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Market Equilibrium with Performance Contracts in the Presence of Exogenous Turnover and Matching Frictions

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Abstract

We consider market equilibrium with renegotiation-proof performance contracts. The contracting model is similar to the model posed by MacLeod and Malcomson (1989). The market model is somewhat different. We impose exogenous turnover as in Shapiro and Stiglitz (1984) and introduce an additional friction in matching that is not considered in either of these papers. We show there is a multiplicity of such equilibria that range in both employment and the distribution of income, with an inextricable link between the two. As the equilibrium match population increases, agents on the long side of the market claim a greater share of the product from a match. Consequently, agents on the short side prefer equilibria that entail a smaller than maximal match population. However, when the turnover probability goes to 0, all but a small fraction of the agents on the short side are matched and agents on the long side have reservation utility near 0, in *any* equilibrium. Thus, the conclusions of MacLeod and Malcomson (1989) depend crucially on the absence of turnover. We also extend our model to the case of heterogeneous firms, and show that our results carry over to this case as well. The heterogeneous firm case points to important empirical implications that can be derived from the model.

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1. Introduction

We consider market equilibrium with renegotiation-proof performance contracts in the presence of exogenous turnover and matching frictions. We show there is a multiplicity of such equilibria that range in both employment and the distribution of income, with an inextricable link between the two. Though a utilitarian social welfare function rates the equilibrium with maximal employment as best, we show there will be disagreement as to this ranking. Agents on the long side have preferences that mirror the utilitarian view but agents on the short side prefer equilibria with fewer matches, since then they earn more from a match.

Thus, selection of the labor market equilibrium can be viewed as a generalized "Battle of the Sexes" game. Our approach supplements recent work on game-theoretic foundations of Keynesian macroeconomics, where strategic complementarities produce mulitple, Pareto rankable equilibria. See for example, Cooper and John (1988), Jones and Manuelli (1992), and Roberts (1987). In our model, which takes a partial equilibrium approach, no feedback or multiplier effect is necessary to achieve an underemployment equilibrium.

Performance contracts are inconsistent with Walrasian market equilibrium. To be self-enforcing, a performance contract must generate sufficient future surplus to offset the immediate gain from malfeasance. This is a result of subgame perfection, since there is no Nash equilibrium of the one-shot contract game in which the employee puts forth effort. But in a Walrasian market equilibrium, the marginal match generates no surplus. Thus, it is necessary to develop an alternative to the Walrasian model to reconcile performance contracts and market equilibrium.

MacLeod and Malcomson (1989), hereafter M&M, produce such a model. They consider the market itself as an infinitely repeated game. In the spirit of the Folk Theorem, they show any individually rational and incentive compatible allocation can be realized as a market equilibrium. This plethora of equilibria is unhelpful. A sharper prediction is desireable. M&M respond by considering a renegotiation-proof refinement suggested by the work of Pearce (1987). In such a renegotiation-proof equilibrium: i) all agents on the short side are matched, ii) the reservation utility of agents on the long side is 0 while the reservation utility of agents on the short side is bid up until there is a unique incentive compatible effort, less than the first-best effort, and iii) any division of the surplus from employment can be attained. Except for determining which agents on the long side are matched, points (i) and (ii) pin down the market allocation. Given this allocation, dividing the surplus is a zero-sum game. Point (iii) can be taken as a rebuttal of Shapiro and Stiglitz (1984), hereafter S&S, who *assume* all the surplus is captured by agents on the long side.

The M&M model differs from that of S&S in several respects. For our purposes, the salient difference is the treatment of turnover. There is no turnover in the M&M model while there is exogenously imposed turnover in the S&S model. Consequently, the labor market is inactive after the initial period in the M&M model but is active in each period in the S&S model. The treatment of turnover has an important effect on market equilibrium via its impact on the reservation utility of agents on the long side.

We introduce a friction not considered by M&M or S&S; unmatched agents face a lag in matching, even when such agents are on the short side. Then, we consider turnover a la S&S in a renegotiation-proof equilibrium a la M&M. We demonstrate a continuum of equilibria, indexed by the match population. The sum of the reservation utilities is invariant to the number of matches, but there is an inextricable link between the (expected) division of the surplus from a match and the match population. However, when the turnover probability goes to 0, all but a small fraction of the agents on the short side are matched and agents on the long side have reservation utility near 0, in *any* equilibrium. But as long as the turnover probability is bounded away from 0, there exist equilibria with a significant fraction of unmatched agents on the short side, even when the lag in matching goes to 0.

When workers are long and there is an equilibrium where all firms are matched, this equilibrium is the efficiency wage equilibrium and is the best equilibrium for workers. This is the worst equilibrium for firms. Firms prefer equilibria with partial bonding because, in addition to capturing some of the surplus, their reservation utility is bid up and the workers' reservation utility is bid down. This preference exists in spite of the fact that such equilibria entail fewer matches.

Though agents on the long side have utilitarian preferences, it is harder to identify the best equilibrium for agents on the short side. A candidate is where agents on the short side capture all the surplus. When firms are short, this is the full bonding equilibrium. In this equilibrium the workers' reservation utility is 0. Thus, the firms' reservation utility is maximal as is the profit a firm earns when matched. However, from a "veil of ignorance" perspective, where firms are not sure whether they will be matched or unmatched initially, firms may prefer equilibria with more matches. But even from the veil of ignorance perspective, it is unambiguous that the efficiency wage equilibrium is worst for firms.

In mapping the set of equilibria into utility imputations (using the veil of ignorance expected utility functions), the Pareto frontier is not a singleton. When the frictions we identify are significant, all equilibria yielding utility imputations on the Pareto frontier are legitimate objects of study. It is a mistake to focus exclusively on the utilitarian ideal, the equilibrium with maximal matching, and to hypothesize that this is necessarily the market outcome in actuality. In this respect our model speaks directly to the debate between Carmichael (1985) and S&S (1985), by showing that the division of the surplus matters for market performance and by showing that firms and workers differ in their views as to what constitutes good market performance. But, in contrast to the view articulated by Carmichael, movement toward increased bonding does not move the market closer to the utilitarian ideal. In fact, it achieves the opposite result.[1]

Our basic model assumes homogeneous firms and homogeneous workers with exogenous firm and worker populations. To better relate the M&M and S&S models and to investigate whether our conclusion is robust, we also consider a model with heterogeneous firms, where we endogenize the number of active firms. We are again able to demonstrate a continuum of equilibria. Interestingly, there is an inverse relationship between the number of active firms and the number of matches. Thus, the heterogeneous firm model necessitates considering whether there are unmatched firms that are more productive than matched firms. We refer to this issue as match quality. The utilitarian measure of market performance accounts for match quality as well as the number of matches. The efficiency wage equilibrium tops out in both respects.[2] Nevertheless, it is the worst equilibrium for firms.

MacLeod and Malcomson (1990) also present an analysis that melds the S&S and M&M approaches, though they do not consider the renegotiation-proof

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[1] This argument presupposes that workers are on the long side. If workers are on the short side, a movement to partial bonding lowers unemployment.

[2] Note that this contradicts Carmichael's view in the specific context of the S&S model.

refinement. In the 1990 paper, MacLeod and Malcomson are primarily concerned with comparing anonymous markets, where the reservation utilities are independent of job histories, to markets where history dependent punishments are possible. They adopt the utilitarian view and conclude that in anonymous markets efficient matching entails all the surplus going to agents on the long side. They argue this is a justification for efficiency wages in actuality.

This argument is too strong. Firms will not cede surplus to workers just to maximize a particular social welfare function. Nor do we believe it has been established empirically that primary sector jobs entail no surplus for the firm. Indeed, recent empirical work in the area suggests strongly that good jobs are associated with more profitable firms. See the discussion in Katz (1986) and the work of Dickens and Katz (1986).[3] But this outcome does not necessitate full efficiency wages. It emerges in our heterogeneous firm model as long as there is not too much bonding, because the total surplus from a match is increasing with firm productivity. The empirical finding is consistent with partial bonding equilibrium.

Our approach follows the one articulated by M&M. There is an important role to be played by firm and worker beliefs in explaining the market equilibrium allocation. The novelty of our paper is in showing that beliefs affect allocational as well as distributional outcomes.

2. The Performance-Matching Model

There are two types of agents, workers and firms. There is a continuum of workers (firms) with Lesbegue measure N (M). Workers have identical preferences. Firms have identical production capabilities.[4]

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[3] Se also Hodson and England (1985), Kwoka (1983), Long and Link (1983), Mellow (1982), Nickell and Wadhawani (1990), Pugel (1980), and Scaramozzino (1991).

[4] M&M assume a finite number of workers and firms. The continuum assumption facilitates performing comparative statics on the set of equilibria.

There are three distinct frictions in the model. First, there is exogenous turnover. Second, there is a lag in detecting malfeasance. Third, there is a lag in getting a draw that might lead to a match. The lags may be of different duration. To model this, we consider two different types of time period, a production interval and a market period. There are T market periods per each production interval, where T is a positive integer taken parametrically. Below, we use the term "period" to refer to a market period.

Workers and firms can either be matched or unmatched. If a production interval starts in period t, there is a probability, $g > 0$, that the match dissolves in period $t + T$ for reasons other than malfeasance. We assume the match cannot dissolve in period $t + \tau$, for $0 < \tau < T$.

A matched worker supplies effort, e, which we assume to be a continuous variable, $e \in [0,\infty)$. An employee's utility is $w - v(e)$ per production interval, when she earns a wage, w, and takes effort, e. We assume $v(0) = 0$ and that v is increasing, strictly convex, and twice differentiable. We also assume $\lim_{e \to 0} v'(e) = 0$ and $\lim_{e \to \infty} v'(e) = \infty$. An unemployed worker's utility is 0 per period.

The firm's output is f(e), when its employee takes effort, e. We assume $f(0) = 0$ and that f is increasing, strictly concave, and twice differentiable. These assumptions imply there is a unique effort level that maximizes the product from a match. This efficient effort level, e^*, is implicitly defined by the equation $f'(e^*) - v'(e^*) = 0$. A matched firm earns profit $f(e) - w$ per production interval, when it pays a wage, w, and its employee takes effort, e. An unmatched firm earns 0 profit per period.

As long as a firm and its employee continue in a match, they are bound by a pre-existing contract. Prior to the start of a new production interval, either party has the option of severing the match. Severance also occurs, with probability g, even if both parties opt to continue in the match.

Matches are divided into cohorts. The cohort for a given match is determined by when the match was formed. If match 1 was formed in period t_1 and match 2 was formed in period t_2 then match 1 and match 2 are in the same cohort if and only if $t_1 \equiv t_2 \bmod T$. At the start of each period there is entry into the pool of unmatched agents from among the cohort of matched agents whose production interval has just ended. Subsequent to this entry, but within the same period, unmatched firms and workers enter into a pairing process. An agent who is successful in the pairing process enters a match which starts that period. An agent who is unsuccessful in the pairing process must wait until the next period to obtain a chance to pair.

We assume the match probabilities at the individual level mimic perfectly the pairing process at the aggregate level. Let P denote the measure of matches in aggregate, determined endogenously. We assume P is time invariant. Consequently, the measure of matches in each cohort is P/T. Let $a(P,H) = \dfrac{gP/T}{H - (1 - g/T)P}$ for $H \geq P$.[5] Then, the probability an unmatched worker (firm) is successfully paired is a(P,N) (a(P,M)). Agents on the short side find a match with probability 1 when P = min {M,N}.

Once a match occurs, a contract is written. We restrict attention to stationary contracts, by which we mean the terms of the contract are the same over each production interval the contract is in force. A contract is a 4 -tuple, z, where $z = (b,w,e,\sigma)$. We have already discussed the wage, w, and the effort, e. We refer to the first component, b, as a performance bond, and the last component, σ, as a severance payment.

[5] Here gP/T is the number of new matches formed each period.

M&M distinguish verifiable components of the contract, by which credible commitments can be made, from unverifiable components of the contract, which are required to satisfy certain incentive compatibility constraints to make the contract self-enforcing. We assume a third party (the courts) can verify whether a new match has been made and whether an old match has been severed or not. The history of payments can also be verified.

The contract mandates the firm pay the worker w - b, per production interval, and mandates the firm pay the worker σ, in the first period of separation. The effort component of the contract should be interpreted as the minimum effort level acceptable to the firm. There is no breach if the worker supplies greater effort than the minimum but also no mechanism for the firm to reward this superior effort. Neither effort nor output is verifiable. The contract specifies the firm keep the performance bond, b, if effort falls below the minimum and specifies return of the performance bond to the worker otherwise. Nonpayment of b means there has been breach, but the courts cannot determine which party has committed malfeasance.

The contract must provide incentive for the worker to put forth the specified effort. Before considering the restrictions imposed by incentive compatibility, we develop a bit of notation. Let ρ denote the discount rate per period held commonly by both workers and firms. Let r denote the discount rate per production interval, $(1 + \rho)^T = 1 + r$. Let U^0 denote the expected discounted value of the worker if she started the current period unmatched. We assume U^0 is constant across workers. It does not depend on whether the worker was previously fired. In this sense, the market is anonymous.

For the moment, let us ignore the possibility of contract breach. Let U(z) denote the value of a match to a worker at the start of a production interval, given contract z:

(1) $$U(z) = w - v(e) + \frac{g[U^0 + \sigma] + (1 - g)U(z)}{1 + r}.$$

The seond term on the right hand side of (1) includes the possibility that the match does not survive into the subsequent production interval and, hence, the worker enters the pool of workers who participate in the pairing process. Note that (1) can be solved to yield:

(2) $$U(z) = \frac{(1 + r)[w - v(e)] + g[U^0 + \sigma]}{g + r}.$$

Though effort is not verifiable, we assume the firm can observe effort perfectly and punishes shirking.[6] The firm has two devices for punishing the worker. First, the worker is made to forfeit the performance bond, b. Second, the firm severs the match at the start of the next production interval. The punishment in this second device is implicit. Firing the worker is only a punishment when the worker expects to lose some surplus she would earn by continuing in the match. There is no turnover cost per se. The expected total punishment is: $b + (1 - g)\frac{U(z) - U^0 - \sigma}{1 + r}$.

The employee will put forth the minimum acceptable effort if the expected punishment from shirking exceeds the disutility of effort. This condition is referred to as the No Shirk Condition, NSC. After rearranging terms, it can be seen that the NSC is given by:

(3) $$\frac{(1 - g)[U(z) - U^0]}{1 + r} \geq v(e) - b + \frac{(1 - g)\sigma}{1 + r}.$$

We proceed in a likewise manner for the firm. The value of the firm, $\Pi(z)$, is given by:

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[6] The punishment is the same regardless of the magnitude of shirking. This is a simplifying assumption. It implies the worker will set $e = 0$, if she decides to shirk.

$$\Pi(z) = \frac{(1 + r)[f(e) - w] + g[\Pi^0 - \sigma]}{g + r}, \tag{4}$$

where Π^0 is the expected lifetime value of a firm that starts the current period unmatched. When $b > 0$, the firm has an incentive to expropriate the performance bond. To deter this malfeasance, the worker punishes by severing the match. The firm will honor the contract if $(1 - g)\frac{\Pi(z) - \Pi^0 + \sigma}{1 + r} \geq b$. We term this condition the No Expropriation Condition, NEC. By rearranging terms, one can see that the NEC is a lower bound constraint on the firm's expected future surplus:

$$\frac{(1 - g)[\Pi(z) - \Pi^0]}{1 + r} \geq b - \frac{(1 - g)\sigma}{1 + r}. \tag{5}$$

Adding (3) to (5) yields:

$$\frac{S(e)}{1 + r} = \frac{(1 - g)[f(e) - v(e) - u^0 - \pi^0]}{g + r} \geq v(e), \tag{6}$$

where $u^0 = rU^0/(1 + r)$, $\pi^0 = r\Pi^0/(1 + r)$, and $S(e)$ is the expected lifetime surplus of a match that calls for effort e. Note that $S(e)$ is independent of the bond, wage, and severance payment. We refer to (6) as the Aggregate Surplus Condition, ASC. The ASC demonstrates that incentive compatible effort must generate sufficient surplus.

M&M have shown that the ASC and individual rationality are sufficient for sustainibility of a self-enforcing contract. That is, for any effort satisfying (6), one can find a contract entailing that effort such that the contract satisfies (3) and (5). Moreover, the effort can be sustained with any division of the surplus from the match. A contract with is individually rational for both worker and firm if and only if:

$$u^0 + v(e) \leq w + g\sigma/(1 + r) \leq f(e) - \pi^0. \tag{7}$$

Proposition 1 (Invariance of the Sustainable Effort within a Match to the Division of Surplus): For any $\tilde{e}$ that satisfies (6) and any $\tilde{w}$ and $\tilde{\sigma}$ that satisfy (7) given $\tilde{e}$, there exists a contract $\tilde{z}$, such that (3) and (5) are satisfied. There exists another contract, $\hat{z}$, with $\hat{e} = \tilde{e}$ and $\hat{\sigma} = 0$, such that (3) and (5) are satisfied and such that both the firm and worker are indifferent between $\tilde{z}$ and $\hat{z}$.

All proofs may be found in the appendix. Hereafter, we restrict attention to contracts with $\sigma = 0$, without loss of generality. Let the constrained efficient effort, $e^C(u^0 + \pi^0)$, solve:

$$(8) \qquad \underset{e \geq 0}{\text{maximize}} \quad f(e) - v(e) \qquad \text{subject to: } (6).$$

We show, in proposition 2, that a contract which is immune from later renegotiation will call for $e = e^C(u^0 + \pi^0)$. Note that when (6) does not bind in (8), $e^C(u^0 + \pi^0) = e^*$, while when (6) binds, $e^C(u^0 + \pi^0) < e^*$, in which case $e^C(u^0 + \pi^0)$ is decreasing in $u^0 + \pi^0$.[7]

We turn to the determination of the reservation utilities in market equilibrium. Let e^R solve:

$$(9) \qquad \underset{e \geq 0}{\text{maximize}} \quad f(e) - (1 + r)v(e)/(1 - g).$$

Note that e^R maximizes the difference between the left and right hand sides of (6) and that $e^R < e^*$. Hence, when (8) has a solution, $e^C(u^0 + \pi^0) \geq e^R$. Let s^R denote the value of (9), $s^R = f(e^R) - (1 + r)v(e^R)/(1 - g)$. Observe that $e^C(s^R) = e^R$. M&M argue that in a renegotiation-proof equilibrium, $P = \min\{M,N\}$. Since $g = 0$ in the M&M model, agents on the long side have reservation

[7] When $u^0 + \pi^0$ is sufficiently large, the feasible set is empty. The feasible set is nonempty when $u^0 + \pi^0$ is sufficiently close to 0.

utility equal to 0, because such agents cannot find an unmatched counterpart. Agents on the short side must get a scarcity rent. M&M argue that when workers are on the short side $\pi^0 = 0$ and $u^0 = s^R$, while when firms are on the short side $\pi^0 = s^R$ and $u^0 = 0$. Considering the case where workers are on the short side, the intuition is that $(1 + r)u^0/r$ represents the most a worker can expect to earn over a lifetime when she renegotiates with an unmatched firm, when the renegotiated contract is itself immune from further renegotiation and, hence, has all the performance incentive provided by the bond.

S&S argue the pairing process should determine the reservation utilities. S&S *assume* $P = \min\{M,N\}$. Since $g > 0$ in the S&S model, unmatched agents on the long side enter into matches with positive probability. Then, their lifetime expected utility is positive, even when they are currently unmatched. For example, the S&S approach yields the following asset equation for an unmatched firm when firms are on the long side $(M > N)$:

$$\Pi^0 = a(N,M)\Pi(z) + [1 - a(N,M)]\,\frac{\Pi^0}{1+\rho}\,. \tag{10}$$

Both the M&M and the S&S approaches treat the long and short sides differently in their determination of the reservation utilities. In contrast, we assume asset equations like (10) are applicable to agents on both sides of the market. This means that an agent who is unsuccessful in being matched has a lower ex post expected utility than lifetime reservation utility. For agents on the short side, this necessitates a lag in pairing.

Conversely, when either the discount rate is 0 or the match probability is 1 in an asset equation like (10), i.e., there is no lag in pairing, it must be that the agent earns no surplus from being matched. This can only be the case for agents on the short side. Then, to be consistent with market equilibrium, all agents on the short side must be matched. Thus, the assumption that there is no lag is tantamount to the assumption that P = min {M,N} and all surplus from a match goes to the agent on the long side, the S&S conclusion. Interestingly, when there is a short lag the latter conclusion does not hold, even approximately, as we show in proposition 6. Both the discount rate and the match probability converge to 0 as the lag shinks. Then, agents on the short side can earn a surplus from being matched and this surplus can be consistent with market equilibrium.

There remains the question of how P is determined in market equilibrium. A minimal requirement is that contracting should be efficient. On the one hand, this means the effort component of the contract equals $e^C(u^0 + \pi^0)$, for parametric reservation utilities. On the other hand, the reservation utilities are increasing in P, for a parametric contract, since the match probabilities are increasing in P. One can identify pairs, (P,z), consistent with efficient contracting. But, a contract with effort component $e^C(0)$ is always consistent with P = 0. Therefore, one is driven to a sterner requirement that forces the reservation utilities to be bid up and, consequently, puts upward pressure on P.

Our approach is to allow an alternative for computing the reservation utility. The alternative, which we refer to as the direct contact method, borrows from M&M. Their derivation of the reservation utility of agents on the short side does not require an asset equation like (10). Instead, it is based on the presumption that agents on the short side can readily find another partner, because there are unmatched agents on the other side of the market. In our model there are unmatched agents on both sides when P < min {M,N}. In this case we assume any agent has the option of earning her lifetime reservation utility by entering into a match formed in the current period. But, to avoid the implications of the S&S logic, we assume an agent who is unsuccessful in the pairing process cannot then find a partner via direct contact in the same period. We also assume participation in the pairing process is voluntary. For the pairing process to be active, as it must be in a nondegenerate market equilibrium, agents must do as well by participating in it as they would by opting for direct contact. This places a stringent lower bound on P.

We define a market equilibrium by a measure of matches, a contract, and a pair of reservation utilities. Let $E = (P, \tilde{z}, u^0, \pi^0)$ denote a market equilibrium.[8]

<u>Definition 1</u>: E is renegotiation proof if:

i) $\tilde{z}$ satisfies (3), (5), and (7);

ii) $u^0 = A(P,N)[\tilde{w} - v(\tilde{e})]$;

iii) $\pi^0 = A(P,M)[f(\tilde{e}) - \tilde{w}]$;

iv) if $P < M$ and $\pi^0 < s^R$, $u^0 \geq \underset{e,w}{\text{maximum}}\ w - v(e)$

subject to: $\dfrac{(1 - g)[f(e) - w - \pi^0]}{g + r} \geq v(e)$;

v) if $P < N$ and $u^0 < s^R$, $\pi^0 \geq \underset{e,w}{\text{maximum}}\ f(e) - w$

subject to: $\dfrac{(1 - g)[w - v(e) - u^0]}{g + r} \geq v(e)$; and where

$$A(P,H) = \frac{ra(P,H)[1 + \rho]}{\rho[a(P,H)r/\rho + g + r - a(P,H)g]} = \frac{gP[1 + \rho]}{gP[1 + \rho] + \rho[1 + g/r][H - P]T}\ .$$

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8 Note that assuming a unique contract is a matter of convenience in the definition of market equilibrium. We could generalize the equilbrium notion by allowing for a lottery over contracts with nondegenerate support. Each contract in the support would specify the same effort, $e^C(u^0 + \pi^0)$. The contracts would differ on the division of the surplus. The important issue is the expected utility, conditional on being matched. The mean division of the surplus for such a lottery is provided by our equilibrium contract.

Condition (i) assures the contract is both incentive compatible and individually rational. Condition (iii) is obtained from the simultaneous solution of (4) and (10). Condition (ii) is obtained similarly. The direct contact method imposes a lower bound on the reservation utilities. This is the upshot of conditions (iv) and (v). When either (iv) or (v) bind, we have a similar renegotiation-proof equilibrium to M&M's (p. 472).

We provide another invariance result in proposition 2. This result generalizes the M&M conclusion to include the possibility that agents on the long side of the market have a positive reservation utility.

<u>Proposition 2</u> (Invariance of the Sum of the Reservation Utilities and the Market Equilibrium Effort to the Match Probability): In any renegotiation-proof market equilibrium that satisfies $M \neq N$ or $P < M = N$: $u^0 + \pi^0 = s^R$ and $\tilde{e} = e^C(u^0 + \pi^0) = e^R$.

Proposition 2 can be restated as follows. In a renegotiation-proof equilibrium, determining the reservation utility of an agent via the direct contact method yields the same result as determining the reservation utility via the pairing process and, consequently, the sum of the reservation utilities is maximal. When $P < \min\{M,N\}$, the indifference between waiting to be paired and directly contacting an agent on the other side of the market necessitates that the surplus from a match be split.

It is important to note that the two invariance results are logically independent. The first invariance result follows because, for any effort and any wage, one can find a bond such that the NSC holds as an equality. Then, the NEC and the ASC are equivalent. The second invariance result follows because the reservation utilities don't appear in the first order condition that implicitly defines e^R. There is a temptation to compound these results, but that temptation should be resisted. In the next section we show that when firms are heterogeneous the second invariance result fails.

Proposition 2 provides an avenue for computing renegotiation-proof market equilibrium. Adding (ii) and (iii) of defintion 1, we get:

$$(11) \qquad s^R = A(P,N)[\tilde{w} - v(e^R)] + A(P,M)[f(e^R) - \tilde{w}].$$

When there is a pair, $(P,\tilde{w})$, which satisfies (11) with $v(e^R) \le \tilde{w} \le f(e^R)$, a renegotiation-proof equilibrium can be constructed. The reservation utilities are given by: $u^0 = A(P,N)[\tilde{w} - v(e^R)]$ and $\pi^0 = A(P,M)[f(e^R) - \tilde{w}]$. Finally, $\tilde{b}$ can be computed from the NSC in equality form.

There always is a solution to (11) with the wage in the acceptable range. When $M \neq N$ there is an interval of P, $[\underline{P},\overline{P}]$, consistent with renegotiation-proof equilibrium. Note that A has the following properties: $A_P > 0$, $A_H < 0$, $A(0,H) = 0$ for all $H > 0$, and, $A(H,H) = 1$ for all $H > 0$. Also note that $s^R < f(e^R) - v(e^R)$. When $P > 0$, the right hand side of (11) is increasing (decreasing) and continuous in $\tilde{w}$, if $M > N$ (if $M < N$). Thus, for any P there is at most one $\tilde{w}$ that solves (11), if $M \neq N$.[9] Values of P that are consistent with equilibrium satisfy:

$$(12) \qquad A(P,M)[f(e^R) - v(e^R)] \le s^R \le A(P,N)[f(e^R) - v(e^R)],$$

when $M > N$. The reverse inequalities apply when $M < N$. Conversely, when $M > N$ and P satisfies (12), there is a $\tilde{w}$ in the acceptable range such that $(P,\tilde{w})$ satisfies (11). When $M > N$, the inequalities given in (12) implicitly define those P consistent with renegotiation-proof equilibrium. The lower bound of this interval, $\underline{P}$, is implicitly defined by: $A(\underline{P},N) = s^R/[f(e^R) - v(e^R)]$, which implies $0 < \underline{P} < N$. The upper bound of this interval, $\overline{P}$, either equals

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[9] When $M = N$, there is a unique P consistent with (11). In this case, $\tilde{w}$ is indeterminant.

N, which occurs when $A(N,M) \le s^R/[f(e^R) - v(e^R)]$, or is implicitly defined by: $A(\overline{P},M) = s^R/[f(e^R) - v(e^R)]$, otherwise. Existence of an equilibrium with all agents on the short side matched necessitates that the long side population be sufficiently large relative to the short side population.

When $M \neq N$ and $P \in [\underline{P},\overline{P}]$, we can solve (11) for the equilibrium wage:

$$\tilde{w}(M,N,P) = \frac{s^R - A(P,M)f(e^R) + A(P,N)v(e^R)}{A(P,N) - A(P,M)} \quad (13)$$

<u>Proposition 3</u>: When $M > N$,

a) If $P = \underline{P}$, then $\tilde{w}(M,N,P) = f(e^R)$, $\pi^0 = 0$, and $u^0 = s^R$.

b) If $P = \overline{P} < N$, then $\tilde{w}(M,N,P) = v(e^R)$, $\pi^0 = s^R$, and $u^0 = 0$.

Likewise, when $N > M$,

c) If $P = \underline{P}$, then $\tilde{w}(M,N,P) = v(e^R)$, $\pi^0 = s^R$, and $u^0 = 0$.

d) If $P = \overline{P} < M$, then $\tilde{w}(M,N,P) = f(e^R)$, $\pi^0 = 0$, and $u^0 = s^R$.

Proposition 3 says that in the equilibrium with minimal match population, agents on the short side capture all the social product from a match, $f(e^R) - v(e^R)$. Also, when the equilibrium with maximal match population entails some unmatched agents on the short side, agents on the long side capture all the social product from a match.

The wage given by (13) is monotonic in P. It turns out that the reservation utilities are also monotonic in P. The reservation utility of agents on the long side increase with P, because the direction of how the wage changes with P is favorable to agents on the long side and because the probability of entering a match increases with P. In addition, agents on the long side claim more of the surplus from a match as P increases, because by the S&S logic, agents on the short side claim less of the surplus as they become more able to form another match.

The reservation utility and the share of the surplus decline with P, for agents on the short side, by proposition 2. Using a "veil of ignorance" utility function, it is possible for the utility of agents on the short side to increase with P over some range, since increases in P raise the likelihood of being matched.[10] But agents on the short side prefer $P < \bar{P}$ to $P = \bar{P}$, because in the latter case their reservation utility is minimal and all the surplus must go to agents on the long side. Although proposition 2 suggests P can be taken as the utilitarian measure of the economy's performance, this measure is not Paretian. The comparative statics of the wage and the workers' reservation utilitiy are given in the following proposition.

<u>Proposition 4</u>: When $\underline{P} < P < \bar{P} \le N < M$: $\tilde{w}_M, \overset{0}{u}_M > 0$ while $\tilde{w}_N, \tilde{w}_P, \overset{0}{u}_N, \overset{0}{u}_P < 0$. When $\underline{P} < P < \bar{P} \le M < N$: $\tilde{w}_N, \tilde{w}_P, \overset{0}{u}_N, \overset{0}{u}_P > 0$ while $\tilde{w}_M, \overset{0}{u}_M < 0$.

At the contract level, the reservation utilities are parametric. Hence, any division of the surplus is possible, by proposition 1. But at the market level, where the reservation utilities are endogenous, in general there is a deterministic relationship between the division of the surplus and the match population, as shown in propositions 3 and 4. This is necessary when the pairing process determines the reservation utilities. This result contrasts sharply with the M&M result, where all renegotiation-proof equilibria entail $P = \min\{M,N\}$.

The explanation for these apparently contradictory results lies in the turnover probability rather than with the lag in matching. Fixing the production interval and the number of periods per production interval, the set of equilibria for our model approximately replicates the set of renegotiation

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[10] The veil of ignorance utility function for firms is $P[f(e^R) - \tilde{w}]/M$ and for workers is $P[\tilde{w} - v(e^R)]/N$. Note that $P/H > A(P,H)$ when $H > P$ and $P/H = A(P,H)$ when $H = P$.

proof equilibria for the M&M model, when the turnover probability becomes small. In contrast, fixing the production interval and the turnover probability, there are equilibria where a significant fraction of the short side is unmatched, as the number of periods per production interval increase indefinitely.

Proposition 5: Fix ρ, r, and T. Suppose $g \to 0$. When $M > N$:

a) $\underline{P} \to N$.

b) In the equilibrium with $P = \overline{P}$, $\pi^0 \to 0$.

Likewise, when $N > M$.

c) $\underline{P} \to M$.

d) In the equilibrium with $P = \overline{P}$, $u^0 \to 0$.

Proposition 5 shows that as the turnover probability nears 0, only a small fraction of the agents on the short side remain unmatched and, in conjunction with proposition 4, shows that the reservation utility of agents on the short side also nears 0, in *any* renegotiation proof equilibrium. Thus, while propositions 3 and 4 demonstrate a linkage between the division of the surplus at the match level and the size of the match population in aggregate, this relationship gets less dramatic as the turnover probability shrinks. In a world with no exogenous turnover, the linkage vanishes and the M&M conclusion is obtained exactly.

We consider the effect of shortening the period while fixing the production interval, i.e., we hold both g and r fixed, which means e^R and s^R are unchanged, but we let $T \to \infty$ which implies $\rho \to 0$, since $(1 + \rho)^T = 1 + r$.

Proposition 6: Hold g and r fixed. Let $T \to \infty$. Then $\lim_{T\to\infty} \underline{P} < \min \{M,N\}$.

In the limit, we have a continuous time model. The time to pair is an expoential random variable. This random variable will have nondegenerate support, even for agents on ths short side, as long as the equilibrium entails $P < \overline{P}$.

The proofs of propositions 5 and 6 point out that the issue is whether $A(P,H)$ converges to 0 when $P < H$. This convergence occurs when the turnover probability gets small but not when the period length gets small.

3. The Heterogenous Firm Performance-Matching Model

We alter the model by assuming firms can be ranked on the basis of their productivity. Letting $f(e,i)$ denote the output of the ith firm when its employee takes effort e, assume $f(\cdot,i)$ satisfies all the restrictions we previously imposed and, in addition, f is jointly differentiable in (e,i) and $f_e(e,i) > f_e(e,j)$ for all $e > 0$ whenever $i < j$.[11] We continue to assume workers are identical.

Our goal is to better relate the S&S model to the M&M model. It is natural to identify positions in the S&S model with firms in the M&M model, since in the S&S model firms are able to employ many workers. The correct analogy requires firms to have differing productivities in the M&M model, since in the S&S model firms operate under diminishing marginal productivity. Then, the determination of the reservation utilities and how the surplus from a match is divided are more interesting, because the number of active firms is endogenous.

The task here is to describe how the reservation utilities are determined and to ascertain how many matches occur in market equilibrium. Since workers are identical, all workers will have the same reservation utility. But since the product generated by a match will vary from firm to firm, their reservation utilities will be firm specific. The pairing process

[11] Since $f(0,i) = 0$ for all i, it follows that $f(e,i) > f(e,j)$ for all $e > 0$, whenever $i < j$.

here is similar to the one in the homogeneous firm model, but is subject to the following caveats. A firm is inactive when, given the workers' reservation utility, it is not able to generate enough surplus to sustain a match. The reservation utility of an inactive firm is 0. Only active firms participate in the pairing process. Also, when in a match, a worker can determine the identity of her partner firm, but she cannot determine the identity of other active firms nor could she determine the identity of other active firms if she were unmatched.[12] These firms appear identical from afar.

Let e_i^R solve:

$$(9') \qquad \underset{e \geq 0}{\text{maximize}} \quad f(e,i) - (1 + r)v(e)/(1 - g).$$

Let s_i^R equal the value of the objective in (9') when evaluated at e_i^R. Note that e_i^R and s_i^R are declining in i. Hence, when firm i is active firm i' is also active, for all i' < i.

Let Q denote the active firm population. Then, a(P,Q) is the match probability for an unmatched active firm. We define a market equilibrium by a match population, an active firm population, a contract for each active firm, the reservation utility for workers, and a reservation utility for each active firm. Let $E = \left(P, Q, (\tilde{z}_i)_{i \leq Q}, u^0, (\pi_i^0)_{i \leq Q}\right)$ be a market equilibrium.

<u>Definition 2</u>: E is renegotiation proof if:

i) $\tilde{z}_i$ satisfies (3), (5), and (7) when the production function is $f(\cdot,i)$ and $\pi^0 = \pi_i^0$, for all $i \leq Q$;

ii) When Q < M, $u^0 > s_i^R$ for all i > Q;

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[12] This approach is consistent with the frictions we modelled in the case of homogeneous firms. At issue is whether, under the direct contact method, a worker can choose her partner, when there are unmatched firms. We assume the worker gets a random draw over firm types, even under direct contact. Otherwise, the worker would always approach the most productive unmatched firm. This would further bid up the worker's reservation utility, reduce the population of active firms and, in effect, force the equilibrium where all active firms are matched.

iii) $u^0 = A(P,N) \int_{i \leq Q} [\tilde{w}_i - v(\tilde{e}_i)]/Q \, di;$

iv) $\pi_i^0 = A(P,Q)[f(\tilde{e}_i, i) - \tilde{w}_i]$ for all $i \leq Q$;

v) if $P < Q$ and $Q^R \equiv \{i: s_i^R - \pi_i^0 > 0\} \neq \emptyset$,

$$u^0 \geq \int_{i \in Q^R} [\tilde{w}_i - v(\tilde{e}_i)]/Q \, di, \quad \text{where}$$

$$(\tilde{w}_i, \tilde{e}_i) = \operatorname*{argmax}_{e,w} \; w - v(e) \text{ subject to:}$$

$$\frac{(1 - g)[f_i(e) - w - \pi_i^0]}{g + r} \geq v(e); \text{ and}$$

vi) if $P < N$ and $u^0 < s_i^R$, $\pi_i^0 \geq \underset{e,w}{\text{maximum}} \; f_i(e) - w$

$$\text{subject to: } \frac{(1 - g)[w - v(e) - u^0]}{g + r} \geq v(e).$$

Definition 2 is analagous to definition 1. The points that are specific to the heterogeneous case are these: Condition (ii) requires firms to be inactive if they can't generate enough surplus. In (iii), an unmatched worker believes it equally likely to be matched with any active firm. In order to make sense of (v) for the case where $\mu(Q^R) < Q$, it is necessary to assume that a worker cannot directly contact more than one unmatched firm per period.[13] We demonstrate, in proposition 7, there is no incentive to contact more than one firm. We also demonstrate that when there are unmatched active firms but all workers are matched in equilibrium, these unmatched firms have no incentive to bid away employees from their less productive rivals.

[13] $\mu(\cdot)$ refers to Lesbegue measure.

<u>Proposition 7</u>: In any renegotiation proof market equilibrium where $P < N$ or $Q < M$ or $M \neq N$: $u^0 + \pi_i^0 = s_i^R$ and $\bar{e}_i = e_i^R$, for all $i \leq Q$, except for a set of Lebesgue measure 0.

<u>Note</u>: Though proposition 7 is clearly analagous to proposition 2, proposition 7 is not an invariance result, because the measure of active firms, Q, is endogenous. Below we show there are equilibria with different values of Q. The sum of the reservation utilities varies at those firms which are active in one equilibrium but are inactive in the other equilibrium.

We turn to computing renegotiation-proof equilibria, following a similar procedure to the one we used in the homogeneous firm case. A necessary condition for existence is that:

$$(12') \qquad A(P,N) \int_{i\leq Q} [f(e_i^R,i) - v(e_i^R)]\, di \leq \int_{i\leq Q} s_i^R\, di$$

$$\leq A(P,Q) \int_{i\leq Q} [f(e_i^R,i) - v(e_i^R)]di,$$

when $Q < N$. After some manipulations we get:

$$(14) \qquad u^0 = \frac{A(P,N)}{A(P,Q) - A(P,N)} \int_{i\leq Q} \frac{A(P,Q)[f(e_i^R,i) - v(e_i^R)] - s_i^R}{Q}\, di.$$

We focus on equilibria where $Q < M$, in which case it is necessary that:

$$(15) \qquad s_Q^R = u^0.$$

When there is a pair, (P,Q), that satisfies (12') with $0 < P \leq Q < N$, (14) gives a nonnegative value of u^0. When (15) is also satisfied, we have a candidate for equilibrium. Then, proposition 7 yields $\pi_i^0 = s_i^R - s_Q^R$ and condition (iv) of definition 2 requires:

$$(16) \qquad \tilde{w}_i = f(e_i^R, i) - \pi_i^0/A(P,Q),$$

for all $i \le Q$. Since $A(P,Q) \le 1$, $\tilde{w}_i$ satisfies individual rationality for firm i. Then, the surplus that goes to a worker employed by firm i is:

$$(17) \qquad \tilde{w}_i - v(e_i^R) - u^0 = f(e_i^R, i) - v(e_i^R) - s_i^R/A(P,Q) + s_Q^R/A(P,Q) - u^0.$$

When $A(P,Q) = 1$, the right hand side of (17) is decreasing in i and is positive at $i = Q$. Thus, there exists $\varepsilon > 0$ such that the worker's surplus at firm i is positive and is decreasing in i when $A(P,Q) > 1 - \varepsilon$. Finally, knowing u^0, π_i^0, and $\tilde{w}_i$, the remaining components of the contract, $\tilde{z}_i$, can be computed using either the NSC or the NEC in equality form.

<u>Lemma 1</u>: There exists $Q^* \in (0,N)$ such that for all $Q \in [Q^*,N)$ there is a unique $P(Q)$, $0 < P(Q) \le Q$, such that (12'), (14), and (15) are satisfied. Moreover, $P'(Q) < 0$ and $P(Q^*) = Q^*$.

This result, in conjunction with the discussion preceding lemma 1, implies the following proposition.

<u>Proposition 8</u>: An efficiency wage equilibrium exists. Partial bonding equilibria in a neighborhood of the efficiency wage equilibrium also exist. These partial bonding equilibria entail more active firms and fewer matches than does the efficiency wage equilibrium.

Partial bonding equilibria have two forms of inefficiency. First, there is underemployment. Second, there is poor match quality, by which we mean there is a positive probability firm i will be matched and firm i' will not be matched, when i' < i. Bonding exacerbates both problems. Among all equilibria where the active firm population is less than the worker population, the efficiency wage equilibrium maximizes the match population! Moreover, the efficiency wage equilibrium entails no deterioration in match quality, since all active firms are matched. But as in the homogeneous firm case, there is not unanimity as to which is the preferred equilibrium. Certainly near marginal firms prefer bonding equilibria, where their reservation utility is positive, to the efficiency wage equilibrium, where their reservation utility is 0.

4. Conclusion

Our modelling approach requires a lag in matching, even for the short side of the market, in order to not rule out a priori the possibility that agents on the short side capture some of the surplus from a match. As we have shown in proposition 5, this assumption may be viewed as a technical device whereby the M&M conclusions can be reconciled with exogenous turnover. However, we do not wish to leave the reader with the impression that this is the sole purpose for taking our approach. We believe the friction in matching is important for real labor markets. Then, our model provides a basis for linking the division of the surplus at the contract level with the behavior of the market in aggregate. Indeed, when such frictions are present, any market characterized by performance moral hazard may exhibit the type of multiple equilibria we have exhibited in our model. Our view is that many markets exhibit this feature.

Though in the section 3 we focused on heterogeneous firms and homogeneous workers, the insights carry over to the case of homogeneous firms and heterogeneous workers, who differ in their marginal rate of substitution between effort and income. Then, the utilitarian ideal is achieved by full bonding contracts. Heterogeneity on both sides may explain performance contracts where the surplus is split, even if the market does maximize a utilitarian social welfare function. Moreover, such heterogeneity obliterates the second invariance result.

Finally, we think it worth pondering whether our approach provides a coherent rationale for government interference in the labor market. Intuitively, government policy should be viewed as an instrument that can steer the beliefs of agents. In so doing, such policy affects equilibrium selection and thereby may be able to move the labor market allocation closer to the utilitarian ideal. For example, suppose a state-imposed minimum wage bids up the reservation utility of workers and, hence, bids down the reservation utility of firms. Our approach suggests this creates jobs when workers are long. Since this view of the minimum wage is so at odds with the traditional approach, it seems worthy of closer examination. Similar consideration should be given to the role of unemployment insurance and other kindred policies.

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Appendix

<u>Proof of Proposition 1</u>: Set $\tilde{b}$ so (3) holds as an equality, given $\tilde{w}$ and $\tilde{\sigma}$. Then, subtract (3) from (6) to get (5). To construct $\hat{z}$, set $\hat{\sigma} = 0$, $\hat{w} = \tilde{w} + g\tilde{\sigma}/(1 + r)$, and $\hat{b} = \tilde{b} - g\tilde{\sigma}/(1 + r)$. Since $(1 + r)\tilde{w} + g\tilde{\sigma} = (1 + r)\hat{w} + g\hat{\sigma}$, $U(\tilde{z}) = U(\hat{z})$ and $\Pi(\tilde{z}) = \Pi(\hat{z})$. Hence, $\hat{z}$ also satisfies (3) and (5). •

<u>Proof of Proposition 2</u>: Suppose $u^0 + \pi^0 < s^R$. Then, the contract $(v(e^R), s^R - \pi^0 + v(e^R), e^R, 0)$ yields a surplus for the worker, because $u^0 + \pi^0 < s^R$. It satisfies (3), since the bond alone is enough to deter worker malfeasance. When operating under this contract, a firm earns surplus equal to $f(e^R) - v(e^R) - s^R$ which, by the definition of e^R and s^R, is positive and large enough that (5) is satisfied. Thus, the contract satisfies (7) and gives a surplus to both parties. The existence of this contract demonstrates that condition (iv) of definition 1 isn't satisfied when $P < M$ and condition (v) isn't satisfied when $P < N$.

Alternatively, suppose $u^0 + \pi^0 > s^R$. Then, from the definition of s^R, there is no effort level that satisfies (6), including the effort specified in the market equilibrium contract. Thus, $\tilde{z}$ doesn't satisfy (3) or (5). This violates condition (i) of definition 1.

Since $u^0 + \pi^0 = s^R$, it must be that $\tilde{e} = e^R$, as this is the only effort level that can satisfy (6). •

<u>Proof of Proposition 3</u>: We consider the case where $M > N$. The case where $N > M$ is handled similarly. Evaluating (13) at $P = \underline{P}$ yields

$$\tilde{w}(M,N,\underline{P}) = \frac{s^R - A(\underline{P},M)f(e^R) + A(\underline{P},N)v(e^R)}{A(\underline{P},N) - A(\underline{P},M)}$$

$$= \frac{s^R - A(\underline{P},N)[f(e^R) - v(e^R)] + [A(\underline{P},N) - A(\underline{P},M)]f(e^R)}{A(\underline{P},N) - A(\underline{P},M)}$$

$$= f(e^R).$$

Then, condition (iii) of definition 1 implies $\pi^0 = 0$. Proposition 2 then requires $u^0 = s^R$. This shows (a).

Evaluating (13) at $P = \bar{P}$ yields

$$\tilde{w}(M,N,\bar{P}) = \frac{s^R - A(\bar{P},M)f(e^R) + A(\bar{P},N)v(e^R)}{A(\bar{P},N) - A(\bar{P},M)}$$

$$= \frac{s^R - A(\bar{P},M)[f(e^R) - v(e^R)] + [A(\bar{P},N) - A(\bar{P},M)]v(e^R)}{A(\bar{P},N) - A(\underline{P},M)}$$

$$= v(e^R).$$

Then, condition (ii) of definition 1 implies $u^0 = 0$. Proposition 2 then requires $\pi^0 = s^R$. This shows (b). •

<u>Proof of Proposition 4</u>: From (13), $\tilde{w}_M = \dfrac{A_H(P,M)\{s^R - A(P,N)[f(e^R) - v(e^R)]\}}{[A(P,N) - A(P,M)]^2}$. Since $A_H(P,M) < 0$, sign $\tilde{w}_M$ = - sign $\{s^R - A(P,N)[f(e^R) - v(e^R)]\}$. When $\underline{P} < P < \bar{P}$ the inequalities in (12) are strict, if $M > N$, while the reverse inequalities are strict, if $M < N$. Thus, $\tilde{w}_M > 0$ when $M > N$ and $\tilde{w}_M < 0$ when $M < N$. Then, sign u^0_M = sign $\tilde{w}_M$, since $u^0 = A(P,N)[\tilde{w} - v(e^R)]$.

Similarly, $\tilde{w}_N = \dfrac{A_H(P,N)\{A(P,M)[f(e^R) - v(e^R)] - s^R\}}{[A(P,N) - A(P,M)]^2}$ and sign $\tilde{w}_N$ = - sign $\{A(P,M)[f(e^R) - v(e^R)] - s^R\}$. Thus, $\tilde{w}_N > 0$ when $M < N$ and $\tilde{w}_N < 0$ when $M > N$. Then, sign π^0_N = - sign $\tilde{w}_N$, since $\pi^0 = A(P,M)[f(e^R) - \tilde{w}]$. Hence, sign u^0_N = sign $\tilde{w}_N$, by proposition 2.

Finally, $\tilde{w}_P = \dfrac{A_P(P,M)\{s^R - A(P,N)[f(e^R) - v(e^R)]\}}{[A(P,N) - A(P,M)]^2} - \dfrac{A_P(P,N)\{A(P,M)[f(e^R) - v(e^R)] - s^R\}}{[A(P,N) - A(P,M)]^2}$. Since $A_P(P,M)$, $A_P(P,N) > 0$ and since sign $\{s^R - A(P,N)[f(e^R) - v(e^R)]\}$ = - sign $\{A(P,M)[f(e^R) - v(e^R)] - s^R\}$, $\tilde{w}_P > 0$

when $M < N$ and $\tilde{w}_P < 0$ when $M > N$. Then, $u_P^0 > 0$ when $M < N$, since $u^0 = A(P,N)[\tilde{w} - v(e^R)]$. Likewise, $\pi_P^0 > 0$ when $M > N$, since $\pi^0 = A(P,M)[f(e^R) - \tilde{w}]$. But then, $u_P^0 < 0$ when $M > N$, by proposition 2. •

Proof of Proposition 5: We consider the case where $M > N$. The case where $N > M$ is handled similarly. By definition:

$$A(\underline{P},N) = \frac{s^R}{f(e^R) - v(e^R)} = 1 - \frac{[r + g]v(e^R)}{[1 - g][f(e^R) - v(e^R)]},$$

where $e^R = \underset{e}{\operatorname{argmax}}\ f(e) - (1 + r)v(e)/(1 - g)$. By our assumptions on f and v, $0 < \lim_{g\to 0} e^R < e^*$ and, consequently, $0 < \lim_{g\to 0} \frac{[r + g]v(e^R)}{[1 - g][f(e^R) - v(e^R)]} < 1$, by the srict concavity of f - g. But $A(\underline{P},N) = \frac{g\underline{P}[1 + \rho]}{g\underline{P}[1 + \rho] + \rho[1 + g/r][N - \underline{P}]T}$

The numerator converges to 0 as g goes to 0. When $N > \underline{P}$, the denominator converges to some positive value. Thus, $\lim_{g\to 0} \underline{P} = N$. This shows (a).

$$\text{Similarly, } A(\overline{P},M) = \frac{g\overline{P}[1 + \rho]}{g\overline{P}[1 + \rho] + [1 + g/r][M - \overline{P}]T}.$$ Since $\overline{P} \le N < M$, $\lim_{g\to 0} A(\overline{P},M) = 0$. Hence, $\lim_{g\to 0} \pi^0 = 0$, because $\pi^0 \le A(\overline{P},M)f(e^R)$. This shows (b). •

Proof of Proposition 6: We consider the case where $M > N$. Since only the period duration changes and $A(\underline{P},N) = \frac{s^R}{f(e^R) - v(e^R)}$, $A(\underline{P},N)$ is constant in T. But since $A(\underline{P},N) = \frac{g\underline{P}[1 + \rho]}{g\underline{P}[1 + \rho] + \rho[1 + g/r][N - \underline{P}]T}$, it must be that $\rho[N - \underline{P}]T/\underline{P} = k$ for some k constant in T, with $k > 0$ since $A(\underline{P},N) < 1$. Thus, $\underline{P} = \frac{\rho TN}{k + \rho T}$. Finally, $\lim_{T\to\infty} \rho T = \ln(1 + r)$. Thus, $\lim_{T\to\infty} \underline{P} < N$. •

Proof of Proposition 7: When $P < N$, the argument is the same as the one given in the proof of proposition 2. To recapitulate, it is not possible that $u^0 + \pi_i^0 > s_i^R$ for any $i \le Q$, because for such an i the set of feasible contracts would be null and, hence, condition (i) of definition 2 would not be satisified. But it is also not possible that $u^0 + \pi_i^0 < s_i^R$ for any $i \le Q$, because this violates condition (vi) of definition 2.

When $N = P \le Q \le M$, it is not possible that $N = P = Q < M$. Were this the case, $a(P,N) = a(P,Q) = 1$. But then by conditions (iii) and (iv) of definition 2, neither workers nor active firms earn a surplus in equilibrium. This is only possible if every contract entails zero effort, in which case workers and active firms have zero reservation utility. This violates condition (ii) of definition 2.

When $N = P < Q$ and $i \in Q^R$, $\hat{w}_i - v(\hat{e}_i) = s_i^R - \pi_i^0$. Then, from condition (v) of definition 2 we have:

$$\int_{j \le Q} [u^0 + \pi_j^0]\, dj \ge \int_{j \le Q} \left[\int_{i \in Q^R} [\hat{w}_i - v(\hat{e}_i)]/Q \, di + \pi_j^0 \right] dj$$

$$= \int_{i \notin Q^R} \pi_i^0 \, di + \int_{i \in Q^R} [\pi_i^0 + \hat{w}_i - v(\hat{e}_i)]\, di \ge \int_{i \le Q} s_i^R \, di.$$

But, it is still not possible that $u^0 + \pi_i^0 > s_i^R$ for any $i \le Q$.

Thus, $u^0 + \pi_i^0 = s_i^R$ for all $i \le Q$, except on a set of Lebesgue measure 0, and $\tilde{e}_i = e_i^R$ whenever $u^0 + \pi_i^0 = s_i^R$, since e_i^R is the unique feasible effort for firm i in that case. $\bullet$

<u>Proof of Lemma 1</u>: Let $I(P,Q) = \int_{i \le Q} \dfrac{A(P,Q)[f_i(e_i^R) - v(e_i^R)] - s_i^R}{Q}$, $J(N,P,Q) = \dfrac{A(P,N)}{A(P,Q) - A(P,N)}$, and $K(N,P,Q) = J(N,P,Q)I(P,Q)$. $I(0,Q) < 0$ when $Q > 0$, since $A(0,Q) = 0$ when $Q > 0$. $I(P,Q)$ is continuous in P for all $P \in [0,Q]$, and for all $Q > 0$. $I(Q,Q) > f(e_Q^R,Q) - v(e_Q^R) - s_Q^R$, since $A(Q,Q) = 1$ and since $f(e_i^R,i) - v(e_i^R) - s_i^R$ is declining in i. But $J(N,P,Q)$ is positive and coninuous in P

for $0 < P \leq Q$, when $Q < N$, and approaches ∞ as Q approaches N. Thus, for Q less than but near N, $K(N,P,Q)$ is less than s_Q^R, when P is small and is greater than s_Q^R when $P = Q$. Then, existence of $P(Q)$ follows from the intermediate value theorem.

Since (12') must be satisifed, $K(N,P,Q)$ is increasing in P in a neighborhood of the solution, by the same argument as in the proof of proposition 3. Thus the solution is unique.

When $P(Q) < Q < N$, K is differentiable at $(N,P(Q),Q)$ and s_i^R is differentiable at $i = Q$. Thus $P(Q)$ is differentiable and:

$$P'(Q) = \left[\frac{ds_Q^R}{dQ} - K_Q(N,P(Q),Q)\right] / K_P(N,P(Q),Q).$$

We have already argued that $K_P(N,P(Q),Q) > 0$ when $Q < N$ and, for similar reasons, $K_Q(N,P(Q),Q) > 0$ as well. Thus, $P'(Q) < 0$.

Let $Q^* = \inf \{Q: (P(Q),Q)$ satisfies (14) and (15)$\}$. Thus, $(P(Q^*),Q^*)$ satisfies (14) and (15) and $P(Q^*) = Q^*$. •

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