MEASUREMENTS OF SOME ACHIEVEMENTS IN ARITHMETIC

BY

CLIFFORD WOODY

Submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the Faculty of Philosophy, Columbia University

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Teachers College, Columbia University
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C. W.

MEASUREMENTS OF SOME ACHIEVEMENTS IN ARITHMETIC

PART I

Section 1. INTRODUCTION

The purpose of this monograph is to set forth the results of an attempt to derive a series of scales in the fundamental operations of arithmetic. Thus the problem is closely related to the general movement for the measurement of educational products by means of objective scales. The method followed in the development of these scales is most clearly related to the methods used by Dr. Buckingham[1] in the development of his Spelling Scale and by Dr. Trabue in the Completion-Test Language Scales.[2] In the development of these scales the fundamental idea was to derive a series of scales which would indicate the type of problems and the difficulty of the problems that a class can solve correctly. Accordingly, each of the scales is composed of as great a variety of problems as the fundamental operations can well permit. These problems, beginning with the easiest that can be found, gradually increase in difficulty until the last ones in each series are so difficult that only a relatively small percentage of the pupils in the eighth grade are able to solve them correctly. In the determination of the relative difficulty of these problems, the relative per cents of correct answers obtained by submitting them to large numbers of school children were taken as a basis.

Two distinct series of scales in each of the fundamental operations have been derived. Series B contains only about half as many problems as Series A. Series A thus has a greater power of diagnosing the weaknesses of a class and is recommended where there is ample time for testing. Series B was derived

[1] Buckingham, B. R., Spelling Ability, Its Measurement and Distribution, 1913.
[2] Trabue, Marion Rex, Completion-Test Language Scales, 1916.

especially for use where the amount of time that can be devoted to measuring is very limited.

Part I of this monograph is devoted especially to the scales and their uses. Specific directions for administering the tests and scoring the results are given in detail. A statement of the values and limitations of the scales is also given in this part.

Part II deals with the history and the method of the derivation of the scales. It also includes many tables of crude data from which the scales were developed.

Section II. THE ARITHMETIC SCALES AND THEIR USES

1. Directions for Administering the Tests

These scales are useful as measures of achievement in the fundamentals of arithmetic either of a class or of a whole school system. Series A is more valuable when the amount of time for testing is plentiful. Series B was especially constructed for use in measuring school systems where the amount of time for testing purposes is limited. Both series of tests are administered in the same way.

The Addition and Subtraction Scales can be used in grades two to eight inclusive; the Multiplication and Division Scales, in grades three to eight inclusive. These scales may be submitted in any order to the pupils. They may be given in immediate succession or with such intervals of time intervening as is most convenient. In the development of the scales subtraction and multiplication were given in succession on one day and addition and division on the next day. The writer recommends that for Series B all tests be given in succession.

If the measurements by these scales are to be valid and comparable, it is necessary that the same standard of procedure be followed in giving the tests and in scoring the results as was followed in the original development of the scales. The same individual should give all of the different tests. He should give the same instructions to every class. He should have the same manner in each class room. In giving the "specific directions" to the class he should use as nearly as possible the same emphasis and intonation. He should not stress one part of the directions more than another part.

It is highly important that the teacher or the one in charge of the room remain silent (saying nothing to the children individually or collectively during the time of giving the tests).

When ready to distribute the tests, place one face downward on each desk. Insist that the pupils do not turn the papers

SERIES A[1]

ADDITION SCALE

Name...

When is your next birthday?..............................How old will you be?...............

Are you a boy or girl?...............................In what grade are you?..........................

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
2	2	17	53	72	60	$3+1=$	$2+5+1=$	20
3	4	2	45	26	37			10
	3							2
								30
								25

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
21	32	43	23	$25+42=$	100	9	199	2563
33	59	1	25		33	24	194	1387
35	17	2	16		45	12	295	4954
		13			201	15	156	2065
					46	19		

(19)	(20)	(21)	(22)	(23)	(24)	(25)
$ .75	$12.50	$8.00	547	$\frac{1}{3}+\frac{1}{3}=$	4.0125	$\frac{3}{8}+\frac{5}{8}+\frac{7}{8}+\frac{1}{8}=$
1.25	16.75	5.75	197		1.5907	
.49	15.75	2.33	685		4.10	
		4.16	678		8.673	
		.94	456			
		6.32	393			
			525			
			240			
			152			

(26)	(27)	(28)	(29)	(30)	(31)	(32)
$12\frac{1}{2}$	$\frac{1}{8}+\frac{1}{4}+\frac{1}{2}=$	$\frac{3}{4}+\frac{1}{4}=$	$4\frac{3}{4}$	$2\frac{1}{2}$	113.46	$\frac{3}{4}+\frac{1}{2}+\frac{1}{4}=$
$62\frac{1}{2}$			$2\frac{1}{4}$	$6\frac{3}{8}$	49.6097	
$12\frac{1}{2}$			$5\frac{1}{4}$	$3\frac{3}{4}$	19.9	
$37\frac{1}{2}$					9.87	
					.0086	
					18.253	
					6.04	

(33)	(34)	(35)	(36)	(37)
.49	$\frac{1}{6}+\frac{3}{8}=$	2 ft. 6 in.	2 yr. 5 mo.	$16\frac{1}{3}$
.28		3 ft. 5 in.	3 yr. 6 mo.	$12\frac{1}{8}$
.63		4 ft. 9 in.	4 yr. 9 mo.	$21\frac{1}{2}$
.95			5 yr. 2 mo.	$32\frac{3}{4}$
1.69			6 yr. 7 mo.	
.22				
.33				
.36				
1.01				
.56				
.88				
.75				
.56				
1.10				
.18				
.56				

(38)

$25.091+100.4+25+98.28+19.3614=$

[1] The scales are printed in large type, on separate sheets, $8\frac{1}{2}''$ x $11''$, with ample space for the insertion of answers.

Series A

SUBTRACTION SCALE

Name..

When is your next birthday?..How old will you be?...............

Are you a boy or girl?............................In what grade are you?............................

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
8	6	2	9	4	11	13	59	78	7 — 4 =	76
5	0	1	3	4	7	8	12	37		60

(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
27	16	50	21	270	393	1000	567482	$2\frac{3}{4} - 1 =$
3	9	25	9	190	178	537	106493	

(21)	(22)	(23)	(24)	(25)	(26)
10.00	$3\frac{1}{2} - \frac{1}{2} =$	80836465	$8\frac{7}{8}$	27	4 yds. 1 ft. 6 in.
3.49		49178036	$5\frac{3}{4}$	$12\frac{5}{8}$	2 yds. 2 ft. 3 in.

(27)	(28)	(29)	(30)
5 yds. 1 ft. 4 in.	10 — 6.25 =	$75\frac{3}{4}$	9.8063 — 9.019 =
2 yds. 2 ft. 8 in.		$52\frac{1}{4}$	

(31)	(32)	(33)	(34)	(35)
7.3 — 3.00081 =	1912 6 mo. 8 da.	$\frac{5}{12} - \frac{2}{10} =$	$6\frac{1}{8}$	$3\frac{7}{8} - 1\frac{5}{8} =$
	1910 7 mo. 15 da.		$2\frac{7}{8}$	

SERIES A

MULTIPLICATION SCALE

Name...

When is your next birthday?.............................How old will you be?...............

Are you a boy or girl?...........................In what grade are you?...........................

(1)	(2)	(3)	(4)	(5)	(6)	(7)
$3 \times 7 =$	$5 \times 1 =$	$2 \times 3 =$	$4 \times 8 =$	23	310	$7 \times 9 =$
				3	4	

(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
50	254	623	1036	5096	8754	165	235
3	6	7	8	6	8	40	23

(16)	(17)	(18)	(19)	(20)	(21)	(22)
7898	145	24	9.6	287	24	$8 \times 5\frac{3}{4} =$
9	206	234	4	.05	$2\frac{1}{2}$	

(23)	(24)	(25)	(26)	(27)	(28)	(29)
$1\frac{1}{4} \times 8 =$	16	$\frac{7}{8} \times \frac{3}{4} =$	9742	6.25	.0123	$\frac{1}{8} \times 2 =$
	$2\frac{5}{8}$		59	3.2	9.8	

(30)	(31)	(32)	(33)	(34)
2.49	$\frac{12}{25} \times \frac{15}{32} =$	6 dollars 49 cents	$2\frac{1}{2} \times 3\frac{1}{2} =$	$\frac{1}{2} \times \frac{1}{2} =$
36		8		

(35)	(36)	(37)	(38)	(39)
$987\frac{3}{4}$	3 ft. 5 in.	$2\frac{1}{4} \times 4\frac{1}{2} \times 1\frac{1}{2} =$	$.0963\frac{1}{8}$	8 ft. $9\frac{1}{2}$ in.
25	5		.084	9

SERIES A

DIVISION SCALE

Name..

When is your next birthday?................................How old will you be?................

Are you a boy or girl?.............................In what grade are you?............................

(1) $3\overline{)6}$

(2) $9\overline{)27}$

(3) $4\overline{)28}$

(4) $1\overline{)5}$

(5) $9\overline{)36}$

(6) $3\overline{)39}$

(7) $4 \div 2 =$

(8) $9\overline{)0}$

(9) $1\overline{)1}$

(10) $6 \times$ .... $= 30$

(11) $2\overline{)13}$

(12) $2 \div 2 =$

(13) $4\overline{)\text{24 lbs. 8 oz.}}$

(14) $8\overline{)5856}$

(15) $\frac{1}{4}$ of $128 =$

(16) $68\overline{)2108}$

(17) $50 \div 7 =$

(18) $13\overline{)65065}$

(19) $248 \div 7 =$

(20) $2.1\overline{)25.2}$

(21) $25\overline{)9750}$

(22) $2\overline{)13.50}$

(23) $23\overline{)469}$

(24) $75\overline{)2250300}$

(25) $2400\overline{)504000}$

(26) $12\overline{)2.76}$

(27) $\frac{7}{8}$ of $624 =$

(28) $.003\overline{).0936}$

(29) $3\frac{1}{2} \div 9 =$

(30) $\frac{3}{4} \div 5 =$

(31) $\dfrac{5}{4} \div \dfrac{3}{5} =$

(32) $9\frac{5}{8} \div 3\frac{3}{4} =$

(33) $52\overline{)3756}$

(34) $62.50 \div 1\frac{1}{4} =$

(35) $531\overline{)37722}$

(36) $9\overline{)\text{69 lbs. 9 oz.}}$

Series B

ADDITION SCALE

Name..

When is your next birthday?..............................How old will you be?...............

Are you a boy or girl?...........................In what grade are you?...........................

(1)
2
3
—

(2)
2
4
3
—

(3)
17
2
—

(5)
72
26
—

(7)
$3 + 1 =$

(10)
21
33
35
—

(13)
23
25
16
—

(14)
$25 + 42 =$

(16)
9
24
12
15
19
—

(19)
$ .75
1.25
.49
———

(20)
$12.50
16.75
15.75
———

(21)
$8.00
5.75
2.33
4.16
.94
6.32
———

(22)
547
197
685
678
456
393
525
240
152
—

(23)
$\frac{1}{3} + \frac{1}{3} =$

(24)
4.0125
1.5907
4.10
8.673
———

(30)
$2\frac{1}{2}$
$6\frac{3}{8}$
$3\frac{3}{4}$
—

(33)
.49
.28
.63
.95
1.69
.22
.33
.36
1.01
.56
.88
.75
.56
1.10
.18
.56
———

(36)
2 yr. 5 mo.
3 yr. 6 mo.
4 yr. 9 mo.
5 yr. 2 mo.
6 yr. 7 mo.
———

(38)
$25.091 + 100.4 + 25 + 98.28 + 19.3614 =$

SERIES B

SUBTRACTION SCALE

Name..

When is your next birthday?..............................How old will you be?...............

Are you a boy or girl?..........................In what grade are you?...........................

(1)	(3)	(6)	(7)
8	2	11	13
5	1	7	8

(9)	(13)	(14)	(17)
78	16	50	393
37	9	25	178

(19)	(20)	(24)	(25)
567482	$2\frac{3}{4} - 1 =$	$8\frac{7}{8}$	27
106493		$5\frac{3}{4}$	$12\frac{5}{8}$

(27)	(31)	(35)
5 yds. 1 ft. 4 in.	$7.3 - 3.00081 =$	$3\frac{7}{8} - 1\frac{5}{8} =$
2 yds. 2 ft. 8 in.		

Series B

MULTIPLICATION SCALE

Name...

When is your next birthday?.................................How old will you be?................

Are you a boy or girl?............................In what grade are you?............................

(1) $3 \times 7 =$	(3) $2 \times 3 =$	(4) $4 \times 8 =$	(5) 23 3
(8) 50 3	(9) 254 6	(11) 1036 8	(12) 5096 6
(13) 8754 8	(16) 7898 9	(18) 24 234	(20) 287 .05
(24) 16 $2\frac{5}{8}$	(26) 9742 59	(27) 6.25 3.2	(29) $\frac{1}{8} \times 2 =$
(33) $2\frac{1}{2} \times 3\frac{1}{2} =$	(35) $987\frac{3}{4}$ 25	(37) $2\frac{1}{4} \times 4\frac{1}{2} \times 1\frac{1}{2} =$	(38) $.0963\frac{1}{8}$ .084

Series B

DIVISION SCALE

Name..

When is your next birthday?...............................How old will you be?...............

Are you a boy or girl?...........................In what grade are you?...........................

(1)
$3\overline{)\,6}$

(2)
$9\overline{)\,27}$

(7)
$4 \div 2 =$

(8)
$9\overline{)\,0}$

(11)
$2\overline{)\,13}$

(14)
$8\overline{)\,5856}$

(15)
$\frac{1}{4}$ of $128 =$

(17)
$50 \div 7 =$

(19)
$248 \div 7 =$

(23)
$23\overline{)\,469}$

(27)
$\frac{7}{8}$ of $624 =$

(28)
$.003\overline{)\,.0936}$

(30)
$\frac{3}{4} \div 5 =$

(34)
$62.50 \div 1\frac{1}{4} =$

(36)
$9\overline{)\,69 \text{ lbs. } 9 \text{ oz.}}$

over until they are told to do so. When all have their pencils in hand, say, " Turn your papers over and answer the questions at the top of the page." (The number of questions to be answered can be determined by the one giving the tests. It will take less time and cause less confusion if the one giving the tests will repeat the question and tell the children what to write. For example say, " The first line asks, 'What is your name?' Write your name," etc.)

When all the questions have been answered repeat the following formula of specific directions. If you should happen to be giving the Addition test say, " Every problem on the sheet which I have given you is an addition problem, an " and problem." Work as many of these problems as you can and be sure that you get them right. Do all of your work on this sheet of paper and don't ask anybody any questions. Begin."

For each scale in Series A, allow twenty minutes; for each in Series B, allow ten minutes. It is important that the time be kept accurately and that all of the children quit work when the signal " Stop " is given. Most of the children will have finished before that time. Those who do not have done, in all probability, all they can; at least they have taken as much time as it takes the average class to complete the test.

The only variation in procedure in giving any of the other tests is the substitution in the formula of specific directions of the expressions subtraction or " take away problems," multiplication or " times problems," and division or " into problems," for the expression addition or " and problems." The expressions " and," " take away," " times," and " into " problems are used so as to make clear to the children what process is to be involved. It is possible that teachers use these expressions in the lower grades instead of " addition, subtraction, multiplication and division problems." There is a great variation in the names applied to the subtraction process. In giving the original tests it was necessary to find out how the teacher designated the process and then use her terminology.

2. Directions for Scoring the Tests

In scoring the tests the standard for marking a problem correct is absolute accuracy, and, wherever possible, reduction to

TABLE I: Answers to Problems

PROBLEM	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION
1	5	3	21	2
2	9	6	5	3
3	19	1	6	7
4	98	6	32	5
5	98	0	69	4
6	97	4	1,240	13
7	4	5	63	2
8	8	47	150	0
9	87	41	1,524	1
10	89	3	4,361	5
11	108	16	8,288	6-1/2 not 6+1
12	59	24	30,576	
13	64	7	70,032	6 lbs. 2 oz. not 6 2
14	67	25	6,600	732
15	425	12	5,405	32
16	79	80	71,082	31
17	844	215	29,870	7-1/7 not 7+1
18	10,966	463	5,616	5,005
19	$2.49	460,989	38.4	35-3/7 not 35+3
20	$45.00	1-3/4	14.35	12
21	$27.50	6.51	60	390
22	3,873	3	46	6.75
23	2/3	31,658,429	10	20-9/23; 20.3, not 20+9
24	18.3762	3-1/8	42	30,004
25	2, not 16/8 nor 2/1	14-3/8	21/32	210
26	125, not 123-4/2 = 2	1 yd. 2 ft. 3 in. not 63 in.	574,778	.23
27	7/8	2 yds. 1 ft. 8 in. not 81 in.	20,000	546
28	1 not 4/4 nor 1/1	3-3/4 or 3.75	.12,054	31.2
29	12-1/4 not 11-3/4 = 1-1/4	23-1/2 not 23-2/4 = 1/2	1/4 not 2/8	7/18
30	12-5/8 not 11-13/8 = 1-5/8	.7873	89.64	3/20 or .15
31	217.1413	4.29919	9/40	2-1/12
32	1-1/2 not 6/4 nor 1-2/4 = 1/2	1 yr. 10 mo. 23 da.	$51.92 or 51 dol. 92 cts.	2-17/30
33	10.55	13/60	8-3/4	72-3/13 or 72.23
34	13/24	3-1/4 not 3-2/8 = 1/4	1/4	50.
35	10 ft. 8 in. or 10-2/3 ft.	2-1/4 not 2-2/8 = 1/4	24693-3/4	71-7/177 or 71.04
36	22 yrs. 5 mo. or 22-5/12 yrs.		17 ft. 1 in.	7 lbs. 11-2/3 oz.; 7 lbs. 9 oz.
37	82-17/24		15-3/16	
38	268.1324		.0080902-1/2 or .00809025	
39			79 ft. 1-1/2 in.	

its lowest terms. If the results are to be comparable with the results and values established in these scales, only those answers should be accepted as correct which are found in Table I. These are the answers which were accepted in the original development of the scales.

A few incorrect answers are also listed in order to offer less chance for variation in the scoring of the results.

3. Directions for Determining the Class Score

For the determination of the class score, two different methods have been derived. The first method was derived especially for use in Series A, where there is no definite attempt to place the problems on a linear scale with equal steps between them. By this method, after the problems have been marked as right or wrong, enter the results on a score sheet similar to the one given in Table II. Thus a complete record of the particular problems solved by each child is obtained.

To complete the class score, find the number of pupils in the class that solved each problem correctly. Divide the number by the total number in the class so as to get for each problem the per cent of the class that solved it correctly. Since, in the development of these scales, that problem which can be solved correctly by just 50 per cent of the class is taken as the best measure of the achievement of the class, select those five problems which come nearest to being solved by just 50 per cent of the class.[1] Table III gives the established value for each problem in the different processes. From Table IV find the amount that must be added or subtracted to the values given in Table III for each of these selected problems to find just what difficulty a problem would need be in order that just 50 per cent of the class could solve it. Take the average of these five determinations and let it represent the class score. This means that a problem of that difficulty can be solved by just 50 per cent of the class in question.

[1] The work of scoring may be greatly economized by omitting the scoring and entering on the score sheet of the problems which will not figure in the determination of the 50 per cent right point. Thus in an eighth grade class the first twenty or more problems can most certainly be neglected. A little experience will teach the scorer what problems he needs to score for a given class.

TABLE II

SAMPLE SCORE SHEET

PUPILS' NAMES	NO. OF PROBLEM																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	ETC.
No. Getting Each Problem																								
% Getting Each Problem																								

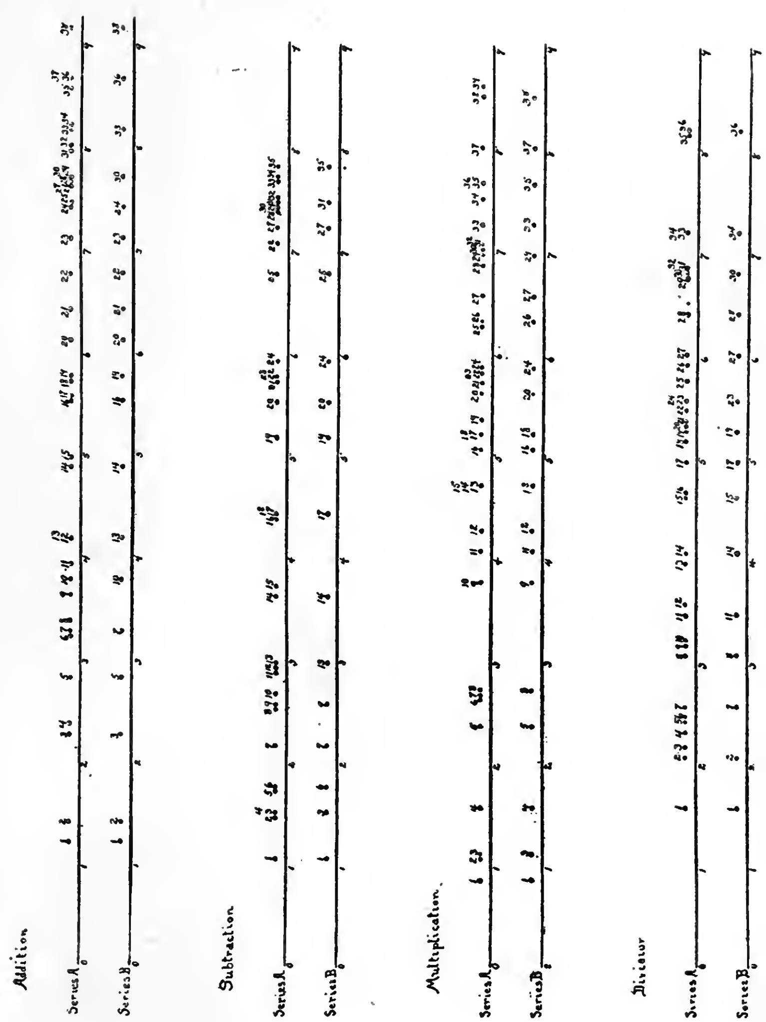
To illustrate the determination of the class score, the five problems in addition which came nearest to being solved by just 50 per cent of the pupils in a certain third grade class of 61 pupils were problems Nos. 14, 17, 16, 18 and 15. These

TABLE III

ESTABLISHED VALUE OF EACH PROBLEM IN EACH SCALE

NO. OF PROBLEM	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION
1	1.23	1.06	.87	1.57
2	1.40	1.48	1.05	2.08
3	2.50	1.50	1.11	2.18
4	2.61	1.50	1.58	2.31
5	2.83	1.70	2.38	2.40
6	3.21	1.75	2.62	2.46
7	3.26	2.18	2.68	2.56
8	3.35	2.51	2.71	3.05
9	3.63	2.57	3.78	3.16
10	3.78	2.65	3.79	3.20
11	3.92	2.88	4.09	3.49
12	4.18	2.90	4.26	3.59
13	4.19	2.96	4.71	3.96
14	4.85	3.64	4.72	4.06
15	4.97	3.70	4.73	4.60
16	5.52	4.35	5.05	4.67
17	5.59	4.41	5.20	4.98
18	5.73	4.42	5.24	5.16
19	5.75	5.18	5.38	5.26
20	6.10	5.52	5.63	5.31
21	6.44	5.70	5.72	5.36
22	6.79	5.75	5.83	5.48
23	7.11	5.76	5.83	5.56
24	7.43	5.91	5.89	5.58
25	7.47	6.77	6.29	5.78
26	7.61	7.07	6.30	5.91
27	7.62	7.21	6.58	6.04
28	7.67	7.38	6.85	6.43
29	7.71	7.41	6.97	6.76
30	7.71	7.41	7.00	6.83
31	7.97	7.49	7.07	6.87
32	8.04	7.52	7.07	6.88
33	8.18	7.69	7.29	7.22
34	8.22	7.72	7.50	7.24
35	8.58	7.84	7.65	8.17
36	8.67		7.66	8.23
37	8.67		8.02	
38	9.19		8.53	
39			8.61	

problems were thus solved correctly by 54, 54, 48, 48 and 68 per cent of the class, respectively. Table IV tells how much to add to or subtract from the established value given in Table III



Linear projection of the difficulty of the problems as shown by Table III.

COMPOSITION OF SCALES IN "SERIES B"

Addition		Subtraction		Multiplication		Division	
NO. OF PROBLEM	VALUE	NO. OF PROBLEM	VALUE	NO. OF PROBLEM	VALUE	NO. OF PROBLEM	VALUE
1	1.23	1	1.06	1	.87	1	1.57
2	1.40	3	1.50	3	1.11	2	2.08
3	2.50	6	1.75	4	1.58	7	2.56
5	2.83	7	2.18	5	2.38	8	3.05
7	3.26	9	2.57	8	2.71	11	3.49
10	3.78	13	2.96	9	3.78	14	4.06
13	4.19	14	3.64	11	4.09	15	4.60
14	4.85	17	4.41	12	4.26	17	4.98
16	5.52	19	5.18	13	4.71	19	5.26
19	5.75	20	5.52	16	5.05	23	5.57
20	6.10	24	5.91	18	5.24	27	6.04
21	6.44	25	6.77	20	5.63	28	6.43
22	6.79	27	7.21	24	5.89	30	6.83
23	7.11	31	7.49	26	6.30	34	7.24
24	7.43	35	7.84	27	6.58	36	8.23
30	7.71			29	6.97		
33	8.18			33	7.29		
36	8.67			35	7.65		
38	9.19			37	8.02		
				38	8.53		

for each of the problems in order to estimate the value of a problem that would be solved correctly by just 50 per cent of the class. Thus

For 54 per cent add .15 to 4.85 = 5.00
" 54 per cent " .15 " 5.59 = 5.74
" 68 per cent " .70 " 4.97 = 5.67
" 48 per cent subtract .07 from 5.52 = 5.45
" 48 per cent " .07 " 5.73 = 5.66
Average 5.50

The average of these 5 determinations (5.50) represents better than either single measurement the degree of difficulty that a problem must have in order that just 50 per cent of this class can solve it correctly. The class score for any other class can be computed in a similar manner.

(2) The second method for the determination of the class score was derived especially for Series B where there was a definite attempt to place the problems on a linear scale with equal steps between them. This method introduces a certain amount of error, but for all practical purposes it is a satisfactory measure. By this method the median number of problems solved cor-

TABLE IV

FOR USE IN ESTIMATING THE DEGREE OF DIFFICULTY REQUIRED IN A PROBLEM SO THAT JUST 50 PER CENT OF THE CLASS CAN SOLVE IT CORRECTLY

	SUBTRACT		ADD
10%	1.90	50%	0.00
11	1.82	51	.03
12	1.74	52	.07
13	1.67	53	.11
14	1.60	54	.15
15	1.54	55	.19
16	1.48	56	.22
17	1.42	57	.26
18	1.36	58	.30
19	1.30	59	.34
20	1.25	60	.38
21	1.20	61	.41
22	1.15	62	.45
23	1.10	63	.49
24	1.05	64	.53
25	1.00	65	.57
26	.95	66	.61
27	.91	67	.65
28	.86	68	.70
29	.82	69	.74
30	.78	70	.78
31	.74	71	.82
32	.70	72	.86
33	.65	73	.91
34	.61	74	.95
35	.57	75	1.00
36	.53	76	1.05
37	.49	77	1.10
38	.45	78	1.15
39	.41	79	1.20
40	.38	80	1.25
41	.34	81	1.30
42	.30	82	1.36
43	.26	83	1.42
44	.22	84	1.48
45	.19	85	1.54
46	.15	86	1.60
47	.11	87	1.67
48	.07	88	1.74
49	.03	89	1.82
		90	1.90

rectly is taken as the measure of the achievement of any class. By the median number of problems solved is meant such a number of problems that there are just as many pupils who solve a greater number as there are those who solve a less number.

In order to determine the median point of the achievement of the class, it is necessary to make a distribution table, showing the number of pupils who were unable to solve a single problem correctly, the number who solved one problem, two problems, three problems, etc. As examples of this sort of distribution we may take the following:

TABLE V

NUMBER OF TIMES EACH ADDITION PROBLEM WAS SOLVED CORRECTLY

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
CLASS I ...	1	0	2	3	0	4	1	2	3	5	6	11	1	5	2	4	1	1	
CLASS II ...	3	3	4	4	7	5	3	4	3		1	1							
CLASS III ...		1	2	4	6	7	4	0	4	5	5	4	3	2	1				

According to these distributions 52 pupils are in Class I, 37 pupils in Class II, and 48 pupils in Class III. Now let us proceed to find the median achievement for each of these class distributions.

Since there are 52 individuals in Class I the median point evidently falls between the achievements of the 26th and 27th pupils. Let us begin with the individual who was unable to solve a single problem correctly and count the two individuals who solved two problems, the three who solved three problems, and so on till we come to the steps that includes the 26th individual. Now if we are to indicate the exact point in the achievement of the pupils where there are just as many pupils who solve a greater number of problems as where there are those who solve a less number, it is necessary to count 5 of the 6 individuals who solved 10 problems correctly. Thus on the assumption that the individuals are distributed over any step at equal distances from one another, the median point is 5/6 of the distance through this step. Hence: the median achievement of this class, i.e., the median number of problems solved, is 10.8 problems correctly solved.

Similarly there are 37 pupils in Class II. The middle case is the 19th pupil, who is the fifth pupil in step 4. There are 18 pupils who solve a greater number of problems and 18 who solve a less number of problems. Thus the exact median point in the achievement of the class lies in the middle of that fraction of a step assigned to the 19th pupil. Thus the median point is $\frac{4.5}{7}$ of the distance through the 4th step. Hence the median achievement for this class is 4.6 problems solved correctly.

The distribution for Class III represents a peculiar difficulty in the calculation of its median. There are 48 pupils in this class and evidently the median point falls between the 24th and the 25th individual. However, it happens that 24 of the pupils solve more than seven problems and 24 of them solve less than seven. Probably the wisest assumption to make is that the 4 pupils on step 6 take up all of that step and the 4 pupils on step 8 take up all of that. If this is assumed, then the median falls on step 7, probably at 7.5 since any given distance on a scale is best represented by its middle point. Thus the median achievement for the Class III is 7.5 problems solved correctly. By similar computations the medians of any distribution can be obtained. By the comparison of the medians thus determined, we get a very satisfactory measure of the achievement of any class on the basis of the total number of problems correctly solved.

4. Tentative Standards of Achievement

While these new scales have not been used in measuring sufficient numbers of children to warrant the establishment of definite standards of achievement, it was thought well to indicate some tentative standards. These tentative standards have been derived from the actual achievements of the children tested with the preliminary tests. The fact that these tests were given during the first part of the school year should be kept in mind when comparison is made with tentative standards shown in Tables VI and VII.

Table VI contains the tentative standards for Series A.

TABLE VI

Tentative Standards of Achievement for Series A

GRADE	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION
II	3.12	1.44		
III	4.99	2.96	1.89	2.54
IV	6.11	4.22	4.05	3.21
V	6.99	5.47	5.53	4.94
VI	7.95	6.46	6.72	5.87
VII	8.65	7.31	7.26	6.59
VIII	9.01	7.64	7.93	7.16

These standards were derived according to the first method given for the determination of the class score. They are based upon the degree of difficulty which the problems must possess in order that just 50 per cent of the class can solve them. Thus, if a problem in addition has 3.12 units of difficulty it will be solved by 50 per cent of the second grade; if it has 4.99 units of difficulty it will be solved by 50 per cent of the third grade, etc.

Table VII contains the tentative standards for Series B.

TABLE VII

TENTATIVE STANDARDS OF ACHIEVEMENT FOR SERIES B

GRADE	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION
II	4.5	3		
III	9	6	3.5	3
IV	11	8	7	5
V	14	10	11	7
VI	16	12	15	10
VII	18	13	17	13
VIII	18.5	14.5	18	14

These standards have been derived according to the second method for determining the class score. They are based upon the total number of problems that were correctly solved in each grade. Thus in the second grade in addition, the median achievement was 4.5 problems, in the third grade, 9 problems correctly solved, etc.

Section III. THE VALUE AND USES OF THE SCALES

1. The scales themselves contain 148 problems which involve many of the fundamental principles of arithmetic. A child who understands and can solve all of these problems correctly probably knows more arithmetic than the average eighth grade child.

2. These scales are useful in that the value of each problem is known, and from these values the value of other problems can easily be determined.

3. The scales are useful in measuring the achievements of any class or of a whole school system. Since all the pupils in all the grades are measured by the same scales, the amount of progress from grade to grade can be definitely determined. Comparisons can be made with similar grades in other buildings or school systems. If the measurements show, for instance, that a certain sixth grade class is unable to solve a greater number of problems correctly than a fifth grade class in the same school system, the cause of this condition should be investigated. In such ways the tests should prove useful to those in charge of school systems.

4. Perhaps the most valuable use of the scales lies in the diagnosing power of the class mistakes. The writer was convinced during the process of scoring these test papers, nearly 20,000 in all, that the mistakes of a class tend to be grouped around some central tendency. The great variety of the problems in these scales and the fact that the problems in each of the various operations proceed from the simplest to the more difficult problems aid greatly in locating the weaknesses of the class. If a large number in a class fail to invert the divisor in the problems in division of fractions, or if a large number in a class fail to locate the decimal point properly in the problems in multiplication of decimal fractions, a teacher should know immediately that these classes need more practice in these particular processes. In a like manner, by locating the particular types of problems missed, one should be able to direct the work of a class more intelligently.

Section IV. LIMITATIONS OF THE SCALES

1. It is possible that with a greater number and variety of pupils the value of some of the problems might be somewhat changed. However, the children tested were from widely separated districts in Indiana, New Jersey, Connecticut, and New York. They represent children from many classes of society and from many nationalities. Moreover, much variation existed in the methods of teaching and in the school room practices. Thus the writer believes the values established are well founded.

2. On the scales as now presented the value of some of the problems may be slightly altered due to the fact that they are located in different positions from those in which they were located on the preliminary lists of problems. The exact amount of this alteration can be determined only by further testing with the scales.

3. The scales as now presented might be slightly bettered if two or three more difficult problems were added to each of them. The scales probably would be bettered if problems could be found of such difficulty as to make the steps between them of exactly equal distance. However, for practical purposes, the effects of these two defects can be disregarded.

4. The value of these scales may be somewhat affected by their more extended use. As teachers become more acquainted with them, they may drill especially upon them. Therefore, it would be much better if several series of such scales of the same difficulty as these should be developed.

5. These scales are not intended to give a definite measure of an individual child. But, if we can measure approximately how difficult a problem a child can solve and then supplement this problem with a large list of problems similar in nature and in difficulty, we can get a fairly accurate measure of the achievement of the child.

6. The relative difficulty of these problems was determined from the achievements of school children in grades two to eight inclusive. It is probable that for adults and teachers the ranking would be in a different order. Only further testing can substantiate this point.

PART II

Section I. DERIVATION OF THE SCALE

1. History of the Scale

The completed scales as shown in Part I of this monograph have been developed from about 20,000 test sheets. The first preliminary series of tests were given to a number of pupils in the public schools of Indiana and New Jersey. The preliminary series of tests consisted of a sheet of problems in addition and likewise one in subtraction, multiplication, and division. In constructing these preliminary lists there was a definite attempt to select problems of as great a variety as the fundamental processes would permit. There was also an attempt to begin the series in each process with the easiest problem that could be found and then gradually to increase the difficulty of each succeeding problem until the last ones in the series would be correctly solved by only a small percentage of the pupils in the eighth grade. By the selection of problems of such varied types and by giving the same lists of problems to pupils in all grades, it was thought that the diagnosing power of the lists would be greater and that the amount and the nature of the progress of one grade over another could best be determined.

The preliminary lists of problems in addition were given to 908 pupils, in subtraction to 916 pupils, in multiplication to 868 pupils, and in division to 696 pupils. The results of these preliminary lists showed that some of the problems were poorly selected and that they should be discarded. When the problems were ranked according to the total percentage of pupils solving them correctly, the results showed large gaps existing between the problems in particular portions of the series.

Guided by the results of these preliminary lists new lists were constructed. Only those problems of the original lists were chosen which were solved by a gradually increasing percentage of the pupils as one proceeded from the lower to the higher grades. If a problem were solved by a higher percentage of the pupils in the lower grades than in the higher grades it was rejected. Wherever there tended to be too large a step between two consecutive problems in the original series an attempt was made to interpose two or three problems of intermediate difficulty.

From the last week in October till the end of the second week in December, 1915, pupils were tested with these new lists of problems. These pupils were from seven different school systems located in Indiana, New Jersey, Connecticut, and New York. The addition problems were given to 4,489 pupils, the subtraction to 4,423 pupils, the multiplication to 3,922 pupils, and the division to 3,660 pupils. These pupils were distributed fairly equally from the second to the eighth grades inclusive.

All of the tests were given by the writer himself with the exception of those given to the pupils in two small school systems in Indiana.[1] The tests were given and the results scored according to the instructions given for administering the tests in Part I of this monograph with the one exception that no time limit for the solution of the problems was used. It was felt to be highly important, if the difficulty of each problem was to be firmly established, that each child should have a chance to solve each problem.

All of the tests were scored by the writer himself and thus the personal element in scoring was reduced to a minimum. The standard for marking a problem right or wrong as presented in Part I of this monograph was arbitrarily adopted. It was decided that a problem to be marked correct must be absolutely accurate and, wherever possible, reduced to its lowest terms. Otherwise, the problem was marked wrong. However, before adopting this arbitrary standard an effort was made to gain from teachers and supervisors of arithmetic the standards by which they marked a problem right or wrong. It was almost unanimously agreed that a problem must be absolutely accurate and reduced to its lowest terms. Thus the arbitrary standard adopted by the writer is in accordance with the best practice exercised in the teaching of arithmetic.

The results of these tests were recorded in two ways:

1. The pupils were distributed according to the number of

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[1] Those giving the tests in these two systems were men who have had experience in giving tests and who could be trusted to carry out the writer's directions.

TABLE VIII

DISTRIBUTION ACCORDING TO THE NUMBER OF ADDITION PROBLEMS SOLVED

	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
38					3	21	41
37					15	37	33
36					30	82	55
35				2	37	96	72
34				4	45	91	70
33				1	51	90	76
32				8	34	75	46
31				13	45	83	45
30			1	13	45	49	27
29			0	26	51	57	20
28			2	35	33	53	18
27			1	32	36	48	19
26		1	2	46	37	34	10
25		0	11	40	34	34	4
24		0	5	54	37	16	4
23		3	33	75	29	27	1
22		6	47	64	25	8	2
21	1	11	42	77	15	7	1
20	0	10	54	54	15	4	
19	0	26	65	49	6	3	
18	0	43	56	43	5	2	
17	4	47	75	28	3		
16	3	64	72	10	2		
15	7	70	42	7			
14	7	54	24	3			
13	13	44	14	0			
12	10	40	18	1			
11	43	39	16	1			
10	31	33	7	0			
9	46	35	5	0			
8	38	23	2	0			
7	35	16	1	0			
6	36	10	2	0			
5	69	14	1	1			
4	48	8	2				
3	43	4	1				
2	17	4	0				
1	13	6	1				
0	25	4					
No. Tested. .	489	615	602	687	633	917	544
Median	6.819	14.509	18.321	23.073	29.774	32.446	33.987
25 per cent. .	4.505	10.902	16.201	20.532	25.625	28.872	31.667
75 " " . .	9.929	16.894	20.694	26.206	33.446	35.070	35.903
Quartile.....	2.712	2.996	2.247	2.837	3.910	3.099	2.118

TABLE IX

NUMBER IN EACH GRADE THAT SOLVED EACH PROBLEM IN ADDITION CORRECTLY

PROBLEM NO.	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
1	388	456	499	654	622	896	541
2	433	582	595	681	630	913	542
3	392	593	595	680	626	911	539
4	326	468	521	659	614	901	540
5	323	501	554	673	629	914	544
6	279	530	565	668	628	911	542
7	259	538	565	679	631	915	542
8	220	474	542	665	624	911	544
9	165	530	568	667	613	880	522
10	152	531	570	663	623	895	539
11	190	543	577	663	608	886	535
12	52	399	541	657	620	896	537
13	32	229	373	627	622	901	537
14	37	405	541	664	627	900	534
15	23	328	499	627	602	876	530
16	6	238	387	567	533	806	500
17	22	288	431	551	500	787	475
18	8	208	386	539	511	801	505
19	1	92	246	399	436	662	457
20	1	87	307	555	586	883	528
21	1	71	276	498	564	839	498
22		49	204	441	528	814	489
23		4	34	308	490	771	500
24		0	4	99	296	521	385
25		3	14	213	397	651	457
26		2	11	192	423	682	483
27		0	10	178	369	678	470
28		0	14	166	414	693	448
29		0	8	131	300	591	409
30		2	33	157	403	684	462
31		3	34	164	317	490	344
32		0	4	157	373	674	421
33		0	3	57	235	461	290
34		15	128	271	338	684	432
35		4	57	169	318	558	392
36		0	2	40	179	537	354
37		0	1	20	176	529	359
38		0	1	9	155	240	274
No. Tested.	489	615	602	687	633	917	544

problems solved correctly. Table VIII represents the distribution for the problems in addition. Beginning at the lower left-hand corner, Table VIII shows that 25 out of 489 pupils in the second grade, and 4 out of 615 pupils in the third grade were unable to solve a single problem, etc. This table also shows the median achievement of each grade distribution. The median achievement of a class is such a number of problems correctly solved that there are just as many pupils who solve a greater number of problems as there are those who solve a less number. This table shows the range in the number of problems correctly solved that will include the middle 50 per cent of the pupils. It also shows the variability in terms of the quartile, or, as it is sometimes designated, the "semi-interquartile range."

2. The results were tabulated in another method so as to record the number of pupils who solved each individual problem correctly. Thus Table IX shows that 388 out of 489 pupils in the second grade solved problem No. 1; 433 pupils solved problem No. 2, etc. From these two crude summaries given in Tables VIII and IX the addition scales have been developed.[1]

2. P.E. as a Unit of Measure

It may be said that we have always measured pupils in the fundamental operations of arithmetic. It may be said that schools and school systems have likewise been measured. No doubt this is true. Whenever a teacher says that one boy is better in addition than another boy, in a certain sense, she measures him. Whenever we compare one individual with another individual, one quality with another quality, or one class with another class, we are measuring. Such standards of measurements as these are no doubt inaccurate and changeable. Whenever a teacher measures a class by means of an examination she tends to have a more constant and more objective measurement. The relation of the different questions of the exam-

[1] Similar tables for the problems in subtraction, multiplication, and division will be found at the end of Part II. In the discussion of the derivation of the scales I shall show in detail the method by which the scale in addition was developed, and shall not discuss the other processes. However, I shall include the final values of each problem in each of the other processes and the most important tables of crude data from which the established values were determined.

ination to one another, however, is unknown. All the questions may be of equal difficulty, or one may be several times as difficult as another. The chief value of a scale as a means of measurement is that it is made up of a number of distinct units whose value is known and remains constant. Such a scale can be used by different people in making similar measurement and the results will be comparable. On the linear rule the unit of measurement is the inch or centimeter; on the thermometer, the degree. Everyone knows what is meant when we speak of an inch, a degree, or any fractional part thereof. These amounts are very definite and always have the same meaning. Moreover almost any one can make reliable measurements with a rule or with a thermometer.

In the building of these arithmetic scales there has been a definite attempt to approximate as closely as possible the accuracy and the constancy of the ruler or the thermometer. The difficulty of each problem has been established and its position above a selected zero point determined. The problems have all been placed in their relative positions on a projected linear scale. In the scales of Series B a definite attempt has been made to select problems with equal amounts of difficulty between them. The unit of measure of difficulty on these arithmetic scales, which corresponds to the inch on the ruler or to the degree on the thermometer, is what is called in statistical terms the Median Deviation or Probable Error. (P.E.)

Before taking up the significance of the median deviation let us discuss the normal surface of frequency. In the construction of these scales, it has been assumed that achievement in the solution of problems in the fundamental processes is distributed according to the normal surface of frequency. Furthermore it has been assumed that the variability of any grade from the second to the eighth is equal to that of any other.

These assumptions are based upon the well-established principle that intellectual abilities are distributed in the same way as are physical traits. If we should arrange one thousand men, selected at random, in a row according to their height, we should find a very large group of men in the center who are about medium height. On one end of the row would be a few very short men and on the other end would be a few very tall men. Likewise if we assume that achievement in the solution of problems in the fundamental processes in any grade is distributed normally, then we should expect to find a large number of the class solving about the same number of problems; furthermore we should expect to find a few dull pupils who can solve but just a few problems and a few bright pupils who can solve more than the average number of problems. The so-called normal curve illustrating such a distribution is reproduced in Fig. 1. The properties of the normal curve have been most accurately determined. Let us assume that Fig. 1 represents the achievement in the solution of problems among a large number of third grade pupils.

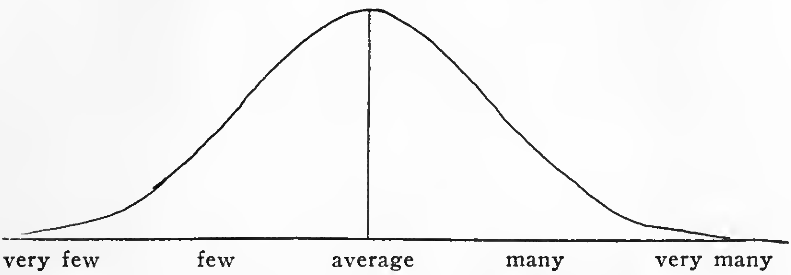

Fig. 1. Normal surface of frequency showing the distribution of achievement in the solution of problems.

The space enclosed between the curve and the base-line represents all of the pupils arranged according to the number of problems solved. The height of the curve above the base-line indicates the number of pupils in the class solving the relative number of problems shown on the base-line. Each pupil is represented by an equal amount of the enclosed area. Thus, at the extreme left the curve is very near the base, which indicates the small number of pupils who were able to solve only a very few problems. In the middle the curve is distant from the base-line representing the large number of pupils who solved an average number of problems; at the extreme right the curve is very near the base, which indicates the small number of pupils who are able to solve many more than the average number of problems.

If our assumption with regard to the achievement in the solution of problems is true, then the graphic representations of the tables of distribution according to the number of problems solved must be similar to Fig. 1.

Figs. 2 to 8 inclusive represent graphically the distribution of the achievement in the solution of the addition problems throughout the various grades. These figures on the whole correspond fairly well to the normal curve of distribution. It will be seen that in the second grade distribution the curve is somewhat skewed to the left. This is probably due to the fact that a great number of the teachers were just beginning to teach the fundamental operations to their classes. It will also be seen that the distributions for grades seven and eight are skewed somewhat to the right. This indicates the need for one or two more difficult problems at the end of the addition series. It will be noted from the distribution tables in the back of this monograph that the distributions for the other processes conform to the normal curve better than the foregoing figures.

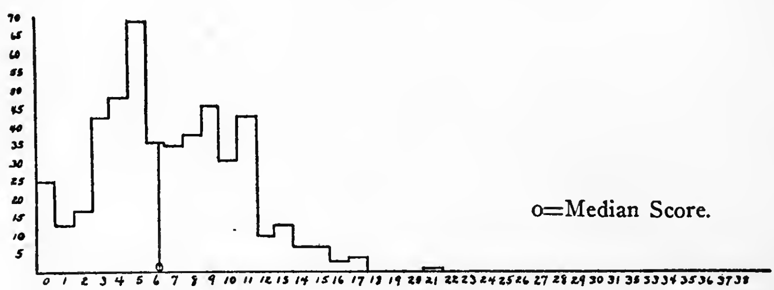

Fig. 2. Distribution according to the number of Addition Problems solved in Second Grade.

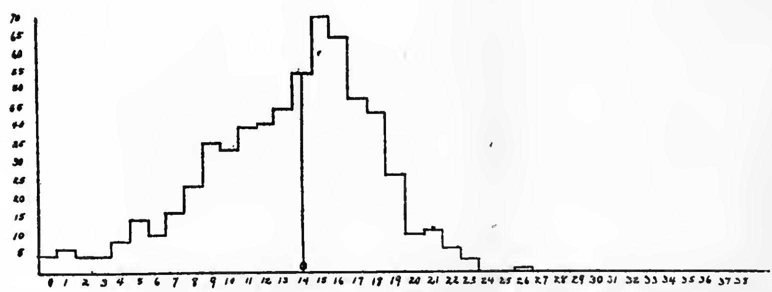

Fig. 3. Distribution according to the number of Addition Problems solved in Third Grade.

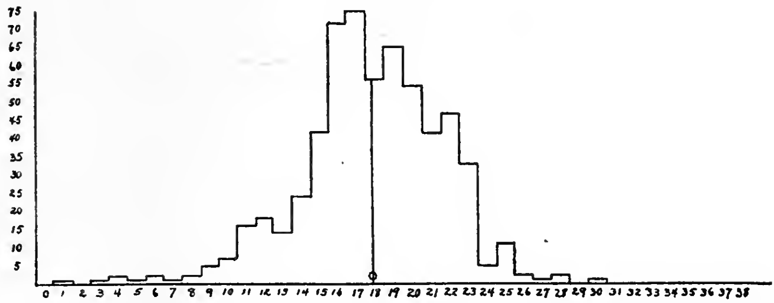

Fig. 4. Distribution according to the number of Addition Problems solved in Fourth Grade.

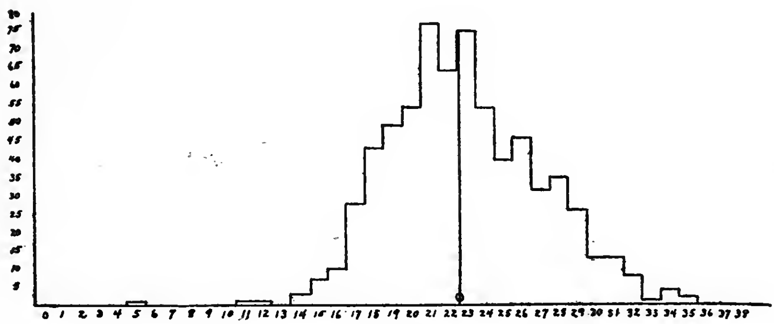

Fig. 5. Distribution according to the number of Addition Problems solved in Fifth Grade.

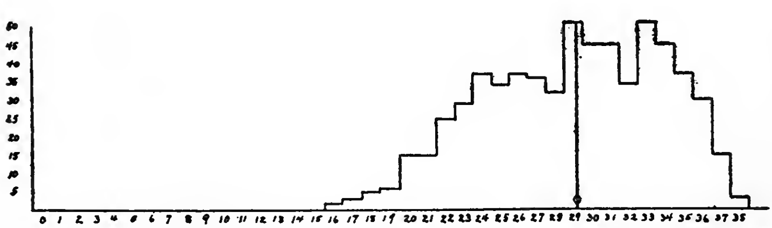

Fig. 6. Distribution according to the number of Addition Problems solved in Sixth Grade.

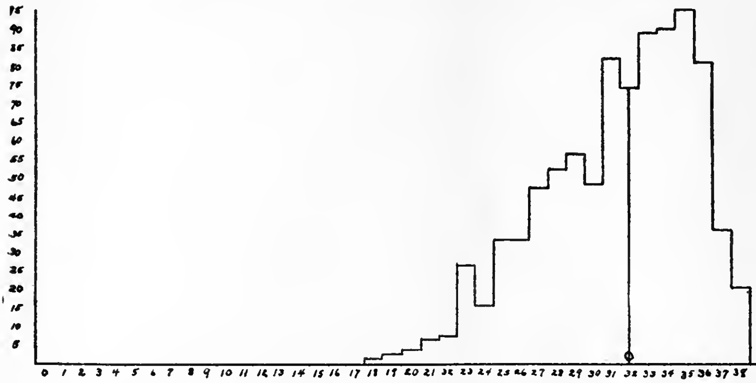

Fig. 7. Distribution according to the number of Addition Problems solved in Seventh Grade.

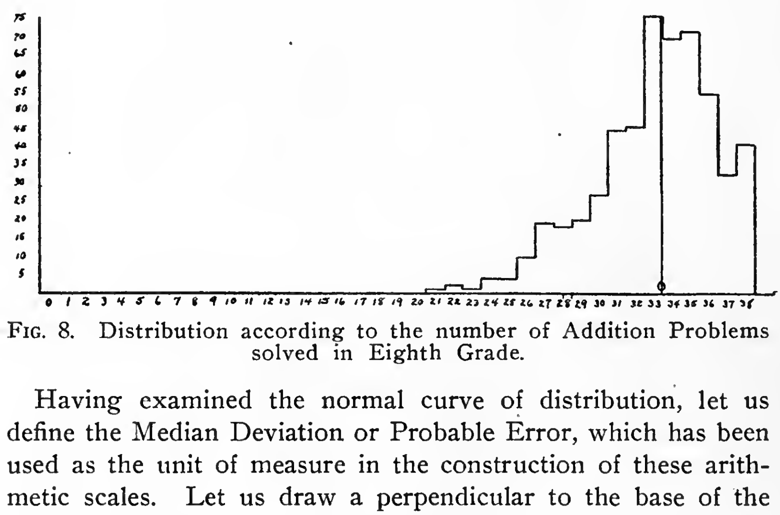

Fig. 8. Distribution according to the number of Addition Problems solved in Eighth Grade.

Having examined the normal curve of distribution, let us define the Median Deviation or Probable Error, which has been used as the unit of measure in the construction of these arithmetic scales. Let us draw a perpendicular to the base of the surface of frequency so that fifty per cent of all the cases lie on one side of the perpendicular and fifty per cent on the other side. The point where the perpendicular cuts the base is the median point. To the left of the median point, draw a perpendicular *f a* so that just 25 per cent of the cases lie between it and the median perpendicular. Draw a similar perpendicular *d c* to the right of the median point. The area *a c d f* cut off by these perpendiculars contains the middle 50 per cent of all the cases. The distance *a m* or *m c* on the base-line of the

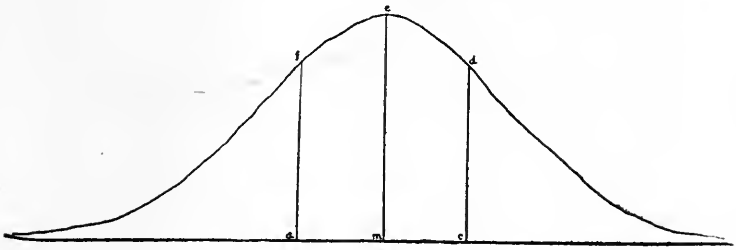

FIG. 9. Normal Surface of Distribution showing the Median and P.E. distance at each side of the Median Point.

surface of frequency is the Median Deviation or the Probable Error. The Probable Error or P.E., as it will be called throughout this monograph, is thus the distance along the base-line of a surface of distribution from the median point to the perpendicular on either side of the median which cuts off 25 per cent of the cases.

Furthermore, it has been established that 2 P.E. is the distance from the median point to the perpendicular on either side of the median which cuts off 41.13 per cent of the cases; 3 P.E., the distance which cuts off 47.85 per cent of the cases; and 4 P.E., the distance which cuts off 49.65 per cent of the cases. Theoretically the curve and the base-line never meet but continually approach one another as the distance from the median point increases. For the purposes of this study we may consider that they meet at a distance of 4.6 P.E. from the median point, for a perpendicular erected here on either side of the median cuts off but 0.1 per cent of all the cases. These facts enable us to locate each problem in its proper position on the base of any grade distribution.

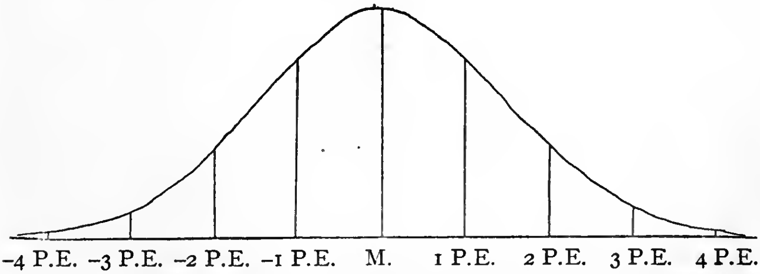

FIG. 10. Normal Surface of Frequency showing P.E. distances from the Median Point.

3. Scaling the Problems in Addition for Each Grade

Since we have assumed that achievement in the solution of problems in the fundamental processes is distributed according to the normal surface of frequency and since we have adopted the P.E. of a grade distribution as the unit of measurement, it is an easy matter to locate each problem on the base-line of each grade distribution. It is evident that a problem which is solved by exactly 50 per cent of pupils in any class represents the median achievement of the class and that it would be located at the median point of the base-line. By definition, P.E. is the distance along the base-line from the median point to the perpendicular on either side of the median which cuts off 25 per cent of the cases. Evidently then a problem that is solved by 75 per cent of the pupils would be 1 P.E. too easy to represent the median achievement of the class and would be located at −1 P.E. distance from the median point. Likewise a problem that is solved by only 25 per cent of the pupils is too difficult to represent the median achievement and would be located at + 1 P.E. distance from the median point. Thus, if we know what per cent of a class solved any problem, it is easy to find the deviation of this per cent from 50 per cent or the median achievement of the class. If this per cent of deviation from the median achievement is known in terms of P.E., we can locate any problem with reference to the median of that distribution. Table X gives the P.E. value for each tenth of a per cent deviation from the median point of a normal distribution (i.e., deviation of 0.0 per cent to 49.9 per cent above or below the median).[1]

Table IX previously given (page 28) shows the number in each grade that solved each problem in addition correctly. Table XI shows these numbers reduced into terms of per cents. Thus in the second grade 79.4 per cent of the pupils solved problem No. 1; 88.6 per cent solved problem No. 2, etc.

Table XII shows the difference between 50 per cent (the median achievement) and the per cents given in Table XI. Table XIII shows the P.E. values for the differences given in Table XII. These P.E. values represent the position of each problem on the base-line of the grade distribution with reference

[1] Table X is taken directly from B. R. Buckingham's Spelling Ability (Table XLVII). It is a modification of the table given in E. L. Thorndike's Mental and Social Measurements (page 200).

TABLE X

P.E. VALUES CORRESPONDING TO GIVEN PER CENTS OF THE NORMAL SURFACE OF FREQUENCY, PER CENTS BEING TAKEN FROM THE MEDIAN

%	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.000	.004	.007	.011	.015	.019	.022	.026	.030	.033
1	.037	.041	.044	.048	.052	.056	.059	.063	.067	.071
2	.074	.078	.082	.085	.089	.093	.097	.100	.104	.108
3	.112	.115	.119	.123	.127	.130	.134	.138	.141	.145
4	.149	.153	.156	.160	.164	.168	.172	.175	.179	.183
5	.187	.190	.194	.198	.201	.205	.209	.213	.216	.220
6	.224	.228	.231	.235	.239	.243	.246	.250	.254	.258
7	.261	.265	.269	.273	.277	.280	.284	.288	.292	.296
8	.299	.303	.307	.311	.315	.318	.322	.326	.330	.334
9	.337	.341	.345	.349	.353	.357	.360	.364	.368	.372
10	.376	.380	.383	.387	.391	.395	.399	.403	.407	.410
11	.414	.418	.422	.426	.430	.434	.437	.441	.445	.449
12	.453	.457	.461	.464	.468	.472	.476	.480	.484	.489
13	.492	.496	.500	.504	.508	.512	.516	.519	.523	.527
14	.531	.535	.539	.543	.547	.551	.555	.559	.563	.567
15	.571	.575	.579	.583	.588	.592	.596	.600	.603	.608
16	.612	.616	.620	.624	.628	.632	.636	.640	.644	.648
17	.652	.656	.660	.665	.669	.673	.677	.681	.685	.689
18	.693	.698	.702	.706	.710	.714	.719	.723	.727	.731
19	.735	.740	.744	.748	.752	.756	.761	.765	.769	.773
20	.778	.782	.786	.790	.795	.799	.803	.807	.812	.816
21	.820	.825	.829	.834	.838	.842	.847	.851	.855	.860
22	.864	.869	.873	.878	.882	.886	.891	.895	.900	.904
23	.909	.913	.918	.922	.927	.931	.936	.940	.945	.949
24	.954	.958	.963	.968	.972	.977	.982	.986	.991	.996
25	1.000	1.005	1.009	1.014	1.019	1.024	1.028	1.033	1.038	1.042
26	1.047	1.052	1.057	1.062	1.067	1.071	1.076	1.081	1.086	1.091
27	1.096	1.101	1.105	1.110	1.115	1.120	1.125	1.130	1.135	1.140
28	1.145	1.150	1.155	1.160	1.165	1.170	1.176	1.181	1.186	1.191
29	1.196	1.201	1.206	1.211	1.217	1.222	1.227	1.232	1.238	1.243
30	1.248	1.253	1.259	1.264	1.269	1.275	1.279	1.286	1.291	1.296
31	1.302	1.307	1.313	1.318	1.324	1.329	1.335	1.340	1.346	1.351
32	1.357	1.363	1.368	1.374	1.380	1.386	1.391	1.397	1.403	1.409
33	1.415	1.421	1.427	1.432	1.438	1.444	1.450	1.456	1.462	1.469
34	1.475	1.481	1.487	1.493	1.499	1.506	1.512	1.518	1.524	1.531
35	1.537	1.543	1.549	1.556	1.563	1.569	1.576	1.582	1.589	1.595
36	1.602	1.609	1.616	1.622	1.629	1.636	1.643	1.649	1.656	1.663
37	1.670	1.677	1.685	1.692	1.699	1.706	1.713	1.720	1.728	1.735
38	1.742	1.749	1.757	1.765	1.772	1.780	1.788	1.795	1.803	1.811
39	1.819	1.827	1.835	1.843	1.851	1.859	1.867	1.875	1.884	1.892
40	1.900	1.909	1.918	1.926	1.935	1.944	1.953	1.962	1.971	1.979
41	1.988	1.997	2.007	2.016	2.026	2.035	2.044	2.054	2.064	2.074
42	2.083	2.093	2.103	2.114	2.124	2.134	2.145	2.155	2.166	2.177
43	2.188	2.199	2.211	2.222	2.234	2.245	2.257	2.269	2.281	2.293
44	2.305	2.318	2.331	2.344	2.357	2.370	2.384	2.397	2.411	2.425
45	2.439	2.453	2.468	2.483	2.498	2.514	2.530	2.546	2.562	2.579
46	2.597	2.614	2.631	2.648	2.667	2.686	2.706	2.726	2.746	2.767
47	2.789	2.811	2.834	2.857	2.881	2.905	2.932	2.958	2.986	3.015
48	3.044	3.077	3.111	3.146	3.182	3.219	3.258	3.300	3.346	3.395
49	3.450	3.506	3.571	3.643	3.725	3.820	3.938	4.083	4.275	4.600
50										

TABLE XI

PER CENT IN EACH GRADE THAT SOLVED EACH PROBLEM IN ADDITION CORRECTLY

PROBLEM NO.	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
1	79.4	74.2	82.9	95.2	98.3	97.7	99.5
2	88.6	94.6	98.9	99.2	99.6	99.6	99.7
3	80.2	96.4	98.9	99.0	98.9	99.4	99.1
4	66.7	76.1	86.6	96.0	97.1	98.3	99.3
5	66.1	81.5	92.1	98.0	99.4	99.7	100.0
6	57.1	86.2	93.9	97.2	99.2	99.4	99.7
7	53.0	87.5	93.9	98.8	99.7	99.8	99.7
8	45.0	77.1	90.1	96.8	98.6	99.4	100.0
9	33.8	86.2	94.4	97.1	96.9	96.0	96.0
10	31.1	86.4	94.7	96.5	98.4	97.6	99.1
11	38.9	88.3	95.9	96.5	96.1	96.6	98.4
12	10.7	64.9	89.9	95.7	97.9	97.7	98.7
13	6.6	37.3	62.0	91.3	98.3	98.3	98.7
14	7.6	65.9	89.9	96.7	99.1	98.2	98.2
15	4.7	53.3	82.9	91.3	95.1	95.5	97.5
16	1.2	38.7	64.3	82.6	84.2	87.9	92.0
17	4.5	46.8	71.6	80.2	79.1	85.8	87.4
18	1.6	33.8	64.2	78.4	80.8	87.4	92.9
19	.2	15.0	40.9	58.1	68.9	72.2	84.0
20	.2	14.2	51.0	80.8	92.6	96.3	97.1
21	.2	11.6	45.9	72.5	89.1	91.5	91.6
22		8.0	33.9	64.2	83.4	88.8	89.9
23		.6	5.6	44.9	77.4	84.1	92.0
24			.6	14.4	46.8	56.8	70.7
25		.5	2.3	31.1	62.7	71.0	84.0
26		.3	1.8	28.0	66.9	74.4	88.8
27			1.6	26.0	58.3	74.0	86.4
28			2.3	24.2	65.4	75.6	82.4
29			1.3	19.1	47.4	64.5	75.2
30		.3	5.5	22.9	63.7	74.6	85.0
31		.5	5.6	23.9	50.1	53.4	63.3
32			.6	22.9	58.9	73.5	77.4
33			.5	8.3	37.1	50.3	53.4
34		2.4	21.3	39.5	53.4	74.6	79.5
35		.6	9.5	24.6	50.3	60.9	72.1
36			.3	5.8	28.3	58.6	65.1
37			.2	2.9	27.9	57.7	66.0
38			.2	1.3	24.5	26.2	50.4

to the median point. These values enable us to scale the problems.

By reference to Table XI it is seen that problem No. 1 was solved by 79.4 per cent of the pupils in the second grade. Table XII shows a difference of 29.4 per cent between the median achievement (i.e., a problem solved by 50 per cent of the class)

TABLE XII

DIFFERENCE BETWEEN FIFTY PER CENT AND THE PER CENT IN EACH GRADE THAT SOLVED EACH PROBLEM IN ADDITION CORRECTLY

PROBLEM NO.	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
1	29.4	24.2	32.9	45.2	48.3	47.7	49.5
2	38.6	44.6	48.9	49.2	49.6	49.6	49.7
3	30.2	46.4	48.9	49.0	48.9	49.4	49.1
4	16.7	26.1	36.6	46.0	47.1	48.3	49.3
5	16.1	31.5	42.1	48.0	49.4	49.7	50.0
6	7.1	36.2	43.9	47.2	49.2	49.4	49.7
7	3.0	37.5	43.9	48.8	49.7	49.8	49.7
8	—5.0	27.1	40.1	46.8	48.6	49.4	50.0
9	—16.2	36.2	44.4	47.1	46.9	46.0	46.0
10	—18.9	36.4	44.7	46.5	48.4	47.6	49.1
11	—11.1	38.3	45.9	46.5	46.1	46.6	48.4
12	—39.3	14.9	39.9	45.7	47.9	47.7	48.7
13	—43.4	—12.7	12.0	41.3	48.3	48.3	48.7
14	—42.4	15.9	39.9	46.7	49.1	48.2	48.2
15	—45.3	3.3	32.9	41.3	45.1	45.5	47.5
16	—48.8	—11.3	14.3	32.6	38.2	37.9	42.0
17	—45.5	—3.2	21.6	30.2	29.1	35.8	37.4
18	—48.4	—16.2	14.2	28.4	30.8	37.4	42.9
19	—49.8	—35.0	—9.1	8.1	18.9	22.2	34.0
20	—49.8	—35.8	1.0	30.8	42.6	46.3	47.1
21	—49.8	—38.4	—4.1	22.5	39.1	41.5	41.6
22		—42.0	—16.1	14.2	33.4	38.8	39.9
23		—49.4	—44.4	—5.1	27.4	34.1	42.0
24		—50.0	—49.4	—35.6	—3.2	6.8	20.7
25		—49.5	—47.7	—18.9	12.7	21.0	34.0
26		—49.7	—48.2	—22.0	16.9	24.4	38.8
27		—50.0	—48.4	—24.0	8.3	24.0	36.4
28		—50.0	—47.7	—25.8	15.4	25.6	32.4
29		—50.0	—48.7	—30.9	—2.6	14.5	25.2
30		—49.7	—44.5	—27.1	13.7	24.6	35.0
31		—49.5	—44.4	—26.1	.1	3.4	13.3
32		—50.0	—49.4	—27.1	8.9	23.5	27.4
33		—50.0	—49.5	—41.7	—12.9	.3	3.4
34		—47.6	—28.7	—10.5	3.4	24.6	29.5
35		—49.4	—40.5	—25.4	.3	10.9	22.1
36		—50.0	—49.7	—44.2	—21.7	8.6	15.1
37		—50.0	—49.8	—47.1	—22.1	7.7	16.0
38		—50.0	—49.8	—48.7	—25.5	—23.8	.4

and the per cent that actually solved this problem. Table XIII shows that this problem would be located at –1.217 P.E. from the median point in the second grade distribution. Table XIII shows that this same problem would be located at –.963 P.E. from the median of the third grade distribution, etc. Table

TABLE XIII

P.E. EQUIVALENT OF DIFFERENCE BETWEEN FIFTY PER CENT AND THE PER CENT IN EACH GRADE THAT SOLVED EACH PROBLEM IN ADDITION CORRECTLY

PROBLEM NO.	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
1	—1.217	— .963	—1.409	—2.468	—3.146	—2.958	—3.820
2	—1.788	—2.384	—3.395	—3.571	—3.938	—3.938	—4.083
3	—1.259	—2.667	—3.395	—3.450	—3.395	—3.725	—3.506
4	— .640	—1.052	—1.643	—2.597	—2.811	—3.146	—3.643
5	— .616	—1.329	—2.093	—3.044	—3.725	—4.083	—4.600
6	— .265	—1.616	—2.293	—2.834	—3.571	—3.725	—4.083
7	— .112	—1.706	—2.293	—3.346	—4.083	—4.275	—4.083
8	.187	—1.101	—1.909	—2.746	—3.258	—3.725	—4.600
9	.620	—1.616	—2.357	—2.811	—2.767	—2.597	—2.597
10	.731	—1.629	—2.397	—2.686	—3.182	—2.932	—3.506
11	.418	—1.765	—2.579	—2.686	—2.614	—2.706	—3.182
12	1.843	— .567	—1.892	—2.546	—3.015	—2.958	—3.300
13	2.234	.480	— .453	—2.016	—3.146	—3.146	—3.300
14	2.124	— .608	—1.892	—2.726	—3.506	—3.111	—3.111
15	2.483	— .123	—1.409	—2.016	—2.453	—2.514	—2.905
16	3.346	.426	— .543	—1.391	—1.757	—1.735	—2.083
17	2.514	.119	— .847	—1.259	—1.201	—1.589	—1.699
18	3.182	.620	— .539	—1.165	—1.291	—1.669	—2.177
19	4.275	1.537	.341	— .303	— .731	— .873	—1.475
20	4.275	1.589	— .037	—1.291	—2.145	—2.648	—2.811
21	4.275	1.772	.153	— .886	—1.827	—2.035	—2.044
22		2.083	.616	— .539	—1.438	—1.803	—1.892
23		3.725	2.357	.190	—1.115	—1.481	—2.083
24			3.725	1.576	.119	— .254	— .807
25		3.820	2.958	.731	— .480	— .820	—1.475
26		4.083	3.111	.864	— .648	— .972	—1.803
27			3.182	.954	— .311	— .954	—1.629
28			2.958	1.038	— .588	—1.028	—1.380
29			3.300	1.296	.097	— .551	—1.009
30		4.083	2.370	1.101	— .519	— .982	—1.537
31		3.820	2.357	1.052	.004	— .164	— .504
32			3.725	1.101	— .334	— .931	—1.115
33			3.820	2.054	.489	— .011	— .127
34		2.932	1.181	.395	— .127	— .982	—1.222
35		3.725	1.944	1.019	— .011	— .410	— .869
36			4.083	2.331	.851	— .322	— .575
37			4.275	2.811	.869	— .288	— .612
38			4.275	3.300	1.024	.945	— .015

XIII thus gives the location of every problem with reference to the median point of each grade distribution. The difficulty of any problem for any grade can be found by reference to this table.

4. Measuring the Distance Between the Grades

Thus far we have located each problem at the proper distance from the median point on the base-line of each grade distribution. We can now locate the difficulty of each problem for each particular grade. We also wish to know how difficult the problems are in general. We wish to know what will be the average position of each problem when placed on one linear scale. Before this can be done, we must determine the distances between the consecutive grade medians and we must establish a common zero point.

Three different methods have been used in this study to determine the interval between the grade medians. After the determinations derived from the three methods were satisfactorily weighted, the average was taken and used as the measure of the intergrade interval. For convenience these three methods will be called the "problem method," the "quartile method," and the "distribution method."

By the "problem method" the distance between the median of two consecutive grades is determined by the difference in position each problem holds with reference to the medians of two consecutive grade distributions. For example, Table XIII shows that problem No. 2 is situated 1.788 P.E. below the median of the second grade and 2.384 P.E. below the median of the third grade. This makes a difference of .596 P.E. between the medians of the second and the third grades so far as this problem is concerned. Each problem will give a similar measure for the interval between any two consecutive grades. Table XIV gives the P.E. intervals between the consecutive grades as determined from each addition problem.

It is interesting to note that as the problems increase in difficulty larger intervals tend to exist between the grade medians. This fact is most clearly brought out by Table XV. In this table the determinations of the intergrade intervals from the various problems are divided into various groups. The group of determinations which is marked below −1.5 P.E. is the average of those determinations from Table XIV which came from values lower than −1.5 P.E. in Table XIII; the group marked −1.5 P.E. to +1.5 P.E. is the average of those determinations obtained from values between −1.5 P.E. and +1.5 P.E.; the

TABLE XIV

P.E. Intervals Shown Between Consecutive Grades by Each Addition Problem

Problem No.	Interval II-III	Interval III-IV	Interval IV-V	Interval V-VI	Interval VI-VII	Interval VII-VIII
1	—.254	.446	1.059	.678	—.188	.862
2	.596	1.011	.176	.367	.000	.145
3	1.408	.728	.055	—.055	.330	—.219
4	.412	.591	.944	.214	.335	.497
5	.713	.764	.951	.681	.358	.517
6	1.351	.677	.541	.737	.154	.358
7	1.594	.587	1.053	.737	.192	—.192
8	1.288	.808	.837	.512	.467	.875
9	2.236	.741	.454	—.044	—.170	.000
10	2.360	.768	.289	.496	—.250	.574
11	2.183	.814	.107	—.072	.092	.476
12	2.410	1.325	.654	.469	—.057	.342
13	1.754	.933	1.563	1.130	.000	.154
14	2.732	1.284	.834	.680	—.395	.000
15	2.606	1.286	.607	.437	.061	.391
16	2.920	.969	.848	.366	—.022	.348
17	2.395	.966	.412	—.058	.388	.110
18	2.562	1.159	.626	.126	.378	.508
19	2.738	1.196	.644	.428	.142	.602
20	2.686	1.626	1.254	.854	.503	.163
21	2.503	1.619	1.039	.941	.206	.009
22		1.467	1.155	.899	.365	.089
23		1.368	2.167	1.305	.366	.602
24			2.149	1.457	.373	.553
25		.862	2.227	1.211	.340	.655
26		.972	2.247	1.512	.324	.831
27			2.228	1.265	.643	.675
28			1.920	1.896	.440	.352
29			2.004	1.199	.648	.458
30		1.730	1.269	1.620	.463	.555
31		1.463	1.305	1.048	.168	.340
32			2.624	1.435	.597	.184
33			1.766	1.565	.500	.116
34		1.751	.788	.522	.855	.240
35		1.781	.925	1.030	.399	.459
36			1.752	1.480	1.173	.253
37			1.464	1.942	1.157	.324
38			.975	2.276	.079	.960

group marked above +1.5 P.E. is the average of the determinations obtained from values larger than +1.5 P.E. The group marked "select group" is the average of the determinations obtained from values between –2 P.E. and +2 P.E. The group marked "compositive" average is the average of the determinations in Table XIV.

TABLE XV

AVERAGES OF GROUPS OF DETERMINATIONS OF INTERGRADE INTERVALS AS MEASURED BY THE ADDITION PROBLEMS

GROUP	II-III INTER-VAL	III-IV INTER-VAL	IV-V INTER-VAL	V-VI INTER VAL	VI-VII -INTER-VAL	VII-VIII INTER-VAL
Below 1.5 P.E.	1.695	.842	.675	.508	.131	.347
—1.5 P.E. to +1.5 P.E.	.539	.959	.846	1.028	.509	.423
Above 1.5 P.E.	2.531	1.456	1.801	1.744	1.522[1]	.600[1]
Compositve Average..	1.866	1.099	1.155	.876	.300	.374
Select Group.........	1.429	1.093	.854	1.011	.456	.437

[1] These two values have been estimated.

Table XV shows so far as this list of problems is concerned that the greatest difference between the medians of the different grades is brought about by the more difficult problems and that the least amount of difference is brought about by the least difficult problems. The one exception to this general statement is found in group --1.5 P.E. to +1.5 P.E. for the interval between the second and third year. The smallness of this determination is due to the small number of cases that happened to fall within those middle limits. These same facts are brought out in similar tables for the other fundamental processes. These results are in conformity with the results found by Dr. Trabue in his measurement with completion-test language scales.

Keeping in mind the fact that the greatest difference between the medians of any two consecutive grades is brought about by the most difficult problems and also that the least difference is brought about by the easier problems, it would seem that the best measure of the interval would be the average of those determinations that come from near the median. It seems rather unfair that those problems at the extreme ends of a distribution should have equal weight with those near the middle of the distribution. In order to give more weight to the problems near the median of the distribution the average of those determinations in Table XIV which were obtained from values –2 P.E. to +2 P.E. in Table XIII will be used in addition to the composite average in computing the final measure of the intervals between the grade medians. This last group of determinations is designated as the " select group " in Table XV. Thus the composite average and the average of the determination of the " select group " are both measures of the intergrade intervals and both are derived from the "problem method" and will be used in the final determination of the intergrade intervals.

The second method of determining the distance between the grades was previously designated as the "quartile" method. If we have a normal surface of distribution, as we have assumed, the quartile of any distribution should be equal to the P.E. of that distribution. Therefore, if we divide the quartile of a distribution into the crude score intervals, we will get the interval between the medians of the grades in terms of P.E. Since for each interval between the grades there are two quartile measures, the average of the two quartiles is used as a divisor of the crude score interval between the grades.

Table XVI shows the intervals obtained by this process.

TABLE XVI

DETERMINATION OF QUARTILE INTERVALS BETWEEN GRADES

	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
Median .	6.819	14.509	18.321	23.073	29.774	32.446	33.987
Quartile.	2.712	2.996	2.247	2.837	3.910	3.099	2.118

Score Interval . . .	7.690	3.812	4.752	6.701	2.672	1.541
Average Interval . . .	2.854	2.621	2.542	3.374	3.505	2.609
Quartile Interval . . .	2.694	1.459	1.869	1.986	.762	.591

This table is made from data taken from Table VIII. The score interval or crude score between the second and third grade medians is the difference between the median number of problems solved in the second and third grades. The average quartile is the average of the quartiles of the second and third grade distributions. The crude score (7.690) divided by the average quartile (2.854) gives the quartile interval (2.694), the distance between the second and third grade medians. By hypothesis this quartile interval is in terms of P.E.

Third measure of the interval between the median of the different grades was by means of the "distribution method." It is based upon the amount of overlapping of the consecutive grade distributions. Table VIII shows that there are pupils in the second grade that excel the median achievement of the pupils in the third grade. On the other hand, there are pupils in the third grade that do not reach the median achievement of the second grade pupils. Between the median of the second grade distribution (6.819) and the median of the third grade (14.509) distribution lie 46.22 per cent of the 489 cases in the second grade distribution; between the same medians lie 42.16 per cent of the 615 cases in the third grade distributions. Since these percentages are deviations from the median or 50 per cent they can be turned into P.E. values by referring to Table X. Thus, as determined from the second grade distribution, the interval between the second and third grade medians is 2.643 P.E. As determined by the third grade distribution the interval between the third and second grade medians is 2.099 P.E. Thus we have two direct measures for this same interval. Similarly, direct measures can be made for the intervals between each of the other succeeding grades.

By the same reasoning, between the second grade median and the fourth grade median lie 49.79 per cent of the second grade distribution; between the third grade median and the fourth grade median lie 35.98 per cent of the third grade distribution. Turning the percentages into P.E. we find that the fourth grade median is 4.256 P.E. above the second grade median and 1.601 P.E. above the third grade median. If we take the distance between the third and fourth grade median from the distance between the second and fourth grade medians, we get an indirect measure of the distance between the second and third grade medians. Thus 4.256 P.E. −1.601 P.E. = 2.655 P.E., the indirect measure of the interval between the second and third grades.

Table XVII shows the percentage with P.E. equivalent of each grade lying between the median and the medians of the neighboring grades.

Various other indirect measures can be obtained in a similar way. Indeed, if one wished he could get a still further remote indirect measure of this same second and third grade interval by taking the distance between the third and fifth grade median from the distance between the second and fifth grade medians. This latter determination would, in the writer's opinion, be influenced so much by the extreme ends of the distributions that it should be given no weight in the final determination of the grade intervals.

In the final determination of the grade intervals by the "distribution method" the direct measures were felt to be the best measures, and hence they were given double weight while the indirect measures were given but single weight.

TABLE XVII

PERCENTAGE WITH P.E. EQUIVALENT OF EACH GRADE LYING BETWEEN THE MEDIAN AND THE MEDIANS OF THE NEIGHBORING GRADES

	II	III	IV	V	VI	VII	VIII
II% P.E.	46.22 2.634	49.79 4.256					
III% P.E.	42.16 2.099		35.98 1.601	49.38 3.709			
IV% P.E.	48.89 3.390	36.34 1.625		41.26 2.012	49.83 4.373		
V% P.E.		49.34 3.676	40.57 1.950		43.17 2.207	48.33 3.157	
VI% P.E.			48.96 3.428	38.44 1.775		18.43 .711	29.35 1.214
VII% P.E.				47.16 2.825	19.45 .754		14.32 .544
VIII% P.E.					36.30 1.622	18.47 .713	

TABLE XVIII

DETERMINATIONS OF THE INTERGRADE INTERVALS FROM OVERLAPPING OF DISTRIBUTIONS IN ADDITION

DETERMINATION	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII
Lower Indirect....		1.291	2.051	1.478	1.050	.868
Lower Direct......	2.099	1.625	1.950	1.775	.754	.713
Lower Direct......	2.099	1.625	1.950	1.775	.754	.713
Upper Direct......	2.634	1.601	2.012	2.207	.711	.544
Upper Direct......	2.634	1.601	2.012	2.207	.711	.544
Upper Indirect....	2.655	1.697	2.166	2.446	.670	
Total.........	12.121	9.440	12.141	11.888	4.650	3.382
AVERAGE.........	2.424	1.573	2.023	1.981	.775	.676

Table XVIII shows the interval between the grades as determined by the distribution method.

Thus we have the results of the intergrade intervals as determined by the three different methods. All of the determinations have about the same general characteristics. The writer felt that the "select group" of determination, which is based upon the individual problems not varying over 2 P.E. in difficulty from the median problem of the grade, was the best measure. He also felt that the second best measure was the composite average based upon the difficulty of all the individual problems. Both the "selected group" determinations and the composite averages take into consideration the fact that the problems are not of the same difficulty. Hence it appeared to the writer that it might be fair to give these determinations increased weight in making the final determination of the intergrade intervals. Thus the "selected group" determinations were given a weight of three, and the "composite average" determinations a weight of two while the determinations from the quartile method and those from the "distribution method" were given but a single

TABLE XIX

FINAL DETERMINATION OF INTERVALS BETWEEN SUCCESSIVE GRADES

METHOD	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII
Prob. select group.	1.429	1.093	.854	1.011	.456	.437
" " "	1.429	1.093	.854	1.011	.456	.437
" " "	1.429	1.093	.854	1.011	.456	.437
" composite av.	1.866	1.099	1.155	.876	.300	.374
" " "	1.866	1.099	1.155	.876	.300	.374
Distribution.......	2.424	1.573	2.023	1.981	.775	.676
Quartile..........	2.694	1.459	1.869	1.986	.762	.591
AVERAGE.........	1.877	1.216	1.252	1.250	.501	.475

weight. Table XIX gives the average of the determinations which is used as the final measure of the intergrade intervals in the construction of the Addition Scales.[1]

[1] The other intergrade intervals as used in the final development of the other scales are as follows:

	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII
Subtraction..........	1.646	1.073	1.392	1.233	.751	.820
Multiplication........		1.748	1.636	1.491	.822	.536
Division.............		1.014	1.554	1.368	.667	.659

5. Location of the Zero Point

Having determined the distance between the various grades, it is easy to locate all of the problems in terms of any grade median. However, if we wish to know the exact relation of one problem to another, if we wish to know how many times more difficult one problem is than another, it is necessary to find the location of each problem with reference to a common zero point.

Professor Thorndike in his "Mental and Social Measurements" (p. 16) in speaking of the definition of a zero point says, "The zero point may be *absolute*, measuring just not any of the thing, or *arbitrary*, meaning a point called zero though actually designating some amount of the thing. Thus the thing being temperature, 20° C is 20° above the arbitrary zero—the melting of ice—and 293° above the supposed absolute zero, just not any molecular motion in a gas."

The zero point in connection with any one of these scales is an arbitrary one. It means simply "the inability to solve correctly a single problem" as presented under the standard conditions for giving these tests. It does not mean that a child has absolutely no ability in addition or subtraction or multiplication or division. It is probable that if the problems had been presented orally to the pupils in the lower grades they would have solved more problems correctly. Some of the pupils who showed "inability to solve correctly a single problem" as the test was presented to them, no doubt would have solved a few problems if presented orally. Moreover, it should be added that zero ability in division (i.e., inability to solve a single problem) does not mean that a child will have zero ability in addition. It should also be added that we cannot say, as the scales are in their present condition, that a problem with a value of 1 in one process is equal in difficulty to a problem with the same value in another process. Each scale has its own zero point. We therefore cannot treat values as equal which have been developed from different zero points.

The method for locating the zero point is the same for all the fundamental processes, but I shall deal in detail only with addition. Table VIII shows that of the 489 pupils in the second grade 44.88 per cent lie between those children who could not get a single problem and the median achievement for that grade

(6.819). Since this represents deviation from the median or 50 percentile, reference to Table X shows that a deviation of 44.88 per cent from the median of a normal distribution represents a distance of 2.322 P.E. This means then that the median of the second grade in addition is 2.422 P.E. above no score at all.

Table VIII also shows that of 615 pupils in the third grade 49.34 per cent lie between those children who could not get a single problem and the median achievement for that grade (14.509). This would locate the median for the third grade 3.676 P.E. above the zero point. We have already determined that the median of the third grade is 1.877 P.E. (Table XIX) above that of the second grade. By subtracting the distance the third grade median is above the second from the distance the third grade median is above the zero point, we get a measure of the distance the second grade median is above the zero point. Thus 3.676 P.E. minus 1.877 P.E. = 1.799 P.E., another measure of the distance that the second median is above the zero point.

Table VIII also shows that the median achievement of the second grade distribution is 6.819 problems and that the quartile is 2.712 problems. Since we have assumed a normal surface of frequency, the quartile is equal to the P.E.; thus by dividing the median achievement by the quartile of the second grade we find that the median of the second grade is 2.514 P.E. above zero.

Similarly the median achievement for the third grade is 14.509 problems and the quartile 2.996 problems. By dividing the median achievement by the quartile we find that the third grade median is 4.843 P.E. above zero. Subtracting the distance the third grade median is above the second (1.877 P.E.) from the distance the third grade is above zero (4.843 P.E.) gives us the distance the second grade median is above zero (2.966 P.E.)

Thus we have four determinations of our zero point as follows:

From the second grade distribution.................	2.422
" " third " "	1.799
" " second " achievement.................	2.514
" " third " "	2.966
Average.....	2.425

The average of the four determinations probably represents a better measure of the distance than any single measure, so for the addition scale we shall use the point 2.425 P.E. below the second grade median as the arbitrary zero point.

Since we have determined the distance between the medians of the various grades (Table XIX) and know that the second grade median is 2.425 P.E. above zero it is easy to determine the distance each grade median is above zero.

TABLE XX

DISTANCE THE MEDIAN OF EACH GRADE IN ADDITION IS ABOVE ZERO

1 = P.E.

GRADE	ABOVE ZERO	BELOW NEXT GRADE
II	2.425	1.877
III	4.302	1.216
IV	5.518	1.252
V	6.770	1.250
VI	8.020	.501
VII	8.521	.475
VIII	8.996	

Figure 11 (page 51) represents graphically the relations of the grade distribution to each other, the relations of the grade medians to each other and to the zero point, as determined by the values of Table XX and based upon the assumption that achievement in the solution of problems is distributed normally.

6. Referring All the Problems in Addition to Zero

Table XIII gives the value of each problem in addition for each grade. It shows, for instance, that problem No. 1 has a negative value of 1.217 P.E. in the second grade and a negative value of .963 P.E. in the third grade, etc. By reference to Table XX one finds that the second grade median is 2.425 P.E. and the third grade median is 4.302 P.E. above zero. By subtracting 1.217 from 2.425 and .903 from 4.302 we find that problem No. 1, in the second and third grades, is respectively 1.208 and 3.339 P.E. above zero. Wherever the value in Table XIII is positive instead of negative, as these just cited have been, add this value to the value which that particular grade median is above zero. For example, to determine the distance problem No. 8 is above zero in the second grade we add .187 P.E. to 2.425 P.E. Thus we find that problem No. 8 is 2.612 P.E. above zero in the second grade. Table XXI shows the location above zero of each problem in addition.

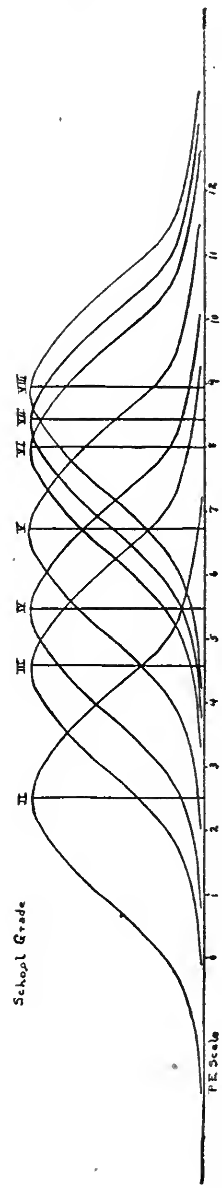

FIG. 11. Relations of Grade Distributions to Each Other in Addition.

Since we have determined the location above zero of each problem in each grade we are now ready to determine the general value of each problem and to locate it on a linear scale. In order to do this we must find the position which best represents the difficulty of each problem.

TABLE XXI

LOCATION ABOVE ZERO OF EACH ADDITION PROBLEM

PROBLEM NO.	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
1	1.208	3.339	4.109	4.302	4.874	5.563	5.176
2	.637	1.918	2.123	3.199	4.082	4.583	4.913
3	1.166	1.635	2.123	3.302	4.625	4.796	5.490
4	1.785	3.250	3.875	4.173	5.209	5.375	5.353
5	1.809	2.973	3.425	3.726	4.295	4.438	4.396
6	2.160	2.686	3.225	3.936	4.449	4.796	4.913
7	2.313	2.596	3.225	3.424	3.937	4.246	4.913
8	2.612	3.201	3.609	4.024	4.762	4.796	4.396
9	3.045	2.686	3.161	3.959	5.253	5.924	6.399
10	3.156	2.673	3.121	4.084	4.838	5.589	5.490
11	2.843	2.537	2.939	4.084	5.406	5.815	5.814
12	4.268	3.735	3.626	4.224	5.005	5.563	5.696
13	4.659	4.782	5.065	4.754	4.874	5.375	5.696
14	4.549	3.694	3.626	4.044	4.514	5.410	5.885
15	4.908	4.179	4.109	4.754	5.567	6.007	6.091
16	5.771	4.728	4.975	5.379	6.263	6.786	6.913
17	4.939	4.421	4.671	5.511	6.819	6.932	7.297
18	5.607	4.922	4.979	5.605	6.729	6.852	6.819
19	6.700	5.839	5.859	6.467	7.289	7.648	7.521
20	6.700	5.891	5.481	5.479	5.875	5.873	6.185
21	6.700	6.074	5.671	5.884	6.193	6.486	6.952
22		6.385	6.134	6.231	6.582	6.718	7.104
23		8.027	7.875	6.960	6.905	7.040	6.913
24			9.243	8.346	8.139	8.267	8.189
25		8.122	8.476	7.501	7.540	7.701	7.521
26		8.385	8.629	7.634	7.372	7.549	7.193
27			8.700	7.724	7.709	7.567	7.367
28			8.476	7.808	7.432	7.493	7.616
29			8.818	8.066	8.117	7.970	7.987
30		8.385	7.888	7.871	7.501	7.539	7.459
31		8.122	7.875	7.822	8.024	8.357	8.492
32			9.243	7.871	7.686	7.590	7.881
33			9.338	8.824	8.509	8.510	8.869
34		7.234	6.699	7.165	7.893	7.539	7.774
35		8.027	7.462	7.789	8.009	8.111	8.127
36			9.601	9.101	8.871	8.199	8.421
37			9.793	9.581	8.889	8 233	8.384
38			9.793	10.070	9.044	9.166	8.981

In making this determination it was felt that the truest value of any problem came from the distribution where the median achievement was nearest the location of that problem. It was also felt that those values which came from those distributions where the median achievements were farthest from the location of that problem should have little or no weight in the determination of the final value of the problem.

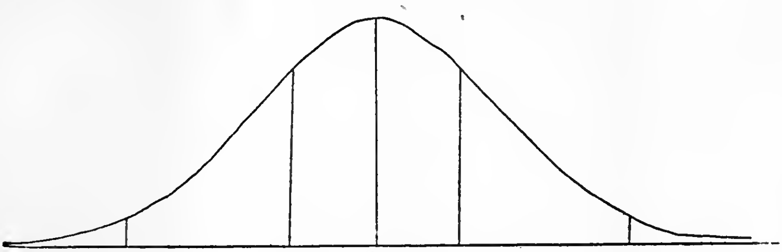

FIG. 12. Showing the influence of various parts of a distribution in determining the general value of a problem.

Thus, as represented by Fig. 12, in computing the general final value of a problem in these various distributions, that value from each grade distribution is given double weight if the problem is less than 1 P.E. distance from the median achievement of that distribution; single weight, if it is more than 1 P.E. but less than 3 P.E. distance from the median achievement of the distribution; and not considered at all if the problem is more than 3 P.E. distance from the median achievement of the distribution. The average of all of these determinations is taken as the general difficulty of a problem.

Therefore, in making this final determination of the general value of any problem reference must be made to both Table XIII and Table XXI. Table XIII is used to determine the weight that shall be given to the value in Table XXI. For example, Table XIII shows that problem No. 1 is located at .963 P.E. from the median achievement of the third grade. It also shows that this same problem is located at more than 1 P.E. but less than 3 P.E. distance from the median achievement in grades two, four, and five. With these facts in mind weight the value given in Table XXI accordingly. The value in the third grade would have double weight, while the values in the second, fourth, and fifth grades would have single weight. The other values are disregarded. The average of the above determinations represents the difficulty of the problem.

Table XXII gives the general value, i.e., distance above the arbitrary zero, for each problem in addition. These values are final and are used in locating the problems on the linear scale.

TABLE XXII

FINAL VALUE OF ADDITION PROBLEMS

RANK	NO. OF PROBLEM	VALUE
1	2	1.23
2	3	1.40
3	5	2.50
4	7	2.61
5	6	2.83
6	8	3.21
7	1	3.26
8	4	3.35
9	10	3.63
10	11	3.78
11	14	3.92
12	9	4.18
13	12	4.19
14	13	4.85
15	15	4.97
16	17	5.52
17	16	5.59
18	18	5.73
19	20	5.75
20	21	6.10
21	22	6.44
22	19	6.79
23	23	7.11
24	34	7.43
25	26	7.47
26	30	7.61
27	27	7.62
28	25	7.67
29	28	7.71
30	32	7.71
31	35	7.97
32	29	8.04
33	31	8.18
34	24	8.22
.35	36	8.58
36	37	8.67
37	33	8.67
38	38	9.19

NOTE.—These values are listed in Part I of this monograph. The graphic representation of the scale, i. e., the problem placed on a linear projection according to these final values, is also given in Part I.

Section II. TABLES OF CRUDE DATA FROM WHICH SCALES WERE DEVELOPED

TABLE XXIII

Distribution According to the Number of Subtraction Problems Solved

	Grade II	Grade III	Grade IV	Grade V	Grade VI	Grade VII	Grade VIII
35					6	14	34
34					8	42	45
33					8	51	56
32					23	76	68
31				2	27	68	80
30				1	25	79	75
29				0	41	86	57
28				2	39	90	48
27				13	38	66	33
26				13	46	76	15
25				19	68	70	18
24				33	70	34	7
23			5	57	63	46	9
22		2	14	61	59	52	0
21		1	13	84	36	23	5
20		7	42	101	36	27	2
19		24	51	72	17	6	2
18	1	26	54	68	24	5	1
17	0	24	54	51	14	4	
16	1	17	51	36	7	3	
15	1	28	58	17	0	0	
14	2	36	49	13	2	0	
13	8	52	60	7	1	0	
12	7	49	38	10	1	0	
11	11	54	27	8	1	1	
10	15	53	20	3	1		
9	23	58	14	2	0		
8	28	29	6	1	0		
7	33	26	11	1	0		
6	31	21	3	2	0		
5	34	13	5	0	0		
4	19	21	2	0	1		
3	13	10	1	2			
2	20	11	4	1			
1	30	21	6	1			
0	104	33	18	3			
No. Tested	381	616	606	684	662	919	555
Median	5.132	11.223	15.672	20.436	24.971	28.517	31.687
25 Per Cent.	.916	6.923	12.908	18.191	22.415	25.411	28.974
75 Per Cent.	8.063	14.306	18.509	22.492	28.295	31.313	32.945
Quartile	3.574	3.692	2.801	2.651	2.940	2.951	1.986

TABLE XXIV

NUMBER IN EACH GRADE THAT SOLVED EACH PROBLEM IN SUBTRACTION CORRECTLY

PROBLEM NO.	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
1	225	464	556	664	658	917	555
2	133	326	449	625	646	906	553
3	222	529	573	675	662	919	555
4	199	496	568	665	654	904	555
5	169	469	551	668	660	917	555
6	184	511	534	671	653	917	555
7	158	468	547	658	651	913	552
8	80	358	445	614	634	908	551
9	113	425	510	646	650	909	553
10	72	391	496	627	639	894	545
11	52	339	466	635	645	907	553
12	53	363	498	649	651	910	552
13	54	364	509	655	655	910	553
14	13	222	362	592	625	885	546
15	9	193	358	588	638	907	551
16	2	141	315	547	570	870	534
17	2	145	334	566	573	840	512
18	0	78	244	498	586	849	542
19	1	61	161	389	496	776	515
20		31	121	326	403	662	441
21		8	75	326	492	772	519
22		14	53	285	434	758	518
23		0	2	27	176	342	223
24		0	1	81	240	417	277
25		0	0	53	197	340	223
26		3	75	228	494	758	518
27			12	204	467	743	494
28			0	125	304	528	420
29			0	24	199	565	472
30			2	17	181	438	438
31				15	140	361	312
32				10	160	462	411
33				17	177	445	359
34				17	199	450	375
35				8	115	336	389
NO. TESTED	381	616	606	684	662	919	555

TABLE XXV

LOCATION ABOVE ZERO OF EACH SUBTRACTION PROBLEM

PROBLEM NO.	GRADE II	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII	FINAL VALUE
1	.782	1.748	1.781	2.416	2.735	2.936	3.431	1.501
2	1.691	2.650	2.877	3.201	3.528	3.953	4.093	2.645
3	.805	1.167	1.451	1.927	1.860	2.611	3.431	1.057
4	1.034	1.487	1.566	2.393	3.114	4.029	3.431	1.502
5	1.325	1.710	1.856	2.269	2.377	2.936	3.431	1.697
6	1.179	1.347	2.086	2.150	3.202	2.936	3.431	1.447
7	1.434	1.355	1.909	2.596	3.314	3.486	4.211	1.745
8	2.312	2.459	2.908	3.343	3.898	3.865	4.388	2.898
9	1.906	2.027	2.348	2.857	3.349	3.761	4.093	2.178
10	2.423	2.250	2.489	3.173	3.774	4.354	4.920	2.959
11	2.738	2.572	2.744	3.061	3.579	3.911	4.093	2.877
12	2.725	2.425	2.467	2.802	3.314	3.705	4.211	2.568
13	2.705	2.421	2.354	2.665	3.010	3.705	4.093	2.513
14	3.822	3.289	3.471	3.584	4.103	4.563	4.849	3.699
15	4.074	3.485	3.494	3.625	3.793	3.911	4.388	3.635
16	4.936	3.863	3.757	3.984	4.851	4.814	5.400	4.346
17	4.936	3.833	3.645	3.824	4.817	5.185	5.917	4.409
18		4.454	4.199	4.327	4.680	5.087	5.073	4.418
19	5.391	4.671	4.762	4.969	5.464	5.712	5.865	5.182
20		5.201	5.088	5.312	6.050	6.342	6.172	5.763
21		6.062	5.548	5.312	5.492	5.730	5.786	5.524
22		5.748	5.851	5.538	5.864	5.825	5.809	5.754
23			7.918	7.841	7.432	7.695	8.399	7.841
24			8.435	6.984	6.979	7.383	8.068	7.406
25				7.341	7.246	7.707	8.399	7.720
26		6.700	5.548	5.867	5.478	5.825	5.809	5.696
27			7.341	6.013	5.661	5.915	6.204	5.911
28				6.567	6.609	6.931	6.998	6.774
29				7.913	7.233	6.777	6.488	7.070
30			7.918	8.159	7.355	7.296	6.840	7.383
31				8.242	7.646	7.614	7.800	7.694
32				8.485	7.498	7.200	7.073	7.208
33				8.159	7.382	7.270	7.472	7.486
34				8.159	7.233	7.248	7.354	7.412
35				8.622	7.851	7.719	7.249	7.517

TABLE XXVI

DISTRIBUTION ACCORDING TO THE NUMBER OF MULTIPLICATION PROBLEMS SOLVED

	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
39					7	9
38					9	19
37				5	25	41
36				10	34	36
35				8	45	51
34				14	72	56
33				27	63	68
32				24	97	41
31				36	84	56
30			1	41	86	44
29			3	45	64	45
28			7	41	66	29
27			10	43	63	24
26			10	45	52	22
25		1	18	42	38	7
24		2	19	42	29	4
23		1	34	45	27	4
22		4	32	44	19	1
21		3	40	29	18	3
20		8	55	27	10	2
19		16	73	14	7	0
18	1	14	62	16	10	1
17	0	24	79	14	5	1
16	2	25	44	16	3	0
15	0	38	37	8	6	1
14	0	38	44	11	1	
13	7	49	27	5	1	
12	12	41	20	3	2	
11	17	42	21	3		
10	20	46	15	1		
9	17	45	5	1		
8	27	37	8	2		
7	27	29	4	0		
6	28	36	3	0		
5	44	36	8	2		
4	91	29	2	0		
3	52	19	0	0		
2	34	4	1	0		
1	22	4	0	1		
0	55	13	7			
No. Tested...	456	604	689	665	943	565
Median.......	4.714	11.095	18.315	26.144	30.587	32.940
25 Per Cent ..	3.058	7.345	15.196	22.301	27.123	29.940
75 Per Cent ..	7.593	14.605	21.044	29.973	33.306	35.289
Quartile......	2.267	3.630	2.924	3.836	3.091	2.674

TABLE XXVII

NUMBER IN EACH GRADE THAT SOLVED EACH PROBLEM IN MULTIPLICATION CORRECTLY

PROBLEM NO.	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
1	372	577	673	660	936	563
2	373	570	671	657	938	561
3	338	545	676	658	936	565
4	281	529	661	652	936	563
5	157	445	644	656	935	559
6	137	421	644	645	937	562
7	139	462	638	648	932	562
8	134	447	651	649	933	562
9	90	349	602	613	898	547
10	82	357	593	615	899	547
11	34	284	571	603	893	548
12	40	273	553	589	868	527
13	18	198	453	526	820	514
14	6	173	433	485	768	506
15	5	149	439	571	880	545
16	8	192	495	555	823	491
17	0	14	228	430	844	541
18	3	100	408	503	770	501
19	2	85	400	531	801	509
20	3	68	251	330	593	402
21	0	9	172	398	820	528
22	0	5	123	418	821	542
23	4	56	248	515	834	547
24	0	36	195	476	834	539
25	0	6	110	442	829	534
26	0	5	100	400	653	454
27	0	0	45	304	541	380
28	0	5	51	348	665	447
29	0	11	93	296	545	326
30	0	1	37	253	535	388
31	0	6	39	296	597	419
32	0	1	32	210	460	363
33	2	18	76	227	452	349
34		0	40	179	373	323
35		0	5	104	271	264
36		3	6	141	377	307
37		1	15	139	406	319
38		0	2	43	212	182
39		0	1	39	186	176
NO. TESTED...	456	604	689	665	943	565

TABLE XXVIII

Location Above Zero of Each Multiplication Problem

Problem No.	Grade III	Grade IV	Grade V	Grade VI	Grade VII	Grade VIII	Final Value
1	.256	.825	2.017	2.823	3.645	3.886	1.050
2	.245	.982	2.094	3.120	3.468	4.181	1.107
3	.633	1.421	1.898	3.016	3.645	3.224	.872
4	1.154	1.626	2.396	3.389	3.645	3.886	1.582
5	2.187	2.399	2.730	3.166	3.717	4.374	2.380
6	2.364	2.574	2.730	3.677	3.563	4.004	2.713
7	2.347	2.268	2.830	3.585	3.942	4.004	2.675
8	2.394	2.385	2.605	3.534	3.838	4.004	2.616
9	2.855	3.047	3.276	4.363	4.820	5.078	3.783
10	2.948	2.998	3.366	4.332	4.805	5.078	3.789
11	3.725	3.451	3.566	4.504	4.891	5.035	4.089
12	3.598	3.518	3.711	4.678	5.205	5.602	4.261
13	4.188	3.999	4.372	5.265	5.618	5.836	4.706
14	4.891	4.177	4.486	5.562	5.964	5.957	5.046
15	4.986	4.353	4.456	4.871	5.066	5.138	4.723
16	4.737	4.041	4.120	5.022	5.596	6.154	4.727
17		6.297	5.623	5.907	5.429	5.262	5.721
18	5.234	4.777	4.630	5.438	5.948	6.029	5.242
19	5.529	4.934	4.672	5.223	5.751	5.915	5.194
20	5.234	5.134	5.491	6.481	6.799	6.995	6.296
21		6.558	5.975	6.094	5.618	5.579	5.889
22		6.910	6.338	5.977	5.611	5.227	5.826
23	5.097	5.301	5.506	5.346	5.516	5.078	5.375
24		5.644	5.826	5.619	5.516	5.326	5.625
25		6.789	6.450	5.834	5.553	5.454	5.825
26		6.910	6.544	6.083	6.540	6.555	6.290
27			7.220	6.626	7.011	7.159	6.973
28		6.910	7.120	6.381	6.489	6.623	6.580
29		6.450	6.611	6.671	6.996	7.536	7.002
30		7.614	7.359	6.915	7.038	7.101	7.066
31		6.789	7.319	6.671	6.784	6.861	6.850
32		7.614	7.458	7.176	7.332	7.281	7.290
33	5.529	6.128	6.786	7.074	7.366	7.379	7.069
34			7.306	7.379	7.679	7.555	7.504
35			8.618	7.965	8.122	7.947	8.020
36		7.159	8.481	7.652	7.668	7.660	7.656
37		7.614	7.961	7.667	7.546	7.581	7.647
38			9.250	8.711	8.408	8.509	8.533
39			9.575	8.784	8.552	8.551	8.609

TABLE XXIX

DISTRIBUTION ACCORDING TO THE NUMBER OF DIVISION PROBLEMS SOLVED

	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
36				2	6	5
35				0	9	21
34				7	25	33
33				10	38	47
32				11	48	59
31			1	18	74	51
30			1	29	67	62
29			0	25	86	63
28			3	42	74	41
27			3	46	77	51
26			2	40	73	30
25			7	45	58	29
24			13	51	58	11
23		1	20	41	60	11
22		1	27	42	38	10
21		1	41	47	28	6
20		3	49	39	29	7
19		2	53	40	26	1
18		4	47	30	8	1
17		4	50	26	19	0
16		14	48	18	14	1
15	1	10	43	16	8	2
14	5	19	77	13	6	
13	14	48	48	9	3	
12	7	47	46	11	2	
11	4	74	33	4	1	
10	12	66	21	2	2	
9	12	67	20	1	1	
8	13	58	11	0	1	
7	15	44	3	2	0	
6	21	37	2	0	0	
5	27	25	4	1	0	
4	12	25	4	2	0	
3	12	17	1		0	
2	10	12	3		1	
1	21	8	3			
0	32	18	1			
NO. TESTED...	218	605	685	670	940	542
MEDIAN.......	5.815	9.873	16.469	23.781	27.442	30.113
25 PER CENT..	2.125	7.211	13.401	19.813	23.800	27.519
75 PER CENT..	9.042	12.585	19.919	27.489	30.478	32.500
QUARTILE......	3.459	2.687	3.259	3.838	3.339	2.491

TABLE XXX

NUMBER IN EACH GRADE THAT SOLVED EACH PROBLEM IN DIVISION CORRECTLY

PROBLEM NO.	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII
1	91	302	506	547	822	470
2	110	325	544	623	912	533
3	98	369	548	578	865	526
4	100	365	571	656	929	541
5	87	451	657	662	934	542
6	138	500	656	661	932	541
7	124	485	656	644	913	532
8	110	428	649	653	930	538
9	76	415	649	655	931	535
10	24	44	315	493	859	527
11	42	271	473	612	917	536
12	96	446	607	582	824	485
13	93	453	595	605	856	511
14	12	144	462	618	893	534
15	4	42	254	496	791	506
16	0	39	415	512	818	502
17	4	31	180	458	779	491
18	4	178	463	578	857	521
19	0	78	313	379	670	433
20	0	37	283	402	621	401
21	0	16	229	433	731	476
22	0	40	267	450	725	485
23	5	25	132	433	675	451
24		0	91	260	640	438
25		8	63	373	638	420
26		1	143	352	658	426
27		6	155	401	678	464
28			18	208	364	323
29			18	221	442	320
30			6	209	491	397
31			18	207	420	313
32			10	179	379	299
33			0	74	265	313
34			5	115	321	254
35			1	30	158	136
36			0	28	123	139
No. Tested...	218	605	685	670	940	542

TABLE XXXI

LOCATION ABOVE ZERO OF EACH DIVISION PROBLEM

PROBLEM NO.	GRADE III	GRADE IV	GRADE V	GRADE VI	GRADE VII	GRADE VIII	FINAL VALUE
1	2.230	2.937	3.538	4.515	4.823	5.532	3.586
2	1.900	1.795	3.270	3.550	3.733	4.035	2.563
3	2.106	2.519	3.244	4.233	4.439	4.392	3.194
4	2.072	2.546	3.049	2.840	3.176	2.906	2.457
5	2.299	1.956	1.908	2.509	2.797	2.581	2.083
6	1.415	1.542	1.925	2.555	3.016	2.906	1.574
7	1.661	1.674	1.925	3.241	3.711	4.104	2.312
8	1.900	2.126	2.090	2.950	3.072	3.538	2.182
9	2.494	2.214	2.090	2.869	3.016	3.881	2.395
10	3.730	5.088	4.636	4.919	4.496	4.347	4.596
11	3.205	3.127	3.752	3.839	3.590	3.786	3.484
12	2.143	1.993	2.699	4.192	4.802	5.322	3.160
13	2.192	1.937	2.824	3.920	4.525	4.837	3.045
14	4.289	3.990	3.818	3.752	4.083	3.962	3.958
15	5.030	5.132	4.976	4.901	5.041	4.947	4.982
16		5.178	4.088	4.788	4.852	5.036	4.671
17	5.030	5.358	5.427	5.145	5.113	5.228	5.263
18	5.030	3.736	3.810	4.233	4.515	4.567	4.058
19		4.610	4.647	5.609	5.688	5.938	5.304
20		5.226	4.813	5.483	5.906	6.227	5.564
21		5.814	5.123	5.300	5.387	5.453	5.357
22		5.167	4.901	5.195	5.421	5.322	5.157
23	4.877	5.512	5.773	5.300	5.667	5.754	5.481
24			6.136	6.277	5.824	5.890	6.038
25		6.233	6.458	5.642	5.833	6.061	5.911
26		7.533	5.688	5.762	5.740	6.005	5.782
27		6.383	5.602	5.483	5.653	5.605	5.579
28			7.368	6.586	6.948	6.821	6.868
29			7.368	6.507	6.634	6.844	6.762
30			7.993	6.582	6.440	6.263	6.428
31			7.368	6.595	6.720	6.893	6.826
32			7.745	6.777	6.882	6.987	6.882
33				7.666	7.377	6.893	7.241
34			8.130	7.258	7.130	7.296	7.222
35				8.369	7.949	8.177	8.168
36				8.417	8.185	8.153	8.227

VITA

The author of this dissertation, CLIFFORD WOODY, was born at Thorntown, Indiana, on June 2, 1884. He was educated in the public schools of Indiana, graduating from the Thorntown High School in 1903. He received the degree of Bachelor of Arts from Indiana University in 1908. He taught in the High School at Gaston, Indiana, from 1908–1912, the last two years of which he was superintendent of the system. He was a graduate student at Indiana University during the Summer Terms of 1910, 1912 and 1913; and during the academic years of 1912–13 and 1913–14. He received the degree of Master of Arts in 1913. He was a Fellow in Education during the year of 1913–1914. He taught in the School of Education during the Spring Terms of 1913 and 1914. He was a graduate Scholar, Teachers College, Columbia University, during the year 1914–15 and Fellow in Education in Teachers College during the year 1915–16.