Machinists' & Draftsmen's Handbook

MACHINISTS'
AND
DRAFTSMEN'S
HANDBOOK:

CONTAINING TABLES, RULES AND FORMULAS,

WITH

NUMEROUS EXAMPLES EXPLAINING THE PRINCIPLES OF MATHEMATICS AND MECHANICS AS APPLIED TO THE MECHANICAL TRADES

INTENDED AS A

REFERENCE BOOK FOR ALL

INTERESTED IN MECHANICAL WORK.

BY

PEDER LOBBEN,
MECHANICAL ENGINEER.

Member of the American Society of Mechanical Engineers and Worcester County Mechanics Association. Honorary Member of the N. A. Stationary Engineers. Non-Resident Member of the Franklin Institute.

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PREFACE.

It is the author's hope and desire that this book, which is the outcome of years of study, work and observation, may be a help to the class of people to which he himself has the honor to belong,—the working mechanics of the world.

This is not intended solely as a reference book, but it may also be studied advantageously by the ambitious young engineer and machinist; and, therefore, as far as believed practical within the scope of the work, the fundamental principles upon which the rules and formulas rest are given and explained.

The use of abstruse theories and complicated formulas is avoided, as it is thought preferable to sacrifice scientific hair-splitting and be satisfied with rules and formulas which will give intelligent approximations within practical limits, rather than to go into intricate and complicated formulas which can hardly be handled except by mathematical and mechanical experts.

In practical work everyone knows it is far more important to understand the correct principles and requirements of the job in hand than to be able to make elaborate scientific demonstrations of the subject; in short, it is only results which count in the commercial world, and every young mechanic must remember that few employers will pay for science only. What they want is practical science. Should, therefore, scientific men, (for whom the author has the greatest respect, as it is to the scientific investigators that the working mechanics are indebted for their progress in utilizing the forces of nature),—find nothing of interest in the book, they will kindly remember that the author does not pretend it to be of scientific interest, and they will therefore, in criticizing both the book and the author, remember that the work was not written with the desire to show the reader how vulgarly or how scientifically he could handle the subject, but with the sole desire to promote and assist the ambitious young working mechanic in the world's march of progress.

P. LOBBEN.

NEW YORK, *October*, 1899.

Notes on Mathematics.

A Unit is any quantity represented by a single thing, as a magnitude, or a number regarded as one undivided whole.

Numbers are the measure of the relation between quantities of things of the same kind and are expressed by figures.

Numbers which are capable of being divided by two without a remainder are called *even numbers*. 2, 4, 6, 8, etc., are even numbers.

Numbers which are not capable of division by two without giving a remainder are called *odd numbers*. 1, 3, 5, 7, 9, etc., are odd numbers.

A number which can not be divided by any whole number but itself and the number 1 without giving a remainder is called a *prime number*. 1, 2, 3, 5, 7, 11, 13, 17, 19, etc , are prime numbers.

All numbers that are not prime are said to be *composite numbers*, because they are composed of two or more factors; 4, 6, 8, 9, 10, 12, etc., are composite numbers.

Whole numbers are called *integers*. Whole numbers are also called *integral numbers*.

A mixed number is the sum of a whole number and a fraction.

The least common multiple of several given numbers is the smallest number that can be divided by each without a remainder. For instance, the least common multiple of 3, 4, 6, and 5 is 60, because 60 is the smallest number that can be divided by those numbers without a remainder.

Signs.

$+$ (plus) is the sign of addition.

$-$ (minus or less) is the sign of subtraction.

The signs $+$ and $-$ are also used to indicate positive and negative quantities.

$\times$ (times or multiply) is the sign of multiplication, but instead of this sign, sometimes a single point (.) is used, especially in formulas; in algebraic expressions very frequently factors are written without any signs at all between them. For instance, $a \times b$ or $a.b$ or ab. All these three expressions indicate that the quantity a is to be multiplied by the quantity b.

$\div$ (divided by) is the sign of division.

$=$ (equal). When this sign is placed between two quantities, it indicates that they are of equal value. For instance:

$$4+5+2=11$$
$$8-3+6-2=9$$
$$8\times 12=96$$
$$100\div 5=20$$

. (decimal point) signifies that the number written after it has some power of 10 for its denominator.

$^\circ$ $'$ $''$ means degrees, minutes and seconds of an angle.

$'$ $''$ means feet and inches.

a' a'' a''' reads a prime, a second, a third.

a_1 a_2 a_3 reads a sub 1, a sub 2, a sub 3, and is always used to designate corresponding values of the same element.

$\sqrt[n]{}$ This is the radical sign and signifies that a root is to be extracted of the quantity coming under the sign; this may be square root, cube root, or any other root, according to what there is signified by the number prefixed in place of the letter ^{n}. For instance: $\sqrt{}$ reads square root, $\sqrt[3]{}$ reads cube root, $\sqrt[4]{}$ reads fourth root, $\sqrt[5]{}$ reads fifth root, $\sqrt{64}=8$, because $8\times 8=64$

$\sqrt[3]{64}=4$, because $4\times 4\times 4=64$

$\sqrt[4]{81}=3$, because $3\times 3\times 3\times 3=81$

The sign that a quantity is to be raised to a certain power is a small number placed at the upper right hand corner of the quantity; this number is called the exponent. For instance, 7^2 signifies that 7 is to be squared or multiplied by itself, that is:

$$7^2=7\times 7=49$$
$$7^3=7\times 7\times 7=343, \text{ etc.}$$

$\{\ \}$ braces, [ ] brackets, ( ) parentheses, signify that the quantities which they include are to be considered as one quantity. For instance: $35-(8+6)$ is equal to $35-14=21$. In this case the parenthesis indicates that not only 8, but the sum of $8+6$ is to be subtracted from 35.

$\overline{}$ (vinculum or bar) is a straight line placed over two or more quantities, indicating that they are to be operated upon as one quantity. For instance, $\sqrt{25+11}$. The vinculum attached to the radical sign indicates that the square root shall be extracted from the sum of $25+11$, which is the same as the square root of 36.

In an expression as $\frac{35+15+22}{3\times 8}$ the bar indicates that the sum of $35+15+22$ shall be divided by the product of 3×8 which is the same as 72 divided by 24.

Whenever a number or a quantity is placed over a line and a number or a quantity is placed under the same line it always indicates that the number or quantity over the line shall be divided by the number or quantity under the line. Such a quantity is called a fraction.

The quantity above the line is called the numerator, and the quantity below the line is called the denominator. A fraction may be either proper or improper. The fraction is proper when the numerator is smaller than the denominator; for instance, $\frac{3}{5}$; but improper if the numerator is larger than the denominator, for instance, $\frac{11}{7} = 1\frac{4}{7}$.

A fraction can always be considered simply as a problem in division.

Formulas.

A formula is an algebraic expression for some general rule, law or principle. Formulas are used in mechanical books, because they are much more convenient than rules. Generally speaking, the knowledge of algebra is not required for the use of formulas, because the numerical values corresponding to the conditions of the problem are inserted for every letter in the formula except the letter representing the unknown quantity, which then is obtained by simple arithmetical calculations. It is generally most convenient to begin the interpretation of formulas from the right-hand side; for instance, the formula for the velocity of water in long pipes is:

$$v = 8.02\sqrt{\frac{h.\ d.}{f.\ l.}}$$

In this formula v represents the velocity of the water in feet per second.

h represents the "head"* in feet.

d represents the diameter of the pipe in feet.

f represents the friction factor determined by experiments.

l represents the length of the pipe in feet, and 8.02 is a constant equal to the square root of twice acceleration due to gravity.

Assume, for instance, that it is required to find the velocity of the flow of water in a pipe of 3 inches diameter (¼ foot); the length of the pipe is 1,440 feet, the "head" is 9 feet, and the friction factor is 0.025.

Inserting in the formula these numerical values, and for convenience writing the diameter of the pipe in decimals, we have:

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* In hydraulics the word "head" means the vertical difference between the level of the water at the receiving end of the pipe and the point of discharge, or its equivalent in pressure. See Hydraulics, page 413.

$$v = 8.02 \times \sqrt{\frac{9 \times 0.25}{0.025 \times 1440}}$$

Solving the problem step by step we have:

$$v = 8.02 \times \sqrt{\frac{2.25}{36}}$$

$$v = 8.02 \times \sqrt{0.0625}$$

$$v = 8.02 \times 0.25.$$

$$v = 2.005 \text{ feet per second.}$$

In mechanical formulas, if not otherwise specified, it is always safe to assume the letter g to mean acceleration due to gravity, usually taken as 32.2 feet or 9.82 meters. In formulas relating to heat the letter J usually signifies the mechanical equivalent of heat = 778 foot pounds of energy; but in formulas relating to strength of materials the letter J usually signifies the polar moment of inertia, and the letter I the least rectangular moment of inertia. The letter x always expresses the unknown quantity. The following Greek letters are also used more or less. The letter π, called pi, is used to signify the ratio of the circumference to the diameter of a circle, and is usually taken as 3.1416. Σ, called sigma, usually signifies the sum of a number of quantities. The letter Δ, called delta, usually signifies small increments of matter.

The letter θ, called theta, or the lefter Φ, called phi, usually signifies some particular angle, sometimes also the coefficient of friction. But all these letters may be employed to express anything, although it is usually safe, if not otherwise specified. to expect their meaning to be as stated. It is always customary to express known quantities by the first letters in the alphabet, such as a, b, c, etc., and unknown quantities by such letters as x, y, z, etc.

Arithmetic.

Addition.

All quantities to be added must be of the same unit; we can not add 3 feet + 8 inches + 2 meters, without first reducing these three terms either to feet, inches or meters. The same also with numbers. Units must be added to units, tens to tens, hundreds to hundreds, etc.

Example.

$$318 + 5 + 38 + 10 + 115 = 486$$

Solution:

$$\begin{array}{r} 318 \\ 5 \\ 38 \\ 10 \\ 115 \\ \hline 486 \end{array} = \text{Sum.}$$

Subtraction.

Two quantities to be subtracted must be of the same unit.

In subtraction, the same as in addition, the units are placed under each other, and units are subtracted from units, tens from tens, hundreds from hundreds, etc.

Example.

$$2543 - 1828 = 715$$

Solution:

$$\begin{array}{rl} 2543 & \text{. . Minuend.} \\ 1828 & \text{. . Subtrahend.} \\ \hline 715 & \text{. . Difference.} \end{array}$$

Subtrahend + Difference = Minuend.

Multiplication.

A quantity is multiplied by a number by adding it to itself as many times as the number indicates.

EXAMPLE.

$$314 \times 3 = 314 + 314 + 314 = 942$$

Solution:

$$\left.\begin{array}{rll} 314 & . \; . & \text{Multiplicand} \\ 3 & . \; . & \text{Multiplier} \end{array}\right\} \text{Factors.}$$

$$\begin{array}{rll} \hline 942 & . \; . & \text{Product.} \end{array}$$

$$\frac{\text{Product}}{\text{Multiplicand}} = \text{Multiplier.}$$

$$\frac{\text{Product}}{\text{Multiplier}} = \text{Multiplicand.}$$

Division.

The quantity or number to be divided is called the dividend. The number by which we divide is called the divisor. The number that shows how many times the divisor is contained in the dividend is called the quotient.

EXAMPLE.

$$6852 \div 3 = 2284$$

Solution:

```
3)6852(2284
  6
  —
   8
   6
   —
   25
   24
   —
    12
    12
    —
     0
```

$$\begin{array}{rll} 6852 & . \; . & \text{Dividend.} \\ 3 & . \; . & \text{Divisor.} \\ 2284 & . \; . & \text{Quotient.} \end{array}$$

$$\text{Divisor} \times \text{Quotient} = \text{Dividend.}$$

FRACTIONS.

Addition.

Fractions to be added must have a common denominator; thus we cannot add $\frac{1}{2} + \frac{2}{3} + \frac{1}{4} + \frac{3}{4}$ unless they be reduced to a common denominator instead of the denominators two, three and four; in other words, we must find the least common multiple of the numbers 2, 3 and 4, which is 12. Thus we have:

$$\begin{aligned} \tfrac{1}{2} &= \tfrac{6}{12} \\ \tfrac{2}{3} &= \tfrac{8}{12} \\ \tfrac{1}{4} &= \tfrac{3}{12} \\ \tfrac{3}{4} &= \tfrac{9}{12} \\ \hline & \tfrac{26}{12} = 2\tfrac{2}{12} = 2\tfrac{1}{6} \end{aligned}$$

EXAMPLE 2.

Add: $\frac{7}{16} + \frac{5}{8} + \frac{1}{4} + \frac{7}{12} + \frac{5}{6} + \frac{5}{7} + \frac{3}{8} + \frac{4}{9}$

The common denominator is found in the following manner: Write in a line all the denominators, and divide with the prime number, 2, as many numbers as can be divided without a remainder. The numbers that cannot be divided without a remainder remain unchanged, and these together with the quotients of the divided numbers, are written in the next line below. Repeat this operation as long as more than one number can be divided without remainder, then try to divide by the next prime number, and so on. These divisors and all those numbers remaining undivided in the last line are multiplied together, and the product is the least common denominator.

2)	$\cancel{16}$	$\cancel{8}$	$\cancel{4}$	$\cancel{12}$	$\cancel{6}$	7	$\cancel{8}$	9
2)	$\cancel{8}$	$\cancel{4}$	$\cancel{2}$	$\cancel{6}$	3	7	$\cancel{4}$	9
2)	$\cancel{4}$	$\cancel{2}$	1	3	3	7	$\cancel{2}$	9
3)	2	1	1	$\cancel{3}$	$\cancel{3}$	7	1	$\cancel{9}$
	2	1	1	1	1	7	1	3

The common denominator is thus:

$$2 \times 2 \times 2 \times 3 \times 2 \times 7 \times 3 = 1008$$

Thus 1008 is the least common multiple of 16, 8, 4, 12, 7 and 9.

The principle of this solution can probably be better understood by resolving these numbers into prime numbers, and also resolving 1008 into prime numbers; we then find that 1008 contains all the prime numbers necessary to make 16, 8, 4, 12, 6, 7 and 9.

Prime numbers	in	1008	are	2	2	2	2	7	3	3
"	"	"	16	"	2	2	2	2			
"	"	"	8	"	2	2	2				
"	"	"	4	"	2	2					
"	"	"	12	"	2	2	3				
"	"	"	6	"	2	3					
"	"	"	7	"	7						
"	"	"	9	"	3	3					

Solution of Example 2:

$$\begin{array}{ll} & \underbrace{1008} \\ \frac{7}{16} & 63 \times 7 = 441 \\ \frac{5}{8} & 126 \times 5 = 630 \\ \frac{1}{4} & 252 \times 1 = 252 \\ \frac{7}{12} & 84 \times 7 = 588 \\ \frac{5}{6} & 168 \times 5 = 840 \\ \frac{5}{7} & 144 \times 5 = 720 \\ \frac{3}{8} & 126 \times 3 = 378 \\ \frac{4}{9} & 112 \times 4 = 448 \\ \hline & 4297 = \frac{4297}{1008} = 4\frac{265}{1008}. \end{array}$$

Subtraction.

When fractions are to be subtracted, they must first be reduced to a common denominator, the same as in addition.

EXAMPLE.

$\frac{5}{9} - \frac{1}{3}$ must be reduced to $\frac{5}{9} - \frac{3}{9} = \frac{2}{9}$

EXAMPLES.

No. 1. $\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

No. 2. $\frac{7}{12} - \frac{1}{5} = \frac{35}{60} - \frac{12}{60} = \frac{23}{60}$

No. 3. $\frac{11}{16} - \frac{5}{32} = \frac{22}{32} - \frac{5}{32} = \frac{17}{32}$

Multiplication.

Fractions are multiplied by fractions, by multiplying numerator by numerator and denominator by denominator; thus:

$$\tfrac{3}{8} \times \tfrac{7}{12} = \tfrac{21}{96} = \tfrac{7}{32}$$

The correctness of this rule can easily be understood if we consider these two fractions as two problems in division. $\frac{3}{8} \times \frac{7}{12}$ will then be 3 divided by 8 and the quotient multiplied by 7 and the product divided by 12; thus, 3 is to be multiplied by 7 and the product is to be divided by 8 times 12. Therefore:

$$\tfrac{3}{8} \times \tfrac{7}{12} = \frac{\cancel{3} \times 7}{8 \times \cancel{12}} = \frac{1 \times 7}{8 \times 4} = \tfrac{7}{32}$$

A mixed number may first be reduced to an improper fraction and then multiplied as a common fraction, numerator by numerator and denominator by denominator. For instance:

$$3\tfrac{1}{2} \times \tfrac{3}{4} = \tfrac{7}{2} \times \tfrac{3}{4} = \tfrac{21}{8} = 2\tfrac{5}{8}$$

A fraction may be multiplied by a whole number by multiplying the numerator and letting the denominator remain unchanged. For instance:

$$\tfrac{7}{12} \times 2 = \tfrac{14}{12} = 1\tfrac{2}{12} = 1\tfrac{1}{6}$$

This must be correct, because we may consider 7 as indicating the quantity and 12 as indicating what kind of quantity in exactly the same sense as we may say 7 dollars or 7 cents; if either of those were multiplied by 2 the product would, of course, be either dollars or cents respectively, and for the same reason 7 twelfths multiplied by 2 must be 14 twelfths.

A fraction may also be multiplied by a whole number, by dividing the denominator by the number and letting the numerator remain unchanged. For instance:

$\frac{7}{12} \times 2 = \frac{7}{6} = 1\frac{1}{6}$, because $\frac{2}{12}$ is equal to $\frac{1}{6}$, so must $\frac{7}{12} \times 2 = \frac{7}{6} = 1\frac{1}{6}$

EXAMPLES.

No. 1. $3\frac{1}{4} \times \frac{5}{8} = \frac{13}{4} \times \frac{5}{8} = \frac{65}{32}$

No. 2. $1\frac{1}{2} \times 1\frac{1}{2} = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$

No. 3. $\frac{3}{16} \times \frac{1}{2} = \frac{3}{32}$

No. 4. $1\frac{5}{8} \times \frac{8}{13} = \frac{13}{8} \times \frac{8}{13} = \frac{13}{1} \times \frac{1}{13} = \frac{1}{1} \times \frac{1}{1} = 1$

Division.

A fraction is divided by a fraction by writing the fractions after each other, then inverting the divisor (that is, changing its numerator to denominator and its denominator to numerator), proceed as in multiplication. For instance:

$$\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{20}{24} = \frac{5}{6}$$

The reason for this rule can very easily be understood, when we consider the fractions as problems in division. That is to say, 5 shall be divided by 8 and the quotient is to be divided by one-fourth of 3. But if the quantity $\frac{5}{8}$ is divided by 3 instead of one-fourth of 3, we must, of course, multiply the quotient by 4 to make the result correct. Therefore:

$$\frac{5}{8} \div \frac{3}{4} = \frac{\frac{5}{8} \times 4}{3} = \frac{20}{8} \div 3 = \frac{20}{24} = \frac{5}{6}$$

A fraction may be divided by a whole number by dividing the numerator by the number and letting the denominator remain unchanged. For instance:

$$\frac{9}{16} \div 3 = \frac{3}{16}$$

A fraction may be divided by a whole number by multiplying the denominator by the whole number and letting the numerator remain unchanged. For instance:

$$\frac{2}{3} \div 3 = \frac{2}{9}$$

Mixed numbers are reduced to improper fractions the same as in multiplication; they are then figured the same as if they were proper fractions.

EXAMPLES.

No. 1. $\frac{7}{16} \div \frac{1}{2} = \frac{7}{16} \times \frac{2}{1} = \frac{14}{16} = \frac{7}{8}$
No. 2. $\frac{5}{18} \div \frac{3}{4} = \frac{5}{18} \times \frac{4}{3} = \frac{20}{54} = \frac{10}{27}$
No. 3. $2\frac{1}{8} \div 1\frac{1}{3} = \frac{17}{8} \times \frac{3}{4} = \frac{51}{32}$
No. 4. $2\frac{1}{3} \div 6 = \frac{7}{3} \div 6 = \frac{7}{18}$
No. 5. $3\frac{1}{5} \div 4 = \frac{16}{5} \div 4 = \frac{4}{5}$

In No. 4 it will be understood that $\frac{7}{3}$ divided by 6 must be $\frac{7}{18}$, because $\frac{1}{18}$ is exactly a sixth of $\frac{1}{3}$.

In No. 5, also, it will be understood that if $\frac{16}{5}$ is divided by 4, the quotient must be $\frac{4}{5}$, because 4 is one-fourth of 16.

To Reduce a Fraction of One Denomination to a Fraction of Another Fixed Denomination, and Approximately of the Same Value.

In mechanical calculations, on drawings, and on other occasions, it is very frequently necessary to reduce fractions of other denominations to eighths, sixteenths, thirty-seconds, or sixty-fourths. This may be done by multiplying the numerator and the denominator of the given fraction by the number which is to be the denominator in the new fraction, then dividing this new numerator and denominator by the denominator of the given fraction.

EXAMPLE.

Reduce $\frac{2}{3}$ to eighths, sixteenths, thirty-seconds, sixty-fourths, or to hundredths.

$$\frac{2}{3} \times \frac{8}{8} = \frac{16}{24} = \frac{5\frac{1}{3}}{8} \text{ or } \frac{5}{8} \text{ approximately.}$$

$$\frac{2}{3} \times \frac{16}{16} = \frac{32}{48} = \frac{10\frac{2}{3}}{16} \text{ or } \frac{11}{16} \text{ approximately.}$$

$$\frac{2}{3} \times \frac{32}{32} = \frac{64}{96} = \frac{21\frac{1}{3}}{32} \text{ or } \frac{21}{32} \text{ approximately.}$$

$$\frac{2}{3} \times \frac{64}{64} = \frac{128}{192} = \frac{42\frac{2}{3}}{64} \text{ or } \frac{43}{64} \text{ approximately.}$$

$$\frac{2}{3} \times \frac{100}{100} = \frac{200}{300} = \frac{66\frac{2}{3}}{100} \text{ or } \frac{67}{100} \text{ approximately.}$$

Thus $\frac{5}{8}$, instead of $\frac{2}{3}$, is considerably too small, namely, $\frac{1}{24}$, but $\frac{43}{64}$ is a great deal nearer, only $\frac{1}{192}$ too large, and 0.67 is $\frac{1}{300}$ too large.

DECIMALS.

In decimal fractions the denominator is always some power of ten, such as tenths, hundredths, thousandths, etc.

The denominator is never written, as it is fixed by the rule that it is 1 with as many ciphers annexed as there are figures on the right-hand side of the decimal point.

$\frac{1}{2} = 0.5 =$ five-tenths $= \frac{5}{10}$
$\frac{1}{4} = 0.25 =$ twenty-five hundredths $= \frac{25}{100}$
$\frac{1}{8} = 0.125 =$ one hundred and twenty-five thousandths $= \frac{125}{1000}$
$1\frac{1}{2} = 1.5 =$ one and five-tenths $= 1\frac{5}{10}$
$1\frac{1}{4} = 1.25 =$ one and twenty-five hundredths $= 1\frac{25}{100}$, etc.

Figures on the left side of the decimal point are whole numbers. When there are no whole numbers, sometimes a cipher is written on the left side of the decimal point, but this is not always done, as it is common with many writers not to write anything on the left side of the decimal point when there is no whole number.

Thus:

½	may	be	written	.5
¼	"	"	"	.25
⅛	"	"	"	.125

It is, however, preferable to fill in a cipher on the left-hand side of the decimal point when there is no whole number, as by so doing the mistake of reading a decimal for a whole number is prevented.

To Reduce a Vulgar Fraction to a Decimal Fraction.

Annex a sufficient number of ciphers to the numerator, divide the numerator by the denominator, and point off as many decimals in the quotient as there are ciphers annexed to the numerator.

EXAMPLE.

Reduce ⅞ to a decimal fraction.

Solution:

```
8 ) 7.000 ( 0.875
    64
    ---
     60
     56
     ---
      40
      40
      ---
      00
```

Thus, ⅞ is equal to the decimal fraction 0.875.

Fractions Reduced to Exact Decimals.

$\frac{1}{64}$	.015625	$\frac{17}{64}$	.265625	$\frac{33}{64}$	.515625	$\frac{49}{64}$	.765625
$\frac{1}{32}$	.03125	$\frac{9}{32}$	.28125	$\frac{17}{32}$	.53125	$\frac{25}{32}$	.78125
$\frac{3}{64}$	.046875	$\frac{19}{64}$	.296875	$\frac{35}{64}$	.546875	$\frac{51}{64}$	.796875
$\frac{1}{16}$	.0625	$\frac{5}{16}$	.3125	$\frac{9}{16}$	.5625	$\frac{13}{16}$	.8125
$\frac{5}{64}$	.078125	$\frac{21}{64}$	.328125	$\frac{37}{64}$	.578125	$\frac{53}{64}$	.828125
$\frac{3}{32}$	.09375	$\frac{11}{32}$	.34375	$\frac{19}{32}$	.59375	$\frac{27}{32}$	.84375
$\frac{7}{64}$	.109375	$\frac{23}{64}$	.359375	$\frac{39}{64}$	.609375	$\frac{55}{64}$	.859375
$\frac{1}{8}$	.125	$\frac{3}{8}$	.375	$\frac{5}{8}$	.625	$\frac{7}{8}$	.875
$\frac{9}{64}$	.140625	$\frac{25}{64}$	.390625	$\frac{41}{64}$	.640625	$\frac{57}{64}$	.890625
$\frac{5}{32}$	.15625	$\frac{13}{32}$	.40625	$\frac{21}{32}$	.65625	$\frac{29}{32}$	.90625
$\frac{11}{64}$	.171875	$\frac{27}{64}$	.421875	$\frac{43}{64}$	.671875	$\frac{59}{64}$	.921875
$\frac{3}{16}$	.1875	$\frac{7}{16}$	.4375	$\frac{11}{16}$	.6875	$\frac{15}{16}$	.9375
$\frac{13}{64}$	.203125	$\frac{29}{64}$	.453125	$\frac{45}{64}$	.703125	$\frac{61}{64}$	.953125
$\frac{7}{32}$	.21875	$\frac{15}{32}$	.46875	$\frac{23}{32}$	.71875	$\frac{31}{32}$	.96875
$\frac{15}{64}$	.234375	$\frac{31}{64}$	.484375	$\frac{47}{64}$	.734375	$\frac{63}{64}$	.984375
$\frac{1}{4}$	.25	$\frac{1}{2}$	.5	$\frac{3}{4}$	.75	1	1.

To Reduce a Decimal Fraction to a Vulgar Fraction.

Write the decimal as the numerator of the fraction and set under it for the denominator the figure one, followed by as many ciphers as there are decimal places; then cancel the fraction thus written, to its smallest possible terms.

EXAMPLE.

Reduce 0.3125 to a vulgar fraction.

Solution:

$0.3125 = \frac{3125}{10000}$, cancelling this by five we have $\frac{625}{2000} = \frac{125}{400} = \frac{25}{80} = \frac{5}{16}$.

To Reduce a Decimal Fraction to a Given Vulgar Fraction of Approximately the Same Value.

Multiply the decimal by the number which is denominator in the fraction to which the decimal shall be reduced, and the product is the numerator in the fraction.

EXAMPLE.

Reduce 0.48437 to sixteenths, thirty-seconds and sixty-fourths.

Solution:

$0.484375 \times 16 = 7.75$, gives $\frac{7.75}{16}$, or $\frac{7}{16}$, approximately.
$0.484375 \times 32 = 15.5$, gives $\frac{15.5}{32}$, or $\frac{15}{32}$, approximately.
$0.484375 \times 64 = 31$, gives $\frac{31}{64}$ exactly.

If the result does not need to be very exact, probably $\frac{7}{16}$, which is $\frac{3}{64}$ too small, is near enough, or the result, $\frac{7.75}{16}$, may be called ½, which is $\frac{1}{64}$ too large. $\frac{15}{32}$ is $\frac{1}{64}$ too small, therefore either ½ or $\frac{15}{32}$ is only $\frac{1}{64}$ different from the true value. The first is $\frac{1}{64}$ too large and the last is $\frac{1}{64}$ too small, and which fraction, if either, should be preferred, will depend entirely upon the purpose for which the problem is solved. $\frac{31}{64}$ is the exact value.

Addition of Decimal Fractions.

In adding decimal fractions, care should be taken to place the decimal points under each other; then add as if they were whole numbers.

EXAMPLE.

Add $50.5 + 5.05 + 0.505 + 0.0505$

Solution:

$$\begin{array}{r} 50.5 \\ 5.05 \\ 0.505 \\ 0.0505 \\ \hline 56.1055 \end{array}$$

To prevent mistakes and mixing up of the figures during addition, it is preferable to make all the decimal fractions in the problem of the same denomination by annexing ciphers.

Thus:

$$\begin{array}{r} 50.5000 \\ 5.0500 \\ 0.5050 \\ 0.0505 \\ \hline 56.1055 \end{array}$$

Subtraction of Decimal Fractions.

The decimal point in the subtrahend must be placed under that in the minuend; the fractions are both brought to the same denomination by annexing ciphers, then the subtraction is performed just as if they were whole numbers, but close attention must be paid to have the decimal point in the same place in the difference as it is in the minuend and subtrahend.

EXAMPLE.

$318.05 - 121.6542$

Solution:	318.0500	Minuend.
	121.6542	Subtrahend.
	196.3958	Difference.

Multiplication of Decimal Fractions.

Multiply the factors as if they were whole numbers. After multiplication is performed, count the number of decimals in both multiplier and multiplicand and point off (from the right) the same number of decimals in the product.

If there are not enough figures in the product to give as many decimals as required, then prefix ciphers on the left until the required number of decimals is obtained.

EXAMPLE 1.

$$0.08 \times 0.065 = 0.00520 = 0.0052$$

In this example it is necessary after the multiplication is performed, to prefix two ciphers to the product in order to obtain the necessary number of decimals, because the product, 520, consists of only three figures, but the two numbers, 0.08 and 0.065, contain five decimals.

EXAMPLE NO. 2.

$$3.1416 \times 5 = 15.7080 = 15.708$$

EXAMPLE NO. 3.

$$3.1416 \times 0.5 = 1.57080 = 1.5708$$

Division of Decimal Fractions.

Divide same as in whole numbers, and point off in the quotient as many decimals as the number of decimals in the dividend exceeds the number of decimals in the divisor.

If the divisor contains more decimals than the dividend, then before dividing annex ciphers (on the right-hand side) in the dividend until dividend and divisor are both of the same denomination, then the quotient will be a whole number.

EXAMPLE.

$$43.62 \div 0.003 = 14{,}540$$

Solution: 0.003) 43.620 (14,540

```
3
——
13
12
——
 16
 15
 ——
  12
  12
  ——
  00
```

In this example the dividend consists of only two decimals, but the divisor has three, therefore we have to annex a cipher to the dividend. This brings divisor and dividend to the same denomination, and the quotient is a whole number.

EXAMPLE 2.

$$43.62 \div 0.3 = 145.4$$

In this example the dividend has one decimal more than the divisor, therefore the quotient has one decimal.

RATIO.

The word ratio causes considerable ambiguity in mechanical books, as it is frequently used with different meaning by different writers.

The common understanding seems to be that the ratio between two quantities is the quotient when the first quantity is divided by the last quantity; for instance, the ratio between 3 and 12 is ¼, but the ratio between 12 and 3 is 4. The ratio between the circumference of a circle and its diameter is π or 3.1416, but the ratio between the diameter and the circumference is $\frac{1}{\pi}$ or 0.3183, etc. This is the sense in which the word is used in this book, as this seems to agree with the common custom with most mechanical writers.

The term ratio is also sometimes applied to the difference of two quantities as well as to their quotient; in which case the former is called arithmetical ratio, and the latter geometrical ratio. (See Progressions, page 68.)

PROPORTION.

In simple proportion there are three known quantities by which we are able to find the fourth unknown quantity; therefore proportion is also called "the rule of three", and it is either direct or inverse proportion.

It is called direct proportion if the terms are in such ratio to one another that if one is doubled then the other will also have to be doubled, or if one is halved the other must also be halved. For instance, if 50 pounds of steel cost \$25, how much will 250 pounds cost?

50 lbs. cost \$25; 250 must cost $\frac{250 \times 25}{50} = \$125.$

This is direct proportion, because the more steel we buy, the more money we have to pay.

In inverse proportion the terms are in such ratio that if one is doubled the other is halved, or if one is halved the other is doubled.

EXAMPLE.

Eight men can finish a certain work in 12 days. How many men are required to do the same work in 3 days?

Here we see that the fewer days in which the work is to be done, the more men are required. Therefore, this example is in inverse proportion.

In 12 days the work was done by 8 men; therefore, in order to do the work in 3 days it will require $\frac{8 \times 12}{3} = 32$ men.

It requires 4 times as many men because the work is to be done in one quarter of the time.

Compound Proportion.

A proportion is called compound, if to the three terms there are combined other terms which must be taken into consideration in solving the problem.

A very easy way to solve a compound proportion is to (same as is shown in the following examples) place the conditional proposition under the interrogative sentence, term for term, and write x for the unknown quantity in the interrogative sentence; draw a vertical line; place x at the top at the left-hand side; then try term for term and see if they are direct or inverse proportionally relative to x, exactly the same way as if each term in the conditional proposition and the corresponding term in the interrogative sentence were terms in a simple rule-of-three problem. Arrange each term in the interrogative sentence either on the right or left of the vertical line, according to whether it is found to be either a multiplier or a divisor, when the problem, independent of the other terms, is considered as a simple rule-of-three problem.

After all the terms in the interrogative sentence are thus arranged, place each corresponding term in the conditional proposition on the opposite side of the vertical line. Then clear away all fractions by reducing them to improper fractions, and let the numerator remain on the same side of the vertical line where it is, but transfer the denominator to the opposite side. Now cancel any term with another on the opposite side of the vertical line; then multiply all the quantities on the right side of the vertical line with each other. Also multiply all the quantities on the left side of the vertical line with each other.

Divide the product on the right side by the product on the left, and the quotient is the answer to the problem.

EXAMPLE 1.

A certain work is executed by 15 men in 6 days, by working 8 hours each day. How many days would it take to do the same amount of work if 12 men are working $7\frac{1}{2}$ hours each day?

Solution:

15 Men 6 Days 8 Hours.
12 " x " 7½ "

		x	$\cancel{6}$		
1	$\cancel{2}$	$\cancel{12}$	$\cancel{15}$	1	
		$\cancel{7}\frac{1}{2}$	8		
	1	$\cancel{15}$	$\cancel{2}$		

8 Days.

EXAMPLE 2.

A steam engine of 25 horse power is using 1500 pounds of coal in 1 day of 9½ working hours. How many pounds of coal in the same proportion will be required for 2 steam engines each having 30 horse power, working 6 days of 12⅔ hours each day?

Solution:

1 Machine 25 Hp. 1,500 pounds 1 Day 9½ hours.
2 " 30 " x " 6 " 12⅔ "

		x	$\cancel{1500}$	300	
		1	2		
1	$\cancel{5}$	$\cancel{25}$	$\cancel{30}$	6	
		1	$\cancel{6}$	2	
1	$\cancel{19}$	$\cancel{9}\frac{1}{2}$	$\cancel{12}\frac{2}{3}$	$\cancel{38}$	2
	1	$\cancel{3}$	2		

28,800 pounds of coal.

EXAMPLE 3.

A piece of composition metal which is 12 inches long, 3½ inches thick and 4½ inches wide, weighs 45 pounds. How many pounds will another piece of the same alloy weigh, if it measures 8 inches long, 1¾ inches thick and 6¾ inches wide?

12″ long, 3½″ thick, 4½″ wide, 45 pounds.
8 " 1¾ " 6¾ " x "

		x	45			
2	$\cancel{4}$	$\cancel{12}$	$\cancel{8}$	$\cancel{2}$		
	$\cancel{7}$	$\cancel{3}\frac{1}{2}$	$\cancel{1}\frac{3}{4}$	$\cancel{7}$		
1	$\cancel{9}$	$\cancel{4}\frac{1}{2}$	$\cancel{6}\frac{3}{4}$	$\cancel{27}$	$\cancel{3}$	1
	1	$\cancel{4}$	$\cancel{2}$	1		
	1	$\cancel{4}$	$\cancel{2}$	1		

2 | 45

22½ pounds.

INTEREST.

The money paid for the use of borrowed capital is called interest. It is usually figured by the year per 100 of the principal.

Simple Interest.

Simple interest is computed by multiplying the principal by the percentage, by the time, and dividing by 100.

What is the interest of $125, for 3 years, at 4% per year?

Solution:

$$\frac{125 \times 4 \times 3}{100} = \$15$$

In Table No. 1, under the given rate per cent., find the interest for the number of years, months, and days; add these together, and multiply by the principal invested, and the product is the interest.

EXAMPLE.

What is the interest of $600, invested at 6%, in 5 years, 3 months, and 6 days?

Solution:

$1.00 in 5 years	at 6%	=	0.30
" " 3 months	" "	=	0.015
" " 6 days	" "	=	0.001
			0.316
			600 = Principal.
			$189.60 = Interest.

TABLE No. 1. Shows the Simple Interest of $1.00 for days, months and years.

Days.	3%	4%	5%	6%	7%	8%	9%	10%
1	0.00008	0.00011	0.00014	0.00017	0.00019	0.00022	0.00025	0.00028
2	0.00016	0.00022	0.00028	0.00033	0.00039	0.00044	0.00049	0.00056
3	0.00025	0.00033	0.00042	0.00050	0.00058	0.00067	0.00075	0.00083
4	0.00033	0.00044	0.00056	0.00067	0.00078	0.00089	0.00100	0.00111
5	0.00041	0.00056	0.00069	0.00083	0.00097	0.00111	0.00125	0.00139
6	0.00050	0.00067	0.00083	0.00100	0.00117	0.00133	0.00150	0.00167
7	0.00058	0.00078	0.00097	0.00117	0.00136	0.00156	0.00175	0.00194
8	0.00066	0.00089	0.00111	0.00133	0.00156	0.00178	0.00200	0.00222
9	0.00075	0.00100	0.00125	0.00150	0.00175	0.00200	0.00225	0.00250
10	0.00083	0.00111	0.00139	0.00167	0.00194	0.00222	0.00250	0.00278
11	0.00092	0.00122	0.00153	0.00183	0.00214	0.00244	0.00275	0.00306
12	0.00100	0.00133	0.00167	0.00200	0.00233	0.00267	0.00300	0.00333
13	0.00108	0.00144	0.00181	0.00217	0.00253	0.00289	0.00325	0.00361
14	0.00116	0.00156	0.00194	0.00233	0.00272	0.00311	0.00350	0.00389
15	0.00125	0.00167	0.00208	0.00250	0.00292	0.00333	0.00375	0.00417
16	0.00133	0.00178	0.00222	0.00267	0.00311	0.00356	0.00400	0.00444
17	0.00141	0.00189	0.00236	0.00283	0.00331	0.00378	0.00425	0.00472
18	0.00150	0.00200	0.00250	0.00300	0.00350	0.00400	0.00450	0.00500
19	0.00158	0.00211	0.00264	0.00317	0.00369	0.00422	0.00475	0.00528
20	0.00166	0.00222	0.00278	0.00333	0.00389	0.00444	0.00500	0.00556
21	0.00175	0.00233	0.00292	0.00350	0.00408	0.00467	0.00525	0.00583
22	0.00183	0.00244	0.00306	0.00367	0.00428	0.00498	0.00550	0.00611
23	0.00191	0.00256	0.00319	0.00383	0.00447	0.00511	0.00575	0.00639
24	0.00200	0.00267	0.00333	0.00400	0.00467	0.00533	0.00600	0.00667
25	0.00208	0.00278	0.00347	0.00417	0.00486	0.00556	0.00625	0.00694
26	0.00216	0.00289	0.00361	0.00433	0.00506	0.00578	0.00650	0.00722
27	0.00225	0.00300	0.00375	0.00450	0.00525	0.00600	0.00675	0.00750
28	0.00233	0.00311	0.00389	0.00467	0.00544	0.00622	0.00700	0.00778
29	0.00241	0.00322	0.00403	0.00483	0.00564	0.00644	0.00725	0.00806

TABLE No. 1.

Months.	3%	4%	5%	6%	7%	8%	9%	10%
1	0.00250	0.00333	0.00417	0.00500	0 00583	0.00667	0.00750	0.00833
2	0.00500	0.00667	0.00833	0.01000	0.01167	0.01333	0.01500	0.01667
3	0.00750	0.01000	0.01250	0.01500	0.01750	0.02000	0.02250	0.02500
4	0.01000	0.01333	0.01667	0.02000	0.02333	0.02667	0.03000	0.03333
5	0.01250	0.01667	0.02083	0.02500	0.02917	0.03333	0.03750	0.04167
6	0.01500	0.02000	0.02500	0.03000	0.03500	0.04000	0.04500	0.05000
7	0.01750	0.02333	0.02917	0.03500	0.04083	0.04667	0.05250	0.05833
8	0.02000	0.02667	0.03333	0.04000	0.04667	0.05333	0.06000	0.06667
9	0.02250	0.03000	0.03750	0.04500	0.05250	0.06000	0.06750	0.07500
10	0.02500	0.03333	0.04167	0.05000	0.05833	0.06667	0.07500	0.08333
11	0.02750	0.03667	0.04583	0.05500	0.06417	0.07333	0.08250	0.09167

Years.	3%	4%	5%	6%	7%	8%	9%	10%
1	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
2	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
3	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.30
4	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40
5	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
6	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60
7	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70
8	0.24	0.32	0.40	0.48	0.56	0.64	0.72	0.80
9	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.90
10	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

Compound Interest Computed Annually.

If the interest is not withdrawn, but added to the principal, so that it will also draw interest, it is called compound interest.

EXAMPLE.

What is the amount of $300, in 3 years, at 5%? The interest is added to the principal at the end of each year.

Solution:

Principal and interest at the end of first year,

$$\frac{105 \times 300}{100} = \$315.$$

Principal and interest at the end of second year,

$$\frac{105 \times 315}{100} = \$330.75.$$

Principal and interest at the end of third year,

$$\frac{105 \times 330.75}{100} = \$347.2875, = \$347.29 = \text{Amount}.$$

When compound interest for a great number of years is to be calculated, the above method of figuring will take too much time, and the following interest tables, No. 2 and No. 3, are computed in order to facilitate such calculations.

In Table No. 2, under the given rate per cent., and opposite the given number of years, find the amount of one dollar invested at that rate for the time taken. Multiply this by the principal invested and the product is the amount.

EXAMPLE.

$400 is invested at 5% compound interest for 17 years, computed annually. What is the amount?

Solution:

In Table No. 2, under 5%, and opposite 17 years, we find 2.292011. Multiply this by the principal.

Thus:

$$\begin{array}{r} 2.292011 \\ 400 \\ \hline 916.8044 \end{array} = \$916.80 = \text{Amount}.$$

TABLE No. 2. Showing the amount of $1 at different rates of Interest, compounded annually for any number of years, from 1 to 25.

Years.	2½%	3%	3½%	4%	4½%	5%	6%	7%
1	1.025	1.03	1.035	1.04	1.045	1.05	1.06	1.07
2	1.050625	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.076891	1.092727	1.108718	1.124864	1.141166	1.157621	1.191016	1.225043
4	1.103813	1.125509	1.147523	1.169858	1.192519	1.215506	1.262477	1.310796
5	1.131409	1.159274	1.187686	1.216652	1.246182	1.276282	1.338226	1.402552
6	1.159694	1.194050	1.229254	1.265318	1.302260	1.340096	1.418518	1.500731
7	1.188686	1.229873	1.272278	1.315931	1.360862	1.407100	1.503631	1.605782
8	1.218404	1.266769	1.316808	1.368568	1.422101	1.477455	1.593849	1.718187
9	1.248863	1.304773	1.362896	1.423311	1.486096	1.551328	1.689481	1.838460
10	1.280086	1.343915	1.410597	1.480243	1.552966	1.628895	1.790845	1.967153
11	1.312082	1.384233	1.459968	1.539452	1.622850	1.710339	1.898300	2.104853
12	1.344890	1.425760	1.511067	1.601031	1.695882	1.795856	2.012199	2.252193
13	1.378512	1.468532	1.563954	1.665072	1.772197	1.885649	2.132930	2.409847
14	1.412975	1.512588	1.618692	1.731674	1.851945	1.979932	2.260906	2.578536
15	1.448300	1.557966	1.675307	1.800941	1.935283	2.078928	2.396561	2.759034
16	1.484507	1.604705	1.733982	1.872978	2.022370	2.182880	2.540355	2.952159
17	1.521620	1.652846	1.794672	1.947897	2.113378	2.292011	2.692776	3.158811
18	1.559661	1.702431	1.857485	2.025813	2.208480	2.406619	2.854343	3.379935
19	1.598653	1.753504	1.922497	2.106841	2.307861	2.526950	3.025604	3.616531
20	1.638619	1.806109	1.989784	2.191119	2.411715	2.653298	3.207141	3.869688
21	1.679585	1.860292	2.059426	2.278764	2.520242	2.785963	3.399570	4.140567
22	1.721574	1.916101	2.131506	2.369914	2.633653	2.925261	3.603544	4.430407
23	1.764614	1.973584	2.206109	2.464710	2.752168	3.071524	3.819757	4.740535
24	1.808730	2.032791	2.283322	2.563299	2.876015	3.225100	4.048942	5.072373
25	1.853948	2.093775	2.363238	2.665830	3.005436	3.386355	4.291880	5.427440

Compound Interest Computed Semi-Annually.

When compound interest is to be computed semi-annually, use Table No. 3. Under the given rate and opposite the given number of years, find the amount of one dollar invested and interest computed semi-annually for the time taken. Multiply this by the principal invested, and the product is the amount.

EXAMPLE.

$350 is put in a savings bank paying 4%, computed semi-annually. What is the amount in 10 years?

Solution:

Under 4%, and opposite 10 years, we find the number 1.4860. This we multiply by the principal invested.

Thus:

$$\begin{array}{r} 1.485949 \\ 350 \\ \hline \end{array}$$

$$520.08215 = \$520.08 = \text{Amount.}$$

To compute compound interest for longer time than is given in the tables, figure the amount for as long a time as the table gives; then consider this amount as a new principal invested, and use the table and figure again for the rest of the time.

EXAMPLE.

What is the amount of $40, left in a savings bank 18 years, at 4%, and the interest computed semi-annually. The table only gives 12 years, therefore we will look opposite 12 years, under 4%, and find the number 1.608440. This we multiply by the principal invested.

Thus:

$$\begin{array}{r} 1.608440 \\ 40 \\ \hline 64.3376 \end{array}$$

But now we have to compute for 6 years more, therefore under 4%, and opposite 6 years, we find the number 1.268243. Multiplying this by the principal, which is now considered as being invested 6 years more, we have:

$$1.268243 \times 64.3376 = \$81.60 = \text{Amount.}$$

Thus, $40, invested at 4% interest, computed semi-annually, will, after 18 years of time, amount to $81.60.

TABLE No. 3. ***Showing the amount of $1 at different rates of Interest, compounded semi-annually for any number of years, from 1 to 12.***

Years.	2½%	3%	3½%	4%	4½%	5%	6%	7%
½	1.0125	1.015	1.0175	1.02	1.0225	1.025	1.03	1.035
1	1.025156	1.030225	1.035306	1.040400	1.045506	1.050625	1.060900	1.071225
1½	1.037970	1.045678	1.053424	1.061208	1.069030	1.076891	1.092727	1.108718
2	1.050945	1.061363	1.071859	1.082432	1.093083	1.103813	1.125509	1.147523
2½	1.064084	1.077283	1.090616	1.104081	1.117677	1.131409	1.159274	1.187685
3	1.077383	1.093442	1.109702	1.126163	1.142825	1.159694	1.194050	1.229254
3½	1.090850	1.109844	1.129122	1.148689	1.168539	1.188686	1.229873	1.272278
4	1.104485	1.126492	1.148815	1.171660	1.194831	1.218404	1.266769	1.316808
4½	1.118292	1.143389	1.168987	1.195093	1.221714	1.248863	1.304773	1.362896
5	1.132228	1.160546	1.189417	1.218996	1.249209	1.280086	1.343916	1.410597
5½	1.146423	1.177948	1.210260	1.243375	1.277310	1.312082	1.384233	1.459968
6	1.160754	1.195617	1.231439	1.268243	1.306050	1.344890	1.425760	1.511067
6½	1.175263	1.213551	1.252989	1.293608	1.335435	1.378512	1.468532	1.563954
7	1.189953	1.231754	1.274916	1.319480	1.365483	1.412975	1.512588	1.618692
7½	1.204828	1.250230	1.297227	1.345870	1.396206	1.448300	1.557966	1.675307
8	1.219888	1.268984	1.319929	1.372787	1.427621	1.484508	1.604705	1.733982
8½	1.235137	1.288018	1.343027	1.400240	1.459742	1.521620	1.652846	1.794672
9	1.250576	1.307338	1.366530	1.428248	1.492586	1.559661	1.702431	1.857485
9½	1.266208	1.326948	1.390444	1.456847	1.526169	1.598653	1.753504	1.922497
10	1.282035	1.346852	1.414777	1.485949	1.560508	1.638619	1.806109	1.989784
10½	1.298035	1.367055	1.439536	1.515668	1.595619	1.679585	1.860292	2.059427
11	1.314286	1.387561	1.464727	1.545982	1.631521	1.721574	1.916101	2.131506
11½	1.330715	1.408374	1.490360	1.576901	1.668230	1.764614	1.973584	2.206109
12	1.347349	1.429499	1.516441	1.608440	1.705765	1.808730	2.032791	2.283322

Table No. 4 gives time in which money will be doubled if it is invested either on simple or compound interest, compounded annually.

TABLE No. 4.

SIMPLE INTEREST.			COMPOUND INTEREST.		
%	Years.	Days.	%	Years.	Days.
2	50		2	35	1
2½	40		2½	28	30
3	33	120	3	23	162
3½	28	206	3½	20	54
4	25		4	17	240
4½	22	80	4½	15	168
5	20		5	14	75
6	16	240	6	11	321
7	14	103	7	10	89
8	12	180	8	9	2
9	11	40	9	8	16
10	10		10	7	98
11	9	33	11	6	231
12	8	120	12	6	42

Results of Saving Small Amounts of Money.

The following shows how easy it is to accumulate a fortune, provided proper steps are taken.

The table gives the result of daily savings, put in a savings bank paying 4 per cent. per year, computed semi-annually:

Savings per Day.	Savings per Mo.	Amount in 5 years.	Amount in 10 years.	Amount in 15 years.	Amount in 20 years.	Amount in 25 years.
.05	$ 1.20	$ 78.84	$ 174.96	$ 292.07	$ 434.88	$ 608.94
.10	2.40	157.68	349.92	584.15	869.76	1,217.88
.25	6.00	394.20	874.80	1,460.37	2,174.40	3,044.74
.50	12.00	788.40	1,749.60	2,920.74	4,348.80	6,089.48
.75	18.00	1,182.60	2,624.40	4,381.11	6,523.20	9,133.22
$1.00	24.00	1,576.80	3,499.20	5,841.48	8,697.60	12,178.96

Nearly every person wastes an amount in twenty or thirty years, which, if saved and carefully invested, would make a family quite independent; but the principle of small savings has been lost sight of in the general desire to become wealthy.

EQUATION OF PAYMENTS.

When several debts are due at different dates the average time when all the debts are due is calculated by the following rule:

Multiply each debt separately by the number of days between its own date of maturity and the date of the debt earliest due. Divide the sum of these products by the sum of the debts; the quotient will express the number of days subsequent to the leading day when the whole debt should be paid in one sum.

EXAMPLE.

A owed to B the following sums: \$250 due May 12, \$120 due July 19, \$410 due August 16, and \$60 due September 21, all in the same year. When should the whole sum be paid at once in order that neither shall lose any interest?

Solution:

May 12	\$250	
May 12 to July 19 is 68 days;	$120 \times 68 =$	8160
May 12 to Aug. 16 is 96 days;	$410 \times 96 =$	39360
May 12 to Sept. 21 is 132 days;	$60 \times 132 =$	7920
	\$840	) 55440 = 65.9

66 days after May 12 will be July 17.

When several debts are due after different lengths of time, the average time is calculated by this rule: Multiply the debt by the time; divide the sum of the products by the sum of the debts, and the quotient is the time when all the debts may be considered due.

EXAMPLE.

A owed B \$600, due in 7 months; \$200 due in one month, and \$700 due in 3 months. When should the whole debt be paid in one sum in order that neither shall lose any interest?

Solution:

$$600 \times 7 = 4200$$
$$700 \times 3 = 2100$$
$$200 \times 1 = 200$$

1500) 6500 = $4\frac{1}{3}$ months.

NOTE: If the debts contain both dollars and cents the cents may, if such refinement is required, be considered as decimal parts of a dollar, but practically in such problems the cents may be omitted in the calculation.

PARTNERSHIP,

or calculating of proportional parts, is the calculation of the parts of a certain quantity in such a way that the ratio between the separate parts is equal to the ratio of certain given numbers.

EXAMPLE 1.

A composition for welding cast steel consists of 9 parts of borax and one part of sal-ammoniac. How much of each, borax and sal-ammoniac, must be taken for a mixture of 5 lbs.?

Solution:

$\frac{9}{10} \times 5 = 4\frac{1}{2}$ lbs. borax.

$\frac{1}{10} \times 5 = \frac{1}{2}$ lb. sal-ammoniac.

EXAMPLE 2.

An alloy shall consist of 160 parts of copper, 15 parts of tin and 5 parts of zinc. How much of each will be used for a casting weighing 360 lbs.?

Solution:

160	$\frac{160}{180} \times 360 = 320$ lbs. of copper.
15	$\frac{15}{180} \times 360 = 30$ lbs. of tin.
5	$\frac{5}{180} \times 360 = 10$ lbs. of zinc.
180	

EXAMPLE 3.

Four persons—A, B, C and D, are buying a certain amount of goods together. A's part is $500, B's, $100, C's, $250, and D's, $150. On the undertaking they are clearing a net profit of $120. How much of this is each to have?

Solution:

500	A's Part $= \frac{500}{1000} \times 120 =$	$60
100	B's " $= \frac{100}{1000} \times 120 =$	12
250	C's " $= \frac{250}{1000} \times 120 =$	30
150	D's " $= \frac{150}{1000} \times 120 =$	18
$1000		

EXAMPLE 4.

Two persons—A and B, are putting money into business, A, $2,000 and B, $3,000, but A has his money invested in the business 2 years and B $2\frac{1}{2}$ years; the net profit of the undertaking is $2,300. How much is each to have of the profit?

Solution:

A, $2000 \times 2 = 4000$
B, $3000 \times 2\frac{1}{2} = 7500$

11500

A's Part is $\frac{4000}{11500} \times 2300 =$ $ 800
B's " " $\frac{7500}{11500} \times 2300 =$ 1500

In cases like this it must be taken into consideration that the time is not equal; B has not only had the largest capital invested but he has also had the capital at work in the business the longest time, namely, 2½ years, while A has only had his capital invested 2 years. The ratio is, therefore, not $2,000 to $3,000 but $4,000 to $7,500, because $2,000 in 2 years is equal to $4,000 in one year, and $3,000 in 2½ years is equal to $7,500 in one year.

SQUARE ROOT.

When the square root is to be extracted the number is divided into periods consisting of two figures, commencing from the extreme right if the number has no decimals, or from the decimal point towards the left for the whole numbers and towards the right for the decimals. (If the last period of decimals should have but one figure then annex a cipher, so that this period also has two figures, but if the period to the extreme left in the integer should happen to have only one figure it makes no difference; leave it as it is.) Ascertain the highest root of the first period and place it to the right of the number as in long division. Square this root and subtract the product of this from the first period. To the remainder annex the next period of numbers. Take for divisor 20 times the part of the root already found* and the quotient is the next figure in the root, if the product of this figure and the divisor added to the square of the figure does not exceed the dividend. To the difference between this sum and the dividend is annexed the next period of numbers. For divisor take again 20 times the part of the root already found, etc. Continue in this manner until the last period is used. If there is any remainder, and a more exact root is required, ciphers may be annexed in pairs and the operation continued until as many decimals in the root are obtained as are wanted.

Example 1.

Extract the square root of 271,441.

Solution:

$$
\begin{array}{rr}
 & \sqrt{27|14|41} = 521 \\
5^2 = & 25 \\
20 \times 5 = 100\ ) & 214 \\
100 \times 2 + 2^2 = & 204 \\
20 \times 52 = 1040\ ) & 1041 \\
1040 \times 1 + 1^2 & 1041 \\
 & 0000
\end{array}
$$

Thus: $\sqrt{271{,}441} = 521$, because $521 \times 521 = 271{,}441$.

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* If this divisor exceeds the dividend, write a cipher in the root; annex the next period of numbers, calculate a new divisor, corresponding to the increased root, and proceed as explained.

EXAMPLE 2.

Extract the square root of 26.6256.

Solution:

$$
\begin{array}{rr}
 & \sqrt{26|62|56} = 5.16 \\
5^2 = & 25 \\
20 \times 5 = 100\ ) & 162 \\
100 \times 1 + 1^2 = & 101 \\
20 \times 51 = 1020\ ) & 6156 \\
1020 \times 6 + 6^2 = & 6156 \\
 & 0000
\end{array}
$$

CUBE ROOT.

When the cube root is to be extracted, the number is divided into periods consisting of three figures. Commencing from the extreme right if the number has no decimals, or from the decimal point, toward the left, for the whole number, and toward the right for the decimals. (If the last period of decimals should not have three figures, then annex ciphers until this period also has three figures, but if the period to the extreme left in the integer should happen to consist of less than three figures it makes no difference; leave it as it is.) Ascertain highest cube root in the first period and place it to the right of the number, the same as in long division. Cube this root and subtract the product from the first period. To the remainder annex next period of numbers. For the divisor in this number take 300 times the square of the part of the root already found,* and the quotient is the next figure in the root, if the product of this figure multiplied by the divisor and added to 30 times the part of the root already found, multiplied by the square of this quotient and added to the cube of the quotient, does not exceed this dividend. To the difference between this sum and the dividend is annexed the next period of numbers. For divisor take again 300 times the square of the part of the root already found, etc. Continue in this manner until the last period is used. If there is any remainder from last period, and a more exact root is required, ciphers may be annexed three at a time, and the operation continued until as many decimals are obtained in the root as are wanted.

---

* If this divisor exceeds the dividend, write a cipher in the root, annex the next period of numbers, calculating a new divisor corresponding to the increased root, and proceed as explained.

EXAMPLE 1.

Extract the cube root of 275,894,451.

Solution:

$$\sqrt[3]{275|894|451} = 651$$

$$\begin{array}{rr}
6^3 = & 216 \\
300 \times 6^2 = 10800\ ) & 59894 \\
10800 \times 5 + 30 \times 6 \times 5^2 + 5^3 = & 58625 \\
300 \times 65^2 = 1267500\ ) & 1269451 \\
1267500 \times 1 + 30 \times 65 \times 1^2 + 1^3 = & 1269451 \\
 & 0000000
\end{array}$$

Thus:

$\sqrt[3]{275,894,451} = 651$, because $651 \times 651 \times 651 = 275,894,451$.

EXAMPLE 2:

Extract the cube root of 551.368.

Solution:

$$\sqrt[3]{551|368} = 8.2$$

$$\begin{array}{rr}
8^3 = & 512 \\
300 \times 8^2 = 19200\ ) & 39368 \\
19200 \times 2 + 30 \times 8 \times 2^2 + 2^3 = & 39368 \\
 & 00000
\end{array}$$

The square root of a number consisting of two figures will never consist of more than one figure, and the square root of a number consisting of four figures will never consist of more than two figures; hence, the rule to divide numbers into periods consisting of two figures.

The cube root of a number consisting of three figures will never consist of more than one figure, and the cube root of a number consisting of six figures will never consist of more than two figures; hence, the rule to divide the numbers into periods consisting of three figures.

There will always be one decimal in the root for each period of decimals in the number of which the root is extracted. This relates to both cube and square root.

The root of a fraction may be found by extracting the separate roots of numerator and denominator, or the fraction may be first reduced to a decimal fraction before the root is extracted.

The root of a mixed number may be extracted by first reducing the number to an improper fraction and then extracting the separate roots of numerator and denominator, or the number may be first reduced to consist of integer and decimal fractions, and the root extracted as usual.

Radical Quantities Expressed without the Radical Sign.

The radical sign is not always used in signifying radical quantities. Sometimes a quantity expressing a root is written as a quantity to be raised into a fractional power. For instance:

$\sqrt{16}$ may be written $16^{\frac{1}{2}}$. This is the same value; thus, $\sqrt{16} = 4$ and $16^{\frac{1}{2}} = 4$.

$\sqrt[3]{27}$ may be written, $27^{\frac{1}{3}} = 3$.

$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$.

The denominator in the exponent always indicates which root is to be extracted. Thus, $8^{\frac{2}{3}}$ will be square 8 and extract the cube root from the product.

EXAMPLE.

$$16^{\frac{3}{4}} = \sqrt[4]{16^3} = \sqrt[4]{4096} = 8.$$

Thus, cube 16 and extract the fourth root of the product.

RECIPROCALS.

The reciprocal of any number is the quotient which is obtained when 1 is divided by the number. For instance, the reciprocal of 4 is $\frac{1}{4} = 0.25$; the reciprocal of 16 is $\frac{1}{16} =$ 0.0625, etc.

Frequently it is a saving of time when performing long division to use the reciprocal, as multiplying the dividend by the reciprocal of the divisor gives the quotient. For instance, divide 4 by 758. In Table No. 6 the reciprocal of 758 is given as 0.0013193. Multiplying 0.0013193 by 4 gives 0.0052772, which is correct to six decimals. When reducing vulgar fractions to decimals the reciprocal may be used with advantage. For instance, reduce $\frac{15}{64}$ to decimals. In Table No. 6 the reciprocal of 64 is given as 0.015625, and $15 \times 0.015625 = 0.234375$, which is the decimal of $\frac{15}{64}$.

IMPORTANT.—Whenever the exact reciprocal is not expressible by decimals the result obtained by its use is, as explained above, only approximate.

TABLE No. 5. Giving Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Fractions and Mixed Numbers, from $\frac{1}{64}$ to 10.

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
$\frac{1}{64}$	0.000244	0.0000038	0.125	0.25	64
$\frac{1}{32}$	0.000977	0.0000305	0.17678	0.31496	32
$\frac{3}{64}$	0.002196	0.000103	0.21651	0.36056	21.3333
$\frac{1}{16}$	0.003906	0.000244	0.25	0.39685	16
$\frac{5}{64}$	0.006104	0.000477	0.27951	0.42750	12.8
$\frac{3}{32}$	0.008789	0.000823	0.30619	0.45428	10.6667
$\frac{7}{64}$	0.011963	0.001308	0.33072	0.47823	9.1428
$\frac{1}{8}$	0.015625	0.001953	0.35355	0.5	8
$\frac{9}{64}$	0.01977	0.00278	0.375	0.52002	7.1111
$\frac{5}{32}$	0.02441	0.00381	0.39528	0.53861	6.4
$\frac{11}{64}$	0.02954	0.00508	0.41458	0.55599	5.8182
$\frac{3}{16}$	0.03516	0.00659	0.43301	0.57236	5.3333
$\frac{13}{64}$	0.04126	0.00838	0.45069	0.58783	4.9231
$\frac{7}{32}$	0.04785	0.01047	0.46771	0.60254	4.5714
$\frac{15}{64}$	0.05493	0.01287	0.48412	0.61655	4.2666
$\frac{1}{4}$	0.06250	0.01562	0.5	0.62996	4
$\frac{17}{64}$	0.07056	0.01874	0.51539	0.64282	3.7647
$\frac{9}{32}$	0.07910	0.02225	0.53033	0.65519	3.5556
$\frac{19}{64}$	0.08813	0.02616	0.54482	0.66709	3.3684
$\frac{5}{16}$	0.09766	0.03052	0.55902	0.67860	3.2
$\frac{21}{64}$	0.10766	0.03533	0.57282	0.68973	3.0476
$\frac{11}{32}$	0.11816	0.04062	0.58630	0.70051	2.9091
$\frac{23}{64}$	0.12915	0.04641	0.59942	0.71097	2.7826
$\frac{3}{8}$	0.14062	0.05273	0.61237	0.72112	2.6667
$\frac{25}{64}$	0.15258	0.05960	0.625	0.73100	2.56
$\frac{13}{32}$	0.16504	0.06705	0.63738	0.74062	2.4615
$\frac{27}{64}$	0.17798	0.07508	0.64952	0.75	2.3703
$\frac{7}{16}$	0.19141	0.08374	0.66144	0.75915	2.2857
$\frac{29}{64}$	0.20522	0.09303	0.67314	0.76808	2.2069
$\frac{15}{32}$	0.21973	0.10300	0.68465	0.77681	2.1333
$\frac{31}{64}$	0.23463	0.11364	0.69597	0.78534	2.0645
$\frac{1}{2}$	0.25	0.12500	0.70711	0.79370	2

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
$\frac{33}{64}$	0.26587	0.13709	0.71807	0.80188	1.9394
$\frac{17}{32}$	0.28225	0.14993	0.72887	0.80990	1.8823
$\frac{35}{64}$	0.29906	0.16356	0.73951	0.81777	1.8286
$\frac{9}{16}$	0.31641	0.17789	0.75000	0.82548	1.7778
$\frac{37}{64}$	0.33423	0.19315	0.76034	0.83306	1.7297
$\frac{19}{32}$	0.35254	0.20932	0.77055	0.84049	1.6842
$\frac{39}{64}$	0.37134	0.22628	0.78062	0.84781	1.6410
$\frac{5}{8}$	0.39062	0.24414	0.79057	0.85499	1.6
$\frac{41}{64}$	0.41040	0.26291	0.80039	0.86205	1.5610
$\frac{21}{32}$	0.43066	0.28262	0.81009	0.86901	1.5238
$\frac{43}{64}$	0.45141	0.30330	0.81968	0.87585	1.4884
$\frac{11}{16}$	0.47266	0.32495	0.82916	0.88259	1.4545
$\frac{45}{64}$	0.49438	0.34761	0.83853	0.88922	1.4222
$\frac{23}{32}$	0.51660	0.37131	0.84779	0.89576	1.3913
$\frac{47}{64}$	0.53931	0.39605	0.85696	0.90221	1.3617
$\frac{3}{4}$	0.56250	0.42187	0.86603	0.90856	1.3333
$\frac{49}{64}$	0.58618	0.44880	0.87500	0.91483	1.3061
$\frac{25}{32}$	0.61035	0.47684	0.88388	0.92101	1.2800
$\frac{51}{64}$	0.63501	0.50602	0.89268	0.92711	1.2549
$\frac{13}{16}$	0.66016	0.53638	0.90139	0.93313	1.2308
$\frac{53}{64}$	0.68579	0.56792	0.91001	0.93907	1.2075
$\frac{27}{32}$	0.71191	0.60068	0.91856	0.94494	1.1852
$\frac{55}{64}$	0.73853	0.63467	0.92702	0.95074	1.1636
$\frac{7}{8}$	0.76562	0.66992	0.93541	0.95647	1.1428
$\frac{57}{64}$	0.79321	0.70646	0.94373	0.96213	1.1228
$\frac{29}{32}$	0.82129	0.74429	0.95197	0.96772	1.1034
$\frac{59}{64}$	0.84985	0.78346	0.96014	0.97325	1.0847
$\frac{15}{16}$	0.87891	0.82397	0.96825	0.97872	1.0667
$\frac{61}{64}$	0.90845	0.86586	0.97628	0.98412	1.0492
$\frac{31}{32}$	0.93848	0.90915	0.98425	0.98947	1.0323
$\frac{63}{64}$	0.96899	0.95385	0.99216	0.99476	1.01587
1	1	1	1	1	1
$1\frac{1}{16}$	1.12891	1.19943	1.03078	1.02041	0.94118
$1\frac{1}{8}$	1.26562	1.42323	1.06066	1.04004	0.88889

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
$1\frac{3}{16}$	1.41016	1.67456	1.08965	1.05896	0.84211
$1\frac{1}{4}$	1.5625	1.953125	1.11803	1.07722	0.8
$1\frac{5}{16}$	1.72266	2.26099	1.14564	1.09488	0.76190
$1\frac{3}{8}$	1.89062	2.59961	1.17260	1.11199	0.72727
$1\frac{7}{16}$	2.06641	2.97046	1.19896	1.12859	0.69565
$1\frac{1}{2}$	2.25	3.375	1.22474	1.14471	0.66667
$1\frac{9}{16}$	2.44141	3.81470	1.25	1.16040	0.64
$1\frac{5}{8}$	2.640625	4.29102	1.27475	1.17567	0.61539
$1\frac{11}{16}$	2.84766	4.80542	1.29904	1.19055	0.59260
$1\frac{3}{4}$	3.0625	5.35937	1.32288	1.20507	0.57143
$1\frac{13}{16}$	3.28516	5.95434	1.34630	1.21925	0.55172
$1\frac{7}{8}$	3.515625	6.59180	1.36931	1.23311	0.53333
$1\frac{15}{16}$	3.75391	7.27319	1.39194	1.24666	0.51613
2	4	8	1.41421	1.25992	0.5
$2\frac{1}{16}$	4.25390	8.77368	1.43614	1.27291	0.48485
$2\frac{1}{8}$	4.515625	9.59582	1.45774	1.28564	0.47059
$2\frac{3}{16}$	4.78516	10.46753	1.47902	1.29812	0.45714
$2\frac{1}{4}$	5.0625	11.390625	1.5	1.31037	0.44444
$2\frac{5}{16}$	5.34766	12.36646	1.52069	1.32239	0.43243
$2\frac{3}{8}$	5.640625	13.39648	1.54110	1.33420	0.42105
$2\frac{7}{16}$	5.94141	14.48217	1.56125	1.34580	0.41026
$2\frac{1}{2}$	6.25	15.625	1.58114	1.35721	0.4
$2\frac{9}{16}$	6.56541	16.82641	1.60078	1.36843	0.39024
$2\frac{5}{8}$	6.890625	18.08789	1.62018	1.37946	0.38095
$2\frac{11}{16}$	7.22266	19.41090	1.63936	1.39032	0.37209
$2\frac{3}{4}$	7.5625	20.79687	1.65831	1.40101	0.36364
$2\frac{13}{16}$	7.91016	22.24731	1.67705	1.41155	0.35555
$2\frac{7}{8}$	8.265625	23.76367	1.69558	1.42193	0.34783
$2\frac{15}{16}$	8.62891	25.34724	1.71391	1.43216	0.34042
3	9	27	1.73205	1.44225	0.33333
$3\frac{1}{8}$	9.765625	30.51758	1.76777	1.46201	0.32
$3\frac{1}{4}$	10.5625	34.32812	1.80278	1.48125	0.3077
$3\frac{3}{8}$	11.390625	38.44336	1.83712	1.5	0.2963

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
$3\frac{1}{2}$	12.25	42.875	1.87083	1.51829	0.28571
$3\frac{5}{8}$	13.140625	47.63476	1.90394	1.53616	0.27586
$3\frac{3}{4}$	14.0625	52.73437	1.93649	1.55362	0.26667
$3\frac{7}{8}$	15.015625	58.18555	1.96850	1.57069	0.25806
4	16	64	2	1.58740	0.25
$4\frac{1}{4}$	18.0625	76.76562	2.06155	1.61981	0.23529
$4\frac{1}{2}$	20.25	91.125	2.12132	1.65096	0.22222
$4\frac{3}{4}$	22.5625	107.17187	2.17945	1.68099	0.21053
5	25	125	2.23607	1.70998	0.2
$5\frac{1}{4}$	27.5625	144.70312	2.291288	1.73801	0.19048
$5\frac{1}{2}$	30.25	166.375	2.34521	1.76517	0.18182
$5\frac{3}{4}$	33.0625	190.10937	2.39792	1.79152	0.17391
6	36	216	2.44949	1.81712	0.16667
$6\frac{1}{4}$	39.0625	244.140625	2.5	1.84202	0.16
$6\frac{1}{2}$	42.25	274.625	2.54951	1.86626	0.15385
$6\frac{3}{4}$	45.5625	307.54687	2.59808	1.88988	0.14815
7	49	343	2.64575	1.91293	0.14286
$7\frac{1}{4}$	52.5625	381.07812	2.69258	1.93544	0.13793
$7\frac{1}{2}$	56.25	421.875	2.73861	1.95743	0.13333
$7\frac{3}{4}$	60.0625	465.48437	2.78388	1.97895	0.12903
8	64	512	2.82843	2	0.125
$8\frac{1}{4}$	68.0625	561.5156	2.87228	2.02062	0.12121
$8\frac{1}{2}$	72.25	614.125	2.91548	2.04083	0.11765
$8\frac{3}{4}$	76.5625	669.92187	2.95804	2.06064	0.11428
9	81	729	3	2.08008	0.111111
$9\frac{1}{4}$	85.5625	791.4531	3.04138	2.09917	0.10811
$9\frac{1}{2}$	90.25	857.375	3.08221	2.11791	0.10526
$9\frac{3}{4}$	95.0625	926.8594	3.1225	2.13633	0.10256
10	100	1000	3.16228	2.15443	0.1

TABLE No. 6. Giving Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers from 0.1 to 1000.

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
0.1	0.01	0.001	0.31623	0.46416	10
0.2	0.04	0.008	0.44721	0.58480	5
0.3	0.09	0.027	0.54772	0.66943	3.3333
0.4	0.16	0.064	0.63245	0.73681	2.5
0.5	0.25	0.125	0.70711	0.79370	2
0.6	0.36	0.216	0.774597	0.84343	1.66667
0.7	0.49	0.343	0.83666	0.88790	1.42857
0.8	0.68	0.512	0.89443	0.92832	1.25
0.9	0.81	0.729	0.94868	0.96549	1.11111
1	1	1	1	1	1
1.1	1.21	1.331	1.04881	1.03228	0.909091
1.2	1.44	1.728	1.09545	1.06266	0.833333
1.3	1.69	2.197	1.14018	1.09134	0.769231
1.4	1.96	2.744	1.18322	1.11869	0.714286
1.5	2.25	3.375	1.22475	1.14471	0.666667
1.6	2.56	4.096	1.26491	1.16961	0.625
1.7	2.89	4.913	1.30384	1.19347	0.588235
1.8	3.24	5.832	1.34164	1.21644	0.555556
1.9	3.61	6.859	1.37840	1.23855	0.526316
2	4	8	1.41421	1.25992	0.5
2.1	4.41	9.261	1.449138	1.28058	0.476190
2.2	4.84	10.648	1.48324	1 30059	0.454545
2.3	5.29	12.167	1.51657	1.32001	0.434783
2.4	5.76	13.824	1.54919	1.33887	0.416667
2.5	6.25	15.625	1.58114	1.35721	0.4
2.6	6.76	17.576	1.61245	1.37508	0.384615
2.7	7.29	19.683	1.64317	1.39248	0.37037
2.8	7.84	21.952	1.67332	1.40946	0.357143
2.9	8.41	24.389	1.70294	1.42604	0.344828
3	9	27	1.73205	1.44225	0.333333
3.2	10.24	32.768	1.78885	1.47361	0.3125
3.4	11.56	39.304	1.84391	1.50369	0.294118
3.6	12.96	46.656	1.89737	1.53262	0.277778
3.8	14.44	54.872	1.94936	1.56089	0.263158
4	16	64	2	1.58740	0.25
4.2	17.64	74.088	2.04939	1.61343	0.238095
4.4	19.36	85.184	2.09762	1.63868	0.227273
4.6	21.16	97.336	2.14476	1.66310	0.217391
4.8	23.04	110.592	2.19089	1.68687	0.208333
5	25	125	2.23607	1.70998	0.2

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
6	36	216	2.44949	1.81712	0.1666667
7	49	343	2.64575	1.91293	0.142857
8	64	512	2.82843	2	0.125
9	81	729	3	2.08008	0.111111
10	100	1000	3.16228	2.15443	0.1
11	121	1331	3.31662	2.22398	0.0909091
12	144	1728	3.46410	2.28943	0.0833333
13	169	2197	3.60555	2.35133	0.0769231
14	196	2744	3.74166	2.41014	0.0714286
15	225	3375	3.87298	2.46621	0.0666667
16	256	4096	4	2.51984	0.0625
17	289	4913	4.12311	2.57128	0.0588235
18	324	5832	4.24264	2.62074	0.0555556
19	361	6859	4.35890	2.66840	0.0526316
20	400	8000	4.47214	2.71442	0.05
21	441	9261	4.58258	2.75892	0.0476190
22	484	10648	4.69042	2.80204	0.0454545
23	529	12167	4.79583	2.84387	0.0434783
24	576	13824	4.89898	2.88450	0.0416667
25	625	15625	5	2.92402	0.04
26	676	17576	5.09902	2.96250	0.0384615
27	729	19683	5.19615	3	0.0370370
28	784	21952	5.29150	3.03659	0.0357143
29	841	24389	5.38516	3.07232	0.0344828
30	900	27000	5.47723	3.10723	0.0333333
31	961	29791	5.56776	3.14138	0.0322581
32	1024	32768	5.65685	3.17480	0.03125
33	1089	35937	5.74456	3.20753	0.0303030
34	1156	39304	5.83095	3.23961	0.0294118
35	1225	42875	5.91608	3.27107	0.0285714
36	1296	46656	6	3.30193	0.0277778
37	1369	50653	6.08276	3.33222	0.0270270
38	1444	54872	6.16441	3.36198	0.0263158
39	1521	59319	6.245	3.39121	0.0256410
40	1600	64000	6.32456	3.41995	0.025
41	1681	68921	6.40312	3.44822	0.0243902
42	1764	74088	6.48074	3.47603	0.0238095
43	1849	79507	6.55744	3.50340	0.0232558
44	1936	85184	6.63325	3.53035	0.0227273

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
45	2025	91125	6.70820	3.55689	0.0222222
46	2116	97336	6.78233	3.58305	0.0217391
47	2209	103823	6.85565	3.60883	0.0212766
48	2304	110592	6.92820	3.63424	0.0208333
49	2401	117649	7	3.65931	0.0204082
50	2500	125000	7.07107	3.68403	0.02
51	2601	132651	7.14143	3.70843	0.0196078
52	2704	140608	7.21110	3.73251	0.0192308
53	2809	148877	7.28011	3.75629	0.0188679
54	2916	157464	7.34847	3.77976	0.0185185
55	3025	166375	7.41620	3.80295	0.0181818
56	3136	175616	7.48331	3.82586	0.0178571
57	3249	185193	7.54983	3.84852	0.0175439
58	3364	195112	7.61577	3.87088	0.0172414
59	3481	205379	7.68115	3.89300	0.0169492
60	3600	216000	7.74597	3.91487	0.0166667
61	3721	226981	7.81025	3.93650	0.0163934
62	3844	238328	7.87401	3.95789	0.0161290
63	3969	250047	7.93725	3.97906	0.0158730
64	4096	262144	8	4	0.0156250
65	4225	274625	8.06226	4.02073	0.0153846
66	4356	287496	8.12404	4.04124	0.0151515
67	4489	300763	8.18535	4.06155	0.0149254
68	4624	314432	8.24621	4.08166	0.0147059
69	4761	328509	8.30662	4.10157	0.0144928
70	4900	343000	8.36660	4.12129	0.0142857
71	5041	357911	8.42615	4.14082	0.0140845
72	5184	373248	8.48528	4.16017	0.0138889
73	5329	389017	8.54400	4.17934	0.0136986
74	5476	405224	8.60233	4.19834	0.0135135
75	5625	421875	8.66025	4.21716	0.0133333
76	5776	438976	8.71780	4.23582	0.0131579
77	5929	456533	8.77496	4.25432	0.0129870
78	6084	474552	8.83176	4.27266	0.0128205
79	6241	493039	8.88819	4.29084	0.0126582
80	6400	512000	8.94427	4.30887	0.0125
81	6561	531441	9	4.32675	0.0123457
82	6724	551368	9.05539	4.34448	0.0121951
83	6889	571787	9.11043	4.36207	0.0120482
84	7056	592704	9.16515	4.37952	0.0119048

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
85	7225	614125	9.21954	4.39683	0.0117647
86	7396	636056	9.27362	4.414	0.0116279
87	7569	658503	9.32738	4.43105	0.0114943
88	7744	681472	9.38083	4.44797	0.0113636
89	7921	704969	9.43398	4.46475	0.0112360
90	8100	729000	9.48683	4.48140	0.0111111
91	8281	753571	9.53939	4.49794	0.0109890
92	8464	778688	9.59166	4.51436	0.0108696
93	8649	804357	9.64365	4.53065	0.0107527
94	8836	830584	9.69536	4.54684	0.0106383
95	9025	857375	9.74679	4.56290	0.0105263
96	9216	884736	9.79796	4.57886	0.0104167
97	9409	912673	9.84886	4.59470	0.0103093
98	9604	941192	9.89949	4.61044	0.0102041
99	9801	970299	9.94987	4.62607	0.0101010
100	10000	1000000	10	4.64159	0.01
101	10201	1030301	10.04988	4.65701	0.0099010
102	10404	1061208	10.09950	4.67233	0.0098039
103	10609	1092727	10.14889	4.68755	0.0097087
104	10816	1124864	10.19804	4.70267	0.0096154
105	11025	1157625	10.24695	4.71769	0.0095238
106	11236	1191016	10.29563	4.73262	0.0094340
107	11449	1225043	10.34408	4.74746	0.0093458
108	11664	1259712	10.39230	4.76220	0.0092593
109	11881	1295029	10.44031	4.77686	0.0091743
110	12100	1331000	10.48809	4.79142	0.0090909
111	12321	1367631	10.53565	4.80590	0.0090090
112	12544	1404928	10.58301	4.82028	0.0089286
113	12769	1442897	10.63015	4.83459	0.0088496
114	12996	1481544	10.67708	4.84881	0.0087719
115	13225	1520875	10.72381	4.86294	0.0086957
116	13456	1560896	10.77033	4.877	0.0086207
117	13689	1601613	10.81665	4.89097	0.0085470
118	13924	1643032	10.86278	4.90487	0.0084746
119	14161	1685159	10.90871	4.91868	0.0084034
120	14400	1728000	10.95445	4.93242	0.0083333
121	14641	1771561	11	4.94609	0.0082645
122	14884	1815848	11.04536	4.95968	0.0081967
123	15129	1860867	11.09054	4.97319	0.0081301
124	15376	1906624	11.13553	4.98663	0.0080645

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
125	15625	1953125	11.18034	5	0.008
126	15876	2000376	11.22497	5.01330	0.0079365
127	16129	2048383	11.26943	5.02653	0.0078740
128	16384	2097152	11.31371	5.03968	0.0078125
129	16641	2146689	11.35782	5.05277	0.0077519
130	16900	2197000	11.40175	5.06580	0.0076923
131	17161	2248091	11.44552	5.07875	0.0076336
132	17424	2299968	11.48913	5.09164	0.0075758
133	17689	2352637	11.53256	5.10447	0.0075188
134	17956	2406104	11.57584	5.11723	0.0074627
135	18225	2460375	11.61895	5.12993	0.0074074
136	18496	2515456	11.66190	5.14256	0.0073529
137	18769	2571353	11.70470	5.15514	0.0072993
138	19044	2628072	11.74734	5.16765	0.0072464
139	19321	2685619	11.78983	5.18010	0.0071942
140	19600	2744000	11.83216	5.19249	0.0071429
141	19881	2803221	11.87434	5.20483	0.0070922
142	20164	2863288	11.91638	5.21710	0.0070423
143	20449	2924207	11.95826	5.22932	0.0069930
144	20736	2985984	12	5.24148	0.0069444
145	21025	3048625	12.04159	5.25359	0.0068966
146	21316	3112136	12.08305	5.26564	0.0068493
147	21609	3176523	12.12436	5.27763	0.0068027
148	21904	3241792	12.16553	5.28957	0.0067568
149	22201	3307949	12.20656	5.30146	0.0067114
150	22500	3375000	12.24745	5.31329	0.0066667
151	22801	3442951	12.28821	5.32507	0.0066225
152	23104	3511808	12.32883	5.33680	0.0065789
153	23409	3581577	12.36932	5.34848	0.0065359
154	23716	3652264	12.40967	5.36011	0.0064935
155	24025	3723875	12.44990	5.37169	0.0064516
156	24336	3796416	12.49	5.38321	0.0064103
157	24649	3869893	12.52996	5.39469	0.0063694
158	24964	3944312	12.56981	5.40612	0.0063291
159	25281	4019679	12.60952	5.41750	0.0062893
160	25600	4096000	12.64911	5.42884	0.00625
161	25921	4173281	12.68858	5.44012	0.0062112
162	26244	4251528	12.72792	5.45136	0.0061728
163	26569	4330747	12.76715	5.46256	0.0061350
164	26896	4410944	12.80625	5.47370	0.0060976

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
165	27225	4492125	12.84523	5.48481	0.0060606
166	27556	4574296	12.88410	5.49586	0.0060241
167	27889	4657463	12.92285	5.50688	0.0059880
168	28224	4741632	12.96148	5.51785	0.0059524
169	28561	4826809	13	5.52877	0.0059172
170	28900	4913000	13.03840	5.53966	0.0058824
171	29241	5000211	13.07670	5.55050	0.0058480
172	29584	5088448	13.11488	5.56130	0.0058140
173	29929	5177717	13.15295	5.57205	0.0057803
174	30276	5268024	13.19091	5.58277	0.0057471
175	30625	5359375	13.22876	5.59344	0.0057143
176	30976	5451776	13.26650	5.60408	0.0056818
177	31329	5545233	13.30413	5.61467	0.0056497
178	31684	5639752	13.34165	5.62523	0.0056180
179	32041	5735339	13.37909	5.63574	0.0055866
180	32400	5832000	13.41641	5.64622	0.0055556
181	32761	5929741	13.45362	5.65665	0.0055249
182	33124	6028568	13.49074	5.66705	0.0054945
183	33489	6128487	13.52775	5.67741	0.0054645
184	33856	6229504	13.56466	5.68773	0.0054348
185	34225	6331625	13.60147	5.69802	0.0054054
186	34596	6434850	13.63818	5.70827	0.0053763
187	34969	6539203	13.67479	5.71848	0.0053476
188	35334	6644672	13.71131	5.72865	0.0053191
189	35721	6751269	13.74773	5.73879	0.0052910
190	36100	6859000	13.78405	5.74890	0.0052632
191	36481	6967871	13.82028	5.75897	0.0052356
192	36864	7077888	13.85641	5.769	0.0052083
193	37249	7189057	13.89244	5.779	0.0051813
194	37636	7301384	13.92839	5.78896	0.0051546
195	38025	7414875	13.96424	5.79889	0.0051282
196	38416	7529536	14	5.80879	0.0051020
197	38809	7645372	14.03567	5.81865	0.0050761
198	39204	7762392	14.07125	5.82848	0.0050505
199	39601	7880599	14.10674	5.83827	0.0050251
200	40000	8000000	14.14214	5.84804	0.005
201	40401	8120601	14.17745	5.45777	0.0049751
202	40804	8242408	14.21267	5.86747	0.0049505
203	41209	8365427	14.24781	5.87713	0.0049261
204	41616	8489664	14.28286	5.88677	0.0049020

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
205	42025	8615125	14.31782	5.89637	0.0048781
206	42436	8741816	14.35270	5.90594	0.0048544
207	42849	8869743	14.38749	5.91548	0.0048309
208	43264	8998912	14.42221	5.92499	0.0048077
209	43681	9129320	14.45683	5.93447	0.0047847
210	44100	9261000	14.49138	5.94392	0.0047619
211	44521	9393931	14.52584	5.95334	0.0047393
212	44944	9528128	14.56022	5.96273	0.0047170
213	45369	9663597	14.59452	5.97209	0.0046948
214	45796	9800344	14.62874	5.98142	0.0046729
215	46225	9938375	14.66288	5.99073	0.0046512
216	46656	10077696	14.69694	6	0.0046296
217	47089	10218313	14.73092	6.00925	0.0046083
218	47524	10360232	14.76482	6.01846	0.0045872
219	47961	10503459	14.79865	6.02765	0.0045662
220	48400	10648000	14.83240	6.03681	0.0045455
221	48841	10793861	14.86607	6.04594	0.0045249
222	49284	10941048	14.89966	6.05505	0.0045045
223	49729	11089567	14.93318	6.06413	0.0044843
224	50176	11239424	14.96663	6.07318	0.0044643
225	50625	11390625	15	6.08220	0.0044444
226	51076	11543176	15.03330	6.09120	0.0044248
227	51529	11697083	15.06652	6.10017	0.0044053
228	51984	11852352	15.09967	6.10911	0.0043860
229	52441	12008989	15.13275	6.11803	0.0043668
230	52900	12167000	15.16575	6.12693	0.0043478
231	53361	12326391	15.19868	6.13579	0.0043290
232	53824	12487168	15.23155	6.14463	0.0043103
233	54289	12649337	15.26434	6.15345	0.0042918
234	54756	12812904	15.29706	6.16224	0.0042735
235	55225	12977875	15.32971	6.17101	0.0042553
236	55696	13144256	15.36229	6.17975	0.0042373
237	56169	13312053	15.39480	6.18846	0.0042194
238	56644	13481272	15.42725	6.19715	0.0042017
239	57121	13651919	15.45962	6.20582	0.0041841
240	57600	13824000	15.49193	6.21447	0.0041667
241	58081	13997521	15.52417	6.22308	0.0041494
242	58564	14172488	15.55635	6.23168	0.0041322
243	59049	14348907	15.58846	6.24025	0.0041152
244	59536	14526784	15.62050	6.24880	0.0040984

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
245	60025	14706125	15.65248	6.25732	0.0040816
246	60516	14886936	15.68439	6.26583	0.0040650
247	61009	15069223	15.71623	6.27431	0.0040486
248	61504	15252992	15.74802	6.28276	0.0040323
249	62001	15438249	15.77973	6.29119	0.0040161
250	62500	15625000	15.81139	6.29961	0.004
251	63001	15813251	15.84298	6.30799	0.0039841
252	63504	16003008	15.87451	6.31636	0.0039683
253	64009	16194277	15.90597	6.32470	0.0039526
254	64516	16387064	15.93738	6.33303	0.0039370
255	65025	16581375	15.96872	6.34133	0.0039216
256	65536	16777216	16	6.34960	0.0039062
257	66049	16974593	16.03122	6.35786	0.0038911
258	66564	17173512	16.06238	6.36610	0.0038760
259	67081	17373979	16.09348	6.37431	0.0038610
260	67600	17576000	16.12452	6.38250	0.0038462
261	68121	17779581	16.15549	6.39068	0.0038314
262	68644	17984728	16.18641	6.39883	0.0038168
263	69169	18191447	16.21727	6.40696	0.0038023
264	69696	18399744	16.24808	6.41507	0.0037879
265	70225	18609625	16.27882	6.42316	0.0037736
266	70756	18821096	16.30951	6.43123	0.0037594
267	71289	19034163	16.34013	6.43928	0.0037453
268	71824	19248832	16.37071	6.44731	0.0037313
269	72361	19465109	16.40122	6.45531	0.0037175
270	72900	19683000	16.43168	6.46330	0.0037037
271	73441	19902511	16.46208	6.47127	0.00369
272	73984	20123648	16.49242	6.47922	0.0036765
273	74529	20346417	16.52271	6.48715	0.0036630
274	75076	20570824	16.55295	6.49507	0.0036496
275	75625	20796875	16.58312	6.50296	0.0036364
276	76176	21024576	16.61325	6.51083	0.0036232
277	76729	21253933	16.64332	6.51868	0.0036101
278	77284	21484952	16.67333	6.52652	0.0035971
279	77841	21717639	16.70329	6.53434	0.0035842
280	78400	21952000	16.73320	6.54213	0.0035714
281	78961	22188041	16.76305	6.54991	0.0035587
282	79524	22425768	16.79286	6.55767	0.0035461
283	80089	22665187	16.82260	6.56541	0.0035336
284	80656	22906304	16.85230	6.57314	0.0035211

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
285	81225	23149125	16.88194	6.58084	0.0035088
286	81796	23393656	16.91153	6.58853	0.0034965
287	82369	23639903	16.94107	6.59620	0.0034843
288	82944	23887872	16.97056	6.60385	0.0034722
289	83521	24137569	17	6.61149	0.0034602
290	84100	24389000	17.02939	6.61911	0.0034483
291	84681	24642171	17.05872	6.62671	0.0034364
292	85264	24897088	17.08801	6.63429	0.0034247
293	85849	25153757	17.11724	6.64185	0.0034130
294	86436	25412184	17.14643	6.64940	0.0034014
295	87025	25672375	17.17556	6.65693	0.0033898
296	87616	25934336	17.20465	6.66444	0.0033784
297	88209	26198073	17.23369	6.67194	0.0033670
298	88804	26463592	17.26268	6.67942	0.0033557
299	89401	26730899	17.29162	6.68688	0.0033445
300	90000	27000000	17.32051	6.69433	0.0033333
301	90601	27270901	17.34935	6.70176	0.0033223
302	91204	27543608	17.37815	6.70917	0.0033113
303	91809	27818127	17.40690	6.71657	0.0033003
304	92416	28094464	17.43560	6.72395	0.0032895
305	93025	28372625	17.46425	6.73132	0.0032787
306	93636	28652616	17.49286	6.73866	0.0032680
307	94249	28934443	17.52142	6.746	0.0032573
308	94864	29218112	17.54993	6.75331	0.0032468
309	95481	29503629	17.57840	6.76061	0.0032362
310	96100	29791000	17.60682	6.76790	0.0032258
311	96721	30080231	17.63519	6.77517	0.0032154
312	97344	30371328	17.66352	6.78242	0.0032051
313	97969	30664297	17.69181	6.78966	0.0031949
314	98596	30959144	17.72005	6.79688	0.0031847
315	99225	31255875	17.74824	6.80409	0.0031746
316	99856	31554496	17.77639	6.81128	0.0031646
317	100489	31855013	17.80449	6.81846	0.0031546
318	101124	32157432	17.83255	6.82562	0.0031447
319	101761	32461759	17.86057	6.83277	0.0031348
320	102400	32768000	17.88854	6.83990	0.0031250
321	103041	33076161	17.91647	6.84702	0.0031153
322	103684	33386248	17.94436	6.85412	0.0031056
323	104329	33698267	17.97220	6.86121	0.0030960
324	104976	34012224	18	6.86829	0.0030864

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
325	105625	34328125	18.02776	6.87534	0.0030769
326	106276	34645976	18.05547	6.88239	0.0030675
327	106929	34965783	18.08314	6.88942	0.0030581
328	107584	35287552	18.11077	6.89643	0.0030488
329	108241	35611289	18.13836	6.90344	0.0030395
330	108900	35937000	18.16590	6.91042	0.0030303
331	109561	36264691	18.19341	6.91740	0.0030211
332	110224	36594368	18.22087	6.92436	0.0030120
333	110889	36926037	18.24829	6.93131	0.0030030
334	111556	37259704	18.27567	6.93823	0.0029940
335	112225	37595375	18.30301	6.94515	0.0029851
336	112896	37933056	18.33030	6.95205	0.0029762
337	113569	38272753	18.35756	6.95894	0.0029674
338	114244	38614472	18.38478	6.96582	0.0029586
339	114921	38958219	18.41195	6.97268	0.0029499
340	115600	39304000	18.43909	6.97953	0.0029412
341	116281	39651821	18.46619	6.98637	0.0029326
342	116964	40001688	18.49324	6.99319	0.0029240
343	117649	40353607	18.52026	7	0.0029155
344	118336	40707584	18.54724	7.00680	0.0029070
345	119025	41063625	18.57418	7.01358	0.0028986
346	119716	41421736	18.60108	7.02035	0.0028902
347	120409	41781923	18.62794	7.02711	0.0028818
348	121104	42144192	18.65476	7.03385	0.0028736
349	121801	42508549	18.68154	7.04059	0.0028653
350	122500	42875000	18.70829	7.04730	0.0028571
351	123201	43243551	18.73499	7.054	0.0028490
352	123904	43614208	18.76166	7.06070	0.0028409
353	124609	43986977	18.78829	7.06738	0.0028329
354	125316	44361864	18.81489	7.07404	0.0028249
355	126025	44738875	18.84144	7.08070	0.0028169
356	126736	45118016	18.86796	7.08734	0.0028090
357	127449	45499293	18.89444	7.09397	0.0028011
358	128164	45882712	18.92089	7.10059	0.0027933
359	128881	46268279	18.94730	7.10719	0.0027855
360	129600	46656000	18.97367	7.11379	0.0027778
361	130321	47045881	19	7.12037	0.0027701
362	131044	47437928	19.02630	7.12694	0.0027624
363	131769	47832147	19.05256	7.13349	0.0027548
364	132496	48228544	19.07878	7.14004	0.0027473

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
365	133225	48627125	19.10497	7.14657	0.0027397
366	133956	49027896	19.13113	7.15309	0.0027322
367	134689	49430863	19.15724	7.15960	0.0027248
368	135424	49836032	19.18333	7.16610	0.0027174
369	136161	50243409	19.20937	7.17258	0.0027100
370	136900	50653000	19.23538	7.17905	0.0027027
371	137641	51064811	19.26136	7.18552	0.0026954
372	138384	51478848	19.28730	7.19197	0.0026882
373	139129	51895117	19.31321	7.19841	0.0026810
374	139876	52313624	19.33908	7.20483	0.0026738
375	140625	52734375	19.36492	7.21125	0.0026667
376	141376	53157376	19.39072	7.21765	0.0026596
377	142129	53582633	19.41649	7.22405	0.0026525
378	142884	54010152	19.44222	7.23643	0.0026455
379	143641	54439939	19.46792	7.23680	0.0026385
380	144400	54872000	19.49359	7.24316	0.0026316
381	145161	55306341	19.51922	7.24950	0.0026247
382	145924	55742968	19.54482	7.25584	0.0026178
383	146689	56181887	19.57039	7.26217	0.0026110
384	147456	56623104	19.59592	7.26848	0.0026042
385	148225	57066625	19.62142	7.27479	0.0025974
386	148996	57512456	19.64688	7.28108	0.0025907
387	149769	57960603	19.67232	7.28736	0.0025840
388	150544	58411072	19.69772	7.29363	0.0025773
389	151321	58863869	19.72308	7.29989	0.0025707
390	152100	59319000	19.74842	7.30614	0.0025641
391	152881	59776471	19.77372	7.31238	0.0025575
392	153664	60236288	19.79899	7.31861	0.0025510
393	154449	60698457	19.82423	7.32483	0.0025445
394	155236	61162984	19.84943	7.33104	0.0025381
395	156025	61629875	19.87461	7.33723	0.0025316
396	156816	62099136	19.89975	7.34342	0.0025253
397	157609	62570773	19.92486	7.34960	0.0025189
398	158404	63044792	19.94994	7.35576	0.0025126
399	159201	63521199	19.97498	7.36192	0.0025063
400	160000	64000000	20	7.36806	0.0025
401	160801	64481201	20.02498	7.37420	0.0024938
402	161604	64964808	20.04994	7.38032	0.0024876
403	162409	65450827	20.07486	7.38644	0.0024814
404	163216	65939264	20.09975	7.39254	0.0024752

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
405	164025	66430125	20.12461	7.39864	0.0024691
406	164836	66923416	20.14944	7.40472	0.0024631
407	165649	67419143	20.17424	7.41080	0.0024570
408	166464	67917312	20.19901	7.41686	0.0024510
409	167281	68417929	20.22375	7.42291	0.0024450
410	168100	68921000	20.24846	7.42896	0.0024390
411	168921	69426531	20.27313	7.43499	0.0024331
412	169744	69934528	20.29778	7.44102	0.0024272
413	170569	70444997	20.32240	7.44703	0.0024213
414	171396	70957944	20.34699	7.45304	0.0024155
415	172225	71473375	20.37155	7.45904	0.0024096
416	173056	71991296	20.39608	7.46502	0.0024038
417	173889	72511713	20.42058	7.471	0.0023981
418	174724	73034632	20.44505	7.47697	0.0023923
419	175561	73560059	20.46949	7.48292	0.0023866
420	176400	74088000	20.49390	7.48887	0.0023810
421	177241	74618461	20.51828	7.49481	0.0023753
422	178084	75151448	20.54264	7.50074	0.0023697
423	178929	75686967	20.56696	7.50666	0.0023641
424	179776	76225024	20.59126	7.51257	0.0023585
425	180625	76765625	20.61553	7.51847	0.0023529
426	181476	77308776	20.63977	7.52437	0.0023474
427	182329	77854483	20.66398	7.53025	0.0023419
428	183184	78402752	20.68816	7.53612	0.0023364
429	184041	78953589	20.71232	7.54199	0.0023310
430	184900	79507000	20.73644	7.54784	0.0023256
431	185761	80062991	20.76054	7.55369	0.0023202
432	186624	80621568	20.78461	7.55953	0.0023148
433	187489	81182737	20.80865	7.56535	0.0023095
434	188356	81746504	20.83267	7.57117	0.0023041
435	189225	82312875	20.85665	7.57698	0.0022989
436	190096	82881856	20.88061	7.58279	0.0022936
437	190969	83453453	20.90455	7.58858	0.0022883
438	191844	84027672	20.92845	7.59436	0.0022831
439	192721	84604519	20.95233	7.60014	0.0022779
440	193600	85184000	20.97618	7.60590	0.0022727
441	194481	85766121	21	7.61166	0.0022676
442	195364	86350888	21.02380	7.61741	0.0022624
443	196249	86938307	21.04757	7.62315	0.0022573
444	197136	87528384	21.07131	7.62888	0.0022523

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
445	198025	88121125	21.09502	7.63461	0.0022472
446	198916	88716536	21.11871	7.64032	0.0022422
447	199809	89314623	21.14237	7.64603	0.0022371
448	200704	89915392	21.16601	7.65172	0.0022321
449	201601	90518849	21.18962	7.65741	0.0022272
450	202500	91125000	21.21320	7.66309	0.0022222
451	203401	91733851	21.23676	7.66877	0.0022173
452	204304	92345408	21.26029	7.67443	0.0022124
453	205209	92959677	21.28380	7.68009	0.0022075
454	206116	93576664	21.30728	7.68573	0.0022026
455	207025	94196375	21.33073	7.69137	0.0021978
456	207936	94818816	21.35416	7.69700	0.0021930
457	208849	95443993	21.37756	7.70262	0.0021882
458	209764	96071912	21.40093	7.70824	0.0021834
459	210681	96702579	21.42429	7.71384	0.0021786
460	211600	97336000	21.44761	7.71944	0.0021739
461	212521	97972181	21.47091	7.72503	0.0021692
462	213444	98611128	21.49419	7.73061	0.0021645
463	214369	99252847	21.51743	7.73619	0.0021598
464	215296	99897344	21.54066	7.74175	0.0021552
465	216225	100544625	21.56386	7.74731	0.0021505
466	217156	101194696	21.58703	7.75286	0.0021459
467	218089	101847563	21.61018	7.75840	0.0021413
468	219024	102503232	21.63331	7.76394	0.0021368
469	219961	103161709	21.65641	7.76946	0.0021322
470	220900	103823000	21.67948	7.77498	0.0021277
471	221841	104487111	21.70253	7.78049	0.0021231
472	222784	105154048	21.72556	7.78599	0.0021186
473	223729	105823817	21.74856	7.79149	0.0021142
474	224676	106496424	21.77154	7.79697	0.0021097
475	225625	107171875	21.79449	7.80245	0.0021053
476	226576	107850176	21.81742	7.80793	0.0021008
477	227529	108531333	21.84033	7.81339	0.0020965
478	228484	109215352	21.86321	7.81885	0.0020921
479	229441	109902239	21.88607	7.82429	0.0020877
480	230400	110592000	21.90890	7.82974	0.0020833
481	231361	111284641	21.93171	7.83517	0.0020790
482	232324	111980168	21.95450	7.84059	0.0020747
483	233289	112678587	21.97726	7.84601	0.0020704
484	234256	113379904	22	7.85142	0.0020661

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
485	235225	114084125	22.02272	7.85683	0.0020619
486	236196	114791256	22.04541	7.86222	0.0020576
487	237169	115501303	22.06808	7.86761	0.0020534
488	238144	116214272	22.09072	7.87299	0.0020492
489	239121	116930169	22.11334	7.87837	0.0020450
490	240100	117649000	22.13594	7.88374	0.0020408
491	241081	118370771	22.15852	7.88909	0.0020367
492	242064	119095488	22.18107	7.89445	0.0020325
493	243049	119823157	22.20360	7.89979	0.0020284
494	244036	120553784	22.22611	7.90513	0.0020243
495	245025	121287375	22.24860	7.91046	0.0020202
496	246016	122023936	22.27106	7.91578	0.0020161
497	247009	122763473	22.29350	7.92110	0.0020121
498	248004	123505992	22.31591	7.92641	0.0020080
499	249001	124251499	22.33831	7.93171	0.0020040
500	250000	125000000	22.36068	7.93701	0.002
501	251001	125751501	22.38303	7.94229	0.0019960
502	252004	126506008	22.40536	7.94757	0.0019920
503	253009	127263527	22.42766	7.95285	0.0019881
504	254016	128024064	22.44994	7.95811	0.0019841
505	255025	128787626	22.47221	7.96337	0.0019802
506	256036	129554216	22.49444	7.96863	0.0019763
507	257049	130323843	22.51666	7.97387	0.0019724
508	258064	131096512	22.53886	7.97911	0.0019685
509	259081	131872229	22.56103	7.98434	0.0019646
510	260100	132651000	22.58318	7.98957	0.0019608
511	261121	133432831	22.60531	7.99479	0.0019569
512	262144	134217728	22.62742	8	0.0019531
513	263169	135005697	22.64950	8.00520	0.0019493
514	264196	135796744	22.67157	8.01040	0.0019455
515	265225	136590875	22.69361	8.01559	0.0019417
516	266256	137388096	22.71563	8.02078	0.0019380
517	267289	138188413	22.73763	8.02596	0.0019342
518	268324	138991832	22.75961	8.03113	0.0019205
519	269361	139798359	22.78157	8.03629	0.0019268
520	270400	140608000	22.80351	8.04145	0.0019231
521	271441	141420761	22.82542	8.04660	0.0019194
522	272484	142236648	22.84732	8.05175	0.0019157
523	273529	143055667	22.86919	8.05689	0.0019120
524	274576	143877824	22.89105	8.06202	0.0019084

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
525	275625	144703125	22.91288	8.06714	0.0019048
526	276676	145531576	22.93469	8.07226	0.0019011
527	277729	146363183	22.95648	8.07737	0.0018975
528	278784	147197952	22.97825	8.08248	0.0018939
529	279841	148035889	23	8.08758	0.0018904
530	280900	148877000	23.02173	8.09267	0.0018868
531	281961	149721291	23.04344	8.09776	0.0018832
532	283024	150568768	23.06513	8.10284	0.0018797
533	284089	151419437	23.08679	8.10791	0.0018762
534	285156	152273304	23.10844	8.11298	0.0018727
535	286225	153130375	23.13007	8.11804	0.0018692
536	287296	153990656	23.15167	8.12310	0.0018657
537	288369	154854153	23.17326	8.12814	0.0018622
538	289444	155720872	23.19483	8.13319	0.0018587
539	290521	156590819	23.21637	8.13822	0.0018553
540	291600	157464000	23.23790	8.14325	0.0018519
541	292681	158340421	23.25941	8.14828	0.0018484
542	293764	159220088	23.28089	8.15329	0.0018450
543	294849	160103007	23.30236	8.15831	0.0018416
544	295936	160989184	23.32381	8.16331	0.0018382
545	297025	161878625	23.34524	8.16831	0.0018349
546	298116	162771336	23.36664	8.17330	0.0018315
547	299209	163667323	23.38803	8.17829	0.0018282
548	300304	164566592	23.40940	8.18327	0.0018248
549	301401	165469149	23.43075	8.18824	0.0018215
550	302500	166375000	23.45208	8.19321	0.0018182
551	303601	167284151	23.47339	8.19818	0.0018149
552	304704	168196608	23.49468	8.20313	0.0018116
553	305809	169112377	23.51595	8.20808	0.0018083
554	306916	170031464	23.53720	8.21303	0.0018051
555	308025	170953875	23.55844	8.21797	0.0018018
556	309136	171879616	23.57965	8.22290	0.0017986
557	310249	172808693	23.60085	8.22783	0.0017953
558	311364	173741112	23.62202	8.23275	0.0017921
559	312481	174676879	23.64318	8.23766	0.0017889
560	313600	175616000	23.66432	8.24257	0.0017857
561	314721	176558481	23.68544	8.24747	0.0017825
562	315844	177504328	23.70654	8.25237	0.0017794
563	316969	178453547	23.72762	8.25726	0.0017762
564	318096	179406144	23.74868	8.26215	0.0017730

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
565	319225	180362125	23.76973	8.26703	0.0017699
566	320356	181321496	23.79075	8.27190	0.0017668
567	321489	182284263	23.81176	8.27677	0.0017637
568	322624	183250432	23.83275	8.28163	0.0017606
569	323761	184220009	23.85372	8.28649	0.0017575
570	324900	185193000	23.87467	8.29134	0.0017544
571	326041	186169411	23.89561	8.29619	0.0017513
572	327184	187149248	23.91652	8.30103	0.0017483
573	328329	188132517	23.93742	8.30587	0.0017452
574	329476	189119224	23.95830	8.31069	0.0017422
575	330625	190109375	23.97916	8.31552	0.0017391
576	331776	191102976	24	8.32034	0.0017361
577	332929	192100033	24.02082	8.32515	0.0017331
578	334084	193100552	24.04163	8.32995	0.0017301
579	335241	194104539	24.06242	8.33476	0.0017271
580	336400	195112000	24.08319	8.33955	0.0017241
581	337561	196122941	24.10394	8.34434	0.0017212
582	338724	197137368	24.12468	8.34913	0.0017182
583	339889	198155287	24.14539	8.35390	0.0017153
584	341056	199176704	24.16609	8.35868	0.0017123
585	342225	200201625	24.18677	8.36345	0.0017094
586	343396	201230056	24.20744	8.36821	0.0017065
587	344569	202262003	24.22808	8.37297	0.0017036
588	345744	203297472	24.24871	8.37772	0.0017007
589	346921	204336469	24.26932	8.38247	0.0016978
590	348100	205379000	24.28992	8.38721	0.0016949
591	349281	206425071	24.31049	8.39194	0.0016920
592	350464	207474688	24.33105	8.39667	0.0016892
593	351649	208527857	24.35159	8.40140	0.0016863
594	352836	209584584	24.37212	8.40612	0.0016835
595	354025	210644875	24.39262	8.41083	0.0016807
596	355216	211708736	24.41311	8.41554	0.0016779
597	356409	212776173	24.43358	8.42025	0.0016750
598	357604	213847192	24.45404	8.42494	0.0016722
599	358801	214921799	24.47448	8.42964	0.0016694
600	360000	216000000	24.49490	8.43433	0.0016667
601	361201	217081801	24.51530	8.43901	0.0016639
602	362404	218167208	24.53569	8.44369	0.0016611
603	363609	219256227	24.55606	8.44836	0.0016584
604	364816	220348864	24.57641	8.45303	0.0016556

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
605	366025	221445125	24.59675	8.45769	0.0016529
606	367236	222545016	24.61707	8.46235	0.0016502
607	368449	223648543	24.63737	8.46700	0.0016474
608	369664	224755712	24.65766	8.47165	0.0016447
609	370881	225866529	24.67793	8.47629	0.0016420
610	372100	226981000	24.69818	8.48093	0.0016393
611	373321	228099131	24.71841	8.48556	0.0016367
612	374544	229220928	24.73863	8.49018	0.0016340
613	375769	230346397	24.75884	8.49481	0.0016313
614	376996	231475544	24.77902	8.49942	0.0016287
615	378225	232608375	24.79919	8.50404	0.0016260
616	379456	233744896	24.81935	8.50864	0.0016234
617	380689	234885113	24.83948	8.51324	0.0016207
618	381924	236029032	24.85961	8.51784	0.0016181
619	383161	237176659	24.87971	8.52243	0.0016155
620	384400	238328000	24.89980	8.52702	0.0016129
621	385641	239483061	24.91987	8.53160	0.0016103
622	386884	240641848	24.93993	8.53618	0.0016077
623	388129	241804367	24.95997	8.54075	0.0016051
624	389376	242970624	24.97999	8.54532	0.0016026
625	390625	244140625	25	8.54988	0.0016000
626	391876	245314376	25.01999	8.55444	0.0015974
627	393129	246491883	25.03997	8.55899	0.0015949
628	394384	247673152	25.05993	8.56354	0.0015924
629	395641	248858189	25.07987	8.56808	0.0015898
630	396900	250047000	25.09980	8.57262	0.0015873
631	398161	251239591	25.11971	8.57715	0.0015848
632	399424	252435968	25.13961	8.58168	0.0015823
633	400689	253636137	25.15949	8.58622	0.0015798
634	401956	254840104	25.17936	8.59072	0.0015773
635	403225	256047875	25.19921	8.59524	0.0015748
636	404496	257259456	25.21904	8.59975	0.0015723
637	405769	258474853	25.23886	8.60425	0.0015699
638	407044	259694072	25.25866	8.60875	0.0015674
639	408321	260917119	25.27845	8.61325	0.0015649
640	409600	262144000	25.29822	8.61774	0.0015625
641	410881	263374721	25.31798	8.62222	0.0015601
642	412164	264609288	25.33772	8.62671	0.0015576
643	413449	265847707	25.35744	8.63118	0.0015552
644	414736	267089984	25.37716	8.63566	0.0015528

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
645	416025	268336125	25.39685	8.64012	0.0015504
646	417316	269586136	25.41653	8.64459	0.0015480
647	418609	270840023	25.43619	8.64904	0.0015456
648	419904	272097792	25.45584	8.65350	0.0015432
649	421201	273359449	35.47548	8.65795	0.0015408
650	422500	274625000	25.49510	8.66239	0.0015385
651	423801	275894451	25.51470	8.66683	0.0015361
652	425104	277167808	25.53429	8.67127	0.0015337
653	426409	278445077	25.55386	8.67570	0.0015314
654	427716	279726264	25.57342	8.68012	0.0015291
655	429025	281011375	25.59297	8.68455	0.0015267
656	430336	282300416	25.61250	8.68896	0.0015244
657	431649	283593393	25.63201	8.69338	0.0015221
658	432964	284890312	25.65151	8.69778	0.0015198
659	434281	286191179	25.67100	8.70219	0.0015175
660	435600	287496000	25.69047	8.70659	0 0015152
661	436921	288804781	25.70992	8.71098	0.0015129
662	438244	290117528	25.72936	8.71537	0.0015106
663	439569	291434247	25.74879	8.71976	0.0015083
664	440896	292754944	25.76820	8.72414	0.0015060
665	442225	294079625	25.78749	8.72852	0.0015038
666	443556	295408296	25.80698	8.73289	0.0015015
667	444889	296740963	25.82634	8.73726	0.0014993
668	446224	298077632	25.84570	8.74162	0.0014970
669	447561	299418309	25.86503	8.74598	0.0014948
670	448900	300763000	25.88436	8.75034	0.0014925
671	450241	302111711	25.90367	8.75469	0.0014903
672	451584	303464448	25.92296	8.75904	0.0014881
673	452929	304821217	25.94224	8.76338	0.0014859
674	454276	306182024	25.96151	8.76772	0.0014837
675	455625	307546875	25.98076	8.77205	0.0014815
676	456976	308915776	26	8.77638	0.0014793
677	458329	310288733	26.01922	8.78071	0.0014771
678	459684	311665752	26.03843	8.78503	0.0014749
679	461041	313046839	26.05763	8.78935	0.0014728
680	462400	314432000	26.07681	8.79366	0.0014706
681	463761	315821241	26.09598	8.79797	0.0014684
682	465124	317214568	26.11513	8.80227	0.0014663
683	466489	318611987	26.13427	8.80657	0.0014641
684	467856	320013504	26.15339	8.81087	0.0014620

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
685	469225	321419125	26.17250	8.81516	0.0014599
686	470596	322828856	26.19160	8.81945	0.0014577
687	471969	324242703	26.21068	8.82373	0.0014556
688	473344	325660672	26.22975	8.82801	0.0014535
689	474721	327082769	26.24881	8.83229	0.0014514
690	476100	328509000	26.26785	8.83656	0.0014493
691	477481	329939371	26.28688	8.84082	0.0014472
692	478864	331373888	26.30589	8.84509	0.0014451
693	480249	332812557	26.32489	8.84934	0.0014430
694	481636	334255384	26.34388	8.85360	0.0014409
695	483025	335702375	26.36285	8.85785	0.0014388
696	484416	337153536	26.38181	8.86210	0.0014368
697	485809	338608873	26.40076	8.86634	0.0014347
698	487204	340068392	26.41969	8.87058	0.0014327
699	488601	341532099	26.43861	8.87481	0.0014306
700	490000	343000000	26.45751	8.87904	0.0014286
701	491401	344472101	26.47640	8.88327	0.0014265
702	492804	345948408	26.49528	8.88749	0.0014245
703	494209	347428927	26.51415	8.89171	0.0014225
704	495616	348913664	26.53300	8.89592	0.0014205
705	497025	350402625	26.55184	8.90013	0.0014184
706	498436	351895816	26.57066	8.90434	0.0014164
707	499849	353393243	26.58947	8.90854	0.0014144
708	501264	354894912	26.60817	8.91274	0.0014124
709	502681	356400829	26.62705	8.91693	0.0014104
710	504100	357911000	26.64583	8.92112	0.0014085
711	505521	359425431	26.66458	8.92531	0.0014065
712	506944	360944128	26.68333	8.92949	0.0014045
713	508369	362467097	26.70206	8.93367	0.0014025
714	509796	363994344	26.72078	8.93784	0.0014006
715	511225	365525875	26.73948	8.94201	0.0013986
716	512656	367061696	26.75818	8.94618	0.0013966
717	514089	368601813	26.77686	8.95034	0.0013947
718	515524	370146232	26.79552	8.95450	0.0013928
719	516961	371694959	26.81418	8.95866	0.0013908
720	518400	373248000	26.83282	8.96281	0.0013889
721	519841	374805361	26.85144	8.96696	0.0013870
722	521284	376367048	26.87006	8.97110	0.0013850
723	522729	377933067	26.88866	8.97524	0.0013831
724	524176	379503424	26.90725	8.97938	0.0013812

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
725	525625	381078125	26.92582	8.98351	0.0013793
726	527076	382657176	26.94439	8.98764	0.0013774
727	528529	384240583	26.96294	8.99176	0.0013755
728	529984	385828352	26.98148	8.99589	0.0013736
729	531441	387420489	27	9	0.0013717
730	532900	389017000	27.01851	9.00411	0.0013699
731	534361	390617891	27.03701	9.00822	0.0013680
732	535824	392223168	27.05550	9.01233	0.0013661
733	537289	393832837	27.07397	9.01643	0.0013643
734	538756	395446904	27.09243	9.02053	0.0013624
735	540225	397065375	27.11088	9.02462	0.0013605
736	541696	398688256	27.12932	9.02871	0.0013587
737	543169	400315553	27.14771	9.03280	0.0013569
738	544644	401947272	27.16616	9.03689	0.0013550
739	546121	403583419	27.18455	9.04097	0.0013532
740	547600	405224000	27.20291	9.04504	0.0013514
741	549081	406869021	27.22132	9.04911	0.0013495
742	550564	408518488	27.23968	9.05318	0.0013477
743	552049	410172407	27.25803	9.05725	0.0013459
744	553536	411830784	27.27636	9.06131	0.0013441
745	555025	413493625	27.29469	9.06537	0.0013423
746	556516	415160936	27.31300	9.06942	0.0013405
747	558009	416832723	27.33130	9.07347	0.0013387
748	559504	418508992	27.34959	9.07752	0.0013369
749	561001	420189749	27.36786	9.08156	0.0013351
750	562500	421875000	27.38613	9.08560	0.0013333
751	564001	423564751	27.40438	9.08964	0.0013316
752	565504	425259008	27.42262	9.09367	0.0013298
753	567009	426957777	27.44085	9.09770	0.0013280
754	568516	428661064	27.45906	9.10173	0.0013263
755	570025	430368875	27.47726	9.10575	0.0013245
756	571536	432081216	27.49545	9.10977	0.0013228
757	573049	433798093	27.51363	9.11378	0.0013210
758	574564	435519512	27.53180	9.11779	0.0013193
759	576081	437245479	27.54995	9.12180	0.0013175
760	577600	438976000	27.56810	9.12581	0.0013158
761	579121	440711081	27.58623	9.12981	0.0013141
762	580644	442450728	27.60435	9.13380	0.0013123
763	582169	444194947	27.62245	9.13780	0.0013106
764	583696	445943744	27.64055	9.14179	0.0013089

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
765	585225	447697125	27.65863	9.14577	0.0013072
766	586756	449455096	27.67671	9.14976	0.0013055
767	588289	451217663	27.69476	9.15374	0.0013038
768	589824	452984832	27.71281	9.15771	0.0013021
769	591361	454756609	27.73085	9.16169	0.0013004
770	592900	456533000	27.74887	9.16566	0.0012987
771	594441	458314011	27.76689	9.16962	0.0012970
772	595984	460099648	27.78489	9.17359	0.0012953
773	597529	461889917	27.80288	9.17754	0.0012937
774	599076	463684824	27.82086	9.18150	0.0012920
775	600625	465484375	27.83882	9.18545	0.0012903
776	602176	467288576	27.85678	9.18940	0.0012887
777	603729	469097433	27.87472	9.19335	0.0012870
778	605284	470910952	27.89265	9.19729	0.0012853
779	606841	472729139	27.91057	9.20123	0.0012837
780	608400	474552000	27.92848	9.20516	0.0012821
781	609961	476379541	27.94638	9.20910	0.0012804
782	611524	478211768	27.96426	9.21303	0.0012788
783	613089	480048687	27.98214	9.21695	0.0012771
784	614656	481890304	28	9.22087	0.0012755
785	616225	483736625	28.01785	9.22479	0.0012739
786	617796	485587656	28.03569	9.22871	0.0012723
787	619369	487443403	28.05352	9.23262	0.0012706
788	620944	489303872	28.07134	9.22653	0.0012690
789	622521	491169069	28.08914	9.24043	0.0012674
790	624100	493039000	28.10694	9.24434	0.0012658
791	625681	494913671	28.12472	9.24823	0.0012642
792	627264	496793088	28.14249	9.25213	0.0012629
793	628849	498677257	28.16026	9.25602	0.0012610
794	630436	500566184	28.17801	9.25991	0.0012594
795	632025	502459875	28.19574	9.26380	0.0012579
796	633616	504358336	28.21347	9.26768	0.0012563
797	635209	506261573	28.23119	9.27156	0.0012547
798	636804	508169592	28.24889	9.27544	0.0012531
799	638401	510082399	28.26659	9.27931	0.0012516
800	640000	512000000	28.28427	9.28318	0.0012500
801	641601	513922401	28.30194	9.28704	0.0012484
802	643204	515849608	28.31960	9.29091	0.0012469
803	644809	517781627	28.33725	9.29477	0.0012453
804	646416	519718464	28.35489	9.29862	0.0012438

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
805	648025	521660125	28.37252	9.30248	0.0012422
806	649636	523606616	28.39014	9.30638	0.0012407
807	651249	525557943	28.40775	9.31018	0.0012392
808	652864	527514112	28.42534	9.31402	0.0012376
809	654481	529475129	28.44293	9.31786	0.0012361
810	656100	531441000	28.46050	9.32170	0.0012346
811	657721	533411731	28.47806	9.32553	0.0012330
812	659344	535387328	28.49561	9.32936	0.0012315
813	660969	537367797	28.51315	9.33319	0.0012300
814	662596	539353144	28.53069	9.33702	0.0012285
815	664225	541343375	28.54820	9.34084	0.0012270
816	665856	543338496	28.56571	9.34466	0.0012255
817	667489	545338513	28.58321	9.34847	0.0012240
818	669124	547343432	28.60070	9.35229	0.0012225
819	670761	549353259	28.61818	9.35610	0.0012210
820	672400	551368000	28.63564	9.35990	0.0012195
821	674041	553387661	28.65310	9.36270	0.0012180
822	675684	555412248	28.67054	9.36751	0.0012165
823	677329	557441767	28.68798	9.37130	0.0012151
824	678976	559476224	28.70540	9.37510	0.0012136
825	680625	561515625	28.72281	9.37889	0.0012121
826	682276	563559976	28.74022	9.38268	0.0012107
827	683929	565609283	28.75761	9.38646	0.0012092
828	685584	567663552	28.77499	9.39024	0.0012077
829	687241	569722789	28.79236	9.39402	0.0012063
830	688900	571787000	28.80972	9.39780	0.0012048
831	690561	573856191	28.82707	9.40157	0.0012034
832	692224	575930368	28.84441	9.40534	0.0012019
833	693889	578009537	28.86174	9.40911	0.0012005
834	695556	580093704	28.87906	9.41287	0.0011990
835	697225	582182875	28.89637	9.41663	0.0011976
836	698896	584277056	28.91366	9.42039	0.0011962
837	700569	586376253	28.93095	9.42414	0.0011947
838	702244	588480472	28.94823	9.42789	0.0011933
839	703921	590589719	28.96550	9.43164	0.0011919
840	705600	592704000	28.98275	9.43538	0.0011905
841	707281	594823321	29	9.43913	0.0011891
842	708964	596947688	29.01724	9.44287	0.0011876
843	710649	599077107	29.03446	9.44661	0.0011862
844	712336	601211584	29.05168	9.45034	0.0011848

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
845	714025	603351125	29.06888	9.45407	0.0011834
846	715716	605495736	29.08608	9.45780	0.0011820
847	717409	607645423	29.10326	9.46152	0.0011806
848	719104	609800192	29.12044	9.46525	0.0011792
849	720801	611960049	29.13760	9.46897	0.0011779
850	722500	614125000	29.15476	9.47268	0.0011765
851	724201	616295051	29.17190	9.47640	0.0011751
852	725904	618470208	29.18904	9.48011	0.0011737
853	727609	620650477	29.20616	9.48381	0.0011723
854	729316	622835864	29.22328	9.48752	0.0011710
855	731025	625026375	29.24038	9.49122	0.0011696
856	732736	627222016	29.25748	9.49492	0.0011682
857	734449	629422793	29.27456	9.49861	0.0011669
858	736164	631628712	29.29164	9.50231	0.0011655
859	737881	633839779	29.30870	9.50600	0.0011641
860	739600	636056000	29.32576	9.50969	0.0011628
861	741321	638277381	29.34280	9.51337	0.0011614
862	743044	640503928	29.35984	9.51705	0.0011601
863	744769	642735647	29.37686	9.52073	0.0011587
864	746496	644972544	29.39388	9.52441	0.0011574
865	748225	647214625	29.41088	9.52808	0.0011561
866	749956	649461896	29.42788	9.53175	0.0011547
867	751689	651714363	29.44486	9.53542	0.0011534
868	753424	653972032	29.46184	9.53908	0.0011521
869	755161	656234909	29.47881	9.54274	0.0011507
870	756900	658503000	29.49576	9.54640	0.0011494
871	758641	660776311	29.51271	9.55006	0.0011481
872	760384	663054848	29.52965	9.55371	0.0011468
873	762129	665338617	29.54657	9.55736	0.0011455
874	763876	667627624	29.56349	9.56101	0.0011442
875	765625	669921875	29.58040	9.56466	0.0011429
876	767376	672221376	29.59730	9.56830	0.0011416
877	769129	674526133	29.61419	9.57194	0.0011403
878	770884	676836152	29.63106	9.57557	0.0011390
879	772641	679151439	29.64793	9.57921	0.0011377
880	774400	681472000	29.66479	9.58284	0.0011364
881	776161	683797841	29.68164	9.58647	0.0011351
882	777924	686128968	29.69848	9.59009	0.0011338
883	779689	688465387	29.71532	9.59372	0.0011325
884	781456	690807104	29.73214	9.59734	0.0011312

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
885	783225	693154125	29.74895	9.60095	0.0011299
886	784996	695506456	29.76575	9.60457	0.0011287
887	786769	697864103	29.78255	9.60818	0.0011274
888	788544	700227072	29.79933	9.61179	0.0011261
889	790321	702595369	29.81610	9.61540	0.0011249
890	792100	704969000	29.83287	9.619	0.0011236
891	793881	707347971	29.84962	9.62260	0.0011223
892	795664	709732288	29.86637	9.62620	0.0011211
893	797449	712121957	29.88311	9.62980	0.0011198
894	799236	714516984	29.89983	9.63339	0.0011186
895	801025	716917375	29.91655	9.63698	0.0011173
896	802816	719323136	29.93326	9.64057	0.0011161
897	804609	721734273	29.94996	9.64415	0.0011148
898	806404	724150792	29.96665	9.64774	0.0011136
899	808201	726572699	29.98333	9.65132	0.0011123
900	810000	729000000	30	9.65489	0.0011111
901	811801	731432701	30.01666	9.65847	0.0011099
902	813604	733870808	30.03331	9.66204	0.0011086
903	815409	736314327	30.04996	9.66561	0.0011074
904	817216	738763264	30.06659	9.66918	0.0011062
905	819025	741217625	30.08322	9.67274	0.0011050
906	820836	743677416	30.09983	9.67630	0.0011038
907	822649	746142643	30.11644	9.67986	0.0011025
908	824464	748613312	30.13304	9.68342	0.0011013
909	826281	751089429	30.14963	9.68697	0.0011001
910	828100	753571000	30.16621	9.69052	0.0010989
911	829921	756058031	30.18278	9.69407	0.0010977
912	831744	758550528	30.19934	9.69762	0.0010965
913	833569	761048497	30.21589	9.70116	0.0010953
914	835396	763551944	30.23243	9.70470	0.0010941
915	837225	766060875	30.24897	9.70824	0.0010929
916	839056	768575296	30.26549	9.71177	0.0010917
917	840889	771095213	30.28201	9.71531	0.0010905
918	842714	773620632	30.29851	9.71884	0.0010893
919	844561	776151559	30.31501	9.72236	0.0010881
920	846400	778688000	30.33150	9.72589	0.0010870
921	848241	781229961	30.34798	9.72941	0.0010858
922	850084	783777448	30.36445	9.73293	0.0010846
923	851929	786330467	30.38092	9.73645	0.0010834
924	853776	788889024	30.39737	9.73996	0.0010823

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
925	855625	791453125	30.41381	9.74348	0.0010811
926	857476	794022776	30.43025	9.74699	0.0010799
927	859329	796597983	30.44667	9.75049	0.0010787
928	861184	799178752	30.46309	9.75400	0.0010776
929	863041	801765089	30.47950	9.75750	0.0010764
930	864900	804357000	30.49590	9.76100	0.0010753
931	866761	806954491	30.51229	9.76450	0.0010741
932	868624	809557568	30.52868	9.76799	0.0010730
933	870489	812166237	30.54505	9.77148	0.0010718
934	872356	814780504	30.56141	9.77497	0.0010707
935	874225	817400375	30.57777	9.77846	0.0010695
936	876096	820025856	30.59412	9.78295	0.0010684
937	877969	822656953	30.61046	9.78543	0.0010672
938	879844	825293672	30.62679	9.78891	0.0010661
939	881721	827936019	30.64311	9.79239	0.0010650
940	883600	830584000	30.65942	9.79586	0 0010638
941	885481	833237621	30.67572	9.79933	0.0010627
942	887364	835896888	30.69202	9.80280	0.0010616
943	889249	838561807	30.70831	9.80627	0.0010604
944	891136	841232384	30.72458	9.80974	0.0010593
945	893025	843908625	30.74085	9.81320	0.0010582
946	894916	846590536	30.75711	9.81666	0.0010571
947	896809	849278123	30.77337	9.82012	0.0010560
948	898704	851971392	30.78961	9.82357	0.0010549
949	900601	854670349	30.80584	9.82703	0.0010537
950	902500	857375000	30.82207	9.83048	0.0010526
951	904401	860085351	30.83829	9.83392	0.0010515
952	906304	862801408	30.85450	9.83737	0.0010504
953	908209	865523177	30.87070	9.84081	0.0010493
954	910116	868250664	30.88689	9.84425	0.0010482
955	912025	870983875	30.90307	9.84769	0.0010471
956	913936	873722816	30.91925	9.85113	0.0010460
957	915849	876467493	30.93542	9.85456	0.0010449
958	917764	879217912	30.95158	9.85799	0.0010438
959	919681	881974079	30.96773	9.86142	0.0010428
960	921600	887436000	30.98387	9.86485	0.0010417
961	923521	887503681	31	9.86827	0.0010406
962	925444	890277128	31.01612	9.87169	0.0010395
963	927369	893056347	31.03224	9.87511	0.0010384
964	929296	895841344	31.04835	9.87853	0.0010373

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
965	931225	898632125	31.06445	9.88195	0.0010363
966	933156	901428696	31.08054	9.88536	0.0010352
967	935089	904231063	31.09662	9.88877	0.0010341
968	937024	907039232	31.11270	9.89217	0.0010331
969	938961	909853209	31.12876	9.89558	0.0010320
970	940900	912673000	31.14482	9.89898	0.0010309
971	942841	915498611	31.16087	9.90238	0.0010299
972	944788	918330048	31.17691	9.90578	0.0010288
973	946729	921167317	31.19295	9.90918	0.0010277
974	948676	924010424	31.20897	9.91257	0.0010267
975	950625	926859375	31.22499	9.91596	0.0010256
976	952576	929714176	31.24100	9.91935	0.0010246
977	954529	932574833	31.25700	9.92274	0.0010235
978	956484	935441352	31.27299	9.92612	0.0010225
979	958441	938313739	31.28898	9.92950	0.0010215
980	960400	941192000	31.30495	9.93288	0.0010204
981	962361	944076141	31.32092	9.93626	0.0010194
982	964324	946966168	31.33688	9.93964	0.0010183
983	966289	949862087	31.35283	9.94301	0.0010173
984	968256	952763904	31.36877	9.94638	0.0010163
985	970225	955671625	31.38471	9.94975	0.0010152
986	972196	958585256	31.40064	9.95311	0.0010142
987	974169	961504803	31.41656	9.95648	0.0010132
988	976144	964430272	31.43247	9.95984	0.0010121
989	978121	967361669	31.44837	9.96320	0.0010111
990	980100	970299000	31.46427	9.96655	0.0010101
991	982081	973242271	31.48015	9.96991	0.0010091
992	984064	976191488	31.49603	9.97326	0.0010081
993	986049	979146657	31.51190	9.97661	0.0010070
994	988036	982107784	31.52777	9.97996	0.0010060
995	990025	985074875	31.54362	9.98331	0.0010050
996	992016	988047936	31.55947	9.98665	0.0010040
997	994009	991026973	31.57531	9.98999	0.0010030
998	996004	994011992	31.59114	9.99333	0.0010020
999	998001	997002999	31.60696	9.99667	0.0010010

Notes on Algebra.

Algebra is that branch of mathematics in which the quantities are denoted by letters and the operations to be performed upon them are indicated by signs. The same signs are used to indicate the same operations as in arithmetic.

Signs of Quantity and Signs of Operation.

If a quantity is written $a + (-b)$, the sign that precedes the parenthesis is called the sign of operation, and the sign within the parenthesis is called the sign of quantity with respect to b, but expressions of this kind can be reduced to have only one sign. Thus, $a + (-b) = a - b$ and this final sign is called the essential sign.

$a + (+b) = a + b.$
$a + (-b) = a - b.$
$a - (+b) = a - b.$
$a - (-b) = a + b.$

Thus, when the sign of operation and the sign of quantity are alike the essential sign is $+$, but if they are unlike the essential sign is $-$.

In multiplying any two quantities, like signs in the two factors give $+$ in the product, but unlike signs in the two factors give $-$ in the product; thus, $(+a) \times (-b) = -ab$, and $(-a) \times (-b) = ab$.

In division, like signs in dividend and divisor give $+$ in the quotient, but unlike signs in dividend and divisor give $-$ in the quotient; thus:

$$\frac{-a}{+b} = -\frac{a}{b}$$

$$\frac{-a}{-b} = +\frac{a}{b}$$

Useful Formulas and Rules in Algebra.

The following rules are very useful to remember in solving practical problems in algebra. Let a and b represent any two quantities; then $a + b$ will represent their sum and $a - b$ their difference; then $(a + b) \times (a + b) = a^2 + 2\ ab + b^2$.

$(a + b) \times (a + b)$ is also written $(a + b)^2$.

This rule reads:

The square of the sum of any two quantities is equal to the square of the first quantity plus double the product of both quantities, plus the square of the second quantity.

$(a - b) \times (a - b) = (a - b)^2 = a^2 - 2\,ab + b^2.$

This rule reads:

The square of the difference of any two quantities is equal to the square of the first quantity minus twice the product of both quantities, plus the square of the second quantity.

$(a + b) \times (a - b) = a^2 - b^2.$

This rule reads:

The sum of any two quantities multiplied by their difference is equal to the difference of their squares.

Extracting Roots.

An even root of a positive quantity is either + or —. An even root cannot be extracted of a negative quantity, as $\sqrt{a^2}$ may be either a or $-a$; but $\sqrt{-a^2}$ is impossible, because $(-a) \times (-a) = a^2$ and $(+a) \times (+a) = a^2$.

An odd root may be extracted as well of a negative quantity as a positive quantity, and the sign of the root is always the same as the sign of the quantity before the root was extracted.

Thus: $\sqrt[3]{a^3} = a$, but $\sqrt[3]{(-a)^3} = (-a)$.

Powers.

When a number or a quantity is to be multiplied by itself a given number of times, the operation is indicated by a small number at the right-hand corner of the quantity; for instance, $a^2 = a \times a$.

A quantity of this kind is called a power; the small number is called the exponent, or the index of the power. Two powers of the same kind may be multiplied by adding the exponents; for instance, $a^2\,a^3 = a^{2+3} = a^5$.

Two powers of the same kind may be divided by subtracting their exponents; for instance,

$$\frac{a^5}{a^2} = a^{5-2} = a^3 = a \times a \times a.$$

$$\frac{a^5}{a^3} = a^{5-3} = a^2 = a \times a.$$

$$\frac{a^5}{a^4} = a^{5-4} = a^1 = a.$$

$$\frac{a^5}{a^5}\ a^{5-5} = a^0 = 1.$$

Thus, any quantity in 0 power must be 1, because always when dividend and divisor are alike the quotient must be 1.

$\frac{a^5}{a^6} = a^{5-6} = a^{-1}$, but a^5 divided by a^5 is equal to 1; therefore $\frac{a^5}{a^6}$ must be $\frac{1}{a}$; $\frac{a^5}{a^7} = a^{-2} = \frac{1}{a^2}$

Thus, any quantity with a negative exponent is one divided by that quantity considering the exponent as positive. We may, therefore, say that as a positive exponent indicates how many times a quantity is to be used as a factor, a negative exponent indicates how many times a quantity should be used as a divisor; for instance,

$$a^{-1} = \frac{1}{a}; \quad a^{-2} = \frac{1}{a \times a}; \quad a^{-3} = \frac{1}{a \times a \times a}$$

Thus:

$$6^{-1} = \tfrac{1}{6}; \quad 6^{-2} = (\tfrac{1}{6})^2 = \tfrac{1}{36}; \quad 3^{-2} = (\tfrac{1}{3})^2 = \tfrac{1}{9}, \text{ etc.}$$

Equations.

An algebraic expression of equality between two quantities is called an equation. The two quantities are connected by the sign of equality, and the quantity on the left-hand side of the sign is called the first member, and the quantity on the right-hand side is called the second member of the equation.

If a part or all of the known quantities are expressed in letters it is called a literal equation: $ax = a + b - 2c$ is a literal equation.

If the equation contains no letters except the unknown quantity, usually expressed by x, it is called a numerical equation; for instance, $5x = 12 - 7 + 9$, is a numerical equation.

In solving equations, we may, without destroying the equality of the equation, add an equal quantity to both members, subtract an equal quantity from both members, multiply both members by an equal quantity, divide both members by an equal quantity, extract the same root of both members, raise both members to the same power.

Quantities inclosed by parentheses, a bar, or under a radical sign, and quantities connected by the sign of multiplication, must always be considered and operated upon as one quantity.

EXAMPLE 1.

$$x = 3 \times 8 + 10 - 3 + 3 \times 10.$$
$$x = 24 + 10 - 3 + 30.$$
$$x = 64 - 3.$$
$$x = 61.$$

EXAMPLE 2.

$$x = \frac{3 + 8 + 10}{10}$$

$$x = \frac{21}{10}$$

$$x = 2\tfrac{1}{10}$$

EXAMPLE 3.

$$x = 12 \times \left(\frac{8 - 3}{3} + \frac{8 + 10}{3}\right)$$

$$x = 12 \times \left(\frac{5}{3} + \frac{18}{3}\right)$$

$$x = 12 \times \left(1\frac{2}{3} + 6\right)$$

$$x = 12 \times \left(7\frac{2}{3}\right)$$

$$x = 92$$

EXAMPLE 4.

$$x = 12 \times \frac{8 - 3}{3} + \frac{8 + 10}{3}$$

$$x = 12 \times \frac{5}{3} + \frac{18}{3}$$

$$x = 20 + 6$$

$$x = 26$$

EXAMPLE 5.

$$x = \left(\frac{3 + 9}{2} \times (8 - 3) + \sqrt{20 + 16}\right) \times 2$$

$$x = \left(\frac{12}{2} \times 5 + \sqrt{36}\right) \times 2$$

$$x = (6 \times 5 + 6) \times 2$$

$$x = 36 \times 2$$

$$x = 72$$

When an equation consists of more than one unknown quantity, as many equations may be arranged as there are unknown quantities, and one equation is solved so that the value of one of its unknown quantities is expressed in terms of the other, and this value is substituted in the other equation.

EXAMPLE.

Two shafts are to be connected by two gears of 16 diametral pitch; the distance between centers is 6¾ inches. The ratio of gearing shall be 1 to 3. How many teeth in each gear?

Call small gear x and large gear y; then $x + y$ must be 108, because $6\frac{3}{4} \times 16 = 108$, which is the number of teeth in both gears added together. The ratio is 1 to 3; therefore $3x = y$. Thus:

$x + y = 108$, transposed to
$x + 3x = 108.$
$4x = 108.$
$x = \frac{108}{4}$
$x = 27$ teeth for small gear.

The large gear $= 27 \times 3 = 81$ teeth.

Quadratic Equations.

Equations containing one or more unknown quantities in the second power are called quadratic equations. If the unknown quantity only exists in the second power the equation may be brought to the form $x^2 = a$ and $x = \sqrt{a}.$

This square root may be either plus or minus.

If the unknown quantity exists in both the first and the second power the equation may be brought to the form $x^2 + ax = b$, or it may be brought to the form $x^2 - ax = b$.

The coefficient a may be any number. After the equation is brought to this form, complete the square of the first member by adding to both members of the equation the square of half the coefficient a; this will make the left member of the equation a complete square.

EXAMPLE.

A coal bin is to hold six tons of coal. Allow 40 cubic feet per ton. (It takes 35 to 40 cubic feet to hold a ton of coal in a bin). Make the width of the bin 6 feet, and the length equal to the width and the depth added together. How deep and how long will the bin be?

Depth $= x$ and length $= y$.
$6xy = 240$, because 6 times 40 equals 240.
$6 + x = y$, because width + depth = length.
Thus:

$$6x(6 + x) = 240.$$
$$6x^2 + 36x = 240.$$

Dividing by 6 we have: $x^2 + 6x = 40.$

Completing the square: $x^2 + 6x + 3^2 = 40 + 3^2.$

$$x + 3 = \sqrt{40 + 9}.$$

Extracting the square root: $x + 3 = \sqrt{49}.$

$$x + 3 = 7.$$
$$x = 7 - 3.$$
$x = 4$ feet deep.

Length = width + depth, $= 6 + 4$, $= 10$ feet.

This satisfies the conditions of the problem, because

$$6 \times 4 \times 10 = 240 \text{ cubic feet,}$$

and the width and depth added together equal the length.

Progressions.

A progression is a series of numbers increasing or decreasing, according to a fixed law.

The successive numbers of which the progression consists are called *terms;* the first and the last terms are called the *extremes* and the others are called the *means*.

ARITHMETICAL PROGRESSION.

An arithmetical progression is a series of numbers which increase or decrease by a constant difference. For instance:

2, 4, 6, 8, 10, 12, 14, 16, is an ascending series;
20, 18, 16, 14, 12, 10, 8, is a descending series.

In each of these series the common difference is 2.

The following are the elements considered in an arithmetical progression:

a = First term; l = last term; d = the common difference; n = the number of terms; s = the sum of all the terms.

When any three of these quantities are known the other two may be calculated:

In the above example of an ascending series:

$$a = 2; \quad l = 16; \quad d = 2; \quad n = 8; \quad s = 72.$$

Formulas:	Examples:
$a = l - (n - 1) \times d$	$a = 16 - (8 - 1) \times 2 = 2$
$l = a + (n - 1) \times d$	$l = 2 + (8 - 1) \times 2 = 16$
$d = \dfrac{l - a}{n - 1}$	$d = \dfrac{16 - 2}{8 - 1} = 2$
$n = \dfrac{l - a}{d} + 1$	$n = \dfrac{16 - 2}{2} + 1 = 8$
$s = \dfrac{(a + l) \times n}{2}$	$s = \dfrac{(2 + 16) \times 8}{2} = 72$

In the above example of a descending series:

$$a = 20; \quad l = 8; \quad d = 2; \quad n = 7; \quad s = 98.$$

Formulas:

$$a = l + (n - 1) \times d$$
$$l = a - (n - 1) \times d$$
$$d = \frac{a - l}{n - 1}$$
$$n = \frac{a - l}{d} + 1$$
$$s = \frac{a + l}{2} \times n$$

Examples:

$$a = 8 + (7 - 1) \times 2 = 20$$
$$l = 20 - (7 - 1) \times 2 = 8$$
$$d = \frac{20 - 8}{7 - 1} = 2$$
$$n = \frac{20 - 8}{2} + 1 = 7$$
$$s = \frac{20 + 8}{2} \times 7 = 98$$

GEOMETRICAL PROGRESSION.

A Geometrical Progression is a series of numbers which increase or decrease by a common constant ratio. For instance:

3, 6, 12, 24, 48, is an ascending series; 48, 24, 12, 6, 3 is a descending series.

The following are the elements considered in a geometrical progression:

a = first term; l = last term; r = ratio; n = number of term; s = sum of the terms.

When three of these are known the other two may be calculated.

In this example of an ascending series:

$$a = 3; \quad l = 48; \quad r = 2; \quad n = 5; \quad s = 93.$$

Formulas:

$$a = \frac{l}{r^{n-1}}$$
$$l = a \times r^{n-1}$$
$$r = \sqrt[n-1]{\frac{l}{a}}$$
$$n = \frac{log.\, l - log.\, a}{log.\, r} + 1$$
$$s = \frac{l \times r - a}{r - 1}$$

Examples:

$$a = \frac{48}{2^{5-1}} = \frac{48}{2^4} = \frac{48}{16} = 3$$
$$l = 3 \times 2^{5-1} = 3 \times 16 = 48$$
$$r = \sqrt[5-1]{\frac{48}{3}} = \sqrt[4]{16} = 2$$
$$n = \frac{log.\, 48 - log.\, 3}{log.\, 2} + 1 = \frac{1.681241 - 0.477121}{0.30103} + 1 = 5$$
$$s = \frac{48 \times 2 - 3}{2 - 1} = 93$$

In this example of a descending series:

$$a = 48;\quad l = 3;\quad r = 2;\quad n = 5;\quad s = 93.$$

Formulas:

$$a = l \times r^{n-1}$$

$$l = \frac{a}{r^{n-1}}$$

$$r = \sqrt[n-1]{\frac{a}{l}}$$

$$n = \frac{log.\ a - log.\ l}{log.\ r} + 1$$

$$s = \frac{a \times r - l}{r - 1}$$

Examples:

$$a = 3 \times 2^{5-1} = 3 \times 16 = 48$$

$$l = \frac{48}{2^{5-1}} = \frac{48}{2^4} = \frac{48}{16} = 3$$

$$r = \sqrt[5-1]{\frac{48}{3}} = \sqrt[4]{16} = 2$$

$$n = \frac{log.\ 48 - log.\ 3}{log.\ 2} + 1 = \frac{1.681241 - 0.477121}{0.30103} + 1 = 5$$

$$s = \frac{48 \times 2 - 3}{2 - 1} = 93$$

The Arithmetical Mean.

The arithmetical mean of two or more quantities is obtained by adding the quantities and dividing the sum by their number. For instance, the arithmetical mean of 14 and 16 is $\frac{14 + 16}{2} = 15$.

Thus: The arithmetical mean is simply the *average*.

The Geometrical Mean.

The geometrical mean of two quantities is the square root of their product. For instance, the geometrical mean of 14 and 16 is $\sqrt{14 \times 16} = 14.9666$.

The geometrical mean of two numbers is also called their *mean proportional.*

When the difference between two numbers is small as compared to either of them, their arithmetical mean is approximately equal to their geometrical mean.

This fact may be used to advantage for calculating approximately a root of any number.

For instance, find the square root of 148.

Knowing that the square of 12 is 144, twelve is used as a divisor, thus:

$$\frac{148}{12} = 12.333, \text{ and } \frac{12.333 + 12}{2} = 12.166,$$

which is correct within 0.005.

Logarithms.

Logarithms are a series of numbers computed in order to facilitate all kinds of laborious calculations, such as evolution, involution, multiplication and division.

Addition takes the place of multiplication, subtraction the place of division; multiplication that of involution, and division of evolution.

The logarithm of any given number is the exponent of the power to which another fixed number, called the *base*, must be raised in order to produce the given number.

There are two systems of logarithms in more or less general use in mechanical calculations: namely, the Napierian system and the Briggs system.

The Napierian system of logarithms was invented and tables published by Baron John Napier, a Scotch mathematician, in 1614, but these tables were improved by John Speidell in 1619.

The *modulus* of any system of logarithms is a constant by which the Napierian logarithm of any given number must be multiplied in order to obtain the logarithm for the same number in the other system.

The base of the Napierian system of logarithms is an incommensurable number expressed approximately by 2.718281828. In mathematical works this base is usually denoted by the letter *e*.

The Napierian logarithms are frequently called *hyperbolic logarithms*, from their relation to certain areas included between the equilateral hyperbola and its asymptotes.

The Napierian logarithms are sometimes called *natural logarithms*.

The Briggs system of logarithms was first invented and computed by Professor Henry Briggs of London in 1615, and is usually termed the *common system of logarithms*. Whenever logarithms in general is mentioned the Briggs system is always the one referred to.

The Briggs system of logarithms has for its modulus 0.4342945, and 10 for its base. Therefore the Briggs logarithm of a number is the exponent of the power to which 10 must be raised in order to give the number. Thus:

Log.	1 = 0	because	$10^0 =$	1.
"	10 = 1	"	$10^1 =$	10.
"	100 = 2	"	$10^2 =$	100.
"	1,000 = 3	"	$10^3 =$	1,000.
"	10,000 = 4	"	$10^4 =$	10,000.

The logarithm of any number between 1 and 10 is a fraction smaller than 1. The logarithm of any number between 10 and 100 is a number between 1 and 2. The logarithm of any number between 100 and 1000 is a number between 2 and 3, etc.

The decimal part of the logarithms is called the *mantissa*, and is given in the table commencing on page 88.

The integer part of a logarithm is called the index or sometimes the characteristic, and is not given in the table, but is obtained by the rule that it is one less than the number of figures in the integer part of the number; thus, the index of a logarithm for any number consisting of two figures must be 1; the index of the logarithm for a number consisting of three figures must be 2, etc.

The index of the logarithm of a decimal fraction is a negative number. Sometimes the negative index is denoted by writing a minus sign over it; for instance, log. 0.5240 = $\overline{1}$.719331, or the negative index is denoted by writing it after the mantissa; thus, *log.* 0.5240 = 9.719331 — 10. This, of course, is of exactly the same value whether written — 1 or 9 — 10. Either of these expressions is *minus one* in value, but it is more convenient in logarithmic calculations to write the negative index after the mantissa; thus, instead of writing $\overline{1}$, write 9 ...... — 10; instead of $\overline{2}$, write 8 ...... — 10, etc. Only the mantissa is given in the table, but the index (as already explained) is obtained by the rule: *One less than the number of figures on the left side of the decimal point.* Therefore, in order to memorize and explain this rule, the following examples are inserted:

Number.	Logarithm.	Number.	Logarithm.
8236	3.915716	0.08236	8.915716 — 10
823.6	2.915716	0.008236	7.915716 — 10
82.36	1.915716	0.0008236	6.915716 — 10
8.236	0.915716	0.00008236	5.915716 — 10
0.8236	9.915716 — 10	0.000008236	4.915716 — 10

Multiplying or dividing a number by any power of 10 does not change the mantissa in the corresponding logarithm, but only the index; for instance:

$$Log.\ 0.5 = 9.698970 - 10,$$
$$\text{and } Log.\ 500 = 2.698970, \text{ etc.}$$

Thus, the mantissa of a logarithm is the same whether the number is 0.5, 5, 50, 500, 5,000, etc. It is only the index that is changed; therefore, when a number consists of three or less figures, its logarithm is found in the tables by taking the mantissa found in the first column to the right of the number; that is, in the column under cipher. The index is found by the same rule as before. For instance, logarithm to 537 will be 2.729974.

To Find the Logarithm of a Number Consisting of Four Figures.

First find the figures in the column headed "N" corresponding to the first three figures of the number; in line with these figures, in the column headed by the fourth figure, will be found the mantissa of the logarithm corresponding to the complete number. By prefixing the index, according to the rule already given, the complete logarithm is obtained.

EXAMPLE.

Find logarithm of 5375.

Solution:

Under the heading "N" find 537; and in the column at the top of the table find "5"; under 5 in the line with 537 is 730378.

This is the mantissa of the logarithm. The index for a number consisting of four integers is 3, therefore the complete logarithm of 5375 is 3.730378.

To Find the Logarithm of a Number Having More Than Four Figures.

EXAMPLE 1.

Find the logarithm to 3658.2.

Solution:

Log. 3658 = 3.563244 and *log.* 3659 = 3.563362; therefore the logarithm for 3658.2 must be somewhere between the two logarithms thus found in the table. The difference between these two logarithms is 0.000118; that is, if the number is increased by 1 the logarithm increases 0.000118, therefore if this number is increased 0.2 the corresponding logarithm must increase 0.2 times, 0.000118 = 0.0000236, which may be taken as 0.000024.

Thus:

$$\begin{array}{rr} & Log.\ 3658 = 3.563244 \\ \text{Difference corresponding} & \underline{\text{to } 0.2 = 0.000024} \\ & Log.\ 3658.2 = 3.563268 \end{array}$$

It is unnecessary to calculate the difference, as the *average* difference between the logarithms in each line is given in the column headed "D" in the tables. The difference in this case is given in the table as 119.

EXAMPLE 2.

Find logarithm to 1892.5.

Solution :

The mantissa of the number 1892 is given in the table as 276921. The difference is given as 229. The index for a number consisting of four integers is 3. Thus:

$$\begin{array}{rr} Log.\ 1892 = & 3.276921 \\ 0.5 \times 0.000229 = 0.0001145 = & 115 \\ \hline Log.\ 1892.5 = & 3.277036 \end{array}$$

EXAMPLE 3.

Find logarithm to 85673.

Solution:

The mantissa for the number 85670 is given in the table as 932829. The difference is given in the table as 51. The index for a number consisting of five integers is 4.

When an increase of 10 in the number increases the logarithm 0.000051 an increase of 3 must increase the corresponding logarithm 0.3 times 0.000051. Thus:

$$\begin{array}{rr} Log.\ 85670 = & 4.932829 \\ 0.3 \times 0.000051 = 0.0000153 = & 15 \\ \hline Log.\ 85673 = & 4.932844 \end{array}$$

These calculations (or interpolations as they are usually called) are based upon the principle that the difference between the numbers and the difference between their corresponding logarithms are directly proportional to each other. This, however, is not strictly true; but within limits, as it is used here, it is near enough for practical results.

To Find the Number Corresponding to a Given Logarithm.

EXAMPLE 1.

Find the number corresponding to the logarithm 2.610979.

Solution:

Always remember when looking for the number *not to consider the index*, but find the mantissa 610979 in the table. In the same line as this mantissa, under the heading "N," is 408, and on the top of the table in the same column as this mantissa is 3; thus, the number corresponding to this mantissa is 4083 and the index of the logarithm is 2; consequently the number is to have three figures on the left-hand side of the decimal point; thus, the number corresponding to the logarithm 2.610979 will be 408.3.

EXAMPLE 2.

Find the number corresponding to the logarithm 3.883991.

Solution:

This mantissa is not in the table. The nearest smaller mantissa is 883945, and to this mantissa corresponds the number 7655. The nearest larger mantissa is 884002, and to this corresponds the number 7656.

Thus, an increment in the mantissa of 57 increases the number by 1, but the difference between the mantissa 883945 and 883991 is 46, therefore the number must increase $\frac{46}{57} = 0.807$.

$$\begin{array}{rl} \text{Number of } Log.\ 3.883945 = & 7655 \\ \text{Difference } \underline{0.000046 =} & \underline{\quad 0.807} \\ \text{Number of } Log.\ 3.883991 = & 7655.807 \end{array}$$

Addition of Logarithms.

(MULTIPLICATION.)

Where the logarithms of the factors have positive indexes, add as if they were decimal fractions, and the sum is the logarithm corresponding to the product.

EXAMPLE 1.

Multiply 81 by 65 by means of logarithms.

Solution:

$$\begin{array}{rl} Log.\ 81 = & 1.908485 \\ Log.\ 65 = & \underline{1.812913} \\ & 3.721398 \end{array}$$

and to this mantissa corresponds the number 5265. The index is 3; therefore the number has no decimals, as it consists of only four figures.

To Add Two Logarithms when One Has a Positive and the Other a Negative Index.

EXAMPLE 2.

Multiply 0.58 by 32.6 by means of logarithms.

Solution:

$$\begin{array}{rl} Log.\ 0.58 = & 9.763428 - 10 \\ Log.\ 32.6 = & \underline{1.513218 \qquad} \\ & 11.276646 - 10 \end{array}$$

This reduces to 1.276646 and to this logarithm corresponds the number 18.908. This mantissa, 276646, cannot be found in the table, but the nearest smaller mantissa is 276462, and the difference between this and the next is found by subtraction to be 230, and the difference between this and the given mantissa is 184.

Thus:

Given logarithm	1.276646		
To the tabulated log.	1.276462	corresponds	18.90
Difference	0.000184	gives	0.008
Thus, logarithm	1.276646	gives number	18.908

To Add Two Logarithms, Both Having a Negative Index.

Add both logarithms in the same manner as decimal fractions, and afterwards subtract 10 from the index on each side of the mantissa.

EXAMPLE.

Multiply 0.82 by 0.082 by means of logarithms.

Solution:

$$\begin{aligned} Log.\ 0.82 &= 9.913814 - 10 \\ Log.\ 0.082 &= \underline{8.913814 - 10} \\ &\quad 18.827628 - 20 \end{aligned}$$

By subtracting 10 on each side of the mantissa this logarithm reduces to 8.827628 — 10 and to the mantissa 827628 corresponds the number 6724, but the negative index 8 . . . . . . — 10 indicates that this first figure 6 is not a whole number, but that it is six-hundredths; therefore a cipher must be placed between this 6 and the decimal point in order to give 6 the right value according to the index; thus, to the logarithm 8.827628 — 10 corresponds the number 0.06724.

Subtraction of Logarithms.

(DIVISION.)

Logarithms are subtracted as common decimal fractions.

To Subtract Two Logarithms, Both Having a Positive Index.

EXAMPLE.

Divide 490 by 70 by means of logarithms.

Solution:

$$\begin{aligned} Log.\ 490 &= 2.690196 \\ Log.\ \ 70 &= \underline{1.845098} \\ &\quad 0.845098 \end{aligned}$$

and to the mantissa of this logarithm corresponds the number 7 or 70 or 700 or 7000, etc., in the table of logarithms, but the index of this logarithm is a cipher; therefore the answer must be a number consisting of one figure, thus it must be 7.

To Subtract a Larger Logarithm From a Smaller One.

This is the same as to divide a smaller number by a larger one. Before the subtraction is commenced add 10 to the index of the smaller logarithm (that is, to the minuend) and place — 10 after the mantissa, then proceed with the subtraction as if they were decimal fractions.

EXAMPLE.

Divide 242 by 367 by means of logarithms.

Solution:

$$\begin{array}{ll} Log.\ 242 = 2.383815 = & 12.383815 - 10 \\ Log.\ 367 = & \underline{2.564666} \\ & 9.819149 - 10 \end{array}$$

and to the mantissa of this logarithm corresponds, according to the table, the number 6594, but the negative index, 9 — 10, indicates it to be 0.6594.

Thus, 242 divided by 367 = 0.6594.

Multiplication of Logarithms.

(INVOLUTION.)

To multiply a logarithm is the same as to raise its corresponding number into the power of the multiplier.

Logarithms having a positive index are multiplied the same as decimal fractions. Thus:

Square 224 by means of logarithms.

Solution:

$$2 \times log.\ 224 = 2 \times 2.350248 = 4.700496 = 50176$$

Logarithms having a negative index are multiplied the same as decimal fractions, but an equal number is subtracted from both the positive and the negative parts of the logarithm, in order to bring the negative part of the index to — 10.

EXAMPLE 1.

Square 0.82 by means of logarithms.

Solution:

$2 \times log.\ 0.82 = 2 \times (9.913814 - 10) = 19.827628 - 10$, and subtracting 10 from both the positive and the negative parts of the logarithm, the result is $9.827628 - 10$; this gives the number 0.6724.

EXAMPLE 2.

Raise 0.9 to the 1.41 power.

Solution:

$$1.41 \times \textit{log.}\ 0.9 = 1.41 \times (9.954243 - 10) = 14.035483 - 14.1$$

In this example 10 cannot be subtracted from both parts of the logarithm, but 4.1 must be subtracted in order to get — 10, after the subtraction is performed. The logarithm will then read 9.935483 — 10, which corresponds to the number 86195, and the negative index, 9 — 10, makes this 0.86195.

Division of Logarithms.

(EVOLUTION.)

To divide a logarithm is the same as to extract a root of the number corresponding to the logarithm.

Logarithms having a positive index are divided the same as common decimal fractions.

EXAMPLE.

Extract the cube root of 512 by means of logarithms.

Solution:

$$\frac{\textit{log.}\ 512}{3} = \frac{2.70927}{3} = 0.90309$$

and the number corresponding to this logarithm is 8, 80, 800, 8,000, etc., but the index of this logarithm is a cipher; therefore the answer must be a number consisting of one integer, consequently it must be 8.

To Divide a Logarithm Having a Negative Index.

Select and add such a number to the index as will give 10 without a remainder for the quotient in the negative index on the right-hand side of the mantissa after division is performed.

EXAMPLE 1.

Extract the square root of 0.64 by means of logarithms.

Solution:

$$\frac{\textit{log.}\ 0.64}{2} = \frac{9.80618-10}{2} = \frac{19.80618-20}{2} = 9.90309-10$$

and to this logarithm corresponds the number 0.8.

EXAMPLE 2.

Extract the cube root of 0.125 by means of logarithms.

Solution:

$$\frac{log.\ 0.125}{3} = \frac{9.09691-10}{3} = \frac{29.09691-30}{3} = 9.69897-10$$

and to this logarithm corresponds the number 0.5.

EXAMPLE 3.

Extract the 1.7 root of 0.78.

Solution:

$$\frac{log.\ 0.78}{1.7} = \frac{9.892095-10}{1.7}$$

We cannot here, as in previous examples, add a multiple of 10 to the index on each side of the mantissa, but 7 must be added in order that the negative quotient shall be — 10 after the division is performed. Thus:

$$\frac{9.892095-10}{1.7} = \frac{16.892095-17}{1.7} = 9.936526-10$$

and to this logarithm corresponds the number 0.864.

Short Rules for Figuring by Logarithms.

MULTIPLICATION.

Add the logarithms of the factors and the sum is the logarithm of the product.

DIVISION.

Subtract divisor's logarithm from the logarithm of the dividend and the difference is the logarithm of the quotient.

INVOLUTION.

Multiply the logarithm of the root by the exponent of the power and the product is the logarithm of the power.

EXAMPLE.

$$Log.\ 86^2 = 2 \times log.\ 86 = 2 \times 1.934498 = 3.868996$$

and to this logarithm corresponds the number 7396.

EVOLUTION.

The logarithm of the number or quantity under the radical sign is divided by the index of the root, and the quotient is the logarithm of the root.

EXAMPLE.

$$Log.\ \sqrt[4]{2401} = \frac{log.\ 2401}{4} = \frac{3.380392}{4} = 0.845098$$

and this logarithm corresponds to the number 7.

EXPONENTS.

The logarithm of a power divided by the logarithm of the root is equal to the exponent of the power.

EXAMPLE.

$$8^x = 64$$

$$x = \frac{log.\ 64}{log.\ 8}$$

$$x = \frac{1.80618}{0.90309}$$

$$x = 2$$

The logarithm of a quantity under the radical sign divided by the logarithm of the root is equal to the index of the root.

EXAMPLE.

$$8 = \sqrt[x]{512}$$

$$x = \frac{log.\ 512}{log.\ 8}$$

$$x = \frac{2.70927}{0.90309}$$

$$x = 3$$

The reason for these last rules may be understood by referring to the rules for Involution and Evolution; for instance:

$86^2 = 7396$, and this expressed by logarithms is:

$$2 \times log.\ 86 = log.\ 7396.$$

Therefore: $\dfrac{log.\ 7396}{log.\ 86} = 2.$

FRACTIONS.

The logarithm of a common fraction is found, either by first reducing the fraction to a decimal fraction, or by taking the logarithm of the numerator and the logarithm of the denominator and subtracting the logarithm of the denominator from the logarithm of the numerator; the difference is the logarithm of the fraction.

EXAMPLE.

$$
\begin{array}{l}
Log.\ \tfrac{3}{4} = log.\ 3 - log.\ 4 \\
Log.\ 3 = 0.477121 = 10.477121 - 10 \\
Log.\ 4 = \qquad\qquad\quad \underline{0.602060\qquad} \\
\qquad \text{Thus, } log.\ \tfrac{3}{4} = 9.875061 - 10
\end{array}
$$

This is also the logarithm of the decimal fraction 0.75.

RECIPROCALS.

Subtract the logarithm of the number from *log.* 1, which is 10.000000 — 10, and the difference is the logarithm of the reciprocal.

EXAMPLE.

Find the reciprocal of 315.

Solution:

$$
\begin{array}{rl}
Log.\ 1 & = 10.000000 - 10 \\
Log.\ 315 & = \underline{\ 2.498311\qquad\quad} \\
Log.\ \text{reciprocal of } 315 & = 7.501689 - 10
\end{array}
$$

To this logarithm corresponds the decimal fraction 0.0031746, which is, therefore, the reciprocal of 315.

Simple Interest by Logarithms.

Add logarithm of principal, logarithm of rate of interest, and logarithm of number of years; from this sum subtract logarithm of 100. The difference is the logarithm of the interest.

EXAMPLE.

Find the interest of \$800 at 4% in 5 years.

Solution:

$$
\begin{array}{rl}
Log.\ 800 & = 2.90309 \\
Log.\ 4 & = 0.60206 \\
Log.\ 5 & = \underline{0.69897} \\
 & \quad 4.20412 \\
Log.\ 100 & = \underline{2.00000} \\
Log.\ \text{interest} & = 2.20412 = \$160 = \text{Interest.}
\end{array}
$$

Compound Interest by Logarithms.

When the interest, at the end of each period of time, is added to the principal the amount will increase at a constant rate; and this rate will be the amount of one dollar invested for one period of the time. For instance: If the periods of time be one year each, then \$30 in 3 years at 5% compound interest will be:

$$\$30 \times 1.05 = \$31.50 \text{ at the end of first year.}$$

$$\$31.50 \times 1.05 = \$33.075 \text{ at the end of second year.}$$

$$\$33.075 \times 1.05 = \$34.73 \text{ at the end of third year.}$$

This calculation may be written:

$$\$30 \times 1.05 \times 1.05 \times 1.05 = \$34.73$$

which also may be written

$$\$30 \times (1.05)^3 = \$34.73.$$

Thus, compound interest is a form of geometrical progression, and may be calculated by the following formulas:

$$a = p \times r^n$$

$$Log.\ a = n \times log.\ r + log.\ p$$

$$p = \frac{a}{r^n}$$

$$Log.\ p = log.\ a - n \times log.\ r$$

$$n = \frac{log.\ a - log.\ p}{log.\ r}$$

$$Log.\ r = \frac{log.\ a - log.\ p}{n}$$

p = Principal invested.

n = The number of periods of time.

a = The amount due after n periods of time.

r = The amount of \$1 invested one period of time.

NOTE.—The quantity r is always obtained by the rule:

Divide the rate of interest per period of time by 100, and add 1 to the quotient.

EXAMPLE.

What is the amount of \$816 invested 6 years at 4% compound interest?

Solution by formula:

$$Log.\ a = n \times log.\ r + log.\ p$$
$$Log.\ a = 6 \times log.\ 1.04 + log.\ 816$$
$$Log.\ a = 6 \times 0.017033 + 2.911690$$
$$Log.\ a = 0.102198 + 2.911690$$
$$Log.\ a = 3.013888$$
$$a = \$1032.49 = \text{Amount.}$$

EXAMPLE.

If $750 is invested at 3% compound interest, how many years will it take before the amount will be $950.

Solution by formula:

$$n = \frac{log.\ a - log.\ p}{log.\ r}$$

$$n = \frac{log.\ 950 - log.\ 750}{log.\ 1.03}$$

$$n = \frac{2.977724 - 2.875061}{0.012837} = 8 \text{ years (nearly).}$$

EXAMPLE.

A principal of $3750 is to be invested so that by compound interest it will amount to $5000 in six years. Find rate of interest.

Solution by formula:

$$Log.\ r = \frac{log.\ a - log.\ p}{n}$$

$$Log.\ r = \frac{3.698970 - 3.574031}{6}$$

$$Log.\ r = 0.020823$$

$$r = 1.0491$$

Rate of interest $= 100r - 100 = 100 \times 1.0491 - 100 = 4.91\%$; or 5% per year (very nearly).

Discount or Rebate.

When calculating discount or rebate, which is a deduction upon money paid before it is due, use formula:

$$p = \frac{a}{r^n}$$

$$Log.\ p = log.\ a - n \times log.\ r$$

EXAMPLE.

A bill of \$500 is due in 3 years. How much cash is it worth if 3% compound interest should be deducted.

$$
\begin{aligned}
Log.\,p &= log.\,a - n \times log.\,r \\
Log.\,p &= 2.698970 - 3 \times 0.012837 \\
Log.\,p &= 2.698970 - 0.038511 \\
Log.\,p &= 2.660459 \\
p &= \$457.57 = \text{Cash payment.}
\end{aligned}
$$

NOTE.— Such examples may be checked to prevent miscalculations, by multiplying the result (the cash payment), by the tabular number given for corresponding number of years and percentage of interest in table on page 23; if calculations are correct, the product will be equal to the original bill. For instance, $457.57 \times 1.092727 = 499.99909339 = \$500.00$. Thus, the calculation in the example is correct.

Sinking Funds and Savings.

If a sum of money denoted by b, set apart or saved during each period of time, is put at compound interest at the end of each period, the amount will be:

$a = b$ at the end of the first period.

$a = b + br$ at the end of the second period.

$a = b + br + br^2$ at the end of the third period.

At the end of n periods the *last term* in this geometrical series is br^{n-1} and the *first term* is b, while the ratio is r. The sum of the series is the amount which according to the rules for geometrical progression (see page 69) will be:

$$a = \frac{r\,(br^{n-1}) - b}{r - 1}$$

$$a = \frac{b\,(r^{n} - 1)}{r - 1}$$

EXAMPLE.

At the end of his first year's business a man sets apart \$1200 for a sinking fund, which he invests at 4% per year. At the end of each succeeding year he sets apart \$1200 which is invested at the same rate. What is the value of the sinking fund after 7 years of business?

Solution:

$$a = \frac{1200 \times (1.04^7 - 1)}{1.04 - 1}$$

$$a = \frac{1200 \times 0.31593}{0.04}$$

$$a = \$9477.90$$

EXAMPLE.

A man 20 years old commences to save 25 cents every working day, and places this in a savings bank at 4% interest, computed semi-annually. How much will he have in the bank when he is 36 years old? (NOTE.—25c. a day = \$1.50 a week = 26 × \$1.50 = \$39 in six months. 4 % per year = 2 % per period of time; 36 — 20 = 16 = 32 periods of time).

Solution by formula:

$$a = \frac{b\ (r^n - 1)}{r - 1}$$

$$a = \frac{39 \times (1.02^{32} - 1)}{1.02 - 1}$$

$$a = \frac{39 \times (1.8845 - 1)}{0.02}$$

$$a = 39 \times 0.8845 \times 50$$

$$a = 1724.775 = \$1724.77 = \text{Amount.}$$

Thus, in 16 years a saving of 25c. a day amounts to \$1724.77.

If the money is paid in advance of the first period of time the terms will be:

$a = br$ at the end of the first period.

$a = br + br^2$ at the end of the second period.

$a = br + br^2 + br^3$ at the end of the third period.

At the end of n years the last term in this geometrical series is br^n and the first term is br, while the ratio is r. The *sum* of the series is the *amount*, which, according to rules for geometrical progressions (see page 69), will be:

$$a = \frac{r\ (br^n) - br}{r - 1}$$

$$a = \frac{br\ (r^n - 1)}{r - 1}$$

EXAMPLE.

Assume that the man mentioned in previous example, instead of commencing to save money when 20 years old, already had \$39 to put in the bank at 4 % the first period of time, and that he always kept up paying \$39 in advance semi-annually. How much money would he then save in 16 years?

Solution:

$$a = \frac{39 \times 1.02 \times (102^{32} - 1)}{1.02 - 1}$$

$$a = \frac{39 \times 1.02 \times 0.8845}{0.02}$$

$$a = 1759.27$$

Thus, by paying the money in advance semi-annually, he will gain $(1759.27 - 1724.77) = \$34.50$.

If a principal denoted by p is invested at a given rate of compound interest, and successive smaller or larger equal payments denoted by b are made at the end of each period of time so that they will commence to draw interest at the beginning of the following period at the same rate as the principal, the formula will be:

$$a = p \times r^n + \frac{b\,(r^n - 1)}{r - 1}$$

but for logarithmic calculations it is more convenient to denote the rate of interest by y % and the formula will read:

$$a = p \times r^n + \frac{b\,(r^n - 1)}{\frac{y}{100}}$$

$$a = p \times r^n + \frac{100\,b\,(r^n - 1)}{y}$$

$$b = \frac{ay - py \times r^n}{100\,r^n - 100}$$

$$r^n = \frac{ay + 100\,b}{py + 100\,b}$$

$$n = \frac{log.\ \frac{ay + 100\,b}{py + 100\,b}}{log.\ r}$$

$$Log.\ r = \frac{log.\ \frac{ay + 100\,b}{py + 100\,b}}{n}$$

NOTE.—Using these formulas it must be understood that n represents the number of periods of time that the *principal is invested*, and that this first period is considered to be the period at the end of which the first payment, b, is made.

EXAMPLE.

A man has \$50 in a savings bank and he also puts in \$25 every month, which goes on interest every 6 months; the bank pays 4% interest, computed semi-annually. How much money can he save in 5 years in this way? (NOTE.—4% per year = 2% per 6 months, or per period of time, and \$25 a month = \$150 every 6 months, or per period of time. The interest is computed semi-annually; therefore 5 years = 10 periods of time).

Solution by formula:

$$a = p \times r^n + \frac{100\,b\ (r^n - 1)}{y}$$

$$a = 50 \times 1.02^{10} + \frac{100 \times 150 \times (102^{10} - 1)}{2}$$

$$a = 50 \times 1.219 + \frac{100 \times 150 \times 0.219}{2}$$

$$a = 60.95 + 1642.50$$

$$a = \$1703.45 = \text{Amount.}$$

The original sum of \$50 has increased to \$60.95, and the monthly payments amounted to \$1500. The last six payments did not draw any interest, as they were deposited in the last six months of the fifth year and would commence to draw interest at the beginning of the sixth year if the amount had not been withdrawn.

EXAMPLE.

A man has \$800 invested at 5%. How much must he save and invest at the same interest every year in order to increase it to \$3000 in five years? Interest is computed annually.

Solution by formula:

$$b = \frac{a\,y - p\,y \times r^n}{100\,r^n - 100}$$

$$b = \frac{3000 \times 5 - 800 \times 5 \times 1.05^5}{100 \times 1.05^5 - 100}$$

$$b = \frac{15000 - 1.2763 \times 4000}{100 \times 1.2763 - 100}$$

$$b = \frac{15000 - 5105.2}{127.63 - 100}$$

$$b = \frac{9894.8}{27.63}$$

$b = 358.118 = \$358.12$ to be paid in each year.

The total payments will be:

$$800 + 5 \times 358.12 = \$800 + \$1790.60 = \$2590.60.$$

The rest of the amount is accumulated interest. The last payment is made at the end of the fifth year; therefore this money does not draw interest.

EXAMPLE.

A man calculates that if he had \$1800 he would start in business. He has only \$120, but is earning \$15 a week and figures that he can save half of his weekly earnings. He puts his money in a savings bank, where it goes on interest every six months, at the rate of 4% a year. How many years will it take him to save the required amount? (NOTE.—\$7.50 a week = 26 × 7½ = \$195 in six months, and 4% per year = 2% per six months, or per period of time).

Solution by formula:

$$n = \frac{\log. \dfrac{a\,y + 100\,b}{p\,y + 100\,b}}{\log.\ r}$$

$$n = \frac{\log. \dfrac{1800 \times 2 + 100 \times 195}{120 \times 2 + 100 \times 195}}{\log.\ 1.02}$$

$$n = \frac{\log. \dfrac{3600 + 19500}{240 + 19500}}{\log.\ 1.02}$$

$$n = \frac{\log.\ 1.1702}{\log.\ 1.02} = \frac{0.0683}{0.0086} = 8 \text{ periods of time (nearly).}$$

One period = 6 months; 8 periods = 4 years; therefore, under these conditions it takes four years to save this amount of money.

If a certain sum of money is withdrawn instead of added, at the end of each period of time, the formula on page 86 will change to:

$$a = p \times r^n - \frac{100\,b\,(r^n - 1)}{y}$$

Every letter denotes the same value as it had in the formula on page 86, except that b represents the sum withdrawn instead of the sum added.

EXAMPLE.

A man has \$5000 invested at 5% interest compounded annually, but at the end of each year he withdraws \$200. How much money has he left after six years?

Solution:

$$a = 1.05^6 \times 5000 - \frac{100 \times 200 \times (1.05^6 - 1)}{5}$$

$$a = 1.34 \times 5000 - \frac{100 \times 200 \times 0.34}{5}$$

$$a = 6700 - 1360$$

$$a = \$5340 = \text{Amount.}$$

If the deducted sum, b, exceeds the interest due at the first period of time, the amount a will become smaller than the principal p, and in time the whole principal will be used up. This will be when:

$$p \times r^n = \frac{100\,b\,(r^n - 1)}{y}$$

This transposes to

$$r^n = \frac{100\,b}{100\,b - py}$$

$$n = \frac{log.\ \frac{100\,b}{100\,b - py}}{log.\ r}$$

EXAMPLE.

A principal of \$5000 is invested at 4% per year, but at the end of each year \$600 is withdrawn. How long will it take to use the whole principal?

$$n = \frac{log.\ \frac{100 \times 600}{100 \times 600 - 5000 \times 4}}{log.\ 1.04}$$

$$n = \frac{log.\ \frac{60000}{60000 - 20000}}{log.\ 1.04}$$

$$n = \frac{log.\ 1.50}{log.\ 1.04}$$

$$n = \frac{0.176091}{0.017033}$$

$$n = 10.3 \text{ years.}$$

Paying a Debt by Instalments.

This same formula applies also in this case; for instance: A man uses \$1500 every year toward paying a debt of \$10,000, and 5% interest per year. How long will it take to pay it?

$$n = \frac{log.\ \frac{100 \times 1500}{100 \times 1500 - 10000 \times 5}}{log.\ 1.05}$$

$$n = \frac{log.\ \frac{150000}{100000}}{log.\ 1.05}$$

$$n = \frac{log.\ 1.5}{log.\ 1.05}$$

$$n = \frac{0.176091}{0.021189}$$

$$n = 8.3 \text{ years.}$$

N.	0	1	2	3	4	5	6	7	8	9	D.
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
101	004321	004751	005181	005609	006038	006466	006894	007321	007748	008174	428
102	008600	009026	009451	009876	010300	010724	011147	011570	011993	012415	424
103	012837	013259	013680	014100	014521	014940	015360	015779	016197	016616	420
104	017033	017451	017868	018284	018700	019116	019532	019947	020361	020775	416
105	021189	021603	022016	022428	022841	023252	023664	024075	024486	024896	412
106	025306	025715	026125	026533	026942	027350	027757	028164	028571	028978	408
107	029384	029789	030195	030600	031004	031408	031812	032216	032619	033021	404
108	033424	033826	034227	034628	035029	035430	035830	036230	036629	037028	400
109	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998	396
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
111	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830	390
112	049218	049606	049993	050380	050766	051153	051538	051924	052309	052694	386
113	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524	383
114	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320	379
115	060698	061075	061452	061829	062206	062582	062958	063333	063709	064083	376
116	064458	064832	065206	065580	065953	066326	066699	067071	067443	067815	373
117	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514	370
118	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182	366
119	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819	363
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
121	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004	357
122	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552	355
123	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
124	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562	349
125	096910	097257	097604	097951	098298	098644	098990	099335	099681	100026	346

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126	100371	100715	101059	101403	101747	102091	102434	102777	103119	103462	343
127	103804	104146	104487	104828	105169	105510	105851	106191	106531	106871	341
128	107210	107549	107888	108227	108565	108903	109241	109579	109916	110253	338
129	110590	110926	111263	111599	111934	112270	112605	112940	113275	113609	335
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
131	117271	117603	117934	118265	118595	118926	119256	119586	119915	120245	330
132	120574	120903	121231	121560	121888	122216	122544	122871	123198	123525	328
133	123852	124178	124504	124830	125156	125481	125806	126131	126456	126781	325
134	127105	127429	127753	128076	128399	128722	129045	129368	129690	130012	323
135	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219	321
136	133539	133858	134177	134496	134814	135133	135451	135769	136086	136403	318
137	136721	137037	137354	137671	137987	138303	138618	138934	139249	139564	316
138	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
139	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818	311
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
141	149219	149527	149835	150142	150449	150756	151063	151370	151676	151982	307
142	152288	152594	152900	153205	153510	153815	154120	154424	154728	155032	305
143	155336	155640	155943	156246	156549	156852	157154	157457	157759	158061	303
144	158362	158664	158965	159266	159567	159868	160168	160469	160769	161068	301
145	161368	161667	161967	162266	162564	162863	163161	163460	163758	164055	299
146	164353	164650	164947	165244	165541	165838	166134	166430	166726	167022	297
147	167317	167613	167908	168203	168497	168792	169086	169380	169674	169968	295
148	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
149	173186	173478	173769	174060	174351	174641	174932	175222	175512	175802	291
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289

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151	178977	179264	179552	179839	180126	180413	180699	180986	181272	181558	287
152	181844	182129	182415	182700	182985	183270	183555	183839	184123	184407	285
153	184691	184975	185259	185542	185825	186108	186391	186674	186956	187239	283
154	187521	187803	188084	188366	188647	188928	189209	189490	189771	190051	281
155	190332	190612	190892	191171	191451	191730	192010	192289	192567	192846	279
156	193125	193403	193681	193959	194237	194514	194792	195069	195346	195623	278
157	195900	196176	196453	196729	197005	197281	197556	197832	198107	198382	276
158	198657	198932	199206	199481	199755	200029	200303	200577	200850	201124	274
159	201397	201670	201943	202216	202488	202761	203033	203305	203577	203848	272
160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206556	271
161	206826	207096	207365	207634	207904	208173	208441	208710	208979	209247	269
162	209515	209783	210051	210319	210586	210853	211121	211388	211654	211921	267
163	212188	212454	212720	212986	213252	213518	213783	214049	214314	214579	266
164	214844	215109	215373	215638	215902	216166	216430	216694	216957	217221	264
165	217484	217747	218010	218273	218536	218798	219060	219323	219585	219846	262
166	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
167	222716	222976	223236	223496	223755	224015	224274	224533	224792	225051	259
168	225309	225568	225826	226084	226342	226600	226858	227115	227372	227630	258
169	227887	228144	228400	228657	228913	229170	229426	229682	229938	230193	256
170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	255
171	232996	233250	233504	233757	234011	234264	234517	234770	235023	235276	253
172	235528	235781	236033	236285	236537	236789	237041	237292	237544	237795	252
173	238046	238297	238548	238799	239049	239299	239550	239800	240050	240300	250
174	240549	240799	241048	241297	241546	241795	242044	242293	242541	242790	249
175	243038	243286	243534	243782	244030	244277	244525	244772	245019	245266	248

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176	245513	245759	246006	246252	246499	246745	246991	247237	247482	247728	246
177	247973	248219	248464	248709	248954	249198	249443	249687	249932	250176	245
178	250420	250664	250908	251151	251395	251638	251881	252125	252368	252610	243
179	252853	253096	253338	253580	253822	254064	254306	254548	254790	255031	242
180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
181	257679	257918	258158	258398	258637	258877	259116	259355	259594	259833	239
182	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
183	262451	262688	262925	263162	263399	263636	263873	264109	264346	264582	237
184	264818	265054	265290	265525	265761	265996	266232	266467	266702	266937	235
185	267172	267406	267641	267875	268110	268344	268578	268812	269046	269279	234
186	269513	269746	269980	270213	270446	270679	270912	271144	271377	271609	233
187	271842	272074	272306	272538	272770	273001	273233	273464	273696	273927	232
188	274158	274389	274620	274850	275081	275311	275542	275772	276002	276232	230
189	276462	276692	276921	277151	277380	277609	277838	278067	278296	278525	229
190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
191	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075	227
192	283301	283527	283753	283979	284205	284431	284656	284882	285107	285332	226
193	285557	285782	286007	286232	286456	286681	286905	287130	287354	287578	225
194	287802	288026	288249	288473	288696	288920	289143	289366	289589	289812	223
195	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
196	292256	292478	292699	292920	293141	293363	293584	293804	294025	294246	221
197	294466	294687	294907	295127	295347	295567	295787	296007	296226	296446	220
198	296665	296884	297104	297323	297542	297761	297979	298198	298416	298635	219
199	298853	299071	299289	299507	299725	299943	300161	300378	300595	300813	218
200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217

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201	303196	303412	303628	303844	304059	304275	304491	304706	304921	305136	216
202	305351	305566	305781	305996	306211	306425	306639	306854	307068	307282	215
203	307496	307710	307924	308137	308351	308564	308778	308991	309204	309417	213
204	309630	309843	310056	310268	310481	310693	310906	311118	311330	311542	212
205	311754	311966	312177	312389	312600	312812	313023	313234	313445	313656	211
206	313867	314078	314289	314499	314710	314920	315130	315340	315551	315760	210
207	315970	316180	316390	316599	316809	317018	317227	317436	317646	317854	209
208	318063	318272	318481	318689	318898	319106	319314	319522	319730	319938	208
209	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
211	324282	324488	324694	324899	325105	325310	325516	325721	325926	326131	205
212	326336	326541	326745	326950	327155	327359	327563	327767	327972	328176	204
213	328380	328583	328787	328991	329194	329398	329601	329805	330008	330211	203
214	330414	330617	330819	331022	331225	331427	331630	331832	332034	332236	202
215	332438	332640	332842	333044	333246	333447	333649	333850	334051	334253	202
216	334454	334655	334856	335057	335257	335458	335658	335859	336059	336260	201
217	336460	336660	336860	337060	337260	337459	337659	337858	338058	338257	200
218	338456	338656	338855	339054	339253	339451	339650	339849	340047	340246	199
219	340444	340642	340841	341039	341237	341435	341632	341830	342028	342225	198
220	342423	342620	342817	343014	343212	343409	343606	343802	343999	344196	197
221	344392	344589	344785	344981	345178	345374	345570	345766	345962	346157	196
222	346353	346549	346744	346939	347135	347330	347525	347720	347915	348110	195
223	348305	348500	348694	348889	349083	349278	349472	349666	349860	350054	194
224	350248	350442	350636	350829	351023	351216	351410	351603	351796	351989	193
225	352183	352375	352568	352761	352954	353147	353339	353532	353724	353916	193

N.	0	1	2	3	4	5	6	7	8	9	D.
226	354108	354301	354493	354685	354876	355068	355260	355452	355643	355834	192
227	356026	356217	356408	356599	356790	356981	357172	357363	357554	357744	191
228	357935	358125	358316	358506	358696	358886	359076	359266	359456	359646	190
229	359835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	188
231	363612	363800	363988	364176	364363	364551	364739	364926	365113	365301	188
232	365488	365675	365862	366049	366236	366423	366610	366796	366983	367169	187
233	367356	367542	367729	367915	368101	368287	368473	368659	368845	369030	186
234	369216	369401	369587	369772	369958	370143	370328	370513	370698	370883	185
235	371068	371253	371437	371622	371806	371991	372175	372360	372544	372728	184
236	372912	373096	373280	373464	373647	373831	374015	374198	374382	374565	184
237	374748	374932	375115	375298	375481	375664	375846	376029	376212	376394	183
238	376577	376759	376942	377124	377306	377488	377670	377852	378034	378216	182
239	378398	378580	378761	378943	379124	379306	379487	379668	379849	380030	181
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
241	382017	382197	382377	382557	382737	382917	383097	383277	383456	383636	180
242	383815	383995	384174	384353	384533	384712	384891	385070	385249	385428	179
243	385606	385785	385964	386142	386321	386499	386677	386856	387034	387212	178
244	387390	387568	387746	387923	388101	388279	388456	388634	388811	388989	178
245	389166	389343	389520	389698	389875	390051	390228	390405	390582	390759	177
246	390935	391112	391288	391464	391641	391817	391993	392169	392345	392521	176
247	392697	392873	393048	393224	393400	393575	393751	393926	394101	394277	176
248	394452	394627	394802	394977	395152	395326	395501	395676	395850	396025	175
249	396199	396374	396548	396722	396896	397071	397245	397419	397592	397766	174
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253	403121	403292	403464	403635	403807	403978	404149	404320	404492	404663	171
254	404834	405005	405176	405346	405517	405688	405858	406029	406199	406370	171
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257	409933	410102	410271	410440	410609	410777	410946	411114	411283	411451	169
258	411620	411788	411956	412124	412293	412461	412629	412796	412964	413132	168
259	413300	413467	413635	413803	413970	414137	414305	414472	414639	414806	167
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
261	416641	416807	416973	417139	417306	417472	417638	417804	417970	418135	166
262	418301	418467	418633	418798	418964	419129	419295	419460	419625	419791	165
263	419956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
264	421604	421768	421933	422097	422261	422426	422590	422754	422918	423082	164
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266	424882	425045	425208	425371	425534	425697	425860	426023	426186	426349	163
267	426511	426674	426836	426999	427161	427324	427486	427648	427811	427973	162
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274	437751	437909	438067	438226	438384	438542	438701	438859	439017	439175	158
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281	448706	448861	449015	449170	449324	449478	449633	449787	449941	450095	154
282	450249	450403	450557	450711	450865	451018	451172	451326	451479	451633	154
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288	459392	459543	459694	459845	459995	460146	460296	460447	460597	460748	151
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292	465383	465532	465680	465829	465977	466126	466274	466423	466571	466719	149
293	466868	467016	467164	467312	467460	467608	467756	467904	468052	468200	148
294	468347	468495	468643	468790	468938	469085	469233	469380	469527	469675	148
295	469822	469969	470116	470263	470410	470557	470704	470851	470998	471145	147
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307	487138	487280	487421	487563	487704	487845	487986	488127	488269	488410	141
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313	495544	495683	495822	495960	496099	496238	496376	496515	496653	496791	139
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317	501059	501196	501333	501470	501607	501744	501880	502017	502154	502291	137
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320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
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323	509203	509337	509471	509606	509740	509874	510009	510143	510277	510411	134
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333	522444	522575	522705	522835	522966	523096	523226	523356	523486	523616	130
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335	525045	525174	525304	525434	525563	525693	525822	525951	526081	526210	129
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345	537819	537945	538071	538197	538322	538448	538574	538699	538825	538951	126
346	539076	539202	539327	539452	539578	539703	539829	539954	540079	540204	125
347	540329	540455	540580	540705	540830	540955	541080	541205	541330	541454	125
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354	549003	549126	549249	549371	549494	549616	549739	549861	549984	550106	123
355	550228	550351	550473	550595	550717	550840	550962	551084	551206	551328	122
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370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
371	569374	569491	569608	569725	569842	569959	570076	570193	570309	570426	117
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389	589950	590061	590173	590284	590396	590507	590619	590730	590842	590953	112
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408	610660	610767	610873	610979	611086	611192	611298	611405	611511	611617	106
409	611723	611829	611936	612042	612148	612254	612360	612466	612572	612678	106
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
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412	614897	615003	615108	615213	615319	615424	615529	615634	615740	615845	105
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415	618048	618153	618257	618362	618466	618571	618676	618780	618884	618989	105
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423	626340	626443	626546	626648	626751	626853	626956	627058	627161	627263	103
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624	795185	795254	795324	795393	795463	795532	795602	795672	795741	795811	70
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637	804139	804208	804276	804344	804412	804480	804548	804616	804685	804753	68
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640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
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644	808886	808953	809021	809088	809156	809223	809290	809358	809425	809492	67
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698	843855	843918	843980	844042	844104	844166	844229	844291	844353	844415	62
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704	847573	847634	847696	847758	847819	847881	847943	848004	848066	848128	62
705	848189	848251	848312	848374	848435	848497	848559	848620	848682	848743	62
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737	867467	867526	867585	867644	867703	867762	867821	867880	857939	867998	59
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754	877371	877429	871487	877544	877602	877659	877717	877774	877832	877889	58
755	877947	878004	878062	878119	878177	878234	878292	878349	878407	878464	57
756	878522	878579	878637	878694	878752	878809	878866	878924	878981	879039	57
757	879096	879153	879211	879268	879325	879383	879440	879497	879555	879612	57
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779	891537	891593	891649	891705	891760	891816	891872	891928	891983	892039	56
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783	893762	893817	893873	893928	893984	894039	894094	894150	894205	894261	55
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791	898176	898231	898286	898341	898396	898451	898506	898561	898615	898670	55
792	898725	898780	898835	898890	898944	898999	899054	899109	899164	899218	55
793	899279	899328	899383	899437	899492	899547	899602	899656	899711	899766	55
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797	901458	901513	901567	901622	901676	901731	901785	901840	901894	901948	54
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803	904716	904770	904824	904878	904932	904986	905040	905094	905148	905202	54
804	905256	905310	905364	905418	905472	905526	905580	905634	905688	905742	54
805	905796	905850	905904	905958	906012	906066	906119	906173	906227	906281	54
806	906335	906389	906443	906497	906551	906604	906658	906712	906766	906820	54
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988	994757	994801	994845	994889	994933	994977	995021	995065	995108	995152	44
989	995196	995240	995284	995328	995372	995416	995460	995504	995547	995591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
991	996074	996117	996161	996205	996249	996293	996337	996380	996424	996468	44
992	996512	996555	996599	996643	996687	996731	996774	996818	996862	996906	44
993	996949	996993	997037	997080	997124	997168	997212	997255	997299	997343	44
994	997386	997430	997474	997517	997561	997605	997648	997692	997736	997779	44
995	997823	997867	997910	997954	997998	998041	998085	998129	998172	998216	44
996	998259	998303	998347	998390	998434	998477	998521	998564	998608	998652	44
997	998695	998739	998782	998826	998869	998913	998956	999000	999043	999087	44
998	999131	999174	999218	999261	999305	999348	999392	999435	999479	999522	44
999	999565	999609	999652	999696	999739	999783	999826	999870	999913	999957	44

HYPERBOLIC LOGARITHMS.

The hyperbolic or Napierian logarithm of any number may be obtained by multiplying the common logarithm by the constant 2.302585; practically 2.3.

Table No. 6 gives the hyperbolic logarithms from 1.01 to 30. The hyperbolic logarithm of numbers intermediate between those which are given in the table may be obtained by interpolating proportional differences.

TABLE No. 6.—Hyperbolic or Napierian Logarithms of Numbers.

N	Log.	N	Log.	N	Log.	N	Log.
1.01	0.0099	1.26	0.2311	1.51	0.4121	1.76	0.5653
1.02	0.0198	1.27	0.2390	1.52	0.4187	1.77	0.5710
1.03	0.0 96	1.28	0.2469	1.53	0.4253	1.78	0.5766
1.04	0.0392	1.29	0.2546	1.54	0.4318	1.79	0.5822
1.05	0.0488	1.30	0.2624	1.55	0.4383	1.80	0.5878
1.06	0.0583	1.31	0.2700	1.56	0.4447	1.81	0.5933
1.07	0.0677	1.32	0.2776	1.57	0.4511	1.82	0.5988
1.08	0.0770	1.33	0.2852	1.58	0.4574	1.83	0.6043
1.09	0.0862	1.34	0.2927	1.59	0.4637	1.84	0.6098
1.10	0.0953	1.35	0.3001	1.60	0.4700	1.85	0.6152
1.11	0.1044	1.36	0.3075	1.61	0.4762	1.86	0.6206
1.12	0.1133	1.37	0.3148	1.62	0.4824	1.87	0.6259
1.13	0.1222	1.38	0.3221	1.63	0.4886	1.88	0.6313
1.14	0.1310	1.39	0.3293	1.64	0.4947	1.89	0.6366
1.15	0.1398	1.40	0.3365	1.65	0.5008	1.90	0.6419
1.16	0.1484	1.41	0.3436	1.66	0.5068	1.91	0.6471
1.17	0.1570	1.42	0.3507	1.67	0.5128	1.92	0.6523
1.18	0.1655	1.43	0.3577	1.68	0.5188	1.93	0.6575
1.19	0.1740	1.44	0.3646	1.69	0.5247	1.94	0.6627
1.20	0.1823	1.45	0.3716	1.70	0.5306	1.95	0.6678
1.21	0.1906	1.46	0.3784	1.71	0.5365	1.96	0.6729
1.22	0.1988	1.47	0.3853	1.72	0.5423	1.97	0.6780
1.23	0.2070	1.48	0.3920	1.73	0.5481	1.98	0.6831
1.24	0.2151	1.49	0.3988	1.74	0.5539	1.99	0.6881
1.25	0.2231	1.50	0.4055	1.75	0.5596	2.00	0.6931

N	Log.	N	Log.	N	Log.	N	Log.
2.01	0.6981	2.41	0.8796	2.81	1.0332	3.21	1.1663
2.02	0.7031	2.42	0.8838	2.82	1.0367	3.22	1.1694
2.03	0.7080	2.43	0.8879	2.83	1.0403	3.23	1.1725
2.04	0.7129	2.44	0.8920	2.84	1.0438	3.24	1.1756
2.05	0.7178	2.45	0.8961	2.85	1.0473	3.25	1.1787
2.06	0.7227	2.46	0.9002	2.86	1.0508	3.26	1.1817
2.07	0.7275	2.47	0.9042	2.87	1.0543	3.27	1.1848
2.08	0.7324	2.48	0.9083	2.88	1.0578	3.28	1.1878
2.09	0.7372	2.49	0.9123	2.89	1.0613	3.29	1.1909
2.10	0.7419	2.50	0.9163	2.90	1.0647	3.30	1.1939
2.11	0.7467	2.51	0.9203	2.91	1.0682	3.31	1.1969
2.12	0.7514	2.52	0.9243	2.92	1.0716	3.32	1.2000
2.13	0.7561	2.53	0.9282	2.93	1.0750	3.33	1.2030
2.14	0.7608	2.54	0.9322	2.94	1.0784	3.34	1.2060
2.15	0.7655	2.55	0.9361	2.95	1.0818	3.35	1.2090
2.16	0.7701	2.56	0.9400	2.96	1.0852	3.36	1.2119
2.17	0.7747	2.57	0.9439	2.97	1.0886	3.37	1.2149
2.18	0.7793	2.58	0.9478	2.98	1.0919	3.38	1.2179
2.19	0.7839	2.59	0.9517	2.99	1.0953	3.39	1.2208
2.20	0.7885	2.60	0.9555	3.	1.0986	3.40	1.2238
2.21	0.7930	2·61	0.9594	3.01	1.1019	3.41	1.2267
2.22	0.7975	2·62	0.9632	3.02	1.1053	3.42	1.2296
2.23	0.8020	2·63	0.9670	3.03	1.1086	3.43	1.2326
2.24	0.8065	2·64	0.9708	3.04	1.1119	3.44	1.2355
2.25	0.8109	2·65	0.9746	3.05	1.1151	3.45	1.2384
2.26	0.8154	2.66	0.9783	3.06	1.1184	3.46	1.2413
2.27	0.8198	2.67	0.9821	3.07	1.1217	3.47	1.2442
2.28	0.8242	2.68	0.9858	3.08	1.1249	3.48	1.2470
2.29	0.8286	2.69	0.9895	3.09	1.1282	3.49	1.2499
2.30	0.8329	2.70	0.9933	3.10	1.1314	3.50	1.2528
2.31	0.8372	2.71	0.9969	3.11	1.1346	3.51	1.2556
2.32	0.8416	2.72	1.0006	3.12	1.1378	3.52	1.2585
2.33	0.8459	2.73	1.0043	3.13	1.1410	3.53	1.2613
2.34	0.8502	2.74	1.0080	3.14	1.1442	3.54	1.2641
2.35	0.8544	2.75	1.0116	3.15	1.1474	3.55	1.2669
2.36	0.8587	2.76	1.0152	3.16	1.1506	3.56	1.2698
2.37	0.8629	2.77	1.0188	3.17	1.1537	3.57	1.2726
2.38	0.8671	2.78	1.0225	3.18	1.1569	3.58	1.2754
2.39	0.8713	2.79	1.0260	3.19	1.1600	3.59	1.2782
2.40	0.8755	2.80	1.0296	3.20	1.1632	3.60	1.2809

N	Log.	N	Log.	N	Log.	N	Log.
3.61	1.2837	4.01	1.3888	4.41	1.4839	4.81	1.5707
3.62	1.2865	4.02	1.3913	4.42	1.4861	4.82	1.5728
3.63	1.2892	4.03	1.3938	4.43	1.4884	4.83	1.5748
3.64	1.2920	4.04	1.3962	4.44	1.4907	4.84	1.5769
3.65	1.2947	4.05	1.3987	4.45	1.4929	4.85	1.5790
3.66	1.2975	4.06	1.4012	4.46	1.4951	4.86	1.5810
3.67	1.3002	4.07	1.4036	4.47	1.4974	4.87	1.5831
3.68	1.3029	4.08	1.4061	4.48	1.4996	4.88	1.5851
3.69	1.3056	4.09	1.4085	4.49	1.5019	4.89	1.5872
3.70	1.3083	4.10	1.4110	4.50	1.5041	4.90	1.5892
3.71	1.3110	4.11	1.4134	4.51	1.5063	4.91	1.5913
3.72	1.3137	4.12	1.4159	4.52	1.5085	4.92	1.5933
3.73	1.3164	4.13	1.4183	4.53	1.5107	4.93	1.5953
3.74	1.3191	4.14	1.4207	4.54	1.5129	4.94	1.5974
3.75	1.3218	4.15	1.4231	4.55	1.5151	4.95	1.5994
3.76	1.3244	4.16	1.4255	4.56	1.5173	4.96	1.6014
3.77	1.3271	4.17	1.4279	4.57	1.5195	4.97	1.6034
3.78	1.3297	4.18	1.4303	4.58	1.5217	4.98	1.6054
3.79	1.3324	4.19	1.4327	4.59	1.5239	4.99	1.6074
3.80	1.3350	4.20	1.4351	4.60	1.5261	5.	1.6094
3.81	1.3376	4.21	1.4375	4.61	1.5282	5.01	1.6114
3.82	1.3403	4.22	1.4398	4.62	1.5304	5.02	1.6134
3.83	1.3429	4.23	1.4422	4.63	1.5326	5.03	1.6154
3.84	1.3455	4.24	1.4446	4.64	1.5347	5.04	1.6174
3.85	1.3481	4.25	1.4469	4.65	1.5369	5.05	1.6194
3.86	1.3507	4.26	1.4493	4.66	1.5390	5.06	1.6214
3.87	1.3533	4.27	1.4516	4.67	1.5412	5.07	1.6233
3.88	1.3558	4.28	1.4540	4.68	1.5433	5.08	1.6253
3.89	1.3584	4.29	1.4563	4.69	1.5454	5.09	1.6273
3.90	1.3610	4.30	1.4586	4.70	1.5476	5.10	1.6292
3.91	1.3635	4.31	1.4609	4.71	1.5497	5.11	1.6312
3.92	1.3661	4.32	1.4633	4.72	1.5518	5.12	1.6332
3.93	1.3686	4.33	1.4656	4.73	1.5539	5.13	1.6351
3.94	1.3712	4.34	1.4679	4.74	1.5560	5.14	1.6371
3.95	1.3737	4.35	1.4702	4.75	1.5581	5.15	1.6390
3.96	1.3762	4.36	1.4725	4.76	1.5602	5.16	1.6409
3.97	1.3788	4.37	1.4748	4.77	1.5623	5.17	1.6429
3.98	1.3813	4.38	1.4770	4.78	1.5644	5.18	1.6448
3.99	1.3838	4.39	1.4793	4.79	1.5665	5.19	1.6467
4.	1.3863	4.40	1.4816	4.80	1.5686	5.20	1.6487

N	Log.	N	Log.	N	Log.	N	Log.
5.21	1.6506	5.61	1.7246	6.01	1.7934	6.41	1.8579
5.22	1.6525	5.62	1.7263	6.02	1.7951	6.42	1.8594
5.23	1.6544	5.63	1.7281	6.03	1.7967	6.43	1.8610
5.24	1.6563	5.64	1.7299	6.04	1.7984	6.44	1.8625
5.25	1.6582	5.65	1.7317	6.05	1.8001	6.45	1.8641
5.26	1.6601	5.66	1.7334	6.06	1.8017	6.46	1.8656
5.27	1.6620	5.67	1.7352	6.07	1.8034	6.47	1.8672
5.28	1.6639	5.68	1.7370	6.08	1.8050	6.48	1.8687
5.29	1.6658	5.69	1.7387	6.09	1.8066	6.49	1.8703
5.30	1.6677	5.70	1.7405	6.10	1.8083	6.50	1.8718
5.31	1.6696	5.71	1.7422	6.11	1.8099	6.51	1.8733
5.32	1.6715	5.72	1.7440	6.12	1.8116	6.52	1.8749
5.33	1.6734	5.73	1.7457	6.13	1.8132	6.53	1.8764
5.34	1.6752	5.74	1.7475	6.14	1.8148	6.54	1.8779
5.35	1.6771	5.75	1.7492	6.15	1.8165	6.55	1.8795
5.36	1.6790	5.76	1.7509	6.16	1.8181	6.56	1.8810
5.37	1.6808	5.77	1.7527	6.17	1.8197	6.57	1.8825
5.38	1.6827	5.78	1.7544	6.18	1.8213	6.58	1.8840
5.39	1.6845	5.79	1.7561	6.19	1.8229	6.59	1.8856
5.40	1.6864	5.80	1.7579	6.20	1.8245	6.60	1.8871
5.41	1.6882	5.81	1.7596	6.21	1.8262	6.61	1.8886
5.42	1.6901	5.82	1.7613	6.22	1.8278	6.62	1.8901
5.43	1.6919	5.83	1.7630	6.23	1.8294	6.63	1.8916
5.44	1.6938	5.84	1.7647	6.24	1.8310	6.64	1.8931
5.45	1.6956	5.85	1.7664	6.25	1.8326	6.65	1.8946
5.46	1.6974	5.86	1.7681	6.26	1.8342	6.66	1.8961
5.47	1.6993	5.87	1.7699	6.27	1.8358	6.67	1.8976
5.48	1.7011	5.88	1.7716	6.28	1.8374	6.68	1.8991
5.49	1.7029	5.89	1.7733	6.29	1.8390	6.69	1.9006
5.50	1.7047	5.90	1.7750	6.30	1.8405	6.70	1.9021
5.51	1.7066	5.91	1.7766	6.31	1.8421	6.71	1.9036
5.52	1.7084	5.92	1.7783	6.32	1.8437	6.72	1.9051
5.53	1.7102	5.93	1.7800	6.33	1.8453	6.73	1.9066
5.54	1.7120	5.94	1.7817	6.34	1.8469	6.74	1.9081
5.55	1.7138	5.95	1.7834	6.35	1.8485	6.75	1.9095
5.56	1.7156	5.96	1.7851	6.36	1.8500	6.76	1.9110
5.57	1.7174	5.97	1.7867	6.37	1.8516	6.77	1.9125
5.58	1.7192	5.98	1.7884	6.38	1.8532	6.78	1.9140
5.59	1.7210	5.99	1.7901	6.39	1.8547	6.79	1.9155
5.60	1.7228	6.	1.7918	6.40	1.8563	6.80	1.9169

N	Log.	N	Log.	N	Log.	N	Log.
6.81	1.9184	7.21	1.9755	7.61	2.0295	8.01	2.0807
6.82	1.9199	7.22	1.9769	7.62	2.0308	8.02	2.0819
6.83	1.9213	7.23	1.9782	7.63	2.0321	8.03	2.0832
6.84	1.9228	7.24	1.9796	7.64	2.0334	8.04	2.0844
6.85	1.9242	7.25	1.9810	7.65	2.0347	8.05	2.0857
6.86	1.9257	7.26	1.9824	7.66	2.0360	8.06	2.0869
6.87	1.9272	7.27	1.9838	7.67	2.0373	8.07	2.0882
6.88	1.9286	7.28	1.9851	7.68	2.0386	8.08	2.0894
6.89	1.9301	7.29	1.9865	7.69	2.0399	8.09	2.0906
6.90	1.9315	7.30	1.9879	7.70	2.0412	8.10	2.0919
6.91	1.9330	7.31	1.9892	7.71	2.0425	8.11	2.0931
6.92	1.9344	7.32	1.9906	7.72	2.0438	8.12	2.0943
6.93	1.9359	7.33	1.9920	7.73	2.0451	8.13	2.0956
6.94	1.9373	7.34	1.9933	7.74	2.0464	8.14	2.0968
6.95	1.9387	7.35	1.9947	7.75	2.0477	8.15	2.0980
6.96	1.9402	7.36	1.9961	7.76	2.0490	8.16	2.0992
6.97	1.9416	7.37	1.9974	7.77	2.0503	8.17	2.1005
6.98	1.9430	7.38	1.9988	7.78	2.0516	8.18	2.1017
6.99	1.9445	7.39	2.0001	7.79	2.0528	8.19	2.1029
7.	1.9459	7.40	2.0015	7.80	2.0541	8.20	2.1041
7.01	1.9473	7.41	2.0028	7.81	2.0554	8.21	2.1054
7.02	1.9488	7.42	2.0042	7.82	2.0567	8.22	2.1066
7.03	1.9502	7.43	2.0055	7.83	2.0580	8.23	2.1078
7.04	1.9516	7.44	2.0069	7.84	2.0592	8.24	2.1090
7.05	1.9530	7.45	2.0082	7.85	2.0605	8.25	2.1102
7.06	1.9544	7.46	2.0096	7.86	2.0618	8.26	2.1114
7.07	1.9559	7.47	2.0109	7.87	2.0631	8.27	2.1126
7.08	1.9573	7.48	2.0122	7.88	2.0643	8.28	2.1138
7.09	1.9587	7.49	2.0136	7.89	2.0656	8.29	2.1150
7.10	1.9601	7.50	2.0149	7.90	2.0669	8.30	2.1163
7.11	1.9615	7.51	2.0162	7.91	2.0681	8.31	2.1175
7.12	1.9629	7.52	2.0176	7.92	2.0694	8.32	2.1187
7.13	1.9643	7.53	2.0189	7.93	2.0707	8.33	2.1199
7.14	1.9657	7.54	2.0202	7.94	2.0719	8.34	2.1211
7.15	1.9671	7.55	2.0215	7.95	2.0732	8.35	2.1223
7.16	1.9685	7.56	2.0229	7.96	2.0744	8.36	2.1235
7.17	1.9699	7.57	2.0242	7.97	2.0757	8.37	2.1247
7.18	1.9713	7.58	2.0255	7.98	2.0769	8.38	2.1258
7.19	1.9727	7.59	2.0268	7.99	2.0782	8.39	2.1270
7.20	1.9741	7.60	2.0281	8.	2.0794	8.40	2.1282

N	Log.	N	Log.	N	Log.	N	Log.
8.41	2.1294	8.81	2.1759	9.21	2.2203	9.61	2.2628
8.42	2.1306	8.82	2.1770	9.22	2.2214	9.62	2.2638
8.43	2.1318	8.83	2.1782	9.23	2.2225	9.63	2.2649
8.44	2.1330	8.84	2.1793	9.24	2.2235	9.64	2.2659
8.45	2.1342	8.85	2.1804	9.25	2.2246	9.65	2.2670
8.46	2.1353	8.86	2.1815	9.26	2.2257	9.66	2.2680
8.47	2.1365	8.87	2.1827	9.27	2.2268	9.67	2.2690
8.48	2.1377	8.88	2.1838	9.28	2.2279	9.68	2.2701
8.49	2.1389	8.89	2.1849	9.29	2.2289	9.69	2.2711
8.50	2.1401	8.90	2.1861	9.30	2.2300	9.70	2.2721
8.51	2.1412	8.91	2.1872	9.31	2.2311	9.71	2.2732
8.52	2.1424	8.92	2.1883	9.32	2.2322	9.72	2.2742
8.53	2.1436	8.93	2.1894	9.33	2.2332	9.73	2.2752
8.54	2.1448	8.94	2.1905	9.34	2.2343	9.74	2.2762
8.55	2.1459	8.95	2.1917	9.35	2.2354	9.75	2.2773
8.56	2.1471	8.96	2.1928	9.36	2.2364	9.76	2.2783
8.57	2.1483	8.97	2.1939	9.37	2.2375	9.77	2.2793
8.58	2.1494	8.98	2.1950	9.38	2.2386	9.78	2.2803
8.59	2.1506	8.99	2.1961	9.39	2.2396	9.79	2.2814
8.60	2.1518	9.	2.1972	9.40	2.2407	9.80	2.2824
8.61	2.1529	9.01	2.1983	9.41	2.2418	9.81	2.2834
8.62	2.1541	9.02	2.1994	9.42	2.2428	9.82	2.2844
8.63	2.1552	9.03	2.2006	9.43	2.2439	9.83	2.2854
8.64	2.1564	9.04	2.2017	9.44	2.2450	9.84	2.2865
8.65	2.1576	9.05	2.2028	9.45	2.2460	9.85	2.2875
8.66	2.1587	9.06	2.2039	9.46	2.2471	9.86	2.2885
8.67	2.1599	9.07	2.2050	9.47	2.2481	9.87	2.2895
8.68	2.1610	9.08	2.2061	9.48	2.2492	9.88	2.2905
8.69	2.1622	9.09	2.2072	9.49	2.2502	9.89	2.2915
8.70	2.1633	9.10	2.2083	9.50	2.2513	9.90	2.2925
8.71	2.1645	9.11	2.2094	9.51	2.2523	9.91	2.2935
8.72	2.1656	9.12	2.2105	9.52	2.2534	9.92	2.2946
8.73	2.1668	9.13	2.2116	9.53	2.2544	9.93	2.2956
8.74	2.1679	9.14	2.2127	9.54	2.2555	9.94	2.2966
8.75	2.1691	9.15	2.2138	9.55	2.2565	9.95	2.2976
8.76	2.1702	9.16	2.2148	9.56	2.2576	9.96	2.2986
8.77	2.1713	9.17	2.2159	9.57	2.2586	9.97	2.2996
8.78	2.1725	9.18	2.2170	9.58	2.2597	9.98	2.3006
8.79	2.1736	9.19	2.2181	9.59	2.2607	9.99	2.3016
8.80	2.1748	9.20	2.2192	9.60	2.2618	10.	2.3026

N	Log.	N	Log.	N	Log.	N	Log.
10.1	2.3126	14.1	2.6462	18.1	2.8959	22.1	3.0956
10.2	2.3225	14.2	2.6532	18.2	2.9014	22.2	3.1001
10.3	2.3322	14.3	2.6602	18.3	2.9069	22.3	3.1046
10.4	2.3419	14.4	2.6672	18.4	2.9123	22.4	3.1090
10.5	2.3515	14.5	2.6741	18.5	2.9178	22.5	3.1135
10.6	2.3609	14.6	2.6810	18.6	2.9231	22.6	3.1179
10.7	2.3703	14.7	2.6878	18.7	2.9285	22.7	3.1224
10.8	2.3796	14.8	2.6946	18.8	2.9338	22.8	3.1267
10.9	2.3888	14.9	2.7013	18.9	2.9391	22.9	3.1311
11	2.3979	15.	2.7080	19.	2.9444	23.	3.1355
11.1	2.4070	15.1	2.7147	19.1	2.9497	23.1	3.1398
11.2	2.4160	15.2	2.7213	19.2	2.9549	23.2	3.1441
11.3	2.4249	15.3	2.6279	19.3	2.9601	23.3	3.1484
11.4	2.4337	15.4	2.7344	19.4	2.9653	23.4	3.1527
11.5	2.4424	15.5	2.7408	19.5	2.9704	23.5	3.1570
11.6	2.4510	15.6	2.7472	19.6	2.9755	23.6	3.1612
11.7	2.4596	15.7	2.7536	19.7	2.9806	23.7	3.1655
11.8	2.4681	15.8	2.7600	19.8	2.9856	23.8	3.1697
11.9	2.4765	15.9	2.7663	19.9	2.9907	23.9	3.1739
12	2.4849	16.	2.7726	20.	2.9957	24.	3.1780
12.1	2.4932	16.1	2.7788	20.1	3.0007	24.1	3.1822
12.2	2.5014	16.2	2.7850	20.2	3.0057	24.2	3.1863
12.3	2.5096	16.3	2.7912	20.3	3.0106	24.3	3.1905
12.4	2.5178	16.4	2.7973	20.4	3.0155	24.4	3.1946
12.5	2.5259	16.5	2.8033	20.5	3.0204	24.5	3.1987
12.6	2.5338	16.6	2.8094	20.6	3.0253	24.6	3.2027
12.7	2.5417	16.7	2.8154	20.7	3.0301	24.7	3.2068
12.8	2.5495	16.8	2.8214	20.8	3.0349	24.8	3.2108
12.9	2.5572	16.9	2.8273	20.9	3.0397	24.9	3.2149
13	2.5649	17.	2.8332	21.	3.0445	25.	3.2189
13.1	2.5726	17.1	2.8391	21.1	3.0493	25.5	3.2387
13.2	2.5802	17.2	2.8449	21.2	3.0540	26.	3.2581
13.3	2.5877	17.3	2.8507	21.3	3.0587	26.5	3.2771
13.4	2.5952	17.4	2.8565	21.4	3.0634	27.	3.2958
13.5	2.6027	17.5	2.8622	21.5	3.0680	27.5	3.3142
13.6	2.6101	17.6	2.8679	21.6	3.0727	28.	3.3322
13.7	2.6174	17.7	2.8735	21.7	3.0773	28.5	3.3499
13.8	2.6247	17.8	2.8792	21.8	3.0819	29.	3.3673
13.9	2.6319	17.9	2.8848	21.9	3.0865	29.5	3.3844
14.	2.6391	18.	2.8904	22.	3.0910	30.	3.4012

Weights and Measures.

The yard is the standard unit for length in the United States and Great Britain. To determine the length of the yard, a pendulum vibrating seconds in a vacuum at the level of the sea in the latitude of London, with the Fahrenheit thermometer at 62°, is supposed to be divided into 391,393 equal parts; 360,000 of these parts is the length of the standard yard. Actually, the standard yard in both the United States and Great Britain is a metallic scale made with great care and kept by the respective governments, and from this standard other measures of length have been produced.

The standard unit of weight in the United States and Great Britain is the Troy pound, which is equal in weight to 22.2157 cubic inches of distilled water at 62° Fahrenheit, the barometer being 30 inches. The Troy pound contains 5,760 Troy grains; the *Pound Avoirdupois*, which is the unit of weight used in commercial transactions and mechanical calculations in the United States and Great Britain, is equal to 7,000 Troy grains.

In the United States the standard unit of liquid measure is the wine gallon, containing 231 cubic inches or 8.3389 pounds avoirdupois of distilled water at a temperature of its greatest density (39°–40° F).

In the United States the standard unit for dry measure is the Winchester Bushel, containing 2150.42 cubic inches.

In Great Britain the standard measure for both liquid and dry substances is the Imperial Gallon, which is defined as the volume of 10 pounds avoirdupois of distilled water, when weighed at 62° Fahrenheit with the barometer at 30 inches. The Imperial Gallon contains 277.463 cubic inches. The Imperial Bushel of 8 gallons contains 2219.704 cubic inches.

Long Measure.

12 inches = 1 foot = 0.30479 meters.
3 feet = 1 yard = 0.91437 meters.
5½ yards = 1 rod or pole = 16½ feet = 198 inches.
40 rods = 1 furlong = 220 yards = 660 feet.
8 furlongs = 1 statute or land mile = 320 rods = 1760 yards.
3 miles = 1 league = 24 furlongs = 960 rods.
5280 feet = 1 statute or land mile = 1.609 kilometer.
1 geographical or nautical mile = 1 minute = $\frac{1}{60}$ degree.

As adopted by the British admiralty,* a nautical mile is 6080 ft.
1 nautical mile = 1.1515 statute or land miles.
1 statute or land mile = 0.869 nautical miles.

Square Measure.

1 square yard = 9 square feet = 0.836 square meters.
1 square foot = 144 square inches = 929 square centimeters.
1 square inch = 6.4514 square centimeters.
A section of land is 1 mile square = 640 acres.
1 acre = 43,560 square feet = 0.40467 hectare.
1 square acre is 208.71 feet on each side.

Cubic Measure.

1 cubic yard = 27 cubic feet = 0.7645 cubic meters.
1 cubic yard = 201.97 (wine) gallons = 7.645 hectoliter.
1 cubic foot = 1728 cubic inches = 28315.3 cubic centimeters.
1 cubic foot = 7.4805 (wine) gallons = 28.315 liters.

NOTE.—1 cubic foot contains 6.2355 imperial (English) gallons.

A cord of wood = 128 cubic feet, being 4 × 4 × 8 feet.
A perch of stone = 24¾ cubic feet, being 16½ × 1½ × 1 foot, but it is generally taken as 25 cubic feet.

Liquid Measure.

1 pint = 28.88 cubic inches.
2 pints = 1 quart = 57.75 cubic inches = 0.9463 liter.
4 quarts = 1 gallon = 231 cubic inches = 3.7852 liters.

NOTE.—1 imperial (English) gallon is 277.274 cubic inches.

Dry Measure.

1 standard U. S. bushel = 2150.42 cubic inches.
1 standard U. S. bushel = 4 pecks.
1 peck = 2 gallons = 8 quarts.
1 gallon = 4 quarts = $268\frac{4}{5}$ cubic inches.
1 quart = 2 pints = $67\frac{1}{5}$ cubic inches.
100 bushels (approximately) = 124½ cubic feet.
80 bushels (approximately) = 100 cubic feet.

Avoirdupois Weight.

(Used in business and mechanical calculations.)

1 pound = 16 ounces = 0.45359 kilograms.
1 ton = 2240 pounds. A short ton is 2000 pounds.

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* See *Machinery*, page 23, Sept., 1897.

Troy Weight.

(Used when weighing gold, silver and jewelry.)

1 pound = 12 ounces = 0.37324 kilogram.
1 ounce = 20 pennyweights = 1.0971 ounces avdp.

Apothecaries' Weight.

1 pound = 1 pound troy weight = 12 ounces.
1 ounce = 8 drachms.
1 drachm = 3 scruples.
1 scruple = 20 grains.

Weights of Produce.

The following are the weights of certain articles of produce:

	Pounds per bushel.		Pounds per bushel.		Pounds per bushel.
Wheat,	60	Oats,	32	White potatoes,	60
Corn in the ear,	70	Peas,	60	Sweet potatoes,	55
Corn shelled,	56	Ground peas,	24	Onions,	57
Rye,	56	Corn meal,	48	Turnips,	57
Buckwheat,	48	Malt,	38	Clover seed,	60
Barley,	48	White beans,	60	Timothy seed,	45

The Metric System of Weights and Measures.

The unit in the metric system is the *meter*. The length of the meter was intended to be $\frac{1}{10000000}$ (one ten-millionth part) of the length of a quadrant of the earth through Paris, which is the same as $\frac{1}{10000000}$ of any quadrant from either pole to equator.

By later calculations it has been ascertained that the meter as first adopted and now used is slightly too short according to this theoretical requirement, but this, of course, makes no difference; because, practically speaking, the length of a meter is the length of a certain standard meter kept at Paris in the care of the French government, and it is from this standard meter (and not from the quadrant of the earth) that all other standard meters kept for reference are derived.

The *gram*, which is the unit of weight, is the weight of 1 cubic centimeter of water at its maximum density, which is at 4° C. (39° to 40° F.)

The commercial denomination used for weight is the *kilogram* = 1000 grams = 1 cubic decimeter = 1 liter of water at maximum density.

The metric system has been adopted by Mexico, Brazil, Chile and Peru, and by all European countries except England and Russia.

Length.

1 Meter	= 10 Decimeters	=	39.37 inches.
1 Decimeter	= 10 Centimeters	=	3.937 inches.
1 Centimeter	= 10 Millimeters	=	0.3937 inches.
1 Millimeter	= $\frac{1}{1000}$ Meter	=	0.03937 inches.
1 Decameter	= 10 Meters	=	32 feet 9.7 inches.
1 Hektometer	= 100 Meters	=	328 " 1 inch.
1 Kilometer	= 1000 Meters	=	0.6214 mile.
1 Myriameter	= 10000 Meters	=	6.214 miles.

Area.

1 square millimeter = 0.00155 square inch.

100 square millimeters	= 1 square centimeter	= 0.155	sq. inch.
100 " centimeters	= 1 " decimeter	= 15.5	sq. inch.
100 " decimeters	= 1 " meter	= 10.764	sq. feet.
1 Centare	= 1 " meter	= 1550	sq. inches.
1 Are	= 100 " meters	= 119.6	sq. yards.
1 Hectare	= 10000 sq. meters	= 2.471	acres.

Solids.

1 cubic millimeter = 0.000061 cubic inch.

1000 cubic millimeters	= 1 cubic centimeter	= 0.061	cubic inch.
1000 " centimeters	= 1 " decimeter	= 61.027	" "
1000 " decimeters	= 1 " meter	= 35.3	" feet.

Liquid.

1 liter	= 10 deciliters	= 1 cubic decimeter.
1 deciliter	= 10 centiliters	= 100 cubic centimeter.
1 centiliter	= 10 milliliters	= 10 cubic centimeters.
1 milliliter	= $\frac{1}{1000}$ liters	= 1 cubic centimeter.
1 decaliter	= 10 liters	= 10 cubic decimeters.
1 hectoliter	= 100 liters	= 100 cubic decimeters.
1 kiloliter	= 1000 liters	= 1 cubic meter.

1 liter = 61.027 cubic inches = 1.0567 quarts.

TABLE No. 7. Reducing Millimeters to Inches.

Mm.	Inches.	Mm.	Inches.	Mm.	Inches.
0.02	0.00079	0.52	0.02047	2	0.07874
0.04	0.00157	0.54	0.02126	3	0.11811
0.06	0.00236	0.56	0.02205	4	0.15748
0.08	0.00315	0.58	0.02283	5	0.19685
0.10	0.00394	0.60	0.02362	6	0.23622
0.12	0.00472	0.62	0.02441	7	0.27559
0.14	0.00551	0.64	0.02520	8	0.31496
0.16	0.00630	0.66	0.02598	9	0.35433
0.18	0.00709	0.68	0.02677	10	0.39370
0.20	0.00787	0.70	0.02756	11	0.43307
0.22	0.00866	0.72	0.02835	12	0.47244
0.24	0.00945	0.74	0.02913	13	0.51181
0.26	0.01024	0.76	0.02992	14	0.55118
0.28	0.01102	0.78	0.03071	15	0.59055
0.30	0.01181	0.80	0.03150	16	0.62992
0.32	0.01260	0.82	0.03228	17	0.66929
0.34	0.01339	0.84	0.03307	18	0.70866
0.36	0.01417	0.86	0.03386	19	0.74803
0.38	0.01496	0.88	0.03465	20	0.78740
0.40	0.01575	0.90	0.03543	21	0.82677
0.42	0.01654	0.92	0.03622	22	0.86614
0.44	0.01732	0.94	0.03701	23	0.90551
0.46	0.01811	0.96	0.03780	24	0.94488
0.48	0.01890	0.98	0.03858	25	0.98425
0.50	0.01969	1.00	0.03937	26	1.02362

TABLE No. 8. Reducing Inches to Millimeters.

Inches.	Mm.	Inches.	Mm.	Inches.	Mm.
$\frac{1}{16}$	1.59	$\frac{13}{16}$	20.64	2¼	57.15
⅛	6.17	⅞	22.22	2½	63.50
$\frac{3}{16}$	4.76	$\frac{15}{16}$	23.81	3	76.20
¼	6.35	1	25.40	4	101.6
$\frac{5}{16}$	7.94	1⅛	28.57	5	127
⅜	9.52	1¼	31.75	6	152.4
$\frac{7}{16}$	11.11	1⅜	34.92	7	177.8
½	12.70	1½	38.10	8	203.2
$\frac{9}{16}$	14.29	1⅝	41.27	9	228.6
⅝	15.87	1¾	44.45	10	254
$\frac{11}{16}$	17.46	1⅞	47.62	11	279.4
¾	19.05	2	50.80	12	304.8

Table of Reduction for Pressure per Unit of Surface.

1 kilogram per sq. centimeter = 14.223 pounds per sq. inch.
1 kilogram per sq. centimeter = 0.968 atmosphere.
1 pound per sq. inch = 0.0703 kilograms per sq. centimeter.
1 pound per sq. inch = 0.068 atmosphere.

Table of Reduction for Length and Weight.

1 kilogram per kilometer	= 3.548 pounds per mile.
1 kilogram per meter	= 0.672 pounds per foot.
1 pound per mile	= 0.282 kilograms per kilometer.
1 pound per foot	= 1.488 kilograms per meter.

Weight of Water (4° C.)

1 cubic cm. weighs 1 gram.	
1 cubic inch weighs 0.036125 pounds	= 16.386 grams.
1 liter weighs 1 kilogram	= 2.2046 pounds.
1 quart weighs 2.0862 pounds	= 0.9463 kilograms.
1 cubic meter weighs 1000 kilograms	= 2204.6 pounds.
1 cubic foot weighs 62.425 pounds	= 28.32 kilograms.

Measure of Water.

1 kilogram measures	1 liter	= 1.057	quarts.
1 kilogram measures	0.353 cubic feet	= 61.03	cubic inches.
1 pound measures	0.01602 cubic ft.	= 0.454	liter.
1 pound measures	27.68 cubic ins.	= 453.59	cubic centimeters.

SPECIFIC GRAVITY.

The specific gravity of a body is its weight as compared with the weight of an equal volume of another body which is adopted as a standard. For all solid substances, water at its maximum density (4° C.) is the usual standard. For instance, the specific gravity of zinc is 7; this simply means that one cubic foot of zinc is 7 times as heavy as one cubic foot of water. One cubic foot of water weighs 62.425 pounds. Therefore, by multiplying the specific gravity of any solid body by 62.425 its weight per cubic foot is obtained. In the metric system of measure and weight, one cubic centimeter of water weighs one gram; therefore the table of specific gravity will also directly give the weight of the material in grams per cubic centimeter, in kilograms per cubic decimeter, or in 1000 kilograms (the so-called metric ton) per cubic meter.

TABLE No. 9. Specific Gravity, Weights and Values.

METALS.	Metric.	American.		Approximate value per pound.
	Kilog. per cubic dec. or specific gravity.	Pounds per cubic inch.	Pounds per cubic foot.	
Water	1	0.036125	62.425	
Gold (24 k) . .	19.361	0.697	1208	$299.70
Platinum . . .	21.531	0.775	1344	122.00
Silver	10.474	0.377	654	12.14
Wrought iron .	7.78	0.28	485	0.015
Cast iron . . .	7.21	0.26	450	0.008
Tool Steel . .	7.85	0.284	490	0.10
Zinc	7	0.252	437	0.10
Antimony . . .	6.72	0.242	419	0.12
Copper	8.607	0.31	537	0.15
Mercury	13.596	0.489	849	
Tin	7.291	0.262	455	0.25
Aluminum . .	2.67	0.096	166	
Lead	11.36	0.408	708	0.05

TABLE No. 10. Specific Gravity and Weight of Medium Dry Wood.

VARIETY.	Metric.	American.
	Kilog. per cubic dec. specific gravity.	Pounds per cubic foot.
Birch	0.60 to 0.80	37.5 to 50
Ash	0.50 to 0.80	31 to 50
Beech	0.60 to 0.80	37.5 to 50
Oak	0.60 to 0.90	37.5 to 56
Ebony	1.19	74
Lignum-vitæ	1.33	83
Spanish mahogany . . .	0.85	53
Hickory	0.50	32
Spruce	0.50	32
Pine	0.40 to 0.80	25 to 50
Pitch pine	0.80	50

TABLE No. 11. Specific Gravity and Weight per Cubic Foot of Various Materials.

(The weight may vary according to the properties of the material).

MATERIALS.	Metric. Kilog. per cubic dec. specific gravity.	American. Pounds per cubic foot.
Asphalt	1.4	87
Brick	1.6 to 2	100 to 125
Gray granite	2.4	150
Red granite	2.5 to 3	157 to 187
Limestone	2.7	168
Sand	1.5	94
Portland cement	1.26	78
Brickwork	1.75	110
Slate	2.8	175
Glass	2.52	157
Emery	4.0	250
Grindstone	2.4	150
Coal	1.5	94
Porcelain	2.4	150
Lime	0.96	60

TABLE No. 12. Specific Gravity and Weight of Liquids.

LIQUIDS.	Specific Gravity.	Metric.		American.	
		Kilog. per cubic dec.	Kilog. per liter.	Pounds per cub. inch.	Pounds per gallon.
Water	1	1	1	0.036125	8.33
Sea water	1.03	1.03	1.03	0.037	8.55
Sulphuric acid	1.841	1.841	1.841	0.067	15.48
Muriatic acid	1.2	1.2	1.2	0.043	9.93
Nitric acid	1.217	1.217	1.217	0.044	10.16
Alcohol	0.833	0.833	0.833	0.03	6.93
Linseed oil	0.94	0.94	0.94	0.034	7.85
Turpentine	0.87	0.87	0.87	0.031	7.16
Petroleum	0.878	0.878	0.878	0.032	7.39
Machine oil	0.9	0.9	0.9	0.0324	7.5

To Calculate Weight of Casting from Weight of Pattern.

When pattern is made from pine and no nails used, the rule is: Multiply the weight of the pattern by 17 and the product is the weight of the castings.

When nails are used in the pattern, multiply its weight by a little less, probably 15 or 16.

When the pattern has core prints, their weight must be calculated and also the weight of what there is to be cored out in the casting, which must all be deducted. This mode of calculating the weight of castings is, of course, only approximation, but it is frequently very useful.

Weight of an Iron Bar of any Shape of Cross Section.

A wrought iron bar of 1 square inch area of cross section and one yard long weighs 10 pounds. Therefore, the weight of wrought iron bars of any shape, as, for instance, railroad rails, I beams, etc., may very conveniently be obtained by first making a correct, full size drawing of the cross section and measuring its area by a planimeter, which gives the area in square inches. Multiply this area by 10 and the product is the weight in pounds per yard; or multiply the area by 3.33 and the product is the weight in pounds per foot.

To Calculate Weight of Sheet Iron of any Thickness.

One square foot of wrought iron, 1 inch thick, weighs very nearly 40 pounds (40.2 pounds) and one square foot $\frac{1}{40}''$, which is $\frac{25}{1000}$ thick, weighs 1 pound. Therefore, a practical rule for quick calculation of the weight of sheet iron is: Divide the thickness of the iron as measured by a micrometer calliper in thousandths of inches by 25, and the quotient is the weight in pounds per square foot.

To Calculate the Weight of Metals Not Given in the Tables.

Find the weight of wrought iron, and multiply by the following constants:

Weight	of	wrought	iron	$\times$ 0.928 =	cast iron.
"	"	"	"	$\times$ 1.014 =	steel.
"	"	"	"	$\times$ 0.918 =	zinc.
"	"	"	"	$\times$ 1.144 =	copper.
"	"	"	"	$\times$ 1.468 =	lead,

To Calculate the Weight of Zinc, Copper, Lead, etc., in Sheets.

First find the weight by the rule given for sheet iron, and multiply by the constant as given in the above table, and the product is the weight of each metal in pounds per square foot.

To Calculate the Weight of Cast Iron Balls.

Multiply the cube of the diameter in inches by 0.1377, and the product is the weight of the ball in pounds.

Thus:

$W = D^3 \times 0.1377. \qquad D = 1.936 \sqrt[3]{W}$

D = diameter of ball in inches.

W = weight of ball in pounds.

In metric measure, multiply the cube of the diameter in centimeters by 0.003775, and the product is the weight of the ball in kilograms.

Thus:

$W = M^3 \times 0.003775. \qquad M = 6.422 \times \sqrt[3]{W}$

W = weight in kilograms.

M = diameter of ball in centimeters.

TABLE No. 13. Weight of Round Steel per Lineal Foot.

Steel weighing 489 pounds per Cubic Foot.

Diameter in inches.	Weight Per Foot.	Diameter in Inches.	Weight Per Foot.	Diameter in Inches.	Weight Per Foot.
$\frac{1}{16}$	.0104	1 $\frac{1}{16}$	3.011	2 ⅛	12.044
⅛	.042	⅛	3.375	¼	13.503
$\frac{3}{16}$	.094	$\frac{3}{16}$	3.761	⅜	15.045
¼	.167	¼	4.168	½	16.67
$\frac{5}{16}$	.261	$\frac{5}{16}$	4.595	⅝	18.379
⅜	.375	⅜	5.043	¾	20.171
$\frac{7}{16}$	.511	$\frac{7}{16}$	5.512	⅞	22.047
½	.667	½	6.001	3	24.005
$\frac{9}{16}$	.844	$\frac{9}{16}$	6.512	⅛	26.048
⅝	1.042	⅝	7.043	¼	28.173
$\frac{11}{16}$	1.261	$\frac{11}{16}$	7.596	⅜	30.382
¾	1.5	¾	8.169	½	32.674
$\frac{13}{16}$	1.761	$\frac{13}{16}$	8.702	⅝	35.05
⅞	2.042	⅞	9.377	¾	37.508
$\frac{15}{16}$	2.344	$\frac{15}{16}$	10.013	⅞	40.05
1	2.667	2	10.669	4	42.675

TABLE No. 14. Weights of Square and Round Bars of Wrought Iron in Pounds per Lineal Foot.

(Iron weighing 480 pounds per cubic foot).

Thickness or Diameter in Inches.	Weight of Square Bar One Foot Long.	Weight of Round Bar One Foot Long.	Thickness or Diameter in Inches.	Weight of Square Bar One Foot Long.	Weight of Round Bar One Foot Long.
1/16	.013	.010	2 9/16	21.89	17.19
1/8	.052	.041	5/8	22.97	18.04
3/16	.117	.092	11/16	24.08	18.91
1/4	.208	.164	3/4	25.21	19.80
5/16	.326	.256	13/16	26.37	20.71
3/8	.469	.368	7/8	27.55	21.64
7/16	.638	.501	15/16	28.76	22.59
1/2	.833	.654	3	30	23.56
9/16	1.055	.828	1/16	31.26	24.55
5/8	1.302	1.023	1/8	32.55	25.57
11/16	1.576	1.237	3/16	33.87	26.60
3/4	1.875	1.473	1/4	35.21	27.65
13/16	2.201	1.728	5/16	36.58	28.73
7/8	2.551	2.004	3/8	37.97	29.82
15/16	2.930	2.301	7/16	39.39	30.94
1	3.333	2.618	1/2	40.83	32.07
1/16	3.763	2.955	9/16	42.30	33.23
1/8	4.219	3.313	5/8	43.80	34.40
3/16	4.701	3.692	11/16	45.33	35.60
1/4	5.208	4.091	3/4	46.88	36.82
5/16	5.742	4.510	13/16	48.45	38.05
3/8	6.302	4.950	7/8	50.05	39.31
7/16	6.888	5.410	15/16	51.68	40.59
1/2	7.5	5.890	4	53.33	41.89
9/16	8.138	6.392	1/16	55.01	43.21
5/8	8.802	6.913	1/8	56.72	44.55
11/16	9.492	7.455	3/16	58.45	45.91
3/4	10.21	8.018	1/4	60.21	47.29
13/16	10.95	8.601	5/16	61.99	48.69
7/8	11.72	9.204	3/8	63.80	50.11
15/16	12.51	9.828	7/16	65.64	51.55
2	13.33	10.47	1/2	67.50	53.01
1/16	14.18	11.14	9/16	69.39	54.50
1/8	15.05	11.82	5/8	71.30	56
3/16	15.95	12.53	11/16	73.24	57.52
1/4	16.88	13.25	3/4	75.21	59.07
5/16	17.83	14	13/16	77.20	60.63
3/8	18.80	14.77	7/8	79.22	62.22
7/16	19.80	15.55	15/16	81.26	63.82
1/2	20.83	16.36	5	83.33	65.45

TABLE No. 14.—(Continued).

Thickness or Diameter in Inches.	Weight of Square Bar One Foot Long.	Weight of Round Bar One Foot Long.	Thickness or Diameter in Inches.	Weight of Square Bar One Foot Long.	Weight of Round Bar One Foot Long.
5 1/16	85.43	67.10	7 1/8	169.2	132.9
1/8	87.55	68.76	1/4	175.2	137.6
3/16	89.70	70.45	3/8	181.3	142.4
1/4	91.88	72.16	1/2	187.5	147.3
5/16	94.08	73.89	5/8	193.8	152.2
3/8	96.30	75.64	3/4	200.2	157.2
7/16	98.55	77.40	7/8	206.7	162.4
1/2	100.8	79.19	8	213.3	167.6
9/16	103.1	81.00	1/4	226.9	178.2
5/8	105.5	82.83	1/2	240.8	189.2
11/16	107.8	84.69	3/4	255.2	200.4
3/4	110.2	86.56	9	270.0	212.1
13/16	112.6	88.45	1/4	285.2	224.0
7/8	115.1	90.36	1/2	300.8	236.3
15/16	117.5	92.29	3/4	316.9	248.9
6	120.0	94.25	10	333.3	261.8
1/8	125.1	98.22	1/4	350.2	275.1
1/4	130.2	192.3	1/2	367.5	288.6
3/8	135.5	106.4	3/4	385.2	302.5
1/2	140.8	110.6	11	403.3	316.8
5/8	146.3	114.9	1/4	421.9	331.3
3/4	151.9	119.3	1/2	440.8	346.2
7/8	157.6	123.7	3/4	460.2	361.4
7	163.3	128.3	12	480.	377.

TABLE No. 15. Weight of Flat Iron in Pounds per Foot.

Inches	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1
1/2	0.11	0.21	0.32	0.42	0.53	0.63	0.73	0.84				
5/8	0.13	0.26	0.40	0.53	0.66	0.79	0.92	1.06	1.31			
3/4	0.16	0.32	0.47	0.63	0.79	0.95	1.11	1.26	1.58	1.90		
7/8	0.18	0.37	0.55	0.74	0.92	1.11	1.29	1.48	1.85	2.22	2.58	
1	0.21	0.42	0.63	0.84	1.05	1.26	1.47	1.68	2.11	2.53	2.95	3.37
1 1/8	0.24	0.47	0.71	0.95	1.18	1.42	1.66	1.90	2.37	2.84	3.32	3 79
1 1/4	0.26	0.53	0.79	1.05	1.32	1.58	1.84	2.11	2.63	3.16	3.68	4.21
1 3/8	0.29	0.58	0.87	1.16	1.45	1.74	2.03	2.32	2.89	3.47	4.05	4.63
1 1/2	0.32	0.63	0.95	1.26	1.58	1.90	2.21	2.53	3.16	3.79	4.42	5.05
1 5/8	0.34	0.68	1.03	1.37	1.71	2.05	2.39	2.74	3.42	4.11	4.79	5.47
1 3/4	0.37	0.74	1.11	1.47	1.84	2.21	2.58	2 95	3.68	4.42	5.16	5.89
1 7/8	0.40	0.79	1.18	1.58	1.97	2.37	2.76	3.16	3.95	4.74	5.53	6.32
2	0.42	0.84	1.26	1.68	2.11	2.53	2.95	3.37	4.21	5.05	5.89	6.74

TABLE No. 16. Sizes of Numbers of the U. S. Standard Gage for Sheet and Plate Iron and Steel.

(Brown & Sharpe Mfg. Co.)

Number of Gage.	Approximate Thickness in Fractions of an Inch.	Approximate Thickness in Decimal Parts of an Inch.	Weight Per Square Foot in Ounces Avoirdupois.	Weight Per Square Foot in Pounds Avoirdupois.
0000000	$\frac{1}{2}$	.5	320	20.00
000000	$\frac{15}{32}$	.46875	300	18.75
00000	$\frac{7}{16}$	.4375	280	17.50
0000	$\frac{13}{32}$	.40625	260	16.25
000	$\frac{3}{8}$	.375	240	15.
00	$\frac{11}{32}$	.34375	220	13.75
0	$\frac{5}{16}$	.3125	200	12.50
1	$\frac{9}{32}$	.28125	180	11.25
2	$\frac{17}{64}$	.265625	170	10.625
3	$\frac{1}{4}$	.25	160	10.
4	$\frac{15}{64}$	.234375	150	9.375
5	$\frac{7}{32}$	.21875	140	8.75
6	$\frac{13}{64}$	.203124	130	8.125
7	$\frac{3}{16}$	.1875	120	7.5
8	$\frac{11}{64}$	.171875	110	6.875
9	$\frac{5}{32}$	.15625	100	6.25
10	$\frac{9}{64}$	.140625	90	5.625
11	$\frac{1}{8}$	.125	80	5.
12	$\frac{7}{64}$	.109375	70	4.375
13	$\frac{3}{32}$	.09375	60	3.75
14	$\frac{5}{64}$	.078125	50	3.125
15	$\frac{9}{128}$	.0703125	45	2.8125
16	$\frac{1}{16}$	.0625	40	2.5
17	$\frac{9}{160}$	.05625	36	2.25
18	$\frac{1}{20}$	.05	32	2.
19	$\frac{7}{160}$	.04375	28	1.75
20	$\frac{3}{80}$	.0375	24	1.50
21	$\frac{11}{320}$	.034375	22	1.375
22	$\frac{1}{32}$	.03125	20	1.25
23	$\frac{9}{320}$	.028125	18	1.125
24	$\frac{1}{40}$	.025	16	1.
25	$\frac{7}{320}$	.021875	14	.875
26	$\frac{3}{160}$	.01875	12	.75
27	$\frac{11}{640}$	.0171875	11	.6875
28	$\frac{1}{64}$	.015625	10	.625
29	$\frac{9}{640}$	.0140625	9	.5625
30	$\frac{1}{80}$	.0125	8	.5
31	$\frac{7}{640}$	.0109375	7	.4375
32	$\frac{13}{1280}$	.01015625	6½	.40625
33	$\frac{3}{320}$	.009375	6	.375
34	$\frac{11}{1280}$	.00859375	5½	.34375
35	$\frac{5}{640}$	.0078125	5	.3125
36	$\frac{9}{1280}$	.00703125	4½	.28125
37	$\frac{17}{2560}$	.006640625	4¼	.265625
38	$\frac{1}{160}$	.00625	4	.25

TABLE No. 17. Different Standards for Wire Gage in Use in the United States.

Dimensions of Sizes in Decimal Parts of an Inch.
(Brown & Sharpe Mfg. Co.)

Number of Wire Gage.	American or Brown & Sharpe.	Birmingham or Stubs' Wire.	Washburn & Moen Mfg. Co. Worcester, Mass.	Trenton Iron Co., Trenton, N. J.	Stubs' Steel Wire.	U. S. Stand. for Plate.	Number of Wire Gage.
000000						.46875	000000
00000				.45		.4375	00000
0000	.46	.454	.3938	.4		.40625	0000
000	.40964	.425	.3625	.36		.375	000
00	.3648	.38	.3310	.33		.34375	00
0	.32486	.34	.3065	.305		.3125	0
1	.2893	.3	.2830	.285	.227	.28125	1
2	.25763	.284	.2625	.265	.219	.265625	2
3	.22942	.259	.2437	.245	.212	.25	3
4	.20431	.238	.2253	.225	.207	.234375	4
5	.18194	.22	.2070	.205	.204	.21875	5
6	.16202	.203	.1920	.19	.201	.203125	6
7	.14428	.18	.1770	.175	.199	.1875	7
8	.12849	.165	.1620	.16	.197	.171875	8
9	.11443	.148	.1483	.145	.194	.15625	9
10	.10189	.134	.1350	.13	.191	.140625	10
11	.090742	.12	.1205	.1175	.188	.125	11
12	.080808	.109	.1055	.105	.185	.109375	12
13	.071961	.095	.0915	.0925	.182	.09375	13
14	.064084	.083	.0800	.08	.180	.078125	14
15	.057068	.072	.0720	.07	.178	.0703125	15
16	.05082	.065	.0625	.061	.175	.0625	16
17	.045257	.058	.0540	.0525	.172	.05625	17
18	.040303	.049	.0475	.045	.168	.05	18
19	.03589	.042	.0410	.04	.164	.04375	19
20	.031961	.035	.0348	.035	.161	.0375	20
21	.028462	.032	.03175	.031	.157	.034375	21
22	.025347	.028	.0286	.028	.155	.03125	22
23	.022571	.025	.0258	.025	.153	.028125	23
24	.0201	.022	.0230	.0225	.151	.025	24
25	.0179	.02	.0204	.02	.148	.021875	25
26	.01594	.018	.0181	.018	.146	.01875	26
27	.014195	.016	.0173	.017	.143	.0171875	27
28	.012641	.014	.0162	.016	.139	.015625	28
29	.011257	.013	.0150	.015	.134	.0140625	29
30	.010025	.012	.0140	.014	.127	.0125	30

TABLE No. 17. — (Continued).

Number of Wire Gage.	American or Brown & Sharpe.	Birmingham or Stubs' Wire.	Washburn & Moen Mfg. Co. Worcester, Mass.	Trenton Iron Co., Trenton, N. J.	Stubs' Steel Wire.	U. S. Stand. for Plate.	Number of Wire Gage.
31	.008928	.01	0132	.013	.120	.0109375	31
32	.00795	.009	.0128	.012	.115	.01015625	32
33	.00708	.008	.0118	.011	.112	.009375	33
34	.006304	.007	.0104	.01	.110	.00859375	34
35	.005614	.005	.0095	.0095	.108	.0078125	35
36	.005	.004	.0090	.009	.106	.00703125	36
37	.004453			.0085	.103	.006640625	37
38	.003965			.008	.101	.00625	38
39	.003531			.0075	.099		39
40	.003144			.007	.097		40

TABLE No. 18. Weight of Iron Wire in Pounds per 100 Feet.

No. of Wire Gage.	American or Brown & Sharpe.	Birmingham or Stubs' Wire.	No. of Wire Gage.	American or Brown & Sharpe.	Birmingham or Stubs' Wire.
0000	56.074	54.620	19	0.341	0.467
000	44.4683	47.865	20	0.270	0.324
00	35.265	38.266	21	0.214	0.271
0	27.966	30.634	22	0.170	0.207
1	22.178	23.850	23	0.135	0.165
2	17.588	21.373	24	0.107	0.128
3	13.948	17.776	25	0.0849	0.106
4	11.061	15.010	26	0.0673	0.0858
5	8.772	12.826	27	0.0534	0.0678
6	6.956	10.920	28	0.0423	0.0519
7	5.516	8.586	29	0.0335	0.0447
8	4.375	7.214	30	0.0266	0.0381
9	3.469	5.804	31	0.0211	0.0265
10	2.751	4.758	32	0.0167	0.0214
11	2.182	3.816	33	0.0132	0.0169
12	1.730	3.148	34	0.0105	0.0129
13	1.372	2.391	35	0.00836	0.00662
14	1.088	1.825	36	0.00662	0.00424
15	0.863	1.372	37	0.00525	
16	0.684	1.119	38	0.00416	
17	0.542	0.891	39	0.00330	
18	0.430	0.636	40	0.00262	

TABLE No. 19.—Decimal Equivalents of the Numbers of Twist Drill and Steel Wire Gage. (Brown & Sharpe Mfg. Co.)

No.	Size in Decimals.	No.	Size in Decimals.	No.	Size in Decimals.	No.	Size in Decimals.	No.	Size in Decimals.	No.	Size in Decimals.
1	.2280	15	.1800	29	.1360	42	.0935	55	.0520	68	.0310
2	.2210	16	.1770	30	.1285	43	.0890	56	.0465	69	.02925
3	.2130	17	.1730	31	.1200	44	.0860	57	.0430	70	.0280
4	.2090	18	.1695	32	.1160	45	.0820	58	.0420	71	.0260
5	.2055	19	.1660	33	.1130	46	.0810	59	.0410	72	.0250
6	.2040	20	.1610	34	.1110	47	.0785	60	.0400	73	.0240
7	.2010	21	.1590	35	.1100	48	.0760	61	.0390	74	.0225
8	.1990	22	.1570	36	.1065	49	.0730	62	.0380	75	.0210
9	.1960	23	.1540	37	.1040	50	.0700	63	.0370	76	.0200
10	.1935	24	.1520	38	.1015	51	.0670	64	.0360	77	.0180
11	.1910	25	.1495	39	.0995	52	.0635	65	.0350	78	.0160
12	.1890	26	.1470	40	.0980	53	.0595	66	.0330	79	.0145
13	.1850	27	.1440	41	.0960	54	.0550	67	.0320	80	.0135
14	.1820	28	.1405								

TABLE No. 20.—Decimal Equivalents of Stubs' Steel Wire Gage. (Brown & Sharpe Mfg. Co.)

Letter.	Size in Decimals.	No. of Wire Gage.	Size in Decimals.	No. of Wire Gage.	Size in Decimals.	No. of Wire Gage.	Size in Decimals.	No. of Wire Gage.	Size in Decimals.	No. of Wire Gage.	Size in Decimals.
Z	.413	H	.266	11	.188	29	.134	47	.077	65	.033
Y	.404	G	.261	12	.185	30	.127	48	.075	66	.032
X	.397	F	.257	13	.182	31	.120	49	.072	67	.031
W	.386	E	.250	14	.180	32	.115	50	.069	68	.030
V	.377	D	.246	15	.178	33	.112	51	.066	69	.029
U	.368	C	.242	16	.175	34	.110	52	.063	70	.027
T	.358	B	.238	17	.172	35	.108	53	.058	71	.026
S	.348	A	.234	18	.168	36	.106	54	.055	72	.024
R	.339	1	.227	19	.164	37	.103	55	.050	73	.023
Q	.332	2	.219	20	.161	38	.101	56	.045	74	.022
P	.323	3	.212	21	.157	39	.099	57	.042	75	.020
O	.316	4	.207	22	.155	40	.097	58	.041	76	.018
N	.302	5	.204	23	.153	41	.095	59	.040	77	.016
M	.295	6	.201	24	.151	42	.092	60	.039	78	.015
L	.290	7	.199	25	.148	43	.088	61	.038	79	.014
K	.281	8	.197	26	.146	44	.085	62	.037	80	.013
J	.277	9	.194	27	.143	45	.081	63	.036		
I	.272	10	.191	28	.139	46	.079	64	.035		

In using the gages known as Stubs' Gages, there should be constantly borne in mind the difference between the Stubs Iron Wire Gage and the Stubs Steel Wire Gage. The Stubs Iron Wire Gage is the one commonly known as the English Standard Wire, or Birmingham Gage, and designates the Stubs *soft* wire sizes. The Stubs Steel Wire Gage is the one that is used in measuring drawn steel wire or drill rods of Stubs' make, and is also used by many makers of American drill rods.

Geometry.

Geometry is the science which teaches the properties of lines, angles, surfaces and solids.

A point indicates only position and has neither length, breadth or thickness. A point has no magnitude.

A line has length, but no breadth or thickness ; it is either straight, curved or mixed.

A straight line is the shortest distance between two points.

A curved line is continuously changing its position.

A mixed line is composed of straight and curved lines.

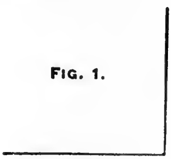
A surface has length and breadth, but no thickness ; it may be either plane or curved.

A solid has length, breadth, and thickness or depth.

An angle is the inclination of two lines which intersect or meet each other. The point of intersection is called the vertex of the angle. An angle is either right, acute or obtuse.

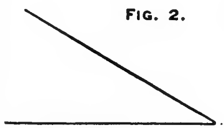
A right angle contains 90 degrees. An acute angle contains less than 90 degrees. An obtuse angle contains more than 90 degrees.

FIG. 1.



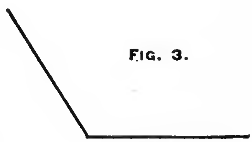
Right Angle.

FIG. 2.



Acute Angle.

FIG. 3.



Obtuse Angle.

* Polygons.

Polygons are plane figures bounded on all sides by straight lines, and are either regular or irregular, according to whether their sides and angles are equal or unequal. The points at which the sides meet are called vertices of the polygon. The distance around any polygon is called the perimeter.

A figure bounded by three straight lines, forming three angles, is called a *triangle*.

The sum of the three angles in any triangle, independent of its size or shape, makes 180 degrees.

All triangles consist of six parts; namely, three sides and three angles. If three of these parts are known, one at least being a side, the other parts may be calculated.

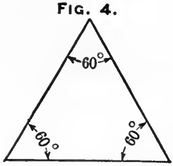
A triangle is called equilateral when all its three sides have equal length. Then all the three angles are equal, namely, 60 degrees, because $60 \times 3 = 180$. (See Fig. 4).

* Some authorities define as polygons only figures having more than four sides.

A triangle is called a right-angled triangle when one angle is 90 degrees; the other two angles will then together consist of 90 degrees, because $90 + 90 = 180$. (See Fig. 5).

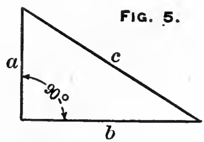
An acute-angled triangle has all its angles acute. (See Fig. 6).

FIG. 4.



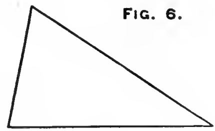
Equilateral Triangle.

FIG. 5.



Right-Angled Triangle.

FIG. 6.



Acute Triangle.

The longest side in a right-angled triangle is called the hypothenuse and the other two sides are called the base and perpendicular. The square of the length of the hypothenuse is equal to the sum of the squares of the lengths of the other two sides. (See Fig. 5).

$$a^2 + b^2 = c^2$$

From this law the third side of a right-angled triangle can always be found, when the length of the other two sides is known. Thus: (See Fig. 5).

$$a = \sqrt{c^2 - b^2} \qquad b = \sqrt{c^2 - a^2} \qquad c = \sqrt{a^2 + b^2}$$

If, instead of the letters a, b, and c, numbers are used, for instance, $a = 3$ and $b = 4$; what then is the length of c?

$$c = \sqrt{3^2 + 4^2} \qquad a = \sqrt{5^2 - 4^2} \qquad b = \sqrt{5^2 - 3^2}$$

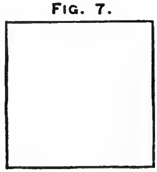
$$c = \sqrt{9 + 16} \qquad a = \sqrt{25 - 16} \qquad b = \sqrt{25 - 9}$$

$$c = \sqrt{25} \qquad a = \sqrt{9} \qquad b = \sqrt{16}$$

$$c = 5 \qquad a = 3 \qquad b = 4$$

A *square* is a plane figure having four right angles and bounded by four straight lines of equal length. (See Fig. 7).

FIG. 7.

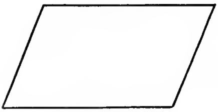


A *parallelogram* is a plane figure whose opposite sides are parallel and of equal length. (See Fig. 8).

A *rectangle* is a parallelogram having all its angles right angles. (See Fig. 9).

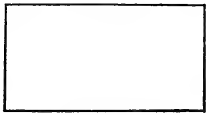
A *trapezoid* is a plane figure bounded by four straight lines, of which only two are parallel. (See Fig. 10).

FIG. 8.



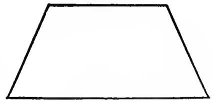
Parallelogram.

FIG. 9.



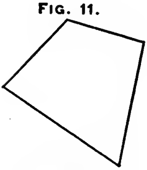
Rectangle.

FIG. 10.



Trapezoid.

FIG. 11.



Trapezium.

A *trapezium* is a plane figure bounded by four sides, all of which have unequal length. (See Fig. 11).

Polygons having four sides, and consequently four angles, are usually called quadrangles. Polygons having more than four sides are named from the number of their sides.

Thus:

A	polygon	having	five	sides	is	called	a pentagon.
"	"	"	six	"	"	"	a hexagon.
"	"	"	seven	"	"	"	a heptagon.
"	"	"	eight	"	"	"	an octagon.
"	"	"	nine	"	"	"	a nonagon.
"	"	"	ten	"	"	"	a decagon.
"	"	"	eleven	"	"	"	an undecagon.
"	"	"	twelve	"	"	"	a dodecagon.

The sum of the degrees of all the angles of any polygon can always be found by subtracting 2 from the number of sides and multiplying the remainder by 180.

For instance:

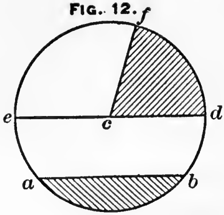
The sum of degrees in any quadrangle is always $(4 - 2) \times 180 = 360$ degrees.

The sum of degrees in any pentagon will always be $(5 - 2) \times 180 = 540$ degrees.

This is a useful fact to remember in making drawings, as it may be used for verifying angles of polygons.

Circles.

FIG. 12.



Circle.

The Circle is a plane figure bounded by a curved line called the circumference or perphery, which is at all points the same distance from a fixed point in the plane, and this point is called the center of the circle. (See point *c*, Fig. 12).

A Diameter is a straight line passing through the center of a circle or a sphere, terminating at the circumference or surface. (See line *e-d*, Fig. 12).

A Radius is a straight line from the center to the circumference of circle or sphere. (See line *c-f*, Fig. 12).

Diameter $= 2 \times$ radius. The ratio of the circumference to the diameter of a circle is usually denoted by the Greek letter π and is expressed approximately by the number 3.1416 or $\frac{22}{7} = 3\frac{1}{7}$.

Thus, if the circumference is required, multiply the diameter by 3.1416. If the diameter is required, divide the circumference by 3.1416.

A Chord is a straight line terminating at the circumference of a circle but not passing through the center of the circle. (See line *a-b*, Fig. 12). The curved line *a-b*, or any other part of the circumference of a circle, is called an *arc*.

Any surface bounded by the chord and an arc, like the shaded surface *a-b*, is called a *segment*.

Any surface bounded by a chord and its two radii, like the shaded surface *c-f-d*, is called a *sector*.

PROPERTIES OF THE CIRCLE.

Circumference $=$ Diameter $\times$ 3.1416

Area $=$ (Diameter)$^2 \times$ 0.7854

Diameter $=$ Circumference $\times$ 0.31831

Diameter $= \sqrt{\frac{\text{area}}{0.7854}}$

Diameter $= 1.1283 \times \sqrt{\text{area}}$

Circumference $= 3.5449 \times \sqrt{\text{area}}$

Length of any arc $=$ Number of degrees $\times$ 0.017453 $\times$ radius.

Length of arc of 1 Degree when radius is 1 is 0.017453.

Length of an arc of 1 Minute when radius is 1 is 0.000290888.

Length of an arc of 1 Second when radius is 1 is 0.000004848.

When the length of the arc is equal to the radius the angle is $57^\circ\ 17'\ 45'' = 57.2957795$ degrees.

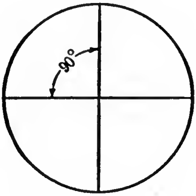
TRIGONOMETRY.

Trigonometry is that branch of geometry which treats of the solution of triangles by means of the trigonometrical functions.

When the circumference of a circle is divided into 360 equal parts each part is called *one degree.*

One fourth of a circle is 90 degrees = right angle, because $4 \times 90 = 360$. (See Fig. 13.)

FIG. 13.



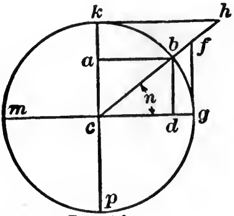


FIG. 14

Circle = 4 right angles.
Circle = 360 degrees (360°.)

1 degree = 60 minutes (60′.)
1 minute = 60 seconds (60″.)

Concerning the angle n (see Fig. 14) the following are the trigonometrical functions:

$c\ g$ radius = 1.
$d\ b$ sine (sin.)
$c\ d$ cosine (cos.)
$c\ f$ secant (sec.)
$c\ h$ cosecant (cosec.)
$g\ f$ tangent (tan.)
$k\ h$ cotangent (cot.)

The complement of an angle is what remains after subtracting the angle from 90°. Thus, the complement of an angle of 30° is 60° because $90 - 30 = 60$.

The supplement of an angle is what remains after subtracting the angle from 180°. Thus, the supplement of an angle of 30° is 150° because $180 - 30 = 150$.

As all circles, regardless of their size, are divided into 360 degrees, the trigonomical functions must always be alike if the radius and the angle that they denote are alike.

It is on this basis that the tables of trigonometrical functions are calculated, and as radius is used the figure 1.

In Table No. 20, the natural sine of 30° is given as 0.5; this means that if the line $c\ g$ (see Fig. 14) is 1 foot, meter, or any other unit, and the angle n is 30 degrees, the line $d\ b$ will be 0.5 of the same unit as the line $c\ g$.

Sine 45° = 0.70711; that is, if the the angle n is 45 degrees and the line $c\ g$ is 1 of any unit, the line $d\ b$ is 0.70711 of the same unit.

Cos. 30° = 0.86603; that is, if the angle n is 30 degrees and the line $c\ g$ is 1 of any unit, the line $a\ b$ or $c\ d$ is 0,86603 of the same unit, Sec. 30° = 1.1547; that is, if the angle n is 30 degrees and the line $c\,g$ is 1 of any unit, the line $c\,f$ is 1.1547 of the same unit.

Cosec. 30° = 2; that is, if the angle n is 30 degrees and the line $c\,g$ is 1 of any unit, the line $c\,h$ is 2 of the same unit.

Tang. 30° = 0.57735; that is, if the angle n is 30 degrees and the line $c\,g$ is 1 of any unit, the line $g\,f$ is 0.57735 of the same unit.

Cot. 30° = 1.73205; that is, if the angle n is 30 degrees and the line $c\,g$ is 1 of any unit, the line $k\,h$ is 1.73205 of the same unit.

Increasing the angle n will increase sine, tangent and secant, but will decrease cosine, cotangent and cosecant.

When the angle n approaches 90°, the tangents $g\,f$ increase more and more to infinite length. When n actually reaches 90° of course $c\,b$ coincides with $c\,k$ and becomes parallel to $g\,f$, so that in an angle of 90° both the secant and the tangent have infinite length, which is denoted by the sign ∞, and cosine and cotangent have vanished.

In the first quadrant (that is when angle n does not exceed 90°) the trigonometrical functions are all considered to be positive and are denoted by + (plus). When the angle n exceeds 90°, only sine and cosecants remain positive; all the other functions have become negative and are denoted by — (minus).

The following table gives the properties of the trigonometrical functions in the four different quadrants:

Degree.	Sine.	Cosine.
0° to 90°	Increase from 0 to radius +	Decrease from radius to 0 +
90° to 180°	Decrease from radius to 0 +	Increase from 0 to radius —
180° to 270°	Increase from 0 to radius —	Decrease from radius to 0 —
270° to 360°	Decrease from radius to 0 —	Increase from 0 to radius +
Degree.	**Secant.**	**Cosecant.**
0° to 90°	Increase from radius to ∞ +	Decrease from ∞ to radius +
90° to 180°	Decrease from ∞ to radius —	Increase from radius to ∞ +
180° to 270°	Increase from radius to ∞ —	Decrease from ∞ to radius —
270° to 360°	Decrease from ∞ to radius +	Increase from radius to ∞ —
Degree.	**Tangent.**	**Cotangent.**
0° to 90°	Increase from 0 to ∞ +	Decrease from ∞ to 0 +
90° to 180°	Decrease from ∞ to 0 —	Increase from 0 to ∞ —
180° to 270°	Increase from 0 to ∞ +	Decrease from ∞ to 0 +
270° to 360°	Decrease from ∞ to 0 —	Increase from 0 to ∞ —

From the rule that the square of the hypothenuse is equal to the sum of the squares of the base and the perpendicular, it also follows that:

$$\begin{aligned} &\text{Sin.}^2 + \text{cos.}^2 = \text{radius}^2. \\ &\text{Tang.}^2 + \text{radius}^2 = \text{secant}^2. \\ &\text{Cot.}^2 + \text{radius}^2 = \text{cosecant}^2. \end{aligned}$$

But the trigonometrical tables are calculated with radius $= 1$, hence,

$$\begin{aligned} &\text{sin.}^2 + \text{cos.}^2 = 1. \\ &\text{tang.}^2 + 1 = \text{sec.}^2 \\ &\text{cotang.}^2 + 1 = \text{cosecant}^2. \end{aligned}$$

$$\text{tang.} = \frac{\text{sin.}}{\text{cosin.}} \qquad \text{tang.} = \frac{1}{\text{cotang.}}$$

$$\text{secant} = \frac{1}{\text{cosin.}} \qquad \text{secant} = \frac{\text{tang.}}{\text{sin.}}$$

$$\text{cotang.} = \frac{\text{cosin.}}{\text{sin.}} \qquad \text{sin.} = \frac{1}{\text{cosec.}}$$

$$\text{cosec.} = \frac{1}{\text{sin.}} \qquad \text{sin.} = \frac{\text{cosin.}}{\text{cotang.}}$$

$$\text{cotang.} = \frac{1}{\text{tang.}} \qquad \text{sin.} = \sqrt{1 - \text{cos.}^2}$$

$$\text{cosin.} = \sqrt{1 - \text{sin.}^2} \qquad \text{cosin.} = \frac{\text{sin.}}{\text{tang.}}$$

FIG. 15.

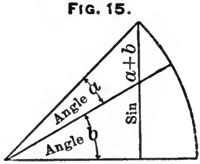


FIG. 16.

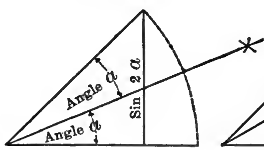
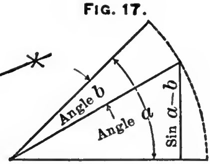


FIG. 17.



Sine and Cosine of the Sum of Two Angles.

(See Fig. 15).

$$\begin{aligned} &\text{Sine } (a + b) = \text{sin. } a \times \text{cos. } b + \text{cos. } a \times \text{sin. } b. \\ &\text{Cos. } (a + b) = \text{cos. } a \times \text{cos. } b - \text{sin. } a \times \text{sin. } b. \end{aligned}$$

Sine and Cosine of Twice any Angle.

(See Fig. 16).

$$\begin{aligned} &\text{Sin. } 2\,a = 2 \times \text{sin. } a \times \text{cos. } a. \\ &\text{Cos. } 2\,a = \text{cos.}^2\, a - \text{sin.}^2\, a. \end{aligned}$$

Sine and Cosine of the Difference of Two Angles.

(See Fig. 17).

$$\begin{aligned} &\text{Sin. } (a - b) = \text{sin. } a \times \text{cos. } b - \text{cos. } a \times \text{sin. } b. \\ &\text{Cosin. } (a - b) = \text{cos, } a \times \text{cos. } b + \text{sin. } a \times \text{sin, } b, \end{aligned}$$

Value of the Trigonometrical Functions for Some of the Most Common Angles.

Angle.	Sine.	Cosine.	Tangent.	Cotangent.	Secant.	Cosecant.
0°	0	1	0	∞	1	∞
30°	$\frac{1}{2} = 0.5000$	$\frac{\sqrt{3}}{2} = 0.8660$	$\frac{1}{\sqrt{3}} = 0.5773$	$\sqrt{3} = 1.731$	$\frac{2}{\sqrt{3}} = 1.1547$	2
45°	$\frac{1}{\sqrt{2}} = 0.7071$	$\frac{1}{\sqrt{2}} = 0.7071$	1	1	$\sqrt{2} = 1.4142$	$\sqrt{2} = 1.4142$
60°	$\frac{\sqrt{3}}{2} = 0.8660$	$\frac{1}{2} = 0.5000$	$\sqrt{3} = 1.732$	$\frac{1}{\sqrt{3}} = 0.5773$	2	$\frac{2}{\sqrt{3}} = 1.1547$
90°	1	0	∞	0	∞	1
120°	$\frac{\sqrt{3}}{2} = 0.8660$	$-\frac{1}{2} = -0.5000$	$-\sqrt{3} = -1.732$	$-\frac{1}{\sqrt{3}} = -0.5773$	$-\frac{2}{\sqrt{3}} = -1.1547$	2
135°	$\frac{1}{\sqrt{2}} = 0.7071$	$-\frac{1}{\sqrt{2}} = -0.7071$	-1	-1	$-\sqrt{2} = -1.4142$	$\sqrt{2} = 1.4142$
150°	$\frac{1}{2} = 0.5000$	$-\frac{\sqrt{3}}{2} = -0.866$	$-\frac{1}{\sqrt{3}} = -0.5773$	$-\sqrt{3} = -1.732$	$-\frac{2}{\sqrt{3}} = -1.1547$	2
180°	0	-1	0	$-\infty$	-1	∞

The Trigonometrical Table and Its Use.

Table No. 21 gives sine, cosine, tangent, and cotang, to angles from 0 to 90 degrees with intervals of 10 minutes.

For sine or tangent find the degree in the left-hand column and find the minutes on the top of the table. For instance, sine to $18^\circ\ 40' = 0.32006$.

If cosine or cotangent is wanted, find the degree in the column at the extreme right and the minutes at the bottom of the table. For instance, cotang $48^\circ\ 10' = 0.89515$.

As the table only gives the angles and their trigonometrical functions with 10-minute intervals, any intermediate angle must be calculated by interpolations. For instance, find sine of 60° $15'\ 10''$.

Solution:

$$\begin{array}{rl} \text{Sine } 60^\circ\ 20'\ 0'' &= 0.86892 \\ \text{Sine } 60^\circ\ 10'\ 0'' &= 0.86748 \\ \hline \text{Difference of } 0^\circ\ 10'\ 0'' &= 0.00144 \end{array}$$

$60^\circ\ 15'\ 10'' - 60^\circ\ 10'\ 0'' = 0^\circ\ 5'\ 10' = 310$ seconds and a difference of $10' = 600''$ increases this sine 0.00144. Therefore a difference of 310 seconds will increase the sine.

$$\frac{310 \times 0.00144}{600} = 0.00074$$

$$\text{and sine } 60^\circ\ 10'\ 0'' = 0.86748$$

$$\text{Therefore sine } 60^\circ\ 15'\ 5'' = 0.86822$$

IMPORTANT.—During all interpolations concerning the trigonometrical functions, remember the fact that if the angle is increasing both sine and tangent are also increasing, and corrections found by interpolations must be added to the number already found; but as the cosine and cotangent decrease when the angle is increased, for these functions the corrections must be subtracted.

Interpolations of this kind are not strictly correct, as neither the trigonometrical functions nor their logarithms differ in proportion to the angle. The error within such small limits as 10 minutes is very slight. When very close calculations of great distances are required, tables are used which give the functions with less difference than 10 minutes; but for mechanical purposes in general these interpolations are correct for all ordinary requirements. It is very seldom in a draughting office or a machine shop that any angle is measured for a difference of less than 10'.

To Find Secant and Cosecant of Any Angle.

Divide 1 by cosine of the angle and the quotient is secant of the same angle.

Divide 1 by sine of the angle and the quotient is cosecant of the same angle.

(Table No. 21) Sines.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	0.00000	0.00291	0.00582	0.00873	0.01164	0.01454	0.01745	89
1	0.01745	0.02036	0.02327	0.02618	0.02909	0.03199	0.03490	88
2	0.03490	0.03781	0.04071	0.04362	0.04653	0.04943	0.05234	87
3	0.05234	0.05524	0.05815	0.06105	0.06395	0.06685	0.06976	86
4	0.06976	0.07266	0.07556	0.07846	0.08136	0.08426	0.08716	85
5	0.08716	0.09005	0.09295	0.09585	0.09874	0.10164	0.10453	84
6	0.10453	0.10742	0.11031	0.11320	0.11609	0.11898	0.12187	83
7	0.12187	0.12476	0.12764	0.13053	0.13341	0.13629	0.13917	82
8	0.13917	0.14205	0.14493	0.14781	0.15069	0.15356	0.15643	81
9	0.15643	0.15931	0.16218	0.16505	0.16792	0.17078	0.17365	80
10	0.17365	0.17651	0.17938	0.18224	0.18510	0.18795	0.19081	79
11	0.19081	0.19366	0.19652	0.19937	0.20222	0.20507	0.20791	78
12	0.20791	0.21076	0.21360	0.21644	0.21928	0.22212	0.22495	77
13	0.22495	0.22778	0.23062	0.23345	0.23627	0.23910	0.24192	76
14	0.24192	0.24474	0.24756	0.25038	0.25320	0.25601	0.25882	75
15	0.25882	0.26163	0.26443	0.26724	0.27004	0.27284	0.27564	74
16	0.27564	0.27843	0.28123	0.28402	0.28680	0.28959	0.29237	73
17	0.29237	0.29515	0.29793	0.30071	0.30348	0.30625	0.30902	72
18	0.30902	0.31178	0.31455	0.31731	0.32006	0.32282	0.32557	71
19	0.32557	0.32832	0.33106	0.33381	0.33655	0.33929	0.34202	70
20	0.34202	0.34475	0.34748	0.35021	0.35293	0.35565	0.35837	69
21	0.35837	0.36108	0.36379	0.36650	0.36921	0.37191	0.37461	68
22	0.37461	0.37730	0.37999	0.38268	0.38537	0.38805	0.39073	67
	60′	50′	40′	30′	20′	10′	0′	Deg.

Cosines (read upwards).

Sines.

Deg.	0′	10′	20′	30′	40′	50	60′	
23	0.39073	0.39341	0.39608	0.39875	0.40142	0.40408	0.40674	66
24	0.40674	0.40939	0.41205	0.41469	0.41734	0.41998	0.42262	65
25	0.42262	0.42525	0.42788	0.43051	0.43314	0.43576	0.43837	64
26	0.43837	0.44098	0.44359	0.44620	0.44880	0.45140	0.45399	63
27	0.45399	0.45658	0.45917	0.46175	0.46433	0.46690	0.46947	62
28	0.46947	0.47204	0.47460	0.47716	0.47971	0.48226	0.48481	61
29	0.48481	0.48735	0.48989	0.49242	0.49495	0.49748	0.50000	60
30	0.50000	0.50252	0.50503	0.50754	0.51004	0.51254	0.51504	59
31	0.51504	0.51753	0.52002	0.52250	0.52498	0.52745	0.52992	58
32	0.52992	0.53238	0.53484	0.53730	0.53975	0.54220	0.54464	57
33	0.54464	0.54708	0.54951	0.55194	0.55436	0.55678	0.55919	56
34	0.55919	0.56160	0.56401	0.56641	0.56880	0.57119	0.57358	55
35	0.57358	0.57596	0.57833	0.58070	0.58307	0.58543	0.58779	54
36	0.58779	0.59014	0.59248	0.59482	0.59716	0.59949	0.60182	53
37	0.60182	0.60414	0.60645	0.60876	0.61107	0.61337	0.61566	52
38	0.61566	0.61795	0.62024	0.62252	0.62479	0.62706	0.62932	51
39	0.62932	0.63158	0.63383	0.63608	0.63832	0.64056	0.64279	50
40	0.64279	0.64501	0.64723	0.64945	0.65166	0.65386	0.65606	49
41	0.65606	0.65825	0.66044	0.66262	0.66480	0.66697	0.66913	48
42	0.66913	0.67129	0.67344	0.67559	0.67773	0.67987	0.68200	47
43	0.68200	0.68412	0.68624	0.68836	0.69046	0.69256	0.69466	46
44	0.69466	0.69675	0.69883	0.70091	0.70298	0.70505	0.70711	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

Cosines (read upwards).

Sines.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	0.70711	0.70916	0.71121	0.71325	0.71529	0.71732	0.71934	44
46	0.71934	0.72136	0.72337	0.72537	0.72737	0.72937	0.73135	43
47	0.73135	0.73333	0.73531	0.73728	0.73924	0.74120	0.74315	42
48	0.74315	0.74509	0.74703	0.74896	0.75088	0.75280	0.75471	41
49	0.75471	0.75662	0.75851	0.76041	0.76229	0.76417	0.76604	40
50	0.76604	0.76791	0.76977	0.77163	0.77347	0.77531	0.77715	39
51	0.77715	0.77897	0.78079	0.78261	0.78442	0.78622	0.78801	38
52	0.78801	0.78980	0.79158	0.79335	0.79512	0.79688	0.79864	37
53	0.79864	0.80038	0.80212	0.80386	0.80558	0.80730	0.80902	36
54	0.80902	0.81072	0.81242	0.81412	0.81580	0.81748	0.81915	35
55	0.81915	0.82082	0.82248	0.82413	0.82577	0.82741	0.82904	34
56	0.82904	0.83066	0.83228	0.83389	0.83549	0.83708	0.83867	33
57	0.83867	0.84025	0.84183	0.84339	0.84495	0.84650	0.84805	32
58	0.84805	0.84959	0.85112	0.85264	0.85416	0.85567	0.85717	31
59	0.85717	0.85866	0.86015	0.86163	0.86310	0.86457	0.86603	30
60	0.86603	0.86748	0.86892	0.87036	0.87178	0.87321	0.87462	29
61	0.87462	0.87603	0.87743	0.87882	0.88020	0.88158	0.88295	28
62	0.88295	0.88431	0.88566	0.88701	0.88835	0.88968	0.89101	27
63	0.89101	0.89232	0.89363	0.89493	0.89623	0.89752	0.89879	26
64	0.89879	0.90007	0.90133	0.90259	0.90383	0.90508	0.90631	25
65	0.90631	0.90753	0.90875	0.90996	0.91116	0.91236	0.91355	24
66	0.91355	0.91473	0.91590	0.91706	0.91822	0.91936	0.92051	23
67	0.92051	0.92164	0.92276	0.92388	0.92499	0.92609	0.92718	22
	60′	50′	40′	30′	20′	10′	0′	Deg.

Cosines (read upwards).

Sines.

Deg.	0′	10′	20′	30′	40′	50′	60′	
68	0.92718	0.92827	0.92935	0.93042	0.93148	0.93253	0.93358	21
69	0.93358	0.93462	0.93565	0.93667	0.93769	0.93869	0.93969	20
70	0.93969	0.94068	0.94167	0.94264	0.94361	0.94457	0.94552	19
71	0.94552	0.94646	0.94740	0.94832	0.94924	0.95015	0.95106	18
72	0.95106	0.95195	0.95284	0.95372	0.95459	0.95545	0.95631	17
73	0.95631	0.95715	0.95799	0.95882	0.95964	0.96046	0.96126	16
74	0.96126	0.96206	0.96285	0.96363	0.96440	0.96517	0.96593	15
75	0.96593	0.96668	0.96742	0.96815	0.96887	0.96959	0.97030	14
76	0.97030	0.97100	0.97169	0.97237	0.97305	0.97371	0.97437	13
77	0.97437	0.97502	0.97566	0.97630	0.97692	0.97754	0.97815	12
78	0 97815	0.97875	0.97934	0.97993	0.98050	0.98107	0.98163	11
79	0.98163	0.98218	0.98272	0.98326	0.98378	0.98430	0.98481	10
80	0.98481	0.98531	0.98580	0.98629	0.98676	0.98723	0.98769	9
81	0.98769	0.98814	0.98858	0.98902	0.98944	0.98986	0.99027	8
82	0.99027	0.99067	0.99106	0.99145	0.99182	0.99219	0.99255	7
83	0.99255	0.99290	0.99324	0.99357	0.99390	0.99421	0.99452	6
84	0.99452	0.99482	0.99511	0.99540	0.99567	0.99594	0.99620	5
85	0.99620	0.99644	0.99669	0.99692	0.99714	0.99736	0.99756	4
86	0.99756	0.99776	0.99795	0.99814	0.99831	0.99847	0.99863	3
87	0.99863	0.99878	0.99892	0.99905	0.99917	0.99929	0.99939	2
88	0.99939	0.99949	0.99958	0.99966	0.99973	0.99979	0.99985	1
89	0.99985	0.99989	0.99993	0.99996	0.99998	0.99999	1.00000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

Cosines (read upwards).

Tangents.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	0.00000	0.00291	0.00582	0.00873	0.01164	0.01455	0.01746	89
1	0.01746	0.02037	0.02328	0.02619	0.02910	0.03201	0.03492	88
2	0.03492	0.03783	0.04075	0,04366	0.04658	0.04949	0.05241	87
3	0.05241	0.05533	0.05824	0.06116	0.06408	0.06700	0.06993	86
4	0.06993	0.07285	0.07578	0.07870	0.08163	0.08456	0.08749	85
5	0.08749	0.09042	0.09335	0.09629	0.09923	0.10216	0.10510	84
6	0.10510	0.10805	0.11099	0.11394	0.11688	0.11983	0.12279	83
7	0.12279	0.12574	0.12869	0.13165	0.13461	0.13758	0.14054	82
8	0.14054	0.14351	0.14648	0.14945	0.15243	0.15540	0.15838	81
9	0.15838	0.16137	0.16435	0.16734	0.17033	0.17333	0.17633	80
10	0.17633	0.17933	0.18233	0.18534	0.18835	0.19136	0.19438	79
11	0.19438	0.19740	0.20043	0.20345	0.20648	0.20952	0.21256	78
12	0.21256	0.21560	0.21865	0.22170	0.22475	0.22781	0.23087	77
13	0.23087	0.23393	0.23700	0.24008	0.24316	0.24624	0.24933	76
14	0.24933	0.25242	0.25552	0.25862	0.26172	0.26483	0.26795	75
15	0.26795	0.27107	0.27419	0,27733	0.28046	0.28360	0.28675	74
16	0.28675	0.28989	0.29305	0.29621	0.29938	0.30255	0.30573	73
17	0.30573	0.30891	0.31210	0.31530	0.31850	0.32171	0.32492	72
18	0.32492	0.32814	0.33136	0.33460	0.33783	0.34108	0.34433	71
19	0.34433	0.34759	0.35085	0.35412	0.35740	0.36068	0.36397	70
20	0.36397	0.36727	0.37057	0.37389	0.37720	0.38053	0.38386	69
21	0.38386	0.38721	0.39055	0.39391	0.39728	0.40065	0.40403	68
22	0.40403	0.40741	0.41081	0.41421	0.41763	0.42105	0.42448	67
	60′	50′	40′	30′	20′	10′	0′	Deg.

Cotangents (read upwards).

Tangents.

Deg.	0′	10′	20′	30′	40′	50′	60′	
23	0.42448	0.42791	0.43136	0.43481	0.43828	0.44175	0.44523	66
24	0.44523	0.44872	0.45222	0.45573	0.45924	0.46277	0.46631	65
25	0.46631	0.46985	0.47341	0.47698	0.48055	0.48414	0.48773	64
26	0.48773	0.49134	0.49496	0.49858	0.50222	0.50587	0.50953	63
27	0.50953	0.51320	0.51688	0.52057	0.52427	0.52798	0.53171	62
28	0.53171	0.53545	0.53920	0.54296	0.54673	0.55051	0.55431	61
29	0.55431	0.55812	0.56194	0.56577	0.56962	0.57348	0.57735	60
30	0.57735	0.58124	0.58514	0.58905	0.59297	0.59691	0.60086	59
31	0.60086	0.60483	0.60881	0.61280	0.61681	0.62083	0.62487	58
32	0.62487	0.62892	0.63299	0.63707	0.64117	0.64528	0.64941	57
33	0.64941	0.65355	0.65771	0.66189	0.66608	0.67028	0.67451	56
34	0.67451	0.67875	0.68301	0.68728	0.69157	0.69588	0.70021	55
35	0.70021	0.70455	0.70891	0.71329	0.71769	0.72211	0.72654	54
36	0.72654	0.73100	0.73547	0.73996	0.74447	0.74900	0.75355	53
37	0.75355	0.75813	0.76272	0.76733	0.77196	0.77661	0.78129	52
38	0.78129	0.78598	0.79070	0.79544	0.80020	0.80498	0.80978	51
39	0.80978	0.81461	0.81946	0.82434	0.82923	0.83416	0.83910	50
40	0.83910	0.84407	0.84906	0.85408	0.85912	0.86419	0.86929	49
41	0.86929	0.87441	0.87955	0.88473	0.88992	0.89515	0.90040	48
42	0.90040	0.90569	0.91099	0.91633	0.92170	0.92709	0.93252	47
43	0 93252	0.93797	0.94345	0.94897	0.95451	0.96008	0.96569	46
44	0.96569	0.97133	0.97700	0.98270	0.98843	0.99420	1.00000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

Cotangents (read upwards).

Tangents.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1.00000	1.00584	1.01170	1.01761	1.02355	1.02952	1.03553	44
46	1.03553	1.04158	1.04766	1.05378	1.05994	1.06613	1.07237	43
47	1.07237	1.07864	1.08496	1.09131	1.09770	1.10414	1.11061	42
48	1.11061	1.11713	1.12369	1.13029	1.13694	1.14363	1.15037	41
49	1.15037	1.15715	1.16398	1.17085	1.17777	1.18474	1.19175	40
50	1.19175	1.19882	1.20593	1.21310	1.22032	1.22758	1.23490	39
51	1.23490	1.24227	1.24969	1.25717	1.26471	1.27230	1.27994	38
52	1.27994	1.28765	1.29541	1.30323	1.31111	1.31904	1.32705	37
53	1.32705	1.33511	1.34323	1.35142	1.35968	1.36800	1.37638	36
54	1.37638	1.38484	1.39336	1.40195	1.41061	1.41934	1.42815	35
55	1.42815	1.43703	1.44598	1.45501	1.46412	1.47330	1.48256	34
56	1.48256	1.49190	1.50133	1.51084	1.52043	1.53010	1.53987	33
57	1.53987	1.54972	1.55966	1.56969	1.57981	1.59002	1.60034	32
58	1.60034	1.61074	1.62125	1.63185	1.64256	1.65337	1.66428	31
59	1.66428	1.67530	1.68643	1.69766	1.70901	1.72047	1.73205	30
60	1.73205	1.74375	1.75556	1.76749	1.77955	1.79174	1.80405	29
61	1.80405	1.81649	1.82906	1.84177	1.85462	1.86760	1.88073	28
62	1.88073	1.89400	1.90742	1.92098	1.93470	1.94858	1.96261	27
63	1.96261	1.97681	1.99116	2.00569	2.02039	2.03526	2.05030	26
64	2.05030	2.06553	2.08094	2.09654	2.11234	2.12832	2.14451	25
65	2.14451	2.16090	2.17749	2.19430	2.21132	2.22857	2.24604	24
66	2.24604	2.26374	2.28167	2.29984	2.31826	2.33693	2.35585	23
67	2.35585	2.37504	2.39449	2.41421	2.43422	2.45451	2.47509	22
	60′	50′	40′	30′	20′	10′	0′	Deg.

Cotangents (read upwards).

Tangents.

Deg.	0′	10′	20′	30′	40′	50′	60′	
68	2.47509	2.49597	2.51715	2.53865	2.56047	2.58261	2.60509	21
69	2.60509	2.62791	2.65109	2.67462	2.69853	2.72281	2.74748	20
70	2.74748	2.77255	2.79802	2.82391	2.85024	2.87700	2.90421	19
71	2.90421	2.93189	2.96004	2.98869	3.01783	3.04749	3.07768	18
72	3.07768	3.10842	3.13972	3.17159	3.20406	3.23714	3.27085	17
73	3.27085	3.30521	3.34023	3.37594	3.41236	3.44951	3.48741	16
74	3.48741	3.52609	3.56558	3.60588	3.64705	3.68909	3.73205	15
75	3.73205	3.77595	3.82083	3.86671	3.91364	3.96165	4.01078	14
76	4.01078	4.06107	4.11256	4.16530	4.21933	4.27471	4.33148	13
77	4.33148	4.38969	4.44942	4.51071	4.57363	4.63825	4.70463	12
78	4.70463	4.77286	4.84301	4.91516	4.98940	5.06584	5.14455	11
79	5.14455	5.22567	5.30928	5.39552	5.48451	5.57638	5.67128	10
80	5.67128	5.76937	5.87080	5.97576	6.08444	6.19703	6.31375	9
81	6.31375	6.43484	6.56055	6.69116	6.82694	6.96823	7.11537	8
82	7.11537	7.26873	7.42871	7.59575	7.77035	7.95302	8.14435	7
83	8.14435	8.34496	8.55555	8.77689	9.00983	9.25530	9.51436	6
84	9.51436	9.78817	10.07803	10.38540	10.71191	11.05943	11.43005	5
85	11.43005	11.82617	12.25051	12.70621	13.19688	13.72674	14.30067	4
86	14.30067	14.92442	15.60478	16.34986	17.16934	18.07500	19.08114	3
87	19.08114	20.20555	21.47040	22.90377	24.54176	26.43160	28.63625	2
88	28.63625	31.24158	34.36777	38.18846	42.96408	49.10388	57.29000	1
89	57.29000	68.75009	85.93979	114.58865	171.88540	343.77371	$+\infty$	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

Cotangents (read upwards).

Logarithms Corresponding to the Trigonometrical Functions.

Table No. 22 gives the logarithms corresponding to sine, cosine, tangent and cotang. for angles from 0 to 90 degrees, with intervals of 10 minutes. For sine and tangent find the degree in the column to the left and the minutes at the top of the table. For instance:

Log. sine 19° 30′ = 9.523495 — 10.

This, of course, is also logarithm to the fraction 0.33381, which is sine of 19° 30′.

For cosine and cotang. find the degree in the column to the extreme right in the table, and find the minutes at the bottom of table. For instance:

Log. cotang. 37° 10′ = 10.120259 — 10 = 0.120259.

NOTE.—In this table the index of the logarithm is increased by 10, therefore — 10 must always be annexed in the logarithm.

Logarithms to angles between those in the table may be obtained by interpolations. For instance, find *log.* sine 25° 45′.

Solution:

Log. sine 25° 50′ = 9.639242 — 10
Log. sine 25° 40′ = 9.636623 — 10
Difference 0.002619

This difference in the logarithm corresponds to a difference in this angle of 10 minutes; therefore a difference of 5 minutes in the angle will make a difference of 0.001309 in the logarithm. Thus:

Log. sine 25° 40′ = 9.636623 — 10
Difference 5′ = 0.001309
Log. sine 25° 45′ = 9.637932 — 10

EXAMPLE 2.

Find angle corresponding to logarithmic sine 9.894246 — 10.

Solution:

In the table of logarithms of sine:

9.894546 — 10 corresponds to 51° 40′
9.893544 — 10 corresponds to 51° 30′
Difference 0.001002 corresponds to 0° 10′

To logarithm 9.894246 — 10 must, therefore, correspond an angle somewhere between 51° 30′ and 51° 40′, which is found thus:

The given logarithm is 9.894246 — 10
Nearest less logarithm 9.893544 — 10 for 51° 30′
Difference 0.000702

Therefore, the correction to be added to the angle already found will be:

$$\frac{0.000702 \times 10}{0.001002} = 0° \, 7'$$

Thus, the logarithmic sine 9.894246 — 10 gives 51° 37′

EXAMPLE 3.

Find *log.* to tangent of $50° 45'$

Solution:

Log. tangent $50° 50' = 0.089049$
Log. tangent $50° 40' = 0.086471$

Difference $0° 10' = 0.002578$ in the logarithm. Therefore a difference of $5'$ in the angle will give 0.001289 in the logarithm.

Thus:

$$\begin{array}{lr} \textit{Log.}\text{ tangent } 50° 40' = & 0.086471 \\ \text{Difference } 5' = & 0.001289 \\ \hline \textit{Log.}\text{ tangent } 50° 45' = & 0.087760 \end{array}$$

EXAMPLE 4.

Find the angle corresponding to *log.* tangent $9.899049 - 10$.

Solution:

Log. tangent $38° 30' = 9.900605 - 10$
Log. tangent $38° 20' = 9.898010 - 10$

Difference $0° 10'$ corresponds to 0.002595

The given logarithm $= 9.899049 - 10$
Nearest less logarithm $= 9.898010 - 10$ gives $38° 20'$
Difference $= 0.001039$

The difference to be added to the angle already found will be $\dfrac{0.001039 \times 10}{0.002595} = 0° 4'$.

The tabulated logarithm $9.898010 - 10$ gives angle $38° 20'$
Difference 0.001039 gives angle $4'$
Logarithm $9.899049 - 10$ gives angle $38° 24'$

To Find Logarithm for Secants and Cosecants.

Logarithm for secants is found by subtracting *log.* cosine from *log.* 1.

For instance, find logarithmic secant 30°.

Solution:

$$\begin{array}{rl} \textit{Log.}\ 1 = & 10.000000 - 10 \\ \textit{Log.}\text{ cosine } 30° = & 9.937531 - 10 \\ \hline \textit{Log.}\text{ secant } 30° = & 0.062469 \end{array}$$

Logarithm for cosecants is found by subtracting *log.* sine from *log.* 1. For instance, find logarithmic cosecant 35°.

Solution:

$$\begin{array}{rl} \textit{Log.}\ 1 = & 10.000000 - 10 \\ \textit{Log.}\text{ sine } 35° = & 9.758591 - 10 \\ \hline \textit{Log.}\text{ co-secant } 35° = & 0.241409 \end{array}$$

NOTE.—What is said concerning interpolations of trigonometrical functions in general in the note headed "Important" on page 157, will also apply to their logarithms.

Table No. 22) Logarithmic Sines.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	— ∞	7.463726	7.764754	7.940842	8.065776	8.162681	8.241855	89
1	8.241855	8.308794	8.366777	8.417919	8,463665	8.505045	8.542819	88
2	8.542819	8.577566	8,609734	8.639680	8,667689	8.693998	8.718800	87
3	8.718800	8,742259	8.764511	8.785675	8,805852	8.825130	8.843585	86
4	8.843585	8,861283	8.878285	8.894643	8.910404	8.925609	8.940296	85
5	8.940296	8,954499	8.968249	8.981573	8,994497	9.007044	9.019235	84
6	9,019235	9.031089	9.042625	9.053859	9.064806	9,075480	9.085894	83
7	9.085894	9.096062	9,105992	9.115698	9.125187	9.134470	9.143555	82
8	9.143555	9.152451	9.161164	9.169702	9.178072	9.186280	9.194332	81
9	9,194332	9.202234	9.209992	9.217609	9.225092	9.232444	9.239670	80
10	9,239670	9.246775	9.253761	9.260633	9.267395	9.274049	9.280599	79
11	9.280599	9.287048	9.293399	9.299655	9.305819	9.311893	9.317879	78
12	9.317879	9.323780	9.329599	9,335337	9.340996	9.346579	9.352088	77
13	9.352088	9.357524	9.362889	9.368185	9,373414	9.378577	9,383675	76
14	9,383675	9.388711	9.393685	9.398600	9.403455	9.408254	9.412996	75
15	9.412996	9.417684	9.422318	9,426899	9.431429	9.435908	9.440338	74
16	9.440338	9.444720	9.449054	9.453342	9.457584	9.461782	9.465935	73
17	9,465935	9.470046	9.474115	9,478142	9.482128	9.486075	9 489982	72
18	9.489982	9.493851	9,497682	9.501476	9,505234	9.508956	9.512642	71
19	9.512642	9.516294	9.519911	9.523495	9.527046	9,530565	9.534052	70
20	9.534052	9.537507	9.540931	9.544325	9.547689	9,551024	9.554329	69
21	9.554329	9.557606	9.560855	9.564075	9.567269	9 570435	9.573575	68
22	9.573575	9.576689	9.579777	9.582840	9.585877	9.588890	9.591878	67
	60′	50′	40′	30′	20′	10′	0′	Deg.

Logarithmic Cosines (read upwards).

Logarithmic Sines.

Deg.	0′	10′	20′	30′	40′	50′	60′	
23	9.591878	9.594842	9.597783	9.600700	9.603594	9.606465	9.609313	66
24	9.609313	9.612140	9.614944	9.617727	9.620488	9.623229	9.625948	65
25	9.625948	9.628647	9.631326	9.633984	9.636623	9.639242	9.641842	64
26	9.641842	9.644423	9.646984	9.649527	9.652052	9.654558	9.657047	63
27	9.657047	9.659517	9.661970	9.664406	9.666824	9.669225	9.671609	62
28	9 671609	9.673977	9.676328	9.678663	9.680982	9.683284	9.685571	61
29	9.685571	9.687843	9.690098	9.692339	9.694564	9.696775	9.698970	60
30	9.698970	9.701151	9.703317	9.705469	9.707606	9.709730	9.711839	59
31	9.711839	9.713935	9.716017	9.718085	9.720140	9.722181	9.724210	58
32	9.724210	9.726225	9.728227	9.730217	9.732193	9.734157	9.736109	57
33	9.736109	9.738048	9.739975	9.741889	9.743792	9.745683	9.747562	56
34	9.747562	9.749429	9.751284	9.753128	9.754960	9.756782	9.758591	55
35	9.758591	9.760390	9.762177	9.763954	9.765720	9.767475	9.769219	54
36	9.769219	9.770952	9.772675	9.774388	9.776090	9.777781	9.779463	53
37	9.779463	9.781134	9.782796	9.784447	9.786089	9.787720	9.789342	52
38	9.789342	9.790954	9.792557	9.794150	9.795733	9.797307	9.798872	51
39	9.798872	9.800427	9.801973	9.803511	9.805039	9.806557	9.808067	50
40	9.808067	9.809569	9.811061	9.812544	9.814019	9.815485	9.816943	49
41	9.816943	9.818392	9.819832	9.821265	9.822688	9.824104	9.825511	48
42	9.825511	9.826910	9.828301	9.829683	9.831058	9.832425	9.833783	47
43	9.833783	9.835134	9.836477	9.837812	9.839140	9.840459	9.841771	46
44	9.841771	9.843076	9.844372	9.845662	9.846944	9.848218	9.849485	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

Logarithmic Cosines (read upwards).

Logarithmic Sines.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	9.849485	9.850745	9.851997	9.853242	9.854480	9.855711	9.856934	**44**
46	9.856934	9.858151	9.859360	9.860562	9.861758	9.862946	9.864127	**43**
47	9.864127	9.865302	9.866470	9.867631	9.868785	9.869933	9.871073	42
48	9.871073	9.872208	9.873335	9.874456	9.875571	9.876678	9.877780	41
49	9.877780	9.878875	9.879963	9.881046	9.882121	9.883191	9.884254	40
50	9.884254	9.885311	9.886362	9.887406	9.888444	9.889477	9.890503	39
51	9.890503	9.891523	9.892536	9.893544	9.894546	9.895542	9.896532	38
52	9.896532	9.897516	9.898494	9.899467	9.900433	9.901394	9.902349	37
53	9.902349	9.903298	9.904241	9.905179	9.906111	9.907037	9.907958	36
54	9.907958	9.908873	9.909782	9.910686	9.911584	9.912477	9.913365	35
55	9.913365	9.914246	9.915123	9.915994	9.916859	9.917719	9.918574	34
56	9.918574	9.919424	9.920268	9.921107	9.921940	9.922768	9.923591	33
57	9.923591	9.924409	9.925222	9.926029	9.926831	9.927629	9.928420	32
58	9.928420	9.929207	9.929989	9.930766	9.931537	9.932304	9.933066	31
59	9.933066	9.933822	9.934574	9.935320	9.936062	9.936799	9.937531	30
60	9.937531	9.938258	9.938980	9.939697	9.940409	9.941117	9.941819	29
61	9.941819	9.942517	9.943210	9.943899	9.944582	9.945261	9.945935	28
62	9.945935	9.946604	9.947269	9.947929	9.948584	9.949235	9.949881	27
63	9.949881	9.950522	9.951159	9.951791	9.952419	9.953042	9.953660	26
64	9.953660	9.954274	9.954883	9.955488	9.956089	9.956684	9.957276	25
65	9.957276	9.957863	9.958445	9.959023	9.959596	9.960165	9.960730	24
66	9.960730	9.961290	9.961846	9.962398	9.962945	9.963488	9.964026	23
67	9.964026	9.964560	9.965090	9.965615	9.966136	9.966653	9.967166	22
	60′	50′	40′	30′	20′	10′	0′	Deg.

Logarithmic Cosines (read upwards).

Logarithmic Sines.

Deg.	0′	10′	20′	30′	40′	50′	60′	
68	9.967166	9.967674	9.968178	9.968678	9.969173	9.969665	9.970152	21
69	9.970152	9.970635	9.971113	9.971588	9.972058	9.972524	9.972986	20
70	9.972986	9.973444	9.973897	9.974347	9.974792	9.975233	9.975670	19
71	9.975670	9.976103	9.976532	9.976957	9.977377	9.977794	9.978206	18
72	9.978206	9.978615	9.979019	9.979420	9.979816	9.980208	9.980596	17
73	9.980596	9.980981	9.981361	9.981737	9.982109	9.982477	9.982842	16
74	9.982842	9.983202	9.983558	9.983911	9.984259	9.984603	9.984944	15
75	9.984944	9.985280	9.985613	9.985942	9.986266	9.986587	9.986904	14
76	9.986904	9.987217	9.987526	9.987832	9.988133	9.988430	9.988724	13
77	9.988724	9.989014	9.989300	9.989582	9.989860	9.990134	9.990404	12
78	9.990404	9.990671	9.990934	9.991193	9.991448	9.991699	9.991947	11
79	9.991947	9.992190	9.992430	9.992666	9.992898	9.993127	9.993351	10
80	9.993351	9.993572	9.993789	9.994003	9.994212	9.994418	9.994620	9
81	9.994620	9.994818	9.995013	9.995203	9.995390	9.995573	9.995753	8
82	9.995753	9.995928	9.996100	9.996269	9.996433	9.996594	9.996751	7
83	9.996751	9.996904	9.997053	9.997199	9.997341	9.997480	9.997614	6
84	9.997614	9.997745	9.997872	9.997996	9.998116	9.998232	9.998344	5
85	9.998344	9.998453	9.998558	9.998659	9.998757	9.998851	9.998941	4
86	9.998941	9.999027	9.999110	9.999189	9.999265	9.999336	9.999404	3
87	9.999404	9.999469	9.999529	9.999586	9.999640	9.999689	9.999735	2
88	9.999735	9.999778	9.999816	9.999851	9.999882	9.999910	9.999934	1
89	9.999934	9.999954	9.999971	9.999983	9.999993	9.999998	10.000000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

Logarithmic Cosines (read upwards).

Logarithmic Tangents.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	— ∞	7.463727	7.764761	7.940858	8.065806	8.162727	8.241921	89
1	8.241921	8.308884	8.366895	8.418068	8.463849	8.505267	8.543084	88
2	8.543084	8.577877	8.610094	8.640093	8.668160	8.694529	8.719396	87
3	8.719396	8.742922	8.765246	8.786486	8.806742	8.826103	8.844644	86
4	8.844644	8.862433	8.879529	8.895984	8.911846	8.927156	8.941952	85
5	8.941952	8.956267	8.970133	8.983577	8.996624	9.009298	9.021620	84
6	9.021620	9.033609	9.045284	9.056659	9.067752	9.078576	9.089144	83
7	9.089144	9.099468	9.109559	9.119429	9.129087	9.138542	9.147803	82
8	9.147803	9.156877	9.165774	9.174499	9.183059	9.191462	9.199713	81
9	9.199713	9.207817	9.215780	9.223607	9.231302	9.238872	9.246319	80
10	9.246319	9.253648	9.260863	9.267967	9.274964	9.281858	9.288652	79
11	9.288652	9.295349	9.301951	9.308463	9.314885	9.321222	9.327475	78
12	9.327475	9.333646	9.339739	9.345755	9.351697	9.357566	9.363364	77
13	9.363364	9.369094	9.374756	9.380354	9.385888	9.391360	9.396771	76
14	9.396771	9.402124	9.407419	9.412658	9.417842	9.422974	9.428052	75
15	9.428052	9.433080	9.438059	9.442988	9.447870	9.452706	9.457496	74
16	9.457496	9.462242	9.466945	9.471605	9.476223	9.480801	9.485339	73
17	9.485339	9.489838	9.494299	9.498722	9.503109	9.507460	9.511776	72
18	9.511776	9.516057	9.520305	9.524520	9.528702	9.532853	9.536972	71
19	9.536972	9.541061	9.545119	9.549149	9.553149	9.557121	9.561066	70
20	9.561066	9.564983	9.568873	9.572738	9.576576	9.580389	9.584177	69
21	9.584177	9.587941	9.591681	9.595398	9.599091	9.602761	9.606410	68
22	9.606410	9.610036	9.613641	9.617224	9.620787	9.624330	9.627852	67
	60′	50′	40′	30′	20′	10′	0′	Deg.

Logarithmic Cotangents (read upwards).

Logarithmic Tangents.

Deg.	0′	10′	20′	30′	40′	50′	60′	
23	9.627852	9.631355	9.634838	9.638302	9.641747	9.645174	9.648583	66
24	9.648583	9.651974	9.655348	9.658704	9.662043	9.665366	9.668673	65
25	9.668673	9.671963	9.675237	9.678496	9.681740	9.684968	9.688182	64
26	9.688182	9.691381	9.694566	9.697736	9.700893	9.704036	9.707166	63
27	9.707166	9.710282	9.713386	9.716477	9.719555	9.722621	9.725674	62
28	9.725674	9.728716	9.731746	9.734764	9.737771	9.740767	9.743752	61
29	9.743752	9.746726	9.749689	9.752642	9.755585	9.758517	9.761439	60
30	9.761439	9.764352	9.767255	9.770148	9.773033	9.775908	9.778774	59
31	9.778774	9.781631	9.784479	9.787319	9.790151	9.792974	9.795789	58
32	9.795789	9.798596	9.801396	9.804187	9.806971	9.809748	9.812517	57
33	9.812517	9.815280	9.818035	9.820783	9.823524	9.826259	9.828987	56
34	9.828987	9.831709	9.834425	9.837134	9.839838	9.842535	9.845227	55
35	9.845227	9.847913	9.850593	9.853268	9.855938	9.858602	9.861261	54
36	9.861261	9.863915	9.866564	9.869209	9.871849	9.874484	9.877114	53
37	9.877114	9.879741	9.882363	9.884980	9.887594	9.890204	9.892810	52
38	9.892810	9.895412	9.898010	9.900605	9.903197	9.905785	9.908369	51
39	9.908369	9.910951	9.913529	9.916104	9.918677	9.921247	9.923814	50
40	9.923814	9.926378	9.928940	9.931499	9.934056	9.936611	9.939163	49
41	9.939163	9.941713	9.944262	9.946808	9.949353	9.951896	9.954437	48
42	9.954437	9.956977	9.959516	9.962052	9.964588	9.967123	9.969656	47
43	9.969656	9.972188	9.974720	9.977250	9.979780	9.982309	9.984837	46
44	9.984837	9.987365	9.989893	9.992420	9.994947	9.997473	10.000000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

Logarithmic Cotangents (read upwards).

Logarithmic Tangents.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	10.000000	10.002527	10.005053	10.007580	10.010107	10.012635	10.015163	44
46	10.015163	10.017691	10.020220	10.022750	10.025280	10.027812	10.030344	43
47	10.030344	10.032877	10.035412	10.037948	10.040484	10.043023	10.045563	42
48	10.045563	10.048104	10.050647	10.053192	10.055738	10.058287	10.060837	41
49	10 060837	10.063389	10.065944	10.068501	10.071060	10.073622	10.076186	40
50	10.076186	10.078753	10.081323	10.083896	10.086471	10.089049	10.091631	39
51	10.091631	10.094215	10.096803	10.099395	10.101990	10.104588	10.107190	38
52	10.107190	10.109796	10.112406	10.115020	10.117637	10.120259	10.122886	37
53	10.122886	10.125516	10.128151	10.130791	10.133436	10.136085	10.138739	36
54	10.138739	10.141398	10.144062	10.146732	10.149407	10.152087	10.154773	35
55	10.154773	10.157465	10.160162	10.162866	10.165575	10.168291	10.171013	34
56	10.171013	10.173741	10.176476	10.179217	10.181965	10.184720	10.187483	33
57	10.187483	10.190252	10.193029	10.195813	10.198604	10.201404	10.204211	32
58	10.204211	10.207026	10.209849	10.212681	10.215521	10.218369	10.221226	31
59	10.221226	10.224092	10.226967	10.229852	10.232745	10.235648	10.238561	30
60	10.238561	10.241483	10.244415	10.247358	10.250311	10.253274	10.256248	29
61	10.256248	10.259233	10.262229	10.265236	10.268254	10.271284	10.274326	28
62	10.274326	10.277379	10.280445	10.283523	10.286614	10.289718	10.292834	27
63	10.292834	10.295964	10.299107	10.302264	10.305434	10.308619	10.311818	26
64	10.311818	10.315032	10.318260	10.321504	10.324763	10.328037	10.331327	25
65	10.331327	10.334634	10.337957	10.341296	10.344652	10.348026	10.351417	24
66	10.351417	10.354826	10.358253	10.361698	10.365162	10 368645	10.372148	23
67	10.372148	10.375670	10.379213	10.382776	10.386359	10.389964	10.393590	22
	60′	50′	40′	30′	20′	10′	0′	Deg.

Logarithmic Cotangents (read upwards).

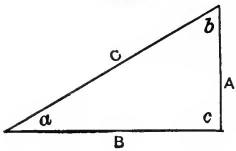
Logarithmic Tangents.

Deg.	0′	10′	20′	30′	40′	50′	60′	
68	10.393590	10.397239	10.400909	10.404602	10.408319	10.412059	10.415823	21
69	10.415823	10.419611	10.423424	10.427262	10.431127	10.435017	10.438934	20
70	10.438934	10.442879	10.446851	10.450851	10.454881	10.458939	10.463028	19
71	10.463028	10.467147	10.471298	10.475480	10.479695	10.483943	10.488224	18
72	10.488224	10.492540	10.496891	10.501278	10.505701	10.510162	10.514661	17
73	10.514661	10.519199	10.523777	10.528395	10.533055	10.537758	10.542504	16
74	10.542504	10.547294	10.552130	10.557012	10.561941	10.566920	10.571948	15
75	10.571948	10.577026	10.582158	10.587342	10.592581	10.597876	10.603229	14
76	10.603229	10.608640	10.614112	10.619646	10.625244	10.630906	10.636636	13
77	10.636636	10.642434	10.648303	10.654245	10.660261	10.666354	10.672525	12
78	10.672525	10.678778	10.685115	10.691537	10.698049	10.704651	10.711348	11
79	10.711348	10.718142	10.725036	10.732033	10.739137	10.746352	10.753681	10
80	10.753681	10.761128	10.768698	10.776393	10.784220	10.792183	10.800287	9
81	10.800287	10.808538	10.816941	10.825501	10.834226	10.843123	10.852197	8
82	10.852197	10.861458	10.870913	10.880571	10.890441	10.900532	10.910856	7
83	10.910856	10.921424	10.932248	10.943341	10.954716	10.966391	10.978380	6
84	10.978380	10.990702	11.003376	11.016423	11.029867	11.043733	11.058048	5
85	11.058048	11.072844	11.088154	11.104016	11.120471	11.137567	11.155356	4
86	11.155356	11.173897	11.193258	11.213514	11.234754	11.257078	11.280604	3
87	11.280604	11.305471	11.331840	11.359907	11.389906	11.422123	11.456916	2
88	11.456916	11.494733	11.536151	11.581932	11.633105	11.691116	11.758079	1
89	11.758079	11.837273	11.934194	12.059142	12.235239	12.536273	$+\infty$	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

Logarithmic Cotangents (read upwards).

Solutions of Right-Angled Triangles.

FIG. 18.



Right-angled triangles (see Fig. 18) may be solved by the following formulas:

Solving for Any Side.

$$A = C \times \text{sine } a = B \times \text{tang. } a = \frac{C}{\text{cosec. } a} = \frac{B}{\text{cot. } a}$$

$$B = C \times \text{cos. } a = A \times \text{cot. } a = \frac{C}{\text{sec. } a} = \frac{A}{\text{tang. } a}$$

$$C = A \times \text{cosec. } a = B \times \text{sec. } a = \frac{A}{\text{sine } a} = \frac{B}{\text{cos. } a}$$

$$A = \sqrt{C^2 - B^2}$$

$$B = \sqrt{C^2 - A^2}$$

$$C = \sqrt{A^2 + B^2}$$

Solving for Any Function or for Any Angle.

$\text{Sin. } a = \frac{A}{C}$	$\text{Tang. } a = \frac{A}{B}$	$\text{Sec. } a = \frac{C}{B}$
$\text{Cos. } a = \frac{B}{C}$	$\text{Cot. } a = \frac{B}{A}$	$\text{Cosec. } a = \frac{C}{A}$
$\text{Sin. } a = \text{cos. } b$	$\text{Tang. } a = \text{cot. } b$	$\text{Sec. } a = \text{cosec. } b$
$\text{Sin. } b = \text{cos. } a$	$\text{Tang. } b = \text{cot. } a$	$\text{Sec. } b = \text{cosec. } a$
$\text{Sin. } b = \frac{B}{C}$	$\text{Tang. } b = \frac{B}{A}$	$\text{Sec. } b = \frac{C}{A}$
$\text{Cos. } b = \frac{A}{C}$	$\text{Cot. } b = \frac{A}{B}$	$\text{Cosec. } b = \frac{C}{B}$

Angle $a = 90° - b$. Angle $b = 90° - a$.

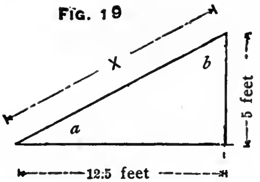
Solving for Area.

$$\text{Area} = \frac{A \times B}{2}$$

$$= \frac{C^2 \times \text{sin. } a \times \text{cos. } a}{2} = \frac{C^2 \times \text{cos. } b \times \text{sin. } b}{2}$$

$$= \frac{B^2 \times \text{tang. } a}{2} = \frac{A^2 \times \text{tang. } b}{2}$$

FIG. 19



EXAMPLE.

Find angles a and b and the side X in the right-angled triangle. (Fig. 19).

$$\text{Tangent corresponding to } a = \frac{5}{12.5} = 0.4$$

$$\text{Tangent corresponding to } b = \frac{12.5}{5} = 2.5$$

By the trigonometrical table the angles are obtained thus:

Tangent 0.40000 gives 21° 48′
Tangent 2.50000 gives 68° 12′

Therefore:
Angle $a = 21° \ 48'$ and angle $b = 68° \ 12'$.

Angle b may also be found by subtracting angle a from 90°, thus:

$$\text{Angle } b = 90° - 21° \ 48' = 68° \ 12'$$

The length of the side X may be found thus:

$$x = \frac{5}{\sin. c}$$

$$x = \frac{5}{0.37137}$$

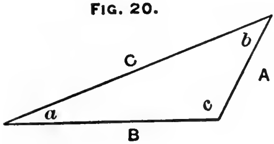
$$x = 13.464 \text{ feet long.}$$

By means of logarithms the length of the side x is obtained thus:

$$Log.\ x = log.\ 5 - log.\ \sin.\ 21° \ 48'$$
$$Log.\ x = 0.698970 - (9.569804 - 10)$$
$$Log.\ x = 1.129166$$
$$x = 13.464 \text{ feet long.}$$

Solution of Oblique-Angled Triangles.

FIG. 20.



Oblique-angled triangles (see Figs. 20-21-22) may be solved by the following formulas:

FIG. 21.

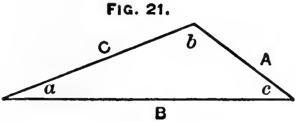
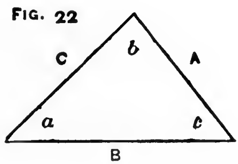


FIG. 22



Solving for Any Side.

$$A = \frac{C \sin. a}{\sin. c} = \frac{B \sin. a}{\sin. b} = \sqrt{B^2 + C^2 - 2\,B\,C \cos. a}$$

$$B = \frac{C \sin. b}{\sin. c} = \frac{A \sin. b}{\sin. a} = \sqrt{C^2 + A^2 - 2\,A\,C \cos. b}$$

$$C = \frac{A \sin. c}{\sin. a} = \frac{B \sin. c}{\sin. b} = \sqrt{A^2 + B^2 - 2\,A\,B \cos. c}$$

Solving for Any Angle.

$$\text{Cos. } a = \frac{B^2 + C^2 - A^2}{2\,B\,C}$$

$$\text{Cos. } b = \frac{A^2 + C^2 - B^2}{2\,A\,C}$$

$$\text{Cos. } c = \frac{A^2 + B^2 - C^2}{2\,A\,B}$$

$$\text{Sin. } a = \sin. b\,\frac{A}{B} = \sin. c\,\frac{A}{C}$$

$$\text{Sin. } b = \sin. c\,\frac{B}{C} = \sin. a\,\frac{B}{A}$$

$$\text{Sin. } c = \sin. a\,\frac{C}{A} = \sin. b\,\frac{C}{B}$$

$$a = 180^\circ - (b + c)$$
$$b = 180^\circ - (a + c)$$
$$c = 180^\circ - (a + b)$$

Solving for Area.

$$\text{Area} = \frac{\sin. c \times A \times B}{2} = \frac{\sin. a \times C \times B}{2} = \frac{\sin. b \times A \times C}{2}$$

EXAMPLE 1.

Find the length of the side C (see Fig. 20) when angle $a = 20^\circ\ 38'\ 12''$, angle $c = 117^\circ\ 48'\ 5''$, and side $A = 12.75$ feet long.

NOTE.—The angle c exceeds 90°, therefore the supplement of the angle must be used, which is $180^\circ - 117^\circ\ 48'\ 5'' = 62^\circ\ 11'\ 55''$.

Thus the solution:

$$C = \frac{12.75 \times \sin. 62^\circ\ 11'\ 55''}{\sin. 20^\circ\ 38'\ 12''}$$

$$C = \frac{12.75 \times 0.88456}{0.35243}$$

$$C = 32 \text{ feet long.}$$

EXAMPLE 2.

Find the length of the side B (see Fig. 20) when angle b is $41^\circ\ 33'\ 43''$, side C is 32 feet and side A is 12.75 feet.

In this example two sides and their included angle are given and the third side is required; therefore the formula

$B = \sqrt{A^2 + C^2 - 2\ A\ C \cos.\ b}$ must be used.

Solution:

$$B = \sqrt{12.75^2 + 32^2 - 2 \times 12.75 \times 32 \times 0.748238}$$

$$B = \sqrt{1186.562 - 610.562}$$

$$B = \sqrt{576} = 24 \text{ feet long.}$$

EXAMPLE 3.

Find the length of the side B when side A is 12.75 feet long, angle b is $41^\circ\ 33'\ 43''$ and angle c is $117^\circ\ 48'\ 5'$. (See Fig. 20).

In this problem one side and its two adjacent angles are given; therefore it can not be solved directly by any of the preceding formulas, but the first thing to do is to find the angle opposite to side A.

Thus: Angle $a = 180^\circ - (41^\circ\ 33'\ 43'' + 117^\circ\ 48'\ 5'') = 20^\circ\ 38'\ 12''$. The side B may be found by the formula $B = \frac{A \sin.\ b}{\sin.\ a}$

Solution:

$$B = \frac{A \sin.\ 41^\circ\ 33'\ 43''}{\sin.\ 20^\circ\ 38'\ 12''}$$

$$B = \frac{12.75 \times 0.66343}{0.35242}$$

$$B = 24 \text{ feet long.}$$

EXAMPLE 4.

Find length of the side C when B is 24 feet long, angle c is $117^\circ\ 48'\ 5''$ and the side A is 12.75 feet long. (See Fig. 20).

Solution:

$$C = \sqrt{A^2 + B^2 - 2\ A\ B \cos.\ c}$$

$$C = \sqrt{12.75^2 - 24^2 - 2 \times 12.75 \times 24 \times (-0.4664)}$$

$$C = \sqrt{162.56 + 576 + 285.44}$$

$$C = \sqrt{1024} = 32 \text{ feet long.}$$

NOTE.—In this example the cos. of $117^\circ\ 48'\ 5''$ is used, which, in numerical value, is equal to cos. of $62^\circ\ 11'\ 55'' = 0.4664$, but cos. in the second quadrant is negative (see page 154); therefore cos. $117^\circ\ 48'\ 5'' = (-0.46645)$ and the essential sign of the last product after it is multiplied by this negative cos. must change from $-$ to $+$. (See Algebra, page 63).

EXAMPLE 5.

Find the length of the side A when C is 32 feet long, angle a is 20° 38′ 12″ and angle c is 117° 48′ 15″.

NOTE.—Supplement to c is 62° 11′ 55″.

Solution:

$$A = \frac{C \sin. a}{\sin. c}$$

$$A = \frac{32 \times 0.35242}{0.88456}$$

$$A = 12.75 \text{ feet long.}$$

In this example, as in the preceding one, we use the supplement of the angle in obtaining its function, but here it has no influence on the signs because sin. is positive as well in the second as in the first quadrant.

EXAMPLE 6.

Find angle a in Fig. 20, when A is 12.75 feet, B is 24 feet and C is 32 feet.

Solution:

$$\cos. a = \frac{B^2 + C^2 - A^2}{2\,B\,C}$$

$$\cos. a = \frac{24^2 + 32^2 - 12.75^2}{2 \times 24 \times 32}$$

$$\cos. a = \frac{576 + 1024 - 162.5625}{1536}$$

$$\cos. a = 0.93583$$

$$\text{Angle } a = 20^\circ\ 38'\ 12''$$

EXAMPLE 7.

Find angle b, Fig. 20, by the same formula.

Solution:

$$\cos. b = \frac{A^2 + C^2 - B^2}{2\,A\,C}$$

$$\cos. b = \frac{12.75^2 + 32^2 - 24^2}{2 \times 12.75 \times 32}$$

$$\cos. b = \frac{610.5625}{816}$$

$$\cos. b = 0.748238$$

$$\text{Angle } b = 41^\circ\ 33'\ 43''$$

EXAMPLE 8.

Find angle c, Fig. 20, by the same formula.

Solution:

$$\cos. c = \frac{A^2 + B^2 - C^2}{2AB}$$

$$\cos. c = \frac{12.75^2 + 24^2 - 32^2}{2 \times 12.75 \times 4}$$

$$\cos. c = \frac{738.5625 - 1024}{612}$$

$$\cos. c = -0.46640$$

Supplement to angle $c = 62° 11' 55''$, and angle $c = 117° 48' 5''$

NOTE.—The negative cosine indicates that it is in the second quadrant, therefore the angle is over 90°.

The angle corresponding to this cosine is the supplement of angle c. To obtain angle c, the angle of its supplement must be subtracted from 180°.

EXAMPLE 9.

Find angles a, b and c in Fig. 20, when side A is 12.75 feet, B 32 feet, and C 24 feet.

$$\cos. a = \frac{B^2 + C^2 - A^2}{2BC}$$

$$\cos. a = \frac{32^2 + 24^2 - 12.75^2}{2 \times 32 \times 24}$$

$$\cos. a = \frac{1437.4375}{1536}$$

$$\cos. a = 0.93583$$

$$\text{Angle } a = 20° 38' 12''$$

Angle b may be found by the formula:

$$\sin. b = \sin. a \frac{B}{A}$$

$$\sin. b = \sin. 20° 38' 12'' \frac{B}{A}$$

$$\sin. b = 0.35244 \times \frac{32}{12.75}$$

$$\sin. b = 0.35244 \times 1.8824$$

$$\sin. b = 0.66343$$

$$\text{Angle } b = 41° 33' 43''$$

Angle c may be found by the formula:

$$c = 180^\circ - (a + b)$$
$$c = 180^\circ - (20^\circ\ 38'\ 12'' + 41^\circ\ 33'\ 43'')$$
$$c = 180^\circ - 62^\circ\ 11'\ 55''$$
$$c = 117^\circ\ 48'\ 5''$$

EXAMPLE 10.

Find the area of a triangle (see Fig. 20), when it is known that side A is 12.75 feet, side B is 24 feet, and the including angle $C = 117^\circ\ 48'\ 5''$.

Solution:

Sin. to supplement of $117^\circ\ 48'\ 5''$ = sin. $62^\circ\ 11'\ 55''$ = 0.88456.

$$\text{Area} = \frac{\sin. C \times A \times B}{2}$$

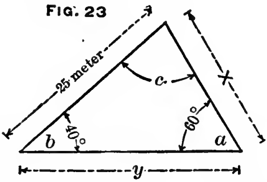
$$\text{Area} = \frac{0.88456 \times 12.75 \times 24}{2} = 135.34 \text{ square feet.}$$

EXAMPLE 11.

Find angle c and the sides X and y in the triangle, Fig. 23.

Solution:

FIG. 23



$$c = 180^\circ - (40^\circ + 60^\circ) = 80^\circ$$

$$\text{The side } X = \frac{25 \times \sin. 40^\circ}{\sin. 60^\circ}$$

$$X = \frac{25 \times 0.64279}{0.86603}$$

$$X = \frac{16.06975}{0.86603}$$

$X = 18.556$ meters long.

By the use of logarithms the side X is solved thus:

Log. X = *log.* 25 + *log.* sin. 40° — *log.* sin. 60°.
Log. X = 1.39794 + (9.808067—10) — (9.937531—10).
Log. X = 1.268476
X = 18.556 meters long.

$$\text{The side } y = \frac{25 \times \sin. 80^\circ}{\sin. 60^\circ}$$

$$y = \frac{25 \times 0.98481}{0.86603}$$

$y = 28.429$ meters long.

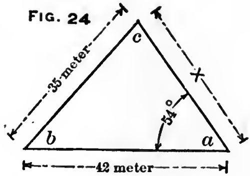
By the use of logarithms the side y is solved thus:

Log. y = *log.* 25 + *log.* sin. 80° — *log.* sin. 60°.
Log. y = 1.39794 + (9.993351—10) — (9.937531—10).
Log. y = 1.45376
y = 28.429 meters long.

EXAMPLE 12.

Find angles c and b and the length of the side X in Fig. 24.

FIG. 24



$$\text{Sin. } c = \frac{42 \times \text{sin. } 54^\circ}{35}$$

$$\text{Sin. } c = \frac{42 \times 0.80902}{35}$$

$$\text{Sin. } c = 0.97082$$

$$\text{Angle } c = 76^\circ\ 7'\ 26''$$

$$\text{Angle } b = 180^\circ - (54^\circ\ 0'\ 0'' + 76^\circ\ 7'\ 26'') = 49^\circ\ 52'\ 34''$$

$$\text{Side } X = \frac{35 \times \text{sin. } 49^\circ\ 52'\ 34''}{\text{sin. } 54^\circ}$$

$$X = \frac{35 \times 0.76465}{0.80901} = 33.08 \text{ meters long.}$$

By means of logarithms the side X is solved thus:

Log. $X =$ *log.* $35 +$ *log.* sin. $49^\circ\ 52'\ 34'' -$ *log.* sin. 54°.

Log. $X = 1.544068 + (9.883463 - 10) - (9.907958 - 10)$.

Log. $X = 1.519573$

$X = 33.08$ meters long.

NOTE.—The angle c is obtained by interpolation thus: In the table of trigonometrical functions the sine 0.97100 corresponds to the angle $76^\circ\ 10'$ and the sine 0.97030 corresponds to the angle 76°. Thus, a difference of 0.00070 in the sine gives a difference of $10' = 600''$ in the angle.

The sine to angle c is 0.97082

The nearest less sine in the table is 0.97030 corresponding to angle $76^\circ\ 0'\ 0''$. Difference, 0.00052

Therefore when an increase in sine of 0.00070 corresponds to an increase of $600''$ in the angle, an increase of 0.00052 will increase the angle $\dfrac{600 \times 0.00052}{0.00070} = 446'' = 0^\circ\ 7'\ 26''$

thus, the angle corresponding to the sine 0.97082 must be $76^\circ\ 7'\ 26''$.

PROBLEMS IN GEOMETRICAL DRAWING.

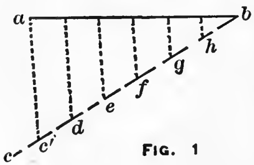


FIG. 1

To divide a straight line into a given number of equal parts. (See Fig. 1).

Given line *a b*, which is to be divided into a given number of equal parts. Draw the line *b c*, of indefinite length, and point off from *b* the required number of equal parts, as *h*, *g*, *f*, *e*, *d*, *c'*; join *c'* and *a*, and draw the other lines parallel to *c' a*.

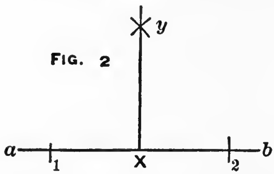


FIG. 2

To erect a perpendicular at a given point on a straight line. (See Fig. 2).

Given line *a b*, and the point *x*. The required perpendicular is *x y*.

Solution:

With *x* as center and any radius, as *x* 1, cut the line *a b* at 1 and 2. With 1 and 2 as centers and with a radius somewhat greater than 1 to *x*, describe arcs intersecting each other at *y*. Draw *x y*. This will be the required perpendicular.

From a given point without a straight line to draw a perpendicular to the line. (See Fig. 3).

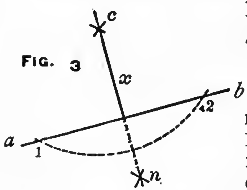


FIG. 3

Given line *a b* and the point *c*. The required perpendicular is *x*.

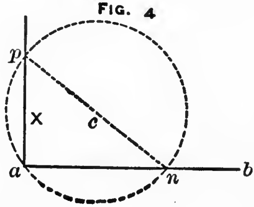
Solution:

With the point *c* as center and any radius as *c* 1, strike the arc 1 to 2. With 1 and 2 as centers and any suitable radius, describe arcs intersecting each other at *n*, lay the straight edge through points *n* and *c* and draw the perpendicular *x*.

To erect a perpendicular at the extremity of a straight line. (See Fig. 4).

Given line *a b*. The required perpendicular is *x*.

FIG. 4

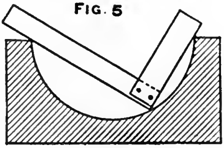


Solution:

From any point, as *c*, with radius as *a c*, draw the circle. From point of intersection, *n*, through center, *c*, draw the diameter *n p*. From the point *a*, through the point of intersection at *p*, draw the perpendicular *x*.

The correctness of this construction is founded on the principle that inside a half circle no other angle but an angle of 90° can simultaneously touch three points in the circumference when two of these points are in the point of intersection with the diameter and the circumference and the third one anywhere on the circumference of the half circle. The pattern maker is making practical use of this geometrical principle, when he by a common carpenter's square is trying the correctness of a semi-circular core box, as shown in Fig. 5.

FIG. 5



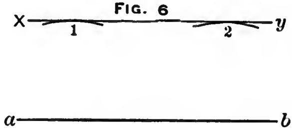
Draw a line parallel to a given line. (See Fig. 6).

Given line *a b*. The required line *x y*.

Solution:

Describe with the compass from the line *a b*, the arcs 1 and 2; draw line *x y*, touching these arcs.

FIG. 6



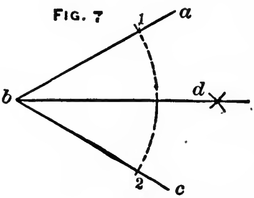
To divide a given angle into two equal angles.

The given angle, *a b c*, is divided by the line *b d*.

Solution:

With *b* as center and any radius, as *b* 1, describe the arc 1 to 2. With 1 and 2 as centers and any suitable radius, describe arcs cutting each other at *d*. Draw line *b d*, which will divide the angle into two equal parts.

FIG. 7

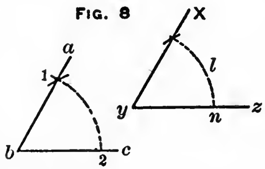


To draw an angle equal to a given angle.

Given angle *a b c*. Construct angle *x y z*.

With *b* as center and any radius, as *b* 1, describe the arc 1 to 2. Using *y* as center and without altering the compass, describe the arc *l*, intersecting *y z*. Measuring the distance from 2 to 1 on the given angle, transfer this measure to the arc *l*, through the point of intersection. Draw the line *y x*, and this angle will be equal to the first angle.

FIG. 8



NOTE.—Angles are usually measured by a tool called a protractor, looking somewhat like Fig. 9 or 10, usually made from metal, and supplied by dealers in draughting instruments. A protractor may also be constructed on paper and used for measuring angles, but it should then always be made on as large a scale as convenient.

FIG. 9

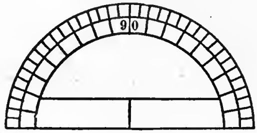
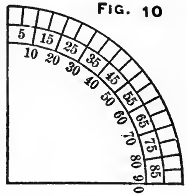


FIG. 10

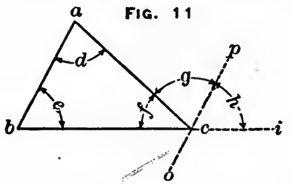


To draw a protractor with a division of 5°. (See Fig. 10).

Construct an angle of exactly 90 degrees, divide the arc into nine equal parts, then each part is 10°; divide each part into two equal parts and each is 5°.

Prove that the sum of the three angles in a triangle consists of 180°. (See Fig. 11).

FIG. 11



Solution:

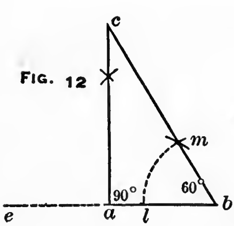
In the triangle *a b c*, extend the base line to *i*. Draw the line *o p*, parallel to the side *a b*, thereby the angle *g* will be equal to the angle *d*, and the angle *h* must be equal to angle *c*. The angle *f* is one angle in the triangle and $f + g + h = 180°$, therefore $f + d + c$ must also be 180°.

To draw on a given base line a triangle having angles 90°, 30° and 60°. (See Fig. 12).

Given line *a b*, required triangle is *a*, *c*, *b*.

Solution:

FIG. 12



Extend the line *a b* to twice its length, to the point *e*. With *e* and *b* as centers strike arcs intersecting each other and erect the perpendicular *a c*. With *b* as center and any radius as *l*, draw the arc *l m*. With *l* as center and with the same radius, describe arc intersecting at *m*. From *b* through point of intersection at *m*, draw line *b* intersecting the perpendicular at *c*. This will complete the triangle.

To draw a square inside a given circle. (See Fig. 13).

Solution:

Draw the line *a b* through the center of the circle. From points of intersection at *a* and *b*, describe with any suitable radius arcs intersecting at *n* and *m*. Draw through the points the line *c d*. Connect the points of intersection on the circle and the required square is constructed.

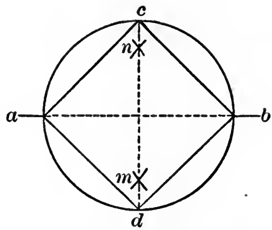


FIG. 13.

To draw a square outside a given circle. (See Fig. 14).

Solution:

Draw lines *a b* and *c d*, and from points of intersection at *b* and *c*, describe half circles; their points of intersection determine the sides of the square.

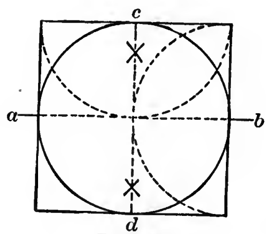


FIG. 14.

To draw a hexagon within a given circle. (See Fig. 15).

Apply the radius as a chord successively about the circle; the resulting figure will be a hexagon.

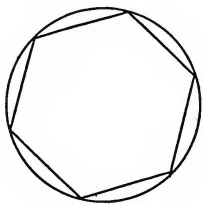
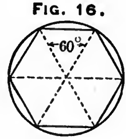


FIG. 15.

To inscribe in a circle a regular polygon of any given number of sides.

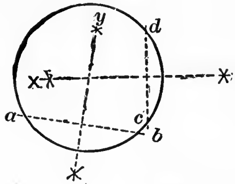
FIG. 16.



Solution:

Divide 360 by the number of sides, and the quotient is the number of degrees, minutes, and seconds contained in the center angle of a triangle, of which one side will make one of the sides in the polygon. For instance, draw a hexagon by this method. (See Fig. 16). $\frac{360}{6} = 60°$

FIG. 17

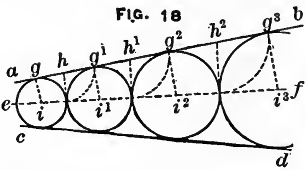


To find the center in a given circle. (See Fig. 17).

Solution:

Draw anywhere on the circumference of the circle two chords at approximately right angles to each other, bisect these by the perpendiculars *x* and *y*, and their point of intersection is the center of the circle.

FIG. 18

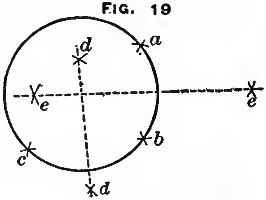


To draw any number of circles between two inclined lines touching themselves and the lines. (See Fig. 18).

Solution:

Draw center line *e f*. Draw first circle on line *i g*. From point of intersection between this circle and the center line draw the line *h*, perpendicular to *a b*. Describe with a radius equal to *h*, the arc intersecting at g^1, draw line g^1 i^1, parallel to *g i*, and its point of intersection with the center line gives the center for the next circle, etc.

FIG. 19



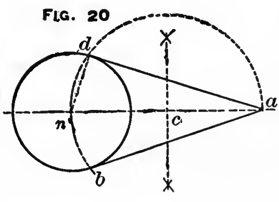
To draw a circle through three given points. (See Fig. 19).

The given points are *a*, *b*, and *c*.

Solution:

From *a* and *b* as centers with suitable radius, describe arcs intersecting at *e e*. Draw a line through these points. From *b* and *c* as centers, describe arcs intersecting at *d d*; draw a line through these points. The point where these two lines intersect is the center of the circle.

FIG. 20



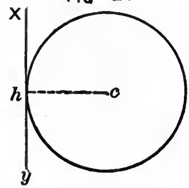
To draw two tangents to a circle from a given point without same circle. (See Fig. 20).

Given point *a*, and the circle with the center *n*. The required tangents are *a d*, and *a b*.

Solution:

Bisect line *n a*. With *c* as center and radius *a c*, describe the arc *b d* through the center of the circle. The points of intersection at *b* and *d* are the points where the required tangents *a b* and *a d* will touch the circle.

FIG 21



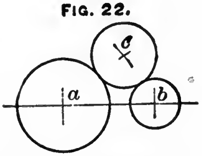
To draw a tangent to a given point in a given circle. (See Fig. 21).

Given circle and the point *h*, *x y* is required.

Solution:

The radius is drawn to the point *h* and a line constructed perpendicular to it at the point *h*. This perpendicular, touching the circle at *h*, is called a tangent.

FIG. 22.



To draw a circle of a certain size that will touch the perphery of two given circles. (See Fig. 22).

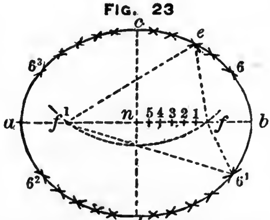
Given the diameter of circles *a*, *b*, and *c*. Locate the center for circle *c*, when centers for *a* and *b* are given.

Solution:

From center of *a*, describe an arc with a radius equal to the sum of radii of *a* and *c*. From *b* as center, describe another arc using a radius equal to the sum of the radii of *b* and *c*. The point of intersection of those two arcs is the center of the circle *c*.

NOTE.—This construction is useful when locating the center for an intermediate gear. For instance, if *a* and *b* are the pitch circles of two gears, *c* would be the pitch circle located in correct position to connect *a* and *b*.

FIG. 23



To draw an ellipse, the longest and shortest diameter being given. The diameters *a b* and *c d* are given. The required ellipse is constructed thus: (See Fig. 23).

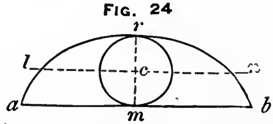
From *c* as center with a radius *a n*, describe an arc f^1 *f*. The points where this arc intersects *a b* are foci. The distance *f n* is divided into any number of parts, as 1, 2, 3, 4, 5. With radius 1 to *b*, and the focus *f* as center, describe arcs 6 and 6^1; with the same radius and with f^1 as center describe arcs 6^2 and 6^3. With radius 1 to *a* and f^1 as center, describe arcs intersecting at 6 and 6^1; with the same radius and with *f* as center, describe arcs intersecting at 6^2 and 6^3. Continue this operation for points 2, 3, etc., and when all the points for the circumference are in this way marked out, draw the ellipse by using a scroll. It is a property with ellipses that the sum of any two lines drawn from the foci to any point in the circumference is equal to the largest diameter. For instance:

$$f^1 e + f e, = a\ b, \text{ or } f\ 6^1 + f^1\ 6^1, = a\ b.$$

Cycloids.

Suppose that a round disc, *c*, rolls on a straight line, *a b*, and that a lead pencil is fastened at the point *r*; it will then describe a curved line, *a, l, r, n, b*. This line is called a cycloid. (See Fig. 24).

FIG. 24



This supposed disk is usually called the generating circle. The line *a b* is the base line of the cycloid and is equal in length to π times *m r*, or practically 3.1416 times the diameter of the generating circle. The length of the curved line *a, l, r, n, b*, is four times *r m*, (four times as long as the diameter of the generating circle).

A circle rolling on a straight line generates a *cycloid*. (See Figs. 24 and 25).

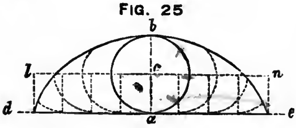
A circle rolling upon another circle is generating an *epicycloid*. (See Fig. 26).

A circle rolling within another circle generates a *hypocycloid*. (See Fig. 27).

To draw a cycloid, the generating circle being given.

Solution:

FIG. 25

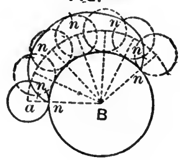


Divide the diameter of the rolling circle in 7 equal parts. Set off 11 of these parts on each side of *a* on the line *d e*. This will give a base line practically equal to the circumference. Divide the base line from the point *a* into any number of equal parts; erect the perpendiculars, with center-line as centers and a radius equal to the radius of the generating circle describe the arcs. On the first arc from *d* or *e* set off one part of the base line. On the second arc set off two parts of the base line; on the third arc three parts, etc. This will give the points through which to draw the cycloid.

To draw an epicycloid (see Fig. 26), the generating circle *a* and the fundamental circle *B* being given.

Solution:

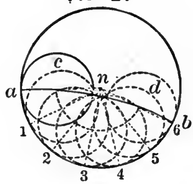
FIG. 26



Concentric with the circle *B*, describe an arc through the center of the generating circle. Divide the circumference of the generating circle into any number of equal parts and set this off on the circumference of the circle *B*. Through those points draw radial lines extending until they intersect the arc passing through the center of the generating circle. These points of intersection give the centers for the different positions of the generating circle, and for the rest, the construction is essentially the same as the cycloids. In Fig. 26, the generating circle is shown in seven different positions, and the point *n*, in the circumference of the generating circle, may be followed from the position at the extreme left for one full rotation, to the position where it again touches the circle *B*.

To draw a hypocycloid. (See Fig. 27).

FIG. 27.



The hypocycloid is the line generated by a point in a circle rolling within another larger circle, and is constructed thus: (See Fig. 27).

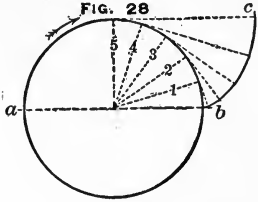
Divide the circumference of the generating circle into any number of equal parts. Set off these on the circumference of the fundamental circle. From each point of division draw radial lines, 1, 2, 3, 4, 5, 6. From *n* as center describe an arc through the center of the generating circle, as the arc *c d*. The point of intersection between this arc and the radial lines are centers for the different positions of the generating circle. The distance from 1 to *a* on the fundamental circle is set off from 1 on the generating circle in its first new position; the distance 2 to *a* on the fundamental circle is set off from 2 on the generating circle in its second position, etc. For the rest, the construction is substantially the same as Figs. 25 and 26.

NOTE.—If the diameter of the generating circle is equal to the radius of the fundamental circle, the hypocycloids will be a straight line, which is the diameter of the fundamental circle.

Involute.

An involute is a curved line which may be assumed to be generated in the following manner: Suppose a string be placed around a cylinder from *a* to *b*, in the direction of the arrow (see Fig. 28), and having a pencil attached at *b*; keep the string tight and move the pencil toward *c*, and the involute, *b c*, is generated.

FIG. 28

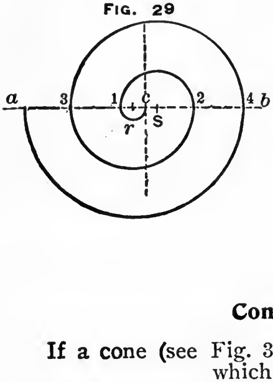


To draw an involute.

Solution:

From the point *b*, (see Fig. 28) set off any number of radial lines at equal distances, as 1, 2, 3, 4, 5. From points of intersection draw the tangents (perpendicular to the radial lines). Set off on the first tangent the length of the arc 1 to *b*; on the second tangent the arc 2 to *b*, etc. This will give the points through which to draw the involute.

FIG. 29



To draw a spiral from a given point, *c*.

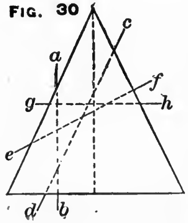
Solution:

Draw the line *a b* through the point *c*. Set off the centers *r* and *S*, one-fourth as far from *c* as the distance is to be between two lines in the spiral. Using *r* as center, describe the arc from *c* to 1, and using *S* as center, describe the arc from 1 to 2; using *r* as center, describe the arc from 2 to 3, etc.

Conical Sections.

If a cone (see Fig. 30), is cut by a plane on the line *a b*, which is parallel to the center line, the section will be a *hyperbola*.

FIG. 30



If cut by a plane on the line *c d*, which is parallel to the side, the section will be a *parabola*.

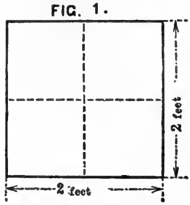
If cut by a plane on the line *g h*, which is parallel to the base line, the section will be a *circle*.

If cut by a line, *e f*, which is neither parallel to the side, the center-line nor the base, the section will be an *ellipse*.

MENSURATION.

If each side in a square (see Fig. 1) is two feet long, the area of the figure will be 4 square feet; that is, it contains four squares, each of which is one square foot. Thus the area of any square or rectangle is calculated by multiplying the length by the width.

FIG. 1.



EXAMPLE 1.

What is the area of a piece of land having right angles and measuring 108 feet long and 20 feet wide?

Solution:

$$108 \times 20 = 2160 \text{ square feet.}$$

EXAMPLE 2.

What is the area in square meters of a square house-lot 30 meters long and 30 meters wide.

Solution:

$$30 \times 30 = 900 \text{ square meters.}$$

(Square meter is frequently written m^2 and cubic meter is written m^3).

A square inscribed in a circle is half in area of a square outside the same circle. Divide the side of a square by 0.8862, and the quotient is the diameter of a circle of the same area as the square.

The Difference between One Square Foot and One Foot Square.

One foot square means one foot long and one foot wide, but one square foot may be any shape, providing the area is one square foot. For instance, Fig. 1 is two feet square, but it contains four square feet. One inch square means one inch long and one inch wide, but one square inch may be any shape, provided the area is one square inch. One mile square means one mile long and one mile wide, but one square mile may have any shape, provided the area is one square mile.

Area of Triangles.

The area of any triangle may be found by multiplying the base by the perpendicular height and dividing the product by 2.

EXAMPLE.

Find the area of a triangle 16 inches long and 5 inches perpendicular height.

Solution:

$$\text{Area} = \frac{5 \times 16}{2} = 40 \text{ square inches.}$$

The perpendicular height in any triangle is equal to the area multiplied by 2 and the product divided by the base.

The area of any triangle is equal to half the base multiplied by the perpendicular height.

The perpendicular height of any equilateral triangle is equal to one of its sides multiplied by 0.866.

The area of any equilateral triangle may be found by multiplying the square of one of the sides by 0.433.

EXAMPLE.

Find the area of an equilateral triangle when the sides are 12 inches long.

Solution:

$$\text{Area} = 12 \times 12 \times 0.433 = 62.352$$

The side of any equilateral triangle multiplied by 0.6582 gives the side of a square of the same area.

The side of any equilateral triangle divided by 1.3468 gives the diameter of a circle of the same area.

To Figure the Area of Any Triangle when Only the Length of the Three Sides is Given.

RULE.

From half the sum of the three sides subtract each side separately; multiply these three remainders with each other and the product by half the sum of the sides, and the square root of this result is the area of the triangle.

EXAMPLE.

Find the area of a triangle having sides 12 inches, 9 inches and 15 inches long.

Solution:

Half the sum of the sides $= 18$

$$\text{Area} = \sqrt{(18-12) \times (18-9) \times (18-15) \times 18}$$

$$\text{Area} = \sqrt{6 \times 9 \times 3 \times 18}$$

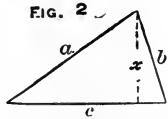
$$\text{Area} = \sqrt{2916}$$

Area $= 54$ square inches.

To Find the Height in any Triangle when the Length of the Three Sides is Given.

(See Fig. 2).

FIG. 2



The base line is to the sum of the other two sides as the difference of the sides is to the difference between the two parts of the base line, on each side of the line measuring the perpendicular height. If half this difference is either added to or subtracted from half the base line, there will be obtained two right-angled triangles, in which the base and hypothenuse are known and the perpendicular may be calculated thus: Using Fig. 2 for an example, and adding half the difference to half the base line, this may be written in the formula:

$$x = \sqrt{a^2 - \left(\frac{(a+b)\times(a-b)}{2c} + \frac{c}{2}\right)^2}$$

RULE.

Multiply the sum of the sides by their difference and divide this product by twice the base; to the quotient add half the base; square this sum (that is, multiply it by itself); subtract this from the square of the longest side, and the square root of the difference is the perpendicular height of the triangle.

EXAMPLE.

In the triangle, Fig. 2, the sides are:

$c = 12$ inches.

$a = 9$ inches.

$b = 6$ inches; find the perpendicular height x.

$$x = \sqrt{9^2 - \left(\frac{(9+6)\times(9-6)}{2\times 12} + \frac{12}{2}\right)^2}$$

$$x = \sqrt{81 - \left(\frac{15\times 3}{24} + 6\right)^2}$$

$$x = \sqrt{81 - (1\tfrac{7}{8} + 6)^2}$$

$$x = \sqrt{81 - 7.875^2}$$

$$x = \sqrt{81 - 62.015}$$

$$x = \sqrt{18.985}$$

$$x = 4.357 \text{ inches.}$$

To Find the Area of a Parallelogram.

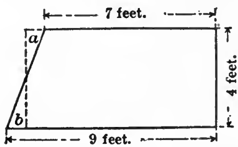
Multiply the length by the width, and the product is the area.

NOTE.—The width must not be measured on the slant side, but perpendicular to its length.

To Find the Area of a Trapezoid.

Add the two parallel sides and divide by two; multiply the quotient by the width, and the product is the area. (See Fig. 3).

FIG. 3.



EXAMPLE.

Find the area of a trapezoid. (Fig. 3).

Solution:

$$\text{Area} = \frac{7+9}{2} \times 4 = 32 \text{ square feet.}$$

NOTE.—The correctness of this may be best understood by assuming the triangle *b* cut off and placed in the position *a*, and the trapezoid will be changed into a rectangle 8 feet long and 4 feet wide.

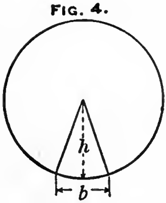
The area of any polygon may be found by dividing it into triangles and calculating the area of each separately, and the sum of the areas of all the triangles is the area of the polygon.

The Area of a Circle.

The area of a circle is equal to the square of the radius multiplied by 3.1416, which written in a formula is,

$$\text{Area} = 3.1416\, r^2.$$

FIG. 4.



The area of a circle is also equal to the square of the diameter multiplied by 0.7854, which may be written,

$$\text{Area} = 0.7854\, d^2$$

The area of a circle is also equal to its circumference multiplied by the radius and the product divided by 2, which may be written,

$$\text{Area} = \frac{c \times r}{2}$$

The correctness of these formulas may be best understood by assuming the circle to be divided into triangles (see Fig. 4), of which the height h = radius and the sum of the bases, b, of all the triangles is equal to the circumference of the circle.

Therefore, according to the formulas,

the area of a triangle $= \frac{\text{base} \times \text{perpendicular height}}{2}$

the area of a circle must be $= \frac{\text{radius} \times \text{circumference}}{2}$

and from this follow all the other formulas.

To Change a Circle into a Square of the Same Area.

RULE.

Multiply the diameter of the circle by the constant 0.8862 and the product is the length of one side in a square of the same area.

EXAMPLE.

A circular water-tank 5 feet in diameter and 3 feet high is to be replaced by a square tank of the same height and volume. How long will each side in the new tank be?

Solution:

Side $= 5 \times 0.8862 = 4.431$ feet long.

To Find the Side of the Largest Square which can be Inscribed in a Circle.

RULE.

Multiply the diameter of the circle by the constant 0.7071; the product is the length of the side of the square.

EXAMPLE.

What is the largest square beam which can be cut from a log 30 inches in diameter.

Solution:

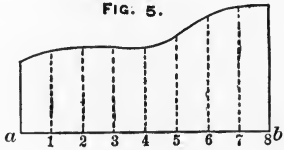
$30 \times 0.7071 = 21.213$ inches square.

NOTE.—A round log of any diameter will always cut into a square beam having sides seven-tenths the diameter of the round log. For instance, a 10-inch log will cut 7 inches square, a 15-inch log will cut 10.5 inches square, a 20-inch log will cut 14 inches square, etc.

To Find the Area of Any Irregular Figure.

(See Fig. 5).

FIG. 5.



Divide the figure into any number of equal parts, as shown by the perpendiculars 1, 2, 3, etc. Measure the width of the figure at the middle of each division; add these measurements together, divide this sum by the number of divisions (in Fig. 5 it is 8), multiply this quotient by the length $a\ b$, and the product is the area, approximately.

NOTE.—Sometimes the figure is of such shape that it is more convenient to divide some of it into squares, rectangles, or triangles, and figure the rest as explained above.

To Find the Area of a Sector of a Circle.

The area of a sector of a circle is to the area of the whole circle as the number of degrees in the arc of the sector is to 360 degrees.

Thus:

$$A = \frac{r^2 \times 3.1416 \times a}{360} = 0.008727 \times r^2 \times a = \frac{r\ l}{2}$$

$$l = \frac{2\ A}{r}$$

$$l = \frac{3.1416 \times a \times r}{180} = 0.01745329 \times a \times r$$

$$r = \sqrt{\frac{360\ A}{3.1416\ a}} = 10.7046 \sqrt{\frac{A}{a}}$$

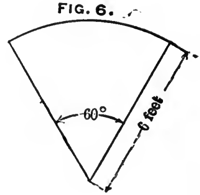
$$a = \frac{180\ l}{3.1416\ r} = \frac{57.2956\ l}{r}$$

A = Area of sector.
r = radius of sector.
a = number of degrees in arc.
l = length of arc in same units as A and r.

EXAMPLE.

The arc of the sector (Fig. 6) is 60° and the radius is 6 feet. Find area.

FIG. 6.



$$\frac{360}{60} = \frac{\pi\ r^2}{\text{Area}}$$

$$\text{Area} = \frac{60\ r^2\ \pi}{360}$$

$$\text{Area} = \frac{60 \times 6 \times 6 \times 3.1416}{360}$$

$$\text{Area} = 18.849 \text{ square feet.}$$

If the length of the arc is known instead of the number of degrees, multiply the length of the arc by the length of the radius, divide product by 2, and the quotient is the area of the sector. The correctness of this rule will be understood by the rule for area of circles, explained under Fig. 4.

To Find the Length of Arc of a Segment of a Circle.

The length of the arc may be calculated by the formula,*

$$l = \frac{8\,c - C}{3}$$

$l =$ Length of arc, $a\,f\,b$
$c =$ Length of chord from a to f (See Fig. 7).
$C =$ Length of chord from a to b

RULE.

Multiply the length of the chord of half the arc by 8; from the product subtract the length of the chord of the arc; divide the remainder by 3, and the quotient is the length of the arc.

When chord and height of segment are known, the chord of half the arc is calculated thus:

Chord of half the arc $= \sqrt{n^2 + h^2}$

$h =$ Height of segment (see $d\,f$, Fig. 7).

$n =$ Half the length of chord (see $a\,d$ or $b\,d$, Fig. 7).

When only the radius and the height of the segment are known, the length of the chord of the whole arc expressed in these terms will be: $2 \times \sqrt{2\,r\,h - h^2}$

The chord of half the arc will be: $\sqrt{2\,r\,h}$

Therefore the length of the arc will be:

$$l = \frac{8 \times \sqrt{2\,r\,h} - 2 \times \sqrt{2\,r\,h - h^2}}{3}$$

$l =$ length of arc ($a\,f\,b$, Fig. 7).
$h =$ height of segment ($d\,f$, Fig. 7).
$r =$ radius of circle ($c\,f$, Fig. 7).

To Find the Area of a Segment of a Circle.

(See Fig. 7).

Ascertain the area of the whole sector and from this area subtract the area of the triangle, and the rest is the area of the segment.

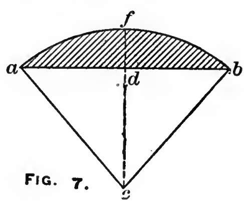


FIG. 7.

EXAMPLE.

Find the the area of the segment when the radius is 9 inches and the arc 60°.

* This formula is called "Huyghens's approximate formula for circular arcs," but it is so close that it may for any practical purpose be considered absolutely correct for arcs having small center angles; for center angles as large as 120°, the result is only *one quarter of one per cent. too small,* and even for half a circle the result is scarcely more than one per cent. small as compared to results calculated by taking π as 3.1416.

Solution:

$$\text{Area of segment} = A = \frac{60 r^2 \pi}{360} - 0.433\ r^2$$

$$A = \frac{60 \times 9 \times 9 \times 3.1416}{360} - 0.433 \times 9 \times 9$$

$$A = 42.4116 - 35.073$$

$$A = 7.3386 \text{ square inches.}$$

In this example the arc was 60°, consequently the triangle is equilateral; therefore its area is found by the formula $0.433\ r^2$. (See area of equilateral triangles, page 194).

NOTE.—When the segment is greater than a semicircle, calculate by preceding rules and formulas the area of the lesser portion of the circle; subtract it from the area of the whole circle. The remainder is the area of the segment.

To Find the Radius Corresponding to the Arc, when the Chord and the Height of the Segment Are Given.

RULE.

Add the square of the height to the square of half the chord; divide this sum by twice the height, and the quotient is the radius. In a formula this may be written:

$$\left.\begin{aligned} x &= \frac{n^2 + h^2}{2\ h} \\ l &= \text{radius} = c\ b \text{ or } c\ f \\ n &= \text{half the chord} = d\ b \\ h &= \text{height} = d\ f \end{aligned}\right\} \text{(See Fig. 7).}$$

The above rule and formula may be proved by rules for right-angled triangles; thus, $c\ b$ or r equals hypothenuse, and n, or half the chord, equals perpendicular, and $c\ d$, which is equal to $r - h$, is the base. From the rule that the square of the hypothenuse is equal to the sum of the square of the base and the square of the perpendicular, we have:

$$r^2 = n^2 + (r - h)^2$$

$$r^2 = n^2 + r^2 - 2\ r\ h + h^2$$

$$r^2 - r^2 + 2\ r\ h = n^2 + h^2$$

$$2\ r\ h = n^2 + h^2$$

$$r = \frac{n^2 + h^2}{2\ h}$$

The perpendicular height of the triangle is always equal to the radius minus the height of the segment. (See triangle $a\ b\ c$, and height, $d\ f$, Fig. 7).

TABLE No. 23.—Areas of Segments of a Circle.

The diameter of a circle = 1, and it is divided into 100 equal parts.

$\frac{h}{D}$	Area.	$\frac{h}{D}$	Area.	$\frac{h}{D}$	Area.
0.01	0.001329	0.18	0.096135	0.35	0.244980
0.02	0.003749	0.19	0.103900	0.36	0.254551
0.03	0.006866	0.20	0.111824	0.37	0.264179
0.04	0.010538	0.21	0.119898	0.38	0.273861
0.05	0.014681	0.22	0.128114	0.39	0.283593
0.06	0.019239	0.23	0.136465	0.40	0.293370
0.07	0.024168	0.24	0.144945	0.41	0.303187
0.08	0.029435	0.25	0.153546	0.42	0.313042
0.09	0.035012	0.26	0.162263	0.43	0.322928
0.10	0.040875	0.27	0.171090	0.44	0.332843
0.11	0.047006	0.28	0.180020	0.45	0.342783
0.12	0.053385	0.29	0.189048	0.46	0.352742
0.13	0.059999	0.30	0.198168	0.47	0.362717
0.14	0.066833	0.31	0.207376	0.48	0.372704
0.15	0.073875	0.32	0.216666	0.49	0.382700
0.16	0.081112	0.33	0.226034	0.50	0.392699
0.17	0.088536	0.34	0.235473		

Table No. 23 gives the areas of segments from 0.01 to 0.5 in height when the diameter of the circle is 1.

The area of any segment is computed by the following rule:

Divide the height of the segment by the diameter of its corresponding circle. Find in the table in column marked $\frac{h}{D}$ the number which is nearest, and multiply the corresponding area by the square of the diameter of the circle, and the product is the area of the segment.

Example.

Figure the area of a segment of a circle, the height of the segment being 12 inches and the diameter of the circle 40 inches.

Solution:

$$12 \text{ divided by } 40 = 0.3$$

In the column marked $\frac{h}{D}$ find 0.3; the corresponding area is 0.198168.

The area of the segment is $40 \times 40 \times 0.198168 = 317.0688$ square inches, or 317 square inches.

To Calculate the Number of Gallons of Oil in a Tank.

EXAMPLE.

A gasoline tank car is standing on a horizontal track, and by putting a stick through its bung-hole on top it is ascertained that the gasoline stands 15 inches high in the tank. The diameter of the tank is 60 inches and the length is 25 feet. How many gallons of gasoline are there in the tank?

Solution:

15 divided by 60 is 0.25

In Table No. 23, the area corresponding to 0.25 is 0.153546. Area of cross section of the gasoline is $60 \times 60 \times 0.153546 = 552.7656$ square inches.

Twenty-five feet is 300 inches; the tank contains $300 \times 552.7656 = 165829.68$ cubic inches. One gallon is 231 cubic inches. The tank contains 165829.68 divided by $231 = 717.88$, or 718 gallons.

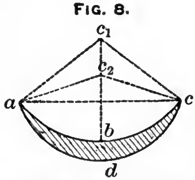
NOTE.—If the tank is more than half full, figure first the cubical contents of the whole tank if full, then figure the cubical contents of the empty space and subtract the last quantity from the first, and the difference is the cubical contents of the fluid in the tank.

Circular Lune.

The circular lune is a crescent-shaped figure bounded by two arcs, as *a b c* and *a d c*. (Fig. 8).

Its area is obtained by first finding the area of the segment *a d c* (having c_2 for center of the circle), then the area of the segment *a b c* (having c_1 for center of circle), then by subtracting the area of the last segment from the area of the first; the difference is the area of the lune.

FIG. 8.

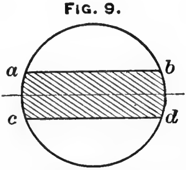


Circular Lune.

A practical example of a circular lune is the area of the opening in a straight-way valve when it is partly shut.

Circular Zone.

FIG. 9.



Circular Zone.

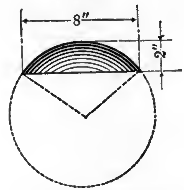
The shaded part, *a b c d*, of the figure is called a circular zone. Its area is obtained by first finding the area of the circle and then subtracting the area of the two segments; the difference is the area of the zone. When the zone is narrow in proportion to the diameter, its area is obtained very nearly by following the rule: Add line *a b* or *c d* to the diameter of the circle, divide the sum by 2 and multiply the quotient by the width of the zone, and the product is the area.

To Compute the Volume of a Segment of a Sphere.

RULE.

Square half the length of its base, and multiply by 3. To this product add square of the height. Multiply the sum by the height and by 0.5236.

FIG. 10.



Segment of a Sphere.

EXAMPLE.

Find volume of the spherical segment shown in Fig. 10; base line is 8″ and height is 2″.

Solution:

$$\text{Volume} = v = (3 \times 4^2 + 2^2) \times 2 \times 0.5236$$
$$v = (3 \times 16 + 4) \times 2 \times 0.5236$$
$$v = 52 \times 2 \times 0.5236$$
$$v = 54.4544 \text{ cubic inches.}$$

To Find the Volume of a Spherical Segment, when the Height of the Segment and the Diameter of the Sphere are Known.

RULE.

Multiply the diameter of sphere by 3, and from this product subtract twice the height of segment. Multiply the remainder by the square of the height and the product by 0.5236.

EXAMPLE.

The segment (Fig. 10) is cut from a sphere 10 inches in diameter and it is 2 inches high. Figure it by this last rule.

Solution:

$$\text{Volume} = v = (10 \times 3 - 2 \times 2) \times 2^2 \times 0.5236$$
$$v = (30 - 4) \times 4 \times 0.5236$$
$$v = 26 \times 4 \times 0.5236$$
$$v = 54.4544 \text{ square inches.}$$

To Find the Surface of a Cylinder.

RULE.

Multiply the circumference by the length, and to this product add the area of the two ends.

A cylinder has the largest volume with the smallest surface when length and diameter are equal to each other.

To Find the Volume of a Cylinder.

RULE.

Multiply area of end by length of cylinder, and the product is the volume of the cylinder.

EXAMPLE.

What is the volume of a cylinder 4 inches in diameter and 9 inches long?

Solution:

Area of end $= r^2 \pi$

Volume $= r^2 \pi l = 2 \times 2 \times 3.1416 \times 9 = 113.0976$ cubic inches.

To Find the Solid Contents of a Hollow Cylinder.

RULE.

Find area of end according to outside diameter; also find area according to inside diameter; subtract the last area from the first and multiply the difference by the length of the cylinder.

Formula:

$$\text{Area} = (R^2 - r^2)\, \pi\, l$$

$R =$ Outside radius.

$r =$ Inside radius.

$l =$ Length of cylinder.

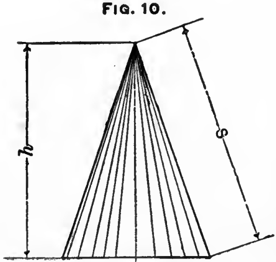
EXAMPLE.

Find the solid contents of a hollow cylinder of 6 feet outside diameter, 4 feet inside diameter and 5 feet long.

Solution:

$$\text{Solid contents} = x = (3^2 - 2^2) \times 3.1416 \times 5$$
$$x = (9 - 4) \times 3.1416 \times 5$$
$$x = 5 \times 3.1416 \times 5$$
$$x = 78.54 \text{ cubic feet.}$$

FIG. 10.



Cone.

To Find the Area of the Curved Surface of a Cone.

(See Fig. 10).

RULE.

Multiply the circumference of the base by the slant height and divide the product by 2; the quotient is the area of the curved surface. If the total surface is wanted, the area of the base is added to the curved area.

If the perpendicular height is known, the length of the slant side or the slant height is found by adding the square of the perpendicular height to the square of the radius and extracting the square root of the sum.

Formula:

$$\text{Curved area} = x = \frac{d \pi \sqrt{r^2 + h^2}}{2} = r \pi \sqrt{r^2 + h^2}$$

$r =$ Radius of base.
$d =$ Diameter of base.
$h =$ Perpendicular height.

To Find the Volume of a Cone.

RULE.

Multiply the area of the base by the perpendicular height, and divide the product by 3.

By formula:

$$\text{Volume} = \frac{r^2 \pi h}{3}$$

To Find the Area of the Curved Surface of a Frustum of a Cone.

(See Fig. 11).

RULE.

Add circumference of small end to circumference of large end, muitiply this sum by the slant height and divide the product by 2.

Formula:

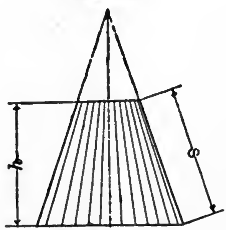
$$\text{Curved area} = (2 R \pi + 2 r \pi) \frac{S}{2}$$

which reduces to

$$\text{Curved area} = (R + r) \pi S$$

If the perpendicular height instead of the slant height is known, we have:

FIG. 11.



Frustum of a Cone.

$$\text{Curved area} = (R + r) \pi \sqrt{(R - r)^2 + h^2}$$

$R =$ Large radius.
$r =$ Small radius.
$h =$ Perpendicular height,
$s =$ Slant height,

To Find the Volume of a Frustum of a Cone.

RULE.

Square the largest radius; square the smallest radius. Multiply largest radius by smallest radius; add these three products and multiply their sum by 3.1416; multiply this last product by one-third of the perpendicular height.

Formula:

$$\text{Volume} = (R^2 + r^2 + Rr)\,\pi\,\frac{h}{3}$$

EXAMPLE.

Find the volume of a frustum of a cone. The largest diameter is 6 feet, the smallest diameter is 4 feet, and perpendicular height is 12 feet.

Solution:

$$\text{Volume} = x = (3^2 + 2^2 + 3 \times 2) \times 3.1416 \times \frac{12}{3}$$

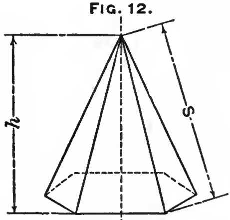
$$x = (9 + 4 + 6) \times 3.1416 \times 4$$

$$x = 19 \times 3.1416 \times 4$$

$$x = 238.7616 \text{ cubic feet.}$$

NOTE.—This rule will also apply for finding the solid contents of wood in a log.

FIG. 12.



Pyramid

To Find the Area of the Slanted Surface of a Pyramid.

(See Fig. 12).

RULE.

Multiply the length of the perimeter of the base by the slant height of the side (not the slant height of the edge). Divide the product by 2, and the quotient is the area.

To Find the Total Area of the Surface of a Pyramid.

RULE.

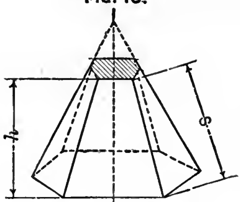
Find area of the slanted surface as explained above, and to this add the area of a polygon equal to the base of the pyramid.

To Find the Volume of a Pyramid.

RULE.

Multiply the area of the base by one-third of the perpendicular height,

FIG. 13.



Frustum of a Pyramid

To Find the Area of the Slanted Surface of a Frustum of a Pyramid.

RULE.

Add perimeter of the small end to the perimeter of the large end. Multiply this sum by the slant height of the side (not slant height of edge). Divide the product by 2.

To Find the Total Area of the Surface of a Frustum of a Pyramid.

RULE.

Find the area of the slanted surface as explained above, and to this area add the area of the two ends. Their areas are obtained in the same way as areas of polygons. (See page 196).

To Compute the Volume of a Frustum of a Pyramid.

RULE.

Multiply the area of the small end by the area of the large end, extract the square root of the product, and to this add the area of the small end and the area of the large end; multiply the sum by one-third of the perpendicular height.

Formula:

$$\text{Volume} = \frac{h}{3}\left(a + A + \sqrt{A\,a}\right)$$

EXAMPLE.

Find volume of a frustum of a pyramid when the area of the small end is 8 square feet, the area of the large end is 18 square feet and the perpendicular height is 30 feet.

$$\text{Volume} = v = \frac{30}{3} \times \left(8 + 18 + \sqrt{18 \times 8}\right)$$

$$v = 10 \times \left(8 + 18 + \sqrt{144}\right)$$

$$v = 10 \times (8 + 18 + 12)$$

$$v = 10 \times 38$$

$$v = 380 \text{ cubic feet.}$$

To Find the Surface of a Sphere.

RULE.

Multiply the circumference by the diameter.

NOTE.—The surface of a sphere is equal to the curved surface of a cylinder having diameter and length equal to the diameter of the sphere.

To Find the Volume of a Sphere.

RULE.

Multiply the cube of the diameter by 3.1416, divide the product by 6 and the quotient is the volume of the sphere. Or, another rule is: Multiply the cube of the diameter by 0.5236 and the product is the volume of the sphere.

EXAMPLE.

Find the volume of a sphere 15″ diameter.

Solution:

$0.5236 \times 15 \times 15 \times 15 = 1767.15$ cubic inches.

A sphere twice as large in diameter as another has twice the circumference, four times the surface, eight times the volume, and if of the same material will weigh eight times as much.

To Compute the Diameter of a Sphere when the Volume is Known.

RULE.

Divide the volume by 0.5236 and the cube root of the quotient is the diameter of the sphere.

To Compute the Circumference of an Ellipse.

RULE.

Add the square of the largest diameter to the square of the smallest diameter and divide the sum by 2; multiply the square root of the quotient by 3.1416.*

EXAMPLE.

Find the circumference of an ellipse. The largest diameter is 24 inches and the smallest diameter is 18 inches.

Solution:

$$\text{Circumference} = c = 3.1416\sqrt{\frac{24^2 + 18^2}{2}}$$

$$c = 3.1416\sqrt{450}$$

$$c = 3.1416 \times 21.2132$$

$$c = 66.643 \text{ inches.}$$

To Compute the Area of an Ellipse.

RULE.

Multiply the smallest diameter by the largest diameter, and this product by 0.7854.

* This Rule gives only approximate results. There is no known rule giving exact results.

TABLE No. 24. — Giving Circumferences and Areas of Circles.

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
1/64	0.0491	0.00019	43/64	2.1108	0.35454
1/32	0.0982	0.00077	11/16	2.1598	0.37122
3/64	0.1473	0.00173	45/64	2.2089	0.38829
1/16	0.1964	0.00307	23/32	2.2580	0.40574
5/64	0.2454	0.00479	47/64	2.3071	0.42357
3/32	0.2945	0.00690	3/4	2.3562	0.44179
7/64	0.3436	0.00940	49/64	2.4053	0.46039
1/8	0.3927	0.01227	25/32	2.4544	0.47937
9/64	0.4418	0.01553	51/64	2.5035	0.49874
5/32	0.4909	0.01918	13/16	2.5525	0.51849
11/64	0.5400	0.02320	53/64	2.6016	0.53862
3/16	0.5890	0.02761	27/32	2.6507	0.55914
13/64	0.6381	0.03241	55/64	2.6998	0.58004
7/32	0.6872	0.03758	7/8	2.7489	0.60132
15/64	0.7363	0.04314	57/64	2.7980	0.62299
1/4	0.7854	0.04909	29/32	2.8471	0.64504
17/64	0.8345	0.05542	59/64	2.8962	0.66747
9/32	0.8836	0.06213	15/16	2.9452	0.69029
19/64	0.9327	0.06922	61/64	2.9943	0.71349
5/16	0.9818	0.07670	31/32	3.0434	0.73708
21/64	1.0308	0.08456	63/64	3.0925	0.76105
11/32	1.0799	0.09281	1	3.1416	0.78540
23/64	1.1290	0.10144	1 1/64	3.1907	0.81013
3/8	1.1781	0.11045	1 1/32	3.2398	0.83525
25/64	1.2272	0.11984	1 3/64	3.2889	0.86075
13/32	1.2763	0.12962	1 1/16	3.3379	0.88664
27/64	1.3254	0.13979	1 5/64	3.3870	0.91291
7/16	1.3744	0.15033	1 3/32	3.4361	0.93956
29/64	1.4235	0.16126	1 7/64	3.4852	0.96660
15/32	1.4726	0.17258	1 1/8	3.5343	0.99402
31/64	1.5217	0.18427	1 9/64	3.5834	1.02182
1/2	1.5708	0.19635	1 5/32	3.6325	1.05001
33/64	1.6199	0.20881	1 11/64	3.6816	1.07858
17/32	1.6690	0.22166	1 3/16	3.7306	1.10753
35/64	1.7181	0.23489	1 13/64	3.7797	1.13687
9/16	1.7671	0.24850	1 7/32	3.8288	1.16659
37/64	1.8162	0.26250	1 15/64	3.8779	1.19670
19/32	1.8653	0.27688	1 1/4	3.9270	1.22718
39/64	1.9144	0.29165	1 17/64	3.9761	1.25806
5/8	1.9635	0.30680	1 9/32	4.0252	1.28931
41/64	2.0126	0.32233	1 19/64	4.0743	1.32095
21/32	2.0617	0.33824	1 5/16	4.1233	1.35297

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
1 21/64	4.1724	1.38538	2 1/8	6.6759	3.5466
1 11/32	4.2215	1.41817	2 3/16	6.8722	3.7584
1 23/64	4.2706	1.45134	2 1/4	7.0686	3.9761
1 3/8	4.3197	1.48489	2 5/16	7.2649	4.2
1 25/64	4.3688	1.51883	2 3/8	7.4613	4.4301
1 13/32	4.4179	1.55316	2 7/16	7.6576	4.6664
1 27/64	4.4670	1.58786	2 1/2	7.8540	4.9087
1 7/16	4.5160	1.62295	2 9/16	8.0503	5.1573
1 29/64	4.5651	1.65843	2 5/8	8.2467	5.4119
1 15/32	4.6142	1.69428	2 11/16	8.4430	5.6727
1 31/64	4.6633	1.73052	2 3/4	8.6394	5.9396
1 1/2	4.7124	1.76715	2 13/16	8.8357	6.2126
1 33/64	4.7615	1.80415	2 7/8	9.0321	6.4918
1 17/32	4.8106	1.84154	2 15/16	9.2284	6.7772
1 35/64	4.8597	1.87932	3	9.4248	7.0686
1 9/16	4.9087	1.91748	3 1/16	9.6211	7.3662
1 37/64	4.9578	1.95602	3 1/8	9.8175	7.6699
1 19/32	5.0069	1.99494	3 3/16	10.0138	7.9798
1 39/64	5.0560	2.03425	3 1/4	10.2102	8.2958
1 5/8	5.1051	2.07394	3 5/16	10.4066	8.6179
1 41/64	5.1542	2.11402	3 3/8	10.6029	8.9462
1 21/32	5.2033	2.15448	3 7/16	10.7992	9.2807
1 43/64	5.2524	2.19532	3 1/2	10.9956	9.6211
1 11/16	5.3014	2.23654	3 9/16	11.1919	9.9678
1 45/64	5.3505	2.27815	3 5/8	11.3883	10.3206
1 23/32	5.3996	2.32015	3 11/16	11.5846	10.6796
1 47/64	5.4487	2.36252	3 3/4	11.7810	11.0447
1 3/4	5.4978	2.40528	3 13/16	11.9773	11.4160
1 49/64	5.5469	2.44843	3 7/8	12.1737	11.7933
1 25/32	5.5960	2.49195	3 15/16	12.3701	12.1768
1 51/64	5.6450	2.53586	4	12.5664	12.5664
1 13/16	5.6941	2.58016	4 1/16	12.7628	12.9622
1 53/64	5.7432	2.62483	4 1/8	12.9591	13.3641
1 27/32	5.7923	2.66989	4 3/16	13.1554	13.7721
1 55/64	5.8414	2.71534	4 1/4	13.3518	14.1863
1 7/8	5.8905	2.76117	4 5/16	13.5481	14.6066
1 57/64	5.9396	2.80738	4 3/8	13.7445	15.0330
1 29/32	5.9887	2.85397	4 7/16	13.9408	15.4656
1 59/64	6.0377	2.90095	4 1/2	14.1372	15.9043
1 15/16	6.0868	2.94831	4 9/16	14.3335	16.3492
1 61/64	6.1359	2.99606	4 5/8	14.5299	16.8002
1 31/32	6.1850	3.04418	4 11/16	14.7262	17.2573
1 63/64	6.2341	3.0927	4 3/4	14.9226	17.7206
2	6.2832	3.1416	4 13/16	15.1189	18.19
2 1/16	6.4795	3.3410	4 7/8	15.3153	18.6655

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
$4\frac{15}{16}$	15.5116	19.1472	$10\frac{1}{2}$	32.9868	86.5903
5	15.7080	19.6350	$10\frac{5}{8}$	33.3795	88.6643
$5\frac{1}{8}$	16.1007	20.6290	$10\frac{3}{4}$	33.7722	90.7625
$5\frac{1}{4}$	16.4934	21.6476	$10\frac{7}{8}$	34.1649	92.8858
$5\frac{3}{8}$	16.8861	22.6907	11	34.5576	95.0334
$5\frac{1}{2}$	17.2788	23.7583	$11\frac{1}{8}$	34.9503	97.2055
$5\frac{5}{8}$	17.6715	24.8505	$11\frac{1}{4}$	35.343	99.4019
$5\frac{3}{4}$	18.0642	25.9673	$11\frac{3}{8}$	35.7357	101.6234
$5\frac{7}{8}$	18.4569	27.1084	$11\frac{1}{2}$	36.1284	103.8691
6	18.8496	28.2744	$11\frac{5}{8}$	36.5211	106.1394
$6\frac{1}{8}$	19.2423	29.4648	$11\frac{3}{4}$	36.9138	108.4338
$6\frac{1}{4}$	19.635	30.6797	$11\frac{7}{8}$	37.3065	110.7537
$6\frac{3}{8}$	20.0277	31.9191	12	37.6992	113.098
$6\frac{1}{2}$	20.4204	33.1831	$12\frac{1}{4}$	38.4846	117.859
$6\frac{5}{8}$	20.8131	34.4717	$12\frac{1}{2}$	39.2700	122.719
$6\frac{3}{4}$	21.2058	35.7848	$12\frac{3}{4}$	40.0554	127.677
$6\frac{7}{8}$	21.5985	37.1224	13	40.8408	132.733
7	21.9912	38.4846	$13\frac{1}{4}$	41.6262	137.887
$7\frac{1}{8}$	22.3839	39.8713	$13\frac{1}{2}$	42.4116	143.139
$7\frac{1}{4}$	22.7766	41.2826	$13\frac{3}{4}$	43.1970	148.490
$7\frac{3}{8}$	23.1693	42.7184	14	43.9824	153.938
$7\frac{1}{2}$	23.5620	44.1787	$14\frac{1}{4}$	44.7678	159.485
$7\frac{5}{8}$	23.9547	45.6636	$14\frac{1}{2}$	45.5532	165.130
$7\frac{3}{4}$	24.3474	47.1731	$14\frac{3}{4}$	46.3386	170.874
$7\frac{7}{8}$	24.7401	48.7071	15	47.1240	176.715
8	25.1328	50.2656	$15\frac{1}{4}$	47 9094	182.655
$8\frac{1}{8}$	25.5255	51.8487	$15\frac{1}{2}$	48.6948	188.692
$8\frac{1}{4}$	25.9182	53.4561	$15\frac{3}{4}$	49.4802	194.828
$8\frac{3}{8}$	26.3109	55.0884	16	50.2656	201.062
$8\frac{1}{2}$	26.7036	56.7451	$16\frac{1}{4}$	51.051	207.395
$8\frac{5}{8}$	27.0963	58.4264	$16\frac{1}{2}$	51.8364	213.825
$8\frac{3}{4}$	27.489	60.1319	$16\frac{3}{4}$	52.6218	220.354
$8\frac{7}{8}$	27.8817	61.8625	17	53.4072	226.981
9	28.2744	63.6174	$17\frac{1}{4}$	54.1926	233.706
$9\frac{1}{8}$	28.6671	65.3968	$17\frac{1}{2}$	54.9780	240.529
$9\frac{1}{4}$	29.0598	67.2008	$17\frac{3}{4}$	55.7634	247.450
$9\frac{3}{8}$	29.4525	69.0293	18	56.5488	254.470
$9\frac{1}{2}$	29.8452	70.8823	$18\frac{1}{4}$	57.3342	261.587
$9\frac{5}{8}$	30.2379	72.7599	$18\frac{1}{2}$	58.1196	268.803
$9\frac{3}{4}$	30.6306	74.6619	$18\frac{3}{4}$	58.905	276.117
$9\frac{7}{8}$	31.0233	76.5888	19	59.6904	283.529
10	31.4160	78.5400	$19\frac{1}{4}$	60.4758	291.040
$10\frac{1}{8}$	31.8087	80.5158	$19\frac{1}{2}$	61.2612	298.648
$10\frac{1}{4}$	32.2014	82.5158	$19\frac{3}{4}$	62.0466	306.355
$10\frac{3}{8}$	32.5941	84.5409	20	62.8320	314.16

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
21	65.9736	346.361	66	207.34	3421.19
22	69.1152	380.134	67	210.49	3525.65
23	72.2568	415.477	68	213.63	3631.68
24	75.3984	452.39	69	216.77	3739.28
25	78.540	490.87	70	219.91	3848.45
26	81.681	530.93	71	223.05	3959.19
27	84.823	572.56	72	226.19	4071.50
28	87.965	615.75	73	229.34	4185.39
29	91.106	660.52	74	232.48	4300.84
30	94.248	706.86	75	235.62	4417.86
31	97.389	754.77	76	238.76	4536.46
32	100.53	804.25	77	241.90	4656.63
33	103.67	855.30	78	245.04	4778.36
34	106.81	907.92	79	248.19	4901.67
35	109.96	962.11	80	251.33	5026.55
36	113.10	1017.88	81	254.47	5153.00
37	116.24	1075.21	82	257.61	5281.02
38	119.38	1134.11	83	260.75	5410.61
39	122.52	1194.59	84	263.89	5541.77
40	125.66	1256.64	85	267.04	5674.50
41	128.81	1320.25	86	270.18	5808.80
42	131.95	1385.44	87	273.32	5944.68
43	135.09	1452.20	88	276.46	6082.12
44	138.23	1520.53	89	279.60	6221.14
45	141.37	1590.43	90	282.74	6361.73
46	144.51	1661.90	91	285.88	6503.88
47	147.65	1734.94	92	289.03	6647.61
48	150.80	1809.56	93	292.17	6792.91
49	153.94	1885.74	94	295.31	6939.78
50	157.08	1963.50	95	298.45	7088.22
51	160.22	2042.82	96	301.59	7238.23
52	163.36	2123.72	97	304.73	7389.81
53	166.50	2206.18	98	307.88	7542.96
54	169.65	2290.22	99	311.02	7697.69
55	172.79	2375.83	100	314.16	7853.98
56	175.93	2463.01	101	317.30	8011.85
57	179.07	2551.76	102	320.44	8171.28
58	182.21	2642.08	103	323.58	8332.29
59	185.35	2733.97	104	326.73	8494.87
60	188.50	2827.43	105	329.87	8659.01
61	191.64	2922.47	106	333.01	8824.73
62	194.78	3019.07	107	336.15	8992.02
63	197.92	3117.25	108	339.29	9160.88
64	201.06	3216.99	109	342.43	9331.32
65	204.20	3318.31	110	345.58	9503.32

Strength of Materials.

The strength of materials may be divided into *Tensile*, *Crushing*, *Transverse*, *Torsional*, or *Shearing*, and besides this, the elasticity of the material or its resistance against deflection must also be taken into consideration in figuring for strength.

Tensile Strength.

From experiments it is known that it it will take from 40,000 to 70,000 pounds to tear off a bar of wrought iron one inch square. Therefore we usually say that the tensile strength of wrought iron is from 40,000 to 70,000 pounds, according to quality. The average is 50,000 to 55,000 pounds. The tensile strength of any body is in proportion to its cross sectional area; thus, if a bar of iron of one square inch area will pull asunder under a load of 40,000 pounds, it will take 80,000 pounds to pull asunder another bar of the same kind of iron but of two square inches area. The tensile strength is independent of the length of the bar, if it is not so long that its own weight must be taken into consideration. Table No. 24 gives the load which will pull asunder one square inch of the most common materials.

No part of any machine should be strained to that limit. A high factor of safety must be used, sometimes from 4 to 30 or even more, which will depend upon the kind of stress the member is exposed to, as dead load, variable load, shocks, etc. Different factors of safety are also used for different kinds of material. (See page 274).

Modulus of Elasticity.

The modulus of elasticity for any kind of material is usually defined as the amount of force which would be required to stretch a straight bar of one square inch area to double its length or compress it to nothing, if this were possible. But a more comprehensive definition is to say that the modulus of elasticity is the reciprocal of the fractional part of the length which one unit of force will, within elastic limit, stretch or compress one unit of area. For instance, if the modulus of elasticity for a certain kind of wrought iron is 25,000,000, it means that it would take 25,000,000 pounds of pulling force to stretch a bar of one square inch area to double its length, if this could possibly be done; but it means also—which is exactly equivalent—that one pound of pulling force will stretch a bar of one square inch area one 25-millionth part of its length, or one pound compressive force will shorten the same bar one 25-millionth part of its length, and that two pounds of force will stretch or compress twice as much, three pounds thrice as much, etc.

Strength of Wrought Iron.

From experiments it is known that wrought iron can not very well be stretched or compressed more than one-thousandth part of its length without destroying its elasticity; therefore if a bar of wrought iron has 25,000,000 as its modulus of elasticity, one pound will stretch it $\frac{1}{25000000}$ of its length and it would take 25,000 pounds to stretch it $\frac{1}{1000}$ of its length. Thus, 25,000 pounds would then be said to be its strength at the limit of elasticity for that kind of iron; 80 to 100 per cent. more will usually be the ultimate breaking load.

The pull or load which such a bar can sustain with safety will depend a great deal on circumstances, but it must never exceed 25,000 pounds per square inch of area. It must not even approach this limit if the structure is of any importance or if the load is to be sustained for any length of time, or if it is, besides the load, also exposed to shocks or jar.

Strength of Cast Iron.

Cast iron of good quality has a modulus of elasticity of 15,000,000 pounds, but if strained so it will stretch $\frac{1}{1500}$ of its length its elasticity is usually destroyed. For instance, a bar of cast iron of one square inch area is exposed to tensile strain, its modulus of elasticity being 15,000,000 pounds and its elasticity being destroyed if it stretches $\frac{1}{1500}$ of its length, what then would be its strength at limit of elasticity? One pound will stretch it $\frac{1}{15000000}$ of its length, therefore it must take 10,000 pounds to stretch it $\frac{1}{1500}$ of its length; thus we would say that 10,000 pounds is its strength at limit of elasticity. It is not always that cast iron is of as good quality as that; very frequently its elasticity is destroyed if it is exposed to a tensile stress of 6,000 pounds per square inch of area; thus the strength of cast iron at its limit of elasticity is often found to be only 6,000 pounds instead of 10,000 pounds. Besides, it is very often found that a pulling force of 10,000 pounds will stretch a bar of one square inch area one twelve-hundredth part of its length, and this, of course, gives the modulus of elasticity 12,000,000 pounds. Frequently cast iron is of such quality that it cannot be stretched over $\frac{1}{2500}$ of its length before its elasticity is destroyed. Cast iron is very variable in quality, and especially so with regard to its tensile strength. Generally speaking, we may say that for cast iron the

Modulus of elasticity is 12,000,000 to 15,000,000 pounds.
Tensile strength at limit of elasticity, 5,000 to 10,000 pounds.
Ultimate tensile strength, 10,000 to 20,000 pounds.

Elongation Under Tension.

The total stretch or elongation of any specimen when exposed to tensile stress within the elastic limit is directly proportional to the length of the specimen, but it is inversely proportional to the modulus of elasticity and the cross sectional area of the specimen. The following formulas may, therefore, be used in such calculations:

$$E = \frac{P \times L}{s \times A} \qquad P = \frac{E \times s \times A}{L}$$

$$s = \frac{P \times L}{E \times A} \qquad A = \frac{P \times L}{s \times E}$$

$$L = \frac{E \times s \times A}{P}$$

$E =$ Modulus of elasticity in pounds per square inch.
$P =$ Load or force in pounds acting to elongate the specimen.
$s =$ Total stretch of specimen in inches in the length L.
$L =$ Original length of specimen in inches before force is applied.
$A =$ Cross-sectional area of specimen in square inches.

EXAMPLE.

From experiments it is known that the modulus of elasticity for a certain kind of wrought iron is 28,000,000; what will then be the total stretch or elongation in a round boiler stay, 1¼ inches in diameter and 6 feet long, when exposed to a stress of 5000 pounds?

Solution:

1¼ inches diameter = 1.227 inches area (see table, page 209)
6 feet long = 72 inches.

$$s = \frac{P \times L}{E \times A}$$

$$s = \frac{5000 \times 72}{28000000 \times 1227}$$

$s = 0.0105$ inches = total stretch in the stay.

NOTE. — As already stated, wrought iron can not be stretched as much as one-thousandth part of its original length without danger of destroying its elasticity; thus, for this stay, which is 72 inches, the limit of elasticity will be at a stretch of 0.072 inches; therefore the stretch produced by a load of 5,000 pounds, which is calculated to be 0.0105 inches, is well within the safe limit.

TABLE No. 25.—Modulus of Elasticity and Ultimate Tensile Strength of Various Materials.

Materials.	Modulus of Elasticity in Pounds per Square Inch.	Ultimate Tensile Strength in Pounds per Square Inch.	Modulus of Elasticity in Kilograms per Square Centimeter.	Ultimate Tensile Strength in Kilograms per Square Centimeter.
Cast steel . . .	30,000,000	100,000	2,200,000	7,000
Bessemer steel . .	28,000,000	70,000	1,970,000	4,930
Wrought iron bars	25,000,000	55,000	1,700,000	3,850
Wrought iron wire	28,000,000	75,000	1,970,000	5,250
Cast iron * . . .	12,000,000 to 15,000,000	10,000 to 20,000	80,0000 to 1,000,000	700 to 1,400
Copper bolts . . .	18,000,000	35,000	1,200,000	2,400
Brass	9,000,000	17,700	630,000	1,200
Oak	1,500,000	17,000	105,000	1,200
Hickory	1,400,000	20,000	98,000	1,400
Maple	1,100,000	15,000	77,000	1,000
Pitch pine	1,600,000	15,000	112,000	1,000
Pine	1,100,000	10,000	77,000	700
Spruce	1,100,000	10,000	77,000	700

The two last columns in above table are calculated by the rule: One pound per sq. inch = 0.07031 kilograms per sq. centimeter and the result is reduced to the nearest round number.

Formulas for Tensile Strength.

The ultimate tensile strength of any specimen is in proportion to its cross-sectional area, and is expressed by the following formula:

$$P = A \times S$$

$$S = \frac{P}{A} \qquad \text{Side of a square bar} = \sqrt{\frac{P}{S}}$$

$$A = \frac{P}{S} \qquad \text{Diameter of a round bar} = \sqrt{\frac{P}{S \times 0.7854}}$$

P = Force in pounds which will pull the specimen asunder.

S = Ultimate tensile strength in pounds per square inch. (See Table No. 25).

A = Cross-sectional area of the specimen in square inches.

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* Very strong cast iron may have an ultimate tensile strength as high as 30,000 pounds per square inch.

Example.

A piece of iron ½ inch square is tested in a testing machine and breaks at a total stress of 14,210 pounds. What is the ultimate tensile strength per square inch?

Solution:

A bar ½ inch square has a cross-sectional area of ½″ × ½″ is ¼ square inch.

$$S = \frac{14210}{\frac{1}{4}} = 56{,}840 \text{ pounds per square inch.}$$

Example.

What will be the breaking load for a wrought iron bar ⅜″ × ⅜″ when exposed to tensile stress, the ultimate tensile strength of the iron being 55,000 pounds per square inch, as given in Table No. 25, page 216?

Solution:

A bar ⅜″ × ⅜″ is $\frac{9}{64}$ square inches in area.

$P = \frac{9}{64} \times 55{,}000 = 7734$ pounds, which will break the bar.

In order to obtain the safe working stress introduce a suitable factor of safety, from 5 to 10, according to circumstances, and calculate by the following formulas:

$$P \times f = A \times S \qquad \text{Side of a square bar} = \sqrt{\frac{P \times f}{S}}$$

$$P = \frac{A \times S}{f}$$

$$A = \frac{P \times f}{S} \qquad \text{Diameter of a round bar} = \sqrt{\frac{P \times f}{0.7854\, S}}$$

$P =$ Load in pounds.

$f =$ Factor of safety.

Example.

A load of 24,000 pounds is suspended on a round wrought iron bar. The ultimate tensile strength of the iron is 55,000 pounds per square inch. What should be the diameter of the bar to sustain the load, with 10 as the factor of safety?

Solution:

$$A = \frac{P \times f}{S}$$

$$A = \frac{24000 \times 10}{55000} = 4.363 \text{ square inches.}$$

In Table No. 24, we find the nearest larger diameter to be 2⅜ inches.

The diameter may also be calculated directly by the following formula:

$$D = \sqrt{\frac{P \times f}{S \times 0.7854}}$$

$$D = \sqrt{\frac{24000 \times 10}{55000 \times 0.7854}}$$

$$D = \sqrt{5.56}$$

$D = 2.358$, or nearly $2\frac{3}{8}$ inches diameter.

To Find the Diameter of a Bolt to Resist a Given Load.

RULE.

Multiply pull in pounds by the factor of safety. Multiply the ultimate tensile strength of the material by 0.7854; divide this first product by the last and extract the square root from the quotient which will then be diameter of bolt at the bottom of the thread.

$$D = \sqrt{\frac{P \times f}{S \times 0.7854}}$$

$$P = \frac{D^2 \times S \times 0.7854}{f}$$

$$f = \frac{D^2 \times S \times 0.7854}{P}$$

$D =$ Diameter of bolt or screw in the bottom of the thread.
$P =$ Load or pull in pounds.
$f =$ Factor of safety.
$S =$ Ultimate tensile strength per square inch.

0.7854 is constant $= \frac{\pi}{4}$

NOTE.—Bolts are frequently exposed to a considerable amount of initial stress, due to the tightening of nuts, which must always be allowed for when deciding upon the load to be considered when calculating their diameter.

EXAMPLE.

Find diameter of a bolt to sustain a load of 4,450 pounds, taking 10 as factor of safety and ultimate tensile strength of the iron to be 50,000 pounds per square inch.

Solution:

$$D = \sqrt{\frac{4450 \times 10}{50000 \times 0.7854}}$$

$$D = \sqrt{1.133}$$

$D = 1.064''$ in the bottom of thread; thus, a $1\frac{1}{4}''$ screw, standard thread, which is $1\frac{1}{16}$ inches in diameter at the bottom of thread, will be the bolt to use.

EXAMPLE 2.

What size of bolt is required to sustain the same load as is mentioned in the previous example, if only 5 is wanted as a factor of safety?

Solution:

$$D = \sqrt{\frac{4450 \times 5}{50000 \times 0.7854}}$$

$$D = \sqrt{0.567}$$

$D = 0.75$ inch diameter in bottom of thread.

Thus a ⅞-inch standard screw is too small, as that is only $\frac{23}{32}''$ in bottom of thread, but a 1-inch standard screw is sufficient, being $\frac{27}{32}''$ in bottom of thread.

To Find the Thickness of a Cylinder to Resist a Given Pressure.

When the walls of cylinders are thin in proportion to their diameters use the formula:

$$P = \frac{S \times t}{R \times f}$$

$$t = \frac{P \times R \times f}{S}$$

$$R = \frac{S \times t}{P \times f}$$

$P =$ Pressure per square inch.
$R =$ Radius of cylinder in inches.
$t =$ Thickness of cylinder wall in inches.
$f =$ Factor of safety.
$S =$ Ultimate tensile strength of material.

When cylinder walls are thick in proportion to the diameter, such as hydraulic cylinders, their thickness is usually figured by the formula:

$$t = \frac{P \times R}{\left(\frac{S}{f}\right) - P}$$

$t =$ Thickness of cylinder wall in inches.
$P =$ Pressure in pounds per square inch.
$R =$ Radius of cylinder.
$S =$ Ultimate tensile strength.
$f =$ Factor of safety.

EXAMPLE.

Find necessary thickness of a hydraulic cylinder of 10-inch inside diameter, made from cast-iron, to stand a pressure of 1000 pounds per square inch, with 4 as factor of safety. The ultimate tensile strength of the iron is, by experiments, found to be 20,000 pounds per square inch. (See Table No. 25).

Solution:

10-inch diameter = 5-inch radius.

$$t = \frac{1000 \times 5}{\frac{20000}{4} - 1000}$$

$$t = \frac{1000 \times 5}{5000 - 1000}$$

$$t = \frac{5000}{4000}$$

$$t = 1\tfrac{1}{4} \text{ inch.}$$

Strength of Flat Cylinder Heads.

The *American Machinist*, in Question No. 147, March 22, 1894, gives the following formula for flat circular heads firmly fixed to the flange of the cylinder:

$$t = \sqrt{\frac{2 \times r^2 \times P}{3 \times S_1}}$$

t = Thickness of cylinder head in inches.

r = Radius of cylinder head in inches.

P = Pressure in pounds per square inch.

S_1 = Allowable working stress in the material.

The allowable working stress may be taken as $\frac{1}{8}$ to $\frac{1}{10}$ of the ultimate tensile strength and may, for cast iron, be from 1500 to 2500 and for wrought iron from 4000 to 6000. The above formula was used to calculate the thickness of a cast iron cylinder head of 30 inches diameter, to resist a pressure of 100 pounds per square inch. This formula is in that case considered to give sufficient thickness, so that no ribs or braces are needed.

The above formula may also be used for wrought iron, by selecting the proper value for S_1. Assuming the tensile strength of wrought iron to be 44,000 pounds, and allowing a factor of safety of 8, the value of S_1 for wrought iron will be 5500.

Strength of Dished Cylinder Heads.

The *American Machinist*, in Question 183, April 12, 1894, gives the following formula for dished circular heads, firmly fixed to the flanges of the cylinder:

$$t = \frac{P \times (R^2 + d^2)}{4 \times S_1 \times d}$$

$t =$ Thickness of cylinder head in inches.
$R =$ Radius of cylinder head in inches.
$P =$ Pressure in pounds per square inch.
$d =$ Depth in inches of dishing of the head at its center.
$S_1 =$ Allowable working stress in the material, which may be the same as given above.

This formula was used to calculate the thickness of a cast iron head 44 inches in diameter, dished 7 inches, steam pressure 75 pounds per square inch.

NOTE.—In these examples the radius of the bolt circle should be considered as the radius of the head when calculating the thickness. For diameter of bolts and spacing of bolts see examples under Steam Engine.

The above formula may also be used for dished cylinder heads of wrought iron or steel, by allowing the proper value for S_1. For soft steel S_1 may be 9000 to 12,000 pounds, and for wrought iron 5000 to 8000 pounds.

CAUTION.—Cast iron is not a desirable material to use for large unribbed cylinder heads; either flat or dished wrought iron or steel is far superior.

Strength of a Hollow Sphere Exposed to Internal Pressure.

The pressure acts on a surface equal to $\frac{d^2 \pi}{4}$ and it is resisted by a metal area equal to $\frac{D + d}{2} \times \pi \times t$.

$D =$ External diameter.
$d =$ Internal diameter.
$t =$ Thickness of metal.

When the difference between inside and outside diameter is small it need not be considered in practice, and the formula will be:

$$P \times \frac{d^2 \pi}{4} = d \times \pi \times t \times S_1$$

which reduces to

$$P = \frac{4 \times t \times S_1}{d}$$

$$t = \frac{P \times d}{4 S_1}$$

$S_1 =$ Allowable tensile stress in the material.

NOTE.—This formula only allows for tensile strength; if it is used for calculating the thickness of the body of a globe valve or anything similar a liberal amount of metal must be added, in order to obtain good results when casting.

Strength of Chains.

The following table gives approximately the weight of wrought iron chains, in pounds per foot and kilograms per meter; and also their strength, with six as factor of safety. Chains ought to be tested with twice the load given in the table. Never lose sight of the fact that a chain in use will wear and consequently become reduced in strength; also, that a chain is no stronger than its weakest link.

Diameter of Links in Inches.	Load in Pounds.	Weight in Pounds Per Foot.	Load in Kilograms.	Weight in Kilograms per Meter.
3/16	280	0.42	125	0.625
¼	500	0.91	225	1.35
⅜	1125	1.5	510	2.22
½	2000	2.5	900	3.72
¾	4500	5.8	3050	8.63
1	8000	10	3600	14.88

Strength of Iron Wire Rope.

The following table gives approximately the strength of iron wire rope, with six as a factor of safety.

Diameter in Inches.	Load in Pounds.	Load in Kilograms.
½	1000	453
⅝	2500	1134
¾	3500	1588

Wire ropes should not be bent over pulleys of very small diameter. When used for hoisting, the diameter of pulley ought at least to be 40 times the diameter of rope. For further information on wire rope see manufacturers' catalogues.

Strength of Manila Rope.

The size of manila rope is measured by the circumference, therefore so-called three-inch rope is about one inch in diameter. New manila rope of three inches circumference will usually break for a load of 7,000 to 9,000 pounds. For common use such ropes may be loaded as given in the following table, and the diameter of the pulley ought to be at least eight times the diameter of the rope.

Size of Rope.	Safe Load in Pounds.	Safe Load in Kilograms.
3 ins. circumference,	500	227
4 " "	800	363
5 " "	1300	590

CRUSHING STRENGTH.

Short posts having square ends well fitted may be considered to give away under pure crushing stress. Their strength is in proportion to their area; therefore, when the length of a post does not exceed four to five times its diameter or smallest side, its strength or size may be calculated by the following formulas:

$$P = \frac{A \times S}{f} \qquad \text{Side of a square post} = \sqrt{\frac{P \times f}{S}}$$

$$A = \frac{P \times f}{S} \qquad \text{Diameter of a round post} = \sqrt{\frac{P \times f}{0.7854\, S}}$$

P = Safe load in pounds to be supported by the post.

A = Area of post in square inches.

S = Ultimate crushing strength of the material in pounds per square inch, given in Table No. 26.

f = Factor of safety.

TABLE No. 26.—Modulus of Elasticity and Ultimate Crushing Strength of Various Materials.

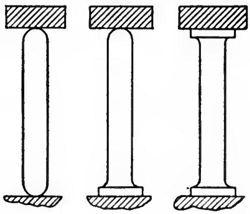
MATERIALS.	Modulus of Elasticity in Pounds per Square Inch.	Ultimate Crushing Strength in Pounds per Square Inch	Modulus of Elasticity in Kilograms per Square Centimeter.	Ultimate Crushing Strength in Kilograms per Sq. Centimeter
Cast steel	30,000,000	150,000	2,200,000	10,500
Bessemer steel . .	28,000,000	50,000	1,970,000	3,500
Wrought iron .	25,000,000	45,000 to 50,000	1,700,000	3,000 to 3,500
Cast iron . . .	12,000,000 to 15,000,000	90,000	800,000 to 1,000,000	6,300
Oak (endwise)	1,500,000	9,000	105,000	630
Pitch pine "	1,600,000	9,000	112,000	630
Pine "	1,100,000	6,000	77,000	420
Spruce "	11,000,000	6,000	77,000	420
Brick	. . . . .	800 to 2,000	. . . .	56 to 140
Brick work laid in 1 part cement and 3 parts sand	. . . . .	600	. . . .	42
Brick work laid in lime and sand	. . . . .	240	. . . .	16 to 17
Granite	. . . . .	10,000	. . . .	700

When a post or column is long compared to its diameter, its strength will decrease as the length is increased. Anyone will, from every-day observation, know that a short post will support with perfect safety a load which will break a long one.

Short columns break under crushing, but long ones break under comparatively light load by the combined effect due to both crushing and flexure. It is, therefore, evident that the strength of long columns follows laws very different from those which apply to short ones.

The form of the ends has also great influence on the strength of a column when under crushing and deflective stress. (See Fig. 1).

FIG. 1.



When both ends are round the column has least strength; if one end is round and one end flat it is stronger, but if both ends are flat and square with the center-line, it is strongest. The proportions are approximately as 1, 2 and 3.

Eccentric loading on columns will also have a very destructive effect upon their strength.

Theoretical calculations regarding the strength of columns and posts are difficult, and such empirical formulas as the well-known Hodgkingson's or Gordon's formulas are usually resorted to.

The Hodgkingson formulas for long columns having square ends well fitted are:

$$P = 99{,}000 \times \frac{D^{3.55}}{L^{1.7}}$$ for solid cast iron columns.

$$P = 99{,}000 \times \frac{D^{3.55} - d^{3.55}}{L^{1.7}}$$ for hollow cast iron columns.

$$P = 285{,}000 \times \frac{D^{3.55}}{L^2}$$ for solid wrought iron columns.

$P =$ Breaking load in pounds.
$D =$ External diameter in inches.
$d =$ Internal diameter in inches.
$L =$ Length in feet.

When the breaking load as calculated by these formulas exceeds one quarter of the crushing load of a short column of the same metal area, the result must be corrected by the formula;

$$P_1 = \frac{P \times C}{P + \frac{3}{4} C \times A}$$

$P_1 =$ Corrected breaking load of column.
$C =$ Crushing strength of material (see Table No. 26).
$A =$ Metal area of column in square inches.

IMPORTANT.— Applying the last formula, the result, P_1, must always be smaller than P.

Table No. 27 was calculated by the following formulas:

Column I. $$\text{Safe Load} = 0.1 \times \left\{ \frac{36000}{1 + (\frac{L^2}{D^2} \times 0.00025)} \right\}$$

Column II. $$\text{Safe Load} = 0.1 \times \left\{ \frac{36000 \times 0.07031}{1 + (\frac{L^2}{D^2} \times 0.00025)} \right\}$$

Column III. $$\text{Safe Load} = 0.1 \times \left\{ \frac{36000}{1 + (\frac{L^2}{D^2} \times 0.0004)} \right\}$$

Column IV. $$\text{Safe Load} = 0.1 \times \left\{ \frac{36000 \times 0.07031}{1 + (\frac{L^2}{D^2} \times 0.0004)} \right\}$$

Column V. $$\text{Safe Load} = 0.1 \times \left\{ \frac{80000}{1 + (\frac{L^2}{D^2} \times 0.0025)} \right\}$$

Column VI. $$\text{Safe Load} = 0.1 \times \left\{ \frac{80000 \times 0.07031}{1 + (\frac{L^2}{D^2} \times 0.0025)} \right\}$$

Column VII. $$\text{Safe Load} = 0.1 \times \left\{ \frac{80000}{1 + (\frac{L^2}{D^2} \times 0.0035)} \right\}$$

Column VIII. $$\text{Safe Load} = 0.1 \times \left\{ \frac{80000 \times 0.07031}{1 + (\frac{L^2}{D^2} \times 0.0035)} \right\}$$

Column IX. $$\text{Safe Load} = 0.1 \times \left\{ \frac{5000}{1 + (\frac{L^2}{D^2} \times 0.004)} \right\}$$

Column X. $$\text{Safe Load} = 0.1 \times \left\{ \frac{5000 \times 0.07031}{1 + (\frac{L^2}{D^2} \times 0.004)} \right\}$$

TABLE No. 27.—Safe Load on Pillars Having Square Ends Well Fitted.

(10 is used as Factor of Safety).

Length divided by diameter or smaller side.	Wrought Iron. Hollow Pillar. Pounds per Square Inch.	Wrought Iron. Hollow Pillar. Kilograms per Square Centimeter.	Wrought Iron. Solid Pillar. Pounds per Square Inch.	Wrought Iron. Solid Pillar. Kilograms per Square Centimeter.	Cast Iron. Hollow Pillar. Pounds per Square Inch.	Cast Iron. Hollow Pillar. Kilograms per Square Centimeter.	Cast Iron. Solid Pillar. Pounds per Square Inch.	Cast Iron. Solid Pillar. Kilograms per Square Centimeter.	Wood. (Spruce or white Pine) Pounds per Square Inch.	Wood. (Spruce or white Pine) Kilograms per Square Centimeter.	Length divided by diameter or smallest side.
$\frac{L}{D}$	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	$\frac{L}{D}$
4	3585	252	3567	252	7692	540	7624	536	476	34	4
6	3567	251	3549	250	7339	512	7105	500	438	30	6
8	3545	250	3510	247	6896	484	6536	460	398	28	8
10	3512	247	3462	243	6400	450	5926	417	357	25	10
12	3475	244	3404	239	5882	413	5314	374	317	22	12
14	3432	241	3338	235	5369	371	4745	333	280	20	14
16	3383	238	3266	230	4878	343	4226	297	248	17	16
18	3332	234	3187	224	4420	311	3749	264	218	15	18
20	3273	230	3103	218	4000	281	3333	234	192	13	20
22	3211	226	3018	212	3620	255	2969	208	170	12	22
24	3147	221	2926	206	3279	230	2653	186	151	10.6	24
26	3080	216	2834	199	2974	211	2376	167	135	9.5	26
28	3010	211	2741	192	2703	190	2137	150	121	8.5	28
30	2938	206	2647	186	2462	173	1928	136	109	7.6	30
32	2866	201	2554	180	2247	158	1745	123	98	6.9	32
34	2793	196	2462	173	2056	145	1585	111	88	6.2	34
36	2719	191	2370	167	1887	133	1440	102	81	5.7	36
38	2645	186	2288	160	1735	122	1321	93	74	5.2	38
40	2571	181	2195	154	1600	112	1212	85	67	4.7	40
42	2498	176	2111	148	1479	104	1113	78	62	4.3	42
44	2426	171	2029	143	1370	96	1028	74	57	4	44
46	2354	166	1950	137	1272	89	952	67	53	3.8	46
48	2284	161	1874	132	1183	83	882	62	49	3.4	48
50	2215	155	1800	127	1103	78	820	58	45	3.2	50

This table is intended, in connection with Table No. 24, to facilitate calculations for pillars of either wood or iron, and may be used with equal advantage for English or metric measures, provided both diameter and length are taken by the same system.

For a round pillar divide the length by the diameter, but for a square or rectangular pillar divide the length by the smallest side. Find the quotient in the column headed $\frac{L}{D}$ and find the safe load per square unit of area in the corresponding column of the table. Multiply this by the metal area of the pillar, and the product is the safe load, with 10 as factor of safety, on a pillar well fitted and having square ends. For any other kind of ends and any other unit of safety, allowance must be made as explained on previous pages.

EXAMPLE 1.

Find the safe load in pounds, according to Table No. 27, for a round, hollow, cast-iron pillar five feet long, five inches outside and four inches inside diameter, having square ends well fitted and being evenly loaded.

Solution:

Five feet equals 60 inches, and 60 divided by 5 gives 12. In the first column, under the heading "Length divided by diameter, or smallest side," is 12, and in that line, in the column headed "pounds per square inch" for hollow cast-iron pillars, is 5882. The metal area of this pillar is obtained by subtracting the area of a circle four inches in diameter from the area of a circle five inches in diameter (see area of circles, page 196; Table, page 209), which is $19.63 - 12.57 = 7.06$, or practically seven square inches, and seven times 5882 equals 41,174 pounds.

EXAMPLE 2.

Find the safe load in kilograms, according to Table No. 27, for a round spruce post 2 meters long and 20 centimeters in diameter.

Solution:

Two meters = 200 centimeters and $\frac{200}{20} = 10$. The corresponding constant in the table is 25 kilograms. The area of a circle 20 centimeters in diameter is 314.2 square centimeters, and 25 times $314.2 = 7855$ kilograms, as safe load.

EXAMPLE 3.

What would be the safe load on the same post if it had been 20 centimeters square, instead of round?

Solution:

The length is the same; therefore the length divided by the side gives 10, as before, and the corresponding constant is 25 kilograms, but as the cross-sectional area in square centimeters is $20 \times 20 = 400$, the corresponding load will be $400 \times 25 = 10{,}000$ kilograms as the safe load.

NOTE.—It will be noticed that in figuring the strength of pillars according to this table, the strength of a square pillar will always be to the strength of a round pillar as 1 to 0.7854, while theoretically the strength of a square pillar compared to that of a round pillar will vary with the length, the extremes being 1 to 0.589 for extremely long pillars and 1 to 0.7854 for very short ones. This discrepancy is frequently unimportant in practical work, because pillars are usually comparatively short, and also because a high factor of safety is always used, but it is well to remember and provide for this fact in cases of very long pillars.

Hollow Cast-Iron Pillars.

By referring to the formulas and considering the laws governing the strength of pillars, it is seen that the strength of pillars increases very fast by increasing their diameter or their sides. In cast-iron pillars this is taken advantage of by making them large in diameter and coring out the stock on the inside.

The thickness of the metal may be about $\frac{1}{12}$ of the diameter of the pillar. In small pillars it must be thicker in order to obtain good results when casting. A flange is cast on each end to form enough bearing surface, and the pillar is squared off very carefully so that both ends are square with the center-line. This is an important point, as the strength is enormously destroyed by squaring the ends carelessly and thereby bringing the load to act corner-ways on the pillar.

Table No. 28 was calculated by the formula:

$$\text{Safe Load} = 0.1 \times \left\{ \frac{80000 \times \text{metal area}}{1 + \left(\frac{L^2}{D^2} \times 0.0025 \right)} \right\}$$

and the result obtained reduced to long tons (2240 pounds). Ten is thus used as a factor of safety; both ends of the pillar are supposed to be square and evenly loaded. For other shapes of ends, mode of loading, or other factors of safety, proportional allowance must be made. For instance, if 15 is required as factor of safety, allow only two-thirds of the load given in the table.

If the pillar has only one square end and one round end, allow only two-thirds as much load. If it has both ends rounded, or, which is the same, if the ends have only a very imperfect bearing, allow only one-third as much load.

Weight of Cast-Iron Pillars.

The weight of a cast-iron pillar may be calculated by the formula:

$$W = (D^2 - d^2) \times L \times 2.45$$

W = Weight of pillar in pounds.

D = Outside diameter in inches.

d = Inside diameter in inches.

L = Length of pillar in feet.

The weight given in Table No. 28 was calculated by this formula, and the length taken as one foot.

TABLE No. 28.—Safe Load on Round Cast-Iron Pillars.

External diameter in inches.	Thickness of Metal in inches	Metal Area in Sq. inches.	Weight in Pounds per Foot in Length.	Length of Pillars in Feet. 6 Feet.	8 Feet.	10 Feet.	12 Feet.	14 Feet.	16 Feet.	18 Feet.	20 Feet.	22 Feet.	24 Feet.
4	½	5.49	17.14	11	8.1	6.1							
4	¾	7.56	23.90	15.2	11.3	8.5							
5	½	7.07	22.06	16.8	13.3	10.4	8.3						
5	¾	10.01	31.23	24	19	15	12						
6	½	8.64	26.95	23	19	15.5	12.6	10.4					
6	¾	12.37	38.59	33	27	22	18	15					
6	⅞	14.09	43.96	37	31	25	21	17					
6	1	15.71	49.01	42	35	28	23	19					
6	1⅛	17.23	53.76	47	40	32	26	22					
7	⅝	12.52	39.06	36	31	26	22	19	16				
7	¾	14.73	45.96	42	36	31	26	22	19				
7	⅞	16.84	52.54	48	41	35	29	25	21				
7	1	18.85	58.90	54	46	39	33	29	24				
7	1⅛	20.76	64.77	60	52	44	37	32	27				
8	¾	17-08	53.29	51	45	39	34	29	25	22			
8	⅞	19.59	61.12	59	52	45	39	34	29	25			
8	1	21.99	68.64	66	58	51	44	38	33	28			
8	1⅛	24.30	75.82	73	64	56	48	42	36	31			
8	1¼	26.51	82.71	79	70	61	52	45	39	34			
8	1⅜	28.62	89.29	86	76	66	57	48	42	37			
9	¾	19.44	60.65	60	54	49	43	37	33	29	24		
9	⅞	22.33	69.67	69	63	56	49	43	38	33	29		
9	1	22.13	78.40	78	71	63	55	48	42	37	33		
9	1⅛	27.83	86.83	87	78	69	62	53	47	41	36		
9	1¼	30.43	94.94	95	85	76	67	58	51	45	39		
9	1⅜	32.94	102.77	102	92	82	72	63	55	48	43		
9	1½	35.34	110.26	110	99	88	78	68	59	52	46		
9	1¾	39.86	124.36	126	113	100	90	78	67	60	51		
10	⅞	25.09	78.28	80	73	67	60	53	47	42	37	34	
10	1	28.28	88.23	90	83	75	67	60	53	47	42	38	
10	1⅛	31.37	97.87	100	92	83	74	66	58	52	47	42	
10	1¼	34.37	107.23	110	101	91	82	73	64	57	51	47	
10	1⅜	37.26	116.25	119	109	98	88	79	69	62	55	51	
10	1½	40.06	124.99	128	117	106	95	85	75	67	59	54	
10	1¾	45.36	141.52	146	133	122	109	97	85	77	67	60	
11	1	31.42	98.04	102	95	87	79	71	64	58	52	48	43
11	1⅛	34.90	108.89	114	105	96	88	79	71	64	58	53	48
11	1¼	38.29	119.46	125	116	106	97	87	78	70	63	58	52
11	1⅜	41.58	129.73	135	126	115	105	94	85	76	68	62	56
11	1½	44.77	139.68	146	136	124	113	102	92	82	74	68	61
11	1¾	50.86	158.68	166	156	142	129	178	106	94	86	79	71
11	2	56.55	176.44	186	176	160	147	134	120	106	98	90	81

TABLE No. 28.—(Continued).

External diameter in inches	Thickness of Metal in inches	Metal Area in Sq. inches.	Weight in Pounds per Foot of Length.	Length of Pillars in Feet.									
				6 Feet.	8 Feet.	10 Feet.	12 Feet.	14 Feet.	16 Feet.	18 Feet.	20 Feet.	22 Feet.	24 Feet.
12	1	34.46	107.51	115	107	99	92	83	76	68	62	58	53
12	1⅛	38.34	119.62	128	119	108	102	92	84	78	69	63	58
12	1¼	42.12	131.41	141	131	119	112	101	93	84	76	70	64
12	1⅜	45.80	142.90	153	142	129	121	110	101	91	82	75	69
12	1½	49.39	154.10	165	154	139	131	119	109	99	89	82	75
12	1¾	56.26	175.53	189	178	159	150	137	125	115	103	94	85
12	2	62.74	195.75	213	201	179	170	155	141	131	117	106	96
13	1	37.97	117.53	127	119	111	104	97	87	79	73	67	61
13	1⅛	41.94	130.85	142	134	126	117	109	101	91	82	75	69
13	1¼	46.11	143.86	158	149	140	130	121	112	101	91	84	77
13	1⅜	50.19	156.59	174	163	154	144	133	122	111	101	93	84
13	1½	54.16	168.98	190	178	168	157	145	133	121	110	101	92
13	1¾	61.82	192.88	214	201	189	176	164	151	137	124	114	104
13	2	69.09	215.56	227	224	210	195	182	168	152	137	126	116
14	1	40.94	127.60	138	131	123	115	109	101	92	85	78	72
14	1⅛	45.50	141.96	153	145	139	130	121	112	103	94	87	80
14	1¼	50.07	156.31	168	160	153	143	133	123	113	104	95	88
14	1⅜	54.54	170.04	183	174	167	156	145	134	123	113	104	95
14	1½	58.78	183.67	198	189	180	168	156	145	133	122	112	103
14	1¾	67.31	210.00	226	216	206	192	179	166	152	140	128	118
14	2	75.36	235.12	254	242	232	216	201	186	171	157	143	133
15	1	43.99	137.28	150	143	136	128	120	112	104	96	90	82
15	1⅛	49.04	153.19	167	159	152	143	136	125	116	108	100	93
15	1¼	54.00	168.48	184	175	167	157	148	138	128	119	110	102
15	1⅜	58.85	183.77	201	191	182	172	161	151	140	130	120	111
15	1½	63.62	198.74	217	207	197	186	175	163	151	141	130	120
15	1¾	72.85	227.45	284	236	225	212	202	190	175	160	148	137
15	2	81.68	254.81	279	265	253	237	229	216	197	179	166	154
16	1	47.12	146.95	160	156	150	140	133	125	117	110	101	93
16	1⅛	52.57	164.11	179	173	167	158	148	139	130	122	116	106
16	1¼	57.92	180.65	198	191	185	174	163	153	143	135	128	117
16	1⅜	63.18	197.18	216	209	202	190	178	167	152	147	139	128
16	1½	68.33	213.10	234	227	220	206	193	181	170	160	151	138
16	1¾	78.34	244.29	268	258	250	237	224	207	199	183	173	158
16	2	87.96	274.56	301	290	281	267	255	233	218	206	194	177

The length of cast-iron pillars, as a rule, ought not to exceed 20 to 25 times their diameter. Cast-iron pillars, when heavily loaded, are apt to be broken if struck by a blow sidewise.

Wrought Iron Pillars.

In important work, cast-iron pillars are rapidly going out of use. Wrought iron pillars are now made which compare favorably in price and are far more reliable than those of cast iron. For full information regarding weight and strength of wrought iron pillars and **Z** bar columns, see manufacturers' catalogues.

Wooden Posts.

Table No. 29 was calculated by the formula:

$$\text{Safe load} = 0.1 \times \left\{ \frac{5000 \times \text{Area}}{1 + \left(\frac{L^2}{D^2} \times 0.004 \right)} \right\}$$

and the result divided by 2240.

$L =$ Length of post in inches.

$D =$ Side of post in inches.

TABLE No. 29.—Safe Load in Tons on Square Pine or Spruce Posts Having Square Ends Well Fitted.

(10 as Factor of Safety).

Length in feet.	Side of Post in Inches.										
	1 in.	2 in.	3 in.	4 in.	5 in.	6 in.	7 in.	8 in.	9 in.	10 in.	12 in.
1	0.14	0.78	1.89	3.45	5.00	8.00	10.00	12.50	16.00	20.00	30.00
2	0.06	0.56	1.59	3.12	5.00	8.00	10.00	12.50	16.00	20.00	30.00
3	0.03	0.39	1.27	2.70	4.62	7.00	10.00	12.50	16.00	20.00	30.00
4	. .	0.32	0.99	2.27	4.08	6.38	9.21	12.50	16.00	20.00	30.00
5	. .	0.19	0.77	1.88	3.54	5.74	8.45	11.21	15.35	19.51	29.22
6	. .	0.14	0.60	1.56	3.05	5.00	7.68	10.80	14.35	18.44	28.10
7	. .	. .	0.49	1.29	2.62	4.50	6.94	9.91	13.41	17.40	26.87
8	. .	. .	0.39	1.08	2.25	4.00	6.24	9.08	12.44	16.32	25.50
10	. .	. .	0.27	0.78	1.69	3.09	5.03	7.53	10.58	14.18	22.95
12	. .	. .	. .	0.58	1.29	2.43	4.06	6.25	8.96	12.22	20.40
14	. .	. .	. .	0.44	1.00	1.94	3.31	5.19	7.57	10.50	18.02
16	. .	. .	. .	. .	0.90	1.58	2.73	4.34	6.43	9.04	15.88
18	. .	. .	. .	. .	0.80	1.30	2.27	3.66	5.49	7.80	14.00
20	. .	. .	. .	. .	. .	1.09	1.92	3.11	4.72	6.76	12.36
22	. .	. .	. .	. .	. .	0.90	1.63	2.66	4.17	5.89	10.95
24	. .	. .	. .	. .	. .	. .	1.41	2.31	3.55	5.17	9.73
26	. .	. .	. .	. .	. .	. .	1.22	2.01	3.11	4.56	8.68
28	. .	. .	. .	. .	. .	. .	. .	1.78	2.68	4.05	7.77
30	. .	. .	. .	. .	. .	. .	. .	1.56	2.38	3.56	6.99
32	. .	. .	. .	. .	. .	. .	. .	. .	2.18	3.23	6.30
34	. .	. .	. .	. .	. .	. .	. .	. .	1.96	2.91	5.71
36	. .	. .	. .	. .	. .	. .	. .	. .	. .	2.64	5.18
38	. .	. .	. .	. .	. .	. .	. .	. .	. .	2.39	4.74
40	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	4.35

The preceding Table gives the safe load in long tons corresponding to a square post of the dimensions of sides given at top of the columns, and lengths given in the first column. For round posts the load should be 0.75 to 0.6 of the given load depending upon the length of post.

EXAMPLE.

What size of post is required, with 10 as factor of safety, to support a load of five tons, when the length of the post is 16 feet?

Solution:

In the column headed "Length of post in feet" find 16, and in line with 16 find the numbers nearest to five tons, which are 4.34 and 6.43. Thus, a post 16 feet long and 8 inches square will support 4.34 tons, and a post 16 feet long and 9 inches square will support 6.43 tons. It is, therefore, best to select a post 9 inches square.

To Calculate the Strength of Rectangular Posts from the Table.

Find, in the Table, the strength of the post according to its smallest side, and increase the tabular value in proportion to the largest side of the post.

EXAMPLE.

What is the strength, with 10 as factor of safety, of a spruce post 10 feet long, 6 inches thick, 8½ inches wide, with square ends well fitted, calculated by Table No. 29.

Solution:

In the Table we find the strength of a post 10 feet long and 6 inches square to be 3.09 tons. Therefore, when the pillar is 6 inches thick and 8½ inches wide its corresponding strength will be $3.09 \times \frac{8\frac{1}{2}}{6} = 4.38$ tons.

It is a waste of material to use a post of rectangular cross-section. For example, this post is $6 \times 8\frac{1}{2}$ inches $= 51$ square inches of cross-section and will support 4.38 tons, but a post of the same length and 7×7 inches $= 49$ square inches of cross-section, will support 5.03 tons. (See Table No. 29).

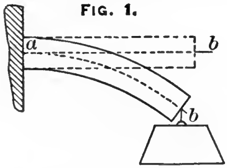
To Obtain the Weight of Pillars in Kilograms per Meter when the Weight in Pounds per Foot is Known.

Multiply the weight in pounds per foot by the constant 1.4882, and the product is the weight in kilograms per meter.

TRANSVERSE STRENGTH.

A beam placed in a horizontal position, fastened at one end and loaded at the other, is exposed to transverse stress, and will usually bend more or less, as shown (exaggerated) in Fig. 1, before it will break. The line *a b* is called the neutral line, and all fibres above the neutral line are exposed to tensile stress, and all fibers below are exposed to crushing stress, but the neutral fiber is neither stretched nor compressed. A line drawn in a horizontal direction, at right angles to, and through the neutral line, is called the *neutral axis* with reference to this particular place of the section of the beam. The neutral axis is considered to pass through the center of gravity of the section, which, for beams of round, square or rectangular section, is always in the geometrical center. Therefore, all beams of such section will have an equal amount of material on the upper and under side of the neutral axis, but it is not always desirable for all materials or for all kinds of load to have an equal amount of material on both the side exposed to compression and that exposed to tension. For instance, cast-iron beams are usually made in I formed section and should always be laid so that the largest web is exposed to tensile stress, because cast-iron offers much more resistance to compression than it does to tension. Cast-iron beams of such section ought, therefore, to be laid in this position (I), if fastened at one end and loaded at the other, but should be laid in this position (I), if they are supported under both ends and loaded between the supports. If this is taken into consideration in placing a cast-iron beam, its ultimate transverse breaking strength is greatly increased, but under a moderate load the deflection will be practically equal in either position, because as long as the load is small, well within the elastic limit, cast-iron will stretch under tensile stress as much as it will compress under an equal amount of crushing stress; therefore, the modulus of elasticity for tension and compression of cast-iron is considered to be equal, but under increased crushing load the compression becomes less in proportion to the load until the point is reached when the cast-iron can not compress more, and the casting will break. The ultimate crushing strength of cast-iron is five to six times as much as its ultimate tensile strength.

FIG. 1.



A beam supported under both ends and loaded in the middle will carry four times as great a load as another beam of the same size and material fixed at one end and loaded at the other. This may be understood by referring to Fig. 2, as when the beam is one foot long and loaded with 100 pounds in the middle, each half of the beam supports only 50 pounds, and this 50 pounds acts only upon an arm ½ foot long, consequently it exerts no more force toward breaking this beam than the 25 pounds would upon the end of the other beam one foot long.

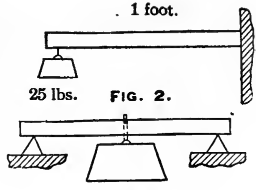


FIG. 2.

A beam twice as wide as another and of the same length, thickness, and material, will carry twice the load, because the wide beam could, of course, be split into two equal beams; consequently it must, as a whole beam, have twice the strength of another one of the same material but of only half the width.

A beam twice as long as another will break under half the load. This is seen by referring to Fig. 3, because 50 pounds on an arm two feet long will balance 100 pounds on an arm one foot long.

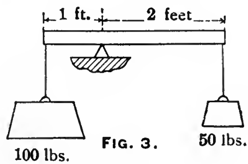
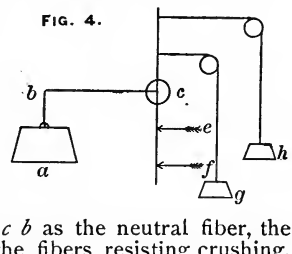


FIG. 3.

A beam twice as thick as another, of the same material, length and width, will carry four times the load. (See Fig. 4).

Suppose the weight *a* is acting on the arm *b*, tending to swing it around the center *c*, and this action being counteracted by the weights *g* and *h*, also by the arrows *e* and *f*. If the weight *h* is taking hold twice as far from the center as the weight *g*, it will offer twice the resistance against swinging the beam that *g* will; and exactly the same with the arrows *f* and *e*. Consider the line *c b* as the neutral fiber, the arrows *e* and *f* as representing the fibers resisting crushing, and the weights *g* and *h* as representing the fibers resisting tensile stress. It will be understood that if the fibers are twice as far above or below the neutral fiber they are in a position to offer twice the resistance to the breaking action of the load; but a beam of twice the thickness has not only its average fiber twice as far from the neutral point, but it has also twice the area or twice as many fibers, consequently the result must be that it can resist four times the load.

FIG. 4.



For instance: The beam a in Figure 5 is four times as strong as the beam b, if placed on the edge, as shown in the figure, and loaded on the top; but a would be only twice as strong as b if it was laid on the side and loaded on top.

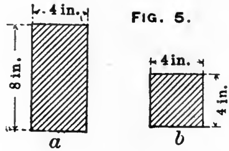


FIG. 5.

Formulas and Rules for Calculating Transverse Strength of Beams.

The fundamental formula for transverse stress in beams is:

BENDING MOMENT = RESISTING MOMENT.

The bending moment for a beam fixed at one end and loaded at the other (see Fig. 1) is obtained by multiplying the load by the horizontal distance from the neutral axis to the point where the load is applied. The distance is taken in inches and the load in pounds.

The resisting moment is obtained by multiplying the moment of inertia by the unit stress, tensile or compressive, upon the fiber most remote from the neutral axis, and dividing the product by the distance from this fiber to the neutral axis.

The theoretical formula for the transverse strength of a beam fastened in a horizontal position at one end and loaded at the extremity of the other end, as shown in Fig. 6, is,

$$P = \frac{S \times I}{L \times a}$$

When the beam is fastened at one end and loaded evenly throughout its whole length, as shown in Fig. 7, the formula will be,

$$P = 2 \times \frac{S \times I}{L \times a}$$

When a beam is placed in a horizontal position and supported under both ends and loaded in the middle (see Fig. 8) the formula is,

$$P = 4 \times \frac{S \times I}{L \times a}$$

When a beam is placed in a horizontal position and supported under both ends and loaded throughout its whole length (see Fig. 9), the formula will be,

$$P = 8 \times \frac{S \times I}{L \times a}$$

When a beam is laid in a horizontal position, fixed at both ends and loaded in the middle between fastenings (see Fig. 10), the formula will be,

$$P = 8 \times \frac{S \times I}{L \times a}$$

When a beam is laid in a horizontal position, fixed at both ends and the load evenly distributed over its whole length (see Fig. 11), the formula will be,

$$P = 12 \times \frac{S \times I}{L \times a}$$

$P =$ Breaking load in pounds.

$S =$ Modulus of rupture, which is 72 times the weight, in pounds, which will break a beam one inch square and one foot long when fixed in a horizontal position, as shown in Fig. 6, and loaded at the extreme end, and which may be taken as follows:

Cast-Iron, 36,000.
Wrought Iron, 50,000.
Spruce and Pine, 9,000.
Pitch Pine, 10,000.

These are the nearest values, in round numbers, of 72 times the average value of the constant given in Table No. 30.

For the safe load, S may be taken as follows:

For timber, 1,000 to 1,200 pounds.
For cast-iron, 3,000 to 5,000 pounds.
For wrought iron, 10,000 to 12,000 pounds.
For steel, 12,000 to 20,000 pounds.

$L =$ Length of beam in inches.

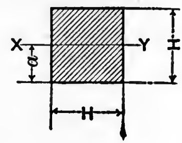
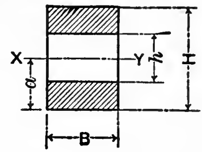
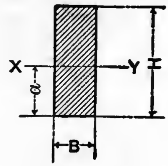
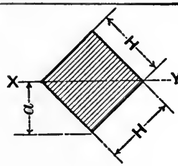
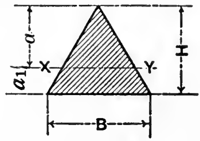
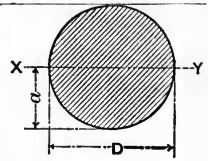
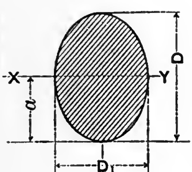
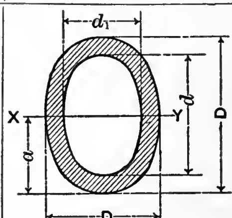
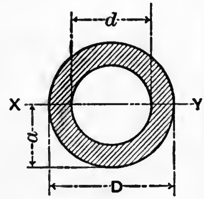
$a =$ The distance in inches from the neutral surface of the section to the most strained fiber.

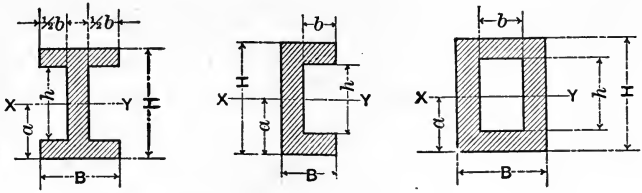
$I =$ Rectangular moment of inertia.

The tables on pages 237 and 238 give the moment of inertia about the neutral axis X Y, and the distance a, for a few of the most common sections:

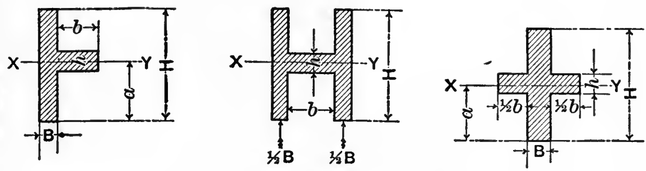
(For explanation of moment of inertia and center of gravity see page 293).

These formulas have the great advantage of being theoretically correct for beams of any shape of cross-section, made from any material, providing the load is within the elastic limit of the beam, and a correct constant is used for S and the correct value obtained for the *moment of inertia.*

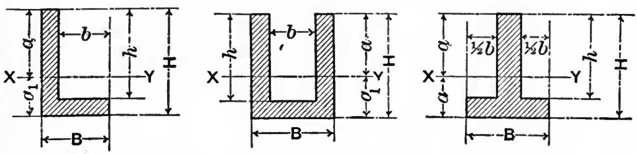
 $I=\frac{H^4}{12}$ $a=\frac{H}{2}$ $\frac{I}{a}=\frac{H^3}{6}$	 $I=\frac{B\times(H^3-h^3)}{12}$ $a=\frac{H}{2}$ $\frac{I}{a}=\frac{B\times(H^3-h^3)}{6\times H}$	 $I=\frac{H^3\times B}{12}$ $a=\frac{H}{2}$ $\frac{I}{a}=\frac{H^2\times B}{6}$
 $I=\frac{H^4}{12}$ $a=\frac{1}{2}H\times\sqrt{2}$ $\frac{I}{a}=0.118\,H^3$	 $I=\frac{B\times H^3}{36}$ $a=\frac{2}{3}H;\ a_1=\frac{1}{3}H$ $\frac{I}{a_1}=\frac{B\times H^2}{12}$ $\frac{I}{a}=\frac{B\times H^2}{24}$	 $I=\frac{D^4\pi}{64}$ $a=\frac{D}{2}$ $\frac{I}{a}=0.0982\times D^3$
 $I=\frac{D_1\,D^3\,\pi}{64}$ $a=\frac{D}{2}$ $\frac{I}{a}=0.0982\,D_1\,D^2$	 $I=\frac{(D_1\,D^3-d_1\,d^3)\,\pi}{64}$ $a=\frac{D}{2}$ $\frac{I}{a}=\frac{(D_1\,D^3-d_1\,d^3)\,\pi}{32\,D}$	 $I=\frac{(D^4-d^4)\,\pi}{64}$ $a=\frac{D}{2}$ $\frac{I}{a}=\frac{(D^4-d^4)\,\pi}{32\,D}$



$$I = \frac{B\ H^3 - b\ h^3}{12} \qquad a = \frac{H}{2} \qquad \frac{I}{a} = \frac{B\ H^3 - b\ h^3}{6\ H}$$



$$I = \frac{B\ H^3 + b\ h^3}{12} \qquad a = \frac{H}{2} \qquad \frac{I}{a} = \frac{B\ H^3 + b\ h^3}{6\ H}$$



$$a_1 = \frac{B\ H^2 - 2\ H\ b\ h + b\ h^2}{2\ (B\ H - b\ h)}$$

$$a = \frac{B\ H^2 - b\ h^2}{2\ (B\ H - b\ h)}$$

$$I = \frac{(B\ H^2 - b\ h^2)^2 - 4\ B\ H\ b\ h\ (H - h)^2}{12\ (B\ H - b\ h)}$$

$$\frac{I}{a_1} = \frac{(B\ H^2 - b\ h^2)^2 - 4\ B\ H\ b\ h\ (H - h)^2}{6\ (B\ H^2 - 2\ H\ b\ h + b\ h^2)}$$

$$\frac{I}{a} = \frac{(B\ H^2 - b\ h^2)^2 - 4\ B\ H\ b\ h\ (H - h)^2}{6\ (B\ H^2 - b\ h^2)}$$

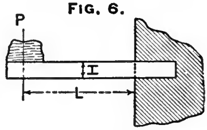
Beams of symmetrical section, as square, round, elliptical, or **H** section, may be calculated on theoretically correct principles in a simpler way, obviating the use of the moment of inertia and the modulus of rupture, as explained below.

For a beam fixed at one end and loaded at the other,

$$P = \frac{C \times H^2 \times B}{L}$$

$$L = \frac{C \times H^2 \times B}{P}$$

FIG. 6.



$$H = \sqrt{\frac{P \times L}{C \times B}}.$$

$$B = \frac{P \times L}{C \times H^2}$$

$$C = \frac{H^2 \times B}{P \times L}$$

When beam is square,

$$Side = \sqrt[3]{\frac{P \times L}{C}}$$

When beam is round,

$$Diameter = \sqrt[3]{\frac{P \times L}{C \times 0.589}}$$

$P =$ Breaking load in pounds.

$H =$ Thickness or height of beam in inches.

$B =$ Width of beam in inches.

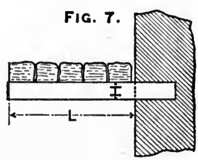
$L =$ Length of beam in feet.

$C =$ Constant which is obtained from experiments, and is the weight in pounds which will break a beam 1 foot long and 1 inch square fixed at one end and loaded at the other. Constant C is given in Table No. 30.

A rectangular beam fixed at one end and loaded evenly throughout its whole length will carry twice the load of a beam fixed at one end and loaded at the other; therefore,

$$P = \frac{2 \times C \times H^2 \times B}{L}$$

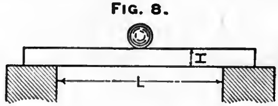
FIG. 7.



For a rectangular beam supported under both ends and loaded at the center,

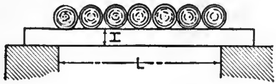
$$P = \frac{4 \times C \times H^2 \times B}{L}$$

FIG. 8.



A rectangular beam supported under both ends and loaded evenly throughout its whole length will carry twice the load of a beam supported under both ends and loaded at the center; therefore,

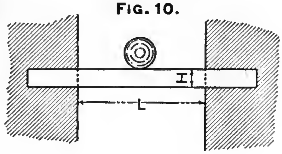
FIG. 9.



$$P = \frac{8 \times C \times H^2 \times B}{L}$$

For a beam fixed at both ends and loaded at the center,

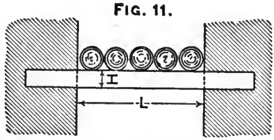
FIG. 10.



$$P = \frac{8 \times C \times H^2 \times B}{L}$$

For a beam fixed at both ends and the load distributed evenly throughout its whole length,

FIG. 11.



$$P = \frac{12 \times C \times H^2 \times B}{L}$$

Each letter in these formulas has the same meaning as in formula for Fig. 6, page 239, and each formula may be transposed the same as that formula. The most convenient way is, in each case, to multiply the numerical value of C from Table No. 30, by its proper coefficient according to mode of loading, before it is inserted in the formula.

NOTE.—A square beam laid in this ■ position has 40 % more transverse strength than the same beam laid in this ◆ position.

TABLE No. 30. — Constant C.

Giving the weight in pounds which will break a beam one foot long and one inch square which is fastened at one end, in a horizontal position, and loaded at the other end.

Material.	Very Good.	Medium.	Poor.
Wrought iron,*	750	600	500
Cast-iron,	650	500	400
Spruce and Pine,	160	125	90
Pitch pine,	225	150	100
Granite,		25	

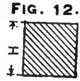
* A wrought iron beam or bar will not actually break under these conditions, but, as it will bend so much that it becomes useless, it is considered to be equivalent to the breaking point.

The following formulas will apply to the strength of beams of the shape shown in the adjacent sectional cuts. These formulas pertain only to the ultimate breaking strength of beams, and have nothing to do with deflection, which follows entirely different laws.

SOLID SQUARE BEAMS.

$$P = \frac{C \times H^3}{L} \qquad C = \frac{P \times L}{H^3}$$

FIG. 12.

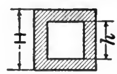


$$L = \frac{C \times H^3}{P} \qquad H = \sqrt[3]{\frac{P \times L}{C}}$$

HOLLOW SQUARE BEAMS.

FIG. 13.

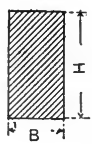
$$P = \frac{C \times (H^4 - h^4)}{L \times H}$$



SOLID RECTANGULAR BEAMS.

FIG. 14

$$P = \frac{C \times B \times H^2}{L} \qquad B = \frac{P \times L}{C \times H^2}$$



$$L = \frac{C \times B \times H^2}{P} \qquad C = \frac{P \times L}{B \times H^2}$$

$$H = \sqrt{\frac{P \times L}{C \times B}}$$

HOLLOW RECTANGULAR BEAMS.

$$P = \frac{C \times (B \times H^3 - b \times h^3)}{H \times L}$$

$$L = \frac{C \times (B \times H^3 - b \times h^3)}{P \times H}$$

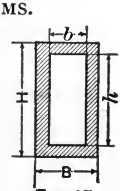


FIG. 15.

SOLID ROUND BEAMS.

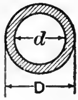
FIG. 16.

$$P = \frac{0.589\ C \times D^3}{L} \qquad D = \sqrt[3]{\frac{P \times L}{0.589\ C}}$$

$$L = \frac{0.589\ C \times D^3}{P}$$

HOLLOW ROUND BEAMS.

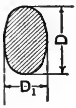
FIG. 17.



$$P = \frac{0.589\ C \times (D^4 - d^4)}{L \times D}$$

SOLID ELLIPTICAL OR OVAL BEAMS.

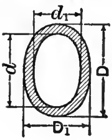
FIG. 18.



$$P = \frac{0.589\ C \times D_1 \times D^2}{L}$$

HOLLOW ELLIPTICAL OR OVAL BEAMS.

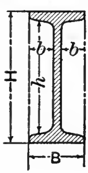
FIG. 19.



$$P = \frac{0.589\ C\,(D_1 \times D^3 - d_1 \times d^3)}{L\,D}$$

I BEAMS.

FIG. 20.



As a general rule, wrought iron I beams should always be selected of such size that their depth is not less than one-twenty-fourth of the span; and their strength may be calculated by the formula:

$$P = \frac{C \times (B \times H^3 - 2b \times h^3)}{L \times H}$$

In the preceding formulas:

P = Breaking load when beam is fastened at one end and loaded at the other.

L = Length of beam in feet.

C = Constant, and is the load in pounds which will break a bar one inch square and one foot long when fastened at one end and loaded at the other, and may be obtained from Table No. 30.

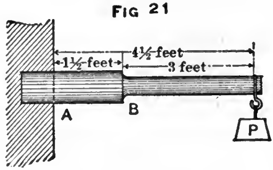
These formulas give the breaking load when the beam is fastened at one end and loaded at the other, but for other fastenings and loads C must be multiplied by either 2, 4, 6, 8, or 12, depending upon conditions. (See pages 239 and 240).

To Find the Transverse Strength of Beams when Their Section is Not Uniform Throughout the Whole Length.

EXAMPLE.

A beam made of wrought iron is fastened at one end and loaded at the other (as shown in Fig. 21). The largest part is 5 inches in diameter and the smallest part is 4 inches in diameter. Where will it break? and what is the breaking load?

FIG 21



NOTE.—Naturally, the beam will break at either A or B; therefore, calculate first the breaking load of a round beam of wrought iron 4½ feet long and 5 inches in diameter, next a beam 3 3 feet long (the distance from P to B).and 4 inches in diameter.

Solving for strength at A:

$$P = \frac{D^3\ 0.6\ C}{L}$$

$$P = \frac{5^3 \times 0.6 \times 600}{4\frac{1}{2}}$$

$$P = 10{,}000 \text{ pounds.}$$

Solving for strength at B:

$$P = \frac{4^3 \times 0.6 \times 600}{3}$$

$$P = \frac{64 \times 360}{3}$$

$$P = 7680 \text{ pounds.}$$

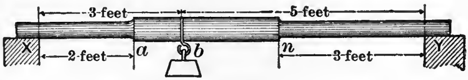
Thus, the weakest point of the beam is at B, where its calculated breaking load is only 7860 pounds, while the calculated breaking load at A is 10,000 pounds.

When a beam is not of uniform section throughout its whole length and is supported under both ends and loaded somewhere between the supports, calculate first the reaction on each support; then consider the beam as if it was fastened by the load, and consider the reaction at each support as a load at the free end of a beam of length and section equal to the length and section between its load and support.

EXAMPLE.

The largest diameter of a round cast-iron shaft is 3 inches and the smallest diameter is 2 inches. The length, mode of loading and support is as shown in Fig. 22. Where will it break? and what is the breaking load?

FIG. 22.



Solution:

The reaction at y will be $\frac{3}{8}$ of the load and the reaction at x will be $\frac{5}{8}$ of the load. The beam will evidently break either at a, b or n. (Find constant C in Table No. 30 and multiply by 0.6, because the beam is round).

Solving for strength at n:

$$\frac{3}{8}\ P = \frac{2^3 \times 500 \times 0.6}{3}$$

$$P = \frac{8 \times 300 \times 8}{3 \times 3}$$

$$P = 2133\tfrac{1}{3} \text{ pounds.}$$

Solving for strength at b:

$$\frac{3}{8}\ P = \frac{3^3 \times 500 \times 0.6}{5}$$

$$P = \frac{27 \times 300 \times 8}{5 \times 3}$$

$$P = 4320 \text{ pounds.}$$

Solving for strength at a:

$$\frac{5}{8}\ P = \frac{2^3 \times 500 \times 0.6}{2}$$

$$P = \frac{8 \times 300 \times 8}{5 \times 2}$$

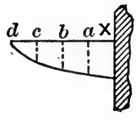
$$P = 1920 \text{ pounds.}$$

Thus, the weakest place in the beam is at a, where it will break when loaded at b with 1920 pounds. If the load is moved nearer n, it will at a certain point exert the same breaking stress on both a and n.

Regular beams of this kind are seldom dealt with, but shafts or spindles of similar shape and loaded in a similar manner are frequently used, and their strength and stiffness may be calculated and their weakest spot ascertained by this way of reasoning, which applies as well to hollow as to solid shafts and spindles made from wrought iron, steel or cast-iron.

Beams fastened at one end and loaded at the other may be reduced in size toward the loaded end and still have the same strength. Suppose the beam to be fastened in the wall at X (Fig. 23) and loaded at the other end with a given load, this load will then have the greatest breaking effect upon the beam at X; at half way between X and d the load has only half the breaking effect, at c only one-quarter the effect. Therefore, the beam may be tapered off toward b in such proportion that the square of the height a is equal to three-quarters the square of the height at X. The square of the thickness at b is one-half the square of the thickness at X, and the square of the thickness at c is one-quarter the square of the height at X.

FIG. 23.



EXAMPLE.

An iron bracket is four feet long, projecting from a wall (as Fig. 23). It is strong enough when 24 inches high at X. How high will it have to be at a, b and c?

Solution:

$X = 24^2 = 576$ Height at $X = 24''$

$a = \sqrt{\frac{3}{4} \times 576} = \sqrt{432}$ Height at $a = 20.78''$

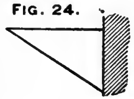
$b = \sqrt{\frac{1}{2} \times 576} = \sqrt{288}$ Height at $b = 16.97''$

$c = \sqrt{\frac{1}{4} \times 576} = \sqrt{144}$ Height at $c = 12''$

The curved boundary line of such a beam is a parabolic curve, because the property of a parabola is that the square of the length of any one of the vertical lines (ordinates) is in proportion as their distance from the extreme point d. By this construction one-third of the material may be saved and the same strength be maintained.

If the load is distributed along the whole length of the bracket instead of at its extreme end, it would have the form shown in Fig. 24.

FIG. 24.



Square and Rectangular Wooden Beams.

The strength increases directly as the width and as the square of the thickness. The strength decreases in the same proportion as the length of the span increases.

EXAMPLE 1.

Find the ultimate breaking load in pounds of a spruce beam 6 inches square and 8 feet long, when supported under both ends and loaded at the center.

Solution:

$$P = \frac{4\ C \times H^3}{L}$$

$$P = \frac{4 \times 125 \times 6 \times 6 \times 6}{8}$$

$$P = 13{,}500 \text{ pounds.}$$

NOTE.—C is obtained from Table No. 30, and is multiplied by 4 because the beam is supported under both ends and loaded at the center. The beam is square; therefore the cube of the thickness is equal to the square of the thickness multiplied by the width. Consequently, for a square beam (thickness)^3 or (width)^3 or square of thickness multiplied by width is the same thing.

EXAMPLE 2.

Find the load which will break a spruce beam 8 inches thick, 4½ inches wide, and 8 feet long, supported under both ends and loaded at the center.

Solution:

$$P = \frac{4\ C \times B \times H^2}{L}$$

$$P = \frac{4 \times 125 \times 4\tfrac{1}{2} \times 8 \times 8}{8}$$

$$P = 18{,}000 \text{ pounds.}$$

EXAMPLE 3.

Find the load which will break the beam mentioned in Example 2, if beam is laid flatwise.

$$P = \frac{4 \times 125 \times 8 \times 4\tfrac{1}{2} \times 4\tfrac{1}{2}}{8}$$

$$P = 10{,}125 \text{ pounds.}$$

In the first example the beam is square, 6″ × 6″ = 36 square inches, and its calculated breaking load is 13,500 pounds. In the second example the beam is rectangular, 8″ × 4½″ = 36 square inches, and laid edgewise its figured breaking load is 18,000 pounds. In the third example the same beam is laid flatwise, and its breaking load is only 10,125 pounds. Thus, by making a beam deep it is possible to secure great strength with only a small quantity of material, but the limit is soon reached where it will not be practical to increase the depth at the expense of the width, because the beam will deflect sidewise and twist and break if it is not prevented by suitable means. The strongest beam which can be cut from a round log is one having the thickness $1\frac{2}{5}$ times the width. The stiffest beam cut from a round log has its thickness $1\frac{7}{10}$ times its width. The best beam for most practical purposes which can be cut from a round log has its thickness $1\frac{1}{2}$ times its width; for instance, 4×6, or 6×9, or 8×12, etc. The largest side in a beam having its thickness $1\frac{1}{2}$ times its width which can be cut from a round log is found by multiplying the diameter by 0.832. The diameter required in a round log to be large enough for such a beam is found by multiplying the largest side of the beam by 1.2; for instance, the diameter of a round log to cut $6'' \times 9''$ will be $9'' \times 1.2 = 10.8$ inches, or the diameter of a round log required to cut $8'' \times 12''$ will be $12'' \times 1.2 = 14.4$ inches, etc.

To Calculate the Size of Beam to Carry a Given Load.

Most frequently the load and the length of span are known and the required size of beam is to be calculated. For a rectangular beam there would then be two unknown quantities, the width and the thickness, but if it is decided to use a beam having its thickness $1\frac{1}{2}$ times its width, the thickness may be expressed in terms of the width.

$H =$ Thickness.

$B =$ Width.

$H = 1\frac{1}{2}\, B$

Use formula for rectangular beams, page 239, and it will read,

$$P = \frac{C \times (1\frac{1}{2}\, B)^2 \times B}{L}$$

This will reduce to,

$$P = \frac{C \times 2\frac{1}{4} \times B^3}{L}$$

This will transpose to,

$$B = \sqrt[3]{\frac{P \times L}{C \times 2\frac{1}{4}}}$$

EXAMPLE.

Find width and thickness of a spruce beam 10 feet long, when fastened at one end and required to carry, with 8 as factor of safety, a load of 1800 pounds at the other end, the thickness to be $1\frac{1}{2}$ times the width.

When the beam is to carry 1800 pounds, with 8 as a factor of safety, its breaking load is $8 \times 1800 = 14{,}400$ pounds.

Solution:

$$B = \sqrt[3]{\frac{14400 \times 10}{2\frac{1}{4} \times 125}}$$

$$B = \sqrt[3]{512}$$

$B = 8$ inches in width.

$H = 1\frac{1}{2} \times 8'' = 12$ inches in thickness.

The weight of the beam itself is not considered in this problem.

To Find the Size of a Beam to Carry a Given Load When Also the Weight of the Beam is to be Considered.

RULE.

Calculate first the size of beam required to carry the load, then figure what such a beam will weigh and add half of this weight to the load, if the beam is fastened at one end and loaded at the other, or supported under both ends and loaded at the center, but add the whole weight of the beam to the weight of the load if the load is distributed along the whole length of the beam. Then figure the size of the required beam for this new load.

EXAMPLE.

Find width and thickness of a pitch pine beam to carry 2000 pounds, with 8 as factor of safety, and a span of 27 feet. The beam is supported under both ends and loaded at the center; its own weight is also to be taken into consideration.

Solution:

Find the constant for pitch pine in Table No. 30 to be 150, and find the weight of pitch pine in Table No. 10 to be 50 pounds per cubic foot. When the beam is supported under both ends and loaded at the center it is four times as strong as if fastened at one end and loaded at the other; therefore, constant 150 is multiplied by 4. The load, 2000 pounds, multiplied by 8 as a factor of safety, gives 16,000 pounds as breaking load of the beam.

$$B = \sqrt[3]{\frac{16000 \times 27}{2\frac{1}{4} \times 150 \times 4}}$$

$$B = \sqrt[3]{320}$$

$B = 6.84'' =$ width, and $1\frac{1}{2} \times 6.84'' = 10.26'' =$ thickness.

The area is $6.84 \times 10.26 = 70$ square inches; the weight per foot is 70 times 50 divided by 144, which equals 24.3 pounds, say 25 pounds. The weight of the beam is $25 \times 27 = 675$ pounds. This weight is distributed along the whole beam and, therefore, it does not have any more effect than if half of it, or 337½ pounds, was placed at the center, but as the beam is to be calculated with 8 as factor of safety, the weight allowed for the beam must be $337\frac{1}{2} \times 8 = 2700$ pounds. Thus, adding this weight to 16,000 pounds gives 18,700 pounds; this new weight is used for calcuing the size of the required beam.

$$B = \sqrt[3]{\frac{18700 \times 27}{2\frac{1}{4} \times 150 \times 4}}$$

$$B = \sqrt[3]{374}$$

$B = 7.2$ inches = width, and $1\frac{1}{2} \times 7.2'' = 10.8$ inches, thickness.

This, of course is also a little too small, as only the weight of a beam 6.84 inches by 10.25 inches is taken into account, but if more exactness should be required the weight of this new beam may be calculated and the whole figured over again, and the result will be closer. This operation may be repeated as many times as is wished, and the result will each time be closer and closer, but never exact; but for all practical purposes one calculation, as shown in this example, is sufficient.

EXAMPLE 2.

Find width and thickness of a spruce beam to carry 4200 pounds distributed along its whole length. The span is 24 feet; use 10 as factor of safety, and also allow for the weight of beam. The thickness of the beam is to be 1½ times its width.

Solution:

$$B = \sqrt[3]{\frac{4200 \times 24 \times 10}{2\frac{1}{4} \times 125 \times 8}}$$

$$B = \sqrt[3]{448}$$

$B = 7.65$ inches, and $H = 11.48$ inches.

$$\text{Weight of beam} = \frac{7.65 \times 11.48 \times 24 \times 32}{144} = 468 \text{ pounds.}$$

Adding ten times the weight of the beam to ten times the weight to be supported, gives 46,680 pounds.

$$B = \sqrt[3]{\frac{46680 \times 24}{2\frac{1}{4} \times 125 \times 8}}$$

$$B = \sqrt[3]{497.9}$$

$B = 7.93$ inches, and $H = 1\frac{1}{2}\ B = 11.9$ inches, or practically, a beam 8 inches by 12 inches is required.

Crushing and Shearing Load of Beams Crosswise on the Fiber.

Too much crushing load must not be allowed at the ends of the beams where they rest on their supports, as all kinds of wood has comparatively low crushing strength when the load is acting crosswise on the fiber.

Approximately, the average ultimate crushing strength of wood, crosswise of the fiber, is as follows :—

White oak, 2000 pounds per square inch.
Pitch pine, 1400 pounds per square inch.
Chestnut, 900 pounds per square inch.
Spruce and pine, 500 to 1000 pounds per square inch.
Hemlock, 500 to 800 pounds per square inch.

The safe load may be from one-tenth to one-fifth of the ultimate crushing load. When the wood is green or water-soaked, its crushing strength is less than is given above.

EXAMPLE.

How much bearing surface must be allowed under each end of the beam mentioned in Example 2, providing it also has 10 as a factor of safety? The crushing strength of spruce crosswise on the fiber is 500 pounds, and using 10 as factor of safety, the load allowed per square inch must be only 50 pounds. The beam is 8 inches wide, and half of 4685 pounds is supported at each end; thus the length of bearing required under each end will be $\frac{2342}{50 \times 8} = 5.85$ inches. Thus, the least bearing allowable should be about 6 inches long.

When beams are heavily loaded and resting on posts, or have supports of small area, either hardwood slabs or cast-iron plates should be placed under their ends, in order to obtain sufficient bearing surface for the soft wood.

The same care must be exercised when a beam is loaded at one point; the bearing surface under the load should at least be as long as the bearing surface of both ends added together.

Short beams are liable to break from shearing at the point of support, especially when loaded throughout their whole length to the limit of their transverse strength.

The ultimate shearing strength for spruce, crosswise of the fiber, is 3000 pounds per square inch (see page 273). Safe load may be 300 pounds per square inch.

In the above example the beam is $8'' \times 12'' = 96$ square inches, and its center load is 4685 pounds, or $2342\frac{1}{2}$ pounds at each end. The shearing stress is $\frac{2342\frac{1}{2}}{96} = 24.6$ pounds per square inch. Hence, the factor of safety against shearing is about 100, and there is not the least danger that this beam will give way under shearing; but such is not always the result.

Round Wooden Beams.

A round beam has 0.589 times the strength of a square beam of same length and material, when the diameter is equal to the side of the square beam. The area of a square beam compared to the area of the round beam is as 0.7854 to 1; therefore it might seem as if that also should be the proportion between their strength, which is the case for tensile, crushing and shearing strength, but not for transverse strength or for deflection, because the material is not applied to such advantage in the round beam as it is in the square one. All preceding formulas for transverse strength of square beams may also be used for round beams if only constant C is multiplied by 0.589, or, say, 0.6.

Thus, the formula for a round beam fastened at one end and loaded at the other will be:

$$P = \frac{0.6\ C \times D^3}{L}$$

NOTE.—In a round beam, of course, it will be D^3 instead of $H^2\ b$ for a rectangular one.

EXAMPLE.

Find the load in pounds which will break a spruce beam 12 feet long and 6 inches in diameter when supported under both ends and loaded at the center. (Find constant C in Table No. 30.)

Solution:

$$P = \frac{4 \times 0.6\ C \times D^3}{L}$$

$$P = \frac{4 \times 0.6 \times 125 \times 6 \times 6 \times 6}{12}$$

$$P = 5400 \text{ pounds.}$$

To Calculate the Size of Round Beams to Carry a Given Load When Span is Known.

Where the load and span are known, the diameter of the beam is calculated, when fastened at one end and loaded at the other, by the formula:

$$D = \sqrt[3]{\frac{P \times L \times \text{factor of safety}}{0.6\ C}}$$

RULE.

Multiply together the load in pounds, factor of safety and length of span in feet, divide this product by six-tenths of the constant in Table No. 30, and the cube root of this quotient is the diameter of the beam.

EXAMPLE.

A round spruce beam is fastened into a wall, and is to carry 1200 pounds on the free end projecting 4 feet from the wall, with 8 as a factor of safety, the weight of the beam not to be considered. Find diameter of beam.

Solution:

$$D = \sqrt[3]{\frac{1200 \times 4 \times 8}{0.6 \times 125}}$$

$$D = \sqrt[3]{\frac{38400}{75}}$$

$$D = \sqrt[3]{512}$$

$D = 8$ inches diameter.

Load Concentrated at Any Point, Not at the Center of a Beam.

If a beam is supported at both ends and loaded anywhere between the supports but not at the center (see Fig. 25), it will carry more load than if it was loaded at the center. With regard to breaking, the carrying capacity is inversely as the square of half the beam to the product of the short and the long ends between the load and the support. For instance, a beam 10 feet long is of such size that when it is supported under both ends and loaded at the center it will carry 1400 pounds. How many pounds will the same beam carry if loaded 3 feet from one end and 7 feet from the other?

Solution:

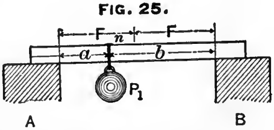
$$X = \frac{1400 \times 5^2}{7 \times 3}$$

$$X = \frac{1400 \times 25}{21}$$

$X = 1666\frac{2}{3}$ pounds.

If weight of beam is also included in its center-breaking-load, the formula will be:

FIG. 25.



$$P_1 = \left(P \times \frac{F^2}{a \times b}\right) - \tfrac{1}{2}\ W$$

P = Breaking load (including weight of beam) if applied at the center in pounds.

F = Half the length of the span.

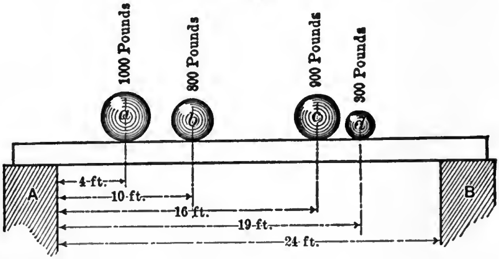
$W =$ Weight of beam.
$P_1 =$ Breaking load applied at n.

$$\text{Load on Pier } A = \frac{\text{Load at } n \times \text{distance } b}{\text{span}}$$

$$\text{Load on Pier } B = \frac{\text{Load at } n \times \text{distance } a}{\text{span}}$$

Beams Loaded at Several Places.

FIG. 26.



When a beam is loaded at several places the equivalent center load and the load on each support may be calculated as shown in the following example: (See Fig. 26).

The equivalent center load for $a = \frac{4 \times 20}{12 \times 12} \times 1000 = 555.6$ lbs.

The equivalent center load for $b = \frac{10 \times 14}{12 \times 12} \times 800 = 777.8$ lbs.

The equivalent center load for $c = \frac{16 \times 8}{12 \times 12} \times 900 = 800$ lbs.

The equivalent center load for $d = \frac{19 \times 5}{12 \times 12} \times 300 = 197.9$ lbs.

The equivalent center load for loads a, b, c and d is 2331.3 pounds.

The load on Pier $A =$

$$\frac{(5 \times 300) + (8 \times 900) + (14 \times 800) + (20 \times 1000)}{24} = 1662\tfrac{1}{2} \text{ lbs.}$$

The load on Pier $B =$

$$\frac{(4 \times 1000) + (10 \times 800) + (16 \times 900) + (19 \times 300)}{24} = 1337\tfrac{1}{2} \text{ lbs.}$$

NOTE.—The sum of the load on supports A and B is always equal to the sum of all the loads; therefore, by subtracting the calculated load on B from the total load the load on A is obtained. By subtracting the calculated load at A from the total load, the load on B is obtained.

To each load as calculated above for each support also add half the weight of the beam.

To Figure Sizes of Beams When Placed in an Inclined Position.

FIG. 23.

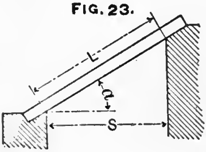


Figure all calculations concerning the transverse strength from the distance S, and leave the length L out of consideration. If the distance S cannot be obtained by measurement it may be found by multiplying L by cosine of angle a.

DEFLECTION IN BEAMS WHEN LOADED TRANSVERSELY.

Experiments and theory both prove that if the span is increased and the width of the beam increased in the same proportion the transverse strength of the beam is unchanged; but such is not the case with its stiffness. If a beam is to have the same stiffness its depth must be increased in the same ratio as the span, providing the width is unchanged. Within the elastic limit of the beam the deflection is directly proportional to the load; that is, half the load produces half the deflection, but doubling the load will double the deflection.

Deflection is proportional to the cube of the span; that is, with twice the length of span the same load will, when the other dimensions of the beam are unchanged, produce eight times as much deflection.

Deflection is inversely as the cube of the depth (thickness) of the beam. For instance, if the depth of a beam is doubled but the length of span and the width of beam is unchanged, the same load will produce only one-eighth as much deflection. Deflection is inversely as the width of the beam; for instance, when a beam is twice as wide as another beam of the same material but all the other dimensions are unchanged, the same load will produce only half as much deflection.

The deflection in a beam caused by various modes of loading is calculated by the following formulas:—

For beams laid in a horizontal position and loaded transversely, fastened at one end and loaded at the other: (See Fig. 6).

$$S = \frac{P \times L^3}{3 \times E \times I}$$

For beams laid in a horizontal position, fastened at one end and loaded thoroughout the whole length: (See Fig. 7.)

$$S = \frac{P \times L^3}{8 \times E \times I}$$

For beams laid in a horizontal position, supported under both ends and loaded at the center: (See Fig. 8).

$$S = \frac{P \times L^3}{48 \times E \times I}$$

This formula may be transposed and used to calculate modulus of elasticity from the results obtained when specimens are tested for transverse stiffness. Deflection should be carefully measured but the specimen must not be bent beyond its elastic limit; the modulus of elasticity is calculated by the transposed formula:

$$E = \frac{P \times L^3}{48 \times S \times I}$$

For a square specimen I is (side of beam)^4 divided by 12. (See moment of inertia, page 237).

(Also see rule for calculating modulus of elasticity, page 265).

For beams laid in a horizontal position, supported under both ends and loaded uniformly throughout their whole length: (See Fig. 9).

$$S = \frac{5 \times P \times L^3}{384 \times E \times I}$$

For beams laid in a horizontal position, fixed at both ends, and loaded at the center: (See Fig. 10).

$$S = \frac{P \times L^3}{192 \times E \times I}$$

For beams laid in a horizontal position, fixed at both ends and loaded uniformly throughout their whole length: (See Fig. 11).

$$S = \frac{P \times L^3}{384 \times E \times I}$$

In these formulas the definitions of the letters are:

S = Deflection in inches.
P = Load in pounds.
L = Length of span in inches.
E = Modulus of elasticity in pounds per square inch.
I = Rectangular moment of inertia. (See pages 237-238).

These formulas are applicable to any shape of section or material, when the load is within the elastic limit.

For beams of symmetrical section it is more convenient to use the following equally correct but more practical formulas, by which the deflection is calculated directly from the size of the beam by simply using a constant obtained by experiment and reduced by calculation to a unit beam one foot long and one inch square, thus avoiding both the use of the modulus of elasticity and the moment of inertia.

When beams are supported under both ends and loaded at the center, and the weight of the beam itself is not considered, the following formulas may be used for solid rectangular beams laid in a horizontal position:

$$S = \frac{L^3 \times P \times c}{H^3 \times B} \qquad L = \sqrt[3]{\frac{H^3 \times B \times S}{P \times c}}$$

$$H = \sqrt[3]{\frac{L^3 \times B \times c}{S \times B}} \qquad c = \frac{S \times H^3 \times B}{L^3 \times P}$$

$$B = \frac{L^3 \times P \times c}{S \times H^3} \qquad P = \frac{H^3 \times B \times S}{L^3 \times c}$$

$S =$ Deflection in inches.
$H =$ Thickness of beam in inches.
$B =$ Width of beam in inches.
$L =$ Length of beam in feet.
$P =$ Load in pounds.
$c =$ Constant obtained by experiment, and is the deflection, in fractions of an inch, which a beam one foot long and one inch square will have if supported under both ends and loaded at the center; the average value for this constant is given in Table No. 31.

For any other mode of loading, see rules and explanations on page 261.

In previous formulas and rules, the weight of the beam itself was not considered. The deflection in a beam caused by its own weight when it is of rectangular shape and uniform size, and laid in a horizontal position, is obtained by the formula,

$$S = \frac{L^3 \times \frac{5}{8} W \times c}{H^3 \times B}$$

When both the weight and the load are to be considered, the deflection in a solid rectangular beam laid in a horizontal position, supported under both ends and loaded at the center, is calculated by the formula,

$$S = \frac{L^3 \left(P + \frac{5}{8} W\right) c}{H^3 \times B}$$

$S =$ Deflection in inches.

$L =$ Length of span in feet.

$P =$ Load in pounds.

$W =$ Weight of beam in pounds.

$c =$ Constant obtained by experiments, and is the deflection in fractions of an inch, which a beam one foot long and one inch square will have if supported under both ends and loaded at the center, and may be found in Table No. 31.

$H =$ Thickness of beam in inches.

$B =$ Width of beam in inches.

RULE.

To the load add five-eighths of the weight of beam, multiply this by the cube of the length of the span in feet, and multiply by a constant from Table No. 31. Divide this product by the product of the cube of the thickness and the width of the beam; the quotient is the deflection in inches.

The deflection in a beam supported under both ends and loaded evenly throughout is five-eighths of that of a beam supported under both ends and loaded at the center. Therefore, in the following formulas, the weight of the beam itself is multiplied by five-eighths to reduce the effect of the weight of the beam to the equivalent of a load placed at its center.

FOR SOLID SQUARE BEAMS.

$$S = \frac{c\,(P + \tfrac{5}{8}\,W)\,L^3}{H^4}$$

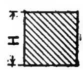


FIG. 28

FOR SOLID RECTANGULAR BEAMS.

$$S = \frac{c\,(P + \tfrac{5}{8}\,W)\,L^3}{B\,H^3}$$

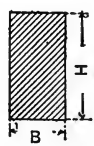


FIG. 29

FOR HOLLOW SQUARE BEAMS.

$$S = \frac{c\,(P + \tfrac{5}{8}\,W)\,L^3}{H^4 - h^4}$$

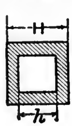


FIG. 30

FOR HOLLOW RECTANGULAR BEAMS.

$$S = \frac{c\,(P + \tfrac{5}{8}\,W)\,L^3}{B\,H^3 - b\,h^3}$$

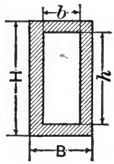


FIG. 31

FOR I BEAMS.

$$S = \frac{c\,(P + \tfrac{5}{8}\,W)\,L^3}{B\,H^3 - 2\,b\,h^3}$$

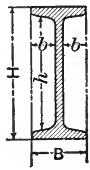


FIG. 32

FOR SOLID ROUND BEAMS.

$$S = \frac{1.7\,c\,(P + \tfrac{5}{8}\,W)\,L^3}{D^4}$$

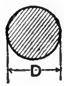


FIG. 33

FOR HOLLOW ROUND BEAMS.

$$S = \frac{1.7\,c\,(P + \tfrac{5}{8}\,W)\,L^3}{D^4 - d^4}$$

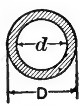


FIG. 34

FOR SOLID ELLIPTICAL OR OVAL BEAMS.

$$S = \frac{1.7\,c\,(P + \tfrac{5}{8}\,W)\,L^3}{D_1 \times D^3}$$

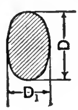


FIG. 35

FOR HOLLOW ELLIPTICAL OR OVAL BEAMS.

$$S = \frac{1.7\,c\,(P + \tfrac{5}{8}\,W)\,L^3}{D_1\,D^3 - d_1\,d^3}$$

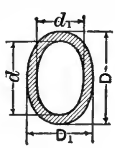


FIG. 36

$S =$ Deflection in inches.
$L =$ Length of span in feet.
$P =$ Load in pounds.
$W =$ Weight of beam in pounds.
$c =$ Constant obtained from experiments, or may be obtained from Table No. 31.

For meaning of the other letters, see figure opposite each formula.

A round beam equal in diameter to the side of a square beam will deflect 1.698 times as much, and for convenience, when the deflection of a square or a rectangular beam, whether solid or hollow, is known, it may be multiplied by 1.7, and the product is the deflection of a corresponding round, oval, or elliptical beam of the same material and diameter and laid in the same relative position and loaded in the same manner as the calculated beam. It is well to remember that a round or elliptical beam weighs a little less than a square or rectangular one, when the sides and diameters are equal, and the deflection due to its own weight is, therefore, a little less.

TABLE No. 31.—Constant c,

Giving deflection in inches per pound of load when the beam is supported under both ends and loaded at the center.

MATERIAL.	Constant c.	MATERIAL.	Constant c.
Cast steel,	0.0000143	Pitch pine,	0.00024
Wrought iron,	0.0000156	Spruce,	0.00035
Machinery steel	0.0000156	Pine,	0.00033
Cast-iron,	0.0000288		

EXAMPLE.

A beam 6″ × 9″ of pitch pine, 10 feet long, supported under both ends, is to be loaded at the center with one-tenth of its breaking load. Find the load and deflection.

Solution:

$$P = \frac{9^2 \times 6 \times 4 \times 150}{10 \times 10}$$

$$P = \frac{291600}{100} = 2916 \text{ pounds.}$$

Deflection will be,

$$S = \frac{10^3 \times 2916 \times 0.00024}{9^3 \times 6} = \frac{699.84}{4374} = 0.16 \text{ inch.}$$

Therefore, if this beam had been curved 0.16 inch upward, by increasing its thickness on the upper side, it would have been straight after the load was applied.*

In this example the weight of the beam itself is not considered either in figuring the strength or the deflection, because the beam is comparatively short in proportion to its width and thickness. The weight of the beam itself will only be about 200 pounds, and this will be of no account in proportion to the load that the beam will carry, with 10 as a factor of safety. The weight of the beam will increase its deflection only 0.006 inch. In such a beam the danger is probably greater from crushing of the ends at the supports, if it has not enough bearing surface. In long beams the weight of the beam must not be neglected, either in calculating safe load or in calculating deflection.

EXAMPLE 2.

A round bar of wrought iron is 5 feet long and 3 inches in diameter, and loaded at the center with 800 pounds. How much will it deflect? A round bar of iron 3 inches in diameter and 5 feet long weighs 119 pounds. (See table of weights of iron, page 143.)

Solution:

$$S = \frac{5^3 \times (800 + \frac{5}{8} \times 119) \times 1.7 \times 0.0000156}{3^4}$$

$S = 0.0359$ inch.

Thus, such a shaft loaded with 800 pounds will deflect $\frac{36}{1000}$ of an inch in the length of 5 feet, or 60 inches. If the deflection must not exceed $\frac{1}{1500}$ of the span (see page 266), then the greatest allowable deflection for this span would be 0.04 inch, and the calculated deflection is within this limit.

NOTE.—$\frac{1}{1500}$ of the span is equal to a deflection of 0.008 inch per foot of length.

EXAMPLE 2.

A shaft of machinery steel, 11 inches in diameter and 6 feet between bearings, carries in the center a 12-ton fly wheel. How much deflection will the weight of the fly wheel cause?

NOTE.—Such shafts are usually considered as a beam supported under both ends. (See formula for deflection in solid round beams, page 258.)

Solution:

12 tons = 24,000 pounds. (Weight of shaft is not taken into consideration.)

* This is a thing frequently done in practice.

$$S = \frac{L^3 P \times 1.7\, c}{D^4}$$

$$S = \frac{6^3 \times 24000 \times 1.7 \times 0.0000156}{11^4}$$

$$S = \frac{216 \times 24000 \times 0.00002652}{14641}$$

$$S = 0.00939 \text{ inches.}$$

Thus, the calculated deflection caused by the fly wheel is a little less than $\frac{1}{100}$ of an inch. The deflection per foot of span will be $\frac{0.00939}{6}$ which equals 0.001565 inch.

EXAMPLE 3.

Calculate the deflection of shaft mentioned in the previous example, when both the weight of fly wheel and the weight of shaft are to be considered.

Solution:

$$S = \frac{6^3 \times (24000 + \frac{5}{8} \times 1920) \times 1.7 \times 0.0000156}{11^4}$$

$$S = \frac{216 \times 25200 \times 0.00002652}{14641}$$

$$S = 0.00986 \text{ inch.}$$

Practically, the deflection is likely to be a little less than what is figured in the two previous examples, because if the hub of the fly wheel fits well on the shaft, it will stiffen it some. (It is a good practice to make such shafts a little larger in diameter in the place where the hub of the wheel is keyed on; this enlargement will then compensate for what the shaft is weakened by cutting the key-way.)

The weight of the shaft may be obtained by considering a cubic foot of machinery steel to weigh 485 pounds, and a shaft 11 inches in diameter will then weigh 320.1 pounds per foot in length, and 6 feet will weigh 1920 pounds. Multiplying this by $\frac{5}{8}$ gives 1200 pounds, to be added to the weight of the fly wheel, which gives 25,200 pounds. The weight of the shaft may also be found in the table of weight of round iron, page 144.

To Calculate Deflection in Beams Under Different Modes of Support and Load.

Constant c in Table No. 31 is the deflection in fractions of an inch per pound of load when a beam one foot long and one inch square is supported under both ends and loaded at the center, and when this constant for any given material is known, the deflection for beams subjected to other modes of fastening and loads may be calculated thus:

For beams supported under both ends with the load distributed evenly throughout their whole length, multiply c by $\frac{5}{8}$.

For beams fixed at both ends and loaded at the center, multiply c by $\frac{1}{4}$.

For beams fixed at both ends with the load distributed evenly throughout their whole length, multiply c by $\frac{1}{8}$.

For beams fixed at one end and loaded at the other, multiply c by 16.

For beams fixed at one end with load distributed evenly throughout their whole length, multiply c by 6.

EXAMPLE.

A square, hollow beam of cast-iron, 8 inches outside and 6 inches inside diameter, and 9-foot span, supported under both ends, is loaded at the center with 8000 pounds. How much will it deflect?

Solution:

Weight of beam $= 9 \times 12 \times (8^2 - 6^2) \times 0.26 = 786$ pounds.

$$S = \frac{9^3 \times (8000 + \frac{5}{8} \times 786) \times 0.0000288}{8^4 - 6^4}$$

$$S = \frac{729 \times 8492 \times 0.0000288}{4096 - 1296}$$

$$S = \frac{178.291}{2800}$$

$S = 0.064$ inches.

EXAMPLE.

How much would this same beam deflect if the load had been distributed evenly throughout its whole span?

Solution:

$$S = \frac{L^3 (P + W) \frac{5}{8} c}{D^4 - d^4}$$

$$S = \frac{9^3 \times 8786 \times \frac{5}{8} \times 0.0000288}{8^4 - 6^4}$$

$$S = \frac{115.289}{2800}$$

$S = 0.041$ inch.

EXAMPLE.

A round cast-iron beam of 7 inches outside and 5 inches inside diameter is 4 feet between supports, with a load of 2000 pounds distributed evenly throughout its span. How much will it deflect, the weight of beam itself not being considered in the calculation?

Solution:

$$S = \frac{4^3 \times 2000 \times 0.0000288 \times 1.7 \times \frac{5}{8}}{7^4 - 5^4}$$

$$S = \frac{256 \times 2000 \times 0.0000288 \times 1.7 \times \frac{5}{8}}{1776}$$

$$S = 0.0085 \text{ inch.}$$

In this example, 1.7 is used as a multiplier because the beam is round, and ⅝ because the load is distributed evenly throughout the length of the span.

EXAMPLE.

A fly wheel weighing 800 pounds is carried on the free end of a 3-inch shaft, 1 foot from the bearing. How much will the shaft deflect?

This is the same as a round beam loaded at one end and fastened at the other; therefore, constant c is multiplied by 16×1.7.

Solution:

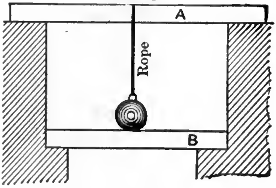
$$S = \frac{L^3\, P\, 1.7\, c \times 16}{D^4}$$

$$S = \frac{1 \times 800 \times 1.7 \times 0.0000156 \times 16}{3^4}$$

$$S = 0.0042 \text{ inch.}$$

Previous calculations for breaking load and also for deflection are based upon a dead load slowly applied and not exposed to jar and vibrations. If the load is applied suddenly it will have greater effect toward breaking the beam than if applied slowly. For instance, imagine a load having its whole weight hanging on a rope, like Fig. 37, just touching the beam but not actually resting upon it. If that rope was cut off suddenly this load would produce twice as much effect toward breaking the beam and would cause twice as much deflection as if it was loaded on gradually. A railroad train running over a bridge will, for the same reason, strain the bridge more when running fast than it would if running slow.

FIG. 37



To Find a Suitable Size of Beam for a Given Limit of Deflection.

For a square beam supported under both ends and loaded at the center, use the formula:

$$\text{Side of the beam} = \sqrt[4]{\frac{L^3 P c}{S}}$$

A round beam supported under both ends and loaded at the center may be calculated by the formula:

$$\text{Diameter of beam} = \sqrt[4]{\frac{L^3 P\, 1.7\, c}{S}}$$

A rectangular beam supported under both ends and loaded at the center, and having its depth 1½ times its width, may be calculated by the formula:

$$\text{Depth or thickness of beam} = \sqrt[4]{\frac{3 L^3 P c}{2 S}}$$

L = Length of span in feet.

P = Center load in pounds.

S = Given deflection in inches.

c = Constant given in Table No. 31.

NOTE.—These three formulas are only approximate, as the weight of the beam itself is not considered; but if necessary, after the size of beam is obtained, its weight may be calculated and five-eighths of it added to the center load, P; and using the same formula again, another beam may be calculated for this new center-load, and this new calculation will give a beam only a mere trifle too small. Constants in Table No. 31 are for beams supported under both ends and loaded at the center. For any other mode of loading or fastening, constant c must be multiplied according to rules on page 261.

To Find the Constant for Deflection.

If experiments are made upon rectangular beams, use formula,

$$c = \frac{S H^3 B}{L^3 (P + \frac{5}{8} W)}$$

EXAMPLE.

Calculate the constant c, or deflection in inches per pound of load, for a beam of 1 foot span and 1 inch square, supported under both ends and loaded at the center, when experiments are made upon a pitch pine beam 40 feet long, 12″ by 8″, weighing 1200 pounds and deflecting 1½ inches for a center-load of 500 pounds.

Solution:

$$c = \frac{1.5 \times 12^3 \times 8}{40^3 \times (500 + \frac{5}{8} \times 1200)}$$

$$c = 0.000259 \text{ inch.}$$

Modulus of Elasticity Calculated from the Transverse Deflection in a Beam.

When experiments are made upon rectangular beams supported under both ends and loaded at the center, the modulus of elasticity may be calculated by the formula,

$$E = \frac{L^3 (P + \frac{5}{8} W)}{4\, S\, T^3\, B}$$

$E =$ Modulus of elasticity.
$L =$ Length of span in inches (not in feet).
$P =$ Load in pounds.
$W =$ Weight of beam in pounds.
$S =$ Deflection of beam in inches.
$T =$ Thickness of beam in inches.
$B =$ Width of beam in inches.

EXAMPLE.

Calculate the modulus of elasticity for a pitch pine rectangular beam weighing 1200 pounds, 40 feet span, and 12″ by 8″, deflecting $1\frac{1}{2}$ inches for a center-load of 500 pounds. (This beam and conditions are the same as mentioned in the previous example for calculating constant c.)

Solution:

$$E = \frac{480^3 \times (500 + \frac{5}{8} \times 1200)}{4 \times 1\frac{1}{2} \times 12^3 \times 8}$$

$$E = \frac{138240000000}{82944}$$

$$E = 1{,}666{,}666 \text{ pounds per square inch.}$$

This deflection was obtained by actual experiments on a pitch pine beam of the dimensions given, and the calculated modulus of elasticity agrees fairly well with what is usually given by different authorities in tables of modulus of elasticity. When experimenting it is necessary to take the average of several experiments with different loads and to try the beam by turning it upside down, as very frequently it will then deflect a different amount under the same load. Care should be taken that the load is not so great as to strain the beam beyond its elastic limit. As long as the deflection increases regularly in proportion to the load, it is a sign that the elastic limit is not reached. It is very difficult to ascertain exactly when deflection will commence to increase faster than the load, because material is never so homogeneous but that the deflection will be more or less irregular, although by care and patience fairly good results may be obtained.

Allowable Deflection.

The greatest amount of deflection which may be allowed in different kinds of construction can only be determined by practical experience and good judgment of the designer. As a rule, in iron work the deflection is seldom allowed to exceed $\frac{1}{1500}$ of the span, which is equal to $\frac{1}{125}$, or 0.008 inch per foot of span. Line shaftings are sometimes allowed to deflect $\frac{1}{1200}$ of the distance between hangers which is equal to 0.01 inch per foot of span, but head shafts carrying large pulleys are generally not allowed to deflect more than 0.005 per foot of span.

In woodwork, considerable more deflection is allowed than in iron structures. Beams in houses are frequently allowed to deflect $\frac{1}{500}$, or even $\frac{1}{480}$ of the span; this is equal to 0.024 to 0.025 inch*, per foot of span. Woodwork to which machinery is to be fastened must never be allowed to deflect so much. Such woodwork must always be so stiff that it supports the machinery, and not *vice versa;* for instance, in beams or posts by which hangers and shafting are supported, it is not all-sufficient that they are strong enough, but they must also always be stiff enough.

In factories it is very important that floor beams as well as beams supporting heavy shafting have sufficient stiffness as well as strength. Floors in factories are frequently loaded up to 300 pounds per square foot of surface. For floors in public buildings, which are never loaded with more than the weight of the people who can get room, the load will hardly exceed 150 pounds per square foot of surface. Floors in tenement houses are seldom loaded more than 60 pounds per square foot.

Slate roofs weigh about 8.5 pounds per square foot of surface. Snow may be reckoned, when newly fallen, to weigh 5 to 15 pounds per cubic foot, and when saturated with water it may weigh 40 to 50 pounds per cubic foot. Usual practice is to allow 15 to 20 pounds per square foot for snow and wind on roofs.

TORSIONAL STRENGTH.

The fundamental formula for torsional strength is,

$$Pm = S\frac{J}{a}$$

Pm = Twisting moment, and is the product of the length of the arm, m, in inches and the force, P, in pounds.

S = Constant computed from experiments, and is sometimes called the modulus of torsion; its value usually agrees closely to the ultimate shearing strength per square inch of the material.

J = Polar moment of inertia (see page 297).

a = The distance in inches from the axis about which the twisting occurs to the most remote part of the cross section.

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* 0.025 is one-fortieth inch per foot of span.

EXAMPLE 1.

A round cast-iron bar 3 inches in diameter, is exposed to torsional stress; the length of the lever, m, is 18 inches. Find the breaking force, P, in pounds when the modulus of torsion for cast-iron is taken as 25,000 pounds.

Polar moment of inertia for a circle of diameter, d, is,

$$\frac{d^4 \pi}{32}$$

The distance $a = \frac{1}{2} d$.

$$P = \frac{25000 \times 3^4 \times 3.1416 \times \frac{1}{32}}{18 \times 1\frac{1}{2}}$$

$$P = \frac{25000 \times 81 \times 3.1416}{27 \times 32}$$

$$P = 7363\tfrac{1}{8} \text{ pounds.}$$

The advantage of the above formula is that it may be used for any form of section, because it takes in the polar moment of inertia of the section; but it is seldom that calculations of torsional strength are required for other than beams of round or square section, either hollow or solid, and the strength of such beams may be more conveniently calculated in an equally correct, but easier way, obviating the use of both the polar moment of inertia and the modulus of torsion, by reasoning thus:

Consider two shafts, a and b, Fig 38. Shaft a has twice the diameter of shaft b, and consequently four times the area; therefore it has, so to say, four times as many fibers to resist the stress, and for this reason it must be four times as strong as shaft b; but, further, the outside fibers in a are twice as far from the center, therefore the fibers in shaft a must also have on an average twice the advantage over the fibers in shaft b to resist the twisting effort of any load exerting a twisting stress, and for this reason shaft a must have twice the strength of b; and taking these two reasons together, shaft a must, consequently, be eight times as strong as shaft b, in resisting torsional stress.

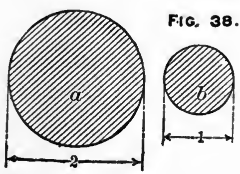


FIG. 38.

Thus, the strength of a solid shaft increases as the cube of the diameter. Shaft a is twice as large in diameter as shaft b, and is, therefore, eight times as strong as b, because $2^3 = 8$. If shaft a had been three times as large as b it would have been 27 times as strong, because $3^3 = 27$; if shaft a had been four times as large as b, it would have been 64 times as strong, because $4^3 = 64$.

Therefore, if the constant corresponding to a load, which, applied to an arm one foot long will twist off or destroy a bar one inch in diameter, is found, the breaking load for any round shaft of the same material when under torsional stress may be easily calculated. The torsional strength (but not the torsional deflection in degrees) is independent of the length of the shaft. The strength depends only upon the kind and the amount of material, and the form of cross-section. A square shaft having its sides equal to the diameter of a round shaft will have approximately 20% more strength than the round one, but it will take nearly 28% more material. A square shaft of the same area as a round shaft has approximately 15% less torsional strength than the round one.

Thus:

Formulas for torsional strength relating to solid round shafts will be:

$$P = \frac{D^3 c}{m} \qquad m = \frac{D^3 c}{P}$$

$$D = \sqrt[3]{\frac{P m}{c}} \qquad c = \frac{P m}{D^3}$$

$P =$ Breaking load in pounds.

$D =$ Diameter of shaft in inches.

$m =$ Length in feet of the arm on which load P is acting.

$c =$ Constant, and it is the load in pounds which, when applied to an arm one foot long, will twist off or destroy a round bar one inch in diameter. This constant is obtained from experiments, and is given in Table No. 32.

RULE.—Multiply the cube of the diameter in inches by the constant c, in pounds, divide this product by the length of the lever m, in feet, and the quotient is the breaking load in pounds.

TABLE No. 32.—Constant c.

The ultimate torsional strength in pounds of a round beam one inch in diameter, when load is acting at the end of a lever one foot long.

MATERIAL.	Very Good.	Medium Good.	Poor.
Cast Steel	2,000	1,000	600
Machinery Steel* . . .	1,200	1,100	700
Wrought Iron	800	580	500
Cast-iron	525	450	350

* Machinery steel or wrought iron may not actually break at this load, but it will deflect and yield so it will become useless.

EXAMPLE 1.

A wrought iron shaft is eight inches in diameter, and the force acts upon a lever two feet long. How much force must be applied in order to twist off or to destroy the shaft?

Solution:

$$P = \frac{8^3 \times 580}{2} = \frac{512 \times 580}{2} = 148{,}480 \text{ pounds.}$$

EXAMPLE 2.

A force of 870 pounds is acting with a leverage of four feet in twisting a wrought iron shaft. What must be the diameter of the shaft in order to resist the twisting stress, with 10 as a factor of safety?

Solution:

$$D = \sqrt[3]{\frac{P\,m \times 10}{c}}$$

$$D = \sqrt[3]{\frac{870 \times 4 \times 10}{580}}$$

$$D = \sqrt[3]{60} = 3.914, \text{ or, practically, a 4-inch shaft.}$$

NOTE.—Ten is used as a multiplier of the twisting moment, $P\,m$, because 10 is the factor of safety. Constant 580 is taken from Table No. 32.

EXAMPLE 3.

A round bar of cast-iron four inches in diameter is to be twisted off by a force of 3200 pounds. How long a leverage is necessary? (c for cast-iron, in Table No. 32, is 450).

Solution:

$$m = \frac{D^3\,c}{P}$$

$$m = \frac{4^3 \times 450}{3200} = \frac{64 \times 450}{3200} = 9 \text{ feet long.}$$

EXAMPLE 4.

Experiments are made upon a cast-iron round bar 2 inches in diameter with a leverage of 5¼ feet; the bar is twisted off at a force of 832 pounds. Calculate constant c, or the force in pounds if acting with a leverage of one foot, which will break a round bar of the same material one inch in diameter.

Solution:

$$c = \frac{P\,m}{D^3}$$

$$c = \frac{832 \times 5\tfrac{1}{4}}{2^3} = \frac{4368}{8} = 546 \text{ pounds.}$$

Hollow Round Shafts.

In proportion to the amount of material used, a round hollow shaft has more torsional strength than a solid shaft of the same diameter. This is because the fibers in any shaft exposed to twisting stress only offer resistance to the load in proportion to their stretch. Therefore, the fibers near the center are always in position to offer less resistance than the fibers more remote from the center.

The formula for torsional strength in round hollow shafts will be:

$$P = \left(\frac{D^4 - d^4}{D \times m}\right) c$$

$P =$ Ultimate breaking load in pounds applied at a leverage of m feet.

$D =$ Outside diameter of shaft in inches.

$d =$ Inside diameter of shaft in inches.

$m =$ Length of lever in feet.

$c =$ Constant (same as for a solid shaft).

Square Beams Exposed to Torsional Stress.

The theoretical formula for twisting strength (on page 266) will apply to square as well as round beams. The proportional strength between a round and a square beam may, therefore, be compared by using that formula. Let S represent the side of a square beam and the polar moment of inertia is $\frac{1}{6} S^4$.

The distance from the center of the beam to the most remote fiber in a square beam is $S\sqrt{\frac{1}{2}}$, and, dividing the polar moment of inertia by this distance, we have,

$$\frac{\frac{1}{6} S^4}{S\sqrt{\frac{1}{2}}} = 0.23\ S^3$$

Let D represent the diameter of a round beam. The polar moment of inertia is $\frac{D^4 \pi}{32} = 0.098\ D^4$

The distance from the center to the most remote fiber in the round beam is $\frac{1}{2} D$. Dividing the polar moment of inertia by this distance, we have $\frac{0.098\ D^4}{\frac{1}{2} D} = 0.196\ D^3$

Suppose, now, that S and D are equal, for instance, one inch; the proportion in torsional strength between the two beams must be 0.23 divided by 0.196, which equals 1.18. Thus, for square beams, use the formulas given for round beams, but multiply constant c, in Table No. 32, by 1.2, and take the side instead of the diameter. The formula for torsional strength in a square beam will be:

$$P = \frac{(\text{Side})^3 \times 1.2 \times c}{\text{Length of leverage.}}$$

$c =$ Constant (same as for a round beam).
$P =$ Load in pounds.
Side is measured in inches.
Length of leverage is measured in feet.

Torsional Deflection.

The torsional deflection in degrees will increase directly with the length of the shaft and the twisting load, and inversely as the fourth power of the diameter of the shaft; therefore, the formula for torsional deflection is:

$$S = \frac{c \times m \times L \times P}{D^4}$$

$S =$ Deflection in degrees for the length of the shaft.
$m =$ Length of lever in feet.
$L =$ Length of shaft in feet.
$P =$ Load in pounds.
$D =$ Diameter of shaft in inches.
$c =$ Constant obtained from experiments for different kinds of material, and is the deflection in degrees for a shaft one inch in diameter and one foot long, when loaded with one pound on the end of a lever one foot long.

The author of this book has made experiments on torsional deflection in wrought iron shafts two inches in diameter. The average deflection was 1½ degrees in 10 feet of length, when a load of 50 pounds was applied on a lever 5¼ feet long. Constant c, as calculated from these experiments, will be 0.00914. Using this constant, the formula for torsional deflection for wrought iron will be:

$$S = \frac{L \times m \times P \times 0.00914}{D^4}$$

Machinery steel and wrought iron will deflect about the same. Cast-iron will deflect twice as much as wrought iron. A square bar will deflect 0.589 times as much as a round bar when side and diameter are alike.

Formula for Torsional Deflection in Hollow Round Shafts.

$$S = \frac{L \times m \times P \times c}{D^4 - d^4}$$

$D =$ Outside diameter in inches.
$d =$ Inside diameter in inches.

All the other letters have the same meaning as explained under formulas for solid shafts.

TABLE No. 33.—Constant c.

The torsional deflection in degrees per foot of length for a shaft of one inch side or diameter when loaded with one pound at the end of a lever one foot long:

Material.	Round Section.	Square Section.
Machinery Steel	0.00914°	0.00538°
Wrought Iron	0.00914°	0.00538°
Cast-iron	0.018°	0.0106°
Oak	0.795°	0.468°
Ash	0.784°	0.460°
Pine and Spruce	1.35°	0.79°

EXAMPLE.

A round bar of wrought iron 16 feet long and 3 inches in diameter is fastened at one end and the other is exposed to a twisting load of 1000 pounds, acting with 5 feet leverage. How many degrees will this load deflect the bar?

Solution:

$$S = \frac{16 \times 5 \times 1000 \times 0.00914}{3^4}$$

$$S = \frac{731.2}{81}$$

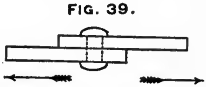
$$S = 9 \text{ degrees.}$$

NOTE.—From Table No. 33, it is seen that only steel and wrought iron are suitable for shafts exposed to torsional stress. Wrought iron is about twice as good as cast-iron, over 80 times better than oak, and about 150 times as good as pine.

SHEARING STRENGTH.

Sometimes force may act in such a manner that the material is sheared off. For instance, the rivets in a steam boiler are exposed to shearing stress (see Fig. 39) when the boiler is under steam pressure.

FIG. 39.



When holes are punched or bars of iron are cut off under punching presses, the action of the punch in cutting off the material is shearing, and the resistance which the material offers is its ultimate shearing strength. The average ultimate shearing strength of wrought iron is 40,000 pounds per square inch, In cast-iron the ultimate shearing strength is usually between 20,000 and 30,000 pounds per square inch. In steel the ultimate shearing strength will vary from 40,000 to 80,000 pounds per square inch.

The resistance offered to shearing is in proportion to the sheared area. Thus, it will take twice as much force to punch a hole two inches in diameter through a three-eighths inch plate as it would to punch a hole only one inch in diameter through the same plate, and it will take four times as much force to shear off a one-inch bolt as it would to shear off a one-half inch bolt, because the area of a one-inch bolt is four times as large as the area of a one-half inch bolt.

EXAMPLE 1.

How much force is required to shear off a wrought iron rivet of one-inch diameter if the shearing strength of the wrought iron is 40,000 pounds.

Solution:

One-inch diameter = 0.7854 square inches; therefore the force required will be $0.7854 \times 40{,}000 = 31{,}416$ pounds.

EXAMPLE 2.

A wrought iron plate is one-quarter of an inch thick and the ultimate shearing strength of the iron is 40,000 pounds per square inch. How much pressure is required to punch a hole three-quarters of an inch in diameter?

Solution:

The circumference of a ¾-inch circle is 2.356 inches. The plate is ¼-inch thick; therefore the area of shearing surface, $2.3562 \times \frac{1}{4} = 0.58905$; thus, the force required will be $40{,}000 \times 0.58905 = 23{,}562$ pounds.

TABLE No. 34.—Shearing Strength Per Square Inch.

MATERIAL.	Pounds Per Square Inch.
Steel	45,000 to 75,000
Wrought Iron Rivets	35,000 to 55,000
Cast-iron	20,000 to 30,000
Oak, crosswise	4,500 to 5,500
Oak, lengthwise	400 to 700
Pitch Pine, crosswise	4,000 to 5,000
Pitch Pine, lengthwise	400 to 600
Spruce, crosswise	3,000 to 4,000
Spruce, lengthwise	300 to 500

FACTOR OF SAFETY.

The factor of safety can only be fixed upon by the experience and good judgment of the designer. It may vary from 4 to 40. In a temporary structure, when the greatest possible load to which it will be exposed is known, a factor of safety of four may be safe enough, but frequently a greater factor is necessary. Different factors of safety are also necessary for different materials; a different factor of safety may also be necessary in different parts of the same machine. The following Table, No. 35, is only offered as a guide in selecting factor of safety:

TABLE No. 35.—Factor of Safety.

Material.	Dead Load, such as buildings containing little or no machinery.	Variable Load, such as bridges and slow-running machinery.	Machinery in General.	Machinery Exposed to hard usage, as Rolling Mills, etc.
Steel,	5	7	10	15
Wrought Iron,	4	6	10	15
Cast-iron,	6	10	15	25
Brickwork,	15	25	30	40
Wood,	8	10	15	20

If a structure is exposed to stress alternately in one direction and then in another, it is necessary to use a higher factor of safety than if it is only exposed to a steady stress one way. A comparatively small load, when applied a sufficient number of times, may break a structure or a machine, although it does not break it the first time. For instance, commence to hammer on a bar of cast-iron and it will break after several blows, although the last blow need not be any more powerful than the first one. It is the same way with anything else; it may break in time, although it is strong enough to resist the stress at the beginning; therefore, within practical limits, the larger the factor of safety the longer time the structure may last.

NOTES ON STRENGTH OF MATERIAL.

In steel, the crushing strength usually exceeds the tensile strength, but wrought iron has usually a little more tensile than crushing strength, and its shearing strength is about 80 per cent. of its tensile strength. Both steel and wrought iron are suitable to resist any kind of stress, and compared to other materials they are especially adapted for anything exposed to twisting and shearing stress.

Cast-iron is variable; it has usually five to six and a-half times as much crushing as tensile strength, and when loaded transversely it will deflect under the same load nearly twice as much as wrought iron. It is especially useful for short pillars or anything exposed to crushing stress, where there is little danger of breakage by flexure; it is very much less reliable when exposed to tensile or torsional stress.

Wood is not adapted to resist torsion, but is useful to resist tensile, crushing and transverse stress, also to resist flexure. It has nearly twice as much tensile as crushing strength; therefore, it would seem specially well adapted, in all kinds of construction, to be the member exposed to tensile stress, but where wood and iron enter into construction together, iron is always used as the member to take the tensile stress and wood as the compressive member, because wood has such low shearing strength lengthwise with its fibers that, with any kind of fastening at the ends, it will tear and split at the holes under comparatively little stress; but this difficulty is easily overcome when wood is used as the compressive member. Wood has comparatively low tensile and crushing strength crosswise on the fiber. This is well to remember with beams loaded transversely and laid on posts. The beams may be sufficiently strong, but under heavy load, if suitable precautions are not taken (see page 250) the top of the post may press into the beam, especially if the lumber is green.

Stone has high crushing strength but low tensile strength, and, in consequence, very low transverse strength. It is very well adapted for foundations when supported and laid in such a way that its crushing strength comes into play, but when laid as a beam to resist transverse stress it is very unreliable, as it will break for a comparatively small load and it may break from a blow or jar.

Brickwork is only suitable for crushing stress, and there is great difference in the strength of different kinds of brick.

In calculating strength and stiffness in any kind of designing, it should be remembered that it is only possible to determine the strength of any material by actual test, and that the tabular and constant numbers here given are only an average approximate.

Mechanics.

The science which treats of the action of forces upon bodies and the effect they produce is called Mechanics.

Newton's Laws of Motion.

The three fundamental principles of the relation between force and motion were first stated by Sir Isaac Newton, and are therefore called Newton's laws of motion.

NEWTON'S FIRST LAW.

All bodies continue in a state of rest or of uniform motion in a straight line, unless acted upon by some external force that compels change.

NEWTON'S SECOND LAW.

Every motion or change of motion is proportional to the acting force, and the motion always takes place in the direction of a straight line in which the force acts.

NEWTON'S THIRD LAW.

To every action there is always an equal and contrary reaction.

Gravity.

The natural attraction of the earth on everything on its surface which will cause any body left free to move to fall in the direction of the center of the earth is called the force of gravity.

Acceleration Due to Gravity.

If a body is left free to fall from a height, its velocity will not be constant throughout the whole fall, but it will increase at a uniform rate. It is this uniform increment in velocity which is called acceleration of gravity. It is usually reckoned in feet per second. A body falling free will at the end of one second have acquired a velocity of 32⅙ feet, or, practically, 32.2 feet per second; but it has fallen through a space of 16.1 feet, because it started from rest and the velocity was increasing at a uniform rate until, at the end of the second, it was 32.2 feet per second; therefore, the average velocity during the first second can only be 16.1 feet. At the end of two seconds the velocity has increased to 64.4 feet per second and the space fallen through is 64.4 feet, because the average velocity per second must be half of the final velocity; therefore, the average velocity is 32.2 feet per second, and, as the time is two seconds the space will be 64.4 feet. At the end of three seconds the final velocity has increased to $3 \times 32.2 = 96.6$ feet per second and the space fallen through is $\frac{96.6}{2} \times 3 = 144.9$ feet, etc. This is supposing the body was falling freely in vacuum, but while the air will offer a resistance and somewhat reduce the actual, motion, the principle is the same. Acceleration due to gravity varies but little at different latitudes of the earth. At the equator it is calculated to be 32.088 and at the pole 32.253 feet. Acceleration due to gravity decreases at higher altitudes,* but all these variations on the earth's surface are so small that they hardly need to be considered in any calculation concerning practical problems in mechanics.

Velocity.

The velocity of falling bodies increases at a uniform rate of 32.2 feet per second; therefore, when commencing from rest, the final velocity in feet per second must be,

$$v = t\,g = \sqrt{2\,g\,h}$$

RULE.

Multiply the time in seconds by 32.2 and the product is the final velocity in feet per second; or, multiply the height of the fall in feet by 64.4 and the square root of the product is the velocity in feet per second.

EXAMPLE.

What final velocity will a body acquire in a free fall during seven seconds?

Solution:

$$v = 7 \times 32.2 = 225.4 \text{ feet per second.}$$

Height of Fall.

The average velocity per second is always half of the final velocity per second. Therefore the space fallen through in a given time is found by multiplying half of the final velocity by the number of seconds which produced that velocity. Thus, the formulas:

$$h = \frac{t\,v}{2} = t\,0.5\,v = v\;0.5\;t = \frac{v^2}{2\,g} = \frac{t^2 g}{2} = t^2\,0.5\,g$$

---

* Above the surface of the earth the weight of a body is inversely proportional to the square of its distance from the center of the earth.

Below the surface of the earth the weight of a body is directly proportional to its distance from the center of the earth.

EXAMPLE.

A fly-wheel has a rim speed of 48 feet per second. From what height must a body drop to acquire the same velocity?

Solution:

$$h = \frac{v^2}{2\,g} = \frac{48^2}{64.4} = \frac{2304}{644} = 35.78 \text{ feet.}$$

Time.

RULE.

Divide the space by 16.1, and the square root of the quotient is the time; or, divide given velocity by 32.2, and the quotient is the time.

$$t = \frac{v}{g} = \sqrt{\frac{h}{0.5\,g}}$$

EXAMPLE.

How long a time does it take before a body in a free fall acquires a velocity of 100 feet per second?

Solution:

$$t = \frac{v}{g} = \frac{100}{32.2} = 3.1 \text{ seconds.}$$

Distance a Body Drops During the Last Second.

The space through which a body will drop in the last second is equal to the final velocity minus half of acceleration due to gravity. Therefore, this space is found by the formula:

$$x = v - \tfrac{1}{2}\,g = g\,(t - \tfrac{1}{2})$$

x = Space in feet which the body drops the last second of the fall.

t = Time in seconds.

v = Final velocity.

g = Acceleration of gravity = 32.2 feet.

h = Height of fall in feet.

EXAMPLE.

A body has in a free fall obtained a final velocity of 40 feet per second. What space did it drop the last second?

Solution:

$$x = v - \tfrac{1}{2}\,g = 40 - \frac{32.2}{2} = 40 - 16.1 = 23.9 \text{ feet.}$$

EXAMPLE.

A body was falling four seconds. How many feet did it drop the last second?

Solution:

$$x = g\,(t - \tfrac{1}{2}) = 32.2 \times (4 - \tfrac{1}{2}) = 32.2 \times 3.5 = 112.7 \text{ feet.}$$

TABLE No. 35.—Time, Velocity and Height.

$g = 32.161$ Feet.

Time in Seconds.	Velocity in Feet at the End of the Time.	Height of Fall in Feet.	Distance in Feet that the Body Drops in the Last Second.
1	32.161	16.08	16.08
2	64.322	64.32	48.24
3	96.483	144.72	80.40
4	128.644	257.28	112.56
5	160.805	402.00	144.72

Upward Motion.

A body thrown perpendicularly upward with a certain velocity will continue the upward movement until it reaches the same height from which it would have to fall in order to get a final velocity equal to the starting velocity. Therefore, a body projected upward with a given velocity will return again with the same velocity. This is theoretical in a vacuum, but actually the body neither continues to the theoretical height nor returns with a final velocity equal to the starting velocity, because the air will always offer considerable resistance. The greater the weight of a body, in proportion to its volume, the nearer the velocity, when it returns, will be equal to its starting velocity.

EXAMPLE.

A body is projected upward with a velocity of 45 feet per second. How high will it go before it stops and commences to drop again, the resistance of the air not being considered?

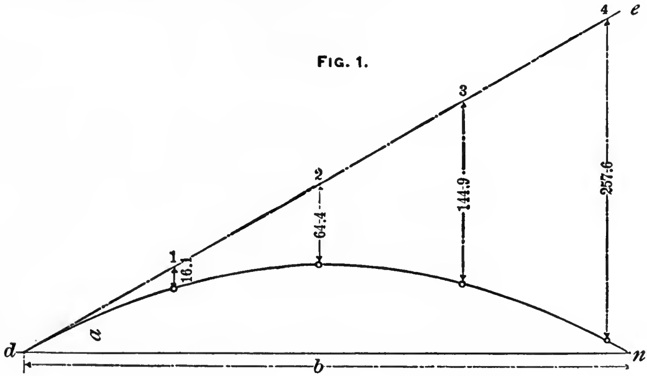
The solution of this problem is simply to find the theoretical height from which a body must drop to attain a final velocity of 45 feet, which is solved by the formula,

$$h = \frac{45^2}{64.4} = \frac{2025}{64.4} = 31.286 \text{ feet.}$$

Body Projected at an Angle.

If a body is projected in the direction of the line $d\ e$ (see Fig. 1), with an initial velocity per second equal to the distance from d to 1. no force acting after the body is started, it will continue to move at constant velocity in a straight line indefinitely; at the end of the first second it would be at 1, at the end of two seconds it would be at 2, at the end of the third second at 3, at the end of the fourth second at 4, etc.; but, on account of the force of gravity, the motion will be entirely different. The force of gravity acts on this body exactly as if it was falling in a vertical line. At the end of the first second the force of gravity has caused this moving body to drop 16.1 feet out of its path; therefore, instead of being at 1 at the end of the first second, it is at a point 16.1 feet vertically under 1; instead of being at 2 at the end of two seconds, it is at a point $2 \times 2 \times 16.1 = 64.4$ feet vertically below 2; instead of being at 3 at the end of the third second, it is at a point $3 \times 3 \times 16.1 = 144.9$ feet vertically below 3; and instead of being at 4 at the end of the fourth second, it is at a point $4 \times 4 \times 16.1 = 257.6$ feet vertically below 4, etc.

FIG. 1.



When a body is projected in a vertical upward direction with an initial velocity of v feet per second, it proceeds to a height $\frac{v^2}{2g}$; therefore, when projected at an angle, a (see Fig. 1), with a velocity of v feet per second, it will proceed to the height $\frac{v^2 \sin.^2 a}{2g}$

When a body is projected in a vertical upward dirction with a velocity of v feet per second, the time for ascent is $\frac{v}{g}$ and the time for descent is equal to the time for ascent; therefore, the total time will be $2\frac{v}{g}$; but when the body is projected upward at an angle of a degrees, the total time for ascent and descent will be $\frac{2\ v \sin. a}{g}$

The horizontal distance, or the range from d to n, will be equal to the velocity in feet per second multiplied by the total number of seconds consumed in the ascent and descent, and this multiplied by cos. of the angle a; therefore,

$$\text{Horizontal range} = v\left(\frac{2\ v \sin. a}{g}\right)\cos. a = \frac{2\ v^2 \sin. a\ \cos. a}{g}$$

but 2 × sin. a × cos. a is always equal to sin. of an angle of twice as many degrees as the angle a. Therefore, the formula reduces to horizontal range $= \dfrac{v^2 \sin. 2\ a}{g}$

Thus, the following formulas will apply to bodies projected at an angle. (See Fig. 1).

The greatest possible height will be,

$$h = \frac{v^2 \sin.^2 a}{2\,g}$$

The greatest possible range will be,

$$b = \frac{v^2 \sin. 2\ a}{g}$$

The time in seconds will be,

$$t = \frac{2\ v \sin. a}{g} = \frac{v \sin. a}{0.5\,g}$$

v = Velocity in feet per second.

g = Acceleration of gravity = 32.2.

TO FIND THE HEIGHT TO WHICH A BODY CAN ASCEND.

RULE.

Multiply the velocity in feet per second by the sine of the angle (to the horizontal line), square this product and divide by 64.4, and the quotient is the height in feet.

TO FIND THE LONGEST POSSIBLE RANGE.

RULE.

Multiply the square of the velocity in feet per second by sine of an angle of twice as many degrees as the angle of the throw (to the horizontal line), and divide by 32.2. The quotient is the longest distance the body can be thrown.

TO FIND THE TIME OF FLIGHT.

RULE.

Multiply the velocity in feet per second by sine of the angle (to the horizontal line), and divide by 16.1. The quotient is the time in seconds.

EXAMPLE.

A body is projected at an angle of 55° to the horizontal line, with an initial velocity of 120 feet per second. How high will it go? How far will it go in a horizontal direction? How many seconds will it take to finish the flight?

Solution for height:

$$h = \frac{v^2 \sin.^2 a}{2g}$$

$$h = \frac{120^2 \times \sin.^2 55^\circ}{64.4}$$

$$h = \frac{120^2 \times 0.81915^2}{64.4}$$

$$h = \frac{14400 \times 0.673}{64.4}$$

$$h = 150.5 \text{ feet.}$$

Solving for horizontal range:

$$b = \frac{v^2 \sin. 2a}{g}$$

Twice the angle of 55° is 110° and sine of 110° will be sine of 70°, because 180° — 110° = 70°; therefore, sine of 110° equals sine of 70° in the second quadrant, and the solution will be:

$$b = \frac{120^2 \times \sin. 70^\circ}{32.2}$$

$$b = \frac{14400 \times 0.93969}{32.2} = 128.4 \text{ feet.}$$

Solving for time of flight:

$$t = \frac{v \sin. a}{0.5g}$$

$$t = \frac{120 \times \sin. 55^\circ}{16.1}$$

$$t = \frac{120 \times 0.81915}{16.1} = 6.1 \text{ seconds.}$$

EXAMPLE.

A nozzle on a hose is placed at an angle of 28° to the horizontal line and the spouting water when leaving the nozzle has a volocity of 36 feet per second. How far will it theoretically reach in a horizontal direction?

Solution:

$$\text{Range} = b = \frac{v^2 \sin. 2a}{g}$$

$$b = \frac{36^2 \times \sin. 56^\circ}{g}$$

$$b = \frac{1296 \times 0.82904}{32.2} = 33.37 \text{ feet.}$$

EXAMPLE 3.

A nozzle on a hose is placed at an angle of 38° to the horizontal line and is spouting water a distance of 40 feet in a horizontal direction. What is, theoretically, the velocity of the water when leaving the nozzle?

Solution:

$$v = \sqrt{\frac{b\ g}{\sin.\ 2\ a}}$$

$$v = \sqrt{\frac{40 \times 32.2}{\sin.\ 76}}$$

$$v = \sqrt{\frac{40 \times 32.2}{0.9703}} = 36.4 \text{ feet per second.}$$

NOTE.—In Example 2 we multiply by sine of 56 degrees, because water is leaving the nozzle at an angle of 28 degrees, and twice 28 equals 56. In Example 3 we multiply by sine of 76 degrees, because twice 38 equals 76. See previous explanations.

The greatest possible height will be reached if the body is thrown perpendicularly upward. The greatest possible range is obtained if the body is thrown at an angle of 45° and will then be:

$$b = \frac{v^2}{g}$$

At an angle of 45° the horizontal range will be twice the greatest possible height which could have been reached if the body had been thrown perpendicularly upward. At this angle the horizontal range is four times the height. For an equal number of degrees over or under 45 degrees the horizontal range will be equal; for instance, if a body is thrown out at an angle of 30 or 60 degrees, the horizontal distance is the same, but the height of ascension will be much more at 60 degrees than at 30 degrees. It is frequently useful to notice this in practical work. For instance, water under pressure is thrown the farthest distance in a horizontal direction from a hose when the nozzle is held at an angle of 45 degrees to the horizontal line. It is possible by the same pressure to throw water twice as far in a horizontal distance as in vertical height.

Motion Down an Inclined Plane.

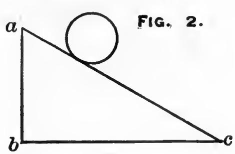


FIG. 2.

A ball rolling along an incline, as $a\ c$ (Fig. 2), will have the same velocity when it gets to c as it would have had if dropping freely from a to b, supposing all friction to be left out of consideration.

The average velocity will also be half of the final velocity, and the time used in the fall will be the distance $a\ c$ (the length of the incline), divided by the average velocity per second.

Body Projected in a Horizontal Direction From an Elevated Place.

When a body is projected in a horizontal direction from a place which is higher than the one where it strikes the ground, the range in feet in a horizontal direction will be equal to the product of velocity in feet per second and the time in seconds which it will take for a body in a free fall to drop a distance equal to the difference in vertical height between the two places. Thus:

$$\text{Horizontal range} = v\sqrt{\frac{2\,h}{g}}$$

v = Initial velocity in feet per second.

h = Vertical height in feet.

g = Acceleration of gravity = 32.2 feet.

EXAMPLE.

Water spouts from a nozzle in a horizontal direction at a velocity of 30 feet per second and the nozzle is placed 12 feet above the ground. What is the horizontal range of the water?

Solution:

$$\text{Horizontal range} = v\sqrt{\frac{2\,h}{g}} = 30\sqrt{\frac{12 \times 2}{32.2}} = 22.45 \text{ feet.}$$

To Calculate the Speed of a Bursted Fly-Wheel from the Distance the Fragments are Thrown.

The angle of 45 degrees is the one most favorable to the range; therefore, suppose the fragments to leave the wheel at that angle and use the formula,

$$\text{Horizontal distance} = b = \frac{v^2}{g} \text{ which transposes to } v = \sqrt{b\,g}$$

RULE.

Multiply the horizontal distance by 32.2, and the square root of the product is the slowest possible rim-speed the wheel could have had at the time of the accident.

EXAMPLE.

A 30-foot fly-wheel bursts from the stress due to centrifugal force, and fragments were thrown a distance of 300 feet from the place of accident. What was the slowest possible speed the wheel could have had at the time the accident occurred? and what was the corresponding number of revolutions per minute?

Solution:

$$v = \sqrt{300 \times 32.2} = \sqrt{9660} = 98.3 \text{ feet per second.}$$

The length of the circumference of a 30-foot wheel is 94.25 feet, therefore the fly-wheel was running at a speed not less than $60 \times \frac{98.3}{94.25} = 62.6$ revolutions per minute. This calculation does not prove that the wheel did not run faster than 62.6 revolutions per minute when it burst; it may have revolved a great deal faster, as it is not at all sure that any fragments left the wheel at an angle of 45 degrees, but it is certain that the speed of the wheel was not slower. Sometimes it may be possible to settle upon the angle at which a certain fragment left the wheel by noticing traces and marks where it went, and, figuring from the angle and the range, a pretty fair idea of the bursting speed may be obtained. (See formula on page 283).

Force, Energy and Power.

Force is a pressure expressed in a push or a pull.

Energy is the ability to do work. It is divided into potential energy and kinetic energy.

Potential energy is the ability of a body to perform work at any time when it is set free to do so.

Kinetic energy is the ability of a moving body to do work when its motion is arrested. Kinetic energy is very frequently called "stored-up energy."

Work is overcoming resistance through space. In the English system of weights and measures the common unit of work is the *foot-pound.*

Power is the rate of doing work. Work is an expression entirely independent of time, but power always takes time into consideration. For instance, to lift one pound one foot is one foot-pound of work, no matter in what time it is done, but it takes 60 times as much power to do it in one second as it would take to do it in one minute.

Inertia.

Inertia is the inability of dead bodies to change either their state of rest or motion. In order to bring about any change, either of motion or rest, dead bodies must always be acted upon by some outside force.

Resistance due to inertia is the resistance which a dead body free to move presents to any external force acting to change either its state of motion or rest.

Mass.

The mass of a body is the quantity of matter which it contains. By common consent the unit of mass is, in mechanics, considered to be that quantity of matter to which one unit of *force* can give one unit of *acceleration in one unit of time;* therefore, when the weight of a body is divided by acceleration of gravity, the quotient is the mass of the body. Thus:

$$m = \frac{W}{g}$$

$$W = m \times g$$

$$g = \frac{W}{m}$$

Momentum.

The product of the mass of a moving body and its velocity is called its *momentum* or, also, its *quantity of motion.* The unit for momentum is the product when unit of mass is multiplied by unit of velocity per second. In mechanical calculations, using English weights and measures, the unit of mass is weight divided by 32.2; therefore, unit of momentum will be: Weight of the moving body in pounds multiplied by velocity in feet per second and the product divided by 32.2. Thus:

$$q = m \times v \qquad m = \text{mass} = \frac{W}{g}; \quad \text{therefore,}$$

$$q = \frac{W}{g} v$$

$$q = \frac{v}{g} W$$

$q =$ Momentum, or quantity of motion.
$W =$ Weight of moving body in pounds.
$v =$ Velocity of moving body in feet per second.
$g =$ Acceleration of gravity.

$\frac{v}{g}$ is the formula by which the time in a free fall is obtained, and, consequently, the momentum of a falling body can also be expressed by the product of the weight of the body in pounds and the time in seconds during the fall. This product is usually called "time effect."

Impulse.

The product of the force and the time in which it is acting as a blow against a body is called impulse, and it is always of the same numerical value as the momentum of the moving body.

Kinetic Energy.

The kinetic energy stored in any moving body is always expressed in foot-pounds, by the product of the *force in pounds* acting to overcome the inertia of the body, and the *distance in feet* through which the force was acting in starting the body, and is always equal to the weight of the body multiplied by the square of the velocity and this product divided by twice the acceleration of gravity. Thus:

$$K = \frac{W \times v^2}{2g}$$

K = Kinetic energy in foot-pounds.
W = Weight of the body in pounds.
v = Velocity of the body in feet per second.
$2g = 64.4$.

In a free fall the height, h, corresponding to a given velocity, is found by the formula, $\frac{v^2}{2g}$; therefore, $K = W \times h$. Thus, multiplying the weight of a moving body by the height which in a free fall corresponds to its velocity, the product will be the kinetic energy stored in the body.

The formula $K = \frac{W \times v^2}{2g}$ transposes to $K = \frac{1}{2} m v^2$.

Hence the simple rule:

Multiply half the mass of a moving body by the square of its velocity in feet per second, and the product is the kinetic energy in foot-pounds stored in the body.

The kinetic energy stored in any moving body always represents a corresponding amount of mechanical work which is required in order to again bring the body to rest.

EXAMPLE.

A body weighing 1610 pounds is moving at a constant velocity of 18 feet per second. How many foot-pounds of kinetic energy is stored in the body?

Solution:

$$K = \frac{W \times v^2}{2g} = \frac{1610 \times 18 \times 18}{64.4} = 8{,}100 \text{ foot-pounds.}$$

If this moving body was brought to rest and all its stored energy could be utilized to do work it could lift 8,100 pounds one foot, or it could lift 81 pounds 100 feet, or any other combination of distance and resistance which, when multiplied by one another, will give 8,100 foot-pounds.

It is very important always to keep in mind a clear distinction between *work* and *power*, as power is the rate of doing work, and time must, therefore, always be considered in the question of power. For instance, when 33,000 foot-pounds of work is performed in one minute it is said to be one horse-power; therefore, if this 32,400 foot-pounds of energy was utilized to do work and used up in one minute, it would do work at a rate of $\frac{32400}{33000} = \frac{54}{55}$ horse-power, but if utilized during a time of two minutes it would only do work at a rate of $\frac{27}{55}$ horse-power, or if utilized in a second the rate of work would be $\frac{54}{55} \times 60 = 58\frac{10}{11}$ horse-power, etc.

To Calculate the Force of a Blow.

The force of a blow may be calculated by the change it produces. For instance, a drop-hammer weighing 800 pounds drops three feet, and compresses the hot iron on the anvil ¼ inch. How much is the average force? (¼ inch = $\frac{1}{48}$ foot).

The kinetic energy stored in the hammer at the moment it commences to compress the iron is $800 \times 3 = 2400$ foot-pounds.

The average force $= \dfrac{2400}{\frac{1}{48}} = 115{,}200$ lbs.

In the above example, friction is neglected.

The shorter the duration of the blow the more intense it will be. Therefore the force of the hammer mentioned above, if, instead of striking against hot iron, compressing it ¼ inch, had been struck against cold iron, compressing it only a few thousandths, the blow would have been as many times more intense as the duration of the blow had been shorter. Therefore it is entirely meaningless to say that a drop-hammer or any other similar machine is giving a blow of any certain number of pounds by falling a certain number of feet, because the intensity of the blow will depend upon its duration.

Formulas for Force, Acceleration and Motion.

From the laws of gravitation, it is known that when one pound of force acts upon one pound of matter it produces an acceleration of 32.2 feet per second each successive second as long as the force continues to act.

From Newton's laws of motion, it is known that the motion is always in proportion to the force by which it is produced; therefore, when *one pound of force* acts for *one second* upon 32.2 pounds of matter, it will produce an acceleration of *one foot per second.*

Hence the following formulas:

m = Mass of the moving body, which is considered to be weight divided by 32.2.

F = Constant force in pounds acting on a body free to move.

G = Constant acceleration in feet per second due to the acting force, F.

T = Time in seconds in which the force F acts upon a body free to move.

v = Final velocity acquired by the moving body in the time of T seconds.

$$F = m\,G \qquad m = \frac{F}{G} \qquad G = \frac{F}{m} \qquad v = T\,G$$

$$G = \frac{v}{T} \qquad T = \frac{v}{G} \qquad \frac{v}{T} = \frac{F}{m} \qquad v\,m = F\,T$$

$$v = \frac{F\,T}{m} \qquad m = \frac{F\,T}{v} \qquad F = \frac{v\,m}{T} \qquad T = \frac{v\,m}{F}$$

When a moving body is arrested the product of the resistance and time is equal to its momentum. Thus:

$$R\,T = v\,m \qquad v = \frac{R\,T}{m} \qquad m = \frac{R\,T}{v}$$

$$R = \frac{v\,m}{T} \qquad T = \frac{v\,m}{R}$$

R = Constant resistance in pounds acting against the moving body.

The average velocity of the moving body is half of the final velocity, and the space passed over by the moving body when acquiring the given velocity is half of the final velocity in feet per second multiplied by the time in seconds. Thus:

$$S = \frac{v}{2} \times T \qquad v = \frac{2\,S}{T} \qquad \frac{F\,T}{m} = \frac{2\,S}{T}$$

$$F\,T^2 = 2\,S\,m \qquad T = \sqrt{\frac{2\,S\,m}{F}} \qquad F = \frac{2\,S\,m}{T^2}$$

$$m = \frac{F\,T^2}{2\,S} \qquad S = \frac{F\,T^2}{2\,m}$$

S = Space in feet.

The work in foot-pounds required to overcome the inertia of a given body when brought from a state of rest to a given velocity is equal to the kinetic energy stored in the moving body. Thus:

$$K = S\,F = \frac{m\,v^2}{2} = \frac{F\,v\,T}{2} = \frac{G\,m\,v\,T}{2}$$

K = Kinetic energy stored in the moving body.

The force required to obtain a given velocity in a given time, when both resistance due to inertia and resistance due to friction is considered, is calculated by the formula:

$$\text{Force} = \left(\frac{\text{Velocity}}{\text{Time}} \times \text{Mass}\right) + (\text{weight} \times \text{coefficient of friction}).$$

which may be written:

$$\text{Force} = \left(\frac{\text{Velocity}}{\text{Time}} \times \text{Mass}\right) + (\text{resistance due to friction}).$$

IMPORTANT.—Always calculate the force required to overcome the resistance due to inertia and the force required to overcome the resistance due to friction separately, and add the two forces in order to obtain the total force required.

It is sometimes assumed that adding so much to the mass, as $\frac{1}{32}$ of the product of weight and coefficient of friction, should give the result in one operation; but such an assumption is erroneous, because the correct value for the required force is:

$$F = \frac{v \times W}{T \times g} + W \times f$$

which cannot be transposed to

$$F = \frac{v \times W + W \times f}{T \times g}$$

$F =$ Required force; $v =$ velocity; $T =$ time; $W =$ weight of moving body in pounds; $g =$ acceleration due to gravity, or 32.2; $f =$ coefficient of friction.

EXAMPLE. 1.

A railroad train weighing 225,400 pounds is started from rest to a velocity of 50 feet per second; the road is straight and level; the resistance due to friction is assumed to remain constant and to be 1000 pounds. What average constant pull in pounds must be exerted by the locomotive at the draw-bar in order to bring the train up to this speed in 40 seconds?

Solution:

For the inertia,

$$\text{Force} = \frac{\text{velocity} \times \text{mass}}{\text{time}} = \frac{50 \times \frac{225400}{32.2}}{40} = 8750 \text{ lbs.}$$

$$\text{For friction the force} = 1000$$

$$\text{Total force,} \quad 9750 \text{ lbs.}$$

NOTE.—This constant force of 9750 pounds has been acting under a uniformly increasing velocity from rest or nothing at the start, to 50 feet per second at the end of 40 seconds; therefore, the average velocity has been half of the final velocity, or 25 feet a second. The average work of the locomotive in starting the train during this 40 seconds was $25 \times 9750 = 243{,}750$ foot-pounds per second, and the horse-power exerted by the locomotive on the draw-bar in starting this train was $\frac{243750}{550} = 443.18$ horse-power, but the power required to keep this train in motion at a speed of 50 feet per second on a level road will be only $\frac{50 \times 1000}{550} = 90.91$ horse-power. From this it may be seen what an immense power there has to be produced in order to start heavy machinery in a short time, in comparison to the power required to keep it going after it is started.

EXAMPLE 2.

How far did the train move before it got up to the required speed of 50 feet per second?

Solution:

$$S = \frac{v\,T}{2} = \frac{50 \times 40}{2} = 1000 \text{ feet.}$$

EXAMPLE 3.

Suppose that after the train had acquired this speed of 50 feet a second, the locomotive was detached and that the resistance due to friction continued to be 1000 pounds. How many seconds would the train be kept in motion by its momentum on a level road?

Solution:

$$\text{Time} = \frac{v\,m}{R} = \frac{50 \times \frac{225400}{32.2}}{1000} = 350 \text{ seconds.}$$

EXAMPLE 4.

How many foot-pounds of kinetic energy is stored in this train, which weighs 225,400 pounds and runs at a constant speed of 50 feet a second?

$$K = \frac{v^2 \times m}{2} = \frac{50^2 \times \frac{225400}{32.2}}{2} = 8{,}750{,}000 \text{ foot-pounds.}$$

EXAMPLE 5.

How far will this kinetic energy drive the train on a horizontal road if we suppose the constant resistance due to friction, as in Example 3, to be 1000 pounds?

Solution:

$$\text{Distance} = \frac{\text{kinetic energy}}{\text{resistance}} = \frac{8750000}{1000} = 8750 \text{ feet.}$$

When a body free to move is acted upon by a constant force the space passed over increases as the square of time.

EXAMPLE 6.

Under the influence of a constant force a body moves five feet the first second. How far will it move in eight seconds, friction not considered?

Solution:

$$\text{Distance} = 8^2 \times 5 = 320 \text{ feet.}$$

Centers.

Center of gravity is the point in a body about which all its parts can be balanced. If a body is supported at its center of gravity the whole body will remain at rest under the action of gravity.

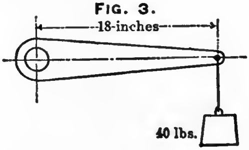
Center of gyration is a point in a rotating body at which the whole mass could be concentrated (theoretically) without altering the resistance, due to the inertia of the body, to angular acceleration or retardation.

Center of oscillation is a point at which, if the whole matter of a suspended body was collected, the time of oscillation would be the same as it is in the actual form of the body.

Center of percussion is a point in a body moving about a fixed axis at which it may strike an obstacle without communicating the shock to the axis.

Moments.

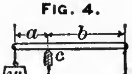
FIG. 3.



The measures of tendency to produce motion about a fixed point or axis, is called moment. The product of the length of a lever and the force acting on the end of it, tending to swing it around its center, is called the moment of force or the statical moment, and may be expressed in either foot-pounds or inch-pounds. In Fig. 3, the arm is 18 inches long and the force is 40 pounds; the moment is $18 \times 40 = 720$ inch-pounds, or $1\frac{1}{2} \times 40 = 60$ foot-pounds.

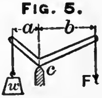
Levers.

FIG. 4.



When a lever is balanced, the distance a, multiplied by the weight w, is always equal to the distance b, multiplied by the force F. In a bent lever (as Fig. 5) it is not the length of the lever but the distance from the fulcrum at right angles to the line in which the force is acting, that must be multiplied. Thus:

FIG. 5.

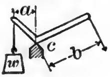


$$a \times w = b \times F.$$

In Fig. 6, the force is acting at a right angle to the lever, and, therefore, the distance a is equal to the length of the long end of the lever.

The force is applied to more advantage in Fig. 6 than in Fig. 5. As a rule, the force should always be applied so as to act at right angles to the lever.

FIG. 6.



Radius of Gyration.

The radius of gyration of a rotating body is the distance from its center of rotation to its center of gyration.

$$\text{Radius of gyration} = \sqrt{\frac{\text{moment of rotation}}{\text{mass of rotating body}}}$$

or, for a plane surface:

$$\text{Radius of gyration} = \sqrt{\frac{\text{moment of inertia}}{\text{area of surface}}}$$

The radius of gyration of a round, solid disc, such as a grindstone, when rotating on its shaft, is equal to its geometrical radius multiplied by $\sqrt{\tfrac{1}{2}}$ or radius multiplied by 0.7071 very nearly. The radius of gyration of a round disc, if indefinitely thin and rotating about one of its diameters, is equal to radius divided by 2. The radius of gyration of a ring, of uniform cross-section, rotating about its center, such as a rim of a fly-wheel when rotating on its shaft, is:

$$\text{Radius of gyration} = \sqrt{\frac{R^2 + r^2}{2}}$$

$R =$ Outside radius.
$r =$ Inside radius.

The radius of gyration of a hollow circle when rotating about one of its diameters is:

$$\text{Radius of gyration} = \sqrt{\frac{R^2 + r^2}{4}}$$

$R =$ Outside radius.
$r =$ Inside radius.

Moment of Inertia.

The moment of inertia is a mathematical expression used in mechanical calculations. It is an expression causing considerable ambiguity, as it is not always used to mean the same thing.

The least rectangular moment of inertia, as used when calculating transverse strength of beams, columns, etc., is the sum of the products of all the elementary areas of cross-sections into the square of their distances from the axis of rotation. The axis of rotation is considered to pass through the center of gravity of the section.

The least rectangular moment of inertia is always equal to the area of surface of cross-section, multiplied by the square of the radius of gyration, when the surface is assumed to rotate about the neutral axis of the section.

Mathematicians calculate the moment of inertia by means of the higher mathematics, but it may also be calculated approximately by dividing the cross-section of the beam into any convenient number of small strips and multiplying the area of each strip by the square of its distance from its center-line to the neutral axis, and the sum of these products is the moment of inertia, very nearly.

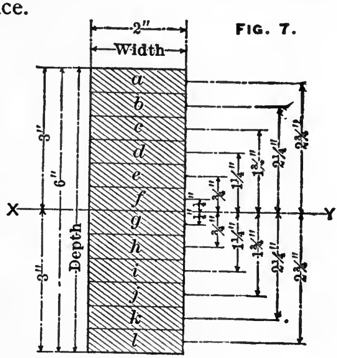
The narrower each strip is taken, the more exact the result will be; but it will always be a trifle too small.

EXAMPLE 1.

Find approximately the rectangular moment of inertia for a surface (or section of a beam) 6″ × 2″, about its axis *x y*. (See Fig. 7.)

Divide the surface into narrow strips, as *a, b, c, d, e, f, g, h, i, j, k, l*, and multiply each strip by the square of its distance from the neutral axis, *x y*, and the sum of these products is the moment of inertia of the surface.

FIG. 7.



$a = 2 \times \frac{1}{2} \times (2\frac{3}{4})^2 = 7.5625$
$b = 2 \times \frac{1}{2} \times (2\frac{1}{4})^2 = 5.0625$
$c = 2 \times \frac{1}{2} \times (1\frac{3}{4})^2 = 3.0625$
$d = 2 \times \frac{1}{2} \times (1\frac{1}{4})^2 = 1.5625$
$e = 2 \times \frac{1}{2} \times (\ \frac{3}{4})^2 = 0.5625$
$f = 2 \times \frac{1}{2} \times (\ \frac{1}{4})^2 = 0.0625$
$g = 2 \times \frac{1}{2} \times (\ \frac{1}{4})^2 = 0.0625$
$h = 2 \times \frac{1}{2} \times (\ \frac{3}{4})^2 = 0.5625$
$i = 2 \times \frac{1}{2} \times (1\frac{1}{4})^2 = 1.5625$
$j = 2 \times \frac{1}{2} \times (1\frac{3}{4})^2 = 3.0625$
$k = 2 \times \frac{1}{2} \times (2\frac{1}{4})^2 = 5.0625$
$l = 2 \times \frac{1}{2} \times (2\frac{3}{4})^2 = 7.5625$
Moment of inertia $= 35.75$ (approximately).

The correct value for the least rectangular moment of inertia for such a surface is obtained by the formula,

$\frac{(\text{Depth})^3 \times \text{width}}{12}$ and for Fig. 7 will be $\frac{6^3 \times 2}{12} = 36$. Thus, the approximate rule gives results a trifle too small, but if the surface had been divided into smaller strips, the result would have been more correct.

Radius of gyration for this surface, when rotating about the axis *x y*, is:

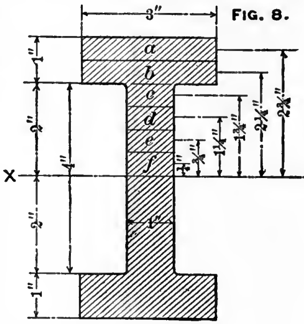
$$\sqrt{\frac{\text{moment of inertia}}{\text{area}}} = \sqrt{\frac{36}{12}} = \sqrt{3} = 1.73 \text{ inches.}$$

EXAMPLE 2.

Find by approximation the rectangular moment of inertia for a surface, as Fig. 8, (the sectional area of an I beam) about the axis $x\,y$.

When the beam is symmetrical, the neutral axis is at an equal distance from the upper and lower side, and the moment of inertia for the upper and lower half of the beam is equal; consequently, when calculating moment of inertia for a surface like Figs. 8 and 7, it is only necessary to calculate the moment of inertia for half the beam, and multiply by 2 in order to get the moment of the whole beam.

FIG. 8.



Solution:

$a = 3 \times \frac{1}{2} \times (2\frac{3}{4})^2 = 11.34375$
$b = 3 \times \frac{1}{2} \times (2\frac{1}{4})^2 = 7.59375$
$c = 1 \times \frac{1}{2} \times (1\frac{3}{4})^2 = 1.53125$
$d = 1 \times \frac{1}{2} \times (1\frac{1}{4})^2 = 0.78125$
$e = 1 \times \frac{1}{2} \times (\frac{3}{4})^2 = 0.28125$
$f = 1 \times \frac{1}{2} \times (\frac{1}{4})^2 = 0.03125$

Moment of inertia = 21.5625 for upper half.
Moment of inertia = 21.5625 for lower half.
Moment of inertia = 43.125 for beam (approximately).

Area of cross-section of beam is 10 square inches.

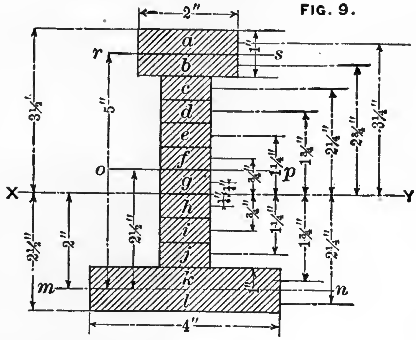
$$\text{Radius of gyration} = \sqrt{\frac{43.125}{10}} = 2.07 \text{ inches.}$$

EXAMPLE 3.

Find approximately the moment of inertia of a surface, as Fig. 9 (usual section for cast-iron beams), about the axis, $x\,y$, passing through the center of gravity of the surface.

In shapes of this kind the axis through the center of gravity is not at an equal distance from the upper and lower side, but it can be obtained experimentally by cutting a templet to the exact shape and size of the surface and balancing it over a knife's edge, or it may be calculated by the principle of moments, as shown in this example. Divide the surfaces into three rectangles, the upper flange, the web and the lower flange. Assume some line as the axis, for instance, the line $n\,m$, which is the center line through the lower flange; multiply the area of each rectangle by the distance of its center of gravity from the axis $n\,m$, and add the products. Divide this sum by the area of the entire section, and the quotient is the distance between the center of gravity of the section and the axis $n\,m$.

FIG. 9.



SOLVING FOR CENTER OF GRAVITY:

	(Area.)	(Distance.)
Area of upper flange $= 2 \times 1 =$	2 square inches	$\times 5 = 10$
Area of web $= 4 \times 1 =$	4 square inches	$\times 2\frac{1}{2} = 10$
Area of lower flange $= 4 \times 1 =$	4 square inches	$\times 0 = 0$
	10	20

and 20 divided by $10 = 2''$ which is the distance from the center of gravity of the lower flange to center of gravity of the section of the beam, or the neutral axis $x\ y$.

SOLVING FOR MOMENT OF INERTIA:

$$a = 2 \times \tfrac{1}{2} \times (3\tfrac{1}{4})^2 = 10.56250$$
$$b = 2 \times \tfrac{1}{2} \times (2\tfrac{3}{4})^2 = 7.56250$$
$$c = 1 \times \tfrac{1}{2} \times (2\tfrac{1}{4})^2 = 2.53175$$
$$d = 1 \times \tfrac{1}{2} \times (1\tfrac{3}{4})^2 = 1.53225$$
$$e = 1 \times \tfrac{1}{2} \times (1\tfrac{1}{4})^2 = 0.78125$$
$$f = 1 \times \tfrac{1}{2} \times (\tfrac{3}{4})^2 = 0.28125$$
$$g = 1 \times \tfrac{1}{2} \times (\tfrac{1}{4})^2 = 0.03125$$
$$h = 1 \times \tfrac{1}{2} \times (\tfrac{1}{4})^2 = 0.03125$$
$$i = 1 \times \tfrac{1}{2} \times (\tfrac{3}{4})^2 = 0.28125$$
$$j = 1 \times \tfrac{1}{2} \times (1\tfrac{1}{4})^2 = 0.78125$$
$$k = 4 \times \tfrac{1}{2} \times (1\tfrac{3}{4})^2 = 6.12500$$
$$l = 4 \times \tfrac{1}{2} \times (2\tfrac{1}{4})^2 = 10.12500$$

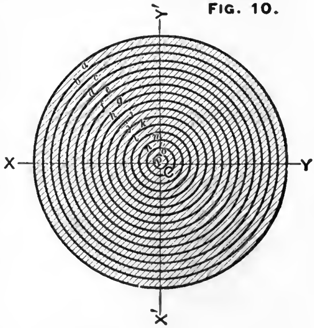
Moment of inertia of beam = 40.6266 (approximately).
Area of cross-section of beam = 10 square inches

$$\text{Radius of gyration of beam} = \sqrt{\frac{40.6266}{10}} = 2.015 \text{ inches.}$$

Polar Moment of Inertia.

The polar moment of inertia is a mathematical expression, used especially when calculating the torsional strength of beams, shafting, etc. It is very frequently denoted by the letter J. The polar moment of inertia is the sum of the products of each elementary area of the surface multiplied by the square of its distance from the center of gravity of surface. Suppose (in Fig. 10) that the area is divided into circular rings, as $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p$, and the area of each ring multiplied by the square of its distance from the center, c; the sum of all these products is the polar moment of inertia. The moment, calculated this way, will always be a trifle too small, but the smaller each ring is taken the more correct the result will be. If each ring could be taken infinitely small the result would be correct.

FIG. 10.



The polar moment of inertia is equal to the square of the radius of gyration about the geometrical center of the shaft, multiplied by the area of cross-section of the shaft; therefore, for a round, solid shaft (as the section shown in Fig. 10), the polar moment of inertia is always expressed by the formula:

$$\frac{(\text{Radius})^4 \times \pi}{2} \quad \text{or,} \quad \frac{(\text{Diameter})^4 \times \pi}{32}$$

For a hollow, round shaft, the polar moment of inertia is expressed by the formula,

$$J = \frac{(D^4 - d^4)}{32} \times \pi$$

$D =$ Outside diameter. $d =$ Inside diameter.

The fundamental principle for the polar moment of inertia for any shape of section is that, if two rectangular moments of inertia are taken, one being the least rectangular moment of inertia, about an axis passing through the center of gravity, and the other, the least rectangular moment, about an axis perpendicular to the first one, also through the center of gravity, the sum of those two rectangular moments is equal to the polar moment.

In Fig. 10, the rectangular moment of inertia about the axis $x\ y$ will be $\frac{(\text{diameter})^4 \times \pi}{64}$ and the rectangular moment about the axis $x'\ y'$ will also be $\frac{(\text{diameter})^4 \times \pi}{64}$; thus the polar moment will be $\frac{(\text{diameter})^4 \times \pi}{32}$

EXAMPLE.

Find the polar moment of inertia and radius of gyration of a round shaft of 4″ diameter.

Solution:

$$J = \frac{D^4 \pi}{32}$$

$$J = \frac{4^4 \times 3.1416}{32} = 25.1328$$

$$\text{Radius of gyration} = \sqrt{\frac{\text{Polar moment of inertia}}{\text{area of section.}}}$$

$$\text{Radius of gyration} = \sqrt{\frac{25.1328}{2^2 \times 3.1416}}$$

Radius of gyration = 1.414 inches.

The term, *moment of inertia*, as used in calculating stored energy in revolving bodies, is frequently and certainly more concisely called *moment of rotation*, and is a mathematical expression by which the effect of the whole mass (theoretically) is transferred to the unit distance from center of rotation. This term (moment of inertia or moment of rotation) is obtained by multiplying the square of radius of gyration by *mass* of moving body.* In English measure, mass is taken as $\frac{1}{32.2}$ of the weight of the revolving body, and the radius of gyration is always taken in feet.

EXAMPLE.

A solid disc of cast-iron, rotating about its geometrical center, is six feet in diameter and of such thickness that it will weigh 4073.3 pounds. What is its moment of rotation or moment of inertia?

Radius of gyration $= 3 \times \sqrt{\tfrac{1}{2}}$ and (radius of gyration)$^2 = 3^2 \times \tfrac{1}{2}$

$$\text{Mass} = \frac{4073.3}{32.2} = 126.5$$

Moment of rotation $= 126.5 \times 3^2 \times \tfrac{1}{2} = 569.25$.

NOTE.—In all such problems relating to stored energy in rotating bodies, the radius of gyration is usually taken in feet and not in inches, as in previous examples of moment of inertia, when relating to strength of material.

* Instead of multiplying the mass of the body by the square of radius of gyration in feet and calling the product moment of inertia, some writers multiply the weight of the body by the square of the radius of gyration in feet and call this product moment of inertia. This last expression for moment of inertia, of course, will have a numerical value of 32.2 times the first one. It does not make any difference in the result of the calculation whether weight or mass is used, but the same unit must be adhered to throughout the whole calculation.

Angular Velocity.

When a body revolves about any axis, the parts furthest from the axis of rotation move the fastest. *The linear velocity at a radius of one foot from the center of rotation is called the angular velocity of the body.* It is usually reckoned in feet per second. The angular velocity of any revolving body is expressed by the formula,

$$V_a = 2 \pi n$$

$V_a =$ Angular velocity in feet per second.

$n =$ Number of revolutions per second.

RULE.

Multiply the number of revolutions per second by 6.2832, and the product is the angular velocity in feet per second.

EXAMPLE.

What is the angular velocity of a fly-wheel making 300 revolutions per minute?

Solution:

300 revolutions per minute = 5 revolutions per second, therefore, angular velocity = 6.2832 × 5 = 31.416 feet per second. Angular velocity expresses the velocity at unit distance from center of rotation and in English measure this unit is feet. As already stated, the moment of rotation is an expression for the mass of the rotating body (theoretically) transferred to unit distance from center of rotation; the product of angular velocity and moment of rotation will, therefore, be the momentum of the rotating body. The constant resistance which has to be exerted at unit radius in order to bring the body to rest in T seconds will be:

$$R = \frac{V_a\ I}{T}$$

The resistance which has to be exerted at any radius of r feet to bring the body to rest in T seconds will be:

$$R = \frac{V_a\ I}{r\ T}$$

$R =$ Resistance in pounds.

$V_a =$ Angular velocity in feet per second.

$I =$ Moment of rotation (also called moment of inertia).

The constant force which has to be exerted at unit radius in order to bring the body from a state of rest to an angular velocity V_a in T seconds will be:

$$F = \frac{V_a\ I}{T}$$

The constant force which has to be exerted at any radius, r, in order to bring the body from a state of rest to an angular velocity V_a in T seconds will be:

$$F = \frac{V_a\ I}{r\ T}$$

R = Constant resistance in pounds.
F = Constant force in pounds.
V_a = Angular velocity in feet per second.
I = Moment of rotation (also called moment of inertia).
r = Radius in feet at which the force is applied.
T = Time in seconds that the force is acting.

EXAMPLE.

A fly-wheel making 120 revolutions per minute and weighing 483 pounds, is brought to rest in two seconds by a resistance acting at a six-inch radius. The radius of gyration of the fly-wheel is 1.2 feet. What is the average force exerted against the resistance during these two seconds?

Solution:

120 revolutions per minute = 2 revolutions per second.

Angular velocity = 6.2832 × 2 = 12.5664 feet per second.

$$\text{Moment of rotation} = 1.2 \times 1.2 \times \frac{483}{32.2} = 21.6$$

Radius of resistance, 6 inches = 0.5 feet.

$$R = \frac{12.5664 \times 21.6}{2 \times 0.5} = 271.43 \text{ pounds.}$$

If a rotating body is not brought to rest, but only reduced in speed to an angular velocity of V_{a}, in T seconds, then the average force or resistance acting at unit radius is:

$$F = \frac{(V_a - V_{a1})\ I}{T}$$

The average force which has to be exerted at any radius at r feet to reduce the angular velocity to V_a, in T seconds will be:

$$F = \frac{(V_a - V_{a1})\ I}{T\ r}$$

EXAMPLE.

A fly-wheel on a punching machine weighs 644 pounds, its radius of gyration is 1½ feet, and it makes at normal speed 300 revolutions per minute, but when the machine is punching the speed is in $\frac{1}{5}$ of a second reduced to a rate of 280 revolutions per minute. What average force has the fly-wheel communicated to the pitch-line of a 6-inch gear on the fly-wheel shaft?

Solution:

$$\text{The mass of the fly-wheel} = \frac{644}{32.2} = 20$$

$$\text{The moment of rotation} = (1\tfrac{1}{2})^2 \times 20 = 45$$

300 revolutions per minute = 5 revolutions per second.

$$\text{Angular velocity} = 5 \times 6.2832 = 31.416$$

280 revolutions per minute = $4\frac{2}{3}$ revolutions per second.

$$\text{Corresponding angular velocity} = 4\tfrac{2}{3} \times 6.2832 = 29.3216$$

6-inch diameter of gear = 3-inch radius = $\frac{1}{4}$ foot.

$$F = \frac{(31.416 - 29.3216) \times 45}{\frac{1}{5} \times \frac{1}{4}}$$

$$F = 2.0944 \times 45 \times 5 \times 4 = 1884.96 \text{ pounds.}$$

The kinetic energy in foot-pounds stored in the revolving body may be obtained by the formula:

$$V_a^2 \times \frac{I}{2} = \text{kinetic energy.}$$

Decreasing the angular velocity to V_{a1}, the stored-up energy will also decrease to

$$V_{a1}^2 \times \frac{I}{2}$$

and the work done by the revolving body will be

$$(V_a^2 - V_{a1}^2) \times \frac{I}{2}$$

EXAMPLE 1.

The moment of rotation in a fly-wheel is 1040; its angular velocity is 5 feet per second. What is the stored-up energy in the wheel?

Solution:

$$\text{Kinetic energy} = 5^2 \times \frac{1040}{2} = 13{,}000 \text{ foot-pounds.}$$

EXAMPLE 2.

At certain intervals, when machinery is started, the angular velocity of this fly-wheel is reduced to $4\frac{1}{2}$ feet per second. How many foot-pounds of energy has the fly-wheel given up in helping to drive the machinery?

Solution:

$$x = (5^2 - (4\tfrac{1}{2})^2) \times 520$$

$$x = (25 - 20\tfrac{1}{4}) \times 520$$

$x = 4\tfrac{3}{4} \times 520 = 2470$ foot-pounds of energy given out by the fly-wheel during this change of speed.

EXAMPLE 3.

How much stored energy is left in the wheel after its angular velocity is reduced to $4\frac{1}{2}$ feet per second?

Solution:

$$K = (V_a)^2 \frac{I}{2}$$

$$K = (4\tfrac{1}{2})^2 \times 520 = 20\tfrac{1}{4} \times 520 = 10{,}530 \text{ foot-pounds.}$$

The same result may be obtained by subtracting, thus:

$$13{,}000 - 2470 = 10{,}530 \text{ foot-pounds.}$$

Centrifugal Force.

The *centrifugal* force is the force with which a revolving body tends to depart from its center of motion and fly in a direction tangent to the path which it describes. The *centripetal* force is the force by which a revolving body is prevented from departing from the center of motion. When the centrifugal force exceeds the centripetal force the body will move away from the center of motion, but if the centripetal force exceeds the centrifugal force, the body will move toward the center of motion. The centrifugal force in any revolving body is equal to the mass of the body (see page 286) multiplied by the square of its velocity, and this product divided by the radius of the revolving body.

$$cf = \frac{W \times v^2}{32.2 \times r} = \frac{m \times v^2}{r}$$

$cf =$ Centrifugal force in pounds.
$r =$ Radius in feet.
$v =$ Velocity in feet per second.
$W =$ Weight of moving body in pounds.
$m =$ Mass of moving body.

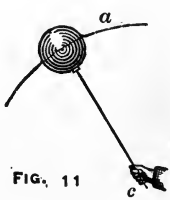


FIG. 11

EXAMPLE.

The weight a, in Fig. 11, is four pounds, and the length of the string is two feet; the weight is made to swing around the center c, three revolutions per second. What is the stress on the string due to centrifugal force?

Solution:

The distance from c, to the center of the ball is two feet, and making three revolutions per second, the velocity will be $2 \times 3 \times 3.1416 \times 2 = 37.7$ feet per second.

$$cf = \frac{4 \times 37.7 \times 37.7}{32.2 \times 2} = 88.2 \text{ pounds.}$$

In metric measure,

$$cf = \frac{W \times v^2}{9.81 \times r}$$

$cf =$ Centrifugal force in kilograms.

$r =$ Radius in meters.

$v =$ Velocity in meters per second.

$W =$ Weight of moving body in kilograms.

Example.

Suppose that the weight a, in Fig. 11, is five kilograms, swinging around the center, c, one revolution per second; the distance from a to c is 1½ meters. What is the stress on the string due to centrifugal force?

Solution:

The velocity will be $1.5 \times 3.1416 \times 2 = 9.4248$ meters per second.

$$cf = \frac{5 \times 9.4248 \times 9.4248}{9.81 \times 1\frac{1}{2}} = 30.2 \text{ kilograms.}$$

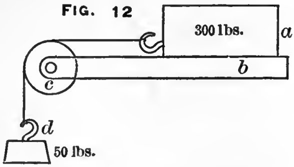
Friction.

The resistance which a body meets with from the surface on which it moves is called friction. It is called sliding friction when one body slides on another; for instance, a sleigh is pulled along on ice—the friction between the runners of the sleigh and the ice is sliding friction. It is said to be rolling friction when one body is rolling on another so that new surfaces continually are coming into contact; for instance, when a wagon is pulled along a road, the friction between the wheels and the road is rolling friction, but the friction between the wheels and their axles is sliding friction. Sliding friction varies greatly between different materials, as everybody knows from daily observation. For instance, a sleigh with iron runners can be pulled with less effort on ice than on sand, even if the road is ever so smooth. This is because the friction between iron and ice is a great deal less than the friction between iron and sand.

Coefficient of Friction.

The ratio between the force required to overcome the resistance due to friction and the weight of a body sliding along a horizontal plane is called coefficient of friction.

FIG. 12



For instance, in Fig. 12 a piece of iron weighing 300 lbs. rests on a horizontal plate *b*. A string fastened to *a*, goes over a pulley, *c*. At the end of the string is applied a weight, *d*. If this weight is increased until the body *a* just starts to move along on *b*, and the weight is found to be 50 pounds, the coefficient of friction will be $\frac{50}{300} = \frac{1}{6} = 0.166$

When the weight of a moving body is multiplied by the coefficient of friction, the product is the force required to keep the body in motion. Of course, any pressure applied to the moving body, perpendicular to its line of motion, may be substituted for its weight. For instance, the frictional resistance of the slide in a slide-valve engine is not due to the weight of the valve, but to the unbalanced steam pressure on the valve. In all cases the rule is:

Multiply the coefficient of friction by the pressure perpendicular to the line of motion, and the product is the force required to overcome the frictional resistance.

EXAMPLE.

The coefficient of friction is 0.1, and the weight of the sliding body is 800 pounds. What force is required to slide it along a horizontal surface?

Solution:

$$\text{Force} = 800 \times 0.1 = 80 \text{ pounds.}$$

Rolling Friction.

If the body, *a*, (see Fig. 12) was lifted up from the plane, *b*, high enough so that two rollers could be placed between *a* and *b*, it would be found that the body would move with much less force than 50 pounds because, instead of sliding friction, as in the first experiment, it would be rolling friction. Suppose it is found that *a* commenced to move when the load, *d*, was four pounds, then the coefficient of friction for this particular case would be $\frac{4}{300} = \frac{1}{75} = 0.0133$

In these experiments the whole force at *d* is not used to move the load *a*, as a small part of it is used to move the pulley at *c*, but in order to make the principle plain, this loss has not been considered.

Axle Friction.

The friction between bearings and shafts is frequently called axle friction. This, of course, is sliding friction, but owing to the fact that the surfaces in question are usually very smooth and well lubricated, the coefficient of friction is smaller than for ordinary slides.

EXAMPLE 2.

A fly-wheel weighs 24,000 pounds, the diameter of the shaft is 10 inches, and the coefficient of friction in the bearings is 0.08. What force must be applied 20 inches from the center in order to keep the wheel turning?

Resistance due to friction $= 24000 \times 0.08 = 1920$ pounds.

This resistance is acting at a radius of 5 inches, but the force is acting at a radius of 20 inches; therefore, the required force necessary to overcome friction will be $\frac{1920 \times 5}{20} = 480$ pounds. How much power is absorbed by this frictional resistance if the wheel is moving 72 revolutions per minute?

Solution:

The space moved through by the force is $\frac{72 \times 20 \times 2 \times 3.1416}{12}$ $= 753.984$ feet, and $753.934 \times 480 = 361{,}912.32$ foot-pounds and $\frac{361912.32}{33000} = 10.97$ horse-power.

Horse-Power Absorbed by Friction in Bearings.

The horse-power absorbed by the friction in the bearings for any shaft may be figured directly by the formula,

$$H\text{-}P = \frac{W \times f \times n \times 3.1416 \times d}{33000 \times 12}$$

This reduces to:

$$H\text{-}P = W \times f \times n \times d \times 0.000008$$

$H\text{-}P =$ Horse-power absorbed by friction.
$W =$ Load on bearings in pounds.
$d =$ Diameter of shaft in inches.
$f =$ Coefficient of friction.
$n =$ Number of revolutions per minute.

Calculating the previous example by this formula, we have:

$H\text{-}P = 24000 \times 0.08 \times 72 \times 10 \times 0.000008 = 11.06$ horse-power, which is practically the same as figured before.

Angle of Friction.

Suppose, instead of using the string and the weight *d* (see Fig. 12), that one end of the plane is lifted until *a* commences to slide; the angle between *b* and the horizontal line, when *a* commences to move, is called the angle of friction. The coefficient of friction may also be calculated from the angle of friction, thus: If the body commences to slide under an angle of *a* degrees, the coefficient of friction will be $\frac{\sin. a}{\cos. a} = \text{tang.}\, a$. Thus, the coefficient of friction is always equal to tangent of the angle of friction.

Rules for Friction.

1. Friction is in direct proportion to the pressure with which the bodies are bearing against each other.
2. Friction is dependent upon the qualities of the surfaces of contact.
3. The velocity has, within ordinary limits, no influence on the value of the coefficient of friction.
4. Sliding friction is greater than rolling friction.
5. Friction offers greater resistance against starting a body than it does after it is set in motion.
6. The area of surfaces of contact has, within ordinary limits, no influence upon the value of the coefficient of friction, but if they are unproportionally large or small the friction will increase.

TABLE No. 36.—Coefficient of Friction.

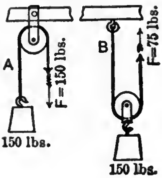
Materials.	Slides.		Bearings.	
	Well Lubricated.	Not well Lubricated.	Well Lubricated.	Not well Lubricated.
Cast-iron on wrought iron	0.08	0.16	0.05	0.075
Cast-iron on cast-iron	0.08	0.16	0.05	0.075
Wrought iron on brass	0.08	0.20	0.05	0.075
Wrought iron on wrought iron	0.10	0.20	0.05	0.075

Friction in Machinery.

When the surfaces are good the frictional resistance for slides may be assumed as 10 per cent., more or less, according to the conditions of the surfaces. It is always well not to take the coefficient of friction too small; it is better to be on the safe side and allow power enough for friction. In bearings for machinery, the frictional resistance ought not to absorb over six per cent. If more is wasted in friction, there is a chance for improvement.

Pulley Blocks.

FIG. 13.



When friction is not considered, the force and the load will be equal in a single fixed pulley (as A, Fig. 13).

Thus, a single fixed pulley does not accomplish anything further than to change the direction of motion. In a single movable pulley (as at B, Fig. 13), the force is equal to only half the load; thus, 75 pounds of force will lift 150 pounds of load, but the force must act through twice the space that the load is moved. The tension in any part of the rope in B is half of the load W; thus, when the load is 150 pounds the tension in the rope is 75 pounds, when arranged at B, but it is 150 pounds when arranged as at A.

FIG. 14.

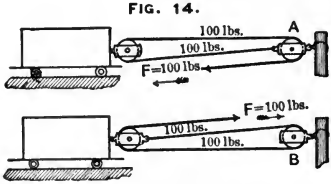


Fig. 14 shows a pair of single sheave pulley blocks in position to pull a car; when the blocks are arranged as at A, and friction is not considered, a force of 100 pounds on the hauling part of the rope exerts a force of 300 pounds on the post, but only 200 pounds on the car; but, turning the blocks end for end, as shown at B, a force of 100 pounds on the hauling part of the rope exerts a force of 300 pounds on the car and 200 pounds on the post. This is a point well worth remembering when using pulley blocks. Suppose, for instance, that a man exerted a force of 100 pounds on the hauling part, and that it required 250 pounds of force to move the car; if he used the pulley blocks as shown at A, his work would be useless, as far as moving the car is concerned, as he could not do it, but turning his blocks end for end he could accomplish the desired result. Always remember whenever it is possible to have the hauling part of the rope coming *from* the movable block and pull in the *same* direction as the load is moving.

Friction in Pulley Blocks.

In practical work, friction will have some influence, and, to a certain extent, change these results, because some of the tension in the rope is lost by friction in each sheave the rope passes over, therefore the tension in each following part of the rope is always less than it was in the preceding part. This loss must be obtained from experiments. In good pulley blocks, having roller bearings, this loss is probably not more than 0.1, and we get a useful effect of 0.9 of the force from one part of the rope to the next; therefore, when friction is considered, the useful effect in the following cases will be:

In single sheave blocks having the hauling part from the movable block (pulling *with* the load as in B, Fig. 14).

$$W = F\,(1 + 0.9 + 0.9^2)$$
$$W = F \times 2.71$$

In single sheave blocks having the hauling part from the fixed block (pulling *against* the load as in A, Fig. 14),

$$W = F\,(0.9 + 0.9^2)$$
$$W = F \times 1.71$$

In double sheave blocks having the hauling part from the *movable* block,

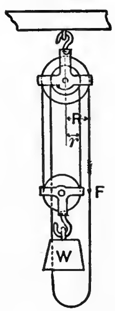
$$W = F\,(1 + 0.9 + 0.9^2 + 0.9^3 + 0.9^4)$$
$$W = F \times 4.1$$

In double sheave blocks having the hauling part from the *fixed* block,

$$W = F\,(0.9 + 0.9^2 + 0.9^3 + 0.9^4)$$
$$W = F \times 3.1$$

Differential Pulley Blocks.

FIG. 15.



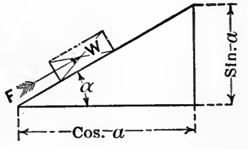
In a differential pulley block (see Fig. 15), the proportion between the force and the weight, when friction is neglected, is expressed by the formula:

$$F = \frac{W \times (R - r)}{2 \times R}$$

The actual force required to lift a weight by such a pulley block is about three times the theoretical force, as calculated above.

Inclined Plane.

FIG. 16.



When a weight is pulled upward on an inclined plane, as shown in Fig. 16, and the force F is acting parallel to the plane, the required force for moving the body will be $F = W \times \sin. a$ plus friction, and the perpendicular pressure P, against the plane will be $W \times \cos. a$.

EXAMPLE 1.

The weight, W, (Fig. 16) is 100 pounds; the angle a is 30°. What force, F, is required to sustain this weight, friction not considered?

Solution:

$$\text{Sin. } 30° = 0.5$$

Thus:

$$F = W \times \sin. 30° = 100 \times 0.5 = 50 \text{ pounds.}$$

EXAMPLE 2.

What is the perpendicular pressure under conditions stated in Example 1?

Solution:

$P = W \times \cos. a = 100 \times 0.86603 = 86.6$ pounds.

Therefore, the frictional resistance between the sliding body and the inclined plane will be only what is due to 86.6 pounds pressure; in other words, the force required to overcome friction will be $W \times f \times \cos. a$.

EXAMPLE 3.

What force is required to move the body mentioned in Example 1 when friction is also considered, taking coefficient of friction, F, as 0.15?

Solution:

$F = W (\sin. a + \cos. a \times f)$
$F = 100 \times (0.5 + 0.86603 \times 0.15) = 100 \times 0.6290 = 62.99$ pounds.

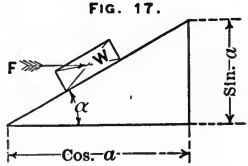
NOTE.—This is the force required for moving the load. In order to put it in motion more force must be applied, varying according to velocity, but after motion is commenced the speed would be, under these conditions, maintained forever by this force of 62.99 pounds.

When a load is moving down an inclined plane the force due to $W \times \sin. a$ will assist in moving the body, and if the product $W \times \sin. a$ exceeds the product $W \times \cos. a \times f$ the body will slide by itself. For instance, in the body mentioned in the previous example, the force required to overcome gravity, regardless of friction, is 50 pounds, and the force required to overcome friction is 12.99 pounds; thus, if the body should be let down the plane instead of pulled up, it would have to be held back with a force of $50 - 12.99 = 37.01$ pounds.

NOTE.—When the incline is less than 1 in 35, cosine is so nearly equal to 1 that it may be neglected, and the force required to overcome friction may be considered to be the same as on a level plane. For instance, a horse is pulling a load and ascending a gradient of 1 in 35; if the tractive force required to pull the load on a level road was 30 pounds and the weight of the load was 1400 pounds, when ascending the hill, the horse will first have to exert a force of 30 pounds, which is all due to friction, but beside that he must also exert a force of $\frac{1}{35}$ times 1400 = 40 pounds; thus the total pull exerted by the horse will be 70 pounds.

Inclined Plane With the Force Acting Parallel to the Base.

FIG. 17.



When the pressure is continually acting in a line parallel to the base of the incline, as F, (see Fig. 17) which is frequently the case in mechanical movements, as for instance, in screws, some kinds of cam motions, etc., it will require more force to move the body than it would if the force was acting parallel to the incline. When force acts parallel to the base, as in Fig. 17, the force required to move the body, if friction is not considered, will be:

$$F = \frac{W \times \sin. a}{\cos. a} = W \times \text{tang. } a$$

EXAMPLE 1.

What force is required to move 100 pounds upward an incline of 30°, as in Example 1, excepting that the force is acting parallel to the base instead of parallel to the incline?

Solution:

$$F = W \times \text{tang. } 30^\circ$$
$$F = W \times 100 \times 0.57735 = 57.74 \text{ pounds.}$$

When both the friction and the weight of the body are considered, the force required to move the body will be:

$$F = W \times \frac{\sin. a + (f \times \cos. a)}{\cos. a - (f \times \sin. a)}$$

EXAMPLE 2.

What force is required to move 100 pounds upward an incline of 30° (as in Example 1) if the force is acting parallel to the base line instead of parallel to the incline; coefficient of friction is supposed to be 0.15?

Solution:

$$F = 100 \times \frac{\sin. 30^\circ + (0.15 \times \cos. 30^\circ)}{\cos. 30^\circ - (0.15 \times \sin. 30^\circ)}$$

$$F = 100 \times \frac{0.5 + (0.15 \times 0.86603)}{0.86603 - (0.15 \times 0.5)}$$

$$F = 100 \times \frac{0.5 + 0.1277045}{0.86603 - 0.075}$$

$$F = 100 \times 0.7936 = 79.36 \text{ pounds.}$$

NOTE.—From these calculations it is seen that it is more advantageous to apply the force parallel to the incline than parallel to the base. When force is applied parallel to the incline:

The force required to overcome gravity = 50 pounds.
The force required to overcome friction = 12.99 pounds.
Total force = 62.99 pounds.

When the force is acting parallel to the base:

The force required to overcome gravity = 57.74 pounds.
The force required to overcome friction = 21.62 pounds.
Total force = 79.36 pounds.

Screws.

When friction is not considered, the force which may be exerted by a screw (see Fig. 18) will be:

$$W = \frac{F \times R \times 2\pi}{P} \qquad F = \frac{W \times P}{R \times 2\pi}$$

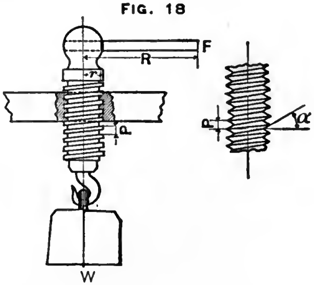
W = Weight of the load lifted, or force exerted, if the screw acts as a press.

F = Acting force.

R = Radius in inches at which the force acts.

P = Pitch of screw in inches.

FIG. 18



Regarding friction in screws, the thread of a screw may be considered as an inclined plane, of which the cos. is the middle circumference of the screw, the sin. is the pitch, and the force is acting parallel to the base. Hence the following formula:

$$F = W \times \frac{P + f \times d\pi}{d\pi - f \times P} \times \frac{r}{R}$$

F = Force, acting at a radius of R inches.

W = Weight.

P = Pitch of screw in inches.

f = Coefficient of friction, usually taken as 0.15.

R = Radius in inches at which the force is acting.

r = Middle radius of screw in inches.

d = Middle diameter of screw in inches.

EXAMPLE.

Find the force required to act on a lever 30 inches long (see Fig. 18) in order to lift the load W, which is 8000 pounds? The screw is ½-inch pitch and 1¼-inch middle radius; coefficient of friction, 0.15.

Solution:

$$F = 8000 \times \frac{0.5 + 0.15 \times 3.1416 \times 2.5}{2.5 \times 3.1416 - 0.15 \times 0.5} \times \frac{1.25}{30}$$

$$F = 8000 \times \frac{1.6781}{7.779} \times 0.0416 = 89.6 \text{ pounds.}$$

When the screw has **V** thread, the frictional resistance will be increased as $\frac{1}{\cos.}$ of the angle a (see Fig. 18), or equal to secant of half the angle of the thread. For United States standard screws the angle of thread is 60°, half the angle is 30°, and secant of 30° is 1.1547, and the formula will, for United States standard thread, become:

$$F = W \times \frac{P + d\pi\, 1.15\, f}{d\pi - 1.15\, f P} \times \frac{r}{R}$$

All the letters having the same meaning as in the formulas for the square-threaded screws.

The following table is calculated for square-threaded screws, the pitch of the screw being double that of the United States standard screw of same diameter. The depth of the thread is equal to its width. We see no good reason why the depth of a square-threaded screw should be, as frequently given in technical books, $\frac{19}{40}$ of the pitch of the screw; $\frac{20}{40}$, as given in previous tables, is more convenient, and also gives a little more wearing surface to the thread. The use of this table is so plain that it needs very little explanation. In the fourth column is the area of the outside diameter of the screw. In the fifth column, the sectional area of the screw at the bottom of the thread, which may be used in calculating the tensile and crushing strength of the screw. Subtracting the fifth column from the fourth gives the sixth column, which is the projected area of one thread; this may be used in calculating the allowable pressure on the thread, etc. The fourteenth column gives the tangential force which is required to act with a leverage of one foot in order to lift one pound by the screw if there was no friction. The fifteenth column gives the total tangential force required per pound of load when both load and friction are included. The sixteenth column gives the difference between the fourteenth and the fifteenth columns, and is the tangential force absorbed by friction alone. The coefficient of friction in both columns is assumed as 0.16. The last four columns in the table give the load or axial pressure which may be allowed on the screw corresponding to 200, 400, 600 and 1000 pounds pressure per square inch of projected area of screw thread when the length of the nut is twice the diameter of the screw.

Diameter of Screw.	Diameter of Screw at Bottom of Thread.	Length of Nut.	Area of Section of Screw.	Area of Section of Screw at Bottom of Thread.	Projected Area of Bearing Surface of One Thread.	Total Area of Bearing Surface of Threads in the Nut.	No. of Threads in a Nut twice as Long as Diam. of Screw.	Number of Threads per Inch.	Inch Pitch.	Thickness of Threading Tool.	Thickness of Threading Tool in Decimals of an Inch.	Depth of Thread.	Force in Decimals of a Pound which will move 1 Pound of Load when acting with 1 Foot Leverage.			Axial pressure corresponding to 200 pounds per square inch of projected area of thread.	Axial pressure corresponding to 400 pounds per square inch of projected area of thread.	Axial pressure corresponding to 600 pounds per square inch of projected area of thread.	Axial pressure corresponding to 1000 pounds per square inch of projected area of thread.
													Tangential Force required to Move Load, Friction not considered.	Tangential Force when both Load and Friction are considered.	Tangential Force Absorbed by Friction.				
½	0.346	1	0.196	0.094	0.102	0.663	6½	6½	2/13	1/13	.077	.077	.00204	.00496	.00292	132			
⅝	0.443	1¼	0.307	0.154	0.153	1.052	6⅞	5½	2/11	1/11	.091	.091	.00241	.00603	.00362	210	420		
¾	0.550	1½	0.442	0.238	0.204	1.530	7½	5	⅕	1/10	.100	.100	.00265	.00714	.00449	306	612	906	
1	0.750	2	0.785	0.442	0.344	2.752	8	4	¼	⅛	.125	.125	.0033	.0093	.0060	550	1100	1650	2752
1¼	0.964	2½	1.227	0.730	0.498	4.357	8¾	3½	2/7	1/7	.143	.143	.0038	.0112	.0074	871	1742	2614	4357
1½	1.166	3	1.767	1.067	0.700	6.300	9	3	⅓	⅙	.167	.167	.0044	.0133	.0089	1260	2520	3780	6300
1¾	1.350	3½	2.405	1.431	0.974	8.520	8¾	2½	⅖	⅕	.200	.200	.0053	.0158	.0105	1704	3408	5112	8520
2	1.554	4	3.142	1.887	1.255	11.295	9	2¼	4/9	2/9	.223	.223	.0059	.0181	.0122	2259	4518	6777	11295
2¼	1.804	4½	3.976	2.545	1.431	16.09	10⅛	2¼	4/9	2/9	.223	.223	.0059	.0198	.0139	3218	6436	9654	16090
2½	2.000	5	4.909	3.142	1.767	17.67	10	2	½	¼	.250	.250	.0066	.0218	.0153	3534	7068	10602	17670
2¾	2.250	5½	5.940	3.976	1.964	21.604	11	2	½	¼	.250	.250	.0066	.0224	.0158	4321	8642	12963	21604
3	2.428	6	7.069	4.634	2.435	25.57	10½	1¾	4/7	2/7	.286	.286	.0076	.0260	.0184	5114	10228	15342	25570
3¼	2.678	6½	8.296	5.641	2.655	30.20	11⅜	1¾	4/7	2/7	.286	.286	.0076	.0277	.0201	6040	12080	18120	30200
3½	2.884	7	9.621	6.514	3.106	35.33	11⅜	1⅝	8/13	4/13	.308	.308	.0082	.0298	.0216	7066	14132	21198	35330
3¾	3.084	7½	11.045	7.451	3.594	40.43	11¼	1½	⅔	⅓	.333	.333	.0089	.0321	.0232	8086	16172	24258	40430
4	3.334	8	12.566	8.709	3.856	46.27	12	1½	⅔	⅓	.333	.333	.0089	.0344	.0255	9254	18508	27762	46270
5	4.200	10	19.635	13.854	5.781	72.26	12½	1¼	⅘	⅖	.400	.400	.0106	.0469	.0363	14452	28904	43356	72260
6	5.108	12	28.274	20.508	7.766	103.84	13½	1⅛	8/9	4/9	.446	.446	.0118	.0493	.0374	20768	40536	62304	103840

The table on page 313 was calculated by the following formulas:

When friction is not considered:

$$\text{Force to balance load} = \frac{\text{Pitch in inches}}{12 \times 2\,\pi} = \frac{\text{Pitch in inches}}{75.4}$$

When both one pound of load and friction are considered,

$$\text{Force} = \left(\frac{\text{pitch in inches} + \text{middle circum.} \times f}{\text{middle circum.} - \text{pitch in inches} \times f}\right) \times \left(\frac{\text{middle radius}}{12}\right)$$

CALCULATIONS BY TABLE ON PRECEDING PAGE.

EXAMPLE 1.

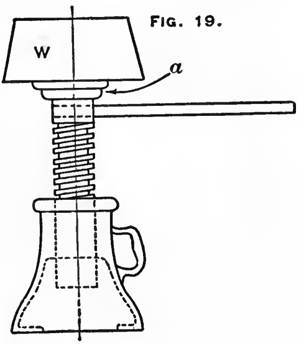


FIG. 19.

A jack screw, as shown in Fig. 19, is 1½″ diameter, three threads per inch. What tangential force is required to act with a leverage of 18 inches in order to lift 5000 pounds? Coefficient of friction in the thread is assumed as 0.16. Tangential force absorbed in friction by the collar at *a* is assumed to be equal to force absorbed by friction in the thread of the screw, and may, therefore, be taken from the thirteenth column in the table.

Solution:

Tangential force per pound at 1 foot radius = 0.0133
Tangential force absorbed by friction in collar = 0.0089

Total force per pound of load at 1 foot radius = 0.0222

The tangential force is acting with 18 inches leverage = 1½ feet, and the load is 5000 pounds; therefore, the required force will be,

$$F = \frac{0.0222 \times 5000}{1\tfrac{1}{2}} = 74 \text{ pounds.}$$

EXAMPLE 2.

A load of 16,000 pounds rests on a slide and is moved back and forth on a horizontal plane by a screw. The coefficient of friction between slide and plane is 0.1, and the screw should not be loaded with more than 400 pounds per square inch of projected area or thread. Find the suitable diameter of screw. If a pulley of 20-inch diameter is attached to the end of the screw, also find the tangential force required to act at the rim of the pulley in order to turn the screw.

Solution:

The coefficient of friction for the slide is 0.10, therefore the axial pressure on the screw will be $16{,}000 \times 1/10 = 1600$ pounds. The allowable force on a $1\frac{1}{4}$-inch screw will be found in the table to be 1742 pounds; therefore, select a screw of $1\frac{1}{4}$ inches diameter and a length of nut of $2\frac{1}{2}$ inches. Assuming the friction due to the reaction of the screw against its collar and bearing to be equal to the friction in the thread, and using the table, we have:

$$\begin{aligned} \text{Force per pound at one foot radius} &= 0.0112 \\ \text{Force absorbed by friction in collar} &= \underline{0.0074} \\ \text{Total force per pound of load at one foot radius} &= 0.0186 \end{aligned}$$

The leverage of a 20-inch pulley is 10 inches $= 10/12$ foot, and the axial force is 1600 pounds; therefore, the tangential force required at the rim of the pulley will be:

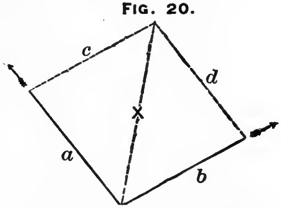
$$F = \frac{0.0186 \times 1600}{10/12} = 36.7 \text{ pounds.}$$

36.7 pounds is really the force required to keep the body in motion after it is started. To start the body from rest requires somewhat additional force, depending on the time used in overcoming its inertia. It is not certain that the friction due to the reaction of the screw against the collar is equal to the friction in the screw. It may be more or it may be less; this will, to a certain extent, depend on the size of the collar, and also on the finish of its surfaces, its means of lubrication, etc. Therefore, instead of assuming this resistance to be equal to the friction of the thread as found in column 16, it may be calculated for each individual case by assuming a proper coefficient of friction and assuming that this friction acts as resistance at a radius equal to the middle radius of the collar. If a screw is acting under the circumstances illustrated in Fig. 18, there is no collar to absorb any of the force by friction; but whenever the screw acts against a shoulder this friction must never be forgotten in calculation. Ball bearings may be used to very good advantage in the thrust collar on a screw. If a screw works a load continuously up and down, and the weight of the load always rests on the screw, it is necessary to be very careful and allow only a limited load on the screw (only a fraction of what is given in the table), because the pressure of the load always acts on the same side of the thread, and this is very disadvantageous for lubrication, as it does not give the oil a good chance to get onto the surfaces which rub against each other; but when the screw works a slide with an alternate push and pull, the wear comes on both sides of the thread, which gives a good chance for lubrication, and an axial pressure of 400 pounds per square inch of projected area of bearing surface in the thread will be safe, although, under certain circumstances, for instance, in a mechanism working continuously, such a load may be too much for the best results with regard to wear.

For anything working like a jack-screw, when the diameter of the screw is over one inch, the load given in the last column is perfectly safe. It is impossible to give rules which will suit all cases; the experience and judgment of the designer are the best guide with regard to the selection of the proper load. It may seem too much to use 0.16 as the coefficient of friction in the thread of the screw, but the author believes, from careful experiments made on common square-thread screws, as used in commercial machinery,—not made for experimental purposes, but for every-day use,—that this coefficient of friction is a safe average. It is well to remember that the surfaces of the thread on screws with cast-iron nuts do not always have the best of finish, and the nut especially is liable to be a little rough when new; therefore, this coefficient of friction may be a little greater than that found in screws in machinery when well lubricated and with surfaces smoothed down and glazed over from wear.

The Parallelogram of Forces.

FIG. 20.



A line may be drawn to such scale that its length represents a given force acting in the direction of the line. Another line is drawn to the same scale, from the same point of application, and its length represents another force acting in the same direction as this line. If these two lines are connected by two auxiliary lines, a parallelogram is formed and the diagonal of the parallelogram will represent both the magnitude and the direction of the resulting force.

EXAMPLE.

Let the lines *a* and *b* in Fig. 20 represent two forces acting in the direction of the arrows. Draw the lines to any scale, for instance, $\frac{1}{16}$ inch to a pound; if the force represented by *a* is 64 pounds, the line *a* will be $64 \times \frac{1}{16} = 4''$ long. If the force represented by *b* is 50 pounds, this line will be $50 \times \frac{1}{16} = 3\frac{1}{8}''$ long. Completing the parallelogram by drawing lines *c* and *d*, the diagonal, *x*, will indicate the magnitude and direction of the resulting force. Suppose these two forces act in such directions that when the parallelogram is completed and the diagonal drawn, it is, by measurement, found to be $4\frac{3}{4}''$ long $= \frac{76}{16}$; then the result of the two forces, *a* and *b*, is a force of 76 pounds. In many cases, the result of force and stress in machinery and structures may very conveniently be obtained in this way with much less labor than by calculation, and with accuracy consistent with good, legitimate practice.

HORSE-POWER.

The term *horse-power*, as applied in mechanical calculations, is 33,000 foot-pounds of work performed per minute, or 550 foot-pounds of work per second.

To Calculate the Horse-Power of a Steam Engine.

RULE.

Multiply the area of piston in square inches by the mean effective steam pressure, and this by the piston speed in feet per minute, and divide this product by 33,000. The quotient is the horse-power of the engine.

Formula:

$$\text{Horse-power} = \frac{0.7854\ D^2 \times p \times 2\ s \times n}{33000}$$

D = Diameter of piston in inches.

p = Mean effective steam pressure in pounds per square inch.

s = Length of stroke in feet.

n = Number of revolutions per minute.

EXAMPLE.

What is the horse-power of a steam engine of the following dimensions?

Cylinder, 20 inches diameter; length of stroke, 3 feet; number of revolutions per minute, 75; mean effective steam pressure in cylinder during the stroke, 60 pounds per square inch.

$$\text{Horse-power} = \frac{20^2 \times 0.7854 \times 2 \times 3 \times 75 \times 60}{33000}$$

$$\text{Horse-power} = \frac{314.16 \times 450 \times 60}{33000}$$

$$\text{Horse-power} = 257.04$$

To Calculate the Horse-Power of a Compound or Triple Expansion Engine.

RULE.

Calculate the mean effective pressure of the steam (according to its number of expansions and initial pressure), and calculate the horse-power exactly as if it was a single cylinder engine of the same size as the size of the last cylinder.

Another way is to take indicator diagrams of each cylinder, and calculate the power of each cylinder separately.

To Judge Approximately the Horse-Power which may be Developed by Any Common Single Cylinder Engine.

RULE.

Square the diameter of the piston in inches and divide by 2; the quotient is the horse-power which the engine may develop.

NOTE.—This rule gives the exact horse-power, if the product of the piston speed in feet and the average pressure per square inch in the cylinder is 21,000.

Horse-Power of Waterfalls.

RULE.

Multiply the quantity of water in cubic feet falling in a minute by 62.5; and multiply this by the height of the fall in feet; divide this product by 33,000, and the quotient is the horse-power of the waterfall. Or, multiply the quantity of water in cubic meters falling in a minute, by 1000, and multiply this by the height of the fall in meters; divide the product by 4500, and the quotient is the horse-power of the waterfall.

NOTE.—The above rules give the gross power of the waterfall, but the useful effect of the fall is a great deal less and will depend on the construction of the motor. It may be only from 40% to 80% of the natural power of the waterfall.

Animal Power.

Under favorable circumstances, a horse can perform 22,000 foot-pounds of work per minute. For instance, a horse walking in a circle turning the lever in a so-called horse-power may exert a pull of 100 pounds, walking at a speed of 220 feet per minute. For the horse to work to advantage, the diameter of the circle ought to be at least 25 feet.

Hauling a Load.

The average speed when horses are used in hauling a load one way and returning without load the other way, allowing for necessary stoppages, may not be more than 175 feet per minute, and, in estimating, time must also be allowed for loading and unloading. Loads may vary from 1000 to 2000 pounds, according to the road. Commonly speaking, the force required to pull a loaded wagon on a good, level road increases in proportion to the load and decreases in proportion to the diameter of the wheels, and on soft roads it is less with wide tires than with narrow ones. The idea that a wagon having small wheels would be easier to pull up-hill than one having larger wheels is a fallacy.

Power of Man.

A man may be able to do work at a rate of 4000 foot-pounds per minute; for instance, in turning a crank on a crane or derrick, a force of 15 pounds may be exerted on a crank, 18 inches long and, with 30 turns per minute, the work would be 4228 foot-pounds per minute.

NOTE.—In derricks, pulley blocks, jack-screws, etc., a large part of the expended power is consumed in overcoming friction.

Power Required to Drive Various Kinds of Machinery.

In the nature of the thing it is impossible from experiments on one machine to tell exactly what power it takes to run another similar machine, as there are so many different factors entering into the problem; for instance, the speed and feed on the machine, the hardness of the stock it works on, the quality of the tools used, the kind of lubrication, etc. Therefore, such assertions are only approximations at the best.

16-inch engine lathe, back geared,	¾ horse-power.
26-inch engine lathe, back geared,	1¼ horse-power.
Planer, 22″ × 22″ × 6 feet,	½ horse-power.
Planer, 32″ × 32″ × 10 feet,	¾ horse-power.
Shaping machine, 10-inch stroke,	¼ horse-power.
20-inch drill press,	½ horse-power.
26-inch drill press, back gear, boring a 3-inch hole, using boring bar,	1 horse-power.
Plain milling machines (Lincoln pattern, No. 2),	1¼ horse-power.
Small Universal milling machines,	½ horse-power.
Circular saws (for wood), 24″ diameter (light work),	3½ horse-power.
Circular saws (for wood), 36″ diameter (light work),	6 horse-power.
Fan blower for cupola, melting four tons of iron per hour,	10 horse-power.
Fan blower for five blacksmith fires,	1 horse-power.
Drop hammer, 800 pounds,	8 horse-power.

In machine shops and similar places, from 40% to 70% of the total power required is consumed in running the line shafting and counter-shafts. An average of from 55% to 60% is probably the most common ratio.

In exceptionally well-arranged establishments, under favorable conditions, in light manufacturing it may be possible that only 30% of the power is consumed in driving line and counter shafting, and that 70% is used for actual work.

SPEED OF MACHINERY.

The peripheral velocity of circular saws ought not to exceed 10,000 feet per minute. Table No. 37 gives the number of revolutions per minute for circular saws of different diameters.

TABLE No. 37.

Diameter of saw in inches.	8	10	12	14	16	20	24	28	32
Number of revolutions per minute.	4500	3600	3000	2585	2222	1800	1500	1285	1125

Band Saws.

Small band saws, such as are usually used in carpenter shops, have a velocity of 3600 feet per minute. The reason why band saws are run so much slower than circular saws is that if the band saw is given too much speed the blade will be pulled to pieces in starting and stopping.

Drilling Machines for Iron.

For drilling steel, the surface speed of a drill should not exceed 15 feet per minute; cast-iron, 22 feet; brass, 27 feet; malleable iron, 25 to 30 feet per minute. The feed will vary according to the hardness of the stock. In cast-iron a ¼″ drill will drill a hole 1″ deep in 125 revolutions. A ½″ drill will drill a hole 1″ deep in 120 revolutions. A 1″ drill will drill a hole 1″ deep in 100 revolutions.

Lathes.

Cast-iron may be turned at a speed of 32 feet per minute when Muchet steel is used for tools. Thus, lathes are usually calculated to have a velocity of about 30 to 32 feet on the slowest speed, supposing that as large a diameter as the lathe will swing is turned.

For wood-turning the surface speed may be from 3000 to 6000 feet per minute; but when the article to be turned is out of balance the speed must be considerably slower.

Planers.

Cast-iron is planed at a speed of 25 to 27 feet per minute; wrought iron, 21 feet; steel, 16 feet per minute. A planer ought to return at least three times as fast as it goes forward.

Milling Machines.

Rotating cutters working on Bessemer steel or other materials of about equal hardness usually have a surface speed of about 40 feet per minute. Oil is used for lubrication. Cast-iron is milled without oil.

Grindstones.

When grindstones are used to grind steel and iron in manufacturing, they work at a surface speed of 2000 to 2500 feet per minute, but grindstones for common shop use, to grind tools, chisels, etc., run at much slower speed.

Emery Wheels and Emery Straps.

Emery wheels and straps do good work at a speed of 5000 to 6000 feet per minute, but all such high-speed machinery, especially grindstones and emery wheels, must be used very carefully and special attention paid to the strength, so that they will not break under the stress of centrifugal force.

Calculating Size of Pulleys.

TO FIND SIZE OF PULLEY ON MAIN SHAFT.

Multiply the diameter of pulley on counter-shaft by its number of revolutions per minute, and divide this product by the number of revolutions of the main shaft, and the quotient is the diameter of the pulley on the main shaft.

EXAMPLE.

A main shaft makes 150 revolutions per minute; the counter-shaft has a pulley 9 inches in diameter and is to make 400 revolutions per minute. What size of pulley is required on the main shaft?

Solution:

$$\text{Diameter of pulley} = \frac{400 \times 9}{150} = 24 \text{ inches.}$$

TO FIND SIZE OF PULLEY ON COUNTER-SHAFT.

RULE.

Multiply the diameter of pulley on the main shaft by its number of revolutions per minute, and divide this product by the number of revolutions of the counter-shaft; the quotient is the diameter of the pulley on the counter-shaft.

EXAMPLE.

The pulley on a main shaft is 36 inches in diameter and it makes 150 revolutions per minute; the counter-shaft is to make 450 revolutions per minute. What size of pulley is required?

Solution:

$$\text{Diameter of pulley} = \frac{36 \times 150}{450} = 12 \text{ inches.}$$

TO FIND THE NUMBER OF REVOLUTIONS OF THE COUNTER SHAFT.

RULE.

Multiply the diameter of pulley on the main shaft by its number of revolutions per minute and divide this product by the diameter of pulley on the counter-shaft, and the quotient is the number of revolutions of the counter-shaft per minute.

EXAMPLE.

The pulley on a main shaft is 24 inches in diameter and makes 150 revolutions per minute, and the pulley on the counter-shaft is 15 inches in diameter. How many revolutions per minute will the counter-shaft make?

$$\text{Number of revolutions} = \frac{24 \times 150}{15} = 240 \text{ revolutions per minute.}$$

To Calculate the Speed of Gearing.

In calculating the speed of gearing, use the same rules as for belting, but take the number of teeth instead of the diameter.

EXAMPLE.

The back gearing on a lathe consists of a gear and pinion of 8 pitch, 96 teeth and 32 teeth, and the other gear and pinion are 10 pitch, 120 teeth and 40 teeth. How many revolutions will the cone pulley make while the spindle makes one revolution?

Solution:

$$\text{Cone pulley makes} = \frac{96 \times 120}{32 \times 40} = 9 \text{ revolutions.}$$

Efficiency of Machinery.

Divide the energy given out by a machine by the energy put into the same machine; multiply the quotient by 100, and the result is the per cent. of efficiency of the machine.

EXAMPLE.

A dynamo requires 15 horse-power, but the electrical power given out is only 12 horse-power. What is the efficiency?

Solution:

$$\text{Efficiency} = \frac{12}{15} \times 100 = 80\%$$

A steam engine is to develop 60 horse-power net. What will be the gross horse-power if the efficiency is 75%?

Solution:

$$\text{Gross power} = \frac{60 \times 100}{75} = 80 \text{ horse-power.}$$

CRANE HOOKS.

FIG. 1.

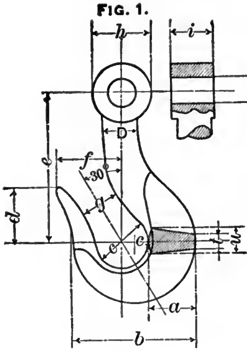


FIG. 2.

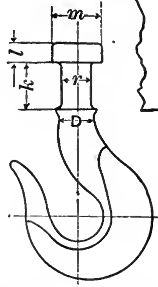
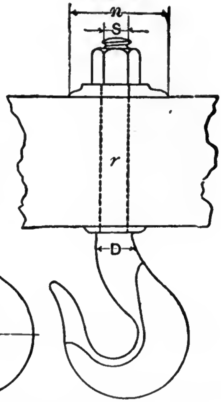


FIG. 3.



Crane hooks, as shown in Figs. 1, 2 and 3, may be designed by the following formulas :

$$P = D^2 \qquad D = \sqrt{P}$$

$P =$ Load in tons.

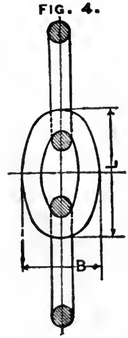
$D =$ Diameter of iron in inches.

$a = 1\frac{1}{2}\ D$	$b = 3\frac{7}{8}\ D$	$c = 1\frac{5}{8}\ D$	$d = 1\frac{1}{2}\ D$
$e = 4\frac{1}{2}\ D$	$f = 2\ D$	$g = 1\frac{1}{4}\ D$	$h = 1\frac{3}{4}\ D$
$i = 1\frac{1}{4}\ D$	$j = \frac{7}{8}\ D$	$t = \frac{1}{4}\ D$	$u = \frac{7}{8}\ D$
$l = \frac{3}{4}\ D$	$m = 1\frac{1}{2}\ D$	$k = 1\frac{3}{8}\ D$	$r = \frac{7}{8}\ D$

$S =$ Standard screw of diameter r $\qquad n = \sqrt{16\ D}$

When a rectangular iron plate is substituted for a washer, the bearing surface of the plate against the wood should at least be equal to the area of the washer, calculated by the above formula.

FIG. 4.



Chain Links.

(See Figure 4.)

$D =$ Diameter of iron.

$L = 4\frac{1}{2}$ to 5 D.

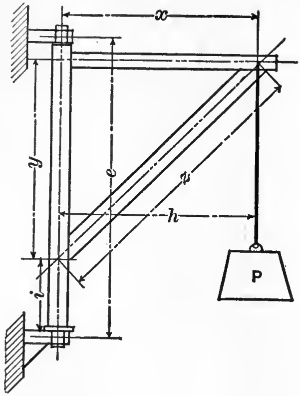
$B = 3\frac{1}{2}\ D$.

(For strength of chains, see page 222).

CRANES.

Cranes and derricks are machines used for raising and lowering heavy weights. In its simplest form, a crane consists of three principal members: The upright post, the horizontal jib and the diagonal brace. (See Fig. 5). The weight P will produce tensile stress in the jib, compressive stress in the brace, and both compressive and transverse stress in the post.

FIG. 5.



$$\text{Tension in jib} = \frac{P \times x}{y}$$

$$\text{Compression in brace} = \frac{P \times z}{y}$$

$$\text{Stress in the upper bearing} = \frac{P \times h}{e}$$

When the post is held at both ends, as in Fig. 5, it may, with regard to transverse strength, be considered as a beam of length i, fastened at one end and loaded at the other with a load equal to the force $\frac{h \times P}{e}$

The compression on the post caused by the load is equal to P.

The downward pressure on the lower bearing is equal to the sum of the weight of the crane and the load which it supports.

Proportions for a Two-Ton Derrick

(Of the construction shown in Fig. 6).

Pulley blocks should be double-sheave (only single are shown in the cut). Circumference of manila rope, 3¼ inches. Mast, 8 × 8 inches, 26 feet long. Boom, 7 × 7 inches, 20 feet long.

Large gear, 72 teeth, 1-inch circular pitch, 2-inch face. Small pinion, 12 teeth, 1-inch circular pitch, 2-inch face. Crank shaft. 1½ inches in diameter. Bearings, 2½ inches long. Crank, 18 inches long, Drum, 7 inches in diameter, 24 inches long. Drum-shaft, 2¼ inches in diameter. The drum and large gear are fitted and keyed to the drum shaft and also bolted together, thereby relieving this shaft from twisting stress.

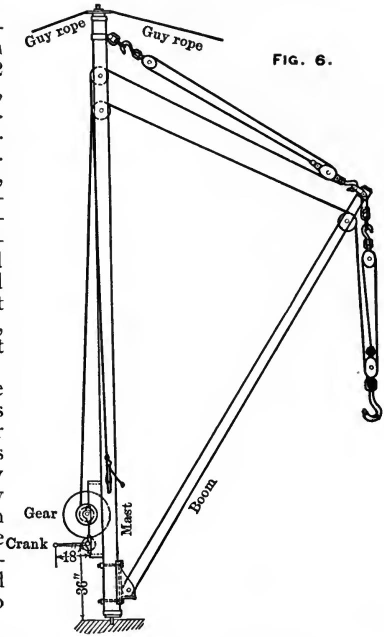


FIG. 6.

The radius of the drum added to the radius of the rope makes four inches, and the force is multiplied five times by the double-sheave pulley block; therefore, when the friction in the mechanism is not considered, the force required on the crank in order to lift 4400 pounds will be:

$$F = \frac{4 \times 12 \times 4400}{18 \times 72 \times 5} = 33 \text{ pounds, very nearly.}$$

Thus, when two men are working the derrick (one at each crank), each man has to exert a force of 16½ pounds, but, including friction, each man probably exerts a force of 20 to 25 pounds, when the derrick is loaded to its full capacity.

For very rapid work it is necessary to have four men (two on each winch-handle) to work the derrick, if it is kept loaded to its maximum capacity, but for ordinary stone work such a derrick is usually worked by two men. Stones as heavy as two tons are seldom handled, except where larger derricks and steam power are used.

When the derrick is to be worked constantly, the limit of the average stress on the crank handle to be allowed for each man is 15 pounds. When working an 18-inch crank, 48 turns per minute, this corresponds to a force of 15 pounds acting through a space of a little over 220 feet = 3300 foot-pounds of work per minute = $\frac{1}{10}$ horse-power.

When the crank swings in a shorter radius a few more turns per minute may be expected, but experience indicates that an 18″ radius is the most practical proportion.

BELTS.

Oak-tanned leather is considered the best for belting. The so-called "short lap" is cut lengthwise from the middle of the back of the hide, where it has the most firmness and strength. Single belting more than three inches in width is about $\frac{3}{16}''$ thick, and weighs 15 to 16 ounces per square foot; when less than three inches in width it is usually $\frac{5}{32}''$ thick and weighs about 13 ounces to the square foot.

Light double belts, as used for dynamos and other machinery having pulleys of comparatively small diameter, are about $\frac{9}{32}''$ thick and weigh about 21 ounces per square foot. Double belting, as used for main belts, is a little heavier and weighs from 25 to 28 ounces per square foot. Belts as heavy as 30 ounces per square foot are frequently used, and are usually termed "heavy double." Large engine belts are sometimes made with three thicknesses of leather.

Belts should be soft, pliable and of even thickness. When a belt is of uneven thickness and has very long joints, so that it looks as if it was partly single and partly double, it is very doubtful if it will do good service, for this is a sure sign that the thin and flimsy parts of the hide have been taken into the stock in making the belt.

The ultimate tensile strength of leather belting is from 2600 to 4800 pounds per square inch of section. Thus, a leather belt $\frac{3}{16}''$ thick will break at a stress of 500 to 900 pounds per inch of width.

The lacing of belts will reduce their strength from 50 to 60 per cent.; therefore, when practicable, belts ought to be made endless by cementing instead of lacing.

A belt will transmit more power, wear better and last longer, if it is run with the grain side next to the pulley.

Belts should never be tighter than is necessary in order to transmit the power without undue slipping; too tight belts cause hot bearings, excessive wear and tear, and loss of power in overcoming friction; but, on the other hand, it is necessary to have a belt tight enough to prevent it from slipping on the pulley, because if a belt slips there is not only a direct loss in velocity, but the belt will wear out in a short time; it is, therefore, very important to use belts of such proportions that the power shall be transmitted with ease.

Belts always run toward the side of the pulley which is largest in diameter (therefore pulleys are crowned, in order to keep the belt running straight).

A belt will always run toward the side where the centers of the shafts are nearest together.

Open belts will cause two shafts to run in the same direction.

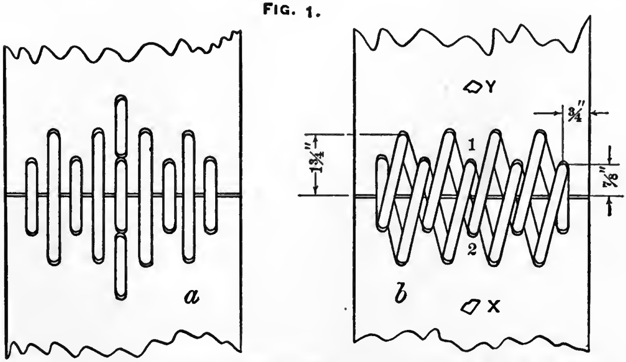
A crossed belt will cause the shafts to run in opposite directions. If the distance between the shafts is short, crossed belts will not work well. A short belt will wear out faster than a long one.

Very long and heavy belts should be supported by idlers as well under the slack as under the working side; if not, the weight of a long belt will cause too much stress on itself and also cause too much pressure on the bearings, as well on the driver as on the driven shaft. Belts should never, when it can be avoided, be run vertically, as the weight of the belt always tends to keep it away from the lower pulley, thereby reducing its transmitting capacity; the longer the belt the worse this is. Belts are most effective when they are run in a horizontal direction and, whenever possible, the lower part of the belt should be the working part, as the slackness in the upper part, by its weight, will cause the belt to lay around the pulley for a longer distance, and this will, in a measure, increase its transmitting capacity; but if the upper part is the working part, the slackness in the lower part tends to keep the belt away from the pulleys, and thereby reduces its transmitting capacity.

Lacing Belts.

Figure 1 shows a good way of lacing belts; *a* is the side running next to the pulley and *b* is the outside. Holes should be punched and not made by an awl, as punched holes are less liable to tear. The lacing is commenced by putting each end of the lace through holes 1 and 2 from the side next to pulley, and then continuing toward the edges, both sides simultaneously,

FIG. 1.



making a double stitch at the edges and sewing back again until holes 1 and 2 are reached; and, lastly, by drawing each end of the lace through *x* and *y*. Each stitch will be double, excepting the middle one. The holes x and y, where the ends of the lacing are finally drawn through for fastening, are made by the belt awl and should always be made small, and the lacing, if laid out rightly, always enters these holes from the *inside* of the belt; after it is pulled through, a small cut is made in the lacing on the outside, which will prevent it from drawing back again, then the ends are cut off about ½″ long, as shown in the figure at x and y. It is a bad practice to leave the lace-ends on the inside of belts, because they will then soon wear off, allowing the joint to rip.

A 1-inch belt ought to have three lace-holes in each end. Length of lacing, 12 inches.

A 2-inch belt ought to have three lace-holes in each end. Length of lacing, 18 inches.

A 3-inch belt ought to have five lace-holes in each end. Length of lacing, 24 inches.

A 4-inch belt ought to have five lace-holes in each end. Length of lacing, 32 inches.

A 5-inch belt ought to have seven lace-holes in each end. Length of lacing, 40 inches.

A 6-inch belt ought to have seven lace-holes in each end. Length of lacing, 48 inches.

An 8-inch belt ought to have nine lace-holes in each end. Length of lacing, 60 inches.

A 10-inch belt ought to have eleven lace-holes in each end. Length of lacing, 72 inches.

A 12-inch belt ought to have thirteen lace-holes in each end. Length of lacing, 84 inches.

Always have the row having the most holes nearest the end of the belt.

Cementing Belts.

When belts are cemented together, a 3-inch belt is lapped four inches and a 4-inch belt 4½ inches. In larger belts the lap is usually made equal to the width of the belt, but it may be made even shorter when the width of the belt is over 12 inches. The two ends are jointed together, so that the thickness is even with the rest of the belt.

The *American Machinist*, in answer to Question No. 430, Dec. 5, 1895, says: "For leather belts take of common glue and American isinglass equal parts; place them in a glue pot and add water sufficient to just cover the whole. Let it soak 10 hours, then bring the whole to a boiling heat, and add pure tannin until the whole appears like the white of an egg. Apply warm. Buff the grain of the leather where it is to be cemented; rub the joint surfaces solidly together, let it dry for a few hours, and the belt will be ready for use. For rubber belts take 16 parts gutta percha, 4 parts India rubber, 2 parts common caulker's pitch, 1 part linseed oil; melt together and use hot. This cement can also be used for leather."

Length of Belts.

Small belts, such as 4 inches wide or less, will work well when the distance between the shafts is from 12 to 15 feet, larger belts when from 20 to 25 feet, and for large main belts 25 to 30 feet distance is satisfactory.

Horse-Power Transmitted by Belting.

A single belt weighing about 15 ounces per square foot is capable of transmitting one horse-power per inch of width, when running at a speed of 800 feet per minute over pulleys of proper size, both of equal diameter. As one horse-power is 33,000 foot-pounds of work per minute, this will make the tension due to the power the belt is transmitting $= \frac{33000}{800} = 41\frac{1}{4}$ lbs. per inch of width, but the total tension in the belt is, of course, considerably more per inch of width, because the belt must be tight enough to prevent its slipping on the pulley. For belts lighter than 15 ounces per square foot it is better to allow 1000 running feet per horse-power per inch of width of belt. For light double belts weighing 21 ounces per square foot, 600 running feet per horse-power per inch of width may be allowed. For double belts weighing 25 ounces per square foot, 500 running feet per horse-power per inch of width may be allowed. Hence the following formulas:

For light single belts weighing less than 15 ounces per square foot,

$$H = \frac{v \times b}{1000} \qquad b = \frac{H \times 1000}{v}$$

For single belts weighing 15 to 16 ounces per square foot,

$$H = \frac{v \times b}{800} \qquad b = \frac{H \times 800}{v}$$

For light double belts weighing about 21 ounces per square foot,

$$H = \frac{v \times b}{600} \qquad b = \frac{H \times 600}{v}$$

For double belts weighing about 25 ounces per square foot,

$$H = \frac{v \times b}{500} \qquad b = \frac{H \times 500}{v}$$

H = Horse-power.

b = Width of belt in inches.

v = Velocity of belt in feet per minute, which will be diameter of pulley in inches multiplied by 3.1416 and by the number of revolutions per minute, and the product divided by 12.

EXAMPLE 1.

A double belt 10 inches wide, weighing 25 ounces per square foot, runs over 50-inch pulleys, making 240 revolutions per minute. How many horse-power will it properly transmit?

Solution:

$$\text{Velocity of belt} = \frac{50 \times 3.1416 \times 240}{12} = 3141.6 \text{ ft. per minute.}$$

$$H = \frac{3141.6 \times 10}{500} = 62.8 \text{ horse-power.}$$

EXAMPLE 2.

One hundred horse-power is to be transmitted by a double belt weighing 25 ounces per square foot. The pulleys are 66 inches in diameter and make 150 revolutions per minute. What is the necessary width of belt?

Solution:

Pulleys of 66 inches diameter, running 150 revolutions per minute, will give a belt speed of $\frac{150 \times 3.1416 \times 66}{12} = 2591.8$; say, 2592 feet per minute.

$b = \frac{100 \times 500}{2592} = 19.3$ inches; thus, a double belt 20 inches wide will do the work.

EXAMPLE 3.

A light single belt 4 inches wide, weighing 13 ounces per square foot, runs over pulleys of 36 inches diameter, making 100 revolutions per minute. How many horse-power may be transmitted?

Solution:

$$\text{Velocity of belt} = \frac{36 \times 3.1416 \times 100}{12} = 942.48 \text{ ft. per minute.}$$

The belt is a light single belt and its transmitting capacity will be, $H = \frac{4 \times 942.48}{1000} = 3.76992$, about $3\frac{3}{4}$ horse-power.

To Calculate Size of Belt for Given Horse-Power when Diameter of Pulley and Number of Revolutions of Shaft Are Known.

The following formulas may be used for calculating belt transmission, and will give results approximately consistent with previously given rules, but they are more convenient for use, as the velocity of the belt does not need to be first calculated, but the velocity of the belt must not exceed the practical limit.

This formula will do for either single or double leather belts with cemented joints (no lacing), of any weight from 12 to 30 ounces per square foot and of any width from one to thirty inches, when the pulleys are of suitable size to correspond with the thickness of the belt, and the diameter of both pulleys is equal or nearly so:

$$H = \frac{d \times n \times b \times w}{50000} \qquad d = \frac{H \times 50000}{n \times b \times w}$$

$$n = \frac{H \times 50000}{d \times b \times w} \qquad b = \frac{H \times 50000}{d \times n \times w} \qquad w = \frac{H \times 50000}{d \times n \times b}$$

H = Horse-power transmitted by the belt.
d = Diameter of pulley in inches.
n = Number of revolutions per minute.
b = Width of belt in inches.
w = Weight of belt in ounces per square foot.
50,000 is constant.

EXAMPLE.

Calculate Example 2 by the above formula.

Solution:

$b = \frac{100 \times 50000}{66 \times 150 \times 25} = 20.2$ inches, which, for all practical purposes, is the same as the result when calculated by the other rule.

Wide and thin belts are unsatisfactory. It is far better when transmitting power to use double and narrow rather than single and wide belts. It is a very bad practice to run at too slow belt speed, and also to use pulleys of too small diameter. The smallest pulley for a light double belt should never be less than 12″ in diameter, for a heavy double belt never less than 20″ in diameter, and for a triple belt the pulley should not be less than 30″ in diameter.

To Calculate Width of Belt when Pulleys are of Unequal Diameter.

When the pulleys are of different diameters the belt will lay around the smallest pulley less than 180 degrees, and the transmitting capacity of the belt is correspondingly reduced. The pressure on the pulley due to the tension of the belt will vary as the sine of half the angle of contact, and the adhesion of the belt to the pulley will vary as the pressure; consequently, also, the transmitting capacity of the belt will vary as the sine of half of the angle of contact, but it is usually advisable in practice to allow a little more on the width of the belt than is called for by this rule. A practical rule is:

First calculate the width of the belt by the above rules and formulas, as though both pulleys had the same diameter, then multiply the result by the following constants, according to the arc of contact between the belt and the small pulley.

When the arc of contact between the belt and the small pulley is 90° multiply by 1.60.

100°	"	"	1.45	140°	multiply	by	1.15
110°	"	"	1.35	150°	"	"	1.10
120°	"	"	1.25	160°	"	"	1.06
130°	"	"	1.20	170°	"	"	1.04

EXAMPLE.

The pulley on a dynamo is 15″ in diameter, and it makes 1200 revolutions per minute. The driving pulley is so large that the belt only lays around the dynamo pulley for a distance of 150 degrees. What is the necessary width of a light double belt, weighing 21 ounces per square foot, when it takes 40 horse-power to run the dynamo?

Solution:

If the arc of contact had been 180 degrees the belt would be $b = \frac{40 \times 50000}{1200 \times 15 \times 21} = 5.3$ inches wide, but as the arc of contact is not 180 degrees, but only 150 degrees, this width is multiplied by the constant 1.10, as given in the preceding table. Thus, the width of the belt will be $5.3 \times 1.1 = 5.83$ inches or, practically, a belt six inches wide is required.

When belts are running in a horizontal direction, and the driven pulley and the driver are of equal diameter and finish, the belt will always, when overloaded, commence to slip on the driver, and when pulleys are of unequal size it is always more favorable for the belt when the driving pulley is the larger than when *vice versa*.

To Find the Arc of Contact of Belts.

Make a scale drawing of the pulleys and the belt, and measure the arc of contact from the drawing by means of a protractor, or the arc of contact in degrees on the small pulley for an open belt may be calculated by the formula:

$$\text{Cosine of half the angle} = \frac{R - r}{l}$$

R = Radius of large pulley in inches.

r = Radius of small pulley in inches.

l = Distance in inches between centers of the shafts.

EXAMPLE.

The distance between centers of two shafts is 16 feet; the large pulley is 60 inches and the small pulley is 20 inches in diameter. What is the arc of contact of the belt?

Solution:

16 feet = 192 inches.

60 inches diameter = 30 inches radius.

20 inches diameter = 10 inches radius.

$$\text{Cos. of half the angle} = \frac{(30 - 10)}{192} = 0.104$$

In tables of natural cosine (page 158), the corresponding angle is found to be 84 degrees, very nearly; thus, the angle for arc of contact will be $2 \times 84 = 168$ degrees on the small pulley. On the large pulley the arc of contact will be $360 - 168 = 192$ degrees.

For a crossed belt the arc of contact is always the same on both pulleys, and it may be calculated by the formula:

$$\text{Cos. of half the angle} = -\frac{R + r}{l}$$

R = Radius of large pulley.

r = Radius of small pulley.

l = Distance between centers.

EXAMPLE.

What will be the arc of contact for the belt on the pulleys in the previous examples if belt is run crossed instead of open?

Solution:

Cosine of half the angle $= -\dfrac{30 + 10}{192} = -0.208$; the corresponding angle will be $180 - 77 = 103$ degrees, and the arc of contact will be $103 \times 2 = 206$ degrees.

Pressure on the Bearings Caused by the Belt.

Approximately, the pressure on the bearings caused by the belt may be considered to be three times the force which the belt is transmitting. Therefore, the pressure may be calculated by the formula:

$$P = \frac{3 \times 33000 \times H}{v}$$

P = Pressure on the bearings due to pull of belt.

H = Number of horse-power transmitted by the belt.

v = Velocity of belt in feet per minute.

EXAMPLE 1.

A belt is transmitting 60 horse-power and its velocity is 900 feet per minute. What is the pressure in the bearings due to the belt?

Solution:

$$P = \frac{3 \times 33000 \times 60}{900} = 6600 \text{ pounds.}$$

EXAMPLE 2.

Suppose the diameters of the pulleys are increased until a belt speed of 3000 feet per minute is obtained. What will then be the pressure in the bearings caused by the belt when transmitting 60 horse-power?

Solution:

$$P = \frac{3 \times 33000 \times 60}{3000} = 1980 \text{ pounds.}$$

By the above examples it is conclusively shown what a great advantage there is in using pulleys so large in diameter that proper belt speed is obtained. (See velocity of belts, page 337).

The approximate pressure may also be very conveniently obtained from the width of the belt, thus: For light single belts, allow 1000 feet of belt speed per horse-power transmitted per inch of width of belt. The effective pull in such a belt will be 33 pounds per inch of width, and the pressure on the bearings due to the belt will accordingly be $33 \times 3 = 99$ pounds per inch of width of belt. For convenience, say 100 pounds pressure in the bearings per inch of width of such belts. For belts where 800 running feet are allowed per horse-power per inch of width of belt, this reasoning will give a pressure on the bearing equal to 123¾ pounds per inch of belt. For convenience, say 125 pounds pressure in the bearings per inch of width of such belts. For belts where 600 running feet are allowed per horse-power per inch of width, the pressure in the bearing is equal to 165 pounds per inch of width of belt, and where the belt is so heavy that only 500 feet of belt speed per horse-power per inch of width is allowed, the pressure in the bearings will be 198 pounds per inch of width. A good, practical rule, which can very easily be remembered, is, (when belts are in good order and have the proper size and the proper tension):

Multiply weight of belt in ounces per square foot by eight times the width of the belt in inches, and the product is approximately the pressure in pounds upon the bearings caused by the belt.

EXAMPLE.

A belt is calculated with regard to the horse-power it has to transmit under a given velocity, and found to be 8-inch double belting, weighing 25 ounces per square foot. What pressure will it cause on the bearings when working at proper tension?

Solution, by the last rule:

$$P = 8 \times 25 \times 8 = 1600 \text{ pounds.}$$

Solution, by the first rule:

At a speed of 3000 feet per minute such a belt will transmit $\frac{3000 \times 8}{500} = 48$ horse-power, and calculating the pressure by the formula:

$$P = \frac{3 \times 33000 \times H}{v}$$

$$P = \frac{3 \times 33000 \times 48}{3000} = 1584 \text{ pounds.}$$

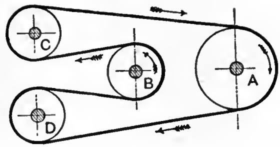
Both rules give nearly the same result, and one is just as correct as the other, as all such figuring is nothing more than approximation at the best. The pressure on the bearings may be a great deal more than calculated above. Sometimes the pulleys are roughly made, belts are poor, and consequently the coefficient of friction between belt and pulley is small, and as the belt has to be a great deal tighter in order to do the work, the pressure on the bearing will be greatly increased. Very frequently, from pure ignorance or carelessness, belts are made very much tighter than necessary, and enormous sums of money may be wasted in this way in large factories, as the steam engines, at the expense of the coal pile, have to furnish power not only to do the useful work, but also to overcome all the friction produced by such over-strained belts, hot bearings, etc. A belt will transmit more power over a good, smooth pulley than over a rough one. When pulleys are covered with leather a belt will transmit about 25% more power than it will when running over bare iron pulleys, and in transmitting the same power a much slacker belt may be used, thereby reducing the friction in the bearings.

Special Arrangement of Belts.

By the use of suitable guide pulleys it is possible to connect with belts shafts at almost any angle to each other. But experience is required and care must be exercised to do it successfully. When guide pulleys are used in order to change the direction of a belt, always remember that when the belt is running the most pressure is thrown on the pulley guiding the working part of the belt. This pulley is, therefore, very liable to heat in its bearings, if not designed to have bearing surface enough and also to have proper means for oiling.

Fig. 2 shows an arrangement by which the direction of motion of two shafts may be reversed, when the distance between the shafts is too short for the use of a crossed belt or when a crossed belt, for any other reason, cannot be used.

FIG. 2.



Suppose pulley A to be the driver and to run in the direction of the arrow. C and D are guide pulleys, and the motion of the driven shaft B is in the opposite direction to the shaft A. In this case the guide pulley C is on the working part of the belt, and is the one to which special attention must be paid in regard to heating. If the direction of shaft A is reversed, guide pulley D will be on the working part of the belt.

Crossed Belts.

If the distance between A and B (Fig. 2) had been long enough, it would have been preferable to reverse the motion of B by means of a crossed belt, instead of by the arrangement shown in Fig. 2.

Crossed belts do not work well when running on pulleys small in diameter as compared to the width of the belt.

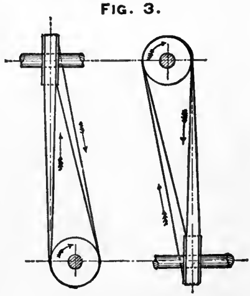
Too short distance between the shafts must be avoided.

Wide crossed belts are very unsatisfactory; therefore, instead of running one wide crossed belt it is preferable to use two belts, each of half the width, and run them on two separate pairs of pulleys. Such belts should be of equal thickness, and the pulleys should be crowned, well finished and of correct size, so that each belt will do its share of the work.

Quarter-Turn Belts.

Fig. 3 shows a so-called quarter-turn belt, used to connect two shafts when running at an angle and laying in different planes. The principal point to look out for is to place the pulleys (as shown in Fig. 3) so that the belt runs straight from the delivering to the receiving side of each pulley.

FIG. 3.

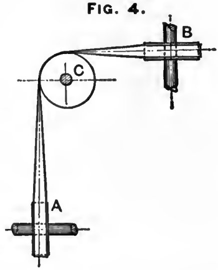


The pulleys shown in Fig. 3 are set right for belts running in the direction of the arrows. If the motion is reversed, the belt will run off the pulleys.

Angle Belts.

The belt arrangement shown in Fig. 4 is usually called an angle belt, and is used to connect two shafts at an angle. Either one, *A* or *B*, may be the driver, and there are two guide pulleys (one for each part of the belt at *C*), one of which, of course, is on the driving part of the belt.

FIG. 4.



Crossed belts, quarter-turn belts, and angle belts must never be wide and thin; much better results are obtained by narrow, double belts than by wide, single ones.

Angle belts and quarter-turn belts are frequently bothersome contrivances. Their running is sometimes improved by making a twist in the belt when joining its ends; that is, lacing the flesh side of one end and the hair side of the other end on the outside. This will prevent one side of the belt from stretching more than the other.

Slipping of Belts.

Owing to the elasticity of belts, there must always be more or less slip or "creep" of the belts on the pulleys. Under favorable conditions it may be as low as 2%, but frequently the slip is more. Therefore, if two shafts are connected by belts, and both should have very nearly the same speed, the diameter of the driver should be at least 2% larger than the diameter of the driven pulley. When the driver is comparatively large in diameter and the driven pulley is small, it is advisable to have the driver from 2 to 5% over size, in order to get the required speed.

Tighteners on Belts.

If tighteners are used they should always be placed on the slack part of the belt.

Velocity of Belts.

Belts are run at almost all velocities from less than 500 to 5000 feet per minute, but good practice indicates that whenever possible main belts having to transmit quantities of power are run most economically at a speed of 3000 to 4000 feet per minute. At a higher speed both practice and theory seem to agree that the loss due to the action of the centrifugal force in the belt when passing around the pulley, and that the wear and tear is so great when the speed is much over 4000 feet per minute that there is not much practical gain in increasing the speed. But, as a general rule, whenever possible the higher the belt speed the more economical is the transmission as long as the belt speed does not exceed the neighborhood of 4000 feet per minute.

Oiling of Belts.

Belts should be kept soft and pliable and are, therefore, usually oiled with either neat's-foot oil or castor oil. Too much oiling is hurtful, but the right amount of oiling at proper times is very beneficial to the action of the belt and will prolong its utility to a great extent.

REMARKS.—All previous rules for calculating belting are founded upon good, legitimate practice, but are only offered as a guide, as no rule can be given which will fit all cases.

For instance, a belt may be amply large to transmit a given horse-power when running in a horizontal direction, but it may fail to do the same work if running in a vertical direction. A belt may be large enough to do its work when running in a vertical direction over pulleys of unequal size with the large pulley on the lower shaft, but it may fail to do the same work satisfactorily with the large pulley on the upper shaft and the small pulley on the lower one.

Leather belts should not be used where it is damp or wet, but rubber belting will usually give good service in such places.

For information regarding rubber belts, see manufacturers' catalogues.

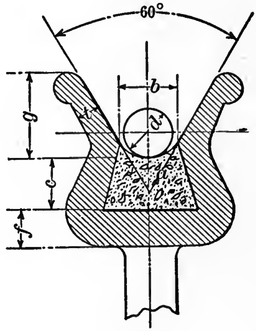
WIRE ROPE TRANSMISSION.

Transmitting power by wire ropes running at a high speed over grooved pulleys, or "telodynamic transmission," as it is also called, is the invention of the brothers Hirn of Switzerland. For long distances this mode of transmission is far cheaper than leather belting or lines of shafting. Fig. 1 shows a section of a pulley as used for this kind of transmission; *a* is an elastic filling, usually made from leather cut out and packed in edgewise. The groove is made wide, so that the rope will rest entirely against the packing and not touch the iron. This is different from transmission with hemp rope, which is made to wedge into the groove of the pulley.

The diameter of the pulley in the groove, where the wire rope runs, ought to be at least 150 times the diameter of the rope; the larger the better, so long as the velocity of the rope does not exceed 5000 feet per minute. The pulleys must run true and be in balance and in exact line with each other, and the shafts must be parallel. The distance between shafts should never be less than 60 feet and should preferably be from 150 to 400 feet.* For distances longer than 400 feet, either carrying pulleys or intermediate jack shafts are generally used, although spans as long as 600 feet or more have been used, but only when it is possible to give the rope the proper deflection without its touching the ground. Usually the speed is from 3000 to 6000 feet per minute. Higher speed would be dangerous from the stress in cast-iron wheels due to centrifugal force.

Diameter of rope in inches.	$b = d + \frac{1}{2}''$	$c = d + \frac{1}{2}''$	$f = \frac{1}{2}d + \frac{3}{8}''$	$g = 3d$	$t = \frac{1}{4}d + \frac{1}{2}''$
d ins.	b ins.	c ins.	f ins.	g ins.	t ins.
$\frac{3}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{9}{16}$	$1\frac{1}{8}$	$\frac{3}{8}$
$\frac{7}{16}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{9}{16}$	$1\frac{5}{16}$	$\frac{3}{8}$
$\frac{1}{2}$	1	1	$\frac{5}{8}$	$1\frac{1}{2}$	$\frac{3}{8}$
$\frac{5}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{11}{16}$	$1\frac{7}{8}$	$\frac{7}{16}$
$\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$	$2\frac{1}{4}$	$\frac{7}{16}$
$\frac{7}{8}$	$1\frac{3}{8}$	$1\frac{3}{8}$	$\frac{13}{16}$	$2\frac{3}{8}$	$\frac{1}{2}$
1	$1\frac{1}{2}$	$1\frac{1}{2}$	$\frac{7}{8}$	3	$\frac{1}{2}$

FIG. 1.



Tightening pulleys should not be used, because if the distance between centers of shaft is too short to give the proper tightness to the rope without a tightening pulley, wire rope transmission is not the form best adapted to the circumstances. Guide pulleys or idlers should be avoided as much as possible, but when necessary they should be as carefully made and put up as the main pulleys, and they ought not to be less than half the diameter of the main pulley if on the slack part, but of the same size if they are on the tight part of the rope. Wire rope for transmission is usually made from the best quality of iron, has seven wires to a strand and consists of six strands laid around a hemp core in the center of the rope. The diameter of the wire rope is from nine to ten times the diameter of each single wire.

Never use galvanized rope for power transmission, but preserve the rope by painting with heavy coats of linseed oil and lampblack.

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* When distance between shafts is less than 60 feet, leather belts are preferable to wire rope.

Transmission Capacity of Wire Ropes.

A one-inch rope running 5000 feet per minute is capable of transmitting 200 horse-power. The transmitting capacity of the rope is in proportion to the square of its diameter, and the power transmitted by the rope when the velocity is less than 5000 feet per minute is practically in proportion to its velocity.* Hence the formula:

$$H = \frac{d^2 \times V \times 200}{5000} \text{ which reduces to } H = 0.04 \times d^2 \times V$$

$H =$ Horse-power transmitted.
$d =$ Diameter of rope in inches.
$V =$ Velocity of rope in feet per minute.

EXAMPLE.

How many horse-power may be transmitted by a wire rope ½ inch in diameter running over proper pulleys at a velocity of 2500 feet per minute?

Solution:

$H = 0.04 \times \frac{1}{2} \times \frac{1}{2} \times 2500 = 25$ horse-power.

The pressure on the bearings will not be less than three times the force transmitted, and may be calculated thus:

$$\text{Pressure on bearings} = \frac{3 \times \text{horse-power} \times 33000}{\text{Velocity in feet per min.}}$$

EXAMPLE.

What will be the least pressure in bearings for a wire rope transmitting 150 horse-power at a velocity of 5000 feet per minute?

$$\text{Pressure on bearings} = \frac{3 \times 150 \times 33000}{5000} = 2970 \text{ pounds.}$$

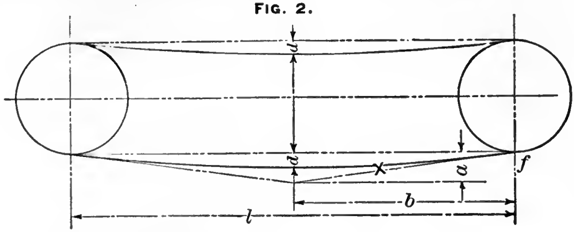
If there is one bearing on each side at an equal distance from the pulley, the pressure on each bearing will be $\frac{2970}{2} =$ 1485 pounds. This is the calculated pressure, and represents what the pressure should be, but it is not certain that this is the actual pressure. It may be greatly increased by having the rope too tight.†

* When the velocity of the rope exceeds 6000 feet per minute the stress caused by centrifugal force when the rope is bending around the pulley considerably reduces its transmitting capacity. This loss increases very fast above this speed, because the centrifugal force increases as the square of the velocity. It is very doubtful if there is practically any gain to run wire ropes at a speed exceeding 6000 feet per minute when wear and tear, loss due to centrifugal force, etc., are considered.

† Sometimes a pulley is put on the free end of a line of shafting projecting through the wall and drawn by a wire rope outside the shop; this will do only when a comparatively small amount of power is to be transmitted.

The tension of the rope may be calculated from its deflection when at rest (see Fig. 2), and for a rope running horizontally the usual formula is:

FIG. 2.



$$P = \frac{W \times \sqrt{a^2 + b^2}}{a}$$

$$P = \frac{W \times b}{a} \text{ (very nearly)}$$

$P =$ Force in pounds at f.

$W =$ Weight of rope in pounds from d to f, which is half the span.

$b =$ Half the span in feet.

$a =$ Twice the deflection in feet.

NOTE.—(See Fig. 2.) If the length of the line a represents the weight of the part of the rope from d to f, the length of the line x represents the tension in the rope at f; therefore the tension will be as many times the weight as the length of line a is contained in the line x.

EXAMPLE.

The horizontal distance between two pulleys is 200 feet; when standing still the deflection in a wire rope of ⅞″ diameter is 5 feet. What is the tension in the rope?

Solution:

In Table No. 38 the weight of ⅞″ wire rope is given as 1.12 pounds per foot; therefore, 100 feet of ⅞″ rope will weigh 112 pounds.

$$P = \frac{112 \times \sqrt{100^2 + 10^2}}{10} = \frac{112 \times 100.5}{10} = 1125.6 \text{ pounds.}$$

This is the tension in each part of the rope; therefore the force against the pulley, due to the weight of the rope, is 1125.6 $\times$ 2 = 2251.2 pounds. If this is supported by a bearing on each side of the pulley, the pressure on each bearing, if both are the same distance from the pulley, will be 1125.6 pounds.

The tension is increased by reducing the deflection. For instance, if the deflection is reduced to 4 feet the tension on the rope will be,

$$P = \frac{112 \times \sqrt{100^2 + 8^2}}{8}$$

$$P = \frac{112 \times 100.3}{8} = 1404.2 \text{ pounds.}$$

Thus, the tension might be increased to any amount within the ultimate breaking strength of the rope.

Deflection in Wire Ropes.

When the rope is in motion the deflection will increase on the slack side and decrease on the tight side; therefore, if the span is long the rope may touch the ground when running if the pulleys are not placed on sufficiently high towers. There is really nothing else which, within practical limits, determines the length of the span, which may just as well be 1000 feet, or even more, providing the proper deflection can be given to the rope without touching the ground. When possible the lower part of the rope should be the working side, but in a long span this is impossible, because, when running, the lower part of the rope would be tight and the upper part slack, causing the two parts of the rope to strike together, which must never be allowed. When the length of the span exceeds 35 times the diameter of the pulleys it is safest to have the upper part of the rope the working side and the lower part the slack side.

When the lower part of the rope is the slack side, the least space allowable for the slack of the rope at the center of the span will (when the rope is as tight as given in Table No. 38), be obtained by the formula:

$$\text{Distance} = 0.00015 \times (\text{span})^2$$

but, to allow for contingencies, it is better to have more room. When the lower side of the rope is the tight side, the rope will be clear from the ground when running if the space is $0.0001 \times (\text{span})^2$. The deflection in the rope when standing still which will produce a pressure on the bearings and give tension enough to transmit the horse-power given in Table No. 38, may be calculated approximately by the formula:

$d = 0.00009 \times l^2$

$d =$ Deflection in feet.

$l =$ Distance between pulleys in feet. (See Fig. 2).

EXAMPLE.

The distance between the pulleys being 400 feet, find the greatest allowable deflection in the rope, when standing still, in order to transmit the horse-power given in Table No. 38.

Solution:

$$d = 0.00009 \times 400 \times 400 = 14.4 \text{ feet.}$$

When the rope is new it is always put on with more tension than is necessary to transmit the power, because new rope will stretch. It is, therefore, very important when designing such transmission to calculate the maximum pressure which the rope will exert on the bearings when put on with the least deflection ever wanted, and calculate size of bearings and shafting for pulleys according to this stress, with due consideration not only for strength but also for heat and wear. (See page 360 and page 367.) The correct amount to allow for stretch will vary with different kinds of rope and also with the temperature. If a rope is spliced on a warm summer day it must be made slacker than if it was spliced on a cold winter day, as the length of the rope will be changed considerably by the difference in temperature; the only guide is practical experience and good judgment. As a general rule, it may be safe to allow about half of the deflection as previously calculated when splicing a new rope, provided that the shafts and bearings are constructed so as to allow such tension. The rope is always strong enough. The splicing of the rope should be done by a man experienced in that kind of work. The splice itself is usually made at least 240 times the diameter of the rope.

TABLE No. 38.—Giving Suitable-Sized Pulleys for Different Sizes of Wire Rope, Weight of Rope, Horse-Power which Different Sizes of Wire Rope May Transmit at Different Velocities, the least Stress at which it may be done and the Least Corresponding Pressure on the Bearings; also, the Ultimate Average Strength of Wire Rope.

Diameter of Smallest Pulley in Feet.	Diameter of Rope in Inches.	Weight of Rope in Pounds per Foot, (hemp core).	Ultimate Tensile Strength in Pounds. (Iron rope having hemp core.)	Driving Force in Pounds.	Tension in Pounds on Slack Side of Rope.	Tension in Pounds on Tight Side of Rope.	Tension in each part of the Rope when at rest.	Pressure on Bearings due to the Given Horse-Power.	Horse-Power Transmitted at Different Velocities. 2500 feet per Minute.	3000 feet per Minute.	4000 feet per Minute.	5000 feet per Minute.
5	3/8	0.21	4500	187	187	374	285.5	561	14	17	22	28
6	7/16	0.23	6000	253	253	506	379.5	759	19	23	31	38
7	1/2	0.31	8000	330	330	660	495	990	25	30	40	50
8	5/8	0.57	12000	517	517	1034	775.5	1551	39	47	62	78
10	3/4	0.92	18000	636	636	1272	954	1908	56	67	90	112
12	7/8	1.12	24000	1012	1012	2024	1518	3036	77	92	122	153
14	1	1.50	32000	1320	1320	2640	1985	3960	100	120	160	200

EXAMPLE.

From a shaft running 150 revolutions per minute 100 horse-power is to be taken off by a wire rope. The velocity of the rope is to be 5000 feet per minute. What size of pulley and rope will be required?

Solution:

In Table No. 38 it will be found that a ¾-inch wire rope, running at 5000 feet per minute, is capable of transmitting 112 horse-power; thus, select a ¾-inch rope. The diameter of the pulley will be $\frac{5000}{150 \times 3.1416} = 10.6$ feet. In the table it will be found that a 10-foot pulley is the smallest advisable to run with a ¾-inch wire rope, therefore the pulley 10.6 feet in diameter is within the requirements. The next step is to calculate the pressure on the bearings. In the table it is found that the least pressure due to the transmission of 112 horse-power is 1908 pounds. This cannot be used in calculating sizes of shafts and bearings, but use the maximum pressure, which is calculated according to the allowable deflection in the rope, as explained on page 341. Also consider weight of pulley and shaft, then calculate size of shaft and bearings, with due consideration to strength, stiffness, wear, heat, etc. (See pages 360–367.)

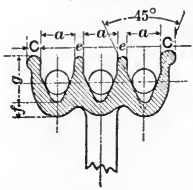
Transmission of Power by Manila Ropes.

Manila ropes are used more or less for transmission of power. In this country one continuous rope, going back and forth in separate grooves over the pulleys several times, is frequently used, and a tightening arrangement is placed on one of the slack parts, which automatically keeps the rope at the proper tension, regardless of changes due to weather or stretch due to wear. This arrangement has its advantages in keeping the rope at more even tension than is possible with the European system, but the disadvantage is that if a break occurs the transmission is entirely disabled until it is repaired. The European practice is to use several single ropes running in separate grooves side by side on the same pulley. This has the advantage that if one of the ropes should break it is usually possible to run undisturbed until there is a chance to repair it, because it is always advisable to have margin enough in the transmission capacity of the ropes so that the shaft will run satisfactorily, even if one rope is taken off. The disadvantage of this system is the difficulty in keeping all the ropes at equal tightness and getting them to pull evenly.

Fig. 3 shows the usual shape of pulley used for manila ropes, which may be made from either wood or iron. The European practice is to use iron, but whichever material is used it is very important to have the sides of the grooves carefully polished, as the rope rubs on the sides in entering and leaving the pulley and will wear out in a short time if the pulley is left as it comes from the lathe tool. Sand and blow-holes must also be avoided. The angle of groove is usually 45°, and the rope is made to wedge into it, as shown in Fig. 3.

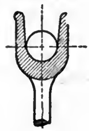
The usual shape of grooves for guide pulleys is shown in Fig. 4.

FIG. 3.



Diameter of rope in inches.	$a = 1\frac{1}{4} d + \frac{1}{8}''$	$c = \frac{1}{2} d$	$e = \frac{1}{4} d + \frac{1}{8}''$	$f = \frac{1}{2} d$	$g = 2 d$
d ins.	*a* ins.	*c* ins.	*e* ins.	*f* ins.	*g* ins.
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1
$\frac{5}{8}$	$\frac{15}{16}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$1\frac{1}{4}$
$\frac{3}{4}$	$1\frac{1}{16}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{3}{8}$	$1\frac{1}{2}$
1	$1\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{2}$	2
$1\frac{1}{4}$	$1\frac{11}{16}$	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{5}{8}$	$2\frac{1}{2}$
$1\frac{1}{2}$	2	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	3
$1\frac{3}{4}$	$2\frac{5}{16}$	$\frac{7}{8}$	$\frac{9}{16}$	$\frac{7}{8}$	$3\frac{1}{2}$
2	$2\frac{5}{8}$	1	$\frac{5}{8}$	1	4

FIG. 4.



The best speed for ropes is from 1500 to 5000 feet per minute. When the velocity of the rope exceeds 6000 feet per minute the loss, due to the centrifugal force, is so great that it will hardly pay to increase the velocity. The diameter of the pulleys ought to be at least 50 times the diameter of the rope.

Transmission Capacity of Manila Rope.

A manila rope two inches in diameter, running over properly-shaped pulleys at a speed of 5000 feet per minute, is capable of transmitting 50 horse-power. The transmitting capacity of the rope is in proportion to the square of its diameter, and the power transmitted by the rope is in proportion to its velocity; therefore, a one-inch rope, running 5000 feet per minute, will transmit 12½ horse-power, and the formula will be:

$$\text{Horse-power} = \frac{d^2 \times v \times 12.5}{5000}$$

which reduces to

$$\text{Horse-power} = 0.0025 \times d^2 \times v$$

$d =$ Diameter of rope in inches.

$v =$ Velocity of rope in feet per minute.

EXAMPLE.

What horse-power may be transmitted by a manila rope $1\frac{1}{2}$ inches in diameter, running over nine-foot pulleys at a speed of 150 revolutions per minute?

Solution:

Nine-foot pulleys, running 150 revolutions per minute, give the rope a velocity of $3.1416 \times 9 \times 150 = 4241$ feet per minute, and the horse-power transmitted will be:

$$H\text{-}P = 0.0025 \times 1\tfrac{1}{2} \times 1\tfrac{1}{2} \times 4241$$

$$H\text{-}P = 0.0025 \times 2\tfrac{1}{4} \times 4241$$

$H\text{-}P = 23.85$; practically, 24 horse-power.

Weight of Manila Rope.

The weight of one foot of manila rope of one-inch diameter is $\frac{3}{10}$ pound; therefore, the weight per foot of any size may be calculated approximately by the formula:

$$W = d^2 \times 0.3$$

d = Diameter of rope in inches.

W = Weight of rope in pounds per foot.

EXAMPLE.

What is the weight of 360 feet of manila rope of $1\frac{1}{2}$-inch diameter?

Solution:

Weight of 360 feet $= 0.3 \times 360 \times 1\frac{1}{2} \times 1\frac{1}{2} = 243$ pounds.

TABLE No. 39,

Giving the Weight of Rope in Pounds per Foot, Driving Force in Pounds, and Corresponding Horse-Power Transmitted at Different Velocities.

Least Diameter of Pulleys in Feet.	Diameter of Rope in inches.	Weight in pounds per Foot.	Driving Force in pounds.	Horse-power Transmitted at Different Velocities.							
				1500 Feet per minute.	2000 Feet per minute.	2500 Feet per minute.	3000 Feet per minute.	3500 Feet per minute.	4000 Feet per minute.	5000 Feet per minute.	6000 Feet per minute.
2	½	0.075	21	0.94	1.25	1.56	1.87	2.18	2.50	3.12	3.75
2¼	⅝	0.12	33	1.45	1.94	2.24	2.90	3.39	3.87	4.48	5.81
2½	¾	0.16	47	2.11	2.81	3.52	4.82	4.92	5.62	7.03	8.44
3½	1	0.30	83	3.75	5	6.25	7.50	8.75	10	12.50	15
4¼	1¼	0.47	132	5.86	7.81	9.77	11.72	13.73	15.62	19.23	23.44
5	1½	0.67	186	8.44	11.25	14.06	16.87	19.69	22.50	28.12	33.75
6½	1¾	0.92	255	11.48	15.31	18.31	22.97	26.79	30.62	36.61	45.94
8	2	1.20	330	15	20	25	30	35	40	50	60

The transmitted horse-power, as given in Table No. 39, is calculated by the formula,

$$H\text{-}P = 0.00025 \times d^2 \times v$$

EXAMPLE. (Showing application of Table No. 39.)

What size of rope is required to transmit 50 horse-power when three independent ropes are used, running over the same pulley at a velocity of 4000 feet per minute?

Solution:

It is always advisable to select ropes having sufficient transmitting capacity to continue the transmission undisturbed, even if one rope breaks; therefore, select ropes of such size that two ropes will transmit nearly 25 horse-power each. In Table No. 39 it is found that a manila rope 1½ inches in diameter, running 4000 feet per minute, will transmit 22.5 horse-power. Thus, this will be the size of rope to use. The small pulley in the transmission must not be less than five feet in diameter. (See Table No. 39.)

The pressure on the bearings, due to tension of the rope, will not exceed three times the driving force, because manila ropes run comparatively slack, as the adhesion to the pulley does not depend so much on the tightness of the rope as it does on its wedging into the groove in the pulley. The driving force of manila rope of 1¼-inch diameter is given in the table as 132 pounds; therefore, the pressure due to one rope will be $3 \times 132 = 396$ pounds, and the pressure due to three ropes will be $3 \times 396 = 1188$ pounds; besides this, the weight of the shaft and the pulley should be considered when calculating the size of shaft and bearings, with due consideration for strength, stiffness, wear, heat, etc. (See page 367.)

Preservation of Manila Rope.

The life of the rope is prolonged by slushing once in a while with tallow mixed with plumbago. The rope will not only wear on the outside but also within itself, because the fibers chafe on each other as the rope bends over the pulleys; hence the preference for pulleys of large diameter. If the rope is not specially prepared for transmission purposes, it ought to be soaked in a mixture of plumbago and melted tallow when new, before it is used. There is on the market manila rope especially manufactured for transmission purposes, having the fibers treated with plumbago and tallow, and, whenever obtainable, such rope should be used, as it will last much longer and give much better service than ordinary manila rope.

PULLEYS.

The following empirical rule gives arms of nice shape and good proportions:

When the diameter of the pulley is at least 4 times its face, use 6 arms for pulleys from 12 to 60 inches. For a 12-inch pulley make the arms $1\frac{1}{4}$ inches wide at the hub and add $\frac{1}{16}$ of an inch to the width of the arm for each inch the pulley is increased in diameter.

Formula:
$$h = \frac{D - 12}{16} + 1\tfrac{1}{4}$$

h = Width of arm in inches projected to center of hub. (See Fig. 1.)

D = Diameter of pulley in inches.

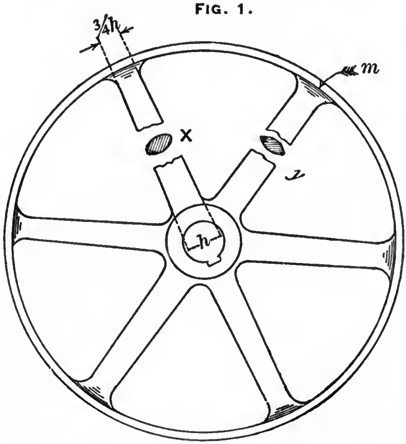
EXAMPLE.

Find width of arms at the hub for a 60-inch pulley.

Solution:
$$h = \frac{60 - 12}{16} + 1\tfrac{1}{4} = 4\tfrac{1}{4} \text{ inches.}$$

The width of the arm at the rim should be three-fourths of the width at the hub, and the thickness should be one-half of the width for arms with segmental sections (see y Fig. 1) and four-tenths of the width for elliptical form of section; (see x Fig. 1.) For double belts multiply these diameters by 1.3.

FIG. 1.



Large, well-rounded fillets must be used where the rim and arms meet at m. (See Fig. 1.) For very wide pulleys it is always better to use two sets of arms. For small pulleys, under 12 inches in diameter, 4 arms are better than 6, as they are less liable to break while being cast. Using 4 arms, the width of the arm at h, in a pulley 10 inches in diameter, may be $1\frac{3}{16}$ in.; pulley 8 inches in diameter, $1\frac{1}{8}$ in.; pulley 6 inches in diameter, 1 inch; and the thickness and taper as given above. When pulley arms crack from shrinkage in casting, the trouble may usually be prevented by either increasing the thickness of the rim of the pattern or by reducing the size of the hub, or both; it will also help the matter to remove the core and the sand from the hub as soon as possible after the pulley is cast, and leave the casting in the sand undisturbed until cool.

When the diameter of the shaft is less than 4 inches, the diameter of the hub is usually made twice the diameter of the shaft. When shafts are over 4 inches in diameter the hub of the pulley is usually made a little less than twice the diameter of the shaft. The length of the hub may be made three-fourths the width of the rim, for a tight pulley, and five-fourths the width of the rim for a loose pulley.

The thickness of rim, measured at the edge, is usually:

For pulleys under 12 inches in diameter, $\frac{3}{16}$ inch.
For pulleys from 12 to 24 inches in diameter, $\frac{1}{4}$ inch.
For pulleys from 24 to 36 inches in diameter, $\frac{5}{16}$ inch.
For pulleys from 36 to 48 inches in diameter, $\frac{7}{16}$ inch.
For pulleys from 48 to 60 inches in diameter, $\frac{1}{2}$ inch.

For double belts increase the thickness of the rim one-eighth of an inch.

The thickness in the middle may be about $1\frac{1}{2}$ times the thickness at the edge.

Pulleys which are to run at high velocity ought to be turned both inside and outside, in order to be in good balance. Pulleys to go on line shafts ought to be made in halves, so that they can be put on and taken off the shaft with convenience. Pulleys on which the belts are to be shifted must be a little over twice as wide as the belt, and they should be turned straight across the face on the outside. Pulleys on which the belts are not to be shifted ought to be only 1.2 times as wide as the belt, and they ought to be turned curved across the face; that is, the outside diameter of the pulley must be largest at the middle. Most frequently a straight taper is turned each way from the middle to the edges, and the following proportions will give good results:

Pulleys under six inches wide, $\frac{3}{4}$-inch taper per foot.
Pulleys from 6 to 12 inches wide, $\frac{1}{2}$-inch taper per foot.
Pulleys from 12 to 18 inches wide, $\frac{3}{8}$-inch taper per foot.

When pulleys are turned in a lathe where the tail-stock can be *set over*, a taper of $\frac{3}{4}$-inch per foot is practically obtained when the tail-stock is *set over* $\frac{1}{32}$-inch per 1 inch length of arbor. For instance, if a crown pulley is to be turned $\frac{3}{4}$-inch per foot, and the arbor is 12 inches long, the back center must be *set over* $\frac{12}{32} = \frac{3}{8}$-inch. If the arbor had been 14 inches long the back center would have had to be *set over* $\frac{14}{32} = \frac{7}{16}$-inch to obtain the same result.

All pulleys must be well rounded on the edges. They must also be carefully balanced, especially if they are to run at high speed.

Loose pulleys ought to have longer hubs than tight pulleys. They ought never to have hubs shorter than the width of the rim, and must always be provided with means for oiling.

Stepped Pulleys.

Stepped pulleys, or cone pulleys, as they are usually called, may be considered as several pulleys of different diameters cast together. Their proportions and sizes are calculated to get the required changes of speed, and the belt must have practically the same tension on all the different changes.

Frequently it is required to have both pulleys of the same size, in order that they may be cast from the same pattern. In such cases the shaft of constant speed (usually a counter-shaft) must be run at a velocity equal to the square root of the product of the fastest and the slowest speed of the shaft of changeable speed (which usually is a spindle in a lathe or a similar machine). For convenience, in the following formulas we will call the driver, which is the shaft of constant speed, a *counter-shaft*, and the shaft of changeable speed, a *spindle*.

The number of revolutions of the counter-shaft per minute is calculated by the formula:

$$N = \sqrt{F \times S}$$

$N =$ Number of revolutions of the counter-shaft per minute.
$F =$ Number of revolutions of the spindle per minute, when run at its fastest speed.
$S =$ Number of revolutions of the spindle per minute, when run at its slowest speed.

The diameter of either the largest or the smallest step is then obtained by choosing one diameter and calculating the other by the formula:

$$D = \frac{d \times N}{n} \qquad d = \frac{D \times n}{N}$$

$D =$ Diameter of largest step on spindle.
$d =$ Diameter of smallest step on counter-shaft.
$n =$ Slowest number of revolutions of the spindle per minute.
$N =$ Revolutions of the counter-shaft per minute.

The intermediate steps may be obtained by drawing a straight line, $a\ b$, and constructing steps within the angle formed by the line $a\ b$ and the center line (see Fig. 2). The sum of the diameters of the two opposite steps will then be equal, and this is the way in which stepped pulleys may primarily be laid out, whether both pulleys are of the same size or not. Afterwards the diameters will have to be slightly changed, in order to give the belt the same tension on any of the different steps, as explained further on.

EXAMPLE 1.

A pair of stepped pulleys, for four changes of speed, both pulleys of the same size, are to be used on a milling machine spindle and its counter, the fastest speed to be 250 revolutions, and the slowest speed 90 revolutions, per minute. The diameter of the largest step is 15 inches. What should be the speed of the counter-shaft, and what is the diameter of each intermediate step?

Solution:

Speed of counter $= \sqrt{90 \times 250} = 150$ revolutions per minute.

The diameter of the largest step is 15″.

$$\text{Diameter of smallest step} = \frac{90 \times 15}{150} = 9 \text{ inches.}$$

By the method as shown in Fig. 2, the intermediate diameters are found to be 11″ and 13″. The speed of spindle will be:

$$\text{First speed} = \frac{150 \times 15}{9} = 250 \text{ revolutions per minute.}$$

$$\text{Second speed} = \frac{150 \times 13}{11} = 177 \text{ revolutions per minute.}$$

$$\text{Third speed} = \frac{150 \times 11}{13} = 127 \text{ revolutions per minute.}$$

$$\text{Fourth speed} = \frac{150 \times 9}{15} = 90 \text{ revolutions per minute.}$$

These calculations are only correct for speed, and must be slightly modified in order to get the proper tension on the belt, if an open belt is used; for a crossed belt the tension is correct if the pulleys are laid out in this manner. (See page 352.)

When the number of revolutions per minute for each change of speed is given, the diameters of the intermediate steps may, with regard to speed, be calculated by the following formulas:

$$D_1 = \frac{(D + d) \times N}{n + N}$$

$D_1 =$ Diameter of any step on spindle.
$D =$ Diameter of largest step on spindle.
$d =$ Diameter of smallest step on counter-shaft.
$n =$ Revolutions of spindle per minute, corresponding to the diameter D_1.
$N =$ Revolutions of counter-shaft per minute.

After the diameter of any step on the spindle is calculated, the diameter of the corresponding step on the counter-shaft may be obtained by subtracting the diameter of the step on the spindle from the value of $(D + d)$.

Example 2.

A lathe spindle is required to run at 40, 120 and 360 revolutions per minute, and the diameter of the largest step is 18 inches. Calculate speed of counter-shaft and diameter of steps.

Solution:

Speed of counter-shaft will be:

$$N = \sqrt{F \times S} = \sqrt{360 \times 40} = 120 \text{ revolutions per minute.}$$

Diameter of smallest step on spindle will be:

$$d = \frac{18 \times 40}{120} = 6 \text{ inches.}$$

Diameter of the intermediate step on spindle will be:

$$D_1 = \frac{(18 + 6) \times 120}{(120 + 120)} = 12 \text{ inches.}$$

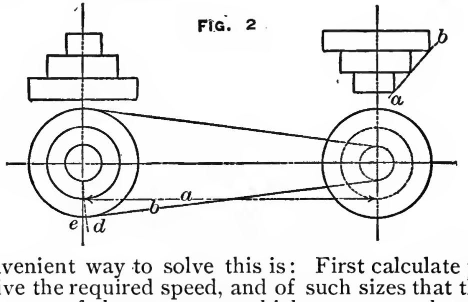
Thus, the sizes of each step, with regard to speed, should be 6, 12 and 18 inches, but with regard to belt tension these sizes have to be slightly altered.

To Correct the Diameter of Stepped Pulleys so that the Belt will have the Same Tension on all the Steps.

At first thought, it may seem as if the belt would have equal tension on each step when the sum of the diameters of the largest and the smallest steps of the two pulleys are equal to the sum of the diameters of the two middle steps; but this is only correct if a crossed belt is used on the pulleys. For a two-step pulley it is also correct for either open or crossed belt, if both pulleys are of the same size; but if the pulleys are of different sizes, the diameter of the steps must be calculated for two steps as well as if there were more.

It is evident from Fig. 2, that an open belt will be tighter over the largest and the smallest pulleys than it would be over the two middle pulleys, as the part *a* of the belt runs parallel to the center line and will be as long as the distance between centers, but the inclined line, *b*, will be as much larger as the distance *d* to *e*. (See Fig. 2).

FIG. 2



A convenient way to solve this is: First calculate pulleys that will give the required speed, and of such sizes that the sum of the diameters of the two steps which are to work together will be equal, then calculate the length of the belt when laying on the largest and smallest steps, with a given distance between the centers of the shafts. Then, by calculating the same way, try the belt on the other steps, which will then have to be corrected until the belt will fit each of the different pairs of steps.

The length of the belt can be most conveniently calculated by the geometrical rule that the square of the perpendicular added to the square of the base is equal to the square of the hypothenuse. (See page 150.) The space between the centers of the shafts is considered as the *base*, and the difference in radius of the two corresponding steps is considered as the *perpendicular*, which are both known, and from this the length of the line b is calculated (see Fig. 2), which is considered as the hypothenuse. Assuming that the belt covers half the circumference of both pulleys, the length of the belt can be found by adding half the circumference of each step to twice the length of b.

NOTE.—This mode of calculation is not exactly correct, but is very well within practical requirements.

The length of half the circumference of the pulley is most conveniently obtained by the use of Table No. 24, page 209, by dividing the circumference of the corresponding circle by 2.

A practical rule is simply to calculate the distance from d to e, and for each $\frac{1}{10}$-inch the belt is found to be too long, add $\frac{1}{32}$-inch to the diameter of the corresponding step on each pulley.

For instance, the stepped pulleys in Example No. 2 are calculated so that they will give the required speed to the machinery when the three steps are 18, 12 and 6 inches in diameter and both pulleys are equal. Assume the distance between centers to be 5 feet. What will be the diameter of the middle step, after it has been corrected so that it will give the right tension to the belt?

Solution:

Five feet = 60 inches, and the difference between the radius of the corresponding steps is $9 - 3 = 6$ inches. The distance from e to d will be:

$$x = \sqrt{60^2 + 6^2} - 60 = \sqrt{3636} - 60 = 60.3 - 60 = 0.3$$

Thus, each part of the belt will be 0.3″ too long, or the whole belt will be 0.6″ too long when on the middle step; therefore, in order to make up for this, the middle step on each pulley must be increased $\frac{6}{32}'' = \frac{3}{16}''$ in diameter. Thus, the middle step on each pulley will be $12\frac{3}{16}$ inches instead of 12 inches in diameter; but, as both pulleys are increased, this does not change the relative speed of the shafts when the belt is on the middle step, and the similarity of the pulleys is also preserved, which will admit that both may be cast from the same pattern.

The square root of 3636 may be obtained by use of logarithms (see page 71), thus:

$$Log. \sqrt{3636} = \frac{log.\ 3636}{2} = \frac{3.560624}{2} = 1.780312$$

and the number corresponding to this logarithm is 60.3.

Stepped Pulleys for Back-Geared Lathes.

On machinery having changeable reducing gearing, such as lathes, milling machines, etc., it is frequently the aim of the designer to arrange the speed of the counter and the diameters of the different steps of the cone pulley in such proportions that the same ratio of speed will be maintained on each step and also from the slowest speed, with back gears *out*, to the fastest speed, with back gears *in*. When the ratio of the back gearing is given, the ratio of speed for each step will be obtained by the formula:

$$S = \sqrt[m]{R}$$

S = Ratio of speed for each step.
m = Number of changes of speed on the cone pulley.
R = Reduction of speed by the back gearing.

EXAMPLE.

The back gearing of a lathe reduces its speed 8 times. The cone pulley has 5 changes of speed. The largest diameter of cone pulley on the spindle is 10½ inches. The cone pulley on the counter-shaft is to be of the same size as the cone pulley on the spindle, and an even ratio of speed is to be maintained throughout the whole range of the ten changes of speed. The slowest speed, when back gears are in, is 6 revolutions per. minute. Calculate the speed of the counter-shaft, the speed of the spindle for each change, and the diameter of each step on the cone pulley of the spindle.

Solution:

The ratio of speed for each step will be:

$$\sqrt[5]{8} \qquad \frac{log.\ 8}{5} = \frac{0.90309}{5} = 0.18062$$

The corresponding number is 1.516.

With back gears in, the speed of spindle:

On first cone is 6 revolutions per minute.
On second cone is $6 \times 1.516 = 9$ revolutions per minute.
On third cone is $9 \times 1.516 = 14$ revolutions per minute.
On fourth cone is $14 \times 1.516 = 21$ revolutions per minute.
On fifth cone is $21 \times 1.516 = 32$ revolutions per minute.

With back gears out, speed of spindle:

On first cone will be $6 \times 8 = 48$ revolutions per minute.
On second cone will be $9 \times 8 = 72$ revolutions per minute.
On third cone will be $14 \times 8 = 112$ revolutions per minute.
On fourth cone will be $21 \times 8 = 168$ revolutions per minute.
On fifth cone will be $32 \times 8 = 256$ revolutions per minute.

The speed of the counter-shaft will be:

$$N = \sqrt{48 \times 256} = 112 \text{ revolutions per minute.}$$

As the speed of main lines in factories usually runs at some multiple of 10, we may, for convenience in getting even-sized pulleys for connections between counter and main shaft, in practical work, decide to run the counter-shaft 110 revolutions per minute.

(When a pair of cone pulleys has an uneven number of steps, and are cast from the same pattern, the speed of the counter should be equal to the speed of the machine when the belt is run on the middle step).

The diameter of the largest step of the cone pulley on the spindle is 10½ inches. The corresponding step on the counter will be $\frac{10\frac{1}{2} \times 48}{110} = 4.581''$; practically, 4½″ diameter.

The largest and smallest step on the counter-shaft will also be 10½ and 4½ inches in diameter.

Any of the intermediate steps on the spindle may be calculated by the formula:

$$D_1 = \frac{(D + d) \times N}{n + N}$$

$$D_1 = \frac{(10\frac{1}{2} + 4\frac{1}{2}) \times 110}{72 + 110} = 9.065\text{; practically, 9 in.}$$

$$D_2 = \frac{(10\frac{1}{2} + 4\frac{1}{2}) \times 110}{110 + 110} = 7\frac{1}{2} \text{ inches.}$$

$$D_3 = \frac{(10\frac{1}{2} + 4\frac{1}{2}) \times 110}{168 + 110} = 5.932\text{; practically, 6 in.}$$

Thus, assuming the counter-shaft to run 110 revolutions per minute, the speed of the spindle, with back gears out, on the five different steps will be:

$$\frac{110 \times 10\frac{1}{2}}{4\frac{1}{2}} = 265 \text{ revolutions per minute.}$$

$$\frac{110 \times 9}{6} = 165 \text{ revolutions per minute.}$$

$$\frac{110 \times 7\frac{1}{2}}{7\frac{1}{2}} = 110 \text{ revolutions per minute.}$$

$$\frac{110 \times 6}{9} = 73 \text{ revolutions per minute.}$$

$$\frac{110 \times 4\frac{1}{2}}{10\frac{1}{2}} = 47 \text{ revolutions per minute.}$$

When the back gears are in action the speed will be:

$\frac{265}{8} = 33\frac{1}{8}$ revolutions per minute.

$\frac{165}{8} = 20\frac{5}{8}$ revolutions per minute.

$\frac{110}{8} = 13\frac{3}{4}$ revolutions per minute.

$\frac{73}{8} = 9\frac{1}{8}$ revolutions per minute.

$\frac{47}{8} = 5\frac{7}{8}$ revolutions per minute.

These speeds are all within the practical requirements of the problem, and now the next operation is to modify the diameters slightly in order to get proper tension on the belt. (See page 352.)

FLY-WHEELS.

Fly-wheels are used to regulate the motion in machinery by storing up energy during increasing velocity, and giving out energy during decreasing velocity. Fly-wheels cannot perform either of these functions without a corresponding change in velocity. The rim of the wheel may be very heavy and moving at a high velocity, the change in speed may be small and hardly perceptible if the energy absorbed and given out is small, but there must always be a change in velocity to enable a fly-wheel to act. The common expression of gaining power by a heavy fly-wheel is very misleading, to say the least. There is no power gained by a fly-wheel but, on the contrary, considerable power is absorbed by friction in the bearings when a shaft is loaded with a heavy fly-wheel, (see example in calculating friction, page 305). Nevertheless, a fly-wheel performs a very useful function in machinery by storing up energy when the supply exceeds the demand and giving it out at the time it is needed to do the work. (For momentum of fly-wheels see example, page 300. For kinetic energy, see example, page 301).

Weight of Rim of a Fly-Wheel.

The weight of a rim of a cast-iron fly-wheel will be:

$W = d^2 \times 0.7854 \times D \times 3.1416 \times 0.26$; this reduces to,

$W = D \times d^2 \times 0.64$

$D =$ Middle diameter of rim in inches.

$d =$ Diameter of section of rim in inches.

$W =$ Weight in pounds.

EXAMPLE.

A round rim of a fly-wheel is 4 inches in diameter and the middle diameter of the wheel is 36 inches. What is the weight of the rim?

Solution:

$$W = 36 \times 4 \times 4 \times 0.64 = 369 \text{ pounds.}$$

For a rim of rectangular section the weight will be:

$W =$ Width $\times$ thickness $\times D \times 3.1416 \times 0.26$

$W =$ Width $\times$ thickness $\times D \times 0.816$

EXAMPLE.

The width of the rim is six inches, the thickness is two inches, and the middle diameter of the rim is 48 inches. What is the weight of the rim?

Solution:

$$W = 2 \times 6 \times 48 \times 0.816 = 470 \text{ pounds.}$$

Centrifugal Force in Fly-Wheels and Pulleys.

Pulleys are not only liable to be broken by the stress due to the action of the driving belt, but in fast-running pulleys and fly-wheels the stress due to centrifugal force is far more dangerous. This stress increases as the square of the velocity and directly as the weight, therefore there is a limit to the velocity at which fly-wheels and pulleys can be run with safety.

Generally speaking, increasing the thickness of the rim does not increase its strength, because the total tensile strength, the total weight of the rim, and, consequently, also the centrifugal force, increase in the same proportion; but it has great influence upon the strength of the wheel to have the material in the rim distributed to the best advantage. At the same time it is very important to construct the rim and arms of such proportions that the initial stress due to uneven cooling in the foundry, is avoided.

The common formula is:

$$\text{Centrifugal force} = \frac{\text{Mass} \times (\text{velocity})^2}{\text{radius}}$$

$$\text{Mass} = \frac{\text{Weight}}{32.2} \qquad \text{Velocity} = \frac{n \times r \times 2\pi}{60}$$

Therefore,

$$cf = \frac{W\left(\frac{n \times r \times 2\pi}{60}\right)^2}{32.2 \times r}$$

$$cf = \frac{W \times n^2 \times r^2 \times 0.01096628}{32.2 \times r}$$

$$cf = W \times n^2 \times r \times 0.00034$$

$cf =$ Centrifugal force in pounds.

W = Weight of revolving body in pounds.
n = Number of revolutions per minute.
r = Middle radius of pulley rim in feet.

Thus, for any body whose center of gravity swings in a circle of one foot radius, at a speed of one revolution per minute, the centrifugal force will be 0.00034 times the weight of the body.

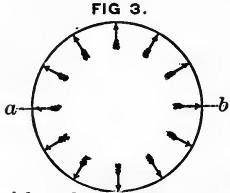
EXAMPLE.

The rim of a fly-wheel is five feet in middle radius and weighs 8000 pounds. It makes 75 revolutions per minute. What is the stress due to centrifugal force?

Solution:

Centrifugal force $= 8000 \times 75^2 \times 5 \times 0.00034 = 76500$ pounds.

FIG 3.



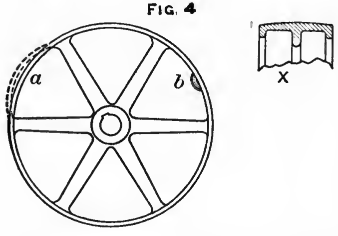
This is the total centrifugal force tending to burst the rim, (see arrows in Fig. 3); the force tending to tear the rim asunder in any two opposite points as a, b, is $\frac{76500}{3.1416 \times 2} = 12175$ pounds.

The next question is: Has the section of the rim tensile strength enough to resist this stress with safety? If not, either decrease the rim speed or make the rim of material having more tensile strength.

The centrifugal force for the same number of revolutions increases as the radius, therefore the average centrifugal force acting in the arms is only about half of the centrifugal force acting in the rim, and as the stretch is in proportion to the stress, the rim tends to stretch more than the arms, and, consequently, it can not yield freely to the action of the centrifugal force, but is to a certain extent held back at the junction with the arms. This action is shown in an exaggerated form at a, Fig. 4.

In regard to this action, the part of the rim between the arms may be considered as a beam fastened at both ends and uniformly loaded throughout its whole length equal to that amount of centrifugal force in the rim which is resisted by the arms; therefore, the rims of large pulleys should always be ribbed on the inside. (See cross-section of rim at X, Fig. 4).

FIG. 4



Another bad feature frequently seen in pulleys is the counter-balance. (See b, Fig. 4). This little piece itself, weighing probably only five pounds, holds the pulley neatly in balance and is very innocent as long as the pulley is standing still, but imagine what stress it will produce on the rim of a 6-foot pulley running at a rim speed of 80 feet per second.

Solution:

$$cf = \frac{80^2 \times 5}{3 \times 32.2} = 331 \text{ pounds.}$$

Thus, when that pulley is running at a speed of 80 feet per second this counter-balance of five pounds will produce the same stress as if it was loaded with 331 pounds when standing still; therefore, it is evident how important it is to turn fast-running pulleys both inside and outside in order to reduce counter-balancing to the least possible amount.

The danger of the rim deflecting or breaking from the stress due to the resistance from the arms (as shown in Fig. 4), can be avoided by running ribs on the inside of the rim, and the danger caused by counter-balancing can be entirely eliminated by making the pulley balance without adding any balancing pieces. Thus, one danger of breaking is avoided by proper designing and the other by good workmanship.

The direct action of the centrifugal force on the rim is calculated by the formula,

$$cf = w \times n^2 \times r \times 0.0034,$$

and the weight of the rim of a cast-iron fly-wheel having one square inch of sectional area and a radius of one foot will be $2 \times \pi \times 12 \times 0.26$ pounds, and a ring of r-foot radius will weigh $r \times 2 \times \pi \times 12 \times 0.26$ pounds. As already stated, the centrifugal force increases as the square of the velocity; that is, if the number of revolutions is doubled the centrifugal force is increased four times; thus is the dangerous limit approached very rapidly under increased speed, and in order to prevent accident, if the speed should happen to increase, it is necessary always to use a high factor of safety in such calculations. Thus, using 15 as a factor of safety* and assuming the tensile strength of cast-iron as 12,000† pounds per square inch, the stress in each cross-section at a and b must not exceed 800 pounds per square inch. The allowable centrifugal force in both sections of the rim is 2×800 pounds, and inserting those values and solving for n the greatest number of revolutions allowable for a cast-iron fly-wheel will be:

$$2\pi \times 12 \times 0.26 \times r \times n^2 \times r \times 0.000340568 = 800 \times 2$$

$$n^2 r^2 = 752891$$

$$nr = \sqrt{752891}$$

$$n = \frac{868}{r} \qquad r = \frac{868}{n}$$

* 15 as factor of safety with regard to strength, is only $\sqrt{15} = 3.873$, or less than 4, as factor of safety with regard to speed.

† This tensile strength for cast-iron may seem very low, but it is dangerous to assume more, because of inside stress in arms or rim already, due to uneven cooling of the casting in the foundry.

Transposing this to diameter in inches, the constant will be $24 \times 868 = 20832$. The formula will be:

$$\text{Number of revolutions per minute} = \frac{20832}{\text{Diameter in inches.}}$$

$$\text{Diameter in inches} = \frac{20832}{\text{Revolutions per minute.}}$$

Rule for Calculating Safe Speed.

Divide 20832 by the diameter of the fly-wheel in inches, and the quotient is the allowable number of revolutions per minute at which a well-constructed fly-wheel may be run with safety.

Rule for Calculating Safe Diameter.

Divide 20832 by the number of revolutions per minute, and the quotient is the safe diameter in inches for a well-constructed fly-wheel.

SHAFTING.

When calculating strength of shafting, both transverse and torsional stress should be considered. Transverse stress is produced by the weight of the shaft itself, the pulleys and the tension of the belts, the effect of which is very severe if the distance between the hangers is too long. Torsional stress is produced by the power which the shaft transmits. Usually the distance between the hangers is made so short that the torsional stress on a shaft is the greater. For transverse stress the shaft may be considered as a round beam, supported under the ends and loaded somewhere between supports. According to Table No. 30 the transverse stress which will destroy a wrought iron beam one inch square, fastened at one end and loaded at the other, is 600 pounds; the strength of a round beam of the same diameter is (see page 251) 0.6 that of a square beam. When the beam is supported under both ends and loaded in the middle, its breaking load will increase four times; therefore, the constant, c, will be $600 \times 4 \times 0.6 = 1440$. Using 10 as factor of safety, the formula for transverse strength of a wrought iron shaft will be:

$$D = \sqrt[3]{\frac{L\ W}{144}} \qquad L = \frac{144\ D^3}{W} \qquad W = \frac{144\ D^3}{L}$$

$D =$ Diameter of shaft in inches.
$L =$ Distance between hangers in feet.
$W =$ Transverse load in pounds, supposed to be at the middle, between the hangers.

144 = Constant for wrought iron, and 100 to 120 may be used as constant for cast-iron, with 10 as factor of safety for transverse strength.

Formula 1, expressed as a rule, will be:

Multiply the distance between hangers, measured in feet, by the transverse load in pounds; divide this product by 144, and the cube root of the quotient will be the diameter of the shaft in inches, calculated with 10 as factor of safety for transverse strength.

Shaft not Loaded at the Middle Between the Hangers.

When a shaft is not loaded at the middle of the span, but somewhere toward one of the hangers, it will carry a heavier load, with the same degree of safety, than it would if loaded in the middle, and the ratio is in inverse proportion as the square of half the distance between hangers to the product of the short and the long ends of the shaft. For instance, a shaft is six feet between hangers and loaded at the middle. What would be the difference in transverse strength if it was loaded two feet from one hanger and four feet from the other?

$3 \times 3 = 9$ and $2 \times 4 = 8$.

Thus, find the transverse load for a shaft when loaded in the middle, multiply by 9 and divide by 8, and the quotient is the load which the same shaft will carry with the same degree of safety against transverse stress, if loaded two feet from one end and four feet from the other.

This rule only applies to the transverse strength, and not to the transverse stiffness of the shaft. For different shapes of shafts and different modes of loading, see beams, pages 243-244. When shafts are heavily loaded near one hanger, and the hanger on the other side of the pulley is further off, most of the load is thrown on the bearing nearest to the pulley, and this bearing is, therefore, liable to heat and to cause trouble, even if the shaft is both stiff and strong enough. (See reaction on the support of beams, page 252).

Transverse Deflection in Shafts.

The transverse deflection in a shaft may be calculated by the formula:

$$D = \sqrt[4]{\frac{L^3\, W\, C}{S}} \qquad L = \sqrt[3]{\frac{S\, D^4}{W\, C}}$$

$$W = \frac{S\, D^4}{L^3\, C} \qquad S = \frac{L^3\, W\, C}{D^4}$$

$S =$ Deflection in inches.
$D =$ Diameter of shaft in inches.
$L =$ Length of span in feet.
$W =$ Load on middle of shaft in pounds.
$C =$ Constant $= 1.7 \times$ constant in Table No. 31, and for wrought iron or Bessemer steel may be taken as 0.00002652.

Constant C may be calculated from experiments by the formula,

$$C = \frac{S\,D^4}{L^3\,W}$$

S = Deflection in inches noted in the specimen, when supported under both ends and loaded transversely at the middle between supports.
D = Diameter of specimen in inches.
L = Distance between supports of specimen in feet.
W = Experimental load in pounds.

EXAMPLE.

A round specimen placed in a testing machine, supported under both ends and loaded at the middle with 2000 pounds, deflects 0.1 inch. The diameter of the specimen is two inches and the distance between supports is three feet. Calculate constant C for this kind of material.

Solution:

$$C = \frac{0.1 \times 2^4}{3^3 \times 2000}$$

$$C = \frac{1.6}{54000} = 0.0000296 \text{ inch.}$$

Thus, the deflection for this kind of material is 0.0000296 inch per pound of load, applied at the middle, between supports, for a round bar one inch in diameter and one foot between supports.

Allowable Deflection in Shafts.

The distance between the hangers must always be determined with due consideration to the allowable transverse deflection in the shafting, especially when the shaft is loaded with large pulleys and heavy belts, remembering that the deflection increases directly with the transverse load and with the cube of the length between the bearings, (see page 254). The allowable transverse deflection in shafting ought not to exceed 0.006 to 0.008 inch per foot of span (see page 266). A beam of wrought iron one foot long and one inch square, when supported under both ends and loaded at the middle, will deflect 0.0000156 inch per pound of load, (see Table No. 31, page 259),and a round beam deflects 1.7 times as much as a square beam, when the diameter and side are equal. A round shaft, one inch in diameter and one foot long, when loaded at the middle with 144 pounds will, therefore, deflect $144 \times 1.7 \times 0.0000156 = 0.00382$ inch.

Thus, this load does not give more than an allowable deflection. But, suppose the distance between bearings is doubled and the load decreased one-half; the ultimate strength of the shaft will be the same, but the deflection will be $72 \times 1.7 \times 2^3 \times 0.0000156 = 0.01528 = 0.0764$ inch per foot.

This calculation shows plainly how very necessary it is to have bearings near the pulleys where shafts are loaded with heavy pulleys and large belts. There is nothing more liable to destroy a shaft than too much deflection, because the shaft is, when running, continually bent back and forth, and at last it must break. The fact must never be lost sight of that strength and stiffness are two entirely different things and follow entirely different laws; therefore, after calculations are made for strength, the stiffness must also be investigated, as stiffness is a very important property in shafting. The best way to overcome too much transverse deflection is to shorten the distance between the bearings. Of course, increasing the diameter of the shaft will also overcome deflection, but shafting should never be larger in diameter than necessary, because the first cost increases with the weight, which increases as the square of the diameter, and the frictional resistance will also increase with the increased diameter; consequently, also, the running expenses.

Torsional Strength of Shafting.

Shafting may be considered as a beam fastened at one end and having a torsional load applied at the other end equal to the pull of the belt on an arm of the same length as the radius of the pulley. In Table No. 32, page 268, constant c is given as 580 pounds for wrought iron.

The formula for twisting stress, as explained under beams (see page 267) is,

$$P = \frac{D^3 c}{m} \qquad D = \sqrt[3]{\frac{P\ m}{c}}$$

m = the length of the lever or arm in feet, and will here be the radius of the pulley and be denoted by r. The length of the shaft has no influence on its torsional strength, but only on its angle of torsional deflection (see page 268). Using 10 as factor of safety, the formula will be:

$$D = \sqrt[3]{\frac{r\ W}{58}}$$

D = Diameter of shaft in inches.
r = Radius of pulley in feet.
W = Pull of belt in pounds.
58 = Constant, with 10 as factor of safety = $\frac{1}{10} \times 580$, taken from Table No. 32, page 268.

Frequently it is more convenient to calculate the torsional strength of shafting according to the number of horse-power the shaft is to transmit (see page 317).

In the above formula, assume W to be 58 pounds, r to be one foot, and D will be one inch. That is, a shaft one inch in diameter is strong enough to resist, with 10 as factor of safety, a torsional load of 58 pounds acting on an arm one foot long. Assuming this 58 pounds to act on the rim of a pulley of one foot radius, two feet in diameter, and making one revolution per minute, it will transmit power at a rate of $58 \times 6\frac{2}{7} = 364\frac{4}{7}$ foot-pounds per minute; but one horse-power is 33,000 foot-pounds per minute, and if the shaft should transmit one horse-power it must make $\frac{33000}{364\frac{4}{7}} = 90.52$ revolutions per minute. Hence the practical formulas for torsional strength of shafting:

$$D = \sqrt[3]{\frac{H \times 90}{n}} \qquad H = \frac{D^3 \times n}{90} \qquad n = \frac{H \times 90}{D^3}$$

D = Diameter of shaft in inches.
H = Number of horse-power transmitted by the shaft.
n = Number of revolutions made by the shaft per minute.
90 = Constant, using 10 as factor of safety, and assuming the torsional strength to be as given in Table No. 32.

Torsional Deflection in Shafting.

In constructing different kinds of machinery it is frequently necessary to consider the torsional deflection. The formula for torsional deflection for wrought iron (see page 271) will be:

$$S = \frac{0.00914 \times m\,L\,P}{D^4}$$ This will transpose to

$$S = \frac{48\,H\,L}{n\,D^4}$$

S = Deflection in degrees.
H = Number of horse-power transmitted.
48 = Constant; calculated thus, $\frac{0.00914 \times 33000}{2 \times 3.1416} = 48$
L = Length of shaft in feet between the force and the resistance.
n = Number of revolutions made by the shaft per minute.
D = Diameter of shaft in inches.

Example.

How many degrees is the deflection of a shaft two inches in diameter, 50 feet long, making 300 revolutions per minute and transmitting 15 horse-power, applied at one end and taken off at the other?

Solution:

$$S = \frac{48\,H\,L}{n\,D^4} = \frac{48 \times 15 \times 50}{300 \times 2^4} = 7\tfrac{1}{2} \text{ degrees.}$$

Classification of Shafting.

Shafting may be divided into three different kinds.

First.— Shafts where the main belts are transmitting the power, or so-called "Jack Shafts." Such shafts must have their boxes as near the pulleys as possible. For torsional strength their diameter may be calculated by the formula,

$$D = \sqrt[3]{\frac{H \times 125}{n}} \qquad \text{(See Table No. 40.)}$$

Second.— Common shafting in shops and factories, where the power is taken off at different places for driving machinery. Such shafts ought to be supported by hangers as given in Table No. 43, and their supports must also be reinforced by extra hangers, if necessary, where an extraordinary large pulley or heavy belt is carried. For torsional strength the diameter of such shafts may be calculated by the formula,

$$D = \sqrt[3]{\frac{H \times 90}{n}} \qquad \text{(See Table No. 41.)}$$

Third.— Shafting having practically no transverse stress, but used simply to transmit power from one place to another. Such shafts ought to be supported by hangers according to Table No. 43, and the diameter may be calculated by the formula,

$$D = \sqrt[3]{\frac{H \times 50}{n}} \qquad \text{(See Table No. 42.)}$$

TABLE No. 40.—Giving Horse-Power of Main Shafting at Various Speeds.

Diameter of Shaft in Inches.	Revolutions per Minute.										
	60	80	100	125	150	175	200	225	250	275	300
1¾	2.6	3.4	4.3	5.4	6.4	7.5	8.6	9.7	10.7	11.8	12.9
2	3.8	5.1	6.4	8	9.6	11.2	12.8	14.4	16	17.6	19
2¼	5.4	7.3	9.1	11	13	16	18	21	23	25	27
2½	7.5	10	12.5	15	18	22	25	28	31	34	37
2¾	10	13	16	21	25	29	33	37	42	46	50
3	13	17	21	27	32	38	43	49	54	59	65
3¼	16	22	27	34	41	48	55	62	69	76	82
3½	20	27	34	43	51	60	68	77	86	94	103
3¾	25	34	42	53	63	74	84	95	105	116	126
4	30	41	51	64	77	90	102	115	128	141	154
4½	43	58	73	91	109	128	146	164	182	201	219
5	60	80	100	125	150	175	200	225	250	275	300

TABLE No. 41.—Giving Horse-Power of Line Shafting at Various Speeds.

Diameter of Shaft in Inches.	Revolutions per Minute.											
	100	125	150	175	200	225	250	275	300	325	350	400
1¾	6	7.4	9	10.4	12	13.4	15	16.4	18	19.4	21	23.8
1⅞	7.3	9.1	10.9	12.7	14.5	16	18	20	22	23.8	25	29
2	8.9	11.1	13	15.5	17.7	20	22	24	27	19	31	35
2⅛	10.6	13.2	16	18.5	21	24	27	29	32	34	37	42
2¼	12.6	15.8	19	22	25	28	32	35	38	41	44	50
2⅜	15	18.6	22	26	30	33	37	41	44	48	52	59
2½	17	22	26	30	35	39	43	48	52	56	61	69
2¾	23	29	34	40	46	52	58	64	69	75	81	92
3	30	37	45	52	60	67	75	82	90	97	105	120
3¼	38	47	57	67	76	86	95	105	114	124	133	152
3½	48	59	71	83	95	107	119	131	143	155	167	190
3¾	58	73	88	102	117	132	146	161	176	190	205	234
4	71	89	107	125	142	160	178	196	213	231	249	284

TABLE No. 42.—Giving Horse-Power of Shafting Used Only for Transmitting Power.

Diameter of Shaft in Inches.	Revolutions per Minute.											
	100	125	150	175	200	225	250	275	300	325	350	400
1½	6.7	8.4	10	11.8	13.5	15.1	16.8	18.5	20.3	22	23	27
1⅝	8.6	10.7	12.8	15	17.1	19.3	21.4	23.6	25.8	28	30	34
1¾	10.7	13.4	16	18.7	21.5	24	26.8	29.4	32.1	35	37	43
1⅞	13.2	16.5	19.7	23	26.4	29.7	33	36.2	39.5	43	46	52
2	16	20	24	28	32	36	40	44	48	52	56	64
2⅛	19	24	29	33	38	42	48	53	57	62	67	76
2¼	23	28	34	40	45	51	57	63	68	74	80	91
2⅜	27	33	40	47	54	60	67	74	80	87	94	107
2½	31	39	47	55	62	70	78	86	94	102	109	125
2¾	41	52	62	73	83	93	104	114	125	132	146	166
3	54	67	81	94	108	121	135	148	162	175	189	216
3¼	69	86	103	120	137	154	172	189	206	223	240	275
3½	86	107	128	150	171	193	114	236	257	279	300	343

Distance Between the Bearings.

Jack shafts should always have bearings as near the pulleys as possible.

Ordinary line shafts, as given in Table No. 41, and shafts for simply transmitting power, may have the distance between the hangers as given in the following table:

TABLE No. 43.

Diameter of Shaft in Inches.	1½ to 1¾	2 to 2½	2½ to 4
Distance between bearings in feet	6½	8	10

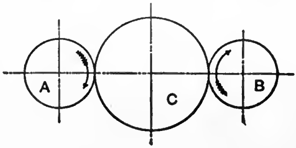
Shafts for Idlers.

Shafts for idlers (see *C*, Fig. 1) have very little torsional stress and the distance between the bearings may also be very short, so that even with a great transverse load such a shaft may be of comparatively small diameter as far as requirements for strength is concerned. In such a shaft there is great danger of trouble from hot bearings; therefore, in designing, it is very important to make its diameter and the length of the bearing of such proportions that excessive pressure per square inch of bearing surface is avoided.

EXAMPLE.

Twenty-five horse-power is to be transmitted from *A* to *B* through idler *C*. (See Fig. 1). The gears on shafts *A* and *B* are 36 inches in diameter and make 40 revolutions per minute. What is the necessary diameter of shaft *C*, which is supported by two bearings one foot apart and carrying a gear 48 inches in diameter placed at the middle between the bearings.

FIG. 1.



Solution:

The velocity on pitch line of gear *A* will be

$$\frac{40 \times 36 \times 3.1416}{12} = 377 \text{ feet per minute.}$$

25 horse-power = 33,000 × 25 = 825,000 foot-pounds.

The pressure at the pitch line of *A* transferred to *C* will be

$$\frac{825000}{377} = 2188 \text{ pounds.}$$

The reaction at the pitch line between C and B, is also 2188 pounds; therefore, the total pressure (besides the weight of C, which is omitted in this calculation) on both bearings will be $2 \times 2188 = 4376$ pounds and the pressure of each bearing of C will be 2188 pounds. Allowing a pressure on the bearings of 100 pounds per square inch, the necessary bearing surface will be

$$\frac{2188}{100} = 21.88 \text{ square inches for each bearing.}$$

Assuming the length of the bearing to be twice its diameter,

$$D \times 2\,D = 21.88$$

$$D^2 = \frac{21.88}{2}$$

$$D = \sqrt{10.94}$$

$$D = 3.3 \text{ inches.}$$

Calculating the size required with regard to transverse strength by the formula on page 360,

$$D = \sqrt[3]{\frac{1 \times 4376}{144}} = 3.12 \text{ inches.}$$

Thus, a shaft 3.3 inches in diameter is of ample size for strength. The surface velocity of this shaft will be,

$$\frac{3.3 \times 3.1416 \times 40}{12} = 34.5 \text{ feet.}$$

per minute, and at that velocity a pressure of 100 pounds per square inch of bearing surface is very safe from liability of heating if the bearing is well made and amply provided with oil.

Proportion of Keys.

The breadth of the key is usually made to be one-fourth of the diameter of the shaft, and the thickness to be one-sixth of the diameter of the shaft.

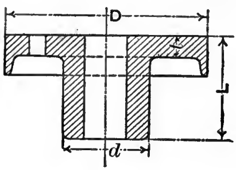
Keys and key-ways are usually made straight and should always be a very good fit sidewise. Frequently set-screws are used on top of keys in mill gearing. Sometimes in heavy machinery keys are made tapering in thickness, usually one-eighth inch per foot of length. A corresponding taper is made in the depth of the key-way in the hub. Key-ways in shafts are always made straight.

For light and fine machinery taper keys are never used.

TABLE No. 44.—Dimensions of Couplings for Shafts.

(All dimensions in inches.)

Diameter of Shaft.	Dimensions of Couplings.				Diameter of Coupling Bolts.	Number of Coupling Bolts.
	d	*D*	*L*	*t*		
1¼	3	7	4	⅝	⅝	4
1½	3⅜	8	4½	⅝	⅝	4
2	4½	9	4¾	¾	¾	4
2½	5	10	5½	¾	¾	4
3	6	11½	6	⅞	¾	5
3½	6⅞	13	6¾	⅞	¾	5
4	7¾	14½	7½	1	⅞	6
4½	8¼	15¾	7⅞	1⅛	1	6
5	9	17	8¼	1¼	1⅛	6



BEARINGS.

A satisfactory rule is to make the length of the bearing for line shafting six times the square root of the diameter of the shaft.

EXAMPLE.

What is a suitable length of bearings for a shaft of four inches diameter?

Solution:

$$\text{Length of bearing} = 6 \times \sqrt{4} = 12 \text{ inches.}$$

Some designers make the length of the bearing four times its diameter.

Area of Bearing Surface.

The projected area of any bearing is always considered as its bearing surface. Thus, the length of the bearing multiplied by the diameter of the shaft gives the area of bearing surface. For instance, the length of the box is twelve inches and the diameter of the shaft is four inches; the area of bearing surface is 12 × 4 = 48 square inches.

Allowable Pressure in Bearings.

The allowable pressure per square inch of bearing surface will depend on the surface speed of the shaft and the condition of the bearing, arrangements for oiling, etc. For common line shafting from two to four inches in diameter, not making over 200 revolutions per minute, a pressure not exceeding forty pounds per square inch ought to work well. Greater pressure or greater speed may make it difficult to keep the bearings cool.

EXAMPLE.

What pressure may be allowed on a bearing twelve inches long and four inches in diameter?

Solution:

$$\text{Pressure} = 4 \times 12 \times 40 = 1920 \text{ pounds.}$$

In well constructed machinery there should not be any trouble from heating, if the surface velocity and the pressure in the bearings does not exceed the values given in the following table:—

METRIC MEASURE.		ENGLISH MEASURE.	
Kilograms per Square Centimeter.	Surface Velocity in Meters per Minute.	Pounds per Square Inch.	Surface Velocity in Feet per Minute.
5	100	75	300
12	50	180	150
20	20	300	60

The bearings for machinery in general are constructed in various ways and of different proportions, according to the designer's judgment, but it is a well-known fact that high-speed machinery must have longer bearings than slow-speed machinery.

The length of the bearing will usually vary from one and one-half to six times the diameter.

When the shafts are small (less than two inches in diameter), and the speed is from 100 to 1000 revolutions per minute, the following empirical formula may be used as a guide:

$$L = d \times \left(1 + \frac{n}{200}\right)$$

L = length of bearing, d = diameter of bearing, n = number of revolutions per minute.

FIG. 1.

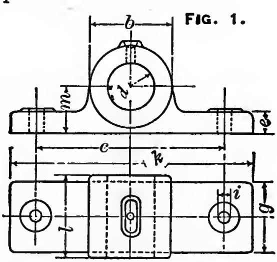


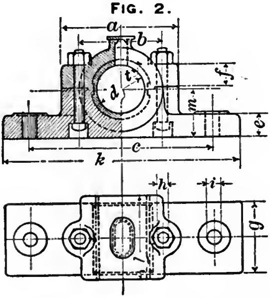
FIGURE 1 shows a cheap, solid cast-iron box used for comparatively small and less important shafts. Dimensions, suitable for bearings from one to two inches in diameter, are given in the following table :—

(All Dimensions in Inches.)

d = Diameter of Shaft.	Length of bearing $l = 1\frac{3}{4}\ d$.	$g = 1\frac{1}{2}\ d$.	$b = 1\frac{3}{4}\ d$.	$c = 3\ d + \frac{3}{4}''$	$k = 4\ d + 1\frac{1}{2}''$	$e = \frac{1}{4}\ d + \frac{1}{4}''$	$i = \frac{1}{4}\ d + \frac{1}{4}''$	$m = d$.
d	l	g	b	c	k	e	i	m
1	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	$3\frac{3}{4}$	$5\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$
$1\frac{1}{4}$	$2\frac{3}{16}$	$1\frac{7}{8}$	$2\frac{3}{16}$	$4\frac{1}{2}$	$6\frac{1}{2}$	$\frac{9}{16}$	$\frac{9}{16}$	$1\frac{1}{4}$
$1\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{1}{4}$	$2\frac{5}{8}$	$5\frac{1}{4}$	$7\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{2}$
$1\frac{3}{4}$	$3\frac{1}{16}$	$2\frac{5}{8}$	$3\frac{1}{16}$	6	$8\frac{1}{2}$	$\frac{11}{16}$	$\frac{11}{16}$	$1\frac{3}{4}$
2	$3\frac{1}{2}$	3	$3\frac{1}{2}$	$6\frac{3}{4}$	$9\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	2

FIGURE 2 shows a babbitted split box suitable for shafts from one to four inches in diameter, and running at a comparatively slow speed.

FIG. 2.



d = Diameter of shaft.

$a = 2\frac{1}{2} \times d$.

$b = 1\frac{3}{4} \times d$.

$c = 3 \times d + \frac{3}{4}$ inch.

$k = 4 \times d + 1\frac{1}{2}$ inches.

$e = \frac{1}{4} \times d + \frac{1}{4}$ inch.

$f = \frac{1}{4} \times d + \frac{1}{4}$ inch.

$g = 1\frac{1}{2} \times d$.

$l = 2 \times d$

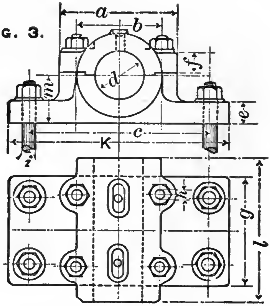
Thickness of babbitt metal, $t = \frac{1}{16}\ d + \frac{1}{8}$ inch.

Diameter of bolt, $h = \frac{1}{5} d + \frac{1}{4}$ inch.

Diameter of bolt, $i = \frac{1}{5} d + \frac{3}{8}$ inch.

FIGURE 3 shows the same general design of box as Figure 2, excepting that the bearing is longer and the base wider. This box is more suitable for comparatively high-speed shafts.

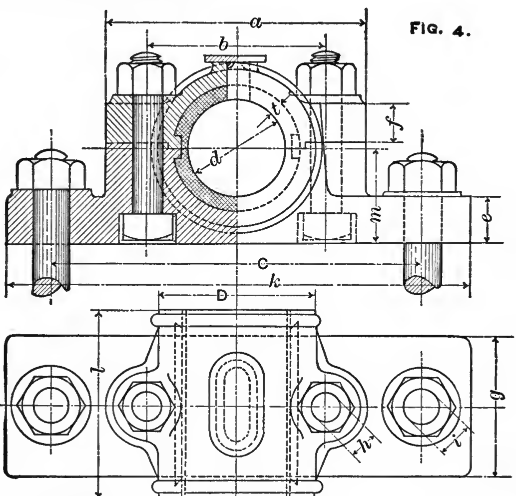
FIG. 3.



$d =$ Diameter of shaft.
$a = 1\frac{3}{4}\,d + 1\frac{1}{4}$ inches.
$b = 1\frac{3}{8}\,d + \frac{5}{8}$ inch.
$c = 2\frac{1}{2}\,d + 2$ inches.
$k = 3\,d + 3$ inches.
$e = \frac{1}{4}d + \frac{1}{4}$ inch.
$f = \frac{1}{4}\,d + \frac{1}{4}$ inch.
$g = \frac{3}{4}\,l$
$l = 5 \times \sqrt{d}$
$m = d$
$h = \frac{1}{8}\,d + \frac{1}{4}$ inch.
$i = \frac{1}{8}\,d + \frac{3}{8}$ inch.

FIGURE 4 shows a babbitted box or pedestal suitable for comparatively heavy-loaded shafts, from three to eight inches in diameter, such as outer bearings for steam engine shafts, bearings for jack shafts, etc.

FIG. 4.



$d =$ Diameter of shaft; $a = 2\,d + 1\frac{1}{4}$ inch; $b = 1\frac{1}{2}\,d + \frac{5}{8}$ inch; $c = 2\frac{3}{4}\,d + 2$ inches; $k = 3\frac{1}{4}\,d + 3$ inches; $e = \frac{1}{4}\,d + \frac{1}{4}$ inch; $f = \frac{1}{4}\,d + \frac{1}{4}$ inch; $g = 1\frac{1}{2}\,d$; $l = 2\,d$; $m = d$; $D = 1\frac{5}{8}\,d$.

Diameter of bolts, $h = 0.2\,d + \frac{1}{4}$ inch (approximately).

Diameter of bolts, $i = 0.2\,d + \frac{3}{8}$ inch (approximately).

FIG. 5.

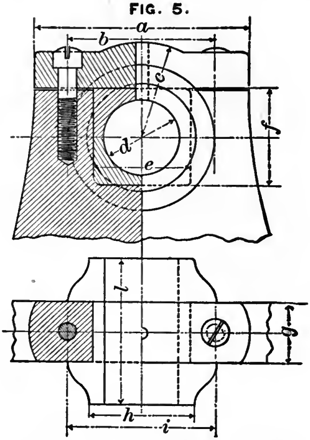


FIGURE 5 shows a bearing fitted into the frame of a machine, suitable for shafts from one to three inches in diameter. The cut shows a part of the head-stock of a speed lathe fitted with this kind of a bearing. The bearing itself, which may be of gun metal or cast iron, is carefully fitted into the frame by planing and scraping.

This kind of a bearing is sometimes lined with babbitt, but more frequently the spindle is carefully fitted into the bearing by scraping.

$d =$ Diameter of bearing; $l = 2d$; $a = 2\frac{1}{4}d + 1$ inch; $b = 1\frac{1}{2}d + \frac{3}{4}$ inch; $c = 1\frac{1}{4}d$; $e = f = 1\frac{1}{4}d + \frac{1}{8}$ inch; $f = e = 1\frac{1}{4}d + \frac{1}{8}$ inch; $g = \frac{3}{4}d + \frac{1}{4}$ inch; $h = 1\frac{3}{8}d + \frac{1}{8}$ inch; $i = 2d$. Diameter of screws $= \frac{3}{16}d + \frac{3}{16}$ inch.

FIGURE 6 shows the form of a self-lining and self-oiling bearing, very suitable for high-speed machinery, and used to a great extent for dynamos and electric motors. The figure shows a part of a dynamo frame with the box in section, cut through the center line of bearing, and also a partly sectional cut from the top. For dimensions see Table No. 46.

The bearing *n n* may be cast in one piece from gun metal, as shown in the cut, or it may be (preferably for the larger size) made in two parts. The seat for this box is turned in spherical form on the outside, and a fit is obtained between this bearing and the frame of the machine by casting in type metal or babbitt metal, as shown at *m m*.

The loose rings, *n n*, are continually dipping into the oil reservoir, and carrying oil to the shaft. (Chains are frequently used instead of rings). The stop-rings should be set so that the spindle has room for a little motion lengthwise in the bearing. This will in a great measure prevent heating and cutting, and by their peculiar shape the stop-rings will, by the action of centrifugal force, throw the oil off at *a a*, to return to the oil reservoir; *h h* are plugs in the oil hole; the screw *i* prevents the box from turning with the shaft, and also forms a convenient projection to take hold of when taking the cap off of the bearing.

FIG. 6.

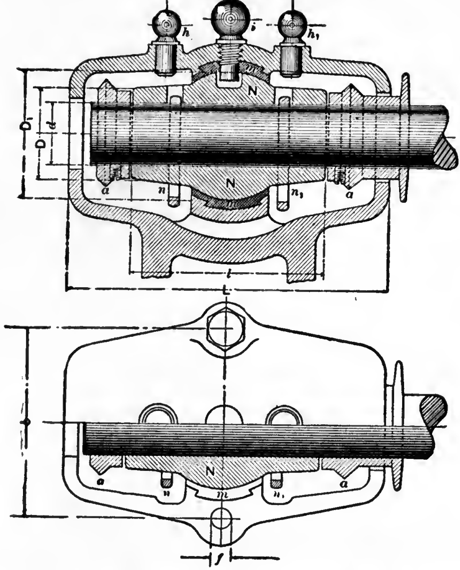


TABLE No. 46.—Giving Dimensions of Fig. 6.

d	D	D_1	L	l	b	f
Inches.	$1\frac{1}{4}d+\frac{1}{4}''$	$1\frac{3}{4}d+\frac{1}{4}''$	$1\frac{1}{2}l+1''$	$4\times\sqrt{d}$	$2\frac{1}{4}d+1''$	Inch.
1	1½	2	7	4	3¼	½
1¼	1 13/16	2 7/16	7¾	4½	3 13/16	½
1½	2⅛	2⅞	8½	5	4⅜	⅝
1¾	2 7/16	3 5/16	8⅞	5¼	4 15/16	⅝
2	2¾	3¾	9 7/16	5⅝	5½	¾
2¼	3 1/16	4 3/16	10	6	6 1/16	¾
2½	3⅜	4⅝	10⅜	6¼	6⅝	¾
2¾	3 11/16	5 1/16	10 15/16	6⅝	7 3/16	⅞
3	4	5½	11½	7	7¾	⅞
3¼	4 5/16	5 15/16	11⅞	7¼	8 5/16	⅞
3½	4⅝	6⅜	12¼	7½	8⅞	⅞
3¾	4 15/16	6 13/16	12⅝	7¾	9 7/16	1
4	5¼	7¼	13	8	10	1

GEAR TEETH.

Circular Pitch.

The length of the pitch circle or pitch line from center of one tooth to the center of the next is the circular pitch of a gear, or a rack.

Cast gear teeth, constructed on the circular pitch system, may be made of the following proportions:

Thickness of tooth on pitch line = $\frac{6}{13}$ pitch.
Space between teeth on pitch line = $\frac{7}{13}$ pitch.
Height of tooth outside of the pitch line = $\frac{3}{10}$ pitch.
Depth of space inside of pitch line = $\frac{4}{10}$ pitch.

$$\text{Pitch diameter of gear} = \frac{\text{circular pitch} \times \text{number of teeth.}}{3.1416}$$

For cut gears, use the following formulas:

Thickness of tooth on pitch line = 0.5 pitch.
Space between teeth on pitch line = 0.5 pitch.
Height of tooth outside pitch line = 0.3183 pitch.
Depth of space inside of pitch line = 0.3683 pitch.

To Calculate Diameter of Gear According to Circular Pitch.

RULE.

Multiply the circular pitch by the number of teeth in the gear and divide the product by 3.1416; the quotient is the diameter of the pitch circle; add $\frac{6}{10}$ of the circular pitch to obtain the whole diameter of the gear.

EXAMPLE.

Find whole diameter of a gear of 48 teeth and three-inch circular pitch.

Solution:

$$\text{Pitch diameter} = \frac{3 \times 48}{3.1416} = 45.84 \text{ inches.}$$

Double the addendum = $3 \times 0.6 =$ 1.80

The whole diameter is 47.64 inches.

Table No. 47 is calculated for one-inch circular pitch; to find the pitch diameter of a gear of any number of teeth given in the table, multiply the diameter given in the table by the circular pitch in the gear, and the product is the pitch diameter of the gear. In order to find the whole diameter, add twice the height of the tooth outside the pitch line, as calculated by the above formula.

TABLE No. 47.—Giving Pitch Diameter of Gears of One Inch Circular Pitch.

Teeth.	Dia.	Teeth.	Dia.	Teeth.	Dia.	Teeth.	Dia.
12	3.82	36	11.46	60	19.10	84	26.74
13	4.14	37	11.78	61	19.42	85	27.06
14	4.46	38	12.10	62	19.74	86	27.38
15	4.78	39	12.42	63	20.06	87	27.70
16	5.09	40	12.73	64	20.37	88	28.01
17	5.41	41	13.05	65	20.69	89	28.33
18	5.73	42	13.37	66	21.01	90	28.65
19	6.05	43	13.69	67	21.33	91	28.97
20	6.37	44	14	68	21.65	92	29.29
21	6.69	45	14.32	69	21.97	93	29.60
22	7	46	14.64	70	22.28	94	29.92
23	7.32	47	14.96	71	22.60	95	30.24
24	7.64	48	15.28	72	22.92	96	30.56
25	7.96	49	15.60	73	23.24	97	30.88
26	8.28	50	15.92	74	23.56	98	31.20
27	8.60	51	16.24	75	23.88	99	31.52
28	8.91	52	16.55	76	24.19	100	31.83
29	9.23	53	16.87	77	24.51	101	32.15
30	9.55	54	17.19	78	24.83	102	32.47
31	9.87	55	17.51	79	25.15	103	32.78
32	10.19	56	17.83	80	25.47	104	33.10
33	10.50	57	18.14	81	25.79	105	33.42
34	10.82	58	18.46	82	26.10	106	33.74
35	11.14	59	18.78	83	26.42	107	34.06

To Calculate Diameter of Gears when Distance Between Centers and Ratio of Speed is Given.

When calculating gears to connect two shafts of given distance between centers and at a given ratio of speed, use the formula,

$$D = \frac{2 \times S \times n}{n + N}$$

$$d = \frac{2 \times S \times N}{n + N}$$

$D =$ Diameter of large gear.
$d =$ Diameter of small gear.
$S =$ Distance between centers in inches.
$N =$ Number of revolutions of large gear per minute.
$n =$ Number of revolutions of small gear per minute.

NOTE.—The small gear is always on the shaft having the greater speed.

EXAMPLE.

What will be the diameter of the gears to connect two shafts when the distance between centers is 32 inches, and one shaft is to make 135 revolutions and the other 105 revolutions per minute?

Solution:

$$D = \frac{2 \times 32 \times 135}{135 + 105} = 36 \text{ inches diameter.}$$

$$d = \frac{2 \times 32 \times 105}{135 + 105} = 28 \text{ inches diameter.}$$

After the diameter of the gears is calculated, the pitch is decided upon according to the power the gears have to transmit.

Frequently the pitch will have to be altered somewhat, and such gears sometimes have teeth of very odd pitch, in order to obtain the right number of teeth to give the required ratio of speed. The ratio between the number of teeth in the gears may always be seen from the ratio of speed between the two shafts. For instance, in the above example, the ratio of speed between the shafts is $^{135}/_{105}$, which, reduced to its lowest terms, is $^{9}/_{7}$; therefore, the number of teeth in the two gears may be any multiple of 9 and 7, respectively.

For instance, $8 \times 9 = 72$ teeth for the large gear, and $8 \times 7 = 56$ teeth for the small gear; or, $10 \times 9 = 90$ teeth for the large gear, and $10 \times 7 = 70$ teeth for the small gear, etc.

The dimensions of teeth may be calculated according to rules given on page 375.

Diametral Pitch.

The *diametral pitch* of a gear is the number of teeth to each inch of its pitch diameter. In cut gearing it is always customary to calculate the gears according to diametral pitch. When gears are calculated according to circular pitch the corresponding circumference of the pitch circle is usually an even number, but the diameter will generally be a number having cumbersome fractions, and therefore the distance between the centers of the gears will be a number having fractions which may be very inconvenient to measure with common scales. This is because the circumference of a circle divided by 3.1416 is equal to its diameter and the diameter multiplied by 3.1416 is equal to the circumference. When gearing is calculated according to diametral pitch this trouble is entirely avoided, as this directly expresses the number of teeth on the circumference of the gear according to its pitch diameter. For instance, "six diametral pitch" means that there are six teeth on the circumference of the gear for each inch of pitch diameter. Thus, a gear of six diametral pitch and forty-eight teeth will be eight inches pitch diameter. A gear of "eight diametral pitch" means that the gear has eight teeth per inch of pitch diameter. A gear of "ten diametral pitch" means that the gear has ten teeth per inch of pitch diameter. A gear of "twelve diametral pitch" means that the gear has twelve teeth per inch of pitch diameter, etc.

Thus, the pitch diameter and, consequently, the distance between the centers, will be a number which may be conveniently measured, and the dimensions of tooth parts are also much more easily calculated by this system.

Rules for Calculating Dimensions of Gears According to Diametral Pitch.

The pitch diameter is obtained by dividing the number of teeth by the diametral pitch.

EXAMPLE.

What is the pitch diameter of a gear of 48 teeth, 16 pitch?

Solution:

48 divided by $16 = 3$, therefore the pitch diameter is 3 inches.

The number of teeth is obtained by multiplying the pitch diameter by the diametral pitch.

EXAMPLE.

What is the number of teeth in a gear of 5 inches pitch diameter and 12 pitch?

Solution:

$5 \times 12 = 60$, therefore the gear has 60 teeth.

The whole diameter of a spur gear is obtained by adding 2 to the number of teeth and dividing the sum by the diametral pitch.

EXAMPLE.

What is the whole diameter of a gear blank for 68 teeth, 10 pitch?

Solution:

$$\text{Whole diameter} = \frac{68 + 2}{10} = 7 \text{ inches.}$$

The number of teeth is obtained by multiplying the whole diameter of the gear by the diametral pitch and subtracting 2 from the product.

EXAMPLE.

The whole diameter of a gear blank is 8 inches; it is to be cut 10 diametral pitch. Find the number of teeth.

Solution:

Number of teeth $= (8 \times 10) - 2 = 78$.

The diametral pitch is obtained by adding 2 to the number of teeth and dividing by the whole diameter.

EXAMPLE.

A gear has 64 teeth and the whole diameter is 16½ inches. What is the diametral pitch?

Solution:

$$\text{Diametral pitch} = \frac{64 + 2}{16\frac{1}{2}} = 4.$$

Thus, the gear is 4 diametral pitch.

NOTE.—The term *diameter* of a gear usually means *diameter of pitch circle.*

The distance between the centers of two spur gears is obtained by dividing half the sum of their teeth by the diametral pitch.

EXAMPLE.

What is the distance between centers of two gears of 48 and 64 teeth and 8 diametral pitch?

Solution:

$$\text{Distance} = \frac{48 + 64}{2 \times 8} = 7 \text{ inches.}$$

The circular pitch is obtained by dividing the constant 3.1416 by the diametral pitch.

EXAMPLE.

What is the circular pitch of a gear of eight diametral pitch?

Solution:

$$\text{Circular pitch} = \frac{3.1416}{8} = 0.393 \text{ inch.}$$

The thickness of the tooth on the pitch line is obtained by dividing the constant 1.5708 by the diametral pitch.

EXAMPLE.

What is the thickness of the tooth on the pitch line of a gear of 6 diametral pitch?

Solution:

$$\text{Thickness of tooth} = \frac{1.5708}{6} = 0.262 \text{ inches.}$$

The working depth of the tooth is obtained by dividing 2 by the diametral pitch. The clearance at the bottom of the teeth is $\frac{1}{10}$ of the thickness of the tooth on the pitch line. The whole depth to cut the gear is obtained by dividing the constant 2.157 by the diametral pitch.

EXAMPLE.

Find the depth to cut a gear of 8 diametral pitch.

Solution:

$$\text{Depth} = \frac{2.157}{8} = 0.27 \text{ inch.}$$

The whole depth is nearly equal to 0.6866 times the circular pitch. The use of the following tables will facilitate calculations regarding dimensions of teeth in diametral pitch.

TABLE No. 48.—Comparing Circular and Diametral Pitch.

Diametral Pitch.	Circular Pitch.	Circular Pitch.	Diametral Pitch.
2	1.571 inch.	1½ inch.	2.094
2½	1.257 "	1 7/16 "	2.185
3	1.047 "	1⅜ "	2.285
3½	0.898 "	1 5/16 "	2.394
4	0.785 "	1¼ "	2.513
5	0.628 "	1 3/16 "	2.646
6	0.524 "	1⅛ "	2.793
7	0.449 "	1 1/16 "	2.957
8	0.393 "	1 "	3.142
9	0.349 "	15/16 "	3.351
10	0.314 "	⅞ "	3.590
11	0.286 "	13/16 "	3.867
12	0.262 "	¾ "	4.189
14	0.224 "	11/16 "	4.570
16	0.196 "	⅝ "	5.027
18	0.175 "	9/16 "	5.585
20	0.157 "	½ "	6.283
22	0.143 "	7/16 "	7.181
24	0.131 "	⅜ "	8.378
26	0.121 "	5/16 "	10.053
28	0.112 "	¼ "	12.566
30	0.105 "	3/16 "	16.755
32	0.098 "	⅛ "	25.133

TABLE No. 49.—Giving Dimensions of Teeth Calculated According to Diametral Pitch.

Diametral Pitch.	Depth to be Cut in Gear.	Thickness of Tooth on Pitch Line.	Diametral Pitch.	Depth to be Cut in Gear.	Thickness of Tooth on Pitch Line.
2	1.078 in.	0.785 in.	12	0.180 in.	0.131 in.
2½	0.863	0.628	14	0.154	0.112
3	0.719	0.523	16	0.135	0.098
3½	0.616	0.448	18	0.120	0.087
4	0.539	0.393	20	0.108	0,079
5	0.431	0.314	22	0.098	0.071
6	0.359	0.262	24	0.090	0.065
7	0.307	0.224	26	0.083	0.060
8	0.270	0.196	28	0.077	0.056
9	0.240	0.175	30	0.072	0.052
10	0.216	0.157	32	0.067	0.049
11	0.196	0.143			

To Calculate the Number of Teeth when Distance Between Centers and Ratio of Speed is Given.

Select for a trial calculation, the diametral pitch which seems most suitable for the work.

Calculate the sum of the number of teeth in both gears corresponding to this pitch by multiplying twice the distance between their centers by the diametral pitch selected.

The number of teeth in each gear is obtained by the following formula:

$$T = \frac{N \times A}{n + N}$$

$$t = \frac{n \times A}{N + n}$$

T = Number of teeth in large gear.
t = Number of teeth in small gear.
N = Number of revolutions of small gear.
n = Number of revolutions of large gear.
A = Number of teeth in both gears.

EXAMPLE.

The center distance between two shafts is 15 inches. The small gear should make 126 and the large gear, 90 revolutions per minute. Calculate the number of teeth in each gear, if 8 diametral pitch is wanted.

Solution:

The number of teeth in both gears is $2 \times 15 \times 8 = 240$.

$$T = \frac{126 \times 240}{126 + 90} = 140 \text{ teeth.}$$

$$t = \frac{90 \times 240}{126 + 90} = 100 \text{ teeth.}$$

Frequently it is impossible to get gears of the desired pitch to fit within the given center distance and to give the exact ratio of speed. Some modifications must then be made; either the exact ratio of speed must be sacrificed, the pitch must be changed, or the distance between centers must be altered.

NOTE.—The ratio of the number of teeth in the gears can be seen from the ratio of the speed. For instance, in the above example the ratio of speed is $^{90}/_{126}$, which, reduced to its lowest terms, is $^{5}/_{7}$; therefore, the number of teeth in the two gears may, with regard to speed ratio, be any multiple of 5 and 7, respectively, but in order to fit the given center distance and also to be 8 pitch, they *must* be **100** and **140**, which is **$20 \times 5 = 100$** and **$20 \times 7 = 140$**.

The shape of gear teeth is usually either Involute or Cycloid (also frequently called Epicycloid). The shape of a cycloid tooth for a rack is four equal cycloid curves, which may be constructed, so to speak, by letting the generating circle a, (see Fig. 1) roll along on the pitch line of the rack, both above and below.

Cycloid gears have the curve outside the pitch circle formed by an Epicycloid (see Fig. 26, page 191) and the curve inside the pitch circle by a Hypocycloid.

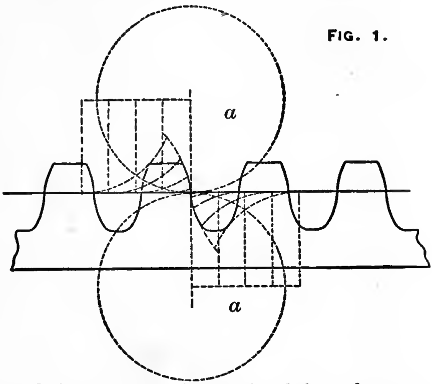


FIG. 1.

The curves always meet on the pitch line in both gears and racks.

The theoretical requirements for correct form of Epicycloid gear teeth are that the face of the teeth of one gear and the flank of the teeth of the other gear must be produced by generating circles of the same diameter.

The diameter of the generating circle is limited by the size of the smallest gear or pinion in the series of gears which are constructed to run together, because if the generating circle is as large in diameter as half the pitch diameter of the gear, the hypocycloid will be a straight line; thus, the flank of the tooth will be a straight radial line. If the generating circle is larger than half the pitch diameter of the gear, the result will be a weak and poor tooth with under-cut flank.

When the same size of generating circle is used for gears of different diameters but of the same pitch, all such gears will work correctly together, and for this reason it is possible to construct interchangeable gears having cycloid teeth. If the diameter of the generating circle is equal to half the diameter of the smallest gear in the set, this gear will have teeth with radial flanks but all the other gears and the rack will have double-curved teeth. Fig. 1 shows a rack drawn to ½-inch circular pitch; the generating circle is 0.98 inch diameter, which is equal to half of the pitch diameter of a gear of 12 teeth and ½ inch circular pitch.

All gears of the same pitch having 12 teeth or more, constructed by the same generating circle in the same manner as the rack, will match and be interchangeable with the rack, and will also match and be interchangeable with each other.

When internal teeth are constructed by the above method, the difference between the number of teeth in the internal gear and its external pinion must never be less than 12; practically, it is better to limit the difference to 15 or 20 teeth.

As interchangeability is seldom required for internal gearing, such gears and their mates are generally constructed together and the designer chooses a generating circle of suitable size to give the shape of tooth he considers best, and he may also vary the size of the driving or the driven gear so as to reduce contacts when the teeth are approaching each other, etc., according to his own judgment and experience.

The difference in pitch diameter of the internal gear and its pinion should never be less than the sum of the diameters of the generating circles, and the diameter of the generating circle of the flanks for the pinion should never be larger than half the pitch diameter, but it should, preferably, be smaller.

As a rule, fillets at the bottom of the teeth are not used in internal gears, but if used they should be very small.

In order that gears constructed with cycloid teeth should run smoothly, it is very important to have the distance between centers correct, so that the pitch lines will exactly meet each other. For this reason, there are many kinds of machinery where cycloid gears should not be used: for instance, for change gears on lathes, involute teeth as far more suitable.

When making patterns, the shape of one tooth is usually carefully drawn on a thin piece of sheet metal, either brass or iron; this is then filed out and used as a templet in tracing the other teeth on the pattern. Sometimes a fly-cutter is made according to this constructed tooth, and all the teeth in the pattern are cut on an index machine or a gear cutting machine; but if such a machine is not available, the next best way is to set out the pitch line of the gear on this templet and also the center line of the tooth, radially towards the center, then draw the pitch line on the pattern, space off each tooth carefully with a pair of dividers and draw the center line on each tooth prolonged across the rim radially in the direction of the center of the gear, then lay the templet carefully on each of these spacings, making the pitch line and the center line of tooth on the templet to exactly match the pitch line and center line of the tooth drawn on the pattern, then trace around the templet and get the shape of one tooth; then move the templet to the next spacing and trace the next tooth, and so on for all the teeth on the gear.

For small patterns it is convenient to fasten the templet to a strip of metal long enough to reach from the teeth to the center of the gear wheel, placing a point in the center of the gear, drilling a hole in the strip and letting it swing around this point, then after all the teeth are spaced off on the pattern the templet is swung from one tooth to the other and all the teeth are traced by the templet. This method has the advantage that it will mark all the teeth exactly alike, because ,the templet being fastened to this strip, can not easily get out of position.

The distance from the pitch line of the templet to the center hole in the strip must be laid off according to the shrinkage rule, and is, of course, in numerical value equal to the pitch radius of the gear, which should always be calculated and given on the drawing. When gear patterns are less than six inches in diameter it is preferable not to allow anything for shrinkage, as the moulder will usually rap the pattern about as much as the casting will shrink in cooling.

When a pair of cycloid gears are constructed without considering interchangeability with other gears of the same pitch, it is customary to choose a generating circle having a diameter equal to three-fourths of the radius of the pitch circle of the small gear, providing this gear has 24 or more teeth. A large generating circle probably reduces the friction in a small measure but gives teeth of less strength. The largest generating circle used ought never to exceed the radius of the pitch circle of the small gear. Decreasing the generating circle will probably increase friction somewhat in the gears, but it gives teeth of greater strength. The smallest generating circle used in practice is equal to half the diameter of the pitch circle of a gear having 12 teeth of the same pitch as the gear to be constructed. Many eminent mechanics consider it preferable never to use a generating circle smaller than half of the pitch diameter of a gear of 15 teeth.

Cycloid gears are mostly used in large cast gears of one-inch circular pitch or more.

Sometimes the driving gear is made of slightly larger diameter, and the teeth spaced at a correspondingly greater pitch than the theoretically calculated size. This is done in order that the teeth shall not rub on each other on the approaching side, but only touch as they are passing the center line and commence to slide away from each other. This will make the gears less noisy, but probably gears made in this manner will wear faster, as there are fewer points of contact, although this may be offset by the fact that the friction between the teeth when they are meeting and pushing onto each other is more injurious than the friction produced when they are sliding away from each other.

The same idea is sometimes employed when constructing bevel gears, in order to make them run quietly.

This mode of sizing gears is not, as a rule, used in modern gear construction, but it is a point well worth remembering, because if either of two gears is over or under size, the gear of over-size should always be used as the driver, and the gear of under-size should always be the driven; never *vice versa*. This will apply as well to involute as to epicycloid gears.

Involute Teeth.

Suppose a strap is fastened at *a* and *b* on the two round discs in Fig. 2. If the disc *b* is turned in the direction of the arrow, the strap will move in a straight line from *c*, toward *d*. This motion will cause the disc *a* to rotate with exactly the same surface speed as the disc *b*, but in the opposite direction.

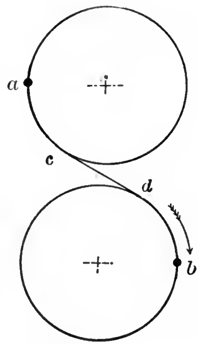


FIG. 2.

Suppose, further, that to the under side of the disc *a* (see Fig. 3) is fastened a piece of sheet brass *p*, or other suitable material of somewhat larger diameter than disc *a*, and that a scratch awl is fastened in the strap at the point *m*; then by turning the disc *b* in the direction of *h* to *b*, and the strap moving with it, being kept tight by the resistance of disc *a*, the scratch awl will trace on the brass plate the curve from *m* to *h*, but if the discs are moving in the opposite direction, the scratch awl will trace the curve from *m* to *K*. Take another brass plate and do the same thing with the other disc, and a similar curve will be produced. In these two brass plates the stock may be filed away carefully, following the curves as shown in Fig. 4. The discs are laid to match each other and free to

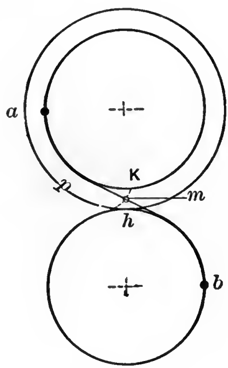


FIG. 3.

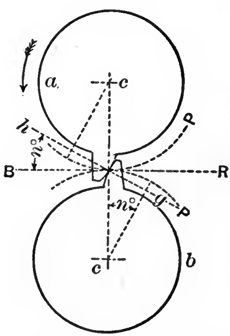


FIG. 4.

swing on their centers; turning the disc *a* in the direction of the arrow, it will give motion to *b*, and both discs will move with the same speed in exactly the same manner as if they were connected by the strap as shown in Fig. 2.

The curve on these two discs represents the form of a gear tooth in the involute system.*

The line $h\,g$, Fig. 4, is called the line of pressure or the line of action. The circles, P and P, are the pitch circles. The line $B\ R$, shows the direction of motion of the teeth at the moment they are passing the center line, $c\ c$.

Approximate Construction of Involute Teeth.

It will be noticed that the line of pressure, $h\ g$, forms an angle with the line $B\ R$. This angle is usually taken as 14½ degrees. This makes the diameter of the base circle, g, (see Fig. 5) equal to 0.968 times the diameter of the pitch circle. The base circle $g\,g$, in Fig. 5, corresponds to the disc in Fig. 3, and the line of pressure in Figs. 5 and 6 corresponds to the strap in Fig. 2. The line of pressure, $h\,g$, Fig. 5, is 75½ degrees to the center line, $f\,c$.

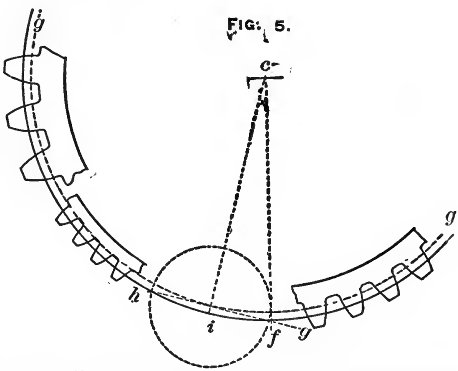


FIG. 5.

A perpendicular is erected from the line h g, through the center, c. Using the point of intersection at i as center, the tooth is drawn simply by a circular arc. This will, in practical work for small gears having more than twenty teeth, correspond nearly enough to the true involute, which was illustrated by means of the strap, disc and scratch awl, as explained in Figs. 2, 3 and 4.

When the gear has less than twenty teeth, and is constructed by circular arcs, as shown in Fig. 5, the top of the tooth will be too thin; but the top of the tooth will be too thick to clear in the rack, if the true involute curve is used.

When the teeth are of true involute curve, a smaller gear than twenty-five teeth will not run freely in a rack having straight teeth slanting 14½ degrees. (See Figs. 6 and 7). Therefore,

*The way to actually draw this curve on paper by means of drawing instruments is explained on page 192. This way explained here, using the disc on the strap, is merely for illustrating and explaining principles, and serves well for that purpose, but would be inconvenient to use in actual construction of gear teeth. In actual work one tooth is carefully constructed, and templets and cutters are made and used, as was explained for Cycloid Gears, page 383.

when a gear has less than twenty-five teeth it is necessary to round the teeth somewhat outside the pitch circle. By making either a drawing or a templet, it is very easy to see how much to round

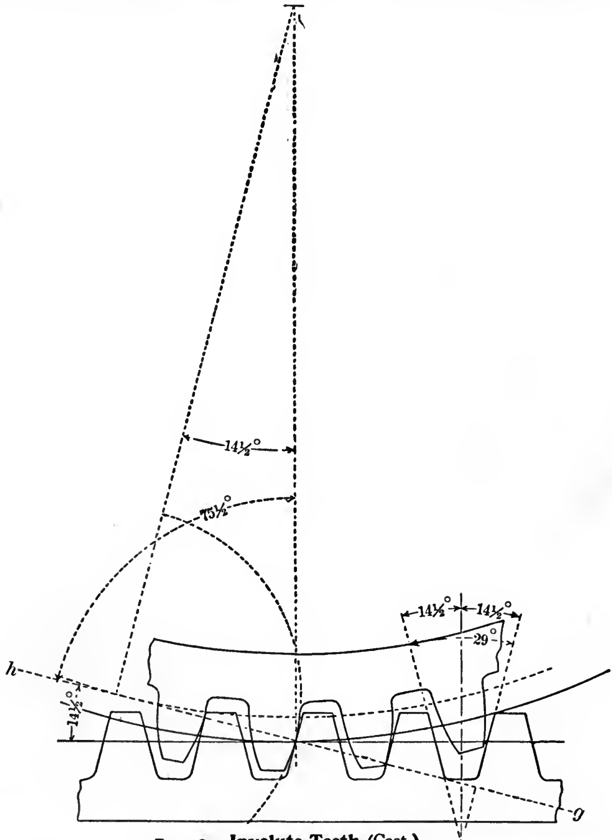


FIG. 6. **Involute Teeth** (Cast.)

the teeth to make them clear in the rack. In interchangeable sets of cut involute gears it is customary to cut the rack with a cutter shaped for a gear of 135 teeth. This will make the teeth in the rack slightly curved instead of straight, as shown in Fig. 6, and this will also make it possible to construct the small gears in an interchangeable set nearer to a true involute, and still have them run freely in the rack.

When gear teeth are constructed as shown in Figs. 6 and 7, the line *g h*, is 75½ degrees to the line *c f*, and the line *c i*, is 14½ degrees to the line *c f*. (See Fig. 5).

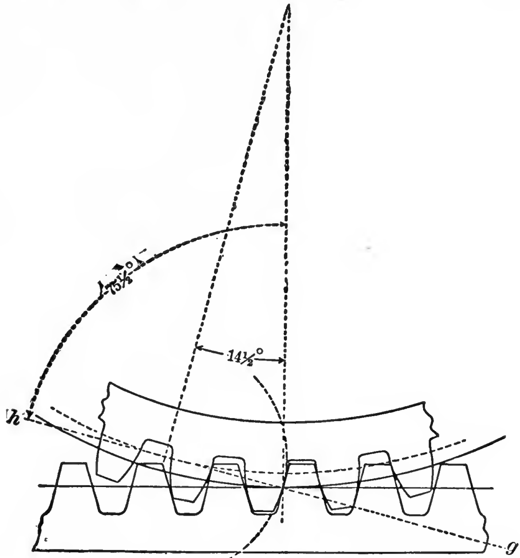


Fig. 7. Involute Teeth (Cut).

The line *h g*, will always be tangent to the base circle which is concentric to the pitch circle. The diameter of the base circle is always 0.968 times the diameter of the pitch circle. The circle forming the shape of the tooth must always have its center on the circumference of the base circle, and its diameter will be one-fourth of the pitch diameter of the gear. As shown in Fig. 2, the same circle gives the form of tooth for coarser or finer pitch. When gears are drawn by this method the pitch circle is divided into as many teeth and spaces as there are to be teeth in the gear; then the form of the tooth is simply struck by the dividers, always using the periphery of the base circle as center, and always taking the distance in the dividers equal to one-fourth of the radius of the pitch circle.

The diameter of the base circle is 0.968 times the diameter of the pitch circle, because cosine of 14½ degrees is 0.96815. The diameter of the circle forming the shape of the tooth is 0.25 times the diameter of the pitch circle of the gear, because sine of 14½ degrees is 0.25038. If the line of pressure is laid at any other angle than 14½ degrees, all these other proportions will also change. Fig. 6 shows a pattern for gears and rack constructed with necessary clearance as used for cast gears. All tooth parts are of the same dimensions as used for cycloid gears as given on page 375. Fig. 7 shows a cut gear and rack constructed in the same manner. The advantages of the involute system of gears are in the strength of teeth, and also that the gears will transmit uniform motion and run satisfactorily, even if the distance between centers should be slightly incorrect.

Width of Gear Wheels.

Gears with cast teeth are usually made narrower than gears with cut teeth. In spur gears with cast teeth it is customary to make the width of the gear four to five times the thickness of the teeth, or twice the circular pitch.

Width of Gears With Cut Teeth.

The following rule is recommended by Brown & Sharpe Mfg. Co. in their "Practical Treatise on Gearing":

Divide eight by the diametral pitch, and add one-fourth inch to the quotient; the sum will be the width of face for the pitch required.

EXAMPLE.

What width of face is required for a gear of four pitch?

Solution:

$$\text{Face} = \tfrac{8}{4} + \tfrac{1}{4} = 2\tfrac{1}{4} \text{ inches.}$$

For change gears on lathes where it is desirable not to have faces very wide, the following rule may be used:

Divide four by the diametral pitch and add one-half inch.

By the latter rule a four-pitch change gear would have but a 1½-inch face.

BEVEL GEARS.

Fig. 8 is a diagram showing how to size bevel gear blanks.

First, lay off the pitch diameters of the two gears, which may be calculated according to diametral pitch or to circular pitch; second, draw the pitch line of teeth; third, lay off on the back of the gear the line *a b*, square to the "pitch line of teeth;" fourth, on the line *a b*, lay off the dimensions of the teeth exactly in the same manner as if it was for a spur gear.

If the gear is calculated according to circular pitch, find dimensions of teeth by formulas on page 375, but if the gear is calculated according to diametral pitch, find dimensions of teeth in Table No. 49.

Make the drawing carefully to scale (full size preferable whenever possible), and measure the outside diameter as shown in the diagram.

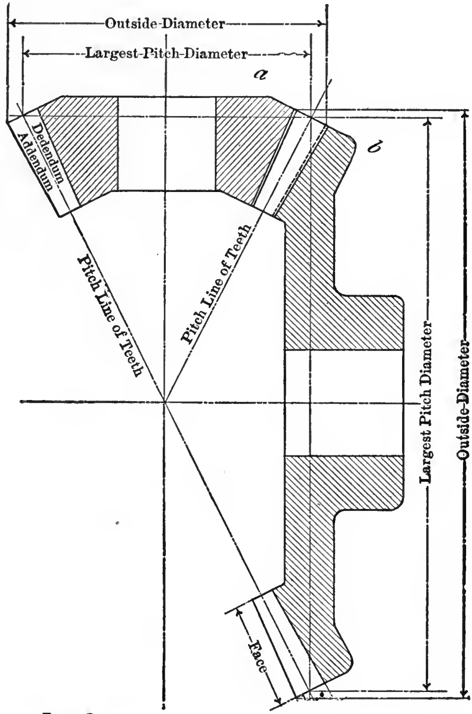


FIG. 8.

To Calculate Size of Bevel Gear for a Given Ratio of Speed.

Ascertain the ratio of speed in its lowest terms. Multiply each term separately by the same number, and the products give the number of teeth in each gear.

EXAMPLE.

Two shafts are to be connected by bevel gears, one shaft to make 80 revolutions and the other 170 revolutions per minute. Find the number of teeth in the gears.

Solution:

$$\text{Ratio} = 80/170 = 8/17.$$

For instance, multiplying by 6, the large gear on the shaft making 80 revolutions will have $17 \times 6 = 102$ teeth. The small gear on the shaft making 170 revolutions will have $8 \times 6 = 48$ teeth.

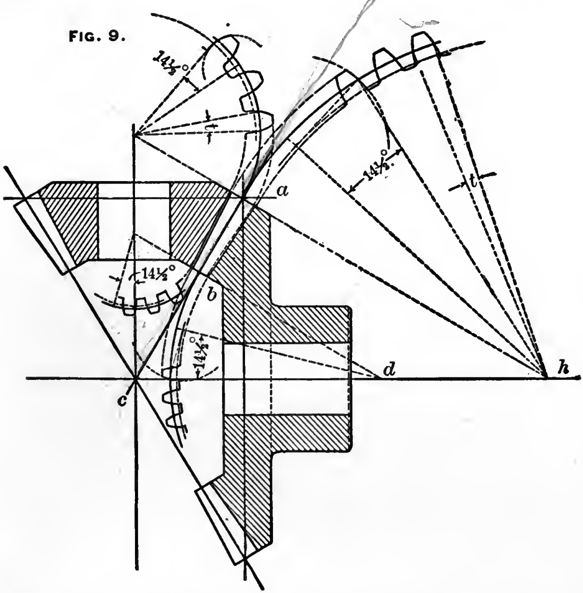
Assuming that on account of room it is necessary to use smaller gears, a smaller multiplier may be used, but if it is desirable to have larger gears, use a larger multiplier.

Decide on the pitch of the gears according to the work they are required to do. Make a scale drawing and get the dimensions as explained on page 390.

Dimensions of Tooth Parts in Bevel Gears.

Fig. 9 shows a sectional drawing of a pair of bevel gears of sixteen diametral pitch, 18 teeth in the small gear and 30 teeth in the large gear. The pitch diameter of the small gear is $\frac{18}{16} = 1\frac{1}{8}$ inches. The pitch diameter of the large gear is $\frac{30}{16} = 1\frac{7}{8}$ inches.

FIG. 9.



The addendum of the teeth on the *back* at *a* is $\frac{1}{16}$ inch, the same as for a spur gear of 16 diametral pitch. The thickness and the total depth to cut the gear at *a* are 0.098 inch and 0.135 inch, respectively.

These dimensions are found in Table No. 49, as if it was a spur gear of 16 diametral pitch. All the dimensions of the tooth decrease gradually toward *b*, as the whole tooth is supposed to vanish in a point in the center at *c*. The dimensions of the teeth at *b* may be calculated and are always in the same proportion to the dimensions at *a* as the distance *c b* is to the distance *c a*; thus, if the length of the tooth from *a* to *b* is made one-third of the length of the distance *c a*, the distance *b c* is two-thirds of the distance *a c*, and, consequently, all the dimensions of the tooth at *b* are two-thirds of the dimensions at *a*. Instead of calculating the size of the teeth at *b*, the dimensions may be obtained by careful drawing. The depth of the tooth at the smallest end is then measured directly at *b*, but the thickness is measured at *t*; the distance *t h* is laid off equal to *b d*.

The length of the tooth from *a* to *b* is to a certain extent arbitrary, but a good rule is seven inches divided by the diametral pitch, but never longer than one-third of the distance from *a* to *c*.

EXAMPLE.

What is the proper length for the teeth of a bevel gear of 8 diametral pitch?

Solution:

Seven inches divided by 8 = ⅞ inch, if the gears are of such diameters that this will not make the length of the teeth more than one-third of the distance from *a* to *c*.

Form of Tooth in Bevel Gears.

Extend the line *a* (see Fig. 9), until it intersects the axial center line of the gear, as at *h*; use *h* as the center, and the shape of tooth at *a* for the large gear is constructed as if it was a spur gear having a pitch radius as large as *a h*.

The shape of the tooth at *b* is constructed in the same way, by extending the line *b*, (which always—the same as line *a*,—is square to the pitch line of the tooth) until it intersects the axial center line of the gear, as at *d*. Using *d* as center, the shape of the tooth is constructed as if it was a spur gear having a pitch radius equal to *d b*. The shape of the teeth of the small gear is obtained in the same way, which is shown by the drawing.

The form of tooth is shown to be approximately involute, constructed as explained for spur gears, page 386.

Measuring the back cone radius, *a h*, of the large gear, it is found to be $\frac{29}{16}$ inch, and the diameter will be $\frac{58}{16}$ inch; thus, the shape of the tooth at *a* for the large gear will be the same as the shape of the tooth in a spur gear of 58 teeth, sixteen diametral pitch.

Measuring the back cone radius of the small gear, it is found to be $\frac{21}{32}$ inch, and the diameter will be $\frac{21}{16}$ inch; consequently the shape of tooth at *a* for the small gear is the same as the shape of tooth in a spur gear of 21 teeth, sixteen diametral pitch.

Therefore, if this pair of gears is to be cut by a rotary cutter having a fixed curve, a different cutter is required for each gear.

When, in a pair of bevel gears, both gears are of the same size and have the same number of teeth, and their axial center lines are at right angles, they are called *miter gears*, and one cutter, of course, will answer for both gears. One cutter will also answer in practice when the difference of the back cone radius of a pair of gears is so small that it comes within the limit of one cutter as used for spur gears of the same size. Bevel gears may also be made with cycloid form of teeth, but whenever cut by rotary cutters, as usually employed in producing small bevel gears of diametral pitch, the involute form of tooth should always be used.

Cutting Bevel Gears.

When bevel gear teeth are correctly formed, the tooth curve will constantly change, from one end of the tooth to the other. Therefore, bevel gears of theoretically correct form cannot be produced by a cutter of fixed curve; but, practically, very satisfactory results are obtained in cutting bevel gears of small and medium size in this way.

When a regular gear-cutting machine is not at hand, the Universal milling machine is a very convenient tool for cutting bevel gears of moderate size, and is used in the following way:

First, see that the gear blank is turned to correct size and angle, and adjust the machine to the angle corresponding to the bottom of the teeth in the gear. The correct index is set according to the number of teeth in the gear. Adjust the cutter to come right to the centre of the gear, cut the correct depth as marked on the gear at *a* (see Fig. 9), according to Table No. 49, and when the machine is adjusted to the correct angle, and the correct depth is cut at *a*, the correct depth at *b* will, as a matter of fact, be obtained.

Second, when a few teeth are cut in the gear (two or three) bring, by means of the index, the first tooth back to the cutter. By means of the index, rotate the gear, moving the tooth toward the cutter; but, by the slide, move the gear sidewise away from the cutter, until the cutter coincides with the space at *b*; then cut through from *a* to *b*. This operation will widen one side of the tooth space at *a*.

Note the position of the machine, and, by the use of the index and slide, return the cutter to its central position and index into the next space, and rotate the other side of the tooth toward the cutter as much as the first side; but, by the slide, the gear is moved sidewise away from the cutter until the cutter coincides with the space at *b*; then cut through on this side from *a* to *b*. Thus, by repeated cutting on each side alternately, one tooth is backed off equally on both sides and measured by a gage, until the correct thickness on the pitch-line at *a*, according to Table No. 49, is obtained.

Be very careful to have the machine *set over* the same amount on each side of the tooth, or else the tooth will be askew.

Third, when one tooth, thus by trial, is correctly cut, note the position of the machine and cut all the teeth through on one side, then *set over* to the other side in exactly the same position as was found to be right for the first tooth; cut through again and the gear is finished. Thus, when the correct position of the machine is obtained, any number of gears of the same size and same pitch may be cut, by simply letting the cutter go through twice.

NOTE.—As already stated, bevel gear cutting in this way is only a compromise at the best, but by careful manipulation and good judgment an experienced man is able to do a very creditable job. A cutter is usually selected of the same curve as is correct for a spur gear corresponding to the *back cone radius* of the gear. Thus, it may be thought that the shape of the tooth should be the shape of the cutter, but by investigation it will be found that, on account of the "backing off," the teeth will be of a little more rounding shape at the large end than corresponds to the cutter; therefore, when the gear has few teeth,—less than 25,—it is usually preferable to make the shape of the cutter to correspond to a gear a little larger than would be called for by the back cone radius of the bevel gear to be cut; but when the gear has more than 25 teeth, a cutter of shape corresponding to the back cone radius of the gear will give good results. For instance, in the pair of bevel gears shown in Fig. 9 the back cone radius of the large gear calls for a cutter corresponding in shape to a cutter for a spur gear of 59 teeth, 16 diametral pitch; and this shape of cutter will, after the teeth are backed off, make the teeth a trifle too round at the large end, and a trifle too straight on the small end, but if the teeth are not too long the job will be very satisfactory.

The back cone radius of the small gear calls for a cutter corresponding in shape to a cutter for a spur gear of 21 teeth, 16 diametral pitch, but when the teeth are backed off they will be a little too rounding on the large end; therefore a better result is obtained by selecting a cutter having a shape corresponding to a little larger spur gear; for instance, a gear of 24 teeth. Such a cutter will give the teeth a better shape on the large end, although it may be necessary to round the teeth a little, outside the pitch line on the small end, by filing.

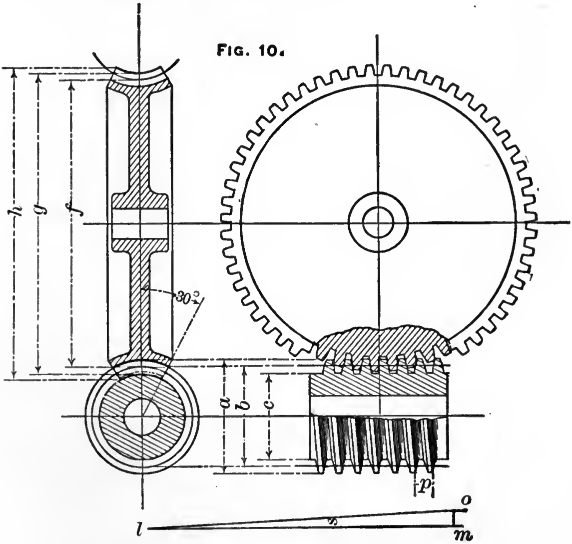
Of course, a spur gear cutter cannot be used for cutting bevel gears, because, although it may have the correct curve, it would be too thick. The thickness of a bevel gear cutter must be at least 0.005 inch thinner than the space between the teeth at their small end.

Large bevel gears are made on theoretically correct principles by planing on specially constructed machines.

WORMS AND WORM GEARS.

Fig. 10 shows a worm and worm gear.

FIG. 10.



f = Pitch diameter of gear.
g = Smallest outside diameter.
h = Largest outside diameter.
a = Outside diameter of worm.
b = Pitch diameter of worm.
c = Diameter of worm at bottom of thread.

The pitch and diameter of worm screws are usually of such proportions that for single-thread the angle of the teeth on the gear is from two to three degrees. This angle is most conveniently obtained by drawing a diagram as shown in Fig. 10.

Draw a line $l\,m$, equal to $3\frac{1}{7}$ times the length of line b; this line will be equal to the length of the circumference of the pitch diameter of the screw. Erect the perpendicular, $m\,o$, equal to the pitch of the screw. Connect the points l and o by the line $l\,o$, and the angle s is the angle of the teeth on the worm gear.

NOTE.—When the pitch diameter of the worm screw is seven times the circular pitch of the worm gear and the worm has single thread, the angle of the thread on the gear is very nearly $2\frac{1}{2}$ degrees.

CAUTION.—When cutting a worm gear, be careful and not lay the angle of the teeth in the wrong direction.

The diameter of worm gears is usually calculated according to circular pitch, for convenience in cutting the worm with the same gears as used for ordinary screw cutting in a lathe.

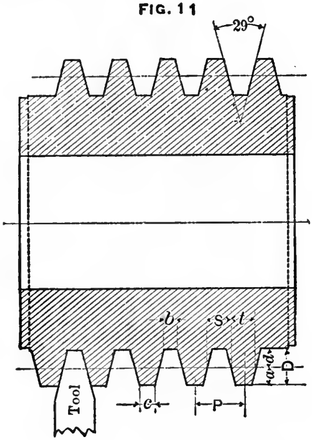
When a worm gear has comparatively few teeth, the flank of the tooth will be undercut by the hob; to prevent this in a measure, it is customary to have the blank somewhat over size, so that from five-eighths to three-fourths of the depth of the tooth may be outside the pitch line.

The form of teeth is usually involute, and the thread on a worm screw is constructed of the same shape as the teeth in a rack. Fig. 11 shows the shape of tooth and the table gives the dimensions of finishing tool for the most common pitches.

The surface speed of a worm screw ought not to exceed 300 feet per minute.

Table No. 50 is calculated by the following formulas. (See Fig. 11.)

FIG. 11



$P =$ Circular pitch.

$$N = \frac{1}{P}$$

$$M = \frac{3.1416}{P}$$

$$t = \frac{P}{2} = S$$

$$a = P \times 0.3183$$

$$d = P \times 0.3683$$

$$D = a + d$$

$$S = P \times 0.5$$

$$b = P \times 0.31$$

$$C = P \times 0.335$$

$$h = D + \frac{k}{2}$$

$$k = P \times 0.1$$

TABLE No. 50.—Giving Proportions of Parts for Worms and Worm Gears, Calculated According to Circular Pitch.

(See Fig. 11.)

WORM GEARS AND WORM SCREWS.											HOB.	
Circular Pitch in inches.	Number of Threads per inch.	Corresponding Diametral Pitch.	Thickness of Thread on Pitch Line.	Width of Space on Pitch Line.	Addendum.	Working Depth of Tooth.	Depth of Space Below Pitch Line.	Whole Depth or Space in Worm on Gear.	Width of Cutting Tool at the Point.	Thickness of Threads on Top.	Whole depth of space in Hob.	Increment in diameter of hob over diameter of worm.
P	N	M	t	S	a	$2a$	d	D	b	C	h	k
2	½	1.5708	1.	1.	0.6366	1.2732	0.7366	1.3732	0.6200	0.6700	1.4732	0.200
1¾	4/7	1.7952	0.875	0.875	0.557	1.1141	0.6445	1.2015	0.5425	0.5863	1.29	0.175
1½	⅔	2.0944	0.750	0.750	0.4775	0.9549	0.5525	1.0299	0.4650	0.5025	1.1049	0.150
1¼	⅘	2.5133	0.625	0.625	0.3979	0.7958	0.4604	0.8583	0.3875	0.4188	0.9208	0.125
1	1	3.1416	0.500	0.500	0.3183	0.6366	0.3683	0.6866	0.3100	0.3350	0.7366	0.1
¾	1⅓	4.1888	0.375	0.375	0.2387	0.4775	0.2762	0.5150	0.2325	0.2513	0.5525	0.075
⅝	1⅗	5.0265	0.3125	0.3125	0.1989	0.3979	0.2301	0.4291	0.1938	0.2094	0.4606	0 063
½	2	6.2832	0.250	0.250	0.1592	0.3183	0.1842	0.3433	0.1550	0.1675	0.3683	0.05
⅖	2½	7.8540	0.200	0.200	0.1273	0.2546	0.1473	0.2746	0.1240	0.1340	0.2946	0.04
⅓	3	9.4248	0.1667	0.1667	0.1061	0.2122	0.1228	0.2289	0.1033	0.1117	0 2454	0.033
2/7	3½	10.9956	0.143	0.143	0.0909	0.1818	0.1052	0.1962	0.0886	0 0957	0.2105	0.028
¼	4	12.5664	0.125	0.125	0.0796	0.1591	0.0921	0.1716	0.0775	0.0838	0.1841	0.025
2/9	4½	14.137	0.111	0.111	0.0707	0.1415	0.0818	0.1526	0.0689	0.0744	0.1637	0.022
⅕	5	15.7080	0.1	0.1	0.0637	0.1273	0.0737	0.1373	0.0620	0.0670	0.1473	0.020
⅙	6	18.8496	0.0833	0.0833	0.0531	0.1061	0.0614	0.1144	0.0517	0.0558	0.1239	0.017
⅛	8	25.1327	0.0625	0.0625	0.0398	0.0796	0.0460	0.0858	0.0388	0.0419	0.0923	0.013
1/10	10	31.416	0.05	0.05	0.0318	0.0636	0.03683	0.06866	0.031	0.0335	0.07366	0.01

TABLE No. 51.—Showing How to Gear Lathes when Cutting Worms of the Pitches Given in Table No. 50.

Circular Pitch in Inches.	Number of Threads per Inch.	Leading Screw of Lathe.											
		2 Threads per inch.		3 Threads per inch.		4 Threads per inch.		5 Threads per inch.		6 Threads per inch.		10 Threads per inch.	
		Stud gear.	Screw gear.	Stud gear.	Screw gear.	Stud gear.	Screw gear.	Stud gear.	Screw gear.	Stud gear.	Screw gear.	Stud gear.	Screw gear.
2	½	160	40										
1¾	4/7	140	40										
1½	⅔	120	40										
1¼	⅘	100	40	120	32								
1	1	80	40	96	32								
¾	1⅓	60	40	72	32	120	40						
⅝	1⅗	50	40	60	32	100	40						
½	2	40	40	48	32	80	40	100	40				
⅖	2½	40	50	48	40	64	40	80	40				
⅓	3	40	60	48	48	64	48	60	36	48	24		
2/7	3½	40	70	48	56	64	56	60	42	48	28		
¼	4	40	80	24	32	40	40	50	40	48	32		
2/9	4½	40	90	24	36	32	36	40	36	48	36		
⅕	5	20	50	24	40	32	40	40	40	48	40	60	30
⅙	6	20	60	24	48	32	48	40	48	48	48	60	36
⅛	8	20	80	24	64	24	48	30	48	48	64	60	48
1/10	10	20	100	24	80	24	60	24	48	24	40	50	50

Reduction of Speed by Worm Gearing.

In a single-threaded worm screw one revolution of the worm moves the gear one tooth; in a double-threaded worm screw one revolution of the worm moves the gear two teeth, and in a triple-threaded worm screw one revolution of the worm moves the gear three teeth. A great deal of work is lost by friction by using worm gearing, frequently from 50 to 75 per cent., some of which could be saved by using a ball bearing to take the end thrust of the worm. The efficiency is also increased by using a worm of double, triple or quadruple thread, because this increases the angle of the teeth in the wheel and the efficiency of the mechanism is increased by increasing the angle until it reaches 20° to 25°, when it rapidly falls off again.

Calculating the Size of Worm Gears.

Example.

Find dimensions of a worm gear having 68 teeth, ¾ inch pitch, cut teeth. Make the pitch diameter of the single-thread worm six times the pitch of the worm. Use Table No. 50.

Solution:

In Table No. 47 the pitch diameter of a gear of 68 teeth of one-inch pitch is given as 21.65 inches; thus, the pitch diameter for a gear of 68 teeth of ¾-inch circular pitch will be 21.65 × 0.75 = 16.738 inches.

In column *a*, of Table No. 50, the addendum for ¾-inch circular pitch is given as 0.2387 inch; this is multiplied by 2, because it is to be added on both sides of the gear.

Thus, the smallest outside diameter of the gear is 16.738 + 0.2387 × 2 = 17.215 inches; or, practically, $17\frac{7}{32}$ inches. If the gear is to be made hollow to correspond to the curve at bottom of thread of the worm, make a scale drawing as shown in Fig. 10, and make line *g*, $17\frac{7}{32}$ inches; from this drawing the largest outside diameter may be obtained by measurement.

The diameter of the worm on the pitch line was to be six times the pitch = 6 × ¾″ = 4½ inches.

The addendum for the thread on the worm can be obtained from Table No. 50, column *a*, and is 0.2387. The outside diameter of the worm will be 4.5 + 2 × 0.2387 = 4.977 inches, or, practically, $4\frac{31}{32}$ inches.

The cutter to be used in roughing out the gear should have a curve of involute form corresponding to a spur gear cutter for 68 teeth, and its thickness ought to be at least 0.005 inch less than the width of space as given in column *S* of Table No. 50. Therefore the thickness on the pitch line of the roughing cutter will be 0.37 inch.

The angle of the teeth may be obtained from a drawing as shown and explained in Fig. 10, or it may be calculated thus:

$$\text{Tangent of angle } S = \frac{\text{circular pitch}}{\text{pitch circumference}}$$

$$\text{Tang. } S = \frac{0.75}{4.5 \times 3.1416} = \frac{0.75}{14.1372} = 0.05305$$

In Table No. 21 the corresponding angle is given as 3 degrees, very nearly.

The depth to which the gear should be cut is given in column *D* as 0.515 inch. The gear is finished with a hob, as described below, which is allowed to cut until it touches the bottom of the spaces in the gear. The outside diameter of the hob should be larger than the outside diameter of the worm, in order that the teeth in the hob may reach the bottom of the spaces in the gear and leave clearance for the worm, and at the same time leave the gear tooth of the proper thickness on the pitch line. This increment is obtained in column *k*, Table No. 50, and for ¾-inch pitch is 0.075 inch; thus, the outside diameter of the hob is 0.075 inch larger than the outside diameter of the worm, or 4.977 + 0.075 = 5.052 inches. The angle of the finishing threading tool for both worm and hob is 14½ degrees, making the angle of space 29°, as shown in Fig. 11. The clearance angle of the threading tool must be a little more than the angle of the thread.

The width of the threading tool at the point is given in Column b, Table No. 50, as 0.2325 inch. The depth of the space to be cut in the worm is given in Column D, as 0.515 inch. The diameter of the worm at the bottom of the thread will be:

$$4.977 - 2 \times 0.515 = 3.947 \text{ inches.}$$

The depth of the space to be cut in the hob is given in column h in Table No. 50 as 0.5525 inch.

The diameter of the hob at bottom of thread will be:

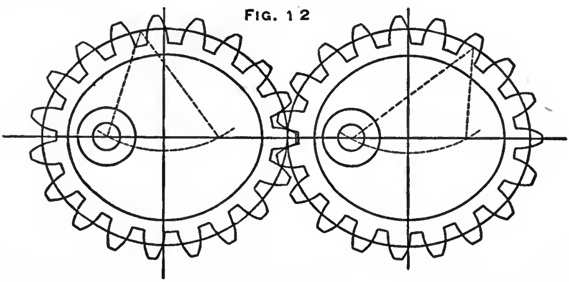
$$5.052 - 2 \times 0.5525 = 3.947 \text{ inches.}$$

Thus the only difference in size between the hob and the worm is in the outside diameter and in the depth of the cut. Both may be finished by the same tool, as the diameter at the bottom of the thread and the thickness of the teeth at the pitch line should be the same for both hob and worm.

Elliptical Gear Wheels.

Elliptical gear wheels are sometimes used in order to change a uniform rotary motion of one shaft to an alternately fast and show motion of the other. See Fig. 12.

FIG. 12



The pitch line is constructed and calculated the same as the circumference of an ellipse. (See page 189.) The gear is constructed involute the same as for spur gears. If the difference between the minor and the major diameters is large it may be necessary to construct the teeth of different shapes at different places on the circumference; in other words, the whole circumference of the gear cannot be cut with the same cutter. A cutter of the same pitch, of course, but corresponding to a larger diameter of gear, must be used where the curve of the pitch line is less sharp.

The centers of the shafts are in the foci of the ellipse. If two elliptical gear wheels, made from the same pattern, or cut together at the same time, on the same arbor, are to work together they must have an uneven number of teeth so that a space will be diametrically opposite a tooth, as will be seen from Fig. 12.

SCREWS.

"Pitch," "Inch Pitch" and "Lead" of Screws and Worms.

The term "pitch of a screw," as commonly used, means its number of threads per inch. while the "inch pitch" is the distance from the center of one thread to that of the next. For instance, a one-inch screw of standard thread is usually said to be an "eight pitch" screw, because it has eight threads per inch of length; but it might more correctly be said to be a screw of $\frac{1}{8}$-inch pitch, because it is $\frac{1}{8}$-inch from the center of one thread to the center of the next.

The "lead" of a worm or a screw means the advancement of the thread in one complete revolution; therefore, in a single-threaded screw, the inch pitch and the lead is the same thing, but in a double or triple-threaded screw the inch pitch and the lead are two different things. The "lead" in a double-threaded screw will be a distance equal to twice the distance from the center of one thread to the center of the next, but in a triple-threaded screw the lead is three times the distance from the center of one thread to the center of the next.

Screw Cutting by the Engine Lathe.

When the stud and the spindle run at the same speed (which they usually do) the ratio between the gears may always be obtained by simply ascertaining the ratio between the number of threads per inch of the lead-screw and the screw to be cut.

EXAMPLE.

The lead-screw on a lathe has four threads per inch and the screw to be cut has $11\frac{1}{2}$ threads per inch (one-inch pipe-thread). Find the gears to be used when the smallest change gear has 24 teeth and the gears advance by four teeth up to 96. The ratio of the number of threads per inch of the two screws is as 4 to $11\frac{1}{2}$.

As the smallest gear has 24 teeth and the gears all advance by four teeth, this ratio of the screws must be multiplied by a number which is a multiple of 4 and which, at least, gives the smallest gear 24 teeth. For instance, multiply by 8 and the result is $8 \times 11\frac{1}{2} = 92$ teeth for the gear on the lead-screw; $8 \times 4 = 32$ teeth for the gear on the stud.

Cutting Multiple-Threaded Screws or Nuts by the Engine Lathe.

Calculate the change gears as if it was a single-threaded screw of the same lead. Cut one thread and move the tool the proper distance and cut the next thread.

The most practical way to move the tool from one thread to another, when cutting double-threaded screws or nuts, is to select a gear for the stud or spindle of the lathe having a number of teeth which is divisible by two, and when one thread is cut make a chalk mark across a tooth in this gear onto the rim of the intermediate gear; count half way around the gear on the stud and make a chalk mark across that tooth; drop the swing plate enough to separate the gears, pull the belt by hand until the opposite mark on the gear on the stud comes in position to match the chalk mark on the intermediate gear; clamp the swing plate again and the tool is in proper position to cut the second thread.

When triple threads are to be cut, select a gear for the spindle or stud whose number of teeth is divisible by three, and in changing the tool from one thread to the next, only turn the lathe enough so that the gear on the stud moves one-third of one revolution.

If, for any reason, it should be inconvenient to make this change by the gear on the stud, the change may be made by the lead-screw gear. The intermediate gear is first released from the gear on the lead-screw, which is then moved ahead the proper number of teeth, and again connected with the intermediate gear. The proper number of teeth to move the gear on the lead-screw is obtained by the following rule:

Multiply the number of teeth in the gear on the lead-screw by the number of threads per inch of the lead-screw; divide this product by the number of threads per inch of the screw to be cut, and the quotient is the number of teeth that the gear on the lead-screw must be moved ahead.

EXAMPLE.

A square-threaded screw is to have ½-inch lead and triple thread. The lead-screw in the lathe has two threads per inch, and the gear on the lead screw has ninety-six teeth. How many teeth must the gear on the lead-screw be moved, when changing from one thread to the next?

Solution:

A screw of ½-inch lead with triple thread has six threads per inch, therefore the gear must be moved $\frac{2 \times 96}{6} = 32$ teeth, in order to change the tool from one thread to the next.

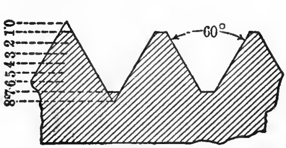


FIG. 1

U. S. Standard Screws.

Fig. 1 shows the shape of thread on United States standard screws. The sides are straight and form an angle of sixty degrees, and the thread is flat at the top and bottom for a distance equal to one-eighth of the pitch, thus the depth of the thread is only three-fourths of a full, sharp thread. (See Fig. 1.)

Fig. 2 shows the shape of the Whitworth (the English) system of thread. As compared with the American system, the principal difference is in the angle between the sides of the thread, which is fifty-five degrees, and one-sixth of the depth of the full, sharp thread is made rounding at the top and bottom. There is also a difference in the pitch of a few sizes.

FIG. 2

The common V-thread screws have the angle of thread of sixty degrees, the same as the United States standard screws, but the thread is sharp at both top and bottom. This style of thread is rapidly, as it should be, going out of use. The principal disadvantages of this thread are that the screw has less tensile strength, and it is also very difficult to keep a sharp-pointed threading tool in order.

Diameter of Screw at Bottom of Thread.

The diameter of screws at the bottom of thread is obtained by the following formulas:—

United States Standard Screws:

$$d = D - \frac{1.299}{n}$$

For V-threaded screws:

$$d = D - \frac{1.733}{n}$$

For Whitworth screws:

$$d = D - \frac{1.281}{n}$$

$d =$ Diameter of screw at bottom of thread.

$D =$ Outside diameter of screw.

$n =$ Number of threads per inch.

1.299 is constant for United States standard thread.

1.733 is constant for sharp V-thread.

1.281 is constant for Whitworth thread.

TABLE No. 52.—Dimensions of Whitworth Screws.

Diameter of Screw in Inches.	Number of Threads per Inch.	Diameter of Screw in Inches.	Number of Threads per Inch.	Diameter of Screw in Inches.	Number of Threads per Inch.
$\frac{1}{8}$	40	$1\frac{1}{4}$	7	$3\frac{1}{2}$	$3\frac{1}{4}$
$\frac{3}{16}$	24	$1\frac{3}{8}$	6	$3\frac{3}{4}$	3
$\frac{1}{4}$	20	$1\frac{1}{2}$	6	4	3
$\frac{5}{16}$	18	$1\frac{5}{8}$	5	$4\frac{1}{4}$	$2\frac{7}{8}$
$\frac{3}{8}$	16	$1\frac{3}{4}$	5	$4\frac{1}{2}$	$2\frac{7}{8}$
$\frac{7}{16}$	14	$1\frac{7}{8}$	$4\frac{1}{2}$	$4\frac{3}{4}$	$2\frac{3}{4}$
$\frac{1}{2}$	12	2	$4\frac{1}{2}$	5	$2\frac{3}{4}$
$\frac{5}{8}$	11	$2\frac{1}{4}$	4	$5\frac{1}{4}$	$2\frac{5}{8}$
$\frac{3}{4}$	10	$2\frac{1}{2}$	4	$5\frac{1}{2}$	$2\frac{5}{8}$
$\frac{7}{8}$	9	$2\frac{3}{4}$	$3\frac{1}{2}$	$5\frac{3}{4}$	$2\frac{1}{2}$
1	8	3	$3\frac{1}{2}$	6	$2\frac{1}{2}$
$1\frac{1}{8}$	7	$3\frac{1}{4}$	$3\frac{1}{4}$		

Diameter of Tap Drill.

The diameter of the drill with which to drill for a tap is, if we want full thread in the nut, equal to the diameter of the screw at the bottom of the thread, and is, therefore, obtained by the same formulas. However, in practical work it is always advisable to use a drill a little larger than the diameter of the screw at the bottom of thread, because in threading wrought iron or steel the thread will swell out more or less, and a few thousandths must be allowed in the size when drilling the hole. In drilling holes for tapping cast-iron, a little larger drill is used, because it is unnecessary in a cast-iron nut to have exactly full thread. Table No. 53 gives sizes of drill for both wrought and cast-iron, which give good practical results for United States standard screws.

Table No. 53 gives sizes of hexagon bolts and nuts. The size of the hexagon is equal to $1\frac{1}{2}$ times the diameter of bolt + $\frac{1}{8}$-inch; the thickness of head is equal to half the hexagon. The thickness of nut is equal to the diameter of the bolt. When heads and nuts are finished they are $\frac{1}{16}$-inch smaller.

The table is calculated by the following formulas:

$$d = D - \frac{1.299}{n} \qquad f = \frac{1}{n} \times \frac{1}{8}$$

$$A = 1\tfrac{1}{2}\, D + \tfrac{1}{8} \qquad B = 1.414\, A$$

$$C = 1.155\, A \qquad E = \tfrac{1}{2}\, A \qquad F = D$$

TABLE No. 53.—Dimensions of U. S. Standard Screws.

Diameter of Bolt.	Diameter of Bolt at Bottom of Thread.	Diameter of Tap Drill.	Area at Bottom of Thread in Square Inches.	Number of Threads per Inch.	Flat at Top and Bottom.	Rough Bolt-Heads and Nuts. Across the Flats of Square or Hexagon.	Rough Bolt-Heads and Nuts. Across the Corners of Square.	Rough Bolt-Heads and Nuts. Across the Corners of Hexagon.	Rough Bolt-Heads and Nuts. Thickness of Bolt-Head.	Rough Bolt-Heads and Nuts. Thickness of Nut.	Finished Bolt-Heads and Nuts. Across the Flats of Square or Hexagon.	Finished Bolt-Heads and Nuts. Thickness of Bolt-Head or Nut.
D	d	d^1	a	n	f	A	B	C	E	F	G	H
1/4	0.185	3/16	0.0269	20	0.0062	1/2	23/32	37/64	1/4	1/4	7/16	3/16
5/16	0.240	1/4	0.0452	18	0.0069	19/32	27/32	11/16	19/64	5/16	17/32	1/4
3/8	0.294	19/64	0.0679	16	0.0078	11/16	31/32	51/64	11/32	3/8	5/8	5/16
7/16	0.345	23/64	0.0935	14	0.0089	25/32	1 7/64	29/32	25/64	7/16	23/32	3/8
1/2	0.400	13/32	0.1257	13	0.0096	7/8	1 15/64	1 1/64	7/16	1/2	13/16	7/16
9/16	0.454	15/32	0.1619	12	0.0104	31/32	1 3/8	1 1/8	31/64	9/16	29/32	1/2
5/8	0.507	33/64	0.2019	11	0.0114	1 1/16	1 1/2	1 15/64	17/32	5/8	1	9/16
3/4	0.620	5/8	0.3019	10	0.0125	1 1/4	1 49/64	1 7/16	5/8	3/4	1 3/16	11/16
7/8	0.731	47/64	0.4197	9	0.0139	1 7/16	2 1/32	1 23/32	23/32	7/8	1 3/8	13/16
1	0.838	27/32	0.5515	8	0.0156	1 5/8	2 19/64	1 7/8	13/16	1	1 9/16	15/16
1 1/8	0.939	61/64	0.6925	7	0.0178	1 13/16	2 9/16	2 3/32	29/32	1 1/8	1 3/4	1 1/16
1 1/4	1.065	1 5/64	0.8892	7	0.0178	2	2 53/64	2 5/16	1	1 1/4	1 15/16	1 3/16
1 3/8	1.158	1 11/64	1.0532	6	0.0208	2 3/16	3 3/32	2 17/32	1 3/32	1 3/8	2 1/8	1 5/16
1 1/2	1.284	1 19/64	1.2928	6	0.0208	2 3/8	3 23/64	2 3/4	1 3/16	1 1/2	2 5/16	1 7/16
1 5/8	1.389	1 25/64	1.5153	5 1/2	0.0227	2 9/16	3 5/8	2 31/32	1 9/32	1 5/8	2 1/2	1 9/16
1 3/4	1.490	1 1/2	1.7437	5	0.0250	2 3/4	3 57/64	3 3/16	1 3/8	1 3/4	2 11/16	1 11/16
1 7/8	1.615	1 5/8	2.0485	5	0.0250	2 15/16	4 5/32	3 13/32	1 15/32	1 7/8	2 7/8	1 13/16
2	1.711	1 23/32	2.2993	4 1/2	0.0278	3 1/8	4 27/64	3 19/32	1 9/16	2	3 1/16	1 15/16
2 1/4	1.961	1 31/32	3.0203	4 1/2	0.0278	3 1/2	4 61/64	4 3/64	1 3/4	2 1/4	3 7/16	2 3/16
2 1/2	2.175	2 3/16	3.7154	4	0.0313	3 7/8	5 31/64	4 31/64	1 15/16	2 1/2	3 13/16	2 7/16
2 3/4	2.425	2 7/16	4.6186	4	0.0313	4 1/4	6	4 29/32	2 1/8	2 3/4	4 3/16	2 11/16
3	2.629	2 41/64	5.4284	3 1/2	0.0357	4 5/8	6 17/32	5 11/32	2 5/16	3	4 9/16	2 15/16
3 1/4	2.879	2 57/64	6.5099	3 1/2	0.0357	5	7 5/64	5 25/32	2 1/2	3 1/4	4 15/16	3 3/16
3 1/2	3.100	3 1/8	7.5477	3 1/4	0.0384	5 3/8	7 39/64	6 13/64	2 11/16	3 1/2	5 5/16	3 7/16
3 3/4	3.317	3 21/64	8.6414	3	0.0417	5 3/4	8 15/64	6 41/64	2 7/8	3 3/4	5 11/16	3 11/16
4	3.567	3 37/64	9.9930	3	0.0417	6 1/8	8 21/32	7 5/64	3 1/16	4	6 1/16	3 15/16
4 1/4	3.798	3 13/16	11.3292	2 7/8	0.0435	6 1/2	9 3/16	7 1/2	3 1/4	4 1/4	6 7/16	4 3/16
4 1/2	4.028	4 3/64	12.7366	2 3/4	0.0455	6 7/8	9 47/64	8	3 7/16	4 1/2	6 13/16	4 7/16
4 3/4	4.255	4 9/32	14.2197	2 5/8	0.0476	7 1/4	10 1/4	8 3/8	3 5/8	4 3/4	7 3/16	4 11/16
5	4.480	4 1/2	15.7633	2 1/2	0.0500	7 5/8	10 25/32	8 13/16	3 13/16	5	7 9/16	4 15/16
5 1/4	4.730	4 3/4	17.5717	2 1/2	0.0500	8	11 5/16	9 1/4	4	5 1/4	7 15/16	5 3/16
5 1/2	4.953	4 63/64	19.2676	2 3/8	0.0526	8 3/8	11 27/32	9 11/16	4 3/16	5 1/2	8 5/16	5 7/16
5 3/4	5.203	5 15/64	21.2617	2 3/8	0.0526	8 3/4	12 3/8	10 7/64	4 3/8	5 3/4	8 11/16	5 11/16
6	5.423	5 29/64	23.0978	2 1/4	0.0556	9 1/8	12 29/32	10 17/32	4 9/16	6	9 1/16	5 15/16

NOTE.—In finished work, the thickness of the head of the bolt and the nut is equal, and is $\frac{1}{16}$ of an inch less than the diameter of the bolt.

Columns *B* and *C* in Table No. 53 are very useful for many purposes; for instance, in selecting size of counter-bore when finishing castings, to give bearing for screw heads; in turning blanks which are afterwards to be cut into square or hexagon heads, etc.

Table No. 54.—Coupling Bolts and Nuts.

(Hexagon).
(All Dimensions in Inches)

Diameter.	Threads per In.	Dimensions of Head and Nut.		
		Across the Flat.	Across the Corner.	Length of Head or Nut.
$\frac{1}{2}$	13	$\frac{7}{8}$	$1\frac{1}{64}$	$\frac{1}{2}$
$\frac{5}{8}$	11	$1\frac{1}{16}$	$1\frac{15}{64}$	$\frac{5}{8}$
$\frac{3}{4}$	10	$1\frac{1}{4}$	$1\frac{7}{16}$	$\frac{3}{4}$
$\frac{7}{8}$	9	$1\frac{7}{16}$	$1\frac{21}{32}$	$\frac{7}{8}$
1	8	$1\frac{5}{8}$	$1\frac{7}{8}$	1
$1\frac{1}{8}$	7	$1\frac{13}{16}$	$2\frac{3}{32}$	$1\frac{1}{8}$
$1\frac{1}{4}$	7	2	$2\frac{5}{16}$	$1\frac{1}{4}$

Table No. 55.—Round and Fillister Head Screws.

(All Dimensions in Inches).

Diameter of Screw.	Number of Threads per Inch.	Diameter of Head.	Length of Head.
$\frac{1}{8}$	40	$\frac{3}{16}$	$\frac{1}{8}$
$\frac{3}{16}$	24	$\frac{1}{4}$	$\frac{3}{16}$
$\frac{1}{4}$	20	$\frac{3}{8}$	$\frac{1}{4}$
$\frac{5}{16}$	18	$\frac{7}{16}$	$\frac{5}{16}$
$\frac{3}{8}$	16	$\frac{9}{16}$	$\frac{3}{8}$
$\frac{7}{16}$	14	$\frac{5}{8}$	$\frac{7}{16}$
$\frac{1}{2}$	13	$\frac{3}{4}$	$\frac{1}{2}$
$\frac{9}{16}$	12	$\frac{13}{16}$	$\frac{9}{16}$
$\frac{5}{8}$	11	$\frac{7}{8}$	$\frac{5}{8}$
$\frac{3}{4}$	10	1	$\frac{3}{4}$
$\frac{7}{8}$	9	$1\frac{1}{8}$	$\frac{7}{8}$
1	8	$1\frac{1}{4}$	1

Round Head Cap Screw.

Fillister Head Screw.

TABLE No. 56.—Dimensions of Hexagon and Square Head Cap Screws.

(All Dimensions in Inches).

Diameter of Screws.	Threads per Inch.	Hexagon Head.		Square Head.		Length of Head.
		Across the Flats.	Across the Corners.	Across the Flats.	Across the Corners.	
1/4	20	7/16	1/2	3/8	17/32	1/4
5/16	18	1/2	37/64	7/16	5/8	5/16
3/8	16	9/16	21/32	1/2	23/32	3/8
7/16	14	5/8	23/32	9/16	51/64	7/16
1/2	13	3/4	55/64	5/8	57/64	1/2
9/16	12	13/16	15/16	11/16	31/32	9/16
5/8	11	7/8	1 1/64	3/4	1 1/16	5/8
3/4	10	1	1 5/32	7/8	1 1/4	3/4
7/8	9	1 1/8	1 19/64	1 1/8	1 19/32	7/8
1	8	1 1/4	1 7/16	1 1/4	1 49/64	1
1 1/8	7	1 3/8	1 19/32	1 3/8	1 15/16	1 1/8
1 1/4	7	1 1/2	1 47/64	1 1/2	2 1/8	1 1/4

Eye Bolts.

It is very customary to weld an eye to a lag screw (see Fig. 3) to use in handling heavy weights in shops.

The following table (No. 57) gives the holding power of lag screws or eye bolts when screwed into spruce timber a little over the full length of thread. The suitable size of bit for the thread is also given in the table.

TABLE No. 57.

Diameter of Screw.	Diameter of Bit.	Load at Which the Screw Pulled Out.	Safe Load.
1 inch	3/4 inch	16,000 lbs.	2,000 lbs
7/8 "	11/16 "	9,000 "	1,125 "
3/4 "	5/8 "	7,000 "	875 "
5/8 "	1/2 "	6,000 "	750 "
1/2 "	3/8 "	3,500 "	437 "
3/8 "	5/16 "	1,900 "	237 "
1/4 "	3/16 "	700 "	87 "

FIG. 3.

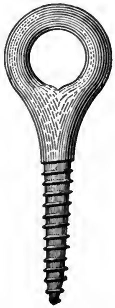


TABLE No. 58.—Giving the Average Weight in Pounds per 100 Square Head Gimlet-Pointed Lag Screws.

LENGTH in Inches.	1/4"	5/16"	3/8"	7/16"	1/2"	9/16"	5/8"	3/4"	7/8"	1"
1 1/2	2 3/4	4 1/2	7	10						
2	3 1/2	5 1/2	8 1/2	12	17	24	27 1/2			
2 1/2	4 1/4	6 1/2	9 3/4	14	19	26	31			
3	4 3/4	7 1/2	11	16	21	28	34	51		
3 1/2	5 1/4	8 1/2	12 1/2	18	24	31	38	55		
4	5 3/4	9 1/2	14	20	26	34	42	60	85	112
4 1/2	6 1/2	10 1/2	15 1/2	22	28	37	46	65	91	121
5	7	11 1/2	17	24	32	40	50	70	97	130
5 1/2	7 1/2	12 1/2	18 1/2	26	34	43	54	76	103	140
6	8	13 1/2	20	28	36	46	58	81	110	150
6 1/2			21 1/2	30	38	49	62	86	117	160
7			23	32	41	52	65	92	125	170
7 1/2			24 1/2	34	44	55	69	97	132	180
8			26	36	47	58	73	103	140	190
8 1/2							77	108	148	200
9							81	113	156	210
9 1/2							85	118	164	220
10							89	123	172	230
Size of Head in Inches.	7/16 × 3/16	1/2 × 1/4	5/8 × 9/32	23/32 × 11/32	13/16 × 3/8	15/16 × 7/16	1 × 1/2	1 3/16 × 5/8	1 3/8 × 3/4	1 9/16 × 13/16

GIMLET-POINTED LAG SCREW.

EXAMPLE.

What is the weight of 8 lag screws 6" long and 1/2-inch in diameter?

Solution:

Under the heading 1/2-inch, in the line with 6 in the column of length, is the number 36. Thus, 100 lag screws of this size will average to weigh 36 pounds, and one such screw will weigh 0.36 pound; 8 such screws will weigh $0.36 \times 8 = 2.88$ pounds.

French System of Standard Threads.

In the French system of standard screws the thread has an angle of 60°, with flat top and bottom. (The French system is in this respect identically the same as the United States standard thread.) The pitch and the diameter are given in millimeters (see Table No. 59). The form of thread is an equilateral triangle. The diameter of tap drill (or diameter of bolt or screw at bottom of the thread) is obtained in the following way: The height of an equilateral triangle is obtained by multiplying its base by 0.86603 (this number is sin. of 60°). Thus, assuming that the base = 1, and taking off one-eighth of the depth at top and bottom, that is reducing the depth of the thread one-fourth, the remaining depth will be $0.86603 - \frac{0.86603}{4} = 0.64952$; multiplying this by 2, to allow for the depth of the thread on both sides of the screw, the constant will be $2 \times 0.64952 = 1.29904$, which for all practical purposes may be reduced to 1.3. Thus, when the pitch of the screw is 1 millimeter, the diameter of the screw at the bottom of the thread is 1.3 millimeters less than the outside diameter of the screw. Therefore, the diameter of the screw at the bottom of the thread may always be calculated by the simple rule:

Multiply the pitch in millimeters by 1.3, and subtract the product from the outside diameter of the screw; the remainder is the diameter of the screw at the bottom of the thread, which is the same as the diameter of the tap drill, given in Table No. 59.

TABLE No. 59.—French Standard Screws.

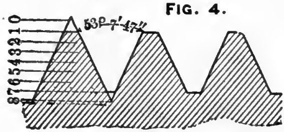
(All Dimensions in Millimeters).

Diameter of Screw.	Pitch of Screw.	Diameter of Tap Drill.	Diameter of Screw.	Pitch of Screw.	Diameter of Tap Drill.
6	1	4.70	36	4	30.80
8	1	6.70	42	4.5	36.15
10	1.5	8.05	48	5	41.50
12	1.5	10.05	56	5.5	48.85
14	2	11.40	64	6	56.20
16	2	13.40	72	6.5	63.55
18	2.5	14.75	80	7	70.90
20	2.5	16.75	88	7.5	78.25
22	2.5	18.75	96	8	85.60
24	3	20.10	106	8.5	94.95
26	3	22.10	116	9	104.30
28	3	24.10	126	9.5	113.65
30	3.5	25.45	136	10	123.00
32	3.5	27.45	148	10.5	134.35

German System of Standard Threads.

In the German system of standard threads the angle is 53° 7′ 47″. The reason for adopting such an odd angle is that the form of thread is a triangle, having its base equal to its height, and the top angle of such a triangle is 53° 7′ 47″. The thread in this system is also made flat at top and bottom equal to one-eighth of the pitch (see Fig. 4). The diameter of the screw at the bottom of the thread is, in this system, calculated by this rule:

FIG. 4.



Multiply the pitch in millimeters by 1.5, subtract this product from the outside diameter of the screw and the remainder is the diameter of the screw at the bottom of the thread, which is the same as the diameter of the tap drill given in Table No. 60.

TABLE No. 60.—German Standard Screws.

(All Dimensions in Millimeters).

Diameter of Screw.	Pitch of Screw.	Diameter of Tap Drill.	Diameter of Screw.	Pitch of Screw.	Diameter of Tap Drill.	Diameter of Screw.	Pitch of Screw.	Diameter of Tap Drill.
1	0.25	0.625	5	0.8	3.8	20	2.4	16.4
1.2	0.25	0.825	5.5	0.9	4.15	22	2.8	17.8
1.4	0.3	0.95	6	1	4.5	24	2.8	19.8
1.7	0.35	1.175	7	1.1	5.35	26	3.2	21.2
2	0.4	1.4	8	1.2	6.2	28	3.2	23.2
2.3	0.4	1.7	9	1.3	7.05	30	3.6	24.6
2.6	0.45	1.125	10	1.4	7.9	32	3.6	26.6
3	0.5	2.25	12	1.6	9.6	36	4	30
3.5	0.6	2.6	14	1.8	11.3	40	4.4	33.4
4	0.7	2.95	16	2	13			
4.5	0.75	3.375	18	2.2	14.7			

International Standard for Metric Screw Threads.

An international standard for metric screw threads was discussed at a congress which met for that purpose at Zurich, in October, 1898. The form of thread adopted is based on the Sellers thread, which it will be remembered has the shape of an equilateral triangle truncated one-eighth of its height at top and bottom.

To insure interchangeability, and to reduce the wear on taps and dies, the congress recommended that the *bottom* of the thread should be rounded off by any suitable curve, which should not deepen the cut more than an amount equal to 1/16 of the pitch beyond the standard Sellers type. The *top* of the thread is to be left flat, as in the Sellers system. The following standard sizes and pitches were decided upon:

TABLE No. 61.—International Standard Thread.

Diameter in Millimeters.	Pitch in Millimeters.	Diameter in Millimeters.	Pitch in Millimeters.
6 and 7	1	30 and 33	3.5
8 and 9	1.25	36 " 39	4
10 and 11	1.5	42 " 45	4.5
12	1.75	48 " 52	5
14 and 16	2	56 " 60	5.5
18, 20 and 22	2.5	64 " 68	6
24 and 27	3	72 " 76	6.5

To Gear a Lathe to Cut Metric Thread when the Lead-Screw is in Inches.

Use two intermediate gears, one having 100 teeth and the other 127 teeth, fasten these two gears together on the same hub, and gear the 100-tooth gear into the gear on the lead-screw and the 127-tooth gear into the gear on the stud (see Fig. 5).

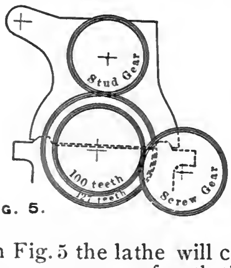


FIG. 5.

The lathe will then cut, practically, one-half the number of threads per centimeter that it originally cut per inch with a common intermediate gear. For instance, the stud gear has 24 teeth, the screw gear has 48 teeth and the lead-screw has four threads per inch; the lathe will then, with a common intermediate gear, cut eight threads per inch, but by using such a double intermediate gear as is shown in Fig. 5 the lathe will cut four threads per centimeter, which is the same as one-fourth times 10 and equals 2½ millimeters pitch, which corresponds to a metric standard screw of 18 millimeters diameter.

To Calculate the Change Gear when Cutting Metric Screws by an English Lead-Screw.

Divide 20 by the pitch in millimeters, and the quotient is the corresponding number of threads per inch to which the lathe must be geared.

EXAMPLE 1.

To gear a lathe in order to cut a metric standard screw 24 millimeters in diameter and of three millimeters pitch, the lead-screw on the lathe having four threads per inch.

Solution:

Twenty divided by three gives $6\frac{2}{3}$, therefore gear the lathe as if it was to cut $6\frac{2}{3}$ threads per inch with a common intermediate gear, and throw in the special intermediate gear as shown in the cut, and the lathe will cut a screw of three millimeters pitch. The gearing is easily obtained, thus:

The ratio between the screw gear and the stud gear is as $6\frac{2}{3}$ to 4, which is the same as 20 to 12, or, reduced to its lowest terms, 5 to 3. Hence, the gears may have any number of teeth providing the ratio is 5 to 3; for instance, multiplying by 9, 45 and 27 could be used, or, multiplying by 10, 50 and 30 could be used, etc.

TABLE No. 62.—How to Gear a Lathe when Cutting Metric Thread, Using Inch-Divided Lead-Screw and Intermediate Gears, as Shown in Fig. 5.

Screw to be Cut. (Pitch in Millimeters).	Lead-Screw on Lathe.											
	2 Threads per Inch.		3 Threads per Inch.		4 Threads per Inch.		5 Threads per Inch.		6 Threads per Inch.		10 Threads per Inch.	
	Stud gear.	Screw gear.	Stud gear.	Screw gear.	Stud gear.	Screw gear.	Stud gear.	Screw gear.	Stud gear.	Screw gear.	Stud gear.	Screw gear.
1	. . .	. . .	. . .	. . .	24	120	20	80	24	80	20	40
1.5	. . .	. . .	. . .	. . .	24	80	30	80	36	80	30	40
2	20	100	24	80	24	60	30	60	24	40	40	40
2.5	20	80	24	64	24	48	30	48	30	40	50	40
3	24	80	27	60	24	40	30	40	36	40	60	40
3.5	28	80	21	40	28	40	35	40	42	40	70	40
4	32	80	24	40	32	40	40	40	48	40	80	40
4.5	27	60	27	40	36	40	45	40	54	40		
5	30	60	30	40	40	40	50	40	60	40		
5.5	33	60	33	40	44	40	55	40	66	40		
6	36	60	36	40	48	40	60	40	72	40		
6.5	39	60	39	40	52	40	65	40				
7	28	40	42	40	56	40	70	40				
7.5	30	40	45	40	60	40						
8	32	40	48	40	64	40						
8.5	34	40	51	40								
9	36	40	54	40								
9.5	38	40	57	40								
10	40	40	60	40								
10.5	42	40										
11	44	40										
11.5	46	40										
12	48	40										

NOTES ON HYDRAULICS.

Hydraulics is the branch of engineering treating on fluid in motion, especially of water, its action in rivers, canals and pipes, the work of machinery for raising water, the work of water as a prime mover, etc.

Pressure of Fluid in a Vessel.

When fluid is kept in a vessel the pressure will vary directly as the perpendicular height, independent of the shape of the vessel. For water, the pressure is 0.434 pounds per square inch, when measured one foot under the surface. The pressure in pounds per square inch may, therefore, always be obtained by multiplying the *head* by 0.434. The head corresponding to a given pressure is obtained by either dividing by 0.434 or multiplying by 2.304.

EXAMPLE.

What head corresponds to a pressure of 80 pounds per square inch?

Solution:

$$80 \times 2.304 = 184 \text{ feet.}$$

Velocity of Efflux.

The velocity of the efflux from a hole in a vessel will vary directly as the square root of the vertical distance between the hole and the surface of the water. For instance, if an opening is made in a vessel four feet, and another 25 feet, below the surface of the water, and the vessel is kept full, the theoretical velocity of the efflux will be nearly 16 feet and 40 feet per second respectively, friction not considered; or, in other words, the velocity will be as 2 to 5, because $\sqrt{4} = 2$ and $\sqrt{25} = 5$.

The velocity of efflux in feet per second may always be calculated theoretically by the formula:

$$v = 8.02 \times \sqrt{h}.$$

Constant 8.02 is $\sqrt{2g} = \sqrt{64.4}$, and $v =$ velocity of efflux.

$h =$ Head in feet.

Table No. 63 gives the theoretical velocity of efflux and the static pressure corresponding to different heads, and is calculated by the following formulas:

$$h = \frac{v^2}{64.4} \qquad v = \sqrt{h \times 64.4} \qquad v = \sqrt{P \times 2.3 \times 64.4}$$

$$v = \sqrt{P \times 148} \qquad P = h \times 0.434 \qquad h = P \times 2.3$$

TABLE No. 63.—Head, Pressure, and Velocity of Efflux of Water.

Head in Feet.	Pressure in Pounds per Square Inch.	Velocity in Feet per Second.	Head in Feet.	Pressure in Pounds per Square Inch.	Velocity in Feet per Second.
h	*P*	*v*	*h*	*P*	*v*
0.1	0.0434	2.54	19	8.246	35
0.2	0.0868	3.59	20	8.68	35.9
0.25	0.1082	4.01	25	10.82	40.1
0.3	0.1302	4.39	30	13.02	43
0.4	0.1736	5.07	35	15.19	47.4
0.5	0.217	5.67	40	17.36	50.7
0.6	0.2604	6.22	45	19.53	53.8
0.7	0.3038	6.71	50	21.7	56.7
0.75	0.3255	6.95	55	23.87	59.5
0.8	0.3472	7.18	60	26.04	62.1
0.9	0.3906	7.61	65	28.21	64.7
1	0.434	8.02	70	30.38	67.1
1.25	0.5425	8.95	75	32.55	69.5
1.5	0.651	9.83	80	34.72	71.8
1.75	0.7595	10.6	85	36.89	73.9
2	0.868	11.4	90	39.06	76.1
2.25	0.9735	12	95	41.23	78.2
2.5	1.082	12.6	100	43.4	80.2
2.75	1.1905	13.3	110	47.74	84.2
3	1.302	13.9	120	52.08	87.68
3.25	1.4102	14.4	130	56.78	91.5
3.5	1.519	15	140	61.06	94.7
3.75	1.6375	15.5	150	65.1	98.3
4	1.736	16	160	69.44	101.2
4.25	1.8445	16.5	170	73.78	104.5
4.5	1.953	17	180	78.12	107.2
4.75	2.0615	17.5	190	82.46	110.4
5	2.17	17.9	200	86.8	113.5
6	2.604	19.6	225	97.35	120
7	3.038	21.2	250	108.2	126
8	3.472	22.8	275	119.05	133
9	3.906	24.1	300	130.2	139
10	4.34	25.4	325	141.05	144
11	4.774	26.6	350	151.9	150
12	5.208	27.8	375	163.75	155
13	5.678	28.9	400	173.6	160
14	6.106	30	425	184.45	165
15	6.51	31.1	450	195.3	170
16	6.944	32.1	475	206.15	174
17	7.378	33.1	500	217	179
18	7.812	34	550	238.7	188

Velocity of Water in Pipes.

The theoretical velocity of water discharged from a pipe is calculated by the same formula as is used in calculating velocities of falling bodies. (See page 277).

$v = \sqrt{2\,g\,h}$

v = Theoretical velocity of efflux per second.

h = Head.

$2\,g = 64.4$ if v and h are reckoned in feet.

$2\,g = 19.64$ if v and h are reckoned in meters.

If the water, besides the pressure due to the head, is also acted upon by some additional pressure, for instance, steam, the theoretical velocity of the discharge is obtained by the formula,

$$v = \sqrt{2\,g\left(h + \frac{P}{0.434}\right)}$$

P = Pressure in pounds per square inch.

The constant 0.434 is used because a column of water one foot high will exert a pressure of 0.434 pounds per square inch; thus, by dividing by 0.434, we actually convert the pressure into its corresponding head in feet.

All other quantities in this formula are, of course, taken in English units.

NOTE.—By *head* is always meant the vertical height in feet, or its equivalent in pressure expressed in feet. Table No. 63 gives the theoretical velocity of the discharge and the pressure corresponding to different heads.

The theoretical velocity is never obtained in practice, because part of the *total head* is used to overcome the resistance at the entrance of the pipe, and part is used to overcome the frictional resistance to the flow of the water in the pipe. Thus, only a part of the *total head* is left to give velocity to the water, therefore the velocity of the water at discharge will only be what is due to the *velocity head*, after deductions are made for resistance at the entrance and for friction in the pipes. In short pipes, the resistance at the entrance to the pipe is comparatively the larger loss, but in long pipes the frictional resistance is the larger.

When both the resistance at the entrance and the friction in the pipe are considered the formula will be :

$$v = \sqrt{\frac{2\,g\,h}{1.5 + f\,\frac{L}{d}}}$$

v = Velocity of discharge in feet per second.

$2\,g = 64.4$

L = Length of pipe in feet.

d = Diameter of pipe in feet.

f = Coefficient of friction, which is obtained from experiments, and will vary according to conditions, from 0.01 to 0.05. It is usually in approximate calculations taken as 0.025.

EXAMPLE.

Find the velocity of discharge from a pipe six inches in diameter. The head is 16 feet and the length of the pipe is 100 feet, and coefficient of friction 0.025.

Solution:

(NOTE. 6 inches = 0.5 foot.)

$$v = \sqrt{\frac{64.4 \times 16}{1.5 + 0.025 \times \frac{100}{0.5}}}$$

$$v = \sqrt{\frac{1030.4}{6.5}}$$

$v = 12.6$ feet per second.

In Table No. 64 the quantity of water discharged per minute by a pipe six inches in diameter, when the velocity is one foot per *second*, is 88.14 gallons. Thus, the quantity of water delivered when the velocity is 12.6 feet per second, is $12.6 \times 88.14 = 1110.6$ gallons per minute.

When the length of the pipe is more than 4,000 diameters the velocity of the water may be calculated by the formula,

$$v = \sqrt{\frac{2\,gh}{f\,\frac{L}{d}}}$$

and the quantity is obtained by multiplying the velocity by the constants given in Table No. 64.

EXAMPLE.

Find the velocity of efflux from a water pipe of three inches diameter and 1200 feet long, having a head of six feet, assuming coefficient of friction as 0.025.

Solution:

$$v = \sqrt{\frac{64.4 \times 6}{0.025 \times \frac{1200}{0.25}}} = 1.79 \text{ feet per second}$$

Discharge in gallons per minute:

$q = 22.03 \times 1.79 = 39.4$ gallons per minute.

TABLE No. 64.—Quantity of Water Discharged Through Pipes in One Minute, when Velocity of Efflux is One Foot per Second.

Internal Diameter of Pipe in Inches.	Internal Diameter of Pipe in Feet.	Internal Area of Pipe in Square Feet.	Discharge in Cubic Feet per Minute at Velocity of One Foot per Second.	Discharge in Gallons per Minute at a Velocity of One Foot per Second.	Internal Diameter of Pipe in Inches.	Internal Diameter of Pipe in Feet.	Internal Area of Pipe in Square Feet.	Discharge in Cubic Feet per Minute at a Velocity of One Foot per Second.	Discharge in Gals. pr. Minute at a Velocity of 1 Foot pr. Sec.
1/8	0.0104	0.00008	0.0048	0.036	20	1.6666	2.182	130.90	979
1/4	0.0208	0.00033	0.0198	0.150	21	1.7500	2.405	144.32	1079
3/8	0.0312	0.00076	0.0456	0.342	22	1.8333	2.640	158.39	1185
1/2	0.0416	0.00136	0.0816	0.612	23	1.9166	2.885	173.11	1294
3/4	0.0624	0.00306	0.1836	1.380	24	2.000	3.142	188.50	1410
1	0.0833	0.00545	0.3272	2.448	25	2.0833	3.409	204.53	1530
1¼	0.1042	0.00852	0.5094	3.828	26	2.1667	3.687	221.22	1655
1½	0.1250	0.01227	0.7362	5.508	27	2.2500	3.976	238.56	1784
2	0.1667	0.0218	1.309	9.792	28	2.3333	4.276	256.56	1919
2½	0.2083	0.0341	2.045	15.30	29	2.4166	4.578	275.22	2058
3	0.2500	0.0491	2.945	22.03	30	2.5000	4.909	294.52	2203
3½	0.2911	0.0668	4.008	29.99	31	2.5822	5.241	314.49	2352
4	0.3333	0.0873	5.238	39.17	32	2.6667	5.585	335.10	2506
4½	0.3750	0.1104	6.626	49.58	33	2.7500	5.939	356.37	2666
5	0.4166	0.1364	8.181	61.20	34	2.8333	6.305	378.30	2829
6	0.5000	0.1963	11.781	88.14	35	2.9166	6.681	400.88	2999
7	0.5822	0.2673	16.035	119.9	36	3.0000	7.069	424.14	3173
8	0.6667	0.3491	20.914	156.7	37	3.0833	7.467	448.02	3351
9	0.7500	0.4418	26.507	198.3	38	3.1667	7.876	472.56	3535
10	0.8333	0.5454	32.725	244.8	39	3.2500	8.296	497.75	3724
11	0.9166	0.6597	39.597	296.2	40	3.3333	8.727	523.60	3918
12	1.0000	0.7854	47.124	352.5	41	3.4166	9.168	550.11	4115
13	1.0833	0.9218	55.305	413.7	42	3.5000	9.621	577.27	4318
14	1.1667	1.069	64.141	479.8	43	3.5822	10.085	605.09	4526
15	1.2500	1.227	73.631	550.8	44	3.6667	10.559	633.56	4739
16	1.3333	1.396	83.776	626.4	45	3.7500	11.045	662.68	4961
17	1.4166	1.576	94.575	707.4	46	3.8333	11.541	692.46	5180
18	1.5000	1.768	106.03	793.2	47	3.9166	12.048	722.90	5408
19	1.5822	1.969	118.14	883.8	48	4	12.566	753.98	5640

NOTES ON STEAM.

When water is heated and converted into steam of atmospheric pressure, one cubic foot of water will make 1646 cubic feet of steam. (The common expression that "a cubic inch of water makes a cubic foot of steam" is not strictly correct, as a cubic foot contains 1728 cubic inches.)

The specific gravity of steam at atmospheric pressure, when compared with water is, therefore, $\frac{1}{1646} = 0.000608$.

The weight of one cubic foot of steam at atmospheric pressure will, therefore, be $0.000608 \times 62.5 = 0.038$ pounds. At any other pressure the weight per cubic foot of steam is given in Table No. 65.

Saturated steam is steam at the temperature of the boiling point which corresponds to its pressure. Saturated steam does not need to be wet steam, as the word *saturated* does not mean that the steam is saturated with water, but it means that it is saturated with heat; that is to say: the temperature under the given pressure cannot possibly be any higher as long as the steam is in contact with water, because if more heat is added more water will be evaporated, and if the volume is kept constant, as in a steam boiler, both the pressure and temperature will increase simultaneously.

High pressure steam is steam the pressure of which greatly exceeds the pressure of the atmosphere.

Low pressure steam is steam the pressure of which is less than the atmosphere, and also steam having a pressure equal to, or not greatly above, the atmospheric pressure.

Wet steam is steam which contains water held in suspension mechanically.

Dry steam is steam which does not contain water held in suspension mechanically.

Super-heated steam is steam which is heated to a temperature higher than the boiling point corresponding to its pressure. It cannot exist in contact with water, nor contain water, and resembles a perfect gas. Vertical boilers with tubes through the steam space (such as the Manning boiler) give slightly super-heated steam; but if steam is to be super-heated to any considerable extent it must be passed through a super-heater, which usually is in the form of a coil of pipes subjected to the hot gases in the uptake from the boiler.

The sensible heat of steam is the temperature which can be measured by a thermometer.

The latent heat of steam is that heat which is absorbed when water of any given temperature is changed into steam of the same temperature.

When water is evaporated under pressure the sensible heat will increase and the latent heat will decrease. For instance, at atmospheric pressure the sensible heat is 212 degrees, and the latent heat of evaporation is 966 B. T. U., but at 100 pounds absolute pressure the sensible heat is 327.9 degrees, while the latent heat of evaporation is only 883.1 B. T. U. (See steam table, No. 65.)

TABLE No. 65.—Properties of Saturated Steam.

Absolute Pressure in Pounds per Square Inch.	Temperature or Boiling Point in Degrees F.	Total Heat in B. T. U. per Pound of Steam from Water at 32 Degrees F.	Latent Heat of Evaporation in B. T. U. per Pound of Steam.	Volume in Cubic Feet per Pound of Steam.	Weight in Pounds per Cubic Foot of Steam.	Cubic Feet of Steam from 1 Cubic Foot of Water at 62 Degrees F.
1	102.1	1112.5	1042.9	330.36	0.0030	20800
2	126.3	1119.7	1025.8	172.80	0.0058	10760
3	141.6	1124.6	1014	117.52	0.0085	7344
4	153.1	1128.1	1006.8	89.36	0.0112	5573
5	162.3	1130.9	1000.3	72.80	0.0138	4524
6	170.2	1133.3	995	61.52	0.0163	3813
7	176.9	1135.3	990	52.62	0.0189	3298
8	182.9	1137.2	985.7	46.66	0.0214	2909
9	188.3	1138.8	982.4	41.79	0.0239	2604
10	193.3	1140.3	978.4	37.84	0.0264	2358
11	197.8	1141.7	975.3	34.63	0.0289	2157
12	202	1143	972.2	31.88	0.0314	1986
13	205.9	1144.2	970	29.57	0.0338	1842
14	209.6	1145.3	968	27.61	0.0362	1720
14.7	212	1146.1	966	26.36	0.0380	1646
15	213.1	1146.4	964.3	25.85	0.0387	1610
16	216.3	1147.7	962.6	24.32	0.0411	1515
17	219.6	1148.3	960.4	22.96	0.0435	1431
18	222.4	1149.2	957.7	21.78	0.0459	1357
19	225.3	1150.1	956.3	20.70	0.0483	1290
20	228	1150.9	952.8	19.72	0.0507	1229
25	240	1154.6	945.3	15.99	0.0625	996
30	250	1157.8	937.9	13.46	0.0743	838
35	259.3	1160.5	931.6	11.65	0.0858	726
40	267.3	1162.9	926	10.27	0.0974	640
45	274.4	1165.1	920.9	9.18	0.1089	572
50	281	1167.1	916.3	8.11	0.1202	518
55	287.1	1169	912	7.61	0.1314	474
60	292.7	1170.7	908	7.01	0.1425	437
65	298	1172.3	904.2	6.49	0 1538	405
70	302.9	1173.8	900.8	6.07	0.1648	378
75	307.5	1175.2	897.5	5.68	0.1759	353
80	312	1176.5	894.3	5.35	0.1869	333
85	316.1	1177.9	891.4	5.05	0.1980	314
90	320.2	1179.1	888.5	4.79	0.2089	298
95	324	1180.3	885.8	4.55	0.2198	283
100	327.9	1181.4	883.1	4.33	0.2307	270
105	331.3	1182.4	881.7	4.14	0.2414	257

TABLE No. 65.—(Continued).

Absolute Pressure in Pounds per Square Inch.	Temperature or Boiling Point in Degrees F.	Total Heat in B. T. U. per Pound of Steam from Water at 32 Degrees F.	Latent Heat of Evaporation in B. T. U. per Pound of Steam.	Volume in Cubic Feet per Pound of Steam.	Weight in Pounds per Cubic Foot of Steam.	Cubic Feet of Steam from 1 Cubic Foot of Water at 62 Degrees F.
110	334.6	1183.5	878.3	3.97	0.2521	247
115	338	1184.5	875.9	3.80	0.2628	237
120	341.1	1185.4	873.7	3.65	0.2738	227
125	344.2	1186.4	871.5	3.51	0.2845	219
130	347.2	1187.3	869.4	3.38	0.2955	211
135	350.1	1188.2	867.4	3.27	0.3060	203
140	352.9	1189	865.4	3.16	0.3162	197
145	355.6	1189.9	863.5	3.06	0.3273	190
150	358.3	1190.7	861.5	2.96	0.3377	184
160	363.4	1192.2	857.9	2.79	0.3590	174
170	368.2	1193.7	854.5	2.63	0.3798	164
180	372.9	1195.1	851.3	2.49	0.4009	155
190	377.5	1196.5	848	2.37	0.4222	148
200	381.7	1197.8	845	2.26	0.4431	141

In the preceding table the first column gives the absolute pressure, which is gage pressure plus 14.7 pounds, or, for ordinary practice, reckon as 15 pounds. For instance, when the gage pressure is 80 pounds per square inch, the corresponding absolute pressure is, for all practical purposes, 95 pounds per square inch, and the corresponding temperature is given in the second column in the table to be 324 degrees Fahr.

The total number of British thermal units (B. T. U.) required to convert each pound of water from 32 degrees Fahr. into steam of any given pressure is given in the third column. For instance, each pound of water of 32 degrees converted into steam of 95 pounds per square inch absolute pressure has received 1180.3 B. T. U.

The fourth column gives the number of British thermal units (B. T. U.) of heat required to change one pound of water of the temperature given in the second column into steam of the same temperature; which also is the number of heat units given up by one pound of steam when it is condensed to water of the same temperature as the temperature of the steam with which it is in contact. For instance, the table gives the latent heat of evaporation of steam at 95 pounds absolute pressure to be 885.8 B. T. U.; therefore, if steam of 95 pounds pressure per square inch is condensing into water in a steam pipe where steam and water are in contact, so that the temperature cannot drop below that due to the pressure, and the pressure is maintained at 95 pounds per square inch, the temperature of the water from the condensed steam will be 324 degrees, the same as the temperature of the steam, but each pound of steam as it is condensing will give out 885.8 British thermal units of heat.

The fifth column gives the number of cubic feet of saturated steam which will weigh one pound at the given pressure and temperature. The sixth column gives the weight of one cubic foot of saturated steam of corresponding given temperature. For instance, one cubic foot of steam at 95 pounds per square inch absolute pressure will weigh 0.2198 pounds, and 100 cubic feet of steam of 95 pounds per square inch absolute pressure will weigh $100 \times 0.2198 = 21.98$ pounds. In other words, it will take 0.2198 pounds of water to give one cubic foot of steam at 95 pounds absolute pressure, and it will require 21.98 pounds of water to make 100 cubic feet of steam of 95 pounds absolute pressure.

The seventh or last column gives the relative volume of steam at the given pressure as compared with water at 32 degrees F. For instance, one cubic foot of water will give 1646 cubic feet of steam at atmospheric pressure, but one cubic foot of water gives only 219 cubic feet of steam at 125 pounds absolute pressure.

Steam Heating.

In the ordinary practice of heating buildings by direct radiation the quantity of heat given off by the radiators or steam pipes will vary from 1¾ to 3 heat units per hour per square foot of radiating surface for each degree of difference in temperature; an average of from 2 to 2¼ is a fair estimate.

One pound of steam at about atmospheric pressure contains 1146 heat units, and if the temperature in the room is to be maintained at 70°, while the temperature of the pipes is 212°, the difference in temperature will be 142 degrees. Multiplying this by 2¼, the emission of heat will be 229½ heat units per hour per square foot of radiating surface. Dividing 229½ by 1146 gives 0.2 pounds of steam condensed per hour, per square foot of radiating surface. From this may be estimated the required size of boiler, as the boiler must always be capable of generating as much steam as the radiators are condensing. A rule frequently given is to have one square foot of heating surface in the boiler for every 8 to 10 square feet of radiating surface and one square foot of grate surface for every 350 to 500 square feet of radiating surface.

One pound of coal is required per hour per 30 to 40 square feet of radiating surface.

When steam is used for heating dwelling-houses, one square foot of radiating surface is required per 40 to 80 cubic feet of space, according to location, number of windows, etc. As a general rule, one square foot of radiating surface is sufficient for heating 40 to 60 cubic feet of air in outer or front rooms, and 80 to 100 cubic feet in inner rooms. The following rule may be used as a guide for different conditions: One square foot of radiating surface is sufficient for heating 60 to 80 cubic feet of space in dwellings, schools and offices; 75 to 100 cubic feet of space in halls, store houses and factories; 150 to 200 cubic feet of space in churches and large auditoriums.

In heating mills 1¼-inch steam pipes are generally used, and one foot of pipe is allowed per 90 cubic feet of space to be heated.

Value of Low Pressure Steam for Heating Purposes.

When steam at atmospheric pressure is condensed into water at a temperature of 212°, each pound of steam gives up 966 B. T. U. of heat, but if steam of 100 pounds gage pressure (115 pounds absolute) is condensed into water at 212 degrees, each pound of steam must give up 1004 B. T. U., which is only 38 heat units more than steam of atmospheric pressure. Hence it is evident that for heating purposes there is no advantage in using steam of high pressure; one pound of exhaust steam, only a pound or two over atmospheric pressure, is almost as valuable an agent for heating purposes as live steam at 100 pounds pressure direct from the boiler.

Hot Water Heating in Dwelling Houses.

One square foot of heating surface is required per 30 to 60 cubic feet of space heated.

Quantity of Water Required to Make any Quantity of Steam at any Pressure.

The weight of water required to make one cubic foot of steam at any pressure is the same as the weight of one cubic foot of steam as given in the sixth column in Table No. 65.

Therefore, the weight of water is obtained by multiplying the number of cubic feet of steam required by the weight of one cubic foot, as given in the table.

Example.

How much water will it take to make 300 cubic feet of steam at 100 pounds absolute pressure?

Solution:

One cubic foot of steam at 100 pounds pressure is given in the table as weighing 0.2307 pounds, therefore 300 cubic feet will weigh $300 \times 0.2307 = 69.21$ pounds of water.

One cubic foot of water may, for any practical purpose, be reckoned to weigh 62½ pounds and one gallon of water may be taken as $8\frac{3}{10}$ pounds. Therefore 69.21 pounds divided by 62.5 gives 1.1 cubic feet, or 69.21 pounds divided by 8.3 gives 8.34 gallons.

At atmospheric pressure one cubic foot of steam has nearly the weight of one cubic inch of water, and the weight increases very nearly as the pressure; therefore, for an approximate estimation, if no steam tables are at hand, it is well to remember the rule:

Multiply the number of cubic feet of steam by the absolute pressure in atmospheres, and the product is the number of cubic inches of water required to give the steam.

NOTE.—In all such calculations for practical purposes, a liberal allowance must be made for loss and leakage.

Weight of Water Required to Condense One Pound of Steam.

The following formula gives the theoretical amount of water required to condense one pound of steam:

$$W = \frac{H + 32 - t_3}{t_2 - t_1}$$

W = Weight of water required per pound of steam condensed.

H = Number of heat units above 32° in one pound of steam at the pressure of exhaust. This temperature is obtained from Table No. 65.

t_1 = Temperature of water when entering the condenser.

t_2 = Temperature of water when leaving the condenser.

t_3 = Temperature of the condensed steam when leaving the condenser and entering the air-pump.

EXAMPLE.

Steam of four pounds absolute pressure is exhausted into a surface condenser. The temperature of the condensed steam when leaving the condenser and entering the air-pump is 120°.

The temperature of the cold water when entering the condenser is 65°.

The temperature when leaving the condenser is 105°. How many pounds of condensing water is needed per pound of steam condensed?

NOTE.—In the steam table, page 419, the total number of heat units above 32° per pound of steam of four pounds absolute pressure is given as 1128.

Solution:

$$W = \frac{1128 + 32 - 120}{105 - 65}$$

$$W = \frac{1040}{40} = 26 \text{ pounds of water per pound of steam.}$$

In a jet condenser the steam and the water are mixed together, and, therefore, the condensed steam and the water when leaving the condenser are of equal temperature, and the formula will change to

$$W = \frac{H + 32 - t_2}{t_2 - t_1}$$

$t_2 =$ Temperature of mixture.

$t_1 =$ Temperature of water when entering condenser.

The other letters have the same meaning as in the previous formula.

EXAMPLE.

Steam of three pounds absolute pressure in exhausted into a jet condenser. The temperature of the cold water entering is 60°. The temperature of the mixture leaving the condenser is 110°. How many pounds of water are needed per pound of steam condensed?

NOTE.—In the steam table, page 419, the total number of heat units above 32° per pound of steam of three pounds pressure is given as 1124.6 or, for convenience, say 1125.

Solution:

$$W = \frac{1125 + 32 - 110}{110 - 60}$$

$$W = \frac{1047}{50} = 20.1 \text{ pounds.}$$

Weight of Steam Required to Boil Water.

An approximate rule is to allow that one pound of steam is condensed for every five pounds of water to be heated to the boiling point.

It does not make much difference about the pressure of the steam, as long as it is a few pounds above atmospheric pressure; for instance, one pound of steam at 10 pounds gage pressure when condensed into water at 212° will give up 973 heat units, and steam of 100 pounds gage pressure will give up 1003 heat units—a difference of only 30 heat units in steam of 10 pounds gage pressure and steam of 100 pounds gage pressure.

More correctly, the weight of steam required to boil one pound of water at 212° may be calculated by the formula,

$$x = \frac{212 - t_1}{H - 180}$$

And the weight of steam required to heat one pound of water to any temperature is obtained by the formula,

$$x = \frac{t_2 - t_1}{H + 32 - t_2}$$

$x =$ Weight of steam required.
$H =$ Number of heat units above 32° in one pound of steam, as given in Table No. 65.
$t_1 =$ Temperature of water before heating.
$t_2 =$ Temperature of water after heating.

Expansion of Steam in Steam Engines.

When steam is expanded without doing work and practically without losing heat by radiation, it will become superheated, but if it is doing work, as in a steam engine, it will lose heat during expansion.

According to the best authorities, the pressure varies inversely as the [redacted] power of the volume, if heat is neither added nor taken away by any outside source during the time the steam is being expanded in the steam engine cylinder. This is called adiabatic expansion of steam.

10/9 <

The pressure varies inversely as the $^{17}/_{16}$ power of the volume, if the steam is kept dry at the temperature of saturation, during expansion, by means of a steam jacket outside the cylinder.

When the pressure is considered to vary inversely as the volume it is called isothermal expansion.

The isothermal curve is not exactly the correct curve to represent the expansion of steam, but it is the theoretical curve usually drawn on the indicator diagram, because it is so easy to handle and is also very nearly correct.

The following formula gives the mean effective pressure according to isothermal expansion.

$$M.E.P. = P_1 \times \left(\frac{1 + hyp.\ log.\ r}{r}\right) - P_2$$

$$\text{Absolute terminal pressure} = P_1 \times \frac{1}{r}$$

$P_1 =$ Absolute initial pressure.
$r =$ Ratio of expansion.
$P_2 =$ Absolute back pressure.
$M.E.P. =$ Mean effective pressure.
Hyp. log. (hyperbolic logarithm), see page 126.

Table No. 66 gives the terminal and the mean effective pressure of steam expanded under any of these three different conditions.

TABLE No. 66.—Constants for Calculating Mean and Terminal Pressure of Expanding Steam.

Cut-Off in Fractions of the Stroke.	Number of Times that the Steam is Expanded: r	At Constant Temperature. (Isothermal Expansion).		Kept Dry at Temperature of Saturation. (Expansion in a Steam Jacketed Cylinder).		Condensing by Working in a Cylinder. (Adiabatic Expansion).	
		Terminal Pressure: $\frac{1}{r}$	Mean Pressure: $\frac{1 + hyp.\ log.\ r}{r}$	Terminal Pressure: $\left(\frac{1}{r}\right)^{1 \cdot 0625}$	Mean Pressure: $\frac{17 - 16\left(\frac{1}{r}\right)^{0 \cdot 0625}}{r}$	Terminal Pressure: $\left(\frac{1}{r}\right)^{1 \cdot 111}$	Mean Pressure: $\frac{10 - 9\left(\frac{1}{r}\right)^{0 \cdot 111}}{r}$
3/4	1 1/3	0.750	0.966	0.737	0.964	0.726	0.962
7/10	1 3/7	0.700	0.950	0.685	0.947	0.673	0.944
2/3	1 1/2	0.667	0.937	0.650	0.933	0.638	0.930
5/8	1 3/5	0.625	0.919	0.607	0.914	0.593	0.911
6/10	1 2/3	0.600	0.906	0.581	0.900	0.567	0.897
1/2	2	0.500	0.846	0.479	0.839	0.463	0.833
4/10	2 1/2	0.400	0.766	0.378	0.756	0.361	0.748
3/8	2 2/3	0.375	0.743	0.353	0.732	0.336	0.723
1/3	3	0.333	0.700	0.311	0.688	0.295	0.678
3/10	3 1/3	0.300	0.662	0.278	0.648	0.262	0.637
1/4	4	0.250	0.596	0.229	0.582	0.214	0.571
1/5	5	0.200	0.522	0.181	0.506	0.167	0.495
1/6	6	0.1667	0.465	0.149	0.449	0.137	0.437
1/7	7	0.1428	0.421	0.127	0.405	0.115	0.393
1/8	8	0.125	0.385	0.110	0.369	0.0992	0.357
1/9	9	0.1111	0.355	0.0968	0.340	0.0862	0.327
1/10	10	0.1000	0.330	0.0865	0.315	0.0774	0.303
1/11	11	0.0909	0.309	0.0782	0.293	0.0696	0.282
1/12	12	0.0833	0.290	0.0713	0.275	0.0631	0.264
1/13	13	0.0769	0.274	0.0655	0.259	0.0578	0.248
1/14	14	0.0714	0.260	0.0605	0.245	0.0533	0.234
1/15	15	0.0667	0.247	0.0563	0.232	0.0494	0.222
1/16	16	0.0625	0.236	0.0526	0.221	0.0459	0.211
1/17	17	0.0588	0.225	0.0493	0.211	0.0429	0.201
1/18	18	0.0556	0.216	0.0463	0.202	0.0403	0.192
1/19	19	0.0526	0.208	0.0438	0.193	0.0379	0.184
1/20	20	0.0500	0.200	0.0415	0.185	0.0359	0.177

To Find the Mean Effective Pressure by the Preceding Table.

Find the constant in the column corresponding to the conditions of expansion, and to the given cut-off. Multiply this by the absolute initial pressure, and the product is the average pressure. Subtract the back pressure and the remainder is the mean effective pressure.

EXAMPLE.

Find the mean effective pressure for isothermal expansion when the engine is cutting off at one-quarter stroke. The initial pressure is 90 pounds absolute. The absolute back pressure is 18 pounds.

Solution :

$M.E.P. = 90 \times 0.596 - 18 = 53.64 - 18 = 35.64$ pounds.

NOTE.—All such calculations *must* be made from absolute pressure (not gage pressure), and when determining the cut-off the clearance *must* be considered.

Clearance.

The clearance of an engine is usually expressed as a percentage of the piston displacement. The space between the piston and the cylinder head at the end of the stroke, also the cavities due to the steam ports, must be included in considering clearance.

In high-class Corliss engines the clearance does not exceed 2½ to 5 per cent., but in common slide-valve engines the clearance may go as high as 5 to 15 per cent. When clearance is taken into account the actual ratio of expansion is

$$R = \frac{1 + c}{\frac{1}{r} + c}$$

$R =$ Actual ratio of expansion.

$r =$ Nominal ratio of expansion.

$c =$ Clearance, expressed as a fractional part of the length of the stroke.

EXAMPLE.

The nominal ratio of expansion is 4, and the clearance is 5 per cent. What is the actual ratio of expansion?

NOTE.—5 per cent. is $\frac{5}{100} = \frac{1}{20} = 0.05$ of the stroke.

Solution:

$$R = \frac{1 + 0.05}{\frac{1}{4} + 0.05}$$

$$R = \frac{1.05}{0.3} = 3.5 = \text{Actual ratio of expansion.}$$

SHOP NOTES.

Weight of a Grindstone.

Multiply the constant 0.064 by the square of the diameter in inches and this product by the thickness in inches; the result is the weight of the grindstone in pounds.

EXAMPLE.

Find the weight of a grindstone 30 inches in diameter and six inches thick.

Solution:

Weight $= 0.064 \times 30 \times 30 \times 6 = 346$ pounds.

Lathe Centers.

In this country lathe centers are universally made 60 degrees, but in Europe the most common practice is to make lathe centers 90 degrees.

Morse Taper.

The Morse Taper, which is so universally used for the shanks of drills and other tools, is given in

TABLE No. 67.—Morse Taper.

No. of Taper.	Standard Plug Depth.	Diameter of Plug at Large End.	Diameter of Plug at Small End.	Taper per Foot.
1	$2\frac{1}{8}$ inch.	0.475 inch.	0.369 inch.	0.600 inch.
2	$2\frac{9}{16}$ "	0.7 "	0.572 "	0.602 "
3	$3\frac{3}{16}$ "	0.938 "	0.778 "	0.602 "
4	$4\frac{1}{16}$ "	1.231 "	1.02 "	0.623 "
5	$5\frac{3}{16}$ "	1.748 "	1.475 "	0.630 "
6	$7\frac{1}{4}$ "	2.494 "	2.116 "	0.626 "

For very complete information regarding the Morse Taper, see *American Machinist*, May 14, 1896.

Jarno Taper.

In the "Jarno Taper" the number of the taper gives the length of the standard plug in half-inches, and it gives the diameter of the small end in tenths of inches and the diameter of the large end in eighths of inches. For instance, a No. 8 "Jarno Taper" is four inches long, one inch diameter at large end, and 0.8 inch diameter at small end. The taper, of course, is 0.6 inch per foot for all numbers. This is a very convenient system, and deserves adoption for its merits. The same taper is also very well adapted to the metric system, as 0.6 inch per foot is equal to 0.05 millimeter per millimeter.

The following table is given to illustrate the system. The table could be extended to as large size tapers as are required for any work.

TABLE No. 68.—Jarno Taper.

Number of Taper.	Length of Taper.	Diameter of Large End of Taper.	Diameter of Small End of Taper.
1	½	⅛ = 0.125	$\frac{1}{10}$ = 0.1
2	1	¼ = 0.250	$\frac{1}{5}$ = 0.2
3	1½	⅜ = 0.375	$\frac{3}{10}$ = 0.3
4	2	½ = 0.500	$\frac{4}{10}$ = 0.4
5	2½	⅝ = 0.625	½ = 0.5
6	3	¾ = 0.750	$\frac{3}{5}$ = 0.6
7	3½	⅞ = 0.875	$\frac{7}{10}$ = 0.7
8	4	1 = 1.000	$\frac{4}{5}$ = 0.8
9	4½	1⅛ = 1.125	$\frac{9}{10}$ = 0.9
10	5	1¼ = 1.250	1 = 1.0

This system of taper is described by "Jarno" in the *American Machinist*, October 31, 1889.

Marking Solution.

Dissolve one ounce of sulphate of copper (blue vitriol) in four ounces of water and half a teaspoonful of nitric acid. When this solution is applied on bright steel or iron, the surface immediately turns copper color, and marks made by a sharp scratch-awl will be seen very distinctly.

A Cheap Lubricant for Milling and Drilling.

Dissolve separately in water 10 pounds of whale-oil soap and 15 pounds of sal-soda. Mix this in 40 gallons of clean water. Add two gallons of best lard oil, stir thoroughly, and the solution is ready for use.

Soda Water for Drilling.

Dissolve three-fourths to one pound of sal-soda in one pail full of water.

Solder.

Ordinary solder is an alloy consisting of two parts of tin and one part of lead, and melts at 360°.

Solder consisting of two parts of lead and one part of tin melts at 475°. For tin work use resin for a flux.

Soldering Fluids.

Add pieces of zinc to muriatic acid until the bubbles cease to rise, and the acid may be used for soldering with soft solder.

Mix one pint of grain alcohol with two tablespoonfuls of chloride of zinc. Shake well. This solution does not rust the joint as acids are liable to do.

When soldering lead use tallow or resin for a flux, and use a solder consisting of one part of tin and 1½ parts of lead.

Spelter.

Hard spelter consists of one part of copper and one part of zinc.

A softer spelter is made from two parts of copper and three parts of zinc.

A spelter which will flow very easily at low heat consists of 46% of Copper, 46% of Zinc, and 8% of Silver. When making any of these different kinds of spelter, melt the copper first in a black lead crucible and then put in the zinc after the copper has cooled enough to furnish just sufficient heat to melt the zinc, but not enough to burn it. Stir with an iron rod and after the metals have compounded and the compound is still molten, pour upon a basin of water. The metal in striking the water will form into small globules or shot and will so cool, leaving a coarse granular spelter ready for use. When pouring the metal let a helper keep stirring the water with an old broom.

Alloy Which Expands in Cooling.

Melt together nine pounds of lead, two pounds of antimony and one pound of bismuth. This alloy may be used in fastening foundation bolts for machinery into foundation stones. In such cases, collars or heads are left on the bolts and after the hole is drilled in the stone a couple of short, small holes are drilled at an angle to the big hole; when the metal is poured in, it will flow around the bolts and also into these small holes, and it is almost impossible for the bolt to pull out.

CAUTION.—When drilling holes in stone, water is always used, but this must be carefully dried out by the use of red-hot iron rods before the melted metal is poured in. If this precaution is not taken the metal will blow out, making a poor job, and it may also cause accident by burning the hands and face of the man who is pouring it in.

Shrinkage of Castings.

General rule:

⅛ inch per foot for iron.

$\frac{3}{16}$ inch per foot for brass.

In small castings the molder generally raps the pattern more than the casting will shrink, therefore no shrinkage is allowed. Frequently castings are of such shape that the pressure of the fluid iron on some part of the mould is liable to make the sand yield a little and thereby cause the casting to be as large as, or even larger than the pattern. All such things a practical pattern maker takes into consideration when allowing for shrinkage in patterns.

Case Hardening Wrought Iron and Soft Steel.

Bone dust specially prepared for the purpose, or burnt leather scrap, is placed in a cast-iron box, together with the article to be hardened. Cover the top of the box with plenty of the hardening material in order to keep the air out. Heat the whole mass slowly in a furnace to a red heat from two to five hours in order that it may be uniformly and thoroughly heated through. A few iron rods about 5/16 inch in diameter are put in when packing the box, one end of the rod reaching about to the middle of the box, and the other end projecting out through the hardening material on top. When the box appears to have the right heat, these rods are pulled out one at a time, in order to judge of the heat in the center of the mass. When the box has been exposed to the fire the desired length of time, its contents are quickly dumped into cool water.

Sieves of iron netting are laid on the bottom of the tub into which the case hardening material is dumped so that the hardened articles may be conveniently taken up from the water by one of the sieves. The case hardening material itself is also taken out by another sieve which is of very fine netting and placed under the first one. The material is dried and used over again, and a little new material is added when repacking the boxes.

When articles are well finished before hardening, this process gives a very fine color to both soft steel and wrought iron.

Case hardening may also be effected by packing the articles in soot, but this process does not give a nice color.

Horn and hoof is also used for case hardening. Malleable iron may also be case hardened, but it requires careful handling in order to prevent its cracking and twisting out of shape.

Case Hardening Boxes

are made from cast-iron and are of various sizes. Small boxes may be made nine inches long, five inches wide, and four inches deep, and about one-fourth inch thick. They should be provided with legs at least one inch high so that the heat may get under the bottom as at the top. An ear having a rectangular hole through it should be cast under the bottom at each end of the box. This gives a chance to handle the box with a fork having flat prongs instead of taking it out of the hardening furnace with a pair of tongs, which is liable to break the box, as cast-iron is very inferior in strength when hot.

To Harden with Cyanide of Potassium.

Heat the cyanide of potassium in a wrought iron pot until cherry red, and keep it so by a steady fire, immerse the pieces to be hardened from three to five minutes, according to their size and degree of hardness required, then plunge into cold water. Large pieces require more time than small ones, and the longer the article remains in the cyanide the deeper the hardening becomes. New cyanide gives the best color and cyanide previously used for hardening produces a harder surface.

BLUE PRINTING.

To Prepare Blue Print Paper.

Dissolve two ounces of citrate of iron and ammonium in 8¼ ounces of soft water. Keep this in a dark bottle. Also dissolve 1⅓ ounces of red prussiate of potash in 8¼ ounces of water and keep in another dark bottle. When about to use, mix (in a dark place) an equal quantity of each solution in a cup and apply with a sponge or a camel's hair brush as evenly as possible on one side of white rag paper (such as used for envelopes). Let it dry and put it away in a dark drawer. The paper must not be prepared in daylight but when taking prints it may be handled then, providing care is used to expose it as little as possible to the light before it is put into the printing frame.

Blue Print Frame.

Make a strong frame similar to a picture frame having a strong and thick glass. Make a loose back, from boards about ½ inch thick, which is held into the frame by four suitable catches so arranged that they press this back firmly and evenly against the glass. The surface next to the glass should be covered by three thicknesses of flannel in order to make a cushion so that the prepared paper and the tracing are kept close together when put in the frame.

Blue Printing.

The drawing must be made on transparent material, for instance, tracing cloth or tracing paper. Place the tracing in the frame with the side on which the drawing is made next to the glass. Place the prepared side of the sensitive paper against the back of the tracing. Put the loose back into the frame with the padded side against the prepared paper, and fasten it up so that both paper and tracing are kept firmly against the glass. Expose to sunlight from three to six minutes, according to the brightness of the sun. Take the sensitive paper out of the frame and quickly put it into a tub of clean cool water and wash it off, and the drawing will appear in white lines on blue ground. Hang the print up by one edge so that the water will run off, and let the print hang until dry.

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