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Mathematical Instruction in France.

By
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XII.—*Mathematical Instruction in France.*

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(Presented by Dr. E. Deville and read in abstract, 28 September, 1910.)

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For Later Publication.

Introductory.

As the result of remarkable progress during the past fifteen years, a vigorous American School of Mathematics, of which the German School may well be considered the parent, has been developing. Many years must elapse before the offspring may exercise the authority and influence made felt by such masters as Gauss, Riemann, Steiner, Weierstrass and Klein. But meanwhile the process of evolution is proceeding in thorough fashion. In preparing for higher mathematical education, America has recognized the fundamental importance of the secondary; organization, discussion, and criticism of home methods as well as study of those of foreign countries, have been widespread in recent years. But here, again, the preponderance of discussion in book and periodical is of German methods. The series of reports of the Carnegie Foundation for the Advancement of Teaching is doing considerable to spread accurate information of a more general character.

Yet in spite of the predominance of German influence in the discussion, I have become convinced, after several months of observation, that just as much might be beneficially acquired by the study of mathematics and methods of mathematical training in France, as in any other country. Has not this country produced Chasles, Monge, Poncelet, Cauchy, LaPlace, Hermite? What city beside Paris has to-day such a large number of mathematicians of the first order? There are Poincaré, Darboux, Goursat, Picard, Painlevé, Appell, Jordan, Humbert, Borel, Tannery, to mention only a few. What other country gives such a course of mathematical training as is provided in the *Classes de Mathématiques Spéciales* of the French Lycées? Where else is the extraordinarily high standard of the *agrégation* demanded of higher teachers in the secondary schools?

Nevertheless when leaving Harvard some ten years ago with a view to further mathematical study in Europe, and although more or less familiar with such classic treatises as those of Darboux, Picard, Tannery, Appell and Goursat, I scarcely even considered France, as a possible place of location. The professors at Harvard who had studied abroad had been trained in Germany and were thoroughly imbued with German methods and ideals. The same was doubtless true of at least ninety-five per cent of the mathematical professors in the larger American colleges, and the same may be said to-day—Why this neglect of France?

For one thing the American student usually looks forward to getting a doctor's degree in one or two years, while the idea is certainly prevalent among us that if the French degree of doctor were at all available for the foreigner, he could only expect to get it at the age of forty-five or fifty, and after writing some monumental or epoch-making treatise. Such ignorance is without doubt due in part to the excessive difficulty of getting any reliable information about French as compared with German universities. Even when the inquirer is in Paris the difficulty does not wholly disappear.

It would seem then, that a service may be rendered to students and university professors who have in prospect a sojourn in Europe for mathematical study, if I should present a general view of the situation in France, along with fuller details on topics which must be of especial interest to every mathematician.

The plan of the paper is indicated by the table of contents.

General Remarks—Educational System and Primary Instruction.

To more thoroughly understand the methods and ends of mathematical instruction in France it will be well to introduce here some brief general remarks.

For educational purposes France is divided geographically into *arrondissements.* The assemblage of government schools (primary, secondary and superior) in each arrondissement, forms an *académie* over which a *recteur* presides. We thus have the 16 académies of Aix-Marseilles, Besançon, Bordeaux, Caen, Chambéry, Clermont, Dijon, Grenoble, Lille, Lyons, Montpellier, Nancy, Paris, Poitiers, Rennes, Toulouse, as well as a seventeenth at Algiers. With the exception of Chambéry these names correspond to the seats of the French universities, which have from two to four faculties (law, science, letters, medicine) each, the faculties of science and letters together corresponding to the German *philosophische Facultät.* In the académie first named above, the faculties of law and letters are at Aix and the faculties of science and medicine at Marseilles.

The assemblage of académies forms the *Université de France*, at the head of which is the Minister of Public Instruction, who is *ex officio* the "Recteur de l'Académie de Paris et Grand Maître de l'Université de Paris." For the Académie de Paris there is a vice-recteur, whose duties are the same as those of the recteurs of other académies. Although nominally lower in rank than the heads of académies in the provinces, he is in reality, the most powerful official in the educational system. The position of the Minister of Public Instruction being so insecure by reason of changing governments, continuity of scheme is assured by three lieutenants who have charge respectively of the primary, secondary and superior education. They in turn have an army of inspectors who report on the work and capabilities of the recteurs and their académies as far as primary and secondary instruction is concerned.

This suffices at present to indicate the remarkably centralized and unique character of the French educational system. It is theoretically possible for the most radical changes in any part of public instruction to be immediately brought about by a stroke of the pen on the part of the Minister of Public Instruction.

In the recuperation of the French nation during the past 40 years, gigantic strides have been made in all departments of education; scores of handsome and spacious new buildings have been erected, new chairs have been endowed, new laboratories established and equipped—while in connection with special schools all over the country, scholarships and prizes call forth and reward the best effort of the nation's youth. Forty years ago the state spent 32 million francs for education. The 1909 budget of the Minister of Public Instruction and Fine Arts called for 293 millions—nearly two-thirds of this amount being allotted to *Primary Instruction.*

Primary and superior instruction are free in France and over five million children are now annually in attendance at the public primary schools. Broadly speaking there are three classes of these schools which give strictly elementary education:—

A.—*Ecoles Maternelles.* A sort of kindergarten for children of both sexes from 2 to 5 or 6 years old.

B.—*Ecoles Primaires Elémentaires* for pupils 7 to 13 years of age. The course is divided as follows:—

		Age
Section Enfantine..		5 or 6-7
Cours Élémentaire........................	Ière Année........	7–8
	IIe "	8–9
Cours Moyen...........................	Ière "	9–10
	IIe "	10–11
Cours Supérieur........................	Ière "	11–12
	IIe "	12–13

On completion of the cours moyen the pupil receives a *certificat d'études primaires élémentaires.* This certificate or its equivalent is required of every child in France. A very small proportion of those receiving it take up further work in the cours supérieur, in the lycées or in

C.—*Ecoles Primaires Supérieures.* These are for children of the labouring class who do not aspire to a classical education, but who wish to prepare themselves for industrial, commercial or agricultural careers.

In what follows it will be supposed that the discussion is limited to the education of boys.

ELEMENTARY MATHEMATICAL INSTRUCTION.

The Lycées.

The present system of secondary education in France dates from the great reform of 1902 and is carried on for the most part in *Lycées* and *Collèges communaux* which are to be found in nearly all cities. Because of their pre-eminence we shall consider the former only, which are under control of the state. Here the boys, who come from families in comfortable circumstances, may enter as *élèves* at the age of five or six years and be led along in their studies till they receive the *Baccalauréat* at the age of 16 or 17. Many lycées have still more advanced courses to prepare for entrance into such schools as the École Normale Supérieure, École Polytechnique, École Centrale, École Navale, École de Saint Cyr, etc. The pupils at the lycées are of four kinds: 1st.—*Externes,* those who come to the lycées for classes but board and lodge outside; 2nd.—*Internes* or *pensionnaires,* élèves who live entirely in the establishment; 3rd.—*Demi-pensionnaires* who usually reside at a distance but take their mid-day meal at the lycée; 4th.—*Externes surveillés,* that is externes who work out their lessons under the eye of the *préparateur* in the *salle d'étude* of the lycée. In the whole of France rather less than one third of the lycée pupils are internes. At lycée Louis le Grand, Paris, in 1907, 275 of the 909 élèves were internes; on the other hand at the Saint Louis, which is quite near, there were 504 internes out of a registration of 854.

The expenses of the pupil vary greatly with the class and the lycée in which he happens to be. The following table exhibits the range of cost (in francs per year), for some of the principal cities of the provinces and for the better lycées of Paris.

	Bordeaux, Lyons, Marseilles, Toulouse.	Paris.
Externes	70–450	90–700
Externes Surveillés	110–540	130–790
Demi-pensionnaires	370–850	500–1200
Pensionnaires	700–1200	900–1700

The lower price in each case is for the classe enfantine, the higher for the special classes open to *bacheliers.*

Instruction in fully equipped lycées may be divided into four sections:—I, Primary; II, Premier Cycle; III, Second Cycle; IV, *Classes de Mathématiques Spéciales.*

I.—*Primary.* The classes in this section are named as follows:—

		Age from
Classes Enfantine	Onzième	5
Preparatory Division	Dixième	6
	Neuvième	7
Elementary Division	Huitième	8
	Septième	9

In a general way this course corresponds to that which leads to the *certificat d'études primaires élémentaires,* but while the latter was designed as a more or less complete unit in itself, the former is laid out on broader lines and has in view further studies which the boy will follow up. According to the *plans d'études,* it would seem as if one essential difference were introduced by instruction in a modern language in the neuvième, huitième and septième. In reality, however, the modern language classes are so conducted in the sixième of the Premier Cycle that both kinds of students are taught together.

II.—*Premier Cycle* (*sixième-troisième*). This cycle of four years constitutes an advanced course for students who have finished their primary studies, and is the first part of secondary education proper. It offers a choice between two lines of study, the one characterised by instruction in Latin with or without Greek, the other in which no dead language is taught. The former is selected by the parent who wishes to prepare his boy for the department of letters in the École Normale Supérieure or for the career of classical professor, lawyer or doctor. The latter is likely to be chosen for the boy who is particularly interested in science or who has a commercial career in view.

III.—*Second Cycle.* This leads, normally, to the *Baccalauréat,* at the end of three years' study, in one of four different sections. The student of Latin and Greek in the quatrième and troisième has passed into the "Latin-Grec" section, the student of modern languages into the "Science-Langues vivantes" section, while this section as well as those of "Latin-Langues vivantes" and "Latin-Science" have been filled by students who have studied Latin (but not Greek), during the four years of the Premier Cycle. The scheme will be clearer in tabular form.

	Pupils who learn Latin, with or without Greek.			Pupils who learn no dead language	Age from
PREMIER CYCLE. 4 years	Sixième A (Latin).			Sixième B	10
	Cinquième A (Latin).			Cinquième B	11
	Quatrième A (Latin Greek)	Quatrième (Latin)		Quatrième B	12
	Troisième A (Latin Greek)	Troisième (Latin) →		Troisième B / Pupils who give up the study of Latin. ↓	13
	LATIN-GREC.	LATIN-LANGUES	LATIN-SCIENCES	SCIENCES-LANGUES	
SECOND CYCLE 3 years	Second A	Second B	Second C	Second D	14
	Première A	Première B	Première C	Première D	15
	Philosophie A	Philosophie B	Mathématiques A	Mathématiques D	16

Let us now observe a little more closely just what is involved in this display, in the matter of studies and demands made upon the élève. As an important examination which we shall presently describe comes at the end of the *Première*, our present analysis will not pass beyond this grade. Here is the programme for a week.

		French.	Latin.	Greek.	Modern Languages.	History and Geography.	Natural Science.	Physics and Chemistry.	Moral Philosophy.	Book-keeping.	Drawing.	Geometrical Drawing.	Arithmetic.	Arithmetic and Geometry.	Algebra and Geometry.	Writing.	Total No. Hours.
VI.	A.	3	7	..	5	3	1	..	..	..	2	..	2	..	..	..	23
	B.	5	..	..	5	3	2	..	..	..	2	1	3	..	..	1	22
V.	A.	3	7	..	5	3	1	..	..	..	2	..	2	..	..	..	23
	B.	5	..	..	5	3	2	..	..	..	2	1	..	3	..	1	22
IV.	A.	3	6	3*	5	3	1	..	1	..	2	..	..	2	..	..	23
	B.	5	..	..	5	3	..	2	1	1	2	1	..	3	..	..	23
III.	A.	4	6	3*	5	3	..	..	1	..	2	..	..	3	..	..	24
	B,	5	..	..	5	3	1	2	1	1	2	1	..	..	4	..	25

* Those who select Greek are exempted from 2 hours in modern languages and one in drawing.

		French.	Latin.	Greek.	Modern Languages.	Ancient History.	Modern History.	Geography.	Physics and Chemistry.	Practical Exs. in Science.	Geology.	Supplementary Exs. in Latin.	Drawing.	Geometrical Drawing.	Algebra and Geometry.	Geometry, Descriptive Geometry, Trig., Algebra.	Total No. Hours.
II.	A.	4	4	5	2	2	2	1	..	..	**	..	2	..	2	....	24
	B.	4	4	..	7	2	2	1	..	..	**	..	2	..	2	....	24
	C.	4	4	..	2	..	2	1	3	2	**	..	2	2	5	....	27
	D.	4	..	..	7	..	2	1	3	2	**	..	2	2	5	....	28
I.	A.	4	3	5	2	2	2	1	..	..	..	2	2†	..	2	....	23
	B.	4	3	..	7	2	2	1	..	..	..	..	2†	..	2	....	21
	C.	4	3	..	2	..	2	1	3	2	..	..	2	2	3	2	26
	D.	4	..	..	7	..	2	1	3	2	..	..	2	2	3	2	28

** 12 lectures of 1 hour.
† Optional.

There are several features of this scheme (we shall refer to four), which are particularly interesting.

1.—The prominence given to the study of French throughout.

2.—That all élèves at the age of 10 or 11 commence the study of modern languages (English, German, Italian, Spanish, Russian, or, in Algiers, Arabic), and continue it during six years at least, before matriculating into schools of university grade. Not only do they get glimpses of the best things in the literature of the language, but also learn to speak the language with considerable freedom and remarkable correctness in pronunciation. How many university graduates with us get a training which leads to this result? The direct method is employed and no word of French is ever spoken in the advanced classes. The majority of the élèves choose German, as this is required of all candidates for entrance into such military schools as the École Polytechnique and École de St. Cyr. On the other hand there is an increasing number taking up the study of English which is required for the École Navale.

3.—The small number of hours devoted to mathematics in the Premier Cycle, in II, A, B, and in I, A, B, of the Second. Yet there can be little doubt that the situation might be summed up in even stronger terms than in the recent report of the trustees of the Carnegie Foundation, when comparing the methods of American and German schools: "... lack of efficient teaching is one of the most expensive national weaknesses, and that the inefficiency of our school system is in great measure due to this lack is evident. For example, mathematics is a subject which has been a standard study in our schools from the beginning. Students who pass through our high schools and enter college spend in the nine years corresponding to the period covered by the German *gymnasium*, seventy-five per cent. more of time of instruction on mathematics and yet receive a training vastly inferior to that of the *gymnasium*." Practically all the *professeurs titulaires* in the French lycées, even those in charge of the very elementary classes, are *agrégés* in the subjects which they instruct. Just what this means in mathematics we shall explain later, but suffice it to remark here that even the demands made upon those who wish to become *gymnasien* professors are nothing like as severe.

Another feature of mathematical instruction which is particularly interesting to us is, that from the troisième on, that is, from the time the boy is 13 or 14 years old, instruction is given entirely by lecture. Indeed, even in classes before the troisième when a text book is generally in the hands of the élève, he is required to take notes "pour préciser" the various topics. By such methods, searching questioning and frequent "tests," on the part of the professor, and rigid inspection, kindly expressed praise or cutting public reprimand on the part of the *proviseur* (director of the lycée), there is no possibility of learning parrot-fashion, as is so prevalent in our schools—no room for the shirker or the boy who does not try his best; reasoning powers and independence of thought must be constantly exercised. The élèves are encouraged to consult the various text-books to be found in all the lycée libraries and for those less bright this may be almost a necessity from time to time; but on personal inspection in different lycées I found the note books of élèves of 14 or 15 alike remarkable for their neatness and completeness. The habits thus gained in the lycée stand in good stead when the student reaches the university. The rapidity of the lecturer and the complexity of his theme seem to make little difference, for at the close of the hour the whole is in the note books as neat as copper-plate. There is surely a lesson to be learned here for improving our secondary and superior education.

4.—The large number of hours in class recitation which may not at first appear very imposing. But we cannot fail to be astonished that 8 hours per day (in class and in preparation of lessons) may be demanded from élèves in the premier cycle, and $10\frac{1}{2}$ in summer, 10 in winter from those in the second cycle. The law further explicitly states that there is no limit to the number of hours which may be demanded of the élèves in the *Classe de Mathématiques Spéciales.* When we later come to look more closely at their programme we shall not be surprised, but nevertheless wonder, how these undoubtedly happy and healthy young men of 17 or 18 have survived the treatment. In more advanced lycée courses as well as at the universities I was also impressed with the almost appalling intensity and seriousness of the auditors—the strife is too strenuous, the competition too keen, to admit of a moment's levity or wandering thought. But when the lesson is over, every care is instantly banished and the national gaiety is once more in evidence.

To return to our table. We remark that the two groups of élèves who elect sciences on entering the Second Cycle have the same number of hours per week in mathematics—indeed the courses are identical. To give greater definiteness to our ideas as to their general attainments let us consider the programme of studies for Première D, when the boy is 15 or 16 years old.

French.—Lectures and questions on the principal French writers of the nineteenth century. Study of selections from prose writers and poets, from moralists, orators, politicians, scientists and historians of the sixteenth, seventeenth, eighteenth and nineteenth centuries.

History.—Political history of Europe in the eighteenth century. Detailed history of France at the close of the eighteenth century.

Geography.—Detailed study of France, its geological constitution, its climatology, physiography, topography, economic and military organization; its colonies, etc.

Physics.—Optics, electricity.

Chemistry.—Of the carbon compounds.

German.—Selections from the dramatic poetry of Schiller, Goethe, Kleist and Grillparzer. Extracts from the prose works of Wieland, Goethe, Schiller, Auerbach, Freytag, Scheffel, etc.

English.—Shakespeare's Julius Caesar and Macbeth, extracts from Milton, Addison, Goldsmith, Wordsworth, Byron, Coleridge, Dickens, Macaulay, Eliot, Tennyson and Thackeray.

Algebra.—Equations and trinomials of the second degree. Calculation of the derivatives of simple functions; study of their variation and graphic representation; study of rectilinear motion by means of the theory of derivatives; velocity and acceleration; uniformly changing motion.

Geometry.—Solid.

Descriptive Geometry.—Elements.

Trigonometry.—Plane, including the use of four or five place logarithim tables, the solution of triangles and trignometric equations.

In passing it may be worth noting that the Latin course for Première A, B, C, includes the study of selections from Cicero's letters and orations, from Livy, Seneca, Tacitus, Lucretius, Virgil and from Horace's Satires and Epodes. The Greek course for Première A, considers extracts from Xenophon's Memorabilia, from Plato, Demosthenes, Homer, Æschylus, Sophocles, Euripides, Aristophanes, etc.

The Baccalauréat.

Having finished the Première, the élève presents himself for examination under conditions which once more emphasise the unity of the French educational system. This is the examination for the first part of the state degree known as the *Baccalauréat.*

A peculiar feature of this examination is that it is not held in the lycées but at the university of the académie to which the particular lycée belongs.[1] As various civil and practically all government positions, except those in post and telegraph offices are only open to *bacheliers,* the state introduces into the body of examiners some who are wholly independent of the lycées. These examiners are the professors in the universities.

Since our future mathematicians are to come from Première C and D we shall give a few particulars concerning their examination. All examinations for the baccalauréat are held in July and October—at the ending of one school year and the beginning of the next. The

[1] As there is no university at Chambéry, the candidate presents himself before a faculty of either Lyons or Grenoble.

examiners of the candidates from Première C are six in number, three of whom are university professors and three professors from the lycées or collèges; for Première D there are but two university professors in addition to three from the lycées. The examinations in all sections are both written and oral. Here is the scheme of examination which practically covers what the élève has studied in earlier years.

Première C (Latin-Sciences).—*Written.* 1st, a French composition (3 hours); (the candidate has a choice of three subjects); 2nd, a Latin translation (3 hours); 3rd, an examination in Mathematics and Physics (4 hours). *Oral.* (about three quarters of an hour). 1st, explanation of a Latin text; 2nd, explanation of a French text; 3rd, examination in a modern language—questions and answers being necessarily in this language. Questions in—4th, History; 5th, Geography; 6th, Mathematics; 7th, Physics; 8th, Chemistry.

Première D (Sciences-Langues Vivantes).—*Written.* 1st, a French composition (3 hours); 2nd, a composition in a modern language (3 hours); 3rd, examination in Mathematics and Physics (4 hours). *Oral* (about three-quarters of an hour). 1st, explanation of a French text; 2nd, two tests in modern languages, one of which must be either English or German. Questions in—3rd, History; 4th, Geography; 5th, Mathematics; 6th, Physics; 7th, Chemistry.

On registering at the secretary's office and paying the fifty francs necessary for the above examinations the student has the option of depositing his *livret scolaire* which contained a full record of his work in the lycée for two or three years previously. If the pupil thus shows a good record but fails to get the necessary fifty per cent on his written paper he is nevertheless admitted to the oral. Otherwise no élève who has not passed the written examination can present himself for the oral. On the other hand if he has passed the written examination, but failed at the oral, he may try another oral examination (within a year), without repeating the written part.

The searching character of the tests prepares us for a large number of failures. Here is the record for 1909.

	Number of Candidates		Number Admitted to Oral		Number Passed		Percentage Passed	
	July.	Oct.	July.	Oct.	July.	Oct.	July.	Oct.
Latin-Grec.	2506	1262	1423	759	1097	537	44	42
Latin-Langues Vivantes. .	3147	1683	1605	930	1293	708	41	42
Latin-Science	2717	1247	1619	729	1350	570	49	46
Science-Langues Vivantes.	4088	1860	2110	915	1741	731	42	39
Philosophie	5824	2639	3764	1911	3144	1572	54	59
Mathématiques.	3163	1128	1995	790	1762	642	56	57

We observe that less than fifty per cent. of the pupils get through on the first examination[1] while a similar percentage of the remainder fail and are required to return to the Première once more or to wait for another year.[2] Those who have been successful return to the lycée to prepare for the second part of the baccalauréat. A choice of two courses (which may be slightly varied), is open to them, the one *Philosophie A* or *B*, the other, *Mathématiques A* or *B*. We shall only refer to the latter which has been supplied with pupils from the Première C and D. There they had 26 and 28 recitation hours per week. This has now been increased to 27½ and 28½. There has been an increase in the number of hours devoted to mathematics, physics and chemistry, but a reduction in the amount of study of modern languages. Latin no longer enters. The programme for Mathématiques A is in outline as follows:—

Philosophy (3 hours).—I. Elements of Scientific Philosophy: introduction, science, method of mathematical sciences, method of the sciences of nature, method of moral and social sciences. II. Elements of moral and social philosophy.

History and Geography (3½ hours). *Modern Languages* (2 hours). *Physics and Chemistry* (5 hours). *Natural Science* (2 hours). *Practical Exercises in Science* (2 hours). *Drawing* (2 hours). *Hygiene* (12 lectures of 1 hour).

Mathematics. (8 hours):

Arithmetic.—Properties of integers; fractions; decimals; square roots; greatest common divisors; theory of errors; etc.

Algebra.—Positive and negative numbers, quadratic equations (without the theory of imaginaries), progressions, logarithms, interest and annuities, graphs—derivatives of a sum, product, quotient, square root of a function, of sin x, cos x, tan x, cot x. Application to the study of the variation and the maxima and minima, of some simple functions, in particular functions of the form

$$\frac{ax^2+bx+c}{a^1x^2+b^1x+c^1}\ ;\quad x^3+px+q$$

when the coefficients have numerical values—Derivative of the area of a curve regarded as function of the abcissa (the notion of area is assumed).[3]

Trigonometry.—Circular functions, solution of triangles, applications of trigonometry to various questions relative to land surveying.

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[1] For some it may have been the third or fourth trial.

[2] There are certain exceptional cases which I shall not consider.

[3] The following note is attached to the résumé in the official programme, "Le professeur laissera de côté toutes les questions subtiles que soulève une exposition rigoureuse de la théorie des dérivées; il aura surtout en vue les applications et ne craindra pas de faire appel à l'intuition."

Geometry.—Translation, rotation, symmetry, homology and similitude, solids, areas, volumes, poles and polars, inversion, stereographic projection, vectors, central projections, etc.

Conics.—Ellipse, hyperbola, parabola, plane sections of a cone or cylinder of revolution, etc.

Descriptive Geometry.—Rabatments—application to distances and angles—projection of a circle—sphere, cone, cylinder, planes, sections, shadows.—application to topographical maps, etc.

Kinematics.—Units of length and time. Rectilinear and curvilinear motion. Translation and rotation of a solid body. Geometric study of the helix, etc.

Dynamics and Statics.—Dynamics of a particle, forces applied to a solid body, simple machines in a state of repose and movement, etc.

Cosmography.—Celestial sphere, earth, sun, moon, planets, comets, stars—Co-ordinate Systems, Kepler's and Newton's laws, etc.

One of the most striking things in this scheme, as compared with American method, is to find arithmetic taught in the last year of the lycée course. Note too, that from the Cinquième on, it has been taken up in connection with instruction in geometry and algebra. Indeed, this method of constantly showing the interdependence or interrelation of the various mathematical subjects was one of the interesting and vaulable characteristics of French education as I observed it. For example, I happened to be present in a classroom when the theory and evaluation of repeating decimals was under discussion. After all the processes had been explained, problems which led similarly to the consideration of infinite series and limits were taken up. By suggestive questioning a pupil found the area under an arc of a semi-cubical parabola and the position of the centre of gravity of a spherical cap. With us it is not till the graduate school of the university that the boy is taught the true inwardness of such processes as long division and extraction of roots; but in France, arithmetic is taught as a science and the élève leaving the lycée has a comprehending and comprehensive grasp of all he has studied.

The increasing general interest in practical education is reflected in the French method of teaching geometry with frequent illustration involving discussion of the form or relation between the parts of objects met with in every day life. Rather curiously, the method employed in at least some German *gymnasien*, of demanding that a pupil demonstrate even the more complicated propositions of geometry without any reference to a figure on a blackboard, does not seem to obtain in France. Curiously, because there can be no doubt of the fine exercise of mental concentration required of the members of the class who first build up in imagination the construction as indicated by one of their number and afterwards follow or criticize his proof; moreover the average French boy could certainly soon become an expert in such mental gymnastics. We remark that most of the mathematical subjects mentioned above are more or less foreign to our secondary education. Instruction in geometrical conics (*courbes usuelles*), is infrequently given by us, even in universities. Again, the ordinary mathematical student who goes up for his doctor's degree in America may have the vaguest idea of what is even meant by Descriptive Geometery. True, it is a regular course for our training of the engineer; but not, unfortunately, of the mathematician. On the other hand the French mathematical student has had at least four years of Descriptive Geometry, two of them before receiving his baccalauréat. The subject is required for admission into many government schools.

We note that the idea of a derivative is familiar to the lycéen during the last two years of his course. Why we so generally shut out the introduction of such an idea into our first courses in analytical geometry and theory of equations is, to me, a mystery. Finally, I would remark that the classes in Mathématiques A last two hours, with the exception of five minutes for recreation at the end of the first hour. The professor thus has sufficient time to amplify and impress his instruction.

At the close of the last year of the Second Cycle, the élève takes the examination for the second part of the baccalauréat. The same general conditions prevail as for the first part. Under no conditions whatever can an élève try the second part till he has passed the first. The jury of four contains two university professors. The written examinations in mathematics, physics and philosophy are each three hours long; the oral covers what has been studied the year previously. If successful, a dipolma now called the *baccalauréat de l'enseignement secondaire*, is granted to the élève by the Minister of Public Instruction. The élève thus becomes a *bachelier*. The diplomas in all four sections are of the same scholastic value. The charge made for diploma and examination is 90 francs. By reference to the foregoing statistical table, it will be noticed that more than forty per cent. of the candidates failed to pass at each of the examinations in 1909.

Because of the similarity of title used in the different countries, the Frenchman does not generally understand what the title Bachelor of Arts implies nor is it easy to make any concise statement in explanation. Little exaggeration can be made, however, in placing the bachelier on a plane of scholastic equality with the Sophomore who has finished his year at one of the best American universities. Certain it is that the bachelier in Latin-Grec has done as much of the dead languages, philosophy and history as is required in the whole pass course of the ordinary

Canadian university. The same may be remarked of the bachelier of Latin-Sciences in modern languages and Latin. When it is further remembered that it is possible for the average Canadian boy to get his B.A. with small effort one inclines to place the baccalauréat, with its rigorous and impartial tests, even higher. No guessing of possible questions and "cramming" for the same, so common in America, can qualify a youth to pass an examination in France.

Another thought which the examinations for the baccalauréat suggest, is the superiority in one respect of Canadian education over that in the United States where a great source of weakness would be removed by the adoption of our plan, under which the examinations for promotion from one grade to the next are conducted by the supervisor of education, not by the teacher. The pressure brought to bear upon teachers to promote ill-prepared pupils is thereby eliminated and this pressure is a fruitful source of demoralization in American public schools.

Finally, does not the French system, as worked out by a great body of educationists, suggest both the kind and method of a much needed reform in our university requirements for the B.A. degree? A large number of bacheliers, as we have seen, have studied no dead language. They may proceed to the Universities and after a time be made doctors in mathematics or natural science without being required to study any dead language. Why may not the same obtain with us? What advantages can be claimed for the study of dead languages, as taught by us, which may not be equally claimed for modern languages? The Harvard authorities apparently see none, as they have not required any dead language after matriculation, for many years past.

The Classes de Mathématiques Spéciales.

IV.—If the bachelier who is proficient in mathematics be not turned aside by circumstances or inclination, to immediately seek a career in civil or government employment, he most probably proceeds to prepare himself for the highly special and exacting examination necessary for entrance into one of the great schools of the government. The method of this preparation exhibits a very peculiar feature of the French system. Whereas with us, or with the German, the boy who has finished his regular course in the secondary school goes directly to some department of a university for his next instruction, the bachelier, who has a perfect right to follow the same course, returns to his old lycée (or enrolls himself at one of the great Paris lycées, such as Saint Louis, Louis le Grand or Henri IV), to enter the *Classe de Mathématiques Spéciales préparatoire* which leads up to the *Classe de Mathématiques Spéciales.* The latter is exactly adapted to prepare students for the École Normale Supérieure, the École Polytechnique and the *bourses de* *licence.* Only a small proportion of the lycées (34 out of the 115), have this *Classe*; but with the exception of Aix they are to be found in all university towns. On the other hand, yet other lycées have classes which prepare specially for the less exacting mathematical entrance examinations of the École Centrale, École de Saint Cyr, École Navale, etc. But the number of élèves who on first starting out deliberately try to pass examinations for these schools is small, in proportion to the number who eventually reach them after repeated but vain effort to get into the École Polytechnique or the École Normale Supérieure. Just what makes these two schools famous and peculiarly attractive will appear in a later section. It has been noticed that when the élève has won his baccalauréat he may immediately matriculate into a university, and although it might be possible for him to keep pace with the courses, in mathematics, at least, it would be a matter of excessive difficulty. There is then in reality, between the baccalauréat and the first courses of the universities, a distinct break, bridged only by the *Classes de Mathématiques Spéciales.*

The élèves who enter the *préparatoire* section of this class are, generally,[1] bacheliers leaving the classes de Mathématiques; in very rare instances, there are those who come from the classe de Philosophie. Natural science, history and geography, philosophy—indeed practically every study except those necessary for the end in view, have been dropped and from this time on to the agrégation and doctorat all energies are bent in the direction of intense specialization. This is the most pronounced characteristic of French education to-day. In mathematics, instruction now occupies 12 instead of 8 hours. New points of view, new topics and broader general principles are developed in algebra and analysis, trigonometry, analytical geometry and mechanics. Physics and chemistry are taught during six hours instead of five. Add to these, German, 2 hours; French literature, one hour; descriptive geometry, 4 hours; drawing, 4 hours. After one year of this preparatory training the élève passes into the remarkable Classe de Mathématiques Spéciales.[1]

Eight years of strenuous training have made this class possible for the young man of 17 or 18 years of age, who is confronted with no less than 34 hours of class and laboratory work per week and no limit as to the number of hours expected in preparing for the classes!

When first I looked over the programme it seemed a well nigh impossible performance for one year. Surely no other country can show anything to compare with it. Although it would be of interest to

[1] Pupils who are not bacheliers, but who are preparing to enter the École Centrale, are also admitted into this class.

reproduce the programme in full, to do so would take up a disproportionate space in a sketch of this kind. Moreover, many parts of it are given in Appendix A in connection with the agrégation examination requirements. I shall, therefore, merely touch on a few of the points of interest. The number of hours per week are distributed as follows:—Mathematics, 15; physics, 7 (2 in laboratory); chemistry, 2; descriptive geometry, 4; drawing, 4; German, 2; French,1. The scope of the mathematical work may be judged from some books which were prepared with the needs of such a class especially in view.

B. Niewenglowski, Cours d'algèbre, I, 382 p.; II, 508 p.; Supplement—*G. Papelier* Précis de géométrie analytique, 696 p.—*Girod* Trigonométrie, 495 p.—*P. Appell* Cours de mécanique, 650 p.—*X. Antomari* Cours de géométrie descriptive, 619 p.

If anything, this list underestimates the work actually covered by those who finally go out from the class. Tannery's Leçons d'algèbre et d'analyse (I, 423 p., II, 636 p.), might well replace Niewenglowski's work while Niewenglowski's Cours de géométrie analytique (1, 483 p.; II, 292 p.; III, 569 p.), represents the standard almost as nearly as Papelier's volume. Another treatise on mechanics widely used is that of Humbert and Antomari.[1]

When we further realize that the books in this list, which represents the work for only one of a half dozen courses, are covered by the professor in about six months—the last three months of the year are given over to drill in review and detail—we begin to get some conception of what the *Classe de Mathématiques Spéciales* really stands for. In his instruction the professor is officially "recommended" " de ne pas charger les cours, de faire grand usage de livres, de ne pas abuser des théories générales, de n'exposer aucune théorie sans en faire de nombreuses applications poussées jusqu'au bout, de commencer habituellement par les cas les plus simples, les plus faciles à comprendre, pour s'élever ensuite aux théorèmes généraux. Parmi les applications d'une théorie mathématique, il conviendra de préférer celles qui se présentent en physique, celles que les jeunes gens rencontreront plus tard dans le cours de leurs études soit théoriques, soit pratiques; c'est ainsi que, dans la construction des courbes, il conviendra de choisir comme exemples des courbes qui se présentent en physique et en mécanique, comme les courbes de Van der Waals, le cycloïde, la chaînette, etc., que, dans la théorie des enveloppes, il conviendra de prendre comme exemples les enveloppes qui se rencontrent dans la théorie des engrenages cylindriques, et ainsi de suite. Les élèves devront être

[1] Further details about these various volumes, as well as of many others, may be found in Appendix D.

interrogés en classe, exercés aux calculs numériques, habitués à raisonner directement sur les cas particuliers et non à appliquer des formules. En résumé, on devra développer leur jugement et leur initiative, non leur mémoire."

In France, as everywhere else, the success of the system depends much on the personality of the professor. A Paris lycée instructor who had a genius for getting hold of his boys has recently died. No less than 35 of his pupils were admitted to the École Polytechnique in a single year. The ordinary professor has to be content with a half or a third of this number. But the success of a class is, by happy arrangement, not left to depend wholly upon a single man. Take, for example, lycée Saint Louis, which is the greatest preparatory school in France for the École Normale Supérieure and the École Polytechnique. There are four Classes de Mathématiques Spéciales and for all the members of these classes, conferences, interrogations and individual examination are organized. These exercises, which complete the daily instruction, are conducted by one of the professors in the lycée itself, or by one of those from the Collège de France, the Sorbonne, the École Polytechnique, the École Normale, from other lycées or from the collèges. Incapables are thus speedily weeded out. Of perhaps greater value than the solidity of the training got in this way is the fact that the interest of the élève is sustained.

Just a word about the calculus course. This is practically equivalent to the first course in the best American universities. The integration of differential equations of the first order in the cases where (1) the variables separate immediately, (2) the equation is linear, as well as of linear differential equations of the second order, constant coefficients, (*a*) without second member, (*b*) when the second member is a polynomial or a sum of exponentials of the form Ae^{ax}—, is taken up.

With the end of the year the élève has his first experience of a *concours*. Previously he has found that it was necessary only to make a certain percentage in order to mount to the next stage in his scholastic career; but now it is quite different. In 1908, 1,078 pupils tried for admission into the École Polytechnique, but only 200, or 19.5 per cent., were received; for the department of science in the École Normale Supérieure, 22 out of 274, or 8 per cent., succeeded. In each case the number was fixed in advance by the Government according to the capacity of the school; the fortunate ones were those who stood highest in the examinations, written and oral. In the case of the École Polytechnique, the written examinations were held in all the lycées which had a Classe de Mathématiques Spéciales. The 387 candidates declared *admissible* were then examined orally at Paris, and from them the 200 were chosen. Similarly for the École Normale, the written examinations are conducted at the seats of the various academies and the oral at Paris. Since 1904 the concours passed by the École Normalians has been that for the *bourses de licence*, open to candidates of at least 18 years of age and not more than 24. Certain dispensations in the matter of age are sometimes granted. The value of the bourse, for the section of science, is from 600–1,200 francs a year and is intended to help the student to prepare for the licence and other examinations required of prospective professors in the lycées and universities. The candidates leading the list in the concours are sent to the École Normale Supérieure for from three to four years. It is necessary for the six or seven other *boursiers* to prepare for future examinations at the various universities of the provinces. Their bourses last regularly for two, and exceptionally for three, years.

But to return to our élèves of the Classe de Mathématiques Spéciales. At the end of the first year, when 18 years old, they usually present themselves for the concours of both the bourse de licence and the École Polytechnique, the examinations in the former being more strenuous and searching. Only from 2 to 5 per cent. succeed on the first trial. The others then go back to the lycée and take another year in the Classe de Mathématiques Spéciales. Many points not fully understood before are now clear, and at the end of the second year from 25 to 28 per cent. are successful. The persevering again return to their *Classe* and try yet a third time (the last permitted for the bourse de licence); but it is a matter of record that less than one-half of those who enter the Classe de Mathématiques Spéciales succeed even with this trial. This is usually the last trial possible for entry into the École Polytechnique, as the young man who has passed the age of 21 on the first of January preceding the concours may not present himself. The remainder of the students either seek for entrance into government schools with less severe admission requirements, and thus give up their aspirations to become mathematicians, or else continue their studies at the Sorbonne. The candidate who heads the list in each of these concours has his name widely published. In the case of the bourse de licence he is called the *cacique,* and he very frequently tops also the École Polytechnique list.

If the work of the Classe de Mathématiques Spéciales is so enormously difficult that only 2 to 5 per cent of its members can, at the end of one year, meet the standard of requirements of the examinations for which it prepares, why is not the instruction spread over two? Since nearly all the mathematical savants who now shed lustre on France's fair fame have passed from this remarkable class on the first trial, there can be no doubt that the answer to this question may be found in the fact that the government ever seeks her servants among the *élite* of the nation's intellectuals.

Higher Mathematical Instruction.

L'enseignement supérieur is carried on in universities, great scientific establishments and special schools. We shall consider in particular, the mathematical instruction as given at the Sorbonne, the École Normale Supérieure, the École Polytechnique and the Collège de France.

The Sorbonne.

The *Université de Paris* consists of the *faculties de droit, de médecine, des sciences, des lettres.* (Faculties of Catholic and Protestant theology were suppressed in the years 1885 and 1906 respectively.) The faculties of science and letters have their offices, lecture rooms, laboratories and special libraries in a building now called the *Sorbonne.* This building contains also the headquarters of the officers of the Académie de Paris and of the university administration, the museums, the main university library, the École Pratique des Hautes Études, the École des Chartres and the great amphitheatre capable of seating 3,500 persons and adorned with a large allegorical painting, "the masterpiece of Puvis de Chavannes and one of the finest decorative works of our time."

The present Sorbonne, completed less than a decade ago, is an immense and magnificent edifice, erected to replace the old Sorbonne (the outlines of which may be seen in the courtyard), dating from the time of Richelieu. To make room for the newer building, the older was (in 1885) torn down, with the exception of the chapel, which picturesquely nestles in the midst of the new structure. The name Sorbonne harks from the time of the confessor of St. Louis, Robert de Sorbon, who in the thirteenth century founded a sort of hostel for the reception of poor students of theology and their teachers. This soon acquired a high reputation as the centre of scholastic theology, and the name came to be applied to the faculty itself, which continued to exercise great influence on French catholicism down through the centuries. It was suppressed, along with some twenty other universities, during the Revolution. But under Napoleon, in 1808, the Sorbonne was re-established as the seat of the monster *Université de France,* which embraced all the universities, secondary schools, etc., in the country. The details of this organization did not prove acceptable, and in 1896 the arrangement explained in the early part of this paper came into effect.

Judged by the number of students, the Université de Paris is the largest university in the world. In January last, 17,512 students had registered. Nearly half of these were law students, while of the remainder, 1,845 were pursuing work in one or more of the twenty-three departments of the faculty of science.[1] The instructors in French universities are of six classes. The *chaires magistrales* are held by *professeurs titulaires*; no less than eleven of these at the Sorbonne, are in the departments of mathematics. Then there are *professeurs adjoints* (of whom there may not be a number greater than one third of the chaires magistrales), *chargés de cours, maîtres de conférences, chargés de conférences* and *maîtres de conférences adjoints.* Just what scholastic status or state recognition is implied in these titles I shall consider later; but it may be remarked here that all *professeurs*, although theoretically appointed only till the age when the law requires them to be pensioned off, are in reality appointed for life. Those at the Sorbonne, at least, are known the world over, because of their eminence in research and exposition. Here are the names of the *chaires*, of the incumbents and of the courses offered during the past year:—

1°—Géométrie supérieure .... Darboux.
I Semester.—30 lectures of 1 hour. "Theory of triply orthogonal systems." Largely as in the new edition of Darboux's work on this subject and as in selected parts of his Théorie des Surfaces.

2°—Analyse supérieure et algèbre supérieure .... Picard.
II Semester.—30 lectures of 1 hour. "Determination of integrals of partial differential equations of the second order with various conditions as to limits."

3°—Calcul differentiel et calcul intégral .... Goursat.
I and II Semesters.—60 lectures of 70 minutes. This course is practically that given in Goursat's Cours d'Analyse Mathématiques, Tomes I-II, new edition.

4°—Applications de l'analyse à la géométrie .... Raffy.
I Semester.—30 lectures of 75 minutes. This course is a slight expansion of Raffy's book on the subject.

5°—Théorie des fonctions .... Borel
I Semester.—15 lectures of 1 hour. The announced subject of this course was "Definite Integrals and some of their Applications," but the treatment was more of series.

6°—Astronomie mathématique et mécanique céleste .... Poincaré.
I Semester.—30 lectures of 1 hour. "Movement of the Celestial Bodies about their centre of Gravity."

7°—Astronomie physique .... Andoyer.
II Semester.—30 lectures.

8°—Physique mathématique et calcul de probabilités ... Boussinesq.
I Semester.—30 lectures of 1 hour. "Mechanical Theory of Light."
II Semester.—30 lectures of 1 hour. "Reflection and refraction of a pencil of light at the limit common to two homogeneous media."

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[1] The total number of students at all the universities in France, in January 1910, was 41,044 as compared with 52,456 in Germany.

9°—Mécanique physique et expérimentale. . . . . Koenigs.
I and II Semesters.—60 lectures of 1 hour. "Moteurs thermiques."
I Semester.—12 lectures of 1 hour. Theoretical Kinematics.

10°—Mécanique rationnelle[1] . . . . [1]Painlevé.
I and II Semesters.—60 lectures of 1½ hours. The lectures in this course practically cover Appell's Traité de Mécanique rationnelle, Tomes I-III.

11°—Mathématiques générales[1] . . . . [1]Appell.
I and II Semesters.—45 lectures of 1 hour. This course is essentially that given in Appell's Éléments d'analyse mathématique.

Physique générale is demanded of all advanced mathematical students. I add the subjects taught.

12°
Physique . . . . Bouty.
I Semester.—30 lectures of 1 hour. Thermodynamics and Electrolysis. Blondelet's text is an equivalent.
Physique . . . . [2]Pellat (Leduc).
I Semester.—15 lectures of 1 hour. "Electrostatics, Ohm's law, Electrodynamics, etc." This course covers Pellat's text.
Physique . . . . Lippmann.
II Semester.—30 conferences of 1 hour. "Gravity, Capillarity, Acoustics, Optics."

Beside these courses, which are open to the public without even the formality of registration, there are certain *conférences et travaux pratiques* —"cours fermés"—for those regularly matriculated. As far as we are interested in them they are:—

13°—Géométrie supérieure . . . . Cartan
I Semester.—15 conferences of 1 hour.

14°—Calcul differentiel et intégral, et ses applications géométriques . . . . Raffy.
I and II Semesters.—60 conferences of 70 minutes.

15°—Astronomie physique : travaux pratiques . . . . Andoyer.
II Semester.—30 conferences.

16°—Mécanique physique et expérimentale: travaux pratiques. Koenigs.
I and II Semesters.—30 conferences.

17°—Mécanique physique: principes de la statique graphigne et de la résistance des materiaux. . . . . Servant.
I and II Semesters.—30 conferences of 1 hour.

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[1] Appell is professor of mécanique rationnelle, and Painlevé of mathématiques générale, but for this year at least they have exchanged the subjects demanded by their chairs. A possible explanation may be found in the fact that Appell has a remarkable gift of clear exposition of elementary subjects.

[2] Pellat died early in the year and Leduc (professeur adjoint) was temporarily given charge of his course.

18°—Mécanique rationnelle .... Cartan.
I and II Semesters.—60 conferences of 1 hour.

19° { Algèbre .... Blutel.
I Semester.—30 lectures of 1 hour.
Exercices de mathématiques générales .... Garnier
I Semester.—20 conferences of 1 hour.
Travaux pratiques de mathématiques générales .... Cartan.
II Semester.—15 conferences of 1 hour.

20°—Physique générale .... Leduc.
I and II Semesters.—45 conferences of 1 hour.

Cartan is a maître de conférence, and Blutel (professor of mathematics at lycée Saint Louis), Garnier (collaborator on the French edition of the *Encyklopädie der Math. Wissenschaften*), and Servant, are chargés de conférences. Unlike the organization at the Collège de France and in German universities where no examinations enter to disturb the serene atmosphere, one of the chief functions of French universities is to provide means for preparing students for two state examinations, the licence and the agrégation. These examinations (but especially the agrégation), demand that exceedingly comprehensive instruction shall each year be given in a large number of special subjects. To this end most of the professors devote themselves. As a consequence there is a great sameness in the courses offered at the different universities from year to year and Lyons is about the only one outside of Paris which attempts to do more than meet the state requirements.[1] At the Sorbonne there is little annual variation in two-thirds of the main courses; these are 3°, 4°, 7°, 8°, 9°, 10°, 11°, 12°. An outgrowth of them is a remarkable series of elegant treatises. But Borel, Darboux, Picard and Poincaré make frequent changes in the subjects on which they lecture. We remark, that no professor gives more than one course (two lectures per week), except in the four cases of those who direct conferences and travaux pratiques; also that Darboux, Picard, Poincaré, Andoyer, Bouty, Lippmann, only lecture during one semester.[2] The maîtres de conférences or chargés de conférences do not give new courses and treat of subjects in which they are especially informed as do the *Privat-docenten* in German universities. But their instruction, as well as that of all others who direct cours fermés, is supplementary to the work of those holding the chaires magistrales. Thus 13° supplements Darboux's course—14° supplements 3° and 4°—15° and 7° go together—so also 16°, 17° and 9°—18° and 10°—19° and 11°—20° and

[1] Compare Appendix B.

[2] Most of Borel's work is at the École Normale Supérieure, which is part of the Université de Paris.

12°. Although there is nothing in the mathematical departments of the universities of France which exactly corresponds to the German *Seminar*,[1] the method of conducting the conferences at travaux pratiques is, I believe, a French specialty. We also find it used in the lycées at the École Normale Supérieure, at the École Polytechnique, etc. Each week the instructor gives out exercises which the students solve and hand in; these are returned with written comment and correction. The hour is employed by calling some student to the board and leading him by means of suggestive questioning to work out, generally in great detail, a piece of analysis or a problem or a theorem—either arising from, or nearly related to, the main course. The manner in which this is carried through, with its exacting demands as to form in statement and black board presentation, is in the highest degree instructive.

The above list does not display all the mathematical courses offered at the Sorbonne this year. Cartan had a special problem course for candidates for the agrégation and Bachelier, gave a *cours libre* of 20 lectures on the calculus of probabilities and its application to financial operations. The number of cours libres varies from year to year.[2]

The Licence.

When a student has finished any one of the groups of studies such as (3°, 4°, 14°) or (10°, 18°), he may pass an examination and receive a *certificat d'études supérieures.* With the third *certificat* is given the diploma *licence ès science.* The choice of subjects is not necessarily limited to those given above but may be selected (at the Sorbonne) from a list of 23[3] which includes general chemistry, zoology, geology, etc. If, however, the student expects to teach in the secondary schools his choice is greatly limited. The mathematician must have *certificats* in calcul différentiel et intégral, in mécanique rationnelle and in physique générale, or a third certificat in mathematics, excepting (11°, 19°). The physicist must have certificats in physique générale (12°, 20°), in chémie générale and in minéralogie or mathématiques générales (11°, 19°), or another subject of mathematical or physical science. The natural scientist must have certificats in zoology or general physiology, in botany and in geology. The examination for certificats may be taken twice in a year, in July or in November. It consists of three parts, *épreuve écrite, épreuve pratique, épreuve orale.*

[1] Other subjects *are* treated in *Seminar* style at the École des Hautes Études which is an off-shoot of the Collège de France.

[2] In July, 1910, the University of Paris accepted the offer of M. Albert Kahn to bear the expense, for a period of five years, of a course on "The Theory of Numbers."

[3] This number varies with the university; at Dijon it is 12.

The first two are written examinations of about four hours each. Theoretical considerations abound in the *écrite* while numerical calculation is characteristic of the *pratique*. The oral lasts for 15–20 minutes and is held before the jury of those professors who have the whole examination in charge. It is necessary to get fifty per cent to pass. The first certificat and examinations cost 35 francs, the second and third 30 francs each and the licence diploma 40 francs.

The Diplôme d'Etudes Supérieures de Mathématiques.

This diploma which was instituted by a decree of 1904 has not yet been awarded to any one, although its equivalent, 4 certificats (one chosen at option), is required of all candidates for the agrégation. It may be considered as a little doctarate. The conditions leading to the diploma are twofold:—

(*a*) That a suitable *travail* be written on a subject agreed upon by the faculty.

(*b*) That satisfactory answers be given to questions on the *travail* and on topics given three months in advance and relating to the same part of mathematics. The *travail* may consist either of original researches, or of the partial or total exposition, of a memoir or of a higher mathematical course. In the latter case by "exposition" is meant either a simplified résumé of the memoir or of the course, or the detailed development, where the result or method that the author or professor presents has only been outlined.

The Agrégation des Sciences Mathématiques.

This examination, unlike that for the baccalauréat and licence, is a concours as in the case for entrance into the École Normale Supérieure and the École Polytechnique. The number who become agrégés each year is fixed in advance by the Minister of Public Instruction according to the needs of the lycées in the country. This number in recent years has generally been 14, but in 1897 as few as seven were chosen. The smallest number of competitors since 1885 was 54, in 1907; in 1909 there were 81; the largest number was in 1893, when there were 134 young men eager for 13 places.[1] The candidate for this examination must have four certificats; those in *calcul différentiel et intégral, mécanique rationnelle, physique générale* and a fourth chosen at pleasure among the remaining mathematical subjects. As an equivalent of the fourth certificate a *diplôme d'études supérieures de mathématiques* may be presented. The subject of the fourth certificate at Paris is usually Picard's Analyse Supérieure et Algèbre Supérieure (2°) or Darboux's Géométrie Supérieure

[1] Compare the analytical table of Appendix A.

(1°, 13° and an épure). Poincaré's course is chosen less frequently and at present Borel's course may not be selected independently of others. It is usually four years after leaving the *Classe de Mathématiques Spéciales* that the young mathematician first presents himself for the agrégation, *i.e.*, when he is about 21 years of age. In this interval he has probably spent a year in military service, worked off the examinations for the first and third of the above mentioned certificates during the second year, for the second and fourth during the third, while the fourth year was spent in general review, study of teaching methods or other special direct preparation for the agrégation. This examination, which is unique in its difficulty and exactions, is organized for selecting the most efficient young men in the country, to take charge of the mathematical classes in the lycées. It consists of *épreuves préparatoires* and *épreuves définitives*. The former are four written examinations, each of seven consecutive hours in length! The first two of these are on subjects chosen from the programme of the lycées in *mathématiques élémentaires* and *mathématiques spéciales*. The last two, based on the work of the candidates in the universities, are a *composition sur l'analyse et ses applications géométriques* and a *composition de mécanique rationnelle*. These épreuves are held at the seats of the various académies of France. Those who have reached a sufficiently high standard are declared *admissible*. Their number is usually a little less than twice the possible number to be finally received. In 1909 it was 27, but in 1905, 20; while in 1887 there were only 15, from which 13 were selected. They must present themselves at the Sorbonne for the *épreuves définitives*. These consist of two written examinations and two *leçons*. The written tests are an *épreuve de géométrie descriptive*, and a *calcul numérique*. Their duration is fixed by the jury, but it is usually four hours for each. The *leçons*, which are supposed to be such as a professor might give (during $\frac{3}{4}$–1 hour) in a lycée, are on subjects from (a) *mathématiques spéciales*, (b) the programmes of the classes, *Secondes*, *Première*, C, D, and *Mathématiques* A, B. The subjects are drawn by lot, and for each lesson the candidate has four hours to think over what he is going to say. No help from any book or other source is permitted. The unfortunate who has little to say is speedily adjourned. As a salve for disappointment and as encouragement to try again, he receives 300 francs a year for three years because he had won a place among those admitted to the second examination.

The names in the list of agrégés are published in order of merit, and those who head the list are likely to get the better positions. Many of the instructors at the Sorbonne were first agrégés. Appell, Picard and Goursat were successively first agrégés, 1876–78; Cartan and Borel 1891–92; Andoyer in 1884. Painlevé was, however, a ninth agrégé;

Blutel a fourth, and Garnier a second. There have been very exceptional cases (in 1885, 1886, 1895), when an agrégé was still in his twenty-first year, but the average age for the past twenty-five years is a little less than 26. There are also those who do not reach the goal of their ambition till after they are 40, and have tried perhaps ten or a dozen times. The difference between the salary of those lycée professors who are agrégés and of those who are not has been emphasized still more by the law just passed, which gives the former an annual bonus of 500 francs.

If the agrégé wishes to become a professor in a university he must pass the examination for the doctorate, but only a very small proportion of the agrégés take this step—during the period 1885–1904, 20 per cent. If, however, he wishes to teach in a lycée he may demand such a position as his right. Among the candidates who are *admissible* but not received are generally selected *professeurs chargés de cours* (or others in positions of inferiority in the lycée), who, however, after 20 years of service may be named *professeurs-titulaires* and be the academic equals of their luckier comrades of years before.

Other details concerning the agrégation, such as the programme for the concours of 1910, the examination papers for the concours of 1909, are given in Appendix A.

The Doctorat.

What is the relative value of the French and German mathematical doctorate? What the relative difficulty of obtaining it? are questions which the average American post-graduate student who is seeking to decide between France and Germany for further study is sure to ask. Small as is the proportion of students in a German university who present themselves for this degree, the number in France is far smaller. In the two years 1906–08 Germany made 87 doctors in mathematics, while France, with but 20 per cent. fewer students, created only 13.[1] This difference in numbers is doubtless principally due to the fact that the end in view in France is entirely different. The Frenchman usually goes up for his doctorate with the expectation of drawing wide attention to his thèse. The step is also necessary for everyone who aspires to be appointed a professor in a university—unless, perchance, he has become a member of the Institut without having the degree. All except three of the French universities offer the degree of doctorate in mathematics, but only eleven of them have ever conferred it. Again, of the 331 degrees which have been conferred by the existing universities, 296 have been granted by the Université de Paris. This is, of course, very different from the results in

[1] There is of course no degree of Doctor of Philosophy in France. The equivalent is explained later.

Germany, where Berlin university turns out a very small fraction of the doctors in any one year—during 1906–08, less than 4 per cent. There is also one other great difference between French and German universities, although the examinations for *licenciés* rather than those for doctors must be chosen to emphasize the point sufficiently. The difficulty of obtaining those degrees common to most French universities is much the same, and although Paris is the principal degree-conferring centre, it is well established that there have been years when it was more difficult to obtain the licence, in some departments, at certain universities of the provinces than at the Université de Paris. That the personality of the professor should play an important role in determining the standard of excellence demanded is only natural; but as it is the ambition of every professor in the provinces to make his department important and to ultimately arrive at Paris, one can be very sure that no one university in France will ever sink to the level of at least two German universities, where, on account of lax demands in study and thesis, even train conductors call out "Twenty minutes wait to get your doctor."

But if such representative universities as Berlin, Göttingen, Munich be selected in Germany and compared with that at Paris, two questions suggest themselves: (1) Does the average doctor's thesis (which in both countries is the essential performance on the part of the candidate for the degree) indicate a higher standard of excellence in one country than the other? (2) Admitting this to be the case, are the minimum requirements in this country as low as the general requirements in the other? By actual study of the theses, I am convinced that the answer to the first question is decidedly in favour of Paris. One could easily cite a number of French thèses which were notable and extensive contributions to mathematical progress, but it is only necessary to refer the reader to the complete list of the thèses, which is given in Appendix C. Before answering the second question, I shall explain more fully the nature of the French doctorate, the general conditions under which it is available for the Frenchman and the possible modifications of those conditions in the case of foreign candidates.

There are two doctor's degrees open to the mathematical student in France: the first, *doctorat ès sciences mathématiques*, conferred by the State (doctorat d'état), the second conferred by the universities —for the Sorbonne, *doctorat de l'Université de Paris*. Only one American, a woman, has won the former degree, which was created in 1810, and only one American has also obtained the latter, which was organized as recently as 1898. In both cases the thèse is the principal requirement, and judging by the eight for which the doctorat de l'Université de Paris has been granted, the standard in this respect is about as high as for the doctorat de l'état. It is in the matter of further requirement that the doctorat d'état is more difficult. For this degree there is no possible way of avoiding the various examinations which lead up to the licence ès science with mention of the certificats: 1st, calcul différentiel et intégral; 2nd, mécanique rationnelle, and 3rd, at the choice of the candidate. For the doctorat de l'Université[1] only two certificats are required, and, in the case of foreigners, very great latitude is permitted the faculty in accepting equivalents for these certificats, in view of work done elsewhere. In both cases only one year of residence is required. We can, then, now answer the second question, proposed above, in the affirmative for the doctorat de l'Université, and in the negative for the doctorat d'état.

The analytical table given in Appendix C clears up misconception as to the age of the French mathematical doctor. During the past 25 years, the average age has been 30, but a large number "sustained" their thèses between the ages of 23 and 25. The youngest doctor was Joseph Louis François Bertrand, aged 17, created in 1839; the oldest, in 1882, aged 55.

Only a small proportion of the agrégés ever become doctors,[2] and in but one case (1894) has a doctor become an agrégé. Which title calls for the greater ability in the getting?[3] The two things are so entirely different, it is perhaps difficult to understand why some say the doctorate ranks the higher. The musician with great technical talent only, may be allowed to have equal ability with the performer less gifted in this direction but endowed with strong temperament—power of perception and interpretation which draws aside the veil for the ordinary observer and discloses formerly hidden heights, beyond. Yet it is the latter who particularly appeals to us. So while the technical skill of the agrégé is admired, and the state gives him certain rights denied the doctor, it is only the latter who, on showing power of disclosing the truths waiting for discovery from the foundations of the worlds, has the opportunity to direct the nation's youth in the great universities of the country. So much the more sought after the man who combines in himself to a high degree both talents, the gift of brilliant exposition and the genius for discovery.

The general procedure toward the doctorate is the same for

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[1] According to a decree of 1906 the insignia of the docteurs de l'université de Paris is, "Epitoge à trois rangs d'hermine, avec les couleurs de Paris (bleu et rouge) dans le sens longitudinal."

[2] *Cf.* Appendix A.

[3] The agrégé in Law and Medicine stands much higher than the doctor in these Departments.

both kinds. A thèse worked out under general supervision of a professor is formally approved by a committee of three professors named by the *doyen*. Birth certificate, diploma as licencié, 148 printed copies of the thèse and 145 francs, are deposited with university officials, and the day fixed for appearing before the committee to publicly answer such general questions on the thèse or other topic which the benevolent ingenuity of the examiners may propound. Compared with those in some other departments of the university, the examinations of the mathematician is a very informal affair. It rarely occupies more than three-quarters of an hour. The candidate is immediately told whether he has got the *note* "honorable" or "très honorable," is congratulated and dismissed. The amount of help which the candidate for the doctorate receives from the professor is much less in France than in Germany. In fact he rarely approaches the professor except when he gets his subject or is reporting progress. It is expected of the Frenchman that the thèse represent his own work and thought. The doctor who has presented a remarkable thèse and passed a brilliant examination may have the full cost of examination and diploma remitted. A similar rule applies to bacheliers.

The École Normale Supérieure.

This great institution is a part of the Université de Paris and its object is to mould the future professors of the secondary schools and universities of France by appropriately supplementing the instruction they receive at the Sorbonne and the Collège de France. There are about 165 pupils, in the departments of science and letters, and practically all are internes. We have already remarked that the élèves in the department of science are the pick of the boursiers de licence. In 1909, 22 out of 270 candidates were thus selected and slightly more than one half devoted themselves to mathematics. All decided to live as internes although it was optional for anyone to attend the École as externe—when the amount of the bourse, 1,200 francs, would have been paid to him. Most of the pupils were 20 years old, had obtained the bourse on second trial and had passed one of the two years military service obligatory for every Frenchman by the *loi de deux ans* of 1905.

The course for mathematicians is three years, and is arranged as follows. During the first year the élèves go to the Sorbonne to hear Goursat's course in calculus and differential equations (3°) and Raffy's applications of analysis to geometry (4°). Instead, however, of following Raffy's conference (14°), which goes to complete the regular university student's training for the *certificat, calcul différentiel et intégral*, they are drilled by Borel and Tannery for three hours per week at the École Normale. They also take physique générale with Bouty, Lipmann and Leduc (12°, 20°), and at the end of the year pass the examinations for the *certificats* in these two subjects. In the second year they take up mécanique rationnelle with Painlevé (10°) at the Sorbonne, and with Hadamard in conference at the École. They also "assist" at such a course as Darboux's and Cartan's in géométrie supérieure (1°, 13°), pass the examinations and receive two more certificats. During the third year the élèves follow their own inclination in the selection of courses at the Sorbonne or Collège de France, while they are well grounded at the École, in descriptive geometry by Roubaudi, and in pedagogy, of algebra and analysis by Tannery, of geometry by Borel. Nearly all those of the first and second year also follow Picard's course (2°), at the Sorbonne, and those of the third year that of Borel (15°).

The drill in conferences (1½ hours each), at the École Normale is unequalled. In addition to the good points of those at the Sorbonne, we here find in much smaller classes a great degree of intimacy between students and professor, and a freedom of question and discussion. When, then, at the end of the third year these élite in intellect present themselves in the terrific competition of the agrégation, we are not surprised that they give a good account of themselves. Some do not succeed at the first trial or for ten, or twelve years afterwards, but 60 per cent. of the 300 agrégés named during the last 25 years were École Normalians; in 1890 there were only 4 out of 12, but of the 96 competitors in 1898, the 8 chosen were from this famous school.[1]

But as the whole end and aim of the École Normale are not only to prepare its élèves for the agrégation and hence for professorships in the lycées, but also to prepare them as university professors, we find many who have been encouraged to take up certain fields of research and who have made good progress toward a thèse for doctorate. Not only this, but from those who have succeeded in the agrégation, are chosen *agrégé préparateurs* who are taken back to the École for still another two years (sometimes three), while they prepare finally for their doctorate with all the attendant advantages, of the counsels from their former masters, and of the great library collections of the city. The *agrégé-préparateur de mathématiques* is officially *chargé de la bibliothèque* of the school.

The life in the École is singularly pleasant and inspiring. Here alone of all the institutions we have considered do we find among the élèves anything approaching the comradeship, so characteristic of the student relations in American colleges. Nor in after years are friendships and interests thus formed easily changed, as the *Association Amicale des Anciens Elèves* serves as a strong bond of sympathy and

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[1] *Cf.* Appendix A.

a constant means of intercourse. The fine old building, its studious atmosphere, three to five years of friendly rivalry with almost equally brilliant companions, daily intercourse with the professors, could hardly fail to have developed a young man's latent talents or to have inspired him to his best effort. Rarely does one find in France a professor such as Tannery[1] who is so generally beloved and respected by his élèves past and present.

All the great privileges of the École are occasionally open to foreigners either as internes or externes. It is now however, somewhat difficult to make arrangements to enter as interne because of the increasing demands made on limited space by the needs of the state. The charge of 1,200 francs per year made for pension complete is exactly the value of the bourse which France gives to students of her own nationality and which she expects to be refunded if after leaving the École, the élève decides not to take up the career of a teacher in her schools. All élèves are assured positions in lycées—those who have become agrégés, as professeurs titulaires, the others as professeurs chargés de cours.

The École Polytechnique.

This École founded by Monge at the close of the Revolution and the most popular of the great schools of France, is under the direct control of the minister of war and not of the minister of public instruction. Its pupils are recruited from the most diverse orders of society solely because of merit determined by concours on leaving the Classe de Mathématiques Spéciales. Its object is to prepare them as military and naval engineers, artillery officers, civil engineers in government employ, telegraphists and officials of the government tobacco manufactories. All élèves are internes. The cost of the *pension* is about 1,100 francs per year, of the *trousseau*, 600-700 francs, but there are an unlimited number of bursaries covering both pension and trousseau so that no poor boy of brilliant attainment is shut out.

As to the past of the school, until the latter part of the nineteenth century it was famous not only by reason of the great engineers it produced but also for its distinguished mathematicians. Poinsot, Poisson, Cauchey, Poncelet, Chasles, Lamé, Leverrier, Bertrand, Duhamel, Liouville, A. Serrett, Laguerre, Halphen, Hermite, Poincaré, not to mention a host of others, were all trained here. But now, the demands made on the engineer are so great, the élèves are only given the merest glimpses of the vistas which open up in modern mathematics.

[1] Tannery occupies one of the 12 chaires magistrales in mathematics at the Université de Paris, and as *sous-directeur* of the école normale supérieure is *directeur des études scientifiques*. [*Later note added in the proof:* Tannery died November 11, 1910, and Borel was appointed to his position.]

From being perhaps the leading school of the time with regard to its output of brilliant mathematicians it has, then, sunk to a position of wholly inconsiderable importance in this respect. Yet each year four times as many talented young mathematicians try to get in here as into the École Normale Supérieure. The number of those who enter the École Polytechnique because of failure to get into the École Normale is not perhaps very large; nevertheless there are certainly some among them who would have made good mathematicians but who do not make good engineers. They form, however, an insignificant fraction in comparison with the hundreds of graduates who by original choice have succeeded to the brilliant careers open to them.

The course at the École Polytechnique is two years and mathematics is taught each year. As at the Sorbonne, but in less effective manner, the instruction is a combination of lecture and conference. Jordan and G. Humbert are the professors but they are assisted by several *interrogateurs* or *répétiteurs* as at Lycée Saint Louis. Humbert's Cours d'Analyse (2 vols.), gives an idea of the course in *analysis* (2 years); then there are also *mechanics and machines* (2 years); *descriptive geometry* (first year); *astronomy*, *geodesy* (second year); *physics*, acoustics and optics (first year); *physics*, thermo-dynamics, electricity and magnetism (second year); etc.

The Collège de France.

This, the highest institution of learning in France, was founded by Frances I, in the sixteenth century. It does not form part of the Académie de Paris, but is under the direct control of the minister of public instruction. No fee or form of matriculation is necessary to attend the lectures, no examinations are held and no degrees are conferred. It is not necessary for its professors to hold any degree or to have passed any specific examination. A man who holds only the degree of bachelier, although not qualified to teach in secondary schools may, if otherwise competent, be appointed professor here. Successive vacancies are filled by the minister of public instruction who chooses between the names of two candidates who have been recommended by the body of professors occupying the 45 chairs. The professors have absolutely no obligations apart from the delivery of lectures, and in some cases, those with untroubled consciences have more or less evaded this requirement. Such abuse of privilege led this year to a law requiring each professor to give 40 lectures, distributed somewhat symmetrically over the two semesters. The purpose of the Collège de France is to advance learning. Within the limits of their chairs, the professors are absolutely free to treat any part of their subjects, no matter how limited or how minute, provided that they go to the bottom of it.

The chairs and foundation which have interest for us are the following:—

Mécanique analytique et mécanique céleste—"Theory of elastic plates." .... Hadamard.
Mathématiques—"Transformation and multiplication of complexes in elliptic functions."...........(Jordan) G. Humbert.
Physique générale et mathématique—"Elasticity of solids and fluids."..Brillouin.
Physique générale at expérimentale — "General phenomena of electricity and magnetism." ....Langevin.
Cours complémentaire — "Hyperelliptic surfaces of the fourth degree." ....Traynard.

This last course was established by a foundation of Mlle. Peccot who wished to be the means of encouraging young mathematicians. The instructor must be a doctor of less than 30 years of age and he may not lecture for more than five years.

Jordan has not lectured for several years and his duties have been performed by the *suppléant* Humbert. We have here another peculiarity of this institution. Jordan continues to draw two-thirds of his salary while Humbert is remunerated with the remaining one-third.

The courses usually represent personal researches of the lecturers and are well attended, particularly by élèves of the École Normale Supérieure.

Concluding Remarks on Mathematical Instruction.

Unless I have greatly failed in my presentation, one thing which may be readily inferred from what has gone before is, that no idea could be more mistaken than the one so prevalent among us, that the French are light-hearted, frivolous and at best superficial. Their struggle for existence is severe and the competition is terribly keen. As far as the mathematician is concerned and his training is by no means exceptional, we have found that from the time the élève leaves the Première, that is when he was 15 years old, onward, he undergoes most exacting examination at almost every turn. The successive stages in his studies are very largely marked out for him and care is constantly exercised to see that he make no false step and that he be properly prepared to pass his examinations. The Université de Paris has appeared to be a great institution, "wonderfully organized, to turn out a certain amount of a certain product, of a certain degree of excellence, with the least possible loss of time and energy." The strenuous directness of method and of achievement in this system cannot help but impress us.

Such are the standards set for those, who have prepared themselves as lycée professors, who direct the boy's education from the time he is ten years old. How woefully low are our standards in comparison! Remark too, that after a boy is 6 years old he is taught by men only.

We are also struck with the breadth of the future mathematician's training. Although it is true that after the age of 16, the humanities are set aside, yet physics, descriptive geometry, pure mathematics, applied mathematics, all continue to occupy positions of importance. I have indicated how, by peculiar method of instruction, all these subjects are welded into a homogeneous whole, how that although knowledge of wide range of fact is fundamental, it is the thorough grasp of broad principles and the powers of ready application of those principles to the most diverse kind of problems, that is made essential. German influence has given us a great respect for fact, but the French, if opportunity were given, would soon convince us that a fact, as a fact, had little of interest, except in so far as it might be contributary to the upbuilding of some system. To the Frenchman learning is not an accomplishment, but "an honourable and arduous profession with all its trials, all its heart-burning competitions, all its pitiless disdain of weakness, all its stimulating rewards." This partly explains the severity of the examinations. Every boy of remarkable intellect, be he rich or poor, has the chance to have his talents developed to the utmost. From the time he is 11 or 12 years old till he is ready to step into a position in a university, bursaries constantly reward his accomplishment. If in time he become professor in a provincial university, his effort is in no whit relaxed; he looks forward to being promoted to Paris. With this advancement accomplished, his intellectual activity does not cease by any means, for he now hopes some day to be numbered among the few members of the Académie des Sciences of the Institut. Great as are the rewards and recognition of merit here, there is still greater for him who, as Poincaré, is pre-eminent, namely, to be numbered among the "immortals" of the Académie Française of the Institut. Men of such calibre and brilliance and unremitting intensity of application and purpose, are the professors at the Collège de France and Université de Paris.

Nothing in French universities takes the place of undergraduate life in England and America; nor would we willingly attempt to adopt their system, though it would certainly silence the frequent criticism of our ordinary B.A. course, namely, that it is, to say the least, a poor training for the future man of business; the student has few obligations to meet, no real obstacles to overcome; if the professors make the courses difficult, he either rises in protest or seeks a college with easier requirements. We prefer to retain the genial, sympathetic relations between the student and professor, to encourage the emotional and sentimental life of the students with one another.

We may, however, still learn from France the advantages of intimate relation, in standards and scheme, of secondary and higher education. How much of the first year in American universities is wasted by getting freshmen into form, in teaching them how to work and how to think for themselves! The French university professors are in constant intercourse with the lycées, are the examiners of all their graduates, are the authors of many of the text-books employed. The mathematical training and equipment of the average writer of secondary texts in France is of far higher order than that of the average American author.[1]

Teaching of Mathematics as a Profession in France.

We have now seen how the mathematician is trained in France. It remains to discuss the nature of the inducements which are offered to young men to prepare themselves for giving mathematical instruction, and to see whether the inducements offered are sufficient to attract the best talent of the nation.

The agrégés are those specially prepared by the State for the positions as *professeurs titulaires* in the lycées. Although this title is not conferred regularly till the agrégé has completed his twenty-fifth year, those who are younger receive temporary appointment. The salaries vary according to the *classe* of the professor. At Paris the lowest salary is 6,000 francs per year, and the highest, 9,500. In this range seven *classes* are represented; six, each differing from the one before by 500 francs, and the *hors classe,* for which the salary is 9,500 francs. Promotion from one class to another takes place by selection and by seniority. From the sixth (the lowest classe) to the third, the number of those who can be advanced each year by selection is equal to the number which can be advanced by seniority. In the second and first classes two advancements may be made by selection to one by seniority. In choosing those for the *hors classe,* selection alone is taken into account. The promotions are made at the end of each calendar year, and take place so that there are always 20 per cent. of them in the sixth class, 18 in the fifth, 18 in the fourth, 16 in the third, 14 in the second, and 14 in the first. This arrangement is obviously a happy one, both by way of recognition of the merits of the unusually successful teacher, as well as those of him whose service is rather characterized by faithfulness.

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[1] **Compare Appendix D.**

In addition to the *professeurs titulaires* there are *professeurs chargé de cours*, who are usually selected from those École Normalians and those *admissible* to the agrégation, who fail to become agrégés. After 20 years of service they may become professeurs titulaires and receive the salaries we have indicated above. The government has, however, this year passed a law which gives the higher reward to the agrégé. It is to the effect that 500 francs per year shall be added to the regular salary of every agrégé. The real range of salaries mentioned above is then 6,500–10,000; in the provinces this reduces to 4,700–6,700. For the professeurs chargé de cours, the salaries at Paris vary from 4,500 to 6,000 francs; in the provinces, from 3,200 to 5,200. In the Premier Cycle the professors have 12 hours of teaching per week, in the second cycle and the Classe de Mathématiques Spéciales, 14–15 hours. Except for correcting exercises and filling out reports the professors have absolutely no obligations outside of class hours. They do not live in the lycées. The superintendence of the study of the élèves is carried on by *répétiteurs*, the more advanced of whom receive at Paris 2,600–4,600 francs for 36 hours service per week.

Only a very small percentage of the agrégés are also doctors,[1] but these few, as well as the more promising of those who are doctors only, usually prefer to seek some of the minor positions in connection with the universities. *Maîtres de conférences adjoints* are selected from among the doctors. *Chargés de conférences* and *maîtres de conférences* are sought for (1) among agrégés, (2) among doctors, but only the latter may receive an appointment for more than one year. A chargé de conférences at Paris receives 5,000–7,000 francs a year[2] for 2–3 hours per week of service. Even this amount is sufficient to enable a man to live well; but when, in addition, the incumbent is professeur agrégé in a Paris lycée (as in two cases at the Université de Paris at present), his income may exceed the regular salary of a university professor. For a good man there are also other sources of income from acting as *suppléant*, *examinateur* or *interrogateur*.

From the chargés de cours or maîtres de conférences, who are at least 30 years of age, who are doctors, who have seen at least two years of service in a school of higher education, and who are distinguished for their services—are appointed the *professeurs adjoints* of the universities. They receive from 6,000–10,000 francs at Paris and 4,500–6,000 francs in the provinces. The salary of *professeurs titulaires* is 12,000–15,000 francs at Paris and 6,000 or 8,000 to 12,000

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[1] Compare Appendices A, C.

[2] According to a decree of June 25, 1910, chargés de cours complémentaires and maîtres de conférences in the provinces, were thence forth to be of four classes, and to receive 4,500-6,000 francs annually.

in the provinces.[1] In recent appointment of professors, selection has been almost exclusively made from those who are both agrégés and doctors. That in exceptional cases the latter only is necessary was illustrated by a recent appointment to Poitiers,[2] but it is quite unlikely that any professor will ever be promoted to Paris who has not passed both examinations. As exceptional, note that any member of the Institut may be appointed professor at a university after six months of service in an establishment of higher education. The professorship of highest honour in the gift of the nation is at the Collège de France. Although the salary here is only 10,000 francs, the duties consist simply in delivering 40 lectures of one hour each. In the universities the professor is expected, in general, to give but one course of lectures, *viz.*, that which is called for by his chair. These lectures are delivered twice a week and last from an hour to an hour and a half each. If the course continue through the whole year, about 60 lectures are given; but we have already remarked that such men as Poincaré, Picard, Darboux, give only half this number. Remember, too, that many courses (practically all in the provinces) are repeated year after year with little change, that the professors are never called upon to arrange hours for conference with members of the class or to correct students' exercises.

One decidedly disagreeable duty does, however, fall to their lot. This is their obligation in the matter of various examinations. The figures given in an earlier section (p. 99) show how formidable this may be in the case of the baccalauréat alone, for the examiners as a whole. At the present time, however, only about one half the work is done by the university professor; and although his time is more or less broken into from June 27 to August 10, and November 1–8, the whole number of hours actually given up to the work by a single individual, in connection with both the baccalauréat and *certificats*, does not exceed 55. The whole number of hours which the professor gives to the State is, then, 85–145 per year. With such insignificant breaks in leisure for research we can no longer wonder at the great productivity of many French mathematical professors. The attractiveness of their positions is still

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[1] Until the recent increase in the salaries of the university instructors in Germany, 70 per cent. of the full professors received less than 15,000 francs. On the other hand there were three who received over 50,000 francs; and in any large German faculty some full professors will generally be found who receive for teaching an income from two to five times as large as some of his colleagues. These larger incomes are due to special allowances from the government, to extra university perquisites and to fees from the large body of students attracted by superior reputation. As distinguished from the rest of the world, in this connection, Germany pays an unusual amount for unusual merit.

[2] Compare Appendix B.

further enhanced by other sources of income. Nearly all those at the Sorbonne are members of the Académie des Sciences of the Institut de France. As such they receive 1,500 francs annually. Since Poincaré is also member of the Académie Française, this amount is presumably doubled. Darboux, as secrétaire perpétuel, receives 6,000 francs. Painlevé, also a member of the Institut, has been elected a member of the Chamber of Deputies, which will bring him in another 15,000 francs a year.

To such professorships the rising young mathematician may aspire; but as there are only fifty chairs in the whole country, the openings are few and the progress toward them slow.

We cannot help but contrast the conditions of the American professor, with at least 10–12 hours of lecturing per week, in several departments of mathematics, not to speak of the demands made on his time in correcting exercises and examination papers, and in administrative work. Yet with all this burden, he is expected not only to keep abreast of the times in his subject, but also to advance knowledge by his own researches.

The American Mathematical Student in Paris.

Many of the attractive features of mathematical study in Paris have been already set forth in the foregoing pages, but I wish here to briefly indicate a few others, as well as to give some special information which may be helpful to the American student.

No one can be wholly insensible to the charm of Paris herself, to the artisticity lavishly displayed by her people in sweeping shaded boulevard, towering monument, imposing building, gorgeous decoration, garden and embowered statuary, far-reaching park. From the time of Cæsar and Roman occupation of the Cité, historic associations have multiplied, and now they cluster about every quarter. Galleries, churches, palaces, Versailles, Saint Cloud, Chantilly, Fontainebleau, illumine and vivify in thrilling fashion the printed accounts of happenings of history. To the sympathetic student of the genius of the people, their customs, their language, their habits of thought—French literature is vested with new dignity and charm and grace and subtle meaning. Preëminent on the stage, strongly influential in the worlds of art, among the foremost in all forms of scholarship—the potentialities of fair France are great, both to educate and to refine.

But to benefit by such influences, as well as by the courses of instruction, which run, for the most part, through the whole year, the American student should plan to stay in Paris at least a year. It would also be well for him to come as early in June as possible. At this time there is no opportunity for attending university lectures, since the second semester closes about the middle of June; but he can look over the ground, get acquainted and prepare generally for the following autumn. The student who does not have friends in Paris should go to the *Bureau des renseignements* at the Sorbonne, where he will find some one who can speak English and give information as to pensions, the various institutions of higher education, etc. Another helpful bureau is the *Comité de patronage des étudiants étrangers*. There are a large number of students' associations, but probably the only one which it will be found worth while to join is the *Association générale des étudiants de Paris*. This association has recently moved into a handsome stone building used for over three centuries as lecture hall for the faculty of medicine. Here may be found reading room, library, fencing room, lounge rooms, etc. Members receive great reductions on tickets for nearly all the theatres and in purchasing books and other supplies. There are numerous social gatherings which professors and alumni occasionally join.

Paris is apt to be uncomfortably warm during the summer months, and unless the student is proficient in both speaking and writing the French language, he will probably wish to seek out more enjoyable quarters to carry on his studies. These may be found at such university towns as Grenoble or Geneva, both beautifully situated, but especially the latter, with endless possibilities in Alpine excursion. In each, excellent summer courses, at small cost, are given specially for foreigners. That at Grenoble lasts from July 1st to October 31st. It would be well, however, to return to Paris a little before this latter date, so as to settle the question of lodging somewhat before the beginning of the scholastic year.

To get the most out of a sojourn in Paris, the American naturally wishes, if possible, to get into a private family where he may enter, to the full, into the spirit of the life and language of the people. This is, however, a matter of much greater difficulty than in Germany, where many are so ready to welcome the stranger to hearth and home. In the case of the French the *foyer* is much more exclusive, and unless mutual friends have intervened, a seat there is almost an impossibility. The next best thing is to be in a small pension, where good French, but no English, is spoken. It is a matter of ever-increasing difficulty to find such a place. The charges near the Sorbonne vary from 150 to 250 francs per month; for 180 francs one may be excellently served. A third method, not less expensive, is to rent a furnished room and dine at a restaurant. A room may be procured for 30 to 65 francs per month. But restaurant cooking and poor French frequently heard are undesirable features of this plan. It sometimes happens, however, that meals alone (that is lunch and dinner) may be arranged for at a good pension for 100–150 francs per month. Pensions on the other side of the Luxembourg from the Sorbonne are usually the cheaper. As some of the courses which the American will likely want to follow at both the École Normale Supérieure and the Sorbonne commence at half-past eight in the morning, it will be well not to live too far away. All necessary expenses of the student who spends the year in Paris, with summers elsewhere, ought not to exceed 750 dollars.

At the Université de Paris the year commences about November 4th and, in contrast to German methods, the lectures start immediately. The student will find the opening addresses of the vice-recteur and doyens, delivered in the great amphitheatre a day or two after the semester has begun, of interest. Matriculation is a very informal, though tiresome, affair. A college diploma and a certified French translation of birth certificate should be taken to the Secretary's office. It is with an air of considerable scepticism that the Frenchman receives the statement that birth certificates are the exceptional possession of the average Canadian or American. The difficulty, in the case of the Canadian, is very easily solved on applying to the Canadian Commissioner, rue de Rome, and doubtless like courtesy awaits the American at the bureau of his Consul General. A passport is not necessary in France, as in Germany. The fee of 30 francs gives the student all rights of library, lecture and conference, for the whole year. To follow the classes at the École Normale it is only necessary to go through the formality of applying for permission to the Vice-recteur of the Université de Paris. The letter received in reply should be presented to M. Tannery, who will most cordially counsel and assist the newcomer. The courses at the Collège de France commence about December 3. The general holidays—one week before and one week after New Year's day, two weeks similarly arranged with reference to Easter—are the same as at the Université de Paris. There is no break between the first and the second semester, which begins March 1. All lectures are freely open to the public.

Just what courses of those offered in these three institutions will particularly appeal to the American student must naturally be both a matter of taste and of previous study. I have given the details of the courses in earlier pages. Appell, Goursat and Picard are especially noted for the elegance and clarity of their presentation. But, except from a pedagogical standpoint, Appell does not, at present, give anything of interest to the American student; while the first half of Goursat's course will be found more or less of a review of earlier work. The courses of Darboux, Poincaré and Painlevé are of a more advanced nature and largely attended. Such conferences as those of Raffy[1] and

[1] Raffy died since I wrote the above; his death occurred June 9, 1910.

Cartan at the Sorbonne, of Tannery, Borel and Hadamard at the École Normale Supérieure, should on no account be neglected. The training and grounding they give is simply invaluable.

If the interests of the student wander into other fields, the opportunity for profit is just as great as in the department of mathematics. There are the chairs in the Faculté des Sciences, such as physics, chemistry, biology, not to mention those held by other world renowned savants in literature, history, philosophy, etc., of the Faculté des Lettres. Any matriculated student in the Faculté des Sciences may have his name inscribed in this Faculté without further charge. The lectures of Reinach, Michel, etc., at the École du Louvre are open to all and are of especial appeal to those interested in the various phases of art. Indeed, as soon as one leaves special for general study, the riches of intellectual treat on every hand lead to embarrassment of choice.

The book treasures and collections available for the student in Paris are unequalled by any other city in the world. Chiefly by cooperation of the *Société Mathématique de France*, the Sorbonne possesses a remarkably complete collection of mathematical periodicals. The officials of the library are exceedingly helpful and most generous to the earnest student; not only do they grant admission to the periodical section, but, occasionally, the privilege of exploring the general stacks as well. Since the catalogue is poor, this facility for the searcher is of inestimable value. The librarian is always ready to purchase any standard work which is not in the library and which the student specially needs. The Bibliothèque Nationale[1] contains nearly all mathematical periodicals lacking at the Sorbonne, a tolerably complete set of French mathematical publications, as well as a representative collection of those of other countries. When the need arises of consulting older mathematical works, this library or the Bibliothèque Mazarine is pretty sure to be able to supply the want. The Bibliothèque Sainte Geneviève has, among others, a good collection of elementary mathematical books.

Finally, under what conditions may an American mathematical student in Paris proceed to the doctorate? The general question has been fully considered in earlier pages, and only a few observations remain to be made here. We have remarked that two degrees are available, the *doctorat d'état* and the *doctorat de l'Université de Paris*. In both cases the candidate must receive permission from the minister of public instruction to present a university diploma as an *Equivalence de Scolarité* of the baccalauréat. In both cases the Thèse (which has

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[1] A card of admission will be granted on presenting a letter from either the Canadian Commissioner or the American Consul General.

always been written in French) is the principal thing required after the student has received the necessary *certificats*. Although the Frenchman works his out independently, the American student will not derive less cordial help or suggestion from a French than, under similar circumstances, from a German professor. He will find, however, that this cordiality is very unlikely to expand in the former case, as in the latter it is almost sure to do, to an invitation to be a guest in the home. The question of *certificats*, with their exacting examinations, makes the doctorat d'état decidedly the more difficult. There is, however, no reason why a student who has the ability to get his doctorate in Germany in two years may not get the doctorat de l'Université de Paris in the same time. Indeed, if he be well equipped, have a thèse well under way before coming to France, and is prepared to "scorn delights and live laborious days," this doctorate is a possibility in one year. But since to 'those who know' no other doctor's degree in mathematics has quite as high a standard as the *doctorat d'état*, why does not some Canadian aspire to be the first in the British Empire to win it? Or why does not some young man from the United States feel spurred to show his equality with one of his countrywomen?

AUTHORITIES.

For the study of primary and secondary education for boys, three publications are essential:

Organisation pédagogique et plan d'études des écoles primaires élémentaires.

Plan d'études et programmes d'enseignement des écoles primaires supérieures de garçons.

Plans d'études et programme d'enseignement dans les lycées et collèges de garçons.

All departments of education are dealt with by the invaluable *Bulletin* [hebdomadaire] *administratif du ministre de l'instruction publique*, 1850– and *Annuaire de l'instruction publique et des beaux-arts*. The budgets may be found in the *Journal Officiel*, and statistics of various kinds in the *Annuaire Statistique* of the *Ministère du travail et de la prévoyance sociale direction du travail.*

Of unofficial publications which I have sometimes found useful are:

Le Nouveau Baccalauréat de l'enseignement secondaire; guide du candidat.

Programmes des certificats d'études supérieures.

L'Université de Paris et les établissements Parisiens d'enseignement supérieur. Livret de l'étudiant, published by the *Bureau des renseignements* at the Sorbonne.

Annuaire de la jeunesse, of H. Vuibert.

Other authorities are indicated in Appendix C.

Interesting comment may be found in Klein's *Vorträge über den mathematischen Unterricht an den höheren Schulen* (1907), and in Klein's *Elementar Mathematik vom höheren Standpunkte aus—Teil II: Geometrie* (1909. Pp. 456–77: "Unterricht in Frankreich").

It is with great pleasure that I have also to acknowledge my indebtedness to a very large number of friends, acquaintances and officials in the Sorbonne, the École Normale Supérieure, the École Polytechnique, in the lycées and in the different departments of the ministère de l'instruction publique. The invariable courtesy and obliging readiness to place all possible material at my disposal, which my innumerable inquiries called forth, constitute a very pleasant memory, among the many, of a delightful year.

Paris, May 2, 1910.

[After my paper had been written, I saw, by chance, a reprint of an interesting article, with the same title as this, published by Professor Pierpont about ten years ago, in the *Bulletin of the American Mathematical Society.* It suggested several improvements (in connection with questions of form and fuller development) which were then made in my paper. Concerning recent literature reference may be given to: "France as a Field for American Students" by S. Newcomb (*Forum,* xxiii., 320-326, May 1897. French translation, *Revue Internationale de l'Enseignement,* xxxiv., 20-27, July 1897. *Cf.* also *Nation,* lxiii., 400-01, Nov. 26, 1896) — The chapter on "The Universities" in B. Wendell's "The France of To-day" (second edition, 1908)—"Life at the Sorbonne" by H. Jones (*Nation,* xci., 576-7, Dec. 15, 1910).]

APPENDIX A.

Agrégation des Sciences Mathématiques.

As there are no mathematical examinations in any other country to compare in difficulty with those to which the candidate for a French *agrégation* is required to submit, it has seemed to me that it would be a matter of interest if fuller details of what is involved were set forth. I therefore subjoin:

I. The programme for the *concours* of 1910 (announced 9–11 months in advance). The examinations will be on topics selected from this programme.

II. The examination papers for 1909. The four written examinations, it will be observed, occurred on consecutive days. The first paper may seem short for the time allowed (seven hours); but not when the enormously high standard in presentation and detail is taken into consideration.

III. A Table giving certain data respecting the *agrégés* named during the last twenty-five years, which show that the impressions prevailing as to the age of the *agrégé*, the number of *Normaliens* who become *agrégés*, and the number of *agrégés* who became doctors are quite erroneous.

PART I.

PROGRAMME FOR THE *CONCOURS* OF 1910.

I.—General Programme in Analysis and Mechanics.

Since the programmes for the *certificats d'études supérieures* vary among the different universities, the jury indicate in the programme below the minimum of general knowledge which is supposed acquired by the candidates in differential calculus, integral calculus and mechanics.

The subjects of the "compositions" in differential calculus, integral calculus and mechanics will be chosen from Nos., 1°, 2°, 3°, 4°, 5°, 7°, 8°, 9°, 14° and 15° of this programme; their scope will not exceed the standard set by the subjects of problems proposed for the corresponding *certificats* for the *Licence.*

Differential Calculus and Integral Calculus.

1°. *Fundamental Operations of Differential and Integral Calculus:* Derivatives and differentials; simple integrals, curvilinear integrals, integrals of total differentials, double and triple integrals.

2°. *Applications of the Differential Calculus:* Study of functions of a real variable (formula of Taylor, maxima and minima, functional determinants, implicit functions); Calculation of derivatives and differentials; change of variables.—Order of contact and *genre* of an area.

3°. *Applications of the Integral Calculus:* Process of integration. Length of an arc of a curve, plane and gauche areas, volumes. Differentiation and change of variables under the sign $\iint$...... Study of the integral $\int_a^b f(x)\,dx$ when one of the limits or the function becomes infinite. Formula of Green.—Study of functions represented by certain series. —Properties of power series.

4°. *Elements of infinitesimal Geometry:* "Infinitesimal properties" of plane and gauche curves (curve envelopes, curvature, torsion). Infinitesimal properties of the surfaces; surface envelopes, summary of the results on the transformations of contact; developable surfaces, ruled surfaces, Meusnier's theorem; principal sections.—Conjugate lines, lines of curvature, asymptotic lines in any curvilinear co-ordinates.

5°. *Elementary Functions of a Complex Variable:* Simple algebraic functions; circular and logarithmic functions.

6°. *Theory of Analytic Functions:* Properties of the integral $\int f(z)\,dz$. Series of Taylor and of Laurent. Poles, essentially singular points, residues.—Reduction of the hyperelliptic integrals.

7°. *Differential Equations of the first order:* General solutions, particular solutions, singular solutions.—Simple types of integrable equations. Integrating factor.—Theorem of Briot and Bouquet on the existence of the solutions in the cases where the known functions are analytic.

8°. *Differential Equations and Systems of Equations of any order:* General solution, particular solutions, first solutions.—Simple types of integrable Equations. Linear Equations.

9°. *Integration of linear partial differential equations of the first order.*

10°. *Integration of differential equations (partial or total) of the first order.*

Mechanics.

11°. *Statics:* Composition of forces applied at a point.—Attraction of a spherical homogeneous shell at an exterior or interior point. Elementary properties of the potential.—Reduction of forces applied to a solid body.—Conditions of equilibrium of a solid body. Applications to simple machines.—Funicular polygon. Suspension bridges. Catenary.—Principal of virtual work.

12°. *Kinematics:* Velocity, acceleration.—Movement of a plane figure in its plane. Representation of the movement by the rolling of a moving curve on a fixed curve.—Movement of a solid body about a fixed point. Representation of the movement by the rolling of a moving cone on a fixed cone.—Movement of a solid body in space. Helicoidal movement.—Relative movements. Theorem of Coriolis.

13°. *Dynamics of a Particle:*—Work. General Theorems.—First integrals of the equations of the motion.—Application to the motions of the planets.—Movement of a point on a curve or on a surface. Pendulum in a vacuum and in a resisting medium. Conical Pendulum. Geodetic Lines.

14°. *Geometry of Masses:* Centres of gravity. Moments of Inertia.

15°. *Dynamics of Systems:* General Theorems. First Integrals.—Energy, Stability of Equilibrium.—Movement of a solid body about a fixed axis. Pressure supported by the axis. Compound Pendulum.—Movement of a solid body about a fixed point.—General movement of a solid body.—Law of friction and slipping.—Application of the principle of *vis viva* to machines.—D'Alembert's Principle.—Lagrange's Equations. —Relative motion.—Percussions.

16°. *Canonical Equations:* Theorem of Jacobi.

17°. *Hydrostatics:* Equilibrium of a fluid mass. "Surfaces de niveau." Pressure on a side plane. Archimedes' principle. Equilibrium of floating bodies.

18°. *Hydrodynamics:* General equations of the movement of a fluid mass. Bernoulli's Theorem. Torricelli's Theorem.

LESSONS.

Parts of the programmes from which are drawn the subjects of the lessons.

1.—MATHÉMATIQUES SPÉCIALES.

Series: Series of positive terms; character of convergence or divergence drawn from the study of the expressions:

$$\frac{U_{n+1}}{U_n}, \sqrt{} U_n, n^p U_n.$$

Absolutely converging series. Convergence of series, with terms alternately positive and negative, of which the general term decreases constantly in absolute value and tends towards zero. Numerical examples.

General Properties of Algebraic Equations: Number of roots of an Equation. Relations between the coefficients and the roots. Every rational and symmetric function of the roots may be expressed rationally as a function of the coefficients.—Elimination of one unknown between two equations by means of symmetric functions.—Condition that an equation has equal roots. Study of the commensurable roots.—Descartes' Theorem.—Complex numbers. De Moivre's Theorem. Trignometric resolution of the binomial equation.

Functions: Function of a real variable, graphic representation, continuity.—Definition and continuity of the exponential function and of the logarithmic function. Limit of $(1 + \frac{1}{m})^m$ when m increases indefinitely in absolute value.—Derivative of a function; slope of the curve represented. Derivative of a sum, of a product, of a quotient, of an integral power; of a function of a function. Derivative of a^x and of log x. — Use of logarithm tables and of the slide rule. —Rolle's Theorem, law of finite increments, graphic representation.—Functions of several independent variables, partial derivatives. Law of finite increments. Derivative of a compound function. Derivative of an implicit function (admitting the existence of this derivative). —Employment of the derivative for the study of the variation of a function; maxima and minima. Primitive functions of a given function, their representation by the area of a curve.

Functions defined by a power series with real coefficients. Interval of Convergence: Addition and multiplication. In the interior of the interval of convergence one obtains the derivative or the primitive functions of the function, on taking the series of derivatives or of the primitive functions (functions which pass to the extremities of the interval are not considered).—Examples:—developments in series of $\frac{1}{1-x}$, $\frac{1}{1+x^2}$, arctan x, log (1—x), log $\frac{1-x}{1+x}$. Exponential series. Binomial series. The equations $y^1 = y$, and $y^1 (1 + x) = my$ serve to determine the sum of two series.—Development into series of a^x, of arcsin x.

Curves whose equation is soluble or insoluble with regard to one of the co-ordinates: Tracing. Equation of the tangent at a point; sub-tangent. Normal, sub-normal. Concavity, convexity, points of inflexion. Asymptotes. Application to simple examples and in particular to the conics and to those curves of which the equation is of the second degree with respect to one of its co-ordinates.

Curves defined by the expression of the co-ordinates of one of their points as function of a prameter: Tracing. Numerical examples. The curves of the second order and those of the third order with a double point are unicursal.

Curves defined by an implicit equation: Equation of the tangent and of the normal at a point. Tangents at the origin in the case where the origin is a simple point or a double point. Discussion of the asymptotes in the case of numerical examples of curves of the second and of the third order.

Curvature. Envelopes. Developables.

Polar Co-ordinates: Their transformation into line co-ordinates. Equation of a right line.—Construction of curves; tangents, asymptotes

Applications (confined to the case when the equation is solved with respect to a radius vector). Case of the conics.

Gauche Curves: Tangent. Osculating plane. Curvature. Applications to the circular helix.

Study of surfaces of the second degree with reduced equation: Condition of the contact of a plane with the surface. Simple problems relative to tangent planes. Normals. Properties of conjugate diameters. Theorems of Apollonius for the ellipsoid and the hyperboloids. Circular sections. Rectilinear generatrices. The surfaces of the second order are unicursal.

DYNAMICS.

1. *Free Material Point:* Principle of inertia. Definition of force and mass.[1] Relation between the mass and the weight. Invariability of the mass. Fundamental units. Derived units. Movement of a point under the action of a force, constant in magnitude and direction or under the action of a force issuing from a fixed centre: 1° proportional to the distance; 2° in the ratio inversely as the square of the distance.—Composition of forces applied at a material point.[2]—Work of a force, work of the resultant of several forces, work of a force for a resulting displacement. Theory of living force. Surfaces de niveau. Fields and lines of force. Kinetic energy and potential energy of a particle placed in a field of force.

2. *Material Point, not free:* Movement of a heavy particle on an inclined plane, with and without friction, the initial veolcity acting along the line of greatest inclination. Total pressure on the plane; reaction of the plane. Small oscillations of a simple pendulum without friction; isochronism.

DESCRIPTIVE GEOMETRY.

Intersection of Surfaces: Two cones or cylinders, cone or cylinder and surface of revolution, two surfaces of revolution of which the axes are in the same plane.

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[1] It is admitted that a force applied at a material point is geometrically equal to the product of the mass of the point by the acceleration that it impresses on the point.

[2] It is admitted that, if several forces act at a point, the acceleration that they impress on the point is the geometric sum of the accelerations that each of them impresses on it, if acting alone.

II.—Lessons on the Subjects of the Programme of the Seconde and Première (C and D) and Mathématiques A.

Seconde (C. and D.).

Algebra: Resolution of equation of the first degree in one unknown. Inequalities of the first degree. Resolution and discussion of two equations of the first degree in two unknowns.—Problems; substitution in equation. Discussion of the results.—Variation of the expression ax+b; graphic representation.—Equations of the second degree in one unknown (theory of imaginaries not discussed). Relations between the coëfficients and the roots.—Existence and signs of the roots. Study of the trinomial of the second degree.—Inequalities of the second degree. Problems of the second degree. Variation of the trinomial of the second degree. Graphic representation. Variation of the Expression $\frac{ax + b}{a^1x + b^1}$; graphic representation.—Notion of derivative; geometrical significance of the derivative. The sign of the derivative indicates the direction of the variation; applications to very simple numerical examples and in particular to the functions studied before.

Geometry: Simple notions of homothetic figures. Similar polygons. Sine, cosine, tangent and cotangent of positive angles less than 2 right angles. Metrical relations in a right triangle and in any triangle. Proportional lines in the circle. Fourth proportional; mean proportional.—Regular polygons. Inscription in a circle of a square, of a hexagon; of an equilateral triangle, of a decagon, of a quindecagon. Two regular polygons of the same number of sides are similar. Ratio of their perimeters. Length of an arc of a circle. Ratio of the circumference to the diameter. Calculation of π (confined to the method of the perimeters).—Area of polygons; area of a circle. Measure of the area of a rectangle, of a parallelogram, of a triangle, of a trapezium, of any polygon.—Ratio of the areas of two similar polygons—Area of a regular convex polygon. Area of a circle, of a sector and of a segment of a circle. Ratio of the areas of two circles.

Première (C. and D.).

Geometry.

Translation: Rotation about an axis. Symmetry with respect to a line. Symmetry with respect to a point. Symmetry with respect to a plane. This second kind of symmetry is equivalent to the first.

Trihedral Angles: Disposition of the elements. Trihedral symmetry. Each face of a trihedral is less than the sum of the other two. Limits of the sum of the faces of a trihedral.—Supplementary trihedrals. Applications.—Inequalities of the trihedrals.

Homology: Parallel plane sections of polyhedral angles. Areas.

Polyhedra: Homothetic polyhedra, similar polyhedra. Prisms, Pyramids.—Summary of notions on the symmetry of the cube and of the regular octahedron.—Volumes of parallelopipeds and of prisms. Volume of the Pyramid.—Volume of a pyramid truncated by parallel sections. Volume of a truncated triangular prism.—Ratio of the volumes of two similar polyhedra.—Two symmetrical polyhedra are equivalent.—Sphere: plane section, poles, tangent plane. Circumscribed cone and cylinder. Area and volume.

Mathématiques A

Arithmetic: Common fractions. Reduction of a fraction to its simplest terms. Reduction of several fractions to a common denominator Least common denominator. Operations with common fractions.—Decimal numbers. Operations (considering the decimal fractions as particular cases of ordinary fractions). Calculation of a quotient to a given decimal approximation.—Reduction of an ordinary fraction to a decimal fraction; condition of possibility. When the reduction is impossible, the ordinary fraction can be regarded as the limit of an unlimited periodic decimal fraction.—Square of a whole number or of a fractional number; nature of the square of the sum of two numbers. The square of a fraction is never equal to a whole number. Definition and extraction of the square root of a whole number or of a fraction to a given decimal approximation.—Definition of absolute error and of relative error. Determination of the upper limit of an error made in a sum, a difference, a product, a quotient, knowing the upper limits of the errors by which the given quantities are affected.—Metric System.

Algebra: Monomials, polynomials; addition, subtraction, multiplication and division of monomials and of polynomials. Equations of the second degree in one unknown. Simple equations which are equivalent. (The theory of imaginaries is not developed).—Problems of the first and second degree.—Arithmetic Progressions. Geometric Progressions. Common Logarithms. Compound Interest, annuities.

Trigonometry: Circular Functions. Addition and Subtraction of arcs. Multiplication and division by 2.—Resolution of triangles. Applications of Trigonometry to various questions relative to the elevation of planes. (The construction of the trigonometric tables is not to be considered).

Geometry: Inversion. Applications. Peaucellier's Cell.—Polar of a point with respect to a circle. Polar plane of a point with respect to a sphere.—Hyperbola: Trace, tangent; asymptotes; simple problems on tangents. Equation of a hyperbola with respect to its axes. Plane sections of a cone and of a cylinder of revolution.

Vectors: Projection of a vector on an axis; linear moment with respect to a point; moment with respect to an axis. Geometric sum of a system of vectors; resultant moment with respect to a point. Sum of the moments with prospect to an axis. Application to a couple of vectors.

Descriptive Geometry: Rabatting. Change of plane of Projection; rotation about an axis perpendicular to a plane of projection.—Application to distances and angles; distance between two points, between a point and a line, between a point and a plane; the shortest distance between two lines of which one is vertical or at right angles to the plane, or of two lines parallel to the same plane of projection; common perpendicular to these lines. Angle between two lines; angle between a line and a plane; angle between two planes.

Kinematics: Units of length and of time.—Motion. Relative motion. Trajectory of a point. Examples of motion.—Rectilinear motion; uniform motion; velocity, its representation by a vector. Varied motion, mean velocity; velocity at a given instant, its representation by a vector; mean acceleration; acceleration at a given instant; its representation by a vector. Uniformly varied movement. —Curvilinear motion. Mean velocity, velocity at a given instant defined as vectors. Algebraic value of velocity. Hodograph. Acceleration.—Uniform circular motion, angular velocity; projection on a diameter. Simple oscillation in a line.—Change of the system of comparison. Resultant of velocities.—Examples and applications. (Purely geometrical applications are not to be insisted upon).—Geometrical study of the helix. Helicoidal motion of a body. Screw and nut.

Dynamics: Work of a force applied to a material point. Unit of work. Work of a constant force, of a variable force. Elementary work, total work. Graphical evaluation. Work of the resultant of several forces. Theorem of forces acting on a material point. Simple examples.

Cosmography: Moon. Apparent proper motion on the celestial sphere. Phases. Rotation. Variation of the apparent diameter. Eclipses of the moon and of the sun.

PART II.

EXAMINATIONS IN THE *CONCOURS* FOR 1909.

(1) *WRITTEN*.

MATHÉMATIQUES ÉLÉMENTAIRES.*

[Time, 7 hours; 7 a.m.-2 p.m].

Given two circles, with centres O and O', radii R and R'; these circles are exterior to one another, and the common exterior tangents are drawn, the points of contact being A and A', B and B', whilst the points of contact of the common interior tangents are C and C', D and D', the points A and C being on either side of the line of centres whereas the contrary takes place for the points A' and C' if we have, as is supposed, $R<R'$. The tangents AA' and CC' cut in E, the tangents BB' and DD' cut in F, and the line EF meets the line OO' in the point G; the tangents AA' and DD' cut in I, the tangents BB' and CC' cut in J and the line IJ meets the line OO' in the point K. Consider the lines AC, BD and $A'C'$, $B'D'$, which cross at the point K.

1°. In order that the lines AC and $B'D'$ become the coincident line r, in which case the lines BD and $A'C'$ become the same line s, it is necessary and sufficient that the orthoptic circles of the two given circles are orthogonal, which is equivalent to the metric relation

$$\overline{OO'}^2 = 2(R^2 + R'^2)$$

(The orthoptic circle of a circle is the circle which is the locus of points from which one sees the given circle under a right angle).—The point G is then the middle of the segment OO'—The preceding condition is supposed fulfilled in all which follows.

2°. If R is a point of the line r, the polars of this point with respect to the two circles O and O' cut in a point S situated on the line s;

3°. The envelope of the line RS is a conic, which is to be determined by metrical elements; determine the principal tangents. The locus of the orthocentre P of the triangle ORS is a conic, of which it is required to find some remarkable points; same question for the triangle $O'RS$.

4°. Suppose RM, RN and RM', RN' the tangents drawn from a point R of the line r to the two circles O and O'; the plane being oriented in the sense $ABCD$, let α, β and γ, δ be the angles, made with an axis r by the half-lines of the tangents, situated on the same side of the line r for each of the circles O and O' (these angles are found again at O and O'); setting

$$\frac{\alpha+\beta}{2} = x, \quad \frac{\gamma+\delta}{2} = y, \quad \frac{\beta-\alpha}{2} = u, \quad \frac{\delta-\gamma}{2} = v.$$

*See for solutions to questions in this paper *Nouvelles Annales de Mathématiques* (4) IX, 455-67, 1909.

MECHANICS.

[Time, 7 hours ; 7 a.m.-2 p.m.]

A kite of weight P is subject to normal action by the wind, represented by a force $\frac{3\sqrt{3}}{2}P$. In its position of equilibrium it is inclined at an angle of 30° to the horizon; it has an axis of symmetry on which is its centre of gravity G and the centre O of the push of the wind; O is above G and OG equals 4 centimetres.

At a point A of the axis, below G, 40 centimetres, is attached a string of length l; two other strings of length l' are attached in two points B and C symmetrical with respect to the axis, the line BC, equal to $2d$ is 29 centimetres above G. In the position of equilibrium these three strings, flexible, inextensible and without mass are tight and united in a point M at which is attached the string which holds the kite.

1°. Find the relation which connects l, l' and d; supposing these lengths known calculate the tensions of the three strings.

2°. The point M being 30 metres above the earth's surface, what is the tension of the other extremity E of the string supposed fixed on the earth, flexible, inextensible and of weight p per unit of length; determine p such that the tangent at E is horizontal (action of wind on the string is to be neglected).

3°. Under these conditions, suppose that the string, lengthened from E, unroll with friction of coefficient f along a helix traced on a fixed cylinder of revolution, of which the axis is perpendicular to the plane of the string, the radius of the cylinder being r and the pitch of the helix h; what will be the necessary force to maintain equilibrium, this force being applied at the new free extremity of the string supposed unrolled for a complete spiral? (The weight of the part unrolled is to be neglected).

4°. The string holding the kite having the form found above (2°) and being supposed indeformable, place at the extremity situated on the earth a runner [*postillon*] subject to a force, the resultant of the weight of the runner and of the action of the wind; this force is constant and is in the plane of the string; what condition must be fulfilled that the runner move, supposing that there is a coefficient of friction 1?

Study the movement of the runner in the case where the force is horizontal. (It is supposed that the runner is a material point moving with coefficient of friction 1 on the material curve represented by the string which is supposed indeformable.

6 July.

Differential and Integral Calculus.

[Time, 7 hours ; 7 a.m.-2 p.m.]

Ox, Oy, Oz being three given rectangular axes, consider a surface S, of a single sheet. Suppose s any portion of S, without any common point with Oz and not having a tangent plane parallel to Oz.

I. Suppose A the area of the projection of s on the plane of xy; B the volume bounded by the area s, its projection A and the projecting cylinder; C, the volume bounded by the area s and the cone having this area for base the origin for vertex; D, the volume bounded by the area s and by the conicoid which has the contour of s for directrix, Oz for axis and xOy for director plane.

The quantities B, C, D representing the volumes in question with suitable signs, show that

$$(1) \qquad 3C = B - 2D$$

as long as the area s is not cut by certain lines situated on S. Show also that the formula is still true without this last restriction, if the elements of B, in magnitude and sign, be always such that

$$B = \int\int z\, dx\, dy$$

(the double integral being applied to the area A), and if at the same time the elements of volumes C, D, are also affected by suitable signs depending on x, y, p, q $\left(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right)$. Indicate as far as possible the geometrical conventions of sign to which we are thus led.

II. The cone (supposed reduced to a single nappe) which bounds the volume C, determines, on the cylinders of revolution of radius 1 which has Oz for axis, an algebraic area of which the elements will be affected by the same signs as the corresponding elements of C, in conformity to the preceding conventions: suppose E this area.

On the other hand turn s about Oz and designate by F the volume of revolution thus generated; by G, the area of the meridian section of this volume, an element of F or of G being equally affected by a sign (the same in the two cases) according to suitable convention.

Determine the surface S such that, for every portion s (without point common with Oz or tangent plane parallel to Oz) taken on this surface, we have the relation

$$(2) \qquad aA + bB + 3cC + eE + \frac{f}{2\pi}F + gG = 0$$

where a, b, c, e, f, g are constants. Show that S will verify a certain partial differential equation of the first order of which the coefficients are rational functions of x, y, z, ρ, where $\rho = \sqrt{x^2+y^2}$ (the radical being taken as positive). Indicate (again geometrically) the determination of the common sign to give to any element of F and to the corresponding element of G such that this equation is the same for the whole surface under consideration.

Volumes of parallelopipeds and prisms; of the pyramid. Unnecessary to consider truncated pyramid or prism.

Notion of the derivative. Geometrical interpretation. Applications.

Homothetic polyhedra. Similar polyhedra. (Programme of the Première).

Problems of the second degree.

First lesson on regular polygons.

Work, kinetic energy for a particle. Simple examples.

Decimal numbers. Operations. Calculation of a quotient to a given decimal approximation.

Resolution of triangles (omitting right angled triangles).

Multiplication and division of arcs by 2.

Summary of notions on the symmetry of the cube and of the regular octahedron.

Rabatting. Applications; angle between two lines, a line and a plane, two planes.

Calculation of Π.

Upper limit of absolute error of a sum, of a difference, of a product of two factors, of a quotient, of a square root.

Volume of a sphere. Spherical segment.

Tangent to a hyperbola. Asymptotes. Simple problems on tangents.

Theory of moments with respect to, a point, an axis.

Mathématiques Spéciales.

Movement of a heavy particle on an inclined plane with and without friction, the initial velocity being zero or directed along the line of greatest inclination.

Power Series. Interval of convergence. Differentiation. Integration.

Series of positive terms. Nature of the convergence or divergence drawn from the study of the expression

$$\frac{u_{n+1}}{u_n},\ \sqrt[n]{u_n},\ n^p u_n,\ \text{numerical examples.}$$

Discussion of the commensurable roots of an equation with integral coefficients. Examples.

Movement of a point attracted by a fixed centre of force in the ratio of the inverse square of the distance.

Functions of several independent variables. Partial derivatives. Formula of finite increments; derivatives of a compound function.

Concavity, convexity. Points of inflexion (rectangular coordinates).

Elimination of one unknown between two algebraic equations by means of symmetric functions.

Normal to an ellipsoid.

Establish the relations,

$$\cos^2 u = \left(1 + \frac{R'^2}{R^2}\right)\cos^2 x, \quad \cos^2 v = \left(1 + \frac{R^2}{R'^2}\right)\cos^2 y,$$

$$\frac{1 + \tan x}{1 + \tan y} = -\frac{R'^2}{R^2}$$

Verify by means of these relations, that the tangents RM, RN and RM', RN' form a harmonic pencil.—The point R of the line r can also be replaced by a point S of the line s.

2 July.

Mathématiques Spéciales.*

[Time, 7 hours; 7 a.m.-2 p.m.]

Given a parabola (P) and a line (D) of which the equations with respect to a system of rectangular coordinate axes, are :

$$(P) \begin{cases} y - 2px = o \\ z = o \end{cases} \qquad (D) \begin{cases} y = o \\ z = a \end{cases}$$

and, suppose the surface (S) generated by a variable line (Δ) which meets (P) in A and (D) in a point B, such that the distance AB is a constant l.

1°. Construct the projection on the plane XOY of a section of the surface by a plane parallel to the plane XOY; construct the tangent in a point of this projection and show that the curve obtained can be regarded as the locus of the middle points of the chords parallel to OX and limited, on the one hand by a parabola of vertex O and axis OX, on the other hand by an ellipse of which the axes are in the direction OX and OY.

2°. Two kinds of lines $\triangle$ can be distinguished, according as the abcissa of A is superior or inferior to that of B; in the preceding sections separate the arcs which correspond to the generatrices of the one system or the other and find the locus of the points which limit these arcs.

3°. Consider the solid limited by the surface (S) and by the planes $z + a = 0$ $z - 2a = 0$; find its volume and construct its apparent contour on the plane ZOX.

4°. Determine the orthogonal trajectories of the lines $(\triangle)$. Through a point A, two lines $(\triangle)$ can be drawn to meet an orthogonal trajectory in two points C and C'; show that this trajectory can be chosen such that the sum $\overline{AC} + \overline{AC'}$ is proportional to the abcissa of A—Can the given constants be chosen such that only one orthogonal trajectory meets all the lines $(\triangle)$ between their points situated on the parabola (P) and on the line (D)?

3 July.

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* The solutions of the questions in this paper are given in *Revue de Mathématiques Spéciales* Juin, 1910; X, 532-540.

Find the characteristic curves of the partial differential equation thus obtained, by employing the semipolar coordinates ρ, ω (polar coordinates of the projection of the point on the plane xOy), z (coordinate of the point).

Study the projections of these characteristic curves on the plane xOy. Show that there exist characteristic curves which are situated on a cylinder of revolution with the axis Oz, and discuss their form.

III. Suppose the constants b, c, bound by the relation

$$(3) \qquad b+3c=0$$

and consider the curvilinear integral,

$$I=\int\left[z\left(c+\frac{e}{\rho^3}\right)-\frac{a}{2}\right](x\partial y-y\partial x)\pm\rho\left(\frac{\rho}{2}f+g\right)\partial z$$

taken from a point M to a point M' of the surface S, along the path L situated entirely on this surface. Show that if S satisfies the condition which has been imposed on it in the II. part, and if, under the sign $\int$, the sign of the term in ∂z has been suitably chosen, the integral I does not change its value, when M and M remaining fixed, we change in a continuous manner on the surface, the line L traced between these two points.

If, instead of the relation (3), the constants b, c are connected by the relation

$$(4) \quad b+6c=0$$

a property analogous to the preceding appertains to the integral

$$J=\int z\frac{(c\rho^3+e)^2}{\rho^3}(x\partial y-y\partial x)-\epsilon gz\frac{c\rho^3+e}{\rho}(x\partial x+y\partial y)+P\left[\epsilon f\rho^2\partial z-a(x\partial y-y\partial x)\right]$$

where P is a suitably chosen polynomial in ρ and ϵ is one of the two quantities $+1$, -1.

IV. Suppose further that the surface S contains a circumference of which the plane passes through Oz and which has no point common with Oz, or with the cylinder of revolution considered above (end of II. part).

On each of the characteristic curves for the different points of this circumference take a finite arc, such that the portion Σ of S thus determined does not contain any singularity.

Supposing given the value of the integral I (in the case of relation (3)) or J (in the case of the relation (4)), the length of a certain path L joining M and M' and situated on Σ, what are the other values that this integral can acquire when L is scccessively replaced by all the other paths which can be traced between the same points on Σ?

Indicate the relation which exists between the radius of the circumference the distance of its centre to Oz and the coefficients of equation (2) in order that the integral considered be unique under these conditions.

5 July.

FINAL EXAMINATIONS.

Numerical Calculation.

Consider the differential equation

$$\frac{d^4y}{dx^4} = m^4y.$$

Find the smallest value to give to m in order that the equation admits a solution of which the representative curve, symmetric with respect to Oy is tangent to Ox at the points A, A' of abscissae $x = +1$, $x = -1$, the value y corresponding to, $x = 0$ being equal to 1 (point B)—Determine the points of inflection between A and A' of the representative curve of y. Find, as exactly as possible, the portion AB of this curve, the unit of length being supposed equal to 40 divisions of the square employed.

Descriptive Geometry (Diagram).*

An equilateral hyperbolic paraboloid has for vertex the point *de cote* 10 cm. and *d'éloignement* 10 cm. projected on the major axis of the sheet; a principal parabola P is horizontal and its focus *de cote* 10 cm. and *d'éloignement* 10 cm. is situated 1 cm. 5 m. to the right of the vertex.

A second hyperbolic paraboloid has director plane, a plane of profile; of rectilinear generativers there are: 1°. the axis of the first paraboloid; 2°. a horizontal *de cote* 13 cm., of which the projection on the plane of the parabola P meets the axis of this parabola 3 cm. to the left of the vertex and the tangent at the vertex 3 cm. in front of the vertex.

Consider, on the one part, the region A of the space limited by the first paraboloid and which corresponds to the interior of the parabola P; on the other part, the region B of the space limited by the second paraboloid and which corresponds to the part of the horizontal plane *de cote* 10 cm. situated in front of the axis of the first paraboloid.

Represent the solid bounded by the two paraboloids, by the horizontal planes *de cote* 17 cm. and 2 cm. and by the plane of the profile situated 10 cm. to the right of the vertex of the first paraboloid, the solid part being always in the regions A, B.

It is supposed that the planes of projection are transparent.

(2) ORAL.

Mathématiques Élémentaires.

Supplementary trihedral angles. Applications.

Symmetry with respect to a line, a point, a plane. (Programme of the Première).

Relations between the coefficients and the roots of the equation of the second degree. Applications.

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*A solution of the problem in this paper is given in *Revue de Mathématiques Spéciales* Nov. 1910, XI, 42-45.

Tangent at a point of a curve of which the coordinates are rational functions of a parameter. Points of inflection. Singular points at a finite distance.

Small oscillations of a simple pendulum without friction (isochronism).

Intersection of a surface of revolution and of a cone.

Theory of envelopes in Plane Geometry.

Conjugate points in connection with a surface of the second order. Conjugate planes. Pole and polar planes. Conjugate lines.

Symmetric and rational functions of the roots of an algebraic equation.

Construction of a curve $\rho = f(\omega)$ in polar coords. (It is supposed that lessons on tangents and asymptotes have been given).

Gauche curves. Tangents. Osculating plane. Curvature. Application to the circular helix.

Number e – limit $\left(1 + \frac{1}{m}\right)^m$.

Field—line of force, function of force, surface de niveau. Theory of kinetic energy at a point.

Theorem of Descartes.

Movement of a point under the action of a force issuing from a fixed centre and proportional to the distance.

Members of the Jury.

Niewenglowski, *Inspector General of Instruction—President.*
Hadamard, *Professor, University of Paris.*
Combette, *Inspector General of Public Instruction.*
Fontené, *Inspector of the Académie.*
Grévy, *Professor, Lycée Saint-Louis.*

PART III.

THE AGRÉGÉS DES SCIENCES MATHÉMATIQUES 1885–1909.

Year	Number of Agrégés fixed by Govt.	Number presenting themselves at Concours	Number admitted to Oral	Number of the Agrégés who became Doctors [1]	Average Age	Number of Agrégés who had been Elèves E.N.S.
1909	14	81	27	0	24+ [21–27]	7
1908	13	75	20	0	25 [22–35]	9
1907	14	54	25	0	27— [23–37]	7
1906	14	58	24	1	27+ [23–37]	7
1905	14	60	20	2	27 [21–33]	7
1904	14	72	26	4	28— [22–33]	10
1903	12	78	25	1	28+ [24–41]	8
1902	12	87	23	1	27—]22–40]	9
1901	10	77	23	2	28 [23–37]	6
1900	8	76	20	0	28— [23–34]	4
1899	8	86	16	1	25— [23–33]	6
1898	8	96	19	2	26— [21–33]	8
1897	7	93	18	3	25— [21–33]	3
1896	12	112	26	1	26+ [23–30]	6
1895	14	125	24	0	24 [20–35]	8
1894	11	126	20	2	26— [21–37]	7
1893	13	134	..	1	26— [22–29]	9
1892	12	125	19	3	24+ [21–27]	7
1891	13	..	22	5	26— [22–30]	7
1890	12	..	16	1	26— [23–30]	4
1889	13	..	..	5	23— [21–32]	7
1888	14	..	23	3	25+ [21–34]	9
1887	13	..	15	5	23— [21–36]	10
1886	13	..	15	6	23+ [20–31]	7
1885	12	..	22	2	27 [20–44]	6
Total....	300			51	$25\frac{4}{5}$	178

[1] These figures were compiled in February, 1910.

The sixth column contains the average age; "24 +" means an age > 24 and $< 24\frac{1}{2}$, "24—" is short for a number < 24 and $\geqq 23\frac{1}{2}$. The figures in square brackets, [ ], show the range of ages for the year.

APPENDIX B.

Mathematical Courses Offered in Universities Outside of Paris

1909–10.

There is nothing in France corresponding to the *Universitäts-Kalendar* of Germany, and it is almost impossible to get any exact information about courses to be offered at even the Uuiversité de Paris, until a couple of days before the Semester commences. The following list is compiled from a variety of sources. It will be observed that Lyons is the only university outside of Paris where any courses, over and above those for the licence and agrégation, are offered.

The letters in brackets after the names of the Academies, indicate the Faculties of the Universities: La. = Law, S. = Science, Le. = Letters, M. = Medicine. The numbers in brackets after the names of Professors, are those in the list of doctors (Appendix C). An "A" added in the brackets is an abbreviation for agrégé.

No information is at hand regarding the Professors in the Université d'Alger which was opened at the beginning of this year, with the Faculties of Science and of Letters, and the mixed Faculty of Medicine and Pharmacy.

Aix—Marseille (La. S. Le. M.)	
Sauvage (142. A)	1. Calculus.
	2. "Cours Complémentaire"
Charve (137)	Mechanics.
Bourget (A)	Astronomy.
Jamet	Cours Complémentaire with Sauvage.
Besançon (S. Le.)	
Lebeuf (231)	Astronomy.
Carrus (272, A)	Calculus.
Andrade (181)	1. Mechanics.
	2. Cours Complémentaire for Engineers.
Franchebois	Préparateur in Mechanics.
Bordeaux (La. S. Le. M.)	
Cousin (209, A)	Calculus.
Delassus (217, A)	1. Mechanics.
	2. Preparatory Mathematics.
Picart (188, A)	Astronomy.
Esclangon (262, A)	Prof. Adjoint and Maître de Conférences.

CAEN (La. S. Le.)	
Riquier (161,A)	Calculus.
Husson (268, A).	Mechanics.
Villat	Maître de Conférences.
CLERMONT (Sc. Le.)	
Pellet (122)	Calculus.
Guichard (151, A)	1. Mechanics. 2. Astronomy.
DIJON (La. Sc. Le.)	
Baire (238, A)	Calculus.
Duport (135, A)	1. Mechanics. 2. Astronomy.
GRENOBLE (La. S. Le.)	
Collet (107)	1. Analysis. 2. Astronomy and Geodesy.
Cotton (242, A)	Mechanics.
Zoretti (264, A)	Maître de Conférences. Cours Complémentaires: 1. Analyse Superieure. 2. Math. Générales.
LILLE (La. S. Le. M.)	
Demartres (156, A)	Calculus.
Petot (171, A)	Mechanics.
Clairin (253, A)	Mathémetiques Générales.
Boulanger (230, A)	Prof. Adjoint and Maître de Conférences: Mechanics.
Traynard (278, A)	Maître de Conférences.
LYON (La. S. Le. M.)	
André (117)	1. Astronomy. 2. Elementary Mathematics (Conférence d'Agrégation.)
Flamme (170)	1. Mechanics. 2. Math. Générales. 3. Mechanics (Conf. d'Agrégation).
Vessiot (192, A)	1. Differential Equations and Calculus of Variations. 2. Theory of Groups of Transformations. 3. Math. Générales: Algebra and Calculus. 4. Higher Geometry (Conf. d'Agrégation).

Le Vavasseur (198, A)	1. Theory of Functions of a Complex Variable and Geometrical Applications of Analysis. 2. Mathématiques Spécial (Conf. d'Agrégation).
Wiernsberger (312)	1. Mechanics. 2. Math. Générales: Analytical Geometry.
Merlin	Chargé de Cours. Astronomy.
MONTPELLIER (La. S. Le. M.)	
Fabry (159, A)	Calculus.
Dautheville (157, A)	Mechanics.
Lattès (274)	Maître de Conférences.
NANCY (La. S. Le. M.)	
Floquet (126, A)	1. Analysis. 2. Calculus.
Vogt (177,A)	Applied Mathematics.
Hahn	Maître de Conférences. Mechanics.
POITIERS (La. S. Le.)	
Lesbesgue (256, A)	1. Calculus. 2. Math. Générales. (Cours Complémentaires)
Boutroux (259)	1. Mechanics. 2. Astronomy.
RENNES (La. S. Le.)	
Lacour (215)	Analysis.
Le Roux (216, A)	Mechanics.
Fréchet (273, A)	Maître de Conférences.
TOULOUSE (La. S. Le. M.)	
Drach (236)	Calculus.
Paraf (193)	Mechanics
Cosserat (176, A)	Astronomy.
Buhl (248)	Math. Générale.
Blondel (A)	Chargé de Conférences.
Saint-Blancat (276)	Assistant Astronomer.

Épreuves écrites et orales du Concours
pour

l'Agrégation des Sciences Mathématiques en 1910

avec les sujets des épreuves finales et les noms des Candidats reçus

suivies du programme pour le Concours de 1911

publiées par la Librairie Croville-Morant, 20, rue de la Sorbonne, Paris.

Compositions

Mathématiques élémentaires

Dans un plan donné, il y a une infinité de cercles (T) orthogonaux à tous les cercles (C) qui passent par deux points donnés P, P' dans ce plan.

A tout point M correspond un point M' tel que M et M' soient conjugués harmoniques par rapport à tout cercle (T).

Donner la construction géométrique de ces points.

1° Trouver le lieu de M et M' quand MM' a une longueur donnée l;

2° Trouver le lieu de M' quand M décrit une droite (D). On pourra supposer successivement que la droite (D) est perpendiculaire à PP', qu'elle passe par l'un des points P ou P'; qu'elle rencontre PP' en un point autre que P ou P', étant oblique à PP' et enfin que (D) est parallèle à PP'.

3° Lieu de M' quand M décrit une parabole passant par P et P' et dont l'axe est perpendiculaire à PP'.

Dans chacun des cas examinés, on étudiera le déplacement de M', celui de M étant supposé connu;

4° On considère dans le plan donné un nombre quelconque de cercles (C), soient (C_1), (C_2), … (C_p), et autant de coefficients numériques donnés a_1, a_2, … a_p; puis q cercles (T): (T_1), (T_2), … (T_q) et des coefficients α_1, α_2, … α_q. Peut-on toujours déterminer, dans le plan donné, deux points S_1, S_2 tels que la somme des puissances d'un point quelconque X du plan par rapport à tous ces cercles, chaque puissance étant multipliée par le coefficient correspondant, soit égale à la somme des carrés des distances de ce point X aux deux points cherchés S_1, S_2, multipliée par la demi-somme des coefficients donnés ? En supposant que les cercles (C) et (T) appartiennent aux deux familles considérées, trouver les conditions pour que S_1 et S_2 soient deux points conjugués M, M'.

1er Juillet

Prix: 1 f.

Mathématiques spéciales

On considère deux paraboloïdes hyperboliques équilatères égaux, P et Q, qui ont même axe et même sommet.

1° On demande de trouver toutes les droites D dont les conjuguées D' et D" par rapport à P et Q sont dans un même plan (on ne considère que des droites réelles situées à distance finie). Montrer que deux droites D' et D" qui correspondent à une même droite D sont à la même distance de l'axe des paraboloïdes, qu'elles font le même angle avec l'axe et que leurs projections sur un plan perpendiculaire à cet axe font un angle constant.

2° On considère une droite D particulière, que l'on désigne par D_1 ; on prend sa conjuguée D_2 par rapport au paraboloïde P, puis on prend la conjuguée D_3 de D_2 par rapport au paraboloïde Q, et ainsi de suite, de telle sorte qu'une droite D_{2p} soit conjuguée de la droite D_{2p-1} par rapport à P et qu'une droite D_{2q+1} soit conjuguée de la droite D_{2q} par rapport à Q ; étudier la distribution de ces droites.

3° Une droite D se déplace en faisant un angle constant avec l'axe des paraboloïdes et de façon que les surfaces engendrées par D' et D" fassent un angle constant au point commun à D' et D" ; quelle surface engendre D ; quelles surfaces engendrent D' et D" et quel est le lieu du point commun à D' et D" ; ce lieu peut-il être situé sur la surface engendrée par D ? (On indiquera un mode de génération simple de ces diverses surfaces.)

2 Juillet.

Calcul différentiel et intégral.

On considère la famille de surfaces du second degré

$$ax^2 + by^2 + cz^2 = \text{Constante},$$

a, b, c étant des nombres positifs distincts et donnés, les axes des coordonnées étant rectangulaires ; on désigne par (E) l'équation aux dérivées partielles des surfaces (S) orthogonales à cette famille de surfaces du second degré et par (γ) les courbes caractéristiques de l'équation (E).

1° Exprimer les coordonnées des points d'une surface (S) à l'aide de deux paramètres u et v ; montrer que l'on peut adopter des expressions de la forme

$$x = f(v) \cdot f_1(u), \quad y = \varphi(v) \cdot \varphi_1(u), \quad z = \psi(v) \cdot \psi_1(u),$$

et que la recherche des lignes asymptotiques de toute surface (S) se ramène aux quadratures.

2° Quelle est la condition pour qu'une courbe imposée distincte d'une caractéristique ;

$$x = f(v), \quad y = \varphi(v), \quad z = \psi(v),$$

soit ligne asymptotique d'une surface ?

La détermination des familles d'asymptotiques distinctes des caractéristiques, des surfaces (S), peut se ramener à la recherche des solutions de trois équations différentielles linéaires analogues.

L'intégration de ces équations linéaires peut-elle se ramener aux quadratures ?

Une famille d'asymptotiques a-t-elle une enveloppe ?

Comment trouvera-t-on les surfaces (S) dont une famille d'asymptotiques est douée d'enveloppe ?

Examiner les particularités de la surface et de ses asymptotiques dans le voisinage de l'enveloppe, en se bornant aux surfaces dont les coordonnées correspondent à des fonctions holomorphes ; donner explicitement des exemples très simples.

3° Déterminer les surfaces (S) dont les asymptotiques d'une famille sont des caractéristiques (γ). Définir géométriquement ces surfaces.

4° Soit (S_1) une surface (S) choisie, et (Γ_1) les développables circonscrites à (S_1) le long des caractéristiques génératrices (γ) supposées non asymptotiques.

On réalise une déformation déterminée de la surface (S_1) de telle sorte que tout point M de cette surface décrive la génératrice rectiligne de la développable (Γ_1) tangente en ce point, et l'on désigne par M' la position de M après la déformation, par (Σ_1) la surface obtenue.

Dans le cas particulier où l'on choisit le point M' sur l'arête de rebroussement de la développable (Γ_1) on obtient une surface (Σ_1) particulière, soit (Σ_0). Montrer qu'au réseau tracé sur (S_1) par les courbes (γ) et leurs conjuguées, correspond sur (Σ_0) un réseau conjugué.

5° Déterminer les déformations (S_1, Σ_1) faisant correspondre une courbe (γ) à toute courbe (γ) de (S_1) et possédant la propriété précédente de la déformation (S_1, Σ_0).

Montrer que ces déformations associent à toute surface (S_1) une famille à un paramètre de surfaces (Σ_1) dont les asymptotiques se correspondent, exception étant faite pour certaines surfaces (S_1) particulières que l'on caractérisera.

Trouver celles de ces familles Σ_1 dont les lignes asymptotiques correspondent aux asymptotiques de la surface (S_1) initiale et les définir géométriquement.

N. B. — On rappelle que l'équation différentielle des lignes asymptotiques, en coordonnées curvilignes est :

$$\begin{vmatrix} x''_{u^2} & x'_u & x'_v \\ y''_{u^2} & y'_u & y'_v \\ z''_{u^2} & z'_u & z'_v \end{vmatrix} du^2 + 2 \begin{vmatrix} x''_{uv} & x'_u & x'_v \\ y''_{uv} & y'_u & y'_v \\ z''_{uv} & z'_u & z'_v \end{vmatrix} du.dv + \begin{vmatrix} x''_{v^2} & x'_u & x'_v \\ y''_{v^2} & y'_u & y'_v \\ z''_{v^2} & z'_u & z'_v \end{vmatrix} dv^2 = 0.$$

4 Juillet

Mécanique

Mouvement d'un corps solide pesant fixé par un de ses points O autour duquel il peut tourner librement et assujetti à toucher un plan horizontal fixe H ; les liaisons sont supposées sans frottement et le solide au-dessus du plan H.

I. — Le solide est de révolution, généralement non homogène, et est suspendu par un point de son axe de figure.

a. Trouver les équations du mouvement et indiquer les circonstances générales de ce mouvement lorsque les conditions initiales sont arbitraires.

Discuter complètement lorsque le solide est abandonné sans vitesses initiales ; examiner les cas particuliers dans lesquels le solide admet, soit un plan passant par l'axe de figure, soit un plan mené par O et perpendiculairement à cet axe, comme plan de symétrie des masses ; indiquer nettement, par des figures, la nature du mouvement du point de contact du solide sur le plan H.

b. Lorsque l'axe de figure du solide est axe de symétrie des masses, discu[ter] le mouvement, les conditions initiales étant arbitraires; examiner le cas particulier où le solide est homogène.

Application. — Le mouvement instantané du solide est une rotation que l'on peut toujours décomposer en une rotation soit ω, portée par l'axe du solide, et en u[ne] rotation verticale.

On suppose, en se plaçant dans le cas (b), que les conditions initiales vérifient la [re]lation.

$$2T = C\omega^2,$$

2 T étant la force vive, C le moment d'inertie du solide par rapport à son axe.

Calculer explicitement le mouvement, noter ses particularités et indiquer, dans [les] deux cas possibles, comment ce mouvement peut être imprimé au solide.

II. — Le solide ayant une forme quelconque, on désigne par (Γ) la courbe située [sur] le solide et lieu des points de contact M avec le plan H; on exprime les coordonnées, pa[r] rapport aux axes principaux d'inertie relatifs au point fixe O, d'un point quelcon[que] de cette courbe à l'aide d'un paramètre u et on définit la position du solide à l'a[ide] du paramètre u et de l'orientation du plan vertical passant par OM.

Trouver les équations donnant ces paramètres et indiquer les circonstances générale[s] du mouvement.

On pourra attacher au solide, en tout point M de la courbe (Γ), un trièdre auxilia[ire] (T) ayant comme arête la normale en M à la surface du solide, l'une des faces passa[nt] par le point fixe O; et regarder le mouvement du solide comme résultant d'un mou[-]vement relatif par rapport au trièdre (T) et d'un mouvement d'entraînement de ce trièd[re].

5 Juillet.

Membres du Jury.

M.M. Niewenglowski, inspecteur général de l'Instruction publique, Président.
Hadamard, professeur au Collège de France, Vice-Président.
Combette, inspecteur général de l'Instruction publique.
Grévy, professeur au lycée Saint-Louis.
Husson, professeur à la Faculté des Sciences de l'Université de Caen.

Candidats admissibles.

A la suite des compositions écrites, ont été déclarés admissibles aux épreuves orales (ordre alphabétique):

M.M. Blum, Bresse, Carron, Cerf, Cotty, Delmas, Dodier, Favre, Franceschini, Gaillard, Gateaux, Gonthiez, Guadet, Humbert, Janet, Lefare, Langlamet, Lagorsse, Nifnecker, Pelissier, Poyet, Raymond, Robert, Sauvigny, Thiry, Turrière

Bulletin administratif du 6 Août 1910.

Sujets de Leçons

Mathématiques élémentaires

M. Thiry. Résolution des triangles quelconques. (Ne pas faire les triangles rectangles

M. Poyet. Symétrie par rapport à un point
Symétrie par rapport à une droite
Symétrie par rapport à un plan } (Classe de première)

M. Blum. Variation de $\frac{ax+b}{a'x+b'}$; représentation graphique. (progr. de seconde).

M. Cotty. Trièdres supplémentaires. – Applications.

M. Lagorsse. Leçon déjà faite par M. Poyet.

M. Sauvigny. Rabattements. – Applications. —— Angle de deux droites, angle d'une droite et d'un plan, angle de deux plans.

M. Gateaux. Conversion d'une fraction ordinaire en fraction décimale. Fractions périodiques. (Mathématiques A).

M. Guadet. Volume des parallélipipèdes et des prismes. (Ne pas faire le tronc de pyramide ni le tronc de prisme.) (Classe de première).

M. Robert. Sections planes d'un cône de révolution (Méthode Dandelin) (Mathém. élémentaires)

M. Carron. Polyèdres homothétiques. Polyèdres semblables. (Programme de première).

M. Langlamet. Relation entre les coefficients et les racines de l'équation du second degré. Applications.

M. Favre. Notions sommaires sur les symétriques du cube et de l'octaèdre (programme de première).

M. Gaillard. Leçon déjà faite par M. Cotty.

M. Bresse. Tangentes à l'hyperbole. Asymptotes. Problèmes simples sur les tangentes.

M. Pélissier. Notion de la dérivée, signification géométrique de la dérivée. – Applications à la variation de fonctions simples. (Programme de seconde).

M. Lefare. Homothétie en géométrie plane.

M. Bodier. Inversion (plan et espace). – Applications.

M. Gonthiez. Leçon déjà faite par M. Guadet.

M. Delmas. Leçon déjà faite par M. Carron.

M. Janet. Leçon déjà faite par M. Bodier.

M. Humbert. Leçon déjà faite par M. Bresse.

M. Turrière. Leçon déjà faite par M. Favre.

M. Raymond. Rabattements. Leçon déjà faite par M. Sauvigny.

M. Cerf. Leçon déjà faite par M. Lefare.

M. Nifnecker. Mouvement de la lune. Phases.

M. Franceschini. Problèmes du second degré (Mathématiques A).

Mathématiques spéciales.

M. Bresse. Petites oscillations d'un pendule sans frottement: Isochronisme.

M. Pélissier. Asymptotes en coordonnées polaires; position de la courbe par rapport à ses asymptotes.

M. Lefare. Théorie des enveloppes en géométrie plane.

M. Bodier. Normales à l'ellipsoïde.

M. Gonthiez. Courbure des courbes planes. Développées. Exemples. (Coordonnées rectilignes).

M. Raymond. Mouvement d'un point sous l'action d'une force issue d'un centre fixe et proportionnelle à la distance.

M. Turrière. Mouvement d'un point pesant sur un plan incliné avec ou sans frottement, la vitesse initiale étant nulle ou dirigée suivant une ligne de plus grande pente.

M. Delmas. Nombre e
Limite de $\left(1+\frac{1}{m}\right)^m$.

M. Janet. Séries à termes positifs. – Caractères de convergence et de divergence tirés de l'étude des expressions:

$$\frac{u_{n+1}}{u_n} \qquad \sqrt[n]{u_n} \qquad n\,u_n^p.$$

Exemples numériques.

M. Humbert. Nombres complexes. $a+bi$. — Addition, soustraction, multiplication, division. – Représentation géométrique.

M. Cerf. Emploi de la dérivée pour l'étude des variations d'une fonction; maxima et minima. — Exemples numériques.

M. Nifnecker. Leçon déjà faite par M. Dodier.

M. Franceschini. Développements en série. Application à la série du binôme et à arc sin x.

M. Thiry. Mouvement d'un point attiré par un centre fixe en raison inverse du carré de la distance.

M. Poyet. Leçon déjà faite par M. Delmas.

M. Blum. Construction d'une courbe ($\rho = f(\omega)$) en coordonnées polaires. (On suppose faites les leçons sur les tangentes et les asymptotes.)

M. Colty. Recherches des racines commensurables d'une équation à coefficients entiers. Exemples.

M. Lagorsse. Leçon déjà faite par M. Humbert.

M. Sauvigny. Multiplication des séries. Applications.

M. Gateaux. Séries entières. — Intervalle de convergences.
Dérivation } ordre à volonté.
Intégration }

M. Guadet. Leçon déjà faite par M.M. Humbert et Lagorsse.

M. Robert. Intersection d'une surface de révolution et d'un cône.

M. Carron. Résolution trigonométrique de l'équation binôme.

M. Langlamet. Branches infinies dans l'intersection des cônes et des cylindres. (Géométrie descriptive).

M. Favre. Intersection de deux surfaces de révolution dont les axes sont dans le même plan.

M. Gaillard. Fonctions symétriques et rationnelles des racines d'une équation algébrique.

Compositions finales

Calcul numérique.

Calculer l'intégrale:

$$I = \int_0^{2\pi} \left(\frac{100 - 10 \cos \alpha}{101 - 20 \cos \alpha} \right)^2 d\alpha$$

On emploiera la méthode directe des fonctions primitives, et l'on comparera aux résultats obtenus par les méthodes d'intégration approchée. On indiquera, pour chacune de ces méthodes, le nombre des chiffres significatifs de I qu'elles auront permis de déterminer avec certitude.

Géométrie descriptive. — (Epure)

Une parabole P située dans un plan horizontal a pour projection horizontale une parabole dont le foyer est à 96 m/m à gauche du grand axe, et à 60 m/m au-dessous du petit axe de la feuille; son sommet est à 102 m/m à gauche du grand axe, et à 54 m/m au-dessous du petit axe.

Une droite de front D est inclinée à 45° sur le plan horizontal et on s'élève sur cette droite, en la parcourant de droite à gauche; elle passe par le point du plan horizontal de projection situé à 130 m/m à droite du grand axe, et à 96 m/m au-dessous du petit axe de la feuille. Cette droite est rencontrée par la parabole P en un point situé à droite du plan de profil qui contient le grand axe de la feuille.

On considère le segment parabolique limité par l'arc de la parabole P qui contient son sommet et par la parallèle à la ligne de terre d'éloignement 144 m/m : on demande de représenter en projection verticale seulement le solide engendré par la rotation de ce segment autour de la droite D.

On prendra pour ligne de terre le petit axe de la feuille.

Candidats reçus

A la suite des épreuves définitives, sur la proposition du Jury, ont été nommés agrégés des sciences mathématiques (ordre de mérite):

MM. 1. Robert
2. Thiry
3. Turrière
4. Cerf
5. Cotty
6. Sauvigny
7. Favre
8. Pélissier

MM. 9. Janet
10. Lefare
11. Gateaux
12. Humbert
13. Franceschini
14. Gaillard
15. Delmas
16. Carron.

Programme pour le Concours de 1911.

Compositions écrites

I. Programme général d'analyse et de mécanique.

Les programmes des certificats d'études supérieures variant d'une Université à l'autre, le jury indique, dans le programme ci-dessous, le minimum des connaissances générales qui sont supposées acquises par les candidats en calcul différentiel, calcul intégral et mécanique.

Les sujets des compositions sur le calcul différentiel, le calcul intégral et la mécanique seront choisis dans les n^os 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 et 15 de ce programme : ils ne dépasseront pas le niveau des sujets de problèmes proposés aux certificats de licence correspondants.

Calcul différentiel et Calcul intégral.

1° Opérations fondamentales du calcul différentiel et du Calcul intégral.

Dérivées et différentielles; intégrales simples, intégrales curvilignes, intégrales de différentielles totales; intégrales doubles et triples.

2° Applications du calcul différentiel.

Etude des fonctions de variables réelles (formule de Taylor, maxima et minima, déterminants fonctionnels, fonctions implicites). Calcul des dérivées et différentielles; changement de variables.

Ordre de connexion et genre d'une aire.

3° Applications du calcul intégral.

Procédés d'intégration. Longueur d'un arc de courbe, aires planes et gauches, volumes.

Différentiation et changement de variables sous le signe $\int\int$... Etude de l'intégrale $\int_a^b f(x)\,dx$ quand une limite ou la fonction devient infinie. Formule de Green.

Etude des fonctions représentées par des séries.

Propriétés des séries entières.

4° Eléments de géométrie infinitésimale.

Propriétés infinitésimales des courbes planes et gauches (courbes enveloppes, courbure, torsion). Propriétés infinitésimales des surfaces : surfaces enveloppes, notions sommaires sur les transformations de contact ; surfaces développables, surfaces réglées ; théorème de Meusnier ; sections principales.

Lignes conjuguées, lignes de courbure, lignes asymptotiques en coordonnées curvilignes quelconques.

5° Fonctions élémentaires d'une variable complexe.

Fonctions algébriques simples ; fonctions circulaires et logarithmiques.

6° Théorie des fonctions analytiques.

Propriétés de l'intégrale $\int f(z)\,dz$. Séries de Taylor et de Laurent.
Pôles, points singuliers essentiels, résidus.
Réduction des intégrales hyperelliptiques.

7° Equations différentielles du premier ordre.

Intégrale générale, intégrales particulières, intégrales singulières. — Types simples d'équations intégrables. Facteur intégrant. — Théorème de Briot et Bouquet sur l'existence des intégrales dans le cas où les données sont analytiques.

8° Equations différentielles et systèmes d'équations d'ordre quelconque.

Intégrale générale, intégrales particulières, intégrales premières.
Types simples d'équations intégrables. Equations linéaires.

9° Intégration de l'équation aux dérivées partielles du premier ordre linéaire.

10° Intégration de l'équation aux dérivées partielles ou aux différentielles totales du premier ordre.

Mécanique.

11° Statique.

Composition des forces appliquées à un même point. — Attraction d'une couche sphérique homogène sur un point extérieur ou intérieur. Propriétés élémentaires du potentiel. — Réduction des forces appliquées à un corps solide. — Conditions d'équilibre d'un corps solide. Application aux machines simples.

Polygone funiculaire. Ponts suspendus. Chaînette. Théorème du travail virtuel.

12° Cinématique.

Vitesse. Accélération.

Mouvement d'une figure plane dans son plan. Représentation du mouvement par le roulement d'une courbe mobile sur une courbe fixe.

Mouvement d'un corps solide autour d'un point fixe. Représentation du mouvement par le roulement d'un cône mobile sur un cône fixe

Mouvement d'un corps solide dans l'espace. Mouvement hélicoïdal.

Mouvements relatifs. Théorème de Coriolis.

13° Dynamique du point.

Travail.
Théorèmes généraux.
Intégrales premières des équations du mouvement. Application au mouvement des planètes.
Mouvement d'un point sur une courbe ou sur une surface.

Pendule dans le vide et dans un milieu résistant. Pendule conique.
Lignes géodésiques.

14° Géométrie des masses.

Centres de gravité. Moments d'inertie.

15° Dynamique des systèmes.

Théorèmes généraux. Intégrales premières. Energie. Stabilité de l'équilibre.
Mouvement d'un corps solide autour d'un axe fixe. Pressions supportées par l'axe.
Pendule composé. Mouvement d'un corps solide autour d'un point fixe.
Mouvement général d'un corps solide.
Lois du frottement de glissement.
Application du théorème des forces vives aux machines.
Principe de d'Alembert. — Equations de Lagrange.
Mouvements relatifs. — Percussions.

16° Equations canoniques.

Théorème de Jacobi.

17° Hydrostatique.

Equilibre d'une masse fluide. Surfaces de niveau. Pression sur une paroi plane.
Principe d'Archimède. Equilibre des corps flottants.

18° Hydrodynamique.

Equations générales du mouvement d'une masse fluide.
Théorème de Bernoulli. Théorème de Torricelli.

Leçons.

Parties des programmes d'où seront tirés les sujets de Leçons.

I — Leçons de Mathématiques spéciales.

Division des polynômes. — Plus grand commun diviseur de deux polynômes.
La condition nécessaire et suffisante pour que deux polynômes $f(x)$ et $g(x)$ de degrés respectifs p et q aient un plus grand commun diviseur de degré n est qu'il existe deux polynômes A et B de degrés respectifs $p-n$ et $q-n$ tels qu'on ait identiquement:

$$A\,g(x) + B\,f(x) = 0.$$

Théorème de Rolle, formule des accroissements finis, représentation graphique.

Propriétés générales des équations algébriques. — Nombre des racines d'une équation. — Relations entre les coefficients et les racines. — Toute fonction rationnelle et symétrique des racines s'exprime rationnellement en fonction des coefficients. — Elimination d'une inconnue entre deux équations au moyen des fonctions symétriques.

Condition pour qu'une équation ait des racines égales.
Recherche des racines commensurables.

Théorème de Descartes. — Nombres complexes. Formule de Moivre.
Résolution trigonométrique de l'équation binôme.

Résolution numérique des équations algébriques ou transcendantes. — Méthode d'approximation de Newton et méthode des parties proportionnelles.

Emploi de la dérivée pour l'étude de la variation d'une fonction; maxima et minima.

Fonctions primitives d'une fonction donnée, leur représentation par l'aire d'une courbe.

Application des quadratures à la rectification des courbes, au calcul d'un volume décomposé en tranches par des plans parallèles, à l'évaluation de l'aire d'une surface de révolution et au calcul des moments d'inertie du cylindre de révolution, de la sphère, et du parallélépipède par rapport à leurs axes de symétrie.

Aires et volumes des solides de la géométrie élémentaire.

Fonction définie par une série entière à coefficients réels. Intervalle de convergence.

Addition et multiplication. A l'intérieur de l'intervalle de convergence, on obtient la dérivée ou les fonctions primitives de la fonction en prenant la série des dérivées ou des fonctions primitives (on ne s'occupera pas de ce qui se passe aux extrémités de l'intervalle).

Exemples : développements en série de :

$$\frac{1}{1-x}, \quad \frac{1}{1+x^2}, \quad \operatorname{arctg} x, \quad L\,(1-x), \quad L\,\frac{1-x}{1+x}.$$

Série exponentielle. Série du binôme. Les équations $y' = y$ et $y'(1+x) = my$ permettent de déterminer les sommes des deux séries.

Développement en série de a^x, de $\arcsin x$.

Intégration des équations différentielles du premier ordre : 1° dans le cas où les variables se séparent immédiatement; 2° dans le cas où l'équation est linéaire.

Intégration de l'équation différentielle linéaire du second ordre à coefficients constants sans second membre; cas où le second membre est un polynôme ou une somme d'expressions de la forme $A\,e^{ax}$.

Courbes dont l'équation est résolue ou résoluble par rapport à l'une des coordonnées. — Tracé. — Équation de la tangente en un point; sous-tangente. — Normale; sous-normale. — Concavité; convexité; points d'inflexion. — Asymptotes. — Application à des exemples simples et en particulier à des coniques et à des courbes dont l'équation est du second degré par rapport à l'une des coordonnées.

Courbes définies par l'expression des coordonnées d'un de leurs points en fonction d'un paramètre. — Tracé. — Exemples numériques. — Les courbes du second ordre et celles du troisième ordre à point double sont unicursales.

Notions succinctes sur les points à l'infini au moyen des coordonnées homogènes, et sur les éléments imaginaires. Relation homographique; relation involutive; rapport anharmonique de quatre nombres. Application au rapport enharmonique de quatre points en ligne droite et de quatre droites appartenant au même faisceau linéaire.

Rapport anharmonique de quatre points d'une conique ou de quatre tangentes à une conique. Divisions homographiques et divisions en involution sur une conique.

Notions succinctes sur les coniques appartenant au faisceau linéaire ponctuel défini par deux coniques données ; les coniques de ce faisceau découpent sur une droite quelconque deux divisions en involution.

Courbure. — Enveloppes. — Développées.

Coordonnées polaires. — Leur transformation en coordonnées rectilignes. Équation de la ligne droite.

Construction des courbes ; tangentes. — Asymptotes. — Applications (on se bornera au cas où l'équation est résolue par rapport au rayon vecteur). — Cas des coniques.

Courbes gauches. — Tangente. Plan osculateur. Courbure. Applications à l'hélice circulaire.

Cinématique du point.

Vitesse. — Accélération.
Composition des vitesses. — Composition des accélérations bornée au cas où le mouvement du système de comparaison est un mouvement de translation.

Dynamique.

I. Point matériel libre. — Mouvement d'un point sous l'action d'une force constante en grandeur et en direction ou sous l'action d'une force issue d'un centre fixe : 1° proportionnelle à la distance ; 2° en raison inverse du carré de la distance.

Travail d'une force, travail de la résultante de plusieurs forces, travail d'une force pour un déplacement résultant. — Théorème de la force vive. — Surfaces de niveau. — Champs et lignes de force.

II. Point matériel non libre. — Mouvement d'un point pesant sur un plan incliné avec et sans frottement, la vitesse initiale étant dirigée suivant une ligne de plus grande pente. Pression totale sur le plan ; réaction du plan. — Petites oscillations d'un pendule simple sans frottement ; isochronisme.

Statique.

Moments. — Moment vectoriel par rapport à un point. — Moment par rapport à un axe.

Statique des systèmes de points matériels. — Démontrer qu'il existe six conditions nécessaires d'équilibre indépendantes des forces intérieures. — Démontrer que, pour les systèmes invariables, ces six conditions sont suffisantes. Cas particuliers.

Equivalence de deux systèmes de forces appliquées à un corps solide. — Application à la réduction d'un système de forces.

Géométrie descriptive.

Intersection de surfaces : deux cônes ou cylindres, cône ou cylindre et surface de révolution, deux surfaces de révolution dont les axes sont dans un même plan.

II. — Leçons sur un sujet du programme de seconde et première (C et D) et de Mathématiques (A).

Seconde (C. D.)

Algèbre.

Résolution des équations du premier degré à une inconnue. Inégalités du premier degré. Résolution et discussion de deux équations du premier degré à deux inconnues.

Variation de l'expression $ax + b$; représentation graphique.

Existence et signes des racines d'une équation du second degré. Variation du trinôme du second degré ; représentation graphique.

Variation de l'expression $\frac{ax+b}{a'x+b'}$; représentation graphique.

Notion de la dérivée ; signification géométrique de la dérivée. Le signe de la dérivée indique le sens de la variation ; application à des exemples numériques très simples et en particulier aux fonctions étudiées précedemment.

Géométrie.

Figures symétriques par rapport à un point ou à une droite. Deux figures planes symétriques sont égales.

Translation d'une figure plane de forme invariable.

Mouvement de rotation autour d'un point (dans le plan). Tout déplacement d'une figure plane de forme invariable dans son plan se ramène à une rotation ou à une translation.

Notions simples sur l'homothétie. Polygones semblables.

Polygones réguliers. Inscription dans le cercle du carré, de l'hexagone, du triangle équilatéral, du décagone, du pentédécagone. Deux polygones réguliers d'un même nombre de côtés sont semblables. Rapport de leurs périmètres. Longueur d'un arc de cercle. Rapport de la circonférence au diamètre. Calcul de π. (On se bornera à la méthode des périmètres).

Aire des polygones ; aire du cercle. — Mesure de l'aire du rectangle, du parallélogramme ; du triangle, du trapèze, d'un polygone quelconque.

Rapport des aires de deux polygones semblables.

Aire d'un polygone régulier convexe. Aire d'un cercle, d'un secteur et d'un segment de cercle. Rapport des aires de deux cercles.

Première C et D.

Trigonométrie.

Résolution de quelques équations trigonométriques simples.

Géométrie.

Translation. Rotation autour d'un axe. Symétrie par rapport à une droite. Symétrie par rapport à un point. Symétrie par rapport à un plan. Ce second mode de symétrie se ramène au premier.

Angles trièdres. Disposition des éléments. Trièdres symétriques. Chaque face d'un trièdre est moindre que la somme des deux autres. Limites de la somme des faces d'un trièdre. Limites de la somme des faces d'un angle polyèdre convexe.

Trièdres supplémentaires. Applications.

Cas d'égalité des trièdres.

Homothétie. Sections planes parallèles d'angles polyèdres Aires.

Polyèdres. Polyèdres homothétiques, polyèdres semblables. Prisme. Pyramide.

Notions sommaires sur les symétries du cube et de l'octaèdre régulier.
Volume du parallélépipède et du prisme. Volume de la pyramide.
Volume du tronc de pyramide à bases parallèles. Volume du tronc de prisme triangulaire.

Rapport des volumes de deux polyèdres semblables.
Deux polyèdres symétriques sont équivalents.

Sphère: section plane, pôles; plan tangent. Cône et cylindre circonscrit. Aire et volume.

Mathématiques A.

Arithmétique.

Division des nombres entiers.
Définition et propriétés élémentaires des nombres premiers.

Fractions ordinaires. — Réduction d'une fraction à sa plus simple expression. Réduction de plusieurs fractions au même dénominateur. Plus petit dénominateur commun. Opérations sur les fractions ordinaires.

Nombres décimaux. Opérations (en considérant les fractions décimales comme cas particulier des fractions ordinaires). Calcul d'un quotient à une approximation décimale donnée.

Réduction d'une fraction ordinaire en fraction décimale; condition de possibilité. Lorsque la réduction est impossible, la fraction ordinaire peut être regardée comme la limite d'une fraction décimale périodique illimitée.

Carré d'un nombre entier ou fractionnaire; composition du carré de la somme de deux nombres. Le carré d'une fraction n'est jamais égal à un nombre entier. Définition et extraction de la racine carrée d'un nombre entier ou fractionnaire à une approximation décimale donnée.

Théorie de la racine carrée d'un entier à une unité près.

Définition de l'erreur absolue et de l'erreur relative. —
Détermination de la limite supérieure de l'erreur commise sur une somme, une différence, un produit, un quotient, connaissant les limites supérieures des erreurs dont les données sont entachées.

Algèbre. — Equation du second degré à une inconnue. Equations simples qui s'y ramènent. (On ne développera pas la théorie des imaginaires).

Problèmes du premier et du second degré.

Application des dérivées à l'étude des variations de

$$\frac{ax^2 + bx + c}{a'x^2 + b'x + c'} \;,\; x^3 + px + q$$

où les coefficients ont des valeurs numériques.

Trigonométrie. — Fonctions circulaires. Addition et soustraction des arcs. Multiplication et division par 2.

Résolution des triangles.

Applications de la trigonométrie aux diverses questions relatives au levé des plans. (On ne parlera pas de la construction des tables trigonométriques).

Géométrie. — Inversion. Applications. Appareil de Peaucellier.

Polaire d'un point par rapport à un cercle. Plan polaire d'un point par rapport à une sphère.

Paraboles. — Tracé, tangente, normale. Problèmes simples sur les tangentes. Equation de la parabole.

Sections planes d'un cône ou d'un cylindre de révolution.

Projections centrales. — Plan du tableau. Perspective d'un point, d'une droite, d'une ligne. Point de fuite d'une droite. Perspective de deux droites parallèles. Ligne de fuite d'un plan.

Conception de la droite à l'infini d'un plan.

Vecteurs. — Projection d'un vecteur sur un axe; moment linéaire par rapport à un point; moment par rapport à un axe.
Somme géométrique d'un système de vecteurs; moment résultant par rapport à un point. Somme des moments par rapport à un axe. Application à un couple de vecteurs.

Géométrie descriptive. — Rabattements. Changement d'un plan de projection; rotation autour d'un axe perpendiculaire à un plan de projection.

Application aux distances et aux angles: distance de deux points, d'un point à une droite, d'un point à un plan; plus courte distance de deux droites, dont l'une est verticale ou de bout ou de deux droites parallèles à un même plan de projection; perpendiculaire commune à ces droites. Angle de deux droites; angle d'une droite et d'un plan; angle de deux plans.

Cinématique. — Mouvement rectiligne. Mouvement uniforme; vitesse, sa représentation par un vecteur.
Mouvement varié; vitesse moyenne; vitesse à un instant donné, sa représentation par un vecteur.
Accélération moyenne; accélération à un instant donné, sa représentation par un vecteur.
Mouvement uniformément varié.

Changement du système de comparaison. Composition des vitesses.
Exemples et applications (ne pas insister sur les applications purement géométriques.

Etude géométrique de l'hélice. Mouvement hélicoïdal d'un corps. Vis et écrou.

Dynamique. — Travail d'une force appliquée à un point matériel. Unité de travail. Travail d'une force constante, d'une force variable. Travail élémentaire; travail total. Evaluation graphique. Travail de la résultante de plusieurs forces.
Théorème des forces vives pour un point matériel.
Exemples simples.

Cosmographie. — Lune. Mouvement propre apparent sur la sphère céleste.
Phases.
Rotation.
Variation du diamètre apparent.
Eclipses de lune et de soleil.

Comètes; étoiles filantes; bolides.

Bulletin administratif du 20 Août 1910

Ouvrages pour la préparation à l'Agrégation des Sciences Mathématiques

d'Adhémar – Exercices et leçons d'Analyse, in-8 . . . . 6f. »
Appell. – Traité de Mécanique rationnelle, Faculté des Sciences de Paris, 3 volumes :
Tome I – Statique. Dynamique du point, in-8 . . . . 20. »
– II – Dynamique des systèmes. Mécanique analytique . . 16. »
– III – Equilibre et mouvement des milieux continus . . 20. »
—— Eléments d'Analyse mathématique, in-8 . . . . 24. »
Appell et Goursat – Théorie des fonctions algébriques et de leurs intégrales, in-8 . . . . 16. »
Appell et Lacour – Principes de la théorie des fonctions elliptiques et applications, in-8 . . . . 12. »
Baire – Leçons sur les Théories générales de l'Analyse :
Tome I – Principes fondamentaux, variables, réelles . . . . 8. »
– II – Variables complexes, Applications géométriques . . 12. »
Borel – Théorie des probabilités, in-8 . . . . 6. »
Boussinesq – Tome I. Calcul différentiel, in-8 . . . . 17. »
– II Calcul intégral 23.50
Brahy – Exercices méthodiques de Calcul différentiel, in-8 . . . . . 5. »
—— — — — intégral, in-8 . . . . 5. »
Cosserat – Théorie des corps déformables, in-8 . . . . 6. »
Darboux – Leçons sur la théorie générale des surfaces et les applications géométriques du Calcul infinitésimal, 4 volumes :
1e Partie : Généralités. Coordonnées curvilignes. Surfaces minimes (Ne se vend plus séparément).
2e Partie : Les congruences et les équations linéaires aux dérivées partielles. Des lignes tracées sur les surfaces . . . . 15. »
3e Partie : Lignes géodésiques et courbures géodésiques. Paramètres différentiels. Déformation des surfaces . . . . 15. »
4e Partie : Déformation infiniment petite et représentation sphérique . . . . 15. »
Leçons sur les systèmes orthogonaux et les coordonnées curvilignes, 2 volumes, Tome I . . . . 10. »
Duporcq – Premiers principes de géométrie moderne, in-8 . . . . . . 3. »

Mathmatical Instruction and the Professors of Mathematics in the French Lycees for Boys.

By R. C. ARCHIBALD.

Professor of Mathematics at Browu University.

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MATHEMATICAL INSTRUCTION AND THE PROFESSORS OF MATHEMATICS IN THE FRENCH LYCÉES FOR BOYS.[1]

By R. C. Archibald,

Professor of Mathematics at Brown University.

The general scheme of the French educational system and the position of the lycée in this system are topics, the consideration of which the title of my paper, strictly speaking, excludes. And yet, to give appropriate setting to the main themes and bases of comparison with our own schools, brief reference to these topics seems not wholly uncalled for in this connection.

For educational purposes France is divided geographically into *arrondissements.* The assemblage of government schools (primary, secondary and superior) in each arrondissement forms an *académie* over which a *recteur* presides. There are thus the 16 académies of Aix-Marseilles, Besançon, Bordeaux, Caen, Chambéry, Clermont, Dijon, Grenoble, Lille, Lyons, Montpellier, Nancy, Paris, Poitiers, Rennes, Toulouse, as well as a seventeenth at Algiers. With the exception of Chambéry these names correspond to the seats of the French universities.

The assemblage of académies forms the *Université de France,* at the head of which is the Minister of Public Instruction, who is *ex officio* the "Recteur de l'Académie de Paris et Grand Maître de l'Université de Paris." For the Académie de Paris there is a vice-recteur, whose duties are the same as those of the recteurs of other académies. Although nominally lower in rank than the heads of académies in the provinces, he is in reality, the most powerful official in the educational system. The position of the Minister of Public Instruction being so insecure by reason of changing governments, continuity of scheme is assured by three lieutenants who have charge respectively of the primary, secondary, and superior education. They in turn have an army of inspectors who report on the work and capabilities of the recteurs and their académies as far as primary and secondary instruction is concerned.

This suffices at present to indicate the remarkably centralized and unique character of the French educational system. It is theoretically possible for the most radical changes in any part of

[1]Abridgment of a paper presented at the mid-winter meeting of the Association of Mathematical Teachers in New England, held at Brown University, Providence, R. I., February 3, 1912.

public instruction to be immediately brought about by a stroke of the pen on the part of the Minister of Public Instruction.

The present system of secondary education in France dates from the great reform of 1902 (important modifications were introducted in 1905 and 1909) and is carried on for the most part in *Lycées* and *Collèges communaux* which are to be found in nearly all cities. Because of their pre-eminence we shall consider the former only, which are under control of the state. Here the boys, who come from families in comfortable circumstances, may enter as *élèves* at the age of five or six years and be led along in their studies till they receive the *Baccalauréat* at the age of 16 or 17.[2] Many lycées have still more advanced courses to prepare for entrance into such schools as the École Normale Supérieure, École Polytechnique, École Centrale, École Navale, École de Saint Cyr, etc.

Instruction in fully equipped lycées may be divided into four sections:—I, *Primary;* II, *Premier Cycle;* III, *Second Cycle;* IV, *Classes de Mathématiques Spéciales.*

I.—*Primary.*[3] The classes in this section are named as follows:—

		Age from
Classes enfantines	Onzième	5
Classes préparatoires	Dixième	6
	Neuvième	7
Classes élémentaires	Huitième	8
	Septième	9

From the Dixième to the Septième 20 hours are devoted to class recitation each week. In the *Classes préparatories* 3 hours a week are taken up with *Calcul,* that is, principles of numeration, elementary operations with integers, notions concerning the metric system; intuitive geometry; simple exercises to enable the pupil to draw the more elementary regular figures (square,

[2]The pupils at the lycées are of four kinds: 1st.—*Externes,* those who come to the lycées for classes but board and lodge outside: 2nd.—*Internes* or *pensionnaires,* élèves who live entirely in the establishment; 3rd.—*Demi-pensionnaires* who usually reside at a distance but take their mid-day meal at the lycée; 4th.—*Externes surveillés,* that is externes who work out their lessons under the eye of the *préparatéur* in the *salle d'étude* of the lycée. The expenses of the pupil vary greatly with the class and the lycée in which he happens to be. The range of cost (in francs per year) (1) for some of the principal cities (Bordeaux, Lyons, Marseilles, Toulouse) of the provinces and (2) for the better lycées of Paris is as follows: *Externes* (1) 70-450, (2) 90-700; *externes surveillés* (1) 110-540, (2) 130-790; *demi-pensionnaires* (1) 370-850, (2) 500-1200; *pensionnaires* (1) 700-1200, (2) 900-1700. The lower price in each case is for the classe enfantine, the higher for the special classes open to *bacheliers.* Primary education (outside of the lycées and superior education) in France, is free.

[3]*Free* primary instruction is given in *Écoles Primaries Élémentaires* for pupils from 6 or 7 to 13 years of age. The course is divided as follows: Cours élémentaire (2 years), cours moyen (2 years), cours supérieur (2 years). On completion of the cours moyen the pupil receives a *certificaté d'études primaries élémentaires.* This certificate or its equivalent is required of every child in France. Many children require considerably more than four years to get the *certificat.*

rectangle, triangle, circle) and different sorts of angles. In the *Classes élémentaires,* 4 hours a week are assigned to revision of the preceding programme; decimal numbers; rules of three; intuitive geometry by the aid of models. One hour a week is given up to drawing.

II.—*Premier Cycle (sixième-troisième).* This cycle of four years constitutes an advanced course for students who have finished their primary studies, and is the first part of secondary education proper. It offers a choice between two lines of study, the one characterised by instruction in Latin with or without Greek, the other in which no dead language is taught. The former is selected by the parent who wishes to prepare his boy for the department of letters in the École Normale Supérieure or for the career of classical professor, lawyer or doctor. The latter is likely to be chosen for the boy who is particularly interested in science or who has a commercial career in view.

III.—*Second Cycle.* This leads, normally, to the *Baccalauréat,* at the end of three years' study, in one of four different sections. The scheme will be clearer in tabular form.

	Pupils who learn Latin, with or without Greek.			Pupils who learn no dead language	Age from
PREMIER CYCLE.	Sixième A (Latin).			Sixième B	10
	Cinquième A (Latin).			Cinquième B	11
4 years	Quatrième A (Latin Greek)	Quatrième (Latin)		Quatrième B	12
	Troisième A (Latin Greek)	Troisième (Latin)		Troisième B	13
			→	Pupils who give up the study of Latin. ↓	
	LATIN-GREC.	LATIN-LANGU.	LATIN-SCIENCES	SCIENCES-LANGUES	
SECOND CYCLE	Second A	Second B	Second C	Second D	14
	Première A	Première B	Première C	Première D	15
3 years	Philosophie A	Philosophie B	Mathématiques A	Mathématiques B	16

Let us now observe a little more closely just what is involved in this display, in the matter of studies and demands made upon the élève. As an important examination which we shall presently describe comes at the end of the *Première,* our present analysis will not pass beyond this grade. Here is the programme for a week.

There are several features of this scheme (we shall refer to

	VI. A.	VI. B.	V. A.	V. B.	IV. A.	IV. B.	III. A.	III. B.
French.	3	5	3	5	3	5	4	5
Latin.	7	..	7	..	6	..	6	..
Greek.	..	..	..	..	3*	..	3*	..
Modern Languages.	5	5	5	5	5	5	5	5
History and Geography.	3	3	3	3	3	3	3	3
Natural Science.	1	2	1	2	1	..	..	1
Physics and Chemistry.	..	..	..	..	..	2	..	2
Moral Philosophy.	..	..	..	..	1	1	1	1
Book-keeping.	..	..	..	..	..	1	..	1
Drawing.	2	2	2	2	2	2	2	2
Geometrical Drawing.	..	1	..	1	..	1	..	1
Arithmetic.	2	3	2	..	..	..	..	..
Arithmetic and Geometry.	..	..	..	3	2	3	3	..
Algebra and Geometry.	..	..	..	..	..	..	..	4
Writing.	..	1	..	1	..	..	..	..
Total No. Hours.	23	22	23	22	23	23	24	25

	II. A.	II. B.	II. C.	II. D.	I. A.	I. B.	I. C.	I. D.
French.	4	4	4	4	4	4	4	4
Latin.	4	4	4	..	3	3	3	..
Greek.	5	..	..	..	5	..	..	..
Modern Languages.	2	7	2	7	2	7	2	7
Ancient History.	2	2	..	..	2	2	..	..
Modern History.	2	2	2	2	2	2	2	2
Geography.	1	1	1	1	1	1	1	1
Physics and Chemistry.	..	..	3	3	..	..	3	3
Practical Exs. in Science.	..	..	2	2	..	..	2	2
Geology.	**	**	**	**	..	..	..	..
Supplementary Exs. in Latin.	..	..	..	..	2	..	..	..
Drawing.	2	2	2	2	2†	2†	2	2
Geometrical Drawing.	..	..	2	2	..	..	2	2
Algebra and Geometry.	2	2	5	5	2	2	..	..
Geometry, Descriptive Geom. Trig., Algebra.	..	..	..	..	..	..	5	5
Total No. Hrs.	24	24	27	28	23	21	26	28

*Those who select Greek are exempted from 2 hours in modern languages and one in drawing.

** 12 lectures of 1 hour.
†Optional.

four), which are particularly interesting.

1.—The prominence given to the study of French throughout.

2.—That all élèves at the age of 10 or 11 commence the study of modern languages (English, German, Italian, Spanish, Russian, or, in Algiers, Arabic), and continue it during six years at least, before matriculating into schools of university grade. Not only do they get glimpses of the best things in the literature of the language, but also learn to speak the language with considerable freedom and remarkable correctness in pronunciation. The direct method is employed and no word of French is ever spoken in the advanced classes. The majority of the élèves choose German, as this is required of all candidates for entrance into such military schools as the École Polytechnique and École de St. Cyr. On the other hand there is an increasing number taking up the study of English which is required for the École Navale.

3.—The proportion of recitation periods devoted to mathematics. As the drawing courses are about equally of a free hand and of a geometrical character, half the time given to drawing may be counted as mathematical and the percentages of recitation periods "required for mathematics during the four years of the Premier Cycle are: In the Latin course 14 per cent, 14 per cent, 22 per cent, 22 per cent; in the Modern Language course, 22.7 per cent, 22 per cent, 22 per cent and 22 per cent. In the Second and Première of the Second Cycle the percentages run: in the Latin-Greek course, 12.5 per cent, 4.5 per cent; in the Latin-Modern-Language course, 12.5 per cent, 6 per cent; in the Latin-Science course, 23 per cent, 24 per cent; and in the Modern-Language-Science course, 30 per cent, 30 per cent." To sum up from the Dixième to the Première the boy has spent 10.5 per cent, 11 per cent, 19.4 per cent, 22.8 per cent of his class hours in mathematical recitation according as he has pursued the courses leading to Première A, B, C or D. This emphasis which the French lay on mathematics is interesting and although the percentages may be somewhat higher than in America the training received is *vastly* superior in France. The fact that practically all the *professeurs titulaires* in the French lycées, even those in charge of the very elementary classes, are *agrégés* in the subjects which they teach means much. Just how much we shall explain later, but suffice it to remark here that no other country imposes as high scholastic standards for its professors of secondary education.

Another feature of mathematical instruction which is particularly interesting to us, is, that from the troisième on, that is, from the time the boy is 13 or 14 years old, instruction is usually given entirely by lecture. Indeed, even in classes before the troisième when a text-book is generally in the hands of the élève, he is required to take notes "pour préciser" the various topics. By such methods, searching questioning and frequent "tests," on the part of the professor, and rigid inspection, kindly expressed praise or cutting public reprimand on the part of the *proviseur* (director of the lycée), there is no possibility of learning parrot-fashion—no room for the shirker or the boy who does not try his best; reasoning powers and independence of thought must be constantly exercised. The élèves are encouraged to consult the various text-books to be found in all the lycée libraries and for those less bright this may be almost a necessity from time to time; but on personal inspection in different lycées I found the note books of élèves of 14 or 15 alike remarkable for their neatness and completeness. The habits thus gained in the lycée stand in good stead when the student reaches the university. The rapidity of the lecturer and the complexity of his theme seem to make little difference, for at the close of the hour the whole is in the note books as neat as copper-plate.

4.—The large number of hours in class recitation may not at first appear very imposing; but we cannot fail to be astonished that 8 hours per day (in class and in preparation of lessons) may be demanded from élèves in the premier cycle, and 10½ in summer, 10 in winter from those in the second cycle. The law further explicitly states that there is no limit to the number of hours which may be demanded of the élèves in the *Classes de Mathématiques Spéciales.* When we later come to look more closely at their programme we shall not be surprised, but nevertheless wonder, how these undoubtedly happy and healthy young men of 17 or 18 have survived the treatment. In more advanced lycée courses as well as at the universities I was also impressed with the almost appalling intensity and seriousness of the auditors —the strife is too strenuous, the competition too keen, to admit of a moment's levity or wandering thought. But when the lesson is over, every care is instantly banished and the national gaiety is once more in evidence.

To return to our table. We remark that the two groups of élèves who elect sciences on entering the second cycle have the same number of hours per week in mathematics—indeed the courses are identical. To give greater definiteness to our ideas as to their general attainments let us consider the programme of studies from Première D, when the boy is 15 or 16 years old.

French.—Lectures and questions on the principal French writers of the nineteenth century. Study of selections from prose writers and poets, from moralists, orators, politicians, scientists and historians of the sixteenth, seventeenth, eighteenth and nineteenth centuries.

History.—Political history of Europe in the eighteenth century. Detailed history of France at the close of the eighteenth century.

Geography.—Detailed study of France, its geological constitution, its climatology, physiography, topography, economic and military organization; its colonies, etc.

Physics.—Optics, electricity.

Chemistry.—Of the carbon compounds.

German.—Selections from the dramatic poetry of Schiller, Goethe, Kleist and Grillparzer. Extracts from the prose works of Wieland, Goethe, Schiller, Auerbach, Freytag, Scheffel, etc.

English.—Shakespeare's Julius Cæsar and Macbeth, extracts from Milton, Addison, Goldsmith, Wordsworth, Byron, Coleridge, Dickens, Macauley, Eliot, Tennyson, and Thackeray.

Algebra.—Equations and trinomials of the second degree.

Calculation of the derivatives of simple functions; study of their variation and graphic representation; study of rectilinear motion by means of the theory of derivatives; velocity and acceleration; uniformly changing motion.

Geometry.—Solid.

Descriptive Geometry.—Elements.

Trigonometry.—Plane, including the use of four or five place logarithm tables, the solution of triangles and trigonometric equations.

Having finished the Première, the élève presents himself for examination under conditions which once more emphasize the unity of the French educational system. This is the examination for the first part of the state degree known as the *Baccalauréat.*

A peculiar feature of this examination is that it is not held in the lycées but at the university of the académie to which the particular lycée belongs.[4] As various civil and practically all government positions, except those in post and telegraph offices are only open to *bacheliers,* the state introduces into the body of examiners some who are wholly independent of the lycées. These examiners are the professors in the universities.

Since our future mathematicians are to come from Première C and D we shall give a few particulars concerning their examination. All examinations for the baccalauréat are held in July and October—at the ending of one school year and the beginning of the next. The examiners of the candidates from Première C are six in number, three of whom are university professors and three professors from the lycées or collèges; for Première D there are but two university professors in addition to three from the lycées. The examinations in all sections are both written and oral. Here is the scheme of examination which practically covers what the élève has studied in earlier years.

Première C (Latin-Sciences).—*Written.* 1st, a French composition (3 hours); (the candidate has a choice of three subjects); 2nd, a Latin translation (3 hours); 3rd, an examination in Mathematics and Physics (4 hours). *Oral* (about three-quarters of an hour). 1st, explanation of a Latin text; 2nd, explanation of a French text; 3rd, examination in a modern language—questions and answers being necessarily in this language. Questions in—4th, History; 5th, Geography; 6th, Mathematics; 7th, Physics; 8th, Chemistry.

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[4]As there is no university at Chambéry, the candidate presents himself before a faculty of either Lyons or Grenoble.

And similarly for Première D.

The searching character of the tests prepares us for a large number of failures. Here is the record of the percentage of candidates passed in (1) July, (2) October, 1909: *Latin-Grec* (1) 44, (2) 42; *Latin-Langues Vivantes* (1) 41, (2) 42; *Latin-Sciences* (1) 49, (2) 46; *Sciences-Langues Vivantes* (1) 42, (2) 39. We observe that less than fifty per cent of the pupils get through on the first examination[5] while a similar percentage of the remainder fail and are required to return to the Première at once or wait for another year.[6] Those who have been successful return to the lycée to prepare for the second part of the baccalauréat. A choice of two courses (which may be slightly varied), is open to them, the one *Philosophie A* or *B*, the other, *Mathématiques A* or *B*. We shall only refer to the latter which has been supplied with pupils from the Première C and D. There they had 26 and 28 recitation hours per week. This has now been increased to 27½ and 28½. There has been an increase in the number of hours devoted to mathematics, physics and chemistry, but a reduction in the amount of study of modern languages. Latin no longer enters. The programme for Mathématiques A is in outline as follows:—

Philosophy (3 hours). *History and Geography* (3½ hours). *Modern Languages* (2 hours). *Physics and Chemistry* (5 hours). *Natural Science* (2 hours). *Practical Exercises in Science* (2 hours). *Drawing* (2 hours). *Hygiene* (12 lectures of 1 hour).

Mathematics. (8 hours): *Arithmetic.*—Properties of integers; fractions; decimals; square roots; greatest common divisors; theory of errors; etc.

Algebra.—Positive and negative numbers, quadratic equations (without the theory of imaginaries), progressions, logarithms, interest and annuities, graphs—derivatives of a sum, product, quotient, square root of a function, of sin x, cos x, tan x, cot x. Application to the study of the variation and the maxima and minima, of some simple functions, etc.

Trigonometry.—Circular functions, solution of triangles, applications of trigonometry to various questions relative to land surveying.

Geometry.—Translation, rotation, symmetry, homology and

[5]For some it may have been the third or fourth trial.

[6]There are certain exceptional cases which I shall not consider.

similitude, solids, areas, volumes, poles and polars, inversion, stereographic projection, central projections, etc.

Conics.—Ellipse, hyperbola, parabola, plane sections of a cone or cylinder of revolution, etc.

Descriptive Geometry.—Rabatments—application to distances and angles—projection of a circle—sphere, cone, cylinder, planes, sections, shadows—application to topographical maps, etc.

Kinematics.—Units of length and time. Rectilinear and curvilinear motion. Translation and rotation of a solid body. Geometric study of the helix, etc.

Dynamics and Statics.—Dynamics of a particle, forces applied to a solid body, simple machines in a state of repose and movement, etc.

Cosmography.—Celestial sphere, earth, sun, moon, planets, comets, stars—Co-ordinate Systems, Kepler's and Newton's laws, etc.

One of the most striking things in this scheme, as compared with the American method, is to find arithmetic taught in the last year of the lycée course. Note, too, that from the Cinquième on, it has been taken up in connection with instruction in geometry and algebra. Indeed, this method of constantly showing the interdependence or interrelation of the various mathematical subjects was one of the interesting and valuable characteristics of French education as I observed it. For example, I happened to be present in a classroom when the theory and evaluation of repeating decimals was under discussion. After all the processes had been explained, problems which led similarly to the consideration of infinite series and limits were taken up. By suggestive questioning a pupil found the area under an arc of a semi-cubical parabola and the position of the centre of gravity of a spherical cap. With us it is not till the graduate school of the university that the boy is taught the true inwardness of such processes as long division and extraction of roots; but in France, arithmetic is taught as a science, not as an art, and the élève leaving the lycée has a comprehending and comprehensive grasp of all he has studied.

We remark that most of the mathematical subjects mentioned above are more or less foreign to our secondary education. Instruction in geometrical conics (*courbes usuelles*), is infrequently given by us, even in universities. Again, the ordinary mathemathical student who goes up for his doctor's degree in America may have the vaguest idea of what is even meant by Descriptive

Geometry. True, it is a regular course for our training of the engineer; but not, unfortunately, of the mathematician. On the other hand the French mathematical student has had at least four years of Descriptive Geometry, two of them before receiving his baccalauréat. The subject is required for admission into many government schools.

We note that the idea of a derivative is familiar to the lycéen during the last two years of his course. Why we so generally shut out the introduction of such an idea into our first courses in analytical geometry and theory of equations is, to me, a mystery. Finally, I would remark that the classes in Mathématiques A last two hours, with the exception of five minutes for recreation at the end of the first hour. The professor thus has sufficient time to amplify and impress his instruction.

At the close of the last year of the Second Cycle, the élève takes the examination for the second part of the baccalauréat. The same general conditions prevail as for the first part. The jury of four contains two university professors. The written examinations in mathematics, physics and philosophy are each three hours long; the oral covers what has been studied the year previously. If successful, a diploma now called the *baccalauréat de l'enseignement secondaire,* is granted to the élève by the Minister of Public Instruction. The élève thus becomes a *bachelier*. Diplomas in all four sections are of the same scholastic value. The charge made for diploma and examination is 90 francs. More than forty per cent of the candidates failed to pass at each of the examinations in 1909.

Because of the similarity of title used in the different countries, the Frenchman does not generally understand what the title Bachelor of Arts implies nor is it easy to make any concise statement in explanation. Little exaggeration can be made, however, in placing the bachelier on a plane of scholastic equality with the Sophomore who has finished his year at one of the best American universities.

Furthermore, his training has been undoubtedly much more thorough. After the age of 6 or 7 French boys are taught by men.[7] These men have all studied at the University and have passed the *examen de licence*. With very few exceptions now, the instructors have also passed the extremely difficult *Examen d' agrégation* in the subjects they propose to teach. By comparison, how woefully deficient our teachers of like grades! The recently

[7]Girls are taught by women. Coëducation does not exist in the lycées.

published reports of the United States sub-committees of the International Commission on the Teaching of Mathematics state the case frankly. That precious years are often lost to our youth by their inferior instruction is obvious to every one. As to Examinations—no guessing of possible questions and "cramming" for the same, so common in America, can qualify a student to pass an examination in France. The rigorous and impartial tests for promotion are conducted, at least in part, by those outside the lycée and pressure brought to bear upon teachers to promote ill-prepared pupils is unknown. According to a recent report of the Carnegie Foundation for the Advancement of Teaching, this is a "great source of weakness" and "a fruitful source of demoralization in American public schools."

I should now like to tell you something of the fourth section of lycée instruction, namely, *the Classes de Mathématiques Spéciales.*

If the bachelier who is proficient in mathematics be not turned aside by circumstances or inclination, to seek immediately a career in civil or government employment, he most probably proceeds to prepare himself for the highly special and exacting examination necessary for entrance into one of the great schools of the government. The method of this preparation exhibits a very peculiar feature of the French system. Whereas with us, or with the German, the boy who has finished his regular course in the secondary school goes directly to some department of a university for his next instruction, the bachelier, who has a perfect right to follow the same course, returns to his old lycée (or enrolls himself at one of the great Paris lycées, such as Saint Louis, Louis le Grand or Henri IV), to enter the *Classe de Mathématiques Spéciales préparatoire* which leads up to the *Classe de Mathématiques Spéciales.* The latter is exactly adapted to prepare students for the École Normale Supérieure, the École Polytechnique and the *bourses de licence.* Only a small proportion of the lycées (36 out of the 115), have this *Classe;* but with the exception of Aix they are to be found in all university towns. On the other hand, yet other lycées have classes which prepare specially for the less exacting mathematical entrance examinations of the École Centrale, École de Saint Cyr, École Navale, etc. But the number of élèves who on first starting out deliberately try to pass examinations for these schools is small, in proportion to the number who eventually reach them after repeated but vain effort to get into the École Polytechnique or the École Normale

Supérieure. Just what makes these two schools famous and peculiarly attractive will appear in a later section. It has been noticed that when the élève has won his baccalauréat he may immediately matriculate into a university, and although it might be possible for him to keep pace with the courses, in mathematics, at least, it would be a matter of excessive difficulty. There is then in reality, between the baccalauréat and the first courses of the universities, a distinct break, bridged only by the *Classes de Mathématiques Spéciales.*[8]

The élèves who enter the *préparatoire* section of this class are, generally, bacheliers leaving the classes de Mathématiques; in very rare instances, there are those who come from the classe de Philosophie. Natural science, history and geography, philosophy—indeed practically every study except those necessary for the end in view, have been dropped and from this time on to the agrégation and doctorat all energies are bent in the direction of intense specialization. This is the most pronounced characteristic of French education to-day. In mathematics, instruction now occupies 12 instead of 8 hours. New points of view, new topics and broader general principles are developed in algebra and analysis, trigonometry, analytical geometry and mechanics. Physics and chemistry are taught during six hours instead of five. Add to these, German, 2 hours; French literature, one hour; descriptive geometry, 4 hours; drawing, 4 hours. After one year of this preparatory training the élève passes into the remarkable Classe de Mathématiques Spéciales.

Eight years of strenuous training have made this class possible for the young man of 17 or 18 years of age, who is confronted with no less than 34 hours of class and laboratory work per week and no limit as to the number of hours expected in preparing for the classes!

When first I looked over the programme it seemed a well nigh impossible performance for one year. Surely no other country can show anything to compare with it.

Did time permit it would be interesting to reproduce in full the mathematical programme as given at the end of the *plan d'étude,* but I shall hastily refer to only a few of the subjects treated: In *Algebra and Analysis* we find developed, the fundamental ideas concerning irrational numbers, convergency and divergency

[8]It is only for mathematical or scientific students that such a break occurs, as no special classes are provided in other subjects except in the case of half a dozen Paris lycées which have classes in "letters" preparatory for entry into the École Normale Supérieure.

of series, the elements of the theory of functions of a real variable, power series, their multiplication and division, their differentiation and integration term by term. Taylor's formula, the theory of algebraic equations, including symmetric functions, but omitting the discussion of infinite roots. The latter part of the course treats of differentials of several variables, elementary ideas concerning definite integrals, integration of such functions as are considered in a first calculus course of the best American colleges, rectification of curves, calculation of volumes, plane areas, moments of inertia, centres of gravity, differential equations of the first order, solutions of simpler differential equations of the second order, which occur in connection with problems of mechanics and physics. Whenever possible in the discussion of these topics the power to work numerical examples is emphasized.

Plane Trigonometry and the discussion of spherical trigonometry through the law of cosines are treated in class and five-place tables are used.

In the course on *Analytical Geometry* is given a thorough discussion of equations of the second degree, of homography and anharmonic ratios as they enter into the discussion of curves and surfaces of the second degree, of points at infinity, asymptotes, foci, trilinear coördinates, curvature, concavity and convexity, envelopes, evolutes. The professor also discusses thoroughly the various questions connected with the treatment of quadric surfaces and less completely, the theory of surfaces in general, of space curves, osculating planes, curvature of surfaces. The elements of the theory of unicursal curves and surfaces and of anallagmatic curves and surfaces are also taken up.

So also, we find broadly arranged programmes mapped out in mechanics and descriptive geometry. The whole number of class hours per week is broken up as follows:

Mathematics, 15; physics, 7 (2 in laboratory); chemistry, 2; descriptive geometry, 4; drawing, 4; German, 2; French, 1. The scope of the mathematical work may be judged from some books which were prepared with the needs of such a class especially in view.

B. Niewenglowski, Cours d' algèbre, I, 382 p.; II, 508 p.; Supplement—*G. Papelier* Précis de géométrie analytique, 696 p. —*Girod* Trigonométrie, 495 p.— *P. Appell* Cours de mécanique, 650 p.—*X. Antomari* Cours de géométrie descriptive, 619 p.

If anything, this list underestimates the work actually covered[9]

[9]That is, much more than what is called for by examination questions is studied. The élèves find truth in the adage: *Qui peut le plus peut le moins.*

by those who finally go out from the class. Tannery's Leçons d'algèbre et d'analyse (I, 423 p., II, 636 p.), might well replace Niewenglowski's work while Niewenglowski's Cours de géométrie analytique (I, 483 p.; II, 292 p.; III, 569 p.), represents the standard almost as nearly as Papelier's volume. Another treatise on mechanics widely used is that of Humbert and Antomari.

When we further realize that the main parts of the books in this list, which represents the work for only one of a half dozen courses, are covered by the professor in about fifteen months—the last three months of the second year are given over to drill in review and detail—we begin to get some conception of what the *Classe de Mathématiques Spéciales* really stands for. In his instruction the professor is officially "recommended" "not to overload the courses, to make considerable use of books, not to abuse general theories, to expound no theory without numerous applications dealt with in detail, to commence invariably with the more simple cases, those most easy to understand, for leading up finally to the general theorems. Among the applications of mathematical theory, those which present themselves in mathematical physics should be given the preference, those which the young people will meet later in the course of their studies either theoretical or practical. Thus in the construction of curves, choose as examples those curves which present themselves in Physics and Mechanics, as the curves of Van der Waals, the Cycloid, the Catenary, etc.—in the theory of envelopes choose those examples of envelopes which are met in the theory of cylindrical gearing—and so on. The pupils should be trained to reason directly on the particular cases and not to apply the formulæ. To sum up, one ought to develop their judgment and their initiative—not their memory."

(*Continued in February Issue.*)

MATHEMATICAL INSTRUCTION AND THE PROFESSORS OF MATHEMATICS IN THE FRENCH LYCEES FOR BOYS.

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(Continued from the January Number).

In France, as everywhere else, the success of the system depends much on the personality of the professor. A renowned Paris lycée instructor who had a genius for getting hold of his boys has recently died. No less than 35 of his pupils were admitted to the École Polytechnique in a single year. The ordinary professor has to be content with a half or a third of this number. But the success of a class is, by happy arrangement, not left to depend wholly upon a single man. Take, for example, lycée Saint Louis, which is the greatest preparatory school for the École Normale Supérieure and the École Polytechnique. There are four Classes de Mathématiques Spéciales and for all the members of these classes, conferences, interrogations and individual examination are organized. These exercises, which complete the daily instruction, are conducted by one of the professors in the lycée itself, or by one of those from the Collège de France, the Sorbonne, the École Polytechnique, the École Normale, from other lycées or from the collèges. Incapable students are thus speedily weeded out. Of perhaps greater value than the solidity of the training got in this way is the fact that the interest of the pupil is sustained.

With the end of the year the élève has his first experience of a *concours.* Previously he has found that it was necessary only to make a certain percentage in order to mount to the next stage in his scholastic career; but now it is quite different. In 1908, 1.078 pupils tried for admission into the École Polytechnique, but only 200, or 19.5 per cent, were received; for the department of science in the École Normale Supérieure, 22 out of 274, or 8 per cent, succeeded. In each case the number was fixed in advance by the Government according to the capacity of the school; the fortunate ones were those who stood highest in the examinations, written and oral. In the case of the École Polytechnique, the written examinations were held in all the lycées which had a Classe de Mathématiques Spéciales. The 387 candidates declared *admissible* were then examined orally at Paris, and from them the 200 were chosen. Similarly for the École Normale, the written examinations are conducted at the seats of the various academies and the oral at Paris. Since 1904 the concours passed by the École Normalians has been that for the *bourses de licence,* open to candidates of at least 18 years of age and not more than 24. Certain dispensations in the matter of age are sometimes granted. The value of the bourse, for the section of science, is from 600-1,200 francs a year and is intended to help the student to prepare for the licence and other examinations required of prospective professors in the lycées and universities. The candidates leading the list in the concours are sent to the École Normale Supérieure for from three to four years. It is necessary for the six or seven other *boursiers* to prepare for future examinations at the various universities of the provinces. Their bourses last regularly for two, and exceptionally for three, years.

But to return to our élèves of the Classes de Mathématiques Spéciales. At the end of the first year, when 18 or 19 years old, they usually present themselves for the concours of both the bourse de licence and the École Polytechnique, the examinations in the former being more strenuous and searching. Only from 2 to 5 per cent, succeed on the first trial. The others then go back to the lycée and take another year in the Classe de Mathématiques Spéciales. Many points not fully understood before are now clear, and at the end of the second year from 25 to 28 per cent are successful. The persevering again return to their *Classe* and try yet a third time (the last permitted for the bourse de licence) ; but it is a matter of record that less than one-half of those who enter the Classe de Mathématiques Spéciales succeed even with this trial. This is usually the last trial possible for entry into the École Polytechnique, as the young man who has passed the age of 21 on the first of January preceding the concours may not present himself. The remainder of the students either seek for entrance into government schools with less severe admission requirements, and thus give up their aspirations to become mathematicians, or else continue their studies at the Sorbonne. The candidate who heads the list in each of these concours has his name widely published. In the case of the bourse de licence he is called the *cacique,* and he very frequently tops also the École Polytechnique list.

If the work in the Classe de Mathématiques Spéciales is so enormously difficult that only 2 to 5 per cent of its members can, at the end of one year, meet the standard of requirements of the examinations for which it prepares, why is not the instruction spread over two? Since nearly all the mathematical savants who now shed lustre on France's fair fame have passed from this remarkable class on the first trial, there can be no doubt that the answer to this question may be found in the fact that the government ever seeks her servants among the *élite* of the nation's intellectuals.

Those who pass from the Classe de Mathématiques Spéciales at the early age of 18 years are not numerous, but Borel and Picard are such men while Goursat entered the École Normale Supérieure at 17 years of age. For the average boy the lycée course is heavy and more than once he may have to halt in order to repeat a year. The system of training is largely formulated to develop to the full the powers of the brilliant boy and to promote his rapid advancement. For such youths poverty is no detriment. Every lycée has a number of bursaries (covering all expenses) which it distributes to just such boys coming with distinguished records from the primary schools. If the boy's record is sustained, renewal of his bursary from year to year is inevitable.

We have now seen something of the nature of the remarkable mathematical training which the French boy may receive in the lycée and have incidentally remarked that the men who have given this training are also exceptional. In conclusion I propose to describe very briefly the preparation necessary to become a lycée professor and the inducements offered by the state to the youth of the country to enter upon this preparation.

I have had occasion to point out the strong influence which the École Normale Supérieure and École Polytechnique exert on the careers of the flower of the French youth; how that instead of entering the university on passing the baccalauréat, as in America or in Germany, "they seek to enter these schools. The reason for this is not difficult to find. The École Polytechnique which prepares its pupils as military and naval engineers, artillery officers, civil engineers, in government employ, telegraphists and officials of the government tobacco manufactories, offers all of its graduates a career which is at once rapid, brilliant and certain. The École Normale practically assures its graduates at least a professorship in a lycée and prepares its élèves for this, or for a university career better and more rapidly than the university can do it."

Let us suppose that our future mathematical professor in the lycée is one of the eleven mathematical students who is successful in getting into the École Normale in a given year. He studies there for three years and receives special drill in pedagogy and in connection with courses of lectures which he hears at the Sorbonne and at the Collège de France. Almost the whole purpose of the drill and instruction is to prepare for two examinations, the *licence* and the *agrégation.*

The diploma *Licence ès Science,* which is necessary for all those who take up secondary teaching is granted to those who have 3 *certificats* in any one of three groups of subjects. Our mathematician is examined in the following subjects: (1)—Differential and Integral Calculus; (2)—Rational Mechanics; (3)—General Physics or some advanced topic in mathematics. The examinations may be taken singly in July or in November; each examination successfully passed entitles the student to a *certificat* for that subject. The examination consists of three parts, *épreuve écrite, épreuve pratique, épreuve orale.* The first two are written examinations of about four hours each. Theoretical considerations abound in the *écrite* while numerical calculation is characteristic of the *pratique.* The *orale* lasts for 15-20 minutes and is held before a jury of those professors who have the whole examination in charge. The pass mark is fifty per cent.

Unlike the *baccalaurèat* and the *licence,* the *agrégation* is a competitive examination and is conducted by the state. The number who become *agrégés* each year is fixed in advance by the Minister of Public Instruction according to the needs of the lycées in the country. This number in recent years has been about 14; the number of candidates is usually about 80. Our candidate for this examination must have four *certificats,* (1)—Differential and Integral Calculus; (2)—Rational Mechanics; (3)—General Physics; and (4)—a subject chosen at pleasure in the advanced mathematical fields in which courses are offered. Just what is implied by the possession of one of these *certificats* we may not pause to consider further than to say that no *one* graduate course in any American university gives the pupil such a comprehensive grasp and mastery of the subject.

To pass the *agrégation* our future professor disposes of his three years as follows: During each of the first two years he passes the examinations for two of the four *certificats.* With these off his hands he turns his whole attention to preparing for the *agrégation* proper. This examination is unique in its difficulty and exactions. As it is organized for selecting the most efficient young men in the country to take charge of the mathematical classes in the lycées, the examination turns largely on the subjects there taught. It consists of *épreuves préparatoires* and *épreuves définitives*. The former are four written examinations each of seven consecutive hours in length (7 a. m. to 2 p. m.)! The first two of these are on subjects chosen from the programme of the lycée in *mathématiques élémentaires* and *mathématiques spéciales*. The last two, based on the work of the candidates in the universities, are a *composition* on Analysis and its geometrical applications and a *composition* on Rational Mechanics. The *épreuves* are held at the seats of the various académies of France. Those who have reached a sufficiently high standard are declared *admissible*. Their number is usually a little less than twice the possible number to be finally received. They must present themselves at Paris for the *épreuves définitives*. These consist of two written examinations and two *leçons*. The written tests are an épreuve de *géométrie descriptive*, and a *calcul numérique*. Their duration is fixed by the jury, but it is usually four hours for each. The *leçons* which are supposed to be such as a professor might give (during ¾-1 hour) in a lycée, are on subjects from the programmes of the classes: (a)—*Mathématiques Spéciales;* (b)— *Seconde, Première* C. D. and *Mathématiques* A. B. The subjects are drawn by lot, and for each lesson the candidate has four hours to think over what he is going to say. No help from book or other source is permitted. The unfortunate who has little to say is speedily "adjourned."

The agrégés are those specially prepared by the State for the positions of *professeurs titulaires* in the lycées. Although this title is not conferred regularly till the agrégé has completed his twenty-fifth year, those who are younger receive temporary appointment for every agrégé may demand a position as his right. The salaries vary according to the *classe* of the professor. At Paris the lowest salary is 6,000 francs per year, and the highest, 9,500. In this range seven *classes* are represented; six, each differing from the one before by 500 francs, and the *hors classe* for which the salary is 9,500 francs. Promotion from one class to another takes place by selection and by seniority. From the sixth (the lowest classe) to the third, the number of those who can be advanced each year by selection is equal to the number which can be advanced by seniority. In the second and first classes two advancements may be made by selection to one by seniority. In choosing those for the *hors classe,* selection alone is taken into account. The promotions are made at the end of each calendar year, and take place so that there are always 20 per cent of them in the sixth classe, 18 in the fifth, 18 in the fourth, 16 in the third, 14 in the second, and 14 in the first. This arrangement is obviously a happy one, both by way of recognition of the merits of the unusually successful teacher, as well as those of him whose service is rather characterized by faithfulness.

In addition to the *professeurs titulaires* there are *professeurs chargé de cours,* who are usually selected from those École Normalians and those *admissible* to the agrégation, who fail to become agrégés. After 20 years of service they may become professeurs titulaires and receive the salaries we have indicated above. The government has, however, recently passed a law which gives the higher reward to the agrégé. It is to the effect that 500 francs per year shall be added to the regular salary of every agrégé. The real range of salaries mentioned above is then 6,500-10,000; in the provinces this reduces to 4,700-6,700. For the professeurs chargé de cours, the salaries at Paris vary from 4,500 to 6,000 francs; in the provinces, from 3,200 to 5,200.[10] In the Premier Cycle the professors have 12 hours of teaching per week, in the second cycle and the Classe de Mathématiques Spéciales, 14-15 hours. Except for correcting exercises and filling out reports the professors have absolutely no obligations outside of class hours. They do not live in the lycées. The superintendence of the study of the élèves is carried on by *répétiteurs,* the more advanced of whom receive at Paris 2,600-4,600 francs for 36 hours service per week.

Attractions connected with a professorship in a lycée are, that the remuneration is ample to live on comfortably, that the work is not onerous but often not a little inspiring, that colleagues are brilliant specialists in the same or other lines of study, that the professorships are positions of honor and prominence in the community and that the incumbents are in demand in many ways which frequently materially increase their regular income. In general, only a professor in the first or *hors classe* has charge of the Classe de Mathématiques Spéciale with its members the pick of the French youth, but to this position all may aspire.

With inducements such as these it is no surprise to learn that, in marked contrast to America, France draws to the development

[10]The tendency of recent legislation is to exclude from the lycées all professors who are not agrégés.

of her system of secondary education much of the best mathematical talent in the country.

In the John Hay Library a few days ago, I came across an old Latin work which turned out to be the course of lectures on Greek Geometry, delivered at Oxford University in 1620,[11] by the erudite Sir Henry Savile. In the course of his concluding remarks he used the following language: "By the grace of God, gentlemen hearers, I have performed my promise; I have redeemed my pledge. I have explained, according to my ability, the definitions, postulates, axioms, and *the first eight propositions* of the *Elements* of Euclid. Here sinking under the weight of years, I lay down my art and my instruments."[12]

It is interesting to speculate on the thoughts which would likely pass through the minds of these "gentlemen hearers" were they privileged to listen for a time to the discussion of questions in geometry, and in other parts of mathematics, as carried on by master and pupil in a *Classe de Mathématiques Spéciales* of a French *lycée.*

BIBLIOGRAPHY.

Of prime importance in the study of French Secondary Education for boys in the *"plan d'études et programmes d'enseignement dans les lycées et collèges de garçons* [Arrêtês du 31 mai 1902, des 27, 28 juillet et 8 Septembre, 1905, 6 janvier, 26, 30 juillet et 5 août 1909]." *Paris* (Librairie Delalain), pp. 264—Next in importance should be mentioned Tomes II and III (Librairie Hachette) of the report of the French sub-committee of the International Commission on Mathematical Teaching. Tome II (159 pages), devoted to secondary education and edited by M. Bioche contains special articles by MM. Bioche, Blutel, Lévy, Guitton, Th. Rousseau, Beghin, Muxart, Lombard. In Tome III (123 pages), Superior Education is discussed and articles by various writers are edited by M. Albert de Saint-Germain. A perusal of this volume will give a good idea of the course of training for lycée teachers. Each year about 100 pages of M. H. Vuibert's *Annuaire de la Jeunesse, Paris* (Librairie Vuibert), is devoted to "Enseignement secondaire des garçons." Information as to the cost of the various classes in the different lycées of France, bursaries offered, regulations with regard to uniform

[11]*Praelectiones tresdecim in principium elementorum Euclidis, Oxonii habitae MDCXX.* Oxonii, . . . 1621.

[12]Cajori represents (*History of Elementary Mathematics,* New York, 1897, p. 281) that the above mentioned lectures were delivered "about 1570." According to this statement Savile was "sinking under the weight of years" about the time that he attained to his majority for he was born in 1549.

—for the manner in which the boys dress is determined in part by the State—*etc.*, are here set forth.

In German, references may be given to Klein's sketch of the Teaching of Geometry in France, *Elementarmathematik vom höheren Standpunkte aus, Teil II, Geometrie,* Leipzig, 1909, pp. 456-476.

In English, the most recent article dealing with our subject is Professor G. W. Myers' "Report on the Teaching of Mathematics in France" (with its curiously inaccurate statements concerning the baccalauréat), *School Review,* September, 1911, XIX, 433-453—Professor F. E. Farrington's book *French Secondary Schools* (Longmans, 1910) contains a chapter, pages 257-287, "Mathematics and Science"; the first six pages form an historical introduction—A pleasant sketch of "Life at the Sorbonne," was given by H. Jones in the *Nation,* XCI, p. 576-577, December 15, 1910—The second edition of Barrett Wendell's *"France of Today"* (Constable, 1908) contains a chapter (p. 1-46) on "The Universities"; attractive in style and treatment, its value is marred by a number of misleading and incorrect statements—In March, 1900, Professor James Pierpont published in *Bulletin of the American Mathematical Society* (VI_2, 225-249) a very interesting article entitled "Mathematical Instruction in France." Although secondary education as it was before the great reform of 1902-5, is analyzed to a certain extent, the article was doubtless intended to be read more particularly by the graduate students or professors in American universities—In 1910 I presented a similar paper to the Royal Society of Canada; it is printed in the *Transactions and Proceedings* for 1910, IV_3, 89-152. Appendix A of this paper contains the programme for the *Concours* of the *Agrégation des Sciences Mathématiques* for 1910 and copies of the examination papers set in 1909. The answers expected (unofficial) to the questions in the annual agrégation examination papers, appear a few months later in *Nouvelles Annales de Mathématiques, Revue de Mathématiques Spéciales* and *Journal de Mathématiques Élementaires.* A study of a series of such examination papers and answers soon leads to adequate appreciation of the mathematical capabilities of the agrégé or lycée professor—The programmes and examination papers for the agrégation from year to year may be purchased at Librairie Croville-Morant, Paris. In a new publication, *Annales du Baccalauréat* (Librairie Vuibert) containing 9 *fasicules* a year, the first and ninth fasicules give full information with regard to the mathematical examination papers of candidates for the *Baccalauréat.*

A List of Mathematical Text Books Used in French Secondary Education.

No student of Secondary Education in France should omit the study of the text-books. Similar preparation for professors in lycées and universities, and the intimate relations between these institutions has for many years inspired French university professors to write school texts. Legendre, Clairaut, Bertrand, Bobillier and Catalan are among others in earlier days, while Goursat, Tannery, Darboux, Hadamard, Méray and Borel are of our time. On the other hand those authors who are lycée professors are brilliant and excellently equipped mathematicians.

The ignorance of the mathematician concerning the books used in connection with elementary or secondary education of a foreign country is very general, and yet, in the case of America, at least, familiarity with works used in France would be a source of great inspiration and help not only to the vast body of text-book writers and teachers in high schools but also to professors in colleges. For, as we have seen, the mathematical student in the lycée is thoroughly conversant with those topics usually treated well on in the undergraduate course of the better American colleges, while those leaving the *Classes de Mathématiques Spéciales* at the age of 18 or 19 years are many times better equipped mathematicians than our sophomore or junior college student of 20 or 21 who has specialized in mathematics.

To refer to the books in the following list as "text-books" will not lead to any misconception when it is recalled that French professors make sparing use of books in classes, since the subject matter of the courses is usually presented by dictation or lecture.[13] But these books are such as the pupils have used in working up the lectures, or such as a professor has encouraged the pupil to consult in the lycée library.

My selection is limited to about 35 works or courses which I have thought would be representative. Many others which would have been of interest to the American mathematician are omitted on account of space limitations. An analysis of the contents and characteristics of a number of the geometrical texts

[13]The reasons for this are clearly set forth in *Rapports,* French Sub-Committee, *l. c.* III. 90.

may be found in M. Th. Rousseau's report (pages 88-110) referred to above.

The arrangement is alphabetical according to authors. The date and place of publication, the name of the publisher, the number of pages, and in most cases the names of the classes for which the books were especially designed, are given.

At the end of the list I have added the names of four defunct and three current periodicals, published in the interests of French elementary and secondary mathematical education. Much of interest in this department may be gleaned from these sets.[14]

A number of French books (such as those by Gelin and Neuberg) by *Belgian* authors are purposely excluded from this list, although they are widely used in France. Translations of books originally written in a language other than French are also excluded; otherwise I might have listed several popular works (such as those by Alexandroff, Faifofer and Petersen).

The editions indicated in the following list are not necessarily the last published—but simply the last I have seen. The size of the works may not be judged by the number of pages only—as the formats vary from 16mo. to royal 8vo.; *e. g.* Borel's course is in 12mo. Darboux's in large 8vo.

Antomari, X. Cours de géométrie descriptive (Mathématiques Spéciales) 3e éd. *Paris* (Vuibert).[15] 1906, 619 p.

Appell, P. Cours de mécanique (Mathématiques Spéciales) 3e éd., entièrement refondue *Paris* (G. V.).[16] 1912, 527 p.

Appell, P. and Chappuis, J. Leçons de mécanique (Mathématiques A. B.). *Paris* (G. V.), 2 volumes.

Tome i: Notions géométrique. Cinematique. 3e éd. 1909, 178 p.

Tome ii: Dynamique et statique du point. Statique des corps solides, machines simples. 2e éd. 1907, 240 p.

Appell, P. *See also* Briot and Bouquet.

Aubert, P. and Papelier, G. Exercices d'algèbre d'analyse et de trigonométrie (Mathématiques Spéciales). *Paris* (Vuibert), Tome i, 1908, 362 p.; tome ii (deuxième année), 1910, 359 p.

Borel, É and Royer, M. "Cours Émile Borel." *Paris* (Colin):—

Arithmétique (I Cycle), 1907, 220 p.

Géométrie (I, II, Cycle), 2e éd. 1908, 383 p.

Note:—Klein (l. c) speaks of this work as "ein sehr interessantes Buch."

Algèbra (I Cycle), 2e éd., 1905, 256 p.

Trigonométrie (II Cycle), 2e éd., 1905, 198 p.+*Fasicule,* 1908, 11 p.

Géométrie cotée par R. Danelle, 1908, 64 p.

Algèbre (II Cycle), 3e éd., 1905, 401 p. + *Fasicule,* 1908, 30 p.

Bourdon, P. L. M. Application de l'algèbre à la géométrie, comprenant la géométrie analytique à deux dimensions. 9e éd., revue et annotée par G. Darboux. *Paris* (G. V.), 1906, 648 p.+10 pl.

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[14]*L'Enseignement Mathématique* (XIVe Année, 1912), published in Geneva, is also of value.

[15]Immediately after the place of publication follows the name of the publisher. The publishing house Vuibert was formerly known as "Vuibert & Nony."

[16]G. V.—Gauthier Villars.

BOURLET, C. Leçons d'algèbre élémentaire (Mathématiques A. B.), 5e éd., *Paris* (Colin), 1907, 566 p. *Cours de Darboux.*

BOURLET, C. (i) Éléments de géométrie, plane et dans l'espace (I et II Cycles A), *Paris* (Hachette), 1908, 378 p. [2e éd., 1910, vi+383 p.][17]

(ii) Corrigés des 773 exercices et problèmes dans les Éléments" avec collaboration de Paul Baudoin, *Paris* (Hachette), 1908, 348 p.

BOURLET, C. Cours abrégé de géométrie—avec nombreux exercices théoriques et pratiques et des applications au dessin géométriques avec la collaboration de M. P. Baudoin, *Paris* (Hachette):

(i) Géométrie plane (VI, V, IV, B), 4e éd., 1909, 408 p.

(ia) Corrigés des exercices théoriques, 1908, 302 p.

(ii) Géométrie dans l'espace (III B), 3e éd., 1911, viii+239 p.

(iia) Corrigés des exercices théoriques, 1908, 172 p.

BOURLET, C. Leçons de trigonométrie rectiligne (Mathématiques A. B.), 3e éd., *Paris* (Colin), 1908, 322 p. *Cours de Darboux.*

BRIOT, CH. Leçons d'algèbre. 2e partie revue par É. Goursat, 18e éd. (Mathématiques Spéciales), *Paris* (Delagrave), [1906], 635 p.

BRIOT, CH. AND BOUQUET, J. C. Leçons de géométrie analytique, 15e éd., revue et annotée par M. Appell. *Paris* (Delagrave), 1893, 758 p.

Note:—The sections of this work devoted to plane analytical geometry (about two-thirds of the whole work) were translated into English by J. H. Boyd in 1896 (Werner School Book Co., Chicago and New York).

DARBOUX, G. Cours complet pour la classe de mathématique A. B., publié sous la direction de G. Darboux—See Bourlet (2), Hadamard, Tannery, Tisserand.

Note:—The title of this course is somewhat of a misnomer as the various topics are treated with an elaboration scarce possible in any lycée class outside of the Classe de Mathématiques Spéciales.

DARBOUX, G. *See also* Bourdon.

DUPORCQ, E. Premier principes de géométrie moderne à l'usage des élèves de Mathématiques Spéciales et des candidate à la licence et à l'agrégation. *Paris* (G. V.), 1899, 160 p. [2e éd., 1912, viii+174 p.]

F. J.[18] Éléments de géométrie. *Tours* (Mame) et *Paris* (Poussielgue) 1909, 523 p.

F. G. M. Exercices de géométrie comprenant l'exposé des méthodes géométriques et 2000 questions résolues. 5e éd *Tours* (Mame) et *Paris* (Gigord), 1912, xxiv+1298 p.

Note:—A livre du maître for the "Éléments." Historical notes abound and 39 pages are devoted to Indexes of various kinds.

F. J. Éléments de géométrie descriptive. *Tours* (Mame) et *Paris* (Poussielgue), 1910, 458 p.

F. G. M. Exercices de géométrie descriptive. 4e éd. *Tours* (Mame) et *Paris* (Poussielgue), 1909, X+1099 p. Compléments, *idem,* 1912, pp. 1100-1161.

Note:—A *livre du maître* for the "Éléments." Indexes, pp. 1073-1099.

GÉOMÉTRIE. Cours supérieure, par une reunion de professeurs. *Tours* (Mame) et *Paris* (Poussielgue), [1908], 323 p.

Note:—Preparatory for F. J. Éléments de géométrie.

GRÉVY, A. Traité d'algèbre (Mathématiques A. B.), 4e éd., *Paris* (Vuibert), 1908, 498 p.

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[17]Information concerning editions I have not seen is added in this way.
[18]The two authors of this and the three following works are *Frères* of the *Écoles Chrétiennes.*

HADAMARD, J. Leçons de géométrie élémentaire (Mathématiques A, B), *Paris* (Colin)—*Cours de Darboux.*

Tome i: Géométrie plane, 2e éd., 1906, 308 p.

Tome ii: Géométrie de l'espace, 1901, 582 p.

HUMBERT, É. Traité d'arithmétique (Baccalauréat et écoles du gouvernement). Avec une préface de Jules Tannery, 4e éd., *Paris* (Vuibert), 1908, 496 p.

KOEHLER, J. Exercices de géométrie analytiques et de géométrie supérieure à l'usage des candidats aux École Polytechnique et Normale et à l'agrégation. Questions et solutions. *Paris* (G. V.). Tome i, 1886, 349 p.; tome ii, 1888, 469 p.

MÉRAY, CH. Nouveaux éléments de géométrie. Nouv. éd refondue et augmentée. Dijon (Jobard), 1903, 450 p.+22 pl. 3e éd., 1906, 309 p.

Note:—Klein (l. c.) devotes more than a page to this highly interesting work of which perhaps the most prominent characteristics are treatment based on the idea of motion and the fusion of planimetry and stereometry from the very first. M. Rousseau gives up over five pages of his report to the discussion of Méray's book.

NIEWENGLOWSKI, B. Cours de géométrie analytique (Mathématiques Spéciales). *Paris* (G. V.), 3 tomes. Tome i: 2e éd., 1911, vi+496 p.; tome ii: Constructions des courbes planes, complément relatifs aux coniques, 2e éd., 1911, iv+324 p.; tome iii: géométrie dans l'espace avec une note de É. Borel sur les transformations en géométrie, 1896, 572 p.

NIEWENGLOWSKI, B. Cours d'algèbre (Mathématiques Spéciales), *Paris* (Colin), 2 tomes et supplément. Tome i, 5e éd., 1902, 391 p.; tome ii, 5e éd., 1902, 488 p.; supplément, 1904, 43 p.

PAPELIER, G. Précis de géométrie analytique (Mathématiques Spéciales), *Paris* (Vuibert), 1907, 696 p.

Note:—Pages 431-696, 3 dimensions.

ROUCHÉ, É. *et* COMBEROUSSE, CH. DE. Éléments de géométrie suivis d'un complément à l'usage des élèves de mathématiques élémentaires et de mathématique spéciales, etc., 7e éd. *Paris* (G. V.), 1904, 651 p.

ROUCHÉ, É. et COMBEROUSSE, CH. DE. Traité de géométrie. 7e éd., *Paris* (G. V.), 1900. Tome i: géométrie plane, 548 p.; tome ii: géométrie dans l'espace, 664 p. 8e éd., 1912.

ROYER, M. *See* Borel, É.

SERRET, J. A. Traité de trigonométrie. 9e éd., *Paris* (G. V.), 1908, 336 p.

TANNERY, JULES. Leçons d'arithmétique théorique et pratique (Mathématiques A, B), 2e éd., *Paris* (Colin), 1911, xvi+545 p. *Cours de Darboux.*

TANNERY, J. Notions de mathématiques avec notions historiques par Paul Tannery, 3e éd, augmentée de notions d'astronomie (Programmes du 1902 et 1905—Classe de philosophie), *Paris* (Delagrave), 1905, 370 p.

TANNERY, J. Leçons d'algèbre et d'analyse (Mathématiques Spéciales), *Paris* (G. V.), 1906; tome i, 423 p.; tome ii, 636 p.

TANNERY, J. *See also* Humbert, É.

TANNERY, P. *See* Tannery J.

TISSERAND AND ANDOYER. Leçons de Cosmographie (Mathématies A. B.), *Paris* (Colin), 1909, 371 p.+12 pl. *Cours de Darboux.*

VACQUANT, CH. AND MACÉ DE LEPINAY. Cours de trigonométrie. *Paris* (Masson). Première Partie (Classes C, D et Mathématiques, A. B). Nouv. éd., 1909, 294 p. Deuxième Partie (Mathématiques Spéciales). Nouv. éd., 1909, 172 p.

L'Education Mathématique publié par A. Durand et H. Vuibert XIVe année, 1911-1912 (Vuibert), 20 numbers a year.

Note:—Very elementary.

Journal de Mathématiques Élémentaires publié par H. Vuibert, XXXVIe Année, 1911-1912 (Vuibert), 20 numbers a year.

Revue de Mathématiques Spéciales redigée par É Humbert, G. Papelier, P. Aubert, P. Lemaire, C. Rivière, H. Vuibert, XXIIe année, 1911-1912 (Vuibert), 10 numbers a year.

Note:—The solutions of the more elementary portions of the examinations for the agrégation are published each year in these last two mentioned periodicals.

Bulletin de Mathématiques Élémentaires dirigée par M. Ch. Michel. Octobre, 1895—Juillet, 1910 (Lamarre).

Bulletin de Mathématiques Spéciales redigée par Niewenglowski at de Longchamps, Octobre, 1894—Juillet, 1900 (Lamarre).

Journal de Mathématiques Élémentaires redigée par Bourlet, de Longchamps, G. Mariaud. Octobre, 1876—Mai, 1901 (Delagrave).

Journal de Mathématiques Spéciales redigée par de Longchamps Mariaud. Octobre, 1879—Mai, 1901 (Delagrave).

SOME MATHEMATICAL BOOKLET SERIES.

R. C. ARCHIBALD

Reprinted from the
BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY
2d Series, Vol. XX., No. 5, pp. 238–243.
New York, February, 1914

[Reprinted from BULL. AMER. MATH. SOCIETY, Vol. 20, No. 5, Feb., 1914]

SOME MATHEMATICAL BOOKLET SERIES.

Matematica dilettevole e curiosa. Di ITALO GHERSI. Con 693 figure originali dell'Autore. Milano, Ulrico Hoepli, 1913. viii+730 pp. Price L. 9.50.

Wo steckt der Fehler? Trugschlüsse und Schülerfehler. Gesammelt von Dr. W. LIETZMANN und V. TRIER. Mathematische Bibliothek, Nr. 10. Leipzig and Berlin, B. G. Teubner, 1913. 57 pp. Price M. 0.80.

ENGLISH and French mathematical literature is entirely lacking in such admirable booklets dealing with elementary topics, as those which have wide circulation in Germany and Italy.† I refer to the Mathematische Bibliothek of the

† It may be suggested that the volumes on Elimination by Laurent and on Geometrography by Lemoine, of the excellent "Scientia" series (Gauthier-Villars, Paris) are elementary, but these are only two of a dozen volumes by Appell, Gibbs, Hadamard, Poincaré, etc., which certainly may not be classed in this way. And even these two brochures are more

Sammlung Göschen,* the Lietzmann-Witting Mathematische Bibliothek † and the mathematical volumes of the Biblioteca degli studenti,‡ and the Manuali Hoepli.

The Mathematische Bibliothek contains about 35 volumes ($4\frac{1}{4}$ x $6\frac{1}{4}$ inches; uniform price, $22\frac{1}{2}$ cents), each neatly bound in cloth and containing from 130 to 230 pages. A. Sturm, H. Schubert, M. Simon, O. Th. Bürklen, K. Doehlemann and E. Beutel are among the authors and the volumes treat of History of mathematics, Plane geometry, Descriptive geometry (2 volumes), Determinants, Analytical geometry of the plane, Analytical geometry of space (notably fine figures), Projective geometry, Algebraic curves (2 volumes),§ Insurance mathematics, Vector analysis, Geodesy, Surveying, Astronomy, etc.

Of the Mathematische Bibliothek herausgegeben von W. Lietzmann und A. Witting a dozen volumes have already appeared. They are bound in boards, contain 41 to 93 pages ($4\frac{3}{4}$ x $7\frac{1}{4}$ inches) each, and are of the same uniform price as the Göschen Sammlung before 1913. In this series Wieleitner has written on the Idea of number in its logical and historical development; O. Meissner is author of Theory of probabilities with applications; M. Zacharias wrote the Introduction to projective geometry; Zühlke, Geometrical constructions in a limited plane; Beutel, Squaring the circle.

In the Biblioteca degli Studenti are nearly a score of volumes (4 x $6\frac{1}{4}$ inches; limp covers; single numbers of about 85 pages, 10 cents, double numbers of about 170 pages, 20 cents). They include, Manual of plane trigonometry, Manual of spherical trigonometry, Exercises of elementary geometry, Guide to the resolution of problems in algebra, Principles of perspective, Repertorium of mathematics and elementary physics, etc., and treat of very elementary topics.

Some 40 of the 1,200 odd volumes ($4\frac{1}{4}$ x 6 inches) in the Manuali Hoepli series are of mathematical content. Perhaps the two best known works are the volumes (658+950

advanced in character than any of those in three, and than many of those of the fourth series, about to be considered. The same may be remarked concerning the Cambridge Tracts in Mathematics and Mathematical Physics.

* G. J. Göschen'sche Verlagshandlung, Berlin und Leipzig.

† B. G. Teubner, Leipzig und Berlin, 1912–1913.

‡ Raffaello Giusti, editore, Livorno.

§ The second volume was reviewed by Professor White in this BULLETIN, vol. 19, pp. 417–419, May, 1913.

pages) of Pascal's Repertorio di matematiche superiori* since translated into German and enlarged,† and Pascal's Determinanti e applicazioni, 1897, which three years later was elaborated into a volume of the Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften. Then there are 4 volumes on Algebra, 4 on Arithmetic, 2 on Astronomy, 4 on the Calculus, including volumes on Calculus of variations and Finite differences,‡ and Critical exercises on the differential and integral calculus; 1 on Mathematical formulæ;§ Saccheri's Euclide emendato; 3 volumes on Functions (analytic, elliptic, polyhedral and modular||); 13 on Geometry, 1 on the Mathematics of economics,¶ 1 on Groups, 4 on Mechanics, and 1 by G. Loria on Exact science in ancient Greece.** The volume of Ghersi under review is the second he has written for this mathematical series, the earlier one having dealt with Methods for resolving problems of elementary geometry.

In recent times English, French, and German writers have published popular works for recreation hours of those who are in any wise interested in mathematics. Ball's Mathematical Recreations and Essays, which has recently reached a fifth edition,†† is almost a classic in its special field. The older works of Lucas, "Récréations mathématiques"‡‡ and "L'Arithmétique amusante"§§ are frequently referred to,

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* Milano, 1898 and 1900. The first volume is reviewed by E. O. Lovett in this BULLETIN, vol. 5, pp. 357–362, April, 1899.

† Two volumes, Leipzig, 1900, 1901. New edition to be completed in 4 volumes; vols. 1_1, 2_1, 1910, reviewed in this BULLETIN by C. H. Sisam, vol. 19, pp. 372–374, April, 1913.

‡ Translated into German by A. Schepp. Leipzig, 1899. Reviewed by J. K. Whittemore in this BULLETIN, vol. 6, pp. 352–4, May, 1900.

§ This volume by Rossotti was reviewed by E. O. Lovett in this BULLETIN, vol. 5, pp. 261–2, Feb., 1899.

|| This volume by Vivanti was reviewed by J. I. Hutchinson in this BULLETIN, vol. 14, pp. 144–5, Dec., 1907. The French edition by A. Cahen was reviewed by G. A. Miller in this BULLETIN, vol. 19, pp. 534–5, July, 1913.

¶ This volume by Virgilii was reviewed by J. M. Gaines in this BULLETIN, vol. 5, pp. 488–9, July, 1899.

** This work, which has just been published, 1914, contains about 1000 pages. The title page with "seconda edizione totalmente riveduta" is misleading, as the original work of over 900 pages of *quarto* format was a reprint of memoirs (in five books) in *Mem. Acc. Modena* (2), vol. 10, pp. 3–168; vol. 11, pp. 3–237; vol. 12_2, pp. 3–411; (1893–1902). Futhermore, "Libro II, Il periodo aureo della geometria Greca" appeared in still another form in *Mem. Reale Acc. d. Sc. di Torino* (2), vol. 40, pp. 369–445; (1890).

†† London, 1911.

‡‡ Paris, T. I, 2e éd., 1891; T. II, 2e éd., 1896; T. III, 1893; T. IV, 1894.

§§ Paris, 1895.

while the circulation of Ahren's Mathematische Unterhaltungen und Spiele* and Schubert's Mathematische Musestunden† is confined more to Germany. Each work has its own peculiar ideals, but Ball is perhaps the most comprehensive in range, while he and Ahrens alone introduce, to an appreciable extent, references to the widely scattered literature of the subject. E. Fourrey's "Curiosités géométriques"‡ is also notably full in exact statement of authorities.

From works such as these, from books like Blythe's on Models of cubic surfaces, Catalan's Théorèmes et problèmes de géométrie élémentaire, Cremona's Elementi di geometria proiettiva, Enriques' Questioni riguardanti la geometria elementare, de Longchamps' Essai sur la géométrie de la règle et de l'équerre, Loria's Spezielle algebraische und transzendente ebene Kurven, and from various periodicals, Ghersi has compiled the present little work on Matematica dilettevole e curiosa.

The first 74 pages are taken up with "Curious and bizarre problems" such as: Euler's problem of the Königsberg bridges, the Hampton Court maze and other unicursal problems, map-coloring problem, and chess problems. Of course little more than the statement of a problem is frequently given.

In the next 100 pages various curious properties of numbers, and problems of arithmetic and arithmetic geometry are set forth. For example, we have properties of perfect and amicable numbers, of the triangle of Pascal, of Lucas's singular products, as well as problems of Benedetti (Speculationes diversæ, 1585) and of Leonardo Pisano (Liber Abaci, 1202).

Fermat's equation and other problems of the theory of numbers are treated in the next 15 pages, then follows a collection of miscellaneous algebraic problems which conclude with graphical solutions of equations of the second, third and fourth degrees and with a sketch of Demanet's and Meslin's hydraulic,‡ and Lucas's electric solution of equations.

Magic squares, magic polygons, and magic polyhedra are illustrated on pages 251–326.

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* Leipzig, 1901.

† Grosse Ausgabe, Leipzig, Bd. I, 3. Aufl., 1907; Bd. II, 2. Aufl., 1900.

‡ 2e éd., Paris, 1906.

§ Cf. "Two hydraulic methods to extract the nth root of any number" and "Hydraulic solution of an algebraic equation of the nth degree," by A. Emch, *American Mathematical Monthly*, January and March, 1901, vol. 8, pp. 10–12 and 58–9.

Then follow 350 pages treating of miscellaneous questions in geometry. On pages 329–367 we find definitions and derivation of properties, of notable transcendental, and cubic, quartic, and other algebraic curves. The next dozen pages contain instruments for tracing by continuous motion such curves as the conic sections, cissoid, and conchoid. Some 20 pages given over to discussion of the solution of problems in elementary geometry, by ruler and compass, and then (pages 407–422) cyclotomy is touched upon. Then come 100 pages occupied with the problems of trisection of an angle, squaring the circle, duplication of the cube. Dissection of figures, geometrical pavements, star-polyhedra, and hyperspace are some of the concluding topics under the head geometry.

In the final sections are paradoxes and other recreations in mechanics.

It will be remarked, as indeed the title implies, that the volume is not confined to so-called recreations, although these occupy the major part of the volume. It is written with light touch and anyone unacquainted with books on mathematical recreations may pass a few pleasant hours in turning over the pages and find some things not met with in other books of the kind. The reader who wishes to learn more of the underlying theory will then naturally turn for guidance to such a book as Ball's or to the article in the Encyklopädie* or to such works in fields other than those of recreations, as mentioned above.

In the Lietzmann-Trier Bändchen, which may be classed as a small addition to the literature of mathematical recreations, Lietzmann collected the 36 fallacies (Trugschlüsse) and Trier the 50 pupils' mistakes. Arithmetic, algebra, elementary geometry (synthetic and analytic), trigonometry are the only subjects illustrated. The errors in the reasoning are not indicated.

Among the fallacies are (1) numerous examples depending for their results upon division of each side of an equality by zero or neglect of consideration of double sign before a radical; (2) a series of geometrical paradoxes, several of which are already familiar through Ball's book.

* Mathematische Spiele, von W. Ahrens, vol. I, 2, pp. 1080–1093, Leipzig, 1902.

Here is an example of a different kind, which appears to be new: "Consider

(1) $\log_e 2 = 1 - 1/2 + 1/3 - 1/4 + 1/5 - \cdots;$

multiplying through by 2 we get

$$2 \log_e 2 = 2 - 1 + 2/3 - 1/2 + 2/5 - 1/3 + 2/7 - \cdots.$$

Collecting terms with common denominators and arranging according to increasing denominators, we get

(2) $2 \log_e 2 = 1 - 1/2 + 1/3 - 1/4 + 1/5 - \cdots.$

This is, however, the same as (1). Therefore

$$\log_e 2 = 2 \log_e 2."$$

The examples of Schülerfehler are taken from the exercises of Danish pupils. The vagaries of American youth suggest that an equally interesting collection could be made on this side of the water. The error in No. 32 is not evident. But here is No. 36: "Given two circles which cut one another in P and Q and touch the sides of an angle, on the same side of the vertex, at the points A, A_1 for one circle and B, B_1 for the other. Prove (1) that PQ produced passes through the middle points of AB and A_1B_1; (2) that AA_1, BB_1 and PQ are parallel to one another." Solution: "PQ cuts AB in C, A_1B_1 in C_1. Then by the power theorem, $CA^2 = CP \cdot CQ = CB^2$. Therefore C is the middle point of AB. In the same way C_1 is the middle point of A_1B_1. AA_1, PQ and BB_1 are parallel to one another because they cut off the equal segments on the lines AB and A_1B_1."

Finally, No. 47: "The sides of a triangle are a, b, c. To express $\sin A$ in terms of the given quantities." Solution: "Of course the following relations hold good:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

In a proportion it is allowable to interchange the means; hence

$$\frac{a}{\sin A} = \frac{b}{c} = \frac{\sin B}{\sin C}. \qquad \therefore \sin A = \frac{ac}{b}."$$

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MATHEMATICAL MODELS.

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MATHEMATICAL MODELS.

Catalog mathematischer Modelle für den höheren mathematischen Unterricht. Veröffentlicht durch die Verlagshandlung von Martin Schilling in Leipzig, mit 106 Abbildungen. Siebente Auflage, Leipzig, 1911, xiv + 172 pp.

Verzeichnis von H. Wieners und P. Treutleins Sammlung mathematischer Modelle für Hochschulen, höhere Lehranstalten und technische Fachschulen. Zweite Ausgabe mit 6 Tafeln. Leipzig und Berlin, Verlag von B. G. Teubner, 1912, 64 pp.

Abhandlungen zur Sammlung mathematischer Modelle. In zwanglosen Heften herausgegeben von HERMANN WIENER. Leipzig, Verlag von B. G. Teubner. 1. Heft von H. Wiener, 1907, 90 pp.; 2. Heft von P. Treutlein, 1911, 20 pp.

Illustrierter Spezialkatalog mathematischer Modelle und Apparate. Entworfen von G. KOEPP und anderen bewährten Fachmännern. New York City, Eimer and Amend, 128 pp.

FOR fifteen years the bulk of the models used by advanced mathematical students all over the country has been procured from Schilling of Leipzig. This firm was developed from the Firma L. Brill of Darmstadt, the foundation of which reaches back some 35 years. Klein and A. von Brill, in those early years professors in the Technische Hochschule at Munich, had more than two score of models made for the Hochschule under their direction. Copies of these (for example: the tractrix of revolution, geodetic lines on an ellipsoid of rotation, Kummer's surface, forms of Dupin's cyclide, the spherical catenary, twisted cubics) in gypsum, wire, and brass form a portion of the great Schilling collection. In more recent times construction of new models has been carried on by the assistance of many other mathematicians. Among them are Professors Dyck, Finsterwalder, Kummer, Schoenflies, H. A. Schwarz, C. und H. Wiener.

As nearly ten years had passed since the sixth edition of the catalogue, the seventh edition† fills a long felt want. It

† Descriptions of models which have been manufactured since this edition was published, have appeared in *Jahresbericht d. Deutsch. Math.-Ver.*, 1913, vol. 22, pp. *75–76, 134–137.*

describes some 400 models. Nothing more than an indication of the subjects illustrated can be given here: surfaces of the second order, algebraic surfaces of the third order, algebraic surfaces of the fourth and higher orders, line geometry, screw surfaces, space curves and developable surfaces, descriptive and projective geometry, analysis situs, algebra, function theory, mechanics and kinematics, mathematical physics, and structure of crystals,

Shortly after the third International Mathematical Congress at Heidelberg in 1904, Teubner offered to the public a selection of about 60 of the mathematical models for Hochschule instruction which had been exhibited at the Congress by the mathematical Institut of the Technische Hochschule of Darmstadt. The construction of the models in the selection was inspired by Professor H. Wiener.* In the new catalogue now before us we find that Professor Wiener has increased his collection by 50 models, while the late Professor Treutlein has contributed about 200 more.† All of the models are designed as aids to instruction in German secondary schools and Hochschulen. For students of higher mathematics the models of twisted curves and deformable quadric surfaces will probably be the only ones of especial interest.

The Abhandlungen are intended to be of value for those using the models. In Heft 1 are 9 Abhandlungen by Wiener: (1) Mathematical models and their use in instruction (pages 3–8); (2) On the projection of some plane figures (9–10); (3) The regular Platonic polyhedra, Regularity in a group (11–14); (4) Regular polygons and closed reflective systems (15–18); (5) The building up of the regular polyhedra (19–51); (6) How shall surfaces, especially those of the second order, be drawn? (52–54); (7) On surfaces of the second order (55–84); (8) Deformable thread models of ruled surfaces of the second order with fixed thread lengths (85–87); (9) Deformable metal-bar models for transforming a surface of the second order into confocal surfaces (88–91).

These Abhandlungen are similar to those which Schilling

* The catalogue (Verzeichnis mathematischer Modelle, 28 pp.) was published in 1905.

† An interesting account of these models written by Prof. H. Wiener, may be seen in *Jahresbericht d. Deutsch. Math.-Ver.*, Nov., 1913, vol. 22, pp. 297–304.

distributes* with his models and some of them are of considerable interest; on the one hand because of the developments of the theory of the surfaces, on the other through the application of the theory to construction of the models. Numerous bibliographical references are given. In illustration of these characteristics note, for example, (5) and (9).

In (5) the first five pages contain an historical review of the subject, then the theoretical considerations are treated under the headings: The notion of a polyhedron (e. g., Idea of a side, of "Vielkant," of "Vielflach," of "Vielzell"); First and second definitions of the regular polyhedron by the group; Transformation of an angle into a neighboring angle (by rotation); Range of the different suppositions; Third definition of the regular polyhedra; Construction of a regular polyhedron from its group.

In (9) we find that the construction of the model was made possible through theorems of Henrici and Greenhill. Among other studies in this connection, those of Mannheim, Darboux, and Schur are also considered.

The second Heft, written by Treutlein, contains Abhandlungen on the following subjects: "On the intuitive method of mathematical instruction" (pages 3–6) ; "On mathematical models and their use in teaching" (7–9); "Explanations in connection with the series, and the single models, of the Treutlein collection" (10–20).

In all of the above mentioned publications, Dyck's Katalog† is frequently referred to.

The Eimer and Amend collection is of use more particularly in connection with elementary work in planimetry, stereom-

* The "Erste Folge, Abhandlungen zu den Serien I–XXIII, mit 71 Figuren auf 6 Tafeln und im Text" have also been published in a single volume, in connected form. In the "Neue Folge" Hefte 1–9 have been already issued between 1899 and 1912. The authors are: Fr. Schilling, H. Wiener, W. Ludwig, H. Grassmann, W. Boy, E. Estanave, R. Hartenstein, and F. Pfeiffer.

† Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente. Unter Mitwirkung zahlreicher Fachgenossen herausgegeben im Auftrage des Vorstandes der Deutschen Mathematiker-Vereinigung von geh. Hofrat Dr. Walther v. Dyck, Professor an der Technischen Hochschule zu München. Teubner, Leipzig, 1892, xvi+430 pp.

Nachtrag, Leipzig, 1893, x+135 pp.

These volumes contain papers by Klein, Voss, Brill, Hauck, v. Braunmühl, Boltzmann, Amsler, and Henrici, beside descriptions of the various models by their respective designers.

etry, trigonometry, and related branches. The models of star-polyhedra, Poinsot polyhedra, and the so-called Archimedian semi-regular solids may be mentioned as desirable for more advanced mathematical considerations.

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