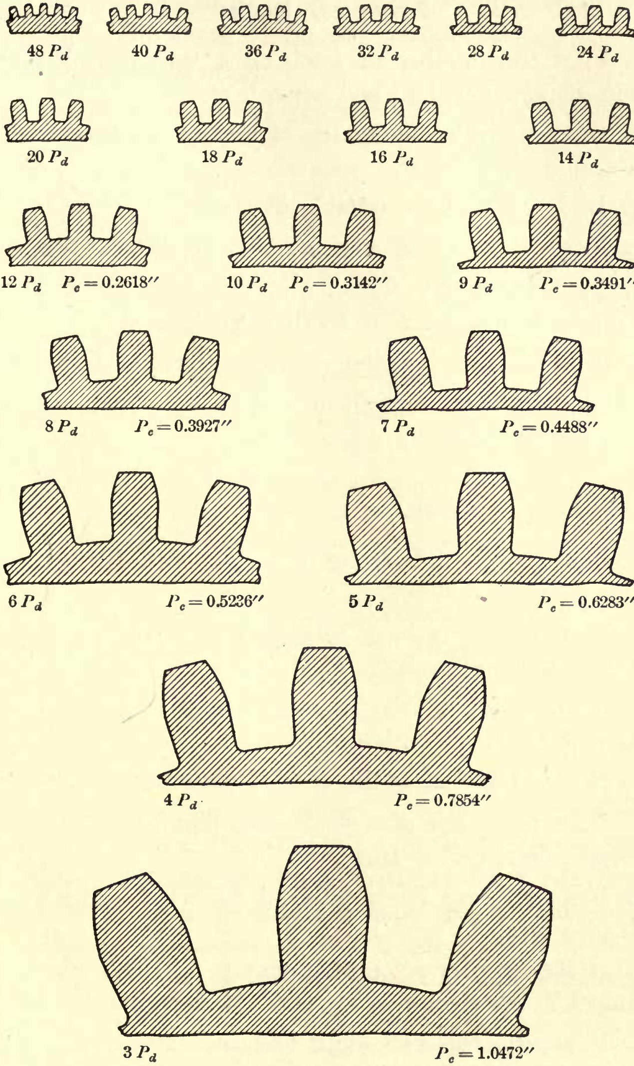
25. If the pitch diameter of a spur gear is 2.8571″ and there are 40 teeth, what is the pitch of the gear?

26. Find the addendum of a $1\frac{1}{4}$-P_d gear tooth.

27. Find the clearance of a $\frac{1}{16}$-inch-P_c gear tooth.

28. Find the whole depth of a 38-P_d gear tooth.

29. Find the working depth of a $\frac{7}{8}$-inch-P_c gear tooth.



COMPARATIVE SIZES OF GEAR TEETH

Exercises. Review

1. Find the number of teeth on a spur gear having an outside diameter of 4.4″ and a pitch of 10.

2. Find the pitch diameter of a spur gear that has 32 teeth and a pitch of 10.

3. In Ex. 2 find the outside diameter.

4. Find the pitch of a gear in which $P_c = 6.2832''$.

5. Find the center distance of two 12-P_d gears in mesh that have 66 teeth and 54 teeth respectively.

6. Find the circular pitch of a gear that has a pitch of 34.

7. Find the whole depth of a tooth on a rack that has a linear pitch of 0.1745″.

When referring to a rack the term "linear pitch" is used in place of the term "circular pitch."

8. Find the pitch diameter of a gear with 42 teeth and a circular pitch of 0.2618″.

9. The velocity ratio of two gears in mesh is 3 to 5 and the center distance is 24″. Find the pitch diameters.

10. If the pitch of the gears in Ex. 9 is 4, what is the number of teeth on each gear?

11. In Ex. 10 what is the outside diameter of each gear?

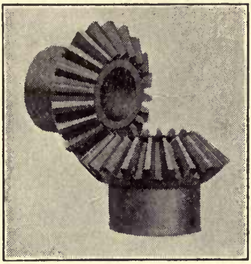
12. Find the pitch of a spur gear that has 80 teeth and an outside diameter of 16.4″.

13. If a spur gear has a pitch of 36, what is the working depth of a tooth?

14. If the circular pitch of a gear is 0.1653″, what is the addendum? the thickness of tooth on the pitch line?

15. If a gear has 140 teeth and an outside diameter of $10\frac{1}{7}''$, what is the pitch?

Bevel Gears. Bevel gears are used for transmitting rotary motion from one shaft to another where the two shafts are not parallel to each other.



BEVEL GEARS

While the two shafts are generally at right angles to each other, this is not essential; gears can be made to allow for any particular angle desired.

As before, one of the two gears, the smaller, if there is a smaller, is called the *pinion*, the other one being then called the *gear*.

The teeth of bevel gears are constructed on imaginary friction cones in the same way that the teeth of spur gears are constructed on imaginary friction cylinders.

Such terms as *diametral pitch*, *circular pitch*, *whole depth*, *pitch diameter*, and *outside diameter* are used with the same general meaning as in the case of spur gears.

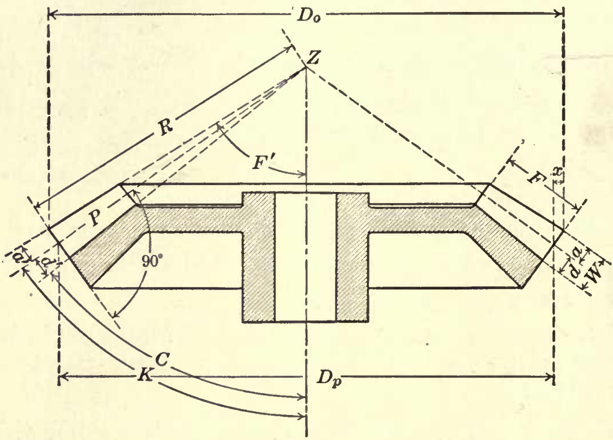
Since the teeth of a bevel gear constantly become thinner as they approach the apex of the cone, in cutting bevel gears with a rotary cutter on a milling machine, it is not possible to obtain perfect finished teeth. The teeth are cut to approximate size by offsetting the cutter and rotating the blank, and the final thinning of the small end of the teeth is generally done by filing. At the large end of the teeth on a bevel gear the circular pitch, the thickness of tooth on the pitch line, and the whole depth of tooth are identical with those of a spur gear of the same size and pitch.

Bevel gears are always figured in pairs, and we shall consider only such cases as those in which the shafts are at right angles to each other.

The method of computing the essential measurements for cutting a pair of bevel gears is shown on page 101.

Terms Used. In addition to the terms used with spur gearing given on page 86, the following terms, illustrated in the figure below, are used in connection with bevel gears:

Addendum angle. The angle (a') between the pitch line and the top line of a tooth extended to the vertex.



TERMS USED WITH BEVEL GEARS

a, addendum; a', addendum angle; C, cutting angle; d, dedendum; d', dedendum angle; D_o, outside diameter; D_p, pitch diameter; F, width of face; F', face angle; K, pitch-cone angle; P, pitch line; R, pitch-cone radius; W, whole depth; x, angular addendum; Z, vertex

Angular addendum. The straight-line distance (x) from the pitch circle to the outside circle.

Twice the angular addendum is called the *diameter increment.*

Cutting angle. The angle (C) between the bottom of a groove and the axis of the gear.

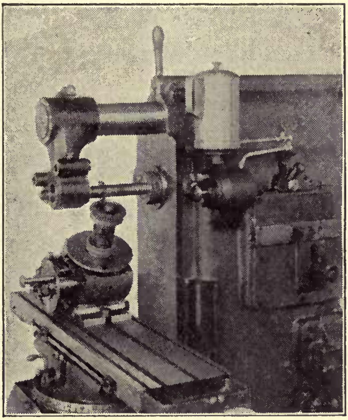
This angle is the same as the angle at which the dividing head is set while cutting the teeth.

Dedendum angle. The angle (d') formed by the bottom of a groove and the pitch line.

Face angle. The angle (F') with the axis to which the gear blank is turned.

Pitch circle. The circle formed by a cross section of the imaginary friction cone, the diameter of the circle being the pitch diameter.

Since bevel gears are constructed on cones they may have several pitch circles, as, for example, a pitch circle at the larger end of the teeth and one at the smaller end. In speaking of the pitch of a bevel gear we always mean the number of teeth per unit of the diameter of the largest pitch circle; that is, the diameter D_p in the figure on page 98.



CUTTING A BEVEL GEAR ON A MILLING MACHINE

Pitch-cone angle. The angle (K) with the axis of the imaginary friction cone.

Pitch-cone radius. The radial distance (R) from the vertex to the largest pitch circle.

This distance is measured from the vertex Z to the circle of which D_p is the diameter in the figure on page 98. If the surface of the imaginary friction cone were laid out flat it would be a sector of a circle. The pitch-cone radius is the radius of this circle, and is sometimes called the *apex distance*, or *vertex distance*.

Pitch line. A line equivalent to the slant height of the imaginary friction cone; more specifically, that part (P) of the line lying within the teeth. Also, as in spur gearing, the circumference of the greatest pitch circle; that is, the circle measured by D_p in the figure on page 98.

Vertex. The apex (Z) of the cone on which the gear is cut.

Width of face. The length (F) of the teeth, usually cut so as to be about one fourth the pitch-cone radius.

Symbols. In the formulas and in computations relating to bevel gears we shall use the following symbols:

a = addendum
a' = addendum angle
C = cutting angle
d = dedendum
d' = dedendum angle
D_o = outside diameter
D_p = pitch diameter
F' = face angle
K = pitch-cone angle
N = number of teeth
P_d = diametral pitch
R = pitch-cone radius
W = whole depth
x = angular addendum

The subscript g for "gear" or p for "pinion" may be added to indicate that a certain symbol applies to the gear or to the pinion. Thus C_g is the cutting angle of the gear and C_p is that of the pinion.

Bevel-Gear Formulas. The following formulas may be referred to in solving the exercises on bevel gears:

$$a = \frac{1}{P_d} \qquad \tan d' = \frac{d}{R} \qquad \tan K_g = \frac{N_g}{N_p}$$

$$\tan a' = \frac{a}{R} \qquad D_o = D_p + \frac{2\cos K}{P_d} \qquad R = \frac{D_p}{2\sin K}$$

$$d = \frac{1.1571}{P_d} \qquad D_p = \frac{N}{P_d} \qquad W = \frac{2.1571}{P_d}$$

$$C = K - d' \qquad F' = K + a' \qquad K_p = 90^\circ - K_g$$

All the formulas in which the subscripts g for "gear" and p for "pinion" are not used, apply to both the gear and the pinion. We may indicate that a formula applies to the gear or to the pinion by adding the proper subscripts, as in the following cases:

$$D_{pg} = \frac{N_g}{P_d} \qquad D_{pp} = \frac{N_p}{P_d}$$

The result found from the formula for d is actually the dedendum plus the clearance, but in bevel gears we treat this as the dedendum.

If the machinist uses formed cutters and the correct indexing, the thickness on the pitch line at the large end of a tooth will be correct. This measurement is usually checked with a table of chordal thicknesses.

Illustrative Problem. Make the calculations for the pair of 12-P_d bevel gears shown below in which $N_g = 48$ and $N_p = 24$.

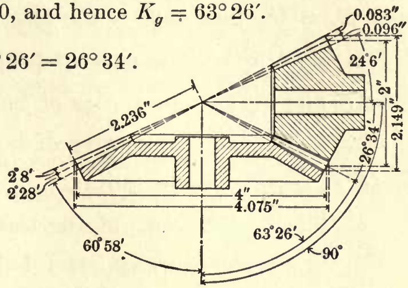
(1) $\tan K_g = \frac{N_g}{N_p} = \frac{48}{24} = 2.0000$, and hence $K_g = 63° 26'$.

$K_p = 90° - K_g = 90° - 63° 26' = 26° 34'$.

(2) $D_{pg} = \frac{N_g}{P_d} = \frac{48}{12} = 4.$

$D_{pp} = \frac{N_p}{P_d} = \frac{24}{12} = 2.$

Hence D_p of gear is 4″ and D_p of pinion is 2″.



(3) $D_{og} = D_{pg} + \frac{2 \cos K_g}{P_d} = 4 + \frac{2 \times 0.4473}{12} = 4.0746.$

$D_{op} = D_{pp} + \frac{2 \cos K_p}{P_d} = 2 + \frac{2 \times 0.8944}{12} = 2.1491.$

Hence D_o of gear is 4.075″ and D_o of pinion is 2.149″.

(4) $R = \frac{D_{pg}}{2 \sin K_g} = \frac{4}{2 \times 0.8945} = 2.2359.$

Hence R for both gear and pinion is 2.236″.

(5) $W = \frac{2.1571}{P_d} = \frac{2.1571}{12} = 0.1798$. Hence W_g and W_p are each 0.180″.

(6) $a = \frac{1}{P_d} = \frac{1}{12} = 0.0833$. Hence a_g and a_p are each 0.083″.

(7) $d = \frac{1.1571}{P_d} = \frac{1.1571}{12} = 0.0964$. Hence d_g and d_p are each 0.096″.

(8) $\tan a' = \frac{a}{R} = \frac{0.0833}{2.2359} = 0.0373$, and a' (both gear and pinion) $= 2° 8'$.

(9) $\tan d' = \frac{d}{R} = \frac{0.0964}{2.2359} = 0.0431$, and d' (both gear and pinion) $= 2° 28'$.

(10) $C_g = K_g - d' = 63° 26' - 2° 28' = 60° 58'$.

$C_p = K_p - d' = 26° 34' - 2° 28' = 24° 6'$.

(11) $F'_g = K_g + a' = 63°26' + 2°8' = 65°34'$.

$F'_p = K_p + a' = 26°34' + 2°8' = 28°42'$.

Exercises. Bevel Gears

For a pair of bevel gears, if it is required that $P_d = 9$, $N_g = 36$, *and* $N_p = 18$, *find each of the following:*

1. The pitch diameters of both the gear and the pinion.
2. The pitch-cone angles of both the gear and the pinion.
3. The outside diameters of both the gear and the pinion.
4. The pitch-cone radius.
5. The whole depth of the teeth.
6. The addendum and the dedendum.
7. The addendum angle and the dedendum angle.
8. The cutting angles of both the gear and the pinion.
9. The face angles of both the gear and the pinion.
10. The thickness of tooth on the pitch line.

This measurement at the large end of the teeth may be calculated from one of the spur-gearing formulas for t given on page 88, but it is not an essential measurement when the gear is cut with formed cutters. As a check, however, the thickness of tooth is usually measured with a special gear-tooth vernier caliper and compared with a table of chordal thicknesses of gear teeth, since the caliper measures the chordal thickness and not the thickness of tooth along the pitch circle.

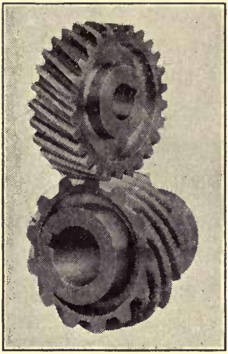
Make all the necessary calculations for cutting each of the following pairs of bevel gears:

11. Gear, 52 teeth; pinion, 36 teeth; pitch, 14.
12. Gear, 45 teeth; pinion, 15 teeth; pitch, 10.
13. Gear, 56 teeth; pinion, 32 teeth; pitch, 8.
14. Gear, 48 teeth; pinion, 12 teeth; pitch, 14.
15. Gear, 60 teeth; pinion, 18 teeth; pitch, 12.

The elements required in making the calculations for cutting a pair of bevel gears are shown on page 101.

Spiral Gears. Gears in which the teeth are wound spirally around the axis instead of being cut parallel to the axis, as in a spur gear, are called *spiral gears* or *helical gears*.

Although it is the common term, the name "spiral gear" is not, strictly speaking, correct, since the teeth are not wound around the axis in a spiral, but in a helix.



SPIRAL GEARS

Spiral gears are essentially cylinders with a succession of equally spaced spiral grooves cut on their periphery, and hence are more nearly like spur gears than bevel gears. Spiral gears may have their axes parallel, as in the case of spur gears, or the axes may be at any desired angle to each other.

A tooth of a spiral gear is much like the thread of a screw, although it does not have the same cross section nor is it meant to do the same kind of work.

In this connection it should be remembered that the meaning of the term "pitch" when applied to gears is different from that when the term is applied to screw threads.

In the treatment of spiral gears we shall consider only those cases in which a variation in the center distance of the gears is permissible; that is, those in which the center distance is said to be approximate.

Uses of Spiral Gears. Spiral gears are used when high speed with smooth and noiseless action is desired. The absence of noise is due to the fact that the engagement of the teeth is gradual, for instead of striking a full line of contact at once, the teeth roll on each other with a sliding motion, one set engaging while the preceding set is still in contact. Thus, the smoothness of action of spiral gears is hardly impaired by wear, and these gears are used extensively in automobiles and many different types of machines.

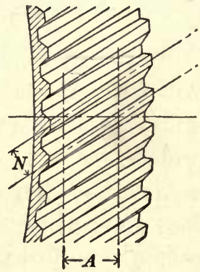
Terms Used. In addition to the terms used in connection with spur gearing, as given on page 86, the following special terms are used with reference to spiral gears:

Axial pitch. The distance from the center of one tooth to the center of the next tooth, measured parallel to the axis of the gear.

Since it is made parallel to the axis, this measurement is similar to the pitch of a screw thread.

Circular pitch. The distance from the center of one tooth to the center of the next tooth, measured on the pitch circle.

This measurement is made at right angles to the axis of the gear as in spur gearing. On account of the different uses of the word "pitch" with spiral gears, this is sometimes called the *real circular pitch*, and the diametral pitch corresponding to this is called the *real pitch*.



SPIRAL-GEAR TEETH

This shows the difference between the ways in which the normal circular pitch (N) and the axial pitch (A) are measured

Lead. The distance that the spiral on which the teeth are cut advances in a single turn.

Normal circular pitch. The distance from the center of one tooth to the center of the next tooth, measured at right angles to the center line of the teeth.

This distance is shorter than the real circular pitch. The section of the teeth which shows the normal circular pitch is called the *true section.*

Normal diametral pitch. The diametral pitch corresponding to the normal circular pitch.

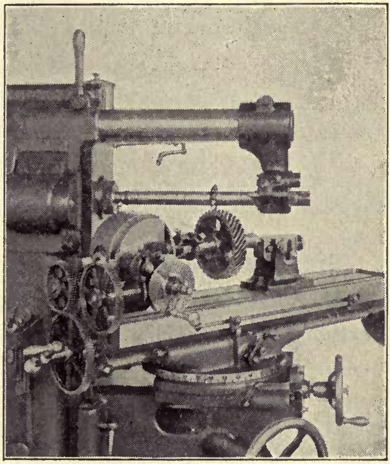
The diametral pitch of the cutter used in cutting the teeth is the same as the normal pitch (normal diametral pitch) of the gear.

Tooth angle. The angle measured at the pitch circle between the center line of the teeth and a line parallel to the axis.

This angle is also called the *spiral angle* of the gear.

Cutting Spiral Gears. The teeth of spiral gears when cut on a milling machine are generally formed with ordinary involute spur-gear cutters, the cutter of the proper pitch and number for the given spiral gear being selected as described on page 107.

The gear cutter is fastened firmly on the arbor of the milling machine. The dividing head and tailstock are set in position on the table of the machine with their centers in alignment and central with the cutter.



CUTTING A SPIRAL GEAR ON A MILLING MACHINE

The change gears needed to give the required spiral lead are placed in their proper positions, as shown on page 74, and the dividing head is set to index the number of teeth to be cut on the gear.

The gear blank is pressed firmly on the mandrel and is mounted between the dividing-head and the tail-stock centers. The blank is brought under the cutter, and the table is set to the required tooth angle and clamped. The blank is then moved clear of the cutter, the knee is raised for the required depth of cut, and the vertical dial is set at zero. The machine is then started and the table moved by hand until the gear blank touches the cutter, when the power feed is thrown in. In returning the blank for another cut, the knee should be lowered until the blank clears the cutter, thus preventing the cutter from dragging through the cut just made.

Symbols. The following symbols are used in the formulas and computations for spiral gears:

A = tooth angle
D_c = center distance
D_o = outside diameter
D_p = pitch diameter
L = lead of spiral
N = number of teeth
P_n = normal pitch
r = ratio of gear to pinion
S = angle of shafts
T_c = number for selecting gear cutter
W = whole depth

In order to distinguish between the symbols for the gear and those for the pinion, we add the subscript g or p as in previous cases. Thus, A_g is the tooth angle of the gear and A_p is the tooth angle of the pinion.

The ratio r of gear to pinion is the ratio of the number of teeth on the gear to the number on the pinion. If the numbers are the same, the ratio is commonly said to be equal.

Spiral-Gear Formulas. The following formulas are used with spiral gears when the ratio is unequal, the center distance is approximate, and the shafts are at any angle:

$$D_c = \frac{D_{pg} + D_{pp}}{2} \qquad N_p = \frac{2\,D_c P_n \cos A_g \cos A_p}{r \cos A_p + \cos A_g}$$

$$D_o = D_p + \frac{2}{P_n} \qquad T_c = \frac{N}{\cos^3 A}$$

$$D_p = \frac{N}{P_n \cos A} \qquad W = \frac{2.1571}{P_n}$$

$$L = \pi D_p \cot A \qquad N_g = rN_p$$

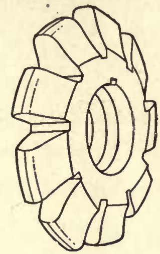
When the subscripts g for "gear" and p for "pinion" are not used, the formulas apply to both gear and pinion, and in distinguishing between the formulas for the gear and those for the pinion the proper subscripts may be added, as in the following cases:

$$L_g = \pi D_{pg} \cot A_g \qquad L_p = \pi D_{pp} \cot A_p$$

In the above formula for N_p, if $A_g = A_p$, we have

$$N_p = \frac{2\,D_c P_n \cos A}{r + 1}.$$

Gear Cutters. In the Brown and Sharpe system of cutters for spur gears with involute teeth, eight different shapes of cutters, as shown in the table below, are made for each pitch. The cutters are numbered according to the number of teeth to be cut on the gear, a #8 8-P_d cutter, for example, being required for cutting 8-P_d spur gears with 12 or 13 teeth. For cutting a spiral gear the proper number of cutter of the normal pitch of the gear is selected from the table according to the number found by the formula for T_c on page 106.



INVOLUTE GEAR CUTTER

If with these cutters the correct indexing and the proper depth of cut are used, the thickness of tooth on the pitch line will be correct.

Number of Cutter	Number of Teeth	Number of Cutter	Number of Teeth
1	135 to rack	5	21 to 25
2	55 to 134	6	17 to 20
3	35 to 54	7	14 to 16
4	26 to 34	8	12 to 13

A #1 cutter is used in cutting gears with 135 or more teeth or in cutting the teeth on racks.

Change Gears. The change gears required to cut the proper lead on a spiral gear may be found by the method described on page 75. Since the spiral leads as calculated from the formula on page 106 usually contain decimals which are difficult to factor, it is customary, in practice, to use tables of spiral leads, which show the leads obtainable with every possible arrangement of the change gears furnished with the dividing head. When an exact lead cannot be found in the table the nearest obtainable lead is generally used.

Illustrative Problem. Make the calculations for cutting a pair of spiral gears in which it is required that $r = 4$, $P_n = 6$, D_c (approximate) $= 12''$, $A_g = 30°$, $A_p = 20°$, and $S = 50°$.

(1) $$N_p = \frac{2\,D_c P_n \cos A_g \cos A_p}{r \cos A_p + \cos A_g} = \frac{2 \times 12 \times 6 \times 0.8660 \times 0.9397}{4 \times 0.9397 + 0.8660} = 25.3.$$

Such a result is taken to the nearest whole number, in this case 25.

$N_g = rN_p = 4 \times 25 = 100.$

The gear and pinion have 100 teeth and 25 teeth respectively.

(2) $$D_{pg} = \frac{N_g}{P_n \cos A_g} = \frac{100}{6 \times 0.8660} = 19.2456.$$

$$D_{pp} = \frac{N_p}{P_n \cos A_p} = \frac{25}{6 \times 0.9397} = 4.4340.$$

The pitch diameters of gear and pinion are 19.246″ and 4.434″.

(3) $$D_{og} = D_{pg} + \frac{2}{P_n} = 19.2456 + \tfrac{2}{6} = 19.5789.$$

$$D_{op} = D_{pp} + \frac{2}{P_n} = 4.4340 + \tfrac{2}{6} = 4.7673.$$

The outside diameters of gear and pinion are 19.579″ and 4.767″.

(4) $L_g = \pi D_{pg} \cot A_g = 3.1416 \times 19.2456 \times 1.7321 = 104.7262.$

$L_p = \pi D_{pp} \cot A_p = 3.1416 \times 4.4340 \times 2.7475 = 38.2724.$

The leads of the spirals on gear and pinion are 104.73″ and 38.27″.

(5) $$T_{cg} = \frac{N_g}{\cos^3 A_g} = \frac{100}{0.8660^3} = \frac{100}{0.6495} = 154.0.$$

$$T_{cp} = \frac{N_p}{\cos^3 A_p} = \frac{25}{0.9397^3} = \frac{25}{0.8298} = 30.1.$$

From the table on page 107 we see that a #1 6-P_d cutter is required for the gear and a #4 6-P_d cutter for the pinion.

(6) $$W = \frac{2.1571}{P_n} = \frac{2.1571}{6} = 0.3595.$$

The whole depth of tooth on both gear and pinion is 0.360″.

(7) $D_c = \frac{1}{2}(D_{pg} + D_{pp}) = \frac{1}{2}(19.2456 + 4.4340) = 11.8398.$

The "exact" center distance of the gears is 11.840″.

Exercises. Spiral Gears at Any Angle

For a pair of spiral gears, if it is required that $r = 2$, $P_n = 10$, D_c *(approximate)* $= 10''$, $A_g = 30°$, $A_p = 20°$, *and* $S = 50°$, *find each of the following:*

1. The number of teeth on the gear and on the pinion.
2. The pitch diameters of both the gear and the pinion.
3. The outside diameters of both the gear and the pinion.
4. The leads of the spirals on both the gear and the pinion.
5. The numbers of the two cutters required.
6. The whole depth of the teeth.
7. The exact center distance of the gears.

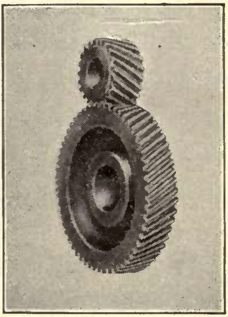
For a pair of spiral gears, if it is required that $r = 3$, $P_n = 4$, D_c *(approximate)* $= 16''$, $A_g = 40°$, $A_p = 20°$, *and* $S = 60°$, *find each of the following:*

8. The number of teeth on the gear and on the pinion.
9. The pitch diameters of both the gear and the pinion.
10. The outside diameters of both the gear and the pinion.
11. The leads of the spirals on both the gear and the pinion.
12. The numbers of the two cutters required.
13. The whole depth of the teeth.
14. The exact center distance of the gears.
15. Make the necessary calculations for cutting a pair of spiral gears, it being required that D_c (approximate) $= 7''$, $r = 1\frac{1}{2}$, $P_n = 8$, $A_g = 25°$, $A_p = 15°$, and $S = 40°$.
16. Make the necessary calculations for cutting a pair of spiral gears, it being required that D_c (approximate) $= 12''$, $r = 4$, $P_n = 8$, $A_g = 30°$, $A_p = 30°$, and $S = 60°$.

In finding N_p use the formula given at the foot of page 106.

Spiral Gears with Parallel Shafts. When the ratio of a pair of spiral gears is unequal, the center distance approximate, and the shafts parallel, as shown in the illustration, the tooth angle of the gear is equal to the tooth angle of the pinion; that is, $A_g = A_p$.

For example, make the calculations for cutting a pair of spiral gears with parallel shafts, if it is required that D_c (approximate) $= 10''$, $N_g = 48$, $N_p = 24$, $P_n = 4$, and $A = 24°$.



SPIRAL GEARS WITH PARALLEL SHAFTS

(1) $D_{pg} = \dfrac{N_g}{P_n \cos A} = \dfrac{48}{4 \times 0.9135} = 13.1363.$

$D_{pp} = \dfrac{N_p}{P_n \cos A} = \dfrac{24}{4 \times 0.9135} = 6.5681.$

The pitch diameters of gear and pinion are 13.136″ and 6.568″ respectively.

(2) $D_{og} = D_{pg} + \dfrac{2}{P_n} = 13.1363 + \frac{2}{4} = 13.6363.$

$D_{op} = D_{pp} + \dfrac{2}{P_n} = 6.5681 + \frac{2}{4} = 7.0681.$

The outside diameters of gear and pinion are 13.636″ and 7.068″.

(3) $L_g = \pi D_{pg} \cot A = 3.1416 \times 13.1363 \times 2.2460 = 92.6902.$

$L_p = \pi D_{pp} \cot A = 3.1416 \times 6.5681 \times 2.2460 = 46.3446.$

The leads of the spirals on gear and pinion are 92.69″ and 46.34″.

(4) $T_{cg} = \dfrac{N_g}{\cos^3 A} = \dfrac{48}{0.9135^3} = \dfrac{48}{0.7623} = 63.0.$

$T_{cp} = \dfrac{N_p}{\cos^3 A} = \dfrac{24}{0.9135^3} = \dfrac{24}{0.7623} = 31.5.$

From the table on page 107 we see that a #2 4-P_d cutter is required for the gear and a #4 4-P_d cutter for the pinion.

(5) $W = \dfrac{2.1571}{P_n} = \dfrac{2.1571}{4} = 0.5393.$

The whole depth of tooth on both gear and pinion is 0.539″.

(6) $D_c = \frac{1}{2}(D_{pg} + D_{pp}) = \frac{1}{2}(13.1363 + 6.5681) = 9.8522.$

The "exact" center distance of the gears is 9.852″.

Exercises. Spiral Gears with Parallel Shafts

1. For a certain pair of spiral gears it is required that the shafts be parallel at an approximate center distance of $2\frac{1}{4}''$, that $N_g = 32$, $N_p = 20$, $P_n = 12$, and $A = 18°$. Find the pitch diameters of both the gear and the pinion.

2. In Ex. 1 find the outside diameters of both the gear and the pinion.

3. In Ex. 1 find the leads of the spirals on both the gear and the pinion.

4. In Ex. 1 find the number of the cutter to be used for cutting the gear and the same for the pinion.

5. In Ex. 1 find the whole depth of the teeth.

6. In Ex. 1 find the exact center distance.

For a pair of spiral gears, if it is required that the shafts be parallel, that D_c (approximate) $= 12\frac{3}{4}''$, $r = 1\frac{3}{4}$, $P_n = 6$, and $A = 20°$, find each of the following:

7. The number of teeth on the gear and on the pinion.

In finding N_p use the formula given at the foot of page 106.

8. The pitch diameters of both the gear and the pinion.

9. The outside diameters of both the gear and the pinion.

10. The leads of the spirals on both the gear and the pinion.

11. The number of the cutter to be used for cutting the gear and the same for the pinion.

12. The whole depth of the teeth.

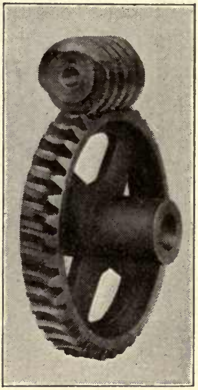
13. The exact center distance of the gears.

14. A spiral gear with a pitch diameter of 6.2121″ has a spiral lead of 72.83″. Find the angle at which the milling-machine table was set in cutting the gear.

Worm Gearing. When the difference between the velocities of two shafts is to be great, *worm gearing* is often employed.

In this case the driver, which is called a *worm*, is a single-thread or multiple-thread screw, the threads of which mesh with a special form of spur gear known as a *worm wheel.*

Uses of Worm Gearing. Worm gearing is generally used when the load is heavy and when smoothness of action and a large reduction in velocity are required in transmitting rotary motion from one shaft to another. Usually the two shafts are at right angles to each other.



WORM GEARING

Terms Used. In general the terms and symbols which are used with spur gearing are also used in connection with worm gearing, together with the following additions, which will be understood from the second figure on page 113:

Face angle. The angle, usually arbitrarily selected, to which the ends of the teeth on the worm wheel are trimmed.

This angle varies from about 55° to 65°, depending upon the kind of worm used. The face angle of the wheel shown on page 113 is 60°.

Gashing angle (G). The angle for swiveling the table of the milling machine when cutting the teeth on the worm wheel to correspond to the spiral angle of the thread on the worm.

Linear pitch (P_c). This is identical with the circular pitch of the worm wheel and is used in speaking of the worm.

Since the two terms are identical, we use the symbol P_c for both.

Throat diameter (D_t). The diameter of the worm wheel measured at the center of the curved teeth.

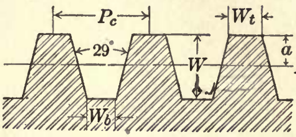
The throat diameter is equivalent to the outside diameter of a spur gear having the same number of teeth and the same pitch.

Worm-Thread Formulas. The following formulas are used with standard worm threads which have an angle of 29°:

$$a = 0.3183 P_c \qquad W = 0.6866 P_c$$

$$D_o = 4 P_c \qquad W_b = 0.3095 P_c$$

$$D_p = D_o - 2a \qquad W_t = 0.3354 P_c$$



STANDARD WORM THREAD

a, addendum; P, pitch line; P_c, linear pitch; W, whole depth of thread; W_b, width at root of thread; W_t, width at top of thread

The formula for D_o gives the outside diameter for a single-thread worm; for a double-thread worm use $5P_c$, and for a triple-thread worm use $6P_c$.

The width (W_b) at the root of the thread is the width of the point of the tool used in cutting the worm thread.

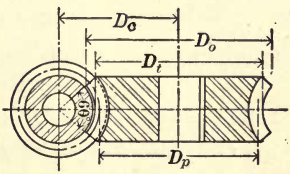
Worm-Wheel Formulas. The following formulas are used with worm wheels:

$$a = \frac{1}{P_d} \qquad \tan G = \frac{P_c}{\pi D_{pw}}$$

$$D_c = \tfrac{1}{2}(D_p + D_{pw}) \qquad D_t = D_p + 2a$$

$$D_p = \frac{N}{P_d} \qquad P_d = \frac{\pi}{P_c}$$

$$D_o = D_p + 4a \qquad t = \tfrac{1}{2} P_c$$



CROSS SECTION OF WORM AND WORM WHEEL

D_c, center distance; D_o, outside diameter; D_p, pitch diameter; D_t, throat diameter

In these formulas D_{pw} is the pitch diameter of the worm and D_p is the pitch diameter of the worm wheel.

The depth of tooth on the worm wheel, when figured for gashing the teeth on a milling machine, is taken as the whole depth of the worm thread. This evidently makes no allowance for clearance, but when the teeth are hobbed, as explained on page 115, an extra depth of tooth is given to the hob to make the necessary allowance.

Illustrative Problem. Make the calculations for cutting a worm and worm wheel with a velocity ratio of 40 to 1, the worm to be single-threaded with a linear pitch of 0.2618″.

(1) $P_d = \frac{\pi}{P_c} = \frac{3.1416}{0.2618} = 12.$

The diametral pitch of the worm wheel is 12.

(2) $a = \frac{1}{P_d} = \frac{1}{12} \doteq 0.0833.$

The addendum of the worm wheel is 0.083″.

(3) Since the velocity ratio is 40 to 1, the wheel must have 40 teeth.

$D_p = \frac{N}{P_d} = \frac{40}{12} = 3.3333.$

The pitch diameter of the worm wheel is 3.333″.

(4) $D_t = D_p + 2a = 3.3333 + (2 \times 0.0833) = 3.4999.$

The throat diameter of the worm wheel is 3.500″.

(5) $D_o = D_p + 4a = 3.3333 + (4 \times 0.0833) = 3.6665''.$

The outside diameter of the worm wheel is 3.667″.

(6) $t = \frac{1}{2} P_c = \frac{1}{2} \times 0.2618 = 0.1309.$

The thickness of tooth on the pitch line of the worm wheel is 0.131″.

(7) $D_o = 4 P_c = 4 \times 0.2618 = 1.0472.$

The outside diameter of the worm is 1.047″.

(8) The addendum of the worm is the same as that of the worm wheel.

$D_p = D_o - 2a = 1.0472 - (2 \times 0.0833) = 0.8806.$

The pitch diameter of the worm is 0.881″.

(9) $D_c = \frac{1}{2}(D_p + D_{pw}) = \frac{1}{2}(3.3333 + 0.8806) = 2.1070.$

The center distance of the worm and worm wheel is 2.107″.

(10) $W = 0.6866 P_c = 0.6866 \times 0.2618 = 0.1798.$

The whole depth of the worm thread is 0.180″.

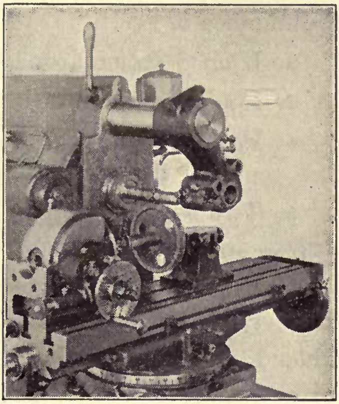
(11) $W_b = 0.3095 P_c = 0.3095 \times 0.2618 = 0.0810.$

The width of the point of the worm-thread tool is 0.081″.

(12) $\tan G = \frac{P_c}{\pi D_{pw}} = \frac{0.2618}{3.1416 \times 0.8806} = 0.0946$, and hence $G = 5^\circ 24'$.

Since the angle for setting a milling-machine table is calculated to the nearest $\frac{1}{4}^\circ$, the gashing angle for the worm-wheel teeth is $5\frac{1}{2}^\circ$.

Cutting Worm Wheels. When cutting worm wheels on a milling machine, the teeth are usually gashed first and then hobbed to give them their final shape. The gashing is done with an involute gear cutter of the proper number and pitch, the table of the machine being first adjusted to center the blank and cutter and then set to the gashing angle.



GASHING A WORM WHEEL ON A MILLING MACHINE

After the teeth are gashed the milling-machine table is set at zero, and the cutter is replaced by a hob. A hob is a tool shaped like a worm and grooved to form cutting teeth, the depth of which is made a little greater than the depth of the worm thread to allow for clearance.

Exercises. Worm Gearing

From the data given, make the calculations for cutting each of the following sets of worm gearing:

1. Worm wheel, 30 teeth; worm, single-threaded with a linear pitch of 0.3927″.

2. Velocity ratio, 20 to 1; worm, double-threaded with a linear pitch of 0.5236″.

Each revolution of this worm will move the worm wheel two teeth.

3. Worm wheel, 24 teeth; worm, single-threaded with a linear pitch of 0.2244″.

4. Worm wheel, 96 teeth; worm, triple-threaded with a lead of 2.3562″.

Exercises. Gears

1. If, in a spur gear, $D_o = 2.228''$ and $P_c = \frac{1}{2}''$, what is the number of teeth?

2. If an 8-P_d spur gear has 203 teeth, what is D_o?

3. Find the whole depth of tooth for a $\frac{3}{4}$-P_d spur gear.

4. Find the circular pitch for a 32-T spur gear in which it is required that $D_p = 16.55''$.

5. Two 8-P_d bevel gears are to have 80 teeth and 20 teeth respectively. Make the calculations for cutting the gears.

6. If a 20-inch spur gear has 198 teeth, what is the pitch?

7. If the circular pitch of a spur gear is $2\frac{1}{4}''$, what is the addendum? the whole depth of tooth?

8. If it is required that a worm wheel is to have 20 teeth and the worm a single thread with a linear pitch of 0.3142″, make the calculations for cutting the gearing.

9. If a 39-T spur gear has a pitch of $3\frac{1}{2}$, what is D_o?

10. Make the necessary calculations for cutting a pair of spiral gears, it being required that D_c (approximate) = 12″, $P_n = 6$, $A_g = 30°$, $A_p = 30°$, $S = 60°$, and $r = 3$.

11. A $1\frac{3}{4}$-P_d spur gear has a pitch diameter of 97.14″. Find the outside diameter and the number of teeth.

12. Make the calculations for cutting a pair of bevel gears, it being required that the pitch be 5, the number of teeth on the pinion 17, and the velocity ratio 3 to 1.

13. A worm wheel is to have 60 teeth and is to mesh with a double-thread worm having a linear pitch of 1.0472″. Make all the calculations for cutting the gearing.

14. Find the pitch diameters of two spur gears in mesh if the center distance between the gears is 12″, and one gear has 40 teeth and the other 20 teeth.

CHAPTER VII

REVIEW PROBLEMS

Exercises. General Applications

Change the following tapers per inch to tapers per foot:

1. 0.0762″. **2.** 0.08175″. **3.** 0.0162″. **4.** 0.0542″.

5. If the outside diameter of a spur gear is $4\frac{1}{2}''$ and the pitch is 18, what is the number of teeth?

6. Find the tap-drill size of a $1\frac{1}{4}$-inch U. S. S. thread.

7. If a quadruple-thread screw has 16 threads per inch, what is the pitch and what is the lead?

8. Find the change gears required on a lathe, which has a lead screw with 5 threads to the inch, when cutting a metric thread with a lead of 1.5 mm.

Assume that the lathe is equipped with a 127-T gear.

9. Two 12-P_d spur gears in mesh have 66 teeth and 40 teeth respectively. Find the center distance.

10. Find the pitch diameter of a spur gear having 37 teeth and a pitch of 10.

11. If a spiral lead of 27″ is to be cut on a $\frac{1}{2}$-inch drill, to what angle should the milling-machine table be set?

12. Find the indexing required on a B. & S. dividing head for milling a ratchet wheel with grooves $13\frac{2}{3}°$ apart.

13. Find the outside diameter of a 96-T spur gear which has a pitch of 14.

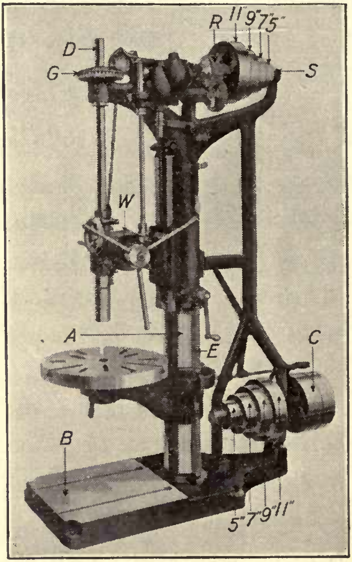
Exercises. Drill Press

1. The part of the cast-iron column marked A in the figure below is $2' \, 4''$ long and was turned in a lathe to its present diameter of $6''$. Find how long it took to machine this casting if a roughing cut with a $\frac{1}{8}$-inch feed and a finishing cut with a $\frac{1}{4}$-inch feed were taken with a high-speed tool.

2. The finished base B is $12'' \times 16''$ and was planed on a 3-to-1 planer at 25 F.P.M., a roughing cut with a $\frac{3}{16}$-inch feed and a finishing cut with a $\frac{3}{8}$-inch feed being taken. Find the length of time consumed in machining this casting.

3. The diameter of the driving pulley C is $10\frac{1}{4}''$ at the center of the crown and $10''$ at the edge, and the width of the face is $3''$. Find the T.P.F. of the pulley.

4. What was the tailstock offset if the pulley in Ex. 3 was turned on a $7\frac{1}{2}$-inch arbor?



DRILL PRESS

5. The diameters of the steps of the cone pulleys are $5''$, $7''$, $9''$, and $11''$ respectively, and pulley C makes 425 R.P.M. Beginning with the belt on the largest step of the driving pulley, find each of the four speeds of shaft S.

6. Spindle D is made of machine steel and has a diameter of $1\frac{13}{16}''$ and a length of $28''$. Find the number of R.P.M. of the lathe for a roughing cut with a carbon-steel tool.

7. The spindle in Ex. 6 was turned from round stock, a roughing cut with a $\frac{1}{16}$-inch feed and a finishing cut with a 0.015-inch feed being taken with a high-speed steel tool. Find the time taken in machining the spindle.

8. If the adjusting screw E has a U. S. S. thread of $\frac{1}{16}$-inch pitch, what is the root diameter of the screw?

9. Find the change gears required on a lathe, which has a lead screw with 5 threads per inch, in cutting screw E.

10. Bevel gear G is a 14-P_d gear with 51 teeth, and the pinion meshing with it has 17 teeth. Make all the necessary calculations for cutting this pair of gears.

11. If spur gear R has 71 teeth and a circular pitch of 0.3491″, what is its outside diameter?

12. In Ex. 11 find the addendum and the whole depth of tooth of gear R.

13. Find the indexing required on a B. & S. dividing head in cutting gear R.

14. Worm W is a single-thread screw with a linear pitch of 0.2244″, and meshes with a worm wheel having 25 teeth. Make the calculations for cutting the worm and wheel.

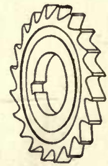
15. If the adjusting screw E has an Acme screw thread with a pitch of 0.1111″, find the depth, the width at the top of the thread, and the number of threads per inch.

16. The circular table of the drill press is made of cast iron and has a diameter of 27″ and a thickness of $1\frac{5}{8}$″. Find the time required for machining the circumference, a roughing cut with a feed of 0.05″ and a finishing cut with a feed of 0.125″ being taken with a high-speed steel tool.

17. Find the number of R.P.M. for boring the center hole in the circular table in Ex. 16 to a diameter of $1\frac{1}{4}$″ with a carbon-steel tool.

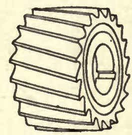
Exercises. Indexing and Milling

1. Using a Cincinnati dividing head, find the indexing required in cutting the teeth on the milling cutter shown in the first figure below.



2. In cutting the teeth on the milling cutter mentioned in Ex. 1 a high-speed steel cutter $1\frac{7}{8}''$ in diameter was used. At how many R.P.M. should this second cutter run in order to give a cutting speed of 55 F.P.M.?

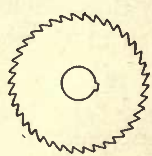
3. If the spiral lead of the teeth is to be $26.25''$ and the diameter of the blank is $2\frac{7}{8}''$, at what angle should the table of the milling machine be set for cutting the spiral grooves in the milling cutter shown in this figure?



4. If the lead of the milling machine in Ex. 3 is $10''$, what change gears should be used?

5. If a milling cutter similar to the one in Ex. 3 is to be made with 24 teeth, find the indexing required on a B. & S. dividing head when cutting the grooves.

6. Using a milling machine equipped with a Cincinnati dividing head, find the indexing required in milling the teeth in the metal-slitting saw here shown.



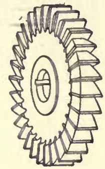
7. If a metal-slitting saw similar to the one mentioned in Ex. 6 is to be made with 79 teeth, find the indexing required on a B. &. S. dividing head.

8. If the table of a milling machine is set at an angle of $7\frac{3}{4}°$ for cutting a spiral with a lead of $62.50''$, what is the diameter of the work?

9. If the lead of the milling machine in Ex. 8 is $10''$, what change gears should be used?

10. Find the different numbers from 1 to 100 that can be indexed by simple indexing with the 21-hole circle of a B. & S. dividing head.

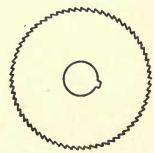
11. Using a Cincinnati dividing head, find the indexing required in cutting the teeth on the side-milling cutter shown in this figure.



12. If the side-milling cutter in Ex. 11 is $4\frac{1}{2}''$ in diameter, what is the cutting speed when the cutter is driven at 21 R.P.M.? at 38 R.P.M.?

Give such results to the nearest unit.

13. Using a Cincinnati dividing head, find the indexing required in milling the screw-slotting saw shown in this figure, the saw having 72 teeth.



14. Consider Ex. 13 if the saw is to have 89 teeth and the milling machine is equipped with a B. & S. dividing head.

15. Using compound indexing on a B. & S. dividing head, find the indexing fractions required for cutting a screw-slotting saw which is to have 93 teeth.

16. Find the change gears required on a B. & S. dividing head for indexing $\frac{1}{4}°$.

It is necessary to index for 4×360, or 1440 divisions. In figuring the ratio of the change gears assume a movement of the index crank of one hole on the 33-hole circle for each division.

17. Find all the circles on a B. & S. dividing head that will index in common the numbers 3, 12, 15, and 60.

18. What is the lead of the spiral that will be cut on a milling machine which has a lead of $10''$ when there is a 56-T gear on the worm, a 32-T gear as the first gear on the stud, a 48-T gear as the second gear on the stud, and a 72-T gear on the feed screw?

Exercises. Tapers

1. Find the T.P.F. of taper A on the automobile pinion shaft shown in the blueprint on page 123.

2. In Ex. 1 find the tailstock offset for cutting taper A.

3. As in Ex. 1, find the T.P.F. of taper B.

4. In Ex. 3 find the tailstock offset for cutting taper B.

5. Find the T.P.I. of the gib key shown in the blueprint.

In the following table supply each missing specification for 2-ball clamping levers of the type shown in the blueprint:

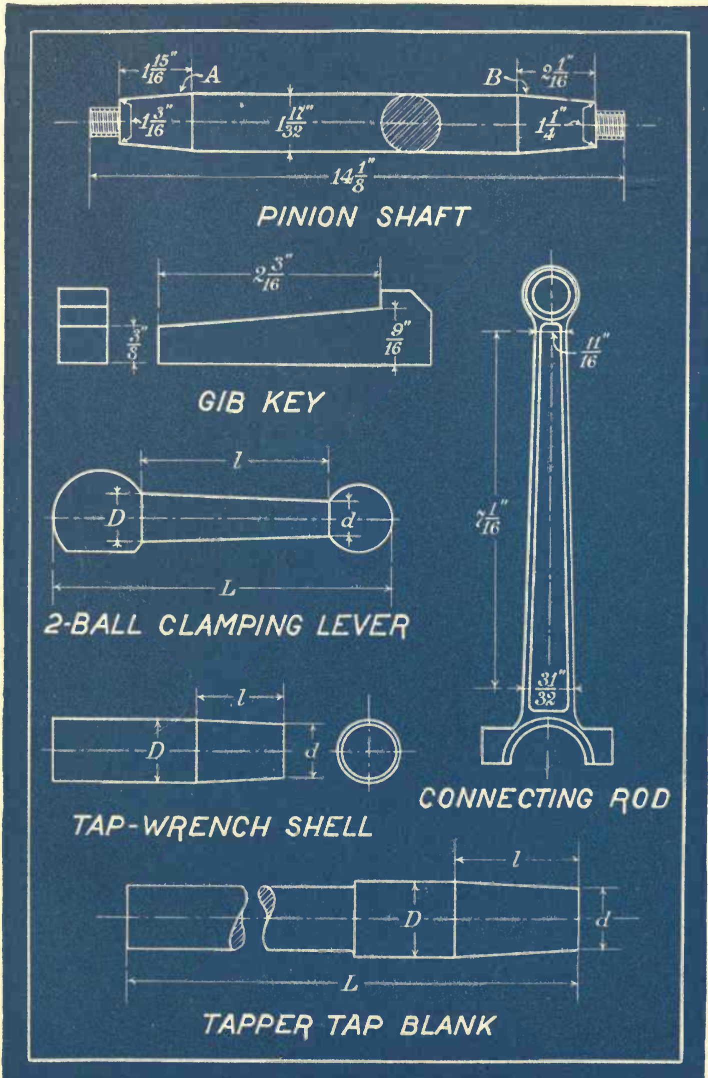
	L	l	D	d	T.P.F.	Tailstock Offset
6.	$4\frac{1}{2}''$	$2\frac{7}{16}''$	$\frac{5}{8}''$	$\frac{7}{16}''$		
7.	$5\frac{1}{2}$	$3\frac{1}{16}$	$\frac{11}{16}$	$\frac{1}{2}$		
8.	$6\frac{1}{2}$	$3\frac{15}{16}$	$\frac{3}{4}$	$\frac{9}{16}$		
9.	$7\frac{1}{2}$	$4\frac{13}{16}$	$\frac{13}{16}$	$\frac{5}{8}$		
10.	$8\frac{1}{2}$	$5\frac{1}{4}$	$\frac{13}{16}$	$\frac{21}{32}$		

11. Find the T.P.F. of the automobile connecting rod.

In the following table supply each missing specification for tap-wrench shells of the type shown in the blueprint:

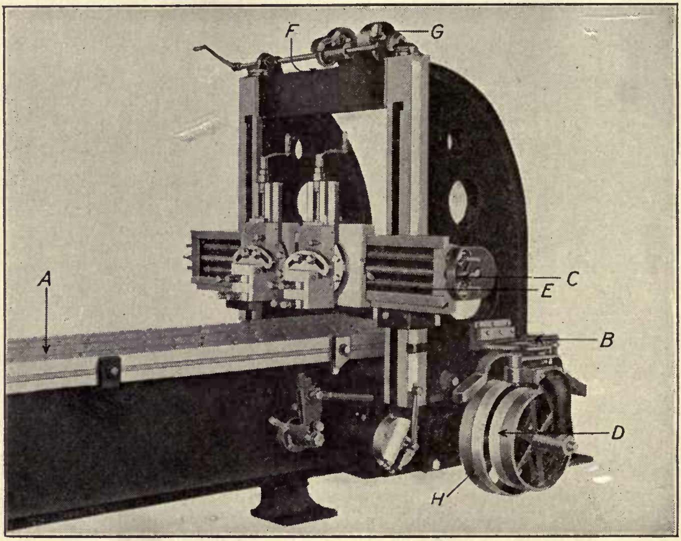
	D	d	l	T.P.F.	Angle with Axis
12.	$\frac{7}{16}''$	$\frac{5}{16}''$	$\frac{17}{32}''$		
13.	$\frac{5}{8}$		$\frac{7}{8}$	$1.5''$	
14.		$\frac{1}{2}$	$\frac{7}{16}$	2.5	

15. In the tapper tap blank shown in the blueprint, if $L = 4\frac{1}{2}''$, $l = \frac{5}{16}''$, $D = \frac{3}{16}''$, and $d = 0.140''$, find the T.P.F. and the included angle of the taper.


$1\frac{15}{16}''$
A
B
$2\frac{1}{16}''$
$1\frac{3}{16}''$
$1\frac{17}{32}''$
$1\frac{1}{4}''$
$14\frac{1}{8}''$
PINION SHAFT
$2\frac{3}{16}''$
$\frac{3}{8}''$
$\frac{9}{16}''$
GIB KEY
l
D
d
L
2-BALL CLAMPING LEVER
$\frac{11}{16}''$
$7\frac{1}{16}''$
$\frac{31}{32}''$
CONNECTING ROD
l
D
d
TAP-WRENCH SHELL
l
D
d
L
TAPPER TAP BLANK

Exercises. Planer

1. The table *A* of the planer shown in the figure below is 4′ wide and 18′ long and was planed on a 2-to-1 planer which had a cutting speed of 25 F.P.M. If a roughing cut with a $\frac{3}{32}$-inch feed and a finishing cut with a $\frac{7}{16}$-inch feed were taken, how long did it take to machine the top of the table?



PLANER

2. The V-shaped ways *B* are planed to an angle of 50°, and the width of the V at the top is $2\frac{9}{16}''$. Find the depth of the V.

Use the formula given on page 28, the included angle being 50°.

3. The spur gear *C* has a pitch diameter of 4″ and a pitch of 12. Find the outside diameter of the gear.

4. In Ex. 3 find the number of teeth on the gear.

5. In Ex. 3 find the whole depth of a tooth.

6. In Ex. 3 find the indexing required in cutting the gear on a milling machine equipped with a B. & S. dividing head.

7. The cast-iron pulley *D* is 22″ in diameter, and the face is 3″ wide. Find the number of R.P.M. of a lathe for finishing the face with a high-speed steel tool.

8. In Ex. 7 how long will it take to make a single cut over the face with a $\frac{1}{32}$-inch feed?

9. Find the depth of the thread on the feed screw *E* which has 6 U. S. S. threads to the inch.

10. In Ex. 9 find the change gears required on a lathe with a lead screw of $\frac{1}{5}$-inch pitch when cutting the thread.

11. The 14-P_d spur gear *F* has 57 teeth. Find the outside diameter of the gear.

12. In Ex. 11 find the thickness of tooth on the pitch line and also find the working depth of a tooth.

13. In Ex. 11 find the indexing required on a B. & S. dividing head in cutting the gear.

Why will two intermediate gears be needed?

14. If the crowned pulley *G* has a $6\frac{1}{2}$-inch face and is $\frac{5}{16}$″ larger in diameter at the center of the crown than at the edges, find the T.P.F. of the pulley.

15. If pulley *G* is turned on a $11\frac{1}{2}$-inch mandrel, find the tailstock offset.

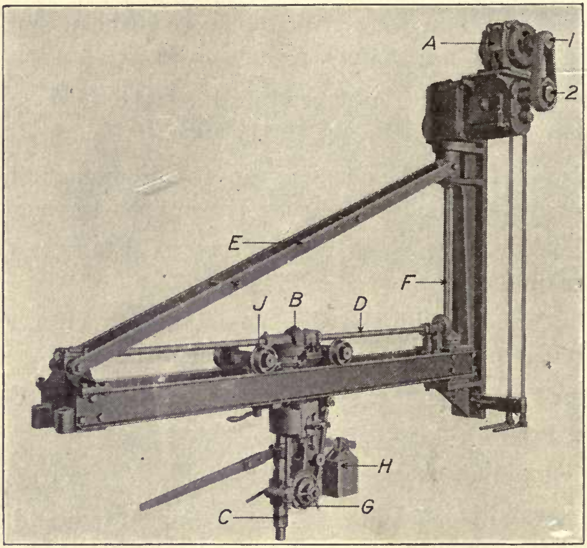
16. If the cast-iron pulley *H* has a diameter of $28\frac{1}{2}$″ and a 3-inch face, find the number of R.P.M. for roughing the face with a tool of carbon steel.

17. In Ex. 16 how long will it take to make a single cut over the face with a $\frac{3}{64}$-inch feed?

18. Find the number of R.P.M. of pulley *H* if it is belted to a 24-inch pulley which has a speed of 125 R.P.M.

Exercises. Drilling Machine

1. If motor *A* in the figure below makes 750 R.P.M., if gear 1 on the motor has 24 teeth, and if gear 2 has 56 teeth, what is the speed of the shaft to which gear 2 is fastened?



WALL RADIAL-DRILLING MACHINE

2. If the gears in Ex. 1 were spur gears with a pitch of 14, what would be the respective outside diameters?

3. What would be the center distance if gears 1 and 2 were spur gears which meshed together instead of being connected by the endless chain?

4. Find the indexing required on a Cincinnati dividing head for cutting the gears in Ex. 3.

5. The 6-P_d bevel gears *B* have 48 teeth and 16 teeth respectively. Make the calculations for cutting these gears.

6. The socket in spindle C is bored to a #4 Morse taper, which has a T.P.F. of 0.623″. To what angle should a compound rest be set to bore this taper?

7. Shaft D, which is made of machine steel, is 10′ 6″ long and has a diameter of $1\frac{3}{8}$″. Find the number of R.P.M. of a lathe for roughing the shaft with a high-speed steel tool.

8. In Ex. 7 find how long it will take to make a single cut over the shaft with a feed of 0.015″.

9. The worm wheel G has 60 teeth and it meshes with a single-thread worm, the linear pitch of which is 0.1963″. Make the calculations for cutting the worm and wheel.

10. If the weight H is made of cast iron and is approximately $2\frac{1}{2}'' \times 5\frac{1}{4}'' \times 8\frac{3}{16}''$, find its approximate weight.

The weight of cast iron may be taken as 450 lb. per cubic foot.

11. If the diameter at the middle of the $1\frac{3}{4}$-inch face of the cast-iron wheel J is $4\frac{29}{32}$″ and that at the edge is $4\frac{7}{8}$″, what is the T.P.F. of the wheel?

12. Using the smaller diameter in Ex. 11, at what speed should the face be finished with a high-speed tool?

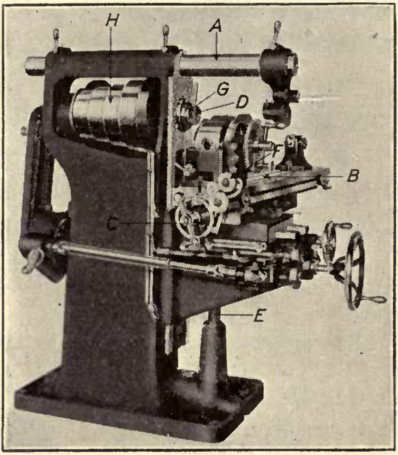
13. In Ex. 11 find the time required for finishing 25 wheels of this type, making a single cut with a $\frac{1}{32}$-inch feed over the face of each wheel.

14. Shaft D makes an angle of 22° with brace E and is perpendicular to shaft F. If the distance between the axis of shaft F and the vertex of the angle formed by D and E is 9′ 8″, find to the nearest 0.1″ the length of E between the vertices of the angles formed by E with D and F.

The ratio $D:E$ is what function of the angle between them? This example and Exs. 3–22 on page 134 may be omitted by those students who have not had the equivalent of Chapter IV (Trigonometry) in "Fundamentals of Practical Mathematics" in this series.

Exercises. Universal Milling Machine

1. In the universal milling machine shown below the diameter of the machine-steel arm A is $4\frac{1}{2}''$ and the length is $3'\ 2''$. Find the number of R.P.M. for a roughing cut with a high-speed tool.



UNIVERSAL MILLING MACHINE

2. Taking a roughing cut with a $\frac{1}{8}$-inch feed and a finishing cut with a $\frac{5}{16}$ inch feed, how long will it take to machine the arm in Ex. 1?

3. If table B is $12\frac{1}{2}''$ wide and $39''$ long, how long will it take, using a $\frac{1}{16}$-inch feed, to plane the top of the table on a 2-to-1 planer which has a cutting speed of 25 F.P.M.?

If spur gear C has an outside diameter of $8\frac{1}{2}''$ and a pitch of 12, find each of the following:

4. Number of teeth.

5. Circular pitch.

6. Addendum and clearance.

7. Whole depth of tooth.

8. In the taper hole in the spindle D the large diameter is $1.260''$, the small diameter is $1.045''$, and the length of the taper is $5''$. Find the T.P.F.

9. In Ex. 8 find the included angle of the taper.

10. Screw E, which regulates the vertical adjustment of the table, has a double Acme screw thread of $\frac{1}{4}$-inch lead. Find the double depth of the thread.

11. In Ex. 10 find the change gears required on a lathe with a lead screw of $\frac{1}{6}$-inch pitch when cutting the thread.

In cutting a double thread the gears are figured as if for a single thread of the required lead. One groove is cut to the double-thread depth, the work is given a half turn, and then the other groove is cut.

12. If gear *F* has 44 teeth, find the indexing required on a B. & S. dividing head in cutting the gear.

13. If the diameters of the steps of the cone pulley *H* are $7\frac{1}{2}''$, $8\frac{3}{4}''$, $10\frac{1}{4}''$, and $12''$ respectively, and the pulley is made of cast iron, find the number of R.P.M. needed in roughing the face of each step with a tool of carbon steel.

14. If each step of the cone pulley in Ex. 13 has a $2\frac{1}{2}$-inch face, find the time taken in making a roughing cut over the four faces with a $\frac{1}{16}$-inch feed.

15. Beginning with the smallest step, a finishing cut was taken over the faces of the steps of the cone pulley in Ex. 13, the speeds of the lathe being as follows: 70 R.P.M., 60 R.P.M., 50 R.P.M., 45 R.P.M. Find the average cutting speed used in finishing the pulley.

16. If the diameter at the center of each step of the cone pulley in Ex. 13 is $\frac{1}{16}''$ greater than that at the edge, find the T.P.F. of the face.

17. The cone pulley in Ex. 13 is driven by a similar cone pulley on a countershaft which makes 98 R.P.M., the $7\frac{1}{2}$-inch steps being opposite the 12-inch steps. Beginning with the lowest speed, find the speed of the shaft to which cone pulley *H* is fastened for each position of the belt on the steps.

18. If the outside diameter of the nose of the spindle *D* is $2\frac{1}{2}''$ and the nose is threaded with a sharp V-thread of $\frac{1}{4}$-inch pitch, to what diameter should the collar *G* be bored?

19. If the lead of the milling machine is $10''$, what change gears are needed in cutting a spiral with a lead of $2.24''$?

Exercises. Tapers

1. Find the T.P.F. of the taper on the automobile stub axle shown in the blueprint on page 131.

2. In Ex. 1 find the tailstock offset for turning the taper.

In the following table supply each missing specification for automobile stub axles of the type shown in the blueprint:

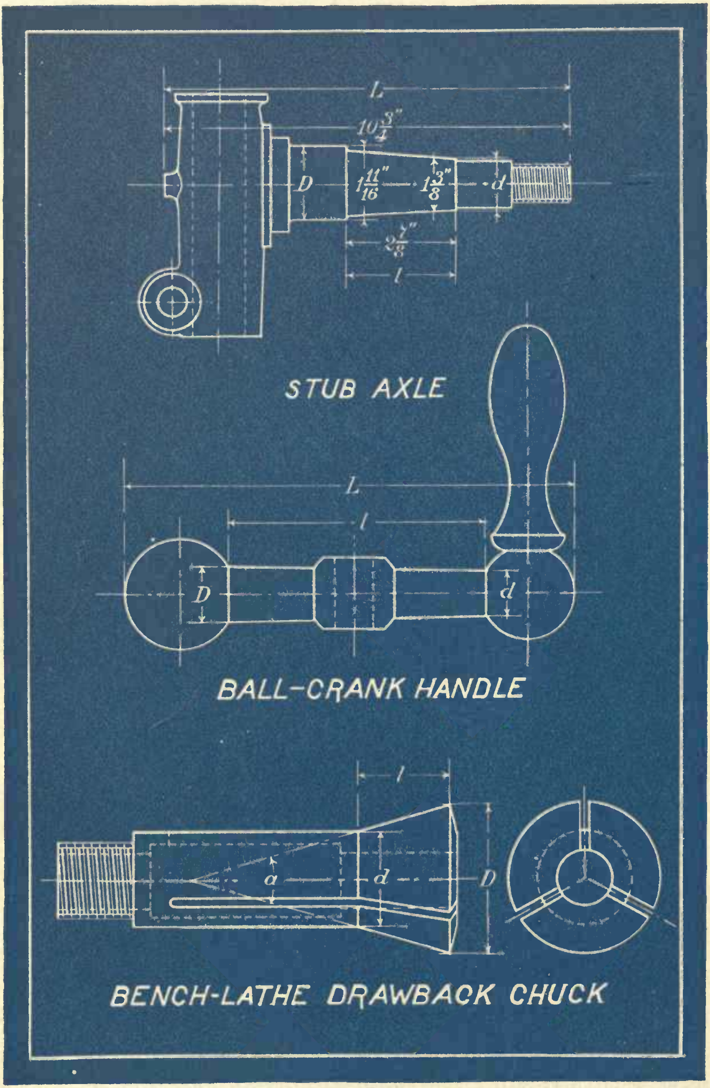
	L	*l*	*D*	*d*	T.P.F.	Tailstock Offset
3.	$11\frac{1}{8}''$	$3''$	$1\frac{5}{8}''$		$1.500''$	
4.	$9\frac{3}{4}$		$1\frac{9}{16}$	$1\frac{1}{8}''$	1.250	
5.	$10\frac{5}{16}$	$2\frac{9}{16}$		$1\frac{1}{4}$	1.4	

In the following table supply each missing specification for ball-crank handles of the type shown in the blueprint:

	L	*l*	*D*	*d*	T.P.F.	Tailstock Offset
6.	$3''$	$1\frac{3}{4}''$	$\frac{3}{8}''$	$\frac{5}{16}''$		
7.	$3\frac{1}{4}$	$1\frac{7}{8}$	$\frac{7}{16}$	$\frac{5}{16}$		
8.	$3\frac{1}{2}$	$1\frac{15}{16}$	$\frac{15}{32}$	$\frac{11}{32}$		
9.	4	$2\frac{1}{8}$	$\frac{17}{32}$	$\frac{13}{32}$		

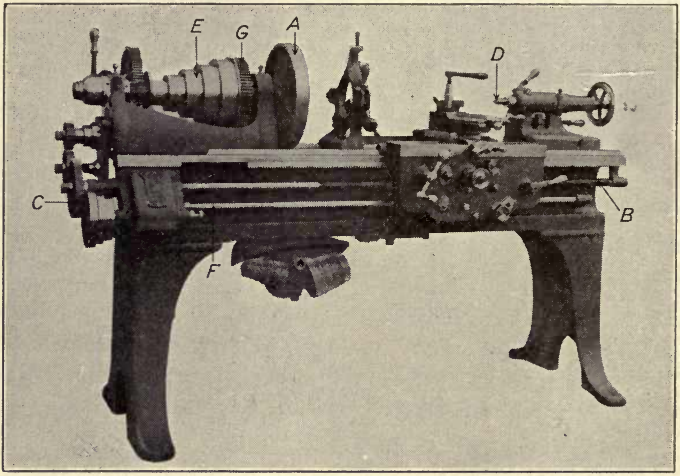
In the following table supply each missing specification for bench-lathe drawback chucks of the type shown in the blueprint:

	l	*D*	*d*	Angle with Axis	T.P.F.
10.	$0.300''$	$0.500''$	$0.335''$		
11.		0.850	0.650	12°	
12.		1.156	0.950		$2.125''$
13.	1.125	1.875	1.625		


L
$10\frac{3}{4}''$
$1\frac{11}{16}''$
$1\frac{3}{8}''$
D
d
$2\frac{7}{8}''$
l
STUB AXLE
L
l
D
d
BALL-CRANK HANDLE
l
a
d
D
BENCH-LATHE DRAWBACK CHUCK

Exercises. Engine Lathe

1. In the lathe here shown the cast-iron faceplate *A* has a diameter of 14″, and the width of the face is $1\frac{1}{2}$″. Using a carbon-steel tool, at what speed should the face be finished?



ENGINE LATHE

2. In Ex. 1 how long will it take, using a $\frac{1}{32}$-inch feed, to make a single cut over the face?

3. If the lead screw *B* has an Acme screw thread with 8 threads to the inch, find the depth of the thread.

In the screw described in Ex. 3 find each of the following:

4. The width of the space at the top of the thread.

5. The width of the flat at the root of the thread.

6. The width of the flat at the top of the thread.

7. The change gears required on a lathe, which has a lead screw with 5 threads per inch, in cutting the thread.

8. Spur gear *C* is a 12-P_d gear with 96 teeth. Find the pitch diameter of the gear.

9. In Ex. 8 find the outside diameter of the gear.

10. In Ex. 8 find the whole depth of tooth.

11. The dead center *D* is 1.475″ in diameter at its small end, 1.790″ in diameter at its large end, and the length of the taper is 6″. Find the T.P.F.

12. In Ex. 11 find the included angle of the taper.

13. In the cast-iron cone pulley *E* the diameters of the steps are $6\frac{1}{8}''$, $7\frac{5}{16}''$, $8\frac{1}{2}''$, $9\frac{13}{16}''$, and 11″ respectively, and the width of each face is $1\frac{7}{8}''$. Beginning with the smallest step, find the number of R.P.M. for finishing each face with a high-speed steel tool.

14. If the diameters at the centers of the steps of the pulley in Ex. 13 are $\frac{3}{32}''$ greater than those at the edges, find the angle which the faces make with the axis of the pulley.

15. If the cone pulley in Ex. 13 is turned on a $7\frac{1}{2}$-inch arbor, find the offset of the tailstock center.

16. If the machine-steel feed rod *F* is $\frac{15}{16}''$ in diameter and 5′ 8″ in length, what is the number of R.P.M. needed in roughing the rod with a tool of carbon steel?

17. In Ex. 16 how long will it take to make a single cut over the rod with a $\frac{1}{8}$-inch feed?

18. If the spur gear *G* has 117 teeth and the pitch is 8, what is the outside diameter?

19. In Ex. 18 find the circular pitch and the thickness of tooth on the pitch line.

20. Using a B. & S. dividing head, find the indexing required in cutting the gear in Ex. 18.

21. A piece of cast iron $1\frac{1}{4}''$ in diameter is turned in the lathe at 175 R.P.M. Find the cutting speed used.

Exercises. General Applications

1. Find the angle a in the lathe center shown in the blueprint on page 135.

2. In Ex. 1 find the length L of the pointed end.

3. The three holes A, B, C are to be drilled on a milling machine in the jig plate shown in the blueprint. Find to the nearest 0.001″ the vertical movement from A for drilling hole C. Find the lateral movement from C for drilling hole B.

In connection with Exs. 3–22 see the note at the foot of page 127.

4. The holes in the cylinder head shown in the blueprint are equally spaced on the circle. Find the straight-line distance between the centers of the holes.

5. Consider Ex. 4 if there are 9 holes on a 12-inch circle; 13 holes on an 18-inch circle; 21 holes on a 16-inch circle.

6. In the pulleys and belting shown in the blueprint find the angle a and also find the distance D between the centers of the pulleys.

7. Find the diameter of the round stock required for making hexagonal heads which are to be $\frac{3}{4}''$ across the flats.

Consider Ex. 7 for each of the following measurements across the flats:

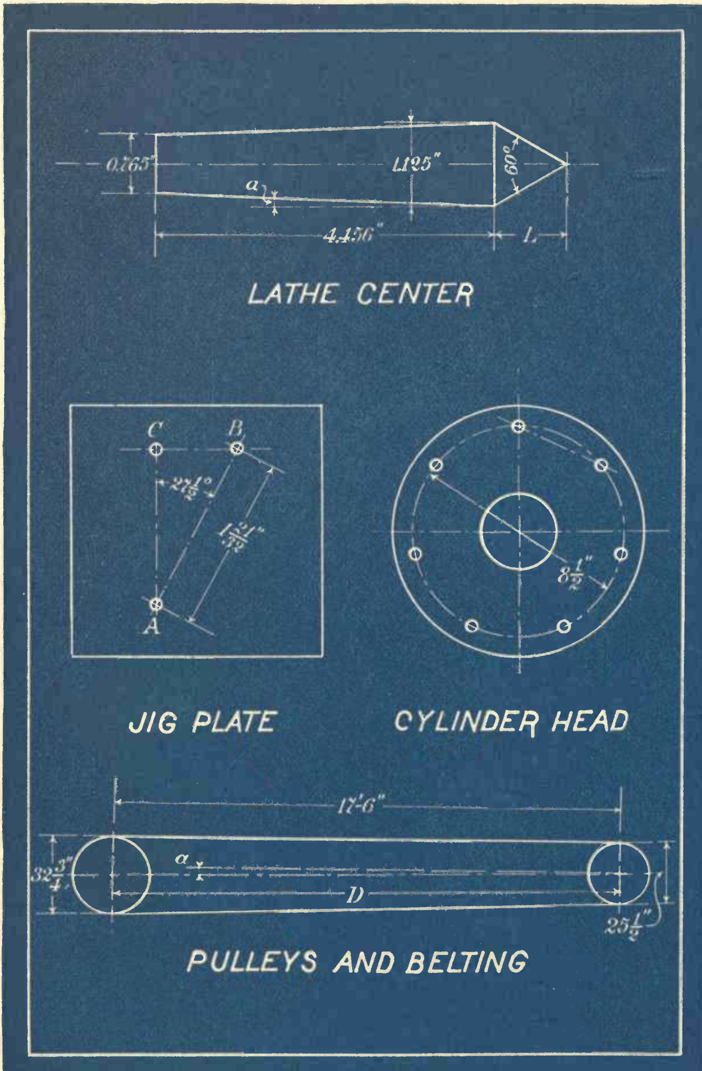
8. $\frac{15}{16}''$. **9.** $1\frac{5}{16}''$. **10.** $1\frac{7}{8}''$. **11.** $1\frac{11}{16}''$. **12.** $2\frac{3}{16}''$.

Find in each case the largest square that can be milled on a round shaft in which the diameter is as follows:

13. $\frac{7}{16}''$. **14.** $\frac{25}{32}''$. **15.** $1\frac{9}{16}''$. **16.** $1\frac{15}{16}''$. **17.** $2\frac{1}{32}''$.

Find in each case the diameter of the round stock required in milling a square in which the length of side is as follows:

18. $\frac{5}{8}''$. **19.** $\frac{11}{16}''$. **20.** $1\frac{1}{16}''$. **21.** $1\frac{7}{8}''$. **22.** $2\frac{1}{8}''$.


0.765"
1.125"
60°
a
4.456"
L
LATHE CENTER
C
B
A
22½°
1 21/32"
8½"
JIG PLATE
CYLINDER HEAD
17'-6"
a
D
32¾"
25½"
PULLEYS AND BELTING

Exercises. Miscellaneous Problems

1. A plug gage $\frac{9}{16}''$ in diameter enters a hole with unknown taper a distance of $3\frac{1}{16}''$, and another plug gage $\frac{11}{16}''$ in diameter enters the same hole a distance of $2\frac{3}{4}''$. Find the T.P.F. of the hole.

2. Find the change gears required on a milling machine with a lead of 10″ in cutting a spiral with a lead of 12.90″.

3. Make the necessary calculations for cutting a pair of 16-P_d bevel gears if the velocity ratio of the gears is to be 3 to 4, if the shafts are to be at right angles to each other, and if the pinion is to have 48 teeth.

4. If a milling machine is geared to cut a spiral with a lead of 0.67″, find the angle at which the dividing head should be elevated in milling a cam which is to have a rise of 0.310″ in 0.47 of the circumference.

5. The center of a 12-P_d spur gear with 54 teeth is $8\frac{1}{2}''$ from the center of a mating gear. Find the number of teeth and the outside diameter of the second gear.

6. Make all the necessary calculations for cutting a pair of spiral gears if it is required that D_c (approximate) $= 12''$, $r = 2$, $A_g = 33°$, $A_p = 27°$, $P_n = 8$, and $S = 60°$.

7. Two spur gears are to mesh with a distance of 27″ between centers, and the velocity ratio is to be 1 to 2. If the gears are to be cut 10 pitch, what should be the outside diameter and the number of teeth on each gear?

8. If an Acme screw thread with 3 threads per inch is cut on a $2\frac{1}{8}$-inch shaft, what is the root diameter of the thread?

9. The tailstock center of a certain lathe can be offset a maximum distance of $2\frac{1}{2}''$ from the center line of the lathe. What is the greatest T.P.F. that can be cut on a piece of work 9″ long? on a piece of work 17″ long?

TABLES FOR REFERENCE

TABLES OF MEASURES

Length

12 inches (in. or ″) = 1 foot (ft. or ′)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel contains 2150.42 cu. in. (approximately $1\frac{1}{4}$ cu. ft.).

Metric Length

1 kilometer (km.) = 1000 meters
Meter (m.)
1 centimeter (cm.) = 0.01 meter
1 millimeter (mm.) = 0.001 meter

In comparing the metric measures of length with those of our common system we usually think of 1 km. as $\frac{3}{5}$ mi., of 1 m. as $39\frac{1}{3}''$, of 1 cm. as $\frac{2}{5}''$, or 0.4″, and of 1 mm. as $\frac{1}{25}''$, or 0.04″. For a higher degree of accuracy, however, we use the following approximate equivalents: 1 km. = 0.62 mi., 1 m. = 39.37″, 1 cm. = 0.3937″, and 1 mm. = 0.0394″. To the machinist the millimeter is the most important of the metric measures on account of its use in connection with imported automobiles and with machines made for foreign trade.

Metric Weight

1 metric ton (t.) = 1000 kilograms
1 kilogram (kg.) = 1000 grams
Gram (g.)

The following approximate equivalents may be used in comparing metric measures of weight with those of our common system: 1 t. = 2204.6 lb., 1 kg. = 2.2 lb., and 1 g. = 15.43 grains.

Metric Capacity

1 hektoliter (hl.) = 100 liters
Liter (l.)
1 centiliter (cl.) = 0.01 liter

The liter is approximately a quart. It is the volume of a cube that is 0.1 m., or about 4″, on an edge.

Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
90 degrees = 1 right angle
360 degrees = 1 circumference

DECIMAL EQUIVALENTS OF COMMON FRACTIONS

Fraction				Decimal	Fraction				Decimal
			1/64	0.015625				33/64	0.515625
		1/32		.03125			17/32		.53125
			3/64	.046875				35/64	.546875
	1/16			.0625		9/16			.5625
			5/64	.078125				37/64	.578125
		3/32		.09375			19/32		.59375
			7/64	.109375				39/64	.609375
1/8				.125	5/8				.625
			9/64	.140625				41/64	.640625
		5/32		.15625			21/32		.65625
			11/64	.171875				43/64	.671875
	3/16			.1875		11/16			.6875
			13/64	.203125				45/64	.703125
		7/32		.21875			23/32		.71875
			15/64	.234375				47/64	.734375
1/4				.25	3/4				.75
			17/64	.265625				49/64	.765625
		9/32		.28125			25/32		.78125
			19/64	.296875				51/64	.796875
	5/16			.3125		13/16			.8125
			21/64	.328125				53/64	.828125
		11/32		.34375			27/32		.84375
			23/64	.359375				55/64	.859375
3/8				.375	7/8				.875
			25/64	.390625				57/64	.890625
		13/32		.40625			29/32		.90625
			27/64	.421875				59/64	.921875
	7/16			.4375		15/16			.9375
			29/64	.453125				61/64	.953125
		15/32		.46875			31/32		.96875
			31/64	.484375				63/64	.984375
1/2				.5	1				1.

CONVENIENT RULES

The circumference of a circle is $\frac{22}{7}$ times the diameter.

For a higher degree of accuracy, $c = 3.1416\, d$.

The diameter of a circle is $\frac{7}{22}$ of the circumference.

For a higher degree of accuracy, $d = 0.3183\, c$.

The area of a circle is $\frac{11}{14}$ of the square of the diameter.

For a higher degree of accuracy, $A = 0.7854\, d^2$.

The height of an equilateral triangle is 0.8660 of the side.

The diagonal of a square is 1.4142 times the side.

The diagonal of a square is also called the "long diameter" or the "distance across the corners."

The long diameter of a regular hexagon is twice the side.

The short diameter, or the perpendicular distance between parallel sides, of a regular hexagon is 1.7321 times the side.

To convert Fahrenheit temperature into centigrade temperature subtract 32° from the Fahrenheit reading and take $\frac{5}{9}$ of the result.

Expressed as a formula, $C = \frac{5}{9}(F - 32)$.

To convert centigrade temperature into Fahrenheit temperature take $\frac{9}{5}$ of the centigrade reading and add 32° to the result.

Expressed as a formula, $F = \frac{9}{5}C + 32$.

COMMON EQUIVALENTS

In ordinary cases the following equivalents may be used:

1 gal. contains 231 cu. in., or 0.134 cu. ft.

1 cu. ft. contains $7\frac{1}{2}$ gal.

1 barrel (bbl.) contains $31\frac{1}{2}$ gal., or $4\frac{1}{5}$ cu. ft.

1 cu. ft. of water weighs 62.425 lb. (approximately $62\frac{1}{2}$ lb.).

1 gal. of water weighs 8.345 lb. (approximately $8\frac{1}{3}$ lb.).

1 l. of water weighs 1 kg.

NATURAL TRIGONOMETRIC FUNCTIONS

Tables of four of the natural functions of an angle, the sine, cosine, tangent, and cotangent, for every 6′, or 0.1°, from 0° to 90° are given on pages 142–149.

The following examples illustrate the use of these tables:

1. Find sin 62° 19′.

On page 143 we look for 62° in the column at the left, and in the same line at the right under 18′ we find that

$$\sin 62° 18' = 0.8854.$$

Under 1′ in the column of differences at the right and in line with 62° we find that the difference for 1′ is 1 (actually this is 0.0001).

Since the column of differences is marked "+ Differences," this difference is added to sin 62° 18′. The addition is done mentally, only the result being written. Therefore

$$\sin 62° 19' = 0.8855.$$

2. Find cot 5° 56′.

From page 148, $\cot 5° 54' = 9.6768.$

Notice that after the 5 at the left the integral part is 11, but since two black numbers intervene, we decrease the 11 by 2. The cotangent is here changing so rapidly that we use a process commonly known as *interpolation*. By subtraction we find that the difference between cot 5° 54′ and cot 6° 0′ is 0.1624. Since 2′ is $\frac{1}{3}$ of 6′, we take $\frac{1}{3}$ of 0.1624, and hence the difference for 2′ is 0.0541.

Since the column of differences is marked "– Differences," this difference is subtracted from cot 5° 54′. Therefore

$$\cot 5° 56' = 9.6227.$$

3. Find the angle of which the tangent is 0.5693.

On page 146 we find that the tangent next smaller than this tangent is 0.5681, which is the tangent of 29° 36′, and by subtraction we see that the difference between 0.5693 and 0.5681 is 0.0012.

In the column of differences we see that 12 is the difference to be added for 3′. Adding 3′ to 29° 36′, we see that

29° 39′ is the angle of which the tangent is 0.5693.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	+Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	.0000	.0017	.0035	.0052	.0070	.0087	.0105	.0122	.0140	.0157	3	6	9	12	15
1	.0175	.0192	.0209	.0227	.0244	.0262	.0279	.0297	.0314	.0332	3	6	9	12	15
2	.0349	.0366	.0384	.0401	.0419	.0436	.0454	.0471	.0488	.0506	3	6	9	12	15
3	.0523	.0541	.0558	.0576	.0593	.0610	.0628	.0645	.0663	.0680	3	6	9	12	15
4	.0698	.0715	.0732	.0750	.0767	.0785	.0802	.0819	.0837	.0854	3	6	9	12	14
5	.0872	.0889	.0906	.0924	.0941	.0958	.0976	.0993	.1011	.1028	3	6	9	12	14
6	.1045	.1063	.1080	.1097	.1115	.1132	.1149	.1167	.1184	.1201	3	6	9	12	14
7	.1219	.1236	.1253	.1271	.1288	.1305	.1323	.1340	.1357	.1374	3	6	9	12	14
8	.1392	.1409	.1426	.1444	.1461	.1478	.1495	.1513	.1530	.1547	3	6	9	12	14
9	.1564	.1582	.1599	.1616	.1633	.1650	.1668	.1685	.1702	.1719	3	6	9	12	14
10	.1736	.1754	.1771	.1788	.1805	.1822	.1840	.1857	.1874	.1891	3	6	9	11	14
11	.1908	.1925	.1942	.1959	.1977	.1994	.2011	.2028	.2045	.2062	3	6	9	11	14
12	.2079	.2096	.2113	.2130	.2147	.2164	.2181	.2198	.2215	.2233	3	6	9	11	14
13	.2250	.2267	.2284	.2300	.2317	.2334	.2351	.2368	.2385	.2402	3	6	8	11	14
14	.2419	.2436	.2453	.2470	.2487	.2504	.2521	.2538	.2554	.2571	3	6	8	11	14
15	.2588	.2605	.2622	.2639	.2656	.2672	.2689	.2706	.2723	.2740	3	6	8	11	14
16	.2756	.2773	.2790	.2807	.2823	.2840	.2857	.2874	.2890	.2907	3	6	8	11	14
17	.2924	.2940	.2957	.2974	.2990	.3007	.3024	.3040	.3057	.3074	3	6	8	11	14
18	.3090	.3107	.3123	.3140	.3156	.3173	.3190	.3206	.3223	.3239	3	6	8	11	14
19	.3256	.3272	.3289	.3305	.3322	.3338	.3355	.3371	.3387	.3404	3	5	8	11	14
20	.3420	.3437	.3453	.3469	.3486	.3502	.3518	.3535	.3551	.3567	3	5	8	11	14
21	.3584	.3600	.3616	.3633	.3649	.3665	.3681	.3697	.3714	.3730	3	5	8	11	14
22	.3746	.3762	.3778	.3795	.3811	.3827	.3843	.3859	.3875	.3891	3	5	8	11	14
23	.3907	.3923	.3939	.3955	.3971	.3987	.4003	.4019	.4035	.4051	3	5	8	11	14
24	.4067	.4083	.4099	.4115	.4131	.4147	.4163	.4179	.4195	.4210	3	5	8	11	13
25	.4226	.4242	.4258	.4274	.4289	.4305	.4321	.4337	.4352	.4368	3	5	8	11	13
26	.4384	.4399	.4415	.4431	.4446	.4462	.4478	.4493	.4509	.4524	3	5	8	10	13
27	.4540	.4555	.4571	.4586	.4602	.4617	.4633	.4648	.4664	.4679	3	5	8	10	13
28	.4695	.4710	.4726	.4741	.4756	.4772	.4787	.4802	.4818	.4833	3	5	8	10	13
29	.4848	.4863	.4879	.4894	.4909	.4924	.4939	.4955	.4970	.4985	3	5	8	10	13
30	.5000	.5015	.5030	.5045	.5060	.5075	.5090	.5105	.5120	.5135	3	5	8	10	13
31	.5150	.5165	.5180	.5195	.5210	.5225	.5240	.5255	.5270	.5284	2	5	7	10	12
32	.5299	.5314	.5329	.5344	.5358	.5373	.5388	.5402	.5417	.5432	2	5	7	10	12
33	.5446	.5461	.5476	.5490	.5505	.5519	.5534	.5548	.5563	.5577	2	5	7	10	12
34	.5592	.5606	.5621	.5635	.5650	.5664	.5678	.5693	.5707	.5721	2	5	7	10	12
35	.5736	.5750	.5764	.5779	.5793	.5807	.5821	.5835	.5850	.5864	2	5	7	9	12
36	.5878	.5892	.5906	.5920	.5934	.5948	.5962	.5976	.5990	.6004	2	5	7	9	12
37	.6018	.6032	.6046	.6060	.6074	.6088	.6101	.6115	.6129	.6143	2	5	7	9	12
38	.6157	.6170	.6184	.6198	.6211	.6225	.6239	.6252	.6266	.6280	2	5	7	9	11
39	.6293	.6307	.6320	.6334	.6347	.6361	.6374	.6388	.6401	.6414	2	4	7	9	11
40	.6428	.6441	.6455	.6468	.6481	.6494	.6508	.6521	.6534	.6547	2	4	7	9	11
41	.6561	.6574	.6587	.6600	.6613	.6626	.6639	.6652	.6665	.6678	2	4	7	9	11
42	.6691	.6704	.6717	.6730	.6743	.6756	.6769	.6782	.6794	.6807	2	4	6	9	11
43	.6820	.6833	.6845	.6858	.6871	.6884	.6896	.6909	.6921	.6934	2	4	6	8	11
44	.6947	.6959	.6972	.6984	.6997	.7009	.7022	.7034	.7046	.7059	2	4	6	8	10

All the above sines are less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	+Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	.7071	.7083	.7096	.7108	.7120	.7133	.7145	.7157	.7169	.7181	2	4	6	8	10
46	.7193	.7206	.7218	.7230	.7242	.7254	.7266	.7278	.7290	.7302	2	4	6	8	10
47	.7314	.7325	.7337	.7349	.7361	.7373	.7385	.7396	.7408	.7420	2	4	6	8	10
48	.7431	.7443	.7455	.7466	.7478	.7490	.7501	.7513	.7524	.7536	2	4	6	8	10
49	.7547	.7559	.7570	.7581	.7593	.7604	.7615	.7627	.7638	.7649	2	4	6	8	9
50	.7660	.7672	.7683	.7694	.7705	.7716	.7727	.7738	.7749	.7760	2	4	6	7	9
51	.7771	.7782	.7793	.7804	.7815	.7826	.7837	.7848	.7859	.7869	2	4	5	7	9
52	.7880	.7891	.7902	.7912	.7923	.7934	.7944	.7955	.7965	.7976	2	4	5	7	9
53	.7986	.7997	.8007	.8018	.8028	.8039	.8049	.8059	.8070	.8080	2	3	5	7	9
54	.8090	.8100	.8111	.8121	.8131	.8141	.8151	.8161	.8171	.8181	2	3	5	7	8
55	.8192	.8202	.8211	.8221	.8231	.8241	.8251	.8261	.8271	.8281	2	3	5	7	8
56	.8290	.8300	.8310	.8320	.8329	.8339	.8348	.8358	.8368	.8377	2	3	5	6	8
57	.8387	.8396	.8406	.8415	.8425	.8434	.8443	.8453	.8462	.8471	2	3	5	6	8
58	.8480	.8490	.8499	.8508	.8517	.8526	.8536	.8545	.8554	.8563	2	3	5	6	8
59	.8572	.8581	.8590	.8599	.8607	.8616	.8625	.8634	.8643	.8652	1	3	4	6	7
60	.8660	.8669	.8678	.8686	.8695	.8704	.8712	.8721	.8729	.8738	1	3	4	6	7
61	.8746	.8755	.8763	.8771	.8780	.8788	.8796	.8805	.8813	.8821	1	3	4	6	7
62	.8829	.8838	.8846	.8854	.8862	.8870	.8878	.8886	.8894	.8902	1	3	4	5	7
63	.8910	.8918	.8926	.8934	.8942	.8949	.8957	.8965	.8973	.8980	1	3	4	5	6
64	.8988	.8996	.9003	.9011	.9018	.9026	.9033	.9041	.9048	.9056	1	3	4	5	6
65	.9063	.9070	.9078	.9085	.9092	.9100	.9107	.9114	.9121	.9128	1	2	4	5	6
66	.9135	.9143	.9150	.9157	.9164	.9171	.9178	.9184	.9191	.9198	1	2	3	5	6
67	.9205	.9212	.9219	.9225	.9232	.9239	.9245	.9252	.9259	.9265	1	2	3	4	6
68	.9272	.9278	.9285	.9291	.9298	.9304	.9311	.9317	.9323	.9330	1	2	3	4	5
69	.9336	.9342	.9348	.9354	.9361	.9367	.9373	.9379	.9385	.9391	1	2	3	4	5
70	.9397	.9403	.9409	.9415	.9421	.9426	.9432	.9438	.9444	.9449	1	2	3	4	5
71	.9455	.9461	.9466	.9472	.9478	.9483	.9489	.9494	.9500	.9505	1	2	3	4	5
72	.9511	.9516	.9521	.9527	.9532	.9537	.9542	.9548	.9553	.9558	1	2	3	4	4
73	.9563	.9568	.9573	.9578	.9583	.9588	.9593	.9598	.9603	.9608	1	2	2	3	4
74	.9613	.9617	.9622	.9627	.9632	.9636	.9641	.9646	.9650	.9655	1	2	2	3	4
75	.9659	.9664	.9668	.9673	.9677	.9681	.9686	.9690	.9694	.9699	1	1	2	3	4
76	.9703	.9707	.9711	.9715	.9720	.9724	.9728	.9732	.9736	.9740	1	1	2	3	3
77	.9744	.9748	.9751	.9755	.9759	.9763	.9767	.9770	.9774	.9778	1	1	2	3	3
78	.9781	.9785	.9789	.9792	.9796	.9799	.9803	.9806	.9810	.9813	1	1	2	2	3
79	.9816	.9820	.9823	.9826	.9829	.9833	.9836	.9839	.9842	.9845	1	1	2	2	3
80	.9848	.9851	.9854	.9857	.9860	.9863	.9866	.9869	.9871	.9874	0	1	1	2	2
81	.9877	.9880	.9882	.9885	.9888	.9890	.9893	.9895	.9898	.9900	0	1	1	2	2
82	.9903	.9905	.9907	.9910	.9912	.9914	.9917	.9919	.9921	.9923	0	1	1	2	2
83	.9925	.9928	.9930	.9932	.9934	.9936	.9938	.9940	.9942	.9943	0	1	1	1	2
84	.9945	.9947	.9949	.9951	.9952	.9954	.9956	.9957	.9959	.9960	0	1	1	1	1
85	.9962	.9963	.9965	.9966	.9968	.9969	.9971	.9972	.9973	.9974	0	0	1	1	1
86	.9976	.9977	.9978	.9979	.9980	.9981	.9982	.9983	.9984	.9985	0	0	1	1	1
87	.9986	.9987	.9988	.9989	.9990	.9990	.9991	.9992	.9993	.9993	0	0	0	1	1
88	.9994	.9995	.9995	.9996	.9996	.9997	.9997	.9997	.9998	.9998	0	0	0	0	0
89	.9998	.9999	.9999	.9999	.9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0

The precise value of all sines except sin 90° is less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	– Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	1.000	1.000	1.000	1.000	1.000	1.000	.9999	.9999	.9999	.9999	0	0	0	0	0
1	.9998	.9998	.9998	.9997	.9997	.9997	.9996	.9996	.9995	.9995	0	0	0	0	0
2	.9994	.9993	.9993	.9992	.9991	.9990	.9990	.9989	.9988	.9987	0	0	0	0	0
3	.9986	.9985	.9984	.9983	.9982	.9981	.9980	.9979	.9978	.9977	0	0	1	1	1
4	.9976	.9974	.9973	.9972	.9971	.9969	.9968	.9966	.9965	.9963	0	0	1	1	1
5	.9962	.9960	.9959	.9957	.9956	.9954	.9952	.9951	.9949	.9947	0	1	1	1	1
6	.9945	.9943	.9942	.9940	.9938	.9936	.9934	.9932	.9930	.9928	0	1	1	1	2
7	.9925	.9923	.9921	.9919	.9917	.9914	.9912	.9910	.9907	.9905	0	1	1	2	2
8	.9903	.9900	.9898	.9895	.9893	.9890	.9888	.9885	.9882	.9880	0	1	1	2	2
9	.9877	.9874	.9871	.9869	.9866	.9863	.9860	.9857	.9854	.9851	0	1	1	2	2
10	.9848	.9845	.9842	.9839	.9836	.9833	.9829	.9826	.9823	.9820	1	1	2	2	3
11	.9816	.9813	.9810	.9806	.9803	.9799	.9796	.9792	.9789	.9785	1	1	2	2	3
12	.9781	.9778	.9774	.9770	.9767	.9763	.9759	.9755	.9751	.9748	1	1	2	3	3
13	.9744	.9740	.9736	.9732	.9728	.9724	.9720	.9715	.9711	.9707	1	1	2	3	3
14	.9703	.9699	.9694	.9690	.9686	.9681	.9677	.9673	.9668	.9664	1	1	2	3	4
15	.9659	.9655	.9650	.9646	.9641	.9636	.9632	.9627	.9622	.9617	1	2	2	3	4
16	.9613	.9608	.9603	.9598	.9593	.9588	.9583	.9578	.9573	.9568	1	2	2	3	4
17	.9563	.9558	.9553	.9548	.9542	.9537	.9532	.9527	.9521	.9516	1	2	3	4	4
18	.9511	.9505	.9500	.9494	.9489	.9483	.9478	.9472	.9466	.9461	1	2	3	4	5
19	.9455	.9449	.9444	.9438	.9432	.9426	.9421	.9415	.9409	.9403	1	2	3	4	5
20	.9397	.9391	.9385	.9379	.9373	.9367	.9361	.9354	.9348	.9342	1	2	3	4	5
21	.9336	.9330	.9323	.9317	.9311	.9304	.9298	.9291	.9285	.9278	1	2	3	4	5
22	.9272	.9265	.9259	.9252	.9245	.9239	.9232	.9225	.9219	.9212	1	2	3	4	6
23	.9205	.9198	.9191	.9184	.9178	.9171	.9164	.9157	.9150	.9143	1	2	3	5	6
24	.9135	.9128	.9121	.9114	.9107	.9100	.9092	.9085	.9078	.9070	1	2	4	5	6
25	.9063	.9056	.9048	.9041	.9033	.9026	.9018	.9011	.9003	.8996	1	3	4	5	6
26	.8988	.8980	.8973	.8965	.8957	.8949	.8942	.8934	.8926	.8918	1	3	4	5	6
27	.8910	.8902	.8894	.8886	.8878	.8870	.8862	.8854	.8846	.8838	1	3	4	5	7
28	.8829	.8821	.8813	.8805	.8796	.8788	.8780	.8771	.8763	.8755	1	3	4	6	7
29	.8746	.8738	.8729	.8721	.8712	.8704	.8695	.8686	.8678	.8669	1	3	4	6	7
30	.8660	.8652	.8643	.8634	.8625	.8616	.8607	.8599	.8590	.8581	1	3	4	6	7
31	.8572	.8563	.8554	.8545	.8536	.8526	.8517	.8508	.8499	.8490	2	3	5	6	8
32	.8480	.8471	.8462	.8453	.8443	.8434	.8425	.8415	.8406	.8396	2	3	5	6	8
33	.8387	.8377	.8368	.8358	.8348	.8339	.8329	.8320	.8310	.8300	2	3	5	6	8
34	.8290	.8281	.8271	.8261	.8251	.8241	.8231	.8221	.8211	.8202	2	3	5	7	8
35	.8192	.8181	.8171	.8161	.8151	.8141	.8131	.8121	.8111	.8100	2	3	5	7	8
36	.8090	.8080	.8070	.8059	.8049	.8039	.8028	.8018	.8007	.7997	2	3	5	7	9
37	.7986	.7976	.7965	.7955	.7944	.7934	.7923	.7912	.7902	.7891	2	4	5	7	9
38	.7880	.7869	.7859	.7848	.7837	.7826	.7815	.7804	.7793	.7782	2	4	5	7	9
39	.7771	.7760	.7749	.7738	.7727	.7716	.7705	.7694	.7683	.7672	2	4	6	7	9
40	.7660	.7649	.7638	.7627	.7615	.7604	.7593	.7581	.7570	.7559	2	4	6	8	9
41	.7547	.7536	.7524	.7513	.7501	.7490	.7478	.7466	.7455	.7443	2	4	6	8	10
42	.7431	.7420	.7408	.7396	.7385	.7373	.7361	.7349	.7337	.7325	2	4	6	8	10
43	.7314	.7302	.7290	.7278	.7266	.7254	.7242	.7230	.7218	.7206	2	4	6	8	10
44	.7193	.7181	.7169	.7157	.7145	.7133	.7120	.7108	.7096	.7083	2	4	6	8	10

The precise value of all cosines except cos 0° is less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	– Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	.7071	.7059	.7046	.7034	.7022	.7009	.6997	.6984	.6972	.6959	2	4	6	8	10
46	.6947	.6934	.6921	.6909	.6896	.6884	.6871	.6858	.6845	.6833	2	4	6	8	11
47	.6820	.6807	.6794	.6782	.6769	.6756	.6743	.6730	.6717	.6704	2	4	6	9	11
48	.6691	.6678	.6665	.6652	.6639	.6626	.6613	.6600	.6587	.6574	2	4	7	9	11
49	.6561	.6547	.6534	.6521	.6508	.6494	.6481	.6468	.6455	.6441	2	4	7	9	11
50	.6428	.6414	.6401	.6388	.6374	.6361	.6347	.6334	.6320	.6307	2	4	7	9	11
51	.6293	.6280	.6266	.6252	.6239	.6225	.6211	.6198	.6184	.6170	2	5	7	9	11
52	.6157	.6143	.6129	.6115	.6101	.6088	.6074	.6060	.6046	.6032	2	5	7	9	12
53	.6018	.6004	.5990	.5976	.5962	.5948	.5934	.5920	.5906	.5892	2	5	7	9	12
54	.5878	.5864	.5850	.5835	.5821	.5807	.5793	.5779	.5764	.5750	2	5	7	9	12
55	.5736	.5721	.5707	.5693	.5678	.5664	.5650	.5635	.5621	.5606	2	5	7	10	12
56	.5592	.5577	.5563	.5548	.5534	.5519	.5505	.5490	.5476	.5461	2	5	7	10	12
57	.5446	.5432	.5417	.5402	.5388	.5373	.5358	.5344	.5329	.5314	2	5	7	10	12
58	.5299	.5284	.5270	.5255	.5240	.5225	.5210	.5195	.5180	.5165	2	5	7	10	12
59	.5150	.5135	.5120	.5105	.5090	.5075	.5060	.5045	.5030	.5015	3	5	8	10	13
60	.5000	.4985	.4970	.4955	.4939	.4924	.4909	.4894	.4879	.4863	3	5	8	10	13
61	.4848	.4833	.4818	.4802	.4787	.4772	.4756	.4741	.4726	.4710	3	5	8	10	13
62	.4695	.4679	.4664	.4648	.4633	.4617	.4602	.4586	.4571	.4555	3	5	8	10	13
63	.4540	.4524	.4509	.4493	.4478	.4462	.4446	.4431	.4415	.4399	3	5	8	10	13
64	.4384	.4368	.4352	.4337	.4321	.4305	.4289	.4274	.4258	.4242	3	5	8	11	13
65	.4226	.4210	.4195	.4179	.4163	.4147	.4131	.4115	.4099	.4083	3	5	8	11	13
66	.4067	.4051	.4035	.4019	.4003	.3987	.3971	.3955	.3939	.3923	3	5	8	11	13
67	.3907	.3891	.3875	.3859	.3843	.3827	.3811	.3795	.3778	.3762	3	5	8	11	13
68	.3746	.3730	.3714	.3697	.3681	.3665	.3649	.3633	.3616	.3600	3	5	8	11	14
69	.3584	.3567	.3551	.3535	.3518	.3502	.3486	.3469	.3453	.3437	3	5	8	11	14
70	.3420	.3404	.3387	.3371	.3355	.3338	.3322	.3305	.3289	.3272	3	5	8	11	14
71	.3256	.3239	.3223	.3206	.3190	.3173	.3156	.3140	.3123	.3107	3	6	8	11	14
72	.3090	.3074	.3057	.3040	.3024	.3007	.2990	.2974	.2957	.2940	3	6	8	11	14
73	.2924	.2907	.2890	.2874	.2857	.2840	.2823	.2807	.2790	.2773	3	6	8	11	14
74	.2756	.2740	.2723	.2706	.2689	.2672	.2656	.2639	.2622	.2605	3	6	8	11	14
75	.2588	.2571	.2554	.2538	.2521	.2504	.2487	.2470	.2453	.2436	3	6	8	11	14
76	.2419	.2402	.2385	.2368	.2351	.2334	.2317	.2300	.2284	.2267	3	6	8	11	14
77	.2250	.2233	.2215	.2198	.2181	.2164	.2147	.2130	.2113	.2096	3	6	9	11	14
78	.2079	.2062	.2045	.2028	.2011	.1994	.1977	.1959	.1942	.1925	3	6	9	11	14
79	.1908	.1891	.1874	.1857	.1840	.1822	.1805	.1788	.1771	.1754	3	6	9	11	14
80	.1736	.1719	.1702	.1685	.1668	.1650	.1633	.1616	.1599	.1582	3	6	9	11	14
81	.1564	.1547	.1530	.1513	.1495	.1478	.1461	.1444	.1426	.1409	3	6	9	12	14
82	.1392	.1374	.1357	.1340	.1323	.1305	.1288	.1271	.1253	.1236	3	6	9	12	14
83	.1219	.1201	.1184	.1167	.1149	.1132	.1115	.1097	.1080	.1063	3	6	9	12	14
84	.1045	.1028	.1011	.0993	.0976	.0958	.0941	.0924	.0906	.0889	3	6	9	12	14
85	.0872	.0854	.0837	.0819	.0802	.0785	.0767	.0750	.0732	.0715	3	6	9	12	14
86	.0698	.0680	.0663	.0645	.0628	.0610	.0593	.0576	.0558	.0541	3	6	9	12	15
87	.0523	.0506	.0488	.0471	.0454	.0436	.0419	.0401	.0384	.0366	3	6	9	12	15
88	.0349	.0332	.0314	.0297	.0279	.0262	.0244	.0227	.0209	.0192	3	6	9	12	15
89	.0175	.0157	.0140	.0122	.0105	.0087	.0070	.0052	.0035	.0017	3	6	9	12	15

All the above cosines are less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	+ Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	0.0000	.0017	.0035	.0052	.0070	.0087	.0105	.0122	.0140	.0157	3	6	9	12	15
1	0.0175	.0192	.0209	.0227	.0244	.0262	.0279	.0297	.0314	.0332	3	6	9	12	15
2	0.0349	.0367	.0384	.0402	.0419	.0437	.0454	.0472	.0489	.0507	3	6	9	12	15
3	0.0524	.0542	.0559	.0577	.0594	.0612	.0629	.0647	.0664	.0682	3	6	9	12	15
4	0.0699	.0717	.0734	.0752	.0769	.0787	.0805	.0822	.0840	.0857	3	6	9	12	15
5	0.0875	.0892	.0910	.0928	.0945	.0963	.0981	.0998	.1016	.1033	3	6	9	12	15
6	0.1051	.1069	.1086	.1104	.1122	.1139	.1157	.1175	.1192	.1210	3	6	9	12	15
7	0.1228	.1246	.1263	.1281	.1299	.1317	.1334	.1352	.1370	.1388	3	6	9	12	15
8	0.1405	.1423	.1441	.1459	.1477	.1495	.1512	.1530	.1548	.1566	3	6	9	12	15
9	0.1584	.1602	.1620	.1638	.1655	.1673	.1691	.1709	.1727	.1745	3	6	9	12	15
10	0.1763	.1781	.1799	.1817	.1835	.1853	.1871	.1890	.1908	.1926	3	6	9	12	15
11	0.1944	.1962	.1980	.1998	.2016	.2035	.2053	.2071	.2089	.2107	3	6	9	12	15
12	0.2126	.2144	.2162	.2180	.2199	.2217	.2235	.2254	.2272	.2290	3	6	9	12	15
13	0.2309	.2327	.2345	.2364	.2382	.2401	.2419	.2438	.2456	.2475	3	6	9	12	15
14	0.2493	.2512	.2530	.2549	.2568	.2586	.2605	.2623	.2642	.2661	3	6	9	12	16
15	0.2679	.2698	.2717	.2736	.2754	.2773	.2792	.2811	.2830	.2849	3	6	9	13	16
16	0.2867	.2886	.2905	.2924	.2943	.2962	.2981	.3000	.3019	.3038	3	6	9	13	16
17	0.3057	.3076	.3096	.3115	.3134	.3153	.3172	.3191	.3211	.3230	3	6	10	13	16
18	0.3249	.3269	.3288	.3307	.3327	.3346	.3365	.3385	.3404	.3424	3	6	10	13	16
19	0.3443	.3463	.3482	.3502	.3522	.3541	.3561	.3581	.3600	.3620	3	7	10	13	16
20	0.3640	.3659	.3679	.3699	.3719	.3739	.3759	.3779	.3799	.3819	3	7	10	13	17
21	0.3839	.3859	.3879	.3899	.3919	.3939	.3959	.3979	.4000	.4020	3	7	10	13	17
22	0.4040	.4061	.4081	.4101	.4122	.4142	.4163	.4183	.4204	.4224	3	7	10	14	17
23	0.4245	.4265	.4286	.4307	.4327	.4348	.4369	.4390	.4411	.4431	3	7	10	14	17
24	0.4452	.4473	.4494	.4515	.4536	.4557	.4578	.4599	.4621	.4642	4	7	11	14	18
25	0.4663	.4684	.4706	.4727	.4748	.4770	.4791	.4813	.4834	.4856	4	7	11	14	18
26	0.4877	.4899	.4921	.4942	.4964	.4986	.5008	.5029	.5051	.5073	4	7	11	15	18
27	0.5095	.5117	.5139	.5161	.5184	.5206	.5228	.5250	.5272	.5295	4	7	11	15	18
28	0.5317	.5340	.5362	.5384	.5407	.5430	.5452	.5475	.5498	.5520	4	8	11	15	19
29	0.5543	.5566	.5589	.5612	.5635	.5658	.5681	.5704	.5727	.5750	4	8	12	15	19
30	0.5774	.5797	.5820	.5844	.5867	.5890	.5914	.5938	.5961	.5985	4	8	12	16	20
31	0.6009	.6032	.6056	.6080	.6104	.6128	.6152	.6176	.6200	.6224	4	8	12	16	20
32	0.6249	.6273	.6297	.6322	.6346	.6371	.6395	.6420	.6445	.6469	4	8	12	16	20
33	0.6494	.6519	.6544	.6569	.6594	.6619	.6644	.6669	.6694	.6720	4	8	13	17	21
34	0.6745	.6771	.6796	.6822	.6847	.6873	.6899	.6924	.6950	.6976	4	9	13	17	21
35	0.7002	.7028	.7054	.7080	.7107	.7133	.7159	.7186	.7212	.7239	4	9	13	18	22
36	0.7265	.7292	.7319	.7346	.7373	.7400	.7427	.7454	.7481	.7508	5	9	14	18	23
37	0.7536	.7563	.7590	.7618	.7646	.7673	.7701	.7729	.7757	.7785	5	9	14	18	23
38	0.7813	.7841	.7869	.7898	.7926	.7954	.7983	.8012	.8040	.8069	5	9	14	19	24
39	0.8098	.8127	.8156	.8185	.8214	.8243	.8273	.8302	.8332	.8361	5	10	15	20	24
40	0.8391	.8421	.8451	.8481	.8511	.8541	.8571	.8601	.8632	.8662	5	10	15	20	25
41	0.8693	.8724	.8754	.8785	.8816	.8847	.8878	.8910	.8941	.8972	5	10	16	21	26
42	0.9004	.9036	.9067	.9099	.9131	.9163	.9195	.9228	.9260	.9293	5	11	16	21	27
43	0.9325	.9358	.9391	.9424	.9457	.9490	.9523	.9556	.9590	.9623	6	11	17	22	28
44	0.9657	.9691	.9725	.9759	.9793	.9827	.9861	.9896	.9930	.9965	6	11	17	23	29

All tangents less than tan 45° are less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	+ Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	1.0000	.0035	.0070	.0105	.0141	.0176	.0212	.0247	.0283	.0319	6	12	18	24	30
46	1.0355	.0392	.0428	.0464	.0501	.0538	.0575	.0612	.0649	.0686	6	12	18	25	31
47	1.0724	.0761	.0799	.0837	.0875	.0913	.0951	.0990	.1028	.1067	6	13	19	25	32
48	1.1106	.1145	.1184	.1224	.1263	.1303	.1343	.1383	.1423	.1463	7	13	20	26	33
49	1.1504	.1544	.1585	.1626	.1667	.1708	.1750	.1792	.1833	.1875	7	14	21	28	34
50	1.1918	.1960	.2002	.2045	.2088	.2131	.2174	.2218	.2261	.2305	7	14	22	29	36
51	1.2349	.2393	.2437	.2482	.2527	.2572	.2617	.2662	.2708	.2753	8	15	23	30	38
52	1.2799	.2846	.2892	.2938	.2985	.3032	.3079	.3127	.3175	.3222	8	16	24	31	39
53	1.3270	.3319	.3367	.3416	.3465	.3514	.3564	.3613	.3663	.3713	8	16	25	33	41
54	1.3764	.3814	.3865	.3916	.3968	.4019	.4071	.4124	.4176	.4229	9	17	26	34	43
55	1.4281	.4335	.4388	.4442	.4496	.4550	.4605	.4659	.4715	.4770	9	18	27	36	45
56	1.4826	.4882	.4938	.4994	.5051	.5108	.5166	.5224	.5282	.5340	10	19	29	38	48
57	1.5399	.5458	.5517	.5577	.5637	.5697	.5757	.5818	.5880	.5941	10	20	30	40	50
58	1.6003	.6066	.6128	.6191	.6255	.6319	.6383	.6447	.6512	.6577	11	21	32	43	53
59	1.6643	.6709	.6775	.6842	.6909	.6977	.7045	.7113	.7182	.7251	11	23	34	45	56
60	1.7321	.7391	.7461	.7532	.7603	.7675	.7747	.7820	.7893	.7966	12	24	36	48	60
61	1.8040	.8115	.8190	.8265	.8341	.8418	.8495	.8572	.8650	.8728	13	26	38	51	64
62	1.8807	.8887	.8967	.9047	.9128	.9210	.9292	.9375	.9458	.9542	14	27	41	55	68
63	1.9626	.9711	.9797	.9883	.9970	**.0057**	.0145	.0233	.0323	.0413	15	29	44	58	73
64	2.0503	.0594	.0686	.0778	.0872	.0965	.1060	.1155	.1251	.1348	16	31	47	63	78
65	2.1445	.1543	.1642	.1742	.1842	.1943	.2045	.2148	.2251	.2355	17	34	51	68	85
66	2.2460	.2566	.2673	.2781	.2889	.2998	.3109	.3220	.3332	.3445	18	37	55	73	92
67	2.3559	.3673	.3789	.3906	.4023	.4142	.4262	.4383	.4504	.4627	20	40	60	79	99
68	2.4751	.4876	.5002	.5129	.5257	.5386	.5517	.5649	.5782	.5916	22	43	65	87	108
69	2.6051	.6187	.6325	.6464	.6605	.6746	.6889	.7034	.7179	.7326	24	47	71	95	119
70	2.7475	.7625	.7776	.7929	.8083	.8239	.8397	.8556	.8716	.8878	26	52	78	104	130
71	2.9042	.9208	.9375	.9544	.9714	.9887	**.0061**	.0237	.0415	.0595	29	58	87	116	144
72	3.0777	.0961	.1146	.1334	.1524	.1716	.1910	.2106	.2305	.2506	32	64	96	129	161
73	3.2709	.2914	.3122	.3332	.3544	.3759	.3977	.4197	.4420	.4646	36	72	108	144	180
74	3,4874	.5105	.5339	.5576	.5816	.6059	.6305	.6554	.6806	.7062	41	81	122	163	204
75	3.7321	.7583	.7848	.8118	.8391	.8667	.8947	.9232	.9520	.9812					
76	4.0108	.0408	.0713	.1022	.1335	.1653	.1976	.2303	.2635	.2972					
77	4.3315	.3662	.4015	.4373	.4737	.5107	.5483	.5864	.6252	.6646					
78	4.7046	.7453	.7867	.8288	.8716	.9152	.9594	**.0045**	.0504	.0970	Use ordinary interpolation.				
79	5.1446	.1929	.2422	.2924	.3435	.3955	.4486	.5026	.5578	.6140					
80	5,6713	.7297	.7894	.8502	.9124	.9758	**.0405**	.1066	.1742	.2432					
81	6.3138	.3859	.4596	.5350	.6122	.6912	.7720	.8548	.9395	**.0264**					
82	7.1154	.2066	.3002	.3962	.4947	.5958	.6996	.8062	.9158	**.0285**					
83	8.1443	.2636	.3863	.5126	.6427	.7769	.9152	**.0579**	.2052	.3572					
84	9.5144	.6768	.8448	**.0187**	.1988	.3854	.5789	.7797	.9882	**.2048**					
85	11.430	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.301	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.081	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.636	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.290	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					

The integral part of tangents in heavy-face type is 1 greater than preceding part.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	— Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	∞	573.0	286.5	191.0	143.2	114.6	95.49	81.85	71.62	63.66					
1	57.290	52.08	47.74	44.07	40.92	38.19	35.80	33.69	31.82	30.14					
2	28.636	27.27	26.03	24.90	23.86	22.90	22.02	21.20	20.45	19.74					
3	19.081	18.46	17.89	17.34	16.83	16.35	15.89	15.46	15.06	14.67	Use ordinary interpolation.				
4	14.301	13.95	13.62	13.30	13.00	12.71	12.43	12.16	11.91	11.66					
5	11.430	.2048	**.9882**	.7797	.5789	.3854	.1988	.0187	**.8448**	.6768					
6	9.5144	.3572	.2052	.0579	**.9152**	.7769	.6427	.5126	.3863	.2636					
7	8.1443	.0285	**.9158**	.8062	.6996	.5958	.4947	.3962	.3002	.2066					
8	7.1154	.0264	**.9395**	.8548	.7720	.6912	.6122	.5350	.4596	.3859					
9	6.3138	.2432	.1742	.1066	.0405	**.9758**	.9124	.8502	.7894	.7297					
10	5.6713	.6140	.5578	.5026	.4486	.3955	.3435	.2924	.2422	.1929					
11	5.1446	.0970	.0504	.0045	**.9594**	.9152	.8716	.8288	.7867	.7453					
12	4.7046	.6646	.6252	.5864	.5483	.5107	.4737	.4373	.4015	.3662					
13	4.3315	.2972	.2635	.2303	.1976	.1653	.1335	.1022	.0713	.0408					
14	4.0108	**.9812**	.9520	.9232	.8947	.8667	.8391	.8118	.7848	.7583					
15	3.7321	.7062	.6806	.6554	.6305	.6059	.5816	.5576	.5339	.5105	41	81	122	163	203
16	3.4874	.4646	.4420	.4197	.3977	.3759	.3544	.3332	.3122	.2914	36	72	108	144	180
17	3.2709	.2506	.2305	.2106	.1910	.1716	.1524	.1334	.1146	.0961	32	64	96	129	161
18	3.0777	.0595	.0415	.0237	.0061	**.9887**	.9714	.9544	.9375	.9208	29	58	87	116	144
19	2.9042	.8878	.8716	.8556	.8397	.8239	.8083	.7929	.7776	.7625	26	52	78	104	130
20	2.7475	.7326	.7179	.7034	.6889	.6746	.6605	.6464	.6325	.6187	24	47	71	95	119
21	2.6051	.5916	.5782	.5649	.5517	.5386	.5257	.5129	.5002	.4876	22	43	65	87	108
22	2.4751	.4627	.4504	.4383	.4262	.4142	.4023	.3906	.3789	.3673	20	40	60	79	99
23	2.3559	.3445	.3332	.3220	.3109	.2998	.2889	.2781	.2673	.2566	18	37	55	73	92
24	2.2460	.2355	.2251	.2148	.2045	.1943	.1842	.1742	.1642	.1543	17	34	51	68	85
25	2.1445	.1348	.1251	.1155	.1060	.0965	.0872	.0778	.0686	.0594	16	31	47	63	78
26	2.0503	.0413	.0323	.0233	.0145	.0057	**.9970**	.9883	.9797	.9711	15	29	44	58	73
27	1.9626	.9542	.9458	.9375	.9292	.9210	.9128	.9047	.8967	.8887	14	27	41	55	68
28	1.8807	.8728	.8650	.8572	.8495	.8418	.8341	.8265	.8190	.8115	13	26	38	51	64
29	1.8040	.7966	.7893	.7820	.7747	.7675	.7603	.7532	.7461	.7391	12	24	36	48	60
30	1.7321	.7251	.7182	.7113	.7045	.6977	.6909	.6842	.6775	.6709	11	23	34	45	56
31	1.6643	.6577	.6512	.6447	.6383	.6319	.6255	.6191	.6128	.6066	11	21	32	43	53
32	1.6003	.5941	.5880	.5818	.5757	.5697	.5637	.5577	.5517	.5458	10	20	30	40	50
33	1.5399	.5340	.5282	.5224	.5166	.5108	.5051	.4994	.4938	.4882	10	19	29	38	48
34	1.4826	.4770	.4715	.4659	.4605	.4550	.4496	.4442	.4388	.4335	9	18	27	36	45
35	1.4281	.4229	.4176	.4124	.4071	.4019	.3968	.3916	.3865	.3814	9	17	26	34	43
36	1.3764	.3713	.3663	.3613	.3564	.3514	.3465	.3416	.3367	.3319	8	16	25	33	41
37	1.3270	.3222	.3175	.3127	.3079	.3032	.2985	.2938	.2892	.2846	8	16	24	31	39
38	1.2799	.2753	.2708	.2662	.2617	.2572	.2527	.2482	.2437	.2393	8	15	23	30	38
39	1.2349	.2305	.2261	.2218	.2174	.2131	.2088	.2045	.2002	.1960	7	14	22	29	36
40	1.1918	.1875	.1833	.1792	.1750	.1708	.1667	.1626	.1585	.1544	7	14	21	28	34
41	1.1504	.1463	.1423	.1383	.1343	.1303	.1263	.1224	.1184	.1145	7	13	20	26	33
42	1.1106	.1067	.1028	.0990	.0951	.0913	.0875	.0837	.0799	.0761	6	13	19	25	32
43	1.0724	.0686	.0649	.0612	.0575	.0538	.0501	.0464	.0428	.0392	6	12	18	25	31
44	1.0355	.0319	.0283	.0247	.0212	.0176	.0141	.0105	.0070	.0035	6	12	18	24	30

The integral part of cotangents in heavy-face type is 1 less than preceding part.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	– Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	1.0000	**.9965**	.9930	.9896	.9861	.9827	.9793	.9759	.9725	.9691	6	11	17	23	29
46	0.9657	.9623	.9590	.9556	.9523	.9490	.9457	.9424	.9391	.9358	6	11	17	22	28
47	0.9325	.9293	.9260	.9228	.9195	.9163	.9131	.9099	.9067	.9036	5	11	16	21	27
48	0.9004	.8972	.8941	.8910	.8878	.8847	.8816	.8785	.8754	.8724	5	10	16	21	26
49	0.8693	.8662	.8632	.8601	.8571	.8541	.8511	.8481	.8451	.8421	5	10	15	20	25
50	0.8391	.8361	.8332	.8302	.8273	.8243	.8214	.8185	.8156	.8127	5	10	15	20	24
51	0.8098	.8069	.8040	.8012	.7983	.7954	.7926	.7898	.7869	.7841	5	9	14	19	24
52	0.7813	.7785	.7757	.7729	.7701	.7673	.7646	.7618	.7590	.7563	5	9	14	18	23
53	0.7536	.7508	.7481	.7454	.7427	.7400	.7373	.7346	.7319	.7292	5	9	14	18	23
54	0.7265	.7239	.7212	.7186	.7159	.7133	.7107	.7080	.7054	.7028	4	9	13	18	22
55	0.7002	.6976	.6950	.6924	.6899	.6873	.6847	.6822	.6796	.6771	4	9	13	17	21
56	0.6745	.6720	.6694	.6669	.6644	.6619	.6594	.6569	.6544	.6519	4	8	13	17	21
57	0.6494	.6469	.6445	.6420	.6395	.6371	.6346	.6322	.6297	.6273	4	8	12	16	20
58	0.6249	.6224	.6200	.6176	.6152	.6128	.6104	.6080	.6056	.6032	4	8	12	16	20
59	0.6009	.5985	.5961	.5938	.5914	.5890	.5867	.5844	.5820	.5797	4	8	12	16	20
60	0.5774	.5750	.5727	.5704	.5681	.5658	.5635	.5612	.5589	.5566	4	8	12	15	19
61	0.5543	.5520	.5498	.5475	.5452	.5430	.5407	.5384	.5362	.5340	4	8	11	15	19
62	0.5317	.5295	.5272	.5250	.5228	.5206	.5184	.5161	.5139	.5117	4	7	11	15	18
63	0.5095	.5073	.5051	.5029	.5008	.4986	.4964	.4942	.4921	.4899	4	7	11	15	18
64	0.4877	.4856	.4834	.4813	.4791	.4770	.4748	.4727	.4706	.4684	4	7	11	14	18
65	0.4663	.4642	.4621	.4599	.4578	.4557	.4536	.4515	.4494	.4473	4	7	11	14	18
66	0.4452	.4431	.4411	.4390	.4369	.4348	.4327	.4307	.4286	.4265	3	7	10	14	17
67	0.4245	.4224	.4204	.4183	.4163	.4142	.4122	.4101	.4081	.4061	3	7	10	14	17
68	0.4040	.4020	.4000	.3979	.3959	.3939	.3919	.3899	.3879	.3859	3	7	10	13	17
69	0.3839	.3819	.3799	.3779	.3759	.3739	.3719	.3699	.3679	.3659	3	7	10	13	17
70	0.3640	.3620	.3600	.3581	.3561	.3541	.3522	.3502	.3482	.3463	3	7	10	13	16
71	0.3443	.3424	.3404	.3385	.3365	.3346	.3327	.3307	.3288	.3269	3	6	10	13	16
72	0.3249	.3230	.3211	.3191	.3172	.3153	.3134	.3115	.3096	.3076	3	6	10	13	16
73	0.3057	.3038	.3019	.3000	.2981	.2962	.2943	.2924	.2905	.2886	3	6	9	13	16
74	0.2867	.2849	.2830	.2811	.2792	.2773	.2754	.2736	.2717	.2698	3	6	9	13	16
75	0.2679	.2661	.2642	.2623	.2605	.2586	.2568	.2549	.2530	.2512	3	6	9	12	16
76	0.2493	.2475	.2456	.2438	.2419	.2401	.2382	.2364	.2345	.2327	3	6	9	12	15
77	0.2309	.2290	.2272	.2254	.2235	.2217	.2199	.2180	.2162	.2144	3	6	9	12	15
78	0.2126	.2107	.2089	.2071	.2053	.2035	.2016	.1998	.1980	.1962	3	6	9	12	15
79	0.1944	.1926	.1908	.1890	.1871	.1853	.1835	.1817	.1799	.1781	3	6	9	12	15
80	0.1763	.1745	.1727	.1709	.1691	.1673	.1655	.1638	.1620	.1602	3	6	9	12	15
81	0.1584	.1566	.1548	.1530	.1512	.1495	.1477	.1459	.1441	.1423	3	6	9	12	15
82	0.1405	.1388	.1370	.1352	.1334	.1317	.1299	.1281	.1263	.1246	3	6	9	12	15
83	0.1228	.1210	.1192	.1175	.1157	.1139	.1122	.1104	.1086	.1069	3	6	9	12	15
84	0.1051	.1033	.1016	.0998	.0981	.0963	.0945	.0928	.0910	.0892	3	6	9	12	15
85	0.0875	.0857	.0840	.0822	.0805	.0787	.0769	.0752	.0734	.0717	3	6	9	12	15
86	0.0699	.0682	.0664	.0647	.0629	.0612	.0594	.0577	.0559	.0542	3	6	9	12	15
87	0.0524	.0507	.0489	.0472	.0454	.0437	.0419	.0402	.0384	.0367	3	6	9	12	15
88	0.0349	.0332	.0314	.0297	.0279	.0262	.0244	.0227	.0209	.0192	3	6	9	12	15
89	0.0175	.0157	.0140	.0122	.0105	.0087	.0070	.0052	.0035	.0017	3	6	9	12	15

All cotangents greater than cot 45° are less than 1.

POWERS AND ROOTS

No.	Squares	Cubes	Square Roots	Cube Roots	No.	Squares	Cubes	Square Roots	Cube Roots
1	1	1	1.000	1.000	51	2 601	132 651	7.141	3.708
2	4	8	1.414	1.260	52	2 704	140 608	7.211	3.733
3	9	27	1.732	1.442	53	2 809	148 877	7.280	3.756
4	16	64	2.000	1.587	54	2 916	157 464	7.348	3.780
5	25	125	2.236	1.710	55	3 025	166 375	7.416	3.803
6	36	216	2.449	1.817	56	3 136	175 616	7.483	3.826
7	49	343	2.646	1.913	57	3 249	185 193	7.550	3.849
8	64	512	2.828	2.000	58	3 364	195 112	7.616	3.871
9	81	729	3.000	2.080	59	3 481	205 379	7.681	3.893
10	100	1 000	3.162	2.154	60	3 600	216 000	7.746	3.915
11	121	1 331	3.317	2.224	61	3 721	226 981	7.810	3.936
12	144	1 728	3.464	2.289	62	3 844	238 328	7.874	3.958
13	169	2 197	3.606	2.351	63	3 969	250 047	7.937	3.979
14	196	2 744	3.742	2.410	64	4 096	262 144	8.000	4.000
15	225	3 375	3.873	2.466	65	4 225	274 625	8.062	4.021
16	256	4 096	4.000	2.520	66	4 356	287 496	8.124	4.041
17	289	4 913	4.123	2.571	67	4 489	300 763	8.185	4.062
18	324	5 832	4.243	2.621	68	4 624	314 432	8.246	4.082
19	361	6 859	4.359	2.668	69	4 761	328 509	8.307	4.102
20	400	8 000	4.472	2.714	70	4 900	343 000	8.367	4.121
21	441	9 261	4.583	2.759	71	5 041	357 911	8.426	4.141
22	484	10 648	4.690	2.802	72	5 184	373 248	8.485	4.160
23	529	12 167	4.796	2.844	73	5 329	389 017	8.544	4.179
24	576	13 824	4.899	2.884	74	5 476	405 224	8.602	4.198
25	625	15 625	5.000	2.924	75	5 625	421 875	8.660	4.217
26	676	17 576	5.099	2.962	76	5 776	438 976	8.718	4.236
27	729	19 683	5.196	3.000	77	5 929	456 533	8.775	4.254
28	784	21 952	5.292	3.037	78	6 084	474 552	8.832	4.273
29	841	24 389	5.385	3.072	79	6 241	493 039	8.888	4.291
30	900	27 000	5.477	3.107	80	6 400	512 000	8.944	4.309
31	961	29 791	5.568	3.141	81	6 561	531 441	9.000	4.327
32	1 024	32 768	5.657	3.175	82	6 724	551 368	9.055	4.344
33	1 089	35 937	5.745	3.208	83	6 889	571 787	9.110	4.362
34	1 156	39 304	5.831	3.240	84	7 056	592 704	9.165	4.380
35	1 225	42 875	5.916	3.271	85	7 225	614 125	9.220	4.397
36	1 296	46 656	6.000	3.302	86	7 396	636 056	9.274	4.414
37	1 369	50 653	6.083	3.332	87	7 569	658 503	9.327	4.431
38	1 444	54 872	6.164	3.362	88	7 744	681 472	9.381	4.448
39	1 521	59 319	6.245	3.391	89	7 921	704 969	9.434	4.465
40	1 600	64 000	6.325	3.420	90	8 100	729 000	9.487	4.481
41	1 681	68 921	6.403	3.448	91	8 281	753 571	9.539	4.498
42	1 764	74 088	6.481	3.476	92	8 464	778 688	9.592	4.514
43	1 849	79 507	6.557	3.503	93	8 649	804 357	9.644	4.531
44	1 936	85 184	6.633	3.530	94	8 836	830 584	9.695	4.547
45	2 025	91 125	6.708	3.557	95	9 025	857 375	9.747	4.563
46	2 116	97 336	6.782	3.583	96	9 216	884 736	9.798	4.579
47	2 209	103 823	6.856	3.609	97	9 409	912 673	9.849	4.595
48	2 304	110 592	6.928	3.634	98	9 604	941 192	9.899	4.610
49	2 401	117 649	7.000	3.659	99	9 801	970 299	9.950	4.626
50	2 500	125 000	7.071	3.684	100	10 000	1 000 000	10.000	4.642

SIZES OF TWIST DRILLS WITH DECIMAL EQUIVALENTS

Size	Decimal Equivalent	Size	Decimal Equivalent	Size	Decimal Equivalent	Size	Decimal Equivalent
$\frac{1}{2}''$	0.5000″	$\frac{1}{4}''$	0.2500″	#26	0.1470″	#56	0.0465″
$\frac{31}{64}''$	.4844	E	.2500	#27	.1440	#57	.0430
$\frac{15}{32}''$	.4688	D	.2460	$\frac{9}{64}''$	.1406	#58	.0420
$\frac{29}{64}''$	.4531	C	.2420	#28	.1405	#59	.0410
$\frac{7}{16}''$	.4375	B	.2380	#29	.1360	#60	.0400
$\frac{27}{64}''$	.4219	$\frac{15}{64}''$	.2344	#30	.1285	#60½	.0390
Z	.4130	A	.2340	$\frac{1}{8}''$	.1250	#61	.0380
$\frac{13}{32}''$	.4063	#1	.2280	#31	.1200	#62	.0370
Y	.4040	#2	.2210	#32	.1160	#63	.0360
X	.3970	$\frac{7}{32}''$	.2188	#33	.1130	#64	.0350
$\frac{25}{64}''$	.3906	#3	.2130	#34	.1110	#65	.0330
W	.3860	#4	.2090	#35	.1100	#66	.0320
V	.3770	#5	.2055	$\frac{7}{64}''$	.1094	$\frac{1}{32}''$	.0313
$\frac{3}{8}''$	.3750	#6	.2040	#36	.1065	#67	.0310
U	.3680	$\frac{13}{64}''$	.2031	#37	.1040	#68	.0300
$\frac{23}{64}''$	.3594	#7	.2010	#38	.1015	#68½	.0295
T	.3580	#8	.1990	#39	.0995	#69	.0290
S	.3480	#9	.1960	#40	.0980	#69½	.0280
$\frac{11}{32}''$	.3438	#10	.1935	#41	.0960	#70	.0270
R	.3390	#11	.1910	$\frac{3}{32}''$	.0938	#71	.0260
Q	.3320	#12	.1890	#42	.0935	#71½	.0250
$\frac{21}{64}''$	.3281	$\frac{3}{16}''$	.1875	#43	.0890	#72	.0240
P	.3230	#13	.1850	#44	.0860	#73	.0230
O	.3160	#14	.1820	#45	.0820	#73½	.0225
$\frac{5}{16}''$	.3125	#15	.1800	#46	.0810	#74	.0220
N	.3020	#16	.1770	#47	.0785	#74½	.0210
$\frac{19}{64}''$	.2969	#17	.1730	$\frac{5}{64}''$	.0781	#75	.0200
M	.2950	$\frac{11}{64}''$	.1719	#48	.0760	#76	.0180
L	.2900	#18	.1695	#49	.0730	#77	.0160
$\frac{9}{32}''$	.2813	#19	.1660	#50	.0700	$\frac{1}{64}''$	.0156
K	.2810	#20	.1610	#51	.0670	#78	.0150
J	.2770	#21	.1590	#52	.0635	#78½	.0145
I	.2720	#22	.1570	$\frac{1}{16}''$	.0625	#79	.0140
H	.2660	$\frac{5}{32}''$	.1563	#53	.0595	#79½	.0135
$\frac{17}{64}''$	.2656	#23	.1540	#54	.0550	#80	.0130
G	.2610	#24	.1520	#55	.0520	. .	. . .
F	.2570	#25	.1495	$\frac{3}{64}''$	.0469	. .	. . .

SPEEDS OF DRILLS

Diameter of Drill	Wrought Iron and Steel	Cast Iron	Brass	Diameter of Drill	Wrought Iron and Steel	Cast Iron	Brass
$\frac{1}{16}''$	1712	2383	3544	$1\frac{1}{16}''$	72	108	180
$\frac{1}{8}$	855	1191	1772	$1\frac{1}{8}$	68	102	170
$\frac{3}{16}$	571	794	1181	$1\frac{3}{16}$	64	97	161
$\frac{1}{4}$	397	565	855	$1\frac{1}{4}$	58	89	150
$\frac{5}{16}$	318	452	684	$1\frac{5}{16}$	55	84	143
$\frac{3}{8}$	265	377	570	$1\frac{3}{8}$	53	81	136
$\frac{7}{16}$	227	323	489	$1\frac{7}{16}$	50	77	130
$\frac{1}{2}$	183	267	412	$1\frac{1}{2}$	46	74	122
$\frac{9}{16}$	163	238	367	$1\frac{9}{16}$	44	71	117
$\frac{5}{8}$	147	214	330	$1\frac{5}{8}$	40	66	113
$\frac{11}{16}$	133	194	300	$1\frac{11}{16}$	38	63	109
$\frac{3}{4}$	112	168	265	$1\frac{3}{4}$	37	61	105
$\frac{13}{16}$	103	155	244	$1\frac{13}{16}$	36	59	101
$\frac{7}{8}$	96	144	227	$1\frac{7}{8}$	33	55	98
$\frac{15}{16}$	89	134	212	$1\frac{15}{16}$	32	53	95
1	76	115	191	2	31	51	92

The speeds in the above table are given in R. P. M.

MACHINE SCREW THREADS, OLD STANDARD SIZES

Number of Screw	Outside Diameter	Threads to 1″	Number of Screw	Outside Diameter	Threads to 1″
1	0.071″	64	12	0.221″	24
$1\frac{1}{2}$	.081	56	13	.234	22
2	.089	56	14	.246	20
3	.101	48	15	.261	20
4	.113	36	16	.272	18
5	.125	36	18	.298	18
6	.141	32	20	.325	16
7	.154	32	22	.350	16
8	.166	32	24	.378	16
9	.180	30	26	.404	16
10	.194	24	28	.430	14
11	.206	24	30	.456	14

The basic form of this thread is the same as that of a U.S.S. thread.

MACHINE SCREW THREADS, A.S.M.E. STANDARD SIZES

Number of Screw	Maximum Diameter	Threads to 1″	Number of Screw	Maximum Diameter	Threads to 1″
0	0.060″	80	12	0.216″	28
1	.073	72	14	.242	24
2	.086	64	16	.268	22
3	.099	56	18	.294	20
4	.112	48	20	.320	20
5	.125	44	22	.346	18
6	.138	40	24	.372	16
7	.151	36	26	.398	16
8	.164	36	28	.424	14
9	.177	32	30	.450	14
10	.190	30	. .	. .	. .

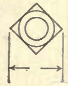
MACHINE SCREW THREADS, A.S.M.E. SPECIAL SIZES

Number of Screw	Maximum Diameter	Threads to 1″	Number of Screw	Maximum Diameter	Threads to 1″
1	0.073″	64	9	0.177″	24
2	.086	56	10	.190	32
3	.099	48	10	.190	24
4	.112	40	12	.216	24
4	.112	36	14	.242	20
5	.125	40	16	.268	20
5	.125	36	18	.294	18
6	.138	36	20	.320	18
6	.138	32	22	.346	16
7	.151	32	24	.372	18
7	.151	30	26	.398	14
8	.164	32	28	.424	16
8	.164	30	30	.450	16
9	.177	30	. .	. .	. .

The A.S.M.E. (American Society of Mechanical Engineers) standard specifies a maximum and a minimum outside diameter. The minimum diameter (not given in these tables) may be found by the formula

$$\text{Minimum } D_o = \text{maximum } D_o - \frac{0.336}{40 + \text{number of threads to } 1''}.$$

U.S.S.-THREAD BOLTS AND NUTS

Outside Diameter	Threads to 1″	Root Diameter	Diameter of Tap Drill	Area in Square Inches		Dimensions of Nuts and Bolt Heads				
				Of Bolt	At Root of Thread					
1/4″	20	0.185″	13/64″	0.049	0.026	1/2″	0.578″	0.707″	1/4″	1/4″
5/16	18	0.240	1/4	0.076	0.045	19/32	0.686	0.840	5/16	19/64
3/8	16	0.294	5/16	0.110	0.068	11/16	0.794	0.972	3/8	11/32
7/16	14	0.345	23/64	0.150	0.093	25/32	0.902	1.105	7/16	25/64
1/2	13	0.400	27/64	0.196	0.126	7/8	1.011	1.237	1/2	7/16
9/16	12	0.454	15/32	0.248	0.162	31/32	1.119	1.370	9/16	31/64
5/8	11	0.507	17/32	0.307	0.202	1 1/16	1.227	1.502	5/8	17/32
3/4	10	0.620	41/64	0.442	0.302	1 1/4	1.444	1.768	3/4	5/8
7/8	9	0.731	3/4	0.601	0.419	1 7/16	1.660	2.033	7/8	23/32
1	8	0.838	55/64	0.785	0.551	1 5/8	1.877	2.298	1	13/16
1 1/8	7	0.939	31/32	0.994	0.694	1 13/16	2.093	2.563	1 1/8	29/32
1 1/4	7	1.064	1 3/32	1.227	0.893	2	2.310	2.828	1 1/4	1
1 3/8	6	1.158	1 7/32	1.485	1.057	2 3/16	2.527	3.093	1 3/8	1 3/32
1 1/2	6	1.283	1 11/32	1.767	1.295	2 3/8	2.743	3.358	1 1/2	1 3/16
1 5/8	5 1/2	1.389	1 27/64	2.074	1.515	2 9/16	2.960	3.623	1 5/8	1 9/32
1 3/4	5	1.490	1 17/32	2.405	1.746	2 3/4	3.176	3.889	1 3/4	1 3/8
1 7/8	5	1.615	1 21/32	2.761	2.051	2 15/16	3.393	4.154	1 7/8	1 15/32
2	4 1/2	1.711	1 49/64	3.142	2.302	3 1/8	3.609	4.419	2	1 9/16
2 1/4	4 1/2	1.961	2 1/64	3.976	3.023	3 1/2	4.043	4.949	2 1/4	1 3/4
2 1/2	4	2.175	2 15/64	4.909	3.719	3 7/8	4.476	5.479	2 1/2	1 15/16
2 3/4	4	2.425	2 31/64	5.940	4.620	4 1/4	4.909	6.010	2 3/4	2 1/8
3	3 1/2	2.629	2 11/16	7.069	5.428	4 5/8	5.342	6.540	3	2 5/16
3 1/4	3 1/2	2.879	2 15/16	8,296	6.510	5	5.775	7.070	3 1/4	2 1/2
3 1/2	3 1/4	3.100	3 11/64	9.621	7.548	5 3/8	6.208	7.600	3 1/2	2 11/16
3 3/4	3	3.317	3 3/8	11.045	8.641	5 3/4	6.641	8.131	3 3/4	2 7/8
4	3	3.567	3 5/8	12.566	9.963	6 1/8	7.074	8.661	4	3 1/16
4 1/4	2 7/8	3.798	3 27/32	14.186	11.340	6 1/2	7.508	9.191	4 1/4	3 1/4
4 1/2	2 3/4	4.028	4 3/32	15.904	12.750	6 7/8	7.941	9.721	4 1/2	3 7/16
4 3/4	2 5/8	4.255	4 5/16	17.721	14.215	7 1/4	8.374	10.252	4 3/4	3 5/8
5	2 1/2	4.480	4 9/16	19.635	15.760	7 5/8	8.807	10.782	5	3 13/16
5 1/4	2 1/2	4.730	4 13/16	21.648	17.570	8	9.240	11.312	5 3/4	4
5 1/2	2 3/8	4.953	5 1/32	23.758	19.260	8 3/8	9.673	11.842	5 1/2	4 3/16
5 3/4	2 3/8	5.203	5 9/32	25.967	21.250	8 3/4	10.106	12.373	5 3/4	4 3/8
6	2 1/4	5.423	5 1/2	28.274	23.090	9 1/8	10.539	12.903	6	4 9/16

TAP DRILLS FOR MACHINE-SCREW TAPS

Number of Tap	Threads to 1″	Number of Drill
2	48	48
2	56	46
2	64	45
3	40	48
3	48	47
3	56	45
4	32	45
4	36	43
4	40	42
5	30	41
5	32	40
5	36	38
5	40	36
6	30	39
6	32	37
6	36	35
6	40	33
7	28	32
7	30	31
7	32	30
8	24	31
8	30	30
8	32	29
9	24	29
9	28	27
9	30	26
9	32	24
10	24	26
10	28	24
10	30	23
10	32	21
11	24	20
11	28	19
11	30	18
12	20	21
12	22	19
12	24	19
13	20	17
13	24	15
14	20	14
14	22	13
14	24	11
15	18	12
15	20	10
15	24	7
16	16	10
16	18	7
16	20	5
16	24	1
17	16	7
17	18	4
17	20	2
18	16	2
18	18	1
18	20	B
19	16	C
19	18	D
19	20	E
20	16	E
20	18	E
20	20	F
22	16	H
22	18	I
24	14	K
24	16	L
24	18	M
26	14	O
26	16	P
28	14	R
28	16	S
30	14	T
30	16	U

TAP DRILLS FOR U.S.S. THREADS

Thread Diameter	Threads to 1″	Diameter of Drill	Thread Diameter	Threads to 1″	Diameter of Drill
$\frac{1}{4}$″	20	0.191″	1″	8	0.854″
$\frac{5}{16}$	18	.248	$1\frac{1}{8}$	7	0.957
$\frac{3}{8}$	16	.302	$1\frac{1}{4}$	7	1.082
$\frac{7}{16}$	14	.354	$1\frac{3}{8}$	6	1.179
$\frac{1}{2}$	13	.409	$1\frac{1}{2}$	6	1.304
$\frac{9}{16}$	12	.465	$1\frac{5}{8}$	$5\frac{1}{2}$	1.412
$\frac{5}{8}$	11	.518	$1\frac{3}{4}$	5	1.515
$\frac{3}{4}$	10	.632	$1\frac{7}{8}$	5	1.640
$\frac{7}{8}$	9	.745	2	$4\frac{1}{2}$	1.739

TAP DRILLS FOR SHARP V-THREADS

Thread Diameter	Threads to 1″	Diameter of Drill	Thread Diameter	Threads to 1″	Diameter of Drill
$\frac{1}{4}$″	20	0.184″	1	8	0.832″
$\frac{5}{16}$	18	.239	$1\frac{1}{8}$	7	0.932
$\frac{3}{8}$	16	.293	$1\frac{1}{4}$	7	1.057
$\frac{7}{16}$	14	.345	$1\frac{3}{8}$	6	1.144
$\frac{1}{2}$	12	.399	$1\frac{1}{2}$	6	1.269
$\frac{9}{16}$	12	.453	$1\frac{5}{8}$	5	1.347
$\frac{5}{8}$	11	.506	$1\frac{3}{4}$	5	1.472
$\frac{3}{4}$	10	.618	$1\frac{7}{8}$	$4\frac{1}{2}$	1.566
$\frac{7}{8}$	9	.728	2	$4\frac{1}{2}$	1.691

TAP DRILLS FOR BRIGGS PIPE THREADS

Nominal Size	Threads to 1″	Diameter of Drill	Nominal Size	Threads to 1″	Diameter of Drill
$\frac{1}{8}$″	27	0.328″	$1\frac{1}{2}$″	$11\frac{1}{2}$	1.719″
$\frac{1}{4}$	18	.453	2	$11\frac{1}{2}$	2.188
$\frac{3}{8}$	18	.594	$2\frac{1}{2}$	8	2.688
$\frac{1}{2}$	14	.719	3	8	3.313
$\frac{3}{4}$	14	.938	$3\frac{1}{2}$	8	3.813
1	$11\frac{1}{2}$	1.188	4	8	4.313
$1\frac{1}{4}$	$11\frac{1}{2}$	1.469	$4\frac{1}{2}$	8	4.813

The diameter of drill allows for reaming the hole before tapping.

TOOTH PARTS OF DIAMETRAL–PITCH GEARS

Diametral Pitch	Circular Pitch	Thickness of Tooth on Pitch Line	Addendum	Working Depth of Tooth	Root of Tooth	Whole Depth of Tooth
$\frac{1}{2}$	6.2832″	3.1416″	2.0000″	4.0000″	2.3142″	4.3142″
$\frac{3}{4}$	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
$1\frac{1}{4}$	2.5133	1.2566	0.8000	1.6000	0.9257	1.7257
$1\frac{1}{2}$	2.0944	1.0472	.6666	1.3333	.7714	1.4381
$1\frac{3}{4}$	1.7952	0.8976	.5714	1.1429	.6612	1.2326
2	1.5708	.7854	.5000	1.0000	.5785	1.0785
$2\frac{1}{4}$	1.3963	.6981	.4444	0.8888	.5143	0.9587
$2\frac{1}{2}$	1.2566	.6283	.4000	.8000	.4628	.8628
$2\frac{3}{4}$	1.1424	.5712	.3636	.7273	.4208	.7844
3	1.0472	.5236	.3333	.6666	.3857	.7190
$3\frac{1}{2}$	0.8976	.4488	.2857	.5714	.3306	.6163
4	.7854	.3927	.2500	.5000	.2893	.5393
5	.6283	.3142	.2000	.4000	.2314	.4314
6	.5236	.2618	.1666	.3333	.1928	.3595
7	.4488	.2244	.1429	.2857	.1653	.3081
8	.3927	.1963	.1250	.2500	.1446	.2696
9	.3491	.1745	.1111	.2222	.1286	.2397
10	.3142	.1571	.1000	.2000	.1157	.2157
12	.2618	.1309	.0833	.1666	.0964	.1798
14	.2244	.1122	.0714	.1429	.0826	.1541
16	.1963	.0982	.0625	.1250	.0723	.1348
18	.1745	.0873	.0555	.1111	.0643	.1198
20	.1571	.0785	.0500	.1000	.0579	.1079
22	.1428	.0714	.0455	.0909	.0526	.0980
24	.1309	.0654	.0417	.0833	.0482	.0898
26	.1208	.0604	.0385	.0769	.0445	.0829
28	.1122	.0561	.0357	.0714	.0413	.0770
30	.1047	.0524	.0333	.0666	.0386	.0719
32	.0982	.0491	.0312	.0625	.0362	.0674
36	.0873	.0436	.0278	.0555	.0321	.0599
40	.0785	.0393	.0250	.0500	.0289	.0539
44	.0714	.0357	.0227	.0455	.0263	.0490
48	.0654	.0327	.0208	.0417	.0241	.0449
50	.0628	.0314	.0200	.0400	.0231	.0431

TOOTH PARTS OF CIRCULAR-PITCH GEARS

Circular Pitch	Diametral Pitch	Thickness of Tooth on Pitch Line	Addendum	Working Depth of Tooth	Root of Tooth	Whole Depth of Tooth
4″	0.7854	2.0000″	1.2732″	2.5464″	1.4732″	2.7464″
$3\frac{1}{2}$	0.8976	1.7500	1.1140	2.2281	1.2890	2.4031
3	1.0472	1.5000	0.9549	1.9098	1.1049	2.0598
$2\frac{3}{4}$	1.1424	1.3750	.8753	1.7506	1.0128	1.8881
$2\frac{1}{2}$	1.2566	1.2500	.7957	1.5915	0.9207	1.7165
$2\frac{1}{4}$	1.3963	1.1250	.7162	1.4323	.8287	1.5448
2	1.5708	1.0000	.6366	1.2732	.7366	1.3732
$1\frac{7}{8}$	1.6755	0.9375	.5968	1.1937	.6906	1.2874
$1\frac{3}{4}$	1.7952	.8750	.5570	1.1141	.6445	1.2016
$1\frac{5}{8}$	1.9333	.8125	.5173	1.0345	.5985	1.1158
$1\frac{1}{2}$	2.0944	.7500	.4775	0.9549	.5525	1.0299
$1\frac{7}{16}$	2.1855	.7187	.4576	.9151	.5294	0.9870
$1\frac{3}{8}$	2.2848	.6875	.4377	.8754	.5064	.9441
$1\frac{5}{16}$	2.3936	.6562	.4178	.8356	.4834	.9012
$1\frac{1}{4}$	2.5133	.6250	.3979	.7958	.4604	.8583
$1\frac{3}{16}$	2.6456	.5937	.3780	.7560	.4374	.8154
$1\frac{1}{8}$	2.7925	.5625	.3581	.7162	.4143	.7724
$1\frac{1}{16}$	2.9568	.5312	.3382	.6764	.3913	.7295
1	3.1416	.5000	.3183	.6366	.3683	.6866
$\frac{15}{16}$	3.3510	.4687	.2984	.5968	.3453	.6437
$\frac{7}{8}$	3.5904	.4375	.2785	.5570	.3223	.6007
$\frac{13}{16}$	3.8666	.4062	.2586	.5173	.2993	.5579
$\frac{3}{4}$	4.1888	.3750	.2387	.4775	.2762	.5150
$\frac{11}{16}$	4.5696	.3437	.2189	.4377	.2532	.4720
$\frac{5}{8}$	5.0265	.3125	.1989	.3979	.2301	.4291
$\frac{9}{16}$	5.5851	.2812	.1790	.3581	.2071	.3862
$\frac{1}{2}$	6.2832	.2500	.1592	.3183	.1842	.3433
$\frac{7}{16}$	7.1808	.2187	.1393	.2785	.1611	.3003
$\frac{3}{8}$	8.3776	.1875	.1194	.2387	.1381	.2575
$\frac{1}{3}$	9.4248	.1666	.1061	.2122	.1228	.2289
$\frac{5}{16}$	10.0531	.1562	.0995	.1989	.1151	.2146
$\frac{1}{4}$	12.5664	.1250	.0796	.1591	.0921	.1716
$\frac{3}{16}$	16.7552	.0937	.0597	.1194	.0690	.1287
$\frac{1}{8}$	25.1327	.0625	.0398	.0796	.0460	.0858
$\frac{1}{16}$	50.2655	.0312	.0199	.0398	.0230	.0429

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