THE

MECHANICAL ENGINEER'S POCKET-BOOK.

A REFERENCE-BOOK OF RULES, TABLES, DATA, AND FORMULÆ, FOR THE USE OF ENGINEERS, MECHANICS, AND STUDENTS.

BY

WILLIAM KENT, A.M., M.E.,

Consulting Engineer,

Member Amer. Soc'y Mechl. Engrs. and Amer. Inst. Mining Engrs.

FIFTH EDITION, REVISED AND ENLARGED.

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WILLIAM KENT.

PASSAIC, N. J., *September*, 1898.

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Weights and Measures.

Braunworth, Munn & Barber
Printers and Binders
Brooklyn, N. Y.

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Locomotives.

Gearing.

NAMES AND ABBREVIATIONS OF PERIODICALS AND TEXT-BOOKS FREQUENTLY REFERRED TO IN THIS WORK.

Am. Mach. American Machinist.
App. Cyl. Mech. Appleton's Cyclopædia of Mechanics, Vols. I and II.
Bull. I. & S. A. Bulletin of the American Iron and Steel Association (Philadelphia).
Burr's Elasticity and Resistance of Materials.
Clark, R. T. D. D. K. Clark's Rules, Tables, and Data for Mechanical Engineers.
Clark, S. E. D. K. Clark's Treatise on the Steam-engine.
Engg. Engineering (London).
Eng. News. Engineering News.
Engr. The Engineer (London).
Fairbairn's Useful Information for Engineers.
Flynn's Irrigation Canals and Flow of Water.
Jour. A. C. I. W. Journal of American Charcoal Iron Workers' Association.
Jour. F. I. Journal of the Franklin Institute.
Kapp's Electric Transmission of Energy.
Lanza's Applied Mechanics.
Merriman's Strength of Materials.
Modern Mechanism. Supplementary volume of Appleton's Cyclopædia of Mechanics.
Proc. Inst. C. E. Proceedings Institution of Civil Engineers (London).
Proc. Inst. M. E. Proceedings Institution of Mechanical Engineers (London).
Peabody's Thermodynamics.
Proceedings Engineers' Club of Philadelphia.
Rankine, S. E. Rankine's The Steam Engine and other Prime Movers.
Rankine's Machinery and Millwork.
Rankine, R. T. D. Rankine's Rules, Tables, and Data.
Reports of U. S. Test Board.
Reports of U. S. Testing Machine at Watertown, Massachusetts.
Rontgen's Thermodynamics.
Seaton's Manual of Marine Engineering.
Hamilton Smith, Jr.'s Hydraulics.
The Stevens Indicator.
Thompson's Dynamo-electric Machinery.
Thurston's Manual of the Steam Engine.
Thurston's Materials of Engineering.
Trans. A. I. E. E. Transactions American Institute of Electrical Engineers.
Trans. A. I. M. E. Transactions American Institute of Mining Engineers.
Trans. A. S. C. E. Transactions American Society of Civil Engineers.
Trans. A. S. M. E. Transactions American Soc'ty of Mechanical Engineers
Trautwine's Civil Engineer's Pocket Book.
The Locomotive (Hartford, Connecticut).
Unwin's Elements of Machine Design.
Weisbach's Mechanics of Engineering.
Wood's Resistance of Materials.
Wood's Thermodynamics.

MATHEMATICS.

Arithmetical and Algebraical Signs and Abbreviations.

$+$ plus (addition).
$+$ positive.
$-$ minus (subtraction).
$-$ negative.
$\pm$ plus or minus.
$\mp$ minus or plus.
$=$ equals.
$\times$ multiplied by.
ab or $a.b = a \times b$.
$\div$ divided by.
$/$ divided by.
$\frac{a}{b} = a/b = a \div b$. $15\text{-}16 = \frac{15}{16}$.
$.2 = \frac{2}{10}$; $.002 = \frac{2}{1000}$.
$\sqrt{\ }$ square root.
$\sqrt[3]{\ }$ cube root.
$\sqrt[4]{\ }$ 4th root.
: is to, :: so is, : to (proportion).
2 : 4 :: 3 : 6, as 2 is to 4 so is 3 to 6.
: ratio; divided by.
2 : 4. ratio of 2 to 4 = 2/4.
$\therefore$ therefore.
$>$ greater than.
$<$ less than.
$\square$ square.
$\bigcirc$ round.
$^\circ$ degrees, arc or thermometer.
$'$ minutes or feet.
$''$ seconds or inches.
$'\ ''\ '''$ accents to distinguish letters, as a', a'', a'''.
a_1, a_2, a_3, a_b, a_c. read a sub 1, a sub b, etc.
( ) [ ] { } $\overline{\quad\quad}$ vincula, denoting that the numbers enclosed are to be taken together; as, $(a + b)c = \overline{4 + 3} \times 5 = 35$.
a^2. a^3, a squared, a cubed.
a^n, a raised to the nth power.
$a^{\frac{2}{3}} = \sqrt[3]{a^2}$, $a^{\frac{3}{2}} = \sqrt{a^3}$.
$a^{-1} = \frac{1}{a}$, $a^{-2} = \frac{1}{a^2}$.
10^9 = 10 to the 9th power = 1,000,000,-000.
sin. a = the sine of a.
sin.$^{-1} a$ = the arc whose sine is a.
sin. $a^{-1} = \frac{1}{\text{sin. } a}$.
log. = logarithm.
$\log_e$ or hyp. log. = hyperbolic logarithm.

$\angle$ angle.
$\llcorner$ right angle.
$\perp$ perpendicular to.
sin., sine.
cos., cosine.
tang., or tan., tangent.
sec., secant.
versin., versed sine.
cot., cotangent.
cosec., cosecant.
covers., co-versed sine.

In Algebra, the first letters of the alphabet, a, b, c, d, etc., are generally used to denote known quantities, and the last letters, w, x, y, z, etc., unknown quantities.

Abbreviations and Symbols commonly used.

d, differential (in calculus).
$\int$, integral (in calculus).
$\int_b^a$, integral between limits a and b.
Δ, delta, difference.
Σ, sigma, sign of summation.
π, pi, ratio of circumference of circle to diameter = 3.14159.
g, acceleration due to gravity = 32.16 ft. per sec.

Abbreviations frequently used in this Book.

L., l., length in feet and inches.
B., b., breadth in feet and inches.
D., d., depth or diameter.
H., h., height, feet and inches.
T., t., thickness or temperature.
V., v., velocity.
F., force, or factor of safety.
f., coefficient of friction.
E., coefficient of elasticity.
R., r., radius.
W., w., weight
P., p., pressure or load.
H.P., horse-power.
I.H.P., indicated horse-power.
B.H.P., brake horse-power.
h. p., high pressure.
i. p., intermediate pressure.
l. p., low pressure.
A.W.G., American Wire Gauge (Brown & Sharpe).
B.W.G., Birmingham Wire Gauge.
r. p. m., or revs. per min., revolutions per minute.

ARITHMETIC.

The user of this book is supposed to have had a training in arithmetic as well as in elementary algebra. Only those rules are given here which are apt to be easily forgotten.

GREATEST COMMON MEASURE, OR GREATEST COMMON DIVISOR OF TWO NUMBERS.

Rule.—Divide the greater number by the less; then divide the divisor by the remainder, and so on, dividing always the last divisor by the last remainder, until there is no remainder, and the last divisor is the greatest common measure required.

LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS.

Rule.—Divide the given numbers by any number that will divide the greatest number of them without a remainder, and set the quotients with the undivided numbers in a line beneath.

Divide the second line as before, and so on, until there are no two numbers that can be divided; then the continued product of the divisors and last quotients will give the multiple required.

FRACTIONS.

To reduce a common fraction to its lowest terms.—Divide both terms by their greatest common divisor: $\frac{18}{24} = \frac{3}{4}$

To change an improper fraction to a mixed number.—Divide the numerator by the denominator; the quotient is the whole number, and the remainder placed over the denominator is the fraction: $\frac{39}{4} = 9\frac{3}{4}$.

To change a mixed number to an improper fraction.—Multiply the whole number by the denominator of the fraction; to the product add the numerator; place the sum over the denominator: $1\frac{7}{8} = \frac{15}{8}$.

To express a whole number in the form of a fraction with a given denominator.—Multiply the whole number by the given denominator, and place the product over that denominator: $18 = \frac{54}{3}$.

To reduce a compound to a simple fraction, also to multiply fractions.—Multiply the numerators together for a new numerator and the denominators together for a new denominator:

$$\frac{2}{3} \text{ of } \frac{4}{3} = \frac{8}{9}, \quad \text{also} \quad \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}.$$

To reduce a complex to a simple fraction.—The numerator and denominator must each first be given the form of a simple fraction; then multiply the numerator of the upper fraction by the denominator of the lower for the new numerator, and the denominator of the upper by the numerator of the lower for the new denominator:

$$\frac{\frac{3}{4}}{1\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{3}{2}} = \frac{6}{12} = \frac{1}{2}.$$

To divide fractions.—Reduce both to the form of simple fractions, invert the divisor, and proceed as in multiplication:

$$\frac{2}{3} \div 1\frac{1}{3} = \frac{2}{3} \div \frac{4}{3} = \frac{2}{3} \times \frac{3}{4} = \frac{6}{12}.$$

Cancellation of fractions.—In compound or multiplied fractions, divide any numerator and any denominator by any number which will divide them both without remainder, striking out the numbers thus divided and setting down the quotients in their stead.

To reduce fractions to a common denominator.—Reduce each fraction to the form of a simple fraction; then multiply each numerator by all the denominators except its own for the new numerators, and all the denominators together for the common denominator:

$$\frac{1}{2},\ \frac{1}{3},\ \frac{3}{7} = \frac{21}{42},\ \frac{14}{42},\ \frac{18}{42}.$$

To add fractions.—Reduce them to a common denominator, then add the numerators and place their sum over the common denominator:

$$\frac{1}{2} + \frac{1}{3} + \frac{3}{7} = \frac{21 + 14 + 18}{42} = \frac{53}{42} = 1\tfrac{11}{42}.$$

To subtract fractions.—Reduce them to a common denominator, subtract the numerators and place the difference over the common denominator:

$$\frac{1}{2} - \frac{3}{7} = \frac{7 - 6}{14} = \frac{1}{14}.$$

DECIMALS.

To add decimals.—Set down the figures so that the decimal points are one above the other, then proceed as in simple addition: $18.75 + .012 = 18.762$.

To subtract decimals.—Set down the figures so that the decimal points are one above the other, then proceed as in simple subtraction: $18.75 - .012 = 18.738$.

To multiply decimals.—Multiply as in multiplication of whole numbers, then point off as many decimal places as there are in multiplier and multiplicand taken together: $1.5 \times .02 = .030 = .03$.

To divide decimals.—Divide as in whole numbers, and point off in the quotient as many decimal places as those in the dividend exceed those in the divisor. Ciphers must be added to the dividend to make its decimal places at least equal those in the divisor, and as many more as it is desired to have in the quotient: $1.5 \div .25 = 6$. $0.1 \div 0.3 = 0.10000 \div 0.3 = 0.3333 +$

Decimal Equivalents of Fractions of One Inch.

1-64	.015625	17-64	.265625	33-64	.515625	49-64	.765625
1-32	.03125	9-32	.28125	17-32	.53125	25-32	.78125
3-64	.046875	19-64	.296875	35-64	.546875	51-64	.796875
1-16	.0625	**5-16**	.3125	**9-16**	.5625	**13-16**	.8125
5-64	.078125	21-64	.328125	37-64	.578125	53-64	.828125
3-32	.09375	11-32	.34375	19-32	.59375	27-32	.84375
7-64	.109375	23-64	.359375	39-64	.609375	55-64	.859375
1-8	.125	**3-8**	.375	**5-8**	.625	**7-8**	.875
9-64	.140625	25-64	.390625	41-64	.640625	57-64	.890625
5-32	.15625	13-32	.40625	21-32	.65625	29-32	.90625
11-64	.171875	27-64	.421875	43-64	.671875	59-64	.921875
3-16	.1875	**7-16**	.4375	**11-16**	.6875	**15-16**	.9375
13-64	.203125	29-64	.453125	45-64	.703125	61-64	.953125
7-32	.21875	15-32	.46875	23-32	.71875	31-32	.96875
15-64	.234375	31-64	.484375	47-64	.734375	63-64	.984375
1-4	.25	**1-2**	.50	**3-4**	.75	**1**	1.

To convert a common fraction into a decimal.—Divide the numerator by the denominator, adding to the numerator as many ciphers prefixed by a decimal point as are necessary to give the number of decimal places desired in the result: $\frac{1}{3} = 1.0000 \div 3 = 0.3333 +$.

To convert a decimal into a common fraction.—Set down the decimal as a numerator, and place as the denominator 1 with as many ciphers annexed as there are decimal places in the numerator; erase the

Product of Fractions Expressed in Decimals.

0	1	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
$\frac{1}{16}$	.0625	.0039															
$\frac{1}{8}$	.1250	.0078	.0156														
$\frac{3}{16}$	.1875	.0117	.0234	.0352													
$\frac{1}{4}$	.2500	.0156	.0313	.0469	.0625												
$\frac{5}{16}$	.3125	.0195	.0391	.0586	.0781	.0977											
$\frac{3}{8}$	.3750	.0234	.0469	.0703	.0937	.1172	.1406										
$\frac{7}{16}$	.4375	.0273	.0547	.0820	.1093	.1367	.1641	.1914									
$\frac{1}{2}$	.5000	.0313	.0625	.0938	.1250	.1562	.1875	.2188	.2500								
$\frac{9}{16}$	.5625	.0352	.0703	.1055	.1406	.1758	.2109	.2461	.2813	.3164							
$\frac{5}{8}$	.6250	.0391	.0781	.1172	.1562	.1953	.2344	.2734	.3125	.3516	.3906						
$\frac{11}{16}$	.6875	.0430	.0859	.1289	.1719	.2148	.2578	.3008	.3438	.3867	.4297	.4727					
$\frac{3}{4}$	.7500	.0469	.0938	.1406	.1875	.2344	.2813	.3281	.3750	.4219	.4688	.5156	.5625				
$\frac{13}{16}$	.8125	.0508	.1016	.1523	.2031	.2539	.3047	.3555	.4063	.4570	.5078	.5586	.6094	.6601			
$\frac{7}{8}$	.8750	.0547	.1094	.1641	.2187	.2734	.3281	.3828	.4375	.4922	.5469	.6016	.6563	.7109	.7656		
$\frac{15}{16}$	.9375	.0586	.1172	.1758	.2344	.2930	.3516	.4102	.4688	.5273	.5859	.6445	.7031	.7617	.8203	.8789	
1	1.000	.0625	.1250	.1875	.2500	.3125	.3750	.4375	.5000	.5625	.6250	.6875	.7500	.8125	.8750	.9375	1.000

decimal point in the numerator, and reduce the fraction thus formed to its lowest terms:

$$.25 = \frac{25}{100} = \frac{1}{4}; \quad .3333 = \frac{3333}{10000} = \frac{1}{3}, \text{ nearly.}$$

To reduce a recurring decimal to a common fraction.—Subtract the decimal figures that do not recur from the whole decimal including one set of recurring figures; set down the remainder as the numerator of the fraction, and as many nines as there are recurring figures, followed by as many ciphers as there are non-recurring figures, in the denominator. Thus:

$$\begin{array}{ll} & .79054054\text{, the recurring figures being 054.} \\ \text{Subtract} & \underline{79} \\ & \dfrac{78975}{99900} = \text{(reduced to its lowest terms) } \dfrac{117}{148}. \end{array}$$

COMPOUND OR DENOMINATE NUMBERS.

Reduction descending.—To reduce a compound number to a lower denomination. Multiply the number by as many units of the lower denomination as makes one of the higher.

3 yards to inches: $3 \times 36 = 108$ inches.

.04 square feet to square inches: $.04 \times 144 = 5.76$ sq. in.

If the given number is in more than one denomination proceed in steps from the highest denomination to the next lower, and so on to the lowest, adding in the units of each denomination as the operation proceeds.

3 yds. 1 ft. 7 in. to inches: $3 \times 3 = 9, + 1 = 10, 10 \times 12 = 120, + 7 = 127$ in.

Reduction ascending.—To express a number of a lower denomination in terms of a higher, divide the number by the number of units of the lower denomination contained in one of the next higher; the quotient is in the higher denomination, and the remainder, if any, in the lower.

127 inches to higher denomination.

$127 \div 12 = 10$ feet $+ 7$ inches; 10 feet $\div 3 = 3$ yards $+ 1$ foot.
Ans. 3 yds. 1 ft. 7 in.

To express the result in decimals of the higher denomination, divide the given number by the number of units of the given denomination contained in one of the required denomination, carrying the result to as many places of decimals as may be desired.

127 inches to yards: $127 \div 36 = 3\frac{19}{36} = 3.5277 +$ yards.

RATIO AND PROPORTION.

Ratio is the relation of one number to another, as obtained by dividing one by the other.

Ratio of 2 to 4, or $2 : 4 = 2/4 = 1/2$.
Ratio of 4 to 2, or $4 : 2 = 2$.

Proportion is the equality of two ratios. Ratio of 2 to 4 equals ratio of 3 to 6, $2/4 = 3/6$: expressed thus, $2 : 4 :: 3 : 6$; read, 2 is to 4 as 3 is to 6.

The first and fourth terms are called the extremes or outer terms, the second and third the means or inner terms.

The product of the means equals the product of the extremes:

$$2 : 4 :: 3 : 6; \quad 2 \times 6 = 12; \quad 3 \times 4 = 12.$$

Hence, given the first three terms to find the fourth, multiply the second and third terms together and divide by the first.

$$2 : 4 :: 3 : \text{what number? Ans. } \frac{4 \times 3}{2} = 6.$$

Algebraic expression of proportion.—$a : b :: c : d$; $\frac{a}{b} = \frac{c}{d}$; $ad = bc$; from which $a = \frac{bc}{d}$; $d = \frac{bc}{a}$; $b = \frac{ad}{c}$; $c = \frac{ad}{b}$.

Mean proportional between two given numbers, 1st and 2d, is such a number that the ratio which the first bears to it equals the ratio which it bears to the second. Thus, $2 : 4 :: 4 : 8$; 4 is a mean proportional between 2 and 8. To find the mean proportional between two numbers, extract the square root of their product.

$$\text{Mean proportional of 2 and 8} = \sqrt{2 \times 8} = 4.$$

Single Rule of Three; or, finding the fourth term of a proportion when three terms are given.—Rule, as above, when the terms are stated in their proper order, multiply the second by the third and divide by the first. The difficulty is to state the terms in their proper order. The term which is of the same kind as the required or fourth term is made the third; the first and second must be like each other in kind and denomination. To determine which is to be made second and which first requires a little reasoning. If an inspection of the problem shows that the answer should be greater than the third term, then the greater of the other two given terms should be made the second term—otherwise the first. Thus, 3 men remove 54 cubic feet of rock in a day; how many men will remove in the same time 10 cubic yards? The answer is to be men—make men third term; the answer is to be more than three men, therefore make the greater quantity, 10 cubic yards, the second term; but as it is not the same denomination as the other term it must be reduced, = 270 cubic feet. The proportion is then stated:

$$54 : 270 :: 3 : x \text{ (the required number)}; \quad x = \frac{3 \times 270}{54} = 15 \text{ men.}$$

The problem is more complicated if we increase the number of given terms. Thus, in the above question, substitute for the words "in the same time" the words "in 3 days." First solve it as above, as if the work were to be done in the same time; then make another proportion, stating it thus: If 15 men do it in the same time, it will take fewer men to do it in 3 days; make 1 day the 2d term and 3 days the first term. $3 : 1 :: 15 \text{ men} : 5 \text{ men}$.

Compound Proportion, or Double Rule of Three.—By this rule are solved questions like the one just given, in which two or more statings are required by the single rule of three. In it as in the single rule, there is one third term, which is of the same kind and denomination as the fourth or required term, but there may be two or more first and second terms. Set down the third term, take each pair of terms of the same kind separately, and arrange them as first and second by the same reasoning as is adopted in the single rule of three, making the greater of the pair the second if this pair considered alone should require the answer to be greater.

Set down all the first terms one under the other, and likewise all the second terms. Multiply all the first terms together and all the second terms together. Multiply the product of all the second terms by the third term, and divide this product by the product of all the first terms. Example: If 3 men remove 4 cubic yards in one day, working 12 hours a day, how many men working 10 hours a day will remove 20 cubic yards in 3 days?

$$\left.\begin{array}{lrcr} \text{Yards} & 4 & : & 20 \\ \text{Days} & 3 & : & 1 \\ \text{Hours} & 10 & : & 12 \end{array}\right| :: 3 \text{ men.}$$

$$\text{Products } 120 : 240 :: 3 : 6 \text{ men. Ans.}$$

To abbreviate by cancellation, any one of the first terms may cancel either the third or any of the second terms; thus, 3 in first cancels 3 in third, making it 1, 10 cancels into 20 making the latter 2, which into 4 makes it 2, which into 12 makes it 6, and the figures remaining are only $1 : 6 :: 1 : 6$.

INVOLUTION, OR POWERS OF NUMBERS.

Involution is the continued multiplication of a number by itself a given number of times. The number is called the root, or first power, and the products are called powers. The second power is called the square and the third power the cube. The operation may be indicated without being performed by writing a small figure called the *index* or *exponent* to the right of and a little above the root; thus, 3^3 = cube of 3, = 27.

To multiply two or more powers of the same number, add their exponents; thus, $2^2 \times 2^3 = 2^5$, or $4 \times 8 = 32 = 2^5$.

To divide two powers of the same number, subtract their exponents; thus, $2^3 \div 2^2 = 2^1 = 2$; $2^2 \div 2^4 = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$. The exponent may thus be negative. $2^2 \div 2^2 = 2^0 = 1$, whence the zero power of any number = 1. The first power of a number is the number itself. The exponent may be fractional, as $2^{\frac{1}{2}}$, $2^{\frac{2}{3}}$, which means that the root is to be raised to a power whose exponent is the numerator of the fraction, and the root whose sign is the denominator is to be extracted (see Evolution). The exponent may be a decimal, as $2^{0.5}$, $2^{1.5}$; read, two to the five-tenths power, two to the one and five-tenths power. These powers are solved by means of Logarithms (which see).

First Nine Powers of the First Nine Numbers.

1st Pow'r	2d Pow'r	3d Power.	4th Power.	5th Power.	6th Power.	7th Power.	8th Power.	9th Power.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

The First Forty Powers of 2.

Power.	Value.	Power.	Value.	Power.	Value.	Power.	Value.	Power.	Value.
0	1	9	512	18	262144	27	134217728	36	68719476736
1	2	10	1024	19	524288	28	268435456	37	137438953472
2	4	11	2048	20	1048576	29	536870912	38	274877906944
3	8	12	4096	21	2097152	30	1073741824	39	549755813888
4	16	13	8192	22	4194304	31	2147483648	40	1099511627776
5	32	14	16384	23	8388608	32	4294967296		
6	64	15	32768	24	16777216	33	8589934592		
7	128	16	65536	25	33554432	34	17179869184		
8	256	17	131072	26	67108864	35	34359738368		

EVOLUTION.

Evolution is the finding of the root (or extracting the root) of any number the power of which is given.

The sign $\sqrt{\ }$ indicates that the square root is to be extracted: $\sqrt[3]{\ }$ $\sqrt[4]{\ }$ $\sqrt[n]{\ }$, the cube root, 4th root, *n*th root.

A fractional exponent with 1 for the numerator of the fraction is also used to indicate that the operation of extracting the root is to be performed; thus, $2^{\frac{1}{2}}, 2^{\frac{1}{3}} = \sqrt{2}, \sqrt[3]{2}$.

When the power of a number is indicated, the involution not being performed, the extraction of any root of that power may also be indicated by dividing the index of the power by the index of the root, indicating the division by a fraction. Thus, extract the square root of the 6th power of 2:

$$\sqrt{2^6} = 2^{\frac{6}{2}} = 2^{\frac{3}{1}} = 2^3 = 8.$$

The 6th power of 2, as in the table above, is 64 ; $\sqrt{64} = 8$.

Difficult problems in evolution are performed by logarithms, but the square root and the cube root may be extracted directly according to the rules given below. The 4th root is the square root of the square root. The 6th root is the cube root of the square root, or the square root of the cube root; the 9th root is the cube root of the cube root; etc.

To Extract the Square Root.—Point off the given number into periods of two places each, beginning with units. If there are decimals, point these off likewise, beginning at the decimal point, and supplying as many ciphers as may be needed. Find the greatest number whose square is less than the first left-hand period, and place it as the first figure in the quotient. Subtract its square from the left-hand period, and to the remainder annex the two figures of the second period for a dividend. Double the first figure of the quotient for a partial divisor; find how many times the latter is contained in the dividend exclusive of the right-hand figure, and set the figure representing that number of times as the second figure in the quotient, and annex it to the right of the partial divisor, forming the complete divisor. Multiply this divisor by the second figure in the quotient and subtract the product from the dividend. To the remainder bring down the next period and proceed as before, in each case doubling the figures in the root already found to obtain the trial divisor. Should the product of the second figure in the root by the completed divisor be greater than the dividend, erase the second figure both from the quotient and from the divisor, and substitute the next smaller figure, or one small enough to make the product of the second figure by the divisor less than or equal to the dividend.

```
       3.1415926536|1.77245 +
       1
    27|214
      |189
   347|2515
      |2429
   3542|8692
       |7084
  35444|160865
       |141776
 354485|1908936
       |1772425
```

To extract the square root of a fraction, extract the root of numerator and denominator separately. $\sqrt{\frac{4}{9}} = \frac{2}{3}$, or first convert the fraction into a decimal, $\sqrt{\frac{4}{9}} = \sqrt{.4444+} = .6666+$.

To Extract the Cube Root.—Point off the number into periods of 3 figures each, beginning at the right hand, or unit's place. Point off decimals in periods of 3 figures from the decimal point. Find the greatest cube that does not exceed the left-hand period; write its root as the first figure in the required root. Subtract the cube from the left-hand period, and to the remainder bring down the next period for a dividend.

Square the first figure of the root; multiply by 300, and divide the product into the dividend for a trial divisor; write the quotient after the first figure of the root as a trial second figure.

Complete the divisor by adding to 300 times the square of the first figure, 30 times the product of the first by the second figure, and the square of the second figure. Multiply this divisor by the second figure; subtract the product from the remainder. (Should the product be greater than the remainder, the last figure of the root and the complete divisor are too large;

substitute for the last figure the next smaller number, and correct the trial divisor accordingly.)

To the remainder bring down the next period, and proceed as before to find the third figure of the root—that is, square the two figures of the root already found; multiply by 300 for a trial divisor, etc.

If at any time the trial divisor is greater than the dividend, bring down another period of 3 figures, and place 0 in the root and proceed.

The cube root of a number will contain as many figures as there are periods of 3 in the number.

Shorter Methods of Extracting the Cube Root.—1. From Wentworth's Algebra:

```
                                 1,881,365,963,625|12345
                                 1
300 × 1²          =        300|   881
 30 × 1 × 2       =         60|
           2² =          {   4|
                         { 364|   728
                         {  64       153365
300 × 12²         =      43200|
 30 × 12 × 3      =       1080|
           3² =              9}
                         44289} 132867
                          1089}    20498963
300 × 123²        =    4538700|
 30 × 123 × 4     =      14760|
           4² =               16}
                       4553476} 18213904
                         14776}    2285059625
300 × 1234²       =  456826800|
 30 × 1234 × 5    =     185100|
           5² =               25|
                     457011925|2285059625
```

After the first two figures of the root are found the next trial divisor is found by bringing down the sum of the 60 and 4 obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers. This method shortens the work in long examples, as is seen in the case of the last two trial divisors, saving the labor of squaring 123 and 1234. A further shortening of the work is made by obtaining the last two figures of the root by division, the divisor employed being three times the square of the part of the root already found; thus, after finding the first three figures:

```
3 × 123² = 45387|20498963|45.1+
                 181548
                 234416
                 226935
                   74813
```

The error due to the remainder is not sufficient to change the fifth figure of the root.

2. By Prof. H. A. Wood (*Stevens Indicator*, July, 1890):

I. Having separated the number into periods of three figures each, counting from the right, divide by the square of the *nearest* root of the first period, or first two periods; the *nearest* root is the *trial* root.

II. To the quotient obtained add twice the *trial* root, and divide by 3. This gives the root, or first approximation.

III. By using the first approximate root as a new trial root, and proceeding as before, a nearer approximation is obtained, which process may be repeated until the root has been extracted, or the approximation carried as far as desired.

Example.—Required the cube root of 20. The nearest cube to 20 is 3^3.

$3^2 = 9)20.0$
$\quad 2.2$
$\quad 6$
$3)8.1$
$\quad 2.7$ 1st T. R.

$2.7^2 = 7.29)20.000$
$\quad 2.743$
$\quad 5.4$
$3)8.143$
$\quad 2.714$, 1st ap. cube root.

$2.714^2 = 7.365796)20.0000000$
$\quad 2.7152534$
$\quad 5.428$
$3)8.1432534$
$\quad 2.7144178$ 2d ap. cube root.

Remark.—In the example it will be observed that the second term, or first two figures of the root, were obtained by using for trial root the root of the first period. Using, in like manner, these two terms for trial root, we obtained four terms of the root; and these four terms for trial root gave seven figures of the root correct. In that example the last figure should be 7. Should we take these eight figures for trial root we should obtain at least fifteen figures of the root correct.

To Extract a Higher Root than the Cube.—The fourth root is the square root of the square root; the sixth root is the cube root of the square root or the square root of the cube root. Other roots are most conveniently found by the use of logarithms.

ALLIGATION

shows the value of a mixture of different ingredients when the quantity and value of each is known.

Let the ingredients be a, b, c, d, etc., and their respective values per unit w, x, y, z, etc.

A = the sum of the quantities $= a + b + c + d$, etc.

P = mean value or price per unit of A.

$AP = aw + bx + cy + dz$, etc.

$$P = \frac{aw + bx + cy + dz}{A}.$$

PERMUTATION

shows in how many positions any number of things may be arranged in a row; thus, the letters a, b, c may be arranged in six positions, viz. abc, acb, cab, cba, bac, bca.

Rule.—Multiply together all the numbers used in counting the things; thus, permutations of 1, 2, and 3 $= 1 \times 2 \times 3 = 6$. In how many positions can 9 things in a row be placed?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880.$$

COMBINATION

shows how many arrangements of a few things may be made out of a greater number. Rule: Set down that figure which indicates the greater number, and after it a series of figures diminishing by 1, until as many are set down as the number of the few things to be taken in each combination. Then beginning under the last one set down said number of few things; then going backward set down a series diminishing by 1 until arriving under the first of the upper numbers. Multiply together all the upper numbers to form one product, and all the lower numbers to form another; divide the upper product by the lower one.

How many combinations of 9 things can be made, taking 3 in each combination?

$$\frac{9 \times 8 \times 7}{1 \times 2 \times 3} = \frac{504}{6} = 84.$$

ARITHMETICAL PROGRESSION,

in a series of numbers, is a progressive increase or decrease in each successive number by the addition or subtraction of the same amount at each step, as 1, 2, 3, 4, 5, etc., or 15, 12, 9, 6, etc. The numbers are called terms, and the equal increase or decrease the difference. Examples in arithmetical progression may be solved by the following formulæ:

Let a = first term, l = last term, d = common difference, n = number of terms, s = sum of the terms:

$$l = a + (n-1)d, \qquad = -\frac{1}{2}d \pm \sqrt{2ds + \left(a - \frac{1}{2}d\right)^2},$$

$$= \frac{2s}{n} - a, \qquad = \frac{s}{n} + \frac{(n-1)d}{2}.$$

$$s = \frac{1}{2}n[2a + (n-1)d], \qquad = \frac{l+a}{2} + \frac{l^2 - a^2}{2d},$$

$$= (l+a)\frac{n}{2}, \qquad = \frac{1}{2}n[2l - (n-1)d].$$

$$a = l - (n-1)d, \qquad = \frac{s}{n} - \frac{(n-1)d}{2},$$

$$= \frac{1}{2}d \pm \sqrt{\left(l + \frac{1}{2}d\right)^2 - 2ds}, \qquad = \frac{2s}{n} - l.$$

$$d = \frac{l-a}{n-1}, \qquad = \frac{2(s-an)}{n(n-1)},$$

$$= \frac{l^2 - a^2}{2s - l - a}, \qquad = \frac{2(nl - s)}{n(n-1)}.$$

$$n = \frac{l-a}{d} + 1, \qquad = \frac{d - 2a \pm \sqrt{(2a-d)^2 + 8ds}}{2d},$$

$$= \frac{2s}{l+a}, \qquad = \frac{2l + d \pm \sqrt{(2l+d)^2 - 8ds}}{2d}.$$

GEOMETRICAL PROGRESSION,

in a series of numbers, is a progressive increase or decrease in each successive number by the same multiplier or divisor at each step, as 1, 2, 4, 8, 16, etc., or 243, 81, 27, 9, etc. The common multiplier is called the ratio.

Let a = first term, l = last term, r = ratio or constant multiplier, n = number of terms, m = any term, as 1st, 2d, etc., s = sum of the terms:

$$l = ar^{n-1}, \qquad = \frac{a + (r-1)s}{r}, \qquad = \frac{(r-1)sr^{n-1}}{r^n - 1},$$

$$\log l = \log a + (n-1)\log r, \qquad l(s-l)^{n-1} - a(s-a)^{n-1} = 0.$$

$$m = ar^{m-1}. \qquad \log m = \log a + (m-1)\log r.$$

$$s = \frac{a(r^n - 1)}{r - 1}, \qquad = \frac{rl - a}{r - 1}, \qquad = \frac{\sqrt[n-1]{l^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{l} - \sqrt[n-1]{a}}, \qquad = \frac{lr^n - l}{r^n - r^{n-1}}$$

$$a = \frac{l}{r^{n-1}}, \qquad = \frac{(r-1)s}{r^n - 1}. \qquad \log a = \log l - (n-1)\log r.$$

$$r = \sqrt[n-1]{\frac{l}{a}}, \qquad = \frac{s-a}{s-l}. \qquad \log r = \frac{\log l - \log a}{n-1}.$$

$$r^n - \frac{s}{a}r + \frac{s-a}{a} = 0. \qquad r^n - \frac{s}{s-l}r^{n-1} + \frac{l}{s-l} = 0.$$

$$n = \frac{\log l - \log a}{\log r} + 1, \qquad = \frac{\log[a + (r-1)s] - \log a}{\log r},$$

$$= \frac{\log l - \log a}{\log(s-a) - \log(s-l)} + 1, \qquad = \frac{\log l - \log[lr - (r-1)s]}{\log r} + 1.$$

Population of the United States.

(A problem in geometrical progression.)

Year.	Population.	Increase in 10 Years, per cent.	Annual Increase, per cent.
1860	31,443,321		
1870	39,818,449*	26.63	2.39
1880	50,155,783	25.96	2.33
1890	62,622,250	24.86	2.25
1900	76,295,220	21.834	1.994
1905	Est. 83,577,000	——	Est. 1.840
1910	" 91,554,000	Est. 20.0	" 1.840

Estimated Population in Each Year from 1870 *to* 1909.

(Based on the above rates of increase, in even thousands.)

1870....	39,818	1880....	50,156	1890....	62,622	1900....	76,295
1871 ..	40,748	1881....	51,281	1891. .	63,871	1901....	77,699
1872. .	41,699	1882 ...	52,433	1892....	65,145	1902 ...	79,129
1873....	42,673	1883....	53,610	1893....	66,444	1903....	80,585
1874....	43,670	1884....	54,813	1894...	67,770	1904....	82,067
1875....	44.090	1885....	56,043	1895 ...	69,122	1905....	83,577
1876....	45,373	1886....	57,301	1896....	70,500	1906....	85,115
1877....	46,800	1887....	58,588	1897....	71,906	1907....	86,681
1878 ..	47,893	1888....	59,903	1898....	73,341	1908....	88,276
1879....	49,011	1889....	61,247	1899....	74,803	1909....	89,900

The above table has been calculated by logarithms as follows :

$\log r = \log l - \log a + (n-1)$, $\qquad \log m = \log a + (m-1)\log r$

Pop. 1900.... 76,295,220 log = 7.8824988 = $\log l$
" 1890... 62,622,250 log = 7.7967285 = $\log a$

diff. = .0857703
$n = 11$, $n - 1 = 10$; diff. + 10 = .00857703 = $\log r$,
add log for 1890 7.7967285 = $\log a$

log for 1891 = 7.80530553 No. = 63,871 . . .
add again .00857703

log for 1892 7.81388256 No. = 65,145 . . .

Compound interest is a form of geometrical progression ; the ratio being 1 plus the percentage.

* Corrected by addition of 1,260,078, estimated error of the census of 1870, Census Bulletin No. 16, Dec. 12, 1890.

INTEREST AND DISCOUNT.

Interest is money paid for the use of money for a given time; the factors are:

p, the sum loaned, or the principal:
t, the time in years;
r, the rate of interest;
i, the amount of interest for the given rate and time;
$a = p + i =$ the amount of the principal with interest at the end of the time.

Formulæ:

$$i = \text{interest} = \text{principal} \times \text{time} \times \text{rate per cent} = i = \frac{ptr}{100};$$

$$a = \text{amount} = \text{principal} + \text{interest} = p + \frac{ptr}{100};$$

$$r = \text{rate} = \frac{100i}{pt};$$

$$p = \text{principal} = \frac{100i}{tr} = a - \frac{ptr}{100};$$

$$t = \text{time} = \frac{100i}{pr}.$$

If the rate is expressed decimally as a per cent,—thus, 6 per cent = .06,—the formulæ become

$$i = prt; \quad a = p(1 + rt); \quad r = \frac{i}{pt}; \quad t = \frac{i}{pr}; \quad p = \frac{i}{tr} = \frac{a}{1 + rt}.$$

Rules for finding Interest.—Multiply the principal by the rate per annum divided by 100, and by the time in years and fractions of a year.

If the time is given in days, interest $= \dfrac{\text{principal} \times \text{rate} \times \text{no. of days}}{365 \times 100}$.

In banks interest is sometimes calculated on the basis of 360 days to a year, or 12 months of 30 days each.

Short rules for interest at 6 per cent, when 360 days are taken as 1 year:
Multiply the principal by number of days and divide by 6000.
Multiply the principal by number of months and divide by 200.
The interest of 1 dollar for one month is ½ cent.

Interest of 100 Dollars for Different Times and Rates.

Time.	2%	3%	4%	5%	6%	8%	10%
1 year	\$2.00	\$3.00	\$4.00	\$5.00	\$6.00	\$8.00	\$10.00
1 month	$.16\frac{2}{3}$	.25	$.33\frac{1}{3}$	$.41\frac{2}{3}$	.50	$.66\frac{2}{3}$	$.83\frac{1}{3}$
1 day = $\frac{1}{360}$ year	$.0055\frac{5}{9}$	$.0083\frac{1}{3}$	$.0111\frac{1}{9}$	$.0138\frac{8}{9}$	$.0166\frac{2}{3}$	$.0222\frac{2}{9}$	$.0277\frac{7}{9}$
1 day = $\frac{1}{365}$ year	.005479	.008219	.010959	.013699	.016438	.0219178	.0273973

Discount is interest deducted for payment of money before it is due.

True discount is the difference between the amount of a debt payable at a future date without interest and its present worth. The present worth is that sum which put at interest at the legal rate will amount to the debt when it is due.

To find the present worth of an amount due at future date, divide the amount by the amount of \$1 placed at interest for the given time. The discount equals the amount minus the present worth.

What discount should be allowed on \$106 paid six months before it is due, interest being 6 per cent per annum?

$$\frac{103}{1 + 1 \times .06 \times \frac{1}{2}} = \$100 \text{ present worth, discount} = 3.00.$$

Bank discount is the amount deducted by a bank as interest on money loaned on promissory notes. It is interest calculated not on the actual sum loaned, but on the gross amount of the note, from which the discount is deducted in advance. It is also calculated on the basis of 360 days in the year, and for 3 (in some banks 4) days more than the time specified in the note. These are called days of grace, and the note is not payable till the last of these days. In some States days of grace have been abolished.

What discount will be deducted by a bank in discounting a note for \$103 payable 6 months hence? Six months = 182 days, add 3 days grace = 185 days, $\frac{103 \times 185}{6000} = \$3.176$.

Compound Interest.—In compound interest the interest is added to the principal at the end of each year, (or shorter period if agreed upon).

Let p = the principal, r = the rate expressed decimally, n = no of years, and a the amount:

$$a = \text{amount} = p\,(1 + r)^n;\quad r = \text{rate} = \sqrt[n]{\frac{a}{p}} - 1,$$

$$p = \text{principal} = \frac{a}{(1 + r)^n};\quad \text{no. of years} = n = \frac{\log a - \log p}{\log\,(1 + r)}.$$

Compound Interest Table.

(Value of one dollar at compound interest, compounded yearly, at 3, 4, 5, and 6 per cent, from 1 to 50 years.)

Years.	3%	4%	5%	6%	Years.	3%	4%	5%	6%
1	1.03	1.04	1.05	1.06	16	1.6047	1.8730	2.1829	2.5403
2	1.0609	1.0816	1.1025	1.1236	17	1.6528	1.9479	2.2920	2.6928
3	1.0927	1.1249	1.1576	1.1910	18	1.7024	2.0258	2.4066	2.8543
4	1.1255	1.1699	1.2155	1.2625	19	1.7535	2.1068	2.5269	3.0256
5	1.1593	1.2166	1.2763	1.3382	20	1.8061	2.1911	2.6533	3.2071
6	1.1941	1.2653	1.3401	1.4185	21	1.8603	2.2787	2.7859	3.3995
7	1.2299	1.3159	1.4071	1.5036	22	1.9161	2.3699	2.9252	3.6035
8	1.2668	1.3686	1.4774	1.5938	23	1.9736	2.4647	3.0715	3.8197
9	1.3048	1.4233	1.5513	1.6895	24	2.0328	2.5633	3.2251	4.0487
10	1.3439	1.4802	1.6289	1.7908	25	2.0937	2.6658	3.3864	4.2919
11	1.3842	1.5394	1.7103	1.8983	30	2.4272	3.2434	4.3219	5.7435
12	1.4258	1.6010	1.7958	2.0122	35	2.8138	3.9460	5.5166	7.6861
13	1.4685	1.6651	1.8856	2.1329	40	3.2620	4.8009	7.0100	10.2858
14	1.5126	1.7317	1.9799	2.2600	45	3.7815	5.8410	8.9850	13.7646
15	1.5580	1.8009	2.0789	2.3965	50	4.3838	7.1064	11.6792	18.4190

At compound interest at 3 per cent money will double itself in 23½ years, at 4 per cent in 17⅔ years, at 5 per cent in 14.2 years, and at 6 per cent in 11.9 years.

EQUATION OF PAYMENTS.

By equation of payments we find the equivalent or average time in which one payment should be made to cancel a number of obligations due at different dates; also the number of days upon which to calculate interest or discount upon a gross sum which is composed of several smaller sums payable at different dates.

Rule.—Multiply each item by the time of its maturity in days from a fixed date, taken as a standard, and divide the sum of the products by the sum of the items: the result is the average time in days from the standard date.

A owes B \$100 due in 30 days, \$200 due in 60 days, and \$300 due in 90 days. In how many days may the whole be paid in one sum of \$600?

$$100 \times 30 + 200 \times 60 + 300 \times 90 = 42{,}000;\quad 42{,}000 \div 600 = 70 \text{ days, } ans.$$

A owes B \$100, \$200, and \$300, which amounts are overdue respectively 30, 60, and 90 days. If he now pays the whole amount, \$600, how many days' interest should he pay on that sum? *Ans.* 70 days.

PARTIAL PAYMENTS.

To compute interest on notes and bonds when partial payments have been made:

United States Rule.—Find the amount of the principal to the time of the first payment, and, subtracting the payment from it, find the amount of the remainder as a new principal to the time of the next payment.

If the payment is less than the interest, find the amount of the principal to the time when the sum of the payments equals or exceeds the interest due, and subtract the sum of the payments from this amount.

Proceed in this manner till the time of settlement.

Note.—The principles upon which the preceding rule is founded are:

1st. That payments must be applied first to discharge accrued interest, and then the remainder, if any, toward the discharge of the principal.

2d. That only unpaid principal can draw interest.

Mercantile Method.—When partial payments are made on short notes or interest accounts, business men commonly employ the following method:

Find the amount of the whole debt to the time of settlement; also find the amount of each payment from the time it was made to the time of settlement. Subtract the amount of payments from the amount of the debt; the remainder will be the balance due.

ANNUITIES.

An **Annuity** is a fixed sum of money paid yearly, or at other equal times agreed upon. The values of annuities are calculated by the principles of compound interest.

1. Let i denote interest on $1 for a year, then at the end of a year the amount will be $1 + i$. At the end of n years it will be $(1 + i)^n$.

2. The sum which in n years will amount to 1 is $\dfrac{1}{(1+i)^n}$ or $(1+i)^{-n}$, or the present value of 1 due in n years.

3. The amount of an annuity of 1 in any number of years n is $\dfrac{(1+i)^n - 1}{i}$.

4. The present value of an annuity of 1 for any number of years n is $\dfrac{1-(1+i)^{-n}}{i}$.

5. The annuity which 1 will purchase for any number of years n is $\dfrac{i}{1-(1+i)^{-n}}$.

6. The annuity which would amount to 1 in n years is $\dfrac{i}{(1+i)^n - 1}$.

Amounts, Present Values, etc., at 5% Interest.

Years	(1) $(1+i)^n$	(2) $(1+i)^{-n}$	(3) $\dfrac{(1+i)^n-1}{i}$	(4) $\dfrac{1-(1+i)^{-n}}{i}$	(5) $\dfrac{i}{1-(1+i)^{-n}}$	(6) $\dfrac{i}{(1+i)^n-1}$
1.....	1.05	.952381	1.	.952381	1.05	1.
2	1.1025	.907029	2.05	1.859410	.537805	.487805
3.....	1.157625	.863838	3.1525	2.723248	.367209	.317209
4.....	1.215506	.822702	4.310125	3.545951	.282012	.232012
5.....	1.276282	.783526	5.525631	4.329477	.230975	.180975
6.....	1.340096	.746215	6.801913	5.075692	.197017	.147018
7.....	1.407100	.710681	8.142008	5.786373	.172820	.122820
8.....	1.477455	.676839	9.549109	6.463213	.154722	.104722
9.....	1.551328	.644609	11.026564	7.107822	.140690	.090690
10.....	1.628895	.613913	12.577893	7.721735	.129505	.079505

Years to run.	Rate of Interest, per cent.												
	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4½	5	5½	[illegible]
2	495.05	494.50	493.78	493.23	492.69	492.05	491.42	490.81	490.20	489.00	487.80	486.62	485.48
3	326.72	325.94	325.14	324.35	323.56	322.75	321.94	321.18	320.36	318.77	317.21	315.63	314.10
4	242.63	241.74	240.84	239.93	239.02	238.14	237.26	236.38	235.50	233.74	232.01	230.29	228.60
5	192.16	191.18	190.24	189.30	188.35	187.42	186.49	185.56	184.63	182.79	180.98	179.18	177.39
6	158.53	157.58	156.56	155.58	154.61	153.64	152.67	151.73	150.79	148.88	147.02	145.18	143.36
7	134.52	133.51	132.49	131.50	130.51	129.54	128.57	127.59	126.61	124.67	122.82	120.96	119.13
8	116.51	115.48	114.47	113.46	112.46	111.47	110.48	109.50	108.53	106.60	104.72	102.86	101.03
9	102.52	101.48	100.46	99.45	98.44	97.44	96.44	95.46	94.49	92.57	90.69	88.83	87.02
10	91.33	90.29	89.26	88.24	87.24	86.21	85.24	84.26	83.29	81.38	79.50	77.67	75.87
11	82.18	81.14	80.11	79.09	78.07	77.08	76.09	75.12	74.15	72.25	70.39	68.57	66.79
12	74.56	73.52	72.49	71.47	70.46	69.47	68.48	67.51	66.55	64.67	62.83	61.03	59.28
13	68.12	67.08	66.05	65.04	64.03	63.05	62.06	61.10	60.14	58.27	56.45	54.68	52.96
14	62.60	61.56	60.54	59.53	58.53	57.55	56.57	55.62	54.67	52.82	51.02	49.28	47.58
15	57.83	56.79	55.77	54.77	53.77	52.79	51.82	50.88	49.94	48.11	46.34	44.62	42.96
16	53.65	52.62	51.60	50.60	49.61	48.64	47.68	46.70	45.82	44.01	42.27	40.58	38.95
17	49.97	48.94	47.98	46.94	45.95	44.99	44.04	43.12	42.20	40.42	38.70	37.04	35.44
18	46.70	45.67	44.67	43.69	42.71	41.76	40.82	39.90	38.99	37.24	35.54	33.92	32.36
19	43.78	42.76	41.76	40.78	39.81	38.87	37.94	37.04	36.14	34.40	32.75	31.15	29.62
20	41.15	40.14	39.14	38.18	37.22	36.29	35.36	34.47	33.58	31.87	30.24	28.68	27.18
25	31.22	30.24	29.27	28.35	27.43	26.55	25.67	24.84	24.01	22.44	20.95	19.55	18.23
30	24.65	23.70	22.78	21.90	21.02	20.19	19.37	18.60	17.83	16.39	15.05	13.80	12.65
35	20.00	19.09	18.20	17.37	16.54	15.77	15.00	14.29	13.58	12.27	11.07	9.97	8.97
40	16.55	15.68	14.84	14.05	13.26	12.54	11.83	11.17	10.52	9.34	8.28	7.32	6.46
45	13.91	13.07	12.27	11.52	10.78	10.12	9.45	8.85	8.26	7.20	6.26	5.43	4.70
50	11.82	11.02	10.26	9.56	8.87	8.25	7.63	7.09	6.55	5.60	4.78	4.06	3.44

TABLES FOR CALCULATING SINKING-FUNDS AND PRESENT VALUES.

Engineers and others connected with municipal work and industrial enterprises often find it necessary to calculate payments to sinking-funds which will provide a sum of money sufficient to pay off a bond issue or other debt at the end of a given period, or to determine the present value of certain annual charges. The accompanying tables were computed by Mr. John W. Hill, of Cincinnati, *Eng'g News*, Jan. 25, 1894.

Table I (opposite page) shows the annual sum at various rates of interest required to net $1000 in from 2 to 50 years, and Table II shows the present value at various rates of interest of an annual charge of $1000 for from 5 to 50 years, at five-year intervals and for 100 years.

Table II.—Capitalization of Annuity of $1000 for from 5 to 100 Years.

Years.	Rate of Interest, per cent.							
	2½	3	3½	4	4½	5	5½	6
5	4,645 83	4,579.60	4,514.92	4,451.68	4,389.91	4,329.45	4,268.09	4,212.40
10	8,752.17	8,530.18	8,316.45	8,110.74	7,912.67	7,721.73	7,537.54	7,360.19
15	12,381.41	11,937.80	11,517.23	11,118.06	10,739.42	10,379.53	10,037.48	9,712.30
20	15,589.215	14,877.27	14,212.12	13,590.21	13,007.88	12,462.13	11,950.26	11,469.96
25	18,424.67	17,413.01	16,481.28	15,621.93	14,828.12	14,093.86	13,413.82	12,783.38
30	20,930.59	19,600.21	18,391.85	17,291.86	16,288.77	15,372.36	14,533.63	13,764.85
35	23,145.31	21,487.04	20,000.43	18,664.87	17,460.89	16,374.36	15,390.48	14,488.65
40	25,103.53	23,114.86	21,354.83	19,792.65	18,401.49	17,159.01	16,044.92	15,046.31
45	26,833.15	24,518.49	22,495.23	20,719.89	19,156.24	17,773.99	16,547.65	15,455.85
50	28,362.48	25,729.58	23,455.21	21,482.08	19,761.93	18,255.86	16,931.97	15,761.87
100	36,614.21	31,598.81	27,655.36	24,504.96	21,949.21	19,847.90	18,095.83	16,612.64

WEIGHTS AND MEASURES.

Long Measure.—Measures of Length.

12 inches	= 1 foot.
3 feet	= 1 yard.
5½ yards, or 16½ feet	= 1 rod, pole, or perch.
40 poles, or 220 yards	= 1 furlong.
8 furlongs, or 1760 yards, or 5280 feet	= 1 mile.
3 miles	= league.

Additional measures of length in occasional use: 1000 mils = 1 inch; 4 inches = 1 hand; 9 inches = 1 span; 2½ feet = 1 military pace; 2 yards = 1 fathom.

Old Land Measure.—7.92 inches = 1 link; 100 links, or 66 feet, or 4 poles = 1 chain; 10 chains = 1 furlong; 8 furlongs = 1 mile; 10 square chains = 1 acre.

Nautical Measure.

6080.26 feet, or 1.15156 statute miles	= 1 nautical mile, or knot.*
3 nautical miles	= 1 league.
60 nautical miles, or 69.168 statute miles	= 1 degree (at the equator).
360 degrees	= circumference of the earth at the equator.

* The British Admiralty takes the round figure of 6080 ft. which is the length of the "measured mile" used in trials of vessels. The value varies from 6080.26 to 6088.44 ft. according to different measures of the earth's diameter. There is a difference of opinion among writers as to the use of the word "knot" to mean length or a distance—some holding that it should be

Square Measure.—Measures of Surface.

144 square inches, or 183.35 circular inches	= 1 square foot.
9 square feet	= 1 square yard.
30¼ square yards, or 272¼ square feet	= 1 square rod, pole, or perch.
40 square poles	= 1 rood.
4 roods, or 10 sq. chains, or 160 sq. poles, or 4840 sq. yards, or 43560 sq. feet,	= 1 acre.
640 acres	= 1 square mile.

An acre equals a square whose side is 208.71 feet.

Circular Inch; Circular Mil.—A circular inch is the area of a circle 1 inch in diameter = 0.7854 square inch.

1 square inch = 1.2732 circular inches.

A circular mil is the area of a circle 1 mil, or .001 inch in diameter. 1000^2 or 1,000,000 circular mils = 1 circular inch.

1 square inch = 1,273,239 circular mils.

The mil, and circular mil are used in electrical calculations involving the diameter and area of wires.

Solid or Cubic Measure.—Measures of Volume.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
1 cord of wood = a pile, 4 × 4 × 8 feet = 128 cubic feet.
1 perch of masonry = 16½ × 1½ × 1 foot = 24¾ cubic feet.

Liquid Measure.

4 gills	= 1 pint.
2 pints	= 1 quart.
4 quarts	= 1 gallon { U. S. 231 cubic inches. Eng. 277.274 cubic inches.
31½ gallons	= 1 barrel.
42 gallons	= 1 tierce.
2 barrels, or 63 gallons	= 1 hogshead.
84 gallons, or 2 tierces	= 1 puncheon.
2 hogsheads, or 126 gallons	= 1 pipe or butt.
2 pipes, or 3 puncheons	= 1 tun.

The U. S. gallon contains 231 cubic inches; 7.4805 gallons = 1 cubic foot. A cylinder 7 in. diam. and 6 in. high contains 1 gallon, very nearly, or 230.9 cubic inches. The British Imperial gallon contains 277.274 cubic inches = 1.20032 U. S. gallon.

The Miner's Inch.—(Western U. S. for measuring flow of a stream of water).

The term Miner's Inch is more or less indefinite, for the reason that California water companies do not all use the same head above the centre of the aperture, and the inch varies from 1.36 to 1.73 cubic feet per minute each; but the most common measurement is through an aperture 2 inches high and whatever length is required, and through a plank 1¼ inches thick. The lower edge of the aperture should be 2 inches above the bottom of the measuring-box, and the plank 5 inches high above the aperture, thus making a 6-inch head above the centre of the stream. Each square inch of this opening represents a miner's inch, which is equal to a flow of 1½ cubic feet per minute.

Apothecaries' Fluid Measure.

60 minims	= 1 fluid drachm.
8 drachms, or 437½ grains, or 1.732 cubic inches	= 1 fluid ounce.

Dry Measure, U. S.

2 pints = 1 quart.
8 quarts = 1 peck.
4 pecks = 1 bushel.

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used only to denote a rate of speed. The length between knots on the log line is $\frac{1}{120}$ of a nautical mile or 50.7 ft. when a half-minute glass is used; so that a speed of 10 knots is equal to 10 nautical miles per hour.

The standard U. S. bushel is the Winchester bushel, which is in cylinder form, 18½ inches diameter and 8 inches deep, and contains 2150.42 cubic inches.

A struck bushel contains 2150.42 cubic inches = 1.2445 cu. ft.; 1 cubic foot = 0.80356 struck bushel. A heaped bushel is a cylinder 18½ inches diameter and 8 inches deep, with a heaped cone not less than 6 inches high. It is equal to 1¼ struck bushels.

The British Imperial bushel is based on the Imperial gallon, and contains 8 such gallons, or 2218.192 cubic inches = 1.2837 cubic feet. The English quarter = 8 Imperial bushels.

Capacity of a cylinder in U. S. gallons = square of diameter, in inches × height in inches × .0034. (Accurate within 1 part in 100,000.)

Capacity of a cylinder in U. S. bushels = square of diameter in inches × height in inches × .0003652.

Shipping Measure.

Register Ton.—For register tonnage or for measurement of the entire internal capacity of a vessel:

100 cubic feet = 1 register ton.

This number is arbitrarily assumed to facilitate computation.

Shipping Ton.—For the measurement of cargo:

40 cubic feet = { 1 U. S. shipping ton.; 31.16 Imp. bushels.; 32.143 U. S. "

42 cubic feet = { 1 British shipping ton.; 32.719 Imp. bushels.; 33.75 U. S. "

Carpenter's Rule.—Weight a vessel will carry = length of keel × breadth at main beam × depth of hold in feet ÷ 95 (the cubic feet allowed for a ton). The result will be the tonnage. For a double-decker instead of the depth of the hold take half the breadth of the beam.

Measures of Weight.—Avoirdupois, or Commercial Weight.

16 drachms, or 437.5 grains	= 1 ounce, oz.
16 ounces, or 7000 grains	= 1 pound, lb.
28 pounds	= 1 quarter, qr.
4 quarters	= 1 hundredweight, cwt. = 112 lbs.
20 hundred weight	= 1 ton of 2240 pounds, or long ton.
2000 pounds	= 1 net, or short ton.
2204.6 pounds	= 1 metric ton.

1 stone = 14 pounds; 1 quintal = 100 pounds.

Troy Weight.

24 grains	= 1 pennyweight, dwt.
20 pennyweights	= 1 ounce, oz. = 480 grains.
12 ounces	= 1 pound, lb. = 5760 grains.

Troy weight is used for weighing gold and silver. The grain is the same in Avoirdupois, Troy, and Apothecaries' weights. A carat, used in weighing diamonds = 3.168 grains = .205 gramme.

Apothecaries' Weight.

20 grains	= 1 scruple, ℈	
3 scruples	= 1 drachm, ʒ	= 60 grains.
8 drachms	= 1 ounce, ℥	= 480 grains.
12 ounces	= 1 pound, lb.	= 5760 grains.

To determine whether a balance has unequal arms.—After weighing an article and obtaining equilibrium, transpose the article and the weights. If the balance is true, it will remain in equilibrium; if untrue, the pan suspended from the longer arm will descend.

To weigh correctly on an incorrect balance.—First, by substitution. Put the article to be weighed in one pan of the balance and counterpoise it by any convenient heavy articles placed on the other pan. Remove the article to be weighed and substitute for it standard weights until equipoise is again established. The amount of these weights is the weight of the article.

Second, by transposition. Determine the apparent weight of the article as usual, then its apparent weight after transposing the article and the weights. If the difference is small, add half the difference to the smaller of the apparent weights to obtain the true weight. If the difference is 2 per cent the error of this method is 1 part in 10,000. For larger differences, or to obtain a perfectly accurate result, multiply the two apparent weights together and extract the square root of the product.

Circular Measure.

60 seconds, $''$ = 1 minute, $'$.
60 minutes, $'$ = 1 degree, $^\circ$.
90 degrees = 1 quadrant.
360 " = circumference.

Time.

60 seconds = 1 minute.
60 minutes = 1 hour.
24 hours = 1 day.
7 days = 1 week.
365 days, 5 hours, 48 minutes, 48 seconds = 1 year.

By the Gregorian Calendar every year whose number is divisible by 4 is a leap year, and contains 366 days, the other years containing 365 days, except that the centesimal years are leap years only when the number of the year is divisible by 400.

The comparative values of mean solar and sidereal time are shown by the following relations according to Bessel:

365.24222 mean solar days = 366.24222 sidereal days, whence
1 mean solar day = 1.00273791 sidereal days;
1 sidereal day = 0.99726957 mean solar day;
24 hours mean solar time = $24^h\ 3^m\ 56^s.555$ sidereal time;
24 hours sidereal time = $23^h\ 56^m\ 4^s.091$ mean solar time,

whence 1 mean solar day is $3^m\ 55^s.91$ longer than a sidereal day, reckoned in mean solar time.

BOARD AND TIMBER MEASURE.

Board Measure.

In board measure boards are assumed to be one inch in thickness. To obtain the number of feet board measure (B. M.) of a board or stick of square timber, multiply together the length in feet, the breadth in feet, and the thickness in inches.

To compute the measure or surface in square feet.—When all dimensions are in feet, multiply the length by the breadth, and the product will give the surface required.

When either of the dimensions are in inches, multiply as above and divide the product by 12.

When all dimensions are in inches, multiply as before and divide product by 144.

Timber Measure.

To compute the volume of round timber.—When all dimensions are in feet, multiply the length by one quarter of the product of the mean girth and diameter, and the product will give the measurement in cubic feet. When length is given in feet and girth and diameter in inches, divide the product by 144; when all the dimensions are in inches, divide by 1728.

To compute the volume of square timber.—When all dimensions are in feet, multiply together the length, breadth, and depth; the product will be the volume in cubic feet. When one dimension is given in inches, divide by 12; when two dimensions are in inches, divide by 144; when all three dimensions are in inches, divide by 1728.

Contents in Feet of Joists, Scantling, and Timber.

Length in Feet.

Size.	12	14	16	18	20	22	24	26	28	30
Feet Board Measure.										
2 × 4	8	9	11	12	13	15	16	17	19	20
2 × 6	12	14	16	18	20	22	24	26	28	30
2 × 8	16	19	21	24	27	29	32	35	37	40
2 × 10	20	23	27	30	33	37	40	43	47	50
2 × 12	24	28	32	36	40	44	48	52	56	60
2 × 14	28	33	37	42	47	51	56	61	65	70
3 × 8	24	28	32	36	40	44	48	52	56	60
3 × 10	30	35	40	45	50	55	60	65	70	75
3 × 12	36	42	48	54	60	66	72	78	84	90
3 × 14	42	49	56	63	70	77	84	91	98	105
4 × 4	16	19	21	24	27	29	32	35	37	40
4 × 6	24	28	32	36	40	44	48	52	56	60
4 × 8	32	37	43	48	53	59	64	69	75	80
4 × 10	40	47	53	60	67	73	80	87	93	100
4 × 12	48	56	64	72	80	88	96	104	112	120
4 × 14	56	65	75	84	93	103	112	121	131	140
6 × 6	36	42	48	54	60	66	72	78	84	90
6 × 8	48	56	64	72	80	88	96	104	112	120
6 × 10	60	70	80	90	100	110	120	130	140	150
6 × 12	72	84	96	108	120	132	144	156	168	180
6 × 14	84	98	112	126	140	154	168	182	196	210
8 × 8	64	75	85	96	107	117	128	139	149	160
8 × 10	80	93	107	120	133	147	160	173	187	200
8 × 12	96	112	128	144	160	176	192	208	224	240
8 × 14	112	131	149	168	187	205	224	243	261	280
10 × 10	100	117	133	150	167	183	200	217	233	250
10 × 12	120	140	160	180	200	220	240	260	280	300
10 × 14	140	163	187	210	233	257	280	303	327	350
12 × 12	144	168	192	216	240	264	288	312	336	360
12 × 14	168	196	224	252	280	308	336	364	392	420
14 × 14	196	229	261	294	327	359	392	425	457	490

FRENCH OR METRIC MEASURES.

The metric unit of length is the metre = 39.37 inches.
The metric unit of weight is the gram = 15.432 grains.
The following prefixes are used for subdivisions and multiples; Milli = $\frac{1}{1000}$, Centi = $\frac{1}{100}$, Deci = $\frac{1}{10}$, Deca = 10, Hecto = 100, Kilo = 1000, Myria = 10,000.

FRENCH AND BRITISH (AND AMERICAN) EQUIVALENT MEASURES.

Measures of Length.

French.	British and U. S.
1 metre	= 39.37 inches, or 3.28083 feet, or 1.09361 yards.
.3048 metre	= 1 foot.
1 centimetre	= .3937 inch.
2.54 centimetres	= 1 inch.
1 millimetre	= .03937 inch, or 1/25 inch, nearly.
25.4 millimetres	= 1 inch.
1 kilometre	= 1093.61 yards, or 0.62137 mile.

Measures of Surface.

FRENCH.	BRITISH and U. S.
1 square metre	= 10.764 square feet, 1.196 square yards.
.836 square metre	= 1 square yard.
.0929 square metre	= 1 square foot.
1 square centimetre	= .155 square inch.
6.452 square centimetres	= 1 square inch.
1 square millimetre	= .00155 square inch.
645.2 square millimetres	= 1 square inch.
1 centiare = 1 sq. metre	= 10.764 square feet.
1 are = 1 sq. decametre	= 1076.41 " "
1 hectare = 100 ares	= 107641 " " = 2.4711 acres.
1 sq. kilometre	= .386109 sq. miles = 247.11 "
1 sq. myriametre	= 38.6109 " "

Of Volume.

FRENCH.	BRITISH and U. S.
1 cubic metre	= 35.314 cubic feet, 1.308 cubic yards.
.7645 cubic metre	= 1 cubic yard.
.02832 cubic metre	= 1 cubic foot.
1 cubic decimetre	= 61.023 cubic inches, .0353 cubic foot.
28.32 cubic decimetres	= 1 cubic foot.
1 cubic centimetre	= .061 cubic inch.
16.387 cubic centimetres	= 1 cubic inch.
1 cubic centimetre = 1 millilitre =	.061 cubic inch.
1 centilitre =	= .610 " "
1 decilitre =	= 6.102 " "
1 litre = 1 cubic decimetre =	61.023 " " = 1.05671 quarts, U. S.
1 hectolitre or decistere =	3.5314 cubic feet = 2.8375 bushels, "
1 stere, kilolitre, or cubic metre =	1.308 cubic yards = 28.37 bushels, "

Of Capacity.

FRENCH.	BRITISH and U. S.
1 litre (= 1 cubic decimetre) =	61.023 cubic inches, .03531 cubic foot, .2642 gallon (American), 2.202 pounds of water at 62° F.
28.317 litres	= 1 cubic foot.
4.543 litres	= 1 gallon (British).
3.785 litres	= 1 gallon (American).

Of Weight.

FRENCH.	BRITISH and U. S.
1 gramme	= 15.432 grains.
.0648 gramme	= 1 grain.
28.35 gramme	= 1 ounce avoirdupois.
1 kilogramme	= 2.2046 pounds.
.4536 kilogramme	= 1 pound.
1 tonne or metric ton = 1000 kilogrammes =	.9842 ton of 2240 pounds, 19.68 cwts., 2204.6 pounds.
1.016 metric tons = 1016 kilogrammes =	1 ton of 2240 pounds.

Mr. O. H. Titmann, in Bulletin No. 9 of the U. S. Coast and Geodetic Survey, discusses the work of various authorities who have compared the yard and the metre, and by referring all the observations to a common standard has succeeded in reconciling the discrepancies within very narrow limits. The following are his results for the number of inches in a metre according to the comparisons of the authorities named:

1817.	Hassler....	39.36994 inches.
1818.	Kater.........................	39.36990 "
1835.	Baily.........................	39.36973 "
1866.	Clarke........................	39.36970 "
1885.	Comstock......................	39.36984 "
	The mean of these is......... ...	39.36982 "

METRIC CONVERSION TABLES.

The following tables, with the subjoined memoranda, were published in 1890 by the United States Coast and Geodetic Survey, office of standard weights and measures, T. C. Mendenhall, Superintendent.

Tables for Converting U. S. Weights and Measures—Customary to Metric.

LINEAR.

	Inches to Millimetres.	Feet to Metres.	Yards to Metres.	Miles to Kilometres.
1 =	25.4001	0.304801	0.914402	1.60935
2 =	50.8001	0.609601	1.828804	3.21869
3 =	76.2002	0.914402	2.743205	4.82804
4 =	101.6002	1.219202	3.657607	6.43739
5 =	127.0003	1.524003	4.572009	8.04674
6 =	152.4003	1.828804	5.486411	9.65608
7 =	177.8004	2.133604	6.400813	11.26543
8 =	203.2004	2.438405	7.315215	12.87478
9 =	228.6005	2.743205	8.229616	14.48412

SQUARE.

	Square Inches to Square Centimetres.	Square Feet to Square Decimetres.	Square Yards to Square Metres.	Acres to Hectares.
1 =	6.452	9.290	0.836	0.4047
2 =	12.903	18.581	1.672	0.8094
3 =	19.355	27.871	2.508	1.2141
4 =	25.807	37.161	3.344	1.6187
5 =	32.258	46.452	4.181	2.0234
6 =	38.710	55.742	5.017	2.4281
7 =	45.161	65.032	5.853	2.8328
8 =	51.613	74.323	6.689	3.2375
9 =	58.065	83.613	7.525	3.6422

CUBIC.

	Cubic Inches to Cubic Centimetres.	Cubic Feet to Cubic Metres.	Cubic Yards to Cubic Metres.	Bushels to Hectolitres.
1 =	16.387	0.02832	0.765	0.35242
2 =	32.774	0.05663	1.529	0.70485
3 =	49.161	0.08495	2.294	1.05727
4 =	65.549	0.11327	3.058	1.40969
5 =	81.936	0.14158	3.823	1.76211
6 =	98.323	0.16990	4.587	2.11454
7 =	114.710	0.19822	5.352	2.46696
8 =	131.097	0.22654	6.116	2.81938
9 =	147.484	0.25485	6.881	3.17181

CAPACITY.

	Fluid Drachms to Millilitres or Cubic Centimetres.	Fluid Ounces to Millilitres.	Quarts to Litres.	Gallons to Litres.
1 =	3.70	29.57	0.94636	3.78544
2 =	7.39	59.15	1.89272	7.57088
3 =	11.09	88.72	2.83908	11.35632
4 =	14.79	118.30	3.78544	15.14176
5 =	18.48	147.87	4.73180	18.92720
6 =	22.18	177.44	5.67816	22.71264
7 =	25.88	207.02	6.62452	26.49808
8 =	29.57	236.59	7.57088	30.28352
9 =	33.28	266.16	8.51724	34.06896

WEIGHT.

	Grains to Milligrammes.	Avoirdupois Ounces to Grammes.	Avoirdupois Pounds to Kilogrammes.	Troy Ounces to Grammes.
1 =	64.7989	28.3495	0.45359	31.10348
2 =	129.5978	56.6991	0.90719	62.20696
3 =	194.3968	85.0486	1.36078	93.31044
4 =	259.1957	113.3981	1.81437	124.41392
5 =	323.9946	141.7476	2.26796	155.51740
6 =	388.7935	170.0972	2.72156	186.62089
7 =	453.5924	198.4467	3.17515	217.72437
8 =	518.3914	226.7962	3.62874	248.82785
9 =	583.1903	255.1457	4.08233	279.93133

1 chain = 20.1169 metres.
1 square mile = 259 hectares.
1 fathom = 1.829 metres.
1 nautical mile = 1853.27 metres.
1 foot = 0.304801 metre.
1 avoir. pound = 453.5924277 gram.
15432.35639 grains = 1 kilogramme.

Tables for Converting U. S. Weights and Measures—Metric to Customary.

LINEAR.

	Metres to Inches.	Metres to Feet.	Metres to Yards.	Kilometres to Miles.
1 =	39.3700	3.28083	1.093611	0.62137
2 =	78.7400	6.56167	2.187222	1.24274
3 =	118.1100	9.84250	3.280833	1.86411
4 =	157.4800	13.12333	4.374444	2.48548
5 =	196.8500	16.40417	5.468056	3.10685
6 =	236.2200	19.68500	6.561667	3.72822
7 =	275.5900	22.96583	7.655278	4.34959
8 =	314.9600	26.24667	8.748889	4.97096
9 =	354.3300	29.52750	9.842500	5.59233

SQUARE.

	Square Centimetres to Square Inches.	Square Metres to Square Feet.	Square Metres to Square Yards.	Hectares to Acres.
1 =	0.1550	10.764	1.196	2.471
2 =	0.3100	21.528	2.392	4.942
3 =	0.4650	32.292	3.588	7.413
4 =	0.6200	43.055	4.784	9.884
5 =	0.7750	53.819	5.980	12.355
6 =	0.9300	64.583	7.176	14.826
7 =	1.0850	75.347	8.372	17.297
8 =	1.2400	86.111	9.568	19.768
9 =	1.3950	96.874	10.764	22.239

CUBIC.

	Cubic Centimetres to Cubic Inches.	Cubic Decimetres to Cubic Inches.	Cubic Metres to Cubic Feet.	Cubic Metres to Cubic Yards.
1 =	0.0610	61.023	35.314	1.308
2 =	0.1220	122.047	70.629	2.616
3 =	0.1831	183.070	105.943	3.924
4 =	0.2441	244.093	141.258	5.232
5 =	0.3051	305.117	176.572	6.540
6 =	0.3661	366.140	211.887	7.848
7 =	0.4272	427.163	247.201	9.156
8 =	0.4882	488.187	282.516	10.464
9 =	0.5492	549.210	317.830	11.771

CAPACITY.

	Millilitres or Cubic Centilitres to Fluid Drachms.	Centilitres to Fluid Ounces.	Litres to Quarts.	Dekalitres to Gallons.	Hektolitres to Bushels.
1 =	0.27	0.338	1.0567	2.6417	2.8375
2 =	0.54	0.676	2.1134	5.2834	5.6750
3 =	0.81	1.014	3.1700	7.9251	8.5125
4 =	1.08	1.352	4.2267	10.5668	11.3500
5 =	1.35	1.691	5.2834	13.2085	14.1875
6 =	1.62	2.029	6.3401	15.8502	17.0250
7 =	1.89	2.368	7.3968	18.4919	19.8625
8 =	2.16	2.706	8.4534	21.1336	22.7000
9 =	2.43	3.043	9.5101	23.7753	25.5375

WEIGHT.

	Milligrammes to Grains.	Kilogrammes to Grains.	Hectogrammes (100 grammes) to Ounces Av.	Kilogrammes to Pounds Avoirdupois.
1 =	0.01543	15432.36	3.5274	2.20462
2 =	0.03086	30864.71	7.0548	4.40924
3 =	0.04630	46297.07	10.5822	6.61386
4 =	0.06173	61729.43	14.1096	8.81849
5 =	0.07716	77161.78	17.6370	11.02311
6 =	0.09259	92594.14	21.1644	13.22773
7 =	0.10803	108026.49	24.6918	15.43235
8 =	0.12346	123458.85	28.2192	17.63697
9 =	0.13889	138891.21	31.7466	19.84159

WEIGHT—(*Continued*).

	Quintals to Pounds Av.	Milliers or Tonnes to Pounds Av.	Grammes to Ounces, Troy.
1 =	220.46	2204.6	0.03215
2 =	440.92	4409.2	0.06430
3 =	661.38	6613.8	0.09645
4 =	881.84	8818.4	0.12860
5 =	1102.30	11023.0	0.16075
6 =	1322.76	13227.6	0.19290
7 =	1543.22	15432.2	0.22505
8 =	1763.68	17636.8	0.25721
9 =	1984.14	19841.4	0.28936

The only authorized material standard of customary length is the Troughton scale belonging to this office, whose length at 59°.62 Fahr. conforms to the British standard. The yard in use in the United States is therefore equal to the British yard.

The only authorized material standard of customary weight is the Troy pound of the mint. It is of brass of unknown density, and therefore not suitable for a standard of mass. It was derived from the British standard Troy pound of 1758 by direct comparison. The British Avoirdupois pound was also derived from the latter, and contains 7000 grains Troy.

The grain Troy is therefore the same as the grain Avoirdupois, and the pound Avoirdupois in use in the United States is equal to the British pound Avoirdupois.

The metric system was legalized in the United States in 1866.

By the concurrent action of the principal governments of the world an International Bureau of Weights and Measures has been established near Paris.

The International Standard Metre is derived from the Mètre des Archives, and its length is defined by the distance between two lines at 0° Centigrade, on a platinum-iridium bar deposited at the International Bureau.

The International Standard Kilogramme is a mass of platinum-iridium deposited at the same place, and its weight *in vacuo* is the same as that of the Kilogramme des Archives.

Copies of these international standards are deposited in the office of standard weights and measures of the U. S. Coast and Geodetic Survey.

The litre is equal to a cubic decimetre of water, and it is measured by the quantity of distilled water which, at its maximum density, will counterpoise the standard kilogramme in a vacuum; the volume of such a quantity of water being, as nearly as has been ascertained, equal to a cubic decimetre.

COMPOUND UNITS.

Measures of Pressure and Weight.

1 lb. per square inch.	=	144 lbs. per square foot.
		2.0355 ins. of mercury at 32° F.
		2.0416 " " " " 62° F.
		2.309 ft. of water at 62° F.
		27.71 ins. " " " 62° F.
1 atmosphere (14.7 lbs. per sq. in.).	=	2116.3 lbs. per square foot.
		33.947 ft. of water at 62° F.
		30 ins. of mercury at 62° F.
		29.922 ins. of mercury at 32° F.
		760 millimetres of mercury at 32° F.
1 inch of water at 62° F.	=	.0361 lb. per square inch.
		5.196 lbs. " " foot.
		.0736 in. of mercury at 62° F.
1 inch of water at 32° F.	=	5.2021 lbs. per square foot.
		.036125 lbs. per " inch.
1 foot of water at 62° F.	=	.433 lb. per square inch.
		62.355 lbs. " " foot.
		.883 in. of mercury at 62° F.
1 inch of mercury at 62° F.	=	.49 lb. per square inch.
		70.56 lbs. " " foot.
		1.132 ft. of water at 62° F.
		13.58 ins. " " " 62° F.

Weight of One Cubic Foot of Pure Water.

At 32° F. (freezing-point)	62.418 lbs.
" 39.1° F. (maximum density)	62.425 "
" 62° F. (standard temperature)	62.355 "
" 212° F. (boiling-point, under 1 atmosphere)	59.76 "

American gallon = 231 cubic ins. of water at 62° F. = 8.3356 lbs.
British " = 277.274 " " " " " " = 10 lbs.

Measures of Work, Power, and Duty.

Work.—The sustained exertion of pressure through space.

Unit of work.—One foot-pound, i.e., a pressure of one pound exerted through a space of one foot.

Horse-power.—The rate of work. Unit of horse-power = 33,000 ft.-lbs. per minute, or 550 ft.-lbs. per second = 1,980,000 ft.-lbs. per hour.

Heat unit = heat required to raise 1 lb. of water 1° F. (from 39° to 40°).

Horse-power expressed in heat units = $\frac{33000}{778}$ = 42.416 heat units per minute = .707 heat unit per second = 2545 heat units per hour.

1 lb. of fuel per H. P. per hour = { 1,980,000 ft.-lbs. per lb. of fuel.
2,545 heat units " " }

1,000,000 ft.-lbs. per lb. of fuel = 1.98 lbs. of fuel per H. P. per hour.

Velocity.—Feet per second = $\frac{5280}{3600} = \frac{22}{15} \times$ miles per hour.

Gross tons per mile = $\frac{1760}{2240} = \frac{11}{14}$ lbs. per yard (single rail.)

French and British Equivalents of Weight and Pressure per Unit of Area.

FRENCH.		BRITISH.
1 gramme per square millimetre	=	1.422 lbs. per square inch.
1 kilogramme per square "	=	1422.32 " " " "
1 " " " centimetre	=	14.223 " " " "
1.0335 kilogrammes per square centimetre (1 atmosphere)	=	14.7 " " " "

0.070308 kilogramme per square centimetre = 1 lb. per square inch.

WIRE AND SHEET-METAL GAUGES COMPARED.

Number of Gauge.	Birmingham (or Stubs' Iron) Wire Gauge.	American or Brown and Sharpe Gauge.	Roebling's and Washburn & Moen's Gauge.	Stubs' Steel Wire Gauge. (See also p. 29.)	British Imperial Standard Wire Gauge. (Legal Standard in Great Britain since March 1, 1884.)		U. S. Standard Gauge for Sheet and Plate Iron and Steel. (Legal Standard since July 1, 1893.)	Number of Gauge.
	inch.	inch.	inch.	inch.	inch.	millim.	inch.	
0000000			.49		.500	12.7	.5	7/0
000000			.46		.464	11.78	.469	6/0
00000			.43		.432	10.97	.438	5/0
0000	.454	.46	.393		.4	10.16	.406	4/0
000	.425	.40964	.362		.372	9.45	.375	3/0
00	.38	.3648	.331		.348	8.84	.344	2/0
0	.34	.32486	.307		.324	8.23	.313	0
1	.3	.2893	.283	.227	.3	7.62	.281	1
2	.284	.25763	.263	.219	.276	7.01	.266	2
3	.259	.22942	.244	.212	.252	6.4	.25	3
4	.238	.20431	.225	.207	.232	5.89	.234	4
5	.22	.18194	.207	.204	.212	5.38	.219	5
6	.203	.16202	.192	.201	.192	4.88	.203	6
7	.18	.14428	.177	.199	.176	4.47	.188	7
8	.165	.12849	.162	.197	.16	4.06	.172	8
9	.148	.11443	.148	.194	.144	3.66	.156	9
10	.134	.10189	.135	.191	.128	3.26	.141	10
11	.12	.09074	.12	.188	.116	2.95	.125	11
12	.109	.08081	.105	.185	.104	2.64	.109	12
13	.095	.07196	.092	.182	.092	2.34	.094	13
14	.083	.06408	.08	.180	.08	2.03	.078	14
15	.072	.05707	.072	.178	.072	1.83	.07	15
16	.065	.05082	.063	.175	.064	1.63	.0625	16
17	.058	.04526	.054	.172	.056	1.42	.0563	17
18	.049	.0403	.047	.168	.048	1.22	.05	18
19	.042	.03589	.041	.164	.04	1.01	.0438	19
20	.035	.03196	.035	.161	.036	.91	.0375	20
21	.032	.02846	.032	.157	.032	.81	.0344	21
22	.028	.02535	.028	.155	.028	.71	.0313	22
23	.025	.02257	.025	.153	.024	.61	.0281	23
24	.022	.0201	.023	.151	.022	.56	.025	24
25	.02	.0179	.02	.148	.02	.51	.0219	25
26	.018	.01594	.018	.146	.018	.45	.0188	26
27	.016	.01419	.017	.143	.0164	.42	.0172	27
28	.014	.01264	.016	.139	.0148	.38	.0156	28
29	.013	.01126	.015	.134	.0136	.35	.0141	29
30	.012	.01003	.014	.127	.0124	.31	.0125	30
31	.01	.00893	.0135	.120	.0116	.29	.0109	31
32	.009	.00795	.013	.115	.0108	.27	.0101	32
33	.008	.00708	.011	.112	.01	.25	.0094	33
34	.007	.0063	.01	.110	.0092	.23	.0086	34
35	.005	.00561	.0095	.108	.0084	.21	.0078	35
36	004	.005	.009	.106	.0076	.19	.007	36
37		.00445	.0085	.103	.0068	.17	.0066	37
38		.00396	.008	.101	.006	.15	.0063	38
39		.00353	.0075	.099	.0052	.13		39
40		.00314	.007	.097	.0048	.12		40
41				.095	.0044	.11		41
42				.092	.004	.10		42
43				.088	.0036	.09		43
44				.085	.0032	.08		44
45				.081	.0028	.07		45
46				.079	.0024	.06		46
47				.077	.002	.05		47
48				.075	.0016	.04		48
49				.072	.0012	.03		49
50				.069	.001	.025		50

EDISON, OR CIRCULAR MIL GAUGE, FOR ELECTRICAL WIRES.

Gauge Number.	Circular Mils.	Diameter in Mils.	Gauge Number.	Circular Mils.	Diameter in Mils.	Gauge Number.	Circular Mils.	Diameter in Mils.
3	3,000	54.78	70	70,000	264.58	190	190,000	435.89
5	5,000	70.72	75	75,000	273.87	200	200,000	447.22
8	8,000	89.45	80	80,000	282.85	220	220,000	469.05
12	12,000	109.55	85	85,000	291.55	240	240,000	489.90
15	15,000	122.48	90	90,000	300.00	260	260,000	509.91
20	20,000	141.43	95	95,000	308.23	280	280,000	529.16
25	25,000	158.12	100	100,000	316.23	300	300,000	547.78
30	30,000	173.21	110	110,000	331.67	320	320,000	565.69
35	35,000	187.09	120	120,000	346.42	340	340,000	583.10
40	40,000	200.00	130	130,000	360.56	360	360,000	600.00
45	45,000	212.14	140	140,000	374.17			
50	50,000	223.61	150	150,000	387.30			
55	55,000	234.53	160	160,000	400.00			
60	60,000	244.95	170	170,000	412.32			
65	65,000	254.96	180	180,000	424.27			

TWIST DRILL AND STEEL WIRE GAUGE.

(Morse Twist Drill and Machine Co.)

No.	Size.	No.	Size.	No.	Size.	No.	Size.	No.	Size.	No.	Size.
	inch.		inch		inch.		inch.		inch.		inch.
1	.2280	11	.1910	21	.1590	31	.1200	41	.0960	51	.0670
2	.2210	12	.1890	22	.1570	32	.1160	42	.0935	52	.0635
3	.2130	13	.1850	23	.1540	33	.1130	43	.0890	53	.0595
4	.2090	14	.1820	24	.1520	34	.1110	44	.0860	54	.0550
5	.2055	15	.1800	25	.1495	35	.1100	45	.0820	55	.0520
6	.2040	16	.1770	26	.1470	36	.1065	46	.0810	56	.0465
7	.2010	17	.1730	27	.1440	37	.1040	47	.0785	57	.0430
8	.1990	18	.1695	28	.1405	38	.1015	48	.0760	58	.0420
9	.1960	19	.1660	29	.1360	39	.0995	49	.0730	59	.0410
10	.1935	20	.1610	30	.1285	40	.0980	50	.0700	60	.0400

STUBS' STEEL WIRE GAUGE.

(For Nos. 1 to 50 see table on page 28.)

No.	Size.	No.	Size.	No.	Size.	No.	Size.	No.	Size.	No.	Size.
	inch.		inch.		inch.		inch.		inch		inch.
Z	.413	P	.323	F	.257	51	.066	61	.038	71	.026
Y	.404	O	.316	E	.250	52	.063	62	.037	72	.024
X	.397	N	.302	D	.246	53	.058	63	.036	73	.023
W	.386	M	.295	C	.242	54	.055	64	.035	74	.022
V	.377	L	.290	B	.238	55	.050	65	.033	75	.020
U	.368	K	.281	A	.234	56	.045	66	.032	76	.018
T	.358	J	.277	1	See	57	.042	67	.031	77	.016
S	.348	I	.272	to	page	58	.041	68	.030	78	.015
R	.339	H	.266	50	28	59	.040	69	.029	79	.014
Q	.332	G	.261			60	.039	70	.027	80	.013

The Stubs' Steel Wire Gauge is used in measuring drawn steel wire or drill rods of Stubs' make, and is also used by many makers of American drill rods.

THE EDISON OR CIRCULAR MIL WIRE GAUGE.

(For table of copper wires by this gauge, giving weights, electrical resistances, etc., see Copper Wire.)

Mr. C. J. Field (*Stevens Indicator*, July, 1887) thus describes the origin of the Edison gauge:

The Edison company experienced inconvenience and loss by not having a wide enough range nor sufficient number of sizes in the existing gauges. This was felt more particularly in the central-station work in making electrical determinations for the street system. They were compelled to make use of two of the existing gauges at least, thereby introducing a complication that was liable to lead to mistakes by the contractors and linemen.

In the incandescent system an even distribution throughout the entire system and a uniform pressure at the point of delivery are obtained by calculating for a given maximum percentage of loss from the potential as delivered from the dynamo. In carrying this out, on account of lack of regular sizes, it was often necessary to use larger sizes than the occasion demanded, and even to assume new sizes for large underground conductors. It was also found that nearly all manufacturers based their calculation for the conductivity of their wire on a variety of units, and that not one used the latest unit as adopted by the British Association and determined from Dr. Matthiessen's experiments; and as this was the unit employed in the manufacture of the Edison lamps, there was a further reason for constructing a new gauge. The engineering department of the Edison company, knowing the requirements, have designed a gauge that has the widest range obtainable and a large number of sizes which increase in a regular and uniform manner. The basis of the graduation is the sectional area, and the number of the wire corresponds. A wire of 100,000 circular mils area is No. 100; a wire of one half the size will be No. 50; twice the size No. 200.

In the older gauges, as the number increased the size decreased. With this gauge, however, the number increases with the wire, and the number multiplied by 1000 will give the circular mils.

The weight per mil-foot, 0.00000302705 pounds, agrees with a specific gravity of 8.889, which is the latest figure given for copper. The ampere capacity which is given was deduced from experiments made in the company's laboratory, and is based on a rise of temperature of 50° F. in the wire.

In 1893 Mr. Field writes, concerning gauges in use by electrical engineers:

The B. and S. gauge seems to be in general use for the smaller sizes, up to 100,000 c. m., and in some cases a little larger. From between one and two hundred thousand circular mils upwards, the Edison gauge or its equivalent is practically in use, and there is a general tendency to designate all sizes above this in circular mils, specifying a wire as 200,000, 400,000, 500,000, or 1,000,000 c. m.

In the electrical business there is a large use of copper wire and rod and other materials of these large sizes, and in ordering them, speaking of them, specifying, and in every other use, the general method is to simply specify the circular milage. I think it is going to be the only system in the future for the designation of wires, and the attaining of it means practically the adoption of the Edison gauge or the method and basis of this gauge as the correct one for wire sizes.

THE U. S. STANDARD GAUGE FOR SHEET AND PLATE IRON AND STEEL, 1893.

There is in this country no uniform or standard gauge, and the same numbers in different gauges represent different thicknesses of sheets or plates. This has given rise to much misunderstanding and friction between employers and workmen and mistakes and fraud between dealers and consumers.

An Act of Congress in 1893 established the Standard Gauge for sheet iron and steel which is given on the next page. It is based on the fact that a cubic foot of iron weighs 480 pounds.

A sheet of iron 1 foot square and 1 inch thick weighs 40 pounds, or 640 ounces, and 1 ounce in weight should be 1/640 inch thick. The scale has been arranged so that each descriptive number represents a certain number of ounces in weight and an equal number of 640ths of an inch in thickness.

The law enacts that on and after July 1, 1893, the new gauge shall be used in determining duties and taxes levied on sheet and plate iron and steel; and that in its application a variation of 2½ per cent either way may be allowed.

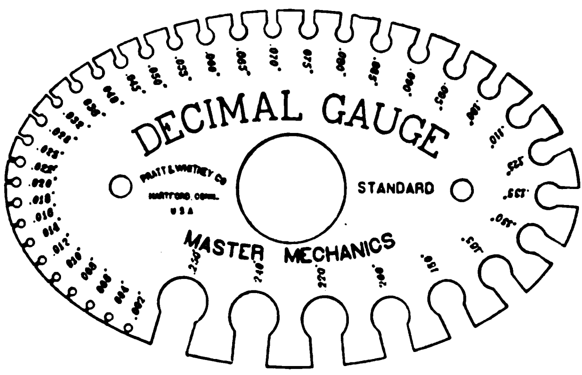
U. S. STANDARD GAUGE FOR SHEET AND PLATE IRON AND STEEL, 1893.

Number of Gauge.	Approximate Thickness in Fractions of an Inch.	Approximate Thickness in Decimal Parts of an Inch.	Approximate Thickness in Millimeters.	Weight per Square Foot in Ounces Avoirdupois.	Weight per Square Foot in Pounds Avoirdupois.	Weight per Square Foot in Kilograms.	Weight per Square Meter in Kilograms.	Weight per Square Meter in Pounds Avoirdupois.
0000000	1-2	0.5	12.7	320	20.	9.072	97.65	215.28
000000	15-32	0.46875	11.90625	300	18.75	8.505	91.55	201.82
00000	7-16	0.4375	11.1125	280	17.50	7.938	85.44	188.37
0000	13-32	0.40625	10.31875	260	16.25	7.371	79.33	174.91
000	3-8	0.375	9.525	240	15.	6.804	73.24	161.46
00	11-32	0.34375	8.73125	220	13.75	6.237	67.13	148.00
0	5-16	0.3125	7.9375	200	12.50	5.67	61.03	134.55
1	9-32	0.28125	7.14375	180	11.25	5.103	54.93	121.09
2	17-64	0.265625	6.746875	170	10.625	4.819	51.88	114.37
3	1-4	0.25	6.35	160	10.	4.536	48.82	107.64
4	15-64	0.234375	5.953125	150	9.375	4.252	45.77	100.91
5	7-32	0.21875	5.55625	140	8.75	3.969	42.72	94.18
6	13-64	0.203125	5.159375	130	8.125	3.685	39.67	87.45
7	3-16	0.1875	4.7625	120	7.5	3.402	36.62	80.72
8	11-64	0.171875	4.365625	110	6.875	3.118	33.57	74.00
9	5-32	0.15625	3.96875	100	6.25	2.835	30.52	67.27
10	9-64	0.140625	3.571875	90	5.625	2.552	27.46	60.55
11	1-8	0.125	3.175	80	5.	2.268	24.41	53.82
12	7-64	0.109375	2.778125	70	4.375	1.984	21.36	47.09
13	3-32	0.09375	2.38125	60	3.75	1.701	18.31	40.36
14	5-64	0.078125	1.984375	50	3.125	1.417	15.26	33.64
15	9-128	0.0703125	1.7859375	45	2.8125	1.276	13.73	30.27
16	1-16	0.0625	1.5875	40	2.5	1.134	12.21	26.91
17	9-160	0.05625	1.42875	36	2.25	1.021	10.99	24.22
18	1-20	0.05	1.27	32	2.	0.9072	9.765	21.53
19	7-160	0.04375	1.11125	28	1.75	0.7938	8.544	18.84
20	3-80	0.0375	0.9525	24	1.50	0.6804	7.324	16.15
21	11-320	0.034375	0.873125	22	1.375	0.6237	6.713	14.80
22	1-32	0.03125	0.793750	20	1.25	0.567	6.103	13.46
23	9-320	0.028125	0.714375	18	1.125	0.5103	5.493	12.11
24	1-40	0.025	0.635	16	1.	0.4536	4.882	10.76
25	7-320	0.021875	0.555625	14	0.875	0.3969	4.272	9.42
26	3-160	0.01875	0.47625	12	0.75	0.3402	3.662	8.07
27	11-640	0.0171875	0.4365625	11	0.6875	0.3119	3.357	7.40
28	1-64	0.015625	0.396875	10	0.625	0.2835	3.052	6.73
29	9-640	0.0140625	0.3571875	9	0.5625	0.2551	2.746	6.05
30	1-80	0.0125	0.3175	8	0.5	0.2268	2.441	5.38
31	7-640	0.0109375	0.2778125	7	0.4375	0.1984	2.136	4.71
32	13-1280	0.01015625	0.25796875	6½	0.40625	0.1843	1.983	4.37
33	3-320	0.009375	0.238125	6	0.375	0.1701	1.831	4.04
34	11-1280	0.00859375	0.21828125	5½	0.34375	0.1559	1.678	3.70
35	5-640	0.0078125	0.1984375	5	0.3125	0.1417	1.526	3.36
36	9-1280	0.00703125	0.17859375	4½	0.28125	0.1276	1.373	3.03
37	17-2560	0.006640625	0.168671875	4¼	0.265625	0.1205	1.297	2.87
38	1-160	0.00625	0.15875	4	0.25	0.1134	1.221	2.69

The Decimal Gauge.—The legalization of the standard sheet-metal gauge of 1893 and its adoption by some manufacturers of sheet iron have only added to the existing confusion of gauges. A joint committee of the American Society of Mechanical Engineers and the American Railway Master Mechanics' Association in 1895 agreed to recommend the use of the decimal gauge, that is, a gauge whose number for each thickness is the number of thousandths of an inch in that thickness, and also to recommend "the abandonment and disuse of the various other gauges now in use, as tending to confusion and error." A notched gauge of oval form, as shown in the cut below, has come into general use as a standard form of the decimal gauge, but for accurate measurement its indications should be checked by the use of a micrometer gauge reading to thousandths of an inch.

Weight of Sheet Iron and Steel. Thickness by Decimal Gauge.

Decimal Gauge.	Approx. Fractions of an Inch.	Approx. Millimetres.	Weight per Square Foot in Pounds.		Decimal Gauge.	Approx. Fractions of an Inch.	Approx. Millimetres.	Weight per Square Foot in Pounds.	
			Iron, 480 Lbs. per Cu. Ft.	Steel, 489.6 Lbs. per Cu. Ft.				Iron, 480 Lbs. per Cu. Ft.	Steel, 489.6 Lbs. per Cu. Ft.
0.002	1/500	0.05	0.08	0.082	0.060	1/16 −	1.52	2.40	2.448
0.004	1/250	0.10	0.16	0.163	0.065	13/200	1.65	2.60	2.652
0.006	3/500	0.15	0.24	0.245	0.070	7/100	1.78	2.80	2.856
0.008	1/125	0.20	0.32	0.326	0.075	3/40	1.90	3.00	3.060
0.010	1/100	0.25	0.40	0.408	0.080	2/25	2.03	3.20	3.264
0.012	3/250	0.30	0.48	0.490	0.085	17/200	2.16	3.40	3.468
0.014	7/500	0.36	0.56	0.571	0.090	9/100	2.28	3.60	3.672
0.016	1/64 +	0.41	0.64	0.653	0.095	19/200	2.41	3.80	3.876
0.018	9/500	0.46	0.72	0.734	0.100	1/10	2.54	4.00	4.080
0.020	1/50	0.51	0.80	0.816	0.110	11/100	2.79	4.40	4.488
0.022	11/500	0.56	0.88	0.898	0.125	1/8	3.18	5.00	5.100
0.025	1/40	0.64	1.00	1.020	0.135	27/200	3.43	5.40	5.508
0.028	7/250	0.71	1.12	1.142	0.150	3/20	3.81	6.00	6.120
0.032	1/32 +	0.81	1.28	1.306	0.165	33/200	4.19	6.60	6.732
0.036	9/250	0.91	1.44	1.469	0.180	9/50	4.57	7.20	7.344
0.040	1/25	1.02	1.60	1.632	0.200	1/5	5.08	8.00	8.160
0.045	9/200	1.14	1.80	1.836	0.220	11/50	5.59	8.80	8.976
0.050	1/20	1.27	2.00	2.040	0.240	6/25	6.10	9.60	9.792
0.055	11/200	1.40	2.20	2.244	0.250	1/4	6.35	10.00	10.200



ALGEBRA.

Addition.—Add a and b. Ans. $a + b$. Add a, b, and $-c$. Ans. $a + b - c$. Add $2a$ and $-3a$. Ans. $-a$. Add $2ab, -3ab, -c, -3c$. Ans. $-ab - 4c$.

Subtraction.—Subtract a from b. Ans. $b - a$. Subtract $-a$ from $-b$. Ans. $-b + a$.

Subtract $b + c$ from a. Ans. $a - b - c$. Subtract $3a^2b - 9c$ from $4a^2b + c$. Ans. $a^2b + 10c$. RULE: Change the signs of the subtrahend and proceed as in addition.

Multiplication.—Multiply a by b. Ans. ab. Multiply ab by $a + b$. Ans. $a^2b + ab^2$.

Multiply $a + b$ by $a + b$. Ans. $(a + b)(a + b) = a^2 + 2ab + b^2$.

Multiply $-a$ by $-b$. Ans. ab. Multiply $-a$ by b. Ans. $-ab$. Like signs give plus, unlike signs minus.

Powers of numbers.—The product of two or more powers of any number is the number with an exponent equal to the sum of the powers: $a^2 \times a^3 = a^5$; $a^2b^2 \times ab = a^3b^3$; $-7ab \times 2ac = -14\,a^2bc$.

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial and add the partial products: $(6a - 3b) \times 3c = 18ac - 9bc$.

To multiply two polynomials, multiply each term of one factor by each term of the other and add the partial products: $(5a - 6b) \times (3a - 4b) = 15a^2 - 38ab + 24b^2$.

The square of the sum of two numbers = sum of their squares + twice their product.

The square of the difference of two numbers = the sum of their squares − twice their product.

The product of the sum and difference of two numbers = the difference of their squares:

$$(a + b)^2 = a^2 + 2ab + b^2; \quad (a - b)^2 = a^2 - 2ab + b^2;$$
$$(a + b) \times (a - b) = a^2 - b^2.$$

The square of half the sums of two quantities is equal to their product plus the square of half their difference: $\left(\frac{a + b}{2}\right)^2 = ab + \left(\frac{a - b}{2}\right)^2$.

The square of the sum of two quantities is equal to four times their products, plus the square of their difference: $(a + b)^2 = 4ab + (a - b)^2$

The sum of the squares of two quantities equals twice their product, plus the square of their difference: $a^2 + b^2 = 2ab + (a - b)^2$.

The square of a trinomial = the square of each term + twice the product of each term by each of the terms that follow it: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$; $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$.

The square of (any number + ½) = square of the number + the number + ¼; = the number × (the number + 1) + ¼; $(a + \frac{1}{2})^2 = a^2 + a + \frac{1}{4}$, $= a(a + 1) + \frac{1}{4}$. $(4\frac{1}{2})^2 = 4^2 + 4 + \frac{1}{4} = 4 \times 5 + \frac{1}{4} = 20\frac{1}{4}$.

The product of any number + ½ by any other number + ½ = product of the numbers + half their sum + ¼. $(a + \frac{1}{2}) \times (b + \frac{1}{2}) = ab + \frac{1}{2}(a + b) + \frac{1}{4}$. $4\frac{1}{2} \times 6\frac{1}{2} = 4 \times 6 + \frac{1}{2}(4 + 6) + \frac{1}{4} = 24 + 5 + \frac{1}{4} = 29\frac{1}{4}$.

Square, cube, 4th power, etc., of a binomial $a + b$.

$$(a + b)^2 = a^2 + 2ab + b^2; \quad (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3;$$
$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$$

In each case the number of terms is one greater than the exponent of the power to which the binomial is raised.

2. In the first term the exponent of a is the same as the exponent of the power to which the binomial is raised, and it decreases by 1 in each succeeding term.

3. b appears in the second term with the exponent 1, and its exponent increases by 1 in each succeeding term.

4. The coefficient of the first term is 1.

5. The coefficient of the second term is the exponent of the power to which the binomial is raised.

6. The coefficient of each succeeding term is found from the next preceding term by multiplying its coefficient by the exponent of a, and dividing the product by a number greater by 1 than the exponent of b. (See Binomial Theorem, below.)

Parentheses.—When a parenthesis is preceded by a plus sign it may be removed without changing the value of the expression: $a + b + (a + b) = 2a + 2b$. When a parenthesis is preceded by a minus sign it may be removed if we change the signs of all the terms within the parenthesis: $1 - (a - b - c) = 1 - a + b + c$. When a parenthesis is within a parenthesis remove the inner one first: $a - \left[b - \left\{c - (d - e)\right\}\right] = a - \left[b - \left\{c - d + e\right\}\right] = a - [b - c + d - e] = a - b + c - d + e$.

A multiplication sign, $\times$, has the effect of a parenthesis, in that the operation indicated by it must be performed before the operations of addition or subtraction. $a + b \times a + b = a + ab + b$; while $(a + b) \times (a + b) = a^2 + 2ab + b^2$, and $(a + b) \times a + b = a^2 + ab + b$.

Division.—The quotient is positive when the dividend and divisor have like signs, and negative when they have unlike signs: $abc \div b = ac$; $abc \div -b = -ac$.

To divide a monomial by a monomial, write the dividend over the divisor with a line between them. If the expressions have common factors, remove the common factors:

$$a^2bx \div aby = \frac{a^2bx}{aby} = \frac{ax}{y}; \quad \frac{a^4}{a^3} = a; \quad \frac{a^3}{a^5} = \frac{1}{a^2} = a^{-2}.$$

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial: $(8ab - 12ac) \div 4a = 2b - 3c$.

To divide a polynomial by a polynomial, arrange both dividend and divisor in the order of the ascending or descending powers of some common letter, and keep this arrangement throughout the operation.

Divide the first term of the dividend by the first term of the divisor, and write the result as the first term of the quotient.

Multiply all the terms of the divisor by the first term of the quotient and subtract the product from the dividend. If there be a remainder, consider it as a new dividend and proceed as before: $(a^2 - b^2) \div (a + b)$.

$$\begin{array}{l|l} a^2 - b^2 & a + b. \\ \underline{a^2 + ab} & \underline{a - b.} \\ \quad - ab - b^2. & \\ \quad \underline{- ab - b^2.} & \end{array}$$

The difference of two equal odd powers of any two numbers is divisible by their difference and also by their sum:

$(a^3 - b^3) \div (a - b) = a^2 + ab + b^2$; $(a^3 - b^3) \div (a + b) = a^2 - ab + b^2$.

The difference of two equal even powers of two numbers is divisible by their difference and also by their sum: $(a^2 - b^2) \div (a - b) = a + b$.

The sum of two equal even powers of two numbers is not divisible by either the difference or the sum of the numbers; but when the exponent of each of the two equal powers is composed of an odd and an even factor, the sum of the given power is divisible by the sum of the powers expressed by the even factor. Thus $x^6 + y^6$ is not divisible by $x + y$ or by $x - y$, but is divisible by $x^2 + y^2$.

Simple equations.—An equation is a statement of equality between two expressions; as, $a + b = c + d$.

A simple equation, or equation of the first degree, is one which contains only the first power of the unknown quantity. If equal changes be made (by addition, subtraction, multiplication, or division) in both sides of an equation, the results will be equal.

Any term may be changed from one side of an equation to another, provided its sign be changed: $a + b = c + d$; $a = c + d - b$. To solve an equation having one unknown quantity, transpose all the terms involving the unknown quantity to one side of the equation, and all the other terms to the other side; combine like terms, and divide both sides by the coefficient of the unknown quantity.

Solve $8x - 29 = 26 - 3x$. $8x + 3x = 29 + 26$; $11x = 55$; $x = 5$, ans.

Simple algebraic problems containing one unknown quantity are solved by making $x =$ the unknown quantity, and stating the conditions of the problem in the form of an algebraic equation, and then solving the equation. What two numbers are those whose sum is 48 and difference 14? Let $x =$ the smaller number, $x + 14$ the greater. $x + x + 14 = 48$. $2x = 34$, $x = 17$; $x + 14 = 31$, ans.

Find a number whose treble exceeds 50 as much as its double falls short of 40. Let $x =$ the number. $3x - 50 = 40 - 2x$; $5x = 90$; $x = 18$, ans. Proving, $54 - 50 = 40 - 36$.

Equations containing two unknown quantities.—If one equation contains two unknown quantities, x and y, an indefinite number of pairs of values of x and y may be found that will satisfy the equation, but if a second equation be given only one pair of values can be found that will satisfy both equations. Simultaneous equations, or those that may be satisfied by the same values of the unknown quantities, are solved by combining the equations so as to obtain a single equation containing only one unknown quantity. This process is called elimination.

Elimination by addition or subtraction.—Multiply the equation by such numbers as will make the coefficients of one of the unknown quantities equal in the resulting equation. Add or subtract the resulting equations according as they have unlike or like signs.

Solve $\begin{cases} 2x + 3y = 7. \\ 4x - 5y = 3. \end{cases}$ Multiply by 2: $4x + 6y = 14$; Subtract: $\underline{4x - 5y = 3}$ $\quad 11y = 11;\ y = 1.$

Substituting value of y in first equation, $2x + 3 = 7;\ x = 2.$

Elimination by substitution.—From one of the equations obtain the value of one of the unknown quantities in terms of the other. Substitute for this unknown quantity its value in the other equation and reduce the resulting equations.

Solve $\begin{cases} 2x + 3y = 8. & (1). \\ 3x + 7y = 7. & (2). \end{cases}$ From (1) we find $x = \dfrac{8 - 3y}{2}$.

Substitute this value in (2): $3\left(\dfrac{8 - 3y}{2}\right) + 7y = 7;\ = 24 - 9y + 14y = 14,$ whence $y = -2$. Substitute this value in (1): $2x - 6 = 8;\ x = 7.$

Elimination by comparison.—From each equation obtain the value of one of the unknown quantities in terms of the other. Form an equation from these equal values, and reduce this equation.

Solve $\begin{cases} 2x - 9y = 11. \quad (1). \quad \text{From (1) we find } x = \dfrac{11 + 9y}{2}. \\ 3x - 4y = 7. \quad (2). \quad \text{From (2) we find } x = \dfrac{7 + 4y}{3}. \end{cases}$

Equating these values of x, $\dfrac{11 + 9y}{2} = \dfrac{7 + 4y}{3};\ 19y = -19;\ y = -1.$

Substitute this value of y in (1): $2x + 9 = 11;\ x = 1.$

If three simultaneous equations are given containing three unknown quantities, one of the unknown quantities must be eliminated between two pairs of the equations; then a second between the two resulting equations.

Quadratic equations.—A quadratic equation contains the square of the unknown quantity, but no higher power. A pure quadratic contains the square only; an affected quadratic both the square and the first power.

To solve a pure quadratic, collect the unknown quantities on one side, and the known quantities on the other; divide by the coefficient of the unknown quantity and extract the square root of each side of the resulting equation.

Solve $3x^2 - 15 = 0.\quad 3x^2 = 15;\ x^2 = 5;\ x = \sqrt{5}$

A root like $\sqrt{5}$, which is indicated, but which can be found only approximately, is called a *surd*.

Solve $3x^2 + 15 = 0.\quad 3x^2 = -15;\ x^2 = -5;\ x = \sqrt{-5}.$

The square root of -5 cannot be found even approximately, for the square of any number positive or negative is positive; therefore a root which is indicated, but cannot be found even approximately, is called *imaginary*.

To solve an affected quadratic.—1. Convert the equation into the form $a^2x^2 \pm 2abx = c$, multiplying or dividing the equation if necessary, so as to make the coefficient of x^2 a square number.

2. Complete the square of the first member of the equation, so as to convert it to the form of $a^2x^2 \pm 2abx + b^2$, which is the square of the binomial $ax \pm b$, as follows: add to each side of the equation the square of the quotient obtained by dividing the second term by twice the square root of the first term.

3. Extract the square root of each side of the resulting equation.

Solve $3x^2 - 4x = 32$. To make the coefficient of x^2 a square number, multiply by 3: $9x^2 - 12x = 96;\ 12x \div (2 \times 3x) = 2;\ 2^2 = 4.$

Complete the square: $9x^2 - 12x + 4 = 100$. Extract the root: $3x - 2 = \pm$

10, whence $x = 4$ or $-2\ 2/3$. The square root of 100 is either $+10$ or -10, since the square of -10 as well as $+10^2 = 100$.

Problems involving quadratic equations have apparently two solutions, as a quadratic has two roots. Sometimes both will be true solutions, but generally one only will be a solution and the other be inconsistent with the conditions of the problem.

The sum of the squares of two consecutive positive numbers is 481. Find the numbers.

Let x = one number, $x + 1$ the other. $x^2 + (x + 1)^2 = 481$. $2x^2 + 2x + 1 = 481$.

$x^2 + x = 240$. Completing the square, $x^2 + x + 0.25 = 240.25$. Extracting the root we obtain $x + 0.5 = \pm 15.5$; $x = 15$ or -16.

The positive root gives for the numbers 15 and 16. The negative root -16 is inconsistent with the conditions of the problem.

Quadratic equations containing two unknown quantities require different methods for their solution, according to the form of the equations. For these methods reference must be made to works on algebra.

Theory of exponents.—$\sqrt[n]{a}$ when n is a positive integer is one of n equal factors of a. $\sqrt[n]{a^m}$ means a is to be raised to the mth power and the nth root extracted.

$(\sqrt[n]{a})^m$ means that the nth root of a is to be taken and the result raised to the mth power.

$\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{\frac{m}{n}}$. When the exponent is a fraction, the numerator indicates a power, and the denominator a root. $a^{\frac{4}{2}} = \sqrt{a^4} = a^2$; $a^{\frac{3}{2}} = \sqrt{a^3} = a^{1.5}$.

To extract the root of a quantity raised to an indicated power, divide the exponent by the index of the required root; as,

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}; \qquad \sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2.$$

Subtracting 1 from the exponent of a is equivalent to dividing by a:

$$a^{2-1} = a^1 = a; \quad a^{1-1} = a^0 = \frac{a}{a} = 1; \quad a^{0-1} = a^{-1} = \frac{1}{a}; \quad a^{-1-1} = a^{-2} = \frac{1}{a^2}$$

A number with a negative exponent denotes the reciprocal of the number with the corresponding positive exponent.

A factor under the radical sign whose root can be taken may, by having the root taken, be removed from under the radical sign:

$$\sqrt{a^2b} = \sqrt{a^2} \times \sqrt{b} = a\sqrt{b}.$$

A factor outside the radical sign may be raised to the corresponding power and placed under it:

$$\sqrt{\frac{a}{b}} = \sqrt{\frac{ab}{b^2}} = \sqrt{ab \times \frac{1}{b^2}} = \frac{1}{b}\sqrt{ab}; \qquad \sqrt{\frac{a}{b^2}} = \frac{1}{b}\sqrt{a}$$

Binomial Theorem.—To obtain any power, as the nth, of an expression of the form $x + a$

$$(a + x)^n = a^n + na^{n-1}x + \frac{n(n-1)a^{n-2}}{1.2}x^2 + \frac{n(n-1)(n-2)a^{n-3}}{1.2.3}x^3 +$$

etc.

The following laws hold for any term in the expansion of $(a + x)^n$.

The exponent of x is less by one than the number of terms.

The exponent of a is n minus the exponent of x.

The last factor of the numerator is greater by one than the exponent of a.

The last factor of the denominator is the same as the exponent of x.

In the rth term the exponent of x will be $r - 1$.

The exponent of a will be $n - (r - 1)$, or $n - r + 1$.

The last factor of the numerator will be $n - r + 2$.

The last factor of the denominator will be $= r - 1$.

Hence the rth term $= \dfrac{n(n-1)(n-2)\ldots(n-r+2)}{1.2.3\ldots(r-1)}a^{n-r+1}x^{r-1}$

GEOMETRICAL PROBLEMS.

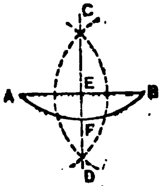


FIG. 1.

1. To bisect a straight line, or an arc of a circle (Fig. 1).—From the ends *A*, *B*, as centres, describe arcs intersecting at *C* and *D*, and draw a line through *C* and *D* which will bisect the line at *E* or the arc at *F*.

2. To draw a perpendicular to a straight line, or a radial line to a circular arc.—Same as in Problem 1. *C D* is perpendicular to the line *A B*, and also radial to the arc.

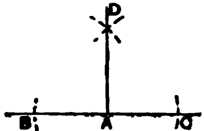


FIG. 2.

3. To draw a perpendicular to a straight line from a given point in that line (Fig. 2).—With any radius, from the given point *A* in the line *B C*, cut the line at *B* and *C*. With a longer radius describe arcs from *B* and *C*, cutting each other at *D*, and draw the perpendicular *D A*.

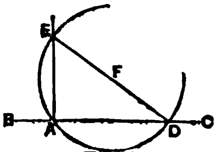


FIG. 3.

4. From the end *A* of a given line *A D* to erect a perpendicular *A E* (Fig. 3).—From any centre *F*, above *A D*, describe a circle passing through the given point *A*, and cutting the given line at *D*. Draw *D F* and produce it to cut the circle at *E*, and draw the perpendicular *A E*.

Second Method (Fig. 4).—From the given point *A* set off a distance *A E* equal to three parts, by any scale; and on the centres *A* and *E*, with radii of four and five parts respectively, describe arcs intersecting at *C*. Draw the perpendicular *A C*.

NOTE.—This method is most useful on very large scales, where straight edges are inapplicable. Any multiples of the numbers 3, 4, 5 may be taken with the same effect as 6, 8, 10, or 9, 12, 15.

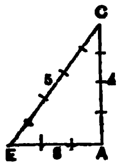


FIG. 4.

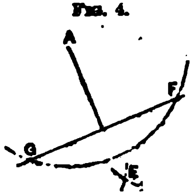


FIG. 5.

5. To draw a perpendicular to a straight line from any point without it (Fig. 5.)—From the point *A*, with a sufficient radius cut the given line at *F* and *G*, and from these points describe arcs cutting at *E*. Draw the perpendicular *A E*.

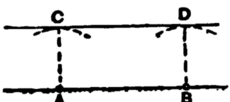


FIG. 6.

6. To draw a straight line parallel to a given line, at a given distance apart (Fig. 6).—From the centres *A*, *B*, in the given line, with the given distance as radius, describe arcs *C*, *D*, and draw the parallel lines *C D* touching the arcs.

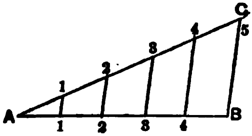


FIG. 7.

7. To divide a straight line into a number of equal parts (Fig. 7).—To divide the line *A B* into, say, five parts, draw the line *A C* at an angle from *A*; set off five equal parts; draw *B* 5 and draw parallels to it from the other points of division in *A C*. These parallels divide *A B* as required.

NOTE.—By a similar process a line may be divided into a number of unequal parts; setting off divisions on *A C*, proportional by a scale to the required divisions, and drawing parallel cutting *A B*. The triangles *A*11, *A*22, *A*33, etc., are *similar triangles.*

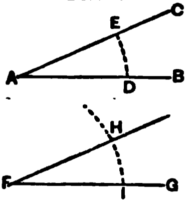


FIG. 8.

8. Upon a straight line to draw an angle equal to a given angle (Fig. 8).—Let *A* be the given angle and *F G* the line. From the point *A* with any radius describe the arc *D E*. From *F* with the same radius describe *I H*. Set off the arc *I H* equal to *D E*, and draw *F H*. The angle *F* is equal to *A*, as required.

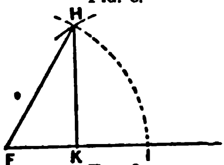


FIG. 9.

9. To draw angles of 60° and 30° (Fig. 9).—From *F*, with any radius *F I*, describe an arc *I H*; and from *I*, with the same radius, cut the arc at *H* and draw *F H* to form the required angle *I F H*. Draw the perpendicular *H K* to the base line to form the angle of 30° *F H K*.

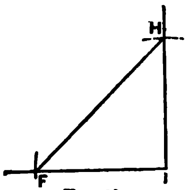


FIG. 10.

10. To draw an angle of 45° (Fig. 10).—Set off the distance *F I*; draw the perpendicular *I H* equal to *I F*, and join *H F* to form the angle at *F*. The angle at *H* is also 45°.

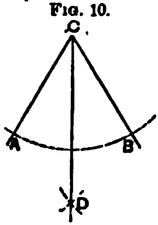


FIG. 11.

11. To bisect an angle (Fig. 11).—Let *A C B* be the angle; with *C* as a centre draw an arc cutting the sides at *A*, *B*. From *A* and *B* as centres, describe arcs cutting each other at *D*. Draw *C D*, dividing the angle into two equal parts.

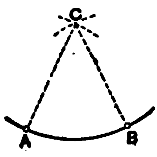


FIG. 12.

12. Through two given points to describe an arc of a circle with a given radius (Fig. 12).—From the points *A* and *B* as centres, with the given radius, describe arcs cutting at *C*; and from *C* with the same radius describe an arc *A B*.

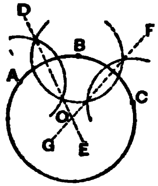


FIG. 13.

13. To find the centre of a circle or of an arc of a circle (Fig. 13).—Select three points, *A*, *B*, *C*, in the circumference, well apart; with the same radius describe arcs from these three points, cutting each other, and draw the two lines, *D E*, *F G*, through their intersections. The point *O*, where they cut, is the centre of the circle or arc.

To describe a circle passing through three given points. —Let *A*, *B*, *C* be the given points, and proceed as in last problem to find the centre *O*, from which the circle may be described.

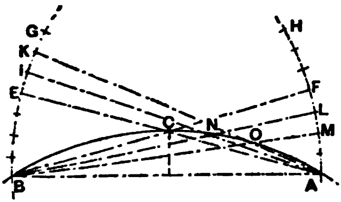


FIG. 14.

14. To describe an arc of a circle passing through three given points when the centre is not available (Fig. 14).—From the extreme points *A*, *B*, as centres, describe arcs *A H*, *B G*. Through the third point *C* draw *A E*, *B F*, cutting the arcs. Divide *A F* and *B E* into any number of equal parts, and set off a series of equal parts of the same length on the upper portions of the arcs beyond the points *E F*. Draw straight lines, *B L*, *B M*, etc., to the divisions in *A F*, and *A I*, *A K*, etc., to the divisions in *E G*. The successive intersections *N*, *O*, etc., of these li es are points in the circle required between the given points *A* and *C*, which may be drawn in; similarly the remaining part of the curve *B C* may be described. (See also Problem 54.)

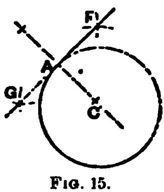


FIG. 15.

15. To draw a tangent to a circle from a given point in the circumference (Fig. 15). —Through the given point *A*, draw the radial line *A C*, and a perpendicular to it, *F G*, which is the tangent required.

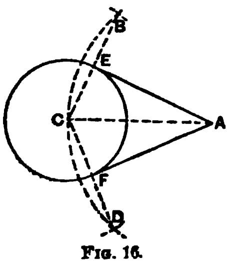


FIG. 16.

16. To draw tangents to a circle from a point without it (Fig. 16).—From *A*, with the radius *A C*, describe an arc *B C D*, and from *C*, with a radius equal to the diameter of the circle, cut the arc at *B D*. Join *B C*, *C D*, cutting the circle at *E F*, and draw *A E*, *A F*, the tangents.

NOTE.—When a tangent is already drawn, the exact point of contact may be found by drawing a perpendicular to it from the centre.

17. Between two inclined lines to draw a series of circles touching these lines and touching each other (Fig. 17). —Bisect the inclination of the given lines *A B*, *C D*, by the line *N O*. From a point *P* in this line draw the perpendicular *P B* to the line *A B*, and

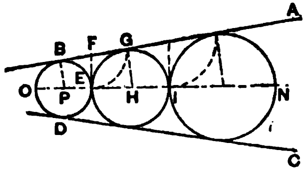


FIG. 17.

on P describe the circle $B\ D$, touching the lines and cutting the centre line at E. From E draw $E\ F$ perpendicular to the centre line, cutting $A\ B$ at F, and from F describe an arc $E\ G$, cutting $A\ B$ at G. Draw $G\ H$ parallel to $B\ P$, giving H, the centre of the next circle, to be described with the radius $H\ E$, and so on for the next circle $I\ N$.

Inversely, the largest circle may be described first, and the smaller ones in succession. This problem is of frequent use in scroll-work.

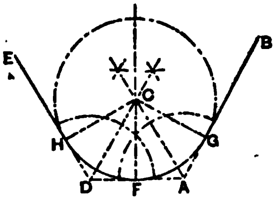


FIG. 18.

18. Between two inclined lines to draw a circular segment tangent to the lines and passing through a point F on the line $F\ C$ which bisects the angle of the lines (Fig. 18).—Through F draw $D\ A$ at right angles to $F\ C$; bisect the angles A and D, as in Problem 11, by lines cutting at C, and from C with radius $C\ F$ draw the arc $H\ F\ G$ required.

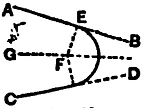


FIG. 19.

19. To draw a circular arc that will be tangent to two given lines $A\ B$ and $C\ D$ inclined to one another, one tangential point E being given (Fig. 19).—Draw the centre line $G\ F$. From E draw $E\ F$ at right to angles $A\ B$; then F is the centre of the circle required.

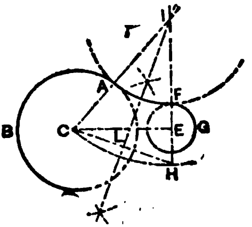


FIG. 20.

20. To describe a circular arc joining two circles, and touching one of them at a given point (Fig. 20).—To join the circles $A\ B$, $F\ G$, by an arc touching one of them at F, draw the radius $E\ F$, and produce it both ways. Set off $F\ H$ equal to the radius $A\ C$ of the other circle; join $C\ H$ and bisect it with the perpendicular $L\ I$, cutting $E\ F$ at I. On the centre I, with radius $I\ F$, describe the arc $F\ A$ as required.

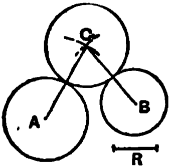


FIG. 21.

21. To draw a circle with a given radius R that will be tangent to two given circles A and B (Fig. 21).—From centre of circle A with radius equal R plus radius of A, and from centre of B with radius equal to R + radius of B, draw two arcs cutting each other in C, which will be the centre of the circle required.

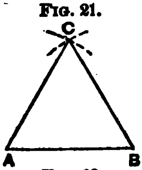


FIG. 22.

22. To construct an equilateral triangle, the sides being given (Fig. 22).—On the ends of one side, A, B, with $A\ B$ as radius, describe arcs cutting at C, and draw $A\ C$, $C\ B$.

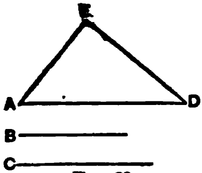


FIG. 23.

23. To construct a triangle of unequal sides (Fig. 23).—On either end of the base *A D*, with the side *B* as radius, describe an arc; and with the side *C* as radius, on the other end of the base as a centre, cut the arc at *E*. Join *A E*, *D E*.

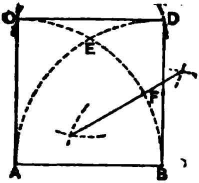


FIG. 24.

24. To construct a square on a given straight line *A B* (Fig. 24).—With *A B* as radius and *A* and *B* as centres, draw arcs *A D* and *B C*, intersecting at *E*. Bisect *E B* at *F*. With *E* as centre and *E F* as radius, cut the arcs *A D* and *B C* in *D* and *C*. Join *A C*, *C D*, and *D B* to form the square.

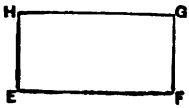


FIG. 25.

25. To construct a rectangle with given base *E F* and height *E H* (Fig. 25).—On the base *E F* draw the perpendiculars *E H*, *F G* equal to the height, and join *G H*.

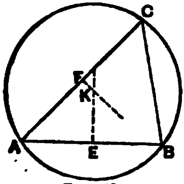


FIG. 26.

26. To describe a circle about a triangle (Fig. 26).—Bisect two sides *A B*, *A C* of the triangle at *E F*, and from these points draw perpendiculars cutting at *K*. On the centre *K*, with the radius *K A*, draw the circle *A B C*.

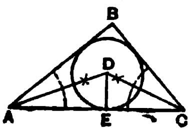


FIG. 27.

27. To inscribe a circle in a triangle (Fig. 27).—Bisect two of the angles *A*, *C*, of the triangle by lines cutting at *D*; from *D* draw a perpendicular *D E* to any side, and with *D E* as radius describe a circle.

When the triangle is equilateral, draw a perpendicular from one of the angles to the opposite side, and from the side set off one third of the perpendicular.

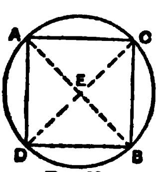


FIG. 28.

28. To describe a circle about a square, and to inscribe a square in a circle (Fig. 28).—To describe the circle, draw the diagonals *A B*, *C D* of the square, cutting at *E*. On the centre *E*, with the radius *A E*, describe the circle.

To inscribe the square.—Draw the two diameters, *A B*, *C D*, at right angles, and join the points *A*, *B*, *C D*, to form the square.

NOTE.—In the same way a circle may be described about a rectangle.

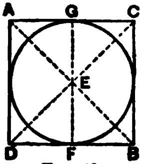


FIG 29.

29. To inscribe a circle in a square (Fig. 29).—To inscribe the circle, draw the diagonals *A B*, *C D* of the square, cutting at *E*; draw the perpendicular *E F* to one side, and with the radius *E F* describe the circle.

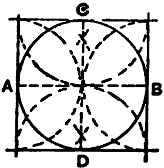


FIG. 30.

30. To describe a square about a circle (Fig. 30).—Draw two diameters *A B*, *C D* at right angles. With the radius of the circle and *A*, *B*, *C* and *D* as centres, draw the four half circles which cross one another in the corners of the square.

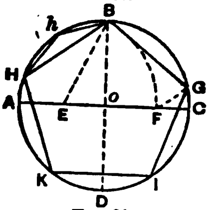


FIG. 31.

31. To inscribe a pentagon in a circle (Fig. 31).—Draw diameters *A C*, *B D* at right angles, cutting at *o*. Bisect *A o* at *E*, and from *E*, with radius *E B*, cut *A C* at *F*; from *B*, with radius *B F*, cut the circumference at *G*, *H*, and with the same radius step round the circle to *I* and *K*; join the points so found to form the pentagon.

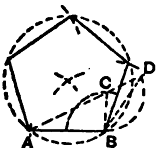


FIG. 32.

32. To construct a pentagon on a given line *A B* (Fig. 32).—From *B* erect a perpendicular *B C* half the length of *A B*; join *A C* and prolong it to *D*, making $C D = B C$. Then *B D* is the radius of the circle circumscribing the pentagon. From *A* and *B* as centres, with *B D* as radius, draw arcs cutting each other in *O*, which is the centre of the circle.

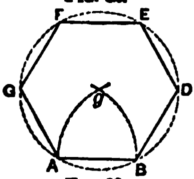


FIG. 33.

33. To construct a hexagon upon a given straight line (Fig. 33).—From *A* and *B*, the ends of the given line, with radius *A B*, describe arcs cutting at *g*; from *g*, with the radius *g A*, describe a circle; with the same radius set off the arcs *A G*, *G F*, and *B D*, *D E*. Join the points so found to form the hexagon. The side of a hexagon = radius of its circumscribed circle.

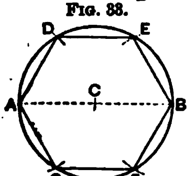


FIG. 34.

34. To inscribe a hexagon in a circle (Fig. 34).—Draw a diameter *A C B*. From *A* and *B* as centres, with the radius of the circle *A C*, cut the circumference at *D*, *E*, *F*, *G*, and draw *A D*, *D E*, etc., to form the hexagon. The radius of the circle is equal to the side of the hexagon; therefore the points *D*, *E*, etc., may also be found by stepping the radius six times round the circle. The angle between the diameter and the sides of a hexagon and also the exterior angle between a side and an adjacent side prolonged is 60 degrees; therefore a hexagon may conveniently be drawn by the use of a 60-degree triangle.

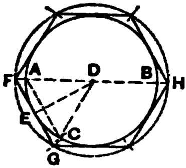


FIG. 35.

35. To describe a hexagon about a circle (Fig. 35).—Draw a diameter *A D B*, and with the radius *A D*, on the centre *A*, cut the circumference at *C*; join *A C*, and bisect it with the radius *D E*; through *E* draw *F G*, parallel to *A C*, cutting the diameter at *F*, and with the radius *D F* describe the circumscribing circle *F H*. Within this circle describe a hexagon by the preceding problem. A more convenient method is by use of a 60-degree triangle. Four of the sides make angles of 60 degrees with the diameter, and the other two are parallel to the diameter.

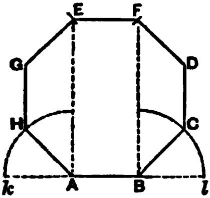


FIG. 36.

36. To describe an octagon on a given straight line (Fig. 36).—Produce the given line *A B* both ways, and draw perpendiculars *A E*, *B F*; bisect the external angles *A* and *B* by the lines *A H*, *B C*, which make equal to *A B*. Draw *C D* and *H G* parallel to *A E*, and equal to *A B*; from the centres *G*, *D*, with the radius *A B*, cut the perpendiculars at *E*, *F*, and draw *E F* to complete the octagon.

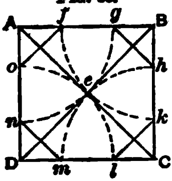


FIG. 37.

37. To convert a square into an octagon (Fig. 37).—Draw the diagonals of the square cutting at *e*; from the corners *A*, *B*, *C*, *D*, with *A e* as radius, describe arcs cutting the sides at *gn*, *fk*, *hm*, and *ol*, and join the points so found to form the octagon. Adjacent sides of an octagon make an angle of 135 degrees.

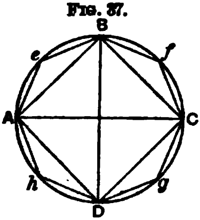


FIG. 38.

38. To inscribe an octagon in a circle (Fig. 38).—Draw two diameters, *A C*, *B D* at right angles; bisect the arcs *A B*, *B C*, etc., at *e f*, etc., and join *A e*, *e B*, etc., to form the octagon.

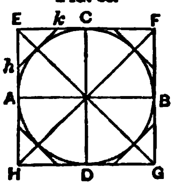


FIG. 39.

39. To describe an octagon about a circle (Fig. 39).—Describe a square about the given circle *A B*; draw perpendiculars *h k*, etc., to the diagonals, touching the circle to form the octagon.

40. To describe a polygon of any number of sides upon a given straight line (Fig. 40).—Produce the given line *A B*, and on *A*,

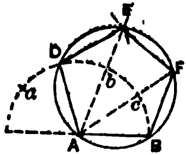


FIG. 40.

with the radius $A\,B$, describe a semicircle; divide the semi-circumference into as many equal parts as there are to be sides in the polygon—say, in this example, five sides. Draw lines from A through the divisional points D, b, and c, omitting one point a; and on the centres B, D, with the radius $A\,B$, cut $A\,b$ at E and $A\,c$ at F. Draw $D\,E$, $E\,F$, $F\,B$ to complete the polygon.

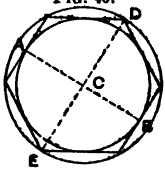


FIG. 41.

41. To inscribe a circle within a polygon (Figs. 41, 42).—When the polygon has an even number of sides (Fig. 41), bisect two opposite sides at A and B; draw $A\,B$, and bisect it at C by a diagonal $D\,E$, and with the radius $C\,A$ describe the circle.

When the number of sides is odd (Fig. 42), bisect two of the sides at A and B, and draw lines $A\,E$, $B\,D$ to the opposite angles, intersecting at C; from C, with the radius $C\,A$, describe the circle.

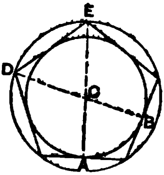


FIG. 42.

42. To describe a circle without a polygon (Figs. 41, 42).—Find the centre C as before, and with the radius $C\,D$ describe the circle.

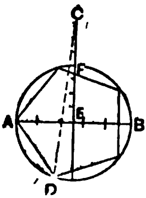


FIG. 43.

43. To inscribe a polygon of any number of sides within a circle (Fig. 43).—Draw the diameter $A\,B$ and through the centre E draw the perpendicular $E\,C$, cutting the circle at F. Divide $E\,F$ into four equal parts, and set off three parts equal to those from F to C. Divide the diameter $A\,B$ into as many equal parts as the polygon is to have sides; and from C draw $C\,D$, through the second point of division, cutting the circle at D. Then $A\,D$ is equal to one side of the polygon, and by stepping round the circumference with the length $A\,D$ the polygon may be completed.

TABLE OF POLYGONAL ANGLES.

Number of Sides.	Angle at Centre.	Number of Sides.	Angle at Centre.	Number of Sides.	Angle at Centre.
No.	Degrees.	No.	Degrees.	No.	Degrees.
3	120	9	40	15	24
4	90	10	36	16	$22\frac{1}{2}$
5	72	11	$32\frac{8}{11}$	17	$21\frac{3}{17}$
6	60	12	30	18	20
7	$51\frac{3}{7}$	13	$27\frac{9}{13}$	19	19
8	45	14	$25\frac{5}{7}$	20	18

In this table the angle at the centre is found by dividing 360 degrees, the number of degrees in a circle, by the number of sides in the polygon; and by setting off round the centre of the circle a succession of angles by means of the protractor, equal to the angle in the table due to a given number of sides, the radii so drawn will divide the circumference into the same number of parts.

44. To describe an ellipse when the length and breadth are given (Fig. 44).—*A B*, transverse axis; *C D*, conjugate axis; *F G*, foci. The sum of the distances from *C* to *F* and *G*, also the sum of the distances from *F* and *G* to any other point in the curve, is equal to the transverse axis. From the centre *C*, with *A E* as radius, cut the axis *A B* at *F* and *G*, the foci; fix a couple of pins into the axis at *F* and *G*, and loop on a thread or cord upon them equal in length to the axis *A B*, so as when stretched to reach to the extremity *C* of the conjugate axis, as shown in dot-lining. Place a pencil inside the cord as at *H*, and guiding the pencil in this way, keeping the cord equally in tension, carry the pencil round the pins *F*, *G*, and so describe the ellipse.

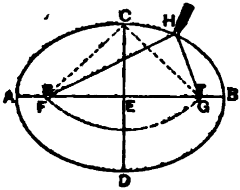


FIG. 44.

NOTE.—This method is employed in setting off elliptical garden-plots, walks, etc.

2d Method (Fig. 45).—Along the straight edge of a slip of stiff paper mark off a distance *a c* equal to *A C*, half the transverse axis; and from the same point a distance *a b* equal to *C D*, half the conjugate axis. Place the slip so as to bring the point *b* on the line *A B* of the transverse axis, and the point *c* on the line *D E*; and set off on the drawing the position of the point *a*. Shifting the slip so that the point *b* travels on the transverse axis, and the point *c* on the conjugate axis, any number of points in the curve may be found, through which the curve may be traced.

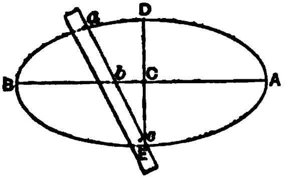


FIG. 45.

3d Method (Fig. 46).—The action of the preceding method may be embodied so as to afford the means of describing a large curve continuously by means of a bar *m k*, with steel points *m*, *l*, *k*, riveted into brass slides adjusted to the length of the semi-axis and fixed with set-screws. A rectangular cross *E G*, with guiding-slots is placed, coinciding with the two axes of the ellipse *A C* and *B H*. By sliding the points *k*, *l* in the slots, and carrying round the point *m*, the curve may be continuously described. A pen or pencil may be fixed at *m*.

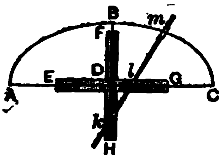


FIG. 46.

4th Method (Fig. 47).—Bisect the transverse axis at *C*, and through *C* draw the perpendicular *D E*, making *C D* and *C E* each equal to half the conjugate axis. From *D* or *E*, with the radius *A C*, cut the transverse axis at *F*, *F′*, for the foci. Divide *A C* into a number of parts at the

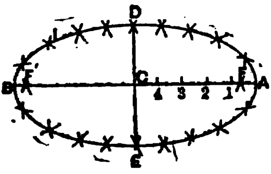


FIG. 47.

points 1, 2, 3, etc. With the radius $A\ I$ on F and F' as centres, describe arcs, and with the radius $B\ I$ on the same centres cut these arcs as shown. Repeat the operation for the other divisions of the transverse axis. The series of intersections thus made are points in the curve, through which the curve may be traced.

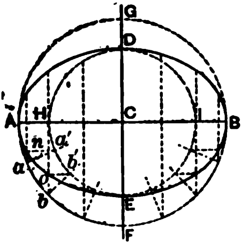


FIG. 48.

5th Method (Fig. 48).—On the two axes $A\ B$, $D\ E$ as diameters, on centre C, describe circles; from a number of points a, b, etc., in the circumference $A\ F\ B$, draw radii cutting the inner circle at a', b', etc. From a, b, etc., draw perpendiculars to AB; and from a', b', etc., draw parallels to $A\ B$, cutting the respective perpendiculars at n, o, etc. The intersections are points in the curve, through which the curve may be traced.

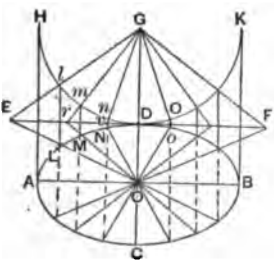


FIG. 49.

6th Method (Fig. 49).—When the transverse and conjugate diameters are given, $A\ B$, $C\ D$, draw the tangent $E\ F$ parallel to $A\ B$. Produce $C\ D$, and on the centre G with the radius of half $A\ B$, describe a semicircle $H\ D\ K$; from the centre G draw any number of straight lines to the points E, r, etc., in the line $E\ F$, cutting the circumference at l, m, n, etc.; from the centre O of the ellipse draw straight lines to the points E, r, etc.; and from the points l, m, n, etc., draw parallels to $G\ C$, cutting the lines $O\ E$, $O\ r$, etc., at L, M, N, etc. These are points in the circumference of the ellipse, and the curve may be traced through them. Points in the other half of the ellipse are formed by extending the intersecting lines as indicated in the figure.

45. To describe an ellipse approximately by means of circular arcs.—*First.*—With arcs of two radii (Fig. 50).—Find the difference of the semi-axes, and set it off from the centre O to a and c on $O\ A$ and $O\ C$; draw $a\ c$, and set off half $a\ c$ to d; draw $d\ i$ parallel to $a\ c$; set off $O\ e$ equal to $O\ d$; join $e\ i$, and draw the parallels $e\ m$, $d\ m$. From m, with radius $m\ C$, describe an arc through C; and from i describe an arc through D; from d and e describe arcs through A and B. The four arcs form the ellipse approximately.

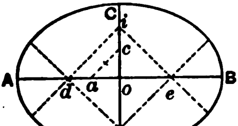


FIG. 50.

NOTE.—This method does not apply satisfactorily when the conjugate axis is less than two thirds of the transverse axis.

2d Method (by Carl G. Barth, Fig. 51).—In Fig. 51 $a\ b$ is the major and $c\ d$ the minor axis of the ellipse to be approximated. Lay off $b\ e$ equal to the semi-minor axis $c\ O$, and use $a\ e$ as radius for the arc at each extremity of the minor axis. Bisect $e\ o$ at f and lay off $e\ g$ equal to $e\ f$, and use $g\ b$ as radius for the arc at each extremity of the major axis.

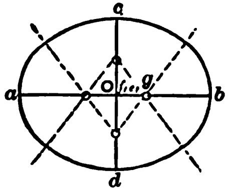


FIG. 51.

The method is not considered applicable for cases in which the minor axis is less than two thirds of the major.

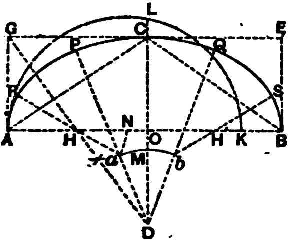


FIG. 52.

3d Method: With arcs of three radii (Fig. 52).—On the transverse axis *A B* draw the rectangle *B G* on the height *O C*; to the diagonal *A C* draw the perpendicular *G H D*; set off *O K* equal to *O C*, and describe a semicircle on *A K*, and produce *O C* to *L*; set off *O M* equal to *C L*, and from *D* describe an arc with radius *D M*; from *A*, with radius *O L*, cut *A B* at *N*; from *H*, with radius *H N*, cut arc *a b* at *a*. Thus the five centres *D*, *a*, *b*, *H*, *H'* are found, from which the arcs are described to form the ellipse.

This process works well for nearly all proportions of ellipses. It is used in striking out vaults and stone bridges.

4th Method (by F. R. Honey, Figs. 53 and 54).—Three radii are employed. With the shortest radius describe the two arcs which pass through the vertices of the major axis, with the longest the two arcs which pass through the vertices of the minor axis, and with the third radius the four arcs which connect the former.

A simple method of determining the radii of curvature is illustrated in Fig. 53. Draw the straight lines *a f* and *a c*, forming any angle at *a*. With *a* as a centre, and with radii *a b* and *a c*, respectively, equal to the semi-minor and semi-major axes, draw the arcs *b e* and *c d*. Join *e d*, and through *b* and *c* respectively draw *b g* and *c f* parallel to *e d*, intersecting *a c* at *g*, and *a f* at *f*; *a f* is the radius of curvature at the vertex of the minor axis; and *a g* the radius of curvature at the vertex of the major axis.

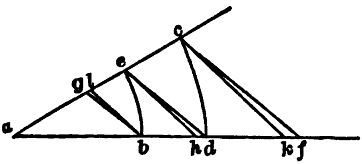


FIG. 53.

Lay off *d h* (Fig. 53) equal to one eighth of *b d*. Join *e h*, and draw *c k* and *b l* parallel to *e h*. Take *a k* for the longest radius (= *R*), *a l* for the shortest radius (= *r*), and the arithmetical mean, or one half the sum of the semi-axes, for the third radius (= *p*), and employ these radii for the eight-centred oval as follows:

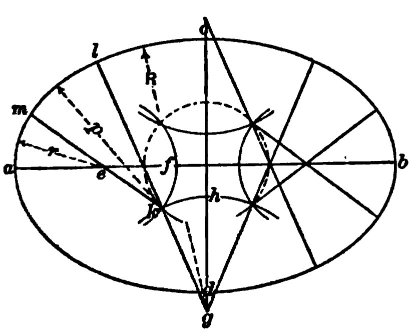


FIG. 54.

Let *a b* and *c d* (Fig. 54) be the major and minor axes. Lay off *a e* equal to *r*, and *a f* equal to *p*; also lay off *c g* equal to *R*, and *c h* equal to *p*. With *g* as a centre and *g h* as a radius, draw the arc *h k*; with the centre *e* and radius *e f* draw the arc *f k*, intersecting *h k* at *k*. Draw the line *g k* and produce it, making *g l* equal to *R*. Draw *k e* and produce it, making *k m* equal to *p*. With the centre *g* and radius *g c* (= *R*) draw the arc *c l*; with the centre *k* and radius *k l* (= *p*) draw the arc *l m*, and with the centre *e* and radius *e m* (= *r*) draw the arc *m a*.

The remainder of the work is symmetrical with respect to the axes.

46. The Parabola.—A parabola ($D\,A\,C$, Fig. 55) is a curve such that every point in the curve is equally distant from the directrix $K\,L$ and the focus F. The focus lies in the axis $A\,B$ drawn from the vertex or head of the curve A, so as to divide the figure into two equal parts. The vertex A is equidistant from the directrix and the focus, or $A\,e = A\,F$. Any line parallel to the axis is a diameter. A straight line, as $E\,G$ or $D\,C$, drawn across the figure at right angles to the axis is a double ordinate, and either half of it is an ordinate. The ordinate to the axis $E\,F\,G$, drawn through the focus, is called the parameter of the axis. A segment of the axis, reckoned from the vertex, is an abscissa of the axis, and it is an abscissa of the ordinate drawn from the base of the abscissa. Thus, $A\,B$ is an abscissa of the ordinate $B\,C$.

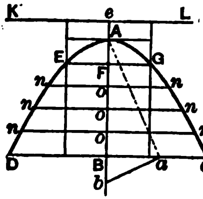


FIG. 55.

Abscissæ of a parabola are as the squares of their ordinates.

To describe a parabola when an abscissa and its ordinate are given (Fig. 55).—Bisect the given ordinate $B\,C$ at a, draw $A\,a$, and then $a\,b$ perpendicular to it, meeting the axis at b. Set off $A\,e$, $A\,F$, each equal to $B\,b$; and draw $K\,e\,L$ perpendicular to the axis. Then $K\,L$ is the directrix and F is the focus. Through F and any number of points, o, o, etc., in the axis, draw double ordinates, $n\,o\,n$, etc., and from the centre F, with the radii $F\,e$, $o\,e$, etc., cut the respective ordinates at E, G, n, n, etc. The curve may be traced through these points as shown.

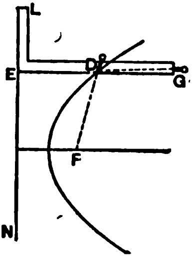


FIG. 56.

2d Method: By means of a square and a cord (Fig. 56).—Place a straight-edge to the directrix $E\,N$, and apply to it a square $L\,E\,G$. Fasten to the end G one end of a thread or cord equal in length to the edge $E\,G$, and attach the other end to the focus F; slide the square along the straight-edge, holding the cord taut against the edge of the square by a pencil D, by which the curve is described.

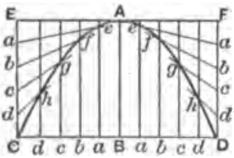


FIG. 57.

3d Method: When the height and the base are given (Fig. 57).—Let $A\,B$ be the given axis, and $C\,D$ a double ordinate or base; to describe a parabola of which the vertex is at A. Through A draw $E\,F$ parallel to $C\,D$, and through C and D draw $C\,E$ and $D\,F$ parallel to the axis. Divide $B\,C$ and $B\,D$ into any number of equal parts, say five, at a, b, etc., and divide $C\,E$ and $D\,F$ into the same number of parts. Through the points a, b, c, d in the base $C\,D$ on each side of the axis draw perpendiculars, and through a, b, c, d in $C\,E$ and $D\,F$ draw lines to the vertex A, cutting the perpendiculars at e, f, g, h. These are points in the parabola, and the curve $C\,A\,D$ may be traced as shown, passing through them.

47. The Hyperbola (Fig. 58).—A hyperbola is a plane curve, such that the difference of the distances from any point of it to two fixed points is equal to a given distance. The fixed points are called the foci.

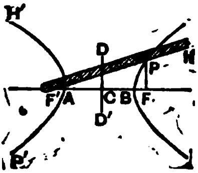


FIG. 58.

To construct a hyperbola.—Let F' and F be the foci, and F' F the distance between them. Take a ruler longer than the distance F' F, and fasten one of its extremities at the focus F'. At the other extremity, H, attach a thread of such a length that the length of the ruler shall exceed the length of the thread by a given distance A B. Attach the other extremity of the thread at the focus F.

Press a pencil, P, against the ruler, and keep the thread constantly tense, while the ruler is turned around F' as a centre. The point of the pencil will describe one branch of the curve.

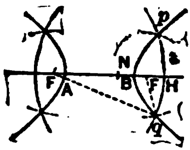


FIG. 59.

2d Method: By points (Fig. 59).—From the focus F' lay off a distance F' N equal to the transverse axis, or distance between the two branches of the curve, and take any other distance, as $F'H$, greater than $F'N$.

With F' as a centre and $F'H$ as a radius describe the arc of a circle. Then with F as a centre and N H as a radius describe an arc intersecting the arc before described at p and q. These will be points of the hyperbola, for F' $q - F$ q is equal to the transverse axis A B.

If, with F as a centre and F' H as a radius, an arc be described, and a second arc be described with F' as a centre and N H as a radius, two points in the other branch of the curve will be determined. Hence, by changing the centres, each pair of radii will determine two points in each branch.

The Equilateral Hyperbola.—The transverse axis of a hyperbola is the distance, on a line joining the foci, between the two branches of the curve. The conjugate axis is a line perpendicular to the transverse axis, drawn from its centre, and of such a length that the diagonal of the rectangle of the transverse and conjugate axes is equal to the distance between the foci. The diagonals of this rectangle, indefinitely prolonged, are the *asymptotes* of the hyperbola, lines which the curve continually approaches, but touches only at an infinite distance. If these asymptotes are perpendicular to each other, the hyperbola is called a *rectangular* or *equilateral hyperbola*. It is a property of this hyperbola that if the asymptotes are taken as axes of a rectangular system of coördinates (see Analytical Geometry), the product of the abscissa and ordinate of any point in the curve is equal to the product of the abscissa and ordinate of any other point; or, if p is the ordinate of any point and v its abscissa, and p_1 and v_1 are the ordinate and abscissa of any other point, $pv = p_1\,v_1$; or pv = a constant.

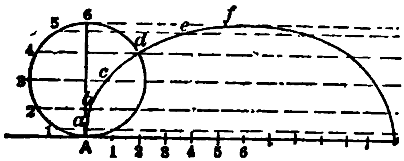


FIG. 60.

48. The Cycloid (Fig. 60).—If a circle $A\,d$ be rolled along a straight line $A\,6$, any point of the circumference as A will describe a curve, which is called a cycloid. The circle is called the generating circle, and A the generating point.

To draw a cycloid.—Divide the circumference of the generating circle into an even number of equal parts, as A 1, 12, etc., and set off these distances on the base. Through the points 1, 2, 3, etc., on the circle draw horizontal lines, and on them set off distances $1a = A1$, $2b = A2$, $3c = A3$, etc. The points A, a, b, c, etc., will be points in the cycloid, through which draw the curve.

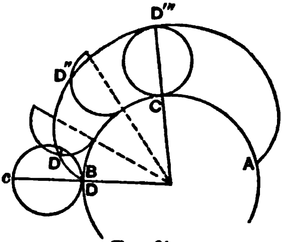


Fig. 61.

49. The Epicycloid (Fig. 61) is generated by a point D in one circle $D\ C$ rolling upon the circumference of another circle $A\ C\ B$, instead of on a flat surface or line; the former being the generating circle, and the latter the fundamental circle. The generating circle is shown in four positions, in which the generating point is successively marked D, D', D'', D'''. $A\ D'''\ B$ is the epicycloid.

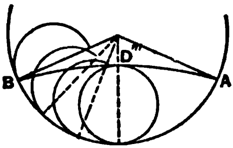


Fig. 62.

50. The Hypocycloid (Fig. 62) is generated by a point in the generating circle rolling on the inside of the fundamental circle.

When the generating circle = radius of the other circle, the hypocycloid becomes a straight line.

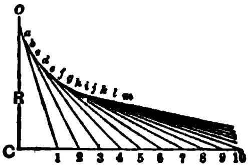


Fig. 63.

51. The Tractrix or Schiele's anti-friction curve (Fig. 63).—R is the radius of the shaft, C, 1, 2, etc., the axis. From O set off on R a small distance, $o\,a$; with radius R and centre a cut the axis at 1, join a 1, and set off a like small distance $a\,b$; from b with radius R cut axis at 2, join b 2, and so on, thus finding points o, a, b, c, d, etc., through which the curve is to be drawn.

52. The Spiral.—The spiral is a curve described by a point which moves along a straight line according to any given law, the line at the same time having a uniform angular motion. The line is called the radius vector. If the radius vector increases directly as the measuring angle, the spires, or parts described in each revolution, thus gradually increasing their distance from each other, the curve is known as the spiral of Archimedes (Fig. 64).

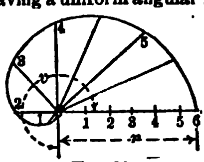


Fig. 64.

This curve is commonly used for cams. To describe it draw the radius vector in several different directions around the centre, with equal angles between them; set off the distances 1, 2, 3, 4, etc., corresponding to the scale upon which the curve is drawn, as shown in Fig. 64.

In the common spiral (Fig. 64) the pitch is uniform; that is, the spires are equidistant. Such a spiral is made by rolling up a belt of uniform thickness.

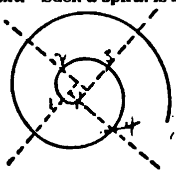


Fig. 65.

To construct a spiral with four centres (Fig. 65).—Given the pitch of the spiral, construct a square about the centre, with the sum of the four sides equal to the pitch. Prolong the sides in one direction as shown; the corners are the centres for each arc of the external angles, forming a quadrant of a spire.

53. To find the diameter of a circle into which a certain number of rings will fit on its inside (Fig. 66).—For instance, what is the diameter of a circle into which twelve ½-inch rings will fit, as per sketch? Assume that we have found the diameter of the required circle, and have drawn the rings inside of it. Join the centres of the rings by straight lines, as shown: we then obtain a regular polygon with 12 sides, each side being equal to the diameter of a given ring. We have now to find the diameter of a circle circumscribed about this polygon, and add the diameter of one ring to it; the sum will be the diameter of the circle into which the rings will fit. Through the centres A and D of two adjacent rings draw the radii CA and CD; since the polygon has twelve sides the angle $ACD = 30°$ and $ACB = 15°$. One half of the side AD is equal to AB. We now give the following proportion: The sine of the angle ACB is to AB as 1 is to the required radius. From this we get the following rule: Divide AB by the sine of the angle ACB; the quotient will be the radius of the circumscribed circle; add to the corresponding diameter the diameter of one ring; the sum will be the required diameter FG.

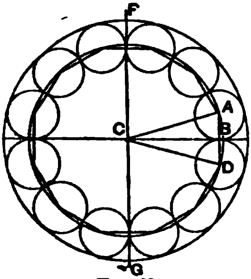


FIG. 66.

54. To describe an arc of a circle which is too large to be drawn by a beam compass, by means of points in the arc, radius being given.—Suppose the radius is 20 feet and it is desired to obtain five points in an arc whose half chord is 4 feet. Draw a line equal to the half chord, full size, or on a smaller scale if more convenient, and erect a perpendicular at one end, thus making rectangular axes of coördinates. Erect perpendiculars at points 1, 2, 3, and 4 feet from the first perpendicular. Find values of y in the formula of the circle, $x^2+y^2=R^2$ by substituting for x the values 0, 1, 2, 3, and 4, etc., and for R^2 the square of the radius, or 400. The values will be $y=\sqrt{R^2-x^2}=\sqrt{400}$, $\sqrt{399}$, $\sqrt{396}$, $\sqrt{391}$, $\sqrt{384}$; = 20, 19.975, 19.90, 19.774, 19.596.

Subtract the smallest, or 19.596, leaving 0.404, 0.379, 0.304, 0.178, 0 feet.

Lay off these distances on the five perpendiculars, as ordinates from the half chord, and the positions of five points on the arc will be found. Through these the curve may be drawn. (See also Problem 14.)

55. The Catenary is the curve assumed by a perfectly flexible cord when its ends are fastened at two points, the weight of a unit length being constant.

The equation of the catenary is

$$y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right),$$

in which e is the base of the Naperian system of logarithms.

To plot the catenary.—Let o (Fig. 67) be the origin of coördinates. Assigning to a any value as 3, the equation becomes

$$y=\frac{3}{2}\left(e^{\frac{x}{3}}+e^{-\frac{x}{3}}\right).$$

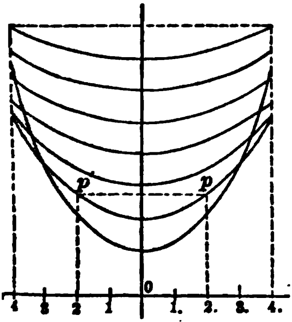


FIG. 67.

To find the lowest point of the curve.

$$\text{Put } x=0;\ \therefore\ y=\frac{3}{2}\left(e^{0}+e^{-0}\right)=\frac{3}{2}(1+1)=3.$$

$$\text{Then put } x = 1;\ \therefore\ y = \frac{3}{2}\left(e^{\frac{1}{3}} + e^{-\frac{1}{3}}\right) = \frac{3}{2}(1.396 + 0.717) = 3.17.$$

$$\text{Put } x = 2;\ \therefore\ y = \frac{3}{2}\left(e^{\frac{2}{3}} + e^{-\frac{2}{3}}\right) = \frac{3}{2}(1.948 + 0.513) = 3.69.$$

Put $x = 3, 4, 5$, etc., etc., and find the corresponding values of y. For each value of y we obtain two symmetrical points, as for example p and p^1.

In this way, by making a successively equal to 2, 3, 4, 5, 6, 7, and 8, the curves of Fig. 67 were plotted.

In each case the distance from the origin to the lowest point of the curve is equal to a; for putting $x = o$, the general equation reduces to $y = a$.

For values of $a = 6, 7$, and 8 the catenary closely approaches the parabola. For derivation of the equation of the catenary see Bowser's Analytic Mechanics. For comparison of the catenary with the parabola, see article by F. R. Honey, *Amer. Machinist*, Feb. 1, 1894.

56. The Involute is a name given to the curve which is formed by the end of a string which is unwound from a cylinder and kept taut; consequently the string as it is unwound will always lie in the direction of a tangent to the cylinder. To describe the involute of any given circle, Fig. 68, take any point A on its circumference, draw a diameter AB, and from B draw Bb perpendicular to AB. Make Bb equal in length to half the circumference of the circle. Divide Bb and the semi-circumference into the same number of equal parts, say six. From each point of division 1, 2, 3, etc., on the circumference draw lines to the centre C of the circle. Then draw $1a$ perpendicular to $C1$; $2a_2$ perpendicular to $C2$; and so on. Make $1a$ equal to bb_1; $2a_2$ equal to bb_2; $3a_3$ equal to bb_3; and so on. Join the points A, $a_1{}^1$, a_2, a_3, etc., by a curve; this curve will be the required involute.

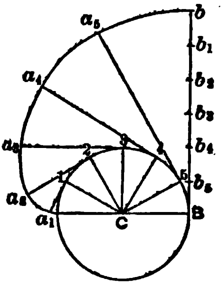


FIG. 68.

57. Method of plotting angles without using a protractor.—The radius of a circle whose circumference is 360 is 57.3 (more accurately 57.296). Striking a semicircle with a radius 57.3 by any scale, spacers set to 10 by the same scale will divide the arc into 18 spaces of 10° each, and intermediates can be measured indirectly at the rate of 1 by scale for each 1°, or interpolated by eye according to the degree of accuracy required. The following table shows the chords to the above-mentioned radius, for every 10 degrees from 0° up to 110°. By means of one of these,

Angle.	Chord.	Angle.	Chord.
1°	0.999	60°	57.296
10°	9.988	70°	65.727
20°	19.899	80°	73.658
30°	29.658	90°	81.029
40°	39.192	100°	87.782
50°	48.429	110°	93.869

a 10° point is fixed upon the paper next less than the required angle, and the remainder is laid off at the rate of 1 by scale for each degree.

GEOMETRICAL PROPOSITIONS.

In a right-angled triangle the square on the hypothenuse is equal to the sum of the squares on the other two sides.

If a triangle is equilateral, it is equiangular, and *vice versa*.

If a straight line from the vertex of an isosceles triangle bisects the base, it bisects the vertical angle and is perpendicular to the base.

If one side of a triangle is produced, the exterior angle is equal to the sum of the two interior and opposite angles.

If two triangles are mutually equiangular, they are similar and their corresponding sides are proportional.

If the sides of a polygon are produced in the same order, the sum of the exterior angles equals four right angles.

In a quadrilateral, the sum of the interior angles equals four right angles.

In a parallelogram, the opposite sides are equal; the opposite angles are equal; it is bisected by its diagonal; and its diagonals bisect each other.

If three points are not in the same straight line, a circle may be passed through them.

If two arcs are intercepted on the same circle, they are proportional to the corresponding angles at the centre.

If two arcs are similar, they are proportional to their radii.

The areas of two circles are proportional to the squares of their radii.

If a radius is perpendicular to a chord, it bisects the chord and it bisects the arc subtended by the chord.

A straight line tangent to a circle meets it in only one point, and it is perpendicular to the radius drawn to that point.

If from a point without a circle tangents are drawn to touch the circle, there are but two; they are equal, and they make equal angles with the chord joining the tangent points.

If two lines are parallel chords or a tangent and parallel chord, they intercept equal arcs of a circle.

If an angle at the circumference of a circle, between two chords, is subtended by the same arc as an angle at the centre, between two radii, the angle at the circumference is equal to half the angle at the centre.

If a triangle is inscribed in a semicircle, it is right-angled.

If an angle is formed by a tangent and chord, it is measured by one half of the arc intercepted by the chord; that is, it is equal to half the angle at the centre subtended by the chord.

If two chords intersect each other in a circle, the rectangle of the segments of the one equals the rectangle of the segments of the other.

And if one chord is a diameter and the other perpendicular to it, the rectangle of the segments of the diameter is equal to the square on half the other chord, and the half chord is a mean proportional between the segments of the diameter.

MENSURATION.

PLANE SURFACES.

Quadrilateral.—A four-sided figure.

Parallelogram.—A quadrilateral with opposite sides parallel.

Varieties.—Square: four sides equal, all angles right angles. Rectangle: opposite sides equal, all angles right angles. Rhombus: four sides equal, opposite angles equal, angles not right angles. Rhomboid: opposite sides equal, opposite angles equal, angles not right angles.

Trapezium.—A quadrilateral with unequal sides.

Trapezoid.—A quadrilateral with only one pair of opposite sides parallel.

Diagonal of a square $= \sqrt{2 \times \text{side}^2} = 1.4142 \times$ side.

Diag. of a rectangle $= \sqrt{\text{sum of squares of two adjacent sides.}}$

Area of any parallelogram = base $\times$ altitude.

Area of rhombus or rhomboid = product of two adjacent sides $\times$ sine of angle included between them.

Area of a trapezium = half the product of the diagonal by the sum of the perpendiculars let fall on it from opposite angles.

Area of a trapezoid = product of half the sum of the two parallel sides by the perpendicular distance between them.

To find the area of any quadrilateral figure.—Divide the quadrilateral into two triangles; the sum of the areas of the triangles is the area.

Or, multiply half the product of the two diagonals by the sine of the angle at their intersection.

To find the area of a quadrilateral inscribed in a circle.—From half the sum of the four sides subtract each side severally; multiply the four remainders together; the square root of the product is the area.

Triangle.—A three-sided plane figure.

Varieties.—Right-angled, having one right angle; obtuse-angled, having one obtuse angle; isosceles, having two equal angles and two equal sides; equilateral, having three equal sides and equal angles.

The sum of the three angles of every triangle = 180°.

The two acute angles of a right-angled triangle are complements of each other.

Hypothenuse of a right-angled triangle, the side opposite the right angle $= \sqrt{\text{sum of the squares of the other two sides.}}$

To find the area of a triangle:

Rule 1. Multiply the base by half the altitude.

Rule 2. Multiply half the product of two sides by the sine of the included angle.

Rule 3. From half the sum of the three sides subtract each side severally; multiply together the half sum and the three remainders, and extract the square root of the product.

The area of an equilateral triangle is equal to one fourth the square of one of its sides multiplied by the square root of 3, $= \frac{a^2\sqrt{3}}{4}$, a being the side; or $a^2 \times .433013$.

Hypothenuse and one side of right-angled triangle given, to find other side, Required side $= \sqrt{\text{hyp}^2 - \text{given side}^2}$.

If the two sides are equal, side = hyp $\div$ 1.4142; or hyp $\times$ .7071.

Area of a triangle given, to find base: Base = twice area $\div$ perpendicular height.

Area of a triangle given, to find height: Height = twice area $\div$ base.

Two sides and base given, to find perpendicular height (in a triangle in which both of the angles at the base are acute).

Rule.—As the base is to the sum of the sides, so is the difference of the sides to the difference of the divisions of the base made by drawing the perpendicular. Half this difference being added to or subtracted from half the base will give the two divisions thereof. As each side and its opposite division of the base constitutes a right-angled triangle, the perpendicular is ascertained by the rule perpendicular = $\sqrt{\text{hyp}^2 - \text{base}^2}$.

Polygon. — A plane figure having three or more sides. Regular or irregular, according as the sides or angles are equal or unequal. Polygons are named from the number of their sides and angles.

To find the area of an irregular polygon.—Draw diagonals dividing the polygon into triangles, and find the sum of the areas of these triangles.

To find the area of a regular polygon:

RULE.—Multiply the length of a side by the perpendicular distance to the centre; multiply the product by the number of sides, and divide it by 2. Or, multiply half the perimeter by the perpendicular let fall from the centre on one of the sides.

The perpendicular from the centre is equal to half of one of the sides of the polygon multiplied by the cotangent of the angle subtended by the half side.

The angle at the centre = 360° divided by the number of sides.

TABLE OF REGULAR POLYGONS.

No. of Sides.	Name of Polygon.	Area, Side = 1.	Radius of Circumscribed Circle. Perpen. from Centre = 1.	Radius of Circumscribed Circle. Side = 1.	Radius of Inscribed Circle, Side = 1.	Length of Side, Radius of Circumsc. Circle = 1.	Angle at Centre.	Angle between Adjacent Sides.
3	Triangle	.4330127	2.	.5773	.2887	1.732	120°	60°
4	Square	1.	1.414	.7071	.5	1.4142	90	90
5	Pentagon	1.7204774	1.238	.8506	.6882	1.1756	72	108
6	Hexagon	2.5980762	1.156	1.	.866	1.	60	120
7	Heptagon	3.6339124	1.11	1.1524	1.0383	.8677	51 26′	128 4-7
8	Octagon	4.8284271	1.083	1.3066	1.2071	.7653	45	135
9	Nonagon	6.1818242	1.064	1.4619	1.3737	.684	40	140
10	Decagon	7.6942088	1.051	1.618	1.5388	.618	36	144
11	Undecagon	9.3656399	1.042	1.7747	1.7028	.5634	32 43′	147 3-11
12	Dodecagon	11.1961524	1.037	1.9319	1.866	.5176	30	150

To find the area of a regular polygon, when the length of a side only is given:

RULE.—Multiply the square of the side by the multiplier opposite to the name of the polygon in the table.

To find the area of an irregular figure (Fig. 69).—Draw ordinates across its breadth at equal distances apart, the first and the last ordinate each being one half space from the ends of the figure. Find the average breadth by adding together the lengths of these lines included between the boundaries of the figure, and divide by the number of the lines added; multiply this mean breadth by the length. The greater the number of lines the nearer the approximation.

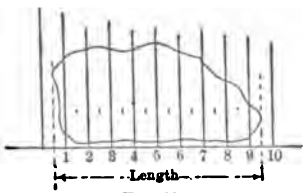


FIG. 69.

In a figure of very irregular outline, as an indicator-diagram from a high-speed steam-engine, mean lines may be substituted for the actual lines of the figure, being so traced as to intersect the undulations, so that the total area of the spaces cut off may be compensated by that of the extra spaces inclosed.

2d Method: THE TRAPEZOIDAL RULE. — Divide the figure into any sufficient number of equal parts; add half the sum of the two end ordinates to the sum of all the other ordinates; divide by the number of spaces (that is, one less than the number of ordinates) to obtain the mean ordinate, and multiply this by the length to obtain the area.

3d Method: SIMPSON'S RULE.—Divide the length of the figure into any even number of equal parts, at the common distance D apart, and draw ordinates through the points of division to touch the boundary lines. Add together the first and last ordinates and call the sum A; add together the even ordinates and call the sum B; add together the odd ordinates, except the first and last, and call the sum C. Then,

$$\text{area of the figure} = \frac{A + 4B + 2C}{3} \times D.$$

4th Method: DURAND'S RULE.—Add together 4/10 the sum of the first and last ordinates, 1 1/10 the sum of the second and the next to the last (or the penultimates), and the sum of all the intermediate ordinates. Multiply the sum thus gained by the common distance between the ordinates to obtain the area, or divide this sum by the number of spaces to obtain the mean ordinate.

Prof. Durand describes the method of obtaining his rule in *Engineering News*, Jan. 18, 1894. He claims that it is more accurate than Simpson's rule, and practically as simple as the trapezoidal rule. He thus describes its application for approximate integration of differential equations. Any definite integral may be represented graphically by an area. Thus, let

$$Q = \int u\, dx$$

be an integral in which u is some function of x, either known or admitting of computation or measurement. Any curve plotted with x as abscissa and u as ordinate will then represent the variation of u with x, and the area between such curve and the axis X will represent the integral in question, no matter how simple or complex may be the real nature of the function u.

Substituting in the rule as above given the word "volume" for "area" and the word "section" for "ordinate," it becomes applicable to the determination of volumes from equidistant sections as well as of areas from equidistant ordinates.

Having approximately obtained an area by the trapezoidal rule, the area by Durand's rule may be found by adding algebraically to the sum of the ordinates used in the trapezoidal rule (that is, half the sum of the end ordinates + sum of the other ordinates) 1/10 of (sum of penultimates — sum of first and last) and multiplying by the common distance between the ordinates.

5th Method.—Draw the figure on cross-section paper. Count the number of squares that are entirely included within the boundary; then estimate the fractional parts of squares that are cut by the boundary, add together these fractions, and add the sum to the number of whole squares. The result is the area in units of the dimensions of the squares. The finer the ruling of the cross-section paper the more accurate the result.

6th Method.—Use a planimeter.

7th Method.—With a chemical balance, sensitive to one milligram, draw the figure on paper of uniform thickness and cut it out carefully; weigh the piece cut out, and compare its weight with the weight per square inch of the paper as tested by weighing a piece of rectangular shape.

THE CIRCLE.

Circumference = diameter × 3.1416, nearly; more accurately, 3.14159265359.
Approximations, $\frac{22}{7} = 3.143$; $\frac{355}{113} = 3.1415929$.
The ratio of circum. to diam. is represented by the symbol π (called *Pi*).

Multiples of π.	Multiples of $\frac{\pi}{4}$.
$1\pi = 3.14159265359$	$\frac{1}{4}\pi = .7853982$
$2\pi = 6.28318530718$	" $\times 2 = 1.5707963$
$3\pi = 9.42477796077$	" $\times 3 = 2.3561945$
$4\pi = 12.56637061436$	" $\times 4 = 3.1415927$
$5\pi = 15.70796326795$	" $\times 5 = 3.9269908$
$6\pi = 18.84955592154$	" $\times 6 = 4.7123890$
$7\pi = 21.99114857513$	" $\times 7 = 5.4977871$
$8\pi = 25.13274122872$	" $\times 8 = 6.2831853$
$9\pi = 28.27433388231$	" $\times 9 = 7.0685835$

Ratio of diam. to circumference = reciprocal of π = 0.3183099.

Reciprocal of $\frac{1}{4}\pi = 1.27324$.	$\frac{7}{\pi} = 2.22817$	$\frac{1}{12}\pi = 0.261799$
Multiples of $\frac{1}{\pi}$.	$\frac{8}{\pi} = 2.54648$	$\frac{\pi}{360} = 0.0087266$
$\frac{1}{\pi} = .31831$	$\frac{9}{\pi} = 2.86479$	$\frac{360}{\pi} = 114.5915$
$\frac{2}{\pi} = .63662$	$\frac{10}{\pi} = 3.18310$	$\pi^2 = 9.86960$
$\frac{3}{\pi} = .95493$	$\frac{12}{\pi} = 3.81972$	$\frac{1}{\pi^2} = 0.101321$
$\frac{4}{\pi} = 1.27324$	$\frac{1}{2}\pi = 1.570796$	$\sqrt{\pi} = 1.772453$
$\frac{5}{\pi} = 1.59155$	$\frac{1}{3}\pi = 1.047197$	$\sqrt{\frac{1}{\pi}} = 0.564189$
$\frac{6}{\pi} = 1.90986$	$\frac{1}{6}\pi = 0.523599$	Log π = 0.49714987
		Log $\frac{1}{4}\pi = \bar{1}.895090$

Diam. in ins. = 13.5405 $\sqrt{\text{area in sq. ft.}}$
Area in sq. ft. = (diam. in inches)^2 × .0054542.
D = diameter, R = radius, C = circumference, A = area.

$$C = \pi D; = 2\pi R; = \frac{4A}{D}; = 2\sqrt{\pi A}; = 3.545\sqrt{A};$$

$$A = D^2 \times .7854; = \frac{CR}{2}; = 4R^2 \times .7854; = \pi R^2; = \frac{1}{4}\pi D^2; = \frac{C^2}{4\pi}; = .07958C^2; = \frac{CD}{4}$$

$$D = \frac{C}{\pi}; = 0.31831C; = 2\sqrt{\frac{A}{\pi}}; = 1.12838\sqrt{A};$$

$$R = \frac{C}{2\pi}; = 0.159155C; = \sqrt{\frac{A}{\pi}}; = 0.564189\sqrt{A}.$$

Areas of circles are to each other as the squares of their diameters.

To find the length of an arc of a circle:

RULE 1. As 360 is to the number of degrees in the arc, so is the circumference of the circle to the length of the arc.

RULE 2. Multiply the diameter of the circle by the number of degrees in the arc, and this product by 0.0087266.

Relations of Arc, Chord, Chord of Half the Arc, Versed Sine, etc.

Let R = radius, D = diameter, Arc = length of arc,
Cd = chord of the arc, ch = chord of half the arc,
V = versed sine, $D - V$ = diam. minus ver. sin.,

$$Arc = \frac{8ch - Cd}{3} \text{ (very nearly)}, \quad = \frac{\sqrt{Cd^2 + 4V^2} \times 10V^2}{15Cd^2 + 33V^2} + 2ch, \text{ nearly.}$$

$$Arc = \frac{2ch \times 10V}{60D - 27V} + 2ch, \text{ nearly.}$$

Chord of the arc $= 2\sqrt{ch^2 - V^2}; \; = \sqrt{D^2 - (D - 2V)^2}; \; = 8ch - 3Arc.$

$$= 2\sqrt{R^2 - (R - V)^2}; \; = 2\sqrt{(D - V) \times V}.$$

Chord of half the arc, $ch = \frac{1}{2}\sqrt{Cd^2 + 4V^2}; \; = \sqrt{D \times V}; \; = \frac{3Arc + Cd}{8}.$

Diameter $= \frac{ch^2}{V}; \; = \frac{\left(\frac{1}{2}Cd\right)^2 + V^2}{V};$

Versed sine $= \frac{ch^2}{D}; = \frac{1}{2}(D - \sqrt{D^2 - Cd^2})$

(or $\frac{1}{2}(D + \sqrt{D^2 - Cd^2})$, if V is greater than radius.

$$= \sqrt{ch^2 - \frac{Cd^2}{4}}.$$

Half the chord of the arc is a mean proportional between the versed sine and diameter minus versed sine:

$$\frac{1}{2}Cd = \sqrt{V \times (D - V)}.$$

Length of a Circular Arc.—Huyghens's Approximation.

Let C represent the length of the chord of the arc and c the length of the chord of half the arc; the length of the arc

$$L = \frac{8c - C}{3}.$$

Professor Williamson shows that when the arc subtends an angle of 30°, the radius being 100,000 feet (nearly 19 miles), the error by this formula is about two inches, or 1/600000 part of the radius. When the length of the arc is equal to the radius, i.e., when it subtends an angle of 57°.3, the error is less than 1/7680 part of the radius. Therefore, if the radius is 100,000 feet, the error is less than $\frac{100000}{7680} = 13$ feet. The error increases rapidly with the increase of the angle subtended.

In the measurement of an arc which is described with a short radius the error is so small that it may be neglected. Describing an arc with a radius of 12 inches subtending an angle of 30°, the error is 1/50000 of an inch. For 57°.3 the error is less than 0″.0015.

In order to measure an arc when it subtends a large angle, bisect it and measure each half as before—in this case making B = length of the chord of half the arc, and b = length of the chord of one fourth the arc; then

$$L = \frac{16b - 2B}{3}.$$

Relation of the Circle to its Equal, Inscribed, and Circumscribed Squares.

Diameter of circle	×	.88623	= side of equal square.
Circumference of circle	×	.28209	= side of equal square.
Circumference of circle	×	1.1284	= perimeter of equal square.

Diameter of circle ×	.7071	= side of inscribed square.
Circumference of circle ×	.22508	= side of inscribed square.
Area of circle × .90031 ÷ diameter		= side of inscribed square.
Area of circle ×	1.2732	= area of circumscribed square.
Area of circle ×	.63662	= area of inscribed square.
Side of square ×	1.4142	= diam. of circumscribed circle.
" " ×	4.4428	= circum. " " "
" " ×	1.1284	= diam. of equal circle.
" " ×	3.5449	= circum. " "
Perimeter of square ×	0.88623	= " " "
Square inches ×	1.2732	= circular inches.

Sectors and Segments.—*To find the area of a sector of a circle.*
RULE 1. Multiply the arc of the sector by half its radius.
RULE 2. As 360 is to the number of degrees in the arc, so is the area of the circle to the area of the sector.
RULE 3. Multiply the number of degrees in the arc by the square of the radius and by .008727.

To find the area of a segment of a circle: Find the area of the sector which has the same arc, and also the area of the triangle formed by the chord of the segment and the radii of the sector.

Then take the sum of these areas, if the segment is greater than a semicircle, but take their difference if it is less.

Another Method: Area of segment $=\frac{R^2}{2}$ (arc $-$ sin A) in which A is the central angle, R the radius, and arc the length of arc to radius 1.

To find the area of a segment of a circle when its chord and height or versed sine only are given. First find radius, as follows:

$$\text{radius} = \frac{1}{2}\left[\frac{\text{square of half the chord}}{\text{height}} + \text{height}\right].$$

2. Find the angle subtended by the arc, as follows: $\frac{\text{half chord}}{\text{radius}}$ = sine of half the angle. Take the corresponding angle from a table of sines, and double it to get the angle of the arc.

3. Find area of the sector of which the segment is a part;

$$\text{area of sector} = \text{area of circle} \times \frac{\text{degrees of arc}}{360}.$$

4. Subtract area of triangle under the segment:

$$\text{Area of triangle} = \frac{\text{chord}}{2} \times (\text{radius} - \text{height of segment}).$$

The remainder is the area of the segment.

When the chord, arc, and diameter are given, to find the area. From the length of the arc subtract the length of the chord. Multiply the remainder by the radius or one-half diameter; to the product add the chord multiplied by the height, and divide the sum by 2.

Another rule: Multiply the chord by the height and this product by .6834 plus one tenth of the square of the height divided by the radius.

To find the chord: From the diameter subtract the height; multiply the remainder by four times the height and extract the square root.

When the chords of the arc and of half the arc and the versed sine are given: To the chord of the arc add four thirds of the chord of half the arc; multiply the sum by the versed sine and the product by .40426 (approximate).

Circular Ring.—*To find the area of a ring included between the circumferences of two concentric circles:* Take the difference between the areas of the two circles; or, subtract the square of the less radius from the square of the greater, and multiply their difference by 3.14159.

The area of the greater circle is equal to πR^2;
and the area of the smaller, πr^2.

Their difference, or the area of the ring, is $\pi(R^2 - r^2)$.

The Ellipse.—Area of an ellipse = product of its semi-axes × 3.14159
= product of its axes × .785398.

The Ellipse.—Circumference (approximate) $= 3.1416\sqrt{\frac{D^2 + d^2}{2}}$, D and d being the two axes.

Trautwine gives the following as more accurate: When the longer axis D is not more than five times the length of the shorter axis, d,

$$\text{Circumference} = 3.1416\sqrt{\frac{D^2+d^2}{2} - \frac{(D-d)^2}{8.8}}.$$

When D is more than $5d$, the divisor 8.8 is to be replaced by the following:

For D/d =	6	7	8	9	10	12	14	16	18	20	30	40	50
Divisor =	9	9.2	9.3	9.35	9.4	9.5	9.6	9.68	9.75	9.8	9.92	9.98	10

An accurate formula is $C = \pi(a+b)\left(1+\frac{A^2}{4}+\frac{A^4}{16}+\frac{A^6}{256}+\frac{25A^8}{16384}+\ldots\right)$, in which $A = \frac{a-b}{a+b}$.—*Ingenieurs Taschenbuch*, 1896.

Carl G. Barth (*Machinery*, Sept., 1900) gives as a very close approximation to this formula

$$C = \pi(a+b)\,\frac{64-3A^4}{64-16A^2}.$$

Area of a segment of an ellipse the base of which is parallel to one of the axes of the ellipse. Divide the height of the segment by the axis of which it is part, and find the area of a circular segment, in a table of circular segments, of which the height is equal to the quotient; multiply the area thus found by the product of the two axes of the ellipse.

Cycloid.—A curve generated by the rolling of a circle on a plane.

Length of a cycloidal curve = 4 × diameter of the generating circle.
Length of the base = circumference of the generating circle.
Area of a cycloid = 3 × area of generating circle.

Helix (Screw).—A line generated by the progressive rotation of a point around an axis and equidistant from its centre.

Length of a helix.—To the square of the circumference described by the generating-point add the square of the distance advanced in one revolution, and take the square root of their sum multiplied by the number of revolutions of the generating point. Or,

$$\sqrt{(c^2+h^2)}\,n = \text{length}, n \text{ being number of revolutions.}$$

Spirals.—Lines generated by the progressive rotation of a point around a fixed axis, with a constantly increasing distance from the axis.

A *plane spiral* is when the point rotates in one plane.

A *conical spiral* is when the point rotates around an axis at a progressing distance from its centre, and advancing in the direction of the axis, as around a cone.

Length of a plane spiral line.—When the distance between the coils is uniform.

RULE.—Add together the greater and less diameters; divide their sum by 2; multiply the quotient by 3.1416, and again by the number of revolutions. Or, take the mean of the length of the greater and less circumferences and multiply it by the number of revolutions. Or,

$$\text{length} = \pi n\,\frac{d+d'}{2}, \; d \text{ and } d' \text{ being the inner and outer diameters.}$$

Length of a conical spiral line.—Add together the greater and less diameters; divide their sum by 2 and multiply the quotient by 3.1416. To the square of the product of this circumference and the number of revolutions of the spiral add the square of the height of its axis and take the square root of the sum.

$$\text{Or, length} = \sqrt{\left(\pi n\frac{d+d'}{2}\right)^2 + h^2}.$$

SOLID BODIES.

The Prism.—*To find the surface of a right prism:* Multiply the perimeter of the base by the altitude for the convex surface. To this add the areas of the two ends when the entire surface is required.

Volume of a prism = area of its base × its altitude.

The pyramid.—Convex surface of a regular pyramid = perimeter of its base × half the slant height. To this add area of the base if the whole surface is required.

Volume of a pyramid = area of base × one third of the altitude.

To find the surface of a frustum of a regular pyramid: Multiply half the slant height by the sum of the perimeters of the two bases for the convex surface. To this add the areas of the two bases when the entire surface is required.

To find the volume of a frustum of a pyramid: Add together the areas of the two bases and a mean proportional between them, and multiply the sum by one third of the altitude. (Mean proportional between two numbers = square root of their product.)

Wedge.—A wedge is a solid bounded by five planes, viz.: a rectangular base, two trapezoids, or two rectangles, meeting in an edge, and two triangular ends. The altitude is the perpendicular drawn from any point in the edge to the plane of the base.

To find the volume of a wedge: Add the length of the edge to twice the length of the base, and multiply the sum by one sixth of the product of the height of the wedge and the breadth of the base.

Rectangular prismoid.—A rectangular prismoid is a solid bounded by six planes, of which the two bases are rectangles, having their corresponding sides parallel, and the four upright sides of the solids are trapezoids.

To find the volume of a rectangular prismoid: Add together the areas of the two bases and four times the area of a parallel section equally distant from the bases, and multiply the sum by one sixth of the altitude.

Cylinder.—Convex surface of a cylinder = perimeter of base × altitude. To this add the areas of the two ends when the entire surface is required.

Volume of a cylinder = area of base × altitude.

Cone.—Convex surface of a cone = circumference of base × half the slant side. To this add the area of the base when the entire surface is required.

$$\text{Volume of a cone} = \text{area of base} \times \frac{1}{3}\text{ altitude.}$$

To find the surface of a frustum of a cone: Multiply half the side by the sum of the circumferences of the two bases for the convex surface; to this add the areas of the two bases when the entire surface is required.

To find the volume of a frustum of a cone: Add together the areas of the two bases and a mean proportional between them, and multiply the sum by one third of the altitude.

Sphere.—*To find the surface of a sphere:* Multiply the diameter by the circumference of a great circle; or, multiply the square of the diameter by 3.14159.

Surface of sphere = 4 × area of its great circle.
" " " = convex surface of its circumscribing cylinder.

Surfaces of spheres are to each other as the squares of their diameters.

To find the volume of a sphere: Multiply the surface by one third of the radius; or, multiply the cube of the diameter by $1/6\pi$; that is, by 0.5236.

Value of $\frac{1}{6}\pi$ to 10 decimal places = .5235987756.

The volume of a sphere = 2/3 the volume of its circumscribing cylinder.

Volumes of spheres are to each other as the cubes of their diameters.

Spherical triangle.—*To find the area of a spherical triangle:* Compute the surface of the quadrantal triangle, or one eighth of the surface of the sphere. From the sum of the three angles subtract two right angles; divide the remainder by 90, and multiply the quotient by the area of the quadrantal triangle.

Spherical polygon.—*To find the area of a spherical polygon:* Compute the surface of the quadrantal triangle. From the sum of all the angles subtract the product of two right angles by the number of sides less two; divide the remainder by 90 and multiply the quotient by the area of the quadrantal triangle.

The prismoid.—The prismoid is a solid having parallel end areas, and may be composed of any combination of prisms, cylinders, wedges, pyramids, or cones or frustums of the same, whose bases and apices lie in the end areas.

Inasmuch as cylinders and cones are but special forms of prisms and pyramids, and warped surface solids may be divided into elementary forms of them, and since frustums may also be subdivided into the elementary forms, it is sufficient to say that all prismoids may be decomposed into prisms, wedges, and pyramids. If a formula can be found which is equally applicable to all of these forms, then it will apply to any combination of them. Such a formula is called

The Prismoidal Formula.

Let A = area of the base of a prism, wedge, or pyramid;
A_1, A_2, A_m = the two end and the middle areas of a prismoid, or of any of its elementary solids;
h = altitude of the prismoid or elementary solid;
V = its volume;

$$V = \frac{h}{6}(A_1 + 4A_m + A_2).$$

For a prism A_1, A_m and A_2 are equal, $= A$; $V = \frac{h}{6} \times 6A = hA$.

For a wedge with parallel ends, $A_2 = 0$, $A_m = \frac{1}{2}A_1$; $V = \frac{h}{6}(A_1 + 2A_1) = \frac{hA}{2}$.

For a cone or pyramid, $A_2 = 0$, $A_m = \frac{1}{4}A_1$; $V = \frac{h}{6}(A_1 + A_1) = \frac{hA}{3}$.

The prismoidal formula is a rigid formula for all prismoids. The only approximation involved in its use is in the assumption that the given solid may be generated by a right line moving over the boundaries of the end areas.

The area of the middle section is never the mean of the two end areas if the prismoid contains any pyramids or cones among its elementary forms. When the three sections are similar in form the *dimensions* of the middle area are always the means of the corresponding end dimensions. This fact often enables the dimensions, and hence the area of the middle section, to be computed from the end areas.

Polyedrons.—A polyedron is a solid bounded by plane polygons. A regular polyedron is one whose sides are all equal regular polygons.

To find the surface of a regular polyedron.—Multiply the area of one of the faces by the number of faces; or, multiply the square of one of the edges by the surface of a similar solid whose edge is unity.

A TABLE OF THE REGULAR POLYEDRONS WHOSE EDGES ARE UNITY.

Names.	No. of Faces.	Surface.	Volume.
Tetraedron	4	1.7320508	0.1178513
Hexaedron	6	6.0000000	1.0000000
Octaedron	8	3.4641016	0.4714045
Dodecaedron	12	20.6457288	7.6631189
Icosaedron	20	8.6602540	2.1816950

To find the volume of a regular polyedron.—Multiply the surface by one third of the perpendicular let fall from the centre on one of the faces; or, multiply the cube of one of the edges by the solidity of a similar polyedron whose edge is unity.

Solid of revolution.—The volume of any solid of revolution is equal to the product of the area of its generating surface by the length of the path of the centre of gravity of that surface.

The convex surface of any solid of revolution is equal to the product of the perimeter of its generating surface by the length of path of its centre of gravity.

Cylindrical ring.—Let d = outer diameter; d' = inner diameter; $\frac{1}{2}(d - d')$ = thickness = t; $\frac{1}{4}\pi t^2$ = sectional area; $\frac{1}{2}(d + d')$ = mean diameter = M; πt = circumference of section; πM = mean circumference of ring; surface $= \pi t \times \pi M$; $= \frac{1}{4}\pi^2(d^2 - d'^2)$; $= 9.86965\, t\, M$; $= 2.46741\,(d^2 - d'^2)$; volume $= \frac{1}{4}\pi t^2 M \pi$; $= 2.46741 t^2 M$.

Spherical zone.—*Surface of a spherical zone* or segment of a sphere = its altitude × the circumference of a great circle of the sphere. A great circle is one whose plane passes through the centre of the sphere.

Volume of a zone of a sphere.—To the sum of the squares of the radii of the ends add one third of the square of the height; multiply the sum by the height and by 1.5708.

Spherical segment.—*Volume of a spherical segment with one base.*—

Multiply half the height of the segment by the area of the base, and the cube of the height by .5236 and add the two products. Or, from three times the diameter of the sphere subtract twice the height of the segment; multiply the difference by the square of the height and by .5236. Or, to three times the square of the radius of the base of the segment add the square of its height, and multiply the sum by the height and by .5236.

Spheroid or ellipsoid.—When the revolution of the spheroid is about the transverse diameter it is *prolate*, and when about the conjugate it is *oblate*.

Convex surface of a segment of a spheroid.—Square the diameters of the spheroid, and take the square root of half their sum; then, as the diameter from which the segment is cut is to this root so is the height of the segment to the proportionate height of the segment to the mean diameter. Multiply the product of the other diameter and 3.1416 by the proportionate height.

Convex surface of a frustum or zone of a spheroid.—Proceed as by previous rule for the surface of a segment, and obtain the proportionate height of the frustum. Multiply the product of the diameter parallel to the base of the frustum and 3.1416 by the proportionate height of the frustum.

Volume of a spheroid is equal to the product of the square of the revolving axis by the fixed axis and by .5236. The volume of a spheroid is two thirds of that of the circumscribing cylinder.

Volume of a segment of a spheroid.—1. When the base is parallel to the revolving axis, multiply the difference between three times the fixed axis and twice the height of the segment, by the square of the height and by .5236. Multiply the product by the square of the revolving axis, and divide by the square of the fixed axis.

2. When the base is perpendicular to the revolving axis, multiply the difference between three times the revolving axis and twice the height of the segment by the square of the height and by .5236. Multiply the product by the length of the fixed axis, and divide by the length of the revolving axis.

Volume of the middle frustum of a spheroid.—1. When the ends are circular, or parallel to the revolving axis: To twice the square of the middle diameter add the square of the diameter of one end; multiply the sum by the length of the frustum and by .2618.

2. When the ends are elliptical, or perpendicular to the revolving axis: To twice the product of the transverse and conjugate diameters of the middle section add the product of the transverse and conjugate diameters of one end; multiply the sum by the length of the frustum and by .2618.

Spindles.—Figures generated by the revolution of a plane area, when the curve is revolved about a chord perpendicular to its axis, or about its double ordinate. They are designated by the name of the arc or curve from which they are generated, as Circular, Elliptic, Parabolic, etc., etc.

Convex surface of a circular spindle, zone, or segment of it.—Rule: Multiply the length by the radius of the revolving arc; multiply this arc by the central distance, or distance between the centre of the spindle and centre of the revolving arc; subtract this product from the former, double the remainder, and multiply it by 3.1416.

Volume of a circular spindle.—Multiply the central distance by half the area of the revolving segment; subtract the product from one third of the cube of half the length, and multiply the remainder by 12.5664.

Volume of frustum or zone of a circular spindle.—From the square of half the length of the whole spindle take one third of the square of half the length of, the frustum, and multiply the remainder by the said half length of the frustum; multiply the central distance by the revolving area which generates the frustum; subtract this product from the former, and multiply the remainder by 6.2832.

Volume of a segment of a circular spindle.—Subtract the length of the segment from the half length of the spindle; double the remainder and ascertain the volume of a middle frustum of this length; subtract the result from the volume of the whole spindle and halve the remainder.

Volume of a cycloidal spindle = five eighths of the volume of the circumscribing cylinder.—Multiply the product of the square of twice the diameter of the generating circle and 3.927 by its circumference, and divide this product by 8.

Parabolic conoid.—*Volume of a parabolic conoid* (generated by the revolution of a parabola on its axis).—Multiply the area of the base by half the height.

Or multiply the square of the diameter of the base by the height and by .3927.

Volume of a frustum of a parabolic conoid.—Multiply half the sum of the areas of the two ends by the height.

Volume of a parabolic spindle (generated by the revolution of a parabola on its base).—Multiply the square of the middle diameter by the length and by .4189.

The volume of a parabolic spindle is to that of a cylinder of the same height and diameter as 8 to 15.

Volume of the middle frustum of a parabolic spindle.—Add together 8 times the square of the maximum diameter, 3 times the square of the end diameter, and 4 times the product of the diameters. Multiply the sum by the length of the frustum and by .05236.

This rule is applicable for calculating the content of casks of parabolic form.

Casks.—*To find the volume of a cask of any form.*—Add together 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the diameters. Multiply the sum by the length, and divide by 31,773 for the content in Imperial gallons, or by 26,470 for U. S. gallons.

This rule was framed by Dr. Hutton, on the supposition that the middle third of the length of the cask was a frustum of a parabolic spindle, and each outer third was a frustum of a cone.

To find the ullage of a cask, the quantity of liquor in it when it is not full.

1. For a *lying cask:* Divide the number of wet or dry inches by the bung diameter in inches. If the quotient is less than .5, deduct from it one fourth part of what it wants of .5. If it exceeds .5, add to it one fourth part of the excess above .5. Multiply the remainder or the sum by the whole content of the cask. The product is the quantity of liquor in the cask, in gallons, when the dividend is *wet* inches; or the empty space, if *dry* inches.

2. For a *standing cask:* Divide the number of wet or dry inches by the length of the cask. If the quotient exceeds .5, add to it one tenth of its excess above .5; if less than .5, subtract from it one tenth of what it wants of .5. Multiply the sum or the remainder by the whole content of the cask. The product is the quantity of liquor in the cask, when the dividend is wet inches; or the empty space, if dry inches.

Volume of cask (approximate) U. S. gallons = square of mean diam. × length in inches × .0034. Mean diam. = half the sum of the bung and head diams.

Volume of an irregular solid.—Suppose it divided into parts, resembling prisms or other bodies measurable by preceding rules. Find the content of each part; the sum of the contents is the cubic contents of the solid.

The content of a small part is found nearly by multiplying half the sum of the areas of each end by the perpendicular distance between them.

The contents of small irregular solids may sometimes be found by immersing them under water in a prismatic or cylindrical vessel, and observing the amount by which the level of the water descends when the solid is withdrawn. The sectional area of the vessel being multiplied by the descent of the level gives the cubic contents.

Or, weigh the solid in air and in water; the difference is the weight of water it displaces. Divide the weight in pounds by 62.4 to obtain volume in cubic feet, or multiply it by 27.7 to obtain the volume in cubic inches.

When the solid is very large and a great degree of accuracy is not requisite, measure its length, breadth, and depth in several different places, and take the mean of the measurement for each dimension, and multiply the three means together.

When the surface of the solid is very extensive it is better to divide it into triangles, to find the area of each triangle, and to multiply it by the mean depth of the triangle for the contents of each triangular portion; the contents of the triangular sections are to be added together.

The mean depth of a triangular section is obtained by measuring the depth at each angle, adding together the three measurements, and taking one third of the sum.

PLANE TRIGONOMETRY.

Trigonometrical Functions.

Every triangle has six parts—three angles and three sides. When any three of these parts are given, provided one of them is a side, the other parts may be determined. By the *solution* of a triangle is meant the determination of the unknown parts of a triangle when certain parts are given.

The *complement* of an angle or arc is what remains after subtracting the angle or arc from 90°.

In general, if we represent any arc by A, its complement is $90° - A$. Hence the complement of an arc that exceeds 90° is negative.

Since the two acute angles of a right-angled triangle are together equal to a right angle, each of them is the complement of the other.

The *supplement* of an angle or arc is what remains after subtracting the angle or arc from 180°. If A is an arc its supplement is $180° - A$. The supplement of an arc that exceeds 180° is negative.

The sum of the three angles of a triangle is equal to 180°. Either angle is the supplement of the other two. In a right-angled triangle, the right angle being equal to 90°, each of the acute angles is the complement of the other.

In all right-angled triangles having the same acute angle, the sides have to each other the same ratio. These ratios have received special names, as follows:

If A is one of the acute angles, a the opposite side, b the adjacent side, and c the hypothenuse.

The sine *of the angle A is the quotient of the opposite side divided by the hypothenuse.* *Sin.* $A = \frac{a}{c}$.

The tangent *of the angle A is the quotient of the opposite side divided by the adjacent side.* *Tang.* $A = \frac{a}{b}$.

The secant *of the angle A is the quotient of the hypothenuse divided by the adjacent side.* *Sec.* $A = \frac{c}{b}$.

The cosine, cotangent, and cosecant of an angle are respectively the sine, tangent, and secant of the complement of that angle. The terms sine, cosine, etc., are called trigonometrical functions.

In a circle whose radius is unity, **the sine** *of an arc, or of the angle at the centre measured by that arc, is the perpendicular let fall from one extremity of the arc upon the diameter passing through the other extremity.*

The tangent *of an arc is the line which touches the circle at one extremity of the arc, and is limited by the diameter (produced) passing through the other extremity.*

The secant *of an arc is that part of the produced diameter which is intercepted between the centre and the tangent.*

The versed sine *of an arc is that part of the diameter intercepted between the extremity of the arc and the foot of the sine.*

In a circle whose radius is not unity, the trigonometric functions of an arc will be equal to the lines here defined, divided by the radius of the circle.

If $I\,C\,A$ (Fig. 70) is an angle in the first quadrant, and $C\,F =$ radius,

$$\text{The sine of the angle} = \frac{F\,G}{\text{Rad.}}.\quad \text{Cos} = \frac{C\,G}{\text{Rad.}} = \frac{K\,F}{\text{Rad.}}$$

$$\text{Tang.} = \frac{I\,A}{\text{Rad.}}.\quad \text{Secant} = \frac{C\,I}{\text{Rad.}}.\quad \text{Cot.} = \frac{D\,L}{\text{Rad.}}.$$

$$\text{Cosec.} = \frac{C\,L}{\text{Rad.}}.\quad \text{Versin.} = \frac{G\,A}{\text{Rad.}}.$$

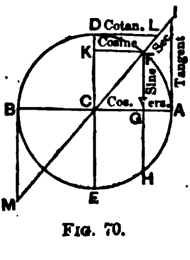


FIG. 70.

If radius is 1, then Rad. in the denominator is omitted, and sine $= F\,G$, *etc.*

The sine of an arc = half the chord of twice the arc.

The sine of the supplement of the arc is the same as that of the arc itself. Sine of arc $B\,D\,F = F\,G =$ sin arc $F\,A$.

The tangent of the supplement is equal to the tangent of the arc, but wi[th] a contrary sign. Tang. $BDF = BM$.

The secant of the supplement is equal to the secant of the arc, but with [a] contrary sign. Sec. $BDF = CM$.

Signs of the functions in the four quadrants.—If w[e] divide a circle into four quadrants by a vertical and a horizontal diameter, the upper right-hand quadrant is called the first, the upper left the second, the lower left the third, and the lower right the fourth. The signs o[f] the functions in the four quadrants are as follows:

	First quad.	*Second* quad.	*Third* quad.	*Fourth* qua[d.]
Sine and cosecant,	+	+	−	−
Cosine and secant,	+	−	−	+
Tangent and cotangent,	+	−	+	−

The values of the functions are as follows for the angles specified:

Angle	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	3[illegible]°
Sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{8}}{2}$	−1	0	1
Tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	−1	$-\frac{1}{\sqrt{8}}$	0	∞	0
Cotangent	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	−1	$-\sqrt{8}$	∞	0	∞
Secant	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	−2	$-\sqrt{2}$	$-\frac{2}{\sqrt{8}}$	−1	∞	1
Cosecant	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	−1	∞
Versed sine	0	$\frac{2-\sqrt{3}}{2}$	$\frac{\sqrt{2}-1}{\sqrt{2}}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{\sqrt{2}+1}{\sqrt{2}}$	$\frac{2+\sqrt{8}}{2}$	2	1	0

TRIGONOMETRICAL FORMULÆ.

The following relations are deduced from the properties of similar t[ri]angles (Radius = 1):

$$\cos A : \sin A :: 1 : \tan A, \text{ whence } \tan A = \frac{\sin A}{\cos A};$$

$$\sin A : \cos A :: 1 : \cot A, \quad \text{"} \quad \text{cotan } A = \frac{\cos A}{\sin A};$$

$$\cos A : 1 :: 1 : \sec A, \quad \text{"} \quad \sec A = \frac{1}{\cos A};$$

$$\sin A : 1 :: 1 : \text{cosec } A, \quad \text{"} \quad \text{cosec } A = \frac{1}{\sin A};$$

$$\tan A : 1 :: 1 : \cot A \quad \text{"} \quad \tan A = \frac{1}{\cot A}.$$

The sum of the square of the sine of an arc and the square of its cosi[ne] equals unity. $\sin^2 A + \cos^2 A = 1$.

Also, $1 + \tan^2 A = \sec^2 A$; $1 + \cot^2 A = \text{cosec}^2 A$.

Functions of the sum and difference of two angles:

Let the two angles be denoted by A and B, their sum $A + B = C$, a[nd] their difference $A - B$ by D.

$$\sin (A + B) = \sin A \cos B + \cos A \sin B; \quad . \; . \; . \; . \; . \; ($$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B; \quad \ldots \quad (2)$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B; \quad \ldots \quad (3)$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B. \quad \ldots \quad (4)$$

From these four formulæ by addition and subtraction we obtain

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B; \quad \ldots \quad (5)$$

$$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B; \quad \ldots \quad (6)$$

$$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B; \quad \ldots \quad (7)$$

$$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B. \quad \ldots \quad (8)$$

If we put $A+B=C$, and $A-B=D$, then $A = \frac{1}{2}(C+D)$ and $B = \frac{1}{2}(C-D)$, and we have

$$\sin C + \sin D = 2 \sin \tfrac{1}{2}(C+D) \cos \tfrac{1}{2}(C-D); \quad \ldots \quad (9)$$

$$\sin C - \sin D = 2 \cos \tfrac{1}{2}(C+D) \sin \tfrac{1}{2}(C-D); \quad \ldots \quad (10)$$

$$\cos C + \cos D = 2 \cos \tfrac{1}{2}(C+D) \cos \tfrac{1}{2}(C-D); \quad \ldots \quad (11)$$

$$\cos D - \cos C = 2 \sin \tfrac{1}{2}(C+D) \sin \tfrac{1}{2}(C-D). \quad \ldots \quad (12)$$

Equation (9) may be enunciated thus: The sum of the sines of any two angles is equal to twice the sine of half the sum of the angles multiplied by the cosine of half their difference. These formulæ enable us to transform a sum or difference into a product.

The sum of the sines of two angles is to their difference as the tangent of half the sum of those angles is to the tangent of half their difference.

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)}{2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}. \quad (13)$$

The sum of the cosines of two angles is to their difference as the cotangent of half the sum of those angles is to the tangent of half their difference.

$$\frac{\cos A + \cos B}{\cos B - \cos A} = \frac{2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)}{2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)} = \frac{\cot \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}. \quad (14)$$

The sine of the sum of two angles is to the sine of their difference as the sum of the tangents of those angles is to the difference of the tangents.

$$\frac{\sin(A+B)}{\sin(A-B)} = \frac{\tan A + \tan B}{\tan A - \tan B}; \quad \ldots \quad (15)$$

$$\frac{\sin(A+B)}{\cos A \cos B} = \tan A + \tan B; \qquad \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B};$$

$$\frac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B; \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B};$$

$$\frac{\cos(A+B)}{\cos A \cos B} = 1 - \tan A \tan B; \qquad \cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A};$$

$$\frac{\cos(A-B)}{\cos A \cos B} = 1 + \tan A \tan B; \qquad \cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}.$$

Functions of twice an angle:

$$\sin 2A = 2 \sin A \cos A; \qquad \cos 2A = \cos^2 A - \sin^2 A;$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}; \qquad \cot 2A = \frac{\cot^2 A - 1}{2 \cot A}.$$

Functions of half an angle:

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}; \qquad \cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}};$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}; \qquad \cot \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{1 - \cos A}}.$$

Solution of Plane Right-angled Triangles.

Let A and B be the two acute angles and C the right angle, and a, b, and c the sides opposite these angles, respectively, then we have

$$1.\ \sin A = \cos B = \frac{a}{c}; \quad 3.\ \tan A = \cot B = \frac{a}{b};$$

$$2.\ \cos A = \sin B = \frac{b}{c}; \quad 4.\ \cot A = \tan B = \frac{b}{a}.$$

1. In any plane right-angled triangle the sine of either of the acute angles is equal to the quotient of the opposite leg divided by the hypothenuse.

2. The cosine of either of the acute angles is equal to the quotient of the adjacent leg divided by the hypothenuse.

3. The tangent of either of the acute angles is equal to the quotient of the opposite leg divided by the adjacent leg.

4. The cotangent of either of the acute angles is equal to the quotient of the adjacent leg divided by the opposite leg.

5. The square of the hypothenuse equals the sum of the squares of the other two sides.

Solution of Oblique-angled Triangles.

The following propositions are proved in works on plane trigonometry. In any plane triangle—

Theorem 1. The sines of the angles are proportional to the opposite sides.

Theorem 2. The sum of any two sides is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.

Theorem 3. If from any angle of a triangle a perpendicular be drawn to the opposite side or base, the whole base will be to the sum of the other two sides as the difference of those two sides is to the difference of the segments of the base.

CASE I. Given two angles and a side, to find the third angle and the other two sides. 1. The third angle = 180° − sum of the two angles. 2. The sides may be found by the following proportion:

The sine of the angle opposite the given side is to the sine of the angle opposite the required side as the given side is to the required side.

CASE II. Given two sides and an angle opposite one of them, to find the third side and the remaining angles.

The side opposite the given angle is to the side opposite the required angle as the sine of the given angle is to the sine of the required angle.

The third angle is found by subtracting the sum of the other two from 180°, and the third side is found as in Case I.

CASE III. Given two sides and the included angle, to find the third side and the remaining angles.

The sum of the required angles is found by subtracting the given angle from 180°. The difference of the required angles is then found by Theorem II. Half the difference added to half the sum gives the greater angle, and half the difference subtracted from half the sum gives the less angle. The third side is then found by Theorem I.

Another method:

Given the sides c, b, and the included angle A, to find the remaining side a and the remaining angles B and C.

From either of the unknown angles, as B, draw a perpendicular $B\,e$ to the opposite side.

Then

$$A\,e = c \cos A, \quad B\,e = c \sin A, \quad e\,C = b - A\,e, \quad B\,e \div e\,C = \tan C.$$

Or, in other words, solve $B\,e$, $A\,e$ and $B\,e\,C$ as right-angled triangles.

CASE IV. Given the three sides, to find the angles.

Let fall a perpendicular upon the longest side from the opposite angle, dividing the given triangle into two right-angled triangles. The two segments of the base may be found by Theorem III. There will then be given the hypothenuse and one side of a right-angled triangle to find the angles.

For areas of triangles, see Mensuration.

ANALYTICAL GEOMETRY.

Analytical geometry is that branch of Mathematics which has for its object the determination of the forms and magnitudes of geometrical magnitudes by means of analysis.

Ordinates and abscissas.—In analytical geometry two intersecting lines YY', XX' are used as *coördinate axes*, XX' being the axis of abscissas or axis of X, and YY' the axis of ordinates or axis of Y. A, the intersection, is called the origin of coördinates. The distance of any point P from the axis of Y measured parallel to the axis of X is called the *abscissa* of the point, as AD or CP, Fig. 71. Its distance from the axis of X, measured parallel to the axis of Y, is called the *ordinate*, as AC or PD. The abscissa and ordinate taken together are called the coördinates of the point P. The angle of intersection is usually taken as a right angle, in which case the axes of X and Y are called *rectangular coördinates*.

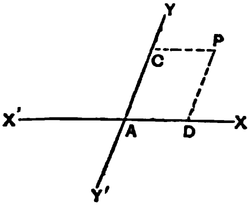


FIG. 71.

The abscissa of a point is designated by the letter x and the ordinate by y.

The *equations* of a point are the equations which express the distances of the point from the axis. Thus $x = a$, $y = b$ are the equations of the point P.

Equations referred to rectangular coördinates.—The equation of a line expresses the relation which exists between the coördinates of every point of the line.

Equation of a straight line, $y = ax \pm b$, in which a is the tangent of the angle the line makes with the axis of X, and b the distance above A in which the line cuts the axis of Y.

Every equation of the first degree between two variables is the equation of a straight line, as $Ay + Bx + C = 0$, which can be reduced to the form $y = ax \pm b$.

Equation of the distance between two points:

$$D = \sqrt{(x'' - x')^2 + (y'' - y')^2},$$

in which $x'y'$, $x''y''$ are the coördinates of the two points.

Equation of a line passing through a given point:

$$y - y' = a(x - x'),$$

in which $x'y'$ are the coördinates of the given point, a, the tangent of the angle the line makes with the axis of x, being undetermined, since any number of lines may be drawn through a given point.

Equation of a line passing through two given points:

$$y - y' = \frac{y'' - y'}{x'' - x'}(x - x').$$

Equation of a line parallel to a given line and through a given point:

$$y - y' = a(x - x').$$

Equation of an angle V included between two given lines:

$$\text{tang } V = \frac{a' - a}{1 + a'a},$$

in which a and a' are the tangents of the angles the lines make with the axis of abscissas.

If the lines are at right angles to each other tang $V = \infty$, and

$$1 + a'a = 0.$$

Equation of an intersection of two lines, whose equations are

$$y = ax + b, \quad \text{and} \quad y = a'x + b',$$

$$x = -\frac{b - b'}{a - a'}, \quad \text{and} \quad y = \frac{ab' - a'b}{a - a'}.$$

Equation of a perpendicular from a given point to a given line:

$$y - y' = -\frac{1}{a}(x - x').$$

Equation of the length of the perpendicular P:

$$P = \frac{y' - ax' - b}{\sqrt{1 + a^2}}.$$

The circle.—Equation of a circle, the origin of coördinates being at the centre, and radius $= R$:

$$x^2 + y^2 = R^2.$$

If the origin is at the left extremity of the diameter, on the axis of X:

$$y^2 = 2Rx - x^2.$$

If the origin is at any point, and the coördinates of the centre are $x'y'$:

$$(x - x')^2 + (y - y')^2 = R^2.$$

Equation of a tangent to a circle, the coördinates of the point of tangency being $x''y''$ and the origin at the centre,

$$yy'' + xx'' = R^2.$$

The ellipse.—Equation of an ellipse, referred to rectangular coördinates with axis at the centre:

$$A^2y^2 + B^2x^2 = A^2B^2,$$

in which A is half the transverse axis and B half the conjugate axis.

Equation of the ellipse when the origin is at the vertex of the transverse axis:

$$y^2 = \frac{B^2}{A^2}(2Ax - x^2).$$

The eccentricity of an ellipse is the distance from the centre to either focus, divided by the semi-transverse axis, or

$$e = \frac{\sqrt{A^2 - B^2}}{A}.$$

The parameter of an ellipse is the double ordinate passing through the focus. It is a third proportional to the transverse axis and its conjugate, or

$$2A : 2B :: 2B : \text{parameter}; \text{ or parameter} = \frac{2B^2}{A}.$$

Any ordinate of a circle circumscribing an ellipse is to the corresponding ordinate of the ellipse as the semi-transverse axis to the semi-conjugate. Any ordinate of a circle inscribed in an ellipse is to the corresponding ordinate of the ellipse as the semi-conjugate axis to the semi-transverse.

Equation of the tangent to an ellipse, origin of axes at the centre:

$$A^2yy'' + B^2xx'' = A^2B^2,$$

$y''x''$ being the coördinates of the point of tangency.

Equation of the normal, passing through the point of tangency, and perpendicular to the tangent:

$$y - y'' = \frac{A^2y''}{B^2x''}(x - x'').$$

The normal bisects the angle of the two lines drawn from the point of tangency to the foci.

The lines drawn from the foci make equal angles with the tangent.

The parabola.—Equation of the parabola referred to rectangular coördinates, the origin being at the vertex of its axis, $y^2 = 2px$, in which $2p$ is the parameter or double ordinate through the focus.

The parameter is a third proportional to any abscissa and its corresponding ordinate, or

$$x : y :: y : 2p.$$

Equation of the tangent:

$$yy'' = p(x + x''),$$

$y''x''$ being coördinates of the point of tangency.

Equation of the normal:

$$y - y'' = -\frac{y''}{p}(x - x'').$$

The sub normal, or projection of the normal on the axis, is constant, and equal to half the parameter.

The tangent at any point makes equal angles with the axis and with the line drawn from the point of tangency to the focus.

The hyperbola.—Equation of the hyperbola referred to rectangular coördinates, origin at the centre:

$$A^2y^2 - B^2x^2 = -A^2B^2,$$

in which A is the semi-transverse axis and B the semi-conjugate axis.

Equation when the origin is at the right vertex of the transverse axis:

$$y^2 = \frac{B^2}{A^2}(2Ax + x^2).$$

Conjugate and equilateral hyperbolas.—If on the conjugate axis, as a transverse, and a focal distance equal to $\sqrt{A^2 + B^2}$, we construct the two branches of a hyperbola, the two hyperbolas thus constructed are called conjugate hyperbolas. If the transverse and conjugate axes are equal, the hyperbolas are called equilateral, in which case $y^2 - x^2 = -A^2$ when A is the transverse axis, and $x^2 - y^2 = -B^2$ when B is the transverse axis.

The parameter of the transverse axis is a third proportional to the transverse axis and its conjugate.

$$2A : 2B :: 2B : \text{parameter}.$$

The tangent to a hyperbola bisects the angle of the two lines drawn from the point of tangency to the foci.

The asymptotes of a hyperbola are the diagonals of the rectangle described on the axes, indefinitely produced in both directions.

In an equilateral hyperbola the asymptotes make equal angles with the transverse axis, and are at right angles to each other.

The asymptotes continually approach the hyperbola, and become tangent to it at an infinite distance from the centre.

Conic sections.—Every equation of the second degree between two variables will represent either a circle, an ellipse, a parabola or a hyperbola. These curves are those which are obtained by intersecting the surface of a cone by planes, and for this reason they are called conic sections.

Logarithmic curve.—A logarithmic curve is one in which one of the coördinates of any point is the logarithm of the other.

The coördinate axis to which the lines denoting the logarithms are parallel is called the *axis of logarithms*, and the other the axis of numbers. If y is the axis of logarithms and x the axis of numbers, the equation of the curve is $y = \log x$.

If the base of a system of logarithms is a, we have $a^y = x$, in which y is the logarithm of x.

Each system of logarithms will give a different logarithmic curve. If $y = 0$, $x = 1$. Hence every logarithmic curve will intersect the axis of numbers at a distance from the origin equal to 1.

DIFFERENTIAL CALCULUS.

The differential of a variable quantity is the difference between any two of its consecutive values; hence it is indefinitely small. It is expressed by writing d before the quantity, as dx, which is read differential of x.

The term $\frac{dy}{dx}$ is called the differential coefficient of y regarded as a function of x.

The differential of a function is equal to its differential coefficient multiplied by the differential of the independent variable; thus, $\frac{dy}{dx}dx = dy$.

The *limit* of a variable quantity is that value to which it continually approaches, so as at last to differ from it by less than any assignable quantity.

The differential coefficient is the limit of the ratio of the increment of the independent variable to the increment of the function.

The differential of a constant quantity is equal to 0.

The differential of a product of a constant by a variable is equal to the constant multiplied by the differential of the variable.

$$\text{If} \quad u = Av, \quad du = Adv.$$

In any curve whose equation is $y = f(x)$, the differential coefficient $\frac{dy}{dx} = \tan a$; hence, the rate of increase of the function, or the ascension of the curve at any point, is equal to the tangent of the angle which the tangent line makes with the axis of abscissas.

All the operations of the Differential Calculus comprise but two objects:

1. To find the rate of change in a function when it passes from one state of value to another, consecutive with it.

2. To find the actual change in the function: The rate of change is the differential coefficient, and the actual change the differential.

Differentials of algebraic functions.—The differential of the sum or difference of any number of functions, dependent on the same variable, is equal to the sum or difference of their differentials taken separately:

$$\text{If} \quad u = y + z - w, \quad du = dy + dz - dw.$$

The differential of a product of two functions dependent on the same variable is equal to the sum of the products of each by the differential of the other:

$$d(uv) = vdu + udv. \qquad \frac{d(uv)}{uv} = \frac{du}{u} + \frac{dv}{v}.$$

The differential of the product of any number of functions is equal to the sum of the products which arise by multiplying the differential of each function by the product of all the others:

$$d(uts) = tsdu + usdt + utds.$$

The differential of a fraction equals the denominator into the differential of the numerator minus the numerator into the differential of the denominator, divided by the square of the denominator:

$$dt = d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2}.$$

If the denominator is constant, $dv = 0$, and $dt = \frac{vdu}{v^2} = \frac{du}{v}$.

If the numerator is constant, $du = 0$, and $dt = -\frac{udv}{v^2}$

The differential of the square root of a quantity is equal to the differential of the quantity divided by twice the square root of the quantity:

$$\text{If} \quad v = u^{\frac{1}{2}}, \quad \text{or} \quad v = \sqrt{u}, \quad dv = \frac{du}{2\sqrt{u}}; = \frac{1}{2}u^{-\frac{1}{2}}du.$$

The differential of any power of a function is equal to the exponent multiplied by the function raised to a power less one, multiplied by the differential of the function, $d(u^n) = nu^{n-1}du$.

Formulas for differentiating algebraic functions.

1. $d(a) = 0$.
2. $d(ax) = adx$.
3. $d(x + y) = dx + dy$.
4. $d(x - y) = dx - dy$.
5. $d(xy) = xdy + ydx$.
6. $d\left(\frac{x}{y}\right) = \frac{ydx - xdy}{y^2}$.
7. $d(x^m) = mx^{m-1}dx$.
8. $d(\sqrt{x}) = \frac{dx}{2\sqrt{x}}$.
9. $d\left(x^{-\frac{r}{s}}\right) = -\frac{r}{s}x^{-\frac{r}{s}-1}dx$.

To find the differential of the form $u = (a + bx^n)^m$:
Multiply the exponent of the parenthesis into the exponent of the variable within the parenthesis, into the coefficient of the variable, into the binomial raised to a power less 1, into the variable within the parenthesis raised to a power less 1, into the differential of the variable.

$$du = d(a + bx^n)^m = mnb(a + bx^n)^{m-1}x^{n-1}dx.$$

To find the rate of change for a given value of the variable:
Find the differential coefficient, and substitute the value of the variable in the second member of the equation.

EXAMPLE.—If x is the side of a cube and u its volume, $u = x^3$, $\frac{du}{dx} = 3x^2$. Hence the rate of change in the volume is three times the square of the edge. If the edge is denoted by 1, the rate of change is 3.

Application. The coefficient of expansion by heat of the volume of a body is three times the linear coefficient of expansion. Thus if the side of a cube expands .001 inch, its volume expands .003 cubic inch. $1.001^3 = 1.003003001$.

A partial differential coefficient is the differential coefficient of a function of two or more variables under the supposition that only one of them has changed its value.

A partial differential is the differential of a function of two or more variables under the supposition that only one of them has changed its value.

The total differential of a function of any number of variables is equal to the sum of the partial differentials.

If $u = f(xy)$, the partial differentials are $\frac{du}{dx}dx$, $\frac{du}{dy}dy$.

If $u = x^2 + y^3 - z$, $du = \frac{du}{dx}dx + \frac{du}{dy}dy + \frac{du}{dz}dz$; $= 2xdx + 3y^2dy - dz$.

Integrals.—An integral is a functional expression derived from a differential. Integration is the operation of finding the primitive function from the differential function. It is indicated by the sign $\int$, which is read "the integral of." Thus $\int 2xdx = x^2$; read, the integral of $2xdx$ equals x^2.

To integrate an expression of the form $mx^{m-1}dx$ or $x^m dx$, add 1 to the exponent of the variable, and divide by the new exponent and by the differential of the variable: $\int 3x^2dx = x^3$. (Applicable in all cases except when $m = -1$. For $\int x^{-1}dx$ see formula 2 page 78.)

The integral of the product of a constant by the differential of a variable is equal to the constant multiplied by the integral of the differential;

$$\int ax^m dx = a\int x^m dx = a\frac{1}{m+1}x^{m+1}.$$

The integral of the algebraic sum of any number of differentials is equal to the algebraic sum of their integrals:

$$du = 2ax^2dx - bydy - z^2dz; \quad \int du = \frac{2}{3}ax^3 - \frac{b}{2}y^2 - \frac{z^3}{3}.$$

Since the differential of a constant is 0, a constant connected with a variable by the sign + or − disappears in the differentiation; thus $d(a + x^m) = dx^m = mx^{m-1}dx$. Hence in integrating a differential expression we must annex to the integral obtained a constant represented by C to compensate for the term which may have been lost in differentiation. Thus if we have $dy = adx$; $\int dy = a\int dx$. Integrating,

$$y = ax \pm C.$$

The constant C, which is added to the first integral, must have such a value as to render the functional equation true for every possible value that may be attributed to the variable. Hence, after having found the first integral equation and added the constant C, if we then make the variable equal to zero, the value which the function assumes will be the true value of C.

An indefinite integral is the first integral obtained before the value of the constant C is determined.

A particular integral is the integral after the value of C has been found.

A definite integral is the integral corresponding to a given value of the variable.

Integration between limits.—Having found the indefinite integral and the particular integral, the next step is to find the definite integral, and then the definite integral between given limits of the variable.

The integral of a function, taken between two limits, indicated by given values of x, is equal to the difference of the definite integrals corresponding to those limits. The expression

$$\int_{x'}^{x''} dy = a \int dx$$

is read: Integral of the differential of y, taken between the limits x' and x''; the least limit, or the limit corresponding to the subtractive integral, being placed below.

Integrate $du = 9x^2dx$ between the limits $x = 1$ and $x = 3$, u being equal to 81 when $x = 0$. $\int du = \int 9x^2dx = 3x^3 + C$; $C = 81$ when $x = 0$, then

$$\int_{x=1}^{x=3} du = 3(3)^3 + 81, \text{ minus } 3(1)^3 + 81 = 78.$$

Integration of particular forms.

To integrate a differential of the form $du = (a + bx^n)^m x^{n-1}dx$.

1. If there is a constant factor, place it without the sign of the integral, and omit the power of the variable without the parenthesis and the differential;

2. Augment the exponent of the parenthesis by 1, and then divide this quantity, with the exponent so increased, by the exponent of the parenthesis, into the exponent of the variable within the parenthesis, into the coefficient of the variable. Whence

$$\int du = \frac{(a + bx^n)^{m+1}}{(m+1)nb} = C.$$

The differential of an arc is the hypothenuse of a right-angle triangle of which the base is dx and the perpendicular dy.

If z is an arc, $dz = \sqrt{dx^2 + dy^2}$ $\quad z = \int \sqrt{dx^2 + dy^2}$.

Quadrature of a plane figure.

The differential of the area of a plane surface is equal to the ordinate into the differential of the abscissa.

$$ds = ydx.$$

To apply the principle enunciated in the last equation, in finding the area of any particular plane surface:

Find the value of y in terms of x, from the equation of the bounding line; substitute this value in the differential equation, and then integrate between the required limits of x.

Area of the parabola.—Find the area of any portion of the common parabola whose equation is

$$y^2 = 2px; \quad \text{whence} \quad y = \sqrt{2px}.$$

Substituting this value of y in the differential equation $ds = ydx$ gives

$$\int ds = \int \sqrt{2pxdx} = \sqrt{2p}\int x^{\frac{1}{2}}dx = \frac{2\sqrt{2p}}{3}x^{\frac{3}{2}} + C;$$

$$\text{or,}\qquad s = \frac{2\sqrt{2px}\times x}{3} = \frac{2}{3}xy + C.$$

If we estimate the area from the principal vertex, $x = 0$. $y = 0$, and $C = 0$; and denoting the particular integral by s', $s' = \frac{2}{3}xy$.

That is, the area of any portion of the parabola, estimated from the vertex, is equal to ⅔ of the rectangle of the abscissa and ordinate of the extreme point. The curve is therefore quadrable.

Quadrature of surfaces of revolution.—The differential of a surface of revolution is equal to the circumference of a circle perpendicular to the axis into the differential of the arc of the meridian curve.

$$ds = 2\pi y\sqrt{dx^2 + dy^2};$$

in which y is the radius of a circle of the bounding surface in a plane perpendicular to the axis of revolution, and x is the abscissa, or distance of the plane from the origin of coördinate axes.

Therefore, to find the volume of any surface of revolution:

Find the value of y and dy from the equation of the meridian curve in terms of x and dx, then substitute these values in the differential equation, and integrate between the proper limits of x.

By application of this rule we may find:

The curved surface of a cylinder equals the product of the circumference of the base into the altitude.

The convex surface of a cone equals the product of the circumference of the base into half the slant height.

The surface of a sphere is equal to the area of four great circles, or equal to the curved surface of the circumscribing cylinder.

Cubature of volumes of revolution.—A volume of revolution is a volume generated by the revolution of a plane figure about a fixed line called the axis.

If we denote the volume by V, $dV = \pi y^2\,dx$.

The area of a circle described by any ordinate y is πy^2; hence the differential of a volume of revolution is equal to the area of a circle perpendicular to the axis into the differential of the axis.

The differential of a volume generated by the revolution of a plane figure about the axis of Y is πx^2dy.

To find the value of V for any given volume of revolution:

Find the value of y^2 in terms of x from the equation of the meridian curve, substitute this value in the differential equation, and then integrate between the required limits of x.

By application of this rule we may find:

The volume of a cylinder is equal to the area of the base multiplied by the altitude.

The volume of a cone is equal to the area of the base into one third the altitude.

The volume of a prolate spheroid and of an oblate spheroid (formed by the revolution of an ellipse around its transverse and its conjugate axis respectively) are each equal to two thirds of the circumscribing cylinder.

If the axes are equal, the spheroid becomes a sphere and its volume $= \frac{2}{3}\pi R^2 \times D = \frac{1}{6}\pi D^3$; R being radius and D diameter.

The volume of a paraboloid is equal to half the cylinder having the same base and altitude.

The volume of a pyramid equals the area of the base multiplied by one third the altitude.

Second, third, etc., differentials.—The differential coefficient being a function of the independent variable, it may be differentiated, and we thus obtain the second differential coefficient:

$d\left(\frac{du}{dx}\right) = \frac{d^2u}{dx}$. Dividing by dx, we have for the second differential coefficient $\frac{d^2u}{dx^2}$, which is read: second differential of u divided by the square of the differential of x (or dx squared).

The third differential coefficient $\frac{d^3u}{dx^3}$ is read: third differential of u divided by dx cubed.

The differentials of the different orders are obtained by multiplying the differential coefficients by the corresponding powers of dx; thus $\frac{d^3u}{dx^3}\,dx^3 =$ third differential of u.

Sign of the first differential coefficient.—If we have a curve whose equation is $y = fx$, referred to rectangular coördinates, the curve will recede from the axis of X when $\frac{dy}{dx}$ is positive, and approach the axis when it is negative, when the curve lies within the first angle of the coördinate axes. For all angles and every relation of y and x the curve will recede from the axis of X when the ordinate and first differential coefficient have the same sign, and approach it when they have different signs. If the tangent of the curve becomes parallel to the axis of X at any point $\frac{dy}{dx} = 0$. If the tangent becomes perpendicular to the axis of X at any point $\frac{dy}{dx} = \infty$.

Sign of the second differential coefficient.—The second differential coefficient has the same sign as the ordinate when the curve is convex toward the axis of abscissa and a contrary sign when it is concave.

Maclaurin's Theorem.—For developing into a series any function of a single variable as $u = A + Bx + Cx^2 + Dx^3 + Ex^4$, etc., in which A, B, C, etc., are independent of x:

$$u = (u)_{x=0} + \left(\frac{du}{dx}\right)_{x=0} x + \frac{1}{1\,.\,2}\left(\frac{d^2u}{dx^2}\right)_{x=0} x^2 + \frac{1}{1\,.\,2\,.\,3}\left(\frac{d^3u}{dx^3}\right)_{x=0} x^3 + \text{etc.}$$

In applying the formula, omit the expressions $x = 0$, although the coefficients are always found under this hypothesis.

EXAMPLES:

$$(a+x)^m = a^m + ma^{m-1}x + \frac{m}{1}\,\frac{(m-1)}{2}a^{m-2}x^2$$
$$+ \frac{m}{1}\,\frac{(m-1)}{2}\,\frac{(m-2)}{3}a^{m-3}x^3 + \text{etc.}$$

$$\frac{1}{a+x} = \frac{1}{a} - \frac{x}{a^2} + \frac{x^2}{a^3} - \frac{x^3}{a^4} + \ldots\ \frac{x^n}{a^{n+1}}, \text{ etc.}$$

Taylor's Theorem.—For developing into a series any function of the sum or difference of two independent variables, as $u' = f(x \pm y)$:

$$u' = u + \frac{du}{dx}y + \frac{d^2u}{dx^2}\,\frac{y^2}{1\,.\,2} + \frac{d^3u}{dx^3}\,\frac{y^3}{1\,.\,2\,.\,3} + \text{etc.},$$

in which u is what u' becomes when $y = 0$, $\frac{du}{dx}$ is what $\frac{du'}{dx}$ becomes when $y = 0$, etc.

Maxima and minima.—To find the maximum or minimum value of a function of a single variable:

1. Find the first differential coefficient of the function, place it equal to 0, and determine the roots of the equation.

2. Find the second differential coefficient, and substitute each real root, in succession, for the variable in the second member of the equation. Each root which gives a negative result will correspond to a maximum value of the function, and each which gives a positive result will correspond to a minimum value.

EXAMPLE.—To find the value of x which will render the function y a maximum or minimum in the equation of the circle, $y^2 + x^2 = R^2$;

$$\frac{dy}{dx} = -\frac{x}{y};\ \text{making } -\frac{x}{y} = 0 \text{ gives } x = 0.$$

The second differential coefficient is: $\frac{d^2y}{dx} = -\frac{x^2+y^2}{y^3}$.

When $x = 0$, $y = R$; hence $\frac{d^2y}{dx^2} = -\frac{1}{R}$, which being negative, y is a maximum for R positive.

In applying the rule to practical examples we first find an expression for the function which is to be made a maximum or minimum.

2. If in such expression a constant quantity is found as a *factor*, it may be omitted in the operation; for the product will be a maximum or a minimum when the variable factor is a maximum or a minimum.

3. Any value of the independent variable which renders a function a maximum or a minimum will render any power or root of that function a maximum or minimum; hence we may square both members of an equation to free it of radicals before differentiating.

By these rules we may find:

The maximum rectangle which can be inscribed in a triangle is one whose altitude is half the altitude of the triangle.

The altitude of the maximum cylinder which can be inscribed in a cone is one third the altitude of the cone.

The surface of a cylindrical vessel of a given volume, open at the top, is a minimum when the altitude equals half the diameter.

The altitude of a cylinder inscribed in a sphere when its convex surface is a maximum is $r\sqrt{2}$. r = radius.

The altitude of a cylinder inscribed in a sphere when the volume is a maximum is $2r \div \sqrt{3}$.

(For maxima and minima without the calculus see Appendix, p. 1070.)

Differential of an exponential function.

$$\text{If} \quad u = a^x. \qquad (1)$$

$$\text{then } du = da^x = a^x k\,dx, \qquad (2)$$

in which k is a constant dependent on a.

The relation between a and k is $a^{\frac{1}{k}} = e$; whence $a = e^k$, ... (3)

in which $e = 2.7182818$ . . . the base of the Naperian system of logarithms.

Logarithms.—The logarithms in the Naperian system are denoted by l, Nap. log or hyperbolic log, hyp. log, or $\log_e$; and in the common system always by log.

$$k = \text{Nap. log } a, \quad \log a = k \log e \qquad (4)$$

The common logarithm of e, = log 2.7182818 . . . = .4342945 . . . , is called the modulus of the common system, and is denoted by M. Hence, if we have the Naperian logarithm of a number we can find the common logarithm of the same number by multiplying by the modulus. Reciprocally, Nap. log = com. log × 2.3025851.

If in equation (4) we make $a = 10$, we have

$$1 = k \log e, \quad \text{or} \quad \frac{1}{k} = \log e = M.$$

That is, the modulus of the common system is equal to 1, divided by the Naperian logarithm of the common base.

From equation (2) we have

$$\frac{du}{u} = \frac{da^x}{a^x} = k dx.$$

If we make $a = 10$, the base of the common system, $x = \log u$, and

$$d(\log u) = dx = \frac{du}{u} \times \frac{1}{k} = \frac{du}{u} \times M.$$

That is, the differential of a common logarithm of a quantity is equal to the differential of the quantity divided by the quantity, into the modulus.

If we make $a = e$, the base of the Naperian system, x becomes the Naperian logarithm of u, and k becomes 1 (see equation (8)); hence $M = 1$, and

$$d(\text{Nap. log } u) = dx = \frac{du}{a^x}; = \frac{du}{u}.$$

That is, the differential of a Naperian logarithm of a quantity is equal to the differential of the quantity divided by the quantity; and in the Naperian system the modulus is 1.

Since k is the Naperian logarithm of a, $du = a^x\ l\ a\ dx$. That is, the differential of a function of the form a^x is equal to the function, into the Naperian logarithm of the base a, into the differential of the exponent.

If we have a differential in a fractional form, in which the numerator is the differential of the denominator, the integral is the Naperian logarithm of the denominator. Integrals of fractional differentials of other forms are given below:

Differential forms which have known integrals; exponential functions. (l = Nap. log.)

1. $$\int a^x\ l\ a\ dx = a^x + C;$$

2. $$\int \frac{dx}{x} = \int dx\, x^{-1} = lx + C;$$

3. $$\int (xy^{x-1}dy + y^x\ l y \times dx) = y^x + C;$$

4. $$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = l(x + \sqrt{x^2 \pm a^2}) + C;$$

5. $$\int \frac{dx}{\sqrt{x^2 \pm 2ax}} = l\left(x \pm a + \sqrt{x^2 \pm 2ax}\right) + C;$$

6. $$\int \frac{2adx}{a^2 - x^2} = l\left(\frac{a+x}{a-x}\right) + C;$$

7. $$\int \frac{2adx}{x^2 - a^2} = l\left(\frac{x-a}{x+a}\right) + C;$$

8. $$\int \frac{2adx}{x\sqrt{a^2 + x^2}} = l\left(\frac{\sqrt{a^2+x^2} - a}{\sqrt{a^2+x^2} + a}\right) + C;$$

9. $$\int \frac{2adx}{x\sqrt{a^2 - x^2}} = l\left(\frac{a - \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}}\right) + C;$$

10. $$\int \frac{x^{-2}dx}{\sqrt{x + x^{-2}}} = -\ l\left(\frac{1 + \sqrt{1 + a^2x^2}}{x}\right) + C.$$

Circular functions.—Let z denote an arc in the first quadrant, y its sine, x its cosine, v its versed sine, and t its tangent; and the following notation be employed to designate an arc by any one of its functions, viz.,

$\sin^{-1} y$ denotes an arc of which y is the sine
$\cos^{-1} x$ " " " " " x is the cosine,
$\tan^{-1} t$ " " " " " t is the tangent

(read "arc whose sine is y," etc.),—we have the following differential forms which have known integrals (r = radius):

$$\int \cos z\, dz = \sin z + C;$$

$$\int - \sin z\, dz = \cos z + C;$$

$$\int \frac{dy}{\sqrt{1 - y^2}} = \sin^{-1} y + C;$$

$$\int \frac{- dx}{\sqrt{1 - x^2}} = \cos^{-1} x + C;$$

$$\int \frac{dv}{\sqrt{2v - v^2}} = \text{ver-sin}^{-1} v + C;$$

$$\int \frac{dt}{1 + t^2} = \tan^{-1} t + C;$$

$$\int \frac{rdy}{\sqrt{r^2 - y^2}} = \sin^{-1} y + C;$$

$$\int \frac{- rdx}{\sqrt{r^2 - x^2}} = \cos^{-1} x + C;$$

$$\int \sin z\, dz = \text{ver-sin}\, z + C;$$

$$\int \frac{dz}{\cos^2 z} = \tan z + C;$$

$$\int \frac{rd\,v}{\sqrt{2rv + v^2}} = \text{ver-sin}^{-1} v + C;$$

$$\int \frac{r^2dt}{r^2 + t^2} = \tan^{-1} t + C;$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C;$$

$$\int \frac{- du}{\sqrt{a^2 - u^2}} = \cos^{-1} \frac{u}{a} + C;$$

$$\int \frac{du}{\sqrt{2au - u^2}} = \text{ver-sin}^{-1} \frac{u}{a} + C;$$

$$\int \frac{adu}{a^2 + u^2} = \tan^{-1} \frac{u}{a} + C.$$

The cycloid.—If a circle be rolled along a straight line, any point of the circumference, as P, will describe a curve which is called a cycloid. The circle is called the generating circle, and P the generating point.

The transcendental equation of the cycloid is

$$x = \text{ver-sin}^{-1} y - \sqrt{2ry - y^2},$$

and the differential equation is $dx = \dfrac{ydx}{\sqrt{2ry - y^2}}$.

The area of the cycloid is equal to three times the area of the generating circle.

The surface described by the arc of a cycloid when revolved about its base is equal to 64 thirds of the generating circle.

The volume of the solid generated by revolving a cycloid about its base is equal to five eighths of the circumscribing cylinder.

Integral calculus.—In the integral calculus we have to return from the differential to the function from which it was derived. A number of differential expressions are given above, each of which has a known integral corresponding to it, and which being differentiated, will produce the given differential.

In all classes of functions any differential expression may be integrated when it is reduced to one of the known forms; and the operations of the integral calculus consist mainly in making such transformations of given differential expressions as shall reduce them to equivalent ones whose integrals are known.

For methods of making these transformations reference must be made to the text-books on differential and integral calculus.

RECIPROCALS OF NUMBERS.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
1	1.00000000	64	.01562500	127	.00787402	190	.00526316	253	.00395257
2	.50000000	5	.01538461	8	.00781250	1	.00523560	4	.00393701
3	.33333333	6	.01515151	9	.00775194	2	.00520833	5	.00392157
4	.25000000	7	.01492537	130	.00769231	3	.00518135	6	.00390625
5	.20000000	8	.01470588	1	.00763359	4	.00515464	7	.00389105
6	.16666667	9	.01449275	2	.00757576	5	.00512820	8	.00387597
7	.14285714	70	.01428571	3	.00751880	6	.00510204	9	.00386100
8	.12500000	1	.01408451	4	.00746269	7	.00507614	260	.00384615
9	.11111111	2	.01388889	5	.00740741	8	.00505051	1	.00383142
10	.10000000	3	.01369863	6	.00735294	9	.00502513	2	.00381679
11	.09090909	4	.01351351	7	.00729927	200	.00500000	3	.00380228
12	.08333333	5	.01333333	8	.00724638	1	.00497512	4	.00378788
13	.07692308	6	.01315789	9	.00719424	2	.00495049	5	.00377358
14	.07142857	7	.01298701	140	.00714286	3	.00492611	6	.00375940
15	.06666667	8	.01282051	1	.00709220	4	.00490196	7	.00374532
16	.06250000	9	.01265823	2	.00704225	5	.00487805	8	.00373134
17	.05882353	80	.01250000	3	.00699301	6	.00485437	9	.00371747
18	.05555556	1	.01234568	4	.00694444	7	.00483092	270	.00370370
19	.05263158	2	.01219512	5	.00689655	8	.00480769	1	.00369004
20	.05000000	3	.01204819	6	.00684931	9	.00478469	2	.00367647
1	.04761905	4	.01190476	7	.00680272	210	.00476190	3	.00366300
2	.04545455	5	.01176471	8	.00675676	11	.00473934	4	.00364964
3	.04347826	6	.01162791	9	.00671141	12	.00471698	5	.00363636
4	.04166667	7	.01149425	150	.00666667	13	.00469484	6	.00362319
5	.04000000	8	.01136364	1	.00662252	14	.00467290	7	.00361011
6	.03846154	9	.01123595	2	.00657895	15	.00465116	8	.00359712
7	.03703704	90	.01111111	3	.00653595	16	.00462963	9	.00358423
8	.03571429	1	.01098901	4	.00649351	17	.00460829	280	.00357143
9	.03448276	2	.01086956	5	.00645161	18	.00458716	1	.00355872
30	.03333333	3	.01075269	6	.00641026	19	.00456621	2	.00354610
1	.03225806	4	.01063830	7	.00636943	220	.00454545	3	.00353357
2	.03125000	5	.01052632	8	.00632911	1	.00452489	4	.00352113
3	.03030303	6	.01041667	9	.00628931	2	.00450450	5	.00350877
4	.02941176	7	.01030928	160	.00625000	3	.00448430	6	.00349650
5	.02857143	8	.01020408	1	.00621118	4	.00446429	7	.00348432
6	.02777778	9	.01010101	2	.00617284	5	.00444444	8	.00347222
7	.02702703	100	.01000000	3	.00613497	6	.00442478	9	.00346021
8	.02631579	1	.00990099	4	.00609756	7	.00440529	290	.00344828
9	.02564103	2	.00980392	5	.00606061	8	.00438596	1	.00343643
40	.02500000	3	.00970874	6	.00602410	9	.00436681	2	.00342466
1	.02439024	4	.00961538	7	.00598802	230	.00434783	3	.00341297
2	.02380952	5	.00952381	8	.00595238	1	.00432900	4	.00340136
3	.02325581	6	.00943396	9	.00591716	2	.00431034	5	.00338983
4	.02272727	7	.00934579	170	.00588235	3	.00429184	6	.00337838
5	.02222222	8	.00925926	1	.00584795	4	.00427350	7	.00336700
6	.02173913	9	.00917431	2	.00581395	5	.00425532	8	.00335570
7	.02127660	110	.00909091	3	.00578035	6	.00423729	9	.00334448
8	.02083333	11	.00900901	4	.00574713	7	.00421941	300	.00333333
9	.02040816	12	.00892857	5	.00571429	8	.00420168	1	.00332226
50	.02000000	13	.00884956	6	.00568182	9	.00418410	2	.00331126
1	.01960784	14	.00877193	7	.00564972	240	.00416667	3	.00330033
2	.01923077	15	.00869565	8	.00561798	1	.00414938	4	.00328947
3	.01886792	16	.00862069	9	.00558659	2	.00413223	5	.00327869
4	.01851852	17	.00854701	180	.00555556	3	.00411523	6	.00326797
5	.01818182	18	.00847458	1	.00552486	4	.00409836	7	.00325733
6	.01785714	19	.00840336	2	.00549451	5	.00408163	8	.00324675
7	.01754386	120	.00833333	3	.00546448	6	.00406504	9	.00323625
8	.01724138	1	.00826446	4	.00543478	7	.00404858	310	.00322581
9	.01694915	2	.00819672	5	.00540540	8	.00403226	11	.00321543
60	.01666667	3	.00813008	6	.00537634	9	.00401606	12	.00320513
1	.01639344	4	.00806452	7	.00534759	250	.00400000	13	.00319489
2	.01612903	5	.00800000	8	.00531914	1	.00398406	14	.00318471
3	.01587302	6	.00793651	9	.00529100	2	.00396825	15	.00317460

No	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
316	.00316456	381	.00262467	446	.00224215	511	.00195695	576	.00173611
17	.00315457	2	.00261780	7	.00223714	12	.00195312	7	.00173310
18	.00314465	3	.00261097	8	.00223214	13	.00194932	8	.00173010
19	.00313480	4	.00260417	9	.00222717	14	.00194552	9	.00172712
320	.00312500	5	.00259740	450	.00222222	15	.00194175	580	.00172414
1	.00311526	6	.00259067	1	.00221729	16	.00193798	1	.00172117
2	.00310559	7	.00258398	2	.00221239	17	.00193424	2	.00171821
3	.00309597	8	.00257732	3	.00220751	18	.00193050	3	.00171527
4	.00308642	9	.00257069	4	.00220264	19	.00192678	4	.00171233
5	.00307692	390	.00256410	5	.00219780	520	.00192308	5	.00170940
6	.00306748	1	.00255754	6	.00219298	1	.00191939	6	.00170648
7	.00305810	2	.00255102	7	.00218818	2	.00191571	7	.00170358
8	.00304878	3	.00254453	8	.00218341	3	.00191205	8	.00170068
9	.00303951	4	.00253807	9	.00217865	4	.00190840	9	.00169779
330	.00303030	5	.00253165	460	.00217391	5	.00190476	590	.00169491
1	.00302115	6	.00252525	1	.00216920	6	.00190114	1	.00169205
2	.00301205	7	.00251889	2	.00216450	7	.00189753	2	.00168919
3	.00300300	8	.00251256	3	.00215983	8	.00189394	3	.00168634
4	.00299401	9	.00250627	4	.00215517	9	.00189036	4	.00168350
5	.00298507	400	.00250000	5	.00215054	530	.00188679	5	.00168067
6	.00297619	1	.00249377	6	.00214592	1	.00188324	6	.00167785
7	.00296736	2	.00248756	7	.00214133	2	.00187970	7	.00167504
8	.00295858	3	.00248139	8	.00213675	3	.00187617	8	.00167224
9	.00294985	4	.00247525	9	.00213220	4	.00187266	9	.00166945
340	.00294118	5	.00246914	470	.00212766	5	.00186916	600	.00166667
1	.00293255	6	.00246305	1	.00212314	6	.00186567	1	.00166389
2	.00292398	7	.00245700	2	.00211864	7	.00186220	2	.00166113
3	.00291545	8	.00245098	3	.00211416	8	.00185874	3	.00165837
4	.00290698	9	.00244499	4	.00210970	9	.00185528	4	.00165563
5	.00289855	410	.00243902	5	.00210526	540	.00185185	5	.00165289
6	.00289017	11	.00243309	6	.00210084	1	.00184843	6	.00165016
7	.00288184	12	.00242718	7	.00209644	2	.00184502	7	.00164745
8	.00287356	13	.00242131	8	.00209205	3	.00184162	8	.00164474
9	.00286533	14	.00241546	9	.00208768	4	.00183823	9	.00164204
350	.00285714	15	.00240964	480	.00208333	5	.00183486	610	.00163934
1	.00284900	16	.00240385	1	.00207900	6	.00183150	11	.00163666
2	.00284091	17	.00239808	2	.00207469	7	.00182815	12	.00163399
3	.00283286	18	.00239234	3	.00207039	8	.00182482	13	.00163132
4	.00282486	19	.00238663	4	.00206612	9	.00182149	14	.00162866
5	.00281690	420	.00238095	5	.00206186	550	.00181818	15	.00162602
6	.00280899	1	.00237530	6	.00205761	1	.00181488	16	.00162338
7	.00280112	2	.00236967	7	.00205339	2	.00181159	17	.00162075
8	.00279330	3	.00236407	8	.00204918	3	.00180832	18	.00161812
9	.00278551	4	.00235849	9	.00204499	4	.00180505	19	.00161551
360	.00277778	5	.00235294	490	.00204082	5	.00180180	620	.00161290
1	.00277008	6	.00234742	1	.00203666	6	.00179856	1	.00161031
2	.00276243	7	.00234192	2	.00203252	7	.00179533	2	.00160772
3	.00275482	8	.00233645	3	.00202840	8	.00179211	3	.00160514
4	.00274725	9	.00233100	4	.00202429	9	.00178891	3	.00160256
5	.00273973	430	.00232558	5	.00202020	560	.00178571	5	.00160000
6	.00273224	1	.00232019	6	.00201613	1	.00178253	6	.00159744
7	.00272480	2	.00231481	7	.00201207	2	.00177936	7	.00159490
8	.00271739	3	.00230947	8	.00200803	3	.00177620	8	.00159236
9	.00271003	4	.00230415	9	.00200401	4	.00177305	9	.00158982
370	.00270270	5	.00229885	500	.00200000	5	.00176991	630	.00158730
1	.00269542	6	.00229358	1	.00199601	6	.00176678	1	.00158479
2	.00268817	7	.00228833	2	.00199203	7	.00176367	2	.00158228
3	.00268096	8	.00228310	3	.00198807	8	.00176056	3	.00157978
4	.00267380	9	.00227790	4	.00198413	9	.00175747	4	.00157729
5	.00266667	440	.00227273	5	.00198020	570	.00175439	5	.00157480
6	.00265957	1	.00226757	6	.00197628	1	.00175131	6	.00157233
7	.00265252	2	.00226244	7	.00197239	2	.00174825	7	.00156986
8	.00264550	3	.00225734	8	.00196850	3	.00174520	8	.00156740
9	.00263852	4	.00225225	9	.00196464	4	.00174216	9	.00156494
380	.00263158	5	.00224719	510	.00196078	5	.00173913	640	.00156250

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
641	.00156006	706	00141643	771	.00129702	836	.00119617	901	.00110988
2	.00155763	7	.00141443	2	.00129534	7	.00119474	2	.00110865
3	.00155521	8	.00141243	3	.00129366	8	.00119332	3	.00110742
4	.00155279	9	.00141044	4	.00129199	9	.00119189	4	.00110619
5	.00155039	710	.00140845	5	.00129032	840	.00119048	5	.00110497
6	.00154799	11	.00140647	6	.00128866	1	.00118906	6	.00110375
7	.00154559	12	.00140449	7	.00128700	2	.00118765	7	.00110254
8	.00154321	13	.00140252	8	.00128535	3	.00118624	8	.00110132
9	.00154083	14	.00140056	9	.00128370	4	.00118483	9	.00110011
650	.00153846	15	.00139860	780	.00128205	5	.00118343	910	.00109890
1	.00153610	16	.00139665	1	.00128041	6	.00118203	11	.00109769
2	.00153374	17	.00139470	2	.00127877	7	.00118064	12	.00109649
3	.00153140	18	.00139276	3	.00127714	8	.00117924	13	.00109529
4	.00152905	19	.00139082	4	.00127551	9	.00117786	14	.00109409
5	.00152672	720	.00138889	5	.00127388	850	.00117647	15	.00109290
6	.00152439	1	.00138696	6	.00127226	1	.00117509	16	.00109170
7	.00152207	2	.00138504	7	.00127065	2	.00117371	17	.00109051
8	.00151975	3	.00138313	8	.00126904	3	00117233	18	.00108932
9	.00151745	4	.00138121	9	.00126743	4	.00117096	19	.00108814
660	.00151515	5	.00137931	790	.00126582	5	.00116959	920	.00108696
1	.00151286	6	.00137741	1	.00126422	6	.00116822	1	.00108578
2	.00151057	7	.00137552	2	.00126263	7	.00116686	2	.00108460
3	.00150830	8	.00137363	3	.00126103	8	.00116550	3	.00108342
4	.00150602	9	.00137174	4	.00125945	9	.00116414	4	.00108225
5	.00150376	730	.00136986	5	.00125786	860	.00116279	5	.00108108
6	.00150150	1	.00136799	6	.00125628	1	.00116144	6	.00107991
7	.00149925	2	.00136612	7	.00125470	2	.00116009	7	.00107875
8	.00149701	3	.00136426	8	.00125313	3	.00115875	8	.00107759
9	.00149477	4	.00136240	9	.00125156	4	.00115741	9	.00107643
670	.00149254	5	.00136054	800	.00125000	5	.00115607	930	.00107527
1	.00149031	6	.00135870	1	.00124844	6	.00115473	1	.00107411
2	.00148809	7	.00135685	2	.00124688	7	.00115340	2	.00107296
3	.00148588	8	.00135501	3	.00124533	8	.00115207	3	.00107181
4	.00148368	9	.00135318	4	.00124378	9	.00115075	4	.00107066
5	.00148148	740	.00135135	5	.00124224	870	.00114942	5	.00106952
6	.00147929	1	.00134953	6	.00124069	1	.00114811	6	.00106838
7	.00147710	2	.00134771	7	.00123916	2	.00114679	7	.00106724
8	.00147493	3	.00134590	8	.00123762	3	.00114547	8	00106610
9	.00147275	4	.00134409	9	.00123609	4	.00114416	9	.00106496
680	.00147059	5	.00134228	810	.00123457	5	.00114286	940	.00106383
1	.00146843	6	.00134048	11	.00123305	6	.00114155	1	.00106270
2	.00146628	7	.00133869	12	.00123153	7	.00114025	2	.00106157
3	.00146413	8	.00133690	13	.00123001	8	.00113895	3	.00106044
4	.00146199	9	.00133511	14	.00122850	9	.00113766	4	.00105932
5	.00145985	750	.00133333	15	.00122699	880	.00113636	5	.00105820
6	.00145773	1	.00133156	16	.00122549	1	00113507	6	.00105708
7	.00145560	2	.00132979	17	.00122399	2	.00113379	7	.00105597
8	.00145349	3	.00132802	18	.00122249	3	.00113250	8	.00105485
9	.00145137	4	.00132626	19	.00122100	4	.00113122	9	.00105374
690	.00144927	5	.00132450	820	.00121951	5	.00112994	950	.00105263
1	00144718	6	.00132275	1	.00121803	6	.00112867	1	.00105152
2	.00144509	7	.00132100	2	.00121654	7	.00112740	2	.00105042
3	.00144300	8	00131926	3	.00121507	8	.00112613	3	.00104932
4	.00144092	9	.00131752	4	.00121359	9	.00112486	4	.00104822
5	.00143885	760	.00131579	5	.00121212	890	.00112360	5	.00104712
6	.00143678	1	00131406	6	.00121065	1	.00112233	6	.00104602
7	.00143472	2	.00131234	7	.00120919	2	.00112108	7	.00104493
8	00143266	3	.00131062	8	.00120773	3	.00111982	8	.00104384
9	00143061	4	.00130890	9	.00120627	4	.00111857	9	.00104275
700	.00142857	5	.00130719	830	.00120482	5	.00111732	960	.00104167
1	00142653	6	.00130548	1	.00120337	6	.00111607	1	.00104058
2	00142450	7	.00130378	2	.00120192	7	.00111483	2	.00103950
3	.00142247	8	.00130208	3	.00120048	8	.00111359	3	.00103842
4	00142045	9	.00130039	4	.00119904	9	.00111235	4	.00103734
5	00141844	770	.00129870	5	.00119760	900	.00111111	5	.00103627

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
966	.00103520	1031	.000969932	1096	.000912409	1161	.000861326	1226	.000815661
7	.00103413	2	.000968992	7	.000911577	2	.000860585	7	.000814996
8	.00103306	3	.000968054	8	.000910747	3	.000859845	8	.000814332
9	.00103199	4	.000967118	9	.000909918	4	.000859106	9	.000813670
970	.00103093	5	.000966184	1100	.000909091	5	.000858369	1230	.000813008
1	.00102987	6	.000965251	1	.000908265	6	.000857633	1	.000812348
2	.00102881	7	.000964320	2	.000907441	7	.000856898	2	.000811688
3	.00102775	8	.000963391	3	.000906618	8	.000856164	3	.000811030
4	.00102669	9	.000962464	4	.000905797	9	.000855432	4	.000810373
5	.00102564	1040	.000961538	5	.000904977	1170	.000854701	5	.000809717
6	.00102459	1	.000960615	6	.000904159	1	.000853971	6	.000809061
7	.00102354	2	.000959693	7	.000903342	2	.000853242	7	.000808407
8	.00102250	3	.000958774	8	.000902527	3	.000852515	8	.000807754
9	.00102145	4	.000957854	9	.000901713	4	.000851789	9	.000807102
980	.00102041	5	.000956938	1110	.000900901	5	.000851064	1240	.000806452
1	.00101937	6	.000956023	11	.000900090	6	.000850340	1	.000805802
2	.00101833	7	.000955110	12	.000899281	7	.000849618	2	.000805153
3	.00101729	8	.000954198	13	.000898473	8	.000848896	3	.000804505
4	.00101626	9	.000953289	14	.000897666	9	.000848176	4	.000803858
5	.00101523	1050	.000952381	15	.000896861	1180	.000847457	5	.000803213
6	.00101420	1	.000951475	16	.000896057	1	.000846740	6	.000802568
7	.00101317	2	.000950570	17	.000895255	2	.000846024	7	.000801925
8	.00101215	3	.000949668	18	.000894454	3	.000845308	8	.000801282
9	.00101112	4	.000948767	19	.000893655	4	.000844595	9	.000800640
990	.00101010	5	.000947867	1120	.000892857	5	.000843882	1250	.000800000
1	.00100908	6	.000946970	1	.000892061	6	.000843170	1	.000799360
2	.00100806	7	.000946074	2	.000891266	7	.000842460	2	.000798722
3	.00100705	8	.000945180	3	.000890472	8	.000841751	3	.000798085
4	.00100604	9	.000944287	4	.000889680	9	.000841043	4	.000797448
5	.00100502	1060	.000943396	5	.000888889	1190	.000840336	5	.000796813
6	.00100402	1	.000942507	6	.000888099	1	.000839631	6	.000796178
7	.00100301	2	.000941620	7	.000887311	2	.000838926	7	.000795545
8	.00100200	3	.000940734	8	.000886525	3	.000838222	8	.000794913
9	.00100100	4	.000939850	9	.000885740	4	.000837521	9	.000794281
1000	.00100000	5	.000938967	1130	.000884956	5	.000836820	1260	.000793651
1	.000999001	6	.000938086	1	.000884173	6	.000836120	1	.000793021
2	.000998004	7	.000937207	2	.000883392	7	.000835422	2	.000792393
3	.000997009	8	.000936330	3	.000882612	8	.000834724	3	.000791766
4	.000996016	9	.000935454	4	.000881834	9	.000834028	4	.000791139
5	.000995025	1070	.000934579	5	.000881057	1200	.000833333	5	.000790514
6	.000994036	1	.000933707	6	.000880282	1	.000832639	6	.000789889
7	.000993049	2	.000932836	7	.000879508	2	.000831947	7	.000789266
8	.000992063	3	.000931966	8	.000878735	3	.000831255	8	.000788643
9	.000991080	4	.000931099	9	.000877963	4	.000830565	9	.000788022
1010	.000990099	5	.000930233	1140	.000877193	5	.000829875	1270	.000787402
11	.000989120	6	.000929368	1	.000876424	6	.000829187	1	.000786782
12	.000988142	7	.000928505	2	.000875657	7	.000828500	2	.000786163
13	.000987167	8	.000927644	3	.000874891	8	.000827815	3	.000785546
14	.000986193	9	.000926784	4	.000874126	9	.000827130	4	.000784929
15	.000985222	1080	.000925926	5	.000873362	1210	.000826446	5	.000784314
16	.000984252	1	.000925069	6	.000872600	11	.000825764	6	.000783699
17	.000983284	2	.000924214	7	.000871840	12	.000825082	7	.000783085
18	.000982318	3	.000923361	8	.000871080	13	.000824402	8	.000782473
19	.000981354	4	.000922509	9	.000870322	14	.000823723	9	.000781861
1020	.000980392	5	.000921659	1150	.000869565	15	.000823045	1280	.000781250
1	.000979432	6	.000920810	1	.000868810	16	.000822368	1	.000780640
2	.000978474	7	.000919963	2	.000868056	17	.000821693	2	.000780031
3	.000977517	8	.000919118	3	.000867303	18	.000821018	3	.000779423
4	.000976562	9	.000918274	4	.000866551	19	.000820344	4	.000778816
5	.000975610	1090	.000917431	5	.000865801	1220	.000819672	5	.000778210
6	.000974659	1	.000916590	6	.000865052	1	.000819001	6	.000777605
7	.000973710	2	.000915751	7	.000864304	2	.000818331	7	.000777001
8	.000972763	3	.000914913	8	.000863558	3	.000817661	8	.000776397
9	.000971817	4	.000914077	9	.000862813	4	.000816993	9	.000775795
1030	.000970874	5	.000913242	1160	.000862069	5	.000816326	1290	.000775194

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
1291	.000774593	1356	.000737463	1421	.000703730	1486	.000672948	1551	.000644745
2	.000773994	7	.000736920	2	.000703235	7	.000672495	2	.000644330
3	.000773395	8	.000736377	3	.000702741	8	.000672043	3	.000643915
4	.000772797	9	.000735835	4	.000702247	9	.000671592	4	.000643501
5	.000772201	1360	.000735294	5	.000701754	1490	.000671141	5	.000643087
6	.000771605	1	.000734754	6	.000701262	1	.000670691	6	.000642673
7	.000771010	2	.000734214	7	.000700771	2	.000670241	7	.000642261
8	.000770416	3	.000733676	8	.000700280	3	.000669792	8	.000641849
9	.000769823	4	.000733138	9	.000699790	4	.000669344	9	.000641437
1300	.000769231	5	.000732601	1430	.000699301	5	.000668896	1560	.000641026
1	.000768639	6	.000732064	1	.000698812	6	.000668449	1	.000640615
2	.000768049	7	.000731529	2	.000698324	7	.000668003	2	.000640205
3	.000767459	8	.000730994	3	.000697837	8	.000667557	3	.000639795
4	.000766871	9	.000730460	4	.000697350	9	.000667111	4	.000639386
5	.000766283	1370	.000729927	5	.000696864	1500	.000666667	5	.000638978
6	.000765697	1	.000729395	6	.000696379	1	.000666223	6	.000638570
7	.000765111	2	.000728863	7	.000695894	2	.000665779	7	.000638162
8	.000764526	3	.000728332	8	.000695410	3	.000665336	8	.000637755
9	.000763942	4	.000727802	9	.000694927	4	.000664894	9	.000637349
1310	.000763359	5	.000727273	1440	.000694444	5	.000664452	1570	.000636943
11	.000762776	6	.000726744	1	.000693962	6	.000664011	1	.000636537
12	.000762195	7	.000726216	2	.000693481	7	.000663570	2	.000636132
13	.000761615	8	.000725689	3	.000693001	8	.000663130	3	.000635728
14	.000761035	9	.000725163	4	.000692521	9	.000662691	4	.000635324
15	.000760456	1380	.000724638	5	.000692041	1510	.000662252	5	.000634921
16	.000759878	1	.000724113	6	.000691563	11	.000661813	6	.000634518
17	.000759301	2	.000723589	7	.000691085	12	.000661376	7	.000634115
18	.000758725	3	.000723066	8	.000690608	13	.000660939	8	.000633714
19	.000758150	4	.000722543	9	.000690131	14	.000660502	9	.000633312
1320	.000757576	5	.000722022	1450	.000689655	15	.000660066	1580	.000632911
1	.000757002	6	.000721501	1	.000689180	16	.000659631	1	.000632511
2	.000756430	7	.000720980	2	.000688705	17	.000659196	2	.000632111
3	.000755858	8	.000720461	3	.000688231	18	.000658761	3	.000631712
4	.000755287	9	.000719942	4	.000687758	19	.000658328	4	.000631313
5	.000754717	1390	.000719424	5	.000687285	1520	.000657895	5	.000630915
6	.000754148	1	.000718907	6	.000686813	1	.000657462	6	.000630517
7	.000753579	2	.000718391	7	.000686341	2	.000657030	7	.000630120
8	.000753012	3	.000717875	8	.000685871	3	.000656599	8	.000629723
9	.000752445	4	.000717360	9	.000685401	4	.000656168	9	.000629327
1330	.000751880	5	.000716846	1460	.000684932	5	.000655738	1590	.000628931
1	.000751315	6	.000716332	1	.000684463	6	.000655308	1	.000628536
2	.000750750	7	.000715820	2	.000683994	7	.000654879	2	.000628141
3	.000750187	8	.000715308	3	.000683527	8	.000654450	3	.000627746
4	.000749625	9	.000714796	4	.000683060	9	.000654022	4	.000627353
5	.000749064	1400	.000714286	5	.000682594	1530	.000653595	5	.000626959
6	.000748503	1	.000713776	6	.000682128	1	.000653168	6	.000626566
7	.000747943	2	.000713267	7	.000681663	2	.000652742	7	.000626174
8	.000747384	3	.000712758	8	.000681199	3	.000652316	8	.000625782
9	.000746826	4	.000712251	9	.000680735	4	.000651890	9	.000625391
1340	.000746269	5	.000711744	1470	.000680272	5	.000651466	1600	.000625000
1	.000745712	6	.000711238	1	.000679810	6	.000651042	2	.000624219
2	.000745156	7	.000710732	2	.000679348	7	.000650618	4	.000623441
3	.000744602	8	.000710227	3	.000678887	8	.000650195	6	.000622665
4	.000744048	9	.000709723	4	.000678426	9	.000649773	8	.000621890
5	.000743494	1410	.000709220	5	.000677966	1540	.000649351	1610	.000621118
6	.000742942	11	.000708717	6	.000677507	1	.000648929	2	.000620347
7	.000742390	12	.000708215	7	.000677048	2	.000648508	4	.000619578
8	.000741840	13	.000707714	8	.000676590	3	.000648088	6	.000618812
9	.000741290	14	.000707214	9	.000676132	4	.000647668	8	.000618047
1350	.000740741	15	.000706714	1480	.000675676	5	.000647249	1620	.000617284
1	.000740192	16	.000706215	1	.000675219	6	.000646830	2	.000616523
2	.000739645	17	.000705716	2	.000674764	7	.000646412	4	.000615763
3	.000739098	18	.000705219	3	.000674309	8	.000645995	6	.000615006
4	.000738552	19	.000704722	4	.000673854	9	.000645578	8	.000614250
5	.000738007	1420	.000704225	5	.000673401	1550	.000645161	1630	.000613497

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
1632	.000612745	1706	.000586166	1780	.000561798	1854	.000539374	1928	.000518672
4	.000611995	8	.000585480	2	.000561167	6	.000538793	1930	.000518135
6	.000611247	1710	.000584795	4	.000560538	8	.000538213	2	.000517599
8	.000610500	12	.000584112	6	.000559910	1860	.000537634	4	.000517063
1640	.000609756	14	.000583430	8	.000559284	2	.000537057	6	.000516528
2	.000609013	16	.000582750	1790	.000558659	4	.000536480	8	.000515996
4	.000608272	18	.000582072	2	.000558035	6	.000535905	1940	.000515464
6	.000607533	1720	.000581395	4	.000557413	8	.000535332	2	.000514933
8	.000606796	2	.000580720	6	.000556793	1870	.000534759	4	.000514403
1650	.000606061	4	.000580046	8	.000556174	2	.000534188	6	.000513874
2	.000605327	6	.000579374	1800	.000555556	4	.000533618	8	.000513347
4	.000604595	8	.000578704	2	.000554939	6	.000533049	1950	.000512820
6	.000603865	1730	.000578035	4	.000554324	8	.000532481	2	.000512295
8	.000603136	2	.000577367	6	.000553710	1880	.000531915	4	.000511770
1660	.000602410	4	.000576701	8	.000553097	2	.000531350	6	.000511247
2	.000601685	6	.000576037	1810	.000552486	4	.000530785	8	.000510725
4	.000600962	8	.000575374	12	.000551876	6	.000530222	1960	.000510204
6	.000600240	1740	.000574713	14	.000551268	8	.000529661	2	.000509684
8	.000599520	2	.000574053	16	.000550661	1890	.000529100	4	.000509165
1670	.000598802	4	.000573394	18	.000550055	2	.000528541	6	.000508647
2	.000598086	6	.000572737	1820	.000549451	4	.000527983	8	.000508130
4	.000597371	8	.000572082	2	.000548848	6	.000527426	1970	.000507614
6	.000596658	1750	.000571429	4	.000548246	8	.000526870	2	.000507099
8	.000595947	2	.000570776	6	.000547645	1900	.000526316	4	.000506585
1680	.000595238	4	.000570125	8	.000547046	2	.000525762	6	.000506073
2	.000594530	6	.000569476	1830	.000546448	4	.000525210	8	.000505561
4	.000593824	8	.000568828	2	.000545851	6	.000524659	1980	.000505051
6	.000593120	1760	.000568182	4	.000545256	8	.000524109	2	.000504541
8	.000592417	2	.000567537	6	.000544662	1910	.000523560	4	.000504032
1690	.000591716	4	.000566893	8	.000544069	12	.000523012	6	.000503524
2	.000591017	6	.000566251	1840	.000543478	14	.000522466	8	.000503018
4	.000590319	8	.000565611	2	.000542888	16	.000521920	1990	.000502513
6	.000589623	1770	.000564972	4	.000542299	18	.000521376	2	.000502008
8	.000588928	2	.000564334	6	.000541711	1920	.000520833	4	.000501504
1700	.000588235	4	.000563698	8	.000541125	2	.000520291	6	.000501002
2	.000587544	6	.000563063	1850	.000540540	4	.000519750	8	.000500501
4	.000586854	8	.000562430	2	.000539957	6	.000519211	2000	.000500000

Use of reciprocals.—Reciprocals may be conveniently used to facilitate computations in long division. Instead of dividing as usual, multiply the dividend by the reciprocal of the divisor. The method is especially useful when many different dividends are required to be divided by the same divisor. In this case find the reciprocal of the divisor, and make a small table of its multiples up to 9 times, and use this as a multiplication-table instead of actually performing the multiplication in each case.

EXAMPLE.—9871 and several other numbers are to be divided by 1638. The reciprocal of 1638 is .000610500.

Multiples of the reciprocal:

1. .0006105
2. .0012210
3. .0018315
4. .0024420
5. .0030525
6. .0036630
7. .0042735
8. .0048840
9. .0054945
10. .0061050

The table of multiples is made by continuous addition of 6105. The tenth line is written to check the accuracy of the addition, but it is not afterwards used.

Operation:

Dividend	9871	
Take from table	1........	.0006105
	7........	0.042735
	8........	00.48840
	9........	005.4945
Quotient........		6.0262455
Correct quotient by direct division........		6.0262515

The result will generally be correct to as many figures as there are significant figures in the reciprocal, less one, and the error of the next figure will in general not exceed one. In the above example the reciprocal has six significant figures, 610500, and the result is correct to five places of figures.

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS OF NUMBERS FROM .1 TO 1600.

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
.1	.01	.001	.3162	.4642	3.1	9.61	29.791	1.761	1.458
.15	.0225	.0034	.3873	.5313	.2	10.24	32.768	1.789	1.474
.2	.04	.008	.4472	.5848	.3	10.89	35.937	1.817	1.489
.25	.0625	.0156	.500	.6300	.4	11.56	39.304	1.844	1.504
.3	.09	027	.5477	.6694	.5	12.25	42.875	1.871	1.518
.35	.1225	.0429	.5916	.7047	.6	12.96	46.656	1.897	1.533
.4	.16	.064	.6325	.7368	.7	13.69	50.653	1.924	1.547
.45	.2025	.0911	.6708	.7663	.8	14.44	54.872	1.949	1.560
.5	.25	.125	.7071	.7937	.9	15.21	59.319	1.975	1.574
.55	.3025	.1664	.7416	.8193	4.	16.	64.	2.	1.5874
.6	.36	.216	.7746	.8434	.1	16.81	68.921	2.025	1.601
.65	.4225	.2746	.8062	.8662	.2	17.64	74.088	2.049	1.613
.7	.49	.343	.8367	.8879	.3	18.49	79.507	2.074	1.626
.75	.5625	.4219	.8660	.9086	.4	19.36	85.184	2.098	1.639
.8	.64	.512	.8944	.9283	.5	20.25	91.125	2.121	1.651
.85	.7225	.6141	.9219	.9473	.6	21.16	97.336	2.145	1.663
.9	.81	.729	.9487	.9655	.7	22.09	103.823	2.168	1.675
.95	.9025	.8574	.9747	.9830	.8	23.04	110.592	2.191	1.687
1.	1.	1.	1.	1.	.9	24.01	117.649	2.214	1.698
1.05	1.1025	1.158	1.025	1.016	5.	25.	125.	2.2361	1.7100
1.1	1.21	1.331	1.049	1.032	.1	26.01	132.651	2.258	1.721
1.15	1.3225	1.521	1.072	1.048	.2	27.04	140.608	2.280	1.732
1.2	1.44	1.728	1.095	1.063	.3	28.09	148.877	2.302	1.744
1.25	1.5625	1.953	1.118	1.077	.4	29.16	157.464	2.324	1.754
1.3	1.69	2.197	1.140	1.091	.5	30.25	166.375	2.345	1.765
1.35	1.8225	2.460	1.162	1.105	.6	31.36	175.616	2.366	1.776
1.4	1.96	2.744	1.183	1.119	.7	32.49	185.193	2.387	1.786
1.45	2.1025	3.049	1.204	1.132	.8	33.64	195.112	2.408	1.797
1.5	2.25	3.375	1.2247	1.1447	.9	34.81	205.379	2.429	1.807
1.55	2.4025	3.724	1.245	1.157	6.	36.	216.	2.4495	1.8171
1.6	2.56	4.096	1.265	1.170	.1	37.21	226.981	2.470	1.827
1.65	2.7225	4.492	1.285	1.182	.2	38.44	238.328	2.490	1.837
1.7	2.89	4.913	1.304	1.193	.3	39.69	250.047	2.510	1.847
1.75	3.0625	5.359	1.323	1.205	.4	40.96	262.144	2.530	1.857
1.8	3.24	5.832	1.342	1.216	.5	42.25	274.625	2.550	1.866
1.85	3.4225	6.332	1.360	1.228	.6	43.56	287.496	2.569	1.876
1.9	3.61	6.859	1.378	1.239	.7	44.89	300.763	2.588	1.885
1.95	3.8025	7.415	1.396	1.249	.8	46.24	314 432	2.608	1.895
2.	4.	8.	1.4142	1.2599	.9	47.61	328.509	2.627	1.904
.1	4.41	9.261	1.449	1.281	7.	49.	343.	2.6458	1.9129
.2	4.84	10 648	1.483	1.301	.1	50.41	357.911	2.665	1.922
.3	5.29	12.167	1.517	1.320	.2	51.84	373.248	2.683	1.931
.4	5.76	13.824	1.549	1.339	.3	53.29	389.017	2.702	1.940
.5	6.25	15.625	1.581	1.357	.4	54.76	405.224	2.720	1.949
.6	6.76	17.576	1.612	1.375	.5	56.25	421.875	2.739	1.957
.7	7.29	19.683	1.643	1.392	.6	57.76	438.976	2.757	1.966
.8	7.84	21.952	1.673	1.409	.7	59.29	456.533	2.775	1.975
.9	8.41	24.389	1.703	1.426	.8	60.84	474.552	2.793	1.983
3.	9.	27.	1.7321	1.4422	.9	62.41	493.039	2.811	1.992

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
8.	64.	512.	2.8284	2.	45	2025	91125	6.7082	3.5569
.1	65.61	531.441	2.846	2.008	46	2116	97336	6.7823	3.5830
.2	67.24	551.368	2.864	2.017	47	2209	103823	6.8557	3.6088
.3	68.89	571.787	2.881	2.025	48	2304	110592	6.9282	3.6342
.4	70.56	592.704	2.898	2.033	49	2401	117649	7.	3.6593
.5	72.25	614.125	2.915	2.041	50	2500	125000	7.0711	3.6840
.6	73.96	636.056	2.933	2.049	51	2601	132651	7.1414	3.7084
.7	75.69	658.503	2.950	2.057	52	2704	140608	7.2111	3.7325
.8	77.44	681.472	2.966	2.065	53	2809	148877	7.2801	3.7563
.9	79.21	704.969	2.983	2.072	54	2916	157464	7.3485	3.7798
9.	81.	729.	3.	2.0801	55	3025	166375	7.4162	3.8030
.1	82.81	753.571	3.017	2.088	56	3136	175616	7.4833	3.8259
.2	84.64	778.688	3.033	2.095	57	3249	185193	7.5498	3.8485
.3	86.49	804.357	3.050	2.103	58	3364	195112	7.6158	3.8709
.4	88.36	830.584	3.066	2.110	59	3481	205379	7.6811	3.8930
.5	90.25	857.375	3.082	2.118	60	3600	216000	7.7460	3.9149
.6	92.16	884.736	3.098	2.125	61	3721	226981	7.8102	3.9365
.7	94.09	912.673	3.114	2.133	62	3844	238328	7.8740	3.9579
.8	96.04	941.192	3.130	2.140	63	3969	250047	7.9373	3.9791
.9	98.01	970.299	3.146	2.147	64	4096	262144	8.	4.
10	100	1000	3.1623	2.1544	65	4225	274625	8.0623	4.0207
11	121	1331	3.3166	2.2240	66	4356	287496	8.1240	4.0412
12	144	1728	3.4641	2.2894	67	4489	300763	8.1854	4.0615
13	169	2197	3.6056	2.3513	68	4624	314432	8.2462	4.0817
14	196	2744	3.7417	2.4101	69	4761	328509	8.3066	4.1016
15	225	3375	3.8730	2.4662	70	4900	343000	8.3666	4.1213
16	256	4096	4.	2.5198	71	5041	357911	8.4261	4.1408
17	289	4913	4.1231	2.5713	72	5184	373248	8.4853	4.1602
18	324	5832	4.2426	2.6207	73	5329	389017	8.5440	4.1793
19	361	6859	4.3589	2.6684	74	5476	405224	8.6023	4.1983
20	400	8000	4.4721	2.7144	75	5625	421875	8.6603	4.2172
21	441	9261	4.5826	2.7589	76	5776	438976	8.7178	4.2358
22	484	10648	4.6904	2.8020	77	5929	456533	8.7750	4.2543
23	529	12167	4.7958	2.8439	78	6084	474552	8.8318	4.2727
24	576	13824	4.8990	2.8845	79	6241	493039	8.8882	4.2908
25	625	15625	5.	2.9240	80	6400	512000	8.9443	4.3089
26	676	17576	5.0990	2.9625	81	6561	531441	9.	4.3267
27	729	19683	5.1962	3.	82	6724	551368	9.0554	4.3445
28	784	21952	5.2915	3.0366	83	6889	571787	9.1104	4.3621
29	841	24389	5.3852	3.0723	84	7056	592704	9.1652	4.3795
30	900	27000	5.4772	3.1072	85	7225	614125	9.2195	4.3968
31	961	29791	5.5678	3.1414	86	7396	636056	9.2736	4.4140
32	1024	32768	5.6569	3.1748	87	7569	658503	9.3276	4.4310
33	1089	35937	5.7446	3.2075	88	7744	681472	9.3808	4.4480
34	1156	39304	5.8310	3.2396	89	7921	704969	9.4340	4.4647
35	1225	42875	5.9161	3.2711	90	8100	729000	9.4868	4.4814
36	1296	46656	6.	3.3019	91	8281	753571	9.5394	4.4979
37	1369	50653	6.0828	3.3322	92	8464	778688	9.5917	4.5144
38	1444	54872	6.1644	3.3620	93	8649	804357	9.6437	4.5307
39	1521	59319	6.2450	3.3912	94	8836	830584	9.6954	4.5468
40	1600	64000	6.3246	3.4200	95	9025	857375	9.7468	4.5629
41	1681	68921	6.4031	3.4482	96	9216	884736	9.7980	4.5789
42	1764	74088	6.4807	3.4760	97	9409	912673	9.8489	4.5947
43	1849	79507	6.5574	3.5034	98	9604	941192	9.8995	4.6104
44	1936	85184	6.6332	3.5303	99	9801	970299	9.9499	4.6261

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
100	10000	1000000	10.	4.6416	155	24025	3723875	12.4499	5.3717
101	10201	1030301	10.0499	4.6570	156	24336	3796416	12.4900	5.3832
102	10404	1061208	10.0995	4.6723	157	24649	3869893	12.5300	5.3947
103	10609	1092727	10.1489	4.6875	158	24964	3944312	12.5698	5.4061
104	10816	1124864	10.1980	4.7027	159	25281	4019679	12.6095	5.4175
105	11025	1157625	10.2470	4.7177	160	25600	4096000	12.6491	5.4288
106	112 6	1191016	10.2956	4.7326	161	25921	4173281	12.6886	5.4401
107	11449	1225043	10.3441	4.7475	162	26244	4251528	12.7279	5.4514
108	11664	1259712	10.3923	4.7622	163	26569	4330747	12.7671	5.4626
109	11881	1295029	10.4403	4.7769	164	26896	4410944	12.8062	5.4737
110	12100	1331000	10.4881	4.7914	165	27225	4492125	12.8452	5.4848
111	12321	1367631	10.5357	4.8059	166	27556	4574296	12.8841	5.4959
112	12544	1404928	10.5830	4.8203	167	27889	4657463	12.9228	5.5069
113	12769	1442897	10.6301	4.8346	168	28224	4741632	12.9615	5.5178
114	12996	1481544	10.6771	4.8488	169	28561	4826809	13.0000	5.5288
115	13225	1520875	10.7238	4.8629	170	28900	4913000	13.0384	5.5397
116	13456	1560896	10.7703	4.8770	171	29241	5000211	13.0767	5.5505
117	13689	1601613	10.8167	4.8910	172	29584	5088448	13.1149	5.5613
118	13924	1643032	10.8628	4.9049	173	29929	5177717	13.1529	5.5721
119	14161	1685159	10.9087	4.9187	174	30276	5268024	13.1909	5.5828
120	14400	1728000	10.9545	4.9324	175	30625	5359375	13.2288	5.5934
121	14641	1771561	11.0000	4.9461	176	30976	5451776	13.2665	5.6041
122	14884	1815848	11.0454	4.9597	177	31329	5545233	13.3041	5.6147
123	15129	1860867	11.0905	4.9732	178	31684	5639752	13.3417	5.6252
124	15376	1906624	11.1355	4.9866	179	32041	5735339	13.3791	5.6357
125	15625	1953125	11.1803	5.0000	180	32400	5832000	13.4164	5.6462
126	15876	2000376	11.2250	5.0133	181	32761	5929741	13.4536	5.6567
127	16129	2048383	11.2694	5.0265	182	33124	6028568	13.4907	5.6671
128	16384	2097152	11.3137	5.0397	183	33489	6128487	13.5277	5.6774
129	16641	2146689	11.3578	5.0528	184	33856	6229504	13.5647	5.6877
130	16900	2197000	11.4018	5.0658	185	34225	6331625	13.6015	5.6980
131	17161	2248091	11.4455	5.0788	186	34596	6434856	13.6382	5.7083
132	17424	2299968	11.4891	5.0916	187	34969	6539203	13.6748	5.7185
133	17689	2352637	11.5326	5.1045	188	35344	6644672	13.7113	5.7287
134	17956	2406104	11.5758	5.1172	189	35721	6751269	13.7477	5.7388
135	18225	2460375	11.6190	5.1299	190	36100	6859000	13.7840	5.7489
136	18496	2515456	11.6619	5.1426	191	36481	6967871	13.8203	5.7590
137	18769	2571353	11.7047	5.1551	192	36864	7077888	13.8564	5.7690
138	19044	2628072	11.7473	5.1676	193	37249	7189057	13.8924	5.7790
139	19321	2685619	11.7898	5.1801	194	37636	7301384	13.9284	5.7890
140	19600	2744000	11.8322	5.1925	195	38025	7414875	13.9642	5.7989
141	19881	2803221	11.8743	5.2048	196	38416	7529536	14.0000	5.8088
142	20164	2863288	11.9164	5.2171	197	38809	7645373	14.0357	5.8186
143	20449	2924207	11.9583	5.2293	198	39204	7762392	14.0712	5.8285
144	20736	2985984	12.0000	5.2415	199	39601	7880599	14.1067	5.8383
145	21025	3048625	12.0416	5.2536	200	40000	8000000	14.1421	5.8480
146	21316	3112136	12.0830	5.2656	201	40401	8120601	14.1774	5.8578
147	21609	3176523	12.1244	5.2776	202	40804	8242408	14.2127	5.8675
148	21904	3241792	12.1655	5.2896	203	41209	8365427	14.2478	5.8771
149	22201	3307949	12.2066	5.3015	204	41616	8489664	14.2829	5.8868
150	22500	3375000	12.2474	5.3133	205	42025	8615125	14.3178	5.8964
151	22801	3442951	12.2882	5.3251	206	42436	8741816	14.3527	5.9059
152	23104	3511808	12.3288	5.3368	207	42849	8869743	14.3875	5.9155
153	23409	3581577	12.3693	5.3485	208	43264	8998912	14.4222	5.9250
154	23716	3652264	12.4097	5.3601	209	43681	9129329	14.4568	5.9345

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
210	44100	9261000	14.4914	5.9439	265	70225	18609625	16.2788	6.4232
211	44521	9393931	14.5258	5.9533	266	70756	18821096	16.3095	6.4312
212	44944	9528128	14.5602	5.9627	267	71289	19034163	16.3401	6.4393
213	45369	9663597	14.5945	5.9721	268	71824	19248832	16.3707	6.4473
214	45796	9800344	14.6287	5.9814	269	72361	19465109	16.4012	6.4553
215	46225	9938375	14.6629	5.9907	270	72900	19683000	16.4317	6.4633
216	46656	10077696	14.6969	6.0000	271	73441	19902511	16.4621	6.4713
217	47089	10218313	14.7309	6.0092	272	73984	20123648	16.4924	6.4792
218	47524	10360232	14.7648	6.0185	273	74529	20346417	16.5227	6.4872
219	47961	10503459	14.7986	6.0277	274	75076	20570824	16.5529	6.4951
220	48400	10648000	14.8324	6.0368	275	75625	20796875	16.5831	6.5030
221	48841	10793861	14.8661	6.0459	276	76176	21024576	16.6132	6.5108
222	49284	10941048	14.8997	6.0550	277	76729	21253933	16.6433	6.5187
223	49729	11089567	14.9332	6.0641	278	77284	21484952	16.6733	6.5265
224	50176	11239424	14.9666	6.0732	279	77841	21717639	16.7033	6.5343
225	50625	11390625	15.0000	6.0822	280	78400	21952000	16.7332	6.5421
226	51076	11543176	15.0333	6.0912	281	78961	22188041	16.7631	6.5499
227	51529	11697083	15.0665	6.1002	282	79524	22425768	16.7929	6.5577
228	51984	11852352	15.0997	6.1091	283	80089	22665187	16.8226	6.5654
229	52441	12008989	15.1327	6.1180	284	80656	22906304	16.8523	6.5731
230	52900	12167000	15.1658	6.1269	285	81225	23149125	16.8819	6.5808
231	53361	12326391	15.1987	6.1358	286	81796	23393656	16.9115	6.5885
232	53824	12487168	15.2315	6.1446	287	82369	23639903	16.9411	6.5962
233	54289	12649337	15.2643	6.1534	288	82944	23887872	16.9706	6.6039
234	54756	12812904	15.2971	6.1622	289	83521	24137569	17.0000	6.6115
235	55225	12977875	15.3297	6.1710	290	84100	24389000	17.0294	6.6191
236	55696	13144256	15.3623	6.1797	291	84681	24642171	17.0587	6.6267
237	56169	13312053	15.3948	6.1885	292	85264	24897088	17.0880	6.6343
238	56644	13481272	15.4272	6.1972	293	85849	25153757	17.1172	6.6419
239	57121	13651919	15.4596	6.2058	294	86436	25412184	17.1464	6.6494
240	57600	13824000	15.4919	6.2145	295	87025	25672375	17.1756	6.6569
241	58081	13997521	15.5242	6.2231	296	87616	25934336	17.2047	6.6644
242	58564	14172488	15.5563	6.2317	297	88209	26198073	17.2337	6.6719
243	59049	14348907	15.5885	6.2403	298	88804	26463592	17.2627	6.6794
244	59536	14526784	15.6205	6.2488	299	89401	26730899	17.2916	6.6869
245	60025	14706125	15.6525	6.2573	300	90000	27000000	17.3205	6.6943
246	60516	14886936	15.6844	6.2658	301	90601	27270901	17.3494	6.7018
247	61009	15069223	15.7162	6.2743	302	91204	27543608	17.3781	6.7092
248	61504	15252992	15.7480	6.2828	303	91809	27818127	17.4069	6.7166
249	62001	15438249	15.7797	6.2912	304	92416	28094464	17.4356	6.7240
250	62500	15625000	15.8114	6.2996	305	93025	28372625	17.4642	6.7313
251	63001	15813251	15.8430	6.3080	306	93636	28652616	17.4929	6.7387
252	63504	16003008	15.8745	6.3164	307	94249	28934443	17.5214	6.7460
253	64009	16194277	15.9060	6.3247	308	94864	29218112	17.5499	6.7533
254	64516	16387064	15.9374	6.3330	309	95481	29503629	17.5784	6.7606
255	65025	16581375	15.9687	6.3413	310	96100	29791000	17.6068	6.7679
256	65536	16777216	16.0000	6.3496	311	96721	30080231	17.6352	6.7752
257	66049	16974593	16.0312	6.3579	312	97344	30371328	17.6635	6.7824
258	66564	17173512	16.0624	6.3661	313	97969	30664297	17.6918	6.7897
259	67081	17373979	16.0935	6.3743	314	98596	30959144	17.7200	6.7969
260	67600	17576000	16.1245	6.3825	315	99225	31255875	17.7482	6.8041
261	68121	17779581	16.1555	6.3907	316	99856	31554496	17.7764	6.8113
262	68644	17984728	16.1864	6.3988	317	100489	31855013	17.8045	6.8185
263	69169	18191447	16.2173	6.4070	318	101124	32157432	17.8326	6.8256
264	69696	18399744	16.2481	6.4151	319	101761	32461759	17.8606	6.8328

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
320	102400	32768000	17.8885	6.8399	375	140625	52734375	19.3649	7.2112
321	103041	33076161	17.9165	6.8470	376	141376	53157376	19.3907	7.2177
322	103684	33386248	17.9444	6.8541	377	142129	53582633	19.4165	7.2240
323	104329	33698267	17.9722	6.8612	378	142884	54010152	19.4422	7.2304
324	104976	34012224	18.0000	6.8683	379	143641	54439939	19.4679	7.2368
325	105625	34328125	18.0278	6.8753	380	144400	54872000	19.4936	7.2432
326	106276	34645976	18.0555	6.8824	381	145161	55306341	19.5192	7.2495
327	106929	34965783	18.0831	6.8894	382	145924	55742968	19.5448	7.2558
328	107584	35287552	18.1108	6.8964	383	146689	56181887	19.5704	7.2622
329	108241	35611289	18.1384	6.9034	384	147456	56623104	19.5959	7.2685
330	108900	35937000	18.1659	6.9104	385	148225	57066625	19.6214	7.2748
331	109561	36264691	18.1934	6.9174	386	148996	57512456	19.6469	7.2811
332	110224	36594368	18.2209	6.9244	387	149769	57960603	19.6723	7.2874
333	110889	36926037	18.2483	6.9313	388	150544	58411072	19.6977	7.2936
334	111556	37259704	18.2757	6.9382	389	151321	58863869	19.7231	7.2999
335	112225	37595375	18.3030	6.9451	390	152100	59319000	19.7484	7.3061
336	112896	37933056	18.3303	6.9521	391	152881	59776471	19.7737	7.3124
337	113569	38272753	18.3576	6.9589	392	153664	60236288	19.7990	7.3186
338	114244	38614472	18.3848	6.9658	393	154449	60698457	19.8242	7.3248
339	114921	38958219	18.4120	6.9727	394	155236	61162984	19.8494	7.3310
340	115600	39304000	18.4391	6.9795	395	156025	61629875	19.8746	7.3372
341	116281	39651821	18.4662	6.9864	396	156816	62099136	19.8997	7.3434
342	116964	40001688	18.4932	6.9932	397	157609	62570773	19.9249	7.3496
343	117649	40353607	18.5203	7.0000	398	158404	63044792	19.9499	7.3558
344	118336	40707584	18.5472	7.0068	399	159201	63521199	19.9750	7.3619
345	119025	41063625	18.5742	7.0136	400	160000	64000000	20.0000	7.3681
346	119716	41421736	18.6011	7.0203	401	160801	64481201	20.0250	7.3742
347	120409	41781923	18.6279	7.0271	402	161604	64964808	20.0499	7.3803
348	121104	42144192	18.6548	7.0338	403	162409	65450827	20.0749	7.3864
349	121801	42508549	18.6815	7.0406	404	163216	65939264	20.0998	7.3925
350	122500	42875000	18.7083	7.0473	405	164025	66430125	20.1246	7.3986
351	123201	43243551	18.7350	7.0540	406	164836	66923416	20.1494	7.4047
352	123904	43614208	18.7617	7.0607	407	165649	67419143	20.1742	7.4108
353	124609	43986977	18.7883	7.0674	408	166464	67917312	20.1990	7.4169
354	125316	44361864	18.8149	7.0740	409	167281	68417929	20.2237	7.4229
355	126025	44738875	18.8414	7.0807	410	168100	68921000	20.2485	7.4290
356	126736	45118016	18.8680	7.0873	411	168921	69426531	20.2731	7.4350
357	127449	45499293	18.8944	7.0940	412	169744	69934528	20.2978	7.4410
358	128164	45882712	18.9209	7.1006	413	170569	70444997	20.3224	7.4470
359	128881	46268279	18.9473	7.1072	414	171396	70957944	20.3470	7.4530
360	129600	46656000	18.9737	7.1138	415	172225	71473375	20.3715	7.4590
361	130321	47045881	19.0000	7.1204	416	173056	71991296	20.3961	7.4650
362	131044	47437928	19.0263	7.1269	417	173889	72511713	20.4206	7.4710
363	131769	47832147	19.0526	7.1335	418	174724	73034632	20.4450	7.4770
364	132496	48228544	19.0788	7.1400	419	175561	73560059	20.4695	7.4829
365	133225	48627125	19.1050	7.1466	420	176400	74088000	20.4939	7.4889
366	133956	49027896	19.1311	7.1531	421	177241	74618461	20.5183	7.4948
367	134689	49430863	19.1572	7.1596	422	178084	75151448	20.5426	7.5007
368	135424	49836032	19.1833	7.1661	423	178929	75686967	20.5670	7.5067
369	136161	50243409	19.2094	7.1726	424	179776	76225024	20.5913	7.5126
370	136900	50653000	19.2354	7.1791	425	180625	76765625	20.6155	7.5185
371	137641	51064811	19.2614	7.1855	426	181476	77308776	20.6398	7.5244
372	138384	51478848	19.2873	7.1920	427	182329	77854483	20.6640	7.5302
373	139129	51895117	19.3132	7.1984	428	183184	78402752	20.6882	7.5361
374	139876	52313624	19.3391	7.2048	429	184041	78953589	20.7123	7.5420

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
430	184900	79507000	20.7364	7.5478	485	235225	114084125	22.0227	7.8568
431	185761	80062991	20.7605	7.5537	486	236196	114791256	22.0454	7.8622
432	186624	80621568	20.7846	7.5595	487	237169	115501303	22.0681	7.8676
433	187489	81182737	20.8087	7.5654	488	238144	116214272	22.0907	7.8730
434	188356	81746504	20.8327	7.5712	489	239121	116930169	22.1133	7.8784
435	189225	82312875	20.8567	7.5770	490	240100	117649000	22.1359	7.8837
436	190096	82881856	20.8806	7.5828	491	241081	118370771	22.1585	7.8891
437	190969	83453453	20.9045	7.5886	492	242064	119095488	22.1811	7.8944
438	191844	84027672	20.9284	7.5944	493	243049	119823157	22.2036	7.8998
439	192721	84604519	20.9523	7.6001	494	244036	120553784	22.2261	7.9051
440	193600	85184000	20.9762	7.6059	495	245025	121287375	22.2486	7.9105
441	194481	85766121	21.0000	7.6117	496	246016	122023936	22.2711	7.9158
442	195364	86350888	21.0238	7.6174	497	247009	122763473	22.2935	7.9211
443	196249	86938307	21.0476	7.6232	498	248004	123505992	22.3159	7.9264
444	197136	87528384	21.0713	7.6289	499	249001	124251499	22.3383	7.9317
445	198025	88121125	21.0950	7.6346	500	250000	125000000	22.3607	7.9370
446	198916	88716536	21.1187	7.6403	501	251001	125751501	22.3830	7.9423
447	199809	89314623	21.1424	7.6460	502	252004	126506008	22.4054	7.9476
448	200704	89915392	21.1660	7.6517	503	253009	127263527	22.4277	7.9528
449	201601	90518849	21.1896	7.6574	504	254016	128024064	22.4499	7.9581
450	202500	91125000	21.2132	7.6631	505	255025	128787625	22.4722	7.9634
451	203401	91733851	21.2368	7.6688	506	256036	129554216	22.4944	7.9686
452	204304	92345408	21.2603	7.6744	507	257049	130323843	22.5167	7.9739
453	205209	92959677	21.2838	7.6800	508	258064	131096512	22.5389	7.9791
454	206116	93576664	21.3073	7.6857	509	259081	131872229	22.5610	7.9843
455	207025	94196375	21.3307	7.6914	510	260100	132651000	22.5832	7.9896
456	207936	94818816	21.3542	7.6970	511	261121	133432831	22.6053	7.9948
457	208849	95443993	21.3776	7.7026	512	262144	134217728	22.6274	8.0000
458	209764	96071912	21.4009	7.7082	513	263169	135005697	22.6495	8.0052
459	210681	96702579	21.4243	7.7138	514	264196	135796744	22.6716	8.0104
460	211600	97336000	21.4476	7.7194	515	265225	136590875	22.6936	8.0156
461	212521	97972181	21.4709	7.7250	516	266256	137388096	22.7156	8.0208
462	213444	98611128	21.4942	7.7306	517	267289	138188413	22.7376	8.0260
463	214369	99252847	21.5174	7.7362	518	268324	138991832	22.7596	8.0311
464	215296	99897344	21.5407	7.7418	519	269361	139798359	22.7816	8.0363
465	216225	100544625	21.5639	7.7473	520	270400	140608000	22.8035	8.0415
466	217156	101194696	21.5870	7.7529	521	271441	141420761	22.8254	8.0466
467	218089	101847563	21.6102	7.7584	522	272484	142236648	22.8473	8.0517
468	219024	102503232	21.6333	7.7639	523	273529	143055667	22.8692	8.0569
469	219961	103161709	21.6564	7.7695	524	274576	143877824	22.8910	8.0620
470	220900	103823000	21.6795	7.7750	525	275625	144703125	22.9129	8.0671
471	221841	104487111	21.7025	7.7805	526	276676	145531576	22.9347	8.0723
472	222784	105154048	21.7256	8.7860	527	277729	146363183	22.9565	8.0774
473	223729	105823817	21.7486	7.7915	528	278784	147197952	22.9783	8.0825
474	224676	106496424	21.7715	7.7970	529	279841	148035889	23.0000	8.0876
475	225625	107171875	21.7945	7.8025	530	280900	148877000	23.0217	8.0927
476	226576	107850176	21.8174	7.8079	531	281961	149721291	23.0434	8.0978
477	227529	108531333	21.8403	7.8134	532	283024	150568768	23.0651	8.1028
478	228484	109215352	21.8632	7.8188	533	284089	151419437	23.0868	8.1079
479	229441	109902239	21.8861	7.8243	534	285156	152273304	23.1084	8.1130
480	230400	110592000	21.9089	7.8297	535	286225	153130375	23.1301	8.1180
481	231361	111284641	21.9317	7.8352	536	287296	153990656	23.1517	8.1231
482	232324	111980168	21.9545	7.8406	537	288369	154854153	23.1733	8.1281
483	233289	112678587	21.9773	7.8460	538	289444	155720872	23.1948	8.1332
484	234256	113379904	22.0000	7.8514	539	290521	156590819	23.2164	8.1382

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
540	291600	157464000	23.2379	8.1433	595	354025	210644875	24.3926	8.4108
541	292681	158340421	23.2594	8.1483	596	355216	211708736	24.4131	8.4155
542	293764	159220088	23.2809	8.1533	597	356409	212776173	24.4336	8.4202
543	294849	160103007	23.3024	8.1583	598	357604	213847192	24.4540	8.4249
544	295936	160989184	23.3238	8.1633	599	358801	214921799	24.4745	8.4296
545	297025	161878625	23.3452	8.1683	600	360000	216000000	24.4949	8.4343
546	298116	162771336	23.3666	8.1733	601	361201	217081801	24.5153	8.4390
547	299209	163667323	23.3880	8.1783	602	362404	218167208	24.5357	8.4437
548	300304	164566592	23.4094	8.1833	603	363609	219256227	24.5561	8.4484
549	301401	165469149	23.4307	8.1882	604	364816	220348864	24.5764	8.4530
550	302500	166375000	23.4521	8.1932	605	366025	221445125	24.5967	8.4577
551	303601	167284151	23.4734	8.1982	606	367236	222545016	24.6171	8.4623
552	304704	168196608	23.4947	8.2031	607	368449	223648543	24.6374	8.4670
553	305809	169112377	23.5160	8.2081	608	369664	224755712	24.6577	8.4716
554	306916	170031464	23.5372	8.2130	609	370881	225866529	24.6779	8.4763
555	308025	170953875	23.5584	8.2180	610	372100	226981000	24.6982	8.4809
556	309136	171879616	23.5797	8.2229	611	373321	228099131	24.7184	8.4856
557	310249	172808693	23.6008	8.2278	612	374544	229220928	24.7386	8.4902
558	311364	173741112	23.6220	8.2327	613	375769	230346397	24.7588	8.4948
559	312481	174676879	23.6432	8.2377	614	376996	231475544	24.7790	8.4994
560	313600	175616000	23.6643	8.2426	615	378225	232608375	24.7992	8.5040
561	314721	176558481	23.6854	8.2475	616	379456	233744896	24.8193	8.5086
562	315844	177504328	23.7065	8.2524	617	380689	234885113	24.8395	8.5132
563	316969	178453547	23.7276	8.2573	618	381924	236029032	24.8596	8.5178
564	318096	179406144	23.7487	8.2621	619	383161	237176659	24.8797	8.5224
565	319225	180362125	23.7697	8.2670	620	384400	238328000	24.8998	8.5270
566	320356	181321496	23.7908	8.2719	621	385641	239483061	24.9199	8.5316
567	321489	182284263	23.8118	8.2768	622	386884	240641848	24.9399	8.5362
568	322624	183250432	23.8328	8.2816	623	388129	241804367	24.9600	8.5408
569	323761	184220009	23.8537	8.2865	624	389376	242970624	24.9800	8.5453
570	324900	185193000	23.8747	8.2913	625	390625	244140625	25.0000	8.5499
571	326041	186169411	23.8956	8.2962	626	391876	245314376	25.0200	8.5544
572	327184	187149248	23.9165	8.3010	627	393129	246491883	25.0400	8.5590
573	328329	188132517	23.9374	8.3059	628	394384	247673152	25.0599	8.5635
574	329476	189119224	23.9583	8.3107	629	395641	248858189	25.0799	8.5681
575	330625	190109375	23.9792	8.3155	630	396900	250047000	25.0998	8.5726
576	331776	191102976	24.0000	8.3203	631	398161	251239591	25.1197	8.5772
577	332929	192100033	24.0208	8.3251	632	399424	252435968	25.1396	8.5817
578	334084	193100552	24.0416	8.3300	633	400689	253636137	25.1595	8.5862
579	335241	194104539	24.0624	8.3348	634	401956	254840104	25.1794	8.5907
580	336400	195112000	24.0832	8.3396	635	403225	256047875	25.1992	8.5952
581	337561	196122941	24.1039	8.3443	636	404496	257259456	25.2190	8.5997
582	338724	197137368	24.1247	8.3491	637	405769	258474853	25.2389	8.6043
583	339889	198155287	24.1454	8.3539	638	407044	259694072	25.2587	8.6088
584	341056	199176704	24.1661	8.3587	639	408321	260917119	25.2784	8.6132
585	342225	200201625	24.1868	8.3634	640	409600	262144000	25.2982	8.6177
586	343396	201230056	24.2074	8.3682	641	410881	263374721	25.3180	8.6222
587	344569	202262003	24.2281	8.3730	642	412164	264609288	25.3377	8.6267
588	345744	203297472	24.2487	8.3777	643	413449	265847707	25.3574	8.6312
589	346921	204336469	24.2693	8.3825	644	414736	267089984	25.3772	8.6357
590	348100	205379000	24.2899	8.3872	645	416025	268336125	25.3969	8.6401
591	349281	206425071	24.3105	8.3919	646	417316	269586136	25.4165	8.6446
592	350464	207474688	24.3311	8.3967	647	418609	270840023	25.4362	8.6490
593	351649	208527857	24.3516	8.4014	648	419904	272097792	25.4558	8.6535
594	352836	209584584	24.3721	8.4061	649	421201	273359449	25.4755	8.6579

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
650	422500	274625000	25.4951	8.6624	705	497025	350402625	26.5518	8.9001
651	423801	275894451	25.5147	8.6668	706	498436	351895816	26.5707	8.9043
652	425104	277167808	25.5343	8.6713	707	499849	353393243	26.5895	8.9085
653	426409	278445077	25.5539	8.6757	708	501264	354894912	26.6083	8.9127
654	427716	279726264	25.5734	8.6801	709	502681	356400829	26.6271	8.9169
655	429025	281011375	25.5930	8.6845	710	504100	357911000	26.6458	8.9211
656	430336	282300416	25.6125	8.6890	711	505521	359425431	26.6646	8.9253
657	431649	283593393	25.6320	8.6934	712	506944	360944128	26.6833	8.9295
658	432964	284890312	25.6515	8.6978	713	508369	362467097	26.7021	8.9337
659	434281	286191179	25.6710	8.7022	714	509796	363994344	26.7208	8.9378
660	435600	287496000	25.6905	8.7066	715	511225	365525875	26.7395	8.9420
661	436921	288804781	25.7099	8.7110	716	512656	367061696	26.7582	8.9462
662	438244	290117528	25.7294	8.7154	717	514089	368601813	26.7769	8.9503
663	439569	291434247	25.7488	8.7198	718	515524	370146232	26.7955	8.9545
664	440896	292754944	25.7682	8.7241	719	516961	371694959	26.8142	8.9587
665	442225	294079625	25.7876	8.7285	720	518400	373248000	26.8328	8.9628
666	443556	295408296	25.8070	8.7329	721	519841	374805361	26.8514	8.9670
667	444889	296740963	25.8263	8.7373	722	521284	376367048	26.8701	8.9711
668	446224	298077632	25.8457	8.7416	723	522729	377933067	26.8887	8.9752
669	447561	299418309	25.8650	8.7460	724	524176	379503424	26.9072	8.9794
670	448900	300763000	25.8844	8.7503	725	525625	381078125	26.9258	8.9835
671	450241	302111711	25.9037	8.7547	726	527076	382657176	26.9444	8.9876
672	451584	303464448	25.9230	8.7590	727	528529	384240583	26.9629	8.9918
673	452929	304821217	25.9422	8.7634	728	529984	385828352	26.9815	8.9959
674	454276	306182024	25.9615	8.7677	729	531441	387420489	27.0000	9.0000
675	455625	307546875	25.9808	8.7721	730	532900	389017000	27.0185	9.0041
676	456976	308915776	26.0000	8.7764	731	534361	390617891	27.0370	9.0082
677	458329	310288733	26.0192	8.7807	732	535824	392223168	27.0555	9.0123
678	459684	311665752	26.0384	8.7850	733	537289	393832837	27.0740	9.0164
679	461041	313046839	26.0576	8.7893	734	538756	395446904	27.0924	9.0205
680	462400	314432000	26.0768	8.7937	735	540225	397065375	27.1109	9.0246
681	463761	315821241	26.0960	8.7980	736	541696	398688256	27.1293	9.0287
682	465124	317214568	26.1151	8.8023	737	543169	400315553	27.1477	9.0328
683	466489	318611987	26.1343	8.8066	738	544644	401947272	27.1662	9.0369
684	467856	320013504	26.1534	8.8109	739	546121	403583419	27.1846	9.0410
685	469225	321419125	26.1725	8.8152	740	547600	405224000	27.2029	9.0450
686	470596	322828856	26.1916	8.8194	741	549081	406869021	27.2213	9.0491
687	471969	324242703	26.2107	8.8237	742	550564	408518488	27.2397	9.0532
688	473344	325660672	26.2298	8.8280	743	552049	410172407	27.2580	9.0572
689	474721	327082769	26.2488	8.8323	744	553536	411830784	27.2764	9.0613
690	476100	328509000	26.2679	8.8366	745	555025	413493625	27.2947	9.0654
691	477481	329939371	26.2869	8.8408	746	556516	415160936	27.3130	9.0694
692	478864	331373888	26.3059	8.8451	747	558009	416832723	27.3313	9.0735
693	480249	332812557	26.3249	8.8493	748	559504	418508992	27.3496	9.0775
694	481636	334255384	26.3439	8.8536	749	561001	420189749	27.3679	9.0816
695	483025	335702375	26.3629	8.8578	750	562500	421875000	27.3861	9.0856
696	484416	337153536	26.3818	8.8621	751	564001	423564751	27.4044	9.0896
697	485809	338608873	26.4008	8.8663	752	565504	425259008	27.4226	9.0937
698	487204	340068392	26.4197	8.8706	753	567009	426957777	27.4408	9.0977
699	488601	341532099	26.4386	8.8748	754	568516	428661064	27.4591	9.1017
700	490000	343000000	26.4575	8.8790	755	570025	430368875	27.4773	9.1057
701	491401	344472101	26.4764	8.8833	756	571536	432081216	27.4955	9.1098
702	492804	345948408	26.4953	8.8875	757	573049	433798093	27.5136	9.1138
703	494209	347428927	26.5141	8.8917	758	574564	435519512	27.5318	9.1178
704	495616	348913664	26.5330	8.8959	759	576081	437245479	27.5500	9.1218

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
760	577600	438976000	27.5681	9.1258	815	664225	541343375	28.5482	9.3408
761	579121	440711081	27.5862	9.1298	816	665856	543338496	28.5657	9.3447
762	580644	442450728	27.6043	9.1338	817	667489	545338513	28.5832	9.3485
763	582169	444194947	27.6225	9.1378	818	669124	547343432	28.6007	9.3523
764	583696	445943744	27.6405	9.1418	819	670761	549353259	28.6182	9.3561
765	585225	447697125	27.6586	9.1458	820	672400	551368000	28.6356	9.3599
766	586756	449455096	27.6767	9.1498	821	674041	553387661	28.6531	9.3637
767	588289	451217663	27.6948	9.1537	822	675684	555412248	28.6705	9.3675
768	589824	452984832	27.7128	9.1577	823	677329	557441767	28.6880	9.3713
769	591361	454756609	27.7308	9.1617	824	678976	559476224	28.7054	9.3751
770	592900	456533000	27.7489	9.1657	825	680625	561515625	28.7228	9.3789
771	594441	458314011	27.7669	9.1696	826	682276	563559976	28.7402	9.3827
772	595984	460099648	27.7849	9.1736	827	683929	565609283	28.7576	9.3865
773	597529	461889917	27.8029	9.1775	828	685584	567663552	28.7750	9.3902
774	599076	463684824	27.8209	9.1815	829	687241	569722789	28.7924	9.3940
775	600625	465484375	27.8388	9.1855	830	688900	571787000	28.8097	9.3978
776	602176	467288576	27.8568	9.1894	831	690561	573856191	28.8271	9.4016
777	603729	469097433	27.8747	9.1933	832	692224	575930368	28.8444	9.4053
778	605284	470910952	27.8927	9.1973	833	693889	578009537	28.8617	9.4091
779	606841	472729139	27.9106	9.2012	834	695556	580093704	28.8791	9.4129
780	608400	474552000	27.9285	9.2052	835	697225	582182875	28.8964	9.4166
781	609961	476379541	27.9464	9.2091	836	698896	584277056	28.9137	9.4204
782	611524	478211768	27.9643	9.2130	837	700569	586376253	28.9310	9.4241
783	613089	480048687	27.9821	9.2170	838	702244	588480472	28.9482	9.4279
784	614656	481890304	28.0000	9.2209	839	703921	590589719	28.9655	9.4316
785	616225	483736625	28.0179	9.2248	840	705600	592704000	28.9828	9.4354
786	617796	485587656	28.0357	9.2287	841	707281	594823321	29.0000	9.4391
787	619369	487443403	28.0535	9.2326	842	708964	596947688	29.0172	9.4429
788	620944	489303872	28.0713	9.2365	843	710649	599077107	29.0345	9.4466
789	622521	491169069	28.0891	9.2404	844	712336	601211584	29.0517	9.4503
790	624100	493039000	28.1069	9.2443	845	714025	603351125	29.0689	9.4541
791	625681	494913671	28.1247	9.2482	846	715716	605495736	29.0861	9.4578
792	627264	496793088	28.1425	9.2521	847	717409	607645423	29.1033	9.4615
793	628849	498677257	28.1603	9.2560	848	719104	609800192	29.1204	9.4652
794	630436	500566184	28.1780	9.2599	849	720801	611960049	29.1376	9.4690
795	632025	502459875	28.1957	9.2638	850	722500	614125000	29.1548	9.4727
796	633616	504358336	28.2135	9.2677	851	724201	616295051	29.1719	9.4764
797	635209	506261573	28.2312	9.2716	852	725904	618470208	29.1890	9.4801
798	636804	508169592	28.2489	9.2754	853	727609	620650477	29.2062	9.4838
799	638401	510082399	28.2666	9.2793	854	729316	622835864	29.2233	9.4875
800	640000	512000000	28.2843	9.2832	855	731025	625026375	29.2404	9.4912
801	641601	513922401	28.3019	9.2870	856	732736	627222016	29.2575	9.4949
802	643204	515849608	28.3196	9.2909	857	734449	629422793	29.2746	9.4986
803	644809	517781627	28.3373	9.2948	858	736164	631628712	29.2916	9.5023
804	646416	519718464	28.3549	9.2986	859	737881	633839779	29.3087	9.5060
805	648025	521660125	28.3725	9.3025	860	739600	636056000	29.3258	9.5097
806	649636	523606616	28.3901	9.3063	861	741321	638277381	29.3428	9.5134
807	651249	525557943	28.4077	9.3102	862	743044	640503928	29.3598	9.5171
808	652864	527514112	28.4253	9.3140	863	744769	642735647	29.3769	9.5207
809	654481	529475129	28.4429	9.3179	864	746496	644972544	29.3939	9.5244
810	656100	531441000	28.4605	9.3217	865	748225	647214625	29.4109	9.5281
811	657721	533411731	28.4781	9.3255	866	749956	649461896	29.4279	9.5317
812	659344	535387328	28.4956	9.3294	867	751689	651714363	29.4449	9.5354
813	660969	537367797	28.5132	9.3332	868	753424	653972032	29.4618	9.5391
814	662596	539353144	28.5307	9.3370	869	755161	656234909	29.4788	9.5427

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
870	756900	658503000	29.4958	9.5464	925	855625	791453125	30.4138	9.7435
871	758641	660776311	29.5127	9.5501	926	857476	794022776	30.4302	9.7470
872	760384	663054848	29.5296	9.5537	927	859329	796597983	30.4467	9.7505
873	762129	665338617	29.5466	9.5574	928	861184	799178752	30.4631	9.7540
874	763876	667627624	29.5635	9.5610	929	863041	801765089	30.4795	9.7575
875	765625	669921875	29.5804	9.5647	930	864900	804357000	30.4959	9.7610
876	767376	672221376	29.5973	9.5683	931	866761	806954491	30.5123	9.7645
877	769129	674526133	29.6142	9.5719	932	868624	809557568	30.5287	9.7680
878	770884	676836152	29.6311	9.5756	933	870489	812166237	30.5450	9.7715
879	772641	679151439	29.6479	9.5792	934	872356	814780504	30.5614	9.7750
880	774400	681472000	29.6648	9.5828	935	874225	817400375	30.5778	9.7785
881	776161	683797841	29.6816	9.5865	936	876096	820025856	30.5941	9.7819
882	777924	686128968	29.6985	9.5901	937	877969	822656953	30.6105	9.7854
883	779689	688465387	29.7153	9.5937	938	879844	825293672	30.6268	9.7889
884	781456	690807104	29.7321	9.5973	939	881721	827936019	30.6431	9.7924
885	783225	693154125	29.7489	9.6010	940	883600	830584000	30.6594	9.7959
886	784996	695506456	29.7658	9.6046	941	885481	833237621	30.6757	9.7993
887	786769	697864103	29.7825	9.6082	942	887364	835896888	30.6920	9.8028
888	788544	700227072	29.7993	9.6118	943	889249	838561807	30.7083	9.8063
889	790321	702595369	29.8161	9.6154	944	891136	841232384	30.7246	9.8097
890	792100	704969000	29.8329	9.6190	945	893025	843908625	30.7409	9.8132
891	793881	707347971	29.8496	9.6226	946	894916	846590536	30.7571	9.8167
892	795664	709732288	29.8664	9.6262	947	896809	849278123	30.7734	9.8201
893	797449	712121957	29.8831	9.6298	948	898704	851971392	30.7896	9.8236
894	799236	714516984	29.8998	9.6334	949	900601	854670349	30.8058	9.8270
895	801025	716917375	29.9166	9.6370	950	902500	857375000	30.8221	9.8305
896	802816	719323136	29.9333	9.6406	951	904401	860085351	30.8383	9.8339
897	804609	721734273	29.9500	9.6442	952	906304	862801408	30.8545	9.8374
898	806404	724150792	29.9666	9.6477	953	908209	865523177	30.8707	9.8408
899	808201	726572699	29.9833	9.6513	954	910116	868250664	30.8869	9.8443
900	810000	729000000	30.0000	9.6549	955	912025	870983875	30.9031	9.8477
901	811801	731432701	30.0167	9.6585	956	913936	873722816	30.9192	9.8511
902	813604	733870808	30.0333	9.6620	957	915849	876467493	30.9354	9.8546
903	815409	736314327	30.0500	9.6656	958	917764	879217912	30.9516	9.8580
904	817216	738763264	30.0666	9.6692	959	919681	881974079	30.9677	9.8614
905	819025	741217625	30.0832	9.6727	960	921600	884736000	30.9839	9.8648
906	820836	743677416	30.0998	9.6763	961	923521	887503681	31.0000	9.8683
907	822649	746142643	30.1164	9.6799	962	925444	890277128	31.0161	9.8717
908	824464	748613312	30.1330	9.6834	963	927369	893056347	31.0322	9.8751
909	826281	751089429	30.1496	9.6870	964	929296	895841344	31.0483	9.8785
910	828100	753571000	30.1662	9.6905	965	931225	898632125	31.0644	9.8819
911	829921	756058031	30.1828	9.6941	966	933156	901428696	31.0805	9.8854
912	831744	758550528	30.1993	9.6976	967	935089	904231063	31.0966	9.8888
913	833569	761048497	30.2159	9.7012	968	937024	907039232	31.1127	9.8922
914	835396	763551944	30.2324	9.7047	969	938961	909853209	31.1288	9.8956
915	837225	766060875	30.2490	9.7082	970	940900	912673000	31.1448	9.8990
916	839056	768575296	30.2655	9.7118	971	942841	915498611	31.1609	9.9024
917	840889	771095213	30.2820	9.7153	972	944784	918330048	31.1769	9.9058
918	842724	773620632	30.2985	9.7188	973	946729	921167317	31.1929	9.9092
919	844561	776151559	30.3150	9.7224	974	948676	924010424	31.2090	9.9126
920	846400	778688000	30.3315	9.7259	975	950625	926859375	31.2250	9.9160
921	848241	781229961	30.3480	9.7294	976	952576	929714176	31.2410	9.9194
922	850084	783777448	30.3645	9.7329	977	954529	932574833	31.2570	9.9227
923	851929	786330467	30.3809	9.7364	978	956484	935441352	31.2730	9.9261
924	853776	788889024	30.3974	9.7400	979	958441	938313739	31.2890	9.9295

No.	Square.	Cube.	Sq. Root.	Cube. Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
980	960400	941192000	31.3050	9.9329	1035	1071225	1108717875	32.1714	10.1153
981	962361	944076141	31.3209	9.9363	1036	1073296	1111934656	32.1870	10.1186
982	964324	946966168	31.3369	9.9396	1037	1075369	1115157653	32.2025	10.1218
983	966289	949862087	31.3528	9.9430	1038	1077444	1118386872	32.2180	10.1251
984	968256	952763904	31.3688	9.9464	1039	1079521	1121622319	32.2335	10.1283
985	970225	955671625	31.3847	9.9497	1040	1081600	1124864000	32.2490	10.1316
986	972196	958585256	31.4006	9.9531	1041	1083681	1128111921	32.2645	10.1348
987	974169	961504803	31.4166	9.9565	1042	1085764	1131366088	32.2800	10.1381
988	976144	964430272	31.4325	9.9598	1043	1087849	1134626507	32.2955	10.1413
989	978121	967361669	31.4484	9.9632	1044	1089936	1137893184	32.3110	10.1446
990	980100	970299000	31.4643	9.9666	1045	1092025	1141166125	32.3265	10.1478
991	982081	973242271	31.4802	9.9699	1046	1094116	1144445336	32.3419	10.1510
992	984064	976191488	31.4960	9.9733	1047	1096209	1147730823	32.3574	10.1543
993	986049	979146657	31.5119	9.9766	1048	1098304	1151022592	32.3728	10.1575
994	988036	982107784	31.5278	9.9800	1049	1100401	1154320649	32.3883	10.1607
995	990025	985074875	31.5436	9.9833	1050	1102500	1157625000	32.4037	10.1640
996	992016	988047936	31.5595	9.9866	1051	1104601	1160935651	32.4191	10.1672
997	994009	991026973	31.5753	9.9900	1052	1106704	1164252608	32.4345	10.1704
998	996004	994011992	31.5911	9.9933	1053	1108809	1167575877	32.4500	10.1736
999	998001	997002999	31.6070	9.9967	1054	1110916	1170905464	32.4654	10.1769
1000	1000000	1000000000	31.6228	10.0000	1055	1113025	1174241375	32.4808	10.1801
1001	1002001	1003003001	31.6386	10.0033	1056	1115136	1177583616	32.4962	10.1833
1002	1004004	1006012008	31.6544	10.0067	1057	1117249	1180932193	32.5115	10.1865
1003	1006009	1009027027	31.6702	10.0100	1058	1119364	1184287112	32.5269	10.1897
1004	1008016	1012048064	31.6860	10.0133	1059	1121481	1187648379	32.5423	10.1929
1005	1010025	1015075125	31.7017	10.0166	1060	1123600	1191016000	32.5576	10.1961
1006	1012036	1018108216	31.7175	10.0200	1061	1125721	1194389981	32.5730	10.1993
1007	1014049	1021147343	31.7333	10.0233	1062	1127844	1197770328	32.5883	10.2025
1008	1016064	1024192512	31.7490	10.0266	1063	1129969	1201157047	32.6036	10.2057
1009	1018081	1027243729	31.7648	10.0299	1064	1132096	1204550144	32.6190	10.2089
1010	1020100	1030301000	31.7805	10.0332	1065	1134225	1207949625	32.6343	10.2121
1011	1022121	1033364331	31.7962	10.0365	1066	1136356	1211355496	32.6497	10.2153
1012	1024144	1036433728	31.8119	10.0398	1067	1138489	1214767763	32.6650	10.2185
1013	1026169	1039509197	31.8277	10.0431	1068	1140624	1218186432	32.6803	10.2217
1014	1028196	1042590744	31.8434	10.0465	1069	1142761	1221611509	32.6956	10.2249
1015	1030225	1045678375	31.8591	10.0498	1070	1144900	1225043000	32.7109	10.2281
1016	1032256	1048772096	31.8748	10.0531	1071	1147041	1228480911	32.7261	10.2313
1017	1034289	1051871913	31.8904	10.0563	1072	1149184	1231925248	32.7414	10.2345
1018	1036324	1054977832	31.9061	10.0596	1073	1151329	1235376017	32.7567	10.2376
1019	1038361	1058089859	31.9218	10.0629	1074	1153476	1238833224	32.7719	10.2408
1020	1040400	1061208000	31.9374	10.0662	1075	1155625	1242296875	32.7872	10.2440
1021	1042441	1064332261	31.9531	10.0695	1076	1157776	1245766976	32.8024	10.2472
1022	1044484	1067462648	31.9687	10.0728	1077	1159929	1249243533	32.8177	10.2503
1023	1046529	1070599167	31.9844	10.0761	1078	1162084	1252726552	32.8329	10.2535
1024	1048576	1073741824	32.0000	10.0794	1079	1164241	1256216039	32.8481	10.2567
1025	1050625	1076890625	32.0156	10.0826	1080	1166400	1259712000	32.8634	10.2599
1026	1052676	1080045576	32.0312	10.0859	1081	1168561	1263214441	32.8786	10.2630
1027	1054729	1083206683	32.0468	10.0892	1082	1170724	1266723368	32.8938	10.2662
1028	1056784	1086373952	32.0624	10.0925	1083	1172889	1270238787	32.9090	10.2693
1029	1058841	1089547389	32.0780	10.0957	1084	1175056	1273760704	32.9242	10.2725
1030	1060900	1092727000	32.0936	10.0990	1085	1177225	1277289125	32.9393	10.2757
1031	1062961	1095912791	32.1092	10.1023	1086	1179396	1280824056	32.9545	10.2788
1032	1065024	1099104768	32.1248	10.1055	1087	1181569	1284365503	32.9697	10.2820
1033	1067089	1102302937	32.1403	10.1088	1088	1183744	1287913472	32.9848	10.2851
1034	1069156	1105507304	32.1559	10.1121	1089	1185921	1291467969	33.0000	10.2883

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
1090	1188100	1295029000	33.0151	10.2914	1145	1311025	1501123625	33.8378	10.4617
1091	1190281	1298596571	33.0303	10.2946	1146	1313316	1505060136	33.8526	10.4647
1092	1192464	1302170688	33.0454	10.2977	1147	1315609	1509003523	33.8674	10.4678
1093	1194649	1305751357	33.0606	10.3009	1148	1317904	1512953792	33.8821	10.4708
1094	1196836	1309338584	33.0757	10.3040	1149	1320201	1516910949	33.8969	10.4739
1095	1199025	1312932375	33.0908	10.3071	1150	1322500	1520875000	33.9116	10.4769
1096	1201216	1316532736	33.1059	10.3103	1151	1324801	1524845951	33.9264	10.4799
1097	1203409	1320139673	33.1210	10.3134	1152	1327104	1528823808	33.9411	10.4830
1098	1205604	1323753192	33.1361	10.3165	1153	1329409	1532808577	33.9559	10.4860
1099	1207801	1327373299	33.1512	10.3197	1154	1331716	1536800264	33.9706	10.4890
1100	1210000	1331000000	33.1662	10.3228	1155	1334025	1540798875	33.9853	10.4921
1101	1212201	1334633301	33.1813	10.3259	1156	1336336	1544804416	34.0000	10.4951
1102	1214404	1338273208	33.1964	10.3290	1157	1338649	1548816893	34.0147	10.4981
1103	1216609	1341919727	33.2114	10.3322	1158	1340964	1552836312	34.0294	10.5011
1104	1218816	1345572864	33.2264	10.3353	1159	1343281	1556862679	34.0441	10.5042
1105	1221025	1349232625	33.2415	10.3384	1160	1345600	1560896000	34.0588	10.5072
1106	1223236	1352899016	33.2566	10.3415	1161	1347921	1564936281	34.0735	10.5102
1107	1225449	1356572043	33.2716	10.3447	1162	1350244	1568983528	34.0881	10.5132
1108	1227664	1360251712	33.2866	10.3478	1163	1352569	1573037747	34.1028	10.5162
1109	1229881	1363938029	33.3017	10.3509	1164	1354896	1577098944	34.1174	10.5192
1110	1232100	1367631000	33.3167	10.3540	1165	1357225	1581167125	34.1321	10.5223
1111	1234321	1371330631	33.3317	10.3571	1166	1359556	1585242296	34.1467	10.5253
1112	1236544	1375036928	33.3467	10.3602	1167	1361889	1589324463	34.1614	10.5283
1113	1238769	1378749897	33.3617	10.3633	1168	1364224	1593413632	34.1760	10.5313
1114	1240996	1382469544	33.3766	10.3664	1169	1366561	1597509809	34.1906	10.5343
1115	1243225	1386195875	33.3916	10.3695	1170	1368900	1601613000	34.2053	10.5373
1116	1245456	1389928896	33.4066	10.3726	1171	1371241	1605723211	34.2199	10.5403
1117	1247689	1393668613	33.4215	10.3757	1172	1373584	1609840448	34.2345	10.5433
1118	1249924	1397415032	33.4365	10.3788	1173	1375929	1613964717	34.2491	10.5463
1119	1252161	1401168159	33.4515	10.3819	1174	1378276	1618096024	34.2637	10.5493
1120	1254400	1404928000	33.4664	10.3850	1175	1380625	1622234375	34.2783	10.5523
1121	1256641	1408694561	33.4813	10.3881	1176	1382976	1626379776	34.2929	10.5553
1122	1258884	1412467848	33.4963	10.3912	1177	1385329	1630532233	34.3074	10.5583
1123	1261129	1416247867	33.5112	10.3943	1178	1387684	1634691752	34.3220	10.5612
1124	1263376	1420034624	33.5261	10.3973	1179	1390041	1638858339	34.3366	10.5642
1125	1265625	1423828125	33.5410	10.4004	1180	1392400	1643032000	34.3511	10.5672
1126	1267876	1427628376	33.5559	10.4035	1181	1394761	1647212741	34.3657	10.5702
1127	1270129	1431435383	33.5708	10.4066	1182	1397124	1651400568	34.3802	10.5732
1128	1272384	1435249152	33.5857	10.4097	1183	1399489	1655595487	34.3948	10.5762
1129	1274641	1439069689	33.6006	10.4127	1184	1401856	1659797504	34.4093	10.5791
1130	1276900	1442897000	33.6155	10.4158	1185	1404225	1664006625	34.4238	10.5821
1131	1279161	1446731091	33.6303	10.4189	1186	1406596	1668222856	34.4384	10.5851
1132	1281424	1450571968	33.6452	10.4219	1187	1408969	1672446203	34.4529	10.5881
1133	1283689	1454419637	33.6601	10.4250	1188	1411344	1676676672	34.4674	10.5910
1134	1285956	1458274104	33.6749	10.4281	1189	1413721	1680914269	34.4819	10.5940
1135	1288225	1462135375	33.6898	10.4311	1190	1416100	1685159000	34.4964	10.5970
1136	1290496	1466003456	33.7046	10.4342	1191	1418481	1689410871	34.5109	10.6000
1137	1292769	1469878353	33.7194	10.4373	1192	1420864	1693669888	34.5254	10.6029
1138	1295044	1473760072	33.7343	10.4404	1193	1423249	1697936057	34.5398	10.6059
1139	1297321	1477648619	33.7491	10.4434	1194	1425636	1702209384	34.5543	10.6088
1140	1299600	1481544000	33.7639	10.4464	1195	1428025	1706489875	34.5688	10.6118
1141	1301881	1485446221	33.7787	10.4495	1196	1430416	1710777536	34.5832	10.6148
1142	1304164	1489355288	33.7935	10.4525	1197	1432809	1715072373	34.5977	10.6177
1143	1306449	1493271207	33.8083	10.4556	1198	1435204	1719374392	34.6121	10.6207
1144	1308736	1497193984	33.8231	10.4586	1199	1437601	1723685399	34.6266	10.6236

No.	Square.	Cube.	Sq. Root.	Cube Root.
1200	1440000	1728000000	34.6410	10.6266
1201	1442401	1732323601	34.6554	10.6295
1202	1444804	1736654408	34.6699	10.6325
1203	1447209	1740992427	34.6843	10.6354
1204	1449616	1745337664	34.6987	10.6384
1205	1452025	1749690125	34.7131	10.6413
1206	1454436	1754049816	34.7275	10.6443
1207	1456849	1758416743	34.7419	10.6472
1208	1459264	1762790912	34.7563	10.6501
1209	1461681	1767172329	34.7707	10.6530
1210	1464100	1771561000	34.7851	10.6560
1211	1466521	1775956931	34.7994	10.6590
1212	1468944	1780360128	34.8138	10.6619
1213	1471369	1784770597	34.8281	10.6648
1214	1473796	1789188344	34.8425	10.6678
1215	1476225	1793613375	34.8569	10.6707
1216	1478656	1798045696	34.8712	10.6736
1217	1481089	1802485313	34.8855	10.6765
1218	1483524	1806932232	34.8999	10.6795
1219	1485961	1811386459	34.9142	10.6824
1220	1488400	1815848000	34.9285	10.6853
1221	1490841	1820316861	34.9428	10.6882
1222	1493284	1824793048	34.9571	10.6911
1223	1495729	1829276567	34.9714	10.6940
1224	1498176	1833767424	34.9857	10.6970
1225	1500625	1838265625	35.0000	10.6999
1226	1503076	1842771176	35.0143	10.7028
1227	1505529	1847284083	35.0286	10.7057
1228	1507984	1851804352	35.0428	10.7086
1229	1510441	1856331989	35.0571	10.7115
1230	1512900	1860867000	35.0714	10.7144
1231	1515361	1865409391	35.0856	10.7173
1232	1517824	1869959168	35.0999	10.7202
1233	1520289	1874516337	35.1141	10.7231
1234	1522756	1879080904	35.1283	10.7260
1235	1525225	1883652875	35.1426	10.7289
1236	1527696	1888232256	35.1568	10.7318
1237	1530169	1892819053	35.1710	10.7347
1238	1532644	1897413272	35.1852	10.7376
1239	1535121	1902014919	35.1994	10.7405
1240	1537600	1906624000	35.2136	10.7434
1241	1540081	1911240521	35.2278	10.7463
1242	1542564	1915864488	35.2420	10.7491
1243	1545049	1920495907	35.2562	10.7520
1244	1547536	1925134784	35.2704	10.7549
1245	1550025	1929781125	35.2846	10.7578
1246	1552516	1934434936	35.2987	10.7607
1247	1555009	1939096223	35.3129	10.7635
1248	1557504	1943764992	35.3270	10.7664
1249	1560001	1948441249	35.3412	10.7693
1250	1562500	1953125000	35.3553	10.7722
1251	1565001	1957816251	35.3695	10.7750
1252	1567504	1962515008	35.3836	10.7779
1253	1570009	1967221277	35.3977	10.7808
1254	1572516	1971935064	35.4119	10.7837
1255	1575025	1976656375	35.4260	10.7865
1256	1577536	1981385216	35.4401	10.7894
1257	1580049	1986121593	35.4542	10.7922
1258	1582564	1990865512	35.4683	10.7951
1259	1585081	1995616979	35.4824	10.7980
1260	1587600	2000376000	35.4965	10.8008
1261	1590121	2005142581	35.5106	10.8037
1262	1592644	2009916728	35.5246	10.8065
1263	1595169	2014698447	35.5387	10.8094
1264	1597696	2019487744	35.5528	10.8122
1265	1600225	2024284625	35.5668	10.8151
1266	1602756	2029089096	35.5809	10.8179
1267	1605289	2033901163	35.5949	10.8208
1268	1607824	2038720832	35.6090	10.8236
1269	1610361	2043548109	35.6230	10.8265
1270	1612900	2048383000	35.6371	10.8293
1271	1615441	2053225511	35.6511	10.8322
1272	1617984	2058075648	35.6651	10.8350
1273	1620529	2062933417	35.6791	10.8378
1274	1623076	2067798824	35.6931	10.8407
1275	1625625	2072671875	35.7071	10.8435
1276	1628176	2077552576	35.7211	10.8463
1277	1630729	2082440933	35.7351	10.8492
1278	1633284	2087336952	35.7491	10.8520
1279	1635841	2092240639	35.7631	10.8548
1280	1638400	2097152000	35.7771	10.8577
1281	1640961	2102071041	35.7911	10.8605
1282	1643524	2106997768	35.8050	10.8633
1283	1646089	2111932187	35.8190	10.8661
1284	1648656	2116874304	35.8329	10.8690
1285	1651225	2121824125	35.8469	10.8718
1286	1653796	2126781656	35.8608	10.8746
1287	1656369	2131746903	35.8748	10.8774
1288	1658944	2136719872	35.8887	10.8802
1289	1661521	2141700569	35.9026	10.8831
1290	1664100	2146689000	35.9166	10.8859
1291	1666681	2151685171	35.9305	10.8887
1292	1669264	2156689088	35.9444	10.8915
1293	1671849	2161700757	35.9583	10.8943
1294	1674436	2166720184	35.9722	10.8971
1295	1677025	2171747375	35.9861	10.8999
1296	1679616	2176782336	36.0000	10.9027
1297	1682209	2181825073	36.0139	10.9055
1298	1684804	2186875592	36.0278	10.9083
1299	1687401	2191933899	36.0416	10.9111
1300	1690000	2197000000	36.0555	10.9139
1301	1692601	2202073901	36.0694	10.9167
1302	1695204	2207155608	36.0832	10.9195
1303	1697809	2212245127	36.0971	10.9223
1304	1700416	2217342464	36.1109	10.9251
1305	1703025	2222447625	36.1248	10.9279
1306	1705636	2227560616	36.1386	10.9307
1307	1708249	2232681443	36.1525	10.9335
1308	1710864	2237810112	36.1663	10.9363
1309	1713481	2242946629	36.1801	10.9391

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
1310	1716100	2248091000	36.1939	10.9418	1365	1863225	2543302125	36.9459	11.0929
1311	1718721	2253243231	36.2077	10.9446	1366	1865956	2548895896	36.9594	11.0956
1312	1721344	2258403328	36.2215	10.9474	1367	1868689	2554497863	36.9730	11.0983
1313	1723969	2263571297	36.2353	10.9502	1368	1871424	2560108032	36.9865	11.1010
1314	1726596	2268747144	36.2491	10.9530	1369	1874161	2565726409	37.0000	11.1037
1315	1729225	2273930875	36.2629	10.9557	1370	1876900	2571353000	37.0135	11.1064
1316	1731856	2279122496	36.2767	10.9585	1371	1879641	2576987811	37.0270	11.1091
1317	1734489	2284322013	36.2905	10.9613	1372	1882384	2582630848	37.0405	11.1118
1318	1737124	2289529432	36.3043	10.9640	1373	1885129	2588282117	37.0540	11.1145
1319	1739761	2294744759	36.3180	10.9668	1374	1887876	2593941624	37.0675	11.1172
1320	1742400	2299968000	36.3318	10.9696	1375	1890625	2599609375	37.0810	11.1199
1321	1745041	2305199161	36.3456	10.9724	1376	1893376	2605285376	37 0945	11.1226
1322	1747684	2310438248	36.3593	10.9752	1377	1896129	2610969633	37.1080	11.1253
1323	1750329	2315685267	36.3731	10.9779	1378	1898884	2616662152	37.1214	11.1280
1324	1752976	2320940224	36.3868	10.9807	1379	1901641	2622362939	37.1349	11.1307
1325	1755625	2326203125	36.4005	10.9834	1380	1904400	2628072000	37.1484	11.1334
1326	1758276	2331473976	36.4143	10.9862	1381	1907161	2633789341	37.1618	11.1361
1327	1760929	2336752783	36.4280	10.9890	1382	1909924	2639514968	37.1753	11.1387
1328	1763584	2342039552	36.4417	10.9917	1383	1912689	2645248887	37.1887	11.1414
1329	1766241	2347334289	36.4555	10.9945	1384	1915456	2650991104	37.2021	11.1441
1330	1768900	2352637000	36.4692	10.9972	1385	1918225	2656741625	37.2156	11.1468
1331	1771561	2357947691	36.4829	11.0000	1386	1920996	2662500456	37.2290	11.1495
1332	1774224	2363266368	36.4966	11.0028	1387	1923769	2668267603	37.2424	11.1522
1333	1776889	2368593037	36.5103	11.0055	1388	1926544	2674043072	37.2559	11.1548
1334	1779556	2373927704	36.5240	11.0083	1389	1929321	2679826869	37.2693	11.1575
1335	1782225	2379270375	36.5377	11.0110	1390	1932100	2685619000	37.2827	11.1602
1336	1784896	2384621056	36.5513	11.0138	1391	1934881	2691419471	37.2961	11.1629
1337	1787569	2389979753	36.5650	11.0165	1392	1937664	2697228288	37.3095	11.1655
1338	1790244	2395346472	36.5787	11 0193	1393	1940449	2703045457	37.3229	11.1682
1339	1792921	2400721219	36.5923	11.0220	1394	1943236	2708870984	37.3363	11.1709
1340	1795600	2406104000	36.6060	11.0247	1395	1946025	2714704875	37.3497	11.1736
1341	1798281	2411494821	36.6197	11.0275	1396	1948816	2720547136	37.3631	11.1762
1342	1800964	2416893688	36.6333	11.0302	1397	1951609	2726397773	37.3765	11.1789
1343	1803649	2422300607	36.6469	11.0330	1398	1954404	2732256792	37.3898	11.1816
1344	1806336	2427715584	36.6606	11.0357	1399	1957201	2738124199	37.4032	11.1842
1345	1809025	2433138625	36.6742	11.0384	1400	1960000	2744000000	37.4166	11.1869
1346	1811716	2438569736	36.6879	11.0412	1401	1962801	2749884201	37.4299	11.1896
1347	1814409	2444008923	36.7015	11.0439	1402	1965604	2755776808	37.4433	11.1922
1348	1817104	2449456192	36.7151	11.0466	1403	1968409	2761677827	37.4566	11.1949
1349	1819801	2454911549	36.7287	11.0494	1404	1971216	2767587264	37.4700	11.1975
1350	1822500	2460375000	36.7423	11.0521	1405	1974025	2773505125	37.4833	11.2002
1351	1825201	2465846551	36.7560	11.0548	1406	1976836	2779431416	37.4967	11.2028
1352	1827904	2471326208	36.7696	11.0575	1407	1979649	2785366143	37.5100	11.2055
1353	1830609	2476813977	36.7831	11.0603	1408	1982464	2791309312	37.5233	11.2082
1354	1833316	2482309864	36.7967	11.0630	1409	1985281	2797260929	37.5366	11.2108
1355	1836025	2487813875	36.8103	11.0657	1410	1988100	2803221000	37.5500	11.2135
1356	1838736	2493326016	36.8239	11.0684	1411	1990921	2809189531	37.5633	11.2161
1357	1841449	2498846293	36.8375	11.0712	1412	1993744	2815166528	37.5766	11.2188
1358	1844164	2504374712	36.8511	11.0739	1413	1996569	2821151997	37.5899	11.2214
1359	1846881	2509911279	36.8646	11.0766	1414	1999396	2827145944	37.6032	11.2240
1360	1849600	2515456000	36.8782	11.0793	1415	2002225	2833148375	37.6165	11.2267
1361	1852321	2521008881	36.8917	11.0820	1416	2005056	2839159296	37.6298	11 2293
1362	1855044	2526569928	36.9053	11.0847	1417	2007889	2845178713	37.6431	11.2320
1363	1857769	2532139147	36.9188	11.0875	1418	2010724	2851206632	37.6563	11.2346
1364	1860496	2537716544	36.9324	11.0902	1419	2013561	2857243059	37.6696	11.2373

No.	Square.	Cube.	Sq. Root.	Cube Root.
1420	2016400	2863288000	37.6829	11.2399
1421	2019241	2869341461	37.6962	11.2425
1422	2022084	2875403448	37.7094	11.2452
1423	2024929	2881473967	37.7227	11.2478
1424	2027776	2887553024	37.7359	11.2505
1425	2030625	2893640625	37.7492	11.2531
1426	2033476	2899736776	37.7624	11.2557
1427	2036329	2905841483	37.7757	11.2583
1428	2039184	2911954752	37.7889	11.2610
1429	2042041	2918076589	37.8021	11.2636
1430	2044900	2924207000	37.8153	11.2662
1431	2047761	2930345991	37.8286	11.2689
1432	2050624	2936493568	37.8418	11.2715
1433	2053489	2942649737	37.8550	11.2741
1434	2056356	2948814504	37.8682	11.2767
1435	2059225	2954987875	37.8814	11.2793
1436	2062096	2961169856	37.8946	11.2820
1437	2064969	2967360453	37.9078	11.2846
1438	2067844	2973559672	37.9210	11.2872
1439	2070721	2979767519	37.9342	11.2898
1440	2073600	2985984000	37.9473	11.2924
1441	2076481	2992209121	37.9605	11.2950
1442	2079364	2998442888	37.9737	11.2977
1443	2082249	3004685307	37.9868	11.3003
1444	2085136	3010936384	38.0000	11.3029
1445	2088025	3017196125	38.0132	11.3055
1446	2090916	3023464536	38.0263	11.3081
1447	2093809	3029741623	38.0395	11.3107
1448	2096704	3036027392	38.0526	11.3133
1449	2099601	3042321849	38.0657	11.3159
1450	2102500	3048625000	38.0789	11.3185
1451	2105401	3054936851	38.0920	11.3211
1452	2108304	3061257408	38.1051	11.3237
1453	2111209	3067586677	38.1182	11.3263
1454	2114116	3073924664	38.1314	11.3289
1455	2117025	3080271375	38.1445	11.3315
1456	2119936	3086626816	38.1576	11.3341
1457	2122849	3092990993	38.1707	11.3367
1458	2125764	3099363912	38.1838	11.3393
1459	2128681	3105745579	38.1969	11.3419
1460	2131600	3112136000	38.2099	11.3445
1461	2134521	3118535181	38.2230	11.3471
1462	2137444	3124943128	38.2361	11.3496
1463	2140369	3131359847	38.2492	11.3522
1464	2143296	3137785344	38.2623	11.3548
1465	2146225	3144219625	38.2753	11.3574
1466	2149156	3150662696	38.2884	11.3600
1467	2152089	3157114563	38.3014	11.3626
1468	2155024	3163575232	38.3145	11.3652
1469	2157961	3170044709	38.3275	11.3677
1470	2160900	3176523000	38.3406	11.3703
1471	2163841	3183010111	38.3536	11.3729
1472	2166784	3189506048	38.3667	11.3755
1473	2169729	3196010817	38.3797	11.3780
1474	2172676	3202524424	38.3927	11.3806

No.	Square.	Cube.	Sq. Root.	Cube Root.
1475	2175625	3209046875	38.4057	11.3832
1476	2178576	3215578176	38.4187	11.3858
1477	2181529	3222118333	38.4318	11.3883
1478	2184484	3228667352	38.4448	11.3909
1479	2187441	3235225239	38.4578	11.3935
1480	2190400	3241792000	38.4708	11.3960
1481	2193361	3248367641	38.4838	11.3986
1482	2196324	3254952168	38.4968	11.4012
1483	2199289	3261545587	38.5097	11.4037
1484	2202256	3268147904	38.5227	11.4063
1485	2205225	3274759125	38.5357	11.4089
1486	2208196	3281379256	38.5487	11.4114
1487	2211169	3288008303	38.5616	11.4140
1488	2214144	3294646272	38.5746	11.4165
1489	2217121	3301293169	38.5876	11.4191
1490	2220100	3307949000	38.6005	11.4216
1491	2223081	3314613771	38.6135	11.4242
1492	2226064	3321287488	38.6264	11.4268
1493	2229049	3327970157	38.6394	11.4293
1494	2232036	3334661784	38.6523	11.4319
1495	2235025	3341362375	38.6652	11.4344
1496	2238016	3348071936	38.6782	11.4370
1497	2241009	3354790473	38.6911	11.4395
1498	2244004	3361517992	38.7040	11.4421
1499	2247001	3368254499	38.7169	11.4446
1500	2250000	3375000000	38.7298	11.4471
1501	2253001	3381754501	38.7427	11.4497
1502	2256004	3388518008	38.7556	11.4522
1503	2259009	3395290527	38.7685	11.4548
1504	2262016	3402072064	38.7814	11.4573
1505	2265025	3408862625	38.7943	11.4598
1506	2268036	3415662216	38.8072	11.4624
1507	2271049	3422470843	38.8201	11.4649
1508	2274064	3429288512	38.8330	11.4675
1509	2277081	3436115229	38.8458	11.4700
1510	2280100	3442951000	38.8587	11.4725
1511	2283121	3449795831	38.8716	11.4751
1512	2286144	3456649728	38.8844	11.4776
1513	2289169	3463512697	38.8973	11.4801
1514	2292196	3470384744	38.9102	11.4826
1515	2295225	3477265875	38.9230	11.4852
1516	2298256	3484156096	38.9358	11.4877
1517	2301289	3491055413	38.9487	11.4902
1518	2304324	3497963832	38.9615	11.4927
1519	2307361	3504881359	38.9744	11.4953
1520	2310400	3511808000	38.9872	11.4978
1521	2313441	3518743761	39.0000	11.5003
1522	2316484	3525688648	39.0128	11.5028
1523	2319529	3532642667	39.0256	11.5054
1524	2322576	3539605824	39.0384	11.5079
1525	2325625	3546578125	39.0512	11.5104
1526	2328676	3553559576	39.0640	11.5129
1527	2331729	3560550183	39.0768	11.5154
1528	2334784	3567549952	39.0896	11.5179
1529	2337841	3574558889	39.1024	11.5204

No.	Square.	Cube.	Sq. Root.	Cube Root.	No.	Square.	Cube.	Sq. Root.	Cube Root.
1530	2340900	3581577000	39.1152	11.5230	1565	2449225	3833037125	39.5601	11.6102
1531	2343961	3588604291	39.1280	11.5255	1566	2452356	3840389496	39.5727	11.6126
1532	2347024	3595640768	39.1408	11.5280	1567	2455489	3847751263	39.5854	11.6151
1533	2350089	3602686437	39.1535	11.5305	1568	2458624	3855123432	39.5980	11.6176
1534	2353156	3609741304	39.1663	11.5330	1569	2461761	3862503009	39.6106	11.6200
1535	2356225	3616805375	39.1791	11.5355	1570	2464900	3869893000	39.6232	11.6225
1536	2359296	3623878656	39.1918	11.5380	1571	2468041	3877292411	39.6358	11.6250
1537	2362369	3630961153	39.2046	11.5405	1572	2471184	3884701248	39.6485	11.6274
1538	2365444	3638052872	39.2173	11.5430	1573	2474329	3892119517	39.6611	11.6299
1539	2368521	3645153819	39.2301	11.5455	1574	2477476	3899547224	39.6737	11.6324
1540	2371600	3652264000	39.2428	11.5480	1575	2480625	3906984375	39.6863	11.6348
1541	2374681	3659383421	39.2556	11.5505	1576	2483776	3914430976	39.6989	11.6373
1542	2377764	3666512088	39.2683	11.5530	1577	2486929	3921887033	39.7115	11.6398
1543	2380849	3673650007	39.2810	11.5555	1578	2490084	3929352552	39.7240	11.6422
1544	2383936	3680797184	39.2938	11.5580	1579	2493241	3936827539	39.7366	11.6447
1545	2387025	3687953625	39.3065	11.5605	1580	2496400	3944312000	39.7492	11.6471
1546	2390116	3695119336	39.3192	11.5630	1581	2499561	3951805941	39.7618	11.6496
1547	2393209	3702294323	39.3319	11.5655	1582	2502724	3959309368	39.7744	11.6520
1548	2396304	3709478592	39.3446	11.5680	1583	2505889	3966822287	39.7869	11.6545
1549	2399401	3716672149	39.3573	11.5705	1584	2509056	3974344704	39.7995	11.6570
1550	2402500	3723875000	39.3700	11.5729	1585	2512225	3981876625	39.8121	11.6594
1551	2405601	3731087151	39.3827	11.5754	1586	2515396	3989418056	39.8246	11.6619
1552	2408704	3738308608	39.3954	11.5779	1587	2518569	3996969003	39.8372	11.6643
1553	2411809	3745539377	39.4081	11.5804	1588	2521744	4004529472	39.8497	11.6668
1554	2414916	3752779464	39.4208	11.5829	1589	2524921	4012099469	39.8623	11.6692
1555	2418025	3760028875	39.4335	11.5854	1590	2528100	4019679000	39.8748	11.6717
1556	2421136	3767287616	39.4462	11.5879	1591	2531281	4027268071	39.8873	11.6741
1557	2424249	3774555693	39.4588	11.5903	1592	2534464	4034866688	39.8999	11.6765
1558	2427364	3781833112	39.4715	11.5928	1593	2537649	4042474857	39.9124	11.6790
1559	2430481	3789119879	39.4842	11.5953	1594	2540836	4050092584	39.9249	11.6814
1560	2433600	3796416000	39.4968	11.5978	1595	2544025	4057719875	39.9375	11.7839
1561	2436721	3803721481	39.5095	11.6003	1596	2547216	4065356736	39.9500	11.6863
1562	2439844	3811036328	39.5221	11.6027	1597	2550409	4073003173	39.9625	11.6888
1563	2442969	3818360547	39.5348	11.6052	1598	2553604	4080659192	39.9750	11.6912
1564	2446096	3845694144	39.5474	11.6077	1599	2556801	4088324799	39.9875	11.6936
					1600	2560000	4096000000	40.0000	11.6961

SQUARES AND CUBES OF DECIMALS.

No.	Square.	Cube.	No.	Square.	Cube.	No.	Square.	Cube.
.1	.01	.001	.01	.0001	.000 001	.001	.00 00 01	.000 000 001
.2	.04	.008	.02	.0004	.000 008	.002	.00 00 04	.000 000 008
.3	.09	.027	.03	.0009	.000 027	.003	.00 00 09	.000 000 027
.4	.16	.064	.04	.0016	.000 064	.004	.00 00 16	.000 000 064
.5	.25	.125	.05	.0025	.000 125	.005	.00 00 25	.000 000 125
.6	.36	.216	.06	.0036	.000 216	.006	.00 00 36	.000 000 216
.7	.49	.343	.07	.0049	.000 343	.007	.00 00 49	.000 000 343
.8	.64	.512	.08	.0064	.000 512	.008	.00 00 64	.000 000 512
.9	.81	.729	.09	.0081	.000 729	.009	.00 00 81	.000 000 729
1.0	1.00	1.000	.10	.0100	.001 000	.010	.00 01 00	.000 001 000
1.2	1.44	1.728	.12	.0144	.001 728	.012	.00 01 44	.000 001 728

Note that the square has twice as many decimal places, and the cube three times as many decimal places, as the root.

FIFTH ROOTS AND FIFTH POWERS.

(Abridged from TRAUTWINE.)

No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.
.10	.000010	3.7	693.440	9.8	90392	21.8	4923597	40	102400000
.15	.000075	3.8	792.352	9.9	95099	22.0	5153632	41	115856201
.20	.000320	3.9	902.242	10.0	100000	22.2	5392186	42	130691232
.25	.000977	4.0	1024.00	10.2	110408	22.4	5639493	43	147008443
.30	.002430	4.1	1158.56	10.4	121665	22.6	5895793	44	164916224
.35	.005252	4.2	1306.91	10.6	133823	22.8	6161327	45	184528125
.40	.010240	4.3	1470.08	10.8	146933	23.0	6436343	46	205962976
.45	.018453	4.4	1649.16	11.0	161051	23.2	6721093	47	229345007
.50	.031250	4.5	1845.28	11.2	176234	23.4	7015834	48	254803968
.55	.050328	4.6	2059.63	11.4	192541	23.6	7320825	49	282475249
.60	.077760	4.7	2293.45	11.6	210084	23.8	7636332	50	312500000
.65	.116029	4.8	2548.04	11.8	228776	24.0	7962624	51	345025251
.70	.168070	4.9	2824.75	12.0	248832	24.2	8299976	52	380204032
.75	.237305	5.0	3125.00	12.2	270271	24.4	8648666	53	418195493
.80	.327680	5.1	3450.25	12.4	293163	24.6	9008978	54	459165024
.85	.443705	5.2	3802.04	12.6	317580	24.8	9381200	55	503284375
.90	.590490	5.3	4181.95	12.8	343597	25.0	9765625	56	550731776
.95	.773781	5.4	4591.65	13.0	371293	25.2	10162550	57	601692057
1.00	1.00000	5.5	5032.84	13.2	400746	25.4	10572278	58	656356768
1.05	1.27628	5.6	5507.32	13.4	432040	25.6	10995116	59	714924299
1.10	1.61051	5.7	6016.92	13.6	465259	25.8	11431377	60	777600000
1.15	2.01135	5.8	6563.57	13.8	500490	26.0	11881376	61	844596301
1.20	2.48832	5.9	7149.24	14.0	537824	26.2	12345437	62	916132832
1.25	3.05176	6.0	7776.00	14.2	577353	26.4	12823886	63	992436543
1.30	3.71293	6.1	8445.96	14.4	619174	26.6	13317055	64	1073741824
1.35	4.48403	6.2	9161.33	14.6	663383	26.8	13825281	65	1160290625
1.40	5.37824	6.3	9924.37	14.8	710082	27.0	14348907	66	1252332576
1.45	6.40973	6.4	10737	15.0	759375	27.2	14888280	67	1350125107
1.50	7.59375	6.5	11603	15.2	811368	27.4	15443752	68	1453933568
1.55	8.94661	6.6	12523	15.4	866171	27.6	16015681	69	1564031349
1.60	10.4858	6.7	13501	15.6	923896	27.8	16604430	70	1680700000
1.65	12.2298	6.8	14539	15.8	984658	28.0	17210368	71	1804229351
1.70	14.1986	6.9	15640	16.0	1048576	28.2	17833868	72	1934917632
1.75	16.4131	7.0	16807	16.2	1115771	28.4	18475309	73	2073071593
1.80	18.8957	7.1	18042	16.4	1186367	28.6	19135075	74	2219006624
1.85	21.6700	7.2	19349	16.6	1260493	28.8	19813557	75	2373046875
1.90	24.7610	7.3	20731	16.8	1338278	29.0	20511149	76	2535525376
1.95	28.1951	7.4	22190	17.0	1419857	29.2	21228253	77	2706784157
2.00	32.0000	7.5	23730	17.2	1505366	29.4	21965275	78	2887174368
2.05	36.2051	7.6	25355	17.4	1594947	29.6	22722628	79	3077056399
2.10	40.8410	7.7	27068	17.6	1688742	29.8	23500728	80	3276800000
2.15	45.9401	7.8	28872	17.8	1786899	30.0	24300000	81	3486784401
2.20	51.5363	7.9	30771	18.0	1889568	30.5	26393634	82	3707398432
2.25	57.6650	8.0	32768	18.2	1996903	31.0	28629151	83	3939040643
2.30	64.3634	8.1	34868	18.4	2109061	31.5	31013642	84	4182119424
2.35	71.6703	8.2	37074	18.6	2226203	32.0	33554432	85	4437053125
2.40	79.6262	8.3	39390	18.8	2348493	32.5	36259082	86	4704270176
2.45	88.2735	8.4	41821	19.0	2476099	33.0	39135393	87	4984209207
2.50	97.6562	8.5	44371	19.2	2609193	33.5	42191410	88	5277319168
2.55	107.820	8.6	47043	19.4	2747949	34.0	45435424	89	5584059449
2.60	118.814	8.7	49842	19.6	2892547	34.5	48875980	90	5904900000
2.70	143.489	8.8	52773	19.8	3043168	35.0	52521875	91	6240321451
2.80	172.104	8.9	55841	20.0	3200000	35.5	56382167	92	6590815232
2.90	205.111	9.0	59049	20.2	3363232	36.0	60466176	93	6956883693
3.00	243.000	9.1	62403	20.4	3533059	36.5	64783487	94	7339040224
3.10	286.292	9.2	65908	20.6	3709677	37.0	69343957	95	7737809375
3.20	335.544	9.3	69569	20.8	3893289	37.5	74157715	96	8153726976
3.30	391.354	9.4	73390	21.0	4084101	38.0	79235168	97	8587340257
3.40	454.354	9.5	77378	21.2	4282322	38.5	84587005	98	9039207968
3.50	525.219	9.6	81537	21.4	4488166	39.0	90224199	99	9509900499
3.60	604.662	9.7	85873	21.6	4701850	39.5	96158012		

CIRCUMFERENCES AND AREAS OF CIRCLES.

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
1	3.1416	0.7854	65	204.20	3318.31	129	405.27	13069.81
2	6.2832	3.1416	66	207.34	3421.19	130	408.41	13273.23
3	9.4248	7.0686	67	210.49	3525.65	131	411.55	13478.22
4	12.5664	12.5664	68	213.63	3631.68	132	414.69	13684.78
5	15.7080	19.635	69	216.77	3739.28	133	417.83	13892.91
6	18.850	28.274	70	219.91	3848.45	134	420.97	14102.61
7	21.991	38.485	71	223.05	3959.19	135	424.12	14313.88
8	25.133	50.266	72	226.19	4071.50	136	427.26	14526.72
9	28.274	63.617	73	229.34	4185.39	137	430.40	14741.14
10	31.416	78.540	74	232.48	4300.84	138	433.54	14957.12
11	34.558	95.033	75	235.62	4417.86	139	436.68	15174.68
12	37.699	113.10	76	238.76	4536.46	140	439.82	15393.80
13	40.841	132.73	77	241.90	4656.63	141	442.96	15614.50
14	43.982	153.94	78	245.04	4778.36	142	446.11	15836.77
15	47.124	176.71	79	248.19	4901.67	143	449.25	16060.61
16	50.265	201.06	80	251.33	5026.55	144	452.39	16286.02
17	53.407	226.98	81	254.47	5153.00	145	455.53	16513.00
18	56.549	254.47	82	257.61	5281.02	146	458.67	16741.55
19	59.690	283.53	83	260.75	5410.61	147	461.81	16971.67
20	62.832	314.16	84	263.89	5541.77	148	464.96	17203.36
21	65.973	346.36	85	267.04	5674.50	149	468.10	17436.62
22	69.115	380.13	86	270.18	5808.80	150	471.24	17671.46
23	72.257	415.48	87	273.32	5944.68	151	474.38	17907.86
24	75.398	452.39	88	276.46	6082.12	152	477.52	18145.84
25	78.540	490.87	89	279.60	6221.14	153	480.66	18385.39
26	81.681	530.93	90	282.74	6361.73	154	483.81	18626.50
27	84.823	572.56	91	285.88	6503.88	155	486.95	18869.19
28	87.965	615.75	92	289.03	6647.61	156	490.09	19113.45
29	91.106	660.52	93	292.17	6792.91	157	493.23	19359.28
30	94.248	706.86	94	295.31	6939.78	158	496.37	19606.68
31	97.389	754.77	95	298.45	7088.22	159	499.51	19855.65
32	100.53	804.25	96	301.59	7238.23	160	502.65	20106.19
33	103.67	855.30	97	304.73	7389.81	161	505.80	20358.31
34	106.81	907.92	98	307.88	7542.96	162	508.94	20611.99
35	109.96	962.11	99	311.02	7697.69	163	512.08	20867.24
36	113.10	1017.88	100	314.16	7853.98	164	515.22	21124.07
37	116.24	1075.21	101	317.30	8011.85	165	518.36	21382.46
38	119.38	1134.11	102	320.44	8171.28	166	521.50	21642.43
39	122.52	1194.59	103	323.58	8332.29	167	524.65	21903.97
40	125.66	1256.64	104	326.73	8494.87	168	527.79	22167.08
41	128.81	1320.25	105	329.87	8659.01	169	530.93	22431.76
42	131.95	1385.44	106	333.01	8824.73	170	534.07	22698.01
43	135.09	1452.20	107	336.15	8992.02	171	537.21	22965.83
44	138.23	1520.53	108	339.29	9160.88	172	540.35	23235.22
45	141.37	1590.43	109	342.43	9331.32	173	543.50	23506.18
46	144.51	1661.90	110	345.58	9503.32	174	546.64	23778.71
47	147.65	1734.94	111	348.72	9676.89	175	549.78	24052.82
48	150.80	1809.56	112	351.86	9852.03	176	552.92	24328.49
49	153.94	1885.74	113	355.00	10028.75	177	556.06	24605.74
50	157.08	1963.50	114	358.14	10207.03	178	559.20	24884.56
51	160.22	2042.82	115	361.28	10386.89	179	562.35	25164.94
52	163.36	2123.72	116	364.42	10568.32	180	565.49	25446.90
53	166.50	2206.18	117	367.57	10751.32	181	568.63	25730.43
54	169.65	2290.22	118	370.71	10935.88	182	571.77	26015.53
55	172.79	2375.83	119	373.85	11122.02	183	574.91	26302.20
56	175.93	2463.01	120	376.99	11309.73	184	578.05	26590.44
57	179.07	2551.76	121	380.13	11499.01	185	581.19	26880.25
58	182.21	2642.08	122	383.27	11689.87	186	584.34	27171.63
59	185.35	2733.97	123	386.42	11882.29	187	587.48	27464.59
60	188.50	2827.43	124	389.56	12076.28	188	590.62	27759.11
61	191.64	2922.47	125	392.70	12271.85	189	593.76	28055.21
62	194.78	3019.07	126	395.84	12468.98	190	596.90	28352.87
63	197.92	3117.25	127	398.98	12667.69	191	600.04	28652.11
64	201.06	3216.99	128	402.12	12867.96	192	603.19	28952.92

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
193	606.33	29255.30	**260**	816.81	53092.92	327	1027.30	83981.84
194	609.47	29559.25	261	819.96	53502.11	328	1030.44	84496.28
195	612.61	29864.77	262	823.10	53912.87	329	1033.58	85012.28
196	615.75	30171.86	263	826.24	54325.21	**330**	1036 73	85529.86
197	618.89	30480.52	264	829.38	54739.11	331	1039.87	86049.01
198	622.04	30790.75	265	832.52	55154.59	332	1043.01	86569.73
199	625.18	31102.55	266	835.66	55571.63	333	1046.15	87092.02
200	628.32	31415.93	267	838.81	55990.25	334	1049.29	87615.88
201	631.46	31730.87	268	841.95	56410.44	335	1052.43	88141.31
202	634.60	32047.39	269	845.09	56832.20	336	1055.58	88668.31
203	637.74	32365.47	**270**	848.23	57255.53	337	1058.72	89196.88
204	640.88	32685.13	271	851.37	57680.43	338	1061 86	89727.03
205	644.03	33006.36	272	854.51	58106.90	339	1065.00	90258.74
206	647.17	33329.16	273	857.65	58534.94	**340**	1068.14	90792.03
207	650.31	33653.53	274	860.80	58964.55	341	1071.28	91326.88
208	653.45	33979.47	275	863.94	59395.74	342	1074.42	91863.31
209	656.59	34306.98	276	867.08	59828.49	343	1077.57	92401 31
210	659.73	34636.06	277	870.22	60262.82	344	1080.71	92940.88
211	662.88	34966.71	278	873.36	60698.71	345	1083.85	93482.02
212	666.02	35298.94	279	876.50	61136.18	346	1086.99	94024.73
213	669.16	35632.73	**280**	879.65	61575.22	347	1090.13	94569.01
214	672.30	35968.09	281	882.79	62015.82	348	1093.27	95114.86
215	675.44	36305.03	282	885.93	62458.00	349	1096.42	95662.28
216	678.58	36643.54	283	889.07	62901.75	**350**	1099.56	96211.28
217	681.73	36983.61	284	892.21	63347.07	351	1102.70	96761.84
218	684.87	37325.26	285	895.35	63793.97	352	1105.84	97313.97
219	688.01	37668.48	286	898.50	64242.43	353	1108.98	97867.68
220	691.15	38013.27	287	901.64	64692.46	354	1112.12	98422.96
221	694.29	38359.63	288	904.78	65144.07	355	1115.27	98979.80
222	697.43	38707.56	289	907.92	65597.24	356	1118.41	99538.22
223	700.58	39057.07	**290**	911.06	66051.99	357	1121.55	100098.21
224	703.72	39408.14	291	914.20	66508.30	358	1124.69	100659.77
225	706.86	39760.78	292	917.35	66966.19	359	1127.83	101222.90
226	710.00	40115.00	293	920.49	67425.65	**360**	1130.97	101787.60
227	713.14	40470.78	294	923.63	67886.68	361	1134.11	102353.87
228	716.28	40828.14	295	926.77	68349.28	362	1137.26	102921.72
229	719.42	41187.07	296	929.91	68813.45	363	1140.40	103491.13
230	722.57	41547.56	297	933.05	69279.19	364	1143.54	104062.12
231	725.71	41909.63	298	936.19	69746.50	365	1146.68	104634.67
232	728.85	42273.27	299	939.34	70215 38	366	1149.82	105208.80
233	731.99	42638.48	**300**	942.48	70685.83	367	1152.96	105784.49
234	735.13	43005.26	301	945.62	71157.86	368	1156.11	106361 76
235	738.27	43373.61	302	948.76	71631.45	369	1159.25	106940.60
236	741.42	43743.54	303	951.90	72106.62	**370**	1162.39	107521.01
237	744.56	44115.03	304	955.04	72583.36	371	1165.53	108102.99
238	747.70	44488.09	305	958.19	73061.66	372	1168.67	108686.54
239	750.84	44862.73	306	961.33	73541.54	373	1171.81	109271.66
240	753 98	45238.93	307	964.47	74022.99	374	1174 96	109858.35
241	757.12	45616.71	308	967.61	74506.01	375	1178.10	110446.62
242	760.27	45996.06	309	970.75	74990.60	376	1181.24	111036.45
243	763.41	46376.98	**310**	973.89	75476.76	377	1184.38	111627.86
244	766.55	46759.47	311	977.04	75964.50	378	1187.52	112220.83
245	769.69	47143.52	312	980.18	76453.80	379	1190.66	112815.38
246	772.83	47529.16	313	983.32	76944.67	**380**	1193.81	113411.49
247	775.97	47916.36	314	986.46	77437.12	381	1196.95	114009.18
248	779.11	48305.13	315	989.60	77931.13	382	1200.09	114608.44
249	782.26	48695.47	316	992.74	78426.72	383	1203.23	115209.27
250	785.40	49087 39	317	995.88	78923.88	384	1206.37	115811.67
251	788.54	49480.87	318	909.03	79422.60	385	1209.51	116415.64
252	791.68	49875.92	319	1002.17	79922.90	386	1212.65	117021.18
253	794.82	50272.55	**320**	1005.31	80424.77	387	1215.80	117628.30
254	797.96	50670.75	321	1008.45	80928.21	388	1218.94	118236.98
255	801.11	51070.52	322	1011.59	81433 22	389	1222.08	118847.24
256	804.25	51471.85	323	1014.73	81939.80	**390**	1225.22	119459.06
257	807.39	51874.76	324	1017.88	82447.96	391	1228.36	120072.46
258	810.53	52279.24	325	1021.02	82957.68	392	1231.50	120687.42
259	813.67	52685.29	326	1024.16	83468.98	393	1234.65	121303.96

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
394	1237.79	121922.07	461	1448.27	166913.60	528	1658.76	218956.44
395	1240.93	122541.75	462	1451.42	167638.53	529	1661.90	219786.61
396	1244.07	123163.00	463	1454.56	168365.02	530	1665.04	220618.34
397	1247.21	123785.82	464	1457.70	169093.08	531	1668.19	221451.65
398	1250.35	124410.21	465	1460.84	169822.72	532	1671.33	222286.53
399	1253.50	125036.17	466	1463.98	170553.92	533	1674.47	223122.98
400	1256.64	125663.71	467	1467.12	171286.70	534	1677.61	223961.00
401	1259.78	126292.81	468	1470.27	172021.05	535	1680.75	224800.59
402	1262.92	126923.48	469	1473.41	172756.97	536	1683.89	225641.75
403	1266.06	127555.73	470	1476.55	173494.45	537	1687.04	226484.48
404	1269.20	128189.55	471	1479.69	174233.51	538	1690.18	227328.79
405	1272.35	128824.93	472	1482.83	174974.14	539	1693.32	228174.66
406	1275.49	129461.89	473	1485.97	175716.35	540	1696.46	229022.10
407	1278.63	130100.42	474	1489.11	176460.12	541	1699.60	229871.12
408	1281.77	130740.52	475	1492.26	177205.46	542	1702.74	230721.71
409	1284.91	131382.19	476	1495.40	177952.37	543	1705.88	231573.86
410	1288.05	132025.43	477	1498.54	178700.86	544	1709.03	232427.59
411	1291.19	132670.24	478	1501.68	179450.91	545	1712.17	233282.89
412	1294.34	133316.63	479	1504.82	180202.54	546	1715.31	234139.76
413	1297.48	133964.58	480	1507.96	180955.74	547	1718.45	234998.20
414	1300.62	134614.10	481	1511.11	181710.50	548	1721.59	235858.21
415	1303.76	135265.20	482	1514.25	182466.84	549	1724.73	236719.79
416	1306.90	135917.86	483	1517.39	183224.75	550	1727.88	237582.94
417	1310.04	136572.10	484	1520.53	183984.23	551	1731.02	238447.67
418	1313.19	137227.91	485	1523.67	184745.28	552	1734.16	239313.96
419	1316.33	137885.29	486	1526.81	185507.90	553	1737.30	240181.83
420	1319.47	138544.24	487	1529.96	186272.10	554	1740.44	241051.26
421	1322.61	139204.76	488	1533.10	187037.86	555	1743.58	241922.27
422	1325.75	139866.85	489	1536.24	187805.19	556	1746.73	242794.85
423	1328.89	140530.51	490	1539.38	188574.10	557	1749.87	243668.99
424	1332.04	141195.74	491	1542.52	189344.57	558	1753.01	244544.71
425	1335.18	141862.54	492	1545.66	190116.62	559	1756.15	245422.00
426	1338.32	142530.92	493	1548.81	190890.24	560	1759.29	246300.86
427	1341.46	143200.86	494	1551.95	191665.43	561	1762.43	247181.30
428	1344.60	143872.38	495	1555.09	192442.18	562	1765.58	248063.30
429	1347.74	144545.46	496	1558.23	193220.51	563	1768.72	248946.87
430	1350.88	145220.12	497	1561.37	194000.41	564	1771.86	249832.01
431	1354.03	145896.35	498	1564.51	194781.89	565	1775.00	250718.73
432	1357.17	146574.15	499	1567.65	195564.93	566	1778.14	251607.01
433	1360.31	147253.52	500	1570.80	196349.54	567	1781.28	252496.87
434	1363.45	147934.46	501	1573.94	197135.72	568	1784.42	253388.30
435	1366.59	148616.97	502	1577.08	197923.48	569	1787.57	254281.29
436	1369.73	149301.05	503	1580.22	198712.80	570	1790.71	255175.86
437	1372.88	149986.70	504	1583.36	199503.70	571	1793.85	256072.00
438	1376.02	150673.93	505	1586.50	200296.17	572	1796.99	256969.71
439	1379.16	151362.72	506	1589.65	201090.20	573	1800.13	257868.99
440	1382.30	152053.08	507	1592.79	201885.81	574	1803.27	258769.85
441	1385.44	152745.02	508	1595.93	202682.99	575	1806.42	259672.27
442	1388.58	153438.53	509	1599.07	203481.74	576	1809.56	260576.26
443	1391.73	154133.60	510	1602.21	204282.06	577	1812.70	261481.83
444	1394.87	154830.25	511	1605.35	205083.95	578	1815.84	262388.96
445	1398.01	155528.47	512	1608.50	205887.42	579	1818.98	263297.67
446	1401.15	156228.26	513	1611.64	206692.45	580	1822.12	264207.94
447	1404.29	156929.62	514	1614.78	207499.05	581	1825.27	265119.79
448	1407.43	157632.55	515	1617.92	208307.23	582	1828.41	266033.21
449	1410.58	158337.06	516	1621.06	209116.97	583	1831.55	266948.20
450	1413.72	159043.13	517	1624.20	209928.29	584	1834.69	267864.76
451	1416.86	159750.77	518	1627.34	210741.18	585	1837.83	268782.89
452	1420.00	160459.99	519	1630.49	211555.63	586	1840.97	269702.59
453	1423.14	161170.77	520	1633.63	212371.66	587	1844.11	270623.86
454	1426.28	161883.13	521	1636.77	213189.26	588	1847.26	271546.70
455	1429.42	162597.05	522	1639.91	214008.43	589	1850.40	272471.12
456	1432.57	163312.55	523	1643.05	214829.17	590	1853.54	273397.10
457	1435.71	164029.62	524	1646.19	215651.49	591	1856.68	274324.66
458	1438.85	164748.26	525	1649.34	216475.37	592	1859.82	275253.78
459	1441.99	165468.47	526	1652.48	217300.82	593	1862.96	276184.48
460	1445.13	166190.25	527	1655.62	218127.85	594	1866.11	277116.75

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
595	1869.25	278050.58	663	2082.88	345236.69	731	2296.50	419686.15
596	1872.39	278985.99	664	2086.02	346278.91	732	2299.65	420835.19
597	1875.53	279922.97	665	2089.16	347322.70	733	2302.79	421985.79
598	1878.67	280861.52	666	2092.30	348368.07	734	2305.93	423137.97
599	1881.81	281801.65	667	2095.44	349415.00	735	2309.07	424291.72
600	1884.96	282743.34	668	2098.58	350463.51	736	2312.21	425447.04
601	1888.10	283686.60	669	2101.73	351513.59	737	2315.35	426603.94
602	1891.24	284631.44	670	2104.87	352565.24	738	2318.50	427762.40
603	1894.38	285577.84	671	2108.01	353618.45	739	2321.64	428922.43
604	1897.52	286525.82	672	2111.15	354673.24	740	2324.78	430084.03
605	1900.66	287475.36	673	2114.29	355729.60	741	2327.92	431247.21
606	1903.81	288426.48	674	2117.43	356787.54	742	2331.06	432411.95
607	1906.95	289379.17	675	2120.58	357847.04	743	2334.20	433578.27
608	1910.09	290333.43	676	2123.72	358908.11	744	2337.34	434746.16
609	1913.23	291289.26	677	2126.86	359970.75	745	2340.49	435915.62
610	1916.37	292246.66	678	2130.00	361034.97	746	2343.63	437086.64
611	1919.51	293205.63	679	2133.14	362100.75	747	2346.77	438259.24
612	1922.65	294166.17	680	2136.28	363168.11	748	2349.91	439433.41
613	1925.80	295128.28	681	2139.42	364237.04	749	2353.05	440609.16
614	1928.94	296091.97	682	2142.57	365307.54	750	2356.19	441786.47
615	1932.08	297057.22	683	2145.71	366379.60	751	2359.34	442965.35
616	1935.22	298024.05	684	2148.85	367453.24	752	2362.48	444145.80
617	1938.36	298992.44	685	2151.99	368528.45	753	2365.62	445327.83
618	1941.50	299962.41	686	2155.13	369605.23	754	2368.76	446511.42
619	1944.65	300933.95	687	2158.27	370683.59	755	2371.90	447696.59
620	1947.79	301907.05	688	2161.42	371763.51	756	2375.04	448883.32
621	1950.93	302881.73	689	2164.56	372845.00	757	2378.19	450071.63
622	1954.07	303857.98	690	2167.70	373928.07	758	2381.33	451261.51
623	1957.21	304835.80	691	2170.84	375012.70	759	2384.47	452452.96
624	1960.35	305815.20	692	2173.98	376098.91	760	2387.61	453645.98
625	1963.50	306796.16	693	2177.12	377186.68	761	2390.75	454840.57
626	1966.64	307778.69	694	2180.27	378276.03	762	2393.89	456036.73
627	1969.78	308762.79	695	2183.41	379366.95	763	2397.04	457234.46
628	1972.92	309748.47	696	2186.55	380459.44	764	2400.18	458433.77
629	1976.06	310735.71	697	2189.69	381553.50	765	2403.32	459634.64
630	1979.20	311724.53	698	2192.83	382649.13	766	2406.46	460837.08
631	1982.35	312714.92	699	2195.97	383746.33	767	2409.60	462041.10
632	1985.49	313706.88	700	2199.11	384845.10	768	2412.74	463246.69
633	1988.63	314700.40	701	2202.26	385945.44	769	2415.88	464453.84
634	1991.77	315695.50	702	2205.40	387047.36	770	2419.03	465662.57
635	1994.91	316692.17	703	2208.54	388150.84	771	2422.17	466872.87
636	1998.05	317690.42	704	2211.68	389255.90	772	2425.31	468084.74
637	2001.19	318690.23	705	2214.82	390362.52	773	2428.45	469298.18
638	2004.34	319691.61	706	2217.96	391470.72	774	2431.59	470513.19
639	2007.48	320694.56	707	2221.11	392580.49	775	2434.73	471729.77
640	2010.62	321699.09	708	2224.25	393691.82	776	2437.88	472947.92
641	2013.76	322705.18	709	2227.39	394804.73	777	2441.02	474167.65
642	2016.90	323712.85	710	2230.53	395919.21	778	2444.16	475388.94
643	2020.04	324722.09	711	2233.67	397035.26	779	2447.30	476611.81
644	2023.19	325732.89	712	2236.81	398152.89	780	2450.44	477836.24
645	2026.33	326745.27	713	2239.96	399272.08	781	2453.58	479062.25
646	2029.47	327759.22	714	2243.10	400392.84	782	2456.73	480289.83
647	2032.61	328774.74	715	2246.24	401515.18	783	2459.87	481518.97
648	2035.75	329791.83	716	2249.38	402639.08	784	2463.01	482749.69
649	2038.89	330810.49	717	2252.52	403764.56	785	2466.15	483981.98
650	2042.04	331830.72	718	2255.66	404891.60	786	2469.29	485215.84
651	2045.18	332852.53	719	2258.81	406020.22	787	2472.43	486451.28
652	2048.32	333875.90	720	2261.95	407150.41	788	2475.58	487688.28
653	2051.46	334900.85	721	2265.09	408282.17	789	2478.72	488926.85
654	2054.60	335927.36	722	2268.23	409415.50	790	2481.86	490166.99
655	2057.74	336955.45	723	2271.37	410550.40	791	2485.00	491408.71
656	2060.88	337985.10	724	2274.51	411686.87	792	2488.14	492651.99
657	2064.03	339016.33	725	2277.65	412824.91	793	2491.28	493896.85
658	2067.17	340049.13	726	2280.80	413964.52	794	2494.42	495143.28
659	2070.31	341083.50	727	2283.94	415105.71	795	2497.57	496391.27
660	2073.45	342119.44	728	2287.08	416248.46	796	2500.71	497640.84
661	2076.59	343156.95	729	2290.22	417392.79	797	2503.85	498891.98
662	2079.73	344196.03	730	2293.36	418538.68	798	2506.99	500144.69

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
799	2510.13	501398.97	867	2723.76	590375.16	935	2937.39	686614.71
800	2513.27	502654.82	868	2726.90	591737.83	936	2940.53	688084.19
801	2516.42	503912.25	869	2730.04	593102.06	937	2943.67	689555.24
802	2519.56	505171.24	**870**	2733.19	594467.87	938	2946.81	691027.86
803	2522.70	506431.80	871	2736.33	595835.25	939	2949.96	692502.05
804	2525.84	507693.94	872	2739.47	597204.20	**940**	2953.10	693977.82
805	2528.98	508957.64	873	2742.61	598574.72	941	2956.24	695455.15
806	2532.12	510222.92	874	2745.75	599946.81	942	2959.38	696934.06
807	2535.27	511489.77	875	2748.89	601320.47	943	2962.52	698414.53
808	2538.41	512758.19	876	2752.04	602695.70	944	2965.66	699896.58
809	2541.55	514028.18	877	2755.18	604072.50	945	2968.81	701380.19
810	2544.69	515299.74	878	2758.32	605450.88	946	2971.95	702865.38
811	2547.83	516572.87	879	2761.46	606830.82	947	2975.09	704352.14
812	2550.97	517847.57	**880**	2764.60	608212.34	948	2978.23	705840.47
813	2554.11	519123.84	881	2767.74	609595.42	949	2981.37	707330.37
814	2557.26	520401.68	882	2770.88	610980.08	**950**	2984.51	708821.84
815	2560.40	521681.10	883	2774.03	612366.31	951	2987.65	710314.88
816	2563.54	522962.08	884	2777.17	613754.11	952	2990.80	711809.50
817	2566.68	524244.63	885	2780.31	615143.48	953	2993.94	713305.68
818	2569.82	525528.76	886	2783.45	616534.42	954	2997.08	714803.43
819	2572.96	526814.46	887	2786.59	617926.93	955	3000.22	716302.76
820	2576.11	528101.73	888	2789.73	619321.01	956	3003.36	717803.66
821	2579.25	529390.56	889	2792.88	620716.66	957	3006.50	719306.12
822	2582.39	530680.97	**890**	2796.02	622113.89	958	3009.65	720810.16
823	2585.53	531972.95	891	2799.16	623512.68	959	3012.79	722315.77
824	2588.67	533266.50	892	2802.30	624913.04	**960**	3015.93	723822.95
825	2591.81	534561.62	893	2805.44	626314.98	961	3019.07	725331.70
826	2594.96	535858.32	894	2808.58	627718.49	962	3022.21	726842.02
827	2598.10	537156.58	895	2811.73	629123.56	963	3025.35	728353.91
828	2601.24	538456.41	896	2814.87	630530.21	964	3028.50	729867.37
829	2604.38	539757.82	897	2818.01	631938.43	965	3031.64	731382.40
830	2607.52	541060.79	898	2821.15	633348.22	966	3034.78	732899.01
831	2610.66	542365.34	899	2824.29	634759.58	967	3037.92	734417.18
832	2613.81	543671.46	**900**	2827.43	636172.51	968	3041.06	735936.93
833	2616.95	544979.15	901	2830.58	637587.01	969	3044.20	737458.24
834	2620.09	546288.40	902	2833.72	639003.09	**970**	3047.34	738981.13
835	2623.23	547599.23	903	2836.86	640420.73	971	3050.49	740505.59
836	2626.37	548911.63	904	2840.00	641839.95	972	3053.63	742031.62
837	2629.51	550225.61	905	2843.14	643260.73	973	3056.77	743559.22
838	2632.65	551541.15	906	2846.28	644683.09	974	3059.91	745088.39
839	2635.80	552858.26	907	2849.42	646107.01	975	3063.05	746619.13
840	2638.94	554176.94	908	2852.57	647532.51	976	3066.19	748151.44
841	2642.08	555497.20	909	2855.71	648959.58	977	3069.34	749685.32
842	2645.22	556819.02	**910**	2858.85	650388.22	978	3072.48	751220.78
843	2648.36	558142.42	911	2861.99	651818.43	979	3075.62	752757.80
844	2651.50	559467.39	912	2865.13	653250.21	**980**	3078.76	754296.40
845	2654.65	560793.92	913	2868.27	654683.56	981	3081.90	755836.56
846	2657.79	562122.03	914	2871.42	656118.48	982	3085.04	757378.30
847	2660.93	563451.71	915	2874.56	657554.98	983	3088.19	758921.61
848	2664.07	564782.96	916	2877.70	658993.04	984	3091.33	760466.48
849	2667.21	566115.78	917	2880.84	660432.68	985	3094.47	762012.93
850	2670.35	567450.17	918	2883.98	661873.88	986	3097.61	763560.95
851	2673.50	568786.14	919	2887.12	663316.66	987	3100.75	765110.54
852	2676.64	570123.67	**920**	2890.27	664761.01	988	3103.89	766661.70
853	2679.78	571462.77	921	2893.41	666206.92	989	3107.04	768214.44
854	2682.92	572803.45	922	2896.55	667654.41	**990**	3110.18	769768.74
855	2686.06	574145.69	923	2899.69	669103.47	991	3113.32	771324.61
856	2689.20	575489.51	924	2902.83	670554.10	992	3116.46	772882.06
857	2692.34	576834.90	925	2905.97	672006.30	993	3119.60	774441.07
858	2695.49	578181.85	926	2909.11	673460.08	994	3122.74	776001.66
859	2698.63	579530.38	927	2912.26	674915.42	995	3125.88	777563.82
860	2701.77	580880.48	928	2915.40	676372.33	996	3129.03	779127.54
861	2704.91	582232.15	929	2918.54	677830.82	997	3132.17	780692.84
862	2708.05	583585.39	**930**	2921.68	679290.87	998	3135.31	782259.71
863	2711.19	584940.20	931	2924.82	680752.50	999	3138.45	783828.15
864	2714.34	586296.59	932	2927.96	682215.69	**1000**	3141.59	785398.16
865	2717.48	587654.54	933	2931.11	683680.46			
866	2720.62	589014.07	934	2934.25	685146.80			

CIRCUMFERENCES AND AREAS OF CIRCLES

Advancing by Eighths.

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
1/64	.04909	.00019	2 3/8	7.4613	4.4301	6 1/8	19.242	29.465
1/32	.09818	.00077	7/16	7.6576	4.6664	1/4	19.635	30.680
3/64	.14726	.00173	1/2	7.8540	4.9087	3/8	20.028	31.919
1/16	.19635	.00307	9/16	8.0503	5.1572	1/2	20.420	33.183
3/32	.29452	.00690	5/8	8.2467	5.4119	5/8	20.813	34.472
1/8	.39270	.01227	11/16	8.4430	5.6727	3/4	21.206	35.785
5/32	.49087	.01917	3/4	8.6394	5.9396	7/8	21.598	37.122
3/16	.58905	.02761	13/16	8.8357	6.2126	7.	21.991	38.485
7/32	.68722	.03758	7/8	9.0321	6.4918	1/8	22.384	39.871
			15/16	9.2284	6.7771	1/4	22.776	41.282
1/4	.78540	.04909				3/8	23.169	42.718
9/32	.88357	.06213	3.	9.4248	7.0686	1/2	23.562	44.179
5/16	.98175	.07670	1/16	9.6211	7.3662	5/8	23.955	45.664
11/32	1.0799	.09281	1/8	9.8175	7.6699	3/4	24.347	47.173
3/8	1.1781	.11045	3/16	10.014	7.9798	7/8	24.740	48.707
13/32	1.2763	.12962	1/4	10.210	8.2958	8.	25.133	50.265
7/16	1.3744	.15033	5/16	10.407	8.6179	1/8	25.525	51.849
15/32	1.4726	.17257	3/8	10.603	8.9462	1/4	25.918	53.456
			7/16	10.799	9.2806	3/8	26.311	55.088
1/2	1.5708	.19635	1/2	10.996	9.6211	1/2	26.704	56.745
17/32	1.6690	.22166	9/16	11.192	9.9678	5/8	27.096	58.426
9/16	1.7671	.24850	5/8	11.388	10.321	3/4	27.489	60.132
19/32	1.8653	.27688	11/16	11.585	10.680	7/8	27.882	61.862
5/8	1.9635	.30680	3/4	11.781	11.045	9.	28.274	63.617
21/32	2.0617	.33824	13/16	11.977	11.416	1/8	28.667	65.397
11/16	2.1598	.37122	7/8	12.174	11.793	1/4	29.060	67.201
23/32	2.2580	.40574	15/16	12.370	12.177	3/8	29.452	69.029
			4.	12.566	12.566	1/2	29.845	70.882
3/4	2.3562	.44179	1/16	12.763	12.962	5/8	30.238	72.760
25/32	2.4544	.47937	1/8	12.959	13.364	3/4	30.631	74.662
13/16	2.5525	.51849	3/16	13.155	13.772	7/8	31.023	76.589
27/32	2.6507	.55914	1/4	13.352	14.186	10.	31.416	78.540
7/8	2.7489	.60132	5/16	13.548	14.607	1/8	31.809	80.516
29/32	2.8471	.64504	3/8	13.744	15.033	1/4	32.201	82.516
15/16	2.9452	.69029	7/16	13.941	15.466	3/8	32.594	84.541
31/32	3.0434	.73708	1/2	14.137	15.904	1/2	32.987	86.590
			9/16	14.334	16.349	5/8	33.379	88.664
1.	3.1416	.7854	5/8	14.530	16.800	3/4	33.772	90.763
1/16	3.3379	.8866	11/16	14.726	17.257	7/8	34.165	92.886
1/8	3.5343	.9940	3/4	14.923	17.721	11.	34.558	95.033
3/16	3.7306	1.1075	13/16	15.119	18.190	1/8	34.950	97.205
1/4	3.9270	1.2272	7/8	15.315	18.665	1/4	35.343	99.402
5/16	4.1233	1.3530	15/16	15.512	19.147	3/8	35.736	101.62
3/8	4.3197	1.4849	5.	15.708	19.635	1/2	36.128	103.87
7/16	4.5160	1.6230	1/16	15.904	20.129	5/8	36.521	106.14
1/2	4.7124	1.7671	1/8	16.101	20.629	3/4	36.914	108.43
9/16	4.9087	1.9175	3/16	16.297	21.135	7/8	37.306	110.75
5/8	5.1051	2.0739	1/4	16.493	21.648	12.	37.699	113.10
11/16	5.3014	2.2365	5/16	16.690	22.166	1/8	38.092	115.47
3/4	5.4978	2.4053	3/8	16.886	22.691	1/4	38.485	117.86
13/16	5.6941	2.5802	7/16	17.082	23.221	3/8	38.877	120.28
7/8	5.8905	2.7612	1/2	17.279	23.758	1/2	39.270	122.72
15/16	6.0868	2.9483	9/16	17.475	24.301	5/8	39.663	125.19
			5/8	17.671	24.850	3/4	40.055	127.68
2.	6.2832	3.1416	11/16	17.868	25.406	7/8	40.448	130.19
1/16	6.4795	3.3410	3/4	18.064	25.967	13.	40.841	132.73
1/8	6.6759	3.5466	13-16	18.261	26.535	1/8	41.233	135.30
3/16	6.8722	3.7583	7/8	18.457	27.109	1/4	41.626	137.89
1/4	7.0686	3.9761	15-16	18.653	27.688	3/8	42.019	140.50
5/16	7.2649	4.2000	6.	18.850	28.274	1/2	42.412	143.14

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
13 5/8	42.804	145.80	21 7/8	68.722	375.83	30 1/8	94.640	712.76
3/4	43.197	148.49	22.	69.115	380.13	1/4	95.033	718.69
7/8	43.590	151.20	1/8	69.508	384.46	3/8	95.426	724.64
14.	43.982	153.94	1/4	69.900	388.82	1/2	95.819	730.62
1/8	44.375	156.70	3/8	70.293	393.20	5/8	96.211	736.62
1/4	44.768	159.48	1/2	70.686	397.61	3/4	96.604	742.64
3/8	45.160	162.30	5/8	71.079	402.04	7/8	96.997	748.69
1/2	45.553	165.13	3/4	71.471	406.49	31.	97.389	754.77
5/8	45.946	167.99	7/8	71.864	410.97	1/8	97.782	760.87
3/4	46.338	170.87	23.	72.257	415.48	1/4	98.175	766.99
7/8	46.731	173.78	1/8	72.649	420.00	3/8	98.567	773.14
15.	47.124	176.71	1/4	73.042	424.56	1/2	98.960	779.31
1/8	47.517	179.67	3/8	73.435	429.13	5/8	99.353	785.51
1/4	47.909	182.65	1/2	73.827	433.74	3/4	99.746	791.73
3/8	48.302	185.66	5/8	74.220	438.36	7/8	100.138	797.98
1/2	48.695	188.69	3/4	74.613	443.01	32.	100.531	804.25
5/8	49.087	191.75	7/8	75.006	447.69	1/8	100.924	810.54
3/4	49.480	194.83	24.	75.398	452.39	1/4	101.316	816.86
7/8	49.873	197.93	1/8	75.791	457.11	3/8	101.709	823.21
16.	50.265	201.06	1/4	76.184	461.86	1/2	102.102	829.58
1/8	50.658	204.22	3/8	76.576	466.64	5/8	102.494	835.97
1/4	51.051	207.39	1/2	76.969	471.44	3/4	102.887	842.39
3/8	51.414	210.60	5/8	77.362	476.26	7/8	103.280	848.83
1/2	51.836	213.82	3/4	77.754	481.11	33.	103.673	855.30
5/8	52.229	217.08	7/8	78.147	485.98	1/8	104.065	861.79
3/4	52.622	220.35	25.	78.540	490.87	1/4	104.458	868.31
7/8	53.014	223.65	1/8	78.933	495.79	3/8	104.851	874.85
17.	53.407	226.98	1/4	79.325	500.74	1/2	105.243	881.41
1/8	53.800	230.33	3/8	79.718	505.71	5/8	105.636	888.00
1/4	54.192	233.71	1/2	80.111	510.71	3/4	106.029	894.62
3/8	54.585	237.10	5/8	80.503	515.72	7/8	106.421	901.26
1/2	54.978	240.53	3/4	80.896	520.77	34.	106.814	907.92
5/8	55.371	243.98	7/8	81.289	525.84	1/8	107.207	914.61
3/4	55.763	247.45	26.	81.681	530.93	1/4	107.600	921.32
7/8	56.156	250.95	1/8	82.074	536.05	3/8	107.992	928.06
18.	56.549	254.47	1/4	82.467	541.19	1/2	108.385	934.82
1/8	56.941	258.02	3/8	82.860	546.35	5/8	108.778	941.61
1/4	57.334	261.59	1/2	83.252	551.55	3/4	109.170	948.42
3/8	57.727	265.18	5/8	83.645	556.76	7/8	109.563	955.25
1/2	58.119	268.80	3/4	84.038	562.00	35.	109.956	962.11
5/8	58.512	272.45	7/8	84.430	567.27	1/8	110.348	969.00
3/4	58.905	276.12	27.	84.823	572.56	1/4	110.741	975.91
7/8	59.298	279.81	1/8	85.216	577.87	3/8	111.134	982.84
19.	59.690	283.53	1/4	85.608	583.21	1/2	111.527	989.80
1/8	60.083	287.27	3/8	86.001	588.57	5/8	111.919	996.78
1/4	60.476	291.04	1/2	86.394	593.96	3/4	112.312	1003.8
3/8	60.868	294.83	5/8	86.786	599.37	7/8	112.705	1010.8
1/2	61.261	298.65	3/4	87.179	604.81	36.	113.097	1017.9
5/8	61.654	302.49	7/8	87.572	610.27	1/8	113.490	1025.0
3/4	62.046	306.35	28.	87.965	615.75	1/4	113.883	1032.1
7/8	62.439	310.24	1/8	88.357	621.26	3/8	114.275	1039.2
20.	62.832	314.16	1/4	88.750	626.80	1/2	114.668	1046.3
1/8	63.225	318.10	3/8	89.143	632.36	5/8	115.061	1053.5
1/4	63.617	322.06	1/2	89.535	637.94	3/4	115.454	1060.7
3/8	64.010	326.05	5/8	89.928	643.55	7/8	115.846	1068.0
1/2	64.403	330.06	3/4	90.321	649.18	37.	116.239	1075.2
5/8	64.795	334.10	7/8	90.713	654.84	1/8	116.632	1082.5
3/4	65.188	338.16	29.	91.106	660.52	1/4	117.024	1089.8
7/8	65.581	342.25	1/8	91.499	666.23	3/8	117.417	1097.1
21.	65.973	346.36	1/4	91.892	671.96	1/2	117.810	1104.5
1/8	66.366	350.50	3/8	92.284	677.71	5/8	118.202	1111.8
1/4	66.759	354.66	1/2	92.677	683.49	3/4	118.596	1119.2
3/8	67.152	358.84	5/8	93.070	689.30	7/8	118.988	1126.7
1/2	67.544	363.05	3/4	93.462	695.13	38.	119.381	1134.1
5/8	67.937	367.28	7/8	93.855	700.98	1/8	119.773	1141.[illegible]
3/4	68.330	371.54	30.	94.248	706.86	1/4	120.166	1149.1

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
38⅜	120.559	1156.6	46⅝	146.477	1707.4	54⅞	172.395	2365.0
½	120.951	1164.2	¾	146.869	1716.5	55.	172.788	2375.8
⅝	121.344	1171.7	⅞	147.262	1725.7	⅛	173.180	2386.6
¾	121.737	1179.3	47.	147.655	1734.9	¼	173.573	2397.5
⅞	122.129	1186.9	⅛	148.048	1744.2	⅜	173.966	2408.3
39.	122.522	1194.6	¼	148.440	1753.5	½	174.358	2419.2
⅛	122.915	1202.3	⅜	148.833	1762.7	⅝	174.751	2430.1
¼	123.308	1210.0	½	149.226	1772.1	¾	175.144	2441.1
⅜	123.700	1217.7	⅝	149.618	1781.4	⅞	175.536	2452.0
½	124.093	1225.4	¾	150.011	1790.8	56.	175.929	2463.0
⅝	124.486	1233.2	⅞	150.404	1800.1	⅛	176.322	2474.0
¾	124.878	1241.0	48.	150.796	1809.6	¼	176.715	2485.0
⅞	125.271	1248.8	⅛	151.189	1819.0	⅜	177.107	2496.1
40.	125.664	1256.6	¼	151.582	1828.5	½	177.500	2507.2
⅛	126.056	1264.5	⅜	151.975	1837.9	⅝	177.893	2518.3
¼	126.449	1272.4	½	152.367	1847.5	¾	178.285	2529.4
⅜	126.842	1280.3	⅝	152.760	1857.0	⅞	178.678	2540.6
½	127.235	1288.2	¾	153.153	1866.5	57.	179.071	2551.8
⅝	127.627	1296.2	⅞	153.545	1876.1	⅛	179.463	2563.0
¾	128.020	1304.2	49.	153.938	1885.7	¼	179.856	2574.2
⅞	128.413	1312.2	⅛	154.331	1895.4	⅜	180.249	2585.4
41.	128.805	1320.3	¼	154.723	1905.0	½	180.642	2596.7
⅛	129.198	1328.3	⅜	155.116	1914.7	⅝	181.034	2608.0
¼	129.591	1336.4	½	155.509	1924.4	¾	181.427	2619.4
⅜	129.983	1344.5	⅝	155.902	1934.2	⅞	181.820	2630.7
½	130.376	1352.7	¾	156.294	1943.9	58.	182.212	2642.1
⅝	130.769	1360.8	⅞	156.687	1953.7	⅛	182.605	2653.5
¾	131.161	1369.0	50.	157.080	1963.5	¼	182.998	2664.9
⅞	131.554	1377.2	⅛	157.472	1973.3	⅜	183.390	2676.4
42.	131.947	1385.4	¼	157.865	1983.2	½	183.783	2687.8
⅛	132.340	1393.7	⅜	158.258	1993.1	⅝	184.176	2699.3
¼	132.732	1402.0	½	158.650	2003.0	¾	184.569	2710.9
⅜	133.125	1410.3	⅝	159.043	2012.9	⅞	184.961	2722.4
½	133.518	1418.6	¾	159.436	2022.8	59.	185.354	2734.0
⅝	133.910	1427.0	⅞	159.829	2032.8	⅛	185.747	2745.6
¾	134.303	1435.4	51.	160.221	2042.8	¼	186.139	2757.2
⅞	134.696	1443.8	⅛	160.614	2052.8	⅜	186.532	2768.8
43.	135.088	1452.2	¼	161.007	2062.9	½	186.925	2780.5
⅛	135.481	1460.7	⅜	161.399	2073.0	⅝	187.317	2792.2
¼	135.874	1469.1	½	161.792	2083.1	¾	187.710	2803.9
⅜	136.267	1477.6	⅝	162.185	2093.2	⅞	188.103	2815.7
½	136.659	1486.2	¾	162.577	2103.3	60.	188.496	2827.4
⅝	137.052	1494.7	⅞	162.970	2113.5	⅛	188.888	2839.2
¾	137.445	1503.3	52.	163.363	2123.7	¼	189.281	2851.0
⅞	137.837	1511.9	⅛	163.756	2133.9	⅜	189.674	2862.9
44.	138.230	1520.5	¼	164.148	2144.2	½	190.066	2874.8
⅛	138.623	1529.2	⅜	164.541	2154.5	⅝	190.459	2886.6
¼	139.015	1537.9	½	164.934	2164.8	¾	190.852	2898.6
⅜	139.408	1546.6	⅝	165.326	2175.1	⅞	191.244	2910.5
½	139.801	1555.3	¾	165.719	2185.4	61.	191.637	2922.5
⅝	140.194	1564.0	⅞	166.112	2195.8	⅛	192.030	2934.5
¾	140.586	1572.8	53.	166.504	2206.2	¼	192.423	2946.5
⅞	140.979	1581.6	⅛	166.897	2216.6	⅜	192.815	2958.5
45.	141.372	1590.4	¼	167.290	2227.0	½	193.208	2970.6
⅛	141.764	1599.3	⅜	167.683	2237.5	⅝	193.601	2982.7
¼	142.157	1608.2	½	168.075	2248.0	¾	193.993	2994.8
⅜	142.550	1617.0	⅝	168.468	2258.5	⅞	194.386	3006.9
½	142.942	1626.0	¾	168.861	2269.1	62.	194.779	3019.1
⅝	143.335	1634.9	⅞	169.253	2279.6	⅛	195.171	3031.3
¾	143.728	1643.9	54.	169.646	2290.2	¼	195.564	3043.5
⅞	144.121	1652.9	⅛	170.039	2300.8	⅜	195.957	3055.7
46.	144.513	1661.9	¼	170.431	2311.5	½	196.350	3068.0
⅛	144.906	1670.9	⅜	170.824	2322.1	⅝	196.742	3080.3
¼	145.299	1680.0	½	171.217	2332.8	¾	197.135	3092.6
⅜	145.691	1689.1	⅝	171.609	2343.5	⅞	197.528	3104.9
½	146.084	1698.2	¾	172.002	2354.3	63.	197.920	3117.2

Diam	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
63 1/8	198.313	3129.6	71 3/8	224.231	4001.1	79 5/8	250.149	4979.5
1/4	198.706	3142.0	1/2	224.624	4015.2	3/4	250.542	4995.2
3/8	199.098	3154.5	5/8	225.017	4029.2	7/8	250.935	5010.9
1/2	199.491	3166.9	3/4	225.409	4043.3	80.	251.327	5026.5
5/8	199.884	3179.4	7/8	225.802	4057.4	1/8	251.720	5042.3
3/4	200.277	3191.9	72.	226.195	4071.5	1/4	252.113	5058.0
7/8	200.669	3204.4	1/8	226.587	4085.7	3/8	252.506	5073.8
64.	201.062	3217.0	1/4	226.980	4099.8	1/2	252.898	5089.6
1/8	201.455	3229.6	3/8	227.373	4114.0	5/8	253.291	5105.4
1/4	201.847	3242.2	1/2	227.765	4128.2	3/4	253.684	5121.2
3/8	202.240	3254.8	5/8	228.158	4142.5	7/8	254.076	5137.1
1/2	202.633	3267.5	3/4	228.551	4156.8	81.	254.469	5153.0
5/8	203.025	3280.1	7/8	228.944	4171.1	1/8	254.862	5168.9
3/4	203.418	3292.8	73.	229.336	4185.4	1/4	255.254	5184.9
7/8	203.811	3305.6	1/8	229.729	4199.7	3/8	255.647	5200.8
65.	204.204	3318.3	1/4	230.122	4214.1	1/2	256.040	5216.8
1/8	204.596	3331.1	3/8	230.514	4228.5	5/8	256.433	5232.8
1/4	204.989	3343.9	1/2	230.907	4242.9	3/4	256.825	5248.9
3/8	205.382	3356.7	5/8	231.300	4257.4	7/8	257.218	5264.9
1/2	205.774	3369.6	3/4	231.692	4271.8	82.	257.611	5281.0
5/8	206.167	3382.4	7/8	232.085	4286.3	1/8	258.003	5297.1
3/4	206.560	3395.3	74.	232.478	4300.8	1/4	258.396	5313.3
7/8	206.952	3408.2	1/8	232.871	4315.4	3/8	258.789	5329.4
66.	207.345	3421.2	1/4	233.263	4329.9	1/2	259.181	5345.6
1/8	207.738	3434.2	3/8	233.656	4344.5	5/8	259.574	5361.8
1/4	208.131	3447.2	1/2	234.049	4359.2	3/4	259.967	5378.1
3/8	208.523	3460.2	5/8	234.441	4373.8	7/8	260.359	5394.3
1/2	208.916	3473.2	3/4	234.834	4388.5	83.	260.752	5410.6
5/8	209.309	3486.3	7/8	235.227	4403.1	1/8	261.145	5426.9
3/4	209.701	3499.4	75.	235.619	4417.9	1/4	261.538	5443.3
7/8	210.094	3512.5	1/8	236.012	4432.6	3/8	261.930	5459.6
67.	210.487	3525.7	1/4	236.405	4447.4	1/2	262.323	5476.0
1/8	210.879	3538.8	3/8	236.798	4462.2	5/8	262.716	5492.4
1/4	211.272	3552.0	1/2	237.190	4477.0	3/4	263.108	5508.8
3/8	211.665	3565.2	5/8	237.583	4491.8	7/8	263.501	5525.3
1/2	212.058	3578.5	3/4	237.976	4506.7	84.	263.894	5541.8
5/8	212.450	3591.7	7/8	238.368	4521.5	1/8	264.286	5558.3
3/4	212.843	3605.0	76.	238.761	4536.5	1/4	264.679	5574.8
7/8	213.236	3618.3	1/8	239.154	4551.4	3/8	265.072	5591.4
68	213.628	3631.7	1/4	239.546	4566.4	1/2	265.465	5607.9
1/8	214.021	3645.0	3/8	239.939	4581.3	5/8	265.857	5624.5
1/4	214.414	3658.4	1/2	240.332	4596.3	3/4	266.250	5641.2
3/8	214.806	3671.8	5/8	240.725	4611.4	7/8	266.643	5657.8
1/2	215.199	3685.3	3/4	241.117	4626.4	85.	267.035	5674.5
5/8	215.592	3698.7	7/8	241.510	4641.5	1/8	267.428	5691.2
3/4	215.984	3712.2	77.	241.903	4656.6	1/4	267.821	5707.9
7/8	216.377	3725.7	1/8	242.295	4671.8	3/8	268.213	5724.7
69.	216.770	3739.3	1/4	242.688	4686.9	1/2	268.606	5741.5
1/8	217.163	3752.8	3/8	243.081	4702.1	5/8	268.999	5758.3
1/4	217.555	3766.4	1/2	243.473	4717.3	3/4	269.392	5775.1
3/8	217.948	3780.0	5/8	243.866	4732.5	7/8	269.784	5791.9
1/2	218.341	3793.7	3/4	244.259	4747.8	86.	270.177	5808.8
5/8	218.733	3807.3	7/8	244.652	4763.1	1/8	270.570	5825.7
3/4	219.126	3821.0	78.	245.044	4778.4	1/4	270.962	5842.6
7/8	219.519	3834.7	1/8	245.437	4793.7	3/8	271.355	5859.6
70.	219.911	3848.5	1/4	245.830	4809.0	1/2	271.748	5876.5
1/8	220.304	3862.2	3/8	246.222	4824.4	5/8	272.140	5893.5
1/4	220.697	3876.0	1/2	246.615	4839.8	3/4	272.533	5910.6
3/8	221.090	3889.8	5/8	247.008	4855.2	7/8	272.926	5927.6
1/2	221.482	3903.6	3/4	247.400	4870.7	87.	273.319	5944.7
5/8	221.875	3917.5	7/8	247.793	4886.2	1/8	273.711	5961.8
3/4	222.268	3931.4	79.	248.186	4901.7	1/4	274.104	5978.9
7/8	222.660	3945.3	1/8	248.579	4917.2	3/8	274.497	5996.0
71.	223.053	3959.2	1/4	248.971	4932.7	1/2	274.889	6013.2
1/8	223.446	3973.1	3/8	249.364	4948.3	5/8	275.282	6030.4
1/4	223.838	3987.1	1/2	249.757	4963.9	3/4	275.675	6047.6

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
87 7/8	276.067	6064.9	92.	289.027	6647.6	96 1/8	301.986	7257.1
88.	276.460	6082.1	1/8	289.419	6665.7	1/4	302.378	7276.0
1/8	276.853	6099.4	1/4	289.812	6683.8	3/8	302.771	7294.9
1/4	277.246	6116.7	3/8	290.205	6701.9	1/2	303.164	7313.8
3/8	277.638	6134.1	1/2	290.597	6720.1	5/8	303.556	7332.8
1/2	278.031	6151.4	5/8	290.990	6738.2	3/4	303.949	7351.8
5/8	278.424	6168.8	3/4	291.383	6756.4	7/8	304.342	7370.8
3/4	278.816	6186.2	7/8	291.775	6774.7	97.	304.734	7389.8
7/8	279.209	6203.7	93.	292.168	6792.9	1/8	305.127	7408.9
89.	279.602	6221.1	1/8	292.561	6811.2	1/4	305.520	7428.0
1/8	279.994	6238.6	1/4	292.954	6829.5	3/8	305.913	7447.1
1/4	280.387	6256.1	3/8	293.346	6847.8	1/2	306.305	7466.2
3/8	280.780	6273.7	1/2	293.739	6866.1	5/8	306.698	7485.5
1/2	281.173	6291.2	5/8	294.132	6884.5	3/4	307.091	7504.5
5/8	281.565	6308.8	3/4	294.524	6902.9	7/8	307.483	7523.7
3/4	281.958	6326.4	7/8	294.917	6921.3	98.	307.876	7543.0
7/8	282.351	6344.1	94.	295.310	6939.8	1/8	308.269	7562.2
90.	282.743	6361.7	1/8	295.702	6958.2	1/4	308.661	7581.5
1/8	283.136	6379.4	1/4	296.095	6976.7	3/8	309.054	7600.8
1/4	283.529	6397.1	3/8	296.488	6995.3	1/2	309.447	7620.1
3/8	283.921	6414.9	1/2	296.881	7013.8	5/8	309.840	7639.5
1/2	284.314	6432.6	5/8	297.273	7032.4	3/4	310.232	7658.9
5/8	284.707	6450.4	3/4	297.666	7051.0	7/8	310.625	7678.3
3/4	285.100	6468.2	7/8	298.059	7069.6	99.	311.018	7697.7
7/8	285.492	6486.0	95.	298.451	7088.2	1/8	311.410	7717.1
91.	285.885	6503.9	1/8	298.844	7106.9	1/4	311.803	7736.6
1/8	286.278	6521.8	1/4	299.237	7125.6	3/8	312.196	7756.1
1/4	286.670	6539.7	3/8	299.629	7144.3	1/2	312.588	7775.6
3/8	287.063	6557.6	1/2	300.022	7163.0	5/8	312.981	7795.2
1/2	287.456	6575.5	5/8	300.415	7181.8	3/4	313.374	7814.8
5/8	287.848	6593.5	3/4	300.807	7200.6	7/8	313.767	7834.4
3/4	288.241	6611.5	7/8	301.200	7219.4	100.	314.159	7854.0
7/8	288.634	6629.6	96.	301.593	7238.2			

DECIMALS OF A FOOT EQUIVALENT TO INCHES AND FRACTIONS OF AN INCH.

Inches.	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0	.01042	.02083	.03125	.04166	.05208	.06250	.07292
1	.0833	.0937	.1042	.1146	.1250	.1354	.1459	.1563
2	.1667	.1771	.1875	.1979	.2083	.2188	.2292	.2396
3	.2500	.2604	.2708	.2813	.2917	.3021	.3125	.3229
4	.3333	.3437	.3542	.3646	.3750	.3854	.3958	.4063
5	.4167	.4271	.4375	.4479	.4583	.4688	.4792	.4896
6	.5000	.5104	.5208	.5313	.5417	.5521	.5625	.5729
7	.5833	.5937	.6042	.6146	.6250	.6354	.6459	.6563
8	.6667	.6771	.6875	.6979	.7083	.7188	.7292	.7396
9	.7500	.7604	.7708	.7813	.7917	.8021	.8125	.8229
10	.8333	.8437	.8542	.8646	.8750	.8854	.8958	.9063
11	.9167	.9271	.9375	.9479	.9583	.9688	.9792	.9896

CIRCUMFERENCE OF CIRCLES FROM 1 INCH TO 32 FT. 11 IN. DIAMETER.

Diam. Feet.	0 In.		1 Inch.		2 In.		3 In.		4 In.		5 In.		6 In.		7 In.		8 In.		9 In.		10 In.		11 In.		Diam. Feet.
	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	
				3⅛		6¼		9⅜	1	0⅝	1	3¾	1	6⅞	1	10	2	1⅛	2	4¼	2	7⅜	2	10½	
1	3	1¾	3	4⅞	3	8	3	11⅛	4	2¼	4	5⅜	4	8½	4	11⅝	5	2⅞	5	6	5	9⅛	6	0¼	1
2	6	3⅜	6	6½	6	9⅝	7	0⅞	7	4	7	7⅛	7	10¼	8	1⅜	8	4½	8	7⅝	8	10¾	9	2	2
3	9	5⅛	9	8¼	9	11⅜	10	2½	10	5⅝	10	8¾	10	11⅞	11	3⅛	11	6¼	11	9⅜	12	0½	12	3⅝	3
4	12	6¾	12	9⅞	13	1⅛	13	4¼	13	7⅜	13	10½	14	1⅝	14	4¾	14	7⅞	14	11	15	2¼	15	5⅜	4
5	15	8½	15	11⅝	16	2¾	16	5⅞	16	9	17	0¼	17	3⅜	17	6½	17	9⅝	18	0¾	18	3⅞	18	7	5
6	18	10⅛	19	1⅜	19	4½	19	7⅝	19	10¾	20	1⅞	20	5	20	8⅛	20	11⅜	21	2½	21	5⅝	21	8¾	6
7	21	11⅞	22	3	22	6⅛	22	9¼	23	0½	23	3⅝	23	6¾	23	9⅞	24	1	24	4⅛	24	7¼	24	10½	7
8	25	1⅝	25	4¾	25	7⅞	25	11	26	2⅛	26	5¼	26	8½	26	11⅝	27	2¾	27	5⅞	27	9	28	0⅛	8
9	28	3¼	28	6⅜	28	9⅝	29	0¾	29	3⅞	29	7	29	10⅛	30	1¼	30	4⅜	30	7½	30	10¾	31	1⅞	9
10	31	5	31	8⅛	31	11¼	32	2⅜	32	5½	32	8¾	32	11⅞	33	3	33	6⅛	33	9¼	34	0⅜	34	3½	10
11	34	6⅝	34	9⅞	35	1	35	4⅛	35	7¼	35	10⅜	36	1½	36	4⅝	36	7¾	36	11	37	2⅛	37	5¼	11
12	37	8⅜	37	11½	38	2⅝	38	5¾	38	9	39	0⅛	39	3¼	39	6⅜	39	9½	40	0⅝	40	3¾	40	6⅞	12
13	40	10⅛	41	1¼	41	4⅜	41	7½	41	10⅝	42	1¾	42	4⅞	42	8⅛	42	11¼	43	2⅜	43	5½	43	8⅝	13
14	43	11¾	44	2⅞	44	6	44	9¼	45	0⅜	45	3½	45	6⅝	45	9¾	46	0⅞	46	4	46	7⅛	46	10⅜	14
15	47	1½	47	4⅝	47	7¾	47	10⅞	48	2	48	5⅛	48	8⅜	48	11½	49	2⅝	49	5¾	49	8⅞	50	0	15
16	50	3⅛	50	6⅜	50	9½	51	0⅝	51	3¾	51	6⅞	51	10	52	1⅛	52	4¼	52	7½	52	10⅝	53	1¾	16
17	53	4⅞	53	8	53	11⅛	54	2¼	54	5½	54	8⅝	54	11¾	55	2⅞	55	6	55	9⅛	56	0¼	56	3½	17
18	56	6⅝	56	9¾	57	0⅞	57	4	57	7⅛	57	10¼	58	1⅜	58	4½	58	7¾	58	10⅞	59	2	59	5⅛	18
19	59	8¼	59	11⅜	60	2½	60	5¾	60	8⅞	61	0	61	3⅛	61	6¼	61	9⅜	62	0½	62	3⅝	62	6⅞	19
20	62	10	63	1⅛	63	4¼	63	7⅜	63	10½	64	1⅝	64	4⅞	64	8	64	11⅛	65	2¼	65	5⅜	65	8½	20
21	65	11⅝	66	2⅞	66	6	66	9⅛	67	0¼	67	3⅜	67	6½	67	9⅝	68	0¾	68	4	68	7⅛	68	10¼	21
22	69	1⅜	69	4½	69	7⅝	69	10¾	70	1⅞	70	5⅛	70	8¼	70	11⅜	71	2½	71	5⅝	71	8¾	71	11⅞	22
23	72	3	72	6¼	72	9⅜	73	0½	73	3⅝	73	6¾	73	9⅞	74	1	74	4¼	74	7⅜	74	10½	75	1⅝	23
24	75	4¾	75	7⅞	75	11	76	2⅛	76	5⅜	76	8½	76	11⅝	77	2¾	77	5⅞	77	9	78	0⅛	78	3⅜	24
25	78	6½	78	9⅝	79	0¾	79	3⅞	79	7	79	10⅛	80	1¼	80	4⅜	80	7⅝	80	10¾	81	1⅞	81	5	25
26	81	8⅛	81	11¼	82	2½	82	5⅝	82	8¾	82	11⅞	83	3	83	6⅛	83	9¼	84	0⅜	84	3⅝	84	6¾	26
27	84	9⅞	85	1	85	4⅛	85	7¼	85	10⅜	86	1⅝	86	4¾	86	7⅞	86	11	87	2⅛	87	5¼	87	8⅜	27
28	87	11⅝	88	2¾	88	5⅞	88	9	89	0⅛	89	3¼	89	6⅜	89	9½	90	0¾	90	3⅞	90	7	90	10⅛	28
29	91	1¼	91	4⅜	91	7½	91	10¾	92	1⅞	92	5	92	8⅛	92	11¼	93	2⅜	93	5½	93	8⅝	93	11⅞	29
30	94	3	94	6⅛	94	9¼	95	0⅜	95	3½	95	6⅝	95	9¾	96	1	96	4⅛	96	7¼	96	10⅜	97	1½	30
31	97	4⅝	97	7¾	97	11	98	2⅛	98	5¼	98	8⅜	98	11½	99	2⅝	99	5¾	99	8⅞	100	0⅛	100	3¼	31
32	100	6⅜	100	9½	101	0⅝	101	3¾	101	6⅞	101	10⅛	102	1¼	102	4⅜	102	7½	102	10⅝	103	1¾	103	4⅞	32

LENGTHS OF CIRCULAR ARCS.
(Degrees being given. Radius of Circle = 1.)

FORMULA.—Length of arc $= \frac{3.1415927}{180} \times$ radius $\times$ number of degrees.

RULE.—Multiply the factor in table for any given number of degrees by the radius.

EXAMPLE.—Given a curve of a radius of 55 feet and an angle of 78° 20′. What is the length of same in feet?

Factor from table for 78°	1.3613568
Factor from table for 20′	.0058178
Factor	1.3671746

1.3671746 × 55 = 75.19 feet.

Degrees.						Minutes.	
1	.0174533	61	1.0646508	121	2.1118484	1	.0002909
2	.0349066	62	1.0821041	122	2.1293017	2	.0005818
3	.0523599	63	1.0995574	123	2.1467550	3	.0008727
4	.0698132	64	1.1170107	124	2.1642083	4	.0011636
5	.0872665	65	1.1344640	125	2.1816616	5	.0014544
6	.1047198	66	1.1519173	126	2.1991149	6	.0017453
7	.1221730	67	1.1693706	127	2.2165682	7	.0020362
8	.1396263	68	1.1868239	128	2.2340214	8	.0023271
9	.1570796	69	1.2042772	129	2.2514747	9	.0026180
10	.1745329	70	1.2217305	130	2.2689280	10	.0029089
11	.1919862	71	1.2391838	131	2.2863813	11	.0031998
12	.2094395	72	1.2566371	132	2.3038346	12	.0034907
13	.2268928	73	1.2740904	133	2.3212879	13	.0037815
14	.2443461	74	1.2915436	134	2.3387412	14	.0040724
15	.2617994	75	1.3089969	135	2.3561945	15	.0043633
16	.2792527	76	1.3264502	136	2.3736478	16	.0046542
17	.2967060	77	1.3439035	137	2.3911011	17	.0049451
18	.3141593	78	1.3613568	138	2.4085544	18	.0052360
19	.3316126	79	1.3788101	139	2.4260077	19	.0055269
20	.3490659	80	1.3962634	140	2.4434610	20	.0058178
21	.3665191	81	1.4137167	141	2.4609142	21	.0061087
22	.3839724	82	1.4311700	142	2.4783675	22	.0063995
23	.4014257	83	1.4486233	143	2.4958208	23	.0066904
24	.4188790	84	1.4660766	144	2.5132741	24	.0069813
25	.4363323	85	1.4835299	145	2.5307274	25	.0072722
26	.4537856	86	1.5009832	146	2.5481807	26	.0075631
27	.4712389	87	1.5184364	147	2.5656340	27	.0078540
28	.4886922	88	1.5358897	148	2.5830873	28	.0081449
29	.5061455	89	1.5533430	149	2.6005406	29	.0084358
30	.5235988	90	1.5707963	150	2.6179939	30	.0087266
31	.5410521	91	1.5882496	151	2.6354472	31	.0090175
32	.5585054	92	1.6057029	152	2.6529005	32	.0093084
33	.5759587	93	1.6231562	153	2.6703538	33	.0095993
34	.5934119	94	1.6406095	154	2.6878070	34	.0098902
35	.6108652	95	1.6580628	155	2.7052603	35	.0101811
36	.6283185	96	1.6755161	156	2.7227136	36	.0104720
37	.6457718	97	1.6929694	157	2.7401669	37	.0107629
38	.6632251	98	1.7104227	158	2.7576202	38	.0110538
39	.6806784	99	1.7278760	159	2.7750735	39	.0113446
40	.6981317	100	1.7453293	160	2.7925268	40	.0116355
41	.7155850	101	1.7627825	161	2.8099801	41	.0119264
42	.7330383	102	1.7802358	162	2.8274334	42	.0122173
43	.7504916	103	1.7976891	163	2.8448867	43	.0125082
44	.7679449	104	1.8151424	164	2.8623400	44	.0127991
45	.7853982	105	1.8325957	165	2.8797933	45	.0130900
46	.8028515	106	1.8500490	166	2.8972466	46	.0133809
47	.8203047	107	1.8675023	167	2.9146999	47	.0136717
48	.8377580	108	1.8849556	168	2.9321531	48	.0139626
49	.8552113	109	1.9024089	169	2.9496064	49	.0142535
50	.8726646	110	1.9198622	170	2.9670597	50	.0145444
51	.8901179	111	1.9373155	171	2.9845130	51	.0148353
52	.9075712	112	1.9547688	172	3.0019663	52	.0151262
53	.9250245	113	1.9722221	173	3.0194196	53	.0154171
54	.9424778	114	1.9896753	174	3.0368729	54	.0157080
55	.9599311	115	2.0071286	175	3.0543262	55	.0159989
56	.9773844	116	2.0245819	176	3.0717795	56	.0162897
57	.9948377	117	2 0420352	177	3.0892328	57	.0165806
58	1.0122910	118	2.0594885	178	3.1066861	58	.0168715
59	1.0297443	119	2.0769418	179	3.1241394	59	.0171624
60	1.0471976	120	2.0943951	180	3.1415927	60	.0174533

LENGTHS OF CIRCULAR ARCS.

(Diameter = 1. Given the Chord and Height of the Arc.)

RULE FOR USE OF THE TABLE.—Divide the height by the chord. Find in the column of heights the number equal to this quotient. Take out the corresponding number from the column of lengths. Multiply this last number by the length of the given chord; the product will be length of the arc.

If the arc is greater than a semicircle, first find the diameter from the formula, Diam. = (square of half chord ÷ rise) + rise; the formula is true whether the arc exceeds a semicircle or not. Then find the circumference. From the diameter subtract the given height of arc, the remainder will be height of the smaller arc of the circle; find its length according to the rule, and subtract it from the circumference.

Hgts.	Lgths.	Hgts.	Lgths.	Hgts.	Lgths.	Hgts.	Lgths.	Hgts.	Lgths.
.001	1.00002	.15	1.05896	.238	1.14480	.326	1.26288	.414	1.40788
.005	1.00007	.152	1.06051	.24	1.14714	.328	1.26588	.416	1.41145
.01	1.00027	.154	1.06209	.242	1.14951	.33	1.26892	.418	1.41503
.015	1.00061	.156	1.06368	.244	1.15189	.332	1.27196	.42	1.41861
.02	1.00107	.158	1.06530	.246	1.15428	.334	1.27502	.422	1.42221
.025	1.00167	.16	1.06693	.248	1.15670	.336	1.27810	.424	1.42583
.03	1.00240	.162	1.06858	.25	1.15912	.338	1.28118	.426	1.42945
.035	1.00327	.164	1.07025	.252	1.16156	.34	1.28428	.428	1.43309
.04	1.00426	.166	1.07194	.254	1.16402	.342	1.28739	.43	1.43673
.045	1.00539	.168	1.07365	.256	1.16650	.344	1.29052	.432	1.44039
.05	1.00665	.17	1.07537	.258	1.16899	.346	1.29366	.434	1.44405
.055	1.00805	.172	1.07711	.26	1.17150	.348	1.29681	.436	1.44773
.06	1.00957	.174	1.07888	.262	1.17403	.35	1.29997	.438	1.45142
.065	1.01123	.176	1.08066	.264	1.17657	.352	1.30315	.44	1.45512
.07	1.01302	.178	1.08246	.266	1.17912	.354	1.30634	.442	1.45883
.075	1.01493	.18	1.08428	.268	1.18169	.356	1.30954	.444	1.46255
.08	1.01696	.182	1.08611	.27	1.18429	.358	1.31276	.446	1.46628
.085	1.01916	.184	1.08797	.272	1.18689	.36	1.31599	.448	1.47002
.09	1.02146	.186	1.08984	.274	1.18951	.362	1.31923	.45	1.47377
.095	1.02389	.188	1.09174	.276	1.19214	.364	1.32249	.452	1.47753
.10	1.02646	.19	1.09365	.278	1.19479	.366	1.32577	.454	1.48131
.102	1.02752	.192	1.09557	.28	1.19746	.368	1.32905	.456	1.48509
.104	1.02860	.194	1.09752	.282	1.20014	.37	1.33234	.458	1.48889
.106	1.02970	.196	1.09949	.284	1.20284	.372	1.33564	.46	1.49269
.108	1.03082	.198	1.10147	.286	1.20555	.374	1.33896	.462	1.49651
.11	1.03196	.20	1.10347	.288	1.20827	.376	1.34229	.464	1.50033
.112	1.03312	.202	1.10548	.29	1.21102	.378	1.34563	.466	1.50416
.114	1.03430	.204	1.10752	.292	1.21377	.38	1.34899	.468	1.50800
.116	1.03551	.206	1.10958	.294	1.21654	.382	1.35237	.47	1.51185
.118	1.03672	.208	1.11165	.296	1.21933	.384	1.35575	.472	1.51571
.12	1.03797	.21	1.11374	.298	1.22213	.386	1.35914	.474	1.51958
.122	1.03923	.212	1.11584	.30	1.22495	.388	1.36254	.476	1.52346
.124	1.04051	.214	1.11796	.302	1.22778	.39	1.36596	.478	1.52736
.126	1.04181	.216	1.12011	.304	1.23063	.392	1.36939	.48	1.53126
.128	1.04313	.218	1.12225	.306	1.23349	.394	1.37283	.482	1.53518
.13	1.04447	.22	1.12444	.308	1.23636	.396	1.37628	.484	1.53910
.132	1.04584	.222	1.12664	.31	1.23926	.398	1.37974	.486	1.54302
.134	1.04722	.224	1.12885	.312	1.24216	.40	1.38322	.488	1.54696
.136	1.04862	.226	1.13108	.314	1.24507	.402	1.38671	.49	1.55091
.138	1.05003	.228	1.13331	.316	1.24801	.404	1.39021	.492	1.55487
.14	1.05147	.23	1.13557	.318	1.25095	.406	1.39372	.494	1.55854
.142	1.05293	.232	1.13785	.32	1.25391	.408	1.39724	.496	1.56282
.144	1.05441	.234	1.14015	.322	1.25689	.41	1.40077	.498	1.56681
.146	1.05591	.236	1.14247	.324	1.25988	.412	1.40432	.50	1.57080
.148	1.05743								

AREAS OF THE SEGMENTS OF A CIRCLE.

(Diameter = 1; Rise or Versed Sine in parts of Diameter being given.)

RULE FOR USE OF THE TABLE.—Divide the rise or height of the segment by the diameter to obtain the versed sine. Multiply the area in the table corresponding to this versed sine by the square of the diameter.

If the segment exceeds a semicircle its area is area of circle – area of segment whose rise is (diam. of circle – rise of given segment).

Given chord and rise, to find diameter. Diam. = (square of half chord ÷ rise) + rise. The half chord is a mean proportional between the two parts into which the chord divides the diameter which is perpendicular to it.

Versed Sine.	Area.	Versed Sine.	Area.	Versed Sine.	Area.	Versed Sine.	Area.	Versed Sine.	Area.
.001	.00004	.054	.01646	.107	.04514	.16	.08111	.213	.12235
.002	.00012	.055	.01691	.108	.04576	.161	.08185	.214	.12317
.003	.00022	.056	.01737	.109	.04638	.162	.08258	.215	.12399
.004	.00034	.057	.01783	.11	.04701	.163	.08332	.216	.12481
.005	.00047	.058	.01830	.111	.04763	.164	.08406	.217	.12563
.006	.00062	.059	.01877	.112	.04826	.165	.08480	.218	.12646
.007	.00078	.06	.01924	.113	.04889	.166	.08554	.219	.12729
.008	.00095	.061	.01972	.114	.04953	.167	.08629	.22	.12811
.009	.00113	.062	.02020	.115	.05016	.168	.08704	.221	.12894
.01	.00133	.063	.02068	.116	.05080	.169	.08779	.222	.12977
.011	.00153	.064	.02117	.117	.05145	.17	.08854	.223	.13060
.012	.00175	.065	.02166	.118	.05209	.171	.08929	.224	.13144
.013	.00197	.066	.02215	.119	.05274	.172	.09004	.225	.13227
.014	.0022	.067	.02265	.12	.05338	.173	.09080	.226	.13311
.015	.00244	.068	.02315	.121	.05404	.174	.09155	.227	.13395
.016	.00268	.069	.02366	.122	.05469	.175	.09231	.228	.13478
.017	.00294	.07	.02417	.123	.05535	.176	.09307	.229	.13562
.018	.0032	.071	.02468	.124	.05600	.177	.09384	.23	.13646
.019	.00347	.072	.02520	.125	.05666	.178	.09460	.231	.13731
.02	.00375	.073	.02571	.126	.05733	.179	.09537	.232	.13815
.021	.00403	.074	.02624	.127	.05799	.18	.09613	.233	.13900
.022	.00432	.075	.02676	.128	.05866	.181	.09690	.234	.13984
.023	.00462	.076	.02729	.129	.05933	.182	.09767	.235	.14069
.024	.00492	.077	.02782	.13	.06000	.183	.09845	.236	.14154
.025	.00523	.078	.02836	.131	.06067	.184	.09922	.237	.14239
.026	.00555	.079	.02889	.132	.06135	.185	.10000	.238	.14324
.027	.00587	.08	.02943	.133	.06203	.186	.10077	.239	.14409
.028	.00619	.081	.02998	.134	.06271	.187	.10155	.24	.14494
.029	.00653	.082	.03053	.135	.06339	.188	.10233	.241	.14580
.03	.00687	.083	.03108	.136	.06407	.189	.10312	.242	.14666
.031	.00721	.084	.03163	.137	.06476	.19	.10390	.243	.14751
.032	.00756	.085	.03219	.138	.06545	.191	.10469	.244	.14837
.033	.00791	.086	.03275	.139	.06614	.192	.10547	.245	.14923
.034	.00827	.087	.03331	.14	.06683	.193	.10626	.246	.15009
.035	.00864	.088	.03387	.141	.06753	.194	.10705	.247	.15095
.036	.00901	.089	.03444	.142	.06822	.195	.10784	.248	.15182
.037	.00938	.09	.03501	.143	.06892	.196	.10864	.249	.15268
.038	.00976	.091	.03559	.144	.06963	.197	.10943	.25	.15355
.039	.01015	.092	.03616	.145	.07033	.198	.11023	.251	.15441
.04	.01054	.093	.03674	.146	.07103	.199	.11102	.252	.15528
.041	.01093	.094	.03732	.147	.07174	.2	.11182	.253	.15615
.042	.01133	.095	.03791	.148	.07245	.201	.11262	.254	.15702
.043	.01173	.096	.03850	.149	.07316	.202	.11343	.255	.15789
.044	.01214	.097	.03909	.15	.07387	.203	.11423	.256	.15876
.045	.01255	.098	.03968	.151	.07459	.204	.11504	.257	.15964
.046	.01297	.099	.04028	.152	.07531	.205	.11584	.258	.16051
.047	.01339	.1	.04087	.153	.07603	.206	.11665	.259	.16139
.048	.01382	.101	.04148	.154	.07675	.207	.11746	.26	.16226
.049	.01425	.102	.04208	.155	.07747	.208	.11827	.261	.16314
.05	.01468	.103	.04269	.156	.07819	.209	.11908	.262	.16402
.051	.01512	.104	.04330	.157	.07892	.21	.11990	.263	.16490
.052	.01556	.105	.04391	.158	.07965	.211	.12071	.264	.16578
.053	.01601	.106	.04452	.159	.08038	.212	.12153	.265	.16666

Versed Sine.	Area.	Versed Sine.	Area.	Versed Sine.	Area.	Versed Sine.	Area.	Versed Sine.	Area.
.266	.16755	.313	.21015	.36	.25455	.407	.30024	.454	.34676
.267	.16843	.314	.21108	.361	.25551	.408	.30122	.455	.34776
.268	.16932	.315	.21201	.362	.25647	.409	.30220	.456	.34876
.269	.17020	.316	.21294	.363	.25743	.41	.30319	.457	.34975
.27	.17109	.317	.21387	.364	.25839	.411	.30417	.458	.35075
.271	.17198	.318	.21480	.365	.25936	.412	.30516	.459	.35175
.272	.17287	.319	.21573	.366	.26032	.413	.30614	.46	.35274
.273	.17376	.32	.21667	.367	.26128	.414	.30712	.461	.35374
.274	.17465	.321	.21760	.368	.26225	.415	.30811	.462	.35474
.275	.17554	.322	.21853	.369	.26321	.416	.30910	.463	.35573
.276	.17644	.323	.21947	.37	.26418	.417	.31008	.464	.35673
.277	.17733	.324	.22040	.371	.26514	.418	.31107	.465	.35773
.278	.17823	.325	.22134	.372	.26611	.419	.31205	.466	.35873
.279	.17912	.326	.22228	.373	.26708	.42	.31304	.467	.35972
.28	.18002	.327	.22322	.374	.26805	.421	.31403	.468	.36072
.281	.18092	.328	.22415	.375	.26901	.422	.31502	.469	.36172
.282	.18182	.329	.22509	.376	.26998	.423	.31600	.47	.36272
.283	.18272	.33	.22603	.377	.27095	.424	.31699	.471	.36372
.284	.18362	.331	.22697	.378	.27192	.425	.31798	.472	.36471
.285	.18452	.332	.22792	.379	.27289	.426	.31897	.473	.36571
.286	.18542	.333	.22886	.38	.27386	.427	.31996	.474	.36671
.287	.18633	.334	.22980	.381	.27483	.428	.32095	.475	.36771
.288	.18723	.335	.23074	.382	.27580	.429	.32194	.476	.36871
.289	.18814	.336	.23169	.383	.27678	.43	.32293	.477	.36971
.29	.18905	.337	.23263	.384	.27775	.431	.32392	.478	.37071
.291	.18996	.338	.23358	.385	.27872	.432	.32491	.479	.37171
.292	.19086	.339	.23453	.386	.27969	.433	.32590	.48	.37270
.293	.19177	.34	.23547	.387	.28067	.434	.32689	.481	.37370
.294	.19268	.341	.23642	.388	.28164	.435	.32788	.482	.37470
.295	.19360	.342	.23737	.389	.28262	.436	.32887	.483	.37570
.296	.19451	.343	.23832	.39	.28359	.437	.32987	.484	.37670
.297	.19542	.344	.23927	.391	.28457	.438	.33086	.485	.37770
.298	.19634	.345	.24022	.392	.28554	.439	.33185	.486	.37870
.299	.19725	.346	.24117	.393	.28652	.44	.33284	.487	.37970
.3	.19817	.347	.24212	.394	.28750	.441	.33384	.488	.38070
.301	.19908	.348	.24307	.395	.28848	.442	.33483	.489	.38170
.302	.20000	.349	.24403	.396	.28945	.443	.33582	.49	.38270
.303	.20092	.35	.24498	.397	.29043	.444	.33682	.491	.38370
.304	.20184	.351	.24593	.398	.29141	.445	.33781	.492	.38470
.305	.20276	.352	.24689	.399	.29239	.446	.33880	.493	.38570
.306	.20368	.353	.24784	.4	.29337	.447	.33980	.494	.38670
.307	.20460	.354	.24880	.401	.29435	.448	.34079	.495	.38770
.308	.20553	.355	.24976	.402	.29533	.449	.34179	.496	.38870
.309	.20645	.356	.25071	.403	.29631	.45	.34278	.497	.38970
.31	.20738	.357	.25167	.404	.29729	.451	.34378	.498	.39070
.311	.20830	.358	.25263	.405	.29827	.452	.34477	.499	.39170
.312	.20923	.359	.25359	.406	.29926	.453	.34577	.5	.39270

For rules for finding the area of a segment see Mensuration, page 59.

SPHERES.

(Some errors of 1 in the last figure only. From TRAUTWINE.)

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
1-32	.00307	.00002	3 ¼	33.183	17.974	9 ⅞	306.36	504.21
1-16	.01227	.00013	5-16	34.472	19.031	10.	314.16	523.60
3-32	.02761	.00043	⅜	35.784	20.129	⅛	322.06	543.48
⅛	.04909	.00102	7-16	37.122	21.268	¼	330.06	563.86
5-32	.07670	.00200	½	38.484	22.449	⅜	338.16	584.74
3-16	.11045	.00345	9-16	39.872	23.674	½	346.36	606.13
7-32	.15033	.00548	⅝	41.283	24.942	⅝	354.66	628.04
¼	.19635	.00818	11-16	42.719	26.254	¾	363.05	650.46
9-32	.24851	.01165	¾	44.179	27.611	⅞	371.54	673.42
5-16	.30680	.01598	13-16	45.664	29.016	11.	380.13	696.91
11-32	.37123	.02127	⅞	47.173	30.466	⅛	388.83	720.95
⅜	.44179	.02761	15-16	48.708	31.965	¼	397.61	745.51
13-32	.51848	.03511	4.	50.265	33.510	⅜	406.49	770.64
7-16	.60132	.04385	⅛	53.456	36.751	½	415.48	796.33
15-32	.69028	.05393	¼	56.745	40.195	⅝	424.50	822.58
½	.78540	.06545	⅜	60.132	43.847	¾	433.73	849.40
9-16	.99403	.09319	½	63.617	47.713	⅞	443.01	876.79
⅝	1.2272	.12783	⅝	67.201	51.801	12.	452.39	904.78
11-16	1.4849	.17014	¾	70.883	56.116	¼	471.44	962.52
¾	1.7671	.22089	⅞	74.663	60.663	½	490.87	1022.7
13-16	2.0739	.28084	5.	78.540	65.450	¾	510.71	1085.3
⅞	2.4053	.35077	⅛	82.516	70.482	13.	530.93	1150.3
15-16	2.7611	.43143	¼	86.591	75.767	¼	551.55	1218.0
1.	3.1416	.52360	⅜	90.763	81.308	½	572.55	1288.3
1-16	3.5466	.62804	½	95.033	87.113	¾	593.95	1361.2
⅛	3.9761	.74551	⅝	99.401	93.189	14.	615.75	1436.8
3-16	4.4301	.87681	¾	103.87	99.541	¼	637.95	1515.1
¼	4.9088	1.0227	⅞	108.44	106.18	½	660.52	1596.3
5-16	5.4119	1.1839	6.	113.10	113.10	¾	683.49	1680.3
⅜	5.9396	1.3611	⅛	117.87	120.31	15.	706.85	1767.2
7-16	6.4919	1.5553	¼	122.72	127.83	¼	730.68	1857.0
½	7.0686	1.7671	⅜	127.68	135.66	½	754.77	1949.8
9-16	7.6699	1.9974	½	132.73	143.79	¾	779.32	2045.7
⅝	8.2957	2.2468	⅝	137.89	152.25	16.	804.25	2144.7
11-16	8.9461	2.5161	¾	143.14	161.03	¼	829.57	2246.8
¾	9.6211	2.8062	⅞	148.49	170.14	½	855.29	2352.1
13-16	10.321	3.1177	7.	153.94	179.59	¾	881.42	2460.6
⅞	11.044	3.4514	⅛	159.49	189.39	17.	907.93	2572.4
15-16	11.793	3.8083	¼	165.13	199.53	¼	934.83	2687.6
2.	12.566	4.1888	⅜	170.87	210.03	½	962.12	2806.2
1-16	13.364	4.5939	½	176.71	220.89	¾	989.80	2928.2
⅛	14.186	5.0243	⅝	182.66	232.13	18.	1017.9	3053.6
3-16	15.033	5.4809	¾	188.69	243.73	¼	1046.4	3182.6
¼	15.904	5.9641	⅞	194.83	255.72	½	1075.2	3315.3
5-16	16.800	6.4751	8.	201.06	268.08	¾	1104.5	3451.5
⅜	17.721	7.0144	⅛	207.39	280.85	19.	1134.1	3591.4
7-16	18.666	7.5829	¼	213.82	294.01	¼	1164.2	3735.0
½	19.635	8.1813	⅜	220.36	307.58	½	1194.6	3882.5
9-16	20.629	8.8103	½	226.98	321.56	¾	1225.4	4033.7
⅝	21.648	9.4708	⅝	233.71	335.95	20.	1256.7	4188.8
11-16	22.691	10.164	¾	240.53	350.77	¼	1288.3	4347.8
¾	23.758	10.889	⅞	247.45	360.02	½	1320.3	4510.9
13-16	24.850	11.649	9.	254.47	381.70	¾	1352.7	4677.9
⅞	25.967	12.443	⅛	261.59	397.83	21.	1385.5	4849.1
15-16	27.109	13.272	¼	268.81	414.41	¼	1418.6	5024.3
3.	28.274	14.137	⅜	270.12	431.44	½	1452.2	5203.7
1-16	29.465	15.039	½	283.53	448.92	¾	1486.2	5387.4
⅛	30.680	15.979	⅝	291.04	466.87	22.	1520.5	5575.3
3-16	31.919	16.957	¾	298.65	485.31	¼	1555.3	5767.6

SPHERES—(*Continued.*)

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
22 ½	1590.4	5964.1	40 ½	5153.1	34783	70 ½	15615	183471
¾	1626.0	6165.2	41.	5281.1	36087	71.	15837	187402
23.	1661.9	6370.6	½	5410.7	37423	½	16061	191389
¼	1698.2	6580.6	42.	5541.9	38792	72.	16286	195433
½	1735.0	6795.2	½	5674.5	40194	½	16513	199532
¾	1772.1	7014.3	43.	5808.8	41630	73.	16742	203689
24.	1809.6	7238.2	½	5944.7	43099	½	16972	207903
¼	1847.5	7466.7	44.	6082.1	44602	74.	17204	212175
½	1885.8	7700.1	½	6221.2	46141	½	17437	216505
¾	1924.4	7938.3	45.	6361.7	47713	75.	17672	220894
25.	1963.5	8181.3	½	6503.9	49321	½	17908	225341
¼	2002.9	8429.2	46.	6647.6	50965	76.	18146	229848
½	2042.8	8682.0	½	6792.9	52645	½	18386	234414
¾	2083.0	8939.9	47.	6939.9	54362	77.	18626	239041
26.	2123.7	9202.8	½	7088.3	56115	½	18869	243728
¼	2164.7	9470.8	48.	7238.3	57906	78.	19114	248475
½	2206.2	9744.0	½	7389.9	59734	½	19360	253284
¾	2248.0	10022	49.	7543.1	61601	79.	19607	258155
27.	2290.2	10306	½	7697.7	63506	½	19856	263088
¼	2332.8	10595	50.	7854.0	65450	80.	20106	268083
½	2375.8	10889	½	8011.8	67433	½	20358	273141
¾	2419.2	11189	51.	8171.2	69456	81.	20612	278263
28.	2463.0	11494	½	8332.3	71519	½	20867	283447
¼	2507.2	11805	52.	8494.8	73622	82.	21124	288696
½	2551.8	12121	½	8658.9	75767	½	21382	294010
¾	2596.7	12443	53.	8824.8	77952	83.	21642	299388
29.	2642.1	12770	½	8992.0	80178	½	21904	304831
¼	2687.8	13103	54.	9160.8	82448	84.	22167	310340
½	2734.0	13442	½	9331.2	84760	½	22432	315915
¾	2780.5	13787	55.	9503.2	87114	85.	22698	321556
30.	2827.4	14137	½	9676.8	89511	½	22966	327264
¼	2874.8	14494	56.	9852.0	91953	86.	23235	333039
½	2922.5	14856	½	10029	94438	½	23506	338882
¾	2970.6	15224	57.	10207	96967	87.	23779	344792
31.	3019.1	15599	½	10387	99541	½	24053	350771
¼	3068.0	15979	58.	10568	102161	88.	24328	356819
½	3117.3	16366	½	10751	104826	½	24606	362935
¾	3166.9	16758	59.	10936	107536	89.	24885	369122
32.	3217.0	17157	½	11122	110294	½	25165	375378
¼	3267.4	17563	60.	11310	113098	90.	25447	381704
½	3318.3	17974	½	11499	115949	½	25730	388102
¾	3369.6	18392	61.	11690	118847	91.	26016	394570
33.	3421.2	18817	½	11882	121794	½	26302	401109
¼	3473.3	19248	62.	12076	124789	92.	26590	407721
½	3525.7	19685	½	12272	127832	½	26880	414405
¾	3578.5	20129	63.	12469	130925	93.	27172	421161
34.	3631.7	20580	½	12668	134067	½	27464	427991
¼	3685.3	21037	64.	12868	137259	94.	27759	434894
½	3739.3	21501	½	13070	140501	½	28055	441871
35.	3848.5	22449	65.	13273	143794	95.	28353	448920
½	3959.2	23425	½	13478	147138	½	28652	456047
36.	4071.5	24429	66.	13685	150533	96.	28953	463248
½	4185.5	25461	½	13893	153980	½	29255	470524
37.	4300.9	26522	67.	14103	157480	97.	29559	477874
½	4417.9	27612	½	14314	161032	½	29865	485302
38.	4536.5	28731	68.	14527	164637	98.	30172	492808
½	4656.7	29880	½	14741	168295	½	30481	500388
39.	4778.4	31059	69.	14957	172007	99.	30791	508047
½	4901.7	32270	½	15175	175774	½	31103	515785
40.	5026.5	33510	70.	15394	179595	100.	31416	523598

CONTENTS IN CUBIC FEET AND U. S. GALLONS OF PIPES AND CYLINDERS OF VARIOUS DIAMETERS AND ONE FOOT IN LENGTH.

1 gallon = 231 cubic inches. 1 cubic foot = 7.4805 gallons.

Diameter in Inches.	For 1 Foot in Length.		Diameter in Inches.	For 1 Foot in Length.		Diameter in Inches.	For 1 Foot in Length.	
	Cubic Ft. also Area in Sq. Ft.	U. S. Gals., 231 Cu. In.		Cubic Ft. also Area in Sq. Ft.	U. S. Gals., 231 Cu. In.		Cubic Ft. also Area in Sq. Ft.	U. S. Gals., 231 Cu. In.
1/4	.0003	.0025	6 3/4	.2485	1.859	19	1.969	14.73
5-16	.0005	.004	7	.2673	1.999	19 1/2	2.074	15.51
3/8	.0008	.0057	7 1/4	.2867	2.145	20	2.182	16.32
7-16	.001	.0078	7 1/2	.3068	2.295	20 1/2	2.292	17.15
1/2	.0014	.0102	7 3/4	.3276	2.45	21	2.405	17.99
9-16	.0017	.0129	8	.3491	2.611	21 1/2	2.521	18.86
5/8	.0021	.0159	8 1/4	.3712	2.777	22	2.640	19.75
11-16	.0026	.0193	8 1/2	.3941	2.948	22 1/2	2.761	20.66
3/4	.0031	.0230	8 3/4	.4176	3.125	23	2.885	21.58
13-16	.0036	.0269	9	.4418	3.305	23 1/2	3.012	22.53
7/8	.0042	.0312	9 1/4	.4667	3.491	24	3.142	23.50
15-16	.0048	.0359	9 1/2	.4922	3.682	25	3.409	25.50
1	.0055	.0408	9 3/4	.5185	3.879	26	3.687	27.58
1 1/4	.0085	.0638	10	.5454	4.08	27	3.976	29.74
1 1/2	.0123	.0918	10 1/4	.5730	4.286	28	4.276	31.99
1 3/4	.0167	.1249	10 1/2	.6013	4.498	29	4.587	34.31
2	.0218	.1632	10 3/4	.6303	4.715	30	4.909	36.72
2 1/4	.0276	.2066	11	.66	4.937	31	5.241	39.21
2 1/2	.0341	.2550	11 1/4	.6903	5.164	32	5.585	41.78
2 3/4	.0412	.3085	11 1/2	.7213	5.396	33	5.940	44.43
3	.0491	.3672	11 3/4	.7530	5.633	34	6.305	47.16
3 1/4	.0576	.4309	12	.7854	5.875	35	6.681	49.98
3 1/2	.0668	.4998	12 1/2	.8522	6.375	36	7.069	52.88
3 3/4	.0767	.5738	13	.9218	6.895	37	7.467	55.86
4	.0873	.6528	13 1/2	.994	7.436	38	7.876	58.92
4 1/4	.0985	.7369	14	1.069	7.997	39	8.296	62.06
4 1/2	.1104	.8263	14 1/2	1.147	8.578	40	8.727	65.28
4 3/4	.1231	.9206	15	1.227	9.180	41	9.168	68.58
5	.1364	1.020	15 1/2	1.310	9.801	42	9.621	71.97
5 1/4	.1503	1.125	16	1.396	10.44	43	10.085	75.44
5 1/2	.1650	1.234	16 1/2	1.485	11.11	44	10.559	78.99
5 3/4	.1803	1.349	17	1.576	11.79	45	11.045	82.62
6	.1963	1.469	17 1/2	1.670	12.49	46	11.541	86.33
6 1/4	.2131	1.594	18	1.768	13.22	47	12.048	90.13
6 1/2	.2304	1.724	18 1/2	1.867	13.96	48	12.566	94.00

To find the capacity of pipes greater than the largest given in the table, look in the table for a pipe of one half the given size, and multiply its capacity by 4; or one of one third its size, and multiply its capacity by 9, etc.

To find the *weight* of water in any of the given sizes multiply the capacity in cubic feet by 62 1/4 or the gallons by 8 1/3, or, if a closer approximation is required, by the weight of a cubic foot of water at the actual temperature in the pipe.

Given the dimensions of a cylinder in inches, to find its capacity in U. S. gallons: Square the diameter, multiply by the length and by .0034. If d = diameter, l = length, gallons $= \frac{d^2 \times .7854 \times l}{231} = .0034d^2l$.

CYLINDRICAL VESSELS, TANKS, CISTERNS, ETC.

Diameter in Feet and Inches, Area in Square Feet, and U. S. Gallons Capacity for One Foot in Depth.

$$1 \text{ gallon} = 231 \text{ cubic inches} = \frac{1 \text{ cubic foot}}{7.4805} = 0.13368 \text{ cubic feet.}$$

Diam. Ft. In.	Area. Sq. ft.	Gals. 1 foot depth.	Diam. Ft. In.	Area. Sq. ft.	Gals. 1 foot depth.	Diam. Ft. In.	Area. Sq. ft.	Gals. 1 foot depth.
1	.785	5.87	5 8	25.22	188.66	19	283.53	2120.9
1 1	.922	6.89	5 9	25.97	194.25	19 3	291.04	2177.1
1 2	1.069	8.00	5 10	26.73	199.92	19 6	298.65	2234.0
1 3	1.227	9.18	5 11	27.49	205.67	19 9	306.35	2291.7
1 4	1.396	10.44	6	28.27	211.51	20	314.16	2350.1
1 5	1.576	11.79	6 3	30.68	229.50	20 3	322.06	2409.2
1 6	1.767	13.22	6 6	33.18	248.23	20 6	330.06	2469.1
1 7	1.969	14.73	6 9	35.78	267.69	20 9	338.16	2529.6
1 8	2.182	16.32	7	38.48	287.88	21	346.36	2591.0
1 9	2.405	17.99	7 3	41.28	308.81	21 3	354.66	2653.0
1 10	2.640	19.75	7 6	44.18	330.48	21 6	363.05	2715.8
1 11	2.885	21.58	7 9	47.17	352.88	21 9	371.54	2779.3
2	3.142	23.50	8	50.27	376.01	22	380.13	2843.6
2 1	3.409	25.50	8 3	53.46	399.88	22 3	388.82	2908.6
2 2	3.687	27.58	8 6	56.75	424.48	22 6	397.61	2974.3
2 3	3.976	29.74	8 9	60.13	449.82	22 9	406.49	3040.8
2 4	4.276	31.99	9	63.62	475.89	23	415.48	3108.0
2 5	4.587	34.31	9 3	67.20	502.70	23 3	424.56	3175.9
2 6	4.909	36.72	9 6	70.88	530.24	23 6	433.74	3244.6
2 7	5.241	39.21	9 9	74.66	558.51	23 9	443.01	3314.0
2 8	5.585	41.78	10	78.54	587.52	24	452.39	3384.1
2 9	5.940	44.43	10 3	82.52	617.26	24 3	461.86	3455.0
2 10	6.305	47.16	10 6	86.59	647.74	24 6	471.44	3526.6
2 11	6.681	49.98	10 9	90.76	678.95	24 9	481.11	3598.9
3	7.069	52.88	11	95.03	710.90	25	490.87	3672.0
3 1	7.467	55.86	11 3	99.40	743.58	25 3	500.74	3745.8
3 2	7.876	58.92	11 6	103.87	776.99	25 6	510.71	3820.3
3 3	8.296	62.06	11 9	108.43	811.14	25 9	520.77	3895.6
3 4	8.727	65.28	12	113.10	846.03	26	530.93	3971.6
3 5	9.168	68.58	12 3	117.86	881.65	26 3	541.19	4048.4
3 6	9.621	71.97	12 6	122.72	918.00	26 6	551.55	4125.9
3 7	10.085	75.44	12 9	127.68	955.09	26 9	562.00	4204.1
3 8	10.559	78.99	13	132.73	992.91	27	572.56	4283.0
3 9	11.045	82.62	13 3	137.89	1031.5	27 3	583.21	4362.7
3 10	11.541	86.33	13 6	143.14	1070.8	27 6	593.96	4443.1
3 11	12.048	90.13	13 9	148.49	1110.8	27 9	604.81	4524.3
4	12.566	94.00	14	153.94	1151.5	28	615.75	4606.2
4 1	13.095	97.96	14 3	159.48	1193.0	28 3	626.80	4688.8
4 2	13.635	102.00	14 6	165.13	1235.3	28 6	637.94	4772.1
4 3	14.186	106.12	14 9	170.87	1278.2	28 9	649.18	4856.2
4 4	14.748	110.32	15	176.71	1321.9	29	660.52	4941.0
4 5	15.321	114.61	15 3	182.65	1366.4	29 3	671.96	5026.6
4 6	15.90	118.97	15 6	188.69	1411.5	29 6	683.49	5112.9
4 7	16.50	123.42	15 9	194.83	1457.4	29 9	695.13	5199.9
4 8	17.10	127.95	16	201.06	1504.1	30	706.86	5287.7
4 9	17.72	132.56	16 3	207.39	1551.4	30 3	718.69	5376.2
4 10	18.35	137.25	16 6	213.82	1599.5	30 6	730.62	5465.4
4 11	18.99	142.02	16 9	220.35	1648.4	30 9	742.64	5555.4
5	19.63	146.88	17	226.98	1697.9	31	754.77	5646.1
5 1	20.29	151.82	17 3	233.71	1748.2	31 3	766.99	5737.5
5 2	20.97	156.83	17 6	240.53	1799.3	31 6	779.31	5829.7
5 3	21.65	161.93	17 9	247.45	1851.1	31 9	791.73	5922.6
5 4	22.34	167.12	18	254.47	1903.6	32	804.25	6016.2
5 5	23.04	172.38	18 3	261.59	1956.8	32 3	816.86	6110.6
5 6	23.76	177.72	18 6	268.80	2010.8	32 6	829.58	6205.7
5 7	24.48	183.15	18 9	276.12	2065.5	32 9	842.39	6301.5

GALLONS AND CUBIC FEET.

United States Gallons in a given Number of Cubic Feet.

1 cubic foot = 7.480519 U. S. gallons; 1 gallon = 231 cu. in. = .13368056 cu. ft.

Cubic Ft.	Gallons.	Cubic Ft.	Gallons.	Cubic Ft.	Gallons.
0.1	0.75	50	374.0	8,000	59,844.2
0.2	1.50	60	448.8	9,000	67,324.7
0.3	2.24	70	523.6	10,000	74,805.2
0.4	2.99	80	598.4	20,000	149,610.4
0.5	3.74	90	673.2	30,000	224,415.6
0.6	4.49	100	748.0	40,000	299,220.8
0.7	5.24	200	1,496.1	50,000	374,025.9
0.8	5.98	300	2,244.2	60,000	448,831.1
0.9	6.73	400	2,992.2	70,000	523,636.3
1	7.48	500	3,740.3	80,000	598,441.5
2	14.96	600	4,488.3	90,000	673,246.7
3	22.44	700	5,236.4	100,000	748,051.9
4	29.92	800	5,984.4	200,000	1,496,103.8
5	37.40	900	6,732.5	300,000	2,244,155.7
6	44.88	1,000	7,480.5	400,000	2,992,207.6
7	52.36	2,000	14,961.0	500,000	3,740,259.5
8	59.84	3,000	22,441.6	600,000	4,488,311.4
9	67.32	4,000	29,922.1	700,000	5,236,363.3
10	74.80	5,000	37,402.6	800,000	5,984,415.2
20	149.6	6,000	44,883.1	900,000	6,732,467.1
30	224.4	7,000	52,363.6	1,000,000	7,480,519.0
40	299.2				

Cubic Feet in a given Number of Gallons.

Gallons.	Cubic Ft.	Gallons.	Cubic Ft.	Gallons.	Cubic Ft.
1	.134	1,000	133.681	1,000,000	133,680.6
2	.267	2,000	267.361	2,000,000	267,361.1
3	.401	3,000	401.042	3,000,000	401,041.7
4	.535	4,000	534.722	4,000,000	534,722.2
5	.668	5,000	668.403	5,000,000	668,402.8
6	.802	6,000	802.083	6,000,000	802,083.3
7	.936	7,000	935.764	7,000,000	935,763.9
8	1.069	8,000	1,069.444	8,000,000	1,069,444.4
9	1.203	9,000	1,203.125	9,000,000	1,203,125.0
10	1.337	10,000	1,336.806	10,000,000	1,336,805.6

NUMBER OF SQUARE FEET IN PLATES 3 TO 32 FEET LONG, AND 1 INCH WIDE.

For other widths, multiply by the width in inches. 1 sq. in. = $.0069\frac{4}{9}$ sq. ft.

Ft. and In. Long.	Ins. Long.	Square Feet.	Ft. and Ins. Long.	Ins. Long.	Square Feet.	Ft. and Ins. Long.	Ins. Long.	Square Feet.
3. 0	36	.25	7.10	94	.6528	12. 8	152	1.056
1	37	.2569	11	95	.6597	9	153	1.063
2	38	.2639	8. 0	96	.6667	10	154	1.069
3	39	.2708	1	97	.6736	11	155	1.076
4	40	.2778	2	98	.6806	13. 0	156	1.083
5	41	.2847	3	99	.6875	1	157	1.09
6	42	.2917	4	100	.6944	2	158	1.097
7	43	.2986	5	101	.7014	3	159	1.104
8	44	.3056	6	102	.7083	4	160	1.114
9	45	.3125	7	103	.7153	5	161	1.118
10	46	.3194	8	104	.7222	6	162	1.125
11	47	.3264	9	105	.7292	7	163	1.132
4. 0	48	.3333	10	106	.7361	8	164	1.139
1	49	.3403	11	107	.7431	9	165	1.146
2	50	.3472	9. 0	108	.75	10	166	1.153
3	51	.3542	1	109	.7569	11	167	1.159
4	52	.3611	2	110	.7639	14. 0	168	1.167
5	53	.3681	3	111	.7708	1	169	1.174
6	54	.375	4	112	.7778	2	170	1.181
7	55	.3819	5	113	.7847	3	171	1.188
8	56	.3889	6	114	.7917	4	172	1.194
9	57	.3958	7	115	.7986	5	173	1.201
10	58	.4028	8	116	.8056	6	174	1.208
11	59	.4097	9	117	.8125	7	175	1.215
5. 0	60	.4167	10	118	.8194	8	176	1.222
1	61	.4236	11	119	.8264	9	177	1.229
2	62	.4306	10. 0	120	.8333	10	178	1.236
3	63	.4375	1	121	.8403	11	179	1.243
4	64	.4444	2	122	.8472	15. 0	180	1.25
5	65	.4514	3	123	.8542	1	181	1.257
6	66	.4583	4	124	.8611	2	182	1.264
7	67	.4653	5	125	.8681	3	183	1.271
8	68	.4722	6	126	.875	4	184	1.278
9	69	.4792	7	127	.8819	5	185	1.285
10	70	.4861	8	128	.8889	6	186	1.292
11	71	.4931	9	129	.8958	7	187	1.299
6. 0	72	.5	10	130	.9028	8	188	1.306
1	73	.5069	11	131	.9097	9	189	1.313
2	74	.5139	11. 0	132	.9167	10	190	1.319
3	75	.5208	1	133	.9236	11	191	1.326
4	76	.5278	2	134	.9306	16. 0	192	1.333
5	77	.5347	3	135	.9375	1	193	1.34
6	78	.5417	4	136	.9444	2	194	1.347
7	79	.5486	5	137	.9514	3	195	1.354
8	80	.5556	6	138	.9583	4	196	1 361
9	81	.5625	7	139	.9653	5	197	1.368
10	82	.5694	8	140	.9722	6	198	1.375
11	83	.5764	9	141	.9792	7	199	1.382
7. 0	84	.5834	10	142	.9861	8	200	1.389
1	85	.5903	11	143	.9931	9	201	1.396
2	86	.5972	12. 0	144	1.000	10	202	1.403
3	87	.6042	1	145	1.007	11	203	1.41
4	88	.6111	2	146	1.014	17. 0	204	1.417
5	89	.6181	3	147	1.021	1	205	1.424
6	90	.625	4	148	1.028	2	206	1.431
7	91	.6319	5	149	1.035	3	207	1.438
8	92	.6389	6	150	1.042	4	208	1.444
9	93	.6458	7	151	1.049	5	209	1.451

SQUARE FEET IN PLATES—(*Continued.*)

Ft. and Ins. Long.	Ins. Long.	Square Feet.	Ft. and Ins. Long.	Ins. Long.	Square Feet.	Ft. and Ins. Long.	Ins. Long.	Square Feet.
17. 0	210	1.458	**22.** 5	269	1.868	**27.** 4	328	2.278
7	211	1.465	6	270	1.875	5	329	2.285
8	212	1.472	7	271	1.882	6	330	2.292
9	213	1.479	8	272	1.889	7	331	2.299
10	214	1.486	9	273	1.896	8	332	2.306
11	215	1.493	10	274	1.903	9	333	2.313
18. 0	216	1.5	11	275	1.91	10	334	2.319
1	217	1.507	**23.** 0	276	1.917	11	335	2.326
2	218	1.514	1	277	1.924	**28.** 0	336	2.333
3	219	1.521	2	278	1.931	1	337	2.34
4	220	1.528	3	279	1.938	2	338	2.347
5	221	1.535	4	280	1.944	3	339	2.354
6	222	1.542	5	281	1.951	4	340	2.361
7	223	1.549	6	282	1.958	5	341	2.368
8	224	1.556	7	283	1.965	6	342	2.375
9	225	1.563	8	284	1.972	7	343	2.382
0	226	1.569	9	285	1.979	8	344	2.389
11	227	1.576	10	286	1.986	9	345	2.396
19. 0	228	1.583	11	287	1.993	10	346	2.403
1	229	1.59	**24.** 0	288	2.	11	347	2.41
2	230	1.597	1	289	2.007	**29.** 0	348	2.417
3	231	1.604	2	290	2.014	1	349	2.424
4	232	1.611	3	291	2.021	2	350	2.431
5	233	1.618	4	292	2.028	3	351	2.438
6	234	1.625	5	293	2.035	4	352	2.444
7	235	1.632	6	294	2.042	5	353	2.451
8	236	1.639	7	295	2.049	6	354	2.458
9	237	1.645	8	296	2.056	7	355	2.465
10	238	1.653	9	297	2.063	8	356	2.472
11	239	1.659	10	298	2.069	9	357	2.479
20. 0	240	1.667	11	299	2.076	10	358	2.486
1	241	1.674	**25.** 0	300	2.083	11	359	2.493
2	242	1.681	1	301	2.09	**30.** 0	360	2.5
3	243	1.688	2	302	2.097	1	361	2.507
4	244	1.694	3	303	2.104	2	362	2.514
5	245	1.701	4	304	2.111	3	363	2.521
6	246	1.708	5	305	2.118	4	364	2.528
7	247	1.715	6	306	2.125	5	365	2.535
8	248	1.722	7	307	2.132	6	366	2.542
9	249	1.729	8	308	2.139	7	367	2.549
10	250	1.736	9	309	2.146	8	368	2.556
11	251	1.743	10	310	2.153	9	369	2.563
21. 0	252	1.75	11	311	2.16	10	370	2.569
1	253	1.757	**26.** 0	312	2.167	11	371	2.576
2	254	1.764	1	313	2.174	**31.** 0	372	2.583
3	255	1.771	2	314	2.181	1	373	2.59
4	256	1.778	3	315	2.188	2	374	2.597
5	257	1.785	4	316	2.194	3	375	2.604
6	258	1.792	5	317	2.201	4	376	2.611
7	259	1.799	6	318	2.208	5	377	2.618
8	260	1.806	7	319	2.215	6	378	2.625
9	261	1.813	8	320	2.222	7	379	2.632
10	262	1.819	9	321	2.229	8	380	2.639
11	263	1.826	10	322	2.236	9	381	2.646
22. 0	264	1.833	11	323	2.243	10	382	2.653
1	265	1.84	**27.** 0	324	2.25	11	383	2.66
2	266	1.847	1	325	2.257	**32.** 0	384	2.667
3	267	1.854	2	326	2.264	1	385	2.674
4	268	1.861	3	327	2.271	2	386	2.681

CAPACITIES OF RECTANGULAR TANKS IN U. S. GALLONS, FOR EACH FOOT IN DEPTH.

1 cubic foot = 7.4805 U. S. gallons.

Width of Tank. ft. in.	Length of Tank. feet. 2	ft. in. 2 6	feet. 3	ft. in. 3 6	feet. 4	ft. in. 4 6	feet. 5	ft. in. 5 6	feet. 6	ft. in. 6 6	feet. 7
2	29.92	37.40	44.88	52.36	59.84	67.32	74.81	82.29	89.77	97.25	104.73
2 6	......	46.75	56.10	65.45	74.80	84.16	93.51	102.86	112.21	121.56	130.91
3	......	......	67.32	78.54	89.77	100.99	112.21	123.43	134.65	145.87	157.09
3 6	......	......	......	91.64	104.73	117.82	130.91	144.00	157.09	170.18	183.27
4	......	......	......	......	119.69	134.65	149.61	164.57	179.53	194.49	209.45
4 6	......	......	......	......	......	151.48	168.31	185.14	201.97	218.80	235.63
5	......	......	......	......	......	......	187.01	205.71	224.41	243.11	261.82
5 6	......	......	......	......	......	......	......	226.28	246.86	267.43	288.00
6	......	......	......	......	......	......	......	......	269.30	291.74	314.18
6 6	......	......	......	......	......	......	......	......	......	316.05	340.36
7	......	......	......	......	......	......	......	......	......	......	366.54

Width of Tank. ft. in.	Length of Tank. ft. in. 7 6	feet. 8	ft. in. 8 6	feet. 9	ft. in. 9 6	feet. 10	ft. in. 10 6	feet. 11	ft. in. 11 6	feet. 12
2	112.21	119.69	127.17	134.65	142.13	149.61	157.09	164.57	172.05	179.53
2 6	140.26	149.61	158.96	168.31	177.66	187.01	196.36	205.71	215.06	224.41
3	168.31	179.53	190.75	202.97	213.19	224.41	235.63	246.86	258.07	269.30
3 6	196.36	209.45	222.54	235.63	248.73	261.82	274.90	288.00	301.09	314.18
4	224.41	239.37	254.34	269.30	284.26	299.22	314.18	329.14	344.10	359.06
4 6	252.47	269.30	286.13	302.96	319.79	336.62	353.45	370.28	387.11	403.94
5	280.52	299.22	317.92	336.62	355.32	374.03	392.72	411.43	430.13	448.83
5 6	308.57	329.14	349.71	370.28	390.85	411.43	432.00	452.57	473.14	493.71
6	336.62	359.06	381.50	403.94	426.39	448.83	471.27	493.71	516.15	538.59
6 6	364.67	388.98	413.30	437.60	461.92	486.23	510.54	534.85	559.16	583.47
7	392.72	418.91	445.09	471.27	497.45	523.64	549.81	575.99	602.18	628.36
7 6	420.78	448.83	476.88	504.93	532.98	561.04	589.08	617.14	645.19	673.24
8	......	478.75	508.67	538.59	568.51	598.44	628.36	658.28	688.20	718.12
8 6	......	......	540.46	572.25	604.05	635.84	667.63	699.42	731.21	763.00
9	......	......	......	605.92	639.58	673.25	706.90	740.56	774.23	807.89
9 6	......	......	......	......	675.11	710.65	746.17	781.71	817.24	852.77
10	......	......	......	......	......	748.05	785.45	822.86	860.26	897.66
10 6	......	......	......	......	......	......	824.73	864.00	903.26	942.56
11	......	......	......	......	......	......	......	905.14	946.27	987.43
11 6	......	......	......	......	......	......	......	......	989.29	1032.3
12	......	......	......	......	......	......	......	......	......	1077.2

NUMBER OF BARRELS (31 1-2 GALLONS) IN CISTERNS AND TANKS.

1 Barrel = 31½ gallons = $\frac{31.5 \times 231}{1728}$ = 4.21094 cubic feet. Reciprocal = .237477.

Depth in Feet.	Diameter in Feet.									
	5	6	7	8	9	10	11	12	13	14
1	4.663	6.714	9.139	11.937	15.108	18.652	22.569	26.859	31.522	36.557
5	23.3	33.6	45.7	59.7	75.5	93.3	112.8	134.3	157.6	182.8
6	28.0	40.3	54.8	71.6	90.6	111.9	135.4	161.2	189.1	219 3
7	32.6	47.0	64.0	83.6	105.8	130.6	158.0	188.0	220.7	255.9
8	37.3	53.7	73.1	95.5	120.9	149.2	180.6	214.9	252.2	292.5
9	42.0	60.4	82.3	107.4	136.0	167.9	203.1	241.7	283.7	329.0
10	46.6	67.1	91.4	119.4	151.1	186.5	225.7	268.6	315.2	365.6
11	51.3	73.9	100.5	131.3	166.2	205.2	248.3	295.4	346.7	402.1
12	56.0	80.6	109.7	143.2	181.3	223.8	270.8	322.3	378.3	438.7
13	60.6	87.3	118.8	155.2	196.4	242.5	293.4	349.2	409.8	475.2
14	65.3	94.0	127.9	167.1	211.5	261.1	316.0	376.0	441.3	511.8
15	69.9	100.7	137.1	179.1	226.6	279.8	338.5	402.9	472.8	548.4
16	74.6	107.4	146.2	191.0	241.7	298.4	361.1	429.7	504.4	584.9
17	79.3	114.1	155.4	202.9	256.8	317.1	383.7	456.6	535.9	621.5
18	83.9	120.9	164.5	214.9	271.9	335.7	406.2	483.5	567.4	658.0
19	88.6	127.6	173.6	226.8	287.1	354.4	428.8	510.3	598.9	694.6
20	93 3	134.3	182.8	238.7	302.2	373.0	451.4	537.2	630.4	731.1

Depth in Feet.	Diameter in Feet.							
	15	16	17	18	19	20	21	22
1	41.966	47.748	53.903	60.431	67.332	74.606	82.253	90.273
5	209.8	238.7	269.5	302.2	336.7	373.0	411.3	451.4
6	251.8	286.5	323.4	362 6	404.0	447.6	493.5	541.6
7	293.8	334.2	377.3	423.0	471.3	522.2	575.8	631.9
8	335.7	382.0	431.2	483.4	538.7	596.8	658.0	722.2
9	377.7	429.7	485 1	543.9	606.0	671.5	740.3	812.5
10	419.7	477.5	539.0	604.3	673.3	746.1	822.5	902.7
11	461.6	525.2	592.9	664.7	740.7	820.7	904.8	993.0
12	503.6	573.0	646.8	725.2	808.0	895.3	987.0	1083.3
13	545.6	620.7	700.7	785.6	875.3	969.9	1069.3	1173.5
14	587.5	668.5	754.6	846.0	942.6	1044.5	1151.5	1263.8
15	629.5	716.2	808.5	906.5	1010.0	1119.1	1233.8	1354.1
16	671.5	764.0	862.4	966.9	1077.3	1193.7	1316.0	1444.4
17	713.4	811.7	916.4	1027.3	1144.6	1268.3	1398.3	1534.5
18	755.4	859.5	970.3	1087.8	1212.0	1342.9	1480.6	1624.9
19	797.4	907.2	1024.2	1148.2	1279.3	1417.5	1562.8	1715.2
20	839.3	955.0	1078.1	1208.6	1346.6	1492.1	1645.1	1805.5

NUMBER OF BARRELS (31 1-2 GALLONS) IN CISTERNS AND TANKS.—*Continued.*

Depth in Feet.	Diameter in Feet.							
	23	24	25	26	27	28	29	30
1	98.666	107.432	116.571	126.083	135.968	146.226	157.858	167.862
5	493.3	537.2	582.9	630.4	679.8	731.1	784.3	839.3
6	592.0	644.6	699.4	756.5	815.8	877.4	941.1	1007.2
7	690.7	752.0	816.0	882.6	951.8	1023.6	1098.0	1175.0
8	789.3	859.5	932.6	1008.7	1087.7	1169.8	1254.9	1342.9
9	888.0	966.9	1049.1	1134.7	1223.7	1316.0	1411.7	1510.8
10	986.7	1074.3	1165.7	1260.8	1359.7	1462.2	1568.6	1678.6
11	1085.3	1181.8	1282.3	1386.9	1495.6	1608.5	1725.4	1846.5
12	1184.0	1289.2	1398.8	1513.0	1631.6	1754.7	1882.3	2014.4
13	1282.7	1396.6	1515.4	1639.1	1767.6	1900.9	2039.2	2182.3
14	1381.3	1504.0	1632.0	1765.2	1903.6	2047.2	2196.0	2350.1
15	1480.0	1611.5	1748.6	1891.2	2039.5	2193.4	2352.9	2517.9
16	1578.7	1718.9	1865.1	2017.3	2175.5	2339.6	2509.7	2685.8
17	1677.3	1826.3	1981.7	2143.4	2311.5	2485.8	2666.6	2853.7
18	1776.0	1933.8	2098.3	2269.5	2447.4	2632.0	2823.4	3021.5
19	1874.7	2041.2	2214.8	2395.6	2583.4	2778.3	2980.3	3189.4
20	1973.3	2148.6	2321.4	2521.7	2719.4	2924.5	3137.2	3357.3

LOGARITHMS.

Logarithms (abbreviation *log*).—The log of a number is the exponent of the power to which it is necessary to raise a fixed number to produce the given number. The fixed number is called the *base*. Thus if the base is 10, the log of 1000 is 3, for $10^3 = 1000$. There are two systems of logs in general use, the *common*, in which the base is 10, and the Naperian, or *hyperbolic*, in which the base is 2.718281828 .... The Naperian base is commonly denoted by e, as in the equation $e^y = x$, in which y is the Nap. log of x.

In any system of logs, the log of 1 is 0; the log of the base, taken in that system, is 1. In any system the base of which is greater than 1, the logs of all numbers greater than 1 are positive and the logs of all numbers less than 1 are negative.

The modulus of any system is equal to the reciprocal of the Naperian log of the base of that system. The modulus of the Naperian system is 1, that of the common system is .4342945.

The log of a number in any system equals the modulus of that system × the Naperian log of the number.

The *hyperbolic* or *Naperian* log of any number equals the common log × 2.3025851.

Every log consists of two parts, an entire part called the *characteristic*, or index, and the decimal part, or *mantissa*. The mantissa only is given in the usual tables of common logs, with the decimal point omitted. The characteristic is found by a simple rule, viz., it is one less than the number of figures to the left of the decimal point in the number whose log is to be found. Thus the characteristic of numbers from 1 to 9.99 + is 0, from 10 to 99.99 + is 1, from 100 to 999 + is 2, from .1 to .99 + is − 1, from .01 to .099 + is − 2, etc. Thus

log of 2000 is 3.30103;		log of .2 is − 1.30103;	
" " 200 " 2.30103;		" " .02 " − 2.30103;	
" " 20 " 1.30103;		" " .002 " − 3.30103;	
" " 2 " 0.30103;		" " .0002 " − 4.30103.	

The minus sign is frequently written above the characteristic thus: log .002 = $\bar{3}$.30103. The characteristic only is negative, the decimal part, or mantissa, being always positive.

When a log consists of a negative index and a positive mantissa, it is usual to write the negative sign over the index, or else to add 10 to the index, and to indicate the subtraction of 10 from the resulting logarithm.

Thus log .2 = $\bar{1}$.30103, and this may be written 9.30103 − 10.

In tables of logarithmic sines, etc., the − 10 is generally omitted, as being understood.

Rules for use of the table of Logarithms.- To find the log of any whole number.—For 1 to 100 inclusive the log is given complete in the small table on page 129.

For 100 to 999 inclusive the decimal part of the log is given opposite the given number in the column headed 0 in the table (including the two figures to the left, making six figures). Prefix the characteristic, or index, 2.

For 1000 to 9999 inclusive: The last four figures of the log are found opposite the first three figures of the given number and in the vertical column headed with the fourth figure of the given number; prefix the two figures under column 0, and the index, which is 3.

For numbers over 10,000 having five or more digits: Find the decimal part of the log for the first four digits as above, multiply the difference figure in the last column by the remaining digit or digits, and divide by 10 if there be only one digit more, by 100 if there be two more, and so on; add the quotient to the log of the first four digits and prefix the index, which is 4 if there are five digits, 5 if there are six digits, and so on. The table of proportional parts may be used, as shown below.

To find the log of a decimal fraction or of a whole number and a decimal.—First find the log of the quantity as if there were no decimal point, then prefix the index according to rule; the index is one less than the number of figures to the left of the decimal point.

Required log of 3.141593.

	log of	3.141	= 0.497068.	Diff. = 138
From proportional parts		5	= 690	
" " "	..	09	= 1242	
" " "		003	= 041	
	log	3.141593	0.4971498	

To find the number corresponding to a given log.—Find in the table the log nearest to the decimal part of the given log and take the first four digits of the required number from the column N and the top or foot of the column containing the log which is the next less than the given log. To find the 5th and 6th digits subtract the log in the table from the given log, multiply the difference by 100, and divide by the figure in the Diff. column opposite the log; annex the quotient to the four digits already found, and place the decimal point according to the rule; the number of figures to the left of the decimal point is one greater than the index.

Find number corresponding to the log..........	0.497150
Next lowest log in table corresponds to 3141......	.497068
Diff. =	82

Tabular diff. = 138; 82 ÷ 138 = .59 +

The index being 0, the number is therefore 3.14159 +.

To multiply two numbers by the use of logarithms. Add together the logs of the two numbers, and find the number whose log is the sum.

To divide two numbers.—Subtract the log of the divisor from the log of the dividend, and find the number whose log is the difference.

To raise a number to any given power.—Multiply the log of the number by the exponent of the power, and find the number whose log is the product.

To find any root of a given number.—Divide the log of the number by the index of the root. The quotient is the log of the root.

To find the reciprocal of a number.—Subtract the decimal part of the log of the number from 0, add 1 to the index and change the sign of the index. The result is the log of the reciprocal.

Required the reciprocal of 3.141593.

Log of 3.141593, as found above	0.4971498
Subtract decimal part from 0 gives	0.5028502
Add 1 to the index, and changing sign of the index gives	$\bar{1}$.5028502

which is the log of 0.31831.

To find the fourth term of a proportion by logarithms.—Add the logarithms of the second and third terms, and from their sum subtract the logarithm of the first term.

When one logarithm is to be subtracted from another, it may be more convenient to convert the subtraction into an addition, which may be done by first subtracting the given logarithm from 10, adding the difference to the other logarithm, and afterwards rejecting the 10.

The difference between a given logarithm and 10 is called its *arithmetical complement*, or *cologarithm*.

To subtract one logarithm from another is the same as to add its complement and then reject 10 from the result. For $a - b = 10 - b + a - 10$.

To work a proportion, then, by logarithms, add the complement of the logarithm of the first term to the logarithms of the second and third terms. The characteristic must afterwards be diminished by 10.

Example in logarithms with a negative index.—Solve by logarithms $\left(\frac{526}{1011}\right)^{2.45}$, which means divide 526 by 1011 and raise the quotient to the 2.45 power.

$$
\begin{array}{rr}
\log 526 = & 2.720986 \\
\log 1011 = & 3.004751 \\
\hline
\text{log of quotient} = & -\ 1.716235 \\
\text{Multiply by} & 2.45 \\
\hline
 & -\ 2.581175 \\
 & -\ 2.8\ 64940 \\
 & -\ 1.43\ 2470 \\
\hline
 & -\ 1.30\ 477575 = .20173,\ \textit{Ans.}
\end{array}
$$

In multiplying − 1.7 by 5, we say: 5 × 7 = 35, 3 to carry; 5 × − 1 = − 5 less + 3 carried = − 2. In adding − 2 + 8 + 3 + 1 carried from previous column, we say: 1 + 3 + 8 = 12, minus 2 = 10, set down 0 and carry 1; 1 + 4 − 2 = 3.

LOGARITHMS OF NUMBERS FROM 1 TO 100.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

No. 100 L. 000.] [No. 109 L. 040.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	0300	0724	1147	1570	1993	2415	424
3	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947	0361	0775	416
5	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	0195	0600	1004	1408	1812	2216	2619	3021	404
8	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	0207	0602	0998	397
	04										

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173.6	217.0	260.4	303.8	347.2	390.6
433	43.3	86.6	129.9	173.2	216.5	259.8	303.1	346.4	389.7
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
431	43.1	86.2	129.3	172.4	215.5	258.6	301.7	344.8	387.9
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
429	42.9	85.8	128.7	171.6	214.5	257.4	300.3	343.2	386.1
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
427	42.7	85.4	128.1	170.8	213.5	256.2	298.9	341.6	384.3
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
425	42.5	85.0	127.5	170.0	212.5	255.0	297.5	340.0	382.5
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
423	42.3	84.6	126.9	169.2	211.5	253.8	296.1	338.4	380.7
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
421	42.1	84.2	126.3	168.4	210.5	252.6	294.7	336.8	378.9
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
419	41.9	83.8	125.7	167.6	209.5	251.4	293.3	335.2	377.1
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
417	41.7	83.4	125.1	166.8	208.5	250.2	291.9	333.6	375.3
416	41.6	83.2	124.8	166.4	208.0	249.6	291.2	332.8	374.4
415	41.5	83.0	124.5	166.0	207.5	249.0	290.5	332.0	373.5
414	41.4	82.8	124.2	165.6	207.0	248.4	289.8	331.2	372.6
413	41.3	82.6	123.9	165.2	206.5	247.8	289.1	330.4	371.7
412	41.2	82.4	123.6	164.8	206.0	247.2	288.4	329.6	370.8
411	41.1	82.2	123.3	164.4	205.5	246.6	287.7	328.8	369.9
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
409	40.9	81.8	122.7	163.6	204.5	245.4	286.3	327.2	368.1
408	40.8	81.6	122.4	163.2	204.0	244.8	285.6	326.4	367.2
407	40.7	81.4	122.1	162.8	203.5	244.2	284.9	325.6	366.3
406	40.6	81.2	121.8	162.4	203.0	243.6	284.2	324.8	365.4
405	40.5	81.0	121.5	162.0	202.5	243.0	283.5	324.0	364.5
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
403	40.3	80.6	120.9	161.2	201.5	241.8	282.1	322.4	362.7
402	40.2	80.4	120.6	160.8	201.0	241.2	281.4	321.6	361.8
401	40.1	80.2	120.3	160.4	200.5	240.6	280.7	320.8	360.9
400	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
399	39.9	79.8	119.7	159.6	199.5	239.4	279.3	319.2	359.1
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	358.2
397	39.7	79.4	119.1	158.8	198.5	238.2	277.9	317.6	357.3
396	39.6	79.2	118.8	158.4	198.0	237.6	277.2	316.8	356.4
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5

No 110 L. 041.] [No. 119 L. 078.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	0380	0766	1153	1538	1924	2309	2694	386
3	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	0320	379
5	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	0038	0407	0776	1145	1514	370
8	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5
394	39.4	78.8	118.2	157.6	197.0	236.4	275.8	315.2	354.6
393	39.3	78.6	117.9	157.2	196.5	235.8	275.1	314.4	353.7
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
391	39.1	78.2	117.3	156.4	195.5	234.6	273.7	312.8	351.9
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
389	38.9	77.8	116.7	155.6	194.5	233.4	272.3	311.2	350.1
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
387	38.7	77.4	116.1	154.8	193.5	232.2	270.9	309.6	348.3
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
385	38.5	77.0	115.5	154.0	192.5	231.0	269.5	308.0	346.5
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
383	38.3	76.6	114.9	153.2	191.5	229.8	268.1	306.4	344.7
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
381	38.1	76.2	114.3	152.4	190.5	228.6	266.7	304.8	342.9
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
379	37.9	75.8	113.7	151.6	189.5	227.4	265.3	303.2	341.1
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
377	37.7	75.4	113.1	150.8	188.5	226.2	263.9	301.6	339.3
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
375	37.5	75.0	112.5	150.0	187.5	225.0	262.5	300.0	337.5
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
373	37.3	74.6	111.9	149.2	186.5	223.8	261.1	298.4	335.7
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
371	37.1	74.2	111.3	148.4	185.5	222.6	259.7	296.8	333.9
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
369	36.9	73.8	110.7	147.6	184.5	221.4	258.3	295.2	332.1
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
367	36.7	73.4	110.1	146.8	183.5	220.2	256.9	293.6	330.3
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
365	36.5	73.0	109.5	146.0	182.5	219.0	255.5	292.0	328.5
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
363	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
361	36.1	72.2	108.3	144.4	180.5	216.6	252.7	288.8	324.9
360	36.0	72.0	108.0	144.0	180.0	216.0	252.0	288.0	324.0
359	35.9	71.8	107.7	143.6	179.5	215.4	251.3	287.2	323.1
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
357	35.7	71.4	107.1	142.8	178.5	214.2	249.9	285.6	321.3
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4

N.	0	1	2	3	4	5	6	7	8	9	Diff.
120	079181	9543	9904	0266	0626	0987	1347	1707	2067	2426	360
1	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	0258	0611	0963	1315	1667	2018	2370	2721	3071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	0026	346
6	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	0253	338
9	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	0245	330
2	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105 13	7429	7753	8076	8399	8722	9045	9368	9690	0012	323

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
355	35.5	71.0	106.5	142.0	177.5	213.0	248.5	284.0	319.5
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
353	35.3	70.6	105.9	141.2	176.5	211.8	247.1	282.4	317.7
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
351	35.1	70.2	105.3	140.4	175.5	210.6	245.7	280.8	315.9
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
349	34.9	69.8	104.7	139.6	174.5	209.4	244.3	279.2	314.1
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
347	34.7	69.4	104.1	138.8	173.5	208.2	242.9	277.6	312.3
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
345	34.5	69.0	103.5	138.0	172.5	207.0	241.5	276.0	310.5
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
343	34.3	68.6	102.9	137.2	171.5	205.8	240.1	274.4	308.7
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
341	34.1	68.2	102.3	136.4	170.5	204.6	238.7	272.8	306.9
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
339	33.9	67.8	101.7	135.6	169.5	203.4	237.3	271.2	305.1
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
337	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
335	33.5	67.0	100.5	134.0	167.5	201.0	234.5	268.0	301.5
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
333	33.3	66.6	99.9	133.2	166.5	199.8	233.1	266.4	299.7
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
331	33.1	66.2	99.3	132.4	165.5	198.6	231.7	264.8	297.9
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
329	32.9	65.8	98.7	131.6	164.5	197.4	230.3	263.2	296.1
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
327	32.7	65.4	98.1	130.8	163.5	196.2	228.9	261.6	294.3
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
325	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
323	32.3	64.6	96.9	129.2	161.5	193.8	226.1	258.4	290.7
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8

No. 135 L. 130.] [No. 149 L. 175.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879	0194	0508	0822	1136	1450	1763	2076	2389	2702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
1	9219	9527	9835	0142	0449	0756	1063	1370	1676	1982	307
2	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868	0168	0469	0769	1068	301
5	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
321	32.1	64.2	96.3	128.4	160.5	192.6	224.7	256.8	288.9
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
319	31.9	63.8	95.7	127.6	159.5	191.4	223.3	255.2	287.1
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	254.4	286.2
317	31.7	63.4	95.1	126.8	158.5	190.2	221.9	253.6	285.3
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
315	31.5	63.0	94.5	126.0	157.5	189.0	220.5	252.0	283.5
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
313	31.3	62.6	93.9	125.2	156.5	187.8	219.1	250.4	281.7
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
311	31.1	62.2	93.3	124.4	155.5	186.6	217.7	248.8	279.9
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
309	30.9	61.8	92.7	123.6	154.5	185.4	216.3	247.2	278.1
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
307	30.7	61.4	92.1	122.8	153.5	184.2	214.9	245.6	276.3
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
305	30.5	61.0	91.5	122.0	152.5	183.0	213.5	244.0	274.5
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
303	30.3	60.6	90.9	121.2	151.5	181.8	212.1	242.4	272.7
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
301	30.1	60.2	90.3	120.4	150.5	180.6	210.7	240.8	270.9
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
299	29.9	59.8	89.7	119.6	149.5	179.4	209.3	239.2	269.1
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
297	29.7	59.4	89.1	118.8	148.5	178.2	207.9	237.6	267.3
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
295	29.5	59.0	88.5	118.0	147.5	177.0	206.5	236.0	265.5
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
293	29.3	58.6	87.9	117.2	146.5	175.8	205.1	234.4	263.7
292	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
291	29.1	58.2	87.3	116.4	145.5	174.6	203.7	232.8	261.9
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
289	28.9	57.8	86.7	115.6	144.5	173.4	202.3	231.2	260.1
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
287	28.7	57.4	86.1	114.8	143.5	172.2	200.9	229.6	258.3
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257.4

N.	0	1	2	3	4	5	6	7	8	9	Diff.
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
1	8977	9264	9552	9839							
					0126	0413	0699	0986	1272	1558	287
2	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7808	8084	8366	8647	8928	9209	9490	9771		
										0051	281
5	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755						
						0029	0303	0577	0850	1124	274
9	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783									
			0051	0319	0586	0653	1121	1388	1654	1921	267
3	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938		
	23									0193	256

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
285	28.5	57.0	85.5	114.0	142.5	171.0	199.5	228.0	256.5
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
283	28.3	56.6	84.9	113.2	141.5	169.8	198.1	226.4	254.7
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
281	28.1	56.2	84.3	112 4	140.5	168.6	196.7	224:8	252.9
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0
279	27.9	55.8	83.7	111.6	139.5	167.4	195.3	223.2	251.1
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
277	27.7	55.4	83.1	110.8	138.5	166.2	193.9	221.6	249.3
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
275	27.5	55.0	82.5	110.0	137.5	165.0	192.5	220.0	247.5
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
273	27.3	54.6	81.9	109.2	136.5	163.8	191.1	218.4	245.7
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
271	27.1	54.2	81.3	108.4	135.5	162.6	189.7	216.8	243.9
270	27.0	54.0	81.0	108.0	135.0	162 0	189.0	216.0	243.0
269	26.9	53.8	80.7	107.6	134.5	161.4	188.3	215.2	242.1
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
267	26.7	53.4	80.1	106.8	133.5	160.2	186.9	213.6	240.3
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
265	26.5	53.0	79.5	106.0	132.5	159.0	185.5	212.0	238.5
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
263	26.3	52.6	78.9	105.2	131.5	157.8	184.1	210.4	236.7
262	26.2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
261	26.1	52.2	78.3	104.4	130.5	156.6	182.7	208.8	234.9
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
259	25.9	51.8	77.7	103.6	129.5	155.4	181.3	207.2	233.1
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
257	25.7	51.4	77.1	102.8	128.5	154.2	179.9	205.6	231.3
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	230.4
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5

No. 170 L. 230.] [No. 180 L. 278.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800	0050	0300	250
4	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	0176	245
8	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	0213	0446	0679	0912	1144	1377	1609	233
7	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
253	25.3	50.6	75.9	101.2	126.5	151.8	177.1	202.4	227.7
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
251	25.1	50.2	75.3	100.4	125.5	150.6	175.7	200.8	225.9
250	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
249	24.9	49.8	74.7	99.6	124.5	149.4	174.3	199.2	224.1
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
247	24.7	49.4	74.1	98.8	123.5	148.2	172.9	197.6	222.3
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
245	24.5	49.0	73.5	98.0	122.5	147.0	171.5	196.0	220.5
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
243	24.3	48.6	72.9	97.2	121.5	145.8	170.1	194.4	218.7
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
241	24.1	48.2	72.3	96.4	120.5	144.6	168.7	192.8	216.9
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
239	23.9	47.8	71.7	95.6	119.5	143.4	167.3	191.2	215.1
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
237	23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
235	23.5	47.0	70.5	94.0	117.5	141.0	164.5	188.0	211.5
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
233	23.3	46.6	69.9	93.2	116.5	139.8	163.1	186.4	209.7
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
231	23.1	46.2	69.3	92.4	115.5	138.6	161.7	184.8	207.9
230	23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0
229	22.9	45.8	68.7	91.6	114.5	137.4	160.3	183.2	206.1
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205.2
227	22.7	45.4	68.1	90.8	113.5	136.2	158.9	181.6	204.3
226	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	203.4

No. 190 L. 278.] [No. 214 L. 332.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
190	278754	8982	9211	9439	9667	9895	0123	0351	0578	0806	228
1	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	0161	0378	0595	0813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542	212
5	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	0008	0211	203
4	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
225	22.5	45.0	67.5	90.0	112.5	135.0	157.5	180.0	202.5
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
223	22.3	44.6	66.9	89.2	111.5	133.8	156.1	178.4	200.7
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
221	22.1	44.2	66.3	88.4	110.5	132.6	154.7	176.8	198.9
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
219	21.9	43.8	65.7	87.6	109.5	131.4	153.3	175.2	197.1
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
217	21.7	43.4	65.1	86.8	108.5	130.2	151.9	173.6	195.3
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
215	21.5	43.0	64.5	86.0	107.5	129.0	150.5	172.0	193.5
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
213	21.3	42.6	63.9	85.2	106.5	127.8	149.1	170.4	191.7
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
211	21.1	42.2	63.3	84.4	105.5	126.6	147.7	168.8	189.9
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
209	20.9	41.8	62.7	83.6	104.5	125.4	146.3	167.2	188.1
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
207	20.7	41.4	62.1	82.8	103.5	124.2	144.9	165.6	186.3
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
205	20.5	41.0	61.5	82.0	102.5	123.0	143.5	164.0	184.5
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
203	20.3	40.6	60.9	81.2	101.5	121.8	142.1	162.4	182.7
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8

No. 215 L. 332.]   [No. 239 L. 38

N.	0	1	2	3	4	5	6	7	8	9	Dif
215	332438	2640	2842	3044	3246	3447	3649	3850	4051	4253	20
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	20
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	20
8	8456	8656	8855	9054	9253	9451	9650	9849	0047	0246	19
9	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	19
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	19
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	19
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	19
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	0054	19
4	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	19
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	19
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	19
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	19
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	19
9	9835	0025	0215	0404	0593	0783	0972	1161	1350	1539	18
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	18
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	18
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	18
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	18
4	9216	9401	9587	9772	9958	0143	0328	0513	0698	0883	18
5	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	18
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	18
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	18
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	18
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	0030	181
	38										

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.
201	20.1	40.2	60.3	80.4	100.5	120.6	140.7	160.8	180.
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.
199	19.9	39.8	59.7	79.6	99.5	119.4	139.3	159.2	179.
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.
197	19.7	39.4	59.1	78.8	98.5	118.2	137.9	157.6	177.
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.
195	19.5	39.0	58.5	78.0	97.5	117.0	136.5	156.0	175.
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.
193	19.3	38.6	57.9	77.2	96.5	115.8	135.1	154.4	173.
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.
191	19.1	38.2	57.3	76.4	95.5	114.6	133.7	152.8	171.
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.
189	18.9	37.8	56.7	75.6	94.5	113.4	132.3	151.2	170.
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.
187	18.7	37.4	56.1	74.8	93.5	112.2	130.9	149.6	168.
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.
185	18.5	37.0	55.5	74.0	92.5	111.0	129.5	148.0	166.
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.
183	18.3	36.6	54.9	73.2	91.5	109.8	128.1	146.4	164.
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.
181	18.1	36.2	54.3	72.4	90.5	108.6	126.7	144.8	162.
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.
179	17.9	35.8	53.7	71.6	89.5	107.4	125.3	143.2	161.

No. 240 L. 380.] [No. 269 L. 431.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	0051	0228	0405	0582	0759	177
6	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
1	9674	9847	0020	0192	0365	0538	0711	0883	1056	1228	173
2	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	.3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	0102	0271	0440	0609	0777	0946	1114	1283	1451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	0121	0286	0451	0616	0781	0945	1110	1275	1439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752 43	9914	0075	0236	0398	0559	0720	0881	1042	1203	161

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
177	17.7	35.4	53.1	70.8	88.5	106.2	123.9	141.6	159.3
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
175	17.5	35.0	52.5	70.0	87.5	105.0	122.5	140.0	157.5
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
173	17.3	34.6	51.9	69.2	86.5	103.8	121.1	138.4	155.7
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
171	17.1	34.2	51.3	68.4	85.5	102.6	119.7	136.8	153.9
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
169	16.9	33.8	50.7	67.6	84.5	101.4	118.3	135.2	152.1
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
167	16.7	33.4	50.1	66.8	83.5	100.2	116.9	133.6	150.3
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
165	16.5	33.0	49.5	66.0	82.5	99.0	115.5	132.0	148.5
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
163	16.3	32.6	48.9	65.2	81.5	97.8	114.1	130.4	146.7
162	16.2	32.4	48.5	64.8	81.0	97.2	113.4	129.6	145.8
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9

No. 270 L. 431.] [No. 299 L. 47

N.	0	1	2	3	4	5	6	7	8	9	Dif
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	16
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	16
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	15
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	15
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	15
5	9333	9491	9648	9806	9964	0122	0279	0437	0594	0752	15
6	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	15
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	15
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	15
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	15
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	15
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	0095	15
2	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	15
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	15
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	15
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	15
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	15
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	15
8	9392	9543	9694	9845	9995	0146	0296	0447	0597	0748	15
9	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	15
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	15
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	14
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	14
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	14
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	14
5	9822	9969	0116	0263	0410	0557	0704	0851	0998	1145	14
6	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	14
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	14
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	14
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	14

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144
159	15.9	31.8	47.7	63.6	79.5	95.4	111.3	127.2	143
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142
157	15.7	31.4	47.1	62.8	78.5	94.2	109.9	125.6	141
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140
155	15.5	31.0	46.5	62.0	77.5	93.0	108.5	124.0	139
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138
153	15.3	30.6	45.9	61.2	76.5	91.8	107.1	122.4	137
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136
151	15.1	30.2	45.3	60.4	75.5	90.6	105.7	120.8	135
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135
149	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133
147	14.7	29.4	44.1	58.8	73.5	88.2	102.9	117.6	132
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131
145	14.5	29.0	43.5	58.0	72.5	87.0	101.5	116.0	130
144	14.4	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129
143	14.3	28.6	42.9	57.2	71.5	85.8	100.1	114.4	128
142	14.2	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127
141	14.1	28.2	42.3	56.4	70.5	84.6	98.7	112.8	126
140	14.0	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126

No. 300 L. 477.] [No. 339 L. 531.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958										
		0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962								
				0099	0236	0374	0511	0648	0785	0922	137
7	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874					
							0009	0143	0277	0411	134
4	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
1	9828	9959									
			0090	0221	0353	0484	0615	0745	0876	1007	131
2	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943		
										0072	128
9	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
139	13.9	27.8	41.7	55.6	69.5	83.4	97.3	111.2	125.1
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
137	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
135	13.5	27.0	40.5	54.0	67.5	81.0	94.5	108.0	121.5
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
133	13.3	26.6	39.9	53.2	66.5	79.8	93.1	106.4	119.7
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
131	13.1	26.2	39.3	52.4	65.5	78.6	91.7	104.8	117.9
130	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
129	12.9	25.8	38.7	51.6	64.5	77.4	90.3	103.2	116.1
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3

No. 340 L. 531.] [N

N.	0	1	2	3	4	5	6	7	8
340	531479	1607	1734	1862	1990	2117	2245	2372	2500
1	2734	2862	3009	3136	3264	3391	3518	3645	3772
2	4026	4153	4280	4407	4534	4661	4787	4914	5041
3	5294	5421	5547	5674	5800	5927	6053	6180	6306
4	6558	6685	6811	6937	7063	7189	7315	7441	7567
5	7819	7945	8071	8197	8322	8448	8574	8699	8825
6	9076	9202	9327	9452	9578	9703	9829	9954	0079
7	540329	0455	0580	0705	0830	0955	1080	1205	1330
8	1579	1704	1829	1953	2078	2203	2327	2452	2576
9	2825	2950	3074	3199	3323	3447	3571	3696	3820
350	4068	4192	4316	4440	4564	4688	4812	4936	5060
1	5307	5431	5555	5678	5802	5925	6049	6172	6296
2	6543	6666	6789	6913	7036	7159	7282	7405	7529
3	7775	7898	8021	8144	8267	8389	8512	8635	8758
4	9003	9126	9249	9371	9494	9616	9739	9861	9984
5	550228	0351	0473	0595	0717	0840	0962	1084	1206
6	1450	1572	1694	1816	1938	2060	2181	2303	2425
7	2668	2790	2911	3033	3155	3276	3398	3519	3640
8	3883	4004	4126	4247	4368	4489	4610	4731	4852
9	5094	5215	5336	5457	5578	5699	5820	5940	6061
360	6303	6423	6544	6664	6785	6905	7026	7146	7267
1	7507	7627	7748	7868	7988	8108	8228	8349	8469
2	8709	8829	8948	9068	9188	9308	9428	9548	9667
3	9907	0026	0146	0265	0385	0504	0624	0743	0863
4	561101	1221	1340	1459	1578	1698	1817	1936	2055
5	2293	2412	2531	2650	2769	2887	3006	3125	3244
6	3481	3600	3718	3837	3955	4074	4192	4311	4429
7	4666	4784	4903	5021	5139	5257	5376	5494	5612
8	5848	5966	6084	6202	6320	6437	6555	6673	6791
9	7026	7144	7262	7379	7497	7614	7732	7849	7967
370	8202	8319	8436	8554	8671	8788	8905	9023	9140
1	9374	9491	9608	9725	9842	9959	0076	0193	0309
2	570543	0660	0776	0893	1010	1126	1243	1359	1476
3	1709	1825	1942	2058	2174	2291	2407	2523	2639
4	2872	2988	3104	3220	3336	3452	3568	3684	3800
5	4031	4147	4263	4379	4494	4610	4726	4841	4957
6	5188	5303	5419	5534	5650	5765	5880	5996	6111
7	6341	6457	6572	6687	6802	6917	7032	7147	7262
8	7492	7607	7722	7836	7951	8066	8181	8295	8410
9	8639	8754	8868	8983	9097	9212	9326	9441	9555

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2
125	12.5	25.0	37.5	50.0	62.5	75.0	87.5
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8
123	12.3	24.6	36.9	49.2	61.5	73.8	86.1
122	12.2	24.4	36.6	48.8	61.0	73.2	85.4
121	12.1	24.2	36.3	48.4	60.5	72.6	84.7
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0
119	11.9	23.8	35.7	47.6	59.5	71.4	83.3

No. 380. L. 579.] [No. 414 L. 617.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	579784	9898	0012	0126	0241	0355	0469	0583	0697	0811	114
1	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	
9	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953	
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	
8	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864	109
9	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	
7	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554	
8	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
118	11.8	23.6	35.4	47.2	59.0	70.8	82.6	94.4	106.2
117	11.7	23.4	35.1	46.8	58.5	70.2	81.9	93.6	105.3
116	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.4
115	11.5	23.0	34.5	46.0	57.5	69.0	80.5	92.0	103.5
114	11.4	22.8	34.2	45.6	57.0	68.4	79.8	91.2	102.6
113	11.3	22.6	33.9	45.2	56.5	67.8	79.1	90.4	101.7
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
111	11.1	22.2	33.3	44.4	55.5	66.6	77.7	88.8	99.9
110	11.0	22.0	33.0	44.0	55.0	66.0	77.0	88.0	99.0
109	10.9	21.8	32.7	43.6	54.5	65.4	76.3	87.2	98.1
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
107	10.7	21.4	32.1	42.8	53.5	64.2	74.9	85.6	96.3
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.4
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6

No. 415 L. 618.]

N.	0	1	2	3	4	5	6	7
415	618048	8153	8257	8362	8466	8571	8676	8780
6	9093	9198	9302	9406	9511	9615	9719	9824
7	620136	0240	0344	0448	0552	0656	0760	0864
8	1176	1280	1384	1488	1592	1695	1799	1903
9	2214	2318	2421	2525	2628	2732	2835	2939
420	3249	3353	3456	3559	3663	3766	3869	3973
1	4282	4385	4488	4591	4695	4798	4901	5004
2	5312	5415	5518	5621	5724	5827	5929	6032
3	6340	6443	6546	6648	6751	6853	6956	7058
4	7366	7468	7571	7673	7775	7878	7980	8082
5	8389	8491	8593	8695	8797	8900	9002	9104
6	9410	9512	9613	9715	9817	9919	0021	0123
7	630428	0530	0631	0733	0835	0936	1038	1139
8	1444	1545	1647	1748	1849	1951	2052	2153
9	2457	2559	2660	2761	2862	2963	3064	3165
430	3468	3569	3670	3771	3872	3973	4074	4175
1	4477	4578	4679	4779	4880	4981	5081	5182
2	5484	5584	5685	5785	5886	5986	6087	6187
3	6488	6588	6688	6789	6889	6989	7089	7189
4	7490	7590	7690	7790	7890	7990	8090	8190
5	8489	8589	8689	8789	8888	8988	9088	9188
6	9486	9586	9686	9785	9885	9984	0084	0183
7	640481	0581	0680	0779	0879	0978	1077	1177
8	1474	1573	1672	1771	1871	1970	2069	2168
9	2465	2563	2662	2761	2860	2959	3058	3156
440	3453	3551	3650	3749	3847	3946	4044	4143
1	4439	4537	4636	4734	4832	4931	5029	5127
2	5422	5521	5619	5717	5815	5913	6011	6110
3	6404	6502	6600	6698	6796	6894	6992	7089
4	7383	7481	7579	7676	7774	7872	7969	8067
5	8360	8458	8555	8653	8750	8848	8945	9043
6	9335	9432	9530	9627	9724	9821	9919	0016
7	650308	0405	0502	0599	0696	0793	0890	0987
8	1278	1375	1472	1569	1666	1762	1859	1956
9	2246	2343	2440	2536	2633	2730	2826	2923
450	3213	3309	3405	3502	3598	3695	3791	3888
1	4177	4273	4369	4465	4562	4658	4754	4850
2	5138	5235	5331	5427	5523	5619	5715	5810
3	6098	6194	6290	6386	6482	6577	6673	6769
4	7056	7152	7247	7343	7438	7534	7629	7725
5	8011	8107	8202	8298	8393	8488	8584	8679
6	8965	9060	9155	9250	9346	9441	9536	9631
7	9916	0011	0106	0201	0296	0391	0486	0581
8	660865	0960	1055	1150	1245	1339	1434	1529
9	1813	1907	2002	2096	2191	2286	2380	2475

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7
105	10.5	21.0	31.5	42.0	52.5	63.0	73.
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8
103	10.3	20.6	30.9	41.2	51.5	61.8	72.1
102	10.2	20.4	30.6	40.8	51.0	61.2	71.4
101	10.1	20.2	30.3	40.4	50.5	60.6	70.7
100	10.0	20.0	30.0	40.0	50.0	60.0	70.0
99	9.9	19.8	29.7	39.6	49.5	59.4	69.3

0	1	2	3	4	5	6	7	8	9	Diff.
662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	
3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	
4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	
6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	
7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	
8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	
9317	9410	9503	9596	9689	9782	9875	9967			
								0060	0153	93
670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	
1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	
2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	
3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	
3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	
5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	
6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	
7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	
8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
9428	9519	9610	9700	9791	9882	9973				
							0063	0154	0245	
680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	
1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	
2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	
3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	
4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	
5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	
6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	
7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	
9309	9398	9486	9575	9664	9753	9841	9930			
								0019	0107	
690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	
1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	
1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	
2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	
4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	
5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	
6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	
7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	
8100	8188	8275	8362	8449	8535	8622	8709	8796	8883	87

Proportional Parts.

1	2	3	4	5	6	7	8	9
9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
9.7	19.4	29.1	38.8	48.5	58.2	67.9	77.6	87.3
9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
9.5	19.0	28.5	38.0	47.5	57.0	66.5	76.0	85.5
9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
9.3	18.6	27.9	37.2	46.5	55.8	65.1	74.4	83.7
9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.9
9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
8.9	17.8	26.7	35.6	44.5	53.4	62.3	71.2	80.1
8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2
8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4

No. 500 L. 698.]

N.	0	1	2	3	4	5	6	7	8
500	698970	9057	9144	9231	9317	9404	9491	9578	96
1	9838	9924	0011	0098	0184	0271	0358	0444	05
2	700704	0790	0877	0963	1050	1136	1222	1309	13
3	1568	1654	1741	1827	1913	1999	2086	2172	22
4	2431	2517	2603	2689	2775	2861	2947	3033	31
5	3291	3377	3463	3549	3635	3721	3807	3893	39
6	4151	4236	4322	4408	4494	4579	4665	4751	48
7	5008	5094	5179	5265	5350	5436	5522	5607	56
8	5864	5949	6035	6120	6206	6291	6376	6462	65
9	6718	6803	6888	6974	7059	7144	7229	7315	74
510	7570	7655	7740	7826	7911	7996	8081	8166	82
1	8421	8506	8591	8676	8761	8846	8931	9015	91
2	9270	9355	9440	9524	9609	9694	9779	9863	99
3	710117	0202	0287	0371	0456	0540	0625	0710	07
4	0963	1048	1132	1217	1301	1385	1470	1554	16
5	1807	1892	1976	2060	2144	2229	2313	2397	24
6	2650	2734	2818	2902	2986	3070	3154	3238	33
7	3491	3575	3659	3742	3826	3910	3994	4078	41
8	4330	4414	4497	4581	4665	4749	4833	4916	50
9	5167	5251	5335	5418	5502	5586	5669	5753	58
520	6003	6087	6170	6254	6337	6421	6504	6588	66
1	6838	6921	7004	7088	7171	7254	7338	7421	75
2	7671	7754	7837	7920	8003	8086	8169	8253	83
3	8502	8585	8668	8751	8834	8917	9000	9083	91
4	9331	9414	9497	9580	9663	9745	9828	9911	99
5	720159	0242	0325	0407	0490	0573	0655	0738	08
6	0986	1068	1151	1233	1316	1398	1481	1563	16
7	1811	1893	1975	2058	2140	2222	2305	2387	24
8	2634	2716	2798	2881	2963	3045	3127	3209	32
9	3456	3538	3620	3702	3784	3866	3948	4030	41
530	4276	4358	4440	4522	4604	4685	4767	4849	49
1	5095	5176	5258	5340	5422	5503	5585	5667	57
2	5912	5993	6075	6156	6238	6320	6401	6483	65
3	6727	6809	6890	6972	7053	7134	7216	7297	73
4	7541	7623	7704	7785	7866	7948	8029	8110	81
5	8354	8435	8516	8597	8678	8759	8841	8922	90
6	9165	9246	9327	9408	9489	9570	9651	9732	98
7	9974	0055	0136	0217	0298	0378	0459	0540	06
8	730782	0863	0944	1024	1105	1186	1266	1347	14
9	1589	1669	1750	1830	1911	1991	2072	2152	22
540	2394	2474	2555	2635	2715	2796	2876	2956	30
1	3197	3278	3358	3438	3518	3598	3679	3759	38
2	3999	4079	4160	4240	4320	4400	4480	4560	46
3	4800	4880	4960	5040	5120	5200	5279	5359	54
4	5599	5679	5759	5838	5918	5998	6078	6157	62

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2
85	8.5	17.0	25.5	34.0	42.5	51.0	59.5
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8

No. 545 L. 736.] [No. 584 L. 767.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
545	736397	6476	6556	6635	6715	6795	6874	6954	7034	7113	
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	
9	9572	9651	9731	9810	9889	9968					
							0047	0126	0205	0284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	
2	9736	9814	9891	9968							
					0045	0123	0200	0277	0354	0431	
3	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	
4	1279	1356	1433	1510	1587	1604	1741	1818	1895	1972	
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	
5	9668	9743	9819	9894	9970						
						0045	0121	0196	0272	0347	
6	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
83	8.3	16.6	24.9	33.2	41.5	49.8	58.1	66.4	74.7
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
81	8.1	16.2	24.3	32.4	40.5	48.6	56.7	64.8	72.9
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
79	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
77	7.7	15.4	23.1	30.8	38.5	46.2	53.9	61.6	69.3
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6

No. 585 L. 767.]

N.	0	1	2	3	4	5	6	7
585	767156	7230	7304	7379	7453	7527	7601	7675
6	7898	7972	8046	8120	8194	8268	8342	8416
7	8638	8712	8786	8860	8934	9008	9082	9156
8	9377	9451	9525	9599	9673	9746	9820	9894
9	770115	0189	0263	0336	0410	0484	0557	0631
590	0852	0926	0999	1073	1146	1220	1293	1367
1	1587	1661	1734	1808	1881	1955	2028	2102
2	2322	2395	2468	2542	2615	2688	2762	2835
3	3055	3128	3201	3274	3348	3421	3494	3567
4	3786	3860	3933	4006	4079	4152	4225	4298
5	4517	4590	4663	4736	4809	4882	4955	5028
6	5246	5319	5392	5465	5538	5610	5683	5756
7	5974	6047	6120	6193	6265	6338	6411	6483
8	6701	6774	6846	6919	6992	7064	7137	7209
9	7427	7499	7572	7644	7717	7789	7862	7934
600	8151	8224	8296	8368	8441	8513	8585	8658
1	8874	8947	9019	9091	9163	9236	9308	9380
2	9596	9669	9741	9813	9885	9957	0029	0101
3	780317	0389	0461	0533	0605	0677	0749	0821
4	1037	1109	1181	1253	1324	1396	1468	1540
5	1755	1827	1899	1971	2042	2114	2186	2258
6	2473	2544	2616	2688	2759	2831	2902	2974
7	3189	3260	3332	3403	3475	3546	3618	3689
8	3904	3975	4046	4118	4189	4261	4332	4403
9	4617	4689	4760	4831	4902	4974	5045	5116
610	5330	5401	5472	5543	5615	5686	5757	5828
1	6041	6112	6183	6254	6325	6396	6467	6538
2	6751	6822	6893	6964	7035	7106	7177	7248
3	7460	7531	7602	7673	7744	7815	7885	7956
4	8168	8239	8310	8381	8451	8522	8593	8663
5	8875	8946	9016	9087	9157	9228	9299	9369
6	9581	9651	9722	9792	9863	9933	0004	0074
7	790285	0356	0426	0496	0567	0637	0707	0778
8	0988	1059	1129	1199	1269	1340	1410	1480
9	1691	1761	1831	1901	1971	2041	2111	2181
620	2392	2462	2532	2602	2672	2742	2812	2882
1	3092	3162	3231	3301	3371	3441	3511	3581
2	3790	3860	3930	4000	4070	4139	4209	4279
3	4488	4558	4627	4697	4767	4836	4906	4976
4	5185	5254	5324	5393	5462	5532	5602	5672
5	5880	5949	6019	6088	6158	6227	6297	6366
6	6574	6644	6713	6782	6852	6921	6990	7060
7	7268	7337	7406	7475	7545	7614	7683	7752
8	7960	8029	8098	8167	8236	8305	8374	8443
9	8651	8720	8789	8858	8927	8996	9065	9134

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8
73	7.3	14.6	21.9	29.2	36.5	43.8	51.1
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4
71	7.1	14.2	21.3	28.4	35.5	42.6	49.7
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0
69	6.9	13.8	20.7	27.6	34.5	41.4	48.3

No. 630 L. 799.] [No. 674 L. 829.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	
1	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	
5	9560	9627	9694	9762	9829	9896	9964	0031	0098	0165	
6	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	
660	9544	9610	9676	9741	9807	9873	9939	0004	0070	0136	
1	820201	0267	0332	0399	0464	0530	0595	0661	0727	0792	
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
67	6.7	13.4	20.1	26.8	33.5	40.2	46.9	53.6	60.3
66	6.6	13.2	19.8	26.4	33.0	39.6	46.2	52.8	59.4
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6

No. 675 L. 829.]

N.	0	1	2	3	4	5	6
675	829304	9368	9432	9497	9561	9625	969
6	9947	0011	0075	0139	0204	0268	033
7	830589	0653	0717	0781	0845	0909	097
8	1230	1294	1358	1422	1486	1550	161
9	1870	1934	1998	2062	2126	2189	225
680	2509	2573	2637	2700	2764	2828	289
1	3147	3211	3275	3338	3402	3466	353
2	3784	3848	3912	3975	4039	4103	416
3	4421	4484	4548	4611	4675	4739	480
4	5056	5120	5183	5247	5310	5373	543
5	5691	5754	5817	5881	5944	6007	607
6	6324	6387	6451	6514	6577	6641	670
7	6957	7020	7083	7146	7210	7273	733
8	7588	7652	7715	7778	7841	7904	796
9	8219	8282	8345	8408	8471	8534	859
690	8849	8912	8975	9038	9101	9164	922
1	9478	9541	9604	9667	9729	9792	985
2	840106	0169	0232	0294	0357	0420	048
3	0733	0796	0859	0921	0984	1046	110
4	1359	1422	1485	1547	1610	1672	173
5	1985	2047	2110	2172	2235	2297	236
6	2609	2672	2734	2796	2859	2921	298
7	3233	3295	3357	3420	3482	3544	360
8	3855	3918	3980	4042	4104	4166	422
9	4477	4539	4601	4664	4726	4788	485
700	5098	5160	5222	5284	5346	5408	547
1	5718	5780	5842	5904	5966	6028	609
2	6337	6399	6461	6523	6585	6646	670
3	6955	7017	7079	7141	7202	7264	732
4	7573	7634	7696	7758	7819	7881	794
5	8189	8251	8312	8374	8435	8497	855
6	8805	8866	8928	8989	9051	9112	917
7	9419	9481	9542	9604	9665	9726	978
8	850033	0095	0156	0217	0279	0340	0401
9	0646	0707	0769	0830	0891	0952	1014
710	1258	1320	1381	1442	1503	1564	162
1	1870	1931	1992	2053	2114	2175	223
2	2480	2541	2602	2663	2724	2785	284
3	3090	3150	3211	3272	3333	3394	345
4	3698	3759	3820	3881	3941	4002	406
5	4306	4367	4428	4488	4549	4610	4670
6	4913	4974	5034	5095	5156	5216	527
7	5519	5580	5640	5701	5761	5822	588
8	6124	6185	6245	6306	6366	6427	648
9	6729	6789	6850	6910	6970	7031	7091

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6
65	6.5	13.0	19.5	26.0	32.5	39
64	6.4	12.8	19.2	25.6	32.0	38
63	6.3	12.6	18.9	25.2	31.5	37
62	6.2	12.4	18.6	24.8	31.0	37
61	6.1	12.2	18.3	24.4	30.5	36
60	6.0	12.0	18.0	24.0	30.0	36

No. 720 L. 857.] [No. 764 L. 883.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	0038	0098	0158	0218	0278	
5	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	
1	9818	9877	9935	9994	0053	0111	0170	0228	0287	0345	
2	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	
8	9669	9726	9784	9841	9898	9956	0013	0070	0127	0185	
9	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	
760	0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
59	5.9	11.8	17.7	23.6	29.5	35.4	41.3	47.2	53.1
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4

No. 765 L. 883.] [No. 809 L. 908.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
765	883661	3718	3775	3832	3888	3945	4002	4059	4115	4172	
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	0030	0086	0141	0197	0253	0309	0365	
7	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	
4	9821	9875	9930	9985	0039	0094	0149	0203	0258	0312	
5	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
55	5.5	11.0	16.5	22.0	27.5	33.0	38.5	44.0	49.5
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

No. 810 L. 908.] [No. 854 L. 931.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
810	908485	8539	8592	8646	8699	8753	8807	8860	8914	8967	
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	0037	
3	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	
1	9601	9653	9706	9758	9810	9862	9914	9967	0019	0071	
2	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	
1	9930	9981	0032	0083	0134	0185	0236	0287	0338	0389	51
2	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
53	5.3	10.6	15.9	21.2	26.5	31.8	37.1	42.4	47.7
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0

No. 855 L. 931.]

N.	0	1	2	3	4	5	6	7	
855	931966	2017	2068	2118	2169	2220	2271	2322	2372
6	2474	2524	2575	2626	2677	2727	2778	2829	2879
7	2981	3031	3082	3133	3183	3234	3285	3335	3386
8	3487	3538	3589	3639	3690	3740	3791	3841	3892
9	3993	4044	4094	4145	4195	4246	4296	4347	4397
860	4498	4549	4599	4650	4700	4751	4801	4852	4902
1	5003	5054	5104	5154	5205	5255	5306	5356	5406
2	5507	5558	5608	5658	5709	5759	5809	5860	5910
3	6011	6061	6111	6162	6212	6262	6313	6363	6413
4	6514	6564	6614	6665	6715	6765	6815	6865	6916
5	7016	7066	7116	7167	7217	7267	7317	7367	7418
6	7518	7568	7618	7668	7718	7769	7819	7869	7919
7	8019	8069	8119	8169	8219	8269	8320	8370	8420
8	8520	8570	8620	8670	8720	8770	8820	8870	8920
9	9020	9070	9120	9170	9220	9270	9320	9369	9419
870	9519	9569	9619	9669	9719	9769	9819	9869	9918
1	940018	0068	0118	0168	0218	0267	0317	0367	0417
2	0516	0566	0616	0666	0716	0765	0815	0865	0915
3	1014	1064	1114	1163	1213	1263	1313	1362	1412
4	1511	1561	1611	1660	1710	1760	1809	1859	1909
5	2008	2058	2107	2157	2207	2256	2306	2355	2405
6	2504	2554	2603	2653	2702	2752	2801	2851	2901
7	3000	3049	3099	3148	3198	3247	3297	3346	3396
8	3495	3544	3593	3643	3692	3742	3791	3841	3890
9	3989	4038	4088	4137	4186	4236	4285	4335	4384
880	4483	4532	4581	4631	4680	4729	4779	4828	4877
1	4976	5025	5074	5124	5173	5222	5272	5321	5370
2	5469	5518	5567	5616	5665	5715	5764	5813	5862
3	5961	6010	6059	6108	6157	6207	6256	6305	6354
4	6452	6501	6551	6600	6649	6698	6747	6796	6845
5	6943	6992	7041	7090	7139	7189	7238	7287	7336
6	7434	7483	7532	7581	7630	7679	7728	7777	7826
7	7924	7973	8022	8070	8119	8168	8217	8266	8315
8	8413	8462	8511	8560	8608	8657	8706	8755	8804
9	8902	8951	8999	9048	9097	9146	9195	9244	9292
890	9390	9439	9488	9536	9585	9634	9683	9731	9780
1	9878	9926	9975	0024	0073	0121	0170	0219	0267
2	950365	0414	0462	0511	0560	0608	0657	0706	0754
3	0851	0900	0949	0997	1046	1095	1143	1192	1240
4	1338	1386	1435	1483	1532	1580	1629	1677	1726
5	1823	1872	1920	1969	2017	2066	2114	2163	2211
6	2308	2356	2405	2453	2502	2550	2599	2647	2696
7	2792	2841	2889	2938	2986	3034	3083	3131	3180
8	3276	3325	3373	3421	3470	3518	3566	3615	3663
9	3760	3808	3856	3905	3953	4001	4049	4098	4146

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0
49	4.9	9.8	14.7	19.6	24.5	29.4	34.3
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6

No 900 L. 954.] [No. 944 L. 975.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	
2	9995	0042	0090	0138	0185	0233	0280	0328	0376	0423	
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	
3	9882	9928	9975	0021	0068	0114	0161	0207	0254	0300	
4	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
47	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	42.3
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4

No. 945 L. 975.] [No. 989 L. 995.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
945	975432	5478	5524	5570	5616	5662	5707	5753	5799	5845	
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	
5	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	
7	9895	9939	9983	0028	0072	0117	0161	0206	0250	0294	
8	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
45	4.5	9.0	13.5	18.0	22.5	27.0	31.5	36.0	40.5
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
43	4.3	8.6	12.9	17.2	21.5	25.8	30.1	34.4	38.7

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

HYPERBOLIC LOGARITHMS.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1.01	.0099	1.45	.3716	1.89	.6366	2.33	.8458	2.77	1.0188
1.02	.0198	1.46	.3784	1.90	.6419	2.34	.8502	2.78	1.0225
1.03	.0296	1.47	.3853	1.91	.6471	2.35	.8544	2.79	1.0260
1.04	.0392	1.48	.3920	1.92	.6523	2.36	.8587	2.80	1.0296
1.05	.0488	1.49	.3988	1.93	.6575	2.37	.8629	2.81	1.0332
1.06	.0583	1.50	.4055	1.94	.6627	2.38	.8671	2.82	1.0367
1.07	.0677	1.51	.4121	1.95	.6678	2.39	.8713	2.83	1.0403
1.08	.0770	1.52	.4187	1.96	.6729	2.40	.8755	2.84	1.0438
1.09	.0862	1.53	.4253	1.97	.6780	2.41	.8796	2.85	1.0473
1.10	.0953	1.54	.4318	1.98	.6831	2.42	.8838	2.86	1.0508
1.11	.1044	1.55	.4383	1.99	.6881	2.43	.8879	2.87	1.0543
1.12	.1133	1.56	.4447	2.00	.6931	2.44	.8920	2.88	1.0578
1.13	.1222	1.57	.4511	2.01	.6981	2.45	.8961	2.89	1.0613
1.14	.1310	1.58	.4574	2.02	.7031	2.46	.9002	2.90	1.0647
1.15	.1398	1.59	.4637	2.03	.7080	2.47	.9042	2.91	1.0682
1.16	.1484	1.60	.4700	2.04	.7129	2.48	.9083	2.92	1.0716
1.17	.1570	1.61	.4762	2.05	.7178	2.49	.9123	2.93	1.0750
1.18	.1655	1.62	.4824	2.06	.7227	2.50	.9163	2.94	1.0784
1.19	.1740	1.63	.4886	2.07	.7275	2.51	.9203	2.95	1.0818
1.20	.1823	1.64	.4947	2.08	.7324	2.52	.9243	2.96	1.0852
1.21	.1906	1.65	.5008	2.09	.7372	2.53	.9282	2.97	1.0886
1.22	.1988	1.66	.5068	2.10	.7419	2.54	.9322	2.98	1.0919
1.23	.2070	1.67	.5128	2.11	.7467	2.55	.9361	2.99	1.0953
1.24	.2151	1.68	.5188	2.12	.7514	2.56	.9400	3.00	1.0986
1.25	.2231	1.69	.5247	2.13	.7561	2.57	.9439	3.01	1.1019
1.26	.2311	1.70	.5306	2.14	.7608	2.58	.9478	3.02	1.1053
1.27	.2390	1.71	.5365	2.15	.7655	2.59	.9517	3.03	1.1086
1.28	.2469	1.72	.5423	2.16	.7701	2.60	.9555	3.04	1.1119
1.29	.2546	1.73	.5481	2.17	.7747	2.61	.9594	3.05	1.1151
1.30	.2624	1.74	.5539	2.18	.7793	2.62	.9632	3.06	1.1184
1.31	.2700	1.75	.5596	2.19	.7839	2.63	.9670	3.07	1.1217
1.32	.2776	1.76	.5653	2.20	.7885	2.64	.9708	3.08	1.1249
1.33	.2852	1.77	.5710	2.21	.7930	2.65	.9746	3.09	1.1282
1.34	.2927	1.78	.5766	2.22	.7975	2.66	.9783	3.10	1.1314
1.35	.3001	1.79	.5822	2.23	.8020	2.67	.9821	3.11	1.1346
1.36	.3075	1.80	.5878	2.24	.8065	2.68	.9858	3.12	1.1378
1.37	.3148	1.81	.5933	2.25	.8109	2.69	.9895	3.13	1.1410
1.38	.3221	1.82	.5988	2.26	.8154	2.70	.9933	3.14	1.1442
1.39	.3293	1.83	.6043	2.27	.8198	2.71	.9969	3.15	1.1474
1.40	.3365	1.84	.6098	2.28	.8242	2.72	1.0006	3.16	1.1506
1.41	.3436	1.85	.6152	2.29	.8286	2.73	1.0043	3.17	1.1537
1.42	.3507	1.86	.6206	2.30	.8329	2.74	1.0080	3.18	1.1569
1.43	.3577	1.87	.6259	2.31	.8372	2.75	1.0116	3.19	1.1600
1.44	.3646	1.88	.6313	2.32	.8416	2.76	1.0152	3.20	1.1632

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.
3.21	1.1663	3.87	1.3533	4.53	1.5107	5.19	1.6467	5.85
3.22	1.1694	3.88	1.3558	4.54	1.5129	5.20	1.6487	5.86
3.23	1.1725	3.89	1.3584	4.55	1.5151	5.21	1.6506	5.87
3.24	1.1756	3.90	1.3610	4.56	1.5173	5.22	1.6525	5.88
3.25	1.1787	3.91	1.3635	4.57	1.5195	5.23	1.6544	5.89
3.26	1.1817	3.92	1.3661	4.58	1.5217	5.24	1.6563	5.90
3.27	1.1848	3.93	1.3686	4.59	1.5239	5.25	1.6582	5.91
3.28	1.1878	3.94	1.3712	4.60	1.5261	5.26	1.6601	5.92
3.29	1.1909	3.95	1.3737	4.61	1.5282	5.27	1.6620	5.93
3.30	1.1939	3.96	1.3762	4.62	1.5304	5.28	1.6639	5.94
3.31	1.1969	3.97	1.3788	4.63	1.5326	5.29	1.6658	5.95
3.32	1.1999	3.98	1.3813	4.64	1.5347	5.30	1.6677	5.96
3.33	1.2030	3.99	1.3838	4.65	1.5369	5.31	1.6696	5.97
3.34	1.2060	4.00	1.3863	4.66	1.5390	5.32	1.6715	5.98
3.35	1.2090	4.01	1.3888	4.67	1.5412	5.33	1.6734	5.99
3.36	1.2119	4.02	1.3913	4.68	1.5433	5.34	1.6752	6.00
3.37	1.2149	4.03	1.3938	4.69	1.5454	5.35	1.6771	6.01
3.38	1.2179	4.04	1.3962	4.70	1.5476	5.36	1.6790	6.02
3.39	1.2208	4.05	1.3987	4.71	1.5497	5.37	1.6808	6.03
3.40	1.2238	4.06	1.4012	4.72	1.5518	5.38	1.6827	6.04
3.41	1.2267	4.07	1.4036	4.73	1.5539	5.39	1.6845	6.05
3.42	1.2296	4.08	1.4061	4.74	1.5560	5.40	1.6864	6.06
3.43	1.2326	4.09	1.4085	4.75	1.5581	5.41	1.6882	6.07
3.44	1.2355	4.10	1.4110	4.76	1.5602	5.42	1.6901	6.08
3.45	1.2384	4.11	1.4134	4.77	1.5623	5.43	1.6919	6.09
3.46	1.2413	4.12	1.4159	4.78	1.5644	5.44	1.6938	6.10
3.47	1.2442	4.13	1.4183	4.79	1.5665	5.45	1.6956	6.11
3.48	1.2470	4.14	1.4207	4.80	1.5686	5.46	1.6974	6.12
3.49	1.2499	4.15	1.4231	4.81	1.5707	5.47	1.6993	6.13
3.50	1.2528	4.16	1.4255	4.82	1.5728	5.48	1.7011	6.14
3.51	1.2556	4.17	1.4279	4.83	1.5748	5.49	1.7029	6.15
3.52	1.2585	4.18	1.4303	4.84	1.5769	5.50	1.7047	6.16
3.53	1.2613	4.19	1.4327	4.85	1.5790	5.51	1.7066	6.17
3.54	1.2641	4.20	1.4351	4.86	1.5810	5.52	1.7084	6.18
3.55	1.2669	4.21	1.4375	4.87	1.5831	5.53	1.7102	6.19
3.56	1.2698	4.22	1.4398	4.88	1.5851	5.54	1.7120	6.20
3.57	1.2726	4.23	1.4422	4.89	1.5872	5.55	1.7138	6.21
3.58	1.2754	4.24	1.4446	4.90	1.5892	5.56	1.7156	6.22
3.59	1.2782	4.25	1.4469	4.91	1.5913	5.57	1.7174	6.23
3.60	1.2809	4.26	1.4493	4.92	1.5933	5.58	1.7192	6.24
3.61	1.2837	4.27	1.4516	4.93	1.5953	5.59	1.7210	6.25
3.62	1.2865	4.28	1.4540	4.94	1.5974	5.60	1.7228	6.26
3.63	1.2892	4.29	1.4563	4.95	1.5994	5.61	1.7246	6.27
3.64	1.2920	4.30	1.4586	4.96	1.6014	5.62	1.7263	6.28
3.65	1.2947	4.31	1.4609	4.97	1.6034	5.63	1.7281	6.29
3.66	1.2975	4.32	1.4633	4.98	1.6054	5.64	1.7299	6.30
3.67	1.3002	4.33	1.4656	4.99	1.6074	5.65	1.7317	6.31
3.68	1.3029	4.34	1.4679	5.00	1.6094	5.66	1.7334	6.32
3.69	1.3056	4.35	1.4702	5.01	1.6114	5.67	1.7352	6.33
3.70	1.3083	4.36	1.4725	5.02	1.6134	5.68	1.7370	6.34
3.71	1.3110	4.37	1.4748	5.03	1.6154	5.69	1.7387	6.35
3.72	1.3137	4.38	1.4770	5.04	1.6174	5.70	1.7405	6.36
3.73	1.3164	4.39	1.4793	5.05	1.6194	5.71	1.7422	6.37
3.74	1.3191	4.40	1.4816	5.06	1.6214	5.72	1.7440	6.38
3.75	1.3218	4.41	1.4839	5.07	1.6233	5.73	1.7457	6.39
3.76	1.3244	4.42	1.4861	5.08	1.6253	5.74	1.7475	6.40
3.77	1.3271	4.43	1.4884	5.09	1.6273	5.75	1.7492	6.41
3.78	1.3297	4.44	1.4907	5.10	1.6292	5.76	1.7509	6.42
3.79	1.3324	4.45	1.4929	5.11	1.6312	5.77	1.7527	6.43
3.80	1.3350	4.46	1.4951	5.12	1.6332	5.78	1.7544	6.44
3.81	1.3376	4.47	1.4974	5.13	1.6351	5.79	1.7561	6.45
3.82	1.3403	4.48	1.4996	5.14	1.6371	5.80	1.7579	6.46
3.83	1.3429	4.49	1.5019	5.15	1.6390	5.81	1.7596	6.47
3.84	1.3455	4.50	1.5041	5.16	1.6409	5.82	1.7613	6.48
3.85	1.3481	4.51	1.5063	5.17	1.6429	5.83	1.7630	6.49
3.86	1.3507	4.52	1.5085	5.18	1.6448	5.84	1.7647	6.50

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
6.51	1.8733	7.15	1.9671	7.79	2.0528	8.66	2.1587	9.94	2.2966
6.52	1.8749	7.16	1.9685	7.80	2.0541	8.68	2.1610	9.96	2.2986
6.53	1.8764	7.17	1.9699	7.81	2.0554	8.70	2.1633	9.98	2.3006
6.54	1.8779	7.18	1.9713	7.82	2.0567	8.72	2.1656	10.00	2.3026
6.55	1.8795	7.19	1.9727	7.83	2.0580	8.74	2.1679	10.25	2.3279
6.56	1.8810	7.20	1.9741	7.84	2.0592	8.76	2.1702	10.50	2.3513
6.57	1.8825	7.21	1.9754	7.85	2.0605	8.78	2.1725	10.75	2.3749
6.58	1.8840	7.22	1.9769	7.86	2.0618	8.80	2.1748	11.00	2.3979
6.59	1.8856	7.23	1.9782	7.87	2.0631	8.82	2.1770	11.25	2.4201
6.60	1.8871	7.24	1.9796	7.88	2.0643	8.84	2.1793	11.50	2.4430
6.61	1.8886	7.25	1.9810	7.89	2.0656	8.86	2.1815	11.75	2.4636
6.62	1.8901	7.26	1.9824	7.90	2.0669	8.88	2.1838	12.00	2.4849
6.63	1.8916	7.27	1.9838	7.91	2.0681	8.90	2.1861	12.25	2.5052
6.64	1.8931	7.28	1.9851	7.92	2.0694	8.92	2.1883	12.50	2.5262
6.65	1.8946	7.29	1.9865	7.93	2.0707	8.94	2.1905	12.75	2.5455
6.66	1.8961	7.30	1.9879	7.94	2.0719	8.96	2.1928	13.00	2.5649
6.67	1.8976	7.31	1.9892	7.95	2.0732	8.98	2.1950	13.25	2.5840
6.68	1.8991	7.32	1.9906	7.96	2.0744	9.00	2.1972	13.50	2.6027
6.69	1.9006	7.33	1.9920	7.97	2.0757	9.02	2.1994	13.75	2.6211
6.70	1.9021	7.34	1.9933	7.98	2.0769	9.04	2.2017	14.00	2.6391
6.71	1.9036	7.35	1.9947	7.99	2.0782	9.06	2.2039	14.25	2.6567
6.72	1.9051	7.36	1.9961	8.00	2.0794	9.08	2.2061	14.50	2.6740
6.73	1.9066	7.37	1.9974	8.01	2.0807	9.10	2.2083	14.75	2.6913
6.74	1.9081	7.38	1.9988	8.02	2.0819	9.12	2.2105	15.00	2.7081
6.75	1.9095	7.39	2.0001	8.03	2.0832	9.14	2.2127	15.50	2.7408
6.76	1.9110	7.40	2.0015	8.04	2.0844	9.16	2.2148	16.00	2.7726
6.77	1.9125	7.41	2.0028	8.05	2.0857	9.18	2.2170	16.50	2.8034
6.78	1.9140	7.42	2.0041	8.06	2.0869	9.20	2.2192	17.00	2.8332
6.79	1.9155	7.43	2.0055	8.07	2.0882	9.22	2.2214	17.50	2.8621
6.80	1.9169	7.44	2.0069	8.08	2.0894	9.24	2.2235	18.00	2.8904
6.81	1.9184	7.45	2.0082	8.09	2.0906	9.26	2.2257	18.50	2.9178
6.82	1.9199	7.46	2.0096	8.10	2.0919	9.28	2.2279	19.00	2.9444
6.83	1.9213	7.47	2.0108	8.11	2.0931	9.30	2.2300	19.50	2.9703
6.84	1.9228	7.48	2.0122	8.12	2.0943	9.32	2.2322	20.00	2.9957
6.85	1.9242	7.49	2.0136	8.13	2.0956	9.34	2.2343	21	3.0445
6.86	1.9257	7.50	2.0149	8.14	2.0968	9.36	2.2364	22	3.0910
6.87	1.9272	7.51	2.0162	8.15	2.0980	9.38	2.2386	23	3.1355
6.88	1.9286	7.52	2.0176	8.16	2.0992	9.40	2.2407	24	3.1781
6.89	1.9301	7.53	2.0189	8.17	2.1005	9.42	2.2428	25	3.2189
6.90	1.9315	7.54	2.0202	8.18	2.1017	9.44	2.2450	26	3.2581
6.91	1.9330	7.55	2.0215	8.19	2.1029	9.46	2.2471	27	3.2958
6.92	1.9344	7.56	2.0229	8.20	2.1041	9.48	2.2492	28	3.3322
6.93	1.9359	7.57	2.0242	8.22	2.1066	9.50	2.2513	29	3.3673
6.94	1.9373	7.58	2.0255	8.24	2.1090	9.52	2.2534	30	3.4012
6.95	1.9387	7.59	2.0268	8.26	2.1114	9.54	2.2555	31	3.4340
6.96	1.9402	7.60	2.0281	8.28	2.1138	9.56	2.2576	32	3.4657
6.97	1.9416	7.61	2.0295	8.30	2.1163	9.58	2.2597	33	3.4965
6.98	1.9430	7.62	2.0308	8.32	2.1187	9.60	2.2618	34	3.5263
6.99	1.9445	7.63	2.0321	8.34	2.1211	9.62	2.2638	35	3.5553
7.00	1.9459	7.64	2.0334	8.36	2.1235	9.64	2.2659	36	3.5835
7.01	1.9473	7.65	2.0347	8.38	2.1258	9.66	2.2680	37	3.6109
7.02	1.9488	7.66	2.0360	8.40	2.1282	9.68	2.2701	38	3.6376
7.03	1.9502	7.67	2.0373	8.42	2.1306	9.70	2.2721	39	3.6636
7.04	1.9516	7.68	2.0386	8.44	2.1330	9.72	2.2742	40	3.6889
7.05	1.9530	7.69	2.0399	8.46	2.1353	9.74	2.2762	41	3.7136
7.06	1.9544	7.70	2.0412	8.48	2.1377	9.76	2.2783	42	3.7377
7.07	1.9559	7.71	2.0425	8.50	2.1401	9.78	2.2803	43	3.7612
7.08	1.9573	7.72	2.0438	8.52	2.1424	9.80	2.2824	44	3.7842
7.09	1.9587	7.73	2.0451	8.54	2.1448	9.82	2.2844	45	3.8067
7.10	1.9601	7.74	2.0464	8.56	2.1471	9.84	2.2865	46	3.8286
7.11	1.9615	7.75	2.0477	8.58	2.1494	9.86	2.2885	47	3.8501
7.12	1.9629	7.76	2.0490	8.60	2.1518	9.88	2.2905	48	3.8712
7.13	1.9643	7.77	2.0503	8.62	2.1541	9.90	2.2925	49	3.8918
7.14	1.9657	7.78	2.0516	8.64	2.1564	9.92	2.2946	50	3.9120

NATURAL TRIGONOMETRICAL FUNCTIONS.

°	M.	Sine.	Co-Vers.	Cosec.	Tang.	Cotan.	Secant.	Ver. Sin.	Cosine.		
0	0	.00000	1.0000	Infinite	.00000	Infinite	1.0000	.00000	1.0000	90	0
	15	.00436	.99564	229.18	.00436	229.18	1.0000	.00001	.99999		45
	30	.00873	.99127	114.59	.00873	114.59	1.0000	.00004	.99996		30
	45	.01309	.98691	76.397	.01309	76.390	1.0001	.00009	.99991		15
1	0	.01745	.98255	57.299	.01745	57.290	1.0001	.00015	.99985	89	0
	15	.02181	.97819	45.840	.02182	45.829	1.0002	.00024	.99976		45
	30	.02618	.97382	38.202	.02618	38.188	1.0003	.00034	.99966		30
	45	.03054	.96946	32.746	.03055	32.730	1.0005	.00047	.99953		15
2	0	.03490	.96510	28.654	.03492	28.636	1.0006	.00061	.99939	88	0
	15	.03926	.96074	25.471	.03929	25.452	1.0008	.00077	.99923		45
	30	.04362	.95638	22.926	.04366	22.904	1.0009	.00095	.99905		30
	45	.04798	.95202	20.843	.04803	20.819	1.0011	.00115	.99885		15
3	0	.05234	.94766	19.107	.05241	19.081	1.0014	.00137	.99863	87	0
	15	.05669	.94331	17.639	.05678	17.611	1.0016	.00161	.99839		45
	30	.06105	.93895	16.380	.06116	16.350	1.0019	.00187	.99813		30
	45	.06540	.93460	15.290	.06554	15.257	1.0021	.00214	.99786		15
4	0	.06976	.93024	14.336	.06993	14.301	1.0024	.00244	.99756	86	0
	15	.07411	.92589	13.494	.07431	13.457	1.0028	.00275	.99725		45
	30	.07846	.92154	12.745	.07870	12.706	1.0031	.00308	.99692		30
	45	.08281	.91719	12.076	.08309	12.035	1.0034	.00348	.99656		15
5	0	.08716	.91284	11.474	.08749	11.430	1.0038	.00381	.99619	85	0
	15	.09150	.90850	10.929	.09189	10.883	1.0042	.00420	.99580		45
	30	.09585	.90415	10.433	.09629	10.385	1.0046	.00460	.99540		30
	45	.10019	.89981	9.9812	.10069	9.9310	1.0051	.00503	.99497		15
6	0	.10453	.89547	9.5668	.10510	9.5144	1.0055	.00548	.99452	84	0
	15	.10887	.89113	9.1855	.10952	9.1309	1.0060	.00594	.99406		45
	30	.11320	.88680	8.8337	.11393	8.7769	1.0065	.00643	.99357		30
	45	.11754	.88246	8.5079	.11836	8.4490	1.0070	.00693	.99307		15
7	0	.12187	.87813	8.2055	.12278	8.1443	1.0075	.00745	.99255	83	0
	15	.12620	.87380	7.9240	.12722	7.8606	1.0081	.00800	.99200		45
	30	.13053	.86947	7.6613	.13165	7.5958	1.0086	.00856	.99144		30
	45	.13485	.86515	7.4156	.13609	7.3479	1.0092	.00913	.99086		15
8	0	.13917	.86083	7.1853	.14054	7.1154	1.0098	.00973	.99027	82	0
	15	.14349	.85651	6.9690	.14499	6.8969	1.0105	.01035	.98965		45
	30	.14781	.85219	6.7655	.14945	6.6912	1.0111	.01098	.98902		30
	45	.15212	.84788	6.5736	.15391	6.4971	1.0118	.01164	.98836		15
9	0	.15643	.84357	6.3924	.15838	6.3138	1.0125	.01231	.98769	81	0
	15	.16074	.83926	6.2211	.16286	6.1402	1.0132	.01300	.98700		45
	30	.16505	.83495	6.0589	.16734	5.9758	1.0139	.01371	.98629		30
	45	.16935	.83065	5.9049	.17183	5.8197	1.0147	.01444	.98556		15
10	0	.17365	.82635	5.7588	.17633	5.6713	1.0154	.01519	.98481	80	0
	15	.17794	.82206	5.6198	.18083	5.5301	1.0162	.01596	.98404		45
	30	.18224	.81776	5.4874	.18534	5.3955	1.0170	.01675	.98325		30
	45	.18652	.81348	5.3612	.18986	5.2672	1.0179	.01755	.98245		15
11	0	.19081	.80919	5.2408	.19438	5.1446	1.0187	.01837	.98163	79	0
	15	.19509	.80491	5.1258	.19891	5.0273	1.0196	.01921	.98079		45
	30	.19937	.80063	5.0158	.20345	4.9152	1.0205	.02008	.97992		30
	45	.20364	.79636	4.9106	.20800	4.8077	1.0214	.02095	.97905		15
12	0	.20791	.79209	4.8097	.21256	4.7046	1.0223	.02185	.97815	78	0
	15	.21218	.78782	4.7130	.21712	4.6057	1.0233	.02277	.97723		45
	30	.21644	.78356	4.6202	.22169	4.5107	1.0243	.02370	.97630		30
	45	.22070	.77930	4.5311	.22628	4.4194	1.0253	.02466	.97534		15
13	0	.22495	.77505	4.4454	.23087	4.3315	1.0263	.02563	.97437	77	0
	15	.22920	.77080	4.3630	.23547	4.2468	1.0273	.02662	.97338		45
	30	.23345	.76655	4.2837	.24008	4.1653	1.0284	.02763	.97237		30
	45	.23769	.76231	4.2072	.24470	4.0867	1.0295	.02866	.97134		15
14	0	.24192	.75808	4.1336	.24933	4.0108	1.0306	.02970	.97030	76	0
	15	.24615	.75385	4.0625	.25397	3.9375	1.0317	.03077	.96923		45
	30	.25038	.74962	3.9939	.25862	3.8667	1.0329	.03185	.96815		30
	45	.25460	.74540	3.9277	.26328	3.7983	1.0341	.03295	.96705		15
15	0	.25882	.74118	3.8637	.26795	3.7320	1.0353	.03407	.96593	75	0
		Cosine.	Ver. Sin.	Secant.	Cotan.	Tang.	Cosec.	Co-Vers.	Sine.	°	M.

From 75° to 90° read from bottom of table upwards.

°	M.	Sine.	Co-Vers.	Cosec.	Tang.	Cotan.	Secant.	Ver. Sin.	Cosine.		
15	0	.25882	.74118	3.8637	.26795	3.7320	1.0353	.03407	.96593	75	0
	15	.26303	.73697	3.8018	.27263	3.6680	1.0365	.03521	.96479		45
	30	.26724	.73276	3.7420	.27732	3.6059	1.0377	.03637	.96363		30
	45	.27144	.72856	3.6840	.28203	3.5457	1.0390	.03754	.96246		15
16	0	.27564	.72436	3.6280	.28674	3.4874	1.0403	.03874	.96126	74	0
	15	.27983	.72017	3.5736	.29147	3.4308	1.0416	.03995	.96005		45
	30	.28402	.71598	3.5209	.29621	3.3759	1.0429	.04118	.95882		30
	45	.28820	.71180	3.4699	.30096	3.3226	1.0443	.04243	.95757		15
17	0	.29237	.70763	3.4203	.30573	3.2709	1.0457	.04370	.95630	73	0
	15	.29654	.70346	3.3722	.31051	3.2205	1.0471	.04498	.95502		45
	30	.30070	.69929	3.3255	.31530	3.1716	1.0485	.04628	.95372		30
	45	.30486	.69514	3.2801	.32010	3.1240	1.0500	.04760	.95240		15
18	0	.30902	.69098	3.2361	.32492	3.0777	1.0515	.04894	.95106	72	0
	15	.31316	.68684	3.1932	.32975	3.0326	1.0530	.05030	.94970		45
	30	.31730	.68270	3.1515	.33459	2.9887	1.0545	.05168	.94832		30
	45	.32144	.67856	3.1110	.33945	2.9459	1.0560	.05307	.94693		15
19	0	.32557	.67443	3.0715	.34433	2.9042	1.0576	.05448	.94552	71	0
	15	.32969	.67031	3.0331	.34921	2.8636	1.0592	.05591	.94409		45
	30	.33381	.66619	2.9957	.35412	2.8239	1.0608	.05736	.94264		30
	45	.33792	.66208	2.9593	.35904	2.7852	1.0625	.05882	.94118		15
20	0	.34202	.65798	2.9238	.36397	2.7475	1.0642	.06031	.93969	70	0
	15	.34612	.65388	2.8892	.36892	2.7106	1.0659	.06181	.93819		45
	30	.35021	.64979	2.8554	.37388	2.6746	1.0676	.06333	.93667		30
	45	.35429	.64571	2.8225	.37887	2.6395	1.0694	.06486	.93514		15
21	0	.35837	.64163	2.7904	.38386	2.6051	1.0711	.06642	.93358	69	0
	15	.36244	.63756	2.7591	.38888	2.5715	1.0729	.06799	.93201		45
	30	.36650	.63350	2.7285	.39391	2.5386	1.0748	.06958	.93042		30
	45	.37056	.62944	2.6986	.39896	2.5065	1.0766	.07119	.92881		15
22	0	.37461	.62539	2.6695	.40403	2.4751	1.0785	.07282	.92718	68	0
	15	.37865	.62135	2.6410	.40911	2.4443	1.0804	.07446	.92554		45
	30	.38268	.61732	2.6131	.41421	2.4142	1.0824	.07612	.92388		30
	45	.38671	.61329	2.5859	.41933	2.3847	1.0844	.07780	.92220		15
23	0	.39073	.60927	2.5593	.42447	2.3559	1.0864	.07950	.92050	67	0
	15	.39474	.60526	2.5333	.42963	2.3276	1.0884	.08121	.91879		45
	30	.39875	60125	2.5078	.43481	2.2998	1.0904	.08294	.91706		30
	45	.40275	.59725	2.4829	.44001	2.2727	1.0925	.08469	.91531		15
24	0	.40674	.59326	2.4586	.44523	2.2460	1.0946	.08645	.91355	66	0
	15	.41072	.58928	2.4348	.45047	2.2199	1.0968	.08824	.91176		45
	30	.41469	.58531	2.4114	.45573	2.1943	1.0989	.09004	.90996		30
	45	41866	.58134	2.3886	.46101	2.1692	1.1011	.09186	.90814		15
25	0	.42262	.57738	2.3662	.46631	2.1445	1.1034	.09369	.90631	65	0
	15	.42657	.57343	2.3443	.47163	2.1203	1.1056	.09554	.90446		45
	30	.43051	.56949	2.3228	.47697	2.0965	1.1079	.09741	.90259		30
	45	.43445	.56555	2.3018	.48234	2.0732	1.1102	.09930	.90070		15
26	0	.43837	.56163	2.2812	.48773	2.0503	1.1126	.10121	.89879	64	0
	15	.44229	.55771	2.2610	.49314	2.0278	1.1150	.10313	.89687		45
	30	.44620	.55380	2.2412	.49858	2.0057	1.1174	.10507	.89493		30
	45	.45010	.54990	2.2217	.50404	1.9840	1.1198	.10702	.89298		15
27	0	.45399	.54601	2.2027	.50952	1.9626	1.1223	.10899	.89101	63	0
	15	.45787	.54213	2.1840	.51503	1.9416	1.1248	.11098	.88902		45
	30	.46175	.53825	2.1657	.52057	1.9210	1.1274	.11299	.88701		30
	45	.46561	.53439	2.1477	.52612	1.9007	1.1300	.11501	.88499		15
28	0	.46947	.53053	2.1300	.53171	1.8807	1.1326	.11705	.88295	62	0
	15	.47332	.52668	2.1127	.53732	1.8611	1.1352	.11911	.88089		45
	30	.47716	.52284	2.0957	.54295	1.8418	1.1379	.12118	.87882		30
	45	.48099	.51901	2.0790	.54862	1.8228	1.1406	.12327	.87673		15
29	0	.48481	.51519	2.0627	.55431	1.8040	1.1433	.12538	.87462	61	0
	15	.48862	.51138	2.0466	.56003	1.7856	1.1461	.12750	.87250		45
	30	.49242	.50758	2.0308	.56577	1.7675	1.1490	.12964	.87036		30
	45	.49622	.50378	2.0152	.57155	1.7496	1.1518	.13180	.86820		15
30	0	.50000	.50000	2.0000	.57735	1.7320	1.1547	.13397	.86603	60	0
		Cosine.	Ver. Sin.	Secant.	Cotan.	Tang.	Cosec.	Co-Vers.	Sine.	°	M.

From 60° to 75° read from bottom of table upwards.

°	M.	Sine.	Co-Vers.	Cosec.	Tang.	Cotan.	Secant.	Ver. Sin.	Cosine.		
30	0	.50000	.50000	2.0000	.57735	1.7320	1.1547	.13397	.86603	60	0
	15	.50377	.49623	1.9850	.58318	1.7147	1.1576	.13616	.86384		45
	30	.50754	.49246	1.9703	.58904	1.6977	1.1606	.13837	.86163		30
	45	.51129	.48871	1.9558	.59494	1.6808	1.1636	.14059	.85941		15
31	0	.51504	.48496	1.9416	.60086	1.6643	1.1666	.14283	.85717	59	0
	15	.51877	.48123	1.9276	.60681	1.6479	1.1697	.14509	.85491		45
	30	.52250	.47750	1.9139	.61280	1.6319	1.1728	.14736	.85264		30
	45	.52621	.47379	1.9004	.61882	1.6160	1.1760	.14965	.85035		15
32	0	.52992	.47008	1.8871	.62487	1.6003	1.1792	.15195	.84805	58	0
	15	.53361	.46639	1.8740	.63095	1.5849	1.1824	.15427	.84573		45
	30	.53730	.46270	1.8612	.63707	1.5697	1.1857	.15661	.84339		30
	45	.54097	.45903	1.8485	.64322	1.5547	1.1890	.15896	.84104		15
33	0	.54464	.45536	1.8361	.64941	1.5399	1 1924	.16133	.83867	57	0
	15	.54829	.45171	1.8238	.65563	1.5253	1.1958	.16371	.83629		45
	30	.55194	.44806	1.8118	.66188	1.5108	1.1992	.16611	.83389		30
	45	.55557	.44443	1.7999	.66818	1.4966	1.2027	.16853	.83147		15
34	0	.55919	.44081	1.7883	.67451	1.4826	1.2062	.17096	.82904	56	0
	15	.56280	.43720	1.7768	.68087	1.4687	1.2098	.17341	.82659		45
	30	.56641	.43359	1.7655	.68728	1.4550	1.2134	.17587	.82413		30
	45	.57000	.43000	1.7544	.69372	1.4415	1.2171	.17835	.82165		15
35	0	.57358	.42642	1.7434	.70021	1.4281	1.2208	.18085	.81915	55	0
	15	.57715	.42285	1.7327	.70673	1.4150	1.2245	.18336	.81664		45
	30	.58070	.41930	1.7220	.71329	1.4019	1.2283	.18588	.81412		30
	45	.58425	.41575	1.7116	.71990	1.3891	1.2322	.18843	.81157		15
36	0	.58779	.41221	1.7013	.72654	1.3764	1.2361	.19098	.80902	54	0
	15	.59131	.40869	1.6912	.73323	1.3638	1.2400	.19356	.80644		45
	30	.59482	.40518	1.6812	.73996	1.3514	1.2440	.19614	.80386		30
	45	.59832	.40168	1.6713	.74673	1.3392	1.2480	.19875	.80125		15
37	0	.60181	.39819	1.6616	.75355	1.3270	1.2521	.20136	.79864	53	0
	15	.60529	.39471	1.6521	.76042	1.3151	1.2563	.20400	.79600		45
	30	.60876	.39124	1.6427	.76733	1.3032	1.2605	.20665	.79335		30
	45	.61222	.38778	1.6334	.77428	1.2915	1.2647	.20931	.79069		15
38	0	.61566	.38434	1.6243	.78129	1.2799	1.2690	.21199	.78801	52	0
	15	.61909	.38091	1.6153	.78834	1.2685	1.2734	.21468	.78532		45
	30	.62251	.37749	1.6064	.79543	1.2572	1.2778	.21739	.78261		30
	45	.62592	.37408	1.5976	.80258	1.2460	1.2822	.22012	.77988		15
39	0	.62932	.37068	1.5890	.80978	1.2349	1.2868	.22285	.77715	51	0
	15	.63271	.36729	1.5805	.81703	1.2239	1.2913	.22561	.77439		45
	30	.63608	.36392	1.5721	.82434	1.2131	1.2960	.22838	.77162		30
	45	.63944	.36056	1.5639	.83169	1.2024	1.3007	.23116	.76884		15
40	0	.64279	.35721	1.5557	.83910	1.1918	1.3054	.23396	.76604	50	0
	15	.64612	.35388	1.5477	.84656	1.1812	1.3102	.23677	.76323		45
	30	.64945	.35055	1.5398	.85408	1.1708	1.3151	.23959	.76041		30
	45	.65276	.34724	1.5320	.86165	1.1606	1.3200	.24244	.75756		15
41	0	.65606	.34394	1.5242	.86929	1.1504	1.3250	.24529	.75471	49	0
	15	.65935	.34065	1.5166	.87698	1.1403	1.3301	.24816	.75184		45
	30	.66262	.33738	1 5092	.88472	1.1303	1.3352	.25104	.74896		30
	45	.66588	.33412	1.5018	.89253	1.1204	1.3404	.25394	.74606		15
42	0	.66913	.33087	1.4945	.90040	1.1106	1.3456	.25686	.74314	48	0
	15	.67237	.32763	1.4873	.90834	1.1009	1.3509	.25978	.74022		45
	30	.67559	.32441	1.4802	.91633	1.0913	1.3563	.26272	.73728		30
	45	.67880	.32120	1.4732	.92439	1.0818	1.3618	.26568	.73432		15
43	0	.68200	.31800	1.4663	.93251	1.0724	1.3673	.26865	.73135	47	0
	15	.68518	.31482	1.4595	.94071	1.0630	1.3729	.27163	.72837		45
	30	.68835	.31165	1.4527	.94896	1.0538	1.3786	.27463	.72537		30
	45	.69151	.30849	1.4461	.95729	1.0446	1.3843	.27764	.72236		15
44	0	.69466	.30534	1.4396	.96569	1.0355	1.3902	.28066	.71934	46	0
	15	.69779	.30221	1.4331	.97416	1.0265	1.3961	.28370	.71630		45
	30	.70091	.29909	1.4267	.98270	1.0176	1.4020	.28675	.71325		30
	45	.70401	.29599	1.4204	.99131	1.0088	1.4081	.28981	.71019		15
45	0	.70711	.29289	1.4142	1.0000	1.0000	1.4142	.29289	.70711	45	0
		Cosine.	Ver. Sin.	Secant.	Cotan.	Tang.	Cosec.	Co-Vers.	Sine.	°	M.

From 45° to 60° read from bottom of table upwards.

LOGARITHMIC SINES, ETC.

Deg.	Sine.	Cosec.	Versin.	Tangent.	Cotan.	Covers.	Secant.	Cosine.	Deg.
0	In.Neg.	Infinite.	In.Neg.	In.Neg.	Infinite.	10.00000	10.00000	10.00000	90
1	8.24186	11.75814	6.18271	8.24192	11.75808	9.99235	10.00007	9.99993	89
2	8.54282	11.45718	6.78474	8.54308	11.45692	9.98457	10.00026	9.99974	88
3	8.71880	11.28120	7.13687	8.71940	11.28060	9.97665	10.00060	9.99940	87
4	8.84358	11.15642	7.38667	8.84464	11.15536	9.96860	10.00106	9.99894	86
5	8.94030	11.05970	7.58039	8.94195	11.05805	9.96040	10.00166	9.99834	85
6	9.01923	10.98077	7.73863	9.02162	10.97838	9.95205	10.00239	9.99761	84
7	9.08589	10.91411	7.87238	9.08914	10.91086	9.94356	10.00325	9.99675	83
8	9.14356	10.85644	7.98820	9.14780	10.85220	9.93492	10.00425	9.99575	82
9	9.19433	10.80567	8.09032	9.19971	10.80029	9.92612	10.00538	9.99462	81
10	9.23967	10.76033	8.18162	9.24632	10.75368	9.91717	10.00665	9.99335	80
11	9.28060	10.71940	8.26418	9.28865	10.71135	9.90805	10.00805	9.99195	79
12	9.31788	10.68212	8.33950	9.32747	10.67253	9.89877	10.00960	9.99040	78
13	9.35209	10.64791	8.40875	9.36336	10.63664	9.88933	10.01128	9.98872	77
14	9.38368	10.61632	8.47282	9.39677	10.60323	9.87971	10.01310	9.98690	76
15	9.41300	10.58700	8.53243	9.42805	10.57195	9.86992	10.01506	9.98494	75
16	9.44034	10.55966	8.58814	9.45750	10.54250	9.85996	10.01716	9.98284	74
17	9.46594	10.53406	8.64043	9.48534	10.51466	9.84981	10.01940	9.98060	73
18	9.48998	10.510·2	8.68969	9.51178	10.48822	9.83947	10.02179	9.97821	72
19	9.51264	10.48736	8.73625	9.53697	10.46303	9.82894	10.02433	9.97567	71
20	9.53405	10.46595	8.78037	9.56107	10.43893	9.81821	10.02701	9.97299	70
21	9.55433	10.44567	8.82230	9.58418	10.41582	9.80729	10.02985	9.97015	69
22	9.57358	10.42642	8.86223	9.60641	10.39359	9.79615	10.03283	9.96717	68
23	9.59188	10.40812	8.90034	9.62785	10.37215	9.78481	10.03597	9.96403	67
24	9.60931	10.39069	8.93679	9.64858	10.35142	9.77325	10.03927	9.96073	66
25	9.62595	10.37405	8.97170	9.66867	10.33133	9.76146	10.04272	9.95728	65
26	9.64184	10.35816	9.00521	9.68818	10.31182	9.74945	10.04634	9.95366	64
27	9.65705	10.34295	9.03740	9.70717	10.29283	9.73720	10.05012	9.94988	63
28	9.67161	10.32839	9.06838	9.72567	10.27433	9.72471	10.05407	9.94593	62
29	9.68557	10.31443	9.09823	9.74375	10.25625	9.71197	10.05818	9.94182	61
30	9.69897	10.30103	9.12702	9.76144	10.23856	9.69897	10.06247	9.93753	60
31	9.71184	10.28816	9.15483	9.77877	10.22123	9.68571	10.06693	9.93307	59
32	9.72421	10.27579	9.18171	9.79579	10.20421	9.67217	10.07158	9.92842	58
33	9.73611	10.26389	9.20771	9.81252	10.18748	9.65836	10.07641	9.92359	57
34	9.74756	10.25244	9.23290	9.82899	10.17101	9.64425	10.08148	9.91857	56
35	9.75859	10.24141	9.25731	9.84523	10.15477	9.62984	10.08664	9.91336	55
36	9.76922	10.23078	9.28099	9.86126	10.13874	9.61512	10.09204	9.90796	54
37	9.77946	10.22054	9.30398	9.87711	10.12289	9.60008	10.09765	9.90235	53
38	9.78934	10.21066	9.32631	9.89281	10.10719	9.58471	10.10347	9.89653	52
39	9.79887	10.20113	9.34802	9.90837	10.09163	9.56900	10.10950	9.89050	51
40	9.80807	10.19193	9.36918	9.92381	10.07619	9.55293	10.11575	9.88425	50
41	9.81694	10.18306	9.38968	9.93916	10.06084	9.53648	10.12222	9.87778	49
42	9.82551	10.17449	9.40969	9.95444	10.04556	9.51966	10.12893	9.87107	48
43	9.83378	10.16622	9.42918	9.96966	10.03034	9.50243	10.13587	9.86413	47
44	9.84177	10.15823	9.44818	9.98484	10.01516	9.48479	10.14307	9.85693	46
45	9.84949	10.15052	9.46671	10.00000	10.00000	9.46671	10.15052	9.84949	45
	Cosine.	Secant.	Covers.	Cotan.	Tangent.	Versin.	Cosec.	Sine.	

From 45° to 90° read from bottom of table upwards.

MATERIALS.

THE CHEMICAL ELEMENTS.

The Common Elements (42).

Chemical Symbol.	Name.	Atomic Weight.	Chemical Symbol.	Name.	Atomic Weight.	Chemical Symbol.	Name.	Atomic Weight.
Al	Aluminum	27.1	F	Fluorine	19.	Pd	Palladium	106.
Sb	Antimony	120.4	Au	Gold	197.2	P	Phosphorus	31.
As	Arsenic	75.1	H	Hydrogen	1.01	Pt	Platinum	194.9
Ba	Barium	137.4	I	Iodine	126.8	K	Potassium	39.1
Bi	Bismuth	208.1	Ir	Iridium	193.1	Si	Silicon	28.4
B	Boron	10.9	Fe	Iron	56.	Ag	Silver	107.9
Br	Bromine	79.9	Pb	Lead	206.9	Na	Sodium	23.
Cd	Cadmium	111.9	Li	Lithium	7.03	Sr	Strontium	87.6
Ca	Calcium	40.1	Mg	Magnesium	24.3	S	Sulphur	32.1
C	Carbon	12.	Mn	Manganese	55.	Sn	Tin	119.
Cl	Chlorine	35.4	Hg	Mercury	200.	Ti	Titanium	48.1
Cr	Chromium	52.1	Ni	Nickel	58.7	W	Tungsten	184.8
Co	Cobalt	59.	N	Nitrogen	14.	Va	Vanadium	51.4
Cu	Copper	63.6	O	Oxygen	16.	Zn	Zinc	65.4

The atomic weights of many of the elements vary in the decimal place as given by different authorities. The above are the most recent values referred to O = 16 and H = 1.008. When H is taken as 1, O = 15.879, and the other figures are diminished proportionately. (See *Jour. Am. Chem. Soc.*, March, 1896.)

The Rare Elements (27).

Beryllium, Be.	Glucinum, G.	Rubidium, Rb.	Thallium, Tl.
Cæsium, Cs.	Indium, In.	Ruthenium, Ru.	Thorium, Th.
Cerium, Ce.	Lanthanum, La.	Samarium, Sm.	Uranium, U.
Didymium, D.	Molybdenum, Mo.	Scandium, Sc.	Ytterbium, Yr.
Erbium, E.	Niobium, Nb.	Selenium, Se.	Yttrium, Y.
Gallium, Ga.	Osmium, Os.	Tantalum, Ta.	Zirconium, Zr.
Germanium, Ge.	Rhodium, R.	Tellurium, Te.	

SPECIFIC GRAVITY.

The specific gravity of a substance is its weight as compared with the weight of an equal bulk of pure water.

To find the specific gravity of a substance.

W = weight of body in air; w = weight of body submerged in water.

$$\text{Specific gravity} = \frac{W}{W - w}.$$

If the substance be lighter than the water, sink it by means of a heavier substance, and deduct the weight of the heavier substance.

Specific-gravity determinations are usually referred to the standard of the weight of water at 62° F., 62.355 lbs. per cubic foot. Some experimenters have used 60° F. as the standard, and others 32° and 39.1° F. There is no general agreement.

Given sp. gr. referred to water at 39.1° F., to reduce it to the standard of 62° F. multiply it by 1.00112.

Given sp. gr. referred to water at 62° F., to find weight per cubic foot multiply by 62.355. Given weight per cubic foot, to find sp. gr. multiply by 0.016037. Given sp. gr., to find weight per cubic inch multiply by .036085.

Weight and Specific Gravity of Metals.

	Specific Gravity. Range according to several Authorities.	Specific Gravity. Approx. Mean Value, used in Calculation of Weight.	Weight per Cubic Foot, lbs.	Weight per Cubic Inch, lbs.
Aluminum	2.56 to 2.71	2.67	166.5	.0963
Antimony	6.66 to 6.86	6.76	421.6	.2439
Bismuth	9.74 to 9.90	9.82	612.4	.3544
Brass: Copper + Zinc 80 20	7.8 to 8.6	8.60	536.3	.3103
70 30		8.40	523.8	.3031
60 40		8.36	521.3	.3017
50 50		8.20	511.4	.2959
Bronze { Copper, 95 to 80; Tin, 5 to 20 }	8.52 to 8.96	8.853	552.	.3195
Cadmium	8.6 to 8.7	8.65	539.	.3121
Calcium	1.58			
Chromium	5.0			
Cobalt	8.5 to 8.6			
Gold, pure	19.245 to 19.361	19.258	1200.9	.6949
Copper	8.69 to 8.92	8.853	552.	.3195
Iridium	22.38 to 23.		1396.	.8076
Iron, Cast	6.85 to 7.48	7.218	450.	.2604
" Wrought	7.4 to 7.9	7.70	480.	.2779
Lead	11.07 to 11.44	11.38	709.7	.4106
Manganese	7. to 8.	8.	499.	.2887
Magnesium	1.69 to 1.75	1.75	109.	.0641
Mercury 32°	13.60 to 13.62	13.62	849.3	.4915
Mercury 60°	13.58	13.58	846.8	.4900
Mercury 212°	13.37 to 13.38	13.38	834.4	.4828
Nickel	8.279 to 8.93	8.8	548.7	.3175
Platinum	20.33 to 22.07	21.5	1347.0	.7758
Potassium	0.865			
Silver	10.474 to 10.511	10.505	655.1	.3791
Sodium	0.97			
Steel	7.69* to 7.932†	7.854	489.6	.2834
Tin	7.291 to 7.409	7.350	458.3	.2652
Titanium	5.3			
Tungsten	17. to 17.6			
Zinc	6.86 to 7.20	7.00	436.5	.2526

* Hard and burned.

† Very pure and soft. The sp. gr. decreases as the carbon is increased.

In the first column of figures the lowest are usually those of cast metals, which are more or less porous; the highest are of metals finely rolled or drawn into wire.

Specific Gravity of Liquids at 60° F.

Acid, Muriatic	1.200	Oil, Olive	.92
" Nitric	1.217	" Palm	.97
" Sulphuric	1.849	" Petroleum	.78 to .88
Alcohol, pure	.794	" Rape	.92
" 95 per cent	.816	" Turpentine	.87
" 50 " "	.934	" Whale	.92
Ammonia, 27.9 per cent	.891	Tar	1.
Bromine	2.97	Vinegar	1.08
Carbon disulphide	1.26	Water	1.
Ether, Sulphuric	.72	" sea	1.026 to 1.03
Oil, Linseed	.94		

Compression of the following Fluids under a Pressure of 15 lbs. per Square Inch.

Water	.00004663	Ether	.00006158
Alcohol	.0000216	Mercury	.00000265

The Hydrometer.

The hydrometer is an instrument for determining the density of liquids. It is usually made of glass, and consists of three parts: (1) the upper part, a graduated stem or fine tube of uniform diameter; (2) a bulb, or enlargement of the tube, containing air; and (3) a small bulb at the bottom, containing shot or mercury which causes the instrument to float in a vertical position. The graduations are figures representing either specific gravities, or the numbers of an arbitrary scale, as in Baumé's, Twaddell's, Beck's, and other hydrometers.

There is a tendency to discard all hydrometers with arbitrary scales and to use only those which read in terms of the specific gravity directly.

Baume's Hydrometer and Specific Gravities Compared.

Degrees Baumé.	Liquids Heavier than Water, sp. gr.	Liquids Lighter than Water, sp. gr.	Degrees Baumé.	Liquids Heavier than Water, sp. gr.	Liquids Lighter than Water, sp. gr.	Degrees Baumé.	Liquids Heavier than Water, sp. gr.	Liquids Lighter than Water, sp. gr.
0	1.000	....	19	1.143	.942	38	1.333	.839
1	1.007	....	20	1.152	.936	39	1.345	.834
2	1.013	....	21	1.160	.930	40	1.357	.830
3	1.020	....	22	1.169	.924	41	1.369	.825
4	1.027	....	23	1.178	.918	42	1.382	.820
5	1.034	....	24	1.188	.913	44	1.407	.811
6	1.041	....	25	1.197	.907	46	1.434	.802
7	1.048	....	26	1.206	.901	48	1.462	.794
8	1.056	....	27	1.216	.896	50	1.490	.785
9	1.063	....	28	1.226	.890	52	1.520	.777
10	1.070	1.000	29	1.236	.885	54	1.551	.768
11	1.078	.993	30	1.246	.880	56	1.583	.760
12	1.086	.986	31	1.256	.874	58	1.617	.753
13	1.094	.980	32	1.267	.869	60	1.652	.745
14	1.101	.973	33	1.277	.864	65	1.747	....
15	1.109	.967	34	1.288	.859	70	1.854	....
16	1.118	.960	35	1.299	.854	75	1.974	....
17	1.126	.954	36	1.310	.849	76	2.000	....
18	1.134	.948	37	1.322	.844			

Specific Gravity and Weight of Wood.

	Specific Gravity.	Avge.	Weight per Cubic Foot, lbs.		Specific Gravity.	Avge.	Weight per Cubic Foot, lbs.
Alder	0.56 to 0.80	.68	42	Hornbeam	.76	.76	47
Apple	.73 to .79	.76	47	Juniper	.56	.56	35
Ash	.60 to .84	.72	45	Larch	.56	.56	35
Bamboo	.31 to .40	.35	22	Lignum vitæ	.65 to 1.33	1.00	62
Beech	.62 to .85	.73	46	Linden	.604		37
Birch	.56 to .74	.65	41	Locust	.728		46
Box	.91 to 1.33	1.12	70	Mahogany	.56 to 1.06	.81	51
Cedar	.49 to .75	.62	39	Maple	.57 to .79	.68	42
Cherry	.61 to .72	.66	41	Mulberry	.56 to .90	.73	46
Chestnut	.46 to .66	.56	35	Oak, Live	.96 to 1.26	1.11	69
Cork	.24	.24	15	" White	.69 to .86	.77	48
Cypress	.41 to .66	.53	33	" Red	.73 to .75	.74	46
Dogwood	.76	.76	47	Pine, White	.35 to .55	.45	28
Ebony	1.13 to 1.33	1.23	76	" Yellow	.46 to .76	.61	38
Elm	.55 to .78	.61	38	Poplar	.38 to .58	.48	30
Fir	.48 to .70	.59	37	Spruce	.40 to .50	.45	28
Gum	.84 to 1.00	.92	57	Sycamore	.59 to .62	.60	37
Hackmatack	.59	.59	37	Teak	.66 to .98	.82	51
Hemlock	.36 to .41	.38	24	Walnut	.50 to .67	.58	36
Hickory	.69 to .94	.77	48	Willow	.49 to .59	.54	34
Holly	.76	.76	47				

Weight and Specific Gravity of Stones, Brick, Cement, etc.

	Pounds per Cubic Foot.	Specific Gravity.
Asphaltum	87	1.39
Brick, Soft	100	1.6
" Common	112	1.79
" Hard	125	2.0
" Pressed	135	2.16
" Fire	140 to 150	2.24 to 2.4
Brickwork in mortar	100	1.6
" " cement	112	1.79
Cement, Rosendale, loose	60	.96
" Portland, "	78	1.25
Clay	120 to 150	1.92 to 2.4
Concrete	120 to 140	1.92 to 2.24
Earth, loose	72 to 80	1.15 to 1.28
" rammed	90 to 110	1.44 to 1.76
Emery	250	4.
Glass	156 to 172	2.5 to 2.75
" flint	180 to 196	2.88 to 3.14
Gneiss / Granite	160 to 170	2.56 to 2.72
Gravel	100 to 120	1.6 to 1.92
Gypsum	130 to 150	2.08 to 2.4
Hornblende	200 to 220	3.2 to 3.52
Lime, quick, in bulk	50 to 55	.8 to .88
Limestone	170 to 200	2.72 to 3.2
Magnesia, Carbonate	150	2.4
Marble	160 to 180	2.56 to 2.88
Masonry, dry rubble	140 to 160	2.24 to 2.56
" dressed	140 to 180	2.24 to 2.88
Mortar	90 to 100	1.44 to 1.6
Pitch	72	1.15
Plaster of Paris	74 to 80	1.18 to 1.28
Quartz	165	2.64
Sand	90 to 110	1.44 to 1.76
Sandstone	140 to 150	2.24 to 2.4
Slate	170 to 180	2.72 to 2.88
Stone, various	135 to 200	2.16 to 3.4
Trap	170 to 200	2.72 to 3.4
Tile	110 to 120	1.76 to 1.92
Soapstone	166 to 175	2.65 to 2.8

Specific Gravity and Weight of Gases at Atmospheric Pressure and 32° F.

(For other temperatures and pressures see pp. 459, 479.)

	Density, Air = 1.	Grammes per Litre.	Lbs. per Cu. Ft.	Cubic Ft. per Lb.
Air	1.0000	1.2931	0.080728	12.387
Oxygen	1.1051	1.4290	0.08921	11.209
Hydrogen	0.0695	0.08987	0.00561	178.23
Nitrogen	0.9714	1.2561	0.07842	12.752
Carbonic oxide, CO	0.9674	1.251	0.07810	12.804
Carbonic acid, CO_2	1.5290	1.977	0.12343	8.103
Marsh-gas, methane, CH_4	0.5560	0.719	0.04488	22.301
Ethylene, C_2H_4	0.9817	1.273	0.07949	12.580

PROPERTIES OF THE USEFUL METALS.

Aluminum, Al.—Atomic weight 27.1. Specific gravity 2.6 to 2.7. The lightest of all the useful metals except magnesium. A soft, ductile, malleable metal, of a white color, approaching silver, but with a bluish cast. Very non-corrosive. Tenacity about one third that of wrought-iron. Formerly a rare metal, but since 1890 its production and use have greatly increased on account of the discovery of cheap processes for reducing it from the ore. Melts at about 1160° F. For further description see Aluminum, under Strength of Materials.

Antimony (Stibium), **Sb.**—At. wt. 120.4. Sp. gr. 6.7 to 6.8. A brittle metal of a bluish-white color and highly crystalline or laminated structure. Melts at 842° F. Heated in the open air it burns with a bluish-white flame. Its chief use is for the manufacture of certain alloys, as type-metal (antimony 1, lead 4), britannia (antimony 1, tin 9), and various anti-friction metals (see Alloys). Cubical expansion by heat from 32° to 212° F., 0.0070. Specific heat .050.

Bismuth, Bi.—At. wt. 208.1. Bismuth is of a peculiar light reddish color, highly crystalline, and so brittle that it can readily be pulverized. It melts at 510° F., and boils at about 2300° F. Sp. gr. 9.823 at 54° F., and 10.055 just above the melting-point. Specific heat about .0301 at ordinary temperatures. Coefficient of cubical expansion from 32° to 212°, 0.0040. Conductivity for heat about 1/56 and for electricity only about 1/80 of that of silver. Its tensile strength is about 6400 lbs. per square inch. Bismuth expands in cooling, and Tribe has shown that this expansion does not take place until after solidification. Bismuth is the most diamagnetic element known, a sphere of it being repelled by a magnet.

Cadmium, Cd.—At. wt. 112. Sp. gr. 8.6 to 8.7. A bluish-white metal, lustrous, with a fibrous fracture. Melts below 500° F. and volatilizes at about 680° F. It is used as an ingredient in some fusible alloys with lead, tin, and bismuth. Cubical expansion from 32° to 212° F., 0.0094.

Copper, Cu.—At. wt. 63.2. Sp. gr. 8.81 to 8.95. Fuses at about 1930° F. Distinguished from all other metals by its reddish color. Very ductile and malleable, and its tenacity is next to iron. Tensile strength 20,000 to 30,000 lbs. per square inch. Heat conductivity 73.6% of that of silver, and superior to that of other metals. Electric conductivity equal to that of gold and silver. Expansion by heat from 32° to 212° F., 0.0051 of its volume. Specific heat .093. (See Copper under Strength of Materials; also Alloys.)

Gold (Aurum). **Au.**—At. wt. 197.2. Sp. gr., when pure and pressed in a die, 19.34. Melts at about 1915° F. The most malleable and ductile of all metals. One ounce Troy may be beaten so as to cover 160 sq. ft. of surface. The average thickness of gold-leaf is 1/282000 of an inch, or 100 sq. ft. per ounce. One grain may be drawn into a wire 500 ft. in length. The ductility is destroyed by the presence of 1/2000 part of lead, bismuth, or antimony. Gold is hardened by the addition of silver or of copper. In U. S. gold coin there are 90 parts gold and 10 parts of alloy, which is chiefly copper with a little silver. By jewelers the fineness of gold is expressed in carats, pure gold being 24 carats, three fourths fine 18 carats, etc.

Iridium.—Iridium is one of the rarer metals. It has a white lustre, resembling that of steel; its hardness is about equal to that of the ruby; in the cold it is quite brittle, but at a white heat it is somewhat malleable. It is one of the heaviest of metals, having a specific gravity of 22.38. It is extremely infusible and almost absolutely inoxidizable.

For uses of iridium, methods of manufacturing it, etc., see paper by W. D. Dudley on the "Iridium Industry," Trans. A. I. M. E. 1884.

Iron (Ferrum), **Fe.**—At. wt. 56. Sp. gr.: Cast, 6.85 to 7.48; Wrought, 7.4 to 7.9. Pure iron is extremely infusible, its melting point being above 3000° F., but its fusibility increases with the addition of carbon, cast iron fusing about 2500° F. Conductivity for heat 11.9, and for electricity 12 to 14.8, silver being 100. Expansion in bulk by heat: cast iron .0033, and wrought iron .0035, from 32° to 212° F. Specific heat: cast iron .1298, wrought iron .1138, steel .1165. Cast iron exposed to continued heat becomes permanently expanded 1½ to 3 per cent of its length. Grate-bars should therefore be allowed about 4 per cent play. (For other properties see Iron and Steel under Strength of Materials.)

Lead (Plumbum), **Pb.**—At. wt. 206.9. Sp. gr. 11.07 to 11.44 by different authorities. Melts at about 625° F., softens and becomes pasty at about 617° F. If broken by a sudden blow when just below the melting-point it is quite brittle and the fracture appears crystalline. Lead is very malleable and ductile, but its tenacity is such that it can be drawn into wire with great difficulty. Tensile strength, 1600 to 2400 lbs. per square inch. Its elasticity is very low, and the metal flows under very slight strain. Lead dissolves to some extent in pure water, but water containing carbonates or sulphates forms over it a film of insoluble salt which prevents further action.

Magnesium, Mg.—At. wt. 24. Sp. gr. 1.69 to 1.75. Silver-white, brilliant, malleable, and ductile. It is one of the lightest of metals, weighing only about two thirds as much as aluminum. In the form of filings, wire, or thin ribbons it is highly combustible, burning with a light of dazzling brilliancy, useful for signal-lights and for flash-lights for photographers. It is nearly non-corrosive, a thin film of carbonate of magnesia forming on exposure to damp air, which protects it from further corrosion. It may be alloyed with aluminum, 5 per cent Mg added to Al giving about as much increase of strength and hardness as 10 per cent of copper. Cubical expansion by heat 0.0083, from 32° to 212° F. Melts at 1200° F. Specific heat .25.

Manganese, Mn.—At. wt. 55. Sp. gr. 7 to 8. The pure metal is not used in the arts, but alloys of manganese and iron, called spiegeleisen when containing below 25 per cent of manganese, and ferro-manganese when containing from 25 to 90 per cent, are used in the manuf cture of steel. Metallic manganese, when alloyed with iron, oxidizes rapidly in the air, and its function in steel manufacture is to remove the oxygen from the bath of steel whether it exists as oxide of iron or as occluded gas.

Mercury (Hydrargyrum), **Hg.**—At. wt. 199.8. A silver-white metal, liquid at temperatures above —39° F., and boils at 680° F. Unchangeable as gold, silver, and platinum in the atmosphere at ordinary temperatures, but oxidizes to the red oxide when near its boiling-point. Sp. gr.: when liquid 13.58 to 13.59, when frozen 14.4 to 14.5. Easily tarnished by sulphur fumes, also by dust, from which it may be freed by straining through a cloth. No metal except iron or platinum should be allowed to touch mercury. The smallest portions of tin, lead, zinc, and even copper to a less extent, cause it to tarnish and lose its perfect liquidity. Coefficient of cubical expansion from 32° to 212° F. .0182; per deg. .000101.

Nickel, Ni.—At. wt. 58.3. Sp. gr. 8.27 to 8.93. A silvery-white metal with a strong lustre, not tarnishing on exposure to the air. Ductile, hard, and as tenacious as iron. It is attracted to the magnet and may be made magnetic like iron. Nickel is very difficult of fusion, melting at about 3000° F. Chiefly used in alloys with copper, as german-silver, nickel silver, etc., and recently in the manufacture of steel to increase its hardness and strength, also for nickel-plating. Cubical expansion from 32° to 212° F., 0.0038. Specific heat .109.

Platinum, Pt.—At. wt. 195. A whitish steel-gray metal, malleable, very ductile, and as unalterable by ordinary agencies as gold. When fused and refined it is as soft as copper. Sp. gr. 21.15. It is fusible only by the oxyhydrogen blowpipe or in strong electric currents. When combined with iridium it forms an alloy of great hardness, which has been used for gun-vents and for standard weights and measures. The most important uses of platinum in the arts are for vessels for chemical laboratories and manufactories, and for the connecting wires in incandescent electric lamps. Cubical expansion from 32° to 212° F., 0.0027, less than that of any other metal except the rare metals, and almost the same as glass.

Silver (Argentum), **Ag.**—At. wt. 107.7. Sp. gr. 10.1 to 11.1, according to condition and purity. It is the whitest of the metals, very malleable and ductile, and in hardness intermediate between gold and copper. Melts at about 1750° F. Specific heat .056. Cubical expansion from 32° to 212° F., 0.0058. As a conductor of electricity it is equal to copper. As a conductor of heat it is superior to all other metals.

Tin (Stannum) **Sn.**—At. wt. 118. Sp. gr. 7.293. White, lustrous, soft, malleable, of little strength, tenacity about 3500 lbs. per square inch. Fuses at 442° F. Not sensibly volatile when melted at ordinary heats. Heat conductivity 14.5, electric conductivity 12.4; silver being 100 in each case. Expansion of volume by heat .0069 from 32° to 212° F. Specific heat .055. Its chief uses are for coating of sheet-iron (called tin plate) and for making alloys with copper and other metals.

Zinc, Zn.—At. wt. 65. Sp. gr. 7.14. Melts at 780° F. Volatilizes and burns in the air when melted, with bluish-white fumes of zinc oxide. It is ductile and malleable, but to a much less extent than copper, and its tenacity, about 5000 to 6000 lbs. per square inch, is about one tenth that of wrought iron. It is practically non corrosive in the atmosphere, a thin film of carbonate of zinc forming upon it. Cubical expansion between 32° and 212° F., 0.0068. Specific heat .096. Electric conductivity 29, heat conductivity 36, silver being 100. Its principal uses are for coating iron surfaces, called "galvanizing," and for making brass and other alloys.

Table Showing the Order of

Malleability.	Ductility.	Tenacity.	Infusibility.
Gold	Platinum	Iron	Platinum
Silver	Silver	Copper	Iron
Aluminum	Iron	Aluminum	Copper
Copper	Copper	Platinum	Gold
Tin	Gold	Silver	Silver
Lead	Aluminum	Zinc	Aluminum
Zinc	Zinc	Gold	Zinc
Platinum	Tin	Tin	Lead
Iron	Lead	Lead	Tin

FORMULÆ AND TABLE FOR CALCULATING THE WEIGHT OF RODS, BARS, PLATES, TUBES, AND SPHERES OF DIFFERENT MATERIALS.

Notation: b = breadth, t = thickness, s = side of square, d = external diameter, d_1 = internal diameter, all in inches.

Sectional areas: of square bars $= s^2$; of flat bars $= bt$; of round rods $= .7854d^2$; of tubes $= .7854(d^2 - d_1^2) = 3.1416(dt - t^2)$.

Volume of 1 foot in length: of square bars $= 12s^2$; of flat bars $= 12bt$; of round bars $= 9.4248d^2$; of tubes $= 9.4248(d^2 - d_1^2) = 37.699(dt - t^2)$, in cu. in.

Weight per foot length = volume × weight per cubic inch of the material.

Weight of a sphere = diam.^{3} × .5236 × weight per cubic inch.

Material.	Specific Gravity.	Weight per cubic foot, lbs.	Weight of Plates 1 inch thick per per sq. ft., lbs.	Weight of Square Bars per foot length, lbs.	Weight of Flat Bars per foot length, lbs.	Weight per cubic inch, lbs.	Relative Weights. Wrought Iron = 1.	Weight of Round Rod per foot length, lbs.	Weight of Spheres or Balls, lbs.
Cast Iron..........	7.218	450.	37.5	$3\frac{1}{8}s^2$	$3\frac{1}{8}bt$	.2604	15–16	$2.454d^2$	$.1363d^3$
Wrought Iron.....	7.7	480.	40.	$3\frac{1}{3}s^2$	$3\frac{1}{3}bt$	.2779	1.	$2.618d^2$	$.1455d^3$
Steel...............	7.854	489.6	40.8	$3.4s^2$	$3.4bt$	.2833	1.02	$2.670d^2$	$.1484d^3$
Copper & Bronze (copper and tin)	8.855	552.	46.	$3.833s^2$	$3.833bt$	.3195	1.15	$3.011d^2$	$.1673d^3$
Brass { 65 Copper.. 35 Zinc.....	8.393	523.2	43.6	$3.633s^2$	$3.633bt$	.3029	1.09	$2.854d^2$	$.1586d^3$
Lead...............	11.38	709.6	59.1	$4.93s^2$	$4.93bt$	.4106	1.48	$3.870d^2$	$.2150d^3$
Aluminum	2.67	166.5	13.9	$1.16s^2$	$1.16bt$	.0963	0.347	$0.908d^2$	$.0504d^3$
Glass....	2.62	163.4	13.6	$1.13s^2$	$1.13bt$	.0945	0.34	$0.891d^2$	$.0495d^3$
Pine Wood, dry...	0.481	30.0	2.5	$0.21s^2$	$0.21bt$	.0174	1–16	$0.164d^2$	$.0091d^3$

Weight per cylindrical in., 1 in. long, = coefficient of d^2 in ninth col. ÷ 12.

For tubes use the coefficient of d^2 in ninth column, as for rods, and multiply it into $(d^2 - d_1^2)$; or take four times this coefficient and multiply it into $(dt - t^2)$.

For hollow spheres use the coefficient of d^3 in the last column and multiply it into $(d^3 - d_1^3)$.

MEASURES AND WEIGHTS OF VARIOUS MATERIALS (APPROXIMATE).

Brickwork.—Brickwork is estimated by the thousand, and for various thicknesses of wall runs as follows:

$8\frac{1}{4}$-in. wall,	or 1 brick in thickness,	14 bricks	per superficial foot.
$12\frac{3}{4}$ " "	" $1\frac{1}{2}$ " "	21 "	" "
17 " "	" 2 " "	28 "	" "
$21\frac{1}{2}$ " "	" $2\frac{1}{2}$ " "	35 "	" "

An ordinary brick measures about $8\frac{1}{4} \times 4 \times 2$ inches, which is equal to 66 cubic inches, or 26.2 bricks to a cubic foot. The average weight is $4\frac{1}{2}$ lbs.

Fuel.—A bushel of bituminous coal weighs 76 pounds and contains 2688 cubic inches = 1.554 cubic feet. 29.47 bushels = 1 gross ton.

A bushel of coke weighs 40 lbs. (35 to 42 lbs.).

One acre of bituminous coal contains 1600 tons of 2240 lbs. per foot of thickness of coal worked. 15 to 25 per cent must be deducted for waste in mining.

41 to 45 cubic feet bituminous coal when broken down	= 1 ton, 2240 lbs.
34 to 41 " " anthracite, prepared for market	= 1 ton, 2240 lbs.
123 " " of charcoal	= 1 ton, 2240 lbs.
70.9 " " " coke	= 1 ton, 2240 lbs.
1 cubic foot of anthracite coal (see also page 625)	= 55 to 66 lbs.
1 " " " bituminous "	= 50 to 55 lbs.
1 " " Cumberland coal	= 53 lbs.
1 " " Cannel coal	= 50.3 lbs.
1 " " charcoal (hardwood)	= 18.5 lbs.
1 " " " (pine)	= 18 lbs.

A bushel of charcoal.—In 1881 the American Charcoal-Iron Workers' Association adopted for use in its official publications for the standard bushel of charcoal 2748 cubic inches, or 20 pounds. A ton of charcoal is to be taken at 2000 pounds. This figure of 20 pounds to the bushel was taken as a fair average of different bushels used throughout the country, and it has since been established by law in some States.

Ores, Earths, etc.

13 cubic feet of ordinary gold or silver ore, in mine	= 1 ton = 2000 lbs.
20 " " " broken quartz	= 1 ton = 2000 lbs.
18 feet of gravel in bank	= 1 ton.
27 cubic feet of gravel when dry	= 1 ton.
25 " " " sand	= 1 ton.
18 " " " earth in bank	= 1 ton.
27 " " " " when dry	= 1 ton.
17 " " " clay	= 1 ton.

Cement.—English Portland, sp. gr. 1.25 to 1.51, per bbl.... 400 to 430 lbs.
Rosendale, U. S., a struck bushel .... 62 to 70 lbs.

Lime.—A struck bushel .... 72 to 75 lbs.

Grain.—A struck bushel of wheat = 60 lbs.; of corn = 56 lbs.; of oats = 30 lbs.

Salt.—A struck bushel of salt, coarse, Syracuse, N. Y. = 56 lbs.; Turk's Island = 76 to 80 lbs.

Weight of Earth Filling.

(From Howe's "Retaining Walls.")

	Average weight in lbs. per cubic foot.
Earth, common loam, loose	72 to 80
" " " shaken	82 to 92
" " " rammed moderately	90 to 100
Gravel	90 to 106
Sand	90 to 106
Soft flowing mud	104 to 120
Sand, perfectly wet	118 to 129

COMMERCIAL SIZES OF IRON BARS.

Flats.

Width.	Thickness.	Width.	Thickness.	Width.	Thickness.
3/4	1/8 to 5/8	1 7/8	1/8 to 1 1/2	4	1/4 to 2
7/8	1/8 to 3/4	2	1/8 to 1 3/4	4 1/2	1/4 to 2
1	1/8 to 15/16	2 1/4	1/4 to 1 3/4	5	1/4 to 2
1 1/8	1/8 to 1	2 3/8	1/4 to 1 1/8	5 1/2	1/4 to 2
1 1/4	1/8 to 1 1/8	2 1/2	3/16 to 1 3/4	6	1/4 to 2
1 3/8	1/8 to 1 1/8	2 5/8	1/4 to 1 1/8	6 1/2	1/4 to 2
1 1/2	1/8 to 1 1/4	2 3/4	1/4 to 1 1/8	7	1/4 to 2
1 5/8	1/4 to 1 1/4	3	1/4 to 2	7 1/2	1/4 to 2
1 3/4	3/16 to 1 1/2	3 1/2	1/4 to 2		

Rounds: ¼ to 1⅝ inches, advancing by 16ths, and 1⅝ to 5 inches by 8ths.

Squares: 5/16 to 1¼ inches, advancing by 16ths, and 1¼ to 3 inches by 8ths.

Half rounds: 7/16, ½, ⅝, 11/16, ¾, 1, 1⅛, 1¼, 1⅜, 1¾, 2 inches.

Hexagons: ¾ to 1⅛ inches, advancing by 8ths.

Ovals: ½ × ¼, ⅝ × 5/16, ¾ × ⅜, ⅞ × 7/16 inch.

Half ovals: ½ × ⅛, ⅝ × 5/32, ¾ × 3/16, ⅞ × 7/32, 1⅛ × ½, 1¾ × ⅝, 1⅞ × ⅝ inch.

Round-edge flats: 1⅛ × ½, 1¾ × ⅝, 1⅞ × ⅝ inch.

Bands: ½ to 1⅛ inches, advancing by 8ths, 7 to 16 B. W. gauge.

1¼ to 5 inches, advancing by 4ths, 7 to 16 gauge up to 3 inches, 4 to 14 gauge, 3¼ to 5 inches.

WEIGHTS OF SQUARE AND ROUND BARS OF WROUGHT IRON IN POUNDS PER LINEAL FOOT.

Iron weighing 480 lbs. per cubic foot. For steel add 2 per cent.

Thickness or Diameter in Inches.	Weight of Square Bar One Foot Long.	Weight of Round Bar One Foot Long.	Thickness or Diameter in Inches.	Weight of Square Bar One Foot Long.	Weight of Round Bar One Foot Long.	Thickness or Diameter in Inches.	Weight of Square Bar One Foot Long.	Weight of Round Bar One Foot Long.
0			11/16	24.08	18.91	⅜	96.30	75.64
1/16	.013	.010	¾	25.21	19.80	7/16	98.55	77.40
⅛	.052	.041	13/16	26.37	20.71	½	100.8	79.19
3/16	.117	.092	⅞	27.55	21.64	9/16	103.1	81.00
¼	.208	.164	15/16	28.76	22.59	⅝	105.5	82.83
5/16	.326	.256	3	30.00	23.56	11/16	107.8	84.69
⅜	.469	.368	1/16	31.26	24.55	¾	110.2	86.56
7/16	.638	.501	⅛	32.55	25.57	13/16	112.6	88.45
½	.833	.654	3/16	33.87	26.60	⅞	115.1	90.36
9/16	1.055	.828	¼	35.21	27.65	15/16	117.5	92.29
⅝	1.302	1.023	5/16	36.58	28.73	6	120.0	94.25
11/16	1.576	1.237	⅜	37.97	29.82	⅛	125.1	98.22
¾	1.875	1.473	7/16	39.39	30.94	¼	130.2	102.3
13/16	2.201	1.728	½	40.83	32.07	⅜	135.5	106.4
⅞	2.552	2.004	9/16	42.30	33.23	½	140.8	110.6
15/16	2.930	2.301	⅝	43.80	34.40	⅝	146.3	114.9
1	3.333	2.618	11/16	45.33	35.60	¾	151.9	119.3
1/16	3.763	2.955	¾	46.88	36.82	⅞	157.6	123.7
⅛	4.219	3.313	13/16	48.45	38.05	7	163.3	128.3
3/16	4.701	3.692	⅞	50.05	39.31	⅛	169.2	132.9
¼	5.208	4.091	15/16	51.68	40.59	¼	175.2	137.6
5/16	5.742	4.510	4	53.33	41.89	⅜	181.3	142.4
⅜	6.302	4.950	1/16	55.01	43.21	½	187.5	147.3
7/16	6.888	5.410	⅛	56.72	44.55	⅝	193.8	152.2
½	7.500	5.890	3/16	58.45	45.91	¾	200.2	157.2
9/16	8.138	6.392	¼	60.21	47.29	⅞	206.7	162.4
⅝	8.802	6.913	5/16	61.99	48.69	8	213.3	167.6
11/16	9.492	7.455	⅜	63.80	50.11	¼	226.9	178.2
¾	10.21	8.018	7/16	65.64	51.55	½	240.8	189.2
13/16	10.95	8.601	½	67.50	53.01	¾	255.2	200.4
⅞	11.72	9.204	9/16	69.39	54.50	9	270.0	212.1
15/16	12.51	9.828	⅝	71.30	56.00	¼	285.2	224.0
2	13.33	10.47	11/16	73.24	57.52	½	300.8	236.3
1/16	14.18	11.14	¾	75.21	59.07	¾	316.9	248.9
⅛	15.05	11.82	13/16	77.20	60.63	10	333.3	261.8
3/16	15.95	12.53	⅞	79.22	62.22	¼	350.2	275.1
¼	16.88	13.25	15/16	81.26	63.82	½	367.5	288.6
5/16	17.83	14.00	5	83.33	65.45	¾	385.2	302.5
⅜	18.80	14.77	1/16	85.43	67.10	11	403.3	316.8
7/16	19.80	15.55	⅛	87.55	68.76	¼	421.9	331.3
½	20.83	16.36	3/16	89.70	70.45	½	440.8	346.2
9/16	21.89	17.19	¼	91.88	72.16	¾	460.2	361.4
⅝	22.97	18.04	5/16	94.08	73.89	12	480.	377.

WEIGHTS OF FLAT ROLLED IRON IN POUNDS PER LINEAL FOOT.
Widths from 1 In, to 12 In.

Iron weighing 480 lbs. per cubic foot. For steel add 2 per cent.

Thickness in Inches.	Widths. 1".	1¼".	1½".	1¾".	2".	2¼".	2½".	2¾".	3".	3¼".	3½".	3¾".	4".	4¼".	4½".	4¾".
1-16	.208	.260	.313	.365	.417	.469	.521	.573	.625	.677	.729	.781	.833	.885	.938	.990
⅛	.417	.521	.625	.729	.833	.938	1.04	1.15	1.25	1.35	1.46	1.56	1.67	1.77	1.88	1.98
3-16	.625	.781	.938	1.09	1.25	1.41	1.56	1.72	1.88	2.03	2.19	2.34	2.50	2.66	2.81	2.97
¼	.833	1.04	1.25	1.46	1.67	1.88	2.08	2.29	2.50	2.71	2.92	3.18	3.33	3.54	3.75	3.96
5-16	1.04	1.30	1.56	1.82	2.08	2.34	2.60	2.86	3.13	3.39	3.65	3.91	4.17	4.43	4.69	4.95
⅜	1.25	1.56	1.88	2.19	2.50	2.81	3.13	3.44	3.75	4.06	4.38	4.69	5.00	5.31	5.63	5.94
7-16	1.46	1.82	2.19	2.55	2.92	3.28	3.65	4.01	4.38	4.74	5.10	5.47	5.83	6.20	6.56	6.93
½	1.67	2.08	2.50	2.92	3.33	3.75	4.17	4.58	5.00	5.42	5.83	6.25	6.67	7.08	7.50	7.92
9-16	1.88	2.34	2.81	3.28	3.75	4.22	4.69	5.16	5.63	6.09	6.56	7.03	7.50	7.97	8.44	8.91
⅝	2.08	2.60	3.13	3.65	4.17	4.69	5.21	5.73	6.25	6.77	7.29	7.81	8.33	8.85	9.38	9.90
11-16	2.29	2.86	3.44	4.01	4.58	5.16	5.73	6.30	6.88	7.45	8.02	8.59	9.17	9.74	10.31	10.89
¾	2.50	3.13	3.75	4.38	5.00	5.63	6.25	6.88	7.50	8.13	8.75	9.38	10.00	10.63	11.25	11.88
13-16	2.71	3.39	4.06	4.74	5.42	6.09	6.77	7.45	8.13	8.80	9.48	10.16	10.83	11.51	12.19	12.86
⅞	2.92	3.65	4.38	5.10	5.83	6.56	7.29	8.02	8.75	9.48	10.21	10.94	11.67	12.40	13.13	13.85
15-16	3.13	3.91	4.69	5.47	6.25	7.08	7.81	8.59	9.38	10.16	10.94	11.72	12.50	13.28	14.06	14.84
1	3.33	4.17	5.00	5.83	6.67	7.50	8.33	9.17	10.00	10.83	11.67	12.50	13.33	14.17	15.00	15.83
1 1-16	3.54	4.43	5.31	6.20	7.08	7.97	8.85	9.74	10.63	11.51	12.40	13.28	14.17	15.05	15.94	16.82
1 ⅛	3.75	4.69	5.63	6.56	7.50	8.44	9.38	10.31	11.25	12.19	13.13	14.06	15.00	15.94	16.88	17.81
1 3-16	3.96	4.95	5.94	6.93	7.92	8.91	9.90	10.89	11.88	12.86	13.85	14.84	15.83	16.82	17.81	18.80
1 ¼	4.17	5.21	6.25	7.29	8.33	9.38	10.42	11.46	12.50	13.54	14.58	15.63	16.67	17.71	18.75	19.79
1 5-16	4.37	5.47	6.56	7.66	8.75	9.84	10.94	12.03	13.13	14.22	15.31	16.41	17.50	18.59	19.69	20.78
1 ⅜	4.58	5.73	6.88	8.02	9.17	10.31	11.46	12.60	13.75	14.90	16.04	17.19	18.33	19.48	20.63	21.77
1 7-16	4.79	5.99	7.19	8.39	9.58	10.78	11.98	13.18	14.38	15.57	16.77	17.97	19.17	20.36	21.56	22.76
1 ½	5.00	6.25	7.50	8.75	10.00	11.25	12.50	13.75	15.00	16.25	17.50	18.75	20.00	21.25	22.50	23.75
1 9-16	5.21	6.51	7.81	9.11	10.42	11.72	13.02	14.32	15.63	16.93	18.23	19.53	20.83	22.14	23.44	24.74
1 ⅝	5.42	6.77	8.13	9.48	10.83	12.19	13.54	14.90	16.25	17.60	18.96	20.31	21.67	23.02	24.38	25.73
1 11-16	5.63	7.03	8.44	9.84	11.25	12.66	14.06	15.47	16.88	18.28	19.69	21.09	22.50	23.91	25.31	26.72
1 ¾	5.83	7.29	8.75	10.21	11.67	13.13	14.58	16.04	17.50	18.96	20.42	21.88	23.33	24.79	26.25	27.71
1 13-16	6.04	7.55	9.06	10.57	12.08	13.59	15.10	16.61	18.13	19.64	21.15	22.66	24.17	25.68	27.19	28.70
1 ⅞	6.25	7.81	9.38	10.94	12.50	14.06	15.63	17.19	18.75	20.31	21.88	23.44	25.00	26.56	28.13	29.69
1 15-16	6.46	8.07	9.69	11.30	12.92	14.53	16.15	17.76	19.38	20.99	22.60	24.22	25.83	27.45	29.06	30.68
2	6.67	8.33	10.00	11.67	13.33	15.00	16.67	18.33	20.00	21.67	23.33	25.00	26.67	28.33	30.00	31.67

Thickness in Inches.	Widths. 5″.	5¼″.	5½″.	5¾″.	6″.	6¼″.	6½″.	6¾″.	7″.	7½″.	8″.	8½″.	9″.	10″.	11″.	12″.
1-16	1.04	1.09	1.15	1.20	1.25	1.30	1.35	1.41	1.46	1.56	1.67	1.77	1.88	2.08	2.29	2.50
⅛	2.08	2.19	2.29	2.40	2.50	2.60	2.71	2.81	2.92	3.13	3.33	3.54	3.75	4.17	4.58	5.00
3-16	3.13	3.28	3.44	3.59	3.75	3.91	4.06	4.22	4.38	4.69	5.00	5.31	5.63	6.25	6.88	7.50
¼	4.17	4.38	4.58	4.79	5.00	5.21	5.42	5.63	5.83	6.25	6.67	7.08	7.50	8.33	9.17	10.00
5-16	5.21	5.47	5.73	5.99	6.25	6.51	6.77	7.08	7.29	7.81	8.33	8.85	9.38	10.42	11.46	12.50
⅜	6.25	6.56	6.88	7.19	7.50	7.81	8.18	8.44	8.75	9.38	10.00	10.63	11.25	12.50	13.75	15.00
7-16	7.29	7.66	8.02	8.39	8.75	9.11	9.48	9.84	10.21	10.94	11.67	12.40	13.18	14.58	16.04	17.50
½	8.33	8.75	9.17	9.58	10.00	10.42	10.83	11.25	11.67	12.50	13.33	14.17	15.00	16.67	18.33	20.00
9-16	9.38	9.84	10.31	10.78	11.25	11.72	12.19	12.66	13.18	14.06	15.00	15.94	16.88	18.75	20.63	22.50
⅝	10.42	10.94	11.46	11.98	12.50	13.02	13.54	14.06	14.58	15.63	16.67	17.71	18.75	20.83	22.92	25.00
11-16	11.46	12.03	12.60	13.18	13.75	14.32	14.90	15.47	16.04	17.19	18.33	19.48	20.63	22.92	25.21	27.50
¾	12.50	13.13	13.75	14.38	15.00	15.63	16.25	16.88	17.50	18.75	20.00	21.25	22.50	25.00	27.50	30.00
13-16	13.54	14.22	14.90	15.57	16.25	16.93	17.60	18.28	18.96	20.31	21.67	23.02	24.38	27.08	29.79	32.50
⅞	14.58	15.31	16.04	16.77	17.50	18.23	18.96	19.69	20.42	21.88	23.33	24.79	26.25	29.17	32.08	35.00
15-16	15.63	16.41	17.19	17.97	18.75	19.53	20.31	21.09	21.88	23.44	25.00	26.56	28.13	31.25	34.38	37.50
1	16.67	17.50	18.33	19.17	20.00	20.83	21.67	22.50	23.33	25.00	26.67	28.33	30.00	33.33	36.67	40.00
1⅛	18.75	19.69	20.63	21.56	22.50	23.44	24.38	25.31	26.25	28.13	30.00	31.88	33.75	37.50	41.25	45.00
1¼	20.83	21.88	22.92	23.96	25.00	26.04	27.08	28.13	29.17	31.25	33.33	35.42	37.50	41.67	45.83	50.00
1⅜	22.92	24.06	25.21	26.35	27.50	28.65	29.79	30.94	32.08	34.38	36.67	38.96	41.25	45.83	50.42	55.00
1½	25.00	26.25	27.50	28.75	30.00	31.25	32.50	33.75	35.00	37.50	40.00	42.50	45.00	50.00	55.00	60.00
1⅝	27.08	28.44	29.79	31.15	32.50	33.85	35.21	36.56	37.92	40.63	43.33	46.04	48.75	54.17	59.58	65.00
1¾	29.17	30.63	32.08	33.54	35.00	36.46	37.92	39.38	40.83	43.75	46.67	49.58	52.50	58.33	64.17	70.00
1⅞	31.25	32.81	34.38	35.94	37.50	39.06	40.63	42.19	43.75	46.88	50.00	53.18	56.25	62.50	68.75	75.00
2	33.33	35.00	36.67	38.33	40.00	41.67	43.33	45.00	46.67	50.00	53.33	56.67	60.00	66.67	73.33	80.00

Other sizes.—Weight of other sizes can easily be obtained from the above table by means of combinations or divisions. Thus, for example,

Weight of 12 × 1¼ equals weight of 12 × 1 plus weight of 12 × ¼................ 50.00
Or, twice weight of 12 × ⅝, as it is twice as thick.................................. 50.00
Weight of 6 × 1 15/16 equals midway weight between 6 × 1⅞ and 6 × 2.............. 38.75
Weight of 24 × 15/16, being twice as wide as 12 × 15/16, weighs....................... 75.00

WEIGHT OF IRON AND STEEL SHEETS.

Weights per Square Foot.

(For weights by Decimal Gauge, see page 32.)

Thickness by Birmingham Gauge.				Thickness by American (Brown and Sharpe's) Gauge.			
No. of Gauge.	Thickness in Inches.	Iron.	Steel.	No. of Gauge.	Thickness in Inches.	Iron.	Steel.
0000	.454	18.16	18.52	0000	.46	18.40	18.77
000	.425	17.00	17.34	000	.4096	16.38	16.71
00	.38	15.20	15.50	00	.3648	14.59	14.88
0	.34	13.60	13.87	0	.3249	13.00	13.26
1	.3	12.00	12.24	1	.2893	11.57	11.80
2	.284	11.36	11.59	2	.2576	10.30	10.51
3	.259	10.36	10.57	3	.2294	9.18	9.36
4	.238	9.52	9.71	4	.2043	8.17	8.34
5	.22	8.80	8.98	5	.1819	7.28	7.42
6	.203	8.12	8.28	6	.1620	6.48	6.61
7	.18	7.20	7.34	7	.1443	5.77	5.89
8	.165	6.60	6.73	8	.1285	5.14	5.24
9	.148	5.92	6.04	9	.1144	4.58	4.67
10	.134	5.36	5.47	10	.1019	4.08	4.16
11	.12	4.80	4.90	11	.0907	3.63	3.70
12	.109	4.36	4.45	12	.0808	3.23	3.30
13	.095	3.80	3.88	13	.0720	2.88	2.94
14	.083	3.32	3.39	14	.0641	2.56	2.62
15	.072	2.88	2.94	15	.0571	2.28	2.33
16	.065	2.60	2.65	16	.0508	2.03	2.07
17	.058	2.32	2.37	17	.0453	1.81	1.85
18	.049	1.96	2.00	18	.0403	1.61	1.64
19	.042	1.68	1.71	19	.0359	1.44	1.46
20	.035	1.40	1.43	20	.0320	1.28	1.31
21	.032	1.28	1.31	21	.0285	1.14	1.16
22	.028	1.12	1.14	22	.0253	1.01	1.03
23	.025	1.00	1.02	23	.0226	.904	.922
24	.022	.88	.898	24	.0201	.804	.820
25	.02	.80	.816	25	.0179	.716	.730
26	.018	.72	.734	26	.0159	.636	.649
27	.016	.64	.653	27	.0142	.568	.579
28	.014	.56	.571	28	.0126	.504	.514
29	.013	.52	.530	29	.0113	.452	.461
30	.012	.48	.490	30	.0100	.400	.408
31	.01	.40	.408	31	.0089	.356	.363
32	.009	.36	.367	32	.0080	.320	.326
33	.008	.32	.326	33	.0071	.284	.290
34	.007	.28	.286	34	.0063	.252	.257
35	.005	.20	.204	35	.0056	.224	.228

	Iron.	Steel.
Specific gravity	7.7	7.854
Weight per cubic foot...........	480.	480.6
" " " inch...........	.2778	.2833

As there are many gauges in use differing from each other, and even the thicknesses of a certain specified gauge, as the Birmingham, are not assumed the same by all manufacturers, orders for sheets and wires should always state the weight per square foot, or the thickness in thousandths of an inch.

WEIGHT OF PLATE IRON, PER LINEAL FOOT, IN POUNDS.

(Based on 480 lbs. per Cubic Foot. For Steel add 2 per cent.)

Width in Inches.	Thickness in Inches.															
	1-16	⅛	3-16	¼	5-16	⅜	7-16	½	9-16	⅝	11-16	¾	13-16	⅞	15-16	1
12	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	25.00	27.50	30.00	32.50	35.00	37.50	40.00
13	2.71	5.42	8.13	10.83	13.54	16.25	18.96	21.67	24.38	27.08	29.79	32.50	35.21	37.92	40.63	43.33
14	2.92	5.83	8.75	11.67	14.58	17.50	20.42	23.33	26.25	29.17	32.08	35.00	37.92	40.83	43.75	46.67
15	3.13	6.25	9.38	12.50	15.63	18.75	21.88	25.00	28.13	31.25	34.38	37.50	40.63	43.75	46.88	50.00
16	3.33	6.67	10.00	13.33	16.67	20.00	23.33	26.67	30.00	33.33	36.67	40.00	43.33	46.67	50.00	53.33
17	3.54	7.08	10.63	14.17	17.71	21.25	24.79	28.33	31.88	35.42	38.96	42.50	46.05	49.59	53.13	56.67
18	3.75	7.50	11.25	15.00	18.75	22.50	26.25	30.00	33.75	37.50	41.25	45.00	48.75	52.50	56.25	60.00
19	3.96	7.92	11.87	15.83	19.79	23.75	27.71	31.67	35.67	39.58	43.54	47.50	51.45	55.41	59.37	63.33
20	4.17	8.33	12.50	16.67	20.83	25.00	29.17	33.33	37.50	41.67	45.83	50.00	54.17	58.33	62.50	66.67
21	4.38	8.75	13.13	17.50	21.88	26.25	30.63	35.00	39.38	43.75	48.13	52.50	56.88	61.25	65.63	70.00
22	4.58	9.17	13.75	18.33	22.92	27.50	32.08	36.67	41.25	45.83	50.42	55.00	59.58	64.17	68.75	73.33
23	4.79	9.58	14.38	19.17	23.96	28.75	33.54	38.33	43.13	47.92	52.71	57.50	62.30	67.09	71.88	76.67
24	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00	55.00	60.00	65.00	70.00	75.00	80.00
25	5.21	10.42	15.62	20.83	26.04	31.25	36.46	41.67	46.88	52.08	57.29	62.50	67.70	72.91	78.13	83.33
26	5.42	10.83	16.25	21.67	27.08	32.50	37.92	43.33	48.75	54.17	59.58	65.00	70.42	75.83	81.25	86.67
27	5.63	11.25	16.88	22.50	28.13	33.75	39.38	45.00	50.63	56.25	61.88	67.50	73.13	78.75	84.38	90.00
28	5.83	11.67	17.50	23.33	29.17	35.00	40.83	46.67	52.50	58.33	64.17	70.00	75.84	81.67	87.50	93.33
29	6.04	12.08	18.13	24.17	30.21	36.25	42.29	48.33	54.38	60.42	66.46	72.50	78.55	84.59	90.63	96.67
30	6.25	12.50	18.75	25.00	31.25	37.50	43.75	50.00	56.25	62.50	68.75	75.00	81.25	87.50	93.75	100.0
32	6.67	13.33	20.00	26.67	33.33	40.00	46.67	53.33	60.00	66.67	73.33	80.00	86.67	93.33	100.0	106.7
34	7.08	14.17	21.25	28.33	35.42	42.50	49.58	56.67	63.75	70.83	77.91	85.00	92.08	99.17	106.8	113.3
36	7.50	15.00	22.50	30.00	37.50	45.00	52.50	60.00	67.50	75.00	82.50	90.00	97.50	105.0	112.5	120.0
38	7.92	15.83	23.75	31.67	39.59	47.50	55.42	63.33	71.25	79.17	87.09	95.00	102.9	110.8	118.8	126.7
40	8.33	16.67	25.00	33.33	41.67	50.00	58.33	66.67	75.00	83.33	91.67	100.0	108.3	116.7	125.0	133.3
42	8.75	17.50	26.25	35.00	43.75	52.50	61.25	70.00	78.75	87.50	96.25	105.0	113.7	122.5	131.3	140.0
44	9.17	18.33	27.50	36.67	45.84	55.00	64.17	73.33	82.50	91.67	100.8	110.0	119.2	128.3	137.5	146.7
46	9.58	19.17	28.75	38.33	47.92	57.50	67.08	76.67	86.25	95.83	105.4	115.0	124.6	134.2	143.8	153.3
48	.0.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00	90.00	100.0	110.0	120.0	130.0	140.0	150.0	160.0
50	10.42	20.83	31.25	41.67	52.08	62.50	72.91	83.33	93.75	104.2	114.6	125.0	135.4	145.8	156.3	166.7
52	10.83	21.67	32.50	43.33	54.17	65.00	75.83	86.67	97.50	108.3	119.2	130.0	140.8	151.7	162.5	173.3
54	11.25	22.50	33.75	45.00	56.25	67.50	78.75	90.00	101.3	112.5	123.8	135.0	146.3	157.5	168.8	180.0
56	11.67	23.33	35.00	46.67	58.33	70.00	81.66	93.33	105.0	116.7	128.3	140.0	151.7	163.3	175.0	186.7
58	12.08	24.17	36.25	48.33	60.42	72.50	84.58	96.67	108.8	120.8	132.9	145.0	157.1	169.2	181.3	193.3
60	12.50	25.00	37.50	50.00	62.50	75.00	87.50	100.0	112.5	125.0	137.5	150.0	162.5	175.0	187.5	200.0

WEIGHTS OF STEEL BLOOMS.

Soft steel. 1 cubic inch = 0.284 lb. 1 cubic foot = 490.75 lbs.

Sizes.		Lengths.											
		1″	6″	12″	18″	24″	30″	36″	42″	48″	54″	60″	66″
12″	× 4″	13.63	82	164	245	327	409	491	573	654	736	818	900
11	× 6	18.75	113	225	338	450	563	675	788	900	1013	1125	1238
	× 5	15.62	94	188	281	375	469	562	656	750	843	937	1031
	× 4	12.50	75	150	225	300	375	450	525	600	675	750	825
10	× 7	19.88	120	239	358	477	596	715	835	955	1074	1193	1312
	× 6	17.04	102	204	307	409	511	613	716	818	920	1022	1125
	× 5	14.20	85	170	256	341	426	511	596	682	767	852	937
	× 4	11.36	68	136	205	273	341	409	477	546	614	682	750
	× 3	8.52	51	102	153	204	255	306	358	409	460	511	562
9	× 7	17.89	107	215	322	430	537	644	751	859	966	1073	1181
	× 6	15.34	92	184	276	368	460	552	644	736	828	920	1012
	× 5	12.78	77	153	230	307	383	460	537	614	690	767	844
	× 4	10.22	61	123	184	245	307	368	429	490	552	613	674
8	× 8	18.18	109	218	327	436	545	655	764	873	982	1091	1200
	× 7	15.9	95	191	286	382	477	573	668	763	859	954	1049
	× 6	13.63	82	164	245	327	409	491	573	654	736	818	900
	× 5	11.36	68	136	205	273	341	409	477	546	614	682	750
	× 4	9.09	55	109	164	218	273	327	382	436	491	545	600
7	× 7	13.92	83	167	251	334	418	501	585	668	752	835	919
	× 6	11.93	72	143	215	286	358	430	501	573	644	716	788
	× 5	9.94	60	119	179	238	298	358	417	477	536	596	656
	× 4	7.95	48	96	143	191	239	286	334	382	429	477	525
	× 3	5.96	36	72	107	143	179	214	250	286	322	358	393
6½	× 6½	12.	72	144	216	288	360	432	504	576	648	720	792
	× 4	7.38	44	89	133	177	221	266	310	354	399	443	487
6	× 6	10.22	61	123	184	245	307	368	429	490	551	613	674
	× 5	8.52	51	102	153	204	255	307	358	409	460	511	562
	× 4	6.82	41	82	123	164	204	245	286	327	368	409	450
	× 3	5.11	31	61	92	123	153	184	214	245	276	307	337
5½	× 5½	8.59	52	103	155	206	258	309	361	412	464	515	567
	× 4	6.25	37	75	112	150	188	225	262	300	337	375	412
5	× 5	7.10	43	85	128	170	213	256	298	341	383	426	469
	× 4	5.68	34	68	102	136	170	205	239	273	307	341	375
4½	× 4½	5.75	35	69	104	138	173	207	242	276	311	345	380
	× 4	5.11	31	61	92	123	153	184	215	246	276	307	338
4	× 4	4.54	27	55	82	109	136	164	191	218	246	272	300
	× 3½	3.97	24	48	72	96	119	143	167	181	215	238	262
	× 3	3.40	20	41	61	82	102	122	143	163	184	204	224
3½	× 3½	3.48	21	42	63	84	104	125	146	167	188	209	230
	× 3	2.98	18	36	54	72	89	107	125	143	161	179	197
3	× 3	2.56	15	31	46	61	77	92	108	123	138	154	169

SIZES AND WEIGHTS OF STRUCTURAL SHAPES.

Minimum, Maximum, and Intermediate Weights and Dimensions of Carnegie Steel I-Beams.

Section Index	Depth of Beam.	Weight per Foot.	Flange Width.	Web Thickness.	Section Index	Depth of Beam.	Weight per Foot.	Flange Width.	Web Thickness.
	ins.	lbs.	ins.	ins.		ins.	lbs.	ins.	ins.
B1	24	100	7.25	0.75	B19	6	17.25	3.58	0.48
"	"	95	7.19	0.69	"	"	14.75	3.45	0.35
"	"	90	7.13	0.63	"	"	12.25	3.33	0.23
"	"	85	7.07	0.57	B21	5	14.75	3.29	0.50
"	"	80	7.00	0.50	"	"	12.25	3.15	0.36
B3	20	75	6.40	0.65	"	"	9.75	3.00	0.21
"	"	70	6.33	0.58	B23	4	10.5	2.88	0.41
"	"	65	6.25	0.50	"	"	9.5	2.81	0.34
B80	18	70	6.26	0.72	"	"	8.5	2.73	0.26
"	"	65	6.18	0.64	"	"	7.5	2.66	0.19
"	"	60	6.10	0.56	B77	3	7.5	2.52	0.36
"	"	55	6.00	0.46	"	"	6.5	2.42	0.26
B7	15	55	5.75	0.66	"	"	5.5	2.33	0.17
"	"	50	5.65	0.56	B2	20	100	7.28	0.88
"	"	45	5.55	0.46	"	"	95	7.21	0.81
"	"	42	5.50	0.41	"	"	90	7.14	0.74
B9	12	35	5.09	0.44	"	"	85	7.06	0.66
"	"	31.5	5.00	0.35	"	"	80	7.00	0.60
B11	10	40	5.10	0.75	B4	15	100	6.77	1.18
"	"	35	4.95	0.60	"	"	95	6.68	1.09
"	"	30	4.81	0.46	"	"	90	6.58	0.99
"	"	20	4.66	0.31	"	"	85	6.48	0.89
B13	9	35	4.77	0.73	"	"	80	6.40	0.81
"	"	30	4.61	0.57	B5	15	75	6.29	0.88
"	"	25	4.45	0.41	"	"	70	6.19	0.78
"	"	21	4.33	0.29	"	"	65	6.10	0.69
B15	8	25.5	4.27	0.54	"	"	60	6.00	0.59
"	"	23	4.18	0.45	B8	12	55	5.61	0.82
"	"	20.5	4.09	0.36	"	"	50	5.49	0.70
"	"	18	4.00	0.27	"	"	45	5.37	0.58
B17	7	20	3.87	0.46	"	"	40	5.25	0.46
"	"	17.5	3.76	0.35					
"	"	15	3.66	0.25					

Sections B2, B4, B5, and B8 are "special" beams, the others are "standard."

Sectional area = weight in lbs. per ft. ÷ 3.4, or × 0.2941.
Weight in lbs. per foot = sectional area × 3.4.

Maximum and Minimum Weights and Dimensions of Carnegie Steel Deck Beams.

Section Index.	Depth of Beam, inches.	Weight per Foot, lbs.		Flange Width.		Web Thickness.		Increase of Web and Flange per lb. increase of Weight.
		Min.	Max.	Min.	Max.	Min.	Max.	
B100	10	27.23	35.70	5.25	5.50	.38	.63	.039
B101	9	26.00	30.00	4.91	5.07	.44	.57	.033
B102	8	20.15	24.48	5.00	5.16	.31	.47	.037
B103	7	18.11	22.46	4.87	5.10	.31	.54	.042
B105	6	15.30	18.36	4.38	4.53	.28	.43	.049

Minimum, Maximum, and Intermediate Weights and Dimensions of Carnegie Standard Channels.

Section Index.	Depth of Channel. Inches.	Weight per Foot. Pounds.	Flange Width. Inches.	Web Thickness. Inches.	Section Index.	Depth of Channel. Inches.	Weight per foot. Pounds.	Flange Width. Inches.	Web Thickness. Inches.
C1	15	55	3.82	0.82	C5	8	16.25	2.44	0.40
"	"	50	3.72	0.72	"	"	13.75	2.35	0.31
"	"	45	3.62	0.62	"	"	11.25	2.26	0.22
"	"	40	3.52	0.52	C6	7	19.75	2.51	0.63
"	"	35	3.43	0.43	"	"	17.25	2.41	0.53
"	"	33	3.40	0.40	"	"	14.75	2.30	0.42
C2	12	40	3.42	0.76	"	"	12.25	2.20	0.32
"	"	35	3.30	0.64	"	"	9.75	2.09	0.21
"	"	30	3.17	0.51	C7	6	15.50	2.28	0.56
"	"	25	3.05	0.39	"	"	13	2.16	0.44
"	"	20.5	2.94	0.28	"	"	10.50	2.04	0.32
C3	10	35	3.18	0.82	"	"	8	1.92	0.20
"	"	30	3.04	0.68	C8	5	11.50	2.04	0.48
"	"	25	2.89	0.53	"	"	9	1.89	0.33
"	"	20	2.74	0.38	"	"	6.50	1.75	0.19
"	"	15	2.60	0.24	C9	4	7.25	1.73	0.33
C4	9	25	2.82	0.62	"	"	6.25	1.65	0.25
"	"	20	2.65	0.45	"	"	5.25	1.58	0.18
"	"	15	2.49	0.29	C72	3	6	1.60	0.36
"	"	13.25	2.43	0.23	"	"	5	1.50	0.26
C5	8	21.25	2.62	0.58	"	"	4	1.41	0.17
"	"	18.75	2.53	0.49					

Weights and Dimensions of Carnegie Steel Z-Bars.

Section Index.	Thickness of Metal.	Size.		Weight. Pounds.	Section Index.	Thickness of Metal.	Size.		Weight. Pounds.
		Flanges.	Web.				Flanges.	Web.	
Z1	3/8	3 1/2	6	15.6	Z6	3/4	3 5/16	5 1/16	26.0
"	7/16	3 9/16	6 1/16	18.3	"	13/16	3 3/8	5 1/8	28.3
"	1/2	3 5/8	6 1/8	21.0	Z7	1/4	3 1/16	4	8.2
Z2	9/16	3 1/2	6	22.7	"	5/16	3 1/8	4 1/16	10.3
"	5/8	3 9/16	6 1/16	25.4	"	3/8	3 3/16	4 1/8	12.4
"	11/16	3 5/8	6 1/8	28.0	Z8	7/16	3 1/16	4	13.8
Z3	3/4	3 1/2	6	29.3	"	1/2	3 1/8	4 1/16	15.8
"	13/16	3 9/16	6 1/16	32.0	"	9/16	3 3/16	4 1/8	17.9
"	7/8	3 5/8	6 1/8	34.6	Z9	5/8	3 1/16	4	18.9
Z4	5/16	3 1/4	5	11.6	"	11/16	3 1/8	4 1/16	20.9
"	3/8	3 5/16	5 1/16	13.9	"	3/4	3 3/16	4 1/8	22.9
"	7/16	3 3/8	5 1/8	16.4	Z10	1/4	2 11/16	3	6.7
Z5	1/2	3 1/4	5	17.8	"	5/16	2 3/4	3 1/16	8.4
"	9/16	3 5/16	5 1/16	20.2	Z11	3/8	2 11/16	3	9.7
"	5/8	3 3/8	5 1/8	22.6	"	7/16	2 3/4	3 1/16	11.4
Z6	11/16	3 1/4	5	23.7	Z12	1/2	2 11/16	3	12.5
					"	9/16	2 3/4	3 1/16	14.2

Pencoyd Steel Angles.

EVEN LEGS.

Size in Inches.	Approximate Weight in Pounds per Foot for Various Thicknesses in Inches.														
	1/8 .125	3/16 .1875	1/4 .25	5/16 .3125	3/8 .375	7/16 .4375	1/2 .50	9/16 .5625	5/8 .625	11/16 .6875	3/4 .75	13/16 .8125	7/8 .875	15/16 .9375	1 1.00
8 × 8							26.4	29.8	33.2	36.6	39.0	42.4	45.8	49.3	52.8
6 × 6					14.8	17.3	19.7	22.0	24.4	26.5	28.8	31.0	33.4	35.9	
5 × 5					12.3	14.3	16.3	18.2	20.1	22.0	23.8	25.6	27.4	29.4	
4 × 4				8.2	9.8	11.3	12.8	14.5	15.8	17.2	18.6				
3½ × 3½				7.1	8.5	9.8	11.1	12.4	13.7						
3 × 3			4.9	6.1	7.2	8.3	9.4	10.4	11.5						
2¾ × 2¾			4.5	5.5	6.6	7.7	8.6								
2½ × 2½		3.1	4.1	5.0	5.9	6.9	7.8								
2¼ × 2¼		2.7	3.6	4.5	5.4										
2 × 2		2.5	3.2	4.0	4.8										
1¾ × 1¾		2.1	2.8	3.5	4.1										
1½ × 1½	1.2	1.8	2.4	2.9	3.5										
1¼ × 1¼	1.0	1.5	2.0												
1 × 1	0.8	1.2	1.5												

UNEVEN LEGS.

Size in Inches.	Approximate Weight in Pounds per Foot for Various Thicknesses in Inches.														
	1/8 .125	3/16 .1875	1/4 .25	5/16 .3125	3/8 .375	7/16 .4375	1/2 .50	9/16 .5625	5/8 .625	11/16 .6875	3/4 .75	13/16 .8125	7/8 .875	15/16 .9375	1 1.00
8 × 6							23.0	25.8	28.7	31.7	33.8	36.6	39.5	42.5	45.6
7 × 3½							17.0	19.0	21.0	23.0	24.8	26.7	28.6	30.5	32.5
6½ × 4					12.9	15.0	17.0	19.0	21.2	23.4	25.6	27.8	29.8	31.9	
6 × 4					12.2	14.3	16.3	18.1	20.1	22.0	23.8	25.6	27.4	29.4	
6 × 3½					11.6	13.6	15.5	17.1	19.0	20.8	22.6	24.5	26.5	28.6	
5½ × 3½					11.0	12.8	14.6	16.2	17.9						
5 × 4					11.0	12.8	14.6	16.2	17.9	19.6	21.3				
5 × 3½				8.7	10.3	12.0	13.6	15.2	16.8	18.4	20.0				
5 × 3				8.2	9.7	11.2	12.8	14.2	15.7	17.2	18.7				
4½ × 3				7.7	9.1	10.5	11.9	13.3	14.7	16.0	17.4				
4 × 3½				7.7	9.1	10.5	11.9	13.3	14.7	16.0	17.4				
4 × 3				7.1	8.5	9.8	11.1	12.4	13.8						
3½ × 3				6.6	7.8	9.1	10.3	11.6	12.9						
3½ × 2½			4.9	6.1	7.2	8.3	9.4								
3½ × 2			4.5	5.5	6.6										
3 × 2½			4.5	5.5	6.6	7.7	8.7								
3 × 2			4.1	5.0	5.9	6.9	7.9								
2½ × 2		2.7	3.6	4.5	5.4	6.2	7.0								
2¼ × 1½		2.8	3.0	3.7	4.4										
2 × 1½		2.1	2.9	3.6	4.3										
2 × 1¼		1.9	2.6	3.3	3.9										

ANGLE-COVERS.

Size in Inches.	3/16	¼	5/16	⅜	7/16	½	9/16	⅝
3 × 3		4.8	5.9	7.1	8.2	9.3	10.4	11.5
2¾ × 2¾		4.4	5.5	6.6	7.7	8.8		
2½ × 2½	3.0	4.0	5.0	6.0	7.0	8.1		
2¼ × 2¼	2.6	3.5	4.4	5.3				
2 × 2	2.4	3.2	4.0	4.8				

SQUARE-ROOT ANGLES.

Size in Inches	Approximate Weight in Pounds per Foot for Various Thicknesses in Inches.							Size in Inches.	Approximate Weight in Pounds per Foot for Various Thicknesses in Inches.				
	¼ .25	5/16 .3125	⅜ .375	7/16 .4375	½ .50	9/16 .5625	⅝ .625		⅛ .125	3/16 .1875	¼ .25	5/16 .3125	⅜ .375
4 × 4			9.8	11.4	13.0	14.6	16.2	2 × 2			3.3	4.1	4.9
3½ × 3½		7.1	8.5	9.9	11.4			1¾ × 1¾			2.9	3.6	4.4
3 × 3	4.9	6.1	7.2	8.3	9.4			1½ × 1½		1.80	2.4	3.0	
2¾ × 2¾	4.5	5.6	6.7	7.8	8.9			1¼ × 1¼		1.53	2.04	2.55	
2½ × 2½	4.1	5.1	6.1	7.1	8.2			1 × 1	0.82	1.16	1.53		
2¼ × 2¼	3.6	4.5	5.4										

Pencoyd Tees.

Section Number.	Size in Inches.	Weight per Foot.
EVEN TEES.		
440T	4 × 4	10.9
441T	4 × 4	13.7
335T	3½ × 3½	7.0
336T	3½ × 3½	9.0
337T	3½ × 3½	11.0
330T	3 × 3	6.5
331T	3 × 3	7.7
225T	2½ × 2½	5.0
226T	2½ × 2½	5.8
227T	2½ × 2½	6.6
222T	2¼ × 2¼	4.0
223T	2¼ × 2¼	4.0
220T	2 × 2	3.5
117T	1¾ × 1¾	2.4
115T	1½ × 1½	2.0
112T	1¼ × 1¼	1.5
110T	1 × 1	1.0
UNEVEN TEES.		
64T	6 × 4	17.4
65T	6 × 5¼	39.0
53T	5 × 3½	17.0
54T	5 × 4	15.3
42T	4 × 2	6.5

Section Number.	Size in Inches.	Weight per Foot.
UNEVEN TEES.		
43T	4 × 3	9.0
44T	4 × 3	10.2
45T	4 × 4½	13.5
38T	3½ × 3	7.0
39T	3½ × 3	8.5
30T	3 × 1½	4.0
31T	3 × 2½	5.0
32T	3 × 2½	6.0
33T	3 × 2½	7.0
34T	3 × 2½	8.0
35T	3 × 3½	8.3
36T	3 × 3½	9.5
28T	2¾ × 1¾	6.6
29T	2¾ × 2	7.2
25T	2½ × 1¼	3.3
26T	2½ × 2¾	5.7
27T	2½ × 3	6.0
24T	2¼ × 9/16	2.2
20T	2 × 9/16	2.0
22T	2 × 1 1/16	2.0
21T	2 × 1	2.5
23T	2 × 1½	3.0
17T	1¾ × 1 1/16	1.9
18T	1¾ × 1½	3.5
15T	1½ × 15/16	1.4
12T	1¼ × 15/16	1.2

Pencoyd Miscellaneous Shapes.

Section Number.	Section.	Size in Inches.	Weight per Foot in Pounds.
217M	Heavy rails.	6	50.0
210M	Floor-bars.	3 1/16 × 4 × 3 1/16 × ¼ to ½	7.1 to 14.3
260M	" "	2½ × 6 × 2½ × ¼ to ⅜	9.8 to 14.7

SIZES AND WEIGHTS OF ROOFING MATERIALS.

Corrugated Iron. (The Cincinnati Corrugating Co.)

SCHEDULE OF WEIGHTS.

U. S. Gauge.	Thickness in decimal parts of an inch. Flat.	Weight per 100 sq. ft. Flat, Painted.	Weight per 100 sq. ft. Corrugated and Painted.	Weight per 100 sq. ft. Corrugated and Galvanized.	Weight in oz. per sq. ft. Flat, Galvanized.
No. 28	.015625	62½ lbs.	70 lbs.	86 lbs.	12½ oz.
No. 26	.01875	75 "	84 "	99 "	14½ "
No. 24	.025	100 "	111 "	127 "	18½ "
No. 22	.03125	125 "	138 "	154 "	22½ "
No. 20	.0375	150 "	165 "	182 "	26½ "
No. 18	.05	200 "	220 "	236 "	34½ "
No. 16	.0625	250 "	275 "	291 "	42½ "

The above table is on the basis of sheets rolled according to the U. S. Standard Sheet-metal Gauge of 1893 (see page 31). It is also on the basis of 2½ × ⅝ in. corrugations.

To estimate the weight per 100 sq. ft. on the roof when lapped one corrugation at sides and 4 in. at ends, add approximately 12½% to the weights per 100 sq. ft., respectively, given above.

Corrugations 2½ in. wide by ½ or ⅝ in. deep are recognized generally as the standard size for both roofing and siding; sheets are manufactured usually in lengths 6, 7, 8, 9, and 10 ft., and have a width of 26½ or 26 in. outside width—ten corrugations,—and will cover 2 ft. when lapped one corrugation at sides.

Ordinary corrugated sheets should have a lap of 1½ or 2 corrugations side-lap for roofing in order to secure water-tight side seams; if the roof is rather steep 1½ corrugations will answer.

Some manufacturers make a special high-edge corrugation on sides of sheets (The Cincinnati Corrugating Co.), and thereby are enabled to secure a water-proof side-lap with one corrugation only, thus saving from 6% to 12% of material to cover a given area.

The usual width of flat sheets used for making the above corrugated material is 28¼ inches.

No. 28 gauge corrugated iron is generally used for applying to wooden buildings; but for applying to iron framework No. 24 gauge or heavier should be adopted.

Few manufacturers are prepared to corrugate heavier than No. 20 gauge, but some have facilities for corrugating as heavy as No. 12 gauge.

Ten feet is the limit in length of corrugated sheets.

Galvanizing sheet iron adds about 2½ oz. to its weight per square foot.

Corrugated Arches.

For corrugated curved sheets for floor and ceiling construction in fire-proof buildings, No. 16, 18, or 20 gauge iron is commonly used, and sheets may be curved from 4 to 10 in. rise—the higher the rise the stronger the arch.

By a series of tests it has been demonstrated that corrugated arches give the most satisfactory results with a base length not exceeding 6 ft., and 5 ft. or even less is preferable where great strength is required.

These corrugated arches are usually made with 2½ × ⅝ in. corrugations, and in same width of sheet as above mentioned.

Terra-Cotta.

Porous terra-cotta roofing 3″ thick weighs 16 lbs. per square foot and 2″ thick, 12 lbs. per square foot.

Ceiling made of the same material 2″ thick weighs 11 lbs. per square foot.

Tiles.

Flat tiles 6¼″ × 10½″ × ⅝″ weigh from 1480 to 1850 lbs. per square of roof (100 square feet), the lap being one-half the length of the tile.

Tiles with grooves and fillets weigh from 740 to 925 lbs. per square of roof.

Pan-tiles 14½″ × 10½″ laid 10″ to the weather weigh 850 lbs. per square.

Tin Plate—Tinned Sheet Steel.

The usual sizes for roofing tin are 14″ × 20″ and 20″ × 28″. Without allowance for lap or waste, tin roofing weighs from 50 to 62 lbs. per square. Tin on the roof weighs from 62 to 75 lbs. per square.

Roofing plates or terne plates (steel plates coated with an alloy of tin and lead) are made only in IC and IX thicknesses (27 and 29 Birmingham gauge). "Coke" and "charcoal" tin plates, old names used when iron made with coke and charcoal was used for the tinned plate, are still used in the trade, although steel plates have been substituted for iron: a coke plate now commonly meaning one made of Bessemer steel, and a charcoal plate one of open-hearth steel. The thickness of the tin coating on the plates varies with different "brands."

For valuable information on Tin Roofing, see circulars of Merchant & Co., Philadelphia.

The thickness and weight of tin plates were formerly designated in the trade, both in the United States and England, by letters, such as I.C., D.C., I.X., D.X., etc. A new system was introduced in the United States in 1898, known as the "American base-box system." The base-box is a package containing 32,000 square inches of plate. The actual boxes used in the trade contain 60, 120, or 240 sheets, according to the size. The number of square inches in any given box divided by 32,000 is known as the "box ratio." This ratio multiplied by the weight or price of the base-box gives the weight or price of the given box. Thus the ratio of a box of 120 sheets 14 × 20 in. is 33,600 ÷ 32,000 = 1.05, and the price at $3.00 base is $3.00 × 1.05 = $3.15. The following tables are furnished by the American Tin Plate Co., Chicago, Ill.

Comparison of Gauges and Weights of Tin Plates.

(Based on U. S. Standard Sheet-metal Gauge.)

American Base-Box (32,000 sq. in.) Weight.	Gauge.	English Base-Box (31,360 sq. in.) Gauge.	Weight.	
55 lbs.	No. 38.00	No. 38.00	54.44 lbs.	
60 "	" 36.72	" 37.00	57.84 "	
65 "	" 35.64	" 36.00	61.24 "	
70 "	" 34.92	" 35.00	68.05 "	
75 "	" 34.20	" 34.00	74.85 "	
80 "	" 33.48	" 33.24	80.00 "	
85 "	" 32.76	" 32.50	85.00 "	
90 "	" 32.04	" 31.77	90.00 "	
95 "	" 31.32	" 31.04	95.00 "	
100 "	" 30.80	" 30.65	100.00 "	I.C.L.
110 "	" 30.08	" 30.06	108.00 "	I.C.
130 "	" 28.64	" 28.74	126.00 "	I.X.L.
140 "	" 27.92	" 28.00	136.00 "	I.X.
160 "	" 26.48	" 26.46	157.00 "	I.2X.
180 "	" 25.52	" 25.46	178.00 "	I.3X.
200 "	" 24.80	" 24.68	199.00 "	I.4X.
220 "	" 24.08	" 23.91	220.00 "	I.5X.
240 "	" 23.36	" 23.14	241.00 "	I.6X.
260 "	" 22.64	" 22.37	262.00 "	I.7X.
280 "	" 21.92	" 21.60	283.00 "	I.8X.
140 "	" 27.92	" 27.86	139.00 "	D.C.
180 "	" 25.52	" 25.38	190.00 "	D.X.
220 "	" 24.08	" 24.24	211.00 "	D.2X.
240 "	" 23.36	" 23.12	212.00 "	D.3X.
280 "	" 21.92	" 22.00	273.00 "	D.4X.

American Packages Tin Plate.

Inches Wide.	Length.	Sheets per Box	Inches Wide.	Length.	Sheets per Box
9 to 16⅞	Square.	240	12 " 12¾	17¼ and longer.	120
17 " 25⅞	Square.	120	13 " 13¾	To 16 in. long, incl.	240
26 " 30	Square.	60	13 to 13¾	16¼ and longer.	120
9 " 10¾	All lengths.	240	14 " 14¾	To 15 in. long, incl	240
11 " 11¾	To 18 in. long, incl.	240	14 " 14¾	15¼ and longer.	120
11 " 11¾	18¼ and longer.	120	15 " 25¾	All lengths.	120
12 " 12¾	To 17 in. long, incl.	240	26 " 30	All lengths.	60

Small sizes of light base weights will be packed in double boxes.

Slate.

Number and superficial area of slate required for one square of roof.
(1 square = 100 square feet.)

Dimensions in Inches.	Number per Square.	Superficial Area in Sq. Ft.	Dimensions in Inches.	Number per Square.	Superficial Area in Sq. Ft.
6 × 12	533	267	12 × 18	160	240
7 × 12	457	....	10 × 20	169	235
8 × 12	400	....	11 × 20	154	
9 × 12	355	....	12 × 20	141	
7 × 14	374	254	14 × 20	121	
8 × 14	327	....	16 × 20	137	
9 × 14	291	....	12 × 22	126	231
10 × 14	261	....	14 × 22	108	
8 × 16	277	246	12 × 24	114	228
9 × 16	246	....	14 × 24	98	
10 × 16	221	....	16 × 24	86	
9 × 18	213	240	14 × 26	89	225
10 × 18	192	....	16 × 26	78	

As slate is usually laid, the number of square feet of roof covered by one slate can be obtained from the following formula:

$$\frac{\text{width} \times (\text{length} - 3 \text{ inches})}{288} = \text{the number of square feet of roof covered.}$$

Weight of slate of various lengths and thicknesses required for one square of roof:

Length in Inches.	Weight in Pounds per Square for the Thickness.							
	1/8″	3-16″	1/4″	3/8″	1/2″	5/8″	3/4″	1″
12	483	724	967	1450	1936	2419	2902	3872
14	460	688	920	1379	1842	2301	2760	3683
16	445	667	890	1336	1784	2229	2670	3567
18	434	650	869	1303	1740	2174	2607	3480
20	425	637	851	1276	1704	2129	2553	3408
22	418	626	836	1254	1675	2093	2508	3350
24	412	617	825	1238	1653	2066	2478	3306
26	407	610	815	1223	1631	2039	2445	3263

The weights given above are based on the number of slate required for one square of roof, taking the weight of a cubic foot of slate at 175 pounds.

Pine Shingles.

Number and weight of pine shingles required to cover one square of roof:

Number of Inches Exposed to Weather.	Number of Shingles per Square of Roof.	Weight in Pounds of Shingle on One-square of Roofs.	Remarks.
4	900	216	The number of shingles per square is for common gable-roofs. For hip-roofs add five per cent. to these figures. The weights per square are based on the number per square.
4½	800	192	
5	720	173	
5½	655	157	
6	600	144	

Skylight Glass.

The weights of various sizes and thicknesses of fluted or rough plate-glass required for one square of roof.

Dimensions in Inches.	Thickness in Inches.	Area in Square Feet.	Weight in Lbs. per Square of Roof.
12 × 48	3-16	3.907	250
15 × 60	¼	6.246	350
20 × 100	⅜	13.880	500
94 × 156	½	101.768	700

In the above table no allowance is made for lap.

If ordinary window-glass is used, single thick glass (about 1-16″) will weigh about 82 lbs. per square, and double thick glass (about ⅛″) will weigh about 164 lbs. per square, *no allowance being made for lap.* A box of ordinary window-glass contains as nearly 50 square feet as the size of the panes will admit of. Panes of any size are made to order by the manufacturers, but a great variety of sizes are usually kept in stock, ranging from 6 × 8 inches to 36 × 60 inches.

APPROXIMATE WEIGHTS OF VARIOUS ROOF-COVERINGS.

For preliminary estimates the weights of various roof coverings may be taken as tabulated below (a square of roof = 10 ft. square = 100 sq. ft.):

Name.	Weight in Lbs. per Square of Roof.
Cast-iron plates (⅜″ thick)	1500
Copper	80- 125
Felt and asphalt	100
Felt and gravel	800-1000
Iron, corrugated	100- 375
Iron, galvanized, flat	100- 350
Lath and plaster	900-1000
Sheathing, pine, 1″ thick yellow, northern	300
" " " " " southern	400
Spruce, 1″ thick	200
Sheathing, chestnut or maple, 1″ thick	400
" ash, hickory, or oak, 1″ thick	500
Sheet iron (1-16″ thick)	300
" " " " and laths	500
Shingles, pine	200
Slates (¼″ thick)	900
Skylights (glass 3-16″ to ½″ thick)	250- 700
Sheet lead	500- 800
Thatch	650
Tin	70- 125
Tiles, flat	1500-2000
" (grooves and fillets)	700-1000
" pan	1000
" with mortar	2000-3000
Zinc	100- 200

Approximate Loads per Square Foot for Roofs of Spans under 75 Feet, Including Weight of Truss.

(Carnegie Steel Co.)

Roof covered with corrugated sheets, unboarded	8 lbs.
Roof covered with corrugated sheets, on boards	11 "
Roof covered with slate, on laths	13 "
Same, on boards, 1¼ in. thick	16 "
Roof covered with shingles, on laths	10 "
Add to above if plastered below rafters	10 "
Snow, light, weighs per cubic foot	5 to 12 "

For spans over 75 feet add 4 lbs. to the above loads per square foot.

It is customary to add 30 lbs. per square foot to the above for snow and wind when separate calculations are not made.

WEIGHT OF CAST-IRON PIPES OR COLUMNS.

In Lbs. per Lineal Foot.

Cast iron = 450 lbs. per cubic foot.

Bore.	Thick. of Metal.	Weight per Foot.	Bore.	Thick. of Metal.	Weight per Foot.	Bore.	Thick. of Metal.	Weight per Foot.
Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
3	3/8	12.4	10	3/4	79.2	22	3/4	167.5
	1/2	17.2	10½	1/2	54.0		7/8	196.5
	5/8	22.2		5/8	68.2	23	3/4	174.9
3½	3/8	14.3		3/4	82.8		7/8	205.1
	1/2	19.6	11	1/2	56.5		1	235.6
	5/8	25.3		5/8	71.3	24	3/4	182.2
4	3/8	16.1		3/4	86.5		7/8	213.7
	1/2	22.1	11½	1/2	58.9		1	245.4
	5/8	28.4		5/8	74.4	25	3/4	189.6
4½	3/8	17.9		3/4	90.2		7/8	222.3
	1/2	24.5	12	1/2	61.3		1	255.3
	5/8	31.5		5/8	77.5	26	3/4	197.0
5	3/8	19.8		3/4	93.9		7/8	230.9
	1/2	27.0	12½	1/2	63.8		1	265.1
	5/8	34.4		5/8	80.5	27	3/4	204.3
5½	3/8	21.6		3/4	97.6		7/8	239.4
	1/2	29.4	13	1/2	66.3		1	274.9
	5/8	37.6		5/8	83.6	28	3/4	211.7
6	3/8	23.5		3/4	101.2		7/8	248.1
	1/2	31.8	14	1/2	71.2		1	284.7
	5/8	40.7		5/8	89.7	29	3/4	219.1
6½	3/8	25.3		3/4	108.6		7/8	256.6
	1/2	34.4	15	5/8	95.9		1	294.5
	5/8	43.7		3/4	116.0	30	7/8	265.2
7	3/8	27.1		7/8	136.4		1	304.3
	1/2	36.8	16	5/8	102.0		1⅛	343.7
	5/8	46.8		3/4	123.3	31	7/8	273.8
7½	3/8	29.0		7/8	145.0		1	314.2
	1/2	39.3	17	5/8	108.2		1⅛	354.8
	5/8	49.9		3/4	130.7	32	7/8	282.4
8	3/8	30.8		7/8	153.6		1	324.0
	1/2	41.7	18	5/8	114.3		1⅛	365.8
	5/8	52.9		3/4	138.1	33	7/8	291.0
8½	1/2	44.2		7/8	162.1		1	333.8
	5/8	56.0	19	5/8	120.4		1⅛	376.9
	3/4	68.1		3/4	145.4	34	7/8	299.6
9	1/2	46.6		7/8	170.7		1	343.7
	5/8	59.1	20	5/8	126.6		1⅛	388.0
	3/4	71.8		3/4	152.8	35	7/8	308.1
9½	1/2	49.1		7/8	179.3		1	353.4
	5/8	62.1	21	5/8	132.7		1⅛	399.0
	3/4	75.5		3/4	160.1	36	7/8	316.6
10	1/2	51.5		7/8	187.9		1	363.1
	5/8	65.2	22	5/8	138.8		1⅛	410.0

The weight of the two flanges may be reckoned = weight of one foot.

WEIGHTS OF CAST-IRON PIPE TO LAY 12 FEET LENGTH.

Weights are Gross Weights, including Hub.

(Calculated by F. H. Lewis.)

Thickness.		Inside Diameter.								
Inches.	Equiv. Decimals.	4″	6″	8″	10″	12″	14″	16″	18″	20″
⅜	.375	209	304	400						
13-32	.40625	228	331	435						
7-16	.4375	247	358	470	581	692	804			
15-32	.4687	266	386	505	624	744	863			
½	.5	286	414	541	668	795	922	1050	1177	
17-32	.53125	306	442	577	712	846	983	1118	1258	
9-16	.5625	327	470	613	756	899	1043	1186	1329	
19-32	.59375	......	498	649	801	951	1103	1254	1405	
⅝	.625	......	......	686	845	1003	1163	1322	1481	1640
11-16	.6875	......	......	......	935	1110	1285	1460	1635	1810
¾	.75	......	......	......	1026	1216	1408	1598	1789	1980
13-16	.8125	......	......	......	......	1324	1531	1738	1945	2152
⅞	.875	......	......	......	......	1432	1656	1879	2101	2324
15-16	.9375	......	......	......	......	......	1783	2021	2259	2496
1	1.	......	......	......	......	......	1909	2163	2418	2672
1⅛	1.125	......	......	......	......	......	......	......	2738	3024
1¼	1.25	......	......	......	......	......	......	......	3062	3330
1⅜	1.375	......	......	......	......	......		......	3389	3739

Thickness.		Inside Diameter.								
Inches.	Equiv. Decimals.	22″	24″	27″	30″	33″	36″	42″	48″	60″
⅝	.625	1799								
11-16	.6875	1985	2160	2422						
¾	.75	2171	2362	2648	2984	3221	3507			
13-16	.8125	2359	2565	2875	3186	3496	3866	4426		
⅞	.875	2547	2769	3103	3437	3771	4105	4773	5442	
15-16	.9375	2737	2975	3332	3690	4048	4406	5122	5839	
1	1.	2927	3180	3562	3942	4325	4708	5472	6236	
1⅛	1 125	3310	3598	4027	4456	4886	5316	6176	7034	
1¼	1.25	3698	4016	4492	4970	5447	5924	6880	7833	9742
1⅜	1.375	......	4439	4964	5491	6015	6540	7591	8640	10740
1½	1.5	......	......	5439	6012	6584	7158	8303	9447	11738
1⅝	1 625	......	......	......	6539	7159	7782	9022	10260	12744
1¾	1.75	......	......	......	......	7737	8405	9742	11076	13750
1⅞	1.875	......	......	......	......	......	......	10468	11898	14762
2	2.	......	......	......	......	......	......	11197	12725	15776
2¼	2.25	......	......	......	......	......	......	......	14883	17821
2½	2.5	......	......	......	......	......	......	......	......	19880
2¾	2.75	......	......	......	......	......	......	......	......	21956

CAST-IRON PIPE FITTINGS.
Approximate Weight.
(Addyston Pipe and Steel Co., Cincinnati, Ohio.)

CROSSES.

Size in Inches.	Weight in Lbs.
2	40
3	110
3 × 2	90
4	120
4 × 3	114
4 × 2	90
6	200
6 × 4	160
6 × 3	160
8	325
8 × 6	280
8 × 4	265
8 × 3	225
10	575
10 × 8	415
10 × 6	450
10 × 4	390
10 × 3	350
12	740
12 × 10	650
12 × 8	620
12 × 6	540
12 × 4	525
12 × 3	495
14 × 10	750
14 × 8	635
14 × 6	570
16	1100
16 × 14	1070
16 × 12	1000
16 × 10	1010
16 × 8	825
16 × 6	700
16 × 4	650
18	1560
20	1790
20 × 12	1370
20 × 10	1225
20 × 8	1000
20 × 6	1000
20 × 4	1000
24	2400
24 × 20	2020
24 × 6	1340
30 × 20	2635
30 × 12	2250
30 × 8	1995

TEES.

Size in Inches.	Weight in Lbs.
2	28
3	80
3 × 2	76
4	100
4 × 3	90
4 × 2	87
6	150
6 × 4	145
6 × 3	145
6 × 2	75
8	300
8 × 6	270
8 × 4	250
8 × 3	220
10	390
10 × 8	380
10 × 6	370
10 × 4	350
10 × 3	310
12	600
12 × 10	555
12 × 8	515
12 × 6	550
12 × 4	525
14 × 12	650
14 × 10	650
14 × 8	575
14 × 6	545
14 × 4	525
14 × 3	490
16	790
16 × 14	850
16 × 12	850
16 × 10	850
16 × 8	755
16 × 6	680
16 × 4	655
18	1235
20	1475
20 × 16	1115
20 × 12	1025
20 × 10	1090
20 × 8	900
20 × 6	875
20 × 4	845
20 × 10	1465
24	2000
24 × 12	1425
24 × 8	1375
24 × 6	1450
30	3025
30 × 24	2640
30 × 20	2200
30 × 12	2035
30 × 10	2050
30 × 6	1825
36	5140
36 × 30	4200
36 × 12	4050

45° BRANCH PIPES.

Size in Inches.	Weight in Lbs.
3	90
4	125
6	205
6 × 6 × 4	145
8	330
8 × 6	330
24	2765
24 × 24 × 20	2145
30	4170
36	10300

SLEEVES.

Size in Inches.	Weight in Lbs.
2	10
3	25
4	45
6	65
8	80
10	140
12	190
14	208
16	350
18	375
20	500
24	710
30	965
36	1200

90° ELBOWS.

Size in Inches.	Weight in Lbs.
2	14
3	34
4	55
6	120
8	150
10	260
12	370
14	450
16	660
18	850
20	900
24	1400
30	3000

⅛ or 45° BENDS.

Size in Inches.	Weight in Lbs.
3	30
4	70
6	95
8	150
10	200
12	290
16	510
18	580
20	780
24	1425
30	2000

1/16 or 22½° BENDS.

Size in Inches.	Weight in Lbs.
6	150
8	155
10	205
12	260
16	450
24	1280
30	2000

REDUCERS.

Size in Inches.	Weight in Lbs.
3 × 2	25
4 × 3	42
4 × 2	40
6 × 4	95
6 × 3	70
8 × 6	126
8 × 4	116
8 × 3	116
10 × 8	212
10 × 6	170
10 × 4	160
12 × 10	320
12 × 8	250
12 × 6	250
12 × 4	250
14 × 12	475
14 × 10	440
14 × 8	390
14 × 6	285
16 × 12	475
16 × 10	435
20 × 16	690
20 × 14	575
20 × 12	540
20 × 8	400
24 × 20	990
30 × 24	1305
30 × 18	1385
36 × 30	1730

ANGLE REDUCERS FOR GAS.

Size in Inches.	Weight in Lbs.
6 × 4	95
6 × 3	70

S PIPES.

Size in Inches.	Weight in Lbs.
4	105
6	190

PLUGS.

Size in Inches.	Weight in Lbs.
2	3
3	10
4	10
6	15
8	30
10	46
12	66
14	90
16	100
18	130
20	150
24	185
30	370

CAPS.

Size in Inches.	Weight in Lbs.
3	20
4	25
6	60
8	75
10	100
12	120

DRIP BOXES.

Size in Inches.	Weight in Lbs.
4	295
6	330
8	375
10	875
20	1420

WEIGHTS OF CAST-IRON WATER- AND GAS-PIPE.

(Addyston Pipe and Steel Co., Cincinnati, Ohio.)

Size in Inches.	Standard Water-pipe.			Size in Inches.	Standard Gas-pipe.		
	Per Foot.	Thickness.	Per Length.		Per Foot.	Thickness.	Per Length.
2	7	5/16	63	2	6	¼	48
3	15	⅜	180	3	12½	5/16	150
3	17	½	204				
4	22	½	264	4	17	⅜	204
6	33	½	396	6	30	7/16	360
8	42	½	504	8	40	7/16	480
8	45	½	540				
10	60	9/16	720	10	50	7/16	600
12	75	9/16	900	12	70	½	840
14	117	¾	1400	14	84	9/16	1000
16	125	¾	1500	16	100	9/16	1200
18	167	⅞	2000	18	134	11/16	1600
20	200	15/16	2400	20	150	11/16	1800
24	250	1	3000	24	184	¾	2200
30	350	1⅛	4200	30	250	¾	3000
36	475	1¼	5700	36	350	⅞	4200
42	600	1⅜	7200	42	417	15/16	5000
48	775	1½	9300	48	542	1⅛	6500
60	1330	2	15960	60	900	1⅜	10800
72	1835	2¼	22020	72	1250	1½	15000

THICKNESS OF CAST-IRON WATER-PIPES.

P. H. Baermann, in a paper read before the Engineers' Club of Philadelphia in 1882, gave twenty different formulas for determining the thickness of cast-iron pipes under pressure. The formulas are of three classes:

1. Depending upon the diameter only.
2. Those depending upon the diameter and head, and which add a constant.
3. Those depending upon the diameter and head, contain an additive or subtractive term depending upon the diameter, and add a constant.

The more modern formulas are of the third class, and are as follows:

$t = .00008hd + .01d + .36$	................Shedd,	No. 1.
$t = .00006hd + .0133d + .296$	.............Warren Foundry,	No. 2.
$t = .000058hd + .0152d + .312$	.............Francis,	No. 3.
$t = .000048hd + .013d + .32$	...............Dupuit,	No. 4.
$t = .00004hd + .1\sqrt{d} + .15$	..............Box,	No. 5.
$t = .000135hd + .4 - .0011d$	..............Whitman,	No. 6.
$t = .00006(h + 230)d + .333 - .0033d$	......Fanning,	No. 7.
$t = .00015hd + .25 - .0052d$	..............Meggs,	No. 8.

In which t = thickness in inches, h = head in feet, d = diameter in inches.

Rankine, "Civil Engineering," p. 721, says: "Cast-iron pipes should be made of a soft and tough quality of iron. Great attention should be paid to moulding them correctly, so that the thickness may be exactly uniform all round. Each pipe should be tested for air-bubbles and flaws by ringing it with a hammer, and for strength by exposing it to double the intended greatest working pressure." The rule for computing the thickness of a pipe to resist a given working pressure is $t = \frac{rp}{f}$, where r is the radius in inches, p the pressure in pounds per square inch, and f the tenacity of the iron per square inch. When $f = 18000$, and a factor of safety of 5 is used, the above expressed in terms of d and h becomes

$$t = \frac{.5d\ .433h}{3600} = \frac{dh}{16628} = .00006dh.$$

"There are limitations, however, arising from difficulties in casting, and by the strain produced by shocks, which cause the thickness to be made greater than that given by the above formula."

Thickness of Metal and Weight per Length for Different Sizes of Cast-iron Pipes under Various Heads of Water.

(Warren Foundry and Machine Co.)

Size.	50 Ft. Head.		100 Ft. Head.		150 Ft. Head.		200 Ft. Head.		250 Ft. Head.		300 Ft. Head.	
	Thickness of Metal.	Weight per Length.	Thickness of Metal.	Weight per Length.	Thickness of Metal.	Weight per Length.	Thickness of Metal.	Weight per Length.	Thickness of Metal.	Weight per Length.	Thickness of Metal.	Weight per Length.
3	.344	144	.353	149	.362	153	.371	157	.380	161	.390	166
4	.361	197	.373	204	.385	211	.397	218	.409	226	.421	235
5	.378	254	.393	265	.408	275	.423	286	.438	298	.453	309
6	.393	315	.411	330	.429	345	.447	361	.465	377	.483	393
8	.422	445	.450	475	.474	502	.498	529	.522	557	.546	584
10	.459	600	.489	641	.519	682	.549	723	.579	766	.609	808
12	.491	768	.527	826	.563	885	.599	944	.635	1004	.671	1064
14	.524	952	.566	1031	.608	1111	.650	1191	.692	1272	.734	1352
16	.557	1152	.604	1253	.652	1360	.700	1463	.748	1568	.796	1673
18	.589	1370	.643	1500	.697	1630	.751	1761	.805	1894	.859	2026
20	.622	1603	.682	1763	.742	1924	.802	2086	.862	2248	.922	2412
24	.687	2120	.759	2349	.831	2580	.903	2811	.975	3045	1.047	3279
30	.785	3020	.875	3376	.965	3735	1.055	4095	1.145	4458	1.235	4822
36	.882	4070	.990	4581	1.098	5096	1.206	5618	1.314	6188	1.422	6656
42	.980	5265	1.106	5658	1.232	6657	1.358	7360	1.484	8070	1.610	8804
48	1.078	6616	1.222	7521	1.366	8431	1.510	9340	1.654	10269	1.798	11195

All pipe cast vertically in dry sand; the 3 to 12 inch in lengths of 12 feet, all larger sizes in lengths of 12 feet 4 inches.

Safe Pressures and Equivalent Heads of Water for Cast-iron Pipe of Different Sizes and Thicknesses.

(Calculated by F. H. Lewis, from Fanning's Formula.)

Thickness.	Size of Pipe. 4″		6″		8″		10″		12″		14″		16″		18″		20″	
	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.
7-16	112	258	49	112	18	42												
1-2	224	516	124	286	74	171	44	101	24	55								
9-16	336	774	199	458	130	300	89	205	62	143	42	97						
5-8	....	....	274	631	186	429	132	304	99	228	74	170	56	129	41	95		
11-16	....	....	....	....	....	....	177	408	137	316	106	244	84	194	66	152	51	118
3-4	....	....	....	....	....	....	224	516	174	401	138	316	112	258	91	210	74	170
13-16	....	....	....	....	....	....	....	....	212	488	170	392	140	323	116	267	96	221
7-8	....	....	....	....	....	....	....	....	249	574	202	465	168	387	141	325	119	274
15-16	....	....	....	....	....	....	....	....	....	....	234	538	196	452	166	382	141	325
1	....	....	....	....	....	....	....	....	....	....	266	612	224	516	191	440	164	378
1 1-8	....	....	....	....	....	....	....	....	....	....	....	....	....	....	216	497	209	481
1 1-4	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	256	589

Safe Pressures, etc., for Cast-iron Pipe.—(*Continued.*)

Thickness.	Size of Pipe. 22″		24″		27″		30″		33″		36″		42″		48″		60″	
	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.	Pressure in Pounds.	Head in Feet.
11-16	40	92	30	69	19	64												
3-4	60	138	49	113	36	83	24	55										
13-16	80	184	68	157	52	120	39	90										
7-8	101	233	86	198	69	159	54	124	42	97	32	74						
15-16	121	279	105	242	85	196	69	159	55	127	44	101						
1	142	327	124	286	102	235	84	194	69	159	57	131	38	88	24	55		
1 1-8	182	419	161	371	135	311	114	263	96	221	82	189	59	136	43	99		
1 1-4	224	516	199	458	169	389	144	332	124	286	107	247	81	187	62	143	[illegible]	78
1 3-8	....	....	237	546	202	465	174	401	151	348	132	304	103	237	81	187	49	113
1 1-2	....	....	....	....	236	544	204	470	178	410	157	362	124	286	99	228	64	147
1 5-8	....	....	....	....	....	....	234	538	205	472	182	419	145	334	118	272	79	[illegible]
1 3-4	....	....	....	....	....	....	....	....	233	537	207	477	167	385	136	313	94	217
1 7-8	....	....	....	....	....	....	....	....	....	....	....	....	188	433	155	357	109	251
2	....	....	....	....	....	....	....	....	....	....	....	....	210	484	174	401	124	[illegible]
2 1-8	....	....	....	....	....	....	....	....	....	....	....	....	....	....	193	445	139	[illegible]
2 1-4	....	....	....	....	....	....	....	....	....	....	....	....	....	....	212	488	154	[illegible]
2 1-2	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	134	424
2 3-4	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	214	[illegible]

NOTE.—The absolute safe static pressure which may be put upon pipe is given by the formula $P = \frac{2T}{D} \times \frac{S}{5}$, in which formula P is the pressure per square inch; T, the thickness of the shell; S, the ultimate strength per square inch of the metal in tension; and D, the inside diameter of the pipe. In the tables S is taken as 18000 pounds per square inch, with a working strain of one fifth this amount or 3600 pounds per square inch. The formula for the absolute safe static pressure then is: $P = \frac{7200T}{D}$.

It is, however, usual to allow for "water-ram" by increasing the thickness enough to provide for 100 pounds additional static pressure, and, to insure sufficient metal for good casting and for wear and tear, a further increase equal to $.333\left(1 - \frac{D}{100}\right)$.

The expression for the thickness then becomes:

$$T = \frac{(P + 100)D}{7200} + .333\left(1 - \frac{D}{100}\right),$$

and for safe working pressure

$$P = \frac{7200}{D}\left(T - .333\left(1 - \frac{D}{100}\right)\right) - 100.$$

The additional section provided as above represents an increased value under static pressure for the different sizes of pipe as follows (see table in margin). So that to test the pipes up to one fifth of the ultimate strength of the material, the pressures in the marginal table should be added to the pressure-values given in the table above.

Size of Pipe.	Lbs.
4″	676
6	476
8	346
10	316
12	276
14	248
16	226
18	209
20	196
22	185
24	176
27	165
30	156
33	149
36	143
42	133
48	126
60	116

SHEET-IRON HYDRAULIC PIPE.

(Pelton Water-Wheel Co.)

Weight per foot, with safe head for various sizes of double-riveted pipe.

Diameter of Pipe.	Area of Pipe.	Thickness of Iron by Wire Gauge.	Safe Head in Feet the Pipe will stand.	Weight of Pipe per Lineal Ft.	Diameter of Pipe.	Area of Pipe.	Thickness of Iron by Wire Gauge.	Safe Head in Feet the Pipe will stand.	Weight of Pipe per Lineal Ft.
in.	sq.in.	B.W.G.	feet.	lbs.	in.	sq.in.	B.G.W.	feet.	lbs.
3	7	18	400	2	18	254	16	165	16½
4	12	18	350	2¼	18	254	14	252	20½
4	12	16	525	3	18	254	12	335	27¼
5	20	18	325	3½	18	254	11	424	30
5	20	16	500	4¼	18	254	10	505	34
5	20	14	675	5	20	314	16	148	18
6	28	18	296	4¼	20	314	14	227	22½
6	28	16	487	5¾	20	314	12	346	30
6	28	14	743	7½	20	314	11	380	32½
7	38	18	251	5¼	20	314	10	456	36½
7	38	16	419	6¾	22	380	16	135	20
7	38	14	640	8½	22	380	14	206	24¾
8	50	16	367	7½	22	380	12	316	32¾
8	50	14	560	9½	22	380	11	347	35¾
8	50	12	854	13	22	380	10	415	40
9	63	16	327	8½	24	452	14	188	27¼
9	63	14	499	10¾	24	452	12	290	35½
9	63	12	761	14¼	24	452	11	318	39
10	78	16	295	9¼	24	452	10	379	43½
10	78	14	450	11¾	24	452	8	466	53
10	78	12	687	15¾	26	530	14	175	29¼
10	78	11	754	17½	26	530	12	267	38½
10	78	10	900	19¼	26	530	11	294	42
11	95	16	269	9¾	26	530	10	352	47
11	95	14	412	13	26	530	8	432	57¼
11	95	12	626	17¼	28	615	14	162	31¼
11	95	11	687	18¾	28	615	12	247	41¼
11	95	10	820	21	28	615	11	278	45
12	113	16	246	11¼	28	615	10	327	50¼
12	113	14	377	14	28	615	8	400	61¼
12	113	12	574	18½	30	706	12	231	44
12	113	11	630	19¾	30	706	11	254	48
12	113	10	758	22¾	30	706	10	304	54
13	132	16	228	12	30	706	8	375	65
13	132	14	348	15	30	706	7		74
13	132	12	540	20	36	1017	11		58
13	132	11	588	22	36	1017	10		67
13	132	10	690	24½	36	1017	8		78
14	153	16	211	13	36	1017	7		88
14	153	14	324	16	40	1256	10		71
14	153	12	494	21½	40	1256	8		86
14	153	11	543	23½	40	1256	7		97
14	153	10	648	26	40	1256	6		108
15	176	16	197	13¾	40	1256	4		126
15	176	14	302	17	42	1385	10		74½
15	176	12	460	23	42	1385	8		91
15	176	11	507	24½	42	1385	7		102
15	176	10	606	28	42	1385	6		114
16	201	16	185	14½	42	1385	4		133
16	201	14	283	17¼	42	1385	¼		187
16	201	12	432	24¼	42	1385	3		145
16	201	11	474	26½	42	1385	5-16		177
16	201	10	567	29½	42	1385	⅜		216

STANDARD PIPE FLANGES.

Adopted August, 1894, at a conference of committees of the American Society of Mechanical Engineers, and the Master Steam and Hot Water Fitters' Association, with representatives of leading manufacturers and users of pipe.—Trans. A. S. M. E., xxi. 29. (The standard dimensions given have not yet, 1901, been adopted by some manufacturers on account of their unwillingness to make a change in their patterns.)

The list is divided into two groups; for medium and high pressures, the first ranging up to 75 lbs. per square inch, and the second up to 200 lbs.

Pipe size, inches.	Pipe Thickness, $\frac{P+100}{.4s}d + .333\left(1-\frac{d}{100}\right)$	Thickness, nearest Fraction, inches.	Stress on Pipe per square inch @ 200 lbs.	Radius of Fillet, inches.	Flange Diameters, inches.		Flange Thicknesses at edge, inches.		Width Flange Face, inches.		Bolt Circle Diameter, inches.		Number of Bolts.	Bolt Diameter, inches.	Bolt Length, inches.	Stress on each Bolt, per square inch, at Bottom of Thread @ 200 lbs.
2	.409	7/16	460	1/8		6		5/8		2		4¾	4	5/8	2	825
2½	.429	7/16	550	1/8		7		11/16		2¼		5½	4	5/8	2¼	1050
3	.448	7/16	690	1/8		7½		¾		2¼		6	4	5/8	2½	1340
3½	.466	½	700	1/8		8½		13/16		2½		7	4	5/8	2½	2530
4	.486	½	800	1/8		9		13/16		2½		7½	4	¾	2¾	2100
4½	.498	½	900	1/8		9¼		15/16		2⅝		7¾	8	¾	3	1430
5	.525	½	1000	1/8		10		15/16		2½		8½	8	¾	3	1630
6	.563	9/16	1060	1/8		11		1		2½		9½	8	¾	3	2360
7	.60	⅝	1120	1/8		12½		1 1/16		2¾		10¾	8	¾	3¼	3200
8	.639	⅝	1280	1/8		13½		1⅛		2¾		11¾	8	¾	3½	4190
9	.678	11/16	1310	3/16		15		1⅛		3		13¼	12	¾	3½	3610
10	.713	¾	1330	3/16		16		1 3/16		3		14¼	12	⅞	3⅝	2970
12	.79	13/16	1470	3/16		19		1¼		3½		17	12	⅞	3¾	4280
14	.864	⅞	1600	3/16		21		1⅜		3½		18¾	12	1	4¼	4280
15	.904	15/16	1600	3/16		22¼		1⅜		3⅝		20	16	1	4¼	3660
16	.946	1	1600	3/16		23½		1 7/16		3¾		21¼	16	1	4¼	4210
18	1.02	1 1/16	1690	3/16		25		1 9/16		3½		22¾	16	1⅛	4¾	4540
20	1.09	1⅛	1780	3/16		27½		1 11/16		3¾		25	20	1⅛	5	4490
22	1.18	1 3/16	1850	¼		29½		1 13/16		3¾		27¼	20	1¼	5½	4320
24	1.25	1¼	1920	¼	31½	32	1¼	1⅞	3¾	4	29¼	29½	20	1¼	5⅝	5130
26	1.30	1 5/16	1980	¼	33¾	34¼	1⅜	2	3⅞	4⅛	31¼	31¾	24	1¼	5¾	5030
28	1.38	1⅜	2040	¼	36	36½	1 7/16	2 1/16	4	4¼	33½	34	28	1¼	6	5000
30	1.18	1½	2000	¼	38	38¾	1½	2⅛	4	4⅜	35½	36	28	1⅜	6¼	4590
36	1.71	1¾	1920	¼	44½	45¾	1¾	2⅜	4¼	4⅞	42	42¾	32	1⅜	6½	5790
42	1.87	2	2100	¼	51	52¾	1⅞	2⅝	4½	5⅜	48½	49½	36	1½	7¼	5700
48	2.17	2¼	2130	¼	57½	59½	2	2¾	4¾	5¾	54¾	56	44	1½	7¾	6090

NOTES.—Sizes up to 24 inches are designed for 200 lbs. or less.

Sizes from 24 to 48 inches are divided into two scales, one for 200 lbs., the other for less.

The sizes of bolts given are for high pressure. For medium pressures the diameters are ⅛ in. less for pipes 2 to 20 in. diameter inclusive, and ¼ in. less for larger sizes, except 48-in. pipe, for which the size of bolt is 1⅜ in.

When two lines of figures occur under one heading, the single columns are for both medium and high pressures. Beginning with 24 inches, the left-hand columns are for medium and the right-hand lines are for high pressures.

The sudden increase in diameters at 16 inches is due to the possible insertion of wrought-iron pipe, making with a nearly constant width of gasket a greater diameter desirable.

When wrought-iron pipe is used, if thinner flanges than those given are sufficient, it is proposed that bosses be used to bring the nuts up to the standard lengths. This avoids the use of a reinforcement around the pipe.

Figures in the 3d, 4th, 5th, and last columns refer only to pipe for high pressure.

In drilling valve flanges a vertical line parallel to the spindles should be midway between two holes on the upper side of the flanges.

DIMENSIONS OF PIPE FLANGES AND CAST-IRON PIPES.

(J. E. Codman, Engineers' Club of Philadelphia, 1889.)

Diameter of Pipe.	Diameter of Flange.	Diameter of Bolt Circle.	Diameter of Bolt	Number of Bolts.	Thickness of Flange.	Thickness of Pipe.		Weight per foot without Flange.	Weight of Flange and Bolts.
						Frac.	Dec.		
2	6¼	4¾	¾	4	⅝	⅜	.373	6.96	4.41
3	7½	5⅞	¾	4	⅝	13-32	.396	11.16	5.93
4	9	7	¾	6	11-16	7-16	.420	15.84	7.66
5	9¾	8	¾	6	¾	7-16	.443	21.00	9.63
6	10¾	9⅛	¾	8	¾	15-32	.466	26.64	11.82
8	13¼	11⅜	¾	8	13-16	½	.511	39.36	16.91
10	15¼	13¼	¾	10	⅞	9-16	.557	54.00	23.00
12	17¾	15¾	⅞	12	15-16	19-32	.603	70.56	30.13
14	20	18	⅞	14	1	21-32	.649	89.04	38.34
16	22	20	⅞	16	1 1-16	11-16	.695	109.44	47.70
18	24	22¼	⅞	16	1⅛	¾	.741	131.76	58.23
20	27	24½	1	18	1 3-16	25-32	.787	156.00	70.00
22	28¾	26½	1	20	1¼	27-32	.833	182.16	83.05
24	31¼	28¾	1	22	1 5-16	⅞	.879	210.24	97.42
26	33¼	31	1	24	1⅜	15-16	.925	240.24	113.18
28	35½	33¼	1	24	1 7-16	31-32	.971	272.16	130.35
30	38	35½	1	26	1 9-16	1	1.017	306.00	149.00
32	40	37½	1⅛	28	1⅝	1 1-16	1.063	341.76	169.17
34	42¼	40	1⅛	30	1 11-16	1⅛	1.109	379.44	190.90
36	45	42	1⅛	32	1¾	1 5-32	1.155	419.04	214.26
38	47	44	1⅛	32	1 13-16	1 3-16	1.201	460.56	239.27
40	49	46	1⅛	34	1⅞	1¼	1.247	504.00	266.00
42	51¼	48¼	1⅛	34	1 15-16	1 5-16	1.293	549.36	294.49
44	53½	50¼	1¼	36	2	1 11-32	1.339	596.64	324.78
46	55¾	52¾	1¼	38	2 1-16	1⅜	1.385	645.84	356.94
48	58	55	1¼	40	2⅛	1 7-16	1.431	696.96	391.00

D = Diameter of pipe. All dimensions in inches.

FORMULÆ.—Thickness of flange = $0.033D + 0.56$.
Thickness of pipe = $0.023D + 0.327$.
Weight of pipe per foot = $0.24D^2 + 3D$.
Weight of flange = $.001D^3 + 0.1D^2 + D + 2$.
Diameter of flange = $1.125D + 4.25$.
Diameter of bolt-circle = $1.092D + 2.566$.
Diameter of bolt = $0.011D + 0.73$.
Number of bolts = $0.78D + 2.56$.

PIPE FLANGES FOR HIGH STEAM-PRESSURE.

(Chapman Valve Mfg. Co.)

Size of Pipe.	Diameter of Flange.	Number of Bolts.	Diameter of Bolts.	Diameter of Bolt Circle.	Length of Pipe-Thread.
Inches.	Inches.		Inches.	Inches.	Inches.
2½	7½	6	⅝	5⅞	1⅛
3	9	6	¾	6⅝	1⅜
3½	9	7	¾	7¼	1 7-16
4	10	8	¾	7⅞	1 9-16
4½	10½	8	¾	8½	1 11-16
5	11	9	¾	9¼	1 13-16
6	13	10	⅞	10⅝	1⅞
7	14	12	⅞	11⅞	1 15-16
8	15	12	⅞	13	2
9	16	13	⅞	14	2
10	17½	15	⅞	15¼	2⅛
12	20	18	⅞	17¾	2¼
14	23	18	1	20¼	2⅜
15	23½	18	1	21¼	2⅝

Standard Dimensions of Wrought-Iron Welded Pipe.

(Morris, Tasker & Co., Incorporated, Philadelphia, Pa.)

Nominal Inside Diam.	Actual Outside Diam.	Actual Inside Diam.	Thickness of Metal.	Internal Circumference.	External Circumference.	Length of Pipe per sq. ft. Inside Surface.	Length of Pipe per sq. ft. Outside Surface.	Internal Area.		External Area.		Length of Pipe cont'g 1 cu. ft.	U. S. Gallons per Ft. of Pipe.	Weight of Pipe per Lin. Ft.	Weight of Water per Lin. Ft. of Pipe.	No. of Threads per Inch.	Length of Perf. Thread.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Feet.	Feet.	Sq. Ins.	Sq. Ft.	Sq. Ins.	Sq. Ft.	Feet.	Galls.	Lbs.	Lbs.	No.	In s.
1/8	.405	.270	.068	.848	1.272	14.151	9.484	.057	.0004	.1288	.0009	2500.0	.0029	.24	.024	27	.19
1/4	.540	.364	.085	1.144	1.696	10.500	7.075	.104	.0007	.2290	.0016	1883.280	.0054	.42	.045	18	.29
3/8	.675	.493	.091	1.552	2.121	7.732	5.658	.191	.0013	.3578	.0025	754.322	.0099	.56	.0[illegible]3	18	.30
1/2	.840	.622	.109	1.957	2.639	6.182	4.547	.304	.0021	.551	.0088	473.840	.0158	.84	.[illegible]32	14	.39
3/4	1.050	.824	.113	2.589	3.299	4.635	3.638	.533	.0037	.866	.0060	270.016	.0277	1.12	.[illegible]81	14	.40
1	1.315	1.048	.134	3.292	4.131	3.645	2.904	.861	.0060	1.358	.0094	167.246	.0447	1.67	.373	11½	.51
1¼	1.660	1.380	.140	4.335	5.215	2.768	2.301	1.496	.0104	2.164	.0150	96.257	.0777	2.24	.648	11½	.54
1½	1.900	1.610	.145	5.058	5.969	2.372	2.010	2.036	.0141	2.885	.0197	70.727	.1058	2.68	.882	11½	.55
2	2.375	2.067	.154	6.494	7.461	1.848	1.608	3.356	.0233	4.430	.0308	42.908	.1743	3.61	1.458	11½	.58
2½	2.875	2.468	.204	7.753	9.032	1.548	1.329	4.780	.0332	6.492	.0451	30.337	.2483	5.74	2.070	8	.89
3	3.500	3.067	.217	9.635	10.996	1.245	1.091	7.393	.0513	9.621	.0668	19.504	.3835	7.54	3.197	8	.95
3½	4.000	3.548	.226	11.146	12.566	1.077	0.955	9.887	.0687	12.566	.0875	14.567	.5136	9.00	4.291	8	1.00
4	4.500	4.026	.237	12.648	14.137	0.949	.849	12.730	.0884	15.904	.1104	11.312	.6613	10.66	5.512	8	1.05
4½	5.000	4.508	.246	14.162	15.708	.847	.764	15.961	.1108	19.635	.1364	9.022	.829	12.34	6.910	8	1.10
5	5.565	5.045	.259	15.849	17.475	.757	.687	19.986	.1388	24.301	.1688	7.205	1.038	14.50	8.652	8	1.16
6	6.625	6.065	.280	19.054	20.813	.630	.577	28.890	.2006	34.472	.2394	4.984	1.500	18.76	12.508	8	1.26
7	7.625	7.023	.301	22.063	23.955	.543	.501	38.738	.2690	45.664	.3171	3.717	2.012	23.27	16.771	8	1.36
8	8.625	7.981	.322	25.076	27.096	.479	.443	50.027	.3474	58.426	.4057	2.876	2.599	28.18	21.664	8	1.46
9	9.625	8.937	.344	28.076	30.238	.427	.397	62.730	.4356	72.760	.5053	2.290	3.259	33.70	27.166	8	1.57
10	10.75	10.018	.366	31.476	33.772	.381	.355	78.823	.5474	90.763	.6303	1.827	4.095	40.06	34.134	8	1.68
11	11.75	11.000	.375	34.558	36.914	.347	.325	95.033	.6600	108.434	.7530	1.515	4.937	45.02	41.153	8	1.78
12	12.75	12.000	.375	37.699	40.055	.318	.300	113.098	.7854	127.677	.8867	1.273	5.875	49.00	48.972	8	1.88
13	14	13.25	.375	41.626	43.982	.288	.273	137.887	.9577	153.938	1.0690	1.044	7.163	54.00	59.708	8	2.09
14	15	14.25	.375	44.768	47.124	.268	.255	159.485	1.1075	176.715	1.2272	0.900	8.285	58.00	69.060	8	2.10
15	16	15.25	.375	47.909	50.266	.250	.239	182.665	1.2685	201.062	1.3963	.793	9.489	62.00	79.097	8	2.20
....	18	17.25	.375	54.193	56.549	.221	.212	239.706	1.6229	254.470	1.7671	.616	12.141	70.00	101.203	....	....
....	20	19.25	.375	60.476	62.832	.198	.191	291.040	2.0211	314.[illegible]59	2.1817	.495	15.119	78.00	126.026	....	....
...	22	21.25	.375	66.759	69.115	.180	.174	354.657	2.4629	380.134	2.6398	.406	18.424	85.00	153.575	....	....
....	24	23.25	.375	73.042	75.398	.164	.159	424.558	2.9483	452.390	3.1416	.339	22.055	98.00	183.842	....	....

Pipe from 1/8" to 1" inclusive is butt-welded, and proved to 300 lbs. per sq. in. Pipe 1¼" and larger is lap-wedded, and proved to 500 lbs per sq. in.

For discussion of the Briggs Standard of Wrought-Iron Pipe Dimensions, see Report of the Committee of the A. S. M. E. in "Standard Pipe and Pipe Threads," 1886. Trans., Vol. VIII, p. 29. The diameter of the bottom of the thread is derived from the formula $D - (0.05D + 1.9) \times \frac{1}{n}$, in which D = outside diameter of the tubes, and n the number of threads to the inch. The diameter of the top of the thread is derived from the formula $0.8\frac{1}{n} \times 2 + d$, or $1.6\frac{1}{n} + d$, in which d is the diameter at the bottom of the thread at the end of the pipe.

Morris, Tasker & Co.'s sizes for the diameters at the bottom and top of the thread at the end of the pipe are as follows:

Diam. of Pipe, Nominal	Diam. at Bottom of Thread.	Diam. at Top of Thread.	Diam. of Pipe, Nominal.	Diam. at Bottom of Thread.	Diam. at Top of Thread.	Diam. of Pipe, Nominal.	Diam. at Bottom of Thread.	Diam. at Top of Thread.
in.	in.	in.	in.	in.	in.	in.	in.	in.
1/8	.334	.393	2 1/2	2.620	2.820	8	8.334	8.534
1/4	.433	.522	3	3.241	3.441	9	9.327	9.527
3/8	.568	.658	3 1/2	3.738	3.938	10	10.445	10.645
1/2	.701	.815	4	4.234	4.434	11	11.439	11.639
3/4	.911	1.025	4 1/2	4.731	4.931	12	12.433	12.633
1	1.144	1.283	5	5.290	5.490	13	13.675	13.875
1 1/4	1.488	1.627	6	6.346	6.546	14	14.669	14.869
1 1/2	1.727	1.866	7	7.340	7.540	15	15.663	15.863
2	2.223	2.339						

Having the taper, length of full-threaded portion, and the sizes at bottom and top of thread at the end of the pipe, as given in the table, taps and dies can be made to secure these points correctly, the length of the imperfect threaded portions on the pipe, and the length the tap is run into the fittings beyond the point at which the size is as given, or, in other words, beyond the end of the pipe, having no effect upon the standard. The angle of the thread is 60°, and it is slightly rounded off at top and bottom, so that, instead of its depth being 0.866 its pitch, as is the case with a full V-thread, it is 4/5 the pitch, or equal to $0.8 \div n$, n being the number of threads per inch.

Taper of conical tube ends, 1 in 32 to axis of tube = 3/4 inch to the foot total taper.

WROUGHT-IRON WELDED TUBES, EXTRA STRONG.
Standard Dimensions.

Nominal Diameter.	Actual Outside Diameter.	Thickness, Extra Strong.	Thickness, Double Extra Strong.	Actual Inside Diameter, Extra Strong.	Actual Inside Diameter, Double Extra Strong.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
1/8	0.405	0.100	....	0.205	....
1/4	0.54	0.123	....	0.294	....
3/8	0.675	0.127	....	0.421	....
1/2	0.84	0.149	0.298	0.542	0.244
3/4	1.05	0.157	0.314	0.736	0.422
1	1.315	0.182	0.364	0.951	0.587
1 1/4	1.66	0.194	0.388	1.272	0.884
1 1/2	1.9	0.203	0.406	1.494	1.088
2	2.375	0.221	0.442	1.933	1.491
2 1/2	2.875	0.280	0.560	2.315	1.755
3	3.5	0.304	0.608	2.892	2.284
3 1/2	4.0	0.321	0.642	3.358	2.716
4	4.5	0.341	0.682	3.818	3.136

STANDARD SIZES, ETC., OF LAP-WELDED CHARCOAL-IRON BOILER-TUBES.

(Morris, Tasker & Co., Inc., Philadelphia, Pa.)

External Diameter.	Internal Diameter.	Standard Thickness.	Internal Circumference.	External Circumference.	Internal Area.		External Area.		Length of Tube per Sq. Ft. of Inside Surface.	Length of Tube per Sq. Ft. of Outside Surface.	Length of Tube per Sq. Ft. of Mean Surface.	Weight per Lineal Foot.
in.	in.	in.	in.	in.	sq. in.	sq. ft.	sq. in.	sq. ft.	ft.	ft.	ft.	lbs.
1	.810	.095	2.545	3.142	.515	.0036	.785	.0055	4.479	3.820	4.149	.90
1 1-4	1.060	.095	3.330	3.927	.882	.0061	1.227	.0085	3.604	3.056	3.310	1.15
1 1-2	1.310	.095	4.115	4.712	1.348	.0094	1.767	.0123	2.916	2.547	2.732	1.40
1 3-4	1.560	.095	4.901	5.498	1.911	.0133	2.405	.0167	2.448	2.183	2.316	1.65
2	1.810	.095	5.686	6.283	2.573	.0179	3.142	.0218	2.110	1.910	2.010	1.91
2 1-4	2.060	.095	6.472	7.069	3.333	.0231	3.976	.0276	1.854	1.698	1.776	2.16
2 1-2	2.282	.109	7.169	7.854	4.090	.0284	4.909	.0341	1.674	1.528	1.601	2.75
2 3-4	2.532	.109	7.955	8.639	5.035	.0350	5.940	.0412	1.508	1.389	1.449	3.04
3	2.782	.109	8.740	9.425	6.079	.0422	7.069	.0491	1.373	1.273	1.322	3.33
3 1-4	3.010	.120	9.456	10.210	7.116	.0494	8.296	.0576	1.269	1.175	1.222	3.96
3 1-2	3.260	.120	10.242	10.996	8.347	.0580	9.621	.0668	1.172	1.091	1.132	4.28
3 3-4	3.510	.120	11.027	11.781	9.676	.0672	11.045	.0767	1.088	1.019	1.054	4.60
4	3.732	.134	11.724	12.566	10.939	.0760	12.566	.0873	1.024	.955	.990	5.47
4 1-2	4.232	.134	13.295	14.137	14.066	.0977	15.904	.1104	.903	.849	.876	6.17
5	4.704	.148	14.778	15.708	17.379	.1207	19.635	.1364	.812	.764	.788	7.58
6	5.670	.165	17.813	18.850	25.250	.1750	28.274	.1963	.674	.637	.656	10.16
7	6.670	.165	20.954	21.991	34.942	.2427	38.485	.2673	.573	.546	.560	11.90
8	7.670	.165	24.096	25.133	46.204	.3209	50.266	.3491	.498	.477	.488	13.65
9	8.640	.180	27.143	28.274	58.630	.4072	63.617	.4418	.442	.424	.433	16.76
10	9.594	.203	30.141	31.416	72.292	.5020	78.540	.5454	.398	.382	.390	21.00
11	10.560	.220	33.175	34.558	87.583	.6082	95.033	.6600	.362	.347	.355	25.00
12	11.542	.229	36.260	37.699	104.629	.7266	113.098	.7854	.331	.318	.325	28.50
13	12.524	.238	39.345	40.841	123.190	.8555	132.733	.9217	.305	.294	.300	32.06
14	13.504	.248	42.424	43.982	143.224	.9946	153.938	1.0690	.283	.273	.278	36.00
15	14.482	.259	45.497	47.124	164.721	1.1439	176.715	1.2272	.264	.255	.260	40.00
16	15.458	.271	48.563	50.266	187.671	1.3033	201.062	1.3963	.247	.239	.243	45.20
17	16.432	.284	51.623	53.407	212.066	1.4727	226.981	1.5763	.232	.225	.229	49.90
18	17.416	.292	54.714	56.549	238.225	1.6543	254.470	1.7671	.219	.212	.216	54.82
19	18.400	.300	57.805	59.690	265.905	1.8466	283.529	1.9690	.208	.201	.205	59.48
20	19.360	.320	60.821	62.832	294.375	2.0443	314.159	2.1817	.197	.191	.194	66.77
21	20.320	.340	63.837	65.974	324.294	2.2520	346.361	2.4053	.188	.182	.185	73.40

In estimating the effective steam-heating or boiler surface of tubes, the surface in contact with air or gases of combustion (whether internal or external to the tubes) is to be taken.

For heating liquids by steam, superheating steam, or transferring heat from one liquid or gas to another, the mean surface of the tubes is to be taken.

To find the square feet of surface, S, in a tube of a given length, L, in feet, and diameter, d, in inches, multiply the length in feet by the diameter in inches and by .2618. Or, $S = \frac{3.1416 dL}{12} = .2618 dL$. For the diameters in the table below, multiply the length in feet by the figures given opposite the diameter.

Inches, Diameter.	Square Feet per Foot Length.	Inches, Diameter.	Square Feet per Foot Length.	Inches, Diameter.	Square Feet per Foot Length.
¼	.0654	2¼	.5890	5	1.3090
½	.1309	2½	.6545	6	1.5708
¾	.1963	2¾	.7199	7	1.8326
1	.2618	3	.7854	8	2.0944
1¼	.3272	3¼	.8508	9	2.3562
1½	.3927	3½	.9163	10	2.6180
1¾	.4581	3¾	.9817	11	2.8798
2	.5236	4	1.0472	12	3.1416

RIVETED IRON PIPE.

(Abendroth & Root Mfg. Co.)

Sheets punched and rolled, ready for riveting, are packed in convenient form for shipment. The following table shows the iron and rivets required for punched and formed sheets.

Number Square Feet of Iron required to make 100 Lineal Feet Punched and Formed Sheets when put together.			Approximate No. of Rivets 1 Inch apart required for 100 Lineal Feet Punched and Formed Sheets.	Number Square Feet of Iron required to make 100 Lineal Feet Punched and Formed Sheets when put together.			Approximate No. of Rivets 1 Inch apart required for 100 Lineal Feet Punched and Formed Sheets.
Diameter in Inches.	Width of Lap in Inches.	Square Feet.		Diameter in Inches.	Width of Lap in Inches.	Square Feet.	
3	1	90	1,600	14	1¼	397	2,800
4	1	116	1,700	15	1¼	428	2,900
5	1⅛	150	1,800	16	1¼	452	3,000
6	1⅛	178	1,900	18	1¼	506	3,200
7	1⅛	206	2,000	20	1¼	562	3,500
8	1⅛	234	2,200	22	1¼	617	3,700
9	1⅛	258	2,300	24	1¼	670	3,900
10	1⅛	289	2,400	26	1¼	725	4,100
11	1⅛	314	2,500	28	1¼	779	4,400
12	1⅛	343	2,600	30	1¼	836	4,600
13	1⅛	369	2,700	36	1¼	998	5,200

WEIGHT OF ONE SQUARE FOOT OF SHEET-IRON FOR RIVETED PIPE.

Thickness by the Birmingham Wire-Gauge.

No. of Gauge.	Thickness in Decimals of an Inch.	Weight in lbs., Black.	Weight in lbs., Galvanized.	No. of Gauge.	Thickness in Decimals of an Inch.	Weight in lbs., Black.	Weight in lbs., Galvanized.
26	.018	.80	.91	18	.049	1.92	2.15
24	.022	1.00	1.16	16	.065	2.50	2.67
22	.028	1.25	1.40	14	.083	3.12	3.34
20	.035	1.56	1.67	12	.109	4.37	4.78

SPIRAL RIVETED PIPE.

(Abendroth & Root Mfg. Co.)

Thickness. B. W. G. No.	Thickness. Inches.	Diameter, Inches.	Approximate Weight in lbs. per Foot in Length.	Approximate Bursting Pressure in lbs. per Square Inch.
26	.018	3 to 6	lbs. =	
24	.022	3 to 12	" = ⅓ of diam. in ins.	
22	.028	3 to 14	" = .4 " "	
20	.035	3 to 24	" = .5 " "	2700 lbs. ÷ diam. in ins.
18	.049	3 to 24	" = .6 " "	3600 " ÷ " "
16	.065	6 to 24	" = .8 " "	4800 " ÷ " "
14	.083	8 to 24	" = 1.1 " "	6400 " ÷ " "
12	.109	9 to 24	" = 1.4 " "	8000 " ÷ " "

The above are black pipes. Galvanized weighs 10 to 30 % heavier.

Double Galvanized Spiral Riveted Flanged Pressure Pipe, tested to 150 lbs. hydraulic pressure.

Inside diameters, inches....	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	20	22	24
Thickness, B. W. G.........	20	20	20	18	18	18	18	16	16	16	16	14	14	14	14	14	12	12
Nominal wt. per foot, lbs...	2¼	3	4	5	6	7	8	11	12	14	15	20	22	24	29	34	40	50

DIMENSIONS OF SPIRAL PIPE FITTINGS.

Inside Diameter.	Outside Diameter Flanges.	Number Bolt-holes.	Diameter Bolt-holes.	Diameter Circles on which Bolt-holes are Drilled.	Sizes of Bolts.
ins.	ins.		ins.	ins.	ins.
3	6	4	½	4¾	7/16 × 1¾
4	7	8	½	5 15/16	7/16 × 1¾
5	8	8	½	6 15/16	7/16 × 1¾
6	8⅞	8	⅝	7⅞	½ × 1¾
7	10	8	⅝	9	½ × 1¾
8	11	8	⅝	10	½ × 2
9	13	8	⅝	11¼	½ × 2
10	14	8	⅝	12¼	½ × 2
11	15	12	⅝	13⅜	½ × 2
12	16	12	⅝	14¼	½ × 2
13	17	12	⅝	15¼	½ × 2
14	17⅞	12	⅝	16¼	½ × 2½
15	19	12	⅝	17 7/16	½ × 2½
16	21 3/16	12	⅝	19¼	½ × 2½
18	23¼	16	11/16	21¼	½ × 2½
20	25⅛	16	11/16	23⅛	½ × 2½
22	28¼	16	¾	26	⅝ × 2½
24	30	16	¾	27¾	⅝ × 2½

SEAMLESS BRASS TUBE. IRON-PIPE SIZES.

(For actual dimensions see tables of Wrought-iron Pipe.)

Nominal Size.	Weight per Foot.	Nom. Size.	Weight per Foot.	Nom. Size.	Weight per Foot.	Nom. Size.	Weight per Foot.
ins.	lbs.	ins.	lbs.	ins.	lbs.	ins.	lbs.
⅛	.25	¾	1.25	2	4.0	4	12.70
¼	.43	1	1.70	2½	5.75	4½	13.90
⅜	.62	1¼	2.50	3	8.30	5	15.75
½	.90	1½	3.	3½	10.90	6	18.31

SEAMLESS DRAWN BRASS TUBING.

(Randolph & Clowes, Waterbury, Conn.)

Outside diameter 3/16 to 7¾ inches. Thickness of walls 8 to 25 Stubs' Gauge, length 12 feet. The following are the standard sizes:

Outside Diameter.	Length Feet.	Stubbs' or Old Gauge.	Outside Diameter.	Length Feet.	Stubbs' or Old Gauge.	Outside Diameter.	Length Feet.	Stubbs' or Old Gauge.
¼	12	20	1⅜	12	14	2⅝	12	11
5-16	12	19	1½	12	14	2¾	12	11
⅜	12	19	1⅝	12	13	3	12	11
½	12	18	1¾	12	13	3¼	12	11
⅝	12	18	1 13-16	12	13	3½	12	11
¾	12	17	1⅞	12	12	4	10 to 12	11
13-16	12	17	1 15-16	12	12	5	10 to 12	11
⅞	12	17	2	12	12	5¼	10 to 12	11
15-16	12	17	2⅛	12	12	5½	10 to 12	11
1	12	16	2¼	12	12	5¾	10 to 12	11
1⅛	12	16	2⅜	12	12	6	10 to 12	11
1¼	12	15	2½	12	11			

BENT AND COILED PIPES.

(National Pipe Bending Co., New Haven, Conn.)

COILS AND BENDS OF IRON AND STEEL PIPE.

Size of pipe............Inches	¼	⅜	½	¾	1	1¼	1½	2	2½	3
Least outside diameter of coil..................Inches	2	2½	3½	4½	6	8	12	16	24	32
Size of pipe............Inches	3½	4	4½	5	6	7	8	9	10	12
Least outside diameter of coil..................Inches	40	48	52	58	66	80	92	105	130	156

Lengths continuous welded up to 3-in. pipe or coupled as desired.

COILS AND BENDS OF DRAWN BRASS AND COPPER TUBING.

Size of tube, outside diameter......Inches	¼	⅜	½	⅝	¾	1	1¼	1⅜
Least outside diameter of coil.....Inches	1	1½	2	2½	3	4	6	7
Size of tube, outside diameter......Inches	1½	1⅝	1¾	2	2¼	2⅜	2½	2¾
Least outside diameter of coil.....Inches	8	9	10	12	14	16	18	20

Lengths continuous brazed, soldered, or coupled as desired.

90° BENDS. EXTRA-HEAVY WROUGHT-IRON PIPE.

Diameter of pipe................Inches	4	4½	5	6	7	8	9	10	12
Radius.........................Inches	22	24	26	30	36	42	48	60	72
Centre to end.................Inches	26	28½	31	36	43	50	57	70	84

The radii given are for the centre of the pipe. "Centre to end" means the perpendicular distance from the centre of one end of the bent pipe to a plane passing across the other end. Standard iron pipes of sizes 4 to 8 in. are bent to radii 8 in. larger than the radii in the above table; sizes 9 to 12 in. to radii 12 in. larger.

Welded Solid Drawn-steel Tubes, imported by P. S. Justice & Co., Philadelphia, are made in sizes from ½ to 4½ in. external diameter, varying by ⅛ths, and with thickness of walls from 1/16 to 11/16 in. The maximum length is 15 feet.

WEIGHT OF BRASS, COPPER, AND ZINC TUBING.
Per Foot.

Thickness by Brown & Sharpe's Gauge.

Brass, No. 17.		Brass, No. 20.		Copper, Lightning-rod Tube, No. 23.	
Inch.	Lbs.	Inch.	Lbs.	Inch.	Lbs.
¼	.107	⅛	.032	½	.162
5-16	.157	3-16	.039	9-16	.176
⅜	.185	¼	.063	⅝	.186
7-16	.234	5-16	.106	11-16	.211
½	.266	⅜	.126	¾	.229
9-11	.318	7-16	.158	**Zinc, No. 20.**	
⅝	.333	½	.189	½	.161
¾	.377	9-16	.208	⅝	.185
⅞	.462	⅝	.220	¾	.234
1	.542	¾	.252	⅞	.273
1⅛	.675	⅞	.284	1	.311
1¼	.740	1	.378	1¼	.380
1⅜	.915	1¼	.500	1½	.452
1½	.980	1½	.580		
2	1.90				
2½	1.506				
3	2.188				

LEAD PIPE IN LENGTHS OF 10 FEET.

In.	3-8 Thick.		5-16 Thick.		¼ Thick.		3-16 Thick.	
	lb.	oz.	lb.	oz.	lb.	oz.	lb.	oz.
2½	17	0	14	0	11	0	8	0
3	20	0	16	0	12	0	9	0
3½	22	0	18	0	15	0	9	8
4	25	0	21	0	16	0	12	8
4½					18	0	14	0
5	31	0			20	0		

LEAD WASTE-PIPE.

1½ in., 2 lbs. per foot.
2 " 3 and 4 lbs. per foot.
3 " 3½ and 5 lbs. per foot.
3½ in., 4 lbs. per foot.
4 " 5, 6, and 8 lbs.
4½ " 6 and 8 lbs.
5 in. 8, 10, and 12 lbs.

LEAD AND TIN TUBING.

⅛ inch. ¼ inch.

SHEET LEAD.

Weight per square foot, 2½, 3, 3½, 4, 4½, 5, 6, 8, 9, 10 lbs. and upwards. Other weights rolled to order.

BLOCK-TIN PIPE.

⅜ in., 4½, 6½, and 8 oz. per foot.
½ " 6, 7½, and 10 oz. "
⅝ " 8 and 10 oz. "
¾ " 10 and 12 oz. "
1 in., 15, and 18 oz. per foot.
1¼ " 1¼ and 1½ lbs. "
1½ " 2 and 2½ lbs. "
2 " 2½ and 3 lbs. "

LEAD AND TIN-LINED LEAD PIPE.

(Tatham & Bros., New York.)

Calibre.	Letter.	Weight per Foot and Rod.	Thickness in 1-100th In.	Calibre.	Letter.	Weight per Foot and Rod.	Thickness in 1-100th In.
3/8 in.	E	7 lbs. per rod		1 in.	E	1½ lbs. per foot	10
"	D	10 oz. per foot	6	"	D	2 " "	11
"	C	12 " "	8	"	C	2½ " "	14
"	B	1 lb. "	12	"	B	3¼ " "	17
"	A	1¼ " "	16	"	A	4 " "	21
"	AA	1½ " "	19	"	AA	4¾ " "	24
"	AAA	1¾ " "	27	"	AAA	6 " "	30
7-16 in.		13 oz. "		1¼ in.	E	2 " "	10
"		1 lb. "		"	D	2½ " "	12
½ in.	E	9 lbs. per rod	7	"	C	3 " "	14
"	D	¾ lb. per foot	9	"	B	3¾ " "	16
"	C	1 " "	11	"	A	4¾ " "	19
"	B	1¼ " "	13	"	AA	5¾ " "	25
"		1½ " "		"	AAA	6¾ " "	
"	A	1¾ " "	16	1½ in.	E	3 " "	12
"	AA	2 " "	19	"	D	3½ " "	14
"		2½ " "	23	"	C	4¼ " "	17
"	AAA	3 " "	25	"	B	5 " "	19
5/8 in.	E	12 " per rod	8	"	A	6½ " "	23
"	D	1 " per foot	9	"	AA	8 " "	27
"	C	1½ " "	13	"	AAA	9 " "	
"	B	2 " "	16	1¾ in.	C	4 " "	13
"	A	2¼ " "	20	"	B	5 " "	17
"	AA	2¾ " "	22	"	A	6½ " "	21
"	AAA	3½ " "	25	"	AA	8½ " "	27
¾ in.	E	1 " per foot	8	2 in.	C	4¾ " "	15
"	D	1¼ " "	10	"	B	6 " "	18
"	C	1¾ " "	12	"	A	7 " "	22
"	B	2¼ " "	16	"	AA	9 " "	27
"	A	3 " "	20	"	AAA	11¾ " "	
"	AA	3½ " "	23				
"	AAA	4¾ " "	30				

WEIGHT OF LEAD PIPE WHICH SHOULD BE USED FOR A GIVEN HEAD OF WATER.

(Tatham & Bros., New York.)

Head or Number of Feet Fall.	Pressure per sq. inch.	Calibre and Weight per Foot.						
		Letter.	3/8 inch.	½ inch.	5/8 inch.	¾ inch.	1 inch.	1¼ in.
30 ft.	15 lbs.	D	10 oz.	¾ lb.	1 lb.	1¼ lbs.	2 lbs.	2½ lbs.
50 ft.	25 lbs.	C	12 oz.	1 lb.	1½ lbs.	1¾ lbs.	2½ lbs.	3 lbs.
75 ft.	38 lbs.	B	1 lb.	1¼ lbs.	2 lbs.	2¼ lbs.	3¼ lbs.	3¾ lbs.
100 ft.	50 lbs.	A	1¼ lbs.	1¾ lbs.	2½ lbs.	3 lbs.	4 lbs.	4¾ lbs.
150 ft.	75 lbs.	AA	1½ lbs.	2 lbs.	2¾ lbs.	3½ lbs.	4¾ lbs.	6 lbs.
200 ft.	100 lbs.	AAA	1¾ lbs.	3 lbs.	3½ lbs.	4½ lbs.	6 lbs.	6¾ lbs.

To find the thickness of lead pipe required when the head of water is given. (Chadwick Lead Works).

RULE.—Multiply the head in feet by size of pipe wanted, expressed decimally, and divide by 750; the quotient will give thickness required, in one-hundredths of an inch.

EXAMPLE.—Required thickness of half-inch pipe for a head of 25 feet.

$$25 \times 0.50 \div 750 = 0.16 \text{ inch.}$$

WEIGHT OF COPPER AND BRASS WIRE AND PLATES.

Brown & Sharpe's Gauge.

(From tables of leading manufacturers.)

No. of Gauge.	Size of Each No.	Weight of Wire per 1000 Lineal Feet.		Weight of Plates per Square Foot.		No. of Gauge.	Size of Each No.	Weight of Wire per 1,000 Lineal Feet.		Weight of Plates per Square Foot.	
		Copper.	Brass.	Copper.	Brass.			Copper.	Brass.	Copper.	Brass.
	Inch.	Lbs.	Lbs.	Lbs.	Lbs.		Inch.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.46000	640.5	605.28	20.84	19.69	21	.028462	2.45	2.317	1.29	1.22
000	.40964	508.0	479.91	18.55	17.53	22	.025347	1.94	1.838	1.15	1.08
00	.36480	402.0	380.77	16.52	15.61	23	.022571	1.54	1.457	1.02	.966
0	.32486	319.5	301.82	14.72	13.90	24	.020100	1.22	1.155	.911	.860
1	.28930	253.3	239.45	13.10	12.38	25	.017900	.970	.916	.811	.766
2	.25763	200.9	189.82	11.67	11.03	26	.01594	.769	.727	.722	.682
3	.22942	159.3	150.52	10.39	9.82	27	.014195	.610	.576	.643	.608
4	.20431	126.4	119.38	9.25	8.74	28	.012641	.484	.457	.573	.541
5	.18194	100.2	94.67	8.24	7.79	29	.011257	.383	.362	.510	.482
6	.16202	79.46	75.08	7.34	6.93	30	.010025	.304	.287	.454	.429
7	.14428	63.01	59.55	6.54	6.18	31	.008928	.241	.228	.404	.382
8	.12849	49.98	47.22	5.82	5.50	32	.007950	.191	.181	.360	.340
9	.11443	39.64	37.44	5.18	4.90	33	.007080	.152	.143	.321	.303
10	.10189	31.43	29.69	4.62	4.36	34	.006304	.120	.114	.286	.270
11	.090742	24.92	23.55	4.11	3.88	35	.005614	.096	.0902	.254	.240
12	.080808	19.77	18.68	3.66	3.46	36	.005000	.0757	.0715	.226	.214
13	.071961	15.65	14.81	3.26	3.08	37	.004453	.0600	.0567	.202	.191
14	.064084	12.44	11.75	2.90	2.74	38	.003965	.0476	.0450	.180	.170
15	.057068	9.86	9.32	2.59	2.44	39	.003531	.0375	.0357	.160	.151
16	.050820	7.82	7.59	2.30	2.18	40	.003144	.0299	.0283	.142	.135
17	.045257	6.20	5.86	2.05	1.94						
18	.040303	4.92	4.65	1.83	1.72	Specific gravity......		8.880	8.386	8.698	8.218
19	.035890	3.90	3.68	1.63	1.54						
20	.031961	3.09	2.92	1.45	1.37	Weight per cubic Ft.		555.	524.16	543.6	513.6

WEIGHT OF ROUND BOLT COPPER.

Per Foot.

Inches.	Pounds.	Inches.	Pounds.	Inches.	Pounds.
⅜	.425	1	3.02	1⅝	7.99
½	.756	1⅛	3.82	1¾	9.27
⅝	1.18	1¼	4.72	1⅞	10.64
¾	1.70	1⅜	5.72	2	12.10
⅞	2.31	1½	6.81		

WEIGHT OF SHEET AND BAR BRASS.

Thickness, Side or Diam.	Sheets per sq. ft.	Square Bars 1 ft. long.	Round Bars 1 ft. long.	Thickness, Side or Diam.	Sheets per sq. ft.	Square Bars 1 ft. long.	Round Bars 1 ft. long.
Inches.				Inches.			
1-16	2.72	.014	.011	1 1-16	46.32	4.10	3.22
⅛	5.45	.056	.045	1⅛	49.05	4.59	3.61
3-16	8.17	.128	.100	1 3-16	51.77	5.12	4.02
¼	10.90	.227	.178	1¼	54.50	5.67	4.45
5-16	13.62	.355	.278	1 5-16	57.22	6.26	4.91
⅜	16.35	.510	.401	1⅜	59.95	6.86	5.39
7-16	19.07	.695	.545	1 7-16	62.67	7.50	5.89
½	21.80	.907	.712	1½	65.40	8.16	6.41
9-16	24.52	1.15	.902	1 9-16	68.12	8.86	6.95
⅝	27.25	1.42	1.11	1⅝	70.85	9.59	7.53
11-16	29.97	1.72	1.35	1 11-16	73.57	10.34	8.12
¾	32.70	2.04	1.60	1¾	76.30	11.12	8.73
13-16	35.42	2.40	1.88	1 13-16	79.02	11.93	9.36
⅞	38.15	2.78	2.18	1⅞	81.75	12.76	10.01
15-16	40.87	3.19	2.50	1 15-16	84.47	13.63	10.70
1	43.60	3.63	2.85	2	87.20	14.52	11.40

COMPOSITION OF VARIOUS GRADES OF ROLLED BRASS, ETC.

Trade Name.	Copper	Zinc.	Tin.	Lead.	Nickel.
Common high brass	61.5	38.5	....	....	....
Yellow metal	60	40	....	....	....
Cartridge brass	66⅔	33⅓	....	....	....
Low brass	80	20	....	....	....
Clock brass	60	40	....	1½	....
Drill rod	60	40	....	1½ to 2	....
Spring brass	66¾	33¼	1½	....	....
18 per cent German silver	61½	20½	....	....	18

The above table was furnished by the superintendent of a mill in Connecticut in 1894. He says: While each mill has its own proportions for various mixtures, depending upon the purposes for which the product is intended, the figures given are about the average standard. Thus, between cartridge brass with 33⅓ per cent zinc and common high brass with 38½ per cent zinc, there are any number of different mixtures known generally as "high brass," or specifically as "spinning brass," "drawing brass," etc., wherein the amount of zinc is dependent upon the amount of scrap used in the mixture, the degree of working to which the metal is to be subjected, etc.

AMERICAN STANDARD SIZES OF DROP-SHOT.

	Diameter.	No. of Shot to the oz.		Diameter.	No. of Shot to the oz.		Diameter.	No. of Shot to the oz.
Fine Dust.	3-100″	10784	No. 8	Trap Shot	472	No. 2....	15-100″	86
Dust......	4-100	4565	" 8	9-100″	399	" 1...	16-100	71
No. 12.....	5-100	2326	" 7	Trap Shot	338	" B...	17-100	59
" 11. ...	6-100	1346	" 7	10-100″	291	" BB.	18-100	50
" 10.....	Trap Shot	1056	" 6	11-100	218	" BBB	19-100	42
" 10.....	7-100″	848	" 5	12-100	168	" T...	20-100	36
" 9.....	Trap Shot	688	" 4	13-100	132	" TT..	21-100	31
" 9.....	8-100″	568	" 3	14-100	106	" F..	22-100	27
						" FF..	23-100	24

COMPRESSED BUCK-SHOT.

	Diameter.	No. of Balls to the lb.		Diameter.	No. of Balls to the lb.
No. 3........	25-100″	284	No 00........	34-100″	115
" 2.........	27-100	232	" 000......	36-100	98
" 1.........	30-100	173	Balls.........	38-100	85
" 0.........	32-100	140	"	44-100	50

SCREW-THREADS, SELLERS OR U. S. STANDARD.

In 1864 a committee of the Franklin Institute recommended the adoption of the system of screw-threads and bolts which was devised by Mr. William Sellers, of Philadelphia. This same system was subsequently adopted as the standard by both the Army and Navy Departments of the United States, and by the Master Mechanics' and Master Car Builders' Associations, so that it may now be regarded, and is in fact is called, the United States Standard.

The rule given by Mr. Sellers for proportioning the thread is as follows: Divide the pitch, or, what is the same thing, the side of the thread, into eight equal parts; take off one part from the top and fill in one part in the bottom of the thread; then the flat top and bottom will equal one eighth of the pitch, the wearing surface will be three quarters of the pitch, and the diameter of screw at bottom of the thread will be expressed by the formula

$$\text{diameter of bolt} - \frac{1.299}{\text{no. threads per inch}}.$$

For a sharp V thread with angle of 60° the formula is

$$\text{diameter of bolt} - \frac{1.733}{\text{no. of threads per inch}}.$$

The angle of the thread in the Sellers system is 60°. In the Whitworth or English system it is 55°, and the point and root of the thread are rounded.

Screw-Threads, United States Standard.

Diam.	Pitch.	Diam.	Pitch.	Diam.	Pitch.	Diam.	Pitch.	Diam.	Pitch.
¼	20	¾	10	1¼	7	1 15-16	5	2 13-16	3½
5-16	18	13-16	10	1 5-16	6	2	4½	3	3½
⅜	16	⅞	9	1⅜	6	2¼	4½	3¼	3½
7-16	14	15-16	9	1½	6	2 5-16	4½	3 5-16	3¼
½	13	1	8	1⅝	5½	2⅜	4	3½	3¼
9-16	12	1 1-16	7	1¾	5	2½	4	3¾	3
⅝	11	1⅛	7	1⅞	5	2¾	4	4	3
11-16	11								

Screw-Threads, Whitworth (English) Standard.

Diam.	Pitch.	Diam.	Pitch.	Diam.	Pitch.	Diam.	Pitch.	Diam.	Pitch.
1/4	20	5/8	11	1	8	1 3/4	5	3	3 1/2
5-16	18	11-16	11	1 1/8	7	1 7/8	4 1/2	3 1/4	3 1/4
3/8	16	3/4	10	1 1/4	7	2	4 1/2	3 1/2	3 1/4
7-16	14	13-16	10	1 3/8	6	2 1/4	4	3 3/4	3
1/2	12	7/8	9	1 1/2	6	2 1/2	4	4	3
9-16	12	15-16	9	1 5/8	5	2 3/4	3 1/2		

U. S. OR SELLERS SYSTEM OF SCREW-THREADS.

BOLTS AND THREADS.						HEX. NUTS AND HEADS.					
Diam. of Bolt.	Threads per Inch.	Diam. of Root of Thread.	Width of Flat.	Area of Bolt Body in Sq. Inches.	Area at Root of Thread in Sq. Inches.	Short Diam., Rough.	Short Diam., Finish.	Long Diam., Rough.	Thickness, Rough.	Thickness, Finish.	Long Diam. Sq. Nuts Rough.
Ins.		Ins.	Ins.			Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
1/4	20	.185	.0062	.049	.027	1/2	7-16	37-64	1/4	3-16	7-10
5-16	18	.240	.0074	.077	.045	19-32	17-32	11-16	5-16	1/4	10-12
3/8	16	.294	.0078	.110	.068	11-16	5/8	51-64	3/8	5-16	63-64
7-16	14	.344	.0089	.150	.093	25-32	23-32	9-10	7-16	3/8	1 7-64
1/2	13	.400	.0096	.196	.126	7/8	13-16	1	1/2	7-16	1 15-64
9-16	12	.454	.0104	.249	.162	31-32	29-32	1 1/8	9-16	1/2	1 23-64
5/8	11	.507	.0113	.307	.202	1 1-16	1	1 7-32	5/8	9-16	1 1/2
3/4	10	.620	.0125	.442	.302	1 1/4	1 3-16	1 7-16	3/4	11-16	1 49-64
7/8	9	.731	.0138	.601	.420	1 7-16	1 3/8	1 21-32	7/8	13-16	2 1-32
1	8	.837	.0156	.785	.550	1 5/8	1 9-16	1 7/8	1	15-16	2 19-64
1 1/8	7	.940	.0178	.994	.694	1 13-16	1 3/4	2 3-32	1 1/8	1 1-16	2 9-16
1 1/4	7	1.065	.0178	1.227	.893	2	1 15-16	2 5-16	1 1/4	1 3-16	2 53-64
1 3/8	6	1.160	.0208	1.485	1.057	2 3-16	2 1/8	2 17-32	1 3/8	1 5-16	3 3-32
1 1/2	6	1.284	.0208	1.767	1.295	2 3/8	2 5-16	2 3/4	1 1/2	1 7-16	3 23-64
1 5/8	5 1/2	1.389	.0227	2.074	1.515	2 9-16	2 1/2	2 31-32	1 5/8	1 9-16	3 5/8
1 3/4	5	1.491	.0250	2.405	1.746	2 3/4	2 11-16	3 3-16	1 3/4	1 11-16	3 57-64
1 7/8	5	1.616	.0250	2.761	2.051	2 15-16	2 7/8	3 13-32	1 7/8	1 13-16	4 5-32
2	4 1/2	1.712	.0277	3.142	2.302	3 1/8	3 1-16	3 5/8	2	1 15-16	4 27-64
2 1/4	4 1/2	1.962	.0277	3.976	3.023	3 1/2	3 7-16	4 1-16	2 1/4	2 3-16	4 61-64
2 1/2	4	2.176	.0312	4.909	3.719	3 7/8	3 13-16	4 1/2	2 1/2	2 7-16	5 31-64
2 3/4	4	2.426	.0312	5.940	4.620	4 1/4	4 3-16	4 29-32	2 3/4	2 11-16	6
3	3 1/2	2.629	.0357	7.069	5.428	4 5/8	4 9-16	5 3/8	3	2 15-16	6 17-32
3 1/4	3 1/2	2.879	.0357	8.296	6.510	5	4 15-16	5 13-16	3 1/4	3 3-16	7 1-16
3 1/2	3 1/4	3.100	.0384	9.621	7.548	5 3/8	5 5-16	6 7-64	3 1/2	3 7-16	7 39-64
3 3/4	3	3.317	.0413	11.045	8.641	5 3/4	5 11-16	6 21-32	3 3/4	3 11-16	8 1/8
4	3	3.567	.0413	12.566	9.993	6 1/8	6 1-16	7 3-32	4	3 15-16	8 41-64
4 1/4	2 7/8	3.798	.0435	14.186	11.329	6 1/2	6 7-16	7 9-16	4 1/4	4 3-16	9 3-16
4 1/2	2 3/4	4.028	.0454	15.904	12.743	6 7/8	6 13-16	7 31-32	4 1/2	4 7-16	9 3/4
4 3/4	2 5/8	4.256	.0476	17.721	14.226	7 1/4	7 3-16	8 13-32	4 3/4	4 11-16	10 1/4
5	2 1/2	4.480	.0500	19.635	15.763	7 5/8	7 9-16	8 27-32	5	4 15-16	10 49-64
5 1/4	2 1/2	4.730	.0500	21.648	17.572	8	7 15-16	9 9-32	5 1/4	5 3-16	11 23-64
5 1/2	2 3/8	4.953	.0526	23.758	19.267	8 3/8	8 5-16	9 23-32	5 1/2	5 7-16	11 7/8
5 3/4	2 3/8	5.203	.0526	25.967	21.262	8 3/4	8 11-16	10 5-32	5 3/4	5 11-16	12 3/8
6	2 1/4	5.423	.0555	28.274	23.098	9 1/8	9 1-16	10 19-32	6	5 15-16	12 15-16

LIMIT GAUGES FOR IRON FOR SCREW THREADS.

In adopting the Sellers, or Franklin Institute, or United States Standard, as it is variously called, a difficulty arose from the fact that it is the habit of iron manufacturers to make iron over-size, and as there are no over-size screws in the Sellers system, if iron is too large it is necessary to cut it away with the dies. So great is this difficulty, that the practice of making taps and dies over-size has become very general. If the Sellers system is adopted it is essential that iron should be obtained of the correct size, or very nearly so. Of course no high degree of precision is possible in rolling iron. and when exact sizes were demanded, the question arose how much allowable variation there should be from the true size. It was proposed to make limit-gauges for inspecting iron with two openings, one larger and the other smaller than the standard size, and then specify that the iron should enter the large end and not enter the small one. The following table of dimensions for the limit-gauges was recommended by the Master Car-Builders' Association and adopted by letter ballot in 1883.

Size of Iron.	Size of Large End of Gauge.	Size of Small End of Gauge.	Difference.	Size of Iron.	Size of Large End of Gauge.	Size of Small End of Gauge.	Difference.
¼ in.	0.2550	0.2450	0.010	⅝ in.	0.6330	0.6170	0.016
5-16	0.3180	0.3070	0.011	¾	0.7585	0.7415	0.017
⅜	0.3810	0.3690	0.012	⅞	0.8840	0.8660	0 018
7-16	0.4440	0.4310	9.013	1	1.0095	0.9905	0.019
½	0.5070	0.4930	0.014	1⅛	1.1350	1.1150	0.020
9-16	0.5700	0.5550	0.015	1¼	1.2605	1.2395	0.021

Caliper gauges with the above dimensions, and standard reference gauges for testing them, are made by The Pratt & Whitney Co.

THE MAXIMUM VARIATION IN SIZE OF ROUGH IRON FOR U. S. STANDARD BOLTS.

Am. Mach., May 12, 1892.

By the adoption of the Sellers or U. S. Standard thread taps and dies keep their size much longer in use when flatted in accordance with this system than when made sharp "V," though it has been found advisable in practice in most cases to make the taps of somewhat larger outside diameter than the nominal size, thus carrying the threads further towards the V-shape and giving corresponding clearance to the tops of the threads when in the nuts or tapped holes.

Makers of taps and dies often have calls for taps and dies, U. S. Standard, "for rough iron."

An examination of rough iron will show that much of it is rolled out of round to an amount exceeding the limit of variation in size allowed.

In view of this it may be desirable to know what the extreme variation in iron may be, consistent with the maintenance of U. S. Standard threads, i.e., threads which are standard when measured upon the angles, the only place where it seems advisable to have them fit closely. Mr. Chas. A. Bauer, the general manager of the Warder, Bushnell & Glessner Co., at Springfield, Ohio, in 1884 adopted a plan which may be stated as follows: All bolts, whether cut from rough or finished stock, are standard size at the bottom and at the sides or angles of the threads, the variation for fit of the nut and allowance for wear of taps being made in the machine taps. Nuts are punched with holes of such size as to give 85 per cent of a full thread, experience showing that the metal of wrought nuts will then crowd into the threads of the taps sufficiently to give practically a full thread, while if punched smaller some of the metal will be cut out by the tap at the bottom of the threads, which is of course undesirable. Machine taps are made enough larger than the nominal to bring the tops of the threads up sharp, plus the amount allowed for fit and wear of taps. This allows the iron to be enough above the nominal diameter to bring the threads up full (sharp) at top, while if it is small the only effect is to give a flat at top of threads; neither condition affecting the actual size of the thread at the point at which it is intended to bear. Limit gauges are furnished to the mills, by which the iron is rolled, the maximum size being shown in the third column of the table. The minimum diameter is not given, the tendency in rolling being nearly always to exceed the nominal diameter.

In making the taps the threaded portion is turned to the size given in the eighth column of the table, which gives 6 to 7 thousandths of an inch allowance for fit and wear of tap. Just above the threaded portion of the tap a place is turned to the size given in the ninth column, these sizes being the same as those of the regular U. S. Standard bolt, at the bottom of the thread, plus the amount allowed for fit and wear of tap; or, in other words, $d' =$ U. S. Standard $d + (D' - D)$. Gauges like the one in the cut, Fig. 72, are furnished for this sizing. In finishing the threads of the tap a tool

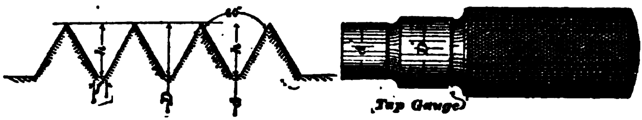


FIG. 72.

is used which has a removable cutter finished accurately to gauge by grinding, this tool being correct U. S. Standard as to angle, and flat at the point. It is fed in and the threads chased until the flat point just touches the portion of the tap which has been turned to size d'. Care having been taken with the form of the tool, with its grinding on the top face (a fixture being provided for this to insure its being ground properly), and also with the setting of the tool properly in the lathe, the result is that the threads of the tap are correctly sized without further attention.

It is evident that one of the points of advantage of the Sellers system is sacrificed, i.e., instead of the taps being flatted at the top of the threads they are sharp, and are consequently not so durable as they otherwise would be; but practically this disadvantage is not found to be serious, and is far overbalanced by the greater ease of getting iron within the prescribed limits; while any rough bolt when reduced in size at the top of the threads, by filing or otherwise, will fit a hole tapped with the U. S. Standard hand taps, thus affording proof that the two kinds of bolts or screws made for the two different kinds of work are practically interchangeable. By this system ¼″ iron can be .005″ smaller or .0108″ larger than the nominal diameter, or, in other words, it may have a total variation of .0158″, while 1¼″ iron can be .0105″ smaller or .0309″ larger than nominal—a total variation of .0414″—and within these limits it is found practicable to procure the iron.

STANDARD SIZES OF SCREW-THREADS FOR BOLTS AND TAPS.

(CHAS. A. BAUER.)

1	2	3	4	5	6	7	8	9	10
A	n	D	d	h	f	$D' - D$	D'	d'	H
		Inches.	Inches	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
¼	20	.2608	.1855	.0379	.0062	.006	.2668	.1915	.2024
5-16	18	.3245	.2403	.0421	.0070	.006	.3305	.2463	.2589
⅜	16	.3885	.2938	.0474	.0078	.006	.3945	.2998	.3139
7-16	14	.4530	.3447	.0541	.0089	.006	.4590	.3507	.3670
½	13	.5166	.4000	.0582	.0096	.006	.5223	.4060	.4236
9-16	12	.5805	.4513	.0631	.0104	.007	.5875	.4613	.4802
⅝	11	.6447	.5069	.0689	.0114	.007	.6517	.5139	.5346
¾	10	.7717	.6201	.0758	.0125	.007	.7787	.6271	.6499
⅞	9	.8991	.7307	.0842	.0139	.007	.9061	.7377	.7630
1	8	1.0271	.8376	.0947	.0156	.007	1.0341	.8446	.8731
1⅛	7	1.1559	.9394	.1083	.0179	.007	1.1629	.9464	.9789
1¼	7	1.2809	1.0644	.1083	.0179	.007	1.2879	1.0714	1.1039

A = nominal diameter of bolt.
D = actual diameter of bolt.
d = diameter of bolt at bottom of thread.
n = number of threads per inch.
f = flat of bottom of thread.
h = depth of thread.
D' and d' = diameters of tap.
H = hole in nut before tapping.

$$D = A + \frac{.2165}{n}.$$

$$d = A - \frac{1.29904}{n}.$$

$$h = \frac{.7577}{n} = \frac{D - d}{2}.$$

$$f = \frac{.125}{n}.$$

$$H = D' - \frac{1.288}{n} = D' - .85(2h.)$$

STANDARD SET-SCREWS AND CAP-SCREWS.

American, Hartford, and Worcester Machine-Screw Companies.
(Compiled by W. S. Dix.)

	(A)	(B)	(C)	(D)	(E)	(F)	(G)
Diameter of Screw....	1/8	3-16	1/4	5-16	3/8	7-16	1/2
Threads per Inch.....	40	24	20	18	16	14	12
Size of Tap Drill*.....	No. 43	No. 30	No. 5	17-64	21-64	3/8	27-64

	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Diameter of Screw....	9-16	5/8	3/4	7/8	1	1 1/8	1 1/4
Threads per Inch....	12	11	10	9	8	7	7
Size of Tap Drill*....	31-64	17-32	21-32	49-64	7/8	63-64	1 1/8

Set Screws.			Hex. Head Cap-screws.			Sq. Head Cap-screws.		
Short Diam. of Head	Long Diam. of Head	Lengths (under Head).	Short Diam. of Head.	Long Diam. of Head.	Lengths (under Head).	Short Diam. of Head.	Long Diam. of Head.	Lengths (under Head).
(C) 1/4	.35	3/4 to 3	7-16	.51	3/4 to 3	3/8	.53	3/4 to 3
(D) 5-16	.44	3/4 to 3 1/4	1/2	.58	3/4 to 3 1/4	7-16	.62	3/4 to 3 1/4
(E) 3/8	.53	3/4 to 3 1/2	9-16	.65	3/4 to 3 1/2	1/2	.71	3/4 to 3 1/2
(F) 7-16	.62	3/4 to 3 3/4	5/8	.73	3/4 to 3 3/4	9-16	.80	3/4 to 3 3/4
(G) 1/2	.71	3/4 to 4	3/4	.87	3/4 to 4	5/8	.89	3/4 to 4
(H) 9-16	.80	3/4 to 4 1/4	13-16	.94	3/4 to 4 1/4	11-16	.98	3/4 to 4 1/4
(I) 5/8	.89	3/4 to 4 1/2	7/8	1.01	1 to 4 1/2	3/4	1.06	1 to 4 1/2
(J) 3/4	1.06	1 to 4 3/4	1	1.15	1 1/4 to 4 3/4	7/8	1.24	1 1/4 to 4 3/4
(K) 7/8	1.24	1 1/4 to 5	1 1/8	1.30	1 1/2 to 5	1 1/8	1.60	1 1/2 to 5
(L) 1	1.42	1 1/2 to 5	1 1/4	1.45	1 3/4 to 5	1 1/4	1.77	1 3/4 to 5
(M) 1 1/8	1.60	1 3/4 to 5	1 3/8	1.59	2 to 5	1 3/8	1.95	2 to 5
(N) 1 1/4	1.77	2 to 5	1 1/2	1.73	2 to 5	1 1/2	2.13	2 1/4 to 5

Round and Filister Head Cap-screws.		Flat Head Cap-screws.		Button-head Cap-screws.	
Diam. of Head.	Lengths (under Head).	Diam. of Head.	Lengths (including Head).	Diam. of Head.	Lengths (under Head).
(A) 3-16	3/4 to 2 1/2	1/4	3/4 to 1 3/4	7-32 (.225)	3/4 to 1 3/4
(B) 1/4	3/4 to 2 3/4	3/8	3/4 to 2	5-16	3/4 to 2
(C) 3/8	3/4 to 3	15-32	3/4 to 2 1/4	7-16	3/4 to 2 1/4
(D) 7-16	3/4 to 3 1/4	5/8	3/4 to 2 3/4	9-16	3/4 to 2 1/2
(E) 9-16	3/4 to 3 1/2	3/4	3/4 to 3	5/8	3/4 to 2 3/4
(F) 5/8	3/4 to 3 3/4	13-16	1 to 3	3/4	3/4 to 3
(G) 3/4	3/4 to 4	7/8	1 1/4 to 3	13-16	1 to 3
(H) 13-16	1 to 4 1/4	1	1 1/2 to 3	15-16	1 1/4 to 3
(I) 7/8	1 1/4 to 4 1/2	1 1/8	1 3/4 to 3	1	1 1/2 to 3
(J) 1	1 1/2 to 4 3/4	1 3/8	2 to 3	1 1/4	1 3/4 to 3
(K) 1 1/8	1 3/4 to 5				
(L) 1 1/4	2 to 5				

* For cast iron. For numbers of twist-drills see p. 29.

Threads are U. S. Standard. Cap screws are threaded 3/4 length up to and including 1″ diam. × 4″ long, and 1/2 length above. Lengths increase by 1/4″ each regular size between the limits given. Lengths of heads, except flat and button, equal diam. of screws.

The angle of the cone of the flat-head screw is 76°, the sides making angles of 52° with the top.

STANDARD MACHINE SCREWS.

No.	Threads per Inch.	Diam. of Body.	Diam. of Flat Head.	Diam. of Round Head.	Diam. of Fillister Head.	Lengths.	
						From	To
2	56	.0842	.1631	.1544	.1332	3-16	½
3	48	.0978	.1894	.1786	.1545	3-16	⅝
4	32, 36, 40	.1105	.2158	.2028	.1747	3-16	¾
5	32, 36, 40	.1236	.2421	.2270	.1985	3-16	⅞
6	30, 32	.1368	.2684	.2512	.2175	3-16	1
7	30, 32	.1500	.2947	.2754	.2392	¼	1⅛
8	30, 32	.1631	.3210	.2936	.2610	¼	1¼
9	24, 30, 32	.1763	.3474	.3238	.2805	¼	1⅜
10	24, 30, 32	.1894	.3737	.3480	.3035	¼	1½
12	20, 24	.2158	.4263	.3922	.3445	⅜	1¾
14	20, 24	.2421	.4790	.4364	.3885	⅜	2
16	16, 18, 20	.2684	.5316	.4866	.4300	⅜	2¼
18	16, 18	.2947	.5842	.5248	.4710	½	2½
20	16, 18	.3210	.6368	.5690	.5200	½	2¾
22	16, 18	.3474	.6894	.6106	.5557	½	3
24	14, 16	.3737	.7420	.6522	.6005	½	3
26	14, 16	.4000	.7420	.6938	.6425	¾	3
28	14, 16	.4263	.7946	.7854	.6920	⅞	3
30	14, 16	.4520	.8473	.7770	.7240	1	3

Lengths vary by 16ths from 3-16 to ½, by 8ths from ½ to 1½, by 4ths from 1½ to 3.

SIZES AND WEIGHTS OF SQUARE AND HEXAGONAL NUTS.

United States Standard Sizes. Chamfered and trimmed. Punched to suit U. S. Standard Taps.

Diam. of Bolt.	Width.	Thickness.	Diam. of Hole.	Long Diam. Sq. Nuts.	Long Diam. Hex. Nuts.	Square.		Hexagon.	
						No. in 100 lbs.	Wt. each in lbs.	No. in 100 lbs.	Wt. each in lbs.
¼	½	¼	13-64	11-16	9-16	7270	.0138	7615	.0181
5-16	19-32	5-16	¼	13-16	11-16	4700	.0231	5200	.0192
⅜	11-16	⅜	19-64	1	13-16	2350	.0426	3000	.0333
7-16	25-32	7-16	11-32	1⅛	⅞	1630	.0613	2000	.050
½	⅞	½	25-64	1¼	1	1120	.0893	1430	.070
9-16	31-32	9-16	29-64	1⅜	1⅛	890	.1124	1100	.091
⅝	1 1-16	⅝	33-64	1½	1¼	640	.156	740	.135
¾	1¼	¾	39-64	1¾	1 7-16	380	.263	450	.222
⅞	1 7-16	⅞	47-64	2 1-16	1 11-16	280	.357	309	.324
1	1⅝	1	53-64	2 5-16	1⅞	170	.588	216	.463
1⅛	1 13-16	1⅛	59-64	2 9-16	2 1-16	130	.769	148	.676
1¼	2	1¼	1 1-16	2 13-16	2 5-16	96	1.04	111	.901
1⅜	2 3-16	1⅜	1 5-32	3⅛	2½	70	1.48	85	1.18
1½	2⅜	1½	1 9-32	3⅜	2¾	58	1.72	68	1.47
1⅝	2 9-16	1⅝	1 13-32	3⅝	2 15-16	44	2.27	56	1.79
1¾	2¾	1¾	1½	3⅞	3 3-16	34	2.94	40	2.50
1⅞	2 15-16	1⅞	1⅝	4⅛	3⅜	30	3.33	37	2.70
2	3⅛	2	1 23-32	4 7-16	3⅝	23	4.35	29	3.45
2¼	3½	2¼	1 15-16	4 15-16	4 1-16	19	5.26	21	4.76
2½	3⅞	2½	2 3-16	5½	4½	12	8.33	15	6.67
2¾	4¼	2¾	2 7-16	6	4 15-16	9	11.11	11	9.09
3	4⅝	3	2⅝	6½	5 5-16	7⅜	13.64	8½	11.76

WEIGHT OF 100 BOLTS WITH SQUARE HEADS.

(Hoopes & Townsend.)

Diam., Inches.	1/4	5/16	3/8	7/16	1/2	9/16	5/8	¾	⅞	1	1⅛	1¼	1⅜	1½	1¾	2
Length.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1½ in.	3.9	6.2	9.7	14.7	20.4	26.0	37.0	58.0	....	....	....	....	....	....	....	....
2 "	4.6	7.2	11.3	16.5	22.4	29.0	39.9	63.2	97.7	145	....	....	....	....	....	....
2½ "	5.4	8.2	12.9	18.5	25.0	32.2	44.1	69.0	105.6	153	....	....	....	....	....	....
3 "	6.2	9.3	14.5	20.5	27.8	35.4	48.3	75.2	113.8	163	240	309	350	480	....	....
3½ "	6.9	10.4	16.1	22.6	30.6	38.7	52.5	81.4	122.0	174	253	325	370	500	....	....
4 "	7.6	11.5	17.7	24.7	33.4	42.0	56.7	87.6	130.2	185	267	342	390	520	800	....
4½ "	8.3	12.6	19.2	26.8	36.2	45.3	60.9	93.8	138.4	196	281	359	410	545	833	....
5 "	9.0	13.7	20.7	28.9	39.0	48.6	65.1	100.0	146.6	207	295	376	430	570	866	1370
5½ "	9.7	14.8	22.2	31.0	41.8	51.9	69.2	106.1	154.9	218	309	394	450	595	900	1414
6 "	10.4	15.9	23.7	33.1	44.6	55.2	73.4	112.2	163.2	229	323	412	470	620	934	1458
6½ "	11.1	17.0	25.2	35.2	47.4	58.5	77.6	118.3	171.5	240	337	430	490	645	968	1502
7 "	11.8	18.1	26.7	37.3	50.2	61.8	81.8	124.4	179.8	251	351	448	510	670	1002	1546
7½ "	12.5	19.2	28.2	39.4	53.1	65.1	86.0	130.5	187.1	262	365	465	530	695	1036	1590
8 "	13.2	20.3	29.7	41.5	56.0	68.5	90.0	136.6	195.4	273	379	484	550	725	1070	1634
9 "	....	....	33.1	45.7	61.5	75.2	98.0	148.8	212.0	295	407	518	590	775	1138	1722
10 "	....	....	36.5	49.9	67.0	81.9	106.3	161.0	229.0	317	435	552	630	825	1206	1810
11 "	....	....	40.0	54.1	72.5	88.7	114.6	173.2	246.0	339	463	586	670	875	1274	1898
12 "	....	....	43.5	58.3	78.0	95.5	122.9	184.4	263.0	361	491	620	710	925	1342	1986
13 "	....	....	....	....	83.5	102.3	131.2	196.6	280.0	383	519	655	751	975	1410	2074
14 "	....	....	....	....	89.0	109.1	139.5	208.8	297.0	405	547	690	793	1025	1478	2162
15 "	....	....	....	....	94.5	116.0	148.0	221.0	314.0	427	575	725	835	1075	1548	2250
16 "	....	....	....	....	100.0	123.0	156.5	233.2	331.0	449	603	760	877	1125	1616	2338
17 "	....	....	....	....	105.5	130.0	165.0	245.4	348.0	471	631	795	919	1175	1684	2426
18 "	....	....	....	....	111.0	137.0	173.5	257.6	365.0	493	659	830	961	1225	1752	2514
19 "	....	....	....	....	116.5	144.0	182.0	269.8	382.0	515	687	865	1003	1275	1820	2602
20 "	....	....	....	....	122.0	151.0	190.5	282.0	399.0	537	715	900	1045	1325	1888	2690
21 "	....	....	....	....	....	....	198.0	294.0	416.0	559	743	965	1087	1375	1956	2778
22 "	....	....	....	....	....	....	206.0	306.0	437.0	581	771	1030	1129	1425	2024	2866
23 "	....	....	....	....	....	....	215.0	318.0	454.0	603	799	1095	1171	1475	2092	2954
24 "	....	....	....	....	....	....	224.0	330.0	470.0	625	827	1160	1213	1525	2160	3042
25 "	....	....	....	....	....	....	....	....	....	....	855	1225	1255	1575	2228	3130

TRACK BOLTS.

With United States Standard Hexagon Nuts.

Rails used.	Bolts.	Nuts.	No. in Keg, 200 lbs.	Kegs per Mile.
45 to 85 lbs...	¾ × 4¼	1¼	230	6.3
	¾ × 4	1¼	240	6.
	¾ × 3¾	1¼	254	5.7
	¾ × 3½	1¼	260	5.5
	¾ × 3¼	1¼	266	5.4
	¾ × 3	1¼	283	5.1
30 to 40 lbs...	⅝ × 3½	1 1-16	375	4.
	⅝ × 3	1 1-16	410	3.7
	⅝ × 2¾	1 1-16	435	3.3
	⅝ × 2½	1 1-16	465	3.1
20 to 30 lbs...	½ × 3	⅞	715	2.
	½ × 2½	⅞	760	2.
	½ × 2¼	⅞	800	2.
	½ × 2	⅞	820	2.

CONE-HEAD BOILER RIVETS, WEIGHT PER 100.

(Hoopes & Townsend.)

Diam., in., Scant.	1/2	9/16	5/8	11/16	¾	13/16	⅞	1	1⅛*	1¼*
Length.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¾ inch	8.75	13.7	16.20							
⅞ "	9.35	14.4	17.22							
1 "	10.00	15.2	18.25	21.70	26.55					
1⅛ "	10.70	16.0	19.28	23.10	28.00					
1¼ "	11.40	16.8	20.31	24.50	29.45	37.0	46	60		
1⅜ "	12.10	17.6	21.34	25.90	30.90	38.6	48	63	95	
1½ "	12.80	18.4	22.37	27.30	32.35	40.2	50	65	98	133
1⅝ "	13.50	19.2	23.40	28.70	33.80	41.9	52	67	101	137
1¾ "	14.20	20.0	24.43	30.10	35.25	43.5	54	69	104	141
1⅞ "	14.90	20.8	25.46	31.50	36.70	45.2	56	71	107	145
2 "	15.60	21.6	26.49	32.90	38.15	47.0	58	74	110	149
2⅛ "	16.30	22.4	27.52	34.30	39.60	48.7	60	77	114	153
2¼ "	17.00	23.2	28.55	35.70	41.05	50.3	62	80	118	157
2⅜ "	17.70	24.0	29.58	37.10	42.50	51.9	64	83	121	161
2½ "	18.40	24.8	30.61	38.50	43.95	53.5	66	86	124	165
2⅝ "	19.10	25.6	31.64	39.90	45.40	55.1	68	89	127	169
2¾ "	19.80	26.4	32.67	41.30	46.85	56.8	70	92	130	173
2⅞ "	20.50	27.2	33.70	42.70	48.30	58.4	72	95	133	177
3 "	21.20	28.0	34.73	44.10	49.75	60.0	74	98	137	181
3¼ "	22.60	29.7	36.79	46.90	52.65	63.3	78	103	144	189
3½ "	24.00	31.5	38.85	49.70	55.55	66.5	82	108	151	197
3¾ "	25.40	33.3	40.91	52.50	58.45	69.8	86	113	158	205
4 "	26.80	35.2	42.97	55.30	61.35	73.0	90	118	165	213
4¼ "	28.20	36.9	45.03	58.10	64.25	76.3	94	124	172	221
4½ "	29.60	38.6	47.09	60.90	67.15	79.5	98	130	179	229
4¾ "	31.00	40.3	49.15	63.70	70.05	82.8	102	136	186	237
5 "	32.40	42.0	51.21	66.50	72.95	86.0	106	142	193	245
5¼ "	33.80	43.7	53.27	69.20	75.85	89.3	110	148	200	254
5½ "	35.20	45.4	55.33	72.00	78.75	92.5	114	154	206	263
5¾ "	36.60	47.1	57.39	74.80	81.65	95.7	118	160	212	272
6 "	38.00	48.8	59.45	77.60	84.55	99.0	122	166	218	281
6½ "	40.80	52.0	63.57	83.30	90.35	105.5	130	177	231	297
7 "	43.60	55.2	67.69	88.90	96.15	112.0	138	188	245	314
Heads......	5.50	8.40	11.50	13.20	18.00	23.0	29.0	38.0	56.0	77.5

* These two sizes are calculated for exact diameter.

Rivets with button heads weigh approximately the same as cone-head rivets.

TURNBUCKLES.

(Cleveland City Forge and Iron Co.)

Standard sizes made with right and left threads. D = outside diameter

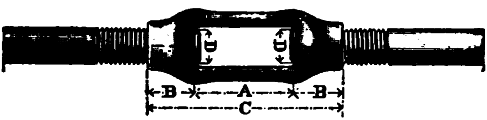


FIG. 73.

of screw. A = length in clear between heads = 6 ins. for all sizes. B = length of tapped heads = 1⅛D nearly. L = 6 ins. + 3D nearly.

SIZES OF WASHERS.

Diameter in inches.	Size of Hole, in inches.	Thickness, Birmingham Wire-gauge.	Bolt in inches.	No. in 100 lbs.
⅝	5-16	No. 16	¼	29,300
¾	⅜	" 16	5-16	18,000
1	7-16	" 14	⅜	7,600
1⅛	9-16	" 11	½	3,300
1¼	⅝	" 11	9-16	2,180
1⅜	11-16	" 11	⅝	2,350
1¾	13-16	" 11	¾	1,680
2	31-32	" 10	⅞	1,140
2½	1⅛	" 8	1	580
2¾	1¼	" 8	1⅛	470
3	1⅜	" 7	1¼	360
3	1½	" 6	1⅜	360

TRACK SPIKES.

Rails used.	Spikes.	Number in Keg, 200 lbs.	Kegs per Mile, Ties 24 in. between Centres.
45 to 85	5½ × 9-16	380	30
40 " 52	5 × 9-16	400	27
35 " 40	5 × ½	490	22
24 " 35	4½ × ½	550	20
24 " 30	4½ × 7-16	725	15
18 " 24	4 × 7-16	820	13
16 " 20	3½ × ⅜	1250	9
14 " 16	3 × ⅜	1350	8
8 " 12	2½ × ⅜	1550	7
8 " 10	2½ × 5-16	2250	6

STREET RAILWAY SPIKES.

Spikes.	Number in Keg, 200 lbs.	Kegs per Mile, Ties 24 in. between Centres.
5½ × 9-16	400	30
5 × ½	575	19
4½ × 7-16	800	13

BOAT SPIKES.

Number in Keg of 200 lbs.

Length.	¼	5-16	⅜	½
4 inch.	2375	....	....	....
5 "	2050	1230	940	....
6 "	1825	1175	800	450
7 "	....	990	650	375
8 "	....	880	600	335
9 "	....	....	525	300
10 "	....	....	475	275

WROUGHT SPIKES.

Number of Nails in Keg of 150 Pounds.

Size.	¼ in.	5-16 in.	⅜ in.	7-16 in.	½ in.
3 inches	2250	....	....	....	....
3½ "	1890	1208	....	....	....
4 "	1650	1135	....	....	....
4½ "	1464	1064	....	....	....
5 "	1380	930	742	....	....
6 "	1292	868	570	....	....
7 "	1161	662	482	445	306
8 "	....	635	455	384	256
9 "	....	573	424	300	240
10 "	....	....	391	270	222
11 "	....	....	....	249	203
12 "	....	....	....	236	180

WIRE SPIKES.

Size.	Approx. Size of Wire Nails.	Ap. No. in 1 lb.	Size.	Approx. Size of Wire Nails.	Ap. No. in 1 lb.
10d Spike	3 in. No. 7	50	60d Spike	6 in. No. 1	10
16d "	3½ " " 6	35	6½ in. "	6½ " " 1	9
20d "	4 " " 5	26	7 " "	7 " " 0	7
30d "	4½ " " 4	20	8 " "	8 " " 00	5
40d "	5 " " 3	15	9 " "	9 " " 00	4½
50d "	5½ " " 2	12			

LENGTH AND NUMBER OF CUT NAILS TO THE POUND.

Size.	Length.	Common.	Clinch.	Fence.	Finishing.	Fine.	Barrel.	Casing.	Brads.	Tobacco.	Cut Spikes.
¾	¾ in.	....	....	....	....	....	800	....	....	....	....
⅞	⅞	....	....	....	....	....	500	....	....	....	....
2d	1	800	....	....	1100	1000	376	....	....	....	....
3d	1¼	480	....	....	720	760	224	....	....	....	....
4d	1½	288	....	....	523	368	180	398	....	....	....
5d	1¾	200	....	....	410	....	....	....	....	130	....
6d	2	168	95	84	268	....	....	224	126	96	....
7d	2¼	124	74	64	188	....	....	....	98	82	....
8d	2½	88	62	48	146	....	....	128	75	68	....
9d	2¾	70	53	36	130	....	....	110	65	....	....
10d	3	58	46	30	102	....	....	91	55	....	28
12d	3¼	44	42	24	76	....	....	71	40	....	....
16d	3½	34	38	20	62	....	....	54	27	....	22
20d	4	23	33	16	54	....	....	40	....	....	14½
30d	4½	18	20	....	....	....	....	33	....	....	12½
40d	5	14	....	....	....	....	....	27	....	....	9½
50d	5½	10	....	....	....	....	....	....	....	....	8
60d	6	8	....	....	....	....	....	....	....	....	6

SIZES, LENGTH, AND NUMBER TO THE POUND OF STANDARD STEEL WIRE NAILS.

(John A. Roebling's Sons Co.)

Sizes.	Length, Inches.	Common Nails and Brads.	Barbed Common.	Clinch.	Fence.	Smooth and Barbed Finishing.	Fine.	Barrel.	Casing, and Smooth and Barbed Box.	Flooring Brads.	Barbed Oval Head Car Nail.		Slating.	Barbed Roofing.	Shingle.	Tobacco.	Lining.	Wire Spikes.	Length, Inches.	Sizes.
											Light.	Heavy.								
....	¾	....	....	....	....	....	....	1500	....	....	....	....	....	714	....	....	2100	....	¾	....
....	⅞	....	....	....	....	....	....	1000	....	....	....	....	....	469	....	....	1780	....	⅞	....
2d........	1	1200	876	710	....	1558	1550	875	1850	....	....	....	411	411	....	....	1500	....	1	2d
3d fine..	1⅛	....	....	....	....	....	1140	775	....	....	....	....	....	....	....	....	....	....	1⅛	3d fine
3d common	1¼	720	568	429	....	960	....	560	913	....	....	....	251	251	....	....	....	....	1¼	3d c'm
....	1⅜	....	....	....	....	....	....	390	....	....	....	....	....	....	....	....	....	....	1⅜	....
4d........	1½	432	357	274	....	760	760	350	584	....	274	165	209	165	....	274	....	....	1½	4d
5d........	1¾	300	235	235	142	575	....	....	410	....	142	118	142	142	270	235	....	....	1¾	5d
6d........	2	252	204	157	124	350	....	....	310	157	124	108	....	108	204	157	....	....	2	6d
7d........	2¼	186	139	139	92	275	....	....	238	139	92	76	....	....	182	139	....	....	2¼	7d
8d........	2½	132	99	99	82	190	....	....	170	99	82	69	....	....	125	99	....	....	2½	8d
9d........	2¾	105	90	90	62	173	....	....	150	90	62	54	....	....	114	90	....	....	2¾	9d
10d........	3	87	69	83	50	137	....	....	121	67	57	50	....	....	83	69	....	50	3	10d
12d........	3¼	66	53	64	38	98	....	....	97	53	47	38	....	....	....	....	....	....	3¼	12d
16d........	3½	51	43	59	30	81	....	....	72	43	48	35	....	....	....	....	....	35	3½	16d
20d........	4	35	31	43	23	71	....	....	54	....	31	26	....	....	....	....	....	26	4	20d
30d........	4½	27	24	....	....	....	....	....	46	....	28	24	....	....	....	....	....	20	4½	30d
40d........	5	21	18	....	....	....	....	....	36	....	21	18	....	....	....	....	....	15	5	40d
50d........	5½	15	....	....	....	....	....	....	....	....	17	14	....	....	....	....	....	12	5½	50d
60d........	6	12	....	....	....	....	....	....	....	....	15	13	....	....	....	....	....	10	6	60d

3⅝ lbs. of 4d Common, or 2⅝ lbs. of 3d Common, will lay 1000 shingles.
3¼ lbs. of 3d Fine will put on 1000 laths—4 nails to the lath.

APPROXIMATE NUMBER OF WIRE NAILS PER POUND.

Wire Gauge. B. W. G.	Length, inches.																						
	¼	⅜	½	⅝	¾	1	1¼	1½	1¾	2	2½	3	3½	4	4½	5	6	7	8	9	10	11	12
00	....	....	....	....	....	....	33	27	23	20	16	14	12	10	9	8	7	6	5	4½	4	3¾	3¼
0	....	....	....	....	....	....	34	29	25	21	17	15	13	11	10	9	8	7	5½	5	4½	4	3½
1	....	....	....	....	....	57	45	33	32	28	23	19	16	14	13	11	10	8	7	6	5½	5	4½
2	....	....	....	....	....	65	52	44	37	32	26	22	19	16	14	13	11	9	8	7	6½	6	5½
3	....	....	....	....	100	76	60	50	43	38	30	25	22	19	17	15	13	11	10	8	7½	....	....
4	....	....	....	....	120	90	72	60	51	45	36	30	26	23	20	18	15	13	11	10	9	....	....
5	....	....	211	169	141	106	85	71	60	53	42	35	30	26	24	21	18	15	....	....	....	....	....
6	....	....	247	197	164	123	99	82	71	62	50	41	35	31	28	25	21	18	....	....	....	....	....
7	....	....	290	239	200	149	120	100	85	75	60	50	43	37	33	30	25	....	....	....	....	....	....
8	....	....	345	275	229	172	187	115	98	86	69	57	49	43	39	35	29	....	....	....	....	....	....
9	....	....	414	331	276	207	165	138	118	103	82	69	59	52	46	41	....	....	....	....	....	....	....
10	....	663	496	397	333	248	198	165	142	124	99	83	71	62	55	50	....	....	....	....	....	....	....
11	....	887	628	502	418	314	251	209	179	157	125	105	90	79	70	....	....	....	....	....	....	....	....
12	....	1096	822	658	548	411	329	274	235	204	164	137	117	108	....	....	....	....	....	....	....	....	....
13	....	1429	1072	857	714	536	429	357	306	268	214	178	158	....	....	....	....	....	....	....	....	....	....
14	2840	1893	1420	1136	947	710	568	473	406	350	284	236	....	....	....	....	....	....	....	....	....	....	....
15	3504	2336	1752	1402	1168	876	701	584	500	438	350												
16	4571	3048	2280	1828	1523	1143	913	761	653	571	....												
17	6233	4156	3116	2495	2077	1558	1246	1038	890	779	....												
18	8276	5517	4138	3310	2758	2069	1655	1379	1182	....	....												
19	10668	7112	5334	4267	3556	2667	2133	1778	....	....	....												
20	15000	10000	7500	6000	5000	3750	3000	....	....	....	....												
21	17777	11850	8888	7111	5926	4444	....	....	....	....	....												
22	22856	15237	11428	9143	7618	....	....	....	....	....	....												

These approximate numbers are an average only, and the figures given may be varied either way by changes in the dimensions of the heads or points. Brads and no-head nails will run more to the pound than the table shows, and large or thick-headed nails will run less.

SIZE, WEIGHT, LENGTH, AND STRENGTH OF IRON WIRE.

(Trenton Iron Co.)

No. by Wire Gauge.	Diam. in Decimals of One Inch.	Area of Section in Decimals of One Inch.	Feet to the Pound.	Weight of One Mile in pounds.	Tensile Strength (Approximate) of Charcoal Iron Wire in Pounds.	
					Bright.	Annealed.
00000	.450	.15904	1.863	2833.248	12598	9449
0000	.400	.12566	2.358	2238.878	9955	7466
000	.360	.10179	2.911	1818.574	8124	6091
00	.330	.08553	3.465	1523.861	6880	5160
0	.305	.07306	4.057	1301.678	5926	4445
1	.285	.06379	4.645	1136.678	5226	3920
2	.265	.05515	5.374	982.555	4570	3425
3	.245	.04714	6.286	839.942	3948	2960
4	.225	.03976	7.454	708.365	3374	2530
5	.205	.03301	8.976	588.189	2839	2130
6	.190	.02835	10.453	505.084	2476	1860
7	.175	.02405	12.322	428.472	2136	1600
8	.160	.02011	14.736	358.3008	1813	1360
9	.145	.01651	17.950	294.1488	1507	1130
10	.130	.01327	22.333	236.4384	1233	925
11	.1175	.01084	27.340	193.1424	1010	758
12	.105	.00866	34.219	154.2816	810	607
13	.0925	.00672	44.092	119.7504	631	473
14	.080	.00503	58.916	89.6016	474	356
15	.070	.00385	76.984	68.5872	372	280
16	.061	.00292	101.488	52.0060	292	220
17	.0525	.00216	137.174	38.4912	222	165
18	.045	.00159	186.835	28.3378	169	127
19	.040	.0012566	235.084	22.3872	137	103
20	.035	.0009621	308.079	17.1389	107	80
21	.031	.0007547	392.772	13.4429		
22	.028	.0006157	481.234	10.9718		
23	.025	.0004909	603.863	8.7437		
24	.0225	.0003976	745.710	7.0805		
25	.020	.0003142	943.396	5.5968		
26	.018	.0002545	1164.689	4.5334		
27	.017	.0002270	1305.670	4.0439		
28	.016	.0002011	1476.869	3.5819		
29	.015	.0001767	1676.989	3.1485		
30	.014	.0001539	1925.321	2.7424		
31	.013	.0001327	2232.653	2.3649		
32	.012	.0001131	2620.607	2.0148		
33	.011	.0000950	3119.092	1.6928		
34	.010	.00007854	3773.584	1.3992		
35	.0095	.00007088	4182.508	1.2624		
36	.009	.00006362	4657.728	1.1336		
37	.0085	.00005675	5222.035	1.0111		
38	.008	.00005027	5896.147	.89549		
39	.0075	.00004418	6724.291	.78672		
40	.007	.00003848	7698.253	.68587		

The above figures on tensile strength are based upon tests made with good charcoal-iron wire from Trenton blooms.
The tensile strength of wire made of—
Good refined iron is about 15% less,
Swedish charcoal iron is about 10 " "
Mild Bessemer steel is about 10 more,
Ordinary crucible steel is about 25 " "
Special crucible steel is from 30 to 150 "
than that of charcoal-iron wire.

GALVANIZED IRON WIRE FOR TELEGRAPH AND TELEPHONE LINES.

(Trenton Iron Co.)

WEIGHT PER MILE-OHM.—This term is to be understood as distinguishing the *resistance of material* only, and means the weight of such material required per mile to give the resistance of one ohm. To ascertain the mileage resistance of any wire, divide the "weight per mile-ohm" by the weight of the wire per mile. Thus in a grade of Extra Best Best, of which the weight per mile-ohm is 5000, the mileage resistance of No. 6 (weight per mile 525 lbs.) would be about 9½ ohms; and No. 14 steel wire, 6500 lbs. weight per mile-ohm (95 lbs. weight per mile), would show about 69 ohms.

Sizes of Wire used in Telegraph and Telephone Lines.

No. 4. Has not been much used until recently; is now used on important lines where the multiplex systems are applied.

No. 5. Little used in the United States.

No. 6. Used for important circuits between cities.

No. 8. Medium size for circuits of 400 miles or less.

No. 9. For similar locations to No. 8, but on somewhat shorter circuits; until lately was the size most largely used in this country.

Nos. 10, 11. For shorter circuits, railway telegraphs, private lines, police and fire-alarm lines, etc.

No. 12. For telephone lines, police and fire-alarm lines, etc.

Nos. 13, 14. For telephone lines and short private lines: steel wire is used most generally in these sizes.

The coating of telegraph wire with zinc as a protection against oxidation is now generally admitted to be the most efficacious method.

The grades of line wire are generally known to the trade as "Extra Best Best" (E. B. B.), "Best Best" (B. B.), and "Steel."

"Extra Best Best" is made of the very best iron, as nearly pure as any commercial iron, soft, tough, uniform, and of very high conductivity, its weight per mile-ohm being about 5000 lbs.

The "Best Best" is of iron, showing in mechanical tests almost as good results as the E. B. B., but not quite as soft, and being somewhat lower in conductivity; weight per mile-ohm about 5700 lbs.

The Trenton "Steel" wire is well suited for telephone or short telegraph lines, and the weight per mile-ohm is about 6500 lbs.

The following are (approximately) the weights per mile of various sizes of galvanized telegraph wire, drawn by Trenton Iron Co.'s gauge:

No.	4,	5,	6,	7,	8,	9,	10,	11,	12,	13,	14.
Lbs.	720,	610,	525,	450,	375,	310,	250,	200,	160,	125,	95.

TESTS OF TELEGRAPH WIRE.

The following data are taken from a table given by Mr. Prescott relating to tests of E. B. B. galvanized wire furnished the Western Union Telegraph Co.:

Size of Wire.	Diam. Parts of One Inch.	Weight.		Length. Feet per pound.	Resistance. Temp. 75.8° Fahr.		Ratio of Breaking Weight to Weight per mile.
		Grains. per foot.	Pounds per mile.		Feet per ohm.	Ohms per mile.	
4	.238	1043.2	886.6	6.00	958	5.51	
5	.220	891.8	673.0	7.85	727	7.26	
6	.203	758.9	572.2	9.20	618	8.54	3.05
7	.180	596.7	449.9	11.70	578	10.86	3.40
8	.165	501.4	378.1	14.00	409	12.92	3.07
9	.148	403.4	304.2	17.4	328	16.10	3.38
10	.134	330.7	249.4	21.2	269	19.60	3.37
11	.120	265.2	200.0	26.4	216	24.42	2.97
12	.109	218.8	165.0	32.0	179	29.60	3.43
14	.083	126.9	95.7	55.2	104	51.00	3.05

JOINTS IN TELEGRAPH WIRES.—The fewer the joints in a line the better. All joints should be carefully made and well soldered over, for a bad joint may cause as much resistance to the electric current as several miles of wire.

TABLE OF DIMENSIONS, WEIGHT, AND RESISTANCE OF COPPER WIRE.
(Birmingham Gauge.)

Gauge Number.	Diameter, Inch.	Sectional Area in Circular Mils. = diam².	Weight. Lbs. per Foot.	Weight. Lbs. per Ohm.	Length. Feet per Lb.	Length. Feet per Ohm.	Resistance. Ohms per Lb.	Resistance. Ohms per Foot.	Gauge Number.
0000	.454	206116	.623985	19486.73	1.0027	19966.55	.00008027	.000050084	0000
000	.425	180625	.54676	9569.13	1.8285	17496.15	.00010450	.000057152	000
00	.38	144400	.437107	6256.59	2.2878	13088.	.00015983	.000071490	00
0	.34	115600	.349928	3918.60	2.8577	11198.17	.00025628	.0000893	0
1	.3	90000	.272435	2375.17	3.6706	8718.3	.00042102	.000114701	1
2	.284	80656	.24415	1952.03	4.0958	7813.15	.00051229	.00012799	2
3	.259	67081	.202964	1318.28	4.927	6495.15	.00075857	.00015396	3
4	.238	56644	.171465	940.862	5.832	5487.11	.00106285	.000182665	4
5	.22	48400	.14651	688.494	6.8255	4688.51	.00145245	.000212796	5
6	.203	41209	.124742	497.631	8.0165	3989.26	.00200029	.00025067	6
7	.18	32400	.098076	307.82	10.1962	3138.59	.00324866	.000318615	7
8	.165	27225	.08241	217.338	12.1345	2637.29	.00460113	.000379178	8
9	.148	21904	.066305	140.754	15.0818	2122.82	.0071046	.000471073	9
10	.134	17956	.054354	94.5433	18.3979	1739.4	.0105772	.00057491	10
11	.12	14400	.04359	60.806	22.941	1394.53	.012446	.00071638	11
12	.109	11881	.035964	43.3914	27.8056	1150.91	.0241596	.000868875	12
13	.095	9025	.027319	23.8837	36.6046	874.253	.0418696	.00114384	13
14	.083	6889	.020853	14.9849	47.9547	667.338	.0718596	.0014985	14
15	.072	5184	.015692	7.88012	63.7267	502.21	.126909	.00199134	15
16	.065	4225	.012789	5.23433	78.1922	409.276	.19106	.00244334	16
17	.058	3364	.01018	3.31726	98.232	325.87	.30144	.0030687	17
18	.049	2401	.007268	1.69043	137.586	232.585	.591566	.0042995	18
19	.042	1764	.005340	.912492	187.266	170.879	1.0959	.0058521	19
20	.035	1225	.003708	.44013	269.687	118.066	2.27256	.008427	20
21	.032	1024	.003099	.307405	322.585	99.195	3.25304	.010087	21
22	.028	784	.002373	.18023	421.407	75.946	5.54876	.0131672	22
23	.025	625	.001892	.114549	528.54	60.5438	8.7299	.016517	23
24	.022	484	.001465	.0686867	682.594	49.885	14.5589	.0213287	24
25	.020	400	.001211	.046924	825.764	38.748	21.3111	.0258078	25
26	.018	324	.0009807	.030786	1019.68	31.386	32.4885	.0318615	26
27	.016	256	.0007749	.0192165	1290.489	24.7967	52.0389	.0403246	27
28	.014	196	.0005933	.0112647	1685.488	18.9866	88.773	.052068	28
29	.013	169	.0005116	.00837544	1954.652	16.371	119.397	.061063	29
30	.012	144	.0004359	.00608035	2294.104	13.949	164.46	.071658	30
31	.01	100	.0003027	.00293966	3303.60	9.687	341.064	.103231	31
32	.009	81	.0002452	.00198596	4078.303	7.8465	519.763	.127446	32
33	.008	64	.0001937	.0012013	5160.78	6.1998	832.486	.1613	33
34	.007	49	.0001483	.000709925	6743.088	4.7466	1420.6	.21068	34
35	.005	25	.00007568	.000183269	13214.16	2.4217	5456.5	.41293	35
36	.004	16	.00004843	.000074894	20647.12	1.5499	13321.406	.6448	36

TABLE OF DIMENSIONS, WEIGHT, AND RESISTANCE OF PURE COPPER WIRE.

(Edison or Circular Mil Gauge.) (See page 30.)

E. & G. Gauge Number.	Circular Mils.	Maximum Amperes. $\sqrt[4]{\left(\frac{C.M.}{104}\right)^3}$	Diameter in Mils. Mil = .001 in.	Weight. Sp. gr. 8.889.		Length.		Resistance. Legal Ohms at 75° Fahr.		E. & G. Gauge Number.
				Lbs. per Foot.	Lbs. per Ohm.	Feet per Lb.	Feet per Ohm.	Ohms per Lb.	Ohms per Ft.	
3	3000	12.5	54.78	.009084	2.597	110.087	285.9	.3850405	.003497600	3
5	5000	18.3	70.72	.015139	7.214	66.054	476.5	.1386225	.002098640	5
8	8000	26.0	89.45	.024220	18.464	41.288	762.3	.0551602	.001311780	8
12	12000	35.2	109.55	.036328	41.538	27.527	1143.4	.0240743	.000874578	12
15	15000	41.6	122.48	.045410	64.902	22.022	1429.2	.0154178	.000699663	15
20	20000	51.6	141.42	.060548	115.372	16.516	1905.7	.0086664	.000524745	20
25	25000	61.0	158.12	.075682	180.278	13.218	2382.0	.0055470	.000419807	25
30	30000	70.0	173.21	.090817	259.722	11.011	2859.9	.0038522	.000349840	30
35	35000	78.6	187.09	.105955	353.340	9.4381	3334.9	.0028301	.000299863	35
40	40000	86.8	200.00	.121088	461.440	8.2589	3811.0	.0021671	.000262400	40
45	45000	94.9	212.14	.136227	584.098	7.3407	4287.7	.0017120	.000233227	45
50	50000	102.7	223.61	.151357	721.096	6.6069	4763.8	.0013868	.000209914	50
55	55000	110.3	234.52	.166501	872.547	6.0060	5240.5	.0011467	.000190821	55
60	60000	117.7	244.95	.181625	1038.258	5.5059	5716.5	.00096315	.000174931	60
65	65000	125.0	254.96	.196772	1218.586	5.0820	6192.9	.00082057	.000161465	65
70	70000	132.1	264.58	.211901	1413.284	4.7192	6669.4	.00070758	.000149937	70
75	75000	139.1	273.87	.227043	1622.457	4.4044	7146.0	.00061635	.000139938	75
80	80000	146.0	282.85	.242176	1845.952	4.1292	7622.3	.00054172	.000131193	80
85	85000	152.8	291.55	.257303	2083.759	3.8865	8098.4	.00047990	.000123480	85
90	90000	159.5	300.00	.272434	2336.405	3.6706	8574.7	.00042807	.000116622	90
95	95000	166.1	308.22	.287587	2603.046	3.4773	9051.6	.00038415	.000110477	95
100	100000	172.6	316.23	.302709	2884.082	3.3035	9527.6	.00034673	.000104960	100
110	110000	185.4	331.67	.332991	3489.958	3.0031	10480.6	.00028656	.000095410	110
120	120000	198.0	346.42	.363267	4153.433	2.7528	11433.6	.00024070	.000084460	120
130	130000	210.2	360.56	.393527	4874.226	2.5411	12386.0	.00020514	.000080730	130
140	140000	222.2	374.17	.423797	5652.899	2.3596	13338.7	.00017690	.000074970	140
150	150000	234.0	387.30	.454061	6484.573	2.2023	14291.3	.00015409	.000069997	150
160	160000	245.6	400.00	.484328	7383.042	2.0647	15243.9	.00013544	.000065600	160
170	170000	257.0	412.32	.514622	8335.525	1.9432	16197.4	.00011995	.000061735	170
180	180000	268.3	424.27	.544884	9344.686	1.8353	17149.9	.00010701	.000058309	180
190	190000	279.4	435.89	.575140	10411.241	1.7387	18102.1	.00009604	.000055242	190
200	200000	290.4	447.22	.605427	11536.681	1.6517	19055.4	.00008667	.000052478	200
220	220000	312.0	469.05	.665975	13959.567	1.5016	20961.1	.00007163	.000047707	220
240	240000	333.0	489.90	.726498	16612.114	1.3765	22866.0	.00006019	.000043733	240
260	260000	353.5	509.91	.787058	19496.997	1.2706	24772.1	.00005129	.000040368	260
280	280000	373.7	529.16	.847605	22612.233	1.1798	26677.8	.00004422	.000037484	280
300	300000	393.6	547.73	.908140	25957.464	1.1012	28583.1	.00003852	.000034986	300
320	320000	413.1	565.69	.968672	29533.696	1.0323	30488.3	.00003386	.000032799	320
340	340000	432.3	583.10	1.029214	33340.181	.9716	32393.8	.00002999	.000030870	340
360	360000	451.3	600.00	1.089738	37376.652	.9177	34298.7	.00002675	.000029155	360

1 Mil Foot = 9.718 B. A. Units at 0° C. (Dr. Matthiessen.)

DIMENSIONS, WEIGHT, AND RESISTANCE OF COPPER WIRE.
(Brown & Sharpe's Gauge.)

Gauge Number.	Diameter Inch.	Sect. Area in Circular Mils.	Weight. Lbs. per Foot.	Weight. Lbs. per Ohm.	Length.—Feet. Per Lb.	Length.—Feet. Per Ohm.	Resistance.—Ohms. Per Foot.	Resistance.—Ohms. Per Lb.	Gauge Number.
0000	.46	211600.	.640525	13129.29	1.56122	20497.7	.000048786	.0000761656	0000
000	.40964	167805.	.507955	8256.95	1.9687	16255.27	.000061519	.00012111	000
00	.3648	133079.	.40284	5193.13	2.4824	12891.37	.0000775713	.000192563	00
0	.32486	105534.	.319457	3265.84	3.1303	10223.08	.000097818	.0003062	0
1	.2893	83694.	.253348	2054.015	3.94714	8107.49	.000123342	.000476866	1
2	.25763	66373.	.200915	1291.80	4.97722	6429.58	.00015553	.000774113	2
3	.22942	52634.	.159325	812.709	6.2765	5098.61	.000196132	.00123102	3
4	.20431	41743.	.126357	522.839	7.9141	4043.6	.000247304	.00191263	4
5	.18194	33102.	.10022	321.309	9.97983	3206.61	.000311856	.00311227	5
6	.16202	26251.	.0794616	202.062	12.5847	2542.89	.000393255	.00494896	6
7	.14428	20817.	.0630134	127.07	15.8696	2015.51	.000495965	.00785156	7
8	.12849	16510.	.0499757	79.9258	20.0097	1599.3	.000625276	.0125116	8
9	.11443	13094.	.039637	50.2886	25.229	1268.44	00078837	.0198852	9
10	.10189	10382.	.0314256	31.6036	31.8212	1055.66	.00099437	.031642	10
11	.090742	8234.	.024925	19.882	40.1202	797.649	.0012537	.0502967	11
12	.080808	6530.	.0197665	12.5034	50.5906	632.555	0015809	.0799783	12
13	.071961	5178.	.0156753	7.86319	63.7948	501.63	.0019935	.127172	13
14	.064084	4107.	.0124314	4.51033	80.4415	397.822	.0025137	.221713	14
15	.057068	3257.	.0098584	3.11015	101.4365	315.482	.00316975	.321523	15
16	.05082	2583.	.0078179	1.95501	127.12	250.184	.00399707	.511507	16
17	.045257	2048.	.0062	1.29013	161.29	198.409	.0050401	.812918	17
18	.040303	1624.	.004917	.773677	203.374	157.35	.0063553	1.29253	18
19	.03589	1288.	.0038991	.486524	256.468	124.777	.00801426	2.0554	19
20	.031961	1021.	.0030922	.305979	323.399	98.9533	.0101058	3.2682	20
21	.028462	810.	.0024522	.192429	407.815	78.473	.0127432	5.19671	21
22	.025347	642.	.0019448	.121037	514.193	62.236	.0160678	8.26197	22
23	.022571	509.	.0015421	.076105	648.452	49.3504	.0202633	13.13974	23
24	.0201	404.	.001223	.0478624	817.688	39.1365	.0255516	20.89323	24
25	.0179	320.	.0009699	.0301038	1031.038	31.0381	.0322184	33.2184	25
26	.01594	254.	.0007691	.0168719	1300.180	24.6131	.0406288	53.8247	26
27	.014195	201.	.0006099	.0119056	1639.49	19.5191	.0512318	83.994	27
28	.012641	159.8	.0004837	.0074748	2067.364	15.4793	.0646023	133.5563	28
29	.011257	126.7	.0003836	.0047087	2606.959	12.2854	.081464	212.373	29
30	.010025	100.5	.0003042	.00296174	3287.084	9.7355	.102717	337.639	30
31	.008928	79.7	.0002413	.0018306	4414.49	7.72143	.12951	536.7515	31
32	.00795	63.	.0001913	.00117133	5226.915	6.12243	.163334	853.739	32
33	.00708	50.1	.0001517	.000736789	6590.41	4.85575	.205942	1357.241	33
34	.006304	39.74	.0001206	.0004631	8312.8	3.84900	.25976	2159.361	34
35	.005614	31.5	.0000954	.000291272	10481.77	3.05305	.327541	3433.21	35
36	.005	25.	.00007568	.000183209	13214.16	2.4217	.41293	5458.45	36
37	.004453	19.8	.00006003	.000115298	16659.97	1.92086	.520601	8673.9	37
38	.003965	15.72	.00004759	.0000724741	21013.25	1.52298	.656635	13798.04	38
39	.003531	12.47	.00003774	.0000455828	26496.237	1.20777	.82797	21938.11	39
40	.003144	9.88	[illegible]	[illegible]	33420.63	0.97984	1.04466	27041.4	40

HARD-DRAWN COPPER TELEGRAPH WIRE.

(J. A. Roebling's Sons Co.)

Furnished in half-mile coils, either bare or insulated.

Size. B. & S. Gauge.	Resistance in Ohms per Mile.	Breaking Strength.	Weight per Mile.	Approximate Size of E. B. B. Iron Wire equal to Copper. (Iron-Wire Gauge)
9	4.30	625	209	2
10	5.40	525	166	3
11	6.90	420	131	4
12	8.70	330	104	6
13	10.90	270	82	6½
14	13.70	213	66	8
15	17.40	170	52	9
16	22.10	130	41	10

In handling this wire the greatest care should be observed to avoid kinks, bends, scratches, or cuts. Joints should be made only with McIntire Connectors.

On account of its conductivity being about five times that of Ex. B. B. Iron Wire, and its breaking strength over three times its weight per mile, copper may be used of which the section is smaller and the weight less than an equivalent iron wire, allowing a greater number of wires to be strung on the poles.

Besides this advantage, the reduction of section materially decreases the electrostatic capacity, while its non-magnetic character lessens the self-induction of the line, both of which features tend to increase the possible speed of signalling in telegraphing, and to give greater clearness of enunciation over telephone lines, especially those of great length.

INSULATED COPPER WIRE, WEATHERPROOF INSULATION.

Numbers, B. & S. Gauge.	Double Braid.			Triple Braid.			Approximate Weights, Pounds.	
	Outside Diameters in 32ds Inch.	Weights, Pounds.		Outside Diameters in 32ds Inch.	Weights, Pounds.			
		1000 Feet.	Mile.		1000 Feet.	Mile.	Reel.	Coil.
0000	20	716	3781	24	775	4092	2000	250
000	18	575	3036	22	630	3326	2000	250
00	17	465	2455	18	490	2587	500	250
0	16	375	1980	17	400	2112	500	250
1	15	285	1505	16	306	1616	500	250
2	14	245	1294	15	268	1415	500	250
3	13	190	1003	14	210	1109	500	250
4	11	152	803	12	164	866	250	125
5	10	120	634	11	145	766	260	130
6	9	98	518	10	112	591	275	140
8	8	66	349	9	78	412	200	100
10	7	45	238	8	55	290	200	100
12	6	30	158	7	35	185	....	25
14	5	20	106	6	26	137	....	25
16	4	14	74	5	20	106	....	25
18	3	10	53	4	16	85	....	25

Power Cables. Lead Incased, Jute or Paper Insulated.

(John A. Roebling's Sons Co.)

Nos., B.&S.G.	Circular Mils.	Outside Diam. Inches.	Weights, 1000 feet. Pounds.	Nos., B.&S.G.	Circular Mils.	Outside Diam. Inches.	Weights, 1000 feet. Pounds.
....	1000000	1 13/16	6685	....	300000	1¼	3060
....	900000	1 23/32	6228	....	250000	1 3/16	2782
....	800000	1 21/32	5773	0000	211600	1 3/32	2533
....	750000	1⅝	5543	000	168100	1 1/16	2300
....	700000	1 19/32	5315	00	133225	1	2021
....	650000	1 9/16	5088	0	105625	15/16	1772
....	600000	1 17/32	4857	1	83521	29/32	1633
....	550000	1½	4630	2	66564	⅞	1482
....	500000	1 7/16	4278	3	52441	25/32	1360
....	450000	1⅜	3923	4	41616	¾	1251
....	400000	1 11/32	3619	6	26244	11/16	1046
....	350000	1 5/16	3416				

Stranded Weather-proof Feed Wire.

Circular Mils.	Outside Diam. Inches.	Weights. Pounds. 1000 feet.	Weights. Pounds. Mile.	Approximate length on reels. Feet.	Circular Mils.	Outside Diam. Inches.	Weights. Pounds. 1000 feet.	Weights. Pounds. Mile.	Approximate length on reels. Feet.
1000000	1⅛	3550	18744	800	550000	1 3/16	2043	10787	1200
900000	1 13/32	3215	16975	800	500000	1⅛	1875	9900	1320
800000	1 11/32	2880	15206	850	450000	1 3/32	1703	8992	1400
750000	1 5/16	2718	14825	850	400000	1 1/16	1530	8078	1450
700000	1 9/32	2545	13438	900	350000	1	1358	7170	1500
650000	¼	2378	12556	900	300000	15/16	1185	6257	1600
600000	1 7/32	2210	11668	1000	250000	29/32	1012	5343	1600

The table is calculated for concentric strands. Rope-laid strands are larger.

GALVANIZED STEEL-WIRE STRAND.

For Smokestack Guys, Signal Strand, etc.

(J. A. Roebling's Sons Co.)

This strand is composed of 7 wires, twisted together into a single strand.

Diameter.	Weight per 100 Feet.	Estimated Breaking Strength.	Diameter.	Weight per 100 Feet.	Estimated Breaking Strength.	Diameter.	Weight per 100 Feet.	Estimated Breaking Strength.
in.	lbs.	lbs.	in.	lbs.	lbs.	in.	lbs.	lbs.
½	51	8,320	9/32	18	2,600	5/32	4½	700
15/32	48	7,500	17/64	15	2,250	9/64	3½	525
7/16	37	6,000	¼	11½	1,750	⅛	2¼	375
⅜	30	4,700	7/32	8¾	1,300	3/32	2	220
5/16	21	3,300	3/16	6½	1,000			

For special purposes these strands can be made of 50 to 100 per cent greater tensile strength. When used to run over sheaves or pulleys the use of soft-iron stock is advisable.

FLEXIBLE STEEL-WIRE CABLES FOR VESSELS.

(Trenton Iron Co., 1886.)

With numerous disadvantages, the system of working ships' anchors with chain cables is still in vogue. A heavy chain cable contributes to the holding-power of the anchor, and the facility of increasing that resistance by paying out the cable is prized as an advantage. The requisite holding-power is obtained, however, by the combined action of a comparatively light anchor and a correspondingly great mass of chain of little service in proportion to its weight or to the weight of the anchor. If the weight and size of the anchor were increased so as to give the greatest holding-power required, and it were attached by means of a light wire cable, the combined weight of the cable and anchor would be much less than the total weight of the chain and anchor, and the facility of handling would be much greater. English shipbuilders have taken the initiative in this direction, and many of the largest and most serviceable vessels afloat are fitted with steel-wire cables. They have given complete satisfaction.

The Trenton Iron Co.'s cables are made of crucible cast-steel wire, and guaranteed to fulfil Lloyd's requirements. They are composed of 72 wires subdivided into six strands of twelve wires each. In order to obtain great flexibility, hempen centres are introduced in the strands as well as in the completed cable.

FLEXIBLE STEEL-WIRE HAWSERS.

These hawsers are extensively used, They are made with six strands of twelve wires each, hemp centres being inserted in the individual strands as well as in the completed rope. The material employed is crucible cast steel, galvanized, and guaranteed to fulfil Lloyd's requirements. They are only one third the weight of hempen hawsers; and are sufficiently pliable to work round any bitts to which hempen rope of equivalent strength can be applied.

13-inch tarred Russian hemp hawser weighs about 39 lbs. per fathom.
10-inch white manila hawser weighs about 20 lbs. per fathom.
1½-inch stud chain weighs about 68 lbs. per fathom.
4-inch galvanized steel hawser weighs about 12 *lbs. per fathom.*
Each of the above named has about the same tensile strength.

SPECIFICATIONS FOR GALVANIZED IRON WIRE.
Issued by the British Postal Telegraph Authorities.

Weight per Mile.			Diameter.			Tests for Strength and Ductility.						Resistance per Mile of the Standard Size at 60° Fahr.	Constant, being Standard Weight × Resistance.
Required Standard.	Allowed.		Required Standard.	Allowed.		Breaking Weight.	No. of Twists in 6 in.	For Breaking Weight not less than—	No. of Twists in 6 in.	For Breaking Weight not less than—	No. of Twists in 6 in.		
	Minimum.	Maximum.		Minimum.	Maximum.	Minimum.	Minimum.		Minimum.		Minimum.	Maximum.	
lbs.	lbs.	lbs.	mils.	mils.	mils.	lbs.		lbs.		lbs.		ohms.	
800	767	833	242	237	247	2480	15	2550	14	2020	13	6.75	5400
600	571	629	209	204	214	1860	17	1910	16	1960	15	9.00	5400
450	424	477	181	176	186	1390	19	1425	18	1460	17	12.00	5400
400	377	424	171	166	176	1240	21	1270	20	1300	19	13.50	5400
200	190	213	121	118	125	620	30	638	28	655	26	27.00	5400

STRENGTH OF PIANO-WIRE.

The average strength of English piano-wire is given as follows by Webster, Horsfals & Lean:

Numbers in Music-wire Gauge.	Equivalents in Fractions of Inches in Diameters.	Ultimate Tensile Strength in Pounds.	Numbers in Music-wire Gauge.	Equivalents in Fractions of Inches in Diameters.	Ultimate Tensile Strength in Pounds.
12	.029	225	18	.041	395
13	.031	250	19	.043	425
14	.033	285	20	.045	500
15	.035	305	21	.047	540
16	.037	340	22	.052	650
17	.039	360			

These strengths range from 300,000 to 340,000 lbs. per sq. in. The composition of this wire is as follows: Carbon, 0.570; silicon, 0.090; sulphur, 0.011; phosphorus, 0.018; manganese, 0.425.

"PLOUGH"-STEEL WIRE.

The term "plough," given in England to steel wire of high quality, was derived from the fact that such wire is used for the construction of ropes used for ploughing purposes. It is to be hoped that the term will not be used in this country, as it tends to confusion of terms. Plough-steel is known here in some steel-works as the quality of plate steel used for the mould-boards of ploughs, for which a very ordinary grade is good enough.

Experiments by Dr. Percy on the English plough-steel (so-called) gave the following results: Specific gravity, 7.814; carbon, 0.828 per cent; manganese, 0.587 per cent; silicon, 0.143 per cent; sulphur, 0.009 per cent; phosphorus, nil; copper, 0.030 per cent. No traces of chromium, titanium, or tungsten were found. The breaking strains of the wire were as follows:

Diameter, inch............	.093	.132	.159	.191
Pounds per sq. inch.........	314,960	257,600	224,000	201,600

The elongation was only from 0.75 to 1.1 per cent.

WIRES OF DIFFERENT METALS AND ALLOYS.

(J. Bucknall Smith's Treatise on Wire.)

Brass Wire is commonly composed of an alloy of 1 3/4 to 2 parts of copper to 1 part of zinc. The tensile strength ranges from 20 to 40 tons per square inch, increasing with the percentage of zinc in the alloy.

German or Nickel Silver, an alloy of copper, zinc, and nickel, is practically brass whitened by the addition of nickel. It has been drawn into wire as fine as .002" diam.

Platinum wire may be drawn into the finest sizes. On account of its high price its use is practically confined to special scientific instruments and electrical appliances in which resistances to high temperature, oxygen, and acids are essential. It expands less than other metals when heated, which property permits its being sealed in glass without fear of cracking. It is therefore used in incandescent electric lamps.

Phosphor-bronze Wire contains from 2 to 6 per cent of tin and from 1/20 to 1/8 per cent of phosphorus. The presence of phosphorus is detrimental to electric conductivity.

"**Delta-metal**" wire is made from an alloy of copper, iron, and zinc. Its strength ranges from 45 to 62 tons per square inch. It is used for some kinds of wire rope, also for wire gauze. It is not subject to deposits of verdigris. It has great toughness, even when its tensile strength is over 60 tons per square inch.

Aluminum Wire.—Specific gravity .268. Tensile strength only about 10 tons per square inch. It has been drawn as fine as 11,400 yards to the ounce, or .042 grains per yard.

Aluminum Bronze, 90 copper, 10 aluminum, has high strength and ductility; is inoxidizable, sonorous. Its electric conductivity is 12.6 per cent

Silicon Bronze, patented in 1882 by L. Weiler of Paris, is made as follows: Fluosilicate of potash, pounded glass, chloride of sodium and calcium, carbonate of soda and lime, are heated in a plumbago crucible, and after the reaction takes place the contents are thrown into the molten bronze to be treated. Silicon-bronze wire has a conductivity of from 40 to 98 per cent of that of copper wire and four times more than that of iron, while its tensile strength is nearly that of steel, or 28 to 55 tons per square inch of section. The conductivity decreases as the tensile strength increases. Wire whose conductivity equals 95 per cent of that of pure copper gives a tensile strength of 28 tons per square inch, but when its conductivity is 34 per cent of pure copper, its strength is 50 tons per square inch. It is being largely used for telegraph wires. It has great resistance to oxidation.

Ordinary Drawn and Annealed Copper Wire has a strength of from 15 to 20 tons per square inch.

SPECIFICATIONS FOR HARD-DRAWN COPPER WIRE.

The British Post Office authorities require that hard-drawn copper wire supplied to them shall be of the lengths, sizes, weights, strengths, and conductivities as set forth in the annexed table.

Weight per Statute Mile.			Approximate Equivalent Diameter.			Minimum Breaking Weight.	Minimum No. of Twists in 3 Inches.	Maximum Resistance per Mile of Wire (when hard) at 60° Fahr.	Minimum Weight of each Piece (or Coil) of Wire.
Required Standard.	Minimum.	Maximum.	Standard.	Minimum.	Maximum.				
lbs.	lbs.	lbs.	mils.	mils.	mils.	lbs.		ohms.	lbs.
100	97½	102½	79	78	80	330	30	9.10	50
150	146¼	153¾	97	95½	98	490	25	6.05	50
200	195	205	112	110½	113¼	650	20	4.53	50
400	390	410	158	155½	160¼	1300	10	2.27	50

WIRE ROPES.

List adopted by manufacturers in 1892. See pamphlets of John A. Roebling's Sons Co., Trenton Iron Co., and other makers.

Pliable Hoisting Rope.

With 6 strands of 19 wires each.

IRON.

Trade Number.	Diameter.	Circumference in inches.	Weight per foot in pounds. Rope with Hemp Centre.	Breaking Strain, tons of 2000 lbs.	Proper Working Load in tons of 2000 lbs.	Circumference of new Manila Rope of equal Strength.	Min. Size of Drum or Sheave in feet.
1	2¼	6¾	8.00	74	15	14	13
2	2	6	6.30	65	13	13	12
3	1¾	5½	5.25	54	11	12	10
4	1⅝	5	4.10	44	9	11	8½
5	1½	4¾	3.65	39	8	10	7½
5½	1⅜	4⅜	3.00	33	6½	9½	7
6	1¼	4	2.50	27	5½	8½	6½
7	1⅛	3½	2.00	20	4	7½	6
8	1	3⅛	1.58	16	3	6½	5¼
9	⅞	2¾	1.20	11.50	2½	5½	4½
10	¾	2¼	0.88	8.64	1¾	4¾	4
10¼	⅝	2	0.60	5.13	1¼	3¾	3½
10½	9-16	1⅝	0.48	4.27	¾	3½	2¾
10¾	½	1½	0.39	3.48	½	3	2¼
10a	7-16	1⅜	0.29	3.00	⅜	2¾	2
10⅞	⅜	1¼	0.23	2.50	¼	2½	1½

CAST STEEL.

Trade Number.	Diameter.	Circumference in inches.	Weight per foot in pounds. Rope with Hemp Centre.	Breaking Strain, tons of 2000 lbs.	Proper Working Load in tons of 2000 lbs.	Circumference of new Manila Rope of equal Strength.	Min. Size of Drum or Sheave in feet.
1	2¼	6¾	8.00	155	31	....	8½
2	2	6	6.30	125	25	....	8
3	1¾	5½	5.25	106	21	....	7¼
4	1⅝	5	4.10	86	17	15	6¼
5	1½	4¾	3.65	77	15	14	5¾
5½	1⅜	4⅜	3.00	63	12	13	5½
6	1¼	4	2.50	52	10	12	5
7	1⅛	3½	2.00	42	8	11	4½
8	1	3⅛	1.58	33	6	9½	4
9	⅞	2¾	1.20	25	5	8½	3½
10	¾	2¼	0.88	18	3½	7	3
10¼	⅝	2	0.60	12	2½	5¾	2¼
10½	9-16	1⅝	0.48	9	1¾	5	1¾
10¾	½	1½	0.39	7	1½	4½	1½
10a	7-16	1⅜	0.29	5½	1¼	3¾	1¼
10⅞	⅜	1¼	0.23	4½	⅞	3½	1

Cable-Traction Ropes.

According to English practice, cable-traction ropes, of about 3½ in. in circumference, are commonly constructed with six strands of seven or fifteen wires, the lays in the strands varying from, say, 3 in. to 3½ in., and the lays in the ropes from, say, 7½ in. to 9 in. In the United States, however, strands of nineteen wires are generally preferred as being more flexible; but, on the other hand, the smaller external wires wear out more rapidly. The Market-street Street Railway Company, San Francisco, has used ropes 1¼ in. in diameter, composed of six strands of nineteen steel wires, weighing 2½ lbs. per foot, the longest continuous length being 24,125 ft. The Chicago City Railroad Company has employed cables of identical construction, the longest length being 27,700 ft. On the New York and Brooklyn Bridge cable-railway steel ropes of 11,500 ft. long, containing 114 wires, have been used.

Transmission and Standing Rope.

With 6 strands of 7 wires each.

IRON.

Trade Number.	Diameter.	Circumference.	Weight per foot in pounds of Rope with Hemp Centre.	Breaking Strain in tons of 2000 lbs	Proper Working Load in tons of 2000 lbs.	Circumference of new Manila Rope of equal Strength.	Min. Size of Drum or Sheave in feet.
11	1½	4¾	3.37	36	9	10	13
12	1⅜	4⅜	2.77	30	7½	9	12
13	1¼	4	2.28	25	6¼	8½	10¾
14	1⅛	3½	1.82	20	5	7½	9½
15	1	3⅛	1.50	16	4	6½	8½
16	⅞	2¾	1.12	12.3	3	5¾	7½
17	¾	2¼	0.92	8.8	2¼	4¾	6¾
18	11-16	2⅛	0.70	7.6	2	4½	6
19	⅝	2	0.57	5.8	1½	4	5½
20	9-16	1⅝	0.41	4.1	1	3¼	4½
21	½	1½	0.31	2.83	¾	2¾	4
22	7-16	1⅜	0.23	2.13	½	2½	3¼
23	⅜	1¼	0.21	1.65	....	2¼	2¾
24	5-16	1	0.16	1.38	....	2	2½
25	9-32	⅞	0.125	1.03	....	1¾	2¼

CAST STEEL.

Trade Number.	Diameter.	Circumference.	Weight per foot in pounds of Rope with Hemp Centre.	Breaking Strain in tons of 2000 lbs	Proper Working Load in tons of 2000 lbs.	Circumference of new Manila Rope of equal Strength.	Min. Size of Drum or Sheave in feet.
11	1½	4¾	3.37	62	13	13	8½
12	1⅜	4⅜	2.77	52	10	12	8
13	1¼	4	2.28	44	9	11	7¼
14	1⅛	3½	1.82	36	7½	10	6½
15	1	3⅛	1.50	30	6	9	5¾
16	⅞	2¾	1.12	22	4½	8	5
17	¾	2¼	0.92	17	3½	7	4½
18	11-16	2⅛	0.70	14	3	6	4
19	⅝	2	0.57	11	2¼	5½	3½
20	9-16	1⅝	0.41	8	1¾	4¾	3
21	½	1½	0.31	6	1½	4	2½
22	7-16	1⅜	0.23	4½	1¼	3½	2¼
23	⅜	1¼	0.21	4	1	3¼	2
24	5-16	1	0.16	3	¾	2¾	1¾
25	9-32	⅞	0.12	2	½	2¼	1½

Plough-Steel Rope.

Wire ropes of very high tensile strength, which are ordinarily called "Plough-steel Ropes," are made of a high grade of crucible steel, which, when put in the form of wire, will bear a strain of from 100 to 150 tons per square inch.

Where it is necessary to use very long or very heavy ropes, a reduction of the dead weight of ropes becomes a matter of serious consideration.

It is advisable to reduce all bends to a minimum, and to use somewhat larger drums or sheaves than are suitable for an ordinary crucible rope having a strength of 60 to 80 tons per square inch. Before using Plough-steel Ropes it is best to have advice on the subject of adaptability.

Plough-Steel Rope.

With 6 strands of 19 wires each.

Trade Number.	Diameter in inches.	Weight per foot in pounds.	Breaking Strain in tons of 2000 lbs.	Proper Working Load.	Min. Size of Drum or Sheave in feet.
1	2¼	8.00	240	46	9
2	2	6.30	189	37	8
3	1¾	5.25	157	31	7½
4	1⅝	4.10	123	25	6
5	1½	3.65	110	22	5½
5½	1⅜	3.00	90	18	5¼
6	1¼	2.50	75	15	5
7	1⅛	2.00	60	12	4½
8	1	1.58	47	9	4¼
9	⅞	1.20	37	7	3¾
10	¾	0.88	27	5	3½
10¼	⅝	0.60	18	3½	3
10½	9-16	0.44	13	2½	2½
10¾	½	0.30	10	2	2

With 7 Wires to the Strand.

Trade Number.	Diameter in inches.	Weight per foot in pounds.	Breaking Strain in tons of 2000 lbs.	Proper Working Load.	Min. Size of Drum or Sheave in feet.
15	1	1.50	45	9	5½
16	⅞	1.12	33	6½	5
17	¾	0.92	25	5	4
18	11-16	0.70	21	4	3½
19	⅝	0.57	16	3¾	3
20	9-16	0.41	12	2½	2¾
21	½	0.31	9	1⅞	2½
22	7-16	0.23	5	1½	2
23	⅜	0.21	4	1	1½

Galvanized Iron Wire Rope.

For Ships' Rigging and Guys for Derricks.

CHARCOAL ROPE.

Circumference in inches.	Weight per Fathom in pounds.	Cir. of new Manila Rope of equal Strength.	Breaking Strain in tons of 2000 pounds	Circumference in inches	Weight per Fathom in pounds.	Cir. of new Manila Rope of equal Strength.	Breaking Strain in tons of 2000 pounds
5½	26½	11	43	2½	5½	5	9
5¼	24½	10½	40	2¼	4½	4¾	8
5	22	10	35	2	3½	4½	7
4¾	21	9½	33	1¾	2½	3¾	5
4½	19	9	30	1½	2	3	3½
4¼	16½	8½	26	1¼	1¾	2½	2½
4	14¼	8	23	1⅛	1¼	2¼	2¼
3¾	12¾	7½	20	1	⅞	2	2
3½	10¾	6½	16	⅞	¾	1¾	1
3¼	9½	6	14	¾	½	1½	¾
3	8	5¾	12	⅝	⅜	1¼	⅝
2¾	6¾	5¼	10	½	¼	1⅛	⅜

Galvanized Cast-steel Yacht Rigging.

Circumference in inches.	Weight per Fathom in pounds.	Cir. of new Manilla Rope of equal Strength.	Breaking Strain in tons of 2000 pounds	Circumference in inches	Weight per Fathom in pounds.	Cir. of new Manilla Rope of equal Strength.	Breaking Strain in tons of 2000 pounds
4	14½	13	66	2	3½	6½	14
3½	10¾	11	43	1¾	2½	5¼	10
3	8	9½	33	1½	2	4¾	8
2¾	6¾	8½	27	1⅜	1⅞	4¼	6½
2½	5½	8	22	1¼	1¾	3¾	5½
2¼	4½	7	18	1	⅞	3	3½

Steel Hawsers.

For Mooring, Sea, and Lake Towing.

Circumference.	Breaking Strength.	Size of Manilla Hawser of equal Strength.	Circumference.	Breaking Strength.	Size of Manilla Hawser of equal Strength.
Inches.	Tons.	Inches.	Inches.	Tons.	Inches.
2½	15	6½	3½	29	9
2¾	18	7	4	35	10
3	22	8½			

Steel Flat Ropes.

(J. A. Roebling's Sons Co.)

Steel-wire Flat Ropes are composed of a number of strands, alternately twisted to the right and left, laid alongside of each other, and sewed together with soft iron wires. These ropes are used at times in place of round ropes in the shafts of mines. They wind upon themselves on a narrow winding-drum, which takes up less room than one necessary for a round rope. The soft-iron sewing-wires wear out sooner than the steel strands, and then it becomes necessary to sew the rope with new iron wires.

Width and Thickness in inches.	Weight per foot in pounds.	Strength in pounds.	Width and Thickness in inches.	Weight per foot in pounds.	Strength in pounds.
⅜ × 2	1.19	35,700	½ × 3	2.38	71,400
⅜ × 2½	1.86	55,800	½ × 3½	2.97	89,000
⅜ × 3	2.00	60,000	½ × 4	3.30	99,000
⅜ × 3½	2.50	75,000	½ × 4½	4.00	120,000
⅜ × 4	2.86	85,800	½ × 5	4.27	128,000
⅜ × 4½	3.12	93,600	½ × 5½	4.82	144,600
⅜ × 5	3.40	100,000	½ × 6	5.10	153,000
⅜ × 5½	3.90	110,000	½ × 7	5.90	177,000

For safe working load allow from one fifth to one seventh of the breaking stress.

"Lang Lay" Rope.

In wire rope, as ordinarily made, the component strands are laid up into rope in a direction opposite to that in which the wires are laid into strands; that is, if the wires in the strands are laid from right to left, the strands are laid into rope from left to right. In the "Lang Lay," sometimes known as "Universal Lay," the wires are laid into strands and the strands into rope in the same direction; that is, if the wire is laid in the strands from right to left, the strands are also laid into rope from right to left. Its use has been found desirable under certain conditions and for certain purposes, mostly for haulage plants, inclined planes, and street railway cables, although it has also been used for vertical hoists in mines, etc. Its advantages are that

GALVANIZED STEEL CABLES.

For Suspension Bridges. (Roebling's.)

Diameter in inches.	Ultimate Strength in tons of 2000 pounds.	Weight per foot.	Diameter in inches.	Ultimate Strength in tons of 2000 pounds.	Weight per foot.	Diameter in inches.	Ultimate Strength in tons of 2000 pounds.	Weight per foot.
2⅝	220	13	2¼	155	8.64	1¾	95	5.6
2½	200	11.3	2	110	6.5	1⅝	75	4.85
2⅜	180	10	1⅞	100	5.8	1½	65	3.7

COMPARATIVE STRENGTHS OF FLEXIBLE GALVANIZED STEEL-WIRE HAWSERS,

With Chain Cable, Tarred Russian Hemp, and White Manila Ropes.

Patent Flexible Steel-wire Hawsers and Cables.				Chain Cable.				Tarred Russian Hemp Rope.			White Manilla Ropes.		
Size, Circumference.	Weight per Fathom.	Guaranteed Breaking Strain, tons.	Diameter of Sheave round which it may be worked, inches.	Size.	Weight per Fathom.	Proof Strain, tons.	Breaking Strain, tons.	Size.	Weight per Fathom.	Breaking Strain, tons.	Size.	Weight per Fathom.	Breaking Strain, tons.
1	¾	1¾	6					2¾		1½	2½	1¼	2
1¼	1	2½	7½	½	14	4½	6	3½	3	2½	3	1¾	2¾
1½	1¾	4	9					4	3⅛	3¼	3¼	2	3½
1¾	2	5½	10½	9-16	17	5⅛	7¼	5	6	5	4	3	5
2	2¾	7	12					5¾	8	7	5	4½	7¾
2¼	3¾	9	13½	10-16	21	7	9½	6½	10	9	5¾	6	10½
2½	4½	12	15					7½	13	11½	6¼	7	12½
2¾	5½	15	16½	11-16	25	8½	12¾	8½	16	14	7	8¾	15
3	7	18	18	12-16	30	10½	15⅛	9	19	16½	7½	10½	18
3¼	8	22	19½	13-16	35	11⅞	17 8-10	10	23	20	8½	18	22¾
3½	9	26	21	15-16	48	15 8-10	23 7-10	11	28	24½	9	14½	25
4	12	33	24	1	54	18	27	12	33	29	10	18	31½
4½	15	39	27	1⅛	68	22¾	34⅛	13	39	34	11	22	38½
5	23½	64	30	1 17-32	112	37⅛	55½	15	56	50	12¾	29¼	51
5½	28	74	33	1⅜	143	47½	66½	17	67	60	13½	35½	62
6	33	88	36	1¾	166	55⅛	77⅛	19	84	72	15	42	73½
6½	37	102	39	1 15-16	204	67½	94⅛	21	106	89			
7	41	116	42	2 1-16	231	76½	107 1-10	23	123	106			
7½	47	130	45	2 3-16	256	86⅛	120½	24	134	115			
8	53	150	48	2 5-16	280	96¼	134¾	25	146	125			

NOTE.—This is an old table, and its authority is uncertain. The figures in the fourth column are probably much too small for durability.

it is somewhat more flexible than rope of the same diameter and composed of the same number of wires laid up in the ordinary manner; and (especially) that owing to the fact that the wires are laid more axially in the rope, longer surfaces of the wire are exposed to wear, and the endurance of the rope is thereby increased. (Trenton Iron Co.)

Notes on the Use of Wire Rope.

(J. A. Roebling's Sons Co.)

Several kinds of wire rope are manufactured. The most pliable variety contains nineteen wires in the strand, and is generally used for hoisting and running rope. The ropes with twelve wires and seven wires in the strand are stiffer, and are better adapted for standing rope, guys, and rigging. Orders should state the use of the rope, and advice will be given. Ropes are made up to three inches in diameter, upon application.

For safe working load, allow one fifth to one seventh of the ultimate strength, according to speed, so as to get good wear from the rope. When substituting wire rope for hemp rope, it is good economy to allow for the former the same weight per foot which experience has approved for the latter.

Wire rope is as pliable as new hemp rope of the same strength: the former will therefore run over the same-sized sheaves and pulleys as the latter. But the greater the diameter of the sheaves, pulleys, or drums, the longer wire rope will last. The minimum size of drum is given in the table.

Experience has demonstrated that the wear increases with the speed. It is, therefore, better to increase the load than the speed.

Wire rope is manufactured either with a wire or a hemp centre. The latter is more pliable than the former, and will wear better where there is short bending. Orders should specify what kind of centre is wanted.

Wire rope must not be coiled or uncoiled like hemp rope.

When mounted on a reel, the latter should be mounted on a spindle or flat turn-table to pay off the rope. When forwarded in a small coil, without reel, roll it over the ground like a wheel, and run off the rope in that way. All untwisting or kinking must be avoided.

To preserve wire rope, apply raw linseed-oil with a piece of sheepskin, wool inside; or mix the oil with equal parts of Spanish brown or lamp-black.

To preserve wire rope under water or under ground, take mineral or vegetable tar, and add one bushel of fresh-slacked lime to one barrel of tar, which will neutralize the acid. Boil it well, and saturate the rope with the hot tar. To give the mixture body, add some sawdust.

The grooves of cast-iron pulleys and sheaves should be filled with well-seasoned blocks of hard wood, set on end, to be renewed when worn out. This end-wood will save wear and increase adhesion. The smaller pulleys or rollers which support the ropes on inclined planes should be constructed on the same plan. When large sheaves run with very great velocity, the grooves should be lined with leather, set on end, or with India rubber. This is done in the case of sheaves used in the *transmission of power* between distant points by means of rope, which frequently runs at the rate of 4000 feet per minute.

Steel ropes are taking the place of iron ropes, where it is a special object to combine lightness with strength.

But in substituting a steel rope for an iron running rope, the object in view should be to gain an increased wear from the rope rather than to reduce the size.

Locked Wire Rope.

Fig. 74 shows what is known as the Patent Locked Wire Rope, made by the Trenton Iron Co. It is claimed to wear two to three times as long as an

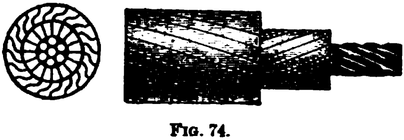


FIG. 74.

ordinary wire rope of equal diameter and of like material. Sizes made are from ½ to 1½ inches diameter.

CRANE CHAINS.

(Pencoyd Iron Works.)

"D. B. G." Special Crane.							Crane.		
Size of Chain, inches.	Pitch Approximately, inches	Weight per Foot in pounds, approximately.	Outside Width, inches.	Proof Test, pounds.	Average Breakage Strain, pounds.	Ordinary Safe Load, General Use, pounds.	Proof Test, pounds.	Average Breaking Strain, pounds.	Ordinary Safe Load, General Use, pounds.
1/4	25-32	7/8	7/8	1932	3864	1288	1680	3360	1120
5-16	27-32	1	1 1-16	2898	5796	1932	2520	5040	1680
3/8	31-32	1 7-10	1 1/4	4186	8372	2790	3640	7280	2427
7-16	1 5-32	2	1 3/8	5796	11592	3864	5040	10080	3360
1/2	1 11-32	2 1/2	1 11-16	7728	15456	5152	6720	13440	4480
9-16	1 15-32	3 2-10	1 7/8	9660	19320	6440	8400	16800	5600
5/8	1 23-32	4 1/8	2 1-16	11914	23828	7942	10360	20720	6907
11-16	1 27-32	5	2 1/4	14490	28980	9660	12600	25200	8400
3/4	1 31-32	5 7/8	2 1/2	17388	34776	11592	15120	30240	10080
13-16	2 3-32	6 7-10	2 11-16	20286	40572	13524	17640	35280	11760
7/8	2 7-32	8	2 7/8	22484	44968	14989	20440	40880	13627
15-16	2 15-32	9	3 1-16	25872	51744	17248	23520	47040	15680
1	2 19-32	10 7-10	3 1/4	29568	59136	19712	26880	53760	17920
1 1-16	2 23-32	11 2-10	3 5-16	33264	66528	22176	30240	60480	20160
1 1/8	2 27-32	12 1/2	3 3/4	37576	75152	25050	34160	68320	22773
1 3-16	3 5-32	13 7-10	3 7/8	41888	83776	27925	38080	76160	25387
1 1/4	3 7-32	16	4 1/8	46200	92400	30800	42000	84000	28000
1 5-16	3 15-32	16 1/2	4 3/8	50512	101024	33674	45920	91840	30613
1 3/8	3 3/8	18 4-10	4 9-16	55748	111496	37165	50680	101360	33787
1 7-16	3 25-32	19 7-10	4 3/4	60368	120736	40245	54880	109760	36587
1 1/2	3 31-32	21 7-10	5	66528	133056	44352	60480	120960	40320

The distance from centre of one link to centre of next is equal to the inside length of link, but in practice 1/32 inch is allowed for weld. This is approximate, and where exactness is required, chain should be made so.

For Chain Sheaves.—The diameter, if possible, should be not less than twenty times the diameter of chain used.

Example.—For 1-inch chain use 20-inch sheaves.

WEIGHTS OF LOGS, LUMBER, ETC.

Weight of Green Logs to Scale 1,000 Feet, Board Measure.

Yellow pine (Southern)	8,000 to 10,000 lbs.
Norway pine (Michigan)	7,000 to 8,000 "
White pine (Michigan) off of stump	6,000 to 7,000 "
White pine (Michigan) out of water	7,000 to 8,000 "
White pine (Pennsylvania), bark off	5,000 to 6,000 "
Hemlock (Pennsylvania), bark off	6,000 to 7,000 "

Four acres of water are required to store 1,000,000 feet of logs.

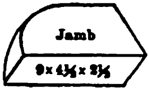
Weight of 1,000 Feet of Lumber, Board Measure.

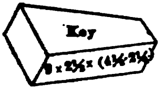
Yellow or Norway pine	Dry, 3,000 lbs.	Green, 5,000 lbs.
White pine	" 2,500 "	" 4,000 "

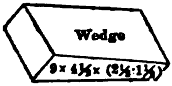
Weight of 1 Cord of Seasoned Wood, 128 Cubic Feet per Cord.

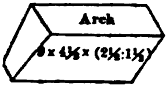
Hickory or sugar maple	4,500 lbs.
White oak	3,850 "
Beech, red oak or black oak	3,250 "
Poplar, chestnut or elm	2,350 "
Pine (white or Norway)	2,000 "
Hemlock bark, dry	2,200 "

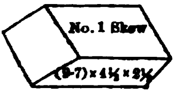
SIZES OF FIRE-BRICK.

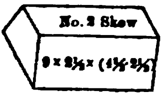


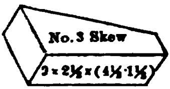




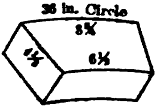


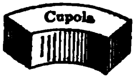






36 in. Circle





9-inch straight............ 9×4½×2½ inches.
Soap...................... 9×2½×2½ "
Checker.................. 9×3 ×3 "
2-inch.. .... 9×4½×2 "
Split .... 9×4½×1¼ "
Jamb..................... 9×4½×2½ "
No. 1 key................. 9×2½ thick×4½ to 4 inches wide.
118 bricks to circle 12 feet inside diam.
No. 2 key .... 9 × 2½ thick × 4½ to 3½ inches wide.
63 bricks to circle 6 ft. inside diam.
No. 3 key................. 9 × 2½ thick × 4½ to 3 inches wide.
38 bricks to circle 3 ft. inside diam.
No. 4 key......... .... 9×2½ thick × 4½ to 2¼ inches wide.
25 bricks to circle 1½ ft. inside diam.
No. 1 wedge (or bullhead). 9×4½ wide × 2½ to 2 in. thick, tapering lengthwise.
98 bricks to circle 5 ft. inside diam.
No. 2 wedge.............. 9×4½×2½ to 1½ in. thick.
60 bricks to circle 2½ ft. inside diam.
No. 1 arch........ .... 9×4½×2½ to 2 in. thick, tapering breadthwise.
72 bricks to circle 4 ft. inside diam.
No. 2 arch................ 9×4½×2½ to 1½.
42 bricks to circle 2 ft. inside diam.
No. 1 skew........ ... ... 9 to 7×4½ to 2½.
Bevel on one end.
No. 2 skew............ ... 9×2½×4½ to 2½.
Equal bevel on both edges.
No. 3 skew................ 9×2½×4½ to 1½.
Taper on one edge.
24 inch circle .... 8¼ to 5½×4½×2½.
Edges curved, 9 bricks line a 24-inch circle.
36-inch circle .... 8¼ to 6½×4½×2½.
13 bricks line a 36-inch circle.
48-inch circle.......... ... 8¼ to 7¼×4½×2½.
17 bricks line a 48-inch circle.
13½-inch straight.........13½×2½×6.
13½-inch key No. 1 .. ....13½×2½×6 to 5 inch.
90 bricks turn a 12-ft. circle.
13½-inch key No. 2.... ...13½×2½×6 to 4½ inch.
52 bricks turn a 6-ft. circle.
Bridge wall, No. 1.........13×6½×6.
Bridge wall, No. 2.........13×6½×3.
Mill tile ... .18, 20, or 24×6×3.
Stock-hole tiles............18, 20, or 24×9×4.
18-inch block ....18×9×6.
Flat back .... 9×6×2½.
Flat back arch............ 9×6×3½ to 2½.
22-inch radius, 56 bricks to circle.
Locomotive tile ....32×10×3.
34×10×3.
34× 8×3.
36× 8×3.
40×10×3.

Tiles, slabs, and blocks, various sizes 12 to 30 inches long, 8 to 30 inches wide, 2 to 6 inches thick.

Cupola brick, 4 and 6 inches high, 4 and 6 inches radial width, to line shells 23 to 66 in diameter.

A 9-inch straight brick weighs 7 lbs. and contains 100 cubic inches. (=120 lbs. per cubic foot. Specific gravity 1.93.)

One cubic foot of wall requires 17 9-inch bricks, one cubic yard requires 460. Where keys, wedges, and other "shapes" are used, add 10 per cent in estimating the number required.

One ton of fire-clay should be sufficient to lay 3000 ordinary bricks. To secure the best results, fire-bricks should be laid in the same clay from which they are manufactured. It should be used as a thin paste, and not as mortar. The thinner the joint the better the furnace wall. In ordering bricks the service for which they are required should be stated.

NUMBER OF FIRE-BRICK REQUIRED FOR VARIOUS CIRCLES.

Diam. of Circle.		KEY BRICKS.					ARCH BRICKS.				WEDGE BRICKS.			
		No. 4.	No. 3.	No. 2.	No. 1.	Total.	No. 2.	No. 1.	9'	Total.	No. 2.	No. 1.	9'	Total.
ft.	in.													
1	6	25	....	....	......	25	....	....	......	......	....	....	......	......
2	0	17	18			30	42	....	.....	42	...	....	......	......
2	6	9	25			34	31	18	......	49	60	....	......	60
3	0	....	38			38	21	36	......	57	48	20	......	68
3	6	.	32	10		42	10	54	.....	64	36	40	......	76
4	0	....	25	21		46	....	72	......	72	24	59	......	83
4	6	....	19	32		51	....	72	8	80	12	79	......	91
5	0	....	13	42		55	....	72	15	87	....	98	......	98
5	6	....	6	53		59	....	72	23	95	. .	98	8	106
6	0	...	....	63		63	....	72	30	102	....	98	15	113
6	6	...	...	58	9	67	....	72	38	110	....	98	23	121
7	0	....	....	52	19	71	...	72	45	117	....	98	30	128
7	6	....	....	47	29	76	....	72	53	125	....	98	38	136
8	0	....	...	42	38	80	...	72	60	132	....	98	46	144
8	6	....	...	37	47	84	....	72	68	140	....	98	53	151
9	0	....	....	31	57	88	....	72	75	147	...	98	61	159
9	6	...	....	26	66	92	....	72	83	155	....	98	68	166
10	0	....	....	21	76	97	....	72	90	162	....	98	76	174
10	6	....	....	16	85	101	....	72	98	170	....	98	83	181
11	0	....	....	11	94	105	...	72	105	177	....	98	91	189
11	6	....	....	5	104	109	....	72	113	185	....	98	98	196
12	0	....	....	....	113	118	....	72	121	193	....	98	106	204
12	6	...	....	....	118	117	....	....	......	......	....	....	... ..	.. ...

For larger circles than 12 feet use 113 No. 1 Key, and as many 9-inch brick as may be needed in addition.

ANALYSES OF MT. SAVAGE FIRE-CLAY.

(1) 1871 Mass. Institute of Technology.	(2) 1877. Report on Clays of New Jersey Prof. G. H. Cook.		(3) 1878. Second Geological Survey of Pennsylvania.	(4) 1885. (2 samples) Dr. Otto Wuth.
50.457	56.80	Silica	44.395	56.15
35.904	30.08	Alumina	33.558	33.295
......	1.15	Titanic acid	1.530	......
1.504	1.12	Peroxide iron	1.080	0.59
0.133	... .	Lime	trace	0.17
0.018	.....	Magnesia	0.108	0.115
trace	0.80	Potash (alkalies)	0.247	......
12.744	10.50	Water and inorg. matter.	14.575	9.68
100.760	100.450		100.493	100.000

MAGNESIA BRICKS.

"Foreign Abstracts" of the Institution of Civil Engineers, 1893, gives a paper by C. Bischof on the production of magnesia bricks. The material most in favor at present is the magnesite of Styria, which, although less pure considered as a source of magnesia than the Greek, has the property of fritting at a high temperature without melting. The composition of the two substances, in the natural and burnt states, is as follows:

Magnesite.	Styrian.	Greek.
Carbonate of magnesia	90.0 to 96.0%	94.46%
" " lime	0.5 to 2.0	4.49
" " iron	3.0 to 6.0	FeO 0.06
Silica	1.0	0.52
Manganous oxide	0.5	Water 0.54
Burnt Magnesite.		
Magnesia	77.6	82.46—95.36
Lime	7.3	0.83—10.92
Alumina and ferric oxide	13.0	0.56— 3.54
Silica	1.2	0.73— 7.96

At a red heat magnesium carbonate is decomposed into carbonic acid and caustic magnesia, which resembles lime in becoming hydrated and recarbonated when exposed to the air, and possesses a certain plasticity, so that it can be moulded when subjected to a heavy pressure. By long-continued or stronger heating the material becomes dead-burnt, giving a form of magnesia of high density, sp. gr. 3.8, as compared with 3.0 in the plastic form, which is unalterable in the air but devoid of plasticity. A mixture of two volumes of dead-burnt with one of plastic magnesia can be moulded into bricks which contract but little in firing. Other binding materials that have been used are: clay up to 10 or 15 per cent; gas-tar, perfectly freed from water, soda, silica, vinegar as a solution of magnesium acetate which is readily decomposed by heat, and carbolates of alkalies or lime. Among magnesium compounds a weak solution of magnesium chloride may also be used. For setting the bricks lightly burnt, caustic magnesia, with a small proportion of silica to render it less refractory, is recommended. The strength of the bricks may be increased by adding iron, either as oxide or silicate. If a porous product is required, sawdust or starch may be added to the mixture. When dead-burnt magnesia is used alone, soda is said to be the best binding material.

See also papers by A. E. Hunt, Trans. A. I. M. E., xvi, 720, and by T. Egleston, Trans. A. I. M. E., xiv, 458.

Asbestos.—J. T. Donald, *Eng. and M. Jour.*, June 27, 1891.

ANALYSIS.

	Italian.	Canadian. Broughton.	Canadian. Templeton.
Silica	40.30%	40.57%	40.52%
Magnesia	43.37	41.50	42.05
Ferrous oxide	.87	2.81	1.97
Alumina	2.27	.90	2.10
Water	13.72	13.55	13.46
	100.53	99.33	100.10

Chemical analysis throws light upon an important point in connection with asbestos, i.e., the cause of the harshness of the fibre of some varieties. Asbestos is principally a hydrous silicate of magnesia, i.e., silicate of magnesia combined with water. When harsh fibre is analyzed it is found to contain less water than the soft fibre. In fibre of very fine quality from Black Lake analysis showed 14.38% of water, while a harsh-fibred sample gave only 11.70%. If soft fibre be heated to a temperature that will drive off a portion of the combined water, there results a substance so brittle that it may be crumbled between thumb and finger. There is evidently some connection between the consistency of the fibre and the amount of water in its composition.

STRENGTH OF MATERIALS.

Stress and Strain.—There is much confusion among writers on strength of materials as to the definition of these terms. An external force applied to a body, so as to pull it apart, is resisted by an internal force, or resistance, and the action of these forces causes a displacement of the molecules, or deformation. By some writers the external force is called a stress, and the internal force a strain; others call the external force a strain, and the internal force a stress: this confusion of terms is not of importance, as the words stress and strain are quite commonly used synonymously, but the use of the word strain to mean molecular displacement, deformation, or distortion, as is the custom of some, is a corruption of the language. See *Engineering News*, June 23, 1892. Definitions by leading authorities are given below.

Stress.—A stress is a force which acts in the interior of a body, and resists the external forces which tend to change its shape. A deformation is the amount of change of shape of a body caused by the stress. The word strain is often used as synonymous with stress and sometimes it is also used to designate the deformation. (Merriman.)

The force by which the molecules of a body resist a strain at any point is called the stress at that point.

The summation of the displacements of the molecules of a body for a given point is called the distortion or strain at the point considered. (Burr).

Stresses are the forces which are applied to bodies to bring into action their elastic and cohesive properties. These forces cause alterations of the forms of the bodies upon which they act. *Strain* is a name given to the kind of alteration produced by the stresses. The distinction between stress and strain is not always observed, one being used for the other. (Wood.)

Stresses are of different kinds, viz.: *tensile*, *compressive*, *transverse*, *torsional*, and *shearing* stresses.

A *tensile stress*, or pull, is a force tending to elongate a piece. A *compressive stress*, or push, is a force tending to shorten it. A *transverse stress* tends to bend it. A *torsional stress* tends to twist it. A *shearing stress* tends to force one part of it to slide over the adjacent part.

Tensile, compressive, and shearing stresses are called simple stresses. Transverse stress is compounded of tensile and compressive stresses, and torsional of tensile and shearing stresses.

To these five varieties of stresses might be added *tearing* stress, which is either tensile or shearing, but in which the resistance of different portions of the material are brought into play in detail, or one after the other, instead of simultaneously, as in the simple stresses.

Effects of Stresses.—The following general laws for cases of simple tension or compression have been established by experiment. (Merriman):

1. When a small stress is applied to a body, a small deformation is produced, and on the removal of the stress the body springs back to its original form. For small stresses, then, materials may be regarded as perfectly elastic.

2. Under small stresses the deformations are approximately proportional to the forces or stresses which produce them, and also approximately proportional to the length of the bar or body.

3. When the stress is great enough a deformation is produced which is partly permanent, that is, the body does not spring back entirely to its original form on removal of the stress. This permanent part is termed a set. In such cases the deformations are not proportional to the stress.

4. When the stress is greater still the deformation rapidly increases and the body finally ruptures.

5. A sudden stress, or shock, is more injurious than a steady stress or than a stress gradually applied.

Elastic Limit.—The elastic limit is defined as that point at which the deformations cease to be proportional to the stresses, or, the point at which the rate of stretch (or other deformation) begins to increase. It is also defined as the point at which the first permanent set becomes visible. The last definition is not considered as good as the first, as it is found that with some materials a set occurs with any load, no matter how small, and that with others a set which might be called permanent vanishes with lapse of time, and as it is impossible to get the point of first set without removing the whole load after each increase of load, which is frequently inconvenient. The elastic limit, defined, however, as the point at which the extensions begin to increase at a higher ratio than the applied stresses, usually corresponds very nearly with the point of first measurable permanent set.

Yield-point.—The term yield-point has recently been introduced into the literature of the strength of materials. It is defined as that point at which the rate of stretch suddenly increases rapidly. The difference between the elastic limit, strictly defined as the point at which the rate of stretch begins to increase, and the yield-point, at which the rate increases suddenly, may in some cases be considerable. This difference, however, will not be discovered in short test-pieces unless the readings of elongations are made by an exceedingly fine instrument, as a micrometer reading to $\frac{1}{10000}$ of an inch. In using a coarser instrument, such as calipers reading to 1/100 of an inch, the elastic limit and the yield-point will appear to be simultaneous. Unfortunately for precision of language, the term yield-point was not introduced until long after the term elastic limit had been almost universally adopted to signify the same physical fact which is now defined by the term yield-point, that is, not the point at which the first change in rate, observable only by a microscope, occurs, but that later point (more or less indefinite as to its precise position) at which the increase is great enough to be seen by the naked eye. A most convenient method of determining the point at which a sudden increase of rate of stretch occurs in short specimens, when a testing-machine in which the pulling is done by screws is used, is to note the weight on the beam at the instant that the beam "drops." During the earlier portion of the test, as the extension is steadily increased by the uniform but slow rotation of the screws, the poise is moved steadily along the beam to keep it in equipoise; suddenly a point is reached at which the beam drops, and will not rise until the elongation has been considerably increased by the further rotation of the screws, the advancing of the poise meanwhile being suspended. This point corresponds practically to the point at which the rate of elongation suddenly increases, and to the point at which an appreciable permanent set is first found. It is also the point which has hitherto been called in practice and in text-books the elastic limit, and it will probably continue to be so called, although the use of the newer term "yield-point" for it, and the restriction of the term elastic limit to mean the earlier point at which the rate of stretch begins to increase, as determinable only by micrometric measurements, is more precise and scientific.

In tables of strength of materials hereafter given, the term elastic limit is used in its customary meaning, the point at which the rate of stress has begun to increase, as observable by ordinary instruments or by the drop of the beam. With this definition it is practically synonymous with yield-point.

Coefficient (or Modulus) of Elasticity.—This is a term expressing the relation between the amount of extension or compression of a material and the load producing that extension or compression.

It may be defined as the load per unit of section divided by the extension per unit of length; or the reciprocal of the fraction expressing the elongation per inch of length, divided by the pounds per square inch of section producing that elongation.

Let P be the applied load, k the sectional area of the piece, l the length of the part extended, λ the amount of the extension, and E the coefficient of elasticity. Then

$$\frac{P}{k} = \text{the load on a unit of section};$$

$$\frac{\lambda}{l} = \text{the elongation of a unit of length.}$$

$$E = \frac{P}{k} \div \frac{\lambda}{l} = \frac{Pl}{k\lambda}.$$

The coefficient of elasticity is sometimes defined as the figure expressing the load which would be necessary to elongate a piece of one square inch section to double its original length, provided the piece would not break, and the ratio of extension to the force producing it remained constant. This definition follows from the formula above given, thus: If $k =$ one square inch, l and λ each $=$ one inch, then $E = P$.

Within the elastic limit, when the deformations are proportional to the stresses, the coefficient of elasticity is constant, but beyond the elastic limit it decreases rapidly.

In cast iron there is generally no apparent limit of elasticity, the deformations increasing at a faster rate than the stresses, and a permanent set being produced by small loads. The coefficient of elasticity therefore is not constant during any portion of a test, but grows smaller as the load increases. The same is true in the case of timber. In wrought iron and steel, however, there is a well-defined elastic limit, and the coefficient of elasticity within that limit is nearly constant.

Resilience, or Work of Resistance of a Material.—Within the elastic limit, the resistance increasing uniformly from zero stress to the stress at the elastic limit, the work done by a load applied gradually is equal to one half the product of the final stress by the extension or other deformation. Beyond the elastic limit, the extensions increasing more rapidly than the loads, and the strain diagram approximating a parabolic form, the work is approximately equal to two thirds the product of the maximum stress by the extension.

The amount of work required to break a bar, measured usually in inch-pounds, is called its resilience; the work required to strain it to the elastic limit is called its elastic resilience.

Under a load applied suddenly the momentary elastic distortion is equal to twice that caused by the same load applied gradually.

When a solid material is exposed to percussive stress, as when a weight falls upon a beam transversely, the work of resistance is measured by the product of the weight into the total fall.

Elevation of Ultimate Resistance and Elastic Limit.—It was first observed by Prof. R. H. Thurston, and Commander L. A. Beardslee, U. S. N., independently, in 1873, that if wrought iron be subjected to a stress beyond its elastic limit, but not beyond its ultimate resistance, and then allowed to "rest" for a definite interval of time, a considerable increase of elastic limit and ultimate resistance may be experienced. In other words, the application of stress and subsequent "rest" increases the resistance of wrought iron.

This "rest" may be an entire release from stress or a simple holding the test-piece at a given intensity of stress.

Commander Beardslee prepared twelve specimens and subjected them to an intensity of stress equal to the ultimate resistance of the material, without breaking the specimens. These were then allowed to rest, entirely free from stress, from 24 to 30 hours, after which period they were again stressed until broken. The gain in ultimate resistance by the rest was found to vary from 4.4 to 17 per cent.

This elevation of elastic and ultimate resistance appears to be peculiar to iron and steel: it has not been found in other metals.

Relation of the Elastic Limit to Endurance under Repeated Stresses (condensed from *Engineering*, August 7, 1891).—When engineers first began to test materials, it was soon recognized that if a specimen was loaded beyond a certain point it did not recover its original dimensions on removing the load, but took a permanent set; this point was called the elastic limit. Since below this point a bar appeared to recover completely its original form and dimensions on removing the load, it appeared obvious that it had not been injured by the load, and hence the working load might be deduced from the elastic limit by using a small factor of safety.

Experience showed, however, that in many cases a bar would not carry safely a stress anywhere near the elastic limit of the material as determined by these experiments, and the whole theory of any connection between the elastic limit of a bar and its working load became almost discredited, and engineers employed the ultimate strength only in deducing the safe working load to which their structures might be subjected. Still, as experience accumulated it was observed that a higher factor of safety was required for a live load than for a dead one.

In 1871 Wöhler published the results of a number of experiments on bars of iron and steel subjected to live loads. In these experiments the stresses were put on and removed from the specimens without impact, but it was, nevertheless, found that the breaking stress of the materials was in every case much below the statical breaking load. Thus, a bar of Krupp's axle steel having a tenacity of 49 tons per square inch broke with a stress of 28.6 tons per square inch, when the load was completely removed and replaced without impact 170,000 times. These experiments were made on a large number of different brands of iron and steel, and the results were concordant in showing that a bar would break with an alternating stress of only, say, one third the statical breaking strength of the material, if the repetitions of stress were sufficiently numerous. At the same time, however, it appeared from the general trend of the experiments that a bar would stand an indefinite number of alternations of stress, provided the stress was kept below the limit.

Prof. Bauschinger defines the elastic limit as the point at which stress ceases to be sensibly proportional to strain, the latter being measured with a mirror apparatus reading to $\frac{1}{5000}$th of a millimetre, or about $\frac{1}{100000}$ in. This limit is always below the yield-point, and may on occasion be zero. On loading a bar above the yield-point, this point rises with the stress, and the rise continues for weeks, months, and possibly for years if the bar is left at rest under its load. On the other hand, when a bar is loaded beyond its true elastic limit, but below its yield-point, this limit rises, but reaches a maximum as the yield-point is approached, and then falls rapidly, reaching even to zero. On leaving the bar at rest under a stress exceeding that of its primitive breaking-down point the elastic limit begins to rise again, and may, if left a sufficient time, rise to a point much exceeding its previous value.

This property of the elastic limit of changing with the history of a bar has done more to discredit it than anything else, nevertheless it now seems as if it, owing to this very property, were once more to take its former place in the estimation of engineers, and this time with fixity of tenure. It had long been known that the limit of elasticity might be raised, as we have said, to almost any point within the breaking load of a bar. Thus, in some experiments by Professor Styffe, the elastic limit of a puddled-steel bar was raised 16,000 lbs. by subjecting the bar to a load exceeding its primitive elastic limit.

A bar has two limits of elasticity, one for tension and one for compression. Bauschinger loaded a number of bars in tension until stress ceased to be sensibly proportional to strain. The load was then removed and the bar tested in compression until the elastic limit in this direction had been exceeded. This process raises the elastic limit in compression, as would be found on testing the bar in compression a second time. In place of this, however, it was now again tested in tension, when it was found that the artificial raising of the limit in compression had lowered that in tension below its previous value. By repeating the process of alternately testing in tension and compression, the two limits took up points at equal distances from the line of no load, both in tension and compression. These limits Bauschinger calls natural elastic limits of the bar, which for wrought iron correspond to a stress of about 8½ tons per square inch, but this is practically the limiting load to which a bar of the same material can be strained alternately in tension and compression, without breaking when the loading is repeated sufficiently often, as determined by Wöhler's method.

As received from the rolls the elastic limit of the bar in tension is above the natural elastic limit of the bar as defined by Bauschinger, having been artificially raised by the deformations to which it has been subjected in the process of manufacture. Hence, when subjected to alternating stresses, the limit in tension is immediately lowered, while that in compression is raised until they both correspond to equal loads. Hence, in Wöhler's experiments, in which the bars broke at loads nominally below the elastic limits of the material, there is every reason for concluding that the loads were really greater than true elastic limits of the material. This is confirmed by tests on the connecting-rods of engines, which of course work under alternating stresses of equal intensity. Careful experiments on old rods show that the elastic limit in compression is the same as that in tension, and that both are far below the tension elastic limit of the material as received from the rolls.

The common opinion that straining a metal beyond its elastic limit injures it appears to be untrue. It is not the mere straining of a metal beyond one elastic limit that injures it, but the straining, many times repeated, beyond its two elastic limits. Sir Benjamin Baker has shown that in bending a shell plate for a boiler the metal is of necessity strained beyond its elastic limit, so that stresses of as much as 7 tons to 15 tons per square inch may obtain in it as it comes from the rolls, and unless the plate is annealed, these stresses will still exist after it has been built into the boiler. In such a case, however, when exposed to the additional stress due to the pressure inside the boiler, the overstrained portions of the plate will relieve themselves by stretching and taking a permanent set, so that probably after a year's working very little difference could be detected in the stresses in a plate built into the boiler as it came from the bending rolls, and in one which had been annealed, before riveting into place, and the first, in spite of its having been strained beyond its elastic limits, and not subsequently annealed, would be as strong as the other.

Resistance of Metals to Repeated Shocks.

More than twelve years were spent by Wöhler at the instance of the Prussian Government in experimenting upon the resistance of iron and steel to repeated stresses. The results of his experiments are expressed in what is known as Wöhler's law, which is given in the following words in Dubois's translation of Weyrauch:

"Rupture may be caused not only by a steady load which exceeds the carrying strength, but also by repeated applications of stresses, none of which are equal to the carrying strength. The differences of these stresses are measures of the disturbance of continuity, in so far as by their increase the minimum stress which is still necessary for rupture diminishes."

A practical illustration of the meaning of the first portion of this law may be given thus: If 50,000 pounds once applied will just break a bar of iron or steel, a stress very much less than 50,000 pounds will break it if repeated sufficiently often.

This is fully confirmed by the experiments of Fairbairn and Spangenberg, as well as those of Wöhler; and, as is remarked by Weyrauch, it may be considered as a long-known result of common experience. It partially accounts for what Mr. Holley has called the "intrinsically ridiculous factor of safety of six."

Another "long-known result of experience" is the fact that rupture may be caused by a succession of *shocks* or *impacts*, none of which alone would be sufficient to cause it. Iron axles, the piston-rods of steam hammers, and other pieces of metal subject to continuously repeated shocks, invariably break after a certain length of service. They have a "life" which is limited.

Several years ago Fairbairn wrote: "We know that in some cases wrought iron subjected to continuous vibration assumes a crystalline structure, and that the cohesive powers are much deteriorated, but we are ignorant of the causes of this change." We are still ignorant, not only of the causes of this change, but of the conditions under which it takes place. Who knows whether wrought iron subjected to very slight continuous vibration will endure forever? or whether to insure final rupture each of the continuous small shocks must amount at least to a certain percentage of single heavy shock (both measured in foot-pounds), which would cause rupture with one application? Wöhler found in testing iron by repeated stresses (not impacts) that in one case 400,000 applications of a stress of 500 centners to the square inch caused rupture, while a similar bar remained sound after 48,000,000 applications of a stress of 300 centners to the square inch (1 centner = 110.2 lbs.).

Who knows whether or not a similar law holds true in regard to repeated shocks? Suppose that a bar of iron would break under a single impact of 1000 foot-pounds, how many times would it be likely to bear the repetition of 100 foot pounds, or would it be safe to allow it to remain for fifty years subjected to a continual succession of blows of even 10 foot-pounds each?

Mr. William Metcalf published in the *Metallurgical Review*, Dec. 1877, the results of some tests of the life of steel of different percentages of carbon under impact. Some small steel pitmans were made, the specifications for which required that the unloaded machine should run 4½ hours at the rate of 1200 revolutions per minute before breaking.

The steel was all of uniform quality, except as to carbon. Here are the results: The

.30 C.	ran	1 h. 21 m.	Heated and bent before breaking.
.49 C.	"	1 h. 28 m.,	" " " " "
.43 C.	"	4 h. 57 m.	Broke without heating.
.65 C.	"	3 h. 50 m.	Broke at weld where imperfect.
.80 C.	"	5 h. 40 m.	
.84 C.	"	18 h.	
.87 C.	Broke in weld near the end.		
.96 C.	Ran 4.55 m., and the machine broke down.		

Some other experiments by Mr. Metcalf confirmed his conclusion, viz., that high-carbon steel was better adapted to resist repeated shocks and vibrations than low-carbon steel.

These results, however, would scarcely be sufficient to induce any engineer to use .84 carbon steel in a car-axle or a bridge-rod. Further experiments are needed to confirm or overthrow them.

(See description of proposed apparatus for such an investigation in the author's paper in Trans. A. I. M. E., vol. viii, p. 76, from which the above extract is taken.)

Stresses Produced by Suddenly Applied Forces and Shocks.

(Mansfield Merriman, *R. R. & Eng. Jour.*, Dec. 1889.)

Let P be the weight which is dropped from a height h upon the end of a bar, and let y be the maximum elongation which is produced. The work performed by the falling weight, then, is

$$W = P(h + y),$$

and this must equal the internal work of the resisting molecular stresses. The stress in the bar, which is at first 0, increases up to a certain limit Q, which is greater than P; and if the elastic limit be not exceeded the elongation increases uniformly with the stress, so that the internal work is equal to the mean stress $1/2Q$ multiplied by the total elongation y, or

$$W = 1/2\,Qy.$$

Whence, neglecting the work that may be dissipated in heat,

$$1/2Qy = Ph + Py.$$

If e be the elongation due to the static load P, within the elastic limit $y = \frac{Q}{P}e$; whence

$$Q = P\left(1 + \sqrt{1 + 2\frac{h}{e}}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which gives the momentary maximum stress. Substituting this value of Q, there results

$$y = e\left(1 + \sqrt{1 + 2\frac{h}{e}}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which is the value of the momentary maximum elongation.

A shock results when the force P, before its action on the bar, is moving with velocity, as is the case when a weight P falls from a height h. The above formulas show that this height h may be small if e is a small quantity, and yet very great stresses and deformations be produced. For instance, let $h = 4e$, then $Q = 4P$ and $y = 4e$; also let $h = 12e$, then $Q = 6P$ and $y = 6e$. Or take a wrought-iron bar 1 in. square and 5 ft. long: under a steady load of 5000 lbs. this will be compressed about 0.012 in., supposing that no lateral flexure occurs; but if a weight of 5000 lbs. drops upon its end from the small height of 0.048 in. there will be produced the stress of 20,000 lbs.

A suddenly applied force is one which acts with the uniform intensity P upon the end of the bar, but which has no velocity before acting upon it. This corresponds to the case of $h = 0$ in the above formulas, and gives $Q = 2P$ and $y = 2e$ for the maximum stress and maximum deformation. Probably the action of a rapidly-moving train upon a bridge produces stresses of this character.

Increasing the Tensile Strength of Iron Bars by Twisting them.—Ernest L. Ransome of San Francisco has obtained an English Patent, No. 16221 of 1888, for an "improvement in strengthening and testing wrought metal and steel rods or bars, consisting in twisting the same in a cold state. . . . Any defect in the lamination of the metal which would otherwise be concealed is revealed by twisting, and imperfections are shown at once. The treatment may be applied to bolts, suspension-rods or bars subjected to tensile strength of any description."

Results of tests of this process were reported by Lieutenant F. P. Gilmore, U. S. N., in a paper read before the Technical Society of the Pacific Coast, published in the Transactions of the Society for the month of December, 1888. The experiments include trials with thirty-nine bars, twenty-nine of which were variously twisted, from three-eighths of one turn to six turns per foot. The test-pieces were cut from one and the same bar, and accurately measured and numbered. From each lot two pieces without twist were tested for tensile strength and ductility. One group of each set was twisted until the pieces broke, as a guide for the amount of twist to be given those to be tested for tensile strain.

The following is the result of one set of Lieut. Gilmore's tests, on iron bars 8 in. long, .719 in. diameter.

No. of Bars.	Conditions.	Twists in Turns.	Twists per ft.	Tensile Strength.	Tensile per sq. in.	Gain per cent.
2	Not twisted.	0	0	22,000	54,180	
2	Twisted cold.	1/2	3/4	23,900	59,020	9
2	" "	1	1 1/2	25,800	63,500	17
2	" "	2	3	26,300	64,750	19
1	" "	2 1/2	3 3/4	26,400	65,000	20

Tests that corroborated these results were made by the University of California in 1889 and by the Low Moor Iron Works, England, in 1890.

TENSILE STRENGTH.

The following data are usually obtained in testing by tension in a testing-machine a sample of a material of construction:

The load and the amount of extension at the elastic limit.

The maximum load applied before rupture.

The elongation of the piece, measured between gauge-marks placed a stated distance apart before the test; and the reduction of area at the point of fracture.

The load at the elastic limit and the maximum load are recorded in pounds per square inch of the original area. The elongation is recorded as a percentage of the stated length between the gauge-marks, and the reduction area as a percentage of the original area. The coefficient of elasticity is calculated from the ratio the extension within the elastic limit per inch of length bears to the load per square inch producing that extension.

On account of the difficulty of making accurate measurements of the fractured area of a test-piece, and of the fact that elongation is more valuable than reduction of area as a measure of ductility and of resilience or work of resistance before rupture, modern experimenters are abandoning the custom of reporting reduction of area. The "strength per square inch of fractured section" formerly frequently used in reporting tests is now almost entirely abandoned. The data now calculated from the results of a tensile test for commercial purposes are: 1. Tensile strength in pounds per square inch of original area. 2. Elongation per cent of a stated length between gauge-marks, usually 8 inches. 3. Elastic limit in pounds per square inch of original area.

The short or grooved test specimen gives with most metals, especially with wrought iron and steel, an apparent tensile strength much higher than the real strength. This form of test-piece is now almost entirely abandoned.

The following results of the tests of six specimens from the same 1 1/4" steel bar illustrate the apparent elevation of elastic limit and the changes in other properties due to change in length of stems which were turned down in each specimen to .798" diameter. (Jas. E. Howard, Eng. Congress 1893, Section G.)

Description of Stem.	Elastic Limit, Lbs. per Sq. In.	Tensile Strength, Lbs. per Sq. In.	Contraction of Area, per cent.
1.00" long............	64,900	94,400	49.0
.50 "	65,320	97,800	43.4
.25 "	68,000	102,420	39.6
Semicircular groove, .4" radius........	75,000	116,380	31.6
Semicircular groove, 1/8" radius..........	86,000, about	134,960	23.0
V-shaped groove......	90,000, about	117,000	Indeterminate.

Tests plate made by the author in 1879 of straight and grooved test-pieces of boiler-plate steel cut from the same gave the following results:

5 straight pieces, 56,605 to 59,012 lbs. T. S. Aver. 57,566 lbs.
4 grooved " 64,341 to 67,400 " " " 65,452 "
Excess of the short or grooved specimen, 21 per cent, or 12,114 lbs.

Measurement of Elongation.—In order to be able to compare records of elongation, it is necessary not only to have a uniform length of section between gauge-marks (say 8 inches), but to adopt a uniform method of measuring the elongation to compensate for the difference between the apparent elongation when the piece breaks near one of the gauge-marks, and when it breaks midway between them. The following method is recommended (Trans. A. S. M. E., vol. xi., p. 622):

Mark on the specimen divisions of 1/2 inch each. After fracture measure from the point of fracture the length of 8 of the marked spaces on each fractured portion (or 7 + on one side and 8 + on the other if the fracture is not at one of the marks). The sum of these measurements, less 8 inches, is the elongation of 8 inches of the original length. If the fracture is so near one end of the specimen that 7 + spaces are not left on the shorter portion, then take the measurement of as many spaces (with the fractional part next to the fracture) as are left, and for the spaces lacking add the measurement of as many corresponding spaces of the longer portion as are necessary to make the 7 + spaces.

Shapes of Specimens for Tensile Tests.—The shapes shown in Fig. 75 were recommended by the author in 1882 when he was connected

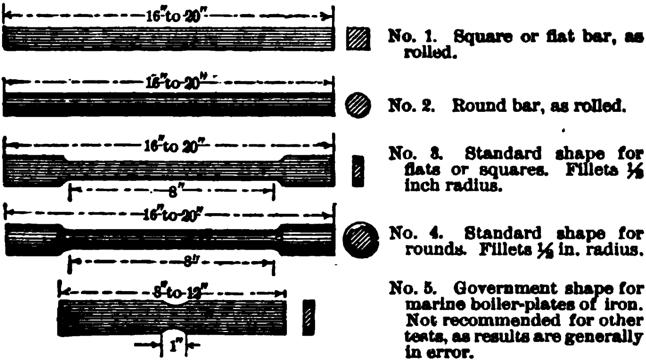


FIG. 75.

with the Pittsburgh Testing Laboratory. They are now in most general use, the earlier forms, with 5 inches or less in length between shoulders, being almost entirely abandoned.

Precautions Required in making Tensile Tests.—The testing-machine itself should be tested, to determine whether its weighing apparatus is accurate, and whether it is so made and adjusted that in the test of a properly made specimen the line of strain of the testing-machine is absolutely in line with the axis of the specimen.

The specimen should be so shaped that it will not give an incorrect record of strength.

It should be of uniform minimum section for not less than five inches of its length.

Regard must be had to the time occupied in making tests of certain materials. Wrought iron and soft steel can be made to show a higher than their actual apparent strength by keeping them under strain for a great length of time.

In testing soft alloys, copper, tin, zinc, and the like, which flow under constant strain their highest apparent strength is obtained by testing them rapidly. In recording tests of such materials the length of time occupied in the test should be stated.

For very accurate measurements of elongation, corresponding to increments of load during the tests, the electric contact micrometer, described in Trans. A. S. M. E., vol. vi., p. 479, will be found convenient. When readings of elongation are then taken during the test, a strain diagram may be plotted from the reading, which is useful in comparing the qualities of different specimens. Such strain diagrams are made automatically by the new Olsen testing-machine, described in *Jour. Frank. Inst.* 1891.

The coefficient of elasticity should be deduced from measurement observed between fixed increments of load per unit section, say between 2000 and 12,000 pounds per square inch or between 1000 and 11,000 pounds instead of between 0 and 10,000 pounds.

COMPRESSIVE STRENGTH.

What is meant by the term "compressive strength" has not yet been settled by the authorities, and there exists more confusion in regard to this term than in regard to any other used by writers on strength of materials. The reason of this may be easily explained. The effect of a compressive stress upon a material varies with the nature of the material, and with the shape and size of the specimen tested. While the effect of a tensile stress is to produce rupture or separation of particles in the direction of the line of strain, the effect of a compressive stress on a piece of material may be either to cause it to fly into splinters, to separate into two or more wedge-shaped pieces and fly apart, to bulge, buckle, or bend, or to flatten out and utterly resist rupture or separation of particles. A piece of speculum metal under compressive stress will exhibit no change of appearance until rupture takes place, and then it will fly to pieces as suddenly as if blown apart by gunpowder. A piece of cast iron or of stone will generally split into wedge-shaped fragments. A piece of wrought iron will buckle or bend. A piece of wood or zinc may bulge, but its action will depend upon its shape and size. A piece of lead will flatten out and resist compression till the last degree; that is, the more it is compressed the greater becomes its resistance.

Air and other gaseous bodies are compressible to any extent as long as they retain the gaseous condition. Water not confined in a vessel is compressed by its own weight to the thickness of a mere film, while when confined in a vessel it is almost incompressible.

It is probable, although it has not been determined experimentally, that solid bodies when confined are at least as incompressible as water. When they are not confined, the effect of a compressive stress is not only to shorten them, but also to increase their lateral dimensions or bulge them. Lateral strains are therefore induced by compressive stresses.

The weight per square inch of original section required to produce any given amount or percentage of shortening of any material is not a constant quantity, but varies with both the length and the sectional area, with the shape of this sectional area, and with the relation of the area to the length. The "compressive strength" of a material, if this term be supposed to mean the weight in pounds per square inch necessary to cause rupture, may vary with every size and shape of specimen experimented upon. Still more difficult would it be to state what is the "compressive strength" of a material which does not rupture at all, but flattens out. Suppose we are testing a cylinder of a soft metal like lead, two inches in length and one inch in diameter, a certain weight will shorten it one per cent, another weight ten per cent, another fifty per cent, but no weight that we can place upon it will rupture it, for it will flatten out to a thin sheet. What, then, is its compressive strength? Again, a similar cylinder of soft wrought iron would probably compress a few per cent, bulging evenly all around; it would then commence to bend, but at first the bend would be imperceptible to the eye and too small to be measured. Soon this bend would be great enough to be noticed, and finally the piece might be bent nearly double, or otherwise distorted. What is the "compressive strength" of this piece of iron? Is it the weight per square inch which compresses the piece one per cent or five per cent, that which causes the first bending (impossible to be discovered), or that which causes a perceptible bend?

As showing the confusion concerning the definitions of compressive strength, the following statements from different authorities on the strength of wrought iron are of interest.

Wood's Resistance of Materials states, "comparatively few experiments have been made to determine how much wrought iron will sustain at the point of crushing. Hodgkinson gives 65,000, Rondulet 70,800, Weisbach 72,000

Rankine 30,000 to 40,000. It is generally assumed that wrought iron will resist about two thirds as much crushing as to tension, but the experiments fail to give a very definite ratio."

Mr. Whipple, in his treatise on bridge-building, states that a bar of good wrought iron will sustain a tensile strain of about 60,000 pounds per square inch, and a compressive strain, in pieces of a length not exceeding twice the least diameter, of about 90,000 pounds.

The following values, said to be deduced from the experiments of Major Wade, Hodgkinson, and Capt. Meigs, are given by Haswell:

American wrought iron	127,720 lbs.
" " " (mean)	85,500 "
English " "	65,200 " 40,000 "

Stoney states that the strength of short pillars of any given material, all having the same diameter, does not vary much, provided the length of the piece is not less than one and does not exceed four or five diameters, and that the weight which will just crush a short prism whose base equals one square inch, and whose height is not less than 1 to 1½ and does not exceed 4 or 5 diameters, is called the crushing strength of the material. It would be well if experimenters would all agree upon some such definition of the term "crushing strength," and insist that all experiments which are made for the purpose of testing the relative values of different materials in compression be made on specimens of exactly the same shape and size. An arbitrary size and shape should be assumed and agreed upon for this purpose. The size mentioned by Stoney is definite as regards area of section, viz., one square inch, but is indefinite as regards length, viz., from one to five diameters. In some metals a specimen five diameters long would bend, and give a much lower apparent strength than a specimen having a length of one diameter. The words "will just crush" are also indefinite for ductile materials, in which the resistance increases without limit if the piece tested does not bend. In such cases the weight which causes a certain percentage of compression, as five, ten, or fifty per cent, should be assumed as the crushing strength.

For future experiments on crushing strength three things are desirable: First, an arbitrary standard shape and size of test specimen for comparison of all materials. Secondly, a standard limit of compression for ductile materials, which shall be considered equivalent to fracture in brittle materials. Thirdly, an accurate knowledge of the relation of the crushing strength of a specimen of standard shape and size to the crushing strength of specimens of all other shapes and sizes. The latter can only be secured by a very extensive and accurate series of experiments upon all kinds of materials, and on specimens of a great number of different shapes and sizes.

The author proposes, as a standard shape and size, for a compressive test specimen for all metals, a cylinder one inch in length, and one half square inch in sectional area, or 0.798 inch diameter; and for the limit of compression equivalent to fracture, ten per cent of the original length. The term "compressive strength," or "compressive strength of standard specimen," would then mean the weight per square inch required to fracture by compressive stress a cylinder one inch long and 0.798 inch diameter, or to reduce its length to 0.9 inch if fracture does not take place before that reduction in length is reached. If such a standard, or any standard size whatever, had been used by the earlier authorities on the strength of materials, we never would have had such discrepancies in their statements in regard to the compressive strength of wrought iron as those given above.

The reasons why this particular size is recommended are: that the sectional area, one-half square inch, is as large as can be taken in the ordinary testing-machines of 100,000 pounds capacity, to include all the ordinary metals of construction, cast and wrought iron, and the softer steels; and that the length, one inch, is convenient for calculation of percentage of compression. If the length were made two inches, many materials would bend in testing, and give incorrect results. Even in cast iron Hodgkinson found as the mean of several experiments on various grades, tested in specimens ¾ inch in height, a compressive strength per square inch of 94,730 pounds, while the mean of the same number of specimens of the same irons tested in pieces 1½ inches in height was only 88,800 pounds. The best size and shape of standard specimen should, however, be settled upon only after consultation and agreement among several authorities.

The Committee on Standard Tests of the American Society of Mechanical Engineers say (vol. xi., p. 624):

"Although compression tests have heretofore been made on diminutive sample pieces, it is highly desirable that tests be also made on long pieces from 10 to 20 diameters in length, corresponding more nearly with actual practice, in order that elastic strain and change of shape may be determined by using proper measuring apparatus.

The elastic limit, modulus or coefficient of elasticity, maximum and ultimate resistances, should be determined, as well as the increase of section at various points, viz., at bearing surfaces and at crippling point.

The use of long compression-test pieces is recommended, because the investigation of short cubes or cylinders has led to no direct application of the constants obtained by their use in computation of actual structures, which have always been and are now designed according to empirical formulæ obtained from a few tests of long columns."

COLUMNS, PILLARS, OR STRUTS.

Hodgkinson's Formula for Columns.

P = crushing weight in pounds; d = exterior diameter in inches; d_1 = interior diameter in inches; L = length in feet.

Kind of Column.	Both ends rounded, the length of the column exceeding 15 times its diameter.	Both ends flat, the length of the column exceeding 30 times its diameter.
Solid cylindrical columns of cast iron.....	$P = 33{,}380\dfrac{d^{3.76}}{L^{1.7}}$	$P = 98{,}920\dfrac{d^{3.55}}{L^{1.7}}$
Hollow cylindrical columns of cast iron.....	$P = 29{,}120\dfrac{d^{3.76} - d_1^{3.76}}{L^{1.7}}$	$P = 99{,}320\dfrac{d^{3.55} - d_1^{3.55}}{L^{1.7}}$
Solid cylindrical columns of wrought iron.	$P = 95{,}850\dfrac{d^{3.76}}{L^2}$	$P = 299{,}600\dfrac{d^{3.55}}{L^2}$
Solid square pillar of Dantzic oak (dry)....	....	$P = 24{,}540\dfrac{d^4}{L^2}$
Solid square pillar of red deal (dry).........	....	$P = 17{,}510\dfrac{d^4}{L^2}$

The above formulæ apply only in cases in which the length is so great that the column breaks by bending and not by simple crushing. If the column be shorter than that given in the table, and more than four or five times its diameter, the strength is found by the following formula:

$$W = \frac{PCK}{P + \frac{3}{4}CK},$$

in which P = the value given by the preceding formulæ, K = the transverse section of the column in square inches, C = the ultimate compressive resistance of the material, and W = the crushing strength of the column.

Hodgkinson's experiments were made upon comparatively short columns, the greatest length of cast-iron columns being 60½ inches, of wrought iron 90¾ inches.

The following are some of his conclusions:

1. In all long pillars of the same dimensions, when the force is applied in the direction of the axis, the strength of one which has flat ends is about three times as great as one with rounded ends.

2. The strength of a pillar with one end rounded and the other flat is an arithmetical mean between the two given in the preceding case of the same dimensions.

3. The strength of a pillar having both ends firmly fixed is the same as one of half the length with both ends rounded.

4. The strength of a pillar is not increased more than one seventh by enlarging it at the middle.

Gordon's formulæ deduced from Hodgkinson's experiments are more generally used than Hodgkinson's own. They are:

Columns with both ends fixed or flat, $P = \dfrac{fS}{1 + a\dfrac{l^2}{r^2}}$;

Columns with one end flat, the other end round, $P = \dfrac{fS}{1 + 1.8a\dfrac{l^2}{r^2}}$;

Columns with both ends round, or hinged, $P = \dfrac{fS}{1 + 4a\dfrac{l^2}{r^2}}$;

S = area of cross-section in inches;
P = ultimate resistance of column, in pounds;
f = crushing strength of the material in lbs. per square inch;
r = least radius of gyration, in inches, $r^2 = \dfrac{\text{Moment of inertia}}{\text{area of section}}$;
l = length of column in inches;
a = a coefficient depending upon the material;

f and a are usually taken as constants; they are really empirical variables, dependent upon the dimensions and character of the column as well as upon the material. (Burr.)

For solid wrought-iron columns, values commonly taken are: f = 36,000 to 40,000; a = 1/36,000 to 1/40,000.

For solid cast-iron columns, f = 80,000, a = 1/6400.

For hollow cast-iron columns, fixed ends, $p = \dfrac{80,000}{1 + \dfrac{1}{800}\dfrac{l^2}{d^2}}$, l = length and d = diameter in the same unit, and p = strength in lbs. per square inch.

The coefficient of l^2/d^2 is given various values, as 1/400, 1/500, 1/600, and 1/800, by different writers. The use of Gordon's formula, with any coefficients derived from Hodgkinson's experiments, for cast-iron columns is to be deprecated. See Strength of Cast-iron Columns, pp. 250, 251.

Sir Benjamin Baker gives,

For mild steel, f = 67,000 lbs., a = 1/22,400.
For strong steel, f = 114,000 lbs., a = 1/14,400

Prof. Burr considers these only loose approximations for the ultimate resistances. See his formulæ on p. 259.

For dry timber Rankine gives f = 7200 lbs., a = 1/3000.

MOMENT OF INERTIA AND RADIUS OF GYRATION.

The moment of inertia of a section is the sum of the products of each elementary area of the section into the square of its distance from an assumed axis of rotation, as the neutral axis.

The radius of gyration of the section equals the square root of the quotient of the moment of inertia divided by the area of the section. If R = radius of gyration, I = moment of inertia and A = area,

$$R = \sqrt{\frac{I}{A}}. \qquad \frac{I}{A} = R^2.$$

The moments of inertia of various sections are as follows:

d = diameter, or outside diameter; d_1 = inside diameter; b = breadth; h = depth; b_1, h_1, inside breadth and diameter;

Solid rectangle $I = 1/12bh^3$; Hollow rectangle $I = 1/12(bh^3 - b_1h_1^3)$;
Solid square $I = 1/12b^4$; Hollow square $I = 1/12(b^4 - b_1^4)$;
Solid cylinder $I = 1/64\pi d^4$; Hollow cylinder $I = 1/64\pi(d^4 - d_1^4)$.

Moments of Inertia and Radius of Gyration for Various Sections, and their Use in the Formulas for Strength of Girders and Columns.—The strength of sections to resist strains, either as girders or as columns, depends not only on the area but also on the form of the section, and the property of the section which forms the basis of the constants used in the formulas for strength of girders and columns to express the effect of the form, is its moment of inertia about its neutral axis. The modulus of resistance of any section to transverse bending is its moment of inertia divided by the distance from the neutral axis to the fibres farthest removed from that axis; or

$$\text{Section modulus} = \frac{\text{Moment of inertia}}{\text{Distance of extreme fibre from axis}}. \quad Z = \frac{I}{y}.$$

Moment of resistance = section modulus × unit stress on extreme fibre.

Moment of Inertia of Compound Shapes. (Pencoyd Iron Works.)—The moment of inertia of any section about any axis is equal to the I about a parallel axis passing through its centre of gravity + (the area of the section × the square of the distance between the axes).

By this rule, the moments of inertia or radii of gyration of any single sections being known, corresponding values may be obtained for any combination of these sections.

Radius of Gyration of Compound Shapes.—In the case of a pair of any shape without a web the value of R can always be found without considering the moment of inertia.

The radius of gyration for any section around an axis parallel to another axis passing through its centre of gravity is found as follows:

Let r = radius of gyration around axis through centre of gravity; R = radius of gyration around another axis parallel to above; d = distance between axes: $R = \sqrt{d^2 + r^2}$.

When r is small, R may be taken as equal to d without material error.

Graphical Method for Finding Radius of Gyration.—Benj. F. La Rue, *Eng. News*, Feb. 2, 1893, gives a short graphical method for finding the radius of gyration of hollow, cylindrical, and rectangular columns, as follows:

For cylindrical columns:

Lay off to a scale of 4 (or 40) a right-angled triangle, in which the base equals the outer diameter, and the altitude equals the inner diameter of the column, or *vice versa*. The hypothenuse, measured to a scale of unity (or 10), will be the radius of gyration sought.

This depends upon the formula

$$G = \sqrt{\frac{\text{Mom. of Inertia}}{\text{Area}}} = \frac{\sqrt{D^2 + d^2}}{4},$$

in which A = area and D = diameter of outer circle, a = area and d = diameter of inner circle, and G = radius of gyration. $\sqrt{D^2 + d^2}$ is the expression for the hypothenuse of a right-angled triangle, in which D and d are the base and altitude.

The sectional area of a hollow round column is $.7854(D^2 - d^2)$. By constructing a right-angled triangle in which D equals the hypothenuse and d equals the altitude, the base will equal $\sqrt{D^2 - d^2}$. Calling the value of this expression for the base B, the area will equal $.7854B^2$.

Value of G for square columns:

Lay off as before, but using a scale of 10, a right-angled triangle of which the base equals D or the side of the outer square, and the altitude equals d, the side of the inner square. With a scale of 8 measure the hypothenuse, which will be, approximately, the radius of gyration.

This process for square columns gives an excess of slightly more than 4%. By deducting 4% from the result, a close approximation will be obtained.

A very close result is also obtained by measuring the hypothenuse with the same scale by which the base and altitude were laid off, and multiplying by the decimal 0.29; more exactly, the decimal is 0.28867.

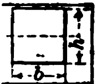
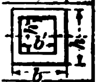
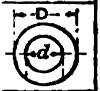
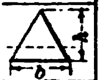
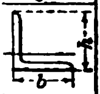
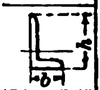
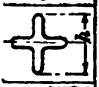
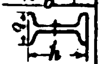
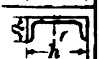
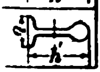
The formula is

$$G = \sqrt{\frac{\text{Mom. of inertia}}{\text{Area}}} = \frac{1}{\sqrt{12}}\sqrt{D^2 + d^2}, = 0.28867\sqrt{D^2 + d^2}$$

This may also be applied to any rectangular column by using the lesser diameters of an unsupported column, and the greater diameters if the column is supported in the direction of its least dimensions.

ELEMENTS OF USUAL SECTIONS.

Moments refer to horizontal axis through centre of gravity. This table is intended for convenient application where extreme accuracy is not important. Some of the terms are only approximate; those marked * are correct. Values for radius of gyration in flanged beams apply to standard minimum sections only. A = area of section; b = breadth; h = depth; D = diameter.

Shape of Section.		Moment of Inertia.	Section Modulus.	Square of Least Radius of Gyration.	Least Radius of Gyration.
	Solid Rectangle.	$\frac{bh^3}{12}$ *	$\frac{bh^2}{6}$ *	$\frac{(\text{Least side})^2}{12}$ *	$\frac{\text{Least side}}{3.46}$ *
	Hollow Rectangle.	$\frac{bh^3-b_1h_1^3}{12}$ *	$\frac{bh^3-b_1h_1^3}{6h}$ *	$\frac{h^2+h_1^2}{12}$ *	$\frac{h+h^1}{4.89}$
	Solid Circle.	$\frac{AD^2}{16}$ *	$\frac{AD}{8}$ *	$\frac{D^2}{16}$ *	$\frac{D}{4}$ *
	Hollow Circle. A, area of large section; a, area of small section.	$\frac{AD^2-ad^2}{16}$	$\frac{AD^2-ad^2}{8D}$	$\frac{D^2+d^2}{16}$ *	$\frac{D+d}{5.64}$
	Solid Triangle.	$\frac{bh^3}{36}$	$\frac{bh^2}{24}$	The least of the two: $\frac{h^2}{18}$ or $\frac{b^2}{24}$	The least of the two: $\frac{h}{4.24}$ or $\frac{b}{4.9}$
	Even Angle.	$\frac{Ah^2}{10.2}$	$\frac{Ah}{7.2}$	$\frac{b^2}{25}$	$\frac{b}{5}$
	Uneven Angle.	$\frac{Ah^2}{9.5}$	$\frac{Ah}{6.5}$	$\frac{(hb)^2}{13(h^2+b^2)}$	$\frac{hb}{2.6(h+b)}$
	Even Cross.	$\frac{Ah^2}{19}$	$\frac{Ah}{9.5}$	$\frac{h^2}{22.5}$	$\frac{h}{4.74}$
	Even Tee.	$\frac{Ah^2}{11.1}$	$\frac{Ah}{8}$	$\frac{b^2}{22.5}$	$\frac{b}{4.74}$
	I Beam.	$\frac{Ah^2}{6.66}$	$\frac{Ah}{3.2}$	$\frac{b^2}{21}$	$\frac{b}{4.58}$
	Channel.	$\frac{Ah^2}{7.34}$	$\frac{Ah}{3.67}$	$\frac{b^2}{12.5}$	$\frac{b}{3.54}$
	Deck Beam.	$\frac{Ah^2}{6.9}$	$\frac{Ah}{4}$	$\frac{b^2}{36.5}$	$\frac{b}{6}$

Distance of base from centre of gravity, solid triangle, $\frac{h}{3}$; even angle, $\frac{h}{3.3}$; uneven angle, $\frac{h}{3.5}$; even tee, $\frac{h}{3.3}$; deck beam, $\frac{h}{2.3}$; all other shapes given in the table, $\frac{h}{2}$ or $\frac{D}{2}$.

The Strength of Cast-iron Columns.

Hodgkinson's experiments (first published in Phil. Trans. Royal Socy., 1840, and condensed in Tredgold on Cast Iron, 4th ed., 1846), and Gordon's formula, based upon them, are still used (1898) in designing cast-iron columns. That they are entirely inadequate as a basis of a practical formula suitable to the present methods of casting columns will be evident from what follows.

Hodgkinson's experiments were made on nine "long" pillars, about 7½ ft. long, whose external diameters ranged from 1.74 to 2.23 in., and average thickness from 0.29 to 0.35 in., the thickness of each column also varying, and on 13 "short" pillars, 0.733 ft. to 2.251 ft. long, with external diameters from 1.08 to 1.26 in., all of them less than ¼ in. thick. The iron used was Low Moor, Yorkshire, No. 3, said to be a good iron, not very hard, earlier experiments on which had given a tensile strength of 14,535 and a crushing strength of 109,801 lbs. per sq. in. The results of the experiments on the "long" pillars were reduced to the equivalent breaking weight of a solid pillar 1 in. diameter and of the same length, 7½ ft., which ranged from 2969 to 3587 lbs. per sq. in., a range of over 12 per cent, although the pillars were made from the same iron and of nearly uniform dimensions. From the 13 experiments on "short" pillars a formula was derived, and from it were obtained the "calculated" breaking weights, the actual breaking weights ranging from about 8 per cent above to about 8 per cent below the calculated weights, a total range of about 16 per cent. Modern cast-iron columns, such as are used in the construction of buildings, are very different in size, proportions, and quality of iron from the slender "long" pillars used in Hodgkinson's experiments. There is usually no check, by actual tests or by disinterested inspection, upon the quality of the material. The tensile, compressive, and transverse strength of cast iron varies through a great range (the tensile strength ranging from less than 10,000 to over 40,000 lbs. per sq. in.), with variations in the chemical composition of the iron, according to laws which are as yet very imperfectly understood, and with variations in the method of melting and of casting. There is also a wide variation in the strength of iron of the same melt when cast into bars of different thicknesses. It is therefore impossible to predict even approximately, from the data given by Hodgkinson of the strength of columns of Low Moor iron in pillars 7½ ft. long, 2 in. diam., and ⅛ in. thick, what will be the strength of a column made of American cast iron, of a quality not stated, in a column 16 ft. long, 12 or 15 in. diam., and from ¾ in. to 1½ in. thick.

Another difficulty in obtaining a practical formula for the strength of cast-iron columns is due to the uncertainty of the quality of the casting, and the danger of hidden defects, such as internal stresses due to unequal cooling, cinder or dirt, blow-holes, "cold-shuts," and cracks on the inner surface, which cannot be discovered by external inspection. Variation in thickness, due to rising of the core during casting, is also a common defect.

In addition to the above theoretical or *a priori* objections to the use of Gordon's formula, based on Hodgkinson's experiments, for cast-iron columns, we have the data of recent experiments on full-sized columns, made by the Building Department of New York City (*Eng'g News*, Jan. 13 and 20, 1898). Ten columns in all were tested, six 15-inch, 190¼ inches long, two 8-inch, 160 inches long, and two 6-inch, 120 inches long. The tests were made on the large hydraulic machine of the Phœnix Bridge Co., of 2,000,000 pounds capacity, which was calibrated for frictional error by the repeated testing within the elastic limit of a large Phœnix column, and the comparison of these tests with others made on the government machine at the Watertown Arsenal. The average frictional error was calculated to be 15.4 per cent, but *Engineering News*, revising the data, makes it 17.1 per cent, with a variation of 3 per cent either way from the average with different loads. The results of the tests of the volumes are given on the opposite page.

Column No. 6 was not broken at the highest load of the testing machine.

Columns Nos. 3 and 4 were taken from the Ireland Building, which collapsed on August 8, 1895; the other four 15-inch columns were made from drawings prepared by the Building Department, as nearly as possible duplicates of Nos. 3 and 4. Nos. 1 and 2 were made by a foundry in New York with no knowledge of their ultimate use. Nos. 5 and 6 were made by a foundry in Brooklyn with the knowledge that they were to be tested. Nos. 7 to 10 were made from drawings furnished by the Department.

TESTS OF CAST-IRON COLUMNS.

Number.	Diam. Inches.	Thickness.			Breaking Load.	
		Max.	Min.	Average.	Pounds.	Pounds per sq. in.
1	15	1	1	1	1,356,000	30,830
2	15	1 5/16	1	1⅛	1,330,000	27,700
3	15	1¼	1	1⅛	1,198,000	24,900
4	15⅛	1 7/32	1	1⅛	1,246,000	25,200
5	15	1 11/16	1	1 11/64	1,632,000	32,100
6	15	1¼	1⅛	1 3/16	2,082,000 +	40,400 +
7	7¾ to 8¼	1¼	⅝	1	651,000	31,900
8	8	1 3/32	1	1 3/64	612,800	26,800
9	6 1/16	1 5/32	1⅛	1 9/64	400,000	22,700
10	6 3/32	1⅛	1 1/16	1 7/64	455,200	26,300

Applying Gordon's formula, as used by the Building Department, $S = \frac{80000a}{1 + \frac{1}{400}\frac{l^2}{d^2}}$, to these columns gives for the breaking strength per square inch of the 15-inch columns 57,143 pounds, for the 8-inch columns 40,000 pounds, and for the 6-inch columns 40,000. The strength of columns Nos. 3 and 4 as calculated is 128 per cent more than their actual strength; their actual strength is less than 44 per cent of their calculated strength; and the factor of safety, supposed to be 5 in the Building Law, is only 2.2 for central loading, no account being taken of the likelihood of eccentric loading.

Prof. Lanza, in his *Applied Mechanics*, p. 372, quotes the records of 14 tests of cast-iron mill columns, made on the Watertown testing-machine in 1887-88, the breaking strength per square inch ranging from 25,100 to 63,310 pounds, and showing no relation between the breaking strength per square inch and the dimensions of the columns. Only 3 of the 14 columns had a strength exceeding 38,500 pounds per square inch. The average strength of the other 11 was 29,600 pounds per square inch. Prof. Lanza says that it is evident that in the case of such columns we cannot rely upon a crushing strength of greater than 25,000 or 30,000 pounds per square inch of area of section.

He recommends a factor of safety of 5 or 6 with these figures for crushing strength, or 5000 pounds per square inch of area of section as the highest allowable safe load, and in addition makes the conditions that the length of the column shall not be greatly in excess of 20 times the diameter, that the thickness of the metal shall be such as to insure a good strong casting, and that the sectional area should be increased if necessary to insure that the extreme fibre stress due to probable eccentric loading shall not be greater than 5000 pounds per square inch.

Prof. W. H. Burr (*Eng'g News*, June 30, 1898) gives a formula derived from plotting the results of the Watertown and Phœnixville tests, above described, which represents the average strength of the columns in pounds per square inch. It is $p = 30{,}500 - 160l/d$. It is to be noted that this is an average value, and that the actual strength of many of the columns was much lower. Prof. Burr says: "If cast-iron columns are designed with anything like a reasonable and real margin of safety, the amount of metal required dissipates any supposed economy over columns of mild steel."

Transverse Strength of Cast-iron Water-pipe. (*Technology Quarterly*, Sept. 1897.)—Tests of 31 cast-iron pipes by transverse stress gave a maximum outside fibre stress, calculated from maximum load, assuming each half of pipe as a beam fixed at the ends, ranging from 12,800 lbs. to 26,300 lbs. per sq. in.

Bars 2 in. wide cut from the pipes gave moduli of rupture ranging from 28,400 to 51,400 lbs. per sq. in. Four of the tests, bars and pipes:

Moduli of rupture of bar....	28,400	34,400	40,000	51,400
Fibre stress of pipe...	18,300	12,800	14,500	26,300

These figures show a great variation in the strength of both bars and pipes, and also that the strength of the bar does not bear any definite relation to the strength of the pipe.

Safe Load, in Tons of 2000 Lbs., for Round Cast-iron Columns, with Turned Capitals and Bases.

Loads being not eccentric, and length of column not exceeding 20 times the diameter. Based on ultimate crushing strength of 25,000 lbs. per sq. in. and a factor of safety of 5. (For eccentric loads see page 254.)

Thickness, inches.	Diameter, inches.										
	6	7	8	9	10	11	12	13	14	15	16	18
5/8	26.4	31.3										
3/4	30.9	36.8	42.7	48.6	54.5							
7/8	35.2	42.1	48.9	55.8	62.7	69.6	76.5					
1	39.2	47.1	55.0	62.8	70.7	78.5	86.4	94.2	102.1	110.0		
1 1/8	....	....	60.8	69.6	78.4	87.2	96.1	104.9	118.8	122.6	131.4	
1 1/4	....	....	....	76.1	85.9	95.7	105.5	115.8	125.2	135.0	144.8	164.4
1 3/8	....	...	....	....	93.1	108.9	114.7	125.5	136.3	147.1	157.9	179.5
1 1/2	....	...	...	....	....	......	128.7	135.5	147.8	159.0	170.8	194.4
1 3/4	....	....	....	....	...	......	......	......	168.4	182.1	195.8	223.3
2	....	....	....	....	....	.. ...	......	......	......	204.2	219.9	251.3

For lengths greater than 20 diameters the allowable loads should be decreased. How much they should be decreased is uncertain, since sufficient data of experiments on full-sized very long columns, from which a formula for the strength of such columns might be derived, are as yet lacking. There is, however, rarely, if ever, any need of proportioning cast-iron columns with a length exceeding 20 diameters.

Safe Loads in Tons of 2000 Pounds for Cast-iron Columns.

(By the Building Laws of New York City, Boston, and Chicago, 1897.)

	New York.	Boston.	Chicago.
Square columns.......	$\dfrac{8a}{1+\dfrac{l^2}{500d^2}}$	$\dfrac{5a}{1+\dfrac{l^2}{1067d^2}}$	$\dfrac{5a}{1+\dfrac{l^2}{800d^2}}$
Round columns.	$\dfrac{8a}{1+\dfrac{l^2}{400d^2}}$	$\dfrac{5a}{1+\dfrac{l^2}{800d^2}}$	$\dfrac{5a}{1+\dfrac{l^2}{600d^2}}$

a = sectional area in square inches; l = unsupported length of column in inches; d = side of square column or diameter of round column in inches.

The safe load of a 15-inch round column 1¼ inches diameter, 16 feet long, according to the laws of these cities would be, in New York, 361 tons; in Boston, 264 tons; in Chicago, 250 tons.

The allowable stress per square inch of area of such a column would be, in New York, 11,850 pounds; in Boston, 8300 pounds; in Chicago, 7850 pounds. A safe stress of 5000 pounds per square inch would give for the safe load on the column 150 tons.

Strength of Brackets on Cast-iron Columns.—The columns tested by the New York Building Department referred to above had brackets cast upon them, each bracket consisting of a rectangular shelf supported by one or two triangular ribs. These were tested after the columns had been broken in the principal tests. In 17 out of 22 cases the brackets broke by tearing a hole in the body of the column, instead of by shearing or transverse breaking of the bracket itself. The results were surprisingly low and very irregular. Reducing them to strength per square inch of the total vertical section through the shelf and rib or ribs, they ranged from 2450 to 5600 lbs., averaging 4200 lbs., for a load concentrated at the end of the shelf, and 4100 to 10,900 lbs., averaging 8000 lbs., for a distributed load. (*Eng'g News*, Jan. 20, 1898.)

Safe Loads, in Tons, for Round Cast Columns.
(In accordance with the Building Laws of Chicago.*)

Diameter in Inches.	Thickness in Inches.	Unsupported Length in Feet.												
		6	8	10	12	14	16	18	20	22	24	26	28	30
6	¾	50	48	37	32	27								
	⅞	57	50	42	36	31								
7	¾	62	56	49	43	38	33							
	⅞	71	64	57	49	43	38							
8	¾	75	69	62	56	50	44	39						
	⅞	86	79	71	64	57	50	44						
	1	97	89	81	72	63	56	50						
9	⅞	101	94	86	78	70	63	57						
	1	113	105	97	88	79	71	64						
	1⅛	126	117	107	97	88	79	71						
10	⅞	116	109	101	93	85	78	71	64					
	1	130	122	114	105	96	88	80	72					
	1⅛	145	136	126	117	107	97	88	80					
	1¼	158	149	139	128	117	107	97	88					
11	1	147	139	131	122	113	104	96	88	80				
	1⅛	163	155	146	136	126	116	106	97	89				
	1¼	179	170	160	149	138	127	117	107	98				
	1⅜	195	185	174	162	150	138	127	117	106				
12	1⅛	181	174	165	155	145	135	125	115	106	98			
	1¼	199	191	181	170	159	148	187	127	117	108			
	1⅜	217	207	197	185	173	161	149	138	127	117			
	1½	234	224	212	200	187	173	161	149	137	126			
13	1⅛	200	192	181	174	164	154	144	134	125	116	107		
	1¼	219	211	202	191	180	169	158	147	137	127	117		
	1⅜	239	230	220	208	196	184	172	160	149	138	128		
	1½	258	248	237	225	212	199	186	173	161	149	138		
14	1¼		232	223	213	202	191	180	168	157	147	137	128	
	1⅜		253	243	232	220	207	195	183	171	160	149	139	
	1½		273	263	251	238	224	211	198	185	173	161	150	
	1⅝		293	282	269	255	241	227	212	198	185	173	161	
15	1⅜			266	255	243	231	219	206	194	182	171	160	150
	1½			287	276	263	250	236	223	210	197	185	173	162
	1⅝			309	296	283	268	254	239	225	211	198	186	174
	1¾			329	316	301	286	271	255	240	225	211	198	185
16	1½				301	288	275	262	248	235	222	209	197	185
	1⅝				323	310	296	282	267	253	239	225	212	199
	1¾				345	331	316	300	285	270	254	239	225	212
18	1⅝					366	351	337	322	307	293	279	264	251
	1¾					391	375	360	344	328	313	298	282	268
	1⅞					415	399	383	366	349	333	317	300	285
20	1¾						435	420	404	389	373	357	341	326
	1⅞						463	447	431	414	397	380	363	347
	2						490	473	456	438	420	402	384	367
	2⅛						517	499	481	462	443	425	406	387
22	1¾							480	464	448	432	416	400	384
	1⅞							511	494	478	461	443	426	409
	2							541	524	506	488	470	452	434
	2⅛							581	562	543	524	504	485	465
24	2⅛								626	608	589	570	550	531
	2¼								668	639	620	600	579	559
	2⅜								691	671	650	629	608	587
	2½								724	703	681	659	637	614

Formula: $w = \dfrac{5a}{1 + \dfrac{l^2}{600d^2}}$.

w = safe load in tons of 2000 pounds;
a = cross-section of column;
l = unsupported length in inches;
d = diameter in inches.

From tables published by The Expanded Metal Co., Chicago, 1897.)

ECCENTRIC LOADING OF COLUMNS.

In a given rectangular cross-section, such as a masonry joint under pressure, the stress will be distributed uniformly over the section only when the resultant passes through the centre of the section; any deviation from such a central position will bring a maximum unit pressure to one edge and a minimum to the other: when the distance of the resultant from one edge is one third of the entire width of the joint, the pressure at the nearer edge is twice the mean pressure, while that at the farther edge is zero, and that when the resultant approaches still nearer to the edge the pressure at the farther edge becomes less than zero: in fact, becomes a tension, if the material (mortar, etc., there is capable of resisting tension. Or, if, as usual in masonry joints, the material is practically incapable of resisting tension, the pressure at the nearer edge, when the resultant approaches it nearer than one third of the width, increases very rapidly and dangerously, becoming theoretically infinite when the resultant reaches the edge.

With a given position of the resultant relatively to one edge of the joint or section, a similar redistribution of the pressures throughout the section may be brought about by simply adding to or diminishing the width of the section.

Let P = the total pressure on any section of a bar of uniform thickness.
w = the width of that section = area of the section, when thickness = 1.
$p = P/w$ = the mean unit pressure on the section.
M = the maximum unit pressure on the section.
m = the minimum unit pressure on the section.
d = the eccentricity of the resultant = its distance from the centre of the section.

Then $M = p\left(1 + \frac{6d}{w}\right)$ and $m = p\left(1 - \frac{6d}{w}\right)$.

When $d = \frac{1}{6}w$ then $M = 2p$ and $m = O$.

When d is greater than $1/6w$, the resultant in that case being less than one third of the width from one edge, p becomes negative. (J. C. Trautwine, Jr., *Engineering News*, Nov. 28, 1893.)

Eccentric Loading of Cast-iron Columns. — Prof. Lanza writes the author as follows: The table on page 252 applies when the resultant of the loads upon the column acts along its central axis, *i.e.*, passes through the centre of gravity of every section. In buildings and other constructions, however, cases frequently occur when the resultant load does not pass through the centre of gravity of the section; and then the pressure is not evenly distributed over the section, but is greatest on the side where the resultant acts. (Examples occur when the loads on the floors are not uniformly distributed.) In these cases the outside fibre stresses of the column should be computed as follows, viz.:

Let P = total pressure on the section;
d = eccentricity of resultant = its distance from the centre of gravity of the section;
A = area of the section, and I its moment of inertia about an axis in its plane, passing through its centre of gravity, and perpendicular to d (see page 267);
c_1 = distance of most compressed and c_2 = that of least compressed fibre from above stated axis;
s_1 = maximum and s_2 = minimum pressure per unit of area. Then

$$s_1 = \frac{P}{A} + \frac{(Pd)c_1}{I} \quad \text{and} \quad s_2 = \frac{P}{A} - \frac{(Pd)c_2}{I}.$$

Having assumed a certain *trial* section for the column to be designed, s_1 should be computed, and, if it exceed the proper safe value, a different section should be used for which s_1 does not exceed this value.

The proper safe value, in the case of cast-iron columns whose ratio of length to diameter does not greatly exceed 20, is 5000 pounds per square inch when the eccentricity used in the computation of s_1 is liable to occur frequently in the ordinary uses of the structure; but when it is one which can only occur in rare cases the value 8000 pounds per square inch may be used.

A long cap on a column is more conducive to the production of eccentricity of loading than a short one, hence a long cap is a source of weakness in a column.

ULTIMATE STRENGTH OF WROUGHT-IRON COLUMNS.

(Pottsville Iron and Steel Co.)

Computed by Gordon's formula, $p = \dfrac{f}{1 + C\left(\dfrac{l}{r}\right)^2}$.

p = ultimate strength in lbs. per square inch;
l = length of column in inches;
r = least radius of gyration in inches;
f = 40,000;
C = 1/40,000 for square end-bearings; 1/30,000 for one pin and one square bearing; 1/20,000 for two pin-bearings.

For safe working load on these columns use a factor of 4 when used in buildings, or when subjected to dead load only; but when used in bridges the factor should be 5.

WROUGHT-IRON COLUMNS.

$\frac{l}{r}$	Ultimate Strength in lbs. per square inch.			$\frac{l}{r}$	Safe Strength in lbs. per square inch—Factor of 5.		
	Square Ends.	Pin and Square End.	Pin Ends.		Square Ends.	Pin and Square End.	Pin Ends.
10	39944	39866	39600	10	7989	7973	7960
15	39776	39702	39554	15	7955	7940	7911
20	39604	39472	39214	20	7921	7894	7843
25	39384	39182	38788	25	7877	7836	7758
30	39118	38834	38278	30	7821	7767	7656
35	38810	38430	37690	35	7762	7686	7538
40	38460	37974	37036	40	7692	7595	7407
45	38072	37470	36322	45	7614	7494	7264
50	37646	36928	35525	50	7529	7386	7105
55	37186	36336	34744	55	7437	7267	6949
60	36697	35714	33898	60	7339	7143	6780
65	36182	34478	33024	65	7236	6896	6605
70	35634	34384	32128	70	7127	6877	6426
75	35076	33682	31218	75	7015	6736	6244
80	34482	32966	30288	80	6896	6593	6058
85	33883	32236	29384	85	6777	6447	5877
90	33264	31496	28470	90	6653	6299	5694
95	32636	30750	27562	95	6527	6150	5512
100	32000	30000	26666	100	6400	6000	5333
105	31357	29250	25786	105	6271	5850	5157

Maximum Permissible Stresses in columns used in buildings. (Building Ordinances of City of Chicago, 1893.)

For riveted or other forms of wrought-iron columns:

$$S = \frac{12000a}{1 + \dfrac{l^2}{36000r^2}}.$$

l = length of column in inches;
r = least radius of gyration in inches;
a = area of column in square inches.

For riveted or other steel columns, if more than $60r$ in length:

$$S = 17{,}000 - \frac{60l}{r}.$$

If less than $60r$ in length: $S = 13{,}500a$.

For wooden posts:

$$S = \frac{ac}{1 + \dfrac{l^2}{250d^2}}.$$

a = area of post in square inches;
d = least side of rectangular post in inches;
l = length of post in inches;
$c = \begin{cases} 600 \text{ for white or Norway pine;} \\ 800 \text{ for oak;} \\ 900 \text{ for long-leaf yellow pine.} \end{cases}$

BUILT COLUMNS.

From experiments by T. D. Lovett, discussed by Burr, the values of f and a in several cases are determined, giving empirical forms of Gordon's formula as follows: p = pounds crushing strength per square inch of section, l = length of column in inches, r = radius of gyration in inches.

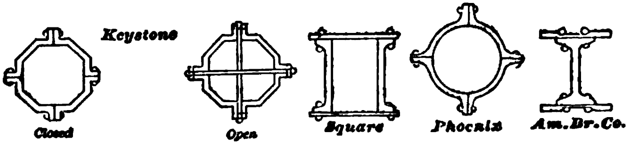


FIG. 76.

	Keystone Columns.	Square Columns.	Phœnix Columns.	American Bridge Co. Columns.
Flat Ends.				
$p =$	$\dfrac{39,500}{1+\dfrac{1}{18,300}\dfrac{l^2}{r^2}}$ (1)	$\dfrac{39,000}{1+\dfrac{1}{35,000}\dfrac{l^2}{r^2}}$ (4)	$\dfrac{42,000}{1+\dfrac{1}{50,000}\dfrac{l^2}{r^2}}$ (6)	$\dfrac{36,000}{1+\dfrac{1}{46,000}\dfrac{l^2}{r^2}}$ (9)
Flat Ends, Swelled.				
$p =$	$\dfrac{36,000}{1+\dfrac{1}{18,300}\dfrac{l^2}{r^2}}$ (2)	....	....	....
Pin Ends.				
$p =$	....	$\dfrac{39,000}{1+\dfrac{1}{17,000}\dfrac{l^2}{r^2}}$ (5)	$\dfrac{42,000}{1+\dfrac{1}{22,700}\dfrac{l^2}{r^2}}$ (7)	$\dfrac{36,000}{1+\dfrac{1}{21,500}\dfrac{l^2}{r^2}}$ (10)
Pin Ends, Swelled.				
$p =$	$\dfrac{36,000}{1+\dfrac{1}{15,000}\dfrac{l^2}{r^2}}$ (3)	....	....	....
Round Ends.				
$p =$	....	....	$\dfrac{42,000}{1+\dfrac{1}{12,500}\dfrac{l^2}{r^2}}$ (8)	$\dfrac{36,000}{1+\dfrac{1}{11,500}\dfrac{l^2}{r^2}}$ (11)

With great variations of stress a factor of safety of as high as 6 or 8 may be used, or it may be as low as 3 or 4, if the condition of stress is uniform or essentially so.

Burr gives the following general principles which govern the resistance of built columns:

The material should be disposed as far as possible from the neutral axis of the cross-section, thereby increasing r;

There should be no initial internal stress;

The individual portions of the column should be mutually supporting;

The individual portions of the column should be so firmly secured to each other that no relative motion can take place, in order that the column may fail as a whole, thus maintaining the original value of r.

Stoney says: "When the length of a rectangular wrought-iron tubular column does not exceed 30 times its least breadth, it fails by the bulging or buckling of a short portion of the plates, not by the flexure of the pillar as a whole."

In Trans. A. S. C. E., Oct. 1880, are given the following formulæ for the ultimate resistance of wrought-iron columns designed by C. Shaler Smith:

Flat Ends.

Square Column.	Phœnix Column.	American Bridge Co. Column.	Common Column.
$p = \dfrac{38,500}{1+\frac{1}{5820}\frac{l^2}{d^2}}$ (12)	$\dfrac{42,500}{1+\frac{1}{4500}\frac{l^2}{d^2}}$ (15)	$\dfrac{36,500}{1+\frac{1}{8750}\frac{l^2}{d^2}}$ (18)	$\dfrac{36,500}{1+\frac{1}{2700}\frac{l^2}{d^2}}$ (21)

One Pin End.

$p = \dfrac{38,500}{1+\frac{1}{3000}\frac{l^2}{d^2}}$ (13)	$\dfrac{40,000}{1+\frac{1}{2250}\frac{l^2}{d^2}}$ (16)	$\dfrac{36,500}{1+\frac{1}{2250}\frac{l^2}{d^2}}$ (19)	$\dfrac{36,500}{1+\frac{1}{1500}\frac{l^2}{d^2}}$ (22)

Two Pin Ends.

$p = \dfrac{37,500}{1+\frac{1}{1900}\frac{l^2}{d^2}}$ (14)	$\dfrac{36,600}{1+\frac{1}{1500}\frac{l^2}{d^2}}$ (17)	$\dfrac{36,500}{1+\frac{1}{1750}\frac{l^2}{d^2}}$ (20)	$\dfrac{36,500}{1+\frac{1}{1200}\frac{l^2}{d^2}}$ (23)

The "common" column consists of two channels, opposite, with flanges outward, with a plate on one side and a lattice on the other.

The formula for "square" columns may be used without much error for the common-chord section composed of two channel-bars and plates, with the axis of the pin passing through the centre of gravity of the cross-section. (Burr).

Compression members composed of two channels connected by zigzag bracing may be treated by formulæ 4 and 5, using $f = 36,000$ instead of 39,000.

Experiments on full-sized Phœnix columns in 1873 showed a close agreement of the results with formulæ 6–8. Experiments on full-sized Phœnix columns on the Watertown testing-machine in 1881 showed considerable discrepancies when the value of $l \div r$ became comparatively small. The following modified form of Gordon's formula gave tolerable results through the whole range of experiments:

$$\text{Phœnix columns, flat end,}\quad p = \frac{40,000\left(1+\frac{2r}{l}\right)}{1+\frac{1}{50,000}\frac{l^2}{r^2}}. \quad \ldots \ldots \ldots \quad (24)$$

Plotting results of three series of experiments on Phœnix columns, a more simple formula than Gordon's is reached as follows:

Phœnix columns, flat ends, $p = 39,640 - 46\frac{l}{r}$, when $l \div r$ is from 30 to 140;

$$p = 64,700 - 4600\sqrt{\frac{l}{r}} \text{ when } l \div r \text{ is less than 30.}$$

Dimensions of Phœnix Columns.

(Phœnix Iron Co.)

The dimensions are subject to slight variations, which are unavoidable in rolling iron shapes.

The weights of columns given are those of the 4, 6, or 8 segments of which they are composed. The *rivet heads* add from 2% *to* 5% to the weights given. Rivets are spaced 3, 4, or 6 in. apart from centre to centre, and somewhat more closely at the ends than towards the centre of the column.

G columns have 8 segments, *E* columns 6 segments, *C*, B^2, B^1, and *A* have 4 segments. *Least radius of gyration* $= D \times .3636$.

The safe loads given are computed as being one-fourth of the breaking load, and as producing a maximum stress, in an axial direction, on a squared-end column of not more than 14,000 lbs. per sq. in. for lengths of 90 radii and under.

Dimensions of Phœnix Steel Columns.

(Least radius of gyration equals $D \times .3636$.)

One Segment.		Diameters in Inches.			One Column.			
Thickness in Inches.	Weight in Lbs. per Yard.	d Inside.	D Outside.	D^1 over Flanges.	Area of Cross Section, Sq. Inches.	Weight per Ft. in Pounds.	Least Radius of Gyration in Inches.	Safe Load in Net Tons for 10-feet Lengths.
3/16	9.7	A	4	6 1/16	3.8	12.9	1.45	18.2
¼	12.2	3⅝	4⅛	6 3/16	4.8	16.3	1.50	23.9
5/16	14.8		4¼	6 5/16	5.8	19.7	1.55	30.0
⅜	17.3		4⅜	6 7/16	6.8	23.1	1.59	35.9
¼	16.3	B.1	5⅜	8⅛	6.4	21.8	1.95	36.4
5/16	19.9	4⅞	5½	8 3/16	7.8	26.5	2.00	45.1
⅜	23.5		5⅝	8 5/16	9.2	31.3	2.04	54.4
7/16	27.0		5¾	8 7/16	10.6	36.0	2.09	63.9
½	30.6		5⅞	8½	12.0	40.8	2.13	73.3
9/16	34.2		6	8 9/16	13.4	45.6	2.18	83.2
⅝	37.7		6⅛	8 11/16	14.8	50.3	2.23	93.1
¼	18.9	B.2	6 9/16	9¼	7.4	25.2	2.39	48.3
5/16	22.9	6 1/16	6 11/16	9⅜	9.0	30.6	2.43	59.5
⅜	27.0		6 13/16	9 7/16	10.6	36.0	2.48	70.7
7/16	31.1		6 15/16	9½	12.2	41.5	2.52	82.3
½	35.2		7 1/16	9⅝	13.8	46.9	2.57	93.9
9/16	39.3		7 3/16	9¾	15.4	52.4	2.61	105.8
⅝	43.3		7 5/16	9 13/16	17.0	57.8	2.66	111.9
¼	25½	C	7 13/16	11 11/16	10.0	34.0	2.84	70.0
5/16	31	7⅜	7 15/16	11¾	12.1	41.3	2.88	85.1
⅜	36		8 1/16	11 13/16	14.1	48.0	2.93	98.8
7/16	41		8 3/16	11⅞	16.0	54.6	2.97	112.5
½	46		8 5/16	11 15/16	18.0	61.3	3.01	126.3
9/16	51		8 7/16	12	19.9	68.0	3.06	140.0
⅝	56		8 9/16	12 1/16	21.9	74.6	3.11	153.7
11/16	62		8 11/16	12 3/16	24.3	82.6	3.16	170.2
¾	68		8 13/16	12 5/16	26.6	90.6	3.20	186.7
13/16	73		8 15/16	12 7/16	28.6	97.3	3.24	200.3
⅞	78		9 1/16	12½	30.6	104.0	3.29	214.2
1	89		9 5/16	12⅝	34.8	118.6	3.34	241.3
1⅛	99		9 9/16	12 13/16	38.8	132.0	3.48	271.7
1¼	109		9 13/16	13	42.7	145.3	3.57	299.2
¼	28	E	11 9/16	15½	16.5	56.0	4.20	115.3
5/16	32½	11 1/16	11 11/16	15⅝	19.1	65.0	4.25	133.8
⅜	37		11 13/16	15¾	21.7	74.0	4.29	152.4
7/16	42		11 15/16	15⅞	24.7	84.0	4.34	173.0
½	47		12 1/16	15 15/16	27.6	94.0	4.38	193.6
9/16	52		12 3/16	16 1/16	30.6	104.0	4.43	214.1
⅝	57		12 5/16	16 3/16	33.5	114.0	4.48	234.7
11/16	62		12 7/16	16 5/16	36.4	124.0	4.52	255.3
¾	68		12 9/16	16 7/16	40.0	136.0	4.56	280.0
13/16	73		12 11/16	16 9/16	43.0	146.0	4.61	300.6
⅞	78		12 13/16	16 11/16	45.9	156.0	4.66	321.2
1	88		13 1/16	16 13/16	51.7	176.0	4.73	362.4
1⅛	98		13 5/16	17 1/16	57.6	196.0	4.84	403.6
1¼	108		13 9/16	17 5/16	63.5	216.0	4.93	444.7
5/16	31	G	15¼	19⅜	24.2	82.6	5.54	170.2
⅜	36	14⅝	15⅜	19½	28.1	96.0	5.59	197.7
7/16	41		15½	19⅝	32.0	109.3	5.64	225.1
½	46		15⅝	19 11/16	36.0	122.6	5.68	252.6

One Segment.		Diameters in Inches.			One Column.			Safe Load in Net Tons for 16-feet Lengths.
Thickness in Inches.	Weight in Lbs. per Yard.	*d* Inside.	*D* Outside.	*D'* over Flanges.	Area of Cross Section, Sq. Inches.	Weight per Ft. in Pounds.	Least Radius of Gyration in Inches.	
9/16	51		15¾	19¾	39.9	136.0	5.73	280.0
⅝	56		15⅞	19⅞	43.8	149.3	5.77	307.4
11/16	61		16	20	47.7	162.6	5.82	334.9
¾	66		16⅛	20⅛	51.7	176.0	5.88	362.4
13/16	71	G	16¼	20¼	55.6	189.3	5.91	389.8
⅞	76	14⅝	16⅜	20⅜	59.6	202.6	5.95	417.3
1	86		16⅝	20⅝	67.4	229.3	6.04	472.1
1⅛	96		16⅞	20⅞	75.3	256.0	6.13	527.3
1¼	106		17⅛	21	83.1	282.6	6.27	582.0
1⅜	116		17⅜	21¼	90.9	309.3	6.32	636.9

Working Formulæ for Wrought-iron and Steel Struts of various Forms.—Burr gives the following practical formulæ, which he believes to possess advantages over Gordon's:

Kind of Strut.	p = Ultimate Strength, lbs. per sq. in. of Section.		p_1 = Working Strength = 1/5 Ultimate, lbs. per sq. in. of Section.	
Flat and fixed end iron angles and tees	$44000-140\frac{l}{r}$	(1)	$8800-28\frac{l}{r}$	(2)
Hinged-end iron angles and tees.......	$46000-175\frac{l}{r}$	(3)	$9200-35\frac{l}{r}$	(4)
Flat-end iron channels and I beams....	$40000-110\frac{l}{r}$	(5)	$8000-22\frac{l}{r}$	(6)
Flat-end mild-steel angles.............	$52000-180\frac{l}{r}$	(7)	$10400-36\frac{l}{r}$	(8)
Flat-end high-steel angles.............	$76000-290\frac{l}{r}$	(9)	$15200-58\frac{l}{r}$	(10)
Pin-end solid wrought iron columns....	$32000-80\frac{l}{r}$	(11)	$6400-16\frac{l}{r}$	(12)
	$32000-277\frac{l}{d}$		$6400-55\frac{l}{d}$	

Equations (1) to (4) are to be used only between $\frac{l}{r}=40$ and $\frac{l}{r}=200$

" (5) and (6) " " " " " " " $=20$ " " $=200$

" (7) to (10) " " " " " " " $=40$ " " $=200$

" (11) and (12) " " " " " " " $=20$ " " $=200$

or $\frac{l}{d}=6$ and $\frac{l}{d}=65$

Steel columns, properly made, of steel ranging in specimens from 65,000 to 73,000 lbs. per square inch should give a resistance 25 to 33 per cent in excess of that of wrought-iron columns with the same value of $l \div r$, provided that ratio does not exceed 140.

The unsupported width of a plate in a compression member should not exceed 30 times its thickness.

In built columns the transverse distance between centre lines of rivets securing plates to angles or channels, etc., should not exceed 35 times the plate thickness. If this width is exceeded, longitudinal buckling of the plate takes place, and the column ceases to fail as a whole, but yields in detail.

The same tests show that the thickness of the leg of an angle to which latticing is riveted should not be less than 1/9 of the length of that leg or side if the column is purely and wholly a compression member. The above limit may be passed somewhat in stiff ties and compression members designed to carry transverse loads.

The panel points of latticing should not be separated by a greater distance than 60 times the thickness of the angle-leg to which the latticing is riveted, if the column is wholly a compression member.

The rivet pitch should never exceed 16 times the thickness of the thinnest metal pierced by the rivet, and if the plates are very thick it should never nearly equal that value.

Merriman's Rational Formula for Columns (*Eng. News*, July 19, 1894).

$$C = \frac{B}{1 - \frac{nB}{\pi^2 E}\frac{l^2}{r^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$B = \frac{C}{1 + \frac{nC}{\pi^2 E}\frac{l^2}{r^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

B = unit-load on the column = total load $P \div$ area of cross-section A; C = maximum compressive unit-stress on the concave side of the column; l = length of the column; r = least radius of gyration of the cross-section E = coefficient of elasticity of the material; n = 1 for both ends round. n = 4/9 for one end round and one fixed; n = ¼ for both ends fixed. This formula is for use with strains within the elastic limit only: it does not hold good when the strain C exceeds the elastic limit.

Prof. Merriman takes the mean value of E for timber = 1,500,000, for cast iron = 15,000,000, for wrought-iron = 25,000,000, and for steel = 30,000,000, and π^2 = 10 as a close enough approximation. With these values he computes the following tables from formula (1):

I.—Wrought-iron Columns with Round Ends.

Unit-load,	Maximum Compressive Unit-stress C.							
$\frac{P}{A}$ or B.	$\frac{l}{r}=20$	$\frac{l}{r}=40$	$\frac{l}{r}=60$	$\frac{l}{r}=80$	$\frac{l}{r}=100$	$\frac{l}{r}=120$	$\frac{l}{r}=140$	$\frac{l}{r}=160$
5,000	5,040	5,170	5,390	5,730	6,250	6,980	8,220	10,250
6,000	6,055	6,240	6,560	7,090	7,890	9,090	11,330	15,560
7,000	7,080	7,330	7,780	8,530	9,720	11,610	15,510	24,720
8,000	8,100	8,430	9,040	10,060	11,660	14,640	21,460	....
9,000	9,130	9,550	10,840	11,690	14,060	18,380	....	....
10,000	10,160	10,680	11,680	13,440	16,670	23,090	....	....
11,000	11,200	11,750	13,070	15,310	19,640	....	....	....
12,000	12,240	13,000	14,500	17,320	23,080	....	....	....
13,000	13,280	14,180	15,990	19,480	....	....	....	....

II.—Wrought-iron Columns with Fixed Ends.

Unit-load.	Maximum Compressive Unit-stress C.							
$\frac{P}{A}$ or B.	$\frac{l}{r} = 20$	$\frac{l}{r} = 40$	$\frac{l}{r} = 60$	$\frac{l}{r} = 80$	$\frac{l}{r} = 100$	$\frac{l}{r} = 120$	$\frac{l}{r} = 140$	$\frac{l}{r} = 160$
6,000	6,010	6,060	6,130	6,240	6,380	6,570	6,800	7,090
7,000	7,020	7,080	7,180	7,330	7,590	7,780	8,110	8,530
8,000	8,025	8,100	8,240	8,430	8,700	9,040	9,490	10,060
9,000	9,030	9,130	9,300	9,550	9,890	10,340	10,990	11,690
10,000	10,040	10,160	10,370	10,710	11,110	11,680	12,440	13,440
11,000	11,050	11,200	11,450	11,830	12,360	13,070	14,020	15,310
12,000	12,060	12,240	12,540	13,000	13,640	14,510	15,690	17,320
13,000	13,070	13,280	13,640	14,210	14,940	15,990	17,440	19,480
14,000	14,080	14,320	14,740	15,380	16,280	17,530	19,290	21,820

III.—Steel Columns with Round Ends.

Unit-load.	Maximum Compressive Unit-stress C.							
$\frac{P}{A}$ or B.	$\frac{l}{r} = 20$	$\frac{l}{r} = 40$	$\frac{l}{r} = 60$	$\frac{l}{r} = 80$	$\frac{l}{r} = 100$	$\frac{l}{r} = 120$	$\frac{l}{r} = 140$	$\frac{l}{r} = 160$
6,000	6,050	6,200	6,470	6,880	7,500	8,430	9,870	12,800
7,000	7,070	7,270	7,650	8,230	9,130	10,540	12,900	17,400
8,000	8,090	8,380	8,770	9,650	10,870	12,990	16,760	24,590
9,000	9,110	9,450	10,090	11,140	12,850	15,850	20,930	....
10,000	10,130	10,560	11,360	12,710	15,000	19,230	28,850	....
11,000	11,160	11,690	12,670	14,870	17,370	23,300	....	....
12,000	12,200	12,820	14,090	16,180	20,000	28,800	....	....
13,000	13,330	13,970	15,400	18,000	22,940	....	....	....
14,000	14,250	15,130	16,830	19,960	26,250	....	....	....

IV.—Steel Columns with Fixed Ends.

Unit-load.	Maximum Compressive Unit-stress C.							
$\frac{P}{A}$ or B.	$\frac{l}{r} = 20$	$\frac{l}{r} = 40$	$\frac{l}{r} = 60$	$\frac{l}{r} = 80$	$\frac{l}{r} = 100$	$\frac{l}{r} = 120$	$\frac{l}{r} = 140$	$\frac{l}{r} = 160$
7,000	7,020	7,070	7,150	7,270	7,430	7,650	7,900	8,280
8,000	8,020	8,090	8,200	8,380	8,570	8,770	9,200	9,650
9,000	9,030	9,110	9,250	9,450	9,730	10,090	10,550	11,140
10,000	10,030	10,130	10,310	10,560	10,910	11,360	11,810	12,710
11,000	11,040	11,160	11,330	11,690	12,110	12,670	13,410	14,370
12,000	12,050	12,200	12,450	12,820	13,330	14,090	14,930	16,130
13,000	13,060	13,230	13,530	13,970	14,580	15,400	16,500	17,990
14,000	14,070	14,250	14,610	15,130	15,850	16,830	18,150	19,960
15,000	15,080	15,310	15,710	16,310	17,140	18,290	19,870	22,060

The design of the cross-section of a column to carry a given load with maximum unit-stress C may be made by assuming dimensions, and then computing C by formula (1). If the agreement between the specified and computed values is not sufficiently close, new dimensions must be chosen, and the computation be repeated. By the use of the above tables the work will be shortened.

The formula (1) may be put in another form which in some cases will abbreviate the numerical work. For B substitute its value $P \div A$, and for Ar^2 write I, the least moment of inertia of the cross-section; then

$$I - \frac{P}{C}r^2 = \frac{nPl^2}{\pi^2 E}, \quad \ldots \ldots \ldots \quad (3)$$

in which I and r^2 are to be determined.

For example, let it be required to find the size of a square oak column with fixed ends when loaded with 24,000 lbs. and 16 ft. long, so that the maximum compressive stress C shall be 1000 lbs. per square inch. Here $I = 24{,}000$, $C = 1000$, $n = \frac{1}{4}$, $\pi^2 = 10$, $E = 1{,}500{,}000$, $l = 16 \times 12$, and (3) becomes

$$I - 24r^2 = 14.75.$$

Now let x be the side of the square; then

$$I = \frac{x^4}{12} \quad \text{and} \quad r^2 = \frac{x^2}{12},$$

so that the equation reduces to $x^4 - 24x^2 = 177$, from which x^2 is found to be 29.92 sq. in., and the side $x = 5.47$ in. Thus the unit-load B is about 802 lbs. per square inch.

WORKING STRAINS ALLOWED IN BRIDGE MEMBERS.

Theodore Cooper gives the following in his Bridge Specifications:

Compression members shall be so proportioned that the maximum load shall in no case cause a greater strain than that determined by the following formula:

$$P = \frac{8000}{1 + \frac{l^2}{40{,}000r^2}} \text{ for square-end compression members;}$$

$$P = \frac{8000}{1 + \frac{l^2}{30{,}000r^2}} \text{ for compression members with one pin and one square end;}$$

$$P = \frac{8000}{1 + \frac{l^2}{20{,}000r^2}} \text{ for compression members with pin-bearings;}$$

(These values may be increased in bridges over 150 ft. span. See Cooper's Specifications.)

P = the allowed compression per square inch of cross-section;

l = the length of compression member, in inches;

r = the least radius of gyration of the section in inches.

No compression member, however, shall have a length exceeding 45 times its least width.

Tension Members.—All parts of the structure shall be so proportioned that the maximum loads shall in no case cause a greater tension than the following (except in spans exceeding 150 feet):

	Pounds per sq. in.
On lateral bracing	15,000
On solid rolled beams, used as cross floor-beams and stringers.	9,000
On bottom chords and main diagonals (forged eye-bars)	10,000
On bottom chords and main diagonals (plates or shapes), net section	8,000
On counter rods and long verticals (forged eye-bars)	8,000
On counter and long verticals (plates or shapes), net section..	6,500
On bottom flange of riveted cross-girders, net section	8,000
On bottom flange of riveted longitudinal plate girders over 20 ft. long, net section	8,000

On bottom flange of riveted longitudinal plate girders under 20 ft. long, net section	7,000
On floor-beam hangers, and other similar members liable to sudden loading (bar iron with forged ends)	6,000
On floor-beam hangers, and other similar members liable to sudden loading (plates or shapes), net section	5,000

Members subject to alternate strains of tension and compression shall be proportioned to resist each kind of strain. Both of the strains shall, however, be considered as increased by an amount equal to 8/10 of the least of the two strains, for determining the sectional area by the above allowed strains.

The Phœnix Bridge Co. (Standard Specifications, 1895) gives the following:

The greatest working stresses in pounds per square inch shall be as follows:

Tension.

Steel. Iron.

$$P = 9{,}000\left[1 + \frac{\text{Min. stress}}{\text{Max. stress}}\right] \quad \text{For bars, forged ends.} \quad P = 7{,}500\left[1 + \frac{\text{Min. stress}}{\text{Max. stress}}\right]$$

$$P = 8{,}500\left[1 + \frac{\text{Min. stress}}{\text{Max. stress}}\right] \quad \text{Plates or shapes net.} \quad P = 7{,}000\left[1 + \frac{\text{Min. stress}}{\text{Max. stress}}\right]$$

Steel		Iron
8,500 pounds.	Floor-beam hangers, forged ends	7,000 pounds.
7,500 "	Floor-beam hangers, plates or shapes, net section	6,000 "
10,000 "	Lower flanges of rolled beams	8,000 "
20,000 "	Outside fibres of pins	15,000 "
30,000 "	Pins for wind-bracing	22,500 "
20,000 "	Lateral bracing	15,000 "

Shearing.

Steel		Iron
9,000 pounds.	Pins and rivets	7,500 pounds.
	Hand-driven rivets 20% less unit stresses. For bracing increase unit stresses 50%.	
6,000 pounds.	Webs of plate girders	5,000 pounds.

Bearing.

Steel		Iron
16,000 pounds.	Projection semi-intrados pins and rivets	12,000 pounds.
	Hand-driven rivets 20% less unit stresses. For bracing increase unit stresses 50%.	

Compression.

Lengths less than forty times the least radius of gyration, P previously found. See Tension.

Lengths more than forty times the least radius of gyration, P reduced by following formulæ:

For both ends fixed, $$b = \frac{P}{1 + \frac{l^2}{36{,}000\, r^2}}.$$

For one end hinged, $$b = \frac{P}{1 + \frac{l^2}{24{,}000\, r^2}}.$$

For both ends hinged, $$b = \frac{P}{1 + \frac{l^2}{18{,}000\, r^2}}.$$

P = permissible stress previously found (see Tension); b = allowable working stress per square inch; l = length of member in inches; r = least radius of gyration of section in inches. No compression member, however, shall have a length exceeding 45 times its least width.

	Pounds per sq. in.
In counter web members	10,500
In long verticals	10,000
In all main-web and lower-chord eye-bars	13,200
In plate hangers (net section)	9,000
In tension members of lateral and transverse bracing	19,000
In steel-angle lateral ties (net section)	15,000

For spans over 200 feet in length the greatest allowed working stresses per square inch, in lower-chord and end main-web eye-bars, shall be taken at

$$10{,}000\left(1+\frac{\text{min. total stress}}{\text{max. total stress}}\right)$$

whenever this quantity exceeds 13,200.

The greatest allowable stress in the main-web eye-bars nearest the centre of such spans shall be taken at 13,200 pounds per square inch; and those for the intermediate eye-bars shall be found by direct interpolation between the preceding values.

The greatest allowable working stresses in steel plate and lattice girders and rolled beams shall be taken as follows:

	Pounds per sq. in.
Upper flange of plate girders (gross section)	10,000
Lower flange of plate girders (net section)	10,000
In counters and long verticals of lattice girders (net section)	9,000
In lower chords and main diagonals of lattice girders (net section)	10,000
In bottom flanges of rolled beams	10,000
In top flanges of rolled beams	10,000

RESISTANCE OF HOLLOW CYLINDERS TO COLLAPSE.

Fairbairn's empirical formula (*Phil. Trans.* 1858) is

$$p = 9{,}675{,}600\,\frac{t^{2.19}}{ld}, \quad \ldots \ldots \quad (1)$$

where p = pressure in lbs. per square inch, t = thickness of cylinder, d = diameter, and l = length, all in inches; or,

$$p = 806{,}600\,\frac{t^{2.19}}{Ld}, \text{ if } L \text{ is in feet.} \quad \ldots \quad (2)$$

He recommends the simpler formula

$$p = 9{,}675{,}600\,\frac{t^2}{ld}. \quad \ldots \ldots \quad (3)$$

as sufficiently accurate for practical purposes, for tubes of considerable diameter and length.

The diameters of Fairbairn's experimental tubes were 4″, 6″, 8″, 10″, and 12″, and their lengths, between the cast-iron ends, ranged between 19 inches and 60 inches.

His formula (3) has been generally accepted as the basis of rules for ascertaining the strength of boiler-flues. In some cases, however, limits are fixed to its application by a supplementary formula.

Lloyd's Register contains the following formula for the strength of circular boiler-flues, viz.,

$$P = \frac{89{,}600t^2}{Ld}. \quad \ldots \ldots \quad (4)$$

The English Board of Trade prescribes the following formula for circular flues, when the longitudinal joints are welded, or made with riveted butt-straps, viz.,

$$P = \frac{90{,}000t^2}{(L+1)d}. \quad \ldots \ldots \quad (5)$$

For lap-joints and for inferior workmanship the numerical factor may be reduced as low as 60,000.

The rules of Lloyd's Register, as well as those of the Board of Trade, prescribe further, that in no case the value of P must exceed the amount given by the following equation, viz.,

$$P = \frac{8000t}{d}. \quad \ldots \ldots \quad (6)$$

In formulæ (4), (5), (6) P is the highest working pressure in pounds per square inch, t and d are the thickness and diameter in inches, L is the length of the flue in feet measured between the strengthening rings, in case it is fitted with such. Formula (4) is the same as formula (3), with a factor of safety of 9. In formula (5) the length L is increased by 1; the influence which this addition has on the value of P is, of course, greater for short tubes than for long ones.

Nystrom has deduced from Fairbairn's experiments the following formula for the collapsing strength of flues:

$$p = \frac{4Tt^2}{d\sqrt{L}}, \quad \ldots \ldots \quad (7)$$

where p, t, and d have the same meaning as in formula (1), L is the length in feet, and T is the tensile strength of the metal in pounds per square inch.

If we assign to T the value 50,000, and express the length of the flue in inches, equation (7) assumes the following form, viz.,

$$p = 692,800 \frac{t^2}{d\sqrt{l}}. \quad \ldots \ldots \quad (8)$$

Nystrom considers a factor of safety of 4 sufficient in applying his formula. (See "A New Treatise on Steam Engineering," by J. W. Nystrom, p. 106.)

Formula (1), (4), and (8) have the common defect that they make the collapsing pressure decrease indefinitely with increase of length, and *vice versa*. M. Love has deduced from Fairbairn's experiments an equation of a different form, which, reduced to English measures, is as follows, viz.,

$$p = 5,358,150 \frac{t^2}{ld} + 41,906 \frac{t^2}{d} + 1323 \frac{t}{d}, \quad \ldots \ldots \quad (9)$$

where the notation is the same as in formula (1).

D. K. Clark, in his "Manual of Rules," etc., p. 696, gives the dimensions of six flues, selected from the reports of the Manchester Steam-Users Association, 1862-69, which collapsed while in actual use in boilers. These flues varied from 24 to 60 inches in diameter, and from 3-16 to 5/8 inch in thickness. They consisted of rings of plates riveted together, with one or two longitudinal seams, but all of them unfortified by intermediate flanges or strengthening rings. At the collapsing pressures the flues experienced compressions ranging from 1.53 to 2.17 tons, or a mean compression of 1.82 tons per square inch of section. From these data Clark deduced the following formula "for the average resisting force of common boiler-flues," viz.,

$$p = t^2\left(\frac{50,000}{d} - 500\right), \quad \ldots \ldots \quad (10)$$

where p is the collapsing pressure in pounds per square inch, and d and t are the diameter and thickness expressed in inches.

C. R. Roelker, in *Van Nostrand's Magazine*, March, 1881, discussing the above and other formulæ, shows that experimental data are as yet insufficient to determine the value of any of the formulæ. He says that Nystrom's formula, (8), gives a closer agreement of the calculated with the actual collapsing pressures in experiments on flues of every description than any of the other formulæ.

Collapsing Pressure of Plain Iron Tubes or Flues.

(Clark, S. E., vol. i. p. 643.)

The resistance to collapse of plain-riveted flues is directly as the square of the thickness of the plate, and inversely as the square of the diameter. The support of the two ends of the flue does not practically extend over a length of tube greater than twice or three times the diameter. The collapsing pressure of long tubes is therefore practically independent of the length.

Instances of collapsed flues of Cornish and Lancashire boilers collated by Clark, showed that the resistance to collapse of flues of 3/8-inch plates, 18 to 43 feet long, and 30 to 50 inches diameter, varied as the 1.75 power of the diameter. Thus,

for diameters of	30	35	40	45	50	inches,
the collapsing pressures were	76	58	45	37	30	lbs. per sq. in;
for 7-16-inch plates the collapsing pressures were	..	..	60	49	42	" " "

For collapsing pressures of plain iron flue-tubes of Cornish and Lancashire steam-boilers, Clark gives:

$$P = \frac{200{,}000t^2}{d^{1.75}}.$$

P = collapsing pressure, in pounds per square inch;
t = thickness of the plates of the furnace tube, in inches.
d = internal diameter of the furnace tube, in inches.

For short lengths the longitudinal tensile resistance may be effective in augmenting the resistance to collapse. Flues efficiently fortified by flange-joints or hoops at intervals of 3 feet may be enabled to resist from 50 lbs. to 60 lbs. or 70 lbs. pressure per square inch more than plain tubes, according to the thickness of the plates.

Strength of Small Tubes.—The collapsing resistance of solid-drawn tubes of small diameter, and from .134 inch to .109 inch in thickness, has been tested experimentally by Messrs. J. Russell & Sons. The results for wrought-iron tubes varied from 14.33 to 20.07 tons per square-inch section of the metal, averaging 18.20 tons, as against 17.57 to 24.28 tons, averaging 22.40 tons, for the bursting pressure.

(For strength of Segmental Crowns of Furnaces and Cylinders see Clark, S. E., vol. i, pp. 649–651 and pp. 627, 628.)

Formula for Corrugated Furnaces (*Eng'g*, July 24, 1891, p. 102).—As the result of a series of experiments on the resistance to collapse of Fox's corrugated furnaces, the Board of Trade and Lloyd's Registry altered their formulæ for these furnaces in 1891 as follows:

Board of Trade formula is altered from

$$\frac{12{,}500 \times T}{D} = WP \text{ to } \frac{14{,}000 \times T}{D} = WP.$$

T = thickness in inches;
D = mean diameter of furnace;
WP = working pressure in pounds per square inch.

Lloyd's formula is altered from

$$\frac{1000 \times (T^2)}{D} = WP \text{ to } \frac{1234 \times (T^2)}{D} = WP.$$

T = thickness in sixteenths of an inch;
D = greatest diameter of furnace;
WP = working pressure in pounds per square inch.

TRANSVERSE STRENGTH.

In transverse tests the strength of bars of rectangular section is found to vary directly as the breadth of the specimen tested, as the square of its depth, and inversely as its length. The deflection under any load varies as the cube of the length, and inversely as the breadth and as the cube of the depth. Represented algebraically, if S = the strength and D the deflection, l the length, b the breadth, and d the depth,

$$S \text{ varies as } \frac{bd^2}{l} \text{ and } D \text{ varies as } \frac{l^3}{bd^3}.$$

For the purpose of reducing the strength of pieces of various sizes to a common standard, the term *modulus of rupture* (represented by R) is used. Its value is obtained by experiment on a bar of rectangular section supported at the ends and loaded in the middle and substituting numerical values in the following formula:

$$R = \frac{3}{2}\frac{Pl}{bd^2},$$

in which P = the breaking load in pounds, l = the length in inches, b the breadth, and d the depth.

The *modulus of rupture* is sometimes defined as the strain at the instant of rupture upon a unit of the section which is most remote from the neutral axis on the side which first ruptures. This definition, however, is based upon a theory which is yet in dispute among authorities, and it is better to define it as a numerical value, or experimental constant, found by the application of the formula above given.

From the above formula, making l 12 inches, and b and d each 1 inch, it follows that the modulus of rupture is 18 times the load required to break a bar one inch square, supported at two points one foot apart, the load being applied in the middle.

$$\text{Coefficient of transverse strength} = \frac{\text{span in feet} \times \text{load at middle in lbs.}}{\text{breadth in inches} \times (\text{depth in inches})^2}.$$

$$= \frac{1}{18}\text{th of the modulus of rupture.}$$

Fundamental Formulæ for Flexure of Beams (Merriman).

Resisting shear = vertical shear;

Resisting moment = bending moment;

Sum of tensile stresses = sum of compressive stresses;

Resisting shear = algebraic sum of all the vertical components of the internal stresses at any section of the beam.

If A be the area of the section and S_s the shearing unit stress, then resisting shear = AS_s; and if the vertical shear = V, then $V = AS_s$.

The vertical shear is the algebraic sum of all the external vertical forces on one side of the section considered. It is equal to the reaction of one support, considered as a force acting upward, minus the sum of all the vertical downward forces acting between the support and the section.

The resisting moment = algebraic sum of all the moments of the internal horizontal stresses at any section with reference to a point in that section, $= \frac{SI}{c}$, in which S = the horizontal unit stress, tensile or compressive as the case may be, upon the fibre most remote from the neutral axis, c = the shortest distance from that fibre to said axis, and I = the moment of inertia of the cross-section with reference to that axis.

The bending moment M is the algebraic sum of the moment of the external forces on one side of the section with reference to a point in that section = moment of the reaction of one support minus sum of moments of loads between the support and the section considered.

$$M = \frac{SI}{c}.$$

The bending moment is a compound quantity = product of a force by the distance of its point of application from the section considered, the distance being measured on a line drawn from the section perpendicular to the direction of the action of the force.

Concerning the above formula, Prof. Merriman, *Eng. News*, July 21, 1894, says: The formula just quoted is true when the unit-stress S on the part of the beam farthest from the neutral axis is within the elastic limit of the material. It is not true when this limit is exceeded, because then the neutral axis does not pass through the centre of gravity of the cross-section, and because also the different longitudinal stresses are not proportional to their distances from that axis, these two requirements being involved in the deduction of the formula. But in all cases of design the permissible unit-stresses should not exceed the elastic limit, and hence the formula applies rationally, without regarding the ultimate strength of the material or any of the circumstances regarding rupture. Indeed so great reliance is placed upon this formula that the practice of testing beams by rupture has been almost entirely abandoned, and the allowable unit-stresses are mainly derived from tensile and compressive tests.

GENERAL FORMULÆ FOR TRANSVERSE STRENGTH OF BEAMS OF UNIFORM CROSS-SECTION.

Beam.	Rectangular Beam.		Beam of any Section.		
	Breaking Load.	Deflection for Load P or W.	Maximum Moment of Stress.	Moment of Rupture.	Deflection. Δ
Fixed at one end, load at the other..........	$P = \frac{1}{6}\frac{Rbd^2}{l}$	$\frac{4Pl^3}{Ebd^3}$	$Pl \quad =$	$\frac{RI}{c}$	$\frac{1}{3}\frac{Pl^3}{EI}$
Same with load distributed uniformly.........	$W = \frac{1}{3}\frac{Rbd^2}{l}$	$\frac{3}{2}\frac{Wl^3}{Ebd^3}$	$\frac{1}{2}Wl \quad =$	$\frac{RI}{c}$	$\frac{1}{8}\frac{Wl^3}{EI}$
Supported at ends, loaded in middle.........	$P = \frac{2}{3}\frac{Rbd^2}{l}$	$\frac{Pl^3}{4Ebd^3}$	$\frac{1}{4}Pl \quad =$	$\frac{RI}{c}$	$\frac{1}{48}\frac{Pl^3}{EI}$
Same loaded uniformly......................	$W = \frac{4}{3}\frac{Rbd^2}{l}$	$\frac{5}{32}\frac{Wl^3}{Ebd^3}$	$\frac{1}{8}Wl \quad =$	$\frac{RI}{c}$	$\frac{5}{384}\frac{Wl^3}{EI}$
Same, loaded at middle, and also with uniform load, ...	$2P+W = \frac{4}{3}\frac{Rbd^2}{l}$	$\frac{1}{4}\left(P+\frac{1}{8}W\right)\frac{l^3}{Ebd^3}$	$\left(\frac{1}{4}P+\frac{1}{8}W\right)l =$	$\frac{RI}{c}$	$\frac{1}{48}\left(P+\frac{5}{8}W\right)\frac{l^3}{EI}$
Fixed at both ends, loaded in middle.........	$P = \frac{4}{3}\frac{Rbd^2}{l}$	$\frac{1}{16}\frac{Pl^3}{Ebd^3}$	$\frac{1}{8}Pl \quad =$	$\frac{RI}{c}$	$\frac{P}{192}\frac{l^3}{EI}$
Same, Barlow's Experiments.................	$P = \frac{Rbd^2}{l}$		$\frac{1}{6}Pl \quad =$	$\frac{RI}{c}$	
Same, uniformly loaded......................	$W = \frac{2Rbd^2}{l}$	$\frac{1}{32}\frac{Wl^3}{Ebd^3}$	$\frac{1}{12}Wl \quad =$	$\frac{RI}{c}$	$\frac{W}{384}\frac{l^3}{EI}$
Fixed at one end, supported at the other, loaded at .634l from fixed end, ...		$\frac{.1148Pl^3}{Ebd^3}$	$\frac{3}{8}\left(2\sqrt{3}-3\right)Pl =$	$\frac{RI}{c}$	$\frac{P}{105}\frac{l^3}{EI}$ (nearly).
Same uniformly loaded......................	$W = \frac{4}{3}\frac{Rbd^2}{l}$	$\frac{.0648Wl^3}{Ebd^3}$	$\frac{1}{8}Wl \quad =$	$\frac{RI}{c}$	$\frac{W}{185}\frac{l^3}{EI}$ (nearly).

Formulæ for Transverse Strength of Beams.—Referring to table on preceding page,

P = load at middle;
W = total load, distributed uniformly;
l = length, b = breadth, d = depth, in inches;
E = modulus of elasticity;
R = modulus of rupture, or stress per square inch of extreme fibre;
I = moment of inertia;
c = distance between neutral axis and extreme fibre.

For breaking load of circular section, replace bd^2 by $0.59d^3$.

For good wrought iron the value of R is about 80,000, for steel about 120,000, the percentage of carbon apparently having no influence. (Thurston, Iron and Steel, p. 491).

For cast iron the value of R varies greatly according to quality. Thurston found 45,740 and 67,980 in No. 2 and No. 4 cast iron, respectively.

For beams fixed at both ends and loaded in the middle, Barlow, by experiment, found the maximum moment of stress $= 1/6Pl$ instead of $\frac{1}{8}Pl$, the result given by theory. Prof. Wood (Resist. Matls. p. 155) says of this case: The phenomena are of too complex a character to admit of a thorough and exact analysis, and it is probably safer to accept the results of Mr. Barlow in practice than to depend upon theoretical results.

APPROXIMATE GREATEST SAFE LOADS IN LBS. ON STEEL BEAMS. (Pencoyd Iron Works.)

Based on fibre strains of 16,000 lbs. for steel. (For iron the loads should be one-eighth less, corresponding to a fibre strain of 14,000 lbs. per square inch.)

L = length in feet between supports;
A = sectional area of beam in square inches;
D = depth of beam in inches.
a = interior area in square inches;
d = interior depth in inches.
w = working load in net tons.

Shape of Section.	Greatest Safe Load in Pounds.		Deflection in Inches.	
	Load in Middle.	Load Distributed.	Load in Middle.	Load Distributed.
Solid Rectangle.	$\frac{890AD}{L}$	$\frac{1780AD}{L}$	$\frac{wL^3}{32AD^2}$	$\frac{wL^3}{52AD^2}$
Hollow Rectangle.	$\frac{890(AD-ad)}{L}$	$\frac{1780(AD-ad)}{L}$	$\frac{wL^3}{32(AD^2-ad^2)}$	$\frac{wL^3}{52(AD^2-ad^2)}$
Solid Cylinder.	$\frac{667AD}{L}$	$\frac{1333AD}{L}$	$\frac{wL^3}{24AD^2}$	$\frac{wL^3}{38AD^2}$
Hollow Cylinder.	$\frac{667(AD-ad)}{L}$	$\frac{1333(AD-ad)}{L}$	$\frac{wL^3}{24(AD^2-ad^2)}$	$\frac{wL^3}{38(AD^2-ad^2)}$
Even-legged Angle or Tee.	$\frac{885AD}{L}$	$\frac{1770AD}{L}$	$\frac{wL^3}{32AD^2}$	$\frac{wL^3}{52AD^2}$
Channel or Z bar.	$\frac{1525AD}{L}$	$\frac{3050AD}{L}$	$\frac{wL^3}{53AD^2}$	$\frac{wL^3}{85AD^2}$
Deck Beam.	$\frac{1380AD}{L}$	$\frac{2760AD}{L}$	$\frac{wL^3}{50AD^2}$	$\frac{wL^3}{80AD^2}$
I Beam.	$\frac{1695AD}{L}$	$\frac{3390AD}{L}$	$\frac{wL^3}{58AD^2}$	$\frac{wL^3}{93AD^2}$
I	II	III	IV	V

GENERAL FORMULÆ FOR TRANSVERSE STRENGTH OF BEAMS OF UNIFORM CROSS-SECTION.

Beam.	Rectangular Beam.		Beam of any Section.		
	Breaking Load.	Deflection for Load P or W.	Maximum Moment of Stress.	Moment of Rupture.	Deflection. Δ
Fixed at one end, load at the other..........	$P=\frac{1}{6}\frac{Rbd^2}{l}$	$\frac{4Pl^3}{Ebd^3}$	$Pl\ =$	$\frac{RI}{c}$	$\frac{1}{3}\frac{Pl^3}{EI}$
Same with load distributed uniformly.........	$W=\frac{1}{3}\frac{Rbd^2}{l}$	$\frac{3}{2}\frac{Wl^3}{Ebd^3}$	$\frac{1}{2}Wl\ =$	$\frac{RI}{c}$	$\frac{1}{8}\frac{Wl^3}{EI}$
Supported at ends, loaded in middle.........	$P=\frac{2}{3}\frac{Rbd^2}{l}$	$\frac{Pl^3}{4Ebd^3}$	$\frac{1}{4}Pl\ =$	$\frac{RI}{c}$	$\frac{1}{48}\frac{Pl^3}{EI}$
Same loaded uniformly......................	$W=\frac{4}{3}\frac{Rbd^2}{l}$	$\frac{5}{32}\frac{Wl^3}{Ebd^3}$	$\frac{1}{8}Wl\ =$	$\frac{RI}{c}$	$\frac{5}{384}\frac{Wl^3}{EI}$
Same, loaded at middle, and also with uniform load, ...	$2P+W=\frac{4}{3}\frac{Rbd^2}{l}$	$\frac{1}{4}\left(P+\frac{1}{8}W\right)\frac{l^3}{Ebd^3}$	$\left(\frac{1}{4}P+\frac{1}{8}W\right)l =$	$\frac{RI}{c}$	$\frac{1}{48}\left(P+\frac{5}{8}W\right)\frac{l^3}{EI}$
Fixed at both ends, loaded in middle.........	$P=\frac{4}{3}\frac{Rbd^2}{l}$	$\frac{1}{16}\frac{Pl^3}{Ebd^3}$	$\frac{1}{8}Pl\ =$	$\frac{RI}{c}$	$\frac{P}{192}\frac{l^3}{EI}$
Same, Barlow's Experiments................	$P=\frac{Rbd^2}{l}$		$\frac{1}{6}Pl\ =$	$\frac{RI}{c}$	
Same, uniformly loaded.....................	$W=\frac{2Rbd^2}{l}$	$\frac{1}{32}\frac{Wl^3}{Ebd^3}$	$\frac{1}{12}Wl\ =$	$\frac{RI}{c}$	$\frac{W}{384}\frac{l^3}{EI}$
Fixed at one end, supported at the other, loaded at .634l from fixed end, ...		$\frac{.1148Pl^3}{Ebd^3}$	$\frac{3}{8}\left(2\sqrt{3}-3\right)Pl =$	$\frac{RI}{c}$	$\frac{P}{105}\frac{l^3}{EI}$ (nearly).
Same uniformly loaded......................	$W=\frac{4}{3}\frac{Rbd^2}{l}$	$\frac{.0648Wl^3}{Ebd^3}$	$\frac{1}{8}Wl\ =$	$\frac{RI}{c}$	$\frac{W}{185}\frac{l^3}{EI}$ (nearly).

Formulæ for Transverse Strength of Beams.—Referring to table on preceding page,

P = load at middle;
W = total load, distributed uniformly;
l = length, b = breadth, d = depth, in inches;
E = modulus of elasticity;
R = modulus of rupture, or stress per square inch of extreme fibre;
I = moment of inertia;
c = distance between neutral axis and extreme fibre.

For breaking load of circular section, replace bd^2 by $0.59d^3$.

For good wrought iron the value of R is about 80,000, for steel about 120,000, the percentage of carbon apparently having no influence. (Thurston, Iron and Steel, p. 491).

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For beams fixed at both ends and loaded in the middle, Barlow, by experiment, found the maximum moment of stress $= 1/6Pl$ instead of $\frac{1}{8}Pl$, the result given by theory. Prof. Wood (Resist. Matls. p. 155) says of this case: The phenomena are of too complex a character to admit of a thorough and exact analysis, and it is probably safer to accept the results of Mr. Barlow in practice than to depend upon theoretical results.

APPROXIMATE GREATEST SAFE LOADS IN LBS. ON STEEL BEAMS. (Pencoyd Iron Works.)

Based on fibre strains of 16,000 lbs. for steel. (For iron the loads should be one-eighth less, corresponding to a fibre strain of 14,000 lbs. per square inch.)

L = length in feet between supports;
A = sectional area of beam in square inches;
D = depth of beam in inches.
a = interior area in square inches;
d = interior depth in inches.
w = working load in net tons.

Shape of Section.	Greatest Safe Load in Pounds.		Deflection in Inches.	
	Load in Middle.	Load Distributed.	Load in Middle.	Load Distributed.
Solid Rectangle.	$\frac{890AD}{L}$	$\frac{1780AD}{L}$	$\frac{wL^3}{32AD^2}$	$\frac{wL^3}{52AD^2}$
Hollow Rectangle.	$\frac{890(AD-ad)}{L}$	$\frac{1780(AD-ad)}{L}$	$\frac{wL^3}{32(AD^2-ad^2)}$	$\frac{wL^3}{52(AD^2-ad^2)}$
Solid Cylinder.	$\frac{667AD}{L}$	$\frac{1333AD}{L}$	$\frac{wL^3}{24AD^2}$	$\frac{wL^3}{38AD^2}$
Hollow Cylinder.	$\frac{667(AD-ad)}{L}$	$\frac{1333(AD-ad)}{L}$	$\frac{wL^3}{24(AD^2-ad^2)}$	$\frac{wL^3}{38(AD^2-ad^2)}$
Even-legged Angle or Tee.	$\frac{885AD}{L}$	$\frac{1770AD}{L}$	$\frac{wL^3}{32AD^2}$	$\frac{wL^3}{52AD^2}$
Channel or Z bar.	$\frac{1525AD}{L}$	$\frac{3050AD}{L}$	$\frac{wL^3}{53AD^2}$	$\frac{wL^3}{85AD^2}$
Deck Beam.	$\frac{1380AD}{L}$	$\frac{2760AD}{L}$	$\frac{wL^3}{50AD^2}$	$\frac{wL^3}{80AD^2}$
I Beam.	$\frac{1695AD}{L}$	$\frac{3390AD}{L}$	$\frac{wL^3}{58AD^2}$	$\frac{wL^3}{93AD^2}$
I	II	III	IV	V

The above formulæ for the strength and stiffness of rolled beams of various sections are intended for convenient application in cases where strict accuracy is not required.

The rules for rectangular and circular sections are correct, while those for the flanged sections are approximate, and limited in their application to the standard shapes as given in the Pencoyd tables. When the section of any beam is increased above the standard minimum dimensions, the flanges remaining unaltered, and the web alone being thickened, the tendency will be for the load as found by the rules to be in excess of the actual; but within the limits that it is possible to vary any section in the rolling, the rules will apply without any serious inaccuracy.

The calculated safe loads will be approximately one half of loads that would injure the elasticity of the materials.

The rules for deflection apply to any load below the elastic limit, or less than double the greatest safe load by the rules.

If the beams are long without lateral support, reduce the loads for the ratios of width to span as follows:

Length of Beam.	Proportion of Calculated Load forming Greatest Safe Load.
20 times flange width.	Whole calculated load.
30 " " "	9-10 " "
40 " " "	8-10 " "
50 " " "	7-10 " "
60 " " "	6-10 " "
70 " " "	5-10 " "

These rules apply to beams supported at each end. For beams supported otherwise, alter the coefficients of the table as described below, referring to the respective columns indicated by number.

Changes of Coefficients for Special Forms of Beams.

Kind of Beam.	Coefficient for Safe Load.	Coefficient for Deflection.
Fixed at one end, loaded at the other.	One fourth of the coefficient, col. II.	One sixteenth of the coefficient of col. IV.
Fixed at one end, load evenly distributed.	One fourth of the coefficient of col. III.	Five forty-eighths of the coefficient of col. V.
Both ends rigidly fixed, or a continuous beam, with a load in middle.	Twice the coefficient of col. II.	Four times the coefficient of col. IV.
Both ends rigidly fixed, or a continuous beam, with load evenly distributed.	One and one-half times the coefficient of col. III.	Five times the coefficient of col. V.

ELASTIC RESILIENCE.

In a rectangular beam tested by transverse stress, supported at the ends and loaded in the middle,

$$P = \frac{2}{3}\,\frac{Rbd^2}{l};$$

$$\Delta = \frac{1}{4}\,\frac{Pl^3}{Ebd^3};$$

in which, if P is the load in pounds at the elastic limit, R = the modulus of transverse strength, or the strain on the extreme fibre, at the elastic limit, E = modulus of elasticity, Δ = deflection, l, b, and d = length, breadth, and depth in inches. Substituting for P in (2) its value in (1), we have

$$\Delta = \frac{1}{6}\,\frac{Rl^2}{Ed}.$$

The elastic resilience = half the product of the load and deflection = $\frac{1}{2}P\Delta$, and the elastic resilience per cubic inch

$$=\frac{1}{2}\frac{P\Delta}{lbd}.$$

Substituting the values of P and Δ, this reduces to elastic resilience per cubic inch $=\frac{1}{18}\frac{R^2}{E}$, which is independent of the dimensions; and therefore the elastic resilience per cubic inch for transverse strain may be used as a modulus expressing one valuable quality of a material.

Similarly for tension:

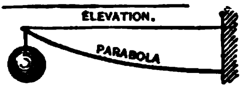
Let P = tensile stress in pounds per square inch at the elastic limit;
e = elongation per unit of length at the elastic limit;
E = modulus of elasticity = $P \div e$; whence $e = P \div E$.

Then elastic resilience per cubic inch $= \frac{1}{2}Pe = \frac{1}{2}\frac{P^2}{E}$.

BEAMS OF UNIFORM STRENGTH THROUGHOUT THEIR LENGTH.

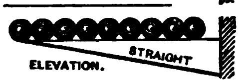
The section is supposed in all cases to be rectangular throughout. The beams shown in plan are of uniform depth throughout. Those shown in elevation are of uniform breadth throughout.

B = breadth of beam. D = depth of beam.

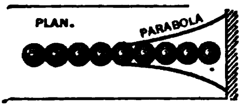


Fixed at one end, loaded at the other; curve parabola, vertex at loaded end; BD^2 proportional to distance from loaded end. The beam may be reversed, so that the upper edge is parabolic, or both edges may be parabolic.

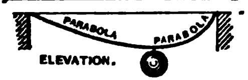
Fixed at one end, loaded at the other; triangle, apex at loaded end; BD^2 proportional to the distance from the loaded end.



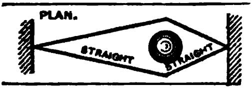
Fixed at one end; load distributed; triangle, apex at unsupported end; BD^2 proportional to square of distance from unsupported end.



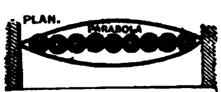
Fixed at one end; load distributed; curves two parabolas, vertices touching each other at unsupported end; BD^2 proportional to distance from unsupported end.



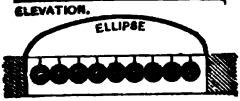
Supported at both ends; load at any one point; two parabolas, vertices at the points of support, bases at point loaded; BD^2 proportional to distance from nearest point of support. The upper edge or both edges may also be parabolic.



Supported at both ends; load at any one point; two triangles, apices at points of support, bases at point loaded; BD^2 proportional to distance from the nearest point of support.



Supported at both ends; load distributed; curves two parabolas, vertices at the middle of the beam; bases centre line of beam; BD^2 proportional to product of distances from points of support.



Supported at both ends; load distributed; curve semi-ellipse; BD^2 proportional to the product of the distances from the points of support.

PROPERTIES OF ROLLED STRUCTURAL STEEL.

Explanation of Tables of the Properties of I Beams, Channels, Angles, Deck-Beams, Bulb Angles, Z Bars, Tees, Trough and Corrugated Plates.

(The Carnegie Steel Co., Limited.)

The tables for I beams and channels are calculated for all standard weights to which each pattern is rolled. The tables for deck-beams and angles are calculated for the minimum and maximum weights of the various shapes, while the properties of Z bars are given for thicknesses differing by 1/16 inch.

For tees, each shape can be rolled to one weight only.

Column 12 in the tables for I beams and channels, and column 9 for deck-beams, give coefficients by the help of which the safe, uniformly distributed load may be readily determined. To do this, divide the coefficient given by the span or distance between supports in feet. If the weight of the deck beams is intermediate between the minimum and maximum weights given, add to the coefficient for the minimum weight the value given for one pound increase of weight multiplied by the number of pounds the section is heavier than the minimum.

If a section is to be selected (as will usually be the case), intended to carry a certain load for a length of span already determined on, ascertain the coefficient which this load and span will require, and refer to the table for a section having a coefficient of this value. The coefficient is obtained by multiplying the load, in pounds uniformly distributed, by the span length in feet.

In case the load is not uniformly distributed, but is concentrated at the middle of the span, multiply the load by 2, and then consider it as uniformly distributed. The deflection will be 8/10 of the deflection for the latter load.

For other cases of loading obtain the bending moment in ft.-lbs.; this multiplied by 8 will give the coefficient required.

If the loads are quiescent, the coefficients for a fibre stress of 16,000 lbs. per square inch for steel may be used; but if moving loads are to be provided for, a coefficient of 12,500 lbs. should be taken. Inasmuch as the effects of impact may be very considerable (the stresses produced in an unyielding inelastic material by a load suddenly applied being double those produced by the same load in a quiescent state), it will sometimes be advisable to use still smaller fibre stresses than those given in the tables. In such cases the coefficients may be determined by proportion. Thus, for a fibre stress of 8,000 lbs. per square inch the coefficient will equal the coefficient for 16,000 lbs. fibre stress, from the table, divided by 2.

The section moduli, column 11, are used to determine the fibre stress per square inch in a beam, or other shape, subjected to bending or transverse stresses, by simply dividing the bending moment expressed in inch-pounds by the section modulus.

In the case of T shapes with the neutral axis parallel to the flange, there will be two section moduli, and the smaller is given. The fibre stress calculated from it will, therefore, give the larger of the two stresses in the extreme fibres, since these stresses are equal to the bending moment divided by the section modulus of the section.

For Z bars the coefficients (C) may be applied for cases where the bars are subjected to transverse loading, as in the case of roof-purlins.

For angles, there will be two section moduli for each position of the neutral axis, since the distance between the neutral axis and the extreme fibres has a different value on one side of the axis from what it has on the other. The section modulus given in the table is the smaller of these two values.

Column 12 in the table of the properties of standard channels, giving the distance of the center of gravity of channel from the outside of web, is used to obtain the radius of gyration for columns or struts consisting of two channels latticed, for the case of the neutral axis passing through the centre of the cross-section parallel to the webs of the channels. This radius of gyration is equal to the distance between the centre of gravity of the channel and the centre of the section, i.e., neglecting the moments of inertia of the channels around their own axes, thereby introducing a slight error on the side of safety.

(For much other important information concerning rolled structural shapes, see the "Pocket Companion" of The Carnegie Steel Co., Limited, Pittsburg, Pa., price $2.)

Properties of Carnegie Standard I Beams—Steel.

1	2	3	4	5	6	7	8	9	10	11	12
Section Index.	Depth of Beam.	Weight per Foot.	Area of Section.	Thickness of Web.	Width of Flange.	Moment of Inertia, Neutral Axis Perpendicular to Web at Centre.	Moment of Inertia, Neutral Axis Coincident with Centre Line of Web.	Radius of Gyration, Neutral Axis Perpendicular to Web at Centre.	Radius of Gyration, Neutral Axis Coincident with Centre Line of Web.	Section Modulus, Neutral Axis Perpendicular to Web at Centre.	Coefficient of Strength for Fibre Stress of 16,000 lbs. per sq. in.
	in.	lbs.	sq. in.	in.	in.	I	I'	r	r'	S	C
B1	24	100	29.41	0.75	7.25	2380.3	48.56	9.00	1.28	198.4	2115800
"	"	95	27.94	0.69	7.19	2309.6	47.10	9.09	1.30	192.5	2052900
"	"	90	26.47	0.63	7.13	2239.1	45.70	9.20	1.31	186.6	1990300
"	"	85	25.00	0.57	7.07	2168.6	44.35	9.31	1.33	180.7	1927600
"	"	80	23.32	0.50	7.00	2087.9	42.86	9.46	1.36	174.0	1855900
B3	20	75	22.06	0.65	6.40	1268.9	30.25	7.58	1.17	126.9	1353500
"	"	70	20.59	0.57	6.32	1219.9	29.04	7.70	1.19	122.0	1301200
"	"	65	19.08	0.50	6.25	1169.6	27.86	7.83	1.21	117.0	1247600
B5	18	70	20.59	0.72	6.26	921.3	24.62	6.69	1.09	102.4	1091900
"	"	65	19.12	0.64	6.18	881.5	23.47	6.79	1.11	97.9	1044800
"	"	60	17.65	0.55	6.09	841.8	22.38	6.91	1.13	93.5	997700
"	"	55	15.93	0.46	6.00	795.6	21.19	7.07	1.15	88.4	943000
B7	15	55	16.18	0.66	5.75	511.0	17.06	5.23	0.95	68.1	726800
"	"	50	14.71	0.56	5.65	483.4	16.04	5.73	1.04	64.5	687500
"	"	45	13.24	0.46	5.55	455.8	15.09	5.87	1.07	60.8	648200
"	"	42	12.48	0.41	5.50	441.7	14.62	5.95	1.08	53.9	628800
B9	12	35	10.29	0.44	5.09	228.3	10.07	4.71	0.99	38.0	405800
"	"	31.5	9.26	0.35	5.00	215.8	9.50	4.83	1.01	36.0	383700
B11	10	40	11.76	0.75	5.10	158.7	9.50	3.67	0.90	31.7	338500
"	"	35	10.29	0.60	4.95	146.4	8.52	3.77	0.91	29.3	312400
"	"	30	8.82	0.45	4.80	134.2	7.65	3.90	0.93	26.8	286300
"	"	25	7.37	0.31	4.66	122.1	6.89	4.07	0.97	24.4	260500
B13	9	35	10.29	0.73	4.77	111.8	7.81	3.29	0.84	24.8	265000
"	"	30	8.82	0.57	4.61	101.9	6.42	3.40	0.85	22.6	241500
"	"	25	7.35	0.41	4.45	91.9	5.65	3.54	0.88	20.4	217900
"	"	21	6.31	0.29	4.33	84.9	5.16	3.67	0.90	18.9	201300
B15	8	25.5	7.50	0.54	4.27	68.4	4.75	3.02	0.80	17.1	182500
"	"	23	6.76	0.45	4.18	64.5	4.39	3.09	0.81	16.1	172000
"	"	20.5	6.03	0.36	4.09	60.6	4.07	3.17	0.82	15.1	161600
"	"	18	5.33	0.27	4.00	56.9	3.78	3.27	0.84	14.2	151700
B17	7	20	5.88	0.46	3.87	42.2	3.24	2.68	0.74	12.1	128600
"	"	17.5	5.15	0.35	3.76	39.2	2.94	2.76	0.76	11.2	119400
"	"	15	4.42	0.25	3.66	36.2	2.67	2.86	0.78	10.4	110400
B19	6	17¼	5.07	0.48	3.58	26.2	2.36	2.27	0.68	8.7	93100
"	"	14¾	4.34	0.35	3.45	24.0	2.09	2.35	0.69	8.0	85300
"	"	12¼	3.61	0.23	3.33	21.8	1.85	2.46	0.72	7.3	77500
B21	5	14¾	4.34	0.50	3.29	15.2	1.70	1.87	0.63	6.1	64600
"	"	12¼	3.60	0.36	3.15	13.6	1.45	1.94	0.63	5.4	58100
"	"	9¾	2.87	0.21	3.00	12.1	1.23	2.05	0.65	4.8	51600
B23	4	10.5	3.09	0.41	2.88	7.1	1.01	1.52	0.57	3.6	38100
"	"	9.5	2.79	0.34	2.80	6.7	0.93	1.55	0.58	3.4	36000
"	"	8.5	2.50	0.26	2.73	6.4	0.85	1.59	0.58	3.2	33900
"	"	7.5	2.21	0.19	2.66	6.0	0.77	1.64	0.59	3.0	31800
B77	3	7.5	2.21	0.36	2.52	2.9	0.60	1.15	0.52	1.9	20700
"	"	6.5	1.91	0.26	2.42	2.7	0.53	1.19	0.52	1.8	19100
"	"	5.5	1.63	0.17	2.33	2.5	0.46	1.23	0.53	1.7	17600

L = safe loads in lbs., uniformly distributed; l = span in feet;
M = moment of forces in ft.-lbs.; C = coefficient given above.

$L = \frac{C}{l}$; $M = \frac{C}{8}$; $C = Ll = 8M = \frac{8fS}{12}$; f = fibre stress.

Properties of Special I Beams - Steel.

1	2	3	4	5	6	7	8	9	10	11	12
Section Index.	Depth of Beam.	Weight per Foot.	Area of Section.	Thickness of Web.	Width of Flange.	Moment of Inertia, Neutral Axis Perpendicular to Web at Centre.	Moment of Inertia, Neutral Axis Coincident with Centre Line of Web.	Radius of Gyration, Neutral Axis Perpendicular to Web at Centre.	Radius of Gyration, Neutral Axis Coincident with Centre Line of Web.	Section Modulus, Neutral Axis Perpendicular to Web at Centre.	Coefficient of Strength for Fibre Stress of 16,000 lbs. per sq. in.
	in.	lbs.	sq. in.	in.	in.	I	I'	r	r'	S	C
B2	20	100	29.41	0.88	7.28	1655.8	52.65	7.50	1.34	165.6	1766100
"	"	95	27.94	0.81	7.21	1606.8	50.78	7.58	1.35	160.7	1713900
"	"	90	26.47	0.74	7.14	1557.8	48.98	7.67	1.36	155.8	1661600
"	"	85	25.00	0.66	7.06	1508.7	47.25	7.77	1.37	150.9	1609100
"	"	80	23.73	0.60	7.00	1466.5	45.81	7.86	1.39	146.7	1564300
B4	15	100	29.41	1.18	6.77	900.5	50.98	5.53	1.31	120.1	1280700
"	"	95	27.94	1.08	6.67	872.9	48.37	5.59	1.32	116.4	1241500
"	"	90	26.47	0.99	6.58	845.4	45.91	5.65	1.32	112.7	1202300
"	"	85	25.00	0.89	6.48	817.8	43.57	5.72	1.32	109.0	1163000
"	"	80	23.81	0.81	6.40	795.5	41.76	5.78	1.32	106.1	1131300
B5	15	75	22.06	0.88	6.29	691.2	30.68	5.60	1.18	92.2	983000
"	"	70	20.59	0.78	6.19	663.6	29.00	5.68	1.19	88.5	943800
"	"	65	19.12	0.69	6.10	636.0	27.42	5.77	1.20	84.8	904600
"	"	60	17.67	0.59	6.00	609.0	25.96	5.87	1.21	81.2	866100
B8	12	55	16.18	0.82	5.61	321.0	17.46	4.45	1.04	53.5	570600
"	"	50	14.71	0.70	5.49	303.3	16.12	4.54	1.05	50.6	539200
"	"	45	13.24	0.58	5.37	285.7	14.89	4.65	1.06	47.6	507900
"	"	40	11.84	0.46	5.25	268.9	13.81	4.77	1.08	44.8	458100

Properties of Carnegie Trough Plates—Steel.

Section Index.	Size, in Inches.	Weight per Foot.	Area of Section.	Thickness in Inches.	Moment of Inertia, Neutral Axis Parallel to Length.	Section Modulus, Axis as before.	Radius of Gyration, Axis as before.
		lbs.	sq. in.		I	S	r
M10	9½ × 3¾	16.32	4.8	½	3.68	1.38	0.91
M11	9½ × 3¾	18.02	5.3	9/16	4.13	1.57	0.91
M12	9½ × 3¾	19.72	5.8	⅝	4.57	1.77	0.90
M13	9½ × 3¾	21.42	6.3	11/16	5.02	1.96	0.90
M14	9½ × 3¾	23.15	6.8	¾	5.46	2.15	0.90

Properties of Carnegie Corrugated Plates - Steel.

Section Index.	Size, in Inches.	Weight per Foot.	Area of Section.	Thickness in Inches.	Moment of Inertia, Neutral Axis Parallel to Length.	Section Modulus, Axis as before.	Radius of Gyration, Axis as before.
		lbs.	sq. in.		I	S	r
M30	8¾ × 1½	8.06	2.4	¼	0.64	0.80	0.52
M31	8¾ × 1½	10.10	3.0	5/16	0.95	1.13	0.57
M32	8¾ × 1½	12.04	3.5	⅜	1.25	1.42	0.62
M33	12 3/16 × 2¾	17.75	5.2	⅜	4.79	3.33	0.96
M34	12 3/16 × 2¾	20.71	6.1	7/16	5.81	3.90	0.98
M35	12 3/16 × 2¾	23.67	7.0	½	6.82	4.46	0.99

Safe Loads, Uniformly Distributed, for Standard and Special I Beams. (The Carnegie Steel Co., Ltd.)

In Tons of 2000 Lbs.

Distance between Supports in Feet.	24″ I.	20″ I.		18″ I.	15″ I.		
	80 lbs.	80 lbs. Special.	65 lbs.	55 lbs.	80 lbs. Special.	60 lbs. Special.	42 lbs.
12	77.33	65.18	51.98	39.29	47.14	36.09	26.18
13	71.38	60.16	47.98	36.27	43.51	33.31	24.17
14	66.28	55.87	44.56	33.68	40.40	30.93	22.44
15	61.86	52.14	41.59	31.43	37.71	28.87	20.94
16	58.00	48.88	38.99	29.47	35.35	27.07	19.63
17	54.58	46.01	36.69	27.74	33.27	25.47	18.48
18	51.56	41.45	34.66	26.19	31.42	24.06	17.45
19	48.84	41.17	32.83	24.82	29.77	22.79	16.53
20	46.40	39.11	31.19	23.58	28.28	21.65	15.71
21	44.19	37.24	29.70	22.45	26.94	20.62	14.96
22	42.18	35.55	28.35	21.43	25.71	19.68	14.28
23	40.35	34.01	27.12	20.50	24.59	18.83	13.66
24	38.67	32.59	25.99	19.65	23.57	18.04	13.09
25	37.12	31.29	24.95	18.86	22.63	17.32	12.57
26	35.69	30.08	23.99	18.14	21.76	16.66	12.08
27	34.37	28.97	23.10	17.46	20.95	16.04	11.64
28	33.14	27.93	22.28	16.84	20.20	15.47	11.22
29	32.00	26.97	21.51	16.26	19.51	14.93	10.83
30	30.93	26.07	20.79	15.72	18.86	14.43	10.47
31	29.94	25.23	20.12	15.21	18.25	13.97	10.13
32	29.00	24.44	19.49	14.73	17.68	13.53	9.82
33	28.12	23.70	18.90	14.29	17.14	13.12	9.52
34	27.29	23.00	18.35	13.87	16.64	12.74	9.24
35	26.51	22.35	17.82	13.47	16.16	12.37	8.98
36	25.78	21.73	17.33	13.10	15.71	12.03	8.73

Distance between Supports in Feet.	12″ I.		10″ I.	9″ I.
	40 lbs. Special.	31.5 lbs.	25 lbs.	21 lbs.
12	19.92	15.99	10.85	8.39
13	18.39	14.76	10.02	7.74
14	17.08	13.70	9.30	7.19
15	15.94	12.79	8.68	6.71
16	14.94	11.99	8.14	6.29
17	14.06	11.29	7.66	5.92
18	13.28	10.66	7.24	5.59
19	12.58	10.10	6.86	5.30
20	11.95	9.59	6.51	5.03
21	11.38	9.14	6.20	4.79
22	10.87	8.72	5.92	4.58
23	10.39	8.34	5.66	4.38
24	9.96	7.99	5.43	4.19
25	9.56	7.67	5.21	4.03
26	9.19	7.38	5.01	3.87
27	8.85	7.11	4.82	3.73
28	8.54	6.85	4.65	3.59
29	8.24	6.62	4.49	3.47
30	7.97	6.40	4.34	3.36

Distance between Supports in Feet.	8″ I.	7″ I.	6″ I.	5″ I.	4″ I.	3″ I.
	18 lbs.	15 lbs.	12.25 lbs.	9.75 lbs.	7.5 lbs.	5.5 lbs.
5	15.17	11.04	7.75	5.16	3.18	1.76
6	12.64	9.20	6.46	4.30	2.65	1.47
7	10.84	7.89	5.54	3.69	2.27	1.26
8	9.48	6.90	4.84	3.23	1.99	1.10
9	8.43	6.13	4.31	2.87	1.77	0.98
10	7.59	5.52	3.88	2.58	1.59	0.88
11	6.90	5.02	3.52	2.35	1.45	0.80
12	6.32	4.60	3.23	2.15	1.33	0.73
13	5.83	4.25	2.98	1.98	1.22	0.68
14	5.42	3.94	2.77	1.84	1.14	0.63
15	5.06	3.68	2.58	1.72	1.06	0.59
16	4.74	3.45	2.42	1.61	0.99	0.55
17	4.46	3.25	2.28	1.52	0.94	0.52
18	4.21	3.07	2.15	1.43	0.88	0.49
19	3.99	2.91	2.04	1.36	0.84	0.46
20	3.79	2.76	1.94	1.29	0.80	0.44
21	3.61	2.63	1.85	1.23	0.76	0.42

Safe loads given include weight of beam. Maximum fibre stress 16,000 lbs. per square inch.

Spacing of Carnegie I-Beams for Uniform Load of 100 lbs. per Square Foot.

STEEL.

(Proper distance in feet, centre to centre of beams.)

Distance between Supports in Feet.	24″ I. 80 lbs.	20″ I. 80 lbs. Special.	20″ I. 65 lbs.	18″ I. 55 lbs.	15″ I. 80 lbs. Special.	15″ I. 60 lbs. Special.	15″ I. 42 lbs.	12″ I. 40 lbs Special.	12″ I. 31.5 lbs.	10″ I. 25 lbs.
12	128.9	108.6	86.6	65.5	78.6	60.1	43.6	33.2	26.6	18.1
13	109.8	92.6	73.8	55.8	67.0	51.8	37.2	28.3	22.7	15.4
14	94.7	79.8	63.7	48.1	57.7	44.2	32.1	24.4	19.6	13.3
15	82.5	69.5	55.5	41.9	50.3	38.5	27.9	21.3	17.1	11.6
16	72.5	61.1	48.7	36.8	44.2	33.8	24.5	18.7	15.0	10.2
17	61.2	54.1	43.2	32.6	39.2	30.0	21.7	16.5	13.3	9.0
18	57.3	48.3	38.5	29.1	34.9	26.7	19.4	14.8	11.8	8.0
19	51.4	43.3	34.6	26.1	31.3	24.0	17.4	13.2	10.6	7.2
20	46.4	39.1	31.2	23.6	28.3	21.7	15.7	12.0	9.6	6.5
21	42.1	35.5	28.3	21.4	25.7	19.6	14.2	10.8	8.7	5.9
22	38.4	32.3	25.8	19.5	23.4	17.9	13.0	9.9	7.9	5.4
23	35.1	29.6	23.6	17.8	21.4	16.4	11.9	9.0	7.3	4.9
24	32.2	27.2	21.7	16.4	19.6	15.0	10.9	8.3	6.7	4.5
25	29.7	25.0	20.0	15.1	18.1	13.9	10.1	7.7	6.1	4.2
26	27.5	23.1	18.5	13.9	16.7	12.8	9.3	7.1	5.7	3.9
27	25.5	21.5	17.1	12.9	15.5	11.9	8.6	6.6	5.3	3.6
28	23.7	20.0	15.9	12.0	14.4	11.0	8.0	6.1	4.9	3.3
29	22.1	18.6	14.8	11.2	13.5	10.3	7.5	5.7	4.6	3.1
30	20.6	17.4	13.9	10.5	12.6	9.6	7.0	5.3	4.3	2.9

Distance between Supports in Feet.	9″ I. 21 lbs.	8″ I. 18 lbs.	7″ I. 15 lbs.	6″ I. 12.25 lbs.	5″ I. 9.75 lbs.	4″ I. 7.5 lbs.	3″ I. 5.5 lbs.
5	80.5	60.7	44.2	31.0	20.6	12.7	7.0
6	55.9	42.1	30.7	21.5	14.3	8.8	4.9
7	41.1	31.0	22.5	15.8	10.5	6.5	3.6
8	31.5	23.7	17.3	12.1	8.1	5.0	2.8
9	24.9	18.7	13.6	9.6	6.4	3.9	2.2
10	20.1	15.2	11.1	7.8	5.2	3.2	1.8
11	16.6	12.5	9.1	6.4	4.3	2.6	1.5
12	14.0	10.5	7.7	5.4	3.6	2.2	1.2
13	11.9	9.0	6.5	4.6	3.1	1.9	1.0
14	10.3	7.7	5.6	4.0	2.6	1.6	0.9
15	9.0	6.7	4.9	3.4	2.3	1.4	....
16	7.9	5.9	4.3	3.0	2.0	1.2	....
17	7.0	5.3	3.8	2.7	1.8	1.1	....
18	6.2	4.7	3.4	2.4	1.6	.98	....
19	5.6	4.2	3.1	2.2	1.4	...	....
20	5.0	3.8	2.8	1.9	1.3	....	..
21	4.6	3.4	2.5	1.8	1.2	....	....
22	3.8	3.1	2.3	1.6	1.1	...	....

For any other load than 100 lbs. per square foot, divide the spacing given by the ratio the given load per square foot bears to 100. Thus for a load of 150 lbs. per square foot divide by 1.5. Maximum fibre stress, 16,000 lbs. per square inch.

Only figures above the cross-lines should be used for plastered ceilings, so that the deflection will not cause cracking of the plaster.

Properties of Standard Channels—Steel.

1	2	3	4	5	6	7	8	9	10	11	12
Depth of Channel.	Weight per Foot.	Area of Section.	Thickness of Web.	Width of Flange.	Moment of Inertia, Neutral Axis Perpendicular to Web at Centre.	Moment of Inertia, Neutral Axis Parallel with Centre Line of Web.	Radius of Gyration, Neutral Axis Perpendicular to Web at Centre.	Radius of Gyration, Neutral Axis Parallel with Centre Line of Web.	Section Modulus, Neutral Axis Perpendic. to Web at Centre.	Coefficient of Strength for Fibre Stress of 16,000 lbs. per sq. in.	Distance of Centre of Gravity from Outside of Web.
in.	lbs.	sq. in.	in.	in.	I	I'	r	r'	S	C	x
15	55.	16.18	0.82	3.82	430.2	12.19	5.16	.868	57.4	611900	.823
"	50.	14.71	0.72	3.72	402.7	11.22	5.23	.873	53.7	572700	.803
"	45.	13.24	0.62	3.62	375.1	10.29	5.32	.882	50.0	533500	.788
"	40.	11.76	0.52	3.52	347.5	9.39	5.43	.893	46.3	494200	.783
"	35.	10.29	0.43	3.43	320.0	8.48	5.58	.908	42.7	455000	.789
"	33.	9.90	0.40	3.40	312.6	8.23	5.62	.912	41.7	444500	.794
12	40.	11.76	0.76	3.42	197.0	6.63	4.09	.751	32.8	350200	.722
"	35.	10.29	0.64	3.30	179.3	5.90	4.17	.757	29.9	318800	.694
"	30.	8.82	0.51	3.17	161.7	5.21	4.28	.768	26.9	287400	.677
"	25.	7.35	0.39	3.05	144.0	4.53	4.43	.785	24.0	256100	.678
"	20.5	6.03	0.28	2.94	128.1	3.91	4.61	.805	21.4	227800	.704
10	35.	10.29	0.82	3.18	115.5	4.66	3.35	.672	23.1	246400	.695
"	30.	8.82	0.68	3.04	103.2	3.99	3.42	.672	20.6	220800	.651
"	25.	7.35	0.53	2.89	91.0	3.40	3.52	.680	18.2	194100	.620
"	20.	5.88	0.38	2.74	78.7	2.85	3.66	.696	15.7	168000	.609
"	15.	4.46	0.24	2.60	66.9	2.30	3.87	.718	13.4	142700	.639
9	25.	7.35	0.61	2.81	70.7	2.98	3.10	.637	15.7	167600	.615
"	20.	5.88	0.45	2.65	60.8	2.45	3.21	.646	13.5	144100	.585
"	15.	4.41	0.29	2.49	50.9	1.95	3.40	.665	11.3	120500	.590
"	13¼	3.89	0.23	2.43	47.3	1.77	3.49	.674	10.5	112200	.607
8	21¼	6.25	0.58	2.62	47.8	2.25	2.77	.600	11.9	127400	.587
"	18¾	5.51	0.49	2.53	43.8	2.01	2.82	.603	11.0	116900	.567
"	16¼	4.78	0.40	2.44	39.9	1.78	2.89	.610	10.0	106400	.556
"	13¾	4.04	0.31	2.35	36.0	1.55	2.98	.619	9.0	96000	.557
"	11¼	3.35	0.22	2.26	32.3	1.33	3.11	.630	8.1	86100	.576
7	19¾	5.81	0.63	2.51	33.2	1.85	2.39	.565	9.5	101100	.583
"	17¼	5.07	0.53	2.41	30.2	1.62	2.44	.561	8.6	92000	.555
"	14¾	4.34	0.42	2.30	27.2	1.40	2.50	.568	7.8	82800	.535
"	12¼	3.60	0.32	2.20	24.2	1.19	2.59	.575	6.9	73700	.528
"	9¾	2.85	0.21	2.09	21.1	0.98	2.72	.586	6.0	66800	.546
6	15.5	4.56	0.56	2.28	19.5	1.28	2.07	.529	6.5	69500	.546
"	13.	3.82	0.44	2.16	17.3	1.07	2.13	.529	5.8	61600	.517
"	10.5	3.09	0.32	2.04	15.1	0.88	2.21	.534	5.0	53800	.503
"	8.	2.38	0.20	1.92	13.0	0.70	2.34	.542	4.3	46200	.517
5	11.5	3.38	0.48	2.04	10.4	0.82	1.75	.493	4.2	44400	.508
"	9.	2.65	0.33	1.89	8.9	0.64	1.83	.493	3.5	37900	.481
"	6.5	1.95	0.19	1.75	7.4	0.48	1.95	.498	3.0	31600	.489
4	7¼	2.13	0.32	1.72	4.6	0.44	1.46	.455	2.3	24400	.463
"	6¼	1.84	0.25	1.65	4.2	0.38	1.51	.454	2.1	22300	.458
"	5¼	1.55	0.18	1.58	3.8	0.32	1.56	.453	1.9	20200	.464
3	6.	1.76	0.36	1.60	2.1	0.31	1.08	.421	1.4	14700	.459
"	5.	1.47	0.26	1.50	1.8	0.25	1.12	.415	1.2	13100	.443
"	4.	1.19	0.17	1.41	1.6	0.20	1.17	.409	1.1	11600	.443

L = safe load in lbs., uniformly distributed; l = span in feet;
M = moment of forces in ft.-lbs.; C = coefficient given above.

$L = \frac{C}{l}$; $M = \frac{C}{8}$; $C = Ll = 8M = \frac{8fS}{12}$; f = fibre stress.

Carnegie Deck-beams.

1	2	3	4	5	6	7	8	9	10	11
Depth of Beam.	Weight per Foot.	Area of Section.	Thickness of Web.	Width of Flange.	Moment of Inertia, Neutral Axis Perpendicular to Web.	Section Modulus, Neutral Axis Perpendicular to Web.	Radius of Gyration, Neutral Axis Perpendicular to Web.	Coefficient of Strength for Fibre Stress of 16,000 lbs. per sq. in.	Moment of Inertia, Neutral Axis Coincident with Centre Line of Web.	Radius of Gyration, Neutral Axis Coincident with Centre Line of Web.
in.	lbs.	sq. in.	in.	in.	I	S	r	C	I'	r'
10	35.70	10.5	.63	5.50	139.9	25.7	3.64	274100	7.41	0.84
10	27.23	8.0	.38	5.25	118.4	21.2	3.83	226100	6.12	0.87
9	30 00	8.8	.57	5.07	93.2	19.6	3.25	208500	5.18	0.75
9	26.00	7.6	.44	4.94	85.2	17.7	3.35	189100	4.61	0.76
8	24.48	7.2	.47	5.16	62.8	14.1	2.97	150100	4.45	0.79
8	20.15	5.9	.31	5.00	55.6	12.2	3.08	129800	3.90	0.82
7	23.46	6.9	.54	5.10	45.5	11.7	2.57	124600	4.30	0.79
7	18.11	5.3	.31	4.87	38.8	9.7	2.70	103000	3.55	0.82
6	18.36	5.4	.43	4.53	26.8	8.2	2.25	87700	2.73	0.72
6	15.30	4.5	.28	4.38	24.0	7.3	2.33	77400	2.38	0.73

Add to coefficient C for every lb. increase in weight of beam, for 10-in. beams, 4900 lbs.; 9-in., 4500 lbs.; 8-in., 4000 lbs.; 7-in., 3400 lbs., 6-in., 3000 lbs.

Carnegie Bulb Angles.

10	26.50	7.80	.48	3.5	104.2	19.9	3.66	211700	....	....
9	21.80	6.41	.44	3.5	69.3	14.5	3.33	154200	....	....
8	19.23	5.66	.41	3.5	48.8	11.7	2.95	124800	....	....
7	18.25	5.37	.44	3.0	34.9	9.6	2.56	102300	....	....
6	17.20	5.06	.50	3.0	23.9	7.6	2.16	80500	....	....
6	13.75	4.04	.38	3.0	20.1	6.6	2.21	70400	....	....
6	12.30	3.62	.31	3.0	18.6	5.7	2.28	60400	....	....
5	10.00	2.94	.31	2.5	10.2	4.1	1.86	43300	....	....

Carnegie T Shapes.

1	2	3	4	5	6	7	8	9	10	11
Size: Flange by Stem.	Weight per foot	Area of Section.	Distance of C. of G. from Outside of Flange.	Mom. of Inertia, Neutral Axis through C. of G. Parallel to Flange.	Least Section Modulus Neut. Axis through C. of G. Parallel to Flange.	Radius of Gyration, Neut. Axis through C. of G. Parallel to Flange.	Mom. of Inertia, Neutral Axis through C. of G. Coincident with Stem.	Section Modulus, Neut. Axis through C. of G. Coincident with Stem.	Radius of Gyration, Neut. Axis through C. of G. Coincident with Stem.	Coefficient of Strength for Fibre Stress of 12,000 lbs. per sq. in., Neutral Axis through C. of G. Parallel to Flange.
in.	lbs.	sq. in.	in.	I	S	r	I'	S'	r'	C
5 ×3	13.6	3.99	0.75	2.6	1.18	0.82	5.6	2.22	1.19	9410
5 ×2½	11.0	2.24	0.65	1.6	0.86	0.71	4.3	1.70	1.16	6900
4½×3½	15.8	4.65	1.11	5.1	2.13	1.04	3.7	1.65	0.90	17020
4½×3	8.5	2.55	0.73	1.8	0.81	0.87	2.0	1.16	1.03	6490
4½×3	10.0	3.00	0.75	2.1	0.94	0.86	3.1	1.38	1.04	7540
4½×2½	8.0	2.40	0.58	1.1	0.56	0.69	2.6	1.16	1.07	4520
4½×2½	9.3	2.79	0.60	1.2	0.65	0.68	3.1	1.38	1.08	5230
4 ×5	15.6	4.56	1.56	10.7	3.10	1.54	2.8	1.41	0.79	24800
4 ×5	12.0	3.54	1.51	8.5	2.43	1.56	2.1	1.06	0.78	19410
4 ×4½	14.6	4.29	1.37	8.0	2.55	1.37	2.8	1.41	0.81	20400
4 ×4½	11.4	3.36	1.31	6.3	1.98	1.38	2.1	1.06	0.80	15840
4 ×4	13.7	4.02	1.18	5.7	2.02	1.20	2.8	1.40	0.84	16190
4 ×4	10.9	3.21	1.15	4.7	1.64	1.23	2.2	1.09	0.84	13100
4 ×3	9.3	2.73	0.78	2.0	0.88	0.86	2.1	1.05	0.88	7070
4 ×2½	8.6	2.52	0.68	1.2	0.62	0.69	2.1	1.05	0.92	4980

Carnegie T Shapes—(*Continued*).

1	2	3	4	5	6	7	8	9	10	11
Size; Flange by Stem.	Weight per foot.	Area of Section.	Distance of C. of G. from Outside of Flange.	Mom. of Inertia, Neutral Axis through C. of G. Parallel to Flange.	Least Section Modulus, Neut. Axis through C. of G. Parallel to Flange.	Radius of Gyration, Neut. Axis through C. of G. Parallel to Flange.	Mom. of Inertia, Neutral Axis through C. of G. Coincident with Stem.	Section Modulus, Neutral Axis through C. of G. Coincident with Stem.	Radius of Gyration, Neut. Axis through C. of G. Coincident with Stem.	Coeff. of Strength for Fibre Stress of 12,000 lbs. per sq. in., Neutral Axis through C. of G. Parallel to Flange.
in.	lbs.	sq. in.	in.	I	S	r	I'	S'	r'	C
4 ×2½	7.3	2.16	0.60	1.0	0.55	0.70	1.8	0.88	0.91	4380
4 ×2½	5.8	1.71	0.56	0.81	0.42	0.71	1.4	0.71	0.94	3350
4 ×2	7.9	2.31	0.48	0.60	0.40	0.52	2.1	1.05	0.96	3180
4 ×2	6.6	1.95	0.51	0.54	0.34	0.51	1.8	0.88	0.95	2700
3½×4	12.8	3.75	1.25	5.5	1.98	1.21	1.89	1.08	0.72	15870
3½×4	9.9	2.91	1.19	4.3	1.55	1.22	1.42	0.81	0.70	12380
3½×3½	11.7	3.45	1.06	3.7	1.52	1.04	1.89	1.08	0.74	12000
3½×3½	9.2	2.70	1.01	3.0	1.19	1.05	1.42	0.81	0.73	9530
3½×3½	6.8	2.04	0.98	2.3	0.93	1.09	1.07	0.61	0.73	7450
3½×3	11.73	3.45	1.01	2.9	1.43	0.92	1.74	1.00	0.72	11470
3½×3	10.9	3.21	0.88	2.4	1.13	0.87	1.88	1.08	0.77	9050
3½×3	8.5	2.49	0.83	1.9	0.88	0.88	1.41	0.81	0.75	7040
3½×3	7.8	2.28	0.78	1.6	0.72	0.89	1.18	0.68	0.76	5790
3 ×4	11.8	3.48	1.32	5.2	1.94	1.23	1.21	0.81	0.59	15480
3 ×4	10.6	3.12	1.32	4.8	1.78	1.25	1.09	0.72	0.60	14270
3 ×4	9.3	2.73	1.29	4.3	1.57	1.26	0.93	0.62	0.59	12540
3 ×3½	10.9	3.21	1.12	3.5	1.49	1.06	1.20	0.80	0.62	11910
3 ×3½	9.8	2.88	1.11	3.3	1.37	1.08	1.31	0.88	0.68	10990
3 ×3½	8.5	2.49	1.09	2.9	1.21	1.09	0.93	0.62	0.61	9680
3 ×3	10.0	2.94	0.93	2.3	1.10	0.88	1.20	0.80	0.64	8780
3 ×3	9.1	2.67	0.92	2.1	1.01	0.90	1.08	0.72	0.64	8110
3 ×3	7.8	2.28	0.88	1.8	0.86	0.90	0.90	0.60	0.63	6900
3 ×3	6.6	1.95	0.86	1.6	0.74	0.90	0.75	0.50	0.62	5900
3 ×2½	7.2	2.10	0.71	1.1	0.60	0.72	0.89	0.60	0.66	4800
3 ×2½	6.1	1.80	0.68	0.94	0.52	0.73	0.75	0.50	0.65	4100
2¾×2	7.4	2.16	0.53	1.1	0.75	0.71	0.62	0.45	0.54	6000
2¾×1¾	6.6	1.95	0.64	0.56	0.50	0.53	0.61	0.44	0.56	4000
2½×3	7.2	2.10	0.97	1.8	0.87	0.92	0.54	0.43	0.51	6960
2½×3	6.1	1.80	0.92	1.6	0.76	0.94	0.44	0.35	0.51	6110
2½×2¾	6.7	1.98	0.87	1.4	0.73	0.84	0.66	0.53	0.58	5860
2½×2¾	5.8	1.71	0.83	1.2	0.60	0.83	0.44	0.35	0.51	4830
2½×2½	6.4	1.89	0.76	1.0	0.59	0.74	0.52	0.42	0.53	4700
2½×2½	5.5	1.62	0.74	0.87	0.50	0.74	0.44	0.35	0.52	4000
2½×1¼	2.9	0.84	0.29	0.094	0.09	0.31	0.29	0.23	0.58	700
2¼×2¼	4.9	1.44	0.69	0.66	0.42	0.68	0.33	0.30	0.48	3360
2¼×2¼	4.1	1.20	0.66	0.51	0.32	0.67	0.25	0.22	0.47	2600
2 ×2	4.3	1.26	0.63	0.45	0.33	0.60	0.23	0.23	0.43	2610
2 ×2	3.7	1.08	0.59	0.36	0.25	0.60	0.18	0.18	0.42	2000
2 ×1½	3.1	0.90	0.42	0.16	0.15	0.42	0.18	0.18	0.45	1200
1¾×1¾	3.1	0.90	0.54	0.23	0.19	0.51	0.12	0.14	0.37	1550
1¾×1¼	3.6	1.05	0.91	0.12	0.15	0.33	0.19	0.22	0.41	1150
1¾×1¼	1.94	0.57	0.33	0.07	0.08	0.35	0.09	0.11	0.40	620
1½×1½	2.6	0.75	0.42	0.15	0.14	0.49	0.08	0.10	0.34	1170
1½×1½	1.84	0.54	0.44	0.11	0.11	0.45	0.06	0.07	0.31	860
1½×1¼	3.0	0.87	0.40	0.10	0.12	0.35	0.10	0.13	0.34	940
1½×1¼	2.24	0.66	0.38	0.09	0.10	0.36	0.08	0.10	0.34	785
1½×1¼	1.73	0.51	0.35	0.07	0.08	0.36	0.06	0.07	0.33	600
1½×1½	1.33	0.39	0.35	0.04	0.05	0.33	0.03	0.04	0.29	420
1½× ¾	1.33	0.39	0.20	0.01	0.03	0.19	0.05	0.07	0.37	210
1¼×1¼	2.04	0.60	0.40	0.08	0.10	0.36	0.05	0.07	0.27	760
1¼×1¼	1.53	0.45	0.38	0.06	0.07	0.37	0.03	0.05	0.26	580
1 ×1½	1.12	0.33	0.50	0.08	0.08	0.48	0.01	0.02	0.19	605
1 ×1	1.23	0.36	0.32	0.03	0.05	0.29	0.02	0.04	0.21	370
1 ×1	0.87	0.26	0.29	0.02	0.03	0.29	0.01	0.02	0.21	270

Properties of Standard and Special Angles of Minimum and Maximum Thicknesses and Weights.

ANGLES WITH EQUAL LEGS.

1	2	3	4	5	6	7	8	9
Dimensions.	Thickness.	Weight per Foot.	Area of Section.	Distance of Centre of Gravity from Back of Flange.	Moment of Inertia, Neutral Axis through Centre of Gravity Parallel to Flange.	Section Modulus, Neutral Axis through Centre of Gravity Parallel to Flange.	Radius of Gyration, Neutral Axis through Centre of Gravity Parallel to Flange.	Least Radius of Gyration, Neutral Axis through Centre of Gravity at Angle of 45° to Flanges.
in.	in.		sq. in.	in.	I	S	r	r'
6 × 6	⅞	33.1	9.74	1.82	31.92	7.64	1.81	1.17
6 × 6	7/16	17.2	5.06	1.66	17.68	4.07	1.87	1.19
*5 × 5	⅞	27.2	7.99	1.57	17.75	5.17	1.49	0.98
*5 × 5	⅜	12.3	3.61	1.39	8.74	2.42	1.56	0.99
4 × 4	13/16	19.9	5.84	1.29	8.14	3.01	1.18	0.80
4 × 4	5/16	8.2	2.40	1.12	3.71	1.29	1.24	0.82
3½ × 3½	13/16	17.1	5.03	1.17	5.25	2.25	1.02	0.69
3½ × 3½	⅜	8.5	2.48	1.01	2.87	1.15	1.07	0.70
3 × 3	⅝	11.4	3.36	0.98	2.62	1.30	0.88	0.59
3 × 3	¼	4.9	1.44	0.84	1.24	0.58	0.93	0.60
*2¾ × 2¾	½	8.5	2.50	0.87	1.67	0.89	0.82	0.54
*2¾ × 2¾	¼	4.5	1.31	0.78	0.93	0.48	0.85	0.55
2½ × 2½	½	7.7	2.25	0.81	1.23	0.73	0.74	0.49
2½ × 2½	¼	4.1	1.19	0.72	0.70	0.40	0.77	0.50
*2¼ × 2¼	½	6.8	2.00	0.74	0.87	0.58	0.66	0.48
*2¼ × 2¼	¼	3.7	1.06	0.66	0.51	0.32	0.69	0.46
2 × 2	7/16	5.3	1.56	0.66	0.54	0.40	0.59	0.39
2 × 2	3/16	2.5	0.72	0.57	0.28	0.19	0.62	0.40
1¾ × 1¾	7/16	4.6	1.30	0.59	0.35	0.30	0.51	0.35
1¾ × 1¾	3/16	2.1	0.62	0.51	0.18	0.14	0.54	0.36
1½ × 1½	⅜	3.4	0.99	0.51	0.19	0.19	0.44	0.31
1½ × 1½	3/16	1.8	0.53	0.44	0.11	0.104	0.46	0.32
1¼ × 1¼	5/16	2.4	0.69	0.42	0.09	0.109	0.36	0.25
1¼ × 1¼	⅛	1.0	0.30	0.35	0.044	0.049	0.38	0.26
*1⅛ × 1⅛	5/16	2.1	0.61	0.39	0.063	0.087	0.32	0.24
*1⅛ × 1⅛	⅛	0.9	0.27	0.32	0.032	0.039	0.34	0.23
1 × 1	¼	1.5	0.44	0.34	0.037	0.056	0.29	0.20
1 × 1	⅛	0.8	0.24	0.30	0.022	0.031	0.31	0.21
*⅞ × ⅞	3/16	1.0	0.29	0.29	0.019	0.033	0.26	0.18
*⅞ × ⅞	⅛	0.7	0.21	0.26	0.014	0.023	0.26	0.19
¾ × ¾	3/16	0.8	0.25	0.26	0.012	0.024	0.22	0.16
¾ × ¾	⅛	0.6	0.17	0.23	0.009	0.017	0.23	0.17
*⅝ × ⅝	⅛	0.5	0.14	0.20	0.005	0.011	0.18	0.13

Angles marked * are special.

Properties of Standard and Special Angles of Minimum and Maximum Thickness and Weights.

ANGLES WITH UNEQUAL LEGS.

1	2	3	4	5	6	7	8	9	10	11
				Moments of Inertia. I		Section Modulus. S		Radii of Gyration. r		
Dimensions.	Thickness.	Weight per Foot.	Area of Section.	Neutral Axis Parallel to Longer Flange.	Neutral Axis Parallel to Shorter Flange.	Neutral Axis Parallel to Longer Flange.	Neutral Axis Parallel to Shorter Flange.	Neutral Axis Parallel to Longer Flange.	Neutral Axis Parallel to Shorter Flange.	Least Radius. Axis diagonal.
Inches.	Inch.	lbs.	sq. in.							
*7 × 3½	1	32.3	9.50	7.53	45.37	2.96	10.58	0.89	2.19	.88
*7 × 3½	7/16	15.0	4.40	3.95	22.56	1.47	5.01	0.95	2.26	.89
6 × 4	⅞	27.2	7.99	9.75	27.73	3.39	7.15	1.11	1.86	.86
6 × 4	⅜	12.3	3.61	4.90	13.47	1.60	3.33	1.17	1.93	.88
6 × 3½	⅞	25.7	7.55	6.55	26.33	2.59	6.98	0.93	1.87	.76
6 × 3½	⅜	11.7	3.42	3.34	12.86	1.23	3.25	0.99	1.94	.77
*5 × 4	⅞	24.2	7.11	9.23	16.42	3.31	4.99	1.14	1.52	.83
*5 × 4	⅜	11.0	3.23	4.67	8.14	1.57	2.34	1.20	1.59	.86
5 × 3½	⅞	22.7	6.67	6.21	15.67	2.52	4.88	0.96	1.53	.77
5 × 3½	⅜	10.4	3.05	3.18	7.78	1.21	2.29	1.02	1.60	.76
5 × 3	13/16	19.9	5.84	3.71	13.98	1.74	4.45	0.80	1.55	.66
5 × 3	5/16	8.2	2.40	1.75	6.26	0.75	1.89	0.85	1.61	.66
*4½ × 3	13/16	18.5	5.43	3.60	10.33	1.71	3.62	0.81	1.38	.67
*4½ × 3	⅜	9.1	2.67	1.98	5.50	0.88	1.83	0.86	1.44	.66
*4 × 3½	13/16	18.5	5.43	5.49	7.77	2.30	2.92	1.01	1.19	.74
*4 × 3½	⅜	9.1	2.67	2.99	4.18	1.18	1.50	1.06	1.25	.73
4 × 3	13/16	17.1	5.03	3.47	7.34	1.68	2.87	0.83	1.21	.66
4 × 3	5/16	7.1	2.09	1.65	3.38	0.74	1.23	0.89	1.27	.65
3½ × 3	13/16	15.7	4.62	3.33	4.98	1.65	2.20	0.85	1.04	.65
3½ × 3	5/16	6.6	1.93	1.56	2.33	0.72	0.96	0.90	1.10	.63
3½ × 2½	11/16	12.4	3.65	1.72	4.13	0.99	1.85	0.67	1.06	.55
3½ × 2½	¼	4.9	1.44	0.78	1.80	0.41	0.75	0.74	1.12	.55
*3¼ × 2	9/16	9.0	2.64	0.75	2.64	0.53	1.30	0.53	1.00	.45
*3¼ × 2	¼	4.3	1.25	0.40	1.36	0.26	0.63	0.57	1.04	.44
3 × 2½	9/16	9.5	2.78	1.42	2.28	0.82	1.15	0.72	0.91	.54
3 × 2½	¼	4.5	1.31	0.74	1.17	0.40	0.56	0.75	0.95	.53
*3 × 2	½	7.7	2.25	0.67	1.92	0.47	1.00	0.55	0.92	.47
*3 × 2	¼	4.0	1.19	0.39	1.09	0.25	0.54	0.56	0.95	.46
2½ × 2	½	6.8	2.00	0.64	1.14	0.46	0.70	0.56	0.75	.44
2½ × 2	3/16	2.8	0.81	0.29	0.51	0.20	0.29	0.60	0.79	.43
*2¼ × 1½	½	5.5	1.62	0.26	0.82	0.26	0.59	0.40	0.71	.39
*2¼ × 1½	3/16	2.3	0.67	0.12	0.34	0.11	0.23	0.43	0.73	.40
*2 × 1⅜	¼	2.7	0.78	0.12	0.37	0.12	0.28	0.39	0.63	.30
*2 × 1⅜	3/16	2.1	0.60	0.09	0.24	0.09	0.18	0.40	0.63	.29
*1¾ × 1	¼	1.8	0.53	0.04	0.09	0.05	0.09	0.27	0.41	.25
*1¾ × 1	⅛	1.0	0.28	0.02	0.05	0.03	0.06	0.29	0.44	.22

Angles marked * are special.

Properties of Carnegie Z Bars.

(For dimensions see table on page 178.)

1	2	3	4	5	6	7	8	9	10	11	12
Section Index.	Weight per Foot.	Area of Section.	Mom. of Inertia, Neutral Axis through C. of Gr. Perpendicular to Web.	Mom. of Inertia, Neutral Axis through C. of Gr. Coincident with Web.	Section Modulus, Neutral Axis through C. of Gr. Perpendicular to Web.	Section Modulus, Neutral Axis through C. of Gr. Coincident with Web.	Radii of Gyration, Neut. Axis through C. of Gr. Perpendicular to Web.	Radii of Gyration, Neut. Axis through C. of Gr. Coincident with Web.	Radii of Gyration, Least Radius, Neutral Axis Diagonal.	**Coeff. of Strength for Fibre Stress of 16,000 lbs. per sq. in., Axis Perpendicular to Web at Centre.**	**Coeff. of Strength for Fibre Stress of 12,000 lbs. per sq. in., Axis Perpendicular to Web at Centre.**
	lbs.	sq. in.	I	I	S	S	r	r	r	**C**	**C'**
Z1	15.6	4.59	25.32	9.11	8.44	2.75	2.35	1.41	0.83	**90,000**	**67,500**
"	18.3	5.39	29.80	10.95	9.83	3.27	2.35	1.43	0.84	**104,800**	**78,600**
"	21.0	6.19	34.36	12.87	11.22	3.81	2.36	1.44	0.84	**119,700**	**89,800**
Z2	22.7	6.68	34.64	12.59	11.55	3.91	2.28	1.37	0.81	**123,200**	**92,400**
"	25.4	7.46	38.86	14.42	12.82	4.43	2.28	1.39	0.82	**136,700**	**102,600**
"	28.0	8.25	43.18	16.34	14.10	4.98	2.29	1.41	0.84	**150,400**	**112,800**
Z3	29.3	8.63	42.12	15.44	14.04	4.94	2.21	1.34	0.81	**149,800**	**112,300**
"	32.0	9.40	46.13	17.27	15.22	5.47	2.22	1.36	0.82	**162,300**	**121,800**
"	34.6	10.17	50.22	19.18	16.40	6.02	2.22	1.37	0.83	**174,900**	**131,200**
Z4	11.6	3.40	13.36	6.18	5.34	2.00	1.98	1.35	0.75	**57,000**	**42,700**
"	13.9	4.10	16.18	7.65	6.39	2.45	1.99	1.37	0.76	**68,200**	**51,100**
"	16.4	4.81	19.07	9.20	7.44	2.92	1.99	1.38	0.77	**79,400**	**59,500**
Z5	17.8	5.25	19.19	9.05	7.68	3.02	1.91	1.31	0.74	**81,900**	**61,400**
"	20.2	5.94	21.83	10.51	8.62	3.47	1.91	1.33	0.75	**91,900**	**69,000**
"	22.6	6.64	24.53	12.06	9.57	3.94	1.92	1.35	0.76	**102,100**	**76,600**
Z6	23.7	6.96	23.68	11.37	9.47	3.91	1.84	1.28	0.73	**101,000**	**75,800**
"	26.0	7.64	26.16	12.83	10.34	4.37	1.85	1.30	0.75	**110,300**	**82,700**
"	28.3	8.33	28.70	14.36	11.20	4.84	1.86	1.31	0.76	**119,500**	**89,600**
Z7	8.2	2.41	6.28	4.23	3.14	1.44	1.62	1.33	0.67	**33,500**	**25,100**
"	10.3	3.03	7.94	5.46	3.91	1.84	1.62	1.34	0.68	**41,700**	**31,300**
"	12.4	3.66	9.63	6.77	4.67	2.26	1.62	1.36	0.69	**49,800**	**37,400**
Z8	13.8	4.05	9.66	6.73	4.83	2.37	1.55	1.29	0.66	**51,500**	**38,600**
"	15.8	4.66	11.18	7.96	5.50	2.77	1.55	1.31	0.67	**58,700**	**44,000**
"	17.9	5.27	12.74	9.26	6.18	3.19	1.55	1.33	0.69	**65,900**	**49,400**
Z9	18.9	5.55	12.11	8.73	6.05	3.18	1.48	1.25	0.66	**64,500**	**48,400**
"	20.9	6.14	13.52	9.95	6.65	3.58	1.48	1.27	0.67	**70,900**	**53,200**
"	22.9	6.75	14.97	11.24	7.26	4.00	1.49	1.29	0.69	**77,400**	**58,100**
Z10	6.7	1.97	2.87	2.81	1.92	1.10	1.21	1.19	0.55	**20,500**	**15,400**
"	8.4	2.48	3.64	3.64	2.38	1.40	1.21	1.21	0.56	**25,400**	**19,000**
Z11	9.7	2.86	3.85	3.92	2.57	1.57	1.16	1.17	0.55	**27,400**	**20,600**
"	11.4	3.36	4.57	4.75	2.98	1.88	1.17	1.19	0.56	**31,800**	**23,800**
Z12	12.5	3.69	4.59	4.85	3.06	1.99	1.12	1.15	0.55	**32,600**	**24,500**
"	14.2	4.18	5.26	5.70	3.43	2.31	1.12	1.17	0.56	**36,600**	**27,400**

Dimensions of lightest weight bars of each size: Z1, Z2, and Z3, depth of web 6 in., width of flange 3½ in., thickness of metal respectively ⅜, 9/16, and ¾ in.; Z4, Z5, Z6, 5 × 3¼ × 5/16, ½, and 11/16 in.; Z7, Z8, Z9, 4 × 3 1/16 × ¼, 7/16, and ⅝ in.; Z10, Z11, Z12, 3 × 2 11/16 × ¼, ⅜, and ½ in. Each dimension is increased 1/16 in. in the next heavier weight.

FLOORING MATERIAL.

For fire-proof flooring, the space between the floor-beams may be spanned with brick arches, or with hollow brick made especially for the purpose, the latter being much lighter than ordinary brick.

Arches 4 inches deep of solid brick weigh about 70 lbs. per square foot, including the concrete levelling material, and substantial floors are thus made up to 6 feet span of arch, or much greater span if the skew backs at the springing of the arch are made deeper, the rise of the arch being preferably not less than 1/10 of the span. Hollow brick for floors are usually in depth about 1/8 of the span, and are used up to, and even exceeding, spans of 10 feet. The weight of the latter material will vary from 20 lbs. per square foot for 3-foot spans up to 60 lbs. per square foot for spans of 10 feet. Full particulars of this construction are given by the manufacturers. For supporting brick floors the beams should be securely tied with rods to resist the lateral pressure.

In the following cases the loads, in addition to the weight of the floor itself, may be assumed as:

For street bridges for general public traffic	80 lbs.	per sq. ft.
For floors of dwellings	40 lbs.	" "
For churches, theatres, and ball-rooms	80 lbs.	" "
For hay-lofts	80 lbs.	" "
For storage of grain	100 lbs.	" "
For warehouses and general merchandise	250 lbs.	" "
For factories	200 to 400 lbs.	" "
For snow thirty inches deep	16 lbs.	" "
For maximum pressure of wind	50 lbs.	" "
For brick walls	112 lbs.	per cu. ft.
For masonry walls	116–144 lbs.	" "

Roofs, allowing thirty pounds per square foot for wind and snow:

For corrugated iron laid directly on the purlins	37 lbs.	per sq. ft.
For corrugated iron laid on boards	40 lbs.	" "
For slate nailed to laths	43 lbs.	" "
For slate nailed on boards	46 lbs.	" "

If plastered below the rafters, the weight will be about ten pounds per square foot additional.

TIE-RODS FOR BEAMS SUPPORTING BRICK ARCHES.

The horizontal thrust of brick arches is as follows:

$$\frac{1.5WS^2}{R} = \text{pressure in pounds per lineal foot of arch:}$$

W = load in pounds per square foot;
S = span of arch in feet;
R = rise in inches.

Place the tie-rods as low through the webs of the beams as possible and spaced so that the pressure of arches as obtained above will not produce a greater stress than 15,000 lbs. per square inch of the least section of the bolt.

TORSIONAL STRENGTH.

Let a horizontal shaft of diameter $= d$ be fixed at one end, and at the other or free end, at a distance $= l$ from the fixed end, let there be fixed a horizontal lever arm with a weight $= P$ acting at a distance $= a$ from the axis of the shaft so as to twist it; then Pa = moment of the applied force.

Resisting moment = twisting moment $= \frac{SJ}{c}$, in which S = unit shearing resistance, J = polar moment of inertia of the section with respect to the axis, and c = distance of the most remote fibre from the axis, in a cross-section. For a circle with diameter d,

$$J = \frac{\pi d^4}{32}; \qquad c = \tfrac{1}{2}d;$$

$$Pa = \frac{SJ}{c} = \frac{\pi d^3 S}{16} = \frac{d^3 S}{5.1} = .1963d^3S; \quad d = \sqrt[3]{\frac{5.1Pa}{S}}.$$

For hollow shafts of external diameter d and internal diameter d_1,

$$Pa = .1963\frac{d^4 - d_1^4}{d}S; \qquad d = \sqrt[3]{\frac{5.1Pa}{\left(1 - \frac{d_1^4}{d^4}\right)S}}.$$

For a square whose side $= d$,

$$J = \frac{d^4}{6}; \quad c = d\sqrt{\tfrac{1}{2}}; \quad \frac{SJ}{c} = Pa = \frac{d^3S}{4.2426} = 0.236d^3S.$$

For a rectangle whose sides are b and d,

$$J = \frac{bd^3}{12} + \frac{b^3d}{12}; \quad c = \tfrac{1}{2}\sqrt{b^2 + d^2}; \quad \frac{SJ}{c} = Pa = \frac{(bd^3 + b^3d)S}{6\sqrt{b^2 + d^2}}.$$

The above formulæ are based on the supposition that the shearing resistance at any point of the cross-section is proportional to its distance from the axis; but this is true only within the elastic limit. In materials capable of flow, while the particles near the axis are strained within the elastic limit those at some distance within the circumference may be strained nearly to the ultimate resistance, so that the total resistance is something greater than that calculated by the formulæ. (See Thurston, "Matls. of Eng.," Part II. p. 527.) Saint Venant finds for square shafts $Pa = 0.208d^3S$ (Cotterill, "Applied Mechanics," pp. 348, 355). For working strength, however, the formulæ may be used, with S taken at the safe working unit resistance.

For a rectangle, sides b (longer) and d (shorter) and area A,

$$Pa = \frac{SA^2}{3b + 1.8d}.$$

The ultimate torsional shearing resistance S is about the same as the direct shearing resistance, and may be taken at 20,000 to 25,000 lbs. per square inch for cast iron, 45,000 lbs. for wrought iron, and 50,000 to 150,000 lbs. for steel, according to its carbon and temper. Large factors of safety should be taken, especially when the direction of stress is reversed, as in reversing engines, and when the torsional stress is combined with other stresses, as is usual in shafting. (See "Shafting.")

Elastic Resistance to Torsion.—Let l = length of bar being twisted, d = diameter, P = force applied at the extremity of a lever arm of length $= a$, Pa = twisting moment, G = torsional modulus of elasticity, θ = angle through which the free end of the shaft is twisted, measured in arc of radius = 1.

For a cylindrical shaft

$$Pa = \frac{\pi\theta Gd^4}{32l}; \qquad \theta = \frac{32Pal}{\pi d^4G}; \qquad G = \frac{32Pal}{\theta\pi d^4}; \qquad \frac{32}{\pi} = 10.186.$$

If a = angle of torsion in degrees,

$$\theta = \frac{a\pi}{180}; \qquad a = \frac{180\theta}{\pi} = \frac{180 \times 32Pal}{\pi^2d^4G} = \frac{583.6Pal}{d^4G}.$$

The value of G is given by different authorities as from $\tfrac{1}{3}$ to 2/5 of E, the modulus of elasticity for tension.

COMBINED STRESSES.

(From Merriman's "Strength of Materials.")

Combined Tension and Flexure.—Let A = the area of a bar subjected to both tension and flexure, P = tensile stress applied at the ends, $P \div A$ = unit tensile stress, S = unit stress at the fibre on the tensile side most remote from the neutral axis, due to flexure alone, then maximum tensile unit stress $= (P \div A) + S$. A beam to resist combined tension and flexure should be designed so that $(P \div A) + S$ shall not exceed the proper allowable working unit stress.

Combined Compression and Flexure.—If $P \div A$ = unit stress due to compression alone, and S = unit compressive stress at fibre most remote from neutral axis, due to flexure alone, then maximum compressive unit stress $= (P \div A) + S$.

Combined Tension (or Compression) and Shear.—If applied tension (or compression) unit stress = p, applied shearing unit stress = v, then from the combined action of the two forces

$$\text{Max. } S = \pm \sqrt{v^2 + \tfrac{1}{4}p^2}, \quad \text{Maximum shearing unit stress;}$$

$$\text{Max. } t = \tfrac{1}{2}p + \sqrt{v^2 + \tfrac{1}{4}p^2}, \quad \text{Maximum tensile (or compressive) unit stress.}$$

Combined Flexure and Torsion.—If S = greatest unit stress due to flexure alone, and S_s = greatest torsional shearing unit stress due to torsion alone, then for the combined stresses

$$\text{Max. tension or compression unit stress } t = \tfrac{1}{2}S + \sqrt{S_s^2 + \tfrac{1}{4}S^2};$$

$$\text{Max. shear } s = \pm \sqrt{S_s^2 + \tfrac{1}{4}S^2}.$$

Formula for diameter of a round shaft subjected to transverse load while transmitting a given horse-power (see also Shafts of Engines):

$$d^3 = \frac{16M}{\pi t} + \frac{16}{t}\sqrt{\frac{M^2}{\pi^2} + \frac{402{,}500{,}000H^2}{n^2}},$$

where M = maximum bending moment of the transverse forces in pound-inches, H = horse-power transmitted, n = No. of revs. per minute, and t = the safe allowable tensile or compressive working strength of the material.

Combined Compression and Torsion.—For a vertical round shaft carrying a load and also transmitting a given horse-power, the resultant maximum compressive unit stress

$$t = \frac{4P}{\pi d^2} + \sqrt{321{,}000^2\frac{H^2}{n^2d^6} + \frac{16P^2}{\pi^2d^4}},$$

in which P is the load. From this the diameter d may be found when t and the other data are given.

Stress due to Temperature.—Let l = length of a bar, A = its sectional area, c = coefficient of linear expansion for one degree, t = rise or fall in temperature in degrees, E = modulus of elasticity, λ the change of length due to the rise or fall t; if the bar is free to expand or contract, $\lambda = ctl$

If the bar is held so as to prevent its expansion or contraction the stress produced by the change of temperature = $S = ActE$. The following are average values of the coefficients of linear expansion for a change in temperature of one degree Fahrenheit:

For brick and stone.... $a = 0.0000050$,
For cast iron........... $a = 0.0000062$,
For wrought iron....... $a = 0.0000067$,
For steel............... $a = 0.0000065$.

The stress due to temperature should be added to or subtracted from the stress caused by other external forces according as it acts to increase or to relieve the existing stress.

What stress will be caused in a steel bar 1 inch square in area by a change of temperature of 100° F.? $S = ActE = 1 \times .0000065 \times 100 \times 30{,}000{,}000 =$ 19,500 lbs. Suppose the bar is under tension of 19,500 lbs. between rigid abutments before the change in temperature takes place, a cooling of 100° F. will double the tension, and a heating of 100° will reduce the tension to zero.

STRENGTH OF FLAT PLATES.

For a circular plate supported at the edge, uniformly loaded, according to Grashof,

$$f = \frac{5}{6}\frac{r^2}{t^2}p; \qquad t = \sqrt{\frac{5r^2p}{6f}}; \qquad p = \frac{6ft^2}{5r^2}.$$

For a circular plate fixed at the edge, uniformly loaded,

$$f = \frac{2}{3}\frac{r^2}{t^2}p; \qquad t = \sqrt{\frac{2}{3}\frac{r^2p}{f}}; \qquad p = \frac{3ft^2}{2r^2};$$

in which f denotes the working stress; r, the radius in inches; t, the thickness in inches; and p, the pressure in pounds per square inch.

For mathematical discussion, see Lanza, "Applied Mechanics," p. 900, etc.
Lanza gives the following table, using a factor of safety of 8, with tensile strength of cast iron 20,000, of wrought iron 40,000, and of steel 80,000:

	Supported.	Fixed.
Cast iron	$t = .0182570r\sqrt{p}$	$t = .0163300r\sqrt{p}$
Wrought iron	$t = .0117850r\sqrt{p}$	$t = .0105410r\sqrt{p}$
Steel	$t = .0091287r\sqrt{p}$	$t = .0081649r\sqrt{p}$

For a circular plate supported at the edge, and loaded with a concentrated load P applied at a circumference the radius of which is r_0:

$$f = \left(\frac{4}{3}\log\frac{r}{r_0} + 1\right)\frac{P}{\pi t^2} = c\frac{P}{\pi t^2};$$

for $\frac{r}{r_0} =$ 10 20 30 40 50;

$c =$ 4.07 5.00 5.53 5.92 6.22;

$$t = \sqrt{\frac{cP}{\pi f}}; \qquad P = \frac{\pi t^2 f}{c}.$$

The above formulæ are deduced from theoretical considerations, and give thicknesses much greater than are generally used in steam-engine cylinder-heads. (See empirical formulæ under Dimensions of Parts of Engines.) The theoretical formulæ seem to be based on incorrect or incomplete hypotheses, but they err in the direction of safety.

The Strength of Unstayed Flat Surfaces.—Robert Wilson (*Eng'g*, Sept. 24, 1877) draws attention to the apparent discrepancy between the results of theoretical investigations and of actual experiments on the strength of unstayed flat surfaces of boiler-plate, such as the unstayed flat crowns of domes and of vertical boilers.

Rankine's "Civil Engineering" gives the following rules for the strength of a circular plate *supported* all round the edge, prefaced by the remark that "the formula is founded on a theory which is only approximately true, but which nevertheless may be considered to involve no error of practical importance:"

$$M = \frac{Wb}{6\pi} = \frac{Pb^2}{24}.$$

Here

M = greatest bending moment;

W = total load uniformly distributed $= \frac{Pb^2\pi}{4}$;

b = diameter of plate in inches;

P = bursting pressure in pounds per square inch.

Calling t the thickness in inches, for a plate *supported* round the edges,

$$M = \frac{1}{6}\,42{,}000bt^2; \qquad \therefore \quad \frac{Pb^2}{24} = 7000t^2.$$

For a plate fixed round the edges,

$$\frac{2}{3}\,\frac{Pb^2}{24} = 7000t^2; \quad \text{whence } P = \frac{t^2 \times 63{,}000}{r^2},$$

where r = radius of the plate.

Dr. Grashof gives a formula from which we have the following rule:

$$P = \frac{t^2 \times 72{,}000}{r^2}.$$

This formula of Grashof's has been adopted by Professor Unwin in his "Elements of Machine Design." These formulæ by Rankine and Grashof may be regarded as being practically the same.

On trying to make the rules given by these authorities agree with the results of his experience of the strength of unstayed flat ends of cylindrical boilers and domes that had given way after long use, Mr. Wilson was led to believe that the above rules give the breaking strength much lower than it actually is. He describes a number of experiments made by Mr. Nichols of Kirkstall, which gave results varying widely from each other, as the method of supporting the edges of the plate was varied, and also varying widely from the calculated bursting pressures, the actual results being in all cases very much the higher. Some conclusions drawn from these results are:

1. Although the bursting pressure has been found to be so high, boiler-makers must be warned against attaching any importance to this, since the plates deflected almost as soon as any pressure was put upon them and sprang back again on the pressure being taken off. This springing of the plate in the course of time inevitably results in grooving or channelling, which, especially when aided by the action of the corrosive acids in the water or steam, will in time reduce the thickness of the plate, and bring about the destruction of an unstayed surface at a very low pressure.

2. Since flat plates commence to deflect at very low pressures, they should never be used without stays; but it is better to dish the plates when they are not stayed by flues, tubes, etc.

3. Against the commonly accepted opinion that the limit of elasticity should never be reached in testing a boiler or other structure, these experiments show that an exception should be made in the case of an unstayed flat end-plate of a boiler, which will be safer when it has assumed a permanent set that will prevent its becoming grooved by the continual variation of pressure in working. The hydraulic pressure in this case simply does what should have been done before the plate was fixed, that is, dishes it.

4. These experiments appear to show that the mode of attaching by flange or by an inside or outside angle-iron exerts an important influence on the manner in which the plate is strained by the pressure.

When the plate is secured to an angle-iron, the stretching under pressure is, to a certain extent, concentrated at the line of rivet-holes, and the plate partakes rather of a beam supported than fixed round the edge. Instead of the strength increasing as the square of the thickness, when the plate is attached by an angle-iron, it is probable that the strength does not increase even directly as the thickness, since the plate gives way simply by stretching at the rivet-holes, and the thicker the plate, the less uniformly is the strain borne by the different layers of which the plate may be considered to be made up. When the plate is flanged, the flange becomes compressed by the pressure against the body of the plate, and near the rim, as shown by the contrary flexure, the inside of the plate is stretched more than the outside, and it may be by a kind of shearing action that the plate gives way along the line where the crushing and stretching meet.

5. These tests appear to show that the rules deduced from the theoretical investigations of Lamé, Rankine, and Grashof are not confirmed by experiment, and are therefore not trustworthy.

The rules of Lamé, etc., apply only within the elastic limit. (*Eng'g*, Dec. 13, 1895.)

Unbraced Wrought-iron Heads of Boilers, etc. (*The Locomotive*, Feb. 1890).—Few experiments have been made on the strength of flat heads, and our knowledge of them comes largely from theory. Experiments have been made on small plates 1-16 of an inch thick, yet the data so obtained cannot be considered satisfactory when we consider the far thicker heads that are used in practice, although the results agreed well with Rankine's formula. Mr. Nichols has made experiments on larger heads, and from them he has deduced the following rule: "To find the proper thickness for a flat unstayed head, multiply the area of the head by the pressure per square inch that it is to bear safely, and multiply this by the desired factor of safety (say 8); then divide the product by ten times the tensile strength of the material used for the head." His rule for finding the bursting pressure when the dimensions of the head are given is: "Multiply the thickness of the end-plate in inches by ten times the tensile strength of the material used, and divide the product by the area of the head in inches."

In Mr. Nichols's experiments the average tensile strength of the iron used for the heads was 44,800 pounds. The results he obtained are given below, with the calculated pressure, by his rule, for comparison.

1. An unstayed flat boiler-head is 34½ inches in diameter and 9-16 inch thick. What is its bursting pressure? The area of a circle 34½ inches in diameter is 935 square inches; then $9\text{-}16 \times 44,800 \times 10 = 252,000$, and $252,000 \div 935 = 270$ pounds, the calculated bursting pressure. The head actually burst at 280 pounds.

2. Head 34½ inches in diameter and ⅜ inch thick. The area = 935 square inches; then, $⅜ \times 44,800 \times 10 = 168,000$, and $168,000 \div 935 = 180$ pounds, calculated bursting pressure. This head actually burst at 200 pounds.

3. Head 26¼ inches in diameter, and 3/8 inch thick. The area 541 square inches. Then, $\frac{3}{8} \times 44,800 \times 10 = 168,000$, and $168,000 \div 541 = 311$ pounds. This head burst at 370 pounds.

4. Head 28½ inches in diameter and 3/8 inch thick. The area = 638 square inches; then, $\frac{3}{8} \times 44,800 \times 10 = 168,000$, and $168,000 \div 638 = 263$ pounds. The actual bursting pressure was 300 pounds.

In the third experiment, the amount the plate bulged under different pressures was as follows:

At pounds per sq. in....	10	20	40	80	120	140	170	200
Plate bulged............	1/32	1/16	1/8	1/4	3/8	1/2	5/8	3/4

The pressure was now reduced to zero, "and the end sprang back 3-16 inch, leaving it with a permanent set of 9-16 inch. The pressure of 200 lbs. was again applied on 36 separate occasions during an interval of five days, the bulging and permanent set being noted on each occasion, but without any appreciable difference from that noted above.

The experiments described were confined to plates not widely different in their dimensions, so that Mr. Nichols's rule cannot be relied upon for heads that depart much from the proportions given in the examples.

Thickness of Flat Cast-iron Plates to resist Bursting Pressures.—Capt. John Ericsson (Church's Life of Ericsson) gave the following rules: The proper thickness of a square cast-iron plate will be obtained by the following: Multiply the side in feet (or decimals of a foot) by ¼ of the pressure in pounds and divide by 850 times the side in inches; the quotient is the square of the thickness in inches.

For a circular plate, multiply 11-14 of the diameter in feet by ¼ of the pressure on the plate in pounds. Divide by 850 times 11-14 of the diameter in inches. [Extract the square root.]

Prof. Wm. Harkness, *Eng'g News*, Sept. 5, 1895, shows that these rules can be put in a more convenient form, thus:

$$\text{For square plates} \quad T = 0.00495S\sqrt{p},$$

and

$$\text{For circular plates} \quad T = 0.00489D\sqrt{p},$$

where T = thickness of plate, S = side of the square, D = diameter of the circle, and p = pressure in lbs. per sq. in. Professor Harkness, however, doubts the value of the rules, and says that no satisfactory theoretical solution has yet been obtained.

Strength of Stayed Surfaces.—A flat plate of thickness t is supported uniformly by stays whose distance from centre to centre is a, uniform load p lbs. per square inch. Each stay supports pa^2 lbs. The greatest stress on the plate is

$$f = \frac{2}{9}\frac{a^2}{t^2}p. \text{ (Unwin).}$$

SPHERICAL SHELLS AND DOMED BOILER-HEADS.

To find the Thickness of a Spherical Shell to resist a given Pressure.—Let d = diameter in inches, and p the internal pressure per square inch. The total pressure which tends to produce rupture around the great circle will be $\frac{1}{4}\pi d^2p$. Let S = safe tensile stress per square inch, and t the thickness of metal in inches; then the resistance to the pressure will be πdtS. Since the resistance must be equal to the pressure,

$$\tfrac{1}{4}\pi d^2p = \pi dtS. \quad \text{Whence } t = \frac{pd}{4S}.$$

The same rule is used for finding the thickness of a hemispherical head to a cylinder, as of a cylindrical boiler.

Thickness of a Domed Head of a Boiler.—If S = safe tensile stress per square inch, d = diameter of the shell in inches, and t = thickness of the shell, $t = pd \div 2S$; but the thickness of a hemispherical head of the same diameter is $t = pd \div 4S$. Hence if we make the *radius* of curvature of a domed head equal to the diameter of the boiler, we shall have $t = \frac{2pd}{4S} = \frac{pd}{2S}$, or the thickness of such a domed head will be equal to the thickness of the shell.

Stresses in Steel Plating due to Water-pressure, as in plating of vessels and bulkheads (*Engineering*, May 22, 1891, page 629).

Mr. J. A. Yates has made calculations of the stresses to which steel plates are subjected by external water-pressure, and arrives at the following conclusions:

Assume $2a$ inches to be the distance between the frames or other rigid supports, and let d represent the depth in feet, below the surface of the water, of the plate under consideration, t = thickness of plate in inches, D the deflection from a straight line under pressure in inches, and P = stress per square inch of section.

For outer bottom and ballast-tank plating, $a = 420\frac{t}{d}$, D should not be greater than $.05\frac{2a}{12}$, and $\frac{P}{2}$ not greater than 2 to 3 tons; while for bulkheads, etc., $a = 2852\frac{t}{d}$, D should not be greater than $.1\frac{2a}{12}$, and $\frac{P}{2}$ not greater than 7 tons. To illustrate the application of these formulæ the following cases have been taken:

For Outer Bottom, etc.			For Bulkheads, etc.		
Thickness of Plating.	Depth below Water.	Spacing of Frames should not exceed	Thickness of Plating	Depth of Water.	Maximum Spacing of Rigid Stiffeners.
in.	ft.	in.	in.	ft.	ft. in.
1/2	20	About 21	1/2	20	9 10
1/2	10	" 42	3/8	20	7 4
3/8	18	" 18	3/8	10	14 8
3/8	9	" 36	1/4	20	4 10
1/4	10	" 20	1/4	10	9 8
1/4	5	" 40	1/8	10	4 10

It would appear that the course which should be followed in stiffening bulkheads is to fit substantially rigid stiffening frames at comparatively wide intervals, and only work such light angles between as are necessary for making a fair job of the bulkhead.

THICK HOLLOW CYLINDERS UNDER TENSION.

Burr, "Elasticity and Resistance of Materials," p. 36, gives

$$t = r\left\{\left(\frac{h+p}{h-p}\right)^{\frac{1}{2}} - 1\right\}.$$

t = thickness; r = interior radius;
h = maximum allowable hoop tension at the interior of the cylinder;
p = intensity of interior pressure.

Merriman gives

s = unit stress at inner edge of the annulus;
r = interior radius; t = thickness;
l = length.

The total stress over the area $2tl = 2sl\dfrac{rt}{r+t}$. . . . . (1)

The total interior pressure which tends to rupture the cylinder is $2rl \times p$. If p be the unit pressure, then $p = \dfrac{st}{r+t}$, from which one of the quantities $s, p, r,$ or t can be found when the other three are given.

$$s = \frac{p(r+t)}{t}; \qquad r = \frac{(s-p)t}{p}; \qquad t = \frac{rp}{s-p}.$$

In eq. (1), if t be neglected in comparison with r, it reduces to $2slt$, which is the same as the formula for thin cylinders. If $t = r$, it becomes slt, or only half the resistance of the thin cylinder.

The formulæ given by Burr and by Merriman are quite different, as will be seen by the following example: Let maximum unit stress at the inner edge of the annulus = 8000 lbs. per square inch, radius of cylinder = 4 inches, interior pressure = 4000 lbs. per square inch. Required the thickness.

By Burr, $$t = 4\left\{\left(\frac{8000 + 4000}{8000 - 4000}\right)^{\frac{1}{2}} - 1\right\} = 4(\sqrt{3} - 1) = 2.928 \text{ inches.}$$

By Merriman, $$t = \frac{4 \times 4000}{8000 - 4000} = 4 \text{ inches.}$$

Limit to Useful Thickness of Hollow Cylinders (*Eng'g*, Jan. 4, 1884).—Professor Barlow lays down the law of the resisting powers of thick cylinders as follows:

"In a homogeneous cylinder, if the metal is incompressible, the tension on every concentric layer, caused by an internal pressure, varies inversely as the square of its distance from the centre."

Suppose a twelve-inch gun to have walls 15 inches thick.

$$\frac{\text{Pressure on exterior}}{\text{Pressure on interior}} = \frac{6^2}{21^2} = 1 : 12.25.$$

So that if the stress on the interior is 12¼ tons per square inch, the stress on the exterior is only 1 ton.

Let s = the stress on the inner layer, and s_1 that at a distance x from the axis; r = internal radius, R = external radius.

$$s_1 : s :: r^2 : x^2, \text{ or } s_1 = s\frac{r^2}{x^2}.$$

The whole stress on a section 1 inch long, extending from the interior to the exterior surface, is $S = sr \times \frac{R - r}{R}$.

In a 12-inch gun, let s = 40 tons, r = 6 in., R = 21 in.

$$S = 40 \times 6 \times \frac{21 - 6}{21} = 172 \text{ tons.}$$

Suppose now we go on adding metal to the gun outside: then R will become so large compared with r, that $R - r$ will approach the value R, so that the fraction $\frac{R - r}{R}$ becomes nearly unity.

Hence for an infinitely thick cylinder the useful strength could never exceed Sr (in this case 240 tons).

Barlow's formula agrees with the one given by Merriman.

Another statement of the gun problem is as follows: Using the formula

$$p = \frac{st}{r + t},$$

s = 40 tons, t = 15 in., r = 6 in., $p = \frac{40 \times 15}{21}$ = 28$\frac{4}{7}$ tons per sq. in., 28$\frac{4}{7}$ × radius = 172 tons, the pressure to be resisted by a section 1 inch long of the thickness of the gun on one side. Suppose thickness were doubled, making t = 30 in.: $p = \frac{40 \times 30}{36}$ = 33⅓ tons, or an increase of only 16 per cent.

For short cast-iron cylinders, such as are used in hydraulic presses, it is doubtful if the above formulæ hold true, since the strength of the cylindrical portion is reinforced by the end. In that case the bursting strength would be higher than that calculated by the formula. A rule used in practice for such presses is to make the thickness = 1/10 of the inner circumference, for pressures of 3000 to 4000 lbs. per square inch. The latter pressure would bring a stress upon the inner layer of 10,350 lbs. per square inch, as calculated by the formula; which would necessitate the use of the best charcoal-iron to make the press reasonably safe.

THIN CYLINDERS UNDER TENSION.

Let p = safe working pressure in lbs. per sq. in.;
d = diameter in inches;
T = tensile strength of the material, lbs. per sq. in.;
t = thickness in inches;
f = factor of safety;
c = ratio of strength of riveted joint to strength of solid plate.

$$fpd = 2Ttc; \quad p = \frac{2Ttc}{df}; \quad t = \frac{fpd}{2Tc}.$$

If $T = 50000$, $f = 5$, and $c = 0.7$; then

$$p = \frac{14000t}{d}; \quad t = \frac{dp}{14000}.$$

The above represents the strength resisting rupture along a longitudinal seam. For resistance to rupture in a circumferential seam, due to pressure on the ends of the cylinder, we have $\frac{p\pi d^2}{4} = \frac{Tt\pi dc}{f}$;

$$\text{whence } p = \frac{4Ttc}{df}.$$

Or the strength to resist rupture around a circumference is twice as great as that to resist rupture longitudinally; hence boilers are commonly single-riveted in the circumferential seams and double-riveted in the longitudinal seams.

HOLLOW COPPER BALLS.

Hollow copper balls are used as floats in boilers or tanks, to control feed and discharge valves, and regulate the water-level.

They are spun up in halves from sheet copper, and a rib is formed on one half. Into this rib the other half fits, and the two are then soldered or brazed together. In order to facilitate the brazing, a hole is left on one side of the ball, to allow air to pass freely in or out; and this hole is made use of afterwards to secure the float to its stem. The original thickness of the metal may be anything up to about 1-16 of an inch, if the spinning is done on a hand lathe, though thicker metal may be used when special machinery is provided for forming it. In the process of spinning, the metal is thinned down in places by stretching; but the thinnest place is neither at the equator of the ball (i.e., along the rib) nor at the poles. The thinnest points lie along two circles, passing around the ball parallel to the rib, one on each side of it, from a third to a half of the way to the poles. Along these lines the thickness may be 10, 15, or 20 per cent less than elsewhere, the reduction depending somewhat on the skill of the workman.

The Locomotive for October, 1891, gives two empirical rules for determining the thickness of a copper ball which is to work under an external pressure, as follows:

$$1.\ \text{Thickness} = \frac{\text{diameter in inches} \times \text{pressure in pounds per sq. in.}}{16{,}000}.$$

$$2.\ \text{Thickness} = \frac{\text{diameter} \times \sqrt[4]{\text{pressure}}}{1240}.$$

These rules give the same result for a pressure of 166 lbs. only. Example: Required the thickness of a 5-inch copper ball to sustain

Pressures of...........	50	100	150	166	200	250	lbs. per sq. in.
Answer by first rule...	.0156	.0312	.0469	.0519	.0625	.0781	inch.
Answer by second rule	.0285	.0403	.0494	.0518	.0570	.0637	"

HOLDING-POWER OF NAILS, SPIKES, AND SCREWS.

(A. W. Wright, Western Society of Engineers, 1881.)

Spikes.—Spikes driven into dry cedar (cut 18 months):

Size of spikes..........................		5 × ¼ in. sq.	6 × ¼	6 × ⅛	5 × ⅜
Length driven in........................		4¼ in.	5 in.	5 in.	4¼ in.
Pounds resistance to drawing.	Av'ge, lbs.	857	821	1691	1202
From 6 to 9 tests each.........	Max. "	1159	923	2129	1556
	Min. "	766	766	1120	687

A. M. Wellington found the force required to draw spikes 9/16 × 9/16 in., driven 4½ inches into seasoned oak, to be 4281 lbs.; same spikes, etc., in unseasoned oak, 6523 lbs.

"Professor W. R. Johnson found that a plain spike ⅜ inch square driven 3⅜ inches into seasoned Jersey yellow pine or unseasoned chestnut required about 2000 lbs. force to extract it; from seasoned white oak about 4000 and from well-seasoned locust 6000 lbs."

Experiments in Germany, by Funk, give from 2465 to 3940 lbs. (mean of many experiments about 3000 lbs.) as the force necessary to extract a plain ½-inch square iron spike 6 inches long, wedge-pointed for one inch and driven 4½ inches into white or yellow pine. When driven 5 inches the force required was about 1/10 part greater. Similar spikes 9/16 inches square, 7 inches long, driven 6 inches deep, required from 3700 to 6745 lbs. to extract them from pine; the mean of the results being 4873 lbs. In all cases about twice as much force was required to extract them from oak. The spikes were all driven across the grain of the wood. When driven with the grain, spikes or nails do not hold with more than half as much force.

Boards of oak or pine nailed together by from 4 to 16 tenpenny common cut nails and then pulled apart in a direction lengthwise of the boards, and across the nails, tending to break the latter in two by a shearing action, averaged about 300 to 400 lbs. per nail to separate them, as the result of many trials.

Resistance of Drift-bolts in Timber.—Tests made by Rust and Coolidge, in 1878.

						Pounds.
1st Test.	1 in. square iron drove	30 in.	in white pine,	15/16-in. hole	.....	26,400
2d "	1 in. round " "	34 "	" "	13/16-in. "	.....	16,800
3d "	1 in. square " "	18 "	" "	15/16-in. "	.....	14,600
4th "	1 in. round " "	22 "	" "	13/16-in. "	.....	13,200
5th "	1 in. round " "	34 "	" Norw'y pine,	13/16-in. "	.....	18,720
6th "	1 in. square " "	30 "	" "	15/16-in. "	.....	19,200
7th "	1 in. square " "	18 "	" "	15/16-in. "	.....	15,600
8th "	1 in. round " "	22 "	" "	13/16-in. "	.....	14,400

NOTE.—In test No. 6 drift-bolts were not driven properly, holes not being in line, and a piece of timber split out in driving.

Force required to draw Screws out of Norway Pine.

½" diam. drive screw 4 in. in wood.	Power required, average	2424 lbs.
" " 4 threads per in. 5 in. in wood.	" " "	2748 "
" " D'ble thr'd, 3 per in., 4 in. in "	" " "	2730 "
" " Lag-screw, 7 per in., 1½ " "	" " "	1465 "
" " " " 6 " " 2½ " "	" " "	2026 "
½ inch R.R. spike... .... 5 " "	" " "	2191 "

Force required to draw Wood Screws out of Dry Wood.—Tests made by Mr. Bevan. The screws were about two inches in length, .22 diameter at the exterior of the threads, .15 diameter at the bottom, the depth of the worm or thread being .035 and the number of threads in one inch equal 12. They were passed through pieces of wood half an inch in thickness and drawn out by the weights stated: Beech, 460 lbs.; ash, 790 lbs.; oak, 760 lbs.; mahogany, 770 lbs.; elm, 665 lbs.; sycamore, 830 lbs.

Tests of Lag-screws in Various Woods were made by A. J. Cox, University of Iowa, 1891:

Kind of Wood.	Size Screw.	Size Hole bored.	Length in Tie.	Max. Resist. lbs.	No. Tests.
Seasoned white oak...............	⅝ in.	½ in.	4½ in.	8037	3
" " "...............	9/16 "	7/16 "	8 "	6480	1
" " "...............	½ "	⅜ "	4½ "	8780	2
Yellow-pine stick...	⅝ "	½ "	4 "	3800	2
White cedar, unseasoned..........	⅝ "	½ "	4 "	3405	2

In figuring area for lag-screws, the surface of a cylinder whose diameter is equal to that of the screw was taken. The length of the screw part in each case was 4 inches.—*Engineering News*, 1891.

Cut *versus* Wire Nails.—Experiments were made at the Watertown Arsenal in 1893 on the comparative direct tensile adhesion, in pine and spruce, of cut and wire nails. The results are stated by Prof. W. H. Burr as follows:

There were 58 series of tests, ten pairs of nails (a cut and a wire nail in each) being used, making a total of 1160 nails drawn. The tests were made in spruce wood in most instances, but some extra ones were made in white pine, with "box nails." The nails were of all sizes, varying from 1⅛ inches to 6 inches in length. In every case the cut nails showed the superior holding strength by a large percentage. In spruce, in nine different sizes of nails, both standard and light weight, the ratio of tenacity of cut to wire nail was about 3 to 2, or, as he terms it, "a superiority of 47.45% of the former." With the "finishing" nails the ratio was roughly 3.5 to 2; superiority 72%. With box nails (1¼ to 4 inches long) the ratio was roughly 3 to 2; superiority 51%. The mean superiority in spruce wood was 61%. In white pine, cut nails, driven with taper along the grain, showed a superiority of 100%, and with taper across the grain of 135%. Also when the nails were driven in the end of the stick, i.e., along the grain, the superiority of cut nails was 100%, or the ratio of cut to wire was 2 to 1. The total of the results showed the ratio of tenacity to be about 3.2 to 2 for the harder wood, and about 2 to 1 for the softer, and for the whole taken together the ratio was 3.5 to 2. We are led to conclude that under these circumstances the cut nail is superior to the wire nail in direct tensile holding-power by 72.74%.

Nail-holding Power of Various Woods.

(Watertown Experiments.)

Kind of Wood.	Size of Nail.	Holding-power per square inch of Surface in Wood, lbs. Wire Nail.	Cut Nail.	Mean.
White pine	8d	167	450	405
	9 "		455	
	20 "		477	
	50 "		347	
			363	
	60 "		340	
Yellow pine	8 "	318	695	662
	10 "		755	
	50 "		596	
	60 "		604	
White oak	8 "	940	1340	1216
	20 "		1292	
	60 "		1018	
Chestnut	50 "		664	683
	60 "		702	
Laurel	9 "	651	1179	1200
	20 "		1221	

Nail-holding Power of Various Woods.

(F. W. Clay's Experiments. *Eng'g News*, Jan. 11, 1894.)

Wood.	Tenacity of 6d nails: Plain.	Barbed.	Blued.	Mean.
White pine	106	94	135	111
Yellow pine	190	130	270	196
Basswood	78	132	219	143
White oak	226	300	555	360
Hemlock	141	201	319	220

Tests made at the University of Illinois gave the resistance of a 1-in. round rod in a 15/16-inch hole perpendicular to the grain, as 6000 lbs. per lin. ft. in pine and 15,600 lbs. in oak. Experiments made at the East River Bridge gave resistances of 12,000 and 15,000 lbs. per lin. ft. for a 1-in. round rod in holes 15/16-in. and 14/16-in. diameter, respectively, in Georgia pine.

Holding-power of Bolts in White Pine.

(*Eng'g News*, September 26, 1891.)

	Round. Lbs.	Square. Lbs.
Average of all plain 1-in. bolts	8224	8200
Average of all plain bolts, ⅝ to 1⅛ in	7805	8110
Average of all bolts	8383	8598

Round drift-bolts should be driven in holes 13/16 of their diameter, and square drift-bolts in holes whose diameter is 14/16 of the side of the square.

STRENGTH OF WROUGHT IRON BOLTS.

(Computed by A. F. Nagle.)

Diameter of Bolt, Inches.	Number of Threads.	Diameter of Bottom of Thread, Inches.	Area at Bottom of Thread, Square Inches.	Stress upon Bolt upon Basis of 3000 lbs. per sq. inch. lbs.	4000 lbs. per sq. inch. lbs.	5000 lbs. per sq. inch. lbs.	7000 lbs. per sq. inch. lbs.	10000 lbs. per sq. inch. lbs.	Probable Breaking Load. lbs.
½	13	.38	.12	350	460	580	810	1160	5800
9-16	12	.44	.15	450	600	750	1050	1500	7500
⅝	11	.49	.19	560	750	930	1310	1870	9000
¾	10	.60	.28	750	1130	1410	1980	2830	14000
⅞	9	.71	.39	1180	1570	1970	2760	3940	19000
1	8	.81	.52	1550	2070	2600	3630	5180	25000
1⅛	7	.91	.65	1950	2600	3250	4560	6510	30000
1¼	7	1.04	.84	2520	3360	4200	5900	8410	39000
1⅜	6	1.12	1.00	3000	4000	5000	7000	10000	46000
1½	6	1.25	1.23	3680	4910	6140	8600	12280	56000
1⅝	5½	1.35	1.44	4300	5740	7180	10000	14360	65000
1¾	5	1.45	1.65	4950	6600	8250	11560	16510	74000
1⅞	5	1.57	1.95	5840	7800	9800	13640	19500	85000
2	4½	1.66	2.18	6540	8720	10900	15260	21800	95000
2¼	4½	1.92	2.88	8650	11530	14400	20180	28800	125000
2½	4	2.12	3.55	10640	14200	17730	24830	35500	150000
2¾	4	2.37	4.43	13290	17720	22150	31000	44300	186000
3	3½	2.57	5.20	15580	20770	26000	36360	52000	213000
3½	3¼	3.04	7.25	21760	29000	36260	50760	72500	250000
4	3	3.50	9.62	28860	38500	48100	67350	96200	385000

When it is known what load is to be put upon a bolt, and the judgment of the engineer has determined what stress is safe to put upon the iron, look down in the proper column of said stress until the required load is found. The area at the bottom of the thread will give the equivalent area of a flat bar to that of the bolt.

Effect of Initial Strain in Bolts.—Suppose that bolts are used to connect two parts of a machine and that they are screwed up tightly before the effective load comes on the connected parts. Let P_1 = the initial tension on a bolt due to screwing up, and P_2 = the load afterwards added. The greatest load may vary but little from P_1 or P_2, according as the former or the latter is greater, or it may approach the value $P_1 + P_2$, depending upon the relative rigidity of the bolts and of the parts connected. Where rigid flanges are bolted together, metal to metal, it is probable that the extension of the bolts with any additional tension relieves the initial tension, and that the total tension is P_1 or P_2, but in cases where elastic packing, as india rubber, is interposed, the extension of the bolts may very little affect the initial tension, and the total strain may be nearly $P_1 + P_2$. Since the latter assumption is more unfavorable to the resistance of the bolt, this contingency should usually be provided for. (See Unwin, "Elements of Machine Design" for demonstration.)

STAND-PIPES AND THEIR DESIGN.

(Freeman C. Coffin, New England Water Works Assoc., *Eng. News*, March 16, 1893.) See also papers by A. H. Howland, Eng. Club of Phil. 1887; B. F. Stephens, Amer. Water Works Assoc., *Eng. News*, Oct. 6 and 13, 1888; W. Kiersted, Rensselaer Soc. of Civil Eng., *Eng'g Record*, April 25 and May 2, 1891, and W. D. Pence, *Eng. News*, April and May, 1894.

The question of diameter is almost entirely independent of that of height. The efficient capacity must be measured by the length from the high-water line to a point below which it is undesirable to draw the water on account of loss of pressure for fire-supply, whether that point is the actual bottom of the stand-pipe or above it. This allowable fluctuation ought not to exceed 50 ft., in most cases. This makes the diameter dependent upon two conditions, the first of which is the amount of the consumption during the ordinary interval between the stopping and starting of the pumps. This should never draw the water below a point that will give a good fire stream and leave a margin for still further draught for fires. The second condition is the maximum number of fire streams and their size which it is considered necessary to provide for, and the maximum length of time which they are liable to have to run before the pumps can be relied upon to reinforce them.

Another reason for making the diameter large is to provide for stability against wind-pressure when empty.

The following table gives the height of stand-pipes beyond which they are not safe against wind-pressures of 40 and 50 lbs. per square foot. The area of surface taken is the height multiplied by one half the diameter.

Heights of Stand-pipe that will Resist Wind-pressure by its Weight alone, when Empty.

Diameter, feet.	Wind, 40 lbs. per sq. ft.	Wind, 50 lbs. per sq. ft.
20	45	35
25	70	55
30	150	80
35	...	160

To have the above degree of stability the stand-pipes must be designed with the outside angle-iron at the bottom connection.

Any form of anchorage that depends upon connections with the side plates near the bottom is unsafe. By suitable guys the wind-pressure is resisted by tension in the guys, and the stand-pipe is relieved from wind strains that tend to overthrow it. The guys should be attached to a band of angle or other shaped iron that completely encircles the tank, and rests upon some sort of bracket or projection, and not be riveted to the tank. They should be anchored at a distance from the base equal to the height of the point at which they are attached, if possible.

The best plan is to build the stand-pipe of such diameter that it will resist the wind by its own stability.

Thickness of the Side Plates.

The pressure on the sides is outward, and due alone to the weight of the water, or pressure per square inch, and increases in direct ratio to the height, and also to the diameter. The strain upon a section 1 inch in height at any point is the total strain at that point divided by two—for each side is supposed to bear the strain equally. The total pressure at any point is equal to the diameter in inches, multiplied by the pressure per square inch, due to the height at that point. It may be expressed as follows:

H = height in feet, and f = factor of safety;
d = diameter in inches;
p = pressure in lbs. per square inch;
.434 = p for 1 ft. in height;
s = tensile strength of material per square inch;
T = thickness of plate.

Then the total strain on each side per vertical inch

$$= \frac{.434Hd}{2} = \frac{pd}{2}; \qquad T = \frac{.434Hdf}{2s} = \frac{pdf}{2s}.$$

Mr. Coffin takes $f = 5$, not counting reduction of strength of joint, equivalent to an actual factor of safety of 3 if the strength of the riveted joint is taken as 60 per cent of that of the plate.

The amount of the wind strain per square inch of metal at any joint can be found by the following formula, in which

H = height of stand-pipe in feet above joint;
T = thickness of plate in inches;
p = wind-pressure per square foot;
W = wind-pressure per foot in height above joint;
$W = Dp$ where D is the diameter in feet;
m = average leverage or movement about neutral axis or central points in the circumference; or,
m = sine of 45°, or .707 times the radius in feet.

Then the strain per square inch of plate

$$= \frac{(Hw)\frac{H}{2}}{\text{circ. in ft.} \times mT}.$$

Mr. Coffin gives a number of diagrams useful in the design of stand-pipes, together with a number of instances of failures, with discussion of their probable causes.

Mr. Kiersted's paper contains the following: Among the most prominent strains a stand-pipe has to bear are: that due to the static pressure of the water, that due to the overturning effect of the wind on an empty stand-pipe, and that due to the collapsing effect, on the upper rings, of violent wind storms.

For the thickness of metal to withstand safely the static pressure of water, let

t = thickness of the plate iron in inches;
H = height of stand-pipe in feet;
D = diameter of stand-pipe in feet.

Then, assuming a tensile strength of 48,000 lbs. per square inch, a factor of safety of 4, and efficiency of double-riveted lap-joint equalling 0.6 of the strength of the solid plate,

$$t = .00036H \times D; \qquad H = \frac{10{,}000t}{3.6D};$$

which will give safe heights for thicknesses up to ⅝ to ¾ of an inch. The same formula may also apply for greater heights and thicknesses within practical limits, if the joint efficiency be increased by triple riveting.

The conditions for the severest overturning wind strains exist when the stand-pipe is empty.

Formula for wind-pressure of 50 pounds per square foot, when

d = diameter of stand-pipe in inches;
x = any unknown height of stand-pipe;
$x = \sqrt{80\pi dt} = 15.85\sqrt{dt}$.

The following table is calculated by these formulæ. The stand-pipe is intended to be self-sustaining; that is, without guys or stiffeners.

Heights of Stand-pipes for Various Diameters and Thicknesses of Plates.

Thickness of Plate in Fractions of an Inch.	Diameters in Feet.												
	5	6	7	8	9	10	12	14	15	16	18	20	25
3-16	50	55	60	65	55	50	35	....	....	....	....	..	...
7-32	55	....	....	....	65	60	50	40	40	....	....	....	....
4-16	60	65	70	75	75	70	55	50	45	40	35	35	25
5-16	70	75	80	85	90	85	70	60	55	50	45	40	35
6-16	75	80	90	95	100	100	85	75	70	65	55	50	40
7-16	80	90	95	100	110	115	100	85	80	75	65	60	45
8-16	85	95	100	110	115	120	115	100	90	85	75	70	55
9-16	...	....	....	115	125	130	130	110	100	95	85	80	60
10-16	....	..	....	...	130	135	145	120	115	105	95	85	65
11-16	....	....	....	....	....	145	155	135	125	120	105	95	75
12-16	....	....	..	....	....	150	165	145	135	130	115	105	80
13-16	....	...	....	...	....	...	....	160	150	140	125	110	90
14-16	....	....	...	...	....	...	....	....	160	150	135	120	95
15-16	....	....	....	....	....	....	....	....	....	160	145	130	105
16-16	...	....	...	....	..	....	....	....	....	...	155	140	110

Heights to nearest 5 feet. Rings are to build 5 feet vertically.

Failures of Stand-pipes have been numerous in recent years. A list showing 23 important failures inside of nine years is given in a paper by Prof. W. D. Pence, *Eng'g. News*, April 5, 12, 19 and 26, May 3, 10 and 24, and June 7, 1894. His discussion of the probable causes of the failures is most valuable.

Kenneth Allen, Engineers Club of Philadelphia, 1886, gives the following rules for thickness of plates for stand pipes.

Assume: Wrought iron plate T. S. 48,000 pounds in direction of fibre, and T. S. 45,000 pounds across the fibre. Strength of single riveted joint .4 that of the plate, and of double riveted joint, .7 that of the plate; wind pressure = 50 pounds per square foot; safety factor = 3.

Let h = total height in feet; r = outer radius in feet; r' = inner radius in feet; p = pressure per square inch; t = thickness in inches; d = outer diameter in feet.

Then for pipe filled and longitudinal seams double riveted

$$t = \frac{pr \times 12}{48{,}000 \times .7 \times \frac{1}{3}} = \frac{hd}{4301};$$

and for pipe empty and lateral seams, single riveted, we have by equating moments:

$$50 \times 2r \left(\frac{h}{2}\right)^2 = 144 \times 6000\,(r^4 - r'^4)\,\frac{.7854}{r}, \text{ whence } r^4 - r'^4 = \frac{h^2 r^2}{27144}.$$

Table showing required Thickness of Bottom Plate.

Height in Feet.	Diameter.					
	5 feet.	10 feet.	15 feet.	20 feet.	25 feet.	30 feet.
	"	"	"	"	"	"
50	† 7-64*	1/8 *	11-64*	15-64	19-64	23-64
60	†11-64*	9-64*	7-32	9-32	23-64	27-64
70	† 7-32	11-64*	1/4	21-64	13-32	31-64
80	†19-64	3-16	9-32	3/8	15-32	9-16
90	† 3/8	7-32	5-16	27-64	17-32	5/8
100	†29-64	†15-64	23-64	15-32	37-64	45-64
125		†23-64	7-16	37-64	47-64	3/4
150		†33-64	17-32	45-64	7/8	1 3-64
175		†11-16	39-64	13-16	1 1-32	1 7-32
200		†29-32	45-64	15-16	1 11-64	1 25-64

* The minimum thickness should = 3-16″.

N.B.—Dimensions marked † determined by wind-pressure.

Water Tower at Yonkers, N. Y.—This tower, with a pipe 122 feet high and 20 feet diameter, is described in *Engineering News*, May 18, 1892. The thickness of the lower rings is 11-16 of an inch, based on a tensile strength of 60,000 lbs. per square inch of metal, allowing 65% for the strength of riveted joints, using a factor of safety of 3½ and adding a constant of ⅛ inch. The plates diminish in thickness by 1-16 inch to the last four plates at the top, which are ¼ inch thick.

The contract for steel requires an elastic limit of at least 33,000 lbs. per square inch; an ultimate tensile strength of from 56,000 to 66,000 lbs. per square inch; an elongation in 8 inches of at least 20%, and a reduction of area of at least 45%. The inspection of the work was made by the Pittsburgh Testing Laboratory. According to their report the actual conditions developed were as follows: Elastic limit from 34,020 to 39,420; the tensile strength from 58,330 to 65,390; the elongation in 8 inches from 22½ to 32%; reduction in area from 52.72 to 71.82%; 17 plates out of 141 were rejected in the inspection.

WROUGHT-IRON AND STEEL WATER-PIPES.

Riveted Steel Water-pipes (*Engineering News*, Oct. 11, 1890, and Aug. 1, 1891.)—The use of riveted wrought-iron pipe has been common in the Pacific States for many years, the largest being a 44-inch conduit in connection with the works of the Spring Valley Water Co., which supplies San Francisco. The use of wrought iron and steel pipe has been necessary in the West, owing to the extremely high pressures to be withstood and the difficulties of transportation. As an example: In connection with the water supply of Virginia City and Gold Hill, Nev., there was laid in 1872 an 11½-inch riveted wrought-iron pipe, a part of which is under a head of 1720 feet.

In the East, the most important example of the use of riveted steel water pipe is that of the East Jersey Water Co., which supplies the city of Newark. The contract provided for a maximum high service supply of 25,000,000 gallons daily. In this case 21 miles of 48-inch pipe was laid, some of it under 340 feet head. The plates from which the pipe is made are about 13 feet long by 7 feet wide, open-hearth steel. Four plates are used to make one section of pipe about 27 feet long. The pipe is riveted longitudinally with a double row, and at the end joints with a single row of rivets of varying diameter, corresponding to the thickness of the steel plates. Before being rolled into the trench, two of the 27-feet lengths are riveted together, thus diminishing still further the number of joints to be made in the trench and the extra excavation to give room for jointing. All changes in the grade of the pipe-line are made by 10° curves and all changes in line by 2½, 5, 7½ and 10° curves. To lay on curved lines a standard bevel was used, and the different curves are secured by varying the number of beveled joints used on a certain length of pipe.

The thickness of the plates varies with the pressure, but only three thicknesses are used, ¼, 5-16, and ⅜ inches, the pipe made of these thicknesses having a weight of 160, 185, and 225 lbs. per foot, respectively. At the works all the pipe was tested to pressure 1½ times that to which it is to be subjected when in place.

Mannesmann Tubes for High Pressures.—At the Mannesmann Works at Komotau, Hungary, more than 600 tons or 25 miles of 3-inch and 4-inch tubes averaging ¼ inch in thickness have been successfully tested to a pressure of 2000 lbs. per square inch. These tubes were intended for a high-pressure water-main in a Chilian nitrate district.

This great tensile strength is probably due to the fact that, in addition to being much more worked than most metal, the fibres of the metal run spirally, as has been proved by microscopic examination. While cast-iron tubes will hardly stand more than 200 lbs. per square inch, and welded tubes are not safe above 1000 lbs. per square inch, the Mannesmann tube easily withstands 2000 lbs. per square inch. The length up to which they can be readily made is shown by the fact that a coil of 3-inch tube 70 feet long was made recently.

For description of the process of making Mannesmann tubes see Trans. A. I. M. E., vol. xix., 384.

STRENGTH OF VARIOUS MATERIALS. EXTRACTS FROM KIRKALDY'S TESTS.

The recent publication, in a book by W. G. Kirkaldy, of the results of many thousand tests made during a quarter of a century by his father, David Kirkaldy, has made an important contribution to our knowledge concerning the range of variation in strength of numerous materials. A condensed abstract of these results was published in the *American Machinist*, May 11 and 18, 1893, from which the following still further condensed extracts are taken:

The figures for tensile and compressive strength, or, as Kirkaldy calls them, pulling and thrusting stress, are given in pounds per square inch of original section, and for bending strength in pounds of actual stress or pounds per BD^2 (breadth × square of depth) for length of 36 inches between supports. The contraction of area is given as a percentage of the original area, and the extension as a percentage in a length of 10 inches, except when otherwise stated. The abbreviations T. S., E. L., Contr., and Ext. are used for the sake of brevity, to represent tensile strength, elastic limit, and percentages of contraction of area, and elongation, respectively.

Cast Iron.—44 tests: T. S. 15,468 to 28,740 pounds; 17 of these were unsound, the strength ranging from 15,468 to 24,357 pounds. Average of all, 23,805 pounds.

Thrusting stress, specimens 2 inches long, 1.34 to 1.5 in. diameter; 43 tests, all sound, 94,352 to 131,912; one, unsound, 93,759; average of all, 113,825.

Bending stress, bars about 1 in. wide by 2 in deep, cast on edge. Ultimate stress 2876 to 3854; stress per BD^2 = 725 to 892; average, 820. Average modulus of rupture, R, = stress per BD^2 × length, = 29,520. Ultimate deflection, .29 to .40 in.; average .34 inch.

Other tests of cast iron, 460 tests, 16 lots from various sources, gave results with total range as follows: Pulling stress, 12,688 to 38,616 pounds; thrusting stress, 66,363 to 175,950 pounds; bending stress, per BD^2, 505 to 1128 pounds; modulus of rupture, R, 18,180 to 40,608. Ultimate deflection, .21 to .45 inch.

The specimen which was the highest in thrusting stress was also the highest in bending, and showed the greatest deflection, but its tensile strength was only 26,502.

The specimen with the highest tensile strength had a thrusting stress of 143,939, and a bending strength, per BD^2, of 979 pounds with 0.41 deflection. The specimen lowest in T. S. was also lowest in thrusting and bending, but gave .38 deflection. The specimen which gave .21 deflection had T. S., 19,188; thrusting, 101,281; and bending, 561.

Iron Castings.—69 tests; tensile strength, 10,416 to 31,652; thrusting stress, ultimate per square inch, 53,502 to 132,031.

Channel Irons.—Tests of 18 pieces cut from channel irons. T. S. 40,693 to 53,141 pounds per square inch; contr. of area from 3.9 to 32.5 %. Ext. in 10 in. from 2.1 to 22.5 %. The fractures ranged all the way from 100 % fibrous to 100 % crystalline. The highest T. S., 53,141, with 8.1 % contr. and 5.3 % ext., was 100 % crystalline; the lowest T. S., 40,693, with 3.9 contr. and 2.1 % ext., was 75 % crystalline. All the fibrous irons showed from 12.2 to 22.5 % ext., 17.3 to 32.5 contr., and T. S. from 43,426 to 49,615. The fibrous irons are therefore of medium tensile strength and high ductility. The crystalline irons are of variable T. S., highest to lowest, and low ductility.

Lowmoor Iron Bars.—Three rolled bars 2½ inches diameter; tensile tests: elastic, 23,200 to 24,200; ultimate, 50,875 to 51,905; contraction, 44.4 to 42.5; extension, 29.2 to 24.3. Three hammered bars, 4½ inches diameter, elastic 25,100 to 24,200; ultimate, 46,810 to 49,223; contraction, 20.7 to 46.5; extension, 10.8 to 31.6. Fractures of all, 100 per cent fibrous. In the hammered bars the lowest T. S. was accompanied by lowest ductility.

Iron Bars, Various.—Of a lot of 80 bars of various sizes, some rolled and some hammered (the above Lowmoor bars included) the lowest T. S. (except one) 40,808 pounds per square inch, was shown by the Swedish "hoop L" bar 3¼ inches diameter, rolled. Its elastic limit was 19,150 pounds; contraction 68.7 % and extension 37.7 % in 10 inches. It was also the most ductile of all the bars tested, and was 100 % fibrous. The highest T. S., 60,780 pounds, with elastic limit, 29,400; contr., 36.6; and ext., 24.3 %, was shown by a "Farnley" 2-inch bar, rolled. It was also 100 % fibrous. The lowest ductility 2.6 % contr., and 4.1 % ext., was shown by a 3¾-inch hammered bar, without brand. It also had the lowest T. S., 40,278 pounds, but rather high elastic limit, 25,700 pounds. Its fracture was 95 % crystalline. Thus of the two bars showing the lowest T. S., one was the most ductile and the other the least ductile in the whole series of 80 bars.

Generally, high ductility is accompanied by low tensile strength, as in the Swedish bars, but the Farnley bars showed a combination of high ductility and high tensile strength.

Locomotive Forgings, Iron.—17 tests: average, E. L., 30,420; T. S., 50,521; contr., 36.5; ext. in 10 inches, 28.8.

Broken Anchor Forgings, Iron.—4 tests: average, E. L., 23,825; T. S., 40,083; contr., 3.0; ext. in 10 inches, 3.8.

Kirkaldy places these two irons in contrast to show the difference between good and bad work. The broken anchor material, he says, is of a most treacherous character, and a disgrace to any manufacturer.

Iron Plate Girder.—Tensile tests of pieces cut from a riveted iron girder after twenty years' service in a railway bridge. Top plate, average of 3 tests, E. L., 26,600; T. S., 40,806; contr. 16.1; ext. in 10 inches, 7.8. Bottom plate, average of 3 tests, E. L., 31,200; T. S., 44,288; contr., 13.3; ext. in 10 inches, 6.3. Web-plate, average of 3 tests, E. L., 28,000; T. S., 45,902; contr., 15.9; ext. in 10 inches, 8.9. Fractures all fibrous. The results of 30 tests from different parts of the girder prove that the iron has undergone no change during twenty years of use.

Steel Plates.—Six plates 100 inches long, 2 inches wide, thickness various, .36 to .97 inch. T. S., 55,485 to 60,805; E. L., 29,600 to 33,200; contr., 52.9 to 59.5; ext., 17.05 to 18.57.

Steel Bridge Links.—40 links from Hammersmith Bridge, 1886.

	T. S.	E. L.	Contr.	Ext. in 100 in.	Fracture. Silky.	Fracture. Granular.
Average of all	67,294	38,294	34.5%	14.11%		
Lowest T. S.	60,753	36,030	30.1	15.51	30%	70%
Highest T. S. and E. L.	75,946	44,166	31.2	12.42	15	85
Lowest E. L.	64,044	32,441	34.7	13.43	30	70
Greatest Contraction	68,745	38,118	52.8	15.46	100	0
Greatest Extension	65,980	36,792	40.8	17.78	35	65
Least Contr. and Ext.	68,960	39,017	6.0	6.62	0	100

The ratio of elastic to ultimate strength ranged from 50.6 to 65.2 per cent; average, 56.9 per cent.

Extension in lengths of 100 inches. At 10,000 lbs. per sq. in., .018 to .024; mean, .020 inch; at 20,000 lbs. per sq. in. .049 to .063; mean, .055 inch; at 30,000 lbs. per sq. in., .083 to .100; mean, .090; set at 30,000 pounds per sq. in., 0 to .002; mean, 0.

The mean extension between 10,000 to 30,000 lbs. per sq. in increased regularly at the rate of .007 inch for each 2000 lbs. per sq. in. increment of strain. This corresponds to a modulus of elasticity of 28,571,429. The least increase of extension for an increase of load of 20,000 lbs. per sq. in., .065 inch, corresponds to a modulus of elasticity of 30,769,231, and the greatest, .076 inch, to a modulus of 26,315,789.

Steel Rails.—Bending tests, 5 feet between supports, 11 tests of flange rails 72 pounds per yard, 4.63 inches high.

	Elastic stress. Pounds.	Ultimate stress. Pounds.	Deflection at 50,000 Pounds.	Ultimate Deflection.
Hardest	34,200	60,960	3.24 ins.	8 ins.
Softest	32,000	56,740	3.76 "	8 "
Mean	32,763	59,209	3.53 "	8 "

All uncracked at 8 inches deflection.

Pulling tests of pieces cut from same rails. Mean results.

	Elastic Stress. per sq. in.	Ultimate Pounds. per sq. in.	Contraction of area of fracture.	Extension in 10 ins.
Top of rails	44,200	83,110	19.9%	13.5%
Botton of rails.	40,900	77,830	30.9%	22.8%

Steel Tires.—Tensile tests of specimens cut from steel tires.

Krupp Steel.—262 Tests.

	E. L.	T. S.	Contr.	Ext. in 5 inches.
Highest	69,250	119,079	31.9	18.1
Mean	52,869	104,112	29.5	19.7
Lowest	41,700	90,523	45.5	23.7

Vickers, Sons & Co.—70 Tests.

	E. L.	T. S.	Contr.	Ext. in 5 inches.
Highest	58,600	120,789	11.8	8.4
Mean	51,066	101,264	17.6	12.4
Lowest	43,700	87,697	24.7	16.0

Note the correspondence between Krupp's and Vickers' steels as to tensile strength and elastic limit, and their great difference in contraction and elongation. The fractures of the Krupp steel averaged 22 per cent silky, 78 per cent granular; of the Vicker steel, 7 per cent silky, 93 per cent granular.

Steel Axles.—Tensile tests of specimens cut from steel axles.

PATENT SHAFT AND AXLE TREE CO.—157 Tests.

	E. L.	T. S.	Contr.	Ext. in 5 inches.
Highest........	49,800	90,009	21.1	16.0
Mean...	36,257	72,099	33.0	23.6
Lowest.	31,800	61,382	34.8	25.3

VICKERS, SONS & CO.—125 Tests.

	E. L.	T. S.	Contr.	Ext. in 5 inches.
Highest........	42,600	83,701	18.9	18.2
Mean.	37,618	70,572	41.6	27.5
Lowest.......	30,250	56,388	49.0	37.2

The average fracture of Patent Shaft and Axle Tree Co. steel was 33 per cent silky, 67 per cent granular.

The average fracture of Vickers' steel was 88 per cent silky, 12 per cent granular.

Tensile tests of specimens cut from locomotive crank axles.

VICKERS'.—88 Tests, 1879.

	E. L.	T. S.	Contr.	Ext. in 5 inches.
Highest........	26,700	68,057	28.3	18.4
Mean	24,146	57,922	32.9	24.0
Lowest	21,700	50,195	52.7	36.2

VICKERS'.—78 Tests, 1884.

	E. L.	T. S.	Contr.	Ext. in 5 inches.
Highest........	27,600	64,873	27.0	20.8
Mean	23,572	56,207	32.7	25.9
Lowest	17,600	47,695	35.0	27.2

FRIED. KRUPP.—48 Tests, 1889.

	E. L.	T. S.	Contr.	Ext. in 5 inches.
Highest........	31,650	66,868	48.6	35.6
Mean	29,491	61,774	47.7	32.3
Lowest	31,950	55,172	55.8	35.6

Steel Propeller Shafts.—Tensile tests of pieces cut from two shafts, mean of four tests each. Hollow shaft, Whitworth. T. S., 61,290; E. L., 30,575; contr., 52.8; ext. in 10 inches, 28.6. Solid Shaft, Vickers', T. S., 46,870; E. L. 20,425; contr., 44.4; ext. in 10 inches, 30.7.

Thrusting tests, Whitworth, ultimate, 56,201; elastic, 29,300; set at 30,000 lbs., 0.18 per cent; set at 40,000 lbs., 2.04 per cent; set at 50,000 lbs., 3.82 per cent.

Thrusting tests, Vickers', ultimate, 44,602; elastic, 22,250; set at 30,000 lbs., 2.29 per cent; set at 40,000 lbs., 4.69 per cent.

Shearing strength of the Whitworth shaft, mean of four tests, was 40,654 lbs. per square inch, or 66.3 per cent of the pulling stress. Specific gravity of the Whitworth steel, 7.867: of the Vickers', 7.856.

Spring Steel.—Untempered, 6 tests, average, E. L., 67,916; T. S., 115,668; contr., 37.8; ext. in 10 inches, 16.6. Spring steel untempered, 15 tests, average, E. L., 36,785; T. S., 69,496; contr., 19.1; ext. in 10 inches, 29.8. These two lots were shipped for the same purpose, viz., railway carriage leaf springs.

Steel Castings.—44 tests, E. L., 31,816 to 35,567; T. S., 54,928 to 63,840; contr., 1.67 to 15.8; ext., 1.45 to 15.1. Note the great variation in ductility. The steel of the highest strength was also the most ductile.

Riveted Joints, Pulling Tests of Riveted Steel Plates, Triple Riveted Lap Joints, Machine Riveted, Holes Drilled.

Plates, width and thickness, inches:

13.50 × .25	13.00 × .51	11.75 × .78	12.25 × 1.01	14.00 × .77

Plates, gross sectional area square inches:

3.375	6.63	9.165	12.372	10.780

Stress, total, pounds:

199,320	332,640	423,180	526,660	455,910

Stress per square inch of gross area, joint:	59,058	50,172	46,173	42,696	42,227
Stress per square inch of plates, solid:	70,765	65,300	64,050	62,280	68,045
Ratio of strength of joint to solid plate:	83.46	76.83	72.09	68.55	62.06
Ratio net area of plate to gross:	73.4	65.5	62.7	64.7	72.9
Where fractured:	plate at holes.	plate at holes.	plate at holes.	plate at holes.	rivets sheared.
Rivets, diameter, area and number:	.45, .159, 24	.64, .321, 21	.95, .708, 12	1.08, .916, 12	.95, .708, 12
Rivets, total area:	3.816	6.741	8.496	10.992	8.496

Strength of Welds.—Tensile tests to determine ratio of strength of weld to solid bar.

IRON TIE BARS.—28 Tests.

Strength of solid bars varied from	43,201 to 57,065 lbs.
Strenth of welded bars varied from	17,816 to 44,586 lbs.
Ratio of weld to solid varied from	37.0 to 79.1%

IRON PLATES.—7 Tests.

Strength of solid plate from	44,851 to 47,481 lbs.
Strength of welded plate from	26,442 to 38,931 lbs.
Ratio of weld to solid	57.7 to 83.9%

CHAIN LINKS.—216 Tests.

Strength of solid bar from	49,122 to 57,875 lbs.
Strength of welded bar from	39,575 to 48,824 lbs.
Ratio of weld to solid	72.1 to 95.4%

IRON BARS.—Hand and Electric Machine Welded.

32 tests, solid iron, average	52,444	
17 " electric welded, average	46,836	ratio 89.1%
19 " hand " "	46,899	" 89.3%

STEEL BARS AND PLATES.—14 Tests.

Strength of solid	54,226 to 64,580
Strength of weld	28,553 to 46,019
Ratio weld to solid	52.6 to 82.1%

The ratio of weld to solid in all the tests ranging from 37.0 to 95.4 is proof of the great variation of workmanship in welding.

Cast Copper.—4 tests, average, E. L., 5900; T. S., 24,781; contr., 24.5; ext., 21.8.

Copper Plates.—As rolled, 22 tests, .26 to .75 in. thick; E. L., 9766 to 18,650; T. S., 30,993 to 34,281; contr., 31.1 to 57.6; ext., 39.9 to 52.2. The variation in elastic limit is due to difference in the heat at which the plates were finished. Annealing reduces the T. S. only about 1000 pounds, but the E. L. from 3000 to 7000 pounds.

Another series, .38 to .52 thick; 148 tests, T. S., 29,099 to 31,924; contr., 26.7 to 56.7; ext. in 10 inches, 28.1 to 41.8. Note the uniformity in tensile strength.

Drawn Copper.—74 tests (0.88 to 1.08 inch diameter); T. S., 31,634 to 40,557; contr., 37.5 to 64.1; ext. in 10 inches, 5.8 to 48.2.

Bronze from a Propeller Blade.—Means of two tests each from centre and edge. Central portion (sp. gr. 8.320). E. L., 7550; T. S., 26,312; contr., 25.4; ext. in 10 inches, 32.8. Edge portion (sp. gr. 8550). E. L., 8950; T. S., 35,960; contr., 37.8; ext. in 10 inches, 47.9.

Cast German Silver.—10 tests: E. L., 13,400 to 29,100; T. S., 23,714 to 46,540; contr., 3.2 to 21.5; ext. in 10 inches, 0.6 to 10.2.

Thin Sheet Metal.—Tensile Strength.

German silver, 2 lots	75,816 to 87,129
Bronze, 4 lots	73,380 to 92,086
Brass, 2 lots	44,398 to 58,188
Copper, 9 lots	30,470 to 48,450
Iron, 13 lots, lengthway	44,331 to 59,484
Iron, 13 lots, crossway	39,838 to 57,350
Steel, 6 lots	49,253 to 73,251
Steel, 6 lots, crossway	55,948 to 80,799

Wire.—Tensile Strength.

German silver, 5 lots	81,735 to 92,224
Bronze, 1 lot	78,049
Brass, as drawn, 4 lots	81,114 to 98,578
Copper, as drawn, 3 lots	37,607 to 46,494
Copper annealed, 3 lots	34,936 to 45,210
Copper (another lot), 4 lots	35,052 to 62,190
Copper (extension 36.4 to 0.6%).	
Iron, 8 lots	59,246 to 97,908
Iron (extension 15.1 to 0.7%).	
Steel, 8 lots	103,272 to 318,823

The Steel of 318,823 T. S. was .047 inch diam., and had an extension of only 0.3 per cent; that of 103,272 T. S. was .107 inch diam. and had an extension of 2.2 per cent. One lot of .044 inch diam. had 267,114 T. S., and 5.2 per cent extension.

Wire Ropes.

Selected Tests Showing Range of Variation.

Description.	Circumference, inches.	Weight per Fathom.	Strands. No. of Strands.	Strands. No. of Wires.	Diameter of Wires, inches.	Hemp Core.	Ultimate Strength, lbs.
Galvanized	7.70	58.00	6	19	.1563	Main	339,780
Ungalvanized	7.00	53.10	7	19	.1495	Main and Strands	314,860
Ungalvanized	6.38	42.50	7	19	.1347	Wire Core	295,920
Galvanized	7.10	37.57	6	30	.1004	Main and Strands	272,750
Ungalvanized	6.18	40.46	7	19	.1302	Wire Core	268,470
Ungalvanized	6.19	40.33	7	19	.1316	Wire Core	221,820
Galvanized	4.92	20.86	6	30	.0728	Main and Strands	190,890
Galvanized	5.36	18.94	6	12	.1104	Main and Strands	136,550
Galvanized	4.82	21.50	6	7	.1693	Main	129,710
Ungalvanized	3.65	12.21	6	19	.0755	Main	110,180
Ungalvanized	3.50	12.65	7	7	.122	Wire Core	101,440
Ungalvanized	3.82	14.12	6	7	.135	Main	98,670
Galvanized	4.11	11.35	6	12	.080	Main and Strands	75,110
Galvanized	3.31	7.27	6	12	.068	Main and Strands	55,095
Ungalvanized	3.02	8.62	6	7	.105	Main	49,555
Ungalvanized	2.68	6.26	6	6	.0963	Main and Strands	41,205
Galvanized	2.87	5.43	6	12	.0560	Main and Strands	38,555
Galvanized	2.46	3.85	6	12	.0472	Main and Strands	28,075
Ungalvanized	1.75	2.80	6	7	.0619	Main	24,552
Galvanized	2.04	2.72	6	12	.0378	Main and Strands	20,415
Galvanized	1.76	1.85	6	12	.0305	Main	14,634

Hemp Ropes, Untarred.—15 tests of ropes from 1.53 to 6.90 inches circumference, weighing 0.42 to 7.77 pounds per fathom, showed an ultimate strength of from 1670 to 33,808 pounds, the strength per fathom weight varying from 2872 to 5534 pounds.

Hemp Ropes, Tarred.—15 tests of ropes from 1.44 to 7.12 inches circumference, weighing from 0.38 to 10.39 pounds per fathom, showed an ultimate strength of from 1046 to 31,549 pounds, the strength per fathom weight varying from 1767 to 5149 pounds.

Cotton Ropes.—5 ropes, 2.48 to 6.51 inches circumference, 1.08 to 8.17 pounds per fathom. Strength 3089 to 23,258 pounds, or 2474 to 3346 pounds per fathom weight.

Manila Ropes.—35 tests: 1.19 to 8.90 inches circumference, 0.20 to 11.40 pounds per fathom. Strength 1280 to 65,550 pounds, or 3003 to 7394 pounds per fathom weight.

Belting.

No. of lots.		Tensile strength per square inch.
11	Leather, single, ordinary tanned	3248 to 4824
4	Leather, single, Helvetia	5631 to 5944
7	Leather, double, ordinary tanned	2160 to 3572
8	Leather, double Helvetia	4078 to 5412
6	Cotton, solid woven	5648 to 8869
14	Cotton, folded, stitched	4570 to 7750
1	Flax, solid, woven	9946
1	Flax, folded, stitched	6389
6	Hair, solid, woven	3852 to 5159
2	Rubber, solid, woven	4271 to 4343

Canvas.—35 lots: Strength, lengthwise, 113 to 408 pounds per inch; crossways, 191 to 468 pounds per inch.

The grades are numbered 1 to 6, but the weights are not given. The strengths vary considerably, even in the same number.

Marbles.—Crushing strength of various marbles. 38 tests, 8 kinds. Specimens were 6-inch cubes, or columns 4 to 6 inches diameter, and 6 and 12 inches high. Range 7542 to 13,720 pounds per square inch.

Granite.—Crushing strength, 17 tests; square columns 4 × 4 and 6 × 4, 4 to 24 inches high, 3 kinds. Crushing strength ranges 10,026 to 13,271 pounds per square inch. (Very uniform.)

Stones.—(Probably sandstone, local names only given.) 11 kinds, 42 tests, 6 × 6, columns 12, 18 and 24 inches high. Crushing strength ranges from 2105 to 12,122. The strength of the column 24 inches long is generally from 10 to 20 per cent less than that of the 6-inch cube.

Stones.—(Probably sandstone) tested for London & Northwestern Railway. 16 lots, 3 to 6 tests in a lot. Mean results of each lot ranged from 3785 to 11,956 pounds. The variation is chiefly due to the stones being from different lots. The different specimens in each lot gave results which generally agreed within 30 per cent.

Bricks.—Crushing strength, 8 lots; 6 tests in each lot; mean results ranged from 1835 to 9209 pounds per square inch. The maximum variation in the specimens of one lot was over 100 per cent of the lowest. In the most uniform lot the variation was less than 20 per cent.

Wood.—Transverse and Thrusting Tests.

	Tests.	Sizes abt. in square.	Span, inches.	Ultimate Stress.	$S = \frac{LW}{4BD^2}$.	Thrusting Stress per sq. in.
Pitch pine	10	11½ to 12½	144	45,856 to 80,520	1096 to 1403	3586 to 5438
Dantzic fir	12	12 to 13	144	37,948 to 54,152	657 to 790	2478 to 3423
English oak	3	4½ × 12	120	32,856 to 39,084	1505 to 1779	2473 to 4437
American white oak	5	4½ × 12	120	23,624 to 26,952	1190 to 1372	2656 to 3899

Demerara greenheart, 9 tests (thrusting)	8169 to 10,785
Oregon pine, 2 tests	5888 and 7284
Honduras mahogany, 1 test	6769
Tobasco mahogany, 1 test	5978
Norway spruce, 2 tests	5259 and 5494
American yellow pine, 2 tests	3875 and 3993
English ash, 1 test	3025

Portland Cement.—(Austrian.) Cross-sections of specimens 2 × 2½ inches for pulling tests only; cubes, 3 × 3 inches for thrusting tests; weight, 96.8 pounds per imperial bushel; residue, 0.7 per cent with sieve 2500 meshes per square inch; 38.8 per cent by volume of water required for mixing; time of setting, 7 days; 10 tests to each lot. The mean results in lbs. per sq. in. were as follows:

Age.	Cement alone, Pulling.	Cement alone, Thrusting.	1 Cement, 2 Sand, Thrusting.	1 Cement, 3 Sand, Thrusting.	1 Cement, 4 Sand, Thrusting.
10 days	376	2910	893	407	228
20 days	420	3342	1023	494	275
30 days	451	3724	1172	594	338

Portland Cement.—Various samples pulling tests, 2 × 2½ inches cross-section, all aged 10 days, 180 tests; ranges 87 to 643 pounds per square inch.

TENSILE STRENGTH OF WIRE.

(From J. Bucknall Smith's Treatise on Wire.)

	Tons per sq. in. sectional area.	Pounds per sq. in. sectional area.
Black or annealed iron wire	25	56,000
Bright hard drawn	35	78,400
Bessemer, steel wire	40	89,600
Mild Siemens-Martin steel wire	60	134,000
High carbon ditto (or "improved")	80	179,200
Crucible cast-steel "improved" wire	100	224,000
"Improved" cast-steel "plough"	120	268,800
Special qualities of tempered and improved cast-steel wire may attain	150 to 170	336,000 to 380,800

MISCELLANEOUS TESTS OF MATERIALS.

Reports of Work of the Watertown Testing-machine in 1883.

TESTS OF RIVETED JOINTS, IRON AND STEEL PLATES.

	Thickness Plate.	Diameter, Rivets, inches.	Diameter, Punched Holes, inches.	Width Plate Tested, inches.	No. Rivets.	Pitch Rivets, inches.	Tensile Strength Joint in Net Section of Plate per square inch, pounds.	Tensile Strength Plate per square inch, pounds.	Efficiency of Joint, Per Cent.
*	⅜	11-16	¾	10½	6	1¾	39,800	47,180	47.0 ‡
*	⅜	11-16	¾	10½	6	1¾	41,000	47,180	49.0 ‡
*	½	¾	13-16	10	5	2	35,650	44,615	45.6 ‡
*	½	¾	13-16	10	5	2	35,150	44,615	44.9 ‡
*	⅜	11-16	¾	10	5	2	46,360	47,180	59.9 §
*	⅜	11-16	¾	10	5	2	46,875	47,180	60.5 §
*	½	¾	13-16	10	5	2	46,400	44,615	59.4 §
*	½	¾	13-16	10	5	2	46,140	44,615	59.2 §
*	⅝	1	1 1-16	10½	4	2⅝	44,260	44,635	57.2 §
*	⅝	1	1 1-16	10½	4	2⅝	42,350	44,635	54.9 §
*	¾	1⅛	1 3-16	11.9	4	2.9	42,310	46,590	52.1 §
*	¾	1⅛	1 3-16	11.9	4	2.9	41,920	46,590	51.7 §
*	⅜	¾	13-16	10½	6	1¾	61,270	53,330	59.5 ‡
†	⅜	¾	13-16	10½	6	1¾	60,830	53,330	59.1 ‡
†	½	15-16	1	10	5	2	47,530	57,215	40.2 ‡
†	½	15-16	1	10	5	2	49,840	57,215	42.8 ‡
†	⅜	11-16	¾	10	5	2	62,770	53,330	71.7 §
†	⅜	11-16	¾	10	5	2	61,210	53,330	69.8 §
†	½	15-16	1	10	5	2	68,920	57,215	57.1 §
†	½	15-16	1	10	5	2	66,710	57,215	55.0 §
†	⅝	1	1 1-16	9½	4	2⅜	62,180	52,445	63.4 §
†	⅝	1	1 1-16	9½	4	2⅜	62,590	52,445	63.8 §
†	¾	1⅛	1 3-16	10	4	2½	51,650	51,545	54.0 §
†	¾	1⅛	1 3-16	10	4	2½	54,200	51,545	53.4 §

* Iron. † Steel. ‡ Lap-joint. § Butt-joint.

The efficiency of the joints is found by dividing the maximum tensile stress on the gross sectional area of plate by the tensile strength of the material.

COMPRESSION TESTS OF 3 × 3 INCH WROUGHT-IRON BARS.

Length, inches.	Tested with Two Pin Ends, Pins 1⅛ inch in Diameter. Ultimate Compressive Strength pounds per square inch.	Tested with Two Flat Ends, Ultimate Compressive Strength, pounds per square inch.	Tested with One Flat and One Pin End, Ultimate Compressive Strength, pounds per square inch.
30	28,260 / 31,990	....	....
60	26,810 / 26,640	....	....
90	24,030 / 25,380	26,780 / 25,580	25,120 / 25,190
120	20,660 / 20,200	23,010 / 22,450	22,450 / 21,870
150	16,520 / 17,840	....	....
180	13,010 / 15,700	....	....

Tested with two pin-ends. Length of bars 120 inches.

Diameter of Pins.	Ult. Comp. Str., per sq. in., lbs.
¾ inch	16,250
1⅛ inches	17,740
1⅞ "	21,400
2¼ "	22,210

TENSILE TEST OF SIX STEEL EYE-BARS.

COMPARED WITH SMALL TEST INGOTS.

The steel was made by the Cambria Iron Company, and the eye-bar heads made by Keystone Bridge Company by upsetting and hammering. All the bars were made from one ingot. Two test pieces, ¾-inch round, rolled from a test-ingot, gave elastic limit 48,040 and 42,210 pounds; tensile strength, 73,150 and 69,470 pounds, and elongation in 8 inches, 22.4 and 25.6 per cent. respectively. The ingot from which the eye-bars were made was 14 inches square, rolled to billet, 7 × 6 inches. The eye-bars were rolled to 6½ × 1 inch. Chemical tests gave carbon .27 to .30; manganese, .64 to .73; phosphorus, .074 to .098.

Gauged Length, inches.	Elastic limit, lbs. per sq. in.	Tensile strength per sq. in., lbs.	Elongation per cent, in Gauged Length.
160	37,480	67,800	15.8
160	36,650	64,000	6.96
160	......	71,560	8.6
200	37,600	68,720	12.3
200	35,810	65,850	13.0
200	33,230	64,410	16.4
200	37,640	68,290	13.9

The average tensile strength of the ¾-inch test pieces was 71,310 lbs., that of the eye-bars 67,230 lbs., a decrease of 5.7%. The average elastic limit of the test pieces was 45,150 lbs., that of the eye-bars 36,402 lbs., a decrease of 19.4%. The elastic limit of the test pieces was 63.3% of the ultimate strength, that of the eye-bars 54.2% of the ultimate strength.

COMPRESSION OF WROUGHT-IRON COLUMNS, LATTICED BOX AND SOLID WEB.

ALL TESTED WITH PIN ENDS.

Columns made of	Length, feet.	Sectional Area, square inch.	Total Weight of Column, pounds.	Ultimate Strength, per square inch, pounds.
6-inch channel, solid web	10.0	9.881	482	30,220
6 " " " "	15.0	9.977	592	21,050
6 " " " "	20.0	9.762	755	16,220
8 " " " "	20.0	16.281	1,290	22,540
8 " " " "	26.8	16.141	1,645	17,570
8-inch channels, with 5-16-in. continuous plates	26.8	19.417	1,940	25,290
5-16-inch continuous plates and angles. Width of plates, 12 in., 1 in. and 7.35 in.	26.8	16.168	1,765	28,020
7-16-inch continuous plates and angles. Plates 12 in. wide.	26.8	20.954	2,242	25,770
8-inch channels, latticed	13.3	7.628	679	33,910
8 " " "	20.0	7.621	924	34,120
8 " " "	26.8	7.673	1,255	29,870
8-inch channels, latticed, swelled sides	13.4	7.624	684	33,530
8 " " " " "	20.0	7.517	921	33,390
8 " " " " "	26.8	7.702	1,280	30,770
10 " " "	16.8	11.944	1,470	33,740
10 " " "	25.0	12.175	1,926	32,440
10-inch channels, latticed, swelled sides.	16.7	12.366	1,549	31,130
" " " " "	25.0	11.932	1,962	32,740
* 10-inch channels, latticed one side; continuous plate one side	25.0	17.622	1,848	26,190
† 10 inch channels, latticed one side; continuous plate one side	25.0	17.721	1,827	17,270

* Pins in centre of gravity of channel bars and continuous plate, 1.63 inches from centre line of channel bars.

† Pins placed in centre of gravity of channel bars.

EFFECT OF COLD-DRAWING ON STEEL.

Three pieces cut from the same bar of hot-rolled steel:

1. Original bar, 2.08 in. diam., gauged length 30 in., tensile strength 55,400 lbs. per square in.; elongation 23.9%.
2. Diameter reduced in compression dies (one pass) .094 in.; T. S. 70,420; el. 2.7% in 20 in.
3. " " " " " " " .222 in.; T. S. 81,890; el. 0.075% in 20 in.

Compression test of cold-drawn bar (same as No. 3), length 4 in., diam. 1.808 in.: Compressive strength per sq. in., 75,000 lbs.; amount of compression .057 in.; set .04 in. Diameter increased by compression to 1.821 in. in the middle; to 1.813 in. at the ends.

Tests of Cold-rolled and Cold-drawn Steel, made by the Cambria Iron Co. in 1897, gave the following results (averages of 12 tests of each)

Before cold-rolling,	E. L. 35,390	T. S. 59,980	El. in 8 in. 28.3%	Red. 58.5%
After " " ,	" 72,530	" 79,830	" " 9.6"	" 34.9"
After cold-drawing,	" 76,350	" 83,860	" " 8.9"	" 34.2"

The original bars were 2 in. and 7/8 in. diameter. The test pieces cut from the bars were 3/4 in. diam., 18 in. long. The reduction in diameter from the hot-rolled to the cold-rolled or cold-drawn bar was 1/16 in. in each case.

TESTS OF AMERICAN WOODS. (See also page 309.)

In all cases a large number of tests were made of each wood. Minimum and maximum results only are given. All of the test specimens had a sectional area of 1.575 × 1.575 inches. The transverse test specimens were 39.37 inches between supports, and the compressive test specimens were 12.60 inches long. Modulus of rupture calculated from formula $R = \frac{3}{2}\frac{Pl}{bd^2}$; P = load in pounds at the middle, l = length in inches, b = breadth, d = depth:

Name of Wood.	Transverse Tests. Modulus of Rupture.		Compression Parallel to Grain, pounds per square inch.	
	Min.	Max.	Min.	Max.
Cucumber tree (*Magnolia acuminata*)	7,440	12,050	4,560	7,410
Yellow poplar white wood (*Liriodendron tulipifera*)	6,500	11,756	4,150	5,790
White wood, Basswood (*Tilia Americana*)	6,720	11,530	3,810	6,480
Sugar-maple, Rock-maple (*Acer saccharinum*)	9,680	20,130	7,460	9,940
Red maple (*Acer rubrum*)	8,610	18,450	6,010	7,500
Locust (*Robinia pseudacacia*)	12,200	21,730	8,330	11,940
Wild cherry (*Prunus serotina*)	8,310	16,800	5,830	9,120
Sweet gum (*Liquidambar styraciflua*)	7,470	11,130	5,630	7,020
Dogwood (*Cornus florida*)	10,190	14,560	6,250	9,400
Sour gum, Pepperidge (*Nyssa sylvatica*)	9,830	14,300	6,240	7,480
Persimmon (*Diospyros Virginiana*)	10,290	18,500	6,650	8,080
White ash (*Fraxunis Americana*)	5,950	15,800	4,520	8,830
Sassafras (*Sassafras officinale*)	5,180	10,150	4,050	5,970
Slippery elm (*Ulmus fulva*)	10,220	13,952	6,980	8,790
White elm (*Ulmus Americana*)	8,250	15,070	4,960	8,040
Sycamore; Buttonwood (*Platanus occidentalis*)	6,720	11,360	4,960	7,340
Butternut; white walnut (*Juglans cinerea*)	4,700	11,740	5,480	6,810
Black walnut (*Juglans nigra*)	8,400	16,320	6,940	8,850
Shellbark hickory (*Carya alba*)	14,870	20,710	7,650	10,280
Pignut (*Carya porcina*)	11,560	19,430	7,460	8,470
White oak (*Quercus alba*)	7,010	18,360	5,810	9,070
Red oak (*Quercus rubra*)	9,760	18,370	4,960	8,970
Black oak (*Quercus tinctoria*)	7,900	18,420	4,540	8,550
Chestnut (*Castanea vulgaris*)	5,950	12,870	3,680	6,650
Beech (*Fagus ferruginea*)	13,850	18,840	5,770	7,840
Canoe-birch, paper-birch (*Betula papyracea*)	11,710	17,610	5,770	8,590
Cottonwood (*Populus monilifera*)	8,390	13,430	3,790	6,510
White cedar (*Thuja occidentalis*)	6,310	9,530	2,660	5,810
Red cedar (*Juniperus Virginiana*)	5,640	15,100	4,400	7,040
Cypress (*Taxodium Distichum*)	8,530	10,030	5,060	7,140
White pine (*Pinus strobus*)	5,610	11,530	3,750	5,600
Spruce pine (*Pinus glabra*)	3,780	10,980	2,580	4,660
Long-leaved pine, Southern pine (*Pinus palustris*)	9,220	21,060	4,010	10,660
White spruce (*Picea alba*)	9,900	11,650	4,150	6,300
Hemlock (*Tsuga Canadensis*)	7,590	14,680	4,500	7,420
Red fir, yellow fir (*Pseudotsuga Douglasii*)	8,220	17,920	4,880	9,800
Tamarack (*Larix Americana*)	10,060	16,770	6,810	10,700

SHEARING STRENGTH OF IRON AND STEEL.

H. V. Loss in *American Engineer and Railroad Journal*, March and April, 1893, describes an extensive series of experiments on the shearing of iron and steel bars in shearing machines. Some of his results are :

Depth of penetration at point of maximum resistance for soft steel bars is independent of the width, but varies with the thickness. If d = depth of penetration and t = thickness, $d = .3t$ for a flat knife, $d = .25\,t$ for a 4° bevel knife, and $d = .16\sqrt{t^3}$ for an 8° bevel knife. The ultimate pressure per inch of width in flat steel bars is approximately 50,000 lbs. $\times\, t$. The energy consumed in foot pounds per inch width of steel bars is, approximately: 1″ thick, 1300 ft.-lbs.; 1¼″, 2500; 1¾″, 3700; 1⅞″, 4500; the energy increasing at a slower rate than the square of the thickness. Iron angles require more energy than steel angles of the same size; steel breaks while iron has to be cut off. For hot-rolled steel the resistance per square inch for rectangular sections varies from 4400 lbs. to 20,500 lbs., depending partly upon its hardness and partly upon the size of its cross-area, which latter element indirectly but greatly indicates the temperature, as the smaller dimensions require a considerably longer time to reduce them down to size, which time again means loss of heat.

It is not probable that the resistance in practice can be brought very much below the lowest figures here given—viz., 4400 lbs. per square inch—as a decrease of 1000 lbs. will henceforth mean a considerable increase in cross-section and temperature.

HOLDING-POWER OF BOILER-TUBES EXPANDED INTO TUBE-SHEETS.

Experiments by Chief Engineer W. H. Shock, U. S. N., on brass tubes, 2½ inches diameter, expanded into plates ¾-inch thick, gave results ranging from 5850 to 46,000 lbs. Out of 48 tests 5 gave figures under 10,000 lbs., 12 between 10,000 and 20,000 lbs., 18 between 20,000 and 30,000 lbs., 10 between 30,000 and 40,000 lbs., and 3 over 40,000 lbs.

Experiments by Yarrow & Co., on steel tubes, 2 to 2¼ inches diameter, gave results similarly varying, ranging from 7900 to 41,715 lbs., the majority ranging from 20,000 to 30,000 lbs. In 15 experiments on 4 and 5 inch tubes the strain ranged from 20,720 to 68,040 lbs. Beading the tube does not necessarily give increased resistance, as some of the lower figures were obtained with beaded tubes. (See paper on Rules Governing the Construction of Steam Boilers, Trans. Engineering Congress, Section G, Chicago, 1893.)

CHAINS.

Weight per Foot, Proof Test and Breaking Weight.

(Pennsylvania Railroad Specifications.)

Nominal Diameter of Wire, inches.	Description.	Specifications.		
		Weight per foot, lbs.	Proof Test, lbs.	Breaking Weight, lbs.
5/32	Lock-chain	0.20		
3/16	Fire-door chain	0.35		
¼	Crossing-gate chain	0.70	1500	3000
5/16	Sprocket-wheel chain	1.10	3000	5500
⅜	Brake-chain	1.50	3500	7000
⅜	Crane-chain	1.50	4000	7500
7/16	Drop-bottom branch chain	1.90	5000	9500
7/16	Crane-chain	1.90	5500	10,000
½	Drop-bottom main chain	2.50	7000	12,500
½	Crane-chain	2.50	7500	13,000
⅝	Safety "	4.00	11,000	20,000
⅝	Crane "	4.00	11,000	20,000
¾	Log "	5.50	16,000	29,000
¾	Crane "	5.50	16,000	29,000
⅞	" "	7.40	22,000	40,000
1	" "	9.50	30,000	55,000
1⅛	" "	12.00	40,000	66,000
1¼	" "	15.00	50,000	82,000
1⅜	" "	21.00	70,000	116,000

Elongation of all sizes, 10 per cent. All chain must stand the prescribed proof test without deformation.

British Admiralty Proving Tests of Chain Cables.—Stud-links. Minimum size in inches and 16ths. Proving test in tons of 2240 lbs.

Min. Size:	$\frac{11}{16}$	$\frac{12}{16}$	$\frac{13}{16}$	$\frac{14}{16}$	$\frac{15}{16}$	1	$1\frac{1}{16}$	$1\frac{2}{16}$	$1\frac{3}{16}$	$1\frac{4}{16}$	$1\frac{5}{16}$	$1\frac{6}{16}$	$1\frac{7}{16}$.
Test, tons:	$8\frac{11}{20}$	$10\frac{2}{20}$	$11\frac{18}{20}$	$13\frac{14}{20}$	$15\frac{14}{20}$	18	$20\frac{6}{20}$	$22\frac{15}{20}$	$25\frac{1}{20}$	$28\frac{4}{20}$	31	34	$37\frac{4}{20}$.

Min. Size:	1^8	1^9	1^{10}	1^{11}	1^{12}	1^{13}	1^{14}	1^{15}	2	2^1	2^2	2^3.
Test, tons:	$40\frac{10}{20}$	$43\frac{13}{20}$	$47\frac{10}{20}$	$51\frac{6}{20}$	$55\frac{3}{20}$	$59\frac{3}{20}$	$63\frac{5}{20}$	$67\frac{11}{20}$	72	$76\frac{13}{20}$	$81\frac{6}{20}$	$91\frac{3}{20}$.

Wrought-iron Chain Cables.—The strength of a chain link is less than twice that of a straight bar of a sectional area equal to that of one side of the link. A weld exists at one end and a bend at the other, each requiring at least one heat, which produces a decrease in the strength. The report of the committee of the U. S. Testing Board, on tests of wrought-iron and chain cables contains the following conclusions. That beyond doubt, when made of American bar iron, with cast-iron studs, the studded link is inferior in strength to the unstudded one.

"That when proper care is exercised in the selection of material, a variation of 5 to 17 per cent of the strongest may be expected in the resistance of cables. Without this care, the variation may rise to 25 per cent.

"That with proper material and construction the ultimate resistance of the chain may be expected to vary from 155 to 170 per cent of that of the bar used in making the links, and show an average of about 163 per cent.

"That the proof test of a chain cable should be about 50 per cent of the ultimate resistance of the weakest link."

The decrease of the resistance of the studded below the unstudded cable is probably due to the fact that in the former the sides of the link do not remain parallel to each other up to failure, as they do in the latter. The result is an increase of stress in the studded link over the unstudded in the proportion of unity, to the secant of half the inclination of the sides of the former to each other.

From a great number of tests of bars and unfinished cables, the committee considered that the average ultimate resistance, and proof tests of chain cables made of the bars, whose diameters are given, should be such as are shown in the accompanying table.

ULTIMATE RESISTANCE AND PROOF TESTS OF CHAIN CABLES.

Diam. of Bar.	Average resist. = 163% of Bar.	Proof Test.	Diam. of Bar.	Average resist. = 163% of Bar.	Proof Test.
Inches.	Pounds.	Pounds.	Inches.	Pounds.	Pounds.
1 1/16	71,172	33,840	1 9/16	162,283	77,159
1 1/16	79,544	37,820	1⅝	174,475	82,956
1⅛	88,445	42,053	1 11/16	187,075	88,947
1 3/16	97,731	46,468	1¾	200,074	95,128
1¼	107,440	51,084	1 13/16	213,475	101,499
1 5/16	117,577	55,903	1⅞	227,271	108,058
1⅜	128,129	60,920	1 15/16	241,463	114,806
1 7/16	139,103	66,138	2	256,040	121,737
1½	150,485	71,550			

STRENGTH OF GLASS.

(Fairbairn's "Useful Information for Engineers," Second Series.)

	Best Flint Glass.	Common Green Glass.	Extra White Crown Glass.
Mean specific gravity	3.078	2.528	2.450
Mean tensile strength, lbs. per sq. in., bars..	2,413	2,896	2,546
do. thin plates.	4,200	4,800	6,000
Mean crush'g strength, lbs. p. sq. in., cyl'drs.	27,582	39,876	31,003
do. cubes.	13,130	20,206	21,867

The bars in tensile tests were about ½ inch diameter. The crushing tests were made on cylinders about ¾ inch diameter and from 1 to 2 inches high, and on cubes approximately 1 inch on a side. The mean transverse strength of glass, as calculated by Fairbairn from a mean tensile strength of 2560 lbs. and a mean compressive strength of 30,150 lbs. per sq. in., is, for a bar supported at the ends and loaded in the middle,

$$w = 3140\frac{bd^2}{l},$$

in which w = breaking weight in lbs., b = breadth, d = depth, and l = length, in inches. Actual tests will probably show wide variations in both directions from the mean calculated strength.

STRENGTH OF COPPER AT HIGH TEMPERATURES.

The British Admiralty conducted some experiments at Portsmouth Dockyard in 1877, on the effect of increase of temperature on the tensile strength of copper and various bronzes. The copper experimented upon was in rods .72 in. diameter.

The following table shows some of the results:

Temperature Fahr.	Tensile Strength in lbs. per sq. in.	Temperature Fahr.	Tensile Strength in lbs. per sq. in.
Atmospheric.	23,115	300°	21,607
100°	23,366	400°	21,105
200°	23,110	500°	19,597

Up to a temperature of 400° F. the loss of strength was only about 10 per cent, and at 500° F. the loss was 16 per cent. The temperature of steam at 200 lbs. pressure is 382° F., so that according to these experiments the loss of strength at this point would not be a serious matter. Above a temperature of 500° the strength is seriously affected.

STRENGTH OF TIMBER.

Strength of Long-leaf Pine (Yellow Pine, *Pinus Palustris*) from Alabama (Bulletin No. 8, Forestry Div., Dept. of Agriculture, 1893. Tests by Prof. J. B. Johnson.)

The following is a condensed table of the range of results of mechanical tests of over 2000 specimens, from 26 trees from four different sites in Alabama; reduced to 15 per cent moisture:

	Butt Logs.	Middle Logs.	Top Logs.	Av'g of all Butt Logs.
Specific gravity	0.449 to 1.039	0.575 to 0.859	0.484 to 0.907	0.767
Transverse strength, $\frac{3\,WL}{2\,bh^2}$	4,762 to 16,200	7,640 to 17,128	4,268 to 15,554	12,614
do do. at elast. limit	4,930 to 13,110	5,540 to 11,790	2,553 to 11,950	9,460
Mod. of elast., thous. lbs.	1,119 to 3,117	1,136 to 2,982	842 to 2,697	1,926
Relative elast. resilience, inch-pounds per cub. in.	0.23 to 4.69	1.34 to 4.21	0.09 to 4.65	2.98
Crushing endwise, str. per sq. in.-lbs.	4,781 to 9,850	5,030 to 9,300	4,587 to 9,100	7,452
Crushing across grain, strength per sq. in., lbs.	675 to 2,094	656 to 1,445	584 to 1,766	1,598
Tensile strength per sq. in.	8,600 to 31,890	6,330 to 29,500	4,170 to 23,280	17,359
Shearing strength (with grain), mean per sq. in.	464 to 1,299	539 to 1,230	484 to 1156	866

Some of the deductions from the tests were as follows:

1. With the exception of tensile strength a reduction of moisture is accompanied by an increase in strength, stiffness, and toughness.
2. Variation in strength goes generally hand-in-hand with specific gravity.
3. In the first 20 or 30 feet in height the values remain constant; then occurs a decrease of strength which amounts at 70 feet to 20 to 40 per cent of that of the butt-log.
4. In shearing parallel with the grain and crushing across and parallel with the grain, practically no difference was found.
5. Large beams appear 10 to 20 per cent weaker than small pieces.
6. Compression tests endwise seem to furnish the best average statement of the value of wood, and if one test only can be made, this is the safest, as was also recognized by Bauschinger.
7. Bled timber is in no respect inferior to unbled timber.

The figures for crushing across the grain represent the load required to cause a compression of 15 per cent. The relative elastic resilience, in inch-pounds per cubic inch of the material, is obtained by measuring the area of the plotted-strain diagram of the transverse test from the origin to the point in the curve at which the rate of deflection is 50 per cent greater than the rate in the earlier part of the test where the diagram is a straight line. This point is arbitrarily chosen since there is no definite "elastic limit" in timber as there is in iron. The "strength at the elastic limit" is the strength taken at this same point. Timber is not perfectly elastic for any load if left on any great length of time.

The long-leaf pine is found in all the Southern coast states from North Carolina to Texas. Prof. Johnson says it is probably the strongest timber in large sizes to be had in the United States. In small selected specimens, other species, as oak and hickory, may exceed it in strength and toughness. The other Southern yellow pines, viz., the Cuban, short-leaf and the loblolly pines are inferior to the long-leaf about in the ratios of their specific gravities; the long-leaf being the heaviest of all the pines. It averages (kiln-dried) 48 pounds per cubic foot, the Cuban 47, the short-leaf 40, and the loblolly 34 pounds.

Strength of Spruce Timber.—The modulus of rupture of spruce is given as follows by different authors: Hatfield, 9900 lbs. per square inch; Rankine, 11,100; Laslett, 9045; Trautwine, 8100; Rodman, 6168. Trautwine advises for use to deduct one-third in the case of knotty and poor timber.

Prof. Lanza, in 25 tests of large spruce beams, found a modulus of rupture from 2995 to 5666 lbs.; the average being 4613 lbs. These were average beams, ordered from dealers of good repute. Two beams of selected stock, seasoned four years, gave 7562 and 8748 lbs. The modulus of elasticity ranged from 897,000 to 1,588,000, averaging 1,294,000.

Time tests show much smaller values for both modulus of rupture and modulus of elasticity. A beam tested to 5800 lbs. in a screw machine was left over night, and the resistance was found next morning to have dropped to about 3000, and it broke at 3500.

Prof. Lanza remarks that while it was necessary to use larger factors of safety, when the moduli of rupture were determined from tests with smaller pieces, it will be sufficient for most timber constructions, except in factories, to use a factor of four. For breaking strains of beams, he states that it is better engineering to determine as the safe load of a timber beam the load that will not deflect it more than a certain fraction of its span, say about 1/300 to 1/400 of its length.

Properties of Timber.

(N. J. Steel & Iron Co.'s Book.)

Description.	Weight per cubic foot, in lbs.	Tensile Strength per sq. inch, in lbs.	Crushing Strength per sq. inch, in lbs.	Relative Strength for Cross Breaking. White Pine = 100.	Shearing Strength with the Grain, lbs. per sq. inch
Ash	48 to 55.8	11,000 to 17,207	4,400 to 9,363	130 to 180	458 to 700
Beech	43 to 53.4	11,500 to 18,000	5,800 to 9,363	100 to 144	....
Cedar	50 to 56.8	10,300 to 11,400	5,600 to 6,000	55 to 63	....
Cherry	....	....	....	130	....
Chestnut	33	10,500	5,350 to 5,600	96 to 123	....
Elm	34 to 36.7	13,400 to 13,489	6,831 to 10,331	96	....
Hemlock	....	8,700	5,700	88 to 95	....
Hickory	....	12,800 to 18,000	8,925	150 to 210	....
Locust	44	20,500 to 24,800	9,113 to 11,700	132 to 227	....
Maple	49	10,500 to 10,584	8,150	122 to 220	367 to 647
Oak, White	45 to 54.5	10,253 to 19,500	4,684 to 9,509	130 to 177	752 to 966
Oak, Live	70	....	6,850	155 to 189	....
Pine, White	30	10,000 to 12,000	5,000 to 6,650	100	225 to 423
Pine, Yellow	28.8 to 33	12,600 to 19,200	5,400 to 9,500	98 to 170	286 to 415
Spruce	....	10,000 to 19,500	5,050 to 7,850	86 to 110	253 to 374
Walnut, Black	42	9,286 to 16,000	7,500	....	....

The above table should be taken with caution. The range of variation in the species is apt to be much greater than the figures indicate. See Johnson's tests on long-leaf pine, and Lanza's on spruce, above. The weight of yellow pine in the table is much less than that given by Johnson. (W. K.)

Compressive Strengths of American Woods, *when slowly and carefully seasoned.*—Approximate averages, deduced from many experiments made with the U. S. Government testing-machine at Watertown, Mass., by Mr. S. P. Sharpless, for the Census of 1880. *Seasoned woods* resist crushing much better than green ones; in many cases, twice as well. Different specimens of the same wood vary greatly. The strengths may readily vary as much as one-third part more or less from the average.

	Endwise,* lbs. per sq. in.	Sidewise,† lbs. per sq. in. .01	Sidewise,† lbs. per sq. in. .1		Endwise,* lbs. per sq. in.	Sidewise,† lbs. per sq. in. .01	Sidewise,† lbs. per sq. in. .1
Ash, red and white	6800	1300	3000	*Maple:*			
Aspen	4400	800	1400	sugar and black	8000	1900	4800
Beech	7000	1100	1900	white and red	6800	1300	2900
Birch	8000	1300	2600	*Oak:*			
Buckeye	4400	600	1400	white, post (or iron), swamp white, red, and black	7000	1600	4000
Butternut	5400	700	1600	scrub and basket	6000	1700	4200
Buttonwood (sycamore)	6000	1300	2600	chestnut and live	7500	1600	4500
Cedar, red	6000	700	1000	pin	6500	1300	3000
Cedar, white (arbor-vitæ)	4400	500	900	*Pine:*			
Catalpa (Ind. bean)	5000	700	1800	white	5400	600	1200
Cherry, wild	8000	1700	2600	red or Norway	6300	600	1400
Chestnut	5300	900	1600	pitch and Jersey scrub	5000	1000	2000
Coffee-tree, Ky	5200	1300	2600	Georgia	8500	1300	2600
Cypress, bald	6000	500	1200	*Poplar*	5000	600	1100
Elm, Am. or white	6800	1300	2600	*Sassafras*	5000	1300	2100
" red	7700	1300	2600	*Spruce*, black	5700	700	1300
Hemlock	5300	600	1100	" white	4500	600	1200
Hickory	8000	2000	4000	*Sycamore* (buttonwood)	6000	1300	2600
Lignum-vitæ	10000	1600	13000	*Walnut:*			
Linden, American	5000	500	900	black	8000	1300	2600
Locust:				white (butternut)	5400	700	1600
black and yellow	9800	1900	4400	*Willow*	4400	700	1400
honey	7000	1600	2600				
Mahogany	9000	1700	5300				
Maple:							
broad-leafed, Ore.	5800	1400	2600				

* Specimens 1.57 ins. square × 12.6 ins. long.

† Specimens 1.57 ins. square × 6.3 ins. long. Pressure applied at mid-length by a punch covering one-fourth of the length. The first column gives the loads producing an indentation of .01 inch, the second those producing an indentation of .1 inch. (See also page 306).

Expansion of Timber Due to the Absorption of Water.

(De Volson Wood, A. S. M. E., vol. x.)

Pieces 36 × 5 in., of pine, oak, and chestnut, were dried thoroughly, and then immersed in water for 37 days.

The mean per cent of elongation and lateral expansion were:

	Pine.	Oak.	Chestnut.
Elongation, per cent	0.065	0.085	0.165
Lateral expansion, per cent	2.6	3.5	3.65

Expansion of Wood by Heat.—Trautwine gives for the expansion of white pine for 1 degree Fahr. 1 part in 440,530, or for 180 degrees 1 part in 2447, or about one-third of the expansion of iron.

Shearing Strength of American Woods, adapted for Pins or Treenails.

J. C. Trautwine (*Jour. Franklin Inst.*). (Shearing across the grain.)

	per sq. in.		per sq. in.
Ash	6280	Hickory	6045
Beech	5223	"	7285
Birch	5595	Maple	6355
Cedar (white)	1372	Oak	4425
" "	1519	Oak (live)	8480
Cedar (Central American)	3410	Pine (white)	2480
Cherry	2945	Pine (Northern yellow	4340
Chestnut	1536	Pine (Southern yellow)	5735
Dogwood	6510	Pine (very resinous yellow)	5053
Ebony	7750	Poplar	4418
Gum	5890	Spruce	3255
Hemlock	2750	Walnut (black)	4728
Locust	7176	Walnut (common)	2830

THE STRENGTH OF BRICK, STONE, ETC.

A great advance has recently been made in the manufacture of brick, in the direction of increasing their strength. Chas. P. Chase, in *Engineering News*, says: "Taking the tests as given in standard engineering books eight or ten years ago, we find in Trautwine the strength of brick given as 500 to 4200 lbs. per sq. in. Now, taking recent tests in experiments made at Watertown Arsenal, the strength ran from 5000 to 22,000 lbs. per sq. in. In the tests on Illinois paving-brick, by Prof. I. O. Baker, we find an average strength in hard paving brick of over 5000 lbs. per square inch. The average crushing strength of ten varieties of paving-brick much used in the West, I find to be 7150 lbs. to the square inch."

A recent test of brick made by the dry-clay process at Watertown Arsenal, according to *Paving*, showed an average compressive strength of 3972 lbs. per sq. in. In one instance it reached 4973 lbs. per sq. in. A test was made at the same place on a "fancy pressed brick." The first crack developed at a pressure of 305,000 lbs., and the brick crushed at 364,300 lbs., or 11,130 lbs. per sq. in. This indicates almost as great compressive strength as granite paving-blocks, which is from 12,000 to 20,000 lbs. per sq. in.

The following notes on bricks are from Trautwine's *Engineer's Pocket-book:*

Strength of Brick.—40 to 300 tons per sq. ft., 622 to 4668 lbs. per sq. in. A soft brick will crush under 450 to 600 lbs. per sq. in., or 30 to 40 tons per square foot, but a first-rate machine-pressed brick will stand 200 to 400 tons per sq. ft. (3112 to 6224 lbs. per sq. in.).

Weight of Bricks.—Per cubic foot, best pressed brick, 150 lbs.; good pressed brick, 131 lbs.; common hard brick, 125 lbs.; good common brick, 118 lbs.; soft inferior brick, 100 lbs.

Absorption of Water.—A brick will in a few minutes absorb 1/2 to 3/4 lb. of water, the last being 1/7 of the weight of a hand-moulded one, or 1/3 of its bulk.

Tests of Bricks, full size, on flat side. (Tests made at Watertown Arsenal in 1883.)—The bricks were tested between flat steel buttresses. Compressed surfaces (the largest surface) ground approximately flat. The bricks were all about 2 to 2.1 inches thick, 7.5 to 8.1 inches long, and 3.5 to 3.76 inches wide. Crushing strength per square inch: One lot ranged from 11,056 to 16,734 lbs.: a second, 12,995 to 22,351; a third, 10,890 to 12,709. Other tests gave results from 5960 to 10,250 lbs. per sq. in.

Crushing Strength of Masonry Materials. (From Howe's "Retaining-Walls.")

	tons per sq. ft.		tons per sq. ft.
Brick, best pressed	40 to 300	Limestones and marbles	250 to 1000
Chalk	20 to 30	Sandstone	150 to 550
Granite	300 to 1200	Soapstone	400 to 800

Strength of Granite.—The crushing strength of granite is commonly rated at 12,000 to 15,000 lbs. per sq. in. when tested in two-inch cubes, and only the hardest and toughest of the commonly used varieties reach a strength above 20,000 lbs. Samples of granite from a quarry on the Connecticut River, tested at the Watertown Arsenal, have shown a strength of 35,963 lbs. per sq. in. (*Engineering News*, Jan. 12, 1893).

Strength of Avondale, Pa., Limestone—(*Engineering News*, Feb. 9, 1893).—Crushing strength of 2-in. cubes: light stone 12,112, gray stone 18,040. lbs. per sq. in.

Transverse test of lintels, tool-dressed, 42 in. between knife-edge bearings, load with knife-edge brought upon the middle between bearings:

Gray stone, section 6 in. wide × 10 in. high, broke under a load of	20,950 lbs.
Modulus of rupture	2,200 "
Light stone, section 8¼ in. wide × 10 in. high, broke under	14,720 "
Modulus of rupture	1,170 "
Absorption.—Gray stone	.051 of 1%
Light stone	.052 of 1%

Transverse Strength of Flagging.

(N. J. Steel & Iron Co.'s Book.)

EXPERIMENTS MADE BY R. G. HATFIELD AND OTHERS.

b = width of the stone in inches; d = its thickness in inches; l = distance between bearings in inches.

The *breaking loads* in tons of 2000 lbs., for a weight placed at the *centre* of the space, will be as follows:

	$\frac{bd^2}{l}\times$		$\frac{bd^2}{l}\times$
Bluestone flagging	.744	Dorchester freestone	.264
Quincy granite	.624	Aubigny freestone	.216
Little Falls freestone	.576	Caen freestone	.144
Belleville, N. J., freestone	.480	Glass	1.000
Granite (another quarry)	.432	Slate	1.2 to 2.7
Connecticut freestone	.312		

Thus a block of Quincy granite 80 inches wide and 6 inches thick, resting on beams 36 inches in the clear, would be broken by a load resting midway between the beams $= \frac{80 \times 36}{36} \times .624 = 49.92$ tons.

STRENGTH OF LIME AND CEMENT MORTAR.

(*Engineering*, October 2, 1891.)

Tests made at the University of Illinois on the effects of adding cement to lime mortar. In all the tests a good quality of ordinary fat lime was used, slaked for two days in an earthenware jar, adding two parts by weight of water to one of lime, the loss by evaporation being made up by fresh additions of water. The cements used were a German Portland, Black Diamond (Louisville), and Rosendale. As regards fineness of grinding, 85 per cent of the Portland passed through a No. 100 sieve, as did 72 per cent of the Rosendale. A fairly sharp sand, thoroughly washed and dried, passing through a No. 18 sieve and caught on a No. 30, was used. The mortar in all cases consisted of two volumes of sand to one of lime paste. The following results were obtained on adding various percentages of cement to the mortar:

Tensile Strength, pounds per square inch.

Age	4 Days.	7 Days.	14 Days.	21 Days.	28 Days.	50 Days.	84 Days.
Lime mortar	4	8	10	13	18	21	26
20 per cent Rosendale	5.	8½	9½	12	17	17	18
20 " " Portland	5	8½	14	20	25	24	26
30 " " Rosendale	7	11	13	18½	21	22½	23
30 " " Portland	8	16	18	22	25	28	27
40 " " Rosendale	10	12	16½	21½	22½	24	36
40 " " Portland	27	39	38	43	47	59	57
60 " " Rosendale	9	13	20	16	22	22½	23
60 " " Portland	45	58	55	68	67	102	78
80 " " Rosendale	12	18½	22½	27	20	31½	33
80 " " Portland	87	91	103	124	94	210	145
100 " " Rosendale	18	23	26	31	34	46	48
100 " " Portland	90	120	146	152	181	205	202

MODULI OF ELASTICITY OF VARIOUS MATERIALS.

The modulus of elasticity determined from a tensile test of a bar of any material is the quotient obtained by dividing the tensile stress in pounds per square inch at any point of the test by the elongation per inch of length produced by that stress; or if P = pounds of stress applied, K = the sectional area, l = length of the portion of the bar in which the measurement is made, and λ = the elongation in that length, the modulus of elasticity $E = \frac{P}{K} \div \frac{\lambda}{l} = \frac{Pl}{K\lambda}$. The modulus is generally measured within the elastic limit only, in materials that have a well-defined elastic limit, such as iron and steel, and when not otherwise stated the modulus is understood to be the modulus within the elastic limit. Within this limit, for such materials the modulus is practically constant for any given bar, the elongation being directly proportional to the stress. In other materials, such as cast iron, which have no well-defined elastic limit, the elongations from the beginning of a test increase in a greater ratio than the stresses, and the modulus is therefore at its maximum near the beginning of the test, and continually decreases. The moduli of elasticity of various materials have already been given above in treating of these materials, but the following table gives some additional values selected from different sources:

Brass, cast	9,170,000	
" wire	14,230,000	
Copper	15,000,000 to 18,000,000	
Lead	1,000,000	
Tin, cast	4,600,000	
Iron, cast	12,000,000 to 27,000,000 (?)	
Iron, wrought	22,000,000 to 29,000,000 (?)	
Steel	28,000,000 to 32,000,000 (see below)	
Marble	25,000,000	
Slate	14,500,000	
Glass	8,000,000	
Ash	1,600,000	
Beech	1,300,000	
Birch	1,250,000 to 1,500,000	
Fir	869,000 to 2,191,000	
Oak	974,000 to 2,283,000	
Teak	2,414,000	
Walnut	306,000	
Pine, long-leaf (butt-logs)	1,119,000 to 3,117,000	Avge. 1,926,000

The maximum figures given by many writers for iron and steel, viz., 40,000,000 and 42,000,000, are undoubtedly erroneous. The modulus of elasticity of steel (within the elastic limit) is remarkably constant, notwithstanding great variations in chemical analysis, temper, etc. It rarely is found below 29,000,000 or above 31,000,000. It is generally taken at 30,000,000 in engineering calculations. Prof. J. B. Johnson, in his report on Long-leaf Pine, 1893, says: "The modulus of elasticity is the most constant and reliable property of all engineering materials. The wide range of value of the modulus of elasticity of the various metals found in public records must be explained by erroneous methods of testing."

In a tensile test of cast iron by the author (Van Nostrand's Science Series, No. 41, page 45), in which the ultimate strength was 23,285 lbs. per sq. in., the measurements of elongation were made to .0001 inch, and the modulus of elasticity was found to decrease from the beginning of the test, as follows: At 1000 lbs. per sq. in., 25,000,000; at 2000 lbs., 16,666,000; at 4000 lbs., 15,384,000; at 6000 lbs., 13,636,000; at 8000 lbs., 12,500,000; at 12,000 lbs., 11,250,000; at 15,000 lbs., 10,000,000; at 20,000 lbs., 8,000,000; at 23,000 lbs., 6,140,000.

FACTORS OF SAFETY.

A factor of safety is the ratio in which the load that is just sufficient to overcome instantly the strength of a piece of material is greater than the greatest safe ordinary working load. (Rankine.)

Rankine gives the following "examples of the values of those factors which occur in machines":

	Dead Load.	Live Load, Greatest.	Live Load, Mean.
Iron and steel	3	6	from 6 to 40
Timber	4 to 5	8 to 10	....
Masonry	4	8	....

The great factor of safety, 40, is for shafts in millwork which transmit very variable efforts.

Unwin gives the following "factors of safety which have been adopted in certain cases for different materials." They "include an allowance for ordinary contingencies."

	Dead Load.	Live Load. In Temporary Structures.	In Permanent Structures.	In Structures subj. to Shocks.
Wrought iron and steel	3	4	4 to 5	10
Cast iron	3	4	5	10
Timber	....	4	10	....
Brickwork	....	....	6	....
Masonry	20	....	20 to 30	....

Unwin says says that "these numbers fairly represent practice based on experience in many actual cases, but they are not very trustworthy."

Prof. Wood in his "Resistance of Materials" says: "In regard to the margin that should be left for safety, much depends upon the character of the loading. If the load is simply a dead weight, the margin may be comparatively small; but if the structure is to be subjected to percussive forces or shocks, the margin should be comparatively large on account of the indeterminate effect produced by the force. In machines which are subjected to a constant jar while in use, it is very difficult to determine the proper margin which is consistent with economy and safety. Indeed, in such cases, economy as well as safety generally consists in making them *excessively* strong, as a single breakage may cost much more than the extra material necessary to fully insure safety."

For discussion of the resistance of materials to repeated stresses and shocks, see pages 238 to 240.

Instead of using factors of safety it is becoming customary in designing to fix a certain number of pounds per square inch as the maximum stress which will be allowed on a piece. Thus, in designing a boiler, instead of naming a factor of safety of 6 for the plates and 10 for the stay-bolts, the ultimate tensile strength of the steel being from 50,000 to 60,000 lbs. per sq. in., an allowable working stress of 10,000 lbs. per sq. in. on the plates and 6000 lbs. per sq. in. on the stay-bolts may be specified instead. So also in Merriman's formula for columns (see page 260) the dimensions of a column are calculated after assuming a maximum allowable compressive stress per square inch on the concave side of the column.

The factors for masonry under dead load as given by Rankine and by Unwin, viz., 4 and 20, show a remarkable difference, which may possibly be explained as follows: If the actual crushing strength of a pier of masonry is known from direct experiment, then a factor of safety of 4 is sufficient for a pier of the same size and quality under a steady load; but if the crushing strength is merely assumed from figures given by the authorities (such as the crushing strength of pressed brick, quoted above from Howe's Retaining Walls, 40 to 300 tons per square foot, average 170 tons), then a factor of safety of 20 may be none too great. In this case the factor of safety is really a "factor of ignorance."

The selection of the proper factor of safety or the proper maximum unit stress for any given case is a matter to be largely determined by the judgment of the engineer and by experience. No definite rules can be given. The customary or advisable factors in many particular cases will be found where these cases are considered throughout this book. In general the following circumstances are to be taken into account in the selection of a factor:

1. When the ultimate strength of the material is known within narrow limits, as in the case of structural steel when tests of samples have been made, when the load is entirely a steady one of a known amount, and there is no reason to fear the deterioration of the metal by corrosion, the lowest factor that should be adopted is 3.

2. When the circumstances of 1 are modified by a portion of the load being variable, as in floors of warehouses, the factor should be not less than 4.

3. When the whole load, or nearly the whole, is apt to be alternately put on and taken off, as in suspension rods of floors of bridges, the factor should be 5 or 6.

4. When the stresses are reversed in direction from tension to compression, as in some bridge diagonals and parts of machines, the factor should be not less than 6.

5. When the piece is subjected to repeated shocks, the factor should be not less than 10.
6. When the piece is subject to deterioration from corrosion the section should be sufficiently increased to allow for a definite amount of corrosion before the piece be so far weakened by it as to require removal.
7. When the strength of the material, or the amount of the load, or both are uncertain, the factor should be increased by an allowance sufficient to cover the amount of the uncertainty.
8. When the strains are of a complex character and of uncertain amount, such as those in the crank-shaft of a reversing engine, a very high factor is necessary, possibly even as high as 40, the figure given by Rankine for shafts in millwork.

THE MECHANICAL PROPERTIES OF CORK.

Cork possesses qualities which distinguish it from all other solid or liquid bodies, namely, its power of altering its volume in a very marked degree in consequence of change of pressure. It consists, practically, of an aggregation of minute air-vessels, having thin, water-tight, and very strong walls, and hence, if compressed, the resistance to compression rises in a manner more like the resistance of gases than the resistance of an elastic solid such as a spring. In a spring the pressure increases in proportion to the distance to which the spring is compressed, but with gases the pressure increases in a much more rapid manner; that is, inversely as the volume which the gas is made to occupy. But from the permeability of cork to air, it is evident that, if subjected to pressure in one direction only, it will gradually part with its occluded air by effusion, that is, by its passage through the porous walls of the cells in which it is contained. The gaseous part of cork constitutes 53% of its bulk. Its elasticity has not only a very considerable range, but it is very persistent. Thus in the better kind of corks used in bottling the corks expand the instant they escape from the bottles. This expansion may amount to an increase of volume of 75%, even after the corks have been kept in a state of compression in the bottles for ten years. If the cork be steeped in hot water, the volume continues to increase till it attains nearly three times that which it occupied in the neck of the bottle.

When cork is subjected to pressure a certain amount of permanent deformation or "permanent set" takes place very quickly. This property is common to all solid elastic substances when strained beyond their elastic limits, but with cork the limits are comparatively low. Besides the permanent set, there is a certain amount of sluggish elasticity—that is, cork on being released from pressure springs back a certain amount at once, but the complete recovery takes an appreciable time.

Cork which had been compressed and released in water many thousand times had not changed its molecular structure in the least, and had continued perfectly serviceable. Cork which has been kept under a pressure of three atmospheres for many weeks appears to have shrunk to from 80% to 85% of its original volume.—*Van Nostrand's Eng'g Mag.* 1886, xxxv. 307.

TESTS OF VULCANIZED INDIA-RUBBER.

Lieutenant L. Vladomiroff, a Russian naval officer, has recently carried out a series of tests at the St. Petersburg Technical Institute with a view to establishing rules for estimating the quality of vulcanized india-rubber. The following, in brief, are the conclusions arrived at, recourse being had to physical properties, since chemical analysis did not give any reliable result: 1. India-rubber should not give the least sign of superficial cracking when bent to an angle of 180 degrees after five hours of exposure in a closed air-bath to a temperature of 125° C. The test-pieces should be 2.4 inches thick. 2. Rubber that does not contain more than half its weight of metallic oxides should stretch to five times its length without breaking. 3. Rubber free from all foreign matter, except the sulphur used in vulcanizing it, should stretch to at least seven times its length without rupture. 4. The extension measured immediately after rupture should not exceed 12% of the original length, with given dimensions. 5. Suppleness may be determined by measuring the percentage of ash formed in incineration. This may form the basis for deciding between different grades of rubber for certain purposes. 6. Vulcanized rubber should not harden under cold. These rules have been adopted for the Russian navy.—*Iron Age*, June 15, 1893.

XYLOLITH, OR WOODSTONE

is a material invented in 1883, but only lately introduced to the trade by Otto Serrig & Co., of Pottschappel, near Dresden. It is made of magnesia cement, or calcined magnesite, mixed with sawdust and saturated with a solution of chloride of calcium. This pasty mass is spread out into sheets and submitted to a pressure of about 1000 lbs. to the square inch, and then simply dried in the air. Specific gravity 1.553. The fractured surface shows a uniform close grain of a yellow color. It has a tensional resistance when dry of 100 lbs. per square inch, and when wet about 66 lbs. When immersed in water for 12 hours it takes up 2.1% of its weight, and 3.8% when immersed 216 hours.

When treated for several days with hydrochloric acid it loses 2.3% in weight, and shows no loss of weight under boiling in water, brine, soda-lye, and solution of sulphates of iron, of copper, and of ammonium. In hardness the material stands between feldspar and quartz, and as a non-conductor of heat it ranks between asbestos and cork.

It stands fire well, and at a red heat it is rendered brittle and crumbles at the edges, but retains its general form and cohesion. This xylolith is supplied in sheets from ¼ in. to 1½ in. thick, and up to one metre square. It is extensively used in Germany for floors in railway stations, hospitals, etc., and for decks of vessels. It can be sawed, bored, and shaped with ordinary woodworking tools. Putty in the joints and a good coat of paint make it entirely water-proof. It is sold in Germany for flooring at about 7 cents per square foot, and the cost of laying adds about 4 cents more.—*Eng'g News*, July 28, 1892, and July 27, 1893.

ALUMINUM—ITS PROPERTIES AND USES.

(By Alfred E. Hunt, Pres't of the Pittsburgh Reduction Co.)

The specific gravity of pure aluminum in a cast state is 2.58; in rolled bars of large section it is 2.6; in very thin sheets subjected to high compression under chilled rolls, it is as much as 2.7. Taking the weight of a given bulk of cast aluminum as 1, wrought iron is 2.90 times heavier; structural steel, 2.95 times; copper, 3.60; ordinary high brass, 3.45. Most wood suitable for use in structures has about one third the weight of aluminum, which weighs 0.092 lb. to the cubic inch.

Pure aluminum is practically not acted upon by boiling water or steam. Carbonic oxide or hydrogen sulphide does not act upon it at any temperature under 600° F. It is not acted upon by most organic secretions.

Hydrochloric acid is the best solvent for aluminum, and strong solutions of caustic alkalies readily dissolve it. Ammonia has a slight solvent action, and concentrated sulphuric acid dissolves aluminum upon heating, with evolution of sulphurous acid gas. Dilute sulphuric acid acts but slowly on the metal, though the presence of any chlorides in the solution allow rapid decomposition. Nitric acid, either concentrated or dilute, has very little action upon the metal, and sulphur has no action unless the metal is at a red heat. Sea-water has very little effect on aluminum. Strips of the metal placed on the sides of a wooden ship corroded less than 1/1000 inch after six months' exposure to sea-water, corroding less than copper sheets similarly placed.

In malleability pure aluminum is only exceeded by gold and silver. In ductility it stands seventh in the series, being exceeded by gold, silver, platinum, iron, very soft steel, and copper. Sheets of aluminum have been rolled down to a thickness of 0.0005 inch, and beaten into leaf nearly as thin as gold leaf. The metal is most malleable at a temperature of between 400° and 600° F., and at this temperature it can be drawn down between rolls with nearly as much draught upon it as with heated steel. It has also been drawn down into the very finest wire. By the Mannesmann process aluminum tubes have been made in Germany.

Aluminum stands very high in the series as an electro-positive metal, and contact with other metals should be avoided, as it would establish a galvanic couple.

The electrical conductivity of aluminum is only surpassed by pure copper, silver, and gold. With silver taken at 100 the electrical conductivity of aluminum is 54.20; that of gold on the same scale is 78; zinc is 29.90; iron is only 16, and platinum 10.60. Pure aluminum has no polarity, and the metal in the market is absolutely non-magnetic.

Sound castings can be made of aluminum in either dry or "green" sand moulds, or in metal "chills." It must not be heated much beyond its melting-point, and must be poured with care, owing to the ready absorption of occluded gases and air. The shrinkage in cooling is 17/64 inch per foot, or a little more than ordinary brass. It should be melted in plumbago crucibles, and the metal becomes molten at a temperature of 1120° F. according to Professor Roberts-Austen, or at 1300° F. according to Richards.

The coefficient of linear expansion, as tested on ⅜-inch round aluminum rods, is 0.00002295 per degree centigrade between the freezing and boiling point of water. The mean specific heat of aluminum is higher than that of any other metal, excepting only magnesium and the alkali metals. From zero to the melting-point it is 0.2185; water being taken as 1, and the latent heat of fusion at 28.5 heat units. The coefficient of thermal conductivity of unannealed aluminum is 37.96; of annealed aluminum, 38.87. As a conductor of heat aluminum ranks fourth, being exceeded only by silver, copper, and gold.

Aluminum, under tension, and section for section, is about as strong as cast iron. The tensile strength of aluminum is increased by cold rolling or cold forging, and there are alloys which add considerably to the tensile strength without increasing the specific gravity to over 3 or 3.25.

The strength of commercial aluminum is given in the following table as the result of many tests:

Form.	Elastic Limit per sq. in. in Tension, lbs.	Ultimate Strength per sq. in. in Tension, lbs.	Percentage of Reduct'n of Area in Tension.
Castings	6,500	15,000	15
Sheet	12,000	24,000	35
Wire	16,000–30,000	30,000–65,000	60
Bars	14,000	28,000	40

The elastic limit per square inch under compression in cylinders, with length twice the diameter, is 3500. The ultimate strength per square inch under compression in cylinders of same form is 12,000. The modulus of elasticity of cast aluminum is about 11,000,000. It is rather an open metal in its texture, and for cylinders to stand pressure an increase in thickness must be given to allow for this porosity. Its maximum shearing stress in castings is about 12,000, and in forgings about 16,000, or about that of pure copper.

Pure aluminum is too soft and lacking in tensile strength and rigidity for many purposes. Valuable alloys are now being made which seem to give great promise for the future. They are alloys containing from 2% to 7% or 8% of copper, manganese, iron, and nickel. As nickel is one of the principal constituents, these alloys have the trade name of "Nickel-aluminum."

Plates and bars of this nickel alloy have a tensile strength of from 40,000 to 50,000 pounds per square inch, an elastic limit of 55% to 60% of the ultimate tensile strength, an elongation of 20% in 2 inches, and a reduction of area of 25%.

This metal is especially capable of withstanding the punishment and distortion to which structural material is ordinarily subjected. Nickel-aluminum alloys have as much resilience and spring as the very hardest of hard-drawn brass.

Their specific gravity is about 2.80 to 2.85, where pure aluminum has a specific gravity of 2.72.

In castings, more of the hardening elements are necessary in order to give the maximum stiffness and rigidity, together with the strength and ductility of the metal; the favorite alloy material being zinc, iron, manganese, and copper. Tin added to the alloy reduces the shrinkage, and alloys of aluminum and tin can be made which have less shrinkage than cast iron.

The tensile strength of hardened aluminum-alloy castings is from 20,000 to 25,000 pounds per square inch.

Alloys of aluminum and copper form two series, both valuable. The first is aluminum-bronze, containing from 5% to 11½% of aluminum; and the second is copper-hardened aluminum, containing from 2% to 15% of copper. Aluminum-bronze is a very dense, fine-grained, and strong alloy, having good ductility as compared with tensile strength. The 10% bronze in forged bars will give 100,000 lbs. tensile strength per square inch, with 60,000 lbs. elastic limit per square inch, and 10% elongation in 8 inches. The 5% to 7½% bronze has a specific gravity of 8 to 8.30, as compared with 7.50 for the 10% to 11½% bronze, a tensile strength of 70,000 to 80,000 lbs., an elastic limit of 40,000 lbs. per square inch, and an elongation of 30% in 8 inches.

Aluminum is used by steel manufacturers to prevent the retention of the occluded gases in the steel, and thereby produce a solid ingot. The proportions of the dose range from ½ lb. to several pounds of aluminum per ton of steel. Aluminum is also used in giving extra fluidity to steel used in castings, making them sharper and sounder. Added to cast iron, aluminum causes the iron to be softer, free from shrinkage, and lessens the tendency to "chill."

With the exception of lead and mercury, aluminum unites with all metals, though it unites with antimony with great difficulty. A small percentage of silver whitens and hardens the metal, and gives it added strength; and this alloy is especially applicable to the manufacture of fine instruments and apparatus. The following alloys have been found recently to be useful in the arts: Nickel-aluminum, composed of 20 parts nickel to 80 of aluminum; rosine, made of 40 parts nickel, 10 parts silver, 30 parts aluminum, and 20 parts tin, for jewellers' work; mettaline, made of 35 parts cobalt, 25 parts aluminum, 10 parts iron, and 30 parts copper. The aluminum-bourbouns metal, shown at the Paris Exposition of 1889, has a specific gravity of 2.9 to 2.96, and can be cast in very solid shapes, as it has very little shrinkage. From analysis the following composition is deduced: Aluminum, 85.74%; tin, 12.94%; silicon, 1.32%; iron, none.

The metal can be readily electrically welded, but soldering is still not satisfactory. The high heat conductivity of the aluminum withdraws the heat of the molten solder so rapidly that it "freezes" before it can flow sufficiently. A German solder said to give good results is made of 80% tin to 20% zinc, using a flux composed of 80 parts stearic acid, 10 parts chloride of zinc, and 10 parts of chloride of tin. Pure tin, fusing at 250° C., has also been used as a solder. The use of chloride of silver as a flux has been patented, and used with ordinary soft solder has given some success. A pure nickel soldering-bit should be used, as it does not discolor aluminum as copper bits do.

ALLOYS.

ALLOYS OF COPPER AND TIN.

(Extract from Report of U. S. Test Board.*)

Number.	Mean Composition by Analysis.		Tensile Strength, lbs. per sq. in.	Elastic Limit, lbs. per sq. in.	Elongation, per cent in 5 inches.	Transverse Test, Modulus of Rupture.	Deflection, 1" sq. Bar 22 in. long, inches.	Crushing Strength, lbs. per sq. in.	Torsion Tests.	
	Copper.	Tin.							Maximum Tor. Moment, ft.-lbs.	Angle of Torsion, degrees.
1	100.	....	27,800	14,000	6.47	29,848	bent.	42,000	143	153
1a	100.	....	12,760	11,000	0.47	21,251	2.31	39,000	65	40
2	97.89	1.90	24,580	10,000	13.33	....	....	34,000	150	317
3	96.06	3.76	32,000	16,000	14.29	33,232	bent.	42,048	157	247
4	94.11	5.43	....	....	....	38,659	"	....	....	....
5	92.11	7.80	28,540	19,000	5.53	43,731	"	42,000	160	126
6	90.27	9.58	26,860	15,750	3.66	49,400	"	38,000	175	114
7	88.41	11.59	....	....	....	60,403	"	....	....	....
8	87.15	12.73	29,430	20,000	3.33	34,531	4.00	53,000	182	100
9	82.70	17.34	....	....	....	67,930	0.63	....	....	....
10	80.95	18.84	32,980	....	0.04	56,715	0.49	78,000	190	16
11	77.56	22.25	....	....	0.	29,926	0.16	....	....	....
12	76.68	23.24	22,010	22,010	0.	32,210	0.19	114,000	122	3.4
13	72.89	26.85	....	....	0.	9,512	0.05	....	....	....
14	69.84	29.88	5,585	5,585	0.	12,076	0.06	147,000	18	1.5
15	68.58	31.26	....	....	0.	9,152	0.04	....	....	....
16	67.87	32.10	....	....	0.	9,477	0.05	....	....	....
17	65.34	34.47	2,201	2,201	0.	4,776	0.02	84,700	16	1
18	56.70	43.17	1,455	1,455	0.	2,126	0.02	....	....	....
19	44.52	55.28	3,010	3,010	0.	4,776	0.03	35,800	23	1
20	34.22	65.80	3,371	3,371	0.	5,924	0.04	19,600	17	2
21	23.35	76.29	6,775	6,775	0.	12,408	0.27	....	....	....
22	15.08	84.62	....	....	....	9,063	0.86	6,500	21	25
23	11.49	88.47	6,380	3,500	4.10	10,706	5.85	10,100	23	62
24	8.57	91.39	6,450	3,300	6.87	5,305	bent.	9,800	23	132
25	3.72	96.31	4,780	2,750	12.32	6,925	"	9,800	23	220
26	0.	100.	3,505	....	35.51	3,740	"	6,400	12	557

* The tests of the alloys of copper and tin and of copper and zinc, the results of which are published in the Report of the U. S. Board appointed to test Iron, Steel, and other Metals, Vols. I and II, 1879 and 1881, were made by the author under direction of Prof. R. H. Thurston, chairman of the Committee on Alloys. See preface to the report of the Committee, in Vol. I.

Nos. 1a and 2 were full of blow-holes.

Tests Nos. 1 and 1a show the variation in cast copper due to varying conditions of casting. In the crushing tests Nos. 12 to 20, inclusive, crushed and broke under the strain, but all the others bulged and flattened out. In these cases the crushing strength is taken to be that which caused a decrease of 10% in the length. The test-pieces were 2 in. long and 5/8 in. diameter. The torsional tests were made in Thurston's torsion-machine, on pieces 5/8 in. diameter and 1 in. long between heads.

Specific Gravity of the Copper-tin Alloys.—The specific gravity of copper, as found in these tests, is 8.874 (tested in turnings from the ingot, and reduced to 39.1° F.). The alloy of maximum sp. gr. 8.956 contained 62.42 copper, 37.48 tin, and all the alloys containing less than 37% tin varied irregularly in sp. gr. between 8.65 and 8.93, the density depending not on the composition, but on the porosity of the casting. It is probable that the actual sp. gr. of all these alloys containing less than 37% tin is about 8.95, and any smaller figure indicates porosity in the specimen.

From 37% to 100% tin, the sp. gr. decreases regularly from the maximum of 8.956 to that of pure tin, 7.293.

Note on the Strength of the Copper-tin Alloys.

The bars containing from 2% to 24% tin, inclusive, have considerable strength, and all the rest are practically worthless for purposes in which strength is required. The dividing line between the strong and brittle alloys is precisely that at which the color changes from golden yellow to silver-white, viz., at a composition containing between 24% and 30% of tin.

It appears that the tensile and compressive strengths of these alloys are in no way related to each other, that the torsional strength is closely proportional to the tensile strength, and that the transverse strength may depend in some degree upon the compressive strength, but it is much more nearly related to the tensile strength. The modulus of rupture, as obtained by the transverse tests, is, in general, a figure between those of tensile and compressive strengths per square inch, but there are a few exceptions in which it is larger than either.

The strengths of the alloys at the copper end of the series increase rapidly with the addition of tin till about 4% of tin is reached. The transverse strength continues regularly to increase to the maximum, till the alloy containing about 17½% of tin is reached, while the tensile and torsional strengths also increase, but irregularly, to the same point. This irregularity is probably due to porosity of the metal, and might possibly be removed by any means which would make the castings more compact. The maximum is reached at the alloy containing 82.70 copper, 17.34 tin, the transverse strength, however, being very much greater at this point than the tensile or torsional strength. From the point of maximum strength the figures drop rapidly to the alloys containing about 27.5% of tin, and then more slowly to 37.5%, at which point the minimum (or nearly the minimum) strength, by all three methods of test, is reached. The alloys of minimum strength are found from 37.5% tin to 52.5% tin. The absolute minimum is probably about 45% of tin.

From 52.5% of tin to about 77.5% tin there is a rather slow and irregular increase in strength. From 77.5% tin to the end of the series, or all tin, the strengths slowly and somewhat irregularly decrease.

The results of these tests do not seem to corroborate the theory given by some writers, that peculiar properties are possessed by the alloys which are compounded of simple multiples of their atomic weights or chemical equivalents, and that these properties are lost as the compositions vary more or less from this definite constitution. It does appear that a certain percentage composition gives a maximum strength and another certain percentage a minimum, but neither of these compositions is represented by simple multiples of the atomic weights.

There appears to be a regular law of decrease from the maximum to the minimum strength which does not seem to have any relation to the atomic proportions, but only to the percentage compositions.

Hardness.—The pieces containing less than 24% of tin were turned in the lathe without difficulty, a gradually increasing hardness being noticed, the last named giving a very short chip, and requiring frequent sharpening of the tool.

With the most brittle alloys it was found impossible to turn the test-pieces in the lathe to a smooth surface. No. 13 to No. 17 (26.85 to 34.47 tin) could not be cut with a tool at all. Chips would fly off in advance of the tool and beneath it, leaving a rough surface: or the tool would sometimes, apparently, crush off portions of the metal, grinding it to powder. Beyond 40% tin the hardness decreased so that the bars could be easily turned.

ALLOYS OF COPPER AND ZINC. (U. S. Test Board).

No.	Mean Composition by Analysis. Copper.	Mean Composition by Analysis. Zinc.	Tensile Strength, lbs. per sq. in.	Elastic Limit % of Breaking Load, lbs. per sq. in.	Elongation % in 5 inches.	Transverse Test Modulus of Rupture.	Deflection 1" sq. bar 22" long, in.	Crushing Str'gth per sq. in., lbs.	Torsional Tests. Max. Tors. Moment ft.-lbs.	Torsional Tests. Angle of Torsion, deg.
1	97.83	1.88	27,240	....	....	....	......	....	130	357
2	82.93	16.98	32,600	26.1	26.7	23,197	Bent	....	155	329
3	81.91	17.99	32,670	30.6	31.4	21,193	"	....	166	345
4	77.39	22.45	35,630	20.0	35.5	25,374	"	....	169	311
5	76.65	23.08	30,520	24.6	35.8	22,325	"	42,000	165	267
6	73.20	26.47	31,580	23.7	38.5	25,894	"	....	168	293
7	71.20	28.54	30,510	29.5	29.2	24,468	"	....	164	269
8	69.74	30.06	28,120	28.7	20.7	26,930	"	....	148	202
9	66.27	33.50	37,800	25.1	37.7	28,459	"	....	176	257
10	63.44	36.36	48,300	32.8	31.7	43,216	"	....	202	230
11	60.94	38.65	41,065	40.1	20.7	38,968	"	75,000	194	202
12	58.49	41.10	50,450	54.4	10.1	63,304	"	....	227	93
13	55.15	44.44	44,280	44.0	15.3	42,463	"	78,000	209	109
14	54.86	44.78	46,400	53.9	8.0	47,955	"	....	223	72
15	49.66	50.14	30,900	54.5	5.0	33,467	1.26	117,400	172	38
16	48.99	50.82	26,050	100.	0.8	40,189	0.61	... .	176	16
17	47.56	52.28	24,150	100.	0.8	48,471	1.17	121,000	155	18
18	43.36	56.22	9,170	100.	....	17,691	0.10	....	88	2
19	41.30	58.12	3,727	100.	....	7,761	0.04	....	18	2
20	32.94	66.23	1,774	100.	....	8,296	0.04	....	29	1
21	29.20	70.17	6,414	100.	....	16,579	0.04	... .	40	2
22	20.81	77.63	9,000	100.	0.2	22,972	0.13	52,152	65	1
23	12.12	86.67	12,413	100.	0.4	35,026	0.31	....	82	8
24	4.35	94.59	18,065	100.	0.5	26,162	0.46	....	81	22
25	Cast	Zinc.	5,400	75.	0.7	7,539	0.12	22,000	87	142

Variation in Strength of Gun-bronze, and Means of Improving the Strength.—The figures obtained for alloys of from 7.8% to 12.7% tin, viz., from 26,860 to 29,430 pounds, are much less than are usually given as the strength of gun-metal. Bronze guns are usually cast under the pressure of a head of metal, which tends to increase the strength and density. The strength of the upper part of a gun casting, or sinking head, is not greater than that of the small bars which have been tested in these experiments. The following is an extract from the report of Major Wade concerning the strength and density of gun-bronze (1850):—Extreme variation of six samples from different parts of the same gun (a 32-pounder howitzer): Specific gravity, 8.487 to 8.835; tenacity, 26,428 to 52,192. Extreme variation of all the samples tested: Specific gravity, 8.308 to 8.850; tenacity, 23,108 to 54,531. Extreme variation of all the samples from the gun heads: Specific gravity, 8.308 to 8.756; tenacity, 23,529 to 35,484.

Major Wade says: The general results on the quality of bronze as it is found in guns are mostly of a negative character. They expose defects in density and strength, develop the heterogeneous texture of the metal in different parts of the same gun, and show the irregularity and uncertainty of quality which attend the casting of all guns, although made from similar materials, treated in like manner.

Navy ordnance bronze containing 9 parts copper and 1 part tin, tested at Washington, D. C., in 1875–6, showed a variation in tensile strength from 29,800 to 51,400 lbs. per square inch, in elongation from 3% to 58%, and in specific gravity from 8.39 to 8.88.

That a great improvement may be made in the density and tenacity of gun-bronze by compression has been shown by the experiments of Mr. S. B. Dean in Boston, Mass., in 1869, and by those of General Uchatius in Austria in 1873. The former increased the density of the metal next the bore of the gun from 8.321 to 8.875, and the tenacity from 27,238 to 41,471 pounds per square inch. The latter, by a similar process, obtained the following figures for tenacity:

	Pounds per sq. in.
Bronze with 10% tin	72,053
Bronze with 8% tin	73,958
Bronze with 6% tin	77,656

ALLOYS OF COPPER, TIN, AND ZINC.

(Report of U. S. Test Board, Vol. II, 1881.)

No. in Report.	Analysis, Original Mixture.			Transverse Strength.		Tensile Strength per square inch.		Elongation per cent in 5 inches.	
	Cu.	Sn.	Zn.	Modulus of Rupture	Deflection, ins.	A.	B.	A.	B.
72	90	5	5	41,334	2.63	23,660	30,740	2.34	9.68
6	88.14	1.86	10	31,986	3.67	32,000	33,000	17.6	19.5
70	85	5	10	44,457	2.85	28,840	28,500	6.80	5.28
71	85	10	5	62,470	2.56	35,680	36,000	2.51	2.35
89	85	12.5	2.5	62,405	2.38	34,500	32,800	1.29	2.79
88	82.5	12.5	5	60,960	1.61	36,000	34,000	.86	.92
77	82.5	15	2.5	69,045	1.09	33,600	33,800	....	.68
67	80	5	15	42,618	3.33	37,560	32,300	11.6	3.59
68	80	10	10	67,117	2.45	32,830	31,950	1.57	1.67
69	80	15	5	54,476	.44	32,350	30,760	.55	.44
86	77.5	10	12.5	68,849	1.19	35,500	36,000	1.00	1.00
87	77.5	12.5	10	71,705	.71	36,000	32,500	.72	.59
63	75	5	20	55,355	2.91	33,140	34,960	2.50	3.19
85	75	7.5	17.5	62,607	1.39	33,700	39,300	1.56	1.32
64	75	10	15	58,345	.73	35,320	34,000	1.13	1.25
65	75	15	10	51,109	.31	35,440	28,000	.59	.54
66	75	20	5	40,235	.21	23,140	27,660	.43	....
83	72.5	7.5	20	51,839	2.86	32,700	34,800	3.73	3.78
84	72.5	10	17.5	58,230	.74	30,000	30,000	.48	.49
59	70	5	25	57,349	1.37	38,000	32,940	2.06	.99
82	70	7.5	22.5	48,836	.36	38,000	22,400	.84	.40
60	70	10	20	36,520	.18	33,140	26,300	.31	....
61	70	15	15	37,924	.20	33,440	27,800	.35	....
62	70	20	10	15,126	.08	17,000	12,900	.08	....
81	67.5	2.5	30	58,345	2.91	34,720	45,850	7.37	8.09
74	67.5	5	27.5	55,976	.49	34,000	34,460	1.06	.48
75	67.5	7.5	25	46,875	.32	29,500	30,000	.36	.26
80	65	2.5	32.5	56,949	2.36	41,350	38,300	3.36	3.03
55	65	5	30	51,369	.56	37,140	36,000	1.31	.61
56	65	10	25	27,075	.14	25,720	22,500	.15	.19
57	65	15	20	18,591	.07	6,820	7,231	....	....
58	65	20	15	11,932	.05	3,765	2,665	....	....
79	62.5	2.5	35	69,255	2.31	44,400	45,000	2.15	2.19
78	60	2.5	37.5	69,508	1.46	57,400	52,900	4.87	3.02
52	60	5	35	46,076	.28	41,160	38,330	.89	.40
53	60	10	30	24,699	.13	21,780	21,240	.15	....
54	60	15	25	18,248	.09	18,020	12,400	....	....
12	58.22	2.30	39.48	95,623	1.99	66,500	67,600	3.13	3.15
3	58.75	8.75	32.5	35,752	.18	Broke before test; very brittle			
4	57.5	21.25	21.25	2,752	.02	725	1,300	....	....
73	55	0.5	44.5	72,308	3.05	68,900	68,900	9.43	8.88
50	55	5	40	38,174	.22	27,400	30,500	.46	.43
51	55	10	35	28,258	.14	25,460	18,500	.29	.10
49	50	5	45	20,814	.11	23,000	31,300	.66	.45

The transverse tests were made in bars 1 in. square, 22 in. between supports. The tensile tests were made on bars 0.798 in. diam. turned from the two halves of the transverse-test bar, one half being marked *A* and the other *B*.

Ancient Bronzes.—The usual composition of ancient bronze was the same as that of modern gun-metal—90 copper, 10 tin; but the proportion of tin varies from 5% to 15%, and in some cases lead has been found. Some ancient Egyptian tools contained 88 copper, 12 tin.

Strength of the Copper-zinc Alloys.—The alloys containing less than 15% of zinc by original mixture were generally defective. The bars were full of blow-holes, and the metal showed signs of oxidation. To insure good castings it appears that copper-zinc alloys should contain more than 15% of zinc.

From No. 2 to No. 8 inclusive, 16.98 to 30.06% zinc the bars show a remarkable similarity in all their properties. They have all nearly the same strength and ductility, the latter decreasing slightly as zinc increases, and are nearly alike in color and appearance. Between Nos. 8 and 10, 30.06 and 36.36% zinc, the strength by all methods of test rapidly increases. Between No. 10 and No. 15, 36.36 and 50.14% zinc, there is another group, distinguished by high strength and diminished ductility. The alloy of maximum tensile, transverse and torsional strength contains about 41% of zinc.

The alloys containing less than 55% of zinc are all yellow metals. Beyond 55% the color changes to white, and the alloy becomes weak and brittle. Between 70% and pure zinc the color is bluish gray, the brittleness decreases and the strength increases, but not to such a degree as to make them useful for constructive purposes.

Difference between Composition by Mixture and by Analysis.—There is in every case a smaller percentage of zinc in the average analysis than in the original mixture, and a larger percentage of copper. The loss of zinc is variable, but in general averages from 1 to 2%.

Liquation or Separation of the Metals.—In several of the bars a considerable amount of liquation took place, analysis showing a difference in composition of the two ends of the bar. In such cases the change in composition was gradual from one end of the bar to the other, the upper end in general containing the higher percentage of copper. A notable instance was bar No. 13, in the above table, turnings from the upper end containing 40.36% of zinc, and from the lower end 48.52%.

Specific Gravity.—The specific gravity follows a definite law, varying with the composition, and decreasing with the addition of zinc. From the plotted curve of specific gravities the following mean values are taken:

Per cent zinc..........	0	10	20	30	40	50	60	70	80	90	100.
Specific gravity.......	8.80	8.72	8.60	8.40	8.36	8.20	8.00	7.72	7.40	7.20	7.14.

Graphic Representation of the Law of Variation of Strength of Copper-Tin-Zinc Alloys.—In an equilateral triangle the sum of the perpendicular distances from any point within it to the three sides is equal to the altitude. Such a triangle can therefore be used to show graphically the percentage composition of any compound of three parts, such as a triple alloy. Let one side represent 0 copper, a second 0 tin, and the third 0 zinc, the vertex opposite each of these sides representing 100 of each element respectively. On points in a triangle of wood representing different alloys tested, wires were erected of lengths proportional to the tensile strengths, and the triangle then built up with plaster to the height of the wires. The surface thus formed has a characteristic topography representing the variations of strength with variations of composition. The cut shows the surface thus made. The vertical section to the left represents the law of tensile strength of the copper-tin alloys, the one to the right that of tin-zinc alloys, and the one at the rear that of the copper-zinc alloys. The high point represents the strongest possible alloys of the three metals. Its composition is copper 55, zinc 43, tin 2, and its strength about 70,000 lbs. The high ridge from this point to the point of maximum height of the section on the left is the line of the strongest alloys, represented by the formula zinc + (3 × tin) = 55.

All alloys lying to the rear of the ridge, containing more copper and less tin or zinc are alloys of greater ductility than those on the line of maximum strength, and are the valuable commercial alloys; those in front on the declivity toward the central valley are brittle, and those in the valley are both brittle and weak. Passing from the valley toward the section at the right the alloys lose their brittleness and become soft, the maximum softness being at tin = 100, but they remain weak, as is shown by the low elevation of the surface. This model was planned and constructed by Prof. Thurston in 1877. (See Trans. A. S. C. E. 1881, Report of the U. S. Board appointed to test Iron, Steel, etc., vol. ii., Washington, 1881, and Thurston's *Materials of Engineering*, vol. iii.)

The best alloy obtained in Thurston's research for the U. S. Testing Board has the composition, Copper 55, Tin 0.5, Zinc 44.5. The tensile strength in a cast bar was 68,900 lbs. per sq. in., two specimens giving the same result; the elongation was 47 to 51 per cent in 5 inches. Thurston's formula for copper-tin-zinc alloys of maximum strength (Trans. A. S. C. E., 1881) is $z + 3t = 55$,

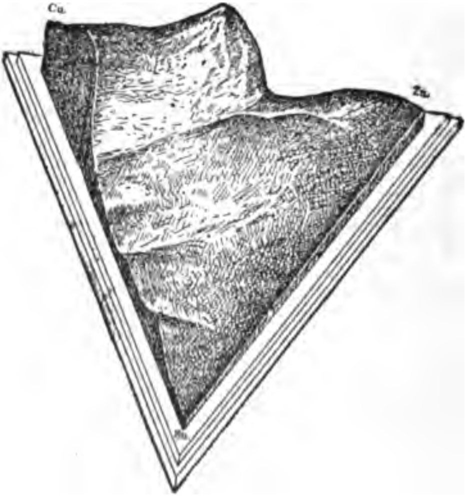


FIG. 77.

in which z is the percentage of zinc and t that of tin. Alloys proportioned according to this formula should have a strength of about 40,000 lbs. per sq. in. $+ 500z$. The formula fails with alloys containing less than 1 per cent of tin.

The following would be the percentage composition of a number of alloys made according to this formula, and their corresponding tensile strength in castings:

Tin.	Zinc.	Copper.	Tensile Strength, lbs. per sq. in.	Tin.	Zinc.	Copper.	Tensile Strength, lbs. per sq. in.
1	52	47	66,000	8	31	61	55,500
2	49	49	64,500	9	28	63	54,000
3	46	51	63,000	10	25	65	52,500
4	43	53	61,500	12	19	69	49,500
5	40	55	60,000	14	13	73	46,500
6	37	57	58,500	16	7	77	43,500
7	34	59	57,000	18	1	81	40,500

These alloys, while possessing maximum tensile strength, would in general be too hard for easy working by machine tools. Another series made on the formula $z + 4t = 50$ would have greater ductility, together with considerable strength, as follows, the strength being calculated as before, tensile strength in lbs. per sq. in. $= 40,000 + 500z$.

Tin.	Zinc.	Copper.	Tensile Strength, lbs. per sq. in.	Tin.	Zinc.	Copper.	Tensile Strength, lbs. per sq. in.
1	46	53	63,000	7	22	71	51,000
2	42	56	61,000	8	18	74	49,000
3	38	59	59,000	9	14	77	47,000
4	34	62	57,000	10	10	80	45,000
5	30	65	55,000	11	6	83	43,000
6	26	68	53,000	12	2	86	41,000

Composition of Alloys in Every-day Use in Brass Foundries. (*American Machinist.*)

	Copper.	Zinc.	Tin.	Lead.	
	lbs.	lbs.	lbs.	lbs.	
Admiralty metal	87	5	8	......	For parts of engines on board naval vessels.
Bell metal	16	......	4	......	Bells for ships and factories.
Brass (yellow)	16	8	......	½	For plumbers, ship and house brass work.
Bush metal	64	8	4	4	For bearing bushes for shafting.
Gun metal	32	1	3	......	For pumps and other hydraulic purposes.
Steam metal	20	1	1½	1	Castings subjected to steam pressure.
Hard gun metal	16	......	2½	......	For heavy bearings.
Muntz metal	60	40	......	......	Metal from which bolts and nuts are forged, valve spindles, etc.
Phosphor bronze	92	......	8 phos. tin		For valves, pumps and general work.
" "	90	......	10 " "		For cog and worm wheels, bushes, axle bearings, slide valves, etc.
Brazing metal	16	3	......	......	Flanges for copper pipes.
" solder	50	50	......	......	Solder for the above flanges.

Gurley's Bronze.—16 parts copper, 1 tin, 1 zinc, ½ lead, used by W. & L. E. Gurley of Troy for the framework of their engineer's transits. Tensile strength 41,114 lbs. per sq. in., elongation 27% in 1 inch, sp. gr. 8.696. (W. J. Keep, Trans. A. I. M. E. 1890.)

Useful Alloys of Copper, Tin, and Zinc.

(Selected from numerous sources.)

	Copper.	Tin.	Zinc.	
U. S. Navy Dept. journal boxes and guide-gibs =	6	1	¼	parts.
	82.8	13.8	3.4	per cent.
Tobin bronze	58.22	2.30	39.48	" "
Naval brass	62	1	37	" "
Composition, U. S. Navy	88	10	2	" "
Brass bearings (J. Rose)	64	8	1	parts.
	87.7	11.0	1.3	per cent.
Gun metal	92.5	5	2.5	" "
" "	91	7	2	" "
" "	87.75	9.75	2.5	" "
" "	85	5	10	" "
" "	83	2	15	" "
Tough brass for engines	13	2	2	parts.
	76.5	11.8	11.7	per cent.
Bronze for rod-boxes (Lafond)	82	16	2	slightly malleable.
" " pieces subject to shock	83	15	1.50	0.50 lead.
Red brass ...parts	20	1	1	1 "
" " ...per cent	87	4.4	4.3	4.3 "
Bronze for pump casings (Lafond)	88	10	2	
" " eccentric straps. "	84	14	2	
" " shrill whistles	80	18	....	2.0 antimony.
" " low-toned whistles	81	17	....	2.0 "

	Copper.	Tin.	Zinc.	
Art bronze, dull red fracture	97	2	1	
Gold bronze	89.5	2.1	5.6	2.8 lead.
Bearing metal	89	8	3	
" "	89	2½	8½	
" "	86	14	....	
" "	85¼	12¾	2	
" "	80	18	2	
" "	79	18	2½	½ lead.
" "	74	9½	9½	7 lead.
English brass of A.D. 1504	64	3	29½	3½ lead.

Copper-Nickel Alloys, German Silver.

	Copper.	Nickel.	Tin.	Zinc.
German silver	51.6	25.8	22.6	......
" "	50.2	14.8	3.1	31.9
" "	51.1	13.8	3.2	31.9
" "	52 to 55	18 to 25	......	20 to 30
Nickel "	75 to 66	25 to 33	......	......

A refined copper-nickel alloy containing 50% copper and 49% nickel, with very small amounts of iron, silicon and carbon, is produced direct from Bessemer matte in the Sudbury (Canada) Nickel Works. German silver manufacturers purchase a ready-made alloy, which melts at a low heat and requires simple addition of zinc, instead of buying the nickel and copper separately. This alloy, "50-50" as it is called, is almost indistinguishable from pure nickel. Its cost is less than nickel, its melting point much lower, it can be cast solid in any form desired, and furnishes a casting which works easily in the lathe or planer, yielding a silvery white surface unchanged by air or moisture. For bullet casings now used in various British and continental rifles, a special alloy of 80% copper and 20% nickel is made.

Special Alloys. (*Engineer*, March 24, 1893.)

JAPANESE ALLOYS for art work:

	Copper.	Silver.	Gold.	Lead.	Zinc.	Iron.
Shaku-do	94.50	1.55	3.73	0.11	trace.	trace.
Shibu-ichi	67.31	32.07	traces.	.52		

GILBERT'S ALLOY for *cera-perduta* process, for casting in plaster-of-paris:
Copper 91.4 Tin 5.7 Lead 2.9 Very fusible.

COPPER-ZINC-IRON ALLOYS.

(F. L. Garrison, *Jour. Frank. Inst.*, June and July, 1891.)

Delta Metal.—This alloy, which was formerly known as *sterro-metal*, is composed of about 60 copper, from 34 to 44 zinc, 2 to 4 iron, and 1 to 2 tin.

The peculiarity of all these alloys is the content of iron, which appears to have the property of increasing their strength to an unusual degree. In making delta metal the iron is previously alloyed with zinc in known and definite proportions. When ordinary wrought-iron is introduced into molten zinc, the latter readily dissolves or absorbs the former, and will take it up to the extent of about 5% or more. By adding the zinc-iron alloy thus obtained to the requisite amount of copper, it is possible to introduce any definite quantity of iron up to 5% into the copper alloy. Garrison gives the following as the range of composition of copper-zinc-iron, and copper-zinc-tin-iron alloys:

I.

	Per cent.
Iron	0.1 to 5
Copper	50 to 65
Zinc	49.9 to 30

II.

	Per cent.
Iron	0.1 to 5
Tin	0.1 to 10
Zinc	1.8 to 45
Copper	98 to 40

The advantages claimed for delta metal are great strength and toughness. It produces sound castings of close grain. It can be rolled and forged hot and can stand a certain amount of drawing and hammering when cold. It takes a high polish, and when exposed to the atmosphere tarnishes less than brass.

When cast in sand delta metal has a tensile strength of about 45,000 pounds per square inch, and about 10% elongation; when rolled, tensile strength of 60,000 to 75,000 pounds per square inch, elongation from 9% to 17% on bars 1.128 inch in diameter and 1 inch area.

Wallace gives the ultimate tensile strength 33,600 to 51,520 pounds per square inch, with from 10% to 20% elongation.

Delta metal can be forged, stamped and rolled hot. It must be forged at a dark cherry-red heat, and care taken to avoid striking when at a black heat.

According to Lloyd's Proving House tests, made at Cardiff, December 20, 1887, a half-inch delta metal-rolled bar gave a tensile strength of 88,400 pounds per square inch, with an elongation of 30% in three inches.

Tobin Bronze.—This alloy is practically a sterro or delta metal with the addition of a small amount of lead, which tends to render copper softer and more ductile.

The following analyses of Tobin bronze were made by Dr. Chas. B. Dudley:

	Pig Metal, per cent.	Test Bar (Rolled), per cent.
Copper	59.00	61.20
Zinc	38.40	37.14
Tin	2.16	0.90
Iron	0.11	0.18
Lead	0.31	0.35

Dr. Dudley writes, "We tested the test bars and found 78,500 tensile strength with 15% elongation in two inches, and 40½% in eight inches. This high tensile strength can only be obtained when the metal is manipulated. Such high results could hardly be expected with cast metal."

The original Tobin bronze in 1875, as described by Thurston, Trans. A. S. C. E 1881, had, composition of copper 58.22, tin 2.30, zinc 39.48. As cast it had a tenacity of 66,000 lbs. per sq. in., and as rolled 79,000 lbs.; cold rolled it gave 104,000 lbs.

A circular of Ansonia Brass & Copper Co. gives the following:—The tensile strength of six Tobin bronze one-inch round rolled rods, turned down to a diameter of ⅝ of an inch, tested by Fairbanks, averaged 79,600 lbs. per sq. in., and the elastic limit obtained on three specimens averaged 54,257 lbs. per sq. in.

At a cherry-red heat Tobin bronze can be forged and stamped as readily as steel. Bolts and nuts can be forged from it, either by hand or by machinery, with a marked degree of economy. Its great tensile strength, and resistance to the corrosive action of sea-water, render it a most suitable metal for condenser plates, steam-launch shafting, ship sheathing and fastenings, nails, hull plates for steam yachts, torpedo and life boats, and ship-deck fittings.

The Navy Department has specified its use for certain purposes in the machinery of the new cruisers. Its specific gravity is 8.071. The weight of a cubic inch is .291 lb.

PHOSPHOR-BRONZE AND OTHER SPECIAL BRONZES.

Phosphor-bronze.—In the year 1868, Montefiore & Kunzel of Liège, Belgium, found by adding small proportions of phosphorus or "phosphoret of tin or copper" to copper that the oxides of that metal, nearly always present as an impurity, more or less, were deoxidized and the copper much improved in strength and ductility, the grain of the fracture became finer, the color brighter, and a greater fluidity was attained.

Three samples of phosphor-bronze tested by Kirkaldy gave:

Elastic limit, lbs. per sq. in	23,800	24,700	16,100
Tensile strength, lbs. per sq. in.	52,625	46,100	44,448
Elongation, per cent	8.40	1.50	33.40

The strength of phosphor-bronze varies like that of ordinary bronze according to the percentages of copper, tin, zinc, lead, etc., in the alloy.

Deoxidized Bronze.—This alloy resembles phosphor bronze somewhat in composition and also delta metal, in containing zinc and iron. The following analysis gives its average composition:

Copper	82.67	Iron	0.10
Tin	12.40	Silver	0.07
Zinc	3.23	Phosphorus	0.005
Lead	2.14		
			100.615

Comparison of Copper, Silicon-bronze, and Phosphor-bronze Wires. (*Engineering*, Nov. 23, 1883.)

Description of Wire.	Tensile Strength.	Relative Conductivity.
Pure copper	39,827 lbs. per sq. in.	100 per cent.
Silicon bronze (telegraph)	41,696 " " " "	96 " "
" " (telephone)	108,080 " " " "	34 " "
Phosphor bronze (telephone)	102,390 " " " "	26 " "

Silicon Bronze. (*Aluminum World*. May, 1897.)

The most useful of the silicon bronzes are the 3% (97% copper, 3% silicon) and the 5% (95% copper, 5% silicon), although the hardness and strength of the alloy can be increased or decreased at will by increasing or decreasing silicon. A 3% silicon bronze has a tensile strength, in a casting, of about 55,000 lbs. per sq. in., and from 50% to 60% elongation. The 5% bronze has a tensile strength of about 75,000 lbs. and about 8% elongation. More than 5% or 5½% of silicon in copper makes a brittle alloy. In using silicon, either as a flux or for making silicon bronze, the rich alloy of silicon and copper which is now on the market should be used. It should be free from iron and other metals if the best results are to be obtained. Ferro-silicon is not suitable for use in copper or bronze mixtures.

ALUMINUM ALLOYS.

Aluminum Bronze. (Cowles Electric Smelting and Al. Co.'s circular.)

The standard A No. 2 grade of aluminum bronze, containing 10% of aluminum and 90% of copper, has many remarkable characteristics which distinguish it from all other metals.

The tenacity of castings of A No. 2 grade metal varies between 75,000 and 90,000 lbs. to the square inch, with from 4% to 14% elongation.

Increasing the proportion of aluminum in bronze beyond 11% produces a brittle alloy; therefore nothing higher than the A No. 1, which contains 11%, is made.

The B, C, D, and E grades, containing 7½%, 5%, 2½%, and 1¼% of aluminum, respectively, decrease in tenacity in the order named, that of the former being about 65,000 pounds, while the latter is 25,000 pounds. While there is also a proportionate decrease in transverse and torsional strengths, elastic limit, and resistance to compression as the percentage of aluminum is lowered and that of copper raised, the ductility on the other hand increases in the same proportion. The specific gravity of the A No. 1 grade is 7.56.

Bell Bros., Newcastle, gave the specific gravity of the aluminum bronzes as follows:

3%, 8.691; 4%, 8.621; 5%, 8.369; 10%, 7.689.

Casting.—The melting point of aluminum bronze varies slightly with the amount of aluminum contained, the higher grades melting at a somewhat lower temperature than the lower grades. The A No. 1 grades melt at about 1700° F., a little higher than ordinary bronze or brass.

Aluminum bronze shrinks more than ordinary brass. As the metal solidifies rapidly it is necessary to pour it quickly and to make the feeders amply large, so that there will be no "freezing" in them before the casting is properly fed. Baked-sand moulds are preferable to green sand, except for small castings, and when fine skin colors are desired in the castings. (See paper by Thos. D. West, Trans. A. S. M. E. 1886, vol. viii.)

All grades of aluminum bronze can be rolled, swedged, spun, or drawn cold except A 1 and A 2. They can all be worked at a bright red heat.

In rolling, swedging, or spinning cold, it should be annealed very often, and at a brighter red heat than is used for annealing brass.

Brazing.—Aluminum bronze will braze as well as any other metal, using one quarter brass solder (zinc 500, copper 500 (and three quarters borax, or, better, three quarters cryolite.

Soldering.—To solder aluminum bronze with ordinary soft (pewter) solder: Cleanse well the parts to be joined free from grease and dirt. Then place the parts to be soldered in a strong solution of sulphate of copper and place in the bath a rod of soft iron touching the parts to be joined. After a while a coppery-like surface will be seen on the metal. Remove from bath, rinse quite clean, and brighten the surfaces. These surfaces can then be tinned by using a fluid consisting of zinc dissolved in hydrochloric acid, in the ordinary way, with common soft solder.

Mierzinski recommends ordinary hard solder, and says that Hulot uses an alloy of the usual half-and-half lead-tin solder, with 12.5%, 25% or 50% of zinc amalgam.

Tests of Aluminum Bronzes.

(By John H. J. Dagger, in a paper read before the British Association, 1889.)

Per cent of Aluminum.	Tensile Strength. Tons per square inch.	Tensile Strength. Pounds per square inch.	Elongation, per cent.	Specific Gravity.
11	40 to 45	89,600 to 100,800	8	7.23
10	33 " 40	73,920 " 89,600	14	7.69
7½	25 " 30	56,000 " 67,200	40	8.00
5-5½	15 " 18	33,600 " 40,320	40	8.37
2½	13 " 15	29,120 " 33,600	50	8.69
1¼	11 " 13	24,640 " 29,120	55	...

Both physical and chemical tests made of samples cut from various sections of 2½%, 5%, 7½%, or 10% aluminized copper castings tend to prove that the aluminum unites itself with each particle of copper with uniform proportion in each case, so that we have a product that is free from liquation and highly homogeneous. (R. C. Cole, *Iron Age*, Jan. 16, 1890.)

Aluminum-Brass (E. H. Cowles, Trans. A. I. M. E., vol. xviii.)—Cowles aluminum-brass is made by fusing together equal weights of A 1 aluminum-bronze, copper, and zinc. The copper and bronze are first thoroughly melted and mixed, and the zinc is finally added. The material is left in the furnace until small test-bars are taken from it and broken. When these bars show a tensile strength of 80,000 pounds or over, with 2 or 3 per cent ductility, the metal is ready to be poured. Tests of this brass, on small bars, have at times shown as high as 100,000 pounds tensile strength.

The screw of the United States gunboat Petrel is cast from this brass, mixed with a trifle less zinc in order to increase its ductility.

Tests of Aluminum-Brass.

(Cowles E. S. & Al. Co.)

Specimen (Castings.)	Diameter of Piece, Inch.	Area. sq. in.	Tensile Strength, lbs. per sq. in.	Elastic Limit, lbs. per sq. in.	Elongation. per ct.	Remarks.
15% *A* grade Bronze. 17% Zinc. 68% Copper.	.465	.1698	41,225	17,668	41½	These test pieces were all 6″ long between the shoulders.
1 part *A* Bronze. 1 part Zinc. 1 part Copper.	.465	.1698	78,327	......	2½	
1 part *A* Bronze. 1 part Zinc. 1 part Copper.	.460	.1661	72,246	....	2½	

The first brass on the above list is an extremely tough metal with low elastic limit, made purposely so as to "upset" easily. The other, which is called Aluminum-brass No. 2, is very hard.

We have not in this country or in England any official standard by which to judge of the physical characteristics of cast metals. There are two conditions that are absolutely necessary to be known before we can make a fair comparison of different materials: namely, whether the casting was made in dry or green sand or in a chill, and whether it was attached to a larger casting or cast by itself. It has also been found that chill-castings give higher results than sand-castings, and that bars cast by themselves purposely for testing almost invariably run higher than test-bars attached to castings. It is also a fact that bars cut out from castings are generally weaker than bars cast alone. (E. H. Cowles.)

Caution as to Reported Strength of Alloys.—The same variation in strength which has been found in tests of gun-metal (copper and tin) noted above, must be expected in tests of aluminum bronze and in fact of all alloys. They are exceedingly subject to variation in density and in grain, caused by differences in method of molding and casting, temperature of pouring, size and shape of casting, depth of "sinking head," etc.

Aluminum Hardened by Addition of Copper Rolled Sheets .04 Inch Thick. (*The Engineer*, Jan 2, 1891.)

Al. Per cent.	Cu. Per cent.	Sp. Gr. Calculated.	Sp. Gr. Determined.	Tensile Strength in pounds per square inch.
100	..	....	2.67	26,535
98	2	2.78	2.71	43,563
96	4	2.90	2.77	44,130
94	6	3.02	2.82	54,773
92	8	3.14	2.85	50,374

Tests of Aluminum Alloys.

(Engineer Harris, U. S. N., Trans. A. I. M. E., vol. xviii.)

Composition.					Tensile Strength, per sq. in. lbs.	Elastic Limit, lbs. per sq. in.	Elongation, per ct.	Reduction of Area, per ct.
Copper.	Aluminum.	Silicon.	Zinc.	Iron.				
91.50%	6.50%	1.75%	....	0.25%	60,760	18,000	23.2	30.7
88.50	9.33	1.66	....	0.50	66,900	27,000	3.6	7.8
91.50	6.50	1.75	....	0.25	67,600	24,000	13.	21.62
90.00	9.00	1.00	....	....	72,830	39,000	2.46	5.78
63.00	2.83	0.83	33.33%	....	82,200	60,000	2.33	9.88
63.00	2.83	0.83	33.33	....	70,400	55,000	0.4	4.23
91.50	6.50	1.75	....	0.25	59,100	19,000	15.1	25.59
93.00	6.50	0.50	....	....	58,000	19,000	6.2	15.5
88.50	9.33	1.66	....	0.50	69,930	33,000	1.33	3.30
92.00	6.50	0.50	....	....	46,530	17,000	7.8	19.19

For comparison with the above 6 tests of "Navy Yard Bronze," Cu 88, Sn 10, Zn 2, are given in which the T. S. ranges from 18,000 to 24,590, E. L. from 10,000 to 18,000, El. 2.5 to 5.8%, Red. 4.7 to 10.89.

Alloys of Aluminum, Silicon and Iron.

M. and E. Bernard have succeeded in obtaining through electrolysis, by treating directly and without previous purification, the aluminum earths (red and white bauxites) the following:

Alloys such as ferro-aluminum, ferro-silicon-aluminum and silicon-aluminum, where the proportion of silicon may exceed 10% which are employed in the metallurgy of iron for refining steel and cast-iron.

Also silicon-aluminum, where the proportion of silicon does not exceed 10%, which may be employed in mechanical constructions in a rolled or hammered condition, in place of steel, on account of their great resistance, especially where the lightness of the piece in construction constitutes one of the main conditions of success.

The following analyses are given:

1. Alloys applied to the metallurgy of iron, the refining of steel and cast iron: No. 1. Al, 70%; Fe, 25%; Si, 5%. No. 2. Al, 70; Fe, 20; Si, 10. No. 3. Al, 70; Fe, 15; Si, 15. No. 4. Al, 70; Fe, 10; Si, 20. No. 5. Al, 70; Fe, 10; Si, 10; Mn, 10. No. 6. Al, 70; Fe, trace; Si, 20; Mn, 10.

2. Mechanical alloys: No. 1. Al, 92; Si, 6.75; Fe, 1.25. No. 2. Al, 90; Si, 9.25; Fe, 0.75. No. 3. Al, 90; Si, 10; Fe, trace. The best results were with alloys where the proportion of iron was very low, and the proportion of silicon in the neighborhood of 10%. Above that proportion the alloy becomes crystalline and can no longer be employed. The density of the alloys of silicon is approximately the same as that of aluminum.—*La Metallurgie*, 1893.

Tungsten and Aluminum.—Mr. Leinhardt Mannesmann says that the addition of a little tungsten to pure aluminum or its alloys communicates a remarkable resistance to the action of cold and hot water, salt water and other reagents. When the proportion of tungsten is sufficient the alloys offer great resistance to tensile strains.

Aluminum, Copper, and Tin.—Prof. R. C. Carpenter, Trans. A. S. M. E., vol. xix., finds the following alloys of maximum strength in a series in which two of the three metals are in equal proportions:

Al. 85; Cu, 7.5; Sn, 7.5; tensile strength, 30,000 lbs. per sq. in.; elongation in 6 in. 4%; sp. gr., 3.02. Al, 6.25; Cu, 87.5; Sn, 6.25; T. S., 63,000; El., 3.8; sp. gr., 7.35. Al, 5; Cu, 5; Sn, 90; T. S., 11,000; El., 10.1; sp. gr., 6.82.

Aluminum and Zinc.—Prof. Carpenter finds that the strongest alloy of these metals consists of two parts of aluminum and one part of zinc. Its tensile strength is 24,000 to 26,000 lbs. per sq. in.; has but little ductility, is readily cut with machine-tools, and is a good substitute for hard cast brass.

Aluminum and Tin.—M. Bourbouze has compounded an alloy of aluminum and tin, by fusing together 100 parts of the former with 10 parts of the latter. This alloy is paler than aluminum, and has a specific gravity of 2.85. The alloy is not as easily attacked by several reagents as aluminum is, and it can also be worked more readily. Another advantage is that it can be soldered as easily as bronze, without further preliminary preparations.

Aluminum-Antimony Alloys.—Dr. C. R. Alder Wright describes some aluminum-antimony alloys in a communication read before the Society of Chemical Industry. The results of his researches do not disclose the existence of a commercially useful alloy of these two metals, and have greater scientific than practical interest. A remarkable point is that the alloy with the chemical composition *Al Sb* has a higher melting point than either aluminum or antimony alone, and that when aluminum is added to pure antimony the melting-point goes up from that of antimony (450° C.) to a certain temperature rather above that of silver (1000° C.).

ALLOYS OF MANGANESE AND COPPER.

Various Manganese Alloys.—E. H. Cowles, in Trans. A. I. M. E., vol. xviii, p. 495, states that as the result of numerous experiments on mixtures of the several metals, copper, zinc, tin, lead, aluminum, iron, and manganese, and the metalloid silicon, and experiments upon the same in ascertaining tensile strength, ductility, color, etc., the most important determinations appear to be about as follows:

1. That pure metallic manganese exerts a bleaching effect upon copper more radical in its action even than nickel. In other words, it was found that 18½% of manganese present in copper produces as white a color in the resulting alloy as 25% of nickel would do, this being the amount of each required to remove the last trace of red.
2. That upwards of 20% or 25% of manganese may be added to copper without reducing its ductility, although doubling its tensile strength and changing its color.
3. That manganese, copper, and zinc when melted together and poured into moulds behave very much like the most "yeasty" German silver, producing an ingot which is a mass of blow-holes, and which swells up above the mould before cooling.
4. That the alloy of manganese and copper by itself is very easily oxidized.
5. That the addition of 1.25% of aluminum to a manganese-copper alloy converts it from one of the most refractory of metals in the casting process into a metal of superior casting qualities, and the non-corrodibility of which is in many instances greater than that of either German or nickel silver.

A "silver-bronze" alloy especially designed for rods, sheets, and wire has the following composition: Manganese, 18; aluminum, 1.20; silicon, 0.5; zinc, 13; and copper, 67.5%. It has a tensile strength of about 57,000 pounds on small bars, and 20% elongation. It has been rolled into thin plate and drawn into wire .003 inch in diameter. A test of the electrical conductivity of this wire (of size No. 32) shows its resistance to be 41.44 times that of pure copper. This is far lower conductivity than that of German silver.

Manganese Bronze. (F. L. Garrison, Jour. F. I., 1891.)—This alloy has been used extensively for casting propeller-blades. Tests of some made by B. H. Cramp & Co., of Philadelphia, gave an average elastic limit of 30,000 pounds per square inch, tensile strength of about 60,000 pounds per square inch, with an elongation of 8% to 10% in sand castings. When rolled, the elastic limit is about 80,000 pounds per square inch, tensile strength 95,000 to 100,000 pounds per square inch, with an elongation of 12% to 15%.

Compression tests made at United States Navy Department from the metal in the pouring-gate of propeller-hub of U. S. S. Maine gave in two tests a crushing stress of 126,450 and 135,750 lbs. per sq. in. The specimens were 1 inch high by 0.7 × 0.7 inch in cross-section = 0.49 square inch. Both specimens gave way by shearing, on a plane making an angle of nearly 45° with the direction of stress.

A test on a specimen 1 × 1 × 1 inch was made from a piece of the same pouring-gate. Under stress of 150,000 pounds it was flattened to 0.72 inch high by about 1¼ × 1¼ inches, but without rupture or any sign of distress.

One of the great objections to the use of manganese bronze, or in fact any alloy except iron or steel, for the propellers of iron ships is on account of the galvanic action set up between the propeller and the stern-posts. This difficulty has in great measure been overcome by putting strips of rolled zinc around the propeller apertures in the stern-frames.

The following analysis of Parsons' manganese bronze No. 2 was made from a chip from the propeller of Mr. W. K. Vanderbilt's yacht Alva.

Copper	88.644
Zinc	1.570
Tin	8.700
Iron	0.720
Lead	0.295
Phosphorus	trace
	99.929

It will be observed there is no manganese present and the amount of zinc is very small.

E. H. Cowles, Trans. A. I. M. E., vol. xviii, says: Manganese bronze, so called, is in reality a manganese brass, for zinc instead of tin is the chief element added to the copper. Mr. P. M. Parsons, the proprietor of this brand of metal, has claimed for it a tensile strength of from 24 to 28 tons on small bars when cast in sand. Mr. W. C. Wallace states that brass-founders of high repute in England will not admit that manganese bronze has more than from 12 to 17 tons tensile strength. Mr. Horace See found tensile strength of 45,000 pounds, and from 6% to 12½% elongation.

GERMAN-SILVER AND OTHER NICKEL ALLOYS.

	Copper.	Nickel.	Zinc.	
Chinese packfong	40.4	31.6	6.5	parts.
" tutenag	8	3	6.5	"
German silver	2	1	1	"
" " (cheaper)	8	2	3.5	"
" " (closely resembles sil).	8	3	3.5	"

For analyses of some German-silvers see page 326.

German Silver.—The composition of German silver is a very uncertain thing and depends largely on the honesty of the manufacturer and the price the purchaser is willing to pay. It is composed of copper, zinc, and nickel in varying proportions. The best varieties contain from 18% to 25% of nickel and from 20% to 30% of zinc, the remainder being copper. The more expensive nickel silver contains from 25% to 33% of nickel and from 75% to 66% of copper. The nickel is used as a whitening element; it also strengthens the alloy and renders it harder and more non-corrodible than the brass made without it, of copper and zinc. Of all troublesome alloys to handle in the foundry or rolling-mill, German silver is the worst. It is unmanageable and refractory at every step in its transition from the crude elements into rods, sheets, or wire. (E. H. Cowles, Trans. A. I. M. E., vol. xviii, p. 494.)

ALLOYS OF BISMUTH.

By adding a small amount of bismuth to lead that metal may be hardened and toughened. An alloy consisting of three parts of lead and two of bismuth has ten times the hardness and twenty times the tenacity of lead. The alloys of bismuth with both tin and lead are extremely fusible, and take fine impressions of casts and moulds. An alloy of one part bismuth, two parts tin, and one part lead is used by pewter-workers as a soft solder, and by soap-makers for moulds. An alloy of five parts bismuth, two parts tin, and three parts lead melts at 199° F., and is somewhat used for stereotyping, and for metallic writing-pencils. Thorpe gives the following proportions for the better-known fusible metals:

Name of Alloy.	Bismuth.	Lead.	Tin.	Cad-mium	Mer-cury.	Melting-point.
Newton's	50	31.25	18.75	......	.....	202° F.
Rose's	50	28.10	24.10	......	.....	203° "
D'Arcet's	50	25.00	25.00	......	.....	201° "
D'Arcet's with mercury.	50	25.00	25.00	.....	250.0	113° "
Wood's	50	25.00	12.50	12.50	.....	149° "
Lipowitz's	50	26.90	12.78	10.40	.....	149° "
Guthrie's "Eutectic"	50	20.55	21.10	14.03	.....	"Very low."

The action of heat upon some of these alloys is remarkable. Thus, Lipowitz's alloy, which solidifies at 149° Fah., contracts very rapidly at first, as it cools from this point. As the cooling goes on the contraction becomes slower and slower, until the temperature falls to 101.8° Fah. From this point the alloy *expands* as it cools, until the temperature falls to about 77° Fah., after which it again contracts, so that at 32° F. a bar of the alloy has the same length as at 115° F.

Alloys of bismuth have been used for making fusible plugs for boilers, but it is found that they are altered by the continued action of heat, so that one cannot rely upon them to melt at the proper temperature. Pure Banca tin is used by the U. S. Government for fusible plugs.

FUSIBLE ALLOYS. (From various sources.)

Sir Isaac Newton's, bismuth 5, lead 3, tin 2, melts at	212° F.
Rose's, bismuth 2, lead 1, tin 1, melts at	200 "
Wood's, cadmium 1, bismuth 4, lead 2, tin 1, melts at	165 "
Guthrie's, cadmium 13.29, bismuth 47.38, lead 19.36, tin 19.97, melts at	160 "
Lead 3, tin 5, bismuth 8, melts at	208 "
Lead 1, tin 3, bismuth 5, melts at	212 "
Lead 1, tin 4, bismuth 5, melts at	240 "
Tin 1, bismuth 1, melts at	286 "
Lead 2, tin 3, melts at	334 "
Tin 2, bismuth 1, melts at	336 "
Lead 1, tin 2, melts at	360 "
Tin 8, bismuth 1, melts at	392 "
Lead 2, tin 1, melts at	475 "
Lead 1, tin 1, melts at	466 "
Lead 1, tin 3, melts at	334 "
Tin 3, bismuth 1, melts at	392 "
Lead 1, bismuth 1, melts at	257 "
Lead 1, Tin 1, bismuth 4, melts at	201 "
Lead 5, tin 3, bismuth 8, melts at	202 "
Tin 3, bismuth 5, melts at	202 "

BEARING-METAL ALLOYS.

(C. B. Dudley, *Jour. F. I.*, Feb. and March, 1892.)

Alloys are used as bearings in place of wrought iron, cast iron, or steel, partly because wear and friction are believed to be more rapid when two metals of the same kind work together, partly because the soft metals are more easily worked and got into proper shape, and partly because it is desirable to use a soft metal which will take the wear rather than a hard metal, which will wear the journal more rapidly.

A good bearing-metal must have five characteristics: (1) It must be strong enough to carry the load without distortion. Pressures on car-journals are frequently as high as 350 to 400 lbs. per square inch.

(2) A good bearing-metal should not heat readily. The old copper-tin bearing, made of seven parts copper to one part tin, is more apt to heat than some other alloys. In general, research seems to show that the harder the bearing-metal, the more likely it is to heat.

3) Good bearing-metal should work well in the foundry. Oxidation while melting causes spongy castings. It can be prevented by a liberal use of powdered charcoal while melting. The addition of 1% to 2% of zinc or a small amount of phosphorus greatly aids in the production of sound castings. This is a principal element of value in phosphor-bronze.

(4) Good bearing-metals should show small friction. It is true that friction is almost wholly a question of the lubricant used; but the metal of the bearing has certainly some influence.

(5) Other things being equal, the best bearing-metal is that which wears slowest.

The principal constituents of bearing-metal alloys are copper, tin, lead, zinc, antimony, iron, and aluminum. The following table gives the constituents of most of the prominent bearing-metals as analyzed at the Pennsylvania Railroad laboratory at Altoona.

Analyses of Bearing-metal Alloys.

Metal.	Copper.	Tin.	Lead.	Zinc.	Antimony.	Iron.
Camelia metal	70.20	4.25	14.75	10.20	....	0.55
Anti-friction metal	1.60	98.13	......	......	....	trace
White metal	....	......	87.92	.....	12.08	....
Car-brass lining	.....	trace	84.87	.....	15.10	....
Salgee anti-friction	4.01	9.91	1.15	85.57	....	....
Graphite bearing-metal	....	14.38	67.73	......	16.73	? (1)
Antimonial lead	.....	......	80.69	......	18.88	... .
Carbon bronze	75.47	9.72	14.57	... ..	....	..... (2)
Cornish bronze	77.83	9.60	12.40	trace	....	trace (3)
Delta metal	92.39	2.37	5.10	......	... ..	0.07
*Magnolia metal	trace	......	83.55	trace	16.45	trace (4)
American anti-friction metal	....	......	78.44	0.98	19.60	0.65
Tobin bronze	59.00	2.16	0.31	38.40	....	0.11
Graney bronze	75.80	9.20	15.06	......	....	....
Damascus bronze	76.41	10.60	12.52	.. ..	....	....
Manganese bronze	90.52	9.58	......	.. ..	....	.. . (5)
Ajax metal	81.24	10.98	7.27	......	....	... . (6)
Anti-friction metal	....	......	88.32	......	11.93	....
Harrington bronze	55.73	0.97	... ..	42.67	....	0.68
Car-box metal	... .	......	84.33	trace	14.38	0.61
Hard lead	....	....	94.40	.....	6.08	.. ....
Phosphor-bronze	79.17	10.22	9.61	......	....	..... (7)
Ex. B. metal	76.80	8.00	15.00	......	....	.... (8)

Other constituents:

(1) No graphite.
(2) Possible trace of carbon.
(3) Trace of phosphorus.
(4) Possible trace of bismuth.
(5) No manganese.
(6) Phosphorus or arsenic, 0.37.
(7) Phosphorus, 0.94.
(8) Phosphorus, 0.20.

* Dr. H. C. Torrey says this analysis is erroneous and that Magnolia metal always contains tin.

As an example of the influence of minute changes in an alloy, the Harrington bronze, which consists of a minute proportion of iron in a copper-zinc alloy, showed after rolling a tensile strength of 75,000 lbs. and 20% elongation in 2 inches.

In experimenting on this subject on the Pennsylvania Railroad, a certain number of the bearings were made of a standard bearing-metal, and the same number were made of the metal to be tested. These bearings were placed on opposite ends of the same axle, one side of the car having the standard bearings, the other the experimental. Before going into service the bearings were carefully weighed, and after a sufficient time they were again weighed.

The standard bearing-metal used is the "S bearing-metal" of the Phosphor-bronze Smelting Co. It contains about 79.70% copper, 9.50% lead, 10% tin, and 0.80% phosphorus. A large number of experiments have shown that the loss of weight of a bearing of this metal is 1 lb. to each 18,000 to 25,000 miles travelled. Besides the measurement of wear, observations were made on the frequency of "hot boxes" with the different metals.

The results of the tests for wear, so far as given, are condensed into the following table:

Metal.	Composition.					Rate of Wear.
	Copper.	Tin.	Lead.	Phos.	Arsenic.	
Standard............	79.70	10.00	9.50	0.80	......	100
Copper-tin..........	87.50	12.50	....	....	......	148
Copper-tin, second experiment, same metal						152
Copper-tin, third experiment, same metal						147
Arsenic-bronze	89.20	10.00	....	....	0.80	142
Arsenic-bronze... ..	79.20	10.00	7.00	....	0.80	115
Arsenic-bronze......	79.70	10.00	9.50	....	0.80	101
"K" bronze	77.00	10.50	12.50	....	....	92
"K" bronze, second experiment, same metal						92.7
Alloy "B"	77.00	8.00	15.00	....	....	86.5

The old copper-tin alloy of 7 to 1 has repeatedly proved its inferiority to the phosphor-bronze metal. Many more of the copper-tin bearings heated than of the phosphor-bronze. The showing of these tests was so satisfactory that phosphor-bronze was adopted as the standard bearing-metal of the Pennsylvania R.R., and was used for a long time.

The experiments, however, were continued. It was found that arsenic practically takes the place of phosphorus in a copper-tin alloy, and three tests were made with arsenic-bronzes as noted above. As the proportion to lead is increased to correspond with the standard, the durability increases as well. In view of these results the "K" bronze was tried, in which neither phosphorus nor arsenic were used, and in which the lead was increased above the proportion in the standard phosphor-bronze. The result was that the metal wore 7.80% slower than the phosphor-bronze. No trouble from heating was experienced with the "K" bronze more than with the standard. Dr. Dudley continues:

At about this time we began to find evidences that wear of bearing-metal alloys varied in accordance with the following law: "That alloy which has the greatest power of distortion without rupture (resilience), will best resist wear." It was now attempted to design an alloy in accordance with this law, taking first the proportions of copper and tin, 9½ parts copper to 1 of tin was settled on by experiment as the standard, although some evidence since that time tends to show that 12 or possibly 15 parts copper to 1 of tin might have been better. The influence of lead on this copper-tin alloy seems to be much the same as a still further diminution of tin. However, the tendency of the metal to yield under pressure increases as the amount of tin is diminished, and the amount of the lead increased, so a limit is set to the use of lead. A certain amount of tin is also necessary to keep the lead alloyed with the copper.

Bearings were cast of the metal noted in the table as alloy "B," and it wore 13.5% slower than the standard phosphor-bronze. This metal is now the standard bearing-metal of the Pennsylvania Railroad, being slightly changed in composition to allow the use of phosphor-bronze scrap. The formula adopted is: Copper, 105 lbs.; phosphor-bronze, 60 lbs.; tin, 9¾ lbs.; lead, 25¼ lbs. By using ordinary care in the foundry, keeping the metal well covered with charcoal during the melting, no trouble is found in casting good bearings with this metal. The copper and the phosphor-bronze can be put in the pot before putting it in the melting-hole. The tin and lead should be added after the pot is taken from the fire.

It is not known whether the use of a little zinc, or possibly some other combination, might not give still better results. For the present, however, this alloy is considered to fulfil the various conditions required for good bearing-metal better than any other alloy. The phosphor-bronze had an ultimate tensile strength of 30,000 lbs., with 6% elongation, whereas the alloy "B" had 24,000 lbs. tensile strength and 11% elongation.

(For other bearing-metals, see Alloys containing antimony, on next page.

ALLOYS CONTAINING ANTIMONY.

VARIOUS ANALYSES OF BABBITT METAL AND OTHER ALLOYS CONTAINING ANTIMONY.

	Tin.	Copper	Antimony.	Zinc.	Lead.	Bismuth.
Babbitt metal for light duty	50 / =89.3	1 / 1.8	5 parts / 8.9 per ct.	....	....	....
Harder Babbitt for bearings*	96 / =88.9	4 / 3.7	8 parts / 7.4 per ct.	....	....	....
Britannia.. ...	85.7	1.0	10.1	2.9	....	....
"	81.9	....	16.2	1.9	....	....
"	81.0	2	16.	1.	....	....
"	70.5	4	25.5	....	....	....
"	22	10	62.	6.	....	....
"Babbitt"	45 5	1.5	13.	....	40.0	....
Plate pewter..	89.3	1.8	7.1	....	......	1.8
White metal...	85	5	10.	Bearings on Ger. locomotives.		

* It is mixed as follows: Twelve parts of copper are first melted and then 36 parts of tin are added; 24 parts of antimony are put in, and then 36 parts of tin, the temperature being lowered as soon as the copper is melted in order not to oxidize the tin and antimony, the surface of the bath being protected from contact with the air. The alloy thus made is subsequently remelted in the proportion of 50 parts of alloy to 100 tin. (Joshua Rose.)

White-metal Alloys.—The following alloys are used as lining metals by the Eastern Railroad of France (1890):

Number.	Lead.	Antimony.	Tin.	Copper.
1............	65	25	0	10
2............	0	11.12	83.33	5.55
3.	70	20	10	0
4............	80	8	12	0

No. 1 is used for lining cross-head slides, rod-brasses and axle-bearings; No. 2 for lining axle-bearings and connecting-rod brasses of heavy engines; No. 3 for lining eccentric straps and for bronze slide-valves; and No. 4 for metallic rod-packing.

Some of the best-known white-metal alloys are the following (Circular of Hoveler & Dieckhaus, London, 1893):

	Tin.	Antimony.	Lead.	Copper.	Zinc.
1. Parsons'	86	1	2	2	27
2. Richards'...............	70	15	10½	4½	0
3. Babbitt's...............	55	18	23½	3½	0
4. Fentons'	16	0	0	5	79
5. French Navy............	7½	0	7	7	87½
6. German Navy...........	85	7½	0	7½	0

"There are engineers who object to white metal containing lead or zinc. This is, however, a prejudice quite unfounded, inasmuch as lead and zinc often have properties of great use in white alloys."

It is a further fact that an "easy liquid" alloy must not contain more than 18% of antimony, which is an invaluable ingredient of white metal for improving its hardness; but in no case must it exceed that margin, as this would reduce the plasticity of the compound and make it brittle.

Hardest alloy of tin and lead: 6 tin, 4 lead. Hardest of all tin alloys (?): 74 tin, 18 antimony, 8 copper.

Alloy for thin open-work, ornamental castings: Lead 2, antimony 1. White metal for patterns: Lead 10, bismuth 6, antimony 2, common brass 8, tin 10.

Type-metal is made of various proportions of lead and antimony, from 17% to 20% antimony according to the hardness desired.

Babbitt Metals. (C. R. Tompkins, *Mechanical News*, Jan. 1891.)

The practice of lining journal-boxes with a metal that is sufficiently fusible to be melted in a common ladle is not always so much for the purpose of securing anti-friction properties as for the convenience and cheapness of forming a perfect bearing in line with the shaft without the necessity of boring them. Boxes that are bored, no matter how accurate, require great care in fitting and attaching them to the frame or other parts of a machine.

It is not good practice, however, to use the shaft for the purpose of casting the bearings, especially if the shaft be steel, for the reason that the hot metal is apt to spring it; the better plan is to use a mandrel of the same size or a trifl· larger for this purpose. For slow-running journals, where the load is moderate, alm st any metal that may be conveniently melted and will run free will answer the purpose. For wearing properties, with a moderate speed, there is probably nothing superior to pure zinc, but when not combined with some other metal it shrinks so much in cooling that it cannot be held firmly in the recess, and soon works loose; and it lacks those anti-friction properties which are necessary in order to stand high speed.

For line-shafting, and all work where the speed is not over 300 or 400 *r. p. m.*, an alloy of 8 parts zinc and 2 parts block-tin will not only wear longer than any composition of this class, but will successfully resist the force of a heavy load. The tin counteracts the shrinkage, so that the metal, if not overheated, will firmly adhere to the box until it is worn out. But this mixture does not possess sufficient anti-friction properties to warrant its use in fast-running journals.

Among all the soft metals in use there are none that possess greater anti-friction properties than pure lead; but lead alone is impracticable, for it is so soft that it cannot be retained in the recess. But when by any process lead can be sufficiently hardened to be retained in the boxes without materially injuring its anti-friction properties, there is no metal that will wear longer in light fast-running journals. With most of the best and most popular anti-friction metals in use and sold under the name of the Babbitt metal, the basis is lead.

Lead and antimony have the property of combining with each other in all proportions without impairing the anti-friction properties of either. The antimony hardens the lead, and when mixed in the proportion of 80 parts lead by weight with 20 parts antimony, no other known composition of metals possesses greater anti-friction or wearing properties, or will stand a higher speed without heat or abrasion. It runs free in its melted state, has no shrinkage, and is better adapted to light high-speeded machinery than any other known metal. Care, however, should be manifested in using it, and it should never be heated beyond a temperature that will scorch a dry pine stick.

Many different compositions are sold under the name of Babbitt metal. Some are good, but more are worthless; while but very little genuine Babbitt metal is sold that is made strictly according to the original formula. Most of the metals sold under that name are the refuse of type-foundries and other smelting-works, melted and cast into fancy ingots with special brands, and sold under the name of Babbitt metal.

It is difficult at the present time to determine the exact formulas used by the original Babbitt, the inventor of the recessed box, as a number of different formulas are given for that composition. Tin, copper, and antimony were the ingredients, and from the best sources of information the original proportions were as follows:

		Another writer gives:
50 parts tin	= 89.3%	83.3%
2 parts copper....................	= 3.6%	8.3%
4 parts antimony.................	= 7.1%	8.3%

The copper was first melted, and the antimony added first and then about ten or fifteen pounds of tin, the whole kept at a dull-red heat and constantly stirred until the metals were thoroughly incorporated, after which the balance of the tin was added, and after being thoroughly stirred again it was then cast into ingots. When the copper is thoroughly melted, and before the antimony is added, a handful of powdered charcoal should be thrown into the crucible to form a flux, in order to exclude the air and prevent the antimony from vaporizing; otherwise much of it will escape in the form of a vapor and consequently be wasted. This metal, when carefully prepared, is probably one of the best metals in use for lining boxes that are subjected to a heavy weight and wear; but for light fast-running journals the copper renders it more susceptible to friction, and it is more liable to heat than the metal composed of lead and antimony in the proportions just given.

SOLDERS.

Common solders, equal parts tin and lead; fine solder, 2 tin to 1 lead; cheap solder, 2 lead, 1 tin.

Fusing-point of tin-lead alloys:

Tin 1 to lead 25......558° F.	Tin 1½ to lead 1......334° F.
" 1 " " 10......541	" 2 " " 1......340
" 1 " " 5......511	" 3 " " 1......356
" 1 " " 3......482	" 4 " " 1......365
" 1 " " 2......441	" 5 " " 1......378
" 1 " " 1......370	" 6 " " 1......381

Common pewter contains 4 lead to 1 tin.

Gold solder: 14 parts gold, 6 silver, 4 copper. Gold solder for 14-carat gold: 25 parts gold, 25 silver, 12½ brass, 1 zinc.

Silver solder: Yellow brass 70 parts, zinc 7, tin 11½. Another: Silver 145 parts, brass (3 copper, 1 zinc) 73, zinc 4.

German-silver solder: Copper 38, zinc 54, nickel 8.

Novel's solders for aluminum:

Tin 100 parts, lead 5;	melts at 536° to 572° F.
" 100 " zinc 5;	" 536 to 612
" 1000 " copper 10 to 15;	" 662 to 842
" 1000 " nickel 10 to 15;	" 662 to 842

Novel's solder for aluminum bronze: Tin 900 parts, copper 100, bismuth 2 to 3. It is claimed that this solder is also suitable for joining aluminum to copper, brass, zinc, iron, or nickel.

ROPES AND CABLES.

STRENGTH OF ROPES.

(A. S. Newell & Co., Birkenhead. Klein's Translation of Weisbach, vol. III, part 1, sec. 2.)

Hemp.		Iron.		Steel.		Tensile Strength.
Girth.	Weight per Fathom.	Girth.	Weight per Fathom.	Girth.	Weight per Fathom.	
Inches.	Pounds.	Inches.	Pounds.	Inches.	Pounds.	Gross tons.
2¾	2	1	1			2
		1⅛	1½	1	1	3
3¾	4	1¼	2			4
		1⅜	2½	1½	1½	5
4½	5	1⅞	3			6
		2	3½	1⅝	2	7
5½	7	2⅛	4	1¾	2½	8
		2¼	4½			9
6	9	2⅜	5	1⅞	3	10
		2½	5½			11
6½	10	2⅝	6	2	3½	12
		2¾	6½	2⅛	4	13
7	12	2⅞	7	2¼	4½	14
		3	7½			15
7½	14	3⅛	8	2⅜	5	16
		3¼	8½			17
8	16	3⅜	9	2½	5½	18
		3½	10	2⅝	6	20
8½	18	3⅝	11	2¾	6½	22
		3¾	12			24
9½	22	3⅞	13	3¼	8	26
10	26	4	14			28
11	30	4¼	15	3⅜	9	30
		4⅜	16			32
		4½	18	3½	10	36
12	34	4⅝	20	3¾	12	40

Flat Ropes.

Hemp.		Iron.		Steel.		Tensile Strength.
Girth.	Weight per Fathom.	Girth.	Weight per Fathom.	Girth.	Weight per Fathom.	
Inches.	Pounds.	Inches.	Pounds.	Inches.	Pounds.	Gross tons.
4 × 1¼	20	2¼ × ½	11			20
5 × 1½	24	2½ × ½	13			23
5½ × 1⅝	26	2¾ × ⅝	15			27
5¾ × 1¾	28	3 × ⅝	16	2 × ½	10	28
6 × 1¾	30	3¼ × ⅝	18	2¼ × ½	11	32
7 × 1⅞	36	3½ × ⅝	20	2¾ × ½	12	36
8¼ × 2⅛	40	3¾ × 11/16	22	2½ × ½	13	40
8½ × 2¼	45	4 × 11/16	25	2¾ × ⅝	15	45
9 × 2¼	50	4¼ × ¾	28	3 × ⅝	16	50
9½ × 2⅜	55	4½ × ¾	32	3¼ × ⅝	18	56
10 × 2½	60	4⅝ × ¾	34	3½ × ⅝	20	60

Working Load, Diameter, and Weight of Ropes and Chains. (Klein's Weisbach, vol. iii, part 1, sec. 2, p. 561.)

Hemp ropes: d = diam. of rope. Wire ropes: d = diam. of wire, n = number of wires, G = weight per running foot, k = permissible load in pounds per square inch of section, P = permissible load on rope or chain.

Oval chains: d = diam. of iron used; inside dimensions of oval $1.5d$ and $2.6d$. Each link is a piece of chain $2.6d$ long. G_1 = weight of a single link = $2.10d^3$ lbs.; G = weight per running foot = $9.73d^2$ lbs.

	Hempen Rope.		Wire Rope.
	Dry and Untarred.	Wet or Tarred.	
k (lbs.) =	1400	1100	17000
d (ins.) =	$0.03\sqrt{P}$	$0.033\sqrt{P}$	$0.0087\sqrt{\frac{P}{n}}$
P (lbs.) =	$1120d^2 = 2855G$	$916d^2 = 1975G$	$13350nd^2 = 4590G$
G (lbs.) =	$1.28d^2 = 0.00035P$	$1.54d^2 = 0.0005P$	$2.91nd^2 = 0.000218P$

	Open-link Chain.	Stud-link Chain.
k (lbs.) =	8500	11400
d (ins.) =	$0.0087\sqrt{P}$	$0.0076\sqrt{P}$
P (lbs.) =	$13350d^2 = 1360G$	$17800d^2 = 1660G$
G (lbs.) =	$9.73d^2 = 0.000737P$	$10.65d^2 = 0.0006P$

Stud Chains 4/3 times as strong as open-link variety. [This is contrary to the statements of Capt. Beardslee, U. S. N., in the report of the U. S. Test Board. He holds that the open link is stronger than the studded link. See p. 308 ante].

STRENGTH AND WEIGHT OF WIRE ROPE, HEMPEN ROPE, AND CHAIN CABLES. (Klein's Weisbach.)

Breaking Load in tons of 2240 lbs.	Kind of Cable.	Girth of Wire Rope and of Hemp Rope Diameter of Iron of Chain, inches.	Weight of One Foot in length. Pounds.
1 Ton	Wire Rope	1.0	0.125
	Hemp Rope	2.0	0.177
	Chain	¼	0.500
8 Tons	Wire Rope	2.0	0.438
	Hemp Rope	5.0	0.978
	Chain	½	2.667
12 Tons	Wire Rope	2.5	0.753
	Hemp Rope	7.0	2.036
	Chain	11/16	4.502
16 Tons	Wire Rope	3.0	1.136
	Hemp Rope	8.0	2.365
	Chain	13/16	6.169
20 Tons	Wire Rope	3.5	1.546
	Hemp Rope	9.0	3.225
	Chain	29/32	7.674
24 Tons	Wire Rope	4.0	2.043
	Hemp Rope	10.0	4.166
	Chain	31/32	8.836
30 Tons	Wire Rope	4.5	2.725
	Hemp Rope	11.0	5.000
	Chain	1.1/16	10.335
36 Tons	Wire Rope	5.0	3.723
	Hemp Rope	12.5	5.940
	Chain	1.3/16	13.01
44 Tons	Wire Rope	5.5	4.50
	Hemp Rope	14.0	6.94
	Chain	1.5/16	16.00
54 Tons	Wire Rope	6.0	5.67
	Hemp Rope	15.0	7.92
	Chain	1.7/16	19.16

Length sufficient to provide the maximum working stress:

Hempen rope, dry and untarred	2855	feet.
" " wet or tarred	1975	"
Wire rope	4590	"
Open-link chain	1360	"
Stud chain	1660	"

Sometimes, when the depths are very great, ropes are given approximately the form of a body of uniform strength, by making them of separate pieces, whose diameters diminish towards the lower end. It is evident that by this means the tensions in the fibres caused by the rope's own weight can be considerably diminished.

Rope for Hoisting or Transmission. Manila Rope. (C. W. Hunt Company, New York.)—Rope used for hoisting or for transmission of power is subjected to a very severe test. Ordinary rope chafes and grinds to powder in the centre, while the exterior may look as though it was little worn.

In bending a rope over a sheave, the strands and the yarns of these strands slide a small distance upon each other, causing friction, and wear the rope internally.

The "Stevedore" rope used by the C. W. Hunt Co. is made by lubricating the fibres with plumbago, mixed with sufficient tallow to hold it in position. This lubricates the yarns of the rope, and prevents internal chafing and wear. After running a short time the exterior of the rope gets compressed and coated with the lubricant.

In manufacturing rope, the fibres are first spun into a yarn, this yarn being twisted in a direction called "right hand." From 20 to 80 of these yarns, depending on the size of the rope, are then put together and twisted in the opposite direction, or "left hand," into a strand. Three of these strands, for a 3-strand, or four for a 4-strand rope, are then twisted together, the twist being again in the "right hand" direction. When the strand is twisted, it untwists each of the threads, and when the three strands are twisted together into rope, it untwists the strands, but again twists up the threads. It is this opposite twist that keeps the rope in its proper form. When a weight is hung on the end of a rope, the tendency is for the rope to untwist, and become longer. In untwisting the rope, it would twist the threads up, and the weight will revolve until the strain of the untwisting strands just equals the strain of the threads being twisted tighter. In making a rope it is impossible to make these strains exactly balance each other. It is this fact that makes it necessary to take out the "turns" in a new rope, that is, untwist it when it is put at work. The proper twist that should be put in the threads has been ascertained approximately by experience.

The amount of work that the rope will do varies greatly. It depends not only on the quality of the fibre and the method of laying up the rope, but also on the kind of weather when the rope is used, the blocks or sheaves over which it is run, and the strain in proportion to the strain put upon the rope. The principal wear comes in practice from defective or badly set sheaves, from excess of load and exposure to storms.

The loads put upon the rope should not exceed those given in the tables, for the most economical wear. The indications of excessive load will be the twist coming out of the rope, or one of the strands slipping out of its proper position. A certain amount of twist comes out in using it the first day or two, but after that the rope should remain substantially the same. If it does not, the load is too great for the durability of the rope. If the rope wears on the outside, and is good on the inside, it shows that it has been chafed in running over the pulleys or sheaves. If the blocks are very small, it will increase the sliding of the strands and threads, and result in a more rapid internal wear. Rope made for hoisting and for rope transmission is usually made with four strands, as experience has shown this to be the most serviceable.

The strength and weight of "stevedore" rope is estimated as follows:

Breaking strength in pounds = 720 (circumference in inches)^2;
Weight in pounds per foot = .032 (circumference in inches)^2.

The Technical Words relating to Cordage most frequently heard are:

Yarn.—Fibres twisted together.
Thread.—Two or more *small yarns* twisted together.
String.—The same as a thread but a little larger *yarns*.
Strand.—Two or more *large yarns* twisted together.
Cord.—Several threads twisted together.
Rope.—Several *strands* twisted together.
Hawser.—A rope of three *strands*.
Shroud-Laid.—A rope of four *strands*.
Cable.—Three hawsers twisted together.
Yarns are laid up left-handed into *strands*.
Strands are laid up right-handed into rope.
Hawsers are laid up left-handed into a cable.

A rope is:
Laid by twisting strands together in making the rope.
Spliced by joining to another rope by interweaving the strands.
Whipped.—By winding a string around the end to prevent untwisting.
Served.—When covered by winding a yarn continuously and tightly around it.
Parcelled.—By wrapping with canvas.
Seized.—When two parts are bound together by a yarn, thread or string.
Payed.—When painted, tarred or greased to resist wet.
Haul.—To pull on a rope.
Taut.—Drawn tight or strained.

Splicing of Ropes.—The splice in a transmission rope is not only the weakest part of the rope but is the first part to fail when the rope is worn out. If the rope is larger at the splice, the projecting part will wear on the pulleys and the rope fail from the cutting off of the strands. The following directions are given for splicing a 4-strand rope.

The engravings show each successive operation in splicing a 1¾ inch manila rope. Each engraving was made from a full-size specimen.

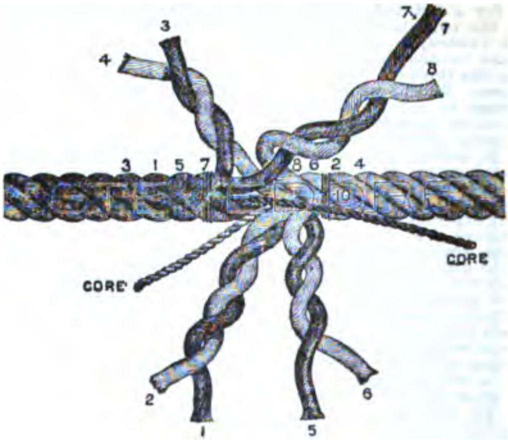


FIG. 78.

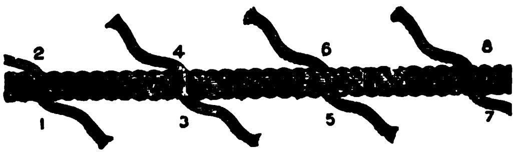


FIG. 79.

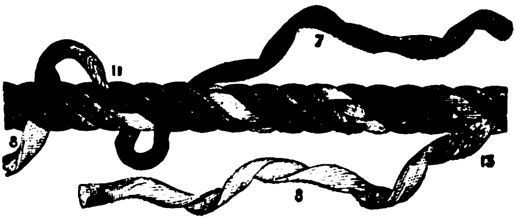


FIG. 80.

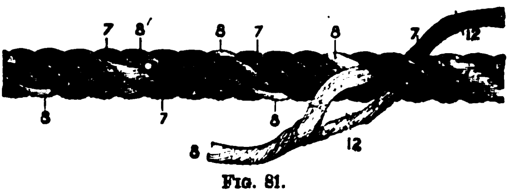


FIG. 81.

SPLICING OF ROPES.

Tie a piece of twine, 9 and 10, around the rope to be spliced, about 6 feet from each end. Then unlay the strands of each end back to the twine.

Butt the ropes together and twist each corresponding pair of strands loosely, to keep them from being tangled, as shown in Fig. 78.

The twine 10 is now cut, and the strand 8 unlaid and strand 7 carefully laid in its place for a distance of four and a half feet from the junction.

The strand 6 is next unlaid about one and a half feet and strand 5 laid in its place.

The ends of the cores are now cut off so they just meet.

Unlay strand 1 four and a half feet, laying strand 2 in its place.

Unlay strand 3 one and a half feet, laying in strand 4.

Cut all the strands off to a length of about twenty inches, for convenience in manipulation.

The rope now assumes the form shown in Fig. 79 with the meeting points of the strands three feet apart.

Each pair of strands is successively subjected to the following operation:

From the point of meeting of the strands 8 and 7, unlay each one three turns; split both the strand 8 and the strand 7 in halves as far back as they are now unlaid and "whip" the end of each half strand with a small piece of twine.

The half of the strand 7 is now laid in three turns and the half of 8 also laid in three turns. The half strands now meet and are tied in a simple knot, 11, Fig. 80, making the rope at this point its original size.

The rope is now opened with a marlin spike and the half strand of 7 worked around the half strand of 8 by passing the end of the half strand 7 through the rope, as shown in the engraving, drawn taut and again worked around this half strand until it reaches the half strand 13 that was not laid in. This half strand 13 is now split, and the half strand 7 drawn through the opening thus made, and then tucked under the two adjacent strands, as shown in Fig. 81. The other half of the strand 8 is now wound around the other half strand 7 in the same manner. After each pair of strands has been treated in this manner, the ends are cut off at 12, leaving them about four inches long. After a few days' wear they will draw into the body of the rope or wear off, so that the locality of the splice can scarcely be detected.

Coal Hoisting. (C. W. Hunt Co.).—The amount of coal that can be hoisted with a rope varies greatly. Under the ordinary conditions of use a rope hoists from 5000 to 8000 tons. Where the circumstances are more favorable, the amounts run up frequently to 12,000 or 15,000 tons, occasionally to 20,000 and in one case 32,400 tons to a single fall.

When a hoisting rope is first put in use, it is likely from the strain put upon it to twist up when the block is loosened from the tub. This occurs in the first day or two only. The rope should then be taken down and the "turns" taken out of the rope. When put up again the rope should give no further trouble until worn out.

It is necessary that the rope should be much larger than is needed to bear the strain from the load.

Practical experience for many years has substantially settled the most economical size of rope to be used which is given in the table below.

Hoisting ropes are not spliced, as it is difficult to make a splice that will not pull out while running over the sheaves, and the increased wear to be obtained in this way is very small.

Coal is usually hoisted with what is commonly called a "double whip;" that is, with a running block that is attached to the tub which reduces the strain on the rope to approximately one half the weight of the load hoisted. The following table gives the usual sizes of hoisting rope and the proper working strain:

Stevedore Hoisting-rope.

C. W. Hunt Co.

Circumference of the rope in ins.	Proper Working Strain on the Rope in lbs.	Nominal size of Coal tubs. Double whip.	Approximate Weight of a Coil, in lbs.
3	350	1/6 to 1/5 tons.	360
3½	500	1/5 " ¼ "	480
4	650	¼ " ⅓ "	650
4½	800	⅓ " ¾ "	830
5	1000	¾ " 1 "	960

Hoisting rope is ordered by circumference, transmission rope by diameter.

Weight and Strength of Manila Rope.

Spencer Miller (*Eng'g News*, Dec. 6, 1890) gives a table of breaking strength of manila rope, which he considers more reliable than the strength computed by Mr. Hunt's formula: Breaking strength $= 720 \times$ (circumference in inches)^2. Mr. Miller's formula is: Breaking weight lbs. $=$ circumference$^2 \times$ a coefficient which varies from 900 for ½" to 700 for 2" diameter rope, as below:

Circumference ...	1½	2	2½	2¾	3	3½	3¾	4¼	4½	5	5½	6
Coefficient	900	845	820	790	780	765	760	745	735	725	712	700

The following table gives the breaking strength of manila rope as calculated by Mr. Hunt's formula, and also by Mr. Miller's, using in the latter the coefficient 900 for sizes below 1½ in. circumference and 700 for sizes above 6 in. The differences between the figures for any given size are probably not greater than the difference in actual strength of samples from different makers. Both sets of figures are considerably lower than those given in tables published by some makers of rope, but they are believed to be more reliable. The figures for weight per 100 ft. are from manufacturers' tables.

Diameter in inches.	Circumference in inches.	Weight of 100 Feet of Rope in lbs.	Ultimate Strength of Rope in lbs.		Diameter in inches.	Circumference in inches.	Weight of 100 Feet of Rope in lbs.	Ultimate Strength of Rope in lbs.	
			Hunt.	Miller.				Hunt.	Miller.
3/16	9/16	2	290	280	1 5/16	4	52	11,500	12,000
¼	¾	3	400	500	1⅜	4¼	58	13,000	13,500
5/16	1	4	630	790	1½	4½	65	14,600	14,900
⅜	1⅛	5	900	1,140	1 9/16	4¾	72½	16,200	16,500
7/16	1¼	6	1,240	1,550	1⅝	5	80	18,000	18,100
½	1½	7⅜	1,620	2,020	1¾	5½	97	21,800	21,500
9/16	1¾	11	2,050	2,480	2	6	113	25,900	25,200
⅝	2	13⅛	2,880	3,880	2⅛	6½	133	30,400	29,600
¾	2¼	16⅛	3,610	4,150	2¼	7	153	35,300	34,300
13/13	2½	20	4,500	5,090	2½	7½	184	40,500	39,400
⅞	2¾	23⅜	5,440	5,970	2⅝	8	211	46,100	44,800
1	3	28⅛	6,480	7,020	2⅞	8½	237	52,000	50,600
1 1/16	3¼	33⅛	7,600	8,160	3	9	262	58,300	56,700
1⅛	3½	38	8,820	9,870	3⅛	9½	293	65,000	63,200
1¼	3¾	45	10,120	10,690	3¼	10	325	72,000	70,000

For rope-driving Mr. Hunt recommends that the working strain should not exceed 1/20 of the ultimate breaking strain. For further data on ropes see "Rope-driving."

Knots.—A great number of knots have been devised of which a few only are illustrated, but those selected are the most frequently used. In the cuts, Fig. 82, they are shown open, or before being drawn taut, in order to show the position of the parts. The names usually given to them are:

A. Bight of a rope.
B. Simple or Overhand knot.
C. Figure 8 knot.
D. Double knot.
E. Boat knot.
F. Bowline, first step.
G. Bowline, second step.
H. Bowline completed.
I. Square or reef knot.
J. Sheet bend or weaver's knot.
K. Sheet bend with a toggle.
L. Carrick bend.
M. Stevedore knot completed.
N. Stevedore knot commenced.
O. Slip knot.
P. Flemish loop.
Q. Chain knot with toggle.
R. Half-hitch.
S. Timber-hitch.
T. Clove hitch.
U. Rolling-hitch.
V. Timber-hitch and half-hitch.
W. Blackwall-hitch.
X. Fisherman's bend.
Y. Round turn and half-hitch.
Z. Wall knot commenced.
A A. " " completed.
B B. Wall knot crown commenced.
C C. " " " completed.

The principle of a knot is that no two parts, which would move in the same direction if the rope were to slip, should lay along side of and touching each other.

The bowline is one of the most useful knots, it will not slip, and after being strained is easily untied. Commence by making a bight in the rope, then put the end through the bight and under the standing part as shown in *G*, then pass the end again through the bight, and haul tight.

The square or reef knot must not be mistaken for the "granny" knot that slips under a strain. Knots *H*, *K* and *M* are easily untied after being under strain. The knot *M* is useful when the rope passes through an eye and is held by the knot, as it will not slip and is easily untied after being strained.

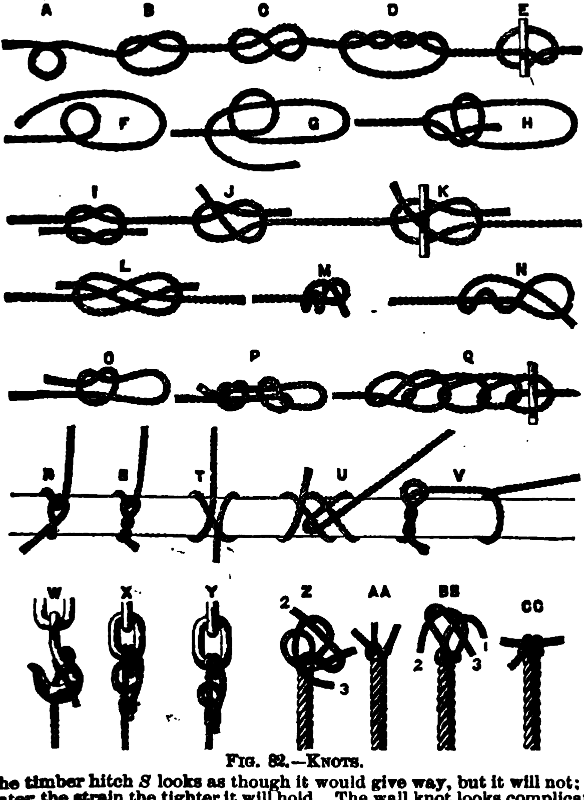


FIG. 82.—KNOTS.

The timber hitch *S* looks as though it would give way, but it will not; the greater the strain the tighter it will hold. The wall knot looks complicated, but is easily made by proceeding as follows: Form a bight with strand 1 and pass the strand 2 around the end of it, and the strand 3 round the end of 2 and then through the bight of 1 as shown in the cut *Z*. Haul the ends taut when the appearance is as shown in *AA*. The end of the strand 1 is now laid over the centre of the knot, strand 2 laid over 1 and 3 over 2, when the end of 3 is passed through the bight of 1 as shown in *BB*. Haul all the strands taut as shown in *CC*.

To Splice a Wire Rope.—The tools required will be a small marline spike, nipping cutters, and either clamps or a small hemp-rope sling with which to wrap around and untwist the rope. If a bench-vise is accessible it will be found convenient.

In splicing rope, a certain length is used up in making the splice. An allowance of not less than 16 feet for ½ inch rope, and proportionately longer for larger sizes, must be added to the length of an endless rope in ordering.

Having measured, carefully, the length the rope should be after splicing, and marked the points *M* and *M′*, Fig. 83, unlay the strands from each end *E* and *E′* to *M* and *M′* and cut off the centre at *M* and *M′*, and then:

(1). Interlock the six unlaid strands of each end alternately and draw them together so that the points *M* and *M′* meet, as in Fig. 84.

(2). Unlay a strand from one end, and following the unlay closely, lay into the seam or groove it opens, the strand opposite it belonging to the other end of the rope, until within a length equal to three or four times the length of one lay of the rope, and cut the other strand to about the same length from the point of meeting as at *A*, Fig. 85.

(3). Unlay the adjacent strand in the opposite direction, and following the unlay closely, lay in its place the corresponding opposite strand, cutting the ends as described before at *B*, Fig. 85.

There are now four strands laid in place terminating at *A* and *B*, with the eight remaining at *M M′*, as in Fig. 85.

It will be well after laying each pair of strands to tie them temporarily at the points *A* and *B*.

Pursue the same course with the remaining four pairs of opposite strands,

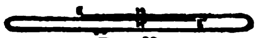


FIG. 83.

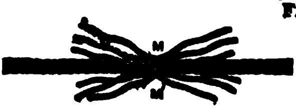


FIG. 84.

FIG. 85.

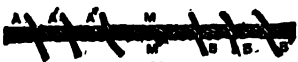


FIG. 86.

FIG. 87.

SPLICING WIRE ROPE.

stopping each pair about eight or ten turns of the rope short of the preceding pair, and cutting the ends as before.

We now have all the strands laid in their proper places with their respective ends passing each other, as in Fig. 86.

All methods of rope-splicing are identical to this point: their variety consists in the method of tucking the ends. The one given below is the one most generally practiced.

Clamp the rope either in a vise at a point to the left of *A*, Fig. 86, and by a hand-clamp applied near *A*, open up the rope by untwisting sufficiently to cut the core at *A*, and seizing it with the nippers, let an assistant draw it out slowly, you following it closely, crowding the strand in its place until it is all laid in. Cut the core where the strand ends, and push the end back into its place. Remove the clamps and let the rope close together around it. Draw out the core in the opposite direction and lay the other strand in the centre of the rope, in the same manner. Repeat the operation at the five remaining points, and hammer the rope lightly at the points where the ends pass each other at *A*, *A*, *B*, *B*, etc., with small wooden mallets, and the splice is complete, as shown in Fig. 87.

If a clamp and vise are not obtainable, two rope slings and short wooden levers may be used to untwist and open up the rope.

A rope spliced as above will be nearly as strong as the original rope and smooth everywhere. After running a few days, the splice, if well made, cannot be found except by close examination.

The above instructions have been adopted by the leading rope manufacturers of America.

SPRINGS.

Definitions.—A spiral spring is one which is wound around a fixed point or centre, and continually receding from it like a watch spring. A helical spring is one which is wound around an arbor, and at the same time advancing like the thread of a screw. An elliptical or laminated spring is made of flat bars, plates, or "leaves," of regularly varying lengths, superposed one upon the other.

Laminated Steel Springs.—Clark (Rules, Tables and Data) gives the following from his work on *Railway Machinery*, 1855:

$$\Delta = \frac{1.66L^3}{bt^3n}; \qquad s = \frac{bt^2n}{11.3L}; \qquad n = \frac{1.66L^3}{\Delta bt^3};$$

Δ = elasticity, or deflection, in sixteenths of an inch per ton of load,
s = working strength, or load, in tons (2240 lbs.),
L = span, when loaded, in inches,
b = breadth of plates, in inches, taken as uniform,
t = thickness of plates, in sixteenths of an inch,
n = number of plates.

NOTE.—The span and the elasticity are those due to the spring when weighted.

2. When extra thick back and short plates are used, they must be replaced by an equivalent number of plates of the ruling thickness, prior to the employment of the first two formulæ. This is found by multiplying the number of extra thick plates by the cube of their thickness, and dividing by the cube of the ruling thickness. Conversely, the number of plates of the ruling thickness given by the third formula, required to be deducted and replaced by a given number of extra thick plates, are found by the same calculation.

3. It is assumed that the plates are similarly and regularly formed, and that they are of uniform breadth, and but slightly taper at the ends.

Reuleaux's Constructor gives for semi-elliptic springs:

$$P = \frac{Snbh^2}{6l} \qquad \text{and} \qquad f = \frac{6Pl^3}{Enbh^3};$$

S = max. direct fibre-strain in plate;
n = number of plates in spring;
l = one half length of spring;
P = load on one end of spring;
b = width of plates;
h = thickness of plates;
f = deflection of end of spring;
E = modulus of direct elasticity.

The above formula for deflection can be relied upon where all the plates of the spring are regularly shortened; but in semi-elliptic springs, as used, there are generally several plates extending the full length of the spring, and the proportion of these long plates to the whole number is usually about one fourth. In such cases $f = \frac{5.5Pl^3}{Enbh^3}$. (G. R. Henderson, Trans. A. S. M. E., vol. xvi.)

In order to compare the formulæ of Reuleaux and Clark we may make the following substitutions in the latter: s in tons $= P$ in lbs. $\div$ 1120; $\Delta s = 16f$; $L = 2l$; $t = 16h$; then

$$\Delta s = 16f = \frac{1.66 \times 8l^3 \times P}{4096 \times 1120 \times nbh^3}, \qquad \text{whence} \qquad f = \frac{Pl^3}{5,527,133},$$

which corresponds with Reuleaux's formula for deflection if in the latter we take E = 33,163,800.

$$\text{Also} \qquad s = \frac{P}{1120} = \frac{256nbh^2}{11.3 \times 2l}, \qquad \text{whence} \qquad P = \frac{12.687nbh^2}{l},$$

which corresponds with Reuleaux's formula for working load when S in the latter is taken at 76,120.

The value of E is usually taken at 30,000,000 and S at 80,000, in which case Reuleaux's formulæ become

$$P = \frac{13,333nbh^2}{l} \qquad \text{and} \qquad f = \frac{Pl^3}{5,000,000nbh^3}.$$

Helical Steel Springs.—Clark quotes the following from the report on Safety Valves (Trans. Inst. Engrs. and Shipbuilders in Scotland, 1874–5):

$$E = \frac{d^3 \times w}{D^4 \times C}.$$

E = compression or extension of one coil in inches,
d = diameter from centre to centre of steel bar constituting the spring, in inches,
w = weight applied, in pounds,
D = diameter, or side of the square, of the steel bar, in sixteenths of an inch,
C = a constant, which may be taken as 22 for round steel and 30 for square steel.

NOTE.—The deflection E for one coil is to be multiplied by the number of free coils, to obtain the total deflection for a given spring.

The relation between the safe load, size of steel, and diameter of coil, may be taken for practical purposes as follows:

$$D = \sqrt[3]{\frac{wd}{3}}, \text{ for round steel;}$$

$$D = \sqrt[3]{\frac{wd}{4.29}}, \text{ for square steel.}$$

Rankine's Machinery and Millwork, p. 390, gives the following:

$$\frac{W}{v} = \frac{cd^4}{64nr^3}; \qquad W_1 = \frac{.196fd^3}{r}; \qquad v_1 = \frac{12.566nfr^2}{cd};$$

$$\frac{W_1}{2} = \text{greatest safe sudden load.}$$

In which d is the diameter of wire in inches; c a co-efficient of transverse elasticity of wire, say 10,500,000 to 12,000,000 for charcoal iron wire and steel; r radius to centre of wire in coil; n effective number of coils; f greatest safe shearing stress, say 30,000; W any load not exceeding greatest safe load; v corresponding extension or compression; W_1 greatest safe load; and v_1 greatest safe steady extension or compression.

If the wire is square, of the dimensions $d \times d$, the load for a given deflection is greater than for a round wire of the diameter d in the ratio of 2.81 to 1.96 or of 1.43 to 1, or of 10 to 7, nearly.

Wilson Hartnell (Proc. Inst. M. E., 1882, p. 426), says: The size of a spiral spring may be calculated from the formula on page 304 of "Rankine's Useful Rules and Tables"; but the experience with Salter's springs has shown that the safe limit of stress is more than twice as great as there given, namely 60,000 to 70,000 lbs. per square inch of section with ⅜ inch wire, and about 50,000 with ½ inch wire. Hence the work that can be done by springs of wire is four or five times as great as Rankine allows.

For ⅜ inch wire and under,

$$\text{Maximum load in lbs.} = \frac{12{,}000 \times (\text{diam. of wire})^3}{\text{Mean radius of springs}};$$

$$\text{Weight in lbs. to deflect spring 1 in.} = \frac{180{,}000 \times (\text{diam.})^4}{\text{Number of coils} \times (\text{rad.})^3}.$$

The work in foot-pounds that can be stored up in a spiral spring would lift it above 50 ft.

In a few rough experiments made with Salter's springs the coefficient of rigidity was noticed to be 12,600,000 to 13,700,000 with ¼ inch wire; 11,000,000 for 11/32 inch; and 10,600,000 to 10,900,000 for ⅜ inch wire.

Helical Springs.—J. Begtrup, in the *American Machinist* of Aug. 18, 1892, gives formulas for the deflection and carrying capacity of helical springs of round and square steel, as follow:

$$\left.\begin{aligned} W &= .3927\frac{Sd^3}{D-d}, \\ F &= 8\frac{P(D-d)^3}{Ed^4}, \end{aligned}\right\} \text{ for round steel.}$$

$$\left.\begin{aligned} W &= .471\frac{Sd^3}{D-d}, \\ F &= 4.712\frac{P(D-d)^3}{Ed^4}, \end{aligned}\right\} \text{ for square steel.}$$

W = carrying capacity in pounds,
S = greatest tensile stress per square inch of material,
d = diameter of steel,
D = outside diameter of coil,
F = deflection of one coil,
E = torsional modulus of elasticity,
P = load in pounds.

From these formulas the following table has been calculated by Mr. Begtrup. A spring being made of an elastic material, and of such shape as to allow a great amount of deflection, will not be affected by sudden shocks or blows to the same extent as a rigid body, and a factor of safety very much less than for rigid constructions may be used.

HOW TO USE THE TABLE.

When designing a spring for continuous work, as a car spring, use a greater factor of safety than in the table; for intermittent working, as in a steam-engine governor or safety valve, use figures given in table; for square steel multiply line W by 1.2 and line F by .59.

Example 1.—How much will a spring of ⅝″ round steel and 3″ outside diameter carry with safety? In the line headed D we find 3, and right underneath 473, which is the weight it will carry with safety. How many coils must this spring have so as to deflect 3″ with a load of 400 pounds? Assuming a modulus of elasticity of 12 millions we find in the centre line headed F the figure .0610; this is deflection of one coil for a load of 100 pounds; therefore $.061 \times 4 = .244''$ is deflection of one coil for 400 pounds load, and $3 \div .244 = 12\frac{1}{3}$ is the number of coils wanted. This spring will therefore be 4¾″ long when closed, counting working coils only, and stretch to 7¾″.

Example 2.—A spring 3¼″ outside diameter of 7/16″ steel is wound close; how much can it be extended without exceeding the limit of safety? We find maximum safe load for this spring to be 702 pounds, and deflection of one coil for 100 pounds load .0405 inches; therefore $7.02 \times .0405 = .284''$ is the greatest admissible opening between coils. We may thus, without knowing the load, ascertain whether a spring is overloaded or not.

Carrying Capacity and Deflection of Helical Springs of Round Steel.

d = diameter of steel. D = outside diameter of coil. W = safe working load in pounds—tensile stress not exceeding 60,000 pounds per square inch. F = deflection by a load of 100 pounds of one coil, and a modulus of elasticity of 10, 12 and 14 millions respectively. The ultimate carrying capacity will be about twice the safe load.

d = .065″ No. 16.	D	.25	.50	.75	1.00	1.25	1.50	1.75	2.00		
	W	85	15	9	7	5	4.5	3.8	3.3		
	F	.0276	.3588	1.433	3.562	7.250	12.88	20.85	31.57		
		.0236	.3075	1.228	3.053	6.214	11.04	17.87	27.06		
		.0197	.2562	1.023	2.544	5.178	9.200	14.89	22.55		
d = .120″ No. 11.	D	.50	.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	
	W	107	65	46	36	29	25	22	19	17	
	F	.0206	.0937	.2556	.5412	.9856	1.624	2.492	3.625	5.056	
		.0176	.0804	.2191	.4639	.8448	1.392	2.136	3.107	4.334	
		.0147	.0670	.182	.3866	.7010	1.160	1.780	2.589	3.612	
d = .180″ No. 7.	D	75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
	W	241	167	128	104	88	75	66	59	53	49
	F	.0137	.0408	.0907	.1703	.2866	.4466	.6571	.9249	1.256	1.660
		.0118	.0350	.0778	.1460	.2457	.3828	.5632	.7928	1.077	1.428
		.0098	.0292	.0648	.1217	.2048	.3190	.4693	.6607	.8975	1.186
d = ¼″	D	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
	W	368	294	245	210	184	164	147	134	123	113
	F	.0199	.0389	.0672	.1067	.1593	.2270	.3109	.4139	.5375	.6835
		.0171	.0333	.0576	.0914	.1365	.1944	.2665	.3548	.4607	.5859
		.0142	.0278	.0480	.0762	.1137	.1610	.2221	.2957	.3839	.4883

Carrying Capacity and Deflection of Helical Springs of Round Steel.—(*Continued*).

$d = 5/16''$	D	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00
	W	605	500	426	371	329	295	267	245	226	209	195
	F	.0136	.0242	.0392	.0593	.0854	.1187	.1583	.2066	.2640	.3312	.4089
		.0117	.0207	.0336	.0508	.0732	.1012	.1357	.1771	.2263	.2839	.3505
		.0097	.0173	.0280	.0424	.0610	.0853	.1131	.1476	.1886	.2366	.2921
$d = 3/8''$	D	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
	W	765	663	589	523	473	433	398	368	343	321	301
	F	.0169	.0259	.0377	.0528	.0711	.0935	.1200	.1513	.1874	.2290	.2761
		.0145	.0222	.0323	.0452	.0610	.0801	.1029	.1297	.1606	1963	.2367
		.0120	.0185	.0269	.0376	.0508	.0668	.0858	.1081	.1338	.1635	.1972
$d = 7/16''$	D	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00
	W	1263	1089	957	853	770	702	644	596	544	466	432
	F	.0081	.0126	.0186	.0262	.0357	.0472	.0617	.0772	.0960	.1423	.2016
		.0069	.0108	.0160	.0225	.0306	.0405	.0529	.0661	.0823	.1220	.1728
		.0058	.0090	.0133	.0187	.0255	.0337	.0441	.0551	.0686	.1017	.1440
$d = 1/2''$	D	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00
	W	1963	1683	1472	1309	1178	1071	982	906	811	785	654
	F	.0042	.0067	.0099	.0141	.0194	.0259	.0336	.0427	.0534	.0796	.1134
		.0036	.0057	.0085	.0121	.0167	.0222	.0288	.0366	.0457	.0683	.0972
		.0030	.0048	.0071	.0101	.0139	.0185	.0240	.0305	.0381	.0569	.0810
$d = 9/16''$	D	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	5.00	5.50
	W	2163	1916	1720	1560	1427	1315	1220	1137	1063	945	849
	F	.0056	.0081	.0112	.0151	.0197	.0252	.0316	.0390	.0474	.0679	.0935
		.0048	.0070	.0096	.0129	.0169	.0216	.0271	.0334	.0406	.0582	.0801
		.0040	.0058	.0080	.0108	.0141	.0180	.0226	.0278	.0339	.0485	.0668
$d = 5/8''$	D	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	5.00	5.50
	W	3068	2707	2422	2191	2001	1841	1704	1587	1484	1315	1180
	F	.0034	.0049	.0068	.0092	.0121	.0155	.0196	.0243	.0297	.0427	.0591
		.0029	.0042	.0058	.0079	.0104	.0133	.0168	.0208	.0254	.0366	.0506
		.0024	.0035	.0049	.0066	.0086	.0111	.0140	.0173	.0212	.0305	.0422
$d = 11/16''$	D	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.50	6.00
	W	3311	2988	2728	2500	2311	2151	2009	1885	1776	1591	1441
	F	.0043	.0058	.0077	.0100	.0127	.0157	.0193	.0233	.0279	.0388	.0522
		.0037	.0050	.0066	.0086	.0108	.0135	.0165	.0200	.0239	.0333	.0447
		.0030	.0042	.0055	.0071	.0090	.0112	.0138	.0167	.0199	.0277	.0373
$d = 3/4''$	D	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.50	6.00
	W	4418	3976	3615	3313	3058	2810	2651	2485	2339	2093	1893
	F	.0028	.0038	.0051	.0066	.0084	.0105	.0129	.0157	.0189	.0264	.0356
		.0024	0033	.0044	.0057	.0072	.0090	.0111	.0135	.0162	.0226	.0306
		.0020	.0027	.0036	.0047	.0060	.0075	.0093	.0113	.0135	.0188	.0254
$d = 7/8''$	D	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	6.00	6.50
	W	6013	5490	5051	4676	4354	4073	3826	3607	3413	3060	2806
	F	.0021	.0027	.0035	.0045	.0055	.0067	.0081	.0097	.0115	.0156	.0207
		.0018	.0024	.0030	.0038	.0047	.0058	.0070	.0083	.0098	.0134	.0177
		.0015	.0020	.0025	.0032	.0039	.0048	.0058	.0069	.0082	.0112	.0148
$d = 1''$	D	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	6.00	6.50
	W	9425	8568	7854	7250	6732	6283	5890	5544	5236	4712	4284
	F	.0012	.0016	.0021	.0026	.0033	.0041	.0049	.0059	.0071	.0097	.0129
		.0010	.0014	.0018	.0023	.0028	.0035	.0043	.0051	.0061	.0083	.0111
		.0008	.0011	.0015	.0019	.0023	.0029	.0035	.0043	.0051	.0069	.0092

The formulæ for deflection or compression given by Clark, Hartnell, and Begtrup, although very different in form, show a substantial agreement when reduced to the same form. Let d = diameter of wire in inches, D_1 = mean diameter of coil, n the number of coils, w the applied weight in pounds, and C a coefficient, then

$$\text{Compression or extension of one coil} = \frac{wD_1^3}{Cd^4};$$

$$\text{Weight in pounds to cause comp. or ext. of 1 in.} = \frac{Cd^4}{nD_1^3}.$$

The coefficient C reduced from Hartnell's formula is $8 \times 180{,}000 = 1{,}440{,}000$; according to Clark, $16^4 \times 22 = 1{,}441{,}792$, and according to Begtrup (using 12,000,000 for the torsional modulus of elasticity) $= 12{,}000{,}000 \div 8 = 1{,}500{,}000$.

Rankine's formula for greatest safe extension, $v_1 = \frac{12.566nfr^2}{cd}$ may take the form $v_1 = \frac{.7854nD_1^2}{100d}$ if we use 30,000 and 12,000,000 as the values for f and c respectively.

The several formulæ for safe load given above may be thus compared, letting d = diameter of wire, and D_1 = mean diameter of coil, Rankine, $W = \frac{.196fd^3}{r}$; Clark, $W = \frac{8(d \times 16)^3}{D_1}$; Begtrup, $W = \frac{.3927Sd^3}{D_1}$; Hartnell, $W = \frac{12000d^3}{r}$. Substituting for f the value 30,000 given by Rankine, and for S, 60,000 as given by Begtrup, we have $W = 11{,}760\frac{d^3}{D_1}$ Rankine; $12{,}288\frac{d^3}{D_1}$ Clark; $23{,}562\frac{d^3}{D_1}$ Begtrup; $24{,}000\frac{d^3}{D_1}$ Hartnell.

Taking from the Pennsylvania Railroad specifications the capacity when closed of the following springs, in which d = diameter of wire, D diameter outside of coil. $D_1 = D - d$, c capacity, H height when free, and h height when closed, all in inches.

No.	d	D	D_1	c	H	h
T.	$d = \frac{1}{4}$	$D = 1\frac{1}{2}$	$D_1 = 1\frac{1}{4}$	$c = 400$	$H = 9$	$h = 6$
S.	$\frac{1}{2}$	3	$2\frac{1}{2}$	1,900	8	5
K.	$\frac{3}{4}$	$5\frac{3}{4}$	5	2,100	7	$4\frac{1}{4}$
D.	1	5	4	8,100	$10\frac{1}{2}$	8
I.	$1\frac{1}{4}$	8	$6\frac{3}{4}$	10,000	9	$5\frac{3}{4}$
C.	$1\frac{1}{2}$	$4\frac{7}{8}$	$3\frac{3}{4}$	16,000	$4\frac{5}{8}$	$3\frac{5}{8}$

and substituting the values of c in the formula $c = W = x\frac{d^3}{D_1}$ we find x, the coefficient of $\frac{d^3}{D_1}$ to be respectively 32,000; 38,000; 32,400; 24,888; 34,560; 42,140, average 34,000.

Taking 12,000 as the coefficient of $\frac{d^3}{D_1}$ according to Rankine and Clark for safe load, and 24,000 as the coefficient according to Begtrup and Hartnell, we have for the safe load on these springs, as we take one or the other coefficient,

	T.	S.	K.	D.	I.	C.	
Rankine and Clark.............	150	600	1,012	3,000	3,750	5,400	lbs.
Hartnel	300	1,200	2,024	6,000	7,500	10,800	"
Capacity when closed, as above	400	1,900	2,100	8,100	10,000	16,000	"

J. W. Cloud (Trans. A. S. M. E., v. 173) gives the following:

$$P = \frac{S\pi d^3}{16R} \quad \text{and} \quad f = \frac{32PR^2l}{G\pi d^4};$$

P = load on spring;
S = maximum shearing fibre-strain in bar;
d = diameter of steel of which spring is made;
R = radius of centre of coil;
l = length of bar before coiling;
G = modulus of shearing elasticity;
f = deflection of spring under load.

Mr. Cloud takes $S = 80{,}000$ and $G = 12{,}600{,}000$.

The stress in a helical spring is almost wholly one of torsion. For method of deriving the formulæ for springs from torsional formula see Mr. Cloud's paper, above quoted.

ELLIPTICAL SPRINGS, SIZES, AND PROOF TESTS.

Pennsylvania Railroad Specifications, 1889.

Class.	Length betw'n centres, in.	Width over all, inches.	Bands, inches.	Width of Plates, inches.	Tests. To stand ins. High.	Tests. With Load of lbs.
A, Triple.......	40	11¾	3 × ⅜	3	3¾ between bands.	4800
					3 " "	5500
					2 " "	A. p. t.*
C, Quadruple..	40	15½	3 × ⅜	3	3¾ " "	6650
					3 " "	8000
					2 " "	A. p. t.*
D, Triple.... ..	36	11¾	3 × ⅜	3	4 " "	6000
					3 " "	8000
E, Single.......	40	3in.	3 × ⅜	3 × 11/32	5 bet. centre of eye and top of leaf.	When free
					3 " "	2350
F, Triple.......	36	11¾	3 × ⅜	3 × 11/32	2½ between bands.	11,800
G, Double.	32	7½	3 × ⅜	3	6¼ " "	When free
					3 " "	8000
H, Double......	36	9½	3 × ⅜	4	3½ " "	5400
					3 " "	6000
K, Double, 6 plates	22	10⅝	3½ × ⅜	4½ × 11/32	13/16 " "	13,800
L, Double, 7 plates	22	10⅝	3½ × ⅜	4½ × 11/32	13/16 " "	15,600
M, Quadruple..	40	15½	3 × ⅜	3	4 " "	8000
					3 " "	10,000
					2 " "	A. p. t.*

* A. p. t., auxiliary plates touching.

PHOSPHOR-BRONZE SPRINGS.

Wilfred Lewis (Engineers' Club, Philadelphia, 1887) made some tests with phosphor-bronze wire, .12 in. diameter, coiled in the form of a spiral spring, 1¼ in. diameter from centre to centre, making 52 coils.

This spring was loaded gradually up to a tension of 30 lbs., but as the load was removed it became evident that a permanent set had taken place. Such a spring of steel, according to the practice of the P. R. R., might be used for 40 lbs. A weight of 21 lbs. was then suspended so as to allow a small amount of vibration, and the length measured from day to day. In 30 hours the spring lengthened from 20⅝ inches to 21⅛ inches, and in 200 hours to 21¼ inches. It was concluded that 21 lbs. was too great for durability, and that probably 10 lbs. was as much as could be depended upon with safety.

For a given load the extension of the bronze spring was just double the extension of a similar steel spring, that is, for the same extension the steel spring is twice as strong.

SPRINGS TO RESIST TORSIONAL FORCE.

(Reuleaux's Constructor.)

Flat spiral or helical spring... $P = \frac{S}{6}\frac{bh^2}{R}$; $f = R\vartheta = 12\frac{PlR^2}{Ebh^3}$.

Round helical spring ....$P = \frac{S\pi}{32}\frac{d^3}{R}$; $f = R\vartheta = \frac{64}{\pi}\frac{Pl}{E}\frac{R^2}{d^4}$.

Round bar, in torsion.......... $P = \frac{S\pi}{16}\frac{d^3}{R}$; $f = R\vartheta = \frac{32}{\pi}\frac{P}{G}\frac{R^2 l}{d^4}$.

Flat bar, in torsion............ $P = \frac{S}{3R}\frac{b^2h^2}{\sqrt{b^2+h^2}}$; $f = R\vartheta = \frac{3PR^2 l}{G}\frac{b^2+h^2}{b^3h^3}$.

P = force applied at end of radius or lever-arm R; ϑ = angular motion at end of radius R; S = permissible maximum stress, = 4/5 of permissible stress in flexure; E = modulus of elasticity in tension; G = torsional modulus, = 2/5 E; l = developed length of spiral, or length of bar; d = diameter of wire; b = breadth of flat bar; h = thickness.

HELICAL SPRINGS FOR CARS AND LOCOMOTIVES. Arranged in Order of Strength.
(Condensed from Specifications of Penna. R. R. Co., 1888 and 1891.)

P. R. R. Co.'s Class.	Bar.			Spring.						
	Diameter.	Length.	Length after Tapering.	Weight. Normal.	Weight. Minimum	Diam. Outside of Coil.	Height. Free.	Height. Closed.	Capacity.	Capacity, partly closed.
	inches.	inches.	inches.	lbs.	lbs.	inches.	inches.	inches.	lbs.	lbs. at height.
Z	9/64	57⅛	....	¼	7/32	1	5¾	8	130	110 at 8¼ in.
E	¼	25¼	....	⅜	....	2¼	2¾	1⅛	240	....
R	11/64	75	....	½	7/16	1	8	5	270	170 at 6
T	¼	94⅞	....	1¼	1 3/16	1⅛	9	6	400	300 at 6¾
A	¼	45½	....	⅝	9/16	1¼	5⅛	3⅝	500	....
S	½	78½	82¾	4⅜	4⅛	3	8	5	1,900	1,200 at 6
K	⅝	88½	96⅞	11	10⅝	5¾	7	4¼	2,100	1,230 at 5⅝
P	15/16	85	92⅞	16⅛	16	6	8	5	6,000	3,600 at 6
J	1	85	92	18½	18	5½	8½	6	7,500	4,500 at 7
D	1	100½	106¾	22	21⅝	5	10½	8	8,100	....
I	1¼	97½	108	33½	32½	8	9	5¾	10,000	4,000 at 7⅛
N	1 ⅝	52¾ 55	60⅝ 60⅛	16⅛	16	6 3⅞	5⅛	3⅝	10,000	6.000 at 4
H	1 5/16	96	106½	36½	35½	8	9	6	11,000	6,000 at 7¼
O	1⅛ ⅝	56 62	63⅝ 66¾	20¾	20	6 3⅝	5¾	4⅛	13,000	6,000 at 4 15/16
Q	1 5/16 13/16	96 99¾	106½ 106½	51	48½	8 5⅛	9	6	14,000	6,800 at 7⅛
C	1⅛	85⅝	41¼	9⅞	9½	4⅞	4⅝	3⅝	16,000	....
L	1⅜ ⅞	87 85	97⅜ 91½	50½	49	8 5	8	5¾	16.000	7,000 at 7
X	1¼ ¾	75¼ 72¼	83⅛ 76¾	35	34	6¼ 3⅝	8	6	19,000	7,000 at 7¾
Y	1¼ ¾	56½ 54¼	64⅝ 58¾	26¼	25½	6¼ 3⅝	6	4½	19,000	7,000 at 5 7/16
B	4 bars 15/16	46	52⅝	35½	34½	5	5⅛	3⅞	28,000	18,000 at 4⅛
U	4 " 15/16	56	62⅝	43¼	42	5	6⅛	4⅝	28,000	18,500 at 5⅛
V	4 " 1 1/16	51¾	58⅝	51½	50	5 5/16	6⅛	4⅝	42,000	16,700 at 5½
G	4 " 15/16 4 " 19/32	72½ 40½	79 48	73	71	....	6⅞ 5 11/16 } 7⅛	over plates 5⅝	42,000	21,000 at 5 27/32
W	4 " 1 1/16	61	67⅛	60½	59	5 5/16	7⅛	5⅝	42,000	18,500 at 6 5/16

Springs N, O, Q, L, X, and Y are made of two coils, one inside of the other.
Springs B, U, G, V, and W are made of four equal coils placed near together and joined by top and bottom cap-pieces.

RIVETED JOINTS.

Fairbairn's Experiments. (From Report of Committee on Riveted Joints, *Proc. Inst. M. E.*, April, 1881.)

The earliest published experiments on riveted joints are contained in the memoir by Sir W. Fairbairn in the Transactions of the Royal Society. Making certain empirical allowances, he adopted the following ratios as expressing the relative strength of riveted joints:

Solid plate	100
Double-riveted joint	70
Single-riveted joint	56

These well-known ratios are quoted in most treatises on riveting, and are still sometimes referred to as having a considerable authority. It is singular, however, that Sir W. Fairbairn does not appear to have been aware that the proportion of metal punched out in the line of fracture ought to be different in properly designed double and single riveted joints. These celebrated ratios would therefore appear to rest on a very unsatisfactory analysis of the experiments on which they were based.

Loss of Strength in Punched Plates.—A report by Mr. W. Parker and Mr. John, made in 1878 to Lloyd's Committee, on the effect of punching and drilling, showed that thin steel plates lost comparatively little from punching, but that in thick plates the loss was very considerable. The following table gives the results for plates punched and not annealed or reamed:

Thickness of Plates.	Material of Plates.	Loss of Tenacity, per cent.
1/4	Steel	8
3/8	"	18
1/2	"	26
3/4	"	33
3/4	Iron	18 to 23

The effect of increasing the size of the hole in the die-block is shown in the following table:

Total Taper of Hole in Plate, inches.	Material of Plates.	Loss of Tenacity due to Punching, per cent.
1-16	Steel	17.8
1/8	"	12.3
1/4	"	(Hole ragged) 24.5

The plates were from 0.675 to 0.712 inch thick. When 5/8-in. punched holes were reamed out to 1 1/8 in. diameter, the loss of tenacity disappeared, and the plates carried as high a stress as drilled plates. Annealing also restores to punched plates their original tenacity.

Strength of Perforated Plates.

(P. D. Bennett, *Eng'g*, Feb. 12, 1886, p. 155.)

Tests were made to determine the relative effect produced upon tensile strength of a flat bar of iron or steel: 1. By a 3/4-inch hole drilled to the required size; 2. by a hole punched 1/8 inch smaller and then drilled to the size of the first hole; and, 3, by a hole punched in the bar to the size of the drilled bar. The relative results in strength per square inch of original area were as follows:

	1. Iron.	2. Iron.	3. Steel.	4. Steel.
Unperforated bar	1.000	1.000	1.000	1.000
Perforated by drilling	1.029	1.012	1.068	1.103
" " punching and drilling	1.030	1.008	1.059	1.110
" " punching only	0.795	0.894	0.935	0.927

In tests 2 and 4 the holes were filled with rivets driven by hydraulic pressure. The increase of strength per square inch caused by drilling is a phenomenon of similar nature to that of the increased strength of a grooved bar over that of a straight bar of sectional area equal to the smallest section of the grooved bar. Mr. Bennett's tests on an iron bar 0.84 in. diameter, 10 in.

long, and a similar bar turned to 0.84 in. diameter at one point only, showed that the relative strength of the latter to the former was 1.823 to 1.000.

Riveted Joints.—Drilling versus Punching of Holes.

The Report of the Research Committee of the Institution of Mechanical Engineers, on Riveted Joints (1881), and records of investigations by Prof. A. B. W. Kennedy (1881, 1882, and 1885), summarize the existing information regarding the comparative effects of punching and drilling upon iron and steel plates. From an examination of the voluminous tables given in Professor Unwin's Report, the results of the greatest number of the experiments made on iron and steel plates lead to the general conclusion that, while thin plates, even of steel, do not suffer very much from punching, yet in those of ¼-inch thickness and upwards the loss of tenacity due to punching ranges from 10% to 23% in iron plates, and from 11% to 33% in the case of mild steel. In drilled plates there is no appreciable loss of strength. It is possible to remove the bad effects of punching by subsequent reaming or annealing; but the speed at which work is turned out in these days is not favorable to multiplied operations, and such additional treatment is seldom practised. The introduction of a practicable method of drilling the plating of ships and other structures, after it has been bent and shaped, is a matter of great importance. If even a portion of the deterioration of tenacity can be prevented, a much stronger structure results from the same material and the same scantling. This has been fully recognized in the modern English practice (1887) of the construction of steam-boilers with steel plates; punching in such cases being almost entirely abolished, and all rivet-holes being drilled after the plates have been bent to the desired form.

Comparative Efficiency of Riveting done by Different Methods.

The Reports of Professors Unwin and Kennedy to the Institution of Mechanical Engineers (*Proc.* 1881, 1882, and 1885) tend to establish the four following points:

1. That the shearing resistance of rivets is not highest in joints riveted by means of the greatest pressure;
2. That the ultimate strength of joints is not affected to an appreciable extent by the mode of riveting; and, therefore,
3. That very great pressure upon the rivets in riveting is not the indispensable requirement that it has been sometimes supposed to be;
4. That the most serious defect of hand-riveted as compared with machine-riveted work consists in the fact that in hand-riveted joints visible slip commences at a comparatively small load, thus giving such joints a low value as regards tightness, and possibly also rendering them liable to failure under sudden strains after slip has once commenced.

The following figures of mean results, taken from Prof. Kennedy's tables (*Proceedings* 1885, pp. 218–225), give a comparative view of hand and hydraulic riveting, as regards their ultimate strengths in joints, and the periods at which in both cases visible slip commenced.

Total Breaking Load.		Load at which Visible Slip began.	
Hand-riveting.	Hydraulic Riveting.	Hand-riveting.	Hydraulic Riveting.
Tons.	Tons.	Tons.	Tons.
86.01	85.75	21.7	47.5
......	77.00	....	35.0
82.16	82.70	25.0	53.7
......	78.58	...	54.0
149.2	145.5	31.7	49.7
......	140.2	....	46.7
193.6	188.1	25.0	56.0
......	183.7	....	....

In these figures hand-riveting appears to be rather better than hydraulic riveting, as far as regards ultimate strength of joint; but is very much inferior to hydraulic work, in view of the small proportion of load borne by it before visible slip commenced.

Some of the Conclusions of the Committee of Research on Riveted Joints.

(*Proc. Inst. M. E.*, Apl. 1885.)

The conclusions all refer to joints made in soft steel plate with steel rivets, the holes all drilled, and the plates in their natural state (unannealed). In every case the rivet or shearing area has been assumed to be that of the holes, not the nominal (or real) area of the rivets themselves. Also, the strength of the metal in the joint has been compared with that of strips cut from the same plates, and not merely with nominally similar material.

The metal between the rivet-holes has a considerably greater tensile resistance per square inch than the unperforated metal. This excess tenacity amounted to more than 20%, both in 3/8-inch and 3/4-inch plates, when the pitch of the rivet was about 1.9 diameters. In other cases 3/8-inch plate gave an excess of 15% at fracture with a pitch of 2 diameters, of 10% with a pitch of 3.6 diameters, and of 6.6%, with a pitch of 3.9 diameters; and 3/4-inch plate gave 7.8% excess with a pitch of 2.8 diameters.

In single-riveted joints it may be taken that about 22 tons per square inch is the shearing resistance of rivet steel, when the pressure on the rivets does not exceed about 40 tons per square inch. In double-riveted joints, with rivets of about 3/4 inch diameter, most of the experiments gave about 24 tons per square inch as the shearing resistance, but the joints in one series went at 22 tons.

The ratio of shearing resistance to tenacity is not constant, but diminishes very markedly and not very irregularly as the tenacity increases.

The size of the rivet heads and ends plays a most important part in the strength of the joints—at any rate in the case of single-riveted joints. An increase of about one third in the weight of the rivets (all this increase, of course, going to the heads and ends) was found to add about 8½% to the resistance of the joint, the plates remaining unbroken at the full shearing resistance of 22 tons per square inch, instead of tearing at a shearing stress of only a little over 20 tons. The additional strength is probably due to the prevention of the distortion of the plates by the great tensile stress in the rivets.

The intensity of bearing pressure on the rivet exercises, with joints proportioned in the ordinary way, a very important influence on their strength. So long as it does not exceed 40 tons per square inch (measured on the projected area of the rivets), it does not seem to affect their strength; but pressures of 50 to 55 tons per square inch seem to cause the rivets to shear in most cases at stresses varying from 16 to 18 tons per square inch. For ordinary joints, which are to be made equally strong in plate and in rivets, the bearing pressure should therefore probably not exceed 42 or 43 tons per square inch. For double-riveted butt-joints perhaps, as will be noted later, a higher pressure may be allowed, as the shearing stress may probably not be more than 16 or 18 tons per square inch when the plate tears.

A margin (or net distance from outside of holes to edge of plate) equal to the diameter of the drilled hole has been found sufficient in all cases hitherto tried.

To attain the maximum strength of a joint, the breadth of lap must be such as to prevent it from breaking zigzag. It has been found that the net metal measured zigzag should be from 30% to 35% in excess of that measured straight across, in order to insure a straight fracture. This corresponds to a diagonal pitch of $2/3\,p + d/3$, if p be the straight pitch and d the diameter of the rivet-hole.

Visible slip or "give" occurs always in a riveted joint at a point very much below its breaking load, and by no means proportional to that load. A collation of the results obtained in measuring the slip indicates that it depends upon the number and size of the rivets in the joint, rather than upon anything else; and that it is tolerably constant for a given size of rivet in a given type of joint. The loads per rivet at which a joint will commence to slip visibly are approximately as follows:

Diameter of Rivet.	Type of Joint.	Riveting.	Slipping Load per Rivet.
3/4 inch	Single-riveted	Hand	2.5 tons
3/4 "	Double-riveted	Hand	3.0 to 3.5 tons
3/4 "	Double-riveted	Machine	7 tons
1 inch	Single-riveted	Hand	3.2 tons
1 "	Double-riveted	Hand	4.3 tons
1 "	Double-riveted	Machine	8 to 10 tons

To find the probable load at which a joint of any breadth will commence to slip, multiply the number of rivets in the given breadth by the proper figure taken from the last column of the table above. It will be understood that the above figures are not given as exact; but they represent very well the results of the experiments.

The experiments point to simple rules for the proportioning of joints of maximum strength. Assuming that a bearing pressure of 43 tons per square inch may be allowed on the rivet, and that the excess tenacity of the plate is 10% of its original strength, the following table gives the values of the ratios of diameter d of hole to thickness t of plate ($d \div t$), and of pitch p to diameter of hole ($p \div d$) in joints of maximum strength in ⅜-inch plate.

For Single-riveted Plates.

Original Tenacity of Plate.		Shearing Resistance of Rivets.		Ratio. $d \div t$	Ratio. $p \div d$	Ratio. $\frac{\text{Plate Area}}{\text{Rivet Area}}$
Tons per sq. in.	Lbs. per sq. in.	Tons per sq. in.	Lbs. per sq. in.			
30	67,200	22	49,200	2.48	2.30	0.667
28	62,720	22	49,200	2.48	2.40	0.785
30	67,200	24	53,760	2.28	2.27	0.713
28	62,720	24	53,760	2.28	2.36	0.690

This table shows that the diameter of the hole (not the diameter of the rivet) should be 2⅓ times the thickness of the plate, and the pitch of the rivets 2⅓ times the diameter of the hole. Also, it makes the mean plate area 71% of the rivet area.

If a smaller rivet be used than that here specified, the joint will not be of uniform, and therefore not of maximum, strength; but with any other size of rivet the best result will be got by use of the pitch obtained from the simple formula

$$p = a\frac{d^2}{t} + d,$$

where, as before, d is the diameter of the hole.

The value of the constant a in this equation is as follows:

For 30-ton plate and 22-ton rivets, $a = 0.524$
" 28 " 22 " " 0.558
" 30 " 24 " " 0.570
" 28 " 24 " " 0.606

Or, in the mean, the pitch $p = 0.56\frac{d^2}{t} + d$.

It should be noticed that with too small rivets this gives pitches often considerably smaller in proportion than 2⅓ times the diameter.

For double-riveted lap-joints a similar calculation to that given above, but with a somewhat smaller allowance for excess tenacity, on account of the large distance between the rivet-holes, shows that for joints of maximum strength the ratio of diameter to thickness should remain precisely as in single-riveted joints; while the ratio of pitch to diameter of hole should be 3.64 for 30-ton plates and 22 or 24 ton rivets, and 3.82 for 28-ton plates with the same rivets.

Here, still more than in the former case, it is likely that the prescribed size of rivet may often be inconveniently large. In this case the diameter of rivet should be taken as large as possible; and the strongest joint for a given thickness of plate and diameter of hole can then be obtained by using the pitch given by the equation

$$p = a\frac{d^2}{t} + d,$$

where the values of the constant a for different strengths of plates and rivets may be taken as follows:

Table of Proportions of Double-riveted Lap-joints,

in which $p = a\frac{d^2}{t} + d$.

Thickness of Plate.	Original tenacity of Plate, Tons per sq. in.	Shearing Resistance of Rivets. Tons per sq. in.	Value of Constant. a
3/8 inch	30	24	1.15
3/8 "	28	24	1.22
3/8 "	30	22	1.05
3/8 "	28	22	1.13
3/4 "	30	24	1.17
3/4 "	28	24	1.25
3/4 "	30	22	1.07
3/4 "	28	23	1.14

Practically, having assumed the rivet diameter as large as possible, we can fix the pitch as follows, for any thickness of plate from 3/8 to 3/4 inch:

For 30-ton plate and 24-ton rivets }
" 28 " " " 22 " " } $p = 1.16\frac{d^2}{t} + d$;

" 30 " " " 22 " " $p = 1.06\frac{d^2}{t} + d$;

" 28 " " " 24 " " $p = 1.24\frac{d^2}{t} + d$.

In double-riveted butt-joints it is impossible to develop the full shearing resistance of the joint without getting excessive bearing pressure, because the shearing area is doubled without increasing the area on which the pressure acts. Considering only the plate resistance and the bearing pressure, and taking this latter as 45 tons per square inch, the best pitch would be about 4 times the diameter of the hole. We may probably say with some certainty that a pressure of from 45 to 50 tons per square inch on the rivets will cause shearing to take place at from 16 to 18 tons per square inch. Working out the equations as before, but allowing excess strength of only 5% on account of the large pitch, we find that the proportions of double-riveted butt-joints of maximum strength, under given conditions, are those of the following table:

Double-riveted Butt-joints.

Original Tenacity of Plate, Tons per sq. in.	Shearing Resistance of Rivets, Tons per sq. in.	Bearing Pressure, Tons per sq. in.	Ratio $\frac{d}{t}$	Ratio $\frac{p}{d}$
30	16	45	1.80	3.85
28	16	45	1.80	4.06
30	18	48	1.70	4.03
28	18	48	1.70	4.27
30	16	50	2.00	4.30
28	16	50	2.00	4.42

Practically, therefore, it may be said that we get a double-riveted butt-joint of maximum strength by making the diameter of hole about 1.8 times the thickness of the plate, and making the pitch 4.1 times the diameter of the hole.

The proportions just given belong to joints of maximum strength. But in a boiler the one part of the joint, the plate, is much more affected by time than the other part, the rivets. It is therefore not unreasonable to estimate the percentage by which the plates might be weakened by corrosion, etc., before the boiler would be unfit for use at its proper steam-pressure, and to add correspondingly to the plate area. Probably the best thing to do in this case is to proportion the joint, not for the actual thickness of plate, but for a nominal thickness less than the actual by the assumed percentage. In this case the joint will be approximately one of uniform strength by the time it has reached its final workable condition; up to which time the joint as a whole will not really have been weakened, the corrosion only gradually bringing the strength of the plates down to that of rivets.

Efficiencies of Joints.

The average results of experiments by the committee gave: For double-riveted lap-joints in ⅜-inch plates, efficiencies ranging from 67.1% to 81.2%. For double-riveted butt-joints (in double shear) 61.4% to 71.3%. These low results were probably due to the use of very soft steel in the rivets. For single-riveted lap-joints of various dimensions the efficiencies varied from 54.8% to 60.8%.

The experiments showed that the shearing resistance of steel did not increase nearly so fast as its tensile resistance. With very soft steel, for instance, of only 26 tons tenacity, the shearing resistance was about 80% of the tensile resistance, whereas with very hard steel of 52 tons tenacity the shearing resistance was only somewhere about 65% of the tensile resistance.

Proportions of Pitch and Overlap of Plates to Diameter of Rivet-Hole and Thickness of Plate.

(Prof. A. B. W. Kennedy, *Proc. Inst. M. E.*, April, 1885.)

t = thickness of plate;
d = diameter of rivet (actual) in parallel hole;
p = pitch of rivets, centre to centre;
s = space between lines of rivets;
l = overlap of plate.

The pitch is as wide as is allowable without imparing the tightness of the joint under steam.

For single-riveted lap-joints in the circular seams of boilers which have double-riveted longitudinal lap joints,

$$d = t \times 2.25;$$
$$p = d \times 2.25 = t \times 5 \text{ (nearly)};$$
$$l = t \times 6.$$

For double-riveted lap-joints:

$$d = 2.25t;$$
$$p = 8t;$$
$$s = 4.5t;$$
$$l = 10.5t.$$

Single-riveted Joints.				Double-riveted Joints.				
t	d	p	l	t	d	p	s	l
3-16	7-16	15-16	1⅛	3-16	7-16	1½	⅞	2
¼	9-16	1¼	1½	¼	9-16	2	1 3-16	2¾
5-16	11-16	1 9-16	1⅞	5-16	11-16	2½	1⅜	3⅜
⅜	13-16	1⅞	2¼	⅜	13-16	3	1¾	4
7-16	1	2 3-16	2⅝	7-16	1	3½	2	4⅝
½	1⅛	2½	3	½	1⅛	4	2¼	5¼
9-16	1¼	2 13-16	3⅜	9-16	1¼	4½	2½	5⅞

With these proportions and good workmanship there need be no fear of leakage of steam through the riveted joint.

The net diagonal area, or area of plate, along a zigzag line of fracture should not be less than 30% in excess of the net area straight across the joint, and 35% is better.

Mr. Theodore Cooper (*R. R. Gazette*, Aug. 22, 1890) referring to Prof. Kennedy's statement quoted above, gives as a sufficiently approximate rule for the proper pitch between the rows in staggered riveting, one half of the pitch of the rivets in a row plus one quarter the diameter of a rivet-hole.

Apparent Excess in Strength of Perforated over Unperforated Plates. (Proc. Inst. M. E., October, 1888.)

The metal between the rivet-holes has a considerably greater tensile resistance per square inch than the unperforated metal. This excess tenacity amounted to more than 20%, both in ⅜-inch and ¾-inch plates, when the pitch of the rivets was about 1.9 diameters. In other cases ⅜-inch plate gave an excess of 15% at fracture with a pitch of 2 diameters, of 10% with a pitch of 3.6 diameters, and of 6.6% with a pitch of 3.9 diameters; and ¾-inch plate gave 7.8% excess with a pitch of 2.8 diameters.

(1) The "excess strength due to perforation" is increased by anything which tends to make the stress in the plate uniform, and to diminish the effect of the narrow strip of metal at the edge of the specimen.

(2) It is diminished by increase in the ratio of p/d, of pitch to diameter of hole, so that in this respect it becomes less as the efficiency of the joint increases.

(3) It is diminished by any increase in hardness of the plate.

(4) For a given ratio p/d, of pitch to diameter of hole, it is also apparently diminished as the thickness of the plate is increased. The ratio of pitch to thickness of plate does not seem to affect this matter directly, at least within the limits of the experiments.

Test of Double-riveted Lap and Butt Joints.

(Proc. Inst. M. E., October, 1888.)

Steel plates of 25 to 26 tons per square inch T. S., steel rivets of 24.6 tons shearing-strength per square inch.

Kind of Joint.	Thickness of Plate.	Diameter of Rivet-holes.	Ratio of Pitch to Diameter.	Comparative Efficiency of Joint.
Lap........ ...	3/8"	0.8"	3.62	75.2
Butt...........	3/8	0.7	3.93	76.5
Lap...........	3/4	1.1	2.82	68.0
"	3/4	1.6	3.41	73.6
Butt...........	3/4	1.1	4.00	72.4
"	3/4	1.6	3.94	76.1
Lap...........	1	1.3	2.42	63.0
"	1	1.75	3.00	70.2
Butt...........	1	1.3	3.92	76.1

Some Rules which have been Proposed for the Diameter of the Rivet in Single Shear. (*Iron*, June 18, 1880.)

Browne....................	$d = 2t$ (with double covers $1\frac{1}{4}t$)	(1)
Fairbairn..................	$d = 2t$ for plates less than 3/8 in.	(2)
"	$d = 1\frac{1}{4}t$ for plates greater than 3/8 in.	(3)
Lemaitre..................	$d = 1.5t + 0.16$	(4)
Antoine....................	$d = 1.1\sqrt{t}$	(5)
Pohlig.............	$d = 2t$ for boiler riveting	(6)
"	$d = 3t$ for extra strong riveting	(7)
Redtenbacher.............	$d = 1.5t$ to $2t$	(8)
Unwin......................	$d = \frac{3}{4}t + 5/16$ to $\frac{7}{8}t + \frac{3}{8}$	(9)
"	$d = 1.2\sqrt{t}$	(10)

The following table contains some data of the sizes of rivets used in practice, and the corresponding sizes given by some of these rules.

Diameter of Rivets for Different Thicknesses of Plates.

Thickness of plate. Inches.	Diameter of Rivets, in inches.									
	Lloyd's Rules.	Liverpool Rules.	English Dock-yards.	French Veritas.	Browne Eq. (1).	Fairbairn (2) and (3).	Lemaitre (4).	Antoine (5).	Unwin (10).	Wilson.
5/16	5/8	5/8	1/2	......	5/8	5/8	5/8	5/8	11/16	5/8
3/8	5/8	5/8	5/8	5/8	3/4	3/4	23/32	11/16	3/4	11/16
7/16	3/4	3/4	3/4	5/8	7/8	21/32	13/16	3/4	13/16	3/4
1/2	3/4	13/16	3/4	......	1	3/4	15/16	3/4	7/8	3/4
9/16	3/4	13/16	7/8	3/4	1 1/8	27/32	1	13/16	7/8	7/8
5/8	3/4	7/8	7/8	......	1 1/4	15/16	1 1/8	7/8	15/16	7/8
11/16	7/8	7/8	7/8	13/16	...	1 1/32	1 3/16	15/16	1	7/8
3/4	7/8	15/16	1	7/8	....	1 1/8	1 1/4	15/16	1 1/16	1
13/16	7/8	1	1	......	....	1 7/32	1 3/8	1	1 3/32	1
7/8	1	1 1/8	1 1/8	1	....	....	....	1	1 1/8	1
15/16	1	1 3/16	1 1/8	......	....	....	....	1 1/16	1 3/16	1 1/8
1	1	1 1/4	1 1/8	1 1/16	....	....	....	1 1/8	1 1/4	1 1/8

Strength of Double-riveted Seams, Calculated. — W. B. Ruggles, Jr., in *Power* for June, 1890, gives tables of relative strength of rivets and parts of sheet between rivets in double-riveted seams, compared with strength of shell, based on the assumption that the shearing strength of rivets and the tensile strength of steel are equal. The following figures show the sizes in his tables which show the nearest approximation to equality of strength of rivets and parts of plates between the rivets, together with the percentage of each relative to the strength of the solid plate.

Thickness of plate, inches.	Pitch of Rivets, inches.	Size of Rivet-holes, inches.	Percentage of Strength of Plate.		Thickness of Plate, inches.	Pitch of Rivets, inches.	Size of Rivet-holes, inches.	Percentage of Strength of Plate.	
			Rivets.	Plate.				Rivets.	Plate.
1/4	2 1/8	1/2	.739	.765	7/16	2 3/4	3/4	.734	.728
1/4	2 1/2	9/16	.795	.775	7/16	3 1/8	13/16	.758	.740
1/4	3 1/8	5/8	.785	.800	7/16	3 5/8	7/8	.758	.759
1/4	3 5/8	11/16	.819	.810	7/16	4 1/8	15/16	.765	.773
5/16	2 1/8	9/16	.749	.785	1/2	2 1/2	3/4	.707	.700
5/16	2 5/8	5/8	.748	.762	1/2	2 7/8	13/16	.721	.718
5/16	3 1/8	11/16	.761	.780	1/2	3 1/4	7/8	.740	.731
5/16	3 5/8	3/4	.780	.793	1/2	3 3/4	15/16	.736	.750
3/8	2 1/4	5/8	.727	.722	1/2	4 1/8	1	.761	.758
3/8	2 5/8	11/16	.755	.738	9/16	2 5/8	13/16	.701	.690
3/8	3 1/8	3/4	.754	.760	9/16	3	7/8	.714	.708
3/8	3 5/8	13/16	.762	.776	9/16	3 3/8	15/16	.727	.722
3/8	4 1/8	7/8	.777	.788	9/16	3 3/4	1	.745	.733
7/16	2 5/8	11/16	.714	.711	9/16	4 1/4	1 1/16	.742	.750

H. De B. Parsons (*Am. Engr. & R. R. Jour.*, 1893) holds that it is an error to assume that the shearing strength of the rivet is equal to the tensile strength. Also, referring to the apparent excess in strength of perforated over unperforated plates, he claims that on account of the difficulty in properly matching the holes, and of the stress caused by forcing, as is too often the case in practice, this additional strength cannot be trusted much more than that of friction.

Adopting the sizes of iron rivets as generally used in American practice for steel plates from 1/4 to 1 inch thick: the tensile strength of the plates as 60,000 lbs.; the shearing strength of the rivets as 40,000 for single-shear and 35,500 for double-shear, Mr. Parsons calculates the following table of pitches, so that the strength of the rivets against shearing will be approximately equal to that of the plate to tear between rivet-holes. The diameter of the rivets has in all cases been taken at 1/16 in. larger than the nominal size, as the rivet is assumed to fill the hole under the power riveter.

Riveted Joints.

LAP OR BUTT WITH SINGLE WELT—STEEL PLATES AND IRON RIVETS.

Thickness of Plates.	Diameter of Rivets.	Pitch.		Efficiency.	
		Single.	Double.	Single.	Double.
in.	in.	in.	in.		
1/4	1/2	1 3/16	1 7/8	55.7%	70.0%
3/8	3/4	1 11/16	2 11/16	52.7	68.6
1/2	7/8	1 7/8	2 3/4	49.0	65.9
5/8	7/8	1 11/16	2 7/16	43.6	60.4
3/4	1	1 7/8	2 5/8	42.0	59.5
7/8	1	1 3/4	2 7/16	38.6	55.4
1	1 1/8	2 3/16	2 5/8	38.1	54.9

Calculated Efficiencies—Steel Plates and Steel Rivets.—The differences between the calculated efficiencies given in the two tables above are notable. Those given by Mr. Ruggles are probably too high, since he assumes the shearing strength of the rivets equal to the tenslie strength of the plates. Those given by Mr. Parsons are probably lower than will be obtained in practice, since the figure he adopts for shearing strength is rather low, and he makes no allowance for excess of strength of the perforated over the unperforated plate. The following table has been calculated by the author on the assumptions that the excess strength of the perforated plate is 10%, and that the shearing strength of the rivets per square inch is four fifths of the tensile strength of the plate. If t = thickness of plate, d = diameter of rivet-hole, p = pitch, and T = tensile strength per square inch, then for single-riveted plates

$$(p - d)t \times 1.10T = \frac{\pi}{4} d^2 \times \frac{4}{5}T, \text{ whence } p = .571\frac{d^2}{t} + d.$$

$$\text{For double-riveted plates, } p = 1.142\frac{d^2}{t} + d.$$

The coefficients .571 and 1.142 agree closely with the averages of those given in the report of the committee of the Institution of Mechanical Engineers, quoted on pages 357 and 358, *ante*.

Thickness.	Diam. of Rivet-hole.	Pitch.		Efficiency.		Thickness.	Diam. of Rivet-hole.	Pitch.		Efficiency.	
		Single Riveting.	Double Riveting.	Single Riveting.	Double Riveting.			Single Riveting.	Double Riveting.	Single Riveting.	Double Riveting.
in.	in.	in.	in.	%	%	in.	in.	in.	in.	%	%
3/16	7/16	1.020	1.603	57.1	72.7	½	¾	1.392	2.035	46.1	63.1
"	½	1.261	2.023	60.5	75.3	"	⅞	1.749	2.624	50.0	66.6
¼	½	1.071	1.642	53.3	69.6	"	1	2.142	3.284	53.3	70.0
"	9/16	1.285	2.008	56.2	72.0	"	1⅛	2.570	4.016	56.2	72.0
5/16	9/16	1.187	1.712	50.5	67.1	9/16	¾	1.321	1.892	43.2	60.3
"	⅝	1.339	2.053	53.3	69.5	"	⅞	1.652	2.429	47.0	64.0
"	11/16	1.551	2.415	55.7	71.5	"	1	2.015	3.090	50.4	67.0
⅜	⅝	1.218	1.810	48.7	65.5	"	1⅛	2.410	3.694	53.3	69.5
"	¾	1.607	2.463	53.3	69.5	"	1¼	2.836	4.422	55.9	71.5
"	⅞	2.041	3.206	57.1	72.7	⅝	¾	1.264	1.778	40.7	57.8
7/16	⅝	1.186	1.647	45.0	62.0	"	⅞	1.575	2.274	44.4	61.5
"	¾	1.484	2.218	49.5	66.2	"	1	1.914	2.827	47.7	64.6
"	⅞	1.869	2.864	53.2	69.4	"	1⅛	2.281	3.488	50.7	67.3
"	1	2.305	3.610	56.6	72.3	"	1¼	2.678	4.105	53.3	69.5

Riveting Pressure Required for Bridge and Boiler Work.

(Wilfred Lewis, Engineers' Club of Philadelphia, Nov., 1893.)

A number of ⅝-inch rivets were subjected to pressures between 10,000 and 60,000 lbs. At 10,000 lbs. the rivet swelled and filled the hole without forming a head. At 20,000 lbs. the head was formed and the plates were slightly pinched. At 30,000 lbs. the rivet was well set. At 40,000 lbs. the metal in the plate surrounding the rivet began to stretch, and the stretching became more and more apparent as the pressure was increased to 50,000 and 60,000 lbs. From these experiments the conclusion might be drawn that the pressure required for cold riveting was about 300,000 lbs. per square inch of rivet section. In hot riveting, until recently there was never any call for a pressure exceeding 60,000 lbs., but now pressures as high as 150,000 lbs. are not uncommon, and even 300,000 lbs. have been contemplated as desirable.

Apparent Shearing Resistance of Rivet Iron and Steel.

(*Proc. Inst. M. E.*, 1879, *Engineering*, Feb. 20, 1880.)

The true shearing resistance of the rivets cannot be ascertained from experiments on riveted joints (1) because the uniform distribution of the load to all the rivets cannot be insured: (2) because of the friction of the plates, which has the effect of increasing the apparent resistance to shearing in an element uncertain in amount. Probably in the case of single-riveted joints the shearing resistance is not much affected by the friction.

	Ultimate Shearing Stress Tons per sq. in.	Ultimate Shearing Stress Lbs. per sq. in.	
Iron, single shear (12 bars)..	24.15	54.096	Clarke.
" double shear (8 bars)..	22.10	49.504	Clarke.
" " " ..	22.62	50.669	Barnaby.
" " " ..	22.30	49.952	Rankine.
" ¾-in. rivets............	23.05 to 25.57	51.632 to 57.277	Riley.
" ⅞-in. rivets............	24.32 to 27.94	54.477 to 62.362	Riley.
" mean value	25.0	56.000	Riley.
" ⅝-in. rivets.	19.01	42.582	Greig and Eyth.
Steel	17 to 26	38.080 to 58.240	Parker.
Landore steel, ¾-in. rivets..	31.67 to 33.69	70.941 to 75.466	Riley.
" " ⅞-in. rivets..	30.45 to 35.73	68.208 to 80.035	Riley.
" " mean value..	33.3	74.592	Riley.
Brown's steel...............	22.18	49.683	Greig and Eyth.

Fairbairn's experiments show that a rivet is 6⅙% weaker in a drilled than in a punched hole. By rounding the edge of the rivet-hole the apparent shearing resistance is increased 12%. Mr. Maynard found the rivets 4% weaker in drilled holes than in punched holes. But these results were obtained with riveted joints, and not by direct experiments on shearing. There is a good deal of difficulty in determining the true diameter of a punched hole, and it is doubtful whether in these experiments the diameter was very accurately ascertained. Messrs. Greig and Eyth's experiments also indicate a greater resistance of the rivets in punched holes than in drilled holes.

If, as appears above, the apparent shearing resistance is less for double than for single shear, it is probably due to unequal distribution of the stress on the two rivet sections.

The shearing resistance of a bar, when sheared in circumstances which prevent friction, is usually less than the tenacity of the bar. The following results show the decrease:

	Tenacity of Bar.	Shearing Resistance.	Ratio.
Harkort, iron..........	26.4	16.5	0.62
Lavalley, iron..........	25.4	20.2	0.79
Greig and Eyth, iron...	22.2	19.0	0.85
" " steel..	28.8	22.1	0.77

In Wöhler's researches (in 1870) the shearing strength of iron was found to be four-fifths of the tenacity. Later researches of Bauschinger confirm this result generally, but they show that for iron the ratio of the shearing resistance and tenacity depends on the direction of the stress relatively to the direction of rolling. The above ratio is valid only if the shear is in a plane perpendicular to the direction of rolling, and if the tension is applied parallel to the direction of rolling. The shearing resistance in a plane parallel to the direction of rolling is different from that in a plane perpendicular to that direction, and again differs according as the plane of shear is perpendicular or parallel to the breadth of the bar. In the former case the resistance is 18 to 20% greater than in a plane perpendicular to the fibres, or is equal to the tenacity. In the latter case it is only half as great as in a plane perpendicular to the fibres.

IRON AND STEEL.

CLASSIFICATION OF IRON AND STEEL.

CLASSIFICATION OF IRON AND STEEL.

(W. Kent, *Railroad & Engineering Journal*, April, 1887.)

Generic Term.	IRON.				
How Obtained.	*CAST*, Or obtained from a fluid mass.			*WROUGHT*, Or welded from a pasty mass.	
Distinguishing Quality.	*Non-malleable.*	*Malleable.*		*Will Not Harden.*	*Will Harden.*
Species.	CAST IRON.		CAST STEEL.	(7) WROUGHT IRON.	(8†) WROUGHT STEEL.
Varieties.	(1) Ordinary castings.	(2) Malleable cast iron, obtained from No. 1 by annealing in oxides.	(3) Crucible, (4) Bessemer, and (5) Open-hearth steels. (6) Mitis.*	*a*. Obtained by direct process from ores, as Catalan, Chenot, and other process irons. *b*. Obtained by indirect process from cast iron, as finery-hearth and puddled irons.	Obtained by direct or indirect process, as German, shear, blister, and puddled steels.

* No. 6. Mitis is the name given to a new product (having the same general properties and produced by the same processes as soft cast steels) made by adding an alloy of aluminum to melted wrought iron or soft steel before pouring.

† No. 8. Wrought steel is almost an obsolete product, having been replaced in commerce by cast steel.

Sub-varieties of Nos. 3, 4, and 5, soft, mild, medium, and hard steels, according to percentage of carbon, the divisions between them not being well defined.

Cast iron usually contains over 3% of carbon; cast steel anywhere from 0.06% to 1.50%, according to the purpose for which it is used; wrought iron from 0.02% to 0.10%. The quality of hardening and tempering which formerly distinguished steel from wrought iron is now no longer the dividing line between them, since soft steels are now produced which, by the ordinary blacksmith's tests, will not harden. All products of the crucible, Bessemer, and open-hearth processes are now commercially known as steel.

CAST IRON.

Grading of Pig Iron.—Pig iron is commonly graded according to its fracture, the number of grades varying in different districts. In Eastern Pennsylvania the principal grades recognized are known as No. 1 and 2 foundry, gray forge or No. 3, mottled or No. 4, and white or No. 5. Intermediate grades are sometimes made, as No. 2 X, between No. 1 and No. 2, and special names are given to irons more highly silicized than No. 1, as No. 1 X, silver-gray, and soft. Charcoal foundry pig iron is graded by numbers 1 to 5, but the quality is very different from the corresponding numbers in anthracite and coke pig. Southern coke pig iron is graded into ten or more grades. Grading by fracture is a fairly satisfactory method of grading irons made from uniform ore mixtures and fuel, but is unreliable as a means of determining quality of irons produced in different sections or from different ores. Grading by chemical analysis, in the latter case, is the only satisfactory method. The following analyses of the five standard grades of northern foundry and mill pig irons are given by J. M. Hartman (*Bull. I. & S. A.*, Feb., 1892):

	No. 1.	No. 2.	No. 3.	No. 4.	No. 4 B.	No. 5.
Iron	92.37	92.31	94.66	94.48	94.08	94.68
Graphitic carbon	3.52	2.99	2.50	2.02	2.02	.....
Combined carbon	.13	.37	1.52	1.98	1.43	3.83
Silicon	2.44	2.52	.72	.56	.92	.41
Phosphorus	1.25	1.08	.26	.19	.04	.04
Sulphur	.02	.02	trace	.08	.04	.02
Manganese	.28	.72	.34	.67	2.02	.98

CHARACTERISTICS OF THESE IRONS.

No. 1. *Gray.*—A large, dark, open-grain iron, softest of all the numbers and used exclusively in the foundry. Tensile strength low. Elastic limit low. Fracture rough. Turns soft and tough.

No. 2. *Gray.*—A mixed large and small dark grain, harder than No. 1 iron, and used exclusively in the foundry. Tensile strength and elastic limit higher than No. 1. Fracture less rough than No. 1. Turns harder, less tough, and more brittle than No. 1.

No. 3. *Gray.*—Small, gray, close grain, harder than No. 2 iron, used either in the rolling-mill or foundry. Tensile strength and elastic limit higher than No. 2. Turns hard, less tough, and more brittle than No. 2.

No. 4. *Mottled.*—White background, dotted closely with small black spots of graphitic carbon; little or no grain. Used exclusively in the rolling-mill. Tensile strength and elastic limit lower than No. 3. Turns with difficulty; less tough and more brittle than No. 3. The manganese in the B pig iron replaces part of the combined carbon, making the iron harder and closing the grain, notwithstanding the lower combined carbon.

No. 5. *White.*—Smooth, white fracture, no grain, used exclusively in the rolling mill. Tensile strength and elastic limit much lower than No. 4. Too hard to turn and more brittle than No. 4.

Southern pig irons are graded as follows, beginning with the highest in silicon: Nos. 1 and 2 silvery, Nos. 1 and 2 soft, all containing over 3% of silicon; Nos. 1, 2, and 3 foundry, respectively about 2.75%, 2.5% and 2% silicon; No. 1 mill, or "foundry forge;" No. 2 mill, or gray forge; mottled; white.

Good charcoal chilling iron for car wheels contains, as a rule, 0.56 to 0.95 silicon, 0.08 to 0.90 manganese, 0.05 to 0.75 phosphorus. The following is an analysis of a remarkably strong car wheel: Si, 0.734; Mn, 0.488; P. 0.428, S, 0.08; Graphitic C, 3.063; Combined C, 1.247; Copper, 0.029. The chill was very hard—¼ in. deep at root of flange, ⅛ in. deep on tread. A good ordnance iron analyzed: Si, 0.30; Graphitic C, 2.20; Combined C, 1.70; P, 0.44; Mn, 3.55 (?). Its specific gravity was 7.22 and tenacity 31,734 lbs. per sq. in.

Influence of Silicon, Phosphorus, Sulphur, and Manganese upon Cast Iron.—W. J. Keep, of Detroit, in several papers (Trans. A. I. M. E., 1889 to 1893), discusses the influence of various chemical elements on the quality of cast iron. From these the following notes have been condensed:

SILICON.—Pig iron contains all the carbon that it could absorb during its reduction in the blast-furnace. Carbon exists in cast iron in two distinct forms. In chemical union, as "combined" carbon, it cannot be discerned, except as it may increase the whiteness of the fracture, in so-called white iron. Carbon mechanically mixed with the iron as graphite is visible, varying in color from gray to black, while the fracture of the iron ranges from a light to a very dark gray.

Silicon will expel carbon, if the iron, when melted, contains all the carbon that it can hold and a portion of silicon be added.

Prof. Turner concludes from his tests that the amount of silicon producing the maximum strength is about 1.80%. But this is only true when a white base is used. If an iron is used as a base which will produce a sound casting to begin with, each addition of silicon will decrease strength. Silicon itself is a weakening agent. Variations in the percentage of silicon added to a pig iron will not insure a given strength or physical structure, but these results will depend upon the physical properties of the original iron.

After enough silicon has been added to cause solid castings, any further addition and consequent increase of graphite weakens the casting. The softness and strength given to castings by a suitable addition of silicon is, by a further increase of silicon, changed to stiffness, brittleness, and weakness.

As strength decreases from increase of graphite and decrease of combined carbon, deflection increases; or, in other words, bending is increased by graphite. When no more graphite can form and silicon still increases, deflection diminishes, showing that high silicon not only weakens iron, but makes it stiff. This stiffness is not the same strength-stiffness which is caused by compact iron and combined carbon. It is a *brittle-stiffness.*

In pig irons which received their silicon while in the blast-furnace the graphite more easily separates, and the shrinkage is less than in any mixture. As silicon increases, shrinkage also increases. Silicon of itself increases shrinkage, though by reason of its action upon the carbon in ordinary practice it is truly said that silicon "takes the shrinkage out of cast-iron." The slower a casting crystallizes, the greater will be the quantity of graphite formed within it.

Silicon of itself, however small the quantity present, hardens cast-iron; but the decrease of hardness from the change of the combined carbon to graphite, caused by the silicon, is so much more rapid than the hardening produced by the increase of silicon, that the total effect is to decrease hardness, until the silicon reaches from 3 to 5%.

As practical foundry-work does not call for more than 3% of silicon, the ordinary use of silicon does reduce the hardness of castings; but this is produced through its influence on the carbon, and not its direct influence on the iron.

When the change from combined to graphite carbon has ceased to diminish hardness, say at from 2% to 5% of silicon, the hardening by the silicon itself becomes more and more apparent as the silicon increases.

Shrinkage and hardness are almost exactly proportional. When silicon varies, and other elements do not vary materially, castings with low shrinkage are soft: as shrinkage increases, the castings grow hard in almost, if not exactly, the same proportion. For ordinary foundry-practice the scale of shrinkage may be made also the scale of hardness, provided variations in sulphur, and phosphorus especially, are not present to complicate the result.

The term "chilling" irons is generally applied to such as, cooled slowly, would be gray, but cooled suddenly, become white either to a depth sufficient for practical utilization (*e.g.*, in car-wheels) or so far as to be detrimental. Many irons chill more or less in contact with the cold surface of the mould in which they are cast, especially if they are thin. Sometimes this is a valuable quality, but for general foundry purposes it is desirable to have all parts of a casting an even gray.

Silicon exerts a powerful influence upon this property of irons, partially or entirely removing their capacity of chilling.

When silicon is mixed with irons previously low in silicon the fluidity is increased.

It is not the percentage of silicon, but the state of the carbon and the action of silicon through other elements, which causes the iron to be fluid.

Silicon irons have always had the reputation of imparting fluidity to other irons. This comes, no doubt, from the fact that up to 3% or 4% they increase the quantity of graphite in the resulting casting.

From the statement of Prof. Turner, that the maximum strength occurs with just such a percentage of silicon, and his statement that a founder can, with silicon, produce just the quality of iron that he may need, and from his naming the composition of what he calls a typical foundry-iron, some founders have inferred that if they knew the percentages of silicon in their irons and in their ferro-silicon, they need only mix so as to get 2% of silicon in order to obtain, always and with certainty, the maximum strength. The solution of the problem is not so simple. Each of the irons which the founder uses will have peculiar tendencies, given them in the blast-furnace, which will exert their influence in the most unexpected ways. However, a white iron which will invariably give porous and brittle castings can be made solid and strong by the addition of silicon; a further addition of silicon will turn the iron gray; and as the grayness increases the iron will grow weaker. Excessive silicon will again lighten the grain and cause a hard and brittle as well as a very weak iron. The only softening and shrinkage-lessening influence of silicon is exerted during the time when graphite is being produced, and silicon of itself is not a softener or a lessener of shrinkage; but through its influence on carbon, and only during a certain stage, does it produce these effects.

PHOSPHORUS.—While phosphorus of itself, in whatever quantity present, weakens cast-iron, yet in quantities less than 1.5% its influence is not sufficiently great to overbalance other beneficial effects, which are exerted before the percentage reaches 1%. Probably no element of itself weakens cast iron as much as phosphorus, especially when present in large quantities.

Shrinkage is decreased when phosphorus is increased. All high-phosphorus pig irons have low shrinkage. Phosphorus does not ordinarily harden cast iron, probably for the reason that it does not increase combined carbon.

The fluidity of the metal is slightly increased by phosphorus, but not to any such great extent as has been ascribed to it.

The property of remaining long in the fluid state must not be confounded with fluidity, for it is not the measure of its ability to make sharp castings, or to run into the very thin parts of a mould. Generally speaking, the statement is justified that, to some extent, phosphorus prolongs the fluidity of the iron while it is filling the mould.

The old Scotch irons contained about 1% of phosphorus. The foundry-irons which are most sought for for small and thin castings in the Eastern States contain, as a general thing, over 1% of phosphorus.

Certain irons which contain from 4% to 7% silicon have been so much used on account of their ability to soften other irons that they have come to be known as "softeners" and as lesseners of shrinkage. These irons are valuable as carriers of silicon; but the irons which are sold most as softeners and shrinkage-lesseners are those containing from 1% to 2% of phosphorus. We must therefore ascribe the reputation of some of them largely to the phosphorus and not wholly to the silicon which they contain.

From ½% to 1% of phosphorus will do all that can be done in a beneficial way, and all above that amount weakens the iron, without corresponding benefit. It is not necessary to search for phosphorus-irons. Most irons contain more than is needed, and the care should be to keep it within limits.

SULPHUR.—Only a small percentage of sulphur can be made to remain in carbonized iron, and it is difficult to introduce sulphur into gray cast iron or into any carbonized iron, although gray cast iron often takes from the fuel as much more sulphur as the iron originally contained. Percentages of sulphur that could be retained by gray cast iron cannot materially injure the iron except through an increase of shrinkage. The higher the carbon, or the higher the silicon, the smaller will be the influence exerted by sulphur.

The influence of sulphur on all cast iron is to drive out carbon and silicon and to increase chill, to increase shrinkage, and, as a general thing, to decrease strength; but if in practice sulphur will not enter such iron, we shall not have any cause to fear this tendency. In every-day work, however, it is found at times that iron which was gray when put into the cupola comes out white, with increased shrinkage and chill, and often with decreased strength. This is caused by decreased silicon, and can be remedied by an increase of silicon.

Mr. Keep's opinion concerning the influence of sulphur, quoted above, is disagreed with by J. B. Nau (*Iron Age*, March 29, 1894). He says:

"Sulphur, in whatever shape it may be present, has a deleterious influence on the iron. It has the tendency to render the iron white by the influence it exercises on the combination between carbon and iron. Pig iron containing a certain percentage of it becomes porous and full of holes, and castings made from sulphurous iron are of inferior quality. This happens especially when the element is present in notable quantities. With foundry-iron containing as high as 0.1% of sulphur, castings of greater strength may be obtained than when no sulphur is present. Thus, in some tests on this element quoted by R. Akerman, it is stated that in the foundry-iron from Finspong, used in the manufacture of cannons, a percentage of 0.1% to 0.14% of sulphur in the iron increased its strength to a considerable extent. The percentage of sulphur found originally in the iron put in the cupola is liable to be further increased by part of the sulphur that is invariably found in the coke used. It is seldom that a coke with a small percentage of sulphur is found, whereas coke containing 1% of it and over is very common. With such a fuel in the cupola, if no special precautions are resorted to, the percentage of sulphur in the metal will in most cases be increased."

That the sulphur contents of pig iron may be increased by the sulphur contained in the coke used, is shown by some experiments in the cupola, reported by Mr. Nau. Seven consecutive heats were made.

The sulphur content of the coke was 1%, and 11.7% of fuel was added to the charge.

Before melting, the silicon ranged from 0.820 to 0.880 in the seven heats; after melting, it was from 0.110 to 0.584, the loss in melting being from .100 to .375. The sulphur before melting was from .076 to .090, and after melting from .132 to .174, a gain from .044 to .098.

From the results the following conclusions were drawn:

1. In all the charges, without exception, sulphur increased in the pig iron after its passage through the cupola. In some cases this increase more than doubled the original amount of sulphnr found in the pig iron.

2. The increase of the sulphur contents in the iron follows the elimination of a greater amount of silicon from that same iron. A larger amount of limestone added to these charges would have produced a more basic cinder, and undoubtedly less sulphur would have been incorporated in the iron.

3. This coke contained 1% of sulphur, and if all its sulphur had passed into the iron there would have been an average increase of 0.12 of sulphur for the seven charges, while the real increase in the pig iron amounted to only 0.081. This shows that two thirds of the sulphur of the coke was taken up by the iron in its passage through the cupola.

MANGANESE.—Manganese is a nearly white metal, having about the same appearance when fractured as white cast iron. Its specific gravity is about 8, while that of white cast iron, reasonably free from impurities, is but a little above 7.5. As produced commercially, it is combined with iron, and with small percentages of silicon, phosphorus, and sulphur.

It is generally produced in the blast-furnace. If the manganese is under 40%, with the remainder mostly iron, and silicon not over 0.50%, the alloy is called spiegeleisen, and the fracture will show flat reflecting surfaces, from which it takes its name.

With manganese above 50%, the iron alloy is called ferro-manganese.

As manganese increases beyond 50%, the mass cracks in cooling, and when it approaches 98% the mass crumbles or falls in small pieces.

Manganese combines with iron in almost any proportion, but if an iron containing manganese is remelted, more or less of the manganese will escape by volatilization, and by oxidation with other elements present in the iron. If sulphur be present, some of the manganese will be likely to unite with it and escape, thus reducing the amount of both elements in the casting.

Cast iron, when free from manganese, cannot hold more than 4.50% of carbon, and 3.50% is as much as is generally present; but as manganese increases, carbon also increases, until we often find it in spiegel as high as 5%, and in ferro-manganese as high as 6%. This effect on capacity to hold carbon is peculiar to manganese.

Manganese renders cast iron less plastic and more brittle.

Manganese increases the shrinkage of cast iron. An increase of 1% raised the shrinkage 26%. Judging from some test records, manganese does not influence chill at all; but other tests show that with a given percentage of silicon the carbon may be a little more inclined to remain in the combined form, and therefore the chill may be a little deeper. Hence, to cause the chill to be the same, it would seem that the percentage of silicon should be a little higher with manganese than without it.

An increase of 1% of manganese increased the hardness 40%. If a hard chill is required, manganese gives it by adding hardness to the whole casting.

J. B. Nau (*Iron Age*, March 29, 1894), discussing the influence of manganese on cast iron, says:

Manganese favors the combination between carbon and iron. Its influence, when present in sufficiently large quantities, is even great enough not only to keep the carbon which would be naturally found in pig iron combined, but it increases the capacity of iron to retain larger amounts of carbon and to retain it all in the combined state.

Manganese iron is often used for foundry purposes when some chill and hardness of surface is required in the casting. For the rolls of steel-rail mills we always put into the mixture a large amount of manganiferous iron, and the rolls so obtained always presented the desired hardness of surface and in general a mottled structure on the outside. The inside, which always cooled much slower, was gray iron. One of the standard mixtures that invariably gave good results was the following:

50% of foundry iron with 1.3% silicon and 1.5% manganese;
35% of foundry iron with 1% silicon and 1.5% manganese;
15% steel (rail ends) with about 0.35% to 0.40% carbon.

The roll resulting from this mixture contained about 1% of silicon and 1% of manganese.

Another mixture, which differed but little from the preceding, was as follows:

45% foundry iron with about 1.3% silicon and 1.5% manganese;
30% foundry iron with about 1% silicon and 1.5% manganese;
10% white or mottled iron with about 0.5% to 0.6% Si. and 1.2% Mn.
15% Bessemer steel-rail ends with about 0.35% to 0.40% C. and 0.6% to 1% Mn.

The pig iron used in the preceding mixtures contained also invariably from 1.5% to 1.8% of phosphorus, so that the rolls obtained therefrom carried about 1.3% to 1.4% of that element. The last mixture used produced rolls containing on the average 0.8% to 1% of silicon and 1% of manganese. Whenever we tried to make those rolls from a mixture containing but 0.2% to 0.3% manganese our rolls were invariably of inferior quality, grayer, and consequently softer. Manganese iron cannot be used indiscriminately for foundry purposes. When greater softness is required in the castings manganese has to be avoided, but when hardness to a certain extent has to be obtained manganese iron can be used with advantage.

Manganese decreases the magnetism of the iron. This characteristic increases with the percentage of manganese that enters into the composition of the iron. The iron loses all its magnetism when manganese reaches 25% of its composition. This peculiarity has been made use of by French metallurgists to draw a clear line between spiegel and ferro-manganese. When the pig contains less than 25% of manganese it is classified as spiegel, and when it contains more than 25% it is classified as ferro-manganese. For this reason manganese iron has to be avoided in castings of dynamo fields and other pieces belonging to electric machinery, where magnetic conductibility is one of the first considerations.

Irregular Distribution of Silicon in Pig Iron.—J. W. Thomas (*Iron Age*, Nov. 12, 1891) finds in analyzing samples taken from every other bed of a cast of pig iron that the silicon varies considerably, the iron coming first from the furnace having generally the highest percentage. In one series of tests the silicon decreased from 2.040 to 1.718 from the first bed to the eleventh. In another case the third bed had 1.260 Si., the seventh 1.718, and the eleventh 1.101. He also finds that the silicon varies in each pig, being higher at the point than at the butt. Some of his figures are: point of pig 2.328 Si., butt of same 2.157; point of pig 1.834, butt of same 1.787.

Some Tests of Cast Iron. (G. Lanza, *Trans. A. S. M. E.*, x., 187.)—The chemical analyses were as follows:

	Gun Iron, per cent.	Common Iron, per cent.
Total carbon	3.51	.....
Graphite	2.80	.....
Sulphur	0.133	0.173
Phosphorus	0.155	0.413
Silicon	1.140	1.89

The test specimens were 26 inches long and square in section; those tested with the skin on being very nearly one inch square, and those tested with the skin removed being cast nearly one and one quarter inches square, and afterwards planed down to one inch square.

		Tensile Strength.	Elastic Limit.	Modulus of Elasticity.
Unplaned common.	20,900 to 23,000 T. S. Av.	= 22,066	6,500	13,194,233
Planed common....	20,300 to 20,800 " "	= 20,520	5,833	11,943,953
Unplaned gun.....	27,000 to 28,775 " "	= 28,175	11,000	16,130,300
Planed gun.........	29,500 to 31,000 " "	= 30,500	8,500	15,932,880

The elastic limit is not clearly defined in cast iron, the elongations increasing faster than the increase of the loads from the beginning of the test. The modulus of elasticity is therefore variable, decreasing as the loads increase. For example, see the results of test of a cast-iron bar on p. 314.

The Strength of Cast Iron depends on many other things besides its chemical composition. Among them are the size and shape of the casting, the temperature at which the metal is poured, and the rapidity of cooling. Internal stresses are apt to be induced by rapid cooling, and slow cooling tends to cause segregation of the chemical constituents and opening of the grain of the metal, making it weak. The relation of these variable conditions to the strength of cast iron is a complex one and as yet but imperfectly understood. (See "Cast-iron Columns," p. 250.)

The author recommends that in making experiments on the strength of cast iron, bars of several different sizes, such as ½, 1, 1½, and 2 in. square (or round), should be taken, and the results compared. Tests of bars of one size only do not furnish a satisfactory criterion of the quality of the iron of which they are made. See Trans. A. I. M. E., xxvi., 1017.

CHEMISTRY OF FOUNDRY IRONS.

(C. A. Meissner, *Columbia College Q'ly*, 1890; *Iron Age*, 1890.)

Silicon is a very important element in foundry irons. Its tendency when not above 2½% is to cause the carbon to separate out as graphite, giving the casting the desired benefits of graphitic iron. Between 2½% and 3½% silicon is best adapted for iron carrying a fair proportion of low silicon scrap and close iron, for ordinarily no mixture should run below 1½% silicon to get good castings.

From 3% to 5% silicon, as occurs in silvery iron, will carry heavy amounts of scrap. Castings are liable to be brittle, however, if not handled carefully as regards proportion of scrap used.

From 1½% to 2% silicon is best adapted for machine work; will give strong clean castings if not much scrap is used with it.

Below 1% silicon seems suited for drills and castings that have to stand great variations in temperature.

Silicon has the effect of making castings fluid, strong, and open-grained; also sound, by its tendency to separate the graphite from the total carbon, and consequent slight expansion of the iron on cooling, causing it to fill out thoroughly. Phosphorus, when high, has a tendency to make iron fluid, retain its heat longer, thereby helping to fill out all small spaces in casting. It makes iron brittle, however, when above ¾% in castings. It is excellent when high to use in a mixture of low-phosphorus irons, up to 1½% giving good results, but, as said before, the casting should be below ¾%. It has a strong tendency when above 1% in pig to make the iron less graphitic, preventing the separation of graphite.

Sulphur in open iron seldom bothers the founder, as it is seldom present to any extent. The conditions causing open iron in the furnace cause low sulphur. A little manganese is an excellent antidote against sulphur in the furnace. Irons above 1% manganese seldom have any sulphur of any consequence.

Graphite is the all-important factor in foundry irons; unless this is present in sufficient amount in the casting, the latter will be liable to be poor. Graphite causes iron to slightly expand on cooling, makes it soft, tough and fluid. (The statement as to expansion on cooling is denied by W. J. Keep.)

Relation of the Appearance of Fracture to the Chemical Composition.—S. H. Chauvenet says when run [from the blast-furnace] the lower bed is almost always close grain, but shows practically the same analysis as the large grain in the rest of the cast. If the iron runs rapidly, the lower bed may have as large grain as any in the cast. If the iron runs rapidly, for, say six beds and some obstruction in the tap-hole causes the seventh bed to fill up slowly and sluggishly, this bed may be close-grain, although the eighth bed, if the obstruction is removed will be open-grain. Neither the graphitic carbon nor the silicon seems to have any influence on the fracture in these cases, since by analysis the graphite and silicon is the same in each. The question naturally arises whether it would not be better to be guided by the analysis than by the fracture. The fracture is a guide, but it is not an infallible guide. Should not the open- and the close-grain iron of the same cast be numbered under the same grade when they have the same analysis?

Mr. Meissner had many analyses made for the comparison of fracture with analysis, and unless the condition of furnace, whether the iron ran fast or slow, and from what part of pig bed the sample is taken, are known, the fracture is often very misleading. Take the following analyses:

	A.	B.	C.	D.	E.	F.
Silicon........	4.315	4.818	4.270	3.328	3.869	3.861
Sulphur	0.008	0.008	0.007	0.033	0.006	0.006
Graphitic car..	3.010	2.757	2.680	2.243	3.070	3.100
Comb. carbon .	....	....	....	....	0.106	0.096

A. Very close-grain iron, dark color, by fracture, gray forge.
B. Open-grain, dark color, by fracture, No. 1.
C. Very close-grain, by fracture, gray forge.
D. Medium-grain, by fracture, No. 2, but much brighter and more open than A, C, or F.
E. Very large, open-grain, dark color, by fracture, No. 1.
F. Very close-grain, by fracture, gray forge.

By comparing analyses A and B, or E and F, it appears that the close-grain iron is in each case the highest in graphitic carbon. Comparing A and E, the graphite is about the same, but the close-grain is highest in silicon.

Analyses of Foundry Irons. (C. A. Meissner.)

SCOTCH IRONS.

Name.	Grade.	Silicon.	Phos-phorus.	Manga-nese.	Sul-phur.	Graph-ite.	Com. Carbon.
Summerlee.......	1	2.70	0.545	1.80	0.01	3.09	0.25
"	1	2.47	0.760	2.51	0.015		
"	1	3.44	1.000	1.70	0.015		
"	2	2.70	0.810	2.90	0.02	2.00	0.80
Eglinton	1	2.15	0.618	2.80	0.025	3.76	0.21
Coltness........ ..	1	2.59	0.840	1.70	0.010	3.75	3.75
Carnbroe.........	1	1.70	1.100	1.83	0.008	3.50	0.40
Glengarnock	1	3.03	1.200	2.85			
Glengarnock said to carry ⅔ scrap	2	4.00	0.900	3.41	0.010	1.78	0.90

AMERICAN SCOTCH IRONS.

No. Sample	Silicon.	Phos-phorus.	Manganese	Sulphur.	No. Grade.	
1	6.00	0.430	1.00	....	1	....
2	1.67	1.920	1.90	....	....	casting.
3	2.40	1.000	1.70	....	2	....
4	1.28	0.690	1.40	....	2	....
5a	3.50	0.613	2.31	....	1	....
5b	2.90	0.733	1.40	....	....	casting.
6a	3.44	1.000	1.70	0.015	1	....
6b	3.35	1.300	1.50	0.012	1	....
7	3.68	0.503	2.96	....	1	....

DESCRIPTION OF SAMPLES.—No. 1. Well known Ohio Scotch iron, almost silvery, but carries two-thirds scrap; made from part black-band ore. Very successful brand. The high silicon gives it its scrap-carrying capacity.

No. 2. Brier Hill Scotch castings, made at scale works; castings demanding more fluidity than strength.

No. 3. Formerly a famous Ohio Scotch brand, not now in the market Made mainly from black-band ore.

No. 4. A good Ohio Scotch, very soft and fluid; made from black-band ore-mixture.

Nos. 5*a* and 5*b*. Brier Hill Scotch iron and casting; made for stove purposes; 350 lbs. of iron used to 150 lbs. scrap gave very soft fluid iron; worked well.

No. 6*a*. Shows comparison between Summerlee (Scotch) (6*a*) and Brier Hill Scotch (6*b*). Drillings came from a Cleveland foundry, which found both irons closely alike in physical and working quality.

No. 7. One of the best southern brands, very hard to compete with, owing to its general qualities and great regularity of grade and general working.

MACHINE IRONS.

Sample No.	Silicon.	Phosphorus.	Manganese.	Sulphur.	Graphite.	Comb. Carbon.	Grade No.
8	2.80	0.492	0.61	0.015	....	....	1
9	1.30	0.262	0.70	0.030	....	....	3
10*a*	2.66	0.770	1.20	0.020	2.51	....	2
10*b*	3.68	0.411	1.25	0.014	3.05	....	1
11	2.10	0.415	0.60	0.050	....	....	2
12	1.37	0.294	1.51	0.080	2.31	0.78	2
13	3.10	0.124	trace	0.021	....	....	2
14	2.12	0.610	0.80	....	....	....	....
15	1.70	0 632	1.60	....	....	....	....
16*a*	1.45	0.470	1.25	0.009	....	....	2
16*b*	1.40	0.316	1.37	0.008	....	....	....
17	3.26	0.426	0.25	....	....	....	1
18	0.80	0.164	0.90	0.015	....	....	1

DESCRIPTION OF SAMPLES.—No. 8. A famous Southern brand noted for fine machine castings.

No. 9. Also a Southern brand, a very good machine iron.

Nos. 10*a* and 10*b*. Formerly one of the best known Ohio brands. Does not shrink; is very fluid and strong. Foundries having used this have reported very favorably on it.

No. 11. Iron from Brier Hill Co., made to imitate No. 8; was stronger than No. 8; did not pull castings; was fluid and soft.

No. 12. Copy of a very strong English machine iron.

No. 13. A Pennsylvania iron, very tough and soft. This is partially Bessemer iron, which accounts for strength, while high silicon makes it soft.

No. 14. Castings made from Brier Hill Co.'s machine brand for scale works, very satisfactory, strong, soft and fluid.

No. 15. Castings made from Brier Hill Co.'s one half machine brand, one half Scotch brand, for scale works, castings desired to be of fair strength, but very fluid and soft.

No. 16*a*. Brier Hill machine brand made to compete with No. 8.

No. 16*b*. Castings (clothes-hooks) from same, said to have worked badly, castings being white and irregular. Analysis proved that some other iron too high in manganese had been used, and probably not well mixed.

No. 17. A Pennsylvania iron, no shrinkage, excellent machine iron, soft and strong.

No. 18. A very good quality Northern charcoal iron.

"Standard Grades" of the Brier Hill Iron and Coal Company.

Brier Hill Scotch Iron.—Standard Analysis, Grade Nos. 1 *and* 2.

Silicon	2.00 to 3.00
Phosphorus	0.50 to 0.75
Manganese	2.00 to 2.50

Used successfully for scales, mowing-machines, agricultural implements, novelty hardware, sounding-boards, stoves, and heavy work requiring no special strength.

Brier Hill Silvery Iron.—Standard Analysis, Grade No. 1.

Silicon	3.50 to 5.50
Phosphorus	1.00 to 1.50
Manganese	2.00 to 2.25

Used successfully for hollow-ware, car-wheels, etc., stoves, bumpers, and similar work, with heavy amounts of scrap in all cases. Should be mainly used where fluidity and no great strength is required, especially for heavy work. When used with scrap or close pig low in phosphorus, castings of considerable strength and great fluidity can be made

Fairly Heavy Machine Iron.—Standard Analysis, Grade No. 1.

Silicon	1.75 to 2.50
Phosphorus	0.50 to 0.60
Manganese	1.20 to 1.40

The best iron for machinery, wagon-boxes, agricultural implements, pump-works, hardware specialties, lathes, stoves, etc., where no large amounts of scrap are to be carried, and where strength, combined with great fluidity and softness, are desired. Should not have much scrap with it.

Regular Machine Iron.—Standard Analysis, Grade Nos. 1 *and* 2.

Silicon	1.50 to 2.00
Phosphorus	0.30 to 0.50
Manganese	0.80 to 1.00

Used for hardware, lawn-mowers, mower and reaper works, oil-well machinery, drills, fine machinery, stoves, etc. Excellent for all small fine castings requiring fair fluidity, softness, and mainly strength. Cannot be well used alone for large castings, but gives good results on same when used with above-mentioned heavy machine grade; also when used with the Scotch in right proportion. Will carry but little scrap, and should be used alone for good strong castings.

For Axles and Materials Requiring Great Strength, Grade No. 2.

Silicon	1.50
Phosphorus	0.200 and less.
Manganese	0.80

This gave excellent results.

A good neutral iron for guns, etc., will run about as follows:

Silicon	1.00
Phosphorus	0.25
Sulphur	0.20
Manganese	none.

It should be open No. 1 iron.

This gives a very tough, elastic metal. More sulphur would make tough but decrease elasticity.

For fine castings demanding elegance of design but no strength, phosphorus to 3.00% is good. Can also stand 1.50% to 2.00% manganese. For work of a hard, abrasive character manganese can run 2.00% in casting.

Analyses of Castings.

Sample No.	Silicon.	Phosphorus.	Manganese	Sulphur.	Graphite.	Comb. Carbon.
31	2.50	1.400	2.20	....	....	....
32	0.85	0.351	0.92	0.080	....	....
33	1.53	0.327	1.06	0.040	3.10	0.58
34*a*	1.84	0.577	1.04	....	....	....
34*b*	2.20	0.742	1.10	....	....	....
34*c*	2.50	1.208	1.16	....	....	....
35*a*	2.80	0.418	0.54	....	....	....
35*b*	3.10	1.260	1.14	....	....	....
35*c*	3.80	0.879	0.80	....	....	....
35*d*	2.88	0.408	1.10	....	....	....
35*e*	4.50	0.660	0.78	....	....	....
36	3.48	1.489	0.90	0.025	....	....
37*a*	2.68	0.900	1.30	....	....	....
37*b*	1.90	0.980	1.20	....	....	....

No. 31. Sewing-machine casting, said to be very fluid and good casting. This is an odd analysis. I should say it would have been too hard and brittle, yet no complaint was made.

No. 32. Very good machine casting, strong, soft, no shrinkage.

No. 33. Drillings from an annealer-box that stood the heat very well.

No. 34*a*. Drillings from door-hinge, very strong and soft.

No. 34*b*. Drillings from clothes-hooks, tough and soft, stood severe hammering.

No. 34*c*. Drillings from window-blind hinge, broke off suddenly at light strain. Too high phosphorus.

No. 35*a*. Casting for heavy ladle support, very strong.

Nos 35*b* and 35*c*. Broke after short usage. Phosphorus too high. Car-bumpers.

No. 35*d*. Elbow for steam heater, very tough and strong.

No. 36. Cog-wheels, very good, shows absolutely no shrinkage.

No. 37. Heater top network, requiring fluidity but no strength.

No. 37*a*. Gray part of above.

No. 37*b*. White, honeycombed part of above. Probably bad mixing and got chilled suddenly.

STRENGTH OF CAST IRON.

Rankine gives the following figures:

Various qualities, T. S......	13,400 to	29,000, average	16,500
Compressive strength......	82,000 to	145,000, "	112,000
Modulus of elasticity.......	14,000,000 to	22,900,000, "	17,000,000

Specific Gravity and Strength. (Major Wade, 1856.)

Third-class guns: Sp. Gr. 7.087, T. S. 20,148. Another lot: least Sp. Gr. 7.163, T. S. 22,402.

Second-class guns: Sp. Gr. 7.154, T. S. 24,767. Another lot: mean Sp. Gr. 7.302, T. S. 27,232.

First class guns: Sp. Gr. 7.204, T. S. 28,805. Another lot: greatest Sp. Gr. 7.402, T. S. 31,027.

Strength of Charcoal Pig Iron.—Pig iron made from Salisbury ores, in furnaces at Wassaic and Millerton, N. Y., has shown over 40,000 lbs. T. S. per square inch, one sample giving 42,281 lbs. Muirkirk, Md., iron tested at the Washington Navy Yard showed: average for No. 2 iron, 21,601 lbs.; No. 3, 23,959 lbs.; No. 4, 41,329 lbs.; average density of No. 4, 7.336 (J. C. I. W., v. p. 44.)

Nos. 3 and 4 charcoal pig iron from Chapinville, Conn., showed a tensile strength per square inch of from 34,761 lbs. to 41,882 lbs. Charcoal pig iron from Shelby, Ala. (tests made in August, 1891), showed a strength of 34,800 lbs. for No. 3; No. 4, 39,675 lbs.; No. 5, 46,450 lbs.; and a mixture of equal parts of Nos. 2, 3, 4, and 5, 41,470 lbs. (*Bull. I. & S. A.*)

Variation of Density and Tenacity of Gun-irons.—An increase of density invariably follows the rapid cooling of cast iron, and as a general rule the tenacity is increased by the same means. The tenacity generally increases quite uniformly with the density, until the latter ascends to some given point; after which an increased density is accompanied by a diminished tenacity.

The turning-point of density at which the best qualities of gun-iron attain their maximum tenacity appears to be about 7.30. At this point of density, or near it, whether in proof-bars or gun-heads, the tenacity is greatest.

As the density of iron is increased its liquidity when melted is diminished. This causes it to congeal quickly, and to form cavities in the interior of the casting. (Pamphlet of Builders' Iron Foundry, 1893.)

Specifications for Cast Iron for the World's Fair Buildings, 1892.—Except where chilled iron is specified, all castings shall be of tough gray iron, free from injurious cold-shuts or blow-holes, true to pattern, and of a workmanlike finish. Sample pieces 1 in. square, cast from the same heat of metal in sand moulds, shall be capable of sustaining on a clear span of 4 feet 6 inches a central load of 500 lbs. when tested in the rough bar.

Specifications for Tests of Cast Iron in 12" B. L. Mortars. (Pamphlet of Builders Iron Foundry, 1893.)—*Charcoal Gun Iron.*—The tensile strength of the metal must average at each end at least 30,000 lbs. per square inch; no specimen to be over 37,000 lbs. per square inch; but one specimen from each end may be as low as 28,000 lbs. per square inch. The long extension specimens will not be considered in making up these averages, but must show a good elongation and an ultimate strength, for each specimen, of not less than 24.000 lbs. The density of the metal must be such as to indicate that the metal has been sufficiently refined, but not carried so high as to impair the other qualities.

Specifications for Grading Pig Iron for Car Wheels by Chill Tests made at the Furnace. (Penna. R. R. Specifications, 1883.)—The chill cup is to be filled, *even full*, at about the middle of every cast from the furnace. The test-piece so made will be 7½ inches long, 3½ inches wide, and 1¾ inches thick, and is to be broken across the centre when entirely cold. The depth of chill will be shown on the bottom of the test-piece, and is to be measured by the clean white portion to the point where gray specks begin to show in the white. The grades are to be by eighths of an inch, viz., ⅛, ¼, ⅜, ½, ⅝, ¾, ⅞, etc., until the iron is mottled; the lowest grade being ⅛ of an inch in depth of chill. The pigs of each cast are to be marked with the depth of chill shown by its test-piece, and each grade is to be kept by itself at the furnace and in forwarding.

Mixture of Cast Iron with Steel.—Car wheels are sometimes made from a mixture of charcoal iron, anthracite iron, and Bessemer steel. The following shows the tensile strength of a number of tests of wheel mixtures, the average tensile strength of the charcoal iron used being 22,000 lbs.:

	lbs. per sq. in.
Charcoal iron with 2½% steel	22,467
" " " 3¾% steel	26,733
" " " 6¼% steel and 6¼% anthracite	24,400
" " " 7½% steel and 7½% anthracite	28,150
" " " 2½% steel, 2½% wro't iron, and 6¼% anth.	25,550
" " " 5 % steel, 5% wro't iron, and 10% anth.	26,500

(*Jour. C. I. W.*, iii. p. 184.)

Cast Iron Partially Bessemerized.—Car wheels made of partially Bessemerized iron (blown in a Bessemer converter for 8½ minutes), chilled in a chill test mould over an inch deep, just as a test of cold blast charcoal iron for car wheels would chill. Car wheels made of this blown iron have run 250,000 miles. (*Jour. C. I. W.*, vi. p. 77.)

Bad Cast Iron.—On October 15, 1891, the cast-iron fly-wheel of a large pair of Corliss engines belonging to the Amoskeag Mfg. Co., of Manchester, N. H., exploded from centrifugal force. The fly-wheel was 30 feet diameter and 110 inches face, with one set of 12 arms, and weighed 116,000 lbs. After the accident, the rim castings, as well as the ends of the arms, were found to be full of flaws, caused chiefly by the drawing and shrinking of the metal. Specimens of the metal were tested for tensile strength, and varied from 15,000 lbs. per square inch in sound pieces to 1000 lbs. in spongy ones. None of these flaws showed on the surface, and a rigid examination of the parts before they were erected failed to give any cause to suspect their true nature. Experiments were carried on for some time after the accident in the Amoskeag Company's foundry in attempting to duplicate the flaws, but with no success in approaching the badness of these castings.

MALLEABLE CAST IRON.

Malleableized cast iron, or malleable iron castings, are castings made of ordinary cast iron which have been subjected to a process of decarbonization, which results in the production of a crude wrought iron. Handles, latches, and other similar articles, cheap harness mountings, plowshares, iron handles for tools, wheels, and pinions, and many small parts of machinery, are made of malleable cast iron. For such pieces charcoal cast iron of the best quality (or other iron of similar chemical composition), should be selected. Coke irons low in silicon and sulphur have been used in place of charcoal irons. The castings are made in the usual way, and are then imbedded in oxide of iron, in the form, usually, of hematite ore, or in peroxide of manganese, and exposed to a full red-heat for a sufficient length of time, to insure the nearly complete removal of the carbon. This decarbonization is conducted in cast-iron boxes, in which the articles, if small, are packed in alternate layers with the decarbonizing material. The largest pieces require the longest time. The fire is quickly raised to the maximum temperature, but at the close of the process the furnace is cooled very slowly. The operation requires from three to five days with ordinary small castings, and may take two weeks for large pieces.

Rules for Use of Malleable Castings, by Committee of Master Carbuilders' Ass'n, 1890.

1. Never run abruptly from a heavy to a light section.
2. As the strength of malleable cast iron lies in the skin, expose as much surface as possible. A star-shaped section is the strongest possible from which a casting can be made. For brackets use a number of thin ribs instead of one thick one.
3. Avoid all round sections; practice has demonstrated this to be the weakest form. Avoid sharp angles.
4. Shrinkage generally in castings will be 3/16 in. per foot.

Strength of Malleable Cast Iron.—Experiments on the strength of malleable cast iron, made in 1891 by a committee of the Master Car-builders' Association. The strength of this metal varies with the thickness, as the following results on specimens from ¼ in. to 1⅛ in. in thickness show:

Dimensions.		Tensile Strength.	Elongation.	Elastic Limit.
in.	in.	lb. per sq. in.	per cent in 4 in.	lb. per sq. in.
1.52 by	.25	34,700	2	21,100
1.52 "	.39	33,700	2	15,200
1.53 "	.5	32,800	2	17,000
1.53 "	.64	32,100	2	19,400
2. "	.78	25,100	1½	15,400
1.54 "	.88	33,600	1½	19,300
1.06 "	1.02	30,600	1	17,600
1.28 "	1.3	27,400	1	
1.52 "	1.54	28,200	1½	

The low ductility of the metal is worthy of notice. The committee gives the following table of the comparative tensile resistance and ductility of malleable cast iron, as compared with other materials:

	Ultimate Strength, lb. per sq. in	Comparative Strength; Cast Iron = 1.	Elongation Per Cent in 4 in.	Comparative Ductility; Malleable Cast Iron = 1.
Cast iron	20,000	1	0.35	0.17
Malleable cast iron.	32,000	1.6	2.00	1
Wrought iron	50,000	2.5	20.00	10
Steel castings	60,000	3	10.00	5

Another series of tests, reported to the Association in 1892, gave the following:

Thickness.	Width.	Area.	Elastic Limit.	Ultimate Strength.	Elongation in 8 in.
in.	in.	sq. in.	lb. per sq.	lb. per sq. in.	per cent.
.271	2.81	.7615	23,520	32,620	1.5
.293	2.78	.8145	22,650	28,160	.6
.39	2.82	1.098	20,595	32,060	1.5
.41	2.79	1.144	20,230	28,850	1.0
.529	2.76	1.46	19,520	27,875	1.1
.661	2.81	1.857	18,840	25,700	.7
.8	2.76	2.208	18,390	25,120	1.1
1.025	2.82	2.890	18,220	28,720	1.5
1.117	2.81	3.138	17,050	25,510	1.3
1.021	2.82	2.879	18,410	26,950	1.3

WROUGHT IRON.

Influence of Chemical Composition on the Properties of Wrought Iron. (Beardslee on Wrought Iron and Chain Cables. Abridgement by W. Kent. Wiley & Sons, 1879.)—A series of 2000 tests of specimens from 14 brands of wrought iron, most of them of high repute, was made in 1877 by Capt. L. A. Beardslee, U.S.N., of the United States Testing Board. Forty-two chemical analyses were made of these irons, with a view to determine what influence the chemical composition had upon the strength, ductility, and welding power. From the report of these tests by A. L. Holley the following figures are taken :

Brand.	Average Tensile Strength.	Chemical Composition.					
		S.	P.	Si.	C.	Mn.	Slag.
L	66,598	trace	0.065	0.080	0.212	0.005	0.192
			0.084	0.105	0.512	0.029	0.452
P	54,363	0.009	0.250	0.182	0.083	0.083	0.848
		0.001	0.095	0.028	0.066	0.009	1.214
B	52,764	0.008	0.231	0.156	0.015	0.017	....
J	51,754	0.003	0.140	0.182	0.027	trace	0.678
		0.005	0.291	0.321	0.051	0.053	1.724
O	51,134	0.004	0.067	0.065	0.045	0.007	1.168
		0.005	0.078	0.073	0.042	0.005	0.974
C	50,765	0.007	0.169	0.154	0.042	0.021	....

Where two analyses are given they are the extremes of two or more analyses of the brand. Where one is given it is the only analysis. Brand L should be classed as a puddled steel.

ORDER OF QUALITIES GRADED FROM No. 1 TO No. 19.

Brand.	Tensile Strength.	Reduction of Area.	Elongation.	Welding Power.
L	1	18	19	most imperfect.
P	6	6	3	badly.
B	12	16	15	best.
J	16	19	18	rather badly.
O	18	1	4	very good.
C	19	12	16	——

The reduction of area varied from 54.2 to 25.9 per cent, and the elongation from 29.9 to 8.3 per cent.

Brand O, the purest iron of the series, ranked No. 18 in tensile strength, but was one of the most ductile; brand B, (quite impure, was below the average both in strength and ductility, but was the best in welding power; P, also quite impure, was one of the best in every respect except welding, while L, the highest in strength, was not the most pure, it had the least ductility, and its welding power was most imperfect. The evidence of the influence of chemical composition upon quality, therefore, is quite contradictory and confusing. The irons differing remarkably in their mechanical properties, it was found that a much more marked influence upon their qualities was caused by different treatment in rolling than by differences in composition.

In regard to slag Mr. Holley says: "It appears that the smallest and most worked iron often has the most slag. It is hence reasonable to conclude that an iron may be dirty and yet thoroughly condensed."

In his summary of "What is learned from chemical analysis," he says: "So far, it may appear that little of use to the makers or users of wrought iron has been learned. . . . The character of steel can be surely predicated on the analyses of the materials; that of wrought iron is altered by subtle and unobserved causes."

Influence of Reduction in Rolling from Pile to Bar on the Strength of Wrought Iron.—The tensile strength of the irons used in Beardslee's tests ranged from 46,000 to 62,700 lbs. per sq. in., brand L, which was really a steel, not being considered. Some specimens of L gave figures as high as 70,000 lbs. The amount of reduction of sectional area in rolling the bars has a notable influence on the strength and elastic limit; the greater the reduction from pile to bar the higher the strength.

The following are a few figures from tests of one of the brands:

Size of bar, in. diam.	4	3	2	1	½	¼
Area of pile, sq. in.:	80	80	72	25	9	3
Bar per cent of pile:	15.7	8.83	4.36	3.14	2.17	1.6
Tensile strength, lb.:	46,322	47,761	48,280	51,128	52,275	59,585
Elastic limit, lb.:	23,430	26,400	31,892	36,467	39,126	——

Specifications for Wrought Iron (F. H. Lewis, Engineers' Club of Philadelphia, 1891).—1. All wrought iron must be tough, ductile, fibrous, and of uniform quality for each class, straight, smooth, free from cinder-pockets, flaws, buckles, blisters, and injurious cracks along the edges, and must have a workmanlike finish. No specific process or provision of manufacture will be demanded, provided the material fulfils the requirements of these specifications.

2. The tensile strength, limit of elasticity, and ductility shall be determined from a standard test-piece not less than ¼ inch thick, cut from the full-sized bar, and planed or turned parallel. The area of cross-section shall not be less than ½ square inch. The elongation shall be measured after breaking on an original length of 8 inches.

3. The tests shall show not less than the following results:

	Ultimate Strength, lbs. per sq. inch.	Limit of Elasticity, lbs. per sq. inch.	Elongation in 8 inches, per cent.
For bar iron in tension............	50,000	26,000	18
For shape iron....................	48,000	26,000	15
For plates under 36 in. wide.......	48,000	26,000	12
For plates over 36 in. wide........	46,000	25,000	10

4. When full-sized tension members are tested to prove the strength of their connections, a reduction in their ultimate strength of (500 × width of bar) pounds per square inch will be allowed.

5. All iron shall bend, cold, 180 degrees around a curve whose diameter is twice the thickness of piece for bar iron, and three times the thickness for plates and shapes.

6. Iron which is to be worked hot in the manufacture must be capable of bending sharply to a right angle at a working heat without sign of fracture.

7. Specimens of tensile iron upon being nicked on one side and bent shall show a fracture nearly all fibrous.

8. All rivet iron must be tough and soft, and be capable of bending cold until the sides are in close contact without sign of fracture on the convex side of the curve.

Pennsylvania Railroad Specifications for Merchant Bar Iron or Steel.—Miscellaneous merchant bar iron or steel for which no special specifications defining shapes and uses are issued, should have a tensile strength of 50,000 to 55,000 lbs. per square inch and an elongation of 20% in a section originally 2 inches long.

No iron or steel will be accepted under this specification if tensile strength falls below 48,000 lbs. or goes above 60,000 lbs. per square inch, nor if elongation is less than 15% in 2 inches, nor if it shows a granular fracture covering more than 50% of the fractured surface, nor if it shows any difficulty in welding.

In preparing test-pieces from round or rectangular bars, they will be turned or shaped so that the tested sections may be the central portion of the bar, in all sizes up to 1¾ inches in any diametrical or side measurement. In larger sizes test-pieces will be made to fall about half-way from centre to circumference.

Bars of iron ½ in. thick or less, or tortured forms of iron, such as angle, tee or channel bars, will be accepted if tensile strength is above 45,000 lbs. and elongation above 12%; but the testing of such sizes and sections is optional.

Specifications for Wrought Iron for the World's Fair Buildings. (*Eng'g News*, March 26, 1892.)—All iron to be used in the tensile members of open trusses, laterals, pins and bolts, except plate iron over 8 inches wide, and shaped iron, must show by the standard test-pieces a tensile strength in lbs. per square inch of:

$$52{,}000 - \frac{7{,}000 \times \text{area of original bar in sq. in.}}{\text{circumference of original bar in inches}},$$

with an elastic limit not less than half the strength given by this formula, and an elongation of 20% in 8 in.

Plate iron 24 inches wide and under, and more than 8 inches wide, must show by the standard test-pieces a tensile strength of 48,000 lbs. per sq. in. with an elastic limit not less than 26,000 lbs. per square inch, and an elongation of not less than 12%. All plates over 24 inches in width must have a tensile strength not less than 46,000 lbs. with an elastic limit not less than 26,000 lbs. per square inch. Plates from 24 inches to 36 inches in width must have an elongation of not less than 10%; those from 36 inches to 48 inches in width, 8%; over 48 inches in width, 5%.

All shaped iron, flanges of beams and channels, and other iron not hereinbefore specified, must show by the standard test-pieces a tensile strength in lbs. per square inch of:

$$50{,}000 - \frac{7{,}000 \times \text{area of original bar}}{\text{circumference of original bar}},$$

with an elastic limit of not less than half the strength given by this formula, and an elongation of 15% for bars 5/8 inch and less in thickness, and of 12% for bars of greater thickness. For webs of beams and channels, specifications for plates will apply.

All rivet iron must be tough and soft, and pieces of the full diameter of the rivet must be capable of bending cold, until the sides are in close contact, without sign of fracture on the convex side of the curve.

Stay-bolt Iron.—Mr. Vauclain, of the Baldwin Locomotive Works, at a meeting of the American Railway Master Mechanics' Association, in 1892, says: Many advocate the softest iron in the market as the best for stay-bolts. He believed in an iron as hard as was consistent with heading the bolt nicely. The higher the tensile strength of the iron, the more vibrations it will stand, for it is not so easily strained beyond the yield-point. The Baldwin specifications for stay-bolt iron call for a tensile strength of 50,000 to 52,000 lbs. per square inch, the upper figure being preferred, and the lower being insisted upon as the minimum.

FORMULÆ FOR UNIT STRAINS FOR IRON AND STEEL IN STRUCTURES.

(F. H. Lewis, Engineers' Club of Philadelphia, 1891.)

The following formulæ for unit strains per square inch of net sectional area shall be used in determining the allowable working stress in each member of the structure. (For definitions of soft and medium steel see Specifications for Steel.)

Tension Members.

	Wrought Iron.	Soft Steel.	Medium Steel.
Floor-beam hangers or suspenders, forged bars	Will not be used	Will not be used	7000
Counter-ties......... .	6000	" " "	7000
Suspenders, hangers and counters, riveted members, net section	5000	5500	7000
Solid rolled beams.....	8000	8000	Will not be used
Riveted truss members and tension flanges of girders, net section	$7000\left(1+\frac{\text{min.}}{\text{max.}}\right)$	8% greater than iron	$9000\left(1+\frac{\text{min.}}{\text{max.}}\right)$
Forged eyebars.......	Will not be used	Will not be used	$9000\left(1+\frac{\text{min}}{\text{max.}}\right)$
Lateral or cross section rods.	15,000	16,000	(For eyebars only, 17,000)

Shearing.

	Wrought Iron.	Soft Steel.	Medium Steel.
On pins and shop rivets	6000	6600	7200
On field rivets.........	4800	5200	Will not be used
In webs of girders.....	Will not be used	5000	6000

Bearing.

	Wrought Iron.	Soft Steel.	Medium Steel.
On projected semi-intrados of main-pin holes	12,000	13,200	14,500
On projected semi-intrados of rivet-holes*	12,000	13,200	14,500
On lateral pins.... ...	15,000	16,500	18,000
Of bed-plates on masonry...	250 lbs. per sq. in.		

* Excepting that in pin-connected members taking alternate stresses, the bearing stress must not exceed 9000 lbs. for iron or steel.

Bending.

On extreme fibres of pins when centres of bearings are considered as points of application of strains:

Wrought Iron, 15,000. Soft Steel, 16,000. Medium Steel, 17,000.

Compression Members.

	Wrought Iron.	Soft Steel.	Medium Steel.
Chord sections:			
Flat ends.................	$7000\left(1+\frac{\text{min.}}{\text{max.}}\right)-30\frac{l}{r}$	10% greater than iron	20% greater than iron
One flat and one pin end..	$7000\left(1+\frac{\text{min.}}{\text{max.}}\right)-35\frac{l}{r}$		
Chords with pin ends and all end-posts	$7000\left(1+\frac{\text{min.}}{\text{max.}}\right)-40\frac{l}{r}$		
All trestle-posts...........	$7000\left(1+\frac{\text{min.}}{\text{max.}}\right)-35\frac{l}{r}$		
Intermediate posts........	$7500-40\frac{l}{r}$		
Lateral struts, and compression in collision struts, stiff suspenders and stiff chords..........	$10{,}500-50\frac{l}{r}$		

In which formulæ l = length of compression member in inches, and r = least radius of gyration of member in inches. No compression member shall have a length exceeding 45 times its least width, and no post should be used in which $l \div r$ exceeds 125.

Members Subject to Alternate Tension and Compression.

	Wrought Iron.	Soft Steel.	Medium Steel.
For compression only...	Use the formulæ above	8% greater than iron	20% greater than iron
For the greatest stress..	$7000\left(1-\frac{\text{max. lesser}}{2\text{ max. greater}}\right)$		

Use the formula giving the greatest area of section.

The compression flanges of beams and plate girders shall have the same cross-section as the tension flanges.

W. H. Burr, discussing the formulæ proposed by Mr. Lewis, says: "Taking the results of experiments as a whole, I am constrained to believe that they indicate at least 15% increase of resistance for soft-steel columns over those of wrought iron, with from 20% to 25% for medium steel, rather than 10% and 20% respectively.

"The high capacity of soft steel for enduring torture fits it eminently for alternate and combined stresses, and for that reason I would give it 15% increase over iron, with about 22% for medium steel.

"Shearing tests on steel seem to show that 15% and 22% increases, for the two grades respectively, are amply justified.

"I should not hesitate to assign 15% and 22% increases over values for iron for bearing and bending of soft and medium steel as being within the safe limits of experience. Provision should also be made for increasing pin-shearing, bending and bearing stresses for increasing ratios of fixed to moving loads."

Maximum Permissible Stresses in Structural Materials used in Buildings. (Building Ordinances of the City of Chicago, 1893.) Cast iron, crushing stress: For plates, 15,000 lbs. per square inch; for lintels, brackets, or corbels, compression 13,500 lbs. per square inch, and tension 3000 lbs. per square inch. For girders, beams, corbels, brackets, and trusses, 16,000 lbs. per square inch for steel and 12,000 lbs. for iron.

For plate girders:

$$\text{Flange area} = \frac{\text{maximum bending moment in ft.-lbs.}}{CD}.$$

D = distance between centre of gravity of flanges in feet.

$$C = \begin{cases} 13{,}500 \text{ for steel.} \\ 10{,}000 \text{ for iron.} \end{cases}$$

$$\text{Web area} = \frac{\text{maximum shear}}{C}. \quad C = \begin{cases} 10{,}000 \text{ for steel,} \\ 6{,}000 \text{ for iron.} \end{cases}$$

For rivets in single shear per square inch of rivet area:

	Steel.	Iron.
If shop-driven	9000 lbs.	7500 lbs.
If field-driven	7500 "	6000 "

For timber girders:

$$S = \frac{cbd^2}{l}.$$

b = breadth of beam in inches.
d = depth of beam in inches.
l = length of beam in feet.

$$c = \begin{cases} 160 \text{ for long-leaf yellow pine,} \\ 120 \text{ for oak,} \\ 100 \text{ for white or Norway pine.} \end{cases}$$

Proportioning of Materials in the Memphis Bridge (Geo. S. Morison, *Trans. A. S. C. E.*, 1893).—The entire superstructure of the Memphis bridge is of steel and it was all worked as steel, the rivet-holes being drilled in all principal members and punched and reamed in the lighter members.

The tension members were proportioned on the basis of allowing the dead load to produce a strain of 20,000 lbs. per square inch, and the live load a strain of 10,000 lbs. per square inch. In the case of the central span, where the dead load was twice the live load, this corresponded to 15,000 lbs. total strain per square inch, this being the greatest tensile strain.

The compression members were proportioned on a somewhat arbitrary basis. No distinction was made between live and dead loads. A maximum strain of 14,000 lbs. per square inch was allowed on the chords and other large compression members where the length did not exceed 16 times the least transverse dimension, this strain being reduced 750 lbs. for each additional unit of length. In long compression members the maximum length was limited to 30 times the least transverse dimension, and the strains limited to 6,000 lbs. per square inch, this amount being increased by 200 lbs. for each unit by which the length is decreased.

Wherever reversals of strains occur the member was proportioned to resist the sum of compression and tension on whichever basis (tension or compression) there would be the greatest strain per square inch; and, in addition, the net section was proportioned to resist the maximum tension, and the gross section to resist the maximum compression.

The floor beams and girders were calculated on the strain being limited to 10,000 lbs. per square inch in extreme fibres. Rivet-holes in cover-plates and flanges were deducted.

The rivets of steel in drilled or reamed holes were proportioned on the basis of a bearing strain of 15,000 lbs. per square inch and a shearing strain of 7500 lbs. per square inch, and special pains were taken to get the double shear in as many rivets as possible. This was the requirement for shop rivets. In the case of field rivets, the number was increased one-half.

The pins were proportioned on the basis of a bearing strain of 18,000 lbs. per square inch and a bending strain of 20,000 lbs. per square inch in extreme fibre, the diameters of the pins being never made more than one inch less than the width of the largest eye-bar attaching to them.

The weight on the rollers of the expansion joint on Pier II is 40,000 lbs. per linear foot of roller, or 3,333 lbs. per linear inch, the rollers being 15 ins. in diameter.

As the sections of the superstructure were unusually heavy, and the strains from dead load greatly in excess of those from moving load, it was thought best to use a slightly higher steel than is now generally used for lighter structures, and to work this steel without punching, all holes being drilled. A somewhat softer steel was used in the floor-system and other lighter parts.

The principal requirements which were to be obtained as the results of tests on samples cut from finished material were as follows:

	Max. Ultimate Strength, lbs. per sq. inch.	Min. Ultimate Strength, lbs. per sq. inch.	Min. Elastic Limit, lbs. per sq. in.	Min. percentage of Elongation in 8 inches.	Min. Percentage of Reduction at Fracture
High-grade steel.	78,500	69,000	40,000	18	38
Eye-bar steel....	75,000	66,000	38,000	20	40
Medium steel....	72,500	64.000	37,000	22	44
Soft steel........	68.000	55,000	30,000	28	50

TENACITY OF METALS AT VARIOUS TEMPERATURES.

The British Admiralty made a series of experiments to ascertain what loss of strength and ductility takes place in gun-metal compositions when raised to high temperatures. It was found that all the varieties of gun-metal suffer a gradual but not serious loss of strength and ductility up to a certain temperature, at which, within a few degrees, a great change takes place, the strength falls to about one half the original, and the ductility is wholly gone. At temperatures above this point, up to 500, there is little, if any, further loss of strength; the temperature at which this great change and loss of strength takes place, although uniform in the specimens cast from the same pot, varies about 100° in the same composition cast at different temperatures, or with some varying conditions in the foundry process. The temperature at which the change took place in No. 1 series was ascertained to be about 370°, and in that of No. 2, at a little over 250°. Whatever may be the cause of this important difference in the same composition, the fact stated may be taken as certain. Rolled Muntz metal and copper are satisfactory up to 500°, and may be used as securing-bolts with safety. Wrought iron, Yorkshire and remanufactured, increase in strength up to 500°, but lose slightly in ductility up to 200°, where an increase begins and continues up to 500°, where it is still less than at the ordinary temperature of the atmosphere. The strength of Landore steel is not affected by temperature up to 500°, but its ductility is reduced more than one half. (*Iron*, Oct. 6, 1877.)

Tensile Strength of Iron and Steel at High Temperatures.—James E. Howard's tests (*Iron Age*, April 10, 1890) show that the tensile strength of steel diminishes as the temperature increases from 0° until a minimum is reached between 200° and 300° F., the total decrease being about 4000 lbs. per square inch in the softer steels, and from 6000 to 8000 lbs. in steels of over 80,000 lbs. tensile strength. From this minimum point the strength increases up to a temperature of 400° to 650° F., the maximum being reached earlier in the harder steels, the increase amounting to from 10,000 to 20,000 lbs. per square inch above the minimum strength at from 200°

to 300°. From this maximum, the strength of all the steel decreases steadily at a rate approximating 10,000 lbs. decrease per 100° increase of temperature. A strength of 20,000 lbs. per square inch is still shown by .10 C. steel at about 1000° F., and by .60 to 1.00 C. steel at about 1600° F.

The strength of wrought iron increases with temperature from 0° up to a maximum at from 400 to 600° F., the increase being from 8000 to 10,000 lbs. per square inch, and then decreases steadily till a strength of only 6000 lbs. per square inch is shown at 1500° F.

Cast iron appears to maintain its strength, with a tendency to increase, until 900° is reached, beyond which temperature the strength gradually diminishes. Under the highest temperatures, 1500° to 1600° F., numerous cracks on the cylindrical surface of the specimen were developed prior to rupture. It is remarkable that cast iron, so much inferior in strength to the steels at atmospheric temperature, under the highest temperatures has nearly the same strength the high-temper steels then have.

Strength of Iron and Steel Boiler-plate at High Temperatures. (Chas. Huston, *Jour. F. I.*, 1877.)

AVERAGE OF THREE TESTS OF EACH.

Temperature F.	68°	575°	925°
Charcoal iron plate, tensile strength, lbs.	55,366	63,080	65,343
" " " contr. of area %	26	23	21
Soft open-hearth steel, tensile strength, lbs.	54,600	66,083	64,350
" " " contr. %	47	38	33
" Crucible steel, tensile strength, lbs.	64,000	69,266	68,600
" " " contr. %	36	30	21

Strength of Wrought Iron and Steel at High Temperatures. (*Jour. F. I.*, cxii., 1881, p. 241.) Kollmann's experiments at Oberhausen included tests of the tensile strength of iron and steel at temperatures ranging between 70° and 2000° F. Three kinds of metal were tested, viz., fibrous iron having an ultimate tensile strength of 52,464 lbs., an elastic strength of 38,280 lbs., and an elongation of 17.5%; fine-grained iron having for the same elements values of 56,892 lbs., 39,113 lbs., and 20%; and Bessemer steel having values of 84,826 lbs., 55,029 lbs., and 14.5%. The mean ultimate tensile strength of each material expressed in per cent of that at ordinary atmospheric temperature is given in the following table, the fifth column of which exhibits, for purposes of comparison, the results of experiments carried on by a committee of the Franklin Institute in the years 1832-36.

Temperature Degrees F.	Fibrous Wrought Iron, p. c.	Fine-grained Iron, per cent.	Bessemer Steel, per cent.	Franklin Institute, per cent.
0	100.0	100.0	100.0	96.0
100	100.0	100.0	100.0	102.0
200	100.0	100.0	100.0	105.0
300	97.0	100.0	100.0	106.0
400	95.5	100.0	100.0	106.0
500	92.5	98.5	98.5	104.0
600	88.5	95.5	92.0	99.5
700	81.5	90.0	68.0	92.5
800	67.5	77.5	44.0	75.5
900	44.5	51.5	36.5	53.5
1000	26.0	36.0	31.0	36.0
1100	20.0	30.5	26.5	.....
1200	18.0	28.0	22.0	.....
1300	16.5	23.0	18.0	.....
1400	13.5	19.0	15.0	.....
1500	10.0	15.5	12.0	.....
1600	7.0	12.5	10.0	.....
1700	5.5	10.5	8.5	.....
1800	4.5	8.5	7.5	.....
1900	3.5	7.0	6.5	.....
2000	3.5	5.0	5.0	.....

The Effect of Cold on the Strength of Iron and Steel.—The following conclusions were arrived at by Mr. Styffe in 1865:

(1) That the absolute strength of iron and steel is not diminished by cold, but that even at the lowest temperature which ever occurs in Sweden it is at least as great as at the ordinary temperature (about 60° F.).

(2) That neither in steel nor in iron is the extensibility less in severe cold than at the ordinary temperature.

(3) That the limit of elasticity in both steel and iron lies higher in severe cold.

(4) That the modulus of elasticity in both steel and iron is increased on reduction of temperature, and diminished on elevation of temperature; but that these variations never exceed 0.05 % for a change of temperature of 1.8° F., and therefore such variations, at least for ordinary purposes, are of no special importance.

Mr. C. P. Sandberg made in 1867 a number of tests of iron rails at various temperatures by means of a falling weight, since he was of opinion that, although Mr. Styffe's conclusions were perfectly correct as regards tensile strength, they might not apply to the resistance of iron to impact at low temperatures. Mr. Sandberg convinced himself that "the breaking strain" of iron, such as was usually employed for rails, "as tested by sudden blows or shocks, is considerably influenced by cold; such iron exhibiting at 10° F. only from one third to one fourth of the strength which it possesses at 84° F." Mr. J. J. Webster (Inst. C. E., 1880) gives reasons for doubting the accuracy of Mr. Sandberg's deductions, since the tests at the lower temperature were nearly all made with 21-ft. lengths of rail, while those at the higher temperatures were made with short lengths, the supports in every case being the same distance apart.

W. H. Barlow (Proc. Inst. C. E.) made experiments on bars of wrought iron, cast iron, malleable cast iron, Bessemer steel, and tool steel. The bars were tested with tensile and transverse strains, and also by impact; one half of them at a temperature of 50° F., and the other half at 5° F. The lower temperature was obtained by placing the bars in a freezing mixture, care being taken to keep the bars covered with it during the whole time of the experiments.

The results of the experiments were summarized as follows:

1. When bars of wrought iron or steel were submitted to a tensile strain and broken, their strength was not affected by severe cold (5° F.), but their ductility was increased about 1% in iron and 3% in steel.

2. When bars of cast iron were submitted to a transverse strain at a low temperature, their strength was diminished about 3% and their flexibility about 16%.

3. When bars of wrought iron, malleable cast iron, steel, and ordinary cast iron were subjected to impact at a temperature of 5° F., the force required to break them, and the extent of their flexibility, were reduced as follows, viz.:

	Reduction of Force of Impact, per cent.	Reduction of Flexibility, per cent.
Wrought iron, about	3	18
Steel (best cast tool), about	3½	17
Malleable cast iron, about	4½	15
Cast iron, about	21	not taken

The experience of railways in Russia, Canada, and other countries where the winter is severe is that the breakages of rails and tires are far more numerous in the cold weather than in the summer. On this account a softer class of steel is employed in Russia for rails than is usual in more temperate climates.

The evidence extant in relation to this matter leaves no doubt that the capability of wrought iron or steel to resist impact is reduced by cold. On the other hand, its static strength is not impaired by low temperatures.

Effect of Low Temperatures on Strength of Railroad Axles. (Thos. Andrews, Proc. Inst. C. E., 1891.)—Axles 6 ft. 6 in. long between centres of journals, total length 7 ft. 8¼ in., diameter at middle 4¼ in., at wheel-sets 5⅛ in., journals 3¾ × 7 in. were tested by impact at temperatures of 0° and 100° F. Between the blows each axle was half turned over, and was also replaced for 15 minutes in the water-bath.

The mean force of concussion resulting from each impact was ascertained as follows:

Let h = height of free fall in feet, w = weight of test ball, $hw = W$ = "energy," or work in foot-tons, x = extent of deflections between bearings,

$$\text{then } F \text{ (mean force)} = \frac{W}{x} = \frac{hw}{x}.$$

The results of these experiments show that whereas at a temperature of 0° F. a total average mean force of 179 tons was sufficient to cause the breaking of the axles, at a temperature of 100° F. a total average mean force of 428 tons was requisite to produce fracture. In other words, the resistance to concussion of the axles at a temperature of 0° F. was only about 42% of what it was at a temperature of 100° F.

The average total deflection at a temperature of 0° F. was 6.48 in., as against 15.06 in. with the axles at 100° F. under the conditions stated; this represents an ultimate reduction of flexibility, under the test of impact, of about 57% for the cold axles at 0° F., compared with the warm axles at 100° F.

EXPANSION OF IRON AND STEEL BY HEAT.

James E. Howard, engineer in charge of the U. S. testing-machine at Watertown, Mass., gives the following results of tests made on bars 35 inches long (*Iron Age*, April 10, 1890):

Metal.	Marks.	Chemical composition.				Coefficient of Expansion.
		C.	Mn.	Si.	Fe by difference.	Per degree F. per unit of length.
Wrought iron	......	......	......	......	......	.0000067302
Steel	1a	.09	.11	......	99.80	.0000067561
"	2a	.20	.45	......	99.35	.0000066259
"	3a	.31	.57	......	99.12	.0000065149
"	4a	.37	.70	......	98.93	.0000066597
"	5a	.51	.58	.02	98.89	.0000066202
"	6a	.57	.93	.07	98.43	.0000063891
"	7a	.71	.58	.08	98.63	.0000064716
"	8a	.81	.56	.17	98.46	.0000062167
"	9a	.89	.57	.19	98.35	.0000062335
"	10a	.97	.80	.28	97.95	.0000061700
Cast (gun) iron	......	......	......	......	......	.0000059261
Drawn copper	......	......	......	......	......	.0000091286

DURABILITY OF IRON, CORROSION, ETC.

Durability of Cast Iron.—Frederick Graff, in an article on the Philadelphia water-supply, says that the first cast-iron pipe used there was laid in 1820. These pipes were made of charcoal iron, and were in constant use for 53 years. They were uncoated, and the inside was well filled with tubercles. In salt water good cast iron, even uncoated, will last for a century at least; but it often becomes soft enough to be cut by a knife, as is shown in iron cannon taken up from the bottom of harbors after long submersion. Close-grained, hard white metal lasts the longest in sea water.—*Eng'g News*, April 23, 1887, and March 26, 1892.

Tests of Iron after Forty Years' Service.—A square link 12 inches broad, 1 inch thick and about 12 feet long was taken from the Kieff bridge, then 40 years old, and tested in comparison with a similar link which had been preserved in the stock-house since the bridge was built. The following is the record of a mean of four longitudinal test-pieces, $1 \times 1\frac{1}{8} \times 8$ inches, taken from each link (*Stahl und Eisen*, 1890):

	Old Link taken from Bridge.	New Link from Store-house.
Tensile strength per square inch, tons	21.8	22.2
Elastic limit " "	11.1	11.9
Elongation, per cent	14.05	13.42
Contraction, per cent	17.35	18.75

Durability of Iron in Bridges. (G. Lindenthal, *Eng'g*, May 2, 1884, p. 139.)—The Old Monongahela suspension bridge in Pittsburgh, built in 1845, was taken down in 1882. The wires of the cables were frequently strained to half of their ultimate strength, yet on testing them after 37 years' use they showed a tensile strength of from 72,700 to 100,000 lbs. per square inch. The elastic limit was from 67,100 to 78,600 lbs. per square inch. Reduction at point of fracture, 35% to 75%. Their diameter was 0.13 inch.

A new ordinary telegraph wire of same gauge tested for comparison showed: T. S., of 100,000 lbs.; E. L., 81,550 lbs.; reduction, 57%. Iron rods used as stays or suspenders showed: T. S., 43,770 to 49,720 lbs. per square inch; E. L., 26,880 to 29,200. Mr. Lindenthal draws these conclusions from his tests:

"The above tests indicate that iron highly strained for a long number of years, but still within the elastic limit, and exposed to slight vibration, will not deteriorate in quality.

"That if subjected to only one kind of strain it will not change its texture, even if strained beyond its elastic limit, for many years. It will stretch and behave much as in a testing-machine during a long test.

"That iron will change its texture only when exposed to alternate severe straining, as in bending in different directions. If the bending is slight but very rapid, as in violent vibrations, the effect is the same."

Corrosion of Iron Bolts.—On bridges over the Thames in London, bolts exposed to the action of the atmosphere and rain-water were eaten away in 25 years from a diameter of 7/8 in. to 1/2 in., and from 5/8 in. diameter to 5/16 inch.

Wire ropes exposed to drip in colliery shafts are very liable to corrosion.

Corrosion of Iron and Steel.—Experiments made at the Riverside Iron Works, Wheeling, W. Va., on the comparative liability to rust of iron and soft Bessemer steel: A piece of iron plate and a similar piece of steel, both clean and bright, were placed in a mixture of yellow loam and sand, with which had been thoroughly incorporated some carbonate of soda, nitrate of soda, ammonium chloride, and chloride of magnesium. The earth as prepared was kept moist. At the end of 33 days the pieces of metal were taken out, cleaned, and weighed, when the iron was found to have lost 0.84% of its weight and the steel 0.72%. The pieces were replaced and after 28 days weighed again, when the iron was found to have lost 2.06% of its original weight and the steel 1.79%. (*Eng'g*, June 26, 1891.)

Corrosive Agents in the Atmosphere.—The experiments of F. Crace Calvert (*Chemical News*, March 3, 1871) show that carbonic acid, in the presence of moisture, is the agent which determines the oxidation of iron in the atmosphere. He subjected perfectly cleaned blades of iron and steel to the action of different gases for a period of four months, with results as follows:

Dry oxygen, dry carbonic acid, a mixture of both gases, dry and damp oxygen and ammonia: no oxidation. Damp oxygen: in three experiments one blade only was slightly oxidized.

Damp carbonic acid: slight appearance of a white precipitate upon the iron, found to be carbonate of iron. Damp carbonic acid and oxygen: oxidation very rapid. Iron immersed in water containing carbonic acid oxidized rapidly.

Iron immersed in distilled water deprived of its gases by boiling rusted the iron in spots that were found to contain impurities.

Galvanic Action is a most active agent of corrosion. It takes place when two metals, one electro-negative to the other, are placed in contact and exposed to dampness.

Sulphurous acid (the product of the combustion of the sulphur in coal) is an exceedingly active corrosive agent, especially when the exposed iron is coated with soot. This accounts for the rapid corrosion of iron in railway bridges exposed to the smoke from locomotives. (See account of experiments by the author on action of sulphurous acid in *Jour Frank Inst.*, June, 1875, p. 437.) An analysis of sooty iron rust from a railway bridge showed the presence of sulphurous, sulphuric, and carbonic acids, chlorine, and ammonia. Bloxam states that ammonia is formed from the nitrogen of the air during the process of rusting.

Corrosion in Steam-boilers.—Internal corrosion may be due either to the use of water containing free acid, or water containing sulphate or chloride of magnesium, which decompose when heated, liberating the acid, or to water containing air or carbonic acid in solution. External corrosion rarely takes place when a boiler is kept hot, but when cold it is apt to corrode rapidly in those portions where it adjoins the brickwork or where it may be covered by dust or ashes, or wherever dampness may lodge. (See Impurities of Water, p. 551, and Incrustation and Corrosion, p. 716.)

PRESERVATIVE COATINGS.

(The following notes have been furnished to the author by Prof. A. H. Sabin.)

Cement.—Iron-work is sometimes protected by bedding in concrete, in which case it is first cleaned and then washed with neat cement before being imbedded.

Asphaltum.—This is applied hot either by dipping (as water-pipe) or by pouring it on (as bridge floors). The asphalt should be slightly elastic when cold, with a high melting-point, not softening much at 100° F., applied at 300° to 400°; surface must be dry and should be hot; coating should be of considerable thickness.

Paint.—Composed of a vehicle or binder, usually linseed oil or some inferior substitute, or varnish (enamel paints); and a pigment which is a more or less inert solid in the form of powder, either mixed or ground together. The principal pigments are white lead (carbonate) and white zinc (oxide), red lead (peroxide), oxides of iron, hydrated and dehydrated, graphite, lamp-black, chrome yellow, ultramarine and Prussian blue, and various tinting colors. White lead has the greatest body or opacity of white pigments; three coats of it equal five of white zinc; zinc is more brilliant and permanent, but it is liable to peel, and it is customary to mix the two. These are the standard white paints for all uses and the basis of all light-colored paints. Anhydrous iron oxides are brown and purplish brown. hydrated iron oxides are yellowish red to reddish yellow, with more or less brown; most iron oxides are mixtures of both sorts. They also contain frequently manganese and clay. They are cheap, and are serviceable paints for wood, and are often used on iron, but for the latter use are falling into disrepute. Graphite used for painting iron contains from 10 to 90% foreign matter, usually silicates and iron oxides. It is very opaque, hence has great covering power, and may be applied in a very thin coat which should be avoided. It retards the drying of oil, hence the necessity of using dryers; these are lead and manganese compounds dissolved in oil and turpentine or benzine, and act as carries of oxygen; they are necessary in most paints, but should be used as little as possible. There are many grades of lamp-black; as a rule the cheaper sorts contain oily matter and are especially hard to dry; all lamp-black is slow to dry in oil. It is the principal black on wood, and is used some on iron, usually in combination with varnish or varnish-like compounds. It is very permanent on wood. A gallon of oil takes only a pound of lamp-black to make a paint, while the same amount of oil requires about 40 lbs. of red lead. On this account red-lead paint, which weighs about 30 lbs. per gallon, is the most expensive of all comon paints. It does not dry slowly like other oil paints, but combines with the oil to make a sort of cement; on this account it is used on the joints of steam-pipes, etc. To prevent the mixture of red lead and oil setting into a cake, and also to cheapen it, it is often adulterated with whiting or sometimes with white zinc, the proportion of adulterant being sometimes double the lead. Red lead has long had a high reputation as a paint for iron and steel and is still used very extensively; but of late years some of the new paints and varnish-like preparations have displaced it to some extent even on the most important work.

Varnishes.—These are made by melting fossil resin, to which is then added from half its weight to three times its weight of refined linseed oil, and the compound is thinned with turpentine; they usually contain a little dryer. They are chiefly used on wood, being more durable and more brilliant than oil, and are often used over paint to preserve it. Asphaltum is sometimes substituted in part or in whole for the fossil resin, and in this way are made varnishes which have been applied to iron and steel with good results. Asphaltum and animal and vegetable tar and pitch have also been simply dissolved in solvents, as benzine or carbon disulphide, and used for the same purpose.

All these preservative coatings are supposed to form impervious films, keeping out air and moisture; but in fact all are somewhat porous. On this account it is necessary to have a film of appreciable thickness, best formed by successive coats, so that the pores of one will be closed by the next. The pigment is used to give an agreeable color, to help fill the pores of the oil film, to make the paint harder so that it will resist abrasion, and to make a thicker film. In varnishes these results are sought to be attained by the resin which is dissolved in the oil. There is no sort of agreement among practical men as to which is the best coating for any particular case; this is probably because so much depends on the preparation of the surface and the care with which the coating is applied, and also because the conditions of exposure vary so greatly.

Methods of Application.—Too much care cannot be given to the preparation of the surface. If it is wood, it should be dry, and the surface of knots should be coated with some preparation which will keep the tarry matter in the wood from the coating. All old paint or varnish should be removed by burning and scraping. Metallic surfaces should be cleaned by wire brushes and scrapers, and if the permanence of the work is of much importance the scale and oxide should be completely removed by acid pickling or by the sand-blast or some equally efficient means. Pickling is usually done with a 10% solution of sulphuric acid; as the solution becomes exhausted it may be made more active by heating. All traces of acid must be removed by washing and the metal must be rapidly dried and painted before it becomes in the slightest degree oxidized. The sand-blast, which has been applied to large work recently and for many years to small work with good results, leaves the surface perfectly clean and dry: the paint must be applied immediately. Plenty of time should always be allowed, usually about a week, for each coat of paint to dry before the next coat is applied; less than two coats should never be used. Two will last three times as long as one coat. Benzine should not be an ingredient in coatings for iron-work, because its rapid evaporation lowers the temperature of the iron and may cause formation of dew on the surface adjacent to the paint which is immediately to be painted.

Cast-iron water-pipes are usually coated by dipping in a hot mixture of coal-tar and coal-tar pitch; riveted steel pipes by dipping in hot asphalt or by a japan enamel which is baked on at about 400° F. Ships' bottoms are usually coated with some sort of paint to prevent rusting, over which is spread, hot, a poisonous, slowly soluble compound, usually a copper soap, to prevent adhesion of marine growths.

Galvanized-iron and tin surfaces should be thoroughly cleaned with benzine and scrubbed before painting. When new they are covered with grease and chemicals used in coating the plates, and these must be removed or the paint will be destroyed.

Quantity of Paint for a Given Surface.—One gallon of paint will cover 250 to 350 sq. ft. as a first coat, depending on the character of the surface, and from 350 to 450 sq. ft. as a second coat.

Qualities of Paints.—*The Railroad and Engineering Journal*, vols. liv and lv, 1890 and 1891, has a series of articles on paint as applied to *wooden structures*, its chemical nature, application, adulteration, etc., by Dr. C. B. Dudley, chemist, and F. N. Pease, assistant chemist, of the Penna. R. R. They give the results of a long series of experiments on paint as applied to railway purposes.

Rustless Coatings for Iron and Steel.—Tinning, enamelling, lacquering, galvanizing, electro-chemical painting, and other preservative methods are discussed in two important papers by M. P. Wood, in Trans. A. S. M. E., vols. xv and xvi.

A Method of Producing an Inoxidizable Surface on iron and steel by means of electricity has been developed by M. A. de Meritens (*Engineering*). The article to be protected is placed in a bath of ordinary or distilled water, at a temperature of from 158° to 176° F., and an electric current is sent through. The water is decomposed into its elements, oxygen and hydrogen, and the oxygen is deposited on the metal, while the hydrogen appears at the other pole, which may either be the tank in which the operation is conducted or a plate of carbon or metal. The current has only sufficient electromotive force to overcome the resistance of the circuit and to decompose the water; for if it be stronger than this, the oxygen combines with the iron to produce a pulverulent oxide, which has no adherence. If the conditions are as they should be, it is only a few minutes after the oxygen appears at the metal before the darkening of the surface shows that the gas has united with the iron to form the magnetic oxide Fe_3O_4, which will resist the action of the air and protect the metal beneath it. After the action has continued an hour or two the coating is sufficiently solid to resist the scratch-brush, and it will then take a brilliant polish.

If a piece of thickly rusted iron be placed in the bath, its sesquioxide (Fe_2O_3) is rapidly transformed into the magnetic oxide. This outer layer has no adhesion, but beneath it there will be found a coating which is actually a part of the metal itself.

In the early experiments M. de Meritens employed pieces of steel only, but in wrought and cast iron he was not successful, for the coating came off with the slightest friction. He then placed the iron at the negative pole of the apparatus, after it had been already applied to the positive pole. Here the oxide was reduced, and hydrogen was accumulated in the pores of the metal. The specimens were then returned to the anode, when it was found that the oxide appeared quite readily and was very solid. But the result was not quite perfect, and it was not until the bath was filled with distilled water, in place of that from the public supply, that a perfectly satisfactory result was attained.

Manganese Plating of Iron as a Protection from Rust.—According to the Italian *Progreso*, articles of iron can be protected against rust by sinking them near the negative pole of an electric bath composed of 10 litres of water, 50 grammes of chloride of manganese, and 200 grammes of nitrate of ammonium. Under the influence of the current the bath deposits on the articles a protecting film of metallic manganese.

A Non-oxidizing Process of Annealing is described by H. P. Jones, in *Eng'g News*, Jan. 2, 1892. The new process uses a non-oxidizing gas, and is the invention of Mr. Horace K. Jones, of Hartford, Conn. Its principal feature consists in keeping the annealing retort in communication with the gas-holder or gas-main during the entire process of heating and cooling, the gas thus being allowed to expand back into the main, and being, therefore, kept at a practically constant pressure.

The retorts are made from wrought-iron tubes. The gas is taken directly from the mains supplying the city with illuminating gas. If metal which has been blued or slightly oxidized is subjected to the annealing process it comes out bright, the oxide being reduced by the action of the gas.

Comparative tests were made of specimens of steel wire annealed in illuminating gas, in nitrogen, and in an open fire and cooled in ashes, and of specimens of the unannealed metal. The wires were .188 in. in diameter and were turned down to .150 in.

The average results were as follows:

Unannealed, two lots, 5 pieces each, tensile strength av. 97,120 and 80,790 lbs. per sq. in., elongation 7.12% and 8.80%. Annealed in open fire, 8 tests, av. t. s. 63,090, el. 26.76%. Annealed in nitrogen, av. of 3 lots, 13 pieces, t. s. 59,820, el. 29.33%. Annealed in illuminating gas, av. of 3 lots, 13 pieces, t. s. 60,180, el. 28.29%. The elongations are referred to an original length of 1.15 ins.

STEEL.

RELATION BETWEEN THE CHEMICAL COMPOSITION AND PHYSICAL CHARACTER OF STEEL.

W. R. Webster (see Trans. A. I. M. E., vols. xxi and xxii, 1893-4) gives results of several hundred analyses and tensile tests of basic Bessemer steel plates, and from a study of them draws conclusions as to the relation of chemical composition to strength, the chief of which are condensed as follows:

The indications are that a pure iron, without carbon, phosphorus, manganese, silicon, or sulphur, if it could be obtained, would have a tensile strength of 34,750 lbs. per square inch, if tested in a 5/8-inch plate. With this as a base, a table is constructed by adding the following hardening effects, as shown by increase of tensile strength, for the several elements named.

Carbon, a constant effect of 800 lbs. for each 0.01%.
Sulphur, " " 500 " " 0.01%.
Phosphorus, the effect is higher in high-carbon than in low-carbon steels.

With carbon hundredths %... ...	9	10	11	12	13	14	15	16	17
Each .01% *P* has an effect of lbs.	900	1000	1100	1200	1300	1400	1500	1500	1500

Manganese, the effect decreases as the per cent of manganese increases.

Mn being per cent......	.00 to .15	.15 to .20	.20 to .25	.25 to .30	.30 to .35	.35 to .40	.40 to .45	.45 to .50	.50 to .55	.55 to .65
Str'gth increases for .01%	240	240	220	200	180	160	140	120	100	100 lbs.
Total incr. from 0 Mn...	3600	4800	5900	6900	7800	8600	9300	9900	10,400	11,400

Silicon is so low in this steel that its hardening effect has not been considered.

With the above additions for carbon and phosphorus the following table has been constructed (abridged from the original by Mr. Webster). To the figures given the additions for sulphur and manganese should be made as above.

Estimated Ultimate Strengths of Basic Bessemer Steel Plates.

For Carbon, .06 to .24; Phosphorus, .00 to .10; Manganese and Sulphur, .00 in all cases.

Carbon.	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24
Phos. .005	39,950	41,550	43,250	44,950	46,650	48,300	49,900	51,500	53,100	54,700
" .01	40,350	41,950	43,750	45,550	47,350	49,050	50,650	52,250	53,850	55,450
" .02	41,150	42,750	44,750	46,750	48,750	50,550	52,150	53,750	55,350	56.950
" .03	41,950	43,550	45,750	47,950	50,150	52,050	53,650	55,250	56,850	58.450
" .04	42,750	44,350	46,750	49,150	51,550	53,550	55,150	56,750	58,350	59,950
" .05	43,550	45,150	47,750	50,350	52,950	55,050	56,650	58,250	59,850	61.450
" .06	44,350	45,950	48,750	51,550	54,350	56,550	58,150	59,750	61,350	62.950
" .07	45,150	46,750	49,750	52,750	55,750	58,050	59,650	61,250	62,850	64.450
" .08	45,950	47,550	50,750	53,950	57,150	59,550	61,150	62,750	64,350	65,950
" .09	46,750	48,350	51,750	55,150	58,550	61,050	62,650	64,250	65,850	67,450
" .10	47,550	49,150	52,750	56,350	59,950	62,550	64,150	65,750	67,350	68.950
.001 Phos =	80 lbs.	80 lbs.	100 lb	120 lb	140 lb	150 lb	150 lb	150 lb	150 lb	150 lb

In all rolled steel the quality depends on the size of the bloom or ingot from which it is rolled, the work put on it, and the temperature at which it is finished, as well as the chemical composition.

The above table is based on tests of plates ⅜ inch thick and under 70 inches wide; for other plates Mr. Webster gives the following corrections for thickness and width. They are made necessary only by the effect of thickness and width on the finishing temperature in ordinary practice. Steel is frequently spoiled by being finished at too high a temperature.

Corrections for Size of Plates.

Plates. Inches thick.	Up to 70 ins. wide. Lbs.	Over 70 ins. wide. Lbs.
¾ and over	— 2000	— 1000
11/16 "	— 1750	— 750
⅝ "	— 1500	— 500
9/16 "	— 1250	— 250
½ "	— 1000	— 0
7/16 "	— 500	± 500
⅜ "	0	+ 1000
5/16 "	+ 3000	+ 5000

Comparing the actual result of tests of 408 plates with the calculated results, Mr. Webster found the variation to range as in the table below.

Summary of the Differences Between Calculated and Actual Results in 408 Tests of Plate Steel.

In the first three columns the effects of sulphur were not considered; in the last three columns the effect of sulphur was estimated at 500 lbs. for each .01% of S.

	Universal Mill.	Sheared.	Both Mills.	Universal Mill.	Sheared.	Both Mills.	Both Mills, Corrected for Thickness and Width.
Per cent within 1000 lbs..	23.4	32.1	28.4	24.6	27.0	26.0	28.4
" " " 2000 " ..	40.9	48.9	45.6	48.5	54.9	52.2	55.1
" " " 3000 " ..	62 5	71.3	67.6	67.8	73.0	70.8	74.7
" " " 4000 " ..	75 5	81.0	78.7	82.5	85.2	84.1	89.9
" " " 5000 " ..	89.5	91.1	90.4	93.0	92.8	92.9	94.9

The last figure in the table would indicate that if specifications were drawn calling for steel plates not to vary more than 5000 lbs. T. S. from a specified figure (equal to a total range of 10,000 lbs.), there would be a probability of the rejection of 5% of the blooms rolled, even if the whole lot was made from steel of identical chemical analysis. In 1000 heats only 2% of the heats failed to meet the requirements of the orders on which they were graded; the loss of plates was much less than 1%, as one plate was rolled from each heat and tested before rolling the remainder of the heat.

R. A. Hadfield (*Jour. Iron and Steel Inst.*, No. 1, 1894) gives the strength of very pure Swedish iron, remelted and tested as cast, 20.1 tons (45,024 lbs.) per sq. in.; remelted and forged, 21 tons (47,040 lbs.). The analysis of the cast bar was: C. 0.08; Si, 0.01; S, 0.02; P, 0.02; Mn. 0.01; Fe, 99.82.

Effect of Oxygen upon Strength of Steel.—A. Lantz, of the Peine works, Germany, in a letter to Mr. Webster, says that oxygen plays an important rôle—such that, given a like content of carbon, phosphorus, and manganese, a blow with greater oxygen content gives a greater hardness and less ductility than a blow with less oxygen content. The method used for determining oxygen is that of Prof. Ledebur, given in *Stahl und Eisen*, May, 1892, p. 193. The variation in oxygen may make a difference in strength of nearly ½ ton per sq. in. (*Jour. Iron and Steel Inst.*, No. 1, 1894.)

RANGE OF VARIATION IN STRENGTH OF BESSEMER AND OPEN-HEARTH STEELS.

The Carnegie Steel Co. in 1888 published a list of 1057 tests of Bessemer and open-hearth steel, from which the following figures are selected:

Kind of Steel.	No. of Tests.	Elastic Limit.		Ultimate Strength.		Elongation per cent in 8 inches.	
		High't.	Lowest	High't.	Lowest	High't.	Lowest
(*a*) Bess. structural...	100	46,570	39,230	71,300	61,450	33.00	28.75
(*b*) " " ...	170	47,690	39,970	73,540	65,200	30.25	23.15
(*c*) Bess. angles......	72	41,890	32,630	63,450	56,130	34.30	26.25
(*d*) O. H. fire-box....	25	....	....	62,790	50,350	36.00	25.62
(*e*) O. H. bridge......	20	....	....	69,940	63,970	30.00	22.75

REQUIREMENTS OF SPECIFICATIONS.

(*a*) Elastic limit, 35,000; tensile strength, 62,000 to 70,000; elong. 22% in 8 in.
(*b*) Elastic limit, 40,000; tensile strength, 67,000 to 75,000.
(*c*) Elastic limit, 30,000; tensile strength, 56,000 to 64,000; elong. 20% in 8 in.
(*d*) Tensile strength 50,000 to 62,000; elong. 26% in 4 in.
(*e*) Tensile strength, 64,000 to 70,000; elong. 20% in 8 in.

Strength of Open-hearth Structural Steel. (Pencoyd Iron Works.)—As a general rule, the percentage of carbon in steel determines its hardness and strength. The higher the carbon the harder the steel, the higher the tenacity, and the lower the ductility will be. The following list exhibits the average physical properties of good open-hearth basic steel:

Per cent Carbon.	Ultimate Strength, lbs. per sq. in.	Elastic Limit, lbs. per sq. in.	Stretch in 8 in., %.	Red. of Area, %.	Per cent Carbon.	Ultimate Strength, lbs. per sq. in.	Elastic Limit, lbs. per sq. in.	Stretch in 8 in., %.	Red. of Area, %.
.08	54000	32500	32	60	.17	61600	37000	27	50
.09	54800	33000	31	58	.18	62500	37500	27	49
.10	55700	33500	31	57	.19	63300	38000	26	48
.11	56500	34000	30	56	.20	64200	38500	26	47
.12	57400	34500	30	55	.21	65000	39000	25	46
.13	58200	35000	29	54	.22	65800	39500	25	45
.14	59100	35500	29	53	.23	66600	40000	24	44
.15	60000	36000	28	52	.24	67400	40500	24	43
.16	60800	36500	28	51	.25	68200	41000	23	42

The coefficient of elasticity is practically uniform for all grades, and is the same as for iron, viz., 29,000,000 lbs. These figures form the average of a numerous series of tests from rolled bars, and can only serve as an approximation in single instances, when the variation from the average may be considerable. Steel below .10 carbon should be capable of doubling flat without fracture, after being chilled from a red heat in cold water. Steel of .15 carbon will occasionally submit to the same treatment, but will usually bend around a curve whose radius is equal to the thickness of the specimen; about 90% of specimens stand the latter bending test without fracture. As the steel becomes harder its ability to endure this bending test becomes more exceptional, and when the carbon ratio becomes .20, little over 25% of specimens will stand the last-described bending test. Steel having about .40% carbon will usually harden sufficiently to cut soft iron and maintain an edge.

Mehrtens gives the following tables in *Stahl und Eisen* (*Iron Age*, April 20, 1893):

Basic Bessemer Steel. 680 Charges.

Elastic Limit, pounds per sq. in.	Charges within Range, per cent of total number.
35,500 to 38,400	15.0
38,400 to 39,800	31.6
39,800 to 41,200	27.5
41,200 to 42,700	16.0
42,700 to 46,400	9.9

Tensile Strength, pounds per sq. in.	Charges within Range, per cent of total number.
55,600 to 56,900	18.67
56,900 to 58,300	38.67
58,300 to 59,700	23.53
59,700 to 61,200	15.60
61,200 to 62,300	3.53

STRUCTURAL STEEL.

Elongation, per cent.	Charges within Range, per cent of total number.
21 to 25	2.65
25 to 26	8.53
26 to 27	17.35
27 to 28	26.76
28 to 29	23.68
29 to 30	14.41
30 to 32.5	6.62

RIVET STEEL.

25.2 to 26	20.0
26 to 27	15.0
27 to 28	25.0
28 to 29	25.0
29 to 29.8	15.0

Basic Open-hearth Structural Steel. 489 Charges.

Elastic Limit, pounds per sq. in.	Charges within Range, per cent of total charges
34,400 to 37,000	12.3
37,000 to 38,400	15.6
38,400 to 39,800	20.3
39,800 to 41,200	17.4
41,200 to 42,700	12.8
42,700 to 44,100	11 4
44,100 to 48,400	8.5
Tensile Strength.	
55,800 to 56,900	8.0
56,900 to 58,300	26.4
58,300 to 59,700	25.4
59,700 to 61,200	19.6
61,200 to 62,600	11.2
62,600 to 65,100	9.04
Elongation, per cent.	
20 to 25	21.7
25 to 26	7.7
26 to 27	10.0
27 to 28	11.0
28 to 29	12.0
29 to 30	13 3
30 to 37.1	24.3

RIVET STEEL, 19 CHARGES.

Tensile Strength.	
51,800	5.3
51,900 to 53,300	26.3
53,300 to 54,900	21.0
54,900 to 56,300	21.0
56,300 to 56,900	26.4

Elongation all above 25 per cent.

In the basic Bessemer steel over 90% was below 0.08 phosphorus, and all were below 0.10; manganese was below 0.6 in over 90%, and below 0.9 in all; sulphur was below 0.05 in 84%, the maximum being 0.071; carbon was below 0.10, and silicon below 0.01 in all. In the basic open-hearth steel phosphorus was below 0.06 in 96%, the maximum being 0.08; manganese below 0.50 in 97%; sulphur below 0.07 in 88%, the maximum being 0.12. The carbon ranged from 0.09 to 0.14.

Low Tensile Strength of Very Pure Steel.—Swedish nail-rod open-hearth steel, tested by the author in 1881, showed a tensile strength of only 42,591 lbs. per sq. in. A piece of American nail-rod steel showed 45,021 lbs. per sq. in. Both steels contained about .10 carbon and .015 phosphorus, and were very low in sulphur, manganese, and silicon. The pieces tested were bars about 2 × 3/8 in. section.

Low Strength Due to Insufficient Work. (A. E. Hunt, Trans. A. I. M. E., 1886.)—Soft steel ingots, made in the ordinary way for boiler plates, have only from 10,000 to 20,000 lbs. tensile strength per sq. in., an elongation of only about 10% in 8 in., and a reduction of area of less than 20%. Such ingots, properly heated and rolled down from 10 in. to 1/2 in.

thickness, will give from 55,000 to 65,000 lbs. tensile strength, an elongation in 8 in. of from 23% to 33%, and a reduction of area of from 55% to 70%. Any work stopping short of the above reduction in thickness ordinarily yields intermediate results in its tensile tests.

Hardening of Soft Steel.—A. E. Hunt (Trans. A. I. M. E., 1883, vol. xii), says that soft steel, no matter how low in carbon, will harden to a certain extent upon being heated red-hot and plunged into water, and that it hardens more when plunged into brine and less when quenched in oil.

An illustration was a heat of open-hearth steel of 0.15% carbon and 0.29% of manganese, which gave the following results upon test-pieces from the same ¼ in. thick plate.

	Maximum Load. lbs. per sq. in.	Elongation in 8 in. Per cent.	Reduction of Area. Per cent.
Unhardened....	55,000	27	62
Hardened in water.....	74,000	25	50
Hardened in brine......	84,000	22	43
Hardened in oil.........	67,700	26	49

While the ductility of such hardened steel does not decrease to the extent that the increased tenacity would indicate, and is much superior to that of normal steel of the high tenacity, still the greatly increased tenacity after hardening indicates that there must be a considerable molecular change in the steel thus hardened, and that if such a hardening should be created locally in a steel plate, there must be very dangerous internal strains caused thereby.

Effect of Cold Rolling.—Cold rolling of iron and steel increases the elastic limit and the ultimate strength, and decreases the ductility. Major Wade's experiments on bars rolled and polished cold by Lauth's process showed an average increase of load required to give a slight permanent set as follows: Transverse, 162%; torsion, 130%; compression, 161% on short columns 1¼ in. long, and 64% on columns 8 in. long; tension, 95%. The hardness, as measured by the weight required to produce equal indentations, was increased 50%; and it was found that the hardness was as great in the centre of the bars as elsewhere. Sir W. Fairbairn's experiments showed an increase in ultimate tensile strength of 50%, and a reduction in the elongation in 10 in. of from 2 in. or 20%, to 0.79 in. or 7.9%.

Comparison of Tests of Full-size Eye-bars and Sample Test-pieces of Same Steel Used in the Memphis Bridge.

(Geo. S. Morison, Trans. A. S. C. E., 1893.)

Full-Sized Eyebars, Sections 10″ wide × 1 to 2 3/16″ thick.					Sample Bars from Same Melts, about 1 in. area.			
Reduction of Area, p.c.	Elongation. Inches.	Elongation. p.c.	Elastic Limit, lbs. per sq. in.	Max. Load, lbs. per sq. in.	Reduction, p. c.	Elongation, p. c.	Elastic Limit, lbs. per sq. in.	Max. Load, lbs. per sq. in.
39.6	20.2	16.8	35,100	67,490	47.5	27.5	41,580	73,050
39.7	26.6	8.2	37,680	70,160	52.6	24.4	42,650	75,620
44.4	36.8	11.8	39,700	65,500	47.9	28.8	40,280	70,280
38.5	38.5	17.3	33,140	65,060	47.5	27.5	41,580	73,050
40.0	22.5	13.5	32,860	65,600	44.5	20.0	43,750	75,000
39.4	36.8	15.3	31,110	61,060	42.7	28.8	42,210	69,730
34.6	32.9	18.7	33,990	68,220	52.2	28.1	40,280	69,720
32.6	13.0	13.5	29,330	63,100	48.3	28.8	38,090	71,300
7.3	20 8	6.9	28,080	55,160	43.2	24.2	38,320	70,220
38.1	28.9	14.1	29,670	62,140	59.6	26.3	40,200	71,080
31.8	24.0	11.8	32,700	65,400	40.3	25.0	39,360	69,360
48.6	39.4	19.3	30,500	58,870	40.3	25.0	40,910	70,360
10.3	11.8	12.3	33,360	73,550	51.5	25.5	40,410	69,900
44.6	32.0	15.7	32,520	60,710	43.6	27.0	40,400	70,490
46.0	35.8	14.9	28,000	58,720	44.4	29.5	40,000	66,800
41.8	28.5	13.1	32,290	62,270	42.8	21.3	40,530	72,240
41.2	47.1	15.1	29,970	58,680	45.7	27.0	40,610	70,480

The average strength of the full-sized eye-bars was about 8000 lbs. per sq. in., or about 12% less than that of the sample test-pieces.

TREATMENT OF STRUCTURAL STEEL.

(James Christie, Trans. A. S. C. E., 1893.)

Effect of Punching and Shearing.—There is no doubt that steel of higher tensile strength than is now accepted for structural purposes should not be punched or sheared, or that the softer material may contain elements prejudicial to its use however treated, but especially if punched. But extensive evidence is on record indicating that steel of good quality, in bars of moderate thickness and below or not much exceeding 80,000 lbs. tensile strength, is not any more, and frequently not as much, injured as wrought iron by the process of punching or shearing.

The physical effects of punching and shearing as denoted by tensile test are for iron or steel:

Reduction of ductility; elevation of tensile strength at elastic limit; reduction of ultimate tensile strength.

In very thin material the superficial disturbance described is less than in thick; in fact, a degree of thinness is reached where this disturbance practically ceases. On the contrary, as thickness is increased the injury becomes more evident.

The effects described do not invariably ensue; for unknown reasons there are sometimes marked deviations from what seems to be a general result.

By thoroughly annealing sheared or punched steels the ductility is to a large extent restored and the exaggerated elastic limit reduced, the change being modified by the temperature of reheating and the method of cooling.

It is probable that the best results combined with least expenditure can be obtained by punching all holes where vital strains are not transferred by the rivets; and by reaming for important joints where strains on riveted joints are vital, or wherever perforation may reduce sections to a minimum. The reaming should be sufficient to thoroughly remove the material disturbed by punching; to accomplish this it is best to enlarge punched holes at least 1/8 in. diameter with the reamer.

Riveting.—It is the current practice to perforate holes 1/16 in. larger than the rivet diameter. For work to be reamed it is also a usual requirement to punch the holes from 1/8 to 3/16 in. less than the finished diameter, the holes being reamed to the proper size after the various parts are assembled.

It is also excellent practice to remove the sharp corner at both ends of the reamed holes, so that a fillet will be formed at the junction of the body and head of the finished rivets.

The rivets of either iron or mild steel should be heated to a bright red or yellow heat and subjected to a pressure of not less than 50 tons per square inch of sectional area.

For rivets of ordinary length this pressure has been found sufficient to completely fill the hole. If, however, the holes and the rivets are exceptionally long, a greater pressure and a slower movement of the closing tool than is used for shorter rivets has been found advantageous in compelling the more sluggish flow of the metal throughout the longer hole.

Welding.—No welding should be allowed on any steel that enters into structures.

Upsetting.—Enlarged ends on tension bars for screw-threads, eyebars, etc., are formed by upsetting the material. With proper treatment and a sufficient increment of enlarged sectional area over the body of the bar the result is entirely satisfactory. The upsetting process should be performed so that the properly heated metal is compelled to flow without folding or lapping.

Annealing.—The object of annealing structural steel is for the purpose of securing homogeneity of structure that is supposed to be impaired by unequal heating, or by the manipulation necessarily attendant on certain processes. The objects to be annealed should be heated throughout to a uniform temperature and uniformly cooled.

The physical effects of annealing, as indicated by tensile tests, depend on the grade of steel, or the amount of hardening elements associated with it; also on the temperature to which the steel is raised, and the method or rate of cooling the heated material.

The physical effects of annealing medium-grade steel, as indicated by tensile test, are reported very differently by different observers, some claiming directly opposite results from others. It is evident, when all the attendant conditions are considered, that the obtained results must vary both in kind and degree.

The temperatures employed will vary from 1000° to 1500° F.; possibly even a wider range is used. In some cases the heated steel is withdrawn at full temperature from the furnace and allowed to cool in the atmosphere; in others the mass is removed from the furnace, but covered under a muffle, to lessen the free radiation; or, again, the charge is retained in the furnace, and the whole mass cooled with the furnace, and more slowly than by either of the other methods.

The best general results from annealing will probably be obtained by introducing the material into a uniformly-heated oven in which the temperature is not so high as to cause a possibility of cracking by sudden and unequal changing of temperature, then gradually raising the temperature of the material until it is uniformly about 1200° F., then withdrawing the material after the temperature is somewhat reduced and cooling under shelter of a muffle, sufficiently to prevent too free and unequal cooling on the one hand or excessively slow cooling on the other.

G. G. Mehrtens, Trans. A. S. C. E. 1893, says: "Annealing is of advantage to all steel above 64,000 lbs. strength per square inch, but it is questionable whether it is necessary in softer steels. The distortions due to heating cause trouble in subsequent straightening, especially of thin plates.

"In a general way all unannealed mild steel for a strength of 56,000 to 64,000 lbs. may be worked in the same way as wrought iron. Rough treatment or working at a blue heat must, however, be prohibited. Shearing is to be avoided, except to prepare rough plates, which should afterwards be smoothed by machine tools or files before using. Drifting is also to be avoided, because the edges of the holes are thereby strained beyond the yield point. Reaming drilled holes is not necessary, particularly when sharp drills are used and neat work is done. A slight countersinking of the edges of drilled holes is all that is necessary. Working the material while heated should be avoided as far as possible, and the engineer should bear this in mind when designing structures. Upsetting, cranking, and bending ought to be avoided, but when necessary the material should be annealed after completion.

"The riveting of a mild-steel rivet should be finished as quickly as possible, before it cools to the dangerous heat. For this reason machine work is the best. There is a special advantage in machine work from the fact that the pressure can be retained upon the rivet until it has cooled sufficiently to prevent elongation and the consequent loosening of the rivet."

Punching and Drilling of Steel Plates. (Proc. Inst. M. E., Aug. 1887, p. 326.)—In Prof. Unwin's report the results of the greater number of the experiments made on iron and steel plates lead to the general conclusion that, while thin plates, even of steel, do not suffer very much from punching, yet in those of ¼ in. thickness and upwards the loss of tenacity due to punching ranges from 10% to 23% in iron plates and from 11% to 33% in the case of mild steel. Mr. Parker found the loss of tenacity in steel plates to be as high as fully one third of the original strength of the plate. In drilled plates, on the contrary, there is no appreciable loss of strength. It is even possible to remove the bad effects of punching by subsequent reaming or annealing.

Working Steel at a Blue Heat.—Not only are wrought iron and steel much more brittle at a blue heat (i.e., the heat that would produce an oxide coating ranging from light straw to blue on bright steel, 430° to 600° F.), but while they are probably not seriously affected by simple exposure to blueness, even if prolonged, yet if they be worked in this range of temperature they remain extremely brittle after cooling, and may indeed be more brittle than when at blueness: this last point, however, is not certain. (Howe, "Metallurgy of Steel," p. 534.)

Tests by Prof. Krohn, for the German State Railways, show that working at blue heat has a decided influence on all materials tested, the injury done being greater on wrought iron and harder steel than on the softer steel. The fact that wrought iron is injured by working at a blue heat was reported by Stromeyer. (*Engineering News*, Jan. 9, 1892.)

A practice among boiler-makers for guarding against failures due to working at a blue heat consists in the cessation of work as soon as a plate which had been red-hot becomes so cool that the mark produced by rubbing a hammer-handle or other piece of wood will not glow. A plate which is not hot enough to produce this effect, yet too hot to be touched by the hand, is most probably blue hot, and should under no circumstances be hammered or bent. (C. E. Stromeyer, Proc. Inst. C. E. 1886.)

Welding of Steel.—A. E. Hunt (A. I. M. E., 1892) says: I have never seen so-called "welded" pieces of steel pulled apart in a testing-machine or otherwise broken at the joint which have not shown a smooth cleavage-plane, as it were, such as in iron would be condemned as an imperfect weld. My experience in this matter leads me to agree with the position taken by Mr. William Metcalf in his paper upon Steel in the Trans. A. S. C. E., vol. xvi., p. 301. Mr. Metcalf says, "I do not believe steel can be welded."

Oil-tempering and Annealing of Steel Forgings.—H. F. J. Porter says (1897) that all steel forgings above 0.1% carbon should be annealed, to relieve them of forging and annealing strains, and that the process of annealing reduces the elastic limit to 47% of the ultimate strength. Oil-tempering should only be practised on thin sections, and large forgings should be hollow for the purpose. This process raises the elastic limit above 50% of the ultimate tensile strength, and in some alloys of steel, notably nickel steel, will bring it up to 60% of the ultimate.

Hydraulic Forging of Steel. (See pages 618 and 619.)

INFLUENCE OF ANNEALING UPON MAGNETIC CAPACITY.

Prof. D. E. Hughes (*Eng'g*, Feb. 8, 1884, p. 130) has invented a "Magnetic Balance," for testing the condition of iron and steel, which consists chiefly of a delicate magnetic needle suspended over a graduated circular index, and a magnet coil for magnetizing the bar to be tested. He finds that the following laws hold with every variety of iron and steel:

1. The magnetic capacity is directly proportional to the softness, or molecular freedom.

2. The resistance to a feeble external magnetizing force is directly as the hardness, or molecular rigidity.

The magnetic balance shows that annealing not only produces softness in iron, and consequent molecular freedom, but it entirely frees it from all strains previously introduced by drawing or hammering. Thus a bar of iron drawn or hammered has a peculiar structure, say a fibrous one, which gives a greater mechanical strength in one direction than another. This bar, if thoroughly annealed at high temperatures, becomes homogeneous in all directions, and has no longer even traces of its previous strains, provided that there has been no actual separation into a distinct series of fibres.

Effect of Annealing upon the Magnetic Capacity of Different Wires; Tests by the Magnetic Balance.

Description.	Magnetic Capacity.	
	Bright as sent.	Annealed.
	deg. on scale.	deg. on scale.
Best Swedish charcoal iron, first variety.	230	525
" " " " second "	236	510
" " " " third "	275	503
Swedish Siemens-Martin iron	165	430
Puddled iron, best best	212	340
Bessemer steel, soft	150	291
" " hard	115	172
Crucible fine cast steel	50	84

Crucible Fine Steel, Tempered.	Magnetic Capacity.
Bright-yellow heat, cooled completely in cold water	28
Yellow-red heat, cooled completely in cold water	32
Bright yellow, let down in cold water to straw color	33
" " " " " " blue	43
" " cooled completely in oil	51
" " let down in water to white	58
Reheat, cooled completely in water	66
" " " " oil	72
Annealed, " " " oil	84

SPECIFICATIONS FOR STEEL.

Structural Steel.—There has been a change during the ten years from 1880 to 1890, in the opinions of engineers, as to the requirements in specifications for structural steel, in the direction of a preference for metal of low tensile strength and great ductility. The following specifications of different dates are given by A. E. Hunt and G. H. Clapp, Trans. A. I. M. E. 1890, xix, 926:

TENSION MEMBERS.	1879.	1881.	1882.	1885.	1887.	1888.
Elastic limit	50,000	40@45,000	40.000	40,000	40,000	38.000
Tensile strength	80,000	70@80,000	70.000	70,000	67@75,000	63@70,000
Elongation in 8 in	12%	18%	18%	18%	20%	22%
Reduction area	20%	30%	45%	42%	42%	45%
Kind of steel	O.H.	O.H. or B.	O.H.	Not spec.	O.H. or B.	O.H. or B.
COMPRESSION MEMBERS:						
Elastic limit	Same	50@55,000	50,000	50,000	Same as tension	
Tensile strength	as	80@90,000	80,000	80.000	members.	
Elongation in 8 in	ten-	12%	15%	15%	"	
Reduction area	sion.	20%	35%	35%	"	

F. H. Lewis (*Iron Age*, Nov. 3, 1892) says: Regarding steel to be used under the same conditions as wrought iron, that is, to be punched without reaming, there seems to be a decided opinion (and a growing one) among engineers, that it is not safe to use steel in this way, when the ultimate tensile strength is above 65,000 lbs. The reason for this is, not so much because there is any marked change in the material of this grade, but because all steel, especially Bessemer steel, has a tendency to segregations of carbon and phosphorus, producing places in the metal which are harder than they normally should be. As long as the percentages of carbon and phosphorus are kept low, the effect of these segregations is inconsiderable; but when these percentages are increased, the existence of these hard spots in the metal becomes more marked, and it is therefore less adapted to the treatment to which wrought iron is subjected.

There is a wide consensus of opinion that at an ultimate of 64,000 to 65,000 lbs. the percentages of carbon and phosphorus (which are the two hardening elements) reach a point where the steel has a tendency to become tender, and to crack when subjected to rough treatment.

A grade of steel, therefore, running in ultimate strength from 54,000 to 62,000 lbs., or in some cases to 64.000 lbs., is now generally considered a proper material for this class of work.

Millard Hunsicker, engineer of tests of Carnegie, Phipps & Co., writes as follows concerning grades of structural steel (*Eng'g News*, June 2, 1892):

Grade of Steel.—Steel shall be of three grades—soft, medium, high.

Soft Steel.—Specimens from finished material for test, cut to size specified above, shall have an ultimate strength of from 54,000 to 62.000 lbs. per sq. in.; elastic limit one half the ultimate strength; minimum elongation of 26% in 8 in.; minimum reduction of area at fracture 50%. This grade of steel to bend cold 180° flat on itself, without sign of fracture on the outside of the bent portion.

Medium Steel.—Specimens from finished material for test, cut to size specified above, shall have an ultimate strength of 60,000 to 68,000 lbs. per sq. in.; elastic limit one half the ultimate strength; minimum elongation 20% in 8 in.; minimum reduction of area at fracture, 40%. This grade of steel to bend cold 180° to a diameter equal to the thickness of the piece tested, without crack or flaw on the outside of the bent portion.

High Steel.—Specimens from finished material for test, cut to size specified above, shall have an ultimate strength of 66 000 to 74.000 lbs. per sq. in.; elastic limit one half the ultimate strength; minimum elongation, 18% in 8 in.; minimum reduction of area at fracture, 35%. This grade of steel to bend cold 180° to a diameter equal to three times the thickness of the test-piece, without crack or flaw on the outside of the bent portion.

F. H. Lewis, Engineers' Club of Phila., 1891, gives specifications for structural steel as follows: The phosphorus in acid open-hearth steel must be less than 0.10%, and in all Bessemer or basic steel must be less than 0.08%.

The material will be tested in specimens of at least one half square inch section, cut from the finished material. Each melt of steel will be tested and each section rolled, and also widely differing gauges of the same section.

Requirements.	Soft Steel.	Medium Steel.
Elastic limit, lbs. per sq. in., at least.........	32,000	35,000
Ultimate strength, lbs. per sq. in.	54,000 to 62,000	60,000 to 70,000
Elongation in 8 in., at least..................	25%	20%
Reduction of area, per cent, at least........	45%	40%

In soft steel for web-plates over 36 in. wide the elongation will be reduced to 20% and the reduction of area to 40%.

It must bend cold 180 degrees and close down on itself without cracking on the outside.

⅝-inch holes pitched ¾ inch from a roll-finished or machined edge and 2 inches between centres must not crack the metal; and ⅝-inch holes pitched 1⅛ inches between centres and 1⅛ inches from the edge must not split the metal between the holes.

Medium steel must bend 180 degrees on itself around a 1½-inch round bar.

Full-sized eye-bars, when tested to destruction, must show an ultimate strength of at least 56,000 lbs., and stretch at least 10% in a length of 10 feet.

A. E. Hunt, in discussing Mr. Lewis's specifications, advises a requirement as to the character of the fracture of tensile tests being entirely silky in sections of less than 7 square inches, and in larger sections the test specimen not to contain over 25% crystalline or granular fracture. He also advises the drifting test as a requirement of both soft and medium steel; the requirement being worded about as follows: "Steel to be capable of having a hole, punched for a ¾" rivet, enlarged by blows of a sledge upon a drift-pin until the hole (which in the first case should be punched 1½" from the roll-finish or machined edge) is 1⅛" diameter in the case of soft steel, and 1¼" diameter in the case of medium steel, without fracture." This drifting test is an excellent requirement, not only as a matter of record, but as a measure of the ductility of the steel.

H. H. Campbell, Trans. A. I. M. E. 1893, says: In adhering to the safest course, engineers are continually calling for a metal with lower phosphorus The limit has been 0.10%; it is now 0.08%; soon it will be 0.06%; it should be 0.04%.

A. E. Hunt, Trans. A. I. M. E. 1892, says: Why should the tests for steel be so much more rigid than for iron destined for the same purpose? Some of the reasons are as follows: Experience shows that the acceptable qualities of one melt of steel offer no absolute guarantee that the next melt to it, even though made of the same stock, will be equally satisfactory.

Again, good wrought iron, in plates and angles, has a narrow range (from 25,000 to 27,000 lbs.) in elastic limit per square inch, and a tensile strength of from 46,000 to 52,000 lbs. per square inch; whereas for steel the range in elastic limit is from 27,000 to 80,000 lbs., and in tensile strength from 48,000 to 120,000 lbs. per square inch, with corresponding variations in ductility. Moreover, steel is much more susceptible than wrought iron to widely varying effects of treatment, by hardening, cold rolling, or overheating.

It is now almost universally recognized that soft steel, if properly made and of good quality, is for many purposes a safe and satisfactory substitute for wrought iron, being capable of standing the same shop-treatment as wrought iron. But the conviction is equally general, that poor steel, or an unsuitable grade of steel, is a very dangerous substitute for wrought iron even under the same unit strains.

For this reason it is advisable to make more rigid requirements in selecting material which may range between the brittleness of glass and a ductility greater than that of wrought iron.

Specifications for Steel for the World's Fair Buildings, Chicago, 1892.—No steel shall contain more than .08% of phosphorus. From three separate ingots of each cast a round sample bar, not less than ¾ in. in diameter, and having a length not less than twelve diameters between jaws of testing machine, shall be furnished and tested by the manufacturer. From these test-pieces alone the quality of the material in the steel works shall be determined as follows:

All the test-bars must have a tensile strength of from 60,000 to 68,000 lbs. per square inch, an elastic limit of not less than half the tensile strength of the test-bar, an elongation of not less than 24%, and a reduction of area of not less than 40% at the point of fracture. In determining the ductility, the elongation shall be measured after breaking on an original length of ten times the shortest dimension of the test-piece.

Rivet steel shall have a tensile strength of from 52,000 to 58,000 lbs. per square inch, and an elastic limit, elongation, and reduction of area at the point of fracture as stated above for test-bars, and be capable of bending double flat, without sign of fracture on the convex surface of the bend.

Boiler, Ship, and Tank Plates. W. F. Mattes (*Iron Age*, July 9, 1896) recommends that the different qualities of steel plates be classified as follows:

	Tank.	Ship.	Shell.	Fire-box.
Tensile test, longitudinal coupon..................	Limit, 75,000	55,000 to 65,000	55,000 to 65,000	55,000 to 60,000
Elongation in 8-in. longitudinal coupon, per cent....	....	20	22½	25
Bending test, longitudinal coupon..................	....	Flat.	Flat	Flat.
Bending test, transverse coupon..................	....	Over 1 in. diam.	Over ½ in. diam.	Flat.
Phosphorus limit............	0.15	0.10	0.06	0.045
Sulphur limit.	....	....	0.065	0.05
Surface inspection..........	Easy.	Careful.	Close.	Rigid.

A steel-manufacturing firm in Pittsburgh advertises six different grades of steel as follows:

Extra fire-box. Fire-box. Extra flange. Flange. Shell. Tank.

The probable average phosphorus content in these grades is, respectively: .02 .03 .04 .06 .08 .10.

Different specifications for steel plates are the following (1889):

United States Navy.—Shell: Tensile strength, 58,000 to 67,000 lbs. per sq. in.; elongation, 22% in 8-in. transverse section, 25% in 8-in. longitudinal section.

Flange: Tensile strength, 50,000 to 58,000 lbs.; elongation. 26% in 8 inches.

Chemical requirements: P. not over .035%; S. not over .040%.

Cold-bending test: Specimen to stand being bent flat on itself.

Quenching test: Steel heated to cherry-red, plunged in water 82° F., and to be bent around curve 1½ times thickness of the plate.

British Admiralty.—Tensile strength, 58,240 to 67,200 lbs.; elongation in 8 in., 20%; same cold-bending and quenching tests as U. S. Navy.

American Boiler-makers' Association.—Tensile strength, 55,000 to 65,000 lbs.; elongation in 8 in., 20% for plates ⅜ in. thick and under; 22% for plates ⅜ in. to ¾ in.; 25% for plates ¾ in. and over.

Cold-bending test: For plates ½ in. thick and under, specimen must bend back on itself without fracture; for plates over ½ in. thick, specimen must withstand bending 180° around a mandril, 1½ times the thickness of the plate.

Chemical requirements: P. not over .040%; S. not over .030%.

American Shipmasters' Association.—Tensile strength, 62,000 to 72,000 lbs.; elongation, 16% on pieces 9 in. long.

Strips cut from plates, heated to a low red and cooled in water the temperature of which is 82° F., to undergo without crack or fracture being doubled over a curve the diameter of which does not exceed three times the thickness of the piece tested.

Boiler Shell-plates, Front Tube-plate, and Butt-strips. (Penna. R. R., 1892.)—The metal desired is a homogeneous steel having a tensile strength of 60,000 lbs. per sq. in., and an elongation of 25% in a section originally 8 in. long. These plates will not be accepted if the test-piece shows—

1. A tensile strength of less than 55,000 lbs. per sq. in.; 2. An elongation in section originally 8 in. long less than 20%; 3. A tensile strength over 65,000 lbs. per sq. in.; should, however, the elongation be 27% or over, plates will not be rejected for high strength.

Inside Fire-box Plates, including Back Tube-plate. (Penna. R. R., 1892.)—The metal should show a tensile strength of 60,000 lbs. per sq. in., and an elongation of 28% in a test section originally 8 in. long.

Chemical Composition.	Desired.	Will be Rejected.
Carbon...	0.18 per cent.	over 0.25, below 0.15
Phosphorus, not above........	0.03 "	over 0.04
Manganese, not above..... ..	0.40 "	over 0.55
Silicon, not above.......... ..	0.02 "	over 0.04
Sulphur, not above............	0.02 "	over 0.05
Copper, not above............	0.03 "	over 0.05

These plates will not be accepted if the test-piece shows: 1. A tensile strength of less than 55,000 lbs. per sq. in.; 2. An elongation in section originally 8 in. long, less than 22% (20% in plates ¼ inch thick); 3. A tensile strength over 65,000 lbs. per sq. in. (68,000 for plates ¼ in. thick); should, however, the elongation be 30% or over, plates will not be rejected for high strength; 4. Any single seam or cavity more than ¼ in. long in either of the three fractures obtained on test for homogeneity, as described below.

Homogeneity test: A portion of the test-piece is nicked with a chisel, or grooved on a machine, transversely about a sixteenth of an inch deep, in three places about 1¼ in. apart. The first groove should be made on one side, 1¼ in. from the square end of the piece; the second, 1¼ in. from it on the opposite side; and the third, 1¼ in. from the last, and on the opposite side from it. The test-piece is then put in a vise, with the first groove about ¼ in. above the jaw, care being taken to hold it firmly. The projecting end of the test-piece is then broken off by means of a hammer, a number of light blows being used, and the bending being away from the groove. The piece is broken at the other two grooves in the same way. The object of this treatment is to open and render visible to the eye any seams due to failure to weld up, or to foreign interposed matter, or cavities due to gas bubbles in the ingot. After rupture, one side of each fracture is examined, a pocket lens being used if necessary, and the length of the seams and cavities is determined. The length of the longest seam or cavity determines the acceptance or rejection of the plate.

Dr. C. B. Dudley, chemist of the Penna. R. R. (Trans. A. I. M. E. 1892, vol. xx. p. 709), gives as an example of the progressive improvement in specifications the following: In the early days of steel boilers the specification in force called for steel of not less than 50,000 lbs. tensile strength and not less than 25% elongation. Some metal was received having 75,000 lbs. tensile strength, and as the elongation was all right it was accepted; but when those plates were being flanged in the boiler-shop they cracked and went to pieces. As a result, an upper limit of 65,000 lbs. tensile strength was established.

Am. Ry. Master Mechanics' Assn., 1894.—Same as Penna. R. R. Specifications of 1892, including homogeneity test.

Plate, Tank, and Sheet Steel. (Penna. R. R., 1888.*)—A test strip taken lengthwise of each plate, ⅛ in. thick and over, without annealing, should have a tensile strength of 60,000 lbs. per sq. in., and an elongation of 25% in a section originally 2 in. long.

Sheets will not be accepted if the tests show the tensile strength less than 55,000 lbs. or greater than 70,000 lbs. per sq. in., nor if the elongation fails below 20%.

Steel Billets for Main and Parallel Rods. (Penna. R. R., 1884.)—One billet from each lot of 25 billets or smaller shipment of steel for main or parallel rods for locomotives will have a piece drawn from it under the hammer and a test-section will be turned down on this piece to ⅝ in. in diameter and 2 in. long. Such test-piece should show a tensile strength of 85,000 lbs. and an elongation of 15%.

No lot will be acceptable if the test shows less than 80,000 lbs. tensile strength or 12% elongation in 2 in.

Locomotive Spring Steel. (Penna. R. R., 1887.)—Bars which vary more than 0.01 in. in thickness, or more than 0.02 in. in width, from the size ordered, or which break where they are not nicked, or which, when properly nicked and held, fail to break square across where they are nicked, will be returned. The metal desired has the following composition: Carbon, 1.00%; manganese, 0.25%; phosphorus, not over 0.03%; silicon, not over 0.15%; sulphur, not over 0.03%; copper, not over 0.03%.

Shipments will not be accepted which show on analysis less than 0.90% or over 1.10% of carbon, or over 0.50% of manganese, 0.05% of phosphorus, 0.25% of silicon, 0.05% of sulphur, and 0.05% of copper.

Steel for Locomotive Driving-axles. (Penna. R. R., 1888.)—Steel for driving-axles should have a tensile strength of 85,000 lbs. per sq. in. and an elongation of 15% in section originally 2 in. long and ⅝ in. diameter, taken midway between centre and circumference of the axle.

Axles will not be accepted if tensile strength is less than 80,000 lbs., nor if elongation is below 12%.

Steel for Crank-pins. (Penna. R. R., 1886.)—Steel ingots for crank-

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* The Penna. R. R. specifications of the several dates given are still in force, July, 1894.

pins must be swaged as per drawings. For each lot of 50 ingots ordered, 51 must be furnished, from which one will be taken at random, and two pieces, with test sections 5/8 in. diameter and 2 in. long, will be cut from any part of it, provided that centre line of test-pieces falls 1 1/8 in. from centre line of ingot. Such test-pieces should have a tensile strength of 85,000 lbs. per sq. in. and an elongation of 15%. Ingots will not be accepted if the tensile strength is less than 80,000 lbs. nor if the elongation is below 12%.

Dr. Chas. B. Dudley, Chemist of the P. R. R. (Trans. A. I. M. E. 1892), referring to this specification, says: In testing a recent shipment, the piece from one side of the pin showed 68,000 lbs. strength and 22% elongation, and the piece from the opposite side showed 106,000 lbs. strength and 14% elongation. Each piece was above the specified strength and ductility, but the lack of uniformity between the two sides of the pin was so marked that it was finally determined not to put the lot of 50 pins in use. To guard against trouble of this sort in future, the specifications are to be amended to require that the difference in ultimate strength of the two specimens shall not be more than 8000 lbs.

Steel Car-axles. (Penna. R. R., 1891)—For each 100 axles ordered 101 must be furnished, from which one will be taken at random, and subjected to tests prescribed.

Axles for passenger cars and passenger locomotive and tender trucks must be made of steel and be rough turned throughout. Two test-pieces will be cut from an axle, and the test sections of 5/8 in. diameter by 2 in. long may fall at any part of the axle provided that the centre line of the test-section is 1 in. from the centre line of the axle. Such test-pieces should have a tensile strength of 80,000 lbs. per sq. in. and an elongation of 20%. Axles will not be accepted if the tensile strength is less than 75,000 lbs. or the elongation below 15%, nor if the fractures are irregular.

Axles for freight cars and freight-locomotive tender trucks must be made of steel, and will be subjected to the following test, which they must stand without fracture:

AXLES 4 IN. DIAMETER AT CENTRE — Five blows at 20 ft. of a 1640-lb. weight, striking midway between supports 3 ft. apart; axle to be turned over after each blow.

AXLES 4 3/8 IN. DIAMETER AT CENTRE—Five blows at 25 ft. of a 1640-lb. weight, striking midway between supports 3 ft. apart: axles to be turned over after each blow.

Steel for Rails.—P. H. Dudley (Trans. A. S. C. E. 1893) recommends the following chemical composition for rails of the weights specified:

Weights per yard.....	60, 65, and 70 lbs.	75 and 80 lbs.	100 lbs.
Carbon................	.45 to .55%	.50 to .60%	.65 to .75%

For all weights: Manganese, .80% to 1.00%; silicon, .10% to .15%; phosphorus, not over .06%; sulphur, not over .07%.

Carbon by itself up to or over 1% increases the hardness and tensile strength of the iron rapidly, and at the same time decreases the elongation. The amount of carbon in the early rails ranged from 0.25 to 0.5 of 1%, while in recent rails and very heavy sections it has been increased to 0.5, 0.6, and 0.75 of 1%. With good irons and suitable sections it can run from 0.55 to 0.75 of 1%, according to the section, and obtain fine-grain tough rails with low phosphorus.

Manganese is a necessary ingredient in the first place to take up the oxide of iron formed in the bath of molten metal during the blow. It also is of great assistance to check red shortness of the ingots during the first passes in the blooming train. In the early rails 0.4 to 0.5 of 1% was sufficient when the ingots were hammered or the reductions in the passes in the trains were very much lighter than to day. With the more rapid rolling of recent years the manganese is very often increased to 1.25% to 1.5%. It makes the rails hard with a coarse crystallization and with a decided tendency to brittleness Rails high in manganese seem to flow quite easily, especially under severe service or the use of sand, and oxidize rapidly in tunnels. From 0.80 to 1.00% seems to be all that is necessary for good rolling at the present time.

Steel Rivets. (H. C. Torrance, Amer. Boiler Mfrs. Assn., 1890.)—The Government requirements for the rivets used in boilers of the cruisers built in 1890 are: For longitudinal seams, 58,000 to 67,000 lbs. tensile strength; elongation, not less than 26% in 8 in., and all others a tensile strength of 50,000 to 58,000 lbs., with an elongation of not less than 30%. They shall be capable of being flattened out cold under the hammer to a thickness of one half the diameter, and of being flattened out hot to a thickness of one third the diameter without showing cracks or flaws. The steel must not contain more than .035 of 1% of phosphorus, nor more than .04 of 1% of sulphur.

A lot of 20 successive tests of rivet steel of the low tensile strength quality and 12 tests of the higher tensile strength gave the following results:

	Low Steel.	Higher.
Tensile strength, lbs. per sq. in....	51,230 to 54,100	59,100 to 61,850
Elastic limit, lbs. per sq. in........	31,050 to 33,190	32,080 to 33,070
Elongation in 8 in., per cent.......	30.5 to 35.25	28.5 to 31.75
Carbon, per cent..................	.11 to .14	.16 to .18
Phosphorus	.027 to .029	.03
Sulphur..........................	.033 to .035	.033 to .035

The safest steel rivets are those of the lowest tensile strength, since they are the least liable to become hardened and fracture by hammering, or to break from repeated concussive and vibratory strains to which they are subjected in practice. For calculations of the strength of riveted joints the tensile strength may be taken as the average of the figures above given, or 52,665 lbs., and the shearing strength at 45,000 lbs. per sq. in.

MISCELLANEOUS NOTES ON STEEL.

May Carbon be Burned Out of Steel?—Experiments made at the Laboratory of the Penna. Railroad Co. (Specifications for Springs, 1888) with the steel of spiral springs, show that the place from which the borings are taken for analysis has a very important influence on the amount of carbon found. If the sample is a piece of the round bar, and the borings are taken from the end of this piece, the carbon is always higher than if the borings are taken from the side of the piece. It is common to find a difference of 0.10% between the centre and side of the bar, and in some cases the difference is as high as 0.23%. Furthermore, experiments made with samples taken from the drawn out end of the bar show, usually, less carbon than samples taken from the round part of the bar, even though the borings may be taken out of the side in both cases.

Apparently during the process of reducing the metal from the ingots to the round bar, with successive heatings, the carbon in the outside of the bar is burned out.

"Recalescence" of Steel.—If we heat a bar of copper by a flame of constant strength, and note carefully the interval of time occupied in passing from each degree to the next higher degree, we find that these intervals increase regularly, i e., that the bar heats more and more slowly, as its temperature approaches that of the flame. If we substitute a bar of steel for one of copper, we find that these intervals increase regularly up to a certain point, when the rise of temperature is suddenly and in most cases greatly retarded or even completely arrested. After this the regular rise of temperature is resumed, though other like retardations may recur as the temperature rises farther. So if we cool a bar of steel slowly the fall of temperature is greatly retarded when it reaches a certain point in dull redness. If the steel contains much carbon, and if certain favoring conditions be maintained, the temperature, after descending regularly, suddenly rises spontaneously very abruptly, remains stationary a while, and then redescends, This spontaneous reheating is known as "recalescence."

These retardations indicate that some change which absorbs or evolves heat occurs within the metal. A retardation while the temperature is rising points to a change which absorbs heat; a retardation during cooling points to some change which evolves heat. (Henry M. Howe, on "Heat Treatment of Steel," Trans. A. I. M. E., vol. xxii.)

Effect of Nicking a Steel Bar.—The statement is sometimes made that, owing to the homogeneity of steel, a bar with a surface crack or nick in one of its edges is liable to fail by the gradual spreading of the nick, and thus break under a very much smaller load than a sound bar. With iron it is contended this does not occur, as this metal has a fibrous structure. Sir Benjamin Baker has, however, shown that this theory, at least so far as statical stress is concerned, is opposed to the facts, as he purposely made nicks in specimens of the mild steel used at the Forth Bridge, but found that the tensile strength of the whole was thus reduced by only about one ton per square inch of section. In an experiment by the Union Bridge Company a full-sized steel counter-bar, with a screw-turned buckle connection, was tested under a heavy statical stress, and at the same time a weight weighing 1040 lbs. was allowed to drop on it from various heights. The bar was first broken by ordinary statical strain, and showed a breaking stress of

66,800 lbs. per square inch. The longer of the broken parts was then placed in the machine and put under the following loads, whilst a weight, as already mentioned, was dropped on it from various heights at a distance of five feet from the sleeve-nut of the turn-buckle, as shown below:

Stress in pounds per sq. in....	50,000	55,000	60,000	63,000	65,000
	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
Height of fall..................	2 1	2 6	3 0	4 0	5 0

The weight was then shifted so as to fall directly on the sleeve-nut, and the test proceeded as follows:

Stress on specimen in lbs. per square inch......	65,350	65,350	68,800
Height of fall, feet..............................	3	6	6

It will be seen that under this trial the bar carried more than when originally tested statically, showing that the nicking of the bar by screwing had not appreciably weakened its power of resisting shocks.—*Eng'g News.*

Electric Conductivity of Steel.—Louis Campredon reports in *Le Génie Civil* the results of experiments on the electric resistance of steel wires of different composition. The wires were 3 mm. diameter. The results are given below, the resistance being that of 1 kilometre of wire 1 square mm. in section.

	Carbon.	Silicon.	Sulphur.	Phosphorus.	Manganese.	Total.	Electric Resistance, Ohms.
1......	0.090	0.020	0.050	0.030	0.210	0.410	127.7
2......	0.100	0.020	0.050	0.040	0.240	0.450	133.0
3......	0.100	0.020	0.060	0.040	0.260	0.480	137.5
4......	0.100	0.020	0.050	0.050	0.310	0.530	140.3
5......	0.120	0.030	0.070	0.050	0.330	0.600	142.7
6......	0.110	0.030	0.060	0.060	0.350	0.610	144.5
7......	0.100	0.020	0.070	0.040	0.400	0.630	149.0
8......	0.120	0.020	0.070	0.070	0.400	0.680	150.3
9......	0.110	0.030	0.060	0.060	0.490	0.750	156.0
10......	0.140	0.030	0.060	0.080	0.540	0.850	173.0

An examination of these series of figures shows that the purer and softer steel the better is its electric conductivity, and, furthermore, that manganese is the element which most influences the conductivity.

Specific Gravity of Soft Steel. (W. Kent, Trans. A. I. M. E., xiv. 585.)—Five specimens of boiler-plate of C. 0.14, P. 0.03 gave an average sp. gr. of 7.932, maximum variation 0.008. The pieces were first planed to remove all possible scale indentations, then filed smooth, then cleaned in dilute sulphuric acid, and then boiled in distilled water, to remove all traces of air from the surface.

The figures of specific gravity thus obtained by careful experiment on bright, smooth pieces of steel are, however, too high for use in determining the weights of rolled plates for commercial purposes. The actual average thickness of these plates is always a little less than is shown by the calipers, on account of the oxide of iron on the surface, and because the surface is not perfectly smooth and regular. A number of experiments on commercial plates, and comparison of other authorities, led to the figure 7.854 as the average specific gravity of open-hearth boiler-plate steel. This figure is easily remembered as being the same figure with change of position of the decimal point (.7854) which expresses the relation of the area of a circle to that of its circumscribed square. Taking the weight of a cubic foot of water at 62° F. as 62.36 lbs. (average of several authorities), this figure gives 489.775 lbs. as the weight of a cubic foot of steel, or the even figure, 490 lbs., may be taken as a convenient figure, and accurate within the limits of the error of observation.

A common method of approximating the weight of iron plates is to consider them to weigh 40 lbs. per square foot one inch thick. Taking this weight and adding 2% gives almost exactly the weight of steel boiler-plate given above ($40 \times 12 \times 1.02 = 489.6$ lbs. per cubic foot).

Occasional Failures of Bessemer Steel.—G. H. Clapp and A. E. Hunt, in their paper on "The Inspection of Materials of Construction in the United States" (Trans. A. I. M. E., vol. xix), say: Numerous instances could be cited to show the unreliability of Bessemer steel for structural purposes. One of the most marked, however, was the following: A 12-in. I-beam weighing 30 lbs. to the foot, 20 feet long, on being unloaded from a car broke in two about 6 feet from one end.

The analyses and tensile tests made do not show any cause for the failure. The cold and quench bending tests of both the original ¾-in. round test-pieces, and of pieces cut from the finished material, gave satisfactory results; the cold-bending tests closing down on themselves without sign of fracture.

Numerous other cases of angles and plates that were so hard in places as to break off short in punching, or, what was worse, to break the punches, have come under our observation, and although makers of Bessemer steel claim that this is just as likely to occur in open-hearth as in Bessemer steel, we have as yet never seen an instance of failure of this kind in open-hearth steel having a composition such as C 0.25%, Mn 0.70%, P 0.80%.

J. W. Wailes, in a paper read before the Chemical Section of the British Association for the Advancement of Science, in speaking of mysterious failures of steel, states that investigation shows that "these failures occur in steel of one class, viz., soft steel made by the Bessemer process."

Segregation in Steel Ingots. (A. Pourcel, Trans. A. I. M. E. 1893.)—H. M. Howe, in his "Metallurgy of Steel," gives a *résumé* of observations, with the results of numerous analyses, bearing upon the phenomena of segregation.

In 1881 Mr. Stubbs, of Manchester, showed the heterogeneous results of analyses made upon different parts of an ingot of large section.

A test-piece taken 24 inches from the head of the ingot 7.5 feet in length gave by analysis very different results from those of a test-piece taken 30 inches from the bottom.

	C.	Mn.	Si.	S.	P.
Top	0.92	0.535	0.043	0.161	0.261
Bottom	0.37	0.498	0.006	0.025	0.096

Windsor Richards says he had often observed in test-pieces taken from different points of one plate variations of 0.05% of carbon. Segregation is specially pronounced in an ingot in its central portion, and around the space of the piping.

It is most observable in large ingots, but in blocks of smaller weight and limited dimensions, subjected to the influence of solidification as rapid as casting within thick walls will permit, it may still be observed distinctly. An ingot of Martin steel, weighing about 1000 lbs., and having a height of 1.10 feet and a section of 10.24 inches square, gave the following:

	C.	S.	P.	Mn.
1. Upper section:				
Border	0.330	0.040	0.033	0.420
Centre	0.530	0.077	0.057	0.430
2. Lower section:	C.	S.	P.	Mn.
Border	0.280	0.029	0.016	0.390
Centre	0.290	0.030	0.038	0.390
3. Middle section:	C.	S.	P.	Mn.
Border	0.320	0.025	0.025	0.400
Centre	0.320	0.048	0.048	0.40[illegible]

Segregation is less marked in ingots of extra-soft metal cast in cast-iron moulds of considerable thickness. It is, however, still important, and explains the difference often shown by the results of tests on pieces taken from different portions of a plate. Two samples, taken from the sound part of a flat ingot, one on the outside and the other in the centre, 7.9 inches from the upper edge, gave:

	C.	S.	P.	Mn.
Centre	0.14	0.053	0.072	0.573
Exterior	0.11	0.036	0.027	0.610

Manganese is the element most uniformly disseminated in hard or soft steel.

For cannon of large calibre, if we reject, in addition to the part cast in sand and called the *masselotte* (sinking-head), one third of the upper part of the ingot, we can obtain a tube practically homogeneous in composition, because the central part is naturally removed by the boring of the tube. With extra-soft steels, destined for ship- or boiler-plates, the solution for practically perfect homogeneity lies in the obtaining of a metal more closely deserving its name of extra-soft metal.

The injurious consequences of segregation must be suppressed by reducing, as far as possible, the elements subject to liquation.

Earliest Uses of Steel for Structural Purposes. (G. G. Mehrtens, Trans. A. S. C. E. 1893).—The Pennsylvania Railroad Company first introduced Bessemer steel in America in locomotive boilers in the year 1863, but the steel was too hard and brittle for such use. The first plates made for steel boilers had a tenacity of 85,000 to 92,000 lbs. and an elongation of but 7% to 10%. The results were not favorable, and the steel works were soon forced to offer a material of less tenacity and more ductility. The requirements were therefore reduced to a tenacity of 78,000 lbs. or less, and the elongation was increased to 15% or more. Even with this, between the years 1870 and 1880, many explosions occurred and many careful examinations were made to determine their cause. It was found on examining the rivet-holes that there were incipient changes in the metal, many cracks around them, and points near them were corroded with rust, all caused by the shock of tools in manufacturing. It was evident that the material was unsuitable, and that the treatment must be changed. In the beginning of 1878, Mr. Parker, chief engineer of the Lloyds, stated that there was then but one English steamer in possession of a steel boiler; a year later there were 120. In 1878 there were but five large English steamers built of steel, while in 1883 there were 116 building. The use of Bessemer steel in bridge-building was tried first on the Dutch State railways in 1863-64, then in England and Austria. In 1874 a bridge was built of Bessemer steel in Austria. The first use of cast steel for bridges was in America, for the St. Louis Arch Bridge and for the wire of the East River Bridge. These gave an impetus to the use of ingot metal, and before 1880 the Glasgow and Plattsmouth Bridges over the Missouri River were also built of ingot metal. Steel eye-bars were applied for the first time in the Glasgow Bridge. Since 1880 the introduction of mild steel in all kinds of engineering structures has steadily increased.

STEEL CASTINGS.

(E. S. Cramp, Engineering Congress, Dept. of Marine Eng'g, Chicago, 1893.)

In 1891 American steel-founders had successfully produced a considerable variety of heavy and difficult castings, of which the following are the most noteworthy specimens:

Bed-plates up to 24,000 lbs.; stern-posts up to 54,000 lbs.; stems up to 21,000 lbs.; hydraulic cylinders up to 11,000 lbs.; shaft-struts up to 32,000 lbs.; hawse-pipes up to 7500 lbs.; stern-pipes up to 8000 lbs.

The percentage of success in these classes of castings since 1890 has ranged from 65% in the more difficult forms to 90% in the simpler ones; the tensile strength has been from 62,000 to 78,000 lbs., elongation from 15% to 25%. The best performance recorded is that of a guide, cast in January, 1893, which developed 84,000 lbs. tensile strength and 15.6% elongation.

The first steel castings of which anything is generally known were crossing-frogs made for the Philadelphia & Reading R. R. in July, 1867, by the William Butcher Steel Works, now the Midvale Steel Co. The moulds were made of a mixture of ground fire-brick, black-lead crucible-pots ground fine, and fire-clay, and washed with a black-lead wash. The steel was melted in crucibles, and was about as hard as tool steel. The surface of these castings was very smooth, but the interior was very much honeycombed. This was before the days when the use of silicon was known for solidifying steel. The sponginess, which was almost universal, was a great obstacle to their general adoption.

The next step was to leave the ground pots out of the moulding mixture and to wash the mould with finely ground fire-brick. This was a great improvement, especially in very heavy castings; but this mixture still clung so strongly to the casting that only comparatively simple shapes could be made with certainty. A mould made of such a mixture became almost as hard as fire-brick, and was such an obstacle to the proper shrinkage of castings, that, when at all complicated in shape, they had so great a tendency to crack as to make their successful manufacture almost impossible. By this time the use of silicon had been discovered, and the only obstacle in the way of making good castings was a suitable moulding mixture. This was ultimately found in mixtures having the various kinds of silica sand as the principal constituent.

One of the most fertile sources of defects in castings is a bad design. Very intricate shapes can be cast successfully if they are so designed as to cool uniformly. Mr. Cramp says while he is not yet prepared to state that anything that can be cast successfully in iron can be cast in steel, indications seem to point that way in all cases where it is possible to put on suitable sinking-heads for feeding the casting.

H. L. Gantt (Trans. A. S. M. E., xii. 710) says: Steel castings not only shrink much more than iron ones, but with less regularity. The amount of shrinkage varies with the composition and the heat of the metal; the hotter the metal the greater the shrinkage; and, as we get smoother castings from hot metal, it is better to make allowance for large shrinkage and pour the metal as hot as possible. Allow 3/16 or ¼ in. per ft. in length for shrinkage, and ¼ in. for finish on machined surfaces, except such as are cast "up." Cope surfaces which are to be machined should, in large or hard castings, have an allowance of from ⅜ to ½ in. for finish, as a large mass of metal slowly rising in a mould is apt to become crusty on the surface, and such a crust is sure to be full of imperfections. On small, soft castings ⅛ in. on drag side and ¼ in. on cope side will be sufficient. No core should have less than ¼ in. finish on a side and very large ones should have as much as ½ in. on a side. Blow-holes can be entirely prevented in castings by the addition of manganese and silicon in sufficient quantities; but both of these cause brittleness, and it is the object of the conscientious steelmaker to put no more manganese and silicon in his steel than is just sufficient to make it solid. The best results are arrived at when all portions of the castings are of a uniform thickness, or very nearly so.

The following table will illustrate the effect of annealing on tensile strength and elongation of steel castings:

Carbon.	Unannealed.		Annealed.	
	Tensile Strength.	Elongation.	Tensile Strength.	Elongation.
.23%	68,738	22.40%	67,210	31.40%
.37	85,540	8.20	82,228	21.80
.53	90,121	2.35	106,415	9.80

The proper annealing of large castings takes nearly a week.

The proper steel for roll pinions, hammer dies, etc., seems to be that containing about .60% of carbon. Such castings, properly annealed, have worn well and seldom broken. Miscellaneous gearing should contain carbon .40% to 60%, gears larger in diameter being softest. General machinery castings should, as a rule, contain less than .40% of carbon, those exposed to great shocks containing as low at .20% of carbon. Such castings will give a tensile strength of from 60,000 to 80,000 lbs. per sq. in. and at least 15% extension in a 2 in. long specimen. Machinery and hull castings for war-vessels for the United States Navy, as well as carriages for naval guns, contain from .20% to .30% of carbon.

The following is a partial list of castings in which steel seems to be rapidly taking the place of iron: Hydraulic cylinders, crossheads and pistons for large engines, roughing rolls, rolling-mill spindles, coupling-boxes, roll pinions, gearing, hammer-heads and dies, riveter stakes, castings for ships, car-couplers, etc.

For description of methods of manufacture of steel castings by the Bessemer, open-hearth, and crucible processes, see paper by P. G. Salom, Trans. A. I. M. E. xiv, 118.

Specifications for steel castings issued by the U. S. Navy Department, 1889 (abridged): Steel for castings must be made by either the open-hearth or the crucible process, and must not show more than .06% of phosphorus. All castings must be annealed, unless otherwise directed. The tensile strength of steel castings shall be at least 60,000 lbs., with an elongation of at least 15% in 8 in. for all castings for moving parts of the machinery, and at least 10% in 8 in. for other castings. Bars 1 in. sq. shall be capable of bending cold, without fracture, through an angle of 90°, over a radius not greater than 1½ in. All castings must be sound, free from injurious roughness, sponginess, pitting, shrinkage, or other cracks, cavities, etc.

Pennsylvania Railroad specifications, 1888: Steel castings should have a tensile strength of 70,000 lbs. per sq. in. and an elongation of 15% in section originally 2 in. long. Steel castings will not be accepted if tensile strength falls below 60,000 lbs., nor if the elongation is less than 12%, nor if castings have blow-holes and shrinkage cracks. Castings weighing 80 lbs. or more must have cast with them a strip to be used as a test-piece. The dimensions of this strip must be ¾ in. sq. by 12 in. long.

MANGANESE, NICKEL, AND OTHER "ALLOY" STEELS.

Manganese Steel. (H. M. Howe, Trans. A. S. M. E., vol. xii.)—Manganese steel is an alloy of iron and manganese, incidentally, and probably unavoidably, containing a considerable proportion of carbon.

The effect of small proportions of manganese on the hardness, strength, and ductility of iron is probably slight. The point at which manganese begins to have a predominant effect is not known: it may be somewhere about 2.5%. As the proportion of manganese rises above 2.5% the strength and ductility diminish, while the hardness increases. This effect reaches a maximum with somewhere about 6% of manganese. When the proportion of this element rises beyond 6% the strength and ductility both increase, while the hardness diminishes slightly, the maximum of both strength and ductility being reached with about 14% of manganese. With this proportion the metal is still so hard that it is very difficult to cut it with steel tools. As the proportion of manganese rises above 15% the ductility falls off abruptly, the strength remaining nearly constant till the manganese passes 18%, when it in turn diminishes suddenly.

Steel containing from 4% to 6.5% of manganese, even if it have but 0.37% of carbon, is reported to be so extremely brittle that it can be powdered under a hand-hammer when cold; yet it is ductile when hot.

Manganese steel is very free from blow-holes; it welds with great difficulty; its toughness is increased by quenching from a yellow heat; its electric resistance is enormous, and very constant with changing temperature; it is low in thermal conductivity. Its remarkable combination of great hardness, which cannot be materially lessened by annealing, and great tensile strength, with astonishing toughness and ductility, at once creates and limits its usefulness. The fact that manganese steel cannot be softened, that it ever remains so hard that it can be machined only with great difficulty, sets up a barrier to its usefulness.

The following comparative results of abrasion tests of manganese and other steel were reported by T. T. Morrell:

ABRASION BY PRESSURE AGAINST A REVOLVING HARDENED-STEEL SHAFT.

Loss of weight of manganese steel	1.0
" blue-tempered hard tool steel	0.4
" annealed hard tool steel	7.5
" hardened Otis boiler-plate steel	7.0
" annealed " " "	14.0

ABRASION BY AN EMERY-WHEEL.

Loss of weight of hard manganese-steel wheels	1.00
" softer " "	1.19
" hardest carbon-steel wheels	1.23
" soft " "	2.85

The hardness of manganese steel seems to be of an anomalous kind. The alloy is hard, but under some conditions not rigid. It is very hard in its resistance to abrasion; it is not always hard in its resistance to impact.

Manganese steel forges readily at a yellow heat, though at a bright white heat it crumbles under the hammer. But it offers greater resistance to deformation, i.e., it is harder when hot, than carbon steel.

The most important single use for manganese-steel is for the pins which hold the buckets of elevator dredges. Here abrasion chiefly is to be resisted.

Another important use is for the links of common chain-elevators.

As a material for stamp-shoes, for horse-shoes, for the knuckles of an automatic car-coupler, manganese steel has not met expectations.

Manganese steel has been regularly adopted for the blades of the Cyclone pulverizer. Some manganese-steel wheels are reported to have run over 300,000 miles each without turning, on a New England railroad.

Nickel Steel.—The remarkable tensile strength and ductility of nickel steel, as shown by the test-bars and the behavior of nickel-steel armor-plate under shot tests, are witness of the valuable qualities conferred upon steel by the addition of a few per cent of nickel.

The following tests were made on nickel steels by Mr. Maunsel White of the Bethlehem Iron Company (*Eng. & M. Jour.*, Sept. 16, 1893.):

Steel	Specimen from—	Diam., in.	Length, in.	Tensile Str'gth, lbs. per sq. in.	Elastic Limit, lbs. per sq. in.	Elongation, %	Reduction of Area, %	
3¼% nickel steel.	Forged bars.*	.625	4	276,800	......	2.75	....	Special treatment.
		"	"	246,595	....	4.25	6.0	
		"	"	105,300	....	19.25	55.0	Annealed.
	1¼-in. round rolled bar.†	.564	4	142,800	74,000	13.0	28.2	
		"	"	143,200	74,000	12.32	27.6	
		"	"	117,600	64,000	17.0	46.0	
		"	"	119,200	65,000	16.66	42.1	
		"	"	91,600	51,000	22.25	53.2	
		"	"	91,200	51,000	21.62	53.4	
		"	"	85,200	53,000	21.82	49.5	
		"	"	86,000	48,000	21.25	47.4	
27% nickel steel.	1¼-in. sq. bar, rolled.‡	.798	8	115,464	51,820	36.25	66.23	
		"	"	112,600	60,000	37.87	62.82	
		"	"	102,010	39,180	41.37	69.59	Annealed.
		"	"	102,510	40,200	44.00	68.34	"
	1-in. round bar, rolled.§	.500	2	114,590	56,020	47.25	68.4	
		"	"	115,610	59,080	45.25	62.3	
		"	"	105,240	45,170	49.65	72.8	Annealed.
		"	"	106,780	45,170	55.50	68.6	"

* Forged from 6-in. ingot to ⅝ in. diam., with conical heads for holding.
† Showing the effect of varying carbon.
‡ Rolled down from 14-in. ingot to 1¼-in. square billet, and turned to size.
§ Rolled down from 14-in. ingot to 1-in. round, and turned to size.

Nickel steel has shown itself to be possessed of some exceedingly valuable properties; these are, resistance to cracking, high elastic limit, and homogeneity. Resistance to cracking, a property to which the name of non fissibility has been given, is shown more remarkably as the percentage of nickel increases. Bars of 27% nickel illustrate this property. A 1¼-in. square bar was nicked ¼ in. deep and bent double on itself without further fracture than the splintering off, as it were, of the nicked portion. Sudden failure or rupture of this steel would be impossible; it seems to possess the toughness of rawhide with the strength of steel. With this percentage of nickel the steel is practically non-corrodible and non-magnetic. The resistance to cracking shown by the lower percentages of nickel is best illustrated in the many trials of nickel-steel armor.

The elastic limit rises in a very marked degree with the addition of about 3% of nickel, the other physical properties of the steel remaining unchanged or perhaps slightly increased.

In such places (shafts, axles, etc.) where failure is the result of the fatigue of the metal this higher elastic limit of nickel steel will tend to prolong indefinitely the life of the piece, and at the same time, through its superior toughness, offer greater resistance to the sudden strains of shock.

Howe states that the hardness of nickel steel depends on the proportion of nickel and carbon jointly, nickel up to a certain percentage increasing the hardness, beyond this lessening it. Thus while steel with 2% of nickel and 0.90% of carbon cannot be machined, with less than 5% nickel it can be worked cold readily, provided the proportion of carbon be low. As the proportion of nickel rises higher, cold-working becomes less easy. It forges easily whether it contain much or little nickel.

The presence of manganese in nickel steel is most important, as it appears that without the aid of manganese in proper proportions, the conditions of treatment would not be successful.

Tests of Nickel Steel.—Two heats of open-hearth steel were made by the Cleveland Rolling Mill Co., one ordinary steel made with 9000 lbs. each scrap and pig, and 165 lbs. ferro-manganese, the other the same with the addition of 3%, or 540 lbs. of nickel. Tests of six plates rolled from each heat., 0.24 to 0.3 in. thick, gave results as follows:

Ordinary steel, T. S. 52,500 to 56,500; E. L. 32,800 to 37,900; elong. 26 to 32%.
Nickel steel, " 63,370 to 67,100; " 47,100 to 48,200; " 23¼ to 26%.

The nickel steel averages 31% higher in elastic limit, 20% higher in ultimate tensile strength, with but slight reduction in ductility. (*Eng. & M. Jour.*, Feb. 25, 1893.)

Aluminum Steel.—R. A. Hadfield (Trans. A. I. M. E. 1890) says: Aluminum appears to be of service as an addition to baths of molten iron or steel unduly saturated with oxides, and this in properly regulated steel manufacture should not often occur. Speaking generally, its *rôle* appears to be similar to that of silicon, though acting more powerfully. The statement that aluminum lowers the melting-point of iron seems to have no foundation in fact. If any increase of heat or fluidity takes place by the addition of small amounts of aluminum, it may be due to evolution of heat, owing to oxidation of the aluminum, as the calorific value of this metal is very high—in fact, higher than silicon. According to Berthollet, the conversion of aluminum to Al_2O_3 equals 7900 cal.; silicon to SiO_2 is stated as 7800.

The action of aluminum may be classed along with that of silicon, sulphur, phosphorus, arsenic, and copper, as giving no increase of hardness to iron, in contradistinction to carbon, manganese, chromium, tungsten, and nickel. Therefore, whilst for some special purposes aluminum may be employed in the manufacture of iron, at any rate with our present knowledge of its properties, this use cannot be large, especially when taking into consideration the fact of its comparatively high price. Its special advantage seems to be that it combines in itself the advantages of both silicon and manganese; but so long as alloys containing these metals are so cheap and aluminum dear, its extensive use seems hardly probable.

J. E. Stead, in discussion of Mr. Hadfield's paper, said: Every one of our trials has indicated that aluminum can kill the most fiery steel, providing, of course, that it is added in sufficient quantity to combine with all the oxygen which the steel contains. The metal will then be absolutely dead, and will pour like dead-melted silicon steel. If the aluminum is added as metallic aluminum, and not as a compound, and if the addition is made just before the steel is cast, 1/10% is ample to obtain perfect solidity in the steel.

Chrome Steel. (F. L. Garrison, *Jour. F. I.*, Sept. 1891.)—Chromium increases the hardness of iron, perhaps also the tensile strength and elastic limit, but it lessens its weldibility.

Ferro chrome, according to Berthier, is made by strongly heating the mixed oxides of iron and chromium in brasqued crucibles, adding powdered charcoal if the oxide of chromium is in excess, and fluxes to scorify the earthy matter and prevent oxidation. Chromium does not appear to give steel the power of becoming harder when quenched or chilled. Howe states that chrome steels forge more readily than tungsten steels, and when not containing over 0.5 of chromium nearly as well as ordinary carbon steels of like percentage of carbon. On the whole the status of chrome steel is not satisfactory. There are other steel alloys coming into use, which are so much better, that it would seem to be only a question of time when it will drop entirely out of the race. Howe states that many experienced chemists have found no chromium, or but the merest traces, in chrome steel sold in the markets.

J. W. Langley (Trans. A. S. C. E. 1892) says: Chromium, like manganese, is a true hardener of iron even in the absence of carbon. The addition of 1% or 2% of chromium to a carbon steel will make a metal which gets excessively hard. Hitherto its principal employment has been in the production of chilled shot and shell. Powerful molecular stresses result during cooling, and the shells frequently break spontaneously months after they are made.

Tungsten Steel—Mushet Steel. (J. B. Nau, *Iron Age*, Feb. 11, 1892.)—By incorporating simultaneously carbon and tungsten in iron, it is possible to obtain a much harder steel than with carbon alone, without danger of an extraordinary brittleness in the cold metal or an increased difficulty in the working of the heated metal.

When a special grade of hardness is required, it is frequently the custom to use a high tungsten steel, known in England as special steel. A specimen from Sheffield, used for chisels, contained 9.3% of tungsten, 0.7% of silver, and 0.6% of carbon. This steel, though used with advantage in its untempered state to turn chilled rolls, was not brittle; nevertheless it was hard enough to scratch glass.

A sample of Mushet's special steel contained 8.3% of tungsten and 1.73% of manganese. The hardness of tungsten steel cannot be increased by the ordinary process of hardening.

The only operation that it can be submitted to when cold is grinding. It has to be given its final shape through hammering at a red heat, and even then, when the percentage of tungsten is high, it has to be treated very carefully; and in order to avoid breaking it, not only is it necessary to reheat it several times while it is being hammered, but when the tool has acquired the desired shape hammering must still be continued gently and with numerous blows until it becomes nearly cold. Then only can it be cooled entirely.

Tungsten is not only employed to produce steel of an extraordinary hardness, but more especially to obtain a steel which, with a moderate hardness, allies great toughness, resistance, and ductility. Steel from Assailly, used for this purpose, contained carbon, 0.52%; silicon, 0.04%; tungsten, 0.3%; phosphorus, 0.04%; sulphur, 0.005%.

Mechanical tests made by Styffe gave the following results:

Breaking load per square inch of original area, pounds..	172,424
Reduction of area, per cent	0.54
Average elongation after fracture, per cent	13

According to analyses made by the Duc de Luynes of ten specimens of the celebrated Oriental damasked steel, eight contained tungsten, two of them in notable quantities (0.518% to 1%), while in all of the samples analyzed nickel was discovered ranging from traces to nearly 4%.

Stein & Schwartz of Philadelphia, in a circular say: It is stated that tungsten steel is suitable for the manufacture of steel magnets, since it retains its magnetism longer than ordinary steel. Mr. Kniesche has made tungsten up to 98% fine a specialty. Dr. Heppe, of Leipsig, has written a number of articles in German publications on the subject. The following instructions are given concerning the use of tungsten: In order to produce cast iron possessing great hardness an addition of one half to one and one half of tungsten is all that is needed. For bar iron it must be carried up to 1% to 2%, but should not exceed 2½%. For puddled steel the range is larger, but an addition beyond 3½% only increases the hardness, so that it is brought up to 1½% only for special tools, coinage dies, drills, etc. For tires 2½% to 5% have proved best, and for axles ½ to 1½%. Cast steel to which tungsten has been added needs a higher temperature for tempering than ordinary steel, and should be hardened only between yellow, red, and white. Chisels made of tungsten steel should be drawn between cherry-red and blue, and stand well on iron and steel. Tempering is best done in a mixture of 5 parts of yellow rosin, 3 parts of tar, and 2 parts of tallow, and then the article is once more heated and then tempered as usual in water of about 15° C.

Fluid-compressed Steel by the "Whitworth Process." (Proc. Inst. M. E., May, 1887, p. 167.)—In this system a gradually increasing pressure up to 6 or 8 tons per square inch is applied to the fluid ingot, and within half an hour or less after the application of the pressure the column of fluid steel is shortened 1½ inch per foot or one-eighth of its length; the pressure is then kept on for several hours, the result being that the metal is compressed into a perfectly solid and homogeneous material, free from blow-holes.

In large gun-ring ingots during cooling the carbon is driven to the centre, the centre containing 0.8 carbon and the outer ring 0.3. The centre is bored out until a test shows that the inside of the ring contains the same percentage of carbon as the outside

Fluid-compressed steel is made by the Bethlehem Iron Co. for gun and other heavy forgings.

CRUCIBLE STEEL.

Selection of Grades by the Eye, and Effect of Heat Treatment. (J. W. Langley, *Amer. Chemist*, November, 1876.)—In 1874, Miller, Metcalf & Parkin, of Pittsburgh, selected eight samples of steel which were believed to form a set of graded specimens, the order being based on the quantity of carbon which they were supposed to contain. They were numbered from one to eight. On analysis, the quantity of carbon was found to follow the order of the numbers, while the other elements present—silicon, phosphorus, and sulphur—did not do so. The method of selection is described as follows:

The steel is melted in black-lead crucibles capable of holding about eighty pounds; when thoroughly fluid it is poured into cast-iron moulds, and when cold the top of the ingot is broken off, exposing a freshly-fractured surface. The appearance presented is that of confused groups of crystals, all appearing to have started from the outside and to have met in the centre; this general form is common to all ingots of whatever composition, but to the trained eye, and only to one long and critically exercised, a minute but indescribable difference is perceived between varying samples of steel, and this difference is now known to be owing almost wholly to variations in the amount of combined carbon, as the following table will show. Twelve samples selected by the eye alone, and analyses of drillings taken direct from the ingot before it had been heated or hammered, gave results as below:

Ingot Nos.	Iron by Diff.	Carbon.	Diff. of Carbon.	Silicon.	Phos.	Sulph.
1	99.614	.302	....	.019	.047	.018
2	99.455	.490	.188	.034	.005	.016
3	99.363	.529	.039	.043	.047	.018
4	99.270	.649	.120	.039	.030	.012
5	99.119	.801	.152	.029	.035	.016
6	99.086	.841	.040	.039	.024	.010
7	99.044	.867	.026	.057	.014	.018
8	99.040	.871	.004	.053	.024	.012
9	98.900	.955	.084	.059	.070	.016
10	98.861	1.005	.050	.068	.084	.012
11	98.752	1.058	.053	.120	.064	.006
12	98.884	1.079	.021	.039	.044	.004

Here the carbon is seen to increase in quantity in the order of the numbers, while the other elements, with the exception of total iron, bear no relation to the numbers on the samples. The mean difference of carbon is .071. In mild steels the discrimination is less perfect.

The appearance of the fracture by which the above twelve selections were made can only be seen in the cold ingot before any operation, except the original one of casting, has been performed upon it. As soon as it is hammered, the structure changes in a remarkable manner, so that all trace of the primitive condition appears to be lost.

Another method of rendering visible to the eye the molecular and chemical changes which go on in steel is by the process of hardening or tempering. When the metal is heated and plunged into water it acquires an increase of hardness, but a loss of ductility. If the heat to which the steel has been raised just before plunging is too high, the metal acquires intense hardness, but it is so brittle as to be worthless; the fracture is of a bright, granular, or sandy character. In this state it is said to be burned, and it cannot again be restored to its former strength and ductility by annealing; it is ruined for all practical purposes, but in just this state it again shows differences of structure corresponding with its content in carbon. The nature of these changes can be illustrated by plunging a bar highly heated at one end and cold at the other into water, and then breaking it off in pieces of equal length, when the fractures will be found to show appearances characteristic of the temperature to which the sample was raised.

The specific gravity of steel is influenced not only by its chemical analysis, but by the heat to which it is subjected, as is shown by the following table (densities referred to 60° F.):

Specific gravities of twelve samples of steel from the ingot; also of six hammered bars, each bar being overheated at one end and cold at the other, in this state plunged into water, and then broken into pieces of equal length.

	1	2	3	4	5	6	7	8	9	10	11	12
Ingot.......	7.855	7.836	7.841	7.829	7.838	7.834	7.819	7.818	7.813	7.807	7.803	7.805
Bar:												
*Burned 1.	....	.....	7.818	7.791	.....	7.789	.....	7.752	....	7.744	.....	7.690
2.	.....	.....	7.814	7.811	.. .	7.784	.....	7.755	.....	7.749	....	7.741
3.	....	.....	7.823	7.880	.....	7.780	.....	7.758	.....	7.755	.....	7.769
4.	. ..	.....	7.826	7.849	.....	7.808	.....	7.773	.....	7.789	.....	7.798
5.	.....	.....	7.831	7.806	.....	7.812	.....	7.790	.....	7.812	.....	7.811
Cold 6.	.....	.....	7.844	7.824	.. .	7.829	.....	7.825	.....	7.826	.....	7.825

* Order of samples from bar.

Effect of Heat on the Grain of Steel. (W. Metcalf,—Jeans on Steel, p. 642.)—A simple experiment will show the alteration produced in a high-carbon steel by different methods of hardening. If a bar of such steel be nicked at about 9 or 10 places, and about half an inch apart, a suitable specimen is obtained for the experiment. Place one end of the bar in a good fire, so that the first nicked piece is heated to whiteness, while the rest of the bar, being out of the fire, is heated up less and less as we approach the other end. As soon as the first piece is at a good white heat, which of course burns a high carbon steel, and the temperature of the rest of the bar gradually passes down to a very dull red, the metal should be taken out of the fire and suddenly plunged in cold water, in which it should be left till quite cold. It should then be taken out and carefully dried. An examination with a file will show that the first piece has the greatest hardness, while the last piece is the softest, the intermediate pieces gradually passing from one condition to the other. On now breaking off the pieces at each nick it will be seen that very considerable and characteristic changes have been produced in the appearance of the metal. The first burnt piece is very open or crystalline in fracture; the succeeding pieces become closer and closer in the grain until one piece is found to possess that perfectly even grain and velvet-like appearance which is so much prized by experienced steel users. The first pieces also, which have been too much hardened, will probably be cracked; those at the other end will not be hardened through. Hence if it be desired to make the steel hard and strong, the temperature used must be high enough to harden the metal through, but not sufficient to open the grain.

Changes in Ultimate Strength and Elasticity due to Hammering, Annealing, and Tempering. (J. W. Langley, Trans. A. S. C. E. 1892.)—The following table gives the result of tests made on some round steel bars, all from the same ingot, which were tested by tensile stresses, and also by bending till fracture took place:

Number.	Treatment.	Angle of cold bend, degrees.	Carbon. Total.	Carbon. Semi-graphite.	Diameter, in.	Elastic limit, pounds per square inch.	Tensile, pounds per square inch.	Elongation, per cent.	Red area, per cent.
1	Cold-hammered bar	153	1.25	.47	.575	92,420	141,500	2.00	2.42
2	Bar drawn black....	75	1.25	.47	.577	114,700	138,400	6.00	12.45
3	Bar annealed	175	1.31	.70	.580	68,110	98,410	10.00	11.69
4	Bar hardened and drawn black	30	1.09	.36	.578	152,800	248,700	8.33	17.9

The total carbon given in the table was found by the color test, which is affected, not only by the total carbon, but by the condition of the carbon.

The analysis of the steel was:

Silicon	.242	Manganese........................	.24
Phosphorus........................	.02	Carbon (true total carbon, by combustion)........................	1.31
Sulphur	.009		

Heating Tool Steel. (Crescent Steel Co., Pittsburg, Pa.)—There are three distinct stages or times of heating: First, for forging; second, for hardening; third, for tempering.

The first requisite for a good heat for forging is a clean fire and plenty of fuel, so that jets of hot air will not strike the corners of the piece; next, the fire should be regular, and give a good uniform heat to the whole part to be forged. It should be keen enough to heat the piece as rapidly as may be, and allow it to be thoroughly heated through, without being so fierce as to overheat the corners.

Steel should not be left in the fire any longer than is necessary to heat it clear through, as "soaking" in fire is very injurious; and, on the other hand, it is necessary that it should be hot through, to prevent surface cracks.

By observing these precautions a piece of steel may always be heated safely, up to even a bright yellow heat, when there is much forging to be done on it.

The best and most economical of welding fluxes is clean, crude borax, which should be first thoroughly melted and then ground to fine powder.

After the steel is properly heated, it should be forged to shape as quickly as possible; and just as the red heat is leaving the parts intended for cutting edges, these parts should be refined by rapid, light blows, continued until the red disappears.

For the second stage of heating, for hardening, great care should be used: first, to protect the cutting edges and working parts from heating more rapidly than the body of the piece; next, that the whole part to be hardened be heated uniformly through, without any part becoming visibly hotter than the other. A uniform heat, as low as will give the required hardness, is the best for hardening.

For every variation of heat, which is great enough to be seen, there will result a variation in grain, which may be seen by breaking the piece; and for every such variation in temperature, there is a very good chance for a crack to be seen. Many a costly tool is ruined by inattention to this point.

The effect of too high heat is to open the grain; to make the steel coarse. The effect of an irregular heat is to cause irregular grain, irregular strains, and cracks.

As soon as the piece is properly heated for hardening, it should be promptly and thoroughly quenched in plenty of the cooling medium, water, brine, or oil, as the case may be.

An abundance of the cooling bath, to do the work quickly and uniformly all over, is very necessary to good and safe work.

To harden a large piece safely a running stream should be used.

Much uneven hardening is caused by the use of too small baths.

For the third stage of heating, to temper, the first important requisite is again uniformity. The next is time; the more slowly a piece is brought down to its temper, the better and safer is the operation.

When expensive tools are to be made it is a wise precaution to try small pieces of the steel at different temperatures, so as to find out how low a heat will give the necessary hardness. The lowest heat is the best for any steel.

Heating to Forge.—The trouble in the forge fire is usually uneven heat, and not too high heat. Suppose the piece to be forged has been put into a very hot fire, and forced as quickly as possible to a high yellow heat, so that it is almost up to the scintillating point. If this be done, in a few minutes the outside will be quite soft and in a nice condition for forging, while the middle parts will not be more than red-hot. Now let the piece be placed under the hammer and forged, and the soft outside will yield so much more readily than the hard inside, that the outer particles will be torn asunder, while the inside will remain sound.

Suppose the case to be reversed and the inside to be much hotter than the outside; that is, that the inside shall be in a state of semi-fusion, while the outside is hard and firm. Now let the piece be forged, and the outside will be all sound and the whole piece will appear perfectly good until it is cropped, and then it is found to be hollow inside

In either case, if the piece had been heated soft all through, or if it had been only red-hot all through, it would have forged perfectly sound.

In some cases a high heat is more desirable to save heavy labor but in every case where a fine steel is to be used for cutting purposes it must be borne in mind that very heavy forging refines the bars as they slowly cool, and if the smith heats such refined bars until they are soft, he raises the grain, makes them coarse, and he cannot get them fine again unless he has a very heavy steam-hammer at command and knows how to use it well.

Annealing. (Crescent Steel Co.)—Annealing or softening is accomplished by heating steel to a red heat and then cooling it very slowly, to prevent it from getting hard again.

The higher the degree of heat, the more will steel be softened, until the limit of softness is reached, when the steel is melted.

It does not follow that the higher a piece of steel is heated the softer it will be when cooled, no matter how slowly it may be cooled; this is proved by the fact that an ingot is always harder than a rolled or hammered bar made from it.

Therefore there is nothing gained by heating a piece of steel hotter than a good, bright, cherry-red; on the contrary, a higher heat has several disadvantages: First. If carried too far, it may leave the steel actually harder than a good red heat would leave it. Second. If a scale is raised on the steel, this scale will be harsh, granular oxide of iron, and will spoil the tools used to cut it. Third. A high scaling heat continued for a little time changes the structure of the steel, makes it brittle, liable to crack in hardening, and impossible to refine.

To anneal any piece of steel, heat it red-hot; heat it uniformly and heat it through, taking care not to let the ends and corners get too hot.

As soon as it is hot, take it out of the fire, the sooner the better, and cool it as slowly as possible. A good rule for heating is to heat it at so low a red that when the piece is cold it will still show the blue gloss of the oxide that was put there by the hammer or the rolls.

Steel annealed in this way will cut very soft; it will harden very hard, without cracking; and when tempered it will be very strong, nicely refined, and will hold a keen, strong edge.

Tempering.—Tempering steel is the act of giving it, after it has been shaped, the hardness necessary for the work it has to do. This is done by first hardening the piece, generally a good deal harder than is necessary, and then toughening it by slow heating and gradual softening until it is just right for work.

A piece of steel properly tempered should always be finer in grain than the bar from which it is made. If it is necessary, in order to make the piece as hard as is required, to heat it so hot that after being hardened the grain will be as coarse as or coarser than the grain in the original bar, then the steel itself is of too low carbon for the desired work.

If a great degree of hardness is not desired, as in the case of taps and most tools of complicated form, and it is found that at a moderate heat the tools are too hard and are liable to crack, the smith should first use a lower heat in order to save the tools already made, and then notify the steelmaker that his steel is too high, so as to prevent a recurrence of the trouble.

For descriptions of various methods of tempering steel, see "Tempering of Metals," by Joshua Rose, in App. Cyc. Mech., vol. ii. p. 863; also, "Wrinkles and Recipes," from the *Scientific American*. In both of these works Mr. Rose gives a "color scale," lithographed in colors, by which the following is a list of the tools in their order on the color scale, together with the approximate color and the temperature at which the color appears on brightened steel when heated in the air:

Scrapers for brass; *very pale yellow*, 430° F.
Steel-engraving tools.
Slight turning tools.
Hammer faces.
Planer tools for steel.
Ivory-cutting tools.
Planer tools for iron.
Paper-cutters.
Wood-engraving tools.
Bone cutting tools.
Milling-cutters; *straw yellow*, 460° F.
Wire-drawing dies.
Boring-cutters.
Leather-cutting dies.
Screw-cutting dies.
Inserted saw-teeth.
Taps.
Rock-drills.
Chasers.
Punches and dies.
Penknives.
Reamers.
Half-round bits.
Planing and moulding cutters.
Stone-cutting tools; *brown yellow*, 500° F.
Gouges.
Hand-plane irons.
Twist-drills.
Flat drills for brass.
Wood-boring cutters.
Drifts.
Coopers' tools.
Edging cutters; *light purple*, 530° F.
Augers.
Dental and surgical instruments.
Cold chisels for steel.
Axes; *dark purple*, 550° F.
Gimlets.
Cold chisels for cast iron.
Saws for bone and ivory.
Needles.
Firmer-chisels.
Hack-saws.
Framing-chisels.
Cold chisels for wrought iron.
Moulding and planing cutters to be filed.
Circular saws for metal.
Screw-drivers.
Springs.
Saws for wood.
Dark blue, 570° F.
Pale blue, 610°.
Blue tinged with green, 630°.

MECHANICS.

FORCE, STATICAL MOMENT, EQUILIBRIUM, ETC.

MECHANICS is the science that treats of the action of force upon bodies.

A Force is anything that tends to change the state of a body with respect to rest or motion. If a body is at rest, anything that tends to put it in motion is a force; if a body is in motion, anything that tends to change either its direction or its rate of motion is a force.

A force should always mean the pull, pressure, rub, attraction (or repulsion) of one body upon another, and always implies the existence of a simultaneous equal and opposite force exerted by that other body on the first body, i.e., the *reaction*. In no case should we call anything a force unless we can conceive of it as capable of measurement by a spring-balance, and are able to say from what other body it comes. (I. P. Church.)

Forces may be divided into two classes, extraneous and molecular: extraneous forces act on bodies from without; molecular forces are exerted between the neighboring particles of bodies.

Extraneous forces are of two kinds, pressures and moving forces: pressures simply tend to produce motion; moving forces actually produce motion. Thus, if gravity act on a fixed body, it creates pressure; if on a free body, it produces motion.

Molecular forces are of two kinds, attractive and repellent: attractive forces tend to bind the particles of a body together; repellent forces tend to thrust them asunder. Both kinds of molecular forces are continually exerted between the molecules of bodies, and on the predominance of one or the other depends the physical state of a body, as solid, liquid, or gaseous.

The Unit of Force used in engineering, by English writers, is the pound avoirdupois. (For some scientific purposes, as in electro-dynamics, forces are sometimes expressed in "absolute units." The absolute unit of force is that force which acting on a unit of mass during a unit of time produces a unit of velocity; in English measures, that force which acting on the mass whose weight is one pound in London will in one second produce a velocity of one foot per second = 1 ÷ 32.187 of the weight of the standard pound avoirdupois at London. In the French C. G. S. or centimetre-gramme second system it is the force which acting on the mass whose weight is one gramme at Paris will produce in one second a velocity of one centimetre per second. This unit is called a "dyne" = 1/981 gramme at Paris.)

Inertia is that property of a body by virtue of which it tends to continue in the state of rest or motion in which it may be placed, until acted on by some force.

Newton's Laws of Motion.—1st Law. If a body be at rest, it will remain at rest; or if in motion, it will move uniformly in a straight line till acted on by some force.

2d Law. If a body be acted on by several forces, it will obey each as though the others did not exist, and this whether the body be at rest or in motion.

3d Law. If a force act to change the state of a body with respect to rest or motion, the body will offer a resistance equal and directly opposed to the force. Or, to every action there is opposed an equal and opposite reaction.

Graphic Representation of a Force.—Forces may be represented geometrically by straight lines, proportional to the forces. A force is given when we know its intensity, its point of application, and the direction in which it acts. When a force is represented by a line, the length of the line represents its intensity; one extremity represents the point of application; and an arrow-head at the other extremity shows the direction of the force.

Composition of Forces is the operation of finding a single force whose effect is the same as that of two or more given forces. The required force is called the resultant of the given forces.

Resolution of Forces is the operation of finding two or more forces whose combined effect is equivalent to that of a given force. The required forces are called components of the given force.

The resultant of two forces applied at a point, and acting in the same direction, is equal to the sum of the forces. If two forces act in opposite directions, their resultant is equal to their difference, and it acts in the direction of the greater.

If any number of forces be applied at a point, some in one direction and others in a contrary direction, their resultant is equal to the sum of those that act in one direction, diminished by the sum of those that act in the opposite direction; or, the resultant is equal to the algebraic sum of the components.

Parallelogram of Forces.—If two forces acting on a point be represented in direction and intensity by adjacent sides of a parallelogram, their resultant will be represented by that diagonal of the parallelogram which passes through the point. Thus *OR*, Fig. 88, is the resultant of *OQ* and *OP*.

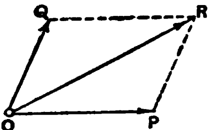

FIG. 88.

Polygon of Forces.—If several forces are applied at a point and act in a single plane, their resultant is found as follows:

Through the point draw a line representing the first force; through the extremity of this draw a line representing the second force; and so on, throughout the system; finally, draw a line from the starting-point to the extremity of the last line drawn, and this will be the resultant required.

Suppose the body *A*, Fig. 89, to be urged in the directions *A*1, *A*2, *A*3, *A*4, and *A*5 by forces which are to each other as the lengths of those lines. Suppose these forces to act successively and the body to first move from *A* to 1; the second force *A*2 then acts and finding the body at 1 would take it to 2′; the third force would then carry it to 3′, the fourth to 4′, and the fifth to 5′. The line *A*5′ represents in magnitude and direction the resultant of all the forces considered. If there had been an additional force, *Ax*, in the group, the body would be returned by that force to its original position, supposing the forces to act successively, but if they had acted simultaneously the body would never have moved at all; the tendencies to motion balancing each other.

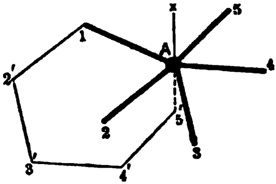

FIG. 89.

It follows, therefore, that if the several forces which tend to move a body can be represented in magnitude and direction by the sides of a closed polygon taken in order, the body will remain at rest; but if the forces are represented by the sides of an open polygon, the body will move and the direction will be represented by the straight line which closes the polygon.

Twisted Polygon.—The rule of the polygon of forces holds true even when the forces are not in one plane. In this case the lines *A*1, 1–2′, 2′–3′, etc., form a twisted polygon, that is, one whose sides are not in one plane.

Parallelopipedon of Forces.—If three forces acting on a point be represented by three edges of a parallelopipedon which meet in a common point, their resultant will be represented by the diagonal of the parallelopipedon that passes through their common point.

Thus *OR*, Fig. 90, is the resultant of *OQ*, *OS*, and *OP*. *OM* is the resultant of *OP* and *OQ*, and *OR* is the resultant of *OM* and *OS*.

Moment of a Force.—The moment of a force (sometimes called statical moment), with respect to a point, is the product of the force by the perpendicular distance from the point to the direction of the force. The fixed point is called the centre of mo-

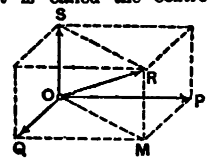

FIG. 90.

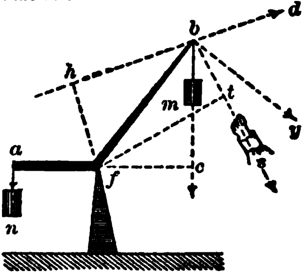

FIG. 91.

ments; the perpendicular distance is the lever-arm of the force; and the moment itself measures the tendency of the force to produce rotation about the centre of moments.

If the force is expressed in pounds and the distance in feet, the moment is expressed in foot-pounds. It is necessary to observe the distinction between foot-pounds of statical moment and foot-pounds of work or energy. (See Work.)

In the bent lever, Fig. 91 (from Trautwine), if the weights n and m represent forces, their moments about the point f are respectively $n \times af$ and $m \times fc$. If instead of the weight m a pulling force to balance the weight n is applied in the direction bs, or by or bd, s, y, and d being the amounts of these forces, their respective moments are $s \times ft$, $y \times fb$, $d \times fh$.

If the forces acting on the lever are in equilibrium it remains at rest, and the moments on each side of f are equal, that is, $n \times af = m \times fc$, or $s \times ft$, or $y \times fb$, or $d \times hf$.

The moment of the resultant of any number of forces acting together in the same plane is equal to the algebraic sum of the moments of the forces taken separately.

Statical Moment. Stability.—The statical moment of a body is the product of its weight by the distance of its line of gravity from some assumed line of rotation. The line of gravity is a vertical line drawn from its centre of gravity through the body. The stability of a body is that resistance which its weight alone enables it to oppose against forces tending to overturn it or to slide it along its foundation.

To be safe against turning on an edge the moment of the forces tending to overturn it, taken with reference to that edge, must be less than the statical moment. When a body rests on an inclined plane, the line of gravity being vertical, falls toward the lower edge of the body, and the condition of its not being overturned by its own weight is that the line of gravity must fall within this edge. In the case of an inclined tower resting on a plane the same condition holds—the line of gravity must fall within the base. The condition of stability against sliding along a horizontal plane is that the horizontal component of the force exerted tending to cause it to slide shall be less than the product of the weight of the body into the coefficient of friction between the base of the body and its supporting plane. This coefficient of friction is the tangent of the angle of repose, or the maximum angle at which the supporting plane might be raised from the horizontal before the body would begin to slide. (See Friction.)

The Stability of a Dam against overturning about its lower edge is calculated by comparing its statical moment referred to that edge with the resultant pressure of the water against its upper side. The horizontal pressure on a square foot at the bottom of the dam is equal to the weight of a column of water of one square foot in section, and of a height equal to the distance of the bottom below water-level; or, if H is the height, the pressure at the bottom per square foot $= 62.4 \times H$ lbs. At the water-level the pressure is zero, and it increases uniformly to the bottom, so that the sum of the pressures on a vertical strip one foot in breadth may be represented by the area of a triangle whose base is $62.4 \times H$ and whose altitude is H, or $62.4H^2 \div 2$. The centre of gravity of a triangle being $\frac{1}{3}$ of its altitude, the resultant of all the horizontal pressures may be taken as equivalent to the sum of the pressures acting at $\frac{1}{3}H$, and the moment of the sum of the pressures is therefore $62.4 \times H^3 \div 6$.

Parallel Forces.—If two forces are parallel and act in the same direction, their resultant is parallel to both, and lies between them, and the intensity of the resultant is equal to the sum of the intensities of the two forces. Thus in Fig. 91 the resultant of the forces n and m acts vertically downward at f, and is equal to $n + m$.

If two parallel forces act at the extremities of a straight line and in the same direction, the resultant divides the line joining the points of application of the components, inversely as the components. Thus in Fig. 91, $m : n :: af : fc$; and in Fig. 92, $P : Q :: SN : SM$.

The resultant of two parallel forces acting in opposite directions is parallel to both, lies without both, on the side and in the direction of the greater, and its intensity is equal to the difference of the intensities of the two forces.

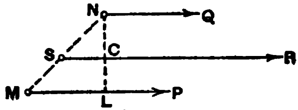

FIG. 92.

Thus the resultant of the two forces Q and P, Fig. 93, is equal to $Q - P = R$. Of any two parallel forces and their resultant each is proportional to the distance between the other two; thus in both Figs. 92 and 93, $P : Q : R :: SN : SM : MN$.

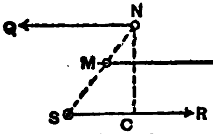

FIG. 93.

Couples.—If P and Q be equal and act in opposite directions, $R = 0$; that is, they have no resultant. Two such forces constitute what is called a couple.

The tendency of a couple is to produce rotation; the measure of this tendency, called *the moment of the couple*, is the product of one of the forces by the distance between the two.

Since a couple has no single resultant, no single force can balance a couple. To prevent the rotation of a body acted on by a couple the application of two other forces is required, forming a second couple. Thus in Fig. 94, P and Q forming a couple, may be balanced by a second couple formed by R and S. The point of application of either R or S may be a fixed pivot or axis.

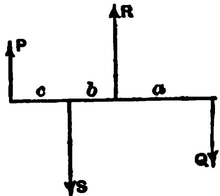

FIG. 94.

Moment of the couple $PQ = P(c + b + a) =$ moment of $RS = Rb$. Also, $P + R = Q + S$.

The forces R and S need not be parallel to P and Q, but if not, then their components parallel to PQ are to be taken instead of the forces themselves.

Equilibrium of Forces.—A system of forces applied at points of a solid body will be in equilibrium when they have no tendency to produce motion, either of translation or of rotation.

The conditions of equilibrium are: 1. The algebraic sum of the components of the forces in the direction of any three rectangular axes must be separately equal to 0.

2. The algebraic sum of the moments of the forces, with respect to any three rectangular axes, must be separately equal to 0.

If the forces lie in a plane: 1. The algebraic sum of the components of the forces, in the direction of any two rectangular axes, must be separately equal to 0.

2. The algebraic sum of the moments of the forces, with respect to any point in the plane, must be equal to 0.

If a body is restrained by a fixed axis, as in case of a pulley, or wheel and axle, the forces will be in a equilibrium when the algebraic sum of the moments of the forces with respect to the axis is equal to 0.

CENTRE OF GRAVITY.

The centre of gravity of a body, or of a system of bodies rigidly connected together, is that point about which, if suspended, all the parts will be in equilibrium, that is, there will be no tendency to rotation. It is the point through which passes the resultant of the efforts of gravitation on each of the elementary particles of a body. In bodies of equal heaviness throughout, the centre of gravity is the centre of magnitude.

(The centre of magnitude of a figure is a point such that if the figure be divided into equal parts the distance of the centre of magnitude of the whole figure from any given plane is the mean of the distances of the centres of magnitude of the several equal parts from that plane.)

If a body be suspended at its centre of gravity, it will be in equilibrium in all positions. If it be suspended at a point out of its centre of gravity, it will swing into a position such that its centre of gravity is vertically beneath its point of suspension.

To find the centre of gravity of any plane figure mechanically, suspend the figure by any point near its edge, and mark on it the direction of a plumb-line hung from that point; then suspend it from some other point, and again mark the direction of the plumb-line in like manner. Then the centre of gravity of the surface will be at the point of intersection of the two marks of the plumb-line.

The Centre of Gravity of Regular Figures, whether plane or solid, is the same as their geometrical centre; for instance, a straight line, parallelogram, regular polygon, circle, circular ring, prism, cylinder, sphere, spheroid, middle frustums of spheroid, etc.

Of a triangle: On a line drawn from any angle to the middle of the opposite side, at a distance of one third of the line from the side; or at the intersection of such lines drawn from any two angles.

Of a trapezium or trapezoid: Draw a diagonal, dividing it into two triangles. Draw a line joining their centres of gravity. Draw the other diagonal, making two other triangles, and a line joining their centres. The intersection of the two lines is the centre of gravity required.

Of a sector of a circle: On the radius which bisects the arc, $2cr \div 3l$ from the centre, c being the chord, r the radius, and l the arc.

Of a semicircle: On the middle radius, $.4244r$ from the centre.

Of a quadrant: On the middle radius, $.6002r$ from the centre.

Of a segment of a circle: $c^3 \div 12a$ from the centre. c = chord, a = area.

Of a parabolic surface: In the axis, 3/5 of its length from the vertex.

Of a semi-parabola (surface): 3/5 length of the axis from the vertex, and 3/8 of the semi-base from the axis.

Of a cone or pyramid: In the axis, 1/4 of its length from the base.

Of a paraboloid: In the axis, 2/3 of its length from the vertex.

Of a cylinder, or regular prism: In the middle point of the axis.

Of a frustum of a cone or pyramid: Let a = length of a line drawn from the vertex of the cone when complete to the centre of gravity of the base, and a' that portion of it between the vertex and the top of the frustum; then distance of centre of gravity of the frustum from centre of gravity of its base $= \frac{a}{4} - \frac{3a'^4}{4(a^2 + aa' + a'^2)}$.

For two bodies, fixed one at each end of a straight bar, the common centre of gravity is in the bar, at that point which divides the distance between their respective centres of gravity in the inverse ratio of the weights. In this solution the weight of the bar is neglected. But it may be taken as a third body, and allowed for as in the following directions:

For more than two bodies connected in one system: Find the common centre of gravity of two of them; and find the common centre of these two jointly with a third body, and so on to the last body of the group.

Another method, by the principle of moments: To find the centre of gravity of a system of bodies, or a body consisting of several parts, whose several centres are known. If the bodies are in a plane, refer their several centres to two rectangular co-ordinate axes. Multiply each weight by its distance from one of the axes, add the products, and divide the sum by the sum of the weights: the result is the distance of the centre of gravity from that axis. Do the same with regard to the other axis. If the bodies are not in a plane, refer them to three planes at right angles to each other, and determine the mean distance of the sum of the weights from each of the three planes.

MOMENT OF INERTIA.

The moment of inertia of the weight of a body with respect to an axis is the algebraic sum of the products obtained by multiplying the weight of each elementary particle by the square of its distance from the axis. If the moment of inertia with respect to any axis $= I$, the weight of any element of the body $= w$, and its distance from the axis $= r$, we have $I = \Sigma(wr^2)$.

The moment of inertia varies, in the same body, according to the position of the axis. It is the least possible when the axis passes through the centre of gravity. To find the moment of inertia of a body, referred to a given axis, divide the body into small parts of regular figure. Multiply the weight of each part by the square of the distance of its centre of gravity from the axis. The sum of the products is the moment of inertia. The value of the moment of inertia thus obtained will be more nearly exact, the smaller and more numerous the parts into which the body is divided.

MOMENTS OF INERTIA OF REGULAR SOLIDS.—Rod, or bar, of uniform thickness, with respect to an axis perpendicular to the length of the rod,

$$I = W\left(\frac{l^2}{3} + d^2\right), \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

W = weight of rod, $2l$ = length, d = distance of centre of gravity from axis.

Thin circular plate, axis in its own plane,

$$I = W\left(\frac{r^2}{4} + d^2\right); \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

r = radius of plate.

Circular plate, axis perpendicular to the plate, $\Big\}\ I = W\left(\frac{r^2}{2} + d^2\right)$. . . . . (3)

Circular ring, axis perpendicular to its own plane, $\Big\}\ I = W\left(\frac{r^2 + r'^2}{2} + d^2\right)$, ... (4)

r and r' are the exterior and interior radii of the ring.

Cylinder, axis perpendicular to the axis of the cylinder, $\Big\}\ I = W\left(\frac{r^2}{4} + \frac{l^2}{3} + d^2\right)$, ... (5)

r = radius of base, $2l$ = length of the cylinder.

By making $d = 0$ in any of the above formulæ we find the moment of inertia for a parallel axis through the centre of gravity.

The moment of inertia, Σwr^2, numerically equals the weight of a body which, if concentrated at the distance unity from the axis of rotation, would require the same work to produce a given increase of angular velocity that the actual body requires. It bears the same relation to angular acceleration which weight does to linear acceleration (Rankine). The term moment of inertia is also used in regard to areas, as the cross-sections of beams under strain. In this case $I = \Sigma ar^2$, in which a is any elementary area, and r its distance from the centre. (See Moment of Inertia, under Strength of Materials, p. 247.)

CENTRE AND RADIUS OF GYRATION.

The centre of gyration, with reference to an axis, is a point at which, if the entire weight of a body be concentrated, its moment of inertia will remain unchanged; or, in a revolving body, the point in which the whole weight of the body may be conceived to be concentrated, as if a pound of platinum were substituted for a pound of revolving feathers, the angular velocity and the accumulated work remaining the same. The distance of this point from the axis is the *radius of gyration*. If W = the weight of a body, $I = \Sigma wr^2$ = its moment of inertia, and k = its radius of gyration,

$$I = Wk^2 = \Sigma wr^2; \quad k = \sqrt{\frac{\Sigma wr^2}{W}}.$$

The moment of inertia = the weight × the square of the radius of gyration.

To find the radius of gyration divide the body into a considerable number of equal small parts—the more numerous the more nearly exact is the result,—then take the mean of all the squares of the distances of the parts from the axis of revolution, and find the square root of the mean square. Or, if the moment of inertia is known, divide it by the weight and extract the square root. For radius of gyration of an area, as a cross-section of a beam, divide the moment of inertia of the area by the area and extract the square root.

The radius of gyration is the least possible when the axis passes through the centre of gravity. This minimum radius is called the principal radius of gyration. If we denote it by k and any other radius of gyration by k', we have for the five cases given under the head of moment of inertia above the following values:

(1) Rod, axis perpen. to length, $\Big\}\ k = l\sqrt{\frac{1}{3}}; \quad k' = \sqrt{\frac{l^2}{3} + d^2}.$

(2) Circular plate, axis in its plane, $\Big\}\ k = \frac{r}{2}; \quad k' = \sqrt{\frac{r^2}{4} + d^2}.$

(3) Circular plate, axis perpen. to plane, $\Big\}\ k = r\sqrt{\frac{1}{2}}; \quad k' = \sqrt{\frac{r^2}{2} + d^2}.$

(4) Circular ring, axis perpen. to plane, $\Big\}\ k = \sqrt{\frac{r^2 + r'^2}{2}}; \quad k' = \sqrt{\frac{r^2 + r'^2}{2} + d^2}.$

(5) Cylinder, axis perpen. to length, $\Big\}\ k = \sqrt{\frac{r^2}{4} + \frac{l^2}{3}}; \quad k' = \sqrt{\frac{r^2}{4} + \frac{l^2}{3} + d^2}$

Principal Radii of Gyration and Squares of Radii of Gyration.

(For radii of gyration of sections of columns, see page 249.)

Surface or Solid.	Rad. of Gyration.	Square of R. of Gyration.
Parallelogram: height h, axis at its base	$.5773h$	$\frac{1}{3}h^2$
Parallelogram: height h, " mid-height	$.2886h$	$1/12h^2$
Straight rod: length l, or thin rectang. plate, axis at end	$.5773l$	$\frac{1}{3}l^2$
Straight rod: length l, or thin rectang. plate, " mid-length	$.2886l$	$1/12l^2$
Rectangular prism: axes $2a$, $2b$, $2c$, referred to axis $2a$	$.577\sqrt{b^2+c^2}$	$(b^2+c^2)\div 3$
Parallelopiped: length l, base b, axis at one end, at mid-breadth	$.289\sqrt{4l^2+b^2}$	$\frac{4l^2+b^2}{12}$
Hollow square tube: out. side h, inn'r h', axis mid-length	$.289\sqrt{h^2+h'^2}$	$(h^2+h'^2)\div 12$
very thin, side $= h$, " "	$.408h$	$h^2\div 6$
Thin rectangular tube: sides b, h, axis mid-length	$.289h\sqrt{\frac{h+3b}{h+b}}$	$\frac{h^2}{12}\cdot\frac{h+3b}{h+b}$
Thin circ. plate: rad. r, diam. h, ax. diam.	$\frac{1}{2}r$	$\frac{1}{4}r^2 = h^2\div 16$
Flat circ. ring: diams. h, h', axis diam.	$\frac{1}{4}\sqrt{h^2+h'^2}$	$(h^2+h'^2)\div 16$
Solid circular cylinder: length l, axis diameter at mid-length	$.289\sqrt{l^2+3r^2}$	$\frac{l^2}{12}+\frac{r^2}{4}$
Circular plate: solid wheel of uniform thickness, or cylinder of any length, referred to axis of cyl.	$.7071r$	$\frac{1}{2}r^2$
Hollow circ. cylinder, or flat ring: l, length; R, r, outer and inner radii. Axis, 1, longitudinal axis; 2, diam. at mid-length	$.7071\sqrt{R^2+r^2}$ $.289\sqrt{l^2+3(R^2+r^2)}$	$(R^2+r^2)\div 2$ $\frac{l^2}{12}+\frac{R^2+r^2}{4}$
Same: very thin, axis its diameter	$.289\sqrt{l^2+6R^2}$	$\frac{l^2}{12}+\frac{R^2}{2}$
" radius r; axis, longitud'l axis	r	r^2
Circumf. of circle, axis its centre	r	r^2
" " " " " diam.	$.7071r$	$\frac{1}{2}r^2$
Sphere: radius r, axis its diam.	$.6325r$	$2/5r^2$
Spheroid: equatorial radius r, revolving polar axis a	$.6325r$	$2/5r^2$
Paraboloid: r = rad. of base, rev. on axis	$.5773r$	$\frac{1}{3}r^2$
Ellipsoid: semi-axes a, b, c; revolving on axis $2a$	$.4472\sqrt{b^2+c^2}$	$\frac{b^2+c^2}{5}$
Spherical shell: radii R, r, revolving on its diam.	$.6325\sqrt{\frac{R^5-r^5}{R^3-r^3}}$	$\frac{2}{5}\frac{R^5-r^5}{R^3-r^3}$
Same: very thin, radius r	$.8165r$	$\frac{2}{3}r^2$
Solid cone: r = rad. of base, rev. on axis	$.5477r$	$0.3r^2$

CENTRES OF OSCILLATION AND OF PERCUSSION.

Centre of Oscillation.—If a body oscillate about a fixed horizontal axis, not passing through its centre of gravity, there is a point in the line drawn from the centre of gravity perpendicular to the axis whose motion is the same as it would be if the whole mass were collected at that point and allowed to vibrate as a pendulum about the fixed axis. This point is called the centre of oscillation.

The Radius of Oscillation, or distance of the centre of oscillation from the point of suspension = the square of the radius of gyration ÷ distance of the centre of gravity from the point of suspension or axis. The centres of oscillation and suspension are convertible.

If a straight line, or uniform thin bar or cylinder, be suspended at one end, oscillating about it as an axis, the centre of oscillation is at ⅔ the length of the rod from the axis. If the point of suspension is at ⅙ the length from the end, the centre of oscillation is also at ⅚ the length from the axis, that is, it is at the other end. In both cases the oscillation will be performed in the same time. If the point of suspension is at the centre of gravity, the length of the equivalent simple pendulum is infinite, and therefore the time of vibration is infinite.

For a sphere suspended by a cord, r = radius, h = distance of axis of motion from the centre of the sphere, h' = distance of centre of oscillation from centre of the sphere, l = radius of oscillation $= h + h' = h + \frac{2}{5}\frac{r^2}{h}$.

If the sphere vibrate about an axis tangent to its surface, $h = r$, and $l = r + 2/5r$. If $h = 10r$, $l = 10r + \frac{r}{25}$.

Lengths of the radius of oscillation of a few regular plane figures or thin plates, suspended by the vertex or uppermost point.

1st. When the vibrations are flatwise, or perpendicular to the plane of the figure:

In an isosceles triangle the radius of oscillation is equal to ¾ of the height of the triangle.

In a circle, ⅝ of the diameter.

In a parabola, 5/7 of the height.

2d. When the vibrations are edgewise, or in the plane of the figure:

In a circle the radius of oscillation is ¾ of the diameter.

In a rectangle suspended by one angle, ⅔ of the diagonal.

In a parabola, suspended by the vertex, 5/7 of the height, plus ⅓ of the parameter.

In a parabola, suspended by the middle of the base, 4/7 of the height plus ½ the parameter.

Centre of Percussion.—The centre of percussion of a body oscillating about a fixed axis is the point at which, if a blow is struck by the body, the percussive action is the same as if the whole mass of the body were concentrated at the point. This point is identical with the centre of oscillation.

THE PENDULUM.

A body of any form suspended from a fixed axis about which it oscillates by the force of gravity is called a *compound pendulum*. The ideal body concentrated at the centre of oscillation, suspended from the centre of suspension by a string without weight, is called a *simple pendulum*. This equivalent simple pendulum has the same weight as the given body, and also the same moment of inertia, referred to an axis passing through the point of suspension, and it oscillates in the same time.

The ordinary pendulum of a given length vibrates in equal times when the angle of the vibrations does not exceed 4 or 5 degrees, that is, 2° or 2½° each side of the vertical. This property of a pendulum is called its isochronism.

The time of vibration of a pendulum varies directly as the square root of the length, and inversely as the square root of the acceleration due to gravity at the given latitude and elevation above the earth's surface.

If T = the time of vibration, l = length of the simple pendulum, g = acceleration = 32.16, $T = \pi\sqrt{\frac{l}{g}}$; since π is constant, $T \propto \frac{\sqrt{l}}{\sqrt{g}}$. At a given location g is constant and $T \propto \sqrt{l}$. If l be constant, then for any location $T \propto \frac{1}{\sqrt{g}}$. If T be constant, $gT^2 = \pi^2 l$; $l \propto g$; $g = \frac{\pi^2 l}{T^2}$. From this equation the force of gravity at any place may be determined if the length of the simple pendulum, vibrating seconds, at that place is known. At New York this length is 39.1017 inches = 3.2585 ft., whence g = 32.16 ft. At London the length is 39.1393 inches. At the equator 39.0152 or 39.0168 inches, according to different authorities.

Time of vibration of a pendulum of a given length at New York

$$= t = \sqrt{\frac{l}{39.1017}} = \frac{\sqrt{l}}{6.253},$$

t being in seconds and l in inches. Length of a pendulum having a given time of vibration, $l = t^2 \times 39.1017$ inches.

The time of vibration of a pendulum may be varied by the addition of a weight at a point above the centre of suspension, which counteracts the lower weight, and lengthens the period of vibration. By varying the height of the upper weight the time is varied.

To find the weight of the upper bob of a compound pendulum, vibrating seconds, when the weight of the lower bob, and the distances of the weights from the point of suspension are given:

$$w = W \frac{(39.1 \dotplus D) - D^2}{(39.1 + d) + d^2}.$$

W = the weight of the lower bob, w = the weight of the upper bob; D = the distance of the lower bob and d = the distance of the upper bob from the point of suspension, in inches.

Thus, by means of a second bob, short pendulums may be constructed to vibrate as slowly as longer pendulums.

By increasing w or d until the lower weight is entirely counterbalanced, the time of vibration may be made infinite.

Conical Pendulum.—A weight suspended by a cord and revolving at a uniform speed in the circumference of a circular horizontal plane whose radius is r, the distance of the plane below the point of suspension being h, is held in equilibrium by three forces—the tension in the cord, the centrifugal force, which tends to increase the radius r, and the force of gravity acting downward. If v = the velocity in feet per second, the centre of gravity of the weight, as it describes the circumference, $g = 32.16$, and r and h are taken in feet, the time in seconds of performing one revolution is

$$t = \frac{2\pi r}{v} = 2\pi \sqrt{\frac{h}{g}}; \quad h = \frac{gt^2}{4\pi^2} = .8146t^2.$$

If $t = 1$ second, $h = .8146$ foot $= 9.775$ inches.

The principle of the conical pendulum is used in the ordinary fly-ball governor for steam-engines. (See Governors.)

CENTRIFUGAL FORCE.

A body revolving in a curved path of radius $= R$ in feet exerts a force, called centrifugal force, F, upon the arm or cord which restrains it from moving in a straight line, or "flying off at a tangent." If W = weight of the body in pounds, N = number of revolutions per minute, v = linear velocity of the centre of gravity of the body, in feet per second, $g = 32.16$, then

$$v = \frac{2\pi RN}{60}; \; F = \frac{Wv^2}{gR} = \frac{Wv^2}{32.16R} = \frac{W4\pi^2RN^2}{3600g} = \frac{WRN^2}{2933} = .0003410WRN^2 \text{ lbs.}$$

If n = number of revolutions per second, $F = 1.2276WRn^2$.

(For centrifugal force in fly-wheels, see Fly-wheels.)

VELOCITY, ACCELERATION, FALLING BODIES.

Velocity is the rate of motion, or the distance passed over by a body in a given time.

If s = space in feet passed over in t seconds, and v = velocity in feet per second, if the velocity is uniform,

$$v = \frac{s}{t}; \quad s = vt; \quad t = \frac{s}{v}.$$

If the velocity varies uniformly, the mean velocity $v_0 = \frac{v_1 + v_2}{2}$, in which v_1 is the velocity at the beginning and v_2 the velocity at the end of the time t.

$$s = \frac{v_1 + v_2}{2}t. \quad \ldots\ldots\ldots\ldots \quad (1)$$

Acceleration is the change in velocity which takes place in a unit of time. Unit of acceleration $= a = 1$ foot per second in one second. For uniformly varying velocity, the acceleration is a constant quantity, and

$$a = \frac{v_2 - v_1}{t}; \quad v_2 = v_1 + at; \quad v_1 = v_2 - at; \quad t = \frac{v_2 - v_1}{a}. \quad \ldots \quad (2)$$

If the body start from rest, $v_1 = 0$; then

$$v_0 = \frac{v^2}{2};\quad v_2 = 2v_0;\quad a = \frac{v_2}{t};\quad v_2 = at;\quad v_2 - at = 0;\quad t = \frac{v_2}{a}.$$

Combining (1) and (2), we have

$$s = \frac{v_2^2 - v_1^2}{2a};\quad s = v_1 t + \frac{at^2}{2};\quad s = v_2 t - \frac{at^2}{2}.$$

If $v_1 = 0$, $s = \frac{v_2}{2}t$.

Retarded Motion.—If the body start with a velocity v_1 and come to rest, $v_2 = 0$; then $s = \frac{v_1}{2}t$.

In any case, if the change in velocity is v,

$$s = \frac{v}{2}t;\quad s = \frac{v^2}{2a};\quad s = \frac{a}{2}t^2.$$

For a body starting from or ending at rest, we have the equations

$$v = at;\quad s = \frac{v}{2}t;\quad s = \frac{at^2}{2};\quad v^2 = 2as.$$

Falling Bodies.—In the case of falling bodies the acceleration due to gravity is 32.16 feet per second in one second, $= g$. Then if $v =$ velocity acquired at the end of t seconds, or final velocity, and $h =$ height or space in feet passed over in the same time,

$$v = gt = 32.16t = \sqrt{2gh} = 8.02\sqrt{h} = \frac{2h}{t};$$

$$h = \frac{gt^2}{2} = 16.08t^2 = \frac{v^2}{2g} = \frac{v^2}{64.32} = \frac{vt}{2};$$

$$t = \frac{v}{g} = \frac{v}{32.16} = \sqrt{\frac{2h}{g}} = \frac{\sqrt{h}}{4.01} = \frac{2h}{v};$$

$u =$ space fallen through in the Tth second $= g(T - \tfrac{1}{2})$.

From the above formula for falling bodies we obtain the following:

During the first second the body starting from a state of rest (resistance of the air neglected) falls $g \div 2 = 16.08$ feet; the acquired velocity is $g =$ 32.16 ft. per sec.; the distance fallen in two seconds is $h = \frac{gt^2}{2} = 16.08 \times 4 =$ 64.32 ft.; and the acquired velocity is $v = gt = 64.32$ ft. The acceleration, or increase of velocity in each second, is constant, and is 32.16 ft. per sec. Solving the equations for different times, we find for

Seconds, t		..	1	2	3	4	5	6
Acceleration, g	32.16	×	1	1	1	1	1	1
Velocity acquired at end of time, v	32.16	×	1	2	3	4	5	6
Height of fall in each second, u	$\frac{32.16}{2}$	×	1	3	5	7	9	11
Total height of fall, h	$\frac{32.16}{2}$	×	1	4	9	16	25	36

Value of g.—The value of g increases with the latitude, and decreases with the elevation. At the latitude of Philadelphia, 40°, its value is 32.16. At the sea-level, Everett gives $g = 32.173 - .082 \cos 2$ lat. $- .000003$ height in feet. At Paris, lat. 48° 50′ N., $g = 980.87$ cm. $= 32.181$ ft.

Values of $\sqrt{2g}$, calculated by an equation given by C. S. Pierce, are given in a table in Smith's Hydraulics, from which we take the following:

Latitude	0°	10°	20°	30°	40°	50°	60°
Value of $\sqrt{2g}$	8.0112	8.0118	8.0137	8.0165	8.0199	8.0235	8.0269

The value of $\sqrt{2g}$ decreases about .0004 for every 1000 feet increase in elevation above the sea-level.

For all ordinary calculations for the United States, g is generally taken at 32.16, and $\sqrt{2g}$ at 8.02. In England $g = 32.2$, $\sqrt{2g} = 8.025$. Practical limiting values of g for the United States, according to Pierce, are:

Latitude 49° at sea-level .... $g = 32.186$
" 25° 10,000 feet above the sea .... $g = 32.089$

Fig. 95 represents graphically the velocity, space, etc., of a body falling for six seconds. The vertical line at the left is the time in seconds, the horizontal lines represent the acquired velocities at the end of each second = $32.16t$. The area of the small triangle at the top represents the height fallen through in the first second = $\frac{1}{2}g$ = 16.08 feet, and each of the other triangles is an equal space. The number of triangles between each pair of horizontal lines represents the height of fall in each second, and the number of triangles between any horizontal line and the top is the total height fallen during the time. The figures under h, u, and v adjoining the cut are to be multiplied by 16.08 to obtain the actual velocities and heights for the given times.

h	u	v	t
1	1	2	1″
4	3	4	2″
9	5	6	3″
16	7	8	4″
25	9	10	5″
36	11	12	6″

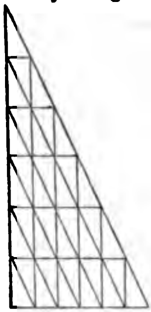

FIG. 95.

Angular and Linear Velocity of a Turning Body.—Let r = radius of a turning body in feet, n = number of revolutions per minute, v = linear velocity of a point on the circumference in feet per second, and $60v$ = velocity in feet per minute.

$$v = \frac{2\pi rn}{60}, \quad 60v = 2\pi rn.$$

Angular velocity is a term used to denote the angle through which any radius of a body turns in a second, or the rate at which any point in it having a radius equal to unity is moving, expressed in feet per second. The unit of angular velocity is the angle which at a distance = radius from the centre is subtended by an arc equal to the radius. This unit angle = $\frac{180}{\pi}$ degrees = 57.3°. $2\pi \times 57.3° = 360°$, or the circumference. If A = angular velocity, $v = Ar$, $A = \frac{v}{r} = \frac{2\pi n}{60}$. The unit angle $\frac{180}{\pi}$ is called a *radian*.

Height Corresponding to a Given Acquired Velocity.

Velocity. feet p. sec.	Height. feet.	Velocity. feet p. sec.	Height. feet.	Velocity. feet p. sec.	Height. feet.	Velocity. feet p. sec.	Height. feet.	Velocity. feet p. sec.	Height. feet.	Velocity. feet p. sec.	Height. feet.
.25	.0010	13	2.62	34	17.9	55	47.0	76	89.8	97	146
.50	.0039	14	3.04	35	19.0	56	48.8	77	92.2	98	149
.75	.0087	15	3.49	36	20.1	57	50.5	78	94.6	99	152
1.00	.016	16	3.98	37	21.3	58	52.3	79	97.0	100	155
1.25	.024	17	4.49	38	22.4	59	54.1	80	99.5	105	171
1.50	.035	18	5.03	39	23.6	60	56.0	81	102.0	110	188
1.75	.048	19	5.61	40	24.9	61	57.9	82	104.5	115	205
2	.062	20	6.22	41	26.1	62	59.8	83	107.1	120	224
2.5	.097	21	6.85	42	27.4	63	61.7	84	109.7	130	263
3	.140	22	7.52	43	28.7	64	63.7	85	112.3	140	304
3.5	.190	23	8.21	44	30.1	65	65.7	86	115.0	150	350
4	.248	24	8.94	45	31.4	66	67.7	87	117.7	175	476
4.5	.314	25	9.71	46	32.9	67	69.8	88	120.4	200	622
5	.388	26	10.5	47	34.3	68	71.9	89	123.2	300	1399
6	.559	27	11.3	48	35.8	69	74.0	90	125.9	400	2488
7	.761	28	12.2	49	37.3	70	76.2	91	128.7	500	3887
8	.994	29	13.1	50	38.9	71	78.4	92	131.6	600	5597
9	1.26	30	14.0	51	40.4	72	80.6	93	134.5	700	7618
10	1.55	31	14.9	52	42.0	73	82.9	94	137.4	800	9952
11	1.88	32	15.9	53	43.7	74	85.1	95	140.3	900	12598
12	2.24	33	16.9	54	45.3	75	87.5	96	143.3	1000	15547

Falling Bodies: Velocity Acquired by a Body Falling a Given Height.

Height.	Velocity.	Height.	Velocity.	Height.	Velocity.	Height.	Velocity.	Height.	Velocity.	Height.	Velocity.
feet.	feet p.sec.	feet.	feet p.sec.	feet.	feet p.sec.	feet.	feet p.sec.	feet.	feet p.sec.	feet.	feet p.sec.
.005	.57	.39	5.01	1.20	8.79	5.	17.9	23.	38.5	72	68.1
.010	.80	.40	5.07	1.22	8.87	.2	18.3	.5	38.9	73	68.5
.015	.98	.41	5.14	1.24	8.94	.4	18.7	24.	39.3	74	69.0
.020	1.13	.42	5.20	1.26	9.01	.6	19.0	.5	39.7	75	69.5
.025	1.27	.43	5.26	1.28	9.08	.8	19.3	25	40.1	76	69.9
.030	1.39	.44	5.32	1.30	9.15	6.	19.7	26	40.9	77	70.4
.035	1.50	.45	5.38	1.32	9.21	.2	20.0	27	41.7	78	70.9
.040	1.60	.46	5.44	1.34	9.29	.4	20.3	28	42.5	79	71.3
.045	1.70	.47	5.50	1.36	9.36	.6	20.6	29	43.2	80	71.8
.050	1.79	.48	5.56	1.38	9.43	.8	20.9	30	43.9	81	72.2
.055	1.88	.49	5.61	1.40	9.49	7.	21.2	31	44.7	82	72.6
.060	1.97	.50	5.67	1.42	9.57	.2	21.5	32	45.4	83	73.1
.065	2.04	.51	5.73	1.44	9.62	.4	21.8	33	46.1	84	73.5
.070	2.12	.52	5.78	1.46	9.70	.6	22.1	34	46.8	85	74.0
.075	2.20	.53	5.84	1.48	9.77	.8	22.4	35	47.4	86	74.4
.080	2.27	.54	5.90	1.50	9.82	8.	22.7	36	48.1	87	74.8
.085	2.34	.55	5.95	1.52	9.90	.2	23.0	37	48.8	88	75.3
.090	2.41	.56	6.00	1.54	9.96	.4	23.3	38	49.4	89	75.7
.095	2.47	.57	6.06	1.56	10.0	.6	23.5	39	50.1	90	76.1
.100	2.54	.58	6.11	1.58	10.1	.8	23.8	40	50.7	91	76.5
.105	2.60	.59	6.16	1.60	10.2	9.	24.1	41	51.4	92	76.9
.110	2.66	.60	6.21	1.65	10.3	.2	24.3	42	52.0	93	77.4
.115	2.72	.62	6.32	1.70	10.5	.4	24.6	43	52.6	94	77.8
.120	2.78	.64	6.42	1.75	10.6	.6	24.8	44	53.2	95	78.2
.125	2.84	.66	6.52	1.80	10.8	.8	25.1	45	53.8	96	78.6
.130	2.89	.68	6.61	1.90	11.1	10.	25.4	46	54.4	97	79.0
.14	3.00	.70	6.71	2.	11.4	.5	26.0	47	55.0	98	79.4
.15	3.11	.72	6.81	2.1	11.7	11.	26.6	48	55.6	99	79.8
.16	3.21	.74	6.90	2.2	11.9	.5	27.2	49	56.1	100	80.2
.17	3.31	.76	6.99	2.3	12.2	12.	27.8	50	56.7	125	89.7
.18	3.40	.78	7.09	2.4	12.4	.5	28.4	51	57.3	150	98.3
.19	3.50	.80	7.18	2.5	12.6	13.	28.9	52	57.8	175	106
.20	3.59	.82	7.26	2.6	12.9	.5	29.5	53	58.4	200	114
.21	3.68	.84	7.35	2.7	13.2	14.	30.0	54	59.0	225	120
.22	3.76	.86	7.44	2.8	13.4	.5	30.5	55	59.5	250	126
.23	3.85	.88	7.53	2.9	13.7	15.	31.1	56	60.0	275	133
.24	3.93	.90	7.61	3.	13.9	.5	31.6	57	60.6	300	139
.25	4.01	.92	7.69	3.1	14.1	16.	32.1	58	61.1	350	150
.26	4.09	.94	7.78	3.2	14.3	.5	32.6	59	61.6	400	160
.27	4.17	.96	7.86	3.3	14.5	17.	33.1	60	62.1	450	170
.28	4.25	.98	7.94	3.4	14.8	.5	33.6	61	62.7	500	179
.29	4.32	1.00	8.02	3.5	15.0	18.	34.0	62	63.2	550	188
.30	4.39	1.02	8.10	3.6	15.2	.5	34.5	63	63.7	600	197
.31	4.47	1.04	8.18	3.7	15.4	19.	35.0	64	64.2	700	212
.32	4.54	1.06	8.26	3.8	15.6	.5	35.4	65	64.7	800	227
.33	4.61	1.08	8.34	3.9	15.8	20.	35.9	66	65.2	900	241
.34	4.68	1.10	8.41	4.	16.0	.5	36.3	67	65.7	1000	254
.35	4.74	1.12	8.49	.2	16.4	21.	36.8	68	66.1	2000	359
.36	4.81	1.14	8.57	.4	16.8	.5	37.2	69	66.6	3000	439
.37	4.88	1.16	8.64	.6	17.2	22.	37.6	70	67.1	4000	507
.38	4.94	1.18	8.72	.8	17.6	.5	38.1	71	67.6	5000	567

Parallelogram of Velocities.—The principle of the composition and resolution of forces may also be applied to velocities or to distances moved in given intervals of time. Referring to Fig. 88, page 416, if a body at *O* has a force applied to it which acting alone would give it a velocity represented by *OQ* per second, and at the same time it is acted on by another force which acting alone would give it a velocity OP per second, the result of the two forces acting together for one second will carry it to R, OR being the diagonal of the parallelogram of OQ and OP, and the resultant velocity. If the two component velocities are uniform, the resultant will be uniform and the line OR will be a straight line; but if either velocity is a varying one, the line will be a curve. Fig. 96 shows the resultant velocities, also the path traversed by a body acted on by two forces, one of which would carry it at a uniform velocity over the intervals 1, 2, 3, B, and the other of which would carry it by an accelerated motion over the intervals a, b, c, D in the same times. At the end of the respective intervals the body will be found at C_1, C_2, C_3, C, and the mean velocity during each interval is represented by the distances between these points. Such a curved path is traversed by a shot, the impelling force from the gun giving it a uniform velocity in the direction the gun is aimed, and gravity giving it an accelerated velocity downward. *The path of a projectile is a parabola.* The distance it will travel is greatest when its initial direction is at an angle 45° above the horizontal.

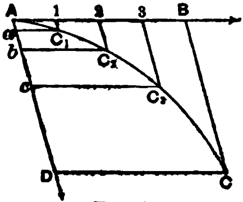

FIG. 96.

Mass—Force of Acceleration.—*The mass* of a body, or the quantity of matter it contains, is a constant quantity, while the weight varies according to the variation in the force of gravity at different places. If g = the acceleration due to gravity, and w = weight, then the mass $m = \frac{w}{g}$, $w = mg$. Weight here means the resultant of the force of gravity on the particles of a body, such as may be measured by a spring-balance, or by the extension or deflection of a rod of metal loaded with the given weight.

Force has been defined as that which causes, or tends to cause, or to destroy, motion. It may also be defined (Kennedy's Mechanics of Machinery) as the cause of acceleration; and the unit of force as the force required to produce unit acceleration in a unit of free mass.

Force equals the product of the mass by the acceleration, or $f = ma$.

Also, if v = the velocity acquired in the time t, $ft = mv$; $f = mv \div t$; the acceleration being uniform.

The force required to produce an acceleration of g (that is, 32.16 ft. per sec.) in one second is $f = mg = \frac{w}{g}g = w$, or the weight of the body. Also, $f = ma = m\frac{v_2 - v_1}{t}$, in which v_2 is the velocity at the end, and v_1 the velocity at the beginning of the time t, and $f = mg = \frac{w}{g}\frac{(v_2 - v_1)}{t} = \frac{w}{g}a$; $\frac{f}{w} = \frac{a}{g}$; or, the force required to give any acceleration to a body is to the weight of the body as that acceleration is to the acceleration produced by gravity. (The weight w is the weight where g is measured.)

EXAMPLE.—Tension in a cord lifting a weight. A weight of 100 lbs. is lifted vertically by a cord a distance of 80 feet in 4 seconds, the velocity uniformly increasing from 0 to the end of the time. What tension must be maintained in the cord? Mean velocity $= v_0 = 20$ ft. per sec.; final velocity $= v_2 = 2v_0 = 40$; acceleration $a = \frac{v_2}{t} = \frac{40}{4} = 10$. Force $f = ma = \frac{wa}{g} = \frac{100}{32.16} \times 10 = 31.1$ lbs. This is the force required to produce the acceleration only; to it must be added the force required to lift the weight without acceleration, or 100 lbs., making a total of 131.1 lbs.

The Resistance to Acceleration is the same as the force required to produce the acceleration $= \frac{w}{g}\frac{(v_2 - v_1)}{t}$.

Formulæ for Accelerated Motion.—For cases of uniformly accelerated motion other than those of falling bodies, we have the formulæ already given, $f = \frac{w}{g}a$, $= \frac{w}{g}\frac{v_2 - v_1}{t}$. If the body starts from rest, $v_1 = 0$, v_2

$= v$, and $f = \frac{w}{g}\frac{v}{t}$, $fgt = wv$. We also have $s = \frac{vt}{2}$. Transforming and substituting for g its value 32.16, we obtain

$$f = \frac{wv^2}{64.32s} = \frac{wv}{32.16t} = \frac{ws}{16.08t^2}; \quad w = \frac{32.16ft}{v} = \frac{64.32fs}{v^2};$$

$$s = \frac{wv^2}{64.32f} = \frac{16.08ft^2}{w} = \frac{vt}{2}; \quad v = 8.02\sqrt{\frac{fs}{w}} = \frac{32.16ft}{w};$$

$$t = \frac{wv}{32.16f} = \frac{1}{4.01}\sqrt{\frac{ws}{f}}$$

For any change in velocity $f = w\left(\frac{v_2^2 - v_1^2}{64.32s}\right)$.

(See also Work of Acceleration, under Work.)

Motion on Inclined Planes.—The velocity acquired by a body descending an inclined plane by the force of gravity (friction neglected) is equal to that acquired by a body falling freely from the height of the plane.

The times of descent down different inclined planes of the same height vary as the length of the planes.

The rules for uniformly accelerated motion apply to inclined planes. If a is the angle of the plane with the horizontal, sin a = the ratio of the height to the length $= \frac{h}{l}$, and the constant accelerating force is g sin a. The final velocity at the end of t seconds is $v = gt \sin a$. The distance passed over in t seconds is $l = \frac{1}{2} gt^2 \sin a$. The time of descent is

$$t = \sqrt{\frac{2l}{g \sin a}} = \frac{l}{4.01\sqrt{h}}.$$

MOMENTUM, VIS-VIVA.

Momentum, or quantity of motion in a body, is the product of the mass by the velocity at any instant $= mv = \frac{w}{g}v$.

Since the moving force = product of mass by acceleration, $f = ma$; and if the velocity acquired in t seconds $= v$, or $a = \frac{v}{t}$, $f = \frac{mv}{t}$; $ft = mv$; that is, the product of a constant force into the time in which it acts equals numerically the momentum.

Since $ft = mv$, if $t = 1$ second $mv = f$, whence momentum might be defined as numerically equivalent to the number of pounds of force that will stop a moving body in 1 second, or the number of pounds of force which acting during 1 second will give it the given velocity.

Vis-viva, or living force, is a term used by early writers on Mechanics to denote the energy stored in a moving body. Some defined it as the product of the mass into the square of the velocity, mv^2, $= \frac{w}{g}v^2$ others as one half of this quantity or $\frac{1}{2}mv^2$, or the same as what is now known as energy. The term is now practically obsolete, its place being taken by the word energy.

WORK, ENERGY, POWER.

Work is the overcoming of resistance through a certain distance. It is measured by the product of the resistance into the space through which it is overcome. It is also measured by the product of the moving force into the distance through which the force acts in overcoming the resistance. Thus in lifting a body from the earth against the attraction of gravity, the resistance is the weight of the body, and the product of this weight into the height the body is lifted is the work done.

The Unit of Work, in British measures, is the *foot-pound,* or the amount of work done in overcoming a pressure or weight equal to one pound through one foot of space.

The work performed by a piston in driving a fluid before it, or by a fluid in driving a piston before it, may be expressed in either of the following ways:

Resistance × distance traversed
= intensity of pressure × area × distance traversed;
= intensity of pressure × volume traversed.

The work performed in lifting a body is the product of the weight of the body into the height through which its centre of gravity is lifted.

If a machine lifts the centres of gravity of several bodies at once to heights either the same or different, the whole quantity of work performed in so doing is the sum of the several products of the weights and heights; but that quantity can also be computed by multiplying the sum of all the weights into the height through which their common centre of gravity is lifted. (Rankine.)

Power is the rate at which work is done, and is expressed by the quotient of the work divided by the time in which it is done, or by units of work per second, per minute, etc., as foot-pounds per second. The most common unit of power is the *horse-power*, established by James Watt as the power of a strong London draught-horse to do work during a short interval, and used by him to measure the power of his steam-engines. This unit is 33,000 foot-pounds per minute = 550 foot-pounds per second = 1,980,000 foot-pounds per hour.

Expressions for Force, Work, Power, etc.

The fundamental conceptions in Dynamics are:

Mass, Force, Time, Space, represented by the letters M, F, T, S.

Mass = weight ÷ g. If the weight of a body is determined by a spring balance standardized at London it will vary with the latitude, and the value of g to be taken in order to find the mass is that of the latitude where the weighing is done. If the weight is determined by a balance or by a platform scale, as is customary in engineering and in commerce, the London value of g, = 32.2, is to be taken.

Velocity = space divided by time, $V = S \div T$, if V be uniform.

Work = force multiplied by space = $FS = \frac{1}{2}MV^2 = FVT$. ($V$ uniform.)

Power = rate of work = work divided by time = $FS \div T = P$ = product of force into velocity = FV.

Power exerted for a certain time produces work; $PT = FS = FVT$.

Effort is a force which acts on a body in the direction of its motion.

Resistance is that which is opposed to a moving force. It is equal and opposite force.

Horse-power Hours, an expression for work measured as the product of a power into the time during which it acts = PT. Sometimes it is the summation of a variable power for a given time, or the average power multiplied by the time.

Energy, or stored work, is the capacity for performing work. It is measured by the same unit as work, that is, in foot-pounds. It may be either *potential*, as in the case of a body of water stored in a reservoir, capable of doing work by means of a water-wheel, or *actual*, sometimes called *kinetic*, which is the energy of a moving body. Potential energy is measured by the product of the weight of the stored body into the distance through which it is capable of acting, or by the product of the pressure it exerts into the distance through which that pressure is capable of acting. Potential energy may also exist as stored heat, or as stored chemical energy, as in fuel, gunpowder, etc., or as electrical energy, the measure of these energies being the amount of work that they are capable of performing. Actual energy of a moving body is the work which it is capable of performing against a retarding resistance before being brought to rest, and is equal to the work which must be done upon it to bring it from a state of rest to its actual velocity.

The measure of actual energy is the product of the weight of the body into the height from which it must fall to acquire its actual velocity. If v = the velocity in feet per second, according to the principle of falling bodies, h, the height due to the velocity = $\frac{v^2}{2g}$, and if w = the weight, the energy = $\frac{1}{2}mv^2 = wv^2 \div 2g = wh$. Since energy is the capacity for performing work, the units of work and energy are equivalent, or $FS = \frac{1}{2}mv^2 = wh$. Energy exerted = work done.

The actual energy of a rotating body whose angular velocity is A and moment of inertia $\Sigma wr^2 = I$ is $\frac{A^2I}{2g}$, that is, the product of the moment of inertia into the height due to the velocity, A, of a point whose distance from the axis of rotation is unity; or it is equal to $\frac{wv^2}{2g}$, in which w is the weight of the body and v is the velocity of the centre of gyration.

Work of Acceleration.—The work done in giving acceleration to a body is equal to the product of the force producing the acceleration, or of the resistance to acceleration, into the distance moved in a given time. This force, as already stated equals the product of the mass into the acceleration, or $f = ma = \frac{w}{g}\frac{v_2 - v_1}{t}$. If the distance traversed in the time $t = s$, then work $= fs = \frac{w}{g}\frac{v_2 - v_1}{t}s$.

EXAMPLE.—What work is required to move a body weighing 100 lbs. horizontally a distance of 80 ft. in 4 seconds, the velocity uniformly increasing, friction neglected?

Mean velocity $v_0 = 20$ ft. per second; final velocity $= v_2 = 2v_0 = 40$; initial velocity $v_1 = 0$; acceleration, $a = \frac{v_2 - v_1}{t} = \frac{40}{4} = 10$; force $= \frac{w}{g}a = \frac{100}{32.16} \times 10 = 31.1$ lbs.; distance 80 ft.; work $= fs = 31.1 \times 80 = 2488$ foot-pounds.

The energy stored in the body moving at the final velocity of 40 ft. per second is

$$\tfrac{1}{2}mv^2 = \frac{1}{2}\frac{w}{g}v^2 = \frac{100 \times 40^2}{2 \times 32.16} = 2488 \text{ foot-pounds},$$

which equals the work of acceleration,

$$fs = \frac{w}{g}\frac{v_2}{t}s = \frac{w}{g}\frac{v_2}{t}\frac{v_2}{2}t = \frac{1}{2}\frac{w}{g}v_2^2.$$

If a body of the weight W falls from a height H, the work of acceleration is simply WH, or the same as the work required to raise the body to the same height.

Work of Accelerated Rotation.—Let A = angular velocity of a solid body rotating about an axis, that is, the velocity of a particle whose radius is unity. Then the velocity of a particle whose radius is r is $v = Ar$. If the angular velocity is accelerated from A_1 to A_2, the increase of the velocity of the particle is $v_2 - v_1 = r(A_1 - A_2)$, and the work of accelerating it is

$$\frac{w}{g} \times \frac{v_2^2 - v_1^2}{2} = \frac{wr^2}{g}\frac{A_2^2 - A_1^2}{2},$$

in which w is the weight of the particle.

The work of acceleration of the whole body is

$$\Sigma\left\{\frac{w}{g} \times \frac{v_2^2 - v_1^2}{2}\right\} = \frac{A_2^2 - A_1^2}{2g} \times \Sigma wr^2.$$

The term Σwr^2 is the moment of inertia of the body.

"Force of the Blow" of a Steam Hammer or Other Falling Weight.—The question is often asked: "With what force does a falling hammer strike?" The question cannot be answered directly, and it is based upon a misconception or ignorance of fundamental mechanical laws. The energy, or capacity of doing work, of a body raised to a given height and let fall cannot be expressed in pounds, simply, but only in foot-pounds, which is the product of the weight into the height through which it falls, or the product of its weight $\div$ 64.32 into the square of the velocity, in feet per second, which it acquires after falling through the given height. If F = weight of the body, M its mass, g the acceleration due to gravity, S the height of fall, and v the velocity at the end of the fall, the energy in the body just before striking, is $FS = \tfrac{1}{2}Mv^2 = Wv^2 \div 2g = Wv^2 \div 64.32$, which is the general equation of energy of a moving body. Just as the energy of the body is a product of a force into a distance, so the work it does when it strikes is not the manifestation of a force, which can be expressed simply in pounds, but it is the overcoming of a resistance through a certain distance, which is expressed as the product of the average resistance into the distance through which it is exerted. If a hammer weighing 100 lbs. falls 10 ft., its energy is 1000 foot-pounds. Before being brought to rest it must do 1000 foot-pounds of work against one or more resistances. These are of various kinds, such as that due to motion imparted to the body struck, penetration against friction, or against resistance to shearing or other deformation, and crushing and heating of both the falling body and the body struck. The distance through which these resisting forces act is generally indeterminate, and therefore the average of the resisting forces, which themselves generally vary with the distance, is also indeterminate.

Impact of Bodies.—If two inelastic bodies collide, they will move on together as one mass, with a common velocity. The momentum of the combined mass is equal to the sum of the momenta of the two bodies before impact. If m_1 and m_2 are the masses of the two bodies and v_1 and v_2 their respective velocities before impact, and v their common velocity after impact, $(m_1 + m_2)v = m_1v_1 \times m_2v_2$,

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}.*$$

If the bodies move in opposite directions $v = \frac{m_1v_1 - m_2v_2}{m_1 + m_2}$, or, the velocity of two inelastic bodies after impact is equal to the algebraic sum of their momenta before impact, divided by the sum of their masses.

If two inelastic bodies of equal momenta impinge directly upon one another from opposite directions they will be brought to rest.

Impact of Inelastic Bodies Causes a Loss of Energy, and this loss is equal to the sum of the energies due to the velocities lost and gained by the bodies, respectively.

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 - \tfrac{1}{2}(m_1 + m_2)v^2 = \tfrac{1}{2}m_1(v_1 - v)^2 + \tfrac{1}{2}m_2(v_2 - v)^2.$$

In which $v_1 - v$ is the velocity lost by m_1 and $v - v_2$ the velocity gained by m_2.

Example—Let $m_1 = 10$, $m_2 = 8$, $v_1 = 12$, $v_2 = 15$.

If the bodies collide they will come to rest, for $v = \frac{10 \times 12 - 8 \times 15}{10 + 8} = 0.$

The energy loss is

$$\tfrac{1}{2}10 \times 144 + \tfrac{1}{2}8 \times 225 - \tfrac{1}{2}18 \times 0 = \tfrac{1}{2}10(12 - 0)^2 + \tfrac{1}{2}8(15 - 0)^2 = 1620 \text{ ft. lbs.}$$

What becomes of the energy lost? Ans. It is used doing internal work on the bodies themselves, changing their shape and heating them.

For imperfectly elastic bodies, let e = the elasticity, that is, the ratio which the force of restitution, or the internal force tending to restore the shape of a body after it has been compressed, bears to the force of compression; and let m_1 and m_2 be the masses, v_1 and v_2 their velocities before impact, and $v_1'v_2'$ their velocities after impact: then

$$v_1' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} - \frac{m_2e(v_1 - v_2)}{m_1 + m_2};$$

$$v_2' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} + \frac{m_1e(v_1 - v_2)}{m_1 + m_2}.$$

If the bodies are perfectly elastic, their relative velocities before and after impact are the same. That is: $v_1' - v_2' = v_2 - v_1$.

In the impact of bodies, the sum of their momenta after impact is the same as the sum of their momenta before impact.

$$m_1v_1' + m_2v_2' = m_1v_1 + m_2v_2.$$

For demonstration of these and other laws of impact, see Smith's Mechanics; also, Weisbach's Mechanics.

Energy of Recoil of Guns.—(*Eng'g*, Jan. 25, 1884, p. 72.)

Let W = the weight of the gun and carriage;
V = the maximum velocity of recoil;
w = the weight of the projectile;
v = the muzzle velocity of the projectile.

Then, since the momentum of the gun and carriage is equal to the momentum of the projectile, we have $WV = wv$, or $V = wv + W$.

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* The statement by Prof. W. D. Marks, in Nystrom's Mechanics, 20th edition, p. 454, that this formula is in error is itself erroneous.

Taking the case of a 10-inch gun firing a 400-lb. projectile with a muzzle velocity of 1400 feet per second, the weight of the gun and carriage being 22 tons = 49,280 lbs., we find the velocity of recoil =

$$V = \frac{1400 \times 400}{49,280} = 11 \text{ feet per second.}$$

Now the energy of a body in motion is $WV^2 \div 2g$.

Therefore the energy of recoil $= \dfrac{49,280 \times 11^2}{2 \times 32.2} = 92,593$ foot-pounds.

The energy of the projectile is $\dfrac{400 \times 1400^2}{2 \times 32.2} = 12,173,913$ foot-pounds.

Conservation of Energy.—No form of energy can ever be produced except by the expenditure of some other form, nor annihilated except by being reproduced in another form. Consequently the sum total of energy in the universe, like the sum total of matter, must always remain the same. (S. Newcomb.) Energy can never be destroyed or lost; it can be transformed, can be transferred from one body to another, but no matter what transformations are undergone, when the total effects of the exertion of a given amount of energy are summed up the result will be exactly equal to the amount originally expended from the source. This law is called the Conservation of Energy. (Cotterill and Slade.)

A heavy body sustained at an elevated position has potential energy. When it falls, just before it reaches the earth's surface it has actual or kinetic energy, due to its velocity. When it strikes it may penetrate the earth a certain distance or may be crushed. In either case friction results by which the energy is converted into heat, which is gradually radiated into the earth or into the atmosphere, or both. Mechanical energy and heat are mutually convertible. Electric energy is also convertible into heat or mechanical energy, and either kind of energy may be converted into the other.

Sources of Energy.—The principal sources of energy on the earth's surface are the muscular energy of men and animals, the energy of the wind, of flowing water, and of fuel. These sources derive their energy from the rays of the sun. Under the influence of the sun's rays vegetation grows and wood is formed. The wood may be used as fuel under a steam boiler, its carbon being burned to carbonic acid. Three tenths of its heat energy escapes in the chimney and by radiation, and seven tenths appears as potential energy in the steam. In the steam-engine, of this seven tenths six parts are dissipated in heating the condensing water and are wasted; the remaining one tenth of the original heat energy of the wood is converted into mechanical work in the steam-engine, which may be used to drive machinery. This work is finally, by friction of various kinds, or possibly after transformation into electric currents, transformed into heat, which is radiated into the atmosphere, increasing its temperature. Thus all the potential heat energy of the wood is, after various transformations, converted into heat, which, mingling with the store of heat in the atmosphere, apparently is lost. But the carbonic acid generated by the combustion of the wood is, again, under the influence of the sun's rays, absorbed by vegetation, and more wood may thus be formed having potential energy equal to the original.

Perpetual Motion.—The law of the conservation of energy, than which no law of mechanics is more firmly established, is an absolute barrier to all schemes for obtaining by mechanical means what is called "perpetual motion," or a machine which will do an amount of work greater than the equivalent of the energy, whether of heat, of chemical combination, of electricity, or mechanical energy, that is put into it. Such a result would be the creation of an additional store of energy in the universe, which is not possible by any human agency.

The Efficiency of a Machine is a fraction expressing the ratio of the useful work to the whole work performed, which is equal to the energy expended. The limit to the efficiency of a machine is unity, denoting the efficiency of a perfect machine in which no work is lost. The difference between the energy expended and the useful work done, or the loss, is usually expended either in overcoming friction or in doing work on bodies surrounding the machine from which no useful work is received. Thus in an engine propelling a vessel part of the energy exerted in the cylinder does the useful work of giving motion to the vessel, and the remainder is spent in overcoming the friction of the machinery and in making currents and eddies in the surrounding water.

ANIMAL POWER.

Work of a Man against Known Resistances. (Rankine.)

Kind of Exertion.	R, lbs.	V, ft. per sec.	$\frac{T'}{3600}$ (hours per day).	RV, ft.-lbs. per sec.	RVT, ft.-lbs. per day.
1. Raising his own weight up stair or ladder	143	0.5	8	72.5	2,088,000
2. Hauling up weights with rope, and lowering the rope unloaded	40	0.75	6	30	648,000
3. Lifting weights by hand	44	0.55	6	24.2	522,720
4. Carrying weights up-stairs and returning unloaded	143	0.13	6	18.5	399,600
5. Shovelling up earth to a height of 5 ft. 3 in	6	1.3	10	7.8	280,800
6. Wheeling earth in barrow up slope of 1 in 12, ½ horiz. veloc. 0.9 ft. per sec. and returning unloaded	132	0.075	10	9.9	356,400
7. Pushing or pulling horizontally (capstan or oar)	26.5	2.0	8	53	1,526,400
8. Turning a crank or winch	12.5	5.0	?	62.5	....
	18.0	2.5	8	45	1,296,000
	20.0	14.4	2 min.	288	....
9. Working pump	13.2	2.5	10	33	1,188,000
10. Hammering	15	?	8?	?	480,000

EXPLANATION.—R, resistance; V, effective velocity = distance through which R is overcome ÷ total time occupied, including the time of moving unloaded, if any; T', time of working, in seconds per day; $T' \div 3600$, same time, in hours per day; RV, effective power, in foot-pounds per second; RVT, daily work.

Performance of a Man in Transporting Loads Horizontally. (Rankine.)

Kind of Exertion.	L, lbs.	V, ft.-sec.	$\frac{T}{3600}$ (hours per day).	LV, lbs. conveyed 1 foot.	LVT, lbs. conveyed 1 foot.
11. Walking unloaded, transporting his own weight	140	5	10	700	25,200,000
12. Wheeling load L in 2-whld. barrow, return unloaded	224	1⅔	10	373	13,428,000
13. Ditto in 1-wh. barrow, ditto	132	1⅔	10	220	7,920,000
14. Travelling with burden	90	2½	7	225	5,670,000
15. Carrying burden, returning unloaded	140	1⅔	6	233	5,032,800
16. Carrying burden, for 30 seconds only	252	0	....	0	....
	126	11.7	....	1474.2	....
	0	23.1	....	0	....

EXPLANATION.—L, load; V, effective velocity, computed as before; T', time of working, in seconds per day; $T' \div 3600$, same time in hours per day; LV, transport per second, in lbs. conveyed one foot; LVT, daily transport.

In the first line only of each of the two tables above is the weight of the man taken into account in computing the work done.

Clark says that the average net daily work of an ordinary laborer at a pump, a winch, or a crane may be taken at 3300 foot-pounds per minute, or one-tenth of a horse-power, for 8 hours a day; but for shorter periods from four to five times this rate may be exerted.

Mr. Glynn says that a man may exert a force of 25 lbs. at the handle of a crane for short periods; but that for continuous work a force of 15 lbs. is all that should be assumed, moving through 220 feet per minute.

Man-wheel.—Fig. 97 is a sketch of a very efficient man-power hoisting-machine which the author saw in Berne, Switzerland, in 1889. The face of the wheel was wide enough for three men to walk abreast, so that nine men could work in it at one time.

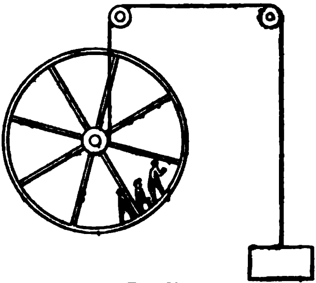

FIG. 97.

Work of a Horse against a Known Resistance. (Rankine.)

Kind of Exertion.	$R.$	$V.$	$\frac{T.}{3600}$	$RV.$	$RVT.$
1. Cantering and trotting, drawing a light railway carriage (thoroughbred)........ ..	min. 22½ mean 30½ max. 50	14⅔	4	447½	6,444,000
2. Horse drawing cart or boat, walking (draught-horse)....	120	3.6	8	432	12,441,600
3. Horse drawing a gin or mill, walking	100	3.0	8	300	8,640,000
4. Ditto, trotting	66	6.5	4½	429	6,950,000

EXPLANATION.—R, resistance, in lbs.; V, velocity, in feet per second; $T \div 3600$, hours work per day; RV, work per second; RVT, work per day.

The average power of a draught-horse, as given in line 2 of the above table, being 432 foot-pounds per second, is 432/550 = 0.785 of the conventional value assigned by Watt to the ordinary unit of the rate of work of prime movers. It is the mean of several results of experiments, and may be considered the average of ordinary performance under favorable circumstances.

Performance of a Horse in Transporting Loads Horizontally. (Rankine.)

Kind of Exertion.	$L.$	$V.$	$T.$	$LV.$	$LVT.$
5. Walking with cart, always loaded......................	1500	3.6	10	5400	194,400,000
6. Trotting, ditto.......	750	7.2	4½	5400	87,480,000
7. Walking with cart, going loaded, returning empty; V, mean velocity..............	1500	2.0	10	3000	108,000,000
8. Carrying burden, walking...	270	3.6	10	972	34,992,000
9. Ditto, trotting	180	7.2	7	1296	32,659,200

EXPLANATION.—L, load in lbs.; V, velocity in feet per second; $T \div 3600$, working hours per day; LV, transport per second; LVT, transport per day.

This table has reference to conveyance on common roads only, and those evidently in bad order as respects the resistance to traction upon them.

Horse Gin.—In this machine a horse works less advantageously than in drawing a carriage along a straight track. In order that the best possible results may be realised with a horse-gin, the diameter of the circular track in which the horse walks should not be less than about forty feet.

Oxen, Mules, Asses.—Authorities differ considerably as to the power of these animals. The following may be taken as an approximative comparison between them and draught-horses (Rankine):

Ox.—Load, the same as that of average draught-horse; best velocity and work, two thirds of horse.

Mule.—Load, one half of that of average draught-horse; best velocity, the same with horse; work one half.

Ass.—Load, one quarter that of average draught-horse; best velocity the same: work one quarter.

Reduction of Draught of Horses by Increase of Grade of Roads. (*Engineering Record*, Prize Essays on Roads, 1892.)—Experiments on English roads by Gayffier & Parnell:

Calling load that can be drawn on a level 100:

On a rise of.	1 in 100.	1 in 50.	1 in 40.	1 in 30.	1 in 26.	1 in 20.	1 in 10.
A horse can draw only	90.	81.	72.	64.	54.	40.	25.

The Resistance of Carriages on Roads is (according to Gen. Morin) given approximately by the following empirical formula:

$$R = \frac{W}{r}[a + b(u - 3.28)].$$

In this formula R = total resistance; r = radius of wheel in inches; W = gross load; u = velocity in feet per second; while a and b are constants, whose values are: For good broken-stone road, a = .4 to .55, b = .024 to .026; for paved roads, a = .27, b = .0684.

Rankine states that on gravel the resistance is about double, and on sand five times, the resistance on good broken-stone roads.

ELEMENTS OF MACHINES.

The object of a machine is usually to transform the work or mechanical energy exerted at the point where the machine receives its motion into work at the point where the final resistance is overcome. The specific end may be to change the character or direction of motion, as from circular to rectilinear, or *vice versa*, to change the velocity, or to overcome a great resistance by the application of a moderate force. In all cases the total energy exerted equals the total work done, the latter including the overcoming of all the frictional resistances of the machine as well as the useful work performed. No increase of power can be obtained from any machine, since this is impossible according to the law of conservation of energy. In a frictionless machine the product of the force exerted at the driving-point into the velocity of the driving-point, or the distance it moves in a given interval of time, equals the product of the resistance into the distance through which the resistance is overcome in the same time.

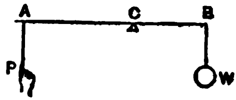

FIG. 98.

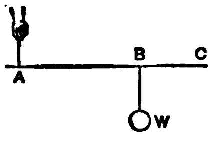

FIG. 99.

The most simple machines, or elementary machines, are reducible to three classes, viz., the Lever, the Cord, and the Inclined Plane.

The first class includes every machine consisting of a solid body capable of revolving on an axis, as the Wheel and Axle.

The second class includes every machine in which force is transmitted by means of flexible threads, ropes, etc., as the Pulley.

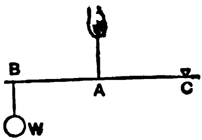

FIG. 100.

The third class includes every machine in which a hard surface inclined to the direction of motion is introduced, as the Wedge and the Screw.

A Lever is an inflexible rod capable of motion about a fixed point, called a fulcrum. The rod may be straight or bent at any angle, or curved. It is generally regarded, at first, as without weight, but its weight may be considered as another force applied in a vertical direction at its centre of gravity.

The arms of a lever are the portions of it intercepted between the force, P, and fulcrum, C, and between the weight, W, and fulcrum.

Levers are divided into three kinds or orders, according to the relative positions of the applied force, weight, and fulcrum.

In a lever of the first order, the fulcrum lies between the points at which the force and weight act. (Fig. 98.)

In a lever of the second order, the weight acts at a point between the fulcrum and the point of action of the force. (Fig. 99.)

In a lever of the third order, the point of action of the force is between that of the weight and the fulcrum. (Fig. 100.)

In all cases of levers the relation between the force exerted or the pull, P, and the weight lifted, or resistance overcome, W, is expressed by the equation $P \times AC = W \times BC$, in which AC is the lever-arm of P, and BC is the lever-arm of W, or moment of the force = the moment of the resistance. (See Moment.)

In cases in which the direction of the force (or of the resistance) is not at right angles to the arm of the lever on which it acts, the "lever-arm" is the length of a perpendicular from the fulcrum to the line of direction of the force (or of the resistance). $W : P :: AC : BC$, or, the ratio of the resistance to the applied force is the inverse ratio of their lever-arms. Also, if Vw is the velocity of W, and Vp is the velocity of P, $W : P :: Vp : Vw$, and $P \times Vp = W \times Vw$.

If Sp is the distance through which the applied force acts, and Sw is the distance the weight is lifted or through which the resistance is overcome, $W : P :: Sp : Sw$; $W \times Sw = P \times Sp$, or the weight into the distance it is lifted equals the force into the distance through which it is exerted.

These equations are general for all classes of machines as well as for levers, it being understood that friction, which in actual machines increases the resistance, is not at present considered.

The Bent Lever.—In the bent lever (see Fig. 91, page 416) the lever-arm of the weight m is cf instead of bf. The lever is in equilibrium when $n \times af = m \times cf$, but it is to be observed that the action of a bent lever may be very different from that of a straight lever. In the latter, so long as the force and the resistance act in lines parallel to each other, the ratio of the lever-arms remains constant, although the lever itself changes its inclination with the horizontal. In the bent lever, however, this ratio changes: thus, in the cut, if the arm bf is depressed to a horizontal direction, the distance cf lengthens while the horizontal projection of af shortens, the latter becoming zero when the direction of af becomes vertical. As the arm af approaches the vertical, the weight m which may be lifted with a given force s is very great, but the distance through which it may be lifted is very small. In all cases the ratio of the weight m to the weight n is the inverse ratio of the horizontal projection of their respective lever-arms.

The Moving Strut (Fig. 101) is similar to the bent lever, except that one of the arms is missing, and that the force and the resistance to be overcome act at the same end of the single arm. The resistance in the case shown in the cut is not the weight W, but its resistance to being moved, R, which may be simply that due to its friction on the horizontal plane, or some other opposing force. When the angle between the strut and the horizontal plane changes, the ratio of the resistance to the applied force changes. When the angle becomes very small, a moderate force will overcome a very great resistance, which tends to become infinite as the angle approaches zero. If a = the angle, $P \times \cos a = R \times \sin a$. If a = 5 degrees, $\cos a = .99619$, $\sin a = .08716$, $R = 11.44\ P$.

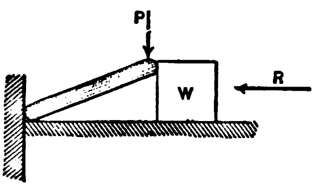

FIG. 101.

The stone-crusher (Fig. 102) shows a practical example of the use of two moving struts.

The Toggle-joint is an elbow or knee-joint consisting of two bars so connected that they may be brought into a straight line and made to produce great endwise pressure when a force is applied to bring them into this position. It is a case of two moving struts placed end to end, the moving force being applied at their point of junction, in a direction at right angles to the direction of the resistance, the other end of one of the struts resting against a fixed abutment, and that of the other against the body to be moved. If a = the angle each strut makes with the straight line joining the points about which their outer ends rotate, the ratio of the resistance to the applied force is $R : P :: \cos a : 2 \sin a$; $2R \sin a = P \cos a$. The

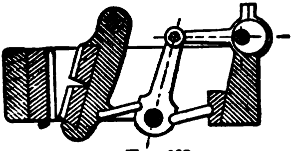

FIG. 102.

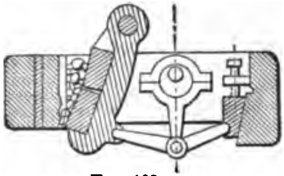

FIG. 103.

ratio varies when the angle varies, becoming infinite when the angle becomes zero.

The toggle-joint is used where great resistances are to be overcome through very small distances, as in stone-crushers (Fig. 103).

The Inclined Plane, as a mechanical element, is supposed perfectly hard and smooth, unless friction be considered. It assists in sustaining a heavy body by its reaction. This reaction, however, being normal to the plane, cannot entirely counteract the weight of the body, which acts vertically downward. Some other force must therefore be made to act upon the body, in order that it may be sustained.

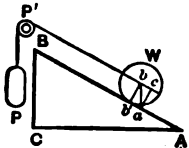

FIG. 104.

If the sustaining force act parallel to the plane (Fig. 104), the force is to the weight as the height of the plane is to its length, measured on the incline.

If the force act parallel to the base of the plane, the power is to the weight as the height is to the base.

If the force act at any other angle, let i = the angle of the plane with the horizon, and e = the angle of the direction of the applied force with the angle of the plane. $P : W :: \sin i : \cos e$; $P \times \cos e = W \sin i$.

Problems of the inclined plane may be solved by the parallelogram of forces thus:

Let the weight W be kept at rest on the incline by the force P, acting in the line bP', parallel to the plane. Draw the vertical line ba to represent the weight: also bb' perpendicular to the plane, and complete the parallelogram $b'c$. Then the vertical weight ba is the resultant of bb', the measure of support given by the plane to the weight, and bc, the force of gravity tending to draw the weight down the plane. The force required to maintain the weight in equilibrium is represented by this force bc. Thus the force and the weight are in the ratio of bc to ba. Since the triangle of forces abc is similar to the triangle of the incline ABC, the latter may be substituted for the former in determining the relative magnitude of the forces, and

$$P : W :: bc : ab :: BC : AB.$$

The Wedge is a pair of inclined planes united by their bases. In the application of pressure to the head or butt end of the wedge, to cause it to penetrate a resisting body, the applied force is to the resistance as the thickness of the wedge is to its length. Let t be the thickness, l the length, W the resistance, and P the applied force or pressure on the head of the wedge. Then, friction neglected, $P : W :: t : l$; $P = \frac{Wt}{l}$; $W = \frac{Pl}{t}$.

The Screw is an inclined plane wrapped around a cylinder in such a way that the height of the plane is parallel to the axis of the cylinder. If the screw is formed upon the internal surface of a hollow cylinder, it is usually called a nut. When force is applied to raise a weight or overcome a resistance by means of a screw and nut, either the screw or the nut may be fixed, the other being movable. The force is generally applied at the end of a wrench or lever-arm, or at the circumference of a wheel. If r = radius of the wheel or lever-arm, and p = pitch of the screw, or distance between threads, that is, the height of the inclined plane for one revolution of the screw. P = the applied force, and W = the resistance overcome, then, neglecting resistance due to friction, $2\pi r \times P = Wp$; $W = 6.283 Pr \div p$. The ratio of P to W is thus independent of the diameter of the screw. In actual screws, much of the power transmitted is lost through friction.

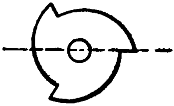

FIG. 105.

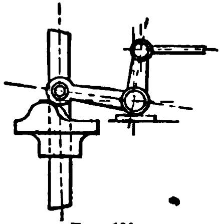

FIG. 106.

The Cam is a revolving inclined plane. It may be either an inclined plane wrapped around a cylinder in such a way that the height of the plane is radial to the cylinder, such as the ordinary lifting-cam, used in stamp-mills (Fig. 105), or it may be an inclined plane curved edgewise, and rotating in a plane parallel to its base (Fig. 106). The relation of the weight to the applied force is calculated in the same manner as in the case of the screw.

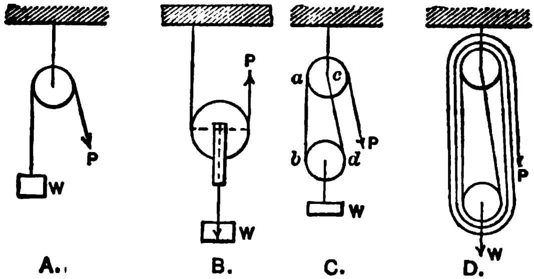

FIG. 107.

Pulleys or Blocks.—P = force applied, or pull; W = weight lifted or resistance. In the simple pulley A (Fig. 107) the point P on the pulling rope descends the same amount that the weight is lifted, therefore $P = W$. In B and C the point P moves twice as far as the weight is lifted, therefore $W = 2P$. In B and C there is one movable block, and two plies of the rope engage with it. In D there are three sheaves in the movable block, each with two plies engaged, or six in all. Six plies of the rope are therefore shortened by the same amount that the weight is lifted, and the point P moves six times as far as the weight, consequently $W = 6P$. In general, the ratio of W to P is equal to the number of plies of the rope that are shortened, and also is equal to the number of plies that engage the lower block. If the lower block has 2 sheaves and the upper 3, the end of the rope is fastened to a hook in the top of the lower block, and then there are 5 plies shortened instead of 6, and $W = 5P$. If V = velocity of W, and v = velocity of P, then in all cases $VW = vP$, whatever the number of sheaves or their arrangement. If the hauling rope, at the pulling end, passes first around a sheave in the upper or stationary block, it makes no difference in what direction the rope is led from this block to the point at which the pull on the rope is applied; but if it first passes around the movable block, it is necessary that the pull be exerted in a direction parallel to the line of action of the resistance, or a line joining the centres of the two blocks, in order to obtain the maximum effect. If the rope pulls on the lower block at an angle, the block will be pulled out of the line drawn between the weight and the upper block, and the effective pull will be less than the actual pull on the rope in the ratio of the cosine of the angle the pulling rope makes with the vertical, or line of action of the resistance, to unity.

Differential Pulley. (Fig. 108.)—Two pulleys, B and C, of different radii, rotate as one piece about a fixed axis, A. An endless chain, $BDECLKH$, passes over both pulleys. The rims of the pulleys are shaped so as to hold the chain and prevent it from slipping. One of the bights or loops in which the chain hangs, DE, passes under and supports the running block F. The other loop or bight, HKL, hangs freely, and is called the hauling part. It is evident that the velocity of the hauling part is equal to that of the pitch-circle of the pulley B.

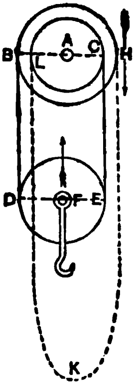

FIG. 108.

In order that the velocity-ratio may be exactly uniform, the radius of the sheave F should be an exact mean between the radii of B and C.

Consider that the point B of the cord BD moves through an arc whose length $= AB$, during the same time the point C or the cord CE will move downward a distance $= AC$. The length of the bight or loop $BDEC$ will be shortened by $AB - AC$, which will cause the pulley F to be raised half of this amount. If $P =$ the pulling force on the cord HK, and W the weight lifted at F, then $P \times AB = W \times \frac{1}{2}(AB - AC)$.

To calculate the length of chain required for a differential pulley, take the following sum: Half the circumference of A + half the circumference of B + half the circumference of F + twice the greatest distance of F from A + the least length of loop HKL. The last quantity is fixed according to convenience.

The Differential Windlass (Fig. 109) is identical in principle with the differential pulley, the difference in construction being that in the differential windlass the running block hangs in the bight of a rope whose two parts are wound round, and have their ends respectively made fast to two barrels of different radii, which rotate as one piece about the axis A. The differential windlass is little used in practice, because of the great length of rope which it requires.

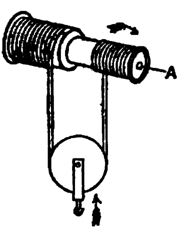

FIG. 109.

The Differential Screw (Fig. 110) is a compound screw of different pitches, in which the threads wind the same way. N_1 and N_2 are the two nuts; S_1S_1, the longer-pitched thread; S_2S_2, the shorter-pitched thread: in the figure both these threads are left-handed. At each turn of the screw the nut N_2 advances relatively to N^2 through a distance equal to the difference of the pitch. The use of the differential screw is to combine the slowness of advance due to a fine pitch with the strength of thread which can be obtained by means of a coarse pitch only.

A Wheel and Axle, or Windlass, resembles two pulleys on one axis, having different diameters. If a weight be lifted by means of a rope wound over the axle, the force being applied at the rim of the wheel, the action is like that of a lever of which the shorter arm is equal to the radius of the axle plus half the thickness of the rope, and the longer arm is equal to the radius of the wheel. A wheel and axle is therefore sometimes classed as a perpetual lever. If $P =$ the applied force, $D =$ diameter of the wheel, $W =$ the weight lifted, and d the diameter of the axle + the diameter of the rope, $PD = Wd$.

FIG. 110.

Toothed-wheel Gearing is a combination of two or more wheels and axles (Fig. 111). If a series of wheels and pinions gear into each other, as in the cut, friction neglected, the weight lifted, or resistance overcome, is to the force applied inversely as the distances through which they act in a given time. If R, R_1, R_2 be the radii of the successive wheels, measured to the pitch-line of the teeth, and r, r_1, r_2 the radii of the corresponding pinions, P the applied force, and W the weight lifted, $P \times$

$R \times R_1 \times R_2 = W \times r \times r_1 \times r_2$, or the applied force is to the weight as the product of the radii of the pinions is to the product of the radii of the wheels; or, as the product of the numbers expressing the teeth in each pinion is to the product of the numbers expressing the teeth in each wheel.

Endless Screw, or Worm-gear. (Fig. 112.)—This gear is commonly used to convert motion at high speed into motion at very slow

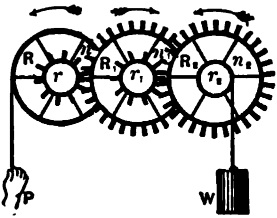

FIG. 111.

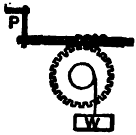

FIG. 112.

speed. When the handle *P* describes a complete circumference, the pitch-line of the cog-wheel moves through a distance equal to the pitch of the screw, and the weight *W* is lifted a distance equal to the pitch of the screw multiplied by the ratio of the diameter of the axle to the diameter of the pitch-circle of the wheel. The ratio of the applied force to the weight lifted is inversely as their velocities, friction not being considered; but the friction in the worm-gear is usually very great, amounting sometimes to three or four times the useful work done.

If v = the distance through which the force *P* acts in a given time, say 1 second, and V = distance the weight *W* is lifted in the same time, r = radius of the crank or wheel through which *P* acts, t = pitch of the screw, and also of the teeth on the cog-wheel, d = diameter of the axle, and D = diameter of the pitch-line of the cog-wheel, $v = \frac{6.283\ r}{t} \frac{D}{d} \times V$; $V = v \times td \div 6.283rd$. $Pv = WV +$ friction.

STRESSES IN FRAMED STRUCTURES.

Framed structures in general consist of one or more triangles, for the reason that the triangle is the one polygonal form whose shape cannot be changed without distorting one of its sides. Problems in stresses of simple framed structures may generally be solved either by the application of the triangle, paralellogram, or polygon of forces, by the principle of the lever, or by the method of moments. We shall give a few examples, referring the student to the works of Burr, Dubois, Johnson, and others for more elaborate treatment of the subject.

1. **A Simple Crane.** (Figs. 113 and 114.)—*A* is a fixed mast, *B* a brace or boom, *T* a tie, and *P* the load. Required the strains in *B* and *T*. The weight *P*, considered as acting at the end of the boom, is held in equilibrium by three forces: first, gravity acting downwards; second, the tension in *T*; and third, the thrust of *B*. Let the length of the line *p* represent the magnitude of the downward force exerted by the load, and draw a parallelogram with sides *bt* parallel, respectively, to *B* and *T*, such that *p* is the diagonal of the parallelogram. Then *b* and *t* are the components drawn to the same scale as *p*, *p* being the resultant. Then if the length *p* represents the load, *t* is the tension in the tie, and *b* is the compression in the brace.

Or, more simply, *T*, *B*, and that portion of the mast included between them or *A′* may represent a triangle of forces, and the forces are proportional to the length of the sides of the triangle; that is, if the height of the triangle *A′* = the load, then *B* = the compression in the brace, and *T* = the tension in the tie; or if *P* = the load in pounds, the tension in $T = P \times \frac{T}{A'}$, and the compression in $B = P \times \frac{B}{A'}$. Also, if a = the angle the inclined member makes with the mast, the other member being horizontal, and the triangle being right-angled, then the length of the inclined member = height of the triangle $\times$ secant a, and the strain in the inclined member = P secant a. Also, the strain in the horizontal member = P tan a.

The solution by the triangle or parallelogram of forces, and the equations Tension in $T = P \times T/A'$, and Compression in $B = P \times B/A'$, hold true even if the triangle is not right-angled, as in Fig. 115; but the trigonometrical rela-

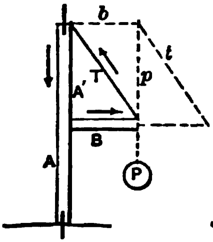

FIG. 113.

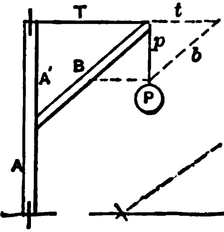

FIG. 114.

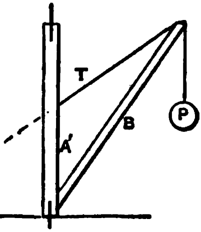

FIG. 115.

tions above given do not hold, except in the case of a right-angled triangle. It is evident that as A' decreases, the strain in both T and B increases, tending to become infinite as A' approaches zero. If the tie T is not attached to the mast, but is extended to the ground, as shown in the dotted line, the tension in it remains the same.

2. **A Guyed Crane or Derrick.** (Fig. 116.)—The strain in B is, as before, $P \times B/A'$, A' being that portion of the vertical included between B and T, wherever T may be attached to A. If, however, the tie T is attached to B beneath its extremity, there may be in addition a bending strain in B due to a tendency to turn about the point of attachment of T as a fulcrum.

The strain in T may be calculated by the principle of moments. The moment of P is Pc, that is, its weight $\times$ its perpendicular distance from the point of rotation of B on the mast. The moment of the strain on T is the product of the strain into the perpendicular distance from the line of its

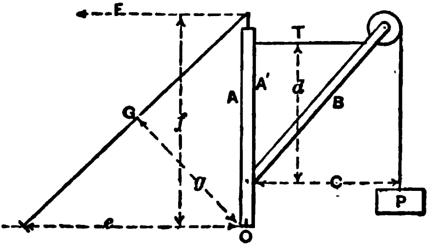

FIG. 116.

direction to the same point of rotation of B, or Td. The strain in T therefore $= Pc \div d$. As d decreases the strain on T increases, tending to infinity as d approaches zero.

The strain on the guy-rope is also calculated by the method of moments. The moment of the load about the bottom of the mast O is, as before, Pc. If the guy is horizontal the strain in it is F and its moment is Ff, and $F = Pc \div f$. If it is inclined, the moment is the strain $G \times$ the perpendicular distance of the line of its direction from O, or Gg, and $G = Pc \div g$.

The guy-rope having the least strain is the horizontal one F, and the strain

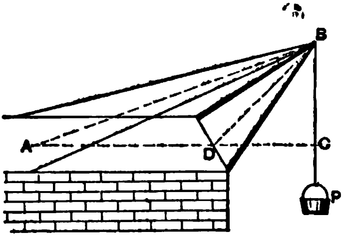

FIG. 117.

in G = the strain in F × the secant of the angle between F and G. As G is made more nearly vertical g decreases, and the strain increases, becoming infinite when $g = 0$.

3. **Shear-poles with Guys.** (Fig. 117.)—First assume that the two masts act as one placed at BD, and the two guys as one at AB. Calculate the strain in BD and AB as in Fig. 115. Multiply half the strain in BD (or AB) by the secant of half the angle the two masts (or guys) make with each other to find the strain in each mast (or guy).

Two Diagonal Braces and a Tie-rod. (Fig. 118.)—Suppose the braces are used to sustain a single load P. Compressive stress on $AD = \frac{1}{2}P \times AD \div AB$; on $CA = \frac{1}{2}P \times CA \div AB$. This is true only if CB and BD are of equal length, in which case $\frac{1}{2}$ of P is supported by each abutment C and D. If they are unequal in length (Fig. 119), then, by the principle of the lever, find the reactions of the abutments R_1 and R_2. If P is the load applied at the point B on the lever CD, the fulcrum being D, then $R_1 \times CD = P \times BD$ and $R_2 \times CD = P \times BC$; $R_1 = P \times BD \div CD$; $R_2 = P \times BC \div CD$.

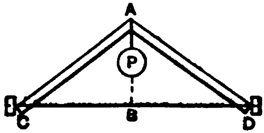

FIG. 118.

The strain on $AC = R_1 \times AC \div AB$, and on $AD = R_2 \times AD \div AB$.

The strain on the tie $= R_1 \times CB \div AB = R_2 \times BD \div AB$.

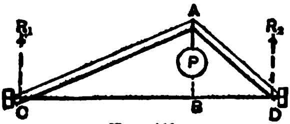

FIG. 119.

When $CB = BD$, $R_1 = R_2$. The strain on CB and BD is the same, whether the braces are of equal length or not, and is equal to $\frac{1}{2}P \times \frac{1}{2}CD \div AB$.

If the braces support a uniform load, as a pair of rafters, the strains caused by such a load are equivalent to that caused by one half of the load applied at the centre. The horizontal thrust of the braces against each other at the apex equals the tensile strain in the tie.

King-post Truss or Bridge. (Fig. 120.)—If the load is distributed over the whole length of the truss, the effect is the same as if half the load were placed at the centre, the other half being carried by the abutments. Let P = one half the load on the truss, then tension in the vertical tie $AB = P$. Compression in each of the inclined braces $= \frac{1}{2}P \times AD \div AB$. Tension in the tie $CD = \frac{1}{2}P \times BD \div AB$. Horizontal thrust of inclined brace AD at D = the tension in the tie. If W = the total load on one truss uniformly distributed, l = its length and d = its depth, then the tension on the horizontal tie $= \frac{Wl}{8d}$.

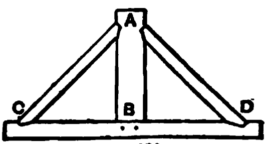

FIG. 120.

Inverted King-post Truss. (Fig. 121.)—If P = a load applied at B, or one half of a uniformly distributed load, then compression on $AB = P$ (the floor-beam CD not being considered to have any resistance to a slight bending). Tension on AC or $AD = \frac{1}{2}P \times AD \div AB$. Compression on $CD = \frac{1}{2}P \times BD \div AB$.

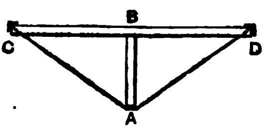

FIG. 121.

Queen-post Truss. (Fig. 122.)—If uniformly loaded, and the queen-posts divide the length into three equal bays, the load may be considered to be divided into three equal parts, two parts of which, P_1 and P_2, are concentrated at the panel joints and the remainder is equally divided between the abutments and supported by them directly. The two parts P_1 and P_2 only are considered to affect the members of the truss. Strain in the vertical ties BE and CF each equals P_1 or P_2. Strain on AB and CD each $= P_1 \times CD \div CF$. Strain on the tie AE or EF or $ED = P_1 \times FD \div CF$. Thrust on $BC =$ tension on EF.

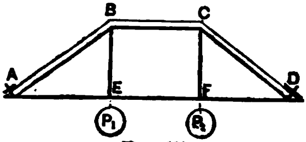

Fig. 122.

For stability to resist heavy unequal loads the queen-post truss should have diagonal braces from B to F and from C to E.

Inverted Queen-post Truss. (Fig. 123.)—Compression on EB and FC each $= P_1$ or P_2. Compression on AB or BC or $CD = P_1 \times AB \div EB$. Tension on AE or $FD = P_1 \times AE \div EB$. Tension on $EF =$ compression on BC. For stability to resist unequal loads, ties should be run from C to E and from B to F.

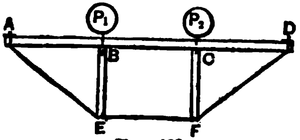

Fig. 123.

Burr Truss of Five Panels. (Fig. 124.)—Four fifths of the load may be taken as concentrated at the points E, K, L and F, the other fifth being

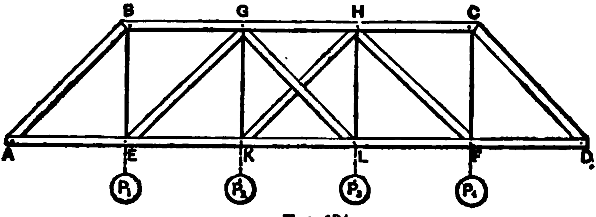

Fig. 124.

supported directly by the two abutments. For the strains in BA and CD the truss may be considered as a queen-post truss, with the loads P_1, P_2 concentrated at E and the loads P_3, P_4 concentrated at F. Then, compressive strain on $AB = (P_1 + P_2) \times AB \div BE$. The strain on CD is the same if the loads and panel lengths are equal. The tensile strain on BE or $CF = P_1 + P_2$. That portion of the truss between E and F may be considered as a smaller queen-post truss, supporting the loads P_2, P_3 at K and L. The strain on EG or $HF = P_2 \times EG \div GK$. The diagonals GL and KH receive no strain unless the truss is unequally loaded. The verticals GK and HL each receive a tensile strain equal to P_2 or P_3.

For the strain in the horizontal members: BG and CH receive a thrust equal to the horizontal component of the thrust in AB or CD, $= (P_1 + P_2) \times \tan$ angle ABE, or $(P_1 + P_2) \times AE \div BE$. GH receives this thrust and also, in addition, a thrust equal to the horizontal component of the thrust in EG or HF, or, in all, $(P_1 + P_2 + P_3) \times AE \div BE$.

The tension in AE or FD equals the thrust in BG or HC, and the tension in EK, KL, and LF equals the thrust in GH.

Pratt or Whipple Truss. (Fig. 125.)—In this truss the diagonals are ties, and the verticals are struts or columns.

Calculation by the method of distribution of strains: Consider first the load P_1. The truss having six bays or panels, 5/6 of the load is transmitted to the abutment H, and 1/6 to the abutment O, on the principle of the lever. As the five sixths must be transmitted through JA and AH, write on these members the figure 5. The one sixth is transmitted successively through JC, CK, KD, DL, etc., passing alternately through a tie and a strut. Write on these members, up to the strut GO inclusive, the figure 1. Then consider the load P_2, of which 4/6 goes to AH and 2/6 to GO. Write on KB, BJ, JA, and AH the figure 4, and on KD, DL, LE, etc., the figure 2. The load P_3

transmit 3/6 in each direction; write 3 on each of the members through which this stress passes, and so on for all the loads, when the figures on the several members will appear as on the cut. Adding them up, we have the following totals:

Tension on diagonals	AJ	BH	BK	CJ	CL	DK	DM	EL	EN	FM	FO	GN
	15	0	10	1	6	3	3	6	1	10	0	15

Compression on verticals	AH	BJ	CK	DL	EM	FN	GO
	15	10	7	6	7	10	15

Each of the figures in the first line is to be multiplied by $1/6P \times$ secant of angle HAJ, or $1/6P \times AJ \div AH$, to obtain the tension, and each figure in the lower line is to be multiplied by $1/6P$ to obtain the compression. The diagonals HB and FO receive no strain.

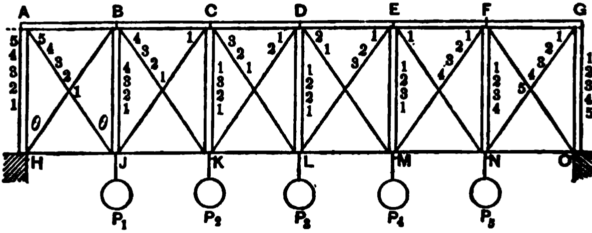

FIG. 125.

It is common to build this truss with a diagonal strut at HB instead of the post HA and the diagonal AJ; in which case 5/6 of the load P is carried through JB and the strut BH, which latter then receives a strain $= 15/6P \times$ secant of HBJ.

The strains in the upper and lower horizontal members or chords increase from the ends to the centre, as shown in the case of the Burr truss. AB receives a thrust equal to the horizontal component of the tension in AJ, or $15/6P \times \tan AJB$. BC receives the same thrust + the horizontal component of the tension in BK, and so on. The tension in the lower chord of each panel is the same as the thrust in the upper chord of the same panel. (For calculation of the chord strains by the method of moments, see below.)

The maximum thrust or tension is at the centre of the chords and is equal to $\frac{WL}{8D}$, in which W is the total load supported by the truss, L is the length, and D the depth. This is the formula for maximum stress in the chords of a truss of any form whatever.

The above calculation is based on the assumption that all the loads P_1, P_2, etc., are equal. If they are unequal the value of each has to be taken into account in distributing the strains. Thus the tension in AJ, with unequal loads, instead of being $15 \times 1/6\ P$ secant θ would be $\sec \theta \times (5/6P_1 + 4/6\ P_2 + 3/6\ P_3 + 2/6\ P_4 + 1/6\ P_5.)$ Each panel load, P_1 etc., includes its fraction of the weight of the truss.

General Formula for Strains in Diagonals and Verticals.—Let n = total number of panels, x = number of any vertical considered from the nearest end, counting the end as 1, r = rolling load for each panel, P = total load for each panel,

$$\text{Strain on verticals} = \frac{[(n-x)+(n-x)^2-(x-1)+(x-1)^2]P}{2n} + \frac{r(x-1)+(x-1)^2}{2n}.$$

For a uniformly distributed load, leave out the last term,

$$[r(x-1)+(x-1)^2] \div 2n.$$

Strain on principal diagonals = strain on verticals $\times$ secant θ, that is secant of the angle the diagonal makes with the vertical.

Strain on the counterbraces: The strain on the counterbrace in the first panel is 0, if the load is uniform. On the 2d, 3d, 4th, etc., it is P secant θ $\times \frac{1}{n}$, $\frac{1+2}{n}$, $\frac{1+2+3}{n}$, etc., P being the total load in one panel.

Strain in the Chords—Method of Moments.—Let the truss be uniformly loaded, the total load acting on it = W. Weight supported at each end, or reaction of the abutment = $W/2$. Length of the truss = L. Weight on a unit of length = W/L. Horizontal distance from the nearest abutment to the point (say M in Fig. 125) in the chord where the strain is to be determined = x. Horizontal strain at that point (tension on the lower chord, compression in the upper) = H. Depth of the truss = D. By the method of moments we take the difference of the moments, about the point M, of the reaction of the abutment and of the load between M and the abutments, and equate that difference with the moment of the resistance, or of the strain in the horizontal chord, considered with reference to a point in the opposite chord, about which the truss would turn if the first chord were severed at M.

The moment of the reaction of the abutment is $Wx/2$. The moment of the load from the abutment to M is $W/Lx \times$ the distance of its centre of gravity from M, which is $x/2$, or moment $= Wx^2 \div 2L$. Moment of the stress in the chord $= HD = \frac{Wx}{2} - \frac{Wx^2}{2L}$, whence $H = \frac{W}{2D}\left(x - \frac{x^2}{L}\right)$. If $x = 0$ or L, $H = 0$. If $x = L/2$, $H = \frac{WL}{8D}$, which is the horizontal strain at the middle of the chords, as before given.

The Howe Truss. (Fig. 126.)—In the Howe truss the diagonals are struts, and the verticals are ties. The calculation of strains may be made

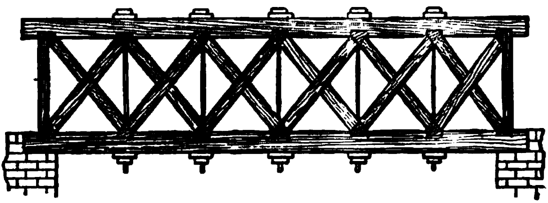

FIG. 126.

in the same method as described above for the Pratt truss.

The Warren Girder. (Fig. 127.)—In the Warren girder, or triangular truss, there are no vertical struts, and the diagonals may transmit either

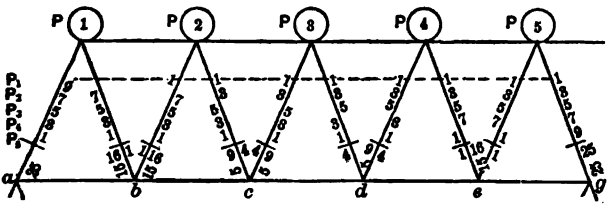

FIG. 127.

tension or compression. The strains in the diagonals may be calculated by the method of distribution of strains as in the case of the rectangular truss.

On the principle of the lever, the load P_1 being 1/10 of the length of the span from the line of the nearest support a, transmits 9/10 of its weight to a and 1/10 to g. Write 9 on the right hand of the strut $1a$, to represent the compression, and 1 on the right hand of $1b$, $2c$, $3d$, etc., to represent compression, and on the left hand of $b2$, $c3$, etc., to represent tension. The load P_2 transmits 7/10 of its weight to a and 3/10 to g. Write 7 on each member from 2 to a and 3 on each member from 2 to g, placing the figures representing compression on the right hand of the member, and those representing tension on the left. Proceed in the same manner with all the loads, then sum up the figures on each side of each diagonal, and write the difference of each sum beneath, and on the side of the greater sum, to show whether the difference represents tension or compression. The results are as follows: Compression, 1*a*, 25; 2*b*, 15; 3*c*, 5; 3*d*, 5; 4*e*, 15; 5*g*, 25. Tension, 1*b*, 15; 2*c*, 5; 4*d*, 5; 5*e*, 15. Each of these figures is to be multiplied by 1/10 of one of the loads as P_1, and by the secant of the angle the diagonals make with a vertical line.

The strains in the horizontal chords may be determined by the method of moments as in the case of rectangular trusses.

Roof-truss.—*Solution by Method of Moments.*—The calculation of strains in structures by the method of statical moments consists in taking a cross-section of the structure at a point where there are not more than three members (struts, braces, or chords).

To find the strain in either one of these members take the moment about the intersection of the other two as an axis of rotation. The sum of the moments of these members must be 0 if the structure is in equilibrium. But the moments of the two members that pass through the point of reference or axis are both 0, hence one equation containing one unknown quantity can be found for each cross-section.

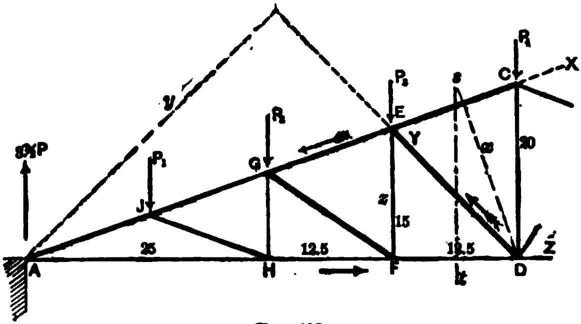

FIG. 128.

In the truss shown in Fig. 128 take a cross-section at *ts*, and determine the strain in the three members cut by it, viz., *CE*, *ED*, and *DF*. Let X = force exerted in direction *CE*, Y = force exerted in direction *DE*, Z = force exerted in direction *FD*.

For X take its moment about the intersection of Y and Z at $D = Xx$. For Y take its moment about the intersection of X and Z at $A = Yy$. For Z take its moment about the intersection of X and Y at $E = Zz$. Let $z = 15$, $x = 18.6$, $y = 38.4$. $AD = 50$, $CD = 20$ ft. Let P_1, P_2, P_3, P_4 be equal loads, as shown, and 3½ P the reaction of the abutment A.

The sum of all the moments taken about D or A or E will be 0 when the structure is at rest. Then $-Xx + 3.5P \times 50 - P_3 \times 12.5 - P_2 \times 25 - P_1 \times 37.5 = 0$.

The + signs are for moments in the direction of the hands of a watch or "clockwise" and − signs for the reverse direction or anti-clockwise. Since $P = P_1 = P_2 = P_3$, $-18.6X + 175P - 75P = 0$; $-18.6X = -100P$; $X = 100P \div 18.6 = 5.376P$.

$-Yy + P_1 \times 37.5 + P_2 \times 25 + P_3 \times 12.5 = 0$; $38.4Y = 75P$; $Y = 75P \div 38.4 = 1.953P$.

$-Zz + 3.5P \times 37.5 - P_1 \times 25 - P_2 \times 12.5 - P_3 \times 0 = 0$; $15Z = 93.75P$; $Z = 6.25P$.

In the same manner the forces exerted in the other members have been found as follows: $EG = 6.73P$; $GJ = 8.07P$; $JA = 9.42P$; $JH = 1.35P$; $GF = 1.59P$; $AH = 8.75P$; $HF = 7.50P$.

The Fink Roof-truss. (Fig. 129.)—An analysis by Prof. P. H. Philbrick (*Van N. Mag.*, Aug. 1880) gives the following results:

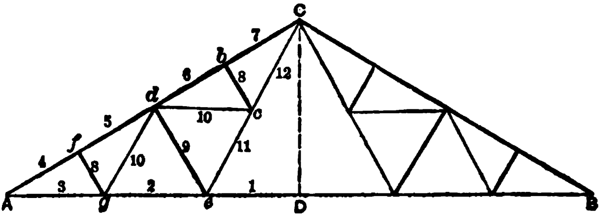

FIG. 129.

W = total load on roof;
N = No. of panels on both rafters;
$W/N = P$ = load at each joint b, d, f, etc.;
V = reaction at $A = \frac{1}{2}W = \frac{1}{2}NP = 4P$;
$AD = S$; $AC = L$; $CD = D$;
t_1, t_2, t_3 = tension on De, eg, gA, respectively;
c_1, c_2, c_3, c_4 = compression on Cb, bd, df, and fA.

Strains in

1, or $De = t_1 = 2PS \div D$;
2, " $eg = t_2 = 3PS \div D$;
3, " $gA = t_3 = 7/2PS \div D$;
4, " $Af = c_4 = 7/2PL \div D$;
5, " $fd = c_3 = 7/2PL/D - PD/L$;
6, " $db = c_2 = 7/2PL/D - 2PD/L$;
7, or $bC = c_1 = 7/2\ PL/D - 3\ PD/L$;
8, " bc or $fg = PS \div L$;
9, " $de = 2PS \div L$;
10, " cd or $dg = \frac{1}{2}PS \div D$;
11, " $ec = PS \div D$;
12, " $cC = 3/2\ PS \div D$.

Example.—Given a Fink roof-truss of span 64 ft., depth 16 ft., with four panels on each side, as in the cut; total load 32 tons, or 4 tons each at the points f, d, b, C, etc. (and 2 tons each at A and B, which transmit no strain to the truss members). Here $W = 32$ tons, $P = 4$ tons, $S = 32$ ft., $D = 16$ ft., $L = \sqrt{S^2 + D^2} = 2.236 \times D$. $L \div D = 2.236$, $D \div L = .4472$, $S \div D = 2$, $S \div L = .8944$. The strains on the numbered members then are as follows:

1,	$2 \times 4 \times 2$	= 16	tons;
2,	$3 \times 4 \times 2$	= 24	"
3,	$7/2 \times 4 \times 2$	= 28	"
4,	$7/2 \times 4 \times 2.236$	= 31.3	"
5,	$31.3 - 4 \times .447$	= 29.52	"
6,	$31.3 - 8 \times .447$	= 27.72	"
7,	$31.3 - 12 \times .447$	= 25.94	tons.
8,	$4 \times .8944$	= 3.58	"
9,	$8 \times .8944$	= 7.16	"
10,	2×2	= 4	"
11,	4×2	= 8	"
12,	6×2	= 12	"

The Economical Angle.—A structure of triangular form, Fig. 129*a*, is supported at a and b. It sustains any load L, the elements $c c$ being in compression and t in tension. Required the angle θ so that the total weight of the structure shall be a minimum. F. R. Honey (*Sci. Am. Supp.*, Jan. 17, 1895) gives a solution of this problem, with the result $\tan \theta = \sqrt{\frac{C + T}{T}}$, in which C and T represent the crushing and the tensile strength respectively of the material employed. It is applicable to any material. For $C = T$, $\tan \theta = 54\frac{3}{4}°$. For $C = 0.4T$ (yellow pine), $\tan \theta = 49\frac{3}{4}°$. For $C = 0.8T$ (soft steel), $\tan \theta = 53\frac{1}{4}°$. For $C = 6T$ (cast iron), $\tan \theta = 69\frac{1}{4}°$.

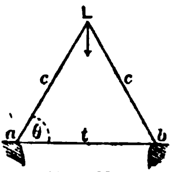

FIG. 129*a*.

HEAT.

THERMOMETERS.

The Fahrenheit thermometer is generally used in English-speaking countries, and the Centigrade, or Celsius thermometer, in countries that use the metric system. In many scientific treatises in English, however, the Centigrade temperatures are also used, either with or without their Fahrenheit equivalents. The Réaumur thermometer is used to some extent on the Continent of Europe.

In the Fahrenheit thermometer the freezing-point of water is taken at 32°, and the boiling-point of water at mean atmospheric pressure at the sea-level, 14.7 lbs. per sq. in., is taken at 212°, the distance between these two points being divided into 180°. In the Centigrade and Réaumur thermometers the freezing-point is taken at 0°. The boiling-point is 100° in the Centigrade scale, and 80° in the Réaumur.

1 Fahrenheit degree	= 5/9 deg. Centigrade	= 4/9 deg. Réaumur.
1 Centigrade degree	= 9/5 deg. Fahrenheit	= 4/5 deg. Réaumur.
1 Réaumur degree	= 9/4 deg. Fahrenheit	= 5/4 deg. Centigrade.
Temperature Fahrenheit	= 9/5 × temp. C. + 32°	= 9/4 R. + 32°.
Temperature Centigrade	= 5/9 (temp. F. − 32°)	= 5/4 R.
Temperature Réaumur	= 4/5 temp. C.	= 4/9 (F. − 32°).

Mercurial Thermometer. (Rankine, S. E., p. 234.)—The rate of expansion of mercury with rise of temperature increases as the temperature becomes higher; from which it follows, that if a thermometer showing the dilatation of mercury simply were made to agree with an air thermometer at 32° and 212°, the mercurial thermometer would show lower temperatures than the air thermometer between those standard points, and higher temperatures beyond them.

For example, according to Regnault, when the air thermometer marked 350° C. (= 662° F.), the mercurial thermometer would mark 362.16° C. (= 683.89° F.), the error of the latter being in excess 12.16° C. (= 21.89° F.).

Actual mercurial thermometers indicate intervals of temperature proportional to the difference between the expansion of mercury and that of glass. The inequalities in the rate of expansion of the glass (which are very different for different kinds of glass) correct, to a greater or less extent, the errors arising from the inequalities in the rate of expansion of the mercury.

For practical purposes connected with heat engines, the mercurial thermometer made of common glass may be considered as sensibly coinciding with the air-thermometer at all temperatures not exceeding 500° F.

PYROMETRY.

Principles Used in Various Pyrometers.—Contraction of clay by heat, as in the Wedgwood pyrometer used by potters. Not accurate, as the contraction varies with the quality of the clay.

Expansion of air, as in the air-thermometers, Wiborgh's pyrometer, Uehling and Steinbart's pyrometer, etc.

Specific heat of solids, as in the copper-ball, platinum-ball, and fire-clay pyrometers.

Relative expansion of two metals or other substances, as copper and iron, as in Brown's and Bulkley's pyrometers, etc.

Melting-points of metals, or other substances, as in approximate determinations of temperature by melting pieces of zinc, lead, etc.

Measurement of strength of a thermo-electric current produced by heating the junction of two metals, as in Le Chatelier's pyrometer.

Changes in electric resistance of platinum, as in the Siemens pyrometer.

Mixture of hot and cold air, as in Hobson's hot-blast pyrometer.

Time required to heat a weighed quantity of water enclosed in a vessel, as in the water pyrometer.

Thermometer for Temperatures up to 950° F.—Mercury with compressed nitrogen in the tube above the mercury. Made by Queen & Co., Philadelphia.

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
-40	-40.	26	78.8	92	197.6	158	316.4	224	435.2	290	554	950	1742
-39	-38.2	27	80.6	93	199.4	159	318.2	225	437.	300	572	960	1760
-38	-36.4	28	82.4	94	201.2	160	320.	226	438.8	310	590	970	1778
-37	-34.6	29	84.2	95	203.	161	321.8	227	440.6	320	608	980	1796
-36	-32.8	30	86.	96	204.8	162	323.6	228	442.4	330	626	990	1814
-35	-31.	31	87.8	97	206.6	163	325.4	229	444.2	340	644	1000	1832
-34	-29.2	32	89.6	98	208.4	164	327.2	230	446.	350	662	1010	1850
-33	-27.4	33	91.4	99	210.2	165	329.	231	447.8	360	680	1020	1868
-32	-25.6	34	93.2	100	212.	166	330.8	232	449.6	370	698	1030	1886
-31	-23.8	35	95.	101	213.8	167	332.6	233	451.4	380	716	1040	1904
-30	-22.	36	96.8	102	215.6	168	334.4	234	453.2	390	734	1050	1922
-29	-20.2	37	98.6	103	217.4	169	336.2	235	455.	400	752	1060	1940
-28	-18.4	38	100.4	104	219.2	170	338.	236	456.8	410	770	1070	1958
-27	-16.6	39	102.2	105	221.	171	339.8	237	458.6	420	788	1080	1976
-26	-14.8	40	104.	106	222.8	172	341.6	238	460.4	430	806	1090	1994
-25	-13.	41	105.8	107	224.6	173	343.4	239	462.2	440	824	1100	2012
-24	-11.2	42	107.6	108	226.4	174	345.2	240	464.	450	842	1110	2030
-23	- 9.4	43	109.4	109	228.2	175	347.	241	465.8	460	860	1120	2048
-22	- 7.6	44	111.2	110	230.	176	348.8	242	467.6	470	878	1130	2066
-21	- 5.8	45	113.	111	231.8	177	350.6	243	469.4	480	896	1140	2084
-20	- 4.	46	114.8	112	233.6	178	352.4	244	471.2	490	914	1150	2102
-19	- 2.2	47	116.6	113	235.4	179	354.2	245	473.	500	932	1160	2120
-18	- 0.4	48	118.4	114	237.2	180	356.	246	474.8	510	950	1170	2138
-17	+ 1.4	49	120.2	115	239.	181	357.8	247	476.6	520	968	1180	2156
-16	3.2	50	122.	116	240.8	182	359.6	248	478.4	530	986	1190	2174
-15	5.	51	123.8	117	242.6	183	361.4	249	480.2	540	1004	1200	2192
-14	6.8	52	125.6	118	244.4	184	363.2	250	482.	550	1022	1210	2210
-13	8.6	53	127.4	119	246.2	185	365.	251	483.8	560	1040	1220	2228
-12	10.4	54	129.2	120	248.	186	366.8	252	485.6	570	1058	1230	2246
-11	12.2	55	131.	121	249.8	187	368.6	253	487.4	580	1076	1240	2264
-10	14.	56	132.8	122	251.6	188	370.4	254	489.2	590	1094	1250	2282
- 9	15.8	57	134.6	123	253.4	189	372.2	255	491.	600	1112	1260	2300
- 8	17.6	58	136.4	124	255.2	190	374.	256	492.8	610	1130	1270	2318
- 7	19.4	59	138.2	125	257.	191	375.8	257	494.6	620	1148	1280	2336
- 6	21.2	60	140.	126	258.8	192	377.6	258	496.4	630	1166	1290	2354
- 5	23.	61	141.8	127	260.6	193	379.4	259	498.2	640	1184	1300	2372
- 4	24.8	62	143.6	128	262.4	194	381.2	260	500.	650	1202	1310	2390
- 3	26.6	63	145.4	129	264.2	195	383.	261	501.8	660	1220	1320	2408
- 2	28.4	64	147.2	130	266.	196	384.8	262	503.6	670	1238	1330	2426
- 1	30.2	65	149.	131	267.8	197	386.6	263	505.4	680	1256	1340	2444
0	32.	66	150.8	132	269.6	198	388.4	264	507.2	690	1274	1350	2462
+ 1	33.8	67	152.6	133	271.4	199	390.2	265	509.	700	1292	1360	2480
2	35.6	68	154.4	134	273.2	200	392.	266	510.8	710	1310	1370	2498
3	37.4	69	156.2	135	275.	201	393.8	267	512.6	720	1328	1380	2516
4	39.2	70	158.	136	276.8	202	395.6	268	514.4	730	1346	1390	2534
5	41.	71	159.8	137	278.6	203	397.4	269	516.2	740	1364	1400	2552
6	42.8	72	161.6	138	280.4	204	399.2	270	518.	750	1382	1410	2570
7	44.6	73	163.4	139	282.2	205	401.	271	519.8	760	1400	1420	2588
8	46.4	74	165.2	140	284.	206	402.8	272	521.6	770	1418	1430	2606
9	48.2	75	167.	141	285.8	207	404.6	273	523.4	780	1436	1440	2624
10	50.	76	168.8	142	287.6	208	406.4	274	525.2	790	1454	1450	2642
11	51.8	77	170.6	143	289.4	209	408.2	275	527.	800	1472	1460	2660
12	53.6	78	172.4	144	291.2	210	410.	276	528.8	810	1490	1470	2678
13	55.4	79	174.2	145	293.	211	411.8	277	530.6	820	1508	1480	2696
14	57.2	80	176.	146	294.8	212	413.6	278	532.4	830	1526	1490	2714
15	59.	81	177.8	147	296.6	213	415.4	279	534.2	840	1544	1500	2732
16	60.8	82	179.6	148	298.4	214	417.2	280	536.	850	1562	1510	2750
17	62.6	83	181.4	149	300.2	215	419.	281	537.8	860	1580	1520	2768
18	64.4	84	183.2	150	302.	216	420.8	282	539.6	870	1598	1530	2786
19	66.2	85	185.	151	303.8	217	422.6	283	541.4	880	1616	1540	2804
20	68.	86	186.8	152	305.6	218	424.4	284	543.2	890	1634	1550	2822
21	69.8	87	188.6	153	307.4	219	426.2	285	545.	900	1652	1600	2912
22	71.6	88	190.4	154	309.2	220	428.	286	546.8	910	1670	1650	3002
23	73.4	89	192.2	155	311.	221	429.8	287	548.6	920	1688	1700	3092
24	75.2	90	194.	156	312.8	222	431.6	288	550.4	930	1706	1750	3182
25	77.	91	195.8	157	314.6	223	433.4	289	552.2	940	1724	1800	3272

F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.
−40	−40.	26	− 3.3	92	33.3	158	70.	224	106.7	290	143.3	360	182.2
−39	−39.4	27	− 2.8	93	33.9	159	70.6	225	107.2	291	143.9	370	187.8
−38	−38.9	28	− 2.2	94	34.4	160	71.1	226	107.8	292	144.4	380	193.3
−37	−38.3	29	− 1.7	95	35.	161	71.7	227	108.3	293	145.	390	198.9
−36	−37.8	30	− 1.1	96	35.6	162	72.2	228	108.9	294	145.6	400	204.4
−35	−37.2	31	− 0.6	97	36.1	163	72.8	229	109.4	295	146.1	410	210.
−34	−36.7	32	0.	98	36.7	164	73.3	230	110.	296	146.7	420	215.6
−33	−36.1	33	+ 0.6	99	37.2	165	73.9	231	110.6	297	147.2	430	221.1
−32	−35.6	34	1.1	100	37.8	166	74.4	232	111.1	298	147.8	440	226.7
−31	−35.	35	1.7	101	38.3	167	75.	233	111.7	299	148.3	450	232.2
−30	−34.4	36	2.2	102	38.9	168	75.6	234	112.2	300	148.9	460	237.8
−29	−33.9	37	2.8	103	39.4	169	76.1	235	112.8	301	149.4	470	243.3
−28	−33.3	38	3.3	104	40.	170	76.7	236	113.3	302	150.	480	248.9
−27	−32.8	39	3.9	105	40.6	171	77.2	237	113.9	303	150.6	490	254.4
−26	−32.2	40	4.4	106	41.1	172	77.8	238	114.4	304	151.1	500	260.
−25	−31.7	41	5.	107	41.7	173	78.3	239	115.	305	151.7	510	265.6
−24	−31.1	42	5.6	108	42.2	174	78.9	240	115.6	306	152.2	520	271.1
−23	−30.6	43	6.1	109	42.8	175	79.4	241	116.1	307	152.8	530	276.7
−22	−30.	44	6.7	110	43.3	176	80.	242	116.7	308	153.3	540	282.2
−21	−29.4	45	7.2	111	43.9	177	80.6	243	117.2	309	153.9	550	287.8
−20	−28.9	46	7.8	112	44.4	178	81.1	244	117.8	310	154.4	560	293.3
−19	−28.3	47	8.3	113	45.	179	81.7	245	118.3	311	155.	570	298.9
−18	−27.8	48	8.9	114	45.6	180	82.2	246	118.9	312	155.6	580	304.4
−17	−27.2	49	9.4	115	46.1	181	82.8	247	119.4	313	156.1	590	310.
−16	−26.7	50	10.	116	46.7	182	83.3	248	120.	314	156.7	600	315.6
−15	−26.1	51	10.6	117	47.2	183	83.9	249	120.6	315	157.2	610	321.1
−14	−25.6	52	11.1	118	47.8	184	84.4	250	121.1	316	157.8	620	326.7
−13	−25.	53	11.7	119	48.3	185	85.	251	121.7	317	158.3	630	332.2
−12	−24.4	54	12.2	120	48.9	186	85.6	252	122.2	318	158.9	640	337.8
−11	−23.9	55	12.8	121	49.4	187	86.1	253	122.8	319	159.4	650	343.3
−10	−23.3	56	13.3	122	50.	188	86.7	254	123.3	320	160.	660	348.9
− 9	−22.8	57	13.9	123	50.6	189	87.2	255	123.9	321	160.6	670	354.4
− 8	−22.2	58	14.4	124	51.1	190	87.8	256	124.4	322	161.1	680	360.
− 7	−21.7	59	15.	125	51.7	191	88.3	257	125.	323	161.7	690	365.6
− 6	−21.1	60	15.6	126	52.2	192	88.9	258	125.6	324	162.2	700	371.1
− 5	−20.6	61	16.1	127	52.8	193	89.4	259	126.1	325	162.8	710	376.7
− 4	−20.	62	16.7	128	53.3	194	90.	260	126.7	326	163.3	720	382.2
− 3	−19.4	63	17.2	129	53.9	195	90.6	261	127.2	327	163.9	730	387.8
− 2	−18.9	64	17.8	130	54.4	196	91.1	262	127.8	328	164.4	740	393.3
− 1	−18.3	65	18.3	131	55	197	91.7	263	128.3	329	165.	750	398.9
0	−17.8	66	18.9	132	55.6	198	92.2	264	128.9	330	165.6	760	404.4
+ 1	−17.2	67	19.4	133	56.1	199	92.8	265	129.4	331	166.1	770	410.
2	−16.7	68	20.	134	56.7	200	93.3	266	130.	332	166.7	780	415.6
3	−16.1	69	20.6	135	57.2	201	93.9	267	130.6	333	167.2	790	421.1
4	−15.6	70	21.1	136	57.8	202	94.4	268	131.1	334	167.8	800	426.7
5	−15.	71	21.7	137	58.3	203	95.	269	131.7	335	168.3	810	432.2
6	−14.4	72	22.2	138	58.9	204	95.6	270	132.2	336	168.9	820	437.8
7	−13.9	73	22.8	139	59.4	205	96.1	271	132.8	337	169.4	830	443.3
8	−13.3	74	23.3	140	60.	206	96.7	272	133.3	338	170.	840	448.9
9	−12.8	75	23.9	141	60.6	207	97.2	273	133.9	339	170.6	850	454.4
10	−12.2	76	24.4	142	61.1	208	97.8	274	134.4	340	171.1	860	460.
11	−11.7	77	25.	143	61.7	209	98.3	275	135.	341	171.7	870	465.6
12	−11.1	78	25.6	144	62.2	210	98.9	276	135.6	342	172.2	880	471.1
13	−10.6	79	26.1	145	62.8	211	99.4	277	136.1	343	172.8	890	476.7
14	−10.	80	26.7	146	63.3	212	100.	278	136.7	344	173.3	900	482.2
15	− 9.4	81	27.2	147	63.9	213	100.6	279	137.2	345	173.9	910	487.8
16	− 8.9	82	27.8	148	64.4	214	101.1	280	137.8	346	174.4	920	493.3
17	− 8.3	83	28.3	149	65.	215	101.7	281	138.3	347	175.	930	498.9
18	− 7.8	84	28.9	150	65.6	216	102.2	282	138.9	348	175.6	940	504.4
19	− 7.2	85	29.4	151	66.1	217	102.8	283	139.4	349	176.1	950	510.
20	− 6.7	86	30.	152	66.7	218	103.3	284	140.	350	176.7	960	515.6
21	− 6.1	87	30.6	153	67.2	219	103.9	285	140.6	351	177.2	970	521.1
22	− 5.6	88	31.1	154	67.8	220	104.4	286	141.1	352	177.8	980	526.7
23	− 5.	89	31.7	155	68.3	221	105.	287	141.7	353	178.3	990	532.2
24	− 4.4	90	32.2	156	68.9	222	105.6	288	142.2	354	178.9	1000	537.8
25	− 3.9	91	32.8	157	69.4	223	106.1	289	142.8	355	179.4	1010	543.3

Platinum or Copper Ball Pyrometer.—A weighed piece of platinum, copper, or iron is allowed to remain in the furnace or heated chamber till it has attained the temperature of its surroundings. It is then suddenly taken out and dropped into a vessel containing water of a known weight and temperature. The water is stirred rapidly and its maximum temperature taken. Let W = weight of the water, w the weight of the ball, t = the original and T the final heat of the water, and S the specific heat of the metal; then the temperature of fire may be found from the formula

$$x = \frac{W(T - t)}{wS} + T.$$

The mean specific heat of platinum between 32° and 446° F. is .03333 or 1/30 that of water, and it increases with the temperature about .000305 for each 100° F. For a fuller description, by J. C. Hoadley, see Trans. A. S. M. E., vi. 702. Compare also Henry M. Howe, Trans. A. I. M. E., xviii. 728.

For accuracy corrections are required for variations in the specific heat of the water and of the metal at different temperatures, for loss of heat by radiation from the metal during the transfer from the furnace to the water, and from the apparatus during the heating of the water; also for the heat-absorbing capacity of the vessel containing the water.

Fire-clay or fire-brick may be used instead of the metal ball.

Le Chatelier's Thermo-electric Pyrometer.—For a very full description see paper by Joseph Struthers, *School of Mines Quarterly*, vol. xii, 1891; also, paper read by Prof. Roberts-Austen before the Iron and Steel Institute, May 7, 1891.

The principle upon which this pyrometer is constructed is the measurement of a current of electricity produced by heating a couple composed of two wires, one platinum and the other platinum with 10% rhodium—the current produced being measured by a galvanometer.

The composition of the gas which surrounds the couple has no influence on the indications.

When temperatures above 2500° F. are to be studied, the wires must have an isolating support and must be of good length, so that all parts of a furnace can be reached.

For a Siemens furnace, about 11½ feet is the general length. The wires are supported in an iron tube, ½ inch interior diameter and held in place by a cylinder of refractory clay having two holes bored through, in which the wires are placed. The shortness of time (five seconds) allows the temperature to be taken without deteriorating the tube.

Tests made by this pyrometer in measuring furnace temperatures under a great variety of conditions show that the readings of the scale uncorrected are always within 45° F. of the correct temperature, and in the majority of industrial measurements this is sufficiently accurate. Le Chatelier's pyrometer is sold by Queen & Co., of Philadelphia.

Graduation of Le Chatelier's Pyrometer.—W. C. Roberts-Austen in his Researches on the Properties of Alloys, Proc. Inst. M. E. 1892, says: The electromotive force produced by heating the thermo-junction to any given temperature is measured by the movement of the spot of light on the scale graduated in millimetres. A formula for converting the divisions of the scale into thermometric degrees is given by M. Le Chatelier; but it is better to calibrate the scale by heating the thermo-junction to temperatures which have been very carefully determined by the aid of the air-thermometer, and then to plot the curve from the data so obtained. Many fusion and boiling-points have been established by concurrent evidence of various kinds, and are now very generally accepted. The following table contains certain of these:

Deg. F.	Deg. C.		Deg. F.	Deg. C.	
212	100	Water boils.	1733	945	Silver melts.
618	326	Lead melts.	1859	1015	Potassium sulphate melts.
676	358	Mercury boils.	1913	1045	Gold melts.
779	415	Zinc melts.	1929	1054	Copper melts.
838	448	Sulphur boils.	2732	1500	Palladium melts.
1157	625	Aluminum melts.	3227	1775	Platinum melts.
1229	665	Selenium boils.			

The Temperatures Developed in Industrial Furnaces.—M. Le Chatelier states that by means of his pyrometer he has discovered that the temperatures which occur in melting steel and in other industrial operations have been hitherto overestimated.

M. Le Chatelier finds the melting heat of white cast iron 1135° (2075° F.), and that of gray cast iron 1220° (2228° F.). Mild steel melts at 1475° (2687° F.), semi-mild at 1455° (2651° F.), and hard steel at 1410° (2570° F.). The furnace for hard porcelain at the end of the baking has a heat of 1370° (2498° F.). The heat of a normal incandescent lamp is 1800° (3272° F.), but it may be pushed to beyond 2100° (3812° F.).

Prof. Roberts-Austen (Recent Advances in Pyrometry, Trans. A. I. M. E., Chicago Meeting, 1893) gives an excellent description of modern forms of pyrometers. The following are some of his temperature determinations.

Gold-melting, Royal Mint.

	Degrees. Centigrade.	Degrees. Fahr.
Temperature of standard alloy, pouring into moulds.	1180	2156
Temperature of standard alloy, pouring into moulds (on a previous occasion, by thermo-couple)	1147	2097
Annealing blanks for coinage, temperature of chamber	890	1634

Silver-melting, Royal Mint.

	Degrees. Centigrade.	Degrees. Fahr.
Temperature of standard alloy, pouring into mould	980	1796

Ten-ton Open-hearth Furnace, Woolwich Arsenal.

	Degrees. Centigrade.	Degrees. Fahr.
Temperature of steel, 0.3% carbon, pouring into ladle	1645	2993
Steel, 0.3% carbon, pouring into large mould	1580	2876
Reheating furnace, interior	930	1706
Cupola furnace, No. 2 cast iron, pouring into ladle	1600	2912

The following determinations have been effected by M. Le Chatelier:

Bessemer Process.

Six-ton Converter.

	Degrees. Centigrade	Degrees Fahr.
A. Bath of slag	1580	2876
B. Metal in ladle	1640	2984
C. Metal in ingot mould	1580	2876
D. Ingot in reheating furnace	1200	2192
E. Ingot under the hammer	1080	1976

Open-hearth Furnace (Siemens).

Semi-Mild Steel.

	Degrees. Centigrade	Degrees Fahr.
A. Fuel gas near gas generator	720	1328
B. Fuel gas entering into bottom of regenerator chamber	400	752
C. Fuel gas issuing from regenerator chamber	1200	2192
Air issuing from regenerator chamber	1000	1832
Chimney gases. Furnace in perfect condition	300	590
End of the melting of pig charge	1420	2588
Completion of conversion	1500	2732
Molten steel. In the ladle—Commencement of casting	1580	2876
End of casting	1490	2714
In the moulds	1520	2768

For very mild (soft) steel the temperatures are higher by 50° C.

Siemens Crucible or Pot Furnace.

1600° C., 2912° F.

Rotary Puddling Furnace.

	Degrees C.	Degrees F
Furnace	1340–1230	2444–2246
Puddled ball—End of operation	1330	2426

Blast-furnace (Gray-Bessemer Pig).

	Degrees C.	Degrees F
Opening in face of tuyere	1930	3506
Molten metal—Commencement of fusion	1400	2552
End, or prior to tapping	1570	2858

Hoffman Red-brick Kiln.

	Degrees C.	Degrees F
Burning temperatures	1100	2012

Hobson's Hot-blast Pyrometer consists of a brass chamber having three hollow arms and a handle. The hot blast enters one of the arms and induces a current of atmospheric air to flow into the second arm. The two currents mix in the chamber and flow out through the third arm, in which the temperature of the mixture is taken by a mercury thermometer. The openings in the arms are adjusted so that the proportion of hot blast to the atmospheric air remains the same.

The Wiborgh Air-pyrometer. (E. Trotz, Trans. A. I. M. E. 1892.)—The inventor using the expansion-coefficient of air, as determined by Gay-Lussac, Dulon, Rudberg, and Regnault, bases his construction on the following theory: If an air-volume, V, enclosed in a porcelain globe and connected through a capillary pipe with the outside air, be heated to the temperature T (which is to be determined) and thereupon the connection be discontinued, and there be then forced into the globe containing V another volume of air V' of known temperature t, which was previously under atmospheric pressure H, the additional pressure h, due to the addition of the air-volume V' to the air-volume V, can be measured by a manometer. But this pressure is of course a function of the temperature T. Before the introduction of V', we have the two separate air-volumes, V at the temperature T and V' at the temperature t, both under the atmospheric pressure H. After the forcing in of V' into the globe, we have, on the contrary, only the volume V of the temperature T, but under the pressure $H + h$.

The Wiborgh Air-pyrometer is adapted for use at blast-furnaces, smelting-works, hardening and tempering furnaces, etc., where determinations of temperature from 0° to 2400° F. are required.

Seger's Fire-clay Pyrometer. (H. M. Howe, *Eng. and Mining Jour.*, June 7, 1890.)—Professor Seger uses a series of slender triangular fire-clay pyramids, about 3 inches high and ⅝ inch wide at the base, and each a little less fusible than the next: these he calls "normal pyramids" ("normal-kegel"). When the series is placed in a furnace whose temperature is gradually raised, one after another will bend over as its range of plasticity is reached; and the temperature at which it has bent, or "wept," so far that its apex touches the hearth of the furnace or other level surface on which it is standing, is selected as a point on Seger's scale. These points may be accurately determined by some absolute method, or they may merely serve to give comparative results. Unfortunately, these pyramids afford no indications when the temperature is stationary or falling.

Mesuré and Nouel's Pyrometric Telescope. (*Ibid.*)—Mesuré and Nouel's pyrometric telescope gives us an immediate determination of the temperature of incandescent bodies, and is therefore much better adapted to cases where a great number of observations are to be made, and at short intervals, than Seger's. Such cases arise in the careful heating of steel. The little telescope, carried in the pocket or hung from the neck, can be used by foreman or heater at any moment.

It is based on the fact that a plate of quartz, cut at right angles to the axis, rotates the plane of polarization of polarized light to a degree nearly inversely proportional to the square of the length of the waves; and, further, on the fact that while a body at dull redness merely emits red light, as the temperature rises, the orange, yellow, green, and blue waves successively appear.

If, now, such a plate of quartz is placed between two Nicol prisms at right angles, "a ray of monochromatic light which passes the first, or polarizer, and is watched through the second, or analyzer, is not extinguished as it was before interposing the quartz. Part of the light passes the analyzer, and, to again extinguish it, we must turn one of the Nicols a certain angle," depending on the length of the waves of light, and hence on the temperature of the incandescent object which emits this light. Hence the angle through which we must turn the analyzer to extinguish the light is a measure of the temperature of the object observed.

For illustrated descriptions of different kinds of pyrometers see circular issued by Queen & Co., Philadelphia.

The Uehling and Steinbart Pyrometer. (For illustrated description see *Engineering*, Aug. 24, 1894.)—The action of the pyrometer is based on a principle which involves the law of the flow of gas through minute apertures in the following manner: If a closed tube or chamber be supplied with a minute inlet and a minute outlet aperture and air be caused by a constant suction to flow in through one and out through the other of these apertures, the tension in the chamber between the apertures will vary with the difference of temperature between the inflowing and outflowing air. If the inflowing air be made to vary with the temperature to be measured, and the outflowing air be kept at a certain constant temperature, then the tension in the space or chamber between the two apertures will be an exact measure of the temperature of the inflowing air, and hence of the temperature to be measured.

In operation it is necessary that the air be sucked into it through the first minute aperture at the temperature to be measured, through the second aperture at a lower but constant temperature, and that the suction be of a constant tension. The first aperture is therefore located in the end of a platinum tube in the bulb of a porcelain tube over which the hot blast sweeps, or inserted into the pipe or chamber containing the gas whose temperature is to be ascertained.

The second aperture is located in a coupling, surrounded by boiling water, and the suction is obtained by an aspirator and regulated by a column of water of constant height.

The tension in the chamber between the apertures is indicated by a manometer.

The Air-thermometer. (Prof. R. C. Carpenter, *Eng'g News*, Jan. 5, 1893.)—Air is a perfect thermometric substance, and if a given mass of air be considered, the product of its pressure and volume divided by its absolute temperature is in every case constant. If the volume of air remain constant, the temperature will vary with the pressure; if the pressure remain constant the temperature will vary with the volume. As the former condition is more easily attained air-thermometers are usually constructed of constant volume, in which case the absolute temperature will vary with the pressure.

If we denote pressure by p and p', the corresponding absolute temperatures by T and T', we should have

$$p : p' :: T : T' \quad \text{and} \quad T' = p'\frac{T}{p}.$$

The absolute temperature T is to be considered in every case 460 higher than the thermometer-reading expressed in Fahrenheit degrees. From the form of the above equation, if the pressure p corresponding to a known absolute temperature T be known, T' can be found. The quotient T/p is a constant which may be used in all determinations with the instrument. The pressure on the instrument can be expressed in inches of mercury, and is evidently the atmospheric pressure b as shown by a barometer, plus or minus an additional amount h shown by a manometer attached to the air thermometer. That is, in general, $p = b \pm h$.

The temperature of 32° F. is fixed as the point of melting ice, in which case $T = 460 + 32 = 492°$ F. This temperature can be produced by surrounding the bulb in melting ice and leaving several minutes, so that the temperature of the confined air shall acquire that of the surrounding ice. When the air is at that temperature, note the reading of the attached manometer h, and that of a barometer; the sum will be the value of p corresponding to the absolute temperature of 492° F. The constant of the instrument, $K = 492 \div p$, once obtained, can be used in all future determinations.

High Temperatures judged by Color.—The temperature of a body can be approximately judged by the experienced eye unaided, and M. Pouillet has constructed a table, which has been generally accepted, giving the colors and their corresponding temperature as below:

	Deg. C.	Deg. F.		Deg. C.	Deg. F.
Incipient red heat..	525	977	Deep orange heat...	1100	2012
Dull red heat.......	700	1292	Clear orange heat..	1200	2192
Incipient cherry-red heat..	800	1472	White heat	1300	2372
Cherry-red heat.....	900	1652	Bright white heat..	1400	2552
Clear cherry-red heat..............	1000	1832	Dazzling white heat	1500 to 1600	2732 to 2912

The results obtained, however, are unsatisfactory, as much depends on the susceptibility of the retina of the observer to light as well as the degree of illumination under which the observation is made.

A bright bar of iron, slowly heated in contact with air, assumes the following tints at annexed temperatures (Claudel):

	Cent.	Fahr.		Cent.	Fahr.
Yellow at	225	437	Indigo at	288	550
Orange at	243	473	Blue at	293	559
Red at	265	509	Green at	332	630
Violet at	277	531	"Oxide-gray"	400	752

BOILING POINTS AT ATMOSPHERIC PRESSURE.

14.7 lbs. per square inch.

Ether, sulphuric	100° F.	Average sea-water	213.2° F.
Carbon bisulphide	118	Saturated brine	226
Ammonia	140	Nitric acid	248
Chloroform	140	Oil of turpentine	315
Bromine	145	Phosphorus	554
Wood spirit	150	Sulphur	570
Alcohol	173	Sulphuric acid	590
Benzine	176	Linseed oil	597
Water	212	Mercury	676

The boiling points of liquids increase as the pressure increases. The boiling point of water at any given pressure is the same as the temperature of saturated steam of the same pressure. (See Steam.)

MELTING-POINTS OF VARIOUS SUBSTANCES.

The following figures are given by Clark (on the authority of Pouillet, Claudel, and Wilson), except those marked *, which are given by Prof. Roberts-Austen in his description of the Le Chatelier pyrometer. These latter are probably the most reliable figures.

Sulphurous acid	− 148° F.	Alloy, 1 tin, 1 lead	370 to 466° F.
Carbonic acid	− 108	Tin	442 to 446
Mercury	− 39	Cadmium	442
Bromine	+ 9.5	Bismuth	504 to 507
Turpentine	14	Lead	608 to 618*
Hyponitric acid	16	Zinc	680 to 779*
Ice	32	Antimony	810 to 1150
Nitro-glycerine	45	Aluminum	1157*
Tallow	92	Magnesium	1200
Phosphorus	112	Calcium	Full red heat.
Acetic acid	113	Bronze	1692
Stearine	109 to 120	Silver	1733* to 1873
Spermaceti	120	Potassium sulphate	1859*
Margaric acid	131 to 140	Gold	1913* to 2282
Potassium	136 to 144	Copper	1929* to 1996
Wax	142 to 154	Cast iron, white	1922 to 2075*
Stearic acid	158	" gray	2012 to 2786 2228*
Sodium	194 to 208	Steel	2372 to 2532
Alloy, 3 lead, 2 tin, 5 bismuth	199	" hard	2570*; mild, 2687*
Iodine	225	Wrought iron	2732 to 2912
Sulphur	239	Palladium	2732*
Alloy, 1½ tin, 1 lead	334	Platinum	3227*

For melting-point of fusible alloys, see Alloys.

Cobalt, nickel, and manganese, fusible in highest heat of a forge. Tungsten and chromium, not fusible in forge, but soften and agglomerate. Platinum and iridium, fusible only before the oxyhydrogen blowpipe.

QUANTITATIVE MEASUREMENT OF HEAT.

Unit of Heat.—The British unit of heat, or British thermal unit (B. T. U.), is that quantity of heat which is required to raise the temperature of 1 lb. of pure water 1° Fahr., at or near 39°.1 F., the temperature of maximum density of water.

The French thermal unit, or *calorie*, is that quantity of heat which is required to raise the temperature of 1 kilogramme of pure water 1° Cent., at or about 4° C., which is equivalent to 39°.1 F.

1 French calorie = 3.968 British thermal units; 1 B. T. U. = .252 calorie. The "pound calorie" is sometimes used by English writers; it is the quan-

tity of heat required to raise the temperature of 1 lb. of water 1° C. 1 lb. calorie = 9/5 B.T.U. = 0.4536 calorie. The heat of combustion of carbon, to CO_2, is said to be 8080 calories. This figure is used either for French calories or for pound calories, as it is the number of pounds of water that can be raised 1° C. by the complete combustion of 1 lb. of carbon, or the number of kilogrammes of water that can be raised 1° C. by the combustion of 1 kilo. of carbon; assumiug in each case that all the heat generated is transferred to the water.

The Mechanical Equivalent of Heat is the number of foot-pounds of mechanical energy equivalent to one British thermal unit, heat and mechanical energy being mutually convertible. Joule's experiments, 1843-50, gave the figure 772, which is known as Joule's equivalent. More recent experiments by Prof. Rowland (*Proc. Am. Acad. Arts and Sciences*, 1880; see also *Wood's Thermodynamics*) give higher figures, and the most probable average is now considered to be 778.

1 heat-unit is equivalent to 778 ft.-lbs. of energy. 1 ft. lb. = 1/778 = .0012852 heat-units. 1 horse-power = 33,000 ft.-lbs. per minute = 2545 heat-uuits per hour = 42.416 + per minute = .70694 per second. 1 lb. carbon burned to CO_2 = 14,544 heat-units. 1 lb. C. per H.P. per hour = 2545 ÷ 14544 = 17½% efficiency (.174986).

Heat of Combustion of Various Substances in Oxygen.

	Heat-units. Cent.	Heat-units. Fahr.	Authority.
Hydrogen to liquid water at 0° C....	34,462	62,032	Favre and Silbermann.
	33,808	60,854	Andrews.
	34,342	61,816	Thomsen.
" to steam at 100° C.	28,732	51,717	Favre and Silbermann.
Carbon (wood charcoal) to carbonic acid, CO_2; ordinary temperatures.	8,080	14,544	" "
	7,900	14,220	Andrews.
	8,137	14,647	Berthelot.
Carbon, diamond to CO_2	7,859	14,146	"
" black diamond to CO_2.	7,861	14,150	"
" graphite to CO_2	7,901	14,222	"
Carbon to carbonic oxide, CO........	2,473	4,451	Favre and Silbermann.
Carbonic oxide to CO_2, per unit of CO	2,403	4,325	" "
	2,431	4,376	Andrews.
	2,385	4,293	Thomsen.
CO to CO_2 per unit of C = 2⅓ × 2403	5,607	10,093	Favre and Silbermann.
Marsh-gas, Methane, CH_4 to water and CO_2	13,120	23,616	Thomsen.
	13,108	23,594	Andrews.
	13,063	23,513	Favre and Silbermann.
Olefiant gas, Ethylene, C_2H_4 to water and CO_2	11,858	21,344	" "
	11,942	21,496	Andrews.
	11,957	21,523	Thomsen.
Benzole gas, C_6H_6 to water and CO_2	10,102	18,184	"
	9,915	17,847	Favre and Silbermann.

In burning 1 pound of hydrogen with 8 pounds of oxygen to form 9 pounds of water, the units of heat evolved are 62,032 (Favre and S.); but if the resulting product is not cooled to the initial temperature of the gases, part of the heat is rendered latent in the steam. The total heat of 1 lb. of steam at 212° F. is 1146.1 heat-units above that of water at 32°, and 9 × 1146 1 = 10,315 heat-units, which deducted from 62,032 gives 51,717 as the heat evolved by the combustion of 1 lb. of hydrogen and 8 lbs. of oxygen at 32° F. to form steam at 212° F.

By the decomposition of a chemical compound as much heat is absorbed or rendered latent as was evolved when the compound was formed. If 1 lb. of carbon is burned to CO_2, generating 14,544 B.T.U., and the CO_2 thus formed is immediately reduced to CO in the presence of glowing carbon, by the reaction $CO_2 + C = 2CO$, the result is the same as if the 2 lbs. C had been burned directly to 2CO, generating 2 × 4451 = 8902 heat-units; consequently 14,544 − 8902 = 5642 heat-units have disappeared or become latent, and the "unburning" of CO_2 to CO is thus a cooling operation. (For heats of combustion of various fuels, see Fuel.)

SPECIFIC HEAT.

Thermal Capacity.—The thermal capacity of a body is the quantity of heat required to raise its temperature one degree. The ratio of the heat required to raise the temperature of a given substance one degree to that required to raise the temperature of water one degree from the temperature of maximum density 39.1 is commonly called the *specific heat* of the substance. Some writers object to the term as being an inaccurate use of the words "specific" and "heat." A more correct name would be "coefficient of thermal capacity."

Determination of Specific Heat.—*Method by Mixture.*—The body whose specific heat is to be determined is raised to a known temperature, and is then immersed in a mass of liquid of which the weight, specific heat, and temperature are known. When both the body and the liquid have attained the same temperature, this is carefully ascertained.

Now the quantity of heat lost by the body is the same as the quantity of heat absorbed by the liquid.

Let c, w, and t be the specific heat, weight, and temperature of the hot body, and c', w', and t' of the liquid. Let T be the temperature the mixture assumes.

Then, by the definition of specific heat, $c \times w \times (t - T)$ = heat-units lost by the hot body, and $c' \times w' \times (T - t')$ = heat-units gained by the cold liquid. If there is no heat lost by radiation or conduction, these must be equal, and

$$cw(t - T) = c'w'(T - t') \quad \text{or} \quad c = \frac{c'w'(T - t')}{w(t - T)}.$$

Specific Heats of Various Substances.

The specific heats of substances, as given by different authorities, show considerable lack of agreement, especially in the case of gases.

The following tables give the mean specific heats of the substances named according to Regnault. (From Rontgen's Thermodynamics, p. 184.) These specific heats are average values, taken at temperatures which usually come under observation in technical application. The actual specific heats of all substances, in the solid or liquid state, increase slowly as the body expands or as the temperature rises. It is probable that the specific heat of a body when liquid is greater than when solid. For many bodies this has been verified by experiment.

SOLIDS.

Antimony	0.0508	Steel (soft)	0.1165
Copper	0.0951	Steel (hard)	0 1175
Gold	0.0324	Zinc	0.0956
Wrought iron	0.1138	Brass	0.0939
Glass	0.1937	Ice	0.5040
Cast iron	0.1298	Sulphur	0.2026
Lead	0.0314	Charcoal	0.2410
Platinum	0.0324	Alumina	0.1970
Silver	0.0570	Phosphorus	0.1887
Tin	0.0562		

LIQUIDS.

Water	1.0000	Mercury	0.0333
Lead (melted)	0.0402	Alcohol (absolute)	0.7000
Sulphur "	0.2340	Fusel oil	0.5640
Bismuth "	0.0308	Benzine	0.4500
Tin "	0.0637	Ether	0.5034
Sulphuric acid	0.3350		

GASES.

	Constant Pressure.	Constant Volume.
Air	0.23751	0.16847
Oxygen	0.21751	0.15507
Hydrogen	3.40900	2.41226
Nitrogen	0.24380	0.17273
Superheated steam	0.4805	0.346
Carbonic acid	0.217	0.1535
Olefiant Gas (CH_2)	0.404	0.173
Carbonic oxide	0.2479	0.1758
Ammonia	0.508	0.299
Ether	0.4797	0.3411
Alcohol	0.4534	0.3200
Acetic acid	0.4125	......
Chloroform	0.1567	....

In addition to the above, the following are given by other authorities. (Selected from various sources.)

METALS.

Platinum, 32° to 446° F. (increased .000305 for each 100° F.)	.0333
Cadmium	.0567
Brass	.0939
Copper, 32° to 212° F	.094
" 32° to 572° F	.1013
Zinc 32° to 212° F	.0927
" 32° to 572° F	.1015
Nickel	.1086
Aluminum, 0° F. to melting-point (A. E. Hunt)	0.2185
Wrought iron (Petit & Dulong). " 32° to 212°	.1098
" 32° to 392°	.115
" 32° to 572°	.1218
" 32° to 662°	.1255
Wrought iron (J. C. Hoadley, A. S. M. E., vi. 713), Wrought iron, 32° to 200°	.1129
" 32° to 600°	.1327
" 32° to 2000°	.2619

OTHER SOLIDS.

Brickwork and masonry, about	.20
Marble	.210
Chalk	.215
Quicklime	.217
Magnesian limestone	.217
Silica	.191
Corundum	.198
Stones generally	.2 to .22
Coal	.20 to .241
Coke	.203
Graphite	.202
Sulphate of lime	.197
Magnesia	.222
Soda	.231
Quartz	.188
River sand	.195

WOODS.

Pine (turpentine)	.467
Fir	.650
Oak	.570
Pear	.500

LIQUIDS.

Alcohol, density .793		.622
Sulphuric acid, density 1.87		.335
" "	1.30	.661
Hydrochloric acid		.600
Olive oil		.310
Benzine		.393
Turpentine, density .872		.472
Bromine		1.111

GASES.

	At Constant Pressure.	At Constant Volume.
Sulphurous acid	.1553.	.1246
Light carburetted hydrogen, marsh gas (CH_4)	.5929	.4683
Blast-furnace gases	.2277	.....

Specific Heat of Salt Solution. (Schuller.)

Per cent salt in solution	5	10	15	20	25
Specific heat	.9306	.8909	.8606	.8490	.8073

Specific Heat of Air.—Regnault gives for the mean value

Between — 30° C. and + 10° C	0.23771
" 0° C. " 100° C	0.23741
" 0° C. " 200° C	0.23751

Hanssen uses 0.1686 for the specific heat of air at constant volume. The value of this constant has never been found to any degree of accuracy by direct experiment. Prof. Wood gives 0.2375 ÷ 1.406 = 0.1689. The ratio of the specific heat of a fixed gas at constant pressure to the sp. ht. at constant volume is given as follows by different writers (*Eng'g*, July 12, 1889): Regnault, 1.3953; Moll and Beck, 1.4085; Szathmari, 1.4027; J. Macfarlane Gray, 1.4. The first three are obtained from the velocity of sound in air. The fourth is derived from theory. Prof. Wood says: The value of the ratio for air, as found in the days of La Place, was 1.41, and we have 0.2377 ÷ 1.41 = 0.1686, the value used by Clausius, Hanssen, and many others. But this ratio is not definitely known. Rankine in his later writings used 1.408, and Tait in a recent work gives 1.404, while some experiments gives less than 1.4 and others more than 1.41. Prof. Wood uses 1.406.

Specific Heat of Gases.—Experiments by Mallard and Le Chatelier indicate a continuous increase in the specific heat at constant volume of steam, CO_2, and even of the perfect gases, with rise of temperature. The variation is inappreciable at 100° C., but increases rapidly at the high temperatures of the gas-engine cylinder. (Robinson's Gas and Petroleum Engines.)

Specific Heat and Latent Heat of Fusion of Iron and Steel. (H. H. Campbell, Trans. A. I. M. E., xix. 181.)

		Åkerman.	Troilius.
Specific heat pig iron,	0 to 1200° C.	0.16	....
" " "	1200 to 1800° C.	0.21	....
" " "	0 to 1500° C.	....	0.18
" " "	1500 to 1800° C.	....	0.20

Calculating by both sets of data we have:

	Åkerman.	Troilius.
Heating from 0 to 1800° C.	318	330 calories per kilo.

Hence probable value is about .... 325 calories per kilo.

Specific heat, steel (probably high carbon)	(Troilius)	.1175
" " soft iron	"	.1081
Hence probable value solid rail steel	...	.1125
" " " melted rail steel		.1275

	Åkerman.	Troilius.
Latent heat of fusion, pig iron, calories per kilo.	46	..
" " " gray pig	..	33
" " " white pig	..	23

From which we may assume that the truth is about: Steel, 20; pig iron, 30.

EXPANSION BY HEAT.

In the centigrade scale the coefficient of expansion of air per degree is 0.003665 = 1/273; that is, the pressure being constant, the volume of a perfect gas increases 1/273 of its volume at 0° C. for every increase in temperature of 1° C. In Fahrenheit units it increases 1/491.2 = .002036 of its volume at 32° F. for every increase of 1° F.

Expansion of Gases by Heat from 32° to 212° F. (Regnault.)

	Increase in Volume, Pressure Constant. Volume at 32° Fahr. = 1.0, for		Increase in Pressure, Volume Constant. Pressure at 32° Fahr. = 1.0, for	
	100° C.	1° F.	100° C.	1° F.
Hydrogen	0.3661	0.002034	0.3667	0.002037
Atmospheric air	0.3670	0.002039	0.3665	0.002036
Nitrogen	0.3670	0.002039	0.3668	0.002039
Carbonic oxide	0.3669	0.002038	0.3667	0.002037
Carbonic acid	0.3710	0.002061	0.3688	0.002039
Sulphurous acid	0.3903	0.002168	0.3845	0.002136

If the volume is kept constant, the pressure varies directly as the absolute temperature.

Lineal Expansion of Solids at Ordinary Temperatures.

(British Board of Trade; from CLARK.)

	For 1° Fahr.	For 1° Cent.	Coefficient of Expansion from 32° to 212° F.	According to Other Authorities.
	Length = 1	Length = 1		
Aluminum (cast)	.00001234	.00002221	.002221	....
Antimony (cryst.)	.00000627	.00001129	.001129	.001083
Brass, cast	.00000957	.00001722	.001722	.001868
" plate	.00001052	.00001894	.001894	....
Brick	.00000306	.00000550	.000550	....
Bronze (Copper, 17; Tin, 2½; Zinc 1)	.00000986	.00001774	.001774	....
Bismuth	.00000975	.00001755	.001755	.001392
Cement, Portland (mixed), pure	.00000594	.00001070	.001070	....
Concrete: cement, mortar, and pebbles	.00000795	.00001430	.001430	....
Copper	.00000887	.00001596	.001596	.001718
Ebonite	.00004278	.00007700	.007700	....
Glass, English flint	.00000451	.00000812	.000812	....
" thermometer	.00000499	.00000897	.000897	....
" hard	.00000397	.00000714	.000714	....
Granite, gray, dry	.00000438	.00000789	.000789	....
" red, dry	.00000498	.00000897	.000897	....
Gold, pure	.00000786	.00001415	.001415	....
Iridium, pure	.00000356	.00000641	.000641	....
Iron, wrought	.00000648	.00001166	.001166	.001235
" cast	.00000556	.00001001	.001001	.001110
Lead	.00001571	.00002828	.002828	....
Magnesium	....	....	....	.002694
Marbles, various { from	.00000308	.00000554	.000554	....
Marbles, various { to	.00000786	.00001415	.001415	....
Masonry, brick { from	.00000256	.00000460	.000460	....
Masonry, brick { to	.00000494	.00000890	.000890	....
Mercury (cubic expansion)	.00009984	.00017971	.017971	.018018
Nickel	.00000695	.00001251	.001251	.001279
Pewter	.00001129	.00002033	.002033	....
Plaster, white	.00000922	.00001660	.001660	....
Platinum	.00000479	.00000863	.000863	....
Platinum, 85 per cent; Iridium, 15 " "	.00000453	.00000815	.000815	.000884
Porcelain	.00000200	.00000360	.000360	....
Quartz, parallel to major axis, t 0° to 40° C	.00000434	.00000781	.000781	....
Quartz, perpendicular to major axis, t 0° to 40° C	.00000788	.00001419	.001419	....
Silver, pure	.00001079	.00001943	.001943	.001908
Slate	.00000577	.00001038	.001038	....
Steel, cast	.00000636	.00001144	.001144	.001079
" tempered	.00000689	.00001240	.001240	....
Stone (sandstone), dry	.00000652	.00001174	.001174	....
" " Rauville	.00000417	.00000750	.000750	....
Tin	.00001163	.00002094	.002094	.001938
Wedgwood ware	.00000489	.00000881	.000881	....
Wood, pine	.00000276	.00000496	.000496	....
Zinc	.00001407	.00002532	.002532	.002942
Zinc, 8; Tin, 1	.00001496	.00002692	.002692	....

Cubical expansion, or expansion of volume = linear expansion × 3.

Absolute Temperature—Absolute Zero.—The absolute zero of a gas is a theoretical consequence of the law of expansion by heat, assuming that it is possible to continue the cooling of a perfect gas until its volume is diminished to nothing.

If the volume of a perfect gas increases 1/273 of its volume at 0° C. for every increase of temperature of 1° C., and decreases 1/273 of its volume for every decrease of temperature of 1° C., then at − 273° C. the volume of the imaginary gas would be reduced to nothing. This point − 273° C., or 491.2° F. below the melting-point of ice on the air thermometer, or 492.66° F. below on a perfect gas thermometer = − 459.2° F. (or − 460.66°), is called the absolute zero; and absolute temperatures are temperatures measured, on either the Fahrenheit or centigrade scale, from this zero. The freezing point, 32° F., corresponds to 491.2° F. absolute. If p_0 be the pressure and v_0 the volume of a gas at the temperature of 32° F. = 491.2° on the absolute scale = T_0, and p the pressure, and v the volume of the same quantity of gas at any other absolute temperature T, then

$$\frac{pv}{p_0v_0} = \frac{T}{T_0} = \frac{t + 459.2}{491.2}; \quad \frac{pv}{T} = \frac{p_0v_0}{T_0}.$$

The value of $p_0v_0 \div T_0$ for air is 53.37, and $pv = 53.37T$, calculated as follows by Prof. Wood:

A cubic foot of dry air at 32° F. at the sea-level weighs 0.080728 lb. The volume of one pound is $v_0 = \frac{1}{.080728} = 12.387$ cubic feet. The pressure per square foot is 2116.2 lbs.

$$\frac{p_0v_0}{T_0} = \frac{2116.2 \times 12.387}{491.13} = \frac{26214}{491.13} = 53.37.$$

The figure 491.13 is the number of degrees that the absolute zero is below the melting-point of ice, by the air thermometer. On the absolute scale, whose divisions would be indicated by a perfect gas thermometer, the calculated value approximately is 492.66, which would make $pv = 53.21T$. Prof. Thomson considers that − 273.1° C., = − 459.4° F., is the most probable value of the absolute zero. See *Heat* in *Ency. Brit.*

Expansion of Liquids from 32° to 212° F.—Apparent expansion in glass (Clark). Volume at 212°, volume at 32° being 1:

Water	1.0466	Nitric acid	1.11
Water saturated with salt	1.05	Olive and linseed oils	1.08
Mercury	1.0182	Turpentine and ether	1.07
Alcohol	1.11	Hydrochlor. and sulphuric acids	1.06

For water at various temperatures, see Water.
For air at various temperatures, see Air.

LATENT HEATS OF FUSION AND EVAPORATION.

Latent Heat means a quantity of heat which has disappeared, having been employed to produce some change other than elevation of temperature. By exactly reversing that change, the quantity of heat which has disappeared is reproduced. Maxwell defines it as the quantity of heat which must be communicated to a body in a given state in order to convert it into another state without changing its temperature.

Latent Heat of Fusion.—When a body passes from the solid to the liquid state, its temperature remains stationary, or nearly stationary, at a certain melting point during the whole operation of melting; and in order to make that operation go on, a quantity of heat must be transferred to the substance melted, being a certain amount for each unit of weight of the substance. This quantity is called the latent heat of fusion.

When a body passes from the liquid to the solid state, its temperature remains stationary or nearly stationary during the whole operation of freezing; a quantity of heat equal to the latent heat of fusion is produced in the body and rejected into the atmosphere or other surrounding bodies.

The following are examples in British thermal units per pound, as given in Landolt & Börnstein's *Physikalische-Chemische Tabellen* (Berlin, 1894).

Substances.	Latent Heat of Fusion.	Substances.	Latent Heat of Fusion.
Bismuth	22.75	Silver	37.93
Cast Iron, gray	41.4	Beeswax	76.14
Cast Iron, white	59.4	Paraffine	63.27
Lead	9.66	Spermaceti	66.56
Tin	25.65	Phosphorus	9.06
Zinc	50.63	Sulphur	16.86

Prof. Wood considers 144 heat units as the most reliable value for the latent heat of fusion of ice. Person gives 142.65.

Latent Heat of Evaporation.—When a body passes from the solid or liquid to the gaseous state, its temperature during the operation remains stationary at a certain boiling point, depending on the pressure of the vapor produced; and in order to make the evaporation go on, a quantity of heat must be transferred to the substance evaporated, whose amount for each unit of weight of the substance evaporated depends on the temperature. That heat does not raise the temperature of the substance, but disappears in causing it to assume the gaseous state, and it is called the latent heat of evaporation.

When a body passes from the gaseous state to the liquid or solid state, its temperature remains stationary, during that operation, at the boiling-point corresponding to the pressure of the vapor: a quantity of heat equal to the latent heat of evaporation at that temperature is produced in the body; and in order that the operation of condensation may go on, that heat must be transferred from the body condensed to some other body.

The following are examples of the latent heat of evaporation in British thermal units, of one pound of certain substances, when the pressure of the vapor is one atmosphere of 14.7 lbs. on the square inch:

Substance.	Boiling-point under one atm. Fahr.	Latent Heat in British units.
Water	212.0	965.7 (Regnault.)
Alcohol	172.2	364.3 (Andrews.)
Ether...........................	95.0	162.8 "
Bisulphide of carbon............	114.8	156.0 "

The latent heat of evaporation of water at a series of boiling-points extending from a few degrees below its freezing-point up to about 375 degrees Fahrenheit has been determined experimentally by M. Regnault. The results of those experiments are represented approximately by the formula in British thermal units per pound,

$$l \text{ nearly} = 1091.7 - 0.7(t - 32°) = 965.7 - 0.7(t - 212°).$$

The Total Heat of Evaporation is the sum of the heat which disappears in evaporating one pound of a given substance at a given temperature (or latent heat of evaporation) and of the heat required to raise its temperature, before evaporation, from some fixed temperature up to the temperature of evaporation. The latter part of the total heat is called the sensible heat.

In the case of water, the experiments of M. Regnault show that the total heat of steam from the temperature of melting ice increases at a uniform rate as the temperature of evaporation rises. The following is the formula in British thermal units per pound:

$$h = 1091.7 + 0.305(t - 32°).$$

For the total heat, latent heat, etc., of steam at different pressures, see table of the Properties of Saturated Steam. For tables of total heat, latent heat, and other properties of steams of ether, alcohol, acetone, chloroform, chloride of carbon, and bisulphide of carbon, see Rontgen's Thermodynamics (Dubois's translation.) For ammonia and sulphur dioxide, see Wood's Thermodynamics; also, tables under Refrigerating Machinery, in this book.

EVAPORATION AND DRYING.

In evaporation, the formation of vapor takes place on the surface; in boiling, within the liquid: the former is a slow, the latter a quick, method of evaporation.

If we bring an open vessel with water under the receiver of an air-pump and exhaust the air the water in the vessel will commence to boil, and if we keep up the vacuum the water will actually boil near its freezing point. The formation of steam in this case is due to the heat which the water takes out of the surroundings.

Steam formed under pressure has the same temperature as the liquid in which it was formed, provided the steam is kept under the same pressure.

By properly cooling the rising steam from boiling water, as in the multiple-effect evaporating systems, we can regulate the pressure so that the water boils at low temperatures.

Evaporation of Water in Reservoirs.—Experiments at the Mount Hope Reservoir, Rochester, N. Y., in 1891, gave the following results:

	July.	Aug.	Sept.	Oct.
Mean temperature of air in shade	70.5	70.8	66.7	53.3
" " " water in reservoir	68.2	70.2	66.1	54.4
" humidity of air, per cent	67.0	74.6	75.2	74.7
Evaporation in inches during month	5.59	4.93	4.05	3.23
Rainfall in inches during month	3.44	2.95	1.44	2.16

Evaporation of Water from Open Channels. (Flynn's Irrigation Canals and Flow of Water.)—Experiments from 1881 to 1885 in Tulare County, California, showed an evaporation from a pan in the river equal to an average depth of one eighth of an inch per day throughout the year.

When the pan was in the air the average evaporation was less than 3/16 of an inch per day. The average for the month of August was 1/3 inch per day, and for March and April 1/12 of an inch per day. Experiments in Colorado show that evaporation ranges from .068 to .16 of an inch per day during the irrigating season.

In Northern Italy the evaporation was from 1/12 to 1/9 inch per day, while in the south, under the influence of hot winds, it was from 1/6 to 1/5 inch per day.

In the hot season in Northern India, with a decidedly hot wind blowing, the average evaporation was ½ inch per day. The evaporation increases with the temperature of the water.

Evaporation by the Multiple System.—A multiple effect is a series of evaporating vessels each having a steam chamber, so connected that the heat of the steam or vapor produced in the first vessel heats the second, the vapor or steam produced in the second heats the third, and so on. The vapor from the last vessel is condensed in a condenser. Three vessels are generally used, in which case the apparatus is called a *Triple Effect*. In evaporating in a triple effect the vacuum is graduated so that the liquid is boiled at a constant and low temperature.

Resistance to Boiling.—Brine. (Rankine.)—The presence in a liquid of a substance dissolved in it (as salt in water) resists ebullition, and raises the temperature at which the liquid boils, under a given pressure; but unless the dissolved substance enters into the composition of the vapor, the relation between the temperature and pressure of saturation of the vapor remains unchanged. A resistance to ebullition is also offered by a vessel of a material which attracts the liquid (as when water boils in a glass vessel), and the boiling take place by starts. To avoid the errors which causes of this kind produce in the measurement of boiling-points, it is advisable to place the thermometer, not in the liquid, but in the vapor, which shows the true boiling-point, freed from the disturbing effect of the attractive nature of the vessel. The boiling-point of saturated brine under one atmosphere is 226° Fahr., and that of weaker brine is higher than the boiling-point of pure water by 1.2° Fahr., for each 1/32 of salt that the water contains. Average sea-water contains 1/32; and the brine in marine boilers is not suffered to contain more than from 2/32 to 3/32.

Methods of Evaporation Employed in the Manufacture of Salt. (F. E. Engelhardt, Chemist Onondaga Salt Springs; Report for 1889.)—1. Solar heat—solar evaporation. 2. Direct fire, applied to the heating surface of the vessels containing brine—kettle and pan methods. 3. The steam-grainer system—steam-pans, steam-kettles, etc. 4. Use of steam and a reduction of the atmospheric pressure over the boiling brine—vacuum system.

When a saturated salt solution boils, it is immaterial whether it is done under ordinary atmospheric pressure at 228° F., or under four atmospheres with a temperature of 320° F., or in a vacuum under 1/10 atmosphere, the result will always be a fine-grained salt.

The fuel consumption is stated to be as follows: By the kettle method, 40 to 45 bu. of salt evaporated per ton of fuel, anthracite dust burned on perforated grates; evaporation, 5.53 lbs. of water per pound of coal. By the pan method, 70 to 75 bu. per ton of fuel. By vacuum pans, single effect, 86 bu. per ton of anthracite dust (2000 lbs.). With a double effect nearly double that amount can be produced.

Solubility of Common Salt in Pure Water. (Andreæ.)

Temp. of brine, F.	32	50	86	104	140	176
100 parts water dissolve parts	35.63	35.69	36.03	36.32	37.06	38.00
100 parts brine contain salt	26.27	26.30	26.49	26.64	27.04	27.54

According to Poggial, 100 parts of water dissolve at 229.66° F., 40.35 parts of salt, or in per cent of brine, 28.749. Gay Lussac found that at 229.72° F., 100 parts of pure water would dissolve 40.38 parts of salt, in per cent of brine, 28.764 parts.

The solubility of salt at 229° F. is only 2.5% greater than at 32°. Hence we cannot, as in the case of alum, separate the salt from the water by allowing a saturated solution at the boiling point to cool to a lower temperature.

Solubility of Sulphate of Lime in Pure Water. (Marignac.)

Temperature F. degrees.	32	64.5	89.6	100.4	105.8	127.4	186.8	212
Parts water to dissolve 1 part gypsum	415	386	371	368	370	375	417	452
Parts water to dissolve 1 part anhydrous $CaSO_4$	525	488	470	466	468	474	528	572

In salt brine sulphate of lime is much more soluble than in pure water. In the evaporation of salt brine the accumulation of sulphate of lime tends to stop the operation, and it must be removed from the pans to avoid waste of fuel.

The average strength of brine in the New York salt districts in 1889 was 69.38 degrees of the salinometer.

Strength of Salt Brines.—The following table is condensed from one given in U. S. Mineral Resources for 1888, on the authority of Dr. Englehardt.

Relations between Salinometer Strength, Specific Gravity, Solid Contents, etc., of Brines of Different Strengths.

Salinometer, degrees.	Baumé, degrees.	Specific gravity.	Per cent of salt.	Weight of a gallon of this brine in pounds.	Pounds of salt in a gallon of brine of 231 cubic inches.	Gallons of brine required for a bushel of salt.	Pounds of water to be evaporated to produce a bushel of salt.	Lbs. of coal required to produce a bushel of salt, 1 lb. coal evaporating 6 lbs. of water.	Bushels of salt that can be made with a ton of coal of 2000 pounds.
1	.26	1.002	.265	8.347	.022	2,531	21,076	3,513	.569
2	.52	1.003	.530	8.356	.044	1,264	10,510	1,752	1.141
4	1.04	1.007	1.060	8.389	.088	629.7	5,227	871.2	2.295
6	1.56	1.010	1.590	8.414	.133	418.6	3,466	577.7	3.462
8	2.08	1.014	2.120	8.447	.179	312.7	2,585	430.9	4.641
10	2.60	1.017	2 650	8.472	.224	249.4	2,057	342.9	5 833
12	3.12	1.021	3.180	8.506	.270	207.0	1,705	284.2	7.038
14	3 64	1.025	3.710	8.539	.316	176.8	1,453	242.2	8.256
16	4.16	1 028	4.240	8.564	.364	154.2	1,265	210.8	9.488
18	4.68	1.032	4.770	8.597	.410	136.5	1,118	186.3	10.73
20	5 20	1.035	5.300	8 622	.457	122.5	1,001	176.8	11.99
30	7.80	1.054	7.950	8.781	.698	80.21	648.4	108.1	18.51
40	10 40	1.073	10.600	8.939	.947	59.09	472.3	78.71	25.41
50	13.00	1.093	13.250	9.105	1.206	46.41	366.6	61.10	32.73
60	15 60	1.114	15.900	9.280	1.475	37.94	296.2	49.36	40.51
70	18 20	1.136	18.550	9.464	1.755	31.89	245.9	40.98	48.80
80	20 80	1.158	21.200	9.647	2.045	27.38	208.1	34.69	57.65
90	23.40	1.182	23.850	9 847	2.348	23.84	178.8	29.80	67.11
100	26.00	1.205	26 500	10.039	2.660	21.04	155.3	25.88	77.26

Concentration of Sugar Solutions.* (From "Heating and Concentrating Liquids by Steam," by John G. Hudson; *The Engineer*, June 13, 1890.)—In the early stages of the process, when the liquor is of low density, the evaporative duty will be high, say two to three (British) gallons per square foot of heating surface with 10 lbs. steam pressure, but will gradually fall to an almost nominal amount as the final stage is approached. As a generally safe basis for designing, Mr. Hudson takes an evaporation of one gallon per hour for each square foot of gross heating surface, with steam of the pressure of about 10 lbs.

As examples of the evaporative duty of a vacuum pan when performing the earlier stages of concentration, during which all the heating surface can be employed, he gives the following:

COIL VACUUM PAN.—4¾ in. copper coils, 528 square feet of surface; steam in coils, 15 lbs.; temperature in pan, 141° to 148°; density of feed, 25° Beaumé, and concentrated to 31° Beaumé.

First Trial.—Evaporation at the rate of 2000 gallons per hour = 3.8 gallons per square foot; transmission, 376 units per degree of difference of temperature.

Second Trial.—Evaporation at the rate of 1508 gallons per hour = 2.8 gallons per square foot; transmission, 265 units per degree.

As regards the total time needed to work up a charge of massecuite from liquor of a given density, the following figures, obtained by plotting the results from a large number of pans, form a guide to practical working. The pans were all of the coil type, some with and some without jackets, the gross heating surface probably averaging, and not greatly differing from, .25 square foot per gallon capacity, and the steam pressure 10 lbs. per square inch. Both plantation and refining pans are included, making various grades of sugar:

	Density of Feed (degs. Beaumé). 10°	15°	20°	25°	30°
Evaporation required per gallon massecuite discharged	6.123	3.6	2.26	1.5	.97
Average working hours required per charge	12.	9.	6½	5.	4.
Equivalent average evaporation per hour per square foot of gross surface, assuming .25 sq. ft. per gallon capacity	2.04	1.6	1.39	1.2	.97
Fastest working hours required per charge	8.5	5.5	3.8	2.75	2.0
Equivalent average evaporation per hour per square foot	2.88	2.6	2.38	2.18	1.9

The quantity of heating steam needed is practically the same in vacuum as in open pans. The advantages proper to the vacuum system are primarily the reduced temperature of boiling, and incidentally the possibility of using heating steam of low pressure.

In a solution of sugar in water, each pound of sugar adds to the volume of the water to the extent of .061 gallon at a low density to .0638 gallon at high densities.

A Method of Evaporating by Exhaust Steam is described by Albert Stearns in Trans. A. S. M. E., vol. viii. A pan 17′ 6″ × 11′ × 1′ 6″, fitted with cast-iron condensing pipes of about 250 sq. ft. of surface, evaporated 120 gallons per hour from clear water, condensing only about one half of the steam supplied by a plain slide-valve engine of 14″ × 32″ cylinder, making 65 revs. per min., cutting off about two thirds stroke, with steam at 75 lbs. boiler pressure.

It was found that keeping the pan-room warm and letting only sufficient air in to carry the vapor up out of a ventilator adds to its efficiency, as the average temperature of the water in the pan was only about 165° F.

Experiments were made with coils of pipe in a small pan, first with no agitator, then with one having straight blades, and lastly with troughed blades; the evaporative results being about the proportions of one, two, and three respectively.

In evaporating liquors whose boiling point is 220° F., or much above that of water, it is found that exhaust steam can do but little more than bring them up to saturation strength, but on weak liquors, syrups, glues, etc., it should be very useful.

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* For other sugar data see Bagasse as Fuel, under *Fuel*.

Drying in Vacuum.—An apparatus for drying grain and other substances in vacuum is described by Mr. Emil Passburg in Proc. Inst. Mech. Engrs., 1889. The three essential requirements for a successful and economical process of drying are: 1. Cheap evaporation of the moisture; 2. Quick drying at a low temperature; 3. Large capacity of the apparatus employed.

The removal of the moisture can be effected in either of two ways: either by slow evaporation, or by quick evaporation—that is, by boiling.

Slow Evaporation.—The principal idea carried into practice in machines acting by slow evaporation is to bring the wet substance repeatedly into contact with the inner surfaces of the apparatus, which are heated by steam, while at the same time a current of hot air is also passing through the substances for carrying off the moisture. This method requires much heat, because the hot-air current has to move at a considerable speed in order to shorten the drying process as much as possible; consequently a great quantity of heated air passes through and escapes unused. As a carrier of moisture hot air cannot in practice be charged beyond half its full saturation; and it is in fact considered a satisfactory result if even this proportion be attained. A great amount of heat is here produced which is not used; while, with scarcely half the cost for fuel, a much quicker removal of the water is obtained by heating it to the boiling point.

Quick Evaporation by Boiling.—This does not take place until the water is brought up to the boiling point and kept there, namely, 212° F., under atmospheric pressure. The vapor generated then escapes freely. Liquids are easily evaporated in this way, because by their motion consequent on boiling the heat is continuously conveyed from the heating surfaces through the liquid, but it is different with solid substances, and many more difficulties have to be overcome, because convection of the heat ceases entirely in solids. The substance remains motionless, and consequently a much greater quantity of heat is required than with liquids for obtaining the same results.

Evaporation in Vacuum.—All the foregoing disadvantages are avoided if the boiling-point of water is lowered, that is, if the evaporation is carried out under vacuum.

This plan has been successfully applied in Mr. Passburg's vacuum drying apparatus, which is designed to evaporate large quantities of water contained in solid substances.

The drying apparatus consists of a top horizontal cylinder, surmounted by a charging vessel at one end, and a bottom horizontal cylinder with a discharging vessel beneath it at the same end. Both cylinders are encased in steam-jackets heated by exhaust steam. In the top cylinder works a revolving cast-iron screw with hollow blades, which is also heated by exhaust steam. The bottom cylinder contains a revolving drum of tubes, consisting of one large central tube surrounded by 24 smaller ones, all fixed in tube-plates at both ends; this drum is heated by live steam direct from the boiler. The substance to be dried is fed into the charging vessel through two manholes, and is carried along the top cylinder by the screw creeper to the back end, where it drops through a valve into the bottom cylinder, in which it is lifted by blades attached to the drum and travels forwards in the reverse direction; from the front end of the bottom cylinder it falls into a discharging vessel through another valve, having by this time become dried. The vapor arising during the process is carried off by an air-pump, through a dome and air-valve on the top of the upper cylinder, and also through a throttle-valve on the top of the lower cylinder; both of these valves are supplied with strainers.

As soon as the discharging vessel is filled with dried material the valve connecting it with the bottom cylinder is shut, and the dried charge taken out without impairing the vacuum in the apparatus. When the charging vessel requires replenishing, the intermediate valve between the two cylinders is shut, and the charging vessel filled with a fresh supply of wet material; the vacuum still remains unimpaired in the bottom cylinder, and has to be restored only in the top cylinder after the charging vessel has been closed again.

In this vacuum the boiling-point of the water contained in the wet material is brought down as low as 110° F. The difference between this temperature and that of the heating surfaces is amply sufficient for obtaining good results from the employment of exhaust steam for heating all the surfaces except the revolving drum of tubes. The water contained in the solid substance to be dried evaporates as soon as the latter is heated to about 110° F.;

and as long as there is any moisture to be removed the solid substance is not heated above this temperature.

Wet grains from a brewery or distillery, containing from 75% to 78% of water, have by this drying process been converted in some localities from a worthless incumbrance into a valuable food-stuff. The water is removed by evaporation only, no previous mechanical pressing being resorted to.

At Messrs. Guinness's brewery in Dublin two of these machines are employed. In each of these the top cylinder is 20′ 4″ long and 2′ 8″ diam., and the screw working inside it makes 7 revs. per min.; the bottom cylinder is 19′ 2″ long and 5′ 4″ diam., and the drum of the tubes inside it makes 5 revs. per min. The drying surfaces of the two cylinders amount together to a total area of about 1000 sq. ft., of which about 40% is heated by exhaust steam direct from the boiler. There is only one air-pump, which is made large enough for three machines; it is horizontal, and has only one air-cylinder, which is double-acting, 17¾ in. diam. and 17¾ in. stroke; and it is driven at about 45 revs. per min. As the result of about eight months' experience, the two machines have been drying the wet grains from about 500 cwt. of malt per day of 24 hours.

Roughly speaking, 3 cwt. of malt gave 4 cwt. of wet grains, and the latter yield 1 cwt. of dried grains; 500 cwt. of malt will therefore yield about 670 cwt. of wet grains, or 335 cwt. per machine. The quantity of water to be evaporated from the wet grains is from 75% to 78% of their total weight, or say about 512 cwt. altogether, being 256 cwt. per machine.

RADIATION OF HEAT.

Radiation of heat takes place between bodies at all distances apart, and follows the laws for the radiation of light.

The heat rays proceed in straight lines, and the intensity of the rays radiated from any one source varies inversely as the square of their distance from the source.

This statement has been erroneously interpreted by some writers, who have assumed from it that a boiler placed two feet above a fire would receive by radiation only one fourth as much heat as if it were only one foot above. In the case of boiler furnaces the side walls reflect those rays that are received at an angle—following the law of optics, that the angle of incidence is equal to the angle of reflection,—with the result that the intensity of heat two feet above the fire is practically the same as at one foot above, instead of only one-fourth as much.

The rate at which a hotter body radiates heat, and a colder body absorbs heat, depends upon the state of the surfaces of the bodies as well as on their temperatures. The rate of radiation and of absorption are increased by darkness and roughness of the surfaces of the bodies, and diminished by smoothness and polish. For this reason the covering of steam pipes and boilers should be smooth and of a light color: uncovered pipes and steam-cylinder covers should be polished.

The quantity of heat radiated by a body is also a measure of its heat-absorbing power, under the same circumstances. When a polished body is struck by a ray of heat, it absorbs part of the heat and reflects the rest. The reflecting power of a body is therefore the complement of its absorbing power, which latter is the same as its radiating power.

The relative radiating and reflecting power of different bodies has been determined by experiment, as shown in the table below, but as far as quantities of heat are concerned, says Prof. Trowbridge (Johnson's Cyclopædia, art. Heat), it is doubtful whether anything further than the said relative determinations can, in the present state of our knowledge, be depended upon, the actual or absolute quantities for different temperatures being still uncertain. The authorities do not even agree on the relative radiating powers. Thus, Leslie gives for tin plate, gold, silver, and copper the figure 12, which differs considerably from the figures in the table below, given by Clark, stated to be on the authority of Leslie, De La Provostaye and Desains, and Melloni.

Relative Radiating and Reflecting Power of Different Substances.

	Radiating or Absorbing Power.	Reflecting Power.		Radiating or Absorbing Power.	Reflecting Power.
Lampblack	100	0	Zinc, polished	19	81
Water	100	0	Steel, polished	17	83
Carbonate of lead	100	0	Platinum, polished	24	76
Writing-paper	98	2	" in sheet	17	83
Ivory, jet, marble	93 to 98	7 to 2	Tin	15	85
Ordinary glass	90	10	Brass, cast, dead polished	11	89
Ice	85	15	Brass, bright polished	7	93
Gum lac	72	28	Copper, varnished	14	86
Silver-leaf on glass	27	73	" hammered	7	93
Cast iron, bright polished	25	75	Gold, plated	5	95
Mercury, about	23	77	" on polished steel	3	97
Wrought iron, polished	23	77	Silver, polished bright	3	97

Experiments of Dr. A. M. Mayer give the following: The relative radiations from a cube of cast iron, having faces rough, as from the foundry, planed, "drawfiled," and polished, and from the same surfaces oiled, are as below (Prof. Thurston, in Trans. A. S. M. E., vol. xvi.):

Surface.	Oiled.	Dry.
Rough	100	100
Planed	60	32
Drawfiled	49	20
Polished	45	18

It here appears that the oiling of smoothly polished castings, as of cylinder-heads of steam-engines, more than doubles the loss of heat by radiation, while it does not seriously affect rough castings.

CONDUCTION AND CONVECTION OF HEAT.

Conduction is the transfer of heat between two bodies or parts of a body which touch each other. Internal conduction takes place between the parts of one continuous body, and external conduction through the surface of contact of a pair of distinct bodies.

The rate at which conduction, whether internal or external, goes on, being proportional to the area of the section or surface through which it takes place, may be expressed in thermal units per square foot of area per hour

Internal Conduction varies with the *heat conductivity*, which depends upon the nature of the substance, and is directly proportional to the difference between the temperatures of the two faces of a layer, and inversely as its thickness. The reciprocal of the conductivity is called the *internal thermal resistance* of the substance. If r represents this resistance, x the thickness of the layer in inches, T' and T the temperatures on the two faces, and q the quantity in thermal units transmitted per hour per square foot of area, $q = \frac{T' - T}{rx}$. (Rankine.)

Péclet gives the following values of r:

Gold, platinum, silver	0.0016	Lead	0.0090
Copper	0.0018	Marble	0.0716
Iron	0.0043	Brick	0.1500
Zinc	0.0045		

Relative Heat-conducting Power of Metals.

(* Calvert & Johnson; † Weidemann & Franz.)

Metals.	*C. & J.	†W. & F.	Metals.	*C. & J.	†W. & F.
Silver	1000	1000	Cadmium	577	....
Gold	981	532	Wrought iron	436	119
Gold, with 1% of silver	840	....	Tin	422	145
Copper, rolled	845	736	Steel	397	116
Copper, cast	811	....	Platinum	380	84
Mercury	677	....	Sodium	365	....
Mercury, with 1.25% of tin	412	....	Cast iron	359	....
Aluminum	665	....	Lead	287	85
Zinc:			Antimony:		
cast vertically	628	....	cast horizontally	215	....
cast horizontally	608	....	cast vertically	192	....
rolled	641	....	Bismuth	61	18

INFLUENCE OF A NON-METALLIC SUBSTANCE IN COMBINATION ON THE CONDUCTING POWER OF A METAL.

Influence of carbon on iron:		Cast copper	811
Wrought iron	436	Copper with 1% of arsenic	570
Steel	397	" with .5% of arsenic	669
Cast iron	359	" with .25% of arsenic	771

The Rate of External Conduction through the bounding surface between a solid body and a fluid is approximately proportional to the difference of temperature, when that is small; but when that difference is considerable the rate of conduction increases faster than the simple ratio of that difference. (Rankine.)

If r, as before, is the coefficient of internal thermal resistance, e and e' the coefficient of external resistance of the two surfaces, x the thickness of the plate, and T' and T the temperatures of the two fluids in contact with the two surfaces, the rate of conduction is $q = \frac{T' - T}{e + e' + rx}$. According to Peclet, $e + e' = \frac{1}{A[1 + B(T' - T)]}$, in which the constants A and B have the following values:

B for polished metallic surfaces	.0028
B for rough metallic surfaces and for non-metallic surfaces	.0037
A for polished metals, about	.90
A for glassy and varnished surfaces	1.34
A for dull metallic surfaces	1.58
A for lamp-black	1.78

When a metal plate has a liquid at each side of it, it appears from experiments by Peclet that $B = .058$, $A = 8.8$.

The results of experiments on the evaporative power of boilers agree very well with the following approximate formula for the thermal resistance of boiler plates and tubes:

$$e + e' = \frac{a}{(T' - T)},$$

which gives for the rate of conduction, per square foot of surface per hour,

$$q = \frac{(T' - T)^2}{a}.$$

This formula is proposed by Rankine as a rough approximation, near enough to the truth for its purpose. The value of a lies between 160 and 200.

Convection, or carrying of heat, means the transfer and diffusion of the heat in a fluid mass by means of the motion of the particles of that mass.

The conduction, properly so called, of heat through a stagnant mass of fluid is very slow in liquids, and almost, if not wholly, inappreciable in gases. It is only by the continual circulation and mixture of the particles of the fluid that uniformity of temperature can be maintained in the fluid mass, or heat transferred between the fluid mass and a solid body.

The free circulation of each of the fluids which touch the side of a solid plate is a necessary condition of the correctness of Rankine's formulæ for the conduction of heat through that plate; and in these formulæ it is implied that the circulation of each of the fluids by currents and eddies is such as to prevent any considerable difference of temperature between the fluid particles in contact with one side of the solid plate and those at considerable distances from it.

When heat is to be transferred by convection from one fluid to another, through an intervening layer of metal, the motions of the two fluid masses should, if possible, be in opposite directions, in order that the hottest particles of each fluid may be in communication with the hottest particles of the other, and that the minimum difference of temperature between the adjacent particles of the two fluids may be the greatest possible.

Thus, in the surface condensation of steam, by passing it through metal tubes immersed in a current of cold water or air, the cooling fluid should be made to move in the opposite direction to the condensing steam.

Steam-pipe Coverings.

(Experiments by Prof. Ordway, Trans. A. S. M. E., vi, 168; also Circular No. 27 of Boston Mfrs. Mutual Fire Ins. Co., 1890.)

Substance 1 inch thick. Heat applied, 310° F.	Pounds of Water heated 10° F., per hour, through 1 sq. ft.	British Thermal Units per sq. ft. per minute.	Solid Matter in 1 sq. ft. 1 inch thick, parts in 1000.	Air included, parts in 1000.
1. *Loose wool*	8.1	1.35	56	944
2. *Live-geese feathers*	9.6	1.60	50	950
3. *Carded cotton wool*	10.4	1.73	20	980
4. *Hair felt*	10.3	1.72	185	815
5. *Loose lampblack*	9.8	1.63	56	944
6. *Compressed lampblack*	10.6	1.77	244	756
7. *Cork charcoal*	11.9	1.98	53	947
8. *White-pine charcoal*	13.9	2.32	119	881
9. *Anthracite-coal powder*	35.7	5.95	506	494
10. Loose calcined magnesia	12.4	2.07	23	977
11. Compressed calcined magnesia	42.6	7.10	285	715
12. Light carbonate of magnesia	13.7	2.28	60	940
13. Compressed carb. of magnesia	15.4	2.57	150	850
14. Loose fossil-meal	14.5	2.42	60	940
15. Crowded fossil-meal	15.7	2.62	112	888
16. Ground chalk (Paris white)	20.6	3.43	253	747
17. Dry plaster of Paris	30.9	5.15	368	632
18. Fine asbestos	49.0	8.17	81	919
19. Air alone	48.0	8.00	0	1000
20. Sand	62.1	10.35	529	471
21. Best slag-wool	13.	2.17	....	....
22. *Paper*	14.	2.33	....	....
23. *Blotting-paper wound tight*	21.	3.50	....	....
24. *Asbestos paper wound tight*	21.7	3.62	....	....
25. *Cork strips bound on*	14.6	2.43	....	....
26. *Straw rope wound spirally*	18.	3.	....	....
27. *Loose rice chaff*	18.7	3.12	....	....
28. Paste of fossil-meal with hair	16.7	2.78	....	....
29. Paste of fossil-meal with asbestos	22.	3.67	....	....
30. Loose bituminous-coal ashes	21.	3.50	....	....
31. Loose anthracite-coal ashes	27.	4.50	....	....
32. Paste of clay and vegetable fibre	30.9	5.15	....	....

It will be observed that several of the incombustible materials are nearly as efficient as wool, cotton, and feathers, with which they may be compared in the preceding table. The materials which may be considered wholly free from the danger of being carbonized or ignited by slow contact with pipes or boilers are printed in Roman type. Those which are more or less liable to be carbonized are printed in italics.

The results Nos. 1 to 20 inclusive were from experiments with the various non-conductors each used in a mass one inch thick, placed on a flat surface of iron kept heated by steam to 310° F. The substances Nos. 21 to

32 were tried as coverings for two-inch steam pipe; the results being reduced to the same terms as the others for convenience of comparison.

Experiments on still air gave results which differ little from those of Nos. 3, 4, and 6. The bulk of matter in the best non-conductors is relatively too small to have any specific effect except to trap the air and keep it stagnant. These substances keep the air still by virtue of the roughness of their fibres or particles. The asbestos, No. 18, had smooth fibres. Asbestos with exceedingly fine fibre made a somewhat better showing, but asbestos is really one of the poorest non-conductors. It may be used advantageously to hold together other incombustible substances, but the less of it the better. A "magnesia" covering, made of carbonate of magnesia with a small percentage of good asbestos fibre and containing 0.25 of solid matter, transmitted 2.5 B. T. U. per square foot per minute, and one containing 0.396 of solid matter transmitted 3.33 B. T. U.

Any suitable substance which is used to prevent the escape of steam heat should not be less than one inch thick.

Any covering should be kept perfectly dry, for not only is water a good carrier of heat, but it has been found that still water conducts heat about eight times as rapidly as still air.

Tests of Commercial Coverings were made by Mr. Geo. M. Brill and reported in Trans. A. S. M. E., xvi. 827. A length of 60 feet of 8-inch steam-pipe was used in the tests, and the heat loss was determined by the condensation. The steam pressure was from 109 to 117 lbs. gauge, and the temperature of the air from 58° to 81° F. The difference between the temperature of steam and air ranged from 263° to 286°, averaging 272°.

The following are the principal results:

Kind of Covering.	Thickness of Covering, inches.	Lbs. Steam condensed per sq. ft. per hour.	B. T. U. per sq. ft. per minute.	B. T. U. per sq. ft. per hour per degree of average difference of temperature.	Saving due to covering lbs. steam per hour per sq. ft.	Ratio of Heat lost, Bare to Covered Pipe, %.	H. P. lost per 100 sq. ft. of pipe (30 lbs. per hour = 1 H. P.).
Bare pipe	....	.846	12.27	2.706	....	100.	2.819
Magnesia	1.25	.120	1.74	.384	.726	14.2	.400
Rock wool	1.60	.080	1.16	.256	.766	9.5	.267
Mineral wool	1.30	.089	1.29	.285	.757	10.5	.297
Fire-felt	1.30	.157	2.28	.502	.689	18.6	.523
Manville sectional	1.70	.109	1.59	.350	.737	12.9	.364
Manv. sect. & hair-felt.	2.40	.066	0.96	.212	.780	7.8	.221
Manville wool-cement.	2.20	.108	1.56	.345	.738	12.7	.359
Champion mineral wool	1.44	.099	1.44	.317	.747	11.7	.330
Hair-felt	.82	.132	1.91	.422	.714	15.6	.439
Riley cement	.75	.298	4.32	.953	.548	35.2	.993
Fossil-meal	.75	.275	3.99	.879	.571	32.5	.916

Transmission of Heat, through Solid Plates, from Water to Water. (Clark, S.E.).—M. Péclet found, from experiments made with plates of wrought iron, cast iron, copper, lead, zinc, and tin, that when the fluid in contact with the surface of the plate was not circulated by artificial means, the rate of conduction was the same for different metals and for plates of the same metal of different thicknesses. But when the water was thoroughly circulated over the surfaces, and when these were perfectly clean, the quantity of transmitted heat was inversely proportional to the thickness, and directly as the difference in temperature of the two faces of the plate. When the metal surface became dull, the rate of transmission of heat through all the metals was very nearly the same.

It follows, says Clark, that the absorption of heat through metal plates is more active whilst evaporation is in progress—when the circulation of the water is more active—than while the water is being heated up to the boiling point.

Transmission from Steam to Water.—M. Péclet's principle is supported by the results of experiments made in 1867 by Mr. Isherwood on the conductivity of different metals. Cylindrical pots, 10 inches in diameter, 21¼ inches deep inside, and ⅛ inch, ¼ inch, and ⅜ inch thick, turned and bored, were formed of pure copper, brass (60 copper and 40 zinc), rolled wrought iron, and remelted cast iron. They were immersed in a steam bath, which was varied from 220° to 320° F. Water at 212° was supplied to the pots, which were kept filled. It was ascertained that the rate of evaporation was in the direct ratio of the difference of the temperatures inside and outside of the pots; that is, that the rate of evaporation per degree of difference of temperatures was the same for all temperatures; and that the rate of evaporation was exactly the same for different thicknesses of the metal. The respective rates of conductivity of the several metals were as follows, expressed in weight of water evaporated from and at 212° F. per square foot of the interior surface of the pots per degree of difference of temperature per hour, together with the equivalent quantities of heat-units:

	Water at 212°.	Heat-units.	Ratio.
Copper	.665 lb.	642.5	1.00
Brass	.577 "	556.8	.87
Wrought iron	.387 "	373.6	.58
Cast iron	.327 "	315.7	.49

Whitham, "Steam Engine Design," p. 283, also Trans. A. S. M. E. ix, 425, in using these data in deriving a formula for surface condensers calls these figures those of perfect conductivity, and multiplies them by a coefficient *C*, which he takes at 0.323, to obtain the efficiency of condenser surface in ordinary use, i.e., coated with saline and greasy deposits.

Transmission of Heat from Steam to Water through Coils of Iron Pipe.—H. G. C. Kopp and F. J. Meystre (*Stevens Indicator*, Jan., 1894), give an account of some experiments on transmission of heat through coils of pipe. They collate the results of earlier experiments as follows, for comparison:

Experimenter.	Character of Surface.	Steam Condensed per Square foot per degree difference of temperature per hour. Heating, pounds.	Evaporating, pounds.	Heat transmitted per square foot per degree difference of temperature per hour. Heating, B. T. U.	Evaporating, B. T. U.	Remarks.
Laurens	Copper coils...	.292	.981	315	974	
"	2 Copper coils.	...	1.20	....	1120	
Havrez..	Copper coil...	.268	1.26	280	1200	
Perkins.	Iron coil.	....	.24	....	215	Steam pressure = 100.
"	" "	....	.22	....	208.2	Steam pressure = 10.
Box.....	Iron tube	.235	.....	230	......	
"	" "	.196	.....	207	......	
"	" "	.206	.....	210	......	
Havrez..	Cast-iron boiler	.077	.105	82	100	

From the above it would appear that the efficiency of iron surfaces is less than that of copper coils, plate surfaces being far inferior.

In all experiments made up to the present time, it appears that the temperature of the condensing water was allowed to rise, a mean between the initial and final temperatures being accepted as the effective temperature. But as water becomes warmer it circulates more rapidly, thereby causing the water surrounding the coil to become agitated and replaced by cooler water, which allows more heat to be transmitted.

Again, in accepting the mean temperature as that of the condensing medium, the assumption is made that the rate of condensation is in direct proportion to the temperature of the condensing water.

In order to correct and avoid any error arising from these assumptions and approximations, experiments were undertaken, in which all the conditions were constant during each test.

The pressure was maintained uniform throughout the coil, and provision was made for the free outflow of the condensed steam, in order to obtain at all times the full efficiency of the condensing surface. The condensing water was continually stirred to secure uniformity of temperature, which was regulated by means of a steam-pipe and a cold-water pipe entering the tank in which the coil was placed.

The following is a condensed statement of the results

HEAT TRANSMITTED PER SQUARE FOOT OF COOLING SURFACE, PER HOUR, PER DEGREE OF DIFFERENCE OF TEMPERATURE. (British Thermal Units.)

Temperature of Condensing Water.	1-in. Iron Pipe; Steam inside, 60 lbs. Gauge Pressure.	1¼ in. Pipe; Steam inside, 10 lbs. Pressure.	1¼ in. Pipe; Steam outside, 10 lbs. Pressure.	1¼ in. Pipe; Steam inside, 60 lbs. Pressure.
80	265	128	200	....
100	269	130	230	239
120	272	137	260	247
140	277	145	267	276
160	281	158	271	306
180	299	174	270	349
200	313	...	....	419

The results indicate that the heat transmitted per degree of difference of temperature in general increases as the temperature of the condensing water is increased.

The amount transmitted is much larger with the steam on the outside of the coil than with the steam inside the coil. This may be explained in part by the fact that the condensing water when inside the coil flows over the surface of conduction very rapidly, and is more efficient for cooling than when contained in a tank outside of the coil.

This result is in accordance with that found by Mr. Thomas Craddock, which indicated that the rate of cooling by transmission of heat through metallic surfaces was almost wholly dependent on the rate of circulation of the cooling medium over the surface to be cooled.

Transmission of Heat in Condenser Tubes. (*Eng'g*, Dec. 10, 1875, p. 449.).—In 1874 B. C. Nichol made experiments for determining the rate at which heat was transmitted through a condenser tube. The results went to show that the amount of heat transmitted through the walls of the tube per estimated degree of mean difference of temperature increased considerably with this difference. For example:

	Vertical Tube.			Horizontal Tube		
Estimated mean difference of temperature between inside and outside of tube, degrees Fahr. .	128	151.9	152.9	111.6	146.2	150.4
Heat-units transmitted per hour per square foot of surface per degree of mean diff. of temp....	422	531	561	610	737	823

These results seem to throw doubt upon Mr. Isherwood's statement that the rate of evaporation per degree of difference of temperature is the same for all temperatures.

Mr. Thomas Craddock found that water was enormously more efficient than air for the abstraction of heat through metallic surfaces in the process of cooling. He proved that the rate of cooling by transmission of heat through metallic surfaces depends upon the rate of circulation of the cooling medium over the surface to be cooled. A tube filled with hot water, moved by rapid rotation at the rate of 59 ft. per second, through air, lost as much heat in one minute as it did in still air in 12 minutes. In water, at a velocity of 3 ft. per second, as much heat was abstracted in half a minute as was abstracted in one minute when it was at rest in the water. Mr. Craddock concluded, further, that the circulation of the cooling fluid became of greater importance as the difference of temperature on the two sides of the plate became less. (Clark, R. T. D., p. 461.)

Heat Transmission through Cast-iron Plates Pickled in Nitric Acid.—Experiments by R. C. Carpenter (Trans. A. S. M. E., xii 179) show a marked change in the conducting power of the plates (from steam to water), due to prolonged treatment with dilute nitric acid.

The action of the nitric acid, by dissolving the free iron and not attacking the carbon, forms a protecting surface to the iron, which is largely composed of carbon. The following is a summary of results:

Character of Plates, each plate 8.4 in. by 5.4 in., exposed surface 27 sq. ft.	Increase in Temperature of 8.125 lbs. of Water each Minute.	Proportionate Thermal Units Transmitted for each Degree of Difference of Temperature per Square Foot per Hour.	Relative Transmission of Heat.
Cast iron—untreated skin on, but clean, free from rust.	13.90	113.2	100.0
Cast iron—nitric acid, 1% sol., 9 days	11.5	97.7	86.3
" " 1% sol., 18 days.	9.7	80.08	70.7
" " 1% sol., 40 days.	9.6	77.8	68.7
" " 5% sol., 9 days	9.98	87.0	76.8
" " 5% sol., 40 days.	10.6	77.4	68.5
Plate of pine wood, same dimensions as the plate of cast iron	0.33	1.9	1.6

The effect of covering cast-iron surfaces with varnish has been investigated by P. M. Chamberlain. He subjected the plate to the action of strong acid for a few hours, and then applied a non-conducting varnish. One surface only was treated. Some of his results are as follows:

Heat units per sq. ft. per hour, for each degree, $T - t$.

- 170. As finished—greasy.
- 152. " " washed with benzine and dried.
- 169. Oiled with lubricating oil.
- 162. After exposure to nitric acid sixteen hours, then oiled (linseed oil.)
- 166 After exposure to hydrochloric acid twelve hours, then oiled (linseed oil.)
- 113. }
- 117. } After exposure to sulphuric acid 1, water 2, for 48 hours, then oiled, varnished, and allowed to dry for 24 hours.

Transmission of Heat through Solid Plates from Air or other Dry Gases to Water. (From Clark on the Steam Engine.) —The law of the transmission of heat from hot air or other gases to water, through metallic plates, has not been exactly determined by experiment. The general results of experiments on the evaporative action of different portions of the heating surface of a steam-boiler point to the general law that the quantity of heat transmitted per degree difference of temperature is practically uniform for various differences of temperature.

The communication of heat from the gas to the plate surface is much accelerated by mechanical impingement of the gaseous products upon the surface.

Clark says that when the surfaces are perfectly clean, the rate of transmission of heat through plates of metal from air or gas to water is greater for copper, next for brass, and next for wrought iron. But when the surfaces are dimmed or coated, the rate is the same for the different metals.

With respect to the influence of the conductivity of metals and of the thickness of the plate on the transmission of heat from burnt gases to water, Mr. Napier made experiments with small boilers of iron and copper placed over a gas-flame. The vessels were 5 inches in diameter and 2½ inches deep. From three vessels, one of iron, one of copper, and one of iron sides and copper bottom, each of them 1/30 inch in thickness, equal quantities of water were evaporated to dryness, in the times as follows ;

Water.	Iron Vessel.	Copper Vessel.	Iron and Copper Vessel.
4 ounces	19 minutes	18.5 minutes	......
11 "	33 "	30.75 "	......
5½ "	50 "	44 "	......
4 "	35.7 "	......	36.83 minutes.

Two other vessels of iron sides 1/30 inch thick, one having a ¼-inch copper bottom and the other a ¼-inch lead bottom, were tested against the iron and copper vessel, 1/30 inch thick. Equal quantities of water were evaporated in 54, 56, and 53½ minutes respectively. Taken generally, the results of these experiments show that there are practically but slight differences between iron, copper, and lead in evaporative activity, and that the activity is not affected by the thickness of the bottom.

Mr. W. B. Johnson formed a like conclusion from the results of his observations of two boilers of 160 horse-power each, made exactly alike, except that one had iron flue-tubes and the other copper flue-tubes. No difference could be detected between the performances of these boilers.

Divergencies between the results of different experimenters are attributable probably to the difference of conditions under which the heat was transmitted, as between water or steam and water, and between gaseous matter and water. On one point the divergence is extreme: the rate of transmission of heat per degree of difference of temperature. Whilst from 400 to 600 units of heat are transmitted from water to water through iron plates, per degree of difference per square foot per hour, the quantity of heat transmitted between water and air, or other dry gas, is only about from 2 to 5 units, according as the surrounding air is at rest or in movement. In a locomotive boiler, where radiant heat was brought into play, 17 units of heat were transmitted through the plates of the fire-box per degree of difference of temperature per square foot per hour.

Transmission of Heat through Plates and Tubes from Steam or Hot Water to Air.—The transfer of heat from steam or water through a plate or tube into the surrounding air is a complex operation, in which the internal and external conductivity of the metal, the radiating power of the surface, and the convection of heat in the surrounding air are all concerned. Since the quantity of heat radiated from a surface varies with the condition of the surface and with the surroundings, according to laws not yet determined, and since the heat carried away by convection varies with the rate of the flow of the air over the surface, it is evident that no general law can be laid down for the total quantity of heat emitted.

The following is condensed from an article on Loss of Heat from Steam-pipes, in *The Locomotive*, Sept. and Oct., 1892.

A hot steam-pipe is radiating heat constantly off into space, but at the same time it is cooling also by convection. Experimental data on which to base calculations of the heat radiated and otherwise lost by steam-pipes are neither numerous nor satisfactory.

In Box's Practical Treatise on Heat a number of results are given for the amount of heat radiated by different substances when the temperature of the air is 1° Fahr. lower than the temperature of the radiating body. A portion of this table is given below. It is said to be based on Péclet's experiments.

HEAT UNITS RADIATED PER HOUR, PER SQUARE FOOT OF SURFACE, FOR 1° FAHRENHEIT EXCESS IN TEMPERATURE.

Copper, polished	.0327	Sheet-iron, ordinary	.5662
Tin, polished	.0440	Glass	.5948
Zinc and brass, polished	.0491	Cast iron, new	.6480
Tinned iron, polished	.0858	Common steam-pipe, inferred	.6400
Sheet-iron, polished	.0920	Cast and sheet iron, rusted	.6868
Sheet lead	.1329	Wood, building stone, and brick	.7358

When the temperature of the air is about 50° or 60° Fahr., and the radiating body is not more than about 30° hotter than the air, we may calculate the radiation of a given surface by assuming the amount of heat given off by it in a given time to be proportional *to the difference in temperature between the radiating body and the air.* This is "Newton's law of cooling." But when the difference in temperature is great, Newton's law does not hold good; the radiation is no longer proportional to the difference in temperature, but must be calculated by a complex formula established experimentally by Dulong and Petit. Box has computed a table from this formula, which greatly facilitates its application, and which is given below:

Factors for Reduction to Dulong's Law of Radiation.

Differences in Temperature between Radiating Body and the Air.	Temperature of the Air on the Fahrenheit Scale.											
	32°	50°	59°	68°	86°	104°	122°	140°	158°	176°	194°	212°
Deg. Fahr.												
18	1.00	1.07	1.12	1.16	1.25	1.36	1.47	1.58	1.70	1.85	1.99	2.15
36	1.03	1.08	1.16	1.21	1.30	1.40	1.52	1.68	1.76	1.91	2.06	2.23
54	1.07	1.16	1.20	1.25	1.35	1.45	1.58	1.70	1.83	1.99	2.14	2.31
72	1.12	1.20	1.25	1.30	1.40	1.52	1.64	1.76	1.90	2.07	2.23	2.40
90	1.16	1.25	1.31	1.36	1.46	1.58	1.71	1.84	1.98	2.15	2.33	2.51
108	1.21	1.31	1.36	1.42	1.52	1.65	1.78	1.92	2.07	2.28	2.42	2.62
126	1.26	1.36	1.42	1.48	1.50	1.72	1.86	2.00	2.16	2.34	2.52	2.72
144	1.32	1.42	1.48	1.54	1.65	1.79	1.94	2.08	2.24	2.44	2.64	2.83
162	1.37	1.48	1.54	1.60	1.73	1.86	2.02	2.17	2.34	2.54	2.74	2.96
180	1.44	1.55	1.61	1.68	1.81	1.95	2.11	2.27	2.46	2.66	2.87	3.10
198	1.50	1.62	1.69	1.75	1.89	2.04	2.21	2.38	2.56	2.78	3.00	3.24
216	1.58	1.69	1.76	1.83	1.97	2.13	2.32	2.48	2.68	2.91	3.18	3.38
234	1.64	1.77	1.84	1.90	2.06	2.28	2.43	2.52	2.80	3.03	3.28	3.46
252	1.71	1.85	1.92	2.00	2.15	2.33	2.52	2.71	2.92	3.18	3.43	3.70
270	1.79	1.93	2.01	2.09	2.22	2.44	2.64	2.84	3.06	3.32	3.58	3.87
288	1.89	2.03	2.12	2.20	2.37	2.56	2.78	2.99	3.22	3.50	3.77	4.07
306	1.98	2.13	2.22	2.31	2.49	2.69	2.90	3.12	3.37	3.66	3.95	4.26
324	2.07	2.23	2.33	2.42	2.62	2.81	3.04	3.28	3.53	3.84	4.14	4.46
342	2.17	2.34	2.44	2.54	2.73	2.95	3.19	3.44	3.70	4.02	4.34	4.68
360	2.27	2.45	2.56	2.66	2.86	3.09	3.35	3.60	3.88	4.22	4.55	4.91
378	2.39	2.57	2.68	2.79	3.00	3.24	3.51	3.78	4.08	4.42	4.77	5.15
396	2.50	2.70	2.81	2.93	3.15	3.40	3.68	3.97	4.28	4.64	5.01	5.40
414	2.63	2.84	2.95	3.07	3.31	3.51	3.87	4.12	4.48	4.87	5.26	5.67
432	2.76	2.98	3.10	3.23	3.47	3.76	4.10	4.32	4.61	5.12	5.33	6.04

The loss of heat by *convection* appears to be independent of the nature of the surface, that is, it is the same for iron, stone, wood, and other materials. It is different for bodies of different shape, however, and it varies with the position of the body. Thus a vertical steam-pipe will not lose so much heat by convection as a horizontal one will; for the air heated at the lower part of the vertical pipe will rise along the surface of the pipe, protecting it to some extent from the chilling action of the surrounding cooler air. For a similar reason the shape of a body has an important influence on the result, those bodies losing most heat whose forms are such as to allow the cool air free access to every part of their surface. The following table from Box gives the number of heat units that horizontal cylinders or pipes lose by convection per square foot of surface per hour, for one degree difference in temperature between the pipe and the air.

Heat Units Lost by Convection from Horizontal Pipes, per Square Foot of Surface per Hour, for a Temperature Difference of 1° Fahr.

External Diameter of Pipe in inches.	Heat Units Lost.	External Diameter of Pipe in inches.	Heat Units Lost.	External Diameter of Pipe in inches.	Heat Units Lost.
2	0.728	7	0.509	18	0.455
3	0.626	8	0.498	24	0.447
4	0.574	9	0.489	36	0.433
5	0.544	10	0.482	48	0.434
6	0.523	12	0.472	..	.....

The loss of heat by convection is nearly proportional to the difference in temperature between the hot body and the air; but the experiments of

Dulong and Péclet show that this is not exactly true, and we may here also resort to a table of factors for correcting the results obtained by simple proportion.

FACTORS FOR REDUCTION TO DULONG'S LAW OF CONVECTION.

Difference in Temp. between Hot Body and Air.	Factor.	Difference in Temp. between Hot Body and Air.	Factor.	Difference in Temp. between Hot Body and Air.	Factor.
18° F.	0.94	180° F.	1.62	342° F.	1.87
36°	1.11	198°	1.65	360°	1.90
54°	1.22	216°	1.68	378°	1.92
72°	1.30	234°	1.72	396°	1.94
90°	1.37	252°	1.74	414°	1.96
108°	1.43	270°	1.77	432°	1.98
126°	1.49	288°	1.80	450°	2.00
144°	1.53	306°	1.83	468°	2.02
162°	1.58	324°	1.85	....	....

EXAMPLE IN THE USE OF THE TABLES.—Required the total loss of heat by both radiation and convection, per foot of length of a steam-pipe 2 11/32 in. external diameter, steam pressure 60 lbs., temperature of the air in the room 68° Fahr.

Temperature corresponding to 60 lbs. equals 307°; temperature difference = 307 − 68 = 239°.

Area of one foot length of steam-pipe = 2 11/32 × 3.1416 ÷ 12 = 0.614 sq. ft.

Heat radiated per hour per square foot per degree of difference, from table, 0.64.

Radiation loss per hour by Newton's law = 239° × .614 ft. × .64 = 93.9 heat units. Same reduced to conform with Dulong's law of radiation: factor from table for temperature difference of 239° and temperature of air 68° = 1.93. 93.9 × 1.93 = 181.2 heat units, total loss by radiation.

Convection loss per square foot per hour from a 2 11/32-inch pipe: by interpolation from table, 2″ = .728. 3″ = .626, 2 11/32″ = .693.

Area, .614 × .693 × 239° = 101.7 heat units. Same reduced to conform with Dulong's law of convection: 101.7 × 1.73 (from table) = 175.9 heat units per hour. Total loss by radiation and convection = 181.2 + 175.9 = 357.1 heat units per hour. Loss per degree of difference of temperature per linear foot of pipe per hour = 357.1 ÷ 239 = 1.494 heat units = 2.433 per sq. ft.

It is not claimed, says *The Locomotive*, that the results obtained by this method of calculation are strictly accurate. The experimental data are not sufficient to allow us to compute the heat-loss from steam-pipes with any great degree of refinement; yet it is believed that the results obtained as indicated above will be sufficiently near the truth for most purposes. An experiment by Prof. Ordway, in a pipe 2 11/32 in. diam. under the above conditions (Trans. A. S. M. E., v. 73), showed a condensation of steam of 181 grammes per hour, which is equivalent to a loss of heat of 358.7 heat units per hour, or within half of one per cent of that given by the above calculation.

According to different authorities, the quantity of heat given off by steam and hot-water radiators in ordinary practice of heating of buildings by direct radiation varies from 1.8 to about 3 heat units per hour per square foot per degree of difference of temperature.

The lowest figure is calculated from the following statement by Robert Briggs in his paper on "American Practice in Warming Buildings by Steam" (Proc. Inst. C. E., 1882, vol. lxxi): "Each 100 sq. ft. of radiating surface will give off 3 Fahr. heat units per minute for each degree F. of difference in temperature between the radiating surface and the air in which it is exposed."

The figure 2 1/2 heat units is given by the Nason Manufacturing Company in their catalogue, and 2 to 2 1/4 are given by many recent writers.

For the ordinary temperature difference in low-pressure steam-heating, say 212° − 70° = 142° F., 1 lb. steam condensed from 212° to water at the same temperature gives up 965.7 heat units. A loss of 2 heat units per sq. ft. per hour per degree of difference, under these conditions, is equivalent to $2 \times 142 \div 965 = 0.3$ lbs. of steam condensed per hour per sq. ft. of heating surface. (See also Heating and Ventilation.)

Transmission of Heat through Walls, etc., of Buildings (Nason Manufacturing Co.). (See also Heating and Ventilation.)—Heat has the remarkable property of passing through moderate thicknesses of air and gases without appreciable loss, so that air is not warmed by radiant heat, but by contact with surfaces that have absorbed the radiation.

POWERS OF DIFFERENT SUBSTANCES FOR TRANSMITTING HEAT.

Window-glass	1000	Bricks, rough	200 to 250
Oak or walnut	66	Bricks, whitewashed	200
White pine	80	Granite or slate	250
Pitch-pine	100	Sheet iron	1030 to 1110
Lath or plaster	75 to 100		

A square foot of glass will cool 1.279 cubic feet of air from the temperature inside to that outside per minute, and outside wall surface is generally estimated at one fifth of the rate of glass in cooling effect.

Box, in his "Practical Treatise on Heat," gives a table of the conducting powers of materials prepared from the experiments of Péclet. It gives the quantity of heat in units transmitted per square foot per hour by a plate 1 inch in thickness, the two surfaces differing in temperature 1 degree:

Fine-grained gray marble	28.00
Coarse-grained white marble	22.4
Stone, calcareous, fine	16.7
Stone, calcareous, ordinary	13.68
Baked clay, brickwork	4.83
Brick-dust, sifted	1.33

Hood, in his "Warming and Ventilating of Buildings," p. 249, gives the results of M. Depretz, which, placing the conducting power of marble at 1.00, give .483 as the value for firebrick.

THERMODYNAMICS.

Thermodynamics, the science of heat considered as a form of energy, is useful in advanced studies of the theory of steam, gas, and air engines, refrigerating machines, compressed air, etc. The method of treatment adopted by the standard writers is severely mathematical, involving constant application of the calculus. The student will find the subject thorougly treated in the recent works by Rontgen (Dubois's translation), Wood, and Peabody.

First Law of Thermodynamics.—Heat and mechanical energy are mutually convertible in the ratio of about 778 foot-pounds for the British thermal unit. (Wood.) Heat is the living force or *vis viva* due to certain molecular motions of the molecules of bodies, and this living force may be stated or measured in units of heat or in foot-pounds, a unit of heat in British measures being equivalent to 772 [778] foot-pounds. (Trowbridge, Trans. A. S. M. E., vii. 727.)

Second Law of Thermodynamics.—The second law has by different writers been stated in a variety of ways, and apparently with ideas so diverse as not to cover a common principle. (Wood, Therm., p. 389.)

It is impossible for a self-acting machine, unaided by any external agency to convert heat from one body to another at a higher temperature. (Clausius.)

If all the heat absorbed be at one temperature, and that rejected be at one lower temperature, then will the heat which is transmuted into work be to the entire heat absorbed in the same ratio as the difference between the absolute temperature of the source and refrigerator is to the absolute temperature of the source. In other words, the second law is an expression for the efficiency of the perfect elementary engine. (Wood.)

The living force, or *vis viva*, of a body (called heat) is always proportional to the absolute temperature of the body. (Trowbridge.)

The expression $\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$ may be called the symbolical or algebraic enunciation of the second law,—the law which limits the efficiency of heat engines, and which does not depend on the nature of the working medium employed. (Trowbridge.) Q_1 and T_1 = quantity and absolute temperature of the heat received, Q_2 and T_2 = quantity and absolute temperature of the heat rejected.

The expression $\frac{T_1 - T_2}{T_1}$ represents the efficiency of a perfect heat engine which receives all its heat at the absolute temperature T_1, and rejects heat at the temperature T_2, converting into work the difference between the quantity received and rejected.

EXAMPLE.—What is the efficiency of a perfect heat engine which receives heat at 388° F. (the temperature of steam of 200 lbs. gauge pressure) and rejects heat at 100° F. (temperature of a condenser, pressure 1 lb. above vacuum).

$$\frac{388 + 459.2 - (100 + 459.2)}{388 + 459.2} = 34\%, \text{ nearly.}$$

In the actual engine this efficiency can never be attained, for the difference between the quantity of heat received into the cylinder and that rejected into the condenser is not all converted into work, much of it being lost by radiation, leakage, etc. In the steam engine the phenomenon of cylinder condensation also tends to reduce the efficiency.

PHYSICAL PROPERTIES OF GASES.

(Additional matter on this subject will be found under Heat, Air, Gas, and Steam.)

When a mass of gas is enclosed in a vessel it exerts a pressure against the walls. This pressure is uniform on every square inch of the surface of the vessel; also, at any point in the fluid mass the pressure is the same in every direction.

In small vessels containing gases the increase of pressure due to weight may be neglected, since all gases are very light; but where liquids are concerned, the increase in pressure due to their weight must always be taken into account.

Expansion of Gases, Marriotte's Law.—The volume of a gas diminishes in the same ratio as the pressure upon it is increased.

This law is by experiment found to be very nearly true for all gases, and is known as Boyle's or Mariotte's law.

If p = pressure at a volume v, and p_1 = pressure at a volume v_1, $p_1v_1 = pv$; $p_1 = \frac{v}{v_1}p$; pv = a constant.

The constant, C, varies with the temperature, everything else remaining the same.

Air compressed by a pressure of seventy-five atmospheres has a volume about 2% less than that computed from Boyle's law, but this is the greatest divergence that is found below 160 atmospheres pressure.

Law of Charles.—The volume of a perfect gas at a constant pressure is proportional to its absolute temperature. If v_0 be the volume of a gas at 32° F., and v_1 the volume at any other temperature, t_1, then

$$v_1 = v_0\left(\frac{t_1 + 459.2}{491.2}\right); \quad v_1 = \left(1 + \frac{t_1 - 32°}{491.2}\right)v_0,$$

$$\text{or} \quad v_1 = [1 + 0.002036(t_1 - 32°)]v_0.$$

If the pressure also change from p_0 to p_1,

$$v_1 = v_0\frac{p_0}{p_1}\left(\frac{t_1 + 459.2}{491.2}\right).$$

The Densities of the elementary gases are simply proportional to their atomic weights. The density of a compound gas, referred to hydrogen as 1, is one-half its molecular weight; thus the relative density of CO_2 is $\frac{1}{2}(12 + 32) = 22$.

Avogadro's Law.—Equal volumes of all gases, under the same conditions of temperature and pressure, contain the same number of molecules.

To find the weight of a gas in pounds per cubic foot at 32° F., multiply half the molecular weight of the gas by .00559. Thus 1 cu. ft. marsh-gas, CH_4,

$$= \frac{1}{2}(12 + 4) \times .00559 = .0447 \text{ lb.}$$

When a certain volume of hydrogen combines with one half its volume of oxygen, there is produced an amount of water vapor which will occupy the same volume as that which was occupied by the hydrogen gas when at the same temperature and pressure.

Saturation-point of Vapors.—A vapor that is not near the saturation-point behaves like a gas under changes of temperature and pressure; but if it is sufficiently compressed or cooled, it reaches a point where it begins to condense: it then no longer obeys the same laws as a gas, but its pressure cannot be increased by diminishing the size of the vessel containing it, but remains constant, except when the temperature is changed. The only gas that can prevent a liquid evaporating seems to be its own vapor.

Dalton's Law of Gaseous Pressures.—Every portion of a mass of gas inclosed in a vessel contributes to the pressure against the sides of the vessel the same amount that it would have exerted by itself had no other gas been present.

Mixtures of Vapors and Gases.—The pressure exerted against the interior of a vessel by a given quantity of a perfect gas enclosed in it is the sum of the pressures which any number of parts into which such quantity might be divided would exert separately, if each were enclosed in a vessel of the same bulk alone, at the same temperature. Although this law is not exactly true for any actual gas, it is very nearly true for many. Thus if 0.080728 lb. of air at 32° F., being enclosed in a vessel of one cubic foot capacity, exerts a pressure of one atmosphere or 14.7 pounds, on each square inch of the interior of the vessel, then will each additional 0.080728 lb. of air which is enclosed, at 32°, in the same vessel, produce very nearly an additional atmosphere of pressure. The same law is applicable to mixtures of gases of different kinds. For example, 0.12344 lb. of carbonic-acid gas, at 32°, being enclosed in a vessel of one cubic foot in capacity, exerts a pressure of one atmosphere; consequently, if 0.080728 lb. of air and 0.12344 lb. of carbonic acid, mixed, be enclosed at the temperature of 32°, in a vessel of one cubic foot of capacity, the mixture will exert a pressure of two atmospheres. As a second example: Let 0.080728 lb. of air, at 212°, be enclosed in a vessel of one cubic foot; it will exert a pressure of

$$\frac{212 + 459.2}{32 + 459.2} = 1.366 \text{ atmospheres.}$$

Let 0.03797 lb. of steam, at 212°, be enclosed in a vessel of one cubic foot; it will exert a pressure of one atmosphere. Consequently, if 0.080728 lb. of air and 0.03797 lb. of steam be mixed and enclosed together, at 212°, in a vessel of one cubic foot, the mixture will exert a pressure of 2.366 atmospheres. It is a common but erroneous practice, in elementary books on physics, to describe this law as constituting a difference between mixed and homogeneous gases; whereas it is obvious that for mixed and homogeneous gases the law of pressure is exactly the same, viz., that the pressure of the whole of a gaseous mass is the sum of the pressures of all its parts This is one of the laws of mixture of gases and vapors.

A second law is that the presence of a foreign gaseous substance in contact with the surface of a solid or liquid does not affect the density of the vapor of that solid or liquid unless there is a tendency to chemical combination between the two substances, in which case the density of the vapor is slightly increased. (Rankine, S. E., p. 239.)

Flow of Gases.—By the principle of the conservation of energy, it may be shown that the velocity with which a gas under pressure will escape into a vacuum is inversely proportional to the square root of its density; that is, oxygen, which is sixteen times as heavy as hydrogen, would, under exactly the same circumstances, escape through an opening only one fourth as fast as the latter gas.

Absorption of Gases by Liquids.—Many gases are readily absorbed by water. Other liquids also possess this power in a greater or less degree. Water will for example, absorb its own volume of carbonic-acid gas, 430 times its volume of ammonia, 2½ times its volume of chlorine, and only about 1/20 of its volume of oxygen.

The weight of gas that is absorbed by a given volume of liquid is proportional to the pressure. But as the volume of a mass of gas is less as the pressure is greater, the volume which a given amount of liquid can absorb at a certain temperature will be constant, whatever the pressure. Water, for example, can absorb its own volume of carbonic-acid gas at atmospheric pressure; it will also dissolve its own volume if the pressure is twice as great, but in that case the gas will be twice as dense, and consequently twice the weight of gas is dissolved.

AIR.

Properties of Air.—Air is a mechanical mixture of the gases oxygen and nitrogen; 20.7 parts O and 79.3 parts N by volume, 23 parts O and 77 parts N by weight.

The weight of pure air at 32° F. and a barometric pressure of 29.92 inches of mercury, or 14.6963 lbs. per sq. in., or 2116.3 lbs. per sq. ft., is .080728 lb. per cubic foot. Volume of 1 lb. = 12.387 cu. ft. At any other temperature and barometric pressure its weight in lbs. per cubic foot is $W = \frac{1.3253 \times B}{459.2 + T}$, where B = height of the barometer, T = temperature Fahr., and 1.3253 = weight in lbs. of 459.2 c. ft. of air at 0° F. and one inch barometric pressure. Air expands 1/491.2 of its volume at 32° F. for every increase of 1° F., and its volume varies inversely as the pressure.

Volume, Density, and Pressure of Air at Various Temperatures. (D. K. Clark.)

Fahr.	Volume at Atmos. Pressure.		Density, lbs. per Cubic Foot at Atmos. Pressure.	Pressure at Constant Volume.	
	Cubic Feet in 1 lb.	Comparative Vol.		Lbs. per Sq. In.	Comparative Pres.
0	11.583	.881	.086331	12.96	.881
32	12.387	.943	.080728	13.86	.943
40	12.586	.958	.079439	14.08	.958
50	12.840	.977	.077884	14.36	.977
62	13.141	1.000	.076097	14.70	1.000
70	13.342	1.015	.074950	14.92	1.015
80	13.593	1.034	.073565	15.21	1.034
90	13.845	1.054	.072230	15.49	1.054
100	14.096	1.073	.070942	15.77	1.073
110	14.344	1.092	.069721	16.05	1.092
120	14.592	1.111	.068500	16.33	1.111
130	14.846	1.130	.067361	16.61	1.130
140	15.100	1.149	.066221	16.89	1.149
150	15.351	1.168	.065155	17.19	1.168
160	15.603	1.187	.064088	17.50	1.187
170	15.854	1.206	.063089	17.76	1.206
180	16.106	1.226	.062090	18.02	1.226
200	16.606	1.264	.060210	18.58	1.264
210	16.860	1.283	.059313	18.86	1.283
212	16.910	1.287	.059135	18.92	1.287

The Air-manometer consists of a long vertical glass tube, closed at the upper end, open at the lower end, containing air, provided with a scale, and immersed, along with a thermometer, in a transparent liquid, such as water or oil, contained in a strong cylinder of glass, which communicates with the vessel in which the pressure is to be ascertained. The scale shows the volume occupied by the air in the tube.

Let v_0 be that volume, at the temperature of 32° Fahrenheit, and mean pressure of the atmosphere, p_0; let v_1 be the volume of the air at the temperature t, and under the absolute pressure to be measured p_1; then

$$p_1 = \frac{(t + 459.2^\circ)p_0 v_0}{491.2^\circ\ v_1}.$$

Pressure of the Atmosphere at Different Altitudes.

At the sea-level the pressure of the air is 14.7 pounds per square inch; at ¼ of a mile above the sea-level it is 14.02 pounds; at ½ mile, 13.33; at ¾ mile, 12.66; at 1 mile, 12.02; at 1¼ mile, 11.42; at 1½ mile, 10.88; and at 2 miles, 9.80 pounds per square inch. For a rough approximation we may assume that the pressure decreases ½ pound per square inch for every 1000 feet of ascent.

It is calculated that at a height of about 3½ miles above the sea-level the weight of a cubic foot of air is only one half what it is at the surface of the earth, at seven miles only one fourth, at fourteen miles only one sixteenth, at twenty-one miles only one sixty-fourth, and at a height of over forty-five miles it becomes so attenuated as to have no appreciable weight.

The pressure of the atmosphere increases with the depth of shafts, equal to about one inch rise in the barometer for each 900 feet increase in depth: this may be taken as a rough-and-ready rule for ascertaining the depth of shafts.

Pressure of the Atmosphere per Square Inch and per Square Foot at Various Readings of the Barometer.

RULE.—Barometer in inches × .4908 = pressure per square inch; pressure per square inch × 144 = pressure per square foot.

Barometer.	Pressure per Sq. In.	Pressure per Sq. Ft.	Barometer.	Pressure per Sq. In.	Pressure per Sq. Ft.
in.	lbs.	lbs.*	in.	lbs.	lbs.*
28.00	13.74	1978	29.75	14.60	2102
28.25	13.86	1995	30.00	14.72	2119
28.50	13.98	2013	30.25	14.84	2136
28.75	14.11	2031	30.50	14.96	2154
29.00	14.23	2049	30.75	15.09	2172
29.25	14.35	2066	31.00	15.21	2190
29.50	14.47	2083			

* Decimals omitted.

For lower pressures see table of the Properties of Steam.

Barometric Readings corresponding with Different Altitudes, in French and English Measures.

Altitude.	Reading of Barometer.	Altitude.	Reading of Barometer.	Altitude.	Reading of Barometer.	Altitude.	Reading of Barometer.
meters.	mm.	feet.	inches.	meters.	mm.	feet.	inches.
0	762	0.	30.	1147	660	3763.2	25.98
21	760	68.9	29.92	1269	650	4163.3	25.59
127	750	416.7	29.52	1393	640	4568.3	25.19
234	740	767.7	29.13	1519	630	4983.1	24.80
342	730	1122.1	28.74	1647	620	5403.2	24.41
453	720	1486.2	28.35	1777	610	5830.2	24.01
564	710	1850.4	27.95	1909	600	6243.	23.62
678	700	2224.5	27.55	2043	590	6702.9	23.22
793	690	2599.7	27.16	2180	580	7152.4	22.83
909	680	2962.1	26.77	2318	570	7605.1	22.44
1027	670	3369.5	26.38	2460	560	8071.	22.04

Levelling by the Barometer and by Boiling Water.

(Trautwine.)—Many circumstances combine to render the results of this kind of levelling unreliable where great accuracy is required. It is difficult to read off from an aneroid (the kind of barometer usually employed for engineering purposes) to within from two to five or six feet, depending on its size. The moisture or dryness of the air affects the results; also winds, the vicinity of mountains, and the daily atmospheric tides, which cause incessant and irregular fluctuations in the barometer. A barometer hanging quietly in a room will often vary 1/4 of an inch within a few hours, corresponding to a difference of elevation of nearly 100 feet. No formula can possibly be devised that shall embrace these sources of error.

To Find the Difference in Altitude of Two Places.—Take from the table the altitudes opposite to the two boiling temperatures, or to the two barometer readings. Subtract the one opposite the lower reading from that opposite the upper reading. The remainder will be the required height, as a rough approximation. To correct this, add together the two thermometer readings, and divide the sum by 2, for their mean. From table of corrections for temperature, take out the number under this mean. Multiply the approximate height just found by this number.

At 70° F. pure water will boil at 1° less of temperature for an average of about 550 feet of elevation above sea-level, up to a height of 1/2 a mile. At the height of 1 mile, 1° of boiling temperature will correspond to about 560 feet of elevation. In the table the mean of the temperatures at the two stations is assumed to be 32°F., at which no correction for temperature is necessary in using the table.

Boiling-point in deg. Fah.	Barom. in.	Altitude above Sea-level, feet.	Boiling-point in deg. Fah.	Barom. in.	Altitude above Sea-level, feet.	Boiling-point in deg. Fah.	Barom. in.	Altitude above Sea-level, feet.
184°	16.79	15,221	196	21.71	8,481	208	27.73	2,063
185	17.16	14,649	197	22.17	7,932	208.5	28.00	1,809
186	17.54	14,075	198	22.64	7,381	209	28.29	1,539
187	17.93	13,498	199	23.11	6,843	209.5°	28.56	1,290
188	18.32	12,934	200	23.59	6,304	210	28.85	1,025
189	18.72	12,367	201	24.08	5,764	210.5	29.15	754
190	19.13	11,799	202	24.58	5,225	211	29.42	512
191	19.54	11,243	203	25.08	4,697	211.5	29.71	255
192	19.96	10,685	204	25.59	4,169	212	30.00	S. L. = 0
193	20.39	10,127	205	26.11	3,642	212.5	30.30	−261
194	20.82	9,579	206	26.64	3,115	213	30.59	−511
195	21.26	9,031	207	27.18	2,589			

CORRECTIONS FOR TEMPERATURE.

Mean temp. F. in shade.	0	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°
Multiply by	.933	.954	.975	.996	1.016	1.036	1.058	1.079	1.100	1.121	1.142

Moisture in the Atmosphere.—Atmospheric air always contains a small quantity of carbonic acid (see Ventilation, p. 528) and a varying quantity of aqueous vapor or moisture. The relative humidity of the air at any time is the percentage of moisture contained in it as compared with the amount it is capable of holding at the same temperature.

The degree of saturation or relative humidity of the air is determined by the use of the dry and wet bulb thermometer. The degree of saturation for a number of different readings of the thermometer is given in the following table, condensed from one published by the U. S. Weather Bureau, 1897:

RELATIVE HUMIDITY, PER CENT.

Dry Thermometer, Deg. F.	Difference between the Dry and Wet Thermometers, Deg. F.																										
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	26	28	30
	Relative Humidity, Saturation being 100.																										
32	90	79	69	59	50	40	31	21	12	3																	
40	92	84	76	68	60	53	45	38	30	22	16	8	1														
50	93	87	80	74	67	61	55	50	44	38	33	27	22	16	11	6	1										
60	94	89	84	78	73	68	63	58	53	48	44	39	34	30	26	22	18	14	10	6	2						
70	95	90	86	81	77	72	68	64	60	55	52	48	44	40	36	33	29	26	23	19	16	13	10	7	1		
80	96	92	87	83	79	75	72	68	64	61	57	54	51	47	44	41	38	35	32	29	26	23	20	18	13	8	3
90	96	92	88	85	81	78	75	71	68	65	62	59	56	53	50	47	44	41	39	36	34	32	29	26	22	17	13
100	97	93	90	86	83	80	77	74	71	68	65	62	59	57	54	51	49	47	44	42	39	37	35	33	29	25	21
110	97	94	90	87	84	81	78	76	73	70	67	65	62	60	57	55	53	50	48	46	44	42	40	38	34	30	27
120	97	94	91	88	85	83	80	77	75	72	70	67	65	62	60	58	56	54	51	49	47	45	44	42	38	35	31
140	97	95	92	89	87	84	82	79	77	75	73	71	68	66	64	62	60	58	56	55	53	51	49	48	44	41	38

Weights of Air, Vapor of Water, and Saturated Mixtures of Air and Vapor at Different Temperatures, under the Ordinary Atmospheric Pressure of 29.921 inches of Mercury.

Temperature, Fahrenheit.	Weight of a Cubic Ft. of Dry Air at Different Temperatures, lbs.	Elastic Force of Vapor, Inches of Mercury.	Mixtures of Air Saturated with Vapor.				
			Elastic Force of the Air in Mixture of Air and Vapor, Inches of Mercury.	Weight of Cubic Foot of the Mixture of Air and Vapor.			Weight of Vapor mixed with 1 lb. of Air, pounds.
				Weight of the Air, lbs.	Weight of the Vapor, pounds.	Total W'ght of Mixture, pounds.	
0°	.0864	.044	29.877	.0863	.000079	.086379	.00092
12	.0842	.074	29.849	.0840	.000130	.084130	.00155
22	.0824	.118	29.803	.0821	.000202	.082302	.00245
32	.0807	.181	29.740	.0802	.000304	.080504	.00379
42	.0791	.267	29.654	.0784	.000440	.078840	.00561
52	.0776	.388	29.533	.0766	.000627	.077227	.00819
62	.0761	.556	29.365	.0747	.000881	.075581	.01179
72	.0747	.785	29.136	.0727	.001221	.073921	.01680
82	.0733	1.092	28.829	.0706	.001667	.072267	.02361
92	.0720	1.501	28.420	.0684	.002250	.070717	.03289
102	.0707	2.036	27.885	.0659	.002997	.068897	.04547
112	.0694	2.731	27.190	.0631	.003946	.067046	.06253
122	.0682	3.621	26.300	.0599	.005142	.065042	.08584
132	.0671	4.752	25.169	.0564	.006639	.063039	.11771
142	.0660	6.165	23.756	.0524	.008473	.060873	.16170
152	.0649	7.930	21.991	.0477	.010716	.058416	.22465
162	.0638	10.099	19.822	.0423	.013415	.055715	.31713
172	.0628	12.758	17.163	.0360	.016682	.052682	.46338
182	.0618	15.960	13.961	.0288	.020536	.049336	.71300
192	.0609	19.828	10.093	.0205	.025142	.045642	1.22643
202	.0600	24.450	5.471	.0109	.030545	.041445	2.80230
212	.0591	29 921	0.000	.0000	.036820	.036820	Infinite.

The weight in lbs. of the vapor mixed with 100 lbs. of pure air at any given temperature and pressure is given by the formula

$$\frac{62.3 \times E}{29.92 - E} \times \frac{29.92}{p},$$

where E = elastic force of the vapor at the given temperature, in inches of mercury; p = absolute pressure in inches of mercury, = 29.92 for ordinary atmospheric pressure.

Specific Heat of Air at Constant Volume and at Constant Pressure.—Volume of 1 lb. of air at 32° F. and pressure of 14.7 lbs. per sq. in. = 12.387 cu. ft. = a column 1 sq. ft. area × 12.387 ft. high. Raising temperature 1° F. expands it $\frac{1}{491.2}$, or to 12.4122 ft. high—a rise of .02522 foot.

Work done = 2116 lbs. per sq. ft. × .02522 = 53.37 foot-pounds, or 53.37 ÷ 778 = .0686 heat units.

The specific heat of air at constant pressure, according to Regnault, is 0.2375; but this includes the work of expansion, or .0686 heat units; hence the specific heat at constant volume = 0.2375 − .0686 = 0.1689.

Ratio of specific heat at constant pressure to specific heat at constant volume = .2375 ÷ .1689 = 1.406. (See Specific Heat, p. 458.)

Flow of Air through Orifices.—The theoretical velocity in feet per second of flow of any fluid, liquid, or gas through an orifice is $v = \sqrt{2gh} = 8.02\sqrt{h}$, in which h = the "head" or height of the fluid in feet required to produce the pressure of the fluid at the level of the orifice. (For gases the formula holds good only for small differences of pressure on the two sides of the orifice.) The quantity of flow in cubic feet per second is equal to the product of this velocity by the area of the orifice, in square feet, multiplied by a "coefficient of flow," which takes into account the contraction of the vein or flowing stream, the friction of the orifice, etc.

For air flowing through an orifice or short tube, from a reservoir of the pressure p_1 into a reservoir of the pressure p_2, Weisbach gives the following values for the coefficient of flow, obtained from his experiments.

Flow of Air through an Orifice.

Coefficient c in formula $v = c\sqrt{2gh}$.

Diameter 1 centimetre.	Ratio of pressures $p_1 \div p_2$	1.05	1.09	1.43	1.65	1.89	2.15
	Coefficient	.555	.589	.692	.724	.754	.788
Diameter 2.14 centimetres	Ratio of pressures	1.05	1.09	1.36	1.67	2.01	....
	Coefficient	.558	.573	.634	.678	.723	....

Flow of Air through a Short Tube.

Diam. 1 cm., Length 3 cm.	Ratio of pressures $p_1 \div p_2$	1.05	1.10	1.30	....	....	....
	Coefficient	.730	.771	.830	....	....	....
Diam. 1.414 cm., Length 4.242 cm.	Ratio of pressures	1.41	1.69	....	....	....	....
	Coefficient	.813	.822	....	....	....	....
Diam. 1 cm., Length 1.6 cm. Orifice rounded.	Ratio of pressures	1.24	1.38	1.59	1.85	2.14	....
	Coefficient	.979	.986	.965	.971	.978	....

Fliegner's Equations for Flow of Air from a Reservoir through an Orifice. (Peabody's Thermodynamics, p. 185.)

$$\text{For } p_1 > 2p_a, \quad G = 0.530\, F \frac{p_1}{\sqrt{T_1}};$$

$$p_1 < 2p_a, \quad G = 1.060\, F \sqrt{\frac{p_a(p_1 - p_a)}{T_1}};$$

G = flow of air through the orifice in lbs. per sec., F = area of orifice in sq. in., p_1 = absolute pressure in reservoir in lbs. per sq. in., p_a = pressure of atmosphere, T_1 = absolute temperature, Fahr., of air in reservoir.

Clark (Rules, Tables, and Data, p. 891) gives, for the velocity of flow of air through an orifice due to small differences of pressure,

$$V = C\sqrt{\frac{2gh}{12} \times 773.2 \times \left(1 + \frac{t - 32}{493}\right) \times \frac{29.92}{p}},$$

or, simplified,

$$V = 352\, C\sqrt{\left(1 + .00203(t - 32)\right)\frac{h}{p}};$$

in which V = velocity in feet per second; $2g = 64.4$; h = height of the column of water in inches, measuring the difference of pressure; t = the temperature Fahr.; and p = barometric pressure in inches of mercury. 773.2 is the volume of air at 32° under a pressure of 29.92 inches of mercury when that of an equal weight of water is taken as 1.

For 62° F., the formula becomes $V = 363C\sqrt{\frac{h}{p}}$, and if $p = 29.92$ inches $V = 66.35C\sqrt{h}$

The coefficient of efflux C, according to Weisbach, is:

For conoidal mouthpiece, of form of the contracted vein, with pressures of from .23 to 1.1 atmospheres	C = .97 to .99
Circular orifices in thin plates	C = .56 to .79
Short cylindrical mouthpieces	C = .81 to .84
The same rounded at the inner end	C = .92 to .93
Conical converging mouthpieces	C = .90 to .99

Flow of Air in Pipes.—Hawksley (Proc. Inst. C. E., xxxiii, 55) states that his formula for flow of water in pipes $v = 48\sqrt{\frac{HD}{L}}$ may also be employed for flow of air. In this case H = height in feet of a column of air required to produce the pressure causing the flow, or the loss of head for a given flow; v = velocity in feet per second, D = diameter in feet, L = length in feet.

If the head is expressed in inches of water, h, the air being taken at 62° F., its weight per cubic foot at atmospheric pressure = .0761 lb. Then $H = \frac{62.36}{.0761 \times 12} = 68.3h$. If d = diameter in inches, $D = \frac{d}{12}$, and the formula becomes $v = 114.5\sqrt{\frac{hd}{L}}$, in which h = inches of water column, d = diameter in inches and L = length in feet; $h = \frac{Lv^2}{18110d}$; $d = \frac{Lv^2}{18110h}$.

The quantity in cubic feet per second is

$$Q = .7854\frac{d^2}{144}v = .6245\sqrt{\frac{hd^5}{L}};\quad d = \sqrt[5]{\frac{Q^2L}{.39h}};\quad h = \frac{Q^2L}{.39d^5}.$$

The horse-power required to drive air through a pipe is the volume Q in cubic feet per second multiplied by the pressure in pounds per square foot and divided by 550. Pressure in pounds per square foot = P = inches of water column × 5.196, whence horse-power =

$$HP. = \frac{QP}{550} = \frac{Qh}{105.9} = \frac{Q^3L}{41.3d^5}.$$

If the head or pressure causing the flow is expressed in pounds per square inch = p, then $h = 27.71p$, and the above formulæ become

$$v = 602.7\sqrt{\frac{pd}{L}};\quad p = \frac{Lv^2}{363{,}300d};\quad d = \frac{Lv^2}{363{,}300p};$$

$$Q = 3.287\sqrt{\frac{pd^5}{L}};\quad p = \frac{Q^2L}{10.806d^5};\quad d = \sqrt[5]{\frac{Q^2L}{10.806p}};$$

$$HP. = \frac{Q144p}{550} = .2618Qp = .02421\frac{Q^3L}{d^5}.$$

Volume of Air Transmitted in Cubic Feet per Minute in Pipes of Various Diameters.

Formula $Q = \frac{.7854}{144}d^2v \times 60.$

Veloc'y of Flow, Feet p. sec.	Actual Diameter of Pipe in Inches.											
	1	2	3	4	5	6	8	10	12	16	20	24
1	.327	1.31	2.95	5.24	8.18	11.78	20.94	32.73	47.12	83.77	130.9	188.5
2	.655	2.62	5.89	10.47	16.36	23.56	41.89	65.45	94.25	167.5	261.8	377
3	.982	3.93	8.84	15.7	24.5	35.3	62.8	98.2	141.4	251.3	392.7	565.5
4	1.31	5.24	11.78	20.9	32.7	47.1	83.8	131	188	335	523	754
5	1.64	6.54	14.7	26.2	41	59	104	163	235	419	654	942
6	1.96	7.85	17.7	31.4	49.1	70.7	125	196	283	502	785	1131
7	2.29	9.16	20.6	36.6	57.2	82.4	146	229	330	586	916	1319
8	2.62	10.5	23.5	41.9	65.4	94	167	262	377	670	1047	1508
9	2.95	11.78	26.5	47	73	106	188	294	424	754	1178	1696
10	3.27	13.1	29.4	52	82	118	209	327	471	838	1309	1885
12	3.93	15.7	35.3	63	98	141	251	393	565	1005	1571	2262
15	4.91	19.6	44.2	78	122	177	314	491	707	1256	1963	2827
18	5.89	23.5	53	94	147	212	377	589	848	1508	2356	3393
20	6.54	26.2	59	105	164	235	419	654	942	1675	2618	3770
24	7.85	31.4	71	125	196	283	502	785	1131	2010	3141	4524
25	8.18	32.7	73	131	204	294	523	818	1178	2094	3272	4712
28	9.16	36.6	82	146	229	330	586	916	1319	2346	3665	5278
30	9.8	39.3	88	157	245	353	628	982	1414	2513	3927	5655

In Hawksley's formula and its derivatives the numerical coefficients are constant. It is scarcely possible, however, that they can be accurate except within a limited range of conditions. In the case of water it is found that the coefficient of friction, on which the loss of head depends, varies with the length and diameter of the pipe, and with the velocity, as well as with the condition of the interior surface. In the case of air and other gases we have, in addition, the decrease in density and consequent increase in volume and in velocity due to the progressive loss of head from one end of the pipe to the other.

Clark states that according to the experiments of D'Aubuisson and those of a Sardinian commission on the resistance of air through long conduits or pipes, the diminution of pressure is very nearly directly as the length, and as the square of the velocity and inversely as the diameter. The resistance is not varied by the density.

If these statements are correct, then the formulæ $h = \frac{Lv^2}{cd}$ and $h = \frac{Q^2L}{c'd^5}$ and their derivatives are correct in form, and they may be used when the numerical coefficients c and c' are obtained by experiment.

If we take the forms of the above formulæ as correct, and let C be a variable coefficient, depending upon the length, diameter, and condition of surface of the pipe, and possibly also upon the velocity, the temperature and the density, to be determined by future experiments, then for h = head in inches of water, d = diameter in inches, L = length in feet, v = velocity in feet per second, and Q = quantity in cubic feet per second:

$$v = C\sqrt{\frac{hd}{L}}; \qquad d = \frac{Lv^2}{C^2h}; \qquad h = \frac{Lv^2}{C^2d};$$

$$Q = .005454C\sqrt{\frac{hd^5}{L}}; \qquad d = \sqrt[5]{\frac{33683Q^2L}{C^2h}}; \qquad h = \frac{33683Q^2L}{C^2d^5}.$$

For difference or loss of pressure p in pounds per square inch,

$$h = 27.71p \qquad \sqrt{h} = 5.264\sqrt{p};$$

$$v = 5.264C\sqrt{\frac{pd}{L}}; \qquad d = \frac{Lv^2}{27.71C^2p}; \qquad p = \frac{Lv^2}{27.71C^2d};$$

$$Q = .02871C\sqrt{\frac{pd^5}{L}}; \qquad d = \sqrt[5]{\frac{1213Q^2L}{C^2p}}; \qquad p = \frac{1213Q^2L}{C^2d^5}.$$

(For other formulæ for flow of air, see Mine Ventilation.)

Loss of Pressure in Ounces per Square Inch.—B. F. Sturtevant Company uses the following formulæ:

$$p_1 = \frac{Lv^2}{25000d}; \quad v = \sqrt{\frac{25000dp_1}{L}}; \quad d = \frac{Lv^2}{25000p_1};$$

in which p_1 = loss of pressure in ounces per square inch, v = velocity of air in feet per second, and L = length of pipe in feet. If p is taken in pounds per square inch, these formulæ reduce to

$$p = .0000025\frac{Lv^2}{d}; \quad v = 632.5\sqrt{\frac{dp_1}{L}}; \quad d = \frac{.0000025Lv^2}{p}.$$

These are deduced from the common formula (Weisbach's), $p = f\frac{l}{d}\frac{v^2}{2g}$, in which f = .0001608.

The following table is condensed from one given in the catalogue of B. F. Sturtevant Company.

Loss of pressure in pipes 100 feet long, in ounces per square inch. For any other length, the loss is proportional to the length.

Velocity of Air, feet per min.	Diameter of Pipe in Inches.											
	1	2	3	4	5	6	7	8	9	10	11	12
	Loss of Pressure in Ounces.											
600	.400	.200	.133	.100	.080	.067	.057	.050	.044	.040	.036	.033
1200	1.600	.800	.533	.400	.320	.267	.229	.200	.178	.160	.145	.133
1800	3.600	1.800	1.200	.900	.720	.600	.514	.450	.400	.360	.327	.300
2400	6.400	3.200	2.133	1.600	1.280	1.067	.914	.800	.711	.640	.582	.533
3000	10.	5.	3.333	2.5	2.	1.667	1 429	1.250	1.111	1.000	.909	.833
3600	14.4	7.2	4.8	3.6	2.88	2.4	2.057	1.8	1.6	1.44	1.309	1.200
4200	......	9.8	6.553	4.9	3.92	3.267	2.8	2.45	2.178	1.96	1.782	1.633
4800	.....	12.8	8.533	6.4	5.12	4.267	3.657	3.2	2.844	2.56	2.327	2.133
6000	.....	20.	18.333	10.0	8.0	6.667	5.714	5.0	4.444	4.0	3 636	3.333
	Diameter of Pipe in Inches.											
	14	16	18	20	22	24	28	32	36	40	44	48
	Loss of Pressure in Ounces.											
600	.029	.026	.022	.020	.018	.017	.014	.012	.011	.010	.009	.008
1200	.114	.100	.089	.080	.073	.067	.057	.050	.044	.040	.036	.033
1800	.257	.225	.200	.180	.164	.156	.129	.112	.100	.090	.082	.075
2400	.457	.400	.356	.320	.291	.267	.239	.200	.178	.160	.145	.133
3600	1.029	.900	.800	.720	.655	.600	.514	.450	.400	.360	.327	.300
4200	1.400	1.225	1.089	.980	.891	.817	.700	.612	.544	.490	.445	.408
4800	1.829	1.600	1.422	1.280	1.164	1.067	.914	.800	.711	640	.582	.533
6000	2.857	2.500	2.222	2.000	1.818	1.667	1.429	1.250	1.111	1.000	.909	.833

Effect of Bends in Pipes. (Norwalk Iron Works Co.)

Radius of elbow, in diameter of pipe =	5	3	2	1½	1¼	1	¾	½
Equivalent lgths. of straight pipe, diams	7.85	8.24	9.08	10.36	12.72	17.51	35.09	121.2

Compressed-air Transmission. (Frank Richards, *Am. Mach.*, March 8, 1894.)—The volume of free air transmitted may be assumed to be directly as the number of atmospheres to which the air is compressed. Thus, if the air transmitted be at 75 pounds gauge-pressure, or six atmospheres, the volume of free air will be six times the amount given in the table (page 486). It is generally considered that for economical transmission the velocity in main pipes should not exceed 20 feet per second. In the smaller distributing pipes the velocity should be decidedly less than this.

The loss of power in the transmission of compressed air in general is not a serious one, or at all to be compared with the losses of power in the operation of compression and in the re-expansion or final application of the air.

The formulas for loss by friction are all unsatisfactory. The statements of observed facts in this line are in a more or less chaotic state, and self-evidently unreliable.

A statement of the friction of air flowing through a pipe involves at least all the following factors: Unit of time, volume of air, pressure of air, diameter of pipe, length of pipe, and the difference of pressure at the ends of the pipe or the head required to maintain the flow. Neither of these factors can be allowed its independent and absolute value, but is subject to modifications in deference to its associates. The flow of air being assumed to be uniform at the entrance to the pipe, the volume and flow are not uniform after that. The air is constantly losing some of its pressure and its volume is constantly increasing. The velocity of flow is therefore also somewhat accelerated continually. This also modifies the use of the length of the pipe as a constant factor.

Then, besides the fluctuating values of these factors, there is the condition of the pipe itself. The actual diameter of the pipe, especially in the smaller sizes, is different from the nominal diameter. The pipe may be straight, or it may be crooked and have numerous elbows. Mr. Richards considers one elbow as equivalent to a length of pipe.

Formulæ for Flow of Compressed Air in Pipes.—The formulæ on pages 486 and 487 are for air at or near atmospheric pressure. For compressed air the density has to be taken into account. A common formula for the flow of air, gas, or steam in pipes is

$$Q = c\sqrt{\frac{pd^5}{wL}},$$

in which Q = volume in cubic feet per minute, p = difference of pressure in lbs. per sq. in. causing the flow, d = diameter of pipe in in., L = length of pipe in ft., w = density of the entering gas or steam in lbs per cu. ft., and c = a coefficient found by experiment. Mr. F. A. Halsey in calculating a table for the Rand Drill Co.'s Catalogue takes the value of c at 58, basing it upon the experiments made by order of the Italian government preliminary to boring the Mt. Cenis tunnel. These experiments were made with pipes of 3281 feet in length and of approximately 4, 8, and 14 in. diameter. The volumes of compressed air passed ranged between 16.64 and 1200 cu. ft. per minute. The value of c is quite constant throughout the range and shows little disposition to change with the varying diameter of the pipe. It is of course probable, says Mr. Halsey, that c would be smaller if determined for smaller sizes of pipe, but to offset that the actual sizes of small commercial pipe are considerably larger than the nominal sizes, and as these calculations are commonly made for the nominal diameters it is probable that in those small sizes the loss would really be less than shown by the table. The formula is of course strictly applicable to fluids which do not change their density, but within the change of density admissible in the transmission of air for power purposes it is probable that the errors introduced by this change are less than those due to errors of observation in the present state of knowledge of the subject. Mr. Halsey's table is condensed below.

Diameter of Pipe, in inches.	Cubic feet of free air compressed to a gauge-pressure of 80 lbs. and passing through the pipe each minute.										
	50	100	200	400	800	1000	1500	2000	3000	4000	5000
	Loss of pressure in lbs. per square inch for each 1000 ft. of straight pipe.										
1¼	3.61										
1½	1.45	5.8									
2	0.20	1.05	4.30								
2½	0.12	0.35	1.41	5.80							
3	......	0.14	0.57	2.28							
3½	......	......	0.26	1.05	4.16	6.4					
4	......	......	0.14	0.54	2.12	3.27	7.60				
5	......	......	......	0.18	0.68	1.08	2.43	4.32	9.6		
6	......	......	......	......	0.28	0.43	1.00	1.75	3.91	7.10	10.7
8	......	......	......	......	0.07	0.10	0.24	0.42	0.93	1.68	2.59
10	......	......	......	......	......	......	0.08	0.14	0.30	0.55	0.84
12	......	......	......	......	......	......	......	......	0.12	0.22	0.34
14	......	......	......	......	......	......	......	......	......	0.10	0.16

To apply the formula given above to air of different pressures it may be given other forms, as follows:

Let Q = the volume in cubic feet per minute of the compressed air; Q_1 = the volume before compression, or "free air," both being taken at mean atmospheric temperature of 62° F.; w_1 = weight per cubic foot of Q_1 = 0.0761 lb.; r = atmospheres, or ratio of absolute pressures, = (gauge-pressure + 14.7) ÷ 14.7; w = weight per cu. ft. of Q; p = difference of pressure, in lbs. per sq. in., causing the flow; d = diam. of pipe in in.; L = length of pipe in ft.; c = experimental constant. Then

$$Q = c\sqrt{\frac{pd^5}{wL}}; \qquad Q_1 = rQ; \qquad w = rw_1 = .0761r;$$

$$Q = 3.625c\sqrt{\frac{pd^5}{rL}}; \qquad Q_1 = 3.625c\sqrt{\frac{pd^5r}{L}};$$

$$d = \sqrt[5]{.0761\frac{LQ^2r}{c^2p}} = 0.597\sqrt[5]{\frac{LQ^2r}{c^2p}} = \sqrt[5]{.0761\frac{LQ_1^2}{c^2pr}} = 0.597\sqrt[5]{\frac{LQ_1^2}{c^2pr}};$$

$$p = .0761\frac{LQ^2r}{c^2d^5} = .0761\frac{LQ_1^2}{c^2d^5r}.$$

The value of c according to the Mt. Cenis experiments is about 58 for pipes 4, 8, and 14 in. diameter, 3381 ft. long. In the St. Gothard experiments it ranged from 62.8 to 73.2 (see table below) for pipes 5.91 and 7.87 in. diameter, 1713 and 15,092 ft. long. Values derived from D'Arcy's formula for flow of water in pipes, ranging from 45.3 for 1 in. diameter to 63.2 for 24 in., are given under "Flow of Steam," p. 671. For approximate calculations the value 60 may be used for all pipes of 4 in. diameter and upwards. Using $c = 60$, the above formulæ become

$$Q = 217.5\sqrt{\frac{pd^5}{rL}}; \qquad Q_1 = 217.5\sqrt{\frac{pd^5r}{L}};$$

$$d = 0.1161\sqrt[5]{\frac{LQ^2r}{p}} = 0.1161\sqrt[5]{\frac{LQ_1^2}{pr}};$$

$$p = 0.00002114\frac{LQ^2r}{d^5} = 0.00002114\frac{LQ_1^2}{d^5r}.$$

Loss of Pressure in Compressed Air Pipe-main, at St. Gothard Tunnel.

(E. Stockalper.)

Experiment.	Air Main Diameter.	Volume per second of free air, or equivalent volume at atmospheric pressure and 32° F.	Volume per second of compressed air at mean density.	Mean density of compressed air. (Water = 1.)	Weight of air flowing per second.	Mean velocity in feet per second.	Observed Pressures. Pressure at beginning of pipe.	Pressure at end of pipe.	Loss of Pressure.		Value of c in formula $Q = c\sqrt{\frac{pd^5}{wL}}$.
No.	in.	cu.ft.	cu.ft.	den.	lbs.	feet.	at.	at.	lbs. per sq.in.	%	
1	7.87	33.056	6.534	.00650	2.609	19.32	5.60	5.24	5.292	6.4	71.3
	5.91		7.063	.00603	2.669	37.14	5.24	5.00	3.528	4.6	61.9
2	7.87	23.002	5.509	.00614	1.776	16.30	4.35	4.13	3.234	5.1	70.7
	5.91		5.863	.00482	1.776	.....	4.13	....	...	...	...
3	7.87	18.364	5.262	.00449	1.483	15.58	3.84	3.65	2.793	5.0	67.6
	5.91		5.580	.00423	1.483	29.34	3.65	3.54	1.617	3.0	62.8

The length of the pipe 7.87 in diameter was 15,092 ft., and of the smaller pipe 1712.6 ft. The mean temperature of the air in the large pipe was 70° F. and in the small pipe 80° F.

Equation of Pipes.—It is frequently desired to know what number of pipes of a given size are equal in carrying capacity to one pipe of a larger size. At the same velocity of flow the volume delivered by two pipes of different sizes is proportional to the squares of their diameters; thus, one 4-inch pipe will deliver the same volume as four 2-inch pipes. With the same head, however, the velocity is less in the smaller pipe, and the volume delivered varies about as the square root of the fifth power (i.e., as the 2.5 power). The following table has been calculated on this basis. The figures opposite the intersection of any two sizes is the number of the smaller-sized pipes required to equal one of the larger. Thus, one 4-inch pipe is equal to 5.7 2-inch pipes.

Diam. In.	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20	24
2	5.7	1														
3	15.6	2.8	1													
4	32	5.7	2.1	1												
5	55.9	9.9	3.6	1.7	1											
6	88.2	15.6	5.7	2.8	1.6	1										
7	130	22.9	8.3	4.1	2.3	1.5	1									
8	181	32	11.7	5.7	3.2	2.1	1.4	1								
9	243	43.	15.6	7.6	4.3	2.8	1.9	1.3	1							
10	316	55.9	20.3	9.9	5.7	3.6	2.4	1.7	1.3	1						
11	401	70.9	25.7	12.5	7.2	4.6	3.1	2.2	1.7	1.3						
12	499	88.2	32	15.6	8.9	5.7	3.8	2.8	2.1	1.6	1					
13	609	108	39.1	19	10.9	7.1	4.7	3.4	2.5	1.9	1.2					
14	733	130	47	22.9	13.1	8.3	5.7	4.1	3.0	2.3	1.5	1				
15	871	154	55.9	27.2	15.6	9.9	6.7	4.8	3.6	2.8	1.7	1.2				
16	...	181	65.7	32	18.3	11.7	7.9	5.7	4.2	3.2	2.1	1.4	1			
17	...	211	76.4	37.2	21.3	13.5	9.2	6.6	4.9	3.8	2.4	1.6	1.2			
18	...	243	88.2	43	24.6	15.6	10.6	7.6	5.7	4.3	2.8	1.9	1.3	1		
19	...	278	101	49.1	28.1	17.8	12.1	8.7	6.5	5	3.2	2.1	1.5	1.1		
20	....	316	115	55.9	32	20.3	13.8	9.9	7.4	5.7	3.6	2.4	1.7	1.3	1	
22	....	401	146	70.9	40.6	25.7	17.5	12.5	9.3	7.2	4.6	3.1	2.2	1.7	1.3	
24	....	499	181	88.2	50.5	32	21.8	15.6	11.6	8.9	5.7	3.8	2.8	2.1	1.6	1
26	...	609	221	108	61.7	39.1	26.6	19.	14.2	10.9	7.1	4.7	3.4	2.5	1.9	1.2
28	....	733	266	130	74.2	47	32	22.9	17.1	13.1	8.3	5.7	4.1	3	2.3	1.5
30	...	871	316	154	88.2	55.9	38	27.2	20.3	15.6	9.9	6.7	4.8	3.6	2.8	1.7
36	..	.	199	243	130	88.2	60	43	32	24.6	15.6	10.6	7.6	5.7	4.3	2.8
42	..	...	733	357	205	130	88.2	63.2	47	36.2	19	15.6	11.2	8.3	6.4	4.1
48	.	..	...	499	286	181	123	88.2	62.7	50.5	32	21.8	15.6	11.6	8.9	5.7
54	...		..	670	383	243	165	118	88.2	67.6	43	29.2	20.9	15.6	12	7.6
60	...		.	871	499	316	215	154	115	88.2	55.9	38	27.2	20.3	15.6	9.9

Measurement of the Velocity of Air in Pipes by an Anemometer.—Tests were made by B. Donkin, Jr. (*Inst. Civil Engrs.* 1892), to compare the velocity of air in pipes from 8 in. to 24 in. diam., as shown by an anemometer 2¾ in. diam. with the true velocity as measured by the time of descent of a gas-holder holding 1622 cubic feet. A table of the results with discussion is given in *Eng'g News*, Dec. 22, 1892. In pipes from 8 in. to 20 in. diam. with air velocities of from 140 to 690 feet per minute the anemometer showed errors varying from 14.5% fast to 10% slow. With a 24-inch pipe and a velocity of 73 ft. per minute, the anemometer gave from 44 to 63 feet, or from 13.6 to 39.6% slow. The practical conclusion drawn from these experiments is that anemometers for the measurement of velocities of air in pipes of these diameters should be used with great caution. The percentage of error is not constant, and varies considerably with the diameter of the pipes and the speeds of air. The use of a baffle, consisting of a perforated plate, which tended to equalise the velocity in the centre and at the sides in some cases diminished the error.

The impossibility of measuring the true quantity of air by an anemometer held stationary in one position is shown by the following figures, given by Wm. Daniel (Proc. Inst. M. E., 1875), of the velocities of air found at different points in the cross-sections of two different airways in a mine.

Differences of Anemometer Readings in Airways.

8 ft. square.

1712	1795	1859	1829
1622	1685	1782	1091
1477	1344	1524	1049
1262	1356	1293	1333

Average 1469.

5 × 8 ft.

1170	1209	1288
948	1104	1177
1134	1049	1106

Average 1132.

WIND.

Force of the Wind.—Smeaton in 1759 published a table of the velocity and pressure of wind, as follows:

Velocity and Force of Wind, in Pounds per Square Inch.

Miles per hour.	Feet per second.	Force per sq. ft. pounds.	Common Appellation of the Force of Wind.	Miles per Hour.	Feet per second.	Force per sq. ft. pounds.	Common Appellation of the Force of Wind.
1	1.47	0.005	Hardly perceptible.	18	26.4	1.55	Very brisk.
2	2.93	0.020	Just perceptible.	20	29.34	1.968	
3	4.4	0.044		25	36.67	3.075	
4	5.87	0.079	Gentle pleasant wind.	30	44.01	4.429	High wind.
5	7.33	0.123		35	51.34	6.027	
6	8.8	0.177		40	58.68	7.873	Very high storm.
7	10.25	0.241		45	66.01	9.963	
8	11.75	0.315	Pleasant brisk gale.	50	73.35	12.30	
9	13.2	0.400		55	80.7	14.9	
10	14.67	0.492		60	88.02	17.71	Great Storm.
12	17.6	0.708		66	95.4	20.85	
14	20.5	0.964		70	102.5	24.1	
15	22.00	1.107		75	110.	27.7	Hurricane.
16	23.45	1.25		80	117.36	31.49	
				100	146.67	49.2	Immense hurricane.

The pressures per square foot in the above table correspond to the formula $P = 0.005V^2$, in which V is the velocity in miles per hour. *Eng'g News*, Feb. 9, 1893, says that the formula was never well established, and has floated chiefly on Smeaton's name and for lack of a better. It was put forward only for surfaces for use in windmill practice. The trend of modern evidence is that it is approximately correct only for such surfaces, and that for large solid bodies it often gives greatly too large results. Observations by others are thus compared with Smeaton's formula:

Old Smeaton formula .... $P = .005V^2$
As determined by Prof. Martin .... $P = .004V^2$
" " Whipple and Dines .... $P = .0029V^2$

At 60 miles per hour these formulas give for the pressure per square foot, 18, 14.4 and 10.44 lbs., respectively, the pressure varying by all of them as the square of the velocity. Lieut. Crosby's experiments (*Eng'g*, June 13, 1890), claiming to prove that $P = fV$ instead of $P = fV^2$, are discredited.

A. R. Wolff (The Windmill as a Prime Mover, p. 9) gives as the theoretical pressure per sq. ft. of surface, $P = \frac{dQv}{g}$, in which d = density of air in pounds per cu. ft. $= \frac{.018743(p + P)}{t}$; p being the barometric pressure per square foot at any level, and temperature of 32° F., t any absolute temperature, Q = volume of air carried along per square foot in one second, v = velocity of the wind in feet per sec., g = 32.16. Since $Q = v$ cu. ft. per sec., $P = \frac{dv^2}{g}$. Multiplying this by a coefficient 0.93 found by experiment, and substituting the above value of d, he obtains $P = \frac{0.017431 \times p}{\frac{t \times 32.16}{v^2} - .018743}$, and when p = 2116.5 lbs. per sq. ft. or average atmospheric pressure at the sea-level, $P = \frac{36.8929}{\frac{t \times 32.16}{v^2} - 0.18743}$, an expression in which the pressure is shown to vary with the temperature; and he gives a table showing the relation between velocity and pressure for temperatures from 0° to 100° F., and velocities from 1 to 80 miles per hour. For a temperature of 45° F. the pressures agree with those in Smeaton's table, for 0° F. they are about 10 per cent greater, and for 100° 10 per cent less. Prof. H. Allen Hazen, *Eng'g News*, July 5, 1890, says that experiments with whirling arms, by exposing plates to direct wind, and on locomotives with velocities running up to 40 miles per hour, have invariably shown the resistance to vary with V^2. In the formula $P = .005SV^2$, in which P = pressure in pounds, S = surface in square feet, V = velocity in miles per hour, the doubtful question is that regarding the accuracy of the first two factors in the second member of this equation. The first factor has been variously determined from .003 to .005 [it has been determined as low as .0014.—Ed. *Eng'g News*].

The second factor has been found in some experiments with very short whirling arms and low velocities to vary with the perimeter of the plate, but this entirely disappears with longer arms or straight line motion, and the only question now to be determined is the value of the coefficient. Perhaps some of the best experiments for determining this value were tried in France in 1886 by carrying flat boards on trains. The resulting formula in this case was, for 44.5 miles per hour, $p = .00535SV^2$.

Mr. Crosby's whirling experiments were made with an arm 5.5 ft. long. It is certain that most serious effects from centrifugal action would be set up by using such a short arm, and nothing satisfactory can be learned with arms less than 20 or 30 ft. long at velocities above 5 miles per hour.

Prof. Kernot, of Melbourne (*Engineering Record*, Feb. 20, 1894), states that experiments at the Forth Bridge showed that the average pressure on surfaces as large as railway carriages, houses, or bridges never exceeded two thirds of that upon small surfaces of one or two square feet, such as have been used at observatories, and also that an inertia effect, which is frequently overlooked, may cause some forms of anemometer to give false results enormously exceeding the correct indication. Experiments of Mr. O. T. Crosby showed that the pressure varied directly as the velocity, whereas all the early investigators, from the time of Smeaton onwards, made it vary as the square of the velocity. Experiments made by Prof. Kernot at speeds varying from 2 to 15 miles per hour agreed with the earlier authorities, and tended to negative Crosby's results. The pressure upon one side of a cube, or of a block proportioned like an ordinary carriage, was found to be .9 of that upon a thin plate of the same area. The same result was obtained for a square tower. A square pyramid, whose height was three times its base, experienced .8 of the pressure upon a thin plate equal to one of its sides, but if an angle was turned to the wind the pressure was increased by fully 20%. A bridge consisting of two plate-girders connected by a deck at the top was found to experience .9 of the pressure on a thin plate equal in size to one girder, when the distance between the girders was equal to their depth, and this was increased by one fifth when the distance between the girders was double the depth. A lattice-work in which the area of the openings was 55% of the whole area experienced a pressure of 80% of that upon a plate of the same area. The pressure upon cylinders and cones was proved to be equal to half that upon the diametral planes, and that upon an octagonal prism to be 20% greater than upon the circumscribing cylinder. A sphere was subject to a pressure of .36 of that upon a thin circular plate of equal diameter. A hemispherical cup gave the same result as the sphere; when its concavity was turned to the wind the pressure was 1.15 of that on a flat plate of equal diameter. When a plane surface parallel to the direction of the wind was brought nearly into contact with a cylinder or sphere, the pressure on the latter bodies was augmented by about 20%, owing to the lateral escape of the air being checked. Thus it is possible for the security of a tower or chimney to be impaired by the erection of a building nearly touching it on one side.

Pressures of Wind Registered in Storms.—Mr. Frizell has examined the published records of Greenwich Observatory from 1849 to 1869, and reports that the highest pressure of wind he finds recorded is 41 lbs. per sq. ft., and there are numerous instances in which it was between 30 and 40 lbs. per sq. ft. Prof. Henry says that on Mount Washington, N. H., a velocity of 150 miles per hour has been observed, and at New York City 60 miles an hour, and that the highest winds observed in 1870 were of 72 and 63 miles per hour, respectively.

Lieut. Dunwoody, U. S. A., says, in substance, that the New England coast is exposed to storms which produce a pressure of 50 lbs. per sq. ft. *Engineering News*, Aug. 20, 1880.

WINDMILLS.

Power and Efficiency of Windmills.—Rankine, S. E., p. 215, gives the following: Let Q = volume of air which acts on the sail, or part of a sail, in cubic feet per second, v = velocity of the wind in feet per second, s = sectional area of the cylinder, or annular cylinder of wind, through which the sail, or part of the sail, sweeps in one revolution, c = a coefficient to be found by experience; then $Q = cvs$. Rankine, from experimental data given by Smeaton, and taking c to include an allowance for friction, gives for a wheel with four sails, proportioned in the best manner, $c = 0.75$. Let A = weather angle of the sail at any distance from the axis, i.e., the angle the portion of the sail considered makes with its plane of revolution. This angle gradually diminishes from the inner end of the sail to the tip; u = the velocity of the same portion of the sail, and E = the efficiency. The efficiency is the ratio of the useful work performed to whole energy of the stream of wind acting on the surface s of the wheel, which energy is $\frac{Dsv^3}{2g}$, D being the weight of a cubic foot of air. Rankine's formula for efficiency is

$$E = \frac{Ru}{\frac{Dsv^3}{2g}} = c\left\{\frac{u}{v}\sin 2A - \frac{u^2}{v^2}(1 - \cos 2A + f) - f\right\},$$

in which $c = 0.75$ and f is a coefficient of friction found from Smeaton's data $= 0.016$. Rankine gives the following from Smeaton's data:

A = weather-angle	$= 7°$	$13°$	$19°$
$V \div v$ = ratio of speed of greatest efficiency, for a given weather-angle, to that of the wind	$= 2.63$	1.83	1.41
E = efficiency	$= 0.24$	0.29	0.31

Rankine gives the following as the best values for the angle of weather at different distances from the axis:

Distance in sixths of total radius	1	2	3	4	5	6
Weather angle	18°	19°	18°	16°	12½°	7°

But Wolff (p. 125) shows that Smeaton did not term these the best angles, but simply says they "answer as well as any," possibly any that were in existence in his time. Wolff says that they "cannot in the nature of things be the most desirable angles." Mathematical considerations, he says, conclusively show that the angle of impulse depends on the relative velocity of each point of the sail and the wind, the angle growing larger as the ratio becomes greater. Smeaton's angles do not fulfil this condition. Wolff develops a theoretical formula for the best angle of weather, and from it calculates a table for different relative velocities of the blades (at a distance of one seventh of the total length from the centre of the shaft) and the wind, from which the following is condensed:

Ratio of the Speed of Blade at 1/7 of Radius to Velocity of Wind.	Distance from the axis of the wheel in sevenths of radius.						
	1	2	3	4	5	6	7
	Best angles of weather.						
0.10	42° 9′	39° 21′	36° 39′	34° 6′	31° 48′	29° 31′	27° 30′
0.15	40 44	36 39	32 53	29 31	26 34	24 0	21 48
0.20	39 21	34 6	29 31	25 40	22 30	19 54	17 46
0.25	37 59	36 43	26 34	22 30	19 20	16 51	14 52
0.30	36 39	29 31	24 0	19 54	16 51	14 32	12 44
0.35	35 21	27 30	21 48	17 46	14 52	12 44	11 6
0.40	34 6	25 40	19 54	16 0	13 17	11 19	9 50
0.45	32 53	24 0	18 16	14 32	11 59	10 10	8 48
0.50	31 48	22 30	16 51	13 17	10 54	9 18	7 58

The effective power of a windmill, as Smeaton ascertained by experiment, varies as s, the sectional area of the acting stream of wind; that is, for similar wheels, as the squares of the radii.

The value 0.75, assigned to the multiplier c in the formula $Q = cvs$, is founded on the fact, ascertained by Smeaton, that the effective power of a windmill with sails of the best form, and about 15¼ ft. radius, with a breeze of 13 ft. per second, is about 1 horse-power. In the computations founded on that fact, the mean angle of weather is made = 13°. The efficiency of this wheel, according to the formula and table given, is 0.29, at its best speed, when the tips of the sails move at a velocity of 2.6 times that of the wind.

Merivale (Notes and Formulæ for Mining Students), using Smeaton's coefficient of efficiency, 0.29, gives the following:

U = units of work in foot-lbs. per sec.;
W = weight, in pounds, of the cylinder of wind passing the sails each second, the diameter of the cylinder being equal to the diameter of the sails;
V = velocity of wind in feet per second;
H.P. = effective horse-power;

$$U = \frac{WV^2}{64}; \quad \text{H.P.} = \frac{0.29\,WV^2}{64 \times 550}.$$

A. R. Wolff, in an article in the *American Engineer*, gives the following (see also his treatise on Windmills):

Let c = velocity of wind in feet per second;
n = number of revolutions of the windmill per minute;
b_0, b_1, b_2, b_x be the breadth of the sail or blade at distances l_0, l_1, l_2, l_3, and l, respectively, from the axis of the shaft;
l_0 = distance from axis of shaft to beginning of sail or blade proper;
l = distance from axis of shaft to extremity of sail proper;
v_0, v_1, v_2, v_3, v_x = the velocity of the sail in feet per second at distances l_0, l_1, l_2, l, respectively, from the axis of the shaft;
a_0, a_1, a_2, a_3, a_x = the angles of impulse for maximum effect at distances l_0, l_1, l_2, l_3, l respectively from the axis of the shaft;
a = the angle of impulse when the sails or blocks are plane surfaces, so that there is but one angle to be considered;
N = number of sails or blades of windmill;
K = .93.
d = density of wind (weight of a cubic foot of air at average temperature and barometric pressure where mill is erected);
W = weight of wind-wheel in pounds;
f = coefficient of friction of shaft and bearings;
D = diameter of bearing of windmill in feet.

The effective horse-power of a windmill with plane sails will equal

$$\frac{(l - l_0)Kc^2dN}{550g} \times \text{mean of}\Big(v_0(\sin a - \frac{v_0}{c} \cos a)b_0 \cos a$$

$$v_x (\sin a - \frac{v_x}{c} \cos a)\, b_x \cos a\Big) - \frac{fW \times .05236nD}{550}.$$

The effective horse-power of a windmill of shape of sail for maximum effect equals

$$\frac{N(l - l_0)Kdc^3}{2200g} \times \text{mean of}\left(\frac{2 \sin^2 a_0 - 1}{\sin^3 a_0} b_0, \quad \frac{2 \sin^2 a_1 - 1}{\sin^3 a_1} b_1 \ldots\right.$$

$$\left.\ldots \frac{2 \sin^2 a_x - 1}{\sin_3 a_x} b_x\right) - \frac{fW \times .05236nD}{550}.$$

The mean value of quantities in brackets is to be found according to Simpson's rule. Dividing l into 7 parts, finding the angles and breadths corresponding to these divisions by substituting them in quantities within brackets will be found satisfactory. Comparison of these formulæ with the only fairly reliable experiments in windmills (Coulomb's) showed a close agreement of results.

Approximate formulæ of simpler form for windmills of present construction can be based upon the above, substituting actual average values for a, c, d, and e, but since improvement in the present angles is possible, it is better to give the formulæ in their general and accurate form.

Wolff gives the following table based on the practice of an American manufacturer. Since its preparation, he says, over 1500 windmills have been sold on its guaranty (1885), and in all cases the results obtained did not vary sufficiently from those presented to cause any complaint. The actual results obtained are in close agreement with those obtained by theoretical analysis of the impulse of wind upon windmill blades.

Capacity of the Windmill.

Designation of Mill.	Velocity of Wind, in miles per hour.	Revolutions of Wheel per minute.	Gallons of Water raised per Minute to an Elevation of—						Equivalent Actual Useful Horse-power developed.	Average No. of Hours per Day during which this Result will be obtained.
			25 feet.	50 feet.	75 feet.	100 feet.	150 feet.	200 feet.		
wheel 8½ ft.	16	70 to 75	6.162	3.016	......	......	......	......	0.04	8
10 "	16	60 to 65	19.179	9.563	6.638	4.750	......	......	0.12	8
12 "	16	55 to 60	33.941	17.952	11.851	8.485	5.680	......	0.21	8
14 "	16	50 to 55	45.139	22.569	15.304	11.246	7.807	4.998	0.28	8
16 "	16	45 to 50	64.600	31.654	19.542	16.150	9.771	8.075	0.41	8
18 "	16	40 to 45	97.682	52.165	32.513	24.421	17.485	12.211	0.61	8
20 "	16	35 to 40	124.950	63.750	40.800	31.248	19.284	15.938	0.78	8
25 "	16	30 to 35	212.381	106.964	71.604	49.725	37.349	26.741	1.34	8

These windmills are made in regular sizes, as high as sixty feet diameter of wheel; but the experience with the larger class of mills is too limited to enable the presentation of precise data as to their performance.

If the wind can be relied upon in exceptional localities to average a higher velocity for eight hours a day than that stated in the above table, the performance or horse-power of the mill will be increased, and can be obtained by multiplying the figures in the table by the ratio of the cube of the higher average velocity of wind to the cube of the velocity above recorded

He also gives the following table showing the economy of the windmill. All the items of expense, including both interest and repairs, are reduced to the hour by dividing the costs per annum by $365 \times 8 = 2920$; the interest, etc., for the twenty-four hours being charged to the eight hours of actual work. By multiplying the figures in the 5th column by 584, the first cost of the windmill, in dollars, is obtained.

Economy of the Windmill.

Designation of Mill.	Gallons of Water raised 25 ft. per hour.	Equivalent Actual Useful Horse-power developed.	Average Number of Hours per Day during which this Quantity will be raised.	Expense of Actual Useful Power Developed, in cents, per hour.					Expense per Horse-power, in cents, per hour.
				For Interest on First Cost (First Cost, including Cost of Windmill, Pump, and Tower, 5% per annum).	For Repairs and Depreciation (5% of First Cost per annum).	For Attendance.	For Oil.	Total.	
8½ ft. wheel	370	0.04	8	0.25	0.25	0.06	0.04	0.60	15.0
10 " "	1151	0.12	8	0.30	0.30	0.06	0.04	0.70	5.8
12 " "	2036	0.21	8	0.36	0.36	0.06	0.04	0.82	3.9
14 " "	2708	0.28	8	0.75	0.75	0.06	0.07	1.63	5.8
16 " "	3876	0.41	8	1.15	1.15	0.06	0.07	2.43	5.9
18 " "	5861	0.61	8	1.35	1.35	0.06	0.07	2.83	4.6
20 " "	7497	0.79	8	1.70	1.70	0.06	0.10	3.56	4.5
25 " "	12743	1.34	8	2.05	2.05	0.06	0.10	4.26	3.2

Lieut. I. N. Lewis (*Eng'g Mag.*, Dec. 1894) gives a table of results of experiments with wooden wheels, from which the following is taken:

Diameter of wheel, Feet.	Velocity of Wind, miles per hour.						
	8	10	12	16	20	25	30
	Actual Useful Horse-power developed.						
12	0	1/8	1/4	1/2	1	1 3/8	2
16	1/8	3/8	3/4	1 1/2	2 1/4	3 1/4	4
20	3/4	1 1/4	2	3	4	5 1/2	7
25	1 1/4	1 3/4	3	4 1/2	6	8	10
30	2	3	4	5 1/2	7	9	12

The wheels were tested by driving a differentially wound dynamo. The "useful horse-power" was measured by a voltmeter and ammeter, allowing 500 watts per horse-power. Details of the experiments, including the means used for obtaining the velocity of the wind, are not given. The results are so far in excess of the capacity claimed by responsible manufacturers that they should not be given credence until established by further experiments.

A recent article on windmills in the *Iron Age* contains the following: According to observations of the United States Signal Service, the average velocity of the wind within the range of its record is 9 miles per hour for the year along the North Atlantic border and Northwestern States, 10 miles on the plains of the West, and 6 miles in the Gulf States.

The horse-powers of windmills of the best construction are proportional to the squares of their diameters and inversely as their velocities; for example, a 10-ft. mill in a 16-mile breeze will develop 0.15 horse-power at 65 revolutions per minute; and with the same breeze

A 20-ft. mill, 40 revolutions, 1 horse-power.
A 25-ft. mill, 35 revolutions, 1¾ horse-power.
A 30-ft. mill, 28 revolutions, 3½ horse-power.
A 40-ft. mill, 22 revolutions, 7½ horse-power.
A 50-ft. mill, 18 revolutions, 12 horse-power.

The increase in power from increase in velocity of the wind is equal to the square of its proportional velocity; as for example, the 25-ft. mill rated above for a 16-mile wind will, with a 32-mile wind, have its horse-power increased to 4 × 1¾ = 7 horse-power, a 40-ft. mill in a 32-mile wind will run up to 30 horse-power, and a 50-ft. mill to 48 horse-power, with a small deduction for increased friction of air on the wheel and the machinery.

The modern mill of medium and large size will run and produce work in a 4-mile breeze, becoming very efficient in an 8 to 16-mile breeze, and increase its power with safety to the running-gear up to a gale of 45 miles per hour.

Prof. Thurston, in an article on modern uses of the windmill, *Engineering Magazine*, Feb. 1893, says: The best mills cost from about $600 for the 10-ft. wheel of ⅙ horse-power to $1200 for the 25-ft. wheel of 1⅙ horse-power or less. In the estimates a working-day of 8 hours is assumed; but the machine, when used for pumping, its most common application, may actually do its work 24 hours a day for days, weeks, and even months together, whenever the wind is "stiff" enough to turn it. It costs, for work done in situations in which its irregularity of action is no objection, only one half or one third as much as steam, hot-air, and gas engines of similar power. At Faversham, it is said, a 15-horse-power mill raises 2,000,000 gallons a month from a depth of 100 ft., saving 10 tons of coal a month, which would otherwise be expended in doing the work by steam.

Electric storage and lighting from the power of a windmill has been tested on a large scale for several years by Charles F. Brush, at Cleveland, Ohio. In 1887 he erected on the grounds of his dwelling a windmill 56 ft. in diameter, that operates with ordinary wind a dynamo at 500 revolutions per minute, with an output of 12,000 watts—16 electric horse-power—charging a storage system that gives a constant lighting capacity of 100 16 to 20 candle-power lamps. The current from the dynamo is automatically regulated to commence charging at 330 revolutions and 70 volts, and cutting the circuit at 75 volts. Thus, by its 24 hours' work, the storage system of 408 cells in 12 parallel series, each cell having a capacity of 100 ampère hours, is kept in constant readiness for all the requirements of the establishment, it being fitted up with 350 incandescent lamps, about 100 being in use each evening. The plant runs at a mere nominal expense for oil, repairs, and attention. (For a fuller description of this plant, and of a more recent one at Marblehead Neck, Mass., see Lieut. Lewis's paper in *Engineering Magazine*, Dec. 1894, p. 475.)

COMPRESSED AIR.

Heating of Air by Compression.—Kimball, in his treatise on Physical Properties of Gases, says: When air is compressed, all the work which is done in the compression is converted into heat, and shows itself in the rise in temperature of the compressed gas. In practice many devices are employed to carry off the heat as fast as it is developed, and keep the temperature down. But it is not possible in any way to totally remove this difficulty. But, it may be objected, if all the work done in compression is converted into heat, and if this heat is got rid of as soon as possible, then the work may be virtually thrown away, and the compressed air can have no more energy than it had before compression. It is true that the compressed gas has no more energy than the gas had before compression, if its temperature is no higher, but the advantage of the compression lies in bringing its energy into more available form.

The total energy of the compressed and uncompressed gas is the same at the same temperature, but the available energy is much greater in the former.

When the compressed air is used in driving a rock-drill, or any other piece of machinery, it gives up energy equal in amount to the work it does, and its temperature is accordingly greatly reduced.

Causes of Loss of Energy in Use of Compressed Air. (Zahner, on Transmission of Power by Compressed Air.)—1. The compression of air always develops heat, and as the compressed air always cools down to the temperature of the surrounding atmosphere before it is used, the mechanical equivalent of this dissipated heat is work lost.

2. The heat of compression increases the volume of the air, and hence it is necessary to carry the air to a higher pressure in the compressor in order that we may finally have a given volume of air at a given pressure, and at the temperature of the surrounding atmosphere. The work spent in effecting this excess of pressure is work lost.

3. Friction of the air in the pipes, leakage, dead spaces, the resistance offered by the valves, insufficiency of valve-area, inferior workmanship, and slovenly attendance, are all more or less serious causes of loss of power.

The first cause of loss of work, namely, the heat developed by compression, is entirely unavoidable. The whole of the mechanical energy which the compressor-piston spends upon the air is converted into heat. This heat is dissipated by conduction and radiation, and its mechanical equivalent is work lost. The compressed air, having again reached thermal equilibrium with the surrounding atmosphere, expands and does work in virtue of its intrinsic energy.

The intrinsic energy of a fluid is the energy which it is capable of exerting against a piston in changing from a given state as to temperature and volume to a total privation of heat and indefinite expansion.

Adiabatic and Isothermal Compression.—Air may be compressed either *adiabatically*, in which all the heat resulting from compression is retained in the air compressed, or *isothermally*, in which the heat is removed as rapidly as produced, by means of some form of refrigerator.

Volumes, Mean Pressures per Stroke, Temperatures, etc., in the Operation of Air-compression from 1 Atmosphere and 60° Fahr. (F. Richards, *Am. Mach.*, March 30, 1893.)

Gauge-pressure.	Atmospheres.	Volume with Air at Constant Temp.	Volume with Air not cooled.	Mean Pressure per Stroke; Air Constant Temp.	Mean Pressure per Stroke; Air not cooled.	Temp. of Air; not cooled.	Gauge-pressure.	Atmospheres.	Volume with Air at Constant Temp.	Volume with Air not cooled.	Mean Pressure per Stroke; Air Constant Temp.	Mean Pressure per Stroke; Air not cooled.	Temp. of Air; not cooled.
1	2	3	4	5	6	7	1	2	3	4	5	6	7
0	1	1	1	0	0	60°	80	6.442	.1552	.266	27.38	36.64	432
1	1.068	.9363	.95	.96	.975	71	85	6.782	.1474	.2566	28.16	37.94	447
2	1.136	.8803	.91	1.87	1.91	80.4	90	7.122	.1404	.248	28.89	39.18	459
3	1.204	.8305	.876	2.72	2.8	88.9	95	7.462	.134	.24	29.57	40.4	472
4	1.272	.7861	.84	3.53	3.67	98	100	7.802	.1281	.2324	30.21	41.6	485
5	1.34	.7462	.81	4.3	4.5	106	105	8.142	.1228	.2254	30.81	42.78	496
10	1.68	.5952	.69	7.62	8.27	145	110	8.483	.1178	.2189	31.39	43.91	507
15	2.02	.495	.606	10.83	11.51	178	115	8.823	.1133	.2129	31.98	44.98	518
20	2.36	.4237	.543	12.62	14.4	207	120	9.163	.1091	.2073	32.54	46.04	529
25	2.7	.3703	.494	14.59	17.01	234	125	9.503	.1052	.2020	33.07	47.06	540
30	3.04	.3289	.4538	16.34	19.4	252	130	9.843	.1015	.1969	33.57	48.1	550
35	3.381	.2957	.42	17.92	21.6	281	135	10.183	.0981	.1922	34.05	49.1	560
40	3.721	.2687	.393	19.32	23.66	302	140	10.523	.095	.1878	34.57	50.02	570
45	4.061	.2462	.37	20.57	25.59	321	145	10.864	.0921	.1837	35.09	51.	580
50	4.401	.2272	.35	21.69	27.39	339	150	11.204	.0892	.1796	35.48	51 89	589
55	4.741	.2109	.331	22.76	29.11	357	160	11.88	.0841	.1722	36.29	53.65	607
60	5.081	.1968	.3144	23.78	30.75	375	170	12.56	.0796	.1657	37.2	55.39	624
65	5.422	.1844	.301	24.75	32.32	389	180	13.24	.0755	.1595	37.96	57.01	640
70	5.762	.1735	.288	25.67	33.83	405	190	13.93	.0718	.154	38.68	58.57	657
75	6.102	.1639	.276	26.55	35.27	420	200	14.61	.0685	.149	39.42	60.14	672

Column 3 gives the volume of air after compression to the given pressure and after it is cooled to its initial temperature. After compression air loses its heat very rapidly, and this column may be taken to represent the volume of air after compression available for the purpose for which the air has been compressed.

Column 4 gives the volume of air more nearly as the compressor has to deal with it. In any compressor the air will lose some of its heat during compression. The slower the compressor runs the cooler the air and the smaller the volume.

Column 5 gives the mean effective resistance to be overcome by the air-cylinder piston in the stroke of compression, supposing the air to remain constantly at its initial temperature. Of course it will not so remain, but this column is the ideal to be kept in view in economical air-compression.

Column 6 gives the mean effective resistance to be overcome by the piston, supposing that there is no cooling of the air. The actual mean effective pressure will be somewhat less than as given in this column; but for computing the actual power required for operating air-compressor cylinders the figures in this column may be taken and a certain percentage added—say 10 per cent—and the result will represent very closely the power required by the compressor.

The mean pressures given being for compression from one atmosphere upward, they will not be correct for computations in compound compression or for any other initial pressure.

Loss Due to Excess of Pressure caused by Heating in the Compression-cylinder.—If the air during compression were kept at a constant temperature, the compression-curve of an indicator-diagram taken from the cylinder would be an isothermal curve, and would follow the law of Boyle and Marriotte, $pv =$ a constant, or $p_1v_1 = p_0v_0$, or $p_1 = p_0\frac{v_0}{v_1}$, p_0 and v_0 being the pressure and volume at the beginning of compression, and p_1v_1 the pressure and volume at the end, or at any intermediate point. But as the air is heated during compression the pressure increases faster than the volume decreases, causing the work required for any given pressure to be increased. If none of the heat were abstracted by radiation or by injection of water, the curve of the diagram would be an adiabatic curve, with the equation $p_1 = p_0\left(\frac{v_0}{v_1}\right)^{1.406}$. Cooling the air during compression, or compressing it in two cylinders, called compounding, and cooling the air as it passes from one cylinder to the other, reduces the exponent of this equation, and reduces the quantity of work necessary to effect a given compression. F. T. Gause (*Am. Mach.*, Oct. 20, 1892), describing the operations of the Popp air-compressors in Paris, says: The greatest saving realized in compressing in a single cylinder was 33 per cent of that theoretically possible. In cards taken from the 2000 H.P. compound compressor at Quai De La Gare, Paris, the saving realized is 85 per cent of the theoretical amount. Of this amount only 8 per cent is due to cooling during compression, so that the increase of economy in the compound compressor is mainly due to cooling the air between the two stages of compression. A compression-curve with exponent 1.25 is the best result that was obtained for compression in a single cylinder and cooling with a very fine spray. The curve with exponent 1.15 is that which must be realized in a single cylinder to equal the present economy of the compound compressor at Quai De La Gare.

Horse-power required to compress and deliver one cubic foot of Free Air per minute to a given pressure with no cooling of the air during the compression; also the horse-power required, supposing the air to be maintained at constant temperature during the compresion.

Gauge-pressure.	Air not cooled.	Air constant temperature.
5	.0196	.0188
10	.0361	.0333
20	.0628	.0551
30	.0846	.0713
40	.1032	.0843
50	.1195	.0946
60	.1342	.1036
70	.1476	.1120
80	.1599	.1195
90	.1710	.1261
100	.1815	.1318

Horse-power required to compress and deliver one cubic foot of Compressed Air per minute at a given pressure with no cooling of the air during the compression; also the horse-power required, supposing the air to be maintained at constant temperature during the compression

Gauge-pressure.	Air not cooled.	Air constant temperature.
5	.0263	.0251
10	.0606	.0559
20	.1483	.1300
30	.2578	.2168
40	.3842	.3138
50	.5261	.4166
60	.6818	.5266
70	.8508	.6456
80	1.0302	.7700
90	1.2177	.8979
100	1.4171	1.0291

The horse-power given above is the theoretical power, no allowance being made for friction of the compressor or other losses, which may amount to 10 per cent or more.

Formulæ for Adiabatic Compression or Expansion of Air (or other sensibly perfect gas).

Let air at an absolute temperature T_1, absolute pressure p_1, and volume v_1 be compressed to an absolute pressure p_2 and corresponding volume v_2 and absolute temperature T_2; or let compressed air of an initial pressure, volume, and temperature p_2, v_2, and T_2 be expanded to p_1, v_1, and T_1, there being no transmission of heat from or into the air during the operation. Then the following equations express the relations between pressure, volume, and temperature (see works on Thermodynamics):

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{0.71}; \qquad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{1.41}, \qquad \frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{2.46};$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{0.41}; \qquad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{0.29}; \qquad \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{3.46}.$$

The exponents are derived from the ratio $cp \div cv = k$ of the specific heats of air at constant pressure and constant volume. Taking $k = 1.406$, $1 \div k = 0.711$; $k - 1 = 0.406$; $1 \div (k - 1) = 2.463$; $k \div (k - 1) = 3.463$; $(k - 1) \div k = 0.289$.

Work of Adiabatic Compression of Air.—If air is compressed in a cylinder without clearance from a volume v_1 and pressure p_1 to a smaller volume v_2 and higher pressure p_2, work equal to p_1v_1 is done by the external air on the piston while the air is drawn into the cylinder. Work is then done by the piston on the air, first, in compressing it to the pressure p_2 and volume v_2, and then in expelling the volume v_2 from the cylinder against the pressure p_2. If the compression is adiabatic, $p_1v_1^k = p_2v_2^k =$ constant. $k = 1.41$.

The work of compression of 1 pound of air is

$$\frac{p_1v_1}{k-1}\left\{\left(\frac{v_1}{v_2}\right)^{k-1} - 1\right\} \qquad = \frac{p_1v_1}{k-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right\}$$

or

$$2.463p_1v_1\left\{\left(\frac{v_1}{v_2}\right)^{0.41} - 1\right\} \qquad = 2.463p_1v_1\left\{\left(\frac{p_2}{p_1}\right)^{0.29} - 1\right\}.$$

The work of expulsion is $p_2v_2 = p_1v_1\left(\frac{p_2}{p_1}\right)^{0.29}$.

The total work is the sum of the work of compression and expulsion less the work done on the piston during admission, and it equals

$$p_1v_1\left\{\frac{k}{k-1}\right\}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right\} \qquad = 3.463\ p_1v_1\left\{\left(\frac{p_2}{p_1}\right)^{0.29} - 1\right\}.$$

The mean effective pressure during the stroke is

$$p_1\frac{k}{k-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right\} \qquad = 3.463\ p_1\left\{\left(\frac{p_2}{p_1}\right)^{0.29} - 1\right\}.$$

p_1 and p_2 are absolute pressures above a vacuum in atmospheres or in pounds per square inch or per square foot.

EXAMPLE.—Required the work done in compressing 1 cubic foot of air per second from 1 to 6 atmospheres, including the work of expulsion from the cylinder.

$p_2 \div p_1 = 6$; $6^{0.29} - 1 = 0.681$; $3.463 \times 0.681 = 2.358$ atmospheres, $\times 14.7 = 34.66$ lbs. per sq. in. mean effective pressure, $\times 144 = 4991$ lbs. per sq. ft., $\times 1$ ft. stroke $= 4991$ ft.-lbs., $\div 550$ ft.-lbs. per second $= 9.08$ H.P.

If R = ratio of pressures = $p_2 \div p_1$, and if $v_1 = 1$ cubic foot, the work done in compressing 1 cubic foot from p_1 to p_2 is in foot-pounds

$$3.463p_1(R^{0.29} - 1),$$

p_1 being taken in lbs. per sq. ft. For compression at the sea-level p_1 may be taken at 14 lbs. per sq. in. = 2016 lbs. per sq. ft., as there is some loss of pressure due to friction of valves and passages.

Indicator-cards from compressors in good condition and under working-speeds usually follow the adiabatic line closely. A low curve indicates piston leakage. Such cooling as there may be from the cylinder-jacket and the re-expansion of the air in clearance-spaces tends to reduce the mean effective pressure, while the "camel-backs" in the expulsion-line, due to resistance to opening of the discharge-valve, tend to increase it.

Work of one stroke of a compressor, with adiabatic compression, in foot-pounds,

$$W = 3.463P_1V_1(R^{0.29} - 1),$$

in which P_1 = initial absolute pressure in lbs. per sq. ft. and V_1 = volume traversed by piston in cubic feet.

The work done during adiabatic compression (or expansion) of 1 pound of air from a volume v_1 and pressure p_1 to another volume v_2 and pressure p_2 is equal to the mechanical equivalent of the heating (or cooling). If t_1 is the higher and t_2 the lower temperature, Fahr., the work done is $c_vJ(t_1 - t_2)$ foot-pounds, c_v being the specific heat of air at constant volume = 0.1689 and $J = 778$, $c_vJ = 131.4$.

The work during compression also equals

$$\frac{c_vJ}{Ra}p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{0.29} - 1\right] = 2.463\,p_1v_1\left\{\left(\frac{p_2}{p_1}\right)^{0.29} - 1\right\},$$

Ra being the value of $pv \div$ absolute temperature for 1 pound of air = 53.37.

The work during expansion is

$$2.463\,p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{0.29}\right] = 2.463\,p_2v_2\left[\left(\frac{p_1}{p_2}\right)^{0.29} - 1\right],$$

in which p_1v_1 are the initial and p_2v_2 the final pressures and volumes.

Compressed-air Engines, Adiabatic Expansion. — Let the initial pressure and volume taken into the cylinder be p_1 lbs. per sq. ft. and v_1 cubic feet; let expansion take place to p_2 and v_2 according to the adiabatic law $p_1v_1^{1.41} = p_2v_2^{1.41}$; then at the end of the stroke let the pressure drop to the back-pressure p_3, at which the air is exhausted. Assuming no clearance, the work done by one pound of air during admission, measured above vacuum, is p_1v_1, the work during expansion is $2.463\ p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{0.29}\right]$, and the negative or back pressure work is $-p_3v_2$. The total work is $p_1v_1 + 2.463p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{0.29}\right] - p_3v_2$, and the mean effective pressure is the total work divided by v_2.

If the air is expanded down to the back-pressure p_3 the total work is

$$3.463p_1v_1\left\{1 - \left(\frac{p_3}{p_1}\right)^{0.29}\right\}$$

or, in terms of the final pressure and volume,

$$3.463p_3v_2\left\{\left(\frac{p_1}{p_3}\right)^{0.29} - 1\right\},$$

and the mean effective pressure is

$$3.463p_3\left\{\left(\frac{p_1}{p_3}\right)^{0.29} - 1\right\}.$$

The actual work is reduced by clearance. When this is considered, the product of the initial pressure p_1 by the clearance volume is to be subtracted from the total work calculated from the initial volume v_1 including clearance. (See p. 744, under "Steam-engine.")

Mean Effective Pressures of Air Compressed Adiabatically.
(F. A. Halsey, *Am. Mach.*, Mar. 10, 1898.)

R	$R^{0.29}$	*MEP* from 14 lbs. Initial.	R	$R^{0.29}$	*MEP* from 14 lbs. Initial.
1.25	1.067	3.24	4.75	1.570	27.5
1.50	1.125	6.04	5.	1.594	28.7
1.75	1.176	8.51	5.25	1.617	29.8
2.	1.223	10.8	5.5	1.639	30.8
2.25	1.265	12.8	5.75	1.660	31.8
2.5	1.304	14.7	6.	1.681	32.8
2.75	1.341	16.4	6.25	1.701	33.8
3.	1.375	18.1	6.5	1.720	34.7
3.25	1.407	19.6	6.75	1.739	35.6
3.5	1.438	21.1	7.	1.757	36.5
3.75	1.467	22.5	7.25	1.775	37.4
4.	1.495	23.9	7.5	1.793	38.3
4.25	1.521	25.2	8.	1.827	39.9
4.5	1.546	26.4			

R = final ÷ initial absolute pressure.
MEP = mean effective pressure, lbs. per sq. in., based on 14 lbs. initial.

Compound Compression, with Air Cooled between the Two Cylinders. (*Am. Mach.*, March 10 and 31, 1898.)—Work in low-pressure cylinder = W_1, in high-pressure cylinder W_2. Total work

$$W_1 + W_2 = 3.46 P_1 V_1 [r_1^{.29} + R^{.29} r_1^{-.29} - 2].$$

r_1 = ratio of pressures in l. p. cyl., r_2 = ratio in h. p. cyl., $R = r_1 r_2$. When $r_1 = r_2 = \sqrt{R}$, the sum $W_1 + W_2$ is a minimum. Hence for a given total ratio of pressures, R, the work of compression will be least when the ratios of the pressures in each of the two cylinders are equal.

The equation may be simplified, when $r_1 = \sqrt{R}$, to the following:

$$W_1 + W_2 = 6.92 P_1 V_1 [R^{0.145} - 1].$$

Dividing by V_1 gives the mean effective pressure reduced to the low-pressure cylinder $MEP = 6.92 P_1 [R^{0.145} - 1]$.

In the above equation the compression in each cylinder is supposed to be adiabatic, but the intercooler is supposed to reduce the temperature of the air to that at which compression began.

Mean Effective Pressures of Air Compressed in Two Stages, assuming the Intercooler to Reduce the Temperature to That at which Compression Began. (F. A. Halsey, *Am. Mach.*, Mar. 31, 1898.)

R	$R^{0.145}$	*MEP* from 14 lbs. Initial.	Ultimate Saving by Compounding, %	R	$R^{0.145}$	*MEP* from 14 lbs. Initial.	Ultimate Saving by Compounding, %
5.0	1.263	25.4	11.5	9.0	1.375	36.3	
5.5	1.280	27.0	12.3	9.5	1.386	37.3	
6.0	1.296	28.6	12.8	10	1.396	38.3	
6.5	1.312	30.1	13.2	11	1.416	40.2	
7.0	1.326	31.5	13.7	12	1.434	41.9	
7.5	1.336	32.8	14.3	13	1.451	43.5	
8.0	1.352	34.0	14.8	14	1.466	45.0	
8.5	1.364	35.2		15	1.481	46.4	

R = final ÷ initial absolute pressure.
MEP = mean effective pressure lbs. per sq. in. based on 14 lbs. absolute initial pressure reduced to the low-pressure cylinder.

To Find the Index of the Curve of an Air-diagram.—If $P_1 V_1$ be pressure and volume at one point on the curve, and PV the pressure and volume at another point, then $\frac{P}{P_1} = \left(\frac{V_1}{V}\right)^x$, in which x is the index to be found. Let $P \div P_1 = R$, and $V_1 \div V = r$; then $R = r^x$ log $R = x$ log r, whence x = log R ÷ log r.

Table for Adiabatic Compression or Expansion of Air.

(Proc. Inst. M.E., Jan. 1881, p. 123.)

Absolute Pressure.		Absolute Temperature.		Volume.	
Ratio of Greater to Less. (Expansion.)	Ratio of Less to Greater. (Compression.)	Ratio of Greater to Less. (Expansion.)	Ratio of Less to Greater. (Compression.)	Ratio of Greater to Less. (Compression.)	Ratio of Less to Greater. (Expansion.)
1.2	.833	1.054	.948	1.138	.879
1.4	.714	1.102	.907	1.270	.788
1.6	.625	1.146	.873	1.396	.716
1.8	.556	1.186	.843	1.518	.659
2.0	.500	1.222	.818	1.636	.611
2.2	.454	1.257	.796	1.750	.571
2.4	.417	1.289	.776	1.862	.537
2.6	.385	1.319	.758	1.971	.507
2.8	.357	1.348	.742	2.077	.481
3.0	.333	1.375	.727	2.182	.458
3.2	.312	1.401	.714	2.284	.438
3.4	.294	1.426	.701	2.384	.419
3.6	.278	1.450	.690	2.483	.403
3.8	.263	1.473	.679	2.580	.388
4.0	.250	1.495	.669	2.676	.374
4.2	.238	1.516	.660	2.770	.361
4.4	.227	1.537	.651	2.863	.349
4.6	.217	1.557	.642	2.955	.338
4.8	.208	1.576	.635	3.046	.328
5.0	.200	1.595	.627	3.135	.319
6.0	.167	1.681	.595	3.569	.280
7.0	.143	1.758	.569	3.981	.251
8.0	.125	1.828	.547	4.377	.228
9.0	.111	1.891	.529	4.759	.210
10.0	.100	1.950	.513	5.129	.195

Mean Effective Pressures for the Compression Part only of the Stroke when compressing and delivering Air from one Atmosphere to given Gauge-pressure in a Single Cylinder. (F. Richards, *Am. Mach.*, Dec. 14, 1893.)

Gauge-pressure.	Adiabatic Compression	Isothermal Compression	Gauge-pressure.	Adiabatic Compression.	Isothermal Compression.
1	.44	.43	45	13.95	12.62
2	.96	.95	50	15.05	13.48
3	1.41	1.4	55	15.98	14.3
4	1.86	1.84	60	16.89	15.05
5	2.26	2.22	65	17.88	15.76
10	4.26	4.14	70	18.74	16.43
15	5.99	5.77	75	19.54	17.09
20	7.58	7.2	80	20.5	17.7
25	9.05	8.49	85	21.22	18.3
30	10.39	9.66	90	22.	18.87
35	11.59	10.72	95	22.77	19.4
40	12.8	11.7	100	23.43	19.92

The mean effective pressure for compression only is always lower than the mean effective pressure for the whole work

Mean and Terminal Pressures of Compressed Air used Expansively for Gauge-pressures from 60 to 100 lbs.

(Frank Richards, *Am. Mach.*, April 13, 1893.)

Initial Pressure.	60.		70.		80.		90.		100.	
Point of Cut-off.	Mean Air-pressure.	Terminal Air-pressure.	Mean Air-pressure.	Terminal Air-pressure.	Mean Air-pressure.	Terminal Air-pressure.	Mean Air-pressure.	Terminal Air-pressure.	Mean Air-pressure.	Terminal Air-pressure.
.25	23.6	*10.65*	28.74	*12.07*	33.89	*13.49*	39.04	*14.91*	44.19	1.33
.30	28.9	*13.77*	34.75	.6	40.61	2.44	46.46	4.27	53.32	6.11
1/3	32.13	.96	38.41	3.09	44.69	5.22	50.98	7.35	57.26	9.48
.35	33.66	2.33	40.15	4.38	46.64	6.66	53.13	8.95	59.62	11.23
3/8	35.85	3.85	42.68	6.36	49.41	7.88	56.2	11.39	62.98	13.89
.40	37.93	5.64	44.99	8.39	52.05	11.14	59.11	13.88	66.16	16.64
.45	41.75	10.71	49.31	12.61	56.9	15.86	64.45	19.11	72.02	22.36
.50	45.14	13.26	53.16	17.	61.18	20.81	69.19	24.56	77.21	28.33
.60	50.75	21.53	59.51	26.4	68.28	31.27	77.05	36.14	85.82	41.01
5/8	51.92	23.69	60.84	28.85	69.76	34.01	78.69	39.16	87.61	44.32
2/3	53.67	27.94	62.83	33.08	71.99	38.68	81.14	44.33	90.32	49.97
.70	54.93	30.89	64.25	36.44	73.57	42.49	82.9	48.54	92.22	54.59
.75	56.52	35.01	66.05	41.68	75.59	48.35	85.12	55.02	94.66	61.69
.80	57.79	39.78	67.5	47.08	77.2	54.38	86.91	61.69	96.61	68.99
7/8	59.15	47.14	69.03	55.43	78.92	63.81	88.81	72.	98.7	80.28
.90	59.46	49.65	69.38	58.27	79.31	66.89	89.24	75.52	99.17	87.82

The pressures in the table are all gauge-pressures except those in italics, which are absolute pressures (above a vacuum).

Mountain or High-altitude Compressors.

(Norwalk Iron Works Co.)

Diameter Air-cylinder.	Length of Stroke.	Diameter of Compressing Cylinder.	Diameter of Steam-cylinder.	Revolutions per minute.	At Sea-level. Capacity, cubic feet.	At Sea-level. Horse-power.	At 2000 feet. Capacity.	At 2000 feet. Horse-power.	At 6000 feet. Capacity.	At 6000 feet. Horse-power.	At 10,000 feet. Capacity.	At 10,000 feet. Horse-power.
12	12	7	10	190	298	35	280	34	244	32	214	30
16	16	9½	14	150	558	70	524	68	462	64	405	60
20	20	13½	18	120	872	110	819	107	722	100	634	94
22	24	13½	20	110	1160	145	1090	140	960	132	843	124
26	30	17½	24	90	1659	215	1560	207	1373	195	1200	184

As the capacity decreases in a greater ratio than the power necessary to compress, it follows that operations at a high altitude are more expensive than at sea-level. At 10,000 feet this extra expense amounts to over 20 per cent.

Compressors at High Altitudes. (Ingersoll-Sergeant Drill Co.)

Alt. above sea-level, ft...	0	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000
Barometer, in. mercury.	30.0	28.9	27.8	26.8	25.8	24.8	23.9	23.0	22.1	21.3	20.5
" lbs. per sq. in.	14.7	14.2	13.7	13.2	12.7	12.2	11.7	11.3	10.9	10.5	10.1
Air delivered, %..........	100	97	93	90	87	84	81	78	76	73	70
Loss of capacity, %......	0	3	7	10	13	16	19	22	24	27	30
Decreased power required, %.............	0	1.8	3.5	5.2	6.9	8.5	10.1	11.6	13.1	14.6	16.1

Air-compressors. Rand Drill Co.

RAND-CORLISS, CLASS "BB-3" (COMPOUND STEAM, CONDENSING; COMPOUND AIR).

FOR STEAM-PRESSURE OF 125 LBS. AND TERMINAL AIR-PRESSURES OF 80 AND 100 LBS.

Capacity in Cu. Ft. of Free Air per Minute.	Cylinder Diameters, Ins.				Stroke, Ins.	Revs. per Min.	Indicated Horse-power.*
	Steam.		Air.				
	h. p.	l. p.	h. p.	l. p.			
670	10	18	10½	17	30	85	102
1196	12	22	13	21	36	88	182
1562	14	26	15	24	36	88	238
1650	14	26	15	24	42	75	252
1920	16	30	17½	28	36	75	298
2242	16	30	17½	28	42	75	342
2395	16	30	17½	28	48	70	365
2520	18	34	20	32	36	75	384
2897	18	34	20	32	42	75	442
3128	18	34	20	32	48	70	475
3960	20	38	22½	36	48	70	604
4100	22	40	24	38	48	65	635
4530	22	42	25	40	48	65	690
5000	24	44	26½	42	48	65	763
6000	26	48	29	46	48	65	915
6820	28	52	30	48	48	65	1040

CLASS "E" (STRAIGHT-LINE, BELT-DRIVEN).

FOR TERMINAL PRESSURES OF 80 AND 100 LBS. PER SQ IN.

Capacity in Cu. Ft. of Free Air per Minute.	Air-Cylinder, Inches.		Revs. per Min.	Indicated H.P. Air-pressure 80 lbs.
	Diam.	Stroke.		
97	8	12	140	17
165	10	14	130	29
251	12	16	120	45
392	14	22	100	69
527	16	24	95	94
633	17½	24	95	112

In the first four sizes (Class "BB-3") the air-cylinders have poppet inlet and outlet valves; in the next six the low-pressure air-cylinders have mechanical inlet-valves and poppet outlet-valves; and in the last six the low-pressure air-cylinders have Corliss inlet-valves and poppet outlet-valves. All high-pressure air-cylinders have poppet inlet and outlet valves.

* Terminal air-pressure at 80 pounds.

CLASS "B-2" (DUPLEX STEAM, NON-CONDENSING, COMPOUND AIR).

FOR STEAM- AND TERMINAL AIR-PRESSURES OF 80 AND 100 LBS.

Capacity in Cu. Ft. of Free Air per Minute.	Cylinder Diameters, Inches.			Stroke, Ins.	Revs. per Min.	Ind. H.P.—Steam and Air Press. at 80 lbs.
	Duplex Steam-cyls.	Air-cyls.				
		h. p.	l. p.			
220	8	7½	12	12	140	35
300	9	9	14	12	140	47
393	10	9½	15	16	120	62
565	12	11	18	16	120	89
770	14	13	21	16	120	121
882	14	13	21	22	100	139
1152	16	15	24	22	100	182
1812	18	17½	28	30	85	285
2085	20	19	30	30	85	328
2356	20	19	30	48	60	370
2848	22	21	33	48	60	446

CLASS "C" (STRAIGHT-LINE, STEAM-DRIVEN).

FOR STEAM- AND TERMINAL AIR-PRESSURES OF 100 LBS. PER SQ IN.

Capacity in Cu. Ft. of Free Air per Minute.	Cyl. Diam., Ins.		Stroke, Ins.	Revs. per Min.	Indicated Horse-power.
	Steam.	Air.			
97	8	8	12	140	20
165	10	10	14	130	33
251	12	12	16	120	52
392	14	14	22	100	82
527	16	16	24	95	110
671	18	18	24	95	140
950	20	20	30	87	200
1335	24	24	30	85	280

All air-cylinders have poppet inlet and outlet valves.

The first six sizes (Class "B-2") have both air-cylinders fitted with poppet-valves (inlet and discharge). The last four have low-pressure air-cylinders fitted with mechanical inlet-valve; high-pressure air-cylinders fitted with poppet inlet and discharge valves.

(The Ingersoll-Sergeant Drill Co., New York City.)

Class and Type.	Diam. of Cyl. Steam. Low.	Steam. High.	Air. Low.	Air. High.	Stroke.	Revs. per min.	Cap'y, Free Air. Cu. ft. per min.	Working Air-pressure.	Space Occupied. Length.	Space Occupied. Width.	Horse-power.
A.* Straight-line, Steam-driven.	10	....	10¼	....	12	160	177	50–100	10′ 2″	3′ 0″	25–35
	12	....	12¼	....	14	155	285	50–100	12 6	3 9	40–56
	14	..	14¼	....	18	120	382	50–100	15 3	4 3	50–76
	16	....	16¼	....	18	120	498	50–100	15 3	4 3	66–100
	18	....	18¼	....	24	94	657	50–100	19 1	5 3	86–131
	20	..	20¼	..	24	94	809	50–100	19 1	5 3	113–160
	22	....	22¼	..	24	94	960	50–100	19 1	5 3	126–192
	24	....	24¼	....	30	80	1225	50–100	22 0	6 0	160–245
B. Straight-line, belt-driven. Same as *A* in sizes up to 16 × 16¼ × 18 ins.											
C.† Duplex Corliss steam, Duplex air.	10½	....	11¼	....	30	90	576	100	31′ 0″	10′ 6″	115
	16	....	16¼	....	36	82	1346	100	36 6	12 6	274
	20	...	20¼	...	42	75	2239	100	41 0	13 6	434
	24	..	24¼	...	42	75	3208	100	43 0	14 6	646
	30	....	30¼	....	48	65	4982	100	41 0	16 6	1011
	32	....	32¼	....	60	62	6717	100	60 0	19 6	1375
C_2. Compound Corliss steam, Compound air.‡	10½	18	16¼	10¼	30	90	615	100	39 6	14 0	97
	14	26	22¼	13¼	36	85	1306	100	43 0	11 6	245
	16	30	24¼	15¼	42	78	1668	100	49 6	15 6	284
	18	34	28¼	17¼	48	75	2137	100	55 6	15 6	367
	22	40	34¼	20¼	48	72	3515	100	56 6	18 6	604
	24	44	36¼	22¼	48	70	3850	100	58 0	19 6	661
F.§ Small straight-line.	6	....	6	....	6	150	28	50–80	5 8	22	4–5¼
	8	....	8	....	8	150	69	50–80	6 8	25	9¾–13
	10	...	10	....	10	150	134	50–80	7 10	30	18¾–25
	12	..	12	....	12	150	237	50–80	8 6	30	33¼–44
	12	....	16¼	....	12	150	415	15–40	10 10	35	24¾–50
E. Belt-driven. Same as *F* in sizes up to 14¼ diam. by 10 ins. stroke.											
G. Steam-actuated, duplex or half duplex.	....	10	....	10¼	12	160	854	100	14′ 6″	7′ 0″	75
	....	12	..	12¼	14	155	570	100	16 6	9 0	121
	....	14	....	14¼	18	120	764	100	20 0	10 0	163
	....	16	....	16¼	18	120	996	100	20 0	10 0	212
	...	18	....	18¼	24	94	1314	100	25 6	11 6	280
	..	20	....	20¼	24	94	1618	100	25 6	12 0	344
G. Duplex st., comp. air.	10	16¼	....	10¼	12	160	446	80–100	16 3	7 3	71–80
	16	24¼	....	15¼	18	120	1130	80–100	23 0	10 0	180–208
	20	30¼	....	18¼	24	100	1963	100	30 0	12 0	353
G. Comp. st., comp. air.	10	17	14¼	9¼	12	160	344	80–100	16 3	7 6	55–62
	16	26	22¼	14¼	18	120	950	80–100	23 0	10 0	152–171
	20	32	28¼	17¼	24	100	1710	80–100	30 0	12 0	274–308
H. Duplex st., duplex air.	....	8	....	8	8	150	138	60–100	8 6	4 6	20–28
	...	10	....	10	10	150	268	70–100	10 0	4 9	43–54
	....	12	....	12	12	150	474	80–100	11 8	5 10	83–95
H. Duplex st., comp. air.	....	8	14	9	8	150	210	80–100	8 6	5 3	32–36
	....	10	16	10	10	150	342	80–100	10 2	5 9	52–58
	....	12	18	12	12	150	519	80–100	11 10	6 9	78–88
J. Belted duplex or compound. 8 to 98 H.P.; 56 to 1059 cu. ft. per m.											

* Classes *A*, *C*, *G*, and *H* are also built in intermediate sizes for lower pressures. † Furnished either duplex or half duplex. ‡ Most economical form of compressor. Compound air-cylinders are two-stage. § Self-contained steam-compressor.

Cubic Feet of Free Air Required to Run from One to Forty Machines with 60 lbs. Pressure. (Ingersoll-Sergeant Drill Co.)

For 75 lbs. Pressure add 1/5. For 90 lbs. add 2/5.

	Rock-drills.								Coal-cutters.	
No. of Machines	A 2 in.	B 2¼ in.	C 2¾ in.	D 3 in.	E 3¼ in.	F 3½ in.	G 4¼ in.	H 5 in.	3½ in.	4 in.
1	65	70	95	110	115	125	140	165	70	93
2	110	120	160	190	200	230	250	280	140	186
3	156	174	234	279	294	333	360	405	210	279
4	196	220	304	356	372	428	460	524	280	372
5	230	260	370	425	445	510	555	635	350	465
6	264	294	426	486	516	588	642	738	420	558
7	294	329	476	546	581	658	721	826	490	651
8	320	360	520	600	640	720	800	920	560	744
9	360	405	585	675	720	810	900	1035	630	837
10	400	450	650	750	800	900	1000	1150	700	930
12	480	540	780	900	960	1080	1200	1380	840	1116
15	......	675	975	1125	1200	1350	1500	1725	1050	1395
20	......	....	1300	1500	1600	1800	2000	2300	1400	1860
25	......	......	1625	1875	2000	2250	2500	2775	1750	2325
30	....	.. ..	1950	2250	2400	2700	3000	3450	2100	2790
40	......	......	2600	3000	3200	3600	4000	4600	2800	3720

Compressed-air Table for Pumping Plants.

(Ingersoll-Sergeant Drill Co.)

For the convenience of engineers and others figuring on pumping plants to be operated by compressed air, we subjoin a table by which the pressure and volume of air required for any size pump can be readily ascertained. Reasonable allowances have been made for loss due to clearances in pump and friction in pipe.

Ratio of Diameters.		Perpendicular Height, in Feet, to which the Water is to be Pumped.										
		25	50	75	100	125	150	175	200	250	300	400
1 to 1	A	13.75	27.5	41.25	55.0	68.25	82.5	96.25	110.0			
	B	0.21	0.45	0.60	0.75	0.89	1.04	1.20	1.34			
1½ to 1	A	.....	12.22	18.33	24.44	30.33	36.66	42.78	48.88	61.11	73.32	97.66
	B	....	0.65	0.80	0.95	1.09	1.24	1.39	1.53	1.88	2.12	2.70
1¾ to 1	A	.....	.....	13.75	19.8	22.8	27.5	32.1	36.66	45.83	55.0	73.23
	B	.....	.....	0.94	1.14	1.24	1.30	1.54	1.69	1.99	2.39	2.88
2 to 1	A	.....	.....	.. ..	13.75	17.19	20.63	24.06	27.5	34.38	41.25	55.0
	B	.....	. ..	.. ..	1.23	1.37	1.52	1.66	1.81	2.11	2.40	2.98
2¼ to 1	A	....	. ...	....	.....	13.75	16.5	19.25	22.0	27.5	33.0	44.0
	B	.....	.....	.....	....	1.533	1.68	1.83	1.97	2.26	2.56	3.15
2½ to 1	A	....	.....	....	.....	.....	13.2	15.4	17.6	22.0	26.4	35.2
	B	... .	. ...	....	.....	... ..	1.79	1.98	2.06	2.34	2.62	3.18

A = air-pressure at pump. B = cubic feet of free air per gallon of water.

To find the amount of air and pressure required to pump a given quantity of water a given height, find the ratio of diameters between water and air cylinders, and multiply the number of gallons of water by the figure found in the column for the required lift. The result is the number of *cubic feet of free air*. The pressure required on the pump will be found directly above in the same column. For example: The ratio between cylinders being 2 to 1, required to pump 100 gallons, height of lift 250 feet. We find under 250 feet at ratio 2 to 1 the figures 2.11; 2.11 × 100 = 211 cubic feet of free air. The pressure required is 34.38 pounds.

Compressed-air Table for Hoisting-engines.

(Ingersoll-Sergeant Drill Co.)

The following table gives an approximate idea of the volume of free air required for operating hoisting-engines, the air being delivered at 60 lbs. gauge-pressure. There are so many variable conditions to the operation of hoisting-engines in common use that accurate computations can only be offered when fixed data are given. In the table the engine is assumed to actually run but one-half of the time for hoisting, while the compressor, of course, runs continuously. If the engine runs less than one-half the time, as it usually does, the volume of air required will be proportionately less, and *vice versa*. The table is computed for maximum loads, which also in practice may vary widely. From the intermittent character of the work of a hoisting-engine the parts are able to resume their normal temperature between the hoists, and there is little probability of the annoyance of freezing up the exhaust-passages.

VOLUME OF FREE AIR REQUIRED FOR OPERATING HOISTING-ENGINES, THE AIR COMPRESSED TO 60 POUNDS GAUGE-PRESSURE.

SINGLE-CYLINDER HOISTING-ENGINE.

Diam. of Cylinder, Inches.	Stroke, Inches.	Revolutions per Minute.	Normal Horse-power.	Actual Horse-power.	Weight Lifted, Single Rope.	Cubic Ft. of Free Air Required.
5	6	200	3	5.9	600	75
5	8	160	4	6.3	1,000	80
6¼	8	160	6	9.9	1,500	125
7	10	125	10	12.1	2,000	151
8¼	10	125	15	16.8	3,000	170
8½	12	110	20	18.9	5,000	238
10	12	110	25	26.2	6,000	330

DOUBLE-CYLINDER HOISTING-ENGINE.

Diam. of Cylinder, Inches.	Stroke, Inches.	Revolutions per Minute.	Normal Horse-power.	Actual Horse-power.	Weight Lifted, Single Rope.	Cubic Ft. of Free Air Required.
5	6	200	6	11.8	1,000	150
5	8	160	8	12.6	1,650	160
6¼	8	160	12	19.8	2,500	250
7	10	125	20	24.2	3,500	302
8¼	10	125	30	33.6	6,000	340
8½	12	110	40	37.8	8,000	476
10	12	110	50	52.4	10,000	660
12¼	15	100	75	89.2	....	1,125
14	18	90	100	125.	....	1,587

Practical Results with Compressed Air.—*Compressed-air System at the Chapin Mines, Iron Mountain, Mich.*—These mines are three miles from the falls which supply the power. There are four turbines at the falls, one of 1000 horse-power and three of 900 horse-power each. The pressure is 60 pounds at 60° Fahr. Each turbine runs a pair of compressors. The pipe to the mines is 24 ins. diameter. The power is applied at the mines to Corliss engines, running pumps, hoists, etc., and direct to rock-drills.

A test made in 1888 gave 1430.27 H.P. at the compressors, and 390.17 H P. as the sum of the horse-power of the engines at the mines. Therefore, only 27% of the power generated was recovered at the mines. This includes the loss due to leakage and the loss of energy in heat, but not the friction in the engines or compressors. (F. A. Pocock, Trans. A. I. M. E., 1890.)

W. L. Saunders (*Jour. F. I.* 1892) says: "There is not a properly designed compressed-air installation in operation to-day that loses over 5% by transmission alone. The question is altogether one of the size of pipe; and if the pipe is large enough, the friction loss is a small item.

"The loss of power in common practice, where compressed air is used to drive machinery in mines and tunnels, is about 70%. In the best practice, with the best air-compressors, and without reheating, the loss is about 60%. These losses may be reduced to a point as low as 20% by combining the best systems of reheating with the best air-compressors."

Gain due to Reheating.—Prof. Kennedy says compressed-air transmission system is now being carried on, on a large commercial scale, in such a fashion that a small motor four miles away from the central station can indicate in round numbers 10 horse-power, for 20 horse-power at the station itself, allowing for the value of the coke used in heating the air.

The limit to successful reheating lies in the fact that air-engines cannot work to advantage at temperatures over 350°.

The efficiency of the common system of reheating is shown by the results obtained with the Popp system in Paris. Air is admitted to the reheater at about 83°, and passes to the engine at about 315°, thus being increased in volume about 42%. The air used in Paris is about 11 cubic feet of free air per minute per horse-power. The ordinary practice in America with cold air is from 15 to 25 cubic feet per minute per horse-power. When the Paris engines were worked without reheating the air consumption was increased to about 15 cubic feet per horse-power per minute. The amount of fuel consumed during reheating is trifling.

Efficiency of Compressed-air Engines.—The efficiency of an air-engine, that is, the percentage which the power given out by the air-engine bears to that required to compress the air in the compressor, depends on the loss by friction in the pipes, valves, etc., as well as in the engine itself. This question is treated at length in the catalogue of the Norwalk Iron Works Co., from which the following is condensed. As the friction increases the most economical pressure increases. In fact, for any given friction in a pipe, the pressure at the compressor must not be carried below a certain limit. The following table gives the lowest pressures which should be used at the compressor with varying amounts of friction in the pipe:

Friction, lbs.	2.9	5.8	8.8	11.7	14.7	17.6	20.5	23.5	26.4	29.4
Lbs. at Compressor	20.5	29.4	38.2	47.	52.8	61.7	70.5	76.4	82.3	88.2
Efficiency %	70.9	64.5	60.6	57.9	55.7	54.0	52.5	51.3	50.2	49.2

An increase of pressure will decrease the bulk of air passing the pipe and its velocity. This will decrease the loss by friction, but we subject ourselves to a new loss, *i.e.* the diminishing efficiencies of increasing pressures. Yet as each cubic foot of air is at a higher pressure and therefore carries more power, we will not need as many cubic feet as before, for the same work. With so many sources of gain or loss, the question of selecting the proper pressure is not to be decided hastily.

The losses are, first, friction of the compressor. This will amount ordinarily to 15 or 20 per cent, and cannot probably be reduced below 10 per cent. Second, the loss occasioned by pumping the air of the engine-room, rather than the air drawn from a cooler place. This loss varies with the season and amounts from 3 to 10 per cent. This can all be saved. The third loss, or series of losses, arises in the compressing cylinder, viz., insufficient supply, difficult discharge, defective cooling arrangements, poor lubrication, etc. The fourth loss is found in the pipe. This loss varies with the situation, and is subject to somewhat complex influences. The fifth loss is chargeable to fall of temperature in the cylinder of the air-engine. Losses arising from leaks are often serious.

Effect of Temperature of Intake upon the Discharge of a Compressor.—Air should be drawn from outside the engine-room, and from as cool a place as possible. The gain amounts to one per cent for every five degrees that the air is taken in lower than the temperature of the engine-room. The inlet conduit should have an area at least 50% of the area of the air-piston, and should be made of wood, brick, or other non-conductor of heat.

Discharge of a compressor having an intake capacity of 1000 cubic feet per minute, and volumes of the discharge reduced to cubic feet at atmospheric pressure and at temperature of 62 degrees Fahrenheit:

Temperature of Intake, F.	0°	32°	62°	75°	80°	90°	100°	110°
Relative volume discharged, cubic ft.	1135	1060	1000	975	966	949	932	916

Requirements of Rock-drills Driven by Compressed Air. (Norwalk Iron Works Co.)—The speed of the drill, the pressure of air, and the nature of the rock affect the consumption of power of drills.

A three-inch drill using air at 30 lbs. pressure made 300 blows per minute and consumed the equivalent of 64 cubic feet of free air per minute. The same drill, with air of 58 lbs. pressure, made 450 blows per minute and consumed 160 cubic feet of free air per minute. At Hell Gate different machines doing the same work used from 80 to 150 cubic feet free air per minute.

An average consumption may be taken generally from 80 to 100 cubic feet per minute, according to the nature of the work.

The Popp Compressed-air System in Paris.—A most extensive system of distribution of power by means of compressed air is that of M. Popp, in Paris. One of the central stations is laid out for 24,000 horse-power. For a very complete description of the system, see *Engineering*, Feb. 15, June 7, 21, and 28, 1889, and March 13 and 20, April 10, and May 1, 1891. Also Proc. Inst. M. E., July, 1889. A condensed description will be found in Modern Mechanism, p. 12.

Utilization of Compressed Air in Small Motors.—In the earliest stages of the Popp system in Paris it was recognized that no good results could be obtained if the air were allowed to expand direct into the motor; not only did the formation of ice due to the expansion of the air rapidly accumulate and choke the exhaust, but the percentage of useful work obtained, compared with that put into the air at the central station, was so small as to render commercial results hopeless.

After a number of experiments M. Popp adopted a simple form of cast-iron stove lined with fire-clay, heated either by a gas jet or by a small coke fire. This apparatus answered the desired purpose until some better arrangement was perfected, and the type was accordingly adopted throughout the whole system. The economy resulting from the use of an improved form was very marked, as will be seen from the following table.

EFFICIENCY OF AIR-HEATING STOVES.

	Cast-iron Box Stoves.		Wrought-iron Coiled Tubes.
Heating surface, sq. ft.	14	14	46.3
Air heated per hour, cu. ft.	20,342	11,054	38,428
Temp. of air admitted to oven, deg. F.	45	45	41
" " " at exit, deg. F.	215	364	347
Total heat absorbed per hour, calories	17,900	17,200	39,200
Do. per sq. ft. of heating surface per hour, cals	1,278	1,228	830
Do. per lb. of coke	2,032	2,058	2,545

The results given in this table were obtained from a large number of trials. From these trials it was found that more than 70% of the total number of calories in the fuel employed was absorbed by the air and transformed into useful work. Whether gas or coal be employed as the fuel, the amount required is so small as to be scarcely worth consideration; according to the experiments carried out it does not exceed 0.2 lb. per horse-power per hour, but it is scarcely to be expected that in regular practice this quantity is not largely exceeded. The efficiency of fuel consumed in this way is at least six times greater than when utilized in a boiler and steam-engine.

According to Prof. Riedler, from 15% to 20% above the power at the central station can be obtained by means at the disposal of the power users, and it has been shown by experiment that by heating the air to 480° F. an increased efficiency of 30% can be obtained.

A large number of motors in use among the subscribers to the Compressed Air Company of Paris are rotary engines developing 1 horse-power and less, and these in the early times of the industry were very extravagant in their consumption. Small rotary engines, working cold air without expansion, used as high as 2330 cu. ft. of air per brake horse-power per hour, and with heated air 1624 cu. ft. Working expansively, a 1 horse-power rotary engine used 1469 cu. ft. of cold air, or 960 cu. ft. of heated air, and a 2-horse-power rotary engine 1059 cu. ft. of cold air, or 847 cu. ft. of air, heated to about 50° C.

The efficiency of this type of rotary motors, with air heated to 50° C., may now be assumed at 43%. With such an efficiency the use of small motors in many industries becomes possible, while in cases where it is necessary to have a constant supply of cold air economy ceases to be a matter of the first importance.

Tests of a small Riedinger rotary engine, used for driving sewing-machines and indicating about 0.1 H.P. showed an air-consumption of 1377 cu. ft. per

H.P. per hour when the initial pressure of the air was 86 lbs. per sq. in. and its temperature 54° F., and 988 cu. ft. when the air was heated to 338° F., its pressure being 72° lbs. With a one-half horse-power variable-expansion rotary engine the air-consumption was from 800 to 900 cu. ft. per H.P. per hour for initial pressures of 54 to 85 lbs. per sq. in. with the air heated from 336° to 388° F., and 1148 cu. ft. with cold air, 46° F., and an initial pressure of 72 lbs. The volumes of air were all taken at atmospheric pressure.

Trials made with an old single-cylinder 80-horse-power Farcot steam-engine, indicating 72 horse-power, gave a consumption of air per brake horse-power as low as 465 cu. ft. per hour. The temperature of admission was 320° F., and of exhaust 95° F.

Prof. Elliott gives the following as typical results of efficiency for various systems of compressors and air-motors:

Simple compressor and simple motor, efficiency	39.1%
Compound compressor and simple motor, "	44.9
" " " compound motor, efficiency.......	50.7
Triple compressor and triple motor, "	55.3

The efficiency is the ratio of the indicated horse-power in the motor cylinders to the indicated horse-power in the steam-cylinders of the compressor. The pressure assumed is 6 atmospheres absolute, and the losses are equal to those found in Paris over a distance of 4 miles.

Summary of Efficiencies of Compressed-air Transmission at Paris, between the Central Station at St. Fargeau and a 10-horse-power Motor Working with Pressure Reduced to 4½ Atmospheres.

(The figures below correspond to mean results of two experiments cold and two heated.)

1 indicated horse-power at central station gives 0.845 indicated horse-power in compressors, and corresponds to the compression of 348 cubic feet of air per hour from atmospheric pressure to 6 atmospheres absolute. (The weight of this air is about 25 pounds.)

0.845 indicated horse-power in compressors delivers as much air as will do 0.52 indicated horse-power in adiabatic expansion after it has fallen in temperature to the normal temperature of the mains.

The fall of pressure in mains between central station and Paris (say 5 kilometres) reduces the possibility of work from 0.52 to 0.51 indicated horse-power.

The further fall of pressure through the reducing valve to 4½ atmospheres (absolute) reduces the possibility of work from 0.51 to 0.50.

Incomplete expansion, wire-drawing, and other such causes reduce the actual indicated horse-power of the motor from 0.50 to 0.39.

By heating the air before it enters the motor to about 320° F., the actual indicated horse-power at the motor is, however, increased to 0.54. The ratio of gain by heating the air is, therefore, 0.54 ÷ 0.39 = 1.38.

In this process additional heat is supplied by the combustion of about 0.39 pounds of coke per indicated horse-power per hour, and if this be taken into account, the real indicated efficiency of the whole process becomes 0.47 instead of 0.54.

Working with cold air the work spent in driving the motor itself reduces the available horse-power from 0.39 to 0.26.

Working with heated air the work spent in driving the motor itself reduces the available horse-power from 0.54 to 0.44.

A summary of the efficiencies is as follows:

Efficiency of main engines 0.845.

Efficiency of compressors 0.52 ÷ 0.845 = 0.61.

Efficiency of transmission through mains 0.51 ÷ 0.52 = 0.98.

Efficiency of reducing valve 0.50 ÷ 0.51 = 0.98.

The combined efficiency of the mains and reducing valve between 5 and 4½ atmospheres is thus 0.98 × 0.98 = 0.96. If the reduction had been to 4, 3½, or 3 atmospheres, the corresponding efficiencies would have been 0.93, 0.89, and 0.85 respectively.

Indicated efficiency of motor 0.39 ÷ 0.50 = 0.78.

Indicated efficiency of whole process with cold air 0.39. Apparent indicated efficiency of whole process with heated air 0.54.

Real indicated efficiency of whole process with heated air 0.47.

Mechanical efficiency of motor, cold, 0.67.

Mechanical efficiency of motor, hot, 0.81.

Most of the compressed air in Paris is used for driving motors, but the work done by these is of the most varied kind. A list of motors driven from St. Fargeau station shows 225 installations, nearly all motors working at from 1/8 horse-power to 50 horse-power, and the great majority of them more than two miles away from the station. The new station at Quai de la Gare is much larger than the one at St. Fargeau. Experiments on the Riedler air-compressors at Paris, made in December, 1891, to determine the ratio between the indicated work done by the air-pistons and the indicated work in the steam-cylinders, showed a ratio of 0.8997. The compressors are driven by four triple-expansion Corliss engines of 2000 horse-power each.

Shops Operated by Compressed Air.—The *Iron Age*, March 2, 1893, describes the shops of the Wuerpel Switch and Signal Co., East St. Louis, the machine tools of which are operated by compressed air, each of the larger tools having its own air engine, and the smaller tools being belted from shafting driven by an air engine. Power is supplied by a compound compressor rated at 55 horse-power. The air engines are of the Kriebel make, rated from 2 to 8 horse-power.

Pneumatic Postal Transmission.—A paper by A. Falkenau, Eng'rs Club of Philadelphia, April 1894, entitled the "First United States Pneumatic Postal System," gives a description of the system used in London and Paris, and that recently introduced in Philadelphia between the main post-office and a substation. In London the tubes are 2¼ and 3 inch lead pipes laid in cast-iron pipes for protection. The carriers used in 2¼-inch tubes are but 1¼ inches diameter, the remaining space being taken up by packing. Carriers are despatched singly. First, vacuum alone was used; later, vacuum and compressed air. The tubes used in the Continental cities in Europe are wrought iron, the Paris tubes being 2⅝ inches diameter. There the carriers are despatched in trains of six to ten, propelled by a piston. In Philadelphia the size of tube adopted is 6⅛ inches, the tubes being of cast iron bored to size. The lengths of the outgoing and return tubes are 2928 feet each. The pressure at the main station is 7 lbs., at the substation 4 lbs., and at the end of the return pipe atmospheric pressure. The compressor has two air-cylinders 18 × 24 in. Each carrier holds about 200 letters, but 100 to 150 are taken as an average. Eight carriers may be despatched in a minute, giving a delivery of 48,000 to 72,000 letters per hour. The time required in transmission is about 57 seconds.

Pneumatic postal transmission tubes were laid in 1898 by the Batcheller Pneumatic Tube Co. between the general post-offices in New York and Brooklyn, crossing the East River on the bridge. The tubes are cast iron, 12-ft. lengths, bored to 8⅛ in. diameter. The joints are bells, calked with lead and yarn. There are two tubes, one operating in each direction. Both lines are operated by air-pressure above the atmospheric pressure. One tube is operated by an air-compressor in the New York office and the other by one located in the Brooklyn office.

The carriers are 24 in. long, in the form of a cylinder 7 in. in diameter, and are made of steel, with fibrous bearing-rings which fit the tube. Each carrier will contain about 600 ordinary letters, and they are despatched at intervals of 10 seconds in each direction, the time of transit between the two offices being 3½ minutes, the carriers travelling at a speed of from 30 to 35 miles per hour.

The air-compressors were built by the Rand Drill Co. and the Ingersoll-Sergeant Drill Co. The Rand Drill Co. compressor is of the duplex type and has two steam-cylinders 10 × 20 in. and two air-cylinders 24 × 20 in., delivering 1570 cu. ft. of free air per minute, at 75 revolutions, the power being about 50 H.P. Corliss valve-gear is on the steam cylinders and the Rand mechanical valve-gear on the air-cylinders.

The Ingersoll-Sergeant Drill Co. furnished two duplex Corliss air-compressors, with mechanically moved valves on air-cylinders. The steam-cylinders are 14 × 18 in. and the air-cylinders 26¼ × 18 in. They are designed for 80 to 90 revs. per min. and to compress to 20 lbs. per sq. in.

Another double line of pneumatic tubes has been laid between the main office and Postal Station H, Lexington Ave. and 44th St., in New York City. This line is about 3½ miles in length. There are three intermediate stations: Third Ave. and 8th St., Madison Square, and Third Ave. and 28th St. The carriers can be so adjusted when they are put into the tube that they will traverse the line and be discharged automatically from the tube at the station for which they are intended. The tubes are of the same size as those of the Brooklyn line and are operated in a similar manner. The initial air-compression is about 12 to 15 lbs. On the Brooklyn line it is about 7 lbs.

There is also a tube system between the New York Post-office and the Produce Exchange. For a very complete description of the system and its machinery see "The Pneumatic Despatch Tube System," by B. C. Batcheller. J. B. Lippincott Co., Philadelphia, 1897.

The Mekarski Compressed-air Tramway at Berne, Switzerland. (*Eng'g News*, April 20, 1893.)—The Mekarski system has been introduced in Berne, Switzerland, on a line about two miles long, with grades of 0.25% to 3.7% and 5.2%. The air is heated by passing it through superheated water at 330° F. It thus becomes saturated with steam, which subsequently partly condenses, its latent heat being absorbed by the expanding air. The pressure in the car reservoirs is 440 lbs. per sq. in.

The engine is constructed like an ordinary steam tramway locomotive, and drives two coupled axles, the wheel-base being 5.2 ft. It has a pair of outside horizontal cylinders, 5.1 × 8.6 in.; four coupled wheels, 27.5 in. diameter. The total weight of the car including compressed air is 7.25 tons, and with 30 passengers, including the driver and conductor, about 9.5 tons.

The authorized speed is about 7 miles per hour. Taking the resistance due to the grooved rails and to curves under unfavorable conditions at 30 lbs. per ton of car weight, the engine has to overcome on the steepest grade, 5%, a total resistance of about 0.63 ton, and has to develop 25 H.P. At the maximum authorized working pressure in cylinders of 176 lbs. per sq. in. the motors can develop a tractive force of 0.64 ton. This maximum is, therefore, just sufficient to take the car up the 5.2% grade, while on the flatter sections of the line the working pressure does not exceed 73 to 147 lbs. per sq. in. Sand has to be frequently used to increase the adhesion on the 2% to 5% grades.

Between the two car frames are suspended ten horizontal compressed-air storage-cylinders, varying in length according to the available space, but of uniform inside diameter of 17.7 in., composed of riveted 0.27-in. sheet iron, and tested up to 588 lbs. per sq. in. These cylinders have a collective capacity of 64.25 cu. ft., which, according to Mr. Mekarski's estimate, should have been sufficient for a double trip, 8¾ miles. The trial trips, however, showed this estimate to be inadequate, and two further small storage-cylinders had therefore to be added of 5.3 cu. ft. capacity each, bringing the total cubic contents of the 12 storage-cylinders per car up to 75 cu. ft., divided into two groups, the working and the reserve battery, the former of 49 cu. ft. the latter of 26 cu. ft. capacity.

From the results of six official trips, the pressure and the mean consumption of air during a double journey per motor car are as follows:

Pressure of air in storage-cylinders at starting 440 lbs. per sq. in.; at end of up-journey 176 lbs., reserve 260 lbs.; at end of down-journey 103 lbs., reserve 176 lbs. Consumption of air during up-journey 92 lbs., during down-journey 81 lbs.

The working experience of 1891 showed that the air consumption per motor car for a double journey was from 103 to 154 lbs., mean 123 lbs., and per car mile from 28 to 42 lbs., mean 35 lbs.

The principal advantages of the compressed-air system for urban and suburban tramway traffic as worked at Berne consist in the smooth and noiseless motion; in the absence of smoke, steam, or heat, of overhead or underground conductors, of the more or less grinding motion of most electric cars, and of the jerky motion to which underground cable traction is subject. On all these grounds the system has vindicated its claims as being preferable to any other so far known system of mechanical traction for street tramways. Its disadvantages, on the other hand, consist in the extremely delicate adjustment of the different parts of the system, in the comparatively small supply of air carried by one motor car, which necessitates the car returning to the depot for refilling after a run of only four miles or 40 minutes, although on the Nogent and Paris lines the cars, which are, moreover, larger, and carry outside passengers on the top, run seven miles, and the loading pressure is 547 lbs. per sq. in. as against only 440 lbs. at Berne.

Longer distances in the same direction would involve either more powerful motors, a larger number of storage-cylinders, and consequently heavier cars, or loading stations every four or seven miles; and in this respect the system is manifestly inferior to electric traction, which easily admits of a line of 10 to 15 miles in length being continuously fed from one central station without the loss of time and expense caused by reloading.

The cost of working the Berne line is compared in the annexed table with some other tramways worked under similar conditions by horse and mechanical traction for the year 1891.

For description of the Mekarski system as used at Nantes, France, see paper by Prof. D. S. Jacobus, Trans. A. I. M. E., xix. 553.

American Experiments on Compressed Air for Street Railways.—Experiments have been made recently in Washington, D. C., and in New York City on the use of compressed air for street-railway traction. The air was compressed to 2000 lbs. per sq. in. and passed through a reducing-valve and a heater before being admitted to the engine. For an extended discussion of the relative merits of compressed air and electric traction, with an account of a test of a four-stage compressor giving a pressure of 2500 lbs. per sq. in., see *Eng'g News*, Oct. 7 and Nov. 4, 1897. A summarized statement of the probable efficiency of compressed-air traction is given as follows: Efficiency of compression to 2000 lbs. per sq. in. 65%. By wire-drawing to 100 lbs. 57.5% of the available energy of the air will be lost, leaving 65 × .425 = 27.625% as the net efficiency of the air. This may be doubled by heating, making 55 25%, and if the motor has an efficiency of 80% the net efficiency of traction by compressed air will be 55.25 × .80 = 44.2%. For a description of the Hardie compressed-air locomotive, designed for street-railway work, see *Eng'g News*, June 24, 1897. For use of compressed air in mine haulage, see *Eng'g News*, Feb. 10, 1898.

Compressed Air for Working Underground Pumps in Mines.—*Eng'g Record*, May 19, 1894, describes an installation of compressors for working a number of pumps in the Nottingham No. 15 Mine, Plymouth, Pa., which is claimed to be the largest in America. The compressors develop above 2300 H.P., and the piping, horizontal and vertical, is 6000 feet in length. About 25,000 gallons of water per hour are raised.

FANS AND BLOWERS.

Centrifugal Fans.—The ordinary centrifugal fan consists of a number of blades fixed to arms, revolving on a shaft at high speed. The width of the blade is parallel to the axis of the shaft. Most engineers' reference books quote the experiments of W. Buckle, Proc. Inst. M.E., 1847, as still standard. Mr. Buckle's conclusions are given below, together with data of more recent experiments.

Experiments were made as to the proper size of the inlet openings and on the proper proportions to be given to the vane. The inlet openings in the sides of the fan-chest were contracted from 17½ in., the original diameter, to 12 and 6 in. diam., when the following results were obtained:

First, that the power expended with the opening contracted to 12 in. diam. was as 2½ to 1 compared with the opening of 17½ in. diam.; the velocity of the fan being nearly the same, as also the quantity and density of air delivered.

Second, that the power expended with the opening contracted to 6 in. diam. was as 2½ to 1 compared with the opening of 17½ in. diam.; the velocity of the fan being nearly the same, and also the area of the efflux pipe, but the density of the air decreased one fourth.

These experiments show that the inlet openings must be made of sufficient size, that the air may have a free and uninterrupted action in its passage to the blades of the fan; for if we impede this action we do so at the expense of power.

With a vane 14 in. long, the tips of which revolve at the rate of 236.8 ft. per second, air is condensed to 9.4 ounces per square inch above the pressure of the atmosphere, with a power of 9.6 H. P.; but a vane 8 inches long, the diameter at the tips being the same, and having, therefore, the same velocity, condenses air to 6 ounces per square inch only, and takes 12 H. P.

Thus the density of the latter is little better than six tenths of the former, while the power absorbed is nearly 1.25 to 1. Although the velocity of the tips of the vanes is the same in each case, the velocities of the heels of the respective blades are very different, for, while the tips of the blades in each case move at the same rate, the velocity of the heel of the 14-inch is in the ratio of 1 to 1.67 to the velocity of the heel of the 8-inch blade. The longer blades approaching nearer the centre, strikes the air with less velocity, and allows it to enter on the blade with greater freedom, and with considerably less force than the shorter one. The inference is, that the short blade must take more power at the same time that it accumulates a less quantity of air. These experiments lead to the conclusion that the length of the vane demands as great a consideration as the proper diameter of the inlet opening. If there were no other object in view, it would be useless to make the vanes of the fan of a greater width than the inlet opening can freely supply. On the proportion of the length and width of the vane and the diameter of the inlet opening rest the three most important points, viz., *quantity* and *density* of air, and expenditure of *power*.

In the 14-inch blade the tip has a velocity 2.6 times greater than the heel; and, by the laws of centrifugal force, the air will have a density 2.6 times greater at the tip of the blade than that at the heel. The air cannot enter on the heel with a density higher than that of the atmosphere; but in its passage along the vane it becomes compressed in proportion to its centrifugal force. The greater the length of the vane, the greater will be the difference of the centrifugal force between the heel and the tip of the blade; consequently the greater the density of the air.

Reasoning from these experiments, Mr. Buckle recommends for easy reference the following proportions for the construction of the fan:

1. Let the width of the vanes be one fourth of the diameter; 2. Let the diameter of the inlet openings in the sides of the fan-chest be one half the diameter of the fan; 3. Let the length of the vanes be one fourth of the diameter of the fan.

In adopting this mode of construction, the area of the inlet openings in the sides of the fan-chest will be the same as the circumference of the heel of the blade, multiplied by its width; or the same area as the space described by the heel of the blade.

Best Proportions of Fans. (Buckle.)

PRESSURE FROM 3 OUNCES TO 6 OUNCES PER SQUARE INCH; OR 5.2 INCHES TO 10.4 INCHES OF WATER.

Diameter of Fan.		Vanes. Width.		Vanes. Length.		Diameter of Inlet Openings.		Diameter of Fan.		Vanes. Width.		Vanes. Length.		Diameter of Inlet Openings.	
ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.
3	0	0	9	0	9	1	6	4	6	1	1½	1	1½	2	3
3	6	0	10½	0	10½	1	9	5	0	1	3	1	3	2	6
4	0	1	0	1	0	2	0	6	0	1	6	1	6	3	0

PRESSURE FROM 6 OUNCES TO 9 OUNCES PER SQUARE INCH, AND UPWARDS, OR 10.4 INCHES TO 15.6 INCHES OF WATER.

Diameter of Fan.		Vanes. Width.		Vanes. Length.		Diameter of Inlet Openings.		Diameter of Fan.		Vanes. Width.		Vanes. Length.		Diameter of Inlet Openings.	
3	0	0	7	1	0	1	0	4	6	0	10½	1	4½	1	9
3	6	0	8½	1	1½	1	3	5	0	1	0	1	6	2	0
4	0	0	9½	1	3½	1	6	6	0	1	2	1	10	2	4

The dimensions of the above tables are not laid down as prescribed limits, but as approximations obtained from the best results in practice.

Experiments were also made with reference to the admission of air into the transit or outlet pipe. By a slide the width of the opening into this pipe was varied from 12 to 4 inches. The object of this was to proportion the opening to the quantity of air required, and thereby to lessen the power necessary to drive the fan. It was found that the less this opening is made, provided we produce sufficient blast, the less noise will proceed from the fan; and by making the tops of this opening level with the tips of the vane, the column of air has little or no reaction on the vanes.

The number of blades may be 4 or 6. The case is made of the form of an arithmetical spiral, widening the space between the case and the revolving blades, circumferentially, from the origin to the opening for discharge.

The following rules deduced from experiments are given in Spretson's treatise on Casting and Founding:

The fan-case should be an arithmetical spiral to the extent of the depth of the blade at least.

The diameter of the tips of the blades should be about double the diameter of the hole in the centre; the width to be about two thirds of the radius of the tips of the blades. The velocity of the tips of the blades should be rather more than the velocity due to the air at the pressure required, say one eighth more velocity.

In some cases, two fans mounted on one shaft would be more useful than one wide one, as in such an arrangement twice the area of inlet opening is obtained as compared with a single wide fan. Such an arrangement may be adopted where occasionally half the full quantity of air is required, as one of them may be put out of gear, thus saving power.

Pressure due to Velocity of the Fan-blades.—"By increasing the number of revolutions of the fan the head or pressure is increased, the law being that the total head produced is equal (in centrifugal fans) to twice the height due to the velocity of the extremities of the blades, or $H = \frac{v^2}{g}$ approximately in practice" (W. P. Trowbridge, Trans. A. S. M. E., vii. 536.) This law is analogous to that of the pressure of a jet striking a plane surface. T. Hawksley, Proc. Inst. M. E., 1882, vol. lxix., says: "The pressure of a fluid striking a plane surface perpendicularly and then escaping at right angles to its original path is that due to twice the height h due the velocity."

(For discussion of this question, showing that it is an error to take the pressure as equal to a column of air of the height $h = v^2 \div 2g$, see Wolff on Windmills, p. 17.)

Buckle says: "From the experiments it further appears that the velocity of the tips of the fan is equal to nine tenths of the velocity a body would acquire in falling the height of a homogeneous column of air equivalent to the density." D. K. Clark (R. T. & D., p. 924), paraphrasing Buckle, apparently, says: "It further appears that the pressure generated at the circumference is one ninth greater than that which is due to the actual circumferential velocity of the fan." The two statements, however, are not in harmony, for if $v = 0.9\sqrt{2gH}$, $H = \frac{v^2}{0.81 \times 2g} = 1.234\frac{v^2}{2g}$ and not $1\frac{1}{9}\frac{v^2}{2g}$.

If we take the pressure as that equal to a head or column of air of twice the height due the velocity, as is correctly stated by Trowbridge, the paradoxical statements of Buckle and Clark—which would indicate that the actual pressure is greater than the theoretical—are explained, and the formula becomes $H = .617\frac{v^2}{g}$ and $v = 1.273\sqrt{gH} = 0.9\sqrt{2gH}$, in which H is the head of a column producing the pressure, which is equal to twice the theoretical head due the velocity of a falling body $\left(\text{or } h = \frac{v^2}{2g}\right)$, multiplied by the coefficient .617. The difference between 1 and this coefficient expresses the loss of pressure due to friction, to the fact that the inner portions of the blade have a smaller velocity than the outer edge, and probably to other causes. The coefficient 1.273 means that the tip of the blade must be given a velocity 1.273 times that theoretically required to produce the head H.

To convert the head H expressed in feet to pressure in lbs. per sq. in. multiply it by the weight of a cubic foot of air at the pressure and temperature of the air expelled from the fan (about .08 lb. usually) and divide by 144. Multiply this by 16 to obtain pressure in ounces per sq. in. or by 2.035 to obtain inches of mercury, or by 27.71 to obtain pressure in inches of water column. Taking .08 as the weight of a cubic foot of air,

p lbs. per sq. in.	$= .00001066v^2$;	$v = 310\sqrt{p}$ nearly;
p_1 ounces per sq. in.	$= .0001706v^2$;	$v = 80\sqrt{p_1}$ "
p_2 inches of mercury	$= .00002169v^2$;	$v = 220\sqrt{p_2}$ "
p_3 inches of water	$= .0002954v^2$;	$v = 60\sqrt{p_3}$ "

in which v = velocity of tips of blades in feet per second.

Testing the above formula by the experiment of Buckle with the vane 14 inches long, quoted above, we have $p = .00001066v^2 = 9.56$ oz. The experiment gave 9.4 oz.

Testing it by the experiment of H. I. Snell, given below, in which the circumferential speed was about 150 ft. per second, we obtain 3.85 ounces, while the experiment gave from 2.38 to 3.50 ounces, according to the amount of opening for discharge. The numerical coefficients of the above formulæ are all based on Buckle's statement that the velocity of the tips of the fan is equal to nine tenths of the velocity a body would acquire in falling the height of a homogeneous column of air equivalent to the pressure. Should other experiments show a different law, the coefficients can be corrected accordingly. It is probable that they will vary to some extent with different proportions of fans and different speeds.

Taking the formula $v = 80\sqrt{p_1}$, we have for different pressures in ounces per square inch the following velocities of the tips of the blades in feet per second:

p_1 = ounces per square inch....	2	3	4	5	6	7	8	10	12	14
v = feet per second............	113	139	160	179	196	212	226	253	277	299

A rule in *App. Cyc. Mech*, article "Blowers," gives the following velocities of circumference for different densities of blast in ounces: 3, 170; 4, 180; 5, 195; 6, 205; 7, 215.

The same article gives the following tables, the first of which shows that the density of blast is not constant for a given velocity, but depends on the ratio of area of nozzle to area of blades:

Velocity of circumference, feet per second.	150	150	150	170	200	200	220
Area of nozzle ÷ area of blades............	2	1	1/3	1/4	1/3	1/5	1/3
Density of blast, oz. per square inch........	1	2	3	4	4	6	6

QUANTITY OF AIR OF A GIVEN DENSITY DELIVERED BY A FAN.

Total area of nozzles in square feet × velocity in feet per minute corresponding to density (see table) = air delivered in cubic feet per minute.

Density, ounces per sq. in.	Velocity, feet per minute.	Density, ounces per sq. in.	Velocity, feet per min.	Density, ounces per sq. in.	Velocity, feet per minute.
1	6000	5	11,000	9	15,000
2	7000	6	12,250	10	15,800
3	8600	7	13,200	11	16,500
4	10,000	8	14,150	12	17,300

Experiments with Blowers. (Henry I. Snell, Trans. A. S. M. E. ix. 51.)—The following tables give velocities of air discharging through an aperture of any size under the given pressures into the atmosphere. The volume discharged can be obtained by multiplying the area of discharge opening by the velocity, and this product by the coefficient of contraction: .65 for a thin plate and .98 when the orifice is a conical tube with a convergence of about 3.5 degrees, as determined by the experiments of Weisbach.

The tables are calculated for a barometrical pressure of 14.69 lbs. (= 235 oz.), and for a temperature of 50° Fahr., from the formula $V = \sqrt{2gh}$.

Allowances have been made for the effect of the compression of the air, but none for the heating effect due to the compression.

At a temperature of 50 degrees, a cubic foot of air weighs .078 lbs., and calling $g = 32.1602$, the above formula may be reduced to

$$V_1 = 60\sqrt{31.5812 \times (235 + P) \times P},$$

where V_1 = velocity in feet per minute.

P = pressure above atmosphere, or the pressure shown by gauge, in oz. per square inch.

Pressure per sq. in. in inches of water.	Corresponding Pressure in oz. per sq. inch.	Velocity due the Pressure in feet per minute.	Pressure per sq. in. in inches of water.	Corresponding Pressure in oz. per sq. inch.	Velocity due the Pressure in feet per minute.
1/32	.01817	696.78	5/8	.36340	3118.38
1/16	.03634	987.66	3/4	.43608	3416.64
1/8	.07268	1393.75	7/8	.50870	3690.62
3/16	.10902	1707.00	1	.58140	3946.17
1/4	.14536	1971.30	1¼	.7267	4362.62
5/16	.18170	2204.16	1½	.8721	4836.06
3/8	.21804	2414.70	1¾	1.0174	5224.98
1/2	.29072	2788.74	2	1.1628	5587.58

Pressure in oz. per sq. inch.	Velocity due the Pressure in ft. per minute.	Pressure in oz. per sq. inch.	Velocity due the Pressure in ft. per minute.	Pressure in oz. per sq. inch.	Velocity due the Pressure in ft. per minute.	Pressure in oz. per sq. in.	Velocity due the Pressure in ft. per minute.
.25	2,582	2.25	7,787	5.50	12,259	11.00	17,534
.50	3,658	2.50	8,218	6.00	12,817	12.00	18,350
.75	4,482	2.75	8,618	6.50	13,354	13.00	19,138
1.00	5,178	3.00	9,006	7.00	13,878	14.00	19,901
1.25	5,792	3.50	9,739	7.50	14,374	15.00	20,641
1.50	6,349	4.00	10,421	8.00	14,861	16.00	21,360
1.75	6,861	4.50	11,055	9.00	15,795		
2.00	7,338	5.00	11,676	10.00	16,684		

Pressure in ounces per square inch.	Velocity in feet per minute.	Pressure in ounces per square inch.	Velocity in feet per minute.
.01	516.90	.06	1266.24
.02	722.64	.07	1367.76
.03	895.26	.08	1462.20
.04	1033.86	.09	1550.70
.05	1155.90	.10	1635.00

Experiments on a Fan with Varying Discharge-opening. Revolutions nearly constant.

Revolutions per minute.	Area of Discharge in square inches.	Observed Pressure in ounces.	Volume of Air discharged per min., cubic feet.	Horse-power.	Actual Number of cu. ft. of Air delivered per H.P.	Theoret. Vol. per min. that may be discharged with 1 H.P. at corresp. Pressure.	Efficiency of Blowers as per Experiment.
1519	0	3.50	0	.80	....	1048	....
1479	6	3.50	406	1.15	353	1048	.337
1480	10	3.50	676	1.30	520	1048	.496
1471	20	3.50	1358	1.95	694	1048	.66
1485	28	3.50	1894	2.56	742	1048	.709
1485	36	3.40	2400	3.10	774	1078	.718
1465	40	3.25	2605	3.30	790	1126	.70
1468	44	3.00	2752	3.55	775	1222	.635
1500	48	3.00	3002	3.80	790	1222	.646
1426	89.5	2.38	3972	4.80	827	1544	.536

The fan wheel was 23 inches in diameter, 6⅝ inches wide at its periphery, and had an inlet of 12½ inches in diameter on either side, which was partially obstructed by the pulleys, which were 5 9/16 inches in diameter. It had eight blades, each of an area of 45.49 square inches.

The discharge of air was through a conical tin tube with sides tapered at an angle of 3½ degrees. The actual area of opening was 7% greater than given in the tables, to compensate for the *vena contracta*.

In the last experiment, 89.5 sq. in. represents the actual area of the mouth of the blower less a deduction for a narrow strip of wood placed across it for the purpose of holding the pressure-gauge. In calculating the volume of air discharged in the last experiment the value of *vena contracta* is taken at .80.

Experiments were undertaken for the purpose of showing the results obtained by running the same fan at different speeds with the discharge-opening the same throughout the series.

The discharge-pipe was a conical tube 8½ inches inside diameter at the end, having an area of 56.74, which is 7% larger than 53 sq. inches; therefore 53 square inches, equal to .368 square feet, is called the area of discharge, as that is the practical area by which the volume of air is computed.

Experiments on a Fan with Constant Discharge-opening and Varying Speed.—The first four columns are given by Mr. Snell, the others are calculated by the author.

Revs. per min.	Pressure in ounces, p	Vol. of Air in cu. ft. per minute, V.	Horse-power.	Velocity of Tips of Blades, ft. per sec.	Velocity due Pressure from Formula $v = 80\sqrt{p}$.	Coefficient of Formula $v = x\sqrt{p}$ from Experiment.	Velocity of Air per minute in Efflux Pipe, $V \div .368$.	Theoretical Horse-power.	Efficiency per cent.
600	.50	1336	.25	60.2	56.6	85.1	3,630	.182	73
800	.88	1787	.70	80.3	75.0	85.6	4,856	.429	61
1000	1.38	2245	1.35	100.4	94.	85.4	6,100	.845	63
1200	2.00	2712	2.20	120.4	113.	85.1	7,370	1.479	67
1400	2.75	3177	3.45	140.5	133.	84.8	8,633	2.283	66
1600	3.80	3670	5.10	160.6	156.	82.4	9,973	3.803	74
1800	4.80	4172	8.00	180.6	175.	82.4	11,337	5.462	68
2000	5.95	4674	11.40	200.7	195.	85.6	12,701	7.586	67

Mr. Snell has not found any practical difference between the efficiencies of blowers with curved blades and those with straight radial ones.

From these experiments, says Mr. Snell, it appears that we may expect to receive back 65% to 75% of the power expended, and no more.

The great amount of power often used to run a fan is not due to the fan itself, but to the method of selecting, erecting, and piping it.

(For opinions on the relative merits of fans and positive rotary blowers, see discussion of Mr. Snell's paper, Trans. A. S. M. E., ix. 66, etc.)

Comparative Efficiency of Fans and Positive Blowers.—(H. M. Howe, Trans. A. I. M. E., x. 482.)—Experiments with fans and positive (Baker) blowers working at moderately low pressures, under 20 ounces, show that they work more efficiently at a given pressure when delivering large volumes (*i.e.*, when working nearly up to their maximum capacity) than when delivering comparatively small volumes. Therefore, when great variations in the quantity and pressure of blast required are liable to arise, the highest efficiency would be obtained by having a number of blowers, always driving them up to their full capacity, and regulating the amount of blast by altering the number of blowers at work, instead of having one or two very large blowers and regulating the amount of blast by the speed of the blowers.

There appears to be little difference between the efficiency of fans and of Baker blowers when each works under favorable conditions as regards quantity of work, and when each is in good order.

For a given speed of fan, any diminution in the size of the blast-orifice decreases the consumption of power and at the same time raises the pressure of the blast; but it increases the consumption of power per unit of orifice for a given pressure of blast. When the orifice has been reduced to the normal size for any given fan, further diminishing it causes but slight elevation of the blast pressure; and, when the orifice becomes comparatively small, further diminishing it causes no sensible elevation of the blast pressure, which remains practically constant, even when the orifice is entirely closed.

Many of the failures of fans have been due to too low speed, to too small pulleys, to improper fastening of belts, or to the belts being too nearly vertical; in brief, to bad mechanical arrangement, rather than to inherent defects in the principles of the machine.

If several fans are used, it is probably essential to high efficiency to provide a separate blast-pipe for each (at least if the fans are of different size or speed), while any number of positive blowers may deliver into the same pipe without lowering their efficiency.

Capacity of Fans and Blowers.

The following tables show the guaranteed air-supply and air-removal of leading forms of blowers and exhaust fans. The figures given are often exceeded in practice, especially when the blowers and fans are driven at higher speeds than stated. The ratings, particularly of the blowers, are below those generally given in catalogues, but it was the desire to present only conservative and assured practice. (A. R. Wolff on Ventilation.)

QUANTITY OF AIR SUPPLIED TO BUILDINGS BY BLOWERS OF VARIOUS SIZES.

Diameter of Wheel in feet.	Ordinary Number of Revs. per min.	Horse-power to Drive Blower.	Capacity cu. ft. per min. against a Pressure of 1 ounce per sq. in.	Diameter of Wheel in feet.	Ordinary Number of Revs. per min.	Horse-power to Drive Blower.	Capacity cu. ft. per min. against a Pressure of 1 ounce per sq. in.
4	350	6.	10,635	9	175	29	56,800
5	325	9.4	17,000	10	160	35.5	70,840
6	275	13.5	29,618	12	130	49.5	102,000
7	230	18.4	42,700	14	110	66	139,000
8	200	24	46,000	15	100	77	160,000

If the resistance exceeds the pressure of one ounce per square inch, of above table, the capacity of the blower will be correspondingly decreased, or power increased, and allowance for this must be made when the distributing ducts are small, of excessive length, and contain many contractions and bends.

QUANTITY OF AIR MOVED BY AN APPROVED FORM OF EXHAUST FAN, THE FAN DISCHARGING DIRECTLY FROM ROOM INTO THE ATMOSPHERE.

Diameter of Wheel in feet.	Ordinary Number of Revs. per min.	Horse-power to Drive Fan.	Capacity in cu. ft. per min.	Diameter of Wheel in feet.	Ordinary Number of Revs. per min.	Horse-power to Drive Fan.	Capacity in cu. ft. per min.
2.0	600	0.50	5,000	4.0	475	3.50	28,000
2.5	550	0.75	8,000	5.0	350	4.50	35,000
3.0	500	1.00	12,000	6.0	300	7.00	50,000
3.5	500	2.50	20,000	7.0	250	9.00	80,000

The capacity of exhaust fans here stated, and the horse-power to drive them, are for free exhaust from room into atmosphere. The capacity decreases and the horse-power increases materially as the resistance, resulting from lengths, smallness and bends of ducts, enters as a factor. The difference in pressures in the two tables is the main cause of variation in the respective records. The fan referred to in the second table could not be used with as high a resistance as one ounce per square inch, the rated resistance of the blowers.

Caution in Regard to Use of Fan and Blower Tables.—Many engineers report that manufacturers' tables overrate the capacity of their fans and underestimate the horse-power required to drive them. In some cases the complaints may be due to restricted air outlets, long and crooked pipes, slipping of belts, too small engines, etc.

CENTRIFUGAL FANS.

Flow of Air through an Orifice.

VELOCITY, VOLUME, AND H.P. REQUIRED WHEN AIR UNDER GIVEN PRESSURE IN OUNCES PER SQ. IN. IS ALLOWED TO ESCAPE INTO THE ATMOSPHERE.

(B. F. Sturtevant Co.)

Pressure in ounces per sq. in.	Velocity, ft. per min.	Volume through 1 sq. in. Effective Area, cu. ft. per min.	Horse-power to move the Given Volume of Air.	Horse-power per 1000 cu. ft. per min.	Pressure in ounces per sq. in.	Velocity, ft. per min.	Volume through 1 sq. in. Effective Area, cu. ft. per min.	Horse-power to move the Given Volume of Air.	Horse-power per 1000 cu. ft. per min.
1/8	1,828	12.69	.00043	.0340	2	7,284	50.59	.02759	.5454
1/4	2,585	17.95	.00122	.0680	2 1/8	7,507	52.13	.03021	.5795
3/8	3,165	21.98	.00225	.1022	2 1/4	7,722	53.63	.03291	.6136
1/2	3,654	25.37	.00346	.1363	2 3/8	7,932	55.08	.03568	.6476
5/8	4,084	28.36	.00483	.1703	2 1/2	8,136	56.50	.03852	.6818
3/4	4,473	31.06	.00635	.2044	2 5/8	8,334	57.88	.04144	.7160
7/8	4,830	33.54	.00800	.2385	2 3/4	8,528	59.22	.04442	.7500
1	5,162	35.85	.00978	.2728	2 7/8	8,718	60.54	.04747	.7841
1 1/8	5,473	38.01	.01166	.3068	3	8,903	61.83	.05058	.8180
1 1/4	5,768	40.06	.01366	.3410	3 1/8	9,084	63.08	.05376	.8522
1 3/8	6,048	42.00	.01575	.3750	3 1/4	9,262	64.32	.05701	.8863
1 1/2	6,315	43.86	.01794	.4090	3 3/8	9,435	65.52	.06031	.9205
1 5/8	6,571	45.63	.02022	.4431	3 1/2	9,606	66.71	.06368	.9546
1 3/4	6,818	47.34	.02260	.4772	3 5/8	9,773	67.87	.06710	.9887
1 7/8	7,055	49.00	.02505	.5112	3 3/4	9,938	69.01	.07058	1.0227
					3 7/8	10,100	70.14	.07412	1.0567

The headings of the 2d and 3d columns in the above table have been abridged from the original, which read as follows: Velocity of dry air, 50° F., escaping into the atmosphere through any shaped orifice in any pipe or reservoir in which the given pressure is maintained. Volume of air in cubic feet which may be discharged in one minute through an orifice having an effective area of discharge of one square inch. The 5th column, not in the original, has been calculated by the author. The figures represent the horse-power theoretically required to move 1000 cu. ft. of air of the given pressures through an orifice, without allowance for the work of compression or for friction or other losses of the fan. These losses may amount to from 60% to 100% of the given horse-power.

The change in density which results from a change in pressure has been taken into account in the calculations of the table. The volume of air at a given velocity discharged through an orifice depends upon its shape, and is always less than that measured by its full area. For a given *effective* area the volume is proportional to the velocity. The power required to move air through an orifice is measured by the product of the velocity and the total resisting pressure. This power for a given orifice varies as the cube of the velocity. For a given volume it varies as the square of the velocity. In the movement of air by means of a fan there are unavoidable resistances which, in proportion to their amount, increase the actual power considerably above the amount here given.

For any size of centrifugal fan there exists a certain maximum area over which a given pressure may be maintained, dependent upon and proportional to the speed at which it is operated. If this area, known as its "capacity area," or square inches of blast, be increased, the pressure is lowered (the volume being increased), but if decreased the pressure remains constant. The revolutions of a given fan necessary to maintain a given pressure under these conditions are given in the table on p. 519, which is based upon the abve table. The pressure produced by a given fan and its effective capacity area being known, its nominal capacity and the horse-power required, without allowance for frictional losses, may be determined from the table above.

In practice the outlet of a fan greatly exceeds the capacity area; hence the volume moved and the horse-power required are in excess of the amounts determined as above.

Steel-plate Full Housing Fans. (Buffalo Forge Co.)

Capacities in cubic feet of air per minute. (See also table on p. 525.)

Size, in.	Revolutions per Minute.										
	100	150	200	250	300	350	400	450	500	550	600
50	1650	2475	3300	4125	4950	5775	6600	7425	8250	9075	9900
60	2480	3720	4960	6200	7440	8680	9920	11160	12400	13640	14880
70	4500	6750	9000	11250	13500	15750	18000	20250	22500		
80	7070	10605	14140	17675	21210	24745	28280	31815			
90	10400	15600	20800	26000	31200	36400	41600				
100	14280	21420	28560	35700	42840	49980	57120				
110	18960	28440	37920	47400	56880	66360					
120	24800	37200	49600	62000	74400						
130	31200	46800	62400	78000	109200						
140	38354	57531	76708	95885							
150	49260	73890	98520	123150							

The Sturtevant Steel Pressure-blower Applied to Cupola Furnaces and Forges.

Number of Blower.	Cupola Furnaces.				Forges.	
	Diameter of Cupola inside of Lining, in.	Melting Capacity of Cupola per hour in lbs.	Blast-pressure required in Wind-box in ounces per sq. in.	Rev. per min. of Blower necessary to produce required pressure.	Number of Forges supplied by Blower.	Rev. per min. Blower necessary to produce pressure for forge fire.
4/0					1	5,648
2/0					2	4,294
0					3	3,645
1	22	1,200	5	3,569	4	3,199
2	26	1,900	6	3,282	6	2,691
3	30	2,900	7	3,030	8	2,306
4	35	4,200	8	2,818	10	2,009
5	40	6,200	10	2,690	14	1,722
6	46	8,900	12	2,670	19	1,567
7	53	12,500	14	2,316	25	1,264
8	60	16,500	14	2,023	35	1,104
9	72	24,000	16	1,854	45	950
10	84	34,000	16	1,627	60	834

The above table relates to common cupolas under ordinary conditions and to forges of medium size. The diameter of cupola given opposite each size blower is the greatest which is recommended; in cases where there is a surplus of power one size larger blower may be used to advantage. The melting capacity per hour is based upon an average of tests on some of the best cupolas found, and is reliable in cases where the cupola is well constructed and carefully operated. The blast-pressure required in wind-box is the maximum under ordinary conditions when coal is used as fuel. When coke is employed the pressure may be lower.

The cupola pressures given are those in the wind-box, while the basis pressure for forges is 4 ounces in the tuyere pipe. The corresponding revolutions of fan given are in each case sufficient to maintain these pressures *at the fan outlet* when the temperature is 50°. The actual speed must be higher than this by an amount proportional to the resistance of pipes and the increase of temperature, and can only be determined by a knowledge of the existing conditions.

(For other data concerning Cupolas see Foundry Practice.)

Diameters of Blast-pipes Required for Steel Pressure-blowers. (B. F. Sturtevant Co.)

Based on the loss of pressure resulting from transmission being limited to one-half ounce per square inch.

Pressure per sq. in.	Length of Pipe in ft.	Number of Blower.												
		4/0	2/0	0	1	2	3	4	5	6	7	8	9	10
4 oz.	100	4¾	5¾	6¼	6⅝	7⅛	8⅛	8⅜	9⅛	10¼	12¼	14⅜	15¼	20¼
	200	5⅜	6½	7¼	7⅝	8⅛	9¼	9⅝	10½	11¾	14⅛	16½	17½	23¼
	300	5⅞	7⅛	7¾	8¼	8⅞	10⅛	10⅜	11¼	12¾	15¼	17⅞	19	25¼
	400	6⅛	7½	8¼	8¾	9⅝	10¾	11	12	13½	16¼	19	20⅝	26¾
8 oz.	100	5⅜	6½	7⅛	7½	8⅛	9¼	9½	10⅜	11¾	14⅛	16⅜	17½	23¼
	200	6⅛	7½	8¼	8¾	9⅜	10⅝	11	12	13½	16⅛	18⅞	20	26⅛
	300	6⅝	8⅛	8⅞	9⅜	10	11½	11⅞	13	14½	17½	20½	21⅝	27⅞
	400	7⅛	8⅝	9½	10	10⅝	12¼	12⅝	13¾	15½	18½	21⅝	23	30½
12 oz.	100	5¾	7⅛	7¾	8⅛	8¾	10	10⅜	11¼	12⅝	15¼	17¾	18⅞	25
	200	6⅝	8⅛	8⅞	9⅜	10	11½	11⅞	12⅞	14½	17⅜	20⅜	21⅝	28¾
	300	7¼	8¾	9⅝	10⅛	10⅞	12½	12⅞	14	15¾	18⅞	22½	23¾	31⅛
	400	7⅝	9⅜	10¼	10¾	11⅝	13¼	13⅝	14⅞	16¾	20⅛	23½	24⅞	33
16 oz.	100	6⅛	7½	8¼	8⅝	9¼	10⅝	10⅞	11⅞	13⅜	16	18¾	19⅞	26⅝
	200	7	8⅝	9⅜	9⅞	10⅝	12⅛	12½	13⅝	15⅝	18⅜	21½	22⅞	30¼
	300	7⅝	9⅜	10¼	10¾	11½	13¼	13⅝	14⅞	16¾	20	23⅜	24⅞	23
	400	8⅛	9⅞	10¾	11⅜	12¼	14½	14½	15¾	17⅝	21¼	24¾	26¼	35

"The above table has been constructed on the following basis: Allowing a loss of pressure of ½ oz. in the process of transmission through any length of pipe of any size as a standard, the increased friction due to lengthening the pipe has been compensated for by an enlargement of the pipe sufficient to keep the loss still at ½ oz. Thus if air under a pressure of 8 oz. is to be delivered by a No. 6 blower, through a pipe 100 ft. in length, with a loss of ½ oz. pressure, the diameter of the pipe must be 11¾ in. If its length is increased to 400 ft. its diameter should also be increased to 15½ in., or if the pressure be increased to 12 oz. the pipe, if 100 ft. long, must be 12⅝ in. in diameter, providing the loss of ½ oz. is not to be exceeded. This loss of ½ oz is to be added to the pressure to be maintained at the fan if the tabulated pressure is to be secured at the other end of the pipe."

Efficiency of Fans.—Much useful information on the theory and practice of fans and blowers, with results of tests of various forms, will be found in *Heating and Ventilation*, June to Dec. 1897, in papers by Prof. R. C. Carpenter and Mr. W. G. Walker. It is shown by theory that the volume of air delivered is directly proportional to the speed of rotation, that the pressure varies as the square of the speed, and that the horse-power varies as the cube of the speed. For a given volume of air moved the horse-power varies as the square of the speed, showing the great advantage of large fans at slow speeds over small fans at high speeds delivering the same volume. The theoretical values are greatly modified by variations in practical conditions. Prof. Carpenter found that with three fans running at a speed of 6200 ft. per minute at the tips of the vanes, and an air-pressure of 2½ in. of water column, the mechanical efficiency, or the horse-power of the air delivered divided by the power required to drive the fan, ranged from 32% to 47%, under different conditions, but with slow speeds it was much less, in some cases being under 20%. Mr. Walker in experiments on disk fans found efficiencies ranging all the way from 7.4% to 48%, the size of the fans and the speed being constant, but the shape and angle of the blades varying. It is evident that there is a wide margin for improvements in the forms of fans and blowers, and a wide field for experiment to determine the conditions that will give maximum efficiency.

Centrifugal Ventilators for Mines.—Of different appliances for ventilating mines various forms of centrifugal machines having proved their efficiency have now almost completely replaced all others. Most if not all of the machines in use in this country are of this class, being either open-periphery fans, or closed, with chimney and spiral casing, of a more or less modified Guibal type. The theory of such machines has been demonstrated by Mr. Daniel Murgue in "Theories and Practices of Centrifugal Ventilating Machines," translated by A. L. Stevenson, and is discussed in a paper by R. Van A. Norris, Trans. A. I. M. E. xx. 687. From this paper the following formulæ are taken:

Let a = area in sq. ft. of an orifice in a thin plate, of such area that its resistance to the passage of a given quantity of air equals the resistance of the mine;
o = orifice in a thin plate of such area that its resistance to the passage of a given quantity of air equals that of the machine;
Q = quantity of air passing in cubic feet per minute;
V = velocity of air passing through a in feet per second;
V_0 = velocity of air passing through o in feet per second;
h = head in feet air-column to produce velocity V;
h_0 = head in feet air-column to produce velocity V_0.

$$Q = 0.65aV; \quad V = \sqrt{2gh}; \quad Q = 0.65a\sqrt{2gh};$$

$$a = \frac{Q}{0.65\sqrt{2gh}} = \text{equivalent orifice of mine;}$$

or, reducing to water-gauge in inches and quantity in thousands of feet per minute,

$$a = \frac{.403Q}{\sqrt{W.G.}}; \quad Q = 0.65oV_0; \quad V_0 = \sqrt{2gh_0}; \quad Q = 0.65o\sqrt{2gh_0};$$

$$o = \sqrt{\frac{Q^2}{0.65^2 h_0 2g}} = \text{equivalent orifice of machine.}$$

The theoretical depression which can be produced by any centrifugal ventilator is double that due to its tangential speed. The formula

$$H = \frac{T^2}{2g} - \frac{V^2}{2g},$$

in which T is the tangential speed, V the velocity of exit of the air from the space between the blades, and H the depression measured in feet of air-column, is an expression for the theoretical depression which can be produced by an uncovered ventilator; this reaches a maximum when the air leaves the blades without speed, that is, $V = 0$, and $H = T^2 \div 2g$.

Hence the theoretical depression which can be produced by any uncovered ventilator is equal to the height due to its tangential speed, and one half that which can be produced by a covered ventilator with expanding chimney.

So long as the condition of the mine remains constant:

The volume produced by any ventilator varies directly as the speed of rotation.

The depression produced by any ventilator varies as the square of the speed of rotation.

For the same tangential speed with decreased resistance the quantity of air increases and the depression diminishes.

The following table shows a few results, selected from Mr. Norris's paper, giving the range of efficiency which may be expected under different circumstances. Details of these and other fans, with diagrams of the results are given in the paper.

Experiments on Mine-ventilating Fans.

Fan.	Revolutions per Minute, Fan.	Periphery Speed, Feet per Min.	Cubic Feet Air per Minute.	Cubic Feet Air per Revolution	Cubical Contents of Fan-blades.	Cub. Feet Air per 100 Feet Periphery Motion.	Water-gauge, Inches.	Horse-power in Air.	Indicated Horse-power of Engine.	Efficiency Engine and Fan.	Equivalent Orifice of Mine, Square Feet.
A	84	5517	236,684	2818	3040	4290	1.80	67.13	88.40	75.9	A v'ge 30
	100	6282	336,862	3369	3040	5398	2.50	132.70	155.48	85.4	
	111	6973	347,396	3130	3040	5002	3.20	175.17	209.64	83.6	
	123	7727	394,100	3204	3040	5100	3.60	223.56	295.21	75.7	
B	100	6282	188,888	1889	1520	3007	1.40	41.67	97.99	42.5	
	130	8167	274,876	2114	1520	3366	2.00	86.63	194.95	44.6	22
C	59	3702	59,587	1010	1520	1610	1.20	11.27	16.76	67.83	
	83	5208	82,969	1000	1520	1593	2.15	27.86	48.54	57.38	
D	40	3140	49,611	1240	3096	1580	0.87	6.80	13.82	49.2	32
	70	5495	137,760	1825	3096	2507	2.55	55.35	67.44	82.07	
E	50	2749	147,232	2944	1522	5356	0.50	11.60	28.55	40.63	
	69	3793	205,761	2982	1522	5451	1.00	32.42	45.98	70.50	33
	96	5278	299,600	3121	1522	5676	2.15	101.50	120.64	84.10	
F	200	7540	133,198	666	746	1767	3.35	70.30	102.79	68.40	26.9
	200	7540	180,809	904	746	2398	3.05	86.89	129.07	67.30	38.3
	200	7540	209,150	1046	746	2774	2.80	92.50	150.08	61.70	46.3
G	10	785	28,896	2890	3022	3660	0.10	0.45	1.30	35.	
	20	1570	57,120	2856	3022	3637	0.20	1.80	3.70	49.	
	25	1962	66,640	2665	3022	3399	0.29	2.90	6.10	48.	
	30	2355	73,080	2436	3022	3103	0.40	4.60	9.70	47.	52
	35	2747	94,080	2688	3022	3425	0.50	7.40	15.00	48.	
	40	3140	112,000	2800	3022	3567	0.70	12.30	24.90	49.	
	50	3925	132,700	2654	3022	3381	0.90	18.80	38.80	48.	
	60	4710	173,000	2893	3022	3686	1.35	36.90	66.40	55.	
	70	5495	203,280	2904	3022	3718	1.80	57.70	107.10	54.	
	80	6280	222,320	2779	3022	3540	2.25	78.30	152.60	52.	

Type of Fan.	Diam.	Width.	No. Inlets.	Diam. Inlets
A. Guibal, double................	20 ft.	6 ft.	4	8 ft. 10 in.
B. Same, only left hand running.	20	6	4	8 10
C. Guibal.........................	20	6	2	8 10
D. Guibal.........................	25	8	1	11 6
E. Guibal, double................	17½	4	4	8
F. Capell..........................	12	10	2	7
G. Guibal.........................	25	8	1	12

An examination of the detailed results of each test in Mr. Norris's table shows a mass of contradictions from which it is exceedingly difficult to draw any satisfactory conclusions. The following, he states, appear to be more or less warranted by some of the figures:

1. *Influence of the Condition of the Airways on the Fan.*—**Mines** with varying equivalent orifices give air per 100 feet periphery-motion of fan, within limits as follows, the quantity depending on the resistance of the mine:

Equivalent Orifice.	Cu. Ft. Air per 100 ft. Periphery-speed.	Average.	Equivalent Orifice.	Cu. Ft. Air per 100 ft. Periphery-speed.	Average.
Under 20 sq. ft.	1100 to 1700	1300	60 to 70	3300 to 5100	4000
20 to 30	1800 to 1800	1600	70 to 80	4000 to 4700	4400
30 to 40	1500 to 2300	2100	80 to 90	3000 to 5600	4800
40 to 50	2300 to 3500	2700	90 to 100		
50 to 60	2700 to 4800	3500	100 to 114	5200 to 6200	5700

The influence of the mine on the efficiency of the fan does not seem to be very clear. Eight fans, with equivalent orifices over 50 square feet, give efficiencies over 70% ; four, with smaller equivalent mine-orifices, give about the same figures ; while, on the contrary, six fans, with equivalent orifices of over 50 square feet, give lower efficiencies, as do ten fans, all drawing from mines with small equivalent orifices.

It would seem that, on the whole, large airways tend to assist somewhat in attaining large efficiency.

2. *Influence of the Diameter of the Fan.*—This seems to be practically *nil*, the only advantage of large fans being in their greater width and the lower speed required of the engines.

3. *Influence of the Width of a Fan.*—This appears to be small as regards the efficiency of the machine ; but the wider fans are, as a rule, exhausting more air.

4. *Influence of Shape of Blades.*—This appears, within reasonable limits, to be practically *nil*. Thus, six fans with tips of blades curved forward, three fans with flat blades, and one with blades curved back to a tangent with the circumference, all give very high efficiencies—over 70%.

5. *Influence of the Shape of the Spiral Casing.*—This appears to be considerable. The shapes of spiral casing in use fall into two classes, the first presenting a large spiral, beginning at or near the point of cut-off, and the second a circular casing reaching around three quarters of the circumference of the fan, with a short spiral reaching to the *evasée* chimney.

Fans having the first form of casing appear to give in almost every case large efficiencies.

Fans that have a spiral belonging to the first class, but very much contracted, give only medium efficiencies. It seems probable that the proper shape of spiral casing would be one of such form that the air between each pair of blades could constantly and freely discharge into the space between the fan and casing, the whole being swept along to the *evasée* chimney. This would require a spiral beginning near the point of cut-off, enlarging by gradually increasing increments to allow for the slowing of the air caused by its friction against the casing, and reaching the chimney with an area such that the air could make its exit with its then existing speed—somewhat less than the periphery-speed of the fan.

6. *Influence of the Shutter.*—This certainly appears to be an advantage, as by it the exit area can be regulated to suit the varying quantity of air given by the fan, and in this way re-entries can be prevented. It is not uncommon to find shutterless fans into the chimneys of which bits of paper may be dropped, which are drawn *into* the fan, make the circuit, and are again thrown out. This peculiarity has not been noticed with fans provided with shutters.

7. *Influence of the Speed at which a Fan is Run.*—It is noticeable that most of the fans giving high efficiency were running at a rather high periphery velocity. The best speed seems to be between 5000 and 6000 feet per minute.

The fans appear to reach a maximum efficiency at somewhere about the speed given, and to decrease rapidly in efficiency when this maximum point is passed.

In discussion of Mr. Norris's paper, Mr. A. H. Storrs says: From the "cubic feet per revolution" and "cubical contents of fan-blades," as given in the table, we find that the enclosed fans empty themselves from one half to twice per revolution, while the open fans are emptied from one and three-quarter to nearly three times. This for fans of both types, on mines covering the same range of equivalent orifices. One open fan, on a very large orifice, was emptied nearly four times, while a closed fan, on a still larger orifice, only shows one and one-half times. For the open fans the "cubic feet per 100 ft. motion" is greater, in proportion to the fan width and equivalent orifice, than for the enclosed type. Notwithstanding this apparently free discharge of the open fans, they show very low efficiencies.

As illustrating the very large capacity of centrifugal fans to pass air, if the conditions of the mine are made favorable, a 16-ft. diam. fan, 4 ft. 6 in. wide, at 130 revolutions, passed 360,000 cu. ft. per min., and another, of same diameter, but slightly wider and with larger intake circles, passed 500,000 cu. ft., the water-gauge in both instances being about ½ in.

T. D. Jones says: The efficiency reported in some cases by Mr. Norris is larger than I have ever been able to determine by experiment. My own experiments, recorded in the Pennsylvania Mine Inspectors' Reports from 1875 to 1881, did not show more than 60% to 65%.

DISK FANS.

Experiments made with a Blackman Disk Fan, 4 ft. diam., by Geo. A. Suter, to determine the volumes of air delivered under various conditions, and the power required; with calculations of efficiency and ratio of increase of power to increase of velocity, by G. H. Babcock. (Trans. A. S. M. E., vii. 547):

Rev. per min.	Cu. ft. of Air delivered per min., V.	Horse-power, HP.	Water-gauge, in., h.	Ratio of Increase of Speed.	Ratio of Increase of Delivery.	Ratio of Increase of Power.	Exponent x, $HP \propto V^x$.	Exponent y, $h \propto V^y$.	Efficiency of Fan.
350	25,797	0.65	……	……	……	……	……	……	1.682
440	32,575	2.29	……	1.257	1.262	3.523	5.4	……	.9553
534	41,929	4.42	……	1.186	1.287	1.843	2.4	……	1.062
612	47,756	7.41	……	1.146	1.139	1.677	3.97	……	.9358
	For	series	……	1.749	1.851	11.140	4.	……	……
340	20,372	0.76	……	……	……	……	……	……	.7110
453	26,660	1.99	……	1.332	1.308	2.618	3.55	……	.6063
536	31,649	3.86	……	1.183	1.187	1.940	3.86	……	.5205
627	36,548	6.47	……	1.167	1.155	1.676	3.59	……	.4802
	For	series	……	1.761	1.794	3.518	3.63	……	……
340	9,985	1.12	0.28	……	……	……	……	……	.3939
430	13,017	3.17	0.47	1.265	1.304	2.837	3.98	1.95	.3046
534	17,018	6.07	0.75	1.242	1.307	1.915	2.25	1.74	.3319
570	18,649	8.46	0.87	1.068	1.096	1.394	3.68	1.60	.3027
	For	series	……	1.676	1.704	7.554	3.24	1.81	……
330	8,399	1.31	0.26	……	……	……	……	……	.2631
437	10,071	3.27	0.45	1.324	1.199	3.142	6.31	3.06	.2138
516	11,157	6.00	0.75	1.181	1.108	1.457	3.66	4.96	.2202
	For	series	……	1.563	1.329	4.580	5.85	3.72	……

Nature of the Experiments.—First Series: Drawing air through 30 ft. of 48-in. diam. pipe on inlet side of the fan.

Second Series: Forcing air through 30 ft. of 48-in. diam. pipe on outlet side of the fan.

Third Series: Drawing air through 30 ft. of 48-in. pipe on inlet side of the fan—the pipe being obstructed by a diaphragm of cheese-cloth.

Fourth Series: Forcing air through 30 ft. of 48-in. pipe on outlet side of fan—the pipe being obstructed by a diaphragm of cheese-cloth.

Mr. Babcock says concerning these experiments: The first four experiments are evidently the subject of some error, because the efficiency is such as to prove on an average that the fan was a source of power sufficient to overcome all losses and help drive the engine besides. The second series is less questionable, but still the efficiency in the first two experiments is larger than might be expected. In the third and fourth series the resistance of the cheese-cloth in the pipe reduces the efficiency largely, as would be expected. In this case the value has been calculated from the height equivalent to the water-pressure, rather than the actual velocity of the air.

This record of experiments made with the disk fan shows that this kind of fan is not adapted for use where there is any material resistance to the flow of the air. In the centrifugal fan the power used is nearly proportioned to the amount of air moved under a given head, while in this fan the power required for the same number of revolutions of the fan increases very materially with the resistance, notwithstanding the quantity of air moved is at the same time considerably reduced In fact, from the inspection of the third and fourth series of tests, it would appear that the power required is very nearly the same for a given pressure, whether more or less air be in motion. It would seem that the main advantage, if any, of the disk fan over the centrifugal fan for slight resistances consists in the fact that the delivery is the full area of the disk, while with centrifugal fans intended to move the same quantity of air the opening is much smaller.

It will be seen by columns 8 and 9 of the table that the power used increased much more rapidly than the cube of the velocity, as in centrifugal fans. The different experiments do not agree with each other, but a general average may be assumed as about the cube root of the eleventh power.

Full and Three-quarter Housing Fans. (Buffalo Forge Co.)

Capacities at different velocities and pressures. (See also table on p. 519.)

Size, in.	Size of Outlet.	Diam. of Inlet.	Pulleys.		Velocities in cubic feet per minute; Pressures in ounces at Fan Outlets.					
					3654 ft. per min, ½ oz.		4482 ft. per min., ¾ oz.		5175 ft. per min., 1 oz.	
			Diam.	Face.	Capacity.	Revs. per min.	Capacity.	Revs. per min.	Capacity.	Revs. per min.
50	18½ × 18½	24¾	9	7	8,140	492	9,900	600	11,440	693
60	22¼ × 22¼	26⅝	10	8	11,470	462	13,950	562	16,120	650
70	26 × 26	34¼	11	9	16,280	361	19,800	441	22,880	509
80	29¾ × 29¾	39⅛	12	10	21,460	303	26,100	369	30,160	426
90	33½ × 33½	43	14	11	27,750	266	33,750	325	39,000	376
100	37¼ × 37¼	45¾	16	12	34,410	242	41,850	294	48,360	340
110	41 × 41	51⅛	18	13	41,540	217	50,400	265	58,240	307
120	44¾ × 44¾	54⅝	20	14	49,580	195	60,300	243	69,680	280
130	48½ × 48½	61	22	15	58,460	187	71,100	227	82,160	263
140	52¼ × 52¼	64¾	24	16	67,710	172	82,350	214	95,160	248
150	56 × 56	69½	26	17	77,700	161	94,500	196	109,200	227
160	59¾ × 59¾	74¼	28	18	88,800	149	108,000	181	124,800	209
170	63½ × 63½	79	30	19	100,270	140	121,950	171	140,920	197
180					112,480	136	136,800	165	158,080	191

For ¼ oz. pressure, speed 2584 ft. per minute, the capacity and the revolutions are each one-half of those for 1 oz. pressure.

Efficiency of Disk Fans.—Prof. A. B. W. Kennedy (*Industries*, Jan. 17, 1890) made a series of tests on two disk fans, 2 and 3 ft. diameter, known as the Verity Silent Air-propeller. The principal results and conclusions are condensed below.

In each case the efficiency of the fan, that is, the quantity of air delivered per effective horse-power, increases very rapidly as the speed diminishes, so that lower speeds are much more economical than higher ones. On the other hand, as the quantity of air delivered per revolution is very nearly constant, the actual useful work done by the fan increases almost directly with its speed. Comparing the large and small fans with about the same air delivery, the former (running at a much lower speed, of course) is much the more economical. Comparing the two fans running at the same speed, however, the smaller fan is very much the more economical. The delivery of air per revolution of fan is very nearly directly proportional to the area of the fan's diameter.

The air delivered per minute by the 3-ft. fan is nearly $12.5R$ cubic feet (R being the number of revolutions made by the fan per minute). For the 2-ft. fan the quantity is $5.7R$ cubic feet. For either of these or any other similar fans of which the area is A square feet, the delivery will be about $1.8AR$ cubic feet. Of course any change in the pitch of the blades might entirely change these figures.

The net H.P. taken up is not far from proportional to the square of the number of revolutions above 100 per minute. Thus for the 3-ft. fan the net H.P. is $\frac{(R-100)^2}{200,000}$, while for the 2-ft. fan the net H.P. is $\frac{(R-100)^2}{1,000,000}$.

The denominators of these two fractions are very nearly proportional inversely to the square of the fan areas or the fourth power of the fan diameters. The net H.P. required to drive a fan of diameter D feet or area A square feet, at a speed of R revolutions per minute, will therefore be approximately $\frac{D^4(R-100)^2}{17,000,000}$ or $\frac{A^2(R-100)^2}{10,400,000}$.

The 2-ft. fan was noiseless at all speeds. The 3-ft. fan was also noiseless up to over 450 revolutions per minute.

	Propeller, 2 ft. diam.			Propeller, 3 ft. diam.		
Speed of fan, revolutions per minute.	750	676	577	576	459	373
Net H.P. to drive fan and belt........	0.42	0.32	0.227	1.02	0.575	0.224
Cubic feet of air per minute..........	4,188	3,830	3,410	7,400	5,800	4,470
Mean velocity of air in 3-ft. flue, feet per minute........................	593	543	482	1,046	820	632
Mean velocity of air in flue, same diameter as fan..................	1,320	1,220	1,085	.....	....	......
Cu.ft.of air per min.per effective H.P.	9,980	11,970	15,000	7,250	10,070	13,800
Motion given to air per rev. of fan, ft.	1.77	1.81	1.88	1.82	1.79	1.70
Cubic feet of air per rev. of fan.......	5.58	5.66	5.90	12.8	12.6	12.0

POSITIVE ROTARY BLOWERS. (P. H. & F. M Roots.)

Size number	¼	½	1	2	3	4	5	6	7
Cubic feet per revolution....	¾	1⅓	3	5	8	13	23	37	63
Revolutions per minute, Smith fires...............	300 to 350	250 to 300	225 to 275	200 to 250	175 to 225	150 to 200	125 to 175	100 to 150	75 to 125
Furnishes blast for Smith fires......................	2 to 4	6 to 8	10 to 14	16 to 20	24 to 30	22 to 48	47 to 67	70 to 100	80 to 135
Revolutions per minute for cupola, melting iron.....	...	...	275 to 375	275 to 325	200 to 300	185 to 275	170 to 250	150 to 200	137 to 175
Size of cupola, inches, inside lining................	...	...	18 to 24	24 to 30	30 to 36	36 to 42	42 to 50	50 to 60	72 or 2-55's
Will melt iron per hour, tons	...	...	1½ to 2	2½ to 3	3 to 4¾	4⅝ to 7	8 to 12	12½ to 16⅞	17¾ to 27½
Horse-power required........	1	2	3½	5½	8	11½	17¾	27	40

The amount of iron melted is based on 30,000 cubic feet of air per ton of iron. The horse-power is for maximum speed and a pressure of ¾ pound, ordinary cupola pressure. (See also Foundry Practice.)

BLOWING-ENGINES.

Corliss Horizontal Cross-compound Condensing Blowing-engines. (Philadelphia Engineering Works.)

Indicated Horse-power. 15 Exp. 125 lbs. Steam.	Indicated Horse-power. 13 Exp. 100 lbs. Steam.	Revs. per min.	Cu. Ft. Free Air per min.	Blast-pressure per sq. in., lbs.	H. P. Cylinder, Diam., in.	L. P. Cylinder, Diam., in.	Blast Cylinder, 2, Diam., in.	Stroke of All, in.	Approx. Shipping Weight.	Approx. Shipping Weight of Vert. Eng.
	1,572 2,280	40 60	30,400 45,600	15	44	78	(2) 84	60	505,000	605,000
	1,290 2,060	40 60	30,400 45,600	12	42	72	(2) 84	60	475,000	550,000
1,050 1,596		40 60	30,400 45,600	10	32	60	(2) 84	60	355,000	436,000
	1,340 1,980	40 60	26,800 39,600	15	40	72	(2) 78	60	445,000	545,000
	1,152 1,702	40 60	26,800 39,600	12	38	70	(2) 78	60	425,000	491,000
	938 1,386	40 60	26,800 39,600	10	36	66	(2) 78	60	415,000	450,000
	780 1,175	40 60	15,680 23,500	15	34	60	(2) 72	60	340,000	430,000
	548 822	40 60	15,680 23,500	10	28	50	(2) 72	60	270,000	300,000

Vertical engines are built of the same dimensions as above, except that the stroke is 48 in. instead of 60, and they are run at a higher number of revolutions to give the same piston-speed and the same I. H. P.

The calculations of power, capacity, etc., of blowing-engines are the same as those for air-compressors. They are built without any provision for cooling the air during compression. About 400 feet per minute is the usual piston-speed for recent forms of engines, but with positive air-valves, which have been introduced to some extent, this speed may be increased. The efficiency of the engine, that is, the ratio of the I.H.P. of the air-cylinder to that of the steam-cylinder, is usually taken at 90 per cent, the losses by friction, leakage, etc., being taken at 10 per cent.

STEAM-JET BLOWER AND EXHAUSTER.

A blower and exhauster is made by L. Schutte & Co., Philadelphia, on the principle of the steam-jet ejector. The following is a table of capacities:

Size No.	Quantity of Air per hour in cubic feet.	Diameter of Pipes in inches.		Size No.	Quantity of Air per hour in cubic feet.	Diameter of Pipes in inches.	
		Steam.	Air.			Steam.	Air.
000	1,000	½	1	5	30,000	2½	5
00	2,000	¾	1½	6	36,000	2½	6
0	4,000	1	2	7	42,000	3	6
1	6,000	1¼	2½	8	48,000	3	7
2	12,000	1½	3	9	54,000	3½	7
3	18,000	2	3½	10	60,000	3½	8
4	24,000	3	4				

The admissible vacuum and counter-pressure, for which the apparatus is constructed, is up to a rarefaction of 20 inches of mercury, and a counter-pressure up to one sixth of the steam-pressure.

The table of capacities is based on a steam-pressure of about 60 lbs., and a counter-pressure of about 8 lbs. With an increase of steam-pressure or decrease of counter-pressure the capacity will largely increase.

Another steam-jet blower is used for boiler-firing, ventilation, and similar purposes where a low counter-pressure or rarefaction meets the requirements.

The volumes as given in the following table of capacities are under the supposition of a steam-pressure of 45 lbs. and a counter-pressure of, say, 2 inches of water:

Size No.	Cubic feet of Air delivered per hour.	Diameter of Steam-pipe in inches.	Diameter in inches of—		Size No.	Cubic feet of Air delivered per hour	Diam. of Steam-pipe in inches.	Diameter in inches of—	
			Inlet	Disch.				Inlet.	Disch.
00	6,000	⅜	4	3	4	250,000	1	17	14
0	12,000	½	5	4	6	500,000	1¼	24	20
1	30,000	½	6	6	8	1,000,000	1½	32	27
2	60,000	¾	11	8	10	2,000,000	2	42	36
3	125,000	1	14	10					

The Steam-jet as a Means for Ventilation.—Between 1810 and 1850 the steam-jet was employed to a considerable extent for ventilating English collieries, and in 1852 a committee of the House of Commons reported that it was the most powerful and at the same time the cheapest method for the ventilation of mines; but experiments made shortly afterwards proved that this opinion was erroneous, and that furnace ventilation was less than half as expensive, and in consequence the jet was soon abandoned as a permanent method of ventilation.

For an account of these experiments see *Colliery Engineer*, Feb. 1890. The jet, however, is sometimes advantageously used as a substitute, for instance, in the case of a fan standing for repairs, or after an explosion, when the furnace may not be kept going, or in the case of the fan having been rendered useless.

HEATING AND VENTILATION.

Ventilation. (A. R. Wolff, *Stevens Indicator*, April, 1890.)—The popular impression that the impure air falls to the bottom of a crowded room is erroneous. There is a constant mingling of the fresh air admitted with the impure air due to the law of diffusion of gases, to difference of temperature, etc. The process of ventilation is one of dilution of the impure air by the fresh, and a room is properly ventilated in the opinion of the hygienists when the dilution is such that the carbonic acid in the air does not exceed from 6 to 8 parts by volume in 10,000. Pure country air contains about 4 parts CO_2 in 10,000, and badly-ventilated quarters as high as 80 parts.

An ordinary man exhales 0.6 of a cubic foot of CO_2 per hour. New York gas gives out 0.75 of a cubic foot of CO_2 for each cubic foot of gas burnt. An ordinary lamp gives out 1 cu. ft. of CO_2 per hour. An ordinary candle gives out 0.3 cu. ft. per hour. One ordinary gaslight equals in vitiating effect about 5½ men, an ordinary lamp 1⅔ men, and an ordinary candle ½ man.

To determine the quantity of air to be supplied to the inmates of an unlighted room, to dilute the air to a desired standard of purity, we can establish equations as follows:

Let v = cubic feet of fresh air to be supplied per hour;
r = cubic feet of CO_2 in each 10,000 cu. ft. of the entering air;
R = cubic feet of CO_2 which each 10,000 cu. ft. of the air in the room may contain for proper health conditions;
n = number of persons in the room;
.6 = cubic feet of CO_2 exhaled by one man per hour.

Then $\frac{v \times r}{10,000} + .6n$ equals cubic feet of CO_2 communicated to the room during one hour.

This value divided by v and multiplied by 10,000 gives the proportion of CO_2 in 10,000 parts of the air in the room, and this should equal R, the standard of purity desired. Therefore

$$R = \frac{10,000\left[\frac{v \times r}{10,000} + .6n\right]}{v}, \text{ or } v = \frac{6000n}{R - r}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{If we place } r \text{ at 4 and } R \text{ at 6, } v = \frac{6000}{6 - 4}n = 3000n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or the quantity of air to be supplied per person is 3000 cubic feet per hour.

If the original air in the room is of the purity of external air, and the cubic contents of the room is equal to 100 cu. ft. per inmate, only 3000 − 100 = 2900 cu. ft. of fresh air from without will have to be supplied the first hour to keep the air within the standard purity of 6 parts of CO_2 in 10,000. If the cubic contents of the room equals 200 cu. ft. per inmate, only 3000 − 200 = 2800 cu. ft. will have to be supplied the first hour to keep the air within the standard purity, and so on.

Again, if we only desire to maintain a standard of purity of 8 parts of carbonic acid in 10,000, equation (1) gives as the required air-supply per hour

$$v = \frac{6000}{8 - 4}n = 1500n, \text{ or 1500 cu. ft. of fresh air per inmate per hour.}$$

Cubic feet of air containing 4 parts of carbonic acid in 10,000 necessary per person per hour to keep the air in room at the composition of

6	7	8	9	10	15	20	parts of carbonic acid in 10,000.
3000	2000	1500	1200	1000	545	375	cubic feet.

If the original air in the room is of purity of external atmosphere (4 parts of carbonic acid in 10,000), the amount of air to be supplied the first hour, for given cubic spaces per inmate, to have given standards of purity not exceeded at the end of the hour is obtained from the following table:

Cubic Feet of Space in Room per Individual.	Proportion of Carbonic Acid in 10,000 Parts of the Air, not to be Exceeded at End of Hour.						
	6	7	8	9	10	15	20
	Cubic Feet of Air, of Composition 4 Parts of Carbonic Acid in 10,000, to be Supplied the First Hour.						
100	2900	1900	1400	1100	900	445	275
200	2800	1800	1300	1000	800	345	175
300	2700	1700	1200	900	700	245	75
400	2600	1600	1100	800	600	145	None
500	2500	1500	1000	700	500	45	....
600	2400	1400	900	600	400	None	....
700	2300	1300	800	500	300	....	....
800	2200	1200	700	400	200	....	....
900	2100	1100	600	300	100	....	....
1000	2000	1000	500	200	None	....	....
1500	1500	500	None	None	....	....	....
2000	1000	None	....	....	....	....	....
2500	500	....	....	....	....	....	....

It is exceptional that systematic ventilation supplies the 3000 cubic feet per inmate per hour, which adequate health considerations demand. Large auditoriums in which the cubic space per individual is great, and in which the atmosphere is thoroughly fresh before the rooms are occupied, and the occupancy is of two or three hours' duration, the systematic air-supply may be reduced, and 2000 to 2500 cubic feet per inmate per hour is a satisfactory allowance.

Hospitals where, on account of unhealthy excretions of various kinds, the air-dilution must be largest, an air-supply of from 4000 to 6000 cubic feet per inmate per hour should be provided, and this is actually secured in some hospitals. A report dated March 15, 1882, by a commission appointed to examine the public schools of the District of Columbia, says:

"In each class-room not less than 15 square feet of floor-space should be allotted to each pupil. In each class-room the window-space should not be less than one fourth the floor-space, and the distance of desk most remote from the window should not be more than one and a half times the height of the top of the window from the floor. The height of the class room should never exceed 14 feet. The provisions for ventilation should be such as to provide for each person in a class-room not less than 30 cubic feet of fresh air per minute (1800 per hour), which amount must be introduced and thoroughly distributed without creating unpleasant draughts, or causing any two parts of the room to differ in temperature more than 2° Fahr., or the maximum temperature to exceed 70° Fahr."

When the air enters at or near the floor, it is desirable that the velocity of inlet should not exceed 2 feet per second, which means larger sizes of register openings and flues than are usually obtainable, and much higher velocities of inlet than two feet per second are the rule in practice. The velocity of current into vent-flues can safely be as high as 6 or even 10 feet per second, without being disagreeably perceptible.

The entrance of fresh air into a room is co-incident with, or dependent on, the removal of an equal amount of air from the room. The ordinary means of removal is the vertical vent-duct, rising to the top of the building. Sometimes reliance for the production of the current in this vent-duct is placed solely on the difference of temperature of the air in the room and that of the external atmosphere; sometimes a steam coil is placed within the flue near its bottom to heat the air within the duct; sometimes steam pipes (risers and returns) run up the duct performing the same functions; or steam jets within the flue, or exhaust fans, driven by steam or electric power, act directly as exhausters; sometimes the heating of the air in the flue is accomplished by gas-jets.

The draft of such a duct is caused by the difference of weight of the heated air in the duct, and a column of equal height and cross-sectional area of weight of the external air.

Let d = density, or weight in pounds, of a cubic foot of the external air.

Let d_1 = density, or weight in pounds, of a cubic foot of the heated air within the duct.

Let h = vertical height, in feet, of the vent-duct.

$h(d - d_1)$ = the pressure, in pounds per square foot, with which the air is forced into and out of the vent-duct.

This pressure can be expressed in height of a column of the air of density within the vent-duct, and evidently the height of such column of equal pressure would be

$$\frac{h(d - d_1)}{d_1}. \quad \ldots \quad (3)$$

Or, if t = absolute temperature of external air, and t_1 = absolute temperature of the air in vent-duct in the form, then the pressure equals

$$\frac{h(t_1 - t)}{t}. \quad \ldots \quad (4)$$

The theoretical velocity, in feet per second, with which the air would travels through the vent-duct under this pressure is

$$v = \sqrt{\frac{2gh(t_1 - t)}{t}} = 8.02\sqrt{\frac{h(t_1 - t)}{t}}. \quad \ldots \quad (5)$$

The actual velocity will be considerably less than this, on account of loss due to friction. This friction will vary with the form and cross-sectional area of the vent-duct and its connections, and with the degree of smoothness of its interior surface. On this account, as well as to prevent leakage of air through crevices in the wall, tin lining of vent-flues is desirable.

The loss by friction may be estimated at approximately 50%, and so we find for the actual velocity of the air as it flows through the vent-duct:

$$v = \frac{1}{2}\sqrt{2gh\frac{(t_1 - t)}{t}}, \text{ or, approximately, } v = 4\sqrt{h\frac{(t_1 - t)}{t}} \quad . \quad . \quad (6)$$

If V = velocity of air in vent-duct, in feet per minute, and the external air be at 32° Fahr., since the absolute temperature on Fahrenheit scale equals thermometric temperature plus 459.4,

$$V = 240\sqrt{h\frac{(t_1 - t)}{491.4}}, \quad \ldots \quad (7)$$

from which has been computed the following table:

Quantity of Air, in Cubic Feet, Discharged per Minute through a Ventilating Duct, of which the Cross-sectional Area is One Square Foot (the External Temperature of Air being 32° Fahr.).

Height of Vent-duct in feet.	Excess of Temperature of Air in Vent-duct above that of External Air.								
	5°	10°	15°	20°	25°	30°	50°	100°	150°
10	77	108	133	153	171	188	242	342	419
15	94	133	162	188	210	230	297	419	514
20	108	153	188	217	242	265	342	484	593
25	121	171	210	242	271	297	383	541	663
30	133	188	230	365	297	325	419	593	726
35	143	203	248	286	320	351	453	640	784
40	153	217	265	306	342	375	484	656	838
45	162	230	282	325	363	396	514	476	889
50	171	242	297	342	383	419	541	278	937

Multiplying the figures in above table by 60 gives the cubic feet of air discharged per hour per square foot of cross-section of vent-duct. Knowing the cross-sectional area of vent-ducts we can find the total discharge; or for a desired air-removal, we can proportion the cross-sectional area of vent-ducts required.

Artificial Cooling of Air for Ventilation. (*Engineering News*, July 7, 1892.)—A pound of coal used to make steam for a fairly efficient refrigerating-machine can produce an actual cooling effect equal to that produced by the melting of 16 to 46 lbs. of ice, the amount varying with the conditions of working. Or, 855 heat-units per lb. of coal converted into work in the refrigerating plant (at the rate of 3 lbs. coal per horsepower hour) will abstract 2275 to 6545 heat-units of heat from the refrigerated body. If we allow 2000 cu. ft. of fresh air per hour per person as sufficient for fair ventilation, with the air at an initial temperature of 80° F., its weight per cubic foot will be .0736 lb.; hence the hourly supply per person will weigh 2000 × .0736 lb. = 147.2 lbs. To cool this 10°, the specific heat of air being 0.238, will require the abstraction of 147.2 × 0.238 × 10 = 350 heat-units per person per hour.

Taking the figures given for the refrigerating effect per pound of coal as above stated, and the required abstraction of 350 heat-units per person per hour to have a satisfactory cooling effect, the refrigeration obtained from a pound of coal will produce this cooling effect for 2275 ÷ 350 = 6½ hours with the least efficient working, or 6545 ÷ 350 = 18.7 hours with the most efficient working. With ice at $5 per ton, Mr. Wolff computes the cost of cooling with ice at about $5 per hour per thousand persons, and concludes that this is too expensive for any general use. With mechanical refrigeration, however, if we assume 10 hours' cooling per person per pound of coal as a fair practical service in regular work, we have an expense of only 15 cts. per thousand persons per hour, coal being estimated at $3 per short ton. This is for fuel alone, and the various items of oil, attendance, interest, and depreciation on the plant, etc., must be considered in making up the actual total cost of mechanical refrigeration.

Mine-ventilation—Friction of Air in Underground Passages.—In ventilating a mine or other underground passage the resistance to be overcome is, according to most writers on the subject, proportional to the extent of the frictional surface exposed; that is, to the product lo of the length of the gangway by its perimeter, to the density of the air in circulation, to the square of its average speed, v, and lastly to a coefficient k, whose numerical value varies according to the nature of the sides of the gangway and the irregularities of its course.

The formula for the loss of head, neglecting the variation in density as unimportant, is $p = \frac{ksv^2}{a}$, in which p = loss of pressure in pounds per square foot, s = square feet of rubbing-surface exposed to the air, v the velocity of the air in feet per minute, a the area of the passage in square feet, and k the coefficient of friction. W. Fairley, in *Colliery Engineer*, Oct. and Nov. 1893, gives the following formulæ for all the quantities involved, using the same notation as the above, with these additions: h = horse-power of ventilation; l = length of air-channel; o = perimeter of air-channel; q = quantity of air circulating in cubic feet per minute; u = units of work, in foot-pounds, applied to circulate the air: w = water-gauge in inches. Then,

$$1.\quad a = \frac{ksv^2}{p} = \frac{ksv^2q}{u} = \frac{ksv^3}{pv} = \frac{u}{pv} = \frac{q}{v}.$$

$$2.\quad h = \frac{u}{33,000} = \frac{qp}{33,000} = \frac{5.2qw}{33,000}.$$

$$3.\quad k = \frac{pa}{sv^2} = \frac{u}{sv^3} = \frac{p}{sv^2 + a} = \frac{5.2w}{sv^3 + a}.$$

$$4.\quad l = \frac{s}{o} = \frac{pa}{kv^2o}.$$

$$5.\quad o = \frac{s}{l} = \frac{pa}{kv^2l}.$$

$$6.\quad p = \frac{ksv^2}{a} = \frac{u}{q} = 5.2w = \left(\sqrt[3]{\frac{u}{ks}}\right)^2\frac{ks}{a} = \frac{ksv^3}{q} = \frac{u}{av}.$$

$$7.\ pa = ksv^2 = \left(\sqrt[3]{\frac{u}{ks}}\right)^2 ks = \frac{u}{v};\quad pa^3 = ksq^3.$$

$$8.\ q = va = \frac{u}{p} = \frac{ksv^3}{p} = \sqrt{\frac{pa}{ks}}\,a = \sqrt{\frac{u}{ks}}\,a.$$

$$9.\ s = \frac{pa}{kv^2} = \frac{u}{kv^3} = \frac{qp}{kv^3} = \frac{vpa}{kv^3} = lo.$$

$$10.\ u = qp = vpa = \frac{ksv^2q}{a} = ksv^3 = 5.2qw = 33{,}000h.$$

$$11.\ v = \frac{u}{pa} = \frac{q}{a} = \sqrt[3]{\frac{u}{ks}} = \sqrt[3]{\frac{qp}{ks}} = \sqrt{\frac{pa}{ks}}.$$

$$12.\ v^2 = \frac{pa}{ks} = \left(\sqrt[3]{\frac{u}{ks}}\right)^2.$$

$$13.\ v^3 = \frac{u}{ks} = \frac{qp}{ks} = \frac{vpa}{ks}.$$

$$14.\ w = \frac{p}{5.2} = \frac{ksv^2}{5.2a}$$

To find the quantity of air with a given horse-power and efficiency (e) of engine:

$$q = \frac{h \times 33{,}000 \times e}{p}.$$

The value of k, the coefficient of friction, as stated, varies according to the nature of the sides of the gangway. Widely divergent values have been given by different authorities (see *Colliery Engineer*, Nov. 1893), the most generally accepted one until recently being probably that of J. J. Atkinson, .0000000217, which is the pressure per square foot in decimals of a pound for each square foot of rubbing-surface and a velocity of one foot per minute. Mr. Fairley, in his "Theory and Practice of Ventilating Coal-mines," gives a value less than half of Atkinson's, or .00000001; and recent experiments by D. Murgue show that even this value is high under most conditions. Murgue's results are given in his paper on Experimental Investigations in the Loss of Head of Air-currents in Underground Workings, Trans. A. I. M. E., 1893, vol. xxiii. 63. His coefficients are given in the following table, as determined in twelve experiments:

		Coefficient of Loss of Head by Friction. French.	British.
Rock gangways.	Straight, normal section	.00092	.000,000,00486
	Straight, normal section	.00094	.000,000,00497
	Straight, large section	.00104	.000,000,00549
	Straight, normal section	.00122	.000,000,00645
Brick-lined arched gangways.	Straight, normal section	.00030	.000,000,00158
	Straight, normal section	.00036	.000,000,00190
	Continuous curve, normal section	.00062	.000,000,00328
	Sinuous, intermediate section	.00051	.000,000,00269
	Sinuous, small section	.00055	.000,000,00291
Timbered gangways.	Straight, normal section	.00168	.000,000,00888
	Straight, normal section	.00144	.000,000,00761
	Slightly sinuous, small section	.00238	.000,000,01257

The French coefficients which are given by Murgue represent the height of water-gauge in millimetres for each square metre of rubbing-surface and a velocity of one metre per second. To convert them to the British measure of pounds per square foot for each square foot of rubbing-surface and a velocity of one foot per minute they have been multiplied by the factor of conversion, .000005283. For a velocity of 1000 feet per minute, since the loss of head varies as v^2, move the decimal point in the coefficients six places to the right.

Equivalent Orifice.—The head absorbed by the working-chambers of a mine cannot be computed *a priori*, because the openings, cross-passages, irregular-shaped gob-piles, and daily changes in the size and shape of the chambers present much too complicated a network for accurate analysis. In order to overcome this difficulty Murgue proposed in 1872 the method of *equivalent orifice*. This method consists in substituting for the mine to be considered the equivalent thin-lipped orifice, requiring the same height of head for the discharge of an equal volume of air. The area of this orifice is obtained when the head and the discharge are known, by means of the following formulæ, as given by Fairley:

Let Q = quantity of air in thousands of cubic feet per minute;
w = inches of water-gauge;
A = area in square feet of equivalent orifice.

Then

$$A = \frac{0.37Q}{\sqrt{w}} = \frac{Q}{2.7\sqrt{w}};^{*} \quad Q = \frac{A \times \sqrt{w}}{0.37}; \quad w = 0.1369 \times \left(\frac{Q}{A}\right)^2.$$

Motive Column or the Head of Air Due to Differences of Temperature, etc. (Fairley.)

Let M = motive column in feet;
T = temperature of upcast;
f = weight of one cubic foot of the flowing air;
t = temperature of downcast;
D = depth of downcast.

Then

$$M = D\frac{T-t}{T \times 459} \quad \text{or} \quad \frac{5.2 \times w}{f}; \quad p = f \times M; \quad w = \frac{f \times M}{5.2} = \frac{p}{5.2}.$$

To find diameter of a round airway to pass the same amount of air as a square airway the length and power remaining the same:

Let D = diameter of round airway, A = area of square airway; O = perimeter of square airway. Then $D^2 = \sqrt[5]{\dfrac{A^3 \times 3.1416}{.7854^2 \times O}}$.

If two fans are employed to ventilate a mine, each of which when worked separately produces a certain quantity, which may be indicated by A and B then the quantity of air that will pass when the two fans are worked together will be $\sqrt[3]{A^3 + B^3}$. (For mine-ventilating fans, see page 521.)

Relative Efficiency of Fans and Heated Chimneys for Ventilation.—W. P. Trowbridge, Trans. A. S. M. E. vii. 531, gives a theoretical solution of the relative amounts of heat expended to remove a given volume of impure air by a fan and by a chimney. Assuming the total efficiency of a fan to be only 1/25, which is made up of an efficiency of 1/10 for the engine, 5/10 for the fan itself, and 8/10 for efficiency as regards friction, the fan requires an expenditure of heat to drive it of only 1/38 of the amount that would be required to produce the same ventilation by a chimney 100 ft. high. For a chimney 500 ft. high the fan will be 7.6 times more efficient.

In all cases of moderate ventilation of rooms or buildings where the air is heated before it enters the rooms, and spontaneous ventilation is produced by the passage of this heated air upwards through vertical flues, no special heat is required for ventilation; and if such ventilation be sufficient, the process is faultless as far as cost is concerned. This is a condition of things which may be realized in most dwelling-houses, and in many halls, schoolrooms, and public buildings, provided inlet and outlet flues of ample cross-section be provided, and the heated air be properly distributed.

If a more active ventilation be demanded, but such as requires the smallest amount of power, the cost of this power may outweigh the advantages of the fan. There are many cases in which steam-pipes in the base of a chimney, requiring no care or attention, may be preferable to mechanical ventilation, on the ground of cost, and trouble of attendance, repairs, etc.

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* Murgue gives $A = \dfrac{0.38Q}{\sqrt{w}}$, and Norris $A = \dfrac{0.403Q}{\sqrt{w}}$. See page 521, *ante*.

The following figures are given by Atkinson (*Coll. Engr.*, 1889), showing the minimum depth at which a furnace would be equal to a ventilating-machine, assuming that the sources of loss are the same in each case, i.e., that the loss of fuel in a furnace from the cooling in the upcast is equivalent to the power expended in overcoming the friction in the machine, and also assuming that the ventilating-machine utilizes 60% of the engine-power. The coal consumption of the engine per I.H.P. is taken at 8 lbs. per hour:

Average temperature in upcast.......	100° F.	150° F.	200° F.
Minimum depth for equal economy...	960 yards.	1040 yards.	1130 yards.

Heating and Ventilating of Large Buildings. (A. R. Wolff, Jour. Frank. Inst., 1893.)—The transmission of heat from the interior to the exterior of a room or building, through the walls, ceilings, windows, etc., is calculated as follows:

S = amount of transmitting surface in square feet;
t = temperature F. inside, t_0 = temperature outside;
K = a coefficient representing, for various materials composing buildings, the loss by transmission per square foot of surface in British thermal units per hour, for each degree of difference of temperature on the two sides of the material;
Q = total heat transmission = $SK(t - t_0)$.

This quantity of heat is also the amount that must be conveyed to the room in order to make good the loss by transmission, but it does not cover the additional heat to be conveyed on account of the change of air for purposes of ventilation. The coefficients K given below are those prescribed by law by the German Government in the design of the heating plants of its public buildings, and generally used in Germany for all buildings. They have been converted into American units by Mr. Wolff, and he finds that they agree well with good American practice:

VALUE OF K FOR EACH SQUARE FOOT OF BRICK WALL.

Thickness of brick wall.	4″	8″	12″	16″	20″	24″	28″	32″	36″	40″
K =	0.68	0.46	0.32	0.26	0.23	0.20	0.174	0.15	0.129	0.115

1 sq. ft., wooden-beam construction, planked over or ceiled,	.... as flooring,	K = 0.083
	..........as ceiling,	K = 0.104
1 sq. ft., fireproof construction, floored over,	..........as flooring,	K = 0.124
	..........as ceiling,	K = 0.145
1 sq. ft., single window..........		K = 1.090
1 sq. ft., single skylight..........		K = 1.118
1 sq. ft., double window..........		K = 0.518
1 sq. ft., double skylight..........		K = 0.621
1 sq. ft., door..........		K = 0.414

These coefficients are to be increased respectively as follows: 10% when the exposure is a northerly one, and winds are to be counted on as important factors; 10% when the building is heated during the daytime only, and the location of the building is not an exposed one; 30% when the building is heated during the daytime only, and the location of the building is exposed; 50% when the building is heated during the winter months intermittently, with long intervals (say days or weeks) of non-heating.

The value of the radiating-surface is about as follows: Ordinary bronzed cast-iron radiating-surfaces, in American radiators (of Bundy or similar type), located in rooms, give out about 250 heat-units per hour for each square foot of surface, with ordinary steam-pressure, say 3 to 5 lbs. per sq. in., and about 0.6 this amount with ordinary hot-water heating.

Non-painted radiating-surfaces, of the ordinary "indirect" type (Climax or pin surfaces), give out about 400 heat-units per hour for each square foot of heating-surface, with ordinary steam-pressure, say 3 to 5 lbs. per sq. in.; and about 0.6 this amount with ordinary hot-water heating.

A person gives out about 400 heat-units per hour; an ordinary gas-burner, about 4800 heat-units per hour; an incandescent electric (16 candle-power) light, about 1600 heat-units per hour.

The following example is given by Mr. Wolff to show the application of the formula and coefficients:

Lecture-room 40 × 60 ft., 20 ft. high, 48,000 cubic feet, to be heated to 69° F.; exposures as follows: North wall, 60 × 20 ft., with four windows, each 14 × 4 feet, outside temperature 0° F. Room beyond west wall and room overhead heated to 69°, except a double skylight in ceiling, 14 × 24 ft., exposed to the outside temperature of 0°. Store-room beyond east wall at 36°. Door 6 × 12 ft. in wall. Corridor beyond south wall heated to 59°. Two doors, 6 × 12, in wall. Cellar below, temperature 36°.

The following table shows the calculation of heat transmission:

$t-t_0$ (Fahr. degrees).	Kind of Transmitting Surface.	Thickness of Wall in inches.	Calculation of Area of Transmitting Surface.	Square feet of Surface.	$K(t-t_0)$.	Thermal Units.
69°	Outside wall..................	36″	68 × 22 − 448	938	9	8,442
69	Four windows (single)......		4 × 8 × 14	448	72	32,256
33	Inside wall (store-room).....	36″	42 × 22 − 72	852	4	3,408
33	Door		6 × 12	72	19	1,368
10	Inside wall (corridor)........	24″	45 × 22 − 72	918	2	1,836
10	Door..........................		6 × 12	72	5	360
10	Inside wall (corridor)........	36″	17 × 22 − 72	302	1	302
10	Door..........................		6 × 12	72	5	360
69	Roof..........................		32 × 42 − 336	1,008	10	10,080
69	Double skylight..............		14 × 24	336	43	14,448
33	Floor.........................		62 × 42	2,604	4	10,416
						83,276
	Supplementary allowance, north outside wall, 10%..........					844
	" " north outside windows, 10%.....					3,226
						87,346
	Exposed location and intermittent day or night use, 30%....					26,204
	Total thermal units					113,550

If we assume that the lecture-room must be heated to 69 degrees Fahr. in the daytime when unoccupied, so as to be at this temperature when first persons arrive, there will be required, ventilation not being considered, and bronzed direct low-pressure steam-radiators being the heating media, about 113,550 ÷ 250 = 455 sq. ft. of radiating-surface. (This gives a ratio of about 105 cu. ft. of contents of room for each sq. ft. of heating-surface.)

If we assume that there are 160 persons in the lecture-room, and we provide 2500 cubic feet of fresh air per person per hour, we will supply 160 × 2500 = 400,000 cubic feet of air per hour (i.e., $\frac{400{,}000}{48{,}000}$ = over eight changes of contents of room per hour).

To heat this air from 0° Fahr. to 69° Fahr. will require 400,000 × 0.0189 × 69 = 521,640 thermal units per hour (0.0189 being the product of a weight of a cubic foot by the specific heat of air). Accordingly there must be provided 521,640 ÷ 400 = 1304 sq. ft. of indirect surface, to heat the air required for ventilation, in zero weather. If the room were to be warmed entirely indirectly, that is, by the air supplied to room (including the heat to be conveyed to cover loss by transmission through walls, etc.), there would have to be conveyed to the fresh-air supply 521,640 + 113,550 = 635,190 heat-units. This would imply the provision of an amount of indirect heating-surface of the "Climax" type of 635,190 ÷ 400 = 1589 sq. ft., and the fresh air entering the room would have to be at a temperature of about 84° Fahr., viz., $69° = \frac{113{,}550}{400{,}000 \times 0.0189}$, or 69 + 15 = 84° Fahr.

The above calculations do not, however, take into account that 160 persons in the lecture-room give out 160 × 400 = 64,000 thermal units per hour; and that, say, 50 electric lights give out 50 × 1600 = 80,000 thermal units per hour; or, say, 50 gaslights, 50 × 4800 = 240,000 thermal units per hour. The presence of 160 people and the gas-lighting would diminish considerably the amount of heat required. Practically, it appears that the heat generated by the presence of 160 people, 64,000 heat-units, and by 50 electric lights, 80,000 heat-units, a total of 144,000 heat-units, more than covers the amount of heat transmitted through walls, etc. Moreover, that if the 50 gaslights give out 240,000 thermal units per hour, the air supplied for ventilation must enter considerably below 69° Fahr., or the room will be heated to an unbearably high temperature. If 400,000 cubic feet of fresh air per hour are supplied, and 240,000 thermal units per hour generated by the gas must be abstracted, it means that the air must, under these conditions, enter $\frac{240,000}{400,000 \times .0189}$ = about 32° less than 84°, or at about 52° Fahr. Furthermore, the additional vitiation due to gaslighting would necessitate a much larger supply of fresh air than when the vitiation of the atmosphere by the people alone is considered, one gaslight vitiating the air as much as five men.

Various Rules for Computing Radiating-surface.—The following rules are compiled from various sources. They are more in the nature of "rule-of-thumb" rules than those given by Mr. Wolff, quoted above, but they may be useful for comparison.

Divide the cubic feet of space of the room to be heated, the square feet of wall surface, and the square feet of the glass surface by the figures given under these headings in the following table, and add the quotients together; the result will be the square feet of radiating-surface required. (F. Schumann.)

SPACE, WALL AND GLASS SURFACE WHICH ONE SQUARE FOOT OF RADIATING-SURFACE WILL HEAT.

Air Change.	Steam-pressure in pounds.	Space in cubic feet.	Exposure of Rooms. All Sides. Wall Surface, sq. ft.	All Sides. Glass Surface, sq. ft.	Northwest. Wall Surface, sq. ft.	Northwest. Glass Surface, sq. ft.	Southeast. Wall Surface, sq. ft.	Southeast. Glass Surface, sq. ft.
Once per hour.	1	190	13.8	7	15.87	8.05	16.56	8.4
	3	210	15.0	7.7	17.25	8.85	18.00	9.24
	5	225	16.5	8.5	18.97	9.77	19.80	10.20
Twice per hour.	1	75	11.1	5.7	12.76	6.55	13.22	6.84
	3	82	12.1	6.2	13.91	7.13	14.52	7.44
	5	90	13.0	6.7	14.52	7.60	15.60	8.04

EMISSION OF HEAT-UNITS PER SQUARE FOOT PER HOUR FROM CAST-IRON PIPES OR RADIATORS. TEMP. OF AIR IN ROOM, 70° F. (F. Schumann.)

Mean Temperature of Heated Pipe, Radiator, etc.	By Contact. Air quiet.	By Contact. Air moving.	By Radiation.	By Radiation and Contact. Air quiet.	By Radiation and Contact. Air moving.
Hot water.........140°	55.51	92.52	59.63	115.14	152 15
" " ....150°	65.45	109.18	69.69	135.14	178 87
" " ....160°	75.68	126.13	80.19	155 87	206.32
" " ....170°	86.18	143.30	91.12	177.30	234.42
" " ....180°	96.93	161.55	102.15	199.43	264.05
" " ....190°	107.90	179.83	114.45	222.35	294.28
" " ....200°	119.13	198.55	127.00	246.13	325.55
" " or steam..210°	130.49	217.48	139.96	270.49	357.48
Steam..............220°	142.20	237.00	155.27	297.47	392.27
" ....230°	153.95	256.58	169.56	323.51	426.14
" ....240°	165.90	279.83	184.58	350.48	464.41
" ....250°	178.00	296.65	200.18	378.18	496.81
" ....260°	189.90	316.50	214.36	404.26	530.86
" ....270°	202.70	337.83	233.42	436.12	571.25
" ....280°	215.30	358.85	251.21	466.51	610.06
" ....290°	228.55	380.91	267.73	496.28	648.64
" ....300°	240.85	401.41	279.12	519.97	680.53

RADIATING-SURFACE REQUIRED FOR DIFFERENT KINDS OF BUILDINGS.

The Nason Mfg. Co.'s catalogue gives the following: One square foot of surface will heat from 40 to 100 cu ft. of space to 75° in − 10° latitudes. This range is intended to meet conditions of exposed or corner rooms of buildings, and those less so, as intermediate ones of a block. As a general rule, 1 sq. ft. of surface will heat 70 cu. ft. of air in outer or front rooms and 100 cu. ft. in inner rooms. In large stores in cities, with buildings on each side, 1 to 100 is ample. The following are approximate proportions:

One square foot radiating-surface will heat:

	In dwellings, schoolrooms, offices, etc.	In hall, stores, lofts, factories, etc.	In churches, large auditoriums, etc.
By direct radiation...	60 to 80 ft.	75 to 100 ft.	150 to 200 ft.
By indirect radiation.	40 to 50 "	50 to 70 "	100 to 140 "

Isolated buildings exposed to prevailing north or west winds should have a generous addition made to the heating-surface on their exposed sides.

The following rule is given in the catalogue of the Babcock & Wilcox Co., and is also recommended by the Nason Mfg. Co.:

Radiating-surface may be calculated by the rule: Add together the square feet of glass in the windows, the number of cubic feet of air required to be changed per minute, and one twentieth the surface of external wall and roof; multiply this sum by the difference between the required temperature of the room and that of the external air at its lowest point, and divide the product by the difference in temperature between the steam in the pipes and the required temperature of the room. The quotient is the required radiating-surface in square feet.

Prof. R. C. Carpenter (*Heating and Ventilation*, Feb. 15, 1897), gives the following handy formula for the amount of heat required for heating buildings by direct radiation:

$$h = \frac{n}{55}C + G + \tfrac{1}{4}W,$$

in which W = wall-surface, G = glass- or window-surface, both in sq. ft., C = contents of building in cu. ft., n = number of times the air must be changed per hour, and h = total heat units required per degree of difference of temperature between the room and the surrounding space. To heat the building to 70° F. when the outside temperature is 0°, 70 times the above quantity of heat will be required. Under ordinary conditions of pressure and temperature 1 sq. ft. of steam-heating surface will supply 280 heat units per hour, and 1 sq. ft. of hot-water heating surface 175 heat units per hour. The square feet of radiating-surface required under these conditions will be $R = 0.25h$ for steam-heating, and $R = 0.4h$ for hot-water heating. Prof. Carpenter says that for residences it is safe to assume that the air of the principal living-rooms will change twice in an hour, that of the halls three times and that of the other rooms once per hour, under ordinary conditions.

Overhead Steam-pipes. (A. R. Wolff, *Stevens Indicator*, 1887.)—When the overhead system of steam-heating is employed, in which system direct radiating-pipes, usually 1¼ in. in diam., are placed in rows overhead, suspended upon horizontal racks, the pipes running horizontally, and side by side, around the whole interior of the building, from 2 to 3 ft. from the walls, and from 2 to 4 ft. from the ceiling, the amount of 1¼ in. pipe required, according to Mr. C. J. H. Woodbury, for heating mills (for which use this system is deservedly much in vogue), is about 1 ft. in length for every 90 cu. ft. of space. Of course a great range of difference exists, due to the special character of the operating machinery in the mill, both in respect to the amount of air circulated by the machinery, and also the aid to warming the room by the friction of the journals.

Indirect Heating-surface.—J. H. Kinealy, in *Heating and Ventilation*, May 15, 1894, gives the following formula, deduced from results of experiments by C. B. Richards, W. J. Baldwin, J. H. Mills, and others, upon indirect heaters of various kinds, supplied with varying amounts of air per hour per square foot of surface:

$$N = \frac{35.04}{\dfrac{T_2 - T_1}{T_0 - T_1} - 0.369}; \quad T_2 = (T_0 - T_1)\left(0.369 + \frac{35.04}{N}\right) + T_1.$$

N = cubic feet of air, reduced to 70° F., supplied to the heater per square foot of heating-surface per hour; T_0 = temperature of the steam or water in the heater; T_1 = temperature of the air when it enters the heater; T_2 = temperature of the air when it leaves the heater.

As the formula is based upon an average of experiments made upon all sorts of indirect heaters, the results obtained by the use of the equation may in some cases be slightly too small and in others slightly too large, although the error will in no case be great. No single formula ought to be expected to apply equally well to all dispositions of heating-surface in indirect heaters, as the efficiency of such heater can be varied between such wide limits by the construction and arrangement of the surface.

In indirect heating, the efficiency of the radiating-surface will increase, and the temperature of the air will diminish, when the quantity of the air caused to pass through the coil increases. Thus 1 sq. ft. radiating-surface, with steam at 212°, has been found to heat 100 cu. ft. of air per hour from zero to 150°, or 300 cu. ft. from zero to 100° in the same time. The best results are attained by using indirect radiation to supply the necessary ventilation, and direct radiation for the balance of the heat. (*Steam*.)

In indirect steam-heating the least flue area should be 1 to 1¼ sq. in. to every square foot of heating-surface, provided there are no long horizontal reaches in the duct, with little rise. The register should have twice the area of the duct to allow for the fretwork. For hot water heating from 25% to 30% more heating-surface and flue area should be given than for low-pressure steam. (*Engineering Record*, May 26, 1894.)

Boiler Heating-surface Required. (A. R. Wolff, *Stevens Indicator*, 1887.)—When the direct system is used to heat buildings in which the street floor is a store, and the upper floors are devoted to sales and stock-rooms and to light manufacturing, and in which the fronts are of stone or iron, and the sides and the rear of building of brick—a safe rule to follow is to supply 1 sq. ft. of boiler heating-surface for each 700 cu. ft., and 1 sq. ft. of radiating-surface for each 100 cu. ft. of contents of building.

For heating mills, shops, and factories, 1 sq. ft. of boiler heating-surface should be supplied for each 475 cu. ft. of contents of building; and the same allowance should also be made for heating exposed wooden dwellings. For heating foundries and wooden shops, 1 sq. ft. of boiler heating-surface should be provided for each 400 cu. ft. of contents; and for structures in which glass enters very largely in the construction—such as conservatories, exhibition buildings, and the like—1 sq. ft. of boiler heating-surface should be provided for each 275 cu. ft. of contents of building.

When the indirect system is employed, the radiator-surface and the boiler capacity to be provided will each have to be, on an average, about 25% more than where direct radiation is used. This percentage also marks approximately the increased fuel consumption in the indirect system.

Steam (Babcock & Wilcox Co.) has the following: 1 sq. ft. of boiler-surface will supply from 7 to 10 sq. ft. of radiating-surface, depending upon the size of boiler and the efficiency of its surface, as well as that of the radiating-surface. Small boilers for house use should be much larger proportionately than large plants. Each horse-power of boiler will supply from 240 to 360 ft. of 1-in. steam-pipe, or 80 to 120 sq. ft. of radiating surface. Cubic feet of space has little to do with amount of steam or surface required, but is a convenient factor for rough calculations. Under ordinary conditions 1 horse-power will heat, approximately, in—

Brick dwellings, in blocks, as in cities	15,000	to	20,000	cu. ft.
" stores " "	10,000	"	15,000	"
" dwellings, exposed all round	10,000	"	15,000	"
" mills, shops, factories, etc.	7,000	"	10,000	"
Wooden dwellings, exposed	7,000	"	10,000	"
Foundries and wooden shops	6,000	"	10,000	"
Exhibition buildings, largely glass, etc.	4,000	"	15,000	"

Steam-consumption in Car-heating.

C., M. & St. Paul Railway Tests. (*Engineering*, June 27, 1890, p. 764.)

Outside Temperature.	Inside Temperature.	Water of Condensation per Car per Hour.
40	70	70 lbs.
30	70	85
10	70	100

Internal Diameters of Steam Supply-mains, with Total Resistance equal to 2 inches of Water-column.*

Steam, Pressure 10 lbs. per square inch above atm., Temperature 239° F.

Formula, $d = 0.5374\sqrt[5]{\frac{Q^2 l}{h}}$; where d = internal diameter in inches;

Q = 9.2 cubic feet of steam per minute per 100 sq. ft. of radiating-surface; l = length of mains in feet; h = 159.3 feet head of steam to produce flow.

Radiating-surface.	Internal Diameters in inches for Lengths of Mains from 1 ft. to 600 ft.										
	1 ft.	10 ft.	20 ft.	40 ft.	60 ft.	80 ft.	100 ft.	200 ft.	300 ft.	400 ft.	600 ft.
sq.ft.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
1	0.075	0.119	0.136	0.157	0.170	0.180	0.189	0.216	0.234	0.248	0.270
10	0.19	0.30	0.34	0.39	0.43	0.45	0.47	0.54	0.59	0.62	0.68
20	0.25	0.39	0.45	0.52	0.56	0.60	0.62	0.72	0.78	0.82	0.89
40	0.33	0.52	0.60	0.69	0.74	0.79	0.82	0.95	1.03	1.09	1.18
60	0.39	0.61	0.71	0.81	0.87	0.93	0.97	1.11	1.21	1.28	1.39
80	0.43	0.68	0.79	0.90	0.96	1.04	1.09	1.25	1.35	1.43	1.55
100	0.47	0.75	0.86	0.99	1.07	1.14	1.19	1.36	1.48	1.57	1.70
200	0.62	0.99	1.14	1.30	1.41	1.50	1.57	1.80	1.95	2.07	2.24
300	0.73	1.16	1.34	1.53	1.66	1.76	1.84	2.12	2.30	2.43	2.64
400	0.82	1.30	1.50	1.72	1.86	1.98	2.07	2.37	2.57	2.73	2.96
500	0.90	1.43	1.64	1.88	2.04	2.16	2.26	2.60	2.81	2.98	3.23
600	0.97	1.53	1.76	2.03	2.20	2.33	2.43	2.79	3.03	3.21	3.48
800	1.09	1.72	1.98	2.27	2.46	2.61	2.73	3.13	3.40	3.60	3.90
1,000	1.19	1.88	2.16	2.48	2.60	2.85	2.98	3.43	3.71	3.94	4.27
1,200	1.28	2.04	2.33	2.67	2.90	3.07	3.21	3.68	4.00	4.23	4.59
1,400	1.36	2.15	2.47	2.84	3.08	3.26	3.41	3.92	4.25	4.50	4.88
1,600	1.43	2.27	2.61	3.00	3.25	3.44	3.60	4.13	4.49	4.75	5.15
1,800	1.50	2.38	2.74	3.14	3.41	3.61	3.78	4.34	4.70	4.98	5.40
2,000	1.57	2.48	2.85	3.28	3.55	3.76	3.93	4.52	4.90	5.19	5.63
3,000	1.84	2.92	3.36	3.85	4.18	4.43	4.63	5.32	5.77	6.11	6.63
4,000	2.07	3.28	3.76	4.32	4.69	4.96	5.19	5.96	6.47	6.85	7.44

* From Robert Briggs's paper on American Practice of Warming Buildings by Steam (Proc. Inst. C. E., 1882, vol. lxxi).

For other resistances and pressures above atmosphere multiply by the respective factors below:

Water col..	6 in.	12 in.	24 in.	Press. ab. atm.	0 lbs.	3 lbs.	30 lbs.	60 lbs.
Multiply by	0.8027	0.6988	0.6084	Multiply by	1.023	1.015	0.973	0.948

Registers and Cold-air Ducts for Indirect Steam Heating. --The *Locomotive* gives the following table of openings for registers and cold-air ducts, which has been found to give satisfactory results. The cold-air boxes should have 1¼ sq. in. area for each square foot of radiator suface, and never less than ¾ the sectional area of the hot-air ducts. The hot-air ducts should have 2 sq. in. of sectional area to each square foot of radiator surface on the first floor, and from 1½ to 2 inches on the second floor.

Heating Surface in Stacks.	Cold-air Supply, First Floor.	Size Register.	Cold-air Supply, 2d Floor.
	inches	inches	inches
30 square feet	45 square inches = 5 by 9	9 by 12	4 by 10
40 " "	60 " " = 6 by 10	10 by 14	4 by 14
50 " "	75 " " = 8 by 10	10 by 14	5 by 15
60 " "	90 " " = 9 by 10	12 by 15	6 by 15
70 " "	108 " " = 9 by 12	12 by 19	6 by 18
80 " "	120 " " = 10 by 12	12 by 22	8 by 15
90 " "	135 " " = 11 by 12	14 by 24	9 by 15
100 " "	150 " " = 12 by 12	16 by 20	12 by 12

The sizes in the table approximate to the rules given, and it will be found that they will allow an easy flow of air and a full distribution throughout the room to be heated.

Physical Properties of Steam and Condensed Water, under Conditions of Ordinary Practice in Warming by Steam. (Briggs.)

A	Steam-pressure per square inch, above atm.	lbs.	0	3	10	30	60
	Steam-pressure per square inch, total	lbs.	14.7	17.7	24.7	44.7	74.7
B	Temperature of steam	Fahr.	212°	222°	239°	274°	307°
C	Temperature of air	Fahr.	60°	60	60°	60°	60°
D	Difference = B − C	Fahr.	152°	162°	179°	214°	247°
E	Heat given out per minute per 100 sq. ft. of radiating-surface = D × 3	units	456	486	537	642	741
F	Latent heat of steam	Fahr.	965°	958°	946°	921°	898°
G	Volume of 1 lb. weight of steam	cu. ft.	26.4	22.1	16.2	9.24	5.70
H	Weight of 1 cubic foot of steam	lb.	0.0380	0.0452	0.0618	0.1082	0.1752
J	Volume Q of steam per minute to give out E units = E × G ÷ F.	cu. ft.	12.48	11.21	9.20	6.44	4.70
K	Weight of 1 cubic foot of condensed water at temperature B.	lbs.	59.64	59.51	59.05	58.07	57.03
L	Volume of condensed water to return to boiler per minute = J × H ÷ K.	cu. ft.	0.0079	0.0085	0.0096	0.0120	0.0144
M	Head of steam equivalent to 12 inches water-column = K ÷ H.	feet	1569	1317	955.5	586.7	325.5
	STEAM-SUPPLY MAINS.						
N	Head h of steam, equivalent to assumed 2 inches water-column for producing steam flow Q, = M ÷ 6.	feet	261.5	219.5	159.3	89.45	54.25
P	Internal diameter d of tube* for flow Q when l = 1 foot,	inch	0.484	0.481	0.474	0.461	0.449
R	Do. do. when l = 100 feet,	inch	1.217	1.207	1.190	1.158	1.128
S	Ratios of values of d.	ratio	1.023	1.015	1.000	0.973	0.948
	WATER-RETURN MAINS.						
T	Head h assumed at ½-inch water-column for producing full-bore water-flow Q.	foot	0.0417	0.0417	0.0417	0.0417	0.0417
U	Internal diameter d of tube* for flow Q when l = 1 foot,	inch	0.147	0.151	0.158	0.173	0.186
V	Do. do. when l = 100 feet,	inch	0.368	0.379	0.398	0.434	0.468
W	Ratios of values of d	ratio	0.926	0.952	1.000	1.092	1.176

* P, R, U, V are each determined from the formula $d = 0.5374 \sqrt[5]{\frac{Q^2 l}{h}}$.

Size of Steam Pipes for Steam Heating. (See also Flow of Steam in Pipes.)—*Sizes of vertical main pipes. Direct radiation.* (J. R. Willett, *Heating and Ventilation*, Feb., 1894.)

Diameter of pipe, inches	1	1¼	1½	2	2½	3	3½	4	5	6
Sq. ft. of radiator surface	40	70	110	220	360	560	810	1110	2000	3000

A horizontal branch pipe for a given extent of radiator surface should be one size larger than a vertical pipe for the same surface.

The Nason Mfg. Co. gives the following:

Diameter of pipe, in	1¼	1½	2	2½	3	3½
Radiator surface sq. ft. (maximum)	125	200	500	1000	1500	2500

When mains and surfaces are very much above the boiler the pipes need not be as large as given above: under very favorable circumstances and conditions a 4-inch pipe may supply from 2000 to 2500 sq. ft. of surface, a 6-inch pipe for 5000 sq. ft., and a 10-inch pipe for 15,000 to 20,000 sq. ft., if the distance of run from boiler is not too great. Less than 1½-inch pipe should not be used horizontally in a main unless for a single radiator connection.

Steam, by the Babcock & Wilcox Co., says: Where the condensed water is returned to the boiler, or where low pressure of steam is used, the diameter of mains leading from the boiler to the radiating-surface should be equal in inches to one tenth the square root of the radiating-surface, mains included, in square feet. Thus a 1-inch pipe will supply 100 square feet of surface, itself included. Return-pipes should be at least ¾ inch in diameter, and never less than one half the diameter of the main—longer returns requiring larger pipe. A thorough drainage of steam-pipes will effectually prevent all cracking and pounding noises therein.

A. R. Wolff's Practice.—Mr. Wolff gives the following figures showing his present practice (1897) in proportioning mains and returns. They are based on an estimated loss of pressure of 2% for a length of 100 ft. of pipe, not including allowance for bends and valves (see p. 678). For longer runs divide the thermal units given in the table by $0.1\sqrt{\text{length in ft.}}$ Besides giving the thermal units the table also indicates the amount of direct radiating surface which the steam-pipes can supply, on the basis of an emission of 250 thermal units per hour for each square foot of direct radiating surface.

Size of Pipes for Steam Heating.

Diam. of Supply.	Diam. of Return.	2 lbs. Pressure		5 lbs. Pressure		Diam. of Supply.	Diam. of Return.	2 lbs. Pressure		5 lbs. Pressure	
		Thermal Units per Hr., Thous'ds	Heating-surface, Sq. Ft.	Thermal Units per Hr., Thous'ds	Heating-surface, Sq. Ft.			Thermal Units, per Hr., Thous'ds	Heating-surface, Sq. Ft.	Thermal Units, per Hr., Thous'ds	Heating-surface, Sq. Ft.
In.	In.					In.	In.				
1	1	9	36	15	60	5	3½	930	3720	1550	6200
1¼	1	18	72	30	120	6	3½	1500	6000	2500	10000
1½	1¼	30	120	50	200	7	4	2250	9000	3750	15000
2	1½	70	280	120	480	8	4	3200	12800	5400	21600
2½	2	132	528	220	880	9	4½	4450	17800	7500	30000
3	2½	225	900	375	1500	10	5	5800	23200	9750	39000
3½	2½	330	1320	550	2200	12	6	9250	37000	15500	62000
4	3	480	1920	800	3200	14	7	13500	54000	23000	92000
4½	3	690	2760	1150	4600	16	8	19000	76000	32500	130000

Heating a Greenhouse by Steam.—Wm. J. Baldwin answers a question in the *American Machinist* as below: With five pounds steam-pressure, how many square feet or inches of heating-surface is necessary to heat 100 square feet of glass on the roof, ends, and sides of a greenhouse in order to maintain a night heat of 55° to 65°, while the thermometer outside ranges at from 15° to 20° below zero; also, what boiler-surface is necessary? Which is the best for the purpose to use—2″ pipe or 1¼″ pipe?

Ans.—Reliable authorities agree that 1.25 to 1.50 cubic feet of air in an enclosed space will be cooled per minute per sq. ft. of glass as many degrees as the internal temperature of the house exceeds that of the air outside. Between + 65° and − 20° there will be a difference of 85°, or, say, one cubic foot of air cooled 127.5° F. for each sq ft. of glass for the most extreme condition mentioned. Multiply this by the number of square feet of glass and by 60, and we have the number of cubic feet of air cooled 1° per hour within the building or house. Divide the number thus found by 48, and it gives the units of heat required, approximately. Divide again by 953, and it will give the number of pounds of steam that must be condensed from a pressure and temperature of five pounds above atmosphere to water at the same temperature in an hour to maintain the heat. Each square foot of surface of pipe will condense from ¼ to nearly ½ lb. of steam per hour, according as the coils are exposed or well or poorly arranged, for which an average of ⅓ lb. may be taken. According to this, it will require 3 sq. ft. of pipe surface per lb. of steam to be condensed. Proportion the heating-surface of the boiler to have about one fifth the actual radiating-surface, if you wish to keep steam over night, and proportion the grate to burn not more than six pounds of coal per sq. ft. of grate per hour. With very slow combustion, such as takes place in base-burning boilers, the grate might be proportioned for four to five pounds of coal per hour. It is cheaper to make coils of 1¼″ pipe than of 2″, and there is nothing to be gained by using 2″ pipe unless the coils are very long. The pipes in a greenhouse should be under or in front of the benches, with every chance for a good circulation of air. "Header" coils are better than "return-bend" coils for this purpose.

Mr. Baldwin's rule may be given the following form: Let H = heat-units transferred per hour, T = temperature inside the greenhouse, t = temperature outside, S = sq. ft. of glass surface; then $H = 1.5S(T - t) \times 60 \div 48 = 1.875S(T - t)$. Mr. Wolff's coefficient K for single skylights would give $H = 1.118S(T - t)$.

Heating a Greenhouse by Hot Water.—W. M. Mackay, of the Richardson & Boynton Co., in a lecture before the Master Plumbers' Association, N. Y., 1889, says: I find that while greenhouses were formerly heated by 4-inch and 3-inch cast-iron pipe, on account of the large body of water which they contained, and the supposition that they gave better satisfaction and a more even temperature, florists of long experience who have tried 4-inch and 3-inch cast-iron pipe, and also 2-inch wrought-iron pipe for a number of years in heating their greenhouses by hot water, and who have also tried steam-heat, tell me that they get better satisfaction, greater economy, and are able to maintain a more even temperature with 2-inch wrought-iron pipe and hot water than by any other system they have used. They attribute this result principally to the fact that this size pipe contains less water and on this account the heat can be raised and lowered quicker than by any other arrangement of pipes, and a more uniform temperature maintained than by steam or any other system.

HOT-WATER HEATING.

(Nason Mfg. Co.)

There are two distinct forms or modifications of hot-water apparatus, depending upon the temperature of the water.

In the first or open-tank system the water is never above 212° temperature, and rarely above 200°. This method always gives satisfaction where the surface is sufficiently liberal, but in making it so its cost is considerably greater than that for a steam-heating apparatus.

In the second method, sometimes called (erroneously) high-pressure hot-water heating, or the closed-system apparatus, the tank is closed. If it is provided with a safety-valve set at 10 lbs. it is practically as safe as the open-tank system.

Law of Velocity of Flow.—The motive power of the circulation in a hot-water apparatus is the difference between the specific gravities of the ascending and the descending pipes. This effective pressure is very small, and is equal to about one grain for each foot in height for each degree difference between the pipes; thus, with a height of 12" in "up" pipe, and a difference between the temperatures of the up and down pipes of 8°, the difference in their specific gravities is equal to 8.16 grains on each square inch of the section of return-pipe, and the velocity of the circulation is proportioned to these differences in temperature and height.

To Calculate Velocity of Flow.—Thus, with a height of ascending pipe equal to 10′ and a difference in temperatures of the flow and return pipes of 8°, the difference in their specific gravities will equal 81.6 grains, or $\div 7000 = .01166$ lbs., or $\times 2.31$ (feet of water in one pound) $= .0269$ ft., and by the law of falling bodies the velocity will be equal to $8\sqrt{.0269} = 1.312$ ft. per second, or $\times 60 = 78.7$ ft. per minute. In this calculation the effect of friction is entirely omitted. Considerable deduction must be made on this account. Even in apparatus where length of pipe is not great, and with pipes of larger areas and with few bends or angles, a large deduction for friction must be made from the theoretical velocity, while in large and complex apparatus with small head, the velocity is so much reduced by friction that sometimes as much as from 50% to 90% must be deducted to obtain the true rate of circulation.

Main flow-pipes from the heater, from which branches may be taken, are to be preferred to the practice of taking off nearly as many pipes from the heater as there are radiators to supply.

It is not necessary that the main flow and return pipes should equal in capacity that of all their branches. The hottest water will seek the highest level, while gravity will cause an even distribution of the heated water if the surface is properly proportioned.

It is good practice to reduce the size of the vertical mains as they ascend, say at the rate of one size for each floor.

As with steam, so with hot water, the pipes must be unconfined to allow for expansion of the pipes consequent on having their temperatures increased.

An expansion tank is required to keep the apparatus filled with water, which latter expands 1/24 of its bulk on being heated from 40° to 212°, and the cistern must have capacity to hold certainly this increased bulk. It is recommended that the supply cistern be placed on level with or above the highest pipes of the apparatus, in order to receive the air which collects in the mains and radiators, and capable of holding at least 1/20 of the water in the entire apparatus.

Approximate Proportions of Radiating-surfaces to Cubic Capacities of Space to be Heated.

One Square Foot of Radiating-surface will heat with—	In Dwellings, School-rooms, Offices, etc.	In Halls, Stores, Lofts, Factories, etc.	In Churches, Large Auditoriums, etc.
High temperature direct hot-water radiation	50 to 70 cu. ft.	65 to 90 cu. ft.	130 to 180 cu. ft.
Low temperature direct hot-water radiation	30 to 50 " "	35 to 65 " "	70 to 130 " "
High temperature indirect hot-water radiation	30 to 60 " "	35 to 75 " "	70 to 150 " "
Low temperature indirect hot-water radiation	20 to 40 " "	25 to 50 " "	50 to 100 " "

Diameter of Main and Branch Pipes and square feet of coil surface they will supply, in a low-pressure hot-water apparatus (212°) for direct or indirect radiation, when coils are at different altitudes for direct radiation or in the lower story for indirect radiation:

Diam. of Pipe, in inches.	Indirect Radiation	Direct Radiation. Height of Coil above Bottom of Boiler, in feet.									
	0	10	20	30	40	50	60	70	80	90	100
	sq. ft.	sq. ft.	sq. ft.	sq. ft.	sq. ft.	sq. ft.	sq. ft.	sq. ft.	sq. ft.	sq. ft.	sq. ft.
¾	40	50	52	53	55	57	59	61	63	65	68
1	87	89	92	95	98	101	103	108	112	116	121
1¼	136	140	144	149	153	158	161	169	175	182	189
1½	196	202	209	214	222	228	235	248	253	261	271
2	249	259	270	280	393	405	418	433	449	465	483
2½	546	561	577	595	613	633	648	678	701	727	755
3	785	807	835	856	883	912	941	974	1009	1046	1086
3½	1069	1099	1132	1166	1203	1241	1283	1327	1374	1435	1480
4	1395	1436	1478	1520	1571	1621	1654	1733	1795	1861	1933
4½	1767	1817	1871	1927	1988	2052	2120	2193	2273	2356	2445
5	2185	2244	2309	2376	2454	2531	2574	2718	2805	2907	3019
6	3140	3228	3341	3424	3552	3648	3763	3897	4036	4184	4344
7	4376	4436	4528	4664	4808	4964	5132	5308	5496	5700	5920
8	5580	5744	5912	6090	6284	6484	6616	6932	7180	7444	7735
9	7068	7268	7484	7706	7952	8208	8482	8774	9088	9424	9780
10	8740	8976	9236	9516	9816	10124	10396	10853	11230	11628	12076
11	10559	10860	11180	11519	11879	12262	12666	13108	13576	14078	14620
12	12560	12912	13364	13696	14208	14592	15052	15588	16144	16736	17476
13	14748	15160	15615	16090	16591	17126	17697	18307	18961	19638	20430
14	17104	17584	18109	18656	19234	19856	20508	21283	21984	22800	23680
15	19634	20195	20789	21419	22089	22801	23561	24373	25244	26179	27168
16	22320	22978	23643	24320	25136	25936	26464	27728	28720	29776	30928

The best forms of hot-water-heating boilers are proportioned about as follows:

1 sq. ft. of grate-surface to about 40 sq. ft. of boiler-surface.
1 " " boiler- " " 5 " " radiating-surface.
1 " " grate- " " 200 " " " "

Rules for Hot-water Heating.—J. L. Saunders (Heating and Ventilation, Dec. 15, 1894) gives the following: Allow 1 sq. ft. of radiating surface for every 3 ft. of glass surface, and 1 sq. ft. for every 30 sq. ft. of wall surface, also 1 sq. ft. for the following numbers of cubic feet of space in the several cases mentioned.

In dwelling-houses:	Libraries and dining-rooms, first floor	35 to 40 cu. ft.
	Reception halls, first floor	40 to 50 " "
	Stair halls, " "	40 to 55 " "
	Chambers above, " "	50 to 65 " "
	Libraries, sewing-rooms, nurseries, etc., above first floor	45 to 55 " "
	Bath-rooms	30 to 40 " "
Public-school rooms		60 to 85 " "
Offices		50 to 65 " "
Factories and stores		65 to 90 " "
Assembly halls and churches		90 to 150 " "

To find the necessary amount of indirect radiation required to heat a room: Find the required amount of direct radiation according to the foregoing method and add 50%. This if wrought-iron pipe coil surface is used; if cast-iron pin indirect-stack surface is used it is advisable to add from 70% to 80%.

Sizes of hot-air flues, cold-air ducts, and registers for indirect work.—Hot-air flues, first floor: Make the net internal area of the flue equal to ¾ sq. in. to every square foot of radiating surface in the indirect stack. Hot-air flues, second floor: Make the net internal area of the flue equal to ⅝ sq. in. to every square foot of radiating surface in the indirect stack.

Cold-air ducts, first floor: Make the net internal area of the duct equal to ⅝ sq. in. to every square foot of radiating surface in the indirect stack. Cold air ducts, second floor: Make the net internal area of the duct equal to ½ sq. in. to every square foot of radiating surface in the indirect stack.

Hot-air registers should have their net area equal in full to the area of the hot-air flues. Multiply the length by the width of the register in inches; ⅔ of the product is the net area of register.

Arrangement of Mains for Hot-water Heating. (W. M. Mackay, Lecture before Master Plumbers' Assoc., N. Y., 1889)—There are two different systems of mains in general use, either of which, if properly placed, will give good satisfaction. One is the taking of a single large-flow main from the heater to supply all the radiators on the several floors, with a corresponding return main of the same size. The other is the taking of a number of 2-inch wrought-iron mains from the heater, with the same number of return mains of the same size, branching off to the several radiators or coils with 1¼-inch or 1-inch pipe, according to the size of the radiator or coil. A 2-inch main will supply three 1¼-inch or four 1-inch branches, and these branches should be taken from the top of the horizontal main with a nipple and elbow, except in special cases where it is found necessary to retard the flow of water to the near radiator, for the purpose of assisting the circulation in the far radiator; in this case the branch is taken from the side of the horizontal main. The flow and return mains are usually run side by side, suspended from the basement ceiling, and should have a gradual ascent from the heater to the radiators of at least 1 inch in 10 feet. It is customary, and an advantage where 2-inch mains are used, to reduce the size of the main at every point where a branch is taken off.

The single or large main system is best adapted for large buildings; but there is a limit as to size of main which it is not wise to go beyond—generally 6-inch, except in special cases.

The proper area of cold-air pipe necessary for 100 square feet of indirect radiation in hot-water heating is 75 square inches, while the hot air pipe should have at least 100 square inches of area. There should be a damper in the cold-air pipe for the purpose of controlling the amount of air admitted to the radiator, depending on the severity of the weather.

THE BLOWER SYSTEM OF HEATING AND VENTILATING.

The system provides for the use of a fan or blower which takes its supply of fresh air from the outside of the building to be heated, forces it over steam coils, located either centrally or divided up into a number of independent groups, and then into the several ducts or flues leading to the various rooms. The movement of the warmed air is positive, and the delivery of the air to the various points of supply is certain and entirely independent of atmospheric conditions. For engines, fans, and steam-coils used with the blower system, see page 519.

Experiments with Radiators of 60 sq. ft. of Surface. (*Mech. News*, Dec., 1893.)—After having determined the volume and temperature of the warm air passing through the flues and radiators from natural causes, a fan was applied to each flue, forcing in air, and new sets of measurements were made. The results showed that more than two and one-third times as much air was warmed with the fans in use, and the falling off in the temperature of this greatly increased air-volume was only about 12.6%. The condensation of steam in the radiators with the forced-air circulation also was only 66⅔% greater than with natural-air draught. One of the several sets of test figures obtained is as follows:

	Natural Draught in Flue.	Forced-air Circulation.
Cubic feet of air per minute	457.5	1227
Condensation of steam per minute in ounces	11.7	19.6
Steam pressure in radiator, pounds	9	9
Temperature of air after leaving radiator	142°	124°
" " " before passing through radiator.	61°	61°
Amount of radiating surface in square feet	60	60
Size of flue in both cases	12 × 18 inches.	

There was probably an error in the determination of the volume of air in these tests, as appears from the following calculation. (W. K.) Assume that 1 lb. of steam in condensing from 9 lbs. pressure and cooling to the temperature at which the water may have been discharged from the radiator gave up 1000 heat-units, or 62.5 h. u. per ounce; that the air weighed .076 lb. per cubic foot, and that its specific heat is .238. We have

	Natural Draught.	Forced Draught.
Heat given up by steam, ounces × 62.5 =	731	1225 H.U.
Heat received by air, cu. ft. × .076 × diff. of tem. × .238 =	673	1399 "

Or, in the case of forced draught the air received 14% more heat than the steam gave out, which is impossible. Taking the heat given up by the steam as the correct measure of the work done by the radiator, the temperature of the steam at 237°, and the average temperature of the air in the case of natural draught at 102° and in the other case at 93°, we have for the temperature difference in the two cases 135° and 144° respectively; dividing these into the heat-units we find that each square foot of radiating surface transmitted 5.4 heat-units per hour per degree of difference of temperature, in the case of natural draught, and 8.5 heat-units in the case of forced draught ($= 8.5 \times 144° = 1224$ heat-units per square foot of surface).

In the Women's Homœopathic Hospital in Philadelphia, 2000 feet of one-inch pipe heats 250,000 cubic feet of space, ventilating as well; this equals one square foot of pipe surface for about 350 cubic feet of space, or less than 3 square feet for 1000 cubic feet. The fan is located in a separate building about 100 feet from the hospital, and the air, after being heated to about 135°, is conveyed through an underground brick duct with a loss of only five or six degrees in cold weather. (H. I. Snell, Trans. A. S. M. E. ix. 106.

Heating a Building to 70° F. Inside when the Outside Temperature is Zero.—It is customary in some contracts for heating to guarantee that the apparatus will heat the interior of the building to 70° in zero weather. As it may not be practicable to obtain zero weather for the purpose of a test, it may be difficult to prove the performance of the guarantee. E. E. Macgovern, in *Engineering Record*, Feb. 3, 1894, gives a calculation tending to show that a test may be made in weather of a higher temperature than zero, if the heat of the interior is raised above 70°. The higher the temperature of the rooms the lower is the efficiency of the radiating-surface, since the efficiency depends upon the difference between the temperature inside of the radiator and the temperature of the room. He concludes that a heating apparatus sufficient to heat a given building to 70° in zero weather with a given pressure of steam will be found to heat the same building, steam-pressure constant, to 110° at 60°, 95° at 50°, 82° at 40°, and 74° at 32°, outside temperature. The accuracy of these figures, however has not been tested by experiment.

The following solution of the question is proposed by the author. It gives results quite different from those of Mr. Macgovern, but, like them, lacks experimental confirmation.

Let S = sq. ft. of surface of the steam or hot-water radiator;
W = sq. ft. of surface of exposed walls, windows, etc.;
T_s = temp. of the steam or hot water, T_1 = temp. of inside of building or room, T_0 = temp. of outside of building or room;
a = heat-units transmitted per sq. ft. of surface of radiator per hour per degree of difference of temperature;
b = average heat-units transmitted per sq. ft. of walls per hour, per degree of difference of temperature, including allowance for ventilation.

It is assumed that within the range of temperatures considered Newton's law of cooling holds good, viz., that it is proportional to the difference of temperature between the two sides of the radiating-surface.

Then $aS(T_s - T_1) = bW(T_1 - T_0)$. Let $\frac{bW}{aS} = C$; then

$$T_s - T_1 = C(T_1 - T_0); \quad T_1 = \frac{T_s + CT_0}{1 + C}; \quad C = \frac{T_s - T_1}{T_1 - T_0}.$$

If $T_1 = 70$, and $T_0 = 0$, $C = \frac{T_s - 70}{70}$.

Let T_s = 140°, 218.5°, 308°;
Then C = 1, 2.05, 3.4.

From these we derive the following:

Temperature of Steam or Hot Water, T_s.	Outside Temperatures, T_0.						
	− 20°	− 10°	0°	10°	20°	30°	40°
	Inside Temperatures, T_1.						
140°	60	65	70	75	80	85	90
218.5	56.6	63.3	70	76.7	83.4	90.2	96.9
308	54.5	62.3	70	77.7	85.5	93.2	100.9

Heating by Electricity.—If the electric currents are generated by a dynamo driven by a steam-engine, electric heating will prove very expensive, since the steam-engine wastes in the exhaust-steam and by radiation about 90% of the heat-units supplied to it. In direct steam-heating, with a good boiler and properly covered supply-pipes, we can utilize about 60% of the total heat value of the fuel. One pound of coal, with a heating value of 13,000 heat-units, would supply to the radiators about 13,000 × .60 = 7800 heat-units. In electric heating, suppose we have a first-class condensing-engine developing 1 H.P. for every 2 lbs. of coal burned per hour. This would be equivalent to 1,980,000 ft.-lbs. ÷ 778 = 2545 heat-units, or 1272 heat-units for 1 lb. of coal. The friction of the engine and of the dynamo and the loss by electric leakage, and by heat radiation from the conducting wires, might reduce the heat-units delivered as electric current to the electric radiator, and these converted into heat to 50% of this, or only 636 heat-units, or less than one twelfth of that delivered to the steam-radiators in direct steam-heating. Electric heating, therefore, will prove uneconomical unless the electric current is derived from water or wind power, which would otherwise be wasted. (See Electrical Engineering.)

WATER.

Expansion of Water.—The following table gives the relative volumes of water at different temperatures, compared with its volume at 4° C. according to Kopp, as corrected by Porter.

Cent.	Fahr.	Volume.	Cent.	Fahr.	Volume.	Cent.	Fahr.	Volume.
4°	39.1°	1.00000	35°	95°	1.00586	70°	158°	1.02241
5	41	1.00001	40	104	1.00767	75	167	1.02548
10	50	1.00025	45	113	1.00967	80	176	1.02872
15	59	1.00083	50	122	1.01186	85	185	1.03213
20	68	1.00171	55	131	1.01423	90	194	1.03570
25	77	1.00286	60	140	1.01678	95	203	1.03943
30	86	1.00425	65	149	1.01951	100	212	1.04332

Weight of 1 cu. ft. at 39.1° F. = 62.4245 lb. ÷ 1.04332 = 59.833, weight of 1 cu. ft. at 212° F.

Weight of Water at Different Temperatures.—The weight of water at maximum density, 39.1°, is generally taken at the figure given by Rankine, 62.425 lbs. per cubic foot. Some authorities give as low as 62.379. The figure 62.5 commonly given is approximate. The highest authoritative figure is 62.425. At 62° F. the figures range from 62.291 to 62.360. The figure 62.355 is generally accepted as the most accurate.

At 32° F. figures given by different writers range from 62.379 to 62.418. Clark gives the latter figure, and Hamilton Smith, Jr., (from Rosetti,) gives 62.416.

Weight of Water at Temperatures above 212° F.—Porter (Richards' "Steam-engine Indicator," p. 52) says that nothing is known about the expansion of water above 212°. Applying formulæ derived from experiments made at temperatures below 212°, however, the weight and volume above 212° may be calculated, but in the absence of experimental data we are not certain that the formulæ hold good at higher temperatures.

Thurston, in his "Engine and Boiler Trials," gives a table from which we take the following (neglecting the third decimal place given by him):

Temperature, deg. F.	Weight, lbs. per cubic foot.	Temperature, deg. F.	Weight, lbs. per cubic foot.	Temperature, deg. F.	Weight, lbs. per cubic foot.	Temperature, deg. F.	Weight, lbs. per cubic foot.	Temperature, deg. F.	Weight, lbs. per cubic foot.
212	59.71	280	57.90	350	55.58	420	52.86	490	50.03
220	59.64	290	57.59	360	55.16	430	52.47	500	49.61
230	59.37	300	57.26	370	54.79	440	52.07	510	49.20
240	59.10	310	56.93	380	54.41	450	51.66	520	48.78
250	58.81	320	56.58	390	54.03	460	51.26	530	48.36
260	58.52	330	56.24	400	53.64	470	50.85	540	47.94
270	58.21	340	55.88	410	53.26	480	50.44	550	47.52

Box on Heat gives the following:

Temperature F........	212°	250°	300°	350°	400°	450°	500°	600°
Lbs. per cubic foot....	59.82	58.85	57.42	55.94	54.34	52.70	51.02	47.64

At 212° figures given by different writers (see Trans. A. S. M. E., xiii. 409) range from 59.56 to 59.845, averaging about 59.77.

Weight of Water per Cubic Foot, from 32° to 212° F., and heat-units per pound, reckoned above 32° F.: The following table, made by interpolating the table given by Clark as calculated from Rankine's formula, with corrections for apparent errors, was published by the author in 1884, Trans. A. S. M. E., vi. 90. (For heat units above 212° see Steam Tables.)

Temp., deg. F.	Weight, lbs. per cubic foot.	Heat-units.	Temperature, deg. F.	Weight, lbs. per cubic foot.	Heat-units.	Temperature, deg. F.	Weight, lbs. per cubic foot.	Heat-units.	Temperature, deg. F.	Weight, lbs. per cubic foot.	Heat-units.
32	62.42	0.	78	62.25	46.03	123	61.68	91.16	168	60.81	136.44
33	62.42	1.	79	62.24	47.03	124	61.67	92.17	169	60.79	137.45
34	62.42	2.	80	62.23	48.04	125	61.65	93.17	170	60.77	138.45
35	62.42	3.	81	62.22	49.04	126	61.63	94.17	171	60.75	139.46
36	62.42	4.	82	62.21	50.04	127	61.61	95.18	172	60.73	140.47
37	62.42	5.	83	62.20	51.04	128	61.60	96.18	173	60.70	141.48
38	62.42	6.	84	62.19	52.04	129	61.58	97.19	174	60.68	142.49
39	62.42	7.	85	62.18	53.05	130	61.56	98.19	175	60.66	143.50
40	62.42	8.	86	62.17	54.05	131	61.54	99.20	176	60.64	144.51
41	62.42	9.	87	62.16	55.05	132	61.52	100.20	177	60.62	145.52
42	62.42	10.	88	62.15	56.05	133	61.51	101.21	178	60.59	146.52
43	62.42	11.	89	62.14	57.05	134	61.49	102.21	179	60.57	147.53
44	62.42	12.	90	62.13	58.06	135	61.47	103.22	180	60.55	148.54
45	62.42	13.	91	62.12	59.06	136	61.45	104.22	181	60.53	149.55
46	62.42	14.	92	62.11	60.06	137	61.43	105.23	182	60.50	150.56
47	62.42	15.	93	62.10	61.06	138	61.41	106.23	183	60.48	151.57
48	62.41	16.	94	62.09	62.06	139	61.39	107.24	184	60.46	152.58
49	62.41	17.	95	62.08	63.07	140	61.37	108.25	185	60.44	153.59
50	62.41	18.	96	62.07	64.07	141	61.36	109.25	186	60.41	154.60
51	62.41	19.	97	62.06	65.07	142	61.34	110.26	187	60.39	155.61
52	62.40	20.	98	62.05	66.07	143	61 32	111.26	188	60.37	156.62
53	62.40	21.01	99	62.03	67.08	144	61.30	112.27	189	60.34	157.63
54	62.40	22.01	100	62.02	68.08	145	61.28	113.28	190	60.32	158.64
55	62.39	23.01	101	62.01	69.08	146	61.26	114.28	191	60.29	159.65
56	62.39	24.01	102	62.00	70.09	147	61.24	115.29	192	60.27	160.67
57	62.39	25.01	103	61.99	71.09	148	61.22	116.29	193	60.25	161.68
58	62.38	26.01	104	61.97	72.09	149	61.20	117.30	194	60.22	162.69
59	62.38	27.01	105	61.96	73.10	150	61.18	118.31	195	60.20	163.70
60	62.37	28.01	106	61.95	74.10	151	61.16	119.31	196	60.17	164.71
61	62.37	29.01	107	61.93	75.10	152	61.14	120.32	197	60.15	165.72
62	62.36	30.01	108	61.92	76.10	153	61.12	121.33	198	60.12	166.73
63	62.36	31.01	109	61.91	77.11	154	61.10	122.33	199	60.10	167.74
64	62.35	32.01	110	61.89	78.11	155	61.08	123.34	200	60.07	168.75
65	62.34	33.01	111	61.88	79.11	156	61.06	124.35	201	60.05	169.77
66	62.34	34.02	112	61.86	80.12	157	61.04	125.35	202	60.02	170.78
67	62.33	35.02	113	61.85	81.12	158	61.02	126.36	203	60.00	171.79
68	62.33	36.02	114	61.83	82.13	159	61.00	127.37	204	59.97	172.80
69	62.32	37.02	115	61.82	83.13	160	60.98	128.37	205	59.95	173.81
70	62.31	38.02	116	61.80	84.13	161	60.96	129.38	206	59.92	174.83
71	62.31	39.02	117	61.78	85.14	162	60.94	130.39	207	59.89	175.84
72	62.30	40.02	118	61.77	86.14	163	60.92	131.40	208	59.87	176.85
73	62.29	41.02	119	61.75	87.15	164	60.90	132.41	209	59.84	177.86
74	62.28	42.03	120	61.74	88.15	165	60.87	133.41	210	59.82	178.87
75	62.28	43.03	121	61.72	89.15	166	60.85	134.42	211	59.79	179.89
76	62.27	44.03	122	61.70	90.16	167	60.83	135.43	212	59.76	180.90
77	62.26	45.03									

Comparison of Heads of Water in Feet with Pressures in Various Units.

One foot of water at 39°.1 Fahr. = 62.425 lbs. on the square foot;
" " " = 0.4335 lbs. on the square inch;
" " " = 0.0295 atmosphere;
" " " = 0.8826 inch of mercury at 32°;
" " " = 773.3 { feet of air at 32° and atmospheric pressure;

One lb. on the square foot, at 39°.1 Fahr........ = 0.01602 foot of water;
One lb. on the square inch " .... = 2.307 feet of water;
One atmosphere of 29.922 inches of mercury.... = 33.9 " " "
One inch of mercury at 32°.1.... .... = 1.133 " " "
One foot of air at 32 deg., and one atmosphere.. = 0.001293 " " "
One foot of average sea-water......... .... = 1.026 foot of pure water;
One foot of water at 62° F....................... = 62.355 lbs. per sq. foot;
" " " " " 62° F............. ... . = 0.43302 lbs. per sq. inch;
One inch of water at 62° F.................. .. .. = 0.036085 " " " "
One pound of water on the square inch at 62° F. = 2.3094 feet of water.

Pressure in Pounds per Square Inch for Different Heads of Water.

At 62° F. 1 foot head = 0.433 lb. per square inch, .433 × 144 = 62.352 lbs. per cubic foot.

Head, feet.	0	1	2	3	4	5	6	7	8	9
0		0.433	0.866	1.299	1.732	2.165	2.598	3.031	3.464	3.897
10	4.330	4.763	5.196	5.629	6.062	6.495	6.928	7.361	7.794	8.227
20	8.660	9.093	9.526	9.959	10 392	10.825	11.258	11.691	12.124	12.557
30	12.990	13.423	13.856	14.289	14.722	15.155	15.588	16.021	16.454	16.887
40	17.320	17.753	18.186	18.619	19.052	19.485	19.918	20.351	20.784	21.217
50	21.650	22.083	22.516	22.949	23.382	23.815	24.248	24.681	25.114	25.547
60	25.980	26.413	26.846	27.279	27.712	28.145	28.578	29.011	29.444	29.877
70	30.310	30.743	31.176	31.609	32.042	32.475	32.908	33.341	33.774	34.207
80	34.640	35.073	35.506	35.939	36.372	36.805	37.238	37.671	38.104	38.537
90	38.970	39.403	39.836	40.269	40.702	41.135	41.568	42.001	42.436	42.867

Head in Feet of Water, Corresponding to Pressures in Pounds per Square Inch.

1 lb. per square inch = 2.30947 feet head, 1 atmosphere = 14.7 lbs. per sq. inch = 33.94 ft. head.

Pressure.	0	1	2	3	4	5	6	7	8	9
0		2.309	4.619	6.928	9.238	11.547	13.857	16.166	18.476	20 785
10	23.0947	25.404	27.714	30.023	32.333	34.642	36.952	39.261	41.570	43.880
20	46.1894	48.499	50.808	53.118	55.427	57.737	60.046	62.356	64.665	66.975
30	69.2841	71.594	73.903	76.213	78.522	80.831	83.141	85.450	87.760	90.069
40	92.3788	94.688	96.998	99.307	101.62	103.93	106.24	108.55	110.85	113.16
50	115.4735	117.78	120.09	122.40	124.71	126.02	129.33	131.64	133.95	136.26
60	138.5682	140.88	143.19	145.50	147.81	150.12	152.42	154.73	157.04	159.35
70	161.6629	163.97	166.28	168.59	170.90	173 21	175.52	177.83	180.14	182.45
80	184.7576	187.07	189.38	191.69	194 00	196.31	198.61	200.92	203.23	205 54
90	207.8523	210.16	212.47	214.78	217.09	219.40	221.71	224.02	226.33	228.64

Pressure of Water due to its Weight.—The pressure of still water in pounds per square inch against the sides of any pipe, channel, or vessel of any shape whatever is due solely to the "head," or height of the level surface of the water above the point at which the pressure is considered, and is equal to .43302 lb. per square inch for every foot of head, or 62.355 lbs. per square foot for every foot of head (at 62° F.).

The pressure per square inch is equal in all directions, downwards, upwards, or sideways, and is independent of the shape or size of the containing vessel.

The pressure against a vertical surface, as a retaining-wall, at any point is in direct ratio to the head above that point, increasing from 0 at the level surface to a maximum at the bottom. The total pressure against a vertical strip of a unit's breadth increases as the area of a right-angled triangle whose perpendicular represents the height of the strip and whose base represents the pressure on a unit of surface at the bottom; that is, it increases as the square of the depth. The sum of all the horizontal pressures is represented by the area of the triangle, and the resultant of this sum is equal to this sum exerted at a point one third of the height from the bottom. (The centre of gravity of the area of a triangle is one third of its height.)

The horizontal pressure is the same if the surface is inclined instead of vertical.

(For an elaboration of these principles see Trautwine's Pocket-Book, or the chapter on Hydrostatics in any work on Physics. For dams, retaining-walls, etc., see Trautwine.)

The amount of pressure on the interior walls of a pipe has no appreciable effect upon the amount of flow.

Buoyancy.—When a body is immersed in a liquid, whether it float or sink, it is buoyed up by a force equal to the weight of the bulk of the liquid displaced by the body. The weight of a floating body is equal to the weight of the bulk of the liquid that it displaces. The upward pressure or buoyancy of the liquid may be regarded as exerted at the centre of gravity of the displaced water, which is called the centre of pressure or of buoyancy. A vertical line drawn through it is called the axis of buoyancy or of flotation. In a floating body at rest a line joining the centre of gravity and the centre of buoyancy is vertical, and is called the axis of equilibrium. When an external force causes the axis of equilibrium to lean, if a vertical line be drawn upward from the centre of buoyancy to this axis, the point where it cuts the axis is called the *metacentre*. If the metacentre is above the centre of gravity the distance between them is called the metacentric height, and the body is then said to be in stable equilibrium, tending to return to its original position when the external force is removed.

Boiling-point.—Water boils at 212° F. (100° C.) at mean atmospheric pressure at the sea-level, 14.696 lbs. per square inch. The temperature at which water boils at any given pressure is the same as the temperature of saturated steam at the same pressure. For boiling-point of water at other pressure than 14.696 lbs. per square inch, see table of the Properties of Saturated Steam.

The Boiling-point of Water may be Raised.—When water is entirely freed of air, which may be accomplished by freezing or boiling, the cohesion of its atoms is greatly increased, so that its temperature may be raised over 50° above the ordinary boiling-point before ebullition takes place. It was found by Faraday that when such air-freed water did boil, the rupture of the liquid was like an explosion. When water is surrounded by a film of oil, its boiling temperature may be raised considerably above its normal standard. This has been applied as a theoretical explanation in the instance of boiler-explosions.

The freezing-point also may be lowered, if the water is perfectly quiet, to − 10° C., or 18° Fahrenheit below the normal freezing-point. (Hamilton Smith, Jr., on Hydraulics, p. 13.) The density of water at 14° F. is .99814, its density at 39°. 1 being 1, and at 32°, .99987.

Freezing-point.—Water freezes at 32° F. at the ordinary atmospheric pressure, and ice melts at the same temperature. In the melting of 1 pound of ice into water at 32° F. about 142 heat-units are absorbed, or become latent; and in freezing 1 lb. of water into ice a like quantity of heat is given out to the surrounding medium.

Sea-water freezes at 27° F. The ice is fresh. (Trautwine.)

Ice and Snow. (From Clark.)—1 cubic foot of ice at 32° F. weighs 57.50 lbs.; 1 pound of ice at 32° F. has a volume of .0174 cu. ft. = 30.067 cu. in.

Relative volume of ice to water at 32° F., 1.0855, the expansion in passing into the solid state being 8.55%. Specific gravity of ice = 0.922, water at 62° F. being 1.

At high pressures the melting-point of ice is lower than 32° F., being at the rate of .0133° F. for each additional atmosphere of pressure.

The specific heat of ice is .504, that of water being 1.

1 cubic foot of fresh snow, according to humidity of atmosphere: 5 lbs. to 12 lbs. 1 cubic foot of snow moistened and compacted by rain: 15 lbs. to 50 lbs. (Trautwine).

Specific Heat of Water. (From Clark's Steam-engine.)—Calculated by means of Regnault's formula, $c = 1 + 0.00004t + 0.0000009t^2$, in which c is the specific heat of water at any temperature t in centigrade degrees, the specific heat at the freezing-point being 1.

Temperatures.		British Thermal Units per pound, above 32° F.	Specific Heat at the given Temperature.	Mean Specific Heat between 32° F. and the given Temp.	Temperatures.		British Thermal Units per pound, above 32° F.	Specific Heat at the given Temperature.	Mean Specific Heat between 32° F. and the given Temp.
Cent.	Fahr.				Cent.	Fahr.			
0°	32°	0.000	1.0000		120°	248°	217.449	1.0177	1.0067
10	50	18.004	1.0005	1.0002	130	266	235.791	1.0204	1.0076
20	68	36.018	1.0012	1.0005	140	284	254.187	1.0232	1.0087
30	86	54.047	1.0020	1.0009	150	302	272.628	1.0262	1.0097
40	104	72.090	1.0030	1.0013	160	320	291.132	1.0294	1.0109
50	122	90.157	1.0042	1.0017	170	338	309.690	1.0328	1.0121
60	140	108.247	1.0056	1.0023	180	356	328.320	1.0364	1.0133
70	158	126.378	1.0072	1.0030	190	374	347.004	1.0401	1.0146
80	176	144.508	1.0089	1.0035	200	392	365.760	1.0440	1.0160
90	194	162.686	1.0109	1.0042	210	410	384.588	1.0481	1.0174
100	212	180.900	1.0130	1.0050	220	428	403.48[illegible]	1.0524	1.0189
110	230	199.152	1.0153	1.0058	230	446	422.47[illegible]	1.0568	1.0204

Compressibility of Water.—Water is very slightly compressible. Its compressibility is from .000040 to .000051 for one atmosphere, decreasing with increase of temperature. For each foot of pressure distilled water will be diminished in volume .0000015 to .0000013. Water is so incompressible that even at a depth of a mile a cubic foot of water will weigh only about half a pound more than at the surface.

THE IMPURITIES OF WATER.

(A. E. Hunt and G. H. Clapp, Trans. A. I. M. E. xvii. 338.)

Commercial analyses are made to determine concerning a given water: (1) its applicability for making steam; (2) its hardness, or the facility with which it will "form a lather" necessary for washing; or (3) its adaptation to other manufacturing purposes.

At the Buffalo meeting of the Chemical Section of the A. A. A. S. it was decided to report all water analyses in parts per thousand, hundred-thousand, and million.

To convert grains per imperial (British) gallons into parts per 100,000, divide by 0.7. To convert parts per 100,000 into grains per U. S. gallon, multiply by 7/12 or .583.

The most common commercial analysis of water is made to determine its fitness for making steam. Water containing more than 5 parts per 100,000 of free sulphuric or nitric acid is liable to cause serious corrosion, not only of the metal of the boiler itself, but of the pipes, cylinders, pistons, and valves with which the steam comes in contact.

The total residue in water used for making steam causes the interior linings of boilers to become coated, and often produces a dangerous hard scale, which prevents the cooling action of the water from protecting the metal against burning.

Lime and magnesia bicarbonates in water lose their excess of carbonic acid on boiling, and often, especially when the water contains sulphuric acid, produce, with the other solid residues constantly being formed by the evaporation, a very hard and insoluble scale. A larger amount than 100 parts per 100,000 of total solid residue will ordinarily cause troublesome scale, and should condemn the water for use in steam-boilers, unless a better supply cannot be obtained.

The following is a tabulated form of the causes of trouble with water for steam purposes, and the proposed remedies, given by Prof. L. M. Norton.

Causes of Incrustation.

1. Deposition of suspended matter.
2. Deposition of deposed salts from concentration.
3. Deposition of carbonates of lime and magnesia by boiling off carbonic acid, which holds them in solution.

4. Deposition of sulphates of lime, because sulphate of lime is but slightly soluble in cold water, less soluble in hot water, insoluble above 270° F.
5. Deposition of magnesia, because magnesium salts decompose at high temperature.
6. Deposition of lime soap, iron soap, etc., formed by saponification of grease.

Means for Preventing Incrustation.

1. Filtration.
2. Blowing off.
3. Use of internal collecting apparatus or devices for directing the circulation.
4. Heating feed-water.
5. Chemical or other treatment of water in boiler.
6. Introduction of zinc into boiler.
7. Chemical treatment of water outside of boiler.'

Tabular View.

Troublesome Substance.	Trouble.	Remedy or Palliation.
Sediment, mud, clay, etc.	Incrustation.	Filtration; blowing off.
Readily soluble salts.	"	Blowing off.
Bicarbonates of lime, magnesia, iron.	"	Heating feed. Addition of caustic soda, lime, or magnesia, etc.
Sulphate of lime.	"	Addition of carb. soda, barium hydrate, etc.
Chloride and sulphate of magnesium.	Corrosion.	Addition of carbonate of soda, etc.
Carbonate of soda in large amounts.	Priming.	Addition of barium chloride, etc.
Acid (in mine waters).	Corrosion.	Alkali.
Dissolved carbonic acid and oxygen.	Corrosion.	Feed milk of lime to the boiler, to form a thin internal coating.
Grease (from condensed water).	Corrosion or incrustation.	Different cases require different remedies. Consult a specialist on the subject.
Organic matter (sewage).	Priming, corrosion, or incrustation.	

' The mineral matters causing the most troublesome boiler-scales are bicarbonates and sulphates of lime and magnesia, oxides of iron and alumina, and silica. The analyses of some of the most common and troublesome boiler-scales are given in the following table:

Analyses of Boiler-scale. (Chandler.)

	Sulphate of Lime.	Magnesia.	Silica.	Peroxide of Iron.	Water.	Carbonate of Lime.
N. Y. C. & H. R. Ry., No. 1	74.07	9.19	0.65	0.08	1.14	14.78
" " " No. 2	71.37	....	1.76	....	....	....
" " " No. 3	62.86	18.96	2.60	0.92	1.28	13.62
" " " No. 4	53.05	....	4.79	....	....	....
" " " No. 5	46.83	....	5.82	....	....	....
" " " No. 6	30.80	31.17	7.75	1.08	2.44	26.93
" " " No. 7	4.95	2.01	2.07	1.08	0.63	86.25
" " " No. 8	0.88	2.84	0.65	0.36	0.15	93.19
" " " No. 9	4.81	....	2.92	....	....	....
" " " No. 10	30.07	....	8.24	....	....	....

Analyses in Parts per 100,000 of Water giving Bad Results in Steam-boilers. (A. E. Hunt.)

	Bicarbonate of Lime deposited on Boiling.	Bicarbonate of Magnesia depos'd on Boil'g.	Total Lime.	Total Magnesia.	Sulphuric Acid.	Chlorine.	Iron.	Organic Matter.	Alumina.	Chloride of Sodium.
Coal-mine water............	110	25	119	39	890	590	780	30	640	......
Salt-well.............	151	38	1.90	48	860	990	88	21	30	18.10
Spring......................	75	89	95	120	310	21	75	10	80	86
Monongahela River.........	130	21	161	33	210	38	70	....	....	......
" "	80	70	94	81	219	210	90	....	....	......
" "	32	32	61	1.04	28	1.90	38	...	...	......
Allegheny R., near Oil-works	30	50	41	68	890	42	23	....	....	......

Many substances have been added with the idea of causing chemical action which will prevent boiler-scale. As a general rule, these do more harm than good, for a boiler is one of the worst possible places in which to carry on chemical reaction, where it nearly always causes more or less corrosion of the metal, and is liable to cause dangerous explosions.

In cases where water containing large amounts of total solid residue is necessarily used, a heavy petroleum oil, free from tar or wax, which is not acted upon by acids or alkalies, not having sufficient wax in it to cause saponification, and which has a vaporizing-point at nearly 600° F., will give the best results in preventing boiler-scale. Its action is to form a thin greasy film over the boiler linings, protecting them largely from the action of acids in the water and greasing the sediment which is formed, thus preventing the formation of scale and keeping the solid residue from the evaporation of the water in such a plastic suspended condition that it can be easily ejected from the boiler by the process of "blowing off." If the water is not blown off sufficiently often, this sediment forms into a "putty" that will necessitate cleaning the boilers. Any boiler using bad water should be blown off every twelve hours.

Hardness of Water.—The hardness of water, or its opposite quality, indicated by the ease with which it will form a lather with soap, depends almost altogether upon the presence of compounds of lime and magnesia. Almost all soaps consist, chemically, of oleate, stearate, and palmitate, of an alkaline base, usually soda and potash. The more lime and magnesia in a sample of water, the more soap a given volume of the water will decompose, so as to give insoluble oleate, palmitate, and stearate of lime and magnesia, and consequently the more soap must be added to a gallon of water in order that the necessary quantity of soap may remain in solution to form the lather. The relative hardness of samples of water is generally expressed in terms of the number of standard soap-measures consumed by a gallon of water in yielding a permanent lather.

The standard soap-measure is the quantity required to precipitate one grain of carbonate of lime.

It is commonly reckoned that one gallon of pure distilled water takes one soap-measure to produce a lather. Therefore one is deducted from the total number of soap-measures found to be necessary to use to produce a lather in a gallon of water, in reporting the number of soap-measures, or "degrees" of hardness of the water sample. In actually making tests for hardness, the "miniature gallon," or seventy cubic centimetres, is used rather than the inconvenient larger amount. The standard measure is made by completely dissolving ten grammes of pure castile soap (containing 60 per cent olive-oil) in a litre of weak alcohol (of about 35 per cent alcohol). This yields a solution containing exactly sufficient soap in one cubic centimeter of the solution to precipitate one milligramme of carbonate of lime, or, in other words, the standard soap solution is reduced to terms of the "miniature gallon" of water taken.

If a water charged with a bicarbonate of lime, magnesia, or iron is boiled, it will, on the excess of the carbonic acid being expelled, deposit a considerable quantity of the lime, magnesia, or iron, and consequently the water will be softer. The hardness of the water after this deposit of lime, after long boiling, is called the *permanent hardness* and the difference between it and the total hardness is called *temporary hardness*.

Lime salts in water react immediately on soap-solutions, precipitating the oleate, palmitate, or stearate of lime at once. Magnesia salts, on the contrary, require some considerable time for reaction. They are, however, more powerful hardeners; one equivalent of magnesia salts consuming as much soap as one and one-half equivalents of lime.

The presence of soda and potash salts softens rather than hardens water. Each grain of carbonate of lime per gallon of water causes an increased expenditure for soap of about 2 ounces per 100 gallons of water. (*Eng'g. News*, Jan. 31, 1885.)

Purifying Feed-water for Steam-boilers. (See also Incrustation and Corrosion, p. 716.)—When the water used for steam-boilers contains a large amount of scale-forming material it is usually advisable to purify it before allowing it to enter the boiler rather than to attempt the prevention of scale by the introduction of chemicals into the boiler. Carbonates of lime and magnesia may be removed to a considerable extent by simple heating of the water in an exhaust-steam feed-water heater or, still better, by a live-steam heater. (See circular of the Hoppes Mfg. Co., Springfield, O.) When the water is very bad it is best treated with chemicals—lime, soda-ash, caustic soda, etc.—in tanks, the precipitates being separated by settling or filtering. For a description of several systems of water purification see a series of articles on the subject by Albert A. Cary in *Eng'g Mag.*, 1897.

Mr. W. B. Coggswell, of the Solvay Process Co.'s Soda Works in Syracuse, N. Y., thus describes the system of purification of boiler feed-water in use at these works (Trans. A. S. M. E., xiii. 255):

For purifying, we use a weak soda liquor, containing about 12 to 15 grams Na_2Co_3 per litre. Say 1½ to 2 M^3 (or 397 to 530 gals.) of this liquor is run into the precipitating tank. Hot water about 60° C. is then turned in, and the reaction of the precipitation goes on while the tank is filling, which requires about 15 minutes. When the tank is full the water is filtered through the Hyatt (4), 5 feet diameter, and the Jewell (1), 10 feet diameter, filters in 30 minutes. Forty tanks treated per 24 hours.

Charge of water purified at once	35 M^3, 9,275 gallons.
Soda in purifying reagent	15 kgs. Na_2CO_3.
Soda used per 1,000 gallons	3.5 lbs.

A sample is taken from each boiler every other day and tested for deg. Baumé, soda and salt. If the deg. B. is more than 2, that boiler is blown to reduce it below 2 deg. B.

The following are some analyses given by Mr. Coggswell:

	Lake Water, grams per litre.	Mud from Hyatt Filter.	Scale from Boiler-tube.	Scale found in Pump.
Calcium sulphate	.261	3.70	51.24	10.9
Calcium chloride	.186	....	....	....
Calcium carbonate	.091	63.37	19.76	87.
Magnesium carbonate	.015	1.11	25.21	....
Magnesium chloride	.067	....	....	....
Salt, NaCl	.63	....	.14	....
Silica	....	15.17	2.29	.8
Iron and aluminum oxide	....	3.75	1.10	1.2
Total	1.270	87.10	99.74	99.9

Softening Hard Water for Locomotive Use.—A water-softening plant in operation at Fossil, in Western Wyoming, on the Union Pacific Railway, is described in *Eng'g News*, June 9, 1892. It is the invention of Arthur Pennell, of Kansas City. The general plan adopted is to first dissolve the chemicals in a closed tank, and then connect this to the supply main so that its contents will be forced into the main tank, the supply-pipe being so arranged that thorough mixture of the solution with the water is obtained. A waste-pipe from the bottom of the tank is opened from time to time to draw off the precipitate. The pipe leading to the tender is arranged to draw the water from near the surface.

A water-tank 24 feet in diameter and 16 feet high will contain about 46,600 gallons of water. About three hours should be allowed for this amount of water to pass through the tank to insure thorough precipitation, giving a permissible consumption of about 15,000 gallons per hour. Should more than this be required, auxiliary settling-tanks should be provided.

The chemicals added to precipitate the scale-forming impurities are sodium carbonate and quicklime, varying in proportions according to the relative proportions of sulphates and carbonates in the water to be treated. Sufficient sodium carbonate is added to produce just enough sodium sulphate to combine with the remaining lime and magnesia sulphate and produce glauberite or its corresponding magnesia salt, thereby to get rid of the sodium sulphate, which produces foaming, if allowed to accumulate.

For a description of a purifying plant established by the Southern Pacific R. R. Co. at Port Los Angeles, Cal., see a paper by Howard Stillmann in Trans. A. S. M. E., vol. xix, Dec. 1897.

HYDRAULICS—FLOW OF WATER.

Formulæ for Discharge of Water though Orifices and Weirs.—For rectangular or circular orifices, with the head measured from centre of the orifice to the surface of the still water in the feeding reservoir.

$$Q = C\sqrt{2gH} \times a. \qquad (1)$$

For weirs with no allowance for increased head due to velocity of approach:

$$Q = C\tfrac{2}{3}\sqrt{2gH} \times LH. \qquad (2)$$

For rectangular and circular or other shaped vertical or inclined orifices; formula based on the proposition that each successive horizontal layer of water passing through the orifice has a velocity due to its respective head:

$$Q = cL\tfrac{2}{3}\sqrt{2g} \times (\sqrt{Hb^3} - \sqrt{Ht^3}). \qquad (3)$$

For rectangular vertical weirs:

$$Q = c\tfrac{2}{3}\sqrt{2gH} \times Lh. \qquad (4)$$

Q = quantity of water discharged in cubic feet per second; C = approximate coefficient for formulas (1) and (2); c = correct coefficient for (3) and (4).

Values of the coefficients c and C are given below.

$g = 32.16$; $\sqrt{2g} = 8.02$; H = head in feet measured from centre of orifice to level of still water; Hb = head measured from bottom of orifice; Ht = head measured from top of orifice; $h = H$, corrected for velocity of approach, Va, $= H + \frac{4}{3}\frac{Va^2}{2g}$; a = area in square feet; L = length in feet.

Flow of Water from Orifices.—The theoretical velocity of water flowing from an orifice is the same as the velocity of a falling body which has fallen from a height equal to the head of water, $= \sqrt{2gH}$. The actual velocity at the smaller section of the *vena contracta* is substantially the same as the theoretical, but the velocity at the plane of the orifice is $C\sqrt{2gH}$, in which the coefficient C has the nearly constant value of .62. The smallest diameter of the *vena contracta* is therefore about .79 of that of the orifice. If C be the approximate coefficient = .62, and c the correct coefficient, the ratio $\frac{C}{c}$ varies with different ratios of the head to the diameter of the vertical orifice, or to $\frac{H}{D}$. Hamilton Smith, Jr., gives the following:

For $\frac{H}{D}$ =	.5	.875	1	1.5	2.	2.5	5.	10.
$\frac{C}{c}$ =	.9604	.9849	.9918	.9965	.9980	.9987	.9997	1.

For vertical rectangular orifices of ratio of head to width W:

For $\frac{H}{W}$ =	.5	.6	.8	1	1.5	2.	3.	4.	5.	8.
$\frac{C}{c}$ =	.9428	.9657	.9823	.9890	.9953	.9974	.9988	.9993	.9996	.9998

For $H \div D$ or $H \div W$ over 8, $C = c$, practically.

Weisbach gives the following values of c for circular orifices in a thin wall. H = measured head from centre of orifice.

D ft.	H ft.						
	.066	.33	.82	2.0	3.0	45.	340.
.033	.711	.665	.637	.628	.641	.632	.600
.066			.629	.621			
.10			.622	.614			
.13			.614	.607			

For an orifice of D = .033 ft. and a well-rounded mouthpiece, H being the effective head in feet,

H =	.066	1.64	11.5	56	338
c =	.959	.967	.975	.994	.994

Hamilton Smith, Jr., found that for great heads, 312 ft. to 336 ft., with converging mouthpieces, c has a value of about one, and for small circular orifices in thin plates, with full contraction, c = about .60. Some of Mr. Smith's experimental values of c for orifices in thin plates discharging into air are as follows. All dimensions in feet.

Circular, in steel, D = .020,	H =	.739	2.43	3.19			
	c =	.6495	.6298	.6264			
Circular, in brass, D = .050,	H =	.185	.536	1.74	2.73	3.57	4.63
	c =	.6525	.6265	.6118	.6070	.6060	.6051
Circular, in brass, D = .100,	H =	.129	.457	.900	1.73	2.05	3.18
	c =	.6337	.6155	.6096	.6042	.6038	.6025
Circular, in iron, D = .100,	H =	1.80	1.81	2.81	4.68		
	c =	.6061	.6041	.6033	.6026		
Square, in brass, .05 × .05,	H =	.313	.877	1.79	2.81	3.70	4.63
	c =	.6410	.6238	.6157	.6127	.6113	.6097
Square, in brass, .10 × .10,	H =	.181	.939	1.71	2.75	3.74	4.59
	c =	.6292	.6139	.6084	.6076	.6060	.6065
Rectangular, in brass, L = .300, W = .050......	H =	.261	.917	1.82	2.83	3.75	4.70
	c =	.6476	.6280	.6203	.6180	.6176	.6168

For the rectangular orifice, L, the length, is horizontal.

Mr. Smith, as the result of the collation of much experimental data of others as well as his own, gives tables of the value of c for vertical orifices, with full contraction, with a free discharge into the air, with the inner face of the plate, in which the orifice is pierced, plane, and with sharp inner corners, so that the escaping vein only touches these inner edges. These tables are abridged below. The coefficient c is to be used in the formulæ (3) and (4) above. For formulæ (1) and (2) use the coefficient C found from the values of the ratios $\frac{C}{c}$ above.

Values of Coefficient *c* for Vertical Orifices with Sharp Edges, Full Contraction, and Free Discharge into Air. (Hamilton Smith, Jr.)

Head from Centre of Orifice *H*.	Square Orifices. Length of the Side of the Square, in feet.												
	.02	.03	.04	.05	.07	.10	.12	.15	.20	.40	.60	.80	1.0
.4			.643	.637	.628	.621	.616	.611					
.6	.660	.645	.636	.630	.623	.617	.613	.610	.605	.601	.598	.596	
1.0	.648	.636	.628	.622	.618	.613	.610	.608	.605	.603	.601	.600	.599
3.0	.632	.622	.616	.612	.609	.607	.606	.606	.605	.605	.604	.603	.603
6.0	.628	.616	.612	.609	.607	.605	.605	.605	.604	.604	.603	.602	.602
10.	.616	.611	.608	.606	.605	.604	.604	.603	.603	.603	.602	.602	.601
20.	.606	.605	.604	.603	.602	.602	.602	.602	.602	.601	.601	.601	.600
100.(?)	.599	.598	.598	.598	.598	.598	.598	.598	.598	.598	.598	.598	.598
H.	Circular Orifices. Diameters, in feet.												
	.02	.03	.04	.05	.07	.10	.12	.15	.20	.40	.60	.80	1.0
.4				.637	.628	.618	.612	.606					
.6	.655	.640	.630	.624	.618	.613	.609	.605	.601	.596	.593	.590	
1.0	.644	.631	.623	.617	.612	.608	.605	.603	.600	.598	.595	.593	.591
2.	.632	.621	.614	.610	.607	.604	.601	.600	.599	.599	.597	.596	.595
4.	.623	.614	.609	.605	.603	.602	.600	.599	.599	.598	.597	.597	.596
6.	.618	.611	.607	.604	.602	.600	.599	.599	.598	.598	.597	.596	.596
10.	.611	.606	.603	.601	.599	.598	.598	.597	.597	.597	.596	.596	.595
20.	.601	.600	.599	.598	.597	.596	.596	.596	.596	.596	.596	.595	.594
50.(?)	.596	.596	.595	.595	.594	.594	.594	.594	.594	.594	.594	.593	.593
100.(?)	.593	.593	.592	.592	.592	.592	.592	.592	.592	.592	.592	.592	.592

HYDRAULIC FORMULÆ.—FLOW OF WATER IN OPEN AND CLOSED CHANNELS.

Flow of Water in Pipes.—The quantity of water discharged through a pipe depends on the "head;" that is, the vertical distance between the level surface of still water in the chamber at the entrance end of the pipe and the level of the centre of the discharge end of the pipe; also upon the length of the pipe, upon the character of its interior surface as to smoothness, and upon the number and sharpness of the bends: but it is independent of the position of the pipe, as horizontal, or inclined upwards or downwards.

The head, instead of being an actual distance between levels, may be caused by pressure, as by a pump, in which case the head is calculated as a vertical distance corresponding to the pressure 1 lb. per sq. in. = 2.309 ft. head, or 1 ft. head = .433 lb. per sq. in.

The total head operating to cause flow is divided into three parts: 1. The *velocity-head*, which is the height through which a body must fall *in vacuo* to acquire the velocity with which the water flows into the pipe = $v^2 \div 2g$, in which v is the velocity in ft. per sec. and $2g = 64.32$; 2. the *entry-head*, that required to overcome the resistance to entrance to the pipe. With sharp-edged entrance the entry-head = about 1/2 the velocity-head; with smooth rounded entrance the entry-head is inappreciable; 3. the *friction-head*, due to the frictional resistance to flow within the pipe.

In ordinary cases of pipes of considerable length the sum of the entry and velocity heads required scarcely exceeds 1 foot. In the case of long pipes with low heads the sum of the velocity and entry heads is generally so small that it may be neglected.

General Formula for Flow of Water in Pipes or Conduits.

$$\text{Mean velocity in ft. per sec.} = c\sqrt{\text{mean hydraulic radius} \times \text{slope}}$$

$$\text{Do. for pipes running full} = c\sqrt{\frac{\text{diameter}}{4} \times \text{slope}},$$

in which c is a coefficient determined by experiment. (See pages 559-564.)

The mean hydraulic radius $= \dfrac{\text{area of wet cross-section}}{\text{wet perimeter.}}$

In pipes running full, or exactly half full, and in semicircular open channels running full it is equal to ¼ diameter.

The slope = the head (or pressure expressed as a head, in feet) ÷ length of pipe measured in a straight line from end to end.

In open channels the slope is the actual slope of the surface, or its fall per unit of length, or the sine of the angle of the slope with the horizon.

If r = mean hydraulic radius, s = slope = head ÷ length, v = velocity in feet per second (all dimensions in feet), $v = c\sqrt{r}\sqrt{s} = c\sqrt{rs}$.

Quantity of Water Discharged.—If Q = discharge in cubic feet per second and a = area of channel, $Q = av = ac\sqrt{rs}$.

$a\sqrt{r}$ is approximately proportional to the discharge. It is a maximum at 308°, corresponding to 19/20 of the diameter, and the flow of a conduit 19/20 full is about 5 per cent greater than that of one completely filled.

Table giving Fall in Feet per Mile, the Distance on Slope corresponding to a Fall of 1 Ft., and also the Values of s and $\sqrt{s}$ for Use in the Formula $v = c\sqrt{rs}$.

$s = H \div L$ = sine of angle of slope = fall of water-surface (H), in any distance (L), divided by that distance.

Fall in Feet per Mi.	Slope, 1 Foot in	Sine of Slope, s.	$\sqrt{s}$.	Fall in Feet per Mi.	Slope, 1 Foot in	Sine of Slope, s.	$\sqrt{s}$.
0.25	21120	.0000473	.006881	17	310.6	.0032197	.056742
.30	17600	.0000568	.007538	18	293.3	.0034091	.058388
.40	13200	.0000758	.008704	19	277.9	.0035985	.059988
.50	10560	.0000947	.009731	20	264	.0037879	.061546
.60	8800	.0001136	.010660	22	240	.0041667	.064549
.702	7520	.0001330	.011532	24	220	.0045455	.067419
.805	6560	.0001524	.012347	26	203.1	.0049242	.070173
.904	5840	.0001712	.013085	28	188.6	.0053030	.072822
1.	5280	.0001894	.013762	30	176	.0056818	.075378
1.25	4224	.0002367	.015386	35.20	150	.0066667	.081650
1.5	3520	.0002841	.016854	40	132	.0075758	.087039
1.75	3017	.0003314	.018205	44	120	.0083333	.091287
2.	2640	.0003788	.019463	48	110	.0090909	.095346
2.25	2347	.0004261	.020641	52.8	100	.010	.1
2.5	2112	.0004735	.021760	60	88	.0113636	.1066
2.75	1920	.0005208	.022822	66	80	.0125	.111803
3.	1760	.0005682	.023837	70.4	75	.0133333	.115470
3.25	1625	.0006154	.024807	80	66	.0151515	.123091
3.5	1508	.0006631	.025751	88	60	.0166667	.1291
3.75	1408	.0007102	.026650	96	55	.0181818	.134839
4	1320	.0007576	.027524	105.6	50	.02	.141421
5	1056	.0009470	.030773	120	44	.0227273	.150756
6	880	.0011364	.03371	132	40	.025	.158114
7	754.3	.0013257	.036416	160	33	.0303030	.174077
8	660	.0015152	.038925	220	24	.0416667	.204124
9	586.6	.0017044	.041286	264	20	.05	.223607
10	528	.0018939	.043519	330	16	.0625	.25
11	443.6	.0020833	.045643	440	12	.0833333	.288675
12	440	.0022727	.047673	528	10	.1	.316228
13	406.1	.0024621	.04962	660	8	.125	.353553
14	377.1	.0026515	.051493	880	6	.1666667	.408248
15	352	.0028409	.0533	1056	5	.2	.447214
16	330	.0030303	.055048	1320	4	.25	.5

Values of $\sqrt{r}$ for Circular Pipes, Sewers, and Conduits of different Diameters.

r = mean hydraulic depth = $\dfrac{\text{area}}{\text{perimeter}}$ = ¼ diam. for circular pipes running full or exactly half full.

Diam., ft. in.	$\sqrt{r}$ in Feet.	Diam., ft. in.	$\sqrt{r}$ in Feet.	Diam., ft. in.	$\sqrt{r}$ in Feet.	Diam., ft. in.	$\sqrt{r}$ in Feet.
⅜	.088	2	.707	4 6	1.061	9	1.500
½	.102	2 1	.722	4 7	1.070	9 3	1.521
¾	.125	2 2	.736	4 8	1.080	9 6	1.541
1	.144	2 3	.750	4 9	1.089	9 9	1.561
1¼	.161	2 4	.764	4 10	1.099	10	1.581
1½	.177	2 5	.777	4 11	1.109	10 3	1.601
1¾	.191	2 6	.790	5	1.118	10 6	1.620
2	.204	2 7	.804	5 1	1.127	10 9	1.639
2½	.228	2 8	.817	5 2	1.137	11	1.658
3	.251	2 9	.829	5 3	1.146	11 3	1.677
4	.290	2 10	.842	5 4	1.155	11 6	1.696
5	.323	2 11	.854	5 5	1.164	11 9	1.714
6	.354	3	.866	5 6	1.173	12	1.732
7	.382	3 1	.878	5 7	1.181	12 3	1.750
8	.408	3 2	.890	5 8	1.190	12 6	1.768
9	.433	3 3	.901	5 9	1.199	12 9	1.785
10	.456	3 4	.913	5 10	1.208	13	1.083
11	.479	3 5	.924	5 11	1.216	13 3	1.820
1	.500	3 6	.935	6	1.225	13 6	1.837
1 1	.520	3 7	.946	6 3	1.250	14	1.871
1 2	.540	3 8	.957	6 6	1.275	14 6	1.904
1 3	.559	3 9	.968	6 9	1.299	15	1.936
1 4	.577	3 10	.979	7	1.323	15 6	1.968
1 5	.595	3 11	.990	7 3	1.346	16	2.
1 6	.612	4	1.	7 6	1.369	16 6	2.031
1 7	.629	4 1	1.010	7 9	1.392	17	2.061
1 8	.646	4 2	1.021	8	1.414	17 6	2.091
1 9	.661	4 3	1.031	8 3	1.436	18	2.121
1 10	.677	4 4	1.041	8 6	1.458	19	2.180
1 11	.692	4 5	1.051	8 9	1.479	20	2.236

Values of the Coefficient c. (Chiefly condensed from P. J. Flynn on Flow of Water.)—Almost all the old hydraulic formulæ for finding the mean velocity in open and closed channels have constant coefficients, and are therefore correct for only a small range of channels. They have often been found to give incorrect results with disastrous effects. Ganguillet and Kutter thoroughly investigated the American, French, and other experiments, and they gave as the result of their labors the formula now generally known as Kutter's formula. There are so many varying conditions affecting the flow of water, that all hydraulic formulæ are only approximations to the correct result.

When the surface-slope measurement is good, Kutter's formula will give results seldom exceeding 7½% error, provided the rugosity coefficient of the formula is known for the site. For small open channels D'Arcy's and Bazin's formulæ, and for cast-iron pipes D'Arcy's formulæ, are generally accepted as being approximately correct.

Kutter's Formula for measures in feet is

$$v = \left\{ \frac{\dfrac{1.811}{n} + 41.6 + \dfrac{.00281}{s}}{1 + \left(41.6 + \dfrac{.00281}{s}\right) \times \dfrac{n}{\sqrt{r}}} \right\} \times \sqrt{rs},$$

in which v = mean velocity in feet per second; $r = \dfrac{a}{p}$ = hydraulic mean depth in feet = area of cross-section in square feet divided by wetted perimeter in lineal feet; s = fall of water-surface (h) in any distance (l) divided by that distance, $= \frac{h}{l}$, = sine of slope; n = the coefficient of rugosity, depending on the nature of the lining or surface of the channel. If we let the first term of the right-hand side of the equation equal c, we have Chezy's formula, $v = c\sqrt{rs} = c \times \sqrt{r} \times \sqrt{s}$.

Values of n in Kutter's Formula.—The accuracy of Kutter's formula depends, in a great measure, on the proper selection of the coefficient of roughness n. Experience is required in order to give the right value to this coefficient, and to this end great assistance can be obtained, in making this selection, by consulting and comparing the results obtained from experiments on the flow of water already made in different channels.

In some cases it would be well to provide for the contingency of future deterioration of channel, by selecting a high value of n, as, for instance, where a dense growth of weeds is likely to occur in small channels, and also where channels are likely not to be kept in a state of good repair.

The following table, giving the value of n for different materials, is compiled from Kutter, Jackson, and Hering, and this value of n applies also in each instance, to the surfaces of other materials equally rough.

VALUE OF n IN KUTTER'S FORMULA FOR DIFFERENT CHANNELS.

n = .009, well-planed timber, in perfect order and alignment; otherwise, perhaps .01 would be suitable.

n = .010, plaster in pure cement; planed timber: glazed, coated, or enamelled stoneware and iron pipes; glazed surfaces of every sort in perfect order.

n = .011, plaster in cement with one third sand, in good condition; also for iron, cement, and terra cotta pipes, well joined. and in best order.

n = .012, unplaned timber, when perfectly continuous on the inside; flumes.

n = .013, ashlar and well-laid brickwork; ordinary metal; earthen and stoneware pipe in good condition, but not new; cement and terra-cotta pipe not well jointed nor in perfect order, plaster and planed wood in imperfect or inferior condition; and, generally, the materials mentioned with n = .010, when in imperfect or inferior condition.

n = .015, second class or rough-faced brickwork; well-dressed stonework; foul and slightly tuberculated iron; cement and terra-cotta pipes, with imperfect joints and in bad order; and canvas lining on wooden frames.

n = .017, brickwork, ashlar, and stoneware in an inferior condition; tuberculated iron pipes; rubble in cement or plaster in good order; fine gravel, well rammed, 1/8 to 3/4 inch diameter; and, generally, the materials mentioned with n = .013 when in bad order and condition.

n = .020, rubble in cement in an inferior condition; coarse rubble, rough set in a normal condition; coarse rubble set dry; ruined brickwork and masonry; coarse gravel well rammed, from 1 to 1½ inch diameter; canals with beds and banks of very firm, regular gravel, carefully trimmed and rammed in defective places; rough rubble with bed partially covered with silt and mud; rectangular wooden troughs, with battens on the inside two inches apart; trimmed earth in perfect order.

n = .0225, canals in earth above the average in order and regimen.

n = .025, canals and rivers in earth of tolerably uniform cross-section; slope and direction, in moderately good order and regimen, and free from stones and weeds.

n = .0275, canals and rivers in earth below the average in order and regimen.

n = .030, canals and rivers in earth in rather bad order and regimen, having stones and weeds occasionally, and obstructed by detritus.

n = .035, suitable for rivers and canals with earthen beds in bad order and regimen, and having stones and weeds in great quantities.

n = .05, torrents encumbered with detritus.

Kutter's formula has the advantage of being easily adapted to a change in the surface of the pipe exposed to the flow of water, by a change in the value of n. For cast-iron pipes it is usual to use n = .013 to provide for the future deterioration of the surface.

Reducing Kutter's formula to the form $v = c \times \sqrt{r} \times \sqrt{s}$, and taking n, the coefficient of roughness in the formula = .011, .012, and .013, and s = .001, we have the following values of the coefficient c for different diameters of conduit.

Values of c in Formula $v = c \times \sqrt{r} \times \sqrt{s}$ for Metal Pipes and Moderately Smooth Conduits Generally.

By KUTTER'S FORMULA. (s = .001 or greater.)

Diameter.		n = .011	n = .012	n = .013	Diameter.	n = .011	n = .012	n = .013
ft.	in.	c =	c =	c =	ft.	c =	c =	c =
0	1	47.1	....	....	7	152.7	139.2	127.9
	2	61.5	....	....	8	155.4	141.9	130.4
	4	77.4	....	....	9	157.7	144.1	132.7
	6	87.4	77.5	69.5	10	159.7	146	134.5
1		105.7	94.6	85.3	11	161.5	147.8	136.2
1	6	116.1	104.3	94.4	12	163	149.3	137.7
2		123.6	111.3	101.1	14	165.8	152	140.4
3		133.6	120.8	110.1	16	168	154.2	142.1
4		140.4	127.4	116.5	18	169.9	156.1	144.4
5		145.4	132.3	121.1	20	171.6	157.7	146
6		149.4	136.1	124.8				

For circular pipes the hydraulic mean depth r equals ¼ of the diameter.

According to Kutter's formula the value of c, the coefficient of discharge, is the same for all slopes greater than 1 in 1000; that is, within these limits c is constant. We further find that up to a slope of 1 in 2640 the value of c is, for all practical purposes, constant, and even up to a slope of 1 in 5000 the difference in the value of c is very little. This is exemplified in the following:

Value of c for Different Values of $\sqrt{r}$ and s in Kutter's Formula, with n = .013.

$$v = c\sqrt{r} \times \sqrt{s}.$$

$\sqrt{r}$	Slopes.				
	1 in 1000	1 in 2500	1 in 3333.3	1 in 5000	1 in 10,000
.6	93.6	91.5	90.4	88.4	83.3
1	116.5	115.2	114.4	113.2	109.7
2	142.6	142.8	143.0	143.1	143.8

The reliability of the values of the coefficient of Kutter's formula for pipes of less than 6 in. diameter is considered doubtful. (See note under table on page 564.)

Values of c for Earthen Channels, by Kutter's Formula, for Use in Formula $v = c\sqrt{rs}$.

	Coefficient of Roughness, n = .0225.					Coefficient of Roughness, n = .035.				
	$\sqrt{r}$ in feet.					$\sqrt{r}$ in feet.				
	0.4	1.0	1.8	2.5	4.0	0.4	1.0	1.8	2.5	4.0
Slope, 1 in	c	c	c	c	c	c	c	c	c	c
1000	35.7	62.5	80.3	89.2	99.9	19.7	37.6	51.6	59.3	69.2
1250	35.5	62.3	80.3	89.3	100.2	19.6	37.6	51.6	59.4	69.4
1667	35.2	62.1	80.3	89.5	100.6	19.4	37.4	51.6	59.5	69.8
2500	34.6	61.7	80.3	89.8	101.4	19.1	37.1	51.6	59.7	70.4
3333	34.	61.2	80.3	90.1	102.2	18.8	36.9	51.6	59.9	71.0
5000	33.	60.5	80.3	90.7	103.7	18.3	36.4	51.6	60.4	72.2
7500	31.6	59.4	80.3	91.5	106.0	17.6	35.8	51.6	60.9	73.9
10000	30.5	58.5	80.3	92.3	107.9	17.1	35.3	51.6	60.5	75.4
15840	28.5	56.7	80.2	93.9	112.2	16.2	34.3	51.6	62.5	78.6
20000	27.4	55.7	80.2	94.8	115.0	15.6	33.8	51.5	63.1	80.6

Mr. Molesworth, in the 22d edition of his "Pocket-book of Engineering Formulæ," gives a modification of Kutter's formula as follows: For flow in cast-iron pipes, $v = c\sqrt{rs}$, in which

$$c = \frac{181 + \frac{.00281}{s}}{1 + \frac{.026}{\sqrt{d}}\left(41.6 + \frac{.00281}{s}\right)},$$

in which d = diameter of the pipe in feet.

(This formula was given incorrectly in Molesworth's 21st edition.)

Molesworth's Formula.—$v = \sqrt{krs}$, in which the values of k are as follows:

Nature of Channel.	Values of k for Velocities.	
	Less than 4 ft. per sec.	More than 4 ft. per sec.
Brickwork	8800	8500
Earth	7200	6800
Shingle	6400	5900
Rough, with bowlders	5300	4700

In very large channels, rivers, etc., the description of the channel affects the result so slightly that it may be practically neglected, and k assumed = from 8500 to 9000.

Flynn's Formula.—Mr. Flynn obtains the following expression of the value of Kutter's coefficient for a slope of .001 and a value of n = .013:

$$c = \frac{183.72}{1 + \left(44.41 \times \frac{.013}{\sqrt{r}}\right)}.$$

The following table shows the close agreement of the values of c obtained from Kutter's, Molesworth's, and Flynn's formulæ:

Diameter.	Slope.	Kutter.	Molesworth.	Flynn.
6 inches	1 in 40	71.50	71.48	69.5
6 inches	1 in 1000	69.50	69.79	69.5
4 feet	1 in 400	117.	117.	116.5
4 feet	1 in 1000	116.5	116.55	116.5
8 feet	1 in 700	130.5	130.68	130.5
8 feet	1 in 2600	129.8	129.93	130.5

Mr. Flynn gives another simplified form of Kutter's formula for use with different values of n as follows:

$$v = \left(\frac{K}{1 + \left(44.41 \times \frac{n}{\sqrt{r}}\right)}\right)\sqrt{rs}.$$

In the following table the value of K is given for the several values of n:

n	K	n	K	n	K	n	K	n	K
.009	245.63	.012	195.33	.015	165.14	.018	145.03	.021	130.65
.010	225.51	.013	183.72	.016	157.6	.019	139.73	.022	126.73
.011	209 05	.014	137.77	.017	150.94	.020	134.96	.0225	124.9

If in the application of Mr. Flynn's formula given above within the limits of n as given in the table, we substitute for n, K, and $\sqrt{r}$ their values, we have a simplified form of Kutter's formula.

For instance, when $n = .011$, and $d = 3$ feet, we have

$$v = \frac{209.05}{1 + \left(44.41 \times \frac{.011}{.866}\right)} \times \sqrt{rs}.$$

Bazin's Formulæ:
For very even surfaces, fine plastered sides and bed, planed planks, etc.,

$$v = \sqrt{1 + .0000045\left(10.16 + \frac{1}{r}\right)} \times \sqrt{rs}.$$

For even surfaces such as cut-stone, brickwork, unplaned planking, mortar, etc.:

$$v = \sqrt{1 + .000013\left(4.354 + \frac{1}{r}\right)} \times \sqrt{rs}.$$

For slightly uneven surfaces, such as rubble masonry:

$$v = \sqrt{1 + .00006\left(1.219 + \frac{1}{r}\right)} \times \sqrt{rs}.$$

For uneven surfaces, such as earth:

$$v = \sqrt{1 + .00035\left(0.2438 + \frac{1}{r}\right)} \times \sqrt{rs}.$$

A modification of Bazin's formula, known as D'Arcy's Bazin's:

$$v = r\sqrt{\frac{1000s}{.08534r + 0.35}}.$$

For small channels of less than 20 feet bed Bazin's formula for earthen channels in good order gives very fair results, but Kutter's formula is superseding it in almost all countries where its accuracy has been investigated.

The last table on p. 561 shows the value of c, in Kutter's formula, for a wide range of channels in earth, that will cover anything likely to occur in the ordinary practice of an engineer.

D'Arcy's Formula for clean iron pipes under pressure is

$$v = \left\{\frac{rs}{.00007726 + \frac{.00000162}{r}}\right\}^{\frac{1}{2}}$$

Flynn's modification of D'Arcy's formula is

$$v = \left(\frac{155256d}{12d + 1}\right)^{\frac{1}{2}} \times \sqrt{rs},$$

in which d = diameter in feet.

D'Arcy's formula, as given by J. B. Francis, C.E., for old cast-iron pipe, lined with deposit and under pressure, is

$$v = \left(\frac{144d^2s}{.0082(12d + 1}\right)^{\frac{1}{2}}.$$

Flynn's modification of D'Arcy's formula for old cast-iron pipe is

$$v = \left(\frac{70243.9d}{12d + 1}\right)^{\frac{1}{2}} \times \sqrt{rs}.$$

For Pipes Less than 5 inches in Diameter, coefficients (c) in the formula $v = c\sqrt{rs}$, from the formula of D'Arcy, Kutter, and Fanning.

Diam. in inches.	D'Arcy, for Clean Pipes.	Kutter, for $n = .011$ $s = .001$	Fanning, for Clean Iron Pipes	Diam. in inches	D'Arcy, for Clean Pipes.	Kutter, for $n = .011$ $s = .001$	Fanning, for Clean Iron Pipes.
3/8	59.4	32.		1¾	90.7	58.8	92.5
½	65.7	36.1		2	92.9	61.5	94.8
¾	74.5	42.6		2½	96.1	66.	
1	80.4	47.4	80.4	3	98.5	70.1	96.6
1¼	84.8	51.9		4	101.7	77.4	103.4
1½	88.1	55.4	88.	5	103.8	82.9	

Mr. Flynn, in giving the above table, says that the facts show that the coefficients diminish from a diameter of 5 inches to smaller diameters, and it is a safer plan to adopt coefficients varying with the diameter than a constant coefficient. No opinion is advanced as to what coefficients should be used with Kutter's formula for small diameters. The facts are simply stated, giving the results of well-known authors.

Older Formulæ.—The following are a few of the many formulæ for flow of water in pipes given by earlier writers. As they have constant coefficients, they are not considered as reliable as the newer formulæ.

Prony, $v = 97\sqrt{rs} - .08;$

Eytelwein, $v = 50\sqrt{\dfrac{dh}{l + 50d}}$, or $v = 108\sqrt{rs} - 0.13;$

Hawksley, $v = 48\sqrt{\dfrac{dh}{l + 54d}}$; Neville, $v = 140\sqrt{rs} - 11\sqrt[3]{rs}.$

In these formulæ d = diameter in feet; h = head of water in feet; l = length of pipe in feet; s = sine of slope $= \dfrac{h}{l}$; r = mean hydraulic depth, = area ÷ wet perimeter $= \dfrac{d}{4}$ for circular pipe.

Mr. Santo Crimp (*Eng'g*, August 4, 1893) states that observations on flow in brick sewers show that the actual discharge is 33% greater than that calculated by Eytelwein's formula. He thinks Kutter's formula not superior to D'Arcy's for brick sewers, the usual coefficient of roughness in the former, viz., .013, being too low for large sewers and far too small in the case of small sewers.

D'Arcy's formula for brickwork is

$$v = \frac{\sqrt{2g}}{m} rs; \quad m = a\left(1 + \frac{B}{r}\right); \quad a = .0037285; \quad B = .229663.$$

VELOCITY OF WATER IN OPEN CHANNELS.

Irrigation Canals.—The minimum mean velocity required to prevent the deposit of silt or the growth of aquatic plants is in Northern India taken at 1½ feet per second. It is stated that in America a higher velocity is required for this purpose, and it varies from 2 to 3½ feet per second. The maximum allowable velocity will vary with the nature of the soil of the bed. A sandy bed will be disturbed if the velocity exceeds 3 feet per second. Good loam with not too much sand will bear a velocity of 4 feet per second. The Cavour Canal in Italy, over a gravel bed, has a velocity of about 5 per second. (Flynn's "Irrigation Canals.")

Mean Surface and Bottom Velocities.—According to the formula of Bazin,

$$v = v_{max} - 25.4\sqrt{rs}; \quad v = v_b + 10.87\sqrt{rs}.$$

$\therefore v_b = v - 10.87\sqrt{rs}$, in which v = mean velocity in feet per second, v_{max} = maximum surface velocity in feet per second, v_b = bottom velocity in feet per second, r = hydraulic mean depth in feet = area of cross-section in square feet divided by wetted perimeter in feet, s = sine of slope.

The least velocity, or that of the particles in contact with the bed, is almost as much less than the mean velocity as the greatest velocity is greater than the mean.

Rankine states that in ordinary cases the velocities may be taken as bearing to each other nearly the proportions of 3, 4, and 5. In very slow currents they are nearly as 2, 3, and 4.

Safe Bottom and Mean Velocities.—Ganguillet & Kutter give the following table of safe bottom and mean velocity in channels, calculated from the formula $v = v_b + 10.87\sqrt{rs}$:

Material of Channel.	Safe Bottom Velocity v_b, in feet per second.	Mean Velocity v, in feet per second.
Soft brown earth	0.249	0.328
Soft loam	0.499	0.656
Sand	1.000	1.312
Gravel	1.998	2.625
Pebbles	2.999	3.938
Broken stone, flint	4.003	5.579
Conglomerate, soft slate	4.988	6.564
Stratified rock	6.006	8.204
Hard rock	10.009	13.127

Ganguillet & Kutter state that they are unable for want of observations to judge how far these figures are trustworthy. They consider them to be rather disproportionately small than too large, and therefore recommend them more confidently.

Water flowing at a high velocity and carrying large quanties of silt is very destructive to channels, even when constructed of the best masonry.

Resistance of Soils to Erosion by Water.—W. A. Burr, *Eng'g News*, Feb. 8, 1894, gives a diagram showing the resistance of various soils to erosion by flowing water.

Experiments show that a velocity greater than 1.1 feet per second will erode sand, while pure clay will stand a velocity of 7.35 feet per second. The greater the proportion of clay carried by any soil, the higher the permissible velocity. Mr. Burr states that experiments have shown that the line describing the power of soils to resist erosion is parabolic. From his diagram the following figures are selected representing different classes of soils:

Pure sand resists erosion by flow of	1.1	feet per second.
Sandy soil, 15% clay	1.2	" "
Sandy loam, 40% clay	1.8	" "
Loamy soil, 65% clay	3.0	" "
Clay loam, 85% clay	4.8	" "
Agricultural clay, 95% clay	6.2	" "
Clay	7.35	" "

Abrading and Transporting Power of Water.—Prof. J. LeConte, in his "Elements of Geology," states:

The erosive power of water, or its power of overcoming cohesion, varies as the square of the velocity of the current.

The transporting power of a current varies as the sixth power of the velocity. * * * If the velocity therefore be increased ten times, the transporting power is increased 1,000,000 times. A current running three feet per second, or about two miles per hour, will bear fragments of stone of the size of a hen's egg, or about three ounces weight. A current of ten miles an hour will bear fragments of one and a half tons, and a torrent of twenty miles an hour will carry fragments of 100 tons.

The transporting power of water must not be confounded with its erosive power. The resistance to be overcome in the one case is weight, in the other, cohesion; the latter varies as the square: the former as the sixth power of the velocity.

In many cases of removal of slightly cohering material, the resistance is a mixture of these two resistances, and the power of removing material will vary at some rate between v^2 and v^6.

Baldwin Latham has found that in order to prevent deposits of sewage silt in small sewers or drains, such as those from 6 inches to 9 inches diameter, a mean velocity of not less than 3 feet per second should be produced. Sewers from 12 to 24 inches diameter should have a velocity of not less than 2½ feet per second, and in sewers of larger dimensions in no case should the velocity be less than 2 feet per second.

The specific gravity of the materials has a marked effect upon the mean velocities necessary to move them. T. E. Blackwell found that coal of a sp. gr. of 1.26 was moved by a current of from 1.25 to 1.50 ft. per second, while stones of a sp. gr. of 2.32 to 3.00 required a velocity of 2.5 to 2.75 ft. per second.

Chailly gives the following formula for finding the velocity required to move rounded stones or shingle:

$$v = 5.67 \sqrt{ag},$$

in which v = velocity of water in feet per second, a = average diameter in feet of the body to be moved, g = its specific gravity.

Geo. Y. Wisner, *Eng'g News*, Jan 10, 1895, doubts the general accuracy of statements made by many authorities concerning the rate of flow of a current and the size of particles which different velocities will move. He says:

The scouring action of any river, for any given rate of current, must be an inverse function of the depth. The fact that some engineer has found that a given velocity of current on some stream of unknown depth will move sand or gravel has no bearing whatever on what may be expected of currents of the same velocity in streams of greater depths. In channels 3 to 5 ft. deep a mean velocity of 3 to 5 ft. per second may produce rapid scouring, while in depths of 18 ft. and upwards current velocities of 6 to 8 ft. per second often have no effect whatever on the channel bed.

Grade of Sewers.—The following empirical formula is given in Baumeister's "Cleaning and Sewerage of Cities," for the minimum grade for a sewer of clear diameter equal to d inches, and either circular or oval in section;

$$\text{Minimum grade, in per cent,} = \frac{100}{5d + 50}.$$

As the lowest limit of grades which can be flushed, 0.1 to 0.2 per cent may be assumed for sewers which are sometimes dry, while 0.3 per cent is allowable for the trunk sewers in large cities. The sewers should run dry as rarely as possible.

Relation of Diameter of Pipe to Quantity Discharged.—In many cases which arise in practice the information sought is the diameter necessary to supply a given quantity of water under a given head. The diameter is commonly taken to vary as the two-fifth power of the discharge. This is almost certainly too large. Hagen's formula, with Prof. Unwin's coefficients, give $d = c\left(\frac{Q}{\left(\frac{h}{l}\right)^{\frac{1}{2}}}\right)^{.387}$, where $c = .239$ when d and Q are in feet and cubic feet per second.

Mr. Thrupp has proposed a formula which makes d vary as the .383 power of the discharge, and the formula of M. Vallot, a French engineer, makes d vary as the .375 power of the discharge. (*Engineering*.)

FLOW OF WATER—EXPERIMENTS AND TABLES.

The Flow of Water through New Cast-iron Pipe was recently measured by S. Bent Russell, of the St. Louis, Mo., Water-works. The pipe was 12 inches in diameter, 1631 feet long, and laid on a uniform grade from end to end. Under an average total head of 3.36 feet the flow was 43,200 cubic feet in seven hours; under an average head of 3.37 feet the flow was the same; under an average total head of 3.41 feet the flow was 46,700 cubic feet in 8 hours and 35 minutes. Making allowance for loss of head due to entrance and to curves, it was found that the value of c in the formula $v = c\sqrt{rs}$ was from 88 to 93. (*Eng'g Record*, April 14, 1894.)

Flow of Water in a 20-inch Pipe 75,000 Feet Long.—A comparison of experimental data with calculations by different formulæ is given by Chas. B. Brush, Trans. A. S. C. E., 1888. The pipe experimented with was that supplying the city of Hoboken, N. J.

RESULTS OBTAINED BY THE HACKENSACK WATER COMPANY, FROM 1882-1887, IN PUMPING THROUGH A 20-IN. CAST-IRON MAIN 75,000 FEET LONG.

Pressure in lbs. per sq. in. at pumping-station:	95	100	105	110	115	120	125	130
Total effective head in feet:	55	66	77	89	100	112	123	135
Discharge in U. S. gallons in 24 hours, 1 = 1000:	2,848	3,165	3,354	3,566	3,804	3,904	4,116	4,255
Actual velocity in main in feet per second;	2.00	2.24	2.36	2.52	2.68	2.76	2.92	3.00
Cost of coal consumed in delivering each million gals. at given velocities.	$8.40	$8.15	$8.00	$8.10	$8.30	$8.60	$9.00	$9.60
Theoretical discharge by D'Arcy's formula:	2,743	3,004	3,244	3,488	3,699	3,915	4,102	4,297

Velocities in Smooth Cast-iron Water-pipes from 1 Foot to 9 Feet in Diameter, on Hydraulic Grades of 0.5 Foot to 8 Feet per Mile; with Corresponding Values of c in $V = c\sqrt{rs}$. (D. M. Greene, in *Eng'g News*, Feb. 24. 1894.)

Diameters, feet. D.	Hydraulic Mean Radii. r.		Hydraulic Grade; Feet per Mile = h. h = 0.5, s = 0.0000947	1.0, 0.0001894	1.5, 0.0002841	2.0, 0.0003788	3.0, 0.0005682	4.0, 0.0007576
1.	0.25	V =	0.4542	0.6673	0.8256	0.9808	1.2277	1.4409
		c =	93.7	97.0	99.1	100.7	103.0	104.7
2.	0.5	V =	0.7359	1.0798	1.3516	1.5256	1.9257	2.2294
		c =	106.6	110.9	113.4	115.2	117.9	119.7
3.	0.75	V =	0.9783	1.4298	1.7906	2.1017	2.6306	3.0360
		c =	115.5	119.9	122.6	124.4	127.5	129.5
4.	1.0	V =	1.1883	1.7456	2.1861	2.5645	3.2116	3.7676
		c =	122.1	126.8	129.7	131.8	134.7	136.9
5.	1.25	V =	1.3872	2.0379	2.5591	2.9689	3.7498	4.3988
		c =	127.5	132.4	135.5	137.6	140.7	142.9
6.	1.5	V =	1.5742	2.3126	2.8961	3.3975	4.2548	4.9913
		c =	132.1	137.3	140.3	142.6	145.8	148.1
7.	1.75	V =	1.7518	2.5736	3.2230	3.7809	4.7350	5.5546
		c =	135.9	141.4	146.0	146.8	150.2	152.5
8.	2.0	V =	1.9218	2.8264	3.5358	4.1479	5.1945	6.0966
		c =	139.7	145.1	148.4	150.7	154.1	156.5
9.	2.25	V =	2.0854	3.0638	3.8368	4.5010	5.6368	6.6125
		c =	142.9	146.4	151.7	154.2	157.6	160.1

The velocities in this table have been calculated by Mr. Greene's modification of the Chezy formula, which modification is found to give results which differ by from 1.29 to − 2.65 per cent (average 0.9 per cent) from very carefully measured flows in pipes from 16 to 48 inches in diameter, on grades from 1.68 feet to 10.296 feet per mile, and in which the velocities ranged from 1.577 to 6.195 feet per second. The only assumption made is that the modified formula for V gives correct results in conduits from 4 feet to 9 feet in diameter, as it is known to do in conduits less than 4 feet in diameter.

Other articles on Flow of Water in long tubes are to be found in *Eng'g News* as follows: G. B. Pearsons, Sept. 23, 1876; E. Sherman Gould, Feb. 16, 23, March 9, 16, and 23, 1889; J. L. Fitzgerald, Sept. 6 and 13, 1890; Jas. Duane, Jan. 2, 1892: J. T. Fanning, July 14, 1892; A. N. Talbot, Aug. 11, 1892.

Flow of Water in Circular Pipes, Sewers, etc., Flowing Full. Based on Kutter's Formula, with $n = .013$.

Discharge in cubic feet per second.

Diameter.	Slope, or Head Divided by Length of Pipe.							
	1 in 40	1 in 70	1 in 100	1 in 200	1 in 300	1 in 400	1 in 500	1 in 600
5 in.	.456	.344	.288	.204	.166	.144	.137	.118
6 "	.762	.576	.482	.341	.278	.241	.220	.197
7 "	1.17	.889	.744	.526	.430	.372	.355	.304
8 "	1.70	1.29	1.08	.765	.624	.54	.516	.441
9 "	2.37	1.79	1.50	1.06	.868	.75	.717	.613
Slope....	1 in 60	1 in 80	1 in 100	1 in 200	1 in 300	1 in 400	1 in 500	1 in 600
10 in.	2.51	2.24	2.01	1.42	1.16	1.00	.90	.82
11 "	3.31	2.94	2.63	1.86	1.52	1.31	1.17	1.07
12 "	4.32	3.74	3.35	2.37	1.93	1.67	1.5	1.37
13 "	5.38	4.66	4.16	2.95	2.40	2.08	1.86	1.70
14 "	6.60	5.72	5.15	3.62	2.95	2.57	2.29	2.09
Slope....	1 in 100	1 in 200	1 in 300	1 in 400	1 in 500	1 in 600	1 in 700	1 in 800
15 in.	6.18	4.37	3.57	3.09	2.77	2.52	2.34	2.19
16 "	7.38	5.22	4.26	3.69	3.30	3.01	2.79	2.61
18 "	10.21	7.22	5.89	5.10	4.56	4.17	3.86	3.61
20 "	13.65	9.65	7.88	6.82	6.10	5.57	5.16	4.83
22 "	17.71	12.52	10.22	8.85	7.92	7.23	6.69	6.26
Slope....	1 in 200	1 in 400	1 in 600	1 in 800	1 in 1000	1 in 1250	1 in 1500	1 in 1800
2 ft.	15.88	11.23	9.17	7.94	7.10	6.35	5.80	5.29
2 ft. 2 in.	19.73	13.96	11.39	9.87	8.82	7.89	7.20	6.58
2 " 4 "	24.15	17.07	13.94	12.07	10.80	9.66	8.82	8.05
2 " 6 "	29.08	20.56	16.79	14.54	13.00	11.63	10.62	9.69
2 " 8 "	34.71	24.54	20.04	17.35	15.52	13.88	12.67	11.57
Slope....	1 in 500	1 in 750	1 in 1000	1 in 1250	1 in 1500	1 in 1750	1 in 2000	1 in 2500
2 ft. 10 in.	25.84	21.10	18.27	16.34	14.92	13.81	12.92	11.55
3 "	30.14	24.61	21.31	19.06	17.40	16.11	15.07	13.48
3 " 2 in.	34.90	28.50	24.68	22.07	20.15	18.66	17.45	15.61
3 " 4 "	40.08	32.72	28.34	25.35	23.14	21.42	20.04	17.93
3 " 6 "	45.66	37.28	32.28	28.87	26.36	24.40	22.83	20.41
Slope....	1 in 500	1 in 750	1 in 1000	1 in 1250	1 in 1500	1 in 1750	1 in 2000	1 in 2500
3 ft. 8 in.	51.74	42.52	36.59	32.72	29.87	27.66	25.87	23.14
3 " 10 "	58.36	47.65	41.27	36.91	33.69	31.20	29.18	26.10
4 "	65.47	53.46	46.30	41.41	37.80	34.50	32.74	29.28
4 " 6 in.	89.75	73.28	63.47	56.76	51.82	47.97	44.88	40.14
5 "	118.9	97.09	84.08	75.21	68.65	63.56	59.46	53.18
Slope ...	1 in 750	1 in 1000	1 in 1500	1 in 2000	1 in 2500	1 in 3000	1 in 3500	1 in 4000
5 ft. 6 in.	125.2	108.4	88.54	76.67	68.58	62.60	57.96	54.21
6 "	157.8	136.7	111.6	96.66	86.45	78.92	73.07	68.35
6 " 6 "	195.0	168.8	137.9	119.4	106.8	97.49	90.26	84.43
7 "	237.7	205.9	168.1	145.6	130.2	118.8	110.00	102.9
7 " 6 "	285.3	247.1	201.7	174.7	156.3	142.6	132.1	123.5
Slope....	1 in 1500	1 in 2000	1 in 2500	1 in 3000	1 in 3500	1 in 4000	1 in 4500	1 in 5000
8 ft.	239.4	207.3	195.4	169.3	156.7	146.6	138.2	131.1
8 " 6 in.	281.1	243.5	217.8	198.8	184.0	172.2	162.3	154.0
9 "	327.0	283.1	253.3	231.2	214.0	200.2	188.7	179.1
9 " 6 "	376.9	326.4	291.9	266.5	246.7	230.8	217.6	206.4
10 "	431.4	373.6	334.1	305.0	282.4	264.2	249.1	236.3

For U. S. gallons multiply the figures in the table by 7.4805.

For a given diameter the quantity of flow varies as the square root of the sine of the slope. From this principle the flow for other slopes than those given in the table may be found. Thus, what is the flow for a pipe 3 feet diameter, slope 1 in 125? From the table take Q = 207.3 for slope 1 in 2000. The given slope 1 in 125 is to 1 in 2000 as 16 to 1, and the square root of this ratio is 4 to 1. Therefore the flow required is 207.3 × 4 = 829.2 cu. ft.

Circular Pipes, Conduits, etc., Flowing Full.

Values of the factor $ac\sqrt{r}$ in the formula $Q = ac\sqrt{r} \times \sqrt{s}$ corresponding to different values of the coefficient of roughness, n. (Based on Kutter's formula.)

Diam.		Value of $ac\sqrt{r}$.					
ft.	in.	n = .010.	n = .011.	n = .012.	n = .013.	n = .015.	n = .017.
	6	6.906	6.0627	5.3800	4.8216	3.9604	3 329
	9	21.25	18.742	16.708	15.029	12.421	10.50
1		46.93	41.487	37.149	33.497	27.803	23 60
1	3	86.05	76.347	68.44	61.867	51.600	43.93
1	6	141.2	125.60	112.79	103.14	85.496	72.99
1	9	214.1	190.79	171.66	155.68	130.58	111.8
2		307.6	274.50	247.33	224.63	188.77	164
2	3	421.9	377.07	340.10	309.23	260.47	223.9
2	6	559.6	500.78	452.07	411.27	347.28	299.3
2	9	723.4	647.18	584.90	532.76	451.23	388.8
3		911.8	817.50	739.59	674.09	570.90	493.3
3	3	1128.9	1013.1	917.41	836.69	709.56	613.9
3	6	1374.7	1234.4	1118.6	1021.1	866.91	750.8
3	9	1652.1	1484.2	1345.9	1229.7	1045	906
4		1962.8	1764.3	1600.9	1463.9	1245.3	1080.7
4	6	2682.1	2418.3	2198	2007	1711.4	1487.3
5		3543	3191.8	2903.6	2659	2272.7	1977
5	6	4557.3	4111.9	3742.7	3429	2934.8	2557.2
6		5731.5	5176.3	4713.9	4322	3702.3	3232.5
6	6	7075.2	6394.9	5825.9	5339	4588.3	4010
7		8595.1	7774.3	7087	6510	5591.6	4893
7	6	10296	9318.8	8501.8	7814	6717	5884.2
8		12196	11044	10083	9272	7978.3	6995.3
8	6	14298	12954	11832	10889	9377.9	8226.8
9		16604	15049	13751	12663	10917	9580.7
9	6	19118	17338	15847	14597	12594	11061
10		21858	19834	18134	16709	14426	12678
10	6	24823	22534	20612	18996	16412	14424
11		28020	25444	23285	21464	18556	16383
11	6	31482	28593	26179	24139	20879	18395
12		35156	31937	29254	26981	23352	20584
12	6	39104	35529	32558	30041	26012	22938
13		43307	39358	36077	33301	28850	25451
13	6	47751	43412	39802	36752	31860	28117
14		52491	47739	43778	40432	35078	30965
14	6	57496	52308	47969	44323	38454	33975
15		62748	57103	52382	48413	42040	37147
16		74191	67557	62008	57343	49823	44073
17		86769	79050	72594	67140	58387	51669
18		100617	91711	84247	77932	67839	60067
19		115769	105570	96991	89759	78201	69301
20		132133	120570	110905	102559	89423	79259

Flow of Water in Circular Pipes, Conduits, etc., Flowing under Pressure.

Based on D'Arcy's formulæ for the flow of water through cast-iron pipes. With comparison of results obtained by Kutter's formula, with n = .013. (Condensed from Flynn on Water Power.)

Values of a, and also the values of the factors $c\sqrt{r}$ and $ac\sqrt{r}$ for use in the formulæ $Q = av$; $v = c\sqrt{r} \times \sqrt{s}$, and $Q = ac\sqrt{r} \times \sqrt{s}$.

Q = discharge in cubic feet per second, a = area in square feet, v = velocity in feet per second, r = mean hydraulic depth, ¼ diam. for pipes running full, s = sine of slope.

(For values of $\sqrt{s}$ see page 558.)

Size of Pipe.			Clean Cast-iron Pipes.		Value of $ac\sqrt{r}$ by Kutter's Formula, when n = .013.	Old Cast-iron Pipes Lined with Deposit.	
d = diam. ft.	in.	a = area in square feet.	For Velocity, $c\sqrt{r}$.	For Discharge, $ac\sqrt{r}$.		For Velocity, $c\sqrt{r}$.	For Discharge, $ac\sqrt{r}$.
	⅜	.00077	5.251	.00403		3.582	.00272
	½	.00136	6.702	.00914		4.507	.00613
	¾	.00307	9.309	.02855		6.261	.01922
	1	.00545	11.61	.06334		7.811	.04257
	1¼	.00852	13.68	.11659		9.255	.07885
	1½	.01227	15.58	.19115		10.48	.12856
	1¾	.01670	17.32	.28936		11.65	.19462
	2	.02182	18.96	.41357		12.75	.27824
	2½	.0341	21.94	.74786		14.76	.50321
	3	.0491	24.63	1.2089		16.56	.81333
	4	.0873	29.37	2.5630		19.75	1.7246
	5	.136	33.54	4.5610		22.56	3.0681
	6	.196	37.28	7.3068	4.822	25.07	4.9147
	7	.267	40.65	10.852		27.34	7.2995
	8	.349	43.75	15.270		29.43	10.271
	9	.442	46.73	20.652	15.08	31.42	13.891
	10	.545	49.45	26.952		33.26	18.129
	11	.660	52.16	34.428		35.09	23.158
1		.785	54 65	42.918	33.50	36.75	28.867
1	2	1.000	59.34	63.435		39 91	42.668
1	4	1.396	63.67	88.886		42.68	59.782
1	6	1.767	67.75	119.72	102.14	45.57	80.531
1	8	2.182	71.71	156.46		48.34	105.25
1	10	2.640	75.32	198.88		50.658	133.74
2		3.142	78.80	247.57	224.63	52.961	166.41
2	2	3.687	82.15	302.90		55.258	203.74
2	4	4.276	85.39	365.14		57.436	245.60
2	6	4.909	88.39	433.92	411.37	59.455	291.87
2	8	5.585	91.51	511.10		61.56	343.8
2	10	6.305	94.40	595.17		63.49	400.3
3		7.068	97.17	686.76	674.09	65.35	461.9
3	2	7.875	99.93	786.94		67.21	529.3
3	4	8.726	102.6	895.7		69	602
3	6	9.621	105.1	1011.2	1021.1	70.70	680.2
3	8	10.559	107.6	1136.5		72.40	764.5
3	10	11.541	110.2	1271.4		74.10	855.2
4		12.566	112.6	1414.7	1463.9	75.73	951.6
4	3	14.186	116.1	1647.6		78.12	1108.2
4	6	15.904	119.6	1901.9	2007	80.43	1279.2
4	9	17.721	122.8	2176.1		82.20	1456.8
5		19.635	126.1	2476.4	2659	84.83	1665.7
5	3	21.648	129.3	2799.7		86.99	1883.2
5	6	23.758	132.4	3146.3	3429	89.07	2116.2
5	9	25.967	135.4	3516		91.08	2365
6		28.274	138.4	3912.8	4322	93.08	2631.7
6	6	33.183	144.1	4782.1	5339	96.93	3216.4
7		38.485	149.6	5757.5	6510	100.61	3872.5
7	6	44.179	154 9	6841.6	7814	104.11	4601.9
8		50.266	160	8043	9272	107.61	5409.9
8	6	56.745	165	9364.7	10889	111	6299.1
9		63.617	169.8	10804	12663	114.2	7267.3
9	6	70.882	174.5	12370	14597	117.4	8320.6
10		78.540	179.1	14066	16709	120.4	9460.9

Size of Pipe.			Clean Cast-iron Pipes.		Value of $ac\sqrt{r}$ by Kutter's Formula, when $n = .013$	Old Cast-iron Pipes Lined with Deposit.	
d = diam. in ft.	in.	a = area in square feet.	For Velocity, $c\sqrt{r}$.	For Discharge, $ac\sqrt{r}$.		For Velocity, $c\sqrt{r}$.	For Discharge, $ac\sqrt{r}$.
10	6	86.590	183.6	15893	18996	123.4	10690
11		95 033	187.9	17855	21464	126.3	12010
11	6	103.869	192.2	19966	24139	129.3	13429
12		113.098	196.3	22204	26981	132	14935
12	6	122 719	200.4	24598	30041	134.8	16545
13		132.733	204.4	27134	33301	137.5	18252
13	6	143.139	208.3	29818	36752	140.1	20056
14		153.938	212.2	32664	40432	142.7	21971
14	6	165.130	216.0	35660	44322	145.2	23986
15		176.715	219.6	38807	48413	147.7	26108
15	6	188.692	223.3	42125	52753	150.1	28335
16		201.062	226.9	45621	57343	152.6	30686
16	6	213.825	230.4	49273	62132	155	33144
17		226.981	233.9	53082	67140	157.3	35704
17	6	240.529	237.3	57074	72409	159.6	38389
18		254.470	240.7	61249	77932	161.9	41199
19		283.529	247.4	70154	89759	166.4	47186
20		314.159	253.8	79736	102559	170.7	53633

Flow of Water in Circular Pipes from 3/8 inch to 12 inches Diameter.

Based on D'Arcy's formula for clean cast-iron pipes. $Q = ac\sqrt{r}\sqrt{s}$.

Value of $ac\sqrt{r}$.	Dia. in.	Slope, or Head Divided by Length of Pipe.							
		1 in 10.	1 in 20.	1 in 40.	1 in 60.	1 in 80.	1 in 100.	1 in 150.	1 in 200.
		Quantity in cubic feet per second.							
.00403	3/8	.00127	.00090	.00064	.00052	.00045	.00040	.00033	.00028
.00914	1/2	.00289	.00204	.00145	.00118	.00102	.00091	.00075	.00065
.02855	3/4	.00903	.00638	.00451	.00369	.00319	.00286	.00233	.00202
.06334	1	.02003	.01416	.01001	.00818	.00708	.00633	.00517	.00448
.11659	1 1/4	.03687	.02607	.01843	.01505	.01303	.01166	.00952	.00824
.19115	1 1/2	.06044	.04274	.03022	.02468	.02137	.01912	.01561	.01352
.28936	1 3/4	.09140	.06470	.04575	.03736	.03235	.02894	.02363	.02046
.41357	2	.13077	.09247	.06539	.05339	.04624	.04136	.03377	.02927
.74786	2 1/2	.23647	.16722	.11824	.09655	.08361	.07479	.06106	.05288
1.2089	3	.38225	.27031	.19113	.15607	.13515	.12089	.09871	.08548
2.5630	4	.81042	.57309	.40521	.33088	.28654	.25630	.20927	.18123
4.5610	5	1.4422	1.0198	.72109	.58882	.50992	.45610	.37241	.32251
7.3068	6	2.3104	1.6338	1.1552	.94331	.81690	.73068	.59660	.51666
10.852	7	3.4314	2.4265	1.7157	1.4110	1.2132	1.0852	.88607	.76734
15.270	8	4.8284	3.4143	2.4141	1.9713	1.7072	1.5270	1.2468	1.0797
20.652	9	6.5302	4.6178	3.2651	2.6662	2.3089	2.0652	1.6862	1.4603
26.952	10	8.5222	6.0265	4.2611	3.4795	3.0132	2.6952	2.2006	1.9058
34.428	11	10.886	7.6981	5.4431	4.4447	3.8491	3.4428	2.8110	2.4344
42.918	12	13.571	9.5965	6.7853	5.5407	4.7982	4.2918	3.5043	3.0347
Value of $\sqrt{s}$ =		.3162	.2236	.1581	.1291	.1118	.1	.08165	.07071

Value of $ac\sqrt{r}$.	Dia. in.	Slope, or Head Divided by Length of Pipe.							
		1 in 250.	1 in 300.	1 in 350.	1 in 400.	1 in 450.	1 in 500.	1 in 550.	1 in 600.
.00403	⅜	.00025	.00023	.00022	.00020	.00019	.00018	.00017	.00016
.00914	½	.00058	.00053	.00049	.00046	.00043	.00041	.00039	.00037
.02855	¾	.00181	.00165	.00153	.00143	.00134	.00128	.00122	.00117
.06334	1	.00400	.00366	.00339	.00317	.00298	.00283	.00270	.00259
.11659	1¼	.00737	.00673	.00623	.00583	.00549	.00521	.00497	.00476
.19115	1½	.01209	.01104	.01022	.00956	.00901	.00855	.00815	.00780
.28936	1¾	.01830	.01671	.01547	.01447	.01363	.01294	.01234	.01181
.41357	2	.02615	.02388	.02211	.02068	.01948	.01849	.01763	.01688
.74786	2½	.04730	.04318	.03997	.03739	.03523	.03344	.03189	.03053
1.2089	3	.07645	.06980	.06462	.06045	.05695	.05406	.05155	.04935
2.5630	4	.16208	.14799	.13699	.12815	.12074	.11461	.10929	.10463
4.5610	5	.28843	.26335	.24379	.22805	.21487	.20397	.19448	.19620
7.3068	6	.46208	.42189	.39055	.36534	.34422	.32676	.31156	.29830
10.852	7	.68628	.62660	.58005	.54260	.51124	.48530	.46273	.44303
15.270	8	.96567	.88158	.81617	.76350	.71936	.68286	.65111	.62340
20.652	9	1.3060	1.1924	1.1038	1.0326	.97292	.92356	.88060	.84310
26.952	10	1.7044	1.5562	1.4405	1.3476	1.2697	1.2053	1.1492	1.1003
34.428	11	2.1772	1.9878	1.8402	1.7214	1.6219	1.5396	1.4680	1.4055
42.918	12	2.7141	2.4781	2.2940	2.1459	2.0219	1.9193	1.8300	1.7521
Value of $\sqrt{s}$ =		.06324	.05774	.05345	.05	.04711	.04472	.04264	.04082

For U. S. gals. per sec., multiply the figures in the table by...... 7.4805
" " " " min., " " " " 448.83
" " " " hour, " " " " 26929.8
" " " " 24 hrs., " " " " ...... 646315.

For any other slope the flow is proportional to the square root of the slope; thus, flow in slope of 1 in 100 is double that in slope of 1 in 400.

Flow of Water in Pipes from ⅜ Inch to 12 Inches Diameter for a Uniform Velocity of 100 Ft. per Min.

Diameter in Inches.	Area in Square Feet.	Flow in Cubic Feet per Minute.	Flow in U. S Gallons per Minute.	Flow in U. S. Gallons per Hour.
⅜	.00077	0.077	.57	34
½	.00136	0.136	1.02	61
¾	.00307	0.307	2.30	138
1	.00545	0.545	4.08	245
1¼	.00852	0.852	6.38	383
1½	.01227	1.227	9.18	551
1¾	.01670	1.670	12.50	750
2	.02182	2.182	16.32	979
2½	.0341	3.41	25.50	1,530
3	.0491	4.91	36.72	2,203
4	.0873	8.73	65.28	3,917
5	.136	13.6	102.00	6,120
6	.196	19.6	146.88	8,813
7	.267	26.7	199.92	11,995
8	.349	34.9	261.12	15,667
9	.442	44.2	330.48	19,829
10	.545	54.5	408.00	24,480
11	.660	66.0	493.68	29,621
12	.785	78.5	587.52	35,251

Given the diameter of a pipe, to find the quantity in gallons it will deliver, the velocity of flow being 100 ft. per minute. Square the diameter in inches and multiply by 4.08.

If Q' = quantity in gallons per minute and d = diameter in inches, then

$$Q' = \frac{d^2 \times .7854 \times 100 \times 7.4805}{144} = 4.08d^2.$$

For any other velocity, V', in feet per minute, $Q' = 4.08d^2\frac{V'}{100} = .0408d^2V'$.

Given diameter of pipe in inches and velocity in feet per second, to find discharge in cubic feet and in gallons per minute.

$$Q' = \frac{d^2 \times .7854 \times v \times 60}{144} = 0.32725d^2v \text{ cubic feet per minute.}$$

$$= .32725 \times 7.4805 \text{ or } 2.448d^2v \text{ U. S. gallons per minute.}$$

To find the capacity of a pipe or cylinder in gallons, multiply the square of the diameter in inches by the length in inches and by .0034. Or multiply the square of the diameter in inches by the length in feet and by .0408.

$$Q = \frac{.7854d^2l}{231} = .0034d^2l \text{ (exact) } .0034 \times 12 = .0408.$$

LOSS OF HEAD.

The loss of head due to friction when water, steam, air, or gas of any kind flows through a straight tube is represented by the formula

$$h = f\frac{4l}{d}\frac{v^2}{2g}; \qquad \text{whence } v = \sqrt{\frac{64.4}{4f}\frac{hd}{l}},$$

in which l = the length and d = the diameter of the tube, both in feet; v = velocity in feet per second, and f is a coefficient to be determined by experiment. According to Weisbach, $f = .00644$, in which case

$$\sqrt{\frac{64.4}{4f}} = 50, \text{ and } v = 50\sqrt{\frac{hd}{l}},$$

which is one of the older formulæ for flow of water (Downing's). Prof. Unwin says that the value of f is possibly too small for tubes of small bore, and he would put f = .006 to .01 for 4-inch tubes, and f = .0084 to .012 for 2-inch tubes. Another formula by Weisbach is

$$h = \left(.0144 + \frac{.01716}{\sqrt{v}}\right)\frac{l}{d}\frac{v^2}{2g}.$$

Rankine gives

$$f = .005\left(1 + \frac{1}{12d}\right).$$

From the general equation for velocity of flow of water $v = c\sqrt{r}\sqrt{s}$, = for round pipes $c\sqrt{\frac{d}{4}}\sqrt{\frac{h}{l}}$, we have $v^2 = c^2\frac{d}{4}\frac{h}{l}$ and $h = \frac{4lv^2}{c^2d}$, in which c is the coefficient c of D'Arcy's, Bazin's, Kutter's, or other formula, as found by experiment. Since this coefficient varies with the condition of the inner surface of the tube, as well as with the velocity, it is to be expected that values of the loss of head given by different writers will vary as much as those of quantity of flow. Two tables for loss of head per 100 ft. in length in pipes of different diameters with different velocities are given below. The first is given by Clark, based on Ellis' and Howland's experiments; the second is from the Pelton Water-wheel Co.'s catalogue, authority not stated. The loss of head as given in these two tables for any given diameter and velocity differs considerably. Either table should be used with caution and the results compared with the quantity of flow for the given diameter and head as given in the tables of flow based on Kutter's and D'Arcy's formulæ.

Relative Loss of Head by Friction for each 100 Feet Length of Clean Cast-iron Pipe.

(Based on Ellis and Howland's experiments.)

Velocity in Feet per Second.	Diameter of Pipes in Inches.									
	3	4	5	6	7	8	9	10	12	14
	Loss of Head in Feet, per 100 Feet Long.									
Feet	Feet of Head	Feet of Head	Feet of Head	Feet of Head	Feet of Head	Feet of Head	Feet of Head	Feet of Head	Feet of Head	Feet of Head
2	.97	.55	.41	.32	.27	.23	.19	.18	.15	.12
2.5	1.49	.92	.64	.50	.43	.36	.30	.27	.23	.19
3	1.9	1.2	.82	.72	.61	.51	.44	.39	.33	.27
3.5	2.6	1.6	1.2	1.0	.7	.71	.61	.52	.45	.37
4	3.3	2.2	1.7	1.3	.9	.92	.79	.69	.59	.49
4.5	.....	.....	......	1.6	1.2	1.2	1.01	.87	.75	.61
5	......	......	.....	......	......	.....	1.2	1.1	.90	.76
5.5	......	.....	....	......	......	......	.....	......	......	.92
6	......	.....	.....	......	.....	......	.....	......	......	.....
	15	18	21	24	27	30	33	36	42	48
2	.11	.095	.075	.065	055	.052	.049	.047	.036	.030
2.5	.17	.147	.117	.109	.088	.065	.076	.067	.056	.046
3	.25	.21	.17	.15	.13	.12	.108	.10	.081	.067
3.5	.34	.29	.23	.20	.18	.16	.15	.14	.111	.093
4	.44	.36	.31	.27	.23	.22	.20	.17	.14	.116
4.5	.56	.46	.39	.34	.30	.28	.25	.22	.18	.15
5	.70	.58	.48	.41	.37	.34	.30	.27	.22	.18
5.5	.84	.70	.59	.50	.44	.39	.36	.33	.27	.22
6	......	......	......	.59	.53	.49	43	.4	.32	.27

Loss of Head in Pipe by Friction.—Loss of head by friction in each 100 feet in length of different diameters of pipe when discharging the following quantities of water per minute (Pelton Water-wheel Co.):

Velocity in Feet per Second.	Inside Diameter of Pipe in Inches.											
	1		2		3		4		5		6	
V	Loss of Head in Feet. h	Cubic Feet per Minute. Q	Loss of Head in Feet. h	Cubic Feet per Minute. Q	Loss of Head in Feet. h	Cubic Feet per Minute. Q	Loss of Head in Feet. h	Cubic Feet per Minute. Q	Loss of Head in Feet. h	Cubic Feet per Minute. Q	Loss of Head in Feet. h	Cubic Feet per Minute. Q
2.0	2.37	.65	1.185	2.62	.791	5.89	.593	10.4	.474	16.3	.395	23.5
3.0	4.89	.99	2.44	3.92	1.62	8.83	1.22	15.7	.978	24.5	.815	35.3
4.0	8.20	1.32	4.10	5.23	2.73	11.80	2.05	20.9	1.64	32.7	1.37	47.1
5.0	12.33	1.65	6.17	6.54	4.11	14.70	3.08	26.2	2.46	40.9	2.05	58.9
6.0	17.23	1.98	8.61	7.85	5.74	17.70	4.31	31.4	3.45	49.1	2.87	70.7
7.0	22.89	2.31	11.45	9.16	7.62	20.6	5.72	36.6	4.57	57.2	3.81	82.4

(Continued on next page.)

Flow of Water in Riveted Steel Pipes.—The laps and rivets tend to decrease the carrying capacity of the pipe. See paper on "New Formulas for Calculating the Flow of Water in Pipes and Channels," by W. E. Foss, *Jour. Assoc. Eng. Soc.*, xiii, 295. Also Clemens Herschel's book on "115 Experiments on the Carrying Capacity of Large Riveted Metal Conduits," John Wiley & Sons, 1897.

	Inside Diameter of Pipe in Inches.											
	7		8		9		10		11		12	
V	h	Q	h	Q	h	Q	h	Q	h	Q	h	Q
2.0	.338	32.0	.296	41.9	.264	53	.237	65.4	.216	79.2	.198	94.2
3.0	.698	48.1	.611	62.8	.544	79.5	.488	98.2	.444	119	.407	141
4.0	1.175	64.1	1.027	83.7	.913	106	.822	131	.747	158	.685	188
5.0	1.76	80.2	1.54	105	1.37	132	1.23	163	1.122	198	1.028	235
6.0	2.46	96.2	2.15	125	1.92	159	1.71	196	1.56	237	1.43	283
7.0	3.26	112.0	2.85	146	2.52	185	2.28	229	2.07	277	1.91	330

	Inside Diameter of Pipe in Inches.											
	13		14		15		16		18		20	
V	h	Q	h	Q	h	Q	h	Q	h	Q	h	Q
2.0	.183	110	.169	128	.158	147	.147	167	.132	212	.119	262
3.0	.375	166	.349	192	.325	221	.306	251	.271	318	.245	393
4.0	.632	221	.587	256	.548	294	.513	335	.456	424	.410	523
5.0	.949	276	.881	321	.822	368	.770	419	.685	530	.617	654
6.0	1.325	332	1.229	385	1.148	442	1.076	502	.957	636	.861	785
7.0	1.75	387	1.63	449	1.52	515	1.43	586	1.27	742	1.148	916

	Inside Diameter of Pipe in Inches.											
	22		24		26		28		30		36	
V	h	Q	h	Q	h	Q	h	Q	h	Q	h	Q
2.0	.108	316	.098	377	.091	442	.084	513	.079	589	.066	848
3.0	.222	475	.204	565	.188	663	.174	770	.163	883	.135	1273
4.0	.373	633	.342	754	.315	885	.293	1026	.273	1178	.228	1697
5.0	.561	792	.513	942	.474	1106	.440	1283	.411	1472	.342	2121
6.0	.782	950	.717	1131	.662	1327	.615	1539	.574	1767	.479	2545
7.0	1.040	1109	.953	1319	.879	1548	.817	1796	.762	2061	.636	2868

EXAMPLE.—Given 200 ft. head and 600 ft. of 11-inch pipe, carrying 119 cubic feet of water per minute. To find effective head: In right-hand column, under 11-inch pipe, find 119 cubic ft.; opposite this will be found the loss by friction in 100 ft. of length for this amount of water, which is .444. Multiply this by the number of hundred feet of pipe, which is 6, and we have 2.66 ft., which is the loss of head. Therefore the effective head is 200 − 2.66 = 197.34.

EXPLANATION.—The loss of head by friction in pipe depends not only upon diameter and length, but upon the quantity of water passed through it. The head or pressure is what would be indicated by a pressure-gauge attached to the pipe near the wheel. Readings of gauge should be taken while the water is flowing from the nozzle.

To reduce heads in feet to pressure in pounds multiply by .433. To reduce pounds pressure to feet multiply by 2.309.

Cox's Formula.—Weisbach's formula for loss of head caused by the friction of water in pipes is as follows:

$$\text{Friction-head} = \left(0.0144 + \frac{0.01716}{\sqrt{V}}\right)\frac{L.V^2}{5.367d},$$

where L = length of pipe in feet;
V = velocity of the water in feet per second;
d = diameter of pipe in inches.

William Cox (*Amer. Mach.*, Dec. 28, 1893) gives a simpler formula which gives almost identical results:

$$H = \text{friction-head in feet} = \frac{L}{d}\,\frac{4V^2 + 5V - 2}{1200}. \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{Hd}{L} = \frac{4V^2 + 5V - 2}{1200}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

He gives a table by means of which the value of $\frac{4V^2+5V-2}{1200}$ is at once obtained when V is known, and *vice versa*.

VALUES OF $\frac{4V^2+5V-2}{1200}$.

V	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	.00583	.00695	.00813	.00938	.01070	.01208	.01353	.01505	.01663	.01828
2	.02000	.02178	.02363	.02555	.02753	.02958	.03170	.03388	.03613	.03845
3	.04083	.04328	.04580	.04838	.05103	.05375	.05653	.05938	.06230	.06528
4	.06833	.07145	.07463	.07788	.08120	.08458	.08803	.09155	.09513	.09878
5	.10250	.10628	.11013	.11405	.11803	.12208	.12620	.13038	.13463	.13895
6	.14333	.14778	.15230	.15688	.16153	.16625	.17103	.17588	.18080	.18578
7	.19083	.19595	.20113	.20638	.21170	.21708	.22253	.22805	.23363	.23928
8	.24500	.25078	.25663	.26255	.26853	.27458	.28070	.28688	.29313	.29945
9	.30583	.31228	.31880	.32538	.33203	.33875	.34553	.35238	.35930	.36628
10	.37333	.38045	.38763	.39488	.40220	.40958	.41703	.42455	.43213	.43978
11	.44750	.45528	.46313	.47105	.47903	.48708	.49520	.50338	.51163	.51995
12	.52833	.53678	.54530	.55388	.56253	.57125	.58003	.58888	.59780	.60678
13	.61583	.62495	.63413	.64338	.65270	.66208	.67153	.68105	.69063	.70028
14	.71000	.71978	.72963	.73955	.74953	.75958	.76970	.77988	.79013	.80045
15	.81083	.82128	.83180	.84238	.85303	.86375	.87453	.88538	.89630	.90728
16	.91833	.92945	.94063	.95188	.96320	.97458	.98603	.99755	1.00913	1.02078
17	1.03250	1.04428	1.05613	1.06805	1.08003	1.09208	1.10420	1.11638	1.12863	1.14095
18	1.15333	1.16578	1.17830	1.19088	1.20353	1.21625	1.22903	1.24188	1.25480	1.26778
19	1.28083	1.29395	1.30713	1.32038	1.33370	1.34708	1.36053	1.37405	1.38763	1.40128
20	1.41500	1.42878	1.44263	1.45655	1.47053	1.48458	1.49870	1.51288	1.52713	1.54145
21	1.55583	1.57028	1.58480	1.59938	1.61403	1.62875	1.64353	1.65838	1.67330	1.68828

The use of the formula and table is illustrated as follows:

Given a pipe 5 inches diameter and 1000 feet long, with 49 feet head, what will the discharge be?

If the velocity V is known in feet per second, the discharge is $0.32725d^2V$ cubic foot per minute.

By equation 2 we have

$$\frac{4V^2+5V-2}{1200} = \frac{Hd}{L} = \frac{49 \times 5}{1000} = 0.245;$$

whence, by table, V = real velocity = 8 feet per second.

The discharge in cubic feet per minute, if V is velocity in feet per second and d diameter in inches, is $0.32725d^2V$, whence, discharge

$$= 0.32725 \times 25 \times 8 = 65.45 \text{ cubic feet per minute.}$$

The velocity due the head, if there were no friction, is $8.025\sqrt{H} = 56.175$ feet per second, and the discharge at that velocity would be

$$0.32725 \times 25 \times 56.175 = 460 \text{ cubic feet per minute.}$$

Suppose it is required to deliver this amount, 460 cubic feet, at a velocity of 2 feet per second, what diameter of pipe will be required and what will be the loss of head by friction?

$$d = \text{diameter} = \sqrt{\frac{Q}{V \times 0.32725}} = \sqrt{\frac{460}{2 \times 0.32725}} = \sqrt{703} = 26.5 \text{ inches.}$$

Having now the diameter, the velocity, and the discharge, the friction-head is calculated by equation 1 and use of the table; thus,

$$H = \frac{L}{d}\,\frac{4V^2+5V-2}{1200} = \frac{1000}{26.5} \times 0.02 = \frac{20}{26.5} = 0.75 \text{ foot,}$$

thus leaving 49 — 0.75 = say 48 feet effective head applicable to power-producing purposes.

Problems of the loss of head may be solved rapidly by means of Cox's Pipe Computer, a mechanical device on the principle of the slide-rule, for sale by Keuffel & Esser, New York.

Frictional Heads at Given Rates of Discharge in Clean Cast-iron Pipes for Each 1000 Feet of Length.

(Condensed from Ellis and Howland's Hydraulic Tables.)

U. S. Gallons Discharged per Minute.	4-inch Pipe.		6-inch Pipe.		8-inch Pipe.		10-inch Pipe.		12-inch Pipe.		14-inch Pipe.	
	Velocity in ft. per sec.	Friction-head, feet.	Velocity in ft. per sec.	Friction-head, feet.	Velocity in ft. per sec.	Friction-head, feet.	Velocity in ft. per sec.	Friction-head, feet.	Velocity, in ft. per sec.	Friction-head, feet.	Velocity, in ft. per sec.	Friction-head, feet.
25	.64	.59	.28	.11	.16	.04	.10	.02	.07	.01	.....	.....
50	1.28	2.01	.57	.32	.32	.10	.20	.04	.14	.02	.10	.01
100	2.55	7.36	1.13	1.08	.64	.29	.41	.11	.28	.05	.21	.03
150	3.83	16.05	1.70	2.28	.96	.60	.61	.22	.43	.10	.31	.05
200	5.11	28.09	2.27	3.92	1.28	1.01	.82	.36	.57	.16	.42	.08
250	6.37	43.47	2.84	6.00	1.60	1.52	1.02	.54	.71	.24	.52	.12
300	7.66	62.20	3.40	8.52	1.91	2.13	1.23	.75	.85	.32	.63	.16
350	8.94	84.26	3.97	11.48	2.23	2.85	1.43	.99	.99	.43	.73	.21
400	10.21	109.68	4.54	14.89	2.55	3.68	1.63	1.27	1.13	.54	.83	.27
500	12.77	170.53	5.67	23.01	3.19	5.64	2.04	1.93	1.42	.81	1.04	.40
600	15.32	244.76	6.81	32.89	3.83	8.03	2.45	2.72	1.70	1.14	1.25	.55
700	17.87	332.36	7.94	44.54	4.47	10.83	2.86	3.66	1.98	1.52	1.46	.73
800	.....	.....	9.08	57.95	5.09	14.05	3.27	4.73	2.27	1.96	1.67	.94
900	.....	.....	10.21	73.12	5.74	17.68	3.68	5.93	2.55	2.45	1.88	1.17
1000	.....	.....	11.35	90.05	6.38	21.74	4.08	7.28	2.84	3.00	2.08	1.43
1200	.....	.....	13.61	129.20	7.66	31.10	4.90	10.38	3.40	4.26	2.50	2.02
1400	.....	.....	15.88	175.38	8.94	42.13	5.72	14.02	3.97	5.74	2.91	2.72
1600	.....	.....	18.15	228.62	10.21	54.84	6.53	18.22	4.54	7.44	3.33	3.51
1800	.....	.....	20.42	288.90	11.47	69.22	7.35	22.96	5.11	9.36	3.75	4.41
2000	.....	.....	22.69	356.22	12.77	85.27	8.17	28.25	5.67	11.50	4.17	5.41
2500	.....	.....	.....	.....	15.96	132.70	10.21	43.87	7.09	17.82	5.21	8.35
3000	.....	.....	.....	.....	.....	.....	12.25	62.92	8.51	25.51	6.25	11.98
4000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	8.34	21.00

U. S. Gallons Discharged per Minute.	16-inch Pipe.		18-inch Pipe.		20-inch Pipe.		24-inch Pipe.		30-inch Pipe.		36-inch Pipe.	
	Velocity in ft. per sec.	Friction-head, feet.	Velocity in ft. per sec.	Friction-head, feet.	Velocity in ft. per sec.	Friction-head, feet.	Velocity in ft. per sec.	Friction-head, feet.	Velocity in ft. per sec.	Friction-head, feet.	Velocity in ft. per sec.	Friction-head, feet.
500	.80	.22	.63	.13	.51	.08	.35	.04	.23	.01	.16	.01
1000	1.60	.76	1.26	.44	1.02	.27	.71	.12	.45	.04	.32	.02
1500	2.39	1.63	1.89	.93	1.53	.56	1.06	.24	.68	.09	.47	.04
2000	3.19	2.82	2.52	1.60	2.04	.96	1.42	.41	.91	.15	.63	.06
2500	3.99	4.34	3.15	2.45	2.55	1.47	1.77	.62	1.13	.22	.79	.09
3000	4.79	6.19	3.78	3.48	3.06	2.09	2.13	.87	1.36	.30	.95	.13
3500	5.59	8.37	4.41	4.70	3.57	2.81	2.48	1.16	1.59	.40	1.10	.17
4000	6.38	10.87	5.04	6.09	4.08	3.64	2.84	1.50	1.82	.52	1.26	.22
4500	7.18	13.70	5.67	7.67	4.59	4.58	3.19	1.88	2.04	.64	1.42	.27
5000	7.98	16.85	6.30	9.43	5.11	5.62	3.55	2.31	2.27	.78	1.58	.33
6000	.....	.....	7.57	13.49	6.13	8.03	4.26	3.28	2.72	1.11	1.89	.46
7000	.....	.....	.....	.....	7.15	10.86	4.96	4.43	3.18	1.49	2.21	.62
8000	.....	.....	.....	.....	.....	.....	5.67	5.75	3.63	1.93	2.52	.80
9000	.....	.....	.....	.....	.....	.....	6.38	7.25	4.08	2.43	2.84	1.00
10000	.....	.....	.....	.....	.....	.....	.....	.....	4.54	2.98	3.15	1.23
12000	.....	.....	.....	.....	.....	.....	.....	.....	5.44	4.25	3.78	1.74
14000	.....	.....	.....	.....	.....	.....	.....	.....	6.36	5.75	4.41	2.35
16000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.05	3.04
18000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.68	3.83
20000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	6.30	4.71

Effect of Bends and Curves in Pipes.—Weisbach's rule for bends: Loss of head in feet $= \left[.131 + 1.847 \left(\frac{r}{R}\right)^{\frac{7}{2}}\right] \times \frac{v^2}{64.4} \times \frac{a}{180}$, in which r = internal radius of pipe in feet, R = radius of curvature of axis of pipe, v = velocity in feet per second, and a = the central angle, or angle subtended by the bend.

Hamilton Smith, Jr., in his work on Hydraulics, says: The experimental data at hand are entirely insufficient to permit a satisfactory analysis of this quite complicated subject; in fact, about the only experiments of value are those made by Bossut and Dubuat with small pipes.

Curves.—If the pipe has easy curves, say with radius not less than 5 diameters of the pipe, the flow will not be materially diminished, provided the tops of all curves are kept below the hydraulic grade-line and provision be made for escape of air from the tops of all curves. (Trautwine.)

Hydraulic Grade-line.—In a straight tube of uniform diameter throughout, running full and discharging freely into the air, the hydraulic grade-line is a straight line drawn from the discharge end to a point immediately over the entry end of the pipe and at a depth below the surface equal to the entry and velocity heads. (Trautwine.)

In a pipe leading from a reservoir, no part of its length should be above the hydraulic grade-line.

Flow of Water in House-service Pipes.

Mr. E. Kuichling, C.E., furnished the following table to the Thomson Meter Co.:

Condition of Discharge.	Pressure in Main, pounds per square inch.	Discharge, or Quantity capable of being delivered, in Cubic Feet per Minute, from the Pipe, under the conditions specified in the first column.								
		Nominal Diameters of Iron or Lead Service-pipe in Inches.								
		½	⅝	¾	1	1½	2	3	4	6
Through 35 feet of service-pipe, no back pressure.	30	1.10	1.92	3.01	6.13	16.58	33.34	88.16	173.85	444.63
	40	1.27	2.22	3.48	7.08	19.14	38.50	101.80	200.75	513.42
	50	1.42	2.48	3.89	7.92	21.40	43.04	113.82	224.44	574.02
	60	1.56	2.71	4.26	8.67	23.44	47.15	124.68	245.87	628.81
	75	1.74	3.03	4.77	9.70	26.21	52.71	139.39	274.89	703.03
	100	2.01	3.50	5.50	11.20	30.27	60.87	160.96	317.41	811.79
	130	2.29	3.99	6.28	12.77	34.51	69.40	183.52	361.91	925.58
Through 100 feet of service-pipe, no back pressure.	30	0.66	1.16	1.84	3.78	10.40	21.30	58.19	118.13	317.23
	40	0.77	1.34	2.12	4.36	12.01	24.59	67.19	136.41	366.30
	50	0.86	1.50	2.37	4.88	13.43	27.50	75.13	152.51	409.54
	60	0.94	1.65	2.60	5.34	14.71	30.12	82.30	167.06	448.63
	75	1.05	1.84	2.91	5.97	16.45	33.68	92.01	186.78	501.58
	100	1.22	2.13	3.36	6.90	18.99	38.89	106.24	215.68	579.18
	130	1.39	2.42	3.83	7.86	21.66	44.34	121.14	245.91	660.36
Through 100 feet of service-pipe and 15 feet vertical rise.	30	0.55	0.96	1.52	3.11	8.57	17.55	47.90	97.17	260.56
	40	0.66	1.15	1.81	3.72	10.24	20.95	57.20	116.01	311.09
	50	0.75	1.31	2.06	4.24	11.67	23.87	65.18	132.20	354.49
	60	0.83	1.45	2.29	4.70	12.94	26.48	72.28	146.61	393.13
	75	0.94	1.64	2.59	5.32	14.64	29.96	81.79	165.90	444.85
	100	1.10	1.92	3.02	6.21	17.10	35.00	95.55	193.82	519.72
	130	1.26	2.20	3.48	7.14	19.66	40.23	109.82	222.75	597.81
Through 100 feet of service-pipe, and 30 feet vertical rise.	30	0.44	0.77	1.22	2.50	6.80	14.11	38.68	78.54	211.54
	40	0.55	0.97	1.53	3.15	8.68	17.79	48.68	98.98	266.59
	50	0.65	1.14	1.79	3.69	10.16	20.82	56.98	115.87	312.08
	60	0.73	1.28	2.02	4.15	11.45	23.47	64.22	130.59	351.73
	75	0.84	1.47	2.32	4.77	13.15	26.95	73.76	149.99	403.98
	100	1.00	1.74	2.75	5.65	15.58	31.93	87.38	177.67	478.55
	130	1.15	2.02	3.19	6.55	18.07	37.02	101.33	206.04	554.96

In this table it is assumed that the pipe is straight and smooth inside; that the friction of the main and meter are disregarded; that the inlet from the main is of ordinary character, sharp, not flaring or rounded, and that the outlet is the full diameter of pipe. The deliveries given will be increased if, first, the pipe between the meter and the main is of larger diameter than the outlet; second, if the main is tapped, say for 1-inch pipe, but is enlarged from the tap to 1¼ or 1½ inch; or, third, if pipe on the outlet is larger than that on the inlet side of the meter. The exact details of the conditions given are rarely met in practice; consequently the quantities of the table may be expected to be decreased, because the pipe is liable to be throttled at the joints, additional bends may interpose, or stop-cocks may be used, or the back-pressure may be increased.

Air-bound Pipes.—A pipe is said to be air-bound when, in consequence of air being entrapped at the high points of vertical curves in the line, water will not flow out of the pipe, although the supply is higher than the outlet. The remedy is to provide cocks or valves at the high points, through which the air may be discharged. The valve may be made automatic by means of a float.

Vertical Jets. (Molesworth.)—H = head of water, h = height of jet, d = diameter of jet, K = coefficient, varying with ratio of diameter of jet to head; then $h = KH$.

If $H = d \times$	300	600	1000	1500	1800	2800	3500	4500,
$K =$	.96	.9	.85	.8	.7	.6	.5	.25

Water Delivered through Meters. (Thomson Meter Co.).—The best modern practice limits the velocity in water-pipes to 10 lineal feet per second. Assume this as a basis of delivery, and we find, for the several sizes of pipes usually metered, the following approximate results:

Nominal diameter of pipe in inches:	⅜	⅝	¾	1	1½	2	3	4	6
Quantity delivered, in cubic feet per minute, due to said velocity:	0.46	1.28	1.85	3.28	7.36	13.1	29.5	52.4	117.9

Prices Charged for Water in Different Cities (National Meter Co.):

Average minimum price for 1000 gallons in 163 places............ 9.4 cents.
" maximum " " " " " " " .... 28 "
Extremes, 2½ cents to ....100 "

FIRE-STREAMS.

Discharge from Nozzles at Different Pressures.

(J. T. Fanning, Am. Water-works Ass'n, 1892, *Eng'g News*, July 14, 1892.)

Nozzle diam., in.	Height of stream, ft.	Pressure at Play-pipe, lbs.	Horizontal Projection of Streams, ft.	Gallons per minute.	Gallons per 24 hours.	Friction per 100 ft. Hose, lbs.	Friction per 100 ft. Hose, Net Head, ft.
1	70	46.5	59.5	203	292,298	10.75	24.77
1	80	59.0	67.0	230	331,200	13.00	31.10
1	90	79.0	76.6	267	384,500	17.70	40.78
1	100	130.0	88.0	311	447,900	22.50	54.14
1⅛	70	44.5	61.3	249	358,520	15.50	35.71
1⅛	80	55.5	69.5	281	404,700	19.40	44.70
1⅛	90	72.0	78.5	324	466,600	25.40	58.52
1⅛	100	103.0	89.0	376	541,500	33.80	77.88
1¼	70	43.0	66.0	306	440,613	23.75	52.42
1¼	80	53.5	72.4	343	493,900	28.40	65.43
1¼	90	68.5	81.0	388	558,800	35.90	82.71
1¼	100	93.0	92.0	460	662,500	57.75	86.98
1⅜	70	41.5	77.0	368	530,149	32.50	74.88
1⅜	80	51.5	74.4	410	590,500	40.00	92.16
1⅜	90	65.5	82.6	468	674,000	51.40	118.43
1⅜	100	88.0	92.0	540	777,700	72.00	165.89

Friction Losses in Hose.—In the above table the volumes of water discharged per jet were for stated pressures at the play-pipe.

In providing for this pressure due allowance is to be made for friction losses in each hose, according to the streams of greatest discharge which are to be used.

The loss of pressure or its equivalent loss of head (h) in the hose may be found by the formula $h = v^2(4m)\frac{l}{2gd}$.

In this formula, as ordinarily used, for friction per 100 ft. of 2½-in. hose there are the following constants: 2½ in. diameter of hose $d = .20833$ ft.; length of hose $l = 100$ ft., and $2g = 64.4$. The variables are: v = velocity in feet per second; h = loss of head in feet per 100 ft. of hose; m = a coefficient found by experiment; the velocity v is found from the given discharges of the jets through the given diameter of hose.

Head and Pressure Losses by Friction in 100-ft. Lengths of Rubber-lined Smooth 2½-in. Hose.

Discharge per minute, gallons.	Velocity per second, ft.	Coefficient, m.	Head Lost, ft.	Pressure Lost, lbs. per sq. in.	Gallons per 24 hours.
200	13.072	.00450	22.89	9.93	288,000
250	16.388	.00446	35.55	15.43	360,000
300	18.858	.00442	46.80	20.31	432,000
347	21.677	.00439	61.53	26.70	499,680
350	22.873	.00439	68.48	29.73	504,000
400	26.144	.00436	88.83	38.55	576,000
450	29.408	.00434	111.80	48.52	648,000
500	32.675	.00432	137.50	59.67	720,000
520	33.982	.00431	148.40	64.40	748,800

These frictions are for given volumes of flow in the hose and the velocities respectively due to those volumes, and are independent of size of nozzle. The changes in nozzle do not affect the friction in the hose if there is no change in velocity of flow, but a larger nozzle with equal pressure at the nozzle augments the discharge and velocity of flow, and thus materially increases the friction loss in the hose.

Loss of Pressure (p) and Head (h) in Rubber-lined Smooth 2½-in. Hose may be found approximately by the formulæ $p = \frac{lq^2}{4150d^5}$ and $h = \frac{lq^2}{1801d^5}$, in which p = pressure lost by friction, in pounds per square inch; l = length of hose in feet; q = gallons of water discharged per minute: d = diam. of the hose in inches, 2½ in.; h = friction-head in feet. The coefficient of d^5 would be decreased for rougher hose.

The loss of pressure and head for a 1⅛-in. stream with power to reach a height of 80 ft. is, in each 100 ft. of 2½-in. hose, approximately 20 lbs., or 45 ft. net, or, say, including friction in the hydrant, ½ ft. loss of head for each foot of hose.

If we change the nozzles to 1¼ or 1⅜ in. diameter, then for the same 80 ft. height of stream we increase the friction losses on the hose to approximately ¾ ft. and 1 ft. head, respectively, for each foot-length of hose.

These computations show the great difficulty of maintaining a high stream through large nozzles unless the hose is very short, especially for a gravity or direct-pressure system.

This single 1⅛-in. stream requires approximately 56 lbs pressure, equivalent to 129 ft. head, at the play-pipe, and 45 to 50 ft. head for each 100 ft. length of smooth 2½-in. hose, so that for 100, 200, and 300 ft. of hose we must have available heads at the hydrant or fire-engine of 106, 156, and 206 ft., respectively. If we substitute 1¼-in. nozzles for same height of stream we must have available heads at the hydrants or engine of 185, 255 and 325 ft., respectively, or we must increase the diameter of a portion at least of the long hose and save friction-loss of head.

Rated Capacities of Steam Fire-engines, which is perhaps one third greater than their ordinary rate of work at fires, are substantially as follows:

3d size,	550 gals. per min., or	792,000 gals. per 24 hours.
2d "	700 " "	1,008,000 " "
1st "	900 " "	1,296,000 " "
1 ext.,	1,100 " "	1,584,000 " "

Pressures required at Nozzle and at Pump, with Quantity and Pressure of Water Necessary to throw Water Various Distances through Different-sized Nozzles—using 2½-inch Rubber Hose and Smooth Nozzles.

(From Experiments of Ellis & Leshure, Fanning's "Water Supply.")

Size of Nozzles.	1 Inch.				1⅛ Inch.			
Pressure at nozzle, lbs. per sq. in.	40	60	80	100	40	60	80	100
* Pressure at pump or hydrant with 100 ft. 2½-inch rubber hose	48	73	97	121	54	81	108	135
Gallons per minute	155	189	219	245	196	240	277	310
Horizontal distance thrown, feet	109	142	168	186	113	148	175	193
Vertical distance thrown, feet	79	108	131	148	81	112	137	157

Size of Nozzles.	1¼ Inch.				1⅜ Inch.			
Pressure at nozzle, lbs. per sq. in.	40	60	80	100	40	60	80	100
* Pressure at pump or hydrant with 100 feet 2½-inch rubber hose	61	92	123	154	71	107	144	180
Gallons per minute	242	297	342	383	293	358	413	462
Horizontal distance thrown, feet	118	156	186	207	124	166	200	224
Vertical distance thrown, feet	82	115	142	164	85	118	146	169

* For greater length of 2½-inch hose the increased friction can be obtained by noting the differences between the above given "pressure at nozzle" and "pressure at pump or hydrant with 100 feet of hose." For instance, if it requires at hydrant or pump eight pounds more pressure than it does at nozzle to overcome the friction when pumping through 100 feet of 2½-inch hose (using 1-inch nozzle, with 40-pound pressure at said nozzle) then it requires 16-pounds pressure to overcome the friction in forcing through 200 feet of same size hose.

Decrease of Flow due to Increase of Length of Hose. (J. R. Freeman's Experiments, Trans. A. S. C. E. 1889.)—If the static pressure is 80 lbs. and the hydrant-pipes of such size that the pressure at the hydrant is 70 lbs., the hose 2½ in. nominal diam., and the nozzle 1⅛ in. diam., the height of effective fire-stream obtainable and the quantity in gallons per minute will be:

	Linen Hose.		Best Rubber-lined Hose.	
	Height, feet.	Gals. per min.	Height, feet.	Gals. per min.
With 50 ft. of 2½-in. hose	73	261	81	282
" 250 " " " "	42	184	61	229
" 500 " " " "	27	146	46	182

With 500 ft. of smoothest and best rubber-lined hose, if diameter be exactly 2½ in., effective height of stream will be 39 ft. (177 gals.); if diameter be ⅛ in. larger, effective height of stream will be 46 ft. (192 gals.)

THE SIPHON.

The Siphon is a bent tube of unequal branches, open at both ends, and is used to convey a liquid from a higher to a lower level, over an intermediate point higher than either. Its parallel branches being in a vertical plane and plunged into two bodies of liquid whose upper surfaces are at different levels, the fluid will stand at the same level both within and without each branch of the tube when a vent or small opening is made at the bend. If the air be withdrawn from the siphon through this vent, the water will rise in the branches by the atmospheric pressure without, and when the two columns unite and the vent is closed, the liquid will flow from the upper reservoir as long as the end of the shorter branch of the siphon is below the surface of the liquid in the reservoir.

If the water was free from air the height of the bend above the supply level might be as great as 33 feet.

If A = area of cross-section of the tube in square feet, H = the difference in level between the two reservoirs in feet, D the density of the liquid in pounds per cubic foot, then ADH measures the intensity of the force which causes the movement of the fluid, and $V = \sqrt{2gH} = 8.02 \sqrt{H}$ is the theoretical velocity, in feet per second, which is reduced by the loss of head for entry and friction, as in other cases of flow of liquids through pipes. In the case of the difference of level being greater than 33 feet, however, the velocity of the water in the shorter leg is limited to that due to a height of 33 feet, or that due to the difference between the atmospheric pressure at the entrance and the vacuum at the bend.

Leicester Allen (*Am. Mach.*, Nov. 2, 1893) says: The supply of liquid to a siphon must be greater than the flow which would take place from the discharge end of the pipe, provided the pipe were filled with the liquid, the supply end stopped, and the discharge end opened when the discharge end is left free, unregulated, and unsubmerged.

To illustrate this principle, let us suppose the extreme case of a siphon having a calibre of 1 foot, in which the difference of level, or between the point of supply and discharge, is 4 inches. Let us further suppose this siphon to be at the sea-level, and its highest point above the level of the supply to be 27 feet. Also suppose the discharge end of this siphon to be unregulated, unsubmerged. It would be inoperative because the water in the longer leg would not be held solid by the pressure of the atmosphere against it, and it would therefore break up and run out faster than it could be replaced at the inflow end under an effective head of only 4 inches.

Long Siphons.—Prof. Joseph Torrey, in the *Amer. Machinist*, describes a long siphon which was a partial failure.

The length of the pipe was 1792 feet. The pipe was 8 inches diameter, and rose at one point 9 feet above the initial level. The final level was 20 feet below the initial level. No automatic air valve was provided. The highest point in the siphon was about one third the total distance from the pond and nearest the pond. At this point a pump was placed, whose mission was to fill the pipe when necessary. This siphon would flow for about two hours and then cease, owing to accumulation of air in the pipe. When in full operation it discharged 43½ gallons per minute. The theoretical discharge from such a sized pipe with the specified head is 55½ gallons per minute.

Siphon on the Water-supply of Mount Vernon, N. Y. (*Eng'g News*, May 4, 1893.)—A 12-inch siphon, 925 feet long, with a maximum lift of 22.12 feet and a 45° change in alignment, was put in use in 1892 by the New York City Suburban Water Co., which supplies Mount Vernon, N. Y.

At its summit the siphon crosses a supply main, which is tapped to charge the siphon.

The air-chamber at the siphon is 12 inches by 16 feet long. A ½-inch tap and cock at the top of the chamber provide an outlet for the collected air.

It was found that the siphon with air-chamber as described would run until 125 cubic feet of air had gathered, and that this took place only half as soon with a 14-foot lift as with the full lift of 22.12 feet. The siphon will operate about 12 hours without being recharged, but more water can be gotten over by charging every six hours. It can be kept running 23 hours out of 24 with only one man in attendance. With the siphon as described above it is necessary to close the valves at each end of the siphon to recharge it.

It has been found by weir measurements that the discharge of the siphon before air accumulates at the summit is practically the same as through a straight pipe.

MEASUREMENT OF FLOWING WATER.

Piezometer.—If a vertical or oblique tube be inserted into a pipe containing water under pressure, the water will rise in the former, and the vertical height to which it rises will be the head producing the pressure at the point where the tube is attached. Such a tube is called a piezometer or pressure measure. If the water in the piezometer falls below its proper level it shows that the pressure in the main pipe has been reduced by an obstruction between the piezometer and the reservoir. If the water rises above its proper level, it indicates that the pressure there has been increased by an obstruction beyond the piezometer.

If we imagine a pipe full of water to be provided with a number of piezometers, then a line joining the tops of the columns of water in them is the hydraulic grade-line.

Pitot Tube Gauge.—The Pitot tube is used for measuring the velocity of fluids in motion. It has been used with great success in measuring the flow of natural gas. (S. W. Robinson, Report Ohio Geol. Survey, 1890.) (See also *Van Nostrand's Mag.*, vol. xxxv.) It is simply a tube so bent that a short leg extends into the current of fluid flowing from a tube, with the plane of the entering orifice opposed at right angles to the direction of the current. The pressure caused by the impact of the current is transmitted through the tube to a pressure-gauge of any kind, such as a column of water or of mercury, or a Bourdon spring-gauge. From the pressure thus indicated and the known density and temperature of the flowing gas is obtained the head corresponding to the pressure, and from this the velocity. In a modification of the Pitot tube described by Prof. Robinson, there are two tubes inserted into the pipe conveying the gas, one of which has the plane of the orifice at right angles to the current, to receive the static pressure plus the pressure due to impact; the other has the plane of its orifice parallel to the current, so as to receive the static pressure only. These tubes are connected to the legs of a *U* tube partly filled with mercury, which then registers the difference in pressure in the two tubes, from which the velocity may be calculated. Comparative tests of Pitot tubes with gas-meters, for measurement of the flow of natural gas, have shown an agreement within 3%.

The Venturi Meter, invented by Clemens Herschel, and described in a pamphlet issued by the Builders' Iron Foundry of Providence, R. I., is named from Venturi, who first called attention, in 1796, to the relation between the velocities and pressures of fluids when flowing through converging and diverging tubes.

It consists of two parts—the tube, through which the water flows, and the recorder, which registers the quantity of water that passes through the tube.

The tube takes the shape of two truncated cones joined in their smallest diameters by a short throat-piece. At the up-stream end and at the throat there are pressure-chambers, at which points the pressures are taken.

The action of the tube is based on that property which causes the small section of a gently expanding frustum of a cone to receive, without material resultant loss of head, as much water at the smallest diameter as is discharged at the large end, and on that further property which causes the pressure of the water flowing through the throat to be less, by virtue of its greater velocity, than the pressure at the up-stream end of the tube, each pressure being at the same time a function of the velocity at that point and of the hydrostatic pressure which would obtain were the water motionless within the pipe.

The recorder is connected with the tube by pressure-pipes which lead to it from the chambers surrounding the up-stream end and the throat of the tube. It may be placed in any convenient position within 1000 feet of the tube. It is operated by a weight and clockwork.

The difference of pressure or head at the entrance and at the throat of the meter is balanced in the recorder by the difference of level in two columns of mercury in cylindrical receivers, one within the other. The inner carries a float, the position of which is indicative of the quantity of water flowing through the tube. By its rise and fall the float varies the time of contact between an integrating drum and the counters by which the successive readings are registered.

There is no limit to the sizes of the meters nor the quantity of water that may be measured. Meters with 24-inch, 36-inch, 48-inch, and even 20-foot tubes can be readily made.

Measurement by Venturi Tubes. (Trans A. S. C. E., Nov., 1887, and Jan., 1888.)—Mr. Herschel recommends the use of a Venturi tube, inserted in the force-main of the pumping engine, for determining the quantity of water discharged. Such a tube applied to a 24-inch main has a total length of about 20 feet. At a distance of 4 feet from the end nearest the engine the inside diameter of the tube is contracted to a throat having a diameter of about 8 inches. A pressure-gauge is attached to each of two chambers, the one surrounding and communicating with the entrance or main pipe, the other with the throat. According to experiments made upon two tubes of this kind, one 4 in. in diameter at the throat and 12 in. at the entrance, and the other about 36 in. in diameter at the throat and 9 feet at its entrance, the quantity of water which passes through the tube is very nearly the theoretical discharge through an opening having an area equal to that of the throat, and a velocity which is that due to the difference in head shown by the two gauges. Mr. Herschel states that the coefficient for these two widely-varying sizes of tubes and for a wide range of velocity through the pipe, was found to be within two per cent. either way, of 98%. In other words, the quantity of water flowing through the tube per second is expressed within two per cent by the formula $W = 0.98 \times A \times \sqrt{2gh}$, in which A is the area of the throat of the tube, h the head, in feet, corresponding to the difference in the pressure of the water entering the tube and that found at the throat, and $g = 32.16$.

Measurement of Discharge of Pumping-engines by Means of Nozzles. (Trans. A. S. M. E., xiii, 557).—The measurement of water by computation from its discharge through orifices, or through the nozzles of fire-hose, furnishes a means of determining the quantity of water delivered by a pumping-engine which can be applied without much difficulty. John R. Freeman, Trans. A. S. C. E., Nov., 1889, describes a series of experiments covering a wide range of pressures and sizes, and the results showed that the coefficient of discharge for a smooth nozzle of ordinary good form was within one half of one per cent, either way, of 0.977; the diameter of the nozzle being accurately calipered, and the pressures being determined by means of an accurate gauge attached to a suitable piezometer at the base of the play-pipe.

In order to use this method for determining the quantity of water discharged by a pumping-engine, it would be necessary to provide a pressure-box, to which the water would be conducted, and attach to the box as many nozzles as would be required to carry off the water. According to Mr. Freeman's estimate, four 1¼-inch nozzles, thus connected, with a pressure of 80 lbs. per square inch, would discharge the full capacity of a two-and a-half-million engine. He also suggests the use of a portable apparatus with a single opening for discharge, consisting essentially of a Siamese nozzle, so-called, the water being carried to it by three or more lines of fire-hose.

To insure reliability for these measurements, it is necessary that the shut-off valve in the force-main, or the several shut-off valves, should be tight, so that all the water discharged by the engine may pass through the nozzles.

Flow through Rectangular Orifices. (Approximate. See p. 556.)

CUBIC FEET OF WATER DISCHARGED PER MINUTE THROUGH AN ORIFICE ONE INCH SQUARE, UNDER ANY HEAD OF WATER FROM 3 TO 72 INCHES.

For any other orifice multiply by its area in square inches.

Formula, $Q' = .624\sqrt{h''} \times a$. Q' = cu. ft. per min.; a = area in sq. in.

Heads in Inches.	Cubic Feet Discharged per min.	Heads in Inches.	Cubic Feet Discharged per min.	Heads in Inches.	Cubic Feet Discharged per min.	Heads in Inches.	Cubic Feet Discharged per min.	Heads in Inches.	Cubic Feet Discharged per min.	Heads in Inches.	Cubic Feet Discharged per min.	Heads in Inches.	Cubic Feet Discharged per min.
3	1.12	13	2.20	23	2.90	33	3.47	43	3.95	53	4.39	63	4.78
4	1.27	14	2.28	24	2.97	34	3.52	44	4.00	54	4.42	64	4.81
5	1.40	15	2.36	25	3.03	35	3.57	45	4.05	55	4.46	65	4.85
6	1.52	16	2.43	26	3.08	36	3.62	46	4.09	56	4.52	66	4.89
7	1.64	17	2.51	27	3.14	37	3.67	47	4.12	57	4.55	67	4.92
8	1.75	18	2.58	28	3.20	38	3.72	48	4.18	58	4.58	68	4.97
9	1.84	19	2.64	29	3.25	39	3.77	49	4.21	59	4.63	69	5.00
10	1.94	20	2.71	30	3.31	40	3.81	50	4.27	60	4.65	70	5.03
11	2.03	21	2.78	31	3.36	41	3.86	51	4.30	61	4.72	71	5.07
12	2.12	22	2.84	32	3.41	42	3.91	52	4.34	62	4.74	72	5.09

Measurement of an Open Stream by Velocity and Cross-section.—Measure the depth of the water at from 6 to 12 points across the stream at equal distances between. Add all the depths in feet together and divide by the number of measurements made; this will be the average depth of the stream, which multiplied by its width will give its area or cross-section. Multiply this by the velocity of the stream in feet per minute, and the result will be the discharge in cubic feet per minute of the stream.

The velocity of the stream can be found by laying off 100 feet of the bank and throwing a float into the middle, noting the time taken in passing over the 100 ft. Do this a number of times and take the average; then, dividing this distance by the time gives the velocity at the surface. As the top of the stream flows faster than the bottom or sides—the average velocity being about 83% of the surface velocity at the middle—it is convenient to measure a distance of 120 feet for the float and reckon it as 100.

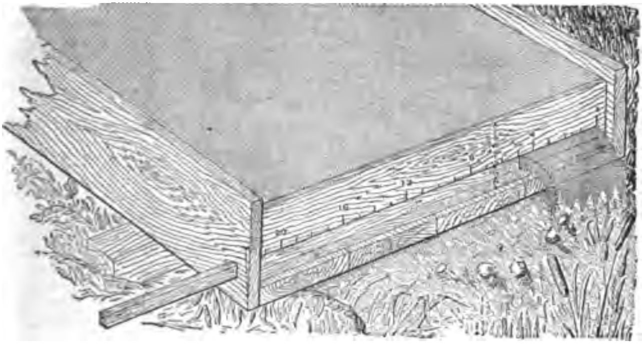

FIG. 180.

Miners' Inch Measurements. (Pelton Water Wheel Co.)

The cut, Fig. 180, shows the form of measuring-box ordinarily used, and the following table gives the discharge in cubic feet per minute of a miner's inch of water, as measured under the various heads and different lengths and heights of apertures used in California.

Length of Opening in inches.	Openings 2 Inches High. Head to Centre, 5 inches.	Openings 2 Inches High. Head to Centre, 6 inches.	Openings 2 Inches High. Head to Centre, 7 inches.	Openings 4 Inches High. Head to Centre, 5 inches.	Openings 4 Inches High. Head to Centre, 6 inches.	Openings 4 Inches High. Head to Centre, 7 inches.
	Cu. ft.	Cu. ft.	Cu. ft.	Cu. ft.	Cu. ft.	Cu. ft.
4	1.348	1.473	1.589	1.320	1.450	1.570
6	1.355	1.480	1.596	1.336	1.470	1.595
8	1.359	1.484	1.600	1.344	1.481	1.608
10	1.361	1.485	1.602	1.349	1.487	1.615
12	1.363	1.487	1.604	1.352	1.491	1.620
14	1.364	1.488	1.604	1.354	1.494	1.623
16	1.365	1.489	1.605	1.356	1.496	1.626
18	1.365	1.489	1.606	1.357	1.498	1.628
20	1.365	1.490	1.606	1.359	1.499	1.630
22	1.366	1.490	1.607	1.359	1.500	1.631
24	1.366	1.490	1.607	1.360	1.501	1.632
26	1.366	1.490	1.607	1.361	1.502	1.633
28	1.367	1.491	1.607	1.361	1.503	1.634
30	1.367	1.491	1.608	1.362	1.503	1.635
40	1.367	1.492	1.608	1.363	1.505	1.637
50	1.368	1.493	1.609	1.364	1.507	1.639
60	1.368	1.493	1.609	1.365	1.508	1.640
70	1.368	1.493	1.609	1.365	1.508	1.641
80	1.368	1.493	1.609	1.366	1.509	1.641
90	1.369	1.493	1.610	1.366	1.509	1.641
100	1.369	1.494	1.610	1.366	1.509	1.642

NOTE.—The apertures from which the above measurements were obtained were through material 1¼ inches thick, and the lower edge 2 inches above the bottom of the measuring-box, thus giving full contraction.

Flow of Water Over Weirs. Weir Dam Measurement. (Pelton Water Wheel Co.)—Place a board or plank in the stream, as shown

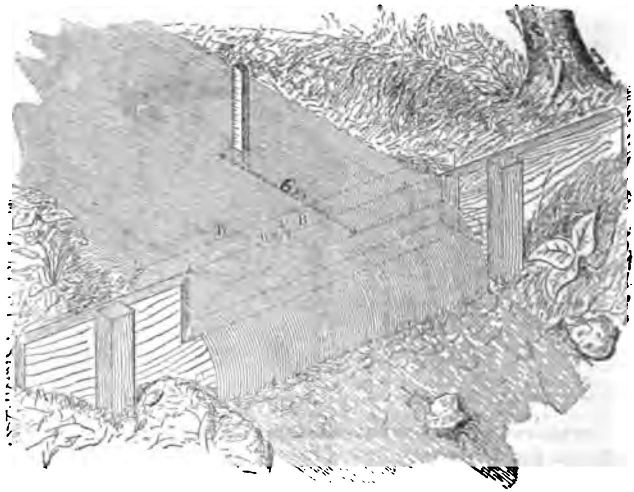

FIG. 131.

in the sketch, at some point where a pond will form above. The length of the notch in the dam should be from two to four times its depth for small quantities and longer for large quantities. The edges of the notch should be bevelled toward the intake side, as shown. The overfall below the notch should not be less than twice its depth. [Francis says a fall below the crest equal to one-half the head is sufficient, but there must be a free access of air under the sheet.]

In the pond, about 6 ft. above the dam, drive a stake, and then obstruct the water until it rises precisely to the bottom of the notch and mark the stake at this level. Then complete the dam so as to cause all the water to flow through the notch, and, after time for the water to settle, mark the stake again for this new level. If preferred the stake can be driven with its top precisely level with the bottom of the notch and the depth of the water be measured with a rule after the water is flowing free, but the marks are preferable in most cases. The stake can then be withdrawn; and the distance between the marks is the theoretical depth of flow corresponding to the quantities in the table on the following page.

Francis's Formulæ for Weirs.

	As given by Francis.	As modified by Smith.
Weirs with both end contractions suppressed	$Q = 3.33lh^{\frac{3}{2}}$	$3.29\left(l + \frac{h}{7}\right)h^{\frac{3}{2}}$
Weirs with one end contraction suppressed....................	$Q = 3.33(l - .1h)h^{\frac{3}{2}}$	$3.29lh^{\frac{3}{2}}$
Weirs with full contraction.......	$Q = 3.33(l - .2h)h^{\frac{3}{2}}$	$3.29\left(l - \frac{h}{10}\right)h^{\frac{3}{2}}$

The greatest variation of the Francis formulæ from the values of c given by Smith amounts to 3½%. The modified Francis formulæ, says Smith, will give results sufficiently exact, when great accuracy is not required, within the limits of h, from .5 ft. to 2 ft., l being not less than 3 h.

Q = discharge in cubic feet per second, l = length of weir in feet, h = effective head in feet, measured from the level of the crest to the level of still water above the weir.

If Q' = discharge in cubic feet per *minute*, and l' and h' are taken in inches, the first of the above formulæ reduces to $Q' = 0.4l'h'^{\frac{3}{2}}$. From this formula the following table is calculated. The values are sufficiently accurate for ordinary computations of water-power for weirs without end contraction, that is, for a weir the full width of the channel of approach, and are approximate also for weirs with end contraction when l = at least $10h$, but about 6% in excess of the truth when $l = 4h$.

Weir Table.

GIVING CUBIC FEET OF WATER PER MINUTE THAT WILL FLOW OVER A WEIR ONE INCH WIDE AND FROM ⅛ TO 20⅞ INCHES DEEP.

For other widths multiply by the width in inches.

		⅛ in.	¼ in.	⅜ in.	½ in.	⅝ in.	¾ in.	⅞ in.
in.	cu. ft.	cu. ft.	cu. ft.	cu. ft.	cu. ft.	cu. ft.	cu. ft.	cu. ft.
0	.00	.01	.05	.09	.14	.19	.26	.32
1	.40	.47	.55	.64	.73	.82	.92	1.02
2	1.13	1.23	1.35	1.46	1.58	1.70	1.82	1.95
3	2.07	2.21	2.34	2.48	2.61	2.76	2.90	3.05
4	3.20	3.35	3.50	3.66	3.81	3.97	4.14	4.30
5	4.47	4.64	4.81	4.98	5.15	5.33	5.51	5.69
6	5.87	6.06	6.25	6.44	6.62	6.82	7.01	7.21
7	7.40	7.60	7.80	8.01	8.21	8.42	8.63	8.83
8	9.05	9.26	9.47	9.69	9.91	10.13	10.35	10.57
9	10.80	11.02	11.25	11.48	11.71	11.94	12.17	12.41
10	12.64	12.88	13.12	13.36	13.60	13.85	14.09	14.34
11	14.59	14.84	15.09	15.34	15.59	15.85	16.11	16.36
12	16.62	16.88	17.15	17.41	17.67	17.94	18.21	18.47
13	18.74	19.01	19.29	19.56	19.84	20.11	20.39	20.67
14	20.95	21.23	21.51	21.80	22.08	22.37	22.65	22.94
15	23.23	23.52	23.82	24.11	24.40	24.70	25.00	25.30
16	25.60	25.90	26.20	26.50	26.80	27.11	27.42	27.72
17	28.03	28.34	28.65	28.97	29.28	29.59	29.91	30.22
18	30.54	30.86	31.18	31.50	31.82	32.15	32.47	32.80
19	33.12	33.45	33.78	34.11	34.44	34.77	35.10	35.44
20	35.77	36.11	36.45	36.78	37.12	37.46	37.80	38.15

For more accurate computations, the coefficients of flow of Hamilton Smith, Jr., or of Bazin should be used. In Smith's hydraulics will be found a collection of results of experiments on orifices and weirs of various shapes made by many different authorities, together with a discussion of their several formulæ. (See also Trautwine's Pocket Book.)

Bazin's Experiments.—M. Bazin (*Annales des Ponts et Chaussées*, Oct., 1888, translated by Marichal and Trautwine, Proc. Engrs. Club of Phila., Jan., 1890), made an extensive series of experiments with a sharp-crested weir without lateral contraction, the air being admitted freely behind the falling sheet, and found values of m varying from 0.42 to 0.50, with variations of the length of the weir from 19¾ to 78¾ in., of the height of the crest above the bottom of the channel from 0.79 to 2.46 ft., and of the head from 1.97 to 23.62 in. From these experiments he deduces the following formula:

$$Q = \left[0.425 + 0.21\left(\frac{H}{P+H}\right)^2\right] LH\sqrt{2gH},$$

in which P is the height in feet of the crest of the weir above the bottom of the channel of approach, L the length of the weir, H the head, both in feet, and Q the discharge in cu. ft. per sec. This formula, says M. Bazin, is entirely practical where errors of 2% to 3% are admissible. The following table is condensed from M. Bazin's paper:

VALUES OF THE COEFFICIENT m IN THE FORMULA $Q = mLH\sqrt{2gH}$, FOR A SHARP-CRESTED WEIR WITHOUT LATERAL CONTRACTION; THE AIR BEING ADMITTED FREELY BEHIND THE FALLING SHEET.

Head, H.		Height of Crest of Weir Above Bed of Channel.									
		Feet ...0.66 Inches 7.87	0.98 11.81	1.31 15.75	1.64 19.69	1.97 23.62	2.62 31.50	3.28 39.38	4.92 59.07	6.56 78.76	∞ ∞
Ft.	In.	m	m	m	m	m	m	m	m	m	m
.164	1.97	0.458	0.453	0.451	0.450	0.449	0.449	0.449	0.448	0.448	0.4481
.230	2.76	0.455	0.448	0.445	0.443	0.442	0.441	0.440	0.440	0.439	0.4391
.295	3.54	0.457	0.447	0.442	0.440	0.438	0.436	0.436	0.435	0.434	0.4340
.394	4.72	0.462	0.448	0.442	0.438	0.436	0.433	0.432	0.430	0.430	0.4291
.525	6.30	0.471	0.453	0.444	0.438	0.435	0.431	0.429	0.427	0.426	0.4246
.656	7.87	0.480	0.459	0.447	0.440	0.436	0.431	0.428	0.425	0.423	0.4215
.787	9.45	0.488	0.465	0.452	0.444	0.438	0.432	0.428	0.424	0.422	0.4194
.919	11.02	0.496	0.472	0.457	0.448	0.441	0.433	0.429	0.424	0.422	0.4181
1.050	12.60	....	0.478	0.462	0.452	0.444	0.436	0.430	0.424	0.421	0.4168
1.181	14.17	....	0.483	0.467	0.456	0.448	0.438	0.432	0.424	0.421	0.4156
1.312	15.75	....	0.489	0.472	0.459	0.451	0.440	0.433	0.424	0.421	0.4144
1.444	17.32	....	0.494	0.476	0.463	0.454	0.442	0.435	0.425	0.421	0.4134
1.575	18.90	....	.....	0.480	0.467	0.457	0.444	0.436	0.425	0.421	0.4122
1.706	20.47	....	.....	0.483	0.470	0.460	0.446	0.438	0.426	0.421	0.4112
1.837	22.05	....	.....	0.487	0.473	0.463	0.448	0.439	0.427	0.421	0.4101
1.969	23.62	....	.....	0.490	0.476	0.466	0.451	0.441	0.427	0.421	0.4092

A comparison of the results of this formula with those of experiments, says M. Bazin, justifies us in believing that, except in the unusual case of a very low weir (which should always be avoided), the preceding table will give the coefficient m in all cases within 1%; provided, however, that the arrangements of the standard weir are exactly reproduced. It is especially important that the admission of the air behind the falling sheet be perfectly assured. If this condition is not complied with, m may vary within much wider limits. The type adopted gives the least possible variation in the coefficient.

WATER-POWER.

Power of a Fall of Water—Efficiency.—The gross power of a fall of water is the product of the weight of water discharged in a unit of time into the total head, i.e., the difference of vertical elevation of the upper surface of the water at the points where the fall in question begins and ends. The term "head" used in connection with water-wheels is the difference in height from the surface of the water in the wheel-pit to the surface in the pen-stock when the wheel is running.

If Q = cubic feet of water discharged per second, D = weight of a cubic foot of water = 62.36 lbs. at 60° F., H = total head in feet; then

$$DQH = \text{gross power in foot-pounds per second,}$$

$$\text{and } DQH \div 550 = .1134QH = \text{gross horse-power.}$$

$$\text{If } Q' \text{ is taken in cubic feet per minute, H. P.} = \frac{Q'H \times 62.36}{33{,}000} = .00189Q'H.$$

A water-wheel or motor of any kind cannot utilize the whole of the head H, since there are losses of head at both the entrance to and the exit from the wheel. There are also losses of energy due to friction of the water in its passage through the wheel. The ratio of the power developed by the wheel to the gross power of the fall is the efficiency of the wheel. For 75% efficiency, net horse-power $= .00142Q'H = \frac{Q'H}{706}$.

A head of water can be made use of in one or other of the following ways viz.:

1st. By its weight, as in the water-balance and overshot-wheel.
2d. By its pressure, as in turbines and in the hydraulic engine, hydraulic press, crane, etc.
3d. By its impulse, as in the undershot-wheel, and in the Pelton wheel.
4th. By a combination of the above.

Horse-power of a Running Stream.—The gross horse-power is, H. P. $= QH \times 62.36 \div 550 = .1134QH$, in which Q is the discharge in cubic feet per second actually impinging on the float or bucket, and H = theoretical head due to the velocity of the stream $= \frac{v^2}{2g} = \frac{v^2}{64.4}$, in which v is the velocity in feet per second. If Q' be taken in cubic feet per minute, H.P. $= .00189Q'H$.

Thus, if the floats of an undershot-wheel driven by a current alone be 5 feet $\times$ 1 foot, and the velocity of stream = 210 ft. per minute, or $3\frac{1}{2}$ ft. per sec., of which the theoretical head is .19 ft., Q = 5 sq. ft. $\times$ 210 = 1050 cu. ft. per minute; H = .19 ft.; H. P. = 1050 $\times$ .19 $\times$ .00189 = .377 H. P.

The wheels would realize only about .4 of this power, on account of friction and slip, or .151 H.P., or about .03 H.P. per square foot of float, which is equivalent to 33 sq. ft. of float per H.P.

Current Motors.—A current motor could only utilize the whole power of a running stream if it could take all the velocity out of the water, so that it would leave the floats or buckets with no velocity at all; or in other words, it would require the backing up of the whole volume of the stream until the actual head was equivalent to the theoretical head due to the velocity of the stream. As but a small fraction of the velocity of the stream can be taken up by a current motor, its efficiency is very small. Current motors may be used to obtain small amounts of power from large streams, but for large powers they are not practicable.

Horse-power of Water Flowing in a Tube.—The head due to the velocity is $\frac{v^2}{2g}$; the head due to the pressure is $\frac{f}{w}$; the head due to actual height above the datum plane is h feet. The total head is the sum of these $= \frac{v^2}{2g} + h + \frac{f}{w}$, in feet, in which v = velocity in feet per second, f = pressure in lbs. per sq. ft., w = weight of 1 cu. ft. of water = 62.36 lbs. If p = pressure in lbs. per sq. in., $\frac{f}{w} = 2.309p$. In hydraulic transmission the velocity and the height above datum are usually small compared with the pressure-head. The work or energy of a given quantity of water under pressure = its volume in cubic feet $\times$ its pressure in lbs. per sq. ft.; or if Q = quantity in cubic feet per second, and p = pressure in lbs. per square inch, $W = 144pQ$, and the H. P. $= \frac{144pQ}{550} = .2618pQ$.

Maximum Efficiency of a Long Conduit.—A. L. Adams and R. C. Gemmell (*Eng'g News*, May 4, 1893), show by mathematical analysis that the conditions for securing the maximum amount of power through a long conduit of fixed diameter, without regard to the economy of water, is that the draught from the pipe should be such that the frictional loss in the pipe will be equal to one third of the entire static head.

Mill-Power.—A "mill-power" is a unit used to rate a water-power for the purpose of renting it. The value of the unit is different in different localities. The following are examples (from Emerson):

Holyoke, Mass.—Each mill-power at the respective falls is declared to be the right during 16 hours in a day to draw 38 cu. ft. of water per second at the upper fall when the head there is 20 feet, or a quantity proportionate to the height at the falls. This is equal to 86.2 horse-power as a maximum.

Lowell, Mass.—The right to draw during 15 hours in the day so much water as shall give a power equal to 25 cu. ft. a second at the great fall, when the fall there is 30 feet. Equal to 85 H. P. maximum.

Lawrence, Mass.—The right to draw during 16 hours in a day so much water as shall give a power equal to 30 cu. ft. per second when the head is 25 feet. Equal to 85 H.P. maximum.

Minneapolis, Minn.—30 cu. ft. of water per second with head of 22 feet. Equal to 74.8 H.P.

Manchester, N. H.—Divide 725 by the number of feet of fall minus 1, and the quotient will be the number of cubic feet per second in that fall. For 20 feet fall this equals 38.1 cu. ft., equal to 86.4 H. P. maximum.

Cohoes, N. Y.—"Mill-power" equivalent to the power given by 6 cu. ft. per second, when the fall is 20 feet. Equal to 13.6 H. P., maximum.

Passaic, N. J.—Mill-power: The right to draw 8½ cu. ft. of water per sec., fall of 22 feet, equal to 21.2 horse-power. Maximum rental $700 per year for each mill-power = $33.00 per H. P.

The horse-power maximum above given is that due theoretically to the weight of water and the height of the fall, assuming the water-wheel to have perfect efficiency. It should be multiplied by the efficiency of the wheel, say 75% for good turbines, to obtain the H. P. delivered by the wheel.

Value of a Water-power.—In estimating the value of a water-power, especially where such value is used as testimony for a plaintiff whose water-power has been diminished or confiscated, it is a common custom for the person making such estimate to say that the value is represented by a sum of money which, when put at interest, would maintain a steam-plant of the same power in the same place.

Mr. Charles T. Main (Trans. A. S. M. E. xiii. 140) points out that this system of estimating is erroneous; that the value of a power depends upon a great number of conditions, such as location, quantity of water, fall or head, uniformity of flow, conditions which fix the expense of dams, canals, foundations of buildings, freight charges for fuel, raw materials and finished product, etc. He gives an estimate of relative cost of steam and water-power for a 500 H. P. plant from which the following is condensed:

The amount of heat required per H. P. varies with different kinds of business, but in an average plain cotton-mill, the steam required for heating and slashing is equivalent to about 25% of steam exhausted from the high-pressure cylinder of a compound engine of the power required to run that mill, the steam to be taken from the receiver.

The coal consumption per H. P. per hour for a compound engine is taken at 1¾ lbs. per hour, when no steam is taken from the receiver for heating purposes. The gross consumption when 25% is taken from the receiver is about 2.06 lbs.

75% of the steam is used as in a compound engine at	1.75 lbs.	=	1.31 lbs.
25% " " " " high-pressure "	3.00 lbs.	=	.75 "
			2.06 "

The running expenses per H. P. per year are as follows:

2.06 lbs. coal per hour = 21.115 lbs. for 10¼ hours or one day = 6503.42 lbs. for 308 days, which, at $3.00 per long ton =	$8 71
Attendance of boilers, one man @ $2.00, and one man @ $1.25 =	2 00
" " engine, " " " $3.50.	2 16
Oil, waste, and supplies.	80
The cost of such a steam-plant in New England and vicinity of 500 H. P. is about $65 per H. P. Taking the fixed expenses as 4% on engine, 5% on boilers, and 2% on other portions, repairs at 2%, interest at 5%, taxes at 1½% on ¾ cost, an insurance at ½% on exposed portion, the total average per cent is about 12½%, or $65 × .12½ =	8 13
Gross cost of power and low-pressure steam per H. P.	$21 80

Comparing this with water-power, Mr. Main says: "At Lawrence the cost of dam and canals was about $650,000, or $65 per H. P. The cost per H. P. of wheel-plant from canal to river is about $45 per H. P. of plant, or about $65 per H. P. used, the additional $20 being caused by making the plant large enough to compensate for fluctuation of power due to rise and fall of river. The total cost per H. P. of developed plant is then about $130 per H. P. Placing the depreciation on the whole plant at 2%, repairs at 1%, interest at 5%, taxes and insurance at 1%, or a total of 9%, gives:

Fixed expenses per H. P. $130 × .09 =	$11 70
Running " " " (Estimated)	2 00
	$13 70

"To this has to be added the amount of steam required for heating purposes, said to be about 25% of the total amount used, but in winter months the consumption is at least 37½%. It is therefore necessary to have a boiler plant of about 37½% of the size of the one considered with the steam-plant, costing about $20 × .375 = $7.50 per H. P. of total power used. The expense of running this boiler-plant is, per H. P. of the the total plant per year:

Fixed expenses 12½% on $7.50	$0.94
Coal	3.26
Labor	1.23
Total	$5.43

Making a total cost per year for water-power, with the auxiliary boiler plant $13.70 + $5.43 = $19.13 which deducted from $21.80 make a difference in favor of water-power of $2.67, or for 10,000 H. P. a saving of $26,700 per year.

"It is fair to say," says Mr. Main, "that the value of this constant power is a sum of money which when put at interest will produce the saving; or if 6% is a fair interest to receive on money thus invested the value would be $26.700 ÷ .06 = $445,000."

Mr. Main makes the following general statements as to the value of a water-power: "The value of an undeveloped variable power is usually nothing if its variation is great, unless it is to be supplemented by a steam-plant. It is of value then only when the cost per horse-power for the double-plant is less than the cost of steam-power under the same conditions as mentioned for a permanent power, and its value can be represented in the same manner as the value of a permanent power has been represented.

"The value of a developed power is as follows: If the power can be run cheaper than steam, the value is that of the power, plus the cost of plant, less depreciation. If it cannot be run as cheaply as steam, considering its cost, etc., the value of the power itself is nothing, but the value of the plant is such as could be paid for it new, which would bring the total cost of running down to the cost of steam-power, less depreciation."

Mr. Samuel Webber, *Iron Age*, Feb. and March, 1893, writes a series of articles showing the development of American turbine wheels, and incidentally criticises the statements of Mr. Main and others who have made comparisons of costs of steam and of water-power unfavorable to the latter. He says: "They have based their calculations on the cost of steam, on large compound engines of 1000 or more H. P. and 120 pounds pressure of steam in their boilers, and by careful 10-hour trials succeeded in figuring down steam to a cost of about $20 per H. P., ignoring the well-known fact that its average cost in practical use, except near the coal mines, is from $40 to $50. In many instances dams, canals, and modern turbines can be all completed for a cost of $100 per H. P.; and the interest on that, and the cost of attendance and oil, will bring water-power up to but about $10 or $12 per annum; and with a man competent to attend the dynamo in attendance, it can probably be safely estimated at not over $15 per H. P."

TURBINE WHEELS.

Proportions of Turbines.—Prof. De Volson Wood discusses at length the theory of turbines in his paper on Hydraulic Reaction Motors, Trans. A. S. M. E. xiv. 266. His principal deductions which have an immediate bearing upon practice are condensed in the following:

Notation.

Q = volume of water passing through the wheel per second,
h_1 = head in the supply chamber above the entrance to the buckets,
h_2 = head in the tail-race above the exit from the buckets,
z_1 = fall in passing through the buckets,
$H = h_1 + z_1 - h_2$, the effective head,
μ_1 = coefficient of resistance along the guides,
μ_2 = coefficient of resistance along the buckets,
r_1 = radius of the initial rim,
r_2 = radius of the terminal rim,
V = velocity of the water issuing from supply chamber,
v_1 = initial velocity of the water in the bucket in reference to the bucket,
v_2 = terminal velocity in the bucket,
ω = angular velocity of the wheel,
α = terminal angle between the guide and initial rim = CAB, Fig. 132,
γ_1 = angle between the initial element of bucket and initial rim = EAD,
γ_2 = GFI, the angle between the terminal rim and terminal element of the bucket,
$a = eb$, Fig. 133 = the arc subtending one gate opening,

a_1 = the arc subtending one bucket at entrance. (In practice a_1 is larger than a.)

$a_2 = gh$, the arc subtending one bucket at exit,

$K = bf$, normal section of passage, it being assumed that the passages and buckets are very narrow,

$k_1 = bd$, initial normal section of bucket,

$k_2 = gi$, terminal normal section,

ωr_1 = velocity of initial rim,

ωr_2 = velocity of terminal rim,

$\theta = HFI$, angle between the terminal rim and actual direction of the water at exit,

Y = depth of K, y_1 of a_1, and y_2 of K_2, then

$K = Ya \sin \alpha$; $K_1 = y_1 a_1 \sin \gamma_1$; $K_2 = y_2 a_2 \sin \gamma_2$.

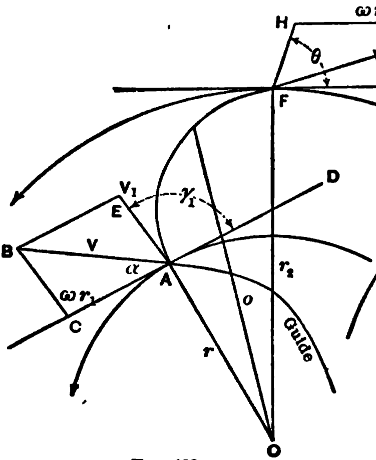

FIG. 182.

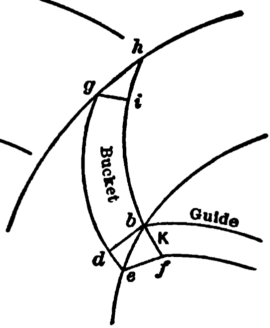

FIG. 183.

Three simple systems are recognized, $r_1 < r_2$, called outward flow; $r_1 > r_2$, called inward flow; $r_1 = r_2$, called parallel flow. The first and second may be combined with the third, making a mixed system.

Value of γ_2 (*the quitting angle*).—The efficiency is increased as γ_2 decreases, and is greatest for $\gamma_2 = 0$. Hence, theoretically, the terminal element of the bucket should be tangent to the quitting rim for best efficiency. This, however, for the discharge of a finite quantity of water, would require an infinite depth of bucket. In practice, therefore, this angle must have a finite value. The larger the diameter of the terminal rim the smaller may be this angle for a given depth of wheel and given quantity of water discharged. In practice γ_2 is from 10° to 20°.

In a wheel in which all the elements except γ_2 are fixed, the velocity of the wheel for best effect must increase as the quitting angle of the bucket decreases.

Values of $\alpha + \gamma_1$ must be less than 180°, but the best relation cannot be determined by analysis. However, since the water should be deflected from its course as much as possible from its entering to its leaving the wheel, the angle α for this reason should be as small as practicable.

In practice, α cannot be zero, and is made from 20° to 30°.

The value $r_1 = 1.4 r_2$ makes the width of the crown for internal flow about the same as for $r_1 = r_2 \sqrt{½}$ for outward flow, being approximately 0.3 of the external radius.

Values of μ_1 *and* μ_2.—The frictional resistances depend upon the construction of the wheel as to smoothness of the surfaces, sharpness of the angles, regularity of the curved parts, and also upon the speed it is run. These values cannot be definitely assigned beforehand, but Weisbach gives for good conditions $\mu_1 = \mu_2 = 0.05$ to 0.10.

They are not necessarily equal, and μ_1 may be from 0.05 to 0.075, and μ_2 from 0.06 to 0.10 or even larger.

Values of γ_1 must be less than $180° - \alpha$.

To be on the safe side, γ_1 may be 20 or 30 degrees less than $180° - 2\alpha$, giving

$$\gamma_1 = 180° - 2\alpha - 25 \quad \text{(say)} \quad = 155 - 2\alpha.$$

Then if $\alpha = 30°$, $\gamma_1 = 95°$. Some designers make γ_1 90°; others more, and still others less, than that amount. Weisbach suggests that it be less, so that the bucket will be shorter and friction less. This reasoning appears to be correct for the inflow wheel, but not for the outflow wheel. In the Tremont turbines, described in the Lowell Hydraulic Experiments, this angle is 90°, the angle α 20°, and γ_2 10°, which proportions insured a positive pressure in the wheel. Fourneyron made $\gamma_1 = 90°$, and α from 30° to 33°, which values made the initial pressure in the wheel near zero.

Form of Bucket —The form of the bucket cannot be determined analytically. From the initial and terminal directions and the volume of the water flowing through the wheel, the area of the normal sections may be found.

The normal section of the buckets will be:

$$K = \frac{Q}{V}; \quad k_1 = \frac{Q}{v_1}; \quad k_2 = \frac{Q}{v_2}.$$

The depths of those sections will be:

$$Y = \frac{K}{a \sin \alpha}; \quad y_1 = \frac{k_1}{a_1 \sin \gamma_1}; \quad y_2 = \frac{k_2}{a_2 \sin \gamma_2}.$$

The changes of curvature and section must be gradual, and the general form regular, so that eddies and whirls shall not be formed. For the same reason the wheel must be run with the correct velocity to secure the best effect. In practice the buckets are made of two or three arcs of circles, mutually tangential.

The Value of ω.—So far as analysis indicates, the wheel may run at any speed; but in order that the stream shall flow smoothly from the supply chamber into the bucket, the velocity V should be properly regulated.

If $\mu_1 = \mu_2 = 0.10$, $r_2 \div r_1 = 1.40$, $\alpha = 25°$, $\gamma_1 = 90°$, $\gamma_2 = 12°$, the velocity of the *initial* rim for outward flow will be for maximum efficiency 0.614 of the velocity due to the head, or $\omega r_1 = 0.614 \sqrt{2gH}$.

The velocity due to the head would be $\sqrt{2gH} = 1.414 \sqrt{gH}$.

For an inflow wheel for the case in which $r_1^2 = 2r_2^2$, and the other dimensions as given above, $\omega r_1 = 0.682 \sqrt{2gH}$.

The highest efficiency of the Tremont turbine, found experimentally, was 0.79375, and the corresponding velocity, 0.62645 of that due to the head, and for all velocities above and below this value the efficiency was less.

In the Tremont wheel $\alpha = 20°$ instead of 25°, and $\gamma_2 = 10°$ instead of 12°. These would make the theoretical efficiency and velocity of the wheel somewhat greater. Experiment showed that the velocity might be considerably larger or smaller than this amount without much diminution of the efficiency.

It was found that if the velocity of the initial (or interior) rim was not less than 44% nor more than 75% of that due to the fall, the efficiency was 75% or more. This wheel was allowed to run freely without any brake except its own friction, and the velocity of the initial rim was observed to be $1.335 \sqrt{2gH}$, half of which is $0.6675 \sqrt{2gH}$, which is not far from the velocity giving maximum effect; that is to say, when the gate is fully raised the coefficient of effect is a maximum when the wheel is moving with about half its maximum velocity.

Number of Buckets.—Successful wheels have been made in which the distance between the buckets was as small as 0.75 of an inch, and others as much as 2.75 inches. Turbines at the Centennial Exposition had buckets from 4½ inches to 9 inches from centre to centre. If too large they will not work properly. Neither should they be too deep. Horizontal partitions are sometimes introduced. These secure more efficient working in case the gates are only partly opened. The form and number of buckets for commercial purposes are chiefly the result of experience.

Ratio of Radii.—Theory does not limit the dimensions of the wheel. In practice,

for outward flow, $r_2 \div r_1$ is from 1.25 to 1.50;
for inward flow, $r_2 \div r_1$ is from 0.66 to 0.80.

It appears that the inflow-wheel has a higher efficiency than the outward-flow wheel. The inflow-wheel also runs somewhat slower for best effect. The centrifugal force in the outward-flow wheel tends to force the water outward faster than it would otherwise flow; while in the inward-flow wheel it has the contrary effect, acting as it does in opposition to the velocity in the buckets.

It also appears that the efficiency of the outward-flow wheel increases slightly as the width of the crown is less and the velocity for maximum efficiency is slower; while for the inflow-wheel the efficiency slightly increases for increased width of crown, and the velocity of the outer rim at the same time also increases.

Efficiency.—The exact value of the efficiency for a particular wheel must be found by experiment.

It seems hardly possible for the effective efficiency to equal, much less exceed, 80%, and all claims of 90 or more per cent for these motors should be discarded as improbable. A turbine yielding from 75% to 80% is extremely good. Experiments with higher efficiencies have been reported.

The celebrated Tremont turbine gave 79¼% without the "diffuser," which might have added some 2%. A Jonval turbine (parallel flow) was reported as yielding 0.75 to 0.90, but Morin suggested corrections reducing it to 0.63 to 0.71. Weisbach gives the results of many experiments, in which the efficiency ranged from 50% to 84%. Numerous experiments give $E = 0.60$ to 0.65. The efficiency, considering only the energy imparted to the wheel, will exceed by several per cent the efficiency of the wheel, for the latter will include the friction of the support and leakage at the joint between the sluice and wheel, which are not included in the former; also as a plant the resistances and losses in the supply-chamber are to be still further deducted.

The Crowns.—The crowns may be plane annular disks, or conical, or curved. If the partitions forming the buckets be so thin that they may be discarded, the law of radial flow will be determined by the form of the crowns. If the crowns be plane, the radial flow (or radial component) will diminish, for the outward flow-wheel, as the distance from the axis increases—the buckets being full—for the angular space will be greater.

Prof. Wood deduces from the formulæ in his paper the tables on page 595.

It appears from these tables: 1. That the terminal angle, α, has frequently been made too large in practice for the best efficiency.

2. That the terminal angle, α, of the guide should be for the inflow less than 10° for the wheels here considered, but when the initial angle of the bucket is 90°, and the terminal angle of the guide is 5° 28′, the gain of efficiency is not 2% greater than when the latter is 25°.

3. That the initial angle of the bucket should exceed 90° for best effect for outflow-wheels.

4. That with the initial angle between 60° and 120° for best effect on inflow wheels the efficiency varies scarcely 1%.

5. In the outflow-wheel, column (9) shows that for the outflow for best effect the direction of the quitting water in reference to the earth should be nearly radial (from 76° to 97°), but for the inflow wheel the water is thrown forward in quitting. This shows that the velocity of the rim should somewhat exceed the relative final velocity backward in the bucket, as shown in columns (4) and (5).

6. In these tables the velocities given are in terms of $\sqrt{2gh}$, and the coefficients of this expression will be the part of the head which would produce that velocity if the water issued freely. There is only one case, column (5), where the coefficient exceeds unity, and the excess is so small it may be discarded; and it may be said that in a properly proportioned turbine with the conditions here given none of the velocities will equal that due to the head in the supply-chamber when running at best effect.

7. The inflow turbine presents the best conditions for construction for producing a given effect, the only apparent disadvantage being an increased first cost due to an increased depth, or an increased diameter for producing a given amount of work. The larger efficiency should, however, more than neutralize the increased first cost.

Outward-flow Turbine.

$r_1 = r_2 \sqrt{\tfrac{1}{2}}$. $\mu_1 = \mu_2 = 0.10$. $\gamma_2 = 12°$. Parallel Crowns. $k_1 v_1 = k_2 v_2 = KV = Q = 1$.

Initial Angle. γ_1	Efficiency. E	Velocity Outer Rim. $r_2\omega'$	Velocity Inner Rim. $r_1\omega' = \sqrt{\tfrac{1}{2}}\, r_2\omega'$	Relative Velocity of Exit. v_2	Relative Velocity of Entrance. v_1	Velocity of Exit from supply-Chamber. V	Terminal Angle of Guide. α	Direction of quitting Water. θ	Head Equivalent of Energy in quitting Water. $\frac{w^2}{2g}$	$k_2 \sqrt{gH}$
1	2	3	4	5	6	7	8	9	10	11
60°	0.804	$0.972 \sqrt{2gH}$	$0.687 \sqrt{2gH}$	$1.048 \sqrt{2gH}$	$0.856 \sqrt{2gH}$	$0.595 \sqrt{2gH}$	31° 17′	76°	$0.051H$	0.67
90°	0.828	$0.874 \sqrt{2gH}$	$0.619 \sqrt{2gH}$	$0.931 \sqrt{2gH}$	$0.274 \sqrt{2gH}$	$0.676 \sqrt{2gH}$	23° 56′	79°	$0.039H$	0.76
120°	0.839	$0.798 \sqrt{2gH}$	$0.565 \sqrt{2gH}$	$0.843 \sqrt{2gH}$	$0.286 \sqrt{2gH}$	$0.749 \sqrt{2gH}$	19° 5′	82°	$0.031H$	0.84
150°	0.921	$0.709 \sqrt{2gH}$	$0.501 \sqrt{2gH}$	$0.707 \sqrt{2gH}$	$0.416 \sqrt{2gH}$	$0.886 \sqrt{2gH}$	13° 31′	97°	$0.022H$	1.00

Inward-flow Turbine.

$r_1 = \sqrt{2}\, r_2$. $\mu_1 = \mu_2 = 0.10$. $\gamma_2 = 12°$. Parallel Crowns. $k_1 v_1 = k_2 v_2 = KV = Q = 1$.

γ_1	E.	Velocity Outer Rim. $r_1\omega'$	Velocity Inner Rim. $r_2\omega'$	v_2	v_1	V	α	θ	$\frac{w^2}{2g}$	$k_2 \sqrt{gH}$
60°	0.920	$0.709 \sqrt{2gH}$	$0.501 \sqrt{2gH}$	$0.476 \sqrt{2gH}$	$0.089 \sqrt{2gH}$	$0.672 \sqrt{2gH}$	7° 0′	110°	$0.010H$	1.48
90°	0.920	$0.688 \sqrt{2gH}$	$0.487 \sqrt{2gH}$	$0.470 \sqrt{2gH}$	$0.069 \sqrt{2gH}$	$0.691 \sqrt{2gH}$	5° 28′	106°	$0.010H$	1.50
120°	0.919	$0.668 \sqrt{2gH}$	$0.473 \sqrt{2gH}$	$0.456 \sqrt{2gH}$	$0.077 \sqrt{2gH}$	$0.709 \sqrt{2gH}$	4° 46′	105°	$0.010H$	1.55
150°	0.918	$0.634 \sqrt{2gH}$	$0.448 \sqrt{2gH}$	$0.429 \sqrt{2gH}$	$0.126 \sqrt{2gH}$	$0.743 \sqrt{2gH}$	3° 08′	107°	$0.009H$	1.65

Tests of Turbines.—Emerson says that in testing turbines it is a rare thing to find two of the same size which can be made to do their best at the same speed. The best speed of one of the leading wheels is invariably wide from the tabled rate. It was found that a 54-in. Leffel wheel under 12 ft. head gave much better results at 78 revolutions per minute than at 60.

Overshot wheels have been known to give 75% efficiency, but the average performance is not over 60%.

A fair average for a good turbine wheel may be taken at 75%. In tests of 18 wheels made at the Philadelphia Water-works in 1859 and 1860, one wheel gave less than 50% efficiency, two between 50% and 60%, six between 60% and 70%, seven between 71% and 77%, two 82%, and one 87.77%. (Emerson.)

Tests of Turbine Wheels at the Centennial Exhibition, 1876. (From a paper by R. H. Thurston on The Systematic Testing of Turbine Wheels in the United States, Trans. A. S. M. E., viii. 359.)—In 1876 the judges at the International Exhibition conducted a series of trials of turbines. Many of the wheels offered for tests were found to be more or less defective in fitting and workmanship. The following is a statement of the results of all turbines entered which gave an efficiency of over 75%. Seven other wheels were tested, giving results between 65% and 75%.

Maker's Name, or Name the Wheel is Known By.	Per Cent at Full Gate or Discharge.	Per Cent at about 9/10 of Full Discharge.	Per Cent at about 7/8 of Full Discharge.	Per Cent at about 3/4 of Full Discharge.	Per Cent at about 5/8 of Full Discharge.	Per Cent at about 1/2 of Full Discharge.	Per Cent at about 4/10 of Full Discharge.
Risdon	87.68	......	86.20	82.41	......	75.35	......
National	88.79	......	......	70.79	......	......	......
Geyelin (single)	83.30	......	......	......	......	......	......
Thos. Tait	82.13	......	......	70.40	66.35	......	55.00
Goldie & McCullough	81.21	......	71.01	55.90	......	......	......
Rodney Hunt Mach. Co.	78.70	71.66	......	68.60	51.03	......	......
Tyler Wheel	79.59	......	81.24	79.92	67.23	69.59	......
Geyelin (duplex)	77.57	......	......	......	......	......	......
Knowlton & Dolan	77.43	74.25	......	......	62.75	......	......
E. T. Cope & Sons	76.94	......	69.92	......	......	......	......
Barber & Harris	76.16	73.33	......	......	70.87	71.74	......
York Manufacturing Co.	75.70	......	67.08	67.57	62.06	......	......
W. F. Mosser & Co.	75.15	74.89	71.90	70.52	......	66.04	......

The limits of error of the tests, says Prof. Thurston, were very uncertain; they are undoubtedly considerable as compared with the later work done in the permanent flume at Holyoke—possibly as much as 4% or 5%.

Experiments with "draught-tubes," or "suction-tubes," which were actually "diffusers" in their effect, so far as Prof. Thurston has analyzed them, indicate the loss by friction which should be anticipated in such cases, this loss decreasing as the tube increased in size, and increasing as its diameter approached that of the wheel—the minimum diameter tried. It was sometimes found very difficult to free the tube from air completely, and next to impossible, during the interval, to control the speed with the brake. Several trials were often necessary before the power due to the full head could be obtained. The loss of power by gearing and by belting was variable with the proportions and arrangement of the gears and pulleys, length of belt, etc., but averaged not far from 30% for a single pair of bevel-gears, uncut and dry, but smooth for such gearing, and but 10% for the same gears, well lubricated, after they had been a short time in operation. The amount of power transmitted was, however, small, and these figures are probably much higher than those representing ordinary practice. Introducing a second pair—spur-gears—the best figures were but little changed, although the difference between the case in which the larger gear was the driver, and the case in which the small wheel was the driver, was perceivable, and was in favor of the former arrangement. A single straight belt gave a loss of but 2% or 3%, a crossed belt 6% to 8%, when transmitting 14 horse-power with maximum tightness and transmitting power. A "quarter turn" wasted about 10% as a maximum, and a "quarter twist" about 5%.

Dimensions of Turbines.—For dimensions, power, etc., of standard makes of turbines consult the catalogues of different manufacturers. The wheels of different makers vary greatly in their proportions for any given capacity.

The Pelton Water-wheel.—Mr. Ross E. Browne (*Eng'g News*, Feb. 20, 1892) thus outlines the principles upon which this water-wheel is constructed:

The function of a water-wheel, operated by a jet of water escaping from a nozzle, is to convert the energy of the jet, due to its velocity, into useful work. In order to utilize this energy fully the wheel-bucket, after catching the jet, must bring it to rest before discharging it, without inducing turbulence or agitation of the particles.

This cannot be fully effected, and unavoidable difficulties necessitate the loss of a portion of the energy. The principal losses occur as follows: First, in sharp or angular diversion of the jet in entering, or in its course through the bucket, causing impact, or the conversion of a portion of the energy into heat instead of useful work. Second, in the so-called frictional resistance offered to the motion of the water by the wetted surfaces of the buckets, causing also the conversion of a portion of the energy into heat instead of useful work. Third, in the velocity of the water, as it leaves the bucket, representing energy which has not been converted into work.

Hence, in seeking a high efficiency: 1. The bucket-surface at the entrance should be approximately parallel to the relative course of the jet, and the bucket should be curved in such a manner as to avoid sharp angular deflection of the stream. If, for example, a jet strikes a surface at an angle and is sharply deflected, a portion of the water is backed, the smoothness of the stream is disturbed, and there results considerable loss by impact and otherwise. The entrance and deflection in the Pelton bucket are such as to avoid these losses in the main. (See Fig. 136.)

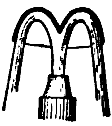

Fig. 134.

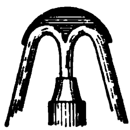

Fig. 135.

2. The number of buckets should be small, and the path of the jet in the bucket short; in other words, the total wetted surface should be small, as the loss by friction will be proportional to this.

3. The discharge end of the bucket should be as nearly tangential to the wheel periphery as compatible with the clearance of the bucket which follows; and great differences of velocity in the parts of the escaping water should be avoided. In order to bring the water to rest at the discharge end of the bucket, it is shown, mathematically, that the velocity of the bucket should be one half the velocity of the jet.

A bucket, such as shown in Fig. 135, will cause the heaping of more or less dead or turbulent water at the point indicated by dark shading. This dead water is subsequently thrown from the wheel with considerable velocity, and represents a large loss of energy. The introduction of the wedge in the Pelton bucket (see Fig. 134) is an efficient means of avoiding this loss.

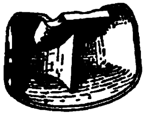

Fig. 136.

A wheel of the form of the Pelton conforms closely in construction to each of these requirements.

In a test made by the proprietors of the Idaho mine, near Grass Valley, Cal., the dimensions and results were as follows: Main supply-pipe, 22 in. diameter, 6900 ft. long, with a head of 386½ feet above centre of nozzle. The loss by friction in the pipe was 1.8 ft., reducing the effective head to 384.7 ft. The Pelton wheel used in the test was 6 ft. in diameter and the nozzle was 1.89 in. diameter. The work done was measured by a Prony brake, and the mean of 13 tests showed a useful effect of 87.3%.

The Pelton wheel is also used as a motor for small powers. A test by M. E. Cooley of a 12-inch wheel, with a ⅜-inch nozzle, under 100 lbs. pressure, gave 1.9 horse-power. The theoretical discharge was .0935 cubic feet per second, and the theoretical horse-power 2.45; the efficiency being 80 per cent. Two other styles of water-motor tested at the same time each gave efficiencies of 55 per cent.

Pelton Water-wheel Tables. (Abridged.)

The smaller figures under those denoting the various heads give the spouting velocity of the water in feet per minute. The cubic-feet measurement is also based on the flow per minute.

Head in ft.	Size of Wheels.	6 in. No. 1	12 in. No. 2	18 in. No. 3	18 in. No. 4	24 in. No. 5	3 ft.	4 ft.	5 ft.	6 ft.
20	Horse-power.	.05	.12	.20	.37	.66	1.50	2.64	4.18	6.00
	Cubic feet....	1.67	3.91	6.62	11.72	20.83	46.93	83.32	130.36	187.72
2151.97	Revolutions..	684	342	228	228	171	114	85	70	57
30	Horse-power.	.10	.23	.38	.69	1.22	2.76	4.88	7.69	11.04
	Cubic feet....	2.05	4.79	8.11	14.36	25.51	57.44	102.04	159.66	229.76
2635.62	Revolutions..	837	418	279	279	209	139	104	83	69
40	Horse-power.	.15	.35	.59	1.06	1.89	4.24	7.53	11.85	16.96
	Cubic feet....	2.37	5.53	9.37	16.59	29.46	66.36	107.84	184.36	265.44
3043.39	Revolutions..	969	484	323	323	242	161	121	96	80
50	Horse-power.	.21	.49	.84	1.49	2.65	5.96	10.60	16.63	23.93
	Cubic feet....	2.64	6.18	10.47	18.54	32.93	74.17	131.72	206.13	296.70
3402.61	Revolutions..	1083	541	361	361	270	180	135	108	90
60	Horse-power.	.28	.65	1.10	1.96	3.48	7.84	13.94	21.77	31.36
	Cubic feet....	2.90	6.77	11.47	20.31	36.08	81.25	144.32	225.80	325.00
3727.37	Revolutions..	1185	592	395	395	296	197	148	118	98
70	Horse-power.	.35	.82	1.39	2.47	4.39	9.88	17.58	27.51	39.52
	Cubic feet....	3.13	7.31	12.39	21.94	38.97	87.76	155.88	243.89	351.04
4026.00	Revolutions..	1281	640	427	427	320	213	160	130	106
80	Horse-power.	.43	1.00	1.70	3.01	5.36	12.04	21.44	33.54	48.16
	Cubic feet....	3.35	7.82	13.25	23.46	41.66	93.84	166.64	260.78	375.36
4303.99	Revolutions..	1368	684	456	456	342	228	171	137	114
90	Horse-power.	.51	1.20	2.03	3.60	6.39	14.40	25.59	40.04	57.60
	Cubic feet....	3.55	8.29	14.05	24.88	44.19	99.52	176.75	276.55	398.08
4565.04	Revolutions..	1452	726	484	484	363	242	181	145	121
100	Horse-power.	.60	1.40	2.32	4.21	7.49	16.84	29.93	46.85	67.36
	Cubic feet....	3.74	8.74	14.81	26.22	46.58	104.88	186.32	291.51	419.52
4812.00	Revolutions..	1530	765	510	510	382	255	191	152	127
120	Horse-power.	.79	1.84	3.12	5.54	9.85	22.18	39.41	61.66	88.75
	Cubic feet....	4.10	9.57	16.21	28.72	51.02	114.91	204.10	319.33	459.64
5271.30	Revolutions..	1677	838	559	559	419	279	209	167	139
140	Horse-power.	.99	2.33	3.94	6.99	12.41	27.96	49.64	77.71	111.85
	Cubic feet....	4.43	10.34	17.53	31.03	55.11	124.12	220.44	344.92	496.48
5693.65	Revolutions..	1812	906	604	604	453	302	226	181	151
160	Horse-power.	1.22	2.84	4.82	8.54	15.17	34.16	60.68	94.94	136.65
	Cubic feet....	4.73	11.05	18.74	33.17	58.92	132.68	235.68	368.73	530.75
6086.74	Revolutions..	1938	969	646	646	484	323	242	193	161
180	Horse power.	1.45	3.39	5.75	10.19	18.10	40.77	72.41	113.30	163.03
	Cubic feet.. .	5.02	11.72	19.87	35.18	62.49	140.74	249.97	391.10	562.96
6455.97	Revolutions..	2049	1024	683	683	513	342	256	206	171
200	Horse-power.	1.70	3.97	6.74	11.93	21.20	47.75	84.81	132.70	191.00
	Cubic feet.. .	5.29	12.36	20.94	37.08	65.87	148.35	263.49	412.25	593.40
6805.17	Revolutions.	2160	1080	720	720	540	360	270	216	180
250	Horse-power.	2.38	5.56	9.42	16.68	29.63	66.74	118.54	185.47	266.96
	Cubic feet....	5.92	13.82	23.42	41.46	73.64	165.86	294.59	460.91	663.45
7608.44	Revolutions..	2418	1209	806	806	605	403	302	241	202

Pelton Water-wheel Tables.—*Continued.*

Head in ft.	Size of Wheels.	6 in. No. 1	12 in. No. 2	18 in. No. 3	18 in. No. 4	24 in. No. 5	3 ft.	4 ft.	5 ft.	6 ft.
300	Horse-pow'r	3.18	7.31	12.38	21.93	38.95	87.78	155.83	243.82	350.94
	Cubic feet...	6.48	15.13	25.66	45.42	80.67	181.69	322.71	504.91	726.76
8334.62	Revolutions	2652	1326	884	884	663	442	331	265	221
350	Horse-pow'r	3.94	9.21	15.61	27.64	49.09	110.56	196.88	307.25	442.27
	Cubic feet...	7.00	16.35	27.71	49.06	87.14	196.25	348.57	545.36	785.00
9002.43	Revolutions	2865	1432	955	955	716	477	358	286	238
400	Horse-pow'r	4.82	11.25	19.0	33.77	59.98	135.08	239.94	375.40	540.35
	Cubic feet...	7.49	17.48	29 63	52.45	93.16	209.80	372.64	583.02	839.20
9624.00	Revolutions	3063	1531	1021	1021	765	510	382	306	255
450	Horse-pow'r	5.75	13.48	22.76	40.29	71.57	161.19	286.31	447.95	644.78
	Cubic feet...	7.94	18.54	31.42	55.63	98.81	222.52	395.24	618.38	890.11
10207.79	Revolutions	3249	1624	1083	1083	812	541	406	324	270
500	Horse-pow'r	6.74	15.73	26.66	47.20	83.83	188.80	335.84	524.66	755.20
	Cubic feet...	8.37	19.54	33.12	58.64	104.15	234.56	416.62	651.83	938.25
10759.96	Revolutions	3426	1713	1142	1142	856	571	428	342	285
600	Horse-pow'r	....	.....	.....	62.04	110.19	248.16	440.77	689.63	992.65
	Cubic feet...	....	.....	.....	64.24	114.09	256.95	456.33	714.05	1027.80
11786.94	Revolutions	..	....	.....	1251	938	625	469	375	312
650	Horse-pow'r	....	.....	....	69.95	124.25	279.82	497.01	777.62	1119.29
	Cubic feet...	....	.....	.....	66.86	118.75	267.44	475.02	743.21	1069.77
12268.24	Revolutions	....	.....	....	1302	976	651	488	390	325
700	Horse-pow'r	....	.....	.....	78.18	138.86	312.78	555.46	869.06	1250.92
	Cubic feet...	...	.....	.....	69.38	123.23	277.54	492.95	771.26	1110.16
12731.34	Revolutions	....	.....	....	1351	1013	675	506	405	337
750	Horse-pow'r	....	....	.....	86.70	154.00	346.83	616.03	963.82	1387.84
	Cubic feet...	...	.....	.....	71.82	127.56	287.28	510.25	798.33	1149.13
13178.19	Revolutions	....	....	.....	1399	1049	699	524	419	349
800	Horse-pow'r	....	....	....	95.52	169.66	382.09	678.66	1061.81	1528.86
	Cubic feet...	...	.....	.....	74.17	131.74	296.70	526.99	824.51	1186.81
13610.40	Revolutions	...	.....	.....	1444	1083	722	542	433	361
900	Horse-pow'r	....	....	.....	113.98	202.45	455.94	809.82	1267.02	1823.76
	Cubic feet...	....	.....	.....	78.67	139.74	314.70	558.96	874.58	1258.81
14436.00	Revolutions	....	....	.....	1532	1149	766	574	459	383
1000	Horse-pow'r	....	.....	.....	133.50	237.12	534.01	948.46	1483.97	2136.04
	Cubic feet...	....	.....	.....	82.93	147.30	331.72	589.19	921.88	1326.91
15216 89	Revolutions	....	.....	.....	1615	1210	807	605	484	403

THE POWER OF OCEAN WAVES.

Albert W. Stahl, U. S. N. (Trans. A. S. M. E., xiii. 438), gives the following formulæ and table, based upon a theoretical discussion of wave motion:

The total energy of one whole wave-length of a wave H feet high, L feet long, and one foot in breadth, the length being the distance between successive crests, and the height the vertical distance between the crest and the trough, is $E = 8LH^2\left(1 - 4.935\,\frac{H^2}{L^2}\right)$ foot-pounds.

The time required for each wave to travel through a distance equal to its own length is $P = \sqrt{\frac{L}{5.123}}$ seconds, and the number of waves passing any given point in one minute is $N = \frac{60}{P} = 60\sqrt{\frac{5.123}{L}}$. Hence the total energy of an indefinite series of such waves, expressed in horse-power per foot of breadth, is

$$\frac{E \times N}{33000} = .0329H^2L\left(1 - 4.935\frac{H^2}{L^2}\right).$$

By substituting various values for $H \div L$, within the limits of such values actually occurring in nature, we obtain the following table of

Total Energy of Deep-sea Waves in Terms of Horse-power per Foot of Breadth.

Ratio of Length of Waves to Height of Waves.	Length of Waves in Feet.							
	25	50	75	100	150	200	300	400
50	.04	.23	.64	1.31	3.62	7.43	20.46	42.01
40	.06	.36	1.00	2.05	5.65	11.59	31.95	65.58
30	.12	.64	1.77	3.64	10.02	20.57	56.70	116.38
20	.25	1.44	3.96	8.13	21.79	45.98	12[illegible].70	260.08
15	.42	2.83	6.97	14.21	39.48	80.94	223.06	457.89
10	.98	5.53	15.24	31.29	86.22	177.00	487.75	1001.25
5	3.30	18.68	51.48	105.68	291.20	597.78	1647.[illegible]1	3381.60

The figures are correct for trochoidal deep-sea waves only, but they give a close approximation for any nearly regular series of waves in deep water and a fair approximation for waves in shallow water.

The question of the practical utilization of the energy which exists in ocean waves divides itself into several parts:

1. The various motions of the water which may be utilized for power purposes.

2. The wave motor proper. That is, the portion of the apparatus in direct contact with the water, and receiving and transmitting the energy thereof; together with the mechanism for transmitting this energy to the machinery for utilizing the same.

3. Regulating devices, for obtaining a uniform motion from the irregular and more or less spasmodic action of the waves, as well as for adjusting the apparatus to the state of the tide and condition of the sea.

4. Storage arrangements for insuring a continuous and uniform output of power during a calm, or when the waves are comparatively small.

The motions that may be utilized for power purposes are the following: 1. Vertical rise and fall of particles at and near the surface. 2. Horizontal to-and-fro motion of particles at and near the surface. 3. Varying slope of surface of wave. 4. Impetus of waves rolling up the beach in the form of breakers. 5. Motion of distorted verticals. All of these motions, except the last one mentioned, have at various times been proposed to be utilized for power purposes; and the last is proposed to be used in apparatus described by Mr. Stahl.

The motion of distorted verticals is thus defined: A set of particles, originally in the same vertical straight line when the water is at rest, does not remain in a vertical line during the passage of the wave; so that the line connecting a set of such particles, while vertical and straight in still water, becomes distorted, as well as displaced, during the passage of the wave, its upper portion moving farther and more rapidly than its lower portion.

Mr. Stahl's paper contains illustrations of several wave-motors designed upon various principles. His conclusions as to their practicability is as follows: "Possibly none of the methods described in this paper may ever prove commercially successful; indeed the problem may not be susceptible of a financially successful solution. My own investigations, however, so far as I have yet been able to carry them, incline me to the belief that wave-power can and will be utilized on a paying basis."

Continuous Utilization of Tidal Power. (P. Decœur, Proc. Inst. C. E. 1890.)—In connection with the training-walls to be constructed in the estuary of the Seine, it is proposed to construct large basins, by means of which the power available from the rise and fall of the tide could be utilized. The method proposed is to have two basins separated by a bank rising above high water, within which turbines would be placed. The upper basin would be in communication with the sea during the higher one third of the tidal range, rising, and the lower basin during the lower one third of the tidal range, falling. If H be the range in feet, the level in the upper basin would never fall below $\frac{5}{6}H$ measured from low water, and the level in the lower basin would never rise above $\frac{1}{6}H$. The available head varies between $0.53H$ and $0.80H$, the mean value being $\frac{2}{3}H$. If S square feet be the area of the lower basin, and the above conditions are fulfilled, a quantity $1/3SH$ cu. ft. of water is delivered through the turbines in the space of $9\frac{1}{4}$ hours. The mean flow is, therefore, $SH \div 99{,}900$ cu. ft. per sec., and, the mean fall being $\frac{2}{3}H$, the available gross horse-power is about $1/30S'H^2$, where S' is measured in acres. This might be increased by about one third if variation of level in the basins amounting to $\frac{1}{6}H$ were permitted. But to reach this end the number of turbines would have to be doubled, the mean head being reduced to $\frac{1}{6}H$, and it would be more difficult to transmit a constant power from the turbines. The turbine proposed is of an improved model designed to utilize a large flow with a moderate diameter. One has been designed to produce 300 horse-power, with a minimum head of 5 ft. 3 in. at a speed of 15 revolutions per minute, the vanes having 13 ft. internal diameter. The speed would be maintained constant by regulating sluices.

PUMPS AND PUMPING ENGINES.

Theoretical Capacity of a Pump.—Let Q' = cu. ft. per min.; G' = Amer. gals. per min. = $7.4805Q'$; d = diam. of pump in inches; l = stroke in inches; N = number of single strokes per min.

$$\text{Capacity in cu. ft. per min.} = Q' = \frac{\pi}{4} \cdot \frac{d^2}{144} \cdot \frac{lN}{12} = .0004545Nd^2l;$$

$$\text{Capacity in gals. per min. } G' = \frac{\pi}{4} \cdot \frac{Nd^2l}{231} \ldots\ldots\ldots = .0034Nd^2l;$$

$$\text{Capacity in gals. per hour} \ldots\ldots\ldots\ldots\ldots\ldots = .204Nd^2l.$$

$$\left.\begin{array}{l}\text{Diameter required for a}\\ \text{given capacity per min.}\end{array}\right\} d = 46.9\sqrt{\frac{Q'}{Nl}} = 17.15\sqrt{\frac{G'}{Nl}}.$$

$$\text{If } v = \text{piston speed in feet per min., } d = 13.54\sqrt{\frac{Q'}{v}} = 4.95\sqrt{\frac{G'}{v}}.$$

If the piston speed is 100 feet per min.:

$$Nl = 1200, \text{ and } d = 1.354\sqrt{Q'} = .495\sqrt{G'}; \quad G' = 4.08d^2 \text{ per min.}$$

The actual capacity will be from 60% to 95% of the theoretical, according to the tightness of the piston, valves, suction-pipe, etc.

Theoretical Horse-power required to raise Water to a given Height.—Horse-power =

$$\frac{\text{Volume in cu. ft. per min.} \times \text{pressure per sq. ft.}}{33{,}000} = \frac{\text{Weight} \times \text{height of lift}}{33{,}000}.$$

Q' = cu. ft. per min.; G' = gals. per min.; W = wt. in lbs.; P = pressure in lbs. per sq. ft.; p = pressure in lbs. per sq. in.; H = height of lift in ft.; $W = 62.36Q'$, $P = 144p$, $p = .433H$, $H = 2.309p$, $G' = 7.4805Q'$.

$$\text{HP} = \frac{Q'P}{33{,}000} = \frac{Q'H \times 144 \times .433}{33{,}000} = \frac{Q'H}{529.2} = \frac{G'H}{3958.7};$$

$$\text{HP} = \frac{WH}{33{,}000} = \frac{Q' \times 62.36 \times 2.309p}{33{,}000} = \frac{Q'p}{229.2} = \frac{G'p}{1714.5}.$$

For the actual horse-power required an allowance must be made for the friction, slips, etc., of engine, pump, valves, and passages.

Depth of Suction.—Theoretically a perfect pump will draw water from a height of nearly 34 feet, or the height corresponding to a perfect vacuum (14.7 lbs. × 2.309 = 33.95 feet); but since a perfect vacuum cannot be obtained, on account of valve-leakage, air contained in the water, and the vapor of the water itself, the actual height is generally less than 30 feet. When the water is warm the height to which it can be lifted by suction decreases, on account of the increased pressure of the vapor. In pumping hot water, therefore, the water must flow into the pump by gravity. The following table shows the theoretical maximum depth of suction for different temperatures, leakage not considered:

Temp. F.	Absolute Pressure of Vapor, lbs. per sq. in.	Vacuum in Inches of Mercury.	Max. Depth of Suction, feet.	Temp. F.	Absolute Pressure of Vapor, lbs. per sq. in.	Vacuum in Inches of Mercury.	Max. Depth of Suction, feet.
101.4	1	27.88	31.6	183.0	8	13.63	15.5
126.2	2	25.85	29.3	188.4	9	11.59	13.2
144.7	3	23.81	27.0	193.2	10	9.55	10.9
153.3	4	21.77	24.7	197.6	11	7.51	8.5
162.5	5	19.74	22.4	201.9	12	5.48	6.2
170.3	6	17.70	20.1	205.8	13	3.44	3.9
177.0	7	15.66	17.8	209.6	14	1.40	1.6

Amount of Water raised by a Single-acting Lift-pump.—It is common to estimate that the quantity of water raised by a single-acting bucket-valve pump per minute is equal to the number of strokes in one direction per minute, multiplied by the volume traversed by the piston in a single stroke, on the theory that the water rises in the pump only when the piston or bucket ascends; but the fact is that the column of water does not cease flowing when the bucket descends, but flows on continuously through the valve in the bucket, so that the discharge of the pump, if it is operated at a high speed, may amount to nearly double that calculated from the displacement multiplied by the number of single strokes in one direction.

Proportioning the Steam-cylinder of a Direct-acting Pump.—Let

A = area of steam-cylinder; a = area of pump-cylinder;
D = diameter of steam-cylinder; d = diameter of pump-cylinder;
P = steam-pressure, lbs. per sq. in.; p = resistance per sq. in. on pumps;
H = head = $2.309p$; $p = .433H$;

$$E = \text{efficiency of the pump} = \frac{\text{work done in pump-cylinder}}{\text{work done by the steam-cylinder}}.$$

$$A = \frac{ap}{EP};\quad a = \frac{EAP}{p};\quad D = d\sqrt{\frac{p}{EP}};\quad d = D\sqrt{\frac{EP}{p}};\quad P = \frac{ap}{EA};\quad p = \frac{EAP}{a}.$$

$$\frac{A}{a} = \frac{p}{EP} = \frac{.433H}{EP};\quad H = 2.309EP\frac{A}{a};\quad \text{If } E = 75\%,\ H = 1.732P\frac{A}{a}.$$

E is commonly taken at 0.7 to 0.8 for ordinary direct-acting pumps. For the highest class of pumping-engines it may amount to 0.9. The steam-pressure P is the mean effective pressure, according to the indicator-diagram; the water-pressure p is the mean total pressure acting on the pump plunger or piston, including the suction, as could be shown by an indicator-diagram of the water-cylinder. The pressure on the pump-piston is frequently much greater than that due to the height of the lift, on account of the friction of the valves and passages, which increases rapidly with velocity of flow.

Speed of Water through Pipes and Pump-passages.—The speed of the water is commonly from 100 to 200 feet per minute. If 200 feet per minute is exceeded, the loss from friction may be considerable.

$$\text{The diameter of pipe required is } 4.95\sqrt{\frac{\text{gallons per minute}}{\text{velocity in feet per minute}}}.$$

For a velocity of 200 feet per minute, diameter $= .35 \times \sqrt{\text{gallons per min.}}$

Sizes of Direct-acting Pumps.—The tables on this and the next page are selected from catalogues of manufacturers, as representing the two common types of direct-acting pump, viz., the single-cylinder and the duplex. Both types are now made by most of the leading manufacturers.

The Deane Single Boiler-feed or Pressure Pump.—Suitable for pumping clear liquids at a pressure not exceeding 150 lbs.

Number.	Sizes.			Gallons per Stroke.	Capacity per min. at Given Speed.		Length in inches.	Width in inches.	Sizes of Pipes.			
	Steam-cylinder.	Water-cylinder.	Length of Stroke.		Strokes.	Gallons.			Steam.	Exhaust.	Suction.	Discharge.
0	8	2	5	.07	150	10	29½	7	½	¾	1¼	1
1	3½	2¼	5	.09	150	13	33½	7½	½	¾	1¼	1
1½	4	2⅜	5	.10	150	15	33½	7½	½	¾	1¼	1
2	4	2½	5	.11	150	16	33½	7½	½	¾	1¼	1
2½	4¾	3	5	.15	150	22	34	8½	½	¾	1½	1¼
3	5	3¼	7	.25	125	31	43½	9¼	¾	1	2	1½
4	5½	3¾	7	.33	125	42	43½	9¼	¾	1	2	1½
4½	7	4¼	8	.49	120	58	51½	12	1	1½	3	2
5	7	4½	10	.69	100	69	55	12	1	1½	3	2
6	7½	5	10	.85	100	85	55	12	1	1½	3	2
6½	8	5	12	1.02	100	102	68	14	1	1½	3	2½
7	10	6	12	1.47	100	147	69	19	1½	2	4	4
8	12	7	12	2.00	100	200	69	19	2	2½	5	4
9	14	8	12	2.61	100	261	69	21	2	2½	5	5

The Deane Single Tank or Light-service Pump.—These pumps will all stand a constant working pressure of 75 lbs. on the water-cylinders.

Sizes.			Gallons per Stroke.	Capacity per min. at Given Speed.		Length in inches.	Width in inches.	Sizes of Pipes.			
Steam-cylinder.	Water-cylinder.	Length of Stroke.		Strokes.	Gallons.			Steam.	Exhaust.	Suction.	Discharge.
4	4	5	.27	130	35	33	9½	½	¾	2	1½
5	4	7	.88	125	48	45½	15	¾	1	3	2½
5½	5½	7	.72	125	90	45½	15	¾	1	3	2½
7½	7½	10	1.91	110	210	58	17	1	1½	5	4
8	6	12	1.46	100	146	67	20½	1	1½	4	4
6	7	12	2.00	100	200	66	17	¾	1	4	4
8	7	12	2 00	100	200	67	20½	1	1½	5	4
8	8	12	2.61	100	261	68	30	1	1½	5	5
10	8	12	2.61	100	261	68½	30	1½	2	5	5
8	10	12	4.08	100	408	68	20½	1	1½	8	8
10	10	12	4.08	100	408	68½	30	1½	2	8	8
12	10	12	4.08	100	408	64	24	2	2½	8	8
10	12	12	5.87	100	587	68½	30	1½	2	8	8
12	12	12	5.87	100	587	64	28½	2	2½	8	8
10	12	18	8.79	70	616	95	25	1½	2	8	8
12	12	18	8.79	70	616	95	28½	2	2½	8	8
12	14	18	12.00	70	840	95	28½	2	2½	8	8
14	16	18	15.66	70	1096	95	34	2	2½	12	10
16	16	18	15.66	70	1096	95	34	2	2½	12	10
18	16	18	15.66	70	1096	97	34	3	3½	12	10
16	18	24	26.42	50	1321	115	40	2	2½	14	12
18	18	24	26.42	50	1321	135	40	3	3½	14	12

Efficiency of Small Direct-acting Pumps.—Chas. E. Emery, in Reports of Judges of Philadelphia Exhibition, 1876, Group xx., says: "Experiments made with steam-pumps at the American Institute Exhibition of 1867 showed that average-sized steam-pumps do not, on the average, utilize more than 50 per cent of the indicated power in the steam-cylinders, the remainder being absorbed in the friction of the engine, but more particularly in the passage of the water through the pump. It may be safely stated that ordinary steam-pumps rarely require less than 120 pounds of steam per hour for each horse-power utilized in raising water, equivalent to a duty of only 15,000,000 foot-pounds per 100 pounds of coal. With larger steam-pumps, particularly when they are proportioned for the work to be done, the duty will be materially increased."

The Worthington Duplex Pump.

STANDARD SIZES FOR ORDINARY SERVICE.

Diameter of Steam-cylinders.	Diameter of Water-plungers.	Length of Stroke.	Displacement in Gallons per Stroke of One Plunger.	Proper Strokes per Minute of One Plunger, varying with kind of work and pressure.	Gallons delivered per Minute by both Plungers at stated Number of Strokes.	Diameter of Plunger required in any single-cylinder pump to do the same work at same speed.	Sizes of Pipes for Short Lengths. To be increased as length increases. Steam-pipe.	Exhaust-pipe.	Suction-pipe.	Discharge-pipe.
3	2	3	.04	100 to 250	8 to 20	2⅞	⅜	½	1¼	1
4½	2¾	4	.10	100 to 200	20 to 40	4	½	¾	2	1½
5¼	3½	5	.20	100 to 200	40 to 80	5	¾	1¼	2½	1½
6	4	6	.33	100 to 150	70 to 100	5⅝	1	1½	3	2
7½	4½	6	.42	100 to 150	85 to 125	6⅜	1¼	2	4	3
7½	5	6	.51	100 to 150	100 to 150	7	1½	2	4	3
7½	4⅝	10	.69	75 to 125	100 to 170	6⅜	1½	2	4	3
9	5¼	10	.93	75 to 125	135 to 230	7½	2	2½	4	3
10	6	10	1.22	75 to 125	160 to 300	8½	2	2½	5	4
10	7	10	1.66	75 to 125	245 to 410	9⅞	2	2½	6	5
12	7	10	1.66	75 to 125	245 to 410	9⅞	2½	3	6	5
14	7	10	1.66	75 to 125	245 to 410	9⅞	2½	3	6	5
12	8½	10	2.45	75 to 125	365 to 610	12	2½	3	6	5
14	8½	10	2.45	75 to 125	365 to 610	12	2½	3	6	5
16	8½	10	2.45	75 to 125	365 to 610	12	2½	3	6	5
18½	8½	10	2.45	75 to 125	365 to 610	12	3	3½	6	5
20	8½	10	2.45	75 to 125	365 to 610	12	4	5	6	5
12	10¼	10	3.57	75 to 125	530 to 890	14¼	2½	3	8	7
14	10¼	10	3.57	75 to 125	530 to 890	14¼	2½	3	8	7
16	10¼	10	3.57	75 to 125	530 to 890	14¼	2½	3	8	7
18½	10¼	10	3.57	75 to 125	530 to 890	14¼	3	3½	8	7
20	10¼	10	3.57	75 to 125	530 to 890	14¼	4	5	8	7
14	12	10	4.89	75 to 125	730 to 1220	17	2½	3	10	8
16	12	10	4.89	75 to 125	730 to 1220	17	2½	3	10	8
18½	12	10	4.89	75 to 125	730 to 1220	17	3	3½	10	8
20	12	10	4.89	75 to 125	730 to 1220	17	4	5	10	8
18½	14	10	6.66	75 to 125	990 to 1660	19¾	3	3½	12	10
20	14	10	6.66	75 to 125	990 to 1660	19¾	4	5	12	10
17	10	15	5.10	50 to 100	510 to 1020	14	3	3½	8	7
20	12	15	7.34	50 to 100	730 to 1460	17	4	5	12	10
20	15	15	11.47	50 to 100	1145 to 2290	21	...	....	......	.. ..
25	15	15	11.47	50 to 100	1145 to 2290	21	...	...	......	.....

Speed of Piston.—A piston speed of 100 feet per minute is commonly assumed as correct in practice, but for short-stroke pumps this gives too high a speed of rotation, requiring too frequent a reversal of the valves. For long-stroke pumps, 2 feet and upward, this speed may be considerably exceeded, if valves and passages are of ample area.

Number of Strokes required to Attain a Piston Speed from 50 to 125 Feet per Minute for Pumps having Strokes from 3 to 18 Inches in Length.

Speed of Piston, in feet per min.	Length of Stroke in Inches.									
	3	4	5	6	7	8	10	12	15	18
	Number of Strokes per Minute.									
50	200	150	120	100	86	75	60	50	40	33
55	220	165	132	110	94	82.5	66	55	44	37
60	240	180	144	120	103	90	72	60	48	40
65	260	195	156	130	111	97.5	78	65	52	43
70	280	210	168	140	120	105	84	70	56	47
75	300	225	180	150	128	112.5	90	75	60	50
80	320	240	192	160	137	120	96	80	64	53
85	340	255	204	170	146	127.5	102	85	68	57
90	360	270	216	180	154	135	108	90	72	60
95	380	285	228	190	163	142.5	114	95	76	63
100	400	300	240	200	171	150	120	100	80	67
105	420	315	252	210	180	157.5	126	105	84	70
110	440	330	264	220	188	165	132	110	88	73
115	460	345	276	230	197	172.5	138	115	92	77
120	480	360	288	240	206	180	144	120	96	80
125	500	375	300	250	214	187.5	150	125	100	83

Piston Speed of Pumping-engines. (John Birkinbine, Trans. A. I. M. E., v. 459.)—In dealing with such a ponderous and unyielding substance as water there are many difficulties to overcome in making a pump work with a high piston speed. The attainment of moderately high speed is, however, easily accomplished. Well-proportioned pumping-engines of large capacity, provided with ample water-ways and properly constructed valves, are operated successfully against heavy pressures at a speed of 250 ft. per minute, without "thug," concussion, or injury to the apparatus, and there is no doubt that the speed can be still further increased.

Speed of Water through Valves.—If areas through valves and water passages are sufficient to give a velocity of 250 ft. per min. or less, they are ample. The water should be carefully guided and not too abruptly deflected. (F. W. Dean, *Eng. News*, Aug. 10, 1893.)

Boiler-feed Pumps.—Practice has shown that 100 ft. of piston speed per minute is the limit, if excessive wear and tear is to be avoided.

The velocity of water through the suction-pipe must not exceed 200 ft. per minute, else the resistance of the suction is too great.

The approximate size of suction-pipe, where the length does not exceed 25 ft. and there are not more than two elbows, may be found as follows:

7/10 of the diameter of the cylinder multiplied by 1/100 of the piston speed in feet. For duplex pumps of small size, a pipe one size larger is usually employed. The velocity of flow in the discharge-pipe should not exceed 500 ft. per minute. The volume of discharge and length of pipe vary so greatly in different installations that where the water is to be forced more than 50 ft. the size of discharge-pipe should be calculated for the particular conditions, allowing no greater velocity than 500 ft. per minute. The size of discharge-pipe is calculated in single-cylinder pumps from 250 to 400 ft. per minute. Greater velocity is permitted in the larger pipes.

In determining the proper size of pump for a steam-boiler, allowances must be made for a supply of water sufficient to cover all the demands of engines, steam-heating, etc., up to the capacity of generator, and should not be calculated simply according to the requirements of the engine. In practice engines use all the way from 12 up to 50, or more, pounds of steam per H.P. per hour when being worked up to capacity. When an engine is overloaded or underloaded more water per H.P. will be required than when operating at its rated capacity. The average run of horizontal tubular boilers will evaporate from 2 to 3 lbs. of water per sq. ft. of heating-surface per hour, but may be driven up to 6 lbs. if the grate-surface is too large or the draught too great for economical working.

Pump-Valves.—A. F. Nagle (Trans. A. S. M. E., x. 521) gives a number of designs with dimensions of double-beat or Cornish valves used in large pumping-engines, with a discussion of the theory of their proportions. The following is a summary of the proportions of the valves described.

SUMMARY OF VALVE PROPORTIONS.

Location of Engine.	Diam. of Valve in inches.	Weight in Water per square inch of Inside Unbalanced Area, in lbs.	Ratio of Seat-area to Inside Unbalanced Area.	Pressure upon Seat per sq. in., in lbs.	Action.
Providence high-service engine	12	1 lb. reduced to .66 lb.	16%	377 lbs.	Good
Providence Cornish-engine	16	1.28	12	680	Good
St. Louis Water Wks.	16	1.86	67	250	Some noise
Milwaukee " "	7	.40	88	120	Some noise at high speed.
Chicago " "	25	1.41	75	151	Noisy
" " "	15	1.31	85	140	"
" " " wood seats.........	15	1.16	94	132	"
Chicago Water Wks.	8	.96	75	151	"

Mr. Nagle says: There is one feature in which the Cornish valves are necessarily defective, namely, the lift must always be quite large, unless great power is sacrificed to reduce it. It is undeniable that a small lift is preferable to a great one, and hence it naturally leads to the substitution of numerous small valves for one or several large ones. To what extreme reduction of size this view might safely lead must be left to the judgment of the engineer for the particular case in hand, but certainly, theoretically, we must adopt small valves. Mr. Corliss at one time carried the theory so far as to make them only 1¾ inches in diameter, but from 3 to 4 inches is the more common practice now. A small valve presents proportionately a larger surface of discharge with the same lift than a larger valve, so that whatever the total area of valve-seat opening, its full contents can be discharged with less lift through numerous small valves than with one large one.

Henry R. Worthington was the first to use numerous small rubber valves in preference to the larger metal valves. These valves work well under all the conditions of a city pumping-engine. A volute spring is generally used to limit the rise of the valve.

In the Leavitt high-duty sewerage-engine at Boston (*Am. Machinist*, May 31, 1884), the valves are of rubber, ⅝-inch thick, the opening in valve-seat being 13½ × 4½ inches. The valves have iron face and back-plates, and form their own hinges.

CENTRIFUGAL PUMPS.

Relation of Height of Lift to Velocity.—The height of lift depends only on the tangential velocity of the circumference, every tangential velocity giving a constant height of lift—sometimes termed "head"—whether the pump is small or large. The quantity of water discharged is in proportion to the area of the discharging orifices at the circumference, or in proportion to the square of the diameter, when the breadth is kept the same. R. H. Buel (App. Cyc. Mech., ii, 606) gives the following:

Let Q represent the quantity of water, in cubic feet, to be pumped per minute, h the height of suction in feet, h' the height of discharge in feet, and d the diameter of suction-pipe, equal to the diameter of discharge-pipe, in feet; then, according to Fink, $d = 0.86\sqrt{\frac{Q}{\sqrt{2g(h+h')}}}$, g being the acceleration due to gravity.

If the suction takes place on one side of the wheel, the inside diameter of the wheel is equal to $1.2d$, and the outside to $2.4d$. If the suction takes place at both sides of the wheel, the inside diameter of the wheel is equal to $0.85d$, and the outside to $1.7d$. Then the suction-pipe will have two branches, the area of each equal to half the area of d. The suction-pipe should be as short as possible, to prevent air from entering the pump. The tangential velocity of the outer edge of wheel for the delivery Q is equal to $1.25\sqrt{2g(h+h')}$ feet per second.

The arms are six in number, constructed as follows: Divide the central angle of 60°, which incloses the outer edges of the two arms, into any number of equal parts by drawing the radii, and divide the breadth of the wheel in the same manner by drawing concentric circles. The intersections of the several radii with the corresponding circles give points of the arm.

In experiments with Appold's pump, a velocity of circumference of 500 ft. per min. raised the water 1 ft. high, and maintained it at that level without discharging any; and double the velocity raised the water to four times the height, as the centrifugal force was proportionate to the square of the velocity; consequently,

500 ft. per min. raised the water	1	ft. without discharge.
1000 " " " " "	4	" " "
2000 " " " " "	16	" " "
4000 " " " " "	64	" " "

The greatest height to which the water had been raised without discharge, in the experiments with the 1-ft. pump, was 67.7 ft., with a velocity of 4158 ft. per min., being rather less than the calculated height, owing probably to leakage with the greater pressure. A velocity of 1128 ft. per min. raised the water 5½ ft. without any discharge, and the maximum effect from the power employed in raising to the same height 5½ ft. was obtained at the velocity of 1678 ft. per min., giving a discharge of 1400 gals. per min. from the 1-ft. pump. The additional velocity required to effect a discharge of 1400 gals. per min., through a 1-ft. pump working at a dead level without any height of lift, is 550 ft. per min. Consequently, adding this number in each case to the velocity given above, at which no discharge takes place, the following velocities are obtained for the maximum effect to be produced in each case:

1050 ft. per min., velocity for	1	ft. height of lift.
1550 " " " "	4	" " "
2550 " " " "	16	" " "
4550 " " " "	64	" " "

Or, in general terms, the velocity in feet per minute for the circumference of the pump to be driven, to raise the water to a certain height, is equal to $550 + 500\sqrt{\text{height of lift in feet}}$.

Lawrence Centrifugal Pumps, Class B—For Lifts from 15 to 35 ft.

No. of Pump.	Suction-pipe, in.	Discharge-pipe, in.	Economical Capacity, gals. per min.	H.P. for each foot of lift.	Weight, lbs.	No. of Pump.	Suction-pipe, in.	Discharge-pipe, in.	Economical Capacity, gals. per min.	H.P. for each foot of lift.	Weight, lbs.
1	1½	1	25	.028	65	10	10	10	3000	1.60	3000
1½	2	1½	70	.05	230	12	12	12	4200	2.15	6800
2	2½	2	100	.08	265	15	15	15	7000	3.50	8840
3	3½	3	250	.15	500	18	18	18	10000	5.00	10000
4	4½	4	450	.27	680	24	24	24	18000	7.60	9000*
5	6	5	700	.36	1032	30	30	30	25000	10.50	20000*
6	6	6	1200	.65	1260	36	36	36	35000	14.75	22000*
8	8	8	2000	1 10	2460						

* Without base.

The economical capacity corresponds to a flow not exceeding 10 ft. per second in the delivery-pipe. Small pipes and high rate of flow cause a great loss of power.

Size of Pulleys, Width of Belts, and Revolutions per Minute Necessary to Raise the Rated Quantity of Water to Different Heights with Pumps of Class B.

Size, in.	Diam. of Pulley, in.	Width of Pulley, in.	Width of Belt, in.	Rated Quantity of Water, gals.	Height in Feet and Revolutions per Minute.									No. of Pump.
					6′	8′	10′	12′	16′	20′	25′	30′	35′	
1½	5	5	3	70	520	590	665	720	885	930	1045	1125	1200	1½
2	6	5	4	100	475	540	605	660	765	850	955	1025	1100	2
3	7½	7	6	250	435	500	560	610	705	790	880	945	1000	3
4	10	7	7	450	400	465	520	570	655	730	815	880	945	4
5	14	11	8	700	355	410	454	595	575	640	715	765	825	5
6	16	11	9	1200	315	365	400	440	510	570	635	685	745	6
8	20	12	10	2000	234	270	300	330	385	425	475	500	555	8
10	22	12	10	3000	234	270	300	330	385	425	475	500	555	10
12	30	14	12	4200	160	185	200	220	255	285	318	340	360	12
15	36	16	15	7000	140	165	180	198	228	255	285	305	330	15
18	40	16	15	10000	125	145	160	173	200	225	250	270	290	18
24	....	....	....	18000	105	125	135	150	170	190	214	230	250	24
30	....	..	...	25000	95	106	118	130	148	165	185	204	215	30
36	...	..	...	35000	95	106	118	130	148	165	185	204	215	36

Efficiencies of Centrifugal and Reciprocating Pumps.—W. O. Webber (Trans. A. S. M. E., vii. 598) gives diagrams showing the relative efficiencies of centrifugal and reciprocating pumps, from which the following figures are taken for the different lifts stated:

Lift, feet:	2	5	10	15	20	25	30	35	40	50	60	80	100	120	160	200	240	280
Efficiency reciprocating pump:	..	.	.30	.45	.55	.61	.66	.68	.71	.75	.77	.82	.85	.87	.90	.89	.88	.85
Efficiency centrifugal pump:	.50	.56	.64	.68	.69	.68	.66	.62	.58	.50	.40	...	...	...	...	...	...	...

The term efficiency here used indicates the value of W. H. P. ÷ I. H. P., or horse-power of the water raised divided by the indicated horse-power of the steam-engine, and does not therefore show the full efficiency of the pump, but that of the combined pump and engine. It is, however, a very simple way of showing the relative values of different kinds of pumping-engines having their motive power forming a part of the plant.

The highest value of this term, given by Mr. Webber, is .9164 for a lift of 170 ft., and 3615 gals. per min. This was obtained in a test of the Leavitt pumping-engine at Lawrence, Mass., July 24, 1879.

With reciprocating pumps, for higher lifts than 170 ft., the curve of efficiencies falls, and from 200 to 300 ft. lift the average value seems about .84. Below 170 ft. the curve also falls reversely and slowly, until at about 90 ft. its descent becomes more rapid, and at 35 ft. .727 appears the best recorded performance. There are not any very satisfactory records below this lift, but some figures are given for the yearly coal consumption and total number of gallons pumped by engines in Holland under a 16-ft. lift, from which an efficiency of .44 has been deduced.

With centrifugal pumps, the lift at which the maximum efficiency is obtained is approximately 17 ft. At lifts from 12 to 18 ft. some makers of large experience claim now to obtain from 65% to 70% of useful effect, but .613 appears to be the best done at a public test under 14.7 ft. head.

The drainage-pumps constructed some years ago for the Haarlem Lake were designed to lift 70 tons per min. 15 ft., and they weighed about 150 tons. Centrifugal pumps for the same work weigh only 5 tons. The weight of a centrifugal pump and engine to lift 10,000 gals. per min. 35 ft. high is 6 tons.

The pumps placed by Gwynne at the Ferrara Marshes, Northern Italy, in 1865, are, it is believed, capable of handling more water than other set of pumping-engines in existence. The work performed by these pumps is the lifting of 2000 tons per min —over 600,000,000 gals. per 24 hours—on a mean lift of about 10 ft. (maximum of 12.5 ft.). (See *Engineering*, 1876.)

The efficiency of centrifugal pumps seems to increase as the size of pump increases, approximately as follows: A 2″ pump (this designation meaning always the size of discharge-outlet in inches of diameter), giving an efficiency of 38%, a 3″ pump 45%, and a 4″ pump 52%, a 5″ pump 60%, and a 6″ pump 64% efficiency.

Tests of Centrifugal Pumps.

W. O. Webber, Trans. A. S. M. E., ix. 237.

Maker.	Andrews.	Andrews.	Andrews.	Heald & Sisco.	Heald & Sisco.	Heald & Sisco.	Berlin. Schwartzkopff.
Size	No. 9.	No. 9.	No. 9.	No. 10.	No. 10.	No. 10.	No. 9.
Diam. discharge	9⅛″	9⅛″	9⅛″	10″	10″	10″	9¼″
" suction	9¾″	9¾″	9¾″	12″	12″	12″	10.3″
" disk	26″	26″	26″	30.5″	30.5″	30.5″	20.5″
Rev. per minute	191.9	195.5	200.5	188.3	202.7	213.7	500
Galls. per minute	1513.12	2023.82	2499.33	1673.37	2044.9	2371.67	1944.8
Height in feet	12.25	12.62	13.08	12.33	12.58	13.0	16.46
Water H.P.	4.69	6.47	8.28	5.22	6.51	7.81	....
Dynam'eter H.P.	10.09	12.2	14.38	8.11	10.74	14.02	11
Efficiency	46.52	53.0	57.57	64.5	60.74	55.72	78.1

Vanes of Centrifugal Pumps.—For forms of pump vanes, see paper by W. O. Webber, Trans. A. S. M. E., ix. 228, and discussion thereon by Profs. Thurston, Wood, and others.

The Centrifugal Pump used as a Suction Dredge.—The Andrews centrifugal pump was used by Gen. Gillmore, U. S. A., in 1871, in deepening the channel over the bar at the mouth of the St. John's River, Florida. The pump was a No. 9, with suction and discharge pipes each 9 inches diam. It was driven at 300 revolutions per minute by belt from an engine developing 26 useful horse-power.

Although 200 revolutions of the pump disk per minute will easily raise 3000 gallons of clear water 12 ft. high, through a straight vertical 9-inch pipe, 300 revolutions were required to raise 2500 gallons of sand and water 11 ft. high, through two inclined suction-pipes having two turns each, discharged through a pipe having one turn.

The proportion of sand that can be pumped depends greatly upon its specific gravity and fineness. The calcareous and argillaceous sands flow more freely than the silicious, and fine sands are less liable to choke the pipe than those that are coarse. When working at high speed, 50% to 55% of sand can be raised through a straight vertical pipe, giving for every 10 cubic yards of material discharged 5 to 5½ cubic yards of compact sand. With the appliances used on the St. John's bar, the proportion of sand seldom exceeded 45%, generally ranging from 30% to 35% when working under the most favorable conditions.

In pumping 2500 gallons, or 12.6 cubic yards of sand and water per minute, there would therefore be obtained from 3.7 to 4.3 cubic yards of sand. During the early stages of the work, before the teeth under the drag had been properly arranged to aid the flow of sand into the pipes, the yield was considerably below this average. (From catalogue of Jos. Edwards & Co., Mfrs. of the Andrews Pump, New York.)

DUTY TRIALS OF PUMPING-ENGINES.

A committee of the A. S. M. E. (Trans., xii. 530) reported in 1891 on a standard method of conducting duty trials. Instead of the old unit of duty of foot-pounds of work per 100 lbs. of coal used, the committee recommend a new unit, foot-pounds of work per million heat-units furnished by the boiler. The variations in quality of coal make the old standard unfit as a basis of duty ratings. The new unit is the precise equivalent of 100 lbs. of coal in cases where each pound of coal imparts 10,000 heat-units to the water in the boiler, or where the evaporation is 10,000 ÷ 965.7 = 10.355 lbs. of water from and at 212° per pound of fuel. This evaporative result is readily obtained from all grades of Cumberland bituminous coal, used in horizontal return tubular boilers, and, in many cases, from the best grades of anthracite coal.

The committee also recommend that the work done be determined by plunger displacement, after making a test for leakage, instead of by measurement of flow by weirs or other apparatus, but advise the use of such apparatus when practicable for obtaining additional data. The following extracts are taken from the report. When important tests are to be made the complete report should be consulted.

The necessary data having been obtained, the duty of an engine, and other quantities relating to its performance, may be computed by the use of the following formulæ:

$$1.\ \text{Duty} = \frac{\text{Foot-pounds of work done}}{\text{Total number of heat-units consumed}} \times 1{,}000{,}000$$

$$= \frac{A(P \pm p + s) \times L \times N}{H} \times 1{,}000{,}000 \text{ (foot-pounds).}$$

$$2.\ \text{Percentage of leakage} = \frac{C \times 144}{A \times L \times N} \times 100 \text{ (per cent).}$$

$$3.\ \text{Capacity} = \text{number of gallons of water discharged in 24 hours}$$

$$= \frac{A \times L \times N \times 7.4805 \times 24}{D \times 144} = \frac{A \times L \times N \times 1.24675}{D} \text{ (gallons).}$$

4. Percentage of total frictions,

$$= \left[\frac{\text{I.H.P.} - \dfrac{A(P \pm p + s) \times L \times N}{D \times 60 \times 33{,}000}}{\text{I.H.P.}}\right] \times 100$$

$$= \left[1 - \frac{A(P \pm p + s) \times L \times N}{A_s \times \text{M.E.P.} \times L_s \times N_s}\right] \times 100 \text{ (per cent);}$$

or, in the usual case, where the length of the stroke and number of strokes of the plunger are the same as that of the steam-piston, this last formula becomes:

$$\text{Percentage of total frictions} = \left[1 - \frac{A(P \pm p + s)}{A_s \times \text{M.E.P.}}\right] \times 100 \text{ (per cent).}$$

In these formulæ the letters refer to the following quantities:

A = Area, in square inches, of pump plunger or piston, corrected for area of piston rod or rods;

P = Pressure, in pounds per square inch, indicated by the gauge on the force main;

p = Pressure, in pounds per square inch, corresponding to indication of the vacuum-gauge on suction-main (or pressure-gauge, if the suction-pipe is under a head). The indication of the vacuum-gauge, in inches of mercury, may be converted into pounds by dividing it by 2.035;

s = Pressure, in pounds per square inch, corresponding to distance between the centres of the two gauges. The computation for this pressure is made by multiplying the distance, expressed in feet, by the weight of one cubic foot of water at the temperature of the pump-well, and dividing the product by 144;

L = Average length of stroke of pump-plunger, in feet;

N = Total number of single strokes of pump-plunger made during the trial;

A_s = Area of steam-cylinder, in square inches, corrected for area of piston-rod. The quantity $A_s \times M.E.P.$, in an engine having more than one cylinder, is the sum of the various quantities relating to the respective cylinders;

L_s = Average length of stroke of steam-piston, in feet;

N_s = Total number of single strokes of steam-piston during trial;

M.E.P. = Average mean effective pressure, in pounds per square inch, measured from the indicator-diagrams taken from the steam-cylinder;

I.H.P. = Indicated horse-power developed by the steam-cylinder;

C = Total number of cubic feet of water which leaked by the pump-plunger during the trial, estimated from the results of the leakage test;

D = Duration of trial in hours:

H = Total number of heat-units (B. T. U.) consumed by engine = weight of water supplied to boiler by main feed-pump × total heat of steam of boiler pressure reckoned from temperature of main feed-water + weight of water supplied by jacket-pump × total heat of steam of boiler-pressure reckoned from temperature of jacket-water + weight of any other water supplied × total heat of steam reckoned from its temperature of supply. The total heat of the steam is corrected for the moisture or superheat which the steam may contain. No allowance is made for water added to the feed water, which is derived from any source, except the engine or some accessory of the engine. Heat added to the water by the use of a flue-heater at the boiler is not to be deducted. Should heat be abstracted from the flue by means of a steam reheater connected with the intermediate receiver of the engine, this heat must be included in the total quantity supplied by the boiler.

Leakage Test of Pump.—The leakage of an inside plunger (the only type which requires testing) is most satisfactorily determined by making the test with the cylinder-head removed. A wide board or plank may be temporarily bolted to the lower part of the end of the cylinder, so as to hold back the water in the manner of a dam, and an opening made in the temporary head thus provided for the reception of an overflow-pipe. The plunger is blocked at some intermediate point in the stroke (or, if this position is not practicable, at the end of the stroke), and the water from the force main is admitted at full pressure behind it. The leakage escapes through the overflow-pipe, and it is collected in barrels and measured. The test should be made, if possible, with the plunger in various positions.

In the case of a pump so planned that it is difficult to remove the cylinder-head, it may be desirable to take the leakage from one of the openings which are provided for the inspection of the suction-valves, the head being allowed to remain in place.

It is assumed that there is a practical absence of valve leakage. Examination for such leakage should be made, and if it occurs, and it is found to be due to disordered valves, it should be remedied before making the plunger test. Leakage of the discharge valves will be shown by water passing down into the empty cylinder at either end when they are under pressure. Leakage of the suction-valves will be shown by the disappearance of water which covers them.

If valve leakage is found which cannot be remedied the quantity of water thus lost should also be tested. One method is to measure the amount of water required to maintain a certain pressure in the pump cylinder when this is introduced through a pipe temporarily erected, no water being allowed to enter through the discharge valves of the pump.

Table of Data and Results.—In order that uniformity may be secured, it is suggested that the data and results, worked out in accordance with the standard method, be tabulated in the manner indicated in the following scheme:

DUTY TRIAL OF ENGINE.

DIMENSIONS.

1. Number of steam-cylinders..
2. Diameter of steam-cylinders.................. .... ins.
3. Diameter of piston-rods of steam-cylinders... .. .. ins.
4. Nominal stroke of steam-pistons............ .. .. ft.
5. Number of water-plungers ....
6. Diameter of plungers.. ins.
7. Diameter of piston-rods of water-cylinders...................... ins.
8. Nominal stroke of plungers.. .... ft.
9. Net area of steam-pistons.......... .... sq. ins.
10. Net area of plungers...... .... sq. ins.
11. Average length of stroke of steam-pistons during trial......... ft.
12. Average length of stroke of plungers during trial .... ft.

(Give also complete description of plant.)

TEMPERATURES.

13. Temperature of water in pump-well.......................... degs.
14. Temperature of water supplied to boiler by main feed-pump.. degs.
15. Temperature of water supplied to boiler from various other sources... degs.

FEED-WATER.

16. Weight of water supplied to boiler by main feed-pump........ lbs.
17. Weight of water supplied to boiler from various other sources. lbs.
18. Total weight of feed-water supplied from all sources........... lbs.

PRESSURES.

19. Boiler pressure indicated by gauge............................ lbs.
20. Pressure indicated by gauge on force main..................... lbs.
21. Vacuum indicated by gauge on suction main.................... ins.
22. Pressure corresponding to vacuum given in preceding line..... lbs.
23. Vertical distance between the centres of the two gauges....... ins.
24. Pressure equivalent to distance between the two gauges....... lbs.

MISCELLANEOUS DATA.

25. Duration of trial .... hrs.
26. Total number of single strokes during trial...................
27. Percentage of moisture in steam supplied to engine, or number of degrees of superheating.................................. % or deg
28. Total leakage of pump during trial, determined from results of leakage test .... lbs.
29. Mean effective pressure, measured from diagrams taken from steam-cylinders .... M.E.P.

PRINCIPAL RESULTS.

30. Duty.. ft. lbs.
31. Percentage of leakage... %
32. Capacity.. gals.
33. Percentage of total friction.................................. %

ADDITIONAL RESULTS.

34. Number of double strokes of steam-piston per minute ....
35. Indicated horse-power developed by the various steam-cylinders I.H.P.
36. Feed-water consumed by the plant per hour.................... lbs.
37. Feed-water consumed by the plant per indicated horse-power per hour, corrected for moisture in steam.................. lbs.
38. Number of heat units consumed per indicated horse-power per hour... B.T.U.
39. Number of heat units consumed per indicated horse-power per minute.. B.T.U.
40. Steam accounted for by indicator at cut-off and release in the various steam-cylinders .... lbs.
41. Proportion which steam accounted for by indicator bears to the feed-water consumption.................................
42. Number of double strokes of pump per minute ....
43. Mean effective pressure, measured from pump diagrams ...... M.E.P.
44. Indicated horse-power exerted in pump-cylinders.............. I.H.P.
45. Work done (or duty) per 100 lbs. of coal .... ft. lbs.

SAMPLE DIAGRAM TAKEN FROM STEAM-CYLINDERS.

(Also, if possible, full measurement of the diagrams, embracing pressures at the initial point, cut-off, release, and compression; also back pressure, and the proportions of the stroke completed at the various points noted.)

SAMPLE DIAGRAM TAKEN FROM PUMP-CYLINDERS.

These are not necessary to the main object, but it is desirable to give them.

DATA AND RESULTS OF BOILER TEST.

(In accordance with the scheme recommended by the Boiler-test Committee of the Society.)

VACUUM PUMPS—AIR-LIFT PUMP.

The Pulsometer.—In the pulsometer the water is raised by suction into the pump-chamber by the condensation of steam within it, and is then forced into the delivery-pipe by the pressure of a new quantity of steam on the surface of the water. Two chambers are used which work alternately, one raising while the other is discharging.

Test of a Pulsometer.—A test of a pulsometer is described by De Volson Wood in Trans. A. S. M. E. xiii. It had a 3½-inch suction-pipe, stood 40 in. high, and weighed 695 lbs.

The steam-pipe was 1 inch in diameter. A throttle was placed about 2 feet from the pump, and pressure gauges placed on both sides of the throttle, and a mercury well and thermometer placed beyond the throttle. The wire drawing due to throttling caused superheating.

The pounds of steam used were computed from the increase of the temperature of the water in passing through the pump.

Pounds of steam × loss of heat = lbs. of water sucked in × increase of temp.

The loss of heat in a pound of steam is the total heat in a pound of saturated steam as found from "steam tables" for the given pressure, plus the heat of superheating, minus the temperature of the discharged water; or

$$\text{Pounds of steam} = \frac{\text{lbs. water} \times \text{increase of temp.}}{H - 0.48t - T.}$$

The results for the four tests are given in the following table:

Data and Results.	Number of Test.			
	1	2	3	4
Strokes per minute	71	60	57	64
Steam press. in pipe before throttl'g	114	110	127	104.3
Steam press. in pipe after throttl'g	19	30	43.8	26.1
Steam temp. after throttling, deg. F.	270.4	277	309.0	270.1
Steam am'nt of superheat'g. deg. F.	8.1	3.4	17.4	1.4
Steam used as det'd from temp., lbs.	1617	931	1518	1019.9
Water pumped, lbs.	404,786	186,362	228,425	248,053
Water temp. before entering pump.	75.15	30.6	76.3	70.25
Water temp., rise of....	4.47	5.5	7.49	4.55
Water head by gauge on lift, ft ...	29.90	54.05	54.05	29.90
Water head by gauge on suction...	12.26	12.26	19.67	19.67
Water head by gauge, total (H)....	42.16	66.31	73.72	49.57
Water head by measure, total (h)	32.8	57.80	66.6	41.60
Coeff. of friction of plant (h) ÷ (H)	0.777	0.877	0.911	0.839
Efficiency of pulsometer..... ...	0.012	0.0155	0.0126	0.0138
Effic. of plant exclusive of boiler...	0.0098	0.0136	0.0115	0.0116
Effic. of plant if that of boiler be 0.7	0.0065	0.0095	0.0080	0.0081
Duty, if 1 lb. evaporates 10 lbs. water	10,511,400	13,391,000	11,059,000	12,036,300

Of the two tests having the highest lift (54.05 ft.), that was more efficient which had the smaller suction (12.26 ft.), and this was also the most efficient of the four tests. But, on the other hand, the other two tests having the same lift (29.9 ft.), that was the more efficient which had the greater suction (19.67), so that no law in this regard was established. The pressures used, 19, 30, 43.8, 26.1, follow the order of magnitude of the total heads, but are not proportional thereto. No attempt was made to determine what pressure would give the best efficiency for any particular head. The pressure used was intrusted to a practical runner, and he judged that when the pump was running regularly and well, the pressure then existing was the proper one. It is peculiar that, in the first test, a pressure of 19 lbs. of steam should produce a greater number of strokes and pump over 50% more water than 26.1 lbs., the lift being the same, as in the fourth experiment.

Chas. E Emery in discussion of Prof. Wood's paper says, referring to tests made by himself and others at the Centennial Exhibition in 1876 (see Report of the Judges, Group xx.), that a vacuum-pump tested by him in 1871 gave a duty of 4.7 millions; one tested by J. F. Flagg, at the Cincinnati Exposition in 1875, gave a maximum duty of 3.25 millions. Several vacuum and small steam-pumps, compared later on the same basis, were reported to have given duties of 10 to 11 millions, the steam-pumps doing no better than the vacuum-pumps. Injectors, when used for lifting water not required to be heated, have an efficiency of 2 to 5 millions; vacuum-pumps vary generally between 3 and 10; small steam-pumps between 8 and 15; larger steam-pumps, between 15 and 30, and pumping-engines between 30 and 140 millions.

A very high record of test of a pulsometer is given in *Eng'g.* Nov. 24, 1893, p. 639, viz.: Height of suction 11.27 ft.; total height of lift, 102.6 ft.; horizontal length of delivery-pipe, 118 ft.; quantity delivered per hour, 26,188 British gallons. Weight of steam used per H. P. per hour, 92.76 lbs.; work done per pound of steam 21,345 foot-pounds, equal to a duty of 21,345,000 foot-pounds per 100 lbs. of coal, if 10 lbs of steam were generated per pound of coal.

The Jet-pump.—This machine works by means of the tendency of a stream or jet of fluid to drive or carry contiguous particles of fluid along with it. The water-jet pump, in its present form, was invented by Prof. James Thomson, and first described in 1852. In some experiments on a small scale as to the efficiency of the jet-pump, the greatest efficiency was found to take place when the depth from which the water was drawn by the suction-pipe was about nine tenths of the height from which the water fell to form the jet; the flow up the suction-pipe being in that case about one fifth of that of the jet, and the efficiency, consequently, $9/10 \times 1/5 = 0.18$. This is but a low efficiency; but it is probable that it may be increased by improvements in proportions of the machine. (Rankine, S. E.)

The Injector when used as a pump has a very low efficiency. (See Injectors, under Steam-boilers.)

Air-lift Pump.—The air-lift pump consists of a vertical water-pipe with its lower end submerged in a well, and a smaller pipe delivering air into it at the bottom. The rising column in the pipe consists of air mingled with water, the air being in bubbles of various sizes, and is therefore lighter than a column of water of the same height; consequently the water in the pipe is raised above the level of the surrounding water. This method of raising water was proposed as early as 1797, by Loescher, of Freiberg, and was mentioned by Collon in lectures in Paris in 1876, but its first practical application probably was by Werner Siemens in Berlin in 1885. Dr. J. G. Pohle experimented on the principle in California in 1886, and U. S. patents on apparatus involving it were granted to Pohle and Hill in the same year. A paper describing tests of the air-lift pump made by Randall, Browne and Behr was read before the Technical Society of the Pacific Coast in Feb. 1890.

The diameter of the pump-column was 3 in., of the air-pipe 0.9 in., and of the air-discharge nozzle 5/8 in. The air-pipe had four sharp bends and a length of 35 ft. plus the depth of submersion.

The water was pumped from a closed pipe-well (55 ft. deep and 10 in. in diameter). The efficiency of the pump was based on the least work theoretically required to compress the air and deliver it to the receiver. If the efficiency of the compressor be taken at 70%, the efficiency of the pump and compressor together would be 70% of the efficiency found for the pump alone.

For a given submersion (h) and lift (H), the ratio of the two being kept within reasonable limits, (H) being not much greater than (h), the efficiency was greatest when the pressure in the receiver did not greatly exceed the head due to the submersion. The smaller the ratio $H \div h$, the higher was the efficiency.

The pump, as erected, showed the following efficiencies:

For $H \div h =$	0.5	1.0	1.5	2.0
Efficiency =	50%	40%	30%	25%

The fact that there are absolutely no moving parts makes the pump especially fitted for handling dirty or gritty water, sewage, mine water, and acid or alkali solutions in chemical or metallurgical works.

In Newark, N. J., pumps of this type are at work having a total capacity of 1,000,000 gallons daily, lifting water from three 8-in. artesian wells. The Newark Chemical Works use an air-lift pump to raise sulphuric acid of 1.72° gravity. The Colorado Central Consolidated Mining Co., in one of its mines at Georgetown, Colo., lifts water in one case 250 ft., using a series of lifts.

For a full account of the theory of the pump, and details of the tests above referred to, see *Eng'g News*, June 8, 1893.

THE HYDRAULIC RAM.

Efficiency.—The hydraulic ram is used where a considerable flow of water with a moderate fall is available, to raise a small portion of that flow to a height exceeding that of the fall. The following are rules given by Eytelwein as the results of his experiments (from Rankine):

Let Q be the whole supply of water in cubic feet per second, of which q is lifted to the height h above the pond, and $Q - q$ runs to waste at the depth H below the pond; L, the length of the supply-pipe, from the pond to the waste-clack; D, its diameter in feet; then

$$D \doteq \sqrt{(1.63Q)}; \quad L = H + h + \frac{h}{H} \times 2 \text{ feet};$$

Volume of air vessel = volume of feed pipe;

$$\text{Efficiency, } \frac{qh}{(Q-q)H} = 1.12 - 0.2\sqrt{\frac{h}{H}} \text{ when } \frac{h}{H} \text{ does not exceed 20.}$$

or

$$1 \div \left(1 + \frac{h}{10H}\right) \text{ nearly, when } \frac{h}{H} \text{ does not exceed 12.}$$

D'Aubuisson gives $$\frac{q(H+h)}{QH} = 1.42 - .28\sqrt{\frac{h}{H}}.$$

Clark, using five sixths of the values given by D'Aubuisson's formula, gives:

Ratio of lift to fall. ...	4	6	8	10	12	14	16	18	20	22	24	26
Efficiency per cent....	72	61	52	44	37	31	25	19	14	9	4	0

Prof. R. C. Carpenter (*Eng'g Mechanics*, 1894) reports the results of four tests of a ram constructed by Rumsey & Co., Seneca Falls. The ram was fitted for pipe connection for 1¼-inch supply and ½-inch discharge. The supply-pipe used was 1¼ inches in diameter, about 50 feet long, with 8 elbows, so that it was equivalent to about 65 feet of straight pipe, so far as resistance is concerned. Each run was made with a different stroke for the waste or clack-valve, the supply and delivery head being constant; the object of the experiment was to find that stroke of clack-valve which would give the highest efficiency.

Length of stroke, per cent..................	100	80	60	40
Number of strokes per minute.............	52	56	61	66
Supply head, feet of water.................	5.67	5.77	5.58	5.65
Delivery head, feet of water...............	19.75	19.75	19.75	19.75
Total water pumped, pounds...............	297	296	301	297.5
Total water supplied, pounds..............	1615	1567	1518	1455.5
Efficiency, per cent........................	64.9	66	74.9	70

The efficiency, 74.9, the highest realized, was obtained when the clack-valve travelled a distance equal to 60% of its full stroke, the full travel being 15/16 of one inch.

Quantity of Water Delivered by the Hydraulic Ram. (Chadwick Lead Works.)—From 80 to 100 feet conveyance, one seventh of supply from spring can be discharged at an elevation five times as high as the fall to supply the ram; or, one fourteenth can be raised and discharged say ten times as high as the fall applied.

Water can be conveyed by a ram 3000 feet, and elevated 200 feet. The drive-pipe is from 25 to 50 feet long.

The following table gives the capacity of several sizes of rams, the dimensions of the pipes to be used, and the size of the spring or brook to which they are adapted:

Size of Ram.	Quantity of Water Furnished per Min. by the Spring or Brook to which the Ram is Adapted.	Caliber of Pipes.		Weight of Pipe (Lead), if Wrought Iron, then of Ordinary Weight.		
		Drive.	Discharge.	Drive-pipe for head or fall not over 10 ft.	Discharge-pipe for not over 50 ft. rise.	Discharge-pipe for over 50 ft. and not exceeding 100 ft. in height.
	Gals. per min.	inch.	inch.	per foot.	per foot.	per foot.
No. 2	¾ to 2	¾	⅜	2 lbs.	10 ozs.	1 lb.
" 3	1½ " 4	1	½	3 "	12 "	1 " 4 ozs.
" 4	3 " 7	1½	½	5 "	12 "	1 " 4 ozs.
" 5	6 " 14	2	¾	8 "	1 lb. 4 "	2 "
" 6	12 " 25	2½	1	13 "	2 "	3 "
" 7	20 " 40	2½	1¼	13 "	3 "	4 "
" 10	25 " 75	4	2	21 "	7 "	8 "

HYDRAULIC-PRESSURE TRANSMISSION.

Water under high pressure (700 to 2000 lbs. per square inch and upwards) affords a very satisfactory method of transmitting power to a distance, especially for the movement of heavy loads at small velocities, as by cranes and elevators. The system consists usually of one or more pumps capable of developing the required pressure; accumulators, which are vertical cylinders with heavily-weighted plungers passing through stuffing-boxes in the upper end, by which a quantity of water may be accumulated at the pressure to which the plunger is weighted; the distributing-pipes; and the presses, cranes, or other machinery to be operated.

The earliest important use of hydraulic pressure probably was in the Bramah hydraulic press, patented in 1796. Sir W. G. Armstrong in 1846 was one of the pioneers in the adaptation of the hydraulic system to cranes. The use of the accumulator by Armstrong led to the extended use of hydraulic machinery. Recent developments and applications of the system are largely due to Ralph Tweddell, of London, and Sir Joseph Whitworth. Sir Henry Bessemer, in his patent of May 18, 1856, No. 1292, first suggested the use of hydraulic pressure for compressing steel ingots while in the fluid state.

The Gross Amount of Energy of the water under pressure stored in the accumulator, measured in foot-pounds, is its volume in cubic feet × its pressure in pounds per square foot. The horse-power of a given quantity steadily flowing is H.P. $= \frac{144pQ}{550} = .2618pQ$, in which Q is the quantity flowing in cubic feet per second and p the pressure in pounds per square inch.

The loss of energy due to velocity of flow in the pipe is calculated as follows (R. G. Blaine, *Eng'g*, May 22 and June 5, 1891):

According to D'Arcy, every pound of water loses $\frac{\lambda 4L}{D}$ times its kinetic energy, or energy due to its velocity, in passing along a straight pipe L feet in length and D feet diameter, where λ is a variable coefficient. For clean cast-iron pipes it may be taken as $\lambda = .005\left(1 + \frac{1}{12D}\right)$, or for diameter in *inches* $= d$.

$d =$	½	1	2	3	4	5	6	7	8	9	10	12
$\lambda =$	.015	.01	.0075	.00667	.00625	.006	.00583	.00571	.00563	.00556	.0055	.00542

The loss of energy per minute is $60 \times 62.36Q \times \frac{\lambda 4L}{D}\frac{v^2}{2g}$, and the horse-power wasted in the pipe is $W = \frac{.6363\lambda L(\text{H.P.})^3}{p^3D^5}$, in which λ varies with the diameter as above. p = pressure at entrance in pounds per square inch. Values of $.6363\lambda$ for different diameters of pipe in inches are:

$d =$	½	1	2	3	4	5	6	7	8	9	10	12
	.00954	.00636	.00477	.00424	.00398	.00382	.00371	.00363	.00358	.00353	.00350	.00345

Efficiency of Hydraulic Apparatus.—The useful effect of a direct hydraulic plunger or ram is usually taken at 93%. The following is given as the efficiency of a ram with chain-and-pulley multiplying gear properly proportioned and well lubricated:

Multiplying....	2 to 1	4 to 1	6 to 1	8 to 1	10 to 1	12 to 1	14 to 1	16 to 1
Efficiency %....	80	76	72	67	63	59	54	50

With large sheaves, small steel pins, and wire rope for multiplying gear the efficiency has been found as high as 66% for a multiplication of 20 to 1.

Henry Adams gives the following formula for effective pressure in cranes and hoists:

P = accumulator pressure in pounds per square inch;
m = ratio of multiplying power;
E = effective pressure in pounds per square inch, including all allowances for friction;

$$E = P(.84 - .02m).$$

J. E. Tuit (*Eng'g*, June 15, 1888) describes some experiments on the friction of hydraulic jacks from 3¼ to 13⅝-inch diameter, fitted with cupped leather packings. The friction loss varied from 5.6% to 18.8% according to the condition of the leather, the distribution of the load on the ram, etc. The friction increased considerably with eccentric loads. With hemp packing a plunger, 14-inch diameter, showed a friction loss of from 11.4% to 8.4%, the load being central, and from 15.0% to 7.6% with eccentric load, the percentage of loss decreasing in both cases with increase of load.

Thickness of Hydraulic Cylinders.—From a table used by Sir W. G. Armstrong we take the following, for cast-iron cylinders, for an interior pressure of 1000 lbs. per square inch:

Diam. of cylinder, inches..	3	4	6	8	10	12	16	20	24
Thickness, inches..........	0.832	1.146	1.552	1.875	2.222	2.578	3.19	3.69	4.11

For any other pressure multiply by the ratio of that pressure to 1000. These figures correspond nearly to the formula $t = 0.175d + 0.48$, in which t = thickness and d = diameter in inches, up to 16 inches diameter, but for 20 inches diameter the addition 0.48 is reduced to 0.19 and at 24 inches it disappears. For formulæ for thick cylinders see page 287, *ante*.

Cast iron should not be used for pressures exceeding 2000 lbs. per square inch. For higher pressures steel castings or forged steel should be used. For working pressures of 750 lbs. per square inch the test pressure should be 2500 lbs. per square inch, and for 1500 lbs. the test pressure should not be less than 3500 lbs.

Speed of Hoisting by Hydraulic Pressure.—The maximum allowable speed for warehouse cranes is 6 feet per second; for platform cranes 4 feet per second; for passenger and wagon hoists, heavy loads, 2 feet per second. The maximum speed under any circumstances should never exceed 10 feet per second.

The Speed of Water Through Valves should never be greater than 100 feet per second.

Speed of Water Through Pipes.—Experiments on water at 1600 lbs. pressure per square inch flowing into a flanging-machine ram, 20-inch diameter, through a ½-inch pipe contracted at one point to ¼-inch, gave a velocity of 114 feet per second in the pipe, and 456 feet at the reduced section. Through a ½-inch pipe reduced to ⅜-inch at one point the velocity was 213 feet per second in the pipe and 381 feet at the reduced section In a ½-inch pipe without contraction the velocity was 355 feet per second.

For many of the above notes the author is indebted to Mr. John Platt, consulting engineer, of New York.

High-pressure Hydraulic Presses in Iron-works are described by R. M. Daelen, of Germany, in Trans. A. I. M. E. 1892. The following distinct arrangements used in different systems of high-pressure hydraulic work are discussed and illustrated:

1. Steam-pump, with fly-wheel and accumulator.
2. Steam-pump, without fly-wheel and with accumulator.
3. Steam-pump, without fly-wheel and without accumulator.

In these three systems the valve-motion of the working press is operated in the high-pressure column. This is avoided in the following:

4. Single-acting steam-intensifier without accumulator.
5. Steam-pump with fly-wheel, without accumulator and with pipe-circuit.
6. Steam-pump with fly-wheel, without accumulator and without pipe-circuit.

The disadvantages of accumulators are thus stated: The weighted plungers which formerly served in most cases as accumulators, cause violent shocks in the pipe-line when changes take place in the movement of the water, so that in many places, in order to avoid bursting from this cause, the pipes are made exclusively of forged and bored steel. The seats and cones of the metallic valves are cut by the water (at high speed), and in such cases only the most careful maintenance can prevent great losses of power.

Hydraulic Power in London.—The general principle involved is pumping water into mains laid in the streets, from which service-pipes are carried into the houses to work lifts or three-cylinder motors when rotatory power is required. In some cases a small Pelton wheel has been tried, working under a pressure of over 700 lbs. on the square inch. Over 55 miles of hydraulic mains are at present laid (1892).

The reservoir of power consists of capacious accumulators, loaded to a pressure of 800 lbs. per square inch, thus producing the same effect as if large supply-tanks were placed at 1700 feet above the street-level. The water is taken from the Thames or from wells, and all sediment is removed therefrom by filtration before it reaches the main engine-pumps.

There are over 1750 machines at work, and the supply is about 6,500,000 gallons per week.

It is essential that the water used should be clean. The storage-tank extends over the whole boiler-house and coal-store. The tank is divided, and a certain amount of mud is deposited here. It then passes through the surface condenser of the engines, and it is turned into a set of filters, eight in number. The body of the filter is a cast-iron cylinder, containing a layer of granular filtering material resting upon a false bottom; under this is the distributing arrangement, affording passage for the air, and under this the real bottom of the tank. The dirty water is supplied to the filters from an overhead tank. After passing through the filters the clean effluent is pumped into the clean-water tank, from which the pumping-engines derive their supply. The cleaning of the filters, which is done at intervals of 24 hours, is effected so thoroughly *in situ* that the filtering material never requires to be removed.

The engine-house contains six sets of triple-expansion engines. The cylinders are 15-inch, 22-inch, 36-inch × 24-inch. Each cylinder drives a single plunger-pump with a 5-inch ram, secured directly to the cross-head, the connecting-rod being double to clear the pump. The boiler-pressure is 150 lbs. on the square inch. Each pump will deliver 300 gallons of water per minute under a pressure of 800 lbs. to the square inch, the engines making about 61 revolutions per minute. This is a high velocity, considering the heavy pressure; but the valves work silently and without perceptible shock.

The consumption of steam is 14.1 pounds per horse per hour.

The water delivered from the main pumps passes into the accumulators. The rams are 20 inches in diameter, and have a stroke of 23 feet. They are each loaded with 110 tons of slag, contained in a wrought-iron cylindrical box suspended from a cross-head on the top of the ram.

One of the accumulators is loaded a little more heavily than the other, so that they rise and fall successively; the more heavily loaded actuates a stop-valve on the main steam-pipe. If the engines supply more water than is wanted, the lighter of the two rams first rises as far as it can go; the other then ascends, and when it has nearly reached the top, shuts off steam and checks the supply of water automatically.

The mains in the public streets are so constructed and laid as to be perfectly trustworthy and free from leakage.

Every pipe and valve used throughout the system is tested to 2500 lbs. per square inch before being placed on the ground and again tested to a reduced pressure in the trenches to insure the perfect tightness of the joints. The jointing material used is gutta-percha.

The average rate obtained by the company is about 3 shillings per thousand gallons. The principal use of the power is for intermittent work in cases where direct pressure can be employed, as, for instance, passenger elevators, cranes, presses, warehouse hoists, etc.

An important use of the hydraulic power is its application to the extinguishing of fire by means of Greathead's injector hydrant. By the use of these hydrants a continuous fire-engine is available.

Hydraulic Riveting-machines.—Hydraulic riveting was introduced in England by Mr. R. H. Tweddell. Fixed riveters were first used about 1868. Portable riveting-machines were introduced in 1872.

The riveting of the large steel plates in the Forth Bridge was done by small portable machines working with a pressure of 1000 lbs. per square inch. In exceptional cases 3 tons per inch was used. (Proc. Inst. M. E., May, 1889.)

An application of hydraulic pressure invented by Andrew Higginson, of Liverpool, dispenses with the necessity of accumulators. It consists of a three-throw pump driven by shafting or worked by steam, and depends partially upon the work accumulated in a heavy fly-wheel. The water in its passage from the pumps and back to them is in constant circulation at a very feeble pressure, requiring a minimum of power to preserve the tube of water ready for action at the desired moment, when by the use of a tap the current is stopped from going back to the pumps, and is thrown upon the piston of the tool to be set in motion. The water is now confined, and the driving-belt or steam-engine, supplemented by the momentum of the heavy fly-wheel, is employed in closing up the rivet, or bending or forging the object subjected to its operation.

Hydraulic Forging.—In the production of heavy forgings from cast ingots of mild steel it is essential that the mass of metal should be operated on as equally as possible throughout its entire thickness. When employing a steam-hammer for this purpose it has been found that the external surface of the ingot absorbs a large proportion of the sudden impact of the blow, and that a comparatively small effect only is produced on the central portions of the ingot, owing to the resistance offered by the inertia of the mass to the rapid motion of the falling hammer—a disadvantage that is entirely overcome by the slow, though powerful, compression of the hydraulic forging-press, which appears destined to supersede the steam-hammer for the production of massive steel forgings.

In the Allen forging-press the force-pump and the large or main cylinder of the press are in direct and constant communication. There are no intermediate valves of any kind, nor has the pump any clack-valves, but it simply forces its cylinder full of water direct into the cylinder of the press, and receives the same water, as it were, back again on the return stroke. Thus, when both cylinders and the pipe connecting them are full, the large ram of the press rises and falls simultaneously with each stroke of the pump, keeping up a continuous oscillating motion, the ram, of course, travelling the shorter distance, owing to the larger capacity of the press cylinder. (Journal Iron and Steel Institute, 1891. See also illustrated article in "Modern Mechanism," page 668.)

For a very complete illustrated account of the development of the hydraulic forging-press, see a paper by R. H. Tweddell in Proc. Inst. C. E., vol. cxvii. 1893-4.

Hydraulic Forging-press.—A 2000-ton forging-press erected at the Couillet forges in Belgium is described in *Eng. and M. Jour.*, Nov. 25, 1893. The press is composed essentially of two parts—the press itself and the compressor. The compressor is formed of a vertical steam-cylinder and a hydraulic cylinder. The piston-rod of the former forms the piston of the latter. The hydraulic piston discharges the water into the press proper. The distribution is made by a cylindrical balanced valve; as soon as the pressure is released the steam-piston falls automatically under the action of gravity. During its descent the steam passes to the other face of the piston to reheat the cylinder, and finally escapes from the upper end.

When steam enters under the piston of the compressor-cylinder the piston rises, and its rod forces the water into the press proper. The pressure thus exerted on the piston of the latter is transmitted through a cross-head to the forging which is upon the anvil. To raise the cross-head two small single-acting steam-cylinders are used, their piston-rods being connected to the cross-head; steam acts only on the pistons of these cylinders from below. The admission of steam to the cylinders, which stand on top of the press frame, is regulated by the same lever which directs the motions of the compressor. The movement given to the dies is sufficient for all the ordinary purposes of forging.

A speed of 80 blows per minute has been attained. A double press on the same system, having two compressors and giving a maximum pressure of 6000 tons, has been erected in the Krupp works, at Essen.

The Aiken Intensifier. (*Iron Age*, Aug. 1890.)—The object of the machine is to increase the pressure obtained by the ordinary accumulator which is necessary to operate powerful hydraulic machines requiring very high pressures, without increasing the pressure carried in the accumulator and the general hydraulic system.

The Aiken Intensifier consists of one outer stationary cylinder and one inner cylinder which moves in the outer cylinder and on a fixed or stationary hollow plunger. When operated in connection with the hydraulic bloom-shear the method of working is as follows: The inner cylinder having been filled with water and connected through the hollow plunger with the hydraulic cylinder of the shear, water at the ordinary accumulator-pressure is admitted into the outer cylinder, which being four times the sectional area of the plunger gives a pressure in the inner cylinder and shear cylinder connected therewith of four times the accumulator-pressure—that is, if the accumulator-pressure is 500 lbs. per square inch the pressure in the intensifier will be 2000 lbs. per square inch.

Hydraulic Engine driving an Air-compressor and a Forging-hammer. (*Iron Age*, May 12, 1892.)—The great hammer in Terni, near Rome, is one of the largest in existence. Its falling weight amounts to 100 tons, and the foundation belonging to it consists of a block of cast iron of 1000 tons. The stroke is 16 feet 4¾ inches; the diameter of the cylinder 6 feet 3½ inches; diameter of piston-rod 13¾ inches; total height of the hammer, 62 feet 4 inches. The power to work the hammer, as well as the two cranes of 100 and 150 tons respectively, and other auxiliary appliances belonging to it, is furnished by four air-compressors coupled together and driven directly by water-pressure engines, by means of which the air is compressed to 73.5 pounds per square inch. The cylinders of the water-pressure engines, which are provided with a bronze lining, have a 13¾-inch bore. The stroke is 47¼ inches, with a pressure of water on the piston amounting to 264.6 pounds per square inch. The compressors are bored out to 31½ inches diameter, and have 47¼-inch stroke. Each of the four cylinders requires a power equal to 280 horse-power. The compressed air is delivered into huge reservoirs, where a uniform pressure is kept up by means of a suitable water-column.

The Hydraulic Forging Plant at Bethlehem, Pa., is described in a paper by R. W. Davenport, read before the Society of Naval Engineers and Marine Architects, 1893. It includes two hydraulic forging-presses complete, with engines and pumps, one of 1500 and one of 4500 tons capacity, together with two Whitworth hydraulic travelling forging-cranes and other necessary appliances for each press; and a complete fluid-compression plant, including a press of 7000 tons capacity and a 125 ton hydraulic travelling crane for serving it (the upper and lower heads of this press weighing respectively about 135 and 120 tons).

A new forging-press has been designed by Mr. John Fritz, for the Bethlehem Works, of 14,000 tons capacity, to be run by engines and pumps of 15,000 horse power. The plant is served by four open-hearth steel furnaces of a united capacity of 120 tons of steel per heat.

Some References on Hydraulic Transmission.—Reuleaux's "Constructor;" "Hydraulic Motors, Turbines, and Pressure-engines," G. Bodmer, London, 1889; Robinson's "Hydraulic Power and Hydraulic Machinery," London, 1888; Colyer's "Hydraulic Steam, and Hand-power Lifting and Pressing Machinery," London, 1881. See also *Engineering* (London), Aug. 1, 1884, p. 99, March 13, 1885, p. 262; May 22 and June 5, 1891, pp. 612, 665; Feb. 19, 1892, p. 25; Feb. 10, 1893, p. 170.

FUEL.

Theory of Combustion of Solid Fuel. (From Rankine, somewhat altered.)—The ingredients of every kind of fuel commonly used may be thus classed: (1) Fixed or free carbon, which is left in the form of charcoal or coke after the volatile ingredients of the fuel have been distilled away. These ingredients burn either wholly in the solid state (C to CO_2), or part in the solid state and part in the gaseous state ($CO + O = CO_2$), the latter part being first dissolved by previously formed carbonic acid by the reaction $CO_2 + C = 2CO$. Carbonic oxide, CO, is produced when the supply of air to the fire is insufficient.

(2) Hydrocarbons, such as olefiant gas, pitch, tar, naphtha, etc., all of which must pass into the gaseous state before being burned.

If mixed on their first issuing from amongst the burning carbon with a large quantity of hot air, these inflammable gases are completely burned with a transparent blue flame, producing carbonic acid and steam. When mixed with cold air they are apt to be chilled and pass off unburned. When raised to a red heat, or thereabouts, before being mixed with a sufficient quantity of air for perfect combustion, they disengage carbon in fine powder, and pass to the condition partly of marsh gas, and partly of free hydrogen; and the higher the temperature, the greater is the proportion of carbon thus disengaged.

If the disengaged carbon is cooled below the temperature of ignition before coming in contact with oxygen, it constitutes, while floating in the gas, smoke, and when deposited on solid bodies, soot.

But if the disengaged carbon is maintained at the temperature of ignition, and supplied with oxygen sufficient for its combustion, it burns while floating in the inflammable gas, and forms red, yellow, or white flame. The flame from fuel is the larger the more slowly its combustion is effected. The flame itself is apt to be chilled by radiation, as into the heating surface of a steam-boiler, so that the combustion is not completed, and part of the gas and smoke pass off unburned.

(3) Oxygen or hydrogen either actually forming water, or existing in combination with the other constituents in the proportions which form water. Such quantities of oxygen and hydrogen are to left be out of account in determining the heat generated by the combustion. If the quantity of water actually or virtually present in each pound of fuel is so great as to make its latent heat of evaporation worth considering, that heat is to be deducted from the total heat of combustion of the fuel.

(4) Nitrogen, either free or in combination with other constituents. This substance is simply inert.

(5) Sulphuret of iron, which exists in coal and is detrimental, as tending to cause spontaneous combustion.

(6) Other mineral compounds of various kinds, which are also inert, and form the ash left after complete combustion of the fuel, and also the clinker or glassy material produced by fusion of the ash, which tends to choke the grate.

Total Heat of Combustion of Fuels. (Rankine.)—The following table shows the total heat of combustion with oxygen of one pound of each of the substances named in it, in British thermal units, and also in lbs. of water evaporated from 212°. It also shows the weight of oxygen required to combine with each pound of the combustible and the weight of air necessary in order to supply that oxygen. The quantities of heat are given on the authority of MM. Favre and Silbermann.

Combustible.	Lbs. Oxygen per lb. Combustible.	Lb. Air (about).	Total British Heat-units.	Evaporative Power from 212° F., lbs.
Hydrogen gas	8	36	62,032	64.2
Carbon imperfectly burned so as to make carbonic oxide	1⅓	6	4,400	4.55
Carbon perfectly burned so as to make carbonic acid	2⅔	12	14,500	15.0
Olefiant gas, 1 lb	3 3/7	15 3/7	21,344	22.1
Various liquid hydrocarbons, 1 lb.	....	......	from 21,700 to 19,000	from 22½ to 20
Carbonic oxide, as much as is made by the imperfect combustion of 1 lb. of carbon, viz., 2⅓ lbs	1⅓	6	10,000	10.45

The imperfect combustion of carbon, making carbonic oxide, produces less than one third of the heat which is yielded by the complete combustion.

The total heat of combustion of any compound of hydrogen and carbon is nearly the sum of the quantities of heat which the constituents would produce separately by their combustion. (Marsh-gas is an exception.)

In computing the total heat of combustion of compounds containing oxygen as well as hydrogen and carbon, the following principle is to be observed: When hydrogen and oxygen exist in a compound in the proper proportion to form water (that is, by weight one part of hydrogen to eight of oxygen), these constituents have no effect on the total heat of combustion. If hydrogen exists in a greater proportion, only the surplus of hydrogen above that which is required by the oxygen is to be taken into account.

The following is a general formula (Dulong's) for the total heat of combustion of any compound of carbon, hydrogen, and oxygen:

Let C, H, and O be the fractions of one pound of the compound, which consists respectively of carbon, hydrogen, and oxygen, the remainder being nitrogen, ash, and other impurities. Let h be the total heat of combustion of one pound of the compound in British thermal units. Then

$$h = 14,500\left\{C + 4.28\left(H - \frac{O}{8}\right)\right\}.$$

The following table shows the composition of those compounds which are of importance, either as furnishing oxygen for combustion, as entering into the composition, or as being produced by the combustion of fuel:

Names.	Symbol of Chemical Composition.	Proportions of Element by Weight.	Chemical Equivalent by Weight.	Proportions of Elements by Volume.
Air	......	N 77 + O 23	100	N 79 + O 21
Water	H_2O	H 2 + O 16	18	H 2 + O
Ammonia	NH_3	H 3 + N 14	17	H 3 + N
Carbonic oxide	CO	C 12 + O 16	28	C + O
Carbonic acid	CO_2	C 12 + O 32	44	C + O 2
Olefiant gas	CH_2	C 12 + H 2	14	C + H 2
Marsh-gas or fire-damp	CH_4	C 12 + H 4	16	C + H 4
Sulphurous acid	SO_2	S 32 + O 32	64	....
Sulphuretted hydrogen	SH_2	S 32 + H 2	34	....
Sulphuret of carbon	S_2C	S 64 + C 12	76	....

Since each lb. of C requires 2⅔ lbs. of O to burn it to CO_2, and air contains 23% of O, by weight, 2⅔ ÷ 0.23 or 11.6 lbs. of air are required to burn 1 lb. of C.

Analyses of Gases of Combustion.—The following are selected from a large number of analyses of gases from locomotive boilers, to show the range of composition under different circumstances (P. H. Dudley, Trans. A. I. M. E., iv. 250):

Test.	CO_2	CO	O	N	
1	13.8	2.5	2.5	81.0	No smoke visible.
2	11.5	..	6	82.5	Old fire, escaping gas white, engine working hard.
3	8.5	...	8	83	Fresh fire, much black gas, " " "
4	2.3	....	17.2	80.5	Old fire, damper closed, engine standing still.
5	5.7	....	14.7	79.6	" " smoke white, engine working hard.
6	8.4	1.2	8.4	82	New fire, engine not working hard.
7	12	1	4.4	82.6	Smoke black, engine not working hard.
8	3.4	...	16.8	76.8	" dark, blower on, engine standing still.
9	6	...	13.5	81.5	" white, engine working hard.

In analyses on the Cleveland and Pittsburgh road, in every instance when the smoke was the blackest, there was found the greatest percentage of unconsumed oxygen in the product, showing that something besides the mere presence of oxygen is required to effect the combustion of the volatile carbon of fuels.

J. C. Hoadley (Trans. A. S. M. E., vi. 749) found as the mean of a great number of analyses of flue gases from a boiler using anthracite coal:

CO_2, 13.10; CO, 0.30; O, 11.94; N, 74.66.

The loss of heat due to burning C to CO instead of to CO_2 was 2.13%. The surplus oxygen averaged 113.3% of the O required for the C of the fuel, the average for different weeks ranging from 88.6% to 137%.

Analyses made to determine the CO produced by excessively rapid firing gave results from 2.54% to 4.81% CO and 5.12 to 8.01% CO_2; the ratio of C in the CO to total carbon burned being from 43.80% to 48.55%, and the number of pounds of air supplied to the furnace per pound of coal being from 33.2 to 19.3 lbs. The loss due to burning C to CO was from 27.84% to 30.86 of the full power of the coal.

Temperature of the Fire. (Rankine, S. E., p. 283.)—By temperature of the fire is meant the temperature of the products of combustion at the instant that the combustion is complete. The elevation of that temperature above the temperature at which the air and the fuel are supplied to the furnace may be computed by dividing the total heat of combustion of one lb. of fuel by the weight and by the mean specific heat of the whole products of combustion, and of the air employed for their dilution under constant pressure. The specific heat under constant pressure of these products is about as follows:

Carbonic-acid gas, 0.217; steam, 0.475; nitrogen (probably), 0.245; air, 0.238; ashes, probably about 0.200. Using these data, the following results are obtained for pure carbon and for olefiant gas burned, respectively, first, in just sufficient air, theoretically, for their combustion, and, second, when an equal amount of air is supplied in addition for dilution.

Fuel.	Products undiluted.		Products diluted.	
	Carbon.	Olefiant Gas.	Carbon.	Olefiant Gas.
Total heat of combustion, per lb...	14,500	21,300	14,500	21,300
Wt. of products of combustion, lbs.	13	16.43	25	31.86
Their mean specific heat..........	0.237	0.257	0.238	0.248
Specific heat × weight..............	3.08	4.22	5.94	7.9
Elevation of temperature, F.........	4580°	5050°	2440°	2710°

[The above calculations are made on the assumption that the specific heats of the gases are constant, but they probably increase with the increase of temperature (see Specific Heat), in which case the temperature would be less than those above given. The temperature would be further reduced by the heat rendered latent by the conversion into steam of any water present in the fuel.]

Rise of Temperature in Combustion of Gases. (*Eng'g*, March 12 and April 2, 1886.)—It is found that the temperatures obtained by experiment fall short of those obtained by calculation. Three theories have been given to account for this: 1. The cooling effect of the sides of the containing vessel; 2. The retardation of the evolution of heat caused by dissociation; 3. The increase of the specific heat of the gases at very high temperatures. The calculated temperatures are obtainable only on the condition that the gases shall combine instantaneously and simultaneously throughout their whole mass. This condition is practically impossible in experiments. The gases formed at the beginning of an explosion dilute the remaining combustible gases and tend to retard or check the combustion of the remainder.

CLASSIFICATION OF SOLID FUELS.

Gruner classifies solid fuels as follows (*Eng'g and M'g Jour.*, July, 1874):

Name of Fuel.	Ratio $\frac{O}{H}$ or $\frac{O+N^*}{H}$.	Proportion of Coke or Charcoal yielded by the Dry Pure Fuel.
Pure cellulose	8	0.28 @ 0.30
Wood (cellulose and encrusting matter)	7	.30 @ .35
Peat and fossil fuel	6 @ 5	.35 @ .40
Lignite,† or brown coal	5	.40 @ .50
Bituminous coals	4 @ 1	.50 @ .90
Anthracite	1 @ 0.75	.90 @ .92

The bituminous coals he divides into five classes as below:

Name of Type.	Elementary Composition. C.	H.	O.	Ratio $\frac{O}{H}$ or $\frac{O+N^*}{H}$	Proportion of Coke yielded by Distillation.	Nature and Appearance of Coke.
1. Long flaming dry coal,	75@80	5.5@4.5	19.5@15	4@3	0.50@.60	Pulverulent.
2. Long flaming fat or coking coals, or gas coals,	80@85	5.8@5	14.2@10	3@2	.60@.68	Melted, but friable.
3. Caking fat coals, or blacksmiths' coals,	84@89	5 @4.5	11 @5.5	2@1	.68@.74	Melted; somewhat compact.
4. Short flaming fat or caking coals, coking coals,	88@91	5.5@4.5	6.5@5.5	1	.74@.82	Melted; very compact.
5. Lean or anthracitic coals,	90@93	4.5@4	5.5@3	1	.82@.90	Pulverulent.

* The nitrogen rarely exceeds 1 per cent of the weight of the fuel.

† Not including bituminous lignites, which resemble petroleums.

Rankine gives the following: The extreme differences in the chemical composition and properties of different kinds of coal are very great. The proportion of free carbon ranges from 30 to 93 per cent; that of hydrocarbons of various kinds from 5 to 58 per cent; that of water, or oxygen and hydrogen in the proportions which form water, from an inappreciably small quantity to 27 per cent; that of ash, from 1½ to 26 per cent.

The numerous varieties of coal may be divided into principal classes as follows: 1, anthracite coal; 2, semi-bituminous coal; 3, bituminous coal; 4, long flaming or cannel coal; 5, lignite or brown coal.

Diminution of H and O in Series from Wood to Anthracite.

(Groves and Thorp's Chemical Technology, vol. i., Fuels, p. 58.)

Substance.	Carbon.	Hydrogen.	Oxygen.
Woody fibre	52.65	5.25	42.10
Peat from Vulcaire	59.57	5.96	34.47
Lignite from Cologne	66.04	5.27	28.69
Earthy brown coal	73.18	5.88	21.14
Coal from Belestat, secondary	75.06	5.84	19.10
Coal from Rive de Gier	89.29	5.05	5.66
Anthracite, Mayenne, transition formation	91.58	3.96	4.46

Progressive Change from Wood to Graphite.

(J. S. Newberry in Johnson's Cyclopedia.)

	Wood.	Loss.	Lignite.	Loss.	Bituminous coal.	Loss.	Anthracite.	Loss.	Graphite.
Carbon	49.1	18.65	30.45	12.35	18.10	3.57	14.53	1.42	13.11
Hydrogen	6.3	3.25	3.05	1.85	1.20	0.93	0.27	0.14	0.13
Oxygen	44.6	24.40	20.20	18.13	2.07	1.32	0.65	0.65	0.00
	100.0	46 30	53.70	32.33	21.37	5.82	15.45	2.21	13.24

Classification of Coals, as Anthracite, Bituminous, etc.—Prof. Persifer Frazer (Trans. A. I. M. E., vi, 430) proposes a classification of coals according to their "fuel ratio," that is, the ratio the fixed carbon bears to the volatile hydrocarbon.

In arranging coals under this classification, the accidental impurities, such as sulphur, earthy matter, and moisture, are disregarded, and the fuel constituents alone are considered.

	Carbon Ratio.	Fixed Carbon.	Volatile Hydrocarbon.
I. Hard dry anthracite	100 to 12	100. to 92.31%	0. to 7 69%
II. Semi-anthracite	12 to 8	92.31 to 88.89	7.69 to 11.11
III. Semi-bituminous	8 to 5	88.89 to 83.33	11.11 to 16.67
IV. Bituminous	5 to 0	83.33 to 0.	16.67 to 100

It appears to the author that the above classification does not draw the line at the proper point between the semi-bituminous and the bituminous coals, viz., at a ratio of C ÷ V.H.C. = 5, or fixed carbon 83.33%, volatile hydrocarbon 16.67%, since it would throw many of the steam coals of Clearfield and Somerset counties, Penn., and the Cumberland, Md., and Pocahontas, Va., coals, which are practically of one class, and properly rated as semi-bituminous coals, into the bituminous class. The dividing line between the semi-anthracite and semi-bituminous coals, C ÷ V.H.C. = 8, would place several coals known as semi-anthracite in the semi-bituminous class. The following is proposed by the author as a better classification:

	Carbon Ratio.	Fixed Carbon.	Vol. H.C.
I. Hard dry anthracite	100 to 12	100 to 92.31%	0 to 7.69%
II. Semi-anthracite	12 to 7	92.31 to 87.5	7.69 to 12.5
III. Semi-bituminous	7 to 3	87.5 to 75	12.5 to 25
IV. Bituminous	3 to 0	75 to 0	25 to 100

Rhode Island Graphitic Anthracite.—A peculiar graphite is found at Cranston, near Providence, R. I. It resembles both graphite and anthracite coal, and has about the following composition (A. E. Hunt, Trans. A. I. M. E., xvii., 678): Graphitic carbon, 78%; volatile matter, 2.60%; silica, 15.06%; phosphorus, .045%. It burns with extreme difficulty.

ANALYSES OF COALS.

Composition of Pennsylvania Anthracites. (Trans. A. I. M. E., xiv., 706.)—Samples weighing 100 to 200 lbs. were collected from lots of 100 to 200 tons as shipped to market, and reduced by proper methods to laboratory samples. Thirty-three samples were analyzed by McCreath, giving results as follows. They show the mean character of the coal of the more important coal-beds in the Northern field in the vicinity of Wilkesbarre, in the Eastern Middle (Lehigh) field in the vicinity of Hazleton, in the Western

Middle field in the vicinity of Shenandoah, and in the Southern field between Mauch Chunk and Tamaqua.

Name of Bed.	Name of Field.	Water.	Volatile Matter.	Fixed Carbon.	Ash.	Sulphur.	Vol. Matter. Per cent of total combustible.	Ratio C ÷ V.H.C.
Wharton...	E. Middle	3.71	3.08	86.40	6.22	.58	3.44	28.07
Mammoth..	E. Middle	4.12	3.08	86.38	5.92	.49	3.45	27.99
Primrose ..	W. Middle	3.54	3.72	81.59	10.65	.50	4.36	21.93
Mammoth .	W. Middle	3.16	3.72	81.14	11.08	.90	4.38	21.88
Primrose F	Southern	3.01	4.13	87.98	4.38	.50	4.48	21.32
Buck Mtn..	W. Middle	3.04	3.95	82.66	9.88	.46	4.56	20.93
Seven Foot	W. Middle	3.41	3.98	80.87	11.23	.51	4.69	20.32
Mammoth .	Southern	3.09	4.28	83.81	8.18	.64	4.85	19.62
Mammoth .	Northern	3.42	4.38	83.27	8.20	.73	5.00	19.00
B. Coal Bed	Loyalsock	1.30	8.10	83.34	6.23	1.03	8.86	10.29

The above analyses were made of coals of all sizes (mixed). When coal is screened into sizes for shipment the purity of the different sizes as regards ash varies greatly. Samples from one mine gave results as follows:

Name of Coal.	Screened Through inches.	Screened Over inches.	Analyses. Fixed Carbon.	Analyses. Ash.
Egg	2.5	1.75	88.49	5.66
Stove.........	1.75	1.25	83.67	10.17
Chestnut......	1.25	.75	80.72	12.67
Pea....... ...	.75	.50	79 05	14.66
Buckwheat...	.50	.25	76.92	16.62

Bernice Basin, Pa., Coals.

	Water.	Vol. H.C.	Fixed C.	Ash.	Sulphur.
Bernice Basin, Sullivan and Lycoming Cos.; range of 8..	0.96 to 1.97	3.56 to 8.56	82.52 to 89.39	3.27 to 9.34	0.24 to 1.04

This coal is on the dividing-line between the anthracites and semi-anthracites, and is similar to the coal of the Lykens Valley district.

More recent analyses (Trans. A. I. M. E., xiv. 721) give:

	Water.	Vol. H.C.	Fixed Carb.	Ash.	Sulphur.
Working seam.......	0 65	9.40	83.69	5.34	0.91
60 ft. below seam....	3.67	15.42	71.34	8.97	0.59

The first is a semi-anthracite, the second a semi-bituminous.

Space Occupied by Anthracite Coal. (J. C. I. W., vol. iii.)—The cubic contents of 2240 lbs. of hard Lehigh coal is a little over 36 feet; an average Schuylkill W. A., 37 to 38 feet; Shamokin, 38 to 39 feet; Lorberry, nearly 41.

According to measurements made with Wilkesbarre anthracite coal from the Wyoming Valley, it requires 32.2 cu. ft. of lump, 33.9 cu. ft. broken, 34.5 cu. ft. egg, 34.8 cu. ft. of stove, 35.7 cu. ft. of chestnut, and 36.7 cu. ft. of pea, to make one ton of coal of 2240 lbs.; while it requires 28.8 cu. ft. of lump, 30.3 cu. ft. of broken, 30.8 cu. ft. of egg, 31.1 cu. ft. of stove, 31.9 cu. ft. of chestnut, and 32.8 cu. ft. of pea, to make one ton of 2000 lbs.

Composition of Anthracite and Semi-bituminous Coals. (Trans. A. I. M. E., vi. 430.)—Hard dry anthracites, 16 analyses by Rogers, show a range from 94.10 to 82.47 fixed carbon, 1.40 to 9.58 volatile matter, and 4.50 to 8.00 ash, water, and impurities. Of the fuel constituents alone, the fixed carbon ranges from 98.53 to 89.63, and the volatile matter from 1.47 to 10.37, the corresponding carbon ratios, or C ÷ Vol. H.C. being from 67.02 to 8.64.

Semi-anthracites.—12 analyses by Rogers show a range of from 90.23 to 74.55 fixed carbon, 7.07 to 13.75 volatile matter, and 2.20 to 12.10 water, ash, and impurities. Excluding the ash, etc., the range of fixed carbon is 92.75 to 84.42, and the volatile combustible 7.27 to 15.58, the corresponding carbon ratio being from 12.75 to 5.41.

Semi-bituminous Coals.—10 analyses of Penna. and Maryland coals give fixed carbon 68.41 to 84.80, volatile matter 11.2 to 17.28, and ash, water, and impurities 4 to 13.99. The percentage of the fuel constituents is fixed carbon 79.84 to 88.80, volatile combustible 11.20 to 20.16, and the carbon ratio 11.41 to 3.96.

American Semi-bituminous and Bituminous Coals.

(Selected chiefly from various papers in Trans. A. I. M. E.)

	Moisture.	Vol. Hydrocarbon.	Fixed Carbon	Ash.	Sulphur.
Penna. Semi-bituminous:					
Broad Top, extremes of 5	.79	18.84	78.46	6.00	.91
	.78	17.88	76.14	4.81	.88
Somerset Co., extremes of 5	1.27	14.83	77.77	6.63	0.66
	1.89	18.51	65.90	10.62	3.08
Blair Co., average of 5	1.07	26.72	60.77	9.45	2.20
Cambria Co., average of 7, lower bed, B.	0.74	21.21	68.94	7.51	1.98
Cambria Co., 1, upper bed, C.	1.14	17.18	73.42	6.58	1.41
Cambria Co., South Fork, 1	....	15.51	78.60	5.84	....
Centre Co., 1	0.60	22.60	68.71	5.40	2.69
Clearfield Co., average of 9, upper bed, C.	0.70	23.94	69.28	4.62	1.42
Clearfield Co., average of 8, lower bed, D.	0.81	21.10	74.08	3.36	0.42
Clearfield Co., range of 17 anal.	0.41 to 1.94	20.09 to 25.19	66.69 to 74.02	2.65 to 7.65	0.43 to 1.79
Bituminous:					
Jefferson Co., average of 26	1.21	32.58	60.99	3.76	1.00
Clarion Co., average of 7	1.97	38.60	54.15	4.10	1.19
Armstrong Co., 1	1.18	42.55	49.69	4.58	2.00
Connellsville Coal	1.26	30.10	59.61	8.23	.78
Coke from Conn'ville (Standard)	.49	0.01	87.46	11.32	.69
Youghiogheny Coal	1.03	36.49	59.05	2.61	.81
Pittsburgh, Ocean Mine	.28	39.09	57.33	3.30	....

The percentage of volatile matter in the Kittaning lower bed B and the Freeport lower bed D increases with great uniformity from east to west; thus

	Volatile Matter.	Fixed Carbon.
Clearfield Co, bed D	20.09 to 25.19	68.78 to 74.76
" " " B	22.56 to 26.13	64.87 to 69.63
Clarion Co., " B	35.70 to 42.55	47.51 to 55.44
" " D	37.15 to 40.60	51.39 to 56.36

Connellsville Coal and Coke. (Trans. A. I. M. E., xiii. [illegible].)—The Connellsville coal-field, in the southwestern part of Pennsylvania, is a strip about 3 miles wide and 60 miles in length. The mine workings are confined to the Pittsburgh seam, which here has its best development as to size, and its quality best adapted to coke-making. It generally affords from 7 to 8 feet of coal.

The following analyses by T. T. Morrell show about its range of composition:

	Moisture.	Vol. Mat.	Fixed C.	Ash.	Sulphur.	Phosph's.
Herold Mine	1.26	28.83	60.79	8.44	.67	.013
Kintz Mine.	0.79	31.91	56.46	9.52	1.32	.02

In comparing the composition of coals across the Appalachian field, in the western section of Pennsylvania, it will be noted that the Connellsville variety occupies a peculiar position between the rather dry semi-bituminous coals eastward of it and the fat bituminous coals flanking it on the west.

Beneath the Connellsville or Pittsburgh coal-bed occurs an interval of from 400 to 600 feet of "barren measures," separating it from the lower productive coal measures of Western Pennsylvania. The following tables show the great similarity in composition in the coals of these upper and lower coal-measures in the same geographical belt or basin.

Analyses from the Upper Coal-measures (Penna.) in a Westward Order.

Localities.	Moisture.	Vol. Mat.	Fixed Carb.	Ash.	Sulphur.
Anthracite	1.35	3.45	89.06	5.81	0.30
Cumberland, Md	0.89	15.52	74.28	9.29	0.71
Salisbury, Pa	1.66	22.35	68.77	5.96	1.24
Connellsville, Pa	....	31.38	60.30	7.24	1.09
Greensburg, Pa	1.02	33.50	61.34	2.28	0.86
Irwin's, Pa	1.41	37.66	54.44	5.86	0.64

Analyses from the Lower Coal-measures in a Westward Order.

Localities.	Moisture.	Vol. Mat.	Fixed Carb.	Ash.	Sulphur.
Anthracite	1.35	3.45	89.06	5.81	0.30
Broad Top	0.77	18.18	73.34	6.69	1.02
Bennington	1.40	27.23	61.84	6.93	2.60
Johnstown	1.18	16.54	74.46	5.96	1.86
Blairsville	0.92	24.36	62.22	7.69	4.92
Armstrong Co	0.96	38.20	52.08	5.14	3.66

Pennsylvania and Ohio Bituminous Coals. Variation in Character of Coals of the same Beds in different Districts.—From 50 analyses in the reports of the Pennsylvania Geological Survey, the following are selected. They are divided into different groups, and the extreme analysis in each group is given, ash and other impurities being neglected, and the percentage in 100 of combustible matter being alone considered.

	No. of Analyses	Fixed Carbon	Vol. H. C.	Carbon Ratio.
Waynesburg coal-bed, upper bench	5			
Jefferson township, Greene Co		59.72	40.28	1.48
Hopewell township, Washington Co		53.22	46.78	1.13
Waynesburg coal-bed, lower bench	9			
Morgan township, Greene Co		60.69	39.31	1.54
Pleasant Valley, Washington Co		54.31	45.69	1.19
Sewickley coal-bed	3			
Whitely Creek, Greene Co		64.39	35.61	1.80
Gray's Bank Creek, Greene Co.		60.35	39.65	1.52
Pittsburgh coal-bed:				
Upper bench, Washington Co		60.87	39.13	1.65
		59.11	40.89	1.20
Lower bench, " "	5	63.54	36.46	1.74
		50.97	49.03	1.04
Main bench, Greene Co	3	61.80	38.20	1.61
		54.33	45.67	1.19
Frick & Co., Washington Co., average		66.44	33.56	1.98
Lower bench, Greene Co	1	57.83	42.17	1.37
Somerset Co., semi-bituminous (showing decrease of vol. mat. to the eastward).	8	79.73	20.27	3.93
		75.47	24.53	3.07
Beaver Co., Pa	7			
Diehl's Bank, Georgetown		40.68	59.32	0.68
Bryan's Bank, Georgetown		62.57	37.43	1.66
OHIO.				
Pittsburgh coal-bed in Ohio:				
Jefferson Co., Ohio		61.45	38.55	1.59
Belmont Co., Ohio		63.46	36.54	1.73
		66.14	33.86	1.95
Harrison Co., Ohio		63.46	36.54	1.73
		64.93	35.07	1.85
Pomeroy Co., Ohio		60.92	39.08	1.55
		62.33	37.67	1.65

Analyses of Southern and Western Coals.

	Moisture.	Vol. Mat.	Fixed C.	Ash.	Sulphur.
OHIO.					
Hocking Valley	5.00	32.80	53.15	9.05	0.44
	7.40	29.20	60.45	2.95	0.93
MARYLAND.					
Cumberland	.95	19.18	72.70	6.40	0.78
	1.23	15.47	73.51	9.09	0.70
VIRGINIA.					
South of James River, 23 analyses, range	from 0.67	27.28	46.70	2.00	0.56
	to 2.46	38.60	67.83	15.76	2.89
Average of 23	1.48	32.24	58.89	7.72	1.45
North of James River, eastern outcrop,	0.40	18.60	71.00	10.00	
	1.79	23.96	59.98	14.28	
Carbonite or Natural Coke	1.57	9.64	79.93	8.86	
	1.56	14.26	81.61	2.24	0.23
Western outcrop, 11 analyses, range	from	21.33	54.97	3.35	
	to ..	30.50	70.80	22.60	
Average of 11		26.06	63.75	10.06	
Pocahontas Flat-top* (Castner & Curran's Circular)	0.52	23.90	74.20	1.38	0.52
	0.62	18.48	75.22	5.68	0.23
WEST VIRGINIA (New River.)					
Quinnimont,† 3 analyses	from 0.76	17.57	75.89	1.11	0.23
	to 0.94	18.19	79.40	4.92	0.30
Nuttalburgh †	0.34	29.59	69.00	1.07	
	1.85	25.35	70.67	2.10	0.06
VIRGINIA and KENTUCKY.					
Big Stone Gap Field,‡ 9 analyses, range	from 0.80	31.44	54.80	1.73	0.56
	to 2.01	36.27	63.50	8.25	1.72
KENTUCKY.					
Pulaski Co., 3 analyses, range	from 1.26	35.15	60.85	1.23	0.40
	to 1.82	39.44	52.48	5.52	1.00
Muhlenberg Co., 4 analyses, range	from 3.60	30.60	58.80	3.40	0.79
	to 7.06	33.70	58.70	6.50	3.16
Pike Co., Eastern Ky., 37 analyses, range	from 1.80	26.80	67.00	3.80	0.97
	to 1.60	41.00	50.37	7.80	0.03
Kentucky Cannel Coals,§ 5 analyses, range	from.....	40.20‖	59.80 coke	8.81	0.96
	to	66.30‖	33.70 coke	4.80	1.32
TENNESSEE.					
Scott Co., Range of several.¶	from .70	32.83	46.61	16.94	3.87
	to 1.83	41.29	61.66	1.11	0.77
Roane Co., Rockwood	1.75	26.62	60.11	11.52	1.49
Hamilton Co., Melville	2.74	26.50	67.08	3.68	.91
Marion Co., Etna	.94	23.72	63.94	11.40	1.19
Sewanee Co., Tracy City	1.60	29.80	61.00	7.80	
Kelly Co., Whiteside	1.30	21.60	74.20	2.70	
GEORGIA.					
Dade Co	1.20	23.05	60.50	15.16	0.84
ALABAMA.					
Warren Field:					
Jefferson Co., Birmingham	3.01	42.76	48.30	3.21	2.72
" " Black Creek	.12	26.11	71.64	2.03	.10
Tuscaloosa Co	1.59	38.33	54.64	5.45	1.33
Cahaba Field, Bibb Co, Helena Vein	2.00	32.90	53.08	11.34	.68
Cahaba Field, Bibb Co, Coke Vein	1.78	30.60	66.58	1.09	.04

* Analyses of Pocahontas Coal by John Pattinson, F.C.S., 1889:

	C.	H.	O.	N.	S.	Ash.	Water.	Coke.	Vol. Mat.
Lumps	86.51	4.44	4.95	0.66	0.61	1.54	1.29	78.8	21.2
Small	83.18	4.29	5.33	0.66	0.56	4.63	1.40	79.8	20.2

† These coals are coked in beehive ovens, and yield from 63% to 64% of coke.

‡ This field covers about 120 square miles in Virginia, and about 30 square miles in Kentucky.

§ The principal use of the cannel coals is for enriching illuminating-gas.

‖ Volatile matter including moisture.

¶ Single analyses from Morgan, Rhea, Anderson, and Roane counties fall within this range.

	Moisture.	Vol Mat.	Fixed C.	Ash.	Sulphur.
Texas.					
Eagle Mine	3.54	30 84	50.69	14.93	......
Sabinas Field, Vein I	1.91	20.04	62.71	15.35	......
" " " II	1.37	16.42	68.18	13.02	......
" " " III	0.84	29.35	50.18	19.63	......
" " " IV	0.45	21.6	45.75	29.1	3.15
Indiana.					
Caking Coals.					
Parke Co	4.50	45.50	45.50	4.50	......
Sullivan Co. coal M	2.35	45.25	51.60	0.80	......
Clay Co	7.00	39.70	47.30	6.00	......
Spencer Co. coal L	3.50	45.00	46.00	2.50	.....
*Block Coals.**					
Clay Co	8.50	31.00	57.50	3.00	......
Martin Co	2.50	44.75	51.25	1.50	......
Daviess Co	5.50	36.00	53.50	5.00	......
Illinois.†					
Bureau Co.: Ladd	12.0	32.3	42.5	13.2	......
Seatonville	10.0	33.8	40.9	15.3	......
Christian Co.: Pana	7.2	36.4	46.9	9.5	......
Clinton Co.: Trenton	13.3	30.4	52.0	4.3	0.9
Fulton Co.: Cuba	4.2	36.4	48.6	10.8	......
Grundy Co.: Morris	7.1	32.1	49.7	11.1	......
Jackson Co.: Big Muddy	6.4	30.6	54.6	8.3	1.5
La Salle Co.: Streator	12.0	35.3	48.8	3.9	2.4
Logan Co.: Lincoln	8.4	35.0	44.5	12.1	......
Macon Co.: Niantic	7.9	36.3	47.4	8.5	......
Macoupin Co.: Gillespie	12.6	30.6	45.3	11.5	1.5
Mt. Olive	10.4	36.7	46.1	6.8	3.5
Staunton	6.3	57.1	26.3	10.3	......
Madison Co.: Collinsville	9.3	29.9	40.8	16.1	3.9
Marion Co.: Centralia	8.3	34.0	45.5	8.0	.....
McLean Co.: Pottstown	4.6	35.5	45.5	14.4	......
Perry Co.: Du Quoin	11.3	30.3	49.9	8.5	0.9
Sangamon Co.: Barclay	10.8	27.3	44.8	17.1	.....
St. Clair Co.: St. Bernard	14.4	30.9	48.4	6.4	1.4
Vermilion Co.: Danville	11.0	32.6	53.0	3.6	......
Will Co.: Wilmington	15.5	32.8	39.9	11.8	......

* *Indiana Block Coal* (J. S. Alexander, Trans. A. I. M. E., iv. 100).—The typical block coal of the Brazil (Indiana) district differs in chemical composition but little from the coking coals of Western Pennsylvania. The physical difference, however, is quite marked; the latter has a cuboid structure made up of bituminous particles lying against each other, so that under the action of heat fusion throughout the mass readily takes place, while block coal is formed of alternate layers of rich bituminous matter and a charcoal-like substance, which is not only very slow of combustion, but so retards the transmission of heat that agglutination is prevented, and the coal burns away layer by layer, retaining its form until consumed.

An ultimate analysis of block coal from Sand Creek by E. T. Cox gave: C, 72.94; H, 4.50; O, 11.77; N, 1.79; ash, 4.50; moisture, 4.50.

† The Illinois coals are generally high in moisture, volatile matter, sulphur and ash, and are consequently low in heating value. The range of quality is a wide one. The Big Muddy coal of Jackson Co., which has a high reputation as a steam coal, has, according to the analysis given above, about 36% of volatile matter in the total combustible, corresponding to the coals of Western Pennsylvania and Ohio, while the Staunton coal has 68%, ranking it among the poorer varieties of lignite. A boiler-test with this coal (see p. 686, also Trans. A. S. M. E., v. 266) gave only 6.19 lbs. water evaporated from and at 212° per lb. combustible. The Staunton coal is remarkable for the high percentage of volatile matter, but it is excelled in this respect by

	Moisture.	Vol. Mat.	Fixed C.	Ash.	Sulphur.
IOWA.*					
Hiteman	4.99	35.27	25.37	34.37	.....
Keb	9.81	37.49	44.75	7.95	.....
Flaglers	9.84	40.16	37.69	12.31	.....
Chisholm	9.18	40.42	39.58	10.82	.....
MISSOURI.*					
Brookfield	4.34	40.27	50.60	4.79	.....
Mendota	9.03	37.48	46.24	7.25	.....
Hamilton	5.06	34.24	47.69	13.01	.....
Lingo	7.33	38.29	47.24	7.14	.....
NEBRASKA.*					
Hastings	0.21	27.82	60.88	11.09	.....
WYOMING.*					
Cambria	4.2	40.6	41.5	13.7	.....
"	2.5	37.4	37.9	22.2	.....
Goose Creek	9.7	40.2	46.3	3.8	.....
" "	13.92	36.78	42.03	7.27	.....
Deek Creek	12.8	35.0	47.7	3.6	.....
Sheridan	6.04	42.37	35.57	16.02	.....
COLORADO.‡					
Sunshine, Colo, average	2.8	36.3	37.1	23.8	.....
Newcastle, " "	1.7	37.95	48.6	11.6	.....
El Moro, " "	1.32	38.23	55.86	4.59	.....
Crested Buttes, "	1.10	23.20	72.60	3.10	.....
UTAH (Southern).					
Castledale	3.43	42.81	47.81†	9.73	.....
Cedar City	3.50	43.66	48.11†	5.95	.....
OREGON.					
Coos Bay	15.45	41.55	34.95	8.05	2.53
"	17.27	44.15	32.40	6.18	1.37
Yaquina Bay	18.03	46.20	32.60	7.10	1.07
John Day River	4.55	40.00	48.19	7.26	.60
" " "	6.54	34.45	52.41	6.95	.65
VANCOUVER ISLAND.					
Comox Coal	1.7	27.17	68.27	2.86	

the Boghead coal of Linlithgowshire, Scotland, an analysis of which by Dr. Penny is as follows: Proximate—moisture 0.84; vol. 67.95; fixed C, 9.81, ash, 21.4; Ultimate—C, 63.94; H, 8.86; O, 4.70; N, 0.96; which is remarkable for the high percentage of H.

* The analyses of Iowa, Missouri, Nebraska, and Wyoming coals are selected from a paper on The Heating Value of Western Coals, by Wm. Forsyth, Mech. Engr. of the C., B. & Q. R. R., *Eng'g News*, Jan. 17, 1895.

† Includes sulphur, which is very high. Coke from Cedar City analyzed: Water and volatile matter, 1.42; fixed carbon, 76.70; ash, 16.61; sulphur, 5.27.

‡ *Colorado Coals.*—The Colorado coals are of extremely variable composition, ranging all the way from lignite to anthracite. G. C. Hewitt (Trans. A. I. M. E., xvii. 377) says: The coal seams, where unchanged by heat and flexure, carry a lignite containing from 5% to 20% of water. In the south-eastern corner of the field the same have been metamorphosed so that in four miles the same seams are an anthracite, coking, and dry coal. In the basin of Coal Creek the coals are extremely fat, and produce a hard, bright, sonorous coke. North of coal basin half a mile of development shows a gradual change from a good coking coal with patches of dry coal to a dry coal that will barely agglutinate in a beehive oven. In another half mile the same seam is dry. In this transition area, a small cross-fault makes the coal fat for twenty or more feet on either side. The dry seams also present wide chemical and physical changes in short distances. A soft and loosely bedded coal has in a hundred feet become compact and hard without the intervention of a fault. A couple of hundred feet has reduced the water of combination from 12% to 5%.

Western Arkansas and Indian Territory. (H. M. Chance, Trans. A. I. M. E. 1890.)—The Choctaw coal-field is a direct westward extension of the Arkansas coal-field, but its coals are not like Arkansas coals, except in the country immediately adjoining the Arkansas line.

The western Arkansas coals are dry semi-bituminous or semi-anthracitic coals, mostly non-coking, or with quite feeble coking properties, ranging from 14% to 16% in volatile matter, the highest percentage yet found, according to Mr. Winslow's Arkansas report, being 17.655.

In the Mitchell basin, about 10 miles west from the Arkansas line, coal recently opened shows 19% volatile matter; the Mayberry coal, about 8 miles farther west, contains 23% volatile matter; and the Bryan Mine coal, about the same distance west, shows 26% volatile matter. About 30 miles farther west, the coal shows from 38% to 41½% volatile matter, which is also about the percentage in coals of the McAlester and Lehigh districts.

Western Lignites. (R. W. Raymond, Trans. A. I. M. E., vol. ii. 1873.)

	C.	H.	N.	O.	S.	Moisture.	Ash.	Calorific Power, calories.
Monte Diabolo	59.72	5.08	1.01	15.69	3.92	8.94	5.64	5757
Weber Cañon, Utah	64.84	4.34	1.29	15.52	1.60	9.41	3.00	5912
Echo Cañon, Utah	69.84	3.90	1.93	10.99	0.77	9.17	3.40	6400
Carbon Station, Wyo	64.99	3.76	1.74	15.20	1.07	11.56	1.68	5738
" " "	69.14	4.36	1.25	9.54	1.03	8.06	6.62	6578
Coos Bay, Oregon	56.24	3.38	0.42	21.82	0.81	13.28	4.05	4565
Alaska	55.79	3.26	0.61	19.01	0.68	16.52	4.18	4610
"	67.67	4.66	1.58	12.80	0.92	3.08	9.28	6428
Canon City, Colo	67.58	7.42	...	13.42	0.63	5.18	5.77	7330
Baker Co., Ore.	60.72	4.30	...	14.42	2.08	14.68	3.80	5602

The calorific power is calculated by Dulong's formula,

$$8080C + 34462\left(H - \frac{O}{8}\right),$$

deducting the heat required to vaporize the moisture and combined water, that is, 537 calories for each unit of water. 1 calorie = 1.8 British thermal units.

Analyses of Foreign Coals. (Selected from D. L. Barnes's paper on American Locomotive Practice, A. S. C. E., 1893.)

	Volatile Matter.	Fixed Carbon.	Ash.	
Great Britain:				
South Wales	8.5	88.3	3.2	
" "	6.2	92.3	1.5	
Lancashire, Eng.	17.2	80.1	2.7	
Derbyshire, "	17.7	79.9	2.4	
Durham, "	15 05	86.8	1.1	Semi-bit. coking coal.
Scotland	17.1	63.1	19.8	Boghead cannel gas coal.
"	17.5	80.1	2.4	Semi-bit. steam-coal.
Staffordshire, Eng.	20.4	78.6	1.0	
South America:				
Chili, Conception Bay	21.93	70.55	7.52	
" Chiroqui	24.11	38.98	36.91	
Patagonia	24.35	62 25	13.4	
Brazil	40.5	57.9	1.6	
Canada:				
Nova Scotia	26.8	60.7	12.5	
Cape Breton	26.9	67.6	5.5	
Australia				
Australian lignite	15.8	64.3	10.0	
Sydney, South Wales	14.98	82.39	2.04	
Borneo	26.5	70.3	14.2	
Van Diemen's Land	6.16	63.4	30.45	

An analysis of Pictou, N. S., coal, in Trans. A. I. M. E., xiv. 560, is: Vol., 29.63; carbon, 56.98; ash, 13.39; and one of Sydney, Cape Breton, coal is: vol., 34.07; carbon, 61.43; ash, 4.50.

Nixon's Navigation Welsh Coal is remarkably pure, and contains not more than 3 to 4 per cent of ashes, giving 88 per cent of hard and lustrous coke. The quantity of fixed carbon it contains would classify it among the dry coals, but on account of its coke and its intensity of combustion it belongs to the class of fat, or long-flaming coals.

Chemical analysis gave the following results: Carbon, 90.27; hydrogen, 4.39; sulphur, .69; nitrogen, .49; oxygen (difference), 4.16.

The analysis showed the following composition of the volatile parts: Carbon, 22.53; hydrogen, 34.96; O + N + S, 42.51.

The heat of combustion was found to be, as a result of several experiments, 8864 calories for the unit of weight. Calculated according to its composition, the heat of combustion would be 8805 calories = 15,849 British thermal units per pound.

This coal is generally used in trial-trips of steam-vessels in Great Britain.

Sampling Coal for Analysis.—J. P. Kimball, Trans. A. I. M. E., xii. 317, says: The unsuitable sampling of a coal-seam, or the improper preparation of the sample in the labóratory, often gives rise to errors in determinations of the ash so wide in range as to vitiate the analysis for all practical purposes; every other single determination, excepting moisture, showing its relative part of the error. The determination of sulphur and ash are especially liable to error, as they are intimately associated in the slates.

Wm. Forsyth, in his paper on The Heating Value of Western Coals (*Eng'g News*, Jan. 17, 1895), says: This trouble in getting a fairly average sample of anthracite coal has compelled the Reading R. R. Co., in getting their samples, to take as much as 300 lbs. for one sample, drawn direct from the chutes, as it stands ready for shipment.

The directions for collecting samples of coal for analysis at the C., B. & Q. laboratory are as follows:

Two samples should be taken, one marked "average," the other "select." Each sample should contain about 10 lbs., made up of lumps about the size of an orange taken from different parts of the dump or car, and so selected that they shall represent as nearly as possible, first, the average lot; second, the best coal.

An example of the difference between an "average" and a "select" sample, taken from Mr. Forsyth's paper, is the following of an Illinois coal:

	Moisture.	Vol. Mat.	Fixed Carbon.	Ash.
Average..........	1.36	27.69	35.41	35.54
Select.............	1.90	34.70	48.23	15.17

The theoretical evaporative power of the former was 9.13 lbs. of water from and at 212° per lb. of coal, and that of the latter 11.44 lbs.

Relative Value of Fine Sizes of Anthracite.—For burning on a grate coal-dust is commercially valueless, the finest commercial anthracites being sold at the following rates per ton at the mines, according to a recent address by Mr. Eckley B. Coxe (1893):

Size.	Range of Size.	Price at Mines.
Chestnut......................	1¼ to ⅞ inch	$2.75
Pea............................	⅞ to 9/16	1.25
Buckwheat....................	9/16 to ⅜	0.75
Rice............................	⅜ to 3/16	0.25
Barley..........................	3/16 to 2/32	0.10

But when coal is reduced to an impalpable dust, a method of burning it becomes possible to which even the finest of these sizes is wholly unadapted; the coal may be blown in as dust, mixed with its proper proportion of air, and no grate at all is then required.

Pressed Fuel. (E. F. Loiseau, Trans. A. I. M. E., viii. 314.)—Pressed fuel has been made from anthracite dust by mixing the dust with ten per cent of its bulk of dry pitch, which is prepared by separating from tar at a temperature of 572° F. the volatile matter it contains. The mixture is kept heated by steam to 212°, at which temperature the pitch acquires its cementing properties, and is passed between two rollers, on the periphery of which are milled out a series of semi-oval cavities. The lumps of the mixture, about the size of an egg, drop out under the rollers on an endless belt which carries them to a screen in eight minutes, which time is sufficient to cool the lumps, and they are then ready for delivery.

The enterprise of making the pressed fuel above described was not commercially successful, on account of the low price of other coal. In France, however, "*briquettes*" are regularly made of coal-dust (bituminous and semi-bituminous).

RELATIVE VALUE OF STEAM COALS.

The heating value of a coal may be determined, with more or less approximation to accuracy, by three different methods.

1st, by chemical analysis; 2d. by combustion in a coal calorimeter; 3d, by actual trial in a steam-boiler. The first two methods give what may be called the theoretical heating value, the third gives the practical value.

The accuracy of the first two methods depends on the precision of the method of analysis or calorimetry adopted, and upon the care and skill of the operator. The results of the third method are subject to numerous sources of variation and error, and may be taken as approximately true only for the particular conditions under which the test is made. Analysis and calorimetry give with considerable accuracy the heating value which may be obtained under the conditions of perfect combustion and complete absorption of the heat produced. A boiler test gives the actual result under conditions of more or less imperfect combustion, and of numerous and variable wastes. It may give the highest practical heating value, if the conditions of grate-bars, draft, extent of heating surface, method of firing, etc., are the best possible for the particular coal tested, and it may give results far beneath the highest if these conditions are adverse or unsuitable to the coal.

The results of boiler tests being so extremely variable, their use for the purpose of determining the relative steaming values of different coals has frequently led to false conclusions. A notable instance is found in the record of Prof. Johnson's tests, made in 1844, the only extensive series of tests of American coals ever made. He reported the steaming value of the Lehigh Coal & Navigation Co.'s coal to be far the lowest of all the anthracites, a result which is easily explained by an examination of the conditions under which he made the test, which were entirely unsuited to that coal. He also reported a result for Pittsburgh coal which is far beneath that now obtainable in every-day practice, his low result being chiefly due to the use of an improper furnace.

In a paper entitled Proposed Apparatus for Determining the Heating Power of Different Coals (Trans. A. I. M. E., xiv. 727) the author described and illustrated an apparatus designed to test fuel on a large scale, avoiding the errors of a steam-boiler test. It consists of a fire-brick furnace enclosed in a water-casing, and two cylindrical shells containing a great number of tubes, which are surrounded by cooling water and through which the gases of combustion pass while being cooled. No steam is generated in the apparatus, but water is passed through it and allowed to escape at a temperature below 200° F. The product of the weight of the water passed through the apparatus by its increase in temperature is the measure of the heating value of the fuel.

There has been much difference of opinion concerning the value of chemical analysis as a means of approximating the heating power of coal. It was found by Scheurer-Kestner and Meunier-Dollfus, in their extensive series of tests, made in Europe in 1868, that the heating power as determined by calorimetric tests was greater than that given to chemical analysis according to Dulong's law.

Recent tests made in Paris by M. Mahler, however, show a much closer agreement of analysis and calorimetric tests. A brief description of these tests, translated from the French, may be found in an article by the author in *The Mineral Industry*, vol. i. page 97.

Dulong's law may be expressed by the formula,

$$\text{Heating Power in British Thermal Units} = 14{,}500C + 62{,}500\left(H - \frac{O}{8}\right),^*$$

in which C, H, and O are respectively the percentage of carbon, hydrogen, and oxygen, each divided by 100. A study of M. Mahler's calorimetric tests shows that the maximum difference between the results of these tests and the calculated heating power by Dulong's law in any single case is only a little over 3%, and the results of 81 tests show that Dulong's formula gives an average of only 47 thermal units less than the calorimetric tests, the average total heating value being over 14,000 thermal units, a difference of less than 4/10 of 1%.

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* Mahler gives Dulong's formula with Berthelot's figure for the heating value of carbon, in British thermal units,

$$\text{Heating Power} = 14{,}650C + 62{,}025\left(H - \frac{(O + N) - 1}{8}\right).$$

Mahler's calorimetric apparatus consists of a strong steel vessel or "bomb" immersed in water, proper precaution being taken to prevent radiation. One gram of the coal to be tested is placed in a platinum boat within this bomb, oxygen gas is introduced under a pressure of 20 to 25 atmospheres, and the coal ignited explosively by an electric spark. Combustion is complete and instantaneous, the heat is radiated into the surrounding water, weighing 2200 grams, and its quantity is determined by the rise in temperature of this water, due corrections being made for the heat capacity of the apparatus itself. The accuracy of the apparatus is remarkable, duplicate tests giving results varying only about 2 parts in 1000.

The close agreement of the results of calorimetric tests when properly conducted, and of the heating power calculated from chemical analysis, indicates that either the chemical or the calorimetric method may be accepted as correct enough for all practical purposes for determining the total heating power of coal. The results obtained by either method may be taken as a standard by which the results of a boiler test are to be compared, and the difference between the total heating power, and the result of the boiler test is a measure of the inefficiency of the boiler under the conditions of any particular test.

In practice with good anthracite coal, in a steam-boiler properly proportioned, and with all conditions favorable, it is possible to obtain in the steam 80% of the total heat of combustion of the coal. This result was nearly obtained in the tests at the Centennial Exhibition in 1876, in five different boilers. An efficiency of 70% to 75% may easily be obtained in regular practice. With bituminous coals it is difficult to obtain as close an approach to the theoretical maximum of economy, for the reason that some of the volatile combustible portion of the coal escapes unburned, the difficulty increasing rapidly as the content of volatile matter increases beyond 20%. With most coals of the Western States it is with difficulty that as much as 60% or 65% of the theoretical efficiency can be obtained without the use of gas-producers.

The chemical analysis heretofore referred to is the ultimate analysis, or the percentage of carbon, hydrogen, and oxygen of the dry coal. It is found, however, from a study of Mahler's tests that the proximate analysis, which gives fixed carbon, volatile matter, moisture, and ash, may be relied on as giving a measure of the heating value with a limit of error of only about 3%. After deducting the moisture and ash, and calculating the fixed carbon as a percentage of the coal dry and free from ash, the author has constructed the following table:

APPROXIMATE HEATING VALUE OF COALS.

Percentage F. C. in Coal Dry and Free from Ash.	Heating Value B.T.U. per lb. Comb'le.	Equiv. Water Evap. from and at 212° per lb. Combustible.	Percentage F. C. in Coal Dry and Free from Ash.	Heating Value B.T.U. per lb. Comb'le.	Equiv. Water Evap. from and at 212° per lb. Combustible.
100	14500	15.00	68	15480	16.03
97	14760	15.28	63	15120	15.65
94	15120	15.65	60	14580	15.09
90	15480	16.03	57	14040	14.53
87	15660	16.21	54	13320	13.79
80	15840	16.40	51	12600	13.04
72	15660	16.21	50	12240	12.67

Below 50% the law of decrease of heating-power shown in the table apparently does not hold, as some cannel coals and lignites show much higher heating-power than would be predicted from their chemical constitution.

The use of this table may be shown as follows:

Given a coal containing moisture 2%, ash 8%, fixed carbon 61%, and volatile matter 29%, what is its probable heating value? Deducting moisture and ash we find the fixed carbon is 61/90 or 68% of the total of fixed carbon and volatile matter. One pound of the coal dry and free from ash would, by the table, have a heating value of 15,480 thermal units, but as the ash and moisture, having no heating value, are 10% of the total weight of the coal, the coal would have 90% of the table value, or 13,932 thermal units. This divided by 966, the latent heat of steam at 212° gives an equivalent evaporation per lb. of coal of 14.42 lbs.

The heating value that can be obtained in practice from this coal would depend upon the efficiency of the boiler, and this largely upon the difficulty of thoroughly burning its volatile combustible matter in the boiler furnace. If a boiler efficiency of 65% could be obtained, then the evaporation per lb. of coal from and at 212° would be 14.42 × .65 = 9.37 lbs.

With the best anthracite coal, in which the combustible portion is, say, 97% fixed carbon and 3% volatile matter, the highest result that can be expected in a boiler-test with all conditions favorable is 12.2 lbs. of water evaporated from and at 212° per lb. of combustible, which is 80% of 15.28 lbs. the theoretical heating-power. With the best semi-bituminous coals, such as Cumberland and Pocahontas, in which the fixed carbon is 80% of the total combustible, 12 5 lbs., or 76% of the theoretical 16.4 lbs., may be obtained. For Pittsburgh coal, with a fixed carbon ratio of 68%, 11 lbs., or 69% of the theoretical 16.03 lbs., is about the best practically obtainable with the best boilers With some good Ohio coals, with a fixed carbon ratio of 60%, 10 lbs., or 66% of the theoretical 15.09 lbs., has been obtained, under favorable conditions, with a fire-brick arch over the furnace. With coals mined west of Ohio, with lower carbon ratios, the boiler efficiency is not apt to be as high as 60%.

From these figures a table of probable maximum boiler-test results from coals of different fixed carbon ratios may be constructed as follows:

Fixed carbon ratio	97	80	68	60	54	50
Evap. from and at 212° per lb. combustible, maximum in boiler-tests:	12.2	12.5	11	10	8.3	7.0
Boiler efficiency, per cent	80	76	69	66	60	55
Loss, chimney, radiation, imperfect combustion, etc:	20	24	31	34	40	45

The difference between the loss of 20% with anthracite and the greater losses with the other coals is chiefly due to imperfect combustion of the bituminous coals, the more highly volatile coals sending up the chimney the greater quantity of smoke and unburned hydrocarbon gases. It is a measure of the inefficiency of the boiler furnace and of the inefficiency of heating-surface caused by the deposition of soot, the latter being primarily caused by the imperfection of the ordinary furnace and its unsuitability to the proper burning of bituminous coal. If in a boiler-test with an ordinary furnace lower results are obtained than those in the above table, it is an indication of unfavorable conditions, such as bad firing, wrong proportions of boiler, defective draft, and the like, which are remediable. Higher results can be expected only with gas-producers, or other styles of furnace especially designed for smokeless combustion.

Kind of Furnace Adapted for Different Coals. (From the author's paper on "The Evaporative Power of Bituminous Coals," Trans. A. S. M. E., iv, 257.)—Almost any kind of a furnace will be found well adapted to burning anthracite coals and semi-bituminous coals containing less than 20% of volatile matter. Probably the best furnace for burning those coals which contain between 20% and 40% volatile matter, including the Scotch, English, Welsh, Nova Scotia, and the Pittsburgh and Monongahela river coals, is a plain grate-bar furnace with a fire-brick arch thrown over it, for the purpose of keeping the combustion-chamber thoroughly hot. The best furnace for coals containing over 40% volatile matter will be a furnace surrounded by fire-brick with a large combustion-chamber, and some special appliance for introducing very hot air to the gases distilled from the coal, or, preferably, a separate gas-producer and combustion-chamber, with facilities for heating both air and gas before they unite in the combustion-chamber. The character of furnace to be especially avoid d in burning all bituminous coals containing over 20% of volatile matter is the ordinary furnace, in which the boiler is set directly above the grate bars, and in which the heating-surfaces of the boiler are directly exposed to radiation from the coal on the grate. The question of admitting air above the grate is still unsettled. The London *Engineer* recently said: "All our experience, extending over many years, goes to show that when the production of smoke is prevented by special devices for admitting air, either there is an increase in the consumption of fuel or a diminution in the production of steam. * * * The best smoke-preventer yet devised is a good fireman."

Downward-draught Furnaces.—Recent experiments show that with bituminous coal considerable saving may be made by causing the draught to go downwards from the freshly-fired coal through the hot coal on the grate. Similar good results are also obtained by the upward draught by feeding the fresh coal under the bed of hot coal instead of on top. (See Boilers.)

Calorimetric Tests of American Coals.—From a number of tests of American and foreign coals, made with an oxygen calorimeter, by Geo. H. Barrus (Trans. A. S. M. E., vol. xiv. 816), the following are selected, showing the range of variation:

	Percentage of Ash.	Total Heat of Combustion. B. T. U.	Total Heat reduced to Fuel free from Ash.
Semi-bituminous.			
George's Cr'k, Cumberl'd, Md., 10 tests	6.1	14,217	15,141
	8.6	12,874	14,085
Pocahontas, Va., 5 tests	3.2	14,603	15,086
	6.2	13,608	14,507
New River, Va., 6 tests	3.5	13,922	14,427
	5.7	13,858	14,696
Elk Garden, Va., 1 test	7.8	13,180	14,295
Welsh, 1 test	7.7	13,581	14,714
Bituminous.			
Youghiogheny, Pa., lump	5.9	12,941	13,752
" " slack	10.2	11,664	12,988
Frontenac, Kansas	17.7	10,506	12,765
Cape Breton, (Caledonia)	8.7	12,420	13,602
Lancashire, Eng	6.8	12,122	13,006
Anthracite, 11 tests	10.5	11,521	12,873
	9.1	13,189	14,509

Evaporative Power of Bituminous Coals.

(Tests with Babcock & Wilcox Boilers, Trans. A. S. M. E., iv. 267.)

Name of Coal.	Duration of Test.	Grate Surface, sq. ft.	Heating Surface, sq. ft.	Percentage of Refuse.	Coal burned per sq. ft. of Grate, pounds.	Water evaporated per sq. ft. of Heating Surface per hour, pounds.	Water per pound Coal from and at 212°, lbs.	Water per pound Combustible from and at 212°.	Rated Horse-power.	Horse-power developed.
1. Welsh	18⅓ hrs	40	1679	7.5	6.3	2.07	11.58	12.46	146	96
2. Anthracite scr's 1/5 Powelton, Pa., Semi-bit. 4/5,	10¼ h	60	3126	8.8	17.6	4.32	11.32	12.42	272	448
3. Pittsbg'h fine slack	4 hrs	33.7	1679	12.3	21.9	4.47	8.12	9.29	146	250
" 3d Pool lump	10 "	43.5	2760	4.8	27.5	4.76	10.47	11.00	240	419
4. Castle Shannon, nr Pittsb'gh, ⅝ nut, ⅜ lump,	42¼ h	69.1	4784	10.5	27.9	4.13	10.00	11.17	416	570
5. Ill. "run of mine"	6 days.	....	1196	....	....	1.41	9.49	....	104	54
" Ind. block, "very good"	3 d'ys	....	1196	....	....	2.95	9.47	......	104	111
6. Jackson, O., nut	8 hrs.	48	3358	9.6	32.1	4.11	8.93	9.86	292	460
" Staunton, Ill., nut	8 "	60	3358	17.7	25.1	2.27	5.09	6.19	292	246
7. Renton screenings.	5 h 50 m	21.2	1564	13.8	31.5	2.95	6.88	7.98	136	151
" Wellington scr'gs	6 h 30 m	21.2	1564	13.3	27	2.98	7.89	9.66	136	150
" Black Diam. scr'gs	5 h 58 m	21.2	1564	19.3	36.4	3.11	6.29	7.80	136	160
" Seattle screenings.	6 h 24 m	21.2	1564	13.4	31.3	2.91	6.86	7.92	136	150
" Wellington lump	6 h 19 m	21.2	1564	13.8	28.2	3.52	9.02	10.46	136	171
" Cardiff lump	6 h 47 m	21.2	1564	11.7	26.7	3.69	10.07	11.40	136	189
" " "	7 h 23 m	21.2	1564	19.1	25.6	3.35	9.62	11.89	136	174
" South Paine lump.	6 h 35 m	21.2	1564	13.9	28.9	3.53	8.96	10.41	136	182
" Seattle lump	6 h 5 m	21.2	1564	9.5	34.1	3.57	7.68	8.49	136	184

Place of Test: 1. London, England; 2. Peacedale, R. I.; 3. Cincinnati, O.; 4. Pittsburgh, Pa.; 5. Chicago, Ill.; 6. Springfield, O.; 7. San Francisco, Cal.

In all the above tests the furnace was supplied with a fire-brick arch for preventing the radiation of heat from the coal directly to the boiler.

Weathering of Coal. (I. P. Kimball, Trans. A. I. M. E., viii. 204.)—The practical effect of the weathering of coal, while sometimes increasing its absolute weight, is to diminish the quantity of carbon and disposable hydrogen and to increase the quantity of oxygen and of indisposable hydrogen. Hence a reduction in the calorific value.

An excess of pyrites in coal tends to produce rapid oxidation and mechanical disintegration of the mass, with development of heat, loss of coking power, and spontaneous ignition.

The only appreciable results of the weathering of anthracite within the ordinary limits of exposure of stocked coal are confined to the oxidation of its accessory pyrites. In coking coals, however, weathering reduces and finally destroys the coking power, while the pyrites are converted from the state of bisulphide into comparatively innocuous sulphates.

Richters found that at a temperature of 158° to 180° Fahr., three coals lost in fourteen days an average of 3.6% of calorific power. (See also paper by R. P. Rothwell, Trans. A. I. M. E., iv. 55.)

COKE.

Coke is the solid material left after evaporating the volatile ingredients of coal, either by means of partial combustion in furnaces called coke ovens, or by distillation in the retorts of gas-works.

Coke made in ovens is preferred to gas coke as fuel. It is of a dark-gray color, with slightly metallic lustre, porous, brittle, and hard.

The proportion of coke yielded by a given weight of coal is very different for different kinds of coal, ranging from 0.9 to 0.35.

Being of a porous texture, it readily attracts and retains water from the atmosphere, and sometimes, if it is kept without proper shelter, from 0.15 to 0.20 of its gross weight consists of moisture.

Analyses of Coke.

(From report of John R. Procter, Kentucky Geological Survey.)

Where Made.	Fixed Carbon	Ash.	Sulphur.
Connellsville, Pa. (Average of 3 samples)........	88.96	9.74	0.810
Chattanooga, Tenn. " " 4 "	80.51	16.34	1.595
Birmingham, Ala. " " 4 "	87.29	10.54	1.195
Pocahontas, Va. " " 3 "	92.53	5.74	0.597
New River, W. Va. " " 8 "	92.38	7.21	0.562
Big Stone Gap, Ky. " " 7 "	93.23	5.69	0.749

Experiments in Coking. Connellsville Region.

(John Fulton, *Amer. Mfr.*, Feb. 10, 1893.)

No. of Test.	Time in Oven.	Coal Charged.	Ash made.	Fine Coke made.	Market Coke made	Total Coke made.	Per cent of Yield. Ash.	Fine Coke.	Market Coke	Total Coke.	Per Cent Lost.
	h. m.	lb.	lb.	lb.	lb.	lb.					
1	67 00	12,420	99	385	7,518	7,903	00.80	3 10	60.53	63.63	35.57
2	68 00	11,090	90	359	6,580	6,939	00.81	3.24	59.33	62.57	36.62
3	45 00	9,120	77	272	5,418	5,690	00.84	2.98	59.41	62.39	36 77
4	45 00	9,020	74	349	5,334	5,683	00.82	3 87	59.13	63.00	36.18
		41,650	340	1365	24,850	26,215	00.82	3.28	59.66	62.94	36.24

These results show, in a general average, that Connellsville coal carefully coked in a modern beehive oven will yield 66.17% of marketable coke, 2.30% of small coke or braize, and 0.82% of ash.

The total average loss in volatile matter expelled from the coal in coking amounts to 30.71%.

The modern beehive coke oven is 12 feet in diameter and 7 feet high at crown of dome. It is used in making 48 and 72 hour coke.

In making these tests the coal was weighed as it was charged into the oven; the resultant marketable coke, small coke or braize and ashes weighed dry as they were drawn from the oven.

Coal Washing.—In making coke from coals that are high in ash and sulphur, it is advisable to crush and wash the coal before coking it. A coal-washing plant at Brookwood, Ala., has a capacity of 50 tons per hour. The average percentage of ash in the coal during ten days' run varied from 14% to 21%, in the washed coal from 4.8% to 8.1%, and in the coke from 6.1% to 10.5%. During three months the average reduction of ash was 60.9%. (*Eng. and Mining Jour.*, March 25, 1893.)

Recovery of By-products in Coke Manufacture.—In Germany considerable progress has been made in the recovery of by-products. The Hoffman-Otto oven has been most largely used, its principal feature being that it is connected with regenerators. In 1884 40 ovens on this system were running, and in 1892 the number had increased to 1209.

A Hoffman-Otto oven in Westphalia takes a charge of 6¼ tons of dry coal and converts it into coke in 48 hours. The product of an oven annually is 1025 tons in the Ruhr district, 1170 tons in Silesia, and 960 tons in the Saar district. The yield from dry coal is 75% to 77% of coke, 2.5% to 3% of tar, and 1.1% to 1.2% of sulphate of ammonia in the Ruhr district; 65% to 70% of coke, 4% to 4.5% of tar, and 1% to 1.25% of sulphate of ammonia in the Upper Silesia region and 68% to 72% of coke, 4% to 4.3% of tar and 1.8% to 1.9% of sulphate of ammonia in the Saar district. A group of 60 Hoffman ovens, therefore, yields annually the following:

District.	Coke, tons.	Tar, tons.	Sulphate Ammonia, tons.
Ruhr	51,200	1860	780
Upper Silesia	48,000	3000	840
Saar	40,500	2400	492

An oven which has been introduced lately into Germany in connection with the recovery of by-products is the Semet-Solvay, which works hotter than the Hoffman-Otto, and for this reason 73% to 77% of gas coal can be mixed with 23% to 27% of coal low in volatile matter, and yet yield a good coke. Mixtures of this kind yield a larger percentage of coke, but, on the other hand, the amount of gas is lessened, and therefore the yield of tar and ammonia is not so great.

The yield of coke by the beehive and the retort ovens respectively is given as follows in a pamphlet of the Solvay Process Co.: Connellsville coal: beehive, 66%, retort, 73%; Pocahontas: beehive, 62%, retort, 83%; Alabama: beehive, 60%, retort, 74%. (See article in *Mineral Industry*, vol. viii., 1900.)

References: F. W. Luerman, Verein Deutscher Eisenhuettenleute 1891, *Iron Age*, March 31, 1892; *Amer. Mfr.*, April 28, 1893. An excellent series of articles on the manufacture of coke, by John Fulton, of Johnstown, Pa., is published in the *Colliery Engineer*, beginning in January, 1893.

Making Hard Coke.—J. J. Fronheiser and C. S. Price, of the Cambria Iron Co., Johnstown, Pa., have made an improvement in coke manufacture by which coke of any degree of hardness may be turned out. It is accomplished by first grinding the coal to a coarse powder and mixing it with a hydrate of lime (air or water slacked caustic lime) before it is charged into the coke-ovens. The caustic lime or other fluxing material used is mechanically combined with the coke, filling up its cell-walls. It has been found that about 5% by weight of caustic lime mixed with the fine coal gives the best results. However, a larger quantity of lime can be added to coals containing more than 5% to 7% of ash. (*Amer. Mfr.*)

Generation of Steam from the Waste Heat and Gases of Coke-ovens. (Erskine Ramsey, *Amer. Mfr.*, Feb. 16, 1894.)—The gases from a number of adjoining ovens of the beehive type are led into a long horizontal flue, and thence to a combustion chamber under a battery of boilers. Two plants are in satisfactory operation at Tracy City, Tenn., and two at Pratt Mines, Ala.

A Bushel of Coal.—The weight of a bushel of coal in Indiana is 70 lbs., in Penna. 76 lbs.; in Ala., Colo., Ga., Ill., Ohio, Tenn., and W. Va. it is 80 lbs.

A Bushel of Coke is almost uniformly 40 lbs., but in exceptional cases, when the coke is very light, 38, 36, and 33 lbs. are regarded as a bushel. In others, from 42 to 50 lbs are given as the weight of a bushel; in this case the coke would be quite heavy.

Products of the Distillation of Coal.—S. P. Sadler's Handbook of Industrial Organic Chemistry gives a diagram showing over 50 chemical products that are derived from distillation of coal. The first derivatives are coal-gas, gas-liquor, coal-tar, and coke. From the gas-liquor are derived ammonia and sulphate, chloride and carbonate of ammonia. The coal-tar is split up into oils lighter than water or crude naphtha, oils heavier than water—otherwise dead oil or tar, commonly called creosote,—and pitch. From the two former are derived a variety of chemical products.

From the coal-tar there comes an almost endless chain of known combinations. The greatest industry based upon their use is the manufacture of dyes, and the enormous extent to which this has grown can be judged from the fact that there are over 600 different coal-tar colors in use, and many more which as yet are too expensive for this purpose. Many medicinal preparations come from the series, pitch for paving purposes, and chemicals for the photographer, the rubber manufacturers and tanners, as well as for preserving timber and cloths.

The composition of the hydrocarbons in a soft coal is uncertain and quite complex; but the ultimate analysis of the average coal shows that it approaches quite nearly to the composition of CH_4 (marsh-gas). (W. H. Blauvelt, Trans. A. I. M. E., xx. 625.)

WOOD AS FUEL.

Wood, when newly felled, contains a proportion of moisture which varies very much in different kinds and in different specimens, ranging between 30% and 50%, and being on an average about 40%. After 8 or 12 months' ordinary drying in the air the proportion of moisture is from 20 to 25%. This degree of dryness, or almost perfect dryness if required, can be produced by a few days' drying in an oven supplied with air at about 240° F. When coal or coke is used as the fuel for that oven, 1 lb. of fuel suffices to expel about 3 lbs. of moisture from the wood. This is the result of experiments on a large scale by Mr. J. R. Napier. If air-dried wood were used as fuel for the oven, from 2 to 2½ lbs. of wood would probably be required to produce the same effect.

The specific gravity of different kinds of wood ranges from 0.3 to 1.2.

Perfectly dry wood contains about 50% of carbon, the remainder consisting almost entirely of oxygen and hydrogen in the proportions which form water. The coniferous family contain a small quantity of turpentine, which is a hydrocarbon. The proportion of ash in wood is from 1% to 5%. The total heat of combustion of all kinds of wood, when dry, is almost exactly the same, and is that due to the 50% of carbon.

The above is from Rankine; but according to the table by S. P. Sharpless in Jour. C. I. W., iv. 36, the ash varies from 0.03% to 1.20% in American woods, and the fuel value, instead of being the same for all woods, ranges from 3667 (for white oak) to 5546 calories (for long-leaf pine) = 6600 to 9883 British thermal units for dry wood, the fuel value of 0.50 lbs. carbon being 7272 B. T. U.

Heating Value of Wood.—The following table is given in several books of reference, authority and quality of coal referred to not stated.

The weight of one cord of different woods (thoroughly air-dried) is about as follows:

Hickory or hard maple....	4500 lbs.	equal to	1800 lbs.	coal.	(Others give	2000.)
White oak....	3850	"	1540	"	("	1715.)
Beech, red and black oak..	3250	"	1300	"	("	1450.)
Poplar, chestnut, and elm..	2350	"	940	"	("	1050.)
The average pine	2000	"	800	"	("	925.)

Referring to the figures in the last column, it is said:

From the above it is safe to assume that 2¼ lbs. of dry wood are equal to 1 lb. average quality of soft coal and that the full value of the same weight of different woods is very nearly the same—that is, a pound of hickory is worth no more for fuel than a pound of pine, assuming both to be dry. It is important that the wood be dry, as each 10% of water or moisture in wood will detract about 12% from its value as fuel.

Taking an average wood of the analysis C 51%, H 6.5%, O 42.0%, ash 0.5%, perfectly dry, its fuel value per pound, according to Dulong's formula, $V =$

$\left[14{,}500\,C + 62{,}000\left(H - \frac{O}{8}\right)\right]$, is 8170 British thermal units. If the wood, as ordinarily dried in air, contains 25% of moisture, then the heating value of a pound of such wood is three quarters of 8170 = 6127 heat-units, less the heat required to heat and evaporate the ¼ lb. of water from the atmospheric temperature, and to heat the steam made from this water to the temperature of the chimney gases, say 150 heat-units per pound to heat the water to 212°, 966 units to evaporate it at that temperature, and 100 heat-units to raise the temperature of the steam to 420° F., or 1216 in all = 304 for ¼ lb., which subtracted from the 6127, leaves 5824 heat-units as the net fuel value of the wood per pound, or about 0.4 that of a pound of carbon.

Composition of Wood.

(Analysis of Woods, by M. Eugene Chevandier.)

Woods.	Composition.				
	Carbon.	Hydrogen.	Oxygen.	Nitrogen.	Ash.
Beech	49.36%	6.01%	42.69%	0.91%	1.06%
Oak	49.64	5.92	41.16	1.29	1.97
Birch	50.20	6.20	41.62	1.15	0.81
Poplar	49.37	6.21	41.60	0.96	1.86
Willow	49.96	5.96	39.56	0.96	3.37
Average	49.70%	6.06%	41.30%	1.05%	1.80%

The following table, prepared by M. Violette, shows the proportion of water expelled from wood at gradually increasing temperatures:

Temperature.	Water Expelled from 100 Parts of Wood.			
	Oak.	Ash.	Elm.	Walnut.
257° Fahr	15.26	14.78	15.32	15.55
302° Fahr	17.93	16.19	17.02	17.43
347° Fahr	32.13	21.22	36.94?	21.00
392° Fahr	35.80	27.51	33.38	41.77?
437° Fahr	44.31	33.38	40.56	36.56

The wood operated upon had been kept in store during two years. When wood which has been strongly dried by means of artificial heat is left exposed to the atmosphere, it reabsorbs about as much water as it contains in its air-dried state.

A cord of wood = 4 × 4 × 8 = 128 cu. ft. About 56% solid wood and 44% interstitial spaces. (Marcus Bull, Phila., 1829. J. C. I. W., vol. i. p. 293.)

B. E. Fernow gives the per cent of solid wood in a cord as determined officially in Prussia (J. C. I. W., vol. iii. p. 20):

Timber cords, 74.07% = 80 cu. ft. per cord;
Firewood cords (over 6″ diam.), 69.44% = 75 cu. ft. per cord;
"Billet" cords (over 3″ diam.), 55.55% = 60 cu. ft. per cord;
"Brush" woods less than 3″ diam., 18.52%; Roots, 37.00%.

CHARCOAL.

Charcoal is made by evaporating the volatile constituents of wood and peat, either by a partial combustion of a conical heap of the material to be charred, covered with a layer of earth, or by the combustion of a separate portion of fuel in a furnace, in which are placed retorts containing the material to be charged.

According to Peclet, 100 parts by weight of wood when charred in a heap yield from 17 to 22 parts by weight of charcoal, and when charred in a retort from 28 to 30 parts.

This has reference to the ordinary condition of the wood used in charcoal-making, in which 25 parts in 100 consist of moisture. Of the remaining 75 parts the carbon amounts to one half, or 37½% of the gross weight of the wood. Hence it appears that on an average nearly half of the carbon in the wood is lost during the partial combustion in a heap, and about one quarter during the distillation in a retort.

To char 100 parts by weight of wood in a retort, 12½ parts of wood must be burned in the furnace. Hence in this process the whole expenditure of wood to produce from 28 to 30 parts of charcoal is 112½ parts; so that if the weight of charcoal obtained is compared with the whole weight of wood expended, its amount is from 25% to 27%; and the proportion lost is on an average 11½ + 87½ = 0.8, nearly.

According to Peclet, good wood charcoal contains about 0.07 of its weight of ash. The proportion of ash in peat charcoal is very variable, and is estimated on an average at about 0.18. (Rankine.)

Much information concerning charcoal may be found in the Journal of the Charcoal-iron Workers' Assn., vols. i. to vi. From this source the following notes have been taken:

Yield of Charcoal from a Cord of Wood.—From 45 to 50 bushels to the cord in the kiln, and from 30 to 35 in the meiler. Prof. Egleston in Trans. A. I. M. E., viii. 395, says the yield from kilns in the Lake Champlain region is often from 50 to 60 bushels for hard wood and 50 for soft wood; the average is about 50 bushels.

The apparent yield per cord depends largely upon whether the cord is a full cord of 128 cu. ft. or not.

In a four months' test of a kiln at Goodrich, Tenn., Dr. H. M. Pierce found results as follows: Dimensions of kiln—inside diameter of base, 28 ft. 8 in.; diam. at spring of arch, 26 ft. 8 in.; height of walls, 8 ft.; rise of arch, 5 ft.; capacity, 30 cords. Highest yield of charcoal per cord of wood (measured) 59.27 bushels, lowest 50.14 bushels, average 53.65 bushels.

No. of charges 12, length of each turn or period from one charging to another 11 days. (J. C. I. W., vol. vi. p. 26.)

Results from Different Methods of Charcoal-making.

Coaling Methods.	Character of Wood used.	Yield. In Volume per cent.	Yield. In Weight per cent.	Bushels of Charcoal per Cord of Wood.	Weight in Lbs. per Bushel of Charcoal.
Odelstjerna's experiments	Birch dried at 230 F.......	....	35.9	......	......
Mathieu's retorts, fuel excluded....................	Air dry, av. good yellow pine weighing abt. 28 lbs. per cu. ft.	77.0	28.3	63.4	15.7
Mathieu's retorts, fuel included....................		65.8	24.2	54.2	15.7
Swedish ovens, av. results	Good dry fir and pine, mixed.	81.0	27.7	66.7	13.8
Swedish ovens, av. results	Poor wood, mixed fir and pine	70.0	25.8	62.0	13.8
Swedish meilers exceptional....................	Fir and white-pine wood, mixed. Av. 25 lbs. per cu. ft.	72.2	24.7	59.5	13.3
Swedish meilers, av. results		52.5	18.3	43.9	13.8
American kilns, av. results	Av. good yellow pine weighing abt. 25 lbs. per cu. ft.	54.7	22.0	45.0	17.5
American meilers, av. results........		42.9	17.1	35.0	17.5

Consumption of Charcoal in Blast-furnaces per Ton of Pig Iron; average consumption according to census of 1880, 1.14 tons charcoal per ton of pig. The consumption at the best furnaces is much below this average. As low as 0.853 ton, is recorded of the Morgan furnace; Bay furnace, 0.858; Elk Rapids, 0.884. (1892.)

Absorption of Water and of Gases by Charcoal.—Svedlius, in his hand-book for charcoal-burners, prepared for the Swedish Government, says: Fresh charcoal, also reheated charcoal, contains scarcely any water but when cooled it absorbs it very rapidly, so that after twenty-four hours, it may contain 4% to 8% of water. After the lapse of a few weeks the moisture of charcoal may not increase perceptibly, and may be estimated at 10% to 15%, or an average of 12%. A thoroughly charred piece of charcoal ought, then, to contain about 84 parts carbon, 12 parts water, 3 parts ash, and 1 part hydrogen.

M. Saussure, operating with blocks of fine boxwood charcoal, freshly burnt, found that by simply placing such blocks in contact with certain gases they absorbed them in the following proportion:

	Volumes.		Volumes.
Ammonia	90.00	Carbonic oxide	9.42
Hydrochloric-acid gas	85.00	Oxygen	9.25
Sulphurous acid	65.00	Nitrogen	6.50
Sulphuretted hydrogen	55.00	Carburetted hydrogen	5.00
Nitrous oxide (laughing-gas)	40.00	Hydrogen	1.75
Carbonic acid	35.00		

It is this enormous absorptive power that renders of so much value a comparatively slight sprinkling of charcoal over dead animal matter, as a preventive of the escape of odors arising from decomposition.

In a box or case containing one cubic foot of charcoal may be stored without mechanical compression a little over nine cubic feet of oxygen, representing a mechanical pressure of one hundred and twenty-six pounds to the square inch. From the store thus preserved the oxygen can be drawn by a small hand-pump.

Composition of Charcoal Produced at Various Temperatures. (By M. Violette.)

	Temperature of Carbonization.		Composition of the Solid Product.				
			Carbon.	Hydrogen.	Oxygen.	Nitrogen and Loss.	Ash.
	Cent.	Fahr.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1	150°	302°	47.51	6.12	46.29	0.08	47.51
2	200	392	51.82	3.99	43.98	0.23	39.88
3	250	482	65.59	4.81	28.97	0.63	32.98
4	300	592	73.24	4.25	21.96	0.57	24.61
5	350	662	76.64	4.14	18.44	0.61	22.42
6	432	810	81.64	4.96	15.24	1.61	15.40
7	1023	1873	81.97	2.30	14.15	1.60	15.30

The wood experimented on was that of black alder, or alder buckthorn, which furnishes a charcoal suitable for gunpowder. It was previously dried at 150 deg. C. = 302 deg. F.

MISCELLANEOUS SOLID FUELS.

Dust Fuel—Dust Explosions.—Dust when mixed in air burns with such extreme rapidity as in some cases to cause explosions. Explosions of flour-mills have been attributed to ignition of the dust in confined passages. Experiments in England in 1876 on the effect of coal-dust in carrying flame in mines showed that in a dusty passage the flame from a blown-out shot may travel 50 yards. Prof. F. A. Abel (Trans. A. I. M. E., xiii. 260) says that coal-dust in mines much promotes and extends explosions, and that it may readily be brought into operation as a fiercely burning agent which will carry flame rapidly as far as its mixture with air extends, and will operate as an explosive agent though the medium of a very small proportion of fire-damp in the air of the mine. The explosive violence of the combustion of dust is largely due to the instantaneous heating and consequent expansion of the air. (See also paper on "Coal Dust as an Explosive Agent," by Dr. R. W. Raymond, Trans. A. I. M. E. 1894.) Experiments made in Germany in 1893, show that pulverized fuel may be burned without smoke, and with high economy. The fuel, instead of being introduced into the fire-box in the ordinary manner, is first reduced to a powder by pulverizers of any construction. In the place of the ordinary boiler fire-box there is a combustion chamber in the form of a closed furnace lined with fire-brick and provided with an air-injector similar in construction to those used in oil-burning furnaces. The nozzle throws a constant stream of the fuel into the chamber. This nozzle is so located that it scatters the powder throughout the whole space of the fire-box. When this powder is once ignited, and it is very readily done by first raising the lining to a high temperature by an open fire, the combustion continues in an intense and regular manner under the action of the current of air which carries it in. (*Mfrs. Record*, April, 1893.)

Powdered fuel was used in the Crompton rotary puddling-furnace at Woolwich Arsenal, England, in 1873. (Jour. I. & S. I., i. 1873, p. 91.)

Peat or Turf, as usually dried in the air, contains from 25% to 30% of water, which must be allowed for in estimating its heat of combustion. This water having been evaporated, the analysis of M. Regnault gives, in 100 parts of perfectly dry peat of the best quality: C 58%, H 6%, O 31%, Ash 5%.

In some examples of peat the quantity of ash is greater, amounting to 7% and sometimes to 11%.

The specific gravity of peat in its ordinary state is about 0.4 or 0.5. It can be compressed by machinery to a much greater density. (Rankine.)

Clark (Steam-engine, i. 61) gives as the average composition of dried Irish peat: C 59%, H 6%, O 30%, N 1.25%, Ash 4%.

Applying Dulong's formula to this analysis, we obtain for the heating value of perfectly dry peat 10,260 heat-units per pound, and for air-dried peat containing 25% of moisture, after making allowance for evaporating the water, 7391 heat-units per pound.

Sawdust as Fuel.—The heating power of sawdust is naturally the same per pound as that of the wood from which it is derived, but if allowed to get wet it is more like spent tan (which see below). The conditions necessary for burning sawdust are that plenty of room should be given it in the furnace, and sufficient air supplied on the surface of the mass. The same applies to shavings, refuse lumber, etc. Sawdust is frequently burned in saw-mills, etc., by being blown into the furnace by a fan-blast.

Horse-manure has been successfully used as fuel by the Cable Railway Co. of Chicago. It was mixed with soft coal and burned in an ordinary urnace provided with a fire-brick arch.

Wet Tan Bark as Fuel.—Tan, or oak bark, after having been used in the processes of tanning, is burned as fuel. The spent tan consists of the fibrous portion of the bark. According to M. Peclet, five parts of oak bark produce four parts of dry tan; and the heating power of perfectly dry tan, containing 15% of ash, is 6100 English units; whilst that of tan in an ordinary state of dryness, containing 30% of water, is only 4284 English units. The weight of water evaporated from and at 212° by one pound of tan, equivalent to these heating powers, is, for perfectly dry tan, 5.46 lbs., for tan with 30% moisture, 3.84 lbs. Experiments by Prof. R. H. Thurston (Jour. Frank. Inst., 1874) gave with the Crockett furnace, the wet tan containing 59% of water, an evaporation from and at 212° F. of 4.24 lbs. of water per pound of the wet tan, and with the Thompson furnace an evaporation of 3.19 lbs. per pound of wet tan containing 55% of water. The Thompson furnace consisted of six fire-brick ovens, each 9 feet × 4 feet 4 inches, containing 234 square feet of grate in all, for three boilers with a total heating surface of 2000 square feet, a ratio of heating to grate surface of 9 to 1. The tan was fed through holes in the top. The Crockett furnace was an ordinary fire-brick furnace, 6 × 4 feet, built in front of the boiler, instead of under it, the ratio of heating surface to grate being 14.6 to 1. According to Prof. Thurston the conditions of success in burning wet fuel are the surrounding of the mass so completely with heated surfaces and with burning fuel that it may be rapidly dried, and then so arranging the apparatus that thorough combustion may be secured, and that the rapidity of combustion be precisely equal to and never exceed the rapidity of desiccation. Where this rapidity of combustion is exceeded the dry portion is consumed completely, leaving an uncovered mass of fuel which refuses to take fire.

Straw as Fuel. (*Eng'g Mechanics*, Feb., 1893, p. 55.)—Experiments in Russia showed that winter-wheat straw, dried at 230° F., had the following composition; C, 46.1; H. 5.6; N, 0.42; O, 43.7; Ash, 4.1. Heating value in British thermal units: dry straw, 6290; with 6% water, 5770; with 10% water, 5448. With straws of other grains the heating value of dry straw ranged from 5590 for buckwheat to 6750 for flax.

Clark (S. E., vol. 1, p. 62) gives the mean composition of wheat and barley straw as C, 36; H. 5; O. 38; O, 0.50; Ash, 4.75; water, 15.75, the two straws varying less than 1%. The heating value of straw of this composition, according to Dulong's formula, and deducting the heat lost in evaporating the water, is 5155 heat units. Clark erroneously gives it as 8144 heat units.

Bagasse as Fuel in Sugar Manufacture.--Bagasse is the name given to refuse sugar-cane, after the juice has been extracted. Prof. L. A.

Becuel, in a paper read before the Louisiana Sugar Chemists' Association, in 1892, says: "With tropical cane containing 12.5% woody fibre, a juice containing 16.18% solids, and 83.37% water, bagasse of, say, 66% and 72% mill extraction would have the following percentage composition:

	Woody Fibre.	Combustible Salts.	Water.
66% bagasse..................	37	10	53
72% bagasse...................... .	45	9	46

"Assuming that the woody fibre contains 51% carbon, the sugar and other combustible matters an average of 42.1%, and that 12,906 units of heat are generated for every pound of carbon consumed, the 66% bagasse is capable of generating 297,834 heat units as against 345,200, or a difference of 47,366 units in favor of the 72% bagasse.

"Assuming the temperature of the waste gases to be 450° F., that of the surrounding atmosphere and water in the bagasse at 86° F., and the quantity of air necessary for the combustion of one pound of carbon at 24 lbs., the lost heat will be as follows: In the waste gases, heating air from 86° to 450° F., and in vaporizing the moisture, etc., the 66% bagasse will require 112,546 heat units, and 116,150 for the 72% bagasse.

"Subtracting these quantities from the above, we find that the 66% bagasse will produce 185,288 available heat units, or nearly 38% less than the 72% bagasse, which gives 299,050 units. Accordingly, one ton of cane of 2000 lbs. at 66% mill extraction will produce 680 lbs. bagasse, equal to 125,995,840 available heat units, while the same cane at 72% extraction will produce 560 lbs. bagasse, equal to 167,468,000 units.

"A similar calculation for the case of Louisiana cane containing 10% woody fibre, and 16% total solids in the juice, assuming 75% mill extraction, shows that bagasse from one ton of cane contains 157,395,640 heat units, from which 56,146,500 have to be deducted.

"This would make such bagasse worth on an average nearly 92 lbs. coal per ton of cane ground. Under fairly good conditions, 1 lb. coal will evaporate 7½ lbs. water, while the best boiler plants evaporate 10 lbs. Therefore, the bagasse from 1 ton of cane at 75% mill extraction should evaporate from 689 lbs. to 919 lbs. of water. The juice extracted from such cane would under these conditions contain 1260 lbs. of water. If we assume that the water added during the process of manufacture is 10% (by weight) of the juice made, the total water handled is 1410 lbs. From the juice represented in this case, the commercial massecuite would be about 15% of the weight of the original mill juice, or say 225 lbs. Said mill juice 1500 lbs., plus 10%, equals 1650 lbs. liquor handled; and 1650 lbs., minus 225 lbs., equals 1425 lbs., the quantity of water to be evaporated during the process of manufacture. To effect a 7½-lb. evaporation requires 190 lbs. of coal, and 142½ lbs. for a 10-lb. evaporation.

"To reduce 1650 lbs. of juice to syrup of, say, 27° Baumé, requires the evaporation of 1770 lbs. of water, leaving 480 lbs. of syrup. If this work be accomplished in the open air, it will require about 156 lbs. of coal at 7½ lbs. boiler evaporation, and 117 at 10 lbs. evaporation.

"With a double effect the fuel required would be from 59 to 78 lbs., and with a triple effect, from 36 to 52 lbs.

"To reduce the above 480 lbs. of syrup to the consistency of commercial massecuite means the further evaporation of 255 lbs. of water, requiring the expenditure of 34 lbs. coal at 7½ lbs. boiler evaporation, and 25½ lbs. with a 10-lb. evaporation. Hence, to manufacture one ton of cane into sugar and molasses, it will take from 145 to 190 lbs. additional coal to do the work by the open evaporator process; from 85 to 112 lbs. with a double effect, and only 7½ lbs. evaporation in the boilers, while with 10 lbs. boiler evaporation the bagasse alone is capable of furnishing 8% more heat than is actually required to do the work. With triple-effect evaporation depending on the excellence of the boiler plant, the 1425 lbs. of water to be evaporated from the juice will require between 62 and 86 lbs. of coal. These values show that from 6 to 30 lbs. of coal can be spared from the value of the bagasse to run engines, grind cane, etc.

"It accordingly appears," says Prof. Becuel, "that with the best boiler plants, those taking up all the available heat generated, by using this heat economically the bagasse can be made to supply all the fuel required by our sugar-houses."

PETROLEUM.

Products of the Distillation of Crude Petroleum.

Crude American petroleum of sp. gr. 0.800 may be split up by fractional distillation as follows (Robinson's Gas and Petroleum Engines):

Temp. of Distillation Fahr.	Distillate.	Percentages.	Specific Gravity.	Flashing Point. Deg. F.
113°	Rhigolene.	traces.	.590 to .625	....
113 to 140°	Chymogene.			
140 to 158°	Gasolene (petroleum spirit)...	1.5	.636 to .657	....
158 to 248°	Benzine, naphtha C, benzolene.	10.	.680 to .700	14
248° to 347°	Benzine, naphtha B.........	2.5	.714 to .718	....
	" " A.........	2.	.725 to .737	32
	Polishing oils.	....	....	....
338° and upwards.	Kerosene (lamp-oil)...........	50.	.802 to .820	100 to 122
482°	Lubricating oil...............	15.	.850 to .915	230
....	Paraffine wax........	2.	....	....
....	Residue and Loss......... ..	16.	....	....

Lima Petroleum, produced at Lima, Ohio, is of a dark green color, very fluid, and marks 48° Baumé at 15° C. (sp. gr., 0.792).

The distillation in fifty parts, each part representing 2% by volume, gave the following results:

Per cent.	Sp. Gr.	Per cent.	Sp. Gr.	Per cent.	Sp. Gr.	Per cent.	Sp. Gr.	Per cent.	Sp. Gr.	Per cent.	Sp. Gr.
2	0.680	18	0.720	84	0.764	50	0.802	68	0.820	88	0.815
4	.683	20	.728	86	.768	52		70	.825	90	.815
6	.685	22	.730	88	.772	to	.806	72	.830		
8	.690	24	.735	40	.778	58		73	.830	92	
10	.694	26	.740	42	.782	60	.800	76	.810	to	Residuum
12	.698	28	.742	44	.788	62	.804	78	.820	100	
14	.700	30	.746	46	.792	64	.808	82	.818		
16	.706	32	.760	48	.800	66	.812	86	.816		

RETURNS.

16 per cent naphtha, 70° Baumé.	6 per cent paraffine oil.
68 " burning oil.	10 " residuum.

The distillation started at 23° C., this being due to the large amount of naphtha present, and when 60% was reached, at a temperature of 310° C., the hydrocarbons remaining in the retort were dissociated, then gases escaped, lighter distillates were obtained, and, as usual in such cases, the temperature decreased from 310° C. down gradually to 200° C., until 75% of oil was obtained, and from this point the temperature remained constant until the end of the distillation. Therefore these hydrocarbons in *statu moriendi* absorbed much heat. (*Jour. Am. Chem. Soc.*)

Value of Petroleum as Fuel.—Thos. Urquhart, of Russia (Proc. Inst. M. E., Jan. 1889), gives the following table of the theoretical evaporative power of petroleum in comparison with that of coal, as determined by Messrs. Favre & Silbermann:

Fuel.	Specific Gravity at 32° F., Water = 1.000.	Chem. Comp.			Heating-power, British Thermal Units.	Theoret. Evap., lbs. Water per lb. Fuel, from and at 212° F.
		C.	H.	O.		
	S. G.	p. c.	p. c.	p. c.	Units.	lbs.
Penna. heavy crude oil....	0.886	84.9	13.7	1.4	20,736	21.48
Caucasian light crude oil..	0.884	86.3	13.6	0.1	22,027	22.79
" heavy " "..	0.938	86.6	12.3	1.1	20,138	20.85
Petroleum refuse..........	0.928	87.1	11.7	1.2	19,832	20.53
Good English Coal, Mean of 98 Samples..........	1.380	80.0	5.0	8.0	14,112	14.61

In experiments on Russian railways with petroleum as fuel Mr. Urquhart obtained an actual efficiency equal to 82% of the theoretical heating-value. The petroleum is fed to the furnace by means of a spray-injector driven by steam. An induced current of air is carried in around the injector-nozzle, and additional air is supplied at the bottom of the furnace.

Oil vs. Coal as Fuel. (*Iron Age*, Nov. 2, 1893.)—Test by the Twin City Rapid Transit Company of Minneapolis and St. Paul. This test showed that with the ordinary Lima oil weighing 6 6/10 pounds per gallon, and costing 2¼ cents per gallon, and coal that gave an evaporation of 7½ lbs. of water per pound of coal, the two fuels were equally economical when the price of coal was $3.85 per ton of 2000 lbs. With the same coal at $2.00 per ton, the coal was 37% more economical, and with the coal at $4.85 per ton, the coal was 20% more expensive than the oil. These results include the difference in the cost of handling the coal, ashes, and oil.

In 1892 there were reported to the Engineers' Club of Philadelphia some comparative figures, from tests undertaken to ascertain the relative value of coal, petroleum, and gas.

	Lbs. Water, from and at 212° F.
1 lb. anthracite coal evaporated	9.70
1 lb. bituminous coal	10.14
1 lb. fuel oil, 36° gravity	16.48
1 cubic foot gas, 20 C. P.	1.28

The gas used was that obtained in the distillation of petroleum, having about the same fuel-value as natural or coal-gas of equal candle-power.

Taking the efficiency of bituminous coal as a basis, the calorific energy of petroleum is more than 60% greater than that of coal; whereas, theoretically, petroleum exceeds coal only about 45%—the one containing 14,500 heat-units, and the other 21,000.

Crude Petroleum vs. Indiana Block Coal for Steam-raising at the South Chicago Steel Works. (E. C. Potter, Trans. A. I. M. E., xvii, 807.)—With coal, 14 tubular boilers 16 ft. × 5 ft. required 25 men to operate them: with fuel oil, 6 men were required, a saving of 19 men at $2 per day, or $38 per day.

For one week's work 2731 barrels of oil were used, against 848 tons of coal required for the same work, showing 3.22 barrels of oil to be equivalent to 1 ton of coal. With oil at 60 cents per barrel and coal at $2.15 per ton, the relative cost of oil to coal is as $1.93 to $2.15. No evaporation tests were made.

Petroleum as a Metallurgical Fuel.—C. E. Felton (Trans. A. I. M. E., xvii, 809) reports a series of trials with oil as fuel in steel-heating and open-hearth steel-furnaces, and in raising steam with results as follows: 1. In a run of six weeks the consumption of oil, partly refined (the paraffine and some of the naphtha being removed), in heating 14-inch ingots in Siemens furnaces was about 6¼ gallons per ton of blooms. 2. In melting in a 30-ton open-hearth furnace 48 gallons of oil were used per ton of ingots. 3. In a six weeks' trial with Lima oil from 47 to 54 gallons of oil were required per ton of ingots. 4. In a six months' trial with Siemens heating-furnaces the consumption of Lima oil was 6 gallons per ton of ingots. Under the most favorable circumstances, charging hot ingots and running full capacity, 4½ to 5 gallons per ton were required. 5. In raising steam in two 100-H.P. tubular boilers, the feed-water being supplied at 160° F., the average evaporation was about 12 pounds of water per pound of oil, the best 12 hours' work being 16 pounds.

In all of the trials the oil was vaporized in the Archer producer, an apparatus for mixing the oil and superheated steam, and heating the mixture to a high temperature. From 0.5 lb. to 0.75 lb. of pea-coal was used per gallon of oil in the producer itself.

FUEL GAS.

The following notes are extracted from a paper by W. J. Taylor on "The Energy of Fuel" (Trans. A. I. M. E., xviii. 205):

Carbon Gas.—In the old Siemens producer, practically, all the heat of primary combustion—that is, the burning of solid carbon to carbon monoxide, or about 30% of the total carbon energy—was lost, as little or no steam was used in the producer, and nearly all the sensible heat of the gas was dissipated in its passage from the producer to the furnace, which was usually placed at a considerable distance.

Modern practice has improved on this plan, by introducing steam with the air blown into the producer, and by utilizing the sensible heat of the gas in the combustion-furnace. It ought to be possible to oxidize one out of every four lbs. of carbon with oxygen derived from water-vapor. The thermic reactions in this operation are as follows:

	Heat-units.
4 lbs. C burned to CO (3 lbs. gasified with air and 1 lb. with water) develop	17,600
1.5 lbs. of water (which furnish 1.33 lbs. of oxygen to combine with 1 lb. of carbon) absorb by dissociation	10,333
The gas, consisting of 9.333 lbs. CO, 0.167 lb. H, and 13.39 lbs. N, heated 600°, absorbs	3,748
Leaving for radiation and loss	3,519
	17,600

The steam which is blown into a producer with the air is almost all condensed into finely-divided water before entering the fuel, and consequently is considered as water in these calculations.

The 1.5 lbs. of water liberates .167 lb. of hydrogen, which is delivered to the gas, and yields in combustion the same heat that it absorbs in the producer by dissociation. According to this calculation, therefore, 60% of the heat of primary combustion is theoretically recovered by the dissociation of steam, and, even if all the sensible heat of the gas be counted, with radiation and other minor items, as loss, yet the gas must carry $4 \times 14,500 - (3748 + 3519) = 50,733$ heat-units, or 87% of the calorific energy of the carbon. This estimate shows a loss in conversion of 13%, without crediting the gas with its sensible heat, or charging it with the heat required for generating the necessary steam, or taking into account the loss due to oxidizing some of the carbon to CO_2. In good producer-practice the proportion of CO_2 in the gas represents from 4% to 7% of the C burned to CO_2, but the extra heat of this combustion should be largely recovered in the dissociation of more water-vapor, and therefore does not represent as much loss as it would indicate. As a conveyer of energy, this gas has the advantage of carrying 4.46 lbs. less nitrogen than would be present if the fourth pound of coal had been gasified with air; and in practical working the use of steam reduces the amount of clinkering in the producer.

Anthracite Gas.—In anthracite coal there is a volatile combustible varying in quantity from 1.5% to over 7%. The amount of energy derived from the coal is shown in the following theoretical gasification made with coal of assumed composition: Carbon, 85%; vol. HC, 5%; ash, 10%; 80 lbs. carbon assumed to be burned to CO; 5 lbs. carbon burned to CO_2; three fourths of the necessary oxygen derived from air, and one fourth from water.

Process.	Products. Pounds.	Cubic Feet.	Anal. by Vol.
80 lbs. C burned to .... CO	186.66	2529.24	33.4
5 lbs. C burned to .... CO_2	18.33	157.64	2.0
5 lbs. vol. HC (distilled)	5.00	116.60	1.6
120 lbs. oxygen are required, of which			
30 lbs. from H_2O liberate .... H	3.75	712.50	9.4
90 lbs. from air are associated with N	301.05	4064.17	53.6
	514.79	7580.15	100.0

Energy in the above gas obtained from 100 lbs. anthracite:

186.66 lbs. CO	807,304	heat-units.
5.00 " CH_4	117,500	"
3.75 " H	232,500	"
	1,157,304	"
Total energy in gas per lb	2,248	"
" " " 100 lbs. of coal..	1,349,500	"

Efficiency of the conversion .... 86%.

The sum of CO and H exceeds the results obtained in practice. The sensible heat of the gas will probably account for this discrepancy, and, therefore, it is safe to assume the possibility of delivering at least 82% of the energy of the anthracite.

Bituminous Gas.—A theoretical gasification of 100 lbs. of coal, containing 55% of carbon and 32% of volatile combustible (which is above the average of Pittsburgh coal), is made in the following table. It is assumed that 50 lbs. of C are burned to CO and 5 lbs. to CO_2; one fourth of the O is derived from steam and three fourths from air; the heat value of the volatile combustible is taken at 20,000 heat-units to the pound. In computing volumetric proportions all the volatile hydrocarbons, fixed as well as condensing, are classed as marsh-gas, since it is only by some such tentative assumption that even an approximate idea of the volumetric composition can be formed. The energy, however, is calculated from weight:

Process.	Products. Pounds.	Cubic Feet.	Anal. by Vol.
50 lbs. C burned to................CO	116.66	1580.7	27.8
5 lbs. C burned to................CO_2	18.33	157.6	2.7
32 lbs. vol. HC (distilled)...............	32.00	746.2	13.2
80 lbs. O are required, of which 20 lbs., derived from H_2O, liberate.......H	2.5	475.0	8.3
60 lbs. O, derived from air, are associated with......................N	200.70	2709.4	47.8
	370.19	5668.9	99.8

Energy in 116.66 lbs. CO.. .... 504,554 heat-units.
" " 32.00 lbs. vol. HC.... 640,000 "
" " 2.50 lbs. H......... 155,000 "

1,299,554 "
Energy in coal..... .... 1,437,500 "
Per cent of energy delivered in gas............ 90.0
Heat-units in 1 lb. of gas......................3,484

Water-gas.—Water-gas is made in an intermittent process, by blowing up the fuel-bed of the producer to a high state of incandescence (and in some cases utilizing the resulting gas, which is a lean producer-gas), then shutting off the air and forcing steam through the fuel, which dissociates the water into its elements of oxygen and hydrogen, the former combining with the carbon of the coal, and the latter being liberated.

This gas can never play a very important part in the industrial field, owing to the large loss of energy entailed in its production, yet there are places and special purposes where it is desirable, even at a great excess in cost per unit of heat over producer-gas; for instance, in small high-temperature furnaces, where much regeneration is impracticable, or where the "blow-up" gas can be used for other purposes instead of being wasted.

The reactions and energy required in the production of 1000 feet of water-gas, composed, theoretically, of equal volumes of CO and H, are as follows:

500 cubic feet of H weigh................................ 2.635 lbs.
500 cubic feet of CO weigh................................ 36.89 "

Total weight of 1000 cubic feet........ .... 39.525 lbs.

Now, as CO is composed of 12 parts C to 16 of O, the weight of C in 36.89 lbs. is 15.81 lbs. and of O 21.08 lbs. When this oxygen is derived from water it liberates, as above, 2.635 lbs. of hydrogen. The heat developed and absorbed in these reactions (roughly, as we will not take into account the energy required to elevate the coal from the temperature of the atmosphere to say 1800°) is as follows:

		Heat-units.
2.635 lbs. H absorb in dissociation from water 2.635 × 62,000..	=	163,370
15.81 lbs. C burned to CO develops 15.81 × 4400...............	=	69,564
Excess of heat-absorption over heat-development	=	93,806

If this excess could be made up from C burnt to CO_2 without loss by radiation, we would only have to burn an additional 4.83 lbs. C to supply this heat, and we could then make 1000 feet of water-gas from 20.64 lbs. of carbon (equal 24 lbs. of 85% coal). This would be the perfection of gas-making, as the gas would contain really the same energy as the coal; but instead, we require in practice more than double this amount of coal, and do not deliver more than 50% of the energy of the fuel in the gas, because the supporting heat is obtained in an indirect way and with imperfect combustion. Besides this, it is not often that the sum of the CO and H exceed 90%, the balance being CO_2 and N. But water-gas should be made with much less loss of energy by burning the "blow-up" (producer) gas in brick regenerators, the stored-up heat of which can be returned to the producer by the air used in blowing-up.

The following table shows what may be considered average volumetric analyses, and the weight and energy of 1000 cubic feet, of the four types of gases used for heating and illuminating purposes:

	Natural Gas.	Coal-gas.	Water-gas.	Producer-gas.	
				Anthra.	Bitu.
CO	0.50	6.0	45.0	27.0	27.0
H	2.18	46.0	45.0	12.0	12.0
CH_4	92.6	40.0	2.0	1.2	2.5
C_2H_4	0.31	4.0	.....	....	0.4
CO_2	0.26	0.5	4.0	2.5	2.5
N	3.61	1.5	2.0	57.0	56.2
O	0.34	0.5	0.5	0.3	0.3
Vapor	....	1.5	1.5	....	....
Pounds in 1000 cubic feet	45.6	32.0	45.6	65.6	65.9
Heat units in 1000 cubic feet	1,100,000	735,000	322,000	137,455	156,917

Natural Gas in Ohio and Indiana.

(*Eng. and M. J.*, April 21, 1894.)

Description.	Ohio.			Indiana.			
	Fostoria.	Findlay	St Mary's.	Muncie.	Anderson.	Kokomo.	Marion.
Hydrogen	1.89	1.64	1.94	2.35	1.86	1.42	1.20
Marsh-gas	92.84	93.35	93.85	92.67	93.07	94.16	93.57
Olefiant gas	.20	.35	.20	.25	.47	.30	.15
Carbon monoxide	.55	.41	.44	.45	.73	.55	.60
Carbon dioxide	.20	.25	.23	.25	.26	.29	.30
Oxygen	.35	.39	.35	.35	.42	.30	.55
Nitrogen	3.82	3.41	2.98	3.53	3.02	2.80	3.42
Hydrogen sulphide	.15	.20	.21	.15	.15	.18	.20

Approximately 30,000 cubic feet of gas have the heating power of one ton of coal.

Producer-gas from One Ton of Coal.

(W. H. Blauvelt, Trans. A. I. M. E., xviii. 614.)

Analysis by Vol.	Per Cent.	Cubic Feet.	Lbs.	Equal to—
CO	25.3	33,213.84	2451.20	1050.51 lbs. C + 1400.7 lbs. O.
H	9.2	12,077.76	63.56	63.56 " H.
CH_4	3.1	4,069.68	174.66	174.66 " CH_4.
C_2H_4	0.8	1,050.24	77.78	77.78 " C_2H_4.
CO_2	3.4	4,463.52	519.02	141.54 " C + 377.44 lbs. O.
N (by difference.	58.2	76,404.96	5659.63	7350.17 " Air.
	100.0	131,280.00	8945.85	

Calculated upon this basis, the 131,280 ft. of gas from the ton of coal contained 20,311.162 B.T.U., or 155 B.T.U. per cubic ft., or 2270 B.T.U. per lb.

The composition of the coal from which this gas was made was as follows: Water, 1.26%; volatile matter, 36.22%; fixed carbon, 57.98%; sulphur, 0.70%; ash, 3.78%. One ton contains 1159.6 lbs. carbon and 724.4 lbs. volatile combustible, the energy of which is 31,302,200 B.T.U. Hence, in the processes of gasification and purification there was a loss of 35.2% of the energy of the coal.

The composition of the hydrocarbons in a soft coal is uncertain and quite complex; but the ultimate analysis of the average coal shows that it approaches quite nearly to the composition of CH_4 (marsh-gas).

Mr. Blauvelt emphasizes the following points as highly important in soft-coal producer-practice:

First. That a large percentage of the energy of the coal is lost when the gas is made in the ordinary low producer and cooled to the temperature of the air before being used. To prevent these sources of loss, the producer should be placed so as to lose as little as possible of the sensible heat of the gas, and prevent condensation of the hydrocarbon vapors. A high fuel-bed should be carried, keeping the producer cool on top, thereby preventing the breaking-down of the hydrocarbons and the deposit of soot, as well as keeping the carbonic acid low.

Second. That a producer should be blown with as much steam mixed with the air as will maintain incandescence. This reduces the percentage of nitrogen and increases the hydrogen, thereby greatly enriching the gas. The temperature of the producer is kept down, diminishing the loss of heat by radiation through the walls, and in a large measure preventing clinkers.

The Combustion of Producer-gas. (H. H. Campbell, Trans. A. I. M. E., xix. 128.)—The combustion of the components of ordinary producer-gas may be represented by the following formulæ:

$$C_2H_4 + 6O = 2CO_2 + 2H_2O; \quad 2H + O = H_2O;$$
$$CH_4 + 4O = CO_2 + 2H_2O; \quad CO + O = CO_2.$$

AVERAGE COMPOSITION BY VOLUME OF PRODUCER-GAS: A, MADE WITH OPEN GRATES, NO STEAM IN BLAST; B, OPEN GRATES, STEAM-JET IN BLAST. 10 SAMPLES OF EACH.

	CO_2.	O.	C_2H_4.	CO.	H.	CH_4.	N.
A min	3.6	0.4	0.2	20.0	5.3	3.0	58.7
A max.......	5.6	0.4	0.4	24.8	8.5	5.2	64.4
A average...	4.84	0.4	0.34	22.1	6.8	3.74	61.78
B min	4.6	0.4	0.2	20.8	6.9	2.2	57.2
B max	6.0	0.8	0.4	24.0	9.8	3.4	62.0
B average...	5.3	0.54	0.36	22.74	8.37	2.56	60.13

The coal used contained carbon 82%, hydrogen 4.7%.

The following are analyses of products of combustion:

	CO_2.	O.	CO.	CH_4.	H.	N.
Minimum.......	15.2	0.2	trace.	trace.	trace.	80.1
Maximum	17.2	1.6	2.0	0.6	2.0	83.6
Average	16.3	0.8	0.4	0.1	0.2	82.2

Use of Steam in Producers and in Boiler-furnaces. (R. W. Raymond, Trans. A. I. M. E., xx. 635.)—No possible use of steam can cause a gain of heat. If steam be introduced into a bed of incandescent carbon it is decomposed into hydrogen and oxygen.

The heat absorbed by the reduction of one pound of steam to hydrogen is much greater in amount than the heat generated by the union of the oxygen thus set free with carbon, forming either carbonic oxide or carbonic acid. Consequently, the effect of steam alone upon a bed of incandescent fuel is to chill it. In every water-gas apparatus, designed to produce by means of the decomposition of steam a fuel-gas relatively free from nitrogen, the loss of heat in the producer must be compensated by some reheating device.

This loss may be recovered if the hydrogen of the steam is subsequently burned, to form steam again. Such a combustion of the hydrogen is contemplated, in the case of fuel-gas, as secured in the subsequent use of that gas. Assuming the oxidation of H to be complete, the use of steam will cause neither gain nor loss of heat, but a simple transference, the heat absorbed by steam decomposition being restored by hydrogen combustion. In practice, it may be doubted whether this restoration is ever complete. But it is certain that an excess of steam would defeat the reaction altogether, and that there must be a certain proportion of steam, which permits the realization of important advantages, without too great a net loss in heat.

The advantage to be secured (in boiler furnaces using small sizes of anthracite) consists principally in the transfer of heat from the lower side of the fire, where it is not wanted, to the upper side, where it is wanted. The decomposition of the steam below cools the fuel and the grate-bars, whereas a blast of air alone would produce, at that point, intense combustion (forming at first CO_2), to the injury of the grate, the fusion of part of the fuel, etc.

The proportion of steam most economical is not easily determined. The temperature of the steam itself, the nature of the fuel mixture, and the use or non-use of auxiliary air-supply, introduced into the gases above or beyond the fire-bed, are factors affecting the problem. (See Trans. A. I. M. E., xx. 625.)

Gas Analyses by Volume and by Weight.—To convert an analysis of a mixed gas by volume into analysis by weight: Multiply the percentage of each constituent gas by the density of that gas (see p. 166). Divide each product by the sum of the products to obtain the percentages by weight.

Gas-fuel for Small Furnaces.—E. P. Reichhelm (*Am. Mach.*, Jan. 10, 1895) discusses the use of gaseous fuel for forge fires, for drop-forging, in annealing-ovens and furnaces for melting brass and copper, for case-hardening, muffle-furnaces, and kilns. Under ordinary conditions, in such furnaces he estimates that the loss by draught, radiation, and the heating of space not occupied by work is, with coal, 80%, with petroleum 70%, and with gas above the grade of producer-gas 25%. He gives the following table of comparative cost of fuels, as used in these furnaces:

Kind of Gas.	No. of Heat-units in 1,000 cu. ft. used.	No. of Heat-units in Furnaces after Deducting 25% Loss.	Average Cost per 1,000 Ft.	Cost of 1,000,000 Heat-units Obtained in Furnaces.
Natural gas	1,000,000	750,000	....	....
Coal-gas, 20 candle-power	675,000	506,250	$1.25	$2.46
Carburetted water-gas	646,000	484,500	1.00	2.06
Gasolene gas, 20 candle-power	690,000	517,500	.90	1.78
Water-gas from coke	313,000	234,750	.40	1.70
Water-gas from bituminous coal	377,000	282,750	.45	1.59
Water-gas and producer-gas mixed	185,000	138,750	.20	1.44
Producer-gas	150,000	112,500	.15	1.33
Naphtha-gas, fuel 2½ gals. per 1000 ft.	306,365	229,774	.15	.65
Coal, $4 per ton, per 1,000,000 heat-units utilized				.73
Crude petroleum, 3 cts. per gal., per 1,000,000 heat-units				.73

Mr. Reichhelm gives the following figures from practice in melting brass with coal and with naphtha converted into gas: 1800 lbs. of metal require 1080 lbs. of coal, at $4.65 per ton, equal to $2.51, or, say, 15 cents per 100 lbs. Mr. T.'s report: 2500 lbs. of metal require 47 gals. of naphtha, at 6 cents per gal., equal to $2.82, or, say, 11¼ cents per 100 lbs.

ILLUMINATING-GAS.

Coal-gas is made by distilling bituminous coal in retorts. The retort is usually a long horizontal semi-cylindrical or ◠ shaped chamber, holding from 160 to 300 lbs. of coal. The retorts are set in "benches" of from 3 to 9, heated by one fire, which is generally of coke. The vapors distilled from the coal are converted into a fixed gas by passing through the retort, which is heated almost to whiteness.

The gas passes out of the retort through an "ascension-pipe" into a long horizontal pipe called the hydraulic main, where it deposits a portion of the tar it contains; thence it goes into a condenser, a series of iron tubes surrounded by cold water, where it is freed from condensable vapors, as ammonia-water, then into a washer, where it is exposed to jets of water, and into a scrubber, a large chamber partially filled with trays made of wood or iron, containing coke, fragments of brick or paving-stones, which are wet with a spray of water. By the washer and scrubber the gas is freed from the last portion of tar and ammonia and from some of the sulphur compounds. The gas is then finally purified from sulphur compounds by passing it through lime or oxide of iron. The gas is drawn from the hydraulic main and forced through the washer, scrubber, etc., by an exhauster or gas pump.

The kind of coal used is generally caking bituminous, but as usually this coal is deficient in gases of high illuminating power, there is added to it a portion of cannel coal or other enricher.

The following table, abridged from one in Johnson's Cyclopedia, shows the analysis, candle power, etc., of some gas-coals and enrichers:

Gas-coals, etc.	Vol. Matter.	Fixed Carb.	Ash.	Gas per ton of 2240 lbs. in cu. ft.	Cand.-pow'r of Gas.	Coke per ton of 2240 lbs.		Gas purified by 1 bush. of lime, in cu. ft.
						lbs.	bush.	
Pittsburgh, Pa	36.76	51.93	7.07	......	.....	....	..	....
Westmoreland, Pa	36.00	58.00	6.00	10,642	16.62	1544	40	6420
Sterling, O..............	37.50	56.90	5.60	10,528	18.81	1480	36	3993
Despard, W. Va........	40.00	53.30	6.70	10,765	20.41	1540	36	2494
Darlington, O...........	43.00	40.00	17.00	9,800	34.98	1320	32	2806
Petonia, W. Va	46.00	41.00	13.00	13,200	42.79	1880	32	4510
Grahamite, W. Va.......	53.50	44.50	2.00	15,000	28.70	1056	44	....

The products of the distillation of 100 lbs. of average gas-coal are about as follows. They vary according to the quality of coal and the temperature of distillation.

Coke, 64 to 65 lbs.; tar, 6.5 to 7.5 lbs.; ammonia liquor, 10 to 12 lbs.; purified gas, 15 to 12 lbs.; impurities and loss, 4.5% to 3.5%.

The composition of the gas by volume ranges about as follows: Hydrogen, 38% to 48%; carbonic oxide, 2% to 14%; marsh-gas (Methane, CH_4), 43% to 31%; heavy hydrocarbons (C_nH_{2n}, ethylene, propylene, benzole vapor, etc.), 7.5% to 4.5%; nitrogen, 1% to 3%.

In the burning of the gas the nitrogen is inert; the hydrogen and carbonic oxide give heat but no light. The luminosity of the flame is due to the decomposition by heat of the heavy hydrocarbons into lighter hydrocarbons and carbon, the latter being separated in a state of extreme subdivision. By the heat of the flame this separated carbon is heated to intense whiteness, and the illuminating effect of the flame is due to the light of incandescence of the particles of carbon.

The attainment of the highest degree of luminosity of the flame depends upon the proper adjustment of the proportion of the heavy hydrocarbons (with due regard to their individual character) to the nature of the diluent mixed therewith.

Investigations of Percy F. Frankland show that mixtures of ethylene and hydrogen cease to have any luminous effect when the proportion of ethylene does not exceed 10% of the whole. Mixtures of ethylene and carbonic oxide cease to have any luminous effect when the proportion of the former does not exceed 20%, while all mixtures of ethylene and marsh-gas have more or less luminous effect. The luminosity of a mixture of 10% ethylene and 90% marsh-gas being equal to about 18 candles, and that of one of 20% ethylene and 80% marsh-gas about 25 candles. The illuminating effect of marsh-gas alone, when burned in an argand burner, is by no means inconsiderable.

For further description, see the Treatises on Gas by King, Richards, and Hughes; also Appleton's Cyc. Mech., vol. i. p. 900.

Water-gas.—Water-gas is obtained by passing steam through a bed of coal, coke, or charcoal heated to redness or beyond. The steam is decomposed, its hydrogen being liberated and its oxygen burning the carbon of the fuel, producing carbonic-oxide gas. The chemical reaction is, $C + H_2O = CO + 2H$, or $2C + 2H_2O = C + CO_2 + 4H$, followed by a splitting up of the CO_2, making $2CO + 4H$. By weight the normal gas $CO + 2H$ is composed of $\underset{12}{C} + \underset{16}{O} + \underset{2}{H}$ = 28 parts CO and 2 parts H, or 93.33% CO and 6.67% H; by volume it is composed of equal parts of carbonic oxide and hydrogen. Water-gas produced as above described has great heating-power, but no illuminating-power. It may, however, be used for lighting by causing it to heat to whiteness some solid substance, as is done in the Welsbach incandescent light.

An illuminating-gas is made from water-gas by adding to it hydrocarbon gases or vapors, which are usually obtained from petroleum or some of its products. A history of the development of modern illuminating water-gas processes, together with a description of the most recent forms of apparatus, is given by Alex. C. Humphreys, in a paper on "Water-gas in the United States," read before the Mechanical Section of the British Association for Advancement of Science, in 1889. After describing many earlier patents, he states that success in the manufacture of water-gas may be said to date from 1874, when the process of T. S. C. Lowe was introduced. All the later most successful processes are the modifications of Lowe's, the essential features of which were "an apparatus consisting of a generator and superheater internally fired; the superheater being heated by the secondary combustion from the generator, the heat so stored up in the loose brick of the superheater being used, in the second part of the process, in the fixing or rendering permanent of the hydrocarbon gases; the second part of the process consisting in the passing of steam through the generator fire, and the admission of oil or hydrocarbon at some point between the fire of the generator and the loose filling of the superheater."

The water-gas process thus has two periods: first the "blow," during which air is blown through the bed coal in the generator, and the partially burned gaseous products are completely burned in the superheater, giving up a great portion of their heat to the fire-brick work contained in it, and then pass out to a chimney; second, the "run" during which the air blast is stopped, the opening to the chimney closed, and steam is blown through the incandescent bed of fuel. The resulting water-gas passing into the carburetting chamber in the base of the superheater is there charged with hydrocarbon vapors, or spray (such as naphtha and other distillates or crude oil) and passes through the superheater, where the hydrocarbon vapors become converted into fixed illuminating gases. From the superheater the combined gases are passed, as in the coal-gas process, through washers, scrubbers, etc., to the gas-holder. In this case, however, there is no ammonia to be removed.

The specific gravity of water-gas increases with the increase of the heavy hydrocarbons which give it illuminating power. The following figures, taken from different authorities, are given by F. H. Shelton in a paper on Water-gas, read before the Ohio Gas Light Association, in 1894:

Candle-power ...	19.5	20.	22.5	24.	25.4	26.3	28.3	29.6	.30 to 31.9
Sp. gr. (Air = 1)..	.571	.630	.589	.60 to .67	.64	.602	.70	.65	.65 to .71

Analyses of Water-gas and Coal-gas Compared.

The following analyses are taken from a report of Dr. Gideon E. Moore on the Granger Water-gas, 1885:

	Composition by Volume.			Composition by Weight.		
	Water-gas.		Coal-gas. Heidelberg.	Water-gas.		Coal-gas.
	Worcester.	Lake.		Worcester.	Lake.	
Nitrogen	2.64	3.85	2.15	0.04402	0.06175	0.04559
Carbonic acid....	0.14	0.30	3.01	0.00365	0.00753	0.09992
Oxygen...........	0.06	0.01	0.65	0.00114	0.00018	0.01569
Ethylene..........	11.29	12.80	2.55	0.18759	0.20454	0.05389
Propylene	0.00	0.00	1.21	....	....	0.03834
Benzole vapor....	1.58	2.68	1.33	0.07077	0.11700	0.07825
Carbonic oxide...	28.26	23.58	8.88	0.46934	0.37664	0.18758
Marsh-gas.... ...	18.88	20.95	34.02	0.17928	0.19133	0.41087
Hydrogen	37.20	35.88	46.20	0.04421	0.04103	0.06987
	100.00	100.00	100.00	1.00000	1.00000	1.00000
Density: Theory.	0.5825	0.6057	0.4580	....	....	....
Practice.	0.5915	0.6018	....	....	....	....
B. T. U. from 1 cu. ft.: Water liquid.	650.1	668.7	642.0	....	....	....
" vapor.	597.0	646.6	577.0	....	....	....
Flame-temp..	5311.2°F.	5281.1°F.	5202.9°F.	....	....	....
Av. candle-power.	22.06	26.31	...	....	....	... .

The heating values (B. T. U.) of the gases are calculated from the analysis by weight, by using the multipliers given below (computed from results of

J. Thomsen), and multiplying the result by the weight of 1 cu. ft. of the gas at 62° F., and atmospheric pressure.

The flame temperatures (theoretical) are calculated on the assumption of complete combustion of the gases in air, without excess of air.

The candle-power was determined by photometric tests, using a pressure of ½-in. water-column, a candle consumption of 120 grains of spermaceti per hour, and a meter rate of 5 cu. ft. per hour, the result being corrected for a temperature of 62° F. and a barometric pressure of 30 in. It appears that the candle-power may be regulated at the pleasure of the person in charge of the apparatus, the range of candle-power being from 20 to 29 candles, according to the manipulation employed.

Calorific Equivalents of Constituents of Illuminating-gas.

	Heat-units from 1 lb. Water Liquid.	Heat-units from 1 lb. Water Vapor.		Heat-units from 1 lb. Water Liquid.	Heat-units from 1 lb. Water Vapor.
Ethylene	21,524.4	20,134.8	Carbonic oxide	4,395.6	4,395.6
Propylene	21,222.0	19,834.2	Marsh-gas	24,021.0	21,592.8
Benzole vapor	18,954.0	17,847.0	Hydrogen	61,524.0	51,804.0

Efficiency of a Water-gas Plant.—The practical efficiency of an illuminating water-gas setting is discussed in a paper by A. G. Glasgow (Proc. Am. Gaslight Assn., 1890), from which the following is abridged:

The results refer to 1000 cu. ft. of unpurified carburetted gas, reduced to 60° F. The total anthracite charged per 1000 cu. ft. of gas was 33.4 lbs., ash and unconsumed coal removed 9.9 lbs., leaving total combustible consumed 23.5 lbs., which is taken to have a fuel-value of 14500 B. T. U. per pound, or a total of 340,750 heat-units.

		Composition by Volume.	Weight per 100 cu. ft.	Composition by Weight.	Specific Heat.
I. Carburetted Water-gas.	$CO_2 + H_2S$	3.8	.465842	.09647	.02088
	C_nH_{2n}	14.6	1.139968	.23607	.08720
	CO	28.0	2.1868	.45285	.11226
	CH_4	17.0	.75854	.15710	.09314
	H	35.6	.1991464	.04124	.14041
	N	1.0	.078596	.01627	.00397
		100.0	4.8288924	1.00000	.45786
II. Uncarburetted gas.	CO_2	3.5	.420065	.1019	.02205
	CO	43.4	3.389540	.8051	.19958
	H	51.8	.289821	.0688	.23424
	N	1.3	.102175	.0242	.00591
		100.0	4.210601	1.0000	.46178
III. Blast products escaping from superheater.	CO_2	17.4	2.138066	.2464	.05342
	O	3.2	.2856096	.0329	.00718
	N	79.4	6.2405224	.7207	.17585
		100.0	8.6591980	1.0000	.23645
IV. Generator blast-gases.	CO_2	9.7	1.189123	.1436	.031075
	CO	17.8	1.390180	.1680	.041647
	N	72.5	5.698210	.6884	.167970
		100.0	8.277518	1.0000	.240692

The heat energy absorbed by the apparatus is 23.5 × 14,500 = 340,750 heat-units = *A*. Its disposition is as follows:

B, the energy of the CO produced;

C, the energy absorbed in the decomposition of the steam;

D, the difference between the sensible heat of the escaping illuminating-gases and that of the entering oil;

E, the heat carried off by the escaping blast products;

F, the heat lost by radiation from the shells;

G, the heat carried away from the shells by convection (air-currents);
H, the heat rendered latent in the gasification of the oil;
I, the sensible heat in the ash and unconsumed coal recovered from the generator.

The heat equation is $A = B + C + D + E + F + G + H + I$; A being known. A comparison of the CO in Tables I and II show that $\frac{280}{434}$, or 64.5% of the volume of carburetted gas is pure water-gas, distributed thus: CO_2, 2.3%; CO, 28.0%; H, 33.4%; N, 0.8%; = 64.5%. 1 lb. of CO at 60° F. = 13.531 cu. ft. CO per 1000 cu. ft. of gas = 280 ÷ 13.531 = 20.694 lbs. Energy of the CO = 20.694 × 4395.6 = 91,043 heat-units, = B. 1 lb. of H at 60° F. = 189.2 cu. ft. H per M of gas = 334 ÷ 189.2 = 1.7653 lbs. Energy of the H per lb. (according to Thomsen, considering the steam generated by its combustion to be condensed to water at 75° F.) = 61,524 B. T. U. In Mr. Glasgow's experiments the steam entered the generator at 331° F.; the heat required to raise the product of combustion of 1 lb. of H, viz., 8.98 lbs. H_2O, from water at 75° to steam at 331° must therefore be deducted from Thomsen's figure, or 61,524 − (8.98 × 1140.2) = 51,285 B. T. U. per lb. of H. Energy of the H, then, is 1.7653 × 51,285 = 90,533 heat-units, = C. The heat lost due to the sensible heat in the illuminating-gases, their temperature being 1450° F., and that of the entering oil 235° F., is 48.29 (weight) × .45786 sp. heat × 1215 (rise of temperature) = 26,864 heat-units = D.

(The specific heat of the entering oil is approximately that of the issuing gas.)

The heat carried off in 1000 cu. ft. of the escaping blast products is 86.592 (weight) × .23645 (sp. heat) × 1474° (rise of temp.) = 30,180 heat-units: the temperature of the escaping blast gases being 1550° F., and that of the entering air 76° F. But the amount of the blast gases, by registration of an anemometer, checked by a calculation from the analyses of the blast gases, was 2457 cubic feet for every 1000 cubic feet of carburetted gas made. Hence the heat carried off per M. of carburetted gas is 30,180 × 2.457 = 74,152 heat-units = E.

Experiments made by a radiometer covering four square feet of the shell of the apparatus gave figures for the amount of heat lost by radiation = 12,454 heat-units = F, and by convection = 15,696 heat-units = G.

The heat rendered latent by the gasefication of the oil was found by taking the difference between all the heat fed into the carburetter and superheater and the total heat dissipated therefrom to be 12,841 heat-units = H. The sensible heat in the ash and unconsumed coal is 9.9 lbs. × 1500° × .25 (sp. ht.) = 3712 heat-units = I.

The sum of all the items $B + C + D + E + F + G + H + I$ = 327,295 heat-units, which substracted from the heat energy of the combustible consumed, 340,750 heat-units, leaves 13,455 heat-units, or 4 per cent, unaccounted for.

Of the total heat energy of the coal consumed, or 340,750 heat-units, the energy wasted is the sum of items D, E, F, G, and I, amounting to 132,878 heat-units, or 39 per cent; the remainder, or 207,872 heat-units, or 61 per cent, being utilized. The efficiency of the apparatus as a heat machine is therefore 61 per cent.

Five gallons, or 35 lbs. of crude petroleum were fed into the carburetter per 1000 cu. ft. of gas made; deducting 5 lbs. of tar recovered, leaves 30 lbs. × 20,000 = 600,000 heat-units as the net heating value of the petroleum used. Adding this to the heating value of the coal, 340,750 B. T. U., gives 940,750 heat-units, of which there is found as heat energy in the carburetted gas, as in the table below, 764,050 heat units, or 81 per cent, which is the commercial efficiency of the apparatus, i.e., the ratio of the energy contained in the finished product to the total energy of the coal and oil consumed.

The heating power per M. cu. ft. of the carburetted gas is

CO_2	38.0				
C_3H_6*	146.0	× .117220	× 21222.0	=	363200
CO	280.0	× .078100	× 4395.6	=	96120
CH_4	170.0	× .044620	× 24021.0	=	182210
H	356.0	× .005594	× 61524.0	=	122520
N	10.0				
	1000.0				764050

The heating power per M. of the uncarburetted gas is

CO_2	35.0				
CO	434.0	× .078100	× 4395.6	=	148991
H	518.0	× .005594	× 61524.0	=	178277
N	13.0				
	1000.0				327268

* The heating value of the illuminants C_nH_{2n} is assumed to equal that of C_3H_6.

The candle-power of the gas is 31, or 6.2 candle-power per gallon of oil used. The calculated specific gravity is .6355, air being 1.

For description of the operation of a modern carburetted water-gas plant, see paper by J. Stelfox, *Eng'g*, July 20, 1894, p. 89.

Space required for a Water-gas Plant.—Mr. Shelton, taking 15 modern plants of the form requiring the most floor-space, figures the average floor-space required per 1000 cubic feet of daily capacity as follows:

Water-gas Plants of Capacity in 24 hours of	Require an Area of Floor-space for each 1000 cu. ft. of about
100,000 cubic feet	4 square feet.
200,000 " "	3.5 " "
400,000 " "	2.75 " "
600,000 " "	2 to 2.5 sq. ft.
7 to 10 million cubic feet	1.25 to 1.5 sq. ft.

These figures include scrubbing and condensing rooms, but not boiler and engine rooms. In coal-gas plants of the most modern and compact forms one with 16 benches of 9 retorts each, with a capacity of 1,500,000 cubic feet per 24 hours, will require 4.8 sq. ft. of space per 1000 cu. ft. of gas, and one of 6 benches of 6 retorts each, with 300,000 cu. ft. capacity per 24 hours will require 6 sq. ft. of space per 1000 cu. ft. The storage-room required for the gas-making materials is: for coal-gas, 1 cubic foot of room for every 232 cubic feet of gas made; for water-gas made from coke, 1 cubic foot of room for every 378 cu. ft. of gas made; and for water-gas made from anthracite, 1 cu. ft. of room for every 645 cu. ft. of gas made.

The comparison is still more in favor of water-gas if the case is considered of a water-gas plant added as an auxiliary to an existing coal-gas plant; for, instead of requiring further space for storage of coke, part of that already required for storage of coke produced and not at once sold can be cut off, by reason of the water-gas plant creating a constant demand for more or less of the coke so produced.

Mr. Shelton gives a calculation showing that a water-gas of .625 sp. gr. would require gas-mains eight per cent greater in diameter than the same quantity coal-gas of .425 sp. gr. if the same pressure is maintained at the holder. The same quantity may be carried in pipes of the same diameter if the pressure is increased in proportion to the specific gravity. With the same pressure the increase of candle-power about balances the decrease of flow. With five feet of coal-gas, giving, say, eighteen candle-power, 1 cubic foot equals 3.6 candle-power; with water-gas of 23 candle-power, 1 cubic foot equals 4.6 candle-power, and 4 cubic feet gives 18.4 candle-power, or more than is given by 5 cubic feet of coal-gas. Water-gas may be made from oven-coke or gas-house coke as well as from anthracite coal. A water-gas plant may be conveniently run in connection with a coal-gas plant, the surplus retort coke of the latter being used as the fuel of the former.

In coal-gas making it is impracticable to enrich the gas to over twenty candle-power without causing too great a tendency to smoke, but water-gas of as high as thirty candle-power is quite common. A mixture of coal-gas and water-gas of a higher C.P. than 20 can be advantageously distributed

Fuel-value of Illuminating-gas.—E. G. Love (School of Mines Qtly, January, 1892) describes F. W. Hartley's calorimeter for determining the calorific power of gases, and gives results obtained in tests of the carburetted water-gas made by the municipal branch of the Consolidated Co. of New York. The tests were made from time to time during the past two years, and the figures give the heat-units per cubic foot at 60° F. and 30 inches pressure: 715, 692, 725, 732, 691, 738, 735, 708, 734, 730, 731, 727. Average, 721 heat units. Similar tests of mixtures of coal- and water-gases made by other branches of the same company give 694, 715, 684, 692, 727, 665, 695, and 686 heat-units per foot, or an average of 694.7. The average of all these tests was 710.5 heat-units, and this we may fairly take as representing the calorific power of the illuminating gas of New York. One thousand feet of this gas, costing $1.25, would therefore yield 710,500 heat-units, which would be equivalent to 568,400 heat-units for $1.00.

The common coal-gas of London, with an illuminating power of 16 to 17 candles, has a calorific power of about 668 units per foot, and costs from 60 to 70 cents per thousand.

The product obtained by decomposing steam by incandescent carbon, as effected in the Motay process, consists of about 40% of CO, and a little over 50% of H.

This mixture would have a heating-power of about 300 units per cubic foot, and if sold at 50 cents per 1000 cubic feet would furnish 600,000 units for $1.00, as compared with 568,400 units for $1.00 from illuminating gas at $1.25 per 1000 cubic feet. This illuminating-gas if sold at $1.15 per thousand would therefore be a more economical heating agent than the fuel-gas mentioned, at 50 cents per thousand, and be much more advantageous than the latter, in that one main, service, and meter could be used to furnish gas for both lighting and heating.

A large number of fuel-gases tested by Mr. Love gave from 184 to 470 heat-units per foot, with an average of 309 units.

Taking the cost of heat from illuminating-gas at the lowest figure given by Mr. Love, viz., $1.00 for 600,000 heat-units, it is a very expensive fuel, equal to coal at $40 per ton of 2000 lbs., the coal having a calorific power of only 12,000 heat-units per pound, or about 83% of that of pure carbon:

$$600{,}000 : (12{,}000 \times 2000) :: \$1 : \$40.$$

FLOW OF GAS IN PIPES.

The rate of flow of gases of different densities, the diameter of pipes required, etc., are given in King's Treatise on Coal Gas, vol. ii. 374, as follows:

If d = diameter of pipe in inches,
Q = quantity of gas in cu. ft. per hour,
l = length of pipe in yards,
h = pressure in inches of water,
s = specific gravity of gas, air being 1,

$$d = \sqrt[5]{\frac{Q^2sl}{(1350)^2h}},$$

$$h = \frac{Q^2sl}{(1350)^2d^5},$$

$$Q = 1350d^2\sqrt{\frac{dh}{sl}} = 1350\sqrt{\frac{d^5h}{sl}}.$$

Moleswortb gives $Q = 1000\sqrt{\frac{d^5h}{sl}}$.

J. P. Gill, *Am. Gas-light Jour.* 1894, gives $Q = 1291\sqrt{\frac{d^5h}{s(l+d)}}$.

This formula is said to be based on experimental data, and to make allowance for obstructions by tar, water, and other bodies tending to check the flow of gas through the pipe.

A set of tables in Appleton's Cyc. Mech. for flow of gas in 2, 6, and 12 in. pipes is calculated on the supposition that the quantity delivered varies as the square of the diameter instead of as $d^2 \times \sqrt{d}$, or $\sqrt{d^5}$.

These tables give a flow in large pipes much less than that calculated by the formulæ above given, as is shown by the following example. Length of pipe 100 yds., specific gravity of gas 0.42, pressure 1-in. water-column

	2-in. Pipe.	6-in. Pipe.	12-in. Pipe.
$Q = 1350\sqrt{\frac{d^5h}{sl}}$	1178	18,368	103,912
$Q = 1000\sqrt{\frac{d^5h}{sl}}$	873	13,606	76,972
$Q = 1291\sqrt{\frac{d^5h}{s(l+d)}}$	1116	16,327	93,845
Table in App. Cyc.............	1290	11,657	46,628

An experiment made by Mr. Clegg, in London, with a 4-in. pipe, 6 miles long, pressure 3 in. of water, specific gravity of gas .398, gave a discharge into the atmosphere of 852 cu. ft. per hour, after a correction of 33 cu. ft. was made for leakage.

Substituting this value, 852 cu. ft., for Q in the formula $Q = C\sqrt{d^5h \div sl}$, we find C, the coefficient, = 997, which corresponds nearly with the formula given by Molesworth.

Services for Lamps. (Molesworth.)

Lamps.	Ft. from Main.	Require Pipe-bore.	Lamps.	Ft. from Main.	Require Pipe-bore.
2	40	3/8 in.	15	130	1 in.
4	40	1/2 in.	20	150	1 1/4 in.
6	50	5/8 in.	25	180	1 1/2 in.
10	100	3/4 in.	30	200	1 3/4 in.

(In cold climates no service less than 3/4 in. should be used.)

Maximum Supply of Gas through Pipes in cu. ft. per Hour, Specific Gravity being taken at .45, calculated from the Formula $Q = 1000 \sqrt{d^5h \div sl}$. (Molesworth.)

LENGTH OF PIPE = 10 YARDS.

Diameter of Pipe in Inches.	Pressure by the Water-gauge in Inches.									
	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
3/8	13	18	22	26	29	31	34	36	38	41
1/2	26	37	46	53	59	64	70	74	79	83
3/4	73	103	126	145	162	187	192	205	218	230
1	149	211	258	298	333	365	394	422	447	471
1 1/4	260	368	451	521	582	638	689	737	781	823
1 1/2	411	581	711	821	918	1006	1082	1162	1232	1299
2	843	1192	1460	1686	1886	2066	2231	2385	2530	2667

LENGTH OF PIPE = 100 YARDS.

	Pressure by the Water-gauge in Inches.										
	.1	.2	.3	.4	.5	.75	1.0	1.25	1.5	2	2.5
1/2	8	12	14	17	19	23	26	29	32	36	42
3/4	23	32	42	46	51	63	73	81	89	103	115
1	47	67	82	94	105	129	149	167	183	211	236
1 1/4	82	116	143	165	184	225	260	291	319	368	412
1 1/2	130	184	225	260	290	356	411	459	503	581	649
2	267	377	462	533	596	730	843	943	1083	1193	1333
2 1/2	466	659	807	932	1042	1276	1473	1647	1804	2083	2329
3	735	1039	1270	1470	1643	2012	2323	2598	2846	3286	3674
3 1/2	1080	1528	1871	2161	2416	2958	3416	3820	4184	4831	5402
4	1508	2133	2613	3017	3373	4131	4770	5333	5842	6746	7542

LENGTH OF PIPE = 1000 YARDS.

	Pressure by the Water-gauge in Inches.						
	.5	.75	1.0	1.5	2.0	2.5	3.0
1	33	41	47	58	67	75	82
1 1/2	92	113	130	159	184	205	226
2	189	231	267	327	377	422	462
2 1/2	329	403	466	571	659	737	807
3	520	636	735	900	1039	1162	1278
4	1067	1306	1508	1847	2133	2385	2613
5	1863	2282	2635	3227	3727	4167	4564
6	2939	3600	4157	5091	5879	6578	7200

LENGTH OF PIPE = 5000 YARDS.

Diameter of Pipe in Inches.	Pressure by the Water-gauge in Inches.				
	1.0	1.5	2.0	2.5	3.0
2	119	146	169	189	207
3	329	402	465	520	569
4	675	826	955	1067	1168
5	1179	1443	1667	1868	2041
6	1859	2277	2629	2939	3220
7	2733	3347	3865	4321	4734
8	3816	4674	5397	6034	6610
9	5123	6274	7245	8100	8873
10	6667	8165	9428	10541	11547
12	10516	12880	14872	16628	18215

Mr. A. O. Humphreys says his experience goes to show that these tables give too small a flow, but it is difficult to accurately check the tables, on account of the extra friction introduced by rough pipes, bends, etc. For bends, one rule is to allow 1/42 of an inch pressure for each right-angle bend.

Where there is apt to be trouble from frost it is well to use no service of less diameter than ¾ in., no matter how short it may be. In extremely cold climates this is now often increased to 1 in., even for a single lamp. The best practice in the U. S. now condemns any service less than ¾ in.

STEAM.

The Temperature of Steam in contact with water depends upon the pressure under which it is generated. At the ordinary atmospheric pressure (14.7 lbs. per sq. in.) its temperature is 212° F. As the pressure is increased, as by the steam being generated in a closed vessel, its temperature, and that of the water in its presence, increases.

Saturated Steam is steam of the temperature due to its pressure—not superheated.

Superheated Steam is steam heated to a temperature above that due to its pressure.

Dry Steam is steam which contains no moisture. It may be either saturated or superheated.

Wet Steam is steam containing intermingled moisture, mist, or spray. It has the same temperature as dry saturated steam of the same pressure.

Water introduced into the presence of superheated steam will flash into vapor until the temperature of the steam is reduced to that due its pressure. Water in the presence of saturated steam has the same temperature as the steam. Should cold water be introduced, lowering the temperature of the whole mass, some of the steam will be condensed, reducing the pressure and temperature of the remainder, until an equilibrium is established.

Temperature and Pressure of Saturated Steam.—The relation between the temperature and the pressure of steam, according to Regnault's experiments, is expressed by the formula (Buchanan's, as given by Clark) $t = \frac{2938.16}{6.1993544 - \log p} - 371.85$, in which p is the pressure in pounds per square inch and t the temperature of the steam in Fahrenheit degrees. It applies with accuracy between 120° F. and 446° F., corresponding to pressures of from 1.68 lbs. to 445 lbs. per square inch. (For other formulæ see Wood's and Peabody's Thermodynamics.)

Total Heat of Saturated Steam (above 32° F.).—According to Regnault's experiments, the formula for total heat of steam is $H = 1091.7 + .305(t - 32°)$, in which t is temperature Fahr., and H the heat-units. (Rankine and many others; Clark gives 1091.16 instead of 1091.7.)

Latent Heat of Steam.—The formula for latent heat of steam, as given by Rankine and others, is $L = 1091.7 - .695(t - 32°)$. Clausius's formula, in Fahrenheit units, as given by Clark, is $L = 1092.6 - .708(t - 32°)$.

The total heat in steam (above 32°) includes three elements:

1st. The heat required to raise the temperature of the water to the temperature of the steam.

2d. The heat required to evaporate the water at that temperature, called internal latent heat.

3d. The latent heat of volume, or the external work done by the steam in making room for itself against the pressure of the superincumbent atmosphere (or surrounding steam if inclosed in a vessel).

The sum of the last two elements is called the latent heat of steam. In Buel's tables (Weisbach, vol. ii., Dubois's translation) the two elements are given separately.

Latent Heat of Volume of Saturated Steam. (External Work.)—The following formulas are sufficiently accurate for occasional use within the given ranges of pressure (Clark, S. E.):

From 14.7 lbs. to 50 lbs. total pressure per square inch... $55.900 + .0772t$.
From 50 lbs. to 200 lbs. total pressure per square inch.... $59.191 + .0655t$.

Heat required to Generate 1 lb. of Steam from water at 32° F.

		Heat-units.
Sensible heat, to raise the water from 32° to 212° =....		180.9
Latent heat, 1, of the formation of steam at 212° =....	894.0	
2, of expansion against the atmospheric pressure, 2116.4 lbs. per sq. ft. × 26.36 cu. ft. = 55,786 foot-pounds ÷ 778 = ...	71.7	965.7
Total heat above 32° F.........		1146.6

The Heat Unit, or British Thermal Unit.—The definition of the heat-unit used in this work is that of Rankine, accepted by most modern writers, viz., the quantity of heat required to raise the temperature of 1 lb. of water 1° F. at or near its temperature of maximum density (39.1° F.). Peabody's definition, the heat required to raise a pound of water from 62° to 63° F. is not generally accepted. (See Thurston, Trans. A. S. M. E., xiii. 351.)

Specific Heat of Saturated Steam.—The specific heat of saturated steam is .305, that of water being 1; or it is 1.281, if that of air be 1. The expression .305 for specific heat is taken in a compound sense, relating to changes both of volume and of pressure which takes place in the elevation of temperature of saturated steam. (Clark, S. E.)

This statement by Clark is not strictly accurate. When the temperature of saturated steam is elevated, water being present and the steam remaining saturated, water is evaporated. To raise the temperature of 1 lb. of water 1° F. requires 1 thermal unit, and to evaporate it at 1° F. higher would require 0.695 *less* thermal unit, the latent heat of saturated steam decreasing 0.695 B.T.U. for each increase of temperature of 1° F. Hence 0.305 is the specific heat of water and its saturated vapor combined.

When a unit weight of saturated steam is increased in temperature and in pressure, the volume decreasing so as to just keep it saturated, the specific heat is negative, and decreases as temperature increases. (See Wood, Therm., p. 147; Peabody, Therm., p. 93.)

Density and Volume of Saturated Steam.—The density of steam is expressed by the weight of a given volume, say one cubic foot; and the volume is expressed by the number of cubic feet in one pound of steam.

Mr. Brownlee's expression for the density of saturated steam in terms of the pressure is $D = \frac{p^{.941}}{330.36}$, or $\log D = .941 \log p - 2.519$, in which D is the density, and p the pressure in pounds per square inch. In this expression, $p^{.941}$ is the equivalent of p raised to the 16/17 power, as employed by Rankine.

The volume v being the reciprocal of the density,

$$v = \frac{330.36}{p^{.941}}, \text{ or } \log v = 2.519 - .941 \log p.$$

Relative Volume of Steam.—The relative volume of saturated steam is expressed by the number of volumes of steam produced from one volume of water, the volume of water being measured at the temperature 39° F. The relative volume is found by multiplying the volume in cu. ft. of one lb. of steam by the weight of a cu. ft. of water at 39° F., or 62.425 lbs.

Gaseous Steam.—When saturated steam is superheated, or surcharged with heat, it advances from the condition of saturation into that of gaseity. The gaseous state is only arrived at by considerably elevating the temperature, supposing the pressure remains the same. Steam thus sufficiently superheated is known as gaseous steam or steam gas.

Total Heat of Gaseous Steam.—Regnault found that the total heat of gaseous steam increased, like that of saturated steam, uniformly with the temperature, and at the rate of .475 thermal units per pound for each degree of temperature, under a constant pressure.

The general formula for the total heat of gaseous steam produced from 1 pound of water at 32° F. is $H = 1074.6 + .475t$. [This formula is for vapor generated at 32°. It is not true if generated at 212°, or at any other temperature than 32°. (Prof. Wood.)]

The Specific Heat of Gaseous Steam is .475, under constant pressure, as found by Regnault. It is identical with the coefficient of increase of total heat for each degree of temperature. [This is at atmospheric pressure and 212° F. He found it not true for any other pressure. Theory indicates that it would be greater at higher temperatures. (Prof. Wood.)]

The Specific Density of Gaseous Steam is .622, that of air being 1. That is to say, the weight of a cubic foot of gaseous steam is about five eighths of that of a cubic foot of air, of the same pressure and temperature.

The density or weight of a cubic foot of gaseous steam is expressible by the same formula as that of air, except that the multiplier or coefficient is less in proportion to the less specific density. Thus,

$$D' = \frac{2.7074p \times .622}{t + 461} = \frac{1.684p}{t + 461},$$

in which D' is the weight of a cubic foot of gaseous steam, p the total pressure per square inch, and t the temperature Fahrenheit.

Superheated Steam.—The above remarks concerning gaseous steam are taken from Clark's Steam-engine. Wood gives for the total heat (above 32°) of superheated steam $H = 1091.7 + 0.48(t - 32°)$.

The following is abridged from Peabody (Therm., p. 115, etc.).

When far removed from the temperature of saturation, superheated steam follows the laws of perfect gases very nearly, but near the temperature of saturation the departure from those laws is too great to allow of calculations by them for engineering purposes.

The specific heat at constant pressure, C_p, from the mean of three experiments by Regnault, is 0.4805.

Values of the ratio of C_p to specific heat at constant volume:

Pressure p, pounds per square inch..	5	50	100	200	300
Ratio $C_p \div C_v = k =$	1.335	1.332	1.330	1.324	1.316

Zeuner takes k as a constant = 1.333.

SPECIFIC HEAT AT CONSTANT VOLUME, SUPERHEATED STEAM.

Pressure, pounds per square inch.....	5	50	100	200	300
Specific heat C_v......................	0.351	.348	.346	.344	.341

It is quite as reasonable to assume that C_v is a constant as to suppose that C_p is constant, as has been assumed. If we take C_v to be constant, then C_p will appear as a variable.

If p = pressure in lbs. per sq. ft., v = volume in cubic feet, and T = temperature in degrees Fahrenheit + 460.7, then $pv = 93.5T - 971p^{\frac{1}{4}}$.

Total heat of superheated steam, $H = 0.4805(T - 10.38p^{\frac{1}{4}}) + 857.2$.

The Rationalization of Regnault's Experiments on Steam. (J. McFarlane Gray, Proc. Inst. M. E., July, 1889.)—The formulæ constructed by Regnault are strictly empirical, and were based entirely on his experiments. They are therefore not valid beyond the range of temperatures and pressures observed.

Mr. Gray has made a most elaborate calculation, based not on experiments but on fundamental principles of thermodynamics, from which he deduces formulæ for the pressure and total heat of steam, and presents tables calculated therefrom which show substantial agreement with Regnault's figures. He gives the following examples of steam-pressures calculated for temperatures beyond the range of Regnault's experiments.

Temperature.		Pounds per sq. in.	Temperature.		Pounds per sq. in.
C.	Fahr.		C.	Fahr	
230	446	406.9	340	644	2156.2
240	464	488.9	360	680	2742.5
250	482	579.9	380	716	3448.1
260	500	691.6	400	752	4300.2
280	536	940.0	415	779	5017.1
300	572	1261.8	427	800.6	5659.9
320	608	1661.9			

These pressures are higher than those obtained by Regnault's formula, which gives for 415° C. only 4067.1 lbs. per square inch.

Table of the Properties of Saturated Steam.—In the table of properties of saturated steam on the following pages the figures for temperature, total heat, and latent heat are taken, up to 210 lbs. absolute pressure, from the tables in Porter's Steam-engine Indicator, which tables have been widely accepted as standard by American engineers. The figures for total heat, given in the original as from 0° F., have been changed to heat above 32° F. The figures for weight per cubic foot and for cubic feet per pound have been taken from Dwelshauvers-Dery's table, Trans. A. S. M. E., vol. xi, as being probably more accurate than those of Porter. The figures for relative volume are from Buel's table, in Dubois's translation of Weisbach, vol. ii. They agree quite closely with the relative volumes calculated from weights as given by Dwelshauvers. From 211 to 219 lbs. the figures for temperature, total heat, and latent heat are from Dwelshauvers' table; and from 220 to 1000 lbs. all the figures are from Buel's table. The figures have not been carried out to as many decimal places as they are in most of the tables given by the different authorities; but any figure beyond the fourth significant figure is unnecessary in practice, and beyond the limit of error of the observations and of the formulæ from which the figures were derived.

Weight of 1 Cubic Foot of Steam in Decimals of a Pound. Comparison of Different Authorities.

Absolute Pressure, lbs. per sq. in.	Weight of 1 cubic foot according to—					Absolute Pressure, lbs. per sq. in.	Weight of 1 cubic foot according to—				
	Porter.	Clark	Buel.	Dery.	Peabody.		Porter.	Clark	Buel.	Dery.	Peabody
1	.0030	.003	.00303	.00299	.00299	120	.27428	.2738	.2735	.2724	.2605
14.7	.03797	.0380	.03798	......	.0376	140	.31386	.3162	.3163	.3147	.3113
20	.0511	.0507	.0507	.0507	.0502	160	.35209	.3590	.3589	.3567	.3530
40	.0994	.0974	.0972	.0972	.0964	180	.38895	.4009	.4012	.3983	.3945
60	.1457	.1425	.1424	.1422	.1409	200	.42496	.4431	.4438	.4400	.4359
80	.19015	.1869	.1866	.1862	.1843	220	.....	.4842	.4852	....	.4772
100	.23302	.2307	.2303	.2296	.2271	240	.....	.5248	.5270	.....	.5186

There are considerable differences between the figures of weight and volume of steam as given by different authorities. Porter's figures are based on the experiments of Fairbairn and Tate. The figures given by the other authorities are derived from theoretical formulæ which are believed to give more reliable results than the experiments. The figures for temperature, total heat, and latent heat as given by different authorities show a practical agreement, all being derived from Regnault's experiments. See Peabody's Tables of Saturated Steam; also Jacobus, Trans. A. S. M. E., vol. xii., 593.

Properties of Saturated Steam.

Vacuum Gauge, Inches of Mercury.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat $L = H - h$. Heat-units.	Relative Volume. Vol. of Water at 39° F. = 1.	Volume. Cu. ft. in 1 lb. of Steam.	Weight of 1 cu. ft. Steam, lb.
29.74	.089	32	0	1091.7	1091.7	208080	3333.3	.00030
29.67	.122	40	8.	1094.1	1086.1	154330	2472.2	.00040
29.56	.176	50	18.	1097.2	1079.2	107630	1724.1	.00058
29.40	.254	60	28.01	1100.2	1072.2	76370	1223.4	.00082
29.19	.359	70	38.02	1103.3	1065.3	54660	875.61	.00115
28.90	.502	80	48.04	1106.3	1058.3	39690	635.80	.00158
28.51	.692	90	58.06	1109.4	1051.3	29290	469.20	.00213
28.00	.943	100	68.08	1112.4	1044.4	21830	349.70	.00286
27.88	1	102.1	70.09	1113.1	1043.0	20623	334.23	.00299
25.85	2	126.3	94.44	1120.5	1026.0	10730	173.23	.00577
23.83	3	141.6	109.9	1125.1	1015.3	7325	117.98	.00848
21.78	4	153.1	121.4	1128.6	1007.2	5588	89.80	.01113
19.74	5	162.3	130.7	1131.4	1000.7	4530	72.50	.01373
17.70	6	170.1	138.6	1133.8	995.2	3816	61.10	.01631
15.67	7	176.9	145.4	1135.9	990.5	3302	53.00	.01887
13.63	8	182.9	151.5	1137.7	986.2	2912	46.60	.02140
11.60	9	188.3	156.9	1139.4	982.4	2607	41.82	.02391
9.56	10	193.2	161.9	1140.9	979.0	2361	37.80	.02641
7.52	11	197.8	166.5	1142.3	975.8	2159	34.61	.02889
5.49	12	202.0	170.7	1143.5	972.8	1990	31.90	.03136
3.45	13	205.9	174.7	1144.7	970.0	1846	29.58	.03381
1.41	14	209.6	178.4	1145.9	967.4	1721	27.59	.03625
Gauge Pressure lbs. per sq. in.	14.7	212	180.9	1146.6	965.7	1646	26.36	.03794
0.304	15	213.0	181.9	1146.9	965.0	1614	25.87	.03868
1.3	16	216.3	185.3	1147.9	962.7	1519	24.33	.04110
2.3	17	219.4	188.4	1148.9	960.5	1434	22.98	.04352
3.3	18	222.4	191.4	1149.8	958.3	1359	21.78	.04592
4.3	19	225.2	194.3	1150.6	956.3	1292	20.70	.04831
5.3	20	227.9	197.0	1151.5	954.4	1231	19.72	.05070
6.3	21	230.5	199.7	1152.2	952.6	1176	18.84	.05308
7.3	22	233.0	202.2	1153.0	950.8	1126	18.03	.05545
8.3	23	235.4	204.7	.7	949.1	1080	17.30	.05782
9.3	24	237.8	207.0	1154.5	947.4	1038	16.62	.06018
10.3	25	240.0	209.3	1155.1	945.8	998.4	15.99	.06253
11.3	26	242.2	211.5	.8	944.3	962.3	15.42	.06487
12.3	27	244.3	213.7	1156.4	942.8	928.8	14.88	.06721
13.3	28	246.3	215.7	1157.1	941.3	897.6	14.38	.06955
14.3	29	248.3	217.8	.7	939.9	868.5	13.91	.07188
15.3	30	250.2	219.7	1158.3	938.9	841.3	13.48	.07420
16.3	31	252.1	221.6	.8	937.2	815.8	13.07	.07652
17.3	32	254.0	223.5	1159.4	935.9	791.8	12.68	.07884
18.3	33	255.7	225.3	.9	934.6	769.2	12.32	.08115
19.3	34	257.5	227.1	1160.5	933.4	748.0	11.98	.08346
20.3	35	259.2	228.8	1161.0	932.2	727.9	11.66	.08576
21.3	36	260.8	230.5	1161.5	931.0	708.8	11.36	.08806
22.3	37	262.5	232.1	1162.0	929.8	690.8	11.07	.09035

Properties of Saturated Steam.

Gauge Pressure, lbs. per sq. in.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat L. $= H - h$. Heat-units.	Relative Volume. Vol. of Water at 39° F. = 1.	Volume. Cu. ft. in 1 lb. of Steam	Weight of 1 cu. ft. Steam, lb.
23.3	38	264.0	233.8	1162.5	928.7	673.7	10.79	.09264
24.3	39	265.6	235.4	.9	927.6	657.5	10.53	.09498
25.3	40	267.1	236.9	1163.4	926.5	642.0	10.28	.09721
26.3	41	268.6	238.5	.9	925.4	627.3	10.05	.09949
27.3	42	270.1	240.0	1164.3	924.4	613.3	9.83	.1018
28.3	43	271.5	241.4	.7	923.3	599.9	9.61	.1040
29.3	44	272.9	242.9	1165.2	922.3	587.0	9.41	.1063
30.3	45	274.3	244.3	.6	921.3	574.7	9.21	.1086
31.3	46	275.7	245.7	1166.0	920.4	563.0	9.02	.1108
32.3	47	277.0	247.0	.4	919.4	551.7	8.84	.1131
33.3	48	278.3	248.4	.8	918.5	540.9	8.67	.1153
34.3	49	279.6	249.7	1167.2	917.5	530.5	8.50	.1176
35.3	50	280.9	251.0	.6	916.6	520.5	8.34	.1198
36.3	51	282.1	252.2	1168.0	915.7	510.9	8.19	.1221
37.3	52	283.3	253.5	.4	914.9	501.7	8.04	.1243
38.3	53	284.5	254.7	.7	914.0	492.8	7.90	.1266
39.3	54	285.7	256.0	1169.1	913.1	484.2	7.76	.1288
40.3	55	286.9	257.2	.4	912.3	475.9	7.63	.1311
41.3	56	288.1	258.3	.8	911.5	467.9	7.50	.1333
42.3	57	289.1	259.5	1170.1	910.6	460.2	7.38	.1355
43.3	58	290.3	260.7	.5	909.8	452.7	7.26	.1377
44.3	59	291.4	261.8	.8	909.0	445.5	7.14	.1400
45.3	60	292.5	262.9	1171.2	908.2	438.5	7.03	.1422
46.3	61	293.6	264.0	.5	907.5	431.7	6.92	.1444
47.3	62	294.7	265.1	.8	906.7	425.2	6.82	.1466
48.3	63	295.7	266.2	1172.1	905.9	418.8	6.72	.1488
49.3	64	296.8	267.2	.4	905.2	412.6	6.62	.1511
50.3	65	297.8	268.3	.8	904.5	406.6	6.53	.1533
51.3	66	298.8	269.3	1173.1	903.7	400.8	6.43	.1555
52.3	67	299.8	270.4	.4	903.0	395.2	6.34	.1577
53.3	68	300.8	271.4	.7	902.3	389.8	6.25	.1599
54.3	69	301.8	272.4	1174.0	901.6	384.5	6.17	.1621
55.3	70	302.7	273.4	.3	900.9	379.3	6.09	.1643
56.3	71	303.7	274.4	.6	900.2	374.3	6.01	.1665
57.3	72	304.6	275.3	.8	899.5	369.4	5.93	.1687
58.3	73	305.6	276.3	1175.1	898.9	364.6	5.85	.1709
59.3	74	306.5	277.2	.4	898.2	360.0	5.78	.1731
60.3	75	307.4	278.2	.7	897.5	355.5	5.71	.1753
61.3	76	308.3	279.1	1176.0	896.9	351.1	5.63	.1775
62.3	77	309.2	280.0	.2	896.2	346.8	5.57	.1797
63.3	78	310.1	280.9	.5	895.6	342.6	5.50	.1819
64.3	79	310.9	281.8	.8	895.0	338.5	5.43	.1840
65.3	80	311.8	282.7	1177.0	894.3	334.5	5.37	.1862
66.3	81	312.7	283.6	.3	893.7	330.6	5.31	.1884
67.3	82	313.5	284.5	.6	893.1	326.8	5.25	.1906
68.3	83	314.4	285.3	.8	892.5	323.1	5.18	.1928
69.3	84	315.2	286.2	1178.1	891.9	319.5	5.13	.1950
70.3	85	316.0	287.0	.3	891.3	315.9	5.07	.1971

Properties of Saturated Steam.

Gauge Pressure, lbs. per sq. in.	Absolute Pressure, lbs per square inch.	Temperature Fahrenheit.	Total Heat above 32° F.		Latent Heat $L = H - h$. Heat-units.	Relative Volume. Vol. of Water at 39° F. = 1.	Volume. Cu. ft. in 1 lb. of Steam	Weight of 1 cu. ft. Steam, lb.
			In the Water h Heat-units.	In the Steam H Heat-units.				
71.3	86	316.8	287.9	1178.6	890.7	312.5	5.02	.1993
72.3	87	317.7	288.7	.8	890.1	309.1	4.96	.2015
73.3	88	318.5	289.5	1179.1	889.5	305.8	4.91	.2036
74.3	89	319.3	290.4	.3	888.9	302.5	4.86	.2058
75.3	90	320.0	291.2	.6	888.4	299.4	4.81	.2080
76.3	91	320.8	292.0	.8	887.8	296.3	4.76	.2102
77.3	92	321.6	292.8	1180.0	887.2	293.2	4.71	.2123
78.3	93	322.4	293.6	.3	886.7	290.2	4.66	.2145
79.3	94	323.1	294.4	.5	886.1	287.3	4.62	.2166
80.3	95	323.9	295.1	.7	885.6	284.5	4.57	.2188
81.3	96	324.6	295.9	1181.0	885.0	281.7	4.53	.2210
82.3	97	325.4	296.7	.2	884.5	279.0	4.48	.2231
83.3	98	326.1	297.4	.4	884.0	276.3	4.44	.2253
84.3	99	326.8	298.2	.6	883.4	273.7	4.40	.2274
85.3	100	327.6	298.9	.8	882.9	271.1	4.36	.2296
86.3	101	328.3	299.7	1182.1	882.4	268.5	4.32	.2317
87.3	102	329.0	300.4	.3	881.9	266.0	4.28	.2339
88.3	103	329.7	301.1	.5	881.4	263.6	4.24	.2360
89.3	104	330.4	301.9	.7	880.8	261.2	4.20	.2382
90.3	105	331.1	302.6	.9	880.3	258.9	4.16	.2403
91.3	106	331.8	303.3	1183.1	879.8	256.6	4.12	.2425
92.3	107	332.5	304.0	.4	879.3	254.3	4.09	.2446
93.3	108	333.2	304.7	.6	878.8	252.1	4.05	.2467
94.3	109	333.9	305.4	.8	878.3	249.9	4.02	.2489
95.3	110	334.5	306.1	1184.0	877.9	247.8	3.98	.2510
96.3	111	335.2	306.8	.2	877.4	245.7	3.95	.2531
97.3	112	335.9	307.5	.4	876.9	243.6	3.92	.2553
98.3	113	336.5	308.2	.6	876.4	241.6	3.88	.2574
99.3	114	337.2	308.8	.8	875.9	239.6	3.85	.2596
100.3	115	337.8	309.5	1185.0	875.5	237.6	3.82	.2617
101.3	116	338.5	310.2	.2	875.0	235.7	3.79	.2638
102.3	117	339.1	310.8	.4	874.5	233.8	3.76	.2660
103.3	118	339.7	311.5	.6	874.1	231.9	3.73	.2681
104.3	119	340.4	312.1	.8	873.6	230.1	3.70	.2703
105.3	120	341.0	312.8	.9	873.2	228.3	3.67	.2724
106.3	121	341.6	313.4	1186.1	872.7	226.5	3.64	.2745
107.3	122	342.2	314.1	.3	872.3	224.7	3.62	.2766
108.3	123	342.9	314.7	.5	871.8	223.0	3.59	.2788
109.3	124	343.5	315.3	.7	871.4	221.3	3.56	.2809
110.3	125	344.1	316.0	.9	870.9	219.6	3.53	.2830
111.3	126	344.7	316.6	1187.1	870.5	218.0	3.51	.2851
112.3	127	345.3	317.2	.3	870.0	216.4	3.48	.2872
113.3	128	345.9	317.8	.4	869.6	214.8	3.46	.2894
114.3	129	346.5	318.4	.6	869.2	213.2	3.43	.2915
115.3	130	347.1	319.1	.8	868.7	211.6	3.41	.2936
116.3	131	347.6	319.7	1188.0	868.3	210.1	3.38	.2957
117.3	132	348.2	320.3	.2	867.9	208.6	3.36	.2978
118.3	133	348.8	320.8	.3	867.5	207.1	3.33	.3000
119.3	134	349.4	321.5	.5	867.0	205.7	3.31	.3021

Properties of Saturated Steam.

Gauge Pressure. lbs. per sq. in.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat $L = H - h$. Heat-units.	Relative Volume. Vol. of water at 39° F. = 1.	Volume. Cu. ft. in 1 lb. of Steam.	Weight of 1 cu. ft. Steam, lb.
120.3	135	350.0	322.1	1188.7	866.6	204.2	3.29	.3042
121.3	136	350.5	322.6	.9	866.2	202.8	3.27	.3063
122.3	137	351.1	323.2	1189.0	865.8	201.4	3.24	.3084
123.3	138	351.6	323.8	.2	865.4	200.0	3.22	.3105
124.3	139	352.2	324.4	.4	865.0	198.7	3.20	.3126
125.3	140	352.8	325.0	.5	864.6	197.3	3.18	.3147
126.3	141	353.3	325.5	.7	864.2	196.0	3.16	.3169
127.3	142	353.9	326.1	.9	863.8	194.7	3.14	.3190
128.3	143	354.4	326.7	1190.0	863.4	193.4	3.11	.3211
129.3	144	355.0	327.2	.2	863.0	192.2	3.09	.3232
130.3	145	355.5	327.8	.4	862.6	190.9	3.07	.3253
131.3	146	356.0	328.4	.5	862.2	189.7	3.05	.3274
132.3	147	356.6	328.9	.7	861.8	188.5	3.04	.3295
133.3	148	357.1	329.5	.9	861.4	187.3	3.02	.3316
134.3	149	357.6	330.0	1191.0	861.0	186.1	3.00	.3337
135.3	150	358.2	330.6	.2	860.6	184.9	2.98	.3358
136.3	151	358.7	331.1	.3	860.2	183.7	2.96	.3379
137.3	152	359.2	331.6	.5	859.9	182.6	2.94	.3400
138.3	153	359.7	332.2	.7	859.5	181.5	2.92	.3421
139.3	154	360.2	332.7	.8	859.1	180.4	2.91	.3442
140.3	155	360.7	333.2	1192.0	858.7	179.2	2.89	.3463
141.3	156	361.3	333.8	.1	858.4	178.1	2.87	.3483
142.3	157	361.8	334.3	.3	858.0	177.0	2.85	.3504
143.3	158	362.3	334.8	.4	857.6	176.0	2.84	.3525
144.3	159	362.8	335.3	.6	857.2	174.9	2.82	.3546
145.3	160	363.3	335.9	.7	856.9	173.9	2.80	.3567
146.3	161	363.8	336.4	.9	856.5	172.9	2.79	.3588
147.3	162	364.3	336.9	1193.0	856.1	171.9	2.77	.3609
148.3	163	364.8	337.4	.2	855.8	171.0	2.76	.3630
149.3	164	365.3	337.9	.3	855.4	170.0	2.74	.3650
150.3	165	365.7	338.4	.5	855.1	169.0	2.72	.3671
151.3	166	366.2	338.9	.6	854.7	168.1	2.71	.3692
152.3	167	366.7	339.4	.8	854.4	167.1	2.69	.3713
153.3	168	367.2	339.9	.9	854.0	166.2	2.68	.3734
154.3	169	367.7	340.4	1194.1	853.6	165.3	2.66	.3754
155.3	170	368.2	340.9	.2	853.3	164.3	2.65	.3775
156.3	171	368.6	341.4	.4	852.9	163.4	2.63	.3796
157.3	172	369.1	341.9	.5	852.6	162.5	2.62	.3817
158.3	173	369.6	342.4	.7	852.3	161.6	2.61	.3838
159.3	174	370.0	342.9	.8	851.9	160.7	2.59	.3858
160.3	175	370.5	343.4	.9	851.6	159.8	2.58	.3879
161.3	176	371.0	343.9	1195.1	851.2	158.9	2.56	.3900
162.3	177	371.4	344.3	.2	850.9	158.1	2.55	.3921
163.3	178	371.9	344.8	.4	850.5	157.2	2.54	.3942
164.3	179	372.4	345.3	.5	850.2	156.4	2.52	.3962
165.3	180	372.8	345.8	.7	849.9	155.6	2.51	.3983
166.3	181	373.3	346.3	.8	849.5	154.8	2.50	.4004
167.3	182	373.7	346.7	.9	849.2	154.0	2.48	.4025
168.3	183	374.2	347.2	1196.1	848.9	153.2	2.47	.4046

Properties of Saturated Steam.

Gauge Pressure, lbs. per sq. in.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat $L. = H - h.$ Heat-units.	Relative Volume. Vol. of water at 39° F. = 1.	Volume. Cu. ft. in 1 lb. of Steam	Weight of 1 cu. ft. Steam, lb.
169.3	184	374.6	347.7	1196.2	848.5	152.4	2.46	.4066
170.3	185	375.1	348.1	.3	848.2	151.6	2.45	.4087
171.3	186	375.5	348.6	.5	847.9	150.8	2.43	.4108
172.3	187	375.9	349.1	.6	847.6	150.0	2.42	.4129
173.3	188	376.4	349.5	.7	847.2	149.2	2.41	.4150
174.3	189	376.9	350.0	.9	846.9	148.5	2.40	.4170
175.3	190	377.3	350.4	1197.0	846.6	147.8	2.39	.4191
176.3	191	377.7	350.9	.1	846.3	147.0	2.37	.4212
177.3	192	378.2	351.3	.3	845.9	146.3	2.36	.4233
178.3	193	378.6	351.8	.4	845.6	145.6	2.35	.4254
179.3	194	379.0	352.2	.5	845.3	144.9	2.34	.4275
180.3	195	379.5	352.7	.7	845.0	144.2	2.33	.4296
181.3	196	380.0	353.1	.8	844.7	143.5	2.32	.4317
182.3	197	380.3	353.6	.9	844.4	142.8	2.31	.4337
183.3	198	380.7	354.0	1198.1	844.1	142.1	2.29	.4358
184.3	199	381.2	354.4	.2	843.7	141.4	2.28	.4379
185.3	200	381.6	354.9	.3	843.4	140.8	2.27	.4400
186.3	201	382.0	355.3	.4	843.1	140.1	2.26	.4420
187.3	202	382.4	355.8	.6	842.8	139.5	2.25	.4441
188.3	203	382.8	356.2	.7	842.5	138.8	2.24	.4462
189.3	204	383.2	356.6	.8	842.2	138.1	2.23	.4482
190.3	205	383.7	357.1	1199.0	841.9	137.5	2.22	.4503
191.3	206	384.1	357.5	.1	841.6	136.9	2.21	.4523
192.3	207	384.5	357.9	.2	841.3	136.3	2.20	.4544
193.3	208	384.9	358.3	.3	841 0	135.7	2.19	.4564
194.3	209	385.3	358.8	.5	840.7	135.1	2.18	.4585
195.3	210	385.7	359.2	.6	840.4	134.5	2.17	.4605
196.3	211	386.1	359.6	.7	840.1	133.9	2.16	.4626
197.3	212	386.5	360.0	.8	839.8	133.3	2.15	.4646
198.3	213	386.9	360.4	.9	839.5	132.7	2.14	.4667
199.3	214	387.3	360.9	1200.1	839.2	132.1	2.13	.4687
200.3	215	387.7	361.3	.2	838.9	131.5	2.12	.4707
201.3	216	388.1	361.7	.3	838.6	130.9	2.12	.4728
202.3	217	388.5	362.1	.4	838.3	130.3	2.11	.4748
203.3	218	388.9	362.5	.6	838.1	129.7	2.10	.4768
204.3	219	389.3	362.9	.7	837.8	129.2	2.09	.4788
205.3	220	389.7	362.2*	1200.8	838.6*	128.7	2.06	.4852
215.3	230	393.6	366.2	1202.0	835.8	123.3	1.98	.5061
225.3	240	397.3	370.0	1203.1	833.1	118.5	1.90	.5270
235.3	250	400.9	373.8	1204.2	830.5	114.0	1.83	.5478
245.3	260	404.4	377.4	1205.3	827.9	109.8	1.76	.5686
255.3	270	407.8	380.9	1206.3	825.4	105.9	1.70	.5894
265.3	280	411.0	384.3	1207.3	823.0	102.3	1.64	.6101
275.3	290	414.2	387.7	1208.3	820.6	99.0	1.585	.6308
285.3	300	417.4	390.9	1209.2	818.3	95.8	1.535	.6515
335.3	350	432.0	406.8	1213.7	807.5	82.7	1.325	.7545

* The discrepancies at 205.3 lbs. gauge are due to the change from Dwelshauvers-Dery's to Buel's figures.

Properties of Saturated Steam.

Gauge Pressure, lbs per sq. in.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	In the Steam H Heat-units.	Latent Heat L. $= H - h$. Heat-units.	Relative Volume. Vol. of water at 39° F. = 1.	Volume. Cu. ft. of Steam in 1 lb.	Weight of 1 cu. ft. Steam, lb.
385.3	400	444.9	419.8	1217.7	797.9	72.8	1.167	.8572
435.3	450	456.6	432.2	1221.3	789.1	65.1	1.042	.9595
485.3	500	467.4	443.5	1224.5	781.0	58.8	.942	1.062
535.3	550	477.5	454.1	1227.6	773.5	53.6	.859	1.164
585.3	600	486.9	464.2	1230.5	766.3	49.3	.790	1.266
635.3	650	495.7	473.6	1233.2	759.6	45.6	.731	1.368
685.3	700	504.1	482.4	1235.7	753.3	42.4	.680	1.470
735.3	750	512.1	490.9	1238.0	747.2	39.6	.636	1.572
785.3	800	519.6	498.9	1240.3	741.4	37.1	.597	1.674
835.3	850	526.8	506.7	1242.5	735.8	34.9	.563	1.776
885.3	900	533.7	514.0	1244.7	730.6	33.0	.533	1.878
935.3	950	540.3	521.3	1246.7	725.4	31.4	.505	1.980
985.3	1000	546.8	528.3	1248.7	720.3	30.0	.480	2.082

FLOW OF STEAM.

Flow of Steam through a Nozzle. (From Clark on the Steam-engine.)—The flow of steam of a greater pressure into an atmosphere of a less pressure increases as the difference of pressure is increased, until the external pressure becomes only 58% of the absolute pressure in the boiler. The flow of steam is neither increased nor diminished by the fall of the external pressure below 58%, or about 4/7ths of the inside pressure, even to the extent of a perfect vacuum. In flowing through a nozzle of the best form, the steam expands to the external pressure, and to the volume due to this pressure, so long as it is not less than 58% of the internal pressure. For an external pressure of 58%, and for lower percentages, the ratio of expansion is 1 to 1.624. The following table is selected from Mr. Brownlee's data exemplifying the rates of discharge under a constant internal pressure, into various external pressures:

Outflow of Steam; from a Given Initial Pressure into Various Lower Pressures.

Absolute initial pressure in boiler, 75 lbs. per sq. in.

Absolute Pressure in Boiler per square inch.	External Pressure per square inch.	Ratio of Expansion in Nozzle.	Velocity of Outflow at Constant Density.	Actual Velocity of Outflow Expanded.	Discharge per square inch of Orifice per minute.
lbs.	lbs.	ratio.	feet per sec.	feet p. sec.	lbs.
75	74	1.012	227.5	230	16.68
75	72	1.037	386.7	401	28.35
75	70	1.063	490	521	35.93
75	65	1.136	660	749	48.38
75	61.62	1.198	736	876	53.97
75	60	1.219	765	933	56.12
75	50	1.434	873	1252	64
75	45	1.575	890	1401	65.24
75	43.46 {58 p. cent}	1.624	890.6	1446.5	65.3
75	15	1.624	890.6	1446.5	65.3
75	0	1.624	890.6	1446.5	65.3

When steam of varying initial pressures is discharged into the atmosphere—the atmospheric pressure being not more than 58% of the initial pressure—the velocity of outflow at constant density, that is, supposing the initial density to be maintained, is given by the formula $V = 3.5953\sqrt{h}$.

V = the velocity of outflow in feet per second, as for steam of the initial density;
h = the height in feet of a column of steam of the given absolute initial pressure of uniform density, the weight of which is equal to the pressure on the unit of base.

The lowest initial pressure to which the formula applies, when the steam is discharged into the atmosphere at 14.7 lbs. per square inch, is ($14.7 \times 100/58 =$) 25.37 lbs. per square inch. Examples of the application of the formula are given in the table below.

From the contents of this table it appears that the velocity of outflow into the atmosphere, of steam above 25 lbs. per square inch absolute pressure, or 10 lbs. effective, increases very slowly with the pressure, obviously because the density, and the weight to be moved, increase with the pressure. An average of 900 feet per second may, for approximate calculations, be taken for the velocity of outflow as for constant density, that is, taking the volume of the steam at the initial volume.

Outflow of Steam into the Atmosphere.—External pressure per square inch 14.7 lbs. absolute. Ratio of expansion in nozzle, 1.624.

Absolute Initial Pressure per square inch.	Velocity of Outflow as at Constant Density.	Actual Velocity of Outflow Expanded.	Discharge per square inch of Orifice per min	Horse-power per sq. in. of Orifice if H. P. = 30 lbs. per hour.	Absolute Initial Pressure per square inch.	Velocity of Outflow as at Constant Density.	Actual Velocity of Outflow Expanded.	Discharge per square inch of Orifice per minute.	Horse-power per sq. in. of Orifice if H. P. = 30 lbs. per hour.
lbs.	feet p. sec.	feet per sec.	lbs.	H.P.	lbs.	feet p. sec.	feet per sec.	lbs.	H.P.
25.37	863	1401	22.81	45.6	90	895	1454	77.94	155.9
30	867	1408	26.84	53.7	100	898	1459	86.34	172.7
40	874	1419	35.18	70.4	115	902	1466	98.76	197.5
50	880	1429	44.06	88.1	135	906	1472	115.61	231.2
60	885	1437	52.59	105.2	155	910	1478	132.21	264.4
70	889	1444	61.07	122.1	165	912	1481	140.46	280.9
75	891	1447	65.30	130.6	215	919	1493	181.58	363.2

Napier's Approximate Rule.—Flow in pounds per second = absolute pressure × area in square inches ÷ 70. This rule gives results which closely correspond with those in the above table, as shown below.

Abs. press., lbs. p. sq. in.	25.37	40	60	75	100	135	165	215
Discharge per min., by table, lbs.	22.81	35.18	52.59	65.30	86.34	115.61	140.46	181.58
By Napier's rule	21.74	34.29	51.43	64.29	85.71	115.71	141.43	184.29

Prof. Peabody, in Trans. A. S. M. E., xi, 187, reports a series of experiments on flow of steam through tubes ¼ inch in diameter, and ¼, ½, and 1⅛ inch long, with rounded entrances, in which the results agreed closely with Napier's formula, the greatest difference being an excess of the experimental over the calculated result of 3.2%. An equation derived from the theory of thermodynamics is given by Prof. Peabody, but it does not agree with the experimental results as well as Napier's rule, the excess of the actual flow being 6.6%.

Flow of Steam in Pipes.—A formula commonly used for velocity of flow of steam in pipes is the same as Downing's for the flow of water in smooth cast-iron pipes, viz., $V = 50\sqrt{\frac{H}{L}D}$, in which V = velocity in feet per second, L = length and D = diameter of pipe in feet, H = height in feet of a column of steam, of the pressure of the steam at the entrance, which would produce a pressure equal to the difference of pressures at the two ends of the pipe. (For derivation of the coefficient 50, see Briggs on "Warming Buildings by Steam," Proc. Inst. C. E. 1882.)

If Q = quantity in cubic feet per minute, d = diameter in inches, L and H being in feet, the formula reduces to

$$Q = 4.7233\sqrt{\frac{H}{L}d^5},\quad H = .0448\frac{Q^2L}{d^5},\quad d = .5874\sqrt[5]{\frac{Q^2L}{H}}.$$

(These formulæ are applicable to air and other gases as well as steam.)

If p_1 = pressure in pounds per square inch of the steam (or gas) at the entrance to the pipe, p_2 = the pressure at the exit, then $144(p_1 - p_2)$ = difference in pressure per square foot. Let w = density or weight per cubic foot of steam at the pressure p_1, then the height of column equivalent to the difference in pressures

$$= H = \frac{144(p_1 - p_2)}{w},\ \text{and}\ Q = 60 \times .7854 \times 50D^2\sqrt{\frac{144(p_1 - p_2)D}{wL}}.$$

If W = weight of steam flowing in pounds per minute = Qw, and d is taken in inches, L being in feet,

$$W = 56.68\sqrt{\frac{w(p_1 - p_2)d^5}{L}};\quad Q = 56.68\sqrt{\frac{(p_1 - p_2)d^5}{Lw}};$$

$$d = 0.199\sqrt[5]{\frac{W^2L}{w(p_1 - p_2)}} = 0.199\sqrt[5]{\frac{Q^2wL}{p_1 - p_2}}.$$

Velocity in feet per minute $= V = Q \div .7854\frac{d^2}{144} = 10392\sqrt{\frac{(p_1 - p_2)d}{wL}}.$

For a velocity of 6000 feet per minute, $d = \frac{wL}{3(p_1 - p_2)}$; $p_1 - p_2 = \frac{wL}{3d}.$

For a velocity of 6000 feet per minute, a steam-pressure of 100 lbs. gauge, or $w = .264$, and a length of 100 feet, $d = \frac{8.8}{p_1 - p_2}$; $p_1 - p_2 = \frac{8.8}{d}$. That is, a pipe 1 inch diameter, 100 feet long, carrying steam of 100 lbs. gauge-pressure at 6000 feet velocity per minute, would have a loss of pressure of 8.8 lbs. per square inch, while steam travelling at the same velocity in a pipe 8.8 inches diameter would lose only 1 lb. pressure.

G. H. Babcock, in "Steam," gives the formula

$$W = 87\sqrt{\frac{w(p_1 - p_2)d^5}{L\left(1 + \frac{3.6}{d}\right)}}.$$

In earlier editions of "Steam" the coefficient is given as 800,—evidently an error,—and this value has been reprinted in Clark's Pocket-Book (1892 edition). It is apparently derived from one of the numerous formulæ for flow of water in pipes, the multiplier of L in the denominator being used for an expression of the increased resistance of small pipes. Putting this formula in the form $W = c\sqrt{\frac{w(p_1 - p_2)d^5}{L}}$, in which c will vary with the diameter of the pipe, we have,

For diameter, inches....	1	2	3	4	6	9	12
Value of c..............	40.7	52.1	58.8	63	68.8	73.7	79.3

instead of the constant value 56.68, given with the simpler formula.

One of the most widely accepted formulæ for flow of water is D'Arcy's, $V = c\sqrt{\frac{HD}{L4}}$, in which c has values ranging from 65 for a ½-inch pipe up to

111.5 for 24-inch. Using D'Arcy's coefficients, and modifying his formula to make it apply to steam, to the form

$$Q = c\sqrt{\frac{(p_1 - p_2)d^5}{wL}}, \quad \text{or} \quad W = c\sqrt{\frac{w(p_1 - p_2)d^5}{L}},$$

we obtain,

For diameter, inches....	½	1	2	3	4	5	6	7	8
Value of c................	36.8	45.3	52.7	56.1	57.8	58.4	59.5	60.1	60.7
For diameter, inches....	9	10	12	14	16	18	20	22	24
Value of c................	61.2	61.8	62.1	62.3	62.6	62.7	62.9	63.2	63.2

In the absence of direct experiments these coefficients are probably as accurate as any that may be derived from formulæ for flow of water.

Loss of pressure in lbs. per sq. in. $= p_1 - p_2 = \dfrac{Q^2wL}{c^2d^5}$.

Loss of Pressure due to Radiation as well as Friction.—E. A. Rudiger (*Mechanics*, June 30, 1888) gives the following formulæ and tables for flow of steam in pipes. He takes into consideration the losses in pressure due both to radiation and to friction.

Loss of power, expressed in heat-units due to friction, $Hf = \dfrac{W^3fl}{10p^2d^5}$.

Loss due to radiation, $Hr = 0.262rld$.

In which W is the weight in lbs. of steam delivered per hour, f the coefficient of friction of the pipe, l the length of the pipe in feet, p the absolute terminal pressure, d the diameter of the pipe in inches, and r the coefficient of radiation. f is taken as from .0165 to .0175, and r varies as follows:

TABLE OF VALUES FOR r.

Pipe Covering.	Absolute Pressure.			
	40 lbs.	65 lbs.	90 lbs.	115 lbs.
Uncovered pipe....................	437	555	620	684
2-inch cement composition.........	146	178	193	209
2 " asbestos	157	192	202	222
2 " asbestos flock................	150	185	197	210
2 " wooden log....................	100	122	145	151
2 " mineral wool..................	61	76	85	93
2 " hair felt.....................	48	58	66	73

The appended table shows the loss due to friction and radiation in a steam-pipe where the quantity of steam to be delivered is 1000 lbs. per hour, l = 1000 feet, the pipe being so protected that loss by radiation r = 64, and the absolute terminal pressure being 90 lbs.:

Diameter of Pipe, inches.	Loss by Friction, Hf.	Loss by Radiation, Hr.	Total Loss, L.	Diam. of Pipe, inches.	Loss by Friction, Hf.	Loss by Radiation, Hr.	Total Loss, L.
1	197,531	16,768	214,300	3½	376	58,688	59,064
1¼	64,727	20,960	85,687	4	193	67,072	67,265
1½	26,012	25,152	51,164	5	63	83,840	83,903
1¾	12,035	29,344	41,379	6	25	100,608	100,623
2	6,173	33,536	39,709	7	12	117,376	117,388
2½	2,023	41,920	43,943	8	6	134,144	134,150
3	813	50,304	51,117				

If the pipes are carrying steam with minimum loss, then for same r, l, and p, the loss of pressure L for pipes of different diameters varies inversely as the diameters.

The general equation for the loss of pressure for the minimal loss from friction and radiation is

$$L = \frac{0.0007023\ drlp}{W}.$$

The loss of pressure for pipes of 1 inch diameter for different absolute terminal pressures when steam is flowing with minimal loss is expressed by the formula $L = Cl\sqrt[4]{r^3}$, in which the coefficient C has the following values:

For 65 lbs. abs. term. pressure	$C = 0.00089387$
" 75 " " " "	0.00093684
" 90 " " " "	0.00099573
" 100 " " " "	0.00103132
" 115 " " " "	0.00108051

In order to find the loss of pressure for any other diameter, divide the loss of pressure in a 1-inch pipe for the given terminal pressure by the given diameter, and the quotient will be the loss of pressure for that diameter.

The following is a general summary of the results of Mr. Rudiger's investigation:

The flow of steam in a pipe is determined in the same manner as the flow of water, the formula for the flow of steam being modified only by substituting the equivalent loss of pressure, divided by the density of the steam, for the loss of head.

The losses in the flow of steam are two in number—the loss due to the friction of flow and that due to radiation from the sides of the pipe. The sum of these is a minimum when the equivalent of the loss due to friction of flow is equal to one fifth of the loss of heat by radiation. For a greater or less loss of pressure—i.e., for a less or greater diameter of pipe—the total loss increases very rapidly.

For delivering a given quantity of steam at a given terminal pressure, with minimal total loss, the better the non-conducting material employed, the larger the diameter of the steam-pipe to be used.

The most economical loss of pressure for a pipe of given diameter is equal to the most economical loss of pressure in a pipe of 1 inch diameter for same conditions, divided by the diameter of the given pipe in inches.

The following table gives the capacity of pipes of different diameters, to deliver steam at different terminal pressures through a pipe one half mile long for loss of pressure of 10 lbs., and a mean value of $f = 0.0175$. Let W denote the number of pounds of steam delivered per hour:

Diameter of Pipe, inches.	Abs. Term. Pressure.			Diameter of Pipe, inches.	Abs. Term. Pressure.		
	65 lbs.	80 lbs.	100 lbs.		65 lbs.	80 lbs.	100 lbs.
	W	W	W		W	W	W
1	102	118	125	4½	4,397	4,872	5,390
1¼	179	198	219	5	5,721	6,339	7,013
1½	282	312	346	6	9,024	10,000	11,063
1¾	415	459	508	7	13,268	14,701	16,265
2	579	641	710	8	18,526	20,528	22,711
2½	1,011	1,121	1,240	9	24,870	27,556	30,488
3	1,595	1,768	1,956	10	32,364	35,860	39,675
3½	2,346	2,599	2,875	11	41,081	45,507	50,349
4	3,275	3,629	4,042	12	51,049	56,564	62,581

Resistance to Flow by Bends, Valves, etc. (From Briggs on Warming Buildings by Steam.)—The resistance at the entrance to a tube when no special bell-mouth is given consists of two parts. The head $v^2 \div 2g$ is expended in giving the velocity of flow; and the head $0.505 \frac{v^2}{2g}$ in overcoming the resistance of the mouth of the tube. Hence the whole loss of head at the entrance is $1.505 \frac{v^2}{2g}$. This resistance is equal to the resistance of a straight tube of a length equal to about 60 times its diameter.

The loss at each sharp right-angled elbow is the same as in flowing through a length of straight tube equal to about 40 times its diameter. For a globe steam stop-valve the resistance is taken to be 1½ times that of the right-angled elbow.

Sizes of Steam-pipes for Stationary Engines.—Authorities on the steam-engine generally agree that steam-pipes supplying engines should be of such size that the mean velocity of steam in them does not exceed 6000 feet per minute, in order that the loss of pressure due to friction may not be excessive. The velocity is calculated on the assumption that the cylinder is filled at each stroke. In very long pipes, 100 feet and upward, it is well to make them larger than this rule would give, and to place a large steam receiver on the pipe near the engine, especially when the engine cuts off early in the stroke.

An article in *Power*, May, 1893, on proper area of supply-pipes for engines gives a table showing the practice of leading builders. To facilitate comparison, all the engines have been rated in horse-power at 40 pounds mean effective pressure. The table contains all the varieties of simple engines, from the slide-valve to the Corliss, and it appears that there is no general difference in the sizes of pipe used in the different types.

The averages selected from this table are as follows:

Diam. of pipe, in......	2	2½	3	3½	4	4½	5	6	7	8	9	10
Av. H.P. of engines....	25	39	56	77	100	126	156	225	306	400	506	625
Calculated, formula (1)	23	36	51	70	91	116	143	206	278	366	468	571
" formula (2)	24	37.5	54	73	96	121	150	216	294	384	486	600

Formula (1) is: 1 H.P. requires .1375 sq. in. of steam-pipe area.

Formula (2) is: Horse-power $= 6d^2$. d = diam. of pipe in inches.

The factor .1375 in formula (1) is thus derived: Assume that the linear velocity of steam in the pipe should not exceed 6000 feet per minute, then pipe area = cyl. area × piston-speed ÷ 6000 (a). Assume that the av. mean effective pressure is 40 lbs. per sq. in., then cyl. area × piston-speed × 40 ÷ 33,000 = horse-power (b). Dividing (a) by (b) and cancelling, we have pipe area ÷ H.P. = .1375 sq. in. If we use 8000 ft. per min. as the allowable velocity, then the factor .1375 becomes .1031; that is, pipe area ÷ H.P. = .1031, or pipe area × 9.7 = horse-power. This, however, gives areas of pipe smaller than are used in the most recent practice. A formula which gives results closely agreeing with practice, as shown in the above table is

$$\text{Horse-power} = 6d^2, \quad \text{or} \quad \text{pipe diameter} = \sqrt{\frac{\text{H.P.}}{6}} = .408\sqrt{\text{H.P.}}$$

DIAMETERS OF CYLINDERS CORRESPONDING TO VARIOUS SIZES OF STEAM-PIPES BASED ON PISTON-SPEED OF ENGINE OF 600 FT. PER MINUTE, AND ALLOWABLE MEAN VELOCITY OF STEAM IN PIPE OF 4000, 6000, AND 8000 FT. PER MIN. (STEAM ASSUMED TO BE ADMITTED DURING FULL STROKE.)

Diam. of pipe, inches.........	2	2½	3	3½	4	4½	5	6
Vel. 4000..................	5.2	6.5	7.7	9.0	10.3	11.6	12.9	15.5
" 6000..................	6.3	7.9	9.5	11.1	12.6	14.2	15.8	19.
" 8000..................	7.3	9.1	10.9	12.8	14.6	16.4	18.3	21.9
Horse-power, approx.........	20	31	45	62	80	100	125	180
Diam. of pipe, inches	7	8	9	10	11	12	13	14
Vel. 4000..................	18.1	20.7	23.2	25.8	28.4	31.0	33.6	36.1
" 6000..................	22.1	25.3	28.5	31.6	34.8	37.9	41.1	44.3
" 8000..................	25.6	29.2	32.9	36.5	40.2	43.8	47.5	51.1
Horse-power, approx.........	245	320	406	500	606	718	845	981

Formula. $\text{Area of pipe} = \dfrac{\text{Area of cylinder} \times \text{piston-speed}}{\text{mean velocity of steam in pipe}}.$

For piston-speed of 600 ft. per min. and velocity in pipe of 4000, 6000, and 8000 ft. per min. area of pipe = respectively .15, .10, and .075 × area of cylinder. Diam. of pipe = respectively .3873, .3162, and .2739 × diam. of cylinder. Reciprocals of these figures are 2.582, 3.162, and 3.651.

The first line in the above table may be used for proportioning exhaust-pipes, in which a velocity not exceeding 4000 ft. per minute is advisable. The last line, approx. H.P. of engine, is based on the velocity of 6000 ft. per min. in the pipe, using the corresponding diameter of piston, and taking H.P. = $\frac{1}{2}$(diam. of piston in inches)^2.

Sizes of Steam-pipes for Marine Engines.—In marine-engine practice the steam-pipes are generally not as large as in stationary practice for the same sizes of cylinder. Seaton gives the following rules:

Main Steam-pipes should be of such size that the mean velocity of flow does not exceed 8000 ft. per min.

In large engines, 1000 to 2000 H.P., cutting off at less than half stroke, the steam-pipe may be designed for a mean velocity of 9000 ft., and 10,000 ft. for still larger engines.

In small engines and engines cutting later than half stroke, a velocity of less than 8000 ft. per minute is desirable.

Taking 8100 ft. per min. as the mean velocity, S speed of piston in feet per min., and D the diameter of the cyl.,

$$\text{Diam. of main steam-pipe} = \sqrt{\frac{D^2S}{8100}} = \frac{D}{90}\sqrt{S}.$$

Stop and Throttle Valves should have a greater area of passages than the area of the main steam-pipe, on account of the friction through the circuitous passages. The shape of the passages should be designed so as to avoid abrupt changes of direction and of velocity of flow as far as possible.

Area of Steam Ports and Passages =

$$\frac{\text{Area of piston} \times \text{speed of piston in ft. per min.}}{6000} = \frac{(\text{Diam.})^2 \times \text{speed}}{7639}.$$

Opening of Port to Steam.—To avoid wire-drawing during admission the area of opening to steam should be such that the mean velocity of flow does not exceed 10,000 ft. per min. To avoid excessive clearance the width of port should be as short as possible, the necessary area being obtained by length (measured at right angles to the line of travel of the valve). In practice this length is usually 0.6 to 0.8 of the diameter of the cylinder, but in long-stroke engines it may equal or even exceed the diameter.

Exhaust Passages and Pipes.—The area should be such that the mean velocity of the steam should not exceed 6000 ft. per min., and the area should be greater if the length of the exhaust-pipe is comparatively long. The area of passages from cylinders to receivers should be such that the velocity will not exceed 5000 ft. per min.

The following table is computed on the basis of a mean velocity of flow of 8000 ft. per min. for the main steam-pipe, 10,000 for opening to steam, and 6000 for exhaust. A = area of piston, D its diameter.

STEAM AND EXHAUST OPENINGS.

Piston-speed, ft. per min.	Diam. of Steam-pipe ÷ D.	Area of Steam-pipe ÷ A.	Diam. of Exhaust ÷ D.	Area of Exhaust ÷ A.	Opening to Steam ÷ A.
300	0.194	0.0375	0.223	0.0500	0.03
400	0.224	0.0500	0.258	0.0667	0.04
500	0.250	0.0625	0.288	0.0833	0.05
600	0.274	0.0750	0.316	0.1000	0.06
700	0.296	0.0875	0.341	0.1167	0.07
800	0.316	0.1000	0.365	0.1333	0.08
900	0.335	0.1125	0.387	0.1500	0.09
1000	0.353	0.1250	0.400	0.1667	0.10

STEAM PIPES.

Bursting-tests of Copper Steam-pipes. (From Report of Chief Engineer Melville, U. S. N., for 1892.)—Some tests were made at the New York Navy Yard which show the unreliability of brazed seams in copper pipes. Each pipe was 8 in. diameter inside and 3 ft. 1⅝ in. long. Both ends were closed by ribbed heads and the pipe was subjected to a hot-water pressure, the temperature being maintained constant at 371° F. Three of the pipes were made of No. 4 sheet copper ("Stubbs" gauge) and the fourth was made of No. 8 sheet.

The following were the results, in lbs. per sq. in., of bursting-pressure:

Pipe number	1	2	3	4	4′
Actual bursting-strength	835	785	950	1225	1275
Calculated " "	1336	1336	1569	1568	1568
Difference	501	551	619	343	293

The theoretical bursting-pressure of the pipes was calculated by using the figures obtained in the tests for the strength of copper sheet with a brazed joint at 350° F. Pipes 1 and 2 are considered as having been annealed.

The tests of specimens cut from the ruptured pipes show the injurious action of heat upon copper sheets; and that, while a white heat does not change the character of the metal, a heat of only slightly greater degree causes it to lose the fibrous nature that it has acquired in rolling, and a serious reduction in its tensile strength and ductility results.

All the brazing was done by expert workmen, and their failure to make a pipe-joint without burning the metal at some point makes it probable that, with copper of this or greater thickness, it is seldom accomplished.

That it is possible to make a joint without thus injuring the metal was proven in the cases of many of the specimens, both of those cut from the pipes and those made separately, which broke with a fibrous fracture.

Rule for Thickness of Copper Steam-pipes. (U. S. Supervising Inspectors of Steam Vessels.)—Multiply the working steam-pressure in lbs. per sq. in. allowed the boiler by the diameter of the pipe in inches, then divide the product by the constant whole number 8000, and add .0625 to the quotient; the sum will give the thickness of material required.

EXAMPLE.—Let 175 lbs. = working steam-pressure per sq. in. allowed the boiler, 5 in. = diameter of the pipe; then $\frac{175 \times 5}{8000} + .0625 = .1718 +$ inch, thickness required.

Reinforcing Steam-pipes. (*Eng.*, Aug. 11, 1893.)—In the Italian Navy copper pipes above 8 in. diam. are reinforced by wrapping them with a close spiral of copper or Delta-metal wire. Two or three independent spirals are used for safety in case one wire breaks. They are wound at a tension of about 1⅛ tons per sq. in.

Wire-wound Steam-pipes.—The system instituted by the British Admiralty of winding all steam-pipes over 8 in. in diameter with 3/16-in. copper wire, thereby about doubling the bursting-pressure, has within recent years been adopted on many merchant steamers using high-pressure steam, says the London *Engineer*. The results of some of the Admiralty tests showed that a wire pipe stood just about the pressure it ought to have stood when unwired, had the copper not been injured in the brazing.

Riveted Steel Steam-pipes have recently been used for high pressures. See paper on A Method of Manufacture of Large Steam-pipes, by Chas. H. Manning, Trans. A. S. M. E., vol. xv.

Valves in Steam-pipes.—Should a globe-valve on a steam-pipe have the steam-pressure on top or underneath the valve is a disputed question. With the steam-pressure on top, the stuffing-box around the valve-stem cannot be repacked without shutting off steam from the whole line of pipe; on the other hand, if the steam-pressure is on the bottom of the valve it all has to be sustained by the screw-thread on the valve-stem, and there is danger of stripping the thread.

A correspondent of the *American Machinist*, 1892, says that it is a very uncommon thing in the ordinary globe-valve to have the thread give out, but by water-hammer and merciless screwing the seat will be crushed down quite frequently. Therefore with plants where only one boiler is used he advises placing the valve with the boiler-pressure underneath it. On plants where several boilers are connected to one main steam-pipe he would reverse the position of the valve, then when one of the valves needs repacking the valve can be closed and the pressure in the boiler whose pipe it controls can be reduced to atmospheric by lifting the safety-valve. The repacking can then be done without interfering with the operation of the other boilers of the plant.

He proposes also the following other rules for locating valves: Place valves with the stems horizontal to avoid the formation of a water-pocket. Never put the junction-valve close to the boiler if the main pipe is above the boiler, but put it on the highest point of the junction-pipe. If the other plan is followed, the pipe fills with water whenever this boiler is stopped and the others are running, and breakage of the pipe may cause serious results. Never let a junction-pipe run into the bottom of the main pipe, but into the side or top. Always use an angle-valve where convenient, as there is more room in them. Never use a gate valve under high pressure unless a by-pass is used with it. Never open a blow-off valve on a boiler a little and then shut it; it is sure to catch the sediment and ruin the valve; throw it well open before closing. Never use a globe-valve on an indicator-pipe. For water, always use gate or angle valves or stop-cocks to obtain a clear passage. Buy if possible valves with renewable disks. Lastly, never let a man go inside a boiler to work, especially if he is to hammer on it, unless you break the joint between the boiler and the valve and put a plate of steel between the flanges.

A Failure of a Brazed Copper Steam-pipe on the British steamer *Prodano* was investigated by Prof. J. O. Arnold. He found that the brazing was originally sound, but that it had deteriorated by oxidation of the zinc in the brazing alloy by electrolysis, which was due to the presence of fatty acids produced by decomposition of the oil used in the engines. A full account of the investigation is given in *The Engineer*, April 15, 1898.

The "Steam Loop" is a system of piping by which water of condensation in steam-pipes is automatically returned to the boiler. In its simplest form it consists of three pipes, which are called the riser, the horizontal, and the drop-leg. When the steam-loop is used for returning to the boiler the water of condensation and entrainment from the steam-pipe through which the steam flows to the cylinder of an engine, the riser is generally attached to a separator; this riser empties at a suitable height into the horizontal, and from thence the water of condensation is led into the drop-leg, which is connected to the boiler, into which the water of condensation is fed as soon as the hydrostatic pressure in drop-leg in connection with the steam-pressure in the pipes is sufficient to overcome the boiler-pressure. The action of the device depends on the following principles: Difference of pressure may be balanced by a water-column; vapors or liquids tend to flow to the point of lowest pressure; rate of flow depends on difference of pressure and mass; decrease of static pressure in a steam-pipe or chamber is proportional to rate of condensation; in a steam-current water will be carried or swept along rapidly by friction. (Illustrated in Modern Mechanism, p. 807.)

Loss from an Uncovered Steam-pipe. (Bjorling on Pumping-engines.)—The amount of loss by condensation in a steam-pipe carried down a deep mine-shaft has been ascertained by actual practice at the Clay Cross Colliery, near Chesterfield, where there is a pipe 7½ in. internal diam., 1100 ft. long. The loss of steam by condensation was ascertained by direct measurement of the water deposited in a receiver, and was found to be equivalent to about 1 lb. of coal per I.H.P. per hour for every 100 ft. of steam-pipe; but there is no doubt that if the pipes had been in the upcast shaft, and well covered with a good non-conducting material, the loss would have been less. (For Steam-pipe Coverings, see p. 469, *ante*.)

THE STEAM-BOILER.

The Horse-power of a Steam-boiler.—The term horse power has two meanings in engineering: *First, an absolute unit or measure of the rate of work*, that is, of the work done in a certain definite period of time, by a source of energy, as a steam-boiler, a waterfall, a current of air or water, or by a prime mover, as a steam-engine, a water-wheel, or a windmill. The value of this unit, whenever it can be expressed in foot-pounds of energy, as in the case of steam-engines, water-wheels, and waterfalls, is 33,000 foot-pounds per minute. In the case of boilers, where the work done, the conversion of water into steam, cannot be expressed in foot-pounds of available energy, the usual value given to the term horse-power is the evaporation of 30 lbs. of water of a temperature of 100° F. into steam at 70 lbs. pressure above the atmosphere. Both of these units are arbitrary; the first, 33,000 foot-pounds per minute, first adopted by James Watt, being considered equivalent to the power exerted by a good London draught-horse, and the 30 lbs. of water evaporated per hour being considered to be the steam requirement per indicated horse-power of an average engine.

The second definition of the term horse-power *is an approximate measure of the size, capacity, value, or "rating"* of a boiler, engine, water-wheel, or other source or conveyer of energy, by which measure it may be described, bought and sold, advertised, etc. No definite value can be given to this measure, which varies largely with local custom or individual opinion of makers and users of machinery. The nearest approach to uniformity which can be arrived at in the term "horse power," used in this sense, is to say that a boiler, engine, water-wheel, or other machine, "rated" at a certain horse-power, should be capable of steadily developing that horse-power for a long period of time under ordinary conditions of use and practice, leaving to local custom, to the judgment of the buyer and seller, to written contracts of purchase and sale, or to legal decisions upon such contracts, the interpretation of what is meant by the term "ordinary conditions of use and practice." (Trans. A. S. M. E., vol. vii. p. 226.)

The committee of the A. S. M. E. on Trials of Steam-boilers in 1884 (Trans., vol. vi. p. 265) discussed the question of the horse-power of boilers as follows:

The Committee of Judges of the Centennial Exhibition, to whom the trials of competing boilers at that exhibition were intrusted, met with this same problem, and finally agreed to solve it, at least so far as the work of that committee was concerned, by the adoption of the unit, 30 lbs. of water evaporated into dry steam per hour from feed-water at 100° F., and under a pressure of 70 lbs. per square inch above the atmosphere, these conditions being considered by them to represent fairly average practice. The quantity of heat demanded to evaporate a pound of water under these conditions is 1110.2 British thermal units, or 1.1496 units of evaporation. The unit of power proposed is thus equivalent to the development of 33,305 heat-units per hour, or 34.488 units of evaporation. . . .

Your committee, after due consideration, has determined to accept the Centennial Standard, the first above mentioned, and to recommend that in all standard trials the commercial horse-power be taken as an evaporation of 30 lbs. of water per hour from a feed-water temperature of 100° F. into steam at 70 lbs. gauge pressure, which shall be considered to be equal to 34½ units of evaporation, that is, to 34½ lbs. of water evaporated from a feed-water temperature of 212° F. into steam at the same temperature. This standard is equal to 33,305 thermal units per hour.

It is the opinion of this committee that a boiler rated at any stated number of horse-powers should be capable of developing that power with easy firing, moderate draught, and ordinary fuel, while exhibiting good economy; and further, that the boiler should be capable of developing at least one third more than its rated power to meet emergencies at times when maximum economy is not the most important object to be attained.

Unit of Evaporation.—It is the custom to reduce results of boiler-tests to the common standard of weight of water evaporated by the unit weight of the combustible portion of the fuel, the evaporation being considered to have taken place at mean atmospheric pressure, and at the temperature due that pressure, the feed-water being also assumed to have been supplied at that temperature. This is, in technical language, said to be the equivalent evaporation from and at the boiling point at atmospheric pressure, or "from and at 212° F." This unit of evaporation, or one pound of water evaporated from and at 212°, is equivalent to 965.7 British thermal units.

Measures for Comparing the Duty of Boilers.—The measure of the efficiency of a boiler is the number of pounds of water evaporated per pound of combustible, the evaporation being reduced to the standard of "from and at 212°;" that is, the equivalent evaporation from feed-water at a temperature of 212° F. into steam at the same temperature.

The measure of the capacity of a boiler is the amount of "boiler horse-power" developed, a horse-power being defined as the evaporation of 30 lbs. of water per hour from 100° F. into steam at 70 lbs. pressure, or 34½ lbs. per hour from and at 212°.

The measure of relative rapidity of steaming of boilers is the number of pounds of water evaporated per hour per square foot of water-heating surface.

The measure of relative rapidity of combustion of fuel in boiler-furnaces is the number of pounds of coal burned per hour per square foot of grate-surface.

STEAM-BOILER PROPORTIONS.

Proportions of Grate and Heating Surface required for a given Horse-power.—The term horse-power here means capacity to evaporate 30 lbs. of water from 100° F., temperature of feed-water, to steam of 70 lbs., gauge-pressure = 34.5 lbs. from and at 212° F.

Average proportions for maximum economy for land boilers fired with good anthracite coal:

Heating surface per horse-power	11.5	sq. ft.
Grate " "	1/3	"
Ratio of heating to grate surface	34.5	"
Water evap'd from and at 212° per sq. ft. H.S. per hour	3	lbs.
Combustible burned per H.P. per hour	3	"
Coal with 1/6 refuse, lbs. per H.P. per hour	3.6	"
Combustible burned per sq. ft. grate per hour	9	"
Coal with 1/6 refuse, lbs. per sq. ft. grate per hour	10.8	"
Water evap'd from and at 212° per lb. combustible	11.5	"
" " " " " " " " coal (1/6 refuse)	9.6	"

The rate of evaporation is most conveniently expressed in pounds evaporated from and at 212° per sq. ft. of water-heating surface per hour, and the rate of combustion in pounds of coal per sq. ft. of grate-surface per hour.

Heating-surface.—For maximum economy with any kind of fuel a boiler should be proportioned so that at least one square foot of heating-surface should be given for every 3 lbs. of water to be evaporated from and at 212° F. per hour. Still more liberal proportions are required if a portion of the heating-surface has its efficiency reduced by: 1. Tendency of the heated gases to short-circuit, that is, to select passages of least resistance and flow through them with high velocity, to the neglect of other passages. 2. Deposition of soot from smoky fuel. 3. Incrustation. If the heating-surfaces are clean, and the heated gases pass over it uniformly, little if any increase in economy can be obtained by increasing the heating-surface beyond the proportion of 1 sq. ft. to every 3 lbs. of water to be evaporated, and with all conditions favorable but little decrease of economy will take place if the proportion is 1 sq. ft. to every 4 lbs. evaporated; but in order to provide for driving of the boiler beyond its rated capacity, and for possible decrease of efficiency due to the causes above named, it is better to adopt 1 sq. ft. to 3 lbs. evaporation per hour as the minimum standard proportion.

Where economy may be sacrificed to capacity, as where fuel is very cheap, it is customary to proportion the heating-surface much less liberally. The following table shows approximately the relative results that may be expected with different rates of evaporation, with anthracite coal.

Lbs. water evapor'd from and at 212° per sq. ft. heating-surface per hour:										
2	2.5	3	3.5	4	5	6	7	8	9	10
Sq. ft. heating-surface required per horse-power:										
17.3	13.8	11.5	9.8	8.6	6.8	5.8	4.9	4.3	3.8	3.5
Ratio of heating to grate surface if 1/3 sq. ft. of G. S. is required per H.P.:										
52	41.4	34.5	29.4	25.8	20.4	17.4	13.7	12.9	11.4	10.5
Probable relative economy:										
100	100	100	95	90	85	80	75	70	65	60
Probable temperature of chimney gases, degrees F.:										
450	450	450	518	585	652	720	787	855	922	990

The relative economy will vary not only with the amount of heating-surface per horse-power, but with the efficiency of that heating-surface as regards its capacity for transfer of heat from the heated gases to the water, which will depend on its freedom from soot and incrustation, and upon the circulation of the water and the heated gases.

With bituminous coal the efficiency will largely depend upon the thoroughness with which the combustion is effected in the furnace.

The efficiency with any kind of fuel will greatly depend upon the amount of air supplied to the furnace in excess of that required to support combustion. With strong draught and thin fires this excess may be very great, causing a serious loss of economy.

Measurement of Heating-surface.—Authorities are not agreed as to the methods of measuring the heating-surface of steam-boilers. The usual rule is to consider as heating-surface all the surfaces that are surrounded by water on one side and by flame or heated gases on the other, but there is a difference of opinion as to whether tubular heating-surface should be figured from the inside or from the outside diameter. Some writers say, measure the heating-surface always on the smaller side—the fire side of the tube in a horizontal return tubular boiler and the water side in a water-tube boiler. Others would deduct from the heating-surface thus measured an allowance for portions supposed to be ineffective on account of being covered by dust, or being out of the direct current of the gases.

It has hitherto been the common practice of boiler-makers to consider all surfaces as heating-surfaces which transmit heat from the flame or gases to the water, making no allowance for different degrees of effectiveness; also, to use the external instead of the internal diameter of tubes, for greater convenience in calculation, the external diameter of boiler-tubes usually being made in even inches or half inches. This method, however, is inaccurate, for the true heating-surface of a tube is the side exposed to the hot gases, the inner surface in a fire-tube boiler and the outer surface in a water-tube boiler. The resistance to the passage of heat from the hot gases on one side of a tube or plate to the water on the other consists almost entirely of the resistance to the passage of the heat from the gases into the metal, the resistance of the metal itself and that of the wetted surface being practically nothing. See paper by C. W. Baker, Trans. A. S. M. E., vol. xix.

Rule for finding the heating-surface of vertical tubular boilers: Multiply the circumference of the fire-box (in inches) by its height above the grate; multiply the combined circumference of all the tubes by their length, and to these two products add the area of the lower tube-sheet; from this sum subtract the area of all the tubes, and divide by 144: the quotient is the number of square feet of heating-surface.

Rule for finding the heating-surface of horizontal tubular boilers: Take the dimensions in inches. Multiply two thirds of the circumference of the shell by its length; multiply the sum of the circumferences of all the tubes by their common length; to the sum of these products add two thirds of the area of both tube-sheets; from this sum subtract twice the combined area of all the tubes; divide the remainder by 144 to obtain the result in square feet.

Rule for finding the square feet of heating-surface in tubes: Multiply the number of tubes by the diameter of a tube in inches, by its length in feet, and by .2618.

Horse-power, Builder's Rating. Heating-surface per Horse-power.—It is a general practice among builders to furnish about 12 square feet of heating-surface per horse-power, but as the practice is not uniform, bids and contracts should always specify the amount of heating-surface to be furnished. Not less than one third square foot of grate-surface should be furnished per horse-power.

Engineering News, July 5, 1894, gives the following rough-and-ready rule for finding approximately the commercial horse-power of tubular or water-tube boilers: Number of tubes × their length in feet × their nominal diameter in inches ÷ 50 = $nLd \div 50$. The number of square feet of surface in the tubes is $\frac{n\pi dL}{12} = \frac{nLd}{3.82}$, and the horse-power at 12 square feet of surface of tubes per horse-power, not counting the shell, $= nLd \div 45.8$. If 15 square feet of surface of tubes be taken, it is $nLd \div 57.3$. Making allowance for the heating-surface in the shell will reduce the divisor to about 50.

Horse-power of Marine and Locomotive Boilers.—The term horse-power is not generally used in connection with boilers in marine practice, or with locomotives. The boilers are designed to suit the engines, and are rated by extent of grate and heating-surface only.

Grate-surface.—The amount of grate-surface required per horse power, and the proper ratio of heating-surface to grate-surface are extremely variable, depending chiefly upon the character of the coal and upon the rate of draught. With good coal, low in ash, approximately equal results may be obtained with large grate-surface and light draught and with small grate-surface and strong draught, the total amount of coal burned per hour being the same in both cases. With good bituminous coal, like Pittsburgh, low in ash, the best results apparently are obtained with strong draught and high rates of combustion, provided the grate-surfaces are cut down so that the total coal burned per hour is not too great for the capacity of the heating-surface to absorb the heat produced.

With coals high in ash, especially if the ash is easily fusible, tending to choke the grates, large grate-surface and a slow rate of combustion are required, unless means, such as shaking grates, are provided to get rid of the ash as fast as it is made.

The amount of grate-surface required per horse-power under various conditions may be estimated from the following table:

	Lbs. Water from and at 212° per lb., Coal.	Lbs. Coal per H.P. per hour.	Pounds of Coal burned per square foot of Grate per hour.								
			8	10	12	15	20	25	30	35	40
			Sq. Ft. Grate per H. P.								
Good coal and boiler,	10	3.45	.43	.35	.28	.23	.17	.14	.11	.10	.09
	9	3.83	.48	.38	.32	.25	.19	.15	.13	.11	.10
Fair coal or boiler,	8.61	4.	.50	.40	.33	.26	.20	.16	.13	.12	.10
	8	4.31	.54	.43	.36	29	.22	.17	.14	.13	.11
	7	4.93	.62	.49	.41	33	.24	.20	.17	.14	.12
Poor coal or boiler,	6.9	5.	.63	.50	.42	.34	.25	.20	.17	.15	.13
	6	5.75	.72	.58	.48	.38	.29	.23	.19	17	.14
	5	6.9	.86	.69	58	.46	.35	.28	.23	.22	.17
Lignite and poor boiler.	3.45	10.	1.25	1.00	.83	.67	.50	.40	.33	.29	.25

In designing a boiler for a given set of conditions, the grate-surface should be made as liberal as possible, say sufficient for a rate of combustion of 10 lbs. per square foot of grate for anthracite, and 15 lbs. per square foot for bituminous coal, and in practice a portion of the grate-surface may be bricked over if it is found that the draught, fuel, or other conditions render it advisable.

Proportions of Areas of Flues and other Gas-passages.—Rules are usually given making the area of gas-passages bear a certain ratio to the area of the grate-surface; thus a common rule for horizontal tubular boilers is to make the area over the bridge wall 1/7 of the grate-surface, the flue area 1/8, and the chimney area 1/9.

For average conditions with anthracite coal and moderate draught, say a rate of combustion of 12 lbs. coal per square foot of grate per hour, and a ratio of heating to grate surface of 30 to 1, this rule is as good as any, but it is evident that if the draught were increased so as to cause a rate of combustion of 24 lbs., requiring the grate-surface to be cut down to a ratio of 60 to 1, the areas of gas-passages should not be reduced in proportion. The amount of coal burned per hour being the same under the changed conditions, and there being no reason why the gases should travel at a higher velocity, the actual areas of the passages should remain as before, but the ratio of the area to the grate-surface would in that case be doubled.

Mr. Barrus states that the highest efficiency with anthracite coal is obtained when the tube area is 1/9 to 1/10 of the grate-surface, and with bituminous coal when it is 1/6 to 1/7, for the conditions of medium rates of combustion, such as 10 to 12 lbs. per square foot of grate per hour, and 12 square feet of heating-surface allowed to the horse-power.

The tube area should be made large enough not to choke the draught, and so lessen the capacity of the boiler; if made too large the gases are apt to select the passages of least resistance and escape from them at a high velocity and high temperature.

This condition is very commonly found in horizontal tubular boilers where the gases go chiefly through the upper rows of tubes; sometimes also in vertical tubular boilers, where the gases are apt to pass most rapidly through the tubes nearest to the centre.

Air-passages through Grate-bars.—The usual practice is, air-opening = 30% to 50% of area of the grate; the larger the better, to avoid stoppage of the air-supply by clinker; but with coal free from clinker much smaller air-space may be used without detriment. See paper by F. A. Scheffler, Trans. A. S. M. E., vol. xv. p. 503.

PERFORMANCE OF BOILERS.

The performance of a steam-boiler comprises both its capacity for generating steam and its economy of fuel. Capacity depends upon size, both of grate-surface and of heating-surface, upon the kind of coal burned, upon the draft, and also upon the economy. Economy of fuel depends upon the completeness with which the coal is burned in the furnace, on the proper regulation of the air-supply to the amount of coal burned, and upon the thoroughness with which the boiler absorbs the heat generated in the furnace. The absorption of heat depends on the extent of heating-surface in relation to the amount of coal burned or of water evaporated, upon the arrangement of the gas-passages, and upon the cleanness of the surfaces. The capacity of a boiler may increase with increase of economy when this is due to more thorough combustion of the coal or to better regulation of the air-supply, or it may increase at the expense of economy when the increased capacity is due to overdriving, causing an increased loss of heat in the chimney gases. The relation of capacity to economy is therefore a complex one, depending on many variable conditions.

Many attempts have been made to construct a formula expressing the relation between capacity, rate of driving, or evaporation per square foot of heating-surface, to the economy, or evaporation per pound of combustible, but none of them can be considered satisfactory, since they make the economy depend only on the rate of driving (a few so-called "constants," however, being introduced in some of them for different classes of boilers, kinds of fuel, or kind of draft), and fail to take into consideration the numerous other conditions upon which economy depends. Such formulæ are Rankine's, Clark's, Emery's, Isherwood's, Carpenter's, and Hale's. A discussion of them all may be found in Mr. R. S. Hale's paper on "Efficiency of Boiler Heating Surface," in Trans. A. S. M. E., vol. xviii. p. 328. Mr. Hale's formula takes into account the effect of radiation, which reduces the economy considerably when the rate of driving is less than 3 lbs. per square foot of heating-surface per hour.

Selecting the highest results obtained at different rates of driving obtained with anthracite coal in the Centennial tests (see p. 685), and the highest results with anthracite reported by Mr. Barrus in his book on Boiler Tests, the author has plotted two curves showing the maximum results which may be expected with anthracite coal, the first under exceptional conditions such as obtained in the Centennial tests, and the second under the best conditions of ordinary practice. (Trans. A. S. M. E., xviii. 354). From these curves the following figures are obtained.

Lbs. water evaporated from and at 212° per sq. ft. heating-surface per hour:	1.5	1.7	2	2.5	3	3.5	4	4.5	5	6	7	8
Lbs. water evaporated from and at 212° per lb. combustible:												
Centennial.	11.8	11.9	12.0	12.1	12.05	12	11.85	11.7	11.5	10.85	9.8	8.5
Barrus.....	11.4	11.5	11.55	11.6	11.6	11.5	11.2	10.9	10.6	9.9	9.2	8.5
Avg. Cent'l	....	.....	12.0	11.6	11.2	10.8	10.4	10.0	9.6	8.8	8.0	7.2

The figures in the last line are taken from a straight line drawn as nearly as possible through the average of the plotting of all the Centennial tests. The poorest results are far below these figures. It is evident that no formula can be constructed that will express the relation of economy to rate of driving as well as do the three lines of figures given above.

For semi-bituminous and bituminous coals the relation of economy to the rate of driving no doubt follows the same general law that it does with anthracite, i.e., that beyond a rate of evaporation of 3 or 4 lbs. per sq. ft. of heating-surface per hour there is a decrease of economy, but the figures obtained in different tests will show a wider range between maximum and average results on account of the fact that it is more difficult with bituminous than with anthracite coal to secure complete combustion in the furnace.

The amount of the decrease in economy due to driving at rates exceeding 4 lbs. of water evaporated per square foot of heating-surface per hour differs greatly with different boilers, and with the same boiler it may differ with different settings and with different coal. The arrangement and size of the gas-passages seem to have an important effect upon the relation of economy to rate of driving. There is a large field for future research to determine the causes which influence this relation.

General Conditions which secure Economy of Steam-boilers.—In general, the highest results are produced where the temperature of the escaping gases is the least. An examination of this question is made by Mr. G. H. Barrus in his book on "Boiler Tests," by selecting those tests made by him, six in number, in which the temperature exceeds the average, that is, 375° F., and comparing with five tests in which the temperature is less than 375°. The boilers are all of the common horizontal type, and all use anthracite coal of either egg or broken size. The average flue temperatures in the two series was 444° and 343° respectively, and the difference was 101°. The average evaporations are 10.40 lbs. and 11.02 lbs. respectively, and the lowest result corresponds to the case of the highest flue temperature. In these tests it appears, therefore, that a reduction of 101° in the temperature of the waste gases secured an increase in the evaporation of 6%. This result corresponds quite closely to the effect of lowering the temperature of the gases by means of a flue-heater where a reduction of 107° was attended by an increase of 7% in the evaporation per pound of coal.

A similar comparison was made on horizontal tubular boilers using Cumberland coal. The average flue temperature in four tests is 450° and the average evaporation is 11.34 lbs. Six boilers have temperatures below 415°, the average of which is 383°, and these give an average evaporation of 11.75 lbs. With 67° less temperature of the escaping gases the evaporation is higher by about 4%.

The wasteful effect of a high flue temperature is exhibited by other boilers than those of the horizontal tubular class. This source of waste was shown to be the main cause of the low economy produced in those vertical boilers which are deficient in heating-surface.

Relation between the Heating-surface and Grate-surface to obtain the Highest Efficiency.—A comparison of three tests of horizontal tubular boilers with anthracite coal, the ratio of heating-surface to grate-surface being 36.4 to 1, with three other tests of similar boilers, in which the ratio was 48 to 1, showed practically no difference in the results. The evidence shows that a ratio of 36 to 1 provides a sufficient quantity of heating-surface to secure the full efficiency of anthracite coal where the rate of combustion is not more than 12 lbs. per sq. ft. of grate per hour.

In tests with bituminous coal an increase in the ratio from 36.8 to 42.8 secured a small improvement in the evaporation per pound of coal, and a high temperature of the escaping gases indicated that a still further increase would be beneficial. Among the high results produced on common horizontal tubular boilers using bituminous coal, the highest occurs where the ratio is 58.1 to 1. This boiler gave an evaporation of 12.47 lbs. A double-deck boiler furnishes another example of high performance, an evaporation of 12.42 lbs. having been obtained with bituminous coal, and in this case the ratio is 65 to 1. These examples indicate that a much larger amount of heating-surface is required for obtaining the full efficiency of bituminous coal than for boilers using anthracite coal. The temperature of the escaping gases in the same boiler is invariably higher when bituminous coal is used than when anthracite coal is used. The deposit of soot on the surfaces when bituminous coal is used interferes with the full efficiency of the surface, and an increased area is demanded as an offset to the loss which this deposit occasions. It would seem, then, that if a ratio of 36 to 1 is sufficient for anthracite coal, from 45 to 50 should be provided when bituminous coal is burned, especially in cases where the rate of combustion is above 10 or 12 lbs. per sq. ft. of grate per hour.

The number of tubes controls the ratio between the area of grate-surface and area of tube-opening. A certain minimum amount of tube-opening is required for efficient work.

The best results obtained with anthracite coal in the common horizontal boiler are in cases where the ratio of area of grate-surface to area of tube-opening is larger than 9 to 1. The conclusion is drawn that the highest efficiency with anthracite coal is obtained when the tube-opening is from 1/9 to 1/10 of the grate-surface.

When bituminous coal is burned the requirements appear to be different. The effect of a large tube opening does not seem to make the extra tubes inefficient when bituminous coal is used. The highest result on any boiler of the horizontal tubular class, fired with bituminous coal, was obtained where the tube-opening was the largest. This gave an evaporation of 12.47 lbs., the ratio of grate-surface to tube-opening being 5.4 to 1. The next highest result was 12.42 lbs., the ratio being 5.2 to 1. Three high results, averaging 12.01 lbs., were obtained when the average ratio was 7.1 to 1. Without going to extremes, the ratio to be desired when bituminous coal is used is that which gives a tube-opening having an area of from 1/6 to 1/7 of the grate-surface. This applies to medium rates of combustion of, say, 10 to 12 lbs. per sq. ft. of grate per hour, 12 sq. ft. of water-heating surface being allowed per horse-power.

A comparison of results obtained from different types of boilers leads to the general conclusion that the economy with which different types of boilers operate depends much more upon their proportions and the conditions under which they work, than upon their type; and, moreover, that when these proportions are suitably carried out, and when the conditions are favorable, the various types of boilers give substantially the same economic result.

Efficiency of a Steam-boiler.—The efficiency of a boiler is the percentage of the total heat generated by the combustion of the fuel which is utilized in heating the water and in raising steam. With anthracite coal the heating-value of the combustible portion is very nearly 14,500 B. T. U. per lb., equal to an evaporation from and at 212° of 14,500 ÷ 966 = 15 lbs. of water. A boiler which when tested with anthracite coal shows an evaporation of 12 lbs. of water per lb. of combustible, has an efficiency of 12 ÷ 15 = 80%, a figure which is approximated, but scarcely ever quite reached, in the best practice. With bituminous coal it is necessary to have a determination of its heating-power made by a coal calorimeter before the efficiency of the boiler using it can be determined, but a close estimate may be made from the chemical analysis of the coal. (See Coal.)

The difference between the efficiency obtained by test and 100% is the sum of the numerous wastes of heat, the chief of which is the necessary loss due to the temperature of the chimney-gases. If we have an analysis and a calorimetric determination of the heating-power of the coal (properly sampled), and an average analysis of the chimney-gases, the amounts of the several losses may be determined with approximate accuracy by the method described below.

Data given:

1. ANALYSIS OF THE COAL. Cumberland Semi-bituminous.

Carbon	80.55
Hydrogen	4.50
Oxygen	2.70
Nitrogen	1.08
Moisture	2.92
Ash	8.25
	100.00

2. ANALYSIS OF THE DRY CHIMNEY-GASES, BY WEIGHT.

		C.	O.	N.
CO_2	= 13.6 =	3.71	9.89	.....
CO	= .2 =	.09	.11	.....
O	= 11.2 =	....	11.20	.....
N	= 75.0 =	....	.....	75.00
	100.0	3.80	21.20	75.00

Heating-value of the coal by Dulong's formula, 14,243 heat-units.

The gases being collected over water, the moisture in them is not determined.

3. Ash and refuse as determined by boiler-test, 10.25, or 2% more than that found by analysis, the difference representing carbon in the ashes obtained in the boiler-test.

4. Temperature of external atmosphere, 60° F.

5. Relative humidity of air, 60%, corresponding (see air tables) to .007 lb. of vapor in each lb. of air.

6. Temperature of chimney-gases, 560° F.

Calculated results:

The carbon in the chimney-gases being 3.8% of their weight, the total weight of dry gases per lb. of carbon burned is 100 ÷ 3.8 = 26.32 lbs. Since the carbon burned is 80.55 − 2 = 78.55% of the weight of the coal, the weight of the dry gases per lb. of coal is 26.32 × 78.55 ÷ 100 = 20.67 lbs.

Each pound of coal furnishes to the dry chimney-gases .7855 lb. C, .0108N, and $\left(2.70 - \frac{4.50}{8}\right) \div 100 = .0214$ lb. O; a total of .8177, say .82 lb. This subtracted from 20.67 lbs. leaves 19.85 lbs. as the quantity of dry air (not including moisture) which enters the furnace per pound of coal, not counting the air required to burn the available hydrogen, that is, the hydrogen minus one eighth of the oxygen chemically combined in the coal. Each lb. of coal burned contained .045 lb. H, which requires .045 × 8 = .36 lb. O for its combustion. Of this, .027 lb. is furnished by the coal itself, leaving .333 lb. to come from the air. The quantity of air needed to supply this oxygen (air containing 23% by weight of oxygen) is .333 ÷ .23 = 1.45 lb., which added to the 19.85 lbs. already found gives 21.30 lbs. as the quantity of dry air supplied to the furnace per lb. of coal burned.

The air carried in as vapor is .0071 lb. for each lb. of dry air, or 21.3 × .0071 = 0.15 lb. for each lb. of coal. Each lb. of coal contained .029 lb. of moisture, which was evaporated and carried into the chimney-gases. The .045 lb. of H per lb. of coal when burned formed .045 × 9 = .405 lb. of H_2O.

From the analysis of the chimney-gas it appears that .09 ÷ 3.80 = 2.37% of the carbon in the coal was burned to CO instead of to CO_2.

We now have the data for calculating the various losses of heat, as follows, for each pound of coal burned:

		Heat-units.	Per cent of Heat-value of the Coal.
20.67 lbs. dry gas × (560° − 60°) × sp. heat 0.24	=	2480.4	17.41
.15 lb. vapor in air × (560° − 60°) × sp. heat .48	=	36.0	0.25
.029 lb. moisture in coal heated from 60° to 212°	=	4.4	0.03
" evaporated from and at 212°; .029 × 966	=	28.0	0.20
" steam (heated from 212° to 560°) × 348 × .48	=	4.8	0.03
.405 lb. H_2O from H in coal × (152 + 966 + 318 × .48)	=	520.4	3.65
.0237 lb. C burned to CO; loss by incomplete combustion, .0237 × (14544 − 4451)	=	239.2	1.68
.02 lb. coal lost in ashes; .02 × 14544	=	290.9	2.04
Radiation and unaccounted for, by difference	=	624.0	4.31
		4228.1	29.68
Utilized in making steam, equivalent evaporation 10.37 lbs. from and at 212° per lb. of coal	=	10,014.9	70.32
		14,243.0	100.00

The heat lost by radiation from the boiler and furnace is not easily determined directly, especially if the boiler is enclosed in brickwork, or is protected by non-conducting covering. It is customary to estimate the heat lost by radiation by difference, that is, to charge radiation with all the heat lost which is not otherwise accounted for.

One method of determining the loss by radiation is to block off a portion of the grate-surface and build a small fire on the remainder, and drive this fire with just enough draught to keep up the steam-pressure and supply the heat lost by radiation without allowing any steam to be discharged, weighing the coal consumed for this purpose during a test of several hours' duration.

Estimates of radiation by difference are apt to be greatly in error, as in this difference are accumulated all the errors of the analyses of the coal and of the gases. An average value of the heat lost by radiation from a boiler set in brickwork is about 4 per cent. When several boilers are in a battery and enclosed in a boiler-house the loss by radiation may be very much less, since much of the heat radiated from the boiler is returned to it in the air supplied to the furnace, which is taken from the boiler-room.

An important source of error in making a "heat balance" such as the one above given, especially when highly bituminous coal is used, may be due to the non-combustion of part of the hydrocarbon gases distilled from the coal immediately after firing, when the temperature of the furnace may be reduced below the point of ignition of the gases. Each pound of hydrogen which escapes burning is equivalent to a loss of heat in the furnace of 62,500 heat-units.

In analyzing the chimney gases by the usual method the percentages of the constituent gases are obtained by volume instead of by weight. To reduce percentages by volume to percentages by weight, multiply the percentage by volume of each gas by its specific gravity as compared with air, and divide each product by the sum of the products.

If O, CO, CO_2, and N represent the per cents by volume of oxygen, carbonic oxide, carbonic acid, and nitrogen, respectively, in the gases of combustion:

$$\left.\begin{array}{c}\text{Lbs. of air required to burn}\\ \text{one pound of carbon}\end{array}\right\} = \frac{3.032\,N}{CO_2 + CO}.$$

$$\text{Ratio of total air to the theoretical requirement} = \frac{N}{N - 3.782\,O}.$$

$$\left.\begin{array}{c}\text{Lbs. of air per pound}\\ \text{of coal}\end{array}\right\} = \left\{\begin{array}{c}\text{Lbs. of air per pound}\\ \text{of carbon}\end{array}\right\} \times \left\{\begin{array}{c}\text{Per cent of carbon}\\ \text{in coal.}\end{array}\right.$$

$$\text{Lbs. dry gas produced per pound of carbon} = \frac{11CO_2 + 8O + 7(CO + N)}{3(CO_2 + CO)}.$$

TESTS OF STEAM-BOILERS.

Boiler-tests at the Centennial Exhibition, Philadelphia, 1876.—(See Reports and Awards Group XX, International Exhibition, Phila., 1876; also, Clark on the Steam-engine, vol. i, page 253.)

Competitive tests were made of fourteen boilers, using good anthracite coal, one boiler, the Galloway, being tested with both anthracite and semi-bituminous coal. Two tests were made with each boiler: one called the capacity trial, to determine the economy and capacity at a rapid rate of driving; and the other called the economy trial, to determine the economy when driven at a rate supposed to be near that of maximum economy and rated capacity. The following table gives the principal results obtained in the economy trial, together with the capacity and economy figures of the capacity trial for comparison.

Name of Boiler.	Economy Tests.										Capacity Tests.	
	Ratio Water-heating Surface to Grate-surface.	Coal burned per sq. ft. Grate per hour.	Per cent Ash and Refuse.	Water evap. from 100° to 70 lbs. p. s.ft. H.S. per hr.	Water evap. from and at 212° p. lb. comb'ble cor. for Quality of Steam.	Temperature in Uptake.	Moisture in Steam.	Superheating of Steam.	Horse-power.		Horse-power.	Water evap. from and at 212° per lb. Combustible.
		lbs.	p.ct	lbs.	lbs.	deg	%	deg	H.P.		H.P.	lbs.
Root	34.6	9.1	10.4	2.25	12.094	393	...	41.4	119.8		148.6	10.441
Firmenich	64.3	12 0	10.4	1.68	11.988	415	. . .	32.6	57.8		68.4	11.064
Lowe	30.6	6.8	11.3	1.87	11.923	333	....	9.4	47.0		69.3	11.163
Smith	45.8	12.1	11.1	2.42	11.906	411	1.3	. . .	99.8		125.0	11.925
Babcock & Wilcox	37.7	10.0	11.0	2.43	11.822	296	2.7	....	135.0		186.6	10.330
Galloway	23.7	9.6	11.1	3.63	11 583	303	...	1.4	103.3		133.8	11.216
Do. semi-bit. coal	23.7	7.9	8.8	3.20	12.125	325	0.3	....	90.9		125.1	11.609
Andrews	15.6	8.0	10.3	2.32	11.039	420	...	71.7	42.6		58.7	9.745
Harrison	27.3	12.4	8.5	2.75	10.930	517	0.9	...	82.4		108.4	9.889
Wiegand	30.7	12.3	9.5	3.30	10.834	524	....	20.5	147.5		162.8	9.145
Anderson	17.5	9.7	9.3	2.64	10.618	417	....	15.7	98.0		132.8	9.568
Kelly	20.9	10.8	9.0	3.82	10.312	....	5.6	...	81.0		99.9	8.397
Exeter	33.5	9.3	11.4	1.38	10.041	430	4.2	....	72.1		108.0	9.974
Pierce	14.0	8.0	11.0	4.44	10.021	374	5.2	....	51.7		67.8	9.865
Rogers & Black	19.0	8.6	9.9	3.43	9.613	572	2.1	....	45.7		67.2	9.429
Averages	....	....	....	2.77	11.123	..		....	85.0		110.8	10.251

The comparison of the economy and capacity trials shows that an average increase in capacity of 30 per cent was attended by a decrease in economy of 8 per cent, but the relation of economy to rate of driving varied greatly in the different boilers. In the Kelly boiler an increase in capacity of 22 per cent was attended by a decrease in economy of over 18 per cent, while the Smith boiler with an increase of 25 per cent in capacity showed a slight increase in economy.

One of the most important lessons gained from the above tests is that there is no necessary relation between the type of a boiler and economy. Of the five boilers that gave the best results, the total range of variation between the highest and lowest of the five being only 2.3%, three were water-tube boilers, one was a horizontal tubular boiler, and the fifth was a combination of the two types. The next boiler on the list, the Galloway, was an internally fired boiler, all of the others being externally fired. The following is a brief description of the principal constructive features of the fourteen boilers:

Root....................	4-in. water-tubes, inclined 20° to horizontal; reversed draught.
Firmenich.............	3-in. water-tubes, nearly vertical; reversed draught.
Lowe....................	Cylindrical shell, multitubular flue.
Smith....................	Cylindrical shell, multitubular flue--water-tubes in side flues.
Babcock & Wilcox....	3½-in. water-tubes, inclined 15° to horizontal; reversed draught.
Galloway......	Cylindrical shell, furnace-tubes and water-tubes.
Andrews....	Square fire-box and double return multitubular flues.
Harrison...............	8 slabs of cast-iron spheres, 8 in. in diameter; reversed draught.
Wiegand...............	4-in. water-tubes, vertical, with internal circulating tubes.
Anderson..............	3-in. flue-tubes, nearly horizontal; return circulation.
Kelly.....................	3-in. water-tubes, slightly inclined; each divided by internal diaphragm to promote circulation.
Exeter..................	27 hollow rectangular cast-iron slabs.
Pierce	Rotating horizontal cylinder, with flue-tubes.
Rogers & Black.......	Vertical cylindrical boiler, with external water-tubes.

Tests of Tubulous Boilers.—The following tables are given by S. H. Leonard, Asst. Engr. U. S. N., in *Jour. Am. Soc. Naval Engrs.* 1890. The tests were made at different times by boards of U. S. Naval Engineers, except the test of the locomotive-torpedo boiler, which was made in England.

No.	Type.	Coal burned per sq. ft. Grate per hour.	Evaporation from and at 212° F.			Weights, lbs.				Air-pressure, in. of Water.	Steam-pressure, lbs.	Coal. A, Anth.; B, Bit.
			Per lb. Com'ble.	Per sq. ft. H. Surface.	Per cu. ft. Space.	E, Empty. S, Steaming Level.	Per I.H.P.	Per sq. ft. H. Surface.	Per lb. Water evaporated.			
1	Belleville ..	12.8	10.42	5.2	6.4	E 40,670 S 42,770	204	53.2	10.1	Nat'l.	111	B.
2	Herreshoff	9.3 25.8	10.23 8.68	3.1 8	9.1 23.8	E 2,945 S 3,050	96 86	14.8	4.8 1.8	Jet. Jet.	120 195	A. A.
3	Towne.....	4.3 24.5	13.4 6.77	2.7 8.2	10 30.4	E 1,380 S 1,640	172 56	21.8	8.1 2.6	Nat'l. 1.14	148 152	A. A.
4	Ward	7.9 15.5 62.5	10.77 10.01 7.01	1.7 3.2 10	5.8 11 34.2	E 1,682 S 1,930	154 82 26	13.2	7.7 4.07 1.3	Nat'l. Jet. Jet.	0 17 161	A. A. B.
5	Scotch.....	24.8 33	9.93 9.06	8.6 12.8	11 16.3	E 18,900 S 30,000	120 80	41.2	4.7 3.1	2.08 4.01	77 78	A. A.
6	Locom'tive torpedo,	93.3 120.8	....	17.1 20.05	30.5 36.2	S 34,990	47.7 33.3	31.3	1.8 1.2	3.13 4.95	125 123	B. B.
7	Ward......	55.04	8.44	9.47	32.1	E 26,533 S 30,474	26	12.3	1.8	2	160	B.
8	Thorny-croft. (U. S.S.Cushing.)	45	. ...	... .	....	E 20,160 S 24,640	*31	10.3	.....	3	245	B.

* Approximate.

Per cent moisture in steam: Belleville, 6.31; Herreshoff (first test), 3.5 Scotch, 1st, 3.44; 2d, 4.29; Ward, 11.6; others not given.

DIMENSIONS OF THE BOILERS.

No.	1	2	3	4	5	6	7	8
Length, ft. and in..	8′ 6″	4′ 9″	2′ 6″	3′ 2″	9′ 0″	16′ 8	10′ 3″*	10′ 0″‡
Width, " " "..	7 0	3 8	2 6	1 7	9 0	6 4	4 6 †	7 0‡
Height, " " "..	11 0	4 0	3 3	7 2	......	7 6	11 8	8 0‡
Space, cu. ft.......	615.5	69.6	20 3	42.7	572.5	630.3	729.3	560‡
Grate-area, sq. ft..	34.17	9	4.25	3.68	31.16	28	66.5	38.3
Heating-surface, sq. ft....	804	205	75	146	727	1116	2490	2375
Ratio H.S. ÷ G ...	23.5	22	17.6	39.5	23.3	39.8	37.4	62

* Diameter. † Diam. of drum. ‡ Approximate.

The weight per I.H.P. is estimated on a basis of 20 lbs. of water per hour for all cases expecting the Scotch boiler, where 25 lbs. have been used, as this boiler was limited to 80 lbs. pressure of steam.

The following approximation is made from the large table, on the assumption that the evaporation varies directly as the combustion, and 25 lbs. of coal per square foot of grate per hour used as the unit.

Type of Boiler.	Combustion.	Evaporation per cu. ft. of Space.	Weight per I.H.P.	Weight per sq. ft. Heating-surface.	Weight per lb. Water Evaporated.
Belleville	0.50	0.50	2.02	2.10	2.50
Herreshoff..............	1.00	0.95	0.72	0.60	0.90
Towne.......	1.00	1.20	1.12	0.87	1.30
Scotch..................	1.00	0.44	2.40	1.64	2.30
Locomotive	3.90	0.31	3.70	1.25	3.50
Ward	2.20	0.58	1.27	0.50	1.53

The Belleville boiler has no practical advantage over the Scotch either in space occupied or weight. All the other tubulous boilers given greatly exceed the Scotch in these advantages of weight and space.

Some High Rates of Evaporation.—*Eng'g*, May 9, 1884, p. 415.

	Locomotive.		Torpedo-boat.	
Water evap. per sq. ft. H.S. per hour. ...	12.57	13.73	12.54	20.74
" " " lb. fuel from and at 212°.	8.22	8.94	8.37	7.04
Thermal units transf'd per sq. ft. of H.S.	12,142	13,263	12,113	20,034
Efficiency	.586	.637	.542	.468

It is doubtful if these figures were corrected for priming.

Economy Effected by Heating the Air Supplied to Boiler-furnaces. (Clark, S. E.)—Meunier and Scheurer-Kestner obtained about 7% greater evaporative efficiency in summer than in winter, from the same boilers under like conditions,—an excess which had been explained by the difference of loss by radiation and conduction. But Mr. Poupardin, surmising that the gain might be due in some degree also to the greater temperature of the air in summer, made comparative trials with two groups of three boilers, each working one week with the heated air, and the next week with cold air. The following were the several efficiencies:

FIRST TRIALS: THREE BOILERS; RONCHAMP COAL.

	Water per lb. of Coal.	Water per lb. of Combustible.
With heated air (128° F.)	7.77 lbs.	8.95 lbs.
With cold air (69°.8)..................	7.33 "	8.63 "
Difference in favor of heated air	0.44 "	0.32 "

SECOND TRIALS: SAME COAL; THREE OTHER BOILERS.

With heated air (120°.4 F.)...........	8.70 lbs.	10.08 lbs.
With cold air (75°.2)..................	8.09 "	9.34 "
Difference in favor of heated air.....	0.61 "	0.64 "

These results show economies in favor of heating the air of 6% and 7½%.

Mr. Poupardin believes that the gain in efficiency is due chiefly to the better combustion of the gases with heated air. It was observed that with heated air the flames were much shorter and whiter, and that there was notably less smoke from the chimney.

An extensive series of experiments was made by J. C. Hoadley (Trans. A. S. M. E., vol. vi., 676) on a "Warm-blast Apparatus," for utilizing the heat of the waste gases in heating the air supplied to the furnace. The apparatus, as applied to an ordinary horizontal tubular boiler 60 in. diameter, 21 feet long, with 65 3½-inch tubes, consisted of 240 2-inch tubes, 18 feet long, through which the hot gases passed while the air circulated around them. The net saving of fuel effected by the warm blast was from 10.7% to 15.5% of the fuel used with cold blast. The comparative temperatures averaged as follows, in degrees F.:

	Cold-blast Boiler.	Warm-blast Boiler.	Difference.
In heat of fire	2498	2798	300
At bridge wall	1340	1600	260
In smoke box	378	375	2
Air admitted to furnace	32	332	300
Steam and water in boiler	300	300	0
Gases escaping to chimney	373	162	211
External air	32	32	0

With anthracite coal the evaporation from and at 212° per lb. combustible was, for the cold-blast boiler, days 10.85 lbs., days and nights 10.51; and for the warm-blast boiler, days 11.83, days and nights 11.08.

Results of Tests of Heine Water-tube Boilers with Different Coals.

(Communicated by E. D. Meier, C.E., 1894.)

Number	1	2	3	4	5	6	7	8
Kind of Coal.	Cumberland, Semi-bitum.	2d Pool, Youghiogheny.		Turkey Hill, Ill.	Carbon Hill, Wash.	Hocking Val., Ohio.	Gillespie, Lump, Ill.	Collinsville, Ill.
Per cent ash	5.1	4.89	......	11.6	16.1	11.5	21.8	12.8
Heating-surface, sq. ft.	2900	2040	2040	2300	1260	3780	1168	2770
Grate-surface, sq. ft.	54	44 8	44.8	50	21	73.8	27.9	50
Ratio H.S. to G.S.	53.7	45.5	45.5	46	60	50.9	41.9	55.4
Coal per sq. ft. G. per hr.	24.7	23.5	22.7	35	33.7	26.2	27.7	36
Water per sq. ft. H.S. per hr. from and at 212°	5.03	5.14	5.24	5.56	4.26	4.28	4.86	5.08
Water evap. from and at 212° per lb. coal	10.91	9.94	10.51	7.31	7.59	8.88	7.36	7.81
Per lb. combustible	11.50	10.48	...	8.27	9.05	9.41	9.41	8.96
Temp. of chimney gases	530°	.....	400	567	571	...	609	707
Calorific value of fuel	13,800	12,936	12,936	10,487	11,785	11,610	9,739	10,359
Efficiency of boiler per c.	77.0	74.3	78.5	67.2	62 5	69.3	78 0	72 6

Tests Nos. 7 and 8 were made with the Hawley Down-draught Furnace, the others with ordinary furnaces.

These tests confirm the statement already made as to the difficulty of obtaining, with ordinary grate-furnaces, as high a percentage of the calorific value of the fuel with the Western as with the Eastern coals.

Test No 3, 78.5% efficiency, is remarkably good for Pittsburgh (Youghiogheny) coal. If the Washington coal had given equal efficiency, the saving of fuel would be $\frac{78.5 - 62.5}{78.5} = 20.2\%$. The results of tests Nos. 7 and 8 indicate that the downward-draught furnace is well adapted for burning Illinois coals.

Maximum Boiler Efficiency with Cumberland Coal.—About 12.5 lbs. of water per lb. combustible from and at 212° is about the highest evaporation that can be obtained from the best steam fuels in the United States, such as Cumberland, Pocahontas, and Clearfield. In exceptional cases 13 lbs. has been reached, and one test is on record (F. W. Dean, *Eng'g News*, Feb. 1, 1894) giving 13.23 lbs. The boiler was internally fired, of the Belpaire type, 82 inches diameter, 31 feet long, with 160 3-inch tubes 12½ feet long. Heating-surface, 1998 square feet; grate-surface, 45 square feet, reduced during the test to 30½ square feet. Double furnace, with fire-brick arches and a long combustion-chamber. Feed-water heater in smoke-box. The following are the principal results:

	1st Test.	2d Test.
Dry coal burned per sq. ft. of grate per hour, lbs.......	8.85	16.06
Water evap. per sq. ft. of heating-surface per hour, lbs	1.63	3.00
Water evap. from and at 212° per lb. combustible, including feed-water heater...........................	13.17	13.23
Water evaporated, excluding feed-water heater.........	12.88	12.90
Temperature of gases after leaving heater, F...........	360°	460°

BOILERS USING WASTE GASES.

Proportioning Boilers for Blast-Furnaces.—(F. W. Gordon, Trans. A. I. M. E., vol. xii., 1883.)

Mr. Gordon's recommendation for proportioning boilers when properly set for burning blast-furnace gas is, for coke practice, 30 sq. ft. of heating-surface per ton of iron per 24 hours, which the furnace is expected to make, calculating the heating-surface thus: For double-flued boilers, all shell-surface exposed to the gases, and half the flue-surface; for the French type, all the exposed surface of the upper boiler and half the lower boiler-surface; for cylindrical boilers, not more than 60 ft. long, all the heating-surface.

To the above must be added a battery for relay in case of cleaning, repairs, etc., and more than one battery extra in large plants, when the water carries much lime.

For anthracite practice add 50% to above calculations. For charcoal practice deduct 20%.

In a letter to the author in May, 1894, Mr. Gordon says that the blast-furnace practice at the time when his article (from which the above extract is taken) was written was very different from that existing at the present time; besides, more economical engines are being introduced, so that less than 30 sq. ft. of boiler-surface per ton of iron made in 24 hours may now be adopted. He says further: Blast-furnace gases are seldom used for other than furnace requirements, which of course is throwing away good fuel. In this case a furnace in an ordinary good condition, and a condition where it can take its maximum of blast, which is in the neighborhood of 200 to 225 cubic ft., atmospheric measurement, per sq. ft. of sectional area of hearth, will generate the necessary H.P. with very small heating-surface, owing to the high heat of the escaping gases from the boilers, which frequently is 1000 degrees.

A furnace making 200 tons of iron a day will consume about 900 H.P. in blowing the engine. About a pound of fuel is required in the furnace per pound of pig metal.

In practice it requires 70 cu ft. of air-piston displacement per lb. of fuel consumed, or 22,400 cu. ft. per minute for 200 tons of metal in 1400 working minutes per day, at, say, 10 lbs. discharge-pressure. This is equal to 9¼ lbs. M.E.P. on the steam-piston of equal area to the blast-piston, or 900 I.H.P. To this add 20% for hoisting, pumping and other purposes for which steam is employed around blast-furnaces, and we have 1100 H.P., or say 5½ H.P. per ton of iron per day. Dividing this into 30 gives approximately 5½ sq. ft. of heating-surface of boiler per H.P.

Water-tube Boilers using Blast-furnace Gases.—D. S. Jacobus (Trans. A. I. M. E., xvii. 50) reports a test of a water-tube boiler using blast-furnace gas as fuel. The heating-surface was 2535 sq. ft. It developed 328 H.P. (Centennial standard), or 5.01 lbs. of water from and at 212° per sq. ft. of heating-surface per hour. Some of the principal data obtained were as follows: Calorific value of 1 lb. of the gas, 1413 B.T.U., including the effect of its initial temperature, which was 650° F. Amount of air used to burn 1 lb. of the gas = 0.9 lb. Chimney draught, 1½ in. of water. Area of gas inlet, 300 sq. in.; of air inlet, 100 sq. in. Temperature of the chimney gases, 775° F. Efficiency of the boiler calculated from the temperatures and analyses of the gases at exit and entrance, 61%. The average analyses were as follows, hydrocarbons being included in the nitrogen:

	By Weight.		By Volume.	
	At Entrance.	At Exit.	At Entrance.	At Exit.
CO_2	10.69	26.37	7.08	18.64
O	.11	3.05	.10	2.96
CO	26.71	1.78	27.80	1.98
Nitrogen	62.48	68.80	65.02	76.42
C in CO_2	2.92	7.19		
C in CO	11.45	.76		
Total C	14.37	7.95		

Steam-boilers Fired with Waste Gases from Puddling and Heating Furnaces.—The *Iron Age*, April 6, 1893, contains a report of a number of tests of steam-boilers utilizing the waste heat from puddling and heating furnaces in rolling-mills. The following principal data are selected: In Nos. 1, 2, and 4 the boiler is a Babcock & Wilcox water-tube boiler, and in No. 3 it is a plain cylinder boiler, 42 in. diam. and 26 ft. long. No. 4 boiler was connected with a heating-furnace, the others with puddling furnaces.

	No. 1.	No. 2.	No. 3.	No. 4.
Heating-surface, sq. ft.	1026	1196	143	1380
Grate-surface, sq. ft.	19.9	18.6	13.6	16.7
Ratio H.S. to G.S.	52	87.2	10.5	82.8
Water evap. per hour, lbs.	3358	2159	1812	3055
" " per sq. ft. H.S. per hr., lbs.	3.3	1.8	12.7	2.2
" " per lb. coal from and at 212°.	5.9	6.24	3.76	6.34
" " " " comb. " " " "	....	7.20	4.31	8.34

In No. 2, 1.38 lbs. of iron were puddled per lb. of coal.
In No. 3, 1.14 lbs. of iron were puddled per lb. of coal.
No. 3 shows that an insufficient amount of heating-surface was provided for the amount of waste heat available.

RULES FOR CONDUCTING BOILER-TESTS.

Code of 1899.

(Reported by the Committee on Boiler Trials, Am. Soc. M. E.*)

I. *Determine at the outset* the specific object of the proposed trial, whether it be to ascertain the capacity of the boiler, its efficiency as a steam-generator, its efficiency and its defects under usual working conditions, the economy of some particular kind of fuel, or the effect of changes of design, proportion, or operation; and prepare for the trial accordingly.

II. *Examine the boiler*, both outside and inside; ascertain the dimensions of grates, heating surfaces, and all important parts; and make a full record, describing the same, and illustrating special features by sketches.

III. *Notice the general condition* of the boiler and its equipment, and record such facts in relation thereto as bear upon the objects in view.

If the object of the trial is to ascertain the maximum economy or capacity of the boiler as a steam-generator, the boiler and all its appurtenances should be put in first-class condition. Clean the heating surface inside and outside, remove clinkers from the grates and from the sides of the furnace. Remove all dust, soot, and ashes from the chambers, smoke-connections, and flues. Close air-leaks in the masonry and poorly fitted cleaning-doors. See that the damper will open wide and close tight. Test for air-leaks by firing a few shovels of smoky fuel and immediately closing the damper, observing the escape of smoke through the crevices, or by passing the flame of a candle over cracks in the brickwork.

* The code is here slightly abridged. The complete report of the Committee may be obtained in pamphlet form from the Secretary of the American Society of Mechanical Engineers, 12 West 31st St., New York.

IV. *Determine the character of the coal* to be used. For tests of the efficiency or capacity of the boiler for comparison with other boilers the coal should, if possible, be of some kind which is commercially regarded as a standard. For New England and that portion of the country east of the Allegheny Mountains, good anthracite egg coal, containing not over 10 per cent. of ash, and semi-bituminous Clearfield (Pa.), Cumberland (Md.), and Pocahontas (Va.) coals are thus regarded. West of the Allegheny Mountains, Pocahontas (Va.) and New River (W. Va.) semi-bituminous, and Youghiogheny or Pittsburg bituminous coals are recognized as standards.*

For tests made to determine the performance of a boiler with a particular kind of coal, such as may be specified in a contract for the sale of a boiler, the coal used should not be higher in ash and in moisture than that specified, since increase in ash and moisture above a stated amount is apt to cause a falling off of both capacity and economy in greater proportion than the proportion of such increase.

V. *Establish the correctness of all apparatus* used in the test for weighing and measuring. These are:

1. Scales for weighing coal, ashes, and water.
2. Tanks or water-meters for measuring water. Water-meters, as a rule, should only be used as a check on other measurements. For accurate work the water should be weighed or measured in a tank.
3. Thermometers and pyrometers for taking temperatures of air, steam, feed-water, waste gases, etc.
4. Pressure-gauges, draught-gauges, etc.

VI. *See that the boiler is thoroughly heated* before the trial to its usual working temperature. If the boiler is new and of a form provided with a brick setting, it should be in regular use at least a week before the trial, so as to dry and heat the walls. If it has been laid off and become cold, it should be worked before the trial until the walls are well heated.

VII. *The boiler and connections* should be proved to be free from leaks before beginning a test, and all water connections, including blow and extra feed-pipes, should be disconnected, stopped with blank flanges, or bled through special openings beyond the valves, except the particular pipe through which water is to be fed to the boiler during the trial. During the test the blow-off and feed pipes should remain exposed to view.

If an injector is used, it should receive steam directly through a felted pipe from the boiler being tested.†

If the water is metered after it passes the injector, its temperature should be taken at the point where it leaves the injector. If the quantity is determined before it goes to the injector, the temperature should be determined on the suction side of the injector, and if no change of temperature occurs other than that due to the injector, the temperature thus determined is properly that of the feed-water. When the temperature changes between the injector and the boiler, as by the use of a heater or by radiation, the temperature at which the water enters and leaves the injector and that at which it enters the boiler should all be taken. In that case the weight to be used is that of the water leaving the injector, computed from the heat units if not directly measured; and the temperature, that of the water entering the boiler.

Let w = weight of water entering the injector;
x = " " steam " " " ;
h_1 = heat-units per pound of water entering injector;
h_2 = " " " " " steam " " ;
h_3 = " " " " " water leaving " .

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* These coals are selected because they are about the only coals which possess the essentials of excellence of quality, adaptability to various kinds of furnaces, grates, boilers, and methods of firing, and wide distribution and general accessibility in the markets.

† In feeding a boiler undergoing test with an injector taking steam from another boiler, or from the main steam-pipe from several boilers, the evaporative results may be modified by a difference in the quality of the steam from such source compared with that supplied by the boiler being tested, and in some cases the connection to the injector may act as a drip for the main steam-pipe. If it is known that the steam from the main pipe is of the same pressure and quality as that furnished by the boiler undergoing the test, the steam may be taken from such main pipe.

Then $w + x =$ weight of water leaving injector,

$$x = w\frac{h_3 - h_1}{h_2 - h_3}.$$

See that the steam-main is so arranged that water of condensation cannot run back into the boiler.

VIII. *Duration of the Test.*—For tests made to ascertain either the maximum economy or the maximum capacity of a boiler, irrespective of the particular class of service for which it is regularly used, the duration should be at least ten hours of continuous running. If the rate of combustion exceeds 25 pounds of coal per square foot of grate-surface per hour, it may be stopped when a total of 250 pounds of coal has been burned per square foot of grate.

IX. *Starting and Stopping a Test.*—The conditions of the boiler and furnace in all respects should be, as nearly as possible, the same at the end as at the beginning of the test. The steam-pressure should be the same; the water-level the same; the fire upon the grates should be the same in quantity and condition; and the walls, flues, etc., should be of the same temperature. Two methods of obtaining the desired equality of conditions of the fire may be used, viz., those which were called in the Code of 1885 "the standard method" and "the alternate method," the latter being employed where it is inconvenient to make use of the standard method.*

X. *Standard Method of Starting and Stopping a Test.*—Steam being raised to the working pressure, remove rapidly all the fire from the grate, close the damper, clean the ash-pit, and as quickly as possible start a new fire with weighed wood and coal, noting the time and the water-level† while the water is in a quiescent state, just before lighting the fire.

At the end of the test remove the whole fire, which has been burned low, clean the grates and ash-pit, and note the water-level when the water is in a quiescent state, and record the time of hauling the fire. The water-level should be as nearly as possible the same as at the beginning of the test. If it is not the same, a correction should be made by computation, and not by operating the pump after the test is completed.

XI. *Alternate Method of Starting and Stopping a Test.*—The boiler being thoroughly heated by a preliminary run, the fires are to be burned low and well cleaned. Note the amount of coal left on the grate as nearly as it can be estimated; note the pressure of steam and the water-level. Note the time, and record it as the starting-time. Fresh coal which has been weighed should now be fired. The ash-pits should be thoroughly cleaned at once after starting. Before the end of the test the fires should be burned low, just as before the start, and the fires cleaned in such a manner as to leave a bed of coal on the grates of the same depth, and in the same condition, as at the start. When this stage is reached, note the time and record it as the stopping-time. The water-level and steam-pressures should previously be brought as nearly as possible to the same point as at the start. If the water-level is not the same as at the start, a correction should be made by computation, and not by operating the pump after the test is completed.

XII. *Uniformity of Conditions.*—In all trials made to ascertain maximum economy or capacity the conditions should be maintained uniformly constant. Arrangements should be made to dispose of the steam so that the rate of evaporation may be kept the same from beginning to end.

XIII. *Keeping the Records.*—Take note of every event connected with the progress of the trial, however unimportant it may appear. Record the time of every occurrence and the time of taking every weight and every observation.

The coal should be weighed and delivered to the fireman in equal proportions, each sufficient for not more than one hour's run, and a fresh portion

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*The Committee concludes that it is best to retain the designations "standard" and "alternate," since they have become widely known and established in the minds of engineers and in the reprints in the Code of 1885. Many engineers prefer the "alternate" to the "standard" method on account of its being less liable to error due to cooling of the boiler at the beginning and end of a test.

†The gauge-glass should not be blown out within an hour before the water-level is taken at the beginning and end of a test, otherwise an error in the reading of the water-level may be caused by a change in the temperature and density to the water in the pipe leading from the bottom of the glass into the boiler.

should not be delivered until the previous one has all been fired. The time required to consume each portion should be noted, the time being recorded at the instant of firing the last of each portion. It is desirable that at the same time the amount of water fed into the boiler should be accurately noted and recorded, including the height of the water in the boiler, and the average pressure of steam and temperature of feed during the time. By thus recording the amount of water evaporated by successive portions of coal, the test may be divided into several periods if desired, and the degree of uniformity of combustion, evaporation, and economy analyzed for each period. In addition to these records of the coal and the feed-water, half-hourly observations should be made of the temperature of the feed-water, of the flue-gases, of the external air in the boiler-room, of the temperature of the furnace when a furnace-pyrometer is used, also of the pressure of steam, and of the readings of the instruments for determining the moisture in the steam. A log should be kept on properly prepared blanks containing columns for record of the various observations.

XIV. *Quality of Steam.*—The percentage of moisture in the steam should be determined by the use of either a throttling or a separating steam-calorimeter. The sampling-nozzle should be placed in the vertical steam-pipe rising from the boiler. It should be made of ½-inch pipe, and should extend across the diameter of the steam-pipe to within half an inch of the opposite side, being closed at the end and perforated with not less than twenty ⅛-inch holes equally distributed along and around its cylindrical surface, but none of these holes should be nearer than ½ inch to the inner side of the steam-pipe. The calorimeter and the pipe leading to it should be well covered with felting. Whenever the indications of the throttling or separating calorimeter show that the percentage of moisture is irregular, or occasionally in excess of three per cent., the results should be checked by a steam-separator placed in the steam-pipe as close to the boiler as convenient, with a calorimeter in the steam-pipe just beyond the outlet from the separator. The drip from the separator should be caught and weighed, and the percentage of moisture computed therefrom added to that shown by the calorimeter.

Superheating should be determined by means of a thermometer placed in a mercury-well inserted in the steam-pipe. The degree of superheating should be taken as the difference between the reading of the thermometer for superheated steam and the readings of the same thermometer for saturated steam at the same pressure as determined by a special experiment, and not by reference to steam-tables.

XV. *Sampling the Coal and Determining its Moisture.*—As each barrow-load or fresh portion of coal is taken from the coal-pile, a representative shovelful is selected from it and placed in a barrel or box in a cool place and kept until the end of the trial. The samples are then mixed and broken into pieces not exceeding one inch in diameter, and reduced by the process of repeated quartering and crushing until a final sample weighing about five pounds is obtained, and the size of the larger pieces is such that they will pass through a sieve with ¼-inch meshes. From this sample two one-quart, air-tight glass preserving-jars, or other air-tight vessels which will prevent the escape of moisture from the sample, are to be promptly filled, and these samples are to be kept for subsequent determinations of moisture and of heating value and for chemical analyses. During the process of quartering, when the sample has been reduced to about 100 pounds, a quarter to a half of it may be taken for an approximate determination of moisture. This may be made by placing it in a shallow iron pan, not over three inches deep, carefully weighing it, and setting the pan in the hottest place that can be found on the brickwork of the boiler-setting or flues, keeping it there for at least 12 hours, and then weighing it. The determination of moisture thus made is believed to be approximately accurate for anthracite and semi-bituminous coals, and also for Pittsburg or Youghiogheny coal; but it cannot be relied upon for coals mined west of Pittsburg, or for other coals containing inherent moisture. For these latter coals it is important that a more accurate method be adopted. The method recommended by the Committee for all accurate tests, whatever the character of the coal, is described as follows:

Take one of the samples contained in the glass jars, and subject it to a thorough air-drying, by spreading it in a thin layer and exposing it for several hours to the atmosphere of a warm room, weighing it before and after, thereby determining the quantity of surface moisture it contains.

Then crush the whole of it by running it through an ordinary coffee-mill adjusted so as to produce somewhat coarse grains (less than $\frac{1}{16}$ inch), thoroughly mix the crushed sample, select from it a portion of from 10 to 50 grams, weigh it in a balance which will easily show a variation as small as 1 part in 1000, and dry it in an air- or sand-bath at a temperature between 240 and 280 degrees Fahr. for one hour. Weigh it and record the loss, then heat and weigh it again repeatedly, at intervals of an hour or less, until the minimum weight has been reached and the weight begins to increase by oxidation of a portion of the coal. The difference between the original and the minimum weight is taken as the moisture in the air-dried coal. This moisture test should preferably be made on duplicate samples, and the results should agree within 0.3 to 0.4 of one per cent., the mean of the two determinations being taken as the correct result. The sum of the percentage of moisture thus found and the percentage of surface moisture previously determined is the total moisture.

XVI. *Treatment of Ashes and Refuse.*—The ashes and refuse are to be weighed in a dry state. If it is found desirable to show the principal characteristics of the ash, a sample should be subjected to a proximate analysis and the actual amount of incombustible material determined. For elaborate trials a complete analysis of the ash and refuse should be made.

XVII. *Calorific Tests and Analysis of Coal.*—The quality of the fuel should be determined either by heat test or by analysis, or by both.

The rational method of determining the total heat of combustion is to burn the sample of coal in an atmosphere of oxygen gas, the coal to be sampled as directed in Article XV of this code.

The chemical analysis of the coal should be made only by an expert chemist. The total heat of combustion computed from the results of the ultimate analysis may be obtained by the use of Dulong's formula (with constants modified by recent determinations), viz.,

$$14{,}600\ C + 62{,}000\left(H - \frac{O}{8}\right) + 4000\ S,$$

in which C, H, O, and S refer to the proportions of carbon, hydrogen, oxygen, and sulphur respectively, as determined by the ultimate analysis.*

It is desirable that a proximate analysis should be made, thereby determining the relative proportions of volatile matter and fixed carbon. These proportions furnish an indication of the leading characteristics of the fuel, and serve to fix the class to which it belongs.

XVIII. *Analysis of Flue-gases.*—The analysis of the flue-gases is an especially valuable method of determining the relative value of different methods of firing or of different kinds of furnaces. In making these analyses great care should be taken to procure average samples, since the composition is apt to vary at different points of the flue. The composition is also apt to vary from minute to minute, and for this reason the drawings of gas should last a considerable period of time. Where complete determinations are desired, the analyses should be intrusted to an expert chemist. For approximate determinations the Orsat or the Hempel apparatus may be used by the engineer.

For the continuous indication of the amount of carbonic acid present in the flue-gases an instrument may be employed which shows the weight of CO_2 in the sample of gas passing through it.

XIX. *Smoke Observations.*—It is desirable to have a uniform system of determining and recording the quantity of smoke produced where bituminous coal is used. The system commonly employed is to express the degree of smokiness by means of percentages dependent upon the judgment of the observer. The actual measurement of a sample of soot and smoke by some form of meter is to be preferred.

XX. *Miscellaneous.*—In tests for purposes of scientific research, in which the determination of all the variables entering into the test is desired, certain observations should be made which are in general unnecessary for ordinary tests. As these determinations are rarely undertaken, it is not deemed advisable to give directions for making them.

XXI. *Calculations of Efficiency.*—Two methods of defining and calculating the efficiency of a boiler are recommended. They are:

* Favre and Silbermann give 14,544 B.T.U. per pound carbon; Berthelot, 14,647 B.T.U. Favre and Silbermann give 62,032 B.T.U. per pound hydrogen; Thomsen, 61,816 B.T.U.

1. Efficiency of the boiler = $\dfrac{\text{Heat absorbed per lb. combustible}}{\text{Calorific value of 1 lb. combustible}}$.

2. Efficiency of the boiler and grate = $\dfrac{\text{Heat absorbed per lb. coal}}{\text{Calorific value of 1 lb. coal}}$.

The first of these is sometimes called the efficiency based on combustible, and the second the efficiency based on coal. The first is recommended as a standard of comparison for all tests, and this is the one which is understood to be referred to when the word "efficiency" alone is used without qualification. The second, however, should be included in a report of a test, together with the first, whenever the object of the test is to determine the efficiency of the boiler and furnace together with the grate (or mechanical stoker), or to compare different furnaces, grates, fuels, or methods of firing.

The heat absorbed per pound of combustible (or per pound coal) is to be calculated by multiplying the equivalent evaporation from and at 212 degrees per pound combustible (or coal) by 965.7.

XXII. *The Heat Balance.*—An approximate "heat balance," may be included in the report of a test when analyses of the fuel and of the chimney-gases have been made. It should be reported in the following form:

HEAT BALANCE, OR DISTRIBUTION OF THE HEATING VALUE OF THE COMBUSTIBLE.

Total Heat Value of 1 lb. of Combustible............ B. T. U.

	B. T. U.	Per Cent.
1. Heat absorbed by the boiler = evaporation from and at 212 degrees per pound of combustible × 965.7		
2. Loss due to moisture in coal = per cent of moisture referred to combustible ÷ 100 × $[(212 - t) + 966 + 0.48(T - 212)]$ (t = temperature of air in the boiler-room, T = that of the flue-gases)..........		
3. Loss due to moisture formed by the burning of hydrogen = per cent of hydrogen to combustible ÷ 100 × 9 × $[(212 - t) + 966 + 0.48(T - 212)]$.....................		
4.* Loss due to heat carried away in the dry chimney-gases = weight of gas per pound of combustible × 0.24 × $(T - t)$		
5.† Loss due to incomplete combustion of carbon $= \dfrac{CO}{CO_2 + CO} \times \dfrac{\text{per cent. C in combustible}}{100} \times 10{,}150$....		
6. Loss due to unconsumed hydrogen and hydrocarbons, to heating the moisture in the air, to radiation, and unaccounted for. (Some of these losses may be separately itemized if data are obtained from which they may be calculated).............................		
Totals		100.00

* The weight of gas per pound of carbon burned may be calculated from the gas analyses as follows:

Dry gas per pound carbon = $\dfrac{11CO_2 + 8O + 7(CO + N)}{3(CO_2 + CO)}$, in which CO_2, CO, O, and N are the percentages by volume of the several gases. As the sampling and analyses of the gases in the present state of the art are liable to considerable errors, the result of this calculation is usually only an approximate one. The heat balance itself is also only approximate for this reason, as well as for the fact that it is not possible to determine accurately the percentage of unburned hydrogen or hydrocarbons in the flue-gases.

The weight of dry gas per pound of combustible is found by multiplying the dry gas per pound of carbon by the percentage of carbon in the combustible, and dividing by 100.

† CO_2 and CO are respectively the percentage by volume of carbonic acid and carbonic oxide in the flue-gases. The quantity 10,150 = number of heat-units generated by burning to carbonic acid one pound of carbon contained in carbonic oxide.

XXIII. *Report of the Trial.*—The data and results should be reported in the manner given in either one of the two following tables [only the "Short Form" of table is given here], omitting lines where the tests have not been made as elaborately as provided for in such tables. Additional lines may be added for data relating to the specific object of the test. The Short Form of Report, Table No. 2, is recommended for commercial tests and as a convenient form of abridging the longer form for publication when saving of space is desirable. For elaborate trials it is recommended that the full log of the trial be shown graphically, by means of a chart.

TABLE NO. 2.

DATA AND RESULTS OF EVAPORATIVE TEST,

Arranged in accordance with the Short Form advised by the Boiler Test Committee of the American Society of Mechanical Engineers. Code of 1899.

Made by....................on....... ....boiler, at....... ....to determine ....
Kind of fuel.......................... ....
Kind of furnace.......................... ....

Method of starting and stopping the test ("standard" or "alternate," Arts. X and XI, Code)......		
Grate surface............................	sq. ft.	
Water-heating surface..................	"	
Superheating surface............	"	
TOTAL QUANTITIES.		
1. Date of trial..........		
2. Duration of trial	hours	
3. Weight of coal as fired *...	lbs.	
4. Percentage of moisture in coal †	per cent.	
5. Total weight of dry coal consumed.......... ...	lbs.	
6. Total ash and refuse..........................	"	
7. Percentage of ash and refuse in dry coal.... ...	per cent.	
8. Total weight of water fed to the boiler ‡	lbs.	
9. Water actually evaporated, corrected for moisture or superheat in steam..................	"	
9a. Factor of evaporation §....................		
10. Equivalent water evaporated into dry steam from and at 212 degrees.‖ (Item 9 × Item 9a.)	"	
HOURLY QUANTITIES.		
11. Dry coal consumed per hour...	"	
12. Dry coal per square foot of grate surface per hour..............	"	
13. Water evaporated per hour corrected for quality of steam	"	
14. Equivalent evaporation per hour from and at 212 degrees ‖	"	
15. Equivalent evaporation per hour from and at 212 degrees per square foot of water-heating surface ‖.......	"	

* Including equivalent of wood used in lighting the fire, not including unburnt coal withdrawn from furnace at times of cleaning and at end of test. One pound of wood is taken to be equal to 0.4 pound of coal, or, in case greater accuracy is desired, as having a heat value equivalent to the evaporation of 6 pounds of water from and at 212 degrees per pound. ($6 \times 965.7 = 5794$ B. T. U.) The term "as fired" means in its actual condition, including moisture.

† This is the total moisture in the coal as found by drying it artificially, as described in Art. XV of Code.

‡ Corrected for inequality of water-level and of steam-pressure at beginning and end of test.

§ Factor of evaporation $= \frac{H - h}{965.7}$, in which H and h are respectively the total heat in steam of the average observed pressure, and in water of the average observed temperature of the feed.

‖ The symbol "U. E.," meaning "units of evaporation," may be con-

AVERAGE PRESSURES, TEMPERATURES, ETC.	
16. Steam pressure by gauge	lbs. per sq. in.
17. Temperature of feed-water entering boiler	deg.
18. Temperature of escaping gases from boiler	"
19. Force of draft between damper and boiler	ins. of water
20. Percentage of moisture in steam, or number of degrees of superheating	per cent. or deg.
HORSE-POWER.	
21. Horse-power developed. (Item 14 ÷ 34½.)¶	H.P.
22. Builders' rated horse-power	"
23. Percentage of builders' rated horse-power developed	per cent.
ECONOMIC RESULTS.	
24. Water apparently evaporated under actual conditions per pound of coal as fired. (Item 8 ÷ Item 3.)	lbs.
25. Equivalent evaporation from and at 212 degrees per pound of coal as fired.‖ (Item 9 ÷ Item 3.)	"
26. Equivalent evaporation from and at 212 degrees per pound of dry coal.‖ (Item 9 ÷ Item 5.)	"
27. Equivalent evaporation from and at 212 degrees per pound of combustible. [Item 9 ÷ (Item 5 − Item 6).]	"
(If Items 25, 26, and 27 are not corrected for quality of steam, the fact should be stated.)	
EFFICIENCY.	
28. Calorific value of the dry coal per pound	B. T. U.
29 Calorific value of the combustible per pound	"
30. Efficiency of boiler (based on combustible)**	per cent.
31. Efficiency of boiler, including grate (based on dry coal)	"
COST OF EVAPORATION.	
32. Cost of coal per ton of —— lbs. delivered in boiler-room	$
33. Cost of coal required for evaporating 1000 pounds of water from and at 212 degrees	$

veniently substituted for the expression "Equivalent water evaporated into dry steam from and at 212 degrees," its definition being given in a foot-note.

¶ Held to be the equivalent of 30 lbs. of water evaporated from 100 degrees Fahr. into dry steam at 70 lbs. gauge-pressure.

** In all cases where the word "combustible" is used, it means the coal without moisture and ash, but including all other constituents. It is the same as what is called in Europe "coal-dry and free from ash."

Factors of Evaporation.—The table on the following pages was originally published by the author in Trans. A. S. M. E., vol. vi., 1884, under the title, Tables for Facilitating Calculations of Boiler-tests. The table gives the factors for every 3° of temperature of feed-water from 32° to 212° F., and for every two pounds pressure of steam within the limits of ordinary working steam-pressures.

The difference in the factor corresponding to a difference of 3° temperature of feed is always either .0031 or .0032. For interpolation to find a factor for a feed-water temperature between 32° and 212°, not given in the table, take the factor for the nearest temperature and add or subtract, as the case may be, .0010 if the difference is .0031, and .0011 if the difference is .0032. As in nearly all cases a factor of evaporation to three decimal places is accurate enough, any error which may be made in the fourth decimal place by interpolation is of no practical importance.

The tables used in calculating these factors of evaporation are those given in Charles T. Porter's Treatise on the Richards' Steam-engine Indicator. The formula is Factor $= \frac{H - h}{965.7}$, in which H is the total heat of steam at the observed pressure, and h the total heat of feed-water of the observed temperature.

Gauge-pressures lbs. / Absolute pressures	0 + / 15	10 + / 25	20 + / 35	30 + / 45	40 + / 55	45 + / 60	50 + / 65	52 + / 67	54 + / 69	56 + / 71
Feed-water Temperature.	Factors of Evaporation.									
212° F.	1.0003	1.0088	1.0149	1.0197	1.0237	1.0254	1.0271	1.0277	1.0283	1.0290
209	35	1.0120	80	1.0228	68	86	1.0302	1.0309	1.0315	1.0321
206	66	51	1.0212	60	99	1.0317	34	40	46	52
203	98	83	43	91	1.0331	49	65	72	78	84
200	1.0129	1.0214	75	1.0323	62	80	97	1.0403	1.0409	1.0415
197	60	46	1.0306	54	94	1.0412	1.0428	34	41	47
194	92	77	38	85	1.0425	43	60	66	72	78
191	1.0223	1.0308	69	1.0417	57	74	91	97	1.0503	1.0510
188	55	40	1.0400	48	88	1.0506	1.0522	1.0528	35	41
185	86	71	32	80	1.0519	37	54	60	66	72
182	1.0317	1.0403	63	1.0511	51	68	85	91	98	1.0604
179	49	34	95	42	82	1.0600	1.0616	1.0623	1.0629	35
176	80	65	1.0526	74	1.0613	31	48	54	60	66
173	1.0411	97	57	1.0605	45	63	79	85	92	98
170	43	1.0528	89	36	76	94	1.0710	1.0717	1.0723	1.0729
167	74	59	1.0620	68	1.0707	1.0725	42	48	54	60
164	1.0505	91	51	99	39	56	73	80	86	92
161	37	1.0622	82	1.0730	70	88	1.0804	1.0811	1.0817	1.0823
158	68	53	1.0714	62	1.0801	1.0819	36	42	48	54
155	99	84	45	93	33	50	67	73	80	86
152	1.0631	1.0716	76	1.0824	64	82	98	1.0905	1.0911	1.0917
149	62	47	1.0808	55	95	1.0913	1.0930	36	42	48
146	93	78	39	87	1.0926	44	61	67	73	79
143	1.0724	1.0810	70	1.0918	58	75	92	98	1.1005	1.1011
140	56	41	1.0901	49	89	1.1007	1.1023	1.1030	36	42
137	87	72	33	80	1.1020	38	55	61	67	73
134	1.0818	1.0903	64	1.1012	51	69	86	92	98	1.1104
131	49	34	95	43	83	1.1100	1.1117	1.1123	1.1130	36
128	81	66	1.1026	74	1.1114	32	48	55	61	67
125	1.0912	97	57	1.1105	45	63	79	86	92	98
122	43	1.1028	89	36	76	94	1.1211	1.1217	1.1223	1.1229
119	74	59	1.1120	68	1.1207	1.1225	42	48	54	60
116	1.1005	90	51	99	39	56	73	79	86	92
113	36	1.1122	82	1.1230	70	88	1.1304	1.1310	1.1317	1.1323
110	68	53	1.1213	61	1.1301	1.1319	35	42	48	54
107	99	84	45	92	32	50	66	73	79	85
104	1.1130	1.1215	76	1.1323	63	81	98	1.1404	1.1410	1.1416
101	61	46	1.1307	55	94	1.1412	1.1429	35	41	47
98	92	77	38	86	1.1426	43	60	66	73	79
95	1.1223	1.1309	69	1.1417	57	75	91	97	1.1504	1.1510
92	55	40	1.1400	48	88	1.1506	1.1522	1.1529	35	41
89	86	71	31	79	1.1519	37	53	60	66	72
86	1.1317	1.1402	63	1.1510	50	68	84	91	97	1.1603
83	48	33	94	41	81	99	1.1616	1.1622	1.1628	34
80	79	64	1.1525	73	1.1612	1.1630	47	53	59	65
77	1.1410	95	56	1.1604	44	61	78	84	90	96
74	41	1.1526	87	35	75	92	1.1709	1.1715	1.1722	1.1728
71	72	58	1.1618	66	1.1706	1.1723	40	46	53	59
68	1.1504	89	49	97	37	55	71	78	84	90
65	35	1.1620	80	1.1728	68	86	1.1802	1.1809	1.1815	1.1821
62	66	51	1.1711	59	99	1.1817	33	40	46	52
59	97	82	43	90	1.1830	48	64	71	77	83
56	1.1628	1.1713	74	1.1821	61	79	96	1.1902	1.1908	1.1914
53	59	44	1.1805	52	92	1.1910	1.1927	33	39	45
50	90	75	36	84	1.1923	41	58	64	70	76
47	1.1721	1.1806	67	1.1915	54	72	89	95	1.2001	1.2007
44	52	37	98	46	86	1.2003	1.2020	1.2026	32	39
41	83	68	1.1929	77	1.2017	34	51	57	64	70
38	1.1814	1.1900	60	1.2008	48	65	82	88	95	1.2101
35	45	31	91	39	79	96	1.2113	1.2119	1.2126	32
32	76	62	1.2022	70	1.2110	1.2128	44	51	57	63

Gauge-press., lbs. 58 + Absolute Pressures, 73.	60 + 75	62 + 77	64 + 79	66 + 81	68 + 83	70 + 85	72 + 87	74 + 89	76 + 91
Feed-water Temp.	Factors of Evaporation.								
212° F.	1.0295	1.0301	1.0307	1.0312	1.0318	1.0323	1.0329	1.0334	1.0339	1.0344
209	1.0327	33	38	44	49	55	60	65	70	75
206	58	64	70	75	81	86	91	97	1.0402	1.0407
203	90	96	1.0401	1.0407	1.0412	1.0418	1.0423	1.0428	33	38
200	1.0421	1.0427	33	38	44	49	54	59	65	69
197	53	58	64	70	75	80	86	91	96	1.0501
194	84	90	96	1.0501	1.0507	1.0512	1.0517	1.0522	1.0527	32
191	1.0515	1.0521	1.0527	33	38	43	49	54	59	64
188	47	53	58	64	69	75	80	85	90	95
185	78	84	90	95	1.0601	1.0606	1.0611	1.0616	1.0622	1.0626
182	1.0610	1.0615	1.0621	1.0627	32	37	43	48	53	58
179	41	47	52	58	63	69	74	79	84	89
176	72	78	84	89	95	1.0700	1.0705	1.0711	1.0716	1.0721
173	1.0704	1.0709	1.0715	1.0721	1.0726	32	37	42	47	52
170	35	41	46	52	57	63	68	73	78	83
167	66	72	78	83	89	94	99	1.0805	1.0810	1.0815
164	98	1.0803	1.0809	1.0815	1.0820	1.0825	1.0831	36	41	46
161	1.0829	35	40	46	51	57	62	67	72	77
158	60	66	72	77	83	88	93	98	1.0904	1.0908
155	92	97	1.0903	1.0909	1.0914	1.0919	1.0925	1.0930	35	40
152	1.0923	1.0929	34	40	45	51	56	61	66	71
149	54	60	66	71	77	82	87	92	97	1.1002
146	85	91	97	1.1002	1.1008	1.1013	1.1018	1.1024	1.1029	34
143	1.1017	1.1022	1.1028	34	39	44	50	55	60	65
140	48	54	59	65	70	76	81	86	91	96
137	79	85	91	96	1.1102	1.1107	1.1112	1.1117	1.1122	1.1127
134	1.1110	1.1116	1.1122	1.1127	33	38	43	49	54	59
131	42	47	53	59	64	69	75	80	85	90
128	73	79	84	90	95	1.1201	1.1206	1.1211	1.1216	1.1221
125	1.1204	1.1210	1.1215	1.1221	1.1226	32	37	42	47	52
122	35	41	47	52	58	63	68	73	78	83
119	66	72	78	83	89	94	99	1.1305	1.1310	1.1315
116	98	1.1303	1.1309	1.1315	1.1320	1.1325	1.1331	36	41	46
113	1.1329	34	40	46	51	57	62	67	72	77
110	60	66	71	77	82	88	93	98	1.1403	1.1408
107	91	97	1.1403	1.1408	1.1414	1.1419	1.1424	1.1429	34	39
104	1.1422	1.1428	34	39	45	50	55	60	65	70
101	53	59	65	70	76	81	86	92	97	1.1502
98	85	90	96	1.1502	1.1507	1.1512	1.1518	1.1523	1.1528	33
95	1.1516	1.1521	1.1527	33	38	43	49	54	59	64
92	47	53	58	64	69	75	80	85	90	95
89	78	84	89	95	1.1600	1.1606	1.1611	1.1616	1.1621	1.1626
86	1.1609	1.1615	1.1621	1.1626	32	37	42	47	52	57
83	40	46	52	57	63	68	73	78	83	88
80	71	77	83	88	94	99	1.1704	1.1710	1.1715	1.1720
77	1.1702	1.1708	1.1714	1.1719	1.1725	1.1730	35	41	46	51
74	34	39	45	51	56	61	67	72	77	82
71	65	70	76	82	87	92	98	1.1803	1.1808	1.1813
68	96	1.1802	1.1807	1.1813	1.1818	1.1824	1.1829	34	39	44
65	1.1827	33	38	44	49	55	60	65	70	75
62	58	64	69	75	80	86	91	96	1.1901	1.1906
59	89	95	1.1901	1.1906	1.1912	1.1917	1.1922	1.1927	32	37
56	1.1920	1.1926	32	37	43	48	53	58	63	68
53	51	57	63	68	74	79	84	89	94	99
50	82	88	94	99	1.2005	1.2010	1.2015	1.2021	1.2026	1.2031
47	1.2013	1.2019	1.2025	1.2030	36	41	46	52	57	62
44	44	50	56	61	67	72	78	83	88	93
41	76	81	87	93	98	1.2103	1.2109	1.2114	1.2119	1.2124
38	1.2107	1.2112	1.2118	1.2124	1.2129	34	40	45	50	55
35	38	43	49	55	60	65	71	76	81	86
32	69	75	80	86	91	97	1.2202	1.2207	1.2212	1.2217

Gauge-pressures lbs., 78 + / Absolute Pressures, 93	80 + / 95	82 + / 97	84 + / 99	86 + / 101	88 + / 103	90 + / 105	92 + / 107	94 + / 109	96 + / 111	98 + / 113
Feed-water Temp.	Factors of Evaporation.									
212	1.0349	1.0353	1.0358	1.0363	1.0367	1.0372	1.0376	1.0381	1.0385	1.0389	1.0393
209	80	85	90	94	99	1.0403	1.0408	1.0412	1.0416	1.0421	1.0425
206	1.0411	1.0416	1.0421	1.0426	1.0430	35	39	43	48	52	56
203	43	48	52	57	62	66	71	75	79	83	88
200	74	79	84	89	93	98	1.0502	1.0506	1.0511	1.0515	1.0519
197	1.0506	1.0511	1.0515	1.0520	1.0525	1.0529	33	38	42	46	50
194	37	42	47	51	56	60	65	69	73	78	82
191	69	73	78	83	87	92	96	1.0601	1.0605	1.0609	1.0613
188	1.0600	1.0605	1.0610	1.0614	1.0619	1.0623	1.0628	32	36	40	45
185	81	86	41	46	50	55	59	63	68	72	76
182	63	68	72	77	81	86	90	95	99	1.0703	1.0707
179	94	99	1.0704	1.0708	1.0713	1.0717	1.0722	1.0726	1.0730	35	39
176	1.0725	1.0730	35	40	44	49	53	57	62	66	70
173	57	62	66	71	75	80	84	89	93	97	1.0801
170	88	93	98	1.0802	1.0807	1.0811	1.0816	1.0820	1.0824	1.0829	33
167	1.0819	1.0824	1.0829	34	38	43	47	51	56	60	64
164	51	56	60	65	69	74	78	83	87	91	95
161	82	87	92	96	1.0901	1.0905	1.0910	1.0914	1.0918	1.0923	1.0927
158	1.0913	1.0918	1.0923	1.0927	32	37	41	45	50	54	58
155	45	49	54	59	63	68	72	77	81	85	89
152	76	81	85	90	95	99	1.1004	1.1008	1.1012	1.1016	1.1021
149	1.1007	1.1012	1.1017	1.1021	1.1026	1.1030	35	39	43	48	52
146	38	43	48	53	57	62	66	70	75	79	83
143	70	74	79	84	88	93	97	1.1102	1.1106	1.1110	1.1114
140	1.1101	1.1106	1.1110	1.1115	1.1120	1.1124	1.1129	33	37	41	46
137	32	37	42	46	51	55	60	64	68	73	77
134	63	68	73	78	82	87	91	95	1.1200	1.1204	1.1208
131	95	99	1.1204	1.1209	1.1213	1.1218	1.1222	1.1227	31	35	39
128	1.1226	1.1231	35	40	45	49	53	58	62	66	71
125	57	62	67	71	76	80	85	89	93	98	1.1302
122	88	93	98	1.1302	1.1307	1.1311	1.1316	1.1320	1.1325	1.1329	33
119	1.1320	1.1324	1.1329	34	38	43	47	51	56	60	64
116	51	55	60	65	69	74	78	83	87	91	95
113	82	87	91	96	1.1401	1.1405	1.1409	1.1414	1.1418	1.1422	1.1426
110	1.1413	1.1418	1.1422	1.1427	32	36	41	45	49	53	58
107	44	49	54	58	63	67	72	76	80	85	89
104	75	80	85	89	94	99	1.1503	1.1507	1.1512	1.1516	1.1520
101	1.1506	1.1511	1.1516	1.1521	1.1525	1.1530	34	38	43	47	51
98	38	42	47	52	56	61	65	70	74	78	82
95	69	74	78	83	87	92	96	1.1601	1.1605	1.1609	1.1613
92	1.1600	1.1605	1.1609	1.1614	1.1619	1.1623	1.1628	32	36	40	45
89	31	36	41	45	50	54	59	63	67	72	76
86	62	67	72	76	81	85	90	94	98	1.1703	1.1707
83	93	98	1.1703	1.1707	1.1712	1.1717	1.1721	1.1725	1.1730	34	38
80	1.1724	1.1729	34	39	43	48	52	56	61	65	69
77	56	60	65	70	74	79	83	88	92	96	1.1800
74	87	91	96	1.1801	1.1805	1.1810	1.1814	1.1819	1.1823	1.1827	31
71	1.1818	1.1823	1.1827	32	36	41	45	50	54	58	62
68	49	54	58	63	68	72	77	81	85	89	94
65	80	85	89	94	99	1.1903	1.1908	1.1912	1.1916	1.1920	1.1925
62	1.1911	1.1916	1.1921	1.1925	1.1930	34	39	43	47	52	56
59	42	47	52	56	61	65	70	74	78	83	87
56	73	78	83	87	92	96	1.2001	1.2005	1.2010	1.2014	1.2018
53	1.2004	1.2009	1.2014	1.2018	1.2023	1.2028	32	36	41	45	49
50	35	40	45	50	54	59	63	67	72	76	80
47	66	71	76	81	85	90	94	98	1.2103	1.2107	1.2111
44	98	1.2102	1.2107	1.2112	1.2116	1.2121	1.2125	1.2130	34	38	42
41	1.2129	33	38	43	47	52	56	61	65	69	73
38	60	64	69	74	78	83	87	92	96	1.2200	1.2204
35	91	96	1.2200	1.2205	1.2209	1.2214	1.2218	1.2223	1.2227	31	35
32	1.2222	1.2227	31	36	41	45	49	54	58	62	67

Gauge-pressures lbs. 100 + Absolute Press. lbs. 115.	105 + 120	110 + 125	115 + 130	120 + 135	125 + 140	130 + 145	135 + 150	140 + 155	145 + 160	150 + 165
Feed-water Temp.	FACTORS OF EVAPORATION.									
212° 1.0397	1.0407	1.0417	1.0427	1.0436	1.0445	1.0453	1.0462	1.0470	1.0478	1.0486
209 1.0429	39	49	58	67	76	85	93	1.0501	1.0509	1.0517
206 60	70	80	89	99	1.0508	1.0516	1.0525	33	41	48
203 92	1.0502	1.0511	1.0521	1.0530	39	48	56	64	72	80
200 1.0523	33	43	52	62	70	79	87	96	1.0604	1.0611
197 55	65	74	84	93	1.0602	1.0610	1.0619	1.0627	35	43
194 86	96	1.0606	1.0615	1.0624	33	42	50	58	66	74
191 1.0617	1.0627	37	47	56	65	73	82	90	98	1.0706
188 49	59	69	78	87	96	1.0705	1.0713	1.0721	1.0729	37
185 80	90	1.0700	1.0709	1.0719	1.0727	36	44	53	61	68
182 1.0712	1.0722	31	41	50	59	67	76	84	92	1.0800
179 43	53	63	72	81	90	99	1.0807	1.0815	1.0823	31
176 74	84	94	1.0803	1.0813	1.0821	1.0830	39	47	55	62
173 1.0806	1.0816	1.0825	35	44	53	61	70	78	86	94
170 37	47	57	66	75	84	93	1.0901	1.0909	1.0917	1.0925
167 68	78	88	97	1.0907	1.0915	1.0924	32	41	49	56
164 1.0900	1.0910	1.0919	1.0929	38	47	55	64	72	80	88
161 31	41	51	60	69	78	87	95	1.1003	1.1011	1.1019
158 62	72	82	91	1.1000	1.1009	1.1018	1.1026	35	43	50
155 93	1.1003	1.1013	1.1023	32	41	49	58	66	74	82
152 1.1025	35	44	54	63	72	81	89	97	1.1105	1.1113
149 56	66	76	85	94	1.1103	1.1112	1.1120	1.1128	36	44
146 87	97	1.1107	1.1116	1.1126	34	43	51	60	68	75
143 1.1118	1.1129	38	48	57	66	74	83	91	99	1.1207
140 50	60	70	79	88	97	1.1206	1.1214	1.1222	1.1230	38
137 81	91	1.1201	1.1210	1.1219	1.1228	37	45	53	61	69
134 1.1212	1.1222	32	41	51	59	68	76	85	93	1.1300
131 43	53	63	73	82	91	99	1.1308	1.1316	1.1324	32
128 75	85	94	1.1304	1.1313	1.1322	1.1331	39	47	55	63
125 1.1306	1.1316	1.1326	35	44	53	62	70	78	86	94
122 37	47	57	66	75	84	93	1.1401	1.1409	1.1417	1.1425
119 68	78	88	97	1.1407	1.1415	1.1424	32	41	49	56
116 99	1.1409	1.1419	1.1429	38	47	55	64	72	80	88
113 1.1431	41	50	60	69	78	86	95	1.1503	1.1511	1.1519
110 62	72	82	91	1.1500	1.1509	1.1518	1.1526	34	42	50
107 93	1.1503	1.1513	1.1522	31	40	49	57	65	73	81
104 1.1524	34	44	53	62	71	80	88	97	1.1605	1.1612
101 55	65	75	84	94	1.1602	1.1611	1.1620	1.1628	36	43
98 86	96	1.1606	1.1616	1.1625	34	42	51	59	67	75
95 1.1618	1.1628	37	47	56	65	73	82	90	98	1.1706
92 49	59	68	78	87	96	1.1705	1.1713	1.1721	1.1729	37
89 80	90	1.1700	1.1709	1.1718	1.1727	36	44	52	60	68
86 1.1711	1.1721	31	40	49	58	67	75	83	91	99
83 42	52	62	71	80	89	98	1.1806	1.1815	1.1823	1.1830
80 73	83	93	1.1802	1.1812	1.1820	1.1829	37	46	54	61
77 1.1804	1.1814	1.1824	34	43	52	60	69	77	85	93
74 35	45	55	65	74	83	91	1.1900	1.1908	1.1916	1.1924
71 67	77	86	96	1.1905	1.1914	1.1922	31	39	47	55
68 98	1.1908	1.1917	1.1927	36	45	54	62	70	78	86
65 1.1929	39	49	58	67	76	85	93	1.2001	1.2009	1.2017
62 60	70	80	89	98	1.2007	1.2016	1.2024	32	40	48
59 91	1.2001	1.2011	1.2020	1.2029	38	47	55	63	71	79
56 1.2022	32	42	51	60	69	78	86	94	1.2102	1.2110
53 53	63	73	82	91	1.2100	1.2109	1.2117	1.2126	34	41
50 84	94	1.2104	1.2113	1.2123	31	40	48	57	65	72
47 1.2115	1.2125	35	44	54	63	71	80	88	96	1.2203
44 46	56	66	76	85	94	1.2202	1.2211	1.2219	1.2227	35
41 77	87	97	1.2207	1.2216	1.2225	33	42	50	58	66
38 1.2208	1.2219	1.2228	38	47	56	64	73	81	89	97
35 40	50	59	69	78	87	95	1.2304	1.2312	1.2320	1.2328
32 71	81	90	1.2300	1.2309	1.2318	1.2326	35	43	51	59

STRENGTH OF STEAM-BOILERS. VARIOUS RULES FOR CONSTRUCTION.

There is a great lack of uniformity in the rules prescribed by different writers and by legislation governing the construction of steam-boilers In the United States, boilers for merchant vessels must be constructed according to the rules and regulations prescribed by the Board of Supervising Inspectors of Steam Vessels; in the U. S. Navy, according to rules of the Navy Department, and in some cases according to special acts of Congress. On land, in some places, as in Philadelphia, the construction of boilers is governed by local laws; but generally there are no laws upon the subject, and boilers are constructed according to the idea of individual engineers and boiler-makers. In Europe the construction is generally regulated by stringent inspection laws. The rules of the U. S. Supervising Inspectors of Steam-vessels, the British Lloyd's and Board of Trade, the French Bureau Veritas, and the German Lloyd's are ably reviewed in a paper by Nelson Foley, M. Inst. Naval Architects, etc., read at the Chicago Engineering Congress, Division of Marine and Naval Engineering. From this paper the following notes are taken, chiefly with reference to the U. S. and British rules: (*Abbreviations.*—T. S., for tensile strength; El., elongation; Contr., contraction of area.)

Hydraulic Tests.—*Board of Trade, Lloyd's, and Bureau Veritas.*—Twice the working pressure.

United States Statutes.—One and a half times the working pressure.

Mr. Foley proposes that the proof pressure should be 1½ times the working pressure + one atmosphere.

Established Nominal Factors of Safety.—*Board of Trade.*—4.5 for a boiler of moderate length and of the best construction and workmanship.

Lloyd's.—Not very apparent, but appears to lie between 4 and 5.

United States Statutes.—Indefinite, because the strength of the joint is not considered, except by the broad distinction between single and double riveting.

Bureau Veritas: 4.4.

German Lloyd's: 5 to 4.65, according to the thickness of the plates.

Material for Riveting.—*Board of Trade.*—Tensile strength of rivet bars between 26 and 30 tons, el. in 10″ not less than 25%, and contr. of area not less than 50%.

Lloyd's.—T. S., 26 to 30 tons; el. not less than 20% in 8″. The material must stand bending to a curve, the inner radius of which is not greater than 1½ times the thickness of the plate, after having been uniformly heated to a low cherry-red, and quenched in water at 82° F.

United States Statutes.—No special provision.

Rules Connected with Riveting.—*Board of Trade.*—The shearing resistance of the rivet steel to be taken at 23 tons per square inch. 5 to be used for the factor of safety independently of any addition to this factor for the plating. Rivets in double shear to have only 1.75 times the single section taken in the calculation instead of 2. The diameter must not be less than the thickness of the plate and the pitch never greater than 8½″. The thickness of double butt-straps (each) not to be less than ⅝ the thickness of the plate; single butt-straps not less than 9/8.

Distance from centre of rivet to edge of hole = diameter of rivet × 1½.

Distance between rows of rivets

$$= 2 \times \text{diam. of rivet or} = [(\text{diam.} \times 4) + 1] \div 2, \text{ if chain, and}$$

$$= \frac{\sqrt{[(\text{pitch} \times 11) + (\text{diam.} \times 4)] \times (\text{pitch} + \text{diam.} \times 4)}}{10} \text{ if zigzag.}$$

Diagonal pitch = (pitch × 6 + diam. × 4) ÷ 10.

Lloyd's.—Rivets in double shear to have only 1.75 times the single section taken in the calculation instead of 2. The shearing strength of rivet steel to be taken at 85% of the T. S. of the material of shell plates. In any case where the strength of the longitudinal joint is satisfactorily shown by experiment to be greater than given by the formula, the actual strength may be taken in the calculation.

United States Statutes.—No rules.

Material for Cylindrical Shells Subject to Internal Pressure.—*Board of Trade.*—T. S. between 27 and 32 tons. In the normal condition, el. not less than 18% in 10″, but should be about 25%; if annealed, not less than 20%. Strips 2" wide should stand bending until the sides are parallel at a distance from each other of not more than three times the plate's thickness.

Lloyd's.—T. S. between the limits of 26 and 30 tons per square inch. El. not less than 20% in 8". Test strips heated to a low cherry-red and plunged into water at 82° F. must stand bending to a curve, the inner radius of which is not greater than 1½ times the plate's thickness.

U. S. Statutes.—Plates of ½" thick and under shall show a contr. of not less than 50%: when over ½" and up to ¾", not less than 45%; when over ¾", not less than 40%.

Mr. Foley's comments: The Board of Trade rules seem to indicate a steel of too high T. S. when a lower and more ductile one can be got: the lower tensile limit should be reduced, and the bending test might with advantage be made after tempering, and made to a smaller radius. Lloyd's rule for quality seems more satisfactory, but the temper test is not severe. The United States Statutes are not sufficiently stringent to insure an entirely satisfactory material.

Mr. Foley suggests a material which would meet the following: 25 tons lower limit in tension; 25% in 8" minimum elongation; radius for bending test after tempering = the plate's thickness.

Shell-plate Formulæ.—*Board of Trade:* $P = \dfrac{T \times B \times t \times 2}{D \times F}$.

D = diameter of boiler in inches;
P = working-pressure in lbs. per square inch;
t = thickness in inches;
B = percentage of strength of joint compared to solid plate;
T = tensile strength allowed for the material in lbs. per square inch;
F = a factor of safety, being 4.5, with certain additions depending on method of construction.

Lloyd's: $P = \dfrac{C \times (t - 2) \times B}{D}$.

t = thickness of plate in sixteenths; B and D as before; C = a constant depending on the kind of joint.

When longitudinal seams have double butt-straps, $C = 20$. When longitudinal seams have double butt-straps of unequal width, only covering on one side the reduced section of plate at the outer line of rivets, $C = 19.5$.

When the longitudinal seams are lap-jointed, $C = 18.5$.

U. S. Statutes.—Using same notation as for Board of Trade,

$$P = \frac{t \times 2 \times T}{D \times 6} \text{ for single-riveting; add 20\% for double-riveting;}$$

where T is the lowest T. S. stamped on any plate.

Mr. Foley criticises the rule of the United States Statutes as follows: The rule ignores the riveting, except that it distinguishes between single and double, giving the latter 20% advantage; the circumferential riveting or class of seam is altogether ignored. The rule takes no account of workmanship or method adopted of constructing the joints. The factor, one sixth, simply covers the actual nominal factor of safety as well as the loss of strength at the joint, no matter what its percentage; we may therefore dismiss it as unsatisfactory.

Rules for Flat Plates.—*Board of Trade;* $P = \dfrac{C(t+1)^2}{S-6}$.

P = working-pressure in lbs. per square inch;
S = surface supported in square inches;
t = thickness in sixteenths of an inch;
C = a constant as per following table:

C = 125 for plates not exposed to heat or flame, the stays fitted with nuts and washers, the latter at least three times the diameter of the stay and ⅔ the thickness of the plate;
C = 187.5 for the same condition, but the washers ⅗ the pitch of stays in diameter, and thickness not less than plate;
C = 200 for the same condition, but doubling plates in place of washers, the width of which is ⅔ the pitch and thickness the same as the plate;
C = 112.5 for the same condition, but the stays with nuts only;
C = 75 when exposed to impact of heat or flame and steam in contact with the plates, and the stays fitted with nuts and washers three times the diameter of the stay and ⅔ the plate's thickness;

$C = 67.5$ for the same condition, but stays fitted with nuts only;
$C = 100$ when exposed to heat or flame, and water in contact with the plates, and stays screwed into the plates and fitted with nuts;
$C = 66$ for the same condition, but stays with riveted heads.

U. S. Statutes.—Using same notation as for Board of Trade. $P = \frac{C \times t^2}{p^2}$, where p = greatest pitch in inches, P and t as above;

$C = 112$ for plates 7/16″ thick and under, fitted with screw stay-bolts riveted over, screw stay-bolts and nuts, or plain bolt fitted with single nut and socket, or riveted head and socket;
$C = 120$ for plates above 7/16″, under the same conditions;
$C = 140$ for flat surfaces where the stays are fitted with nuts inside and outside;
$C = 200$ for flat surfaces under the same condition, but with the addition of a washer riveted to the plate at least ½ plate's thickness, and of a diameter equal to ⅖ of the pitch of the stay-bolts.

N.B.—Plates fitted with double angle-irons and riveted to plate, with leaf at least ⅔ the thickness of plate and depth at least ¼ of pitch, would be allowed the same pressure as determined by formula for plate with washer riveted on.

N.B.—No brace or stay-bolt used in marine boilers to have a greater pitch than 10½″ on fire-boxes and back connections.

Certain experiments were carried out by the Board of Trade which showed that the resistance to bulging does not vary as the square of the plate's thickness. There seems also good reason to believe that it is not inversely as the square of the greatest pitch. Bearing in mind, says Mr. Foley, that mathematicians have signally failed to give us true theoretical foundations for calculating the resistance of bodies subject to the simplest forms of stresses, we therefore cannot expect much from their assistance in the matter of flat plates.

The Board of Trade rules for flat surfaces, being based on actual experiment, are especially worthy of respect; sound judgment appears also to have been used in framing them.

Furnace Formulæ.—Board of Trade.—*Long Furnaces.*—

$P = \frac{C \times t^2}{(L+1) \times D}$, but not where L is shorter than $(11.5t - 1)$, at which length the rule for short furnaces comes into play.

P = working-pressure in pounds per square inch; t = thickness in inches; D = outside diameter in inches; L = length of furnace in feet up to 10 ft.; C = a constant, as per following table, for drilled holes:

$C = 99{,}000$ for welded or butt-jointed with single straps, double-riveted;
$C = 88{,}000$ for butts with single straps, single-riveted;
$C = 99{,}000$ for butts with double straps, single-riveted.

Provided always that the pressure so found does not exceed that given by the following formulæ, which apply also to short furnaces:

$P = \frac{C \times t}{D}$ for all the patent furnaces named;

$P = \frac{C \times t}{3 \times D}\left(5 - \frac{L \times 12}{67.5 \times t}\right)$ when with Adamson rings.

$C = 8{,}800$ for plain furnaces;
$C = 14{,}000$ for Fox; minimum thickness 5/16″, greatest ⅝″; plain part not to exceed 6″ in length;
$C = 13{,}500$ for Morison; minimum thickness 5/16″, greatest ⅝″; plain part not to exceed 6″ in length;
$C = 14{,}000$ for Purves-Brown; limits of thickness 7/16″ and ⅝″; plain part 9″ in length;
$C = 8{,}800$ for Adamson rings; radius of flange next fire 1½″.

U. S. Statutes.—*Long Furnaces.*—Same notation.

$P = \frac{89{,}600 \times t^2}{L \times D}$, but L not to exceed 8 ft.

N.B.—If rings of wrought iron are fitted and riveted on properly around and to the flue in such a manner that the tensile stress on the rivets shall not exceed 6000 lbs. per sq. in., the distance between the rings shall be taken as the length of the flue in the formulæ.

Short Furnaces, Plain and Patent.—P, as before, when not 8 ft. long $= \frac{89.600 \times t^2}{L \times D}$;

$$P = \frac{t \times C}{D} \text{ when}$$

$C = 14,000$ for Fox corrugations where $D =$ mean diameter;
$C = 14,000$ for Purves-Brown where $D =$ diameter of flue;
$C = 5677$ for plain flues over 16″ diameter and less than 40″, when not over 3 ft. lengths.

Mr. Foley comments on the rules for long furnaces as follows: The Board of Trade general formula, where the length is a factor, has a very limited range indeed, viz., 10 ft. as the extreme length, and 135 thicknesses − 12″, as the short limit. The original formula, $P = \frac{C \times t^2}{L \times D}$, is that of Sir W. Fairbairn, and was, I believe, never intended by him to apply to short furnaces. On the very face of it, it is apparent, on the other hand, that if it is true for moderately long furnaces, it cannot be so for very long ones. We are therefore driven to the conclusion that any formula which includes simple L as a factor must be founded on a wrong basis.

With Mr. Traill's form of the formula, namely, substituting $(L + 1)$ for L, the results appear sufficiently satisfactory for practical purposes, and indeed, as far as can be judged, tally with the results obtained from experiment as nearly as could be expected. The experiments to which I refer were six in number, and of great variety of length to diameter; the actual factors of safety ranged from 4.4 to 6.2, the mean being 4.78, or practically 5. It seems to me, therefore, that, within the limits prescribed, the Board of Trade formula may be accepted as suitable for our requirements.

The United States Statutes give Fairbairn's rule pure and simple, except that the extreme limit of length to which it applies is fixed at 8 feet. As far as can be seen, no limit for the shortest length is prescribed, but the rules to me are by no means clear, flues and furnaces being mixed or not well distinguished.

Material for Stays.—The qualities of material prescribed are as follows:

Board of Trade.—The tensile strength to lie between the limits of 27 and 32 tons per square inch, and to have an elongation of not less than 20% in 10″. Steel stays which have been welded or worked in the fire should not be used.

Lloyd's.—26 to 30 ton steel, with elongation not less than 20% in 8″.

U. S. Statutes.—The only condition is that the reduction of area must not be less than 40% if the test bar is over ¾″ diameter.

Loads allowed on Stays.—*Board of Trade.*—9000 lbs. per square inch is allowed on the net section, provided the tensile strength ranges from 27 to 32 tons. Steel stays are not to be welded or worked in the fire.

Lloyd's.—For screwed and other stays, not exceeding 1½″ diameter effective, 8000 lbs. per square inch is allowed; for stays above 1½″, 9000 lbs. No stays are to be welded.

U. S. Statutes.—Braces and stays shall not be subjected to a greater stress than 6000 lbs. per square inch.

[Rankine, S. E., p. 459, says: "The iron of the stays ought not to be exposed to a greater working tension than 3000 lbs. on the square inch, in order to provide against their being weakened by corrosion. This amounts to making the factor of safety for the working pressure about 20." It is evident, however, that an allowance in the factor of safety for corrosion may reasonably be decreased with increase of diameter. W. K.]

Girders.—*Board of Trade.* $P = \frac{C \times d^2 \times t}{(W - p)D \times L}$. $P =$ working pressure in lbs. per sq. in.; $W =$ width of flame-box in inches; $L =$ length of girder in inches; $p =$ pitch of bolts in inches; $D =$ distance between girders from centre to centre in inches; $d =$ depth of girder in inches; $t =$ thickness of sum of same in inches; $C =$ a constant $= 6600$ for 1 bolt, 9900 for 2 or 3 bolts, and 11,220 for 4 bolts.

Lloyd's.—The same formula and constants, except that $C = 11,000$ for 4 or 5 bolts, 11,550 for 6 or 7, and 11,880 for 8 or more.

U. S. Statutes.—The matter appears to be left to the designers.

Tube-Plates.—*Board of Trade.* $P = \frac{t(D-d) \times 20,000}{W \times D}$. D = least horizontal distance between centres of tubes in inches; d = inside diameter of ordinary tubes; t = thickness of tube-plate in inches; W = extreme width of combustion-box in inches from front tube-plate to back of fire-box, or distance between combustion-box tube-plates when the boiler is double-ended and the box common to both ends.

The crushing stress on tube-plates caused by the pressure on the flame-box top is to be limited to 10,000 lbs. per square inch.

Material for Tubes.—Mr. Foley proposes the following: If iron, the quality to be such as to give at least 22 tons per square inch as the minimum tensile strength, with an elongation of not less than 15% in 8″. If steel, the elongation to be not less than 26% in 8″ for the material before being rolled into strips; and after tempering, the test bar to stand completely closing together. Provided the steel welds well, there does not seem to be any object in providing tensile limits.

The ends should be annealed after manufacture, and stay-tube ends should be annealed before screwing.

Holding-power of Boiler-tubes.—Experiments made in Washington Navy Yard show that with 2½ in. brass tubes in no case was the holding-power less, roughly speaking, than 6000 lbs., while the average was upwards of 20,000 lbs. It was further shown that with these tubes nuts were superfluous, quite as good results being obtained with tubes simply expanded into the tube-plate and fitted with a ferrule. When nuts were fitted it was shown that they drew off without injuring the threads.

In Messrs. Yarrow's experiments on iron and steel tubes of 2″ to 2¼″ diameter the first 5 tubes gave way on an average of 23,740 lbs., which would appear to be about ⅝ the ultimate strength of the tubes themselves. In all these cases the hole through the tube-plate was parallel with a sharp edge to it, and a ferrule was driven into the tube.

Tests of the next 5 tubes were made under the same conditions as the first 5, with the exception that in this case the ferrule was omitted, the tubes being simply expanded into the plates. The mean pull required was 15,270 lbs., or considerably less than half the ultimate strength of the tubes.

Effect of beading the tubes, the holes through the plate being parallel and ferrules omitted. The mean of the first 3, which are tubes of the same kind, gives 26,876 lbs. as their holding-power, under these conditions, as compared with 23,740 lbs. for the tubes fitted with ferrules only. This high figure is, however, mainly due to an exceptional case where the holding-power is greater than the average strength of the tubes themselves.

It is disadvantageous to cone the hole through the tube-plate unless its sharp edge is removed, as the results are much worse than those obtained with parallel holes, the mean pull being but 16,031 lbs., the experiments being made with tubes expanded and ferruled but not beaded over.

In experiments on tubes expanded into tapered holes, beaded over and fitted with ferrules, the net result is that the holding-power is, for the size experimented on, about ¾ of the tensile strength of the tube, the mean pull being 28,797 lbs.

With tubes expanded into tapered holes and simply beaded over, better results were obtained than with ferrules; in these cases, however, the sharp edge of the hole was rounded off, which appears in general to have a good effect.

In one particular the experiments are incomplete, as it is impossible to reproduce on a machine the racking the tubes get by the expansion of a boiler as it is heated up and cooled down again, and it is quite possible, therefore, that the fastening giving the best results on the testing-machine may not prove so efficient in practice.

N.B.—It should be noted that the experiments were all made under the cold condition, so that reference should be made with caution, the circumstances in practice being very different, especially when there is scale on the tube-plates, or when the tube-plates are thick and subject to intense heat.

Iron versus Steel Boiler-tubes. (Foley.)—Mr. Blechynden prefers iron tubes to those of steel, but how far he would go in attributing the leaky-tube defect to the use of steel tubes we are not aware. It appears, however, that the results of his experiments would warrant him in going a considerable distance in this direction. The test consisted of heating and cooling two tubes, one of wrought iron and the other of steel. Both tubes were 2¾ in. in diameter and .16 in. thickness of metal. The tubes were put in the same furnace, made red-hot, and then dipped in water. The length was gauged at a temperature of 46° F.

This operation was twice repeated, with results as follows:

	Steel.	Iron.
Original length	55.495 in.	55.495 in.
Heated to 186° F.; increase	.052 "	.048 "
Coefficient of expansion per degree F.	.0000067	.0000062
Heated red-hot and dipped in water; decrease	.007 in.	.003 in.
Second heating and cooling, decrease	.031 in.	.004 in.
Third heating and cooling, decrease	.017 in.	.006 in.
Total contraction	.055 in.	.013 in.

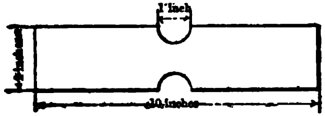
Mr. A. C. Kirk writes: That overheating of tube ends is the cause of the leakage of the tubes in boilers is proved by the fact that the ferrules at present used by the Admiralty prevent it. These act by shielding the tube ends from the action of the flame, and consequently reducing evaporation, and so allowing free access of the water to keep them cool.

Although many causes contribute, there seems no doubt that thick tube-plates must bear a share of causing the mischief.

Rules for Construction of Boilers in Merchant Vessels in the United States.

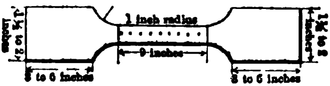
(Extracts from General Rules and Regulations of the Board of Supervising Inspectors of Steam-vessels (as amended 1898).)

Tensile Strength of Plate. (Section 3.)—To ascertain the tensile strength and other qualities of iron plate there shall be taken from each sheet to be used in shell or other parts of boiler which are subject to tensile strain a test piece prepared in form according to the following diagram, viz.: 10 inches in length, 2 inches in width, cut out in the centre in the manner indicated.



To ascertain the tensile strength and other qualities of *steel plate*, there shall be taken from each sheet to be used in shell or other parts of boiler which are subject to tensile strain a test-piece prepared in form according to the following diagram:

The straight part in centre shall be 9 inches in length and 1 inch in width, marked with light prick-punch marks at distances 1 inch apart, as shown, spaced so as to give 8 inches in length.



The sample must show when tested an elongation of at least 25% in a length of 2 in. for thickness up to ¼ in., inclusive; in a length of 4 in. for over ¼ to 7/16, inclusive; in a length of 6 in., for all plates over 7/16 in. and under 1¼ in. thickness.

The reduction of area shall be the same as called for by the rules of the Board. No plate shall contain more than .06% of phosphorus and .04% of sulphur.

The samples shall also be capable of being bent to a curve, of which the inner radius is not greater than 1½ times the thickness of the plates after having been heated uniformly to a low cherry-red and quenched in water of 82° F.

[Prior to 1894 the shape of test-piece for steel was the same as that for iron, viz., the grooved shape. This shape has been condemned by authorities on strength of materials for over twenty years. It always gives results which are too high, the error sometimes amounting to 25 per cent. See pages 242, 243, *ante*; also, Strength of Materials, W. Kent, Van N. Science Series No. 41, and Beardslee on Wrought-iron and Chain Cables.]

Ductility. (Section 6.)—To ascertain the ductility and other lawful qualities, iron of 45,000 lbs. tensile strength shall show a contraction of area of 15 per cent, and each additional 1000 lbs. tensile strength shall show 1 per cent additional contraction of area, up to and including 55,000 tensile strength. Iron of 55,000 tensile strength and upwards, showing 25 per cent reduction of area, shall be deemed to have the lawful ductility. All steel plate of ½ inch thickness and under shall show a contraction of area of not less than 50 per cent. Steel plate over ½ inch in thickness, up to ¾ inch in thickness, shall show a reduction of not less than 45 per cent. All steel plate over ¾ inch thickness shall show a reduction of not less than 40 per cent.

Bumped Heads of Boilers. (Section 17 as amended 1894.)—*Pressure Allowed on Bumped Heads.*—Multiply the thickness of the plate by one sixth of the tensile strength, and divide by six tenths of the radius to which head is bumped, which will give the pressure per square inch of steam allowed.

Pressure Allowable for Concaved Heads of Boilers.—Multiply the pressure per square inch allowable for bumped heads attached to boilers or drums convexly, by the constant .6, and the product will give the pressure per square inch allowable in concaved heads.

The pressure on unstayed flat-heads on steam-drums or shells of boilers, when flanged and made of wrought iron or steel or of cast steel, shall be determined by the following rule:

The thickness of plate in inches multiplied by one sixth of its tensile strength in pounds, which product divided by the area of the head in square inches multiplied by .09 will give pressure per square inch allowed. The material used in the construction of flat-heads when tensile strength has not been officially determined shall be deemed to have a tensile strength of 45,000 lbs.

Table of Pressures allowable on Steam-boilers made of Riveted Iron or Steel Plates.

(Abstract from a table published in Rules and Regulations of the U. S. Board of Supervising Inspectors of Steam-vessels.)

Plates ¼ inch thick. For other thicknesses, multiply by the ratio of the thickness to ¼ inch.

Diameter of Boiler, ins.	50,000 Tensile Strength.		55,000 Tensile Strength.		60,000 Tensile Strength.		65,000 Tensile Strength.		70,000 Tensile Strength.	
	Pressure.	20% Additional.	Pressure.	20% Additional.	Pressure.	20% Additional.	Pressure.	20% Additional.	Pressure.	20% Additional.
36	115.74	138.88	127.31	152.77	138.88	166.65	150.46	180.55	162.03	194.43
38	109.64	131.56	120.61	144.73	131.57	157.88	142.54	171.04	158.5	184.20
40	104.16	124.99	114.58	137.49	125	150	135.41	162.49	145.83	174.99
42	99.2	119.04	109.12	130.94	119.04	142.81	128.96	154.75	138.88	166.65
44	94.69	113.62	104.16	124.99	113.63	136.35	123.1	147.72	132.56	159.07
46	90.57	108.68	99.63	119.55	108.69	130.42	117.75	141.3	126.8	152.16
48	86.8	104.16	95.48	114.57	104.16	124.99	112.84	135.4	121.52	145.82
54	77.16	92.59	84.87	101.84	92.59	111.10	100.3	120.36	108.02	129.62
60	69.44	83.32	76.38	91.65	83.33	99.99	90.27	108.32	97.22	116.66
66	63.13	75.75	69.44	83.32	75.75	90.90	82.07	98.48	88.37	106.04
72	57.87	69.44	63.65	76.38	69.44	83.32	75.22	90.26	81.01	97.21
78	53.41	64.09	58.76	70.5	64.4	76.92	69.44	83.33	74.78	89.73
84	49.6	59.52	54.56	65.47	59.52	71.42	64.48	77.37	69.44	83.32
90	46.29	55.44	50.92	61.1	55.55	66.66	60.18	72.21	64.81	77.77
96	43.4	52.08	47.74	57.28	52.08	62.49	56.42	67.67	60.76	72.91

The figures under the columns headed "pressure" are for single-riveted boilers. Those under the columns headed "20% Additional" are for double-riveted.

U. S. RULE FOR ALLOWABLE PRESSURES.

The pressure of any dimension of boilers not found in the table annexed to these rules must be ascertained by the following rule:

Multiply one sixth of the lowest tensile strength found stamped on any plate in the cylindrical shell by the thickness (expressed in inches or parts of an inch) of the thinnest plate in the same cylindrical shell, and divide by the radius or half diameter (also expressed in inches), the quotient will be the pressure allowable per square inch of surface for single-riveting, to which add twenty per centum for double-riveting when all the rivet-holes in the shell of such boiler have been "fairly drilled" and no part of such hole has been punched.

The author desires to express his condemnation of the above rule, and of the tables derived from it, as giving too low a factor of safety. (See also criticism by Mr. Foley, page 701, *ante.*)

If Pb = bursting-pressure, t = thickness, T = tensile strength, c = coefficient of strength of riveted joint, that is, ratio of strength of the joint to that of the solid plate, d = diameter, $Pb = \frac{2tTc}{d}$, or if c be taken for double-riveting at 0.7, then $Pb = \frac{1.4tT}{d}$.

By the U. S. rule the allowable pressure $Pa = \frac{1/6tT}{\frac{1}{2}d} \times 1.20 = \frac{0.4tT}{d}$; whence $Pb = 3.5Pa$; that is, the factor of safety is only 3.5, provided the "tensile strength found stamped in the plate" is the real tensile strength of the material. But in the case of iron plates, since the stamped T.S. is obtained from a grooved specimen, it may be greatly in excess of the real T.S., which would make the factor of safety still lower. According to the table, a boiler 40 in. diam., ¼ in. thick, made of iron stamped 60,000 T.S., would be licensed to carry 150 lbs. pressure if double-riveted. If the real T.S. is only 50,000 lbs. the calculated bursting-strength would be

$$P = \frac{2tTc}{d} = \frac{2 \times 50{,}000 \times .25 \times .70}{40} = 437.5 \text{ lbs.},$$

and the factor of safety only $437.5 \div 150 = 2.91$!

The author's formula for safe working-pressure of externally-fired boilers with longitudinal seams double-riveted, is $P = \frac{14000t}{d}$; $t = \frac{Pd}{14000}$; P = gauge-pressure in lbs. per sq. in.; t = thickness and d = diam. in inches.

This is derived from the formula $P = \frac{2tTc}{fd}$, taking c at 0.7 and $f = 5$ for steel of 50,000 lbs. T.S., or 6 for 60,000 lbs. T.S.; the factor of safety being increased in the ratio of the T.S., since with the higher T.S. there is greater danger of cracking at the rivet-holes from the effect of punching and riveting and of expansion and contraction caused by variations of temperature. For external shells of internally-fired boilers, these shells not being exposed to the fire, with rivet-holes drilled or reamed after punching, a lower factor of safety and steel of a higher T.S. may be allowable.

If the T.S. is 60,000, a working pressure $P = \frac{16000t}{d}$ would give a factor of safety of 5.25.

The following table gives safe working pressures for different diameters of shell and thicknesses of plate calculated from the author's formula.

Safe Working Pressures in Cylindrical Shells of Boilers, Tanks, Pipes, etc., in Pounds per Square Inch.

Longitudinal seams double-riveted.
(Calculated from formula $P = 14{,}000 \times$ thickness $\div$ diameter.)

Thickness in 16ths of an Inch.	Diameter in Inches.										
	24	30	36	38	40	42	44	46	48	50	52
1	36.5	29.2	24.3	23.0	21.9	20.8	19.9	19.0	18.2	17.5	16.8
2	72.9	58.3	48.6	46.1	43.8	41.7	39.8	38.0	36.5	35.0	33.7
3	109.4	87.5	72.9	69.1	65.6	62.5	59.7	57.1	54.7	52.5	50.5
4	145.8	116.7	97.2	92.1	87.5	83.3	79.5	76.1	72.9	70.0	67.3
5	182.3	145.8	121.5	115.1	109.4	104.2	99.4	95.1	91.1	87.5	84.1
6	218.7	175.0	145.8	138.2	131.3	125.0	119.3	114.1	109.4	105.0	101.0
7	255.2	204.1	170.1	161.2	153.1	145.9	139.2	133.2	127.6	122.5	117.8
8	291.7	233.3	194.4	184.2	175.0	166.7	159.1	152.2	145.8	140.0	134.6
9	328.1	262.5	218.8	207.2	196.9	187.5	179.0	171.2	164.1	157.5	151.4
10	364.6	291.7	243.1	230.3	218.8	208.3	198.9	190.2	182.3	175.0	168.3
11	401.0	320.8	267.4	253.3	240.6	229.2	218.7	209.2	200.5	192.5	185.1
12	437.5	350.0	291.7	276.3	262.5	250.0	238.6	228.3	218.7	210.0	201.9
13	473.9	379.2	316.0	299.3	284.4	270.9	258.5	247.3	337.0	227.5	218.8
14	410.4	408.3	340.3	322.4	306.3	291.7	278.4	266.3	255.2	245.0	235.6
15	546.9	437.5	364.6	345.4	328.1	312.5	298.3	285.3	273.4	266.5	252.4
16	583.3	466.7	388.9	368.4	350.0	333.3	318.2	304.4	291.7	280.0	269.2

Thickness in 16ths of an Inch.	Diameter in Inches.											
	54	60	66	72	78	84	90	96	102	108	114	120
1	16.2	14.6	13.3	12.2	11.2	10.4	9.7	9.1	8.6	8.1	7.7	7.3
2	32.4	29.2	26.5	24.3	22.4	20.8	19.4	18.2	17.2	16.2	15.4	14.6
3	48.6	43.7	39.8	36.5	33.7	31.3	29.2	27.3	25.7	24.3	23.0	21.9
4	64.8	58.3	53.0	48.6	44.9	41.7	38.9	36.5	34.3	32.4	30.7	29.2
5	81.0	72.9	66.3	60.8	56.1	52.1	48.6	45.6	42.9	40.5	38.4	36.5
6	97.2	87.5	79.5	72.9	67.3	62.5	58.3	54.7	51.5	48.6	46.1	43.8
7	113.4	102.1	92.8	85.1	78.5	72.9	68.1	63.8	60.0	56.7	53.7	51.0
8	129.6	116.7	106.1	97.2	89.7	83.3	77.8	72.9	68.6	64.8	61.4	58.3
9	145.8	131.2	119.3	109.4	101.0	93.8	87.5	82.0	77.2	72.9	69.1	65.6
10	162.0	145.8	132.6	121.5	112.2	104.2	97.2	91.1	85.8	81.0	76.8	72.9
11	178.2	160.4	145.8	133.7	123.4	114.6	106.9	100.3	94.4	89.1	84.4	80.2
12	194.4	175.0	159.1	145.8	134.6	125.0	116.7	109.4	102.9	97.2	92.1	87.5
13	210.7	189.6	172.4	158.0	145.8	135.4	126.4	118.5	111.5	105.3	99.8	94.8
14	226.9	204.2	185.6	170.1	157.1	145.8	136.1	127.6	120.1	113.4	107.5	102.1
15	243.1	218.7	198.9	182.3	168.3	156.3	145.8	136.7	128.7	121.5	115.1	109.4
16	259.3	233.3	212.1	194.4	179.5	166.7	155.6	145.8	137.3	129.6	122.8	116.7

Rules governing Inspection of Boilers in Philadelphia.

In estimating the strength of the longitudinal seams in the cylindrical shells of boilers the inspector shall apply two formulæ, A and B:

$$\text{A,}\quad \frac{\text{Pitch of rivets} - \text{diameter of holes punched to receive the rivets}}{\text{pitch of rivets}} =$$

percentage of strength of the sheet at the seam.

$$\text{B,}\quad \frac{\text{Area of hole filled by rivet} \times \text{No. of rows of rivets in seam} \times \text{shearing strength of rivet}}{\text{pitch of rivets} \times \text{thickness of sheet} \times \text{tensile strength of sheet}} =$$

percentage of strength of the rivets in the seam.

Take the lowest of the percentages as found by formulæ A and B and apply that percentage as the "strength of the seam" in the following formula C, which determines the strength of the longitudinal seams:

$$\text{C,}\quad \frac{\text{Thickness of sheet in parts of inch} \times \text{strength of seam as obtained by formula A or B} \times \text{ultimate strength of iron stamped on plates}}{\text{internal radius of boiler in inches} \times 5 \text{ as a factor of safety}} =$$

safe working pressure.

TABLE OF PROPORTIONS AND SAFE WORKING PRESSURES WITH FORMULÆ A AND C, @ 50,000 LBS., T.S.

Diameter of rivet.	⅝″	11/16	¾	13/16	⅞
Diameter of rivet-hole.	11/16″	¾	13/16	⅞	15/16
Pitch of rivets.	2″	2 1/16	2⅛	2 3/16	2¼
Strength of seam, %.	.656	.636	.62	.60	.58
Thickness of plate.	¼″	5/16	⅜	7/16	½
Diameter of boiler, in.	Safe Working Pressure with Longitudinal Seams Single-riveted.				
24	137	165	193	220	242
30	109	132	154	176	194
32	102	124	144	165	182
34	96	117	136	155	171
36	91	110	129	147	161
38	86	104	122	139	153
40	82	99	116	132	145
44	74	91	105	120	132
48	68	83	96	110	121
54	60	73	86	98	107
60	55	66	77	88	97

Diameter of rivet........	5/8"	11/16	3/4	13/16	7/8
Diameter of rivet-hole...	11/16"	3/4	13/16	7/8	15/16
Pitch of rivets...........	3"	3 1/8	3 1/4	3 3/8	3 1/2
Strength of seam, %.	.77	.76	.75	.74	.73
Thickness of plate..	1/4"	5/16	3/8	7/16	1/2
Diameter of boiler, in...	Safe Working Pressure with Longitudinal Seams, Double-riveted.				
24	160	198	235	269	305
30	127	158	188	215	243
32	119	148	176	202	228
34	112	140	166	190	215
36	106	132	156	179	203
38	101	125	148	170	192
40	96	119	141	161	183
44	87	108	128	147	166
48	79	99	118	135	152
54	70	88	104	120	135
60	64	79	94	108	122

Flues and Tubes for Steam-boilers.—(From Rules of U. S. Supervising Inspectors. Steam-pressures per square inch allowable on riveted and lap-welded flues made in sections. Extract from table in Rules of U. S. Supervising Inspectors.)

T = least thickness of material allowable, D = greatest diameter in inches, P = allowable pressure. For thickness greater than T with same diameter P is increased in the ratio of the thickness.

D = in.	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
T = in.	.18	.20	.21	.21	.22	.22	.23	.24	.25	.26	.27	.28	.29	.30	.31	.32	.33
P = lbs.	189	184	179	174	172	158	152	147	143	139	136	134	131	129	126	125	123
D = in.	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
T = in.	.34	.35	.36	.37	.38	.39	.40	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
P = lbs.	121	120	119	117	116	115	115	114	112	112	110	110	109	109	108	108	107

For diameters not over 10 inches the greatest length of section allowable is 5 feet; for diameters 10 to 22 inches, 3 feet; for diameters 23 to 40 inches, 30 inches. If lengths of sections are greater than these lengths, the allowable pressure is reduced proportionately.

The U. S. rule for corrugated flues, as amended in 1894, is as follows: Rule II, Section 14. The strength of all corrugated flues, when used for furnaces or steam chimneys (corrugation not less than 1 1/4 inches deep and not exceeding 8 inches from centres of corrugation), and provided that the plain parts at the ends do not exceed 6 inches in length, and the plates are not less than 5/16 inch thick, when new, corrugated, and practically true circles, to be calculated from the following formula:

$$\frac{14,000}{D} \times T = \text{pressure}.$$

T = thickness, in inches; D = mean diameter in inches.

Ribbed Flues.—The same formula is given for ribbed flues, with rib projections not less than 1 3/8 inches deep and not more than 9 inches apart.

Flat Stayed Surfaces in Steam-boilers.—Rule II., Section 6, of the rules of the U. S. Supervising Inspectors provides as follows:

No braces or stays hereafter employed in the construction of boilers shall be allowed a greater strain than 6000 lbs. per square inch of section.

Clark, in his treatise on the Steam-engine, also in his Pocket-book, gives the following formula: $p = 407ts \div d$, in which p is the internal pressure in pounds per square inch that will strain the plates to their elastic limit, t is the thickness of the plate in inches, d is the distance between two rows of stay-bolts in the clear, and s is the tensile stress in the plate in tons of 2240 lbs. per square inch, at the elastic limit. Substituting values of s for iron, steel, and copper, 12, 14, and 8 tons respectively, we have the following:

FORMULÆ FOR ULTIMATE ELASTIC STRENGTH OF FLAT STAYED SURFACES.

	Iron.	Steel.	Copper.
Pressure....................	$p = 5000\frac{t}{d}$	$p = 5700\frac{t}{d}$	$p = 3300\frac{t}{d}$
Thickness of plate.........	$t = \frac{p \times d}{5000}$	$t = \frac{p \times d}{5700}$	$t = \frac{p \times d}{3300}$
Pitch of bolts..............	$d = \frac{5000t}{p}$	$d = \frac{5700t}{p}$	$d = \frac{3300t}{p}$

For Diameter of the Stay-bolts, Clark gives $d' = .0024\sqrt{\frac{PP'p}{s}}$, in which d' = diameter of screwed bolt at bottom of thread, P = longitudinal and P' transverse pitch of stay-bolts between centres, p = internal pressure in lbs. per sq. in. that will strain the plate to its elastic limit, s = elastic strength of the stay-bolts in lbs. per sq. in. Taking s = 12, 14, and 8 tons, respectively for iron, steel, and copper, we have

For iron, $d' = .00069\sqrt{PP'p}$, or if $P = P'$, $d' = .00069P\sqrt{p}$;
For steel, $d' = .00064\sqrt{PP'p}$, " " $d' = .00064P\sqrt{p}$;
For copper, $d' = .00084\sqrt{PP'p}$, " " $d' = .00084P\sqrt{p}$.

In using these formulæ a large factor of safety should be taken to allow for reduction of size by corrosion. Thurston's Manual of Steam-boilers, p. 144, recommends that the factor be as large as 15 or 20. The Hartford Steam Boiler Insp. & Ins. Co. recommends not less than 10.

Strength of Stays.—A. F. Yarrow (*Engr.*, March 20, 1891) gives the following results of experiments to ascertain the strength of water-space stays:

Description.	Length between Plates.	Diameter of Stay over Threads.	Ultimate Stress.
			lbs.
Hollow stays screwed into plates and hole expanded	4.75 in.	1 in. (hole 7/16 in. and 5/16 in.	25,457
	4.64 in.	1 in. (hole 9/16 in. and 7/16 in.	20,992
Solid stays screwed into plates and riveted over.	4.80 in.	⅞ in.	22,008
	4.80 in.	⅞ in.	23,070

The above are taken as a fair average of numerous tests.

Stay-bolts in Curved Surfaces, as in Water-legs of Vertical Boilers.—The rules of the U. S. Supervising Inspectors provide as follows: All vertical boiler-furnaces constructed of wrought iron or steel plates, and having a diameter of over 42 in. or a height of over 40 in. shall be stayed with bolts as provided by § 6 of Rule II, for flat surfaces; and the thickness of material required for the shells of such furnaces shall be determined by the distance between the centres of the stay-bolts in the furnace and not in the shell of the boiler; and the steam-pressure allowable shall be determined by the distance from centre of stay-bolts in the furnace and the diameter of such stay-bolts at the bottom of the thread.

The Hartford Steam-boiler Insp. & Ins. Co. approves the above rule (*The Locomotive*, March, 1892) as far as it states that curved surfaces are to be computed the same as flat ones, but prefers Clark's formulæ for flat stayed surfaces to the rules of the U. S. Supervising Inspectors.

Fusible-plugs.—Fusible-plugs should be put in that portion of the heating-surface which first becomes exposed from lack of water. The rules of the U. S. Supervising Inspectors specify Banca tin for the purpose. Its melting-point is about 445° F. The rule says: All steamers shall have inserted in their boilers plugs of Banca tin, at least ½ in. in diameter at the smallest end of the internal opening, in the following manner, to wit: Cylinder-boilers with flues shall have one plug inserted in one flue of each boiler; and also one plug inserted in the shell of each boiler from the inside, immediately before the fire line and not less than 4 ft. from the forward end of the boiler. All fire-box boilers shall have one plug inserted in the crown of the back connection, or in the highest fire-surface of the boiler.

All upright tubular boilers used for marine purposes shall have a fusible plug inserted in one of the tubes at a point at least 2 in. below the lower gauge-cock, and said plug may be placed in the upper head sheet when deemed advisable by the local inspectors.

Steam-domes.—Steam domes or drums were formerly almost universally used on horizontal boilers, but their use is now generally discontinued, as they are considered a useless appendage to a steam-boiler, and unless properly designed and constructed are an element of weakness.

Height of Furnace.—Recent practice in the United States makes the height of furnace much greater than it was formerly. With large sizes of anthracite there is no serious objection to having the furnace as low as 12 to 18 in., measured from the surface of the grate to the nearest portion of the heating surface of the boiler, but with coal containing much volatile matter and moisture a much greater distance is desirable. With very volatile coals the distance may be as great as 4 or 5 ft. Rankine (S. E., p. 457) says: The clear height of the "crown" or roof of the furnace above the grate-bars is seldom less than about 18 in., and often considerably more. In the fire-boxes of locomotives it is on an average about 4 ft. The height of 18 in. is suitable where the crown of the furnace is a brick arch. Where the crown of the furnace, on the other hand, forms part of the heating-surface of the boiler, a greater height is desirable in every case in which it can be obtained; for the temperature of the boiler-plates, being much lower than that of the flame, tends to check the combustion of the inflammable gases which rise from the fuel. As a general principle a high furnace is favorable to complete combustion.

IMPROVED METHODS OF FEEDING COAL.

Mechanical Stokers. (William R. Roney, Trans. A. S. M. E., vol. xii.)—Mechanical stokers have been used in England to a limited extent since 1785. In that year one was patented by James Watt. It was a simple device to push the coal, after it was coked at the front end of the grate, back towards the bridge. It was worked intermittently by levers, and was designed primarily to prevent smoke from bituminous coal. (See D. K. Clark's Treatise on the Steam-engine.)

After the year 1840 many styles of mechanical stokers were patented in England, but nearly all were variations and modifications of the two forms of stokers patented by John Jukes in 1841, and by E. Henderson in 1843.

The Jukes stoker consisted of longitudinal fire-bars, connected by links, so as to form an endless chain, similar to the familiar treadmill horse-power. The small coal was delivered from a hopper on the front of the boiler, on to the grate, which slowly moving from front to rear, gradually advanced the fuel into the furnace and discharged the ash and clinker at the back.

The Henderson stoker consists primarily of two horizontal fans revolving on vertical spindles, which scatter the coal over the fire.

Numerous faults in mechanical construction and in operation have limited the use of these and other mechanical stokers. The first American stoker was the Murphy stoker, brought out in 1878. It consists of two coal magazines placed in the side walls of the boiler furnace, and extending back from the boiler front 6 or 7 feet. In the bottom of these magazines are rectangular iron boxes, which are moved from side to side by means of a rack and pinion, and serve to push the coal upon the grates, which incline at an angle of about 35° from the inner edge of the coal magazines, forming a V-shaped receptacle for the burning coal. The grates are composed of narrow parallel bars, so arranged that each alternate bar lifts about an inch at the lower end, while at the bottom of the V, and filling the space between the ends of the grate-bars, is placed a cast-iron toothed bar, arranged to be turned by a crank. The purpose of this bar is to grind the clinker coming in contact with it. Over this V-shaped receptacle is sprung a fire-brick arch.

In the Roney mechanical stoker the fuel to be burned is dumped into a hopper on the boiler front. Set in the lower part of the hopper is a "pusher" to which is attached the "feed-plate" forming the bottom of the hopper. The "pusher," by a vibratory motion, carrying with it the "feed-plate," gradually forces the fuel over the "dead-plate" and on the grate. The grate-bars, in their normal condition form a series of steps, to the top step of which coal is fed from the "dead-plate." Each bar rests in a concave seat in the bearer, and is capable of a rocking motion through an adjustable angle. All the grate-bars are coupled together by a "rocker-bar." A variable back-and-forth motion being given to the "rocker-bar," through a connecting-rod, the grate-bars rock in unison, now forming a series of steps, and now approximating to an inclined plane, with the grates partly overlapping, like shingles on a roof. When the grate-bars rock forward the fire will tend to work down in a body. But before the coal can move too far the bars rock back to the stepped position, checking the downward motion, breaking up the cake over the whole surface, and admitting a free volume of air through the fire. The rocking motion is slow, being from 7 to 10 strokes per minute, according to the kind of coal. This alternate starting and checking motion is continuous, and finally lands the cinder and ash on the dumping-grate below.

Mr. Roney gives the following record of six tests to determine the comparative economy of the Roney mechanical stoker and hand-firing on return tubular boilers, 60 inches × 20 feet, burning Cumberland coal with natural draught. Rating of boiler at 12.5 square feet, 105 H. P.

	Three tests, hand-firing.			Three tests, Stoker.		
Evaporation per pound, dry coal from and at 212° lbs	10.36	10.44	11.00	11.89	12.25	12.54
H.P. developed above rating, %	5.8	13.5	68	54.6	66.7	84.3

Results of comparative tests like the above should be used with caution in drawing generalizations. It by no means follows from these results that a stoker will always show such comparative excellence, for in this case the results of hand-firing are much below what may be obtained under favorable circumstances from hand-firing with good Cumberland coal.

The Hawley Down-draught Furnace.—A foot or more above the ordinary grate there is carried a second grate composed of a series of water-tubes, opening at both ends into steel drums or headers, through which water is circulated. The coal is fed on this upper grate, and as it is partially consumed falls through it upon the lower grate, where the combustion is completed in the ordinary manner. The draught through the coal on the upper grate is downward through the coal and the grate. The volatile gases are therefore carried down through the bed of coal, where they are thoroughly heated, and are burned in the space beneath, where they meet the excess of hot air drawn through the fire on the lower grate. In tests in Chicago, from 30 to 45 lbs. of coal were burned per square foot of grate upon this system, with good economical results. (See catalogue of the Hawley Down Draught Furnace Co., Chicago.)

Under-feed Stokers.—Results similar to those that may be obtained with downward draught are obtained by feeding the coal at the bottom of the bed, pushing upward the coal already on the bed which has had its volatile matter distilled from it. The volatile matter of the freshly fired coal then has to pass through a body of ignited coke, where it meets a supply of hot air. (See circular of The American Stoker Co., New York, 1898.)

SMOKE PREVENTION.

A committee of experts was appointed in St. Louis in 1891 to report on the smoke problem. A summary of its report is given in the *Iron Age* of April 7, 1892. It describes the different means that have been tried to prevent smoke, such as gas-fuel, steam-jets, fire-brick arches and checker-work, hollow walls for preheating air, coking arches or chambers, double combustion furnaces, and automatic stokers. All of these means have been more or less effective in diminishing smoke, their effectiveness depending largely upon the skill with which they are operated; but none is entirely satisfactory. Fuel-gas is objectionable chiefly on account of its expense. The average quality of fuel-gas made from a trial run of several car-loads of Illinois coal, in a well-designed fuel-gas plant, showed a calorific value of 243,391 heat-units per 1000 cubic feet. This is equivalent to 5052.8 heat units per lb. of coal, whereas by direct calorimeter test an average sample of the coal gave 11,172 heat-units. One lb. of the coal showed a theoretical evaporation of 11.56 lbs. water, while the gas from 1 lb. showed a theoretical evaporation of 5.23 lbs. 48.17 lbs. of coal were required to furnish 1000 cubic feet of the gas. In 30 tests the smoke-preventing furnaces showed only 74% of the capacity of the common furnaces, reduced the work of the boilers 28%, and required about 2% more fuel to do the same work. In one case with steam-jets the fuel consumption was increased 12% for the same work.

Prof. O. H. Landreth, in a report to the State Board of Health of Tennessee (*Engineering News*, June 8, 1893), writes as follows on the subject of smoke prevention:

As pertains to steam-boilers, the object must be attained by one or more of the following agencies:

1. Proper design and setting of the boiler-plant. This implies proper grate area, sufficient draught, the necessary air-space between grate-bars and through furnace, and ample combustion-room under boilers.

2. That system of firing that is best adapted to each particular furnace to secure the perfect combustion of bituminous coal. This may be either: (*a*) "coke-firing," or charging all coal into the front of the furnace until partially coked, then pushing back and spreading; or (*b*) "alternate side-firing"; or (*c*) "spreading," by which the coal is spread over the whole grate area in thin, uniform layers at each charging.

3. The admission of air through the furnace-door, bridge-wall, or side walls.

4. Steam-jets and other artificial means for thoroughly mixing the air and combustible gases.

5. Preventing the cooling of the furnace and boilers by the inrush of cold air when the furnace-doors are opened for charging coal and handling the fire.

6. Establishing a gradation of the several steps of combustion so that the coal may be charged, dried, and warmed at the coolest part of the furnace, and then moved by successive steps to the hottest place, where the final combustion of the coked coal is completed, and compelling the distilled combustible gases to pass through this hottest part of the fire.

7. Preventing the cooling by radiation of the unburned combustible gases until perfect mixing and combustion have been accomplished.

8. Varying the supply of air to suit the periodic variation in demand.

9. The substitution of a continuous uniform feeding of coal instead of intermittent charging.

10. Down-draught burning or causing the air to enter above the grate and pass down through the coal, carrying the distilled products down to the high temperature plane at the bottom of the fire.

The number of smoke-prevention devices which have been invented is legion. A brief classification is:

(*a*) Mechanical stokers. They effect a material saving in the labor of firing, and are efficient smoke-preventers when not pushed above their capacity, and when the coal does not cake badly. They are rarely susceptible to the sudden changes in the rate of firing frequently demanded in service.

(*b*) Air-flues in side walls, bridge-wall, and grate-bars, through which air when passing is heated. The results are always beneficial, but the flues are difficult to keep clean and in order.

(*c*) Coking arches, or spaces in front of the furnace arched over, in which the fresh coal is coked, both to prevent cooling of the distilled gases, and to force them to pass through the hottest part of the furnace just beyond the arch. The results are good for normal conditions, but ineffective when the fires are forced. The arches also are easily burned out and injured by working the fire.

(*d*) Dead-plates, or a portion of the grate next the furnace-doors, reserved for warming and coking the coal before it is spread over the grate. These give good results when the furnace is not forced above its normal capacity. This embodies the method of "coke-firing" mentioned before.

(*e*) Down-draught furnaces, or furnaces in which the air is supplied to the coal above the grate, and the products of combustion are taken away from beneath the grate, thus causing a downward draught through the coal, carrying the distilled gases down to the highly heated incandescent coal at the bottom of the layer of coal on the grate. This is the most perfect manner of producing combustion, and is absolutely smokeless.

(*f*) Steam jets to draw air in or inject air into the furnace above the grate, and also to mix the air and the combustible gases together. A very efficient smoke-preventer, but one liable to be wasteful of fuel by inducing too rapid a draught.

(*g*) Baffle-plates placed in the furnace above the fire to aid in mixing the combustible gases with the air.

(*h*) Double furnaces, of which there are two different styles; the first of which places the second grate below the first grate; the coal is coked on the first grate, during which process the distilled gases are made to pass over the second grate, where they are ignited and burned; the coke from the first grate is dropped onto the second grate: a very efficient and economical smoke-preventer, but rather complicated to construct and maintain. In the second form the products of combustion from the first furnace pass through the grate and fire of the second, each furnace being charged with fresh fuel when needed, the latter generally with a smokeless coal or coke: an irrational and unpromising method.

Mr. C. F. White, Consulting Engineer to the Chicago Society for the Prevention of Smoke, writes under date of May 4, 1893:

The experience had in Chicago has shown plainly that it is perfectly easy to equip steam-boilers with furnaces which shall burn ordinary soft coal in such a manner that the making of smoke dense enough to obstruct the vision shall be confined to one or two intervals of perhaps a couple of minutes' duration in the ordinary day of 10 hours.

Gas-fired Steam-boilers.—Converting coal into gas in a separate producer, before burning it under the steam-boiler, is an ideal method of smoke-prevention, but its expense has hitherto prevented its general introduction. A series of articles on the subject, illustrating a great number of devices, by F. J. Rowan, is published in the *Colliery Engineer*, 1889–90. See also Clark on the Steam-engine.

FORCED COMBUSTION IN STEAM-BOILERS.

For the purpose of increasing the amount of steam that can be generated by a boiler of a given size, forced draught is of great importance. It is universally used in the locomotive, the draught being obtained by a steam-jet in the smoke-stack. It is now largely used in ocean steamers, especially in ships of war, and to a small extent in stationary boilers. Economy of fuel is generally not attained by its use, its advantages being confined to the securing of increased capacity from a boiler of a given bulk, weight, or cost. The subject of forced draught is well treated in a paper by James Howden, entitled, "Forced Combustion in Steam-boilers" (Section G, Engineering Congress at Chicago, in 1893), from which we abstract the following:

Edwin A. Stevens at Bordentown, N. J., in 1827, in the steamer "North America," fitted the boilers with closed ash-pits, into which the air of combustion was forced by a fan. In 1828 Ericsson fitted in a similar manner the steamer "Victory," commanded by Sir John Ross.

Messrs. E. A. and R. L. Stevens continued the use of forced draught for a considerable period, during which they tried three different modes of using the fan for promoting combustion: 1, blowing direct into a closed ash-pit; 2, exhausting the base of the funnel by the suction of the fan; 3, forcing air into an air-tight boiler-room or stoke-hold. Each of these three methods was attended with serious difficulties.

In the use of the closed ash-pit the blast-pressure would frequently force the gases of combustion, in the shape of a serrated flame, from the joint around the furnace doors in so great a quantity as to affect both the efficiency and health of the firemen.

The chief defect of the second plan was the great size of the fan required to produce the necessary exhaustion. The size of fan required grows in a rapidly increasing ratio as the combustion increases, both on account of the greater air-supply and the higher exit temperature enlarging the volume of the waste gases.

The third method, that of forcing cold air by the fan into an air-tight boiler-room—the present closed stoke-hold system—though it overcame the difficulties in working belonging to the two forms first tried, has serious defects of its own, as it cannot be worked, even with modern high-class boiler-construction, much, if at all, above the power of a good chimney draught, in most boilers, without damaging them.

In 1875 John I. Thornycroft & Co., of London, began the construction of torpedo-boats with boilers of the locomotive type, in which a high rate of combustion was attained by means of the air-tight boiler-room, into which air was forced by means of a fan.

In 1882 H.B.M. ships "Satellite" and "Conqueror" were fitted with this system, the former being a small ship of 1500 I.H.P., and the latter an iron-clad of 4500 I.H.P. On the trials with forced draught, which lasted from two to three hours each, the highest rates of combustion gave 16.9 I.H.P. per square foot of fire-grate in the "Satellite," and 13.41 I.H.P. in the "Conqueror."

None of the short trials at these rates of combustion were made without injury to the seams and tubes of the boilers, but the system was adopted, and it has been continued in the British Navy to this day (1893).

In Mr. Howden's opinion no advantage arising from increased combustion over natural-draught rates is derived from using forced draught in a closed ash-pit sufficient to compensate the disadvantages arising from difficulties in working, there being either excessive smoke from bituminous coal or reduced evaporative economy.

In 1880 Mr. Howden designed an arrangement intended to overcome the defects of both the closed ash-pit and closed stoke-hold systems.

An air-tight reservoir or chamber is placed on the front end of the boiler and surrounding the furnaces. This reservoir, which projects from 8 to 10 inches from the end of the boiler, receives the air under pressure, which is passed by the valves into the ash-pits and over the fires in proportions suited to the kind of fuel used and the rate of combustion required. The air used above the fires is admitted to a space between the outer and inner furnace-doors, the inner having perforations and an air-distributing box through which the air passes under pressure.

By means of the balance of air-pressure above and below the fires all tendency for the fire to blow out at the furnace-door is removed.

By regulating the admission of the air by the valves above and below the fires, the highest rate of combustion possible by the air-pressure used can be effected, and in same manner the rate of combustion can be reduced to far below that of natural draught, while complete and economical combustion at all rates is secured.

A feature of the system is the combination of the heating of the air of combustion by the waste gases with the controlled and regulated admission of air to the furnaces. This arrangement is effected most conveniently by passing the hot fire-gases after they leave the boiler through stacks of vertical tubes enclosed in the uptake, their lower ends being immediately above the smoke-box doors.

Installations on Howden's system have hitherto been arranged for a rate of combustion to give at full sea-power an average of from 18 to 22 I.H.P. per square foot of fire-grate with fire-bars from 5′ 0″ to 5′ 6″ in length.

It is believed that with suitable arrangement of proportions even 30 I.H.P. per square foot can be obtained.

For an account of recent uses of exhaust-fans for increasing draught, see paper by W. R. Roney, Trans. A. S. M. E., vol. xv.

FUEL ECONOMIZERS.

Green's Fuel Economizer.—Clark gives the following average results of comparative trials of three boilers at Wigan used with and without economizers:

	Without Economizers.	With Economizers.
Coal per square foot of grate per hour......	21.6	21.4
Water at 100° evaporated per hour..........	73.55	79.32
Water at 212° per pound of coal	9.60	10.56

Showing that in burning equal quantities of coal per hour the rapidity of evaporation is increased 9.3% and the efficiency of evaporation 10% by the addition of the economizer.

The average temperatures of the gases and of the feed-water before and after passing the economizer were as follows:

	With 6-ft. grate.		With 4-ft. grate.	
	Before.	After.	Before.	After.
Average temperature of gases.......	649	340	501	312
Average temperature of feed-water.	47	157	41	137

Taking averages of the two grates, to raise the temperature of the feed-water 100° the gases were cooled down 250°.

Performance of a Green Economizer with a Smoky Coal.—The action of Green's Economizer was tested by M. W. Grosseteste for a period of three weeks. The apparatus consists of four ranges of vertical pipes, 6½ feet high, 3¾ inches in diameter outside, nine pipes in each range, connected at top and bottom by horizontal pipes. The water enters all the tubes from below, and leaves them from above. The system of pipes is enveloped in a brick casing, into which the gaseous products of combustion are introduced from above, and which they leave from below. The pipes are cleared of soot externally by automatic scrapers. The capacity for water is 24 cubic feet, and the total external heating-surface is 290 square feet. The apparatus is placed in connection with a boiler having 355 square feet of surface.

This apparatus had been at work for seven weeks continuously without having been cleaned, and had accumulated a ⅛-inch coating of soot and ash, when its performance, in the same condition, was observed for one week. During the second week it was cleaned twice every day; but during the third week, after having been cleaned on Monday morning, it was worked continuously without further cleaning. A smoke-making coal was used. The consumption was maintained sensibly constant from day to day.

Green's Economizer.—Results of Experiments on its Efficiency as affected by the State of the Surface. (W. Grosseteste.)

Time (February and March).	Temperature of Feed-water.			Temperature of Gaseous Products.		
	Entering Feed-heater.	Leaving Feed-heater.	Difference.	Entering Feed-heater.	Leaving Feed-heater.	Difference.
	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.
1st Week	73.5°	161.5°	88.0°	649°	261°	388°
2d Week	77.0	230.0	153.0	862	297	565
3d Week—Monday	78.4	196.0	117.6	861	294	547
Tuesday	78.4	181.4	103.0	871	309	562
Wednesday	79.0	178.0	99.0	—	—	—
Thursday	80.6	170.6	90.0	952	329	623
Friday	80.6	169.0	88.4	869	368	551
Saturday	79.0	172.4	93.4	901	351	550

	1st Week.	2d Week.	3d Week.
Coal consumed per hour	214 lbs.	216 lbs.	218 lbs.
Water evaporated from 32° F. per hour	1424	1525	1468
Water per pound of coal	6.65	7.06	6.70

It is apparent that there is a great advantage in cleaning the pipes daily—the elevation of temperature having been increased by it from 88° to 153°. In the third week, without cleaning, the elevation of temperature relapsed in three days to the level of the first week; even on the first day it was quickly reduced by as much as half the extent of relapse. By cleaning the pipes daily an increased elevation of temperature of 65° F., was obtained, whilst a gain of 6% was effected in the evaporative efficiency.

INCRUSTATION AND CORROSION.

Incrustation and Scale.—Incrustation (as distinguished from mere sediments due to dirty water, which are easily blown out, or gathered up, by means of sediment-collectors) is due to the presence of salts in the feed-water (carbonates and sulphates of lime and magnesia for the most part), which are precipitated when the water is heated, and form hard deposits upon the boiler-plates. (See Impurities in Water, p. 551, *ante*.)

Where the quantity of these salts is not very large (12 grains per gallon, say) scale preventives may be found effective. The chemical preventives either form with the salts other salts soluble in hot water; or precipitate them in the form of soft mud, which does not adhere to the plates, and can be washed out from time to time. The selection of the chemical must depend upon the composition of the water, and it should be introduced regularly with the feed.

Examples.—Sulphate-of-lime scale prevented by carbonate of soda: The sulphate of soda produced is soluble in water; and the carbonate of lime falls down in grains, does not adhere to the plates, and may therefore be blown out or gathered into sediment-collectors. The chemical reaction is:

Sulphate of lime + Carbonate of soda = Sulphate of soda + Carbonate of lime
$$CaSO_4 + Na_2CO_3 = Na_2SO_4 + CaCO_3$$

Sodium phosphate will decompose the sulphates of lime and magnesia:

Sulphate of lime + Sodium phosphate = Calcium phos. + Sulphate of soda.
$$CaSO_4 + Na_2HPO_4 = CaHPO_4 + Na_2SO_4$$

Sul. of magnesia + Sodium phosphate = Phosphate of magnesia + Sul. of soda.
$$MgSO_4 + Na_2HPO_4 = MgHPO_4 + Na_2SO_4$$

Where the quantity of salts is large, scale preventives are not of much use. Some other source of supply must be sought, or the bad water purified before it is allowed to enter the boilers. The damage done to boilers by unsuitable water is enormous.

Pure water may be obtained by collecting rain, or condensing steam by means of surface condensers. The water thus obtained should be mixed with a little bad water, or treated with a little alkali, as undiluted, pure water corrodes iron; or, after each periodic cleaning, the bad may be used for a day or two to put a skin upon the plates.

Carbonate of lime and magnesia may be precipitated either by heating the water or by mixing milk of lime (Porter Clark process) with it, the water being then filtered.

Corrosion may be produced by the use of pure water, or by the presence of acids in the water, caused perhaps in the engine-cylinder by the action of high-pressure steam upon the grease, resulting in the production of fatty acids. Acid water may be neutralized by the addition of lime.

Amount of Sediment which may collect in a 100-H.P. steam-boiler, evaporating 3000 lbs. of water per hour, the water containing different amounts of impurity in solution, provided that no water is blown off:

Grains of solid impurities per U. S. gallon:	5	10	20	30	40	50	60	70	80	90	100
Equivalent parts per 100,000:	8.57	17.14	34.28	51.42	68.56	85.71	102.85	120	137.1	154.3	171.4
Sediment deposited in 1 hour, pounds:	.257	.514	1.028	1.542	2.056	2.571	3.085	3.6	4.11	4.63	5.14
In one day of 10 hours, pounds:	2.57	5.14	10.28	15.42	20.56	25.71	30.85	36.0	41.1	46.3	51.4
In one week of 6 days, pounds:	15.42	30.85	61.7	92.55	123.4	154.3	185.1	216.0	246.8	277.6	308.5

If a 100-H.P. boiler has 1200 sq. ft. heating-surface, one week's running without blowing off, with water containing 100 grains of solid matter per gallon in solution, would make a scale nearly .02 in. thick, if evenly deposited all over the heating-surface, assuming the scale to have a sp. gr. of $2.5 = 156$ lbs. per cu. ft.; $.02 \times 1200 \times 156 \times 1/12 = 312$ lbs.

Boiler-scale Compounds.—The Bavarian Steam-boiler Inspection Assn. in 1885 reported as follows:

Generally the unusual substances in water can be retained in soluble form or precipitated as mud by adding caustic soda or lime. This is especially desirable when the boilers have small interior spaces.

It is necessary to have a chemical analysis of the water in order to fully determine the kind and quantity of the preparation to be used for the above purpose.

All secret compounds for removing boiler-scale should be avoided. (A list of 27 such compounds manufactured and sold by German firms is then given which have been analyzed by the association.)

Such secret preparations are either nonsensical or fraudulent, or contain either one of the two substances recommended by the association for removing scale, generally soda, which is colored to conceal its presence, and sometimes adulterated with useless or even injurious matter.

These additions as well as giving the compound some strange, fanciful name, are meant simply to deceive the boiler owner and conceal from him the fact that he is buying colored soda or similar substances, for which he is paying an exorbitant price.

The Chicago, Milwaukee & St. P. R. R. uses for the prevention of scale in locomotive-boilers an alkaline compound consisting of 3750 gals. of water, 2600 lbs. of 70% caustic soda, and 1600 lbs. of 58% soda-ash. Between Milwaukee and Madison the water-supply contains from 1 to 4½ lbs. of incrusting solids per 1000 gals., principally calcium carbonate and sulphate and magnesium sulphate. The amount of compound necessary to prevent the incrustation is 1½ to 7 pints per 1000 gals. of water. This is really only one fourth of the quantity needed for chemical combination, but the action of the compound is regenerative. The soda-ash (sodium carbonate) extracts carbonic acid from the carbonates of lime and magnesia and precipitates them in a granular form. The bicarbonate of soda thus formed, however, loses its carbonic acid by the heat, and is again changed to the active carbonate form. Theoretically this action might continue indefinitely; but as account of the loss by blowing off and the presence of other impurities in the water, it is found that the soda-ash will precipitate only about four times the theoretical quantity. Scaling is entirely prevented. One engine made 122,000 miles, and inspection of the boiler showed that it was as clean as when new. This compound precipitates the impurities in a granular form, and careful attention must be paid to washing out the precipitate. The practice is to change the water every 600 miles and wash out the boiler every 1200 miles, using the blow-off cocks also whenever there is any indication of foaming, which seems to be caused by the precipitate in the water, but not by the alkali itself. (*Eng'g News*, Dec. 5, 1891.)

Kerosene and other Petroleum Oils; Foaming.—Kerosene has recently been highly recommended as a scale preventive. See paper by L. F. Lyne (Trans. A. S. M. E., ix. 247). The *Am. Mach.*, May 22, 1890, says: Kerosene used in moderate quantities will not make the boiler foam; it is recommended and used for loosening the scale and for preventing the formation of scale. Neither will a small quantity of common oil always cause foaming; it is sometimes injected into small vertical boilers to prevent priming, and is supposed to have the same effect on the disturbed surface of the water that oil has when poured on the rough sea. Yet oil in boilers will not have the same effect, and give the desired results in all cases. The presence of oil in combination with other impurities increases the tendency of many boilers to foam, as the oil with the impurities impedes the free escape of steam from the water surface. The use of common oil not only tends to cause foaming, but is dangerous otherwise. The grease appears to combine with the impurities of the water, and when the boiler is at rest this compound sinks to the plates and clings to them in a loose, spongy mass, preventing the water from coming in contact with the plates, and thereby producing overheating, which may lead to an explosion. Foaming may also be caused by forcing the fire, or by taking the steam from a point over the furnace or where the ebullition is violent; the greasy and dirty state of new boilers is another good cause for foaming. Kerosene should be used at first in small quantities, the effect carefully noted, and the quantity increased if necessary for obtaining the desired results.

R. C. Carpenter (Trans. A. S. M. E., vol. xi.) says: The boilers of the State Agricultural College at Lansing, Mich., were badly incrusted with a hard scale. It was fully three eighths of an inch thick in many places. The first application of the oil was made while the boilers were being but little used, by inserting a gallon of oil, filling with water, heating to the boiling-point and allowing the water to stand in the boiler two or three weeks before removal. By this method fully one half the scale was removed during the warm season and before the boilers were needed for heavy firing. The oil was then added in small quantities when the boiler was in actual use. For boilers 4 ft. in diam. and 12 ft. long the best results were obtained by the use of 2 qts. for each boiler per week, and for each boiler 5 ft. in diam. 3 qts. per week. The water used in the boilers has the following analysis:

$CaCO_3$ (carbonate calcium)	206 parts in 1,000,000.
$MgCO_3$ (carbonate magnesium)	78 " " "
F_2CO_3 (carbonate iron)	22 " " "

Traces of sulphates and chlorides of potash and soda.
Total solid parts, 825 to 1,000,000.

Tannate of Soda Compound.—T. T. Parker writes to *Am. Mach.*: Should you find kerosene not doing any good, try this recipe: 50 lbs. sal-soda, 35 lbs. japonica; put the ingredients in a 50-gal. barrel, fill half full of water, and run a steam hose into it until it dissolves and boils. Remove the hose, fill up with water, and allow to settle. Use one quart per day of ten hours for a 40-H.P. boiler, and, if possible, introduce it as you do cylinder oil to your engine. Barr recommends tannate of soda as a remedy for scale composed of sulphate and carbonate of lime. As the japonica yields the tannic acid, I think the resultant equivalent to the tannate of soda.

Petroleum Oils heavier than kerosene have been used with good results. Crude oil should never be used. The more volatile oils it contains make explosive gases, and its tarry constituents are apt to form a spongy incrustation.

Removal of Hard Scale.—When boilers are coated with a hard scale difficult to remove the addition of ¼ lb. caustic soda per horse-power, and steaming for some hours, according to the thickness of the scale, just before cleaning, will greatly facilitate that operation, rendering the scale soft and loose. This should be done, if possible, when the boilers are not otherwise in use. (*Steam.*)

Corrosion in Marine Boilers. (Proc. Inst. M. E., Aug. 1884).—The investigations of the Committee on Boilers served to show that the internal corrosion of boilers is greatly due to the combined action of air and sea-water when under steam, and when not under steam to the combined action of air and moisture upon the unprotected surfaces of the metal. There are other deleterious influences at work, such as the corrosive action of fatty acids, the galvanic action of copper and brass, and the inequalities of temperature; these latter, however, are considered to be of minor importance.

Of the several methods recommended for protecting the internal surfaces of boilers, the three found most effectual are: First, the formation of a thin layer of hard scale, deposited by working the boiler with sea-water; second, the coating of the surfaces with a thin wash of Portland cement, particularly wherever there are signs of decay; third, the use of zinc slabs suspended in the water and steam spaces.

As to general treatment for the preservation of boilers in store or when laid up in the reserve, either of the two following methods is adopted, as may be found most suitable in particular cases. First, the boilers are dried as much as possible by airing-stoves, after which 2 to 3 cwt. of quick-lime, according to the size of the boiler, is placed on suitable trays at the bottom of the boiler and on the tubes. The boiler is then closed and made as air-tight as possible. Periodical inspection is made every six months, when if the lime be found slacked it is renewed. Second, the other method is to fill the boilers up with sea or fresh water, having added soda to it in the proportion of 1 lb. of soda to every 100 or 120 lbs. of water. The sufficiency of the saturation can be tested by introducing a piece of clean new iron and leaving it in the boiler for ten or twelve hours; if it shows signs of rusting, more soda should be added. It is essential that the boilers be entirely filled, to the complete exclusion of air.

Great care is taken to prevent sudden changes of temperature in boilers. Directions are given that steam shall not be raised rapidly, and that care shall be taken to prevent a rush of cold air through the tubes by too suddenly opening the smoke-box doors. The practice of emptying boilers by blowing out is also prohibited, except in cases of extreme urgency. As a rule the water is allowed to remain until it becomes cool before the boilers are emptied.

Mineral oil has for many years been exclusively used for internal lubrication of engines, with the view of avoiding the effects of fatty acid, as this oil does not readily decompose and possesses no acid properties.

Of all the preservative methods adopted in the British service, the use of zinc properly distributed and fixed has been found the most effectual in saving the iron and steel surfaces from corrosion, and also in neutralizing by its own deterioration the hurtful influences met with in water as ordinarily supplied to boilers. The zinc slabs now used in the navy boilers are 12 in. long, 6 in. wide, and ½ inch thick; this size being found convenient for general application. The amount of zinc used in new boilers at present is one slab of the above size for every 20 I.H.P., or about one square foot of zinc surface to two square feet of grate surface. Rolled zinc is found the most suitable for the purpose. To make the zinc properly efficient as a protector especial care must be taken to insure perfect metallic contact between the slabs and the stays or plates to which they are attached. The slabs should be placed in such positions that all the surfaces in the boiler shall be protected. Each slab should be periodically examined to see that its connection remains perfect, and to renew any that may have decayed; this examination is usually made at intervals not exceeding three months. Under ordinary circumstances of working these zinc slabs may be expected to last in fit condition from sixty to ninety days, immersed in hot sea-water; but in new boilers they at first decay more rapidly. The slabs are generally secured by means of iron straps 2 in. wide and ⅜ inch thick, and long enough to reach the nearest stay, to which the strap is firmly attached by screw-bolts.

To promote the proper care of boilers when not in use the following order has been issued to the French Navy by the Government: On board all ships in the reserve, as well as those which are laid up, the boilers will be completely filled with fresh water. In the case of large boilers with large tubes there will be added to the water a certain amounts of milk of lime, or a solution of soda may be used instead. In the case of tubulous boilers with small tubes milk of lime or soda may be added, but the solution will not be so strong as in the case of the larger tube, so as to avoid any danger of contracting the effective area by deposit from the solution; but the strength of the solution will be just sufficient to neutralize any acidity of the water. (*Iron Age*, Nov. 2, 1893.)

Use of Zinc.—Zinc is often used in boilers to prevent the corrosive action of water on the metal. The action appears to be an electrical one, the iron being one pole of the battery and the zinc being the other. The hydrogen goes to the iron shell and escapes as a gas into the steam. The oxygen goes to the zinc.

On account of this action it is generally believed that zinc will always prevent corrosion, and that it cannot be harmful to the boiler or tank. Some experiences go to disprove this belief, and in numerous cases zinc has not only been of no use, but has even been harmful. In one case a tubular boiler had been troubled with a deposit of scale consisting chiefly of organic matter and lime, and zinc was tried as a preventive. The beneficial action of the zinc was so obvious that its continued use was advised, with frequent opening of the boiler and cleaning out of detached scale until all the old scale should be removed and the boiler become clean. Eight or ten months later the water-supply was changed, it being now obtained from another stream supposed to be free from lime and to contain only organic matter. Two or three months after its introduction the tubes and shell were found to be coated with an obstinate adhesive scale, and composed of zinc oxide and the organic matter or sediment of the water used. The deposit had become so heavy in places as to cause overheating and bulging of the plates over the fire. (*The Locomotive.*)

Effect of Deposit on Flues. (Rankine.)—An external crust of a carbonaceous kind is often deposited from the flame and smoke of the furnaces in the flues and tubes, and if allowed to accumulate seriously impairs the economy of fuel. It is removed from time to time by means of scrapers and wire brushes. The accumulation of this crust is the probable cause of the fact that in some steamships the consumption of coal per indicated horse-power per hour goes on gradually increasing until it reaches one and a half times its original amount, and sometimes more.

Dangerous Steam-boilers discovered by Inspection.—The Hartford Steam-boiler Inspection and Insurance Co. reports that its inspectors during 1893 examined 163,328 boilers, inspected 66,698 boilers, both internally and externally, subjected 7861 to hydrostatic pressure, and found 597 unsafe for further use. The whole number of defects reported was 122,893, of which 12,390 were considered dangerous. A summary is given below. (*The Locomotive*, Feb. 1894.)

SUMMARY, BY DEFECTS, FOR THE YEAR 1893.

Nature of Defects.	Whole No.	Dangerous.	Nature of Defects.	Whole No.	Dangerous.
Deposit of sediment	9,774	548	Leakage around tubes	21,211	2,909
Incrustation and scale	18,369	865	Leakage at seams	5,424	482
Internal grooving	1,249	148	Water-gauges defective	3,670	660
Internal corrosion	6,252	397	Blow-outs defective	1,620	425
External corrosion	8,600	536	Deficiency of water	204	107
Def'tive braces and stays	1,966	485	Safety-valves overloaded	723	203
Settings defective	3,094	852	Safety-valves defective	942	300
Furnaces out of shape	4,575	254	Pressure-gauges def'tive	5,953	552
Fractured plates	3,532	640	Boilers without pressure-gauges	115	115
Burned plates	2,762	325	Unclassified defects	758	4
Blistered plates	3,381	164			
Defective rivets	17,415	1,569			
Defective heads	1,357	350	Total	122,893	12,390

The above-named company publishes annually a classified list of boiler-explosions, compiled chiefly from newspaper reports, showing that from 200 to 300 explosions take place in the United States every year, killing from 200 to 300 persons, and injuring from 300 to 450. The lists are not pretended to be complete, and may include only a fraction of the actual number of explosions.

Steam-boilers as Magazines of Explosive Energy.—Prof. R. H. Thurston (Trans. A. S. M. E., vol. vi.), in a paper with the above title, presents calculations showing the stored energy in the hot water and steam of various boilers. Concerning the plain tubular boiler of the form and dimensions adopted as a standard by the Hartford Steam-boiler

Insurance Co., he says: It is 60 inches in diameter, containing 66 3-inch tubes, and is 15 feet long. It has 850 feet of heating and 30 feet of grate surface; is rated at 60 horse-power, but is oftener driven up to 75; weighs 9500 pounds, and contains nearly its own weight of water, but only 21 pounds of steam when under a pressure of 75 pounds per square inch, which is below its safe allowance. It stores 52,000,000 foot-pounds of energy, of which but 4 per cent is in the steam, and this is enough to drive the boiler just about one mile into the air, with an initial velocity of nearly 600 feet per second.

SAFETY-VALVES.

Calculation of Weight, etc., for Lever Safety-valves.

Let W = weight of ball at end of lever, in pounds;
w = weight of lever itself, in pounds;
V = weight of valve and spindle, in pounds;
L = distance between fulcrum and centre of ball, in inches;
l = " " " " " " valve, in inches;
g = " " " " " " gravity of lever, in in.;
A = area of valve, in square inches;
P = pressure of steam, in lbs. per sq. in., at which valve will open.

Then $PA \times l = W \times L + w \times g + V \times l$;

$$\text{whence} \quad P = \frac{WL + wg + Vl}{Al};$$

$$W = \frac{PAl - wg - Vl}{L};$$

$$L = \frac{PAl - wg - Vl}{W}.$$

EXAMPLE.—Diameter of valve, 4″; distance from fulcrum to centre of ball, 36″; to centre of valve, 4″; to centre of gravity of lever, 15½″; weight of valve and spindle, 3 lbs.; weight of lever, 7 lbs.; required the weight of ball to make the blowing-off pressure 80 lbs. per sq. in.; area of 4″ valve = 12.566 sq. in. Then

$$W = \frac{PAl - wg - Vl}{L} = \frac{80 \times 12.566 \times 4 - 7 \times 15\tfrac{1}{2} - 3 \times 4}{36} = 108.4 \text{ lbs.}$$

The following rules governing the proportions of lever-valves are given by the U. S. Supervisors. The distance from the fulcrum to the valve-stem must in no case be less than the diameter of the valve-opening; the length of the lever must not be more than ten times the distance from the fulcrum to the valve-stem; the width of the bearings of the fulcrum must not be less than three quarters of an inch; the length of the fulcrum-link must not be less than four inches; the lever and fulcrum-link must be made of wrought iron or steel, and the knife-edged fulcrum points and the bearings for these points must be made of steel and hardened; the valve must be guided by its spindle, both above and below the ground seat and above the lever, through supports either made of composition (gun-metal) or bushed with it; and the spindle must fit loosely in the bearings or supports.

Rules for Area of Safety-valves.

(Rule of U. S. Supervising Inspectors of Steam-vessels (as amended 1891).)

Lever safety-valves to be attached to marine boilers shall have an area of not less than 1 sq. in. to 2 sq. ft. of the grate surface in the boiler, and the seats of all such safety-valves shall have an angle of inclination of 45° to the centre line of their axes.

Spring-loaded safety-valves shall be required to have an area of not less than 1 sq. in. to 3 sq. ft. of grate surface of the boiler, except as hereinafter otherwise provided for water-tube or coil and sectional boilers, and each spring-loaded valve shall be supplied with a lever that will raise the valve from its seat a distance of not less than that equal to one eighth the diameter of the valve-opening, and the seats of all such safety-valves shall have an angle of inclination to the centre line of their axes of 45°. All spring-loaded safety-valves for water-tube or coil and sectional boilers required to carry a steam-pressure exceeding 175 lbs. per square inch shall be required to have an area of not less than 1 sq. in. to 6 sq. ft. of the grate surface of the boiler. Nothing herein shall be construed so as to prohibit the use of two safety-valves on one water-tube or coil and sectional boiler, provided the combined area of such valves is equal to that required by rule for one such valve.

Rule in Philadelphia Ordinances: Bureau of Steam-engine and Boiler Inspection.—Every boiler when fired separately, and every set or series of boilers when placed over one fire, shall have attached thereto, without the interposition of any other valve, two or more safety-valves, the aggregate area of which shall have such relations to the area of the grate and the pressure within the boiler as is expressed in schedule A.

SCHEDULE A.—Least aggregate area of safety-valve (being the least sectional area for the discharge of steam) to be placed upon all stationary boilers with natural or chimney draught [see note *a*].

$$A = \frac{22.5G}{P + 8.62},$$

in which A is area of combined safety-valves in inches; G is area of grate in square feet; P is pressure of steam in pounds per square inch to be carried in the boiler above the atmosphere.

The following table gives the results of the formula for one square foot of grate, as applied to boilers used at different pressures:

Pressures per square inch:	10	20	30	40	50	60	70	80	90	100	110	120
Area corresponding to one square foot of grate:	1.21	0.79	0.58	0.46	0.38	0.33	0.29	0.25	0.23	0.21	0.19	0.17

[Note *a*.] Where boilers have a forced or artificial draught, the inspector must estimate the area of grate at the rate of one square foot of grate-surface for each 16 lbs. of fuel burned on the average per hour.

Comparison of Various Rules for Area of Lever Safety-valves. (From an article by the author in *American Machinist*, May 24, 1894, with some alterations and additions.)—Assume the case of a boiler rated at 100 horse-power; 40 sq. ft. grate; 1200 sq. ft. heating-surface; using 400 lbs. of coal per hour, or 10 lbs. per sq. ft. of grate per hour, and evaporating 3600 lbs. of water, or 3 lbs. per sq. ft. of heating-surface per hour; steam-pressure by gauge, 100 lbs. What size of safety-valve, of the lever type, should be required?

A compilation of various rules for finding the area of the safety-valve disk, from *The Locomotive* of July, 1892, is given in abridged form below, together with the area calculated by each rule for the above example.

	Disk Area in sq. in.
U. S. Supervisors, heating-surface in sq. ft. ÷ 25*	48
English Board of Trade, grate-surface in sq. ft. ÷ 2	20
Molesworth, four fifths of grate-surface in sq. ft	32
Thurston, 4 times coal burned per hour × (gauge pressure + 10)	14.5
Thurston, $\frac{1}{2}\frac{(5 \times \text{heating-surface})}{\text{gauge pressure} + 10}$	27.3
Rankine, .006 × water evaporated per hour	21.6
Committee of U. S. Supervisors, .005 × water evaporated per hour	18

Suppose that, other data remaining the same, the draught were increased so as to burn 13⅓ lbs. coal per square foot of grate per hour, and the grate-surface cut down to 30 sq. ft. to correspond, making the coal burned per hour 400 lbs., and the water evaporated 3600 lbs., the same as before; then the English Board of Trade rule and Molesworth's rule would give an area of disk of only 15 and 24 sq. in., respectively, showing the absurdity of making the area of grate the basis of the calculation of disk area.

Another rule by Prof. Thurston is given in *American Machinist*, Dec. 1877, viz.:

$$\text{Disk area} = \frac{\frac{1}{2}\text{ max. wt. of water evap. per hour}}{\text{gauge pressure} + 10}.$$

This gives for the example considered 16.4 sq. in.

* The edition of 1893 of the Rules of the Supervisors does not contain this rule, but gives the rule grate-surface ÷ 2.

One rule by Rankine is 1/150 to 1/180 of the number of pounds of water evaporated per hour, equals for the above case 27 to 20 sq. in. A communition in *Power*, July, 1890, gives two other rules:

1st. 1 sq. in. disk area for 3 sq. ft. grate, which would give 13.3 sq. in.

2d. ¾ sq. in. disk area for 1 sq. ft. grate, which would give 30 sq. in.; but if the grate-surface were reduced to 30 sq. ft. on account of increased draught, these rules would make the disk area only 10 and 22.5 sq. in., respectively.

The Philadelphia rule for 100 lbs. gauge pressure gives a disk area of 0.21 sq. in. for each sq. ft. of grate area, which would give an area of 8.4 sq. in. for 40 sq. ft. grate, and only 6.3 sq. in. if the grate is reduced to 30 sq. ft.

According to the rule this aggregate area would have to be divided between two valves. But if the boiler was driven by forced draught, then the inspector "must estimate the area of grate at 1 sq. ft. for each 16 lbs. of fuel burned per hour."

Under this condition the actual grate-surface might be cut down to 400 ÷ 16 = 25 sq. ft., and by the rule the combined area of the two safety-valves would be only 25 × 0.21 = 5.25 sq. in.

Nystrom's Pocket-book, edition of 1891, gives ¾ sq. in. for 1 sq. ft. grate; also quoting from Weisbach, vol. ii, 1/3000 of the heating-surface. This in the case considered is 1200/3000 = .4 sq. ft. or 57.6 sq. in.

We thus have rules which give for the area of safety-valve of the same 100-horse-power boiler results ranging all the way from 5.25 to 57.6 sq. in.

All of the rules above quoted give the area of the disk of the valve as the thing to be ascertained, and it is this area which is supposed to bear some direct ratio to the grate-surface, to the heating-surface, to the water evaporated, etc. It is difficult to see why this area has been considered even approximately proportional to these quantities, for with small lifts the area of actual opening bears a direct ratio, not to the area of disk, but to the circumference.

Thus for various diameters of valve:

Diameter	1	2	3	4	5	6	7
Area	.785	3.14	7.07	12.57	19.64	28.27	38.48
Circumference	3.14	6.28	9.42	12.57	15.71	18.85	21.99
Circum. × lift of 0.1 in.	.31	.63	.94	1.26	1.57	1.89	2.20
Ratio to area	.4	.2	.13	.1	.08	.067	.057

The apertures, therefore, are therefore directly proportional to the diameter or to the circumference, but their relation to the area is a varying one.

If the lift = ¼ diameter, then the opening would be equal to the area of the disk, for circumference × ¼ diameter = area, but such a lift is far beyond the actual lift of an ordinary safety-valve.

A correct rule for size of safety-valves should make the product of the diameter and the lift proportional to the weight of steam to be discharged.

A "logical" method for calculating the size of safety-valve is given in *The Locomotive*, July, 1892, based on the assumption that the actual opening should be sufficient to discharge all the steam generated by the boiler. Napier's rule for flow of steam is taken, viz., flow through aperture of one sq. in. in lbs. per second = absolute pressure ÷ 70, or in lbs. per hour = 51.43 × absolute pressure.

If the angle of the seat is 45°, as specified in the rules of the U. S. Supervisors, the area of opening in sq. in. = circumference of the disk × the lift × .71, .71 being the cosine of 45°; or diameter of disk × lift × 2.23.

A. G. Brown in his book on The Indicator and its Practical Working (London, 1894) gives the following as the lift of the ordinary lever safety-valve for 100 lbs. gauge-pressure:

Diam. of valve	2	2½	3	3½	4	4½	5	6	inches.
Rise of valve	.0583	.0523	.0507	.0492	.0478	.0462	.0446	.0430	inch.

The lift decreases with increase of steam-pressure; thus for a 4-inch valve:

Abs. pressure, lbs.	45	65	85	105	115	135	155	175	195	215
Gauge-press., lbs.	30	50	70	90	100	120	140	160	180	200
Rise, inch	.1034	.0775	.0620	.0517	.0478	.0413	.0365	.0327	.0296	.0270

The effective area of opening Mr. Brown takes at 70% of the rise multiplied by the circumference.

An approximate formula corresponding to Mr. Brown's figures for diameters between 2½ and 6 in. and gauge-pressures between 70 and 200 lbs. is

$$\text{Lift} = (.0603 - .0031d) \times \frac{115}{\text{abs. pressure}},$$ in which d = diam. of valve in in.

If we combine this formula with the formulæ
Flow in lbs. per hour = area of opening in sq. in. × 51.43 × abs. pressure, and
Area = diameter of valve × lift × 2.23, we obtain the following, which the author suggests as probably a more correct formula for the discharging capacity of the ordinary lever safety-valve than either of those above given.

Flow in lbs. per hour $= d(.0603 - .0031d) \times 115 \times 2.23 \times 51.43 = d(795 - 41d)$.

From which we obtain:

Diameter, inches....	1	1½	2	2½	3	3½	4	5	6	7
Flow, lbs. per hour..	754	1100	1426	1733	2016	2282	2524	2950	3294	3556
Horse-power........	25	37	47	58	67	76	84	98	110	119

the horse-power being taken as an evaporation of 30 lbs. of water per hour.

If we solve the example, above given, of the boiler evaporating 3600 lbs. of water per hour by this table, we find it requires one 7-inch valve, or a 2½- and a 3-inch valve combined. The 7-inch valve has an area of 38.5 sq. in., and the two smaller valves taken together have an area of only 12 sq. in.; another evidence of the absurdity of considering the area of disk as the factor which determined the capacity of the valve.

It is customary in practice not to use safety-valves of greater diameter than 4 in. If a greater diameter is called for by the rule that is adopted, then two or more valves are used instead of one.

Spring-loaded Safety-valves.—Instead of weights, springs are sometimes employed to hold down safety-valves. The calculations are similar to those for lever safety-valves, the tension of the spring corresponding to a given rise being first found by experiment (see Springs, page 347).

The rules of the U. S. Supervisors allow an area of 1 sq. in. of the valve to 3 sq. ft. of grate, in the case of spring-loaded valves, except in water-tube, coil, or sectional boilers, in which 1 sq. in. to 6 sq. ft. of grate is allowed.

Spring-loaded safety-valves are usually of the reactionary or "pop" type, in which the escape of the steam is opposed by a lip above the valve-seat, against which the escaping steam reacts, causing the valve to lift higher than the ordinary valve.

A. G. Brown gives the following for the rise, effective area, and quantity of steam discharged per hour by valves of the "pop" or Richardson type. The effective is taken at only 50% of the actual area due to the rise, on account of the obstruction which the lip of the valve offers to the escape of steam.

Dia. value, in	1	1½	2	2½	3	3½	4	4½	5	6
Lift, inches.	.125	.150	.175	.200	.225	.250	.275	.300	.325	.375
Area, sq. in.	.196	.354	.550	.785	1.061	1.375	1.728	2.121	2.553	3.535
Gauge-pres.,	Steam discharged per hour, lbs.									
30 lbs.	474	856	1330	1897	2568	3325	4178	5128	6178	8578
50	669	1200	1878	2680	3620	4695	5901	7242	8718	12070
70	861	1556	2417	3450	4660	6144	7596	9324	11220	15535
90	1050	1897	2947	4207	5680	7370	9260	11365	13685	18945
100	1144	2065	3206	4580	6185	8332	10080	12375	14895	20625
120	1332	2405	3736	5332	7202	9342	11735	14410	17340	24015
140	1516	2738	4254	6070	8200	10635	13365	16405	19745	27340
160	1696	3064	4760	6794	9175	11900	14955	18355	22095	30595
180	1883	3400	5288	7540	10180	13250	16595	20370	24520	33950
200	2062	3724	5786	8258	11150	14465	18175	22310	26855	37185

If we take 30 lbs. of steam per hour, at 100 lbs. gauge-pressure = 1 H.P., we have from the above table:

Diameter, inches...	1	1½	2	2½	3	3½	4	4½	5	6
Horse-power.......	38	69	107	153	206	277	336	412	496	687

A safety-valve should be capable of discharging a much greater quantity of steam than that corresponding to the rated horse-power of a boiler, since a boiler having ample grate surface and strong draught may generate more than double the quantity of steam its rating calls for.

The Consolidated Safety-valve Co.'s circular gives the following rated capacity of its nickel-seat "pop" safety-valves:

Size, in	....	1	1¼	1½	2	2½	3	3½	4	4½	5	5½
Boiler H.P.	from	8	10	20	35	60	75	100	125	150	175	200
	to	10	15	30	50	75	100	125	150	175	200	275

The figures in the lower line from 2 inch to 5 inch, inclusive, correspond to the formula H.P. = 50(diameter − 1 inch).

THE INJECTOR.

Equation of the Injector.

Let S be the number of pounds of steam used;
- W the number of pounds of water lifted and forced into the boiler;
- h the height in feet of a column of water, equivalent to the absolute pressure in the boiler;
- h_0 the height in feet the water is lifted to the injector;
- t_1 the temperature of the water before it enters the injector;
- t_2 the temperature of the water after leaving the injector;
- H the total heat above 32° F. in one pound of steam in the boiler, in heat-units;
- L the lost work in friction and the equivalent lost work due to radiation and lost heat;
- 778 the mechanical equivalent of heat.

Then

$$S[H-(t_2-32^\circ)] = W(t_2-t_1) + \frac{(W+S)h + Wh_0 + L}{778}.$$

An equivalent formula, neglecting $Wh_0 + L$ as small, is

$$S = \left[W(t_2-t_1) + \frac{W+S}{d}\cdot p\cdot\frac{144}{778}\right]\frac{1}{H-(t_2-32^\circ)},$$

or $$S = \frac{W[(t_2-t_1)d + .1851p]}{H-(t_2-32^\circ)d - .1851p},$$

in which d = weight of 1 cu. ft. of water at temperature t_2; p = absolute pressure of steam, lbs. per sq. in.

The rule for finding the proper sectional area for the narrowest part of the nozzles is given as follows by Rankine, S. E. p. 477:

$$\text{Area in square inches} = \frac{\text{cubic feet per hour gross feed-water}}{800\sqrt{\text{pressure in atmospheres}}}.$$

An important condition which must be fulfilled in order that the injector will work is that the supply of water must be sufficient to condense the steam. As the temperature of the supply or feed-water is higher, the amount of water required for condensing purposes will be greater.

The table below gives the calculated value of the maximum ratio of water to the steam, and the values obtained on actual trial, also the highest admissible temperature of the feed-water as shown by theory and the highest actually found by trial with several injectors.

Gauge-pressure, pounds per sq. in.	Maximum Ratio Water to Steam.				Gauge-pressure, pounds per sq. in.	Maximum Temperature of Feed-Water.					
	Calculated from Theory.	Actual Experiment.				Theoretical.		Experi'tal Results.			
		H.	P.	M.		Temp. discharge 180°.	Temp. discharge 212°.	H.	P.	M.	S.
10	36.5	30.9	....	....	10	....	....	....	....	....	132°
20	25.6	22.5	19.9	21.5	20	142°	178°	135°	120°	130°	134
30	20.9	19.0	17.2	19.0	30	132	162	....	....	...	134
40	17.87	15.8	15.0	15.86	40	126	156	140	113	125	132
50	16.2	13.3	14.0	13.3	50	120	150	....	....	...	131
60	14.7	11.2	11.2	12.6	60	114	143	....	115	128	130
70	13.7	12.3	11.7	12.9	70	109	139	141*	...	123	130
80	12.9	11.4	11.2	.....	80	105	134	141*	118	122	131
90	12.1	....	....	....	90	99	129	.....	...	....	132*
100	11.5	....	....	....	100	95	125	.....	....	...	132*
					120	87	117	.....	...	..	134*
					150	77	107	.	....	....	121*

* Temperature of delivery above 212°. Waste-valve closed.

H, Hancock inspirator; P, Park injector; M, Metropolitan injector; S, Sellers 1876 injector.

Efficiency of the Injector.—Experiments at Cornell University, described by Prof. R. C. Carpenter, in *Cassier's Magazine*, Feb. 1892, show that the injector, when considered merely as a pump, has an exceedingly low efficiency, the duty ranging from 161,000 to 2,752,000 under different circumstances of steam and delivery pressure. Small direct-acting pumps, such as are used for feeding boilers, show a duty of from 4 to 8 million lbs., and the best pumping-engines from 100 to 140 million. When used for feeding water into a boiler, however, the injector has a thermal efficiency of 100%, less the trifling loss due to radiation, since all the heat rejected passes into the water which is carried into the boiler.

The loss of work in the injector due to friction reappears as heat which is carried into the boiler, and the heat which is converted into useful work in the injector appears in the boiler as stored-up energy.

Although the injector thus has a perfect efficiency as a boiler-feeder, it is nevertheless not the most economical means for feeding a boiler, since it can draw only cold or moderately warm water, while a pump can feed water which has been heated by exhaust steam which would otherwise be wasted.

Performance of Injectors.—In *Am. Mach.*, April 13, 1893, are a number of letters from different manufacturers of injectors in reply to the question: "What is the best performance of the injector in raising or lifting water to any height?" Some of the replies are tabulated below.

W. Sellers & Co.—25.51 lbs. water delivered to boiler per lb. of steam; temperature of water, 64°; steam pressure, 65 lbs.

Schaeffer & Budenberg—1 gal. water delivered to boiler for 0.4 to 0.8 lb. steam.

Injector will lift by suction water of	140° F.	136° to 133°	122° to 118°	113° to 107°
If boiler pressure is	30 to 60 lbs.	60 to 90 lbs.	90 to 120 lbs.	120 to 150 lbs.

If the water is not over 80° F., the injector will force against a pressure 75 lbs. higher than that of the steam.

Hancock Inspirator Co.:

Lift in feet	22	22	22	11
Boiler pressure, absolute, lbs.	75.8	54.1	95.5	75.4
Temperature of suction	84.9°	35.4°	47.3°	53.2°
Temperature of delivery	134°	117.4°	173.7°	131.1
Water fed per lb. of steam, lbs.	11.02	13.67	8.18	13.3

The theory of the injector is discussed in Wood's, Peabody's, and Rontgen's treatises on Thermodynamics. See also "Theory and Practice of the Injector," by Strickland L. Kneass, New York, 1895.

Boiler-feeding Pumps.—Since the direct-acting pump, commonly used for feeding boilers, has a very low efficiency, or less than one tenth that of a good engine, it is generally better to use a pump driven by belt from the main engine or driving shaft. The mechanical work needed to feed a boiler may be estimated as follows: If the combination of boiler and engine is such that half a cubic foot, say 32 lbs. of water, is needed per horse-power, and the boiler-pressure is 100 lbs. per sq. in., then the work of feeding the quantity of water is 100 lbs. × 144 sq. in. × ½ ft.-lbs. per hour = 120 ft.-lbs. per min. = 120/33,000 = .0036 H.P., or less than 4/10 of 1% of the power exerted by the engine. If a direct-acting pump, which discharges its exhaust steam into the atmosphere, is used for feeding, and it has only 1/10 the efficiency of the main engine, then the steam used by the pump will be equal to nearly 4% of that generated by the boiler.

The following table by Prof. D. S. Jacobus gives the relative efficiency of steam and power pumps and injector, with and without heater, as used upon a boiler with 80 lbs. gauge-pressure, the pump having a duty of 10,000,000 ft.-lbs. per 100 lbs. of coal when no heater is used; the injector heating the water from 60° to 150° F.

Direct-acting pump feeding water at 60°, without a heater	1.000
Injector feeding water at 150°, without a heater	.985
Injector feeding water through a heater in which it is heated from 150° to 200°	.938
Direct-acting pump feeding water through a heater, in which it is heated from 60° to 200°	.879
Geared pump, run from the engine, feeding water through a heater, in which it is heated from 60° to 200°	.868

FEED-WATER HEATERS.

Percentage of Saving for Each Degree of Increase in Temperature of Feed-water Heated by Waste Steam.

Initial Temp. of Feed.	Pressure of Steam in Boiler, lbs. per sq. in. above Atmosphere.											Initial Temp.
	0	20	40	60	80	100	120	140	160	180	200	
32°	.0872	.0861	.0855	.0851	.0847	.0844	.0841	.0839	.0837	.0835	.0833	32
40	.0878	.0867	.0861	.0856	.0853	.0850	.0847	.0845	.0843	.0841	.0839	40
50	.0886	.0875	.0868	.0864	.0860	.0857	.0854	.0852	.0850	.0848	.0846	50
60	.0894	.0883	.0876	.0872	.0867	.0864	.0862	.0859	.0856	.0855	.0853	60
70	.0902	.0890	.0884	.0879	.0875	.0872	.0869	.0867	.0864	.0862	.0860	70
80	.0910	.0898	.0891	.0887	.0883	.0879	.0877	.0874	.0872	.0870	.0868	80
90	.0919	.0907	.0900	.0895	.0888	.0887	.0884	.0883	.0879	.0877	.0875	90
100	.0927	.0915	.0908	.0903	.0899	.0895	.0892	.0890	.0887	.0885	.0883	100
110	.0936	.0923	.0916	.0911	.0907	.0903	.0900	.0898	.0895	.0893	.0891	110
120	.0945	.0932	.0925	.0919	.0915	.0911	.0908	.0906	.0903	.0901	.0899	120
130	.0954	.0941	.0934	.0928	.0924	.0920	.0917	.0914	.0912	.0909	.0907	130
140	.0963	.0950	.0943	.0937	.0932	.0929	.0925	.0923	.0920	.0918	.0916	140
150	.0973	.0959	.0951	.0946	.0941	.0937	.0934	.0931	.0929	.0926	.0924	150
160	.0982	.0968	.0961	.0955	.0950	.0946	.0943	.0940	.0937	.0935	.0933	160
170	.0992	.0978	.0970	.0964	.0959	.0955	.0952	.0949	.0946	.0944	.0941	170
180	.1002	.0988	.0981	.0973	.0969	.0965	.0961	.0958	.0955	.0953	.0951	180
190	.1012	.0998	.0989	.0983	.0978	.0974	.0971	.0968	.0964	.0962	0960	190
200	.1022	.1008	.0999	.0993	.0988	.0984	.0980	.0977	.0974	.0972	.0969	200
210	.1033	.1018	.1009	.1003	.0998	.0994	.0990	.0987	.0984	.0981	.0979	210
220	.....	.1029	.1019	.1013	.1008	.1004	.1000	.0997	.0994	.0991	.0989	220
230	.....	.1039	.1031	.1024	.1018	.1012	.1010	.1007	.1003	.1001	.0999	230
240	.....	.1050	.1041	.1034	.1029	.1024	.1020	.1017	.1014	.1011	.1009	240
250	.....	.1062	.1052	.1045	.1040	.1035	.1031	.1027	.1025	.1022	.1019	250

An approximate rule for the conditions of ordinary practice is a saving of 1% is made by each increase of 11° in the temperature of the feed-water. This corresponds to .0909% per degree.

The calculation of saving is made as follows: Boiler-pressure, 100 lbs. gauge; total heat in steam above 32° = 1185 B.T.U. Feed-water, original temperature 60°, final temperature 209° F. Increase in heat-units, 150. Heat-units above 32° in feed-water of original temperature = 28. Heat-units in steam above that in cold feed-water, 1185 − 28 = 1157. Saving by the feed-water heater = 150/1157 = 12.96%. The same result is obtained by the use of the table. Increase in temperature 150° × tabular figure .0864 = 12.96%. Let total heat of 1 lb. of steam at the boiler-pressure = H; total heat of 1 lb. of feed-water before entering the heater = h_1, and after passing through the heater = h_2; then the saving made by the heater is $\frac{h_2 - h_1}{H - h_1}$.

Strains Caused by Cold Feed-water.—A calculation is made in *The Locomotive* of March, 1893, of the possible strains caused in the section of the shell of a boiler by cooling it by the injection of cold feed-water. Assuming the plate to be cooled 200° F., and the coefficient of expansion of steel to be .0000067 per degree, a strip 10 in. long would contract .013 in., if it were free to contract. To resist this contraction, assuming that the strip is firmly held at the ends and that the modulus of elasticity is 29,000,000, would require a force of 37,700 lbs. per sq. in. Of course this amount of strain cannot actually take place, since the strip is not firmly held at the ends, but is allowed to contract to some extent by the elasticity of the surrounding metal. But, says *The Locomotive*, we may feel pretty confident that in the case considered a longitudinal strain of somewhere in the neighborhood of 8000 or 10,000 lbs. per sq. in. may be produced by the feed-water striking directly upon the plates; and this, in addition to the normal strain produced by the steam-pressure, is quite enough to tax the girth-seams beyond their elastic limit, if the feed-pipe discharges anywhere near them. Hence it is not surprising that the girth-seams develop leaks and cracks in 99 cases out of every 100 in which the feed discharges directly upon the fire-sheets.

STEAM SEPARATORS.

If moist steam flowing at a high velocity in a pipe has its direction suddenly changed, the particles of water are by their momentum projected in their original direction against the bend in the pipe or wall of the chamber in which the change of direction takes place. By making proper provision for drawing off the water thus separated the steam may be dried to a greater or less extent.

For long steam-pipes a large drum should be provided near the engine for trapping the water condensed in the pipe. A drum 3 feet diameter, 15 feet high, has given good results in separating the water of condensation of a steam-pipe 10 inches diameter and 800 feet long.

Efficiency of Steam Separators.—Prof. R. C. Carpenter, in 1891, made a series of tests of six steam separators, furnishing them with steam containing different percentages of moisture, and testing the quality of steam before entering and after passing the separator. A condensed table of the principal results is given below.

Make of Separator.	Test with Steam of about 10% of Moisture.			Tests with Varying Moisture.		
	Quality of Steam before.	Quality of Steam after.	Efficiency per cent.	Quality of Steam before.	Quality of Steam after.	Av'ge Efficiency.
B	87.0%	98.8%	90.8	66.1 to 97.5%	97.8 to 99%	87.6
A	90.1	98.0	80.0	51.9 " 98	97.9 " 99.1	76.4
D	89.6	95.8	59.6	72.2 " 96.1	95.5 " 98.2	71.7
C	90.6	93.7	33.0	67.1 " 96.8	93.7 " 98.4	63.4
E	88.4	90.2	15.5	68.6 " 96.1	79.3 " 98.5	36.9
F	88.9	92.1	28.8	70.4 " 97.7	84.1 " 97.9	28.4

Conclusions from the tests were: 1. That no relation existed between the volume of the several separators and their efficiency.

2. No marked decrease in pressure was shown by any of the separators, the most being 1.7 lbs. in E.

3. Although changed direction, reduced velocity, and perhaps centrifugal force are necessary for good separation, still some means must be provided to lead the water out of the current of the steam.

The high efficiency obtained from B and A was largely due to this feature. In B the interior surfaces are corrugated and thus catch the water thrown out of the steam and readily lead it to the bottom.

In A, as soon as the water falls or is precipitated from the steam, it comes in contact with the perforated diaphragm through which it runs into the space below, where it is not subjected to the action of the steam.

Experiments made by Prof. Carpenter on a "Stratton" separator in 1894 showed that the moisture in the steam leaving the separator was less than 1% when that in the steam supplied ranged from 6% to 21%.

DETERMINATION OF THE MOISTURE IN STEAM—STEAM CALORIMETERS.

In all boiler-tests it is important to ascertain the quality of the steam, i.e., 1st, whether the steam is "saturated" or contains the quantity of heat due to the pressure according to standard experiments; 2d, whether the quantity of heat is deficient, so that the steam is wet; and 3d, whether the heat is in excess and the steam superheated. The best method of ascertaining the quality of the steam is undoubtedly that employed by a committee which tested the boilers at the American Institute Exhibition of 1871-2, of which Prof. Thurston was chairman, i.e., condensing all the water evaporated by the boiler by means of a surface condenser, weighing the condensing water, and taking its temperature as it enters and as it leaves the condenser; but this plan cannot always be adopted.

A substitute for this method is the barrel calorimeter, which with careful operation and fairly accurate instruments may generally be relied on to give results within two per cent of accuracy (that is, a sample of steam which gives the apparent result of 2% of moisture may contain anywhere between 0 and 4%). This calorimeter is described as follows: A sample of the steam is taken by inserting a perforated ½-inch pipe into and through the main pipe near the boiler, and led by a hose, thoroughly felted, to a barrel, holding preferably 400 lbs. of water, which is set upon a platform scale and provided with a cock or valve for allowing the water to flow to waste, and with a small propeller for stirring the water.

To operate the calorimeter the barrel is filled with water, the weight and temperature ascertained, steam blown through the hose outside the barrel until the pipe is thoroughly warmed, when the hose is suddenly thrust into the water, and the propeller operated until the temperature of the water is increased to the desired point, say about 110° usually. The hose is then withdrawn quickly, the temperature noted, and the weight again taken.

An error of 1/10 of a pound in weighing the condensed steam, or an error of ½ degree in the temperature, will cause an error of over 1% in the calculated percentage of moisture. See Trans. A. S. M. E., vi. 293.

The calculation of the percentage of moisture is made as below:

$$Q = \frac{1}{H - T}\left[\frac{W}{w}(h_1 - h) - (T - h_1)\right].$$

Q = quality of the steam, dry saturated steam being unity.
H = total heat of 1 lb. of steam at the observed pressure.
T = " " " " " water at the temperature of steam of the observed pressure.
h = " " " " " condensing water, original.
h_1 = " " " " " " " final.
W = weight of condensing water, corrected for water-equivalent of the apparatus.
w = weight of the steam condensed.
Percentage of moisture $= 1 - Q$.

If Q is greater than unity, the steam is superheated, and the degrees of superheating $= 2.0833\ (H - T)\ (Q - 1)$.

Difficulty of Obtaining a Correct Sample.—Recent experiments by Prof. D. S. Jacobus, Trans. A. S. M. E., xvi. 1017, show that it is practically impossible to obtain a true average sample of the steam flowing in a pipe. For accurate determinations all the steam made by the boiler should be passed through a separator, the water separated should be weighed, and a calorimeter test made of the steam just after it has passed the separator.

Coil Calorimeters.—Instead of the open barrel in which the steam is condensed, a coil acting as a surface-condenser may be used, which is placed in the barrel, the water in coil and barrel being weighed separately. For description of an apparatus of this kind designed by the author, which he has found to give results with a probable error not exceeding ½ per cent of moisture, see Trans. A. S. M. E., vi. 294. This calorimeter may be used continuously, if desired, instead of intermittently. In this case a continuous flow of condensing water into and out of the barrel must be established, and the temperature of inflow and outflow and of the condensed steam read at short intervals of time.

Throttling Calorimeter.—For percentages of moisture not exceeding 3 per cent the throttling calorimeter is most useful and convenient and remarkably accurate. In this instrument the steam which reaches it in a ½-inch pipe is throttled by an orifice 1/16 inch diameter, opening into a chamber which has an outlet to the atmosphere. The steam in this chamber has its pressure reduced nearly or quite to the pressure of the atmosphere, but the total heat in the steam before throttling causes the steam in the chamber to be superheated more or less according to whether the steam before throttling was dry or contained moisture. The only observations required are those of the temperature and pressure of the steam on each side of the orifice.

The author's formula for reducing the observations of the throttling calorimeter is as follows (Experiments on Throttling Calorimeters, *Am. Mach.*, Aug. 4, 1892): $w = 100 \times \frac{H - h - K(T - t)}{L}$, in which w = percentage of moisture in the steam; H = total heat, and L = latent heat of steam in the main pipe; h = total heat due the pressure in the discharge side of the calorimeter, = 1146.6 at atmospheric pressure; K = specific heat of superheated steam; T = temperature of the throttled and superheated steam in the calorimeter; t = temperature due the pressure in the calorimeter, = 212° at atmospheric pressure.

Taking K at 0.48 and the pressure in the discharge side of the calorimeter as atmospheric pressure, the formula becomes

$$w = 100 \times \frac{H - 1146.6 - 0.48(T - 212°)}{L}.$$

From this formula the following table is calculated:

MOISTURE IN STEAM—DETERMINATIONS BY THROTTLING CALORIMETER.

Degree of Superheating $T - 212°$.	Gauge-pressures.											
	5	10	20	30	40	50	60	70	75	80	85	90
	Per Cent of Moisture in Steam.											
0°	0.51	0.90	1.54	2.06	2.50	2.90	3.24	3.56	3.71	3.86	3.99	4.13
10°	0.01	0.39	1.02	1.54	1.97	2.36	2.71	3.02	3.17	3.32	3.45	3.58
20°	.....	.....	.51	1.02	1.45	1.83	2.17	2.48	2.63	2.77	2.90	3.03
30°	.....	.....	.00	.50	.92	1.30	1.64	1.94	2.09	2.23	2.35	2.49
40°	.....	.....	.....	.....	.39	.77	1.10	1.40	1.55	1.69	1.80	1.94
50°	.....	.....	.....	.....	.....	.24	.57	.87	1.01	1.15	1.26	1.40
60°	.....	.....	.....	.....	.....	.....	.03	.33	.47	.60	.72	.85
70°	.....	.....	.....	.....	.....	.....	.....	.....	.....	.06	.17	.31
Dif.p.deg	.0503	.0507	.0515	.0521	.0526	.0531	.0535	.0539	.0541	.0542	.0544	.0546

Degree of Superheating $T - 212°$.	Gauge-pressures.											
	100	110	120	130	140	150	160	170	180	190	200	250
	Per Cent of Moisture in Steam.											
0°	4.39	4.63	4.85	5.08	5.29	5.49	5.68	5.87	6.05	6.22	6.39	7.16
10°	3.84	4.08	4.29	4.52	4.73	4.93	5.12	5.30	5.48	5.65	5.82	6.58
20°	3.29	3.52	3.74	3.96	4.17	4.37	4.56	4.74	4.91	5.08	5.25	6.00
30°	2.74	2.97	3.18	3.41	3.61	3.80	3.99	4.17	4.34	4.51	4.67	5.41
40°	2.19	2.42	2.63	2.85	3.05	3.24	3.43	3.61	3.78	3.94	4.10	4.83
50°	1.64	1.87	2.08	2.29	2.49	2.68	2.87	3.04	3.21	3.37	3.53	4.25
60°	1.09	1.32	1.52	1.74	1.93	2.12	2.30	2.48	2.64	2.80	2.96	3.67
70°	.55	.77	.97	1.18	1.38	1.56	1.74	1.91	2.07	2.23	2.38	3.09
80°	.00	.22	.42	.63	.82	1.00	1.18	1.34	1.50	1.66	1.81	2.51
90°	.....	.....	.....	.07	.26	.44	.61	.78	.94	1.09	1.24	1.93
100°	.....	.....	.....	.....	.....	.....	.05	.21	.37	.52	.67	1.34
110°	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.10	.76
Dif.p.deg	.0549	.0551	.0554	.0556	.0559	.0561	.0564	.0566	.0568	.0570	.0572	.0581

Separating Calorimeters.—For percentages of moisture beyond the range of the throttling calorimeter the separating calorimeter is used, which is simply a steam separator on a small scale. An improved form of this calorimeter is described by Prof. Carpenter in *Power*, Feb. 1893.

For fuller information on various kinds of calorimeters, see papers by Prof. Peabody, Prof. Carpenter, and Mr. Barrus in Trans. A. S. M. E., vols. x, xi, xii, 1889 to 1891; Appendix to Report of Com. on Boiler Tests, A. S. M. E., vol. vi, 1884; Circular of Schaeffer & Budenberg, N. Y., "Calorimeters, Throttling and Separating," 1894.

Identification of Dry Steam by Appearance of a Jet.—Prof. Denton (Trans. A. S. M. E., vol. x.) found that jets of steam show unmistakable change of appearance to the eye when steam varies less than 1% from the condition of saturation either in the direction of wetness or superheating.

If a jet of steam flow from a boiler into the atmosphere under circumstances such that very little loss of heat occurs through radiation, etc., and the jet be transparent close to the orifice, or be even a grayish-white color, the steam may be assumed to be so nearly dry that no portable condensing calorimeter will be capable of measuring the amount of water in the steam. If the jet be strongly white, the amount of water may be roughly judged up to about 2%, but beyond this a calorimeter only can determine the exact amount of moisture.

A common brass pet-cock may be used as an orifice, but it should, if possible, be set into the steam-drum of the boiler and never be placed further away from the latter than 4 feet, and then only when the intermediate reservoir or pipe is well covered.

Usual Amount of Moisture in Steam Escaping from a Boiler.—In the common forms of horizontal tubular land boilers and water-tube boilers with ample horizontal drums, and supplied with water free from substances likely to cause foaming, the moisture in the steam does not generally exceed 2% unless the boiler is overdriven or the water-level is carried too high.

CHIMNEYS.

Chimney Draught Theory.—The commonly accepted theory of chimney draught, based on Peclet's and Rankine's hypotheses (see Rankine, S. E.), is discussed by Prof. De Volson Wood in Trans. A. S. M. E., vol. xi.

Peclet represented the law of draught by the formula

$$h = \frac{u^2}{2g}\left(1 + G + \frac{fl}{m}\right),$$

in which h is the "head," defined as such a height of hot gases as, if added to the column of gases in the chimney, would produce the same pressure at the furnace as a column of outside air, of the same area of base, and a height equal to that of the chimney;
u is the required velocity of gases in the chimney;
G a constant to represent the resistance to the passage of air through the coal;
l the length of the flues and chimney;
m the mean hydraulic depth or the area of a cross-section divided by the perimeter;
f a constant depending upon the nature of the surfaces over which the gases pass, whether smooth, or sooty and rough.

Rankine's formula (Steam Engine, p. 288), derived by giving certain values to the constants (so-called) in Peclet's formula, is

$$h = \frac{\frac{\tau_0}{\tau_2}\left(0.0807\right)}{\frac{\tau_0}{\tau_1}\left(0.084\right)} H - H = \left(0.96\frac{\tau_1}{\tau_2} - 1\right)H;$$

in which H = the height of the chimney in feet;
τ_0 = 493° F., absolute (temperature of melting ice);
τ_1 = absolute temperature of the gases in the chimney;
τ_2 = absolute temperature of the external air.

Prof. Wood derives from this a still more complex formula which gives the height of chimney required for burning a given quantity of coal per second, and from it he calculates the following table, showing the height of chimney required to burn respectively 24, 20, and 16 lbs. of coal per square foot of grate per hour, for the several temperatures of the chimney gases given.

Outside Air. τ_2	Chimney Gas. τ_1 Absolute.	Chimney Gas. Temp. Fahr.	Coal per sq. ft. of grate per hour, lbs. 24 — Height H, feet.	20	16
520° absolute or 59° F.	700	239	250.9	157.6	67.8
	800	339	172.4	115.8	55.7
	1000	539	149.1	100.0	48.7
	1100	639	148.8	98.9	48.2
	1200	739	152.0	100.9	49.1
	1400	939	159.9	105.7	51.2
	1600	1139	168.8	111.0	53.5
	2000	1539	206.5	132.2	63.0

Rankine's formula gives a maximum draught when $\tau = 2\,1/12\tau_2$, or 622° F., when the outside temperature is 60°. Prof. Wood says: "This result is not a fixed value, but departures from theory in practice do not affect the result largely. There is, then, in a properly constructed chimney, properly working, a temperature giving a maximum draught,* and that temperature is not far from the value given by Rankine, although in special cases it may be 50° or 75° more or less."

All attempts to base a practical formula for chimneys upon the theoretical formula of Peclet and Rankine have failed on account of the impossibility of assigning correct values to the so-called "constants" G and f. (See Trans. A. S. M. E., xi. 984.)

Force or Intensity of Draught.—The force of the draught is equal to the difference between the weight of the column of hot gases inside of the chimney and the weight of a column of the external air of the same height. It is measured by a draught-gauge, usually a U-tube partly filled with water, one leg connected by a pipe to the interior of the flue, and the other open to the external air.

If D is the density of the air outside, d the density of the hot gas inside, in lbs. per cubic foot, h the height of the chimney in feet, and .192 the factor for converting pressure in lbs. per sq. ft. into inches of water column, then the formula for the force of draught expressed in inches of water is,

$$F = .192h(D - d).$$

The density varies with the absolute temperature (see Rankine).

$$d = \frac{\tau_0}{\tau_1}\,0.084; \quad D = 0.0807\,\frac{\tau_0}{\tau_2},$$

where τ_0 is the absolute temperature at 32° F., = 493., τ_1 the absolute temperature of the chimney gases and τ_2 that of the external air. Substituting these values the formula for force of draught becomes

$$F = .192h\left(\frac{39.79}{\tau_2} - \frac{41.41}{\tau_1}\right) = h\left(\frac{7.64}{\tau_2} - \frac{7.95}{\tau_1}\right).$$

To find the maximum intensity of draught for any given chimney, the heated column being 600° F., and the external air 60°, multiply the height above grate in feet by .0078, and the product is the draught in inches of water.

Height of Water Column Due to Unbalanced Pressure in Chimney 100 Feet High. (*The Locomotive*, 1884.)

Temp. in the Chimney.	Temperature of the External Air—Barometer, 14.7 lbs. per sq. in.										
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°
200	.453	.419	.384	.353	.321	.292	.263	.234	.209	.182	.157
220	.488	.453	.419	.388	.355	.326	.298	.269	.244	.217	.192
240	.520	.488	.451	.421	.388	.359	.330	.301	.276	.250	.225
260	.555	.528	.484	.453	.420	.392	.363	.334	.309	.282	.257
280	.584	.549	.515	.482	.451	.422	.394	.365	.340	.318	.288
300	.611	.576	.541	.511	.478	.449	.420	.392	.367	.340	.315
320	.637	.603	.568	.538	.505	.476	.447	.419	.394	.367	.342
340	.662	.638	.593	.563	.530	.501	.472	.443	.419	.392	.367
360	.687	.653	.618	.588	.555	.526	.497	.468	.444	.417	.392
380	.710	.676	.641	.611	.578	.549	.520	.493	.467	.440	.415
400	.732	.697	.662	.632	.598	.570	.541	.513	.488	.461	.436
420	.753	.718	.684	.653	.620	.591	.563	.534	.509	.482	.457
440	.774	.739	.705	.674	.641	.612	.584	.555	.530	.503	.478
460	.793	.758	.724	.694	.660	.632	.603	.574	.549	.522	.497
480	.810	.776	.741	.710	.678	.649	.620	.591	.566	.540	.515
500	.829	.791	.760	.730	.697	.669	.639	.610	.586	.559	.534

* Much confusion to students of the theory of chimneys has resulted from their understanding the words maximum draught to mean maximum intensity or pressure of draught, as measured by a draught-gauge. It here means maximum quantity or weight of gases passed up the chimney. The maximum intensity is found only with maximum temperature, but after the temperature reaches about 622° F. the density of the gas decreases more rapidly than its velocity increases, so that the weight is a maximum about 622° F., as shown by Rankine.—W. K.

For any other height of chimney than 100 ft. the height of water-column is found by simple proportion, the height of water column being directly proportioned to the height of chimney.

The calculations have been made for a chimney 100 ft. high, with various temperatures outside and inside of the flue, and on the supposition that the temperature of the chimney is uniform from top to bottom. This is the basis on which all calculations respecting the draught-power of chimneys have been made by Rankine and other writers, but it is very far from the truth in most cases. The difference will be shown by comparing the reading of the draught-gauge with the table given. In one case a chimney 123 ft. high showed a temperature at the base of 320°, and at the top of 230°.

Box, in his "Treatise on Heat," gives the following table:

DRAUGHT POWERS OF CHIMNEYS, ETC., WITH THE INTERNAL AIR AT 552°, AND THE EXTERNAL AIR AT 62°, AND WITH THE DAMPER NEARLY CLOSED.

Height of Chimney in feet.	Draught Power in ins. of water.	Theoretical Velocity in feet per second.		Height of Chimney in feet.	Draught Power in ins. of water.	Theoretical Velocity in feet per second.	
		Cold Air Entering.	Hot Air at Exit.			Cold Air Entering.	Hot Air at Exit.
10	.073	17.8	35.6	80	.585	50.6	101.2
20	.146	25.3	50.6	90	.657	53.7	107.4
30	.219	31.0	62.0	100	.730	56.5	113.0
40	.292	35.7	71.4	120	.876	62.0	124.0
50	.365	40.0	80.0	150	1.095	69.3	138.6
60	.438	43.8	87.6	175	1.277	74.8	149 6
70	.511	47.3	94.6	200	1.460	80.0	160 0

Rate of Combustion Due to Height of Chimney.—Trowbridge's "Heat and Heat Engines" gives the following table showing the heights of chimney for producing certain rates of combustion per sq. ft. of section of the chimney. It may be approximately true for anthracite in moderate and large sizes, but greater heights than are given in the table are needed to secure the given rates of combustion with small sizes of anthracite, and for bituminous coal smaller heights will suffice if the coal is reasonably free from ash—5% or less.

Heights in feet.	Lbs. of Coal Burned per Hour per Sq. Ft. of Section of Chimney.	Lbs. of Coal Burned per Sq. Ft. of Grate, the Ratio of Grate to Section of Chimney being 8 to 1.	Heights in feet.	Lbs. of Coal Burned per Hour per Sq. Ft. of Section of Chimney.	Lbs. of Coal Burned per Sq. Ft. of Grate, the Ratio of Grate to Section of Chimney being 8 to 1.
20	60	7.5	70	126	15.8
25	68	8.5	75	131	16 4
30	76	9.5	80	135	16.9
35	84	10.5	85	139	17.4
40	93	11.6	90	144	18.0
45	99	12.4	95	148	18.5
50	105	13.1	100	153	19.0
55	111	13.8	105	156	19.5
60	116	14.5	110	160	20 0
65	121	15.1			

Thurston's rule for rate of combustion effected by a given height of chimney (Trans. A. S. M. E., xi. 991) is: Subtract 1 from twice the square root of the height, and the result is the rate of combustion in pounds per square foot of grate per hour, for anthracite. Or rate $= 2\sqrt{h} - 1$, in which h is the height in feet. This rule gives the following:

$h =$	50	60	70	80	90	100	110	125	150	175	200
$2\sqrt{h} - 1 =$	13.14	14.49	15.73	16.89	17.97	19	19.97	21.36	23.49	25.45	27.28

The results agree closely with Trowbridge's table given above. In practice the high rates of combustion for high chimneys given by the formula are not generally obtained, for the reason that with high chimneys there are usually long horizontal flues, serving many boilers, and the friction and the interference of currents from the several boilers are apt to cause the intensity of draught in the branch flues leading to each boiler to be much less than that at the base of the chimney. The draught of each boiler is also usually restricted by a damper and by bends in the gas-passages. In a battery of several boilers connected to a chimney 150 ft. high, the author found a draught of ¾-inch water-column at the boiler nearest the chimney, and only ¼-inch at the boiler farthest away. The first boiler was wasting fuel from too high temperature of the chimney-gases, 900°, having too large a grate-surface for the draught, and the last boiler was working below its rated capacity and with poor economy, on account of insufficient draught.

The effect of changing the length of the flue leading into a chimney 60 ft. high and 2 ft. 9 in. square is given in the following table, from Box on "Heat":

Length of Flue in feet.	Horse-power.	Length of Flue in feet.	Horse-power.
50	107.6	800	56.1
100	100.0	1,000	51.4
200	85.3	1,500	43.3
400	70.8	2,000	38.2
600	62.5	3,000	31.7

The temperature of the gases in this chimney was assumed to be 552° F., and that of the atmosphere 62°.

High Chimneys not Necessary.—Chimneys above 150 ft. in height are very costly, and their increased cost is rarely justified by increased efficiency. In recent practice it has become somewhat common to build two or more smaller chimneys instead of one large one. A notable example is the Spreckels Sugar Refinery in Philadelphia, where three separate chimneys are used for one boiler-plant of 7500 H.P. The three chimneys are said to have cost several thousand dollars less than a single chimney of their combined capacity would have cost. Very tall chimneys have been characterized by one writer as "monuments to the folly of their builders."

Heights of Chimney required for Different Fuels.—The minimum height necessary varies with the fuel, wood requiring the least, then good bituminous coal, and fine sizes of anthracite the greatest. It also varies with the character of the boiler—the smaller and more circuitous the gas-passages the higher the stack required; also with the number of boilers, a single boiler requiring less height than several that discharge into a horizontal flue. No general rule can be given.

SIZE OF CHIMNEYS.

The formula given below, and the table calculated therefrom for chimneys up to 96 in. diameter and 200 ft. high, were first published by the author in 1884 (Trans. A. S. M. E. vi., 81). They have met with much approval since that date by engineers who have used them, and have been frequently published in boiler-makers' catalogues and elsewhere. The table is now extended to cover chimneys up to 12 ft. diameter and 300 ft. high. The sizes corresponding to the given commercial horse-powers are believed to be ample for all cases in which the draught areas through the boiler-flues and connections are sufficient, say not less than 20% greater than the area of the chimney, and in which the draught between the boilers and chimney is not checked by long horizontal passages and right-angled bends.

Note that the figures in the table correspond to a coal consumption of 5 lbs. of coal per horse-power per hour. This liberal allowance is made to cover the contingencies of poor coal being used, and of the boilers being driven beyond their rated capacity. In large plants, with economical boilers and engines, good fuel and other favorable conditions, which will reduce the maximum rate of coal consumption at any one time to less than 5 lbs. per H. P. per hour, the figures in the table may be multiplied by the ratio of 5 to the maximum expected coal consumption per H.P. per hour. Thus, with conditions which make the maximum coal consumption only 2.5 lbs. per hour, the chimney 300 ft. high × 12 ft. diameter should be sufficient for 6155 × 2 = 12,310 horse-power. The formula is based on the following data:

Size of Chimneys for Steam-boilers.

Formula, H.P. = $3.33(A - 0.6\sqrt{A})\sqrt{H}$. (Assuming 1 H. P. = 5 lbs. of coal burned per hour.)

Diam. inches.	Area A. sq. ft.	Effective Area. $E = A - 0.6\sqrt{A}$. sq. ft.	Height of Chimney. 50 ft.	60 ft.	70 ft.	80 ft.	90 ft.	100 ft.	110 ft.	125 ft.	150 ft.	175 ft.	200 ft.	225 ft.	250 ft.	300 ft.	Equivalent Square Chimney. Side of Square $\sqrt{E}+4$ inches.
			Commercial Horse-power of Boiler.														
18	1.77	.97	23	25	27	29	......	......	......	......	......	......	......	......	......	......	16
21	2.41	1.47	35	38	41	44	......	......	......	......	......	......	......	......	......	......	19
24	3.14	2.08	49	54	58	62	66	......	......	......	......	......	......	......	......	......	22
27	3.98	2.78	65	72	78	83	88	......	......	......	......	......	......	......	......	......	24
30	4.91	3.58	84	92	100	107	113	119	......	......	......	......	......	......	......	......	27
33	5.94	4.48	......	115	125	133	141	149	156	......	......	......	......	......	......	......	30
36	7.07	5.47	......	141	152	163	173	182	191	204	......	......	......	......	......	......	32
39	8.30	6.57	......	......	183	196	208	219	229	245	......	......	......	......	......	......	35
42	9.62	7.76	......	......	216	231	245	258	271	289	316	......	......	......	......	......	38
48	12.57	10.44	......	......	......	311	330	348	365	389	426	......	......	......	......	......	43
54	15.90	13.51	......	......	......	......	427	449	472	503	551	595	......	......	......	......	48
60	19.64	16.98	......	......	......	......	536	565	593	632	692	748	......	......	......	......	54
66	23.76	20.83	......	......	......	......	......	694	728	776	849	918	981	......	......	......	59
72	28.27	25.08	......	......	......	......	......	835	876	934	1023	1105	1181	1253	......	......	64
78	33.18	29.73	......	......	......	......	......	......	1038	1107	1212	1310	1400	1485	1565	......	70
84	38.48	34.76	......	......	......	......	......	......	1214	1294	1418	1531	1637	1736	1830	2005	75
90	44.18	40.19	......	......	......	......	......	......	......	1496	1639	1770	1893	2008	2116	2318	80
96	50.27	46.01	......	......	......	......	......	......	......	1712	1876	2027	2167	2298	2423	2654	86
102	56.75	52.23	......	......	......	......	......	......	......	1944	2130	2300	2459	2600	2750	3012	91
108	63.62	58.83	......	......	......	......	......	......	......	2090	2399	2592	2771	2939	3098	3393	96
114	70.88	65.83	......	......	......	......	......	......	......	......	2685	2900	3100	3288	3466	3797	101
120	78.54	73.22	......	......	......	......	......	......	......	......	2986	3296	3448	3657	3855	4223	107
132	95.03	89.18	......	......	......	......	......	......	......	......	3637	3929	4200	4455	4696	5144	117
144	113.10	106.72	......	......	......	......	......	......	......	......	4352	4701	5026	5331	5618	6155	128

For pounds of coal burned per hour for any given size of chimney, multiply the figures in the table by 5.

1. The draught power of the chimney varies as the square root of the height.

2. The retarding of the ascending gases by friction may be considered as equivalent to a diminution of the area of the chimney, or to a lining of the chimney by a layer of gas which has no velocity. The thickness of this lining is assumed to be 2 inches for all chimneys, or the diminution of area equal to the perimeter × 2 inches (neglecting the overlapping of the corners of the lining). Let D = diameter in feet, A = area, and E = effective area in square feet.

$$\text{For square chimneys, } E = D^2 - \frac{8D}{12} = A - \frac{2}{3}\sqrt{A}.$$

$$\text{For round chimeys, } \quad E = \frac{\pi}{4}\left(D^2 - \frac{8D}{12}\right) = A - 0.591\sqrt{A}.$$

For simplifying calculations, the coefficient of $\sqrt{A}$ may be taken as 0.6 for both square and round chimneys, and the formula becomes

$$E = A - 0.6\sqrt{A}.$$

3. The power varies directly as this effective area E.

4. A chimney should be proportioned so as to be capable of giving sufficient draught to cause the boiler to develop much more than its rated power, in case of emergencies, or to cause the combustion of 5 lbs. of fuel per rated horse-power of boiler per hour.

5. The power of the chimney varying directly as the effective area, E, and as the square root of the height, H, the formula for horse-power of boiler for a given size of chimney will take the form H.P. = $CE\sqrt{H}$, in which C is a constant, the average value of which, obtained by plotting the results obtained from numerous examples in practice, the author finds to be 3.33.

The formula for horse-power then is

$$\text{H.P.} = 3.33E\sqrt{H}, \quad \text{or} \quad \text{H.P.} = 3.33(A - .6\sqrt{A})\sqrt{H}.$$

If the horse-power of boiler is given, to find the size of chimney, the height being assumed,

$$E = \frac{0.3\ \text{H.P.}}{\sqrt{H}}; = A - 0.6\sqrt{A}.$$

For round chimneys, diameter of chimney = diam. of $E + 4''$.

For square chimneys, side of chimney = $\sqrt{E} + 4''$.

If effective area E is taken in square feet, the diameter in inches is $d = 13.54\sqrt{E} + 4''$, and the side of a square chimney in inches is $s = 12\sqrt{E} + 4''$.

If horse-power is given and area assumed, the height $H = \left(\frac{0.3\ \text{H.P.}}{E}\right)^2$.

In proportioning chimneys the height is generally first assumed, with due consideration to the heights of surrounding buildings or hills near to the proposed chimney, the length of horizontal flues, the character of coal to be used, etc., and then the diameter required for the assumed height and horse-power is calculated by the formula or taken from the table.

The Protection of Tall Chimney-shafts from Lightning. —C. Molyneux and J. M. Wood (*Industries*, March 28, 1890) recommend for tall chimneys the use of a coronal or heavy band at the top of the chimney, with copper points 1 ft. in height at intervals of 2 ft. throughout the circumference. The points should be gilded to prevent oxidation. The most approved form of conductor is a copper tape about ¾ in. by ⅛ in. thick, weighing 6 ozs. per ft. If iron is used it should weigh not less than 2¼ lbs. per ft. There must be no insulation, and the copper tape should be fastened to the chimney with holdfasts of the same material, to prevent voltaic action. An allowance for expansion and contraction should be made, say 1 in. in 40 ft. Slight bends in the tape, not too abrupt, answer the purpose. For an earth terminal a plate of metal at least 3 ft. sq. and 1/16 in. thick should be buried as deep as possible in a damp spot. The plate should be of the same metal as the conductor, to which it should be soldered. The best earth terminal is water, and when a deep well or other large body of water is at hand, the conductor should be carried down into it. Right-angled bends in the conductor should be avoided. No bend in it should be over 30°.

Some Tall Brick Chimneys.

	Height.	Internal Diam.	Outside Diameter. Base.	Top.	Capacity by the Author's Formula. H. P.	Pounds Coal per hour.
1. Hallsbrückner Hütte, Sax.	460	15 7′	33′	16′	13,221	66,105
2. Townsend's, Glasgow.. ..	454	....	32			
3. Tennant's, Glasgow........	435	13′ 6″	40		9,795	48,975
4. Dobson & Barlow, Bolton, Eng....	367½	13′ 2″	33′10″		8,245	41,225
5. Fall River Iron Co., Boston	350	11	30	21	5,558	27,790
6. Clark Thread Co., Newark, N. J................. ...	335	11	28′ 6″	14	5,435	27,175
7. Merrimac Mills, Low'l, Mass	282′9″	12			5,980	29,900
8. Washington Mills, Lawrence, Mass.............	250	10			3,839	19,195
9. Amoskeag Mills, Manchester, N. H................ ..	250	10			3,839	19,195
10. Narragansett E. L. Co., Providence, R. I.........	238	14			7,515	37,575
11. Lower Pacific Mills, Lawrence, Mass.........	214	8			2,248	11,240
12. Passaic Print Works, Passaic, N. J	200	9			2,771	13,855
13. Edison Sta, B'klyn, Two e'ch	150	50″ × 120″		each	1,541	7,705

NOTES ON THE ABOVE CHIMNEYS.—1. This chimney is situated near Freiberg, on the right bank of the Mulde, at an elevation of 219 feet above that of the foundry works, so that its total height above the sea will be 711¾ feet. The works are situated on the bank of the river, and the furnace-gases are conveyed across the river to the chimney on a bridge, through a pipe 3227 feet in length. It is built throughout of brick, and will cost about $40,000.—*Mfr. and Bldr.*

2. Owing to the fact that it was struck by lightning, and somewhat damaged, as a precautionary measure a copper extension subsequently was added to it, making its entire height 488 feet.

1, 2, 3, and 4 were built of these great heights to remove deleterious gases from the neighborhood, as well as for draught for boilers.

5. The structure rests on a solid granite foundation, 55 × 30 feet, and 16 feet deep. In its construction there were used 1,700,000 bricks, 2000 tons of stone, 2000 barrels of mortar, 1000 loads of sand, 1000 barrels of Portland cement, and the estimated cost is $40,000. It is arranged for two flues, 9 feet 6 inches by 6 feet, connecting with 40 boilers, which are to be run in connection with four triple-expansion engines of 1350 horse-power each.

6. It has a uniform batter of 2.85 inches to every 10 feet. Designed for 21 boilers of 200 H. P. each. It is surmounted by a cast-iron coping which weighs six tons, and is composed of thirty-two sections, which are bolted together by inside flanges, so as to present a smooth exterior. The foundation is in concrete, composed of crushed limestone 6 parts, sand 3 parts, and Portland cement 1 part. It is 40 feet square and 5 feet deep. Two qualities of brick were used; the outer portions were of the first quality North River, and the backing up was of good quality New Jersey brick. Every twenty feet in vertical measurement an iron ring, 4 inches wide and ¾ to ½ inch thick, placed edgewise, was built into the walls about 8 inches from the outer circle. As the chimney starts from the base it is double. The outer wall is 5 feet 2 inches in thickness, and inside of this is a second wall 20 inches thick and spaced off about 20 inches from main wall. From the interior surface of the main wall eight buttresses are carried, nearly touching this inner or main flue wall in order to keep it in line should it tend to sag. The interior wall, starting with the thickness described, is gradually reduced until a height of about 90 feet is reached, when it is diminished to 8 inches. At 165 feet it ceases,

and the rest of the chimney is without lining. The total weight of the chimney and foundation is 5000 tons. It was completed in September, 1888.

7. Connected to 12 boilers, with 1200 square feet of grate-surface. Draught-gauge 1 9/16 inches.

8. Connected to 8 boilers, 6′ 8″ diameter × 18 feet. Grate-surface 448 square feet.

9. Connected to 64 Manning vertical boilers, total grate surface 1810 sq. ft. Designed to burn 18,000 lbs. anthracite per hour.

10. Designed for 12,000 H.P. of engines; (compound condensing).

11. Grate-surface 434 square feet; H.P. of boilers (Galloway) about 2500.

13. Eight boilers (water-tube) each 450 H.P.; 12 engines, each 300 H.P. Plant designed for 36,000 incandescent lights. For the first 60 feet the exterior wall is 28 inches thick, then 24 inches for 20 feet, 20 inches for 30 feet, 16 inches for 20 feet, and 12 inches for 20 feet. The interior wall is 9 inches thick of fire-brick for 50 feet, and then 8 inches thick of red brick for the next 30 feet. Illustrated in *Iron Age*, January 2, 1890.

A number of the above chimneys are illustrated in *Power*, Dec., 1890.

Chimney at Knoxville, Tenn., illustrated in *Eng'g News*, Nov. 2, 1893. 6 feet diameter, 120 feet high, double wall:

Exterior wall,	height	20 feet,	30 feet,	30 feet,	40 feet;
" "	thickness	21½ in.,	17 in.,	13 in.,	8½ in.;
Interior wall,	height	35 ft.,	35 ft.,	29 ft.,	21 ft.;
" "	thickness	13½ in.,	8½ in.,	4 in.,	0.

Exterior diameter, 15′ 6″ at bottom; batter, 7/16 inch in 12 inches from bottom to 8 feet from top. Interior diameter of inside wall, 6 feet uniform to top of interior wall. Space between walls, 16 inches at bottom, diminishing to 0 at top of interior wall. The interior wall is of red brick except a lining of 4 inches of fire-brick for 20 feet from bottom.

Stability of Chimneys.—Chimneys must be designed to resist the maximum force of the wind in the locality in which they are built, (see Weak Chimneys, below). A general rule for diameter of base, of brick chimneys, approved by many years of practice in England and the United States, is to make the diameter of the base one tenth of the height. If the chimney is square or rectangular, make the diameter of the inscribed circle of the base one tenth of the height. The "batter" or taper of a chimney should be from 1/16 to ¼ inch to the foot on each side. The brickwork should be one brick (8 or 9 inches) thick for the first 25 feet from the top, increasing ½ brick (4 or 4½ inches) for each 25 feet from the top downwards. If the inside diameter exceed 5 feet, the top length should be 1½ bricks; and if under 3 feet, it may be ½ brick for ten feet.

(From *The Locomotive*, 1884 and 1886.) For chimneys of four feet in diameter and one hundred feet high, and upwards, the best form is circular, with a straight batter on the outside. A circular chimney of this size, in addition to being cheaper than any other form, is lighter, stronger, and looks much better and more shapely.

Chimneys of any considerable height are not built up of uniform thickness from top to bottom, nor with a uniformly varying thickness of wall, but the wall, heaviest of course at the base, is reduced by a series of steps.

Where practicable the load on a chimney foundation should not exceed two tons per square foot in compact sand, gravel, or loam. Where a solid rock-bottom is available for foundation, the load may be greatly increased. If the rock is sloping, all unsound portions should be removed, and the face dressed to a series of horizontal steps, so that there shall be no tendency to slide after the structure is finished.

All boiler-chimneys of any considerable size should consist of an outer stack of sufficient strength to give stability to the structure, and an inner stack or core independent of the outer one. This core is by many engineers extended up to a height of but 50 or 60 feet from the base of the chimney, but the better practice is to run it up the whole height of the chimney; it may be stopped off, say, a couple feet below the top, and the outer shell contracted to the area of the core, but the better way is to run it up to about 8 or 12 inches of the top and *not* contract the outer shell. But under no circumstances should the core at its upper end be built into or connected with the outer stack. This has been done in several instances by bricklayers, and the result has been the expansion of the inner core which lifted the top of the outer stack squarely up and cracked the brickwork.

For a height of 100 feet we would make the outer shell in three steps, the first 20 feet high, 16 inches thick, the second 30 feet high, 12 inches thick, the this 50 feet high and 8 inches thick. These are the minimum thicknesses admissible for chimneys of this height, and the batter should be not less than 1 in 36 to give stability. The core should also be built in three steps each of which may be about one third the height of the chimney, the lowest 12 inches, the middle 8 inches, and the upper step 4 inches thick. This will insure a good sound core. The top of a chimney may be protected by a cast-iron cap; or perhaps a cheaper and equally good plan is to lay the ornamental part in some good cement, and plaster the top with the same material.

Weak Chimneys.—James B. Francis, in a report to the Lawrence Mfg. Co. in 1873 (*Eng'g News*, Aug. 28, 1880), gives some calculations concerning the probable effects of wind on that company's chimney as then constructed. Its outer shell is octagonal. The inner shell is cylindrical, with an air-space between it and the outer shell; the two shells not being bonded together, except at the openings at the base, but with projections in the brickwork, at intervals of about 20 ft. in height, to afford lateral support by contact of the two shells. The principal dimensions of the chimney are as follows:

Height above the surface of the ground........................	211 ft.
Diameter of the inscribed circle of the octagon near the ground.	15 "
Diameter of the inscribed circle of the octagon near the top...	10 ft. 1½ in.
Thickness of the outer shell near the base, 6 bricks, or.........	23½ in.
Thickness of the outer shell near the top, 3 bricks, or...........	11½ "
Thickness of the inner shell near the base, 4 bricks, or..........	15 "
Thickness of the inner shell near the top, 1 brick, or	3¾ "

One tenth of the height for the diameter of the base is the rule commonly adopted. The diameter of the inscribed circle of the base of the Lawrence Manufacturing Company's chimney being 15 ft., it is evidently much less than is usual in a chimney of that height.

Soon after the chimney was built, and before the mortar had hardened, it was found that the top had swayed over about 29 in. toward the east. This was evidently due to a strong westerly wind which occurred at that time. It was soon brought back to the perpendicular by sawing into some of the joints, and other means.

The stability of the chimney to resist the force of the wind depends mainly on the weight of its outer shell, and the width of its base. The cohesion of the mortar may add considerably to its strength; but it is too uncertain to be relied upon. The inner shell will add a little to the stability, but it may be cracked by the heat, and its beneficial effect, if any, is too uncertain to be taken into account.

The effect of the joint action of the vertical pressure due to the weight of the chimney, and the horizontal pressure due to the force of the wind is to shift the centre of pressure at the base of the chimney, from the axis toward one side, the extent of the shifting depending on the relative magnitude of the two forces. If the centre of pressure is brought too near the side of the chimney, it will crush the brickwork on that side, and the chimney will fall. A line drawn through the centre of pressure, perpendicular to the direction of the wind, must leave an area of brickwork between it and the side of the chimney, sufficient to support half the weight of the chimney; the other half of the weight being supported by the brickwork on the windward side of the line.

Different experimenters on the strength of brickwork give very different results. Kirkaldy found the weights which caused several kinds of bricks, laid in hydraulic lime mortar and in Roman and Portland cements, to fail slightly, to vary from 19 to 60 tons (of 2000 lbs.) per sq. ft. If we take in this case 25 tons per sq. ft., as the weight that would cause it to begin to fail, we shall not err greatly. To support half the weight of the outer shell of the chimney, or 322 tons, at this rate, requires an area of 12.88 sq. ft. of brickwork. From these data and the drawings of the chimney, Mr. Francis calculates that the area of 12.88 sq. ft. is contained in a portion of the chimney extending 2.428 ft. from one of its octagonal sides, and that the limit to which the centre of pressure may be shifted is therefore 5.072 ft. from the axis. If shifted beyond this, he says, on the assumption of the strength of the brickwork, it will crush and the chimney will fall.

Calculating that the wind-pressure can affect only the upper 141 ft. of the chimney, the lower 70 ft. being protected by buildings, he calculates that a wind-pressure of 44.02 lbs. per sq. ft. would blow the chimney down.

Rankine, in a paper printed in the transactions of the Institution of Engineers, in Scotland, for 1867–68, says: "It had previously been ascertained by observation of the success and failure of actual chimneys, and especially of those which respectively stood and fell during the violent storms of 1856, that, in order that a round chimney may be sufficiently stable, its weight should be such that a pressure of wind, of about 55 lbs. per sq. ft. of a plane surface, directly facing the wind, or 27½ lbs. per sq. ft. of the plane projection of a cylindrical surface, . . . shall not cause the resultant pressure at any bed-joint to deviate from the axis of the chimney by more than one quarter of the outside diameter at that joint."

According to Rankine's rule, the Lawrence Mfg. Co.'s chimney is adapted to a maximum pressure of wind on a plane acting on the whole height of 18.80 lbs. per sq. ft., or of a pressure of 21.70 lbs. per sq. ft. acting on the uppermost 141 ft. of the chimney.

Steel Chimneys are largely coming into use, especially for tall chimneys of iron-works, from 150 to 300 feet in height. The advantages claimed are: greater strength and safety; smaller space required; smaller cost, by 30 to 50 per cent, as compared with brick chimneys; avoidance of infiltration of air and consequent checking of the draught, common in brick chimneys. They are usually made cylindrical in shape, with a wide curved flare for 10 to 25 feet at the bottom. A heavy cast-iron base-plate is provided, to which the chimney is riveted, and the plate is secured to a massive foundation by holding-down bolts. No guys are used. F. W. Gordon, of the Phila. Engineering Works, gives the following method of calculating their resistance to wind pressure (*Power*, Oct. 1893):

In tests by Sir William Fairbairn we find four experiments to determine the strength of thin hollow tubes. In the table will be found their elements, with their breaking strain. These tubes were placed upon hollow blocks, and the weights suspended at the centre from a block fitted to the inside of the tube.

	Clear Span, ft.	in.	Thickness Iron, in.	Outside Diameter, in.	Sectional Area, in.	Breaking Weight, lbs.	Breaking W't, lbs., by Clarke's Formula, Constant 1.2.
I.	17		.037	12	1.3901	2,704	2,627
II.	15	7½	.113	12.4	4.3669	11,440	9,184
III.	23	5	.0631	17.68	3.487	6,400	7,302
IV.	23	5	.119	18.18	6.74	14,240	13,910

Edwin Clarke has formulated a rule from experiments conducted by him during his investigations into the use of iron and steel for hollow tube bridges, which is as follows:

$$\left.\begin{array}{l}\text{Center break-}\\\text{ing load, in tons.}\end{array}\right\} = \frac{\text{Area of material in sq. in.} \times \text{Mean depth in in.} \times \text{Constant}}{\text{Clear span in feet}}.$$

When the constant used is 1.2, the calculation for the tubes experimented upon by Mr. Fairbairn are given in the last column of the table. D. K. Clark's "Rules, Tables, and Data," page 513, gives a rule for hollow tubes as follows: $W = 3.14D^2TS \div L$. W = breaking weight in pounds in centre; D = extreme diameter in inches; T = thickness in inches; L = length between supports in inches; S = ultimate tensile strength in pounds per sq. in.

Taking S, the strength of a square inch of a riveted joint, at 35,000 lbs. per. sq. in., this rule figures as follows for the different examples experimented upon by Mr. Fairbairn: I, 2670; II, 10,190; III, 7700; IV, 15,320.

This shows a close approximation to the breaking weight obtained by experiments and that derived from Edwin Clarke's and D. K. Clark's rules. We therefore assume that this system of calculation is practically correct, and that it is eminently safe when a large factor of safety is provided, and from the fact that a chimney may be standing for many years without receiving anything like the strain taken as the basis of the calculation, viz., fifty pounds per square foot. Wind pressure at fifty pounds per square foot may be assumed to be travelling in a horizontal direction, and be of the same velocity from the top to the bottom of the stack. This is the extreme assumption. If, however, the chimney is round, its effective area would be only half of its diameter plane. We assume that the entire force may be concentrated in the centre of the height of the section of the chimney under consideration.

Taking as an example a 125-foot iron chimney at Poughkeepsie, N. Y., the average diameter of which is 90 inches, the effective surface in square feet upon which the force of the wind may play will therefore be 7½ times 125 divided by 2, which multiplied by 50 gives a total wind force of 23,437 pounds. The resistance of the chimney to breaking across the top of the foundation would be 3.14 × 168² (that is, diameter of base) × .25 × 35,000 ÷ (750 × 4) = 258,486, or 10.6 times the entire force of the wind. We multiply the half height above the joint in inches, 750, by 4, because the chimney is considered a fixed beam with a load suspended on one end. In calculating its strength half way up, we have a beam of the same character. It is a fixed beam at a line half way up the chimney, where it is 90 inches in diameter and .187 inch thick. Taking the diametrical section above this line, and the force as concentrated in the centre of it, or half way up from the point under consideration, its breaking strength is: 3.14 × 90² × .187 × 35,000 ÷ (381 × 4) = 109,220; and the force of the wind to tear it apart through its cross-section, 7¼ × 62½ × 50 ÷ 2 = 11,352, or a little more than one tenth of the strength of the stack.

The Babcock & Wilcox Co.'s book "Steam" illustrates a steel chimney at the works of the Maryland Steel Co., Sparrow's Point, Md. It is 225 ft. in height above the base, with internal brick lining 13' 9" uniform inside diameter. The shell is 25 ft. diam. at the base, tapering in a curve to 17 ft. 25 ft. above the base, thence tapering almost imperceptibly to 14' 8" at the top. The upper 40 feet is of ¼-inch plates, the next four sections of 40 ft. each are respectively 9/32, 5/16, 11/32, and ⅜ inch.

Sizes of Foundations for Steel Chimneys.

(Selected from circular of Phila. Engineering Works.)

HALF-LINED CHIMNEYS.

Diameter, clear, feet	3	4	5	6	7	9	11
Height, feet	100	100	150	150	150	150	150
Least diameter foundation	15'9"	16'4"	20'4"	21'10"	22'7"	23'8"	24'8"
Least depth foundation	6'	6'	9'	8'	9'	10'	10'
Height, feet	....	125	200	200	250	275	300
Least diameter foundation	....	18'5"	23'8"	25'	29'8"	33'6"	36'
Least depth foundation	....	7'	10'	10'	12'	12'	14'

Weight of Sheet-iron Smoke-stacks per Foot.

(Porter Mfg. Co.)

Diam., inches.	Thickness W. G.	Weight per ft.	Diam., inches.	Thickness W. G.	Weight per ft.	Diam. inches.	Thickness W. G.	Weight per ft.
10	No. 16	7.20	26	No. 16	17.50	20	No. 14	18.33
12	"	8.66	28	"	18.75	22	"	20.00
14	"	9.58	30	"	20.00	24	"	21.66
16	"	11.68	10	No. 14	9.40	26	"	23.33
20	"	13.75	12	"	11.11	28	"	25.00
22	"	15.00	14	"	13.69	30	"	26.66
24	"	16.25	16	"	15.00			

Sheet-iron Chimneys. (Columbus Machine Co.)

Diameter Chimney, inches.	Length Chimney, feet.	Thickness Iron, B. W. G.	Weight, lbs.	Diameter Chimney, inches.	Length Chimney, feet.	Thickness Iron, B. W. G.	Weight, lbs.
10	20	No. 16	160	30	40	No. 15	950
15	20	" 16	240	32	40	" 15	1,020
20	20	" 16	320	34	40	" 14	1,170
22	20	" 16	350	36	40	" 14	1,240
24	40	" 16	760	38	40	" 12	1,600
26	40	" 16	826	40	40	" 12	1,690
28	40	" 15	900				

THE STEAM-ENGINE.

Expansion of Steam. Isothermal and Adiabatic.—According to Mariotte's law, the volume of a perfect gas, the temperature being kept constant, varies inversely as its pressure, or $p \propto \frac{1}{v}$; $pv =$ a constant. The curve constructed from this formula is called the *isothermal* curve, or curve of equal temperatures, and is a common or rectangular hyperbola. The relation of the pressure and volume of saturated steam, as deduced from Regnault's experiments, and as given in Steam tables, is approximately, according to Rankine (S. E., p. 403), for pressures not exceeding 120 lbs., $p \propto \frac{1}{v^{\frac{17}{16}}}$, or $p \propto v^{-\frac{17}{16}}$, or $pv^{\frac{17}{16}} = pv^{1.0625} =$ a constant. Zeuner has found that the exponent 1.0646 gives a closer approximation.

When steam expands in a closed cylinder, as in an engine, according to Rankine (S. E., p. 385), the approximate law of the expansion is $p \propto \frac{1}{v^{\frac{10}{9}}}$, or $p \propto v^{-\frac{10}{9}}$, or $pv^{1.111} =$ a constant. The curve constructed from this formula is called the *adiabatic* curve, or curve of no transmission of heat.

Peabody (Therm., p. 112) says: "It is probable that this equation was obtained by comparing the expansion lines on a large number of indicator-diagrams. . . . There does not appear to be any good reason for using an exponential equation in this connection, . . . and the action of a lagged steam-engine cylinder is far from being adiabatic. . . . For general purposes the hyperbola is the best curve for comparison with the expansion curve of an indicator-card. . . ." Wolff and Denton, Trans. A. S. M. E., ii. 175, say: "From a number of cards examined from a variety of steam-engines in current use, we find that the actual expansion line varies between the 10/9 adiabatic curve and the Mariotte curve."

Prof. Thurston (A. S. M. E., ii. 203), says he doubts if the exponent ever becomes the same in any two engines, or even in the same engines at different times of the day and under varying conditions of the day.

Expansion of Steam according to Mariotte's Law and to the Adiabatic Law. (Trans. A. S. M. E., ii. 156.)—Mariotte's law $pv = p_1v_1$; values calculated from formula $\frac{p_m}{p_1} = \frac{1}{R}(1 + \text{hyp log } R)$, in which $R = v_2 \div v_1$, $p_1 =$ absolute initial pressure, $p_m =$ absolute mean pressure, $v_1 =$ initial volume of steam in cylinder at pressure p_1, $v_2 =$ final volume of steam at final pressure. Adiabatic law: $pv^{\frac{10}{9}} = p_1v_1^{\frac{10}{9}}$; values calculated from formula $\frac{p_m}{p_1} = 10R^{-\frac{1}{9}} - 9R^{-\frac{10}{9}}$.

Ratio of Expansion R.	Ratio of Mean to Initial Pressure.		Ratio of Expansion R.	Ratio of Mean to Initial Pressure.		Ratio of Expansion R.	Ratio of Mean to Initial Pressure.	
	Mar.	Adiab.		Mar.	Adiab.		Mar.	Adiab.
1.00	1.000	1.000	3.7	.624	.600	6.	.465	.438
1.25	.978	.976	3.8	.614	.590	6.25	.453	.425
1.50	.937	.931	3.9	.605	.580	6.5	.442	.413
1.75	.891	.881	4.	.597	.571	6.75	.431	.403
2.	.847	.834	4.1	.588	.562	7.	.421	.393
2.2	.813	.798	4.2	.580	.554	7.25	.411	.383
2.4	.781	.765	4.3	.572	.546	7.5	.402	.374
2.5	.766	.748	4.4	.564	.538	7.75	.393	.365
2.6	.752	.733	4.5	.556	.530	8.	.385	.357
2.8	.725	.704	4.6	.549	.523	8.25	.377	.349
3.	.700	.678	4.7	.542	.516	8.5	.369	.342
3.1	.688	.666	4.8	.535	.509	8.75	.362	.335
3.2	.676	.654	4.9	.528	.502	9.	.355	.328
3.3	.665	.642	5.05	.522	.495	9.25	.349	.321
3.4	.654	.630	5.2	.506	.479	9.5	.342	.315
3.5	.644	.620	5.5	.492	.464	9.75	.336	.309
3.6	.634	.610	5.75	.478	.450	10.	.330	.303

Mean Pressure of Expanded Steam.—For calculations of engines it is generally assumed that steam expands according to Mariotte's law, the curve of the expansion line being a hyperbola. The mean pressure, measured above vacuum, is then obtained from the formula

$$P_m = p_1 \frac{1 + \text{hyp log } R}{R}, \quad \text{or} \quad P_m = P_t(1 + \text{hyp log } R),$$

in which P_m is the absolute mean pressure, p_1 the absolute initial pressure taken as uniform up to the point of cut-off, P_t the terminal pressure, and R the ratio of expansion. If l = length of stroke to the cut-off, L = total stroke.

$$P_m = \frac{p_1 l + p_1 l \text{ hyp log} \frac{L}{l}}{L}; \quad \text{and if } R = \frac{L}{l}, \quad P_m = p_1 \frac{1 + \text{hyp log } R}{R}.$$

Mean and Terminal Absolute Pressures.—Mariotte's Law.—The values in the following table are based on Mariotte's law, except those in the last column, which give the mean pressure of superheated steam, which, according to Rankine, expands in a cylinder according to the law $p \propto v^{-\frac{17}{16}}$. These latter values are calculated from the formula $\frac{P_m}{p_1} = \frac{17 - 16R^{-\frac{1}{16}}}{R}$. $R^{-\frac{1}{16}}$ may be found by extracting the square root of $\frac{1}{R}$ four times. From the mean absolute pressures given deduct the mean back pressure (absolute) to obtain the mean effective pressure.

Rate of Expansion.	Cut-off.	Ratio of Mean to Initial Pressure.	Ratio of Mean to Terminal Pressure.	Ratio of Terminal to Mean Pressure.	Ratio of Initial to Mean Pressure.	Ratio of Mean to Initial Dry Steam.
30	0.033	0.1467	4.40	0.227	6.83	0.136
28	0.036	0.1547	4.33	0.231	6.46	....
26	0.038	0.1638	4.26	0.235	6.11	....
24	0.042	0.1741	4.18	0.239	5.75	....
22	0.045	0.1860	4.09	0.244	5.33	....
20	0.050	0.1996	4.00	0.250	5.00	0.186
18	0.055	0.2161	3.89	0.256	4.63	....
16	0.062	0.2358	3.77	0.265	4.24	....
15	0.066	0.2472	3.71	0.269	4.05	....
14	0.071	0.2599	3.64	0.275	3.85	....
13.33	0.075	0.2690	3.59	0.279	3.72	0.254
13	0.077	0.2742	3.56	0.280	3.65	....
12	0.083	0.2904	3.48	0.287	3.44	....
11	0.091	0.3069	3.40	0.294	3.24	....
10	0.100	0.3303	3.30	0.303	3.03	0.314
9	0.111	0.3552	3.20	0.312	2.81	....
8	0.125	0.3849	3.08	0.321	2.60	0.370
7	0.143	0.4210	2.95	0.339	2.37	....
6.66	0.150	0.4347	2.90	0.345	2.30	0.417
6.00	0.166	0.4653	2.79	0.360	2.15	....
5.71	0.175	0.4807	2.74	0.364	2.08	....
5.00	0.200	0.5218	2.61	0.383	1.92	0.506
4.44	0.225	0.5608	2.50	0.400	1.78	....
4.00	0.250	0.5965	2.39	0.419	1.68	0.582
3.63	0.275	0.6308	2.29	0.437	1.58	....
3.33	0.300	0.6615	2.20	0.454	1.51	0.648
3.00	0.333	0.6995	2.10	0.476	1.43	....
2.86	0.350	0.7171	2.05	0.488	1.39	0.707
2.66	0.375	0.7440	1.98	0.505	1.34	....
2.50	0.400	0.7664	1.91	0.523	1.31	0.756
2.22	0.450	0.8095	1.80	0.556	1.24	0.800
2.00	0.500	0.8465	1.69	0.591	1.18	0.840
1.82	0.550	0.8786	1.60	0.626	1.14	0.874
1.66	0.600	0.9066	1.51	0.662	1.10	0.900
1.60	0.625	0.9187	1.47	0.680	1.09	....
1.54	0.650	0.9292	1.43	0.699	1.07	0.936
1.48	0.675	0.9405	1.39	0.718	1.06	....

Calculation of Mean Effective Pressure, Clearance and Compression Considered.—In the above tables no account is taken of clearance, which in actual steam-engines modifies the ratio of expansion and the mean pressure; nor of compression and back-pressure, which diminish the mean effective pressure. In the following calculation these elements are considered.

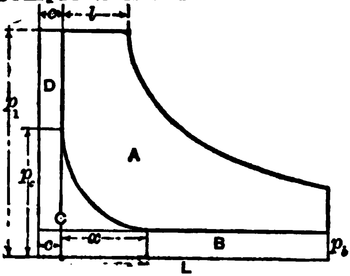


FIG. 187.

L = length of stroke, l = length before cut-off, x = length of compression part of stroke, c = clearance, p_1 = initial pressure, p_b = back pressure, p_c = pressure of clearance steam at end of compression. All pressures are absolute, that is, measured from a perfect vacuum.

$$\text{Area of ABCD} = p_1(l + c)\left(1 + \text{hyp log}\,\frac{L + c}{l + c}\right);$$

$$\text{B} = p_b(L - x);$$

$$\text{C} = p_c c\left(1 + \text{hyp log}\,\frac{x + c}{c}\right) = p_b(x + c)\left(1 + \text{hyp log}\,\frac{x + c}{c}\right);$$

$$\text{D} = (p_1 - p_c)c = p_1 c - p_b(x + c).$$

$$\text{Area of A} = \text{ABCD} - (\text{B} + \text{C} + \text{D})$$

$$= p_1(l + c)\left(1 + \text{hyp log}\,\frac{L + c}{l + c}\right)$$

$$- \left[p_b(L - x) + p_b(x + c)\left(1 + \text{hyp log}\,\frac{x + c}{c}\right) + p_1 c - p_b(x + c)\right]$$

$$= p_1(l + c)\left(1 + \text{hyp log}\,\frac{L + c}{l + c}\right)$$

$$- p_b\left[(L - x) + (x + c)\,\text{hyp log}\,\frac{x + c}{c}\right] - p_1 c.$$

$$\text{Mean effective pressure} = \frac{\text{area of A}}{L}.$$

EXAMPLE.—Let $L = 1$, $l = 0.25$, $x = 0.25$, $c = 0.1$, $p_1 = 60$ lbs., $p_b = 2$ lbs.

$$\text{Area A} = 60(.25 + .1)\left(1 + \text{hyp log}\,\frac{1.1}{.35}\right)$$

$$- 2\left[(1 - .25) + .35\ \text{hyp log}\,\frac{.35}{.1}\right] - 60 \times .1$$

$$= 21(1 + 1.145) - 2[.75 + 35 \times 1.253] - 6$$

$$= 45.045 - 2.377 - 6 = 36.668 = \text{mean effective pressure.}$$

The actual indicator-diagram generally shows a mean pressure considerably less than that due to the initial pressure and the rate of expansion. The causes of loss of pressure are: 1. Friction in the stop-valves and steam-pipes. 2. Friction or wire-drawing of the steam during admission and cut-off, due chiefly to defective valve-gear and contracted steam-passages. 3. Liquefaction during expansion. 4. Exhausting before the engine has completed its stroke. 5. Compression due to early closure of exhaust. 6. Friction in the exhaust-ports, passages, and pipes.

Re-evaporation during expansion of the steam condensed during admission, and valve-leakage after cut-off, tend to elevate the expansion line of the diagram and increase the mean pressure.

If the theoretical mean pressure be calculated from the initial pressure and the rate of expansion on the supposition that the expansion curve follows Mariotte's law, pv = a constant, and the necessary corrections are made for clearance and compression, the expected mean pressure in practice may be found by multiplying the calculated results by the factor in the following table, according to Seaton.

Particulars of Engine.	Factor.
Expansive engine, special valve-gear, or with a separate cut-off valve, cylinder jacketed	0.94
Expansive engine having large ports, etc., and good ordinary valves, cylinders jacketed	0.9 to 0.92
Expansive engines with the ordinary valves and gear as in general practice, and unjacketed	0.8 to 0.85
Compound engines, with expansion valve to h.p. cylinder; cylinders jacketed, and with large ports, etc	0.9 to 0.92
Compound engines, with ordinary slide-valves, cylinders jacketed, and good ports, etc	0.8 to 0.85
Compound engines as in general practice in the merchant service, with early cut-off in both cylinders, without jackets and expansion-valves	0.7 to 0.8
Fast-running engines of the type and design usually fitted in war-ships	0.6 to 0.8

If no correction be made for clearance and compression, and the engine is in accordance with general modern practice, the theoretical mean pressure may be multiplied by 0.96, and the product by the proper factor in the table, to obtain the expected mean pressure.

Given the Initial Pressure and the Average Pressure, to Find the Ratio of Expansion and the Period of Admission.

P = initial absolute pressure in lbs. per sq. in.;
p = average total pressure during stroke in lbs. per sq. in.;
L = length of stroke in inches;
l = period of admission measured from beginning of stroke;
c = clearance in inches;

$$R = \text{actual ratio of expansion} = \frac{L+c}{l+c}. \qquad (1)$$

$$p = \frac{P(1 + \text{hyp log } R)}{R}.$$

To find average pressure p, taking account of clearance,

$$p = \frac{P(l+c) + P(l+c)\text{ hyp log } R - Pc}{L}, \qquad (2)$$

whence $$pL + Pc = P(l+c)(1 + \text{hyp log } R);$$

$$\text{hyp log } R = \frac{pL + Pc}{Pl + Pc} - 1 = \frac{\frac{p}{P}L + c}{l + c} - 1. \qquad (3)$$

Given p and P, to find R and l (by trial and error).—There being two unknown quantities R and l, assume one of them, viz., the period of admission l, substitute it in equation (3) and solve for R. Substitute this value of R in the formula (1), or $l = \frac{L+c}{R} - c$, obtained from formula (1), and find l. If the result is greated than the assumed value of l, then the assumed value of the period of admission is too long; if less, the assumed value is too short. Assume a new value of l, substitute it in formula (3) as before, and continue by this method of trial and error till the required values of R and l are obtained.

EXAMPLE.—$P = 70$, $p = 42.78$, $L = 60''$, $c = 3''$, to find l. Assume $l = 21$ in.

$$\text{hyp log } R = \frac{\frac{p}{P}L + c}{l + c} - 1 = \frac{\frac{42.78}{70} \times 60 + 3}{21 + 3} - 1 = 1.653 - 1 = .653;$$

$$\text{hyp log } R = .653, \quad \text{whence } R = 1.92,$$

$$l = \frac{L+c}{R} - c = \frac{63}{1.92} - 3 = 29.8,$$

which is greater than the assumed value, 21 inches.

Now assume $l = 15$ inches:

$$\text{hyp log } R = \frac{\frac{42.78}{70} \times 60 + 3}{15 + 3} - 1 = 1.204, \text{ whence } R = 3.5;$$

$$l = \frac{L+c}{R} - c = \frac{63}{3.5} - 3 = 18 - 3 = 15 \text{ inches, the value assumed.}$$

Therefore $R = 3.5$, and $l = 15$ inches.

Period of Admission Required for a Given Actual Ratio of Expansion:

$$l = \frac{L+c}{R} - c, \text{ in inches} \quad . \quad . \quad . \quad . \quad (4)$$

$$\text{In percentage of stroke, } l = \frac{100 + \text{p.ct. clearance}}{R} - \text{p. ct. clearance.} \quad . \quad (5)$$

$$\textit{Terminal pressure} = \frac{P(l+c)}{L+c} = \frac{P}{R}. \quad . \quad . \quad . \quad . \quad (6)$$

Pressure at any other Point of the Expansion.—Let L_1 = length of stroke up to the given point.

$$\text{Pressure at the given point} = \frac{P(l+c)}{L_1 + c}. \quad . \quad . \quad . \quad . \quad (7)$$

WORK OF STEAM IN A SINGLE CYLINDER.

To facilitate calculations of steam expanded in cylinders the table on the next page is abridged from Clark on the Steam-engine. The actual ratios of expansion, column 1, range from 1.0 to 8.0, for which the hyperbolic logarithms are given in column 2. The 3d column contains the periods of admission relative to the actual ratios of expansion, as percentages of the stroke, calculated by formula (5) above. The 4th column gives the values of the mean pressures relative to the initial pressures, the latter being taken as 1, calculated by formula (2). In the calculation of columns 3 and 4, clearance is taken into account, and its amount is assumed at 7% of the stroke. The final pressures, in the 5th column, are such as would be arrived at by the continued expansion of the whole of the steam to the end of the stroke, the initial pressure being equal to 1. They are the reciprocals of the ratios of expansion, column 1. The 6th column contains the relative total performances of equal weights of steam worked with the several actual ratios of expansion; the total performance, when steam is admitted for the whole of the stroke, without expansion, being equal to 1. They are obtained by dividing the figures in column 4 by those in column 5.

The pressures have been calculated on the supposition that the pressure of steam, during its admission into the cylinder, is uniform up to the point of cutting off, and that the expansion is continued regularly to the end of the stroke. The relative performances have been calculated without any allowance for the effect of compressive action.

The calculations have been made for periods of admission ranging from 100%, or the whole of the stroke, to 6.4%, or 1/16 of the stroke. And though, nominally, the expansion is 16 times in the last instance, it is actually only 8 times, as given in the first column. The great difference between the nominal and the actual ratios of expansion is caused by the clearance, which is equal to 7% of the stroke, and causes the nominal volume of steam admitted, namely, 6.4%, to be augmented to 6.4 + 7 = 13.4% of the stroke, or, say, double, for expansion. When the steam is cut off at 1/9, the actual expansion is only 6 times; when cut off at 1/5, the expansion is 4 times; when cut off at ⅓, the expansion is 2⅔ times; and to effect an actual expansion to twice the initial volume, the steam is cut off at 46½% of the stroke, not at half-stroke.

Expansive Working of Steam—Actual Ratios of Expansion, with the Relative Periods of Admission, Pressures, and Performance.

Steam-pressure 100 lbs. absolute. Clearance at each end of the cylinder 7% of the stroke.

(SINGLE CYLINDER.)

1 Actual Ratio of Expansion, or No. of Volumes to which the Initial Volume is Expanded.	2 Hyperbolic Logarithm of Actual Ratio of Expansion.	3 Period of Admission or Cut-off, 7% Clearance.	4 Average Total Pressure. Initial Pressure = 1.	5 Total Final Pressure. Initial Pressure = 1.	6 Ratio of Total Performance of Equal Weights of Steam. (Col. 4 ÷ Col 5.)	7 Actual Work done by 1 lb. of 100 lbs. Steam. Ft.-lbs.	8 Quantity of Steam Consumed per H.P. of Actual Work done per hour	9 Net Capacity of Cylinder per lb. of 100 lbs. Steam admitted in 1 stroke. Cubic feet.
1	.0000	100	1.000	1.000	1.000	58,273	34.0	4.05
1.1	.0953	90.3	.996	.909	1.096	63,850	31.0	4.45
1.18	.1698	83.3	.986	.847	1.164	67,836	29.2	4.78
1.23	.2070	80	.980	.813	1.206	70,246	28.2	4.98
1.3	.2624	75.3	.969	.769	1.261	73,513	26.9	5.26
1.39	.3293	70	.953	.719	1.325	77,242	25.6	5.63
1.45	.3716	66.8	.942	.690	1.365	79,555	24.9	5.87
1.54	.4317	62.5	.925	.649	1.425	83,055	23.8	6.23
1.6	.4700	59.9	.913	.625	1.461	85,125	23.3	6.47
1.75	.5595	54.1	.883	.571	1.546	90,115	22.0	7.08
1.88	.6314	50	.860	.532	1.616	94,200	21.0	7.61
2	.6931	46.5	.836	.5	1.672	97,432	20.8	8.09
2.28	.8241	40	.787	.439	1.793	104,466	19.0	9.23
2.4	.8755	37.6	.766	.417	1.837	107,050	18.5	9.71
2.65	.9745	33.3	.726	.377	1.925	112,220	17.7	10.72
2.9	1.065	29.9	.692	.345	2.006	116,885	16.9	11.74
3.2	1.163	26.4	.652	.313	2.083	121,386	16.3	12.95
3.35	1.209	25	.637	.298	2.129	124,066	16.0	13.56
3.6	1.281	22.7	.608	.278	2.187	127,450	15.5	14.57
3.8	1.335	21.2	.589	.263	2.240	130,533	15.2	15.38
4	1.386	19.7	.569	.250	2.278	132,770	14.9	16.19
4.2	1.435	18.5	.551	.238	2.315	134,900	14.7	17.00
4.5	1.504	16.8	.526	.222	2.370	138,130	14.34	18.21
4.8	1.569	15.3	.503	.208	2.418	140,920	14.05	19.43
5	1.609	14.4	.488	.200	2.440	142,180	13.92	20.23
5.2	1.649	13.6	.476	.193	2.466	143,720	13.78	21.04
5.5	1.705	12.5	.457	.182	2.511	146,325	13.53	22.25
5.8	1.758	11.4	.438	.172	2.547	148,390	13.34	23.47
5.9	1.775	11.1	.432	.169	2.556	148,940	13.29	23.87
6.2	1.825	10.3	.419	.161	2.585	150,630	13.14	25.09
6.3	1.841	10	.413	.159	2.597	151,370	13.08	25.49
6.6	1.887	9.2	.398	.152	2.619	152,595	12.98	26.71
7	1.946	8.3	.381	.143	2.664	155,200	12.75	28.33
7.3	1.988	7.7	.369	.137	2.693	156,960	12.61	29.54
7.6	2.028	7.1	.357	.132	2.711	157,975	12.53	30.76
7.8	2.054	6.7	.348	.128	2.719	158,414	12.50	31.57
8	2.079	6.4	.342	.125	2.736	159,433	11.83	32.38

ASSUMPTIONS OF THE TABLE.—That the initial pressure is uniform; that the expansion is complete to the end of the stroke; that the pressure in expansion varies inversely as the volume; that there is no back-pressure of exhaust or of compression, and that clearance is 7% of the stroke at each end of the cylinder. No allowance has been made for loss of steam by cylinder-condensation or leakage.

Volume of 1 lb. of steam of 100 lbs. pressure per sq. in., or 14,400 lbs. per sq. ft. .... 4.33 cu. ft.
Product of initial pressure and volume .... 62,352 ft.-lbs.

Though a uniform clearance of 7% at each end of the stroke has been assumed as an average proportion for the purpose of compiling the table, the clearance of cylinders with ordinary slides varies considerably—say from 5% to 10%. (With Corliss engines it is sometimes as low as 2%.) With the clearance, 7%, that has been assumed, the table gives approximate results sufficient for most practical purposes, and more trustworthy than results deduced by calculations based on simple tables of hyperbolic logarithms, where clearance is neglected.

Weight of steam of 100 lbs. total initial pressure admitted for one stroke, per cubic foot of net capacity of the cylinder, in decimals of a pound = reciprocal of figures in column 9.

Total actual work done by steam of 100 lbs. total initial pressure in one stroke per cubic foot of net capacity of cylinder, in foot-pounds = figures in column 7 ÷ figures in column 9.

RULE 1: To find the net capacity of cylinder for a given weight of steam admitted for one stroke, and a given actual ratio of expansion. (Column 9 of table.)—Multiply the volume of 1 lb. of steam of the given pressure by the given weight in pounds, and by the actual ratio of expansion. Multiply the product by 100, and divide by 100 plus the percentage of clearance. The quotient is the net capacity of the cylinder.

RULE 2: To find the net capacity of cylinder for the performance of a given amount of total actual work in one stroke, with a given initial pressure and actual ratio of expansion.—Divide the given work by the total actual work done by 1 lb. of steam of the same pressure, and with the same actual ratio of expansion; the quotient is the weight of steam necessary to do the given work, for which the net capacity is found by Rule 1 preceding.

NOTE.—1. Conversely, the weight of steam admitted per cubic foot of net capacity for one stroke is the reciprocal of the cylinder-capacity per pound of steam, as obtained by Rule 1.

2. The total actual work done per cubic foot of net capacity for one stroke is the reciprocal of the cylinder-capacity per foot-pound of work done, as obtained by Rule 2.

3. The total actual work done per square inch of piston per foot of the stroke is 1/144th part of the work done per cubic foot.

4. The resistance of back pressure of exhaust and of compression are to be added to the net work required to be done, to find the total actual work.

APPENDIX TO ABOVE TABLE—MULTIPLIERS FOR NET CYLINDER-CAPACITY, AND TOTAL ACTUAL WORK DONE.

(For steam of other pressures than 100 lbs. per square inch.)

Total Pressures per square inch.	Multipliers.		Total Pressures per square inch.	Multipliers.	
	For Col. 7. Total Work by 1 lb. of Steam.	For Col. 9. Capacity of Cylinder.		For Col. 7. Total Work by 1 lb. of Steam.	For Col. 9. Capacity of Cylinder.
lbs.			lbs.		
65	.975	1.50	100	1.000	1.00
70	.981	1.40	110	1.009	.917
75	.986	1.31	120	1.011	.843
80	.988	1.24	130	1.015	.781
85	.991	1.17	140	[illegible]	.730
90	.995	1.11	150	1.025	.683
95	.998	1.05	160	1.031	.644

The figures in the second column of this table are derived by multiplying the total pressure per square foot of any given steam by the volume in cubic feet of 1 lb. of such steam, and dividing the product by 62,352, which is the product in foot-pounds for steam of 100 lbs. pressure. The quotient is the multiplier for the given pressure.

The figures in the third column are the quotients of the figures in the second column divided by the ratio of the pressure of the given steam to 100 lbs.

Measures for Comparing the Duty of Engines.—Capacity is measured in horse-powers, expressed by the initials, H.P.: 1 H.P. = 33,000 ft.-lbs. per minute, = 550 ft.-lbs. per second, = 1,980,000 ft.-lbs. per hour.

1 ft.-lb. = a pressure of 1 lb. exerted through a space of 1 ft. Economy is measured, 1, in pounds of coal per horse-power per hour; 2, in pounds of steam per horse-power per hour. The second of these measures is the more accurate and scientific, since the engine uses steam and not coal, and it is indepndent of the economy of the boiler.

In gas-engine tests the common measure is the number of cubic feet of gas (measured at atmospheric pressure) per horse-power, but as all gas is not of the same quality, it is necessary for comparison of tests to give the analysis of the gas. When the gas for one engine is made in one gas-producer, then the number of pounds of coal used in the producer per hour per horse-power of the engine is the proper measure of economy.

Economy, or duty of an engine, is also measured in the number of foot-pounds of work done per pound of fuel. As 1 horse-power is equal to 1,980,000 ft.-lbs. of work in an hour, a duty of 1 lb. of coal per H.P. per hour would be equal to 1,980,000 ft.-lbs. per lb. of fuel; 2 lbs. per H.P. per hour equals 990,000 ft.-lbs. per lb. of fuel, etc.

The duty of pumping-engines is commonly expressed by the number of foot-pounds of work done per 100 lbs. of coal.

When the duty of a pumping-engine is thus given, the equivalent number of pounds of fuel consumed per horse-power per hour is found by dividing 198 by the number of millions of foot-pounds of duty. Thus a pumping-engine giving a duty of 99 millions is equivalent to 198/99 = 2 lbs. of fuel per horse-power per hour.

Efficiency Measured in Thermal Units per Minute.—Some writers express the efficiency of an engine in terms of the number of thermal units used by the engine per minute for each indicated horse-power, instead of by the number of pounds of steam used per hour.

The heat chargeable to an engine per pound of steam is the difference between the total heat in a pound of steam at the boiler-pressure and that in a pound of the feed-water entering the boiler. In the case of condensing engines, suppose we have a temperature in the hot-well of 101° F., corresponding to a vacuum of 28 in. of mercury, or an absolute pressure of 1 lb. per sq. in. above a perfect vacuum: we may feed the water into the boiler at that temperature. In the case of a non-condensing-engine, by using a portion of the exhaust steam in a good feed-water heater, at a pressure a trifle above the atmosphere (due to the resistance of the exhaust passages through the heater), we may obtain feed-water at 212°. One pound of steam used by the engine then would be equivalent to thermal units as follows:

Pressure of steam by gauge:	50	75	100	125	150	175	200
Total heat in steam above 32°:	1172.8	1179.6	1185.0	1189.5	1193.5	1197.0	1200.2

Subtracting 69.1 and 180.9 heat-units, respectively, the heat above 32° in feed-water of 101° and 212° F., we have—

Heat given by boiler:							
Feed at 101°.....	1103.7	1110.5	1115.9	1120.4	1124.4	1127.9	1131.1
Feed at 212°.....	991.9	998.7	1004.1	1008.6	1012.6	1016.1	1019.3

Thermal units per minute used by an engine for each pound of steam used per indicated horse-power per hour:

Feed at 101°.....	18.40	18.51	18.60	18.67	18.74	18.80	18.85
Feed at 212°.....	16.53	16.65	16.74	16.81	16.88	16.94	16.99

EXAMPLES.—A triple-expansion engine, condensing, with steam at 175 lbs. gauge and vacuum 28 in., uses 13 lbs. of water per I.H.P. per hour, and a high-speed non-condensing engine, with steam at 100 lbs. gauge, uses 30 lbs. How many thermal units per minute does each consume?

Ans.—13 × 18.80 = 244.4, and 30 × 16.74 = 502.2 thermal units per minute.

A perfect engine converting all the heat-energy of the steam into work would require 33,000 ft.-lbs. ÷ 778 = 42.4164 thermal units per minute per indicated horse-power. This figure, 42.4164, therefore, divided by the number of thermal units per minute per I.H.P. consumed by an engine, gives its efficiency as compared with an ideally perfect engine. In the examples above, 42.4164 divided by 244.4 and by 502.2 gives 17.35% and 8.45% efficiency, respectively.

Total Work Done by One Pound of Steam Expanded in a Single Cylinder. (Column 7 of table.)—If 1 pound of water be converted into steam of atmospheric pressure = 2116.8 lbs. per sq. ft., it occupies a volume equal to 26.36 cu. ft. The work done is equal to 2116.8 lbs.

× 26.36 ft. = 55,788 ft.-lbs. The heat equivalent of this work is (55,788 ÷ 778 =) 71.7 units. This is the work of 1 lb. of steam of one atmosphere acting on a piston without expansion.

The gross work thus done on a piston by 1 lb. of steam generated at total pressures varying from 15 lbs. to 100 lbs. per sq. in. varies in round numbers from 56,000 to 62,000 ft.-lbs., equivalent to from 72 to 80 units of heat.

This work of 1 lb. of steam without expansion is reduced by clearance according to the proportion it bears to the net capacity of the cylinder. If the clearance be 7% of the stroke, the work of a given weight of steam without expansion, admitted for the whole of the stroke, is reduced in the ratio of 107 to 100.

Having determined by this ratio the quantity of work of 1 lb. of steam without expansion, as reduced by clearance, the work of the same weight of steam for various ratios of expansion may be found by multiplying it by the relative performance of equal weights of steam, given in the 6th column of the table.

Quantity of Steam Consumed per Horse-power of Total Work per Hour. (Column 8 of table.)—The measure of a horse-power is the performance of 33,000 ft.-lbs. per minute, or 1,980,000 ft.-lbs. per hour. This work, divided by the work of 1 lb of steam, gives the weight of steam required per horse-power per hour. For example, the total actual work done in the cylinder by 1 lb. of 100 lbs. steam, without expansion and with 7% of clearance, is 58,273 ft.-lbs.; and $\frac{1,980,000}{58,273} = 34$ lbs. of steam, is the weight of steam consumed for the total work done in the cylinder per horse-power per hour. For any shorter period of admission with expansion the weight of steam per horse-power is less, as the total work of 1 lb. of steam is more, and may be found by dividing 1,980,000 ft.-lbs. by the respective total work done; or by dividing 34 lbs. by the ratio of performance, column 6 in the table.

ACTUAL EXPANSIONS.

With Different Clearances and Cut-offs.

Computed by A. F. Nagle.

Cut-off.	Per Cent of Clearance.										
	0	1	2	3	4	5	6	7	8	9	10
.01	100.00	50.5	34.0	25.75	20.8	17.5	15.14	13.38	12.00	10.9	10
.02	50.00	33.67	25.50	20.60	17.58	15.00	13.25	11.89	10.80	9.91	8.17
.03	33.33	25.25	20.40	17.16	14.86	13.12	11.78	10.70	9.82	9.08	8.46
.04	25.00	20.20	17.00	14.71	13.00	11.66	10.60	9.73	9.00	8.39	7.86
.05	20.00	16.83	14.57	12.87	11.55	10.50	9.64	8.92	8.31	7.79	7.33
.06	16.67	14.43	12.75	11.44	10.40	9.55	8.83	8.23	7.71	7.27	6.88
.07	14.28	12.62	11.83	10.30	9.46	8.75	8.15	7.64	7.20	6.81	6.47
.08	12.50	11.22	10.2	9.36	8.67	8.08	7.57	7.13	6.75	6.41	6.11
.09	11.11	10.10	9.27	8.58	8.00	7.50	7.07	6.69	6.35	6.06	5.79
.10	10.00	9.18	8.50	7.92	7.43	7.00	6.62	6.30	6.00	5.74	5.50
.11	9.09	8.42	7.84	7.36	6.93	6.56	6.24	5.94	5.68	5.45	5.24
.12	8.33	7.78	7.29	6.86	6.50	6.18	5.89	5.63	5.40	5.19	5.00
.14	7.14	6.73	6.37	6.06	5.78	5.53	5.30	5.10	4.91	4.74	4.58
.16	6.25	5.94	5.67	5.42	5.20	5.00	4.82	4.65	4.50	4.36	4.23
.20	5.00	4.81	4.64	4.48	4.33	4.20	4.08	3.96	3.86	3.76	3.67
.25	4.00	3.88	3.77	3.68	3.58	3.50	3.42	3.34	3.27	3.21	3.14
.30	3.33	3.26	3.19	3.12	3.06	3.00	2.94	2.90	2.84	2.80	2.75
.40	2.50	2.46	2.43	2.40	2.36	2.33	2.30	2.28	2.25	2.22	2.20
.50	2.00	1.98	1.96	1.94	1.92	1.90	1.89	1.88	1.86	1.85	1.83
.60	1.67	1.66	1.65	1.64	1.63	1.615	1.606	1.597	1.588	1.580	1.571
.70	1.43	1.42	1.42	1.41	1.41	1.400	1.395	1.390	1.385	1.380	1.375
.80	1.25	1.25	1.244	1.241	1.238	1.235	1.233	1.230	1.227	1.224	1.222
.90	1.111	1.11	1.109	1.108	1.106	1.105	1.104	1.103	1.102	1.101	1.100
1.00	1.00	1 00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Relative Efficiency of 1 lb. of Steam with and without Clearance; back pressure and compression not considered.

$$\text{Mean total pressure} = p = \frac{P(l+c) + P(l+c) \text{ hyp. log. } R - Pc}{L}.$$

Let $P = 1$; $L = 100$; $l = 25$; $c = 7$.

$$p = \frac{32 + 32 \text{ hyp. log. } \frac{107}{32} - 7}{100} = \frac{32 + 32 \times 1.209 - 7}{100} = .637.$$

If the clearance be added to the stroke, so that clearance becomes zero, the same quantity of steam being used, admission l being then $= l + c = 32$, and stroke $L + c = 107$.

$$p_1 = \frac{32 + 32 \text{ hyp. log. } \frac{107}{32} - 0}{107} = \frac{32 + 32 \times 1.209}{107} = .707.$$

That is, if the clearance be reduced to 0, the amount of the clearance 7 being added to both the admission and the stroke, the same quantity of steam will do more work than when the clearance is 7 in the ratio 707 : 637, or 11% more.

Back Pressure Considered.—If back pressure $= .10$ of P, this amount has to be subtracted from p and p_1 giving $p = .537$, $p_1 = .607$, the work of a given quantity of steam used without clearance being greater than when clearance is 7 per cent in the ratio of 607 : 537, or 13% more.

Effect of Compression.—By early closure of the exhaust, so that a portion of the exhaust-steam is compressed into the clearance-space, much of the loss due to clearance may be avoided. If expansion is continued down to the back pressure, if the back pressure is uniform throughout the exhaust-stroke, and if compression begins at such point that the exhaust-steam remaining in the cylinder is compressed to the initial pressure at the end of the back stroke, then the work of compression of the exhaust-steam equals the work done during expansion by the clearance-steam. The clearance-space being filled by the exhaust-steam thus compressed, no new steam is required to fill the clearance-space for the next forward stroke, and the work and efficiency of the steam used in the cylinder are just the same as if there were no clearance and no compression. When, however, there is a drop in pressure from the final pressure of the expansion, or the terminal pressure, to the exhaust or back pressure (the usual case), the work of compression to the initial pressure is greater than the work done by the expansion of the clearance-steam, so that a loss of efficiency results. In this case a greater efficiency can be attained by inclosing for compression a less quantity of steam than that needed to fill the clearance-space with steam of the initial pressure. (See Clark, S. E., p. 399, *et seq.*; also F. H. Ball, Trans. A. S. M. E., xiv. 1067.) It is shown by Clark that a somewhat greater efficiency is thus attained whether or not the pressure of the steam be carried down by expansion to the back exhaust-pressure. As a result of calculations to determine the most efficient periods of compression for various percentages of back pressure, and for various periods of admission, he gives the table on the next page:

Clearance in Low- and High-speed Engines. (Harris Tabor, *Am. Mach.*, Sept. 17, 1891.)—The construction of the high-speed engine is such, with its relatively short stroke, that the clearance must be much larger than in the releasing-valve type. The short-stroke engine is, of necessity, an engine with large clearance, which is aggravated when a variable compression is a feature. Conversely, the releasing-valve gear is, from necessity, an engine of slow rotative speed, where great power is obtainable from long stroke, and small clearance is a feature in its construction. In one case the clearance will vary from 8% to 12% of the piston-displacement, and in the other from 2% to 3%. In the case of an engine with a clearance equalling 10% of the piston-displacement the waste room becomes enormous when considered in connection with an early cut-off. The system of compounding reduces the waste due to clearance in proportion as the steam is expanded to a lower pressure. The farther expansion is carried through a train of cylinders the greater will be the reduction of waste due to clearance. This is shown from the fact that the high-speed engine, expanding steam much less than the Corliss, will show a greater gain when changed from simple to compound than its rival under similar conditions.

Compression of Steam in the Cylinder.

Best Periods of Compression; Clearance 7 per cent.

Cut-off in Percentages of the Stroke.	Total Back Pressure, in percentages of the total initial pressure.							
	2½	5	10	15	20	25	30	35
	Periods of Compression, in parts of the stroke.							
10%	65%	57%	44%	32%	....	......	....	....
15	58	52	40	29	23%	.....	....	....
20	52	47	37	27	22	......	....	....
25	47	42	34	26	21	17%	....	......
30	42	39	32	25	20	16	14%	12%
35	39	35	29	23	19	15	13	11
40	36	32	27	21	18	14	13	11
45	33	30	25	20	17	14	12	10
50	30	27	23	18	16	13	12	10
55	27	24	21	17	15	13	11	9
60	24	22	19	15	14	12	11	9
65	22	20	17	15	14	12	10	8
70	19	17	16	14	14	12	10	8
75	17	16	14	13	12	11	9	8

Notes to Table.—1. For periods of admission, or percentages of back pressure, other than those given, the periods of compression may be readily found by interpolation.

2. For any other clearance, the values of the tabulated periods of compression are to be altered in the ratio of 7 to the given percentage of clearance.

Cylinder-condensation may have considerable effect upon the best point of compression, but it has not yet (1893) been determined by experiment. (Trans. A. S. M. E., xiv. 1078.)

Cylinder-condensation.—Rankine, S. E., p. 421, says: Conduction of heat to and from the metal of the cylinder, or to and from liquid water contained in the cylinder, has the effect of lowering the pressure at the beginning and raising it at the end of the stroke, the lowering effect being on the whole greater than the raising effect. In some experiments the quantity of steam wasted through alternate liquefaction and evaporation in the cylinder has been found to be greater than the quantity which performed the work.

Percentage of Loss by Cylinder-condensation, taken at Cut-off.

(From circular of the Ashcroft Mfg. Co. on the Tabor Indicator, 1889.)

Percentage of Stroke completed at Cut-off.	Percent. of Feed-water accounted for by the Indicator diagram.			Percent. of Feed-water Consumption due to Cylinder-condensat'n.		
	Simple Engines.	Compound Engines, h.p. cyl.	Triple-expansion Engines, h.p. cyl.	Simple Engines.	Compound Engines, h.p. cyl.	Triple-expansion Engines, h.p. cyl.
5	58	....	....	42	... .	... .
10	66	74	....	34	26	....
15	71	76	78	29	24	22
20	74	78	80	26	22	20
30	78	82	84	22	18	16
40	82	85	87	18	15	13
50	86	88	90	14	12	10

Theoretical Compared with Actual Water-consumption, Single-cylinder Automatic Cut-off Engines. (From the catalogue of the Buckeye Engine Co.)—The following table has been prepared on the basis of the pressures that result in practice with a constant boiler-pressure of 80 lbs. and different points of cut-off, with Buckeye engines and others with similar clearance. Fractions are omitted, except in the percentage column, as the degree of accuracy their use would seem to imply is not attained or aimed at.

Cut-off Part of Stroke.	Mean Effective Pressure.	Total Terminal Pressure.	Indicated Rate, lbs. Water, per I.H.P. per hour.	Assumed.	
				Act'l Rate.	Per ct. Loss.
.10	18	11	20	32	58
.15	27	15	19	27	41
.20	35	20	19	25	31.5
.25	42	25	20	25	25
.30	48	30	20	24	21.8
.35	53	35	21	25	19
.40	57	38	22	26	16.7
.45	61	43	23	27	15
.50	64	48	24	27	13.6

It will be seen that while the best indicated economy is when the cut-off is about at .15 or .20 of the stroke, giving about 30 lbs. M.E.P., and a terminal 3 or 4 lbs. above atmosphere, when we come to add the percentages due to a constant amount of unindicated loss, as per sixth column, the most economical point of cut-off is found to be about .30 of the stroke, giving 48 lbs. M.E.P. and 30 lbs. terminal pressure. This showing agrees substantially with modern experience under automatic cut-off regulation.

Experiments on Cylinder-condensation.—Experiments by Major Thos. English (*Eng'g*, Oct. 7, 1887, p. 386) with an engine 10 × 14 in., jacketed in the sides but not on the ends, indicate that the net initial condensation (or excess of condensation over re-evaporation) by the clearance surface varies directly as the initial density of the steam, and inversely as the square root of the number of revolutions per unit of time. The mean results gave for the net initial condensation by clearance-space per sq. ft. of surface at one rev. per second 6.06 thermal units in the engine when run non-condensing and 5.75 units when condensing.

G. R. Bodmer (*Eng'g*, March 4, 1892, p. 299) says: Within the ordinary limits of expansion desirable in one cylinder the expansion ratio has practically no influence on the amount of condensation per stroke, which for simple engines can be expressed by the following formula for the weight of water condensed [per minute, probably; the original does not state]: $W = C\frac{S(T-t)}{L\sqrt[3]{N^2}}$, where T denotes the mean admission temperature, t the mean exhaust temperature, S clearance-surface (square feet), N the number of revolutions per second, L latent heat of steam at the mean admission temperature, and C a constant for any given type of engine.

Mr. Bodmer found from experimental data that for high-pressure non-jacketed engines C = about 0.11, for condensing non-jacketed engines 0.085 to 0.11, for condensing jacketed engines 0.085 to 0.053. The figures for jacketed engines apply to those jacketed in the usual way, and not at the ends. C varies for different engines of the same class, but is practically constant for any given engine. For simple high-pressure non-jacketed engines it was found to range from 0.1 to 0.112.

Applying Mr. Bodmer's formula to the case of a Corliss non-jacketed non-condensing engine, 4-ft. stroke, 24 in. diam., 60 revs. per min., initial pressure 90 lbs. gauge, exhaust pressure 2 lbs., we have $T - t = 112°$, $N = 1$, $L = 880$, $S = 7$ sq. ft.; and, taking $C = .112$ and W = lbs. water condensed per minute, $W = \frac{.112 \times 112 \times 7}{1 \times 880} = .09$ lb. per minute, or 5.4 lbs. per hour. If the steam used per I.H.P. per hour according to the diagram is 20 lbs., the actual water consumption is 25.4 lbs., corresponding to a cylinder condensation of 27%.

INDICATOR-DIAGRAM OF A SINGLE-CYLINDER ENGINE.

Definitions.—*The Atmospheric Line*, *AB*, is a line drawn by the pencil of the indicator when the connections with the engine are closed and both sides of the piston are open to the atmosphere.

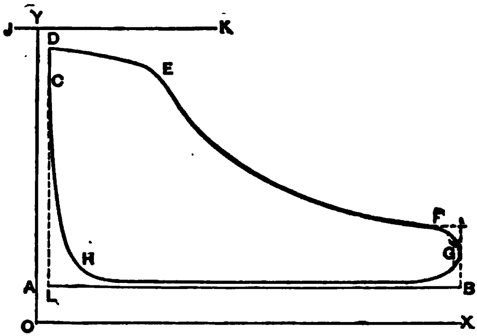


FIG. 138.

The Vacuum Line, *OX*, is a reference line usually drawn about 14 7/10 pounds by scale below the atmospheric line.

The Clearance Line, *OY*, is a reference line drawn at a distance from the end of the diagram equal to the same per cent of its length as the clearance and waste room is of the piston-displacement.

The Line of Boiler-pressure, *JK*, is drawn parallel to the atmospheric line, and at a distance from it by scale equal to the boiler-pressure shown by the gauge.

The Admission Line, *CD*, shows the rise of pressure due to the admission of steam to the cylinder by opening the steam-valve.

The Steam Line, *DE*, is drawn when the steam-valve is open and steam is being admitted to the cylinder.

The Point of Cut-off, *E*, is the point where the admission of steam is stopped by the closing of the valve. It is often difficult to determine the exact point at which the cut-off takes place. It is usually located where the outline of the diagram changes its curvature from convex to concave.

The Expansion Curve, *EF*, shows the fall in pressure as the steam in the cylinder expands doing work.

The Point of Release, *F*, shows when the exhaust-valve opens.

The Exhaust Line, *FG*, represents the change in pressure that takes place when the exhaust-valve opens.

The Back-pressure Line, *GH*, shows the pressure against which the piston acts during its return stroke.

The Point of Exhaust Closure, *H*, is the point where the exhaust-valve closes. It cannot be located definitely, as the change in pressure is at first due to the gradual closing of the valve.

The Compression Curve, *HC*, shows the rise in pressure due to the compression of the steam remaining in the cylinder after the exhaust-valve has closed.

The Mean Height of the Diagram equals its area divided by its length.

The Mean Effective Pressure is the mean net pressure urging the piston forward = the mean height $\times$ the scale of the indicator-spring.

To find the Mean Effective Pressure from the Diagram.—Divide the length, *LB*, into a number, say 10, equal parts, setting off half a part at *L*, half a part at *B*, and nine other parts between; erect ordinates perpendicular to the atmospheric line at the points of division of *LB*, cutting the diagram; add together the lengths of these ordinates intercepted between the upper and lower lines of the diagram and divide by their number. This gives the mean height, which multiplied by the scale of the indicator-spring gives the M.E.P. Or find the area by a planimeter, or other means (see Mensuration, p. 55), and divide by the length LB to obtain the mean height.

The Initial Pressure is the pressure acting on the piston at the beginning of the stroke.

The Terminal Pressure is the pressure above the line of perfect vacuum that would exist at the end of the stroke if the steam had not been released earlier. It is found by continuing the expansion-curve to the end of the diagram.

INDICATED HORSE-POWER OF ENGINES, SINGLE-CYLINDER.

$$\text{Indicated Horse-power I.H.P.} = \frac{PLan}{33,000},$$

in which P = mean effective pressure in lbs. per sq. in.; L = length of stroke in feet; a = area of piston in square inches. For accuracy, one half of the sectional area of the piston-rod must be subtracted from the area of the piston if the rod passes through one head, or the whole area of the rod if it passes through both heads; n = No. of single strokes per min. = 2 × No. of revolutions.

$$\text{I.H.P.} = \frac{PaS}{33,000}, \text{ in which } S = \text{piston speed in feet per minute.}$$

$$\text{I.H.P.} = \frac{PLd^2n}{42,017} = \frac{Pd^2S}{42,017} = .0000238PLd^2n = .0000238Pd^2S,$$

in which d = diam. of cyl. in inches. (The figures 238 are exact, since 7854 ÷ 33 = 23.8 exactly.) If product of piston-speed × mean effective pressure = 42,017, then the horse-power would equal the square of the diameter in inches.

Handy Rule for Estimating the Horse-power of a Single-cylinder Engine.—Square the diameter and divide by 2. This is correct whenever the product of the mean effective pressure and the piston-speed = ½ of 42,017, or, say, 21,000, viz., when M.E.P. = 30 and S = 700; when M.E.P. = 35 and S = 600; when M.E.P. = 38.2 and S = 550; and when M.E.P. = 42 and S = 500. These conditions correspond to those of ordinary practice with both Corliss engines and shaft-governor high-speed engines.

Given Horse-power, Mean Effective Pressure, and Piston-speed, to find Size of Cylinder.—

$$\text{Area} = \frac{33,000 \times \text{I.H.P.}}{PLn}. \quad \text{Diameter} = 205\sqrt{\frac{\text{I.H.P.}}{PS}}. \text{ (Exact.)}$$

Brake Horse-power is the actual horse-power of the engine as measured at the fly-wheel by a friction-brake or dynamometer. It is the indicated horse-power minus the friction of the engine.

Table for Roughly Approximating the Horse-power of a Compound Engine from the Diameter of its Low-pressure Cylinder.—The indicated horse-power of an engine being $\frac{Psd^2}{42,017}$, in which P = mean effective pressure per sq. in., s = piston-speed in ft. per min., and d = diam. of cylinder in inches; if s = 600 ft. per min., which is approximately the speed of modern stationary engines, and P = 35 lbs., which is an approximately average figure for the M.E.P. of single-cylinder engines, and of compound engines referred to the low-pressure cylinder, then I.H.P. = ½d^2; hence the rough-and-ready rule for horse-power given above: Square the diameter in inches and divide by 2. This applies to triple and quadruple expansion engines as well as to single cylinder and compound. For most economical loading, the M.E.P. referred to the low-pressure cylinder of compound engines is usually not greater than that of simple engines; for the greater economy is obtained by a greater number of expansions of steam of higher pressures, and the greater the number of expansions for a given initial pressure the lower the mean effective pressure. The following table gives approximately the figures of mean total and effective pressures for the different types of engines, together with the factor by which the square of the diameter is to be multiplied to obtain the horse-power at most economical loading, for a piston-speed of 600 ft. per minute.

Type of Engine.	Initial Absolute Steam-pressure.	Number of Expansions.	Terminal Absolute Press., lbs.	Ratio Mean Total to Initial Pressure.	Mean Total Pressure, lbs.	Total Back Pressure, Mean, lbs.	Mean Effective Pressure, lbs.	Piston-speed, ft. per min.	Horse-power = $diam.^2 \times$
Non-condensing.									
Single Cylinder.	100	5.	20	.522	52.2	15.5	36.7	600	.524
Compound......	120	7.5	16	.402	48.2	15.5	32.7	"	.467
Triple.........	160	10.	16	.330	52.8	15.5	37.3	"	.553
Quadruple.....	200	12.5	16	.282	56.4	15.5	40.9	"	.584
Condensing Engines.									
Single Cylinder.	100	10.	10	.330	33.0	2	31.0	600	.443
Compound.....	120	15.	8	.247	29.6	2	27.6	"	.390
Triple....... ...	160	20.	8	.200	32.0	2	30.0	"	.429
Quadruple......	200	25.	8	.169	33.8	2	31.8	"	.454

For any other piston-speed than 600 ft. per min., multiply the figures in the last column by the ratio of the piston-speed to 600 ft.

Nominal Horse-power.—The term "nominal horse-power" originated in the time of Watt, and was used to express approximately the power of an engine as calculated from its diameter, estimating the mean pressure in the cylinder at 7 lbs. above the atmosphere. It has long been obsolete in America, and is nearly obsolete in England.

Horse-power Constant of a given Engine for a Fixed Speed = product of its area of piston in square inches, length of stroke in feet, and number of single strokes per minute divided by 33,000, or $\frac{Lan}{33,000} = C$. The product of the mean effective pressure as found by the diagram and this constant is the indicated horse-power.

Horse-power Constant of a given Engine for Varying Speeds = product of its area of piston and length of stroke divided by 33,000. This multiplied by the mean effective pressure and by the number of single strokes per minute is the indicated horse-power.

Horse-power Constant of any Engine of a given Diameter of Cylinder, whatever the length of stroke = area of piston ÷ 33,000 = square of the diameter of piston in inches × .0000238. A table of constants derived from this formula is given below.

The constant multiplied by the piston-speed in feet per minute and by the M.E.P. gives the I.H.P.

Errors of Indicators.—The most common error is that of the spring, which may vary from its normal rating; the error may be determined by proper testing apparatus and allowed for. But after making this correction, even with the best work, the results are liable to variable errors which may amount to 2 or 3 per cent. See Barrus, Trans. A. S. M. E., v. 310; Denton, A. S. M. E., xi. 329; David Smith, U. S. N., Proc. Eng'g Congress, 1893, Marine Division.

Indicator "Rigs," or Reducing-motions; Interpretation of Diagrams for Errors of Steam-distribution, etc. For these see circulars of manufacturers of Indicators; also works on the Indicator.

Table of Engine Constants for Use in Figuring Horse-power.—"Horse-power constant" for cylinders from 1 inch to 60 inches in diameter, advancing by 8ths, for one foot of piston-speed per minute and one pound of M.E.P. Find the diameter of the cylinder in the column at the side. If the diameter contains no fraction the constant will be found in the column headed Even Inches. If the diameter is not in even inches, follow the line horizontally to the column corresponding to the required fraction.

The constants multiplied by the piston-speed and by the M.E.P. give the horse-power.

Diameter of Cylinder.	Even Inches.	+ 1/8 or .125.	+ 1/4 or .25.	+ 3/8 or .375.	+ 1/2 or .5.	+ 5/8 or .625.	+ 3/4 or .75.	+ 7/8 or .875.
1	.0000238	.0000301	.0000372	.0000450	.0000535	.0000628	.0000729	.0000837
2	.0000952	.0001074	.0001205	.0001342	.0001487	.0001640	.0001800	0001967
3	.0002142	.0002324	.0002514	.0002711	.0002915	.0003127	.0003347	.0003574
4	.0003808	.0004050	.0004299	.0004554	.0004819	.0005091	.0005370	.0005656
5	.0005950	.0006251	.0006560	.0006876	.0007199	.0007530	.0007869	.0008215
6	.0008568	.0008929	.0009297	.0009672	.0010055	.0010445	.0010844	.0011249
7	.0011662	.0012082	.0012510	.0012944	.0013387	.0013837	.0014295	.0014759
8	.0015232	.0015711	.0016198	.0016693	.0017195	.0017705	.0018222	.0018746
9	.0019278	.0019817	.0020363	.0020916	.0021479	.0022048	.0022625	.0023209
10	.0023800	.0024398	.0025004	.0025618	.0026239	.0026867	.0027502	.0028147
11	.0028798	.0029456	.0030121	.0030794	.0031475	.0032163	.0032859	.0033561
12	.0034272	.0034990	.0035714	.0036447	.0037187	.0037934	.0038690	.0039452
13	.0040222	.0040999	.0041783	.0042576	.0043375	.0044182	.0044997	.0045819
14	.0046648	.0047484	.0048328	.0049181	.0050039	.0050906	.0051780	.0052661
15	.0053550	.0054446	.0055349	.0056261	.0057179	.0058105	.0059039	.0059979
16	.0060928	.0061884	.0062847	.0063817	.0064795	.0065780	.0066774	0067774
17	.0068782	.0069797	.0070819	.0071850	.0072887	.0073932	.0074985	.0076044
18	.0077112	.0078187	.0079268	.0080360	.0081452	.0082560	.0083672	.0084791
19	.0085918	0087052	.0088193	.0089343	.0090499	.0091663	.0092835	.0094013
20	.0095200	.0096393	.0097594	.0098803	.0100019	.0101243	.0102474	.0103712
21	.0104958	.0106211	.0107472	.0108739	.0110015	.0111299	.0112589	.0113886
22	.0115192	.0116505	.0117825	.0119152	.0120487	.0121830	.0123179	.0124537
23	.0125902	.0127274	.0128654	.0130040	.0131435	.0132837	.0134247	.0135664
24	.0137088	.0138519	.0139959	.0141405	.0142859	.0144321	.0145789	.0147266
25	.0148750	.0150241	.0151740	.0153246	.0154759	.0156280	.0157809	.0159345
26	.0160888	.0162439	.0163997	.0165568	.0167135	.0168716	.0170304	.0171899
27	.0173502	.0175112	.0176729	.0178355	.0179988	.0181627	.0183275	.0184929
28	.0186592	.0188262	.0189939	.0191624	.0193316	.0195015	.0196722	.0198436
29	.0200158	.0201887	.0203624	.0205368	.0207119	.0208879	.0210645	.0212418
30	.0214200	.0215988	.0217785	.0219588	.0221399	.0223218	.0225044	.0226877
31	.0228718	.0230566	.0232422	.0234285	.0236155	.0238033	.0239919	.0241812
32	.0243712	.0245619	.0247535	.0249457	.0251387	.0253325	.0255269	.0257222
33	.0259182	.0261149	.0263124	.0265106	.0267095	.0269092	.0271097	.0273109
34	.0275128	.0277155	.0279189	.0281231	.0283279	.0285336	.0287399	.0289471
35	.0291550	.0293636	.0295729	.0297831	.0299939	.0302056	.0304179	.0306309
36	.0308448	.0310594	.0312747	.0314908	.0317075	.0319251	.0321434	.0323624
37	.0325822	.0328027	.0330239	.0332460	.0334687	.0336922	.0339165	.0341415
38	.0343672	.0345937	.0348209	.0350489	.0352775	.0355070	.0357372	.0359681
39	.0361998	.0364322	.0366654	.0368993	.0371339	.0373694	.0376055	.0378424
40	.0380800	.0383184	.0385575	.0387973	.0390379	.0392793	.0395214	.0397642
41	.0400078	.0402521	.0404972	.0407430	.0409895	.0412368	.0414849	.0417337
42	.0419832	0422335	.0424845	.0427362	.0429887	.0432420	.0434959	.0437507
43	.0440062	.0442624	.0445194	.0447771	.0450355	.0452947	.0455547	.0458154
44	.0460768	.0463389	0466019	.0468655	.0471299	.0473951	.0476609	.0479276
45	.0481950	.0484631	.0487320	.0490016	.0492719	.0495430	.0498149	.0500875
46	.0503608	.0506349	.0509097	.0511853	.0514615	.0517386	.0520164	.0522949
47	.0525742	.0528542	.0531349	.0534165	.0536988	.0539818	.0542655	.0545499
48	.0548352	.0551212	.0554079	.0556953	.0559835	.0562725	.0565622	.0568526
49	.0571438	.0574357	.0577284	.0580218	.0583159	.0586109	.0589065	.0592029
50	.0595000	.0597979	.0600965	.0603959	.0606959	.0609969	.0612984	.0616007
51	.0619038	.0622076	.0625122	.0628175	.0631235	.0634304	.0637379	.0640462
52	.0643552	.0646649	.0649755	.0652867	.0655987	.0659115	.0662250	.0665392
53	.0668542	.0671699	.0674864	.0678036	.0681215	.0684402	.0687597	.0690799
54	.0694008	.0697225	.0700449	.0703681	.0705293	.0710166	.0713419	0716681
55	.0719950	.0723226	.0726510	.0729801	.0733099	.0736406	.0739719	.0743039
56	.0746368	.0749704	.0753047	.0756398	.0759755	.0763120	.0766494	.0769874
57	.0773262	.0776657	.0780060	.0783476	.0786887	.0790312	.0793745	.0797185
58	.0800632	.0804087	.0807549	.0811019	.0814495	.0817980	.0821472	.0824971
59	.0828478	.0831992	.0835514	.0839048	0842579	.0846123	.0849675	.0853234
60	.0856800	.0860374	.0863955	.0867548	.0871139	.0874743	.0878354	.0881978

Horse-power per Pound Mean Effective Pressure.

Formula, $\frac{\text{Area in sq. in.} \times \text{piston-speed}}{33.000}$.

Diam. of Cylinder, inches.	Speed of Piston in feet per minute.								
	100	200	300	400	500	600	700	800	900
4	.0381	.0762	.1142	.1523	.1904	.2285	.2666	.3046	.3427
4½	.0482	.0964	.1446	.1928	.2410	.2892	.3374	.3856	.4338
5	.0595	.1190	.1785	.2380	.2975	.3570	.4165	.4760	.5355
5½	.0720	.1440	.2160	.2880	.3600	.4320	.5040	.5760	.6480
6	.0857	.1714	.2570	.3427	.4284	.5141	.5998	.6854	.7711
6½	.1006	.2011	.3017	.4022	.5028	.6033	.7039	.8044	.9050
7	.1166	.2332	.3499	.4665	.5831	.6997	.8163	.9330	1.0496
7½	.1339	.2678	.4016	.5355	.6694	.8033	.9371	1.0710	1.2049
8	.1523	.3046	.4570	.6093	.7616	.9139	1.0662	1.2186	1.3709
8½	.1720	.3439	.5159	.6878	.8598	1.0317	1.2037	1.3756	1.5476
9	.1928	.3856	.5783	.7711	.9639	1.1567	1.3495	1.5422	1.7350
9½	.2148	.4296	.6444	.8592	1.0740	1.2888	1.5036	1.7184	1.9532
10	.2380	.4760	.7140	.9520	1.1900	1.4280	1.6660	1.9040	2.1420
11	.2880	.5760	.8639	1.1519	1.4399	1.7279	2.0159	2.3038	2.5818
12	.3427	.6854	1.0282	1.3709	1.7136	2.0563	2.3990	2.7418	3.0845
13	.4022	.8044	1.2067	1.6089	2.0111	2.4133	2.8155	3.2178	3.6200
14	.4665	.9330	1.3994	1.8659	2.3324	2.7989	3.2654	3.7318	4.1983
15	.5355	1.0710	1.6065	2.1420	2.6775	3.2130	3.7485	4.2840	4.8195
16	.6093	1.2186	1.8278	2.4371	3.0464	3.6557	4.2650	4.8742	5.4835
17	.6878	1.2756	1.9635	2.6513	3.3391	4.0269	4.6147	5.4026	6.1904
18	.7711	1.5422	2.3134	3.0845	3.8556	4.6267	5.3978	6.1690	6.9401
19	.8592	1.7184	2.5775	3.4367	4.2959	5.1551	6.0143	6.8734	7.7326
20	.9520	1.9040	2.8560	3.8080	4.7600	5.7120	6.6640	7.6160	8.5680
21	1.0496	2.0992	3.1488	4.1983	5.2479	6.2975	7.3471	8.3966	9.4462
22	1.1519	2.3038	3.4558	4.6077	5.7596	6.9115	8.0634	9.2154	10.367
23	1.2590	2.5180	3.7771	5.0361	6.2951	7.5541	8.8131	10.072	11.331
24	1.3709	2.7418	4.1126	5.4835	6.8544	8.2253	9.5962	10.967	12.338
25	1.4875	2.9750	4.4625	5.9500	7.4375	8.9250	10.413	11.900	13.388
26	1.6089	3.2178	4.8266	6.4355	8.0444	9.6534	11.262	12.871	14.480
27	1.7350	3.4700	5.2051	6.9401	8.6751	10.410	12.145	13.880	15.615
28	1.8659	3.7318	5.5978	7.4637	9.3296	11.196	13.061	14.927	16.793
29	2.0016	4.0032	6.0047	8.0063	10.008	12.009	14.011	16.013	18.014
30	2.1420	4.2840	6.4260	8.5680	10.710	12.852	14.994	17.136	19.278
31	2.2872	4.5744	6.8615	9.1487	11.436	13.723	16.010	18.297	20.585
32	2.4371	4.8742	7.3114	9.7485	12.186	14.623	17.060	14.497	21.934
33	2.5918	5.1836	7.7755	10.367	12.959	15.551	18.143	20.735	23.326
34	2.7513	5.5026	8.2538	11.005	13.756	16.508	19.259	22.010	24.762
35	2.9155	5.8310	8.7465	11.662	14.578	17.493	20.409	23.324	26.240
36	3.0845	6.1690	9.2534	12.338	15.422	18.507	21.591	24.676	27.760
37	3.2582	6.5164	9.7747	13.033	16.291	19.549	22.808	26.066	29.324
38	3.4367	6.8734	10.310	13.747	17.184	20.620	24.057	27.494	30.930
39	3.6200	7.2400	10.860	14.480	18.100	21.720	25.340	28.960	32.580
40	3.8080	7.6160	11.424	15.232	19.040	22.848	26.656	30.464	34.272
41	4.0008	8.0016	12.002	16.003	20.004	24.005	28.005	32.006	36.007
42	4.1983	8.3866	12.585	16.783	20.982	25.180	29.378	33.577	37.775
43	4.4006	8.8012	13.202	17.602	22.003	26.404	30.804	35.205	39.606
44	4.6077	9.2154	13.823	18.431	23.038	27.646	32.254	36.861	41.469
45	4.8195	9.6390	14.459	19.278	24.098	28.917	33.737	38.556	43.376
46	5.0361	10.072	15.108	20.144	25.180	30.216	35.253	40.289	45.325
47	5.2574	10.515	15.772	21.030	26.287	31.545	36.802	42.059	47.317
48	5.4835	10.967	16.451	21.934	27.418	32.901	38.385	43.868	49.352
49	5.7144	11.429	17.143	22.858	28.572	34.286	40.001	45.715	51.429
50	5.9500	11.900	17.850	23.800	29.750	35.700	41.650	47.600	53.550
51	6.1904	12.381	18.571	24.762	30.952	37.142	43.333	49.523	55.713
52	6.4355	12.871	19.307	25.742	32.178	38.613	45.049	51.484	57.920
53	6.6854	13.371	20.056	26.742	33.427	40.113	46.798	53.483	60.169
54	6.9401	13.880	20.820	27.760	34.700	41.640	48.581	55.521	62.461
55	7.1995	14.399	21.599	28.798	35.998	43.197	50.397	57.596	64.796
56	7.4637	14.927	22.391	29.855	37.318	44.782	52.246	59.709	67.173
57	7.7326	15.465	23.198	30.930	38.663	46.396	54.128	61.861	69.594
58	8.0063	16.013	24.019	32.025	40.032	48.038	56.044	64.051	72.057
59	8.2849	16.570	24.854	33.139	41.424	49.709	57.993	66.278	74.563
60	8.5680	17.136	25.704	34.272	42.840	51.408	59.976	68.544	77.112

To draw the Clearance-line on the Indicator-diagram, the actual clearance not being known.—The clearance-line may be obtained approximately by drawing a straight line, *cbad*, across the compression curve, first having drawn *OX* parallel to the atmospheric line and 14.7 lbs. below. Measure from *a* the distance *ad*, equal to *cb*, and draw *YO* perpendicular to *OX* through *d*; then will *TB* divided by *AT* be the percentage of

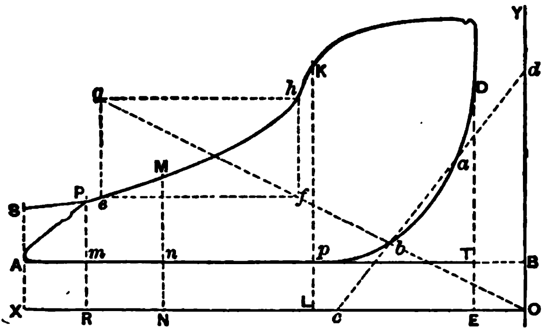


FIG. 139.

clearance. The clearance may also be found from the expansion-line by constructing a rectangle *efhg*, and drawing a diagonal *gf* to intersect the line *XO*. This will give the point *O*, and by erecting a perpendicular to *XO* we obtain a clearance-line *OY*.

Both these methods for finding the clearance require that the expansion and compression curves be hyperbolas. Prof. Carpenter (*Power*, Sept., 1893) says that with good diagrams the methods are usually very accurate, and give results which check substantially.

The Buckeye Engine Co., however, say that, as the results obtained are seldom correct, being sometimes too little, but more frequently too much, and as the indications from the two curves seldom agree, the operation has little practical value, though when a clearly defined and apparently undistorted compression curve exists of sufficient extent to admit of the application of the process, it may be relied on to give much more correct results than the expansion curve.

To draw the Hyperbolic Curve on the Indicator-diagram.—Select any point *I* in the actual curve, and from this point draw a line perpendicular to the line *JB*, meeting the latter in the point *J*. The line *JB* may be the line of boiler-pressure, but this is not material; it may be drawn at any convenient height near the top of diagram and parallel to the atmospheric line. From *J* draw a diagonal to *K*, the latter point being the intersection of the vacuum and clearance lines; from *I* draw *IL* parallel with the atmospheric line. From *L*, the point of intersection of the diagonal *JK* and the horizontal line *IL*, draw the vertical line *LM*. The point *M* is the theoretical point of cut-off, and *LM* the cut-off line. Fix upon any number of points 1, 2, 3, etc., on the line *JB*, and from these points draw diagonals to *K*. From the intersection of these diagonals with *LM* draw horizontal lines, and from 1, 2, 3, etc., vertical lines. Where these lines meet will be points in the hyperbolic curve.

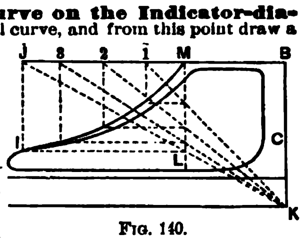


FIG. 140.

Pendulum Indicator Rig.—*Power* (Feb. 1893) gives a graphical representation of the errors in indicator-diagrams, caused by the use of incorrect form of the pendulum rigging. It is shown that the "brumbo" pulley on the pendulum, to which the cord is attached, does not generally give as good a reduction as a simple pin attachment. When the end of the pendulum is slotted, working in a pin on the crosshead, the error is apt to be considerable at both ends of the card. With a vertical slot in a plate fixed to the crosshead, and a pin on the pendulum working in this slot, the reduction is perfect, when the cord is attached to a pin on the pendulum, a slight error being introduced if the brumbo pulley is used. With the connection between the pendulum and the crosshead made by means of a horizontal link, the reduction is nearly perfect, if the construction is such that the connecting link vibrates equally above and below the horizontal, and the cord is attached by a pin. If the link is horizontal at mid-stroke a serious error is introduced, which is magnified if a brumbo pulley also is used. The adjoining figures show the two forms recommended.

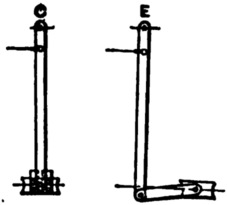


FIG. 141.

Theoretical Water-consumption calculated from the Indicator-card.—The following method is given by Prof. Carpenter (*Power*, Sept. 1893): p = mean effective pressure, l = length of stroke in feet, a = area of piston in square inches, $a \div 144$ = area in square feet, c = percentage of clearance to the stroke, b = percentage of stroke at point where water rate is to be computed, n = number of strokes per minute, $60n$ = number per hour, w = weight of a cubic foot of steam having a pressure as shown by the diagram corresponding to that at the point where water rate is required, w' = that corresponding to pressure at end of compression.

Number of cubic feet per stroke $= l\left(\frac{b+c}{100}\right)\frac{a}{144}$.

Corresponding weight of steam per stroke in lbs. $= l\left(\frac{b+c}{100}\right)\frac{a}{144}w$.

Volume of clearance $= \frac{lca}{14,400}$.

Weight of steam in clearance $= \frac{lcaw'}{14,400}$.

$$\left.\begin{array}{r}\text{Total weight of}\\ \text{steam per stroke}\end{array}\right\} = l\left(\frac{b+c}{100}\right)\frac{wa}{144} - \frac{lcaw'}{14,400} = \frac{la}{14,400}\Big[(b+c)w - cw'\Big].$$

$$\left.\begin{array}{r}\text{Total weight of steam}\\ \text{from diagram per hour}\end{array}\right\} = \frac{60nla}{14,400}\Big[(b+c)w - cw'\Big].$$

The indicated horse-power is $p\,l\,a\,n \div 33,000$. Hence the steam-consumption per indicated horse-power is

$$= \frac{\frac{60nla}{14,400}\Big[(b+c)w - cw'\Big]}{\frac{p\,l\,a\,n}{33,000}} = \frac{137.50}{p}\Big[(b+c)w - cw'\Big].$$

Changing the formula to a rule, we have: To find the water rate from the indicator diagram at any point in the stroke.

RULE.—To the percentage of the entire stroke which has been completed by the piston at the point under consideration add the percentage of clearance. Multiply this result by the weight of a cubic foot of steam, having a pressure of that at the required point. Subtract from this the product of percentage of clearance multiplied by weight of a cubic foot of steam having a pressure equal to that at the end of the compression. Multiply this result by 137.50 divided by the mean effective pressure.*

NOTE.—This method only applies to points in the expansion curve or between cut-off and release.

* For compound or triple-expansion engines read: divided by the equivalent mean effective pressure, on the supposition that all work is done in one cylinder.

The beneficial effect of compression in reducing the water-consumption of an engine is clearly shown by the formula. If the compression is carried to such a point that it produces a pressure equal to that at the point under consideration, the weight of steam per cubic foot is equal, and $w = w'$. In this case the effect of clearance entirely disappears, and the formula becomes $\frac{137.5}{p}(bw)$.

In case of no compression, w' becomes zero, and the water-rate =

$$\frac{137.5}{p}[(b + c)w].$$

Prof. Denton (Trans. A. S. M. E., xiv. 1363) gives the following table of theoretical water-consumption for a perfect Mariotte expansion with steam at 150 lbs. above atmosphere, and 2 lbs. absolute back pressure:

Ratio of Expansion, r.	M.E.P., lbs. per sq. in.	Lbs. of Water per hour per horse-power, W.
10	52.4	9.68
15	38.7	8.74
20	30.9	8.20
25	25.9	7.84
30	22.2	7.63
35	19.5	7.45

The difference between the theoretical water-consumption found by the formula and the actual consumption as found by test represents "water not accounted for by the indicator," due to cylinder condensation, leakage through ports, radiation, etc.

Leakage of Steam.—Leakage of steam, except in rare instances, has so little effect upon the lines of the diagram that it can scarcely be detected. The only satisfactory way to determine the tightness of an engine is to take it when not in motion, apply a full boiler-pressure to the valve, placed in a closed position, and to the piston as well, which is blocked for the purpose at some point away from the end of the stroke, and see by the eye whether leakage occurs. The indicator-cocks provide means for bringing into view steam which leaks through the steam-valves, and in most cases that which leaks by the piston, and an opening made in the exhaust-pipe or observations at the atmospheric escape-pipe, are generally sufficient to determine the fact with regard to the exhaust-valves.

The steam accounted for by the indicator should be computed for both the cut-off and the release points of the diagram. If the expansion-line departs much from the hyperbolic curve a very different result is shown at one point from that shown at the other. In such cases the extent of the loss occasioned by cylinder condensation and leakage is indicated in a much more truthful manner at the cut-off than at the release. (Tabor Indicator Circular.)

COMPOUND ENGINES.

Compound, Triple- and Quadruple-expansion Engines.—A compound engine is one having two or more cylinders, and in which the steam after doing work in the first or high-pressure cylinder completes its expansion in the other cylinder or cylinders.

The term "compound" is commonly restricted, however, to engines in which the expansion takes place in two stages only—high and low pressure, the terms triple-expansion and quadruple-expansion engines being used when the expansion takes place respectively in three and four stages. The number of cylinders may be greater than the number of stages of expansion, for constructive reasons; thus in the compound or two-stage expansion engine the low-pressure stage may be effected in two cylinders so as to obtain the advantages of nearly equal sizes of cylinders and of three cranks at angles of 120°. In triple-expansion engines there are frequently two low-pressure cylinders, one of them being placed tandem with the high-pressure, and the other with the intermediate cylinder, as in mill engines with two cranks at 90°. In the triple-expansion engines of the steamers *Campania* and *Lucania*, with three cranks at 120°, there are five cylinders, two high, one intermediate, and two low, the high-pressure cylinders being tandem with the low.

Advantages of Compounding.—The advantages secured by dividing the expansion into two or more stages are twofold: 1. Reduction of wastes of steam by cylinder-condensation, clearance, and leakage; 2. Dividing the pressures on the cranks, shafts, etc., in large engines so as to avoid excessive pressures and consequent friction. The diminished loss by cylinder-condensation is effected by decreasing the range of temperature of the metal surfaces of the cylinders, or the difference of temperature of the steam at admission and exhaust. When high-pressure steam is admitted into a single-cylinder engine a large portion is condensed by the comparatively cold metal surfaces; at the end of the stroke and during the exhaust the water is re-evaporated, but the steam so formed escapes into the atmosphere or into the condenser, doing no work; while if it is taken into a second cylinder, as in a compound engine, it does work. The steam lost in the first cylinder by leakage and clearance also does work in the second cylinder. Also, if there is a second cylinder, the temperature of the steam exhausted from the first cylinder is higher than if there is only one cylinder, and the metal surfaces therefore are not cooled to the same degree. The difference in temperatures and in pressures corresponding to the work of steam of 150 lbs. gauge-pressure expanded 20 times, in one, two, and three cylinders, is shown in the following table, by W. H. Weightman, *Am. Mach.*, July 28, 1892:

	Single Cylinder.	Compound Cylinders.		Triple-expansion Cylinders.		
Diameter of cylinders, in..	60	33	61	28	46	61
Area ratios........	.. ....	1	3.416	1	2.70	4.747
Expansions.....	20	5	4	2.714	2.714	2.714
Initial steam-pressures—absolute—pounds	165	165	33	165	60.8	22.4
Mean pressures, pounds. .	32.96	86.11	19.68	121.44	44.75	16.49
Mean effective pressures, pounds....	28.96	53.11	15.68	60.64	22.35	12.49
Steam temperatures into cylinders.......	366°	366°	259°.9	366°	293°.5	234°.1
Steam temperatures out of the cylinders....	184°.2	259°.9	184°.2	293°.5	234°.1	184°.2
Difference in temperatures	181.8	106.1	75.7	72.5	59.4	49.9
Horse-power developed...	800	399	408	269	268	264
Speed of piston........ ...	322	290	290	238	238	238
Total initial pressures on pistons, pounds....... ..	455,218	112,900	84,752	64,162	63,817	53,773

"Woolf" and Receiver Types of Compound Engines.—The compound steam-engine, consisting of two cylinders, is reducible to two forms, 1, in which the steam from the h.p. cylinder is exhausted direct into the l. p. cylinder, as in the Woolf engine; and 2, in which the steam from the h. p. cylinder is exhausted into an intermediate reservoir, whence the steam is supplied to, and expanded in, the l. p. cylinder, as in the "receiver-engine."

If the steam be cut off in the first cylinder before the end of the stroke, the total ratio of expansion is the product of the ratio of expansion in the first cylinder, into the ratio of the volume of the second to that of the first cylinder; that is, the product of the two ratios of expansion.

Thus, let the areas of the first and second cylinders be as 1 to 3½, the strokes being equal, and let the steam be cut off in the first at ½ stroke; then

Expansion in the 1st cylinder..	1 to 2
" " " 2d "	1 to 3½
Total or combined expansion, the product of the two ratios...	1 to 7

Woolf Engine, without Clearance—Ideal Diagrams.—The diagrams of pressure of an ideal Woolf engine are shown in Fig. 142, as they would be described by the indicator, according to the arrows. In these diagrams *pq* is the atmospheric line, *mn* the vacuum line, *cd* the admission line, *dg* the hyperbolic curve of expansion in the first cylinder, and *gh* the consecutive expansion-line of back pressure for the return-stroke of the first piston, and of positive pressure for the steam-stroke of the second piston. At the point *h*, at the end of the stroke of the second piston, the steam is exhausted into the condenser, and the pressure falls to the level of perfect vacuum, *mn*.

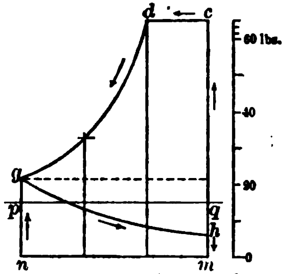


FIG. 142.—WOOLF ENGINE—IDEAL INDICATOR-DIAGRAMS.

The diagram of the second cylinder, below *gh*, is characterized by the absence of any specific period of admission; the whole of the steam-line *gh* being expansional, generated by the expansion of the initial body of steam contained in the first cylinder into the second. When the return-stroke is completed, the whole of the steam transferred from the first is shut into the second cylinder. The final pressure and volume of the steam in the second cylinder are the same as if the whole of the initial steam had been admitted at once into the second cylinder, and then expanded to the end of the stroke in the manner of a single-cylinder engine.

The net work of the steam is also the same, according to both distributions.

Receiver-engine, without Clearance—Ideal Diagrams.—In the ideal receiver-engine the pistons of the two cylinders are connected to cranks at right angles to each other on the same shaft. The receiver takes the steam exhausted from the first cylinder and supplies it to the second, in which the steam is cut off and then expanded to the end of the stroke. On the assumption that the initial pressure in the second cylinder is equal to the final pressure in the first, and of course equal to the pressure in the receiver, the volume cut off in the second cylinder must be equal to the volume of the first cylinder, for the second cylinder must admit as much steam at each stroke as is discharged from the first cylinder.

In Fig. 143 *cd* is the line of admission and *hg* the exhaust-line for the first

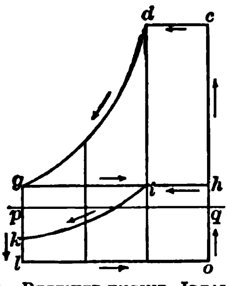


FIG. 143.—RECEIVER-ENGINE, IDEAL INDICATOR-DIAGRAMS.

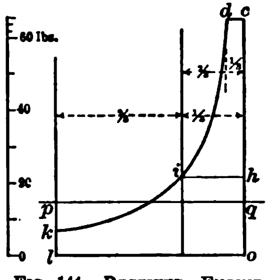


FIG. 144.—RECEIVER ENGINE, IDEAL DIAGRAMS REDUCED AND COMBINED.

cylinder; and *dg* is the expansion-curve and *pq* the atmospheric line. In the region below the exhaust-line of the first cylinder, between it and the line of perfect vacuum, *ol*, the diagram of the second cylinder is formed; *hi*, the second line of admission, coincides with the exhaust-line *hg* of the first cylinder, showing in the ideal diagram no intermediate fall of pressure, and *ik* is the expansion-curve. The arrows indicate the order in which the diagrams are formed.

In the action of the receiver-engine, the expansive working of the steam, though clearly divided into two consecutive stages, is, as in the Woolf engine, essentially continuous from the point of cut-off in the first cylinder to the end of the stroke of the second cylinder, where it is delivered to the condenser; and the first and second diagrams may be placed together and combined to form a continuous diagram. For this purpose take the second diagram as the basis of the combined diagram, namely, *hiklo*, Fig. 144. The period of admission, *hi*, is one third of the stroke, and as the ratios of the cylinders are as 1 to 3, *hi* is also the proportional length of the first diagram as applied to the second. Produce *oh* upwards, and set off *oc* equal to the total height of the first diagram above the vacuum-line; and, upon the shortened base *hi*, and the height *hc*, complete the first diagram with the steam-line *cd*, and the expansion-line *di*.

It is shown by Clark (S. E., p. 432, *et seq.*) in a series of arithmetical calculations, that the receiver-engine is an elastic system of compound engine, in which considerable latitude is afforded for adapting the pressure in the receiver to the demands of the second cylinder, without considerably diminishing the effective work of the engine. In the Woolf engine, on the contrary, it is of much importance that the intermediate volume of space between the first and second cylinders, which is the cause of an intermediate fall of pressure, should be reduced to the lowest practicable amount.

Supposing that there is no loss of steam in passing through the engine, by cooling and condensation, it is obvious that whatever steam passes through the first cylinder must also find its way through the second cylinder. By varying, therefore, in the receiver-engine, the period of admission in the second cylinder, and thus also the volume of steam admitted for each stroke, the steam will be measured into it at a higher pressure and of a less bulk, or at a lower pressure and of a greater bulk; the pressure and density naturally adjusting themselves to the volume that the steam from the receiver is permitted to occupy in the second cylinder. With a sufficiently restricted admission, the pressure in the receiver may be maintained at the pressure of the steam as exhausted from the first cylinder. On the contrary, with a wider admission, the pressure in the receiver may fall or "drop" to three fourths or even one half of the pressure of the exhaust-steam from the first cylinder.

(For a more complete discussion of the action of steam in the Woolf and receiver engines, see Clark on the Steam-engine.)

Combined Diagrams of Compound Engines.—The only way of making a correct combined diagram from the indicator-diagrams of the several cylinders in a compound engine is to set off all the diagrams on the same horizontal scale of volumes, adding the clearances to the cylinder ca-

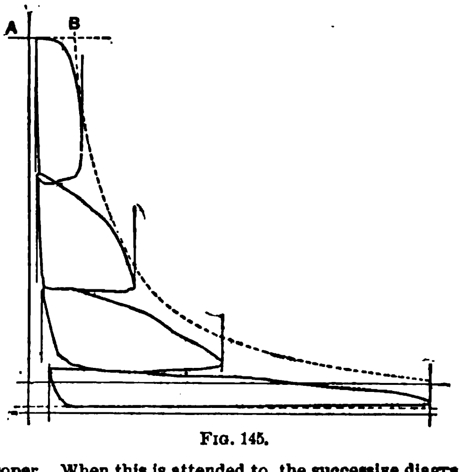


FIG. 145.

pacities proper. When this is attended to, the successive diagrams fall exactly into their right places relatively to one another, and would compare properly with any theoretical expansion-curve. (Prof. A. B. W. Kennedy, Proc. Inst. M. E., Oct. 1886.)

This method of combining diagrams is commonly adopted, but there are objections to its accuracy, since the whole quantity of steam consumed in the first cylinder at the end of the stroke is not carried forward to the second, but a part of it is retained in the first cylinder for compression. For a method of combining diagrams in which compression is taken account of, see discussions by Thomas Mudd and others, in Proc. Inst. M. E., Feb., 1887, p. 48. The usual method of combining diagrams is also criticised by Frank H. Ball as inaccurate and misleading (*Am. Mach.*, April 12, 1894; Trans. A. S. M. E., xiv. 1405, and xv. 403).

Figure 145 shows a combined diagram of a quadruple-expansion engine, drawn according to the usual method, that is, the diagrams are first reduced in length to relative scales that correspond with the relative piston-displacement of the three cylinders. Then the diagrams are placed at such distances from the clearance-line of the proposed combined diagram as to correctly represent the clearance in each cylinder.

Calculated Expansions and Pressures in Two-cylinder Compound Engines. (James Tribe, *Am. Mach.*, Sept. & Oct. 1891.)

TWO-CYLINDER COMPOUND NON-CONDENSING.

Back pressure ½ lb. above atmosphere.

Initial gauge-pressure		100	110	120	130	140	150	160	170	175
Initial absolute pressure		115	125	135	145	155	165	175	185	190
Total expansion		7.39	7.84	8.41	9	9.61	10.24	10.89	11.56	11.9
Expansions in each cylinder		2.7	2.8	2.9	3	3.10	3.2	3.3	3.4	3.45
Hyp. log. plus 1		1.993	2.029	2.064	2.028	2.131	2.163	2.193	2.223	2.238
Forward pressures	High	84.8	90.5	96	101.4	106.5	111.5	116.8	120.9	123.2
	Low	31.3	32.3	33.1	33.7	34.3	34.8	35.2	35.6	35.7
Back pressures	High	42.5	44.6	46.5	48.3	50	51.5	53	54.4	55
	Low	15.5	15.5	15.5	15.5	15.5	15.5	15.5	15.5	15.5
Mean effective pressures	High	42.3	45.9	49.5	53.1	56.5	60	63.3	66.5	68.2
	Low	15.8	16.8	17.6	18.2	18.8	19.3	19.7	20.1	20.3
Ratio-cylinder areas		2.67	2.73	2.81	2.91	3	3.11	3.21	3.31	3.37

TWO-CYLINDER COMPOUND CONDENSING.

Back pressure, 6.5 lbs. above vacuum.

Initial gauge-pressures		90	100	110	120	130	140	150
Initial absolute pressures		105	115	125	135	145	155	165
Probable per cent of loss		2.6	2.9	3.3	3.6	3.8	4.0	4.3
Total expansions		15.7	17	18.5	20	21.5	22.7	24.2
Exps. in each cylinder		3.96	4.13	4.3	4.47	4.64	4.77	4.92
Hyp. log. plus 1		2.376	2.418	2.458	2.497	2.534	2.562	2.593
Mean forward pressures	High	62.9	67.3	71.4	75.4	79.3	83.2	87
	Low	15.25	15.55	15.9	16.2	16.5	16.75	17.05
Mean back pressures	High	26.5	27.8	29	30.2	31.4	32.4	33.5
	Low	4.3	4.3	4.3	4.3	4.3	4.3	4.3
Mean effective pressures	High	36.4	39.5	42.4	45.2	47.9	50.8	53.5
	Low	10.95	11.25	11.6	11.9	12.2	12.45	12.75
Terminal pressures	High	26.5	27.8	29.0	30.2	31.4	32.4	33.5
	Low	6.4	6.45	6.45	6.5	6.55	6.55	6.6
Initial pressure in l. p. cyl.		25.3	26.6	27.8	29	30.2	31.4	32.4
Ratio of cylinder areas		3.32	3.51	3.66	3.8	3.92	4.08	4.19

The probable percentage of loss, line 3, is thus explained: There is always a loss of heat due to condensation, and which increases with the pressure of steam. The exact percentage cannot be predetermined, as it depends largely upon the quality of the non-conducting covering used on the cylinder, receiver, and pipes, etc., but will probably be about as shown.

Proportions of Cylinders in Compound Engines.—Authorities differ as to the proportions by volume of the high and low pressure cylinders v and V. Thus Grashof gives $V + v = 0.85 \sqrt{r}$; Hrabak, $0.90 \sqrt{r}$;

Werner, $\sqrt{r}$; and Rankine, $\sqrt[9]{r^3}$, r being the ratio of expansion. Busley makes the ratio dependent on the boiler-pressure thus:

Lbs. per sq. in.	60	90	105	120
$V \div v$ =	3	4	4.5	5

(See Seaton's Manual, p. 95, etc., for analytical method; Sennett, p. 496, etc.; Clark's Steam-engine, p. 445, etc; Clark, Rules, Tables, Data, p. 849, etc.)

Mr. J. McFarlane Gray states that he finds the mean effective pressure in the compound engine reduced to the low-pressure cylinder to be approximately the square root of 6 times the boiler-pressure.

Approximate Horse-power of a Modern Compound Marine-engine. (Seaton.)—The following rule will give approximately the horse-power developed by a compound engine made in accordance with modern marine practice. Estimated H.P. $= \frac{D^2 \times \sqrt{p} \times R \times S}{8500}$.

D = diameter of l.p. cylinder; p = boiler-pressure by gauge;
R = revs. per min.; S = stroke of piston in feet.

Ratio of Cylinder Capacity in Compound Marine Engines. (Seaton.)—The low-pressure cylinder is the measure of the power of a compound engine, for so long as the initial steam-pressure and rate of expansion are the same, it signifies very little, so far as total power only is concerned, whether the ratio between the low and high-pressure cylinders is 3 or 4; but as the power developed should be nearly equally divided between the two cylinders, in order to get a good and steady working engine, there is a necessity for exercising a considerable amount of discretion in fixing on the ratio.

In choosing a particular ratio the objects are to divide the power evenly and to avoid as much as possible "drop" and high initial strain.

If increased economy is to be obtained by increased boiler-pressures, the rate of expansion should vary with the initial pressure, so that the pressure at which the steam enters the condenser should remain constant. In this case, with the ratio of cylinders constant, the cut-off in the high-pressure cylinder will vary inversely as the initial pressure.

Let R be the ratio of the cylinders; r, the rate of expansion; p_1 the initial pressure: then cut-off in high-pressure cylinder $= R \div r$; r varies with p_1, so that the terminal pressure p_n is constant and consequently $r = p_1 \div p_n$; therefore, cut-off in high-pressure cylinder $= R \times p_n \div p_1$.

Ratios of Cylinders as Found in Marine Practice.—The rate of expansion may be taken at one tenth of the boiler-pressure (or about one twelfth the absolute pressure), to work economically at full speed. Therefore, when the diameter of the low-pressure cylinder does not exceed 100 inches, and the boiler-pressure 70 lbs., the ratio of the low-pressure to the high-pressure cylinder should be 3.5; for a boiler-pressure of 80 lbs., 3.75; for 90 lbs., 4.0; for 100 lbs., 4.5. If these proportions are adhered to, there will be no need of an expansion-valve to either cylinder. If, however, to avoid "drop," the ratio be reduced, an expansion-valve should be fitted to the high-pressure cylinder.

Where economy of steam is not of first importance, but rather a large power, the ratio of cylinder capacities may with advantage be decreased, so that with a boiler-pressure of 100 lbs. it may be 3.75 to 4.

In tandem engines there is no necessity to divide the work equally. The ratio is generally 4, but when the steam-pressure exceeds 90 lbs. absolute 4.5 is better, and for 100 lbs. 5.0.

When the power requires that the l. p. cylinder shall be more than 100 in. diameter, it should be divided in two cylinders. In this case the ratio of the combined capacity of the two l. p. cylinders to that of the h. p. may be 3.0 for 85 lbs. absolute, 3.4 for 95 lbs., 3.7 for 105 lbs., and 4.0 for 115 lbs.

Receiver Space in Compound Engines should be from 1 to 1.5 times the capacity of the high-pressure cylinder, when the cranks are at an angle of from 90° to 120°. When the cranks are at 180° or nearly this, the space may be very much reduced. In the case of triple-compound engines, with cranks at 120°, and the intermediate cylinder leading the high-pressure, a very small receiver will do. The pressure in the receiver should never exceed half the boiler-pressure. (Seaton.)

Formula for Calculating the Expansion and the Work of Steam in Compound Engines.

(Condensed from Clark on the "Steam-engine.")

a = area of the first cylinder in square inches;
a' = area of the second cylinder in square inches;
r = ratio of the capacity of the second cylinder to that of the first;
L = length of stroke in feet, supposed to be the same for both cylinders;
l = period of admission to the first cylinder in feet, excluding clearance;
c = clearance at each end of the cylinders, in parts of the stroke, in feet;
L' = length of the stroke plus the clearance, in feet;
l' = period of admission plus the clearance, in feet;
s = length of a given part of the stroke of the second cylinder, in feet;
P = total initial pressure in the first cylinder, in lbs. per square inch, supposed to be uniform during admission;
P' = total pressure at the end of the given part of the stroke s;
p = average total pressure for the whole stroke;
R = nominal ratio of expansion in the first cylinder, or $L \div l$;
R' = actual ratio of expansion in the first cylinder, or $L' \div l'$;
R'' = actual combined ratio of expansion, in the first and second cylinders together;
n = ratio of the final pressure in the first cylinder to any intermediate fall of pressure between the first and second cylinders;
N = ratio of the volume of the intermediate space in the Woolf engine, reckoned up to, and including the clearance of, the second piston, to the capacity of the first cylinder plus its clearance. The value of N is correctly expressed by the actual ratio of the volumes as stated, on the assumption that the intermediate space is a vacuum when it receives the exhaust-steam from the first cylinder. In point of fact, there is a residuum of unexhausted steam in the intermediate space, at low pressure, and the value of N is thereby practically reduced below the ratio here stated. $N = \frac{n}{n-1} - 1.$
w = whole net work in one stroke, in foot-pounds.

Ratio of expansion in the second cylinder:

$$\text{In the Woolf engine, } \frac{\left(r\frac{L}{L'}\right) + N}{1 + N};$$

$$\text{In the receiver-engine, } \frac{(n-1)r}{n}.$$

Total actual ratio of expansion = product of the ratios of the three consecutive expansions, in the first cylinder, in the intermediate space, and in the second cylinder,

$$\text{In the Woolf engine, } R'\left(r\frac{L}{L'} + N\right);$$

$$\text{In the receiver-engine, } r\frac{L'}{l'}, \text{ or } rR'.$$

$$\text{Combined ratio of expansion behind the pistons} = \frac{n-1}{n} rR' = R''.$$

Work done in the two cylinders for one stroke, with a given cut-off and a given combined actual ratio of expansion:

$$\text{Woolf engine, } w = aP[l'(1 + \text{hyp log } R'') - c],$$

$$\text{Receiver engine, } w = aP\left[l'(1 + \text{hyp log } R'') - c\left(1 + \frac{r-1}{R'}\right)\right],$$

when there is no intermediate fall of pressure.

When there is an intermediate fall, when the pressure falls to ¾, ⅔, ½ of the final pressure in the 1st cylinder, the reduction of work is 0.2%, 1.0%, 4.6% of that when there is no fall.

Total work in the two cylinders of a receiver-engine, for one stroke for any intermediate fall of pressure,

$$w = aP\left[r\left(\frac{n+1}{n} + \text{hyp log } R''\right) - c\left(1 + \frac{(n-1)(r-1)}{nR'}\right)\right].$$

EXAMPLE.—Let $a = 1$ sq. in., $P = 63$ lbs., $l' = 2.42$ ft., $n = 4$, $R'' = 5.969$, $c = .42$ ft., $r = 3$, $R' = 2.658$;

$$w = 1 \times 63\left[2.42(5/4 \text{ hyp log } 5.969) - .42\left(1 + \frac{3 \times 2}{4 \times 2.658}\right)\right] = 421.55 \text{ ft.-lbs.}$$

Calculation of Diameters of Cylinders of a compound condensing engine of 2000 H.P. at a speed of 700 feet per minute, with 100 lbs. boiler-pressure.

100 lbs. gauge-pressure = 115 absolute, less drop of 5 lbs. between boiler and cylinder = 110 lbs. initial absolute pressure. Assuming terminal pressure in l. p. cylinder = 6 lbs., the total expansion of steam in both cylinders = 110 ÷ 6 = 18.33 lbs. Hyp log 18.33 = 2.909. Back pressure in l. p. cylinder, 3 lbs. absolute.

The following formulæ are used in the calculation of each cylinder:

(1) Area of cylinder $= \dfrac{\text{H.P.} \times 33{,}000}{\text{M.E.P.} \times \text{piston-speed}}$.

(2) Mean effective pressure = mean total pressure − back pressure.

(3) Mean total pressure = terminal pressure × (1 + hyp log R).

(4) Absolute initial pressure = absolute terminal pressure × ratio of expansion.

First calculate the area of the low-pressure cylinder as if all the work were done in that cylinder.

From (3), mean total pressure = 6 × (1 + hyp log 18.33) = 23.454 lbs.

From (2), mean effective pressure = 23.454 − 3 = 20.454 lbs.

From (1), area of cylinder $= \dfrac{2000 \times 33{,}000}{20.454 \times 700} = 4610$ sq. ins. = 76.6 ins. diam.

If half the work, or 1000 H.P., is done in the l. p. cylinder the M.E.P. will be half that found above, or 10.227 lbs., and the mean total pressure 10.227 + 3 = 13.227 lbs.

From (3), 1 + hyp log R = 13.227 ÷ 6 = 2.2045.

Hyp log R = 1.2045, whence R in l. p. cyl. = 3.335.

From (4), 3.335 × 6 = 20.01 lbs. initial pressure in l. p. cyl. and terminal pressure in h. p. cyl., assuming no drop between cylinders.

110 ÷ 22.01 = 18.33 ÷ 3.335 = 5.497, R in h. p. cyl.

From (3), mean total pres. in h. p. cyl. = 20.01 × (1 + hyp log 5.497) = 54.11.

From (2), 54.11 − 20.01 = 34.10, M.E.P. in h. p. cyl.

From (1), area of h. p. cyl. $= \dfrac{1000 \times 33{,}000}{700 \times 34.1} = 1382$ sq. ins. = 42 ins. diam.

Cylinder ratio = 4610 ÷ 1382 = 3.336.

The area of the h. p. cylinder may be found more directly by dividing the area of the l. p. cyl. by the ratio of expansion in that cylinder. 4610 ÷ 3.335 = 1382 sq. ins.

In the above calculation no account is taken of clearance, of compression, of drop between cylinders, nor of area of piston-rods. It also assumes that the diagram in each cylinder is the full theoretical diagram, with a horizontal steam-line and a hyperbolic expansion line, with no allowance for rounding of the corners. To make allowance for these, the mean effective pressure in each cylinder must be multiplied by a diagram factor, or the ratio of the area of an actual diagram of the class of engine considered, with the given initial and terminal pressures, to the area of the theoretical diagram. Such diagram factors will range from 0.6 to 0.94, as in the table on p. 745.

Best Ratios of Cylinders.—The question what is the best ratio of areas of the two cylinders of a compound engine is still (1901), a disputed one, but there appears to be an increasing tendency in favor of large ratios, even as great as 7 or 8 to 1, with considerable terminal drop, in the high-pressure cylinder. A discussion of the subject, together with a description of a new method of drawing theoretical diagrams of multiple-expansion engines, taking into consideration drop, clearance, and compression, will be found in a paper by Bert C. Ball, in Trans. A. S. M. E., xxi. 1002.

TRIPLE-EXPANSION ENGINES.

Proportions of Cylinders.—H. H. Suplee, *Mechanics*, Nov. 1887, gives the following method of proportioning cylinders of triple-expansion engines:

As in the case of compound engines the diameter of the low-pressure cylinder is first determined, being made large enough to furnish the entire power required at the mean pressure due to the initial pressure and expansion ratio given; and then this cylinder is only given pressure enough to perform one third of the work, and the other cylinders are proportioned so as to divide the other two thirds between them.

Let us suppose that an initial pressure of 150 lbs. is used and that 900 H.P. is to be developed at a piston-speed of 800 ft. per min., and that an expansion ratio of 16 is to be reached with an absolute back pressure of 2 lbs.

The theoretical M.E.P. with an absolute initial pressure of $150 + 14.7 = 164.7$ lbs. initial at 16 expansions is

$$\frac{P(1 + \text{hyp log } 16)}{16} = 164.7 \times \frac{3.7726}{16} = 38.83,$$

less 2 lbs. back pressure, $= 38.83 - 2 = 36.83$.

In practice only about 0.7 of this pressure is actually attained, so that $36.83 \times 0.7 = 25.781$ lbs. is the M.E.P. upon which the engine is to be proportioned.

To obtain 900 H.P. we must have $33,000 \times 900 = 29,700,000$ foot-pounds, and this divided by the mean pressure (25.78) and by the speed in feet (800) will give

$$\frac{33000 \times 900}{800 \times 25.78} = 1440 \text{ sq. in.}$$

for the area of the l. p. cylinder, which is about equivalent to 43 in. diam.

Now as one third of the work is to be done in the l. p. cylinder, the M.E.P. in it will be $25.78 \div 3 = 8.59$ lbs.

The cut-off in the high-pressure cylinder is generally arranged to cut off at 0.6 of the stroke, and so the ratio of the h. p. to the l. p. cylinder is equal to $16 \times 0.6 = 9.6$, and the h. p. cylinder will be $1440 \div 9.6 = 150$ sq. in. area, or about 14 in. diameter, and the M.E.P. in the h. p. cylinder is equal to $9.6 \times 8.59 = 82.46$ lbs.

If the intermediate cylinder is made a mean size between the other two, its size would be determined by dividing the area of the l. p. cylinder by the square root of the ratio between the low and the high; but in practice this is found to give a result too large to equalize the stresses, so that instead the area of the int. cylinder is found by dividing the area of the l. p. piston by 1.1 times the square root of the ratio of l. p. to h. p. cylinder, which in this case is $1440 \div (1.1\sqrt{9.6}) = 422.5$ sq. in., or a little more than 23 in. diam.

To put the above into the form of rules, we have

$$\text{Area h. p. cyl.} = \frac{\text{Area of low-pressure piston}}{\text{Cut-off in h. p. cyl.} \times \text{rate of expansion.}}$$

$$\text{Area intermediate cyl.} = \frac{\text{Area of low-pressure piston}}{1.1 \times \sqrt{\text{ratio of l. p. to h. p. cyl.}}}$$

The choice of expansion ratio is governed by the initial pressure, and is generally chosen so that the terminal pressure in the l. p. cylinder shall be about 10 lbs. absolute.

Annular Ring Method.—Jay M. Whitham, Trans. A. S. M. E., x. 577, gives the following method of ascertaining the diameter of pistons of triple expansion engines:

Lay down a theoretical indicator-diagram of a simple engine for the particular expansion desired. By trial find (with the polar planimeter or otherwise) the position of horizontal lines, parallel to the back-pressure line, such that the three areas into which they divide the diagram, representing low, intermediate, and high pressure diagrams, marked respectively A, B, and C, are equal.

Find the mean ordinate of each area: that of "C" will be the mean unbalanced pressure on the small piston; that of "B" will be the mean unbalanced pressure on the area remaining after subtracting the area of the small piston from that of the intermediate; and that of the area "A" will denote the mean unbalanced pressure on a square inch of the annular ring of the large piston obtained by subtracting the intermediate from the large piston We thus see that the mean ordinates of the two lower cards act on annular rings.

Let H = area of small piston in square inches;
I = " " intermediate piston in square inches;
L = " " large piston in square inches;
p_h = mean unbalanced pressure per square inch from card "C";
p_i = " " " " " " " " "B";
p_l = " " " " " " " " "A";
S = piston-speed in feet per minute;
(I.H.P.) = indicated horse-power of engine.

Then for equal work in each cylinder we have:

$$\text{Area of small piston} = H = 33{,}000 \times \frac{\text{I.H.P.}}{3} \div (p_h \times S); \quad . \quad . \quad . \quad . \quad (1$$

$$\left.\begin{array}{c}\text{Area of annular ring of}\\ \text{intermediate cylinder}\end{array}\right\} = 33{,}000 \times \frac{\text{I.H.P.}}{3} \div (p_i \times S);$$

$$\left.\begin{array}{c}\text{Area of interme-}\\ \text{diate piston}\end{array}\right\} = I = H + 33{,}000 \times \frac{\text{I.H.P.}}{3} \div (p_i \times S); \quad . \quad . \quad (2)$$

$$\text{Area of annular ring of large piston} = 33{,}000 \times \frac{\text{I.H.P.}}{3} \div (p \times S).$$

$$\text{Area of large piston} = L = I + 33{,}000 \times \frac{\text{I.H.P.}}{3} \div (p_l \times S); \quad . \quad . \quad (3)$$

This method is illustrated by the following example: Given I.H.P. = 3000, piston-speed S = 900 ft. per min., ratio of expansion 10, initial steam-pressure at cylinder 127 lbs. absolute, and back-pressure in large cylinder 4 lbs. absolute. Find cylinder diameters for equal work in each.

The mean ordinate of "C" is found to be p_h = 37.414 lbs. per sq. in.
" " " " "B" " " " p_i = 15.782 " " "
" " " " "A" " " " p_l = 11.730 " " "

Then by (1), (2), and (3) we have:

$$H = 33{,}000 \times \frac{3000}{3} \div 37.414 \times 900 = 980 \text{ sq. in., diam. } 35\tfrac{3}{8}'';$$

$$I = 980 + 33{,}000 \times \frac{3000}{3} \div 15.782 \times 900 = 3308 \text{ sq. in., diam. } 65'';$$

$$L = 3308 + 33{,}000 \times \frac{3000}{3} \div 11.730 \times 900 = 6432 \text{ sq. in., diam. } 90\tfrac{1}{2}$$

Mr. Whitham recommends the following cylinder ratios when the piston-speed is from 750 to 1000 ft. per min., the terminal pressure in the large cylinder being about 10 lbs. absolute.

CYLINDER RATIOS RECOMMENDED FOR TRIPLE-EXPANSION ENGINES.

Boiler-pressure (Gauge).	Small.	Intermediate.	Large.
130	1	2.25	5.00
140	1	2.40	5.85
150	1	2.55	6.90
160	1	2.70	7.25
170 and upwards—quadruple-expansion engine to be used.			

He gives the following ratios from examination of a number of actual engines:

No. of Engines Averaged.	Steam-boiler Pressure.	Cylinder Ratios. h. p.	int.	l. p.
9	130	1	2.10	4.38
3	135	1	2.07	5.00
11	140	1	2.40	5.84
2	145	1	2.35	5.23
23	150	1	2.54	6.00
27	160	1	2.66	7.24

A Common Rule for Proportioning the Cylinders of multiple-expansion engines is: for two-cylinder compound engines, the cylinder ratio is the square root of the number of expansions, and for triple-expansion engines the ratios of the high to the intermediate and of the intermediate to the low are each equal to the cube root of the number of expansions, the ratio of the high to the low being the product of the two ratios, that is, the square of the cube root of the number of expansions. Applying this rule to the pressures above given, assuming a terminal pressure (absolute) of 10 lbs. and 8 lbs. respectively, we have, for triple-expansion engines:

Boiler-pressure (Absolute).	Terminal Pressure, 10 lbs.		Terminal Pressure, 8 lbs.	
	No. of Expansions.	Cylinder Ratios, areas.	No. of Expansions.	Cylinder Ratios, areas.
130	13	1 to 2.35 to 5.53	16¼	1 to 2.53 to 6.42
140	14	1 to 2.41 to 5.81	17½	1 to 2.60 to 6.74
150	15	1 to 2.47 to 6.08	18¾	1 to 2.66 to 7.06
160	16	1 to 2.52 to 6.35	20	1 to 2.71 to 7.37

The ratio of the diameters is the square root of the ratios of the areas, and the ratio of the diameters of the first and third cylinders is the same as the ratio of the areas of first and second.

Seaton, in his Marine Engineering, says: When the pressure of steam employed exceeds 115 lbs. absolute, it is advisable to employ three cylinders, through each of which the steam expands in turn. The ratio of the low-pressure to high-pressure cylinder in this system should be 5, when the steam-pressure is 125 lbs. absolute; when 135 lbs. absolute, 5.4; when 145 lbs. absolute, 5.8; when 155 lbs. absolute, 6.2; when 165 lbs. absolute, 6.6. The ratio of low-pressure to intermediate cylinder should be about one half that between low-pressure and high-pressure, as given above. That is, if the ratio of l. p. to h. p. is 6, that of l. p. to int. should be about 3, and consequently that of int. to h. p. about 2. In practice the ratio of int. to h. p. is nearly 2.25, so that the diameter of the int. cylinder is 1.5 that of the h. p. The introduction of the triple-compound engine has admitted of ships being propelled at higher rates of speed than formerly obtained without exceeding the consumption of fuel of similar ships fitted with ordinary compound engines; in such cases the higher power to obtain the speed has been developed by decreasing the rate of expansion, the low-pressure cylinder being only 6 times the capacity of the high-pressure, with a working pressure of 170 lbs. absolute. It is now a very general practice to make the diameter of the low pressure cylinder equal to the sum of the diameters of the h. p. and int. cylinders; hence,

Diameter of int. cylinder = 1.5 diameter of h. p. cylinder;
Diameter of l. p. cylinder = 2.5 diameter of h. p. cylinder.

In this case the ratio of l. p. to h. p. is 6.25; the ratio of int. to h. p. is 2.25; and ratio of l. p. to int. is 2.78.

Ratios of Cylinders for Different Classes of Engines. (Proc. Inst. M. E., Feb. 1887, p. 36.)—As to the best ratios for the cylinders in a triple engine there seems to be great difference of opinion. Considerable latitude, however, is due to the requirements of the case, inasmuch as it would not be expected that the same ratio would be suitable for an economical land engine, where the space occupied and the weight were of minor importance, as in a war-ship, where the conditions were reversed. In the land engine, for example, a theoretical terminal pressure of about 7 lbs. above absolute vacuum would probably be aimed at, which would give a ratio of capacity of high pressure to low pressure of 1 to 8½ or 1 to 9; whilst in a war-ship a terminal pressure would be required of 12 to 13 lbs. which would need a ratio of capacity of 1 to 5; yet in both these instances the cylinders were correctly proportioned and suitable to the requirements of the case. It is obviously unwise, therefore, to introduce any hard-and-fast rule.

Types of Three-stage Expansion Engines.—1. Three cranks at 120 deg. 2. Two cranks with 1st and 2d cylinders tandem. 3. Two cranks with 1st and 3d cylinders tandem. The most common type is the first, with cylinders arranged in the sequence high, intermediate, low.

Sequence of Cranks.—Mr. Wyllie (Proc. Inst. M. E., 1887) favors the sequence high, low, intermediate, while Mr. Mudd favors high, intermediate, low. The former sequence, high, low, intermediate, gave an approximately horizontal exhaust-line, and thus minimizes the range of temperature and the initial load; the latter sequence, high, intermediate, low, increased the range and also the load.

Mr. Morrison, in discussing the question of sequence of cranks, presented a diagram shewing that with the cranks arranged in the sequence high, low, intermediate, the mean compression into the receiver was 19½ per cent of the stroke; with the sequence high, intermediate, low, it was 57 per cent. In the former case the compression was just what was required to keep the receiver-pressure practically uniform; in the latter case the compression caused a variation in the receiver-pressure to the extent sometimes of 22½ lbs.

Velocity of Steam through Passages in Compound Engines. (Proc. Inst. M. E., Feb. 1887.)—In the SS. *Para*, taking the area of the cylinder multiplied by the piston-speed in feet per second and dividing by the area of the port the velocity of the initial steam through the high-pressure cylinder port would be about 100 feet per second; the exhaust would be about 90. In the intermediate cylinder the initial steam had a velocity of about 180, and the exhaust of 120. In the low-pressure cylinder the initial steam entered through the port with a velocity of 250, and in the exhaust-port the velocity was about 140 feet per second.

QUADRUPLE-EXPANSION ENGINES.

H. H. Suplee (Trans. A. S. M. E., x. 583) states that a study of 14 different quadruple-expansion engines, nearly all intended to be operated at a pressure of 180 lbs. per sq. in., gave average cylinder ratios of 1 to 2, to 3.78, to 7.70, or nearly in the proportions 1, 2, 4, 8.

If we take the ratio of areas of any two adjoining cylinders as the fourth root of the number of expansions, the ratio of the 1st to the 4th will be the cube of the fourth root. On this basis the ratios of areas for different pressures and rates of expansion will be as follows:

Gauge-pressures.	Absolute Pressures.	Terminal Pressures.	Ratio of Expansion.	Ratios of Areas of Cylinders.
		12	14.6	1 : 1.95 : 3.81 : 7.43
160	175	10	17.5	1 : 2.05 : 4.18 : 8.55
		8	21.9	1 : 2.16 : 4.68 : 10.12
		12	16.2	1 : 2.01 : 4.02 : 8.07
180	195	10	19.5	1 : 2.10 : 4.42 : 9.28
		8	24.4	1 : 2.22 : 4.94 : 10.98
		12	17.9	1 : 2.06 : 4.23 : 8.70
200	215	10	21.5	1 : 2.15 : 4.64 : 9.98
		8	26.9	1 : 2.28 : 5.19 : 11.81
		12	19.6	1 : 2.10 : 4.43 : 9.31
220	235	10	23.5	1 : 2.20 : 4.85 : 10.67
		8	29.4	1 : 2.33 : 5.42 : 12.62

Seaton says: When the pressure of steam employed exceeds 190 lbs. absolute, four cylinders should be employed, with the steam expanding through each successively; and the ratio of l. p. to h. p. should be at least 7.5, and if economy of fuel is of prime consideration it should be 8; then the ratio of first intermediate to h. p. should be 1.8, that of second intermediate to first int. 2, and that of l. p. to second int. 2.2.

In a paper read before the North East Coast Institution of Engineers and Shipbuilders, 1890, William Russell Cummins advocates the use of a four-cylinder engine with four cranks as being more suitable for high speeds than the three-cylinder three-crank engine. The cylinder ratios, he claims, should be designed so as to obtain equal initial loads in each cylinder. The ratios determined for the triple engine are 1, 2.04, 6.54, and for the quadruple 1, 2.08, 4.46, 10.47. He advocates long stroke, high piston-speed, 180 revolutions per minute, and 250 lbs. boiler-pressure, unjacketed cylinders, and separate steam and exhaust valves.

Diameters of Cylinders of Recent Triple-expansion Engines, Chiefly Marine.

Compiled from several sources, 1890-1893.

Diam. in inches: H = high pressure, I = intermediate, L = low pressure.

H	*I*	*L*	*H*	*I*	*L*	*H*	*I*	*L*	*H*	*I*	*L*
3	5	8	16	25.6	41	22	36	40	36	58	94
4¾	7.5	13	16¼	23⅞	38.5			40	38	61.5	100
5	8	12	16.5	24.5	31	23	38	61	28	56	86
6.5	10.5	16.5			31	23.5	38	60	28		
7	9	12.5	17	27	44	24	37	56	39	61	97
7.1	11.8	18.9	17	28.5	42	25	40	64	40	59	88
7.5	12	19	17	28	45	26	42	69	40	67	106
8	11.5	16	18	27	40	26	42.5	70	40	66	100
9	14.5	22.5	18	29	48	28	44	72	41	66	101
9.8	15.7	25.6	18	305.	51	29⅝	44	70	41⅝	67	106¾
10	16	25	18.7	29.5	43.3	29.5	48	78	42	59	92
11	16	24	18¾	23.6	35.4	30	48	77	43	66	92
11	18	25	19.7	29.6	47.3	32	46	70	43	68	110
11	18	30	20	30	45	32	51	82	43⅝	67	106¼
11.5	18	28.5	20	32.5	36	32	54	82	45	71	113
11.5	17.5	30.5			36	33	58	88	32.5	68	85.7
12	19.2	30.7	20	33	52	33.9	55.1	84.6	32.5		85.7
13	22	33.5	21	32	48	34	54	85	47	75	81.5
14	22.4	36	21	36	51	34	50	90			81.5
14.5	24	39	21.7	38.5	49.2	34.5	51	85	37	79	98
15	24	39	21.9	34	57	34.5	57	92	37		98
15	24.5	38	22	34	51						

Where the figures are bracketed there are two cylinders of a kind. Two 28″ = one 39.6″, two 31″ = one 43.8″, two 32.5″ = one 46.0″, two 36″ = one 50.9″, two 37″ = one 52.3″, two 40″ = one 56.6″, two 81.5″ = one 115″, two 85.7″ = one 121″, two 98″ = one 140″. The average ratio of diameters of cylinders of all the engines in the above table is nearly 1 to 1.60 to 2.56 and the ratio of areas nearly 1 to 2.56 to 6.55.

The Progress in Steam-engines between 1876 and 1893 is shown in the following comparison of the Corliss engine at the Centennial Exhibition in 1876 and the Allis-Corliss quadruple-expansion engine at the Chicago Exhibition.

	1893.	1876.
Engine	Quadruple-expansion.	Simple
Cylinders, number	4	2
" diameter	24, 40, 60, 70 in.	40 in.
" stroke	72 in.	120 in.
Fly-wheel, diameter	30 ft.	30 ft.
" width of face	76 in.	24 in.
" weight	136,000 lbs.	125,440 lbs.
Revolutions per minute	60	36
Capacity, economical	2000 H.P.	1400 H.P.
" maximum	3000 H.P.	2500 H.P.
Total weight	650,000 lbs.	1,360,588 lbs.

The crank-shaft body or wheel-seat of the Allis engine has a diameter of 21 inches, journals 19 inches, and crank bearings 18 inches, with a total length of 18 feet. The crank-disks are of cast iron and are 8 feet in diameter. The crank-pins are 9 inches in diameter by 9 inches long.

A Double-tandem Triple-expansion Engine, built by Watts, Campbell & Co., Newark, N. J., is described in *Am. Mach.*, April 26, 1894. It is two three-cylinder tandem engines coupled to one shaft, cranks at 90°, cylinders 21, 32 and 48 by 60 in. stroke, 65 revolutions per minute, rated H.P. 2000; fly-wheel 28 feet diameter, 12 ft. face, weight 174,000 lbs.; main shaft 22 in. diameter at the swell; main journals 19 × 38 in.; crank-pins 9½ × 10 in.; distance between centre lines of two engines 24 ft. 7⅛ in.; Corliss valves, with separate eccentrics for the exhaust-valves of the l.p. cylinder.

Name of Engine and where Built.	Type of Engine.	Horizontal or Vertical.	Cylinders, ins. Diameters and Stroke.	I.H.P. Maximum Economy.	I.H.P. Maximum Load.	Driv. Pulley, Diameter in. Face, in.	Revolutions per Minute.	Size of Steam-pipe.	Size of Exhaust-pipe.	Weight of Engine, lbs.
E. P. Allis Co., Milwaukee	Quad. exp. condensing	H.	24, 40, 60, 70 x 72	2,000	3,000	360 x 76	60	10	22	650,000
Fraser & Chalmers, Chicago	4-cyl. trip. exp. cond	"	20, 34, 2-34 x 60	1,000	1,250	336 x 68	64 1-2	7	16	260,000
McIntosh-Seymour, Auburn, N. Y.	Double tand. comp. cond.	"	18, 32 x 36 double	1,000	1,500	192 x 78	112	7	14	250,000
Buckeye Engine Co., Salem, O	4-cyl. trip. exp. cond	"	20, 30, 2-36 x 48	1,000	1,300	240 x 75	85	8	12	120,000
Atlas, Indianapolis, Ind	Comp. condensing	"	14, 24 x 30	1,000	1,300	144 x 75	150	5	8	114,000
Westinghouse, Pittsburg, Pa.	Comp. cond., 6 engines	V.	21 1-2, 37 x 22	1,000	......	132 x 14	200	10	14	195,000*
" " "	" " 2 "	"	18, 30 x 16	350	......	96 x 26	250	8	10	......
Russell, Massillon, O.	Double tand. comp. cond.	H.	15, 24 x 24	506	897	120 x 60	125	7	12	83,298
Atlas, Indianapolis, Ind	Tandem comp. cond	"	14, 24 x 30	500	650	144 x 36	150	5	8	57,000
Bass, Ft. Wayne, Ind	Cross compound cond.	"	16, 30 x 42	224	440	192 x 43	70	5	10	61,000
Lane & Bodley, Cincinnati, O.	" " "	"	16, 30 x 42	300	350	192 x 43	70	5	10	80,000
" " " " "	Tandem comp. cond.	"	16, 28 x 42	300	350	216 x 36	80	5	10	75,000
Watertown, Watertown, N. Y.	Double tand. comp. cond.	"	9, 16 x 14	250	......	60 x 12	250	4 1-2	10	21,200
Buckeye, Salem, O.	Cross compound cond.	"	14, 28 x 24	300	360	120 x 33	50	......	......	33,000
Ball & Wood, Elizabeth, N. J.	Cross compound.	"	14, 22 x 12	200	250	66	280	5	8	......
N. Y. Safety Steam-Power Co.	Simple.	"	15 1-2 x 16	150	200	72 x 12	250	4 1-2	6	......
Schichau, Germany	Triple exp. condensing	V.	23, 37, 57 x 27	1,000	......	......	100	......	......	......
Stearns Mfg. Co., Erie, Pa.	Tandem comp. cond.	H.	19, 31 x 24	600	800	102 x 31	165	7	12	75,000
Phœnix Iron Works Co., Meadville, Pa.	4-cyl. trip. exp. cond.	"	15, 24, 2-26 x 18	500	......	2-108 x 26 1-108 x 38	200	6	14	......
" " " " " "	Tandem compound cond.	"	13, 24 x 18	250	......	108 x 26	200	5	9	......
A. L. Ide & Son, Springfield, Ill.	" " "	"	13, 22 x 16	225	250	72 x 16 1-2	245	6	8	24,000
Allis, Milwaukee, Wis.	Cross compound cond.	"	19, 36 x 48	500	800	192	90	7	12	85,000
General Electric Co., N. Y.	Triple exp. condensing	V.	......	1,000	......	......	......	8	18	......
Armington & Sims, Providence, R. I.	Simple	H.	18 x 21	......	......	84 x 25	225	7	10	......
Ball Engine Co., Erie, Pa.	Cross compound	"	18, 36 x 18	480	570	86	225	7	18	65,000
McEwen, Ridgway, Pa.	Tandem compound cond.	"	14, 23 x 20	220	325	84 x 29	200	5	10	24,000
Sioux City, Sioux City, Ia.	Simple	"	24 x 48	350	425	192 x 44	......	8	9	49,000
Gold St. Min. Iron Wks., San Fran'co, Cal.	Tandem compound	"	12, 22 x 30	200	270	132 x 25	115	4	7 1-2	35,000
Bates Machine Co., Joliet, Ill.	Simple	"	20 x 48	297	346	192 x 27	60	6	7	43,000
Harrisburg, Harrisburg, Pa.	Tandem compound	"	17, 28 x 18	300	370	102 x 31	176	6	11	50,000
Providence Steam-Engine Works, R. I.	" "	"	12, 20 x 42	225	300	144 x 25	125	5	8	49,600
Erie City Iron Works, Erie, Pa.	Simple.	"	18 x 22	300	390	84 x 24 1-2	160	6	7	18,000
Galloway, England	Comp. condensing.	"	17, 30 x 45	350	......	282	70	......	......	......
Willans & Robinson, England	Comp. cond., 3-throw	V.	3-14, 20 x 8	......	360	58 x 40	350	4	5	......
Schichau, Germany	Triple exp. condensing	"	12, 19, 29 1-2 x 14	150	......	......	150	......	......	......

* Engine and dynamo.

ECONOMIC PERFORMANCE OF STEAM-ENGINES.

Economy of Expansive Working under Various Conditions, Single Cylinder.

(Abridged from Clark on the Steam Engine.)

1. SINGLE CYLINDERS WITH SUPERHEATED STEAM, NONCONDENSING.—Inside cylinder locomotive, cylinders and steam-pipes enveloped by the hot gases in the smoke-box. Net boiler pressure 100 lbs.; net maximum pressure in cylinders 80 lbs. per sq. in.

Cut-off, per cent.........	20	25	30	35	40	50	60	70	80
Actual ratio of expansion	3.91	3.31	2.87	2.53	2.26	1.86	1.59	1.39	1.23
Water per I.H.P. per hour, lbs..................	18.5	19.4	20	21.2	22.2	24.5	27	30	33

2. SINGLE CYLINDERS WITH SUPERHEATED STEAM, CONDENSING.—The best results obtained by Hirn, with a cylinder 23¾ × 67 in. and steam superheated 150° F., expansion ratio 3¾ to 4½, total maximum pressure in cylinder 63 to 69 lbs. were 15.63 and 15.69 lbs. of water per I.H.P. per hour.

3. SINGLE CYLINDERS OF SMALL SIZE, 8 OR 9 IN. DIAM., JACKETED, NONCONDENSING.—The best results are obtained at a cut-off of 20 per cent, with 75 lbs. maximum pressure in the cylinder; about 25 lbs. of water per I.H.P. per hour.

4. SINGLE CYLINDERS, NOT STEAM-JACKETED, CONDENSING.—Best results.

Engine.	Cylinder, Diam. and Stroke.	Cut-off.	Actual Expansion Ratio.	Total Maximum Pressure in Cylinder per sq. in.	Water as Steam per I.H.P. per hour.
	ins.	per cent.	ratio.	lbs.	lbs.
Corliss and Wheelock...	18 × 48	12.5	6.95	104.4	19.58
Hirn, No. 6..............	23¾ × 67	16.8	5.84	61.5	19.93
Mair, M.................	32 × 66	24.6	3.84	54.5	26.46
Bache..................	25 × 24	15.5	5.32	87.7	26.25
Dexter.................	26 × 36	18.3	4.46	80.4	23.86
Dallas..................	36 × 30	13 3	5.07	46.9	26.69
Gallatin.................	30.1 × 30	15.0	4.94	81.7	21.89

SAME ENGINES, AVERAGE RESULTS.

Long Stroke.	Inches.	Cut-off, Per cent.	Lbs.	Lbs.
Corliss and Wheelock...	18 × 48	12.5	104.4	19.58
Hirn	23¾ × 67	16.3	61.5	19.93
Short Stroke.				
Bache	25 × 24	15.5	87.7	26.25
Dexter, Nos. 20, 21, 22, 23	26 × 36	18.3 to 33.3, average 25	79.0	24.05
Dallas, Nos. 27, 28, 29	36 × 30	13.3 to 26.4, average 19.8	46.8	26.86
Gallatin, Nos. 24, 25, 22, 26	30.1 × 30	12.3 to 18.5, average 15.8	78.2	23.50

Feed-water Consumption of Different Types of Engines.—The following tables are taken from the circular of the *Tabor Indicator* (Ashcroft Mfg. Co., 1889). In the first of the two columns under Feed-water required, in the tables for simple engines, the figures are obtained by computation from nearly perfect indicator diagrams, with allowance for cylinder condensation according to the table on page 752, but without allowance for leakage, with back-pressure in the non-condensing table taken at 16 lbs. above zero, and in the condensing table at 3 lbs. above zero. The compression curve is supposed to be hyperbolic, and commences at 0.91 of the return-stroke, with a clearance of 3% of the piston-displacement.

Table No. 2 gives the feed-water consumption for jacketed compound-condensing engines of the best class. The water condensed in the jackets is included in the quantities given. The ratio of areas of the two cylinders are as 1 to 4 for 120 lbs. pressure; the clearance of each cylinder is 3%; and the cut off in the two cylinders occurs at the same point of stroke. The initial pressure in the l. p. cylinder is 1 lb. per sq. in. below the back-pressure of the h. p. cylinder. The average back pressure of the whole stroke in the l. p. cylinder is 4.5 lbs. for 10% cut-off; 4.75 lbs. for 20% cut-off; and 5 lbs. for 30% cut-off. The steam accounted for by the indicator at cut-off in the h. p. cylinder (allowing a small amount for leakage) is .74 at 10% cut-off, .78 at 20%, and .82 at 30% cut-off. The loss by condensation between the cylinders is such that the steam accounted for at cut-off in the l. p. cylinder, expressed in proportion of that shown at release in the h. p. cylinder, is .85 at 10% cut-off, .87 at 20% cut-off, and .89 at 30% cut-off.

The data upon which table No. 3 is calculated are not given, but the feed-water consumption is somewhat lower than has yet been reached (1894), the lowest steam consumption of a triple-exp. engine yet recorded being 11.7 lbs.

TABLE No. 1.

FEED-WATER CONSUMPTION, SIMPLE ENGINES.

Non-condensing Engines.					Condensing Engines.				
Per Cent Cut-off.	Initial Pressure above Atmosphere, lbs.	Mean Effective Pressure, lbs.	Feed-water Required per I.H.P. per Hour.		Per Cent Cut-off.	Initial Pressure above Atmosphere, lbs.	Mean Effective Pressure, lbs.	Feed-water Required per I.H.P. per Hour.	
			Corresponding to Diagrams with no Leakage, lbs.	Corresponding to Actual Results Attained in Practice, assuming Slight Leakage.				Corresponding to Diagrams with no Leakage, lbs.	Corresponding to Actual Results Attained in Practice, assuming Slight Leakage.
10	60	8.70	37.26	40.95	5	60	14.42	18.22	20.00
	70	12.89	30.99	33.68		70	16.96	17.96	19.69
	80	16.07	27.61	29.88		80	19.50	17.76	19.47
	90	19.76	25.43	27.43		90	22.04	17.57	19.27
	100	23.45	23.90	25.73		100	24.58	17.41	19.07
20	60	21.12	27.55	29.43	10	60	22.34	17.68	19.34
	70	26.57	25.44	27.04		70	26.08	17 47	19.09
	80	32.02	21.04	25.68		80	29.72	17.30	18.89
	90	37.47	23.00	24.57		90	33.41	17.15	18.70
	100	42.92	22.25	23.77		100	37.10	17.02	18.56
30	60	30.47	27.24	29.10	15	60	29.00	17.93	19.51
	70	37.21	25.76	27.43		70	33.65	17.75	19.27
	80	43.97	24.71	26.29		80	38.28	17.60	19.08
	90	50.73	23.91	25.38		90	42.92	17.45	18.91
	100	57.49	23.27	24.68		100	47.56	17.32	18 74
40	60	37.75	27.92	29.63	20	60	34.73	18.58	20 09
	70	45.50	26.66	28.18		70	40.18	18.40	19.85
	80	53.25	25.76	27.17		80	45.63	18.27	19.69
	90	61.01	25.03	26.35		90	51.08	18.14	19.51
	100	68.76	24.47	25.73		100	56.53	18.02	19.36
50	60	43.42	28.94	30.66	30	60	44.06	20.19	21.64
	70	51.94	27.79	29.31		70	50.81	20.04	21.41
	80	60.44	26.99	28.38		80	57.57	19.91	21.25
	90	68.96	26.32	27.62		90	64.32	19.78	21.06
	100	77.48	25.78	26.99		100	71.08	19.67	20.93
					40	60	51.35	21.63	22.93
						70	59.10	21.49	22.74
						80	66.85	21.30	22.56
						90	74.60	21.24	22.41
						100	82.36	21.18	22.24

TABLE No. 2.

FEED-WATER CONSUMPTION FOR COMPOUND CONDENSING ENGINES.

Cut-off, per cent.	Initial Pressure above Atmosphere. H.P. Cyl., lbs.	Initial Pressure above Atmosphere. L.P. Cyl., lbs.	Mean Effective Press- Atmosphere. H.P. Cyl., lbs.	Mean Effective Press- Atmosphere. L.P. Cyl., lbs.	Feed-water Required per I.H.P. per Hour, Lbs.
10	80	4.0	11.67	2.65	16.92
	100	7.3	15.33	3.87	15.00
	120	11.0	18.54	5.23	13.86
20	80	4.3	26.73	5.48	14.60
	100	8.1	33.13	7.56	13.67
	120	12.1	39.29	9.74	13.09
30	80	4.6	37.61	7.48	14.99
	100	8.5	46.41	10.10	14.21
	120	11.7	56.00	12.26	13.87

TABLE No. 3.

FEED-WATER CONSUMPTION FOR TRIPLE-EXPANSION CONDENSING ENGINES.

Cut-off. per cent.	Initial Pressure above Atmosphere. H.P. Cyl., lbs.	I. Cyl., lbs.	L.P. Cyl., lbs.	Mean Effective Pressure. H.P. Cyl., lbs.	I. Cyl., lbs.	L.P. Cyl., lbs.	Feed-water Required per I.H.P. per Hour, lbs.
30	120	37.8	1.3	38.5	17.1	6.5	12.06
	140	43.3	2.8	46.5	18.6	7.1	11.4
	160	49.3	3.8	55.0	20.0	8.0	10.75
40	120	38.8	2.8	51.5	22.8	8.6	11.65
	140	45.8	3.9	59.5	23.7	9.1	11.4
	160	51.3	5.3	70.0	25.5	10.0	10.85
50	120	39.8	3.7	60.5	26.7	10.1	12.2
	140	46.8	4.8	70.5	28.0	10.8	11.6
	160	52.8	6.3	82.5	30.0	11.8	11.15

Most Economical Point of Cut-off in Steam-engines. (See paper by Wolff and Denton, Trans. A. S. M. E., vol. ii. p. 147-281; also, Ratio of Expansion at Maximum Efficiency, R. H. Thurston, vol. ii. p. 128.) —The problem of the best ratio of expansion is not one of economy of consumption of fuel and economy of cost of boiler alone. The question of interest on cost of engine, depreciation of value of engine, repairs of engine, etc., enters as well; for as we increase the rate of expansion, and thus, within certain limits fixed by the back-pressure and condensation of steam, decrease the amount of fuel required and cost of boiler per unit of work, we have to increase the dimensions of the cylinder and the size of the engine, to attain the required power. We thus increase the cost of the engine, etc., as we increase the rate of expansion, while at the same time we decrease the fuel consumption, the cost of boiler, etc. So that there is in every engine some point of cut-off, determinable by calculation and graphical construction, which will secure the greatest efficiency for a given expenditure of money, taking into consideration the cost of fuel, wages of engineer and firemen, interest on cost, depreciation of value, repairs to and insurance of boiler and engine, and oil, waste, etc., used for engine. In case of freight-carrying vessels, the value of the room occupied by fuel should be considered in estimating the cost of fuel.

Sizes and Calculated Performances of Vertical High-speed Engines.—The following tables are taken from a circular of the Field Engineering Co., New York, describing the engines made by the Lake Erie Engineering Works, Buffalo, N. Y. The engines are fair representatives of the type now coming largely into use for driving dynamos directly without belts. The tables were calculated by E. F. Williams, designer of the engines. They are here somewhat abridged to save space:

Simple Engines—Non-condensing.

Diam. of Cylinder, inches.	Stroke, inches.	Revs. per Minute.	H.P. when Cutting off at 1/5 stroke. 70 lbs.	80 lbs.	90 lbs.	H.P. when Cutting off at ¼ stroke. 70 lbs.	80 lbs.	90 lbs.	H.P. when Cutting off at ⅓ stroke. 70 lbs.	80 lbs.	90 lbs.	Dimensions of Wheels, diam. Ft.	face In.	Steam-pipe, in.	Exhaust-pipe.
7½	10	370	20	25	30	26	31	36	32	37	43	4	4	2½	3
8½	12	318	27	32	39	34	41	47	41	48	56	4½	5	2¾	3½
10½	14	277	41	49	60	52	62	71	63	74	85	5′9″	6½	3½	4
12	16	246	53	64	77	67	81	93	82	96	111	6′8″	9	4	4½
13½	18	222	66	80	96	84	100	116	102	120	138	7½	11	4	5
16	20	181	95	115	138	120	144	166	146	172	198	8′4″	15	4½	6
18	24	158	119	144	173	151	181	208	183	215	248	10	19	5	7
22	28	138	179	216	261	227	272	313	276	324	373	11′8″	28	6	8
24½	32	120	221	267	322	281	336	386	340	400	460	13′4″	34	7	9
27	34	112	269	325	392	342	409	470	414	487	560	14′2″	41	8	10
Mean eff. press. lb.			24	29	35	30.5	36.5	42	37	43.5	50				
Ratio of expans'n.			5			4			3						
Terminal pressure (about) ... lbs.			17.9	20	22.3	22.4	25	27.6	29.8	33.3	36.8				
Cyl. condensat'n, %			26	26	26	24	24	24	21	21	21				
Steam per I.H.P. per hour. ... lbs			32.9	30	27.4	31.2	29.0	27.9	32	31.4	30				

NOTE.—The nominal-power rating of the engines is at 80 lbs. gauge pressure, steam cut-off at ¼ stroke.

Compound Engines — Non-condensing — High-pressure Cylinder and Receiver Jacketed.

Diam. Cylinder, inches. H.P.	H.P.	L.P.	Stroke, inches.	Revolutions per Minute.	H.P. when cutting off at ¼ Stroke in h.p. Cylinder. Cyl. Ratio, 3⅓ : 1. 80 lbs.	90 lbs.	Cyl. Ratio, 4⅓ : 1. 130 lbs.	150 lbs.	H.P. when cutting off at ⅓ Stroke in h.p. Cylinder. Cyl. Ratio, 3⅓ : 1. 80 lbs.	90 lbs.	Cyl. Ratio, 4⅓ : 1. 130 lbs.	150 lbs.	H.P. when cutting off at ½ Stroke in h.p. Cylinder. Cyl. Ratio, 3⅓ : 1. 80 lbs.	90 lbs.	Cyl. Ratio, 4⅓ : 1. 130 lbs.	150 lbs.
5¾	6½	12	10	370	7	15	19	32	23	31	35	46	44	55	64	79
6¾	7½	13½	12	318	9	19	24	40	29	39	45	59	56	70	81	101
7¾	9	16½	14	277	14	28	36	60	43	58	67	87	83	104	121	159
9	10½	19	16	246	18	37	47	78	57	76	87	114	109	136	158	196
10½	12	22½	18	222	26	53	68	112	81	109	125	164	156	195	226	281
12	13½	25	20	185	32	65	84	139	100	135	154	202	192	241	279	346
13½	15½	28½	24	158	43	88	112	186	135	181	206	271	258	323	374	464
16	18½	33½	28	138	57	118	151	249	180	242	277	363	346	433	502	623
18	20½	38	32	120	74	152	194	321	232	312	357	468	446	558	647	803
20	22½	43	34	112	94	194	249	412	297	400	457	601	572	715	829	1030
24½	28½	52	42	93	188	285	335	603	436	587	670	880	838	1048	1215	1508
28½	33	60	48	80	180	374	477	789	570	767	877	1151	1096	1370	1589	1973
Mean effec. press...lbs					3.3	6.8	8.7	14.4	10.4	14.0	16	21	20	25	20	36
Ratio of expansion....					13⅓		18¼		10¼		13¾		6¾		9⅓	
Cyl. condensation, %...					14	14	16	16	12	12	13	13	10	10	11	11
Ter. press. (about). lbs.					7.3	7.7	7.9	9	9.2	10.4	10.5	12	14	15.5	14.6	17.8
Loss from expanding below atmosphere, %					34	15	17	3	5	0	0	0	0	0	0	0
St. per I.H.P. p. hr. lbs.					55	42	47	29	33.3	27.7	28.7	25.4	30	26.2	21	20

The original table contains figures of horse-power, etc., for 110 and 120 lbs., cylinder ratio of 4 to 1; and 140 lbs., ratio 4⅓ to 1.

Compound-engines—Condensing—Steam-jacketed.

Diam. Cylinder, Inches. H.P.	H.P.	L.P.	Stroke, inches.	Revolutions per Minute.	H.P. when cutting off at ¼ Stroke in h.p. Cylinder. Cyl. Ratio, 3⅓ : 1. 80 lbs.	110 lbs.	Cyl. Ratio, 4 : 1. 115 lbs.	125 lbs.	H.P. when cutting off at ⅓ Stroke in h.p. Cylinder. Cyl. Ratio, 3⅓ : 1. 80 lbs.	110 lbs.	Cyl. Ratio, 4 : 1. 115 lbs.	125 lbs.	H.P. when cutting off at ½ Stroke in h.p. Cylinder. Cyl. Ratio, 3⅓ : 1. 80 lbs.	110 lbs.	Cyl. Ratio, 4 : 1. 115 lbs.	125 lbs.
6	6½	12	10	370	44	59	53	62	55	70	68	75	70	97	95	106
6½	7½	13½	12	318	56	76	67	78	70	90	87	95	90	123	120	134
8¼	9	16½	14	277	83	112	100	116	104	133	129	141	133	183	179	200
9½	10½	19	16	246	109	147	131	152	136	174	169	185	174	239	234	261
11	12	22½	18	222	156	210	187	218	195	250	242	265	250	343	335	374
12½	13½	25	20	185	192	260	231	269	241	308	298	327	308	423	414	462
14	15½	28½	24	158	258	348	310	361	323	413	400	439	413	568	555	619
17	18½	33½	28	138	346	467	415	484	433	554	536	588	554	761	744	830
19	20½	38	32	120	446	602	535	624	558	714	691	758	714	981	959	1070
21	22½	43	34	112	572	772	686	801	715	915	887	972	915	1258	1230	1373
26	28½	52	42	93	838	1131	1006	1174	1048	1341	1299	1425	1341	1844	1801	2012
30	33	60	48	80	1096	1480	1316	1534	1370	1757	1699	1863	1757	2411	2356	2632
Mean effec. press..lbs.					20	27	24	28	25	32	31	34	32	44	43	48
Ratio of Expansion...					13½		16¼		10		12¼		6¾		8¼	
Cyl. condensation, %...					18	18	20	20	15	15	18	18	12	12	14	14
St. per I.H.P. p. hr.lbs.					17.3	16.6	16.6	15.2	17.0	16.4	16.3	15.8	17.5	17.0	16.8	16.0

The original table contains figures for 95 lbs., cylinder ratio 3⅓ to 1; and 120 lbs , ratio 4 to 1.

Triple-expansion Engines, Non-condensing.—Receiver only Jacketed.

Diameter Cylinders, inches. H. P.	I. P.	L. P.	Stroke, inches.	Revolutions per Minute.	Horse-power when Cutting off at 42 per cent of Stroke in First Cylinder. 180 lbs.	200 lbs.	Horse-power when Cutting off at 50 per cent of Stroke in First Cylinder. 180 lbs.	200 lbs.	Horse-power when Cutting off at 67 per cent of Stroke in First Cylinder. 180 lbs.	200 lbs.
4¾	7½	12	10	370	55	64	70	84	95	108
5½	8½	13½	12	318	70	81	90	106	120	137
6½	10½	16½	14	277	104	121	133	158	179	204
7½	12	19	16	246	136	158	174	207	234	267
9	14½	22½	18	222	195	226	250	296	335	382
10	16	25	20	185	241	279	308	366	414	471
11½	18	28½	24	158	323	374	413	490	555	632
13	22	33½	28	138	433	502	554	657	744	848
15	24½	38	32	120	558	647	714	847	959	1093
17	27	43	34	112	715	829	915	1089	1230	1401
20	33	52	42	93	1048	1215	1341	1592	1801	2058
23½	38	60	48	80	1370	1589	1754	2082	2356	2685
Mean effective press., lbs.					25	29	32	38	43	49
No. of expansions.......					16		13		10	
Per cent cyl. condens....					14		12		10	
Steam p. I.H.P. p.hr., lbs.					20.76	19.36	19.25	17.00	17.89	17.20
Lbs. coal at 8 lb. evap. lbs.					2.59	2.39	2.40	2.12	2.23	2.15

Triple-expansion Engines—Condensing—Steam-Jacketed.

Diameter Cylinders, inches. H.P.	I.P.	L.P.	Stroke, inches.	Revolutions per Minute.	Horse-power when Cutting off at ¼ Stroke in First Cylinder. 120 lbs.	140 lbs.	160 lbs.	Horse-power when Cutting off at ⅕ Stroke in First Cylinder. 120 lbs.	140 lbs.	160 lbs.	Horse-power when Cutting off at ⅙ Stroke in First Cylinder. 120 lbs.	140 lbs.	160 lbs.	Horse-power when Cutting off at ⅛ Stroke in First Cylinder. 120 lbs.	140 lbs.	160 lbs.
4¾	7½	12	10	370	35	42	48	44	53	59	57	72	84	81	97	110
5½	8½	13½	12	318	45	53	62	56	67	76	73	92	107	104	123	140
6½	10½	16½	14	277	67	79	92	83	100	112	108	137	159	154	183	[illegible]
7½	12	19	16	246	87	103	120	109	131	147	141	180	208	201	239	[illegible]
9	14½	22½	18	222	125	148	172	156	187	211	203	257	299	289	343	[illegible]
10	16	25	20	185	154	183	212	192	231	260	250	317	368	356	423	481
11½	18	28½	24	158	206	245	284	258	310	348	335	426	494	477	568	645
13	22	33½	28	138	277	329	381	346	415	467	450	571	663	640	761	865
15	24½	38	32	120	357	424	491	446	535	602	580	736	854	825	981	1115
17	27	43	34	112	458	543	629	572	686	772	744	944	1095	1058	1258	1490
20	33	52	42	93	670	796	922	838	1006	1131	1089	1383	1605	1551	1844	[illegible]
23½	38	60	48	80	877	1041	1206	1096	1316	1480	1424	1808	2099	2028	2411	2740
Mean effec. press., lbs.					16	19	22	20	24	27	26	33	38.3	37	44	50
No. of expansions....					26.8			20.1			13.4			8.9		
Per cent cyl. condens.					19	19	19	16	16	16	12	12	12	8	8	8
St. p. I.H.P. p. hr., lbs.					14.7	13.9	13.3	14.3	13.98	13.2	14.3	13.6	13.0	15.7	14.9	14.2
Coal at 8 lb. evap., lbs.					1.8	1.73	1.66	1.78	1.74	1.65	1.78	1.70	1.62	1.96	1.86	1.77

Type of Engine to be used where Exhaust-steam is needed for Heating.—In many factories more or less of the steam exhausted from the engines is utilized for boiling, drying, heating, etc. Where all the exhaust-steam is so used the question of economical use of steam in the engine itself is eliminated, and the high-pressure simple engine is entirely suitable. Where only part of the exhaust-steam is used, and the quantity so used varies at different times, the question of adopting a simple, a condensing, or a compound engine becomes more complex. This problem is treated by C. T. Main in Trans. A. S. M. E., vol. x. p. 48. He shows that the ratios of the volumes of the cylinders in compound engines should vary according to the amount of exhaust-steam that can be used for heating. A case is given in which three different pressures of steam are required or could be used, as in a worsted dye-house: the high or boiler pressure for the engine, an intermediate pressure for crabbing, and low-pressure for boiling, drying, etc. If it did not make too much complication of parts in the engine, the boiler-pressure might be used in the high-pressure cylinder, exhausting into a receiver from which steam could be taken for running small engines and crabbing, the steam remaining in the receiver passing into the intermediate cylinder and expanded there to from 5 to 10 lbs. above the atmosphere and exhausted into a second receiver. From this receiver is drawn the low-pressure steam needed for drying, boiling, warming mills, etc., the steam remaining in receiver passing into the condensing cylinder.

Comparison of the Economy of Compound and Single-cylinder Corliss Condensing Engines, each expanding about Sixteen Times. (D. S. Jacobus, Trans. A. S. M. E., xii. 943.)

The engines used in obtaining comparative results are located at Stations I. and II. of the Pawtucket Water Co.

The tests show that the compound engine is about 30% more economical than the single-cylinder engine. The dimensions of the two engines are as follows: Single 20″ × 48″; compound 15″ and 30½″ × 30″. The steam used per horse-power per hour was: single 20.35 lbs., compound 13.73 lbs.

Both of the engines are steam-jacketed, practically on the barrels only, with steam at full boiler-pressure, viz. single 106.3 lbs., compound 127.5 lbs.

The steam-pressure in the case of the compound engine is 127 lbs., or 21 lbs. higher than for the single engine. If the steam-pressure be raised this amount in the case of the single engine, and the indicator-cards be increased accordingly, the consumption for the single-cylinder engine would be 19.97 lbs. per hour per horse-power.

Two-cylinder vs. Three-cylinder Compound Engine.—A Wheelock triple-expansion engine, built for the Merrick Thread Co., Holyoke, Mass., is constructed so that the intermediate cylinder may be cut out of the circuit and the high-pressure and low-pressure cylinders run as a two-cylinder compound, using the same conditions of initial steam-pressure and load. The diameters of the cylinders are 12, 16, and 24 1/32 inches, the stroke of the first two being 36 in. and that of the low-pressure cylinder 48 in. The results of a test reported by S. M. Green and G. I. Rockwood, Trans. A. S. M. E., vol. xiii. 647, are as follows: In lbs. of dry steam used per I.H.P. per hour, 12 and 24 1/32 in. cylinders only used, two tests 13.06 and 12.76 lbs., average 12.91. All three cylinders used, two tests 12.67 and 12.90 lbs., average 12.79. The difference is only 1%, and would indicate that more than two cylinders are unnecessary in a compound engine, but it is pointed out by Prof. Jacobus, that the conditions of the test were especially favorable for the two-cylinder engine, and not relatively so favorable for the three cylinders. The steam-pressure was 142 lbs. and the number of expansions about 25. (See also discussion on the Rockwood type of engine, Trans. A. S. M. E., vol. xvi.)

Effect of Water contained in Steam on the Efficiency of the Steam-engine. (From a lecture by Walter C. Kerr, before the Franklin Institute, 1891.)—Standard writers make little mention of the effect of entrained moisture on the expansive properties of steam, but by common consent rather than any demonstration they seem to agree that moisture produces an ill effect simply to the percentage amount of its presence. That is, 5% moisture will increase the water rate of an engine 5%.

Experiments reported in 1893 by R. C. Carpenter and L. S. Marks, Trans. A. S. M. E., xv., in which water in varying quantity was introduced into the steam-pipe, causing the quality of the steam to range from 99% to 58% dry, showed that throughout the range of qualities used the consumption of dry steam per indicated horse-power per hour remains practically constant, and indicated that the water was an inert quantity, doing neither good nor harm.

It appears that the extra work done by the heat of the entrained water during expansion is sensibly equal to the extra negative work which it does during exhaust and compression, that the heat carried in by the entrained water performs no useful function, and that a fair measure of the economy of an engine is the consumption of dry and saturated steam.

Relative Commercial Economy of Best Modern Types of Compound and Triple-expansion Engines. (J. E. Denton, *American Machinist*, Dec. 17, 1891.)—The following table and deductions show the relative commercial economy of the compound and triple type for the best stationary practice in steam plants of 500 indicated horse-power. The table is based on the tests of Prof. Schröter, of Munich, of engines built at Augsburg, and those of Geo. H. Barrus on the best plants of America, and of detailed estimates of cost obtained from several first-class builders.

Trip motion, or Corliss engines of the twin-compound-receiver condensing type, expanding 16 times. Boiler pressure 120 lbs.	Lbs. water per hour per H.P., by measurement.	13.6	14.0
	Lbs. coal per hour per H.P., assuming 8.5 lbs. actual evaporation.	1.60	1.65
Trip motion, or Corliss engines of the triple-expansion four-cylinder-receiver condensing type, expanding 22 times. Boiler pressure, 150 lbs.	Lbs. water per hour per H.P., by measurement.	12.56	12.80
	Lbs. coal per hour per H.P., assuming 8.5 lbs. actual evaporation.	1.48	1.50

The figures in the first column represent the best recorded performance (1891), and those in the second column the probable reliable performance.

Increased cost of triple-expansion plant per horse-power, including boilers, chimney, heaters, foundations, piping and erection.......... $4.50

The following table shows the total annual cost of operation, with coal at $4.00 per ton, the plant running 300 days in the year, for 10 hours and for 24 hours per day:

Hours running per day	10	24
	Per H.P.	Per H.P.
Expense for coal. Compound plant	$9.90	$28.50
Expense for coal. Triple plant	9.00	25.92
Annual saving of triple plant in fuel	0.90	2.60
Annual interest at 5% on $4.50	$0.23	$0.23
Annual depreciation at 5% on $4.50	0.23	0.23
Annual extra cost of oil, 1 gallon per 24-hour day, at $0.50, or 15% of extra fuel cost	0.15	0.36
Annual extra cost of repairs at 3% on $4.50 per 24 hours	0.06	0.14
	$0.67	$0.96
Annual saving per H.P.	$0.23	$1.64

The saving between the compound and triple types is much less than that involved in the step from the single-expansion condensing to the compound engine. The increased cost per horse-power of the triple plant over the compound is due almost entirely to the extra cost of the triple engine and its foundations, the boilers costing the same or slightly more, owing to their extra strength. In the case of the single *versus* the compound, however, about one third of the increased cost of the compound engine is offset by the less cost of the latter's boilers.

Taking the total cost of the plants at $33.50, $36.50 and $41 per horse-power respectively, the figures in the table imply that the total annual saving is as follows for coal at $4 per ton:

1. A compound 500 horse-power plant costs $18,250, and saves about $1630 for 10 hours' service, and $4885 for 24 hours' service, per year over a single plant costing $16,750. That is, the compound saves its extra cost in 10-hour service in about one year, or in 24-hour service in four months.

2. A triple 500 horse-power plant costs $20,500, and saves about $114 per year in 10-hour service, or $826 in 24-hour service, over a compound plant, thereby saving its extra cost in 10-hour service in about 19¾ years, or in 24-hour service in about 2¾ years.

Triple-expansion Pumping-engine at Milwaukee—Highest Economy on Record, 1893. (See paper on "Maximum Contemporary Economy of the Steam-engine," by R. H. Thurston, Trans. A. S. M. E., xv. 313.)—Cylinders 28, 48 and 74 in. by 60 in. stroke; ratios of volumes 1 to 3 to 7; total number of expansions 19.55; clearances, h.p. 1.4%; int. 1.5%; l. p. 0.77%; volume of receivers: 1st, 101.3 cu. ft.; 2d, 181 cu. ft.; steam-pressure gauge during test, average 121.5 lbs.; vacuum 13.84 lbs. absolute; revolutions 20.3 per minute; indicated horse-power, h.p. 175.4, int. 169.6, l. p. 228.9; total, 573.9; total friction, horse-power 52.91 = 9.22%; dry steam per I.H.P. per hour 11.678; B.T.U. per I.H.P. per min. 217.6; duty in foot-pounds per 100 lbs. of coal, 143,306,000; per million B.T.U., 137,656,000.

Steam per I.H.P. per hour, from diagram, at cut-off	9.35	9.12	8.37
" " " " " " " " release	10.1	10.0	8.92
Steam accounted for by indicator at cut-off, per cent	87.1	85.0	78.2
" " " " " " release, "	94.0	93.2	88.2
Per cent of total steam used by jackets	9.25		

Highest Economy of the Two-cylinder Compound Pumping-engines.—Repeated tests of the Pawtucket-Corliss engine, 15 and 30⅛ by 30 in. stroke, gave a water consumption of 13.69 to 14.16 lbs. per I.H.P. per hour. Steam-pressure 123 lbs.; revolutions per min. 48; expansions about 16. Cylinders jacketed. The lowest water rate was with jackets in use; both jackets supplied with steam of boiler pressure. The average saving due to jackets was only about 2⅛ per cent. (Trans. A. S. M. E., xi. 328 and 1038; xiii. 176.)

This record was beaten in 1894 by a Leavitt pumping-engine at Louisville, Ky. (Trans. A. S. M. E. xvi.) Cylinders 27.21 and 54.13 in. diam. by 10 ft stroke; revolutions per min. 18.57; piston speed 371.5 ft.; expansions 20.4; steam-pressure, gauge, 140 lbs. Cylinders and receiver jacketed. Steam used per I.H.P. per hour, 12.223 lbs. Duty per million B.T.U. = 138,126,000 ft.-lbs.

Test of a Triple-expansion Pumping-engine with and without Jackets, at Laketon, Ind., by Prof. J. E. Denton (Trans. A. S. M. E., xiv. 1340).—Cylinders 24, 34 and 54 in. by 36 in. stroke; 28 revs. per min.; H.P. developed about 320; boiler-pressure 150 lbs. Tests made on eight different days with different sets of conditions in jackets. At 150 lbs. boiler-pressure, and about 20 expansions, with any pressure above 48 lbs. in all of the jackets and reheaters, or with no pressure in the high jacket, the performance was as follows: With 2.5% of moisture in the steam entering the engine, the jackets used 16% of the total feed-water. About 20% of the latter was condensed during admission to the high cylinder. and about 13.85 lbs. of feed-water was consumed per hour per indicated horse-power. With no jackets or reheaters in action the feed-water consumption was 14.99 lbs., or 8.3% more than with jackets and reheaters. The consumption of lubricating oil was two thirds of a gallon of machine oil and one and three quarter gallons of cylinder oil per 24 hours. The friction of the engine in eight tests on different days varied from 5.1% to 8.7%.

If we regard the measurements of indicated horse-power and water as liable to an error of one per cent, which is probably a minimum allowance for the most careful determinations, the steam economy is the same for the following conditions:

(*a*) Any pressure from 48 to 131 in the intermediate and low jackets and receivers.

(*b*) Any pressure from 0 to 151 in the jacket of high cylinder.

(*c*) Any cut-off from 21% to 23% in high cylinder, from 39% to 43% in intermediate cylinder, from 40% to 53% in low cylinder.

Water Consumption of Three Types of Sulzer Engines.

(B. Donkin, Jr., *Eng'g*, Jan. 15, 1892, p. 77.)

SUMMARY AND AVERAGES OF TWENTY-ONE PUBLISHED EXPERIMENTS OF THE SULZER TYPE OF STEAM-ENGINE. ALL HORIZONTAL CONDENSING AND STEAM-JACKETED. From 1872 to 1891.

Type of Engine.	Steam-pressure above Atmosphere.	Piston-speed.	Indicated Horse-power.	Steam Consumption, pounds per I.H.P. per hour, including Steam-pipe water and Jacket Water.	Steam Consumption, pounds per I.H.P. per hour, exclud'g Steam-pipe water, but including Jacket Water.	Remarks.
	lbs.	ft. per min		lbs.	lbs.	
Single Cyl.	72 to 95	272 to 433	157 to 400	18.7 to 19 8 Mean 19.4	17.9 to 19.2 Mean 18.95	5 exp. 1872–78
Compound.	84 to 104	384 to 689	133 to 524	13.35 to 16.0 Mean 14.44	13.4 to 15.5 Mean 14.3	10 exp. 1888–91
Triple..	104 to 156	444 to 607	198 to 615	11.85 to 12.86 Mean 12.36	11.7 to 12.7 Mean 12.18	6 exp. 1888–89

Triple-expansion Corliss engine at Narragansett E. L. Co., Providence, R. I., built by E. P. Allis Co. Cylinder 14, 25 and 33 in. by 48 in. stroke tested at 99 revs. per min.; 125 lbs. steam-pressure; steam per I.H.P. per hour 12.91 lbs.; H.P. 516. A full account of this engine, with records of tests is given by J. T. Henthorn, in Trans. A. S. M. E., xii. 643.

Buckeye-cross compound engine, tested at Chicago Exposition, by Geo. H. Barrus (*Eng'g Record*, Feb. 17, 1894). Cylinder 14 and 28 by 24 in. stroke; tested at 165 r. p. m.; 120 lbs. steam-pressure. I.H.P. in four tests condensing and one non-condensing

I.H.P. in four tests condensing and one non-condensing	295	224	123	277	267
Steam per horse-power per hour	16.07	15.71	17.22	16.07	23.24

Relative Economy of Compound Non-condensing Engines under Variable Loads.—F. M. Rites, in a paper on the Steam Distribution in a Form of Single-acting Engine (Trans. A. S. M. E. xiii. 537), discusses an engine designed to meet the following problem: Given an extreme range of conditions as to load or steam-pressure, either or both, to fluctuate together or apart, violently or with easy gradations, to construct an engine whose economical performance should be as good as though the engine were specially designed for a momentary condition—the adjustment to be complete and automatic. In the ordinary non-condensing compound engine with light loads the high-pressure cylinder is frequently forced to supply all the power and in addition drag along with it the low-pressure piston, whose cylinder indicates negative work. Mr. Rites shows the peculiar value of a receiver of predetermined volume which acts as a clearance chamber for compression in the high-pressure cylinder. The Westinghouse compound single-acting engine is designed upon this principle. The following results of tests of one of these engines rated at 175 H.P. for most economical load are given:

WATER RATES UNDER VARYING LOADS, LBS. PER H.P. PER HOUR.

Horse-power	210	170	140	115	100	80	50
Non-condensing	22.6	21.9	22.2	22.2	22.4	24.6	28.8
Condensing	18.4	18.1	18.2	18.2	18.8	18.3	20.4

Efficiency of Non-condensing Compound Engines. (W. Lee Church, *Am. Mach.*, Nov. 19, 1891.)—The compound engine, non-condensing, at its best performance will exhaust from the low-pressure cylinder at a pressure 2 to 6 pounds above atmosphere. Such an engine will be limited in its economy to a very short range of power, for the reason that its valve-motion will not permit of any great increase beyond its rated power, and any material decrease below its rated power at once brings the expansion curve in the low-pressure cylinder below atmosphere. In other words, decrease of load tells upon the compound engine somewhat sooner, and much more severely, than upon the non-compound engine. The loss commences the moment the expansion line crosses a line parallel to the atmospheric line, and at a distance above it representing the mean effective pressure necessary to carry the frictional load of the engine. When expansion falls to this point the low-pressure cylinder becomes an air-pump over more or less of its stroke, the power to drive which must come from the high-pressure cylinder alone. Under the light loads common in many industries the low-pressure cylinder is thus a positive resistance for the greater portion of its stroke. A careful study of this problem revealed the functions of a fixed intermediate clearance, always in communication with the high-pressure cylinder, and having a volume bearing the same ratio to that of the high-pressure cylinder that the high-pressure cylinder bears to the low-pressure. Diagrams were laid out on this principle and justified until the best theoretical results were obtained. The designs were then laid down on these lines, and the subsequent performance of the engines, of which some 600 have been built, have fully confirmed the judgment of the designers.

The effect of this constant clearance is to supply sufficient steam to the low-pressure cylinder under light loads to hold its expansion curve up to atmosphere, and at the same time leave a sufficient clearance volume in the high-pressure cylinder to permit of governing the engine on its compression under light loads.

Economy of Engines under Varying Loads. (From Prof. W. C. Unwin's lecture before the Society of Arts, London, 1892.)—The general result of numerous trials with large engines was that with a constant load an indicated horse-power should be obtained with a consumption of 1½ pounds of coal per indicated horse-power for a condensing engine, and 1¾ pounds for a non-condensing engine, figures which correspond to about 1¾ pounds to 2½ pounds of coal per effective horse-power. It was much more difficult to ascertain the consumption of coal in ordinary every-day work, but such facts as were known showed it was more than on trial.

In electric-lighting stations the engines work under a very fluctuating load, and the results are far more unfavorable. An excellent Willans non-condensing engine, which on full-load trials worked with under 2 pounds per effective horse-power hour, in the ordinary daily working of the station used 7½ pounds per effective horse-power hour in 1886, which was reduced to 4.3 pounds in 1890 and 3.8 pounds in 1891. Probably in very few cases were the engines at electric-light stations working under a consumption of 4½ pounds per effective horse-power hour. In the case of small isolated motors working with a fluctuating load, still more extravagant results were obtained.

ENGINES IN ELECTRIC CENTRAL STATIONS.

Year	1886.	1890.	1892.
Coal used per hour per effective H.P.	8.4	5.6	4.9
" " " " " indicated "	6.5	4.35	3.8

At electric-lighting stations the load factor, viz., the ratio of the average load to the maximum, is extremely small, and the engines worked under very unfavorable conditions, which largely accounted for the excessive fuel consumption at these stations.

In steam-engines the fuel consumption has generally been reckoned on the indicated horse-power. At full-power trials this was satisfactory enough, as the internal friction is then usually a small fraction of the total.

Experiment has, however, shown that the internal friction is nearly constant, and hence, when the engine is lightly loaded, its mechanical efficiency is greatly reduced. At full load small engines have a mechanical efficiency of 0.8 to 0.85, and large engines might reach at least 0.9, but if the internal friction remained constant this efficiency would be much reduced at low powers. Thus, if an engine working at 100 indicated horse-power had an efficiency of 0.85, then when the indicated horse-power fell to 50 the effective horse-power would be 35 horse-power and the efficiency only 0.7. Similarly, at 25 horse-power the effective horse-power would be 10 and the efficiency 0.4.

Experiments on a Corliss engine at Creusot gave the following results:

Effective power at full load	1.0	0.75	0.50	0.25	0.125
Condensing, mechanical efficiency	0.82	0.79	0.74	0.63	0.48
Non condensing, " "	0.86	0.83	0.78	0.67	0.52

At light loads the economy of gas and liquid fuel engines fell off even more rapidly than in steam-engines. The engine friction was large and nearly constant, and in some cases the combustion was also less perfect at light loads. At the Dresden Central Station the gas-engines were kept working at nearly their full power by the use of storage-batteries. The results of some experiments are given below:

Brake-load, per cent of full Power.	Gas-engine, cu. ft. of Gas per Brake H.P. per hour.	Petroleum Eng., Lbs. of Oil per B.H.P. per hr.	Petroleum Eng., Lbs. of Oil per B.H.P. per hr.
100	22.2	0.96	0.88
75	23.8	1.11	0.99
50	28.0	1.44	1.20
20	40.8	2.38	1.82
12½	66.3	4.25	3.07

Steam Consumption of Engines of Various Sizes.—W. C. Unwin (Cassier's *Magazine*, 1894) gives a table showing results of 49 tests of engines of different types. In non-condensing simple engines, the steam consumption ranged from 65 lbs. per hour in a 5-horse-power engine to 29 lbs. in a 134-H.P. Harris-Corliss engine. In non-condensing compound engines, the only type tested was the Willans, which ranged from 27 lbs. in a 10 H.P. slow-speed engine, 122 ft. per minute, with steam-pressure of 84 lbs. to 19.2 lbs. in a 40-H.P. engine, 401 ft. per minute, with steam-pressure 165 lbs. A Willans triple-expansion non-condensing engine, 39 H.P., 172 lbs. pressure, and 400 ft. piston speed per minute, gave a consumption of 18.5 lbs. In condensing engines, nine tests of simple engines gave results ranging only from 18.4 to 22 lbs., and, leaving out a beam pumping-engine running at slow speed (240 ft. per minute) and low steam-pressure (45 lbs.), the range is only from 18.4 to 19.8 lbs. In compound-condensing engines over 100 H.P., in 13 tests the range is from 13.9 to 20 lbs. In three triple-expansion engines the figures are 11.7, 12.2, and 12.45 lbs., the lowest being a Sulzer engine of 360 H.P. In marine compound engines, the Fusiyama and Colchester, tested by Prof. Kennedy, gave steam consumption of 21.2 and 21.7 lbs.; and the Meteor and Tartar triple-expansion engines gave 15.0 and 19.8 lbs.

Taking the most favorable results which can be regarded as not exceptional, it appears that in test trials, with constant and full load, the expenditure of steam and coal is about as follows:

Kind of Engine.	Per Indicated Horse-power Hour. Coal, lbs.	Per Indicated Horse-power Hour. Steam, lbs.	Per Effective Horse-power Hour. Coal, lbs.	Per Effective Horse-power Hour. Steam, lbs.
Non-condensing	1.80	16.5	2.00	18.0
Condensing	1.50	13.5	1.75	15.8

These may be regarded as minimum values, rarely surpassed by the most efficient machinery, and only reached with very good machinery in the favorable conditions of a test trial.

Small Engines and Engines with Fluctuating Loads are usually very wasteful of fuel. The following figures, illustrating their low economy, are given by Prof. Unwin, Cassier's *Magazine*, 1894.

COAL CONSUMPTION PER INDICATED HORSE-POWER IN SMALL ENGINES.

In Workshops in Birmingham, Eng.

Probable I.H.P. at full load...	12	45	60	45	75	60	60
Average I.H.P. during observation..	2.96	7.37	8.2	8.6	23.64	19.08	20.08
Coal per I.H.P. per hour during observation, lbs.........	36.0	21.25	22.61	18.13	11.68	9.53	8.50

It is largely to replace such engines as the above that power will be distributed from central stations.

Steam Consumption in Small Engines.

Tests at Royal Agricultural Society's show at Plymouth, Eng. *Engineering*, June 27, 1890.

Rated H.P.	Compound or Simple.	Diam. of Cylinders.		Stroke, ins.	Max. Steam-pressure.	Per Brake H.P., per hour.		Water per lb. Coal.
		h.p.	l.p.			Coal.	Water.	
5	simple	7	.....	10	75	12.12	78.1 lbs.	6.1 lb.
3	compound	3	6	6	110	4.82	42.03 "	8.72 "
2	simple	4½	.....	7½	75	11.77	89.9 "	7.64 "

Steam-consumption of Engines at Various Speeds.

(Profs. Denton and Jacobus, Trans. A. S. M. E., x. 722)—17 × 30 in. engine, non-condensing, fixed cut-off, Meyer valve.

STEAM-CONSUMPTION, LBS. PER I.H.P. PER HOUR.

Figures taken from plotted diagram of results.

Revs. per min.....	8	12	16	20	24	32	40	48	56	72	88
⅛ cut-off, lbs.....	39	35	32	30	29.3	29	28.7	28.5	28.3	28	27.7
¼ " ".....	39	34	31	29.5	29	28.4	28	27.5	27.1	26.3	25.6
½ " ".....	39	36	34	33	32	30.8	29.8	29.2	28.8	28.7	....

STEAM-CONSUMPTION OF SAME ENGINE; FIXED SPEED, 60 REVS. PER MIN.

Varying cut-off compared with throttling-engine for same horse-power and boiler-pressures:

Cut-off, fraction of stroke	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.8
Boiler-pressure, 90 lbs...	29	27.5	27	27	27.2	27.8	28.5	...	...	...
" 60 lbs...	39	34.2	32.2	31.5	31.4	31.6	32.2	34.1	36.5	39

THROTTLING-ENGINE, ⅞ CUT-OFF, FOR CORRESPONDING HORSE-POWERS.

Boiler-pressure, 90 lbs...	42	37	33.8	31.5	29.8	....	....	...	..	...
" 60 lbs...	....	50.1	49	46.8	44.6	41	....	....	....	...

Some of the principal conclusions from this series of tests are as follows:

1. There is a distinct gain in economy of steam as the speed increases for ½, ⅛, and ¼ cut-off at 90 lbs. pressure. The loss in economy for about ½ cut-off is at the rate of 1/12 lb. of water per H.P. for each decrease of a revolution per minute from 86 to 26 revolutions, and at the rate of ⅝ lb. of water below 26 revolutions. Also, at all speeds the ¼ cut-off is more economical than either the ½ or ⅛ cut-off.

2. At 90 lbs. boiler-pressure and above ⅛ cut-off, to produce a given H.P. requires about 20% less steam than to cut off at ⅞ stroke and regulate by the throttle.

3. For the same conditions with 60 lbs. boiler-pressure, to obtain, by throttling, the same mean effective pressure at ⅞ cut-off that is obtained by

cutting off about 1/6, requires about 30% more steam than for the latter condition.

High Piston-speed in Engines. (Proc. Inst. M. E., July, 1883, p. 321).—The torpedo boat is an excellent example of the advance towards high speeds, and shows what can be accomplished by studying lightness and strength in combination. In running at 22½ knots an hour, an engine with cylinders of 16 in. stroke will make 480 revolutions per minute, which gives 1280 ft. per minute for piston-speed; and it is remarked that engines running at that high rate work much more smoothly than at lower speeds, and that the difficulty of lubrication diminishes as the speed increases.

A High-speed Corliss Engine.—A Corliss engine, 20 × 42 in., has been running a wire-rod mill at the Trenton Iron Co.'s works since 1877, at 160 revolutions or 1120 ft. piston-speed per minute (Trans. A. S. M. E., ii. 72). A piston-speed of 1200 ft. per min. has been realized in locomotive practice.

The Limitation of Engine-speed. (Chas. T. Porter, in a paper on the Limitation of Engine-speed, Trans. A. S. M. E., xiv. 806.)—The practical limitation to high rotative speed in stationary reciprocating steam-engines is not found in the danger of heating or of excessive wear, nor, as is generally believed, in the centrifugal force of the fly-wheel, nor in the tendency to knock in the centres, nor in vibration. He gives two objections to very high speeds: First, that "engines ought not to be run as fast as they can be;" second, the large amount of waste room in the port, which is required for proper steam distribution. In the important respect of economy of steam, the high-speed engine has thus far proved a failure. Large gain was looked for from high speed, because the loss by condensation on a given surface would be divided into a greater weight of steam, but this expectation has not been realized. For this unsatisfactory result we have to lay the blame chiefly on the excessive amount of waste room. The ordinary method of expressing the amount of waste room in the percentage added by it to the total piston displacement, is a misleading one. It should be expressed as the percentage which it adds to the length of steam admission. For example, if the steam is cut off at 1/5 of the stroke, 8% added by the waste room to the total piston displacement means 40% added to the volume of steam admitted. Engines of four, five and six feet stroke may properly be run at from 700 to 800 ft. of piston travel per minute, but for ordinary sizes, says Mr. Porter, 600 ft. per minute should be the limit.

Influence of the Steam-jacket.—Tests of numerous engines with and without steam-jackets show an exceeding diversity of results, ranging all the way from 30% saving down to zero, or even in some cases showing an actual loss. The opinions of engineers at this date (1894) is also as diverse as the results, but there is a tendency towards a general belief that the jacket is not as valuable an appendage to an engine as was formerly supposed. An extensive *résumé* of facts and opinions on the steam-jacket is given by Prof. Thurston, in Trans. A. S. M. E., xiv. 462. See also Trans. A. S. M. E., xiv. 873 and 1340; xiii. 176; xii. 426 and 1340; and Jour. F. I., April, 1891, p. 276. The following are a few statements selected from these papers.

The results of tests reported by the research committee on steam-jackets appointed by the British Institution of Mechanical Engineers in 1886, indicate an increased efficiency due to the use of the steam-jacket of from 1% to over 30%, according to varying circumstances.

Sennett asserts that "it has been abundantly proved that steam-jackets are not only advisable but absolutely necessary, in order that high rates of expansion may be efficiently carried out and the greatest possible economy of heat attained."

Isherwood finds the gain by its use, under the conditions of ordinary practice, as a general average, to be about 20% on small and 8% or 9% on large engines, varying through intermediate values with intermediate sizes, it being understood that the jacket has an effective circulation, and that both heads and sides are jacketed.

Professor Unwin considers that "in all cases and on all cylinders the jacket is useful; provided, of course, ordinary, not superheated, steam is used; but the advantages may diminish to an amount not worth the interest on extra cost."

Professor Cotterill says: Experience shows that a steam-jacket is advantageous, but the amount to be gained will vary according to circumstances. In many cases it may be that the advantage is small. Great caution is necessary in drawing conclusions from any special set of experiments on the influence of jacketing.

Mr. E. D. Leavitt has expressed the opinion that, in his practice, steam-jackets produce an increase of efficiency of from 15% to 20%.

In the Pawtucket pumping engine, 15 and 30⅛ × 30 in., 50 revs. per min., steam-pressure 125 lbs. gauge, cut-off ¼ in h.p. and ⅛ in l.p. cylinder, the barrels only jacketed, the saving by the jackets was from 1% to 4%.

The superintendent of the Holly Mfg. Co. (compound pumping-engines) says: "In regard to the benefits derived from steam-jackets on our steam-cylinders, I am somewhat of a skeptic. From data taken on our own engines and tests made I am yet to be convinced that there is any practical value in the steam-jacket." . . . "You might practically say that there is no difference."

Professor Schröter from his work on the triple-expansion engines at Augsburg, and from the results of his tests of the jacket efficiency on a small engine of the Sulzer type in his own laboratory, concludes: (1) The value of the jacket may vary within very wide limits, or even become negative. (2) The shorter the cut-off the greater the gain by the use of a jacket. (3) The use of higher pressure in the jacket than in the cylinder produces an advantage. The greater this difference the better. (4) The high-pressure cylinder may be left unjacketed without great loss, but the others should always be jacketed.

The test of the Laketon triple-expansion pumping-engine showed a gain of 8.3% by the use of the jackets, but Prof. Denton points out (Trans. A. S. M. E., xiv. 1412) that all but 1.9% of the gain was ascribable to the greater range of expansion used with the jackets.

Test of a Compound Condensing Engine with and without Jackets at different Loads. (R. C. Carpenter, Trans. A. S. M. E., xiv. 428.)—Cylinders 9 and 16 in.×14 in. stroke; 112 lbs. boiler-pressure; rated capacity 100 H.P.; 265 revs. per min. Vacuum, 23 in. From the results of several tests curves are plotted, from which the following principal figures are taken.

Indicated H.P.	30	40	50	60	70	80	90	100	110	120	125
Steam per I.H.P. per hour:											
With jackets, lbs.	22.6	21.4	20.3	19.6	19	18.7	18.6	18.9	19.5	20.4	21.0
Without jackets, lbs.	....	....	....	22.	20.5	19.6	19.2	19.1	19.3	20.1	....
Saving by jacket, p. c.	....	....	....	10.9	7.3	4.6	3.1	1.0	−1.0	−1.5	....

This table gives a clue to the great variation in the apparent saving due to the steam-jacket as reported by different experimenters. With this particular engine it appears that when running at its most economical rate of 100 H.P., without jackets, very little saving is made by use of the jackets. When running light the jacket makes a considerable saving, but when overloaded it is a detriment.

At the load which corresponds to the most economical rate, with no steam in jackets, or 100 H.P., the use of the jacket makes a saving of only 1%; but at a load of 60 H.P. the saving by use of the jacket is about 11%, and the shape of the curve indicates that the relative advantage of the jacket would be still greater at lighter loads than 60 H.P.

Counterbalancing Engines.—Prof. Unwin gives the formula for counterbalancing vertical engines:

$$W_1 = W_2\frac{r}{p}; \quad \ldots\ldots\ldots \quad (1)$$

in which W_1 denotes the weight of the balance weight and p the radius to its centre of gravity, W_2 the weight of the crank-pin and half the weight of the connecting-rod, and r the length of the crank. For horizontal engines:

$$W_1 = \tfrac{2}{3}(W_2 + W_3)\frac{r}{p} \quad \text{to} \quad \tfrac{3}{4}(W_2 + W_3)\frac{r}{p}, \quad \ldots\ldots \quad (2)$$

in which W_3 denotes the weight of the piston, piston-rod, cross-head, and the other half of the weight of the connecting-rod.

The *American Machinist*, commenting on these formulæ, says: For horizontal engines formula (2) is often used; formula (1) will give a counterbalance too light for vertical engines. We should use formula (2) for computing the counterbalance for both horizontal and vertical engines, excepting locomotives, in which the counterbalance should be heavier.

Preventing Vibrations of Engines.—Many suggestions have been made for remedying the vibration and noise attendant on the working of the big engines which are employed to run dynamos. A plan which has given great satisfaction is to build hair-felt into the foundations of the engine. An electric company has had a 90-horse-power engine removed from its foundations, which were then taken up to the depth of 4 feet. A layer of felt 5 inches thick was then placed on the foundations and run up 2 feet on all sides, and on the top of this the brickwork was built up.—*Safety Valve.*

Steam-engine Foundations Embedded in Air.—In the sugar-refinery of Claus Spreckels, at Philadelphia, Pa., the engines are distributed practically all over the buildings, a large proportion of them being on upper floors. Some are bolted to iron beams or girders, and are consequently innocent of all foundation. Some of these engines ran noiselessly and satisfactorily, while others produced more or less vibration and rattle. To correct the latter the engineers suspended foundations from the bottoms of the engines, so that, in looking at them from the lower floors, they were literally hanging in the air.—*Iron Age*, Mar. 13, 1890.

Cost of Coal for Steam-power.—The following table shows the amount and the cost of coal per day and per year for various horse-powers, from 1 to 1000, based on the assumption of 4 lbs. of coal being used per hour per horse-power. It is useful, among other things, in estimating the saving that may be made in fuel by substituting more economical boilers and engines for those already in use. Thus with coal at $3.00 per ton, a saving of $9000 per year in fuel may be made by replacing a steam plant of 1000 H.P., requiring 4 lbs. of coal per hour per horse-power, with one requiring only 2 lbs.

Horse-power.	Coal Consumption, at 4 lbs. per H.P. per hour; 10 hours a day; 300 days in a Year.					$1.50.		$2.00.		$3.00.		$4.00.	
	Lbs.	Long Tons.		Short Tons.		Per Short Ton.		Per Short Ton.		Per Short Ton.		Per Short Ton.	
						Cost in Dollars.		Cost in Dollars.		Cost in Dollars.		Cost in Dollars.	
	Per Day.	Per Day.	Per Year.	Per Day.	Per Year	Per Day.	Per Year	Per Day.	Per Year.	Per Day.	Per Year.	Per Day.	Per Year
1	40	.0179	5.357	.02	6	.03	9	.04	12	.06	18	.08	24
10	400	.1786	53.57	.20	60	.30	90	.40	120	.60	180	.80	240
25	1,000	.4464	133.92	.50	150	.75	225	1.00	300	1.50	450	2.00	600
56	2,000	.8928	267.85	1.00	300	1.50	450	2.00	600	3.00	900	4.00	1,200
75	3,000	1.3393	401.78	1.50	450	2.25	675	3.00	900	4.50	1,350	6.00	1,800
100	4,000	1.7857	535.71	2.00	600	3.00	900	4.00	1,200	6.00	1,800	8.00	2,400
150	6,000	2.6785	803.56	3.00	900	4.50	1,350	6.00	1,800	9.00	2,700	12.00	3,600
200	8,000	3.5714	1,071.42	4.00	1,200	6.00	1,800	8.00	2,400	12.00	3,600	16.00	4,800
250	10,000	4.4642	1,339.27	5.00	1,500	7.50	2,250	10.00	3,000	15.00	4,500	20.00	6,000
300	12,000	5.3571	1,607.13	6.00	1,800	9.00	2,700	12.00	3,600	18.00	5,400	24.00	7,200
350	14,000	6.2500	1,874.98	7.00	2,100	10.50	3,150	14.00	4,200	21.00	6,300	28.00	8,400
400	16,000	7.1428	2,142.84	8.00	2,400	12.00	3,600	16.00	4,800	24.00	7,200	32.00	9,600
450	18,000	8.0356	2,410.69	9.00	2,700	13.50	4,050	18.00	5,400	27.00	8,100	36.00	10,800
500	20,000	8.9285	2,678.55	10.00	3,000	15.00	4,500	20.00	6,000	30.00	9,000	40.00	12,000
600	24,000	10.7142	3,214.26	12.00	3,600	18.00	5,400	24.00	7,200	36.00	10,800	48.00	14,400
700	28,000	12.4999	3,749.97	14.00	4,200	21.00	6,300	28.00	8,400	42.00	11,600	56.00	16,800
800	32,000	14.2856	4,285.68	16.00	4,800	24.00	7,200	32.00	9,600	48.00	12,400	64.00	19,200
900	36,000	16.0713	4,821.39	18.00	5,400	27.00	8,100	36.00	10,800	54.00	14,200	72.00	21,600
1,000	40,000	17.8570	5,357.10	20.00	6,000	30.00	9,000	40.00	12,000	60.00	18,000	80.00	24,000

Storing Steam Heat.—There is no satisfactory method for equalizing the load on the engines and boilers in electric-light stations. Storage-batteries have been used, but they are expensive in first cost, repairs, and attention. Mr. Halpin, of London, proposes to store heat during the day in specially constructed reservoirs. As the water in the boilers is raised to 250 lbs. pressure, it is conducted to cylindrical reservoirs resembling English horizontal boilers, and stored there for use when wanted. In this way a comparatively small boiler-plant can be used for heating the water to 250 lbs. pressure all through the twenty-four hours of the day, and the stored water may be drawn on at any time, according to the magnitude of the demand. The steam-engines are to be worked by the steam generated by the release of pressure from this water, and the valves are to be arranged in such a way that the steam shall work at 130 lbs. pressure. A reservoir 8 ft. in diameter and 30 ft. long, containing 84,000 lbs. of heated water at 250 lbs. pressure, would supply 5250 lbs. of steam at 130 lbs. pressure. As the steam consumption of a condensing electric-light engine is about 18 lbs. per horse-power hour, such a reservoir would supply 286 effective horse-power hours. In 1878, in France, this method of storing steam was used on a tramway. M. Francq, the engineer, designed a smokeless locomotive to work by steam-power supplied by a reservoir containing 400 gallons of water at 220 lbs. pressure. The reservoir was charged with steam from a stationary boiler at one end of the tramway.

Cost of Steam-power. (Chas. T. Main, A. S. M. E., x. 48.)—Estimated costs in New England in 1888, per horse-power, based on engines of 1000 H.P.

	Compound Engine.	Condensing Engine.	Non-condensing Engine.
1. Cost engine and piping, complete	$25.00	$20.00	$17.50
2. Engine-house	8.00	7.50	7.50
3. Engine foundations	7.00	5.50	4.50
4. Total engine plant	40.00	33.00	29.50
5. Depreciation, 4% on total cost	1.60	1.32	1.18
6. Repairs, 2% " " "	0.80	0.66	0.59
7. Interest, 5% " " "	2.00	1.65	1.475
8. Taxation, 1.5% on ¾ cost	0.45	0.371	0.332
9. Insurance on engine and house	0.165	0.138	0.125
10. Total of lines 5, 6, 7, 8, 9	5.015	4.139	3.702
11. Cost boilers, feed-pumps, etc.	9.33	13.33	16.00
12. Boiler-house	2.92	4.17	5.00
13. Chimney and flues	6.11	7.30	8.00
14. Total boiler-plant	18.36	24.80	29.00
15. Depreciation, 5% on total cost	0.918	1.240	1.450
16. Repairs, 2% " " "	.367	.496	.580
17. Interest, 5% " " "	.918	1.240	1.450
18. Taxation, 1.5% on ¾ cost	.207	.279	.326
19. Insurance, 0.5% on total cost	.092	.124	.145
20. Total of lines 15 to 19	2.502	3.379	3.951
21. Coal used per I.H.P. per hour, lbs	1.75	2.50	3.00
22. Cost of coal per I.H.P. per day of 10¼ hours at $5.00 per ton of 2240 lbs	cts. 4.00	cts. 5.72	cts. 6.86
23. Attendance of engine per day	0.60	0.40	0.35
24. " " boilers " "	0.53	0.75	0.90
25. Oil, waste, and supplies, per day	0.25	0.22	0.20
26. Total daily expense	5.38	7.09	8.31
27. Yearly running expense, 308 days, per I.H.P	$16.570	$21.837	$25.595
28. Total yearly expense, lines 10, 20, and 27	24.087	29.355	33.248
29. Total yearly expense per I.H.P. for power if 50% of exhaust-steam is used for heating	12.597	14.907	16.663
30. Total if all ex.-steam is used for heating	8.624	7.916	7.700

When exhaust-steam or a part of the receiver-steam is used for heating, or if part of the steam in a condensing engine is diverted from the condenser, and used for other purposes than power, the value of such steam should be deducted from the cost of the total amount of steam generated in order to arrive at the cost properly chargeable to power. The figures in lines 29 and 30 are based on an assumption made by Mr. Main of losses of heat amounting to 25% between the boiler and the exhaust-pipe, an allowance which is probably too large.

See also two papers by Chas. E. Emery on "Cost of Steam Power," Trans. A. S. C. E., vol. xii, Nov. 1883, and Trans. A. I. E. E., vol. x, Mar. 1893.

ROTARY STEAM-ENGINES.

Steam Turbines.—The steam turbine is a small turbine wheel which runs with steam as the ordinary turbine does with water. (For description of the Parsons and the Dow steam turbines see Modern Mechanism, p. 298, etc.) The Parsons turbine is a series of parallel-flow turbines mounted side by side on a shaft; the Dow turbine is a series of radial outward-flow turbines, placed like a series of concentric rings in a single plane, a stationary guide-ring being between each pair of movable rings. The speeds of the steam turbines enormously exceed those of any form of engine with reciprocating piston, or even of the so-called rotary engines. The three- and four-cylinder engines of the Brotherhood type, in which the several cylinders are usually grouped radially about a common crank and shaft, often exceed 1000 revolutions per minute, and have been driven, experimentally, above 2000; but the steam turbine of Parsons makes 10,000 and even 20,000 revolutions, and the Dow turbine is reputed to have attained 25,000. (See Trans. A. S. M. E., vol. x. p. 680, and xii. p. 888; Trans. Assoc. of Eng'g Societies, vol. viii. p. 583; *Eng'g*, Jan. 13, 1888, and Jan. 8, 1892; *Eng'g News*, Feb. 27, 1892.) A Dow turbine, exhibited in 1889, weighed 68 lbs., and developed 10 H.P., with a consumption of 47 lbs. of steam per H.P. per hour, the steam pressure being 70 lbs. The Dow turbine is used to spin the fly-wheel of the Howell torpedo. The dimensions of the wheel are 13.8 in. diam., 0.5 in. width, radius of gyration 5.57 in. The energy stored in it at 10,000 revs. per min. is 500,000 ft.-lbs.

The De Laval Steam Turbine, shown at the Chicago exhibition, 1893, is a reaction wheel somewhat similar to the Pelton water-wheel. The steam jet is directed by a nozzle against the plane of the turbine at quite a small angle and tangentially against the circumference of the medium periphery of the blades. The angle of the blades is the same at the side of admission and discharge. The width of the blade is constant along the entire thickness of the turbine.

The steam is expanded to the pressure of the surroundings before arriving at the blades. This expansion takes place in the nozzle, and is caused simply by making its sides diverging. As the steam passes through this channel its specific volume is increased in a greater proportion than the cross section of the channel, and for this reason its velocity is increased, and also its momentum, till the end of the expansion at the last sectional area of the nozzle. The greater the expansion in the nozzle the greater its velocity at this point. A pressure of 75 lbs. and expansion to an absolute pressure of one atmosphere give a final velocity of about 2625 ft. per second.

Expansion is carried further in this steam turbine than in ordinary steam-engines. This is on account of the steam expanding completely during its work to the pressure of the surroundings.

For obtaining the greatest possible effect the admission to the blades must be free from blows and the velocity of discharge as low as possible. These conditions would require in the steam turbine an enormous velocity of periphery—as high as 1300 to 1650 ft. per second. The centrifugal force, nevertheless, puts a limit to the use of very high velocities. In the 5 horse-power turbine the velocity of periphery is 574 ft. per second, and the number of revolutions 30,000 per minute.

However carefully the turbine may be manufactured it is impossible, on account of unevenness of the material, to get its centre of gravity to correspond exactly to its geometrical axle of revolution; and however small this difference may be, it becomes very noticeable at such high velocities. De Laval has succeeded in solving the problem by providing the turbine with a flexible shaft. This yielding shaft allows the turbine at the high rate of speed to adjust itself and revolve around its true centre of gravity, the centre line of the shaft meanwhile describing a surface of revolution.

In the gearing-box the speed is reduced from 30,000 revolutions to 3000 by means of a driver on the turbine shafts, which sets in motion a cog-wheel of 10 times its own diameter. These gearings are provided with spiral cogs placed at an angle of about 45°. The shaft of the larger cog-wheel, running at a speed of 3000 revolutions, is provided at its outer end with a pulley for the further transmission of the power.

Rotary Steam-engines, other than steam turbines, have been invented by the thousands, but not one has attained a commercial success. The possible advantages, such as saving of space, to be gained by a rotary engine are overbalanced by its waste of steam.

The Tower Spherical Engine, one of the most recent forms of rotary-engine, is described in Proc. Inst. M. E., 1885, also in Modern Mechanism, p. 296.

DIMENSIONS OF PARTS OF ENGINES.

The treatment of this subject by the leading authorities on the steam-engine is very unsatisfactory, being a confused mass of rules and formulæ based partly upon theory and partly upon practice. The practice of builders shows an exceeding diversity of opinion as to correct dimensions. The treatment given below is chiefly the result of a study of the works of Rankine, Seaton, Unwin, Thurston, Marks, and Whitham, and is largely a condensation of a series of articles by the author published in the *American Machinist*, in 1894, with many alterations and much additional matter. In order to make a comparison of many of the formulæ they have been applied to the assumed cases of six engines of different sizes, and in some cases this comparison has led to the construction of new formulæ.

Cylinder. (Whitham.)—Length of bore = stroke + breadth of piston-ring − 1/8 to 1/4 in; length between heads = stroke + thickness of piston + sum of clearances at both ends; thickness of piston = breadth of ring + thickness of flange on one side to carry the ring + thickness of follower-plate.

Thickness of flange or follower....	3/8 to 1/2 in.	3/4 in.	1 in.
For cylinder of diameter...........	8 to 10 in.	36 in.	60 to 100 in.

Clearance of Piston. (Seaton.)—The clearance allowed varies with the size of the engine from 1/8 to 3/8 in. for roughness of castings and 1/16 to 1/8 in. for each working joint. Naval and other very fast-running engines have a larger allowance. In a vertical direct-acting engine the parts which wear so as to bring the piston nearer the bottom are three, viz., the shaft journals, the crank-pin brasses, and piston-rod gudgeon-brasses.

Thickness of Cylinder. (Thurston.)—For engines of the older types and under moderate steam-pressures, some builders have for many years restricted the stress to about 2550 lbs. per sq. in.

$$t = ap_1D + b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is a common proportion; t, D, and b being thickness, diam., and a constant added quantity varying from 0 to 1/2 in., all in inches; p_1 is the initial unbalanced steam-pressure per sq. in. In this expression b is made larger for horizontal than for vertical cylinders, as, for example, in large engines 0.5 in the one case and 0.2 in the other, the one requiring re-boring more than the other. The constant a is from 0.0004 to 0.0005; the first value for vertical cylinders, or short strokes; the second for horizontal engines, or for long strokes.

Thickness of Cylinder and its Connections for Marine Engines. (Seaton).—D = the diam. of the cylinder in inches; p = load on the safety-valves in lbs. per sq. in.; f, a constant multiplier = thickness of barrel + .25 in.

Thickness of metal of cylinder barrel or liner, not to be less than $p \times D \div 8000$ when of cast iron.* . . . . (2)

$$\text{Thickness of cylinder-barrel} = \frac{p \times D}{5000}^{*} + 0.6 \text{ in.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{" " liner} = 1.1 \times f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thickness of liner when of steel $p \times D \div 6000 + 0.5$
" metal of steam-ports $= 0.6 \times f$.
" " valve-box sides $= 0.65 \times f$.

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* When made of exceedingly good material, at least twice melted, the thickness may be 0.8 of that given by the above rules.

Thickness of metal of valve-box covers $= 0.7 \times f$.
" " cylinder bottom $= 1.1 \times f$, if single thickness.
" " " " $= 0.65 \times f$, if double "
" " " covers $= 1.0 \times f$, if single "
" " " " $= 0.6 \times f$, if double "
" cylinder flange $= 1.4 \times f$.
" " cover-flange $= 1.3 \times f$.
" " valve-box " $= 1.0 \times f$.
" " door-flange $= 0.9 \times f$.
" " face over ports $= 1.2 \times f$.
" " " " $= 1.0 \times f$, when there is a false-face.
" " false-face $= 0.8 \times f$, when cast iron.
" " " $= 0.6 \times f$, when steel or bronze.

Whitham gives the following from different authorities:

Van Buren: $\begin{cases} t = 0.0001Dp + 0.15\sqrt{D}; & (5) \\ t = 0.03\sqrt{Dp}. & (6) \end{cases}$

Tredgold: $t = \dfrac{(D + 2.5)p}{1900}$. . . . . (7)

Weisbach: $t = 0.8 + 0.00033pD$. . . . . (8)

Seaton: $t = 0.5 + 0.0004pD$. . . . . (9)

Haswell: $\begin{cases} t = 0.0004pD + \frac{1}{4} \text{ (vertical)}; & (10) \\ t = 0.0005pD + \frac{1}{4} \text{ (horizontal)}. & (11) \end{cases}$

Whitham recommends (6) where provision is made for the reboring, and where ample strength and rigidity are secured, for horizontal or vertical cylinders of large or small diameter; (9) for large cylinders using steam under 100 lbs. gauge-pressure, and

$t = 0.003D\sqrt{p}$ for small cylinders. . . . . (12)

Marks gives $t = 0.00028pD$. . . . . (13)

This is a smaller value than is given by the other formulæ quoted; but Marks says that it is not advisable to make a steam-cylinder less than 0.75 in. thick under any circumstances.

The following table gives the calculated thickness of cylinders of engines of 10, 30, and 50 in. diam., assuming p the maximum unbalanced pressure on the piston = 100 lbs. per sq. in. As the same engines will be used for calculation of other dimensions, other particulars concerning them are here given for reference.

DIMENSIONS, ETC., OF ENGINES.

Engine No.	1 and 2.		3 and 4.		5 and 6.	
Indicated horse-power . . . . . . I.H.P.	50		450		1250	
Diam. of cyl., in . . . . D	10		30		50	
Stroke, feet L	1	2	2½	5	4	8
Revs. per min. . . . . r	250	125	130	65	90	45
Piston speed, ft. per min. . . . . S	500		650		700	
Area of piston, sq. in . . . . . . a	78.54		706.86		1963.5	
Mean effective pressure . . . M.E.P.	42		32.3		30	
Max. total unbalanced press. . . . . P	7854		70,686		196,350	
Max. total per sq. in. . . . . . . p	100		100		100	

THICKNESS OF CYLINDER BY FORMULA.	1 and 2.	3 and 4.	5 and 6.
(1) $.0004pD + 0.5$, short stroke...	.90	1.70	2.50
(1) $.0005pD + 0.5$, long stroke....	1.00	2.00	3.00
(2) $.00033pD$....................	.33	.99	1.67
(3) $.0002pD + 0.6$	.80	1.40	1.60
(5) $.0001pD + .15\sqrt{D}$	.57	1.12	1.56
(6) $.03\sqrt{Dp}$....................	.95	1.64	2.12
(7) $\frac{(D + 2.5)}{1900}p$....................	.66	1.71	2.76
(8) $.00033pD + 0.8$..................	1.13	1.79	2.45
(9) $.0004pD + 0.5$..................	.90	1.70	2.50
(10) $.0004pD + \frac{1}{8}$ (vertical)	.53	1.33	2.13
(11) $.0005pD + \frac{1}{8}$ (horizontal).....	.63	1.63	2.63
(12) $.003D\sqrt{p}$ (small engines)......	.30(?)	....	....
(13) $.00028pD$....................	.28(?)	.84(?)	1.40(?)
Average of first eleven.	.76	1.48	2.26

The average corresponds nearly to the formula $t = .00037Dp + 0.4$ in. A convenient approximation is $t = .0004Dp + 0.3$ in., which gives for

Diameters.............	10	20	30	40	50	60 in.
Thicknesses...........	.70	1.10	1.50	1.90	2.30	2.70 in.

The last formula corresponds to a tensile strength of cast iron of 12,500 lbs., with a factor of safety of 10 and an allowance of 0.3 in. for reboring.

Cylinder-heads.—Thurston says: Cylinder-heads may be given a thickness, at the edges and in the flanges, exceeding somewhat that of the cylinder. An excess of not less than 25% is usual. It may be thinner in the middle. Where made, as is usual in large engines, of two disks with intermediate radiating, connecting ribs or webs, that section which is safe against shearing is probably ample. An examination of the designs of experienced builders, by Professor Thurston, gave

$$t = \frac{Dp}{3000} + \frac{1}{4} \text{ inch}, \qquad (1)$$

D being the diameter of that circle in which the thickness is taken.

Thurston also gives $\quad t = .005D\sqrt{p} + 0.25. \qquad (2)$

Marks gives $\quad t = 0.003D\sqrt{p}. \qquad (3)$

He also says a good practical rule for pressures under 100 lbs. per sq. in. is to make the thickness of the cylinder-heads $1\frac{1}{4}$ times that of the walls; and applying this factor to his formula for thickness of walls, or $.00028pD$, we have

$$t = .00035pD. \qquad (4)$$

Whitham quotes from Seaton,

$$t = \frac{pD + 500}{2000}, \text{ which is equal to } .0005pD + .25 \text{ inch.} \qquad (5)$$

Seaton's formula for cylinder bottoms, quoted above, is

$$t = 1.1f, \text{ in which } f = .0002pD + .85 \text{ inch, or } t = .00022pD + .93. \qquad (6)$$

Applying the above formulæ to the engines of 10, 30, and 50 inches diameter, with maximum unbalanced steam-pressure of 100 lbs. per sq. in., we have

Cylinder diameter, inches =	10	30	50
(1) $t = .00033Dp + .25$ =	.58	1.25	1.82
(2) $t = .005D\sqrt{p} + .25$ =	.75	1.75	2.75
(3) $t = .003D\sqrt{p}$ =	.30	.90	1.50
(4) $t = .00035Dp$ =	.35	1.05	1.75
(5) $t = .0005Dp + .25$ =	.75	1.75	2.75
(6) $t = .00022Dp + .93$ =	1.15	1.59	2.03
Average of 6	.65	1.38	2.10

The average is expressed by the formula $t = .00066Dp + .31$ inch.

Meyer's "Modern Locomotive Construction," p. 24, gives for locomotive cylinder-heads for pressures up to 120 lbs.:

For diameters, in.........	19 to 22	16 to 18	14 to 15	11 to 13	9 to 10
Thickness, in...	$1\frac{1}{4}$	1	1	$\frac{7}{8}$	$\frac{3}{4}$

Taking the pressure at 120 lbs. per sq. in., the thicknesses $1\frac{1}{4}$ in. and $\frac{3}{4}$ in. for cylinders 22 and 10 in. diam., respectively, correspond to the formula $t = .00035Dp + .33$ inch.

Web-stiffened Cylinder-covers.—Seaton objects to webs for stiffening cast-iron cylinder-covers as a source of danger. The strain on the web is one of tension, and if there should be a nick or defect in the outer edge of the web the sudden application of strain is apt to start a crack. He recommends that high-pressure cylinders over 24 in. and low-pressure cylinders over 40 in. diam. should have their covers cast hollow, with two thicknesses of metal. The depth of the cover at the middle should be about $\frac{1}{4}$ the diam. of the piston for pressures of 80 lbs. and upwards, and that of the low-pressure cylinder-cover of a compound engine equal to that of the high-pressure cylinder. Another rule is to make the depth at the middle not less than 1.3 times the diameter of the piston-rod. In the British Navy the cylinder-covers are made of steel castings, $\frac{3}{4}$ to $1\frac{1}{4}$ in. thick, generally cast without webs, stiffness being obtained by their form, which is often a series of corrugations.

Cylinder-head Bolts.—Diameter of bolt-circle for cylinder-head = diameter of cylinder + 2 × thickness of cylinder + 2 × diameter of bolts. The bolts should not be more than 6 inches apart (Whitham).

Marks gives for number of bolts $b = \frac{.7854D^2p}{5000c} = .0001571\frac{D^2p}{c}$, in which c = area of a single bolt, p = boiler-pressure in lbs. per sq. in.; 5000 lbs. is taken as the safe strain per sq. in. on the nominal area of the bolt.

Seaton says: Cylinder-cover studs and bolts, when made of steel, should be of such a size that the strain in them does not exceed 5000 lbs. per sq. in. When of less than $\frac{7}{8}$ inch diameter it should not exceed 4500 lbs. per sq. in. When of iron the strain should be 20% less.

Thurston says: Cylinder flanges are made a little thicker than the cylinder, and usually of equal thickness with the flanges of the heads. Cylinder-bolts should be so closely spaced as not to allow springing of the flanges and leakage, say, 4 to 5 times the thickness of the flanges. Their diameter should be proportioned for a maximum stress of not over 4000 to 5000 lbs. per square inch.

If D = diameter of cylinder, p = maximum steam-pressure, b = number of bolts, s = size or diameter of each bolt, and 5000 lbs. be allowed per sq. in. of nominal area of the bolt, $.7854D^2p = 3927bs^2$; whence $bs^2 = .0002D^2p$; $b = .0002\frac{D^2p}{s^2}$; $s = .01414D\sqrt{\frac{p}{b}}$. For the three engines we have:

Diameter of cylinder, inches........	10	30	50
Diameter of bolt-circle, approx.....	13	35	57.5
Circumference of circle, approx....	40.8	110	180
Minimum No. of bolts, circ. ÷ 6.....	7	18	30
Diam. of bolts, $s = .01414D\sqrt{\frac{p}{b}}$......	$\frac{3}{4}$ in.	1.00	1.20

The diameter of bolt for the 10-inch cylinder is 0.54 in. by the formula, but $\frac{3}{4}$ inch is as small as should be taken, on account of possible overstrain by the wrench in screwing up the nut.

The Piston. Details of Construction of Ordinary Pistons. (Seaton.)—Let D be the diameter of the piston in inches, p the effective pressure per square inch on it, x a constant multiplier, found as follows:

$$x = \frac{D}{50} \times \sqrt{p} + 1,$$

The thickness of front of piston near the boss	$= 0.2 \times x$
" " " " rim	$= 0.17 \times x$.
" back "	$= 0.16 \times x$.
" boss around the rod	$= 0.3 \times x$.
" flange inside packing-ring	$= 0.23 \times x$.
" " at edge	$= 0.25 \times x$.
" packing-ring	$= 0.15 \times x$.
" junk-ring at edge	$= 0.23 \times x$.
" " inside packing-ring	$= 0.21 \times x$.
" " at bolt-holes	$= 0.35 \times x$.
" metal around piston edge	$= 0.25 \times x$.
The breadth of packing-ring	$= 0.63 \times x$.
" depth of piston at centre	$= 1.4 \times x$.
" lap of junk-ring on the piston	$= 0.45 \times x$.
" space between piston body and packing-ring	$= 0.3 \times x$.
" diameter of junk-ring bolts	$= 0.1 \times x + 0.25$ in.
" pitch " " "	= 10 diameters.
" number of webs in the piston	$= (D + 20) \div 12$.
" thickness " " "	$= 0.18 \times x$.

Marks gives the approximate rule: Thickness of piston-head $= \sqrt[4]{ld}$, in which l = length of stroke, and d = diameter of cylinder in inches. Whitham says in a horizontal engine the rings support the piston, or at least a part of it, under ordinary conditions. The pressure due to the weight of the piston upon an area equal to 0.7 the diameter of the cylinder $\times$ breadth of ring-face should never exceed 200 lbs. per sq. in. He also gives a formula much used in this country: Breadth of ring-face = 0.15 $\times$ diameter of cylinder.

For our engines we have diameter = ...	10	30	50
	Thickness of piston-head.		
Marks, $\sqrt[4]{lD}$; long stroke..	3.31	5.48	7.00
Marks, " ; short stroke.............................	3.94	6.51	8.32
Seaton, depth at centre = $1.4x$....................	4.30	9.80	15.40
Seaton, breadth of ring = $.63x$.....................	1.89	4.41	6.93
Whitham, breadth of ring = $.15D$...................	1.50	4.50	7.50

Diameter of Piston Packing-rings. — These are generally turned, before they are cut, about ¼ inch diameter larger than the cylinder, for cylinders up to 20 inches diameter, and then enough is cut out of the ring to spring them to the diameter of the cylinder. For larger cylinders the rings are turned proportionately larger. Seaton recommends an excess of 1% of the diameter of the cylinder.

Cross-section of the Rings.—The thickness is commonly made 1/30th of the diam. of cyl. + ⅛ inch, and the width = thickness + ⅛ inch. For an eccentric ring the mean thickness may be the same as for a ring of uniform thickness, and the minimum thickness = ⅔ the maximum.

A circular issued by J. H. Dunbar, manufacturer of packing-rings, Youngstown, O., says: Unless otherwise ordered, the thickness of rings will be made equal to .03 $\times$ their diameter. This thickness has been found to be satisfactory in practice. It admits of the ring being made about 3/16" to the foot larger than the cylinder, and has, when new, a tension of about two pounds per inch of circumference, which is ample to prevent leakage, if the surface of the ring and cylinder are smooth.

As regards the width of rings, authorities "scatter" from very narrow to very wide, the latter being fully ten times the former. For instance, Unwin gives $W = d\ .014 + .08$. Whitham's formula is $W = d\ .15$. In both formulæ W is the width of the ring in inches, and d the diameter of the cylinder in inches. Unwin's formula makes the width of a 20" ring $W = 20 \times .014 + .08 = .36"$, while Whitham's is $20 \times .15 = 3"$ for the same diameter of ring. There is much less difference in the practice of engine-builders in this respect, but there is still room for a standard width of ring. It is believed that for cylinders over 16" diameter ¾" is a popular and practical width, and ½" for cylinders of that size and under.

Fit of Piston-rod into Piston. (Seaton.)—The most convenient and reliable practice is to turn the piston-rod end with a shoulder of 1/16 inch for small engines, and ⅛ inch for large ones, make the taper 3 in. to the foot until the section of the rod is three fourths of that of the body, then turn the remaining part parallel; the rod should then fit into the piston so as to leave ¼ inch between it and the shoulder for large pistons, and 1/16 in. for small. The shoulder prevents the rod from splitting the piston, and allows of the rod being turned true after long wear without encroaching on the taper.

The piston is secured to the rod by a nut, and the size of the rod should be such that the strain on the section at the bottom of the thread does not exceed 5500 lbs. per sq. in. for iron, 7000 lbs. for steel. The depth of this nut need not exceed the diameter which would be found by allowing these strains. The nut should be locked to prevent its working loose.

Diameter of Piston-rods.—Unwin gives

$$d'' = bD\sqrt{p}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which D is the cylinder diameter in inches, p is the maximum unbalanced pressure in lbs. per sq. in., and the constant $b = 0.0167$ for iron, and $b = 0.0144$ for steel. Thurston, from an examination of a considerable number of rods in use, gives

$$d'' = \sqrt[4]{\frac{D^2pL^2}{a}} + \frac{D}{80}, \text{ nearly}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(L in feet, D and d in inches), in which $a = 10,000$ and upward in the various types of engines, the marine screw engines or ordinary fast engines on shore given the lowest values, while "low-speed engines" being less liable to accident from shock give $a = 15,000$, often.

Connections of the piston-rod to the piston and to the crosshead should have a factor of safety of at least 8 or 10. Marks gives

$$d'' = 0.0179D\sqrt{p}, \text{ for iron}; \quad \text{for steel } d'' = 0.0105D\sqrt{p}; \quad . \quad . \quad (3)$$

$$\text{and } d'' = 0.03901\sqrt[4]{D^2l^2p}, \text{ for iron}; \quad \text{for steel } d'' = 0.03525\sqrt[4]{D^2l^2p}, \quad (4)$$

in which l is the length of stroke, all dimensions in inches. Deduce the diameter of piston-rod by (3), and if this diameter is less than $1/12l$, then use (4).

Seaton gives: Diameter of piston-rod $= \dfrac{\text{Diameter of cylinder}}{F}\sqrt{p}$.

The following are the values of F:

Naval engines, direct-acting	....	$F = 60$
" " return connnecting-rod, 2 rods	....	$F = 80$
Mercantile ordinary stroke, direct-acting	....	$F = 50$
" long " "	....	$F = 48$
" very long " "	....	$F = 45$
" medium stroke, oscillating	....	$F = 45$

NOTE.—Long and very long, as compared with the stroke usual for the power of engine or size of cylinder.

In considering an expansive engine p, the effective pressure should be taken as the absolute working pressure, or 15 lbs. above that to which the boiler safety-valve is loaded; for a compound engine the value of p for the high-pressure piston should be taken as the absolute pressure, less 15 lbs., or the same as the load on the safety-valve; for the medium-pressure the load may be taken as that due to half the absolute boiler-pressure; and for the low-pressure cylinder the pressure to which the escape-valve is loaded + 15 lbs., or the maximum absolute pressure, which can be got in the receiver, or about 25 lbs. It is an advantage to make all the rods of a compound engine alike, and this is now the rule.

Applying the above formulæ to the engines of 10, 30, and 50 in. diameter, both short and long stroke, we have:

Diameter of Piston-rods.

Diameter of Cylinder, inches	10		30		50	
Stroke, inches	12	24	30	60	48	96
Unwin, iron, $.0167 D \sqrt{p}$	1.67	1.67	5.01	5.01	8.35	8.35
Unwin, steel, $.0144 D \sqrt{p}$	1.44	1.44	4.32	4.32	7.20	7.20
Thurston $\sqrt[4]{\frac{D^2pL^2}{10,000}} + \frac{D}{80}$ (L in feet).	1.13	......	3.12	......	5.10	......
Thurston, same with $a = 15,000$	......	1.40	......	3.88	......	6.35
Marks, iron, $.0179 D \sqrt{p}$	1.79	......	5.37	5.37	8.95	8.95
Marks, iron, $.03901 \sqrt[4]{D^2l^2p}$	1.35	1.91	3.70	5.13	6.04	8.54
Marks, steel, $.0105 D \sqrt{p}$	(1.05)	......	(3.15)	......	(5.25)	......
Marks, steel, $.03525 \sqrt[4]{D^2l^2p}$	1.22	1.73	3.34	4.72	5.46	7.72
Seaton, naval engines, $\frac{D}{60} \sqrt{p}$	1.67	......	5.01	.....	8.35	.....
Seaton, land engine, $\frac{D}{45} \sqrt{p}$	......	2.22	......	6.67	......	11.11
Average of four for iron	1.49	1.82	4.30	5.26	7.11	8.74

The figures in brackets opposite Marks' third formula would be rejected since they are less than 1/8 of the stroke, and the figures derived by his fourth formula would be taken instead. The figure 1.79 opposite his first formula would be rejected for the engine of 24-inch stroke.

An empirical formula which gives results approximating the above averages is $d'' = .013 \sqrt{Dlp}$.

The calculated results from this formula, for the six engines, are, respectively, 1.42, 1.88, 3.90, 5.61, 6.37, 9.01.

Piston-rod Guides.—The thrust on the guide, when the connecting-rod is at its maximum angle with the line of the piston-rod, is found from the formula: Thrust = total load on piston × tangent of maximum angle of connecting-rod $= p \tan \theta$. This angle, θ, is the angle whose sine = half stroke of piston ÷ length of connecting-rod.

Ratio of length of connecting-rod to stroke	2	2½	3
Maximum angle of connecting-rod with line of piston-rod	14° 29′	11° 33′	9° 36′
Tangent of the angle	.258	.204	.169
Secant of the angle	1.0327	1.0206	1.014

Seaton says: The area of the guide-block or slipper surface on which the thrust is taken should in no case be less than will admit of a pressure of 400 lbs. on the square inch; and for good working those surfaces which take the thrust when going ahead should be sufficiently large to prevent the maximum pressure exceeding 100 lbs. per sq. in. When the surfaces are kept well lubricated this allowance may be exceeded.

Thurston says: The rubbing surfaces of guides are so proportioned that if V be their relative velocity in feet per minute, and p be the intensity of pressure on the guide in lbs. per sq. in., $pV < 60,000$ and $pV > 40,000$.

The lower is the safer limit; but for marine and stationary engines it is allowable to take $p = 60,000 \div V$. According to Rankine, for locomotives, $p = \frac{44800}{V + 20}$, where p is the pressure in lbs. per sq. in. and V the velocity of rubbing in feet per minute. This includes the sum of all pressures forcing the two rubbing surfaces together.

Some British builders of portable engines restrict the pressure between the guides and cross-heads to less than 40, sometimes 35 lbs. per square inch.

For a mean velocity of 600 feet per minute, Prof. Thurston's formulas give, $p < 100$, $p > 66.7$; Rankine's gives $p = 72.2$ lbs. per sq. in.

Whitham gives,

$$A = \text{area of slides in square inches} = \frac{P}{p_e\sqrt{n^2-1}} = \frac{.7854d^2p_1}{p_e\sqrt{n^2-1}},$$

in which P = total unbalanced pressure, p_1 = pressure per square inch on piston, d = diameter of cylinder, p_e = pressure allowable per square inch on slides, and n = length of connecting-rod ÷ length of crank. This is equivalent to the formula, $A = P \tan \theta \div p_0$. For $n = 5$, $p_1 = 100$ and $p_e = 80$, $A = .2004d^2$. For the three engines 10, 30 and 50 in. diam., this would give for area of slides, A = 20, 180 and 500 sq. in., respectively. Whitham says: The normal pressure on the slide may be as high as 500 lbs. per sq. in., but this is when there is good lubrication and freedom from dust. Stationary and marine engines are usually designed to carry 100 lbs. per sq. in., and the area in this case is reduced from 50% to 60% by grooves. In locomotive engines the pressure ranges from 40 to 50 lbs. per sq. in. of slide, on account of the inaccessibility of the slide, dirt, cinder, etc.

There is perfect agreement among the authorities as to the formula for area of the slides, $A = P \tan \theta \div p_0$; but the value given to p_e, the allowable pressure per square inch, ranges all the way from 35 lbs. to 500 lbs.

The Connecting-rod. *Ratio of length of connecting-rod to length of stroke.*—Experience has led generally to the ratio of 2 or 2½ to 1, the latter giving a long and easy-working rod, the former a rather short, but yet a manageable one (Thurston). Whitham gives the ratio of from 2 to 4½, and Marks from 2 to 4.

Dimensions of the Connecting-rod.—The calculation of the diameter of a connecting-rod on a theoretical basis, considering it as a strut subject to both compressive and bending stresses, and also to stress due to its inertia, in high-speed engines, is quite complicated. See Whitham, Steam-engine Design, p. 217; Thurston, Manual of S. E., p. 100. Empirical formulas are as follows: For circular rods, largest at the middle, D = diam. of cylinder, l = length of connecting-rod in inches, p = maximum steam-pressure per sq. in.

(1) Whitham, diam. at middle, $d'' = 0.0272\sqrt{Dl\sqrt{p}}$.
(2) Whitham, diam. at necks, d'' = 1.0 to 1.1 × diam. of piston-rod.

(3) Sennett, diam. at middle, $d'' = \frac{D}{55}\sqrt{p}$.

(4) Sennett, diam. at necks, $d'' = \frac{D}{60}\sqrt{p}$.

(5) Marks, diam., $d'' = 0.0179D\sqrt{p}$, if diam. is greater than 1/24 length.

(6) Marks, diam., $d'' = 0.02758\sqrt{Dl\sqrt{p}}$ if diam. found by (5) is less than 1/24 length.

(7) Thurston, diam. at middle, $d'' = a\sqrt{DL\sqrt{p}} + C$, D in inches, L in feet, $a = 0.15$ and $C = \frac{1}{8}$ inch for fast engines, $a = 0.08$ and $C = \frac{3}{4}$ inch for moderate speed.

(8) Seaton says: The rod may be considered as a strut free at both ends, and, calculating its diameter accordingly,

$$\text{diameter at middle} = \frac{\sqrt{R(1+4ar^2)}}{48.5},$$

where R = the total load on piston P multiplied by the secant of the maximum angle of obliquity of the connecting-rod.

For wrought iron and mild steel a is taken at 1/3000. The following are the values of r in practice:

Naval	engines—	Direct-acting	r = 9 to 11;
"	"	Return connecting-rod	r = 10 to 13, old;
"	"	" "	r = 8 to 9, modern;
"	"	Trunk	r = 11.5 to 13.
Mercantile	"	Direct-acting, ordinary	r = 12.
"	"	" long stroke	r = 13 to 16.

(9) The following empirical formula is given by Seaton as agreeing closely with good modern practice:

Diameter of connecting-rod at middle $= \sqrt{lK} + 4$, l = length of rod in inches, and $K = 0.08\sqrt{\text{effective load on piston in pounds}}$.

The diam. at the ends may be 0.875 of the diam. at the middle.

Seaton's empirical formula when translated into terms of D and p is the same as the second one by Marks, viz., $d'' = 0.02758 \sqrt{Dl\sqrt{p}}$. Whitham's (1) is also practically the same.

(10) Taking Seaton's more complex formula, with length of connecting-rod = 2.5 × length of stroke, and r = 12 and 16, respectively, it reduces to: Diam. at middle = $.02294 \sqrt{P}$ and $.02411 \sqrt{P}$ for short and long stroke engines, respectively.

Applying the above formulas to the engines of our list, we have

Diameter of Connecting-rods.

Diameter of Cylinder, inches	10		30		50	
Stroke, inches	12	24	30	60	48	96
Length of connecting-rod l	30	60	75	150	120	240
(3) $d'' = \frac{D}{55}\sqrt{p} = .0182D\sqrt{p}$	1.82	1.82	5.46	5.46	9.09	9.09
(5) $d'' = .0179D\sqrt{p}$	1.79	......	5.37	......	8.95	......
(6) $d'' = .02758\sqrt{Dl\sqrt{p}}$	......	2.14	......	5.85	......	9.51
(7) $d'' = 0.15\sqrt{DL\sqrt{p}} + \frac{1}{8}$	2.87	......	7.00	......	11.11	......
(7) $d'' = 0.08\sqrt{DL\sqrt{p}} + \frac{3}{8}$	......	2.54	......	5.65	......	8.75
(9) $d'' = .03\sqrt{P}$	2.67	2.67	7.97	7.97	13.29	13.29
(10) $d'' = .02294\sqrt{P}$; $.02411\sqrt{P}$	2.03	2.14	6.09	6.41	10.16	10.68
Average	2.24	2.26	6.38	6.27	10.52	10.26

Formulæ 5 and 6 (Marks), and also formula 10 (Seaton), give the larger diameters for the long-stroke engine; formulæ 7 give the larger diameters for the short-stroke engines. The average figures show but little difference in diameter between long- and short-stroke engines; this is what might be expected, for while the connecting-rod, considered simply as a column, would require an increase of diameter for an increase of length, the load remaining the same, yet in an engine generally the shorter the connecting-rod the greater the number of revolutions, and consequently the greater the strains due to inertia. The influences tending to increase the diameter therefore tend to balance each other, and to render the diameter to some extent independent of the length. The average figures correspond nearly to the simple formula $d'' = .021D\sqrt{p}$. The diameters of rod for the three diameters of engine by this formula are, respectively, 2.10, 6.30, and 10.50 in. Since the total pressure on the piston $P = .7854D^2p$, the formula is equivalent to $d' = .0237\sqrt{P}$.

Connecting-rod Ends.—For a connecting-rod end of the marine type, where the end is secured with two bolts, each bolt should be proportioned for a safe tensile strength equal to two thirds the maximum pull or thrust in the connecting-rod.

The cap is to be proportioned as a beam loaded with the maximum pull of the connecting-rod, and supported at both ends. The calculation should be made for rigidity as well as strength, allowing a maximum deflection of 1/100 inch. For a strap-and-key connecting-rod end the strap is designed for tensile strength, considering that two thirds of the pull on the connecting-rod may come on one arm. At the point where the metal is slotted for the key and gib, the straps must be thickened to make the cross-section equal to that of the remainder of the strap. Between the end of the strap and the slot the strap is liable to fail in double shear, and sufficient metal must be provided at the end to prevent such failure.

The breadth of the key is generally one fourth of the width of the strap, and the length, parallel to the strap, should be such that the cross-section will have a shearing strength equal to the tensile strength of the section of the strap. The taper of the key is generally about ⅝ inch to the foot.

Tapered Connecting-rods.—In modern high-speed engines it is customary to make the connecting-rods of rectangular instead of circular section, the sides being parallel, and the depth increasing regularly from the crosshead end to the crank-pin end. According to Grashof, the bending action on the rod due to its inertia is greatest at 6/10 the length from the crosshead end, and, according to this theory, that is the point at which the section should be greatest, although in practice the section is made greatest at the crank-pin end.

Professor Thurston furnishes the author with the following rule for tapered connecting-rod of rectangular section: Take the section as computed by the formula $d'' = 0.1\sqrt{DL\sqrt{p}} + 3/4$ for a circular section, and for a rod 4/3 the actual length, placing the computed section at 2/3 the length from the small end, and carrying the taper straight through this fixed section to the large end. This brings the computed section at the surge point and makes it heavier than the rod for which a tapered form is not required.

Taking the above formula, multiplying L by 4/3, and changing it to l in inches, it becomes $d = 1/30\sqrt{Dl\sqrt{p}} + 3/4''$. Taking a rectangular section of the same area as the round section whose diameter is d, and making the depth of the section h = twice the thickness t, we have $.7854d^2 = ht = 2t^2$, whence $t = .627d = .0209\sqrt{Dl\sqrt{p}} + .47''$, which is the formula for the thickness or distance between the parallel sides of the rod. Making the depth at the crosshead end $= 1.5t$, and at 2/3 the length $= 2t$, the equivalent depth at the crank end is $2.25t$. Applying the formula to the short-stroke engines of our examples, we have

Diameter of cylinder, inches	10	30	50
Stroke, inches	12	30	48
Length of connecting-rod	30	75	120
Thickness, $t = .0209\sqrt{Dl\sqrt{p}} + .47 =$	1.61	3.60	5.59
Depth at crosshead end, $1.5t =$	2.42	5.41	8.39
Depth at crank end, $2\frac{1}{4}t$	3.62	8.11	12.58

The thicknesses t, found by the formula $t = .0209\sqrt{Dl\sqrt{p}} + .47$, agree closely with the more simple formula $t = .01D\sqrt{p} + .60''$, the thicknesses calculated by this formula being respectively 1.6, 3.6, and 5.6 inches.

The Crank-pin.—A crank-pin should be designed (1) to avoid heating, (2) for strength, (3) for rigidity. The heating of a crank-pin depends on the pressure on its rubbing-surface, and on the coefficient of friction, which latter varies greatly according to the effectiveness of the lubrication. It also depends upon the facility with which the heat produced may be carried away: thus it appears that locomotive crank-pins may be prevented to some degree from overheating by the cooling action of the air through which they pass at a high speed.

Marks gives $l = .0000247fpND^2 = 1.038f\dfrac{\text{(I.H.P.)}}{L}$ . . . . (1)

Whitham gives $l = 0.9075f\dfrac{\text{(I.H.P.)}}{L}$, ... (2)

in which l = length of crank-pin journal in inches, f = coefficient of friction, which may be taken at .03 to .05 for perfect lubrication, and .08 to .10 for imperfect; p = mean pressure in the cylinder in pounds per square inch; D = diameter of cylinder in inches; N = number of single strokes per minute; I.H.P. = indicated horse-power; L = length of stroke in feet. These formulæ are independent of the diameter of the pin, and Marks states as a general law, within reasonable limits as to pressure and speed of rubbing, the longer a bearing is made, for a given pressure and number of revolutions, the cooler it will work; and its diameter has no effect upon its heating. Both of the above formulæ are deduced empirically from dimensions of crank-pins of existing marine engines. Marks says that about one-fourth the length required for crank-pins of propeller engines will serve for the pins of side-wheel engines, and one tenth for locomotive engines, making the formula for locomotive crank-pins $l = .0000247 fpND^2$, or if $p = 150$, $f = .06$, and $N = 600$, $l = .018D^2$.

Whitham recommends for pressure per square inch of projected area, for naval engines 500 pounds, for merchant engines 400 pounds, for paddle-wheel engines 800 to 900 pounds.

Thurston says the pressure should, in the steam-engine, never exceed 500 or 600 pounds per square inch for wrought-iron pins, or about twice that figure for steel. He gives the formula for length of a steel pin, in inches,

$$l = PR \div 600,000, \quad \ldots \ldots \ldots \quad (3)$$

in which P and R are the mean total load on the pin in pounds, and the number of revolutions per minute. For locomotives, the divisor may be taken as 500,000. Where iron is used this figure should be reduced to 300,000 and 250,000 for the two cases taken. Pins so proportioned, if well made and well lubricated, may always be depended upon to run cool: if not well formed, perfectly cylindrical, well finished, and kept well oiled, no crank-pin can be relied upon. It is assumed above that good bronze or white-metal bearings are used.

Thurston also says: The size of crank-pins required to prevent heating of the journals may be determined with a fair degree of precision by either of the formulæ given below:

$$l = \frac{P(V + 20)}{44,800d} \text{ (Rankine, 1865)}; \quad \ldots \ldots \ldots \quad (4)$$

$$l = \frac{PV}{60,000d} \text{ (Thurston, 1863)}; \quad \ldots \ldots \ldots \quad (5)$$

$$l = \frac{PN}{850,000} \text{ (Van Buren, 1866)}. \quad \ldots \ldots \ldots \quad (6)$$

The first two formulæ give what are considered by their authors fair working proportions, and the last gives minimum length for iron pins. (V = velocity of rubbing-surface in feet per minute.)

Formula (1) was obtained by observing locomotive practice in which great liability exists of annoyance by dust, and great risk occurs from inaccessibility while running, and (2) by observation of crank-pins of naval screw-engines. The first formula is therefore not well suited for marine practice.

Steel can usually be worked at nearly double the pressure admissible with iron running at similar speed.

Since the length of the crank-pin will be directly as the power expended upon it and inversely as the pressure, we may take it as

$$l = a\frac{\text{I.H.P.}}{L}, \quad \ldots \ldots \ldots \ldots \quad (7)$$

in which a is a constant, and L the stroke of piston, in feet. The values of the constant, as obtained by Mr. Skeel, are about as follows: $a = 0.04$ where water can be constantly used; $a = 0.045$ where water is not generally used; $a = 0.05$ where water is seldom used; $a = 0.06$ where water is never needed. Unwin gives

$$l = a\frac{\text{I.H.P.}}{r}, \quad \ldots \ldots \ldots \ldots \quad (8$$

in which r = crank radius in inches, $a = 0.3$ to $a = 0.4$ for iron and for marine engines, and $a = 0.066$ to $a = 0.1$ for the case of the best steel and for locomotive work, where it is often necessary to shorten up outside pins as much as possible.

J. B. Stanwood (*Eng'g*, June 12, 1891), in a table of dimensions of parts of American Corliss engines from 10 to 30 inches diameter of cylinder, gives sizes of crank-pins which approximate closely to the formula

$$l = .275D'' + .5 \text{ in.}; \quad d = .25D''. \quad \ldots \ldots \ldots \quad (9)$$

By calculating lengths of iron crank-pins for the engines 10, 30, and 50 inches diameter, long and short stroke, by the several formulæ above given, it is found that there is a great difference in the results, so that one formula in certain cases gives a length three times as great as another. Nos. (4), (5), and (6) give lengths much greater than the others. Marks (1), Whitham (2), Thurston (7), $l = .06$ I.H.P. $\div L$, and Unwin (8), $l = 0.4$ I.H.P. $\div r$, give results which agree more closely.

The calculated lengths of iron crank-pins for the several cases by formulæ (1), (2), (7), and (8) are as follows:

Length of Crank-pins.

Diameter of cylinder .... D	10	10	30	30	50	50
Stroke .... L (ft.)	1	2	2½	5	4	8
Revolutions per minute .... R	250	125	130	65	90	45
Horse-power .... I.H.P.	50	50	450	450	1,250	1,250
Maximum pressure .... lbs.	7,854	7,854	70,686	70,686	196,350	196,350
Mean pressure per cent of max	42	42	32.3	32.3	30	30
Mean pressure .... P.	3,299	3,299	22,832	22,832	58,905	58,905
Length of crank-pin						
(1) Whitham, $l = .9075 \times .05$ I.H.P. $\div L$.	2.18	1.09	8.17	4.08	14.18	7.09
(2) Marks, $l = 1.038 \times .05$ I.H.P. $\div L$.	2.59	1.30	9.34	4.67	16.22	8.11
(7) Thurston, $l = .06$ I.H.P. $\div L$	3.00	1.50	10.80	5.40	18.75	9.38
(8) Unwin, $l = .4$ I.H.P. $\div r$	3.33	1.67	12.0	6.0	20.83	10.42
(8) " $l = .3$ I.H.P. $\div r$	2.50	1.25	9.0	4.5	15.62	7.81
Average	2.72	1.36	9.86	4.93	17.12	8.56
(8) Unwin, best steel, $l = .1\frac{\text{I.H.P.}}{r}$	.83	.42	3.0	1.5	5.21	2.61
(3) Thurston, steel, $l = \frac{PR}{600,000}$	1.37	.69	4.95	2.47	8.84	4.42

The calculated lengths for the long-stroke engines are too low to prevent excessive pressures. See "Pressures on the Crank-pins," below.

The Strength of the Crank-pin is determined substantially as is that of the crank. In overhung cranks the load is usually assumed as carried at its extremity, and, equating its moment with that of the resistance of the pin,

$$\tfrac{1}{2}Pl = 1/32 t\pi d^3, \quad \text{and} \quad d = \sqrt[3]{\frac{5.1 Pl}{t}},$$

in which d = diameter of pin in inches, P = maximum load on the piston, t = the maximum allowable stress on a square inch of the metal. For iron it may be taken at 9000 lbs. For steel the diameters found by this formula may be reduced 10%. (Thurston.)

Unwin gives the same formula in another form, viz.:

$$d = \sqrt[3]{\frac{5.1}{t}}\sqrt[3]{Pl} = \sqrt{\frac{5.1}{t}}\sqrt{P\frac{l}{d}},$$

the last form to be used when the ratio of length to diameter is assumed.

For wrought iron, t = 6000 to 9000 lbs. per sq. in.,

$$\sqrt[3]{\frac{5.1}{t}} = .0947 \text{ to } .0827; \qquad \sqrt{\frac{5.1}{t}} = .0291 \text{ to } .0238.$$

For steel, t = 9000 to 13,000 lbs. per sq. in.,

$$\sqrt[3]{\frac{5.1}{t}} = .0827 \text{ to } .0723; \qquad \sqrt{\frac{5.1}{t}} = .0238 \text{ to } .0194.$$

Whitham gives $d = 0.0827\sqrt[3]{Pl} = 2.1058\sqrt[3]{\frac{l \times \text{I.H.P.}}{LR}}$ for strength, and $d = 0.405\sqrt[4]{Pl^3}$ for rigidity, and recommends that the diameter be calculated by both formulæ, and the largest result taken. The first is the same as Unwin's formula, with t taken at 9000 lbs. per sq. in. The second is based upon an erroneous assumption.

Marks, calculating the diameter for rigidity, gives

$$d = 0.066\sqrt[4]{pl^3D^2} = 0.945\sqrt[4]{\frac{(\text{H.P.})l^3}{LN}};$$

p = maximum steam-pressure in pounds per square inch, D = diameter of cylinder in inches, L = length of stroke in feet, N = number of single strokes per minute. He says there is no need of an investigation of the strength of a crank-pin, as the condition of rigidity gives a great excess of strength.

Marks's formula is based upon the assumption that the whole load may be concentrated at the outer end, and cause a deflection of .01 inch at that point.

It is serviceable, he says, for steel and for wrought iron alike.

Using the average lengths of the crank-pins already found, we have the following for our six engines:

Diameter of Crank-pins.

Diameter of cylinder	10	10	30	30	50	50
Stroke, ft.	1	2	2½	5	4	3
Length of crank-pin	2.72	1.36	9.86	4.93	17.12	8.56
Unwin, $d = \sqrt[3]{\frac{5.1Pl}{t}}$	2.29	1.82	7.34	5.82	12.40	9.84
Marks, $d = .066\sqrt[4]{pl^3D^2}$	1.39	.85	6.44	3.78	12.41	7.39

Pressures on the Crank-pins.—If we take the mean pressure upon the crank-pin = mean pressure on piston, neglecting the effect of the varying angle of the connecting-rod, we have the following, using the average lengths already found, and the diameters according to Unwin and Marks:

Engine No.	1	2	3	4	5	6
Diameter of cylinder, inches	10	10	30	30	50	50
Stroke, feet	1	2	2½	5	4	3
Mean pressure on pin, pounds	3,299	3,299	22,832	22,832	58,905	58,905
Projected area of pin, Unwin	6.23	2.36	72.4	28.7	212.3	84.2
" " " " Marks	3.78	1.16	63.5	18.6	212.5	63.3
Pressure per square inch, Unwin	530	1,398	315	796	277	700
" " " " Marks	873	2,845	360	1,228	277	930

The results show that the application of the formulæ for length and diameter of crank-pins give quite low pressures per square inch of projected area for the short-stroke high-speed engines of the larger sizes, but too high pressures for all the other engines. It is therefore evident that after calculating the dimensions of a crank-pin according to the formulæ given that the results should be modified, if necessary, to bring the pressure per square inch down to a reasonable figure.

In order to bring the pressures down to 500 pounds per square inch, we divide the mean pressures by 500 to obtain the projected area, or product of length by diameter. Making $l = 1.5d$ for engines Nos. 1, 2, 4 and 6, the revised table for the six engines is as follows:

Engine, No.	1	2	3	4	5	6
Length of crank-pin, inches	3.15	3.15	9.86	8.37	17.12	13.30
Diameter of crank-pin	2.10	2.10	7.34	5.58	12.40	8.87

Crosshead-pin or Wrist-pin.—Whitham says the bearing surface for the wrist-pin is found by the formula for crank-pin design. Seaton says the diameter at the middle must, of course, be sufficient to withstand the bending action, and generally from this cause ample surface is provided for good working; but in any case the area, calculated by multiplying the diameter of the journal by its length, should be such that the pressure does not exceed 1200 lbs. per sq. in., taking the maximum load on the piston as the total pressure on it.

For small engines with the gudgeon shrunk into the jaws of the connecting-rod, and working in brasses fitted into a recess in the piston-rod end and secured by a wrought-iron cap and two bolts, Seaton gives:

Diameter of gudgeon = 1.25 × diam. of piston-rod.
Length of gudgeon = 1.4 × diam. of piston-rod.

If the pressure on the section, as calculated by multiplying length by diameter, exceeds 1200 lbs. per sq. in., this length should be increased.

J. B. Stanwood, in his "Ready Reference" book, gives for length of crosshead-pin 0.25 to 0.3 diam. of piston, and diam. = 0.18 to 0.2 diam. of piston. Since he gives for diam. of piston-rod 0.14 to 0.17 diam. of piston, his dimensions for diameter and length of crosshead-pin are about 1.25 and 1.8 diam. of piston-rod respectively. Taking the maximum allowable pressure at 1200 lbs. per sq. in. and making the length of the crosshead-pin = 4/3 of its diameter, we have $d = \sqrt{P} \div 40$, $l = \sqrt{P} \div 30$, in which P = maximum total load on piston in lbs., d = diam. and l = length of pin in inches. For the engines of our example we have:

Diameter of piston, inches	10	30	50
Maximum load on piston, lbs	7854	70,686	196,350
Diameter of crosshead-pin, inches	2.22	6.65	11.09
Length of crosshead-pin, inches	2.96	8.86	14.77
Stanwood's rule gives diameter, inches	1.8 to 2	5.4 to 6	9.9 to 10
Stanwood's rule gives length, inches	2.5 to 3	7.5 to 9	12.5 to 15
Stanwood's largest dimensions give pressure per sq. in., lbs	1309	1329	1309

Which pressures are greater than the maximum allowed by Seaton.

The Crank-arm.—The crank-arm is to be treated as a lever, so that if a is the thickness in direction parallel to the shaft-axis and b its breadth at a section x inches from the crank-pin centre, then, bending moment M at that section = Px, P being the thrust of the connecting-rod, and f the safe strain per square inch,

$$Px = \frac{fab^2}{6} \text{ and } \frac{a \times b^2}{6} = \frac{T}{f}, \text{ or } a = \frac{6T}{b^2 \times f};\ b = \sqrt{\frac{6T}{fa}}.$$

If a crank-arm were constructed so that b varied as $\sqrt{x}$ (as given by the above rule) it would be of such a curved form as to be inconvenient to manufacture, and consequently it is customary in practice to find the maximum value of b and draw tangent lines to the curve at the points; these lines are generally, for the same reason, tangential to the boss of the crank-arm at the shaft.

The shearing strain is the same throughout the crank-arm; and, consequently, is large compared with the bending strain close to the crank-pin; and so it is not sufficient to provide there only for bending strains. The section at this point should be such that, in addition to what is given by the calculation from the bending moment, there is an extra square inch for every 3000 lbs. of thrust on the connecting-rod (Seaton).

The length of the boss h into which the shaft is fitted is from 0.75 to 1.0 of the diameter of the shaft D, and its thickness e must be calculated from the twisting strain PL. (L = length of crank.)

For different values of length of boss h, the following values of thickness of boss e are given by Seaton:

When $h = D$, then $e = 0.35\ D$; if steel, 0.3.
$h = 0.9\ D$, then $e = 0.38\ D$, if steel, 0.32.
$h = 0.8\ D$, then $e = 0.40\ D$, if steel, 0.33.
$h = 0.7\ D$, then $e = 0.41\ D$, if steel, 0.34.

The crank-eye or boss into which the pin is fitted should bear the same relation to the pin that the boss does to the shaft.

The diameter of the shaft-end onto which the crank is fitted should be 1.1 × diameter of shaft.

Thurston says: The empirical proportions adopted by builders will commonly be found to fall well within the calculated safe margin. These proportions are, from the practice of successful designers, about as follows:

For the wrought-iron crank, the hub is 1.75 to 1.8 times the least diameter of that part of the shaft carrying full load; the eye is 2.0 to 2.25 the diameter of the inserted portion of the pin, and their depths are, for the hub, 1.0 to 1.2 the diameter of shaft, and for the eye, 1.25 to 1.5 the diameter of pin.

Marks, calculating the diameter for rigidity, gives

$$d = 0.066\sqrt[4]{pl^3D^2} = 0.945\sqrt[4]{\frac{(\text{H.P.})l^3}{LN}}$$

p = maximum steam-pressure in pounds per square ... cylinder in inches, L = length of stroke in feet, N = ... per minute. He says there is no need of an inve... a crank-pin, as the condition of rigidity gives ...

Marks's formula is based upon the assumptio... concentrated at the outer end, and cause a... point.

It is serviceable, he says, for steel and fo...

Using the average lengths of the cran... following for our six engines:

...ng in ...ers of ...depth ...se. ...out 1.1 ...ow for ...of one ...ives an

...y agree. ...fixed at ...ng to the ...are as fol

Diameter of ...

Diameter of cylinder.......					50	50
Stroke, ft..........................					48	96
Length of crank-pin........						
Unwin, $d = \sqrt[3]{\frac{5.1Pl}{t}}$........				10.686 5.58	196,350 12.40	196,350 8.87
Marks, $d = .066\sqrt[4]{pl^3D^2}$....			7.70	9.70	12.55	15.82

Pressures on the ... the crank-pin = mean ... ing angle of the conr... lengths already foun...

		..77	6.16	7.76	10.04	12.65
		1.39	8.08	8.88	5.02	6.32
	..3	6.23	13.86	17.46	22.59	28.47
	..76	1.76	5.87	4.46	9.92	7.10
Engine No........ ...nce ...centre	.88	.88	2.94	2.23	4.46	3.55
Diameter of cyl...	37,149	80,661	788,149	1,848,439	3,479,322	7,871,671
Stroke, feet... ...ank-arm						
Mean pressur...	2.05	2.60	5.78	7.28	9.41	11.87
Projected ar... ...dth,						
Pressure p... $b = \sqrt{\frac{6T}{9000a}}$	3.48	4.55	9.54	13.0	15.7	21.0
T_0 at distance ...ntre of pin $= Pd$ ...breadth,	16,493	16,498	528,835	394,428	2,434,740	1,741,625
$b_1 = \sqrt{\frac{6T_0}{9000a}}$	2.32	2.06	7.81	6.01	13.13	9.89

The ... eter c... area ... pres... lat... re... i...

...e Shaft.—Twisting Resistance.—From the general formula for torsion, we have: $T = \frac{\pi}{16}d^3S = .19635d^3S$, whence $d = \sqrt[3]{\frac{5.1T}{S}}$, in which T = torsional moment in inch-pounds, d = diameter in inches, and S = the shearing resistance of the material in pounds per square inch.

If a constant force P were applied to the crank-pin tangentially to its path, the work done per minute would be

$$P \times L \times \frac{2\pi}{12} \times R = 33{,}000 \times \text{I.H.P.},$$

in which L = length of crank in inches, and R = revs. per min., and the mean twisting moment $T = \frac{\text{I.H.P.}}{R} \times 63{,}025$. Therefore

$$d = \sqrt[3]{\frac{5.1T}{S}} = \sqrt[3]{\frac{321{,}427\,\text{I.H.P.}}{RS}}.$$

· take the form

$$d = \sqrt[3]{\frac{\text{I.H.P.}}{R} \times F}, \text{ or } d = a\sqrt[3]{\frac{\text{I.H.P.}}{R}}.$$

…e factors that depend on the strength of the material …afety. Taking S at 45,000 pounds per square inch for …000 for steel, we have, for simple twisting by a uni-

8	8	10		5	6	8	10
8	57.1	71.4	$a =$	3.3	3.5	3.85	4.15
	42.8	53.5	$a =$	3.0	3.18	3.5	3.77

· strength of wrought iron 9000 lbs., steel …ves $a = 3.294$ for wrought iron, 2.877 for …on, for crank-axles of wrought iron,

…, f, the safe strain per square inch, should …the shafts are more than 10 inches diameter, …rom the ingot and of good materials, will ad-…s. for small shafts, and 10,000 lbs. for those above

…e allowance between large and small shafts is to com-…ective material observable in the heart of large shafting, …mmering failing to affect it.

…ula $d = a\sqrt[3]{\frac{\text{I.H.P.}}{R}}$ assumes the tangential force to be uniform . that it is the only acting force. For engines, in which the tangential …orce varies with the angle between the crank and the connecting-rod, and with the variation in steam-pressure in the cylinder, and also is influenced by the inertia of the reciprocating parts, and in which also the shaft may be subjected to bending as well as torsion, the factor a must be increased, to provide for the maximum tangential force and for bending.

Seaton gives the following table showing the relation between the maximum and mean twisting moments of engines working under various conditions, the momentum of the moving parts being neglected, which is allowable:

Description of Engine.	Steam Cut-off at	Max. Twist Divided by Mean Twist. Mome't	Cube Root of the Ratio.
Single-crank expansive	0.2	2.625	1.38
" "	0.4	2.125	1.29
" "	0.6	1.835	1.22
" "	0.8	1.698	1.20
Two-cylinder expansive, cranks at 90°	0.2	1.616	1.17
" " "	0.3	1.415	1.12
" " "	0.4	1.298	1.09
" " "	0.5	1.256	1.08
" " "	0.6	1.270	1.08
" " "	0.7	1.329	1.10
" " "	0.8	1.357	1.11
Three-cylinder compound, cranks 120°	h.p. 0.5, l.p. 0.66	1.40	1.12
" " l. p. cranks opposite one another, and h.p. midway	" "	1.26	1.08

Seaton also gives the following rules for ordinary practice for ordinary two-cylinder marine engines:

$$\text{Diameter of the tunnel-shafts} = \sqrt[3]{\frac{\text{I.H.P}}{R} \times F}, \text{ or } a\sqrt[3]{\frac{\text{I.H.P.}}{R}}.$$

The web is made 0.7 to 0.75 the width of adjacent hub or eye, and is given a depth of 0.5 to 0.6 that of adjacent hub or eye.

For the cast-iron crank the hub and eye are a little larger, ranging in diameter respectively from 1.8 to 2 and from 2 to 2.2 times the diameters of shaft and pin. The flanges are made at either end of nearly the full depth of hub or eye. Cast-iron has, however, fallen very generally into disuse.

The crank-shaft is usually enlarged at the seat of the crank to about 1.1 its diameter at the journal. The size should be nicely adjusted to allow for the shrinkage or forcing on of the crank. A difference of diameter of one fifth of 1%, will usually suffice; and a common rule of practice gives an allowance of but one half of this, or .001.

The formulæ given by different writers for crank-arms practically agree, since they all consider the crank as a beam loaded at one end and fixed at the other. The relation of breadth to thickness may vary according to the taste of the designer. Calculated dimensions for our six engines are as follows:

Dimensions of Crank-arms.

Diam. of cylinder, ins.	10	10	30	30	50	50
Stroke S, ins.	12	24	30	60	48	96
Max. pressure on pin P, (approx.) lbs	7854	7854	70,686	70,686	196,350	196,350
Diam. crank-pin d.	2.10	2.10	7.34	5.58	12.40	8.87
Diam. shaft, $a\sqrt[3]{\frac{\text{I.H.P.}}{R}}D$ ($a = 4.69$, 5.09 and 5.22).	2.74	3.46	7.70	9.70	12.55	15.82
Length of boss, $.8D$.	2.19	2.77	6.16	7.76	10.04	12.65
Thickness of boss, $.4D$.	1.10	1.39	3.08	3.88	5.02	6.32
Diam. of boss, $1.8D$.	4.93	6.23	13.86	17.46	22.59	28.47
Length crank-pin eye, $.8d$	1.76	1.76	5.87	4.46	9.92	7.10
Thickness of crank-pin eye, $.4d$.	.88	.88	2.94	2.23	4.46	3.55
Max. mom. T at distance $\frac{1}{2}S - \frac{1}{2}D$ from centre of pin, inch-lbs.	37,149	80,661	788,149	1,848,439	3,479,322	7,871,671
Thickness of crank-arm $a = .75D$	2.05	2.60	5.78	7.28	9.41	11.87
Greatest breadth, $b = \sqrt{\frac{6T}{9000a}}$	3.48	4.55	9.54	13.0	15.7	21.0
Min. mom. T_0 at distance d from centre of pin $= Pd$	16,493	16,493	528,835	394,428	2,434,740	1,741,625
Least breadth, $b_1 = \sqrt{\frac{6T_0}{9000a}}$	2.32	2.06	7.81	6.01	13.13	9.89

The Shaft.—Twisting Resistance.—From the general formula for torsion, we have: $T = \frac{\pi}{16}d^3S = .19635d^3S$, whence $d = \sqrt[3]{\frac{5.1T}{S}}$, in which T = torsional moment in inch-pounds, d = diameter in inches, and S = the shearing resistance of the material in pounds per square inch.

If a constant force P were applied to the crank-pin tangentially to its path, the work done per minute would be

$$P \times L \times \frac{2\pi}{12} \times R = 33{,}000 \times \text{I.H.P.},$$

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This may take the form

$$d = \sqrt[3]{\frac{\text{I.H.P.}}{R} \times F}, \text{ or } d = a\sqrt[3]{\frac{\text{I.H.P.}}{R}},$$

in which F and a are factors that depend on the strength of the material and on the factor of safety. Taking S at 45,000 pounds per square inch for wrought iron, and at 60,000 for steel, we have, for simple twisting by a uniform tangential force,

Factor of safety =	5	6	8	10		5	6	8	10
Iron...... F =	35.7	42.8	57.1	71.4	a =	3.3	3.5	3.85	4.15
Steel..... F =	26.8	32.1	42.8	53.5	a =	3.0	3.18	3.5	3.77

Unwin, taking for safe working strength of wrought iron 9000 lbs., steel 13,500 lbs., and cast iron 4500 lbs., gives $a = 3.294$ for wrought iron, 2.877 for steel, and 4.15 for cast iron. Thurston, for crank-axles of wrought iron, gives $a = 4.15$ or more.

Seaton says: For wrought iron, f, the safe strain per square inch, should not exceed 9000 lbs., and when the shafts are more than 10 inches diameter, 8000 lbs. Steel, when made from the ingot and of good materials, will admit of a stress of 12,000 lbs. for small shafts, and 10,000 lbs. for those above 10 inches diameter.

The difference in the allowance between large and small shafts is to compensate for the defective material observable in the heart of large shafting, owing to the hammering failing to affect it.

The formula $d = a\sqrt[3]{\frac{\text{I.H.P.}}{R}}$ assumes the tangential force to be uniform and that it is the only acting force. For engines, in which the tangential force varies with the angle between the crank and the connecting-rod, and with the variation in steam-pressure in the cylinder, and also is influenced by the inertia of the reciprocating parts, and in which also the shaft may be subjected to bending as well as torsion, the factor a must be increased, to provide for the maximum tangential force and for bending.

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" " "	0.4	1.298	1.09
" " "	0.5	1.256	1.08
" " "	0.6	1.270	1.08
" " "	0.7	1.329	1.10
" " "	0.8	1.357	1.11
Three-cylinder compound, cranks 120°....	h.p. 0.5, l.p. 0.66	1.40	1.12
" " l. p. cranks opposite one another, and h.p. midway	" "	1.26	1.08

Seaton also gives the following rules for ordinary practice for ordinary two-cylinder marine engines:

$$\text{Diameter of the tunnel-shafts} = \sqrt[3]{\frac{\text{I.H.P}}{R} \cdot \times F}, \text{ or } a\sqrt[3]{\frac{\text{I.H.P.}}{R}}.$$

Compound engines, cranks at right angles:

Boiler pressure 70 lbs., rate of expansion 6 to 7, $F = 70$, $a = 4.12$.
Boiler pressure 80 lbs., rate of expansion 7 to 8, $F = 72$, $a = 4.16$.
Boiler pressure 90 lbs., rate of expansion 8 to 9, $F = 75$, $a = 4.22$.

Triple compound, three cranks at 120 degrees:

Boiler pressure 150 lbs., rate of expansion 10 to 12, $F = 62$, $a = 3.96$.
Boiler pressure 160 lbs., rate of expansion 11 to 13, $F = 64$, $a = 4$.
Boiler pressure 170 lbs., rate of expansion 12 to 15, $F = 67$, $a = 4.06$.

Expansive engines, cranks at right angles, and the rate of expansion 5, boiler-pressure 60 lbs., $F = 90$, $a = 4.48$.

Single-crank compound engines, pressure 80 lbs., $F = 96$, $a = 4.58$.

For the engines we are considering it will be a very liberal allowance for ratio of maximum to mean twisting moment if we take it as equal to the ratio of the maximum to the mean pressure on the piston. The factor a, then, in the formula for diameter of the shaft will be multiplied by the cube root of this ratio, or $\sqrt[3]{\frac{100}{42}} = 1.34$, $\sqrt[3]{\frac{100}{32.3}} = 1.45$, and $\sqrt[3]{\frac{100}{30}} = 1.49$ for the 10, 30, and 50-in. engines, respectively. Taking $a = 3.5$, which corresponds to a shearing strength of 60,000 and a factor of safety of 8 for steel, or to 45,000 and a factor of 6 for iron, we have for the new coefficient a_1 in the formula $d_1 = a_1\sqrt[3]{\frac{\text{I.H.P.}}{R}}$, the values 4.69, 5.08, and 5.22, from which we obtain the diameters of shafts of the six engines as follows:

Engine No.	1	2	3	4	5	6
Diam. of cyl.	10	10	30	30	50	50
Horse-power, I.H.P.	50	50	450	450	1250	1250
Revs. per min., R	250	125	130	65	90	45
Diam. of shaft $d = a_1\sqrt[3]{\frac{\text{I.H.P.}}{R}}$	2.74	3.46	7.67	9.70	12.55	15.82

These diameters are calculated for twisting only. When the shaft is also subjected to bending strain the calculation must be modified as below:

Resistance to Bending.—The strength of a circular-section shaft to resist bending is one half of that to resist twisting. If B is the bending moment in inch-lbs., and d the diameter of the shaft in inches,

$$B = \frac{\pi d^3}{32} \times f; \text{ and } d = \sqrt[3]{\frac{B}{f} \times 10.2};$$

f is the safe strain per square inch of the material of which the shaft is composed, and its value may be taken as given above for twisting (Seaton).

Equivalent Twisting Moment.—When a shaft is subject to both twisting and bending simultaneously, the combined strain on any section of it may be measured by calculating what is called the *equivalent twisting moment;* that is, the two strains are so combined as to be treated as a twisting strain only of the same magnitude and the size of shaft calculated accordingly. Rankine gave the following solution of the combined action of the two strains.

If T = the twisting moment, and B = the bending moment on a section of a shaft, then the equivalent twisting moment $T_1 = B + \sqrt{B^2 + T^2}$.

Seaton says: Crank-shafts are subject always to twisting, bending, and shearing strains; the latter are so small compared with the former that they are usually neglected directly, but allowed for indirectly by means of the factor f.

The two principal strains vary throughout the revolution, and the maximum equivalent twisting moment can only be obtained accurately by a series of calculations of bending and twisting moments taken at fixed intervals, and from them constructing a curve of strains.

Considering the engines of our examples to have overhung cranks, the maximum bending moment resulting from the thrust of the connecting-rod on the crank-pin will take place when the engine is passing its centres (neglecting the effect of the inertia of the reciprocating parts), and it will be the product of the total pressure on the piston by the distance between two parallel lines passing through the centres of the crank-pin and of the shaft bearing, at right angles to their axes; which distance is equal to ½ length of crank-pin bearing + length of hub + ½ length of shaft-bearing + any clearance that may be allowed between the crank and the two bearings. For our six engines we may take this distance as equal to ½ length of crank-pin + thickness of crank-arm + 1.5 × the diameter of the shaft as already found by the calculation for twisting. The calculation of diameter is then as below:

Engine No.	1	2	3	4	5	6
Diam. of cyl., in. .	10	10	30	30	50	50
Horse-power.......	50	50	450	450	1250	1250
Revs. per min.. ..	250	125	130	65	90	45
Max. press. on pis, P	7,854	7,854	70,686	70,686	196,350	196,350
Leverage,* L in....	6.32	7.94	22.20	26.00	36.80	42.25
Bd. mo. $PL = B$ in.-lb	49,637	62,361	1,569,222	1,837,836	7,225,680	8,295,788
Twist. mom. T......	47,124	94,248	1,060,290	2,120,580	4,712,400	9,424,800
Equiv. Twist. mom. $T_1 = B + \sqrt{B^2+T^2}$ (approx.)........	118,000	175,000	3,463,000	4,647,000	15,840,000	20,850,000

* Leverage = distance between centres of crank-pin and shaft bearing = $\frac{1}{2}l + 2.25d$.

Having already found the diameters, on the assumption that the shafts were subjected to a twisting moment T only, we may find the diameter for resisting combined bending and twisting by multiplying the diameters already found by the cube roots of the ratio $T_1 \div T$, or

	1.40	1.27	1.46	1.34	1.64	1.36
Giving corrected diameters d_1 =...	3.84	4.39	11.35	12.99	20.58	21.52

By plotting these results, using the diameters of the cylinders for abscissas and diameters of the shafts for ordinates, we find that for the long-stroke engines the results lie almost in a straight line expressed by the formula, diameter of shaft = .43 × diameter of cylinder; for the short-stroke engines the line is slightly curved, but does not diverge far from a straight line whose equation is, diameter of shaft = .4 diameter of cylinder. Using these two formulas, the diameters of the shafts will be 4.0, 4.3, 12.0, 12.9, 20.0, 21.5.

J. B. Stanwood, in *Engineering*, June 12, 1891, gives dimensions of shafts of Corliss engines in American practice for cylinders 10 to 30 in. diameter. The diameters range from 4 15/16 to 14 15/16, following precisely the equation, diameter of shaft = ½ diameter of cylinder − 1/16 inch.

Fly-wheel Shafts.—Thus far we have considered the shaft as resisting the force of torsion and the bending moment produced by the pressure on the crank-pin. In the case of fly-wheel engines the shaft on the opposite side of the bearing from the crank-pin has to be designed with reference to the bending moment caused by the weight of the fly-wheel, the weight of the shaft itself, and the strain of the belt. For engines in which there is an outboard bearing, the weight of fly-wheel and shaft being supported by two bearings, the point of the shaft at which the bending moment is a maximum may be taken as the point midway between the two bearings or at the middle of the fly-wheel hub, and the amount of the moment is the product of the weight supported by one of the bearings into the distance from the centre of that bearing to the middle point of the shaft. The shaft is thus to be treated as a beam supported at the ends and loaded in the middle. In the case of an overhung fly-wheel, the shaft having only one bearing, the point of maximum moment should be taken as the middle of the bearing, and its amount is very nearly the product of half the weight of the fly-wheel and the shaft into the distance from the middle of its hub from the middle of the bearing. The bending moment should be calculated and combined with the twisting moment as above shown, to obtain the equivalent twisting moment, and the diameter necessary at the point of maximum moment calculated therefrom.

In the case of our six engines we assume that the weights of the fly-wheels, together with the shaft, are double the weight of fly-wheel rim obtained from the formula, $W = 785,400 \frac{d^2s}{R^2D^2}$ (given under Fly-wheels);

that the shaft is supported by an outboard bearing, the distance between the two bearings being 2½, 5, and 10 feet for the 10-in., 30-in., and 50-in. engines, respectively. The diameters of the fly-wheels are taken such that their rim velocity will be a little less than 6000 feet per minute.

Engine No.	1	2	3	4	5	6
Diam. of cyl., inches	10	10	30	30	50	50
Diam. of fly-wheel, ft.	7.5	15	14.5	29	21	42
Revs. per min	250	125	130	65	90	45
Half wt. fly-wh'l and shaft, lb.	268	536	5,968	11,936	26,381	52,769
Lever arm for max. mom., in.	15	15	30	30	60	60
Max. bending moment, in.-lb.	4020	8040	179,040	358,080	1,583,070	3,166,140

As these are very much less than the bending moments calculated from the pressures on the crank-pin, the diameters already found are sufficient for the diameter of the shaft at the fly-wheel hub.

In the case of engines with heavy band fly-wheels and with long fly-wheel shafts it is of the utmost importance to calculate the diameter of the shaft with reference to the bending moment due to the weight of the fly-wheel and the shaft.

B. H. Coffey (*Power*, October, 1892) gives the formula for combined bending and twisting resistance, $T_1 = .196d^3S$, in which $T_1 = B + \sqrt{B^2 + T^2}$; T being the maximum, not the mean twisting moment; and finds empirical working values for $.196S$ as below. He says: Four points should be considered in determining this value: First, the nature of the material; second, the manner of applying the loads, with shock or otherwise; third, the ratio of the bending moment to the torsional moment—the bending moment in a revolving shaft produces reversed strains in the material, which tend to rupture it; fourth, the size of the section. Inch for inch, large sections are weaker than small ones. He puts the dividing line between large and small sections at 10 in. diameter, and gives the following safe values of $S \times .196$ for steel, wrought iron, and cast iron, for these conditions.

VALUE OF $S \times .196$.

Ratio.	Heavy Shafts with Shock.			Light shafts with Shock. Heavy Shafts No Shock.			Light Shafts No Shock.		
B to *T*.	Steel.	Wro't Iron.	Cast Iron.	Steel.	Wro't Iron.	Cast Iron.	Steel.	Wro't Iron.	Cast Iron.
3 to 10 or less	1045	880	440	1566	1320	660	2090	1760	880
3 to 5 or less	941	785	393	1410	1179	589	1882	1570	785
1 to 1 or less	855	715	358	1281	1074	537	1710	1430	715
B greater than *T*	784	655	328	1176	984	492	1568	1310	655

Mr. Coffey gives as an example of improper dimensions the fly-wheel shaft of a 1500 H.P. engine at Willimantic, Conn., which broke while the engine was running at 425 H.P. The shaft was 17 ft. 5 in. long between centres of bearings, 18 in. diam. for 8 ft. in the middle, and 15 in. diam. for the remainder, including the bearings. It broke at the base of the fillet connecting the two large diameters, or 56⅛ in. from the centre of the bearing. He calculates the mean torsional moment to be 446,654 inch-pounds, and the maximum at twice the mean; and the total weight on one bearing at 87,530 lbs., which, multiplied by 56⅛ in., gives 4,945,445 in.-lbs. bending moment at the fillet. Applying the formula $T_1 = B + \sqrt{B^2 + T^2}$, gives for equivalent twisting moment 9,971,045 in.-lbs. Substituting this value in the formula $T_1 = .196, Sd^3$ gives for S the shearing strain 15,070 lbs. per sq. in., or if the metal had a shearing strength of 45,000 lbs., a factor of safety of only 3. Mr. Coffey considers that 6000 lbs. is all that should be allowed for S under these circumstances. This would give $d = 20.35$ in. If we take from Mr. Coffey's table a value of $.196S = 1100$, we obtain $d^3 = 9000$ nearly, or $d = 20.8$ in., instead of 15 in., the actual diameter.

Length of Shaft-bearings.—There is as great a difference of opinion among writers, and as great a variation in practice concerning length of journal-bearings, as there is concerning crank-pins. The length of a journal being determined from considerations of its heating, the observations concerning heating of crank-pins apply also to shaft-bearings, and the formulæ for length of crank-pins to avoid heating may also be used, using for the total load upon the bearing the resultant of all the pressures brought upon it, by the pressure on the crank, by the weight of the fly-wheel, and by the pull of the belt. After determining this pressure, however, we must resort to empirical values for the so-called constants of the formulæ, really variables, which depend on the power of the bearing to carry away heat, and upon the quantity of heat generated, which latter depends on the pressure, on the number of square feet of rubbing surface passed over in a minute, and upon the coefficient of friction. This coefficient is an exceedingly variable quantity, ranging from .01 or less with perfectly polished journals, having end-play, and lubricated by a pad or oil-bath, to .10 or more with ordinary oil-cup lubrication.

For shafts resisting torsion only, Marks gives for length of bearing $l = .0000247fpND^2$, in which f is the coefficient of friction, p the mean pressure in pounds per square inch on the piston, N the number of single strokes per minute, and D the diameter of the piston. For shafts under the combined stress due to pressure on the crank-pin, weight of fly-wheel, etc., he gives the following: Let Q = reaction at bearing due to weight, S = stress due steam pressure on piston, and R_1 = the resultant force; for horizontal engines, $R_1 = \sqrt{Q^2 + S^2}$, for vertical engines $R_1 = Q + S$, when the pressure on the crank is in the same direction as the pressure of the shaft on its bearings, and $R_1 = Q - S$ when the steam pressure tends to lift the shaft from its bearings. Using empirical values for the work of friction per square inch of projected area, taken from dimensions of crank-pins in marine vessels, he finds the formula for length of shaft-journals $l = .0000325fR_1N$, and recommends that to cover the defects of workmanship, neglect of oiling, and the introduction of dust, f be taken at .16 or even greater. He says that 500 lbs. per sq. in. of projected area may be allowed for steel or wrought-iron shafts in brass bearings with good results if a less pressure is not attainable without inconvenience. Marks says that the use of empirical rules that do not take account of the number of turns per minute has resulted in bearings much too long for slow-speed engines and too short for high-speed engines.

Whitham gives the same formula, with the coefficient .00002575.

Thurston says that the maximum allowable mean intensity of pressure may be, for all cases, computed by his formula for journals, $l = \frac{PV}{60{,}000d}$, or by Rankine's, $l = \frac{P(V+20)}{44{,}800d}$, in which P is the mean total pressure in pounds, V the velocity of rubbing surface in feet per minute, and d the diameter of the shaft in inches. It must be borne in mind, he says, that the friction work on the main bearing next the crank is the sum of that due the action of the piston on the pin, and that due that portion of the weight of wheel and shaft and of pull of the belt which is carried there. The outboard bearing carries practically only the latter two parts of the total. The crank-shaft journals will be made longer on one side, and perhaps shorter on the other, than that of the crank-pin, in proportion to the work falling upon each, i.e., to their respective products of mean total pressure, speed of rubbing surfaces, and coefficients of friction.

Unwin says: Journals running at 150 revolutions per minute are often only one diameter long. Fan shafts running 150 revolutions per minute have journals six or eight diameters long. The ordinary empirical mode of proportioning the length of journals is to make the length proportional to the diameter, and to make the ratio of length to diameter increase with the speed. For wrought-iron journals:

Revs. per min. =	50	100	150	200	250	500	1000	$\frac{l}{d} = .004R + 1.$
Length ÷ diam. =	1.2	1.4	1.6	1.8	2.0	3.0	5.0.	

Cast-iron journals may have $l \div d$ = 9/10, and steel journals $l \div d$ = 1¼, of the above values.

Unwin gives the following, calculated from the formula $l = \frac{0.4 \text{ H.P.}}{r}$, in which r is the crank radius in inches, and H.P. the horse-power transmitted to the crank-pin.

THEORETICAL JOURNAL LENGTH IN INCHES.

Load on Journal in pounds.	Revolutions of Journal per minute.					
	50	100	200	300	500	1000
1,000	.2	.4	.8	1.2	2.	4.
2,000	.4	.8	1.6	2.4	4.	8.
4,000	.8	1.6	3.2	4.8	8.	16.
5,000	1.0	2.	4.	6.	10.	20.
10,000	2.	4.	8.	12.	20.	40.
15,000	3.	6.	12.	18.	30.	....
20,000	4.	8.	16.	24.	40.	....
30,000	6.	12.	24.	36.	....	....
40,000	8.	16.	32.	....	....	....
50,000	10.	20.	40.	....	....	....

Applying these different formulæ to our six engines, we have:

Engine No.	1	2	3	4	5	6
Diam. cyl.	10	10	30	30	50	50
Horse-power	50	50	450	450	1,250	1,250
Revs. per min.	250	125	130	65	90	45
Mean pressure on crank-pin $= S$	3,299	3,299	23,185	23,185	58,905	58,905
Half wt. of fly-wheel and shaft $= Q$	268	536	5,968	11,936	26,470	52,940
Resultant press. on bearing $\sqrt{Q^2 + S^2} = R_1$	3,310	3,335	23,924	26,194	64,580	79,200
Diam. of shaft journal	3.84	4.39	11.35	12.99	20.58	21.52
Length of shaft journal:						
Marks, $l = .0000325 f R_1 N (f = .10)$	5.88	2.71	20.87	11.07	37.78	23.17
Whitham, $l = .0000515 f R_1 R (f = 10)$.	4.27	2.15	16.53	8.77	29.95	18.35
Thurston, $l = \frac{PV}{60,000d}$	3.61	1.82	14.00	7.43	25.86	15.55
Rankine, $l = \frac{P(V + 20)}{44,800d}$	5.22	3.78	21.70	10.88	35.16	22.47
Unwin, $l = (.004R + 1)d$	7.68	6.50	17.25	16.36	27.99	25.39
Unwin, $l = \frac{0.4 \text{ H.P.}}{r}$	3.33	1.60	12.00	6.00	20.83	10.40
Average	4.22	2.99	17.05	10.00	29.54	19.22

If we divide the mean resultant pressure on the bearing by the projected area, that is, by the product of the diameter and length of the journal, using the greatest and smallest length out of the seven lengths for each journal given above, we obtain the pressure per square inch upon the bearing, as follows:

Engine No.	1	2	3	4	5	6
Pressure per sq. in., shortest journal.	259	455	176	236	151	353
Longest journal	112	115	97	123	83	145
Average journal	175	254	124	209	106	191
Journal of length = diam.	.....	173	.....	155	.....	175

Many of the formulæ give for the long-stroke engines a length of journal less than the diameter, but such short journals are rarely used in practice. The last line in the above table has been calculated on the supposition that the journals of the long-stroke engines are made of a length equal to the diameter.

In the dimensions of Corliss engines given by J. B. Stanwood (*Eng.*, June 12, 1891), the lengths of the journals for engines of diam. of cyl. 10 to 20 in. are the same as the diam. of the cylinder, and a little more than twice the diam. of the journal. For engines above 20 in. diam. of cyl. the ratio of length to diam. is decreased so that an engine of 30 in. diam. has a journal 26 in. long, its diameter being 14½ in. These lengths of journal are greater than those given by any of the formulæ above quoted.

There thus appears to be a hopeless confusion in the various formulæ for length of shaft journals, but this is no more than is to be expected from the variation in the coefficient of friction, and in the heat-conducting power of journals in actual use, the coefficient varying from .10 (or even .16 as given by Marks) down to .01, according to the condition of the bearing surfaces and the efficiency of lubrication. Thurston's formula, $l = \frac{PV}{60,000d}$, reduces to the form $l = .0000043\!63PR$, in which P = mean total load on journal, and R = revolutions per minute. This is of the same form as Marks' and Whitham's formulæ, in which, if f the coefficient of friction be taken at .10, the coefficients of PR are, respectively, .0000065 and .00000515. Taking the mean of these three formulæ, we have $l = .0000053PR$, if $f = .10$ or $l = .000053fPR$ for any other value of f. The author believes this to be as safe a formula as any for length of journals, with the limitation that if it brings a result of length of journal less than the diameter, then the length should be made equal to the diameter. Whenever with $f = .10$ it gives a length which is inconvenient or impossible of construction on account of limited space, then provision should be made to reduce the value of the coefficient of friction below .10 by means of forced lubrication, end play, etc., and to carry away the heat, as by water-cooled journal-boxes. The value of P should be taken as the resultant of the mean pressure on the crank, and the load brought on the bearing by the weight of the shaft, fly-wheel, etc., as calculated by the formula already given, viz., $R_1 = \sqrt{Q^2 + S^2}$ for horizontal engines, and $R_1 = Q + S$ for vertical engines.

For our six engines the formula $l = .0000053PR$ gives, with the limitation for the long-stroke engines that the length shall not be less than the diameter, the following:

Engine No.	1	2	3	4	5	6
Length of journal	4.39	4.39	16.48	12.99	30.80	21.62
Pressure per square inch on journal	196	173	128	155	102	171

Crank-shafts with Centre-crank and Double-crank Arms.—In centre-crank engines, one of the crank-arms, and its adjoining journal, called the after journal, usually transmit the power of the engine to the work to be done, and the journal resists both twisting and bending moments, while the other journal is subjected to bending moment only. For the after crank-journal the diameter should be calculated the same as for an overhung crank, using the formula for combined bending and twisting moment, $T_1 = B + \sqrt{B^2 + T^2}$, in which T_1 is the equivalent twisting moment, B the bending moment, and T the twisting moment. This value of T_1 is to be used in the formula diameter $= \sqrt[3]{\frac{5.1T}{S}}$. The bending moment is taken as the maximum load on piston multiplied by one fourth of the length of the crank-shaft between middle points of the two journal bearings, if the centre crank is midway between the bearings, or by one half the distance measured parallel to the shaft from the middle of the crank-pin to the middle of the after bearing. This supposes the crank-shaft to be a beam loaded at its middle and supported at the ends, but Whitham would make the bending moment only one half of this, considering the shaft to be a beam secured or fixed at the ends, with a point of contraflexure one fourth of the length from the end. The first supposition is the safer, but since the bending moment will in any case be much less than the twisting moment, the resulting diameter will be but little greater than if Whitham's supposition is used. For the forward journal, which is subjected to bending moment only, diameter of shaft $= \sqrt[3]{\frac{10.2B}{S}}$, in which B

THEORETICAL JOURNAL LENGTH IN INCHES.

Load on Journal in pounds.	Revolutions of Journal per minute.					
	50	100	200	300	500	1000
1,000	.2	.4	.8	1.2	2.	4.
2,000	.4	.8	1.6	2.4	4.	8.
4,000	.8	1.6	3.2	4.8	8.	16.
5,000	1.0	2.	4.	6.	10.	20.
10,000	2.	4.	8.	12.	20.	40.
15,000	3.	6.	12.	18.	30.	....
20,000	4.	8.	16.	24.	40.	....
30,000	6.	12.	24.	36.	....	....
40,000	8.	16.	32.	....	....	....
50,000	10.	20.	40.	....	....	....

Applying these different formulæ to our six engines, we have:

Engine No.	1	2	3	4	5	6
Diam. cyl.	10	10	30	30	50	50
Horse-power	50	50	450	450	1,250	1,250
Revs. per min.	250	125	130	65	90	45
Mean pressure on crank-pin $= S$	3,299	3,299	23,185	23,185	58,905	58,905
Half wt. of fly-wheel and shaft $= Q$	268	536	5,968	11,936	26,470	52,940
Resultant press. on bearing $\sqrt{Q^2 + S^2} = R_1$	3,310	3,335	23,924	26,194	64,580	79,200
Diam. of shaft journal	3.84	4.39	11.25	12.99	20.58	21.52
Length of shaft journal:						
Marks, $l = .0000325fR_1N(f = .10)$	5.38	2.71	20.87	11.07	37.78	23.17
Whitham, $l = .0000515fR_1R(f = 10)$.	4.27	2.15	16.58	8.77	29.95	18.35
Thurston, $l = \frac{PV}{60,000d}$	3.61	1.82	14.00	7.42	25.26	15.55
Rankine, $l = \frac{P(V+20)}{44,800d}$	5.22	2.78	21.70	10.86	35.16	22.47
Unwin, $l = (.004R + 1)d$	7.68	6.50	17.25	16.36	27.99	28.39
Unwin, $l = \frac{0.4\ \text{H.P.}}{r}$	3.33	1.60	12.00	6.00	20.83	10.42
Average	4.92	2.99	17.05	10.00	29.54	19.22

If we divide the mean resultant pressure on the bearing by the projected area, that is, by the product of the diameter and length of the journal, using the greatest and smallest length out of the seven lengths for each journal given above, we obtain the pressure per square inch upon the bearing, as follows:

Engine No	1	2	3	4	5	6
Pressure per sq. in., shortest journal.	259	455	176	326	151	353
Longest journal	112	115	97	123	83	145
Average journal	175	254	124	209	106	191
Journal of length = diam	.....	173	.....	155	.....	175

Many of the formulæ give for the long-stroke engines a length of journal less than the diameter, but such short journals are rarely used in practice. The last line in the above table has been calculated on the supposition that the journals of the long-stroke engines are made of a length equal to the diameter.

In the dimensions of Corliss engines given by J. B. Stanwood (*Eng.*, June 12, 1891), the lengths of the journals for engines of diam. of cyl. 10 to 20 in. are the same as the diam. of the cylinder, and a little more than twice the diam. of the journal. For engines above 20 in. diam. of cyl. the ratio of length to diam. is decreased so that an engine of 30 in. diam. has a journal 28 in. long, its diameter being 14½ in. These lengths of journal are greater than those given by any of the formulæ above quoted.

There thus appears to be a hopeless confusion in the various formulæ for length of shaft journals, but this is no more than is to be expected from the variation in the coefficient of friction, and in the heat-conducting power of journals in actual use, the coefficient varying from .10 (or even .16 as given by Marks) down to .01, according to the condition of the bearing surfaces and the efficiency of lubrication. Thurston's formula, $l = \frac{PV}{60,000d}$, reduces to the form $l = .0000043 63PR$, in which P = mean total load on journal, and R = revolutions per minute. This is of the same form as Marks' and Whitham's formulæ, in which, if f the coefficient of friction be taken at .10, the coefficients of PR are, respectively, .0000065 and .00000515. Taking the mean of these three formulæ, we have $l = .0000053PR$, if $f = .10$ or $l = .000053fPR$ for any other value of f. The author believes this to be as safe a formula as any for length of journals. with the limitation that if it brings a result of length of journal less than the diameter, then the length should be made equal to the diameter. Whenever with $f = .10$ it gives a length which is inconvenient or impossible of construction on account of limited space, then provision should be made to reduce the value of the coefficient of friction below .10 by means of forced lubrication, end play, etc., and to carry away the heat, as by water-cooled journal-boxes. The value of P should be taken as the resultant of the mean pressure on the crank, and the load brought on the bearing by the weight of the shaft, fly-wheel, etc., as calculated by the formula already given, viz., $R_1 = \sqrt{Q^2 + S^2}$ for horizontal engines, and $R_1 = Q + S$ for vertical engines.

For our six engines the formula $l = .0000053PR$ gives, with the limitation for the long-stroke engines that the length shall not be less than the diameter, the following:

Engine No.	1	2	3	4	5	6
Length of journal	4.39	4.39	16.48	12.99	30.80	21.52
Pressure per square inch on journal	196	173	128	155	102	171

Crank-shafts with Centre-crank and Double-crank Arms.—In centre-crank engines, one of the crank-arms, and its adjoining journal, called the after journal, usually transmit the power of the engine to the work to be done, and the journal resists both twisting and bending moments, while the other journal is subjected to bending moment only. For the after crank-journal the diameter should be calculated the same as for an overhung crank, using the formula for combined bending and twisting moment, $T_1 = B + \sqrt{B^2 + T^2}$, in which T_1 is the equivalent twisting moment, B the bending moment, and T the twisting moment. This value of T_1 is to be used in the formula diameter $= \sqrt[3]{\frac{5.1T}{S}}$. The bending moment is taken as the maximum load on piston multiplied by one fourth of the length of the crank-shaft between middle points of the two journal bearings, if the centre crank is midway between the bearings, or by one half the distance measured parallel to the shaft from the middle of the crank-pin to the middle of the after bearing. This supposes the crank-shaft to be a beam loaded at its middle and supported at the ends, but Whitham would make the bending moment only one half of this, considering the shaft to be a beam secured or fixed at the ends, with a point of contraflexure one fourth of the length from the end. The first supposition is the safer, but since the bending moment will in any case be much less than the twisting moment, the resulting diameter will be but little greater than if Whitham's supposition is used. For the forward journal, which is subjected to bending moment only, diameter of shaft $= \sqrt[3]{\frac{10.2B}{S}}$, in which B

is the maximum bending moment and S the safe shearing strength of the metal per square inch.

For our six engines, assuming them to be centre-crank engines, and considering the crank-shaft to be a beam supported at the ends and loaded in the middle, and assuming lengths between centres of shaft bearings as given below, we have:

Engine No.	1	2	3	4	5	6
Length of shaft, assumed, inches, L	20	24	48	60	76	96
Max. press. on crank-pin, P	7,854	7,854	70,686	70,686	196,350	196,350
Max. bending moment, $B = \frac{1}{4}PL$, inch-lbs.	39,270	49,637	848,232	1,060,290	3,729,750	4,712,400
Twisting moment, T	47,124	94,248	1,060,290	2,120,580	4,712,400	9,424,800
Equiv. twisting moment, $B + \sqrt{B^2 + T^2}$	101,000	156,000	2,208,000	3,430,000	9,740,000	15,240,000
Diameter of after journal, $d = \sqrt[3]{\frac{5.1T_1}{8000}}$	3.98	4.60	11.15	13.00	18.35	21.20
Diam. of forward journal, $d_1 = \sqrt[3]{\frac{10.2B}{8000}}$	3.68	3.99	10.28	11.16	16.82	18.18

The lengths of the journals would be calculated in the same manner as in the case of overhung cranks, by the formula $l = .000063fPR$, in which P is the resultant of the mean pressure due to pressure of steam on the piston, and the load of the fly-wheel, shaft, etc., on each of the two bearings. Unless the pressures are equally divided between the two bearings, the calculated lengths of the two will be different; but it is usually customary to make them both of the same length, and in no case to make the length less than the diameter. The diameters also are usually made alike for the two journals, using the largest diameter found by calculation.

The crank-pin for a centre crank should be of the same length as for an overhung crank, since the length is determined from considerations of heating, and not of strength. The diameter also will usually be the same, since it is made great enough to make the pressure per square inch on the projected area (product of length by diameter) small enough to allow of free lubrication, and the diameter so calculated will be greater than is required for strength.

Crank-shaft with Two Cranks coupled at 90°.—If the whole power of the engine is transmitted through the after journal of the after crank-shaft, the greatest twisting moment is equal to 1.414 times the maximum twisting moment due to the pressure on one of the crank-pins. If T = the maximum twisting moment produced by the steam-pressure on one of the pistons, then T_1 the maximum twisting moment on the after part of the crank-shaft, and on the line-shaft, produced when each crank makes an angle of 45° with the centre line of the engine, is $1.414T$. Substituting this value in the formula for diameter to resist simple torsion, viz., $d = \sqrt[3]{\frac{5.1T}{S}}$, we have $d = \sqrt[3]{\frac{5.1 \times 1.414T}{S}}$, or $d = 1.932\sqrt[3]{\frac{T}{S}}$, in which T is the maximum twisting moment produced by one of the pistons, d = diameter in inches, and S = safe working shearing strength of the material. For the forward journal of the after crank, and the after journal of the forward crank, the torsional moment is that due to the pressure of steam on the forward piston only, and for the forward journal of the forward crank, if none of the power of the engine is transmitted through it, the torsional moment is zero, and its diameter is to be calculated for bending moment only.

For Combined Torsion and Flexure.—Let B_1 = bending moment on either journal of the forward crank due to maximum pressure on forward piston, B_2 = bending moment on either journal of the after crank due to maximum pressure on after piston, T_1 = maximum twisting moment on after journal of forward crank, and T_2 = maximum twisting moment on after journal of after crank due to pressure on the after piston.

Then equivalent twisting moment on after journal of forward crank $= B_1 + \sqrt{B_1^2 + T_1^2}$.

On forward journal of after crank $= B_2 + \sqrt{B_2^2 + T_1^2}$.

On after journal of after crank $= B_2 + \sqrt{B_2^2 + (T_1 + T_2)^2}$.

These values of equivalent twisting moment are to be used in the formula for diameter of journals $d = \sqrt[3]{\frac{5.1T}{S}}$. For the forward journal of the forward crank-shaft $d = \sqrt[3]{\frac{10.2B_1}{S}}$.

It is customary to make the two journals of the forward crank of one diameter, viz., that calculated for the after journal.

For a Three-cylinder Engine with cranks at 120°, the greatest twisting moment on the after part of the shaft, if the maximum pressures on the three pistons are equal, is equal to twice the maximum pressure on any one piston, and it takes place when two of the cranks make angles of 30° with the centre line, the third crank being at right angles to it. (For demonstration, see Whitham's "Steam-engine Design," p. 252.) For combined torsion and flexure the same method as above given for two crank engines is adopted for the first two cranks; and for the third, or after crank, if all the power of the three cylinders is transmitted through it, we have the equivalent twisting moment on the forward journal $= B_3 + \sqrt{B_3^2+(T_1+T_2)^2}$, and on the after journal $= B_3 + \sqrt{B_3^2 + (T_1 + T_2 + T_3)^2}$, B_3 and T_3 being respectively the bending and twisting moments due to the pressure on the third piston.

Crank-shafts for Triple-expansion Marine Engines, according to an article in *The Engineer*, April 25, 1890, should be made larger than the formulæ would call for, in order to provide for the stresses due to the racing of the propeller in a sea-way, which can scarcely be calculated. A kind of unwritten law has sprung up for fixing the size of a crank-shaft, according to which the diameter of the shaft is made about $0.45D$, where D is the diameter of the high-pressure cylinder. This is for solid shafts. When the speeds are high, as in war-ships, and the stroke short, the formula becomes $0.4D$, even for hollow shafts.

The Valve-stem or Valve-rod.—The valve-rod should be designed to move the valve under the most unfavorable conditions, which are when the stem acts by thrusting, as a long column, when the valve is unbalanced (a balanced valve may become unbalanced by the joint leaking) and when it is imperfectly lubricated. The load on the valve is the product of the area into the greatest unbalanced pressure upon it per square inch, and the coefficient of friction may be as high as 20%. The product of this coefficient and the load is the force necessary to move the valve, which equals the maximum thrust on the valve-rod. From this force the diameter of the valve-rod may be calculated by Hodgkinson's formula for columns. An empirical formula given by Seaton is: Diam. of rod $= d = \sqrt{\frac{lbp}{F}}$, in which l = length and b = breadth of valve, in inches; p = maximum absolute pressure on the valve in lbs. per sq. in., and F a coefficient whose values are, for iron: long rod 10,000, short 12,000; for steel: long rod 12,000, short 14,500.

Whitham gives the short empirical rule: Diam. of valve-rod = 1/30 diam. of cyl. = ⅛ diam of piston-rod.

Size of Slot-link. (Seaton.)—Let D be the diam. of the valve rod

$$D = \sqrt{\frac{lbp}{12,000}};$$

then Diameter of block-pin when overhung		$= D$.
" " " secured at both ends		$= 0.75 \times D$.
" eccentric-rod pins		$= 0.7 \times D$.
" suspension-rod pins		$= 0.55 \times D$.
" " " pin when overhung		$= 0.75 \times D$.

Breadth of link	$= 0.8$ to $0.9 \times D$.
Length of block	$= 1.8$ to $1.6 \times D$.
Thickness of bars of link at middle	$= 0.7 \times D$.

If a single suspension rod of round section, its diameter $= 0.7 \times D$.
If two suspension rods of round section, their diameter $= 0.55 \times D$.

Size of Double-bar Links.—When the distance between centres of eccentric pins = 6 to 8 times throw of eccentrics (throw = eccentricity = half-travel of valve at full gear) D as before:

Depth of bars	$= 1.25 \times D + \frac{3}{4}$ in.
Thickness of bars	$= 0.5 \times D + \frac{1}{4}$ in.
Length of sliding-block	$= 2.5$ to $3 \times D$.
Diameter of eccentric-rod pins	$= 0.8 \times D + \frac{1}{4}$ in.
" centre of sliding-block	$= 1.3 \times D$.

When the distance between eccentric-rod pins = 5 to 5½ times throw of eccentrics:

Depth of bars	$= 1.25 \times D + \frac{1}{2}$ in.
Thickness of bars	$= 0.5 \times D + \frac{1}{4}$ in.
Length of sliding-block	$= 2.5$ to $3 \times D$.
Diameter of eccentric-rod pins	$= 0.75 \times D$.

Diameter of eccentric bolts (top end) at bottom of thread $= 0.42 \times D$ when of iron, and $0.38 \times D$ when of steel.

The Eccentric.—Diam. of eccentric-sheave $= 2.4 \times$ throw of eccentric $+ 1.2 \times$ diam. of shaft. D as before

Breadth of the sheave at the shaft	$= 1.15 \times D + 0.65$ inch
Breadth of the sheave at the strap	$= D + 0.6$ inch.
Thickness of metal around the shaft	$= 0.7 \times D + 0.5$ inch.
Thickness of metal at circumference	$= 0.6 \times D + 0.4$ inch.
Breadth of key	$= 0.7 \times D + 0.5$ inch.
Thickness of key	$= 0.25 \times D + 0.5$ inch.
Diameter of bolts connecting parts of strap	$= 0.6 \times D + 0.1$ inch.

THICKNESS OF ECCENTRIC-STRAP.

When of bronze or malleable cast iron:

Thickness of eccentric-strap at the middle	$= 0.4 \times D + 0.6$ inch.
" " " " " " sides	$= 0.3 \times D + 0.5$ inch.

When of wrought iron or cast steel:

Thickness of eccentric-strap at the middle	$= 0.4 \times D + 0.5$ inch.
" " " " " " sides	$= 0.27 \times D + 0.4$ inch

The Eccentric-rod.—The diameter of the eccentric-rod in the body and at the eccentric end may be calculated in the same way as that of the connecting-rod, the length being taken from centre of strap to centre of pin. Diameter at the link end $= 0.8D + 0.2$ inch.

This is for wrought-iron; no reduction in size should be made for steel.

Eccentric-rods are often made of rectangular section.

Reversing-gear should be so designed as to have more than sufficient strength to withstand the strain of both the valves and their gear at the same time under the most unfavorable circumstances; it will then have the stiffness requisite for good working.

Assuming the work done in reversing the link-motion, W, to be only that due to overcoming the friction of the valves themselves through their whole travel, then, if T be the travel of valves in inches; for a compound engine

$$W = \frac{T}{12}\left(\frac{l \times b \times p}{5}\right) + \frac{T}{12}\left(\frac{l^1 \times b^1 \times p^1}{5}\right);$$

l^1, b^1 and p^1 being length, breadth and maximum steam-pressure on valve of the second cylinder; and for an expansive engine

$$W = 2 \times \frac{T}{12}\left(\frac{l \times b \times p}{5}\right); \quad \text{or} \quad \frac{T}{30}(l \times b \times p).$$

To provide for the friction of link-motion, eccentrics and other gear, and for abnormal conditions of the same, take the work at one and a half times the above amount.

To find the strain at any part of the gear having motion when reversing, divide the work so found by the space moved through by that part in feet; the quotient is the strain in pounds, and the size may be found from the ordinary rules of construction for any of the parts of the gear. (Seaton.)

Engine-frames or Bed-plates.—No definite rules for the design of engine-frames have been given by authors of works on the steam-engine. The proportions are left to the designer who uses "rule of thumb," or copies from existing engines. F. A. Halsey (*Am. Mach.*, Feb. 14, 1895) has made a comparison of proportions of the frames of horizontal Corliss engines of several builders. The method of comparison is to compute from the measurements the number of square inches in the smallest cross-section of the frame, that is, immediately behind the pillow-block, also to compute the total maximum pressure upon the piston, and to divide the latter quantity by the former. The result gives the number of pounds pressure upon the piston allowed for each square inch of metal in the frame. He finds that the number of pounds per square inch of smallest section of frame ranges from 217 for a 10 × 30-in. engine up to 575 for a 28 × 48-inch. A 30 × 60-inch engine shows 350 lbs., and a 32-inch engine which has been running for many years shows 667 lbs. Generally the strains increase with the size of the engine, and more cross-section of metal is allowed with relatively long strokes than with short ones.

From the above Mr. Halsey formulates the general rule that in engines of moderate speed, and having strokes up to one and one-half times the diameter of the cylinder, the load per square inch of smallest section should be for a 10-inch engine 300 pounds, which figure should be increased for larger bores up to 500 pounds for a 30-inch cylinder of same relative stroke. For high speeds or for longer strokes the load per square inch should be reduced.

FLY-WHEELS.

The function of a fly-wheel is to store up and to restore the periodical fluctuations of energy given to or taken from an engine or machine, and thus to keep approximately constant the velocity of rotation. Rankine calls the quantity $\frac{\Delta E}{2E_0}$ the coefficient of fluctuation of speed or of unsteadiness, in which E_0 is the mean actual energy, and ΔE the excess of energy received or of work performed, above the mean, during a given interval. The ratio of the periodical excess or deficiency of energy ΔE to the whole energy exerted in one period or revolution General Morin found to be from 1/6 to 1/4 for single-cylinder engines using expansion; the shorter the cut-off the higher the value. For a pair of engines with cranks coupled at 90° the value of the ratio is about 1/4, and for three engines with cranks at 120°, 1/12 of its value for single-cylinder engines. For tools working at intervals, such as punching, slotting and plate-cutting machines, coining-presses, etc., ΔE is nearly equal to the whole work performed at each operation.

A fly-wheel reduces the coefficient $\frac{\Delta E}{2E_0}$ to a certain fixed amount, being about 1/32 for ordinary machinery, and 1/50 or 1/60 for machinery for fine purposes.

If m be the reciprocal of the intended value of the coefficient of fluctuation of speed, ΔE the fluctuation or energy, I the moment of inertia of the fly-wheel alone, and a_0 its mean angular velocity, $I = \frac{mg\Delta E}{a_0}$. As the rim of a fly-wheel is usually heavy in comparison with the arms, I may be taken to equal Wr^2, in which W = weight of rim in pounds, and r the radius of the wheel; then $W = \frac{mg\Delta E}{a_0^2 r^2} = \frac{mg\Delta E}{v^2}$, if v be the velocity of the rim in feet per second. The usual mean radius of the fly-wheel in steam-engines is from three to five times the length of the crank. The ordinary values of the product mg, the unit of time being the second, lie between 1000 and 2000 feet. (Abridged from Rankine, S E., p. 62.)

Thurston gives for engines with automatic valve-gear $W = 250{,}000 \frac{ASp}{R^2D^2}$, in which A = area of piston in square inches, S = stroke in feet, p = mean steam-pressure in lbs. per sq. in., R = revolutions per minute, D = outside diameter of wheel in feet. Thurston also gives for ordinary forms of non-condensing engine with a ratio of expansion between 3 and 5, $W = \frac{aAS}{R^3D^2}$, in which a ranges from 10,000,000 to 15,000,000, averaging 12,000,000. For gas-engines, in which the charge is fired with every revolution, the *American Machinist* gives this latter formula, with a doubled, or 24,000,000. Presumably, if the charge is fired every other revolution, a should be again doubled.

Rankine ("Useful Rules and Tables," p. 247) gives $W = 475{,}000 \frac{ASp}{VD^2R^3}$, in which V is the variation of speed per cent. of the mean speed. Thurston's first rule above given corresponds with this if we take V at 1.9 per cent.

Hartnell (Proc. Inst., M. E. 1882, 427) says: The value of V, or the variation permissible in portable engines, should not exceed 3 per cent. with an ordinary load, and 4 per cent when heavily loaded. In fixed engines, for ordinary purposes, $V = 2\frac{1}{2}$ to 3 per cent. For good governing or special purposes, such as cotton-spinning, the variation should not exceed $1\frac{1}{2}$ to 2 per cent.

F. M. Rites (Trans. A. S. M. E., xiv. 100) develops a new formula for weight of rim, viz., $W = \frac{C \times \text{I.H.P.}}{R^3D^2}$, and weight of rim per horse-power $= \frac{C}{R^3D^2}$, in which C varies from 10,000,000,000 to 20,000,000,000; also using the latter value of C, he obtains for the energy of the fly-wheel $\frac{Mv^2}{2} = \frac{W}{64.4} \frac{3.14^2D^2R^2}{3600} = \frac{C \times \text{H.P.}(3.14)^2D^2R^2}{R^3D^2 \times 64.4 \times 3600} = \frac{850{,}000\ \text{H.P.}}{R}$. Fly-wheel energy per H.P. $= \frac{850{,}000}{R}$.

The limit of variation of speed with such a weight of wheel from excess of power per fraction of revolution is less than .0023.

The value of the constant C given by Mr. Rites was derived from practice of the Westinghouse single-acting engines used for electric-lighting. For double-acting engines in ordinary service a value of $C = 5{,}000{,}000{,}000$ would probably be ample.

From these formulæ it appears that the weight of the fly-wheel for a given horse-power should vary inversely with the cube of the revolutions and the square of the diameter.

J. B. Stanwood (*Eng'g*, June 12, 1891) says: Whenever 480 feet is the lowest piston-speed probable for an engine of a certain size, the fly-wheel weight for that speed approximates closely to the formula

$$W = 700{,}000 \frac{d^2s}{D^2R^2}.$$

W = weight in pounds, d = diameter of cylinder in inches, s = stroke in inches, D = diameter of wheel in feet, R = revolutions per minute, corresponding to 480 feet piston-speed.

In a Ready Reference Book published by Mr. Stanwood, Cincinnati, 1892, he gives the same formula, with coefficients as follows: For slide-valve engines, ordinary duty, 850,000; same, electric-lighting, 700,000; for automatic high-speed engines, 1,000,000; for Corliss engines, ordinary duty 700,000, electric-lighting 1,000,000.

Thurston's formula above given, $W = \frac{aAS}{R^3D^2}$, with $a = 12{,}000{,}000$, when reduced to terms of d and s in inches, becomes $W = 785{,}400 \frac{d^2s}{R^3D^2}$.

If we reduce it to terms of horse-power, we have I.H.P. $= \frac{2ASPR}{33{,}000}$, in which P = mean effective pressure. Taking this at 40 lbs., we obtain $W = 5{,}000{,}000{,}000 \frac{\text{I.H.P.}}{R^3D^2}$. If mean effective pressure = 30 lbs., then $W = 6{,}666{,}000{,}000 \frac{\text{I.H.P.}}{R^3D^2}$.

Emil Theiss (*Am. Mach.*, Sept. 7 and 14, 1893) gives the following values of δ, the coefficient of steadiness, which is the reciprocal of what Rankine calls the coefficient of fluctuation:

For engines operating—

Hammering and crushing machinery.............	$d = 5$
Pumping and shearing machinery................	$d = 20$ to 30
Weaving and paper-making machinery...........	$d = 40$
Milling machinery................................	$d = 50$
Spinning machinery..............................	$d = 50$ to 100
Ordinary driving-engines (mounted on bed-plate), belt transmission..............................	$d = 35$
Gear-wheel transmission........................	$d = 50$

Mr. Theiss's formula for weight of fly-wheel in pounds is $W = i \times \frac{d \times \text{I.H.P.}}{V^2 \times n}$, where d is the coefficient of steadiness, V the mean velocity of the fly-wheel rim in feet per second, n the number of revolutions per minute, i = a coefficient obtained by graphical solution, the values of which for different conditions are given in the following table. In the lines under "cut-off," p means "compression to initial pressure," and O "no compression":

VALUES OF *i*. SINGLE-CYLINDER NON-CONDENSING ENGINES.

Piston-speed, ft. per min.	Cut-off, 1/6.		Cut-off, 1/4.		Cut-off, 1/3.		Cut-off, 1/2.	
	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*
200	272,690	218,580	242,010	209,170	220,760	201,920	193,340	182,840
400	240,810	187,430	208,200	179,460	188,510	170,040	174,680	167,860
600	194,670	145,400	168,590	136,460	165,210	146,610	....	....
800	158,200	108,690	162,070	135,260	... ...	....	....	....

SINGLE-CYLINDER CONDENSING ENGINES.

Piston-speed, ft. per min.	Cut-off, 1/8.		Cut-off, 1/6.		Cut-off, 1/4.		Cut-off, 1/3.		Cut-off, 1/2.	
	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*
200	265,560	176,560	234,160	173,660	204,210	167,140	189,600	161,830	172,690	156,990
400	194,550	117,870	174,880	118,350	164,720	133,080	174,630	151,680	....	....
600	148,780	140,090	....	... ..	......	......	... ...	....	....	....

TWO-CYLINDER ENGINES, CRANKS AT 90°.

Piston-speed, ft. per min.	Cut-off, 1/6.		Cut-off, 1/4.		Cut-off, 1/3.		Cut-off, 1/2.	
	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*
200	71,980		59,420		49,272			
400	70,160	Mean 60,140	57,000	Mean 54,340	49,150	Mean 50,000	37,920	Mean 36,950
600	70,040		57,480		49,220		35,500	
800	70,040		60,140					

THREE-CYLINDER ENGINES, CRANKS AT 120°.

Piston-speed, ft. per Min.	Cut-off, 1/6.		Cut-off, 1/4.		Cut-off, 1/3.		Cut-off, 1/2.	
	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*	Comp. *p*	*O*
200	33,810	32,240	33,810	35,500	34,540	33,450	35,260	32,370
800	30,190	31,570	35,140	33,810	36,470	32,850	33,810	32,370

As a mean value of i for these engines we may use 33,810.

Centrifugal Force in Fly-wheels.—Let W = weight of rim in pounds; R = mean radius of rim in feet; r = revolutions per minute, g = 32.16; v = velocity of rim in feet per second = $2\pi Rr \div 60$.

$$\text{Centrifugal force of whole rim} = F = \frac{Wv^2}{gR} = \frac{4W\pi^2Rr^2}{3600g} = .000341WRr^2.$$

The resultant, acting at right angles to a diameter of half of this force, tends to disrupt one half of the wheel from the other half, and is resisted by the section of the rim at each end of the diameter. The resultant of half the radial forces taken at right angles to the diameter is $1 \div \frac{1}{2}\pi = \frac{2}{\pi}$ of the sum of these forces; hence the total force F is to be divided by $2 \times 2 \times 1.5708 = 6.2832$ to obtain the tensile strain on the cross-section of the rim, or, total strain on the cross-section $= S = .0000542WRr^2$. The weight W_1 of a rim of cast iron 1 inch square in section is $2\pi R \times 3.125 = 19.635R$ pounds, whence strain per square inch of sectional area of rim $= S_1 = .0010656R^2r^2 = .0002664D^2r^2 = .0000270V^2$, in which D = diameter of wheel in feet, and V is velocity of rim in feet per minute. $S_1 = .0972v^2$, if v is taken in feet per second.

For wrought iron.......... $S_1 = .0011366R^2r^2 = .0002842D^2r^2 = .0000288V^2$.
For steel......... .... $S_1 = .0011593R^2r^2 = .0002901D^2r^2 = .0000294V^2$.
For wood.................. $S_1 = .0000888R^2r^2 = .0000222D^2r^2 = .00000225V^2$.

The specific gravity of the wood being taken at 0.6 = 37.5 lbs. per cu. ft., or 1/12 the weight of cast iron.

Example.—Required the strain per square inch in the rim of a cast-iron wheel 30 ft. diameter, 60 revolutions per minute.
Answer. $15^2 \times 60^2 \times .0010656 = 863.1$ lbs.

Required the strain per square inch in a cast-iron wheel-rim running a mile a minute. *Answer.* $.000027 \times 5280^2 = 752.7$ lbs.

In cast-iron fly-wheel rims, on account of their thickness, there is difficulty in securing soundness, and a tensile strength of 10,000 lbs. per sq. in. is as much as can be assumed with safety. Using a factor of safety of 10 gives a maximum allowable strain in the rim of 1000 lbs. per sq. in., which corresponds to a rim velocity of 6085 ft. per minute.

For any given material, as cast iron, the strength to resist centrifugal force depends only on the velocity of the rim, and not upon its bulk or weight.

Chas. E. Emery (*Cass. Mag.*, 1892) says: By calculation half the strength of the arms is available to strengthen the rim, or a trifle more if the fly-wheel centres are relatively large. The arms, however, are subject to transverse strains, from belts and from changes of speed, and there is, moreover, no certainty that the arms and rim will be adjusted so as to pull exactly together in resisting disruption, so the plan of considering the rim by itself and making it strong enough to resist disruption by centrifugal force within safe limits, as is assumed in the calculations above, is the safer way.

It does not appear that fly-wheels of customary construction should be unsafe at the comparatively low speeds now in common use if proper materials are used in construction. The cause of rupture of fly-wheels that have failed is usually either the "running away" of the engine, such as may be caused by the breaking or slackness of a governor-belt, or incorrect design or defective materials of the fly-wheel.

Chas. T. Porter (Trans. A. S. M. E., xiv. 808) states that no case of the bursting of a fly-wheel with a solid rim in a high-speed engine is known. He attributes the bursting of wheels built in segments to insufficient strength of the flanges and bolts by which the segments are held together. (See also Thurston, "Manual of the Steam-engine," Part II, page 413, etc.)

Arms of Fly-wheels and Pulleys.—Professor Torrey (*Am. Mach.*, July 30, 1891) gives the following formula for arms of elliptical cross-section of cast-iron wheels:

W = load in pounds acting on one arm; S = strain on belt in pounds per inch of width, taken at 56 for single and 112 for double belts; v = width of belt in inches; n = number of arms; L = length of arm in feet; b = breadth of arm at hub; d = depth of arm at hub, both in inches: $W = \frac{Sv}{n}$; $b = \frac{WL}{20d^2}$. The breadth of the arm is its least dimension = minor axis of the ellipse, and the depth the major axis. This formula is based on a factor of safety of 10.

In using the formula, first assume some depth for the arm, and calculate the required breadth to go with it. If it gives too round an arm, assume the breadth a little greater, and repeat the calculation. A second trial will almost always give a good section.

The size of the arms at the hub having been calculated, they may be somewhat reduced at the rim end. The actual amount cannot be calculated, as there are too many unknown quantities. However, the depth and breadth can be reduced about one third at the rim without danger, and this will give a well-shaped arm.

Pulleys are often cast in halves, and bolted together. When this is done the greatest care should be taken to provide sufficient metal in the bolts. This is apt to be the very weakest point in such pulleys. The combined area of the bolts at each joint should be about 28/100 the cross-section of the pulley at that point. (Torrey.)

Unwin gives $$d = 0.6337\sqrt[3]{\frac{BD}{n}} \text{ for single belts;}$$

$$d = 0.798\sqrt[3]{\frac{BD}{n}} \text{ for double belts;}$$

D being the diameter of the pulley, and B the breadth of the rim, both in inches. These formulæ are based on an elliptical section of arm in which $b = 0.4d$ or $d = 2.5b$ on a width of belt $= 4/5$ the width of the pulley rim, a maximum driving force transmitted by the belt of 56 lbs. per inch of width for a single belt and 112 lbs. for a double belt, and a safe working stress of cast iron of 2250 lbs. per square inch.

If in Torrey's formula we make $b = 0.4d$, it reduces to

$$b = \sqrt[3]{\frac{WL}{187.5}}; \quad d = \sqrt[3]{\frac{WL}{12}}.$$

Example.—Given a pulley 10 feet diameter; 8 arms, each 4 feet long; face, 36 inches wide; belt, 30 inches: required the breath and depth of the arm at the hub. According to Unwin,

$$d = 0.6337\sqrt[3]{\frac{BD}{n}} = 0.633\sqrt[3]{\frac{36 \times 120}{8}} = 5.16 \text{ for single belt, } b = 2.06;$$

$$d = 0.798\sqrt[3]{\frac{BD}{n}} = 0.798\sqrt[3]{\frac{36 \times 120}{8}} = 6.50 \text{ for double belt, } b = 2.60.$$

According to Torrey, if we take the formula $b = \frac{WL}{80d^2}$ and assume $d = 5$ and 6.5 inches, respectively, for single and double belts, we obtain $b = 1.08$ and 1.33, respectively, or practically only one half of the breadth according to Unwin, and, since transverse strength is proportional to breadth, an arm only one half as strong.

Torrey's formula is said to be based on a factor of safety of 10, but this factor can be only apparent and not real, since the assumption that the strain on each arm is equal to the strain on the belt divided by the number of arms, is, to say the least, inaccurate. It would be more nearly correct to say that the strain of the belt is divided among half the number of arms. Unwin makes the same assumption in developing his formula, but says it is only in a rough sense true, and that a large factor of safety must be allowed. He therefore takes the low figure of 2250 lbs. per square inch for the safe working strength of cast iron. Unwin says that his equations agree well with practice.

Diameters of Fly-wheels for Various Speeds.—If 6000 feet per minute be the maximum velocity of rim allowable, then $6000 = \pi RD$, in which $R =$ revolutions per minute, and $D =$ diameter of wheel in feet, whence $D = \frac{6000}{\pi R} = \frac{1910}{R}$.

MAXIMUM DIAMETER OF FLY-WHEEL ALLOWABLE FOR DIFFERENT NUMBERS OF REVOLUTIONS.

Revolutions per minute.	Assuming Maximum Speed of 5000 feet per minute.		Assuming Maximum Speed of 6000 feet per minute.	
	Circum. ft.	Diam. ft.	Circum. ft.	Diam. ft.
40	125	39.8	150.	47.7
50	100	31.8	120.	38.2
60	83.3	26.5	100.	31.8
70	71.4	22.7	85.72	27.3
80	62.5	19.9	75.00	23.9
90	55.5	17.7	66.66	21.2
100	50.	15.9	60.00	19.1
120	41.67	13.3	50.00	15.9
140	35.71	11.4	42.86	13.6
160	31.25	9.9	37.5	11.9
180	27.77	8.8	33.33	10.6
200	25.00	8.0	30.00	9.6
220	22.73	7.2	27.27	8.7
240	20.83	6.6	25.00	8.0
260	19.23	6.1	23.08	7.3
280	17.86	5.7	21.43	6.8
300	16.66	5.3	20.00	6.4
350	14.29	4.5	17.14	5.5
400	12.5	4.0	15.00	4.8
450	11.11	3.5	13.33	4.2
500	10.00	3.2	12.00	3.8

Strains in the Rims of Fly-band Wheels Produced by Centrifugal Force. (James B. Stanwood, Trans. A. S. M. E., xiv. 251.) —Mr. Stanwood mentions one case of a fly-band wheel where the periphery velocity on a 17′ 9″ wheel is over 7500 ft. per minute.

In band saw-mills the blade of the saw is operated successfully over wheels 8 and 9 ft. in diameter, at a periphery velocity of 9000 to 10,000 ft. per minute. These wheels are of cast iron throughout, of heavy thickness, with a large number of arms.

In shingle-machines and chipping-machines where cast-iron disks from 2 to 5 ft. in diameter are employed, with knives inserted radially, the speed is frequently 10,000 to 11,000 ft. per minute at the periphery.

If the rim of a fly-wheel alone be considered, the tensile strain in pounds per square inch of the rim section is $T = \frac{V^2}{10}$ nearly, in which V = velocity in feet per second; but this strain is modified by the resistance of the arms, which prevent the uniform circumferential expansion of the rim, and induce a bending as well as a tensile strain. Mr. Stanwood discusses the strains in band-wheels due to transverse bending of a section of the rim between a pair of arms.

When the arms are few in number, and of large cross-section, the ring will be strained transversely to a greater degree than with a greater number of lighter arms. To illustrate the necessary rim thicknesses for various rim velocities, pulley diameters, number of arms, etc., the following table is given, based upon the formula

$$t = \frac{.475d}{N^2\left(\frac{F}{V^2} - \frac{1}{10}\right)},$$

in which t = thickness of rim in inches, d = diameter of pulley in inches, N = number of arms, V = velocity of rim in feet per second, and F = the greatest strain in pounds per square inch to which any fibre is subjected. The value of F is taken at 6000 lbs. per sq. in.

Thickness of Rims in Solid Wheels.

Diameter of Pulley in inches.	Velocity of Rim in feet per second.	Velocity of Rim in feet per minute.	No. of Arms.	Thickness in inches.
24	50	3,000	6	2/10
24	88	5,280	6	15/32
48	88	5,280	6	15/16
108	184	11,040	16	2½
108	184	11,040	36	½

If the limit of rim velocity for all wheels be assumed to be 88 ft. per second, equal to 1 mile per minute, $F = 6000$ lbs., the formula becomes

$$t = \frac{.475d}{.67N^2} = 0.7\frac{d}{N^2}.$$

When wheels are made in halves or in sections, the bending strain may be such as to make t greater than that given above. Thus, when the joint comes half way between the arms, the bending action is similar to a beam supported simply at the ends, uniformly loaded, and t is 50% greater. Then the formula becomes

$$t = \frac{.712d}{N^2\left(\frac{F}{V^2} - \frac{1}{10}\right)}.$$

or for a fixed maximum rim velocity of 88 ft. per second and $F = 6000$ lbs., $t = \frac{1.05d}{N^2}$. In segmental wheels it is preferable to have the joints opposite the arms. Wheels in halves, if very thin rims are to be employed, should have double arms along the line of separation.

Attention should be given to the proportions of large receiving and tightening pulleys. The thickness of rim for a 48-in. wheel (shown in table) with a rim velocity of 88 ft. per second, is 15/16 in. Many wrecks have been caused by the failure of receiving or tightening pulleys whose rims have been too thin. Fly-wheels calculated for a given coefficient of steadiness are frequently lighter than the minimum safe weight. This is true especially of large wheels. A rough guide to the minimum weight of wheels can be deduced from our formulæ. The arms, hub, lugs, etc., usually form from one quarter to one third the entire weight of the wheel. If b represents the face of a wheel in inches, the weight of the rim (considered as a simple annular ring) will be $w = .82dtb$ lbs. If the limit of speed is 88 ft. per second, then for solid wheels $t = 0.7d \div N^2$. For sectional wheels (joint between arms) $t = 1.05d \div N^2$. Weight of rim for solid wheels, $w = .57d^2b \div N^2$ in pounds. Weight of rim in sectional wheels with joints between arms, $w = .86d^2b \div N^2$ in pounds. Total weight of wheel: for solid wheel, $W = .76d^2b \div N^2$ to $.86d^2b \div N^2$, in pounds. For segmental wheels with joint between arms, $W = 1.05d^2b \div N^2$ to $1.3d^2b \div N^2$, in pounds.

(This subject is further discussed by Mr. Stanwood, in vol. xv., and by Prof. Gaetano Lanza, in vol. xvi., Trans. A. S. M. E.)

A Wooden Rim Fly-wheel, built in 1891 for a pair of Corliss engines at the Amoskeag Mfg. Co.'s mill, Manchester, N. H., is described by C. H. Manning in Trans. A. S. M. E., xiii. 618. It is 30 ft. diam. and 108 in. face. The rim is 12 inches thick, and is built up of 44 courses of ash plank, 2, 3, and 4 inches thick, reduced about ⅛ inch in dressing, set edgewise, so as to break joints, and glued and bolted together. There are two hubs and two sets of arms, 12 in each, all of cast iron. The weights are as follows:

Weight (calculated) of ash rim	31,855	lbs.
" of 24 arms (foundry 45,020)	40,849	"
" " 2 hubs (" 35,030)	31,394±	"
Counter-weights in 6 arms	664	"
Total, excluding bolts and screws	104,262±	"

The wheel was tested at 76 revs. per min., being a surface speed of nearly 7200 feet per minute.

MAXIMUM DIAMETER OF FLY-WHEEL ALLOWABLE FOR ... OF REVOLUTIONS.

Revolutions per minute.	Assuming Maximum Speed of 5000 feet per minute.	
	Circum. ft.	Diam[illegible]
40	125	[illegible]
50	100	[illegible]
60	83.3	[illegible]
70	71.4	[illegible]
80	62.5	[illegible]
90	55.5	[illegible]
100	50.	[illegible]
120	4[illegible]	[illegible]
140	[illegible]	[illegible]
160	[illegible]	[illegible]
180	[illegible]	[illegible]
200	[illegible]	[illegible]
220	[illegible]	[illegible]
240	[illegible]	[illegible]
260	[illegible]	[illegible]
280	[illegible]	[illegible]
300	[illegible]	[illegible]
350	[illegible]	[illegible]
40[illegible]	[illegible]	[illegible]

... iron and of wooden ... being the same in both ... ss-section would be directly ... o stand the strain directly as ... e the tensile strengths divided ... f different materials. Cast iron ... a tensile strength of 1,440,000 lbs. ... 1,440,000 ÷ 450 = 3200, whilst ash, of ... lbs. per cubic foot, and with 1,152,000 ... gives a result 1,152,000 ÷ 84 = 33,882 ... wood-rimmed pulley is ten times safer ... ngs are good. This would allow the wood- ... speed to $\sqrt{10.58} = 3.25$ times that of a sound ...

... of the Willimantic Linen Co. (Illus- ...)—Rim 28 ft. diam., 110 in. face. The rim is ... arms, one under the centre of each belt, with 12 ... is ordinary whitewood, ⅜ in. in thickness, cut into ... 4 feet in length, and either 5 or 8 inches in width. ... by building a complete circle 13 inches in width, first ... and the 5-inch outside, and then beside it another cir- ... reversed, so as to break joints. Each piece as it was ... over with glue and nailed with three-inch wire nails to ... in position. The nails pass through three and into the ... At the end of each arm four 14-inch bolts secure the ... being covered by wooden plugs glued and driven into the face ...

... wound Fly-wheels for Extreme Speeds. (*Eng'g News,* ... 1890.)—The power required to produce the Mannesmann tubes is ... varying from 2000 to 10,000 H.P., according to the dimensions of ... since this power is only needed for a short time (it takes only 30 seconds to convert a bar 10 to 12 ft. long and 4 in. in diameter into a ... and then some time elapses before the next bar is ready, an engine of ... H.P. provided with a large fly-wheel for storing the energy will supply ... power enough for one set of rolls. These fly-wheels are so large and run at such great speeds that the ordinary method of constructing them cannot be followed. A wheel at the Mannesmann Works, made in Komotau, Hungary, in the usual manner, broke at a tangential velocity of 125 ft. per second. The fly-wheels designed to hold at more than double this speed consist of a cast-iron hub to which two steel disks, 20 ft. in diameter, are bolted; around the circumference of the wheel thus formed 70 tons of No. 5 wire are wound under a tension of 50 lbs. In the Mannesmann Works at Landore, Wales, such a wheel makes 240 revolutions a minute, corresponding to a tangential velocity of 15,080 ft. or 2.85 miles per minute.

THE SLIDE-VALVE.

Definitions.—*Travel* = total distance moved by the valve.

Throw of the Eccentric = eccentricity of the eccentric = distance from the centre of the shaft to the centre of the eccentric disk = ½ the travel of the valve. (Some writers use the term "throw" to mean the whole travel of the valve.)

Lap of the valve, also called outside lap or steam-lap = distance the outer or steam edge of the valve extends beyond or laps over the steam edge of the port when the valve is in its central position.

Inside lap, or *exhaust-lap* = distance the inner or exhaust edge of the valve extends beyond or laps over the exhaust edge of the port when the valve is in its central position. The inside lap is sometimes made zero, or even negative, in which latter case the distance between the edge of the valve and the edge of the port is sometimes called exhaust clearance, or inside clearance.

Lead of the valve = the distance the steam-port is opened when the engine is on its centre and the piston is at the beginning of the stroke.

Lead-angle = the angle between the position of the crank when the valve begins to be opened and its position when the piston is at the beginning of the stroke.

The valve is said to have lead when the steam-port opens before the piston begins its stroke. If the piston begins its stroke before the admission of steam begins the valve is said to have negative lead, and its amount is the lap of the edge of the valve over the edge of the port at the instant when the piston stroke begins.

Lap-angle = the angle through which the eccentric must be rotated to cause the steam edge to travel from its central position the distance of the lap.

Angular advance of the eccentric = lap-angle + lead angle.

Linear advance = lap + lead.

Effect of Lap, Lead, etc., upon the Steam Distribution.—Given valve-travel 2¾ in., lap ¾ in., lead 1/16 in., exhaust-lap ⅛ in., required crank position for admission, cut-off, release and compression, and greatest port-opening. (Halsey on Slide-valve Gears.) Draw a circle of diameter *fh* = travel of valve. From *O* the centre set off *Oa* = lap and *ab* = lead, erect perpendiculars *Oe*, *ac*, *bd*; then *ec* is the lap-angle and *cd* the lead-angle, measured as arcs. Set off *fg* = *cd*, the lead-angle, then *Og* is the position of the crank for steam admission. Set off 2*ec* + *cd* from *h* to *i*; then *Oi* is the crank-angle for cut-off, and *fk* ÷ *fh* is the fraction of stroke completed at cut-off. Set off *Ol* = exhaust-lap and draw *lm*; *em* is the exhaust-lap angle. Set off *hn* = *ec* + *cd* − *em*, and *On* is the position of crank at release. Set off *fp* = *ec* + *cd* + *em*, and *Op* is the position of crank for compression, *fo* ÷ *fh* is the fraction of stroke completed at release, and *hq* ÷ *hf* is the fraction of the return stroke completed when compression begins; *Oh*, the throw of the eccentric, minus *Oa* the lap, equals *ah* the maximum port-opening.

If a valve has neither lap nor lead, the line joining the centre of the eccen-

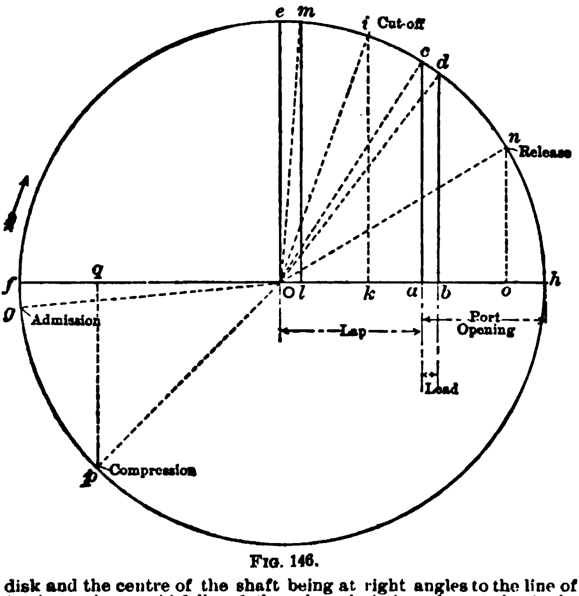

FIG. 146.

tric disk and the centre of the shaft being at right angles to the line of the crank, the engine would follow full stroke, admission of steam beginning at the beginning of the stroke and ending at the end of the stroke.

Adding lap to the valve enables us to cut off steam before the end of the stroke; the eccentric being advanced on the shaft an amount equal to the lap-angle enables steam to be admitted at the beginning of the stroke, as before lap was added, and advancing it a further amount equal to the lead angle causes steam to be admitted before the beginning of the stroke.

Having given lap to the valve, and having advanced the eccentric on the shaft from its central position at right angles to the crank, through the angular advance = lap-angle and lead-angle, the four events, admission, cut-off, release or exhaust-opening, and compression or exhaust-closure, take place as follows: Admission, when the crank lacks the lead-angle of having reached the centre; cut-off, when the crank lacks two lap-angles and one lead-angle of having reached the centre. During the admission of steam the crank turns through a semicircle less twice the lap-angle. The greatest port-opening is equal to half the travel of the valve less the lap. Therefore for a given port-opening the travel of the valve must be increased if the lap is increased. When exhaust-lap is added to the valve it delays the opening of the exhaust and hastens its closing by an angle of rotation equal to the exhaust-lap angle, which is the angle through which the eccentric rotates from its middle position while the exhaust edge of the valve uncovers its lap. Release then takes place when the crank lacks one lap-angle and one lead-angle minus one exhaust-lap angle of having reached the centre, and compression when the crank lacks lap-angle + lead-angle + exhaust-lap angle of having reached the centre.

The above discussion of the relative position of the crank, piston, and valve for the different points of the stroke is accurate only with a connecting-rod of infinite length.

For actual connecting-rods the angular position of the rod causes a distortion of the position of the valve, causing the events to take place too late in the forward stroke and too early in the return. The correction of this distortion may be accomplished to some extent by setting the valve so as to give equal lead on both forward and return stroke, and by altering the exhaust-lap on one end so as to equalize the release and compression. F. A. Halsey, in his Slide-valve Gears, describes a method of equalizing the cut-off without at the same time affecting the equality of the lead. In designing slide-valves the effect of angularity of the connecting-rod should be studied on the drawing-board, and preferably by the use of a model.

Sweet's Valve-diagram.—To find outside and inside lap of valve for different cut-offs and compressions (see Fig. 147): Draw a circle whose

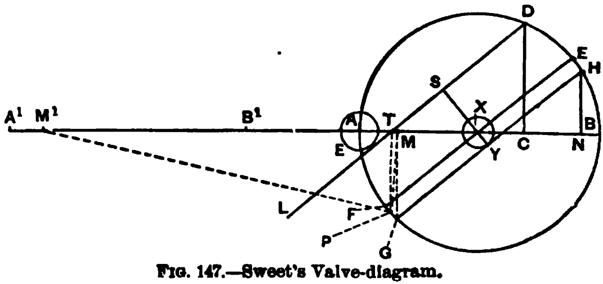

FIG. 147.—Sweet's Valve-diagram.

diameter equals travel of valve. Draw diameter BA and continue to A^1, so that the length AA^1 bears the same ratio to XA as the length of connecting-rod does to length of engine-crank. Draw small circle E with a radius equal to lead. Lay off AC so that ratio of AC to AB = cut-off in parts of the stroke. Erect perpendicular CD. Draw DL tangent to E; draw XS perpendicular to DL; XS is then outside lap of valve.

To find release and compression: If there is no inside lap, draw FE through X parallel to DL. F and E will be position of crank for release and compression. If there is an inside lap, draw a circle about X, in which radius XY equals inside lap. Draw HG tangent to this circle and parallel to DL; then H and G are crank position for release and compression. Draw HN and MG, then AN is piston position at release and AM piston position at compression, AB being considered stroke of engine.

To make compression alike on each stroke it is necessary to increase the inside lap on crank end of valve, and to decrease by the same amount the inside lap on back end of valve. To determine this amount, through M with a radius $MM^1 = AA^1$, draw arc MP, from P draw PT perpendicular to AB, then TM is the amount to be added to inside lap on crank end, and to be deducted from inside lap on back end of valve, inside lap being XY.

For the *Bilgram Valve Diagram*, see Halsey on Slide-valve Gears.

The Zeuner Valve-diagram is given in most of the works on the steam-engine, and in treatises on valve-gears, as Zeuner's, Peabody's, and

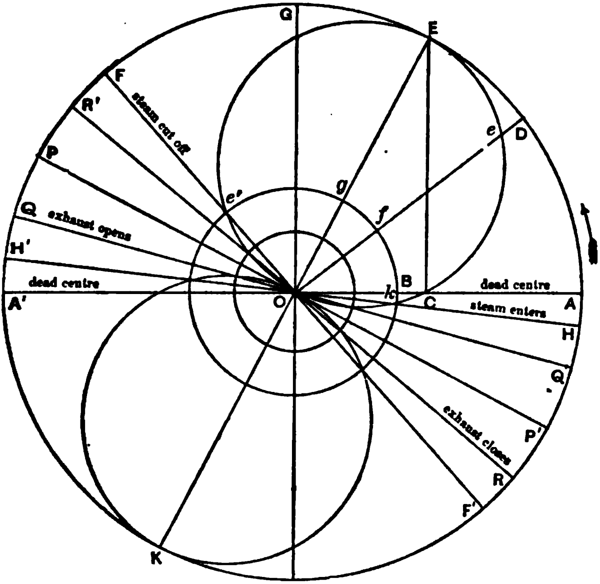

FIG. 148.—Zeuner's Valve-diagram.

Spangler's. The following is condensed from Holmes on the Steam-engine: Describe a circle, with radius OA equal to the half travel of the valve. From O measure off OB equal to the outside lap, and BC equal to the lead. When the crank-pin occupies the dead centre A, the valve has already moved to the right of its central position by the space $OB + BC$. From C erect the perpendicular CE and join OE. Then will OE be the position occupied by the line joining the centre of the eccentric with the centre of the crank-shaft at the commencement of the stroke. On the line OE as diameter describe the circle OCE; then any chords, as Oe, OE, Oe', will represent the spaces travelled by the valve from its central position when the crank-pin occupies respectively the positions opposite to D, E, and F. Before the port is opened at all the valve must have moved from its central position by an amount equal to the lap OB. Hence, to obtain the space by which the port is opened, subtract from each of the arcs Oe, OE, etc., a length equal to OB. This is represented graphically by describing from centre O a circle with radius equal to the lap OB; then the spaces fe, gE, etc., intercepted between the circumferences of the lap-circle Bfe' and the valve-circle OCE, will give the extent to which the steam-port is opened.

At the point k, at which the chord Ok is common to both valve and lap circles, it is evident that the valve has moved to the right by the amount of the lap, and is consequently just on the point of opening the steam-port. Hence the steam is admitted before the commencement of the stroke, when the crank occupies the position OH, and while the portion HA of the revolution still remains to be accomplished. When the crank-pin reaches the position A, that is to say, at the commencement of the stroke, the port is already opened by the space $OC - OB = BC$, called the lead. From this point forward till the crank occupies the position OE the port continues to open, but when the crank is at OE the valve has reached the furthest limit of its travel to the right, and then commences to return, till when in the position OF the edge of the valve just covers the steam-port, as is shown by the chord Oe', being again common to both lap and valve circles. Hence when the crank occupies the position OF the cut-off takes place and the steam commences to expand, and continues to do so till the exhaust opens. For the return stroke the steam-port opens again at H' and closes at F'.

There remains the exhaust to be considered. When the line joining the centres of the eccentric and crank-shaft occupies the position opposite to OG at right angles to the line of dead centres, the crank is in the line OP at right angles to OE; and as OP does not intersect either valve-circle the valve occupies its central position, and consequently closes the port by the amount of the inside lap. The crank must therefore move through such an angular distance that its line of direction OQ must intercept a chord on the valve-circle OK equal in length to the inside lap before the port can be opened to the exhaust. This point is ascertained precisely in the same manner as for the outside lap, namely, by drawing a circle from centre O, with a radius equal to the inside lap; this is the small inner circle in the figure. Where this circle intersects the two valve-circles we get four points which show the positions of the crank when the exhaust opens and closes during each revolution. Thus at Q the valve opens the exhaust on the side of the piston which we have been considering, while at R the exhaust closes and compression commences and continues till the fresh steam is readmitted at H.

Thus the diagram enables us to ascertain the exact position of the crank when each critical operation of the valve takes place. Making a *résumé* of these operations of one side of the piston, we have: Steam admitted before the commencement of the stroke at H. At the dead centre A the valve is already opened by the amount BC. At E the port is fully opened, and valve has reached one end of its travel. At F steam is cut off, consequently admission lasted from H to F. At P valve occupies central position, and ports are closed both to steam and exhaust. At Q exhaust opened, consequently expansion lasted from F to Q. At K exhaust opened to maximum extent, and valve reached the end of its travel to the left. At R exhaust closed, and compression begins and continues till the fresh steam is admitted at H.

PROBLEM.—The simplest problem which occurs is the following: Given the length of throw, the angle of advance of the eccentric, and the laps of the valve, find the angles of the crank at which the steam is admitted and cut off and the exhaust opened and closed. Draw the line OE, representing the half-travel of the valve or the throw of the eccentric at the given angle of advance with the perpendicular OG. Produce OE to K. On OE and OK as diameters describe the two valve-circles. With centre and radii equal to the given laps describe the outside and inside lap-circles. Then the intersection of these circles with the two valve-circles give points through which the lines OH, OF, OQ, and OR can be drawn. These lines give the required positions of the crank.

Numerous other problems will be found in Holmes on the Steam-engine, including problems in valve-setting and the application of the Zeuner diagram to link motion and to the Meyer valve-gear.

Port Opening.—The area of port opening should be such that the velocity of the steam in passing through it should not exceed 6000 ft. per min. The ratio of port area to piston area will then vary with the piston-speed as follows:

For speed of piston, ft. per min.	100	200	300	400	500	600	700	800	900	1000	1200
Port area = piston area ×	.017	.033	.05	.067	.083	.1	.107	.133	.15	.167	.2

For a velocity of 6000 ft. per min.,

$$\text{Port area} = \frac{\text{sq. of diam. of cyl.} \times \text{piston speed}}{7639}.$$

The length of the port opening may be equal to or something less than the diameter of the cylinder, and the width = area of port opening ÷ its length.

The bridge between steam and exhaust ports should be wide enough to prevent a leak of steam into the exhaust due to overtravel of the valve.

Auchincloss gives: Width of exhaust port = width of steam port + ¼ travel of valve − width of bridge.

Lead. (From Peabody's Valve-gears.)—The lead, or the amount that the valve is open when the engine is on a dead point, varies, with the type and size of the engine, from a very small amount, or even nothing, up to ⅜ of an inch or more. Stationary-engines running at slow speed may have from 1/64 to 1/16 inch lead. The effect of compression is to fill the waste space at the end of the cylinder with steam; consequently, engines having much compression need less lead. Locomotive-engines having the valves controlled by the ordinary form of Stephenson link-motion may have a small lead when running slowly and with a long cut-off, but when at speed with a short cut-off the lead is at least ¼ inch; and locomotives that have valve-gear which gives constant lead commonly have ¼ inch lead. The lead-angle is the angle the crank makes with the line of dead points at admission. It may vary from 0° to 8°.

Inside Lead.—Weisbach (vol. ii. p. 296) says: Experiment shows that the earlier opening of the exhaust ports is especially of advantage, and in the best engines the lead of the valve upon the side of the exhaust, or the inside lead; is 1/25 to 1/15; i.e., the slide-valve at the lowest or highest position of the piston has made an opening whose height is 1/25 to 1/15 of the whole throw of the slide-valve. The outside lead of the slide-valve or the lead on the steam side, on the other hand, is much smaller, and is often only 1/100 of the whole throw of the valve.

Effect of Changing Outside Lap, Inside Lap, Travel and Angular Advance. (Thurston.)

	Admission	Expansion	Exhaust	Compression
Incr. O.L.	is later, ceases sooner	occurs earlier, continues longer	is unchanged	begins at same point
Incr. I.L.	unchanged	begins as before, continues longer	occurs later, ceases earlier	begins sooner, continues longer
Incr. T.	begins sooner, continues longer	begins later, ceases sooner	begins later, ceases later	begins later, ends sooner
Incr. A.A.	begins earlier, period unaltered	begins sooner, per. the same	begins earlier, per. unchanged	begins earlier, per. the same

Zeuner gives the following relations (Weisbach-Dubois, vol. ii. p. 307):

If S = travel of valve, p = maximum port opening;
L = steam-lap, l = exhaust-lap;

$$R = \text{ratio of steam-lap to half travel} = \frac{L}{.5S},\quad L = \frac{R}{2} \times S;$$

$$r = \text{ratio of exhaust lap to half travel} = \frac{l}{.5S},\quad l = \frac{r}{2} \times S;$$

$$S = 2p + 2L = 2p + 2R + S;\quad S = \frac{2p}{1 - R}.$$

If α = angle HOF between positions of crank at admission and at cut-off, and β = angle QOR between positions of crank at release and at compression, then $R = \frac{1}{2}\frac{\sin(180° - \alpha)}{\sin \frac{1}{2}\alpha}$; $r = \frac{1}{2}\frac{\sin(180° - \beta)}{\sin \frac{1}{2}\beta}$.

Ratio of Lap and of Port-opening to Valve-travel.—The table on page 831, giving the ratio of lap to travel of valve and ratio of travel to port opening, is abridged from one given by Buel in Weisbach-Dubois, vol. ii. It is calculated from the above formulæ. Intermediate values may be found by the formulæ, or with sufficient accuracy by interpolation from the figures in the table. By the table on page 830 the crank-angle may be found, that is, the angle between its position when the engine is on the centre and its position at cut-off, release, or compression, when these are known in fractions of the stroke. To illustrate the use of the tables the following example is given by Buel: width of port = 2.2 in.; width of port opening = width of port + 0.3 in.; overtravel = 2.5 in.; length of connecting-rod = 2½ times stroke; cut-off = 0.75 of stroke; release = 0.95 of stroke; lead-angle, 10°. From the first table we find crank-angle = 114.6.

add lead-angle, making 124.6.° From the second table, for angle between admission and cut-off, 125°, we have ratio of travel to port-opening = 3.72, or for 124.6° = 3.74, which, multiplied by port-opening 2.5, gives 9.45 in travel. The ratio of lap to travel, by the table, is .2324, or 9.45 × .2324 = 2.2 in. lap. For exhaust-lap we have, for release at .95, crank-angle = 151.3; add lead-angle 10° = 161.3°. From the second table, by interpolation, ratio of lap to travel = .0811, and .0811 × 9.45 = 0.77 in., the exhaust-lap.

Lap-angle = ½ (180° − lead-angle − crank-angle at cut-off);
= ½ (180° − 10 − 114.6) = 27.7°.
Angular advance = lap-angle + lead-angle = 27.7 + 10 = 37.7°.
Exhaust lap-angle = crank-angle at release + lap-angle + lead-angle − 180°;
= 151.3 + 27.7 + 10 − 180° = 9°.

Crank-angle at compression measured on return stroke } = 180° − lap-angle − lead-angle − exhaust lap-angle;
= 180 − 27.7 − 10 − 9 = 133.3° ; corresponding, by table, to a piston position of .81 of the return stroke; or
Crank-angle at compression = 180° − (angle at release − angle at cut-off) + lead-angle;
= 180 − (151.3 − 114.6) + 10 = 133.3°.

The positions determined above for cut-off and release are for the forward stroke of the piston. On the return stroke the cut-off will take place at the same angle, 114.6°, corresponding by table to 66.6% of the return stroke, instead of 75%. By a slight adjustment of the angular advance and the length of the eccentric rod the cut-off can be equalized. The width of the bridge should be at least 2.5 + 0.25 − 2.2 = 0.55 in.

Crank Angles for Connecting-rods of Different Length.

FORWARD AND RETURN STROKES.

Fraction of Stroke from Commencement.	Ratio of Length of Connecting-rod to Length of Stroke.												
	2		2½		3		3½		4		5		Infinite.
	For.	Ret.	For.	Ret.	For.	Ret.	For.	Ret.	For.	Ret.	For.	Ret.	For. or Ret.
.01	10.3	13.2	10.5	12.8	10.6	12.6	10.7	12.4	10.8	12.3	10.9	12.1	11.5
.02	14.6	18.7	14.9	18.1	15.1	17.8	15.2	17.5	15.3	17.4	15.5	17.1	16.3
.03	17.9	22.9	18.2	22.2	18.5	21.8	18.7	21.5	18.8	21.3	19.0	21.0	19.9
.04	20.7	26.5	21.1	25.7	21.4	25.2	21.6	24.9	21.8	24.6	22.0	24.3	23.1
.05	23.2	29.6	23.6	28.7	24.0	28.2	24.2	27.8	24.4	27.5	24.7	27.2	25.8
.10	33.1	41.9	33.8	40.8	34.3	40.1	34.6	39.6	34.9	39.2	35.2	38.7	36.9
.15	41	51.5	41.9	50.2	42.4	49.3	42.9	48.7	43.2	48.3	43.6	47.7	45.6
.20	48	59.6	48.9	58.2	49.6	57.3	50.1	56.6	50.4	56.2	50.9	55.5	53.1
.25	54.3	66.9	55.4	65.4	56.1	64.4	56.6	63.7	57.0	63.3	57.6	62.6	60.0
.30	60.3	73.5	61.5	72.0	62.2	71.0	62.8	70.3	63.3	69.8	63.9	69.1	66.4
.35	66.1	79.8	67.3	78.3	68.1	77.3	68.8	76.6	69.2	76.1	69.9	75.3	72.5
.40	71.7	85.8	73.0	84.3	73.9	83.3	74.5	82.6	75.0	82.0	75.7	81.3	78.5
.45	77.2	91.5	78.6	90.1	79.6	89.1	80.2	88.4	80.7	87.9	81.4	87.1	84.3
.50	82.8	97.2	84.3	95.7	85.2	94.8	85.9	94.1	86.4	93.6	87.1	92.9	90.0
.55	88.5	102.8	89.9	101.4	90.9	100.4	91.6	99.8	92.1	99.3	92.9	98.6	95.7
.60	94.2	108.3	95.7	107.0	96.7	106.1	97.4	105.5	98.0	105.0	98.7	104.3	101.5
.65	100.2	113.9	101.7	112.7	102.7	111.9	103.4	111.2	103.9	110.8	104.7	110.1	107.5
.70	106.5	119.7	108.0	118.5	109.0	117.8	109.7	117.2	110.2	116.7	110.9	116.1	113.6
.75	113.1	125.7	114.6	124.6	115.6	123.9	116.3	123.4	116.7	123.0	117.4	122.4	120.0
.80	120.4	132	121.8	131.1	122.7	130.4	123.4	129.9	123.8	129.6	124.5	129.1	126.9
.85	128.5	139	129.8	138.1	130.7	137.6	131.3	137.1	131.7	136.8	132.3	136.4	134.4
.90	138.1	146.9	139.2	146.2	139.9	145.7	140.4	145.4	140.8	145.1	141.3	144.8	143.1
.95	150.4	156.8	151.3	156.4	151.8	156.0	152.2	155.8	152.5	155.6	152.8	155.3	154.2
.96	153.5	159.3	154.3	158.9	154.8	158.6	155.1	158.4	155.4	158.2	155.7	158.0	156.9
.97	157.1	162.1	157.8	161.8	158.2	161.5	158.5	161.3	158.7	161.2	159.0	161.0	160.1
.98	161.3	165.4	161.9	165.1	162.2	164.9	162.5	164.8	162.6	164.7	162.9	164.5	163.7
.99	166.8	169.7	167.2	169.5	167.4	169.4	167.6	169.3	167.7	169.2	167.9	169.1	168.5
1.00	180	180	180	180	180	180	180	180	180	180	180	180	180

Relative Motions of Cross-head and Crank.—If L = length of connecting-rod, R = length of crank, θ = angle of crank with centre line of engine, D = displacement of cross-head from the beginning of its stroke,

$$D = R(1 - \cos\theta) + L - \sqrt{L^2 - R^2 \sin^2\theta}.$$

Lap and Travel of Valve.

Angle between Positions of Crank at Points of Admission and Cut-off, or Release and Compression.	Ratio of Lap to Travel of Valve.	Ratio of Travel of Valve to Width of Port-opening.	Angle between Positions of Crank at Points of Admission and Cut-off, or Release and Compression.	Ratio of Lap to Travel of Valve.	Ratio of Travel of Valve to Width of Port-opening.	Angle between Positions of Crank at Points of Admission and Cut-off, or Release and Compression.	Ratio of Lap to Travel of Valve.	Ratio of Travel of Valve to Width of Port-opening.
30°	.4830	58.70	85°	.3686	7.61	135°	.1913	3.24
35	.4769	43.22	90	.3536	6.83	140	.1710	3.04
40	.4699	33.17	95	.3378	6.17	145	.1504	2.86
45	.4619	26.27	100	.3214	5.60	150	.1294	2.70
50	.4532	21.34	105	.3044	5.11	155	.1082	2.55
55	.4435	17.70	110	.2868	4.69	160	.0868	2.42
60	.4330	14.93	115	.2687	4.32	165	.0653	2.30
65	.4217	12.77	120	.2500	4.00	170	.0436	2.19
70	.4096	11.06	125	.2309	3.72	175	.0218	2.09
75	.3967	9.68	130	.2113	3.46	180	.0000	2.00
80	.3830	8.55						

PERIODS OF ADMISSION, OR CUT-OFF, FOR VARIOUS LAPS AND TRAVELS OF SLIDE-VALVES.

The two following tables are from Clark on the Steam-engine. In the first table are given the periods of admission corresponding to travels of valve of from 12 in. to 2 in., and laps of from 2 in. to 3/8 in., with 1/4 in. and 1/8 in. of lead. With greater leads than those tabulated, the steam would be cut off earlier than as shown in the table.

The influence of a lead of 5/16 in. for travels of from 1 5/8 in. to 6 in., and laps of from 1/8 in. to 1 1/8 in., as calculated for in the second table, is exhibited by comparison of the periods of admission in the table, for the same lap and travel. The greater lead shortens the period of admission, and increases the range for expansive working.

Periods of Admission, or Points of Cut-off, for Given Travels and Laps of Slide-valves.

Travel of Valve.	Lead.	Periods of Admission, or Points of Cut-off, for the following Laps of Valves in inches.									
		2	1 3/4	1 1/2	1 1/4	1	7/8	3/4	5/8	1/2	3/8
in.	in.	%	%	%	%	%	%	%	%	%	%
12	1/4	88	90	93	95	96	97	98	98	99	99
10	1/4	83	87	89	92	95	96	97	98	98	99
8	1/4	72	78	84	88	92	94	95	96	98	98
6	1/4	50	62	71	79	86	89	91	94	96	97
5 1/2	1/8	43	56	68	77	85	88	91	94	96	97
5	1/8	32	47	61	72	82	86	89	92	95	97
4 1/2	1/8	14	35	51	66	78	83	87	90	94	96
4	1/8	......	17	39	57	72	78	83	88	92	95
3 1/2	1/8	......	......	20	44	63	71	79	84	90	94
3	1/8	......	......	......	23	50	61	71	79	86	91
2 1/2	1/8	......	......	......	......	27	43	57	70	80	88
2	1/8	......	......	......	......	......	......	33	52	70	81

Periods of Admission, or Points of Cut-off, for given Travels and Laps of Slide-valves.

Constant lead, 5/16.

Travel.	Lap.								
Inches.	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
1⅝	19	......	......	......	......	......	......	......	......
1¾	39	......	......	......	......	......	......	......	......
1⅞	47	17	......	......	......	......	......	......	......
2	55	34	......	......	......	......	......	......	......
2⅛	61	42	14	......	......	......	......	......	......
2¼	65	50	30	......	......	......	......	......	......
2⅜	68	55	38	13	......	......	......	......	......
2½	71	59	45	27	......	......	......	......	......
2⅝	74	63	49	36	12	......	......	......	......
2¾	76	67	56	43	26	......	......	......	......
2⅞	78	70	59	47	32	11	......	......	......
3	80	73	63	50	38	23	......	......	......
3⅛	81	74	65	55	44	30	10	......	......
3¼	83	76	68	59	48	34	22	......	......
3⅜	84	78	71	62	51	40	29	9	......
3½	85	80	73	64	53	45	34	20	......
3⅝	86	81	75	66	57	49	38	26	9
3¾	87	82	76	68	60	52	42	32	19
3⅞	87	83	78	70	63	55	46	36	25
4	88	84	79	72	66	58	49	40	29
4¼	89	86	81	76	70	63	56	47	37
4½	90	87	83	79	73	67	61	54	45
4¾	92	89	85	81	76	70	65	58	51
5	93	90	87	83	78	73	67	62	56
5½	94	92	89	86	82	78	73	68	63
6	95	93	91	88	85	82	78	74	69

Diagram for Port-opening, Cut-off, and Lap.—The diagram on the opposite page was published in *Power*, Aug., 1893. It shows at a glance the relations existing between the outside lap, steam port-opening, and cut-off in slide valve engines.

In order to use the diagram to find the lap, having given the cut-off and maximum port-opening, follow the ordinate representing the latter, taken on the horizontal scale, until it meets the oblique line representing the given cut-off. Then read off this height on the vertical lap scale. Thus, with a port-opening of 1¼ inch and a cut-off of .50, the intersection of the two lines occurs on the horizontal 3. The required lap is therefore 3 in.

If the cut-off and lap are given, follow the horizontal representing the latter until it meets the oblique line representing the cut-off. Then vertically below this read the corresponding port-opening on the horizontal scale.

If the lap and port-opening are given, the resulting cut-off may be ascertained by finding the point of intersection of the ordinate representing the port-opening with the horizontal representing the lap. The oblique line passing through the point of intersection will give the cut-off.

If it is desired to take lead into account, multiply the lead in inches by the numbers in the following table corresponding to the cut-off, and deduct the result from the lap as obtained from the diagram:

Cut-off.	Multiplier.	Cut-off.	Multiplier.
.20	4.717	.60	1.358
.25	3.731	.625	1.288
.30	3.048	.65	1.222
.33	2.717	.70	1.108
.375	2.381	.75	1.000
.40	2.171	.80	0.904
.45	1.930	.85	0.815
.50	1.706	.875	0.772
.55	1.515	.90	0.731

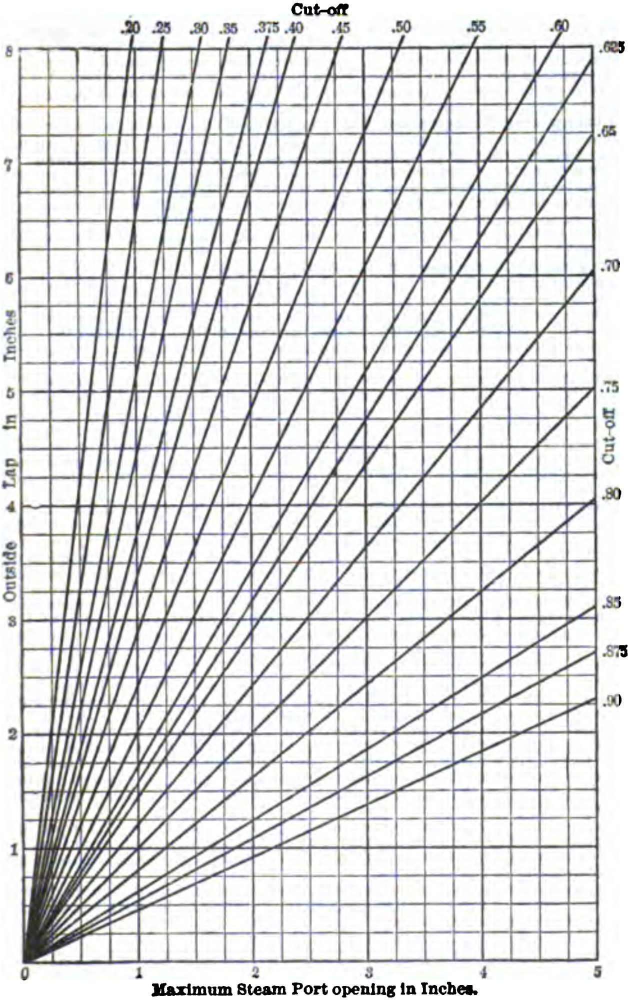

DIAGRAM FOR SLIDE VALVES.

FIG. 149.

Piston-valve.—The piston-valve is a modified form of the slide-valve. The lap, lead, etc., are calculated in the same manner as for the common slide-valve. The diameter of valve and amount of port-opening are calculated on the basis that the most contracted portion of the steam-passage between the valve and the cylinder should have an area such that the velocity of steam through it will not exceed 6000 ft. per minute. The area of the opening around the circumference of the valve should be about double the area of the steam-passage, since that portion of the opening that is opposite from the steam-passage is of little effect.

Setting the Valves of an Engine.—The principles discussed above are applicable not only to the designing of valves, but also to adjustment of valves that have been improperly set; but the final adjustment of the eccentric and of the length of the rod depend upon the amount of lost motion, temperature, etc., and can be effected only after trial. After the valve has been set as accurately as possible when cold, the lead and lap for the forward and return strokes being equalized, indicator diagrams should be taken and the length of the eccentric-rod adjusted, if necessary, to correct slight irregularities.

To Put an Engine on its Centre.—Place the engine in a position where the piston will have nearly completed its outward stroke, and opposite some point on the cross-head, such as a corner, make a mark upon the guide. Against the rim of the pulley or crank-disk place a pointer and mark a line with it on the pulley. Then turn the engine over the centre until the cross-head is again in the same position on its inward stroke. This will bring the crank as much below the centre as it was above it before. With the pointer in the same position as before make a second mark on the pulley-rim. Divide the distance between the marks in two and mark the middle point. Turn the engine until the pointer is opposite this middle point, and it will then be on its centre. To avoid the error that may arise from the looseness of crank-pin and wrist-pin bearings, the engine should be turned a little above the centre and then be brought up to it, so that the crank-pin will press against the same brass that it does when the first two marks are made.

Link-motion.—Link-motions, of which the Stephenson link is the most commonly used, are designed for two purposes: first, for reversing the motion of the engine, and second, for varying the point of cut-off by varying the travel of the valve. The Stephenson link-motion is a combination of two eccentrics, called forward and back eccentrics, with a link connecting the extremities of the eccentric-rods; so that by varying the position of the link the valve-rod may be put in direct connection with either eccentric, or may be given a movement controlled in part by one and in part by the other eccentric. When the link is moved by the reversing lever into a position such that the block to which the valve-rod is attached is at either end of the link, the valve receives its maximum travel, and when the link is in mid-gear the travel is the least and cut-off takes place early in the stroke.

In the ordinary shifting-link with open rods, that is, not crossed, the lead of the valve increases as the link is moved from full to mid-gear, that is, as the period of steam admission is shortened. The variation of lead is equalized for the front and back strokes by curving the link to the radius of the eccentric-rods concavely to the axles. With crossed eccentric-rods the lead decreases as the link is moved from full to mid-gear. In a valve-motion with stationary link the lead is constant. (For illustration see Clark's Steam-engine, vol. ii. p. 22.)

The linear advance of each eccentric is equal to that of the valve in full gear, that is, to lap + lead of the valve, when the eccentric-rods are attached to the link in such position as to cause the half-travel of the valve to equal the eccentricity of the eccentric.

The angle between the two eccentric radii, that is, between lines drawn from the centre of the eccentric disks to the centre of the shaft equals 180° less twice the angular advance.

Buel, in Appleton's Cyclopedia of Mechanics, vol. ii. p. 316, discusses the Stephenson link as follows: "The Stephenson link does not give a perfectly correct distribution of steam; the lead varies for different points of cut-off. The period of admission and the beginning of exhaust are not alike for both ends of the cylinder, and the forward motion varies from the backward.

"The correctness of the distribution of steam by Stephenson's link-motion depends upon conditions which, as much as the circumstances will permit, ought to be fulfilled, namely: 1. The link should be curved in the arc of a circle whose radius is equal to the length of the eccentric-rod. 2. The eccentric-rods ought to be long; the longer they are in proportion to the eccentricity the more symmetrical will the travel of the valve be on both sides of the centre of motion. 3. The link ought to be short. Each of its points describes a curve in a vertical plane, whose ordinates grow larger the farther the considered point is from the centre of the link; and as the horizontal motion only is transmitted to the valve, vertical oscillation will cause irregularities. 4. The link-hanger ought to be long. The longer it is the nearer will be the arc in which the link swings to a straight line, and thus the less its vertical oscillation. If the link is suspended in its centre, the curves that are described by points equidistant on both sides from the centre are not alike, and hence results the variation between the forward and backward gear. If the link is suspended at its lower end, its lower half will have less vertical oscillation and the upper half more. 5. The centre from which the link-hanger swings changes its position as the link is lowered or raised, and also causes irregularities. To reduce them to the smallest amount the arm of the lifting-shaft should be made as long as the eccentric-rod, and the centre of the lifting-shaft should be placed at the height corresponding to the central position of the centre on which the link-hanger swings."

All these conditions can never be fulfilled in practice, and the variations in the lead and the period of admission can be somewhat regulated in an artificial way, but for one gear only. This is accomplished by giving different lead to the two eccentrics, which difference will be smaller the longer the eccentric-rods are and the shorter the link, and by suspending the link not exactly on its centre line but at a certain distance from it, giving what is called "the offset."

For application of the Zeuner diagram to link-motion, see Holmes on the Steam-engine, p. 290. See also Clark's Railway Machinery (1855), Clark's Steam-engine, Zeuner's and Auchincloss's Treatises on Slide-valve Gears, and Halsey's Locomotive Link Motion. (See Appendix, p. 1077.)

The following rules are given by the *American Machinist* for laying out a link for an upright slide-valve engine. By the term radius of link is meant the radius of the link-arc *ab*, Fig. 150, drawn through the centre of the slot;

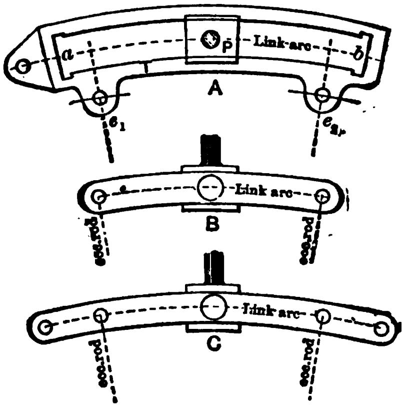

FIG. 150.

this radius is generally made equal to the distance from the centre of shaft to centre of the link-block pin *P* when the latter stands midway of its travel. The distance between the centres of the eccentric-rod pins e_1 e_2 should not be less than 2½ times, and, when space will permit, three times the throw of the eccentric. By the throw we mean twice the eccentricity of the eccentric. The slot link is generally suspended from the end next to the forward eccentric at a point in the link-arc prolonged. This will give comparatively a small amount of slip to the link-block when the link is in forward gear; but this slip will be increased when the link is in backward gear. This increase of slip is, however, considered of little importance, because marine engines, as a rule, work but very little in the backward gear. When it is necessary that the motion shall be as efficient in backward gear as in forward gear, then the link should be suspended from a point midway between the two eccentric-rod pins; in marine engine practice this point is generally located on the link-arc; for equal cut-offs it is better to move the point of suspension a small amount towards the eccentrics.

For obtaining the dimensions of the link in inches: Let L denote the length of the valve, B the breadth, p the absolute steam-pressure per sq. in., and R a factor of computation used as below; then $R = .01\sqrt{L \times B \times p}$.

Breadth of the link	$= R \times 1.6$
Thickness T of the bar	$= R \times .8$
Length of sliding-block	$= R \times 2.5$
Diameter of eccentric-rod pins	$= (R \times .7) + \frac{1}{4}$
Diameter of suspension-rod pin	$= (R \times .6) + \frac{1}{4}$
Diameter of suspension-rod pin when overhung	$= (R \times .8) + \frac{1}{4}$
Diameter of block-pin when overhung	$= R + \frac{1}{4}$
Diameter of block-pin when secured at both ends	$= (R \times .8) + \frac{1}{4}$

The length of the link, that is, the distance from a to b, measured on a straight line joining the ends of the link-arc in the slot, should be such as to allow the centre of the link-block pin P to be placed in a line with the eccentric-rod pins, leaving sufficient room for the slip of the block. Another type of link frequently used in marine engines is the double-bar link, and this type is again divided into two classes: one class embraces those links which have the eccentric-rod ends as well as the valve-spindle end between the bars, as shown at B (with these links the travel of the valve is less than the throw of the eccentric); the other class embraces those links, shown at C, for which the eccentric-rods are made with fork-ends, so as to connect to studs on the outside of the bars, allowing the block to slide to the end of the link, so that the centres of the eccentric-rod ends and the block-pin are in line when in full gear, making the travel of the valve equal to the throw of the eccentric. The dimensions of these links when the distance between the eccentric-rod pins is $2\frac{1}{2}$ to $2\frac{3}{4}$ times the throw of eccentrics can be found as follows:

Depth of bars	$= (R \times 1.25) + \frac{1}{2}''$
Thickness of bars	$= (R \times .5) + \frac{1}{4}''$
Diameter of centre of sliding-block	$= R \times 1.3$

When the distance between the eccentric-rod pins is equal to 3 or 4 times the throw of the eccentrics, then

Depth of bars	$= (R \times 1.25) + \frac{3}{4}''$
Thickness of bars	$= (R \times .5) + \frac{1}{4}''$

All the other dimensions may be found by the first table. These are empirical rules, and the results may have to be slightly changed to suit given conditions. In marine engines the eccentric-rod ends for all classes of links have adjustable brasses. In locomotives the slot-link is usually employed, and in these the pin-holes have case-hardened bushes driven into the pin-holes, and have no adjustable brasses in the ends of the eccentric-rods. The link in B is generally suspended by one of the eccentric-rod pins; and the link in C is suspended by one of the pins in the end of the link, or by one of the eccentric-rod pins. (See note on Locomotive Link Motion in Appendix, p. 1077.)

Other Forms of Valve-Gear, as the Joy, Marshall, Hackworth, Bremme, Walschaert, Corliss, etc., are described in Clark's Steam-engine, vol. ii. The design of the Reynolds-Corliss valve-gear is discussed by A. H. Eldridge in *Power*, Sep. 1893. See also Henthorn on the Corliss engine. Rules for laying down the centre lines of the Joy valve-gear are given in *American Machinist*, Nov. 13, 1890. For Joy's "Fluid-pressure Reversing-valve," see *Eng'g*, May 25, 1894.

GOVERNORS.

Pendulum or Fly-ball Governor.—The inclination of the arms of a revolving pendulum to a vertical axis is such that the height of the point of suspension h above the horizontal plane in which the centre of gravity of the balls revolve (assuming the weight of the rods to be small compared with the weight of the balls) bears to the radius r of the circle described by the centres of the balls the ratio

$$\frac{h}{r} = \frac{\text{weight}}{\text{centrifugal force}} = \frac{w}{\frac{wv^2}{gr}} = \frac{gr}{v^2},$$

which ratio is independent of the weight of the balls, v being the velocity of the centres of the balls in feet per second.

If T = number of revolutions of the balls in 1 second, $v = 2\pi rT = ar$, in which a = the angular velocity, or $2\pi T$, and

$$h = \frac{gr^2}{v^2} = \frac{g}{4\pi^2T^2}, \text{ or } h = \frac{0.8146}{T^2} \text{ feet} = \frac{9.775}{T^2} \text{ inches},$$

g being taken at 32.16. If N = number of revs. per minute, $h = \frac{35190}{N^2}$ inches

For revolutions per minute..... ...	40	45	50	60	75
The height in inches will be..... ...	21.99	17.38	14.08	9.775	6.256

Number of turns per minute required to cause the arms to take a given angle with the vertical axis: Let l = length of the arm in inches from the centre of suspension to the centre of gyration, and a the required angle; then

$$N = \sqrt{\frac{35190}{l \cos a}} = 187.6\sqrt{\frac{1}{l \cos a}} = 187.6\sqrt{\frac{1}{h}}.$$

The simple governor is not isochronous; that is, it does not revolve at a uniform speed in all positions, the speed changing as the angle of the arms changes. To remedy this defect loaded governors, such as Porter's, are used. From the balls of a common governor whose collective weight is A let there be hung by a pair of links of lengths equal to the pendulum arms a load B capable of sliding on the spindle, having its centre of gravity in the axis of rotation. Then the centrifugal force is that due to A alone, and the effect of gravity is that due to $A + 2B$; consequently the altitude for a given speed is increased in the ratio $(A + 2B) : A$, as compared with that of a simple revolving pendulum, and a given absolute variation in altitude produces a smaller proportionate variation in speed than in the common governor. (Rankine, S. E., p. 551.)

For the weighted governor let l = the length of the arm from the point of suspension to the centre of gravity of the ball, and let the length of the suspending-link, l_1 = the length of the portion of the arm from the point of suspension of the arm to the point of attachment of the link; G = the weight of one ball, Q = half the weight of the sliding weight, h = the height of the governor from the point of suspension to the plane of revolution of the balls, a = the angular velocity = $2\pi T$, T being the number of revolutions per second; then $a = \sqrt{\frac{32.16}{h}\left(1 + \frac{2l_1}{l}\frac{Q}{G}\right)}$; $h = \frac{32.16}{a^2}\left(1 + \frac{2l_1}{l}\frac{Q}{G}\right)$ in feet, or $h = \frac{35190}{N^2}\left(1 + \frac{2l_1}{l}\frac{Q}{G}\right)$ in inches, N being the number of revolutions per minute.

For various forms of governor see App. Cycl. Mech., vol. ii. 61, and Clark's Steam-engine, vol. ii. p. 65.

To Change the Speed of an Engine Having a Fly-ball Governor.—A slight difference in the speed of a governor changes the position of its weights from that required for full load to that required for no load. It is evident therefore that, whatever the speed of the engine, the normal speed of the governor must be that for which the governor was designed; i.e., the speed of the governor must be kept the same. To change the speed of the engine the problem is to so adjust the pulleys which drive the governor that the engine at its new speed shall drive it just as fast as it was driven at its original speed. In order to increase the engine-speed we must decrease the pulley upon the shaft of the engine, i.e., the driver, or increase that on the governor, i.e., the driven, in the proportion that the speed of the engine is to be increased.

Fly-wheel or Shaft Governors.—At the Centennial Exhibition in 1876 there were shown a few steam-engines in which the governors were contained in the fly-wheel or band-wheel, the fly-balls or weights revolving around the shaft in a vertical plane with the wheel and shifting the eccentric so as automatically to vary the travel of the valve and the point of cut-off. This form of governor has since come into extensive use, especially for high-speed engines. In its usual form two weights are carried on arms the ends of which are pivoted to two points on the pulley near its circumference, 180° apart. Links connect these arms to the eccentric. The eccentric is not rigidly keyed to the shaft but is free to move transversely across it for a certain distance, having an oblong hole which allows of this movement. Centrifugal force causes the weights to fly towards the circumference of the wheel and to pull the eccentric into a position of minimum eccentricity. This force is resisted by a spring attached to each arm which tends to pull the weights towards the shaft and shift the eccentric to the position of maximum eccentricity. The travel of the valve is thus varied, so that it tends to cut off earlier in the stroke as the engine increases its speed. Many modifications of this general form are in use. For discussions of this form of governor see Hartnell, Proc. Inst. M. E., 1882, p. 408; Trans. A. S. M. E., ix. 300; xi. 1081; xiv. 92; xv. 929; Modern Mechanism, p. 399; Whitham's Constructive Steam Engineering; J. Begtrup, *Am. Mach.*, Oct. 19 and Dec. 14, 1893, Jan. 18 and March 1, 1894.

Calculation of Springs for Shaft-governors. (Wilson Hartnell, Proc. Inst. M. E., Aug. 1882.)—The springs for shaft-governors may be conveniently calculated as follows, dimensions being in inches:

Let W = weight of the balls or weights, in pounds;
- r_1 and r_2 = the maximum and minimum radial distances of the centre of the balls or of the centre of gravity of the weights;
- l_1 and l_2 = the leverages, i.e., the perpendicular distances from the centre of the weight-pin to a line in the direction of the centrifugal force, drawn through the centre of gravity of the weights or balls at radii r_1 and r_2;
- m_1 and m_2 = the corresponding leverages of the springs;
- C_1 and C_2 = the centrifugal forces, for 100 revolutions per minute, at radii r_1 and r_2;
- P_1 and P_2 = the corresponding pressures on the spring;
- (It is convenient to calculate these and note them down for reference.)
- C_3 and C_4 = maximum and minimum centrifugal forces;
- S = mean speed (revolutions per minute);
- S_1 and S_2 = the maximum and minimum number of revolutions per minute;
- P_3 and P_4 = the pressures on the spring at the limiting number of revolutions (S_1 and S_2);
- $P_4 - P_3 = D$ = the difference of the maximum and minimum pressures on the springs;
- V = the percentage of variation from the mean speed, or the sensitiveness;
- t = the travel of the spring;
- u = the initial pressure on the spring;
- v = the stiffness in pounds per inch;
- w = the maximum pressure $= u + t$.

The mean speed and sensitiveness desired are supposed to be given. Then

$$S_1 = S - \frac{SV}{100}; \qquad S_2 = S + \frac{SV}{100};$$

$$C_1 = 0.28 \times r_1 \times W; \qquad C_2 = 0.28 \times r_2 \times W;$$

$$P_1 = C_1 \times \frac{l_1}{m_1}; \qquad P_2 = C_2 \times \frac{l_2}{m_2};$$

$$P_3 = P_1 \times \left(\frac{S_1}{100}\right)^2; \qquad P_4 = P_2 \times \left(\frac{S_2}{100}\right)^2;$$

$$v = \frac{D}{t}, \quad u = \frac{P_3}{v}, \quad w = \frac{P_4}{v}.$$

It is usual to give the spring-maker the values of P_4 and of v or w. To ensure proper space being provided, the dimensions of the spring should be calculated by the formulæ for strength and extension of springs, and the least length of the spring as compressed be determined.

$$\text{The governor-power} = \frac{P_3 + P_4}{2} \times \frac{t}{12}.$$

With a straight centripetal line, the governor-power

$$= \frac{C_3 + C_4}{2} \times \left(\frac{r_2 - r_1}{12}\right).$$

For a preliminary determination of the governor-power it may be taken as equal to this in all cases, although it is evident that with a curved centripetal line it will be slightly less. The difference D must be constant for the same spring, however great or little its initial compression. Let the spring be screwed up until its minimum pressure is P_5. Then to find the speed $P_6 = P_5 + D$,

$$S_5 = 100\sqrt{\frac{P_5}{P_1}}; \qquad S_6 = 100\sqrt{\frac{P_6}{P_2}}.$$

The speed at which the governor would be isochronous would be

$$100\sqrt{\frac{D}{P_2 - P_1}}.$$

Suppose the pressure on the spring with a speed of 100 revolutions, at the maximum and minimum radii, was 200 lbs. and 100 lbs., respectively, then the pressure of the spring to suit a variation from 95 to 105 revolutions will be $100 \times \left(\frac{95}{100}\right)^2 = 90.2$ and $200 \times \left(\frac{105}{100}\right)^2 = 220.5$. That is, the increase of resistance from the minimum to the maximum radius must be $220 - 90 = 130$ lbs.

The extreme speeds due to such a spring, screwed up to different pressures, are shown in the following table:

Revolutions per minute, balls shut	80	90	95	100	110	120
Pressure on springs, balls shut	64	81	90	100	121	144
Increase of pressure when balls open fully	130	130	130	130	130	130
Pressure on springs, balls open fully	194	211	220	230	251	274
Revolutions per minute, balls open fully	98	102	105	107	112	117
Variation, per cent of mean speed	10	6	5	3	1	−1

The speed at which the governor would become isochronous is 114.

Any spring will give the right variation at some speed; hence in experimenting with a governor the correct spring may be found from any wrong one by a very simple calculation. Thus, if a governor with a spring whose stiffness is 50 lbs. per inch acts best when the engine runs at 95, 90 being its proper speed, then $50 \times \left(\frac{90}{95}\right)^2 = 45$ lbs. is the stiffness of spring required.

To determine the speed at which the governor acts best, the springs may be screwed up until it begins to "hunt" and then slackened until the governor is as sensitive as is compatible with steadiness.

CONDENSERS, AIR-PUMPS CIRCULATING-PUMPS, ETC.

The Jet Condenser. (Chiefly abridged from Seaton's Marine Engineering.)—The jet condenser is now uncommon in marine practice, being generally supplanted by the surface condenser. It is commonly used where fresh water is available for boiler feed. With the jet condenser a vacuum of 24 in. was considered fairly good, and 25 in. as much as was possible with most condensers; the temperature corresponding to 24 in. vacuum, or 3 lbs. pressure absolute, is 140°. In practice the temperature in the hot-well varies from 110° to 120°, and occasionally as much as 130° is maintained. To find the quantity of injection-water per pound of steam to be condensed: Let T_1 = temperature of steam at the exhaust pressure; T_0 = temperature of the cooling-water; T_2 = temperature of the water after condensation, or of the hot-well; Q = pounds of the cooling-water per lb. of steam condensed; then

$$Q = \frac{1114^\circ + 0.3T_1 - T_2}{T_2 - T_0}.$$

Another formula is: $Q = \frac{WH}{R}$, in which W is the weight of steam condensed, H the units of heat given up by 1 lb. of steam in condensing, and R the rise in temperature of the cooling-water.

This is applicable both to jet and to surface condensers. The allowance made for the injection-water of engines working in the temperate zone is usually 27 to 30 times the weight of steam, and for the tropics 30 to 35 times; 30 times is sufficient for ships which are occasionally in the tropics, and this is what was usual to allow for general traders.

Area of injection orifice = weight of injection-water in lbs. per min. ÷ 650 to 750.

A rough rule sometimes used is: Allow one fifteenth of a square inch for every cubic foot of water condensed per hour.

Another rule: Area of injection orifice = area of piston ÷ 250.

The volume of the jet condenser is from one fourth to one half of that of the cylinder. It need not be more than one third, except for very quick-running engines.

Ejector Condensers.—For ejector or injector condensers (Bulkley's, Schutte's, etc.) the calculations for quantity of condensing-water is the same as for jet condensers.

The Surface Condenser—Cooling Surface.—Peclet found that with cooling water of an initial temperature of 68° to 77°, one sq. ft. of copper plate condensed 21.5 lbs. of steam per hour, while Joule states that 100 lbs. per hour can be condensed. In practice, with the compound engine, brass condenser-tubes, 18 B.W.G thick, 13 lbs. of steam per sq. ft. per hour, with the cooling-water at an initial temperature of 60°, is considered very fair work when the temperature of the feed-water is to be maintained at 120°. It has been found that the surface in the condenser may be half the heating surface of the boiler, and under some circumstances considerably less than this. In general practice the following holds good when the temperature of sea-water is about 60°:

Terminal pres., lbs., abs....	30	20	15	12½	10	8	6
Sq. ft. per I H.P............	3	2.50	2.25	2.00	1.80	1.60	1.50

For ships whose station is in the tropics the allowance should be increased by 20%. and for ships which occasionally visit the tropics 10% increase will give satisfactory results. If a ship is constantly employed in cold climates 10% less suffices

Whitham (Steam-engine Design, p. 283, also Trans. A. S. M. E., ix 431) gives the following: $S = \frac{WL}{ck(T_1 - t)}$, in which S = condensing-surface in sq. ft.; T_1 = temperature Fahr. of steam of the pressure indicated by the vacuum-gauge; t = mean temperature of the circulating water, or the arithmetical mean of the initial and final temperatures; L = latent heat of saturated steam at temperature T_1; k = perfect conductivity of 1 sq. ft. of the metal used for the condensing-surface for a range of 1° F. (or 557 B T U. per hour for brass, according to Isherwood's experiments); c = fraction denoting the efficiency of the condensing surface; W = pounds of steam condensed per hour. From experiments by Loring and Emery, on U.S.S *Dallas*, c is found to be 0.323, and ck = 180; making the equation $S = \frac{WL}{180(T_1 - t)}$.

Whitham recommends this formula for designing engines having independent circulating pumps. When the pump is worked by the main engine the value of S should be increased about 10%.

Taking T_1 at 135° F., and L = 1020, corresponding to 25 in. vacuum, and t for summer temperatures at 75°, we have: $S = \frac{1020W}{180(135 - 75)} = \frac{17W}{180}$.

For a mathematical discussion of the efficiency of surface condensers see a paper by T. E. Stanton in Proc. Inst. C. E., cxxxvi, June 1899, p. 321.

Condenser Tubes are generally made of solid drawn brass tubes, and tested both by hydraulic pressure and steam. They are usually made of a composition of 68% of best selected copper and 32% of best Silesian spelter.

The Admiralty, however, always specify the tubes to be made of 70% of best selected copper and to have 1% of tin in the composition, and test the tubes to a pressure of 300 lbs. per sq. in. (Seaton.)

The diameter of the condenser tubes varies from ¼ inch in small condensers, when they are very short, to 1 inch in very large condensers and long tubes. In the mercantile marine the tubes are, as a rule, ¾ inch diameter externally, and 18 B.W.G. thick (0.049 inch); and 16 B.W.G. (0.065), under some exceptional circumstances. In the British Navy the tubes are also, as a rule, ¾ inch diameter, and 18 to 19 B.W.G. thick, tinned on both sides; when the condenser is made of brass the Admiralty do not require the tubes to be tinned. Some of the smaller engines have tubes ⅝ inch diameter, and 19 B.W.G. thick. The smaller the tubes, the larger is the surface which can be got in a certain space.

In the merchant service the almost universal practice is to circulate the water through the tubes.

Whitham says the velocity of flow through the tubes should not be less than 400 nor more than 700 ft. per min.

Tube-plates are usually made of brass. Rolled-brass tube-plates should be from 1.1 to 1.5 times the diameter of tubes in thickness, depending on the method of packing. When the packings go completely through the plates the latter, but when only partly through the former, is sufficient. Hence, for ¾-inch tubes the plates are usually ⅞ to 1 inch thick with glands and tape-packings, and 1 to 1¼ inch thick with wooden ferrules.

The tube-plates should be secured to their seatings by brass studs and nuts, or brass screw-bolts; in fact there must be no wrought iron of any kind inside a condenser. When the tube-plates are of large area it is advisable to stay them by brass-rods, to prevent them from collapsing.

Spacing of Tubes, etc.—The holes for ferrules, glands, or india-rubber are usually ¼ inch larger in diameter than the tubes; but when absolutely necessary the wood ferrules may be only 3/32 inch thick.

The pitch of tubes when packed with wood ferrules is usually ¼ inch more than the diameter of the ferrule-hole. For example, the tubes are generally arranged zigzag, and the number which may be fitted into a square foot of plate is as follows:

Pitch of Tubes.	No. in a sq. ft.	Pitch of Tubes.	No. in a sq. ft.	Pitch of Tubes.	No. in a sq. ft.
1″	172	1 5/32″	128	1¼″	110
1 1/16″	150	1 3/16″	121	1 9/32″	106
1⅛″	137	1 7/32″	116	1 5/16″	99

Quantity of Cooling Water.—The quantity depends chiefly upon its initial temperature, which in Atlantic practice may vary from 40° in the winter of temperate zone to 80° in subtropical seas. To raise the temperature to 100° in the condenser will require three times as many thermal units in the former case as in the latter, and therefore only one third as much cooling-water will be required in the former case as in the latter.

T_1 = temperature of steam entering the condenser;
T_0 = " " circulating-water entering the condenser;
T_2 = " " " " leaving the condenser;
T_3 = " " water condensed from the steam;

$$Q = \text{quantity of circulating water in lbs.} = \frac{1114 + 0.3(T_1 - T_3)}{T_2 - T_0}.$$

It is usual to provide pumping power sufficient to supply 40 times the weight of steam for general traders, and as much as 50 times for ships stationed in subtropical seas, when the engines are compound. If the circulating-pump is double-acting, its capacity may be 1/58 in the former and 1/42 in the latter case of the capacity of the low-pressure cylinder.

Air-pump.—The air-pump in all condensers abstracts the water condensed and the air originally contained in the water when it entered the boiler. In the case of jet-condensers it also pumps out the water of condensation and the air which it contained. The size of the pump is calculated from these conditions, making allowance for efficiency of the pump.

Ordinary sea-water contains, mechanically mixed with it, 1/20 of its volume of air when under the atmospheric pressure. Suppose the pressure in the condenser to be 2 lbs. and the atmospheric pressure 15 lbs., neglecting the effect of temperature, the air on entering the condenser will be expanded to 15/2 times its original volume; so that a cubic foot of sea-water, when it has entered the condenser, is represented by 19/20 of a cubic foot of water and 15/40 of a cubic foot of air.

Let q be the volume of water condensed per minute, and Q the volume of sea-water required to condense it; and let T_2 be the temperature of the condenser, and T_1 that of the sea-water.

Then 19/20 $(q + Q)$ will be the volume of water to be pumped from the condenser per minute,

$$\text{and } \frac{15}{40}(q + Q) \times \frac{T_2 + 461^\circ}{T_1 + 461^\circ} \text{ the quantity of air.}$$

If the temperature of the condenser be taken at 120°, and that of sea-water at 60°, the quantity of air will then be $.418(q + Q)$, so that the total volume to be abstracted will be

$$.95(q + Q) + .418(q + Q) = 1.368(q + Q).$$

If the average quantity of injection-water be taken at 26 times that condensed, $q + Q$ will equal $27q$. Therefore, volume to be pumped from the condenser per minute $= 37q$, nearly.

In surface condensation allowance must be made for the water occasionally admitted to the boilers to make up for waste, and the air contained in it, also for slight leak in the joints and glands, so that the air-pump is made about half as large as for jet-condensation.

The efficiency of a single-acting air-pump is generally taken at 0.5, and that of a double-acting pump at 0.35. When the temperatur of the sea is 60°, and that of the (jet) condenser is 120°, Q being the volume of the cooling water and q the volume of the condensed water in cubic feet, and n the number of strokes per minute,

$$\text{The volume of the single-acting pump} = 2.74\left(\frac{Q+q}{n}\right).$$

$$\text{The volume of the double-acting pump} = 4\left(\frac{Q+q}{n}\right).$$

The following table gives the ratio of capacity of cylinder or cylinders to that of the air-pump; in the case of the compound engine, the low-pressure cylinder capacity only is taken.

Description of Pump.	Description of Engine.	Ratio.
Single-acting vertical......	Jet-condensing, expansion 1½ to 2....	6 to 8
" "	Surface " " 1½ to 2....	8 to 10
" "	Jet " " 3 to 5....	10 to 12
" "	Surface " " 3 to 5...	12 to 15
" "	Surface " compound...........	15 to 18
Double-acting horizontal..	Jet " expansion 1½ to 2....	10 to 13
" " ..	Surface " " 1½ to 2....	13 to 16
" " ..	Jet " " 3 to 5 ...	16 to 19
" " ..	Surface " " 3 to 5 ...	19 to 24
" " ..	Surface " compound	24 to 28

The Area through Valve-seats and past the valves should not be less than will admit the full quantity of water for condensation at a velocity not exceeding 400 ft. per minute. In practice the area is generally in excess of this.

Area through foot-valves $= D^2 \times S \div 1000$ square inches.
Area through head-valves $= D^2 \times S \div 800$ square inches.
Diameter of discharge-pipe $= D \times \sqrt{S} \div 35$ inches.
$D =$ diam. of air-pump in inches, $S =$ its speed in ft. per min.

James Tribe (*Am. Mach.*, Oct. 8, 1891) gives the following rule for air-pumps used with jet-condensers: Volume of single-acting air-pump driven by main engine = volume of low-pressure cylinder in cubic feet, multiplied by 3.5 and divided by the number of cubic feet contained in one pound of exhaust-steam of the given density. For a double-acting air-pump the same rule will apply, but the volume of steam for each stroke of the pump will be but one half. Should the pump be driven independently of the engine, then the relative speed must be considered. Volume of jet-condenser = volume of air-pump × 4. Area of injection valve = vol. of air-pump in cubic inches ÷ 520.

Circulating-pump.—Let Q be the quantity of cooling water in cubic feet, n the number of strokes per minute, and S the length of stroke in feet.

Capacity of circulating-pump = $Q \div n$ cubic feet.

Diameter " " " $= 13.55\sqrt{\dfrac{Q}{n \times S}}$ inches.

The following table gives the ratio of capacity of steam-cylinder or cylinders to that of the circulating-pump:

Description of Pump.	Description of Engine.	Ratio.
Single-acting.	Expansive 1½ to 2 times.	13 to 16
" "	" 3 to 5 "	20 to 25
" "	Compound.	25 to 30
Double "	Expansive 1½ to 2 times.	25 to 30
" "	" 3 to 5 "	36 to 46
" "	Compound.	46 to 56

The clear area through the valve-seats and past the valves should be such that the mean velocity of flow does not exceed 450 feet per minute. The flow through the pipes should not exceed 500 ft. per min. in small pipes and 600 in large pipes.

For Centrifugal Circulating-pumps, the velocity of flow in the inlet and outlet pipes should not exceed 400 ft. per min. The diameter of the fan-wheel is from 2½ to 3 times the diam. of the pipe, and the speed at its periphery 450 to 500 ft. per min. If W = quantity of water per minute, in American gallons, d = diameter of pipes in inches, R = revolutions of wheel per min., $d = \sqrt{\dfrac{W}{16.44}}$; diam. of fan-wheel = not less than $\dfrac{1700}{R}$. Breadth of blade at tip $= \dfrac{W}{36d}$. Diam. of cylinder for driving the fan = about $2.8\sqrt{\text{diam. of pipe}}$, and its stroke = 0.28 × diam. of fan.

Feed-pumps for Marine Engines.—With surface-condensing engines the amount of water to be fed by the pump is the amount condensed from the main engine plus what may be needed to supply auxiliary engines and to supply leakage and waste. Since an accident may happen to the surface-condenser, requiring the use of jet-condensation, the pumps of engines fitted with surface-condensers must be sufficiently large to do duty under such circumstances. With jet-condensers and boilers using salt water the dense salt water in the boiler must be blown off at intervals to keep the density so low that deposits of salt will not be formed. Sea-water contains about 1/32 of its weight of solid matter in solution. The boiler of a surface-condensing engine may be worked with safety when the quantity of salt is four times that in sea-water. If Q = net quantity of feed-water required in a given time to make up for what is used as steam, n = number of times the saltness of the water in the boiler is to that of sea-water, then the gross feed-water $= \dfrac{n}{n-1}Q$. In order to be capable of filling the boiler rapidly each feed-pump is made of a capacity equal to twice the gross feed-water. Two feed-pumps should be supplied, so that one may be kept in reserve to be used while the other is out of repair. If Q be the quantity of net feed-water in cubic feet, l the length of stroke of feed-pump in feet, and n the number of strokes per minute,

Diameter of each feed-pump plunger in inches $= \sqrt{\dfrac{550 \times Q}{n \times l}}$.

If W be the n.' feed-water in pounds,

$$\text{Diameter of each feed-pump plunger in inches} = \sqrt{\frac{8.9 \times W}{n \times l}}.$$

An Evaporative Surface Condenser built at the Virginia Agricultural College is described by James H. Fitts (Trans. A. S. M. E., xiv. 690). It consists of two rectangular end chambers connected by a series of horizontal rows of tubes, each row of tubes immersed in a pan of water. Through the spaces between the surface of the water in each pan and the bottom of the pan above air is drawn by means of an exhaust-fan. At the top of one of the end-chambers is an inlet for steam, and a horizontal diaphragm about midway causes the steam to traverse the upper half of the tubes and back through the lower. An outlet at the bottom leads to the air-pump. The condenser, exclusive of connection to the exhaust-fan, occupies a floor space of 5′ 4½″ × 1′ 9¾″, and 4′ 1½″ high. There are 27 rows of tubes, 8 in some and 7 in others; 210 tubes in all. The tubes are of brass, No. 20 B.W G., ⅝″ external diameter and 4′ 9½″ in length. The cooling surface (internal) is 176.5 sq. ft. There are 27 cooling pans, each 4′ 9½″ × 1′ 9⅝″, and 1 7/16″ deep. These pans have galvanized iron bottoms which slide into horizontal grooves ¼″ wide and ¼″ deep, planed into the tube-sheets. The total evaporating surface is 234.8 sq. ft. Water is fed to every third pan through small cocks, and overflow-pipes feed the rest. A wood casing connects one side with a 30″ Buffalo Forge Co.'s disk-wheel. This wheel is belted to a 8″ × 4″ vertical engine The air-pump is 5¾″ diameter with a 6″ stroke, is vertical and single-acting.

The action of this condenser is as follows: The passage of air over the water surfaces removes the vapor as it rises and thus hastens evaporation. The heat necessary to produce evaporation is obtained from the steam in the tubes, causing the steam to condense. It was designed to condense 800 lbs steam per hour and give a vacuum of 22 in., with a terminal pressure in the cylinder of 20 lbs. absolute.

Results of tests show that the cooling-water required is practically equal in amount to the steam used by the engine. And since consumption of steam is reduced by the application of a condenser, its use will actually reduce the total quantity of water required. From a curve showing the rate of evaporation per square foot of surface in still air, and also one showing the rate when a current of air of about 2300 ft. per min. velocity is passed over its surface, the following approximate figures are taken:

Temp. F.	Evaporation, lbs. per sq. ft. per hour.		Temp. F.	Evaporation, lbs. per sq. ft. per hour.	
	Still Air.	Current.		Still Air.	Current.
100°	0.2	1.1	140°	0.8	5.0
110	0.25	1.6	150	1.1	6.7
120	0.4	2.5	160	1.5	9.5
130	0 6	3.5	170	2.0	...

The Continuous Use of Condensing-water is described in a series of articles in *Power*, Aug.-Dec., 1892. It finds its application in situations where water for condensing purposes is expensive or difficult to obtain.

In San Francisco J. C. H. Stut cools the water after it has left the hot-well by means of a system of pans upon the roof. These pans are shallow troughs of galvanized iron arranged in tiers, on a slight incline, so that the water flows back and forth for 1500 o. 1800 ft., cooling by evaporation and radiation as it flows. The pans are about 5 ft. in width, and the water as it flows has a depth of about half an inch, the temperature being reduced from about 140° to 90°. The water from the hot-well is pumped up to the highest point of the cooling system and allowed to flow as above described, discharging finally into the main tank or reservoir, whence it again flows to the condenser as required. As the water in the reservoir lowers from evaporation, an auxiliary feed from the city mains to the condenser is operated, thereby keeping the amount of water in circulation practically constant. An accumulation of oil from the engines, with dust from the surrounding streets, makes a cleaning necessary about once in six weeks or two months. It is found by comparative trials, running condensing and non condensing, that about 50% less water is taken from the city mains when the whole apparatus is in use than when the engine is run non-condensing. 22 to 23 in. of vacuum are maintained. A better vacuum is obtained on a warm day with a brisk breeze blowing than on a cold day with but a slight movement of the air.

In another plant the water from the hot-well is sprayed from a number of fountains, and also from a pipe extending around its border, into a large pond, the exposure cooling it sufficiently for the obtaining of a good vacuum by its continuous use.

In the system patented by Messrs. See, of Lille, France, the water is discharged from a pipe laid in the form of a rectangle and elevated above a pond through a series of special nozzles, by which it is projected into a fine spray. On coming into contact with the air in this state of extreme division the water is cooled 40° to 50°, with a loss by evaporation of only one tenth of its mass, and produces an excellent vacuum. A 3000-H.P. cooler upon this system has been erected at Lannoy, one of 2500 H.P. at Madrid, and one of 1200 H.P. at Liege, as well as others at Roubaix and Tourcoing. The system could be used upon a roof if ground space were limited.

In the "self-cooling" system of H. R. Worthington the injection-water is taken from a tank, and after having passed through the condenser is discharged in a heated condition to the top of a cooling tower, where it is scattered by means of distributing-pipes and trickles down through a cellular structure made of 6-in. terra-cotta pipes, 2 ft. long, stood on end. The water is cooled by a blast of air furnished by a disk fan at the bottom of the tower and the absorption of heat caused by a portion of the water being vaporized, and is led to the tank to be again started on its circuit. (*Eng'g News*, March 5, 1896.)

In the evaporative condenser of T. Ledward & Co. of Brockley, London, the water trickles over the pipes of the large condenser or radiator, and by evaporation carries away the heat necessary to be abstracted to condense the steam inside. The condensing pipes are fitted with corrugations mounted with circular ribs, whereby the radiating or cooling surface is largely increased. The pipes, which are cast in sections about 76 in. long by 3½ in. bore, have a cooling surface of 26 sq. ft., which is found sufficient under favorable conditions to permit of the condensation of 20 to 30 lbs. of steam per hour when producing a vacuum of 13 lbs. per sq. in. In a condenser of this type at Rixdorf, near Berlin, a vacuum ranging from 24 to 26 in. of mercury was constantly maintained during the hottest weather of August. The initial temperature of the cooling-water used in the apparatus under notice ranged from 80° to 85° F., and the temperature in the sun, to which the condenser was exposed, varied each day from 100° to 115° F. During the experiments it was found that it was possible to run one engine under a load of 100 horse-power and maintain the full vacuum without the use of any cooling-water at all on the pipes, radiation afforded by the pipes alone sufficing to condense the steam for this power.

In Klein's condensing water-cooler, the hot water coming from the condenser enters at the top of a wooden structure about twenty feet in height, and is conveyed into a series of parallel narrow metal tanks. The water overflowing from these tanks is spread as a thin film over a series of wooden partitions suspended vertically about 3½ inches apart within the tower. The upper set of partitions, corresponding to the number of metal tanks, reaches half-way down the tower. From there down to the well is suspended a second set of partitions placed at right angles to the first set. This impedes the rapidity of the downflow of the water, and also thoroughly mixes the water, thus affording a better cooling. A fan-blower at the base of the tower drives a strong current of air with a velocity of about twenty feet per second against the thin film of water running down over the partitions. It is estimated that for an effectual cooling two thousand times more air than water must be forced through the apparatus. With such a velocity the air absorbs about two per cent of aqueous vapor. The action of the strong air-current is twofold: first, it absorbs heat from the hot water by being itself warmed by radiation; and, secondly, it increases the evaporation, which process absorbs a great amount of heat. These two cooling effects are different during the different seasons of the year. During the winter months the direct cooling effect of the cold air is greater, while during summer the heat absorption by evaporation is the more important factor. Taking all the year round, the effect remains very much the same. The evaporation is never so great that the deficiency of water would not be supplied by the additional amount of water resulting from the condensed steam, while in very cold winter months it may be necessary to occasionally rid the cistern of surplus water. It was found that the vacuum obtained by this continual use of the same condensing-water varied during the year between 27.5 and 28.7 inches. The great saving of space is evident from the fact that only the five-hundredth part of the floor-space is required as if cooling tanks or ponds were used. For a 100-horse-power engine the floor-space required is about four square yards by a height of twenty feet. For one horse-power 3 6 square yards cooling-surface is necessary. The vertical suspension of the partitions is very essential. With a ventilator 50 inches in diameter and a tower 6 by 7 feet and 20 feet high, 10,500 gallons of water per hour were cooled from 104° F. to 68° F. The following record was made at Mannheim, Germany: Vacuum in condenser, 28.1 inches; temperature of condensing-water entering at top of tower, 104° to 108° F.; temperature of water leaving the cooler, 66.2° to 71.6° F. The engine was of the Sulzer compound type, of 120 horse-power. The amount of power necessary for the arrangement amounts to about three per cent of the total horse-power of the engine for the ventilator, and from one and one half to three per cent for the lifting of the water to the top of the cooler, the total being four and one half to six per cent.

A novel form of condenser has been used with considerable success in Germany and other parts of the Continent. The exhaust-steam from the engine passes through a series of brass pipes immersed in water, to which it gives up its heat. Between each section of tubes a number of galvanized disks are caused to rotate. These disks are cooled by a current of air supplied by a fan and pass down into the water, cooling it by abstracting the heat given out by the exhaust-steam and carrying it up where it is driven off by the air-current. The disks serve also to agitate the water and thus aid it in abstracting the heat from the steam. With 85 per cent vacuum the temperature of the cooling water was about 130° F., and a consumption of water for condensing is guaranteed to be less than a pound for each pound of steam condensed. For an engine 40 in. × 50 in., 70 revolutions per minute, 90 lbs. pressure, there is about 1150 sq. ft. of condensing-surface. Another condenser, 1600 sq. ft. of condensing-surface, is used for three engines, 32 in. × 48 in., 27 in. × 40 in., and 30 in. × 40 in., respectively. —*The Steamship.*

The Increase of Power that may be obtained by adding a condenser giving a vacuum of 26 inches of mercury to a non-condensing engine may be approximated by considering it to be equivalent to a net gain of 12 pounds mean effective pressure per square inch of piston area. If A = area of piston in square inches, S = piston-speed in ft. per minute, then $\frac{12AS}{33,000} = \frac{AS}{2750}$ = H.P. made available by the vacuum. If the vacuum = 13.2 lbs. per sq. in. = 27.9 in. of mercury, then H.P. = $AS \div 2500$.

The saving of steam for a given horse-power will be represented approximately by the shortening of the cut-off when the engine is run with the condenser. Clearance should be included in the calculation. To the mean effective pressure non-condensing, with a given actual cut-off, clearance considered, add 8 lbs. to obtain the approximate mean *total* pressure, condensing. From tables of expansion of steam find what actual cut-off will give this mean total pressure. The difference between this and the original actual cut-off, divided by the latter and by 100, will give the percentage of saving.

The following diagram (from catalogue of H. R. Worthington) shows the percentage of power that may be gained by attaching a condenser to a non-condensing engine, assuming that the vacuum is 12 lbs. per sq. in. The diagram also shows the mean pressure in the cylinder for a given initial pressure and cut-off, clearance and compression not considered.

The pressures given in the diagram are absolute pressures above a vacuum.

To find the mean effective pressure produced in an engine-cylinder with 90 lbs. gauge (= 105 lbs. absolute) pressure, cut-off at ¼ stroke: find 105 in the left-hand or initial-pressure column, follow the horizontal line to the right until it intersects the oblique line that corresponds to the ¼ cut-off, and read the mean total pressure from the row of figures directly above the point of intersection, which in this case is 63 lbs. From this subtract the mean absolute back pressure (say 3 lbs. for a condensing engine and 15 lbs. for a non-condensing engine exhausting into the atmosphere) to obtain the mean effective pressure, which in this case, for a non-condensing engine, gives 48 lbs. To find the gain of power by the use of a condenser with this engine read on the lower scale the figures that correspond in position to 48 lbs. in the upper row, in this case 25%. As the diagram does not take into consideration clearance or compression, the results are only approximate.

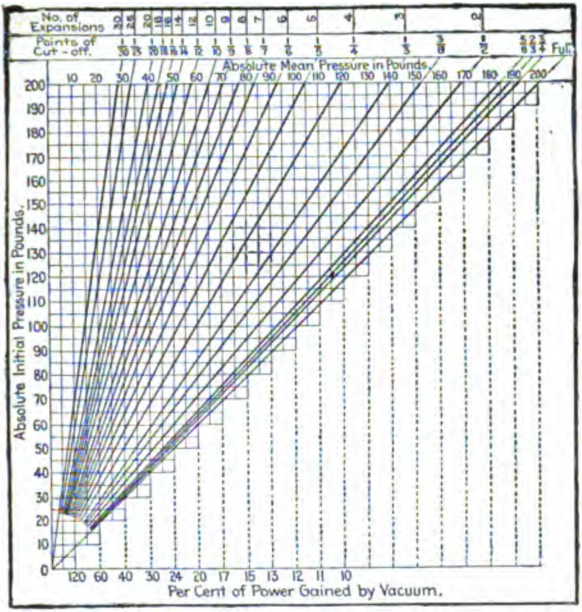

FIG. 151.

Evaporators and Distillers are used with marine engines for the purpose of providing fresh water for the boilers or for drinking purposes.

Weir's Evaporator consists of a small horizontal boiler, contrived so as to be easily taken to pieces and cleaned. The water in it is evaporated by the steam from the main boilers passing through a set of tubes placed in its bottom. The steam generated in this boiler is admitted to the low-pressure valve-box, so that there is no loss of energy, and the water condensed in it is returned to the main boilers.

In *Weir's Feed-heater* the feed-water before entering the boiler is heated up very nearly to boiling-point by means of the waste water and steam from the low-pressure valve-box of a compound engine.

GAS, PETROLEUM, AND HOT-AIR ENGINES.

Gas-engines.—For theory of the gas-engine, see paper by Dugald Clerk, Proc. Inst. C. E. 1882, vol. lxix.; and Van Nostrand's Science Series, No. 62. See also Wood's Thermodynamics. Three standard works on gas-engines are "A Practical Treatise on the 'Otto' Cycle Gas-engine," by Wm. Norris; "A Text-book on Gas, Air, and Oil Engines," by Bryan Donkin; and "The Gas and Oil Engine," by Dugald Clerk (6th edition, 1896).

In the ordinary type of single-cylinder gas-engine (for example the Otto) known as a four-cycle engine one ignition of gas takes place in one end of the cylinder every two revolutions of the fly-wheel, or every two double strokes. The following sequence of operations takes place during four consecutive strokes: (*a*) inspiration during an entire stroke; (*b*) compression during the second (return) stroke; (*c*) ignition at the dead-point, and expansion during the third stroke; (*d*) expulsion of the burnt gas during the fourth (return) stroke. Beau de Rochas in 1862 laid down the law that there are four conditions necessary to realize the best results from the elastic force of gas: (1) The cylinders should have the greatest capacity with the smallest circumferential surface; (2) the speed should be as high as possible; (3) the cut-off should be as early as possible; (4) the initial pressure should be as high as possible. In modern engines it is customary for ignition to take place, not at the dead point, as proposed by Beau de Rochas, but somewhat later, when the piston has already made part of its forward stroke. At first sight it might be supposed that this would entail a loss of power, but experience shows that though the area of the diagram is diminished, the power registered by the friction-brake is greater. Starting is also made easier by this method of working. (The Simplex Engine, Proc. Inst. M. E. 1889.)

In the Otto engine the mixture of gas and air is compressed to about 3 atmospheres. When explosion takes place the temperature suddenly rises to somewhere about 2900° F. (Robinson.)

The two great sources of waste in gas-engines are: 1. The high temperature of the rejected products of combustion; 2. Loss of heat through the cylinder walls to the water-jacket. As the temperature of the water-jacket is increased the efficiency of the engine becomes higher.

With ordinary coal-gas the consumption may be taken at 20 cu. ft. per hour per I.H.P., or 24 cu. ft. per brake H.P. The consumption will vary with the quality of the gas. When burning Dowson producer-gas the consumption of anthracite (Welsh) coal is about 1.3 lbs. per I.H.P. per hour for ordinary working. With large twin engines, 100 H.P., the consumption is reduced to about 1.1 lb. The mechanical efficiency or B.H.P. ÷ I.H.P. in ordinary engines is about 85%; the friction loss is less in larger engines.

Efficiency of the Gas-engine. (Thurston on Heat as a Form of Energy.)

Heat transferred into useful work		17%
" " to the jacket-water	52	
" lost in the exhaust-gas	16	
" " by conduction and radiation	15	
	—	83%

This represents fairly the distribution of heat in the best forms of gas-engine. The consumption of gas in the best engines ranges from a minimum of 18 to 20 cu. ft. per I.H.P. per hour to a maximum exceeding in the smaller engines 25 cu. ft. or 30 cu. ft. In small engines the consumption per brake horse-power is one third greater than these figures.

The report of a test of a 170-H.P. Crossley (Otto) gas-engine in England, 1892, using producer-gas, shows a consumption of but .85 lb. of coal per H.P. hour, or an absolute combined efficiency of 21.3% for the engine and producer. The efficiency of the engine alone is in the neighborhood of 25%.

The Taylor gas-producer is used in connection with the Otto gas-engine at the Otto Gas-engine Works in Philadelphia. The only loss is due to radiation through the walls of the producer and a small amount of heat carried off in the water from the scrubber. Experiments on a 100-H.P. engine show a consumption of 97/100 lb. of carbon per I.H.P. per hour. This result is superior to any ever obtained on a steam-engine. (*Iron Age*, 1893.)

Tests of the Simplex Gas-engine. (Proc. Inst. M. E. 1889.)—Cylinder 7⅞ × 15¾ in., speed 160 revs. per min. Trials were made with town gas of a heating value of 607 heat-units per cubic foot, and with Dowson gas, rich in CO, of about 150 heat-units per cubic foot.

	Town Gas.			Dowson Gas.		
	1.	2.	3.	1.	2.	3.
Effective H.P.	6.70	8.67	9.28	7.12	8.61	5.26
Gas per H.P. per hour, cu. ft.	21.55	20.12	20.73	88.08	114.85	97.88
Water per H.P. per hour, lbs.	54.7	44.4	43.8	58.3		
Temp. water entering, F.	51°	51°	51°	48°		
" " effluent	185°	144°	172°	144°		

The gas volume is reduced to 32° F. and 30 in barometer. A 50-H.P. engine working 35 to 40 effective H.P. with Dowson generator consumed 51 lbs. English anthracite per hour, equal to 1.48 to 1.3 lbs. per effective H.P. A 16-H.P. engine working 12 H.P. used 19.4 cu. ft. of gas per effective H.P.

A 320-H.P. Gas-engine.—The flour-mills of M. Leblanc, at Pantin, France, have been provided with a 320-horse-power fuel-gas engine of the Simplex type. With coal-gas the machine gives 450 horse-power. There is one cylinder, 34.8 in. diam.; the piston-stroke is 40 in.; and the speed 100 revs.

per min. Special arrangements have been devised in order to keep the different parts of the machine at appropriate temperatures. The coal used is 0.812 lb. per indicated or 1.08 lb. per brake horse-power. The water used is 8¾ gallons per brake horse-power per hour.

Test of an Otto Gas-engine. (*Jour. F. I.*, Feb. 1890, p. 115.)—Engine 7 H.P. nominal; working capacity of cylinder .2594 cu. ft.; clearance space .1796 cu. ft.

	° F.
Temperature of gas supplied..	62.2
" " " exhaust...	774.3
" " entering water	50.4
" " exit water....	89.2
Pressure of gas, in. of water..	3.06
Revolution per min., av'ge....	161.6
Explosions missed per min., average..................	6.8
Mean effective pressure, lbs. per sq. in.	59.
Horse-power, indicated.......	4.94
Work per explosion, foot-pounds	2204.
Explosions per minute........	74.
Gas per I.H.P. per hour, cu. ft.	28.4

Heat-units.	Per cent.
Transferred into work.........	23.34
Taken by jacket-water........	49.94
" " exhaust..............	27.22

Composition of the gas:

	By Volume.	By Weight.
CO_2............	0.50%	1.922%
C_2H_4...........	4.32	10.520
O..............	1.00	2.797
CO.............	5.83	15.410
CH_4...........	27.18	38.042
H	51.57	9.021
N...	9.06	22.278
	99.96	99.995

Test of the Clerk Gas-engine. (Proc. Inst. C. E. 1882, vol. lxix.)—Cylinder 6 × 12 in., 150 revs. per min.; mean available pressure, 70.1 lbs., 9 I.H.P.; maximum pressure, 220 lbs. per sq. in. above atmosphere; pressure before ignition, 41 lbs. above atm.; temperature before compression, 60° F., after compression, 313° F.; temperature after ignition calculated from pressure, 2800° F.; gas required per I.H.P. per hour, 22 cu. ft.

More Recent Tests of gas-engines, 1898, have given higher economical results than those above quoted. The gas-consumption (city gas) has been as low as 15 cu. ft. per I.H.P. per hour, and the efficiency as high as 27% of the heating value of the gas. The principal improvement in practice has been the use of much higher compression of the working charge.

Combustion of the Gas in the Otto Engine.—John Imray, in discussion of Mr. Clerk's paper on Theory of the Gas-engine, says: The change which Mr. Otto introduced, and which rendered the engine a success, was that, instead of burning in the cylinder an explosive mixture of gas and air, he burned it in company with, and arranged in a certain way in respect of, a large volume of incombustible gas which was heated by it, and which diminished the speed of combustion. W. R. Bousfield, in the same discussion, says: In the Otto engine the charge varied from a charge which was an explosive mixture at the point of ignition to a charge which was merely an inert fluid near the piston. When ignition took place there was n explosion close to the point of ignition that was gradually communicated throughout the mass of the cylinder. As the ignition got farther away from the primary point of ignition the rate of transmission became slower, and if the engine were not worked too fast the ignition should gradually catch up to the piston during its travel, all the combustible gas being thus consumed. This theory of slow combustion is, however, disputed by Mr. Clerk, who holds that the whole quantity of combustible gas is ignited in an instant.

Temperatures and Pressures developed in a Gas-engine. (Clerk on the Gas-engine.)—Mixtures of air and Oldham coal-gas. Temperature before explosion, 17° C.

Mixture.		Max. Press above Atmos., lbs. per sq. in.	Temp. of Explosion calculated from observed Pressure.	Theoretical Temp. of Explosion if all Heat were evolved.
Gas.	Air.			
1 vol.	14 vols.	40.	806° C.	1786° C.
1 "	13 "	51.5	1033	1912
1 "	12 "	60.	1202	2058
1 "	11 "	61.	1220	2228
1 "	9 "	78.	1557	2670
1 "	7 "	87.	1733	3334
1 "	6 "	90.	1792	3808
1 "	5 "	91.	1812	....
1 "	4 "	80.	1595	....

Use of Carburetted Air in Gas-engines.—Air passed over gasoline or volatile petroleum spirit of low sp. gr., 0.65 to 0.70, liberates some of the gasoline, and the air thus saturated with vapor is equal in heating or lighting power to ordinary coal-gas. It may therefore be used as a fuel for gas-engines. Since the vapor is given off at ordinary temperatures gasoline is very explosive and dangerous, and should be kept in an underground tank out of doors. A defect in the use of carburetted air for gas-engines is that the more volatile products are given off first, leaving an oily residue which is often useless. Some of the substances in the oil that are taken up by the air are apt to form troublesome deposits and incrustations when burned in the engine cylinder.

The Otto Gasoline-engine. (*Eng'g News*, May 4, 1893.)—It is claimed that where but a small gasoline-engine is used and the gasoline bought at retail the liquid fuel will be on a par with a steam-engine using 6 lbs. of coal per horse-power per hour, and coal at $3.50 per ton, and will besides save all the handling of the solid fuel and ashes, as well as the attendance for the boilers. As very few small steam-engines consume less than 6 lbs. of coal per hour, this is an exceptional showing for economy. At 8 cts. per gallon for gasoline and 1/10 gal. required per H.P. per hour, the cost per H.P. per hour will be 0.8 cent.

Gasoline-engines are coming into extensive use (1898). In these engines the gasoline is pumped from an underground tank, located at some distance outside the engine-room, and led through carefully soldered pipes to the working cylinder. In the combustion chamber the gasoline is sprayed into a current of air, by which it is vaporized. The mixture is then compressed and ignited by an electric spark. At no time does the gasoline come in contact with the air outside of the engine, nor is there any flame or burning gases outside of the cylinder.

Naphtha-engines are in use to some extent in small yachts and launches. The naphtha is vaporized in a boiler, and the vapor is used expansively in the engine-cylinder, as steam is used; it is then condensed and returned to the boiler. A portion of the naphtha vapor is used for fuel under the boiler. According to the circular of the builders, the Gas Engine and Power Co. of New York, a 2-H.P. engine requires from 3 to 4 quarts of naphtha per hour, and a 4-H.P. engine from 4 to 6 quarts. The chief advantages of the naphtha-engine and boiler for launches are the saving of weight and the quickness of operation. A 2-H.P. engine weighs 200 lbs., a 4-H.P. 300 lbs. It takes only about two minutes to get under headway. (Modern Mechanism, p. 270.)

Hot-air (or Caloric) Engines.—Hot-air engines are used to some extent, but their bulk is enormous compared with their effective power. For an account of the largest hot-air engine ever built (a total failure) see Church's Life of Ericsson. For theoretical investigaton, see Rankine's Steam-engine and Rontgen's Thermodynamics. For description of constructions, see Appleton's Cyc. of Mechanics and Modern Mechanism, and Babcock on Substitutes for Steam, Trans. A. S. M. E., vii., p. 693.

Test of a Hot-air Engine (Robinson).—A vertical double-cylinder (Caloric Engine Co.'s) 12 nominal H.P. engine gave 20.19 I.H.P. in the working cylinder and 11.38 I.H.P. in the pump, leaving 8.81 net I.H.P.; while the effective brake H.P. was 5.9, giving a mechanical efficiency of 67%. Consumption of coke, 3.7 lbs. per brake H.P. per hour. Mean pressure on pistons 15.37 lbs. per square inch, and in pumps 15.9 lbs., the area of working cylinders being twice that of the pumps. The hot air supplied was about 1150° F. and that rejected at end of stroke about 890° F.

The Priestman Petroleum-engine. (*Jour. Frank. Inst.*, Feb. 1893.)—The following is a description of the operation of the engine: Any ordinary high-test (usually 150° test) oil is forced under air-pressure to an atomizer, where the oil is met by a current of air and broken up into atoms and sprayed into a mixer, where it is mixed with the proper proportion of supplementary air and sufficiently heated by the exhaust from the cylinder passing around this chamber. The mixture is then drawn by suction into the cylinder, where it is compressed by the piston and ignited by an electric spark, a governor controlling the supply of oil and air proportionately to the work performed. The burnt products are discharged through an exhaust-valve which is actuated by a cam. Part of the air supports the combustion of the oil, and the heat generated by the combustion of the oil expands the air that remains and the products resulting from the explosion, and thus develops its power from air that it takes in while running. In other words, the engine exerts its power by inhaling air, heating that air, and expelling the products of combustion when done with. In the largest engines only the 1/250 part of a pint of oil is used at any one time, and in the smallest sizes the fuel is prepared in correct quantities varying from 1/7000 of a pint upward, according to whether the engine is running on light or full duty. The cycle of operations is the same as that of the Otto gas-engine.

Trials of a 5-H.P. Priestman Petroleum-engine. (Prof. W. C. Unwin, Proc. Inst. C. E. 1892.)—Cylinder, 8½ × 12 in., making normally 200 revs. per min. Two oils were used, Russian and American. The more important results were given in the following table:

	Trial V. Full Power.	Trial I. Full Power.	Trial IV. Full Power.	Trial II. Half Power.	Trial III. Light.
Oil used	Day-light.	Russo-lene.	Russo-lene.	Russo-lene.	Russo-lene.
Brake H.P	7.722	6.763	6.882	3.62	......
I.H.P	9.369	7.408	8.332	4.70	0.889
Mechanical efficiency	0.824	0.91	0.876	0.769	....
Oil used per brake H.P. hour, lb	0.842	0.946	0.988	1.381	....
Oil used per indicated H.P. hour, lb	0.694	0.864	0.816	1.063	5.734
Lb. of air per lb. of oil	33.4	31.7	48.2	21.7	10.1
Mean explosion pressure, lbs. per sq. in	151.4	134.3	128.5	48.5	9.6
Mean compression pressure, lbs. per sq. in	35.0	27.6	26.0	14.8	6.0
Mean terminal pressure, lbs. per sq in	35.4	23.7	25.5	15.6	....

To compare the fuel consumption with that of a steam-engine, 1 lb. of oil might be taken as equivalent to 1½ lbs. of coal. Then the consumption in the oil-engine was equivalent, in Trials I., IV., and V., to 1.42 lbs., 1.48 lbs., and 1.26 lbs. of coal per brake horse-power per hour. From Trial IV. the following values of the expenditure of heat were obtained:

	Per cent.
Useful work at brake	13.31
Engine friction	2.81
Heat shown on indicator-diagram	16.12
Rejected in jacket-water	47.54
" in exhaust-gases	26.72
Radiation and unaccounted for	9.61
Total	99.99

LOCOMOTIVES.

Efficiency of Locomotives and Resistance of Trains. (George R. Henderson, Proc. Engrs. Club of Phila. 1886.)—The efficiency of locomotives can be divided into two principal parts: the first depending upon the size of the cylinders and wheels, the valve-gear, boiler and steam-passages, of which the tractive power is a function; and the second upon the speed, grade, curvature, and friction, which combine to produce the resistance.

The tractive power may be determined as follows:

Let P = tractive power;
p = average effective pressure in cylinder;
S = stroke of piston;
d = diameter of cylinders;
D = diameter of driving-wheels. Then

$$P = \frac{4\pi d^2 pS}{4\pi D} = \frac{d^2 pS}{D}.$$

The average effective pressure can be obtained from an indicator-diagram, or by calculation, when the initial pressure and ratio of expansion are known, together with the other properties of the valve-motion. The subjoined table from "Auchincloss" gives the proportion of mean effective pressure to boiler-pressure above atmosphere for various proportions of cut-off.

Stroke, Cut off at—	M.E.P. (Boiler-pres. = 1).	Stroke, Cut off at—	M.E.P. (Boiler-pres. = 1).	Stroke, Cut off at—	M.E.P. (Boiler-pres. = 1).
.1	.15	.333 = ⅓	.5 = ½	.625 = ⅝	.79
.125 = ⅛	.2	.375 = ⅜	.55	.666 = ⅔	.82
.15	.24	.4	.57	.7	.85
.175	.28	.45	.62	.75 = ¾	.89
.2	.32	.5 = ½	.67	.8	.93
.25 = ¼	.4	.55	.72	.875 = ⅞	.96
.3	.46				

These values were deduced from experiments with an English locomotive by Mr. Gooch. As diagrams vary so much from different causes, this table will only fairly represent practical cases. It is evident that the cut-off must be such that the boiler will be capable of supplying sufficient steam at the given speed.

In the following calculations it is assumed that the adhesion of the engine is at least equal to the tractive power, which is generally the case—if the engine be well designed—except when starting, or running at a very low rate of speed, with a small expansive ratio. When running faster, economy, and also the size of the boiler, necessitate a higher ratio of expansion, thus reducing the tractive power below the adhesion. If the adhesion be less than the tractive power, substitute it for the latter in the following formulæ.

The resistances can be computed in the following manner, first considering the train:

There is a resistance due to friction of the journals, pressure of wind, etc., which increases with the speed. Most of the experiments made with a view of determining the resistance of trains have been with European rolling-stock and on European railways. The few trials that have been made here seem to prove that with American systems this resistance is less.

The following table gives the resistance at different speeds, assumed for American practice:

Speed in miles per hour:												
$s =$	5	10	15	20	25	30	35	40	45	50	55	60
Resistance in pounds per ton of 2240 lbs.:												
$y =$	3.1	3.4	4.	4.8	5 8	7.1	8.6	10.2	12.1	14.3	16.8	19.2
Coefficient of resistance in terms of load:												
$l =$	.0015	.0017	.0020	.0024	.0029	.0035	.0043	.0051	.0060	.0071	.0084	.0096

$$l = .0015\left(1 + \frac{s^2}{650}\right).$$

The resistance due to curvature is about .5 lb. per ton per degree of curvature, or the coefficient $= .00025c$, where $c =$ the curvature in degrees

The effect of grades may be determined by the theory of the inclined plane.

Consider a load L on a grade of m feet per mile. The component of the weight L acting in the line of traction, or parallel to the track, is

$$L \sin \theta = \frac{Lm}{5280} = .00019Lm.$$

To combine these coefficients in one equation representing the resistance of the train:

Let $L =$ weight of train, exclusive of engine, in pounds;
$R =$ resistance of train, in pounds.
s, c, and m, as above. Then

$$R = L\left[.0015\left(1 + \frac{s^2}{650}\right) + .00025c \pm .00019m\right],$$

the ± sign meaning that this coefficient is positive for ascending and negative for descending grades.

To find a grade upon which a train would descend by itself, take the last coefficient minus and make $R = O$, whence

$$m = 7.9\left(1+\frac{s^2}{650}\right) + 1.3c.$$

As locomotives usually have a long rigid wheel-base, the coefficient for curvature had better be doubled. The resistance due to the friction of the working parts will be considered as being proportional to the tractive power, so that the effective tractive power will be represented by uP, the resistance being $(1 - u)P$.

Combining all these values, there results the equation between the tractive power and the weight of the train and engine:

$$uP - W(.0005c \pm .00019m) = Ll + .00025c \pm .00019m,$$

W being weight of engine and tender, and u being probably about .8.

Transforming, we have

$$L = \frac{uP - W(.0005c \pm .00019m)}{l + .00025c \pm .00019m},$$

and

$$P = \frac{L(l + .00025c \pm .00019m) + W(.0005c \pm .00019m)}{u}.$$

These deductions, says Mr. Henderson, agree well with railroad practice.

The figures given above for resistances are very much less than those given by the old formulæ (which were certainly wrong), but even Mr. Henderson's figures for high speed are too high, according to a diagram given by D. L. Barnes in *Eng'g Mag.*, June, 1894, from which the following figures are derived:

Speed, miles per hour..............	50	60	70	80	90	100
Resistance, pounds per gross ton..	12	12.4	13.5	15	17	20

Eng'g News, March 8, 1894, gives a formula which for high speeds gives figures for resistance between those of Mr. Barnes and Mr. Henderson. See tests reported in *Eng'g News* of June 9, 1892. The formula is, resistance in pounds per ton = ¼ velocity in miles per hour + 2. This gives for

Speed	5	10	15	20	25	30	35	40	45	50	60	70	80	90	100
Resistance..	3¼	4.5	5¾	7	8¼	9.5	10¾	12	13¼	14.5	17	19.5	22	24.5	27

For tables showing that the resistance varies with the area exposed to the resistance and friction of the air per ton of load, see Dashiell, Trans. A. S. M. E., vol. xiii. p. 371.

Inertia and Resistances of Railroad Trains at Increasing Speeds.—A series of tables and diagrams is given in *R. R. Gaz.*, Oct. 31, 1890, to show the resistances due to inertia in starting trains and accelerating their speeds.

The mechanical principles and formulæ from which these data were calculated are as follows:

S = speed in miles per hour to be acquired at the end of a mile.

$S \div 2$ = average speed in miles per hour during the first mile run.

V = velocity in feet per second at the end of a mile; then $V \div 2$ = average velocity in feet per second during the first mile run.

$5280 \div V/2$ = time in seconds required to run first mile = $10560 \div V$.

$V \div (10560 \div V) = V^2 \div 10560 = .0000947V^2$ = Constant gain in velocity or acceleration in feet per second necessary to the acquirement of a velocity V at the end of a mile.

g = acceleration due to the force of gravity, i.e., 32.2 feet per second.

The forces required to accelerate a given mass in a given time to different velocities are in proportion to those velocities. The weight of a body is the measure of the force which accelerates it in the case of gravity, and as we are considering 1 lb., or the unit of weight, as the mass to be accelerated, we have $g : (V^2 \div 10560) :: 1$ is to the force required to accelerate 1 lb. to the velocity V at the end of a mile run, or, what is the same, to accelerate it at the rate of $V^2 \div 10560$ feet per second.

From this the pull on the drawbar—it is the same as the force just mentioned, and is properly termed the inertia—in pounds per pound of train weight is $V^2 \div (10560 \times 32.2)$, which equals $.00000294V^2$.

This last formula also gives the grade in per cent which will give a resistance equal to the inertia due to acceleration.

The grade in feet per mile is $.00000294 V^2 \times 5280 = .015531 V^2$.

The resistance offered in pounds per ton is 2000 times as much as per pound, or $.00588 V^2$.

When the adhesion of locomotive drivers is 600 lbs. per ton of weight thereon—this is about the maximum—then the tons on drivers necessary to overcome the inertia of each ton of total train load are $.00588 V^2 \div 600 = .0000098 V^2$. In this determination of resistances no account has been taken of the rotative energy of the wheels.

Efficiency of the Mechanism of a Locomotive.—Druitt Halpin (Proc. Inst. M. E., January, 1889,) writes as follows, concerning the tractive efficiency of locomotives; With simple two-cylinder engines, having four wheels coupled, experiments have been made by the late locomotive superintendent of the Eastern Railway of France, M. Regray, with the greatest possible care and with the best apparatus, and the result arrived at was that out of 100 I.H.P in the cylinders 48 H.P. only was available on the draw-bar. Moreover, the loss of 57% had been confirmed independently on the Pennsylvania Railroad, with an engine having 18¼ × 24-in. cylinders and 6 ft. 6 in. wheels four-coupled; up to 65 miles an hour, the power on the draw-bar was found to be only 42% of that in the cylinders.

Frank C. Wagner (Proc. A. A. A. S., 1900, p. 140), commenting on the above tests, says it does not seem possible that they fairly represent average conditions. He gives an account of some dynamometer tests which indicate that in ordinary freight service the power used to drive the locomotive and tender and to overcome the friction of the mechanism is from 10% to 35% of the total power developed in the steam-cylinder. In one test the weight of the locomotive and tender was 16% of the total weight of the train, while the power consumed in the locomotive and tender was from 30% to 33% of the indicated horse power.

The Size of Locomotive Cylinders is usually taken to be such that the engine will just overcome the adhesion of its wheels to the rails under favorable circumstances.

The adhesion of the wheel is about one third the weight when the rail is clean and sanded, but is usually assumed at 0.25. (Thurston.)

A committee of the American Association of Master Mechanics, after studying the performance reports of the best engines, proposes the following formula for weight on driving-wheels: $W = \frac{0.85 C d^2 PS}{D}$ in which the mean pressure in the cylinder is taken at 0.85 of the boiler-pressure at starting, C is a numerical coefficient of adhesion, d the diameter of cylinder in inches, D that of the drivers in inches, P the pressure in the boiler in pounds per square inch, S the stroke of piston in inches. C is taken as 0.25 for passenger engines, 0.24 for freight, and 0.22 for "switching" engines.

The common builder's rule for determining the size of cylinders for the locomotive is the following, in which we accept Mr. Forney's assumption that the steam-pressure at the engine may be taken as nine tenths that in the boiler: The tractive force is, approximately, $F = \frac{0.9 p_1 \times A \times 4S}{C}$ where C is the circumference of tires of driving-wheels, S = the stroke in inches, p_1 = the initial unbalanced steam-pressure in the cylinder in pounds per square inch, and A = the area of one cylinder in square inches. If D = diameter of driving wheel and d = diameter of cylinder, $F = \frac{0.9 p_1 \times d^2 S}{D}$.

Taking the adhesion at one fourth the weight W,

$$F = 0.25 W = \frac{0.9 p_1 \times A \times 4S}{C} = \frac{0.9 p_1 d^2 S}{D};$$

whence the area of each piston is

$$A = \frac{0.25 C W}{0.9 \times 4 \times p_1 S}; \quad d = \sqrt{\frac{0.25 D W}{0.9 p_1 S}}.$$

The above formulæ give the maximum tractive force; for the mean tractive force substitute for p_1 in the formulæ the mean effective pressure.

Von Borries's rule for the diameter of the low-pressure cylinder of a compound locomotive is $d^2 = \frac{2ZD}{ph}$,

where d = diameter of l.p. cylinder in inches;
D = diameter of driving-wheel in inches;
p = mean effective pressure per sq. in., after deducting internal machine friction;
h = stroke of piston in inches;
Z = tractive force required, usually 0.14 to 0.16 of the adhesion.

The value of p depends on the relative volume of the two cylinders, and from indicator experiments may be taken as follows:

Class of Engine.	Ratio of Cylinder Volumes.	p in percentage of Boiler-pressure.	p for Boiler-pressure of 176 lbs.
Large-tender eng's	1 : 2 or 1 : 2.05	42	74
Tank-engines......	1 : 2 or 1 : 2.2	40	71

The Size of Locomotive Boilers. (Forney's Catechism of the Locomotive.)—They should be proportioned to the amount of adhesive weight and to the speed at which the locomotive is intended to work. Thus a locomotive with a great deal of weight on the driving-wheels could pull a heavier load, would have a greater cylinder capacity than one with little adhesive weight, would consume more steam, and therefore should have a larger boiler.

The weight and dimensions of locomotive boilers are in nearly all cases determined by the limits of weight and space to which they are necessarily confined. It may be stated generally that *within these limits a locomotive boiler cannot be made too large.* In other words, boilers for locomotives should always be made as large as is possible under the conditions that determine the weight and dimensions of the locomotives.

Wootten's Locomotive. (Clark's Steam-engine; see also Jour. Frank. Inst. 1891, and Modern Mechanism, p. 485.)—J. E. Wootten designed and constructed a locomotive boiler for the combustion of anthracite and lignite, though specially for the utilization as fuel of the waste produced in the mining and preparation of anthracite. The special feature of the engine is the fire-box, which is made of great length and breadth, extending clear over the wheels, giving a grate-area of from 64 to 85 sq. ft. The draught diffused over these large areas is so gentle as not to lift the fine particles of the fuel. A number of express-engines having this type of boiler are engaged on the fast trains between Philadelphia and Jersey City. The fire-box shell is 8 ft. 8 in. wide and 10 ft. 5 in. long; the fire-box is 8×9½ ft., making 76 sq. ft. of grate-area. The grate is composed of bars and water-tubes alternately. The regular types of cast-iron shaking grates are also used. The height of the fire-box is only 2 ft. 5 in. above the grate. The grate is terminated by a bridge of fire-brick, beyond which a combustion-chamber, 27 in. long, leads to the flue-tubes, about 184 in number, 1¾ in. diam. The cylinders are 21 in. diam., with a stroke of 22 inches. The driving-wheels, four-coupled, are 5 ft. 8 in. diam. The engine weighs 44 tons, of which 29 tons are on driving wheels. The heating-surface of the fire-box is 135 sq. ft., that of the flue-tubes is 982 sq. ft.; together, 1117 sq. ft., or 14.7 times the grate-area. Hauling 15 passenger-cars, weighing with passengers 360 tons, at an average speed of 42 miles per hour, over ruling gradients of 1 in 89, the engine consumes 62 lbs. of fuel per mile, or 34¼ lbs. per sq. ft. of grate per hour.

Qualities Essential for a Free-steaming Locomotive. (From a paper by A. E. Mitchell, read before the N. Y. Railroad Club; *Eng'g News*, Jan. 24, 1891.)—Square feet of boiler-heating surface for bituminous coal should not be less than 4 times the square of the diameter in inches of a cylinder 1 inch larger than the cylinder to be used. One tenth of this should be in the fire-box. On anthracite locomotives more heating-surface is required in the fire-box, on account of the larger grate-area required, but the heating-surface of the flues should not be materially decreased.

Grate-surface, Smoke-stacks, and Exhaust-nozzles for Locomotives. (*Am. Mach.*, Jan. 8, 1891.)—For grate-surface for anthracite coal: Multiply the displacement in cubic feet of one piston during a stroke by 8.5; the product will be the area of the grate in square feet.

For bituminous coal: Multiply the displacement in feet of one piston during a stroke by 6¼; the product will be the grate-area in square feet for engines with cylinders 12 in. in diameter and upwards. For engines with smaller cylinders the ratio of grate-area to piston-displacement should be 7½ to 1, or even more, if the design of the engine will admit this proportion.

The grate-areas in the following table have been found by the foregoing rules, and agree very closely with the average practice:

Smoke-stacks.—The internal area of the smallest cross-section of the stack should be 1/17 of the area of the grate in soft-coal-burning engines.

A. E. Mitchell, Supt. of Motive Power of the N. Y. L. E. & W. R. R., says that recent practice varies from this rule. Some roads use the same size of stack, 13½ in. diam. at throat, for all engines up to 20 in. diam. of cylinder.

The area of the orifices in the exhaust-nozzles depends on the quantity and quality of the coal burnt, size of cylinder, construction of stack, and the condition of the outer atmosphere. It is therefore impossible to give rules for computing the exact diameter of the orifices. All that can be done is to give a rule by which an approximate diameter can be found. The exact diameter can only be found by trial. Our experience leads us to believe that the area of each orifice in a double exhaust-nozzle should be equal to 1/400 part of the grate-surface, and for single nozzles 1/200 of the grate-surface. These ratios have been used in finding the diameters of the nozzles given in the following table. The same sizes are often used for either hard or soft coal-burners.

Size of Cylinders, in inches.	Grate-area for Anthracite Coal, in sq. in.	Grate-area for Bituminous Coal, in sq. in.	Diameter of Stacks, in inches.	Double Nozzles. Diam. of Orifices, in inches.	Single Nozzles. Diam. of Orifices, in inches.
12 × 20	1501	1217	9½	2	2 13/16
13 × 20	1878	1482	10½	2⅛	3
14 × 20	2179	1666	11¼	2 5/16	3¼
15 × 22	2742	2097	12½	2 9/16	3 11/16
16 × 24	3415	2611	14	2⅞	4 1/16
17 × 24	3856	2948	15	3 1/16	4 5/16
18 × 24	4321	3304	15¾	3¼	4⅝
19 × 24	4810	3678	16½	3 7/16	4 13/16
20 × 24	5337	4081	17½	3⅝	5 1/16

Exhaust-nozzles in Locomotive Boilers.—A committee of the Am. Ry. Master Mechanics' Assn. in 1890 reported that they had, after two years of experiment and research, come to the conclusion that, owing to the great diversity in the relative proportions of cylinders and boilers, together with the difference in the quality of fuel, any rule which does not recognize each and all of these factors would be worthless.

The committee was unable to devise any plan to determine the size of the exhaust-nozzle in proportion to any other part of the engine or boiler, and believes that the best practice is for each user of locomotives to adopt a nozzle that will make steam freely and fill the other desired conditions, best determined by an intelligent use of the indicator and a check on the fuel account. The conditions desirable are: That it must create draught enough on the fire to make steam, and at the same time impose the least possible amount of work on the pistons in the shape of back pressure. It should be large enough to produce a nearly uniform blast without lifting or tearing the fire, and be economical in its use of fuel.

Fire-brick Arches in Locomotive Fire-boxes.—A committee of the Am. Ry. Master Mechanics' Assn. in 1890 reported strongly in favor of the use of brick arches in locomotive fire-boxes. They say: It is the unanimous opinion of all who use bituminous coal and brick arch, that it is most efficient in consuming the various gases composing black smoke, and by impeding and delaying their passage through the tubes, and mingling and subjecting them to the heat of the furnace, greatly lessens the volume ejected, and intensifies combustion, and does not in the least check but rather augments draught, with the consequent saving of fuel and increased steaming capacity that might be expected from such results. This in particular when used in connection with extension front.

Size, Weight, Tractive Power, etc., of Different Sizes of Locomotives. (J. G. A. Meyer, Modern Locomotive Construction, *Am.*

Mach., Aug. 8, 1885.)—The tractive power should not be more or less than the adhesion. In column 3 of each table the adhesion is given, and since the adhesion and tractive power are expressed by the same number of pounds, these figures are obtained by finding the tractive power of each engine, for this purpose always using the small diameter of driving-wheels given in column 2. The weight on drivers is shown in column 4, which is obtained by multiplying the adhesion by 5 for all classes of engines. Column 5 gives the weights on the trucks, and these are based upon observations. Thus, the weight on the truck for an eight-wheeled engine is about one half of that placed on the drivers.

For Mogul engines we multiply the total weight on drivers by the decimal .2, and the product will be the weight on the truck.

For ten-wheeled engines the total weight on the drivers, multiplied by the decimal .32, will be equal to the weight on the truck.

And lastly, for consolidation engines, the total weight on drivers multiplied by the decimal .16, will determine the weight on the truck.

In column 6 the total weight of each engine is given, which is obtained by adding the weight on the drivers to the weight on the truck. Dividing the adhesion given in column 1 by 7½ gives the tons of 2000 lbs. that the engine is capable of hauling on a straight and level track column 7, at slow speed.

The weight of engines given in these tables will be found to agree generally with the actual weights of locomotives recently built, although it must not be expected that these weights will agree in every case with the actual weights, because the different builders do not build the engines alike.

The actual weight on trucks for eight-wheeled or ten-wheeled engines will not differ much from those given in the tables, because these weights depend greatly on the difference between the total and rigid wheel-base, and these are not often changed by the different builders. The proportion between the rigid and total wheel-base is generally the same.

The rule for finding the tractive power is:

$$\frac{\left\{\begin{matrix}\text{Square of dia. of}\\ \text{piston in inches}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Mean effect. steam}\\ \text{press. per sq. in.}\end{matrix}\right\} \times \left\{\begin{matrix}\text{stroke}\\ \text{in feet}\end{matrix}\right\}}{\text{Diameter of wheel in feet.}} = \text{tractive power.}$$

EIGHT-WHEELED LOCOMOTIVES.

Cylinders—Diameter, Stroke.	Diameter of Driving-wheels.	Adhesion.	Weight on Drivers.	Weight on Truck.	Total Weight.	Hauling Capacity on Level Track in tons of 2000 lbs., including Tender.
1	2	3	4	5	6	7
in.	in.	lbs.	lbs.	lbs.	lbs.	
10×20	45-51	4000	20000	10000	30000	533
11×22	45-51	5324	26620	13310	39930	709
12×22	48-54	5940	29700	14850	44550	792
13×22	49-57	6828	34140	17070	51210	910
14×24	55-61	7697	38485	19242	57727	1026
15×24	55-66	8836	44180	22090	66270	1178
16×24	58-66	9533	47665	23832	71497	1271
17×24	60-66	10404	52020	26010	78030	1387
18×24	61-66	11472	57360	28680	86040	1529

TEN-WHEELED ENGINES.

Cylinders—Diameter, Stroke.	Diameter of Driving-wheels.	Adhesion.	Weight on Drivers.	Weight on Truck.	Total Weight, with Water and Fuel.	Hauling Capacity on Level Track in tons of 2000 lbs., including Tender.
1	2	3	4	5	6	7
in.	in.	lbs.	lbs.	lbs.	lbs.	
12×18	39-43	5981	29907	9570	39477	797
13×18	41-45	6677	33387	10683	44070	890
14×20	43-47	8205	41023	13127	54150	1093
15×22	45-50	9900	49500	15840	65340	1320
16×24	48-54	11520	57600	18432	76032	1536
17×24	51-56	12240	61200	19584	80784	1632
18×24	51-56	13722	68611	21955	90566	1829
19×24	54-60	14440	72200	23104	95304	1925

MOGUL ENGINES.

in.	in.	lbs.	lbs.	lbs.	lbs.	
11×16	35-40	4978	24891	4978	29869	663
12×18	36-41	6480	32400	6480	38880	864
13×18	37-42	7399	36997	7399	44396	986
14×20	39-43	9046	45230	9046	54276	1206
15×22	42-47	10607	53035	10607	63642	1414
16×24	45-51	12288	61440	12288	73738	1638
17×24	49-54	12739	63697	12739	76436	1698
18×24	51-56	13722	68611	13722	82333	1829
19×24	54-60	14440	72200	14440	86640	1925

CONSOLIDATION ENGINES.

in.	in.	lbs.	lbs.	lbs.	lbs.	
14×16	36-38	7840	39200	6272	45472	1045
15×18	36-38	10125	50625	8100	58725	1350
20×24	48-50	18000	90000	14400	104400	2400
22×24	50-52	20909	104544	16727	121271	2787

Leading American Types of Locomotive for Freight and Passenger Service.

1. The eight-wheel or "American" passenger type, having four coupled driving-wheels and a four-wheeled truck in front.

2. The "ten-wheel" type, for mixed traffic, having six coupled drivers and a leading four-wheel truck.

3. The "Mogul" freight type, having six coupled driving-wheels and a pony or two-wheel truck in front.

4. The "Consolidation" type, for heavy freight service, having eight coupled driving-wheels and a pony truck in front.

Besides these there is a great variety of types for special conditions of service, as four-wheel and six-wheel switching-engines, without trucks; the Forney type used on elevated railroads, with four coupled wheels under the engine and a four-wheeled rear truck carrying the water-tank and fuel; locomotives for local and suburban service with four coupled driving-wheels, with a two-wheel truck front and rear, or a two-wheel truck front and a four-wheel truck rear, etc. "Decapod" engines for heavy freight service have ten coupled driving-wheels and a two-wheel truck in front.

Steam-distribution for High-speed Locomotives.

(C. H. Quereau, *Eng'g News*, March 8, 1894.)

Balanced Valves.—Mr. Philip Wallis, in 1886, when Engineer of Tests for the C., B. & Q. R. R., reported that while 6 H.P. was required to work unbalanced valves at 40 miles per hour, for the balanced valves 2.2 H.P. only was necessary.

Effect of Speed on Average Cylinder-pressure.—Assume that a locomotive has a train in motion, the reverse lever is placed in the running notch, and the track is level; by what is the maximum speed limited? The resistance of the train and the load increase, and the power of the locomotive decreases with increasing speed till the resistance and power are equal, when the speed becomes uniform. The power of the engine depends on the average pressure in the cylinders. Even though the cut-off and boiler-pressure remain the same, this pressure decreases as the speed increases; because of the higher piston-speed and more rapid valve-travel the steam has a shorter time in which to enter the cylinders at the higher speed. The following table, from indicator-cards taken from a locomotive at varying speeds, shows the decrease of average pressure with increasing speed:

Miles per hour	46	51	51	53	54	57	60	66
Speed, revolutions	224	248	248	258	263	277	292	321
Average pressure per sq. in.:								
Actual	51.5	44.0	47.3	43.0	41.3	42.5	37.3	36.3
Calculated	....	46.5	46.5	44.7	43.8	41.6	39.5	35.9

The "average pressure calculated" was figured on the assumption that the mean effective pressure would decrease in the same ratio that the speed increased. The main difference lies in the higher steam-line at the lower speeds, and consequent higher expansion-line, showing that more steam entered the cylinder. The back pressure and compression-lines agree quite closely for all the cards, though they are slightly better for the slower speeds. That the difference is not greater may safely be attributed to the large exhaust-ports, passages, and exhaust tip, which is 5 in. diameter. These are matters of great importance for high speeds.

Boiler-pressure.—The increase of train resistance with increased speed is not as the square of the velocity, as is commonly supposed. It is more likely that it increases as the speed after about 20 miles an hour is reached. Assuming that the latter is true, and that an average of 50 lbs. per square inch is the greatest that can be realized in the cylinders of a given engine at 40 miles an hour, and that this pressure furnishes just sufficient power to keep the train at this speed, it follows that, to increase the speed to 50 miles, the mean effective pressure must be increased in the same proportion. To increase the capacity for speed of any locomotive its power must be increased, and at least by as much as the speed is to be increased. One way to accomplish this is to increase the boiler-pressure. That this is generally realized, is shown by the increase in boiler-pressure in the last ten years. For twenty-three single-expansion locomotives described in the railway journals this year the steam-pressures are as follows: 3, 160 lbs.; 4, 165 lbs.; 2, 170 lbs.; 13, 180 lbs.; 1, 190 lbs.

Valve-travel. — An increased average cylinder-pressure may also be obtained by increasing the valve-travel without raising the boiler-pressure, and better results will be obtained by increasing both. The longer travel gives a higher steam-pressure in the cylinders, a later exhaust-opening, later exhaust-closure, and a larger exhaust-opening—all necessary for high speeds and economy. I believe that a 20-in. port and 6½-in. (or even 7-in.) travel could be successfully used for high-speed engines, and that frequently by so doing the cylinders could be economically reduced and the counter-balance lightened. Or, better still, the diameter of the drivers increased, securing lighter counterbalance and better steam-distribution.

Size of Drivers.—Economy will increase with increasing diameter of drivers, provided the work at average speed does not necessitate a cut-off longer than one fourth the stroke. The piston-speed of a locomotive with 62-in. drivers at 55 miles per hour is the same as that of one with 68-in. drivers at 61 miles per hour.

Steam-ports.—The length of steam-ports ranges from 15 in. to 23 in., and has considerable influence on the power, speed, and economy of the locomotive. In cards from similar engines the steam-line of the card from the engine with 23-in. ports is considerably nearer boiler-pressure than that of the card from the engine with 17¼-in. ports. That the higher steam-line is due to the greater length of steam-port there is little room for doubt. The 23-in. port produced 531 H.P. in an 18½-in. cylinder at a cost of 23.5 lbs. of indicated water per I.H.P. per hour. The 17¼-in. port, 424 H.P., at the rate of 22.9 lbs. of water, in a 19-in. cylinder.

Allen Valves.—There is considerable difference of opinion as to the advantage of the Allen ported-valve (See *Eng. News*, July 6, 1893.)

Speed of Railway Trains.—In 1834 the average speed of trains on the Liverpool and Manchester Railway was twenty miles an hour; in 1838 it was twenty-five miles an hour. But by 1840 there were engines on the Great Western Railway capable of running fifty miles an hour with a train, and eighty miles an hour without. A speed of 86 miles per hour was made in England with the T. W. Worsdell compound locomotive. The total weight of the engine, tender, and train was 695,000 lbs.; indicator-cards were taken showing 1068.6 H.P. on the level. At a speed of 75 miles per hour on a level, and the same train, the indicator-cards showed 1040 H.P. developed. (Trans. A. S. M. E., vol. xiii., 363.)

The limitation to the increase of speed of heavy locomotives seems at present to be the difficulty of counterbalancing the reciprocating parts. The unbalanced vertical component of the reciprocating parts causes the pressure of the driver on the rail to vary with every revolution. Whenever the speed is high, it is of considerable magnitude, and its change in direction is so rapid that the resulting effect upon the rail is not inappropriately called a "hammer blow." Heavy rails have been kinked, and bridges have been shaken to their fall under the action of heavily balanced drivers revolving at high speeds. The means by which the evil is to be overcome has not yet been made clear. See paper by W. F. M. Goss, Trans. A. S. M. E., vol. xvi.

Engine No. 999 of the New York Central Railroad ran a mile in 32 seconds equal to 112 miles per hour, May 11, 1893.

$$\left.\begin{array}{l}\textit{Speed in miles}\\ \textit{per hour}\end{array}\right\} = \frac{\text{circum. of driving-wheels in in.} \times \text{no. of rev. per min.} \times 60}{63{,}360}$$

$$= \text{diam. of driving-wheels in in.} \times \text{no. of rev. per min.} \times .003$$

(approximate, giving result 8/10 of 1 per cent too great).

DIMENSIONS OF SOME LARGE AMERICAN LOCOMOTIVES, 1893.

The four locomotives described below were exhibited at the Chicago Exposition in 1893. The dimensions are from *Engineering News*, June, 1893. The first, or Decapod engine, has ten-coupled driving-wheels. It is one of the heaviest and most powerful engines ever built for freight service. The Philadelphia & Reading engine is a new type for passenger service, with four-coupled drivers. The Rhode Island engine has six drivers, with a 4-wheel leading truck and a 2-wheel trailing truck. These three engines have all compound cylinders. The fourth is a simple engine, of the standard American 8 wheel type, 4 driving-wheels, and a 4-wheel truck in front. This engine holds the world's record for speed (1893) for short distances, having run a mile in 32 seconds.

	Baldwin. N. Y., L. E. & W. R. R. Decapod Freight.	Baldwin. Phila. & Read. R. R. Express Passenger.	Rhode Isl. Locomoti've Works. Heavy Express.	N. Y. C. & H. R. R. Empire State Express, No. 999.
Running-gear:				
Driving-wheels, diam	4 ft. 2 in.	6 ft. 6 in.	6 ft. 6 in.	7 ft. 2 in.
Truck " "	2 " 6 "	4 " 0 "	2 " 9 "	3 " 4 "
Journals, driving-axles...	9 × 10 in.	8½ × 12 in.	8 × 8¾ in.	9 × 12½ in.
" truck- " ...	5 × 10 "	6½ × 10 "	5½ × 10 "	6¼ × 10 "
" tender- " ...	4½ × 9 "	4½ × 8 "	4¼ × 8 "	4½ × 8 "
Wheel-base:				
Driving..................	18 ft. 10 in.	6 ft. 10 in.	13 ft. 6 in.	8 ft. 6 in.
Total engine............	27 " 3 "	23 " 4 "	29 " 9¼ "	23 " 11 "
" tender...............	16 " 8 "	16 " 0 "	15 " 0 "	15 ft. 2½ "
" engine and tender...	58 " 4 "	47 " 3 "	50 " 6¾ "	47 " 8⅝ "
Wt. in working-order:				
On drivers..............	170,000 lbs.	82,700 lbs.	88,500 lbs.	84,000 lbs.
On truck-wheels.........	22,500 "	47,000 "	54,500 "	40,000 "
Engine, total...........	192,500 "	129,700 "	143,000 "	124,000 "
Tender "	117,500 "	80,573 "	75,000 "	80,000 "
Engine and tender, loaded	310,000 "	210,273 "	218,000 "	204,000 "
Cylinders:				
h.p. (2)................	16 × 28 in.	13 × 24 in.	one 21 × 26	19 × 24 in.
l.p. (2)................	27 × 28 "	22 × 24 "	one 31 × 26	....
Distance centre to centre.	7 ft. 5 "	7 ft. 4½ in.	7 ft. 1 in.	6 ft. 5 in.
Piston-rod, diam...... ...	4 in.	3½ in.	3½ in.	3⅝ in.
Connecting-rod, length...	9′ 8 7/16″	8 ft. 0½ in.	10 ft. 3½ in.	8 ft. 1½ in.
Steam-ports....	28½ × 2 in.	24 × 1½ in.	1½ × 20 and 1½ × 25	1½ × 18 in.
Exhaust-ports...........	28½ × 8 "	24 × 4½ "	3 × 20 in.	2¾ × 18 "
Slide-valves, out. lap, h.p.	⅞ in.	⅞ in.	1¼ in.	1 in.
" " out. lap, l.p..	⅝ "	⅝ "	1 in.	....
" " in. lap, h.p...	....	(neg.) ⅛ in.	....	1/10 in.
" " in. lap, l.p...	....	None	....	....
" " max. travel..	6 in.	5 in.	6½ in.	5½ in.
" " lead, h.p.....	1/16 in.	⅛ "	3/32 "	....
" " lead, l.p.....	5/16 "	⅜ "	....	....
Boiler—Type............	Straight	Straight	Wagon top	Wagon top
Diam. of barrel inside....	6 ft. 2½ in.	4 ft. 8¼ in.	5 ft. 2 in.	4 ft. 9 in.
Thickness of barrel-plates	¾ in.	⅝ in.	⅝ in.	9/16 in.
Height from rail to centre line	8 ft. 0 in.	....	8 ft. 11 in.	7 ft. 11½ in.
Length of smoke-box.....	5 " 7⅞ "	....	6 " 1 "	4 " 8 "
Working steam-pressure..	180 lbs.	180 lbs.	200 lbs.	190 lbs.
Firebox—type.........	Wootten	Wootten	Radial stay	Buchanan
Length inside...........	10′ 11 9/16″	9 ft. 6 in.	10 ft. 0 in.	9 ft. 6⅜ in.
Width "	8 ft. 2⅛ in.	8 " 0½ "	3 " 9⅝ "	3 " 4⅞ "
Depth at front	4 " 6 "	3 " 2¾ "	6 " 10¾ "	6 " 1¼ "
Thickness of side plates..	5/16 in.	5/16 in.	5/16 in.	5/16 in.
" " back plate...	5/16 "	5/16 "	⅜ "	5/16 "
Thickness of crown-sheet.	⅜ "	5/16 "	⅜ "	⅜ "
" " tube " .	½ "	½ "	½ "	½ "
Grate-area..............	89.6 sq. ft.	76.8 sq. ft.	28 sq. ft.	30.7 sq. ft.
Stay-bolts, diam., 1⅛ in..	pitch, 4¼ in.	....	4 in.	4 in.
Tubes—iron.............	354	324	272	268
Pitch....	2¾ in.	2 1/16 in.	2¾ in.	....
Diam., outside	2 "	1½ in.	2 "	2 in.
Length betw'n tube-plates	11 ft. 11 in.	10 ft. 0 in.	12 ft. 8⅝ in.	12 ft. 0 in.
Heating-surface:				
Tubes, exterior	2,208.8 ft.	1,262 sq. ft.	....	1,697 sq. ft.
Fire-box................	234.3 "	178 " "	....	238 " "
Miscellaneous:				
Exhaust-nozzle, diam....	5 in.	5½ in.	....	3¼ in.
Smokestack, smal'st diam.	1 ft. 6 "	1 ft. 6 in.	1 ft. 3 in.	1 ft. 3¼ in.
" height from rail to top....	15 " 6½ "	14 ft. 0¾ in.	15 " 2 "	14 " 10 "

Dimensions of American Locomotives. (D. L. Barnes, Trans. A. S. C. E., 1893.)

Name of Railroad.	Passenger or Freight Engine	No. of Drivers.	No. of Front Truck-wheels.	Diam. of Driving-wheels, in.	Size of Cylinders, Inches.	Total Weight of Engine, lbs.	Total Weight on Driving-wheels, lbs.	Area of Grate, sq. ft.	Firebox Heating-surface, sq. ft.	Tube Heating-surface, sq. ft.	Steam-pressure per sq. in. Atmospheric, lbs.	Length of Tubes, ft. and in.	Diam. of Tubes, in.	Ratio of Cylinder-power to Weight, Available for Adhesion.
C. M. & St. P.	P.	4	4	62	16 × 24	86,000	54,000	15.5	115	801	160	11 0	2	0.461
C. R. R. of N. J.	"	4	4	68	19 × 24	105,900	75,900	27	......	......	160	11 9¾	2	0.421
B. & O.	"	6	4	62	21 × 26	133,000	103,300	28.2	188.5	1846.8	160	13 2¾	2¼	0.450
C., C., C. & St. L.	"	6	4	66	18½ × 24	131,400	102,800	18.2	147	1883.4	160	13 4⅛	2	0.304
Penn. R. R.	"	4	4	78	19 × 24	128,000	81,000	26.2	144.3	1672.2	180	12 0	2	0.387
N. Y. C. & H. R.	"	4	4	84½	19 × 24	123,000	82,200	27.3	......	......	180	13 0	2	0.352
C. B. & Q.	"	4	4	68⅞	18 × 24	......	66,600	24.5	111	1301	160	......	...	0.426
N. Y. C. & H. R.	"	4	4	78	19 × 24	126,150	81,400	27.3	147.7	1670.7	180	12 0	2	0.385
C., C., C. & St. L.	"	4	4	68	18½ × 24	129,280	86,500	31.8	148.7	1426.3	180	11 4	2	0.395
N. Y., L. E. & W.	"	6	4	68	20 × 24	127,600	100,000	39.5	186	1885	160	13 2	2	0.355
M. C.	"	6	4	74	20 and 29 × 24	135,000	99,000	28.2	141.2	1839.5	180	13 0	....	0.389
Penn. R. R.	"	6	4	74	20 and 30 × 24	138,000	102,000	26.2	141.7	1811.5	180	13 0	2	0.404
C. R. R. of N. J.	"	4	4	78	13 and 22 × 24	123,800	88,400	33.5	......	......	180	11 10	2	0.476
Penn. R. R.	"	6	4	72	14 and 24 × 24	133,000	100,000	26.3	......	......	180	14 0	2	0.542
Philadelphia & Reading	"	4	2	78	13 and 22 × 24	129,000	83,000	76	128.9	1261.7	170	10 0	1⅛	0.428
C., B. & Q.	"	6	2	62	20 and 29 × 24	115,000	98,000	27.1	126	1380	160	......	...	0.417
C., M. & St. P.	"	6	4	78	21 and 31 × 26	143,000	88,500	......	......	......	200	12 8⅝	2	0.554
W. N. Y. & Pa.	F.	8	2	50¼	19 × 26	126,370	110,650	29	......	......	160	12 1½	2	0.494
B. & M.	"	10	0	50	22 × 28	150,300	150,300	37.5	180	2171	160	14 6	2⅛	0.458
C., C., C. & St. L.	"	6	4	56	19 × 24	128,200	90,600	18.2	147	1883.4	160	13 4⅛	2	0.391
B. & O.	"	8	2	50	21 × 26	125,000	113,400	28.7	155	1584	160	13 7	2¼	0.507
"	"	6	4	60	20 × 26	127,500	101,500	32.5	146.2	1608	180	13 0	2¼	0.482
D. & I. R.	"	8	4	54	19 × 24	129,000	96,000	34.5	189.7	2212.6	180	13 6	2¼	0.472
W. N. Y. & Pa.	"	8	2	50¼	13 and 21 × 26	129,000	116,550	29	......	......	175	12 11½	2	0.538
So. Pacific	"	6	4	55	20 and 28 × 26	126,000	99,500	28.6	141.2	1788.3	180	13 0	2¼	0.525
N. Y., L. E. & W.	"	10	2	50	16 and 27 × 28	200,550	173,700	89.6	182.5	2308	180	13 0	2	0.660
Cornwall & Lebanon	"	8	2	50	14 and 24 × 28	150,000	135,000	35.8	......	......	180	13 6	2¼	0.675
C. M. & St. P.	"	4	4	62	12 and 20 × 26	122,400	87,970	18.2	......	......	175	13 7⅛	2¼	0.525
Illinois Central	"	6	2	56½	20 and 29 × 26	138,500	107,300	26.1	......	......	180	11 0	2	0.512

Dimensions of Some American Locomotives.—The table on page 861 is condensed from one given by D. L. Barnes, in his paper on "Distinctive Features and Advantages of American Locomotive Practice," Trans. A.S.C.E., 1893. The formula from which column marked "Ratio of cylinder-power to weight available for adhesion" is calculated as follows:

$$\frac{2 \times \text{cylinder area} \times \text{boiler-pressure} \times \text{stroke}}{\text{Weight on drivers} \times \text{diameter of driving-wheel}}.$$

(Ratio of cylinder-power of compound engines cannot be compared with that of the single-expansion engines.)

Where the boiler-pressure could not be determined from the description of the locomotives, as given by the builders and operators of the locomotives, it has been assumed to be 160 lbs. per sq. in. above the atmosphere.

For compound locomotives the figures in the last column of ratios are based on the capacity of the low-pressure cylinders only, the volume of the high-pressure being omitted. This has been done for the purpose of comparison, and because there is no accurate simple way of comparing the cylinder-power of single-expansion and compound locomotives.

Dimensions of Standard Locomotives on the N. Y. C. & H. R. R. and Penna. R. R., 1882 and 1893.

C. H. Quereau, *Eng'g News*, March 8, 1894.

	N. Y. C. & H. R. R.				Pennsylvania R. R.			
	Through Passenger.		Through Freight.		Through Passenger.		Through Freight.	
	1882.	1893.	1882.	1893.	1882.	1893.	1882.	1893.
Grate surface, sq. ft....	17.87	27.3	17.87	29.8	17.6	33.2	23.	31.5
Heating surface, sq. ft..	1353	1821	1353	1763	1057	1583	1260	1498
Boiler, diam., in.........	50	58	50	58	50	57	54	60
Driver, diam., in	70	78, 86	64	67	62	78	50	50
Steam-pressure, lbs.. ..	150	180	150	160	125	175	125	140
Cylin., diam. and stroke.	17×24	19×24	17×24	19×26	17×24	18½×24	20×24	20×24
Valve-travel, ins	5¼	5½	5¼	5¾	5	5½	5	5
Lead at full gear, ins....	1/16	1/16	1/16	1/16	1/16	0	⅛	1/16
Outside lap.............	⅞	1	⅞	⅞	¾	1	¾	¾
Inside lap or clearance..	0	0	1/16*l*	3/32*l*	0	⅛*cl*	1/32*l*	1/32*l*
Steam-ports, length.....	15½	18	15⅛	18	16	17¼	16	16
" " width......	1¼	1½	1¼	1¼	1¼	1¼	1¼	1⅝
Type of engine...... ..	Am.	Am.	Am.	Mog.	Am.	Am.	Cons.	Cons.

Indicated Water Consumption of Single and Compound Locomotive Engines at Varying Speeds.

C. H. Quereau, *Eng'g News*, March 8, 1894.

Two-cylinder Compound.			Single-expansion.		
Revolutions.	Speed, miles per hour.	Water per I.H.P. per hour.	Revolutions.	Miles per Hour.	Water.
100 to 150	21 to 31	18.33 lbs.	151	31	21.70
150 " 200	31 " 41	18.9 "	219	45	20.91
200 " 250	41 " 51	19.7 "	253	52	20.52
250 " 275	51 " 56	21.4 "	307	63	20.23
			321	66	20.01

It appears that the compound engine is the more economical at low speeds, the economy decreasing as the speed increases, and that the single engine increases in economy with increase of speed within ordinary limits, becoming more economical than the compound at speeds of more than 50 miles per hour.

The C., B. & Q. two-cylinder compound, which was about 30% less economical than simple engines of the same class when tested in passenger service, has since been shown to be 15% more economical in freight service than the best single-expansion engine, and 29% more economical than the average record of 40 simple engines of the same class on the same division.

Indicator-tests of a Locomotive at High Speed. (*Locomotive Eng'g*, June, 1893.)—Cards were taken by Mr. Angus Sinclair on the locomotive drawing the Empire State Express.

RESULTS OF INDICATOR-DIAGRAMS.

Card No.	Revs.	Miles per hour.	I.H.P.	Card No.	Revs.	Miles per hour.	I.H.P.
1	160	37.1	648.3	7	304	70.5	977
2	260	60.8	728	8	296	68.6	972
3	190	44	551	9	300	69.6	1,045
4	250	58	891	10	304	70.5	1,059
5	260	60	960	11	340	78.9	1,120
6	298	69	983	12	310	71.9	1,026

The locomotive was of the eight-wheel type, built by the Schenectady Locomotive Works, with 19 × 24 in. cylinders, 78-in. drivers, and a large boiler and fire-box. Details of important dimensions are as follows: Heating-surface of fire-box, 150.8 sq. ft.; of tubes, 1670.7 sq. ft.; of boiler, 1821.5 sq ft. Grate area, 27.3 sq. ft. Fire-box: length, 8 ft.; width, 3 ft. 4⅞ in. Tubes, 268; outside diameter, 2 in. Ports: steam, 18 × 1¼ in.; exhaust, 18 × 2¾ in. Valve-travel, 5½ in. Outside lap, 1 in.; inside lap, 1/64 in. Journals: driving-axle, 8½ × 10½ in.; truck-axle, 6 × 10 in.

The train consisted of four coaches, weighing, with estimated load, 340,000 lbs. The locomotive and tender weighed in working order 200,000 lbs., making the total weight of the train about 270 tons. During the time that the engine was first lifting the train into speed diagram No. 1 was taken. It shows a mean cylinder-pressure of 59 lbs. According to this, the power exerted on the rails to move the train is 6558 lbs., or 24 lbs. per ton. The speed is 37 miles an hour. When a speed of nearly 60 miles an hour was reached the average cylinder-pressure is 40.7 lbs., representing a total traction force of 4520 lbs., without making deductions for internal friction. If we deduct 10% for friction, it leaves 15 lbs. per ton to keep the train going at the speed named. Cards 6, 7, and 8 represent the work of keeping the train running 70 miles an hour. They were taken three miles apart, when the speed was almost uniform. The average cylinder-pressure for the three cards is 47.6 lbs. Deducting 10% again for friction, this leaves 17.6 lbs. per ton as the power exerted in keeping the train up to a velocity of 70 miles. Throughout the trip 7 lbs. of water were evaporated per lb. of coal. The work of pulling the train from New York to Albany was done on a coal consumption of about 3½ lbs. per H.P. per hour. The highest power recorded was at the rate of 1120 H.P.

Locomotive-testing Apparatus at the Laboratory of Purdue University. (W. F. M. Goss, Trans. A. S. M. E., vol. xiv. 826.)—The locomotive is mounted with its drivers upon supporting wheels which are carried by shafts turning in fixed bearings, thus allowing the engine to be run without changing its position as a whole. Load is supplied by four friction-brakes fitted to the supporting shafts and offering resistance to the turning of the supporting wheels. Traction is measured by a dynamometer attached to the draw-bar. The boiler is fired in the usual way, and an exhaust-blower above the engine, but not in pipe connection with it, carries off all that may be given out at the stack.

A Standard Method of Conducting Locomotive-tests is given in a report by a Committee of the A. S. M. E. in vol. xiv. of the Transactions, page 1312.

Waste of Fuel in Locomotives.—In American practice economy of fuel is necessarily sacrificed to obtain greater economy due to heavy train-loads. D. L. Barnes, in *Eng. Mag.*, June, 1894, gives a diagram showing the reduction of efficiency of boilers due to high rates of combustion, from which the following figures are taken:

Lbs. of coal per sq. ft. of grate per hour......	12	40	80	120	160	200
Per cent efficiency of boiler......	80	75	67	59	51	43

A rate of 12 lbs. is given as representing stationary-boiler practice, 40 lbs. is English locomotive practice, 120 lbs. average American, and 200 lbs. maximum American, locomotive practice.

Advantages of Compounding.—Report of a Committee of the American Railway Master Mechanics' Association on Compound Locomotives (*Am. Mach.*, July 3, 1890) gives the following summary of the advantages gained by compounding: (*a*) It has achieved a saving in the fuel burnt averaging 18% at reasonable boiler-pressures, with encouraging possibilities of further improvement in pressure and in fuel and water economy. (*b*) It has lessened the amount of water (dead weight) to be hauled, so that (*c*) the tender and its load are materially reduced in weight. (*d*) It has increased the possibilities of speed far beyond 60 miles per hour, without unduly straining the motion, frames, axles, or axle-boxes of the engine. (*e*) It has increased the haulage-power at full speed, or, in other words, has increased the continuous H.P. developed, per given weight of engine and boiler. (*f*) In some classes has increased the starting-power. (*g*) It has materially lessened the slide-valve friction per H.P. developed. (*h*) It has equalized or distributed the turning force on the crank-pin, over a longer portion of its path, which, of course, tends to lengthen the repair life of the engine. (*i*) In the two-cylinder type it has decreased the oil consumption, and has even done so in the Woolf four-cylinder engine. (*j*) Its smoother and steadier draught on the fire is favorable to the combustion of all kinds of soft coal; and the sparks thrown being smaller and less in number, it lessens the risk to property from destruction by fire. (*k*) These advantages and economies are gained without having to improve the man handling the engine, less being left to his discretion (or careless indifference) than in the simple engine. (*l*) Valve-motion, of every locomotive type, can be used in its best working and most effective position. (*m*) A wider elasticity in locomotive design is permitted; as, if desired, side-rods can be dispensed with, or articulated engines of 100 tons weight, with independent trucks, used for sharp curves on mountain service, as suggested by Mallet and Brunner.

Of 27 compound locomotives in use on the Phila. and Reading Railroad (in 1892), 12 are in use on heavy mountain grades, and are designed to be the equivalent of 22 × 24 in. simple consolidations; 10 are in somewhat lighter service and correspond to 20 × 24 in. consolidations; 5 are in fast passenger service. The monthly coal record shows:

Class of Engine.	No.	Gain in Fuel Economy.
Mountain locomotives..........................	12	25% to 30%
Heavy freight service..........................	10	12% to 17%
Fast passenger..................................	5	9% to 11%

(Report of Com. A. R. M. M. Assn. 1892.) For a description of the various types of compound locomotive, with discussion of their relative merits, see paper by A. Von Borries, of Germany, The Development of the Compound Locomotive, Trans. A. S. M. E. 1893, vol. xiv., p. 1172.

Counterbalancing Locomotives.—The following rules, adopted by different locomotive-builders, are quoted in a paper by Prof. Lanza (Trans. A. S. M. E., x, 302):

A. "For the main drivers, place opposite the crank-pin a weight equal to one half the weight of the back end of the connecting-rod plus one half the weight of the front end of the connecting-rod, piston, piston-rod, and cross-head. For balancing the coupled wheels, place a weight opposite the crank-pin equal to one half the parallel rod plus one half of the weights of the front end of the main-rod, piston, piston-rod, and cross-head. The centres of gravity of the above weights must be at the same distance from the axles as the crank-pin."

B. The rule given by D. K. Clark: "Find the separate revolving weights of crank-pin boss, coupling-rods, and connecting-rods for each wheel, also the reciprocating weight of the piston and appendages, and one half the connecting-rod, divide the reciprocating weight equally between each wheel and add the part so allotted to the revolving weight on each wheel: the sums thus obtained are the weights to be placed opposite the crank-pin, and at the same distance from the axis. To find the counterweight to be used when the distance of its centre of gravity is known, multiply the above weight by the length of the crank in inches and divide by the given distance." This rule differs from the preceding in that the same weight is placed in each wheel.

C. "$W = \dfrac{S \times \left(w - \frac{w}{f}\right)}{G}$, in which S = one half the stroke, G = distance from centre of wheel to centre of gravity in counterbalance, w = weight at crank-pin to be balanced, W = weight in counterbalance, f = coefficient of friction so called, = 5 in ordinary practice. The reciprocating weight is found by adding together the weights of the piston, piston-rod, cross-head, and one half of the main rod. The revolving weight for the main wheel is found by adding together the weights of the crank-pin hub, crank-pin, one half of the main rod, and one half of each parallel-rod connecting to this wheel; to this add the reciprocating weight divided by the number of wheels. The revolving weight for the remainder of the wheels is found in the same manner as for the main wheel, except one half of the main rod is not added. The weight of the crank-pin hub and the counterbalance does not include the weight of the spokes, but of the metal inclosing them. This calculation is based for one cylinder and its corresponding wheels."

D. "Ascertain as nearly as possible the weights of crank-pin, additional weight of wheel boss for the same, add side rod, and main connections, piston-rod and head, with cross-head on one side: the sum of these multiplied by the distance in inches of the centre of the crank-pin from the centre of the wheel, and divided by the distance from the centre of the wheel to the common centre of gravity of the counterweights, is taken for the total counterweight for that side of the locomotive which is to be divided among the wheels on that side."

E. "Balance the wheels of the locomotive with a weight equal to the weights of crank-pin, crank-pin hub, main and parallel rods, brasses, etc., plus two thirds of the weight of the reciprocating parts (cross-head, piston and rod and packing)."

F. "Balance the weights of the revolving parts which are attached to each wheel with exactness, and divide equally two thirds of the weights of the reciprocating parts between all the wheels. One half of the main rod is computed as reciprocating, and the other as revolving weight."

See also articles on Counterbalancing Locomotives, in *R. R. & Eng. Jour.*, March and April, 1890, and a paper by W. F. M. Goss, in Trans. A. S. M. E., vol xvi

Maximum Safe Load for Steel Tires on Steel Rails. (A. S. M. E., vii., p. 786.)—Mr. Chanute's experiments led to the deduction that 12,000 lbs. should be the limit of load for any one driving-wheel. Mr. Angus Sinclair objects to Mr. Chanute's figure of 12,000 lbs., and says that a locomotive tire which has a light load on it is more injurious to the rail than one which has a heavy load. In English practice 8 and 10 tons are safely used. Mr. Oberlin Smith has used steel castings for cam-rollers 4 in. diam. and 3 in. face, which stood well under loads of from 10,000 to 20,000 lbs. Mr. C. Shaler Smith proposed a formula for the rolls of a pivot-bridge which may be reduced to the form: Load $= 1760 \times$ face $\times \sqrt{\text{diam.}}$, all in lbs. and inches.

See dimensions of some large American locomotives on pages 860 and 861. On the "Decapod" the load on each driving-wheel is 17,000 lbs., and on "No. 999," 21.000 lbs.

Narrow-gauge Railways in Manufacturing Works.—A tramway of 18 inches gauge, several miles in length, is in the works of the Lancashire and Yorkshire Railway. Curves of 13 feet radius are used. The locomotives used have the following dimensions (Proc. Inst. M. E., July, 1888): The cylinders were 5 in. diameter with 6 in. stroke, and 2 ft. 3¼ in. centre to centre. The wheels were 16¼ in. diameter, the wheel-base 2 ft. 9 in.; the frame 7 ft. 4½ in. long, and the extreme width of the engine 3 feet. The boiler, of steel, 2 ft. 3 in. outside diameter and 2 ft. long between tube-plates, containing 55 tubes of 1⅜ in. outside diameter; the fire-box, of iron and cylindrical, 2 ft. 3 in. long and 17 in. inside diameter. The heating-surface 10.42 sq. ft. in the fire-box and 36.12 in the tubes, total 46.54 sq. ft.; the grate-area, 1.78 sq. ft.; capacity of tank, 26½ gallons; working-pressure, 170 lbs. per sq. in.; tractive power, say, 1412 lbs., or 9.22 lbs. per lb. of effective pressure per sq. in. on the piston. Weight, when empty, 2.80 tons; when full and in working order, 3.19 tons.

For description of a system of narrow-gauge railways for manufactories, see circular of the C. W. Hunt Co., New York.

Light Locomotives.—For dimensions of light ocomotives used for. mining, etc., and for much valuable information concerning them, see catalogue of H. K. Porter & Co., Pittsburgh.

Petroleum-burning Locomotives. (From Clark's Steam-engine.)—The combustion of petroleum refuse in locomotives has been successfully practised by Mr. Thos. Urquhart, on the Grazi and Tsaritsin Railway, Southeast Russia. Since November, 1884, the whole stock of 143 locomotives under his superintendence has been fired with petroleum refuse. The oil is injected from a nozzle through a tubular opening in the back of the fire-box, by means of a jet of steam, with an induced current of air.

A brickwork cavity or "regenerative or accumulative combustion-chamber" is formed in the fire-box, into which the combined current breaks as spray against the rugged brickwork slope. In this arrangement the brickwork is maintained at a white heat, and combustion is complete and smokeless. The form, mass, and dimensions of the brickwork are the most important elements in such a combination.

Compressed air was tried instead of steam for injection, but no appreciable reduction in consumption of fuel was noticed.

The heating-power of petroleum refuse is given as 19,832 heat-units, equivalent to the evaporation of 20.53 lbs. of water from and at 212° F., or to 17.1 lbs. at 8½ atmospheres, or 125 lbs. per sq. in., effective pressure. The highest evaporative duty was 14 lbs. of water under 8½ atmospheres per lb. of the fuel, or nearly 82% efficiency.

There is no probability of any extensive use of petroleum as fuel for locomotives in the United States, on account of the unlimited supply of coal and the comparatively limited supply of petroleum.

Fireless Locomotive.—The principle of the Francq locomotive is that it depends for the supply of steam on its spontaneous generation from a body of heated water in a reservoir. As steam is generated and drawn off the pressure falls; but by providing a sufficiently large volume of water heated to a high temperature, at a pressure correspondingly high, a margin of surplus pressure may be secured, and means may thus be provided for supplying the required quantity of steam for the trip.

The fireless locomotive designed for the service of the Metropolitan Railway of Paris has a cylindrical reservoir having segmental ends, about 5 ft. 7 in. in diameter, 26¼ ft. in length, with a capacity of about 620 cubic feet. Four fifths of the capacity is occupied by water, which is heated by the aid of a powerful jet of steam supplied from stationary boilers. The water is heated until equilibrium is established between the boilers and the reservoir. The temperature is raised to about 390° F., corresponding to 225 lbs. per sq. in. The steam from the reservoir is passed through a reducing-valve, by which the steam is reduced to the required pressure. It is then passed through a tubular superheater situated within the receiver at the upper part, and thence through the ordinary regulator to the cylinders. The exhaust-steam is expanded to a low pressure, in order to obviate noise of escape. In certain cases the exhaust-steam is condensed in closed vessels, which are only in part filled with water. In the upper free space a pipe is placed, into which the steam is exhausted. Within this pipe another pipe is fixed, perforated, from which cold water is projected into the surrounding steam, so as to effect the condensation as completely as may be. The heated water falls on an inclined plane, and flows off without mixing with the cold water. The condensing water is circulated by means of a centrifugal pump driven by a small three-cylinder engine.

In working off the steam from a pressure of 225 lbs. to 67 lbs., 530 cubic feet of water at 390° F. is sufficient for the traction of the trains, for working the circulating-pump for the condensers, for the brakes, and for electric-lighting of the train. At the stations the locomotive takes from 2200 to 3300 lbs. of steam—nearly the same as the weight of steam consumed during the run between two consecutive charging stations. There is 210 cubic feet of condensing water. Taking the initial temperature at 60° F., the temperature rises to about 180° F. after the longest runs underground.

The locomotive has ten wheels, on a base 24 ft. long, of which six are coupled, 4⅓ ft. in diameter. The extreme wheels are on radial axles. The cylinders are 23⅓ in. in diameter, with a stroke of 23⅓ in.

The engine weighs, in working order, 53 tons, of which 36 tons are on the coupled wheels. The speed varies from 15 miles to 25 miles per hour. The trains weigh about 140 tons.

Compressed-air Locomotives.—For an account of the Mekarski system of compressed-air locomotives see page 510 *ante*.

SHAFTING.

(See also TORSIONAL STRENGTH; also SHAFTS OF STEAM-ENGINES.)

For diameters of shafts to resist torsional strains only, Molesworth gives $d = \sqrt[3]{\frac{Pl}{K}}$, in which d = diameter in inches, P = twisting force in pounds applied at the end of a lever-arm whose length is l in inches, K = a coefficient whose values are, for cast iron 1500, wrought iron 1700, cast steel 3200, gun-bronze 460, brass 425, copper 380, tin 220, lead 170. The value given for cast steel probably applies only to high-carbon steel.

Thurston gives:

For head shafts well supported against springing (bearings close to pulleys or gears):

$$\text{H.P.} = \frac{d^3R}{125};\ d = \sqrt[3]{\frac{125\ \text{H.P.}}{R}}, \text{ for iron;}$$

$$\text{H.P.} = \frac{d^3R}{75};\ d = \sqrt[3]{\frac{100\ \text{H.P.}}{R}}, \text{ for cold-rolled iron.}$$

For line shafting, hangers 8 ft. apart:

$$\text{H.P.} = \frac{d^3R}{90};\ d = \sqrt[3]{\frac{90\ \text{H.P.}}{R}}, \text{ for iron;}$$

$$\text{H.P.} = \frac{d^3R}{55};\ d = \sqrt[3]{\frac{55\ \text{H.P.}}{R}}, \text{ for cold-rolled iron.}$$

For transmission simply, no pulleys:

$$\text{H.P.} = \frac{d^3R}{62.5};\ d = \sqrt[3]{\frac{62.5\ \text{H.P.}}{R}}, \text{ for iron;}$$

$$\text{H.P.} = \frac{d^3R}{35};\ d = \sqrt[3]{\frac{35\ \text{H.P.}}{R}}, \text{ for cold-rolled iron.}$$

H.P. = horse-power transmitted, d = diameter of shaft in inches, R = revolutions per minute.

J. B. Francis gives for turned-iron shafting $d = \sqrt[3]{\frac{100\ \text{H.P.}}{R}}$.

Jones and Laughlins give the same formulæ as Prof. Thurston, with the following exceptions: For line shafting, hangers 8 ft. apart:

$$\text{cold-rolled iron, H.P.} = \frac{d^3R}{50},\ d = \sqrt[3]{\frac{50\ \text{H.P.}}{R}}.$$

For simply transmitting power and short counters:

$$\text{turned iron, H.P.} = \frac{d^3R}{50},\ d = \sqrt[3]{\frac{50\ \text{H.P.}}{R}};$$

$$\text{cold-rolled iron, H.P.} = \frac{d^3R}{30},\ d = \sqrt[3]{\frac{30\ \text{H.P.}}{R}}.$$

They also give the following notes: Receiving and transmitting pulleys should always be placed as close to bearings as possible; and it is good practice to frame short "headers" between the main tie-beams of a mill so as to support the *main receivers*, carried by the head shafts, with a bearing close to each side as is contemplated in the formulæ. But if it is preferred, or necessary, for the shaft to span the full width of the "bay" without intermediate bearings, or for the pulley to be placed away from the bearings towards or at the middle of the bay, the size of the shaft must be largely increased to secure the *stiffness* necessary to support the load without undue deflection. Shafts may not deflect more than 1/80 of an inch to each foot of clear length with safety.

To find the diameter of shaft necessary to carry safely the main pulley at the centre of a bay: Multiply the fourth power of the diameter obtained by above formulæ by the length of the "bay," and divide this product by the distance from centre to centre of the bearings when the shaft is supported as required by the formula. The fourth root of this quotient will be the diameter required.

The following table, computed by this rule, is practically correct and safe.

Diameter of Shaft given by the Formulæ for Head Shafts.	Diameter of Shaft necessary to carry the Load at the Centre of a Bay, which is from Centre to Centre of Bearings							
	2½ ft.	3 ft.	3½ ft.	4 ft.	5 ft.	6 ft.	8 ft.	10 ft.
in.	in.	in.	in.	in.	in.	in.	in.	in.
2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	3
2½	2½	2⅝	2¾	2⅞	3	3⅛	3⅜	3⅝
3	3	3⅛	3¼	3⅜	3½	3¾	4	4¼
3½	....	3½	3⅝	3¾	4	4¼	4½	4¾
4	....	4	4⅛	4¼	4½	4¾	5⅛	5⅜
4½	....	....	4½	4⅝	4⅞	5⅛	5½	5⅞
5	....	....	5	5⅛	5⅜	5⅝	6	6⅛
5½	....	....	....	5½	5¾	6	6¼	6⅞
6	....	....	....	6	6⅜	6⅝	7⅛	7⅜

As the strain upon a shaft from a load upon it is proportional to the product of the parts of the shaft multiplied into each other, therefore, should the load be applied near one end of the span or bay instead of at the centre, multiply the fourth power of the diameter of the shaft required to carry the load at the centre of the span or bay by the product of the two parts of the shaft when the load is near one end, and divide this product by the product of the two parts of the shaft when the load is carried at the centre. The fourth root of this quotient will be the diameter required.

The shaft in a line which carries a receiving-pulley, or which carries a transmitting-pulley to drive another line, should always be considered a head-shaft, and should be of the size given by the rules for shafts carrying main pulleys or gears.

Deflection of Shafting. (Pencoyd Iron Works.)—As the deflection of steel and iron is practically alike under similar conditions of dimensions and loads, and as shafting is usually determined by its transverse stiffness rather than its ultimate strength, nearly the same dimensions should be used for steel as for iron.

For continuous line-shafting it is considered good practice to limit the deflection to a maximum of 1/100 of an inch per foot of length. The weight of bare shafting in pounds $= 2.6d^2L = W$, or when as fully loaded with pulleys as is customary in practice, and allowing 40 lbs. per inch of width for the vertical pull of the belts, experience shows the load in pounds to be about $13d^2L = W$. Taking the modulus of transverse elasticity at 26,000,000 lbs., we derive from authoritative formulæ the following:

$$L = \sqrt[3]{873d^2},\quad d = \sqrt{\frac{L^3}{873}},\text{ for bare shafting;}$$

$$L = \sqrt[3]{175d^2},\quad d = \sqrt{\frac{L^3}{175}},\text{ for shafting carrying pulleys, etc.;}$$

L being the maximum distance in feet between bearings for continuous shafting subjected to bending stress alone, d = diam. in inches.

The torsional stress is inversely proportional to the velocity of rotation, while the bending stress will not be reduced in the same ratio. It is therefore impossible to write a formula covering the whole problem and sufficiently simple for practical application, but the following rules are correct within the range of velocities usual in practice.

For continuous shafting so proportioned as to deflect not more than 1/100 of an inch per foot of length, allowance being made for the weakening effect of key-seats,

$$d = \sqrt[3]{\frac{50\,\text{H.P.}}{R}}, \quad L = \sqrt[3]{720d^2}, \text{ for bare shafts;}$$

$$d = \sqrt[3]{\frac{70\,\text{H.P.}}{R}}, \quad L = \sqrt[3]{140d^2}, \text{ for shafts carrying pulleys, etc.}$$

d = diam. in inches, L = length in feet, R = revs. per min.

The following table (by J. B. Francis) gives the greatest admissible distances between the bearings of continuous shafts subject to no transverse strain except from their own weight, as would be the case were the power given off from the shaft equal on all sides, and at an equal distance from the hanger-bearings.

Diam. of Shaft, in inches.	Distance between Bearings, in ft. Wrought-iron Shafts.	Steel Shafts.	Diam. of Shaft, in inches.	Distance between Bearings, in ft. Wrought-iron Shafts.	Steel Shafts.
2	15.46	15.89	6	22.30	22.92
3	17.70	18.19	7	23.48	24.13
4	19.48	20.02	8	24.55	25.23
5	20.99	21.57	9	25.53	26.24

These conditions, however, do not usually obtain in the transmission of power by belts and pulleys, and the varying circumstances of each case render it impracticable to give any rule which would be of value for universal application.

For example, the theoretical requirements would demand that the bearings be nearer together on those sections of shafting where most power is delivered from the shaft, while considerations as to the location and desired contiguity of the driven machines may render it impracticable to separate the driving-pulleys by the intervention of a hanger at the theoretically required location. (Joshua Rose.)

Horse-power Transmitted by Turned Iron Shafting at Different Speeds.

As Prime Mover or Head Shaft carrying Main Driving-pulley or Gear, well supported by Bearings. Formula: H.P. = $d^3R \div 125$.

Diam. of Shaft.	Number of Revolutions per Minute. 60	80	100	125	150	175	200	225	250	275	300
Ins.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
1¾	2.6	3.4	4.3	5.4	6.4	7.5	8.6	9.7	10.7	11.8	12.9
2	3.8	5.1	6.4	8	9.6	11.2	12.8	14.4	16	17.6	19.2
2¼	5.4	7.3	8.1	10	12	14	16	18	20	22	24
2½	7.5	10	12.5	15	18	22	25	28	31	34	37
2¾	10	13	16	20	24	28	32	36	40	44	48
3	13	17	20	25	30	35	40	45	50	55	60
3¼	16	22	27	34	40	47	54	61	67	74	81
3½	20	27	34	42	51	59	68	76	85	93	102
3¾	25	33	42	52	63	73	84	94	105	115	126
4	30	41	51	64	76	89	102	115	127	140	153
4½	43	58	72	90	108	126	144	162	180	198	216
5	60	80	100	125	150	175	200	225	250	275	300
5½	80	106	133	166	199	233	266	299	333	366	400

As Second Movers or Line-shafting, Bearings 8 ft. apart.
Formula: $H.P. = d^3R \div 90$.

Diam. of Shaft.	Number of Revolutions per Minute.										
	100	125	150	175	200	225	250	275	300	325	350
Ins.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
1¾	6	7.4	8.9	10.4	11.9	13.4	14.9	16.4	17.9	19.4	20.9
1⅞	7.3	9.1	10.9	12.7	14.5	16.3	18.2	20	21.8	23.6	25.4
2	8.9	11.1	13.3	15.5	17.7	20	22.2	24.4	26.6	28.8	31
2⅛	10.6	13.2	15.9	18.5	21.2	23.8	26.5	29.1	31.8	34.4	37
2¼	12.6	15.8	19	22	25	28	31	35	38	41	44
2⅜	15	18	22	26	29	33	37	41	44	48	52
2½	17	21	26	30	34	39	43	47	52	56	60
2¾	23	29	34	40	46	52	58	64	69	75	81
3	30	37	45	52	60	67	75	82	90	97	105
3¼	38	47	57	66	76	85	95	104	114	123	133
3½	47	59	71	83	95	107	119	131	143	155	167
3¾	58	73	88	102	117	132	146	162	176	190	205
4	71	89	107	125	142	160	178	196	213	231	249

For Simply Transmitting Power.
Formula: $H.P. = d^3R \div 50$.

Diam. of Shaft.	Number of Revolutions per Minute.										
	100	125	150	175	200	233	267	300	333	367	400
Ins.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
1½	6.7	8.4	10.1	11.8	13.5	15.7	17.9	20.3	22.5	24.8	27.0
1⅝	8.6	10.7	12.8	15	17.1	20	22.8	25.8	28.6	31.5	34.3
1¾	10.7	13.4	16	18.7	21.5	25	28	32	36	39	43
1⅞	13.2	16.5	19.7	23	26.4	31	35	39	44	48	53
2	16	20	24	28	32	37	42	48	53	58	64
2⅛	19	24	29	33	38	44	51	57	63	70	76
2¼	22	28	34	39	45	52	60	68	75	83	90
2⅜	27	33	40	47	53	62	70	79	88	96	105
2½	31	39	47	54	62	73	83	93	104	114	125
2¾	41	52	62	73	83	97	111	125	139	153	167
3	54	67	81	94	108	126	144	162	180	198	216
3¼	68	86	103	120	137	160	182	205	228	250	273
3½	85	107	128	150	171	200	228	257	285	313	342

Horse-power Transmitted by Cold-rolled Iron Shafting at Different Speeds.

As Prime Mover or Head Shaft carrying Main Driving-pulley or Gear, well supported by Bearings. Formula: $H.P. = d^3R \div 75$.

Diam. of Shaft.	Number of Revolutions per Minute.										
	60	80	100	125	150	175	200	225	250	275	300
Ins.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
1½	2.7	3.6	4.5	5.6	6.7	7.9	9.0	10	11	12	13
1¾	4.3	5.6	7.1	8.9	10.6	12.4	14.2	16	18	19	21
2	6.4	8.5	10.7	13	16	19	21	24	26	29	32
2¼	9	12	15	19	23	26	30	34	38	42	46
2½	12	17	21	26	31	36	41	47	52	57	62
2¾	16	22	27	35	41	48	55	62	70	76	82
3	21	29	36	45	54	63	72	81	90	98	108
3¼	27	36	45	57	68	80	91	103	114	126	136
3½	34	45	57	71	86	100	114	129	142	157	172
3¾	42	56	70	87	105	123	140	158	174	193	210
4	51	69	85	106	128	149	170	192	212	234	256
4½	73	97	121	151	182	212	243	273	302	333	364

AS SECOND MOVERS OR LINE-SHAFTING, BEARINGS 8 FT. APART.

Formula: H.P. $= d^3R \div 50$.

Diam. of Shaft.	Number of Revolutions per Minute.										
	100	125	150	175	200	225	250	275	300	325	350
Ins.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
1 1/2	6.7	8.4	10.1	11.8	13.5	15.2	16.8	18.5	20.2	21.9	23.6
1 5/8	8.6	10.7	12.8	15	17.1	19.3	21.5	23.6	25.7	28.9	31
1 3/4	10.7	13.4	16	18.7	21.5	24.2	26.8	29.5	32.1	34.8	39
1 7/8	13.2	16.5	19.7	23	26.4	29.6	32.9	36.2	39.5	42.8	46
2	16	20	24	28	32	36	40	44	48	52	56
2 1/8	19	24	29	33	38	43	48	52	57	62	67
2 1/4	22	28	34	39	45	50	56	61	68	74	80
2 3/8	27	33	40	47	53	60	67	73	80	86	94
2 1/2	31	39	47	54	62	69	78	86	93	101	109
2 3/4	41	52	62	73	83	93	104	114	125	135	145
3	54	67	81	94	108	121	134	148	162	175	189
3 1/4	68	86	103	120	137	154	172	188	205	222	240
3 1/2	85	107	128	150	171	192	214	235	257	278	300

FOR SIMPLY TRANSMITTING POWER AND SHORT COUNTERS.

Formula: H.P. $= d^3R \div 30$.

Diam. of Shaft.	Number of Revolutions per Minute.										
	100	125	150	175	200	233	267	300	333	367	400
Ins.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
1 1/4	6.5	8.1	9.7	11.3	13	15.2	17.4	19.5	21.7	23.9	26
1 3/8	8.5	10.7	12.8	15	17	19.8	22.7	25.5	28.4	31	34
1 1/2	11.2	14	16.8	19.6	22.5	26	30	33	37	41	45
1 5/8	14.2	17.7	21.2	24.8	28.4	33	38	42	47	52	57
1 3/4	18	22	27	31	35	41	47	53	59	65	71
1 7/8	22	27	33	38	44	51	58	65	72	79	87
2	26	33	40	46	53	62	71	80	88	97	106
2 1/8	32	40	47	55	63	73	84	95	105	116	127
2 1/4	38	47	57	66	76	89	101	114	127	139	152
2 3/8	44	55	66	77	88	103	118	133	148	163	178
2 1/2	52	65	78	91	104	121	138	155	172	190	207
2 3/4	69	84	99	113	138	161	184	207	231	254	277
3	90	112	135	157	180	210	240	270	300	330	360

SPEED OF SHAFTING.—Machine shops .... 120 to 180
Wood-working................. 250 to 300
Cotton and woollen mills...... 300 to 400

There are in some factories lines 1000 ft. long, the power being applied at the middle.

Hollow Shafts.—Let d be the diameter of a solid shaft, and $d_1 d_2$ the external and internal diameters of a hollow shaft of the same material. Then the shafts will be of equal torsional strength when $d^3 = \frac{d_1^4 - d_2^4}{d_1}$. A 10-inch hollow shaft with internal diameter of 4 inches will weigh 16% less than a solid 10-inch shaft, but its strength will be only 2.56% less. If the hole were increased to 5 inches diameter the weight would be 25% less than that of the solid shaft, and the strength 6.25% less.

Table for Laying Out Shafting.—The table on the opposite page (from the *Stevens Indicator*, April, 1892) is used by Wm. Sellers & Co. to facilitate the laying out of shafting.

The wood-cuts at the head of this table show the position of the hangers and position of couplings, either for the case of extension in both directions from a central head-shaft or extension in one direction from that head-shaft.

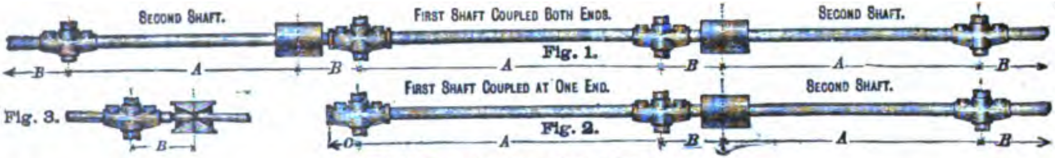

Table for Laying Out Shafting.

Length of Collared End for Fast Coll., ins.	Nominal Size of 2d Shaft. / Nominal Size of 1st Shaft, ins.	1½″	1¾″	2″	2¼″	2½″	2¾″	3″	3¼″	3½″	4″	4½″	5″	5½″	6″	6½″	7″	7½″	8″	Length of Bearing, or Box, ins.	Double Cone-vise Coupling. Length, inches.	Double Cone-vise Coupling. Diameter, inches.
		Distance from Centre of Bearing to End of Shaft for Coupling. See B, Figs. 1, 2, and 3.																				
3½	1½	8½																		6	5½	3¾
4	1¾	9	9½																	7	6½	4⅝
4½	2	9½	10½	11																8	7⅞	5⅜
5	2¼		11	11½	12															9	8¼	5⅞
5⅝	2½		11½	12	12½	13														10	9½	6 9/16
6¼	2¾		12	12½	13	13½	14													11	10	7
6¾	3			13	13½	14	14½	15												12	11	7¾
7¼	3¼			13½	14	14½	15	15½	15½											13	12	8¼
7⅞	3½			14	14½	15	15½	16	16	16½										14	13	9 1/16
8 15/16	4			15	15½	16	16½	17	17	17½	18									16	14¼	10
10	4½				16½	17	17½	18	18	18½	19½	20								18	16	11¾
11½	5					18½	18½	19	19½	20	20½	21½	22							20	18	12 9/16
12 3/16	5½						20	20½	21	21	22	22½	23½	24						22	19	13½
13¼	6						21½	21½	22	22½	23	24	24½	25	26					24	21	14½
14½	6½							23	23½	24	24½	25	26	26½	27½	28½				26	25	17⅜
15½	7								25	25	26	26½	27½	28	29	30	30½			28	25	17¾
16½	7½									26½	27	28	28½	29½	30	31	31½	32		30	28½	19 5/16
17¾	8										28½	29½	30	31	[illegible]	32½	33	34	34½	32	28¾	19 5/16

Use of Table. — Look for size of first shaft in left-hand column, under the head of Size of first shaft, and in the top line of table, marked Size of second shaft, find the size of the shaft to be coupled to it. The intersection gives the length B; this added to the length A, or distance from centre to centre of bearing, and in cases similar to Fig. 2, to the length C, gives the length of the first shaft, thus: as in Fig. 1, $B + A + B$ = length; Fig. 2, $C + A + B$ = length.

Make bearings at equal distances from each other, when practicable, and always put two bearings on the first, which is the collared shaft. See Figs. 1 and 2.

In coupling shafts of different sizes, either reduce the end of the large shaft in diameter, and use a small coupling, or use a coupling to suit the larger shaft, with one cone bored for the smaller nominal shaft.

PULLEYS.

Proportions of Pulleys. (See also Fly-wheels, pages 820 to 823.)—Let n = number of arms, D = diameter of pulley, S = thickness of belt, t = thickness of rim at edge, T = thickness in middle, B = width of rim, β = width of belt, h = breadth of arm at hub, h_1 = breadth of arm at rim, e = thickness of arm at hub e_1 = thickness of arm at rim, c = amount of crowning; dimensions in inches.

	Unwin.	Reuleaux.
B = width of rim..................	$9/8\,(\beta + 0.4)$	$9/8\beta$ to $5/4\beta$
t = thickness at edge of rim.......	$0.7S + .005D$	(thick. of rim.) $1/5h$ to $\frac{1}{4}h$
T = " " middle of rim.....	$2t + c$	....
h = breadth of arm at hub.....	For single belts $= .6337\sqrt[3]{\frac{BD}{n}}$; For double belts $= .798\sqrt[3]{\frac{BD}{n}}$	$\frac{1}{4}'' + \frac{B}{4} + \frac{D}{20n}$
h_1 = " " " " rim.........	$\frac{2}{3}h$	$0.8h$
e = thickness of arm at hub.	$0.4h$	$0.5h$
e_1 = " " " " rim.......	$0.4h_1$	$0.5h_1$
n = number of arms, for a single set, ...	$3 + \frac{BD}{150}$	$\frac{1}{2}\left(5 \times \frac{D}{2B}\right)$
L = length of hub	not less than $2.5S$, is often $\frac{2}{3}B$.	B for sin.-arm pulleys. $2B$ " double-arm "
M = thickness of metal in hub.......	....	h to $\frac{3}{4}h$
c = crowning of pulley.............	$1/24B$	....

The number of arms is really arbitrary, and may be altered if necessary. (Unwin.)

Pulleys with two or three sets of arms may be considered as two or three separate pulleys combined in one, except that the proportions of the arms should be 0.8 or 0.7 time that of single-arm pulleys. (Reuleaux.)

EXAMPLE.—Dimensions of a pulley 60″ diam., 16″ face, for double belt $\frac{1}{2}''$ thick.

Solution by....	n	h	h_1	e	e_1	t	T	L	M	c
Unwin.........	9	3.79	2.53	1.52	1.01	.65	1.97	10.7	3.8	.67
Reuleaux.. ...	4	5.0	4.0	2.5	2.0	1.25		16	5	

The following proportions are given in an article in the *Amer. Machinist*, authority not stated:

$h = .0625D + .5$ in., $h_1 = .04D + .3125$ in., $e = .025D + .2$ in., $e_1 = .016D + .125$ in.

These give for the above example: $h = 4.25$ in., $h_1 = 2.71$ in., $e = 1.7$ in. $e_1 = 1.09$ in. The section of the arms in all cases is taken as elliptical.

The following solution for breadth of arm is proposed by the author: Assume a belt pull of 45 lbs. per inch of width of a single belt, that the whole strain is taken in equal proportions on one half of the arms, and that the arm is a beam loaded at one end and fixed at the other. We have the formula for a beam of elliptical section $fP = .0982\frac{Rbd^2}{l}$, in which P = the load, R = the modulus of rupture of the cast iron, b = breadth, d = depth, and l = length of the beam, and f = factor of safety. Assume a modulus of rupture of 36,000 lbs., a factor of safety of 10, and an additional allowance for safety in taking $l = \frac{1}{2}$ the diameter of the pulley instead of $\frac{1}{2}D$ less the radius of the hub.

Take $d = h$, the breadth of the arm at the hub, and $b = e = 0.4h$, the thickness. We then have $fP = 10 \times \frac{45B}{n \div 2} = 900\frac{B}{n} = \frac{3535 \times 0.4h^3}{\frac{1}{2}D}$, whence $h = \sqrt[3]{\frac{900BD}{3535n}} = .683\sqrt[3]{\frac{BD}{n}}$, which is practically the same as the value reached by Unwin from a different set of assumptions.

Convexity of Pulleys.—Authorities differ. Morin gives a rise equal to 1/10 of the face; Molesworth, 1/24; others from 1/6 to 1/96. Scott A. Smith says the crown should not be over 1/8 inch for a 24-inch face. Pulleys for shifting belts should be "straight," that is, without crowning.

CONE OR STEP PULLEYS.

To find the diameters for the several steps of a pair of cone-pulleys:

1. **Crossed Belts.**—Let D and d be the diameters of two pulleys connected by a crossed belt, L = the distance between their centres, and β = the angle either half of the belt makes with a line joining the centres of the pulleys: then total length of belt $= (D+d)\frac{\pi}{2} + (D+d)\frac{\pi\beta}{180} + 2L \cos \beta$.

β = angle whose sine is $\frac{D+d}{2L}$. $\cos \beta = \sqrt{L^2 - \left(\frac{D+d}{2}\right)}$. The length of the belt is constant when $D+d$ is constant; that is, in a pair of step-pulleys the belt tension will be uniform when the sum of the diameters of each opposite pair of steps is constant. Crossed belts are seldom used for cone-pulleys, on account of the friction between the rubbing parts of the belt.

To design a pair of tapering speed-cones, so that the belt may fit equally tight in all positions: When the belt is crossed, use a pair of equal and similar cones tapering opposite ways.

2. **Open Belts.**—When the belt is uncrossed, use a pair of equal and similar conoids tapering opposite ways, and bulging in the middle, according to the following formula: Let L denote the distance between the axes of the conoids; R the radius of the larger end of each; r the radius of the smaller end; then the radius in the middle, r_0, is found as follows:

$$r_0 = \frac{R+r}{2} + \frac{(R-r)^2}{6.28L}. \quad \text{(Rankine.)}$$

If D_0 = the diameter of equal steps of a pair of cone-pulleys, D and d = the diamèters of unequal opposite steps, and L = distance between the axes, $D_0 = \frac{D+d}{2} + \frac{(D-d)^2}{12.566L}$.

If a series of differences of radii of the steps, $R-r$, be assumed, then for each pair of steps $\frac{R+r}{2} = r_0 - \frac{(R-r)^2}{6.28L}$, and the radii of each may be computed from their half sum and half difference, as follows:

$$R = \frac{R+r}{2} + \frac{R-r}{2}; \qquad r = \frac{R+r}{2} - \frac{R-r}{2}.$$

A. J. Frith (Trans. A. S. M. E., x. 298) shows the following application of Rankine's method: If we had a set of cones to design, the extreme diameters of which, including thickness of belt, were 40″ and 10″, and the ratio desired 4, 3, 2, and 1, we would make a table as follows, L being 100″:

Trial Sum of $D+d$.	Ratio.	Trial Diameters.		Values of $\frac{(D-d)^2}{12.56L}$	Amount to be Added.	Corrected Values.	
		D	d			D	d
50	4	40	10	.7165	.0000	40	10
50	3	37.5	12.5	.4975	.2190	37.7190	12.7190
50	2	33.333	16.666	.2212	.4953	33.8286	17.1619
50	1	25	25	.0000	.7165	25.7165	25.7165

The above formulæ are approximate, and they do not give satisfactory results when the difference of diameters of opposite steps is large and when the axes of the pulleys are near together, giving a large belt-angle. The following more accurate solution of the problem is given by C. A. Smith (Trans. A. S. M. E., x. 269) (Fig 152):

Lay off the centre distance C or EF, and draw the circles D_1 and d_1 equal to the first pair of pulleys, which are always previously determined by known conditions. Draw HI tangent to the circles D_1 and d_1. From B, midway between E and F, erect the perpendicular BG, making the length

$BG = .314C$. With G as a centre, draw a circle tangent to HI. Generally this circle will be outside of the belt-line, as in the cut, but when C is short and the first pulleys D_1 and d_1 are large, it will fall on the inside of the belt-line. The belt-line of any other pair of pulleys must be tangent to the circle G; hence any line, as JK or LM, drawn tangent to the circle G, will give

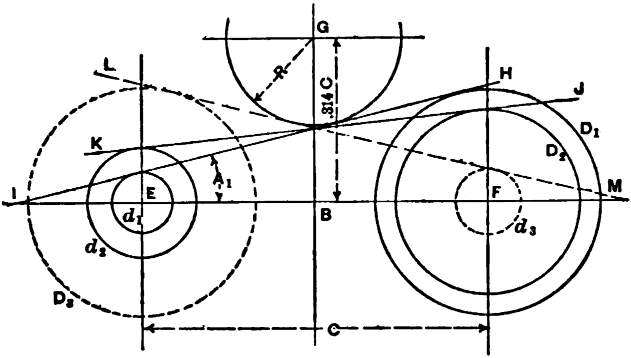

FIG. 152.

the diameters D_2, d_2 or D_3, d_3 of the pulleys drawn tangent to these lines from the centres E and F.

The above method is to be used when the belt-angle A does not exceed 18°. When it is between 18° and 30° a slight modification is made. In that case, in addition to the point G, locate another point m on the line BG .298 C above B. Draw a tangent line to the circle G, making an angle of 18° to the line of centres EF, and from the point m draw an arc tangent to this tangent line. All belt-lines with angles greater than 18° are tangent to this arc. The following is the summary of Mr. Smith's mathematical method:

A = angle in degrees between the centre line and the belt of any pair of pulleys;
a = .314 for belt-angles less than 18°, and .298 for angles between 18° and 30°;
$B°$ = an angle depending on the velocity ratio;
C = the centre distance of the two pulleys;
D, d = diameters of the larger and smaller of the pair of pulleys;
$E°$ = an angle depending on $B°$;
L = the length of the belt when drawn tight around the pulleys;
$r = D \div d$, or the velocity ratio (larger divided by smaller).

$$(1)\ \operatorname{Sin} A = \frac{D - d}{2C}; \qquad (2)\ \tan B° = \frac{2a(r-1)}{r+1};$$

$$(3)\ \operatorname{Sin} E° = \sin B°\left(\cos A - \frac{D+d}{4aC}\right);$$

(4) $A = B° - E°$ when $\sin E°$ is positive; $= B° + E°$ when $\sin E°$ is negative;

(5) $d = \dfrac{2C \sin A}{r-1}$; $= .3183(L - 2C)$ when $A = 0$ and $r = 1$;

(6) $D = rd$;

(7) $L = 2C \cos A + .01745d[180 + (r-1)(90 + A)]$.

Equation (1) is used only once for any pair of cones to obtain the constant $\cos A$, by the aid of tables of sines and cosines, for use in equation (3).

BELTING.

Theory of Belts and Bands.—A pulley is driven by a belt by means of the friction between the surfaces in contact. Let T_1 be the tension on the driving side of the belt, T_2 the tension on the loose side; then $S_1 = T_1 - T_2$, is the total friction between the band and the pulley, which is equal to the tractive or driving force. Let f = the coefficient of friction, θ the ratio of the length of the arc of contact to the length of the radius, a = the angle of the arc of contact in degrees, e = the base of the Naperian logarithms = 2.71828, m = the modulus of the common logarithms = 0.434295. The following formulæ are derived by calculus (Rankine's Mach'y & Millwork, p. 351; Carpenter's Exper. Eng'g, p. 173):

$$\frac{T_1}{T_2} = e^{f\theta}; \quad T_2 = \frac{T_1}{e^{f\theta}}; \quad T_1 - T_2 = T_1 - \frac{T_1}{e^{f\theta}} = T_1(1 - e^{-f\theta}).$$

$$T_1 - T_2 = T_1(1 - e^{-f\theta}) = T_1(1 - 10^{-f\theta m}) = T_1(1 - 10^{-.00758fa});$$

$$\frac{T_1}{T_2} = 10^{.00758fa}; \quad T_1 = T_2 \times 10^{.00758fa}; \quad T_2 = \frac{T_1}{10^{.00758fa}}.$$

If the arc of contact between the band and the pulley expressed in turns and fractions of a turn = n, $\theta = 2\pi n$; $e^{f\theta} = 10^{2.7288fn}$; that is, $e^{f\theta}$ is the natural number corresponding to the common logarithm $2.7288fn$.

The value of the coefficient of friction f depends on the state and material of the rubbing surfaces. For leather belts on iron pulleys, Morin found f = .56 when dry, .36 when wet, .23 when greasy, and .15 when oily. In calculating the proper mean tension for a belt, the smallest value, f = .15, is to be taken if there is a probability of the belt becoming wet with oil. The experiments of Henry R. Towne and Robert Briggs, however (Jour. Frank. Inst., 1868), show that such a state of lubrication is not of ordinary occurrence; and that in designing machinery we may in most cases safely take f = 0.42. Reuleaux takes f = 0.25. The following table shows the values of the coefficient $2.7288f$, by which n is multiplied in the last equation, corresponding to different values of f; also the corresponding values of various ratios among the forces, when the arc of contact is half a circumference:

f =	0.15	0.25	0.42	0.56
$2.7288f$ =	0.41	0.68	1.15	1.53
Let $\theta = \pi$ and $n = \frac{1}{2}$, then				
$T_1 \div T_2$ =	1.603	2.188	3.758	5.821
$T_1 \div S$ =	2.66	1.84	1.36	1.21
$T_1 + T_2 \div 2S$ =	2.16	1.34	0.86	0.71

In ordinary practice it is usual to assume $T_2 = S$; $T_1 = 2S$; $T_1 + T_2 \div 2S = 1.5$. This corresponds to f = 0.22 nearly.

For a wire rope on cast iron f may be taken as 0.15 nearly; and if the groove of the pulley is bottomed with gutta-percha, 0.25. (Rankine.)

Centrifugal Tension of Belts.—When a belt or band runs at a high velocity, centrifugal force produces a tension in addition to that existing when the belt is at rest or moving at a low velocity. This centrifugal tension diminishes the effective driving force.

Rankine says: If an endless band, of any figure whatsoever, runs at a given speed, the centrifugal force produces a uniform tension at each cross-section of the band, equal to the weight of a piece of the band whose length is twice the height from which a heavy body must fall, in order to acquire the velocity of the band. (See Cooper on Belting, p. 101.)

If T_c = centrifugal tension;
V = velocity in feet per second;
g = acceleration due to gravity = 32.2;
W = weight of a piece of the belt 1 ft. long and 1 sq. in. sectional area,—

Leather weighing 56 lbs. per cubic foot gives $W = 56 \div 144 = .388$.

$$T_c = \frac{WV^2}{g} = \frac{.388V^2}{32.2} = .012V^2.$$

Belting Practice. Handy Formulæ for Belting. — Since in the practical application of the above formulæ the value of the coefficient of friction must be assumed, its actual value varying within wide limits (15% to 135%), and since the values of T_1 and T_2 also are fixed arbitrarily, it is customary in practice to substitute for these theoretical formulæ more simple empirical formulæ and rules, some of which are given below.

Let d = diam. of pulley in inches; πd = circumference;
V = velocity of belt in ft. per second; v = vel. in ft. per minute;
a = angle of the arc of contact;
L = length of arc of contact in feet $= \pi da \div (12 \times 360)$;
F = tractive force per square inch of sectional area of belt;
w = width in inches; t = thickness;
S = tractive force per inch of width $= F \div t$;
rpm. = revs. per minute; rps. = revs. per second = rpm. ÷ 60.

$$V = \frac{\pi d}{12} \times \text{rps.} = \frac{\pi d}{12} \times \frac{\text{rpm.}}{60} = .004363d \times \text{rpm.} = \frac{d \times \text{rpm.}}{229.2};$$

$$v = \frac{\pi d}{12} \times \text{rpm.}; = .2618d \times \text{rpm.}$$

$$\text{Horse-power, H.P.} = \frac{Svw}{33000} = \frac{SVw}{550} = \frac{Swd \times \text{rpm.}}{126050} = .000007933Swd \times \text{rpm.}$$

If F = working tension per square inch = 275 lbs., and t = 7/32 inch, S = 60 lbs. nearly, then

$$\text{H.P.} = \frac{vw}{550} = .109Vw = .000476wd \times \text{rpm.} = \frac{wd \times \text{rpm.}}{2101}. \quad . \qquad (1)$$

If F = 180 lbs. per square inch, and t = 1/6 inch, S = 30 lbs., then

$$\text{H.P.} = \frac{vw}{1100} = .055Vw = .000238wd \times \text{rpm.} = \frac{wd \times \text{rpm.}}{4202}. \quad . \qquad (2)$$

If the working strain is 60 lbs. per inch of width, a belt 1 inch wide travelling 550 ft. per minute will transmit 1 horse-power. If the working strain is 30 lbs. per inch of width, a belt 1 inch wide, travelling 1100 ft. per minute, will transmit 1 horse-power. Numerous rules are given by different writers on belting which vary between these extremes. A rule commonly used is: 1 inch wide travelling 1000 ft. per min. = I.H.P.

$$\text{H.P.} = \frac{vw}{1000} = .06Vw = .000262wd \times \text{rpm.} = \frac{wd \times \text{rpm.}}{3820}. \quad . \quad . \qquad (3)$$

This corresponds to a working strain of 33 lbs. per inch of width.

Many writers give as safe practice for single belts in good condition a working tension of 45 lbs. per inch of width. This gives

$$\text{H.P.} = \frac{wv}{733} = .0818Vw = .000357wd \times \text{rpm.} = \frac{wd \times \text{rpm.}}{2800}. \quad . \qquad (4)$$

For double belts of average thickness, some writers say that the transmitting efficiency is to that of single belts as 10 to 7, which would give

$$\text{H.P. of double belts} = \frac{wv}{513} = .1169Vw = .00051wd \times \text{rpm.} = \frac{wd \times \text{rpm.}}{1960}. \qquad (5)$$

Other authorities, however, make the transmitting-power of double belts twice that of single belts, on the assumption that the thickness of a double belt is twice that of a single belt.

Rules for horse-power of belts are sometimes based on the number of square feet of surface of the belt which pass over the pulley in a minute. Sq. ft. per min. $= wv \div 12$. The above formulæ translated into this form give:

(1)	For	S = 60 lbs.	per inch	wide;	H.P. = 46	sq. ft.	per minute.	
(2)	"	S = 30	"	"	H.P. = 92	"	"	
(3)	"	S = 33	"	"	H.P. = 83	"	"	
(4)	"	S = 45	"	"	H.P. = 61	"	"	
(5)	"	S = 64.3	"	"	H.P. = 43	"	"	(double belt).

The above formulæ are all based on the supposition that the arc of contact is 180° For other arcs, the transmitting power is approximately proportional to the ratio of the degrees of arç to 180°.

Some rules base the horse-power on the length of the arc of contact in feet. Since $L = \frac{\pi da}{12 \times 360}$ and $\text{H P.} = \frac{Svw}{33000} = \frac{Sw}{33000} \times \frac{\pi d}{12} \times \text{rpm.} \times \frac{a}{180}$, we obtain by substitution $\text{H.P.} = \frac{Sw}{16500} \times L \times \text{rpm.}$, and the five formulæ then take the following form for the several values of S:

$$\text{H.P} = \frac{wL \times \text{rpm.}}{275}\ (1);\quad \frac{wL \times \text{rpm.}}{550}\ (2);\quad \frac{wL \times \text{rpm.}}{500}\ (3);\quad \frac{wL \times \text{rpm.}}{367}\ (4);$$

$$\text{H.P. (double belt)} = \frac{wL \times \text{rpm.}}{257}\ (5).$$

None of the handy formulæ take into consideration the centrifugal tension of belts at high velocities. When the velocity is over 3000 ft. per minute the effect of this tension becomes appreciable, and it should be taken account of as in Mr. Nagle's formula, which is given below.

Horse-power of a Leather Belt One Inch wide. (NAGLE.)

Formula: $\text{H.P.} = CVtw(S - .012V^2) \div 550.$

For $f = .40$, $a = 180°$, $C = .715$, $w = 1$.

Velocity in ft. per sec.	Laced Belts, $S = 275$. Thickness in inches $= t$.							Velocity in ft. per sec.	Riveted Belts, $S = 400$. Thickness in inches $= t$.						
	1/7 .143	1/6 .167	3/16 .187	7/32 .219	1/4 .250	5/16 .312	1/3 .333		7/32 .219	1/4 .250	5/16 .312	1/3 .333	3/8 .375	7/16 .437	1/2 .500
10	.51	.59	.63	.73	.84	1.05	1.18	15	1.69	1.94	2.42	2.58	2.91	3.29	3.87
15	.75	.88	1.00	1.10	1.32	1.66	1.77	20	2.24	2.57	3.21	3.42	3.85	4.49	5.13
20	1.00	1.17	1.32	1.54	1.75	2.19	2.34	25	2.79	3.19	3.98	4.25	4.78	5.57	6.37
25	1.23	1.43	1.61	1.88	2.16	2.69	2.86	30	3.31	3.79	4.74	5.05	5.67	6.62	7.58
30	1.47	1.72	1.93	2.25	2.58	3.22	3.44	35	3.82	4.37	5.46	5.83	6.56	7.65	8.75
35	1.69	1.97	2.22	2.59	2.96	3.70	3.94	40	4.33	4.95	6.19	6.60	7.42	8.66	9.99
40	1.90	2.22	2.49	2.90	3.32	4.15	4.44	45	4.85	5.49	6.86	7.32	8.43	9.70	10.98
45	2.09	2.45	2.75	3.21	3.67	4.58	4.89	50	5.26	6.01	7.51	8.02	9.02	10.52	12.05
50	2.27	2.65	2.98	3.48	3.98	4.97	5.30	55	5.68	6.50	8.12	8.66	9.74	11.36	13.00
55	2.41	2.84	3.19	3.72	4.26	5.32	5.69	60	6.09	6.96	8.70	9.28	10.43	12.17	13.91
60	2.58	3.01	3.38	3.95	4.51	5.64	6.02	65	6.45	7.37	9.22	9.83	11.06	12.90	14.75
65	2.71	3.16	3.55	4.14	4.74	5.92	6.32	70	6.78	7.75	9.69	10.33	11.62	13.56	15.50
70	2.81	3.27	3.68	4.29	4.91	6.14	6.54	75	7.09	8.11	10.13	10.84	12.16	14.18	16.21
75	2.89	3.37	3.79	4.42	5.05	6.31	6.73	80	7.36	8.41	10.51	11.21	12.61	14.71	16.81
80	2.94	3.43	3.86	4.50	5.15	6.44	6.86	85	7.58	8.66	10.82	11.55	13.00	15.16	17.32
85	2.97	3.47	3.90	4.55	5.20	6.50	6.93	90	7.74	8.85	11.06	11.80	13.27	15.48	17.69
90	2.97	3.47	3.90	4.55	5.20	6.50	6.93	100	7.96	9.10	11.37	12.13	13.65	15.92	18.20
The H.P. becomes a maximum at 87.41 ft. per sec., = 5245 ft. p. min.								The H.P. becomes a maximum at 105.4 ft. per sec. = 6324 ft. per min.							

In the above table the angle of subtension, a, is taken at 180°.

Should it be..	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°	200°
Multiply above values by	.65	.70	.75	.79	.83	.87	.91	.94	.97	1	1.05

A. F. Nagle's Formula (Trans. A. S. M. E., vol. ii., 1881, p. 91. Tables published in 1882.)

$$\text{H.P.} = CVtw\left(\frac{S - .012V^2}{550}\right);$$

$C = 1 - 10^{-.00758fa}$;
a = degrees of belt contact;
f = coefficient of friction;
w = width in inches;
t = thickness in inches;
V = velocity in feet per second;
S = stress upon belt per square inch.

Taking S at 275 lbs. per sq. in. for laced belts and 400 lbs. per sq. in. for lapped and riveted belts, the formula becomes

$$H.P. = CVtw(.50 - .0000218V^2) \text{ for laced belts;}$$
$$H.P. = CVtw(.727 - .0000218V^2) \text{ for riveted belts.}$$

VALUES OF $C = 1 - 10^{-.00758fa}$. (NAGLE.)

f = coefficient of friction.	Degrees of contact = a.										
	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°	200°
.15	.210	.230	.250	.270	.288	.307	.325	.342	.359	.376	.408
.20	.270	.295	.319	.342	.364	.386	.408	.428	.448	.467	.503
.25	.325	.354	.381	.407	.432	.457	.480	.503	.524	.544	.582
.30	.376	.408	.438	.467	.494	.520	.544	.567	.590	.610	.649
.35	.423	.457	.489	.520	.548	.575	.600	.624	.646	.667	.705
.40	.467	.502	.536	.567	.597	.624	.649	.673	.695	.715	.753
.45	.507	.544	.579	.610	.640	.667	.692	.715	.737	.757	.792
.55	.578	.617	.652	.684	.713	.739	.763	.785	.805	.822	.853
.60	.610	.649	.684	.715	.744	.769	.792	.813	.832	.848	.877
1.00	.792	.825	.853	.877	.897	.913	.927	.937	.947	.956	.969

The following table gives a comparison of the formulæ already given for the case of a belt one inch wide, with arc of contact 180°.

Horse-power of a Belt One Inch wide, Arc of Contact 180°.

COMPARISON OF DIFFERENT FORMULÆ.

Velocity in ft. per sec.	Velocity in ft. p. min.	Sq. ft. of Belt p. min.	Form. 1 H.P. = $\frac{wv}{550}$.	Form. 2 H.P. = $\frac{wv}{1100}$.	Form. 3 H.P. = $\frac{wv}{1000}$.	Form. 4 H.P. = $\frac{wv}{733}$.	Form. 5 dbl. belt H.P. = $\frac{wv}{513}$.	Nagle's Form. 7/32" single belt Laced.	Nagle's Form. 7/32" single belt Riveted
10	600	50	1.09	.55	.60	.82	1.17	.73	1.14
20	1200	100	2.18	1.09	1.20	1.64	2.34	1.54	2.24
30	1800	150	3.27	1.64	1.80	2.46	3.51	2.25	3.31
40	2400	200	4.36	2.18	2.40	3.27	4.68	2.90	4.33
50	3000	250	5.45	2.73	3.00	4.09	5.85	3.48	5.26
60	3600	300	6.55	3.27	3.60	4.91	7.02	3.95	6.09
70	4200	350	7.63	3.82	4.20	5.73	8.19	4.29	6.78
80	4800	400	8.73	4.36	4.80	6.55	9.36	4.50	7.36
90	5400	450	9.82	4.91	5.40	7.37	10.53	4.55	7.74
100	6000	500	10.91	5.45	6.00	8.18	11.70	4.41	7.96
110	6600	550	....	....	....	....	....	4.05	7.97
120	7200	600	....	....	....	....	....	3.49	7.75

Width of Belt for a Given Horse-power.—The width of belt required for any given horse-power may be obtained by transposing the formulæ for horse-power so as to give the value of w. Thus:

From formula (1), $w = \frac{550 \text{ H.P.}}{v} = \frac{9.17 \text{ H.P.}}{V} = \frac{2101 \text{ H.P.}}{d \times \text{rpm.}} = \frac{275 \text{ H.P.}}{L \times \text{rpm.}}$.

From formula (2), $w = \frac{1100 \text{ H.P.}}{v} = \frac{18.33 \text{ H.P.}}{V} = \frac{4202 \text{ H.P.}}{d \times \text{rpm.}} = \frac{550 \text{ H.P.}}{L \times \text{rpm.}}$.

From formula (3), $w = \frac{1000 \text{ H.P.}}{v} = \frac{16.67 \text{ H.P.}}{V} = \frac{3820 \text{ H.P.}}{d \times \text{rpm.}} = \frac{500 \text{ H.P.}}{L \times \text{rpm.}}$.

From formula (4), $w = \frac{733 \text{ H.P.}}{v} = \frac{12.22 \text{ H.P.}}{V} = \frac{2800 \text{ H.P.}}{d \times \text{rpm.}} = \frac{350 \text{ H.P.}}{L \times \text{rpm.}}$.

From formula (5),* $w = \frac{513 \text{ H.P.}}{v} = \frac{8.56 \text{ H.P.}}{V} = \frac{1960 \text{ H.P.}}{d \times \text{rpm.}} = \frac{257 \text{ H.P.}}{L \times \text{rpm.}}$.

* For double belts.

Many authorities use formula (1) for double belts and formula (2) or (3) for single belts.

To obtain the width by Nagle's formula, $w = \frac{550 \text{ H.P.}}{CVt(S - .012V^2)}$, or divide the given horse-power by the figure in the table corresponding to the given thickness of belt and velocity in feet per second.

The formula to be used in any particular case is largely a matter of judgment. A single belt proportioned according to formula (1), if tightly stretched, and if the surface is in good condition, will transmit the horse-power calculated by the formula, but one so proportioned is objectionable, first, because it requires so great an initial tension that it is apt to stretch, slip, and require frequent restretching and relacing; and second, because this tension will cause an undue pressure on the pulley-shaft, and therefore an undue loss of power by friction. To avoid these difficulties, formula (2), (3), or (4,) or Mr. Nagle's table, should be used; the latter especially in cases in which the velocity exceeds 4000 ft. per min.

Taylor's Rules for Belting.—F. W. Taylor (Trans. A. S. M. E., xv. 204) describes a nine years' experiment on belting in a machine-shop, giving results of tests of 42 belts running night and day. Some of these belts were run on cone pulleys and others on shifting, or fast-and-loose, pulleys. The average net working load on the shifting belts was only 4/10 of that of the cone belts.

The shifting belts varied in dimensions from 39 ft. 7 in. long, 3.5 in. wide, .25 in. thick, to 51 ft. 5 in. long, 6.5 in. wide, .37 in. thick. The cone belts varied in dimensions from 24 ft. 7 in. long, 2 in. wide, .25 in. thick, to 31 ft. 10 in. long, 4 in. wide, .37 in. thick.

Belt-clamps were used having spring-balances between the two pairs of clamps, so that the exact tension to which the belt was subjected was accurately weighed when the belt was first put on, and each time it was tightened.

The tension under which each belt was spliced was carefully figured so as to place it under an initial strain—while the belt was at rest immediately after tightening—of 71 lbs. per inch of width of double belts. This is equivalent, in the case of

Oak tanned and fulled belts,	to 192	lbs. per sq. in. section;
Oak tanned, not fulled belts,	to 229	" " " "
Semi-raw-hide belts,	to 253	" " " "
Raw-hide belts,	to 284	" " " "

From the nine years' experiment Mr. Taylor draws a number of conclusions, some of which are given in an abridged form below.

In using belting so as to obtain the greatest economy and the most satisfactory results, the following rules should be observed:

	Oak Tanned and Fulled Leather Belts.	Other Types of Leather Belts and 6- to 7-ply Rubber Belts.
A double belt, having an arc of contact of 180°, will give an effective pull on the face of a pulley per inch of width of belt of....	35 lbs.	30 lbs.
Or, a different form of same rule: The number of sq. ft. of double Belt passing around a pulley per minute required to transmit one horse-power is..............	80 sq. ft.	90 sq. ft.
Or: The number of lineal feet of double belting 1 in. wide passing around a pulley per minute required to transmit one horse-power is.......	950 ft.	1100 ft.
Or: A double belt 6 in. wide, running 4000 to 5000 ft. per min., will transmit...........	30 H.P.	25 H.P.

The terms "initial tension," "effective pull," etc., are thus explained by Mr. Taylor: When pulleys upon which belts are tightened are at rest, both strands of the belt (the upper and lower) are under the same stress per in. of width. By "tension," "initial tension," or "tension while at rest," we mean the stress per in. of width, or sq. in. of section, to which one of the strands of the belt is tightened, when at rest. After the belts are in motion and transmitting power, the stress on the slack side, or strand, of the belt becomes less, while that on the tight side—or the side which does the pulling—becomes greater than when the belt was at rest. By the term "total load" we mean the total stress per in. of width, or sq. in. of section, on the tight side of belt while in motion.

The difference between the stress on the tight side of the belt and its slack side, while in motion, represents the effective force or pull which is transmitted from one pulley to another. By the terms "working load," "net working load," or "effective pull," we mean the difference in the tension of the tight and slack sides of the belt per in. of width, or sq. in. section, while in motion, or the net effective force that is transmitted from one pulley to another per in. of width or sq. in. of section.

The discovery of Messrs. Lewis and Bancroft (Trans. A. S. M. E., vii. 749) that the "sum of the tension on both sides of the belt does not remain constant," upsets all previous theoretical belting formulæ.

The belt speed for maximum economy should be from 4000 to 4500 ft. per minute.

The best distance from centre to centre of shafts is from 20 to 25 ft.

Idler pulleys work most satisfactorily when located on the slack side of the belt about one quarter way from the driving-pulley.

Belts are more durable and work more satisfactorily made narrow and thick, rather than wide and thin.

It is safe and advisable to use: a double belt on a pulley 12 in. diameter or larger; a triple belt on a pulley 20 in. diameter or larger; a quadruple belt on a pulley 30 in. diameter or larger.

As belts increase in width they should also be made thicker.

The ends of the belt should be fastened together by splicing and cementing, instead of lacing, wiring, or using hooks or clamps of any kind.

A V-splice should be used on triple and quadruple belts and when idlers are used. Stepped splice, coated with rubber and vulcanized in place, is best for rubber belts.

For double belting the rule works well of making the splice for all belts up to 10 in. wide, 10 in. long; from 10 in. to 18 in. wide the splice should be the same width as the belt, 18 in. being the greatest length of splice required for double belting.

Belts should be cleaned and greased every five to six months.

Double leather belts will last well when repeatedly tightened under a strain (when at rest) of 71 lbs. per in. of width, or 240 lbs. per sq. in. section. They will not maintain this tension for any length of time, however.

Belt-clamps having spring-balances between the two pairs of clamps should be used for weighing the tension of the belt accurately each time it is tightened.

The stretch, durability, cost of maintenance, etc., of belts proportioned (A) according to the ordinary rules of a total load of 111 lbs. per inch of width corresponding to an effective pull of 65 lbs. per inch of width, and (B) according to a more economical rule of a total load of 54 lbs., corresponding to an effective pull of 26 lbs. per inch of width, are found to be as follows:

When it is impracticable to accurately weigh the tension of a belt in tightening it, it is safe to shorten a double belt one half inch for every 10 ft. of length for (A) and one inch for every 10 ft. for (B), if it requires tightening.

Double leather belts, when treated with great care and run night and day at moderate speed, should last for 7 years (A); 18 years (B).

The cost of all labor and materials used in the maintenance and repairs of double belts, added to the cost of renewals as they give out, through a term of years, will amount on an average per year to 37% of the original cost of the belts (A); 14% or less (B).

In figuring the total expense of belting, and the manufacturing cost chargeable to this account, by far the largest item is the time lost on the machines while belts are being relaced and repaired.

The total stretch of leather belting exceeds 6% of the original length.

The stretch during the first six months of the life of belts is 36% of their entire stretch (A); 15% (B).

A double belt will stretch 47/100 of 1% of its length before requiring to be tightened (A); 81/100 of 1% (B).

The most important consideration in making up tables and rules for the use and care of belting is how to secure the minimum of interruptions to manufacture from this source.

The average double belt (A), when running night and day in a machine-shop, will cause at least 26 interruptions to manufacture during its life, or 5 interruptions per year, but with (B) interruptions to manufacture will not average oftener for each belt than one in sixteen months.

The oak-tanned and fulled belts showed themselves to be superior in all respects except the coefficient of friction to either the oak-tanned not fulled, the semi-raw-hide, or raw-hide with tanned face.

Belts of any width can be successfully shifted backward and forward on tight and loose pulleys. Belts running between 5000 and 6000 ft. per min. and driving 300 H.P. are now being daily shifted on tight and loose pulleys, to throw lines of shafting in and out of use.

The best form of belt-shifter for wide belts is a pair of rollers twice the width of belt, either of which can be pressed onto the flat surface of the belt on its slack side close to the driven pulley, the axis of the roller making an angle of 75° with the centre line of the belt.

Remarks on Mr. Taylor's Rules. (Trans. A. S. M. E., xv., 242.) —The most notable feature in Mr. Taylor's paper is the great difference between his rules for proper proportioning of belts and those given by earlier writers. A very commonly used rule is, one horse-power may be transmitted by a single belt 1 in. wide running x ft. per min., substituting for x various values, according to the ideas of different engineers, ranging usually from 550 to 1100.

The practical mechanic of the old school is apt to swear by the figure 600 as being thoroughly reliable, while the modern engineer is more apt to use the figure 1000. Mr. Taylor, however, instead of using a figure from 550 to 1100 for a single belt, uses 950 to 1100 for double belts. If we assume that a double belt is twice as strong, or will carry twice as much power, as a single belt, then he uses a figure at least twice as large as that used in modern practice, and would make the cost of belting for a given shop twice as large as if the belting were proportioned according to the most liberal of the customary rules.

This great difference is to some extent explained by the fact that the problem which Mr. Taylor undertakes to solve is quite a different one from that which is solved by the ordinary rules with their variations. The problem of the latter generally is, "How wide a belt must be used, or how narrow a belt may be used, to transmit a given horse-power?" Mr. Taylor's problem is: "How wide a belt must be used so that a given horse-power may be transmitted with the minimum cost for belt repairs, the longest life to the belt, and the smallest loss and inconvenience from stopping the machine while the belt is being tightened or repaired?"

The difference between the old practical mechanic's rule of a 1-in.-wide single belt, 600 ft. per min., transmits one horse-power, and the rule commonly used by engineers, in which 1000 is substituted for 600, is due to the belief of the engineers, not that a horse-power could not be transmitted by the belt proportioned by the older rule, but that such a proportion involved undue strain from overtightening to prevent slipping, which strain entailed too much journal friction, necessitated frequent tightening, and decreased the length of the life of the belt.

Mr. Taylor's rule substituting 1100 ft. per min. and doubling the belt is a further step, and a long one, in the same direction. Whether it will be taken in any case by engineers will depend upon whether they appreciate the extent of the losses due to slippage of belts slackened by use under overstrain, and the loss of time in tightening and repairing belts, to such a degree as to induce them to allow the first cost of the belts to be doubled in order to avoid these losses.

It should be noted that Mr. Taylor's experiments were made on rather narrow belts, used for transmitting power from shafting to machinery, and his conclusions may not be applicable to heavy and wide belts, such as engine fly-wheel belts.

MISCELLANEOUS NOTES ON BELTING.

Formulæ are useful for proportioning belts and pulleys, but they furnish no means of estimating how much power a particular belt may be transmitting at any given time, any more than the size of the engine is a measure of the load it is actually drawing, or the known strength of a horse is a measure of the load on the wagon. The only reliable means of determining the power actually transmitted is some form of dynamometer. (See Trans. A. S. M. E., vol. xii. p. 707.)

If we increase the thickness, the power transmitted ought to increase in proportion; and for double belts we should have half the width required for a single belt under the same conditions. With large pulleys and moderate velocities of belt it is probable that this holds good. With small pulleys, however, when a double belt is used, there is not such perfect contact between the pulley-face and the belt, due to the rigidity of the latter, and more work is necessary to bend the belt-fibres than when a thinner and more pliable belt is used. The centrifugal force tending to throw the belt from the pulley also increases with the thickness, and for these reasons the width of a double belt required to transmit a given horse-power when used with small pulleys is generally assumed not less than seven tenths the width of a single belt to transmit the same power. (Flather on "Dynamometers and Measurement of Power.")

F. W. Taylor, however, finds that great pliability is objectionable, and favors thick belts even for small pulleys. The power consumed in bending the belt around the pulley he considers inappreciable. According to Rankine's formula for centrifugal tension, this tension is proportional to the sectional area of the belt, and hence it does not increase with increase of thickness when the width is decreased in the same proportion, the sectional area remaining constant.

Scott A. Smith (Trans. A. S. M. E., x. 765) says: The best belts are made from all oak-tanned leather, and curried with the use of cod oil and tallow, all to be of superior quality. Such belts have continued in use thirty to forty years when used as simple driving-belts, driving a proper amount of power, and having had suitable care. The flesh side should not be run to the pulley-face, for the reason that the wear from contact with the pulley should come on the grain side, as that surface of the belt is much weaker in its tensile strength than the flesh side; also as the grain is hard it is more enduring for the wear of attrition; further, if the grain is actually worn off, then the belt may not suffer in its integrity from a ready tendency of the hard grain side to crack.

The most intimate contact of a belt with a pulley comes, first, in the smoothness of a pulley-face, including freedom from ridges and hollows left by turning-tools; second, in the smoothness of the surface and evenness in the texture or body of a belt; third, in having the crown of the driving and receiving pulleys exactly alike,—as nearly so as is practicable in a commercial sense; fourth, in having the crown of pulleys not over 1/8" for a 24" face, that is to say, that the pulley is not to be over 1/4" larger in diameter in its centre; fifth, in having the crown other than two planes meeting at the centre; sixth, the use of any material on or in a belt, in addition to those necessarily used in the currying process, to keep them pliable or increase their tractive quality, should wholly depend upon the exigencies arising in the use of belts; non-use is safer than over-use; seventh, with reference to the lacing of belts, it seems to be a good practice to cut the ends to a convex shape by using a former, so that there may be a nearly uniform stress on the lacing through the centre as compared with the edges. For a belt 10" wide, the centre of each end should recede 1/10".

Lacing of Belts.—In punching a belt for lacing, use an oval punch, the longer diameter of the punch being parallel with the sides of the belt. Punch two rows of holes in each end, placed zigzag. In a 3-in. belt there should be four holes in each end—two in each row. In a 6-inch belt, seven holes—four in the row nearest the end. A 10-inch belt should have nine holes. The edge of the holes should not come nearer than 3/4 of an inch from the sides, nor 7/8 of an inch from the ends of the belt. The second row should be at least 1 3/4 inches from the end. On wide belts these distances should be even a little greater.

Begin to lace in the centre of the belt and take care to keep the ends exactly in line, and to lace both sides with equal tightness. The lacing should not be crossed on the side of the belt that runs next the pulley. In taking up belts, observe the same rules as putting on new ones.

Setting a Belt on Quarter-twist.—A belt must run squarely on to the pulley. To connect with a belt two horizontal shafts at right angles with each other, say an engine-shaft near the floor with a line attached to the ceiling, will require a quarter-turn. First, ascertain the central point on the face of each pulley at the extremity of the horizontal diameter where the belt will leave the pulley, and then set that point on the driven pulley plumb over the corresponding point on the driver. This will cause the belt to run squarely on to each pulley, and it will leave at an angle greater or less, according to the size of the pulleys and their distance from each other.

In quarter-twist belts, in order that the belt may remain on the pulleys, the central plane on each pulley must pass through the point of delivery of the other pulley. This arrangement does not admit of reversed motion.

To find the Length of Belt required for two given Pulleys.—When the length cannot be measured directly by a tape-line, the following approximate rule may be used: Add the diameter of the two pulleys together, divide the sum by 2, and multiply the quotient by 3¼, and add the product to twice the distance between the centres of the shafts. (See accurate formula below.)

To find the Angle of the Arc of Contact of a Belt.—Divide the difference between the radii of the two pulleys in inches by the distance between their centres, also in inches, and in a table of natural sines find the angle most nearly corresponding with the quotient. Multiply this angle by 2, and add the product to 180° for the angle of contact with the larger pulley, or subtract it from 180° for the smaller pulley.

Or, let R = radius of larger pulley, r = radius of smaller;
L = distance between centres of the pulleys;
a = angle whose sine is $(R - r) \div L$.
Arc of contact with smaller pulley $= 180° - 2a$;
" " " larger pulley $= 180° + 2a$.

To find the Length of Belt in Contact with the Pulley.—For the larger pulley, multiply the angle a, found as above, by .0349, to the product add 3.1416, and multiply the sum by the radius of the pulley. Or length of belt in contact with the pulley

$$= \text{radius} \times (\pi + .0349a) = \text{radius} \times \pi\left(1 + \frac{a}{90}\right).$$

For the smaller pulley, length $= \text{radius} \times (\pi - .0349a) = \text{radius} \times \pi\left(1 - \frac{a}{90}\right)$.

The above rules refer to **Open Belts**. The accurate formula for length of an open belt is,

$$\text{Length} = \pi R\left(1 + \frac{a}{90}\right) + \pi r\left(1 - \frac{a}{90}\right) + 2L \cos a$$

$$= R(\pi + .0349a) + r(\pi - .0349a) + 2L \cos a,$$

in which R = radius of larger pulley, r = radius of smaller pulley, L = distance between centres of pulleys, and a = angle whose sine is $(R - r) \div L$; $\cos a = \sqrt{L^2 - (R - r)^2} \div L$.

For Crossed Belts the formula is

$$\text{Length of belt} = \pi R\left(1 + \frac{\beta}{90}\right) + \pi r\left(1 + \frac{\beta}{90}\right) + 2L \cos \beta,$$

$$= (R + r) \times (\pi + .0349\beta) + 2L \cos \beta,$$

in which β = angle whose sine is $(R + r) \div L$; $\cos \beta = \sqrt{L^2 - (R + r)^2} \div L$.

To find the Length of Belt when Closely Rolled.—The sum of the diameter of the roll, and of the eye in inches, × the number of turns made by the belt and by .1309, = length of the belt in feet

To find the Approximate Weight of Belts.—Multiply the length of belt, in feet, by the width in inches, and divide the product by 13 for single, and 8 for double belt.

Relations of the Size and Speeds of Driving and Driven Pulleys.—The driving pulley is called the driver, D, and the driven pulley the driven, d. If the number of teeth in gears is used instead of diameter, in these calculations, number of teeth must be substituted wherever diameter occurs. R = revs. per min. of driver, r = revs. per min. of driven.

$$D = dr \div R;$$

Diam. of driver = diam. of driven × revs. of driven ÷ revs. of driver.

$$d = DR \div r;$$

Diam. of driven = diam. of driver × revs. of driver ÷ revs. of driven.

$$R = dr \div D;$$

Revs. of driver = revs. of driven × diam. of driven ÷ diam. of driver.

$$r = DR \div d;$$

Revs. of driven = revs. of driver × diam. of driver ÷ diam. of driven.

Evils of Tight Belts. (Jones and Laughlins.)—Clamps with powerful screws are often used to put on belts with extreme tightness, and with most injurious strain upon the leather. They should be very judiciously used for horizontal belts, which should be allowed sufficient slackness to move with a loose undulating vibration on the returning side, as a test that they have no more strain imposed than is necessary simply to transmit the power.

On this subject a New England cotton-mill engineer of large experience, says: I believe that three quarters of the trouble experienced in broken pulleys, hot boxes, etc., can be traced to the fault of tight belts. The enormous and useless pressure thus put upon pulleys must in time break them, if they are made in any reasonable proportions, besides wearing out the whole outfit, and causing heating and consequent destruction of the bearings. Below are some figures showing the power it takes in average modern mills with first-class shafting, to drive the shafting alone:

Mill, No.	Whole Load, H.P.	Shafting Alone.		Mill, No.	Whole Load, H.P.	Shafting Alone.	
		Horse-power.	Per cent of whole.			Horse-power.	Per cent of whole.
1	199	51	25.6	5	759	172.6	22.7
2	472	111.5	23.6	6	235	84.8	36.1
3	486	134	27.5	7	670	262.9	39.2
4	677	190	28.1	8	677	182	26.8

These may be taken as a fair showing of the power that is required in many of our best mills to drive shafting. It is unreasonable to think that all that power is consumed by a legitimate amount of friction of bearings and belts. I know of no cause for such a loss of power but tight belts. These, when there are hundreds or thousands in a mill, easily multiply the friction on the bearings, and would account for the figures.

Sag of Belts.—In the location of shafts that are to be connected with each other by belts, care should be taken to secure a proper distance one from the other. This distance should be such as to allow of a gentle sag to the belt when in motion.

A general rule may be stated thus: Where narrow belts are to be run over small pulleys 15 feet is a good average, the belt having a sag of 1½ to 2 inches.

For larger belts, working on larger pulleys, a distance of 20 to 25 feet does well, with a sag of 2½ to 4 inches.

For main belts working on very large pulleys, the distance should be 25 to 30 feet, the belts working well with a sag of 4 to 5 inches.

If too great a distance is attempted, the belt will have an unsteady flapping motion, which will destroy both the belt and machinery.

Arrangement of Belts and Pulleys.—If possible to avoid it, connected shafts should never be placed one directly over the other, as in such case the belt must be kept very tight to do the work. For this purpose belts should be carefully selected of well-stretched leather.

It is desirable that the angle of the belt with the floor should not exceed 45°. It is also desirable to locate the shafting and machinery so that belts should run off from each shaft in opposite directions, as this arrangement will relieve the bearings from the friction that would result when the belts all pull one way on the shaft.

In arranging the belts leading from the main line of shafting to the counters, those pulling in an opposite direction should be placed as near each other as practicable, while those pulling in the same direction should be separated. This can often be accomplished by changing the relative positions of the pulleys on the counters. By this procedure much of the friction on the journals may be avoided.

If possible, machinery should be so placed that the direction of the belt motion shall be from the top of the driving to the top of the driven pulley, when the sag will increase the arc of contact.

The pulley should be a little wider than the belt required for the work.

The motion of driving should run with and not against the laps of the belts.

Tightening or guide pulleys should be applied to the slack side of belts and near the smaller pulley.

Jones & Laughlins, in their Useful Information, say: The diameter of the pulleys should be as large as can be admitted, provided they will not produce a speed of more than 4750 feet of belt motion per minute.

They also say: It is better to gear a mill with small pulleys and run them at a high velocity, than with large pulleys and to run them slower. A mill thus geared costs less and has a much neater appearance than with large heavy pulleys.

M. Arthur Achard (Proc. Inst. M. E., Jan. 1881, p. 62) says: When the belt is wide a partial vacuum is formed between the belt and the pulley at a high velocity. The pressure is then greater than that computed from the tensions in the belt, and the resistance to slipping is greater. This has the advantage of permitting a greater power to be transmitted by a given belt, and of diminishing the strain on the shafting.

On the other hand, some writers claim that the belt entraps air between itself and the pulley, which tends to diminish the friction, and reduce the tractive force. On this theory some manufacturers perforate the belt with numerous holes to let the air escape.

Care of Belts.—Leather belts should be well protected against water, loose steam, and all other moisture, with which they should not come in contact. But where such conditions prevail fairly good results are obtained by using a special dressing prepared for the purpose of water-proofing leather, though a positive water-proofing material has not yet been discovered.

Belts made of coarse, loose-fibred leather will do better service in dry and warm places, but if damp or moist conditions exist then the very finest and firmest leather should be used. (Fayerweather & Ladew.)

Do not allow oil to drip upon the belts. It destroys the life of the leather.

Leather belting cannot safely stand above 110° of heat.

Strength of Belting.—The ultimate tensile strength of belting does not generally enter as a factor in calculations of power transmission.

The strength of the solid leather in belts is from 2000 to 5000 lbs. per square inch; at the lacings, even if well put together, only about 1000 to 1500. If riveted, the joint should have half the strength of the solid belt. The working strain on the driving side is generally taken at not over one third of the strength of the lacing, or from one eighth to one sixteenth of the strength of the solid belt. Dr. Hartig found that the tension in practice varied from 30 to 532 lbs. per square inch, averaging 273 lbs.

Adhesion Independent of Diameter. (Schultz Belting Co.)—

1. The adhesion of the belt to the pulley is the same—the arc or number of degrees of contact, aggregate tension or weight being the same—without reference to width of belt or diameter of pulley.

2. A belt will slip just as readily on a pulley four feet in diameter as it will on a pulley two feet in diameter, provided the conditions of the faces of the pulleys, the arc of contact, the tension, and the number of feet the belt travels per minute are the same in both cases.

3. A belt of a given width, and making any given number of feet per minute, will transmit as much power running on pulleys two feet in diameter as it will on pulleys four feet in diameter, provided the arc of contact, tension, and conditions of pulley faces are the same in both cases.

4. To obtain a greater amount of power from belts the pulleys may be covered with leather; this will allow the belts to run very slack and give 25% more durability.

Endless Belts.—If the belts are to be endless, they should be put on and drawn together by "belt clamps" made for the purpose. If the belt is made endless at the belt factory, it should never be run on to the pulleys, lest the irregular strain spring the belt. Lift out one shaft, place the belt on the pulleys, and force the shaft back into place.

Belt Data.—A fly-wheel at the Amoskeag Mfg. Co., Manchester, N. H., 30 feet diameter, 110 inches face, running 61 revolutions per minute, carried two heavy double-leather belts 40 inches wide each, and one 24 inches wide. The engine indicated 1950 H.P., of which probably 1850 H.P. was transmitted by the belts. The belts were considered to be heavily loaded, but not overtaxed.

$$\frac{30 \times 3.14 \times 104 \times 61}{1850} = 323 \text{ feet per minute for 1 H.P. per inch of width.}$$

Samuel Webber (*Am. Mach.*, Feb. 22, 1894) reports a case of a belt 30 inches wide, 5/16 inch thick, running for six years at a velocity of 3900 feet per minute, on to a pulley 5 feet diameter, and transmitting 556 H.P. This gives a velocity of 210 feet per minute for 1 H.P. per inch of width. By Mr. Nagle's table of riveted belts this belt would be designed for 332 H.P. By Mr. Taylor's rule it would be used to transmit only 128 H.P.

The above may be taken as examples of what a belt may be made to do, but they should not be used as precedents in designing. It is not stated how much power was lost by the journal friction due to over-tightening of these belts.

Belt Dressings.—We advise that no belt dressing should be used except when the belt becomes dry and husky, and in such instances we recommend the use of Post's Belt Dressing. Where this is not used beef tallow at blood-warm temperature should be applied and then dried in either by artificial heat or the sun. The oil of the tallow passes into the leather, serving to soften it, and the stearine is left on the outside to fill the pores and leave a smooth surface. The addition of beeswax to the tallow will be of some service if the belts are used in wet or damp places. Belts which have become dry and hard should have an application of Post's belt oil or neats's-foot oil of the purest quality. Our experience convinces us that resin should never be used on leather belting in any form. (Fayerweather & Ladew.)

Belts should not be soaked in water before oiling, and penetrating oils should but seldom be used, except occasionally when a belt gets very dry and husky from neglect. It may then be moistened a little, and have neat's-foot oil applied. Frequent applications of such oils to a new belt render the leather soft and flabby, thus causing it to stretch, and making it liable to run out of line. A composition of tallow and oil, with a little resin or beeswax, is better to use. Prepared castor-oil dressing is good, and may be applied with a brush or rag while the belt is running. (Alexander Bros.)

Cement for Cloth or Leather. (Molesworth.)—16 parts gutta-percha, 4 india-rubber, 2 pitch, 1 shellac, 2 linseed-oil, cut small, melted together and well mixed.

Rubber Belting.—The advantages claimed for rubber belting are perfect uniformity in width and thickness; it will endure a great degree of heat and cold without injury; it is also specially adapted for use in damp or wet places, or where exposed to the action of steam; it is very durable, and has great tensile strength, and when adjusted for service it has the most perfect hold on the pulleys, hence is less liable to slip than leather.

Never use animal oil or grease on rubber belts, as it will greatly injure and soon destroy them.

Rubber belts will be improved, and their durability increased, by putting on with a painter's brush, and letting it dry, a composition made of equal parts of red lead, black lead, French yellow, and litharge, mixed with boiled linseed-oil and japan enough to make it dry quickly. The effect of this will be to produce a finely polished surface. If, from dust or other cause, the belt should slip, it should be lightly moistened on the side next the pulley with boiled linseed-oil. (From circulars of manufacturers.)

GEARING.

TOOTHED-WHEEL GEARING.

Pitch, Pitch-circle, etc.—If two cylinders with parallel axes are pressed together and one of them is rotated on its axis, it will drive the other by means of the friction between the surfaces. The cylinders may be considered as a pair of spur-wheels with an infinite number of very small teeth. If actual teeth are formed upon the cylinders, making alternate elevations and depressions in the cylindrical surfaces, the distance between the axes remaining the same, we have a pair of gear-wheels which will drive one another by pressure upon the faces of the teeth, if the teeth are properly shaped. In making the teeth the cylindrical surface may entirely disappear, but the position it occupied may still be considered as a cylindrical surface, which is called the "pitch-surface," and its trace on the end of the wheel, or on a plane cutting the wheel at right angles to its axis, is called the "pitch-circle" or "pitch-line." The diameter of this circle is called the pitch-diameter, and the distance from the face of one tooth to the corresponding face of the next tooth on the same wheel, measured on an arc of the pitch-circle, is called the "pitch of the tooth," or the circular pitch.

If two wheels having teeth of the same pitch are geared together so that their pitch-circles touch, it is a property of the pitch-circles that their diameters are proportional to the number of teeth in the wheels, and *vice versa*;

thus, if one wheel is twice the diameter (measured on the pitch-circle) of the other, it has twice as many teeth. If the teeth are properly shaped the linear velocity of the two wheels are equal, and the angular velocities, or speeds of rotation, are inversely proportional to the number of teeth and to the diameter. Thus the wheel that has twice as many teeth as the other will revolve just half as many times in a minute.

The "pitch," or distance measured on an arc of the pitch-circle from the face of one tooth to the face of the next, consists of two parts—the "thickness" of the tooth and the "space" between it and the next tooth. The space is larger than the thickness by a small amount called the "backlash," which is allowed for imperfections of workmanship. In finely cut gears the backlash may be almost nothing.

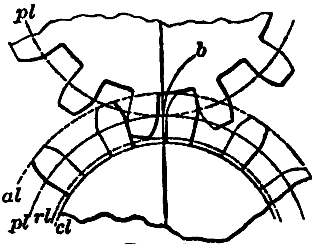

Fig. 153.

The length of a tooth in the direction of the radius of the wheel is called the "depth," and this is divided into two parts: First, the "addendum," the height of the tooth above the pitch-line; second, the "dedendum," the depth below the pitch line, which is an amount equal to the addendum of the mating gear. The depth of the space is usually given a little "clearance" to allow for inaccuracies of workmanship, especially in cast gears.

Referring to Fig. 153, *pl*, *pl* are the pitch-lines, *al* the addendum-line, *rl* the root-line or dedendum-line, *cl* the clearance-line, and *b* the backlash. The addendum and dedendum are usually made equal to each other.

$$\textit{Diametral pitch} = \frac{\text{No. of teeth}}{\text{diam. of pitch-circle in inches}} = \frac{3.1416}{\text{circular pitch}};$$

$$\text{Circular pitch} = \frac{\text{diam.} \times 3.1416}{\text{No. of teeth}} = \frac{3.1416}{\text{diametral pitch}}.$$

Some writers use the term diametral pitch to mean $\frac{\text{diam.}}{\text{No. of teeth}} = \frac{\text{circular pitch}}{3.1416}$, but the first definition is the more common and the more convenient. A wheel of 12 in. diam. at the pitch-circle, with 48 teeth is 48/12 = 4 diametral pitch, or simply 4 pitch. The circular pitch of the same wheel is $\frac{12 \times 3.1416}{48} = .7854$, or $\frac{3.1416}{4} = .7854$ in.

Relation of Diametral to Circular Pitch.

Diametral Pitch.	Circular Pitch.	Diametral Pitch.	Circular Pitch.	Circular Pitch.	Diametral Pitch.	Circular Pitch.	Diametral Pitch.
1	3.142 in.	11	.286 in.	3	1.047	15/16	3.351
1½	2.094	12	.262	2½	1.257	⅞	3.590
2	1.571	14	.224	2	1.571	13/16	3.867
2¼	1.396	16	.196	1⅞	1.676	¾	4.189
2½	1.257	18	.175	1¾	1.795	11/16	4.570
2¾	1.142	20	.157	1⅝	1.933	⅝	5.027
3	1.047	22	.143	1½	2.094	9/16	5.585
3½	.898	24	.131	1 7/16	2.185	½	6.283
4	.785	26	.121	1⅜	2.285	7/16	7.181
5	.628	28	.112	1 5/16	2.394	⅜	8.378
6	.524	30	.105	1¼	2.513	5/16	10.053
7	.449	32	.098	1 3/16	2.646	¼	12.566
8	.393	36	.087	1⅛	2.793	3/16	16.755
9	.349	40	.079	1 1/16	2.957	⅛	25.133
10	.314	48	.065	1	3.142	1/16	50.266

Since circular pitch $= \frac{\text{diam.} \times 3.1416}{\text{No. of teeth}}$, diam. $= \frac{\text{circ. pitch} \times \text{No. of teeth}}{3.1416}$, which always brings out the diameter as a number with an inconvenient fraction if the pitch is in even inches or simple fractions of an inch. By the diametral-pitch system this inconvenience is avoided. The diameter may be in even inches or convenient fractions, and the number of teeth is usually an even multiple of the number of inches in the diameter.

Diameter of Pitch-line of Wheels from 10 to 100 Teeth of 1 in. Circular Pitch.

No. Teeth.	Diam., in.	No. Teeth.	Diam., in.	No Teeth.	Diam., in.	No. Teeth.	Diam., in.	No. Teeth.	Diam., in.	No. Teeth.	Diam., in.
10	3.183	26	8.276	41	13.051	56	17.825	71	22.600	86	27.375
11	3.501	27	8.594	42	13.369	57	18.144	72	22.918	87	27.693
12	3.820	28	8.913	43	13.687	58	18.462	73	23.236	88	28.011
13	4.138	29	9.231	44	14.006	59	18 781	74	23.555	89	28.329
14	4.456	30	9.549	45	14.324	60	19.099	75	23.873	90	28.648
15	4.775	31	9.868	46	14.642	61	19.417	76	24.192	91	28 966
16	5.093	32	10.186	47	14.961	62	19.735	77	24.510	92	29.285
17	5.411	33	10.504	48	15.279	63	20 054	78	24.828	93	29.603
18	5.730	34	10.823	49	15.597	64	20.372	79	25.146	94	29.921
19	6.048	35	11.141	50	15.915	65	20.690	80	25.465	95	30.239
20	6.366	36	11.459	51	16.234	66	21 008	81	25.783	96	30.558
21	6.685	37	11.777	52	16.552	67	21.327	82	26.101	97	30.876
22	7.003	38	12.096	53	16.870	68	21.645	83	26.419	98	31.194
23	7.321	39	12 414	54	17.189	69	21.963	84	26.738	99	31.512
24	7.639	40	12.732	55	17.507	70	22.282	85	27.056	100	31.831
25	7.958										

For diameter of wheels of any other pitch than 1 in., multiply the figures in the table by the pitch. Given the diameter and the pitch, to find the number of teeth. Divide the diameter by the pitch, look in the table under diameter for the figure nearest to the quotient, and the number of teeth will be found opposite.

Proportions of Teeth. Circular Pitch = 1.

	1.	2.	3.	4.	5.	6.
Depth of tooth above pitch-line...	.35	.30	.37	.33	.30	.30
" " " below pitch-line...	.40	.40	.43	...	.40	.35
Working depth of tooth..........	.70	.60	.73	.66	...	...
Total depth of tooth	.75	.70	.80	.75	.70	.65
Clearance at root	.05	.10	.07	...	...	...
Thickness of tooth..............	.45	.45	.47	.45	.475	.485
Width of space..	.54	.55	.53	.55	.525	.515
Backlash...	.09	.10	.06	.10	.05	.03
Thickness of rim...	....	....	.47	.45	.70	.65

	7.	8.	9.	10.*
Depth of tooth above pitch-line...	.25 to .33	.30	.318	1÷P
" " " below pitch-line..	.35 to .42	.35+.08″	.369	1.157÷P
Working depth of tooth..........	....	....	.637	2÷P
Total depth of tooth......	.6 to .75	.65+.08″	.687	2.157÷P
Clearance at root..................	....	....	.04 to .05	.157÷P
Thickness of tooth...	.48 to .485	.48−.03″	.48 to .5	1.51 ÷P to 1.57 ÷P
Width of space	.52 to .515	.52+.03″	.52 to .5	1.57 ÷P to 1.63 ÷P
Backlash	.04 to .03	.04+.06″	.0 to .04	0 to 0 6÷P

* In terms of diametral pitch.

AUTHORITIES.—1. Sir Wm. Fairbairn. 2, 3. Clark, R. T. D.; "used by engineers in good practice." 4. Molesworth. 5, 6. Coleman Sellers: 5 for cast, 6 for cut wheels. 7, 8. Unwin. 9, 10. Leading American manufacturers of cut gears.

The Chordal Pitch (erroneously called "true pitch" by some authors) is the length of a straight line or chord drawn from centre to centre of two adjacent teeth. The term is now but little used.

Chordal pitch = diam. of pitch-circle $\times$ sine of $\frac{180°}{\text{No. of teeth}}$. Chordal pitch of a wheel of 10 in. pitch diameter and 10 teeth, $10 \times \sin 18° = 3.0902$ in. Circular pitch of same wheel = 3.1416. Chordal pitch is used with chain or sprocket wheels, to conform to the pitch of the chain.

Formulæ for Determining the Dimensions of Small Gears.

(Brown & Sharpe Mfg. Co.)

P = diametral pitch, or the number of teeth to one inch of diameter of pitch-circle;

D' = diameter of pitch circle D = whole diameter N = number of teeth V = velocity	Larger Wheel.	These wheels run together.
d' = diameter of pitch-circle d = whole diameter n = number of teeth v = velocity	Smaller Wheel.	

a = distance between the centres of the two wheels;
b = number of teeth in both wheels;
t = thickness of tooth or cutter on pitch-circle;
s = addendum;
D'' = working depth of tooth;
f = amount added to depth of tooth for rounding the corners and for clearance;
$D''+f$ = whole depth of tooth;
π = 3.1416.
P' = circular pitch, or the distance from the centre of one tooth to the centre of the next measured on the pitch-circle.

Formulæ for a single wheel:

$$P = \frac{N+2}{D}; \quad D' = \frac{D \times N}{N+2}; \quad D'' = \frac{2}{P} = 2s; \quad s = \frac{1}{P} = \frac{P'}{\pi} = .3183P';$$

$$P = \frac{N}{D'}; \quad D' = \frac{N}{P}; \quad N = PD'; \quad N = PD - 2; \quad s = \frac{D'}{N} = \frac{D}{N+2};$$

$$P' = \frac{\pi}{P}; \quad D = \frac{N+2}{P}; \quad f = \frac{t}{10}; \quad s + f = \frac{1}{P}\left(1 + \frac{\pi}{20}\right) = .3685P'$$

$$P = \frac{\pi}{P'}; \quad D = D' + \frac{2}{P}; \quad t = \frac{1.57}{P} = \tfrac{1}{2}P'.$$

Formulæ for a pair of wheels:

$$b = 2aP; \quad n = \frac{PD'V}{v} \quad D = \frac{2a(N+2)}{b};$$

$$N = \frac{nv}{V}; \quad v = \frac{PD'V}{n}; \quad d = \frac{2a(n+2)}{b};$$

$$n = \frac{NV}{v}; \quad v = \frac{NV}{n}; \quad a = \frac{b}{2P};$$

$$N = \frac{bv}{v+V}; \quad V = \frac{nv}{N}; \quad a = \frac{D'+d'}{2};$$

$$n = \frac{bV}{v+V}; \quad D' = \frac{2av}{v+V}; \quad d' = \frac{2aV}{v+V}.$$

The following proportions of gear-wheels are recommended by Prof. Coleman Sellers. (*Stevens Indicator*, April, 1892.)

Proportions of Gear-wheels.

Diametral Pitch.	Circular Pitch.	Outside of Pitch-line. $P \times .3$	Inside of Pitch-line. For Cast or Cut Bevels or for Cast Spurs. $P \times .4$	Inside of Pitch-line. For Cut Spurs. $P \times .35$	Width of Space. For Cast Spurs or Bevels. $P \times .525$	Width of Space. For Cut Bevels or Spurs. $P \times .51$
	¼	.075	.100	.088	.131	.128
12	.2618	.079	.105	.092	.137	.134
10	.31416	.094	.126	.11	.165	.16
	⅜	.113	.150	.131	.197	.191
8	.3927	.118	.157	.137	.206	.2
7	.4477	.134	.179	.157	.235	.228
	½	.15	.20	.175	.263	.255
6	.5236	.157	.209	.183	.275	.267
	9/16	.169	.225	.197	.295	.287
	⅝	.188	.25	.219	.328	.319
5	.62832	.188	.251	.22	.33	.32
	¾	.225	.3	.263	.394	.383
4	.7854	.236	.314	.275	.412	.401
	⅞	.263	.35	.307	.459	.446
	1	.3	.4	.35	.525	.51
3	1.0472	.314	.419	.364	.55	.534
	1⅛	.338	.45	.394	.591	.574
2¾	1.1424	.343	.457	.40	.6	.583
	1¼	.375	.5	.438	.656	.638
2½	1.25664	.377	.503	.44	.66	.641
	1⅜	.413	.55	.481	.722	.701
	1½	.45	.6	.525	.788	.765
2	1.5708	.471	.628	.55	.825	.801
	1¾	.525	.7	.613	.919	.893
	2	.6	.8	.7	1.05	1.02
1½	2.0944	.628	.838	.733	1.1	1.068
	2¼	.675	.9	.788	1.181	1.148
	2½	.75	1.0	.875	1.313	1.275
	2¾	.825	1.1	.963	1.444	1.403
	3	.9	1.2	1.05	1.575	1.53
1	3.1416	.942	1.257	1.1	1.649	1.602
	3¼	.975	1.3	1.138	1.706	1.657
	3½	1.05	1.4	1.225	1.838	1.785

Thickness of rim below root = depth of tooth.

Width of Teeth.—The width of the faces of teeth is generally made from 2 to 3 times the circular pitch — from 6.28 to 9.42 divided by the diametral pitch. There is no standard rule for width.

The following sizes are given in a stock list of cut gears in "Grant's Gears:"

Diameter pitch.....	3	4	6	8	12	16
Face, inches........	3 and 4	2½	1¾ and 2	1¼ and 1½	¾ and 1	½ and ⅝

The Walker Company give:

Circular pitch, in..	½	⅝	¾	⅞	1	1½	2	2½	3	4	5	6
Face, in............	1¼	1½	1¾	2	2½	4½	6	7½	9	12	16	20

Rules for Calculating the Speed of Gears and Pulleys.—The relations of the size and speed of driving and driven gear wheels are the same as those of belt pulleys. In calculating for gears, multiply or divide by the diameter of the pitch-circle or by the number of teeth, as may be required. In calculating for pulleys, multiply or divide by their diameter in inches.

If D = diam. of driving wheel, d = diam. of driven, R = revolutions per minute of driver, r = revs. per min. of driven.

$$R = rd \div D;\quad r = RD \div d;\quad D = dr \div R;\quad d = DR \div r.$$

If N = number of teeth of driver and n = number of teeth of driven,

$$N = nr \div R;\quad n = NR \div r;\quad R = rn \div N;\quad r = RN \div n.$$

To find the number of revolutions of the last wheel at the end of a train of spur-wheels, all of which are in a line and mesh into one another, when the revolutions of the first wheel and the number of teeth or the diameter of the first and last are given: Multiply the revolutions of the first wheel by its number of teeth or its diameter, and divide the product by the number of teeth or the diameter of the last wheel.

To find the number of teeth in each wheel for a train of spur-wheels, each to have a given velocity: Multiply the number of revolutions of the driving-wheel by its number of teeth, and divide the product by the number of revolutions each wheel is to make.

To find the number of revolutions of the last wheel in a train of wheels and pinions, when the revolutions of the first or driver, and the diameter, the teeth, or the circumference of all the drivers and pinions are given: Multiply the diameter, the circumference, or the number of teeth of all the driving-wheels together, and this continued product by the number of revolutions of the first wheel, and divide this product by the continued product of the diameter, the circumference, or the number of teeth of all the driven wheels, and the quotient will be the number of revolutions of the last wheel.

EXAMPLE.—1. A train of wheels consists of four wheels each 12 in. diameter of pitch-circle, and three pinions 4, 4, and 3 in. diameter. The large wheels are the drivers, and the first makes 36 revs. per min. Required the speed of the last wheel.

$$\frac{36 \times 12 \times 12 \times 12}{4 \times 4 \times 3} = 1296 \text{ rpm.}$$

2. What is the speed of the first large wheel if the pinions are the drivers, the 3-in. pinion being the first driver and making 36 revs. per min.?

$$\frac{36 \times 3 \times 4 \times 4}{12 \times 12 \times 12} = 1 \text{ rpm. } Ans.$$

Milling Cutters for Interchangeable Gears.—The Pratt & Whitney Co. make a series of cutters for cutting epicycloidal teeth. The number of cutters to cut from a pinion of 12 teeth to a rack is 24 for each pitch coarser than 10. The Brown & Sharpe Mfg. Co. make a similar series, and also a series for involute teeth, in which eight cutters are made for each pitch, as follows:

No.	1.	2.	3.	4.	5.	6.	7.	8.
Will cut from	135	55	35	26	21	17	14	12
to	Rack	134	54	34	25	20	16	13

FORMS OF THE TEETH.

In order that the teeth of wheels and pinions may run together smoothly and with a constant relative velocity, it is necessary that their working faces shall be formed of certain curves called odontoids. The essential property of these curves is that when two teeth are in contact the common normal to the tooth curves at their point of contact must pass through the pitch-point, or point of contact of the two pitch circles. Two such curves are in common use—the cycloid and the involute.

The Cycloidal Tooth.—In Fig. 154 let *PL* and *pl* be the pitch-circles of two gear-wheels; *GC* and *gc* are two equal generating-circles, whose radii should be taken as not greater than one half of the radius of the smaller pitch-circle. If the circle *gc* be rolled to the left on the larger pitch-circle *PL*, the point *O* will describe an epicycloid, *oefgh*. If the other generating-circle *GC* be rolled to the right on *PL*, the point *O* will describe a hypocycloid *oabcd*. These two curves, which are tangent at *O*, form the two parts of a tooth curve for a gear whose pitch-circle is *PL*. The upper part *oh* is called the face and the lower part *od* is called the flank. If the same circles be rolled on the other pitch-circle *pl*, they will describe the curve for a tooth of the gear *pl*, which will work properly with the tooth on *PL*.

The cycloidal curves may be drawn without actually rolling the generating-circle, as follows: On the line *PL*, from *O*, step off and mark equal distances, as 1, 2, 3, 4, etc. From 1, 2, 3, etc., draw radial lines toward the centre of *PL*, and from 6, 7, 8, etc., draw radial lines from the same centre, but beyond *PL*. With the radius of the generating-circle, and with centres successively placed on these radial lines, draw arcs of circles tangent to *PL* at 1 2 3, 6 7 8, etc. With the dividers set to one of the equal divisions, as O_1, step off $1a$ and $6e$; step off two such divisions on the circle from 2 to b, and from 7 to f; three such divisions from 3 to c, and from 8 to g; and so on, thus locating the several points $abcdH$ and $efgk$, and through these points draw the tooth curves.

The curves for the mating tooth on the other wheel may be found in like manner by drawing arcs of the generating-circle tangent at equidistant points on the pitch-circle pl.

The tooth curve of the face oh is limited by the addendum-line r or r_1,

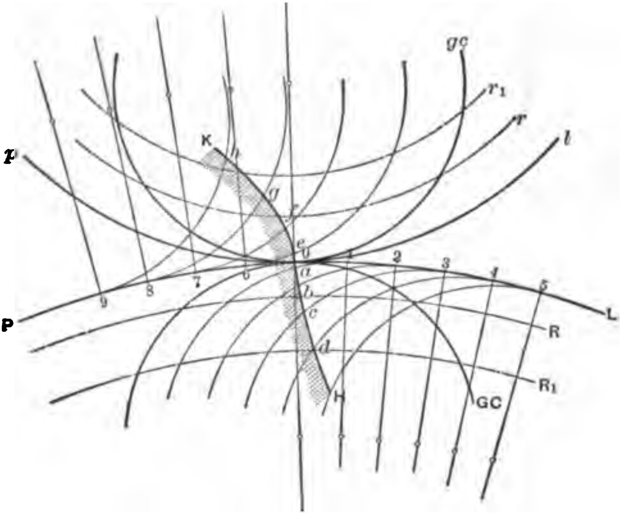

FIG. 154.

and that of the flank oH by the root curve R or R_1. R and r represent the root and addendum curves for a large number of small teeth, and R_1r the like curves for a small number of large teeth. The form or appearance of the tooth therefore varies according to the number of teeth, while the pitch-circle and the generating-circle may remain the same.

In the cycloidal system, in order that a set of wheels of different diameters but equal pitches shall all correctly work together, it is necessary that the generating-circle used for the teeth of all the wheels shall be the same, and it should have a diameter not greater than half the diameter of the pitch-line of the smallest wheel of the set. The customary standard size of the generating-circle of the cycloidal system is one having a diameter equal to the radius of the pitch-circle of a wheel having 12 teeth. (Some gear-makers adopt 15 teeth.) This circle gives a radial flank to the teeth of a wheel having 12 teeth. A pinion of 10 or even a smaller number of teeth can be made, but in that case the flanks will be undercut, and the tooth will not be as strong as a tooth with radial flanks. If in any case the describing circle be half the size of the pitch-circle, the flanks will be radial; if it be less, they will spread out toward the root of the tooth, giving a stronger form; but if greater, the flanks will curve in toward each other, whereby the teeth become weaker and difficult to make.

In some cases cycloidal teeth for a pair of gears are made with the generating-circle of each gear, having a radius equal to half the radius of its pitch-circle. In this case each of the gears will have radial flanks. This method makes a smooth working gear, but a disadvantage is that the wheels are not interchangeable with other wheels of the same pitch but different numbers of teeth.

The rack in the cycloidal system is equivalent to a wheel with an infinite number of teeth. The pitch is equal to the circular pitch of the mating gear. Both faces and flanks are cycloids formed by rolling the generating-circle of the mating gear-wheel on each side of the straight pitch-line of the rack.

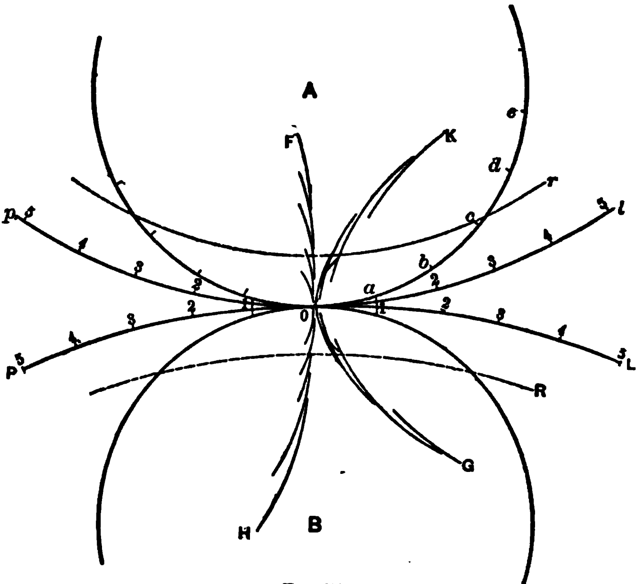

FIG. 155.

Another method of drawing the cycloidal curves is shown in Fig. 155. It is known as the method of tangent arcs. The generating-circles, as before, are drawn with equal radii, the length of the radius being less than half the radius of *pl*, the smaller pitch-circle. Equal divisions 1, 2, 3, 4, etc., are marked off on the pitch circles and divisions of the same length stepped off on one of the generating-circles, as *oabc*, etc. From the points 1, 2, 3, 4, 5 on the line *po*, with radii successively equal to the chord distances *oa*, *ob*, *oc*, *od*, *oe*, draw the five small arcs *F*. A line drawn through the outer edges of these small arcs, tangent to them all, will be the hypocycloidal curve for the flank of a tooth below the pitch-line *pl*. From the points 1, 2, 3, etc., on the line *ol*, with radii as before, draw the small arcs *G*. A line tangent to these arcs will be the epicycloid for the face of the same tooth for which the flank curve has already been drawn. In the same way, from centres on the line *Po* and *oL*, with the same radii, the tangent arcs *H* and *K* may be drawn, which will give the tooth for the gear whose pitch-circle is *PL*.

If the generating-circle had a radius just one half of the radius of *pl*, the hypocycloid *F* would be a straight line, and the flank of the tooth would have been radial.

The Involute Tooth.—In drawing the involute tooth curve, the angle of obliquity, or the angle which a common tangent to the teeth, when they are in contact at the pitch-point, makes with a line joining the centres of the wheels, is first arbitrarily determined. It is customary to take it at 15°. The pitch-lines *pl* and *PL* being drawn in contact at *O*, the line of obliquity *AB* is drawn through *O* normal to a common tangent to the tooth curves, or at the given angle of obliquity to a common tangent to the pitch-circles. In the cut the angle is 20°. From the centres of the pitch-circles draw circles *c* and *d* tangent to the line *AB*. These circles are called base-lines or base-circles, from which the involutes *F* and *K* are drawn. By laying off convenient distances, 0, 1, 2, 3, which should each be less than 1/10 of the diameter of the base-circle, small arcs can be drawn with successively increasing radii, which will form the involute. The involute extends from the points *F*

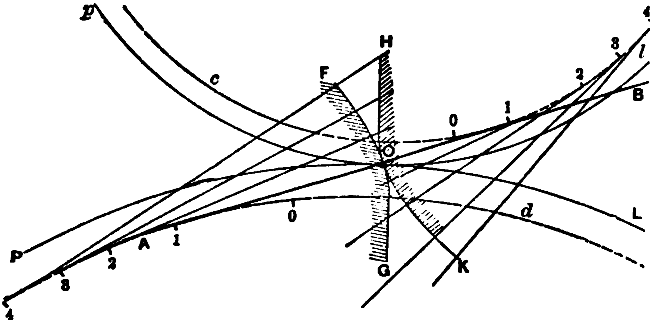

Fig. 156.

and *K* down to their respective base-circles, where a tangent to the involute becomes a radius of the circle, and the remainders of the tooth curves, as *G* and *H*, are radial straight lines.

In the involute system the customary standard form of tooth is one having an angle of obliquity of 15° (Brown and Sharpe use 14½°), an addendum of about one third the circular pitch, and a clearance of about one eighth of the addendum. In this system the smallest gear of a set has 12 teeth, this being the smallest number of teeth that will gear together when made with this angle of obliquity. In gears with less than 30 teeth the points of the teeth must be slightly rounded over to avoid interference (see Grant's Teeth of Gears). All involute teeth of the same pitch and with the same angle of obliquity work smoothly together. The rack to gear with an involute-toothed wheel has straight faces on its teeth, which make an angle with the middle line of the tooth equal to the angle of obliquity, or in the standard form the faces are inclined at an angle of 30° with each other.

To draw the teeth of a rack which is to gear with an involute wheel (Fig. 157).—Let *AB* be the pitch-line of the rack and *AI*=*II'*=the pitch. Through

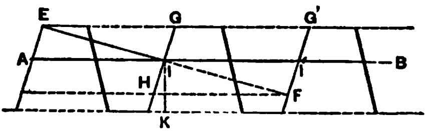

Fig. 157.

the pitch-point *I* draw *EF* at the given angle of obliquity. Draw *AE* and *I'F* perpendicular to *EF*. Through *E* and *F* draw lines *EGG'* and *FH* parallel to the pitch-line. *EGG'* will be the addendum-line and *HF* the flank-line. From *I* draw *IK* perpendicular to *AB* equal to the greatest addendum in the set of wheels of the given pitch and obliquity plus an allowance for clearance equal to ⅛ of the addendum. Through *K*, parallel to *AB*, draw the clearance-line. The fronts of the teeth are planes perpendicular to *EF*, and the backs are planes inclined at the same angle to *AB* in the contrary direction. The outer half of the working face *AE* may be slightly curved. Mr. Grant makes it a circular arc drawn from a centre on the pitch-line with a radius = 2.. inches divided by the diametral pitch, or .67 in. × circular pitch.

To Draw an Angle of 15° without using a Protractor.—From *C*, on the line *AC*, with radius *AC*, draw an arc *AB*, and from *A*, with the same radius, cut the arc at *B*. Bisect the arc *BA* by drawing small arcs at *D* from *A* and *B* as centres, with the same radius, which must be greater than one half of *AB*. Join *DC*, cutting *BA* at *E*. The angle *ECA* is 30°. Bisect the arc *AE* in like manner, and the angle *FCA* will be 15°.

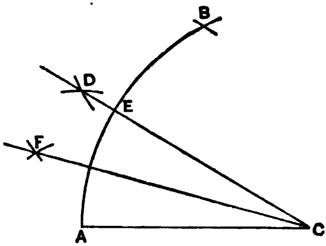

FIG. 158.

A property of involute-toothed wheels is that the distance between the axes of a pair of gears may be altered to a considerable extent without interfering with their action. The backlash is therefore variable at will, and may be adjusted by moving the wheels farther from or nearer to each other, and may thus be adjusted so as to be no greater than is necessary to prevent jamming of the teeth.

The relative merits of cycloidal and involute-shaped teeth are still a subject of dispute, but there is an increasing tendency to adopt the involute tooth for all purposes.

Clark (R. T. D., p. 734) says: Involute teeth have the disadvantage of being too much inclined to the radial line, by which an undue pressure is exerted on the bearings.

Unwin (Elements of Machine Design, 8th ed., p. 265) says: The obliquity of action is ordinarily alleged as a serious objection to involute wheels. Its importance has perhaps been overrated.

George B. Grant (*Am. Mach.*, Dec. 26, 1885) says:

1. The work done by the friction of an involute tooth is always less than the same work for any possible epicycloidal tooth.

2. With respect to work done by friction, a change of the base from a gear of 12 teeth to one of 15 teeth makes an improvement for the epicycloid of less than one half of one per cent.

3. For the 12-tooth system the involute has an advantage of 1 1/5 per cent, and for the 15-tooth system an advantage of 3/4 per cent.

4. That a maximum improvement of about one per cent can be accomplished by the adoption of any possible non-interchangeable radial flank tooth in preference to the 12-tooth interchangeable system.

5. That for gears of very few teeth the involute has a decided advantage.

6. That the common opinion among millwrights and the mechanical public in general in favor of the epicycloid is a prejudice that is founded on long-continued custom, and not on an intimate knowledge of the properties of that curve.

Wilfred Lewis (Proc. Engrs. Club of Phila., vol. x., 1893) says a strong reaction in favor of the involute system is in progress, and he believes that an involute tooth of 22½° obliquity will finally supplant all other forms.

Approximation by Circular Arcs.—Having found the form of the actual tooth-curve on the drawing-board, circular arcs may be found by trial which will give approximations to the true curves, and these may be

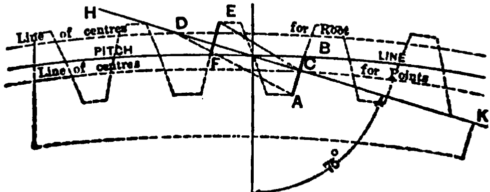

FIG. 159.

ised in completing the drawing and the pattern of the gear-wheels. The
oot of the curve is connected to the clearance by a fillet, which should be
is large aspossible to give increased strength to the tooth, provided it is not
arge enough to cause interference.

Molesworth gives the following method of construction by circular arcs:
From the radial line at the edge of the tooth on the pitch-line, lay off the
ine *HK* at an angle of 75° with the radial line; on this line will be the cen-
res of the root *AB* and the point *EF*. The lines struck from these centres
are shown in thick lines. Circles drawn through centres thus found will
give the lines in which the remaining centres will be. The radius *DA* for
striking the root *AB* is = pitch + the thickness of the tooth. The radius
CE for striking the point of the tooth *EF* = the pitch.

George B. Grant says: It is sometimes attempted to construct the curve
by some handy method or empirical rule, but such methods are generally
worthless.

Stepped Gears.—Two gears of the same pitch and diameter mounted
side by side on the same shaft will act as a single gear. If one gear is keyed
on the shaft so that the teeth of the two wheels are not in line, but the
teeth of one wheel slightly in advance of the other, the two gears form a
stepped gear. If mated with a similar stepped gear on a parallel shaft the
number of teeth in contact will be twice as great as in an ordinary gear,
which will increase the strength of the gear and its smoothness of action.

Twisted Teeth.—If a great number of very thin gears were placed
together, one slightly in advance of the other, they would still act as a
stepped gear. Continuing the subdivision until the
thickness of each separate gear is infinitesimal, the
faces of the teeth instead of being in steps take the
form of a spiral or twisted surface, and we have a
twisted gear. The twist may take any shape, and if it is
in one direction for half the width of the gear and in the
opposite direction for the other half, we have what is
known as the herring-bone or double helical tooth. The
obliquity of the twisted tooth if twisted in one direction
causes an end thrust on the shaft, but if the herring-
bone twist is used, the opposite obliquities neutralize
each other. This form of tooth is much used in heavy
rolling-mill practice, where great strength and resistance
to shocks are necessary. They are frequently made of
steel castings (Fig. 160). The angle of the tooth with a
line parallel to the axis of the gear is usually 30°.

Fig. 160.

Spiral Gears.—If a twisted gear has a uniform twist it becomes a
spiral gear. The line in which the pitch-surface intersects the face of the
tooth is part of a helix drawn on the pitch-surface. A spiral wheel may be
made with only one helical tooth wrapped around the cylinder several
times, in which it becomes a screw or worm. If it has two or three teeth
so wrapped, it is a double- or triple-threaded screw or worm. A spiral-gear
meshing into a rack is used to drive the table of some forms of planing-
machine.

Worm-gearing.—When the axes of two spiral gears are at right
angles, and a wheel of one, two, or three threads works with a larger wheel
of many threads, it becomes a worm-gear, or endless screw, the smaller

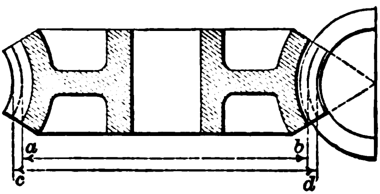

Fig. 161.

wheel or driver being called the worm, and the larger, or driven wheel, the
worm-wheel. With this arrangement a high velocity ratio may be obtained
with a single pair of wheels. For a one-threaded wheel the velocity ratio is

the number of teeth in the worm-wheel. The worm and wheel are commonly so constructed that the worm will drive the wheel, but the wheel will not drive the worm.

To find the diameter of a worm-wheel at the throat, number of teeth and pitch of the worm being given: Add 2 to the number of teeth, multiply the sum by 0.3183, and by the pitch of the worm in inches.

To find the number of teeth, diameter at throat and pitch of worm being given: Divide 3.1416 times the diameter by the pitch, and subtract 2 from the quotient.

In Fig. 161 *ab* is the diam. of the pitch-circle, *cd* is the diam. at the throat.

EXAMPLE.—Pitch of worm ¼ in., number of teeth 70, required the diam. at the throat. $(70 + 2) \times .3183 \times .25 = 5.73$ in.

Teeth of Bevel-wheels. (Rankine's Machinery and Millwork.)—The teeth of a bevel-wheel have acting surfaces of the conical kind, generated by the motion of a line traversing the apex of the conical pitch surface, while a point in it is carried round the traces of the teeth upon a spherical surface described about that apex.

The operations of drawing the traces of the teeth of bevel-wheels exactly, whether by involutes or by rolling curves, are in every respect analogous to those for drawing the traces of the teeth of spur-wheels; except that in the case of bevel-wheels all those operations are to be performed on the surface of a sphere described about the apex, instead of on a plane, substituting poles for centres and great circles for straight lines.

In consideration of the practical difficulty, especially in the case of large wheels, of obtaining an accurate spherical surface, and of drawing upon it when obtained, the following approximate method, proposed originally by Tredgold, is generally used:

Let *O*, Fig. 162, be the common apex of the pitch-cones, *OBI*, *OB'I*, of a pair of bevel-wheels; *OC*, *OC'*, the axes of those cones; *OI* their line of contact. Perpendicular to *OI* draw *AIA'*, cutting the axes in *A*, *A'*; make the outer rims of the patterns and of the wheels portions of the cones *ABI*, *A'B'I*, of which the narrow zones occupied by the teeth will be sufficiently near for practical purposes to a spherical surface described about *O*. As the cones *ABI*, *A'B'I* cut the pitch-cones at right angles in the outer pitch-circles *IB*, *IB'*, they may be called the normal cones. To find the traces of the teeth upon the normal cones, draw on a flat surface circular arcs, *ID*, *ID'*, with the radii *AI*, *A'I*; those arcs will be the developments of arcs of the pitch-circles *IB*, *IB'* when the conical surfaces *ABI*, *A'B'I* are spread out flat. Describe the traces of teeth for the developed arcs as for a pair of spur-wheels, then wrap the developed arcs on the normal cones, so as to make them coincide with the pitch-circles, and trace the teeth on the conical surfaces.

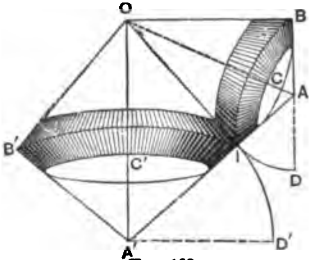

FIG. 162.

For formulæ and instructions for designing bevel-gears, and for much other valuable information on the subject of gearing, see "Practical Treatise on Gearing," and "Formulas in Gearing," published by Brown & Sharpe Mf'g Co.; and "Teeth of Gears," by George B. Grant, Lexington, Mass. The student may also consult Rankine's Machinery and Millwork, Reuleaux's Constructor, and Unwin's Elements of Machine Design. See also article on Gearing, by C. W. MacCord in App. Cyc. Mech., vol. ii.

Annular and Differential Gearing. (S. W. Balch., *Am. Mach.*, Aug. 24, 1893.)—In internal gears the sum of the diameters of the describing circles for faces and flanks should not exceed the difference in the pitch diameters of the pinion and its internal gear. The sum may be equal to this difference or it may be less; if it is equal, the faces of the teeth of each wheel will drive the faces as well as the flanks of the teeth of the other wheel. The teeth will therefore make contact with each other at two points at the same time.

Cycloidal tooth-curves for interchangeable gears are formed with describing circles of about ⅝ the pitch diameter of the smallest gear of the series. To admit two such circles between the pitch-circles of the pinion and internal gear the number of teeth in the internal gear should exceed the number in the pinion by 12 or more, if the teeth are of the customary proportions and curvature used in interchangeable gearing.

Very often a less difference is desirable, and the teeth may be modified in several ways to make this possible.

First. The tooth curves resulting from smaller describing circles may be employed. These will give teeth which are more rounding and narrower at their tops, and therefore not as desirable as the regular forms.

Second. The tips of the teeth may be rounded until they clear. This is a cut-and-try method which aims at modifying the teeth to such outlines as smaller describing circles would give.

Third. One of the describing circles may be omitted and one only used, which may be equal to the difference between the pitch-circles. This will permit the meshing of gears differing by six teeth. It will usually prove inexpedient to put wheels in inside gears that differ by much less than 12 teeth.

If a regular diametral pitch and standard tooth forms are determined on, the diameter to which the internal gear-blank is to be bored is calculated by subtracting 2 from the number of teeth, and dividing the remainder by the diametral pitch.

The tooth outlines are the match of a spur-gear of the same number of teeth and diametral pitch, so that the spur-gear will fit the internal gear as a punch fits its die, except that the teeth of each should fail to bottom in the tooth spaces of the other by the customary clearance of one tenth the thickness of the tooth.

Internal gearing is particularly valuable when employed in differential action. This is a mechanical movement in which one of the wheels is mounted on a crank so that its centre can move in a circle about the centre of the other wheel. Means are added to the device which restrain the wheel on the crank from turning over and confine it to the revolution of the crank.

The ratio of the number of teeth in the revolving wheel compared with the difference between the two will represent the ratio between the revolving wheel and the crank-shaft by which the other is carried. The advantage in accomplishing the change of speed with such an arrangement, as compared with ordinary spur-gearing, lies in the almost entire absence of friction and consequent wear of the teeth.

But for the limitation that the difference between the wheels must not be too small, the possible ratio of speed might be increased almost indefinitely, and one pair of differential gears made to do the service of a whole train of wheels. If the problem is properly worked out with bevel-gears this limitation may be completely set aside, and external and internal bevel-gears, differing by but a single tooth if need be, made to mesh perfectly with each other.

Differential bevel-gears have been used with advantage in mowing-machines. A description of their construction and operation is given by Mr. Balch in the article from which the above extracts are taken.

EFFICIENCY OF GEARING.

An extensive series of experiments on the efficiency of gearing, chiefly worm and spiral gearing, is described by Wilfred Lewis in Trans. A. S. M. E., vii. 273. The average results are shown in a diagram, from which the following approximate average figures are taken :

EFFICIENCY OF SPUR, SPIRAL, AND WORM GEARING.

Gearing.	Pitch.	Velocity at Pitch line in feet per min.				
		3	10	40	100	200
Spur pinion		.90	.935	.97	.98	.985
Spiral pinion	45°	.81	.87	.93	.955	.965
" "	30	.75	.815	.89	.93	.945
" "	20	.67	.75	.845	.90	.92
" "	15	.61	.70	.805	.87	.90
Spiral pinion or worm	10	.51	.615	.74	.82	.86
" " "	7	.43	.53	.72	.765	.815
" " "	5	.34	.43	.60	.70	.765

The experiments showed the advantage of spur-gearing over all other kinds in both durability and efficiency. The variation from the mean results rarely exceeded 5% in either direction, so long as no cutting occurred, but the variation became much greater and very irregular as soon as cutting began. The loss of power varies with the speed, the pressure, the temperature, and the condition of the surfaces. The excessive friction of worm and spiral gearing is largely due to thee nd thrust on the collars of the shaft. This may be considerably reduced by roller-bearings for the collars.

When two worms with opposite spirals run in two spiral worm-gears that also work with each other, and the pressure on one gear is opposite that on the other, there is no thrust on the shaft. Even with light loads a worm will begin to heat and cut if run at too high a speed, the limit for safe working being a velocity of the rubbing surfaces of 200 to 300 ft. per minute, the former being preferable where the gearing has to work continuously. The wheel teeth will keep cool, as they form part of a casting having a large radiating surface; but the worm itself is so small that its heat is dissipated slowly. Whenever the heat generated increases faster than it can be conducted and radiated away, the cutting of the worm may be expected to begin. A low efficiency for a worm-gear means more than the loss of power, since the power which is lost reappears as heat and may cause the rapid destruction of the worm.

Unwin (Elements of Machine Design, p. 294) says: The efficiency is greater the less the radius of the worm. Generally the radius of the worm = 1.5 to 3 times the pitch of the thread of the worm or the circular pitch of the worm-wheel. For a one-threaded worm the efficiency is only 2/5 to 1/4; for a two-threaded worm, 4/7 to 2/5; for a three-threaded worm, 5/8 to 1/2. Since so much work is wasted in friction it is not surprising that the wear is excessive. The following table gives the calculated efficiencies of worm-wheels of 1, 2, 3, and 4 threads and ratios of radius of worm to pitch of teeth of from 1 to 6, assuming a coefficient of friction of 0.15:

No. of Threads.	Radius of Worm ÷ Pitch.								
	1	1¼	1½	1¾	2	2½	3	4	6
1	.50	.44	.40	.36	.33	.28	.25	.20	.14
2	.67	.62	.57	.53	.50	.44	.40	.33	.25
3	.75	.70	.67	.63	.60	.55	.50	.43	.33
4	.80	.76	.73	.70	.67	.62	.57	.50	.40

STRENGTH OF GEAR-TEETH.

The strength of gear-teeth and the horse-power that may be transmitted by them depend upon so many variable and uncertain factors that it is not surprising that the formulas and rules given by different writers show a wide variation. In 1879 John H. Cooper (*Jour. Frank. Inst.*, July, 1879) found that there were then in existence about 48 well-established rules for horse-power and working strength, differing from each other in extreme cases about 500%. In 1886 Prof. Wm. Harkness (Proc. A. A. A. S. 1886), from an examination of the bibliography of the subject, beginning in 1796, found that according to the constants and formulæ used by various authors there were differences of 15 to 1 in the power which could be transmitted by a given pair of geared wheels. The various elements which enter into the constitution of a formula to represent the working strength of a toothed wheel are the following: 1. The strength of the metal, usually cast iron, which is an extremely variable quantity. 2. The shape of the tooth, and especially the relation of its thickness at the root or point of least strength to the pitch and to the length. 3. The point at which the load is taken to be applied, assumed by some authors to be at the pitch-line, by others at the extreme end, along the whole face, and by still others at a single outer corner. 4. The consideration of whether the total load is at any time received by a single tooth or whether it is divided between two teeth. 5. The influence of velocity in causing a tendency to break the teeth by shock. 6. The factor of safety assumed to cover all the uncertainties of the other elements of the problem.

Prof. Harkness, as a result of his investigation, found that all the formulæ on the subject might be expressed in one of three forms, viz.:

$$\text{Horse-power} = CVpf, \quad \text{or} \quad CVp^2, \quad \text{or} \quad CVp^2f;$$

in which C is a coefficient, V = velocity of pitch-line in feet per second, p = pitch in inches, and f = face of tooth in inches.

From an examination of precedents he proposed the following formula for cast-iron wheels:

$$\text{H.P.} = \frac{0.910Vpf}{\sqrt{1 + 0.65V}}.$$

He found that the teeth of chronometer and watch movements were subject to stresses four times as great as those which any engineer would dare to use in like proportion upon cast-iron wheels of large size.

It appears that all of the earlier rules for the strength of teeth neglected the consideration of the variations in their form; the breaking strength, as said by Mr. Cooper, being based upon the thickness of the teeth at the pitch-line or circle, as if the thickness at the root of the tooth were the same in all cases as it is at the pitch-line.

Wilfred Lewis (Proc. Eng'rs Club, Phila., Jan. 1893; *Am. Mach.*, June 22, 1893) seems to have been the first to use the form of the tooth in the construction of a working formula and table. He assumes that in well-constructed machinery the load can be more properly taken as well distributed across the tooth than as concentrated in one corner, but that it cannot be safely taken as concentrated at a maximum distance from the root less than the extreme end of the tooth. He assumes that the whole load is taken upon one tooth, and considers the tooth as a beam loaded at one end, and from a series of drawings of teeth of the involute, cycloidal, and radial flank systems, determines the point of weakest cross-section of each, and the ratio of the thickness at that section to the pitch. He thereby obtains the general formula,

$$W = spfy;$$

in which W is the load transmitted by the teeth, in pounds; s is the safe working stress of the material, taken at 8000 lbs. for cast iron, when the working speed is 100 ft. or less per minute; p = pitch; f = face, in inches; y = a factor depending on the form of the tooth, whose value for different cases is given in the following table:

No. of Teeth.	Factor for Strength, y.			No. of Teeth.	Factor for Strength, y.		
	Involute 20° Obliquity.	Involute 15° and Cycloidal	Radial Flanks.		Involute 20° Obliquity.	Involute 15° and Cycloidal	Radial Flanks.
12	.078	.067	.052	27	.111	.100	.064
13	.083	.070	.053	30	.114	102	.065
14	.088	.072	.054	34	.118	.104	.066
15	.092	.075	.055	38	.122	.107	.067
16	.094	.077	.056	43	126	.110	.068
17	.096	.080	.057	50	.130	.112	.069
18	.098	.083	.058	60	134	.114	.070
19	.100	.087	.059	75	.138	.116	.071
20	.102	.090	.060	100	.142	.118	.072
21	.104	.092	.061	150	146	.120	.073
23	.106	.094	.062	300	.150	122	.074
25	.108	.097	.063	Rack.	.154	.124	.075

SAFE WORKING STRESS, s, FOR DIFFERENT SPEEDS.

Speed of Teeth in ft. per minute.	100 or less.	200	300	600	900	1200	1800	2400
Cast iron................	8000	6000	4800	4000	3000	2400	2000	1700
Steel......................	20000	15000	12000	10000	7500	6000	5000	4300

The values of s in the above table are given by Mr. Lewis tentatively, in the absence of sufficient data upon which to base more definite values, but they have been found to give satisfactory results in practice.

Mr. Lewis gives the following example to illustrate the use of the tables: Let it be required to find the working strength of a 12-toothed pinion of 1-inch pitch, 2½-inch face, driving a wheel of 60 teeth at 100 feet or less per minute, and let the teeth be of the 20-degree involute form. In the formula $W = spfy$ we have for a cast-iron pinion $s = 8000$, $pf = 2.5$, and $y = .078$; and multiplying these values together, we have $W = 1560$ pounds. For the wheel we have $y = .134$ and $W = 2680$ pounds.

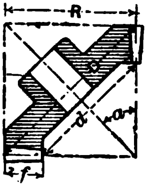

FIG. 168.

The cast-iron pinion is, therefore, the measure of strength; but if a steel pinion be substituted we have $s = 20{,}000$ and $W = 3900$ pounds, in which combination the wheel is the weaker, and it therefore becomes the measure of strength.

For bevel-wheels Mr. Lewis gives the following, referring to Fig. 168: D = large diameter of bevel; d = small diameter of bevel; p = pitch at large diameter; n = actual number of teeth; f = face of bevel; N = formative number of teeth = $n \times$ secant a, or the number corresponding to radius R; y = factor depending upon shape of teeth and formative number N; W = working load on teeth.

$$W = spfy\frac{D^3 - d^3}{3D^2(D - d)}; \text{ or, more simply, } W = spfy\frac{d}{D},$$

which gives almost identical results when d is not less than ⅔ D, as is the case in good practice.

In *Am. Mach.*, June 22, 1893, Mr. Lewis gives the following formulæ for the working strength of the three systems of gearing, which agree very closely with those obtained by use of the table:

For involute, 20° obliquity, $W = spf\left(.154 - \frac{.912}{n}\right)$;

For involute 15°, and cycloidal, $W = spf\left(.124 - \frac{.684}{n}\right)$;

For radial flank system, $W = spf\left(.075 - \frac{.276}{n}\right)$;

in which the factor within the parenthesis corresponds to y in the general formula. For the horse-power transmitted, Mr. Lewis's general formula $W = spfy, = \frac{33{,}000 \text{ H.P.}}{v}$, may take the form $\text{H.P.} = \frac{spfyv}{33{,}000}$, in which v = velocity in feet per minute; or since $v = d\pi \times \text{rpm.} \div 12 = .2618d \times \text{rpm.}$, in which d = diameter in inches and rpm. = revolutions per minute,

$$\text{H.P.} = \frac{Wv}{33{,}000} = \frac{spfy \times d \times \text{rpm.}}{126{,}050} = .000007933dspfy \times \text{rpm.}$$

It must be borne in mind, however, that in the case of machines which consume power intermittently, such as punching and shearing machines, the gearing should be designed with reference to the maximum load W, which can be brought upon the teeth at any time, and not upon the average horse-power transmitted.

Comparison of the Harkness and Lewis Formulas.—Take an average case in which the safe working strength of the material, $s = 6000$, $v = 200$ ft. per min., and $y = .100$, the value in Mr. Lewis's table for an involute tooth of 15° obliquity, or a cycloidal tooth, the number of teeth in the wheel being 27.

$$\text{H.P.} = \frac{spfyv}{33{,}000} = \frac{6000pfv \times .100}{33{,}000} = \frac{pfv}{55} = 1.091pfV,$$

if V is taken in feet per second.

Prof. Harkness gives $\text{H.P.} = \frac{0.910Vpf}{\sqrt{1 + 0.65V}}$. If the V in the denominator be taken at $200 \div 60 = 3\frac{1}{3}$ feet per second, $\sqrt{1 + 0.65V} = \sqrt{3.167} = 1.78$, and H.P. $= \frac{.910}{1.78} Vpf = .571pfV$, or about 52% of the result given by Mr. Lewis's formula. This is probably as close an agreement as can be expected, since Prof. Harkness derived his formula from an investigation of ancient precedents and rule-of-thumb practice, largely with common cast gears, while Mr. Lewis's formula was derived from considerations of modern practice with machine-moulded and cut gears.

Mr. Lewis takes into consideration the reduction in working strength of a tooth due to increase in velocity by the figures in his table of the values of the safe working stress s for different speeds. Prof. Harkness gives expression to the same reduction by means of the denominator of his formula, $\sqrt{1 + 0.65V}$. The decrease in strength as computed by this formula is somewhat less than that given in Mr. Lewis's table, and as the figures given in the table are not based on accurate data, a mean between the values given by the formula and the table is probably as near to the true value as may be obtained from our present knowledge. The following table gives the values for different speeds according to Mr. Lewis's table and Prof. Harkness's formula, taking for a basis a working stress s, for cast-iron 8000, and for steel 20,000 lbs. at speeds of 100 ft. per minute and less:

v = speed of teeth, ft. per min.. V = " " ft. per sec..	100 $1\frac{2}{3}$	200 $3\frac{1}{3}$	300 5	600 10	900 15	1200 20	1800 30	2400 40
Safe stress s, cast-iron, Lewis...	8000	6000	4800	4000	3000	2400	2000	1700
Relative do.. $s \div 8000$............	1	.75	.6	.5	.375	.3	.25	.2125
$c = 1 \div \sqrt{1 + 0.65V}$..........	.6930	.5621	.4850	.3650	.3050	.2672	.2208	.1924
Relative val. $c \div .693$...........	1	.811	.700	.526	.439	.385	.318	.277
$s_1 = 8000 \times (c \div .693)$...........	8000	6488	5600	4208	3512	3080	2544	2216
Mean of s and s_1, cast-iron $= s_2$.	8000	6200	5200	4100	3300	2700	2300	2000
" " " for steel $= s_3$.	20000	15500	13000	10300	8100	6800	5700	4900
Safe stress for steel, Lewis.. ...	20000	15000	12000	10000	7500	6000	5000	4800

Comparing the two formulæ for the case of $s = 8000$, corresponding to a speed of 100 ft. per min., we have

Harkness: $\text{H.P.} = 1 \div \sqrt{1 + 0.65V} \times .910Vpf = .693 \times .91 \times 1\frac{2}{3}pf = 1.051pf$

Lewis: $\text{H.P.} = \frac{spfyv}{33{,}000} = \frac{spfyV}{550} = \frac{8000 \times 1\frac{2}{3}pfy}{550} = 24.24pfy$,

in which y varies according to the shape and number of the teeth.

For radial-flank gear with 12 teeth $y = .052$; $24.24pfy = 1.260pf$;
For 20° involute, 19 teeth, or 15° inv., 27 teeth $y = .100$; $24.24pfy = 2.424pf$;
For 15° involute, 300 teeth $y = .150$; $24.24pfy = 3.636pf$.

Thus the weakest-shaped tooth, according to Mr. Lewis, will transmit 20 per cent more horse-power than is given by Prof. Harkness's formula, in which the shape of the tooth is not considered, and the average-shaped tooth, according to Mr. Lewis, will transmit more than double the horse-power given by Prof. Harkness's formula.

Comparison of Other Formulæ.—Mr. Cooper, in summing up his examination, selected an old English rule, which Mr. Lewis considers as a passably correct expression of good general averages, viz.: $X = 2000pf$, X = breaking load of tooth in pounds, p = pitch, f = face. If a factor of safety of 10 be taken, this would give for safe working load $W = 200pf$.

George B. Grant, in his Teeth of Gears, page 33, takes the breaking load at $3500pf$, and, with a factor of safety of 10, gives $W = 350pf$.

Nystrom's Pocket-Book, 20th ed., 1891, says: "The strength and durability of cast-iron teeth require that they shall transmit a force of 80 lbs. per inch of pitch and per inch breadth of face." This is equivalent to $W = 80pf$, or only 40% of that given by the English rule.

F. A. Halsey (Clark's Pocket Book) gives a table calculated from the formula H.P. $= pfd \times$ rpm. $\div$ 850.

Jones & Laughlins give H.P. $= pfd \times$ rpm. $\div$ 550.

These formulæ transformed give $W = 128pf$ and $W = 218pf$, respectively.

Unwin, on the assumption that the load acts on the corners of the teeth, derives a formula $p = K\sqrt{W}$, in which K is a coefficient derived from existing wheels, its values being: for slowly moving gearing not subject to much vibration or shock $K = .04$; in ordinary mill-gearing, running at greater speed and subject to considerable vibration, $K = .05$; and in wheels subjected to excessive vibration and shock, and in mortise gearing, $K = .06$. Reduced to the form $W = Cpf$, assuming that $f = 2p$, these values of K give $W = 262pf$, $200pf$, and $139pf$, respectively.

Unwin also gives the following formula, based on the assumption that the pressure is distributed along the edge of the tooth: $p = K_1\sqrt{\frac{p}{f}}\sqrt{W}$, where K_1 = about .0707 for iron wheels and .0848 for mortise wheels when the breadth of face is not less than twice the pitch. For the case of $f = 2p$ and the given values of K_1 this reduces to $W = 200pf$ and $W = 139pf$, respectively.

Box, in his Treatise on Mill Gearing, gives H.P. $= \frac{12p^2f\sqrt{dn}}{1000}$, in which n = number of revolutions per minute. This formula differs from the more modern formulæ in making the H.P. vary as p^2f, instead of as pf, and in this respect it is no doubt incorrect.

Making the H.P. vary as $\sqrt{dn}$ or as $\sqrt{v}$, instead of directly as v, makes the velocity a factor of the working strength as in the Harkness and Lewis formulæ, the relative strength varying as $\frac{\sqrt{v}}{v}$, or as $\frac{1}{\sqrt{v}}$, which for different velocities is as follows:

Speed of teeth in ft. per min., v =	100	200	300	600	900	1200	1800	2400
Relative strength =	1	.707	.574	.408	.333	.289	.236	.204

Showing a somewhat more rapid reduction than is given by Mr. Lewis.

For the purpose of comparing different formulæ they may in general be reduced to either of the following forms:

$$\text{H.P.} = Cpfv, \qquad \text{H.P.} = C_1pfd \times \text{rpm.}, \qquad W = cpf,$$

in which p = pitch, f = face, d = diameter, all in inches; v = velocity in feet per minute, rpm. revolutions per minute, and C, C_1 and c coefficients. The formulæ for transformation are as follows:

$$\text{H.P.} = \frac{Wv}{33000} = \frac{W \times d \times \text{rpm.}}{126{,}050};$$

$$W = \frac{33{,}000\ \text{H.P.}}{v} = \frac{126{,}050\ \text{H.P.}}{d \times \text{rpm.}} = 33{,}000Cpf;\ pf = \frac{\text{H.P.}}{Cv} = \frac{\text{H.P.}}{C_1d \times \text{rpm.}} = \frac{W}{c}.$$

$$C_1 = .2618C;\quad c = 33{,}000C;\quad C = 3.82C_1,\ = \frac{c}{33{,}000};\quad c = 126{,}050C_1.$$

In the Lewis formula C varies with the form of the tooth and with the speed, and is equal to $sy \div 33{,}000$, in which y and s are the values taken from the table, and $c = sy$.

In the Harkness formula C varies with the speed and is equal to $\frac{910}{\sqrt{1+0.65V}}$ (V being in feet per second), $= \frac{.01517}{\sqrt{1+.011v}}$.

In the Box formula C varies with the pitch and also with the velocity, and equals $\frac{12p\sqrt{d \times \text{rpm.}}}{1000v} = .02345\frac{p}{\sqrt{v}}$. $c = 33{,}000C = 774\frac{p}{\sqrt{v}}$.

For v = 100 ft. per min. $C = 77.4p$; for v = 600 ft. per minute $c = 31.6p$.

In the other formulæ considered C, C_1, and c are constants. Reducing the several formulæ to the form $W = cpf$, we have the following:

COMPARISON OF DIFFERENT FORMULÆ FOR STRENGTH OF GEAR-TEETH.

Safe working pressure per inch pitch and per inch of face, or value of c in formula $W = cpf$:

		$v = 100$ ft. per min.	$v = 600$ ft. per min.
Lewis: Weak form of tooth, radial flank, 12 teeth...	$c =$	416	208
Medium tooth, inv. 15°, or cycloid, 27 teeth..	$c =$	800	400
Strong form of tooth, or cycloid, 300 teeth...	$c =$	1200	600
Harkness: Average tooth.............	$c =$	347	184
Box: Tooth of 1 inch pitch.............	$c =$	77.4	31.6
" " " 3 inches pitch.............	$c =$	232	95

Various, in which c is independent of form and speed: Old English rule, $c = 200$; Grant, $c = 350$; Nystrom, $c = 80$; Halsey, $c = 128$; Jones & Laughlins, $c = 218$; Unwin, $c = 262$, 200, or 139, according to speed, shock, and vibration.

The value given by Nystrom and those given by Box for teeth of small pitch are so much smaller than those given by the other authorities that they may be rejected as having an entirely unnecessary surplus of strength. The values given by Mr. Lewis seem to rest on the most logical basis, the form of the teeth as well as the velocity being considered; and since they are said to have proven satisfactory in an extended machine practice, they may be considered reliable for gears that are so well made that the pressure bears along the face of the teeth instead of upon the corners. For rough ordinary work the old English rule $W = 200pf$ is probably as good as any, except that the figure 200 may be too high for weak forms of tooth and for high speeds.

The formula $W = 200pf$ is equivalent to H.P. $= \frac{pfd \times \text{rpm.}}{630} = \frac{pfv}{165}$, or H.P. $= .0015873pfd \times$ rpm. $= .006063pfv$.

Maximum Speed of Gearing.—A. Towler, *Eng'g*, April 19, 1889, p. 388, gives the maximum speeds at which it was possible under favorable conditions to run toothed gearing safely as follows:

	Ft. per min.
Ordinary cast-iron wheels............	1800
Helical " " "	2400
Mortise " " "	2400
Ordinary cast-steel wheels............	2600
Helical " " "	3000
Special cast-iron machine-cut wheels............	3000

Prof. Coleman Sellers (*Stevens Indicator*, April, 1892) recommends that gearing be not run over 1200 ft. per minute, to avoid great noise. The Walker Company, Cleveland, O., say that 2200 ft. per min. for iron gears and 3000 ft. for wood and iron (mortise gears) are excessive, and should be avoided if possible. The Corliss engine at the Philadelphia Exhibition (1876) had a fly wheel 30 ft. in diameter running 35 rpm. geared into a pinion 12 ft. diam. The speed of the pitch-line was 3300 ft. per min.

A Heavy Machine-cut Spur-gear was made in 1891 by the Walker Company, Cleveland, O., for a diamond mine in South Africa, with dimensions as follows: Number of teeth, 192; pitch diameter, 30′ 6.66″; face, 30″; pitch, 6″; bore, 27″; diameter of hub, 9′ 2″; weight of hub, 15 tons; and total weight of gear, 66¾ tons. The rim was made in 12 segments, the joints of the segments being fastened with two bolts each. The spokes were bolted to the middle of the segments and to the hub with four bolts in each end.

Frictional Gearing.—In frictional gearing the wheels are toothless, and one wheel drives the other by means of the friction between the two surfaces which are pressed together. They may be used where the power to be transmitted is not very great; when the speed is so high that toothed wheels would be noisy; when the shafts require to be frequently put into and out of gear or to have their relative direction of motion reversed; or when it is desired to change the velocity-ratio while the machinery is in motion, as in the case of disk friction-wheels for changing the feed in machine tools.

Let $P =$ the normal pressure in pounds at the line of contact by which two wheels are pressed together, $T =$ tangential resistance of the driven wheel at the line of contact, $f =$ the coefficient of friction, $V =$ the velocity of the pitch-surface in feet per second, and H.P. = horse-power; then T may be equal to or less than fP; H.P. $= TV \div 550$. The value of f for metal on metal may be taken at .15 to .20; for wood on metal, .25 to .30; and for wood on compressed paper, .20. The tangential driving force T may be as high as 80 lbs. per inch width of face of the driving surface, but this is accompanied by great pressure and friction on the journal-bearings.

In frictional grooved gearing circumferential wedge-shaped grooves are cut in the faces of two wheels in contact. If $P =$ the force pressing the wheels together, and $N =$ the normal pressure on all the grooves, $P = N(\sin a + f \cos a)$, in which $2a =$ the inclination of the sides of the grooves, and the maximum tangential available force $T = fN$. The inclination of the sides of the grooves to a plane at right angles to the axis is usually 30°.

Frictional Grooved Gearing.—A set of friction-gears for transmitting 150 H.P. is on a steam-dredge described in Proc. Inst. M. E., July, 1888. Two grooved pinions of 54 in. diam., with 9 grooves of 1¾ in. pitch and angle of 40° cut on their face, are geared into two wheels of 127¼ in diam. similarly grooved. The wheels can be thrown in and out of gear by levers operating eccentric bushes on the large wheel-shaft. The circumferential speed of the wheels is about 500 ft. per min. Allowing for engine-friction, if half the power is transmitted through each set of gears the tangential force at the rims is about 3960 lbs., requiring, if the angle is 40° and the coefficient of friction 0.18, a pressure of 7524 lbs. between the wheels and pinion to prevent slipping.

The wear of the wheels proving excessive, the gears were replaced by spur-gear wheels and brake-wheels with steel brake-bands, which arrangement has proven more durable than the grooved wheels. Mr. Daniel Adamson states that if the frictional wheels had been run at a higher speed the results would have been better, and says they should run at least 30 ft. per second.

HOISTING.

Approximate Weight and Strength of Cordage. (Boston and Lockport Block Co.)—See also pages 339 to 345.

Size in Circumference.	Size in Diameter.	Weight of 100 ft. Manila, in lbs.	Strength of Manila Rope, in lbs.	Size in Circumference.	Size in Diameter.	Weight of 100 ft. Manila, in lbs.	Strength of Manila Rope, in lbs.
inch.	inch.			inch.	inch.		
2	⅝	13	4,000	4¾	1 9/16	72	22,500
2¼	¾	16	5,000	5	1⅝	80	25,000
2½	13/16	20	6,250	5½	1¾	97	30,250
2¾	⅞	24	7,500	6	2	113	36,000
3	1	28	9,000	6½	2⅛	133	42,250
3¼	1 1/16	33	10,500	7	2¼	153	49,000
3½	1⅛	38	12,250	7½	2½	184	56,250
3¾	1¼	45	14,000	8	2⅝	211	64,000
4	1 5/16	51	16,000	8½	2⅞	236	72,250
4¼	1⅜	58	18,062	9	3	262	81,000
4½	1½	65	20,250				

Working Strength of Blocks. (B. & L. Block Co.)

Regular Mortise-blocks Single and Double, or Two Double Iron-strapped Blocks, will hoist about—		Wide Mortise and Extra Heavy Single and Double, or Two Double, Iron-strapped Blocks, will hoist about—	
inch.	lbs.	inch.	lbs.
5	250	8	2,000
6	350	10	6,000
7	600	12	12,000
8	1,200	14	24,000
9	2,000	16	36,000
10	4,000	18	50,000
12	10,000	20	90,000
14	16,000		

Where a double and triple block are used together, a certain extra proportioned amount of weight can be safely hoisted, as larger hooks are used.

Comparative Efficiency in Chain-blocks both in Hoisting and Lowering.

(Tests by Prof. R. H. Thurston, *Hoisting*, March, 1892.)

Number of Block.	Work of Hoisting. Load of 2000 lbs.				Work of Lowering. Load of 2000 lbs., lowered 7 ft. in each case. Exclusive of Factor of Time.				Inclusive of Time.	
	Waste by Friction, per cent.	Actual Efficiency, per cent.	Relative Efficiency.	Velocity-ratio.	Pull on Hand Chain, lbs.	Length of Hand Chain, feet.	Work performed, ft.-lbs.	Relative Force expended by Operator.	Time in Min.	Relative Efficiency.
1	20.50	79.50	1.00	82.50	8.00	227.	1,816	1.00	0.75	1.000
2	68.00	32.00	.40	62.44	14.00	436.	6,104	3.33	1 20	.186
3	69.00	31.00	.39	30 00	92.30	196.	18,090	10.00	1.50	.050
4	71.20	28.80	.36	28.00	92.60	168.	15,556	8.60	2.50	.035
5	73.96	26.04	.33	48.00	73.30	17.5	1,282	0.71	2 80	.330
6	75.66	24.34	.31	53.00	56.60	370.	20,942	11.60	1.80	.036
7	77.00	23.00	.29	44.30	55.00	310.	17,050	9.40	2.75	.029
8	81.03	18.97	.24	61.00	48.50	426.	20,000	11.60	3.75	.018

No. 1 was Weston's triplex block; No. 3, Weston's differential; No. 4, Weston's imported. The others were from different makers, whose names are not given. All the blocks were of one-ton capacity.

Proportions of Hooks.—The following formulæ are given by Henry R. Towne, in his Treatise on Cranes, as a result of an extensive experimental and mathematical investigation. They apply to hooks of capacities from 250 lbs. to 20,000 lbs. Each size of hook is made from some commercial size of round iron. The basis in each case is, therefore, the size of iron of which the hook is to be made, indicated by A in the diagram. The dimension D is arbitrarily assumed. The other dimensions, as given by the formulæ, are those which, while preserving a proper bearing-face on the interior of the hook for the ropes or chains which may be passed through it, give the greatest resistance to spreading and to ultimate rupture, which the amount of material in the original bar admits of. The symbol Δ is used to indicate the nominal capacity of the hook in tons of 2000 lbs. The formulæ which determine the lines of the other parts of the hooks of the several sizes are as follows, the measurements being all expressed in inches:

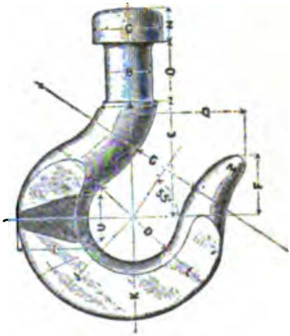

FIG. 164.

$D = .5\ \Delta + 1.25$ $\quad G = .75D.$ $\quad H = 1.08A$ $\quad L = 1.05A$

$E = .64\ \Delta + 1.60$ $\quad O = .363\ \Delta + .66$ $\quad I = 1.33A$ $\quad M = .50A$

$F = .33\ \Delta + .85$ $\quad Q = .64\ \Delta + 1.60$ $\quad J = 1.20A$ $\quad N = .85B - .16$

$K = 1.13A$ $\quad U = .866A$

The dimensions A are necessarily based upon the ordinary merchant sizes of round iron. The sizes which it has been found best to select are the following:

Capacity of hook:											
1/8	1/4	1/2	1	1 1/2	2	3	4	5	6	8	10 tons.
Dimension A:											
5/8	11/16	3/4	1 1/16	1 1/4	1 3/8	1 3/4	2	2 1/4	2 1/2	2 7/8	3 1/4 in.

Experiment has shown that hooks made according to the above formulæ will give way first by opening of the jaw, which, however, will not occur except with a load much in excess of the nominal capacity of the hook. This yielding of the hook when overloaded becomes a source of safety, as it constitutes a signal of danger which cannot easily be overlooked, and which must proceed to a considerable length before rupture will occur and the load be dropped.

POWER OF HOISTING-ENGINES.

Horse-power required to raise a Load at a Given Speed. — H.P. $= \frac{\text{Gross weight in lbs}}{33,000} \times$ speed in ft. per min. To this add 25% to 50% for friction, contingencies, etc. The gross weight includes the weight of cage, rope, etc. In a shaft with two cages balancing each other use the net load + weight of one rope, instead of the gross weight.

To find the load which a given pair of engines will start.—Let A = area of cylinder in square inches, or total area of both cylinders, if there are two; P = mean effective pressure in cylinder in lbs. per sq. in.; S = stroke of cylinder in inches; C = circumference of hoisting-drum in inches; L = load lifted by hoisting-rope in lbs.; F = friction, expressed as a diminution of the load. Then $L = \frac{AP2S}{C} - F$.

An example in *Coll'y Engr.*, July, 1891, is a pair of hoisting-engines 24″ × 40″, drum 12 ft. diam., average steam-pressure in cylinder = 59.5 lbs.; A = 904.8; P = 59.5; S = 40; C = 452.4. Theoretical load, not allowing for friction, $AP2S \div C$ = 9589 lbs. The actual load that could just be lifted on trial was 7988 lbs., making friction loss F = 1601 lbs., or 20 + per cent of the actual load lifted, or 16⅔% of the theoretical load.

The above rule takes no account of the resistance due to inertia of the load, but for all ordinary cases in which the acceleration of speed of the cage is moderate, it is covered by the allowance for friction, etc. The resistance due to inertia is equal to the force required to give the load the velocity acquired in a given time, or, as shown in Mechanics, equal to the product of the mass by the acceleration, or $R = \frac{WV}{gT}$, in which R = resistance in lbs. due to inertia; W = weight of load in lbs.; V = maximum velocity in feet per second; T = time in seconds taken to acquire the velocity V; g = 32.16.

Effect of Slack Rope upon Strain in Hoisting.—A series of tests with a dynamometer are published by the Trenton Iron Co., which show that a dangerous extra strain may be caused by a few inches of slack rope In one case the cage and full tubs weighed 11,300 lbs.; the strain when the load was lifted gently was 11,525 lbs.; with 3 in. of slack chain it was 19,025 lbs., with 6 in. slack 25,750 lbs., and with 9 in. slack 27,950 lbs.

Limit of Depth for Hoisting.—Taking the weight of a cast-steel hoisting-rope of 1½ inches diameter at 2 lbs. per running foot, and its breaking strength at 84,000 lbs., it should, theoretically, sustain itself until 42,000 feet long before breaking from its own weight. But taking the usual factor of safety of 7, then the safe working length of such a rope would be only 6000 feet. If a weight of 3 tons is now hung to the rope, which is equivalent to that of a cage of moderate capacity with its loaded cars, the maximum length at which such a rope could be used, with the factor of safety of 7, is 3000 feet, or

$$2x + 6000 = \frac{84,000}{7}; \qquad \therefore x = 3000 \text{ feet.}$$

This limit may be greatly increased by using special steel rope of higher strength, by using a smaller factor of safety, and by using taper ropes. (See paper by H. A. Wheeler, Trans. A. I. M. E., xix. 107.)

Large Hoisting Records.—At a colliery in North Derbyshire during the first week in June, 1890, 6309 tons were raised from a depth of 509 yards, the time of winding being from 7 a.m. to 8.30 p.m.

At two other Derbyshire pits, 170 and 140 yards in depth, the speed of winding and changing has been brought to such perfection that tubs are drawn and changed three times in one minute. (Proc. Inst. M. E., 1890.)

At the Nottingham Colliery near Wilkesbarre, Pa., in Oct. 1891, 70,152 tons were shipped in 24.15 days, the average hoist per day being 1818 mine cars. The depth of hoist was 470 feet, and all coal came from one opening. The engines were fast motion, 22 × 48 inches, conical drums 4 feet 1 inch long, 7 feet diameter at small end and 9 feet at large end. (*Eng'g News*, Nov. 1891.)

Pneumatic Hoisting. (H. A. Wheeler, Trans. A. I. M. E., xix. 107.)—A pneumatic hoist was installed in 1876 at Epinac, France, consisting of two continuous air-tight iron cylinders extending from the bottom to the top of the shaft. Within the cylinder moved a piston from which was hung the cage. It was operated by exhausting the air from above the piston, the lower side being open to the atmosphere. Its use was discontinued on account of the failure of the mine. Mr. Wheeler gives a description of the system, but criticises it as not being equal on the whole to hoisting by steel ropes.

Pneumatic hoisting-cylinders using compressed air have been used at blast-furnaces, the weighted piston counterbalancing the weight of the cage, and the two being connected by a wire rope passing over a pulley-sheave above the top of the cylinder. In the more modern furnaces steam-engine hoists are generally used.

Counterbalancing of Winding-engines. (H. W. Hughes, *Columbia Coll. Q'ly.*)—Engines running unbalanced are subject to enormous variations in the load; for let W = weight of cage and empty tubs, say 6270 lbs.; c = weight of coal, say 4480 lbs.; r = weight of hoisting rope, say 6000 lbs.; r' = weight of counterbalance rope hanging down pit, say 6000 lbs. The weight to be lifted will be:

	If weight of rope is unbalanced.	If weight of rope is balanced.	
At beginning of lift:	$W+c+r-W$ or 10,480 lbs.	$W+c+r-(W+r')$,	or 4480 lbs.
At middle of lift:	$W+c+\frac{r}{2}-\left(W+\frac{r}{2}\right)$ or 4480 lbs.	$W+c+\frac{r}{2}+\frac{r'}{2}-\left(W+\frac{r}{2}+\frac{r'}{2}\right)$,	
At end of lift:	$W+c-(W+r)$ or *minus* 1520 lbs.	$W+c+r'-(W+r)$,	

That counterbalancing materially affects the size of winding-engines is shown by a formula given by Mr. Robert Wilson, which is based on the fact that the greatest work a winding-engine has to do is to get a given mass into a certain velocity uniformly accelerated from rest, and to raise a load the distance passed over during the time this velocity is being obtained.

Let W = the weight to be set in motion: one cage, coal, number of empty tubs on cage, one winding rope from pit head-gear to bottom, and one rope from banking level to bottom.
v = greatest velocity attained, uniformly accelerated from rest;
g = gravity = 32.2;
t = time in seconds during which v is obtained;
L = unbalanced load on engine;
R = ratio of diameter of drum and crank circles;
P = average pressure of steam in cylinders;
N = number of cylinders;
S = space passed over by crank-pin during time t;
C = ⅔, constant to reduce angular space passed through by crank, to the distance passed through by the piston during the time t;
A = area of one cylinder, without margin for friction. To this an addition for friction, etc., of engine is to be made, varying from 10 to 30% of A.

1st. Where load is balanced,

$$A=\frac{\left\{\left(\frac{Wv^2}{2g}\right)+\left(L\frac{vt}{2}\right)\right\}R}{PNSC}.$$

2d. Where load is unbalanced:

The formula is the same, with the addition of another term to allow for the variation in the lengths of the ascending and descending ropes. In this case

h_1 = reduced length of rope in t attached to ascending cage;
h_2 = increased length of rope in t attached to descending cage;
w = weight of rope per foot in pounds. Then

$$A = \frac{\left[\left(\frac{Wv^2}{2g}\right) + \left\{\left(L\frac{vt}{2}\right) - \frac{h_1 w + h_2 w}{2}\right\}\right]R}{PNSC}.$$

Applying the above formula when designing new engines, Mr. Wilson found that 30 inches diameter of cylinders would produce equal results, when balanced, to those of the 36-inch cylinder in use, the latter being unbalanced.

Counterbalancing may be employed in the following methods:

(*a*) *Tapering Rope.*—At the initial stage the tapering rope enables us to wind from greater depths than is possible with ropes of uniform section. The thickness of such a rope at any point should only be such as to safely bear the load on it at that point.

With tapering ropes we obtain a smaller difference between the initial and final load, but the difference is still considerable, and for perfect equalization of the load we must rely on some other resource. The theory of taper ropes is to obtain a rope of uniform strength, thinner at the cage end where the weight is least, and thicker at the drum end where it is greatest.

(*b*) *The Counterpoise System* consists of a heavy chain working up and down a staple pit, the motion being obtained by means of a special small drum placed on the same axis as the winding drum. It is so arranged that the chain hangs in full length down the staple pit at the commencement of the winding; in the centre of the run the whole of the chain rests on the bottom of the pit, and, finally, at the end of the winding the counterpoise has been rewound upon the small drum, and is in the same condition as it was at the commencement.

(*c*) *Loaded-wagon System.*—A plan, formerly much employed, was to have a loaded wagon running on a short incline in place of this heavy chain; the rope actuating this wagon being connected in the same manner as the above to a subsidiary drum. The incline was constructed steep at the commencement, the inclination gradually decreasing to nothing. At the beginning of a wind the wagon was at the top of the incline, and during a portion of the run gradually passed down it till, at the meet of cages, no pull was exerted on the engine—the wagon by this time being at the bottom. In the latter part of the wind the resistance was all against the engine, owing to its having to pull the wagon up the incline, and this resistance increased from nothing at the meet of cages to its greatest quantity at the conclusion of the lift.

(*d*) *The Endless-rope System* is preferable to all others, if there is sufficient sump room and the shaft is free from tubes, cross timbers, and other impediments. It consists in placing beneath the cages a tail rope, similar in diameter to the winding rope, and, after conveying this down the pit, it is attached beneath the other cage.

(*e*) *Flat Ropes Coiling on Reels.*—This means of winding allows of a certain equalization, for the radius of the coil of ascending rope continues to increase, while that of the descending one continues to diminish. Consequently, as the resistance decreases in the ascending load the leverage increases, and as the power increases in the other, the leverage diminishes. The variation in the leverage is a constant quantity, and is equal to the thickness of the rope where it is wound on the drum.

By the above means a remarkable uniformity in the load may be obtained, the only objection being the use of flat ropes, which weigh heavier and only last about two thirds the time of round ones.

(*f*) *Conical Drums.*—Results analogous to the preceding may be obtained by using round ropes coiling on conical drums, which may either be smooth, with the successive coils lying side by side, or they may be provided with a spiral groove. The objection to these forms is, that perfect equalization is not obtained with the conical drums unless the sides are very steep, and consequently there is great risk of the rope slipping; to obviate this, scroll drums were proposed. They are, however, very expensive, and the lateral displacement of the winding rope from the centre line of pulley becomes very great, owing to their necessary large width.

(*g*) *The Koepe System of Winding.*—An iron pulley with a single circular groove takes the place of the ordinary drum. The winding rope passes from one cage, over its head-gear pulley, round the drum, and, after pass ing over the other head-gear pulley, is connected with the second cage. The winding rope thus encircles about half the periphery of the drum in the same manner as a driving-belt on an ordinary pulley. There is a balance rope beneath the cages, passing round a pulley in the sump; the arrangement may be likened to an endless rope, the two cages being simply points of attachment.

BELT-CONVEYORS.

Grain-elevators. — American Grain-elevators are described in a paper by E. Lee Heidenreich, read at the International Engineering Congress at Chicago (Trans. A. S. C. E. 1893). See also Trans. A. S. M. E. vii, 660.

Bands for carrying Grain. — Flexible-rubber bands are extensively used for carrying grain in and around elevators and warehouses. An article on the grain-storage warehouses of the Alexandria Dock, Liverpool (Proc. Inst. M. E., July, 1891), describes the performance of these bands, aggregating three miles in length. A band 16½ inches wide, 1270 feet long, running 9 to 10 feet per second has a carrying capacity of 50 tons per hour. See also paper on Belts as Grain Conveyors, by T. W. Hugo, Trans. A. S. M. E., vi. 400.

Carrying-bands or Belts are used for the purpose both of sorting coal and of removing impurities. These carrying-bands may be said to be confined to two descriptions, namely, the wire belt, which consists of an endless length of woven wire; and the steel-plate belt, which consists of two or three endless chains, carrying steel plates varying in width from 6 inches to 14 inches. (Proc. Inst. M. E., July, 1890.)

CRANES.

Classification of Cranes. (Henry R. Towne, Trans. A. S. M. E., iv. 288. Revised in *Hoisting*, published by The Yale & Towne Mfg. Co.)

A Hoist is a machine for raising and lowering weights. A Crane is a hoist with the added capacity of moving the load in a horizontal or lateral direction.

Cranes are divided into two classes, as to their motions, viz., *Rotary* and *Rectilinear*, and into four groups, as to their source of motive power, viz.:

Hand.—When operated by manual power.

Power.—When driven by power derived from line shafting.

Steam, Electric, Hydraulic, or Pneumatic.—When driven by an engine or motor attached to the crane, and operated by steam, electricity, water, or air transmitted to the crane from a fixed source of supply.

Locomotive.—When the crane is provided with its own boiler or other generator of power, and is self-propelling; usually being capable of both rotary and rectilinear motions.

Rotary and Rectilinear Cranes are thus subdivided:

Rotary Cranes.

(1) *Swing-cranes.*—Having rotation, but no trolley motion.

(2) *Jib-cranes.*—Having rotation, and a trolley travelling on the jib.

(3) *Column-cranes.*—Identical with the jib-cranes, but rotating around a fixed column (which usually supports a floor above).

(4) *Pillar-cranes.*—Having rotation only; the pillar or column being supported entirely from the foundation.

(5) *Pillar Jib-cranes.*—Identical with the last, except in having a jib and trolley motion.

(6) *Derrick-cranes.*—Identical with jib-cranes, except that the head of the mast is held in position by guy-rods, instead of by attachment to a roof or ceiling.

(7) *Walking-cranes.*—Consisting of a pillar or jib-crane mounted on wheels and arranged to travel longitudinally upon one or more rails.

(8) *Locomotive-cranes.*—Consisting of a pillar crane mounted on a truck, and provided with a steam-engine capable of propelling and rotating the crane, and of hoisting and lowering the load.

Rectilinear Cranes.

(9) *Bridge-cranes.*—Having a fixed bridge spanning an opening, and a trolley moving across the bridge.

(10) *Tram-cranes.*—Consisting of a truck, or short bridge, travelling longitudinally on overhead rails, and without trolley motion.

(11) *Travelling-cranes.*—Consisting of a bridge moving longitudinally on overhead tracks, and a trolley moving transversely on the bridge.

(12) *Gantries.*—Consisting of an overhead bridge, carried at each end by a trestle travelling on longitudinal tracks on the ground, and having a trolley moving transversely on the bridge.

(13) *Rotary Bridge-cranes.*—Combining rotary and rectilinear movements and consisting of a bridge pivoted at one end to a central pier or post, and supported at the other end on a circular track; provided with a trolley moving transversely on the bridge.

For descriptions of these several forms of cranes see Towne's "Treatise on Cranes."

Stresses in Cranes.—See Stresses in Framed Structures, p. 440, *ante.*

Position of the Inclined Brace in a Jib-crane.—The most economical arrangement is that in which the inclined brace intersects the jib at a distance from the mast equal to four fifths the effective radius of the crane. (*Hoisting.*)

A Large Travelling-crane, designed and built by the Morgan Engineering Co., Alliance, O., for the 12-inch-gun shop at the Washington Navy Yard, is described in *American Machinist*, June 12, 1890. Capacity, 150 net tons; distance between centres of inside rails, 59 ft. 6 in.; maximum cross travel, 44 ft. 2 in.; effective lift, 40 ft.; four speeds for main hoist, 1, 2, 4, and 8 ft. per min.; loads for these speeds, 150, 75, 37½, and 18¾ tons respectively; traversing speeds of trolley on bridge, 25 and 50 ft. per minute; speeds of bridge on main track, 30 and 60 ft. per minute. Square shafts are employed for driving.

A 150-ton Pillar-crane was erected in 1893 on Finnieston Quay, Glasgow. The jib is formed of two steel tubes, each 39 in. diam. and 90 ft. long. The radius of sweep for heavy lifts is 65 ft. The jib and its load are counterbalanced by a balance-box weighted with 100 tons of iron and steel punchings. In a test a 130-ton load was lifted at the rate of 4 ft. per minute, and a complete revolution made with this load in 5 minutes. *Eng'g News*, July 20, 1893.

Compressed-air Travelling-cranes.—Compressed-air overhead travelling-cranes have been built by the Lane & Bodley Co., of Cincinnati. They are of 20 tons nominal capacity, each about 50 ft. span and 400 ft. length of travel, and are of the triple-motor type, a pair of simple reversing-engines being used for each of the necessary operations, the pair of engines for the bridge and the pair for the trolley travel being each 5-inch bore by 7-inch stroke, while the pair for hoisting is 7-inch bore by 9-inch stroke. Air is furnished by a compressor having steam and air cylinders each 10-in. diam. and 12-in. stroke, which with a boiler-pressure of about 80 pounds gives an air-pressure when required of somewhat over 100 pounds. The air-compressor is allowed to run continuously without a governor, the speed being regulated by the resistance of the air in a receiver. From a pipe extending from the receiver along one of the supporting trusses communication is continuously maintained with an auxiliary receiver on each traveller by means of a one-inch hose, the object of the auxiliary receiver being to provide a supply of air near the engines for immediate demands and independent of the hose connection, which may thus be of small dimension. Some of the advantages said to be possessed by this type of crane are: simplicity; absence of all moving parts, excepting those required for a particular motion when that motion is in use; no danger from fire, leakage, electric shocks, or freezing; ease of repair; variable speeds and reversal without gearing; almost entire absence of noise; and moderate cost.

Quay-cranes.—An illustrated description of several varieties of stationary and travelling cranes, with results of experiments, is given in a paper on Quay-cranes in the Port of Hamburg by Chas. Nehls, Trans. A. S. C. E., Chicago Meeting, 1893.

Hydraulic Cranes, Accumulators, etc.—See Hydraulic Pressure Transmission, page 616, *ante.*

Electric Cranes.—Travelling-cranes driven by electric motors have largely supplanted cranes driven by square shafts or flying-ropes. Each of the three motions, viz., longitudinal, traversing and hoisting, is usually accomplished by a separate motor carried upon the crane.

WIRE-ROPE HAULAGE.

Methods for transporting coal and other products by means of wire rope, though varying from each other in detail, may be grouped in five classes:

I. The Self-acting or Gravity Inclined Plane.

II. The Simple Engine-plane.

III. The Tail-rope System.
IV. The Endless-rope System.
V. The Cable Tramway.

The following brief description of these systems is abridged from a pamphlet on Wire-rope Haulage, by Wm. Hildenbrand, C.E., published by John A. Roebling's Sons Co., Trenton, N. J.

I. The Self-acting Inclined Plane.—The motive power for the self-acting inclined plane is gravity; consequently this mode of transporting coal finds application only in places where the coal is conveyed from a higher to a lower point and where the plane has sufficient grade for the loaded descending cars to raise the empty cars to an upper level.

At the head of the plane there is a drum, which is generally constructed of wood, having a diameter of seven to ten feet. It is placed high enough to allow men and cars to pass under it. Loaded cars coming from the pit are either singly or in sets of two or three switched on the track of the plane, and their speed in descending is regulated by a brake on the drum.

Supporting rollers, to prevent the rope dragging on the ground, are generally of wood, 5 to 6 inches in diameter and 18 to 24 inches long, with ¾- to ⅞-inch iron axles. The distance between the rollers varies from 15 to 30 feet, steeper planes requiring less rollers than those with easy grades. Considering only the reduction of friction and what is best for the preservation of rope, a general rule may be given to use rollers of the greatest possible diameter, and to place them as close as economy will permit.

The smallest angle of inclination at which a plane can be made self-acting will be when the motive and resisting forces balance each other. The motive forces are the weights of the loaded car and of the descending rope. The resisting forces consist of the weight of the empty car and ascending rope, of the rolling and axle friction of the cars, and of the axle friction of the supporting rollers. The friction of the drum, stiffness of rope, and resistance of air may be neglected. A general rule cannot be given, because a change in the length of the plane or in the weight of the cars changes the proportion of the forces; also, because the coefficient of friction, depending on the condition of the road, construction of the cars, etc., is a very uncertain factor.

For working a plane with a ⅝-inch steel rope and lowering from one to four pit cars weighing empty 1400 lbs. and loaded 4000 lbs., the rise in 100 feet necessary to make the plane self-acting will be from about 5 to 10 feet, decreasing as the number of cars increase, and increasing as the length of plane increases.

A gravity inclined plane should be slightly concave, steeper at the top than at the bottom. The maximum deflection of the curve should be at an inclination of 45 degrees, and diminish for smaller as well as for steeper inclinations.

II. The Simple Engine-plane.—The name "Engine-plane" is given to a plane on which a load is raised or lowered by means of a single wire rope and stationary steam-engine. It is a cheap and simple method of conveying coal underground, and therefore is applied wherever circumstances permit it.

Under ordinary conditions such as prevail in the Pennsylvania mine region, a train of twenty-five to thirty loaded cars will descend, with reasonable velocity, a straight plane 5000 feet long on a grade of 1¾ feet in 100, while it would appear that 2¼ feet in 100 is necessary for the same number of empty cars. For roads longer than 5000 feet, or when containing sharp curves, the grade should be correspondingly larger.

III. The Tail-rope System.—Of all methods for conveying coal underground by wire rope, the tail-rope system has found the most application. It can be applied under almost any condition. The road may be straight or curved, level or undulating, in one continuous line or with side branches. In general principle a tail-rope plane is the same as an engine-plane worked in both directions with two ropes. One rope, called the "main rope," serves for drawing the set of full cars outward; the other, called the "tail-rope," is necessary to take back the empty set, which on a level or undulating road cannot return by gravity. The two drums may be located at the opposite ends of the road, and driven by separate engines, but more frequently they are on the same shaft at one end of the plane. In the first case each rope would require the length of the plane, but in the second case the tail rope must be twice as long, being led from the drum around a sheave at the other end of the plane and back again to its starting-point. When the main rope draws a set of full cars out, the tail-rope drum runs loose on the shaft, and the rope, being attached to the rear car, unwinds itself steadily. Going in, the reverse takes place. Each drum is provided with a brake to check the speed of the train on a down grade and prevent its overrunning the forward rope. As a rule, the tail rope is strained less than the main rope, but in cases of heavy grades dipping outward it is possible that the strain in the former may become as large, or even larger, than in the latter, and in the selection of the sizes reference should be had to this circumstance.

IV. The Endless-rope System.—The principal features of this system are as follows:

1. The rope, as the name indicates, is endless.

2. Motion is given to the rope by a single wheel or drum, and friction is obtained either by a grip-wheel or by passing the rope several times around the wheel.

3. The rope must be kept constantly tight, the tension to be produced by artificial means. It is done in placing either the return-wheel or an extra tension wheel on a carriage and connecting it with a weight hanging over a pulley, or attaching it to a fixed post by a screw which occasionally can be shortened.

4. The cars are attached to the rope by a grip or clutch, which can take hold at any place and let go again, starting and stopping the train at will, without stopping the engine or the motion of the rope.

5. On a single-track road the rope works forward and backward, but on a double track it is possible to run it always in the same direction, the full cars going on one track and the empty cars on the other.

This method of conveying coal, as a rule, has not found as general an introduction as the tail-rope system, probably because its efficacy is not so apparent and the opposing difficulties require greater mechanical skill and more complicated appliances. Its advantages are, first, that it requires one third less rope than the tail-rope system. This advantage, however, is partially counterbalanced by the circumstance that the extra tension in the rope requires a heavier size to move the same load than when a main and tail rope are used. The second and principal advantage is that it is possible to start and stop trains at will without signalling to the engineer. On the other hand, it is more difficult to work curves with the endless system, and still more so to work different branches, and the constant stretch of the rope under tension or its elongation under changes of temperature frequently causes the rope to slip on the wheel, in spite of every attention, causing delay in the transportation and injury to the rope.

V. Wire-rope Tramways.—The methods of conveying products on a suspended rope tramway find especial application in places where a mine is located on one side of a river or deep ravine and the loading station on the other. A wire rope suspended between the two stations forms the track on which material in properly constructed "carriages" or "buggies" is transported. It saves the construction of a bridge or trestlework, and is practical for a distance of 2000 feet without an intermediate support.

There are two distinct classes of rope tramways:

1. The rope is stationary, forming the track on which a bucket holding the material moves forward and backward, pulled by a smaller endless wire rope.

2. The rope is movable, forming itself an endless line, which serves at the same time as supporting track and as pulling rope.

Of these two the first method has found more general application, and is especially adapted for long spans, steep inclinations, and heavy loads. The second method is used for long distances, divided into short spans, and is only applicable for light loads which are to be delivered at regular intervals.

For detailed descriptions of the several systems of wire-rope transportation, see circulars of John A. Roebling's Sons Co., The Trenton Iron Co., and other wire-rope manufacturers. See also paper on Two-rope Haulage Systems, by R. Van A. Norris, Trans. A. S. M. E., xii. 626.

In the Bleichert System of wire-rope tramways, in which the track rope is stationary, loads of 1000 pounds each and upward are carried. While the average spans on a level are from 150 to 200 feet, in crossing rivers, ravines, etc., spans up to 1500 feet are frequently adopted. In a tramway on this system at Granite, Montana, the total length of the line is 9750 feet, with a fall of 1225 feet. The descending loads, amounting to a constant weight of about 11 tons, develop over 14 horse-power, which is sufficient to haul the empty buckets as well as about 50 tons of supplies per day up the line, and also to run the ore crusher and elevator. It is capable of delivering 250 tons of material in 10 hours.

Suspension Cableways or Cable Hoist-conveyors.

(Trenton Iron Co.)

In quarrying, rock-cutting, stripping, piling, dam-building, and many other operations where it is necessary to hoist and convey large individual loads economically, it frequently happens that the application of a system of derricks is impracticable, by reason of the limited area of their efficiency and the room which they occupy.

To meet such conditions cable hoist-conveyors are adapted, as they can be operated in clear spans up to 1500 feet, and in lifting individual loads up to 15 tons. Two types are made—one in which the hoisting and conveying are done by separate running ropes, and the other applicable only to inclines, in which the carriage descends by gravity, and but one running rope is required. The moving of the carriage in the former is effected by means of an endless rope, and these are commonly known as "endless-rope" hoist-conveyors to distinguish them from the latter, which are termed "inclined" hoist-conveyors.

The general arrangement of the endless-rope hoist-conveyors consists of a main cable passing over towers, A frames or masts, as may be most convenient, and anchored firmly to the ground at each end, the requisite tension in the cable being maintained by a turnbuckle at one anchorage.

Upon this cable travels the carriage, which is moved back and forth over the line by means of the endless rope. The hoisting is done by a separate rope, both ropes being operated by an engine specially designed for the purpose, which may be located at either end of the line, and is constructed in such a way that the hoisting-rope is coiled up or paid out automatically as the carriage is moved in and out. Loads may be picked up or discharged at any point along the line. Where sufficient inclination can be obtained in the main cable for the carriage to descend by gravity, and the loading and unloading is done at fixed points, the endless rope can be dispensed with. The carriage, which is similar in construction to the carriage used in the endless-rope cableways, is arrested in its descent by a stop-block, which may be clamped to the main cable at any desired point, the speed of the descending carriage being under control of a brake on the engine-drum.

Stress in Hoisting-ropes on Inclined Planes.

(Trenton Iron Co.)

Rise per 100 ft. horizontal.	Angle of inclination.	Stress in lbs. per ton of 2000 lbs.	Rise per 100 ft. horizontal.	Angle of inclination.	Stress in lbs. per ton of 2000 lbs.	Rise per 100 ft. horizontal.	Angle of inclination.	Stress in lbs. per ton of 2000 lbs.
ft.			ft.			ft.		
5	2° 52′	140	55	28° 49′	1008	110	47° 44′	1516
10	5° 43′	240	60	30° 58′	1067	120	50° 12′	1578
15	8° 32′	336	65	33° 02′	1128	130	52° 26′	1620
20	11° 10′	432	70	35° 00′	1185	140	54° 28′	1663
25	14° 03′	527	75	36° 53′	1238	150	56° 19′	1699
30	16° 42′	618	80	38° 40′	1287	160	58° 00′	1730
35	19° 18′	700	85	40° 22′	1332	170	59° 33′	1758
40	21° 49′	782	90	42° 00′	1375	180	60° 57′	1782
45	24° 14′	860	95	43° 32′	1415	190	62° 15′	1804
50	26° 34′	933	100	45° 00′	1450	200	63° 27′	1823

The above table is based on an allowance of 40 lbs. per ton for rolling friction, but an additional allowance must be made for stress due to the weight of the rope proportional to the length of the plane. A factor of safety of 5 to 7 should be taken.

In hoisting the slack-rope should be taken up gently before beginning the lift, otherwise a severe extra strain will be brought on the rope.

A Double-suspension Cableway, carrying loads of 15 tons, erected near Williamsport, Pa., by the Trenton Iron Co., is described by E. G. Spilsbury in Trans. A. I. M. E. xx. 766. The span is 733 feet, crossing the Susquehanna River. Two steel cables, each 2 in. diam., are used. On these cables runs a carriage supported on four wheels and moved by an endless cable 1 inch in diam. The load consists of a cage carrying a railroad-car loaded with lumber, the latter weighing about 12 tons. The power is furnished by a 50-H.P. engine, and the trip across the river is made in about three minutes.

A hoisting cableway on the endless-rope system, erected by the Lidgerwood Mfg. Co., at the Austin Dam, Texas, had a single span 1350 ft. in length, with main cable 2½ in. diam., and hoisting-rope 1¾ in. diam. Loads of 7 to 8 tons were handled at a speed of 600 to 800 ft. per minute.

Another, of still longer span, 1650 ft., was erected by the same company at Holyoke, Mass., for use in the construction of a dam. The main cable is the Elliott or locked wire cable, having a smooth exterior. In the construction of the Chicago Drainage Canal twenty cableways, of 700 ft. span and 8 tons capacity, were used, the towers travelling on rails.

Tension required to Prevent Slipping of Rope on Drum. (Trenton Iron Co.)—The amount of artificial tension to be applied in an endless rope to prevent slipping on the driving-drum depends on the character of the drum, the condition of the rope and number of laps which it makes. If T and S represent respectively the tensions in the taut and slack lines of the rope; W, the necessary weight to be applied to the tail-sheave; R, the resistance of the cars and rope, allowing for friction; n, the number of half-laps of the rope on the driving-drum; and f, the coefficient of friction, the following relations must exist to prevent slipping:

$$T = Se^{fn\pi}, \quad W = T + S, \quad \text{and} \quad R = T - S;$$

from which we obtain $$W = \frac{e^{fn\pi} + 1}{e^{fn\pi} - 1}R,$$

in which $e = 2.71828$, the base of the Naperian system of logarithms.

The following are some of the values of f:

	Dry.	Wet.	Greasy.
Wire-rope on a grooved iron drum.......	.120	.085	.070
Wire-rope on wood-filled sheaves	235	.170	.140
Wire-rope on rubber and leather filling..	.495	.400	.205

The values of the coefficient $\frac{e^{fn\pi} + 1}{e^{fn\pi} - 1}$, corresponding to the above values of f, for one up to six half-laps of the rope on the driving-drum or sheaves, are as follows:

f	n = Number of Half-laps on Driving-wheel.					
	1	2	3	4	5	6
.070	9.180	4.623	3.141	2.418	1.999	1.729
.085	7.536	3.833	2.629	2.047	1.714	1.505
.120	5.345	2.777	1.953	1.570	1.358	1.232
.140	4.623	2.418	1.729	1.416	1.249	1.154
.170	3.833	2.047	1.505	1.268	1.149	1.085
.205	3.212	1.762	1.338	1.165	1.083	1.043
.235	2.831	1.592	1.245	1.110	1.051	1.024
.400	1.795	1.176	1.047	1.018	1.004	1.001
.495	1.538	1.093	1.010	1.004	1.001	....

The importance of keeping the rope dry is evident from these figures.

When the rope is at rest the tension is distributed equally on the two lines of the rope, but when running there will be a difference in the tensions of the taut and slack lines equal to the resistance, and the values of T and S may be readily computed from the foregoing formulæ.

Taper Ropes of Uniform Tensile Strength.—The true form of rope is not a regular taper but follows a logarithmic curve, the girth rapidly increasing toward the upper end. Mr. Chas. D. West gives the following formula, based on a breaking strain of 80,000 lbs. per sq. in. of the rope, core included, and a factor of safety of 10: $\log G = F/3680 + \log g$, in which F = length in fathoms, and G and g the girth in inches at any two sections F fathoms apart. The girth g is first calculated for a safe strain of 8000 lbs. per sq. in., and then G is obtained by the formula. For a mathematical investigation see *The Engineer*, April, 1890, p. 267.

TRANSMISSION OF POWER BY WIRE ROPE.

The following notes have been furnished to the author by Mr. Wm. Hewitt, Vice-President of the Trenton Iron Co. (See also circulars of the Trenton Iron Co. and of the John A. Roebling's Sons Co., Trenton, N. J.; "Transmission of Power by Wire Ropes," by A. W. Stahl, Van Nostrand's Science Series, No. 28; and Reuleaux's Constructor.)

The force transmitted should not exceed the difference between the elastic limit of the wires and the bending stress as determined by the following tables, taking the elastic limit of tempered steel, such as is used in the best rope, at 57,000 lbs. per sq. in., and that of Swedish iron at half this, or 28,500 lbs. (The el. lim. of fine steel wires may be higher than 57,000 lbs.)

Elastic Limit of Wire Ropes.

7-Wire Rope.	Diam. of Wires.	Aggregate Area of Wires.	Elastic Limit. Steel.	Elastic Limit. Iron.
diam., in.	ins.	sq. in.	lbs.	lbs.
1/4	.028	.025862	1,474	737
5/16	.035	.040409	2,303	1,152
3/8	.042	.058189	3,317	1,659
7/16	.049	.079201	4,514	2,257
1/2	.055	.099785	5,688	2,844
9/16	.0625	.128855	7,345	3,672
5/8	.070	.161635	9,213	4,607
11/16	.076	.190532	10,860	5,430
3/4	.083	.227246	12,953	6,477
7/8	.097	.310378	17,691	8,846
1	.111	.406430	23,167	11,583
19-Wire Rope.				
1/4	.017	.025876		
5/16	.021	.039485		
3/8	.024	.051573		
7/16	.029	.075299		
1/2	.033	.097504		
9/16	.0375	.125909		
5/8	.042	.157941		
11/16	.046	.189458		
3/4	.050	.223839		
7/8	.058	.301198		
1	.067	.401925		

The elastic limit of 19-wire rope may be taken the same as for 7-wire rope since the ultimate strength of the wires is 7 to 10 per cent greater.

The working tension may be greater, therefore, as the bending stress is less; but since the tension in the slack portion of the rope cannot be less than a certain proportion of the tension in the taut portion, to avoid slipping, a ratio exists between the diameter of sheave and the wires composing the rope, corresponding to a maximum safe working tension. This ratio depends upon the number of laps that the rope makes about the sheaves, and the kind of filling in the rims or the character of the material upon which the rope tracks.

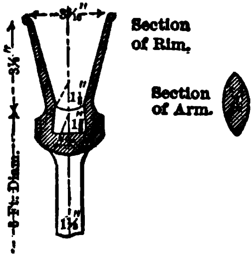

FIG. 165.

The sheaves (Fig. 165) are usually of cast iron, and are made as light as possible consistent with the requisite strength. Various materials have been used for filling the bottom of the groove, such as tarred oakum, jute yarn, hard wood, India-rubber, and leather. The filling which gives the best satisfaction, however, in ordinary transmissions consists of segments of leather and blocks of India-rubber soaked in tar and packed alternately in the groove. Where the working tension is very great, however, the wood filling is to be preferred, as in the case of long-distance transmissions where the rope makes several laps about the sheaves, and is run at a comparatively slow speed.

Bending Stresses, 7-Wire Rope.

$$k = \frac{Ea}{2.06\frac{R}{\delta} + 27.54}.$$

k = Bending stress; E = Modulus of elasticity = 28,500,000;
a = Aggregate area of wires; R = Radius of bend; δ = Diam. of wires (lbs. and inches).

Diam. Bend.	24	36	48	60	72	84	96	108	120	132
Diam. Rope.										
1/4	810	545	411							
5/16	1,569	1,060	800	642						
3/8	......	1,822	1,377	1,106	925					
7/16	......	2,878	2,178	1,751	1,465	1,259				
1/2	......	4,053	3,070	2,470	2,067	1,777				
9/16	......	......	4,486	3,613	3,025	2,601	2,282			
5/8	......	......	6,278	5,060	4,238	3,646	3,199			
11/16	......	......	......	6,459	5,412	4,657	4,087	3,641		
3/4	......	......	......	8,388	7,032	6,053	5,314	4,735		
7/8	......	......	......	......	11,168	9,620	8,449	7,532	6,795	
1	......	......	......	......	16,651	14,354	12,613	11,249	10,151	
1 1/8	......	......	......	......	......	20,411	17,986	16,011	14,458	13,172
1 1/4	......	......	......	......	......	......	24,582	21,942	19,814	18,062
1 3/8	......	......	......	......	......	......	......	29,164	26,344	24,021
1 1/2	......	......	......	......	......	......	......	......	34,155	31,151

Bending Stresses, 19-Wire Rope.

$$k = \frac{Ea}{2.06\frac{R}{\delta} + 45.9}.$$

Diam. Bend.	12	24	36	48	60	72	84	96	108	120
Diam. Rope.										
1/4	965	495								
5/16	1,774	920	621							
3/8	......	1,366	924	698						
7/16	......	2,389	1,620	1,226						
1/2	......	3,495	2,376	1,800	1,448					
9/16	......	5,089	3.468	2,630	2,118					
5/8	......	......	4,847	3,680	2,967	2,485				
11/16	......	......	6,201	4,818	3,886	3,257				
3/4	......	......	8,101	6.165	4.977	4,173	3,591			
7/8	......	......	12,528	9,556	7.724	6,481	5.583			
1	......	......	......	14,614	11,830	9,937	8,566	7,528		
1 1/8	......	......	......	......	16,500	13,872	11,966	10,528	9,387	
1 1/4	......	......	......	......	22,239	18,713	16.153	14,209	12,682	
1 3/8	......	......	......	......	......	25,350	21,897	19,272	17,209	15,545
1 1/2	......	......	......	......	......	32,403	28,008	24,662	22,030	19,906
1 5/8	......	......	......	......	......	......	35,140	30,957	27,664	23,005
1 3/4	......	......	......	......	......	......	44,476	39,203	35,048	31,689
1 7/8	......	......	......	......	......	......	......	47,689	42,606	38,534
2	......	......	......	......	......	......	......	57,183	51,160	46,285
2 1/4	......	......	......	......	......	......	......	......	72,908	66,002

Horse-Power Transmitted.—The general formula for the amount of power capable of being transmitted is as follows:

$$H.P. = [cd^2 - .000006\,(w + g_1 + g_2)]v;$$

in which d = diameter of the rope in inches, v = velocity of the rope in feet per second, w = weight of the rope, g_1 = weight of the terminal sheaves and shafts, g_2 = weight of the intermediate sheaves and shafts (all in lbs.), and c = a constant depending on the material of the rope, the filling in the grooves of the sheaves, and the number of laps about the sheaves or drums, a single lap meaning a half-lap at each end. The values of c for one up to six laps for steel rope are given in the following table:

c = for steel rope on	Number of Laps about Sheaves or Drums.					
	1	2	3	4	5	6
Iron	5.61	8.81	10.62	11.65	12.16	12.56
Wood	6.70	9.93	11.51	12.26	12.66	12.83
Rubber and leather	9.29	11.95	12.70	12.91	12.97	13.00

The values of c for iron rope are one half the above.

When more than three laps are made, the character of the surface in contact is immaterial as far as slippage is concerned.

From the above formula we have the general rule, that *the actual horse-power capable of being transmitted by any wire rope approximately equals c times the square of the diameter of the rope in inches, less six millionths the entire weight of all the moving parts, multiplied by the speed of the rope, in feet per second.*

Instead of grooved drums or a number of sheaves, about which the rope makes two or more laps, it is sometimes found more desirable, especially where space is limited, to use grip-pulleys. The rim is fitted with a continuous series of steel jaws, which bite the rope in contact by reason of the pressure of the same against them, but as soon as relieved of this pressure they open readily, offering no resistance to the egress of the rope.

In the ordinary or "flying" transmission of power, where the rope makes a single lap about sheaves lined with rubber and leather or wood, the ratio between the diameter of the sheaves and the wires of the rope, corresponding to a maximum safe working tension, is one resulting in a working tension of one third and bending stress of two thirds of the elastic limit of the material. The diameters of sheaves are as follows:

Diameters of Minimum Sheaves in Inches, Corresponding to a Maximum Safe Working Tension.

Diameter of Rope. In.	Steel.			Iron.		
	7-Wire.	12-Wire.	19-Wire.	7-Wire.	12-Wire.	19-Wire.
1/4	19	14	12	40	30	24
5/16	24	18	14	50	38	30
3/8	29	22	17	60	45	36
7/16	34	25	20	70	53	42
1/2	38	29	23	80	60	48
9/16	43	32	26	90	68	54
5/8	48	36	29	100	75	60
11/16	53	40	32	110	83	66
3/4	58	43	35	120	90	72
7/8	67	50	40	140	105	84
1	77	57	46	160	120	96

Assuming the sheaves to be of equal diameter, and of the sizes in the above table, the *horse-power that may be transmitted by a steel rope making a single lap on wood-filled sheaves* is given in the table on the next page.

The transmission of greater horse-powers than 250 is impracticable with filled sheaves, as the tension would be so great that the filling would quickly cut out, and the adhesion on a metallic surface would be insufficient where the rope makes but a single lap. In this case it becomes necessary to use the Reuleaux method, in which the rope is given more than one lap, as referred to below, under the caption "Long-distance Transmissions."

Horse-power Transmitted by a Steel Rope on Wood-filled Sheaves.

Diameter of Rope. In.	Velocity of Rope in Feet per Second.									
	10	20	30	40	50	60	70	80	90	100
1/4	4	8	13	17	21	25	28	32	37	40
5/16	7	13	20	26	33	40	44	51	57	62
3/8	10	19	28	38	47	56	64	73	80	89
7/16	13	26	38	51	63	75	88	99	109	121
1/2	17	34	51	67	83	99	115	130	144	159
9/16	22	43	65	86	106	128	147	167	184	203
5/8	27	53	79	104	130	155	179	203	225	247
11/16	32	63	95	126	157	186	217	245		
3/4	38	76	103	150	186	223				
7/8	52	104	156	206						
1	68	135	202							

The horse-power that may be transmitted by iron ropes is one half of the above.

This table gives the amount of horse-power transmitted by wire ropes under maximum safe working tensions. In using wood-lined sheaves, therefore, it is well to make some allowance for the stretching of the rope, and to advocate somewhat heavier equipments than the above table would give; that is, if it is desired to transmit 20 horse-power, for instance, to put in a plant that would transmit 25 to 30 horse-power, thus avoiding the necessity of having to take up a comparatively small amount of stretch. On rubber and leather filling, however, the amount of power capable of being transmitted is 40 per cent greater than for wood, so that this filling is generally used, and in this case no allowance need be made for stretch, as such sheaves will likely transmit the power given by the table, under all possible deflections of the rope.

Under ordinary conditions, ropes of seven wires to the strand, laid about a hemp core, are best adapted to the transmission of power, but conditions often occur where 12- or 19-wire rope is to be preferred, as stated below.

Deflections of the Rope.—The tension of the rope is measured by the amount of sag or deflection at the centre of the span, and the deflection corresponding to the maximum safe working tension is determined by the following formulæ, in which S represents the span in feet:

	Steel Rope.	Iron Rope.
Def. of still rope at centre, in feet....	$h = .00004S^2$	$h = .00008S^2$
" driving " " "	$h_1 = .000025S^2$	$h_1 = .00005S^2$
" slack " " "	$h_2 = .0000875S^2$	$h_2 = .000175S^2$

Limits of Span.—On spans of less than sixty feet, it is impossible to splice the rope to such a degree of nicety as to give exactly the required deflection, and as the rope is further subject to a certain amount of stretch, it becomes necessary in such cases to apply mechanical means for producing the proper tension, in order to avoid frequent splicing, which is very objectionable; but care should always be exercised in using such tightening devices that they do not become the means, in unskilled hands, of overstraining the rope. The rope also is more sensitive to every irregularity in the sheaves and the fluctuations in the amount of power transmitted, and is apt to sway to such an extent beyond the narrow limits of the required deflections as to cause a jerking motion, which is very injurious. For this reason on very short spans it is found desirable to use a considerably heavier rope than that actually required to transmit the power; or in other words, instead of a 7-wire rope corresponding to the conditions of maximum tension, it is better to use a 19-wire rope of the same size wires, and to run this under a tension considerably below the maximum. In this way is obtained the advantages of increased weight and no stretch, without having to use larger sheaves, while the wear will be greater in proportion to the increased surface.

In determining the maximum limit of span, the contour of the ground and the available height of the terminal sheaves must be taken into consideration. It is customary to transmit the power through the lower portion of the rope, as in this case the greatest deflection in this portion occurs when the rope is at rest. When running, the lower portion rises and the upper portion sinks, thus enabling obstructions to be avoided which otherwise would have to be removed, or make it necessary to erect very high towers. The maximum limit of span in this case is determined by the maximum deflection that may be given to the upper portion of the rope when running, which for sheaves of 10 ft. diameter is about 600 feet.

Much greater spans than this, however, are practicable where the contour of the ground is such that the upper portion of the rope may be the driver, and there is nothing to interfere with the proper deflection of the under portion. Some very long transmissions of power have been effected in this way without an intervening support, one at Lockport, N. Y., having a clear span of 1700 feet.

Long-distance Transmissions.—When the distance exceeds the limit for a clear span, intermediate supporting sheaves are used, with plain grooves (not filled), and as a rule the taut portion of the rope requires fewer than the slack portion. The size of these sheaves will depend on the angle of the bend, gauged by the tangents to the curves of the rope at the points of inflection. If the curvature due to the tension, regardless of the size of the sheave, is less than that of the minimum sheave corresponding to a maximum safe working tension, the intermediate sheaves should be equal in size to the terminal sheaves or minimum sheave corresponding to the rope used (see table of minimum sheaves), but if it is greater, smaller intermediate sheaves may be used. (See Bending Curvature of Wire Ropes, below.)

In very long transmissions of power, requiring numerous intermediate supports, it is found impracticable to run the rope at the high speeds maintained in "flying transmissions." The rope therefore is run under a higher working tension, made practicable by wrapping it several times about grooved terminal drums, with a lap about a sheave on a take-up or counterweighted carriage, which preserves a constant tension in the slack portion.

Inclined Transmissions.—When the terminal sheaves are not on the same elevation, the tension at the upper sheave will be greater than that at the lower, but this difference is so slight, in most cases, that it may be ignored. The span to be considered is the horizontal distance between the sheaves, and the principles governing the limits of span will hold good in this case, so that for very steep inclinations it becomes necessary to resort to tightening devices for maintaining the requisite tension in the rope. The limiting case of inclined transmissions occurs when one wheel is directly above the other. The rope in this case produces no tension whatever on the lower wheel, while the upper is subject only to the weight of the rope, which is usually so insignificant that it may be neglected altogether, and tightening sheaves are therefore an absolute necessity.

Bending Curvature of Wire Ropes.—The curvature due to any bend in a wire rope is dependent on the tension, and is not always the same as the sheave in contact, but may be greater, which explains how it is that large ropes are frequently run around comparatively small sheaves without detriment, since it is possible to place these so close that the bending angle on each will be such that the resulting curvature will not overstrain the wires. This curvature may be ascertained from the formula and table on the next page, which give the theoretical radii of curvature in inches for various sizes of ropes and different angles for one pound tension in the rope. Dividing these figures by the actual tension in pounds, gives the radius of curvature assumed by the rope in cases where this exceeds the curvature of the sheave. The rigidity of the rope or internal friction of the wires and core has not been taken into account in these figures, but the effect of this is insignificant, and it is on the safe side to ignore it. By the "angle of bend" is meant the angle between the tangents to the curves of the rope at the points of inflection. When the rope is straight the angle is 180°. For angles less than 180° the radius of curvature in most cases will be less than that corresponding to the safe working tension, and the proper size of sheave to use in such cases will be governed by the table headed "Diameters of Minimum Sheaves Corresponding to a Maximum Safe Working Tension."

Radius of Curvature of Wire Ropes in Inches for 1-lb. Tension.

Formula: $R = E\delta^4 n \div 5.25 t \cos \frac{1}{2}\theta$; in which R = radius of curvature; E = modulus of elasticity = 28,500,000; δ = diameter of wires; n = no. of wires; θ = angle of bend; t = working stress (lbs. and ins.).

Divide by stress in pounds to obtain radius in inches.

	Diam. of wire.	160°	165°	170°	172°	174°	176°	178°
19-Wire Rope	½	4,226	5,623	8,421	10,949	14,593	21,884	43,762
	⅝	11,090	14,753	22,095	26,731	35,628	53,429	106,841
	¾	22,274	29,633	45,412	54,417	72,580	108,767	217,502
	⅞	43,184	57,451	86,040	102,688	136,869	205,251	410,440
	1	71,816	95,541	148,085	175,182	233,492	350,150	700,193
	1⅛	112,763	150,016	224,667	280,607	374,010	560,872	1,121,574
	1¼	169,135	225,012	336,982	427,669	570,050	854,858	1,709,456
7-Wire Rope	½	12,914	17,179	25,727	31,125	41,485	62,212	124,405
	⅝	29,762	39,594	59,297	75,988	101,282	151,884	303,727
	¾	62,318	82,899	124,151	157,570	210,018	314,948	629,800
	⅞	116,289	154,641	231,593	291,917	389,085	583,479	1,164,092
	1	199,323	265,173	397,129	497,998	663,767	995,390	1,990,478
	1⅛	320,556	426,459	638,674	797,607	1,063,217	1,594,422	3,188,359
	1¼	504,402	671,041	1,004,965	1,215,817	1,620,518	2,430,151	4,859,561

ROPE-DRIVING.

The transmission of power by cotton or manila ropes is a competitor with gearing and leather belting when the amount of power is large, or the distance between the power and the work is comparatively great. The following is condensed from a paper by C. W. Hunt, Trans. A. S. M. E., xii. 230:

But few accurate data are available, on account of the long period required in each experiment, a rope lasting from three to six years. Installations which have been successful, as well as those in which the wear of the rope was destructive, indicate that 200 lbs. on a rope one inch in diameter is a safe and economical working strain. When the strain is materially increased, the wear is rapid.

In the following equations

C = circumference of rope in inches; g = gravity;
D = sag of the rope in inches; H = horse-power;
F = centrifugal force in pounds; L = distance between pulleys in feet;
P = pounds per foot of rope; w = working strain in pounds;
R = force in pounds doing useful work;
S = strain in pounds on the rope at the pulley;
T = tension in pounds of driving side of the rope;
t = tension in pounds on slack side of the rope;
v = velocity of the rope in feet per second;
W = ultimate breaking strain in pounds.

$$W = 720C^2; \quad P = .032C^2; \quad w = 20C^2.$$

This makes the normal working strain equal to 1/36 of the breaking strength, and about 1/25 of the strength at the splice. The actual strains are ordinarily much greater, owing to the vibrations in running, as well as from imperfectly adjusted tension mechanism.

For this investigation we assume that the strain on the driving side of a rope is equal to 200 lbs. on a rope one inch in diameter, and an equivalent strain for other sizes, and that the rope is in motion at various velocities of from 10 to 140 ft. per second.

The centrifugal force of the rope in running over the pulley will reduce the amount of force available for the transmission of power. The centrifugal force $F = Pv^2 \div g$.

At a speed of about 80 ft. per second, the centrifugal force increases faster than the power from increased velocity of the rope, and at about 140 ft. per second equals the assumed allowable tension of the rope. Computing this force at various speeds and then subtracting it from the assumed maximum tension, we have the force available for the transmission of power. The whole of this force cannot be used, because a certain amount of tension on the slack side of the rope is needed to give adhesion to the pulley. What tension should be given to the rope for this purpose is uncertain, as there are no experiments which give accurate data. It is known from considerable experience that when the rope runs in a groove whose sides are inclined toward each other at an angle of 45° there is sufficient adhesion when the ratio of the tensions $T \div t = 2$.

For the present purpose, T can be divided into three parts: 1. Tension doing useful work; 2. Tension from centrifugal force; 3. Tension to balance the strain for adhesion.

The tension t can be divided into two parts: 1. Tension for adhesion; 2. Tension from centrifugal force.

It is evident, however, that the tension required to do a given work should not be materially exceeded during the life of the rope.

There are two methods of putting ropes on the pulleys; one in which the ropes are single and spliced on, being made very taut at first, and less so as the rope lengthens, stretching until it slips, when it is respliced. The other method is to wind a single rope over the pulley as many turns as needed to obtain the necessary horse-power and put a tension pulley to give the necessary adhesion and also take up the wear. The tension t required to transmit the normal horse-power for the ordinary speeds and sizes of rope is computed by formula (1), below. The total tension T on the driving side of the rope is assumed to be the same at all speeds. The centrifugal force, as well as an amount equal to the tension for adhesion on the slack side of the rope, must be taken from the total tension T to ascertain the amount of force available for the transmission of power.

It is assumed that the tension on the slack side necessary for giving adhesion is equal to one half the force doing useful work on the driving side of the rope; hence the force for useful work is $R = \frac{2(T - F)}{3}$; and the tension on the slack side to give the required adhesion is $\frac{1}{3}(T - F)$. Hence

$$t = \frac{(T - F)}{3} + F. \quad (1)$$

The sum of the tensions T and t is not the same at different speeds, as the equation (1) indicates.

As F varies as the square of the velocity, there is, with an increasing speed of the rope, a decreasing useful force, and an increasing total tension, t, on the slack side.

With these assumptions of allowable strains the horse-power will be

$$H = \frac{2v(T - F)}{3 \times 550}. \quad (2)$$

Transmission ropes are usually from 1 to 1¾ inches in diameter. A computation of the horse-power for four sizes at various speeds and under ordinary conditions, based on a maximum strain equivalent to 200 lbs. for a rope one inch in diameter, is given in Fig. 166. The horse-power of other sizes is readily obtained from these. The maximum power is transmitted, under the assumed conditions, at a speed of about 80 feet per second.

The wear of the rope is both internal and external; the internal is caused by the movement of the fibres on each other, under pressure in bending over the sheaves, and the external is caused by the slipping and the wedging in the grooves of the pulley. Both of these causes of wear are, within the limits of ordinary practice, assumed to be directly proportional to the speed. Hence, if we assume the coefficient of the wear to be k, the wear will be kv, in which the wear increases directly as the velocity, but the horse-power that can be transmitted, as equation (2) shows, will not vary at the same rate.

The rope is supposed to have the strain T constant at all speeds on the driving side, and in direct proportion to the area of the cross-section; hence the catenary of the driving side is not affected by the speed or by the diameter of the rope.

The deflection of the rope between the pulleys on the slack side varies with each change of the load or change of the speed, as the tension equation (1) indicates.

The deflection of the rope is computed for the assumed value of T and t

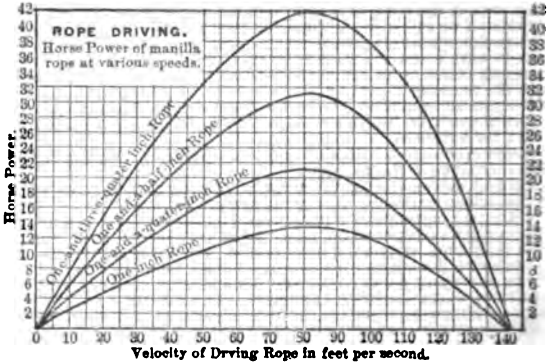

FIG. 166.

by the parabolic formula $S = \frac{PL^2}{8D} + PD$, S being the assumed strain T on the driving side, and t, calculated by equation (1), on the slack side. The tension t varies with the speed.

Horse-power of Transmission Rope at Various Speeds.

Computed from formula (2), given above.

Diam. of Ropes.	Speed of the Rope in feet per minute.											Smallest Diam. of Pulleys in Inches.
	1500	2000	2500	3000	3500	4000	4500	5000	6000	7000	8000	
½	1.45	1.9	2.3	2.7	3	3.2	3.4	3.4	3.1	2.2	0	20
⅝	2.3	3.2	3.6	4.2	4.6	5.0	5.3	5.3	4.9	3.4	0	24
¾	3.3	4.3	5.2	5.8	6.7	7.2	7.7	7.7	7.1	4.9	0	30
⅞	4.5	5.9	7.0	8.2	9.1	9.8	10.8	10.8	9.3	6.9	0	36
1	5.8	7.7	9.2	10.7	11.9	12.8	13.6	13.7	12.5	8.8	0	42
1¼	9.2	12.1	14.3	16.8	18.6	20.0	21.2	21.4	19.5	13.8	0	54
1½	13.1	17.4	20.7	23.1	26.8	28.8	30.6	30.8	28.2	19.8	0	60
1¾	18	23.7	28.2	32.8	36.4	39.2	41.5	41.8	37.4	27.6	0	72
2	23.2	30.8	36.8	42.8	47.6	51.2	54.4	54.8	50	35.2	0	84

The following notes are from the circular of the C. W. Hunt Co., New York:

For a temporary installation, when the rope is not to be long in use, it might be advisable to increase the work to double that given in the table.

For convenience in estimating the necessary clearance on the driving and on the slack sides, we insert a table showing the sag of the rope at different speeds when transmitting the horse-power given in the preceding table. When at rest the sag is not the same as when running, being greater on the driving and less on the slack sides of the rope. The sag of the driving side when transmitting the normal horse-power is the same no matter what size of rope is used or what the speed driven at, because the assumption is that the strain on the rope shall be the same at all speeds when transmitting the assumed horse-power, but on the slack side the strains, and consequently the sag, vary with the speed of the rope and also with the horse-power. The table gives the sag for three speeds. If the actual sag is less than given in the table, the rope is strained more than the work requires.

This table is only approximate, and is exact only when the rope is running at its normal speed, transmitting its full load and strained to the assumed amount. All of these conditions are varying in actual work, and the table must be used as a guide only.

Sag of the Rope between Pulleys.

Distance between Pulleys in feet.	Driving Side.	Slack Side of Rope.		
	All Speeds.	80 ft. per sec.	60 ft. per sec.	40 ft. per sec.
40	0 feet 4 inches	0 feet 7 inches	0 feet 9 inches	0 feet 11 inches
60	0 " 10 "	1 " 5 "	1 " 8 "	1 " 11 "
80	1 " 5 "	2 " 4 "	2 " 10 "	3 " 8 "
100	2 " 0 "	3 " 8 "	4 " 5 "	5 " 2 "
120	2 " 11 "	5 " 8 "	6 " 8 "	7 " 4 "
140	3 " 10 "	7 " 2 "	8 " 9 "	9 " 9 "
160	5 " 1 "	9 " 3 "	11 " 8 "	14 " 0 "

The size of the pulleys has an important effect on the wear of the rope—the larger the sheaves, the less the fibres of the rope slide on each other, and consequently there is less internal wear of the rope. The pulleys should not be less than forty times the diameter of the rope for economical wear, and as much larger as it is possible to make them. This rule applies also to the idle and tension pulleys as well as to the main driving pulley.

The angle of the sides of the grooves in which the rope runs varies, with different engineers, from 45° to 60°. It is very important that the sides of these grooves should be carefully polished, as the fibres of the rope rubbing on the metal as it comes from the lathe tools will gradually break fibre by fibre, and so give the rope a short life. It is also necessary to carefully avoid all sand or blow holes, as they will cut the rope out with surprising rapidity.

Much depends also upon the arrangement of the rope on the pulleys, especially where a tension weight is used. Experience shows that the increased wear on the rope from bending the rope first in one direction and then in the other is similar to that of wire rope. At mines where two cages are used, one being hoisted and one lowered by the same engine doing the same work, the wire ropes, cut from the same coil, are usually arranged so that one rope is bent continuously in one direction and the other rope is bent first in one direction and then in the other, in winding on the drum of the engine. The rope having the opposite bends wears much more rapidly than the other, lasting about three quarters as long as its mate. This difference in wear shows in manila rope, both in transmission of power and in coal-hoisting. The pulleys should be arranged, as far as possible, to bend the rope in one direction.

The wear of the rope is independent of the distance apart of the shafts, since the wear takes place only on the pulleys; hence in transmitting power any distance within the limits of rope-driving, the life of the rope will be the same whether the distance is small or great, but the first cost will be in proportion to the distance.

Tension on the Slack Part of the Rope.

Speed of Rope, in feet per second.	Diameter of the Rope and Pounds Tension on the Slack Rope.								
	½	⅝	¾	⅞	1	1¼	1½	1¾	2
20	10	27	40	54	71	110	162	216	283
30	14	29	42	56	74	115	170	226	296
40	15	31	45	60	79	123	181	240	315
50	16	33	49	65	85	132	195	259	339
60	18	36	53	71	93	145	214	285	373
70	19	39	50	78	101	158	236	310	406
80	21	43	64	85	111	173	255	340	445
90	24	48	70	93	122	190	279	372	487

For large amounts of power it is common to use a number of ropes lying side by side in grooves, each spliced separately. For lighter drives some engineers use one rope wrapped as many times around the pulleys as is necessary to get the horse-power required, with a tension pulley to take up the slack as the rope wears when first put in use. The weight put upon this tension pulley should be carefully adjusted, as the overstraining of the rope from this cause is one of the most common errors in rope driving. We therefore give a table showing the proper strain on the rope for the various sizes, from which the tension weight to transmit the horse-power in the tables is easily deduced. This strain can be still further reduced if the horse-power transmitted is usually less than the nominal work which the rope was proportioned to do, or if the angle of groove in the pulleys is acute.

DIAMETER OF PULLEYS AND WEIGHT OF ROPE.

Diameter of Rope, in inches.	Smallest Diameter of Pulleys, in inches.	Length of Rope to allow for Splicing, in feet.	Approximate Weight, in lbs. per foot of rope.
½	20	6	.12
⅝	24	6	.18
¾	30	7	.24
⅞	36	8	.32
1	42	9	.49
1¼	54	10	.60
1½	60	12	.83
1¾	72	13	1.10
2	84	14	1.40

With a given velocity of the driving-rope, the weight of rope required for transmitting a given horse-power is the same, no matter what size rope is adopted. The smaller rope will require more parts, but the weight will be the same.

Miscellaneous Notes on Rope-driving.—W. H. Booth communicates to the *Amer. Machinist* the following data from English practice with cotton ropes. The calculated figures are based on a total allowable tension on a 1¾-inch rope of 600 lbs., and an initial tension of 1/10 the total allowed stress, which corresponds fairly with practice.

Diameter of rope	1¼"	1⅜"	1½"	1⅝"	1¾"	1⅞"	2"
Weight per foot, lbs	.5	.6	.72	.844	.98	1.125	1.3
Centrifugal tension = V^2 divided by	64	53	44	38	33	28	25
" for $V = 80$ ft. per sec., lbs.	100	121	145	170	193	228	256
Total tension allowable	300	360	430	500	600	675	780
Initial tension	30	36	43	50	60	67	78
Net working tension at 80 ft. velocity	170	203	242	280	347	380	446
Horse-power per rope " "	24	28	34	41	49	54	63

The most usual practice in Lancashire is summed up roughly in the following figures: 1¾-inch cotton ropes at 5000 ft. per minute velocity = 50 H.P. per rope. The most common sizes of rope now used are 1¾ and 1⅝ in. The maximum horse-power for a given rope is obtained at about 80 to 82 feet per second. Above that speed the power is reduced by centrifugal tension. At a speed of 2500 ft. per minute four ropes will do about the same work as three at 5000 ft. per min.

Cotton ropes do not require much lubrication in the sense that it is required by ropes made of the rough fibre of manila hemp. Merely a slight surface dressing is all that is required. For small ropes, common in spinning machinery, from ½ to ¾ inch diameter, it is the custom to prevent the fluffing of the ropes on the surface by a light application of a mixture of black-lead and molasses,—but only enough should be used to lay the fibres,—put upon one of the pulleys in a series of light dabs.

Reuleaux's Constructor gives as the "specific capacity" of hemp rope in actual practice, that is, the horse-power transmitted per square inch of cross-section for each foot of linear velocity per minute, .004 to .002, the cross-section being taken as that due to the full outside diameter of the rope. For a 1¾-in. rope, with a cross-section of 2.405 sq. in., at a velocity of 5000 ft. per min., this gives a horse-power of from 24 to 48, as against 41.8 by Mr. Hunt's table and 49 by Mr. Booth's.

Reuleaux gives formulæ for calculating sources of loss in hemp-rope transmission due to (1) journal friction, (2) stiffness of ropes, and (3) creep of ropes. The constants in these formulæ are, however, uncertain from lack of experimental data. He calculates an average case giving loss of power due to journal friction = 4%, to stiffness 7.8%, and to creep 5%, or 16.8% in all, and says this is not to be considered higher than the actual loss.

Spencer Miller, in a paper entitled "A Problem in Continuous Rope-driving" (Trans. A. S. C. E., 1897), reviews the difficulties which occur in rope-driving, with a continuous rope from a large to a small pulley. He adopts the angle of 45° as a minimum angle to use on the smaller pulley, and recommends that the larger pulley be grooved with a wider angle to a degree such that the resistance to slipping is equal in both wheels. By doing this the effect of the tension weight is felt equally throughout all the slack strands of the rope-drive, hence the tight ropes pull equally. It is shown that when the wheels are grooved alike the strains in the various ropes may differ greatly, and to such a degree that danger is introduced, for while one-half the tension weight should represent the maximum strain on the slack rope, it is demonstrated in the paper that the actual maximum strain may be even four or six times as great.

In a drive such as is recommended, with a wide angle in the large sheave with the larger arc of contact, the conditions governing the ropes are the same as if the wheels were of the same diameter; and where the wheels are of the same diameter, with a proper tension weight, the ropes pull alike. It is claimed that by widening the angle of the large sheave not only is there no power lost, but there is actually a great gain in power transmitted. An example is given in which it is shown that in that instance the power transmitted is nearly doubled. Mr. Miller refers to a 250-horse-power drive which has been running ten years, the large pulley being grooved 60° and the smaller 45°. This drive was designed to use a 1¼-in. manila rope, but the grooves were made deep enough so that a ⅞-in. rope would not bottom. In order to determine the value of the drive a common ⅞-in. rope was put in at first, and lasted six years, working under a factor of safety of only 14. He recommends, however, the employment in continuous rope-driving of a factor of safety of not less than 20.

The Walker Company adopts a curved form of groove instead of one with straight sides inclined to each other at 45°. The curves are concave to the rope. The rope rests on the sides of the groove in driving and driven pulleys. In idler pulleys the rope rests on the bottom of the groove, which is semicircular. The Walker Company also uses a "differential" drum for heavy rope-drives, in which the grooves are contained each in a separate ring which is free to slide on the turned surface of the drum in case one rope pulls more than another.

A heavy rope-drive on the separate, or English, rope system is described and illustrated in *Power*, April, 1892. It is in use at the India Mill at Darwen, England. This mill was originally driven by gears, but did not prove successful, and rope-driving was resorted to. The 85,000 spindles and preparation are driven by a 2000-horse-power tandem compound engine, with cylinders 23 and 44 inches in diameter and 72-inch stroke, running at 54 revolutions per minute. The fly-wheel is 30 feet in diameter, weighs 65 tons, and is arranged with 30 grooves for 1¾-inch ropes. These ropes lead off to receiving-pulleys upon the several floors, so that each floor receives its power direct from the fly-wheel. The speed of the ropes is 5089 feet per minute, and five 7-foot receivers are used, the number of ropes upon each being proportioned to the amount of power required upon the several floors. Lambeth cotton ropes are used. (For much other information on this subject see "Rope-Driving," by J. J. Flather, John Wiley & Sons, 1895.)

FRICTION AND LUBRICATION.

Friction is defined by Rankine as that force which acts between two bodies at their surface of contact so as to resist their sliding on each other, and which depends on the force with which the bodies are pressed together.

Coefficient of Friction.—The ratio of the force required to slide a body along a horizontal plane surface to the weight of the body is called the coefficient of friction. It is equivalent to the tangent of the *angle of repose*, which is the angle of inclination to the horizontal of an inclined plane on which the body will just overcome its tendency to slide. The angle is usually denoted by θ, and the coefficient by f. $f = \tan \theta$.

Friction of Rest and of Motion.—The force required to start a body sliding is called the friction of rest, and the force required to continue its sliding after having started is called the friction of motion.

Rolling Friction is the force required to roll a cylindrical or spherical body on a plane or on a curved surface. It depends on the nature of the surfaces and on the force with which they are pressed together, but is essentially different from ordinary, or sliding, friction.

Friction of Solids.—Rennie's experiments (1829) on friction of solids, usually unlubricated and dry, led to the following conclusions:

1. The laws of sliding friction differ with the character of the bodies rubbing together.
2. The friction of fibrous material is increased by increased extent of surface and by time of contact, and is diminished by pressure and speed.
3. With wood, metal, and stones, within the limit of abrasion, friction varies only with the pressure, and is independent of the extent of surface, time of contact and velocity.
4. The limit of abrasion is determined by the hardness of the softer of the two rubbing parts.
5. Friction is greatest with soft and least with hard materials.
6. The friction of lubricated surfaces is determined by the nature of the lubricant rather than by that of the solids themselves.

Friction of Rest. (Rennie.)

Pressure, lbs. per square inch.	Values of f.			
	Wrought iron on Wrought Iron.	Wrought on Cast Iron.	Steel on Cast Iron.	Brass on Cast Iron.
187	.25	.28	.30	.23
224	.27	.29	.33	.22
336	.31	.33	.35	.21
448	.38	.37	.35	.21
560	.41	.37	.36	.23
672	Abraded	.38	.40	.23
784	"	Abraded	Abraded	.23

Law of Unlubricated Friction.—A. M. Wellington, *Eng'g News*, April 7, 1888, states that the most important and the best determined of all the laws of unlubricated friction may be thus expressed:

The coefficient of unlubricated friction decreases materially with velocity, is very much greater at minute velocities of 0 +, falls very rapidly with minute increases of such velocities, and continues to fall much less rapidly with higher velocities up to a certain varying point, following closely the laws which obtain with lubricated friction.

Friction of Steel Tires Sliding on Steel Rails. (Westinghouse & Galton.)

Speed, miles per hour..........	10	15	25	38	45	50
Coefficient of friction..........	0.110	.087	.080	.051	.047	.040
Adhesion, lbs. per ton (2240 lbs.)	246	195	179	128	114	90

Rolling Friction is a consequence of the irregularities of form and the roughness of surface of bodies rolling one over the other. Its laws are not yet definitely established in consequence of the uncertainty which exists in experiment as to how much of the resistance is due to roughness of surface, how much to original and permanent irregularity of form, and how much to distortion under the load. (Thurston.)

Coefficients of Rolling Friction.—If R = resistance applied at the circumference of the wheel, W = total weight, r = radius of the wheel, and f = a coefficient, $R = fW \div r$. f is very variable. Coulomb gives .06 for wood, .005 for metal, where W is in pounds and r in feet. Tredgold made the value of f for iron on iron .002.

For wagons on soft soil Morin found f = .065, and on hard smooth roads .02.

A Committee of the Society of Arts (Clark, R. T. D.) reported a loaded omnibus to exhibit a resistance on various loads as below:

Pavement	Speed per hour.	Coefficient.	Resistance.
Granite	2.87 miles.	.007	17.41 per ton.
Asphalt....	3.56 "	.0121	27.14 "
Wood	3.34 "	.0185	41.60 "
Macadam, gravelled.....	3.45 "	.0199	44.48 "
" granite, new..	3.51 "	.0451	101.09 "

Thurston gives the value of f for ordinary railroads, .003, well-laid railroad track, .002; best possible railroad track, .001.

The few experiments that have been made upon the coefficients of rolling friction, apart from axle friction, are too incomplete to serve as a basis for practical rules. (Trautwine).

Laws of Fluid Friction.—For all fluids, whether liquid or gaseous, the resistance is (1) independent of the pressure between the masses in contact; (2) directly proportional to the area of rubbing-surface; (3) proportional to the square of the relative velocity at moderate and high speeds, and to the velocity nearly at low speeds; (4) independent of the nature of the surfaces of the solid against which the stream may flow, but dependent to some extent upon their degree of roughness; (5) proportional to the density of the fluid, and related in some way to its viscosity. (Thurston.)

The Friction of Lubricated Surfaces approximates to that of solid friction as the journal is run dry, and to that of fluid friction as it is flooded with oil.

Angles of Repose and Coefficients of Friction of Building Materials. (From Rankine's Applied Mechanics.)

	θ.	$f = \tan\theta$.	$\frac{1}{\tan\theta}$.
Dry masonry and brickwork..	31° to 35°	.6 to .7	1.67 to 1.4
Masonry and brickwork with damp mortar	36½°	.74	1.35
Timber on stone...............	22°	about .4	2.5
Iron on stone.	35° to 16¾°	.7 to .3	1.43 to 3.3
Timber on timber.	26½° to 11½°	.5 to .2	2 to 5
" " metals.	31° to 11½°	.6 to .2	1.67 to 5
Metals on metals	14° to 8½°	.25 to .15	4 to 6.67
Masonry on dry clay	27°	.51	1.96
" " moist clay.	18¼°	.33	3.
Earth on earth	14° to 45°	.25 to 1.0	4 to 1
" " " dry sand, clay, and mixed earth.....	21° to 37°	.38 to .75	2.63 to 1.33
Earth on earth, damp clay....	45°	1.0	1
" " " wet clay......	17°	.31	3.23
" " " shingle and gravel	39° to 48°	.81	1.23 to 0.9

Friction of Motion.—The following is a table of the angle of repose θ, the coefficient of friction $f = \tan\theta$, and its reciprocal, $1 \div f$, for the materials of mechanism—condensed from the tables of General Morin (1831), and other sources, as given by Rankine:

No.	Surfaces.	θ.	f.	1 ÷ f.
1	Wood on wood, dry.. ..	14° to 26½°	.25 to .5	4 to 2
2	" " " soaped..	11½° to 2°	.2 to .04	5 to 25
3	Metals on oak, dry......	26½° to 31°	.5 to .6	2 to 1.67
4	" " " wet......	13½° to 14°	.24 to .26	4.17 to 3.85
5	" " " soapy.. .	11½°	.2	5
6	" " elm, dry	11½° to 14°	.2 to .25	5 to 4
7	Hemp on oak, dry.......	28°	.53	1.89
8	" " " wet......	18½°	.33	3
9	Leather on oak...... ...	15° to 19½°	.27 to .38	3.7 to 2.86
10	" " metals, dry..	29½°	.56	1.79
11	" " " wet..	20°	.36	2.78
12	" " " greasy	13°	.23	4.35
13	" " " oily...	8½°	.15	6.67
14	Metals on metals, dry...	8½° to 11°	.15 to .2	6.67 to 5
15	" " " wet...	16½°	.3	3.33
16	Smooth surfaces, occasionally greased.......	4° to 4½°	.07 to .08	14.3 to 12.5
17	Smooth surfaces, continuously greased.....	3°	.05	20
18	Smooth surfaces, best results.............. ...	1¾° to 2°	.03 to .036	....
19	Bronze on lignum vitæ, constantly wet........	3° ?	.05 ?	....

Coefficients of Friction of Journals. (Morin.)

Material.	Unguent.	Lubrication.	
		Intermittent.	Continuous.
Cast iron on cast iron....	Oil, lard tallow.	.07 to .08	.03 to .054
	Unctuous and wet.	.14	
Cast iron on bronze......	Oil, lard, tallow.	.07 to .08	.03 to .054
	Unctuous and wet.	.16	
Cast iron on lignum-vitæ..	Oil, lard.	....	.09
Wrought iron on cast iron / " " " bronze..	Oil, lard, tallow.	.07 to .08	.03 to .054
Iron on lignum vitæ.....	Oil, lard.	.11	
	Unctuous.	.19	
Bronze on bronze........	Olive-oil.	.10	
	Lard.	.09	

Prof. Thurston says concerning the above figures that much better results are probably obtained in good practice with ordinary machinery. Those here given are so greatly modified by variations of speed, pressure, and temperature, that they cannot be taken as correct for general purposes.

Average Coefficients of Friction. Journal of cast iron in bronze bearing; velocity 720 feet per minute; temperature 70° F.; intermittent feed through an oil-hole. (Thurston on Friction and Lost Work.)

Oils.	Pressures, pounds per square inch.			
	8	16	32	48
Sperm, lard, neat's-foot, etc.	.159 to .250	.138 to .192	.086 to .141	.077 to .144
Olive, cotton-seed, rape, etc.	.160 " .283	.107 " .245	.101 " .168	.079 " .131
Cod and menhaden.	.248 " .278	.124 " .167	.097 " .102	.081 " .122
Mineral lubricating-oils. ...	.154 " .261	.145 " .233	.086 " .178	.094 " .222

With fine steel journals running in bronze bearings and continuous lubrication, coefficients far below those above given are obtained. Thus with sperm-oil the coefficient with 50 lbs. per square inch pressure was .0034; with 200 lbs., .0051; with 300 lbs., .0057.

For very low pressures, as in spindles, the coefficients are much higher. Thus Mr. Woodbury found, at a temperature of 100° and a velocity of 600 feet per minute,

Pressures, lbs. per sq. in......	1	2	3	4	5
Coefficient	.38	.27	.22	.18	.17

These high coefficients, however, and the great decrease in the coefficient at increased pressures are limited as a practical matter only to the smaller pressures which exist especially in spinning machinery, where the pressure is so light and the film of oil so thick that the viscosity of the oil is an important part of the total frictional resistance.

Experiments on Friction of a Journal Lubricated by an Oil-bath (reported by the Committee on Friction, Proc. Inst. M. E., Nov. 1883) show that the absolute friction, that is, the absolute tangential force per square inch of bearing, required to resist the tendency of the brass to go round with the journal, is nearly a constant under all loads, within ordinary working limits. Most certainly it does not increase in direct proportion to the load, as it should do according to the ordinary theory of solid friction. The results of these experiments seem to show that the friction of a perfectly lubricated journal follows the laws of liquid friction much more closely than those of solid friction. They show that under these circumstances the friction is nearly independent of the pressure per square inch, and that it increases with the velocity, though at a rate not nearly so rapid as the square of the velocity.

The experiments on friction at different temperatures indicate a great diminution in the friction as the temperature rises. Thus in the case of lard-oil, taking a speed of 450 revolutions per minute, the coefficient of friction at a temperature of 120° is only one third of what it was at a temperature of 60.

The journal was of steel, 4 inches diameter and 6 inches long, and a gun-metal brass, embracing somewhat less than half the circumference of the journal, rested on its upeer side, on which the load was applied. When the bottom of the journal was immersed in oil, and the oil therefore carried under the brass by rotation of the journal, the greatest load carried with rape-oil was 573 lbs per square inch, and with mineral oil 625 lbs.

In experiments with ordinary lubrication, the oil being fed in at the centre of the top of the brass, and a distributing groove being cut in the brass parallel to the axis of the journal, the bearing would not run cool with only 100 lbs. per square inch, the oil being pressed out from the bearing-surface and through the oil-hole, instead of being carried in by it. On introducing the oil at the sides through two parallel grooves, the lubrication appeared to be satisfactory, but the bearing seized with 380 lbs. per square inch.

When the oil was introduced through two oil-holes, one near each end of the brass, and each connected with a curved groove, the brass refused to take its oil or run cool, and seized with a load of only 200 lbs. per square inch.

With an oil-pad under the journal feeding rape-oil, the bearing fairly carried 551 lbs. Mr. Tower's conclusion from these experiments is that the friction depends on the quantity and uniformity of distribution of the oil, and may be anything between the oil-bath results and seizing, according to the perfection or imperfection of the lubrication. The lubrication may be very small, giving a coefficient of 1/100; but it appeared as though it could not be diminished and the friction increased much beyond this point without imminent risk of heating and seizing. The oil-bath probably represents the most perfect lubrication possible, and the limit beyond which friction cannot be reduced by lubrication; and the experiments show that with speeds of from 100 to 200 feet per minute, by properly proportioning the bearing-surface to the load, it is possible to reduce the coefficient of friction to as low as 1/1000. A coefficient of 1/1500 is easily attainable, and probably is frequently attained, in ordinary engine-bearings in which the direction of the force is rapidly alternating and the oil given an opportunity to get between the surfaces, while the duration of the force in one direction is not sufficient to allow time for the oil film to be squeezed out.

Observations on the behavior of the apparatus gave reason to believe that with perfect lubrication the speed of minimum friction was from 100 to 150 feet per minute, and that this speed of minimum friction tends to be higher with an increase of load, and also with less perfect lubrication. By the speed of minimum friction is meant that speed in approaching which from rest the friction diminishes, and above which the friction increases.

Coefficients of Friction of Journal with Oil-bath.—Abstract of results of Tower's experiments on friction (Proc. Inst. M. E., Nov. 1883). Journal, 4 in. diam., 6 in. long; temperature, 90° F.

Lubricant in Bath.	Nominal Load, in pounds per square inch.						
	625	520	415	310	205	153	100
	Coefficients of Friction.						
Lard-oil:							
157 ft. per min.	....	.0009	.0012	.0014	.0020	.0027	.0042
471 " "	....	.0017	.0021	.0029	.0042	.0052	.009
Mineral grease:							
157 ft. per min.	.001	.0014	.0016	.0023	.0034	.0038	.0076
471 " "	.002	.0022	.0027	.004	.0066	.0083	.0151
Sperm-oil:							
157 ft. per min.	....	seiz'd	.0015	.0011	.0016	.0019	.003
471 " "	....	......	.0021	.0019	.0027	0037	.0064
Rape-oil:	(573 lb.)						
157 ft. per min.	.001	.001	.0009	.0008	.0014	.002	.004
471 " "	....	.0015	.0016	.0016	.0024	.004	.007
Mineral-oil:							
157 ft. per min.	.0018	.0012	.0012	.0014	.0021	.....	.004
471 " "	....	.0018	.002	.0024	.0035	.....	.007
Rape-oil fed by syphon lubricator:							
157 ft. per min.	....	......	.....	.0056	.0098	.....	.0125
314 " "	....	......	.....	.0068	.0077	.....	.0152
Rape-oil, pad under journal:							
157 ft. per min.	....	......	.....	.0099	.0105	.....	.0099
314 " "	....	......	.....	.0099	.0078	.....	.0133

Comparative friction of different lubricants under same circumstances, temperature 90°, oil-bath:

Sperm-oil	100 per cent.	Lard	135 per cent.
Rape-oil	106 "	Olive-oil	135 "
Mineral oil	129 "	Mineral grease	217 "

Coefficients of Friction of Motion and of Rest of a Journal.—A cast-iron journal in steel boxes, tested by Prof. Thurston at a speed of rubbing of 150 feet per minute, with lard and with sperm oil, gave the following:

Pressures per sq. in., lbs.	50	100	250	500	750	1000
Coeff., with sperm	.013	.008	.005	.004	.0043	.009
" " lard	.02	.0137	.0085	.0053	.0066	.0125

The coefficients at starting were:

With sperm	.07	.135	.14	.15	.185	.18
With lard	.07	.11	.11	.10	.12	.12

The coefficient at a speed of 150 feet per minute decreases with increase of pressure until 500 lbs. per sq. in. is reached; above this it increases. The coefficient at rest or at starting increases with the pressure throughout the range of the tests.

Value of Anti-friction Metals. (Denton.)—The various white metals available for lining brasses do not afford coefficients of friction lower than can be obtained with bare brass, but they are less liable to "overheating," because of the superiority of such material over bronze in ability to permit of abrasion or crushing, without excessive increase of friction.

Thurston (Friction and Lost Work) says that gun-bronze, Babbitt, and other soft white alloys have substantially the same friction; in other words, the friction is determined by the nature of the unguent and not by that of the rubbing-surfaces, when the latter are in good order. The soft metals run at higher temperatures than the bronze. This, however, does not necessarily indicate a serious defect, but simply deficient conductivity. The value of the white alloys for bearings lies mainly in their ready reduction to a smooth surface after any local or general injury by alteration of either surface or form.

Cast-iron for Bearings. (Joshua Rose.)—Cast iron appears to be an exception to the general rule, that the harder the metal the greater the resistance to wear, because cast iron is softer in its texture and easier to cut with steel tools than steel or wrought iron, but in some situations it is far more durable than hardened steel; thus when surrounded by steam it will wear better than will any other metal. Thus, for instance, experience has demonstrated that piston-rings of cast iron will wear smoother, better, and equally as long as those of steel, and longer than those of either wrought iron or brass, whether the cylinder in which it works be composed of brass, steel, wrought iron, or cast iron; the latter being the more noteworthy, since two surfaces of the same metal do not, as a rule, wear or work well together. So also slide-valves of brass are not found to wear so long or so smoothly as those of cast iron, let the metal of which the seating is composed be whatever it may; while, on the other hand, a cast iron slide-valve will wear longer of itself and cause less wear to its seat, if the latter is of cast iron, than if of steel, wrought iron, or brass.

Friction of Metals under Steam-pressure.—The friction of brass upon iron under steam-pressure is double that of iron upon iron. (G. H. Babcock, Trans. A. S. M. E., i. 151.)

Morin's "Laws of Friction."—1. The friction between two bodies is directly proportioned to the pressure; i.e., the coefficient is constant for all pressures.

2. The coefficient and amount of friction, pressure being the same, is independent of the areas in contact.

3 The coefficient of friction is independent of velocity, although static friction (friction of rest) is greater than the friction of motion.

Eng'g News, April 7, 1888, comments on these "laws" as follows: From 1831 till about 1876 there was no attempt worth speaking of to enlarge our knowledge of the laws of friction, which during all that period was assumed to be complete, although it was really worse than nothing, since it was for the most part wholly false. In the year first mentioned Morin began a series of experiments which extended over two or three years, and which resulted in the enunciation of these three "fundamental laws of friction," no one of which is even approximately true.

For fifty years these laws were accepted as axiomatic, and were quoted as such without question in every scientific work published during that whole period. Now that they are so thoroughly discredited it has been attempted to explain away their defects on the ground that they cover only a very limited range of pressures, areas, velocities, etc., and that Morin himself only announced them as true within the range of his conditions. It is now clearly established that there are no limits or conditions within which any one of them even approximates to exactitude, and that there are many conditions under which they lead to the wildest kind of error, while many of the constants were as inaccurate as the laws. For example, in Morin's "Table of Coefficients of Moving Friction of Smooth Plane Surfaces, perfectly lubricated," which may be found in hundreds of text-books now in use, the coefficient of wrought iron on brass is given as .075 to .103, which would make the rolling friction of railway trains 15 to 20 lbs. per ton instead of the 3 to 6 lbs. which it actually is.

General Morin, in a letter to the Secretary of the Institution of Mechanical Engineers, dated March 15, 1879, writes as follows concerning his experiments on friction made more than forty years before: "The results furnished by my experiments as to the relations between pressure, surface, and speed on the one hand, and sliding friction on the other, have always been regarded by myself, not as mathematical laws, but as close approximations to the truth, within the limits of the data of the experiments themselves. The same holds, in my opinion, for many other laws of practical mechanics, such as those of rolling resistance, fluid resistance, etc."

Prof J. E. Denton (*Stevens Indicator*, July, 1890) says: It has been generally assumed that friction between lubricated surfaces follows the simple law that the amount of the friction is some fixed fraction of the pressure between the surfaces, such fraction being independent of the intensity of the pressure per square inch and the velocity of rubbing, between certain limits of practice, and that the fixed fraction referred to is represented by the coefficients of friction given by the experiments of Morin or obtained from experimental data which represent conditions of practical lubrication, such as those given in Webber's Manual of Power.

By the experiments of Thurston, Woodbury, Tower, etc., however, it appears that the friction between lubricated metallic surfaces, such as machine bearings, is not directly proportional to the pressure, is not independent of the speed, and that the coefficients of Morin and Webber are about tenfold too great for modern journals.

Prof. Denton offers an explanation of this apparent contradiction of authorities by showing, with laboratory testing-machine data, that Morin's laws hold for bearings lubricated by a restricted feed of lubricant, such as is afforded by the oil-cups common to machinery; whereas the modern experiments have been made with a surplus feed or superabundance of lubricant, such as is provided only in railroad-car journals, and a few special cases of practice.

That the low coefficients of friction obtained under the latter conditions are realized in the case of car-journals, is proved by the fact that the temperature of car-boxes remains at 100° at high velocities; and experiment shows that this temperature is consistent only with a coefficient of friction of a fraction of one per cent. Deductions from experiments on train resistance also indicate the same low degree of friction. But these low co-efficients do not account for the internal friction of steam-engines as well as do the coefficients of Morin and Webber.

In *American Machinist*, Oct. 23, 1890. Prof. Denton says: Morin's measurement of friction of lubricated journals did not extend to light pressures. They apply only to the conditions of general shafting and engine work.

He clearly understood that there was a frictional resistance. due solely to the viscosity of the oil, and that therefore, for very light pressures, the laws which he enunciated did not prevail.

He applied his dynamometers to ordinary shaft-journals without special preparation of the rubbing-surfaces, and without resorting to artificial methods of supplying the oil.

Later experimenters have with few exceptions devoted themselves exclusively to the measurement of resistance practically due to viscosity alone. They have eliminated the resistance to which Morin confined his measurements, namely, the friction due to such contact of the rubbing-surfaces as prevail with a very thin film of lubricant between comparatively rough surfaces.

Prof. Denton also says (Trans. A. S. M. E., x. 518): "I do not believe there is a particle of proof in any investigation of friction ever made, that Morin's laws do not hold for ordinary practical oil-cups or restricted rates of feed."

Laws of Friction of well-lubricated Journals.—John Goodman (Trans. Inst. C. E. 1886, *Eng'g News*, Apr. 7 and 14, 1888), reviewing the results obtained from the testing-machines of Thurston, Tower, and Stroudley, arrives at the following laws:

LAWS OF FRICTION: WELL-LUBRICATED SURFACES.
(Oil-bath.)

1. The coefficient of friction with the surfaces efficiently lubricated is from 1/6 to 1/10 that for dry or scantily lubricated surfaces.
2. The coefficient of friction for moderate pressures and speeds varies approximately inversely as the normal pressure; the frictional resistance varies as the area in contact, the normal pressure remaining constant.
3. At very low journal speeds the coefficient of friction is abnormally high; but as the speed of sliding increases from about 10 to 100 ft. per min., the friction diminishes, and again rises when that speed is exceeded, varying approximately as the square root of the speed.
4. The coefficient of friction varies approximately inversely as the temperature, within certain limits, namely, just before abrasion takes place.

The evidence upon which these laws are based is taken from various modern experiments. That relating to Law 1 is derived from the "First Report on Friction Experiments," by Mr. Beauchamp Tower.

Method of Lubrication.	Coefficient of Friction.	Comparative Friction.
Oil-bath	.00139	1.00
Siphon lubricator	.0098	7.06
Pad under journal	.0090	6.48

With a load of 293 lbs. per sq. in. and a journal speed of 314 ft. per min. Mr. Tower found the coefficient of friction to be .0016 with an oil-bath, and

.0097, or six times as much, with a pad. The very low coefficients obtained by Mr. Tower will be accounted for by Law 2, as he found that the frictional resistance per square inch under varying loads is nearly constant, as below:

Load in lbs. per sq. in.....	529	468	415	363	310	258	205	153	100
Frictional resist. per sq. in.	.416	.514	.498	.472	.464	.438	.43	.458	.45

The frictional resistance per square inch is the product of the coefficient of friction into the load per square inch on horizontal sections of the brass. Hence, if this product be a constant, the one factor must vary inversely as the other, or a high load will give a low coefficient, and *vice versa*.

For ordinary lubrication, the coefficient is more constant under varying loads; the frictional resistance then varies directly as the load, as shown by Mr. Tower in Table VIII of his report (Proc. Inst. M. E. 1883).

With respect to Law 3, A. M. Wellington (Trans. A. S. C. E. 1884), in experiments on journals revolving at very low velocities, found that the friction was then very great, and nearly constant under varying conditions of the lubrication, load, and temperature. But as the speed increased the friction fell slowly and regularly, and again returned to the original amount when the velocity was reduced to the same rate. This is shown in the following table:

Speed, feet per minute:									
0+	2.16	3.33	4.86	8.82	21.42	35.37	53.01	89.28	106.02
Coefficient of friction:									
.118	.094	.070	.069	.055	.047	.040	.035	.030	.026

It was also found by Prof. Kimball that when the journal velocity was increased from 6 to 110 ft. per minute, the friction was reduced 70%; in another case the friction was reduced 67% when the velocity was increased from 1 to 100 ft. per minute; but after that point was reached the coefficient varied approximately with the square root of the velocity.

The following results were obtained by Mr. Tower:

Feet per minute...	209	262	314	366	419	471	Nominal Load per sq. in.
Coeff. of friction..	.0010	.0012	.0013	.0014	.0015	.0017	520 lbs.
" "	.0013	.0014	.0015	.0017	.0018	.002	468 "
" "	.0014	.0015	.0017	.0019	.0021	.0024	415 "

The variation of friction with temperature is approximately in the inverse ratio, Law 4. Take, for example, Mr. Tower's results, at 262 ft. per minute:

Temp. F.	110°	100°	90°	80°	70°	60°
Observed......	.0044	.0051	.006	.0073	.0092	.0119
Calculated....	.00451	.00518	.00608	.00733	.00964	.01252

This law does not hold good for pad or siphon lubrication, as then the coefficient of friction diminishes more rapidly for given increments of temperature, but on a gradually decreasing scale, until the normal temperature has been reached; this normal temperature increases directly as the load per sq. in. This is shown in the following table taken from Mr. Stroudley's experiments with a pad of rape oil:

Temp. F....... ...	105°	110°	115°	120°	125°	130°	135°	140°	145°
Coefficient.........	.022	.0180	.0160	.0140	.0125	.0115	.0110	.0106	.0102
Decrease of coeff..	......	.0040	.0020	.0020	.0015	.0010	.0005	.0004	.0002

In the Galton-Westinghouse experiments it was found that with velocities below 100 ft. per min., and with low pressures, the frictional resistance varied directly as the normal pressure; but when a velocity of 100 ft. per min. was exceeded, the coefficient of friction greatly diminished; from the same experiments Prof. Kennedy found that the coefficient of friction for high pressures was sensibly less than for low.

Allowable Pressures on Bearing-surfaces. (Proc. Inst. M. E., May, 1888.)—The Committee on Friction experimented with a steel ring of rectangular section, pressed between two cast-iron disks, the annular bearing-surfaces of which were covered with gun-metal, and were 12 in. inside diameter and 14 in. outside. The two disks were rotated together, and the steel ring was prevented from rotating by means of a lever, the holding force of which was measured. When oiled through grooves cut in each face of the ring and tested at from 50 to 130 revs. per min., it was found that a pressure of 75 lbs. per sq. in. of bearing-surface was as much as it would bear safely at the highest speed without seizing, although it carried 90 lbs. per sq. in. at the lowest speed. The coefficient of friction is also much higher than for a cylindrical bearing, and the friction follows the law of the friction of solids much more nearly than that of liquids. This is doubtless due to the much less perfect lubrication applicable to this form of bearing compared with a cylindrical one. The coefficient of friction appears to be about the same with the same load at all speeds, or, in other words, to be independent of the speed; but it seems to diminish somewhat as the load is increased, and may be stated approximately as 1/20 at 15 lbs. per sq. in., diminishing to 1/30 at 75 lbs. per sq. in.

The high coefficients of friction are explained by the difficulty of lubricating a collar-bearing. It is similar to the slide-block of an engine, which can carry only about one tenth the load per sq. in. that can be carried by the crank-pins.

In experiments on cylindrical journals it has been shown that when a cylindrical journal was lubricated from the side on which the pressure bore, 100 lbs. per sq. in. was the limit of pressure that it would carry; but when it came to be lubricated on the lower side and was allowed to drag the oil in with it, 600 lbs. per sq. in. was reached with impunity; and if the 600 lbs. per sq. in., which was reckoned upon the full diameter of the bearing, came to be reckoned on the sixth part of the circle that was taking the greater proportion of the load, it followed that the pressure upon that part of the circle amounted to about 1200 lbs. per sq. in.

In connection with these experiments Mr. Wicksteed states that in drilling-machines the pressure on the collars is frequently as high as 336 lbs. per sq. in., but the speed of rubbing in this case is lower than it was in any of the experiments of the Research Committee. In machines working very slowly and intermittently, as in testing-machines, very much higher pressures are admissible.

Mr. Adamson mentions the case of a heavy upright shaft carried upon a small footstep-bearing, where a weight of at least 20 tons was carried on a shaft of 5 in. diameter, or, say, 20 sq. in. area, giving a pressure of 1 ton per sq. in. The speed was 190 to 200 revs. per min. It was necessary to force the oil under the bearing by means of a pump. For heavy horizontal shafts, such as a fly-wheel shaft, carrying 100 tons on two journals, his practice for getting oil into the bearings was to flatten the journal along one side throughout its whole length to the extent of about an eighth of an inch in width for each inch in diameter up to 8 in. diameter; above that size rather less flat in proportion to the diameter. At first sight it appeared alarming to get a continuous flat place coming round in every revolution of a heavily loaded shaft; yet it carried the oil effectually into the bearing, which ran much better in consequence than a truly cylindrical journal without a flat side.

In thrust-bearings on torpedo-boats Mr. Thornycroft allows a pressure of never more than 50 lbs. per sq. in.

Prof. Thurston (Friction and Lost Work, p. 240) says 7000 to 9000 lbs. pressure per square inch is reached on the slow-working and rarely-moved pivots of swing-bridges.

Mr. Tower says (Proc. Inst. M. E., Jan. 1884): In eccentric-pins of punching and shearing-machines very high pressures are sometimes used without seizing. In addition to the alternation in the direction, the pressure is applied for only a very short space of time in these machines, so that the oil has no time to be squeezed out.

In the discussion on Mr. Tower's paper (Proc. Inst. M. E. 1885) it was stated that it is well known from practical experience that with a constant load on an ordinary journal it is difficult and almost impossible to have more than 200 lbs. per square inch, otherwise the bearing would get hot and the oil go out of it; but when the motion was reciprocating, so that the load was alternately relieved from the journal, as with crank-pins and similar journals, much higher loads might be applied than even 700 or 800 lbs. per square inch.

Mr. Goodman (Proc. Inst. C. E. 1886) found that the total frictional resistance is materially reduced by diminishing the width of the brass.

The lubrication is most efficient in reducing the friction when the brass subtends an angle of from 120° to 60°. The film is probably at its best between the angles 80° and 110°.

In the case of a brass of a railway axle-bearing where an oil-groove is cut along its crown and an oil-hole is drilled through the top of the brass into it, the wear is invariably on the off side, which is probably due to the oil escaping as soon as it reaches the crown of the brass, and so leaving the off side almost dry, where the wear consequently ensues.

In railway axles the brass wears always on the forward side. The same observation has been made in marine engine journals, which always wear in exactly the reverse way to what they might be expected. Mr. Stroudley thinks this peculiarity is due to a film of lubricant being drawn in from the under side of the journal to the aft part of the brass, which effectually lubricates and prevents wear on that side; and that when the lubricant reaches the forward side of the brass it is so attenuated down to a wedge shape that there is insufficient lubrication, and greater wear consequently follows.

Prof. J. E. Denton (*Am. Mach.*, Oct. 30, 1890) says: Regarding the pressure to which oil is subjected in railroad car-service, it is probably more severe than in any other class of practice. Car brasses, when used bare, are so imperfectly fitted to the journal, that during the early stages of their use the area of bearing may be but about one square inch. In this case the pressure per square inch is upwards of 6000 lbs. But at the slowest speeds of freight service the wear of a brass is so rapid that, within about thirty minutes the area is either increased to about three inches, and is thereby able to relieve the oil so that the latter can successfully prevent overheating of the journal, or else overheating takes place with *any oil*, and measures of relief must be taken which eliminate the question of differences of lubricating power among the different lubricants available. A brass which has been run about fifty miles under 5000 lbs. load may have extended the area of bearing-surface to about three square inches. The pressure is then about 1700 lbs. per square inch. It may be assumed that this is an average minimum area for car-service where no violent and unmanageable overheating has occurred during the use of a brass for a short time. This area will very slowly increase with any lubricant.

C. J. Field (*Power*, Feb. 1893) says: One of the most vital points of an engine for electrical service is that of main bearings. They should have a surface velocity of not exceeding 350 feet per minute, with a mean bearing-pressure per square inch of projected area of journal of not more than 80 lbs. This is considerably within the safe limit of cool performance and easy operation. If the bearings are designed in this way, it would admit the use of grease on all the main wearing-surface, which in a large type of engines for this class of work we think advisable.

Oil-pressure in a Bearing.—Mr. Beauchamp Tower (Proc. Inst. M. E., Jan. 1885) made experiments with a brass bearing 4 inches diameter by 6 inches long, to determine the pressure of the oil between the brass and the journal. The bearing was half immersed in oil, and had a total load of 8008 lbs. upon it. The journal rotated 150 revolutions per minute. The pressure of the oil was determined by drilling small holes in the bearing at different points and connecting them by tubes to a Bourdon gauge. It was found that the pressure varied from 310 to 625 lbs. per square inch, the greatest pressure being a little to the "off" side of the centre line of the top of the bearing, in the direction of motion of the journal. The sum of the upward force exerted by these pressures for the whole lubricated area was nearly equal to the total pressure on the bearing. The speed was reduced from 150 to 20 revolutions, but the oil-pressure remained the same, showing that the brass was as completely oil-borne at the lower speed as at the higher. The following was the observed friction at the lower speed:

Nominal load, lbs. per square inch...	443	333	211	89
Coefficient of friction	.00132	.00168	.00247	.0044

The nominal load per square inch is the total load divided by the product of the diameter and length of the journal. At the same low speed of 20 revolutions per minute it was increased to 676 lbs. per square inch without any signs of heating or seizing.

Friction of Car-journal Brasses. (J. E. Denton, Trans. A. S. M. E., xii. 405.)—A new brass dressed with an emery-wheel, loaded with 5000 lbs., may have an actual bearing-surface on the journal, as shown by the polish of a portion of the surface, of only 1 square inch. With this pressure of 5000 lbs. per square inch, the coefficient of friction may be 6%, and the brass may be overheated, scarred and cut but, on the contrary, it may wear down evenly to a smooth bearing, giving a highly polished area of contact of 3 square inches, or more, inside of two hours of running, gradually decreasing the pressure per square inch of contact, and a coefficient of friction of less than 0.5%. A reciprocating motion in the direction of the axis is of importance in reducing the friction. With such polished surfaces any oil will lubricate, and the coefficient of friction then depends on the viscosity of the oil. With a pressure of 1000 lbs per square inch, revolutions from 170 to 320 per minute, and temperatures of 75° to 113° F. with both sperm and parraffine oils, a coefficient of as low as 0.11% has been obtained, the oil being fed continuously by a pad.

Experiments on Overheating of Bearings.—Hot Boxes. (Denton.)—Tests with car brasses loaded from 1100 to 4500 lbs. per square inch gave 7 cases of overheating out of 82 trials. The tests show how purely a matter of chance is the overheating, as a brass which ran hot at 5000 lbs. load on one day would run cool on a later date at the same or higher pressure. The explanation of this apparently arbitrary difference of behavior is that the accidental variations of the smoothness of the surfaces, almost infinitesimal in their magnitude, cause variations of friction which are always tending to produce overheating, and it is solely a matter of chance when these tendencies preponderate over the lubricating influence of the oil. There is no appreciable advantage shown by sperm-oil, when there is no tendency to overheat—that is, paraffine can lubricate under the highest pressures which occur, as well as sperm, when the surfaces are within the conditions affording the minimum coefficients of friction.

Sperm and other oils of high heat-resisting qualities, like vegetable oil and petroleum cylinder stocks, only differ from the more volatile lubricants, like paraffine, in their ability to reduce the chances of the continual accidental infinitesimal abrasion producing overheating.

The effect of emery or other gritty substance in reducing overheating of a bearing is thus explained:

The effect of the emery upon the surfaces of the bearings is to cover the latter with a series of parallel grooves, and apparently after such grooves are made the presence of the emery does not practically increase the friction over the amount of the latter when pure oil only is between the surfaces. The infinite number of grooves constitute a very perfect means of insuring a uniform oil supply at every point of the bearings. As long as grooves in the journal match with those in the brasses the friction appears to amount to only about 10% to 15% of the pressure. But if a smooth journal is placed between a set of brasses which are grooved, and pressure be applied, the journal crushes the grooves and becomes brazed or coated with brass, and then the coefficient of friction becomes upward of 40%. If then emery is applied, the friction is made very much less by its presence, because the grooves are made to match each other, and a uniform oil supply prevails at every point of the bearings, whereas before the application of the emery many spots of the latter receive no oil between them.

Moment of Friction and Work of Friction of Sliding-surfaces, etc.

	Moment of Friction, inch-lbs.	Energy lost by Friction in ft.-lbs. per min.
Flat surfaces	....	fWS
Shafts and journals	$\frac{1}{2}fWd$	$.2618fWdn$
Flat pivots	$\frac{2}{3}fWr$	$.1745fWrn$
Collar-bearing	$\frac{2}{3}fW\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$	$.1745fWn\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$
Conical pivot	$\frac{2}{3}fWr \operatorname{cosec} a$	$.1745fWrn \operatorname{cosec} a$
Conical journal	$\frac{2}{3}fWr \sec a$	$.1745fWrn \sec a$
Truncated-cone pivot	$\frac{2}{3}fW\frac{r_2^3 - r_1^3}{r_2 \sin a}$	$.1745fW\frac{r_2^3 - r_1^3}{r_2 \sin a}$
Hemispherical pivot	fWr	$.2618fWr$
Tractrix, or Schiele's "anti-friction" pivot	fWr	$.2618fWr.$

In the above f = coefficient of friction;
W = weight on journal or pivot in pounds;
r = radius, d = diameter, in inches;
S = space in feet through which sliding takes place;
r_2 = outer radius, r_1 = inner radius;
n = number of revolutions per minute;
a = the half-angle of the cone, i.e., the angle of the slope with the axis.

To obtain the horse-power, divide the quantities in the last column by 33,000. Horse-power absorbed by friction of a shaft $= \frac{fWdn}{126050}$.

The formula for energy lost by shafts and journals is approximately true for loosely fitted bearings. Prof. Thurston shows that the correct formula varies according to the character of fit of the bearing; thus for loosely fitted journals, if U = the energy lost,

$$U = \frac{2f\pi r}{\sqrt{1+f^2}} Wn \text{ inch-pounds} = \frac{.2618fWdn}{\sqrt{1+f^2}} \text{ foot-lbs.}$$

For perfectly fitted journals $U = 2.54f\pi rWn$ inch-lbs. $= .3325fWdn$, ft.-lbs.

For a bearing in which the journal is so grasped as to give a uniform pressure throughout, $U = f\pi^2 rWn$ inch-lbs. $= .4112fWdn$. ft.-lbs.

Resistance of railway trains and wagons due to friction of trains:

$$\text{Pull on draw-bar} = \frac{f \times 2240}{R} \text{ pounds per gross ton,}$$

in which R is the ratio of the radius of the wheel to the radius of journal.

A cylindrical journal, perfectly fitted into a bearing, and carrying a total load, distributes the pressure due to this load unequally on the bearing, the maximum pressure being at the extremity of the vertical radius, while at the extremities of the horizontal diameter the pressure is zero. At any point of the bearing-surface at the extremity of a radius which makes an angle θ with the vertical radius the normal pressure is proportional to cos θ. If p = normal pressure on a unit of surface, w = total load on a unit of length of the journal, and r = radius of journal,

$$w \cos \theta = 1.57rp, \quad p = \frac{w \cos \theta}{1.57r}.$$

PIVOT-BEARINGS.

The Schiele Curve.—W. H. Harrison, in a letter to the *Am. Machinist*, 1891, says the Schiele curve is not as good a form for a bearing as the segment of a sphere. He says: A mill-stone weighing a ton frequently bears its whole weight upon the flat end of a hard-steel pivot 1⅛" diameter, or one square inch area of bearing; but to carry a weight of 3000 lbs. he advises an end bearing about 4 inches diameter, made in the form of a segment of a sphere about ⅛ inch in height. The die or fixed bearing should be dished to fit the pivot. This form gives a chance for the bearing to adjust itself, which it does not have when made flat, or when made with the Schiele curve. If a side bearing is necessary it can be arranged farther up the shaft. The pivot and die should be of steel, hardened; cross-gutters should be in the die to allow oil to flow, and a central oil-hole should be made in the shaft.

The advantage claimed for the Schiele bearing is that the pressure is uniformly distributed over its surface, and that it therefore wears uniformly. Wilfred Lewis (*Am. Mach.*, April 19, 1894) says that its merits as a thrust-bearing have been vastly overestimated; that the term "anti-friction" applied to it is a misnomer, since its friction is greater than that of a flat step or collar of the same diameter. He advises that flat thrust-bearings should always be annular in form, having an inside diameter one half of the external diameter

Friction of a Flat Pivot-bearing.—The Research Committee on Friction (Proc. Inst. M. E. 1891) experimented on a step-bearing, flat-ended, 3 in. diam., the oil being forced into the bearing through a hole in its centre and distributed through two radial grooves, insuring thorough lubrication. The step was of steel and the bearing of manganese-bronze.

At revolutions per min...........	50	128	194	290	353
The coefficient of friction varied between	.0131	.0053	.0051	.0044	.0053
	and .0221	.0113	.0102	.0178	.0167

With a white-metal bearing at 128 revolutions the coefficient of friction was a little larger than with the manganese-bronze. At the higher speeds the coefficient of friction was less, owing to the more perfect lubrication, as shown by the more rapid circulation of the oil. At 128 revolutions the bronze bearing heated and seized on one occasion with a load of 260 pounds and on another occasion with 300 pounds per square inch. The white-metal bearing under similar conditions heated and seized with a load of 240 pounds per square inch. The steel footstep on manganese-bronze was afterwards tried, lubricating with three and with four radial grooves; but the friction was from one and a half times to twice as great as with only the two grooves. (See also Allowable Pressures, page 936.)

Mercury-bath Pivot.—A nearly frictionless step-bearing may be obtained by floating the bearing with its superincumbent weight upon mercury. Such an apparatus is used in the lighthouses of La Heve, Havre. It is thus described in *Eng'g*, July 14, 1893, p. 41:

The optical apparatus, weighing about 1 ton, rests on a circular cast-iron table, which is supported by a vertical shaft of wrought iron 2.36 in. diameter.

This is kept in position at the top by a bronze ring and outer iron support, and at the bottom in the same way, while it rotates on a removable steel pivot resting in a steel socket, which is fitted to the base of the support. To the vertical shaft there is rigidly fixed a floating cast-iron ring 17.1 in. diameter and 11.8 in. in depth, which is plunged into and rotates in a mercury bath contained in a fixed outer drum or tank, the clearance between the vertical surfaces of the drum and ring being only 0.2 in., so as to reduce as much as possible the volume of mercury (about 220 lbs.), while the horizontal clearance at the bottom is 0.4 in.

BALL-BEARINGS, FRICTION ROLLERS, ETC.

A. H. Tyler (*Eng'g*, Oct. 20, 1893, p. 483), after experiments and comparison with experiments of others arrives at the following conclusions:

That each ball must have two points of contact only.

The balls and race must be of glass hardness, and of absolute truth.

The balls should be of the largest possible diameter which the space at disposal will admit of.

Any one ball should be capable of carrying the total load upon the bearing.

Two rows of balls are always sufficient.

A ball-bearing requires no oil, and has no tendency to heat unless overloaded.

Until the crushing strength of the balls is being neared, the frictional resistance is proportional to the load.

The frictional resistance is inversely proportional to the diameter of the balls, but in what exact proportion Mr. Tyler is unable to say. Probably it varies with the square.

The resistance is independent of the number of balls and of the speed.

No rubbing action will take place between the balls, and devices to guard against it are unnecessary, and usually injurious.

The above will show that the ball-bearing is most suitable for high speeds and light loads. On the spindles of wood-carving machines some make as much as 30,000 revolutions per minute. They run perfectly cool, and never have any oil upon them. For heavy loads the balls should not be less than two thirds the diameter of the shaft, and are better if made equal to it.

Ball-bearings have not been found satisfactory for thrust-blocks, for the reason apparently that the tables crowd together. Better results have been obtained from coned rollers. A combined system of rollers and balls is described in *Eng'g*, Oct. 6, 1893, p. 429.

Friction-rollers.—If a journal instead of revolving on ordinary bearings be supported on friction-rollers the force required to make the journal revolve will be reduced in nearly the same proportion that the diameter of the axles of the rollers is less than the diameter of the rollers themselves. In experiments by A. M. Wellington with a journal 3½ in. diam. supported on rollers 8 in. diam., whose axles were 1¾ in. diam., the friction in starting from rest was ¼ the friction of an ordinary 3½-in. bearing, but at a car speed of 10 miles per hour it was ½ that of the ordinary bearing. The ratio of the diam. of the axle to diam. of roller was 1¾ : 8, or as 1 to 4.6.

Bearings for Very High Rotative Speeds. (Proc. Inst. M. E., Oct. 1888, p. 482.)—In the Parsons steam-turbine, which has a speed of as high as 18,000 rev. per min., as it is impossible to secure absolute accuracy of balance, the bearings are of special construction so as to allow of a certain very small amount of lateral freedom. For this purpose the bearing is surrounded by two sets of steel washers 1/16 inch thick and of different diameters, the larger fitting close in the casing and about 1/32 inch clear of the bearing, and the smaller fitting close on the bearing and about 1/32 inch clear of the casing. These are arranged alternately, and are pressed together by a spiral spring. Consequently any lateral movement of the bearing causes them to slide mutually against one another, and by their friction to check or damp any vibrations that may be set up in the spindle. The tendency of the spindle is then to rotate about its axis of mass, or principal axis as it is called; and the bearings are thereby relieved from excessive pressure, and the machine from undue vibration. The finding of the centre of gyration, or rather allowing the turbine itself to find its own centre of gyration, is a well-known device in other branches of mechanics: as in the instance of the centrifugal hydro-extractor, where a mass very much out of balance is allowed to find its own centre of gyration; the faster it ran the more steadily did it revolve and the less was the vibration. Another illustration is to be found in the spindles of spinning machinery, which run at about 10,000 or 11,000 revolutions per minute: they are made of hardened and tempered steel, and although of very small dimensions, the outside diameter of the largest portion or driving whorl being perhaps not more than 1¼ in., it is found impracticable to run them at that speed in what might be called a hard-and-fast bearing. They are therefore run with some elastic substance surrounding the bearing, such as steel springs, hemp, or cork. Any elastic substance is sufficient to absorb the vibration, and permit of absolutely steady running.

FRICTION OF STEAM-ENGINES.

Distribution of the Friction of Engines.—Prof. Thurston in his "Friction and Lost Work," gives the following:

	1.	2.	3.
Main bearings	47.0	35.4	35.0
Piston and rod	32.9	25.0	21.0
Crank-pin	6.8	5.1	13.0
Cross-head and wrist-pin	5.4	4.1	
Valve and rod	2.5	26.4	22.0
Eccentric strap	5.3	4.0	
Link and eccentric	....	....	9.01
Total	100.0	100.0	100.0

No. 1, Straight-line, 6″ × 12″, balanced valve; No. 2, Straight-line, 6″ × 12″, unbalanced valve; No. 3, 7″ × 10″, Lansing traction locomotive valve-gear.

Prof. Thurston's tests on a number of different styles of engines indicate that the friction of any engine is practically constant under all loads. (Trans. A. S. M. E., viii. 86; ix. 74.)

In a Straight-line engine, 8″ × 14″, I.H.P. from 7.41 to 57.54, the friction H.P. varied irregularly between 1.97 and 4.02, the variation being independent of the load. With 50 H.P. on the brake the I.H.P. was only 52.6, the friction being only 2.6 H.P., or about 5%.

In a compound condensing-engine, tested from 0 to 102.6 brake H.P., gave I.H.P. from 14.92 to 117.8 H.P., the friction H.P. varying only from 14.92 to 17.42. At the maximum load the friction was 15.2 H.P., or 12.9%.

The friction increases with increase of the boiler-pressure from 30 to 70 lbs., and then becomes constant. The friction generally increases with increase of speed, but there are exceptions to this rule.

Prof. Denton (*Stevens Indicator*, July, 1890), comparing the calculated friction of a number of engines with the friction as determined by measurement, finds that in one case, a 75-ton ammonia ice-machine, the friction of the compressor, 17½ H.P., is accounted for by a coefficient of friction of 7½% on all the external bearings, allowing 6% of the entire friction of the machine for the friction of pistons, stuffing-boxes, and valves. In the case of the Pawtucket pumping-engine, estimating the friction of the external bearings with a coefficient of friction of 6% and that of the pistons, valves, and stuffing-boxes as in the case of the ice-machine, we have the total friction distributed as follows:

	Horse-power.	Per cent of Whole.
Crank-pins and effect of piston-thrust on main shaft..	0.71	11.4
Weight of fly-wheel and main shaft..................	1.95	32.4
Steam-valves..........	0.23	3.7
Eccentric........	0.07	1.2
Pistons..	0.43	7.2
Stuffing-boxes, six altogether	0.72	11.3
Air-pump..	2.10	32.8
Total friction of engine with load........	6.21	100.0
Total friction per cent of indicated power ...	4.27	

The friction of this engine, though very low in proportion to the indicated power, is satisfactorily accounted for by Morin's law used with a coefficient of friction of 5%. In both cases the main items of friction are those due to the weight of the fly-wheel and main shaft and to the piston-thrust on crank-pins and main-shaft bearings. In the ice-machine the latter items are the larger owing to the extra crank-pin to work the pumps, while in the Pawtucket engine the former preponderates, as the crank-thrusts are partly absorbed by the pump-pistons, and only the surplus effect acts on the crank-shaft.

Prof. Denton describes in Trans. A. S. M. E., x. 392, an apparatus by which he measured the friction of a piston packing-ring. When the parts of the piston were thoroughly devoid of lubricant, the coefficient of friction was found to be about 7½%; with an oil-feed of one drop in two minutes the coefficient was about 5%; with one drop per minute it was about 3%. These rates of feed gave unsatisfactory lubrication, the piston groaning at the ends of the stroke when run slowly, and the flow of oil left upon the surfaces was found by analysis to contain about 50% of iron. A feed of two drops per minute reduced the coefficient of friction to about 1%, and gave practically perfect lubrication, the oil retaining its natural color and purity.

LUBRICATION.

Measurement of the Durability of Lubricants. (J. E. Denton, Trans. A. S. M. E., xi. 1013.)—Practical differences of durability of lubricants depend not on any differences of inherent ability to resist being "worn out" by rubbing, but upon the rate at which they flow through and away from the bearing-surfaces. The conditions which control this flow are so delicate in their influence that all attempts thus far made to measure durability of lubricants may be said to have failed to make distinctions of lubricating value having any practical significance. In some kinds of service the limit to the consumption of oil depends upon the extent to which dust or other refuse becomes mixed with it, as in railroad-car lubrication and in the case of agricultural machinery. The economy of one oil over another, so far as the quality used is concerned—that is, so far as durability is concerned—is simply proportional to the rate at which it can insinuate itself into and flow out of minute orifices or cracks. Oils will differ in their ability to do this, first, in proportion to their viscosity, and, second, in proportion to the capillary properties which they may possess by virtue of the particular ingredients used in their composition. Where the thickness of film between rubbing-surfaces must be so great that large amounts of oil pass through bearings in a given time, and the surroundings are such as to permit oil to be fed at high temperatures or applied by a method not requiring a perfect fluidity, it is probable that the least amount of oil will be used when the viscosity is as great as in the petroleum cylinder stocks. When, however, the oil must flow freely at ordinary temperatures and the feed of oil is restricted, as in the case of crank-pin bearings, it is not practicable to feed such heavy oils in a satisfactory manner. Oils of less viscosity or of a fluidity approximating to lard-oil must then be used.

Relative Value of Lubricants. (J. E. Denton, *Am. Mach.*, Oct 30, 1890.)—The three elements which determine the value of a lubricant are the cost due to consumption of lubricants, the cost spent for coal to overcome the frictional resistance caused by use of the lubricant, and the cost due to the metallic wear on the journal and the brasses. In cotton-mills the cost of the power is alone to be considered; in rolling-mills and marine engines the cost of the quantity of lubricant used is the only important factor; but in railroads not only do both these elements enter the problem as tangible factors, but the cost of the wearing away of the metallic parts enters in addition, and furthermore, the latter is the greatest element of cost in the case.

The Qualifications of a Good Lubricant, as laid down by W. H. Bailey, in Proc Inst. C. E., vol. xlv., p. 372, are: 1. Sufficient body to keep the surfaces free from contact under maximum pressure. 2. The greatest possible fluidity consistent with the foregoing condition. 3. The lowest possible coefficient of friction, which in bath lubrication would be for fluid friction approximately. 4. The greatest capacity for storing and carrying away heat. 5. A high temperature of decomposition. 6. Power to resist oxidation or the action of the atmosphere. 7. Freedom from corrosive action on the metals upon which used.

Best Lubricants for Different Purposes. (Thurston.)

Low temperatures, as in rock-drills driven by compressed air:	Light mineral lubricating-oils.
Very great pressures, slow speed..	Graphite, soapstone, and other solid lubricants.
Heavy pressures, with slow speed...	The above, and lard, tallow, and other greases.
Heavy pressures and high speed....	Sperm-oil, castor-oil, and heavy mineral oils.
Light pressures and high speed.....	Sperm, refined petroleum, olive, rape, cotton-seed.
Ordinary machinery	Lard-oil, tallow-oil, heavy mineral oils, and the heavier vegetable oils.
Steam-cylinders......................	Heavy mineral oils, lard, tallow.
Watches and other delicate mechanism:	Clarified sperm, neat's-foot, porpoise, olive, and light mineral lubricating oils.

For mixture with mineral oils, sperm is best; lard is much used; olive and cotton-seed are good.

Amount of Oil needed to Run an Engine.—The Vacuum Oil Co. in 1892, in response to an inquiry as to cost of oil to run a 1000-H.P. Corliss engine, wrote: The cost of running two engines of equal size of the same make is not always the same. Therefore while we could furnish figures showing what it is costing some of our customers having Corliss engines of 1000 H.P., we could only give a general idea, which in itself might be considerably out of the way as to the probable cost of cylinder- and engine-oils per year for a particular engine. Such an engine ought to run readily on less than 8 drops of 600 W oil per minute. If 3000 drops are figured to the quart, and 8 drops used per minute, it would take about two and one half barrels (52.5 gallons) of 600 W cylinder-oil, at 65 cents per gallon, or about $85 for cylinder-oil per year, running 6 days a week and 10 hours a day. Engine-oil would be even more difficult to guess at what the cost would be, because it would depend upon the number of cups required on the engine, which varies somewhat according to the style of the engine. It would doubtless be safe, however, to calculate at the outside that not more than twice as much engine-oil would be required as of cylinder-oil.

The Vacuum Oil Co. in 1892 published the following results of practice with "600 W" cylinder-oil:

Corliss compound engine,	20 and 33 × 48; 83 revs. per min.; 1 drop of oil per min. to 1 drop in two minutes.
" triple exp. "	20, 33, and 46 × 48; 1 drop every 2 minutes.
Porter-Allen "	20 and 36 × 36; 143 revs. per min.; 2 drops of oil per min., reduced afterwards to 1 drop per min.
Ball "	15 × 25 × 16; 240 revs. per min.; 1 drop every 4 minutes.

Results of tests on ocean-steamers communicated to the author by Prof. Denton in 1892 gave: for 1200-H.P. marine engine, 5 to 6 English gallons (6 to 7.2 U. S. gals.) of engine-oil per 24 hours for external lubrication; and for a 1500-H.P. marine engine, triple expansion, running 75 revs. per min., 6 to 7 English gals. per 24 hours. The cylinder-oil consumption is exceedingly variable,—from 1 to 4 gals. per day on different engines, including cylinder-oil used to swab the piston-rods.

Quantity of Oil used on a Locomotive Crank-pin.—Prof. Denton, Trans. A. S. M. E., xi. 1020, says: A very economical case of practical oil-consumption is when a locomotive main crank-pin consumes about six cubic inches of oil in a thousand miles of service. This is equivalent to a consumption of one milligram to seventy square inches of surface rubbed over.

The Examination of Lubricating-oils. (Prof. Thos. B. Stillman, *Stevens Indicator*, July, 1890.)—The generally accepted conditions of a good lubricant are as follows:

1. "Body" enough to prevent the surfaces, to which it is applied, from coming in contact with each other. (Viscosity.)
2. Freedom from corrosive acid, either of mineral or animal origin.
3. As fluid as possible consistent with "body."
4. A minimum coefficient of friction.
5. High "flash" and burning points.
6. Freedom from all materials liable to produce oxidation or "gumming."

The examinations to be made to verify the above are both chemical and mechanical, and are usually arranged in the following order;

1. Identification of the oil, whether a simple mineral oil, or animal oil, or a mixture. 2. Density. 3. Viscosity. 4. Flash-point. 5. Burning-point. 6. Acidity. 7. Coefficient of friction. 8. Cold test.

Detailed directions for making all of the above tests are given in Prof. Stillman's article.

Weights of Oil per Gallon.—The following are approximately the weights per gallon of different kinds of oil (Penn. R. R. Specifications):

Lard-oil, tallow-oil, neat's-foot oil, bone-oil, colza-oil, mustard-seed oil, rape-seed oil, paraffine-oil, 500° fire-test oil, engine-oil, and cylinder lubricant, 7½ pounds per gallon.

Well-oil and passenger-car oil, 7.4 pounds per gallon; navy sperm-oil, 7.2 pounds per gallon; signal-oil, 7.1 pounds per gallon; 300° burning-oil, 6.9 pounds per gallon; and 150° burning-oil, 6.6 pounds per gallon.

Penna. R. R. Specifications for Petroleum Products. 1895.—Five different grades of petroleum products will be used.

The materials desired under this specification are the products of the distillation and refining of petroleum unmixed with any other substances.

150° *Fire-test Oil.*—This grade of oil will not be accepted if sample (1) is not "water-white" in color; (2) flashes below 130° Fahrenheit; (3) burns below 151° Fahrenheit; (4) is cloudy or shipment has cloudy barrels when received, from the presence of glue or suspended matter; (5) becomes opaque or shows cloud when the sample has been 10 minutes at a temperature of 0° Fahrenheit.

300° *Fire-test Oil.*—This grade of oil will not be accepted if sample (1) is not "water-white" in color; (2) flashes below 249° Fahrenheit; (3) burns below 298° Fahrenheit; (4) is cloudy or shipment has cloudy barrels when received, from the presence of glue or suspended matter; (5) becomes opaque or shows cloud when the sample has been 10 minutes at a temperature of 32° Fahrenheit; (6) shows precipitation when some of the sample is heated to 450° F. The precipitation test is made by having about two fluid ounces of the oil in a six-ounce beaker, with a thermometer suspended in the oil, and then heating slowly until the thermometer shows the required temperature. The oil changes color, but must show no precipitation.

Paraffine and Neutral Oils.—These grades of oil will not be accepted if the sample from shipment (1) is so dark in color that printing with long-primer type cannot be read with ordinary daylight through a layer of the oil ½ inch thick; (2) flashes below 298° F.; (3) has a gravity at 60° F., below 24° or above 35° Baumé; (4) from October 1st to May 1st has a cold test above 10° F., and from May 1st to October 1st has a cold test above 32° F.

The color test is made by having a layer of the oil of the prescribed thickness in a proper glass vessel, and then putting the printing on one side of the vessel and reading it through the layer of oil with the back of the observer toward the source of light.

Well Oil.—This grade of oil will not be accepted if the sample from shipment (1) flashes, from May 1st to October 1st, below 298° F., or, from October 1st to May 1st, below 249° F.; (2) has a gravity at 60° F., below 25° or above 31° Baumé; (3) from October 1st to May 1st has a cold test above 10° F., and from May 1st to October 1st has a cold test above 32° F.; (4) shows any precipitation when 5 cubic centimetres are mixed with 95 c. c. of gasoline. The precipitation test is to exclude tarry and suspended matter. It is made by putting 95 c.c. of 88° B gasoline, which must not be above 80° F. in temperature, into a 100 c. c.

graduate, then adding the prescribed amount of oil and shaking thoroughly. Allow to stand ten minutes. With satisfactory oil no separated or precipitated material can be seen.

500° *Fire-test Oil.*—This grade of oil will not be accepted if sample from shipment (1) flashes below 494° F.; (2) shows precipitation with gasoline when tested as described for well oil.

Printed directions for determining flashing and burning tests and for making cold tests and taking gravity are furnished by the railroad company. The specifications of 1889 contained the following:

150° *Fire-test Oil.*—The flashing and burning points are determined by heating the oil in an open vessel, not less than 12° per minute, and applying the test-flame every 7°, beginning at 123° Fahrenheit. The cold test may be conveniently made by having an ounce of the oil, in a four-ounce sample bottle, with a thermometer suspended in the oil, and exposing this to a freezing mixture of ice and salt. It is advisable to stir with the thermometer while the oil is cooling. The oil must remain transparent in the freezing mixture ten minutes after it has cooled to zero.

300° *Fire-test Oil.*—The flashing and burning points are determined the same as for 150° fire-test oil, except that the oil is heated 15° per minute, test-flame being applied first at 242° Fahrenheit. The cold test is made the same as above, except that ice and water are used.

Paraffine-oil.—The flashing-point is determined same as for 300° fire-test oil. The cold test is determined as follows: A couple of ounces of oil is put in a four-ounce sample bottle, and a thermometer placed in it. The oil is then frozen, a freezing mixture of ice and salt being used if necessary. When the oil has become hard, the bottle is removed from the freezing mixture and the frozen oil allowed to soften, being stirred and thoroughly mixed at the same time by means of the thermometer, until the mass will run from one end of the bottle to the other. The reading of the thermometer when this is the case is regarded as the cold test of the oil.

Well Oil.—For summer oil the flashing-point is determined the same as for paraffine-oil; and for winter oil the same, except that the test-flame is applied first at 193° Fahrenheit. The cold test is made the same as for paraffine-oil.

500° *Fire-test Oil.*—In the flashing-test the flame is first applied at 438° F.

SOLID LUBRICANTS.

Graphite in a condition of powder and used as a *solid lubricant*, so called, to distinguish it from a liquid lubricant, has been found to do well where the latter has failed.

Rennie, in 1829, says: "Graphite lessened friction in all cases where it was used." General Morin, at a later date, concluded from experiments that it could be used with advantage under heavy pressures; and Prof. Thurston found it well adapted for use under both light and heavy pressures when mixed with certain oils. It is especially valuable to prevent abrasion and cutting under heavy loads and at low velocities.

Soapstone, also called talc and steatite, in the form of powder and mixed with oil or fat, is sometimes used as a lubricant. Graphite or soapstone, mixed with soap, is used on surfaces of wood working against either iron or wood.

Fibre-graphite.—A new self-lubricating bearing known as fibre-graphite is described by John H. Cooper in Trans. A. S. M. E., xiii. 374, as the invention of P. H. Holmes, of Gardiner, Me. This bearing material is composed of selected natural graphite, which has been finely divided and freed from foreign and gritty matter, to which is added wood-fibre or other growth mixed in water in various proportions, according to the purpose to be served, and then solidified by pressure in specially prepared moulds; after removal from which the bearings are first thoroughly dried, then saturated with a drying oil, and finally subjected to a current of hot, dry air for the purpose of oxidizing the oil, and hardening the mass. When finished, they may be "machined" to size or shape with the same facility and means employed on metals. (Holmes Fibre-Graphite Mfg. Co., Philadelphia.)

Metaline is a solid compound, usually containing graphite, made in the form of small cylinders which are fitted permanently into holes drilled in the surface of the bearing. The bearing thus fitted runs without any other lubrication.

THE FOUNDRY.

CUPOLA PRACTICE.

The following notes, with the accompanying table, are taken from an article by Simpson Bolland in *American Machinist*, June 30, 1892. The table shows heights, depth of bottom, quantity of fuel on bed, proportion of fuel and iron in charges, diameter of main blast-pipes, number of tuyeres, blast-pressure, sizes of blowers and power of engines, and melting capacity per hour, of cupolas from 24 inches to 84 inches in diameter.

Capacity of Cupola.—The accompanying table will be of service in determining the capacity of cupola needed for the production of a given quantity of iron in a specified time.

First, ascertain the amount of iron which is likely to be needed at each cast, and the length of time which can be devoted profitably to its disposal; and supposing that two hours is all that can be spared for that purpose, and that ten tons is the amount which must be melted, find in the column, Melting Capacity per hour in Pounds, the nearest figure to five tons per hour, which is found to be 10,760 pounds per hour, opposite to which in the column Diameter of Cupolas, Inside Lining, will be found 48 inches; this will be the size of cupola required to furnish ten tons of molten iron in two hours.

Or suppose that the heats were likely to average 6 tons, with an occasional increase up to ten, then it might not be thought wise to incur the extra expense consequent on working a 48-inch cupola, in which case, by following the directions given, it will be found that a 40-inch cupola would answer the purpose for 6 tons, but would require an additional hour's time for melting whenever the 10-ton heat came along.

The quotations in the table are not supposed to be all that can be melted in the hour by some of the very best cupolas, but are simply the amounts which a common cupola under ordinary circumstances may be expected to melt in the time specified.

Height of Cupola.—By height of cupola is meant the distance from the base to the bottom side of the charging hole.

Depth of Bottom of Cupola.—Depth of bottom is the distance from the sand-bed, after it has been formed at the bottom of the cupola, up to the under side of the tuyeres.

All the amounts for fuel are based upon a bottom of 10 inches deep, and any departure from this depth must be met by a corresponding change in the quantity of fuel used on the bed; more in proportion as the depth is increased, and less when it is made shallower.

Amount of Fuel Required on the Bed.—The column "Amount of Fuel required on Bed, in Pounds" is based on the supposition that the cupola is a straight one all through, and that the bottom is 10 inches deep. If the bottom be more, as in those of the Colliau type, then additional fuel will be needed.

The amounts being given in pounds, answer for both coal and coke, for, should coal be used, it would reach about 15 inches above the tuyeres; the same weight of coke would bring it up to about 22 inches above the tuyeres, which is a reliable amount to stock with.

First Charge of Iron.—The amounts given in this column of the table are safe figures to work upon in every instance, yet it will always be in order, after proving the ability of the bed to carry the load quoted, to make a slow and gradual increase of the load until it is fully demonstrated just how much burden the bed will carry.

Succeeding Charges of Fuel and Iron.—In the columns relating to succeeding charges of fuel and iron, it will be seen that the highest proportions are not favored, for the simple reason that successful melting with any greater proportion of iron to fuel is not the rule, but, rather, the exception. Whenever we see that iron has been melted in prime condition in the proportion of 12 pounds of iron to one of fuel, we may reasonably expect that the talent, material, and cupola have all been up to the highest degree of excellence.

Diameter of Main Blast-pipe.—The table gives the diameters of main blast-pipes for all cupolas from 24 to 84 inches diameter. The sizes given opposite each cupola are of sufficient area for all lengths up to 100 feet.

Cupola Practice.

Diam. of Cupola inside lining.	Height of Cupola from bot'm plate to bottom of charging door.	Depth, from sand-bed to under side of Tuyeres.	Am't of Fuel required in bed.	First charge of Iron.	Succeeding charges of Fuel.	Succeeding charges of Iron.	Diam. Main Blast-pipe when not exceed'g 100ft. in length.	Number of Tuyeres 6 inches diam. required for each Cup'la.	Number and Dimensions of flat Tuyeres equivalent to the 6-inch round ones.		Blast-press. required.	Sizes of Root Blower required.	Revolutions per minute	H.P. of Engine to drive Root Bl'wer.	Sizes of Sturtevant Blower.	H.P. of Engine to drive Sturtevant Blower.	Melting Capacity per hour.
Inches.	Feet.	Inches.	Pounds.	Pounds.	Pounds.	Pounds.	Inches.	No.	No.	Dimensions.	Oz.	No.		H.P.	No.	H.P.	Pounds.
24	10	10	300	900	50	500	10	1.5	2	10″ × 2″	6	½	250	¾	2	1	1,500
26	10	10	390	1,170	92	828	10	2.	2	14½″ × 2″	6	½	335	1	2	1	2,000
28	10	10	480	1,440	134	1,206	10	2.6	3	12½″ × 2″	6	1	210	1¼	2	1	2,500
30	12	10	570	1,710	176	1,584	12	2.6	3	12½″ × 2″	7	1	250	1½	3	2	3,000
32	12	10	660	1,980	218	1,962	12	3.3	4	12″ × 2″	7	1	292	1¾	3	2	3,500
34	12	10	750	2,250	260	2,340	12	3.3	4	12″ × 2″	7	1	333	2	3	2	4,000
36	12	10	840	2,520	302	2,718	14	3.7	4	13½″ × 2″	8	2	241	2½	4	3	4,820
38	12	10	930	2,790	344	3,096	14	5.	6	12″ × 2″	8	2	282	3½	4	3	5,640
40	13	10	1,020	3,060	386	3,474	14	5.	6	12″ × 2″	8	2	325	4	4	3	6,460
42	13	10	1,110	3,330	428	3,852	16	5.8	6	11½″ × 2½″	10	3	235	6	5	5½	7,550
44	13	10	1,200	3,600	470	4,230	16	5.8	6	11½″ × 2½″	10	3	270	7½	5	5½	8,640
46	13	10	1,290	3,870	512	4,608	16	6.8	6	13½″ × 2½″	10	3	304	8½	5	5½	9,730
48	13	10	1,380	4,140	554	4,986	18	6.8	6	13½″ × 2½″	12	4	212	10	6	9¾	10,760
50	14	10	1,470	4,410	596	5,364	18	8.	8	11½″ × 2½″	12	4	236	12	6	9¾	11,790
52	14	10	1,560	4,680	638	5,742	18	10.7	8	15½″ × 2½″	12	4	256	13	6	9¾	12,820
54	14	10	1,650	4,950	680	6,120	20	10.7	8	15½″ × 2½″	14	4	277	14	7	16	13,850
56	14	10	1,740	5,220	722	6,498	20	12.2	8	17½″ × 2½″	14	5	170	16	7	16	14,880
58	14	10	1,830	5,490	764	6,876	20	12.2	8	17½″ × 2½″	14	5	180	17½	7	16	15,910
60	15	10	1,920	5,760	806	7,254	22	13.7	8	16½″ × 3″	14	5	192	18½	8	22	16,940
62	15	10	2,010	6,030	848	7,632	22	13.7	8	16½″ × 3″	14	5	208	20	8	22	18,340
64	15	10	2,100	6,300	890	8,110	22	15.4	8	18½″ × 3″	14	5	225	22	8	22	19,770
66	15	10	2,190	6,570	932	8,388	22	15.4	8	18½″ × 3″	14	5	240	23	8	22	21,200
68	15	10	2,280	6,840	974	8,766	22	17.1	10	16½″ × 3″	14	6	142	25	8	22	22,630
70	16	10	2,370	7,110	1,016	9,144	22	19.	10	18½″ × 3″	14	6	150	27	8	22	24,060
72	16	10	2,460	7,380	1,058	9,522	24	19.	10	18½″ × 3″	16	6	163	33	9	35	26,070
74	16	10	2,550	7,650	1,100	9,900	24	23.9	12	16½″ × 3½″	16	6	175	35	9	35	27,980
76	16	10	2,640	7,920	1,142	10,278	24	23.9	12	16½″ × 3½″	16	6	187	37	9	35	29,890
78	16	10	2,730	8,190	1,184	10,656	24	26.	12	18½″ × 3½″	16	6	200	40	9	35	31,800
80	16	10	2,820	8,460	1,226	11,134	24	26.	12	18½″ × 3½″	16	7	140	42	9	35	33,710
82	16	10	2,910	8,730	1,268	11,412	24	28.	14	17″ × 3½″	16	7	148	45	9	35	35,620
84	16	10	3,000	9,000	1,310	11,790	26	31.	16	16″ × 3½″	16	7	160	47	10	48	37,530

Tuyeres for Cupola.—Two columns are devoted to the number and sizes of tuyeres requisite for the successful working of each cupola; one gives the number of pipes 6 inches diameter, and the other gives the number and dimensions of rectangular tuyeres which are their equivalent in area.

From these two columns any other arrangement or disposition of tuyeres may be made, which shall answer in their totality to the areas given in the table.

When cupolas exceed 60 inches in diameter, the increase in diameter should begin somewhere above the tuyeres. This method is necessary in all common cupolas above 60 inches, because it is not possible to force the blast to the middle of the stock, effectively, at any greater diameter.

On no consideration must the tuyere area be reduced; thus, an 84-inch cupola must have tuyere area equal to 31 pipes 6 inches diameter, or 16 flat tuyeres 16 inches by 13½ inches.

If it is found that the given number of flat tuyeres exceed in circumference that of the diminished part of the cupola, they can be shortened, allowing the decreased length to be added to the depth, or they may be built in on end; by so doing, we arrive at a modified form of the Blakeney cupola.

Another important point in this connection is to arrange the tuyeres in such a manner as will concentrate the fire at the melting-point into the smallest possible compass, so that the metal in fusion will have less space to traverse while exposed to the oxidizing influence of the blast.

To accomplish this, recourse has been had to the placing of additional rows of tuyeres in some instan es—the "Stewart rapid cupola" having three rows, and the "Colliau cupola furnace" having two rows, of tuyeres.

Blast-pressure.—Experiments show that about 30,000 cubic feet of air are consumed in melting a ton of iron, which would weigh about 2400 pounds, or more than both iron and fuel. When the proper quantity of air is supplied, the combustion of the fuel is perfect, and carbonic-acid gas is the result. When the supply of air is insufficient, the combustion is imperfect, and carbonic-oxide gas is the result. The amount of heat evolved in these two cases is as 15 to 4½, showing a loss of over two thirds of the heat by imperfect combustion.

It is not always true that we obtain the most rapid melting when we are forcing into the cupola the largest quantity of air. Some time is required to elevate the temperature of the air supplied to the point that it will enter into combustion. If more air than this is supplied, it rapidly absorbs heat, reduces the temperature, and retards combustion, and the fire in the cupola may be extinguished with too much blast.

Slag in Cupolas.—A certain amount of slag is necessary to protect the molten iron which has fallen to the bottom from the action of the blast; if it was not there, the iron would suffer from decarbonization.

When slag from any cause forms in too great abundance, it should be led away by inserting a hole a little below the tuyeres, through which it will find its way as the iron rises in the bottom.

In the event of clean iron and fuel, slag seldom forms to any appreciable extent in small heats; this renders any preparation for its withdrawal unnecessary, but when the cupola is to be taxed to its utmost capacity it is then incumbent on the melter to flux the charges all through the heat, carrying it away in the manner directed.

The best flux for this purpose is the chips from a white marble yard. About 6 pounds to the ton of iron will give good results when all is clean.

When fuel is bad, or iron is dirty, or both together, it becomes imperative that the slag be kept running all the time.

Fuel for Cupolas.—The best fuel for melting iron is coke, because it requires less blast, makes hotter iron, and melts faster than coal. When coal must be used, care should be exercised in its selection. All anthracites which are bright, black, hard, and free from slate, will melt iron admirably. The size of the coal used affects the melting to an appreciable extent, and, for the best results, small cupolas should be charged with the size called "egg," a still larger grade for medium-sized cupolas, and what is called "lump" will answer for all large cupolas, when care is taken to pack it carefully on the charges.

Charging a Cupola.—Chas. A. Smith (*Am. Mach.*, Feb. 12, 1891) gives the following: A 28-in. cupola should have from 300 to 400 pounds of coke on bottom bed; a 36-in. cupola, 700 to 800 pounds; a 48-in. cupola, 1500 lbs.; and a 60-in. cupola should have one ton of fuel on bottom bed. To every pound of fuel on the bed, three, and sometimes four pounds of metal can be added with safety, if the cupola has proper blast; in after-charges, to every pound of fuel add 8 to 10 pounds of metal; any well-constructed cupola will stand ten.

F. P. Wolcott (*Am. Mach.*, Mar. 5, 1891) gives the following as the practice of the Colwell Iron-works, Carteret, N. J.: "We melt daily from twenty to forty tons of iron, with an average of 11.2 pounds of iron to one of fuel. In a 36-in. cupola seven to nine pounds is good melting, but in a cupola that lines up 48 to 60 inches, anything less than nine pounds shows a defect in arrangement of tuyeres or strength of blast, or in charging up."

"The Moulder's Text-book," by Thos. D. West, gives forty-six reports in tabular form of cupola practice in thirty States, reaching from Maine to Oregon.

Cupola Charges in Stove-foundries. (*Iron Age*, April 14, 1892.) No two cupolas are charged exactly the same. The amount of fuel on the bed or between the charges differs, while varying amounts of iron are used in the charges. Below will be found charging-lists from some of the prominent stove-foundries in the country:

	lbs.		lbs.
A—Bed of fuel, coke	1,500	Four next charges of coke, each	150
First charge of iron	5,000	Six next charges of coke, each	120
All other charges of iron	1,000	Nineteen next charges of coke, each	100
First and second charges of coke, each	200		

Thus for a melt of 18 tons there would be 5120 lbs. of coke used, giving a ratio of 7 to 1. Increase the amount of iron melted to 24 tons, and a ratio of 8 pounds of iron to 1 of coal is obtained.

	lbs.		lbs.
B—Bed of fuel, coke	1,600	Second and third charges of fuel	180
First charge of iron	1,800	All other charges of fuel, each	100
First charge of fuel	150		
All other charges of iron, each	1,000		

For an 18-ton melt 5060 lbs. of coke would be necessary, giving a ratio of 7.1 lbs. of iron to 1 pound of coke.

	lbs.		lbs.
C—Bed of fuel, coke	1,600	All other charges of iron	2,000
First charge of iron	4,000	All other charges of coke	150
First and second charges of coke	200		

In a melt of 18 tons 4100 lbs. of coke would be used, or a ratio of 8.5 to 1.

	lbs.		lbs.
D—Bed of fuel, coke	1,800	All charges of coke, each	200
First charge of iron	5,600	All other charges of iron	2,900

In a melt of 18 tons, 3900 lbs. of fuel would be used, giving a ratio of 9.4 pounds of iron to 1 of coke. Very high, indeed, for stove-plate.

	lbs.		lbs.
E—Bed of fuel, coal	1,900	All other charges of iron, each	2,000
First charge of iron	5,000	All other charges of coal, each	175
First charge of coal	200		

In a melt of 18 tons 4700 lbs. of coal would be used, giving a ratio of 7.7 lbs. of iron to 1 lb. of coal.

These are sufficient to demonstrate the varying practices existing among different stove-foundries. In all these places the iron was proper for stove-plate purposes, and apparently there was little or no difference in the kind of work in the sand at the different foundries.

Results of Increased Driving. (Erie City Iron-works, 1891.)—May—Dec. 1890: 60-in. cupola, 100 tons clean castings a week, melting 8 tons per hour; iron per pound of fuel, 7⅛ lbs.; per cent weight of good castings to iron charged, 75¾. Jan.-May, 1891: Increased rate of melting to 11⅛ tons per hour; iron per lb. fuel, 9⅛; per cent weight of good castings, 75; one week, 13¼ tons per hour, 10.3 lbs. iron per lb. fuel; per cent weight of good castings, 75.8. The increase was made by putting in an additional row of tuyeres and using stronger blast, 14 ounces. Coke was used as fuel. (W. O. Webber, Trans. A. S. M. E. xii. 1045.)

Buffalo Steel Pressure-blowers. Speeds and Capacities as applied to Cupolas.

No. of Blower.	Square inches Blast.	Diam. inside of Cupola in inches.	Pressure in oz.	Speed—No. of Revs. per minute.	Melting Capacity in lbs. per hour.	Cubic Feet of Air required per minute.	Pressure in oz.	Speed—No. of Revs. per min	Melting Capacity in lbs. per hour.	Cubic Feet of Air required per minute.
4	4	20	8	4782	1545	606	9	5030	1647	717
5	6	25	8	4209	2321	773	10	4736	2600	867
6	8	30	8	3660	3093	951	10	4108	3671	1067
7	14	35	8	3244	4218	1486	10	3542	4777	1668
8	18	40	8	2948	5425	2199	10	3310	6082	2469
9	26	45	10	2785	7818	3208	12	3260	8598	3523
10	36	55	10	2195	11295	4938	12	2413	12878	5431
11	45	65	12	1952	16955	7707	14	2116	18357	8358
11½	55	72	12	1647	22607	10276	14	1797	25176	11144
12	75	84	12	1625	25836	11744	14	1775	28019	12736

In the table are given two different speeds and pressures for each size of blower, and the quantity of iron that may be melted, per hour, with each. In all cases it is recommended to use the lowest pressure of blast that will do the work. Run up to the speed given for that pressure, and regulate quantity of air by the blast-gate. The tuyere area should be at least one ninth of the area of cupola in square inches, with not less than four tuyeres at equal distances around cupola, so as to equalize the blast throughout. Variations in temperature affect the working of cupolas materially, hot weather requiring increase in volume of air.

(For tables of the Sturtevant blower see pages 519 and 520.)

Loss in Melting Iron in Cupolas.—G. O. Vair, *Am. Mach.*, March 5, 1891, gives a record of a 45-in. Colliau cupola as follows:

Ratio of fuel to iron, 1 to 7.42.

Good castings	21,314 lbs.
New scrap	3,005 "
Millings	200 "
Loss of metal	1,481 "
Amount melted	26,000 lbs.

Loss of metal, 5.69%. Ratio of loss, 1 to 17.55.

Use of Softeners in Foundry Practice. (W. Graham, *Iron Age*, June 27, 1889.)—In the foundry the problem is to have the right proportions of combined and graphitic carbon in the resulting casting; this is done by getting the proper proportion of silicon. The variations in the proportions of silicon afford a reliable and inexpensive means of producing a cast iron of any required mechanical character which is possible with the material employed. In this way, by mixing suitable irons in the right proportions, a required grade of casting can be made more cheaply than by using irons in which the necessary proportions are already found.

If a strong machine casting were required, it would be necessary to keep the phosphorus, sulphur, and manganese within certain limits. Professor Turner found that cast iron which possessed the maximum of the desired qualities contained, graphite, 2.59%; silicon, 1.42%; phosphorus, 0.39%; sulphur, 0.06%; manganese, 0.58%.

A strong casting could not be made if there was much increase in the amount of phosphorus, sulphur, or manganese. Irons of the above percentages of phosphorus, sulphur, and manganese would be most suitable for this purpose, but they could be of different grades, having different percentages of silicon, combined and graphitic carbon. Thus hard irons, mottled and white irons, and even steel scrap, all containing low percentages of silicon and high percentages of combined carbon, could be employed if an iron having a large amount of silicon were mixed with them in sufficient amount. This would bring the silicon to the proper proportion and would cause the combined carbon to be forced into the graphitic state, and the resulting casting would be soft. High-silicon irons used in this way are called "softeners."

The following are typical analyses of softeners:

	Ferro-silicon.				Softeners, American.			Scotch Irons, No. 1.	
	Foreign.		American.		Well-ston.	Globe	Belle-fonte.	Eg-linton	Colt-ness.
Silicon	10.55	9.80	12.08	10.34	6.67	5.89	3 to 6	2.15	2.59
Combined C.	1.84	0.69	0.06	0.07	....	0.30	0.25	0.21	......
Graphitic C.	0.52	1.12	1.52	1.92	2.57	2.85	3.	3.76	......
Manganese	3.86	1.95	0.76	0.52	. .	1.00	0.53	2.80	1.70
Phosphorus	0.04	0.21	0.48	0.45	0.50	1.10	0.35	0.62	0.85
Sulphur	0.03	0.04	Trace	Trace	Trace	0.02	0.03	0.03	0.01

(For other analyses, see pages 371 to 373.)

Ferro-silicons contain a low percentage of total carbon and a high percentage of combined carbon. Carbon is the most important constituent of cast iron, and there should be about 3.4% total carbon present. By adding ferro-silicon which contains only 2% of carbon the amount of carbon in the resulting mixture is lessened.

Mr. Keep found that more silicon is lost during the remelting of pig of over 10% silicon than in remelting pig iron of lower percentages of silicon. He also points out the possible disadvantage of using ferro-silicons containing as high a percentage of combined carbon as 0.70% to overcome the bad effects of combined carbon in other irons.

The Scotch irons generally contain much more phosphorus than is desired in irons to be employed in making the strongest castings. It is a mistake to mix with strong low-phosphorus irons an iron that would increase the amount of phosphorus for the sake of adding softening qualities, when softness can be produced by mixing irons of the same low phosphorus.

(For further discussion of the influence of silicon see page 365.)

Shrinkage of Castings.—The allowance necessary for shrinkage varies for different kinds of metal, and the different conditions under which they are cast. For castings where the thickness runs about one inch, cast under ordinary conditions, the following allowance can be made:

For cast-iron,	1/8 inch per foot.		For zinc,	5/16 inch per foot.
" brass,	3/16 " " "		" tin,	1/12 " " "
" steel,	1/4 " " "		" aluminum,	3/16 " " "
" mal. iron,	1/8 " " "		" Britannia,	1/32 " " "

Thicker castings, under the same conditions, will shrink less, and thinner ones more, than this standard. The quality of the material and the manner of moulding and cooling will also make a difference.

Numerous experiments by W. J. Keep (see Trans. A. S. M. E., vol. xvi.) showed that the shrinkage of cast iron of a given section decreases as the percentage of silicon increases, while for a given percentage of silicon the shrinkage decreases as the section is increased. Mr. Keep gives the following table showing the approximate relation of shrinkage to size and percentage of silicon:

Percentage of Silicon.	Sectional Area of Casting.					
	1/2″ □	1″ □	1″ × 2″	2″ □	3″ □	4″ □
	Shrinkage in Decimals of an inch per foot of Length.					
1.	.183	.158	.146	.130	.113	.102
1.5	.171	.145	.133	.117	.098	.087
2.	.159	.133	.121	.104	.085	.074
2.5	.147	.121	.108	.092	.073	.060
3.	.135	.108	.095	.077	.059	.045
3.5	.123	.095	.082	.065	.046	.032

Mr. Keep also gives the following "approximate key for regulating foundry mixtures" so as to produce a shrinkage of 1/8 in. per ft. in castings of different sections:

Size of casting...............	1/2	1	2	3	4	in. sq.
Silicon required, per cent.....	3.25	2.75	2.25	1.75	1.25	per cent.
Shrinkage of a 1/2-in. test-bar.	.125	.135	.145	.155	.165	in. per ft.

Weight of Castings determined from Weight of Pattern.

(Rose's Pattern-maker's Assistant.)

A Pattern weighing One Pound, made of—	Will weigh when cast in				
	Cast Iron.	Zinc.	Copper.	Yellow Brass.	Gun-metal.
	lbs.	lbs.	lbs.	lbs.	lbs.
Mahogany—Nassau...............	10.7	10.4	12.8	12.2	12.5
" Honduras...........	12.9	12.7	15.3	14.6	15.
" Spanish.............	8.5	8.2	10.1	9.7	9.9
Pine, red.........................	12.5	12.1	14.9	14.2	14.6
" white.........................	16.7	16.1	19.8	19.0	19.5
" yellow......................	14.1	13.6	16.7	16.0	16.5

Moulding Sand. (From a paper on "The Mechanical Treatment of Moulding Sand," by Walter Bagshaw, Proc. Inst. M. E. 1891.)—The chemical composition of sand will affect the nature of the casting, no matter what treatment it undergoes. Stated generally, good sand is composed of 94 parts silica, 5 parts alumina, and traces of magnesia and oxide of iron. Sand containing much of the metallic oxides, and especially lime, is to be avoided. Geographical position is the chief factor governing the selection of sand; and whether weak or strong, its deficiencies are made up for by the skill of the moulder. For this reason the same sand is often used for both heavy and light castings, the proportion of coal varying according to the nature of the casting. A common mixture of facing-sand consists of six parts by weight of old sand, four of new sand, and one of coal-dust. Floor-sand requires only half the above proportions of new sand and coal-dust to renew it. German founders adopt one part by measure of new sand to two of old sand; to which is added coal-dust in the proportion of one tenth of the bulk for large castings, and one twentieth for small castings. A few founders mix street-sweepings with the coal in order to get porosity when the metal in the mould is likely to be a long time before setting. Plumbago is effective in preventing destruction of the sand; but owing to its refractory nature, it must not be dusted on in such quantities as to close the pores and prevent free exit of the gases. Powdered French chalk, soapstone, and other substances are sometimes used for facing the mould; but next to plumbago, oak charcoal takes the best place, notwithstanding its liability to float occasionally and give a rough casting.

For the treatment of sand in the moulding-shop the most primitive method is that of hand-riddling and treading. Here the materials are roughly proportioned by volume, and riddled over an iron plate in a flat heap, where the mixture is trodden into a cake by stamping with the feet; it is turned over with the shovel, and the process repeated. Tough sand can be obtained in this manner, its toughness being usually tested by squeezing a handful into a ball and then breaking it; but the process is slow and tedious. Other things being equal, the chief characteristics of a good moulding-sand are toughness and porosity, qualities that depend on the manner of mixing as well as on uniform ramming.

Toughness of Sand.—In order to test the relative toughness, sand mixed in various ways was pressed under a uniform load into bars 1 in. sq. and about 12 in. long, and each bar was made to project further and further over the edge of a table until its end broke off by its own weight. Old sand from the shop floor had very irregular cohesion, breaking at all lengths of projections from 1/2 in. to 1 1/2 in. New sand in its natural state held together until an overhang of 2 3/4 in. was reached. A mixture of old sand, new sand, and coal-dust

Mixed under rollers....................	broke at 2	to 2 1/4	in.	of	overhang.
" in the centrifugal machine.......	" " 2	" 2 1/4	"	"	"
" through a riddle..................	" " 1 3/4	" 2 1/8	"	"	"

showing as a mean of the tests only slight differences between the last three methods, but in favor of machine-work. In many instances the fractures were so uneven that minute measurements were not taken.

Dimensions of Foundry Ladles.—The following table gives the dimensions, inside the lining, of ladles from 25 lbs. to 16 tons capacity. All the ladles are supposed to have straight sides. (*Am. Mach.*, Aug. 4, 1892.)

Capacity.	Diam.	Depth.	Capacity.	Diam.	Depth.
	in.	in.		in.	in.
16 tons	54	56	¾ ton	20	20
14 "	52	53	½ "	17	17
12 "	49	50	¼ "	13½	13½
10 "	46	48	300 pounds	11½	11½
8 "	43	44	250 "	10¾	11
6 "	39	40	200 "	10	10½
4 "	34	35	150 "	9	9½
3 "	31	32	100 " ...	8	8½
2 "	27	28	75 "	7	7½
1½ "	24½	25	50 "	6½	6½
1 "	22	22	35 " ...	5½	6

THE MACHINE-SHOP.

SPEED OF CUTTING-TOOLS IN LATHES, MILLING MACHINES, ETC.

Relation of diameter of rotating tool or piece, number of revolutions, and cutting-speed:

Let d = diam. of rotating piece in inches, n = No. of revs. per min.;
S = speed of circumference in feet per minute;

$$S = \frac{\pi d n}{12} = .2618 dn; \quad n = \frac{S}{.2618 d} = \frac{3.82 S}{d}; \quad d = \frac{3.82 S}{n}.$$

Approximate rule: No. of revs. per min. = 4 × speed in ft. per min. ÷ diam. in inches.

Speed of Cut for Lathes and Planers. (Prof. Coleman Sellers, *Stevens' Indicator*, April, 1892.)—*Brass* may be turned at high speed like wood.

Bronze.—A speed of 18 feet per minute can be used with the soft alloys—say 8 to 1, while for hard mixtures a slow speed is required—say 6 feet per minute.

Wrought Iron can be turned at 40 feet per minute, but planing-machines that are used for both cast and forged iron are operated at 18 feet per minute.

Machinery Steel.—Ordinary, 14 feet per minute; car-axles, etc., 9 feet per minute.

Wheel Tires.—6 feet per minute: the tool stands well, but many prefer to run faster, say 8 to 10 feet, and grind the tool more frequently.

Lathes.—The speeds obtainable by means of the cone-pulley and the back gearing are in geometrical progression from the slowest to the fastest. In a well-proportioned machine the speeds hold the same relation through all the steps. Many lathes have the same speed on the slowest of the cone and the fastest of the back-gear speeds.

The Speed of Counter-shaft of the lathe is determined by an assumption of a slow speed with the back gear, say 6 feet per minute, on the largest diameter that the lathe will swing.

EXAMPLE.—A 30-inch lathe will swing 30 inches =, say, 90 inches circumference = 7′ 6″; the lowest triple gear should give a speed of 5 or 6 per minute.

In turning or planing, if the cutting-speed exceed 30 ft. per minute, so much heat will be produced that the temper will be drawn from the tool. The speed of cutting is also governed by the thickness of the shaving, and by the hardness and tenacity of the metal which is being cut; for instance, in cutting mild steel, with a traverse of ⅜ in. per revolution or stroke, and with a shaving about ⅝ in. thick, the speed of cutting must be reduced to about 8 ft. per minute. A good average cutting-speed for wrought or cast iron is 20 ft. per minute, whether for the lathe, planing, shaping, or slotting machine. (Proc. Inst. M. E., April, 1883, p. 248.)

Table of Cutting-speeds.

Diameter, inches.	Feet per minute.									
	5	10	15	20	25	30	35	40	45	50
	Revolutions per minute.									
1/4	76.4	152.8	229.2	305.6	382.0	458.4	534.8	611.2	687.6	764.0
3/8	50.9	101.9	152.8	203.7	254.6	305.6	356.5	407.4	458.3	509.3
1/2	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8	382.0
5/8	30.6	61.1	91.7	122.2	152.8	183.4	213.9	244.5	275.0	305.6
3/4	25.5	50.9	76.4	101.8	127.3	152.8	178.2	203.7	229.1	254.6
7/8	21.8	43.7	65.5	87.3	109.1	130.9	152.8	174.6	196.4	218.3
1	19.1	38.2	57.3	76.4	95.5	114.6	133.7	152.8	171.9	191.0
1 1/8	17.0	34.0	50.9	67.9	84.9	101.8	118.8	135.8	152.8	169.7
1 1/4	15.3	30.6	45.8	61.1	76.4	91.7	106.9	122.2	137.5	152.8
1 3/8	13.9	27.8	41.7	55.6	69.5	83.3	97.2	111.1	125.0	138.9
1 1/2	12.7	25.5	38.2	50.9	63.6	76.4	89.1	101.8	114.5	127.3
1 3/4	10.9	21.8	32.7	43.7	54.6	65.5	76.4	87.3	98.2	109.2
2	9.6	19.1	28.7	38.2	47.8	57.3	66.9	76.4	86.0	95.5
2 1/4	8.5	17.0	25.5	34.0	42.5	50.9	59.4	67.9	76.4	84.9
2 1/2	7.6	15.3	22.9	30.6	38.2	45.8	53.5	61.1	68.8	76.4
2 3/4	6.9	13.9	20.8	27.8	34.7	41.7	48.6	55.6	62.5	69.5
3	6.4	12.7	19.1	25.5	31.8	38.2	44.6	50.9	57.3	63.7
3 1/2	5.5	10.9	16.4	21.8	27.3	32.7	38.2	43.7	49.1	54.6
4	4.8	9.6	14.3	19.1	23.9	28.7	33.4	38.2	43.0	47.8
4 1/2	4.2	8.5	12.7	17.0	21.2	25.5	29.7	34.0	38.2	42.5
5	3.8	7.6	11.5	15.3	19.1	22.9	26.7	30.6	34.4	38.1
5 1/2	3.5	6.9	10.4	13.9	17.4	20.8	24.3	27.8	31.2	34.7
6	3.2	6.4	9.5	12.7	15.9	19.1	22.3	25.5	28.6	31.8
7	2.7	5.5	8.2	10.9	13.6	16.4	19.1	21.8	24.6	27.3
8	2.4	4.8	7.2	9.6	11.9	14.3	16.7	19.1	21.5	23.9
9	2.1	4.2	6.4	8.5	10.6	12.7	14.8	17.0	19.1	21.2
10	1.9	3.8	5.7	7.6	9.6	11.5	13.3	15.3	17.2	19.1
11	1.7	3.5	5.2	6.9	8.7	10.4	12.2	13.9	15.6	17.4
12	1.6	3.2	4.8	6.4	8.0	9.5	11.1	12.7	14.3	15.9
13	1.5	2.9	4.4	5.9	7.3	8.8	10.3	11.8	13.2	14.7
14	1.4	2.7	4.1	5.5	6.8	8.2	9.5	10.9	12.3	13.6
15	1.3	2.5	3.8	5.1	6.4	7.6	8.9	10.2	11.5	12.7
16	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.5	10.7	11.9
18	1.1	2.1	3.2	4.2	5.3	6.4	7.4	8.5	9.5	10.6
20	1.0	1.9	2.9	3.8	4.8	5.7	6.7	7.6	8.6	9.6
22	.9	1.7	2.6	3.5	4.3	5.2	6.1	6.9	7.8	8.7
24	.8	1.6	2.4	3.2	4.0	4.8	5.6	6.4	7.2	8.0
26	.7	1.5	2.2	2.9	3.7	4.4	5.1	5.9	6.6	7.3
28	.7	1.4	2.0	2.7	3.4	4.1	4.8	5.5	6.1	6.8
30	.6	1.3	1.9	2.5	3.2	3.8	4.5	5.1	5.7	6.4
36	.5	1.1	1.6	2.1	2.7	3.2	3.7	4.2	4.8	5.3
42	.5	.9	1.4	1.8	2.3	2.7	3.2	3.6	4.1	4.5
48	.4	.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0
54	.4	.7	1.1	1.4	1.8	2.1	2.5	2.8	3.2	3.5
60	.3	.6	1.0	1.3	1.6	1.9	2.2	2.5	2.9	3.2

Speed of Cutting with Turret Lathes.—Jones & Lamson Machine Co. give the following cutting-speeds for use with their flat turret lathe on diameters not exceeding two inches:

		Ft. per minute.
Threading	Tool steel and taper on tubing	10
	Machinery	15
	Very soft steel	20
Turning machinery steel	Cut which reduces the stock to 1/8 of its original diam.	20
	Cut which reduces the stock to 3/4 of its original diam.	25
	Cut which reduces the stock to 7/8 of its original diam.	30 to 35
	Cut which reduces the stock to 15/16 of its original diam.	40 to 45
Turning very soft machinery steel, light cut and cool work		50 to 60

Forms of Metal-cutting Tools.—"*Hutte*," the German Engineers' Pocket-book, gives the following cutting-angles for using least power:

	Top Rake.	Angle of Cutting-edge.
Wrought iron	3°	51°
Cast iron	4°	51°
Bronze	4°	66°

The *American Machinist* comments on these figures as follows: We are not able to give the best nor even the generally used angles for tools, because these vary so much to suit different circumstances, such as degree of hardness of the metal being cut, quality of steel of which the tool is made, depth of cut, kind of finish desired, etc. The angles that cut with the least expenditure of power are easily determined by a few experiments, but the best angles must be determined by good judgment, guided by experience. In nearly all cases, however, we think the best practical angles are greater than those given.

For illustrations and descriptions of various forms of cutting-tools, see articles on Lathe Tools in App. Cyc. App. Mech., vol. ii., and in Modern Mechanism.

Cold Chisels.—Angle of cutting-faces (Joshua Rose): For cast steel, about 65 degrees; for gun-metal or brass, about 50 degrees; for copper and soft metals, about 30 to 35 degrees.

Rule for Gearing Lathes for Screw-cutting. (Garvin Machine Co.)—Read from the lathe index the number of threads per inch cut by equal gears, and multiply it by any number that will give for a product a gear on the index; put this gear upon the stud, then multiply the number of threads per inch to be cut by the same number, and put the resulting gear upon the screw.

EXAMPLE.—To cut 11½ threads per inch. We find on the index that 48 into 48 cuts 6 threads per inch, then 6 × 4 = 24, gear on stud, and 11½ × 4 = 46, gear on screw. Any multiplier may be used so long as the products include gears that belong with the lathe. For instance, instead of 4 as a multiplier we may use 6. Thus, 6 × 6 = 36, gear upon stud, and 11½ × 6 = 69, gear upon screw.

Rules for Calculating Simple and Compound Gearing where there is no Index. (*Am Mach.*)—If the lathe is simple-geared, and the stud runs at the same speed as the spindle, select some gear for the screw, and multiply its number of teeth by the number of threads per inch in the lead-screw, and divide this result by the number of threads per inch to be cut. This will give the number of teeth in the gear for the stud. If this result is a fractional number, or a number which is not among the gears on hand, then try some other gear for the screw. Or, select the gear for the stud first, then multiply its number of teeth by the number of threads per inch to be cut, and divide by the number of threads per inch on the lead-screw. This will give the number of teeth for the gear on the screw. If the lathe is compound, select at random all the driving-gears, multiply the numbers of their teeth together, and this product by the number of threads to be cut. Then select at random all the driven gears except one; multiply the numbers of their teeth together, and this product by the number of threads per inch in the lead-screw. Now divide the first result by the second, to obtain the number of teeth in the remaining driven gear. Or, select at random all the driven gears. Multiply the numbers of their teeth together, and this product by the number of threads per inch in the lead-screw. Then select at random all the driving-gears except one. Multiply the numbers of their teeth together, and this result by the number of threads per inch of the screw to be cut. Divide the first result by the last, to obtain the number of teeth in the remaining driver. When the gears on the compounding stud are fast together, and cannot be changed, then the driven one has usually twice as many teeth as the other, or driver, in which case in the calculations consider the lead-screw to have twice as many threads per inch as it actually has, and then ignore the compounding entirely. Some lathes are so constructed that the stud on which the first driver is placed revolves only half as fast as the spindle. This can be ignored in the calculations by doubling the number of threads of the lead-screw. If both the last conditions are present ignore them in the calculations by multiplying the number of threads per inch in the lead-screw by four. If the thread to be cut is a fractional one, or if the pitch of the lead-screw is fractional, or if both are fractional, then reduce the fractions to a common denominator, and use the numerators of these fractions as if they equalled the pitch of the screw to be cut, and of the lead-screw, respectively. Then use that part of the rule given above which applies to the lathe in question. For instance, suppose it is desired to cut a thread of 25/32-inch pitch, and the lead-screw has 4 threads per inch. Then the pitch of the lead-screw will be ¼ inch, which is equal to 8/32 inch. We now have two fraction, 25/32 and 8/32, and the two screws will be in the proportion of 25 to 8, and the gears can be figured by the above rule, assuming the number of threads to be cut to be 8 per inch, and those on the lead-screw to be 25 per inch. But this latter number may be further modified by conditions named above, such as a reduced speed of the stud, or fixed compound gears. In the instance given, if the lead-screw had been 2½ threads per inch, then its pitch being 4/10 inch, we have the fractions 4/10 and 25/32, which, reduced to a common denominator, are 64/160 and 125/160, and the gears will be the same as if the lead-screw had 125 threads per inch, and the screw to be cut 64 threads per inch.

On this subject consult also "Formulas in Gearing," published by Brown & Sharpe Mfg. Co., and Jamieson's Applied Mechanics.

Change-gears for Screw-cutting Lathes.—There is a lack of uniformity among lathe-builders as to the change-gears provided for screw-cutting. W. R. Macdonald, in *Am. Mach.*, April 7, 1892, proposes the following series, by which 33 whole threads (not fractional) may be cut by changes of only nine gears:

Screw.	Spindle.									Whole Threads.			
	20	30	40	50	60	70	110	120	130				
20	....	8	6	4 4/5	4	3 3/7	2 2/11	2	1 11/13	2	11	22	44
30	18	....	9	7 1/5	6	5 1/7	3 3/11	3	2 10/13	3	12	24	48
40	24	16	12	9 3/5	8	6 6/7	4 4/11	4	3 9/13	4	13	26	52
50	30	20	15	.....	10	8 4/7	5 5/11	5	4 8/13	5	14	28	66
60	36	24	18	14 2/5	....	10 2/7	6 6/11	6	5 7/13	6	15	30	72
70	42	28	21	16 4/5	14	......	7 7/11	7	6 6/13	7	16	33	78
110	66	44	33	26 2/5	22	18 6/7	....	11	10 2/13	8	18	36	
120	72	48	36	28 4/5	24	20 4/7	13 1/11	....	11 1/13	9	20	39	
130	78	52	39	31 1/5	26	22 3/7	14 2/11	13	......	10	21	42	

Ten gears are sufficient to cut all the usual threads, with the exception of perhaps 11½, the standard pipe-thread; in ordinary practice any fractional thread between 11 and 12 will be near enough for the customary short pipe-thread; if not, the addition of a single gear will give it.

In this table the pitch of the lead-screw is 12, and it may be objected to as too fine for the purpose. This may be rectified by making the real pitch 6 or any other desirable pitch, and establishing the proper ratio between the lathe spindle and the gear-stud.

Metric Screw-threads may be cut on lathes with inch-divided lead-ing-screws, by the use of change-wheels with 50 and 127 teeth; for 127 centimetres = 50 inches (127 × 0.3937 = 49.9999 in.).

Rule for Setting the Taper in a Lathe. (*Am. Mach.*)—No rule can be given which will produce exact results, owing to the fact that the centres enter the work an indefinite distance. If it were not for this circumstance the following would be an exact rule, and it is an approximation as it is. To find the distance to set the centre over: Divide the difference in the diameters of the large and small end of the taper by 2, and multiply this quotient by the ratio which the total length of the shaft bears to the length of the tapered portion. Example: Suppose a shaft three feet long is to have a taper turned on the end one foot long, the large end of the taper being two inches and the small end one inch diameter. $\frac{2-1}{2} \times \frac{3}{1} = 1\frac{1}{2}$ inches.

Electric Drilling-machines - Speed of Drilling Holes in Steel Plates. (Proc. Inst. M. E., Aug. 1887, p. 329)—In drilling holes in the shell of the S.S. "Albania," after a very small amount of practice the men working the machines drilled the ⅞-inch holes in the shell with great rapidity, doing the work at the rate of one hole every 69 seconds, inclusive of the time occupied in altering the position of the machines by means of differential pulley-blocks, which were not conveniently arranged as slings for this purpose. Repeated trials of these drilling-machines have also shown that, when using electrical energy in both holding-on magnets and motor amounting to about ¾ H.P., they have drilled holes of 1 inch diameter through 1½ inch thickness of solid wrought iron, or through 1⅝ inch of mild steel in two plates of 13/16 inch each, taking exactly 1¾ min. for each hole.

Speed of Drills. (Morse Twist-drill and Machine Company.)—The following table gives the revolutions per minute for drills from 1/16 in. to 2 in. diameter, as usually applied:

Diameter of Drills, in.	Speed for Wrought Iron and Steel.	Speed for Cast Iron.	Speed for Brass.	Diameter of Drills, in.	Speed for Wrought Iron and Steel.	Speed for Cast Iron.	Speed for Brass.
1/16	1712	2883	3544	1 1/16	72	108	180
⅛	855	1191	1772	1⅛	68	102	170
3/16	571	794	1181	1 3/16	64	97	161
¼	397	565	855	1¼	58	89	150
5/16	318	452	684	1 5/16	55	84	143
⅜	265	377	570	1⅜	53	81	136
7/16	227	328	489	1 7/16	50	77	130
½	183	267	412	1½	46	74	122
9/16	163	238	367	1 9/16	44	71	117
⅝	147	214	330	1⅝	40	66	113
11/16	133	194	300	1 11/16	38	63	109
¾	112	168	265	1¾	37	61	105
13/16	103	155	244	1 13/16	36	59	101
⅞	96	144	227	1⅞	33	55	98
15/16	89	134	212	1 15/16	32	53	95
1	76	115	191	2	31	51	92

One inch to be drilled in soft cast iron will usually require: for ¼-in. drill, 160 revolutions; for ½-in. drill, 140 revolutions; for ¾-in. drill, 100 revolutions; for 1-in. drill, 95 revolutions. These speeds should seldom be exceeded. Feed per revolution for ¼-in. drill, .005 inch; for ½-in. drill, .007 inch; for ¾-in. drill .010 inch.

The rates of feed for twist drills are thus given by the same company:

Diameter of drill...........	1/16	¼	⅜	½	¾	1	1½
Revs. per inch depth of hole.	125	125	120 to 140		1 inch feed per min.		

MILLING-CUTTERS.

George Addy, (Proc. Inst. M. E., Oct. 1890, p. 537), gives the following:

Analyses of Steel.—The following are analyses of milling-cutter blanks, made from best quality crucible cast steel and from self-hardening "Ivanhoe" steel:

	Crucible Cast Steel, per cent.	Ivanhoe Steel, per cent.
Carbon	1.2	1.67
Silicon..........................	0.112	0.252
Phosphorus	0.018	0.051
Manganese	0.36	2.557
Sulphur	0.02	0.01
Tungsten..............	......	4.65
Iron, by difference......... ...	98.29	90.81
	100.000	100.000

The first analysis is of a cutter 14 in. diam., 1 in. wide, which gave very good service at a cutting-speed of 60 ft. per min. Large milling-cutters are sometimes built up, the cutting-edges only being of tool steel. A cutter 22 in. diam. by 5½ in. wide has been made in this way, the teeth being clamped between two cast-iron flanges. Mr. Addy recommends for this form of tooth one with a cutting-angle of 70°, the face of the tooth being set 10° back of a radial line on the cutter, the clearance-angle being thus 10°. At the Clarence Iron-works, Leeds, the face of the tooth is set 10° back of the radial line for cutting wrought iron and 20° for steel.

Pitch of Teeth.—For obtaining a suitable pitch of teeth for milling-cutters of various diameters there exists no standard rule, the pitch being usually decided in an arbitrary manner, according to individual taste.

For estimating the pitch of teeth in a cutter of any diameter from 4 in. to 15 in., Mr. Addy has worked out the following rule, which he has found capable of giving good results in practice:

$$\text{Pitch in inches} = \sqrt{(\text{diam. in inches} \times 8)} \times 0.0625 = .177\sqrt{\text{diam.}}$$

J. M. Gray gives a rule for pitch as follows: The number of teeth in a milling-cutter ought to be 100 times the pitch in inches; that is, if there were 27 teeth, the pitch ought to be 0.27 in. The rules are practically the same, for if d = diam., n = No. of teeth, p = pitch, c = circumference, $c = pn$; $d = \frac{pn}{\pi} = \frac{100p^2}{\pi} = 31.83p^2$; $p = \sqrt{.0314d} = .177\sqrt{d}$; No. of teeth, n, $= 3.14d \div p$.

Number of Teeth in Mills or Cutters. (Joshua Rose.)—The teeth of cutters must obviously be spaced wide enough apart to admit of the emery-wheel grinding one tooth without touching the next one, and the front faces of the teeth are always made in the plane of a line radiating from the axis of the cutter. In cutters up to 3 in. in diam. it is good practice to provide 8 teeth per in. of diam., while in cutters above that diameter the spacing may be coarser, as follows:

Diameter of cutter, 6 in.; number of teeth in cutter, 40
" " " 7 " " " " " " 45
" " " 8 " " " " " " 50

Speed of Cutters.—The cutting speed for milling was originally fixed very low; but experience has shown that with the improvements now in use it may with advantage be considerably increased, especially with cutters of large diameter. The following are recommended as safe speeds for cutters of 6 in. and upwards, provided there is not any great depth of material to cut away:

	Steel.	Wrought iron.	Cast iron.	Brass.
Feet per minute......	36	48	60	120
Feed, inch per min...	$\frac{1}{2}$	1	$1\frac{5}{8}$	$2\frac{5}{8}$

Should it be desired to remove any large quantity of material, the same cutting-speeds are still recommended, but with a finer feed. A simple rule for cutting-speed is: Number of revolutions per minute which the cutter spindle should make when working on cast iron = 240, divided by the diameter of the cutter in inches.

Speed of Milling-cutters. (Proc. Inst. M. E., April, 1883, p. 248.)—The cutting-speed which can be employed in milling is much greater than that which can be used in any of the ordinary operations of turning in the lathe, or of planing, shaping, or slotting. A milling-cutter with a plentiful supply of oil, or soap and water, can be run at from 80 to 100 ft. per min. when cutting wrought iron. The same metal can only be turned in a lathe, with a tool-holder having a good cutter, at the rate of 30 ft. per min., or at about one third the speed of milling. A milling-cutter will cut cast steel at the rate of 25 to 30 ft. per min.

The following extracts are taken from an article on speed and feed of milling-cutters in *Eng'g*, Oct. 22, 1891: Milling-cutters are successfully employed on cast iron at a speed of 250 ft. per min.; on wrought iron at from 80 ft. to 100 ft. per min. The latter materials need a copious supply of good lubricant, such as oil or soapy water. These rates of speed are not approached by other tools. The usual cutting-speeds on the lathe, planing, shaping, and slotting machines rarely exceed about one third of those given above, and frequently average about a fifth, the time lost in back strokes not being reckoned.

The feed in the direction of cutting is said by one writer to vary, in ordinary work, from 40 to 70 revs. of a 4-in. cutter per in. of feed. It must always to an extent depend on the character of the work done, but the above gives shavings of extreme thinness. For example, the circumference of a 4-in. cutter being, say, $12\frac{1}{2}$ in., and having, say, 60 teeth, the advance corresponding to the passage of one cutting-tooth over the surface, in the coarser of the above-named feed-motions, is $1/40 \times 1/60 = 1/2400$ in.; the finer feed gives an advance for each tooth of only $1/70 \times 1/60 = 1/4200$ in. Such fine feeds as these are used only for light finishing cuts, and the same authority recommends, also for finishing, a cutter about 9 in. in circumference, or nearly 3 in. in diameter, which should be run at about 60 revs. per min. to cut tough wrought steel, 120 for ordinary cast iron, about 80 for wrought iron, and from 140 to 160 for the various qualities of gun-metal and brass. With cutters smaller or larger the rates of revolution are increased or diminished to accord with the following table, which gives these rates of cutting-speeds and shows the lineal speed of the cutting-edge:

	Steel.	Wrought Iron.	Cast Iron.	Gun-metal.	Brass.
Feet per minute...	45	60	90	105	120

These speeds are intended for very light finishing cuts, and they must be reduced to about one half for heavy cutting.

The following results have been found to be the highest that could be attained in ordinary workshop routine, having due consideration to economy and the time taken to change and grind the cutters when they become dull: Wrought iron—36 ft. to 40 ft. per min.; depth of cut, 1 in.; feed, 5/8 in. per min. Soft mild steel—About 30 ft. per min.; depth of cut, 1/4 in.; feed, 3/4 in. per min. Tough gun-metal—80 ft. per min.; depth of cut, 1/8 in.; feed, 3/4 in. per min. Cast-iron gear-wheels—26 1/2 ft. per min.; depth of cut, 1/8 in.; feed, 3/4 in. per min. Hard, close-grained cast iron—30 ft. per min.; depth of cut, 2 1/8 in.; feed, 5/16 in. per min. Gun-metal joints, 58 ft. per min.; depth of cut, 1 3/8 in.; feed, 5/8 in. per min. Steel-bars—21 ft. per min.; depth of cut, 1/32 in.; feed, 3/4 in. per min.

A stepped milling-cutter, 4 in. in diam. and 12 in. wide, tested under two conditions of speed in the same machine, gave the following results: The cutter in both instances was worked up to its maximum speed before it gave way, the object being to ascertain definitely the relative amount of work done by a high speed and a light feed, as compared with a low speed and a heavy cut. The machine was used single-geared and double-geared, and in both cases the width of cut was 10 1/2 in.

Single-gear, 42 ft. per min.; 5/16 in. depth of cut; feed, 1.3 in. per min. = 4.16 cu. in. per min. Double-gear, 19 ft. per min.; 3/8 in. depth of cut; feed, 5/8 in. per min. = 2.40 cu. in. per min.

Extreme Results with Milling-machines.—Horace L. Arnold (*Am. Mach.*, Dec. 28, 1893) gives the following results in flat-surface milling, obtained in a Pratt & Whitney milling-machine: The mills for the flat cut were 5″ diam., 12 teeth, 40 to 50 revs. and 4 7/8″ feed per min. One single cut was run over this piece at a feed of 9″ per min., but the mills showed plainly at the end that this rate was greater than they could endure. At 50 revs. for these mills the figures are as follows, with 4 7/8″ feed: Surface speed, 64 ft., nearly; feed per tooth, 0.00812″: cuts per inch, 123. And with 9″ feed per min.: Surface speed, 64 ft. per min.; feed per tooth, 0.015″; cuts per inch, 66 2/3.

At a feed of 4 7/8″ per min. the mills stood up well in this job of cast-iron surfacing, while with a 9″ feed they required grinding after surfacing one piece; in other words, it did not damage the mill-teeth to do this job with 123 cuts per in. of surface finished, but they would not endure 66 2/3 cuts per inch. In this cast-iron milling the surface speed of the mills does not seem to be the factor of mill destruction: it is the increase of feed per tooth that prohibits increased production of finished surface. This is precisely the reverse of the action of single-pointed lathe and planer tools in general: with such tools there is a surface-speed limit which cannot be economically exceeded for dry cuts, and so long as this surface-speed limit is not reached, the cut per tooth or feed can be made anything up to the limit of the driving power of the lathe or planer, or to the safe strain on the work itself, which can in many cases be easily broken by a too great feed.

In wrought metal extreme figures were obtained in one experiment made in cutting keyways 5/16″ wide by 1/8″ deep in a bank of 8 shafts 1 1/4″ diam. at once, on a Pratt & Whitney No. 3 column milling-machine. The 8 mills were successfully operated with 45 ft. surface speed and 19 1/2 in. per min. feed; the cutters were 5″ diam., with 28 teeth, giving the following figures, in steel: Surface speed, 45 ft. per min.; feed per tooth, 0.02024″; cuts per inch, 50, nearly. Fed with the revolution of mill. Flooded with oil, that is, a large stream of oil running constantly over each mill. Face of tooth radial. The resulting keyway was described as having a heavy wave or cutter-mark in the bottom, and it was said to have shown no signs of being heavy work on the cutters or on the machine. As a result of the experiment it was decided for economical steady work to run at 17 revs., with a feed of 4″ per min., flooded cut, work fed with mill revolution, giving the following figures: Surface speed, 22 1/4 ft. per min.; feed per tooth, 0.0081″; cuts per inch, 119.

An experiment in milling a wrought-iron connecting-rod of a locomotive on a Pratt & Whitney double-head milling-machine is described in the *Iron Age*, Aug. 27, 1891. The amount of metal removed at one cut measured 3¼ in. wide by 1 3/16 in. deep in the groove, and across the top ⅛ in. deep by 4¾ in. wide. This represented a section of nearly 4⅛ sq. in. This was done at the rate of 1¾ in. per min. Nearly 8 cu. in. of metal were cut up into chips every minute. The surface left by the cutter was very perfect. The cutter moved in a direction contrary to that of ordinary practice; that is, it cut down from the upper surface instead of up from the bottom.

Milling "with" or "against" the Feed.—Tests made with the Brown & Sharpe No. 5 milling-machine (described by H. L. Arnold, in *Am. Mach.*, Oct. 18, 1894) to determine the relative advantage of running the milling-cutter with or against the feed—"with the feed" meaning that the teeth of the cutter strike on the top surface or "scale" of cast-iron work in process of being milled, and "against the feed" meaning that the teeth begin to cut in the clean, newly cut surface of the work and cut upwards toward the scale—showed a decided advantage in favor of running the cutter against the feed. The result is directly opposite to that obtained in tests of a Pratt & Whitney machine, by experts of the P. & W. Co.

In the tests with the Brown & Sharpe machine the cutters used were 6 inches face by 4½ and 8 inches diameter respectively, 15 teeth in each mill. 42 revolutions per minute in each case, or nearly 50 feet per minute surface speed for the 4½-inch and 88 feet per minute for the 8-inch mill. The revolution marks were 6 to the inch, giving a feed of 7 inches per minute, and a cut per tooth of .011″. When the machine was forced to the limit of its driving the depth of cut was 11/32 inch when the cutter ran in the "old" way, or against the feed, and only ¼ inch when it ran in the "new" way, or with the feed. The endurance of the milling-cutters was much greater when they were run in the "old" way.

Spiral Milling-cutters.—There is no rule for finding the angle of the spiral; from 10° to 15° is usually considered sufficient; if much greater the end thrust on the spindle will be increased to an extent not desirable for some machines.

Milling-cutters with Inserted Teeth.—When it is required to use milling-cutters of a greater diameter than about 8 in., it is preferable to insert the teeth in a disk or head, so as to avoid the expense of making solid cutters and the difficulty of hardening them, not merely because of the risk of breakage in hardening them, but also on account of the difficulty in obtaining a uniform degree of hardness or temper.

Milling-machine versus Planer.—For comparative data of work done by each see paper by J. J. Grant, Trans. A. S. M. E., ix. 259. He says: The advantages of the milling machine over the planer are many, among which are the following: Exact duplication of work; rapidity of production — the cutting being continuous; cost of production, as several machines can be operated by one workman, and he not a skilled mechanic; and cost of tools for producing a given amount of work.

POWER REQUIRED FOR MACHINE TOOLS.

Resistance Overcome in Cutting Metal. (Trans. A. S. M. E., viii. 308.)—Some experiments made at the works of William Sellers & Co. showed that the resistance in cutting steel in a lathe would vary from 180,000 to 700,000 pounds per square inch of section removed, while for cast iron the resistance is about one third as much. The power required to remove a given amount of metal depends on the shape of the cut and on the shape and the sharpness of the tool used. If the cut is nearly square in section, the power required is a minimum; if wide and thin, a maximum. The dulness of a tool affects but little the power required for a heavy cut.

Heavy Work on a Planer.—Wm. Sellers & Co. write as follows to the *American Machinist*: The 120″ planer table is geared to run 18 ft. per minute under cut, and 72 feet per minute on the return, which is equivalent, without allowance for time lost in reversing, to continuous cut of 14.4 feet per minute. Assuming the work to be 28 feet long, we may take 14 feet as the continuous cutting speed per minute, the .8 of a foot being much more than sufficient to cover time loss in reversing and feeding. The machine carries four tools. At ⅛″ feed per tool, the surface planed per hour would be 35 square feet. The section of metal cut at ¾″ depth would be .75″ × .125″ × 4 = .375 square inch, which would require approximately 30,000 lbs.

pressure to remove it. The weight of metal removed per hour would be $14 \times 12 \times .375 \times .26 \times 60 = 1082.8$ lbs. Our earlier form of 36″ planer has removed with one tool on ¾″ cut on work 200 lbs. of metal per hour, and the 120″ machine has more than five times its capacity. The total pulling power of the planer is 45,000 lbs.

Horse-power Required to Run Lathes. (J. J. Flather, *Am. Mach.*, April 23, 1891.)—The power required to do useful work varies with the depth and breadth of chip, with the shape of tool, and with the nature and density of metal operated upon; and the power required to run a machine empty is often a variable quantity.

For instance, when the machine is new, and the working parts have not become worn or fitted to each other as they will be after running a few months, the power required will be greater than will be the case after the running parts have become better fitted.

Another cause of variation of the power absorbed is the driving-belt; a tight belt will increase the friction, hence to obtain the greatest efficiency of a machine we should use wide belts, and run them just tight enough to prevent slip. The belts should also be soft and pliable, otherwise power is consumed in bending them to the curvature of the pulleys.

A third cause is the variation of journal-friction, due to slacking up or tightening the cap-screws, and also the end-thrust bearing screw.

Hartig's investigations show that it requires less total power to turn off a given weight of metal in a given time than it does to plane off the same amount; and also that the power is less for large than for small diameters.

The following table gives the actual horse-power required to drive a lathe empty at varying numbers of revolutions of main spindle.

HORSE-POWER FOR SMALL LATHES.

Without Back Gears.		With Back Gears.		Remarks.
Revs. of Spindle per min.	H.P. required to drive empty.	Revs. of Spindle per min.	H.P. required to drive empty.	
132.72 219.08 365.00	.145 .197 .310	14.6 24.33 38.42	.126 .141 .274	20″ Fitchburg lathe.
47.4 125.0 188	.159 .259 .339	4.84 12.8 19.2	.132 .187 .230	Smallila the (13½″), Chemnitz. Germany. New machine.
54.6 122 183	.206 .339 .455	6.61 14.8 22.1	.157 .206 .249	17½″ lathe do. New machine.
18.8 54.6 82.2	.086 .210 .326	2.31 6.72 10.8	.085 .063 .087	26″ lathe do.

If $H.P._0$ = horse-power necessary to drive lathe empty, and N = number of revolutions per minute, then the equation for average small lathes is $H.P._0 = 0.095 + 0.0012N$.

For the power necessary to drive the lathes empty when the back gears are in, an average equation for lathes under 20″ swing is

$$H.P._0 = 0.10 + 0.006N.$$

The larger lathes vary so much in construction and detail that no general rule can be obtained which will give, even approximately, the power required to run them, and although the average formula shows that at least 0.095 horse-power is needed to start the small lathes, there are many American lathes under 20″ swing working on a consumption of less than 05 horse-power.

The amount of power required to remove metal in a machine is determinable within more accurate limits.

Referring to Dr. Hartig's researches, $H.P._1 = CW$, where C is a constant, and W the weight of chips removed per hour.

Average values of C are .030 for cast-iron, .032 for wrought-iron, .047 for steel.

The size of lathe, and, therefore, the diameter of work, has no apparent effect on the cutting power. If the lathe be heavy, the cut can be increased, and consequently the weight of chips increased, but the value of C appears to be about the same for a given metal through several varying sizes of lathes.

HORSE-POWER REQUIRED TO REMOVE CAST IRON IN A 20-INCH LATHE.
(J. J. Hobart.)

Descriptive No.	Number of Trials.	Tool used.	Average Cutting-speed in feet per minute.	Depth of Cut in inches.	Average Breadth of Cut in inches.	Average H.P. required to remove Metal.	Average pounds Metal turned off per hour.	Value of Constant C.
1	22	Side tool...........	37.90	.125	.015	.342	13.30	.025
2	15	Diamond....... ..	30.50	.125	.015	.218	10.70	.020
3	17	Round nose........	42.61	.125	.015	.352	14.95	.023
4	2	Left-hand round nose..............	26.29	.125	.015	.237	9.22	.026
5	4	Square-faced tool ⅛" broad........	25.82	.015	.125	.255	9.06	.028
6	1	"	25.27	.048	.048	.200	10.89	.018
7	1	"	25.64	.125	.015	.246	8.99	.027

The above table shows that an average of .26 horse-power is required to turn off 10 pounds of cast-iron per hour, from which we obtain the average value of the constant $C = .024$.

Most of the cuts were taken so that the metal would be reduced ¼" in diameter; with a broad surface cut and a coarse feed, as in No. 5, the power required per pound of chips removed in a given time was a maximum; the least power per unit of weight removed being required when the chip was square, as in No. 6.

HORSE-POWER REQUIRED TO REMOVE METAL IN A 29-INCH LATHE.
(R. H. Smith.)

Number of Experiments.	Metal.	Cutting-speed. ft. per min.	Depth of Cut, in.	Average Breadth of Cut, in.	Avereage H.P. required to remove Metal.	Average pounds Metal removed per hour.	Value of C.
4	Cast iron	12.7	.05	.046	.105	5.49	.019
4	Cast iron	11.1	.135	.046	.217	12.96	.017
2	Cast iron	12.85	.04	.033	.098	3.66	.027
4	Wrought iron	9.6	.03	.046	.059	2.49	.023
4	Wrought iron	9.1	.06	.046	.138	4.72	.029
4	Wrought iron	7.9	.14	.046	.186	9.56	.019
2	Wrought iron	9.35	.045	.038	.092	2.99	.031
4	Steel	6.00	.02	.046	.048	1.08	.042
4	Steel	5.8	.04	.046	.085	2.00	.042
4	Steel	5.1	.06	.046	.108	2.64	.040

The small values of C, .017 and .019, obtained for cast iron are probably due to two reasons: the iron was soft and of fine quality, known as pulley metal, requiring less power to cut; and, as Prof. Smith remarks, a lower cutting-speed also takes less horse-power.

Hardness of metals and forms of tools vary, otherwise the amount of chips turned out per hour per horse-power would be practically constant, the higher cutting-speeds decreasing but slightly the visible work done.

Taking into account these variations, the weight of metal removed per hour, multiplied by a certain constant, is equal to the power necessary to do the work.

This constant, according to the above tests, is as follows:

	Cast Iron.	Wrought Iron.	Steel.
Hartig	.030	.062	.047
Smith	.023	.098	.042
Hobart	.024		
Average	.026	.080	.044

The power necessary to run the lathe empty will vary from about .05 to .3 H.P., which should be ascertained and added to the useful horse-power, to obtain the total power expended.

Power used by Machine-tools. (R. E. Dinsmore, from the *Electrical World.*)

1. Shop shafting 2 3/16″ × 180 ft. at 160 revs., carrying 26 pulleys from 6″ diam. to 36″, and running 20 idle machine belts	1.32 H.P.
2. Lodge-Davis upright back-geared drill-press with table, 28″ swing, drilling ⅝″ hole in cast iron, with a feed of 1 in. per minute	0.78 H.P.
3. Morse twist-drill grinder No. 2, carrying 2″ × 6″ wheels at 3200 revs	0.29 H.P.
4. Pease planer 30″ × 36″, table 6 ft., planing cast iron, cut ¼″ deep, planing 6 sq. in. per minute, at 9 reversals	1.06 H.P.
5. Shaping-machine 22″ stroke, cutting steel die, 6″ stroke, ⅛″ deep, shaping at rate of 1.7 square inch per minute	0.37 H.P.
6. Engine-lathe 17″ swing, turning steel shaft 2⅜″ diam., cut 3/16 deep, feeding 7.92 inch per minute	0.43 H.P.
7. Engine-lathe 21″ swing, boring cast-iron hole 5″ diam., cut 3/16 diam., feeding 0.8″ per minute	0.23 H.P.
8. Sturtevant No. 2, monogram blower at 1800 revs. per minute, no piping	0.8 H.P.
9. Heavy planer 28″ × 28″ × 14 ft. bed, stroke 8″, cutting steel, 22 reversals per minute	3.2 H.P.

The table on the next page compiled from various sources, principally from Hartig's researches, by Prof. J. J. Flather (*Am. Mach.*, April 12, 1894), may be used as a guide in estimating the power required to run a given machine; but it must be understood that these values, although determined by dynamometric measurements for the individual machines designated, are not necessarily representative, as the power required to drive a machine itself is dependent largely on its particular design and construction. The character of the work to be done may also affect the power required to operate; thus a machine to be used exclusively for brass work may be speeded from 10% to 15% higher than if it were to be used for iron work of similar size, and the power required will be proportionately greater.

Where power is to be transmitted to the machines by means of shafting and countershafts, an additional amount, varying from 30% to 50% of the total power absorbed by the machines, will be necessary to overcome the friction of the shafting.

Horse-power required to drive Shafting.—Samuel Webber, in his "Manual of Power" gives among numerous tables of power required to drive textile machinery, a table of results of tests of shafting. A line of 2⅛″ shafting, 342 ft. long, weighing 4098 lbs., with pulleys weighing 5331 lbs., or a total of 9429 lbs., supported on 47 bearings, 216 revolutions per minute, required 1.858 H.P. to drive it. This gives a coefficient of friction of 5.52%. In seventeen tests the coefficient ranged from 3.34% to 11.4%, averaging 5.73%.

Horse-power Required to Drive Machinery.

Name of Machine.	Observed Horse-power. Total Work.	Running Light.
Small screw-cutting lathe 13½″ swing, B. G.	0.41	0.18; 0.15*-0.34†
Screw-cutting lathe 17½″, B. G.	0.367	0.207; 0.16-0.466
Screw-cutting lathe 20″ (Fitchburg), B. G.	0.47	0.12; 0.12 to 0.31
Screw-cutting lathe 26″, B. G.	0.462	0.05; 0.08 to 0.33
Lathe, 30″ face plate, will swing 108″, T. G.	0.53	0.187; 0.12 to 0.66
Large facing lathe, will swing 68″, T. G.	0.91	0.37; 0.39 to 0.81
Wheel lathe 60″ swing		0.28 to 3.40
Small shaper (stroke 4″, traverse 11″)	0.16	0.066 to 0.26
Small shaper, Richards (9½″ × 22″)	0.24	0.07; 0.07 to 0.12
Shaper (15″ stroke Gould & Eberhardt)	0.63	0.21; 0.01 to 0.47
Large shaper, Richards (29″ × 91″)	1.14	0.26; 0.15 to 0.73
Crank planer (capacity 22″ × 27″ × 28½″ stroke)	0.24	0.12; 0.12 to 0.40
Planer (capacity 36″ × 36″ × 11 feet)	0.84	0.27
Large planer (capacity 76″ × 76″ × 57 feet	1.47	0.60
Small drill press	0.62	0.39
Upright slot drilling mach. (will drill 2½″ diam.)	0.41	0.15; 0.15 to 0.43
Medium drill press	1.33	0.62
Large drill press	1.24	0.62
Radial drill 6 feet swing	0.53	0.44; 0.1*-0.44†
Radial drill 8½ feet swing	0.67	0.30; 0.12*-0.60†
Radial drill press	1.08	0.46
Slotter (8″ stroke)	0.28	0.09; 0.05 to 0.25
Slotter (9½″ stroke)	0.44	0.22; 0.15 to 0.65
Slotter (15″ stroke)	0.95	0.57; 0.43 to 0.94
Universal milling mach (Brown & Sharpe No. 1)	0.28	0.01; 0.003-0.13
Milling machine (18″ cutter-head, 12 cutters)	0.66	0.26; 0.26 to 0.55
Small head traversing milling machine (cutter-head 11″ diameter, 16 cutters)	0.18	0.10
Gear cutter will cut 20″ diameter	0.28	0.11
Horizontal boring machine for iron, 22½″ swing	0.98	0.12; 0.10-0.12*; 0.10 to 0.25†
Hydraulic shearing machine	1.52	0.87
Large plate shears—knives 28″ long, 3″ stroke	7.12	0.67
Large punch press, over-reach 28″, 3″ stroke, 1½″ stock can be punched	4.41	1.00
Small punch and shear comb'd, 7½″ knives, 1½″ str.	0.79	0.16
Circular saw for hot iron (30½″ diameter of saw)	4.12	0.61
Plate-bending rolls, diam. of rolls 13″, length 9½ ft.	2.70	.54
Wood planer 13½″ (rotary knives, 2 hor'l 2 vert.	4.34	3.35
Wood planer 24″ (rotary knives)	3.03	1.42
Wood planer 17½″ (rotary knives)	4.63	1.25
Wood planer 28″ (rotary knives)	5.00	0.74‡-0.17§
Wood planer 28″ (Daniel's pattern)	3.20	1.45
Wood planer and matcher (capacity 14½ × 4¾″)	6.91	4.18
Circular saw for wood (23″ diameter of saw)	3.23	0.70
Circular saw for wood (35″ diameter of saw)	5.64	1.16
Band saw for wood (34″ band wheel)	0.96	0.19
Wood-mortising and boring machine	0.49	0.34
Hor'l wood-boring and mortising machine, drill 4″ diam., mortise 8½ deep × 11½″ long	3.68	1.67; 0.05 to 2.0
Tenon and mortising machine	2.11	1.42
Tenon and mortising machine	2.73	0.61
Tenon and mortising machine	2.25	2.17
Edge-molder and shaper. (Vertical spindle)	2.00	1.30
Wood-molding mach. (cap. 7½ × 2½). Hor. spindle	2.45	2.00
Grindstone for tools, 31″ diam., 6″ face. Velocity 680 ft. per minute	1.55	0.32
Grindstone for stock, 42″×12″. Vel. 1680 ft. per min.	3.11	0.34
Emery wheel 11½″ diameter × ¼″. Saw grinder.	0.56	0.40

* With back gears. † Without back gears. ‡ For surface cutters. § With side cutters. B. G., back-geared. T. G., triple-geared.

Horse-power consumed in Machine-shops.—How much power is required to drive ordinary machine-tools? and how many men can be employed per horse-power? are questions which it is impossible to answer by any fixed rule. The power varies greatly according to the conditions in each shop. The following table given by J. J. Flather in his work on Dynamometers gives an idea of the variation in several large works. The percentage of the total power required to drive the shafting varies from 15 to 80, and the number of men employed per total H.P. varies from 0.62 to 6.04.

Horse-power; Friction; Men Employed.

Name of Firm.	Kind of Work.	Horse-power. Total.	Required to drive Shafting.	Required to drive Machinery.	Per cent to drive Shafting.	Number of Men.	No. of Men per Total H.P.	No. of Men per Effective H.P.
Lane & Bodley	E. & W. W.	58				132	2.37	
J. A. Fay & Co	W. W.	100	15	85	15	300	3.00	3.53
Union Iron Works	E., M. M.	400	95	305	23	1600	4.00	5.24
Frontier Iron & Brass W'ks	M. E., etc.	25	8	17	32	150	6.00	8.83
Taylor Mfg. Co	E.	95				230	2.42	
Baldwin Loco. Works	L.	2500	2000	500	80	4100	1.64	8.20
W. Sellers & Co. (one department)	H. M.	102	41	61	40	300	2.93	4.87
Pond Machine Tool Co	M. T.	180	75	105	41	432	2.40	4.11
Pratt & Whitney Co	"	120				725	6.04	
Brown & Sharpe Co	"	230				900	3.91	
Yale & Towne Co	C. & L.	135	67	68	49	700	5.11	10.25
Ferracute Machine Co.	P. & D.	35	11	24	31	90	2.57	3.75
T. B. Wood's Sons	P. & S.	12				30	2.50	
Bridgeport Forge Co	H. F.	150	75	75	50	130	.86	1.73
Singer Mfg. Co	S. M.	1300				3500	2.69	
Howe Mfg. Co.	"	350				1500	4.28	
Worcester Mach. Screw Co	M. S.	40				80	2.00	
Hartford " " "	"	400	100	300	25	250	0.62	0.83
Nicholson File Co.	F.	350				400	1.14	
Averages		346.4			38.6%	818.3	2.96	5.13

Abbreviations: E., engine; W.W., wood-working machinery; M. M., mining machinery; M. E., marine engines; L., locomotives; H. M., heavy machinery; M. T., machine tools; C. & L., cranes and locks; P. & D., presses and dies; P. & S., pulleys and shafting; H. F., heavy forgings; S. M., sewing-machines; M. S., machine-screws; F., files.

J. T. Henthorn states (Trans. A. S. M. E., vi. 462) that in print-mills which he examined the friction of the shafting and engine was in 7 cases below 20% and in 35 cases between 20% and 30%, in 11 cases from 30% to 35% and in 2 cases above 35%, the average being 25.9%. Mr. Barrus in eight cotton-mills found the range to be between 15% and 25.7%, the average being 22%. Mr. Flather believes that for shops using heavy machinery the percentage of power required to drive the shafting will average from 40% to 50% of the total power expended. This presupposes that under the head of shafting are included elevators, fans, and blowers.

ABRASIVE PROCESSES.

Abrasive cutting is performed by means of stones, sand, emery, glass, corundum, carborundum, crocus, rouge, chilled globules of iron, and in some cases by soft, friable iron alone. (See paper by John Richards, read before the Technical Society of the Pacific Coast, *Am. Mach.*, Aug. 20, 1891, and *Eng. & M. Jour.*, July 25 and Aug. 15, 1891.)

The "Cold Saw."—For sawing any section of iron while cold the cold saw is sometimes used. This consists simply of a plain soft steel or iron disk without teeth, about 42 inches diameter and 3/16 inch thick. The velocity of the circumference is about 15,000 feet per minute. One of these saws will saw through an ordinary steel rail cold in about one minute. In this saw the steel or iron is ground off by the friction of the disk, and is not cut as with the teeth of an ordinary saw. It has generally been found more profitable, however, to saw iron with disks or band-saws fitted with cutting-teeth, which run at moderate speeds, and cut the metal as do the teeth of a milling-cutter.

Reese's Fusing-disk.—Reese's fusing-disk is an application of the cold saw to cutting iron or steel in the form of bars, tubes, cylinders, etc., in which the piece to be cut is made to revolve at a slower rate of speed than the saw. By this means only a small surface of the bar to be cut is presented at a time to the circumference of the saw. The saw is about the same size as the cold saw above described, and is rotated at a velocity of about 25,000 feet per minute. The heat generated by the friction of this saw against the small surface of the bar rotated against it is so great that the particles of iron or steel in the bar are actually fused, and the "sawdust" welds as it falls into a solid mass. This disk will cut either cast iron, wrought iron, or steel. It will cut a bar of steel 1⅜ inch diameter in one minute, including the time of setting it in the machine, the bar being rotated about 200 turns per minute.

Cutting Stone with Wire.—A plan of cutting stone by means of a wire cord has been tried in Europe. While retaining sand as the cutting agent, M. Paulin Gay, of Marseilles, has succeeded in applying it by mechanical means, and as continuously as formerly the sand-blast and band-saw, with both of which appliances his system—that of the "helicoidal wire cord"—has considerable analogy. An engine puts in motion a continuous wire cord (varying from five to seven thirty-seconds of an inch in diameter, according to the work), composed of three mild-steel wires twisted at a certain pitch, that is found to give the best results in practice, at a speed of from 15 to 17 feet per second.

The Sand-blast.—In the sand-blast, invented by B. F. Tilghman, of Philadelphia, and first exhibited at the American Institute Fair, New York, in 1871, common sand, powdered quartz, emery, or any sharp cutting material is blown by a jet of air or steam on glass, metal, or other comparatively brittle substance, by which means the latter is cut, drilled, or engraved. To protect those portions of the surface which it is desired shall not be abraded it is only necessary to cover them with a soft or tough material, such as lead, rubber, leather, paper, wax, or rubber-paint. (See description in App. Cyc. Mech.; also U. S. report of Vienna Exhibition, 1873, vol. iii. 316.)

A "jet of sand" impelled by steam of moderate pressure, or even by the blast of an ordinary fan, depolishes glass in a few seconds; wood is cut quite rapidly; and metals are given the so-called "frosted" surface with great rapidity. With a jet issuing from under 300 pounds pressure, a hole was cut through a piece of corundrum 1½ inches thick in 25 minutes.

The sand-blast has been applied to the cleaning of metal castings and sheet metal, the graining of zinc plates for lithographic purposes, the frosting of silverware, the cutting of figures on stone and glass, and the cutting of devices on monuments or tombstones, the recutting of files, etc. The time required to sharpen a worn-out 14-inch bastard file is about four minutes. About one pint of sand, passed through a No. 120 sieve, and four horse-power of 60-lb. steam are required for the operation. For cleaning castings compressed air at from 8 to 10 pounds pressure per square inch is employed. Chilled-iron globules instead of quartz or flint-sand are used with good results, both as to speed of working and cost of material, when the operation can be carried on under proper conditions. With the expenditure of 2 horse-power in compressing air, 2 square feet of ordinary scale on the surface of steel and iron plates can be removed per minute. The surface thus prepared is ready for tinning, galvanizing, plating, bronzing, painting, etc. By continuing the operation the hard skin on the surface of castings, which is so destructive to the cutting edges of milling and other tools, can be removed. Small castings are placed in a sort of slowly rotating barrel, open at one or both ends, through which the blast is directed downward against them as they tumble over and over. No portion of the surface escapes the action of the sand. Plain cored work, such as valve-bodies, can be cleaned perfectly both inside and out. 100 lbs. of castings can be cleaned in from 10 to 15 minutes with a blast created by 2 horse-power. The same weight of small forgings and stampings can be scaled in from 20 to 30 minutes.—*Iron Age*, March 8, 1894.

EMERY-WHEELS AND GRINDSTONES.

The Selection of Emery-wheels.—A pamphlet entitled "Emery-wheels, their Selection and Use," published by the Brown & Sharpe Mfg. Co., after calling attention to the fact that too much should not be expected of one wheel, and commenting upon the importance of selecting the proper wheel for the work to be done, says:

Wheels are numbered from coarse to fine; that is, a wheel made of No. 60 emery is coarser than one made of No. 100. Within certain limits, and other things being equal, a coarse wheel is less liable to change the temperature of the work and less liable to glaze than a fine wheel. As a rule, the harder the stock the coarser the wheel required to produce a given finish. For example, coarser wheels are required to produce a given surface upon hardened steel than upon soft steel, while finer wheels are required to produce this surface upon brass or copper than upon either hardened or soft steel.

Wheels are graded from soft to hard, and the grade is denoted by the letters of the alphabet, A denoting the softest grade. A wheel is soft or hard chiefly on account of the amount and character of the material combined in its manufacture with emery or corundum. But other characteristics being equal, a wheel that is composed of fine emery is more compact and harder than one made of coarser emery. For instance, a wheel of No. 100 emery, grade B, will be harder than one of No. 60 emery, same grade.

The softness of a wheel is generally its most important characteristic. A soft wheel is less apt to cause a change of temperature in the work, or to become glazed, than a harder one. It is best for grinding hardened steel, cast-iron, brass, copper, and rubber, while a harder or more compact wheel is better for grinding soft steel and wrought iron. As a rule, other things being equal, the harder the stock the softer the wheel required to produce a given finish.

Generally speaking, a wheel should be softer as the surface in contact with the work is increased. For example, a wheel 1/16-inch face should be harder than one ½-inch face. If a wheel is hard and heats or chatters, it can often be made somewhat more effective by turning off a part of its cutting surface; but it should be clearly understood that while this will sometimes prevent a hard wheel from heating or chattering the work, such a wheel will not prove as economical as one of the full width and proper grade, for it should be borne in mind that the grade should always bear the proper relation to the width. (See the pamphlet referred to for other information. See also lecture by T. Dunkin Paret, Pres't of The Tanite Co., on Emery-wheels, Jour. Frank. Inst., March, 1890.)

Speed of Emery-wheels.—The following speeds are recommended by different makers:

Diameter of Wheel, inches	Revolutions per minute.				Diameter of Wheel, inches	Revolutions per minute.			
	Waltham E. W. Co.	The Tanite Co.	Grant Corundum Wheel Co.	Norton E. W. Co.		Waltham E. W. Co.	The Tanite Co.	Grant Corundum Wheel Co.	Norton E. W. Co.
1	19,000	....	....	....	10	1,950	2,160	2,200	2,200
1½	12,500	14,400	... .	12,000	12	1,600	1,800	1,800	1,850
2	9,500	10,800	....	10,000	14	1,400	1,570	1,600	1,600
2½	7,600	8,640	.. ....	8,500	16	1,200	1,350	1,400	1,400
3	6,400	7,200	7,400	7,400	18	1,050	1,222	1,250	1,250
4	4,800	5,400	5,400	5,450	20	950	1,080	1,100	1,100
5	3,800	4,320	4,400	4,400	22	875	1,000	1,000	1,000
6	3,200	3,600	3,600	3,600	24	800	917	925	925
7	2,700	3,080	3,200	3,150	26	750	....	600	825
8	2,400	2,700	2,700	2,750	30	675	733	500	735
9	2,150	2,400	2,400	2,450	36	550	611	400	550

"We advise the regular speed of 5500 feet per minute." (Detroit Emery-wheel Co.)

"Experience has demonstrated that there is no advantage in running

solid emery-wheels at a higher rate than 5500 feet per minute peripheral speed." (Springfield E. W. Mfg. Co.)

"Although there is no exactly defined limit at which a wheel must be run to render it effective, experience has demonstrated that, taking into account safety, durability, and liability to heat, 5500 feet per minute at the periphery gives the best results. All first-class wheels have the number of revolutions necessary to give this rate marked on their labels, and a column of figures in the price-list gives a corresponding rate. Above this speed all wheels are unsafe. If run much below it they wear away rapidly in proportion to what they accomplish." (Northampton E. W. Co.)

Grades of Emery.—The numbers representing the grades of emery run from 8 to 120, and the degree of smoothness of surface they leave may be compared to that left by files as follows:

8 and 10	represent the cut of	a wood rasp.
16 " 20	" " " "	a coarse rough file.
24 " 30	" " " "	an ordinary rough file.
36 " 40	" " " "	a bastard file.
46 " 60	" " " "	a second-cut file.
70 " 80	" " " "	a smooth "
90 " 100	" " " "	a superfine "
120 F and FF	" " " "	a dead-smooth file.

Speed of Polishing-wheels.

Wood covered with leather, about	7000 ft. per minute
" " " a hair brush, about	2500 revs. for largest
" " 1½″ to 8″ diam., hair 1″ to 1¼″ long, ab.	4500 " " smallest
Walrus-hide wheels, about......	8000 ft. per minute
Rag-wheels, 4 to 8 in. diameter, about......	7000 " " "

Safe Speeds for Grindstones and Emery-wheels.—G. D. Hiscox (*Iron Age*, April 7, 1892), by an application of the formula for centrifugal force in fly-wheels (see Fly-wheels), obtains the figures for strains in grindstones and emery-wheels which are given in the tables below. His formulæ are:

Stress per sq. in. of section of a grindstone $= (.7071D \times N)^2 \times .0000795$

" " " " " " " an emery-wheel $= (.7071D \times N)^2 \times .0001026$

D = diameter in feet, N = revolutions per minute.

He takes the weight of sandstone at .078 lb. per cubic inch, and that of an emery-wheel at 0.1 lb. per cubic inch; Ohio stone weighs about .081 lb. and Huron stone about .089 lb. per cubic inch. The Ohio stone will bear a speed at the periphery of 2500 to 3000 ft. per min., which latter should never be exceeded. The Huron stone can be trusted up to 4000 ft., when properly clamped between flanges and not excessively wedged in setting. Apart from the speed of grindstones as a cause of bursting, probably the majority of accidents have really been caused by wedging them on the shaft and over-wedging to true them. The holes being square, the excessive driving of wedges to true the stones starts cracks in the corners that eventually run out until the centrifugal strain becomes greater than the tenacity of the remaining solid stone. Hence the necessity of great caution in the use of wedges, as well as the holding of large quick-running stones between large flanges and leather washers.

Strains in Grindstones.

LIMIT OF VELOCITY AND APPROXIMATE ACTUAL STRAIN PER SQUARE INCH OF SECTIONAL AREA FOR GRINDSTONES OF MEDIUM TENSILE STRENGTH.

Diameter.	Revolutions per minute.						
	100	150	200	250	300	350	400
feet.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
2	1.58	3.57	6.35	9.93	14.30	18.36	25.42
2½	2.47	5.57	9.88	15.49	22.29	28.64	39.75
3	3.57	8.04	14.28	22.34	32.16	....	....
3½	4.86	10.93	19.44	30.38	....	....	....
4	6.35	14.30	27.37	... .	....	... .	....
4½	8.04	18.08	32.16				
5	9.93	22.34	... ...				
6	14.30	32.17	....				
7	19.44	....	....				

Approximate breaking strain ten times the strain for size opposite the bottom figure in each column.

The figures at the bottom of columns designate the limit of velocity (in revolutions per minute), at the head of the columns for stones of the diameter in the first column opposite the designating figure.

A general rule of safety for any size grindstone that has a compact and strong grain is to limit the peripheral velocity to 47 feet per second.

There is a large variation in the listed speeds of emery-wheels by different makers—4000 as a minimum and 5600 maximum feet per minute, while others claim a maximum speed of 10,000 feet per minute as the safe speed of their best emery-wheels. Rim wheels and iron centre wheels are specialties that require the maker's guarantee and assignment of speed.

Strains in Emery-wheels.

ACTUAL STRAIN PER SQUARE INCH OF SECTION IN EMERY-WHEELS AT THE VELOCITIES AT HEAD OF COLUMNS FOR SIZES IN FIRST COLUMN.

Diam., inches.	Revolutions per minute.										
	600	800	1000	1200	1400	1600	1800	2000	2200	2400	2600
4	......	......	......	......	......	......	......	22.67	27.48	32.64	38.31
6	......	......	......	......	......	......	......	51.18	61.86	73.62	86.40
8	......	......	22.67	32.65	44.45	58.06	73.47	90.71	109.76	130.62	153.30
10	......	......	35.47	51.08	69.51	90.81	114.94	141.90	171.71	......	......
12	18.40	32.72	51.12	73.62	100.21	130.88	165.65	......	......	......	......
14	24.80	43.90	68.70	99.21	134.65	175.60	......	......			
16	32.57	57.65	90.24	130.31	177.80	......	......	......			
18	41.41	73.62	115.03	165.65	......	......	......	......			
20	50.98	90.23	141.22	......	......	......	......	......			
22	61.81	109.41	171.23	......	......	......	......	......			
24	73.62	130.88	......	......	......	......	......	......			
26	86.36	152.85	......	......	......	......	......	......			
30	115.04	......	......	......	......	......	......	......			
36	165.64	......	......	......	......	......	......	......			

Diam	Revs. per min.	
in.	2800	3000
4	44.43	51.12
6	100.21	115.03
8	177.80	......

Joshua Rose (Modern Machine-shop Practice) says: The average speed of grindstones in workshops may be given as follows:

	Circumferential Speed of Stone.
For grinding machinists' tools, about	900 feet per minute.
" " carpenters' " "	600 " " "

The speeds of stones for file-grinding, and other similar rapid grinding is thus given in the "Grinders' List."

Diam. ft.......	8	7½	7	6½	6	5½	5	4½	4	3½	3
Revs. per min.	135	144	154	166	180	196	216	240	270	308	360

The following table, from the *Mechanical World*, is for the diameter of stones and the number of revolutions they should run per minute (not to be exceeded), with the diameter of change of shift-pulleys required, varying each shift or change 2½ inches, 2¼ inches, or 2 inches in diameter for each reduction of 6 inches in the diameter of the stone.

Diameter of Stone.		Revolutions per minute.	Shift of Pulleys, in inches.		
			2½	2¼	2
ft.	in.				
8	0	135	40	36	32
[illegible]	6	144	37½	33¾	30
7	0	154	35	31½	28
6	[illegible]	166	32½	29¼	26
[illegible]	0	180	30	27	24
5	6	196	27½	24¾	22
5	0	216	25	22½	20
4	6	240	22½	20¼	18
4	0	270	20	18	16
3	6	308	17½	15¾	14
3	0	360	15	13½	12
1		2	3	4	5

Columns 3, 4, and 5 are given to show that if we start an 8-foot stone with, say, a countershaft pulley driving a 40-inch pulley on the grindstone spindle, and the stone makes the right number (185) of revolutions per minute, the reduction in the diameter of the pulley on the grinding-stone spindle, when the stone has been reduced 6 inches in diameter, will require to be also reduced 2½ inches in diameter, or to shift from 40 inches to 37½ inches, and so on similarly for columns 4 and 5. Any other suitable dimensions of pulley may be used for the stone when eight feet in diameter, but the number of inches in each shift named, in order to be correct, will have to be proportional to the numbers of revolutions the stone should run, as given in column 2 of the table.

Varieties of Grindstones.

(Joshua Rose.)

For Grinding Machinists' Tools.

Name of Stone.	Kind of Grit.	Texture of Stone.	Color of Stone.
Nova Scotia,	All kinds, from finest to coarsest	All kinds, from hardest to softest	Blue or yellowish gray
Bay Chaleur (New Brunswick),	Medium to finest	Soft and sharp	Uniformly light blue
Liverpool or Melling.	Medium to fine	Soft, with sharp grit	Reddish.

For Wood-working Tools.

Wickersley..........	Medium to fine	Very soft	Grayish yellow
Liverpool or Melling.	Medium to fine	Soft, with sharp grit	Reddish
Bay Chaleur (New Brunswick),	Medium to finest	Soft and sharp	Uniform light blue
Huron, Michigan ...	Fine	Soft and sharp	Uniform light blue

For Grinding Broad Surfaces, as Saws or Iron Plates.

Newcastle............	Coarse to med'm	The hard ones	Yellow
Independence........	Coarse	Hard to medium	Grayish white
Massillon	Coarse	Hard to medium	Yellowish white

TAP DRILLS.

Taps for Machine-screws. (The Pratt & Whitney Co.)

Approx. Diameter, fractions of an inch.	Wire Gauge.	No. of Threads to inch.	Approx. Diameter, fractions of an inch.	Wire Gauge.	No. of Threads to inch.
	No. 1	60, 72		No. 13	20, 24
	2	48, 56, 64	¼	14	16, 18, 20, 22, 24
	3	40, 48, 56		15	18, 20, 24
7/64	4	32, 36, 40	17/64	16	16, 18, 20, 22
	5	30, 32, 36, 40	9/32	18	16, 18, 20
9/64	6	30, 32, 36, 40		19	16, 18, 20
	7	24, 30, 32	5/16	20	16, 18, 20
5/32	8	24, 30, 32, 36, 40		22	16, 18
	9	24, 28, 30, 32	⅜	24	14, 16, 18
3/16	10	20, 22, 24, 30, 32		26	16
	11	22, 24		28	16
7/32	12	20, 22, 24		30	16

The Morse Twist Drill and Machine Co. gives the following table showing the different sizes of drills that should be used when a suitable thread is to be tapped in a hole. The sizes given are practically correct.

Tap Drills.

(The Morse Twist Drill and Machine Co.)

Diam. of Tap.	No. Threads to inch.			Drill for V Thread.			Drill for U. S. S. Thread.			Diam. of Tap.	No. Threads to inch.		Drill for V Thread.		Drill for U. S. S. Thread.	
¼	16	18	20	5/32	11/64	15	.....	.....	3/16	1 ⅛	7	8	59/64	61/64	61/64	.....
9/32	16	18	20	3/16	13/64	8	.....	.....	.....	1 5/32	7	8	61/64	63/64	.....	.....
5/16	16	18	..	7/32	15/64	.....	.....	C	.....	1 3/16	7	8	63/64	1 1/64	.....	.....
11/32	16	18	..	¼	17/64	.....	.....	.....	.....	1 7/32	7	8	1 1/64	1 3/64	.....	.....
⅜	14	16	18	17/64	9/32	M	.....	N	.....	1 ¼	7	..	1 3/64	.....	1 5/64	.....
13/32	14	16	18	N	5/16	P	.....	.....	.....	1 9/32	7	..	1 5/64	.....	.....	.....
7/16	14	16	..	Q	11/32	.....	S	.....	.....	1 5/16	7	..	1 7/64	.....	.....	.....
15/32	14	16	..	23/64	⅜	.....	.....	.....	.....	1 11/32	7	..	1 9/64	.....	.....	.....
½	12	13	14	⅜	W	25/64	.....	13/32	.....	1 ⅜	6	..	1 ⅛	.....	1 11/64	.....
17/32	12	13	14	13/32	27/64	7/16	.....	.....	.....	1 13/32	6	..	1 5/32	.....	.....	.....
9/16	12	14	..	7/16	29/64	.....	29/64	.....	.....	1 7/16	6	..	1 3/16	.....	.....	.....
19/32	12	14	..	31/64	½	.....	.....	.....	.....	1 15/32	6	..	1 7/32	.....	.....	.....
⅝	10	11	12	31/64	½	33/64	.....	33/64	.....	1 ½	6	..	1 17/64	.....	1 19/64	.....
21/32	10	11	12	33/64	17/32	35/64	.....	.....	.....	1 17/32	6	..	1 19/64	.....	.....	.....
11/16	11	12	..	9/16	37/64	.....	.....	.....	.....	1 9/16	6	..	1 21/64	.....	.....	.....
23/32	11	12	..	19/32	39/64	.....	.....	.....	.....	1 19/32	6	..	1 23/64	.....	.....	.....
¾	10	11	12	39/64	⅝	41/64	⅝	.....	.....	1 ⅝	5	5½	1 21/64	1 23/64	.....	1 25/64
25/32	10	11	12	41/64	21/32	43/64	.....	.....	.....	1 21/32	5	5½	1 23/64	1 25/64	.....	.....
13/16	10	..	..	43/64	.....	.....	.....	.....	.....	1 11/16	5	5½	1 25/64	1 27/64	.....	.....
27/32	10	..	..	45/64	.....	.....	.....	.....	.....	1 23/32	5	5½	1 27/64	1 29/64	.....	.....
⅞	9	10	..	23/32	47/64	.....	47/64	.....	.....	1 ¾	5	..	1 29/64	.....	1 ½	.....
29/32	9	10	..	¾	49/64	.....	.....	.....	.....	1 25/32	5	..	1 31/64	.....	.....	.....
15/16	9	..	..	25/32	.....	.....	.....	.....	.....	1 13/16	5	..	1 33/64	.....	.....	.....
31/32	9	..	..	13/16	.....	.....	.....	.....	.....	1 27/32	5	..	1 35/64	.....	.....	.....
1	8	..	..	53/64	.....	.....	27/32	.....	.....	1 ⅞	4½	5	1 35/64	1 37/64	.....	1 ⅝
1 1/32	8	..	..	55/64	.....	.....	.....	.....	.....	1 29/32	4½	5	1 37/64	1 39/64	.....	.....
1 1/16	8	..	..	57/64	.....	.....	.....	.....	.....	1 15/16	4½	5	1 39/64	1 41/64	.....	.....
1 3/32	8	..	..	59/64	.....	.....	.....	.....	.....	1 31/32	4½	5	1 41/64	1 43/64	.....	.....
										2	4½	..	1 43/64	.....	1 23/32	.....

TAPER BOLTS, PINS, REAMERS, ETC.

Taper Bolts for Locomotives.—Bolt-threads, U. S. standard, except stay-bolts and boiler-studs, V threads, 12 per inch; valves, cocks, and plugs, V threads, 14 per inch, and ⅛-inch taper per 1 inch. Standard bolt taper 1/16 inch per foot.

Taper Reamers.—The Pratt & Whitney Co. makes standard taper reamers for locomotive work taper 1/16 inch per foot from ¼ inch diam.; 4 in. length of flute to 2 in. diam.; 18 in. length of flute, diameters advancing by 16ths and 32ds. P. & W. Co.'s standard taper pin reamers taper ¼ in. per foot, are made in 14 sizes of diameters, 0.135 to 1.009 in.; length of flute 1 5/16 in. to 12 in.

DIMENSIONS OF THE PRATT & WHITNEY COMPANY'S REAMERS FOR MORSE STANDARD-TAPER SOCKET.

No.	Diameter Small End, inches.	Diameter Large End, inches.	Gauge Diam., la'ge end, inches	Gauge L'ngth, inches.	Length Flute, inches.	Total L'ngth.	Taper per foot, inches.
1	0.365	0.525	0.475	2⅛	3	5¼	0.600
2	0.573	0.749	0.699	2½	3½	6¼	0.602
3	0.779	0.982	0.930	3 5/16	4	7½	0.602
4	1.026	1.283	1.231	4	5	8¾	0.623
5	1.486	1.796	1.746	5	6	10	0.630
6	2.117	2.566	2.500	7¼	8½	12½	0.625

Standard Steel Taper-pins.—The following sizes are made by The Pratt & Whitney Co.:

Number:	0	1	2	3	4	5	6	7	8	9	10
Diameter large end:	.156	.172	.193	.219	.250	.289	.341	.409	.492	.591	.706
Approximate fractional sizes:	5/32	11/64	3/16	7/32	¼	19/64	11/32	13/32	½	19/32	23/32
Lengths from	¾	¾	¾	¾	¾	¾	¾	1	1¼	1¼	1½
To*	1	1¼	1½	1¾	2	2¼	3¼	3¾	4½	5¼	6
Diameter small end of standard taper-pin reamer:†	.135	.146	.162	.183	.208	.240	.279	.331	.398	.482	.581

Standard Steel Mandrels. (The Pratt & Whitney Co.)—These mandrels are made of tool-steel, hardened, and ground true on their centres. Centres are also ground to true 60° cones. The ends are of a form best adapted to resist injury likely to be caused by driving. They are slightly taper. Sizes, ¼ in. diameter by 3⅝ in. long to 3 in. diam. by 14⅝ in. long, diameters advancing by 16ths.

PUNCHES AND DIES, PRESSES, ETC.

Clearance between Punch and Die.—For computing the amount of clearance that a die should have, or, in other words, the difference in size between die and punch, the general rule is to make the diameter of die-hole equal to the diameter of the punch, plus 2/10 the thickness of the plate. Or, $D = d + .2t$, in which D = diameter of die-hole, d = diameter of punch, and t = thickness of plate. For very thick plates some mechanics prefer to make the die-hole a little smaller than called for by the above rule. For ordinary boiler-work the die is made from 1/10 to 3/10 of the thickness of the plate larger than the diameter of the punch; and some boiler-makers advocate making the punch fit the die accurately. For punching nuts, the punch fits in the die. (*Am. Machinist.*)

Kennedy's Spiral Punch. (The Pratt & Whitney Co.)—B. Martell, Chief Surveyor of Lloyd's Register, reported tests of Kennedy's spiral punches in which a ⅞-inch spiral punch penetrated a ⅝-inch plate at a pressure of 22 to 25 tons, while a flat punch required 33 to 35 tons. Steel boiler-plates punched with a flat punch gave an average tensile strength of 58,579

* Lengths vary by ¼″ each size. † Taken ⅛″ from extreme end. Each size overlaps smaller one about ⅛″. Taper ¼″ to the foot.

lbs. per square inch, and an elongation in two inches across the hole of 5.9%, while plates punched with a spiral punch gave 68,920 lbs., and 10.6% elongation.

The spiral shear form is not recommended for punches for use in metal of a thickness greater than the diameter of the punch. This form is of greatest benefit when the thickness of metal worked is less than two thirds the diameter of punch.

Size of Blanks used in the Drawing-press. Oberlin Smith (Jour. Frank. Inst., Nov. 1886) gives three methods of finding the size of blanks. The first is a tentative method, and consists simply in a series of experiments with various blanks, until the proper one is found. This is for use mainly in complicated cases, and when the cutting portions of the die and punch can be finally sized after the other work is done. The second method is by weighing the sample piece, and then, knowing the weight of the sheet metal per square inch, computing the diameter of a piece having the required area to equal the sample in weight. The third method is by computation, and the formula is $x = \sqrt{d^2 + 4dh}$ for sharp-cornered cup, where x = diameter of blank, d = diameter of cup, h = height of cup. For round-cornered cup where the corner is small, say radius of corner less than ¼ height of cup, the formula is $x = (\sqrt{d^2 + 4dh}) - r$, about; r being the radius of the corner. This is based upon the assumption that the thickness of the metal is not to be altered by the drawing operation.

Pressure attainable by the Use of the Drop-press. (R. H. Thurston, Trans. A. S. M. E., v. 58.)—A set of copper cylinders was prepared, of pure Lake Superior copper; they were subjected to the action of presses of different weights and of different heights of fall. Companion specimens of copper were compressed to exactly the same amount, and measures were obtained of the loads producing compression, and of the amount of work done in producing the compression by the drop. Comparing one with the other it was found that the work done with the hammer was 90% of the work which should have been done with perfect efficiency. That is to say, 90% of the work done in the testing-machine was equal to that due the weight of the drop falling the given distance.

$$\text{Formula: Mean pressure in pounds} = \frac{\text{Weight of drop} \times \text{fall} \times \text{efficiency}}{\text{compression}}.$$

For pressures per square inch, divide by the mean area opposed to crushing action during the operation.

Flow of Metals. (David Townsend, Jour. Frank. Inst., March, 1878.)—In punching holes 7/16 inch diameter through iron blocks 1¾ inches thick, it was found that the core punched out was only 1 1/16 inch thick, and its volume was only about 32% of the volume of the hole. Therefore, 68% of the metal displaced by punching the hole flowed into the block itself, increasing its dimensions.

FORCING AND SHRINKING FITS.

Forcing Fits of Pins and Axles by Hydraulic Pressure.—A 4-inch axle is turned .015 inch diameter larger than the hole into which it is to be fitted. They are pressed on by a pressure of 30 to 35 tons. (Lecture by Coleman Sellers, 1872.)

For forcing the crank-pin into a locomotive driving-wheel, when the pin-hole is perfectly true and smooth, the pin should be pressed in with a pressure of 6 tons for every inch of diameter of the wheel fit. When the hole is not perfectly true, which may be the result of shrinking the tire on the wheel centre after the hole for the crank-pin has been bored, or if the hole is not perfectly smooth, the pressure may have to be increased to 9 tons for every inch of diameter of the wheel-fit. (*Am. Machinist.*)

Shrinkage Fits.—In 1886 the American Railway Master Mechanics' Association recommended the following shrinkage allowances for tires of standard locomotives. The tires are uniformly heated by gas-flames, slipped over the cast-iron centres, and allowed to cool. The centres are turned to the standard sizes given below, and the tires are bored smaller by the amount of the shrinkage designated for each:

Diameter of centre, in ...	38	44	50	56	62	66
Shrinkage allowance, in .	.040	.047	.053	.060	.066	.070

This shrinkage allowance is approximately 1/80 inch per foot, or 1/960. A common allowance is 1/1000. Taking the modulus of elasticity of steel at

30,000,000, the strain caused by shrinkage would be 30,000 lbs. per square inch, which is well within the elastic limit of machinery steel.

SCREWS, SCREW-THREADS, ETC.*

Efficiency of a Screw.—Let a = angle of the thread, that is, the angle whose tangent is the pitch of the screw divided by the circumference of a circle whose diameter is the mean of the diameters at the top and bottom of the thread. Then for a square thread

$$\text{Efficiency} = \frac{1 - f \tan a}{1 + f \operatorname{cotan} a},$$

in which f is the coefficient of friction. (For demonstration, see Cotterill and Slade, Applied Mechanics, p. 146.) Since cotan = 1 ÷ tan, we may substitute for cotan a the reciprocal of the tangent, or if p = pitch, and c = mean circumference of the screw,

$$\text{Efficiency} = \frac{1 - f\dfrac{p}{c}}{1 + f\dfrac{c}{p}}.$$

EXAMPLE.—Efficiency of square-threaded screws of ½ in. pitch.

Diameter at bottom of thread, in....	1	2	3	4
" " top " " ".....	1½	2½	3½	4½
Mean circumference " " ".....	3.927	7.069	10.21	13.35
Cotangent $a = c \div p$.............. =	7.854	14.14	20.42	26.70
Tangent $a = p \div c$................ =	.1273	.0661	.0490	.0375
Efficiency if f = .10............... =	55.3%	41.2%	32.7%	27.2%
" " f = .15............... =	45%	31.7%	24.4%	19.9%

The efficiency thus increases with the steepness of the pitch.

The above formulæ and examples are for square-threaded screws, and consider the friction of the screw-thread only, and not the friction of the collar or step by which end thrust is resisted, and which further reduces the efficiency. The efficiency is also further reduced by giving an inclination to the side of the thread, as in the V-threaded screw. For discussion of this subject, see paper by Wilfred Lewis, Jour. Frank. Inst. 1880; also Trans. A. S. M. E., vol. xii. 784.

Efficiency of Screw-bolts.—Mr. Lewis gives the following approximate formula for ordinary screw-bolts (V threads, with collars): p = pitch of screw, d = outside diameter of screw, F = force applied at circumference to lift a unit of weight, E = efficiency of screw. For an average case, in which the coefficient of friction may be assumed at .15,

$$F = \frac{p + d}{3d}, \qquad E = \frac{p}{p + d}.$$

For bolts of the dimensions given above, ½-in. pitch, and outside diameters 1½, 2½, 3½, and 4½ in., the efficiencies according to this formula would be, respectively, .25, .167, .125, and .10.

James McBride (Trans. A. S. M. E., xii. 781) describes an experiment with an ordinary 2-in screw-bolt, with a V thread, 4½ threads per inch, raising a weight of 7500 lbs., the force being applied by turning the nut. Of the power applied 89.8% was absorbed by friction of the nut on its supporting washer and of the threads of the bolt in the nut. The nut was not faced, and had the flat side to the washer.

Prof. Ball in his "Experimental Mechanics" says: "Experiments showed in two cases respectively about ⅔ and ¾ of the power was lost."

Trautwine says: "In practice the friction of the screw (which under heavy loads becomes very great) make the theoretical calculations of but little value."

Weisbach says: "The efficiency is from 19% to 30%."

Efficiency of a Differential Screw.—A correspondent of the *American Machinist* describes an experiment with a differential screw-punch, consisting of an outer screw 2 in. diam., 3 threads per in., and an inner screw 1⅜ in. diam., 3½ threads per inch. The pitch of the outer screw

* For U. S. Standard Screw-threads, see page 204.

being ⅛ in. and that of the inner screw 2/7 in., the punch would advance in one revolution ⅛ − 2/7 = 1/21 in. Experiments were made to determine the force required to punch an 11/16-in. hole in iron ¼ in. thick, the force being applied at the end of a lever-arm of 47¾ in. The leverage would be $47\frac{3}{4} \times 2\pi \times 21 = 6300$. The mean force applied at the end of the lever was 95 lbs., and the force at the punch, if there was no friction, would be $6300 \times 95 = 598,500$ lbs. The force required to punch the iron, assuming a shearing resistance of 50,000 lbs. per sq. in., would be $50,000 \times 11/16 \times \pi \times \frac{1}{4} = 27,000$ lbs., and the efficiency of the punch would be $27,000 \div 598,500 =$ only 4.5%. With the larger screw only used as a punch the mean force at the end of the lever was only 82 lbs. The leverage in this case was $47\frac{3}{4} \times 2\pi \times 3 = 900$, the total force referred to the punch, including friction, $900 \times 82 = 73,800$, and the efficiency $27,000 \div 73,800 = 36.7\%$. The screws were of tool-steel, well fitted, and lubricated with lard-oil and plumbago.

Powell's New Screw-thread.—A. M. Powell (*Am. Mach.*, Jan. 24, 1895) has designed a new screw-thread to replace the square form of thread, giving the advantages of greater ease in making fits, and provision for "take up" in case of wear. The dimensions are the same as those of square-thread screws, with the exception that the sides of the thread, instead of being perpendicular to the axis of the screw, are inclined 14½° to such perpendicular; that is, the two sides of a thread are inclined 29° to each other. The formulæ for dimensions of the thread are the following: Depth of thread = ½ ÷ pitch; width of top of thread = width of space at bottom = .3707 ÷ pitch; thickness at root of thread = width of space at top = .6293 ÷ pitch. The term pitch is the number of threads to the inch.

PROPORTIONING PARTS OF MACHINES IN A SERIES OF SIZES.

(*Stevens Indicator*, April, 1892.)

The following method was used by Coleman Sellers while at William Sellers & Co.'s to get the proportions of the parts of machines, based upon the size obtained in building a large machine and a small one to any series of machines. This formula is used in getting up the proportion-book and arranging the set of proportions from which any machine can be constructed of intermediate size between the largest and smallest of the series.

Rule to Establish Construction Formulæ.—Take difference between the nominal sizes of the largest and the smallest machines that have been designed of the same construction. Take also the difference between the sizes of similar parts on the largest and smallest machines selected. Divide the latter by the former, and the result obtained will be a "factor," which, multiplied by the nominal capacity of the intermediate machine, and increased or diminished by a constant "increment," will give the size of the part required. To find the "increment:" Multiply the nominal capacity of some known size by the factor obtained, and subtract the result from the size of the part belonging to the machine of nominal capacity selected.

EXAMPLE.—Suppose the size of a part of a 72-in. machine is 3 in., and the corresponding part of a 42-in. machine is 1⅞, or 1.875 in.: then 72 − 42 = 30, and 3 in. − 1⅞ in. = 1⅛ in. = 1.125. 1.125 ÷ 30 = .0375 = the "factor," and .0375 × 42 = 1.575. Then 1.875 − 1.575 = .3 = the "increment" to be added. Let D = nominal capacity; then the formula will read: $x = D \times .0375 + .3$.

Proof: $42 \times .0375 + .3 = 1.875$, or 1⅞, the size of one of the selected parts.

Some prefer the formula: $aD + c = x$, in which D = nominal capacity in inches or in pounds, c is a constant increment, a is the factor, and x = the part to be found.

KEYS.

Sizes of Keys for Mill-gearing. (Trans. A. S. M. E., xiii. 229.)—E. G. Parkhurst's rule: Width of key = ⅛ diam. of shaft, depth = 1/9 diam. of shaft; taper ⅛ in. to the foot.

Custom in Michigan saw-mills: Keys of square section, side = ¼ diam. of shaft, or as nearly as may be in even sixteenths of an inch.

J. T. Hawkins's rule: Width = ⅛ diam. of hole; depth of side abutment in shaft = ⅛ diam. of hole.

W. S. Huson's rule: ¼-inch key for 1 to 1¼ in. shafts, 5/16 key for 1¼ to 1½ in. shafts, ⅜ in. key for 1½ to 1¾ in. shafts, and so on. Taper ⅛ in. to the foot. Total thickness at large end of splice, 4/5 width of key.

Unwin (Elements of Machine Design) gives: Width = ¼d + ⅛ in. Thickness = ⅛d + ⅛ in., in which d = diam. of shaft in inches. When wheels or pulleys transmitting only a small amount of power are keyed on large shafts, he says, these dimensions are excessive. In that case, if H.P. = horse-power transmitted by the wheel or pulley, N = revs. per min., P = force acting at the circumference, in lbs., and R = radius of pulley in inches, take

$$d = \sqrt[3]{\frac{100\,\text{H.P.}}{N}} \quad \text{or} \quad \sqrt[3]{\frac{PR}{630}}.$$

Prof. Coleman Sellers (*Stevens Indicator*, April, 1892) gives the following: The size of keys, both for shafting and for machine tools, are the proportions adopted by William Sellers & Co., and rigidly adhered to during a period of nearly forty years. Their practice in making keys and fitting them is, that the keys shall always bind tight sidewise, but not top and bottom; that is, not necessarily touch either at the bottom of the key-seat in the shaft or touch the top of the slot cut in the gear-wheel that is fastened to the shaft; but in practice keys used in this manner depend upon the fit of the wheel upon the shaft being a forcing fit, or a fit that is so tight as to require screw-pressure to put the wheel in place upon the shaft.

Size of Keys for Shafting.

Diameter of Shaft, in.					Size of Key, in.
1¼	1 7/16	1 11/16			5/16 × ⅜
1 15/16	2 3/16				7/16 × ½
2 7/16					9/16 × ⅝
2 11/16	2 15/16	3 3/16	3 7/16		11/16 × ¾
3 15/16	4 7/16	4 15/16			13/16 × ⅞
5 7/16	5 15/16	6 7/16			15/16 × 1
6 15/16	7 7/16	7 15/16	8 7/16	8 15/16	1 1/16 × 1⅛

Length of key-seat for coupling = 1⅛ × nominal diameter of shaft.

Size of Keys for Machine Tools.

Diam. of Shaft, in.	Size of Key, in. sq.	Diam. of Shaft, in.	Size of Key, in. sq.
15/16 and under	⅛	4 to 5 7/16	13/16
1 to 1 3/16	3/16	5½ to 6 15/16	15/16
1¼ to 1 7/16	¼	7 to 8 15/16	1 1/16
1½ to 1 11/16	5/16	9 to 10 15/16	1 3/16
1¾ to 2 3/16	7/16	11 to 12 15/16	1 5/16
2¼ to 2 11/16	9/16	13 to 14 15/16	1 7/16
2¾ to 3 15/16	11/16		

John Richards, in an article in *Cassier's Magazine*, writes as follows: There are two kinds or system of keys, both proper and necessary, but widely different in nature. 1. The common fastening key, usually made in width one fourth of the shaft's diameter, and the depth five eighths to one third the width. These keys are tapered and fit on all sides, or, as it is commonly described, "bear all over." They perform the double function in most cases of driving or transmitting and fastening the keyed-on member against movement endwise on the shaft. Such keys, when properly made, drive as a strut, diagonally from corner to corner.

2. The other kind or class of keys are not tapered and fit on their sides only, a slight clearance being left on the back to insure against wedge action or radial strain. These keys drive by shearing strain.

For fixed work where there is no sliding movement such keys are commonly made of square section, the sides only being planed, so the depth is more than the width by so much as is cut away in finishing or fitting.

For sliding bearings, as in the case of drilling-machine spindles, the depth should be increased, and in cases where there is heavy strain there should be two keys or feathers instead of one.

The following tables are taken from proportions adopted in practical use.

Flat keys, as in the first table, are employed for fixed work when the parts are to be held not only against torsional strain, but also against movement endwise; and in case of heavy strain the strut principle being the strongest and most secure against movement when there is strain each way, as in the case of engine cranks and first movers generally. The objections to the system for general use are, straining the work out of truth, the care and expense required in fitting, and destroying the evidence of good or bad fitting of the keyed joint. When a wheel or other part is fastened with a tapering key of this kind there is no means of knowing whether the work is well fitted or not. For this reason such keys are not employed by machine-tool-makers, and in the case of accurate work of any kind, indeed, cannot be, because of the wedging strain, and also the difficulty of inspecting completed work.

I. DIMENSIONS OF FLAT KEYS, IN INCHES.

Diam. of shaft	1	1¼	1½	1¾	2	2½	3	3½	4	5	6	7	8
Breadth of keys	¼	5/16	⅜	7/16	½	⅝	¾	⅞	1	1⅛	1¼	1½	1¾
Depth of keys	5/32	3/16	¼	9/32	5/16	⅜	7/16	½	⅝	11/16	13/16	⅞	1

II. DIMENSIONS OF SQUARE KEYS, IN INCHES.

Diam. of shaft	1	1¼	1½	1¾	2	2½	3	3½	4
Breadth of keys	5/32	7/32	9/32	11/32	13/32	15/32	17/32	9/16	11/16
Depth of keys	3/16	¼	5/16	⅜	7/16	½	9/16	⅝	¾

III. DIMENSIONS OF SLIDING FEATHER-KEYS, IN INCHES.

Diam. of shaft	1¼	1½	1¾	2	2¼	2½	3	3½	4	4½
Breadth of keys	¼	¼	5/16	5/16	⅜	⅜	½	9/16	9/16	⅝
Depth of keys	⅜	⅜	7/16	7/16	½	½	⅝	¾	¾	⅞

P. Pryibil furnishes the following table of dimensions to the *Am. Machinist*. He says: On special heavy work and very short hubs we put in two keys in one shaft 90° apart. With special long hubs, where we cannot use keys with noses, the keys should be thicker than the standard.

Diameter of Shafts, inches.	Width, inches.	Thickness, in.	Diameter of Shafts, inches.	Width, inches.	Thickness, in.
¾ to 1 1/16	3/16	3/16	3 7/16 to 3 11/16	⅞	⅝
1⅛ to 1 5/16	5/16	¼	3 15/16 to 4 3/16	1	11/16
1 7/16 to 1 11/16	⅜	5/16	4 7/16 to 4 11/16	1⅛	¾
1 15/16 to 2 3/16	½	⅜	4⅞ to 5⅝	1¼	15/16
2 7/16 to 2 11/16	⅝	½	5⅞ to 6⅝	1½	1
2 15/16 to 3 3/16	¾	9/16	6⅞ to 7⅞	1¾	1⅛

Keys longer than 10 inches, say 14 to 16″, 1/16″ thicker; keys longer than 10 inches, say 18 to 20″, ⅛″ thicker; and so on. Special short hubs to have two keys.

For description of the Woodruff system of keying, see circular of the Pratt & Whitney Co.; also Modern Mechanism, page 455.

HOLDING-POWER OF KEYS AND SET-SCREWS.

Tests of the Holding-power of Set-screws in Pulleys. (G. Lanza, Trans. A. S. M. E., x. 230.)—These tests were made by using a pulley fastened to the shaft by two set-screws with the shaft keyed to the holders; then the load required at the rim of the pulley to cause it to slip was determined, and this being multiplied by the number 6.037 (obtained by adding to the radius of the pulley one-half the diameter of the wire rope, and dividing the sum by twice the radius of the shaft, since there were two set-screws in action at a time) gives the holding-power of the set-screws. The set-screws used were of wrought-iron, ⅝ of an inch in diameter, and ten threads to the inch; the shaft used was of steel and rather hard, the set-screws making but little impression upon it. They were set up with a force of 75 lbs. at the end of a ten-inch monkey-wrench. The set-screws used were of four kinds, marked respectively A, B, C, and D. The results were as follows:

A, ends perfectly flat, 9/16-in. diameter,	1412 to 2294 lbs.; average	2064.
B, radius of rounded ends about 1/8 inch,	2747 " 3079 " "	2912.
C, " " " " " 1/4 "	1902 " 3079 " "	2573.
D ends cup-shaped and case-hardened,	1962 " 2958 " "	2470.

REMARKS.—A. The set-screws were not entirely normal to the shaft; hence they bore less in the earlier trials, before they had become flattened by wear.

B. The ends of these set-screws, after the first two trials, were found to be flattened, the flattened area having a diameter of about 1/4 inch.

C. The ends were found, after the first two trials, to be flattened, as in B.

D. The first test held well because the edges were sharp, then the holding-power fell off till they had become flattened in a manner similar to B, when the holding-power increased again.

Tests of the Holding-power of Keys. (Lanza.)—The load was applied as in the tests of set-screws, the shaft being firmly keyed to the holders. The load required at the rim of the pulley to shear the keys was determined, and this, multiplied by a suitable constant, determined in a similar way to that used in the case of set-screws, gives us the shearing strength per square inch of the keys.

The keys tested were of eight kinds, denoted, respectively, by the letters A, B, C, D, E, F, G and H, and the results were as follows: A, B, D and F, each 4 tests; E, 3 tests; C, G, and H, each 2 tests.

A, Norway iron, 2″ × 1/4″ × 15/32″,	40,184 to 47,760 lbs.;	average,	42,726.
B, refined iron, 2″ × 1/4″ × 15/32″,	36,482 " 39,254;	"	38,059.
C, tool steel, 1″ × 1/4″ × 15/32″,	91,344 & 100,056.		
D, machinery steel, 2″ × 1/4″ × 15/32″,	64,630 to 70,186;	"	66,875.
E, Norway iron, 1 1/8″ × 3/8″ × 7/16″,	36,850 " 37,222;	"	37,036.
F, cast-iron, 2″ × 1/4″ × 15/32″,	30,278 " 36,944;	"	33,034.
G, cast-iron, 1 1/8″ × 3/8″ × 7/16″,	37,222 & 38,700.		
H, cast-iron, 1″ × 1/2″ × 7/16″,	29,814 & 38,978.		

In A and B some crushing took place before shearing. In E, the keys being only 7/16 in. deep, tipped slightly in the key-way. In H, in the first test, there was a defect in the key-way of the pulley.

DYNAMOMETERS.

Dynamometers are instruments used for measuring power. They are of several classes, as: 1. Traction dynamometers, used for determining the power required to pull a car or other vehicle, or a plough or harrow. 2. Brake or absorption dynamometers, in which the power of a rotating shaft or wheel is absorbed or converted into heat by the friction of a brake; and 3. Transmission dynamometers, in which the power in a rotating shaft is measured during its transmission through a belt or other connection to another shaft, without being absorbed.

Traction Dynamometers generally contain two principal parts: (1) A spring or series of springs, through which the pull is exerted, the extension of the spring measuring the amount of the pulling force; and (2) a paper-covered drum, rotated either at a uniform speed by clockwork, or at a speed proportional to the speed of the traction, through gearing, on which the extension of the spring is registered by a pencil. From the average height of the diagram drawn by the pencil above the zero-line the average pulling force in pounds is obtained, and this multiplied by the distance traversed, in feet, gives the work done, in foot-pounds. The product divided by the time in minutes and by 33,000 gives the horse-power.

The Prony brake is the typical form of absorption dynamometer. (See Fig. 167, from Flather on Dynamometers and the Measurement of Power.)

Primarily this consists of a lever connected to a revolving shaft or pulley in such a manner that the friction induced between the surfaces in contact will tend to rotate the arm in the direction in which the shaft revolves. This rotation is counterbalanced by weights P, hung in the scale-pan at the end of the lever. In order to measure the power for a given number of revolutions of pulley, we add weights to the scale-pan and screw up on bolts bb, until the friction induced balances the weights and the lever is maintained in its horizontal position while the revolutions of shaft per minute remain constant.

For small powers the beam is generally omitted—the friction being measured by weighting a band or strap thrown over the pulley. Ropes or cords are often used for the same purpose.

Instead of hanging weights in a scale-pan, as in Fig. 167, the friction may be weighed on a platform-scale; in this case, the direction of rotation being the same, the lever-arm will be on the opposite side of the shaft.

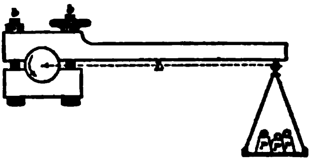

FIG. 167.

In a modification of this brake, the brake-wheel is keyed to the shaft, and its rim is provided with inner flanges which form an annular trough for the retention of water to keep the pulley from heating. A small stream of water constantly discharges into the trough and revolves with the pulley—the centrifugal force of the particles of water overcoming the action of gravity; a waste-pipe with its end flattened is so placed in the trough that it acts as a scoop, and removes all surplus water. The brake consists of a flexible strap to which are fitted blocks of wood forming the rubbing-surface; the ends of the strap are connected by an adjustable bolt-clamp, by means of which any desired tension may be obtained.

The horse-power or work of the shaft is determined from the following:

Let W = work of shaft, equals power absorbed, per minute;
P = unbalanced pressure or weight in pounds, acting on lever-arm at distance L;
L = length of lever-arm in feet from centre of shaft;
V = velocity of a point in feet per minute at distance L, if arm were allowed to rotate at the speed of the shaft;
N = number of revolutions per minute;
H.P. = horse-power.

Then will $W = PV = 2\pi LNP$.

Since H.P. $= PV \div 33{,}000$, we have H.P. $= 2\pi LNP \div 33{,}000$.

If $L = \frac{33}{2\pi}$, we obtain H.P. $= \frac{NP}{1000}$. $33 \div 2\pi$ is practically 5 ft. 3 in., a value often used in practice for the length of arm.

If the rubbing-surface be too small, the resulting friction will show great irregularity—probably on account of insufficient lubrication—the jaws being allowed to seize the pulley, thus producing shocks and sudden vibrations of the lever-arm.

Soft woods, such as bass, plane-tree, beech, poplar, or maple are all to be preferred to the harder woods for brake-blocks. The rubbing-surface should be well lubricated with a heavy grease.

The Alden Absorption-dynamometer. (G. I. Alden, Trans. A. S. M. E., vol. xi. 958; also xii, 700 and xiii. 429.)—This dynamometer is a friction-brake, which is capable in quite moderate sizes of absorbing large powers with unusual steadiness and complete regulation. A smooth cast-iron disk is keyed on the rotating shaft. This is enclosed in a cast-iron shell, formed of two disks and a ring at their circumference, which is free to revolve on the shaft. To the interior of each of the sides of the shell is fitted a copper plate, enclosing between itself and the side a water-tight space. Water under pressure from the city pipes is admitted into each of these spaces, forcing the copper plate against the central disk. The chamber enclosing the disk is filled with oil. To the outer shell is fixed a weighted arm, which resists the tendency of the shell to rotate with the shaft, caused by the friction of the plates against the central disk. Four brakes of this type, 56 in. diam., were used in testing the experimental locomotive at Purdue University (Trans. A. S. M. E., xiii. 429). Each was designed for a maximum moment of 10,500 foot-pounds with a water-pressure of 40 lbs. per sq. in.

The area in effective contact with the copper plates on either side is represented by an annular surface having its outer radius equal to 28 inches, and its inner radius equal to 10 inches. The apparent coefficient of friction between the plates and the disk was 3½%.

W. W. Beaumont (Proc. Inst. C. E. 1889) has deduced a formula by means of which the relative capacity of brakes can be compared, judging from the amount of horse-power ascertained by their use.

If W = width of rubbing-surface on brake-wheel in inches; V = vel. of point on circum. of wheel in feet per minute; K = coefficient; then

$$K = WV \div \text{H.P.}$$

Capacity of Friction-brakes.—Prof. Flather obtains the values of K given in the last column of the subjoined table:

Horse-power.	R. P. M. Brake-pulley.	Brake-pulley. Face, in inches.	Brake-pulley. Diameter, in feet.	Length of Arm.	Design of Brake.	Value of K.
21	150	7	5	33″	Royal Ag. Soc., compensating........	785
19	148.5	7	5	33.38″	McLaren, compensating	858
20	146	7	5	32.19″	" water-cooled and comp.....	802
40	180	10.5	5	32″	Garrett, " " "	741
33	150	10.5	5	32″	" " " "	749
150	150	10	9	......	Schoenheyder, water-cooled..........	282
24	142	12	6	38.31″	Balk....................................	1385
180	100	24	5	126.1″	Gately & Kletsch, water-cooled.......	209
475	76.2	24	7	191″	Webber, water-cooled	84.7
125 } 250 }	290 } 250 }	24	4	63″	Westinghouse, water-cooled..........	465
40 } 125 }	322 } 290 }	13	4	27¾″	" "	847

The above calculations for eleven brakes give values of K varying from 84.7 to 1385 for actual horse-powers tested, the average being $K = 655$.

Instead of assuming an average coefficient, Prof. Flather proposes the following:

Water-cooled brake, non-compensating, $K = 400$; $W = 400\ \text{H.P.} \div V$.

Water-cooled brake, compensating, $K = 750$; $W = 750\ \text{H.P.} \div V$.

Non-cooling brake, with or without compensating device, $K = 900$; $W = 900\ \text{H.P.} \div V$.

Transmission Dynamometers are of various forms, as the Batchelder dynamometer, in which the power is transmitted through a "train-arm" of bevel gearing, with its modifications, as the one described by the author in Trans. A. I. M. E., viii. 177, and the one described by Samuel Webber in Trans. A. S. M. E., x. 514; belt dynamometers, as the Tatham; the Van Winkle dynamometer, in which the power is transmitted from a revolving shaft to another in line with it, the two almost touching, through the medium of coiled springs fastened to arms or disks keyed to the shafts; the Brackett and the Webb cradle dynamometers, used for measuring the power required to run dynamo-electric machines. Descriptions of the four last named are given in Flather on Dynamometers.

Much information on various forms of dynamometers will be found in Trans. A. S. M. E., vol. vii. to xv., inclusive, indexed under Dynamometers

ICE-MAKING OR REFRIGERATING MACHINES.

References.—An elaborate discussion of the thermodynamic theory of the action of the various fluids used in the production of cold was published by M. Ledoux in the *Annales des Mines*, and translated in *Van Nostrand's Magazine* in 1879. This work, revised and additions made in the light of recent experience by Professors Denton, Jacobus, and Riesenberger, was reprinted in 1892. (Van Nostrand's Science Series, No. 46.) The work is largely mathematical, but it also contains much information of immediate practical value, from which some of the matter given below is taken. Other references are Wood's Thermodynamics, Chap. V., and numerous papers by Professors Wood, Denton, Jacobus, and Linde in Trans. A. S. M. E., vols. x. to xiv.; Johnson's Cyclopædia, article on Refrigerating-machines; also *Eng'g*, June 18, July 2 and 9, 1886; April 1, 1887; June 15, 1888; July 31, Aug. 28, 1889; Sept. 11 and Dec. 4, 1891; May 6 and July 8, 1892. For properties of Ammonia and Sulphur Dioxide, see papers by Professors Wood and Jacobus, Trans. A. S. M. E., vols. x. and xii.

For illustrated articles describing refrigerating-machines, see *Am. Mach.*, May 29 and June 26, 1890, and *Mfrs. Record*, Oct. 7, 1892; also catalogues of builders, as Frick & Co., Waynesboro, Pa.; De La Vergne Refrigerating-machine Co., New York; and others.

Operations of a Refrigerating-machine.—Apparatus designed for refrigerating is based upon the following series of operations:

Compress a gas or vapor by means of some external force, then relieve it of its heat so as to diminish its volume; next, cause this compressed gas or vapor to expand so as to produce mechanical work, and thus lower its temperature. The absorption of heat at this stage by the gas, in resuming its original condition, constitutes the refrigerating effect of the apparatus.

A refrigerating-machine is a heat-engine reversed.

From this similarity between heat-motors and freezing-machines it results that all the equations deduced from the mechanical theory of heat to determine the performance of the first, apply equally to the second.

The efficiency depends upon the difference between the extremes of temperature.

The useful effect of a refrigerating-machine depends upon the ratio between the heat-units eliminated and the work expended in compressing and expanding.

This result is independent of the nature of the body employed.

Unlike the heat-motors, the freezing-machine possesses the greatest efficiency when the range of temperature is small, and when the final temperature is elevated.

If the temperatures are the same, there is no theoretical advantage in employing a gas rather than a vapor in order to produce cold.

The choice of the intermediate body would be determined by practical considerations based on the physical characteristics of the body, such as the greater or less facility for manipulating it, the extreme pressures required for the best effects, etc.

Air offers the double advantage that it is everywhere obtainable, and that we can vary at will the higher pressures, independent of the temperature of the refrigerant. But to produce a given useful effect the apparatus must be of larger dimensions than that required by liquefiable vapors.

The maximum pressure is determined by the temperature of the condenser and the nature of the volatile liquid: this pressure is often very high.

When a change of volume of a saturated vapor is made under constant pressure, the temperature remains constant. The addition or subtraction of heat, which produces the change of volume, is represented by an increase or a diminution of the quantity of liquid mixed with the vapor.

On the other hand, when vapors, even if saturated, are no longer in contact with their liquids, and receive an addition of heat either through compression by a mechanical force, or from some external source of heat, they comport themselves nearly in the same way as permanent gases, and become superheated.

It results from this property, that refrigerating-machines using a liquefiable gas will afford results differing according to the method of working, and depending upon the state of the gas, whether it remains constantly saturated, or is superheated during a part of the cycle of working.

The temperature of the condenser is determined by local conditions. The interior will exceed by 9° to 18° the temperature of the water furnished to the exterior. This latter will vary from about 52° F., the temperature of water from considerable depth below the surface, to about 95° F., the temperature of surface-water in hot climates. The volatile liquid employed in the machine ought not at this temperature to have a tension above that which can be readily managed by the apparatus.

On the other hand, if the tension of the gas at the minimum temperature is too low, it becomes necessary to give to the compression-cylinder large dimensions, in order that the weight of vapor compressed by a single stroke of the piston shall be sufficient to produce a notably useful effect.

These two conditions, to which may be added others, such as those depending upon the greater or less facility of obtaining the liquid, upon the dangers incurred in its use, either from its inflammability or unhealthfulness, and finally upon its action upon the metals, limit the choice to a small number of substances.

The gases or vapors generally available are: sulphuric ether, sulphurous oxide, ammonia, methylic ether, and carbonic acid.

The following table, derived from Regnault, shows the tensions of the vapors of these substances at different temperatures between − 22° and + 104°.

Pressures and Boiling-points of Liquids available for Use in Refrigerating-machines.

Temp. of Ebullition.	Tension of Vapor, in lbs. per sq. in., above Zero.					
Deg. Fahr.	Sulphuric Ether.	Sulphur Dioxide.	Ammonia.	Methylic Ether.	Carbonic Acid.	Pictet Fluid.
− 40	....	....	10.22	....	....	....
− 31	....	....	13.23	....	....	....
− 22	....	5.56	16.95	11.15	....	....
− 13	....	7.23	21.51	13.85	251.6	....
− 4	1.30	9.27	27.04	17.06	292.9	13.5
5	1.70	11.76	33.67	20.84	340.1	16.2
14	2.19	14.75	41.58	25.27	393.4	19.3
23	2.79	18.31	50.91	30.41	453.4	22.9
32	3.55	22.53	61.85	36.34	520.4	26.9
41	4.45	27.48	74.55	43.13	594.8	31.2
50	5.54	33.26	89.21	50.84	676.9	36.2
59	6.84	39.93	105.99	59.56	766.9	41.7
68	8.38	47.62	125.08	69.35	864.9	48.1
77	10.19	56.39	146.64	80.28	971.1	55.6
86	12.31	66.37	170.83	92.41	1085.6	64.1
95	14.76	77.64	197.83	....	1207.9	73.2
104	17.59	90.32	227.76	....	1338.2	82.9

The table shows that the use of ether does not readily lead to the production of low temperatures, because its pressure becomes then very feeble.

Ammonia, on the contrary, is well adapted to the production of low temperatures.

Methylic ether yields low temperatures without attaining too great pressures at the temperature of the condenser. Sulphur dioxide readily affords temperatures of − 14 to − 5, while its pressure is only 3 to 4 atmospheres at the ordinary temperature of the condenser. These latter substances then lend themselves conveniently for the production of cold by means of mechanical force.

The "Pictet fluid" is a mixture of 97% sulphur dioxide and 3% carbonic acid. At atmospheric pressure it affords a temperature 14° lower than sulphur dioxide.

Carbonic acid is as yet (1895) in use but to a limited extent, but the relatively greater compactness of compressor that it requires, and its inoffensive character, are leading to its recommendation for service on shipboard, where economy of space is important.

Certain ammonia plants are operated with a surplus of liquid present during compression, so that superheating is prevented. This practice is known as the "cold system" of compression.

Nothing definite is known regarding the application of methylic ether or of the petroleum product chymogene in practical refrigerating service. The inflammability of the latter and the cumbrousness of the compressor required are objections to its use.

"Ice-melting Effect."—It is agreed that the term "ice-melting effect" means the cold produced in an insulated bath of brine, on the assumption that each 142.2 B.T.U.* represents one pound of ice, this being the latent heat of fusion of ice, or the heat required to melt a pound of ice at 32° to water at the same temperature.

The performance of a machine, expressed in pounds or tons of "ice-melting capacity," does not mean that the refrigerating-machine would make the same amount of actual ice, but that the cold produced is equivalent to the effect of the melting of ice at 32° to water of the same temperature.

In making artificial ice the water frozen is generally about 70° F. when submitted to the refrigerating effect of a machine; second, the ice is chilled from 12° to 20° below its freezing-point; third, there is a dissipation of cold, from the exposure of the brine tank and the manipulation of the ice-cans: therefore the weight of actual ice made, multiplied by its latent heat of fusion, 142.2 thermal units, represents only about three fourths of the cold produced in the brine by the refrigerating fluid per I.H.P. of the engine driving the compressing-pumps. Again, there is considerable fuel consumed to operate the brine-circulating pump, the condensing-water and feed-pumps, and to reboil, or purify, the condensed steam from which the ice is frozen. This fuel, together with that wasted in leakage and drip water, amounts to about one half that required to drive the main steam-engine. Hence the pounds of actual ice manufactured from distilled water is just about half the equivalent of the refrigerating effect produced in the brine per indicated horsepower of the steam-cylinders.

When ice is made directly from natural water by means of the "plate system," about half of the fuel, used with distilled water, is saved by avoiding the reboiling, and using steam expansively in a compound engine.

Ether-machines, used in India, are said to have produced about 6 lbs. of actual ice per pound of fuel consumed.

The ether machine is obsolete, because the density of the vapor of ether, at the necessary working-pressure, requires that the compressing-cylinder shall be about 6 times larger than for sulphur dioxide, and 17 times larger than for ammonia.

Air-machines require about 1.2 times greater capacity of compressing cylinder, and are, as a whole, more cumbersome than ether machines, but they remain in use on ship-board. In using air the expansion must take place in a cylinder doing work, instead of through a simple expansion-cock which is used with vapor machines. The work done in the expansion-cylinder is utilized in assisting the compressor.

Ammonia Compression-machines.—*"Cold" vs. "Dry" Systems of Compression.*—In the "cold" system or "humid" system some of the ammonia entering the compression-cylinder is liquid, so that the heat developed in the cylinder is absorbed by the liquid and the temperature of the ammonia thereby confined to the boiling-point due to the condenser-pressure. No jacket is therefore required about the cylinder.

In the "dry" or "hot" system all ammonia entering the compressor is gaseous, and the temperature becomes by compression several hundred degrees greater than the boiling-point due to the condenser-pressure. A water-jacket is therefore necessary to permit the cylinder to be properly lubricated.

Relative Performance of Ammonia Compression- and Absorption-machines, assuming no Water to be Entrained with the Ammonia-gas in the Condenser. (Denton and Jacobus, Trans. A. S. M. E., xiii.)—It is assumed in the calculation for both machines that 1 lb. of coal imparts 10,000 B.T.U. to the boiler. The

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* The latent heat of fusion of ice is 144 thermal units (*Phil. Mag.*, 1871, xli., 182); but it is customary to use 142. (Prof. Wood, Trans. A. S. M. E., xi. 834.)

condensed steam from the generator of the absorption-machine is assumed to be returned to the boiler at the temperature of the steam entering the generator. The engine of the compression-machine is assumed to exhaust through a feed-water heater that heats the feed-water to 212° F. The engine is assumed to consume 26¼ lbs. of water per hour per horse-power. The figures for the compression-machine include the effect of friction, which is taken at 15% of the net work of compression.

Condenser.		Refrigerating Coils.			Pounds of Ice-melting Effect per lb. of Coal.				
					Compress. Machine.		Absorption-machine.*		
Temp. in degrees Fahr.	Absolute pressure, lbs. per sq. in.	Temp. in degrees Fahr.	Absolute pressure, lbs. per sq. in.	Temp. of Absorber, degrees F.	Using 3 lbs. of coal per hour per I.H.P.	Using 1.6 lbs. of coal per hour per I.H.P.	Absorption-machine in which the ammonia circulating-pump exhausts into the generator.	In which the amm. circ. pump exhausts into the atmosphere through a heater, yielding 212° temp. to the feed-water.	Heat furnished to generator of absorption-machine, B.T.U. per lb. of ammonia circulated.
61.2	110.6	5	33.7	61.2	38.1	71.4	38.1	33.5	969
59.0	106.0	5	33.7	59.0	39.8	74.6	38.3	33.9	967
59.0	106.0	5	33.7	130.0	39.8	74.6	39.8	35.1	931
59.0	106.0	−22	16.9	59.0	23.4	43.9	36.8	31.5	1000
86.0	170.8	5	33.7	86.0	25.0	46.9	35.4	28.6	968
86.0	170.8	5	33.7	130.0	25.0	46.9	36.2	29.2	966
86.0	170.8	−22	16.9	86.0	16.5	30.8	33.8	26.5	1025
86.0	170.8	−22	16.9	130.0	16.5	30.8	34.1	27.0	1002
104.0	227.7	5	33.7	104.0	19.6	36.8	33.4	25.1	1002
104.0	227.7	−22	16.9	104.0	13.5	25.8	31.4	23.4	1041

The Ammonia Absorption-machine comprises a generator which contains a concentrated solution of ammonia in water; this generator is heated either directly by a fire, or indirectly by pipes leading from a steam-boiler. The condenser communicates with the upper part of the generator by a tube; it is cooled externally by a current of cold water. The cooler or brine-tank is so constructed as to utilize the cold produced; the upper part of it is in communication with the lower part of the condenser.

An absorption-chamber is filled with a weak solution of ammonia; a tube puts this chamber in communication with the cooling-tank.

The absorption-chamber communicates with the boiler by two tubes: one leads from the bottom of the generator to the top of the chamber, the other leads from the bottom of the chamber to the top of the generator. Upon the latter is mounted a pump, to force the liquid from the absorption chamber, where the pressure is maintained at about one atmosphere, into the generator, where the pressure is from 8 to 12 atmospheres.

To work the apparatus the ammonia solution in the generator is first heated. This releases the gas from the solution, and the pressure rises. When it reaches the tension of the saturated gas at the temperature of the condenser there is a liquefaction of the gas, and also of a small amount of steam. By means of a cock the flow of the liquefied gas into the refrigerating coils contained in the cooler is regulated. It is here vaporized by absorbing the heat from the substance placed there to be cooled. As fast as it is vaporized it is absorbed by the weak solution in the absorbing-chamber.

Under the influence of the heat in the boiler the solution is unequally saturated, the stronger solution being uppermost.

The weaker portion is conveyed by the pipe entering the top of the absorbing-chamber, the flow being regulated by a cock, while the pump sends an equal quantity of strong solution from the chamber back to the boiler.

* 5% of water entrained in the ammonia will lower the economy of the absorption-machine about 15% to 20% below the figures given in the table.

The working of the apparatus depends upon the adjustment and regulation of the flow of the gas and liquid; by these means the pressure is varied, and consequently the temperature in the cooler may be controlled.

The working is similar to that of compression-machines. The absorption-chamber fills the office of aspirator, and the generator plays the part of compressor.

The mechanical force producing exhaustion is here replaced by the affinity of water for ammonia gas; and the mechanical force required for compression is replaced by the heat which severs this affinity and sets the gas at liberty.

(For discussion of the efficiency of the absorption system, see Ledoux's work; paper by Prof. Linde, and discussion on the same by Prof. Jacobus, Trans. A. S. M. E., xiv. 1416, 1436; and papers by Denton and Jacobus, Trans. A. S. M. E. x. 792; xiii. 507.

Sulphur-Dioxide Machines.—Results of theoretical calculations are given in a table by Ledoux showing an ice-melting capacity per hour per horse-power ranging from 134 to 63 lbs., and per pound of coal ranging from 44.7 to 21.1 lbs., as the temperature corresponding to the pressure of the vapor in the condenser rises from 59° to 104° F. The theoretical results do not represent the actual. It is necessary to take into account the loss occasioned by the pipes, the waste spaces in the cylinder, loss of time in opening of the valves, the leakage around the piston and valves, the reheating by the external air, and finally, when the ice is being made, the quantity of the ice melted in removing the blocks from their moulds. Manufacturers estimate that practically the sulphur-dioxide apparatus using water at 55° or 60° F. produces 56 lbs. of ice, or about 10,000 heat-units, per hour per horse-power, measured on the driving-shaft, which is about 55% of the theoretical useful effect. In the commercial manufacture of ice about 7 lbs. are produced per pound of coal. This includes the fuel used for reboiling the water, which, together with that wasted by the pumps and lost by radiation, amounts to a considerable portion of that used by the engine.

Prof. Denton says concerning Ledoux's theoretical results: The figures given are higher than those obtained in practice, because the effect of superheating of the gas during admission to the cylinder is not considered. This superheating may cause an increase of work of about 25%. There are other losses due to superheating the gas at the brine-tank, and in the pipe leading from the brine-tank to the compressor, so that in actual practice a sulphur-dioxide machine, working under the conditions of an absolute pressure in the condenser of 56 lbs. per sq. in. and the corresponding temperature of 77° F., will give about 22 lbs. of ice-melting capacity per pound of coal, which is about 60% of the theoretical amount neglecting friction, or 70% including friction. The following tests, selected from those made by Prof. Schröter on a Pictet ice-machine having a compression-cylinder 11.8 in. bore and 24.4 in. stroke, show the relation between the theoretical and actual ice-melting capacity.

No. of Test.	Temp. in degrees Fahr. corresponding to pressure of vapor.		Ice-melting capacity per pound of coal, assuming 3 lbs. per hour per H.P.		
	Condenser.	Suction.	Theoretical friction included.*	Actual.	Per cent loss due to cylinder superheating, or difference between cols. 4 and 5.
11	77.3	28.5	41.3	33.1	19.9
12	76.2	14.4	31.2	24.1	22.8
13	75.2	−2.5	23.0	17.5	23.9
14	80.6	−15.9	16.6	10.1	39.2

The Refrigerating Coils of a Pictet ice-machine described by Ledoux had 79 sq. ft. of surface for each 100,000 theoretic negative heat-units produced per hour. The temperature corresponding to the pressure of the dioxide in the coils is 10.4° F., and that of the bath (calcium chloride solution) in which they were immersed is 19.4°.

* Friction taken at figure observed in the test, which ranged from 23% to 26% of the work of the steam-cylinder.

Ammonia Compression-machines.—Ammonia gas possesses the advantage of affording about three times the useful effect of sulphur dioxide for the same volume described by the piston.

The perfection of ammonia apparatus now renders it so convenient and reliable that no practical advantage results from the lower pressures afforded by sulphur dioxide.

The results of the calculations for ammonia are given in the table below:

PERFORMANCE OF AMMONIA COMPRESSION-MACHINES.

Gas superheated during compression as in ordinary practice. Temperature of condenser, 64.4° Fahr. Pressure in condenser, 117.44 lbs per sq. in. (Ledoux.)

Temperature Corresponding to Pressure of Vapor in Refrigerating-coils. t_2	Absolute Pressure in Refrigerating-coils. $P_2 + 144$	Temperature of Gas at End of Compression. t_1	Per Cubic Foot of Piston Displacement. Weight of Gas Compressed. m	Per Cubic Foot of Piston Displacement. Heat Abstracted at Condenser. Q_1	Per Cubic Foot of Piston Displacement. Number of Negative Thermal Units Developed. Q	Per Cubic Foot of Piston Displacement. Work of Compression. Without Friction. W_r	Per Cubic Foot of Piston Displacement. Work of Compression. With Friction, or Indicated Steam-power. $1.15W_r$	Performance in British Thermal Units. Per ft.-lb. of Work of Compression. With Friction. $\frac{Q}{1.15W_r}$	Performance in British Thermal Units. Per hour per Horse-power. With Friction.	Ice-melting Capacity per Cubic Foot of Piston Displacement.	Ice-melting Capacity per lb. of Coal, assuming 3 lbs. of Coal per hour per H.P. of Steam-cylinder. With Friction.	Condensing-water. Per Ton of Ice-melting Capacity, assuming 30° F. Range of Temperature
Deg. F.	Lbs. per sq. in.	Deg. F.	Lbs.	B.T.U.	B.T.U.	Ft.-lbs.	Ft.-lbs.			Tons.	Lbs.	Gals.
9.66	37.76	158.9	.1829	78.56	69.41	7070	8130	.00854	16,900	.000244	39.6	1290
5.00	33.67	170.1	.1208	71.98	62.77	7120	8190	.00766	15,170	.000221	35.6	1310
−22.00	16.95	241.8	.0639	40.45	32.58	6080	6990	.00466	9,230	.000115	21.6	1410

The theoretical results for ammonia are higher than the actual, for the same reasons that have been stated for sulphur dioxide. In the case of ammonia the action of the cylinder-walls in superheating the entering vapor has been determined experimentally by Prof. Denton, and the amount found to agree with that indicated by theory. In these experiments the ammonia circulated in a 75-ton refrigerating machine was measured directly by means of a special meter, so that in addition to determining the effect of superheating, the latent heats can be calculated at the suction and condenser pressure.

Economy of Ammonia Compression-machines at Various Condenser Temperatures. (Ledoux.)

Refrigerating Effect of 1 cu. ft. or .12061 lb. of Ammonia Expanded through a Simple Cock to 33.67 lbs. Absolute Pressure per sq. in., and taken into the Compressor at this Pressure and the Corresponding Temperature of 5° F.

Temp. Due to Press. of Vapor in Condenser.	Absolute Pressure in Condenser.	Temperature at End of Compression.	Heat Carried away from Condenser.	Refrigerating Effect in Heat Units.	Ratio of Refrigerating Effect to Heat Expended.	Work of Compression, without Friction.	Work of Compression, with Friction, or Indicated Steam-power.	Refrigerating Effect in Heat Units. Per ft.-lb. of Work Expended, without Friction.	Refrigerating Effect in Heat Units. Per ft.-lb. of Work Expended, including Friction.	Refrigerating Effect in Heat Units. Per Hour per H.P., including Friction.	Ice-melting Capacity. Per Hour per H.P. Without Friction.	Ice-melting Capacity. Per Hour per H.P. With Friction.	Ice-melting Capacity. Per Pound of Coal. Without Friction.	Ice-melting Capacity. Per Pound of Coal. With Friction.	Ice-melting Capacity. Per cu. ft. of Piston Displacement.	Condensing-water. Per cu. ft. of Piston Displacement, assuming 30° Range of Temp.	Condensing-water. Per Ton of Ice-melting Capacity.	Condensing-water. Per Minute per Ton of Ice-melting Capacity in 24 hours.
Deg. F.	Lbs. per sq. in.	Deg. F.	B.T.U.	B.T.U.		Ft.-lbs.	Ft.-lbs.	B.T.U.	B.T.U.	B.T.U.	Lbs.	Lbs.	Lbs.	Lbs.	Tons.	Gals.	Gals.	Gals.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
59	106.0	154.6	71.81	63.47	7.61	6,450	7,410	.00984	.00857	16,960	137.0	119.8	45.8	39.8	.000223	.2872	1290	.89
68	125.1	179.9	72.05	62.31	6.40	7,530	8,660	.00827	.00719	14,250	115.2	100.2	38.4	33.4	.000219	.2882	1320	.92
77	146.6	205.1	72.26	61.13	5.49	8,600	9,890	.00711	.00618	12,240	99.0	86.1	33.0	28.7	.000215	.2890	1350	.94
86	170.8	230.3	72.46	59.93	4.78	9,680	11,130	.00619	.00538	10,660	86.2	75.0	28.7	25.0	.000211	.2898	1380	.96
95	197.8	255.4	72.61	58.70	4.22	10,750	12,360	.00546	.00475	9,400	76.0	66.1	25.3	22.0	.000206	.2904	1410	.98
104	227.8	280.3	72.74	57.45	3.76	11,820	13,590	.00486	.00423	8,380	67.7	58.9	22.6	19.6	.000202	.2910	1440	1.00

Refrigerating Effect of 1 cu. ft. or .06386 lbs. of Ammonia Expanded through a Simple Cock to 16.95 lbs. Absolute Pressure per sq. in., and taken into the Compressor at this Pressure and the Corresponding Temperature of − 22° F.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
59	106.0	224.1	40.28	32.93	4.48	5,680	6,530	.00580	.00504	9,960	80.7	70.2	26.9	23.4	.000116	.1611	1,390	.97
68	125.1	252.2	40.50	32.31	3.95	6,330	7,280	.00510	.00444	8,790	71.0	61.8	23.7	20.6	.000114	.1620	1,420	.99
77	146.6	280.2	40.70	31.69	3.52	6,960	8,000	.00455	.00396	7,840	63.3	55.1	21.1	18.4	.000111	.1628	1,470	1.02
86	170.8	308.3	40.90	31.05	3.15	7,610	8,750	.00408	.00355	7,080	56.8	49.4	18.9	16.5	.000109	.1636	1,500	1.04
95	197.8	336.2	41.07	30.41	2.85	8,240	9,480	.00369	.00321	6,360	51.4	44.7	17.1	14.9	.000107	.1643	1,540	1.07
104	227.8	364.0	41.23	29.75	2.59	8,870	10,200	.00335	.00292	5,780	46.6	40.6	15.5	13.5	.000105	.1649	1,570	1.09

The following is a comparison of the theoretical ice-melting capacity of an ammonia compression machine with that obtained in some of Prof. Schröter's tests on a Linde machine having a compression-cylinder 9.9-in. bore and 16.5 in. stroke, and also in tests by Prof. Denton on a machine having two single-acting compression cylinders 12 in. × 30 in.:

No. of Test.		Temp. in Degrees F. Corresponding to Pressure of Vapor.		Ice-melting Capacity per lb. of Coal, assuming 3 lbs per hour per Horse-power.		
		Condenser.	Suction.	Theoretical, Friction * included.	Actual.	Per Cent of Loss Due to Cylinder Superheating.
Schröter	1	72.3	26.6	50.4	40.6	19.4
	2	70.5	14.3	37.6	30.0	20.2
	3	69.2	0.5	29.4	22.0	25.2
	4	68.5	−11.8	22.8	16.1	29.4
Denton	24	84.2	15.0	27.4	24.2	11.7
	26	82.7	− 3.2	21.6	17.5	19.0
	25	84.6	−10.8	18.8	14.5	22.9

Refrigerating Machines using Vapor of Water. (Ledoux.)—In these machines, sometimes called vacuum machines, water, at ordinary temperatures, is injected into, or placed in connection with, a chamber in which a strong vacuum is maintained. A portion of the water vaporizes, the heat to cause the vaporization being supplied from the water not vaporized, so that the latter is chilled or frozen to ice. If brine is used instead of pure water, its temperature may be reduced below the freezing-point of water. The water vapor is compressed from, say, a pressure of one tenth of a pound per square inch to one and one half pounds, and discharged into a condenser. It is then condensed and removed by means of an ordinary air-pump. The principle of action of such a machine is the same as that of volatile-vapor machines.

A theoretical calculation for ice-making, assuming a lower temperature of 32° F., a pressure in the condenser of 1½ lbs. per square inch, and a coal consumption of 3 lbs. per I.H.P. per hour, gives an ice-melting effect of 34.5 lbs. per pound of coal, neglecting friction. Ammonia for ice-making conditions gives 40.9 lbs. The volume of the compressing cylinder is about 150 times the theoretical volume for an ammonia machine for these conditions.

Relative Efficiency of a Refrigerating Machine.—The efficiency of a refrigerating machine is sometimes expressed as the quotient of the quantity of heat received by the ammonia from the brine, that is, the quantity of useful work done, divided by the heat equivalent of the mechanical work done in the compressor. Thus in column 1 of the table of performance of the 75-ton machine (page 998) the heat given by the brine to the ammonia per minute is 14,776 B.T.U. The horse-power of the ammonia cylinder is 65.7, and its heat equivalent = 65.7 × 33,000 ÷ 778 = 2786 B.T.U. Then 14,776 ÷ 2786 = 5.304, efficiency. The apparent paradox that the efficiency is greater than unity, which is impossible in any machine, is thus explained. The working fluid, as ammonia, receives heat from the brine and rejects heat into the condenser. (If the compressor is jacketed, a portion is rejected into the jacket-water.) The heat rejected into the condenser is greater than that received from the brine; the difference (plus or minus a small difference radiated to or from the atmosphere) is heat received by the ammonia from the compressor. The work to be done by the compressor is not the mechanical equivalent of the refrigeration of the brine, but only that necessary to supply the difference between the heat rejected by the ammonia into the condenser and that received from the brine. If cooling water colder than the brine were available, the brine might transfer its heat directly into the cooling water, and there would be no need of ammonia or of a compressor; but

* Friction taken at figures observed in the tests, which range from 14% to 20% of the work of the steam-cylinder.

since such cold water is not available, the brine rejects its heat into the colder ammonia, and then the compressor is required to heat the ammonia to such a temperature that it may reject heat into the cooling water.

The efficiency of a refrigerating plant referred to the amount of fuel consumed is

$$\left.\begin{array}{l}\text{Ice-melting capacity}\\ \text{per pound of fuel.}\end{array}\right\} = \frac{\left\{\begin{array}{l}\text{Pounds circulated per hour}\\ \times\text{ specific heat}\times\text{range}\\ \text{of temperature}\end{array}\right\}\begin{array}{l}\text{of brine or other}\\ \text{circulating fluid.}\end{array}}{142.2\times\text{pounds of fuel used per hour.}}$$

The ice-melting capacity is expressed as follows:

$$\left.\begin{array}{l}\text{Tons (of 2000 lbs.)}\\ \text{ice-melting ca-}\\ \text{pacity per 24 hours}\end{array}\right\} = \frac{\left\{\begin{array}{l}24\times\text{pounds}\\ \times\text{ specific heat}\\ \times\text{ range of temp.}\end{array}\right\}\text{of brine circulated per hour.}}{142.2\times 2000}$$

The analogy between a heat-engine and a refrigerating-machine is as follows: A steam-engine receives heat from the boiler, converts a part of it into mechanical work in the cylinder, and throws away the difference into the condenser. The ammonia in a compression refrigerating machine receives heat from the brine-tank or cold-room, receives an additional amount of heat from the mechanical work done in the compression-cylinder, and throws away the sum into the condenser. The efficiency of the steam-engine = work done ÷ heat received from boiler. The efficiency of the refrigerating-machine = heat received from the brine-tank or cold-room ÷ heat required to produce the work in the compression-cylinder. In the ammonia

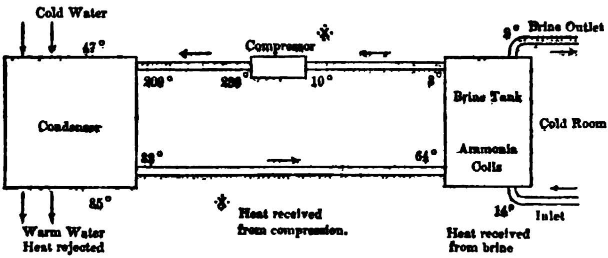


DIAGRAM OF AMMONIA COMPRESSION MACHINE.

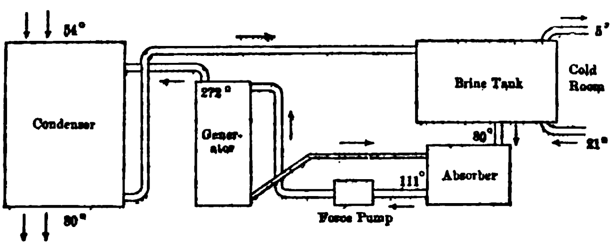


DIAGRAM OF AMMONIA ABSORPTION MACHINE.

absorption-apparatus, the ammonia receives heat from the brine-tank and additional heat from the boiler or generator, and rejects the sum into the condenser and into the cooling water supplied to the absorber. The efficiency = heat received from the brine ÷ heat received from the boiler.

TEST-TRIALS OF REFRIGERATING-MACHINES.

(G. Linde, Trans. A. S. M. E., xiv. 1414.)

The purpose of the test is to determine the ratio of consumption and production, so that there will have to be measured both the refrigerative effect and the heat (or mechanical work) consumed, also the cooling water. The refrigerative effect is the product of the number of heat-units (Q) abstracted from the body to be cooled, and the quotient $\frac{T_c - T}{T}$; in which T_c = absolute temperature at which heat is transmitted to the cooling water, and T = absolute temperature at which heat is taken from the body to be cooled.

The determination of the quantity of cold will be possible with the proper exactness only when the machine is employed during the test to refrigerate a liquid; and if the cold be found from the quantity of liquid circulated per unit of time, from its range of refrigeration, and from its specific heat. Sufficient exactness cannot be obtained by the refrigeration of a current of circulating air, nor from the manufacture of a certain quantity of ice, nor from a calculation of the fluid circulating within the machine (for instance, the quantity of ammonia circulated by the compressor). Thus the refrigeration of brine will generally form the basis for tests making any pretension to accuracy. The degree of refrigeration should not be greater than necessary for allowing the range of temperature to be measured with the necessary exactness; a range of temperature of from 5° to 6° Fahr. will suffice.

The condenser measurements for cooling water and its temperatures will be possible with sufficient accuracy only with submerged condensers.

The measurement of the quantity of brine circulated, and of the cooling water, is usually effected by water-meters inserted into the conduits. If the necessary precautions are observed, this method is admissible. For quite precise tests, however, the use of two accurately gauged tanks must be advised, which are alternately filled and emptied.

To measure the temperatures of brine and cooling water at the entrance and exit of refrigerator and condenser respectively, the employment of specially constructed and frequently standardized thermometers is indispensable; no less important is the precaution of using at each spot simultaneously two thermometers, and of changing the position of one such thermometer series from inlet to outlet (and *vice versa*) after the expiration of one half of the test, in order that possible errors may be compensated.

It is important to determine the specific heat of the brine used in each instance for its corresponding temperature range, as small differences in the composition and the concentration may cause considerable variations.

As regards the measurement of consumption, the programme will not have any special rules in cases where only the measurement of steam and cooling water is undertaken, as will be mainly the case for trials of absorption-machines. For compression-machines the steam consumption depends both on the quality of the steam-engine and on that of the refrigerating-machine, while it is evidently desirable to know the consumption of the former separately from that of the latter. As a rule steam-engine and compressor are coupled directly together, thus rendering a direct measurement of the power absorbed by the refrigerating-machine impossible, and it will have to suffice to ascertain the indicated work both of steam-engine and compressor. By further measuring the work for the engine running empty, and by comparing the differences in power between steam-engine and compressor resulting for wide variations of condenser-pressures, the effective consumption of work L_e for the refrigerating-machine can be found very closely. In general, it will suffice to use the indicated work found in the steam-cylinder, especially as from this observation the expenditure of heat can be directly determined. Ordinarily the use of the indicated work in the compressor-cylinder, for purposes of comparison, should be avoided; firstly, because there are usually certain accessory apparatus to be driven (agitators, etc.), belonging to the refrigerating-machine proper; and secondly, because the external friction would be excluded.

Heat Balance.—We possess an important aid for checking the correctness of the results found in each trial by forming the balance in each case for the heat received and rejected. Only such tests should be regarded as correct beyond doubt which show a sufficient conformity in the heat balance. It is true that in certain instances it may not be easy to account fully for the transmission of heat between the several parts of the machine and its environment by radiation and convection, but generally (particularly for compression-machines) it will be possible to obtain for the heat received and rejected a balance exhibiting small discrepancies only.

Report of Test.—Reports intended to be used for comparison with the figures found for other machines will therefore have to embrace at least the following observations:

Refrigerator:
- Quantity of brine circulated per hour........ ....
- Brine temperature at inlet to refrigerator........................ ....
- Brine temperature at outlet of refrigerator .. ..t
- Specific gravity of brine (at 64° Fahr.) ....
- Specific heat of brine............... ... .
- Heat abstracted (cold produced)........................ ....Q_e
- Absolute pressure in the refrigerator.......................... ... .

Condenser:
- Quantity of cooling water per hour ... .
- Temperature at inlet to condenser............................... ....
- Temperature at outlet of condenser.... ....t
- Heat abstracted........ .... Q_1
- Absolute pressure in the condenser.. ....
- Temperature of gases entering the condenser.... ....

ABSORPTION-MACHINE.

Still:
- Steam consumed per hour......
- Abs. pressure of heating steam.
- Temperature of condensed steam at outlet.... ....
- Heat imparted to still........Q'_e

Absorber:
- Quantity of cooling water per hour.......... ....
- Temperature at inlet ....
- Temperature at outlet.........
- Heat removed ....Q_2

Pump for Ammonia Liquor:
- Indicated work of steam-engine
- Steam-consumption for pump..
- Thermal equivalent for work of pump.................. ALp
- Total sum of losses by radiation and convection.......... $\pm Q_3$

Heat Balance:

$$Q_e + Q'_e = Q_1 + Q_2 \pm Q_3.$$

COMPRESSION-MACHINE.

Compressor:
- Indicated work........... ....Lt
- Temperature of gases at inlet..
- Temperature of gases at exit..

Steam-engine:
- Feed-water per hour........ ...
- Temperature of feed-water....
- Absolute steam-pressure before steam-engine.................
- Indicated work of steam-engine Le
- Condensing water per hour....
- Temperature of do..........
- Total sum of losses by radiation and convection... ...... $\pm Q_3$

Heat Balance:

$$Q_e + ALe = Q_1 \pm Q_3.$$

For the calculation of efficiency and for comparison of various tests, the actual efficiencies must be compared with the theoretical maximum of efficiency $\left(\frac{Q}{AL}\right)$ max. $= \frac{T}{Tc - T}$ corresponding to the temperature range.

Temperature Range. — As temperatures (T and Tc) at which the heat is abstracted in the refrigerator and imparted to the condenser, it is correct to select the temperature of the brine leaving the refrigerator and that of the cooling water leaving the condenser, because it is in principle impossible to keep the refrigerator pressure higher than would correspond to the lowest brine temperature, or to reduce the condenser pressure below that corresponding to the outlet temperature of the cooling water.

Prof. Linde shows that the *maximum theoretical* efficiency of a compression-machine may be expressed by the formula

$$\frac{Q}{AL} = \frac{T}{Tc - T},$$

in which Q = quantity of heat abstracted (cold produced);
AL = thermal equivalent of the mechanical work expended;
L = the mechanical work, and $A = 1 \div 778$;
T = absolute temperature of heat abstraction (refrigerator);
Tc = " " " " rejection (condenser).

If u = ratio between the heat equivalent of the mechanical work AL, and the quantity of heat Q' which must be imparted to the motor to produce the work L, then

$$\frac{AL}{Q'} = u, \text{ and } \frac{Q'}{Q} = \frac{T_0 - T}{uT}.$$

It follows that the expenditure of heat Q' necessary for the production of the quantity of cold Q in a compression-machine will be the smaller, the smaller the difference of temperature $T_0 - T$.

Metering the Ammonia.—For a complete test of an ammonia refrigerating-machine it is advisable to measure the quantity of ammonia circulated, as was done in the test of the 75-ton machine described by Prof. Denton. (Trans. A. S. M. E., xii. 326.)

PROPERTIES OF SULPHUR DIOXIDE AND AMMONIA GAS.

Ledoux's Table for Saturated Sulphur-dioxide Gas.

Heat-units expressed in B.T.U. per pound of sulphur dioxide.

Temperature of Ebullition in deg. F. t	Absolute Pressure in lbs. per sq. in. $\frac{P}{144}$	Total Heat reckoned from 32° F. λ	Heat of Liquid reckoned from 32° F. q	Latent Heat of Evaporation r	Heat Equivalent of External Work. APu	Internal Latent Heat. ρ	Increase of Volume during Evaporation. u	Density of Vapor or Weight of 1 cu. ft. $\frac{1}{v}$
Deg. F.	Lbs.	B.T.U.	B.T.U.	B.T.U.	B.T.U.	B.T.U.	Cu. ft.	Lbs.
−22	5.56	157.43	−19.56	176.99	13.59	163.39	13.17	.076
−13	7.23	158.64	−16.30	174.95	13.83	161.12	10.27	.097
− 4	9.27	159.84	−13.05	172.89	14.05	158.84	8.12	.123
5	11.76	161.03	− 9.79	170.82	14.26	156.56	6.50	.153
14	14.74	162.20	− 6.53	168.73	14.46	154.27	5.25	.190
23	18.31	163.36	− 3.27	166.63	14.66	151.97	4.29	.232
32	22.53	164.51	0.00	164.51	14.84	149.68	3.54	.282
41	27.48	165.65	3.27	162.38	15.01	147.37	2.93	.340
50	33.25	166.78	6.55	160.23	15.17	145.06	2.45	.407
59	39.93	167.90	9.83	158.07	15.32	142.75	2.07	.483
68	47.61	168.99	13.11	155.89	15.46	140.43	1.75	.570
77	56.39	170.09	16.39	153.70	15.59	138.11	1.49	.669
86	66.36	171.17	19.69	151.49	15.71	135.78	1.27	.780
95	77.64	172.24	22.98	149.26	15.82	133.45	1.09	.906
104	90.31	173.30	26.28	147.02	15.91	131.11	91	1.046

Density of Liquid Ammonia. (D'Andreff, Trans. A. S. M. E., x. 641.)

At temperature C......	−10	−5	0	5	10	15	20
" " F.....	+14	23	32	41	50	59	68
Density...............	.6492	.6429	.6364	.6298	.6230	.6160	.6089

These may be expressed very nearly by

$$\delta = 0.6364 - 0.0014t^\circ \text{ Centigrade};$$
$$\delta = 0.6502 - 0.000777T^\circ \text{ Fahr.}$$

Latent Heat of Evaporation of Ammonia. (Wood, Trans. A. S. M. E., x. 641.)

$$h_e = 555.5 - 0.613T - 0.000219T^2 \text{ (in B.T.U., Fahr. deg.)};$$

Ledoux found $h_e = 538.33 - 0.5499T - 0.0001173T^2$.

For experimental values at different temperatures determined by Prof. Denton, see Trans. A. S. M. E., xii. 356. For calculated values, see vol. x. 646.

Density of Ammonia Gas.—Theoretical, 0.5894; experimental 0.596. Regnault (Trans. A. S. M. E., x. 633).

Specific Heat of Liquid Ammonia. (Wood, Trans. A. S. M. E., x 645.)—The specific heat is nearly constant at different temperatures, and about equal to that of water, or unity. From 0° to 100° F., it is

$$c = 1.096 - .0012T, \text{ nearly.}$$

In a later paper by Prof. Wood (Trans. A. S. M. E., xii. 136) he gives a higher value, viz., $c = 1.12136 + 0.000438T$.

L. A. Elleau and Wm. D. Ennis (*Jour. Franklin Inst.*, April, 1898) give the results of nine determinations, made between 0° and 20° C., which range from 0.983 to 1.056, averaging 1.0206. Von Strombeck (*Jour. Franklin Inst.*, Dec. 1890) found the specific heat between 62° and 31° C. to be 1.22876. Ludeking and Starr (*Am. Jour. Science*, iii, 45, 200) obtained 0.886. Prof. Wood deduced from thermodynamic equations $c = 1.093$ at $-34°$ F. or $-38°$ C., and Ledoux in like manner finds $c = 1.0058 + .003658t°$ C. Elleau and Ennis give Ledoux's equation with a new constant derived from their experiments, thus $c = 0.9834 + 0.003658t°$ C.

Properties of the Saturated Vapor of Ammonia.

(Wood's Thermodynamics.)

Temperature.		Pressure, Absolute.		Heat of Vaporization, thermal units.	Volume of Vapor per lb., cu. ft.	Volume of Liquid per lb., cu. ft.	Weight of a cu. ft. of Vapor, lbs.
Degs. F.	Absolute, F.	Lbs. per sq. ft.	Lbs. per sq. in.				
− 40	420.66	1540.7	10.69	579.67	24.372	.0234	.0410
− 35	425.66	1773.6	12.31	576.69	21.319	.0236	.0468
− 30	430.66	2035.8	14.18	573.69	18.697	.0237	.0535
− 25	435.66	2329.5	16.17	570.68	16.445	.0238	.0608
− 20	440.66	2657.5	18.45	567.67	14.507	.0240	.0689
− 15	445.66	3022.5	20.99	564.64	12.834	.0242	.0779
− 10	450.66	3428.0	23.80	561.61	11.384	.0243	.0878
− 5	455.66	3877.2	26.93	558.56	10.125	.0244	.0988
0	460.66	4373.5	30.37	555.50	9.027	.0246	.1108
+ 5	465.66	4920.5	34.17	552.43	8.069	.0247	.1239
+ 10	470.66	5522.2	38.34	549.35	7.229	.0249	.1383
+ 15	475.66	6182.4	42.93	546.26	6.492	.0250	.1544
+ 20	480.66	6905.3	47.95	543.15	5.842	.0252	.1712
+ 25	485.66	7695.2	53.43	540.03	5.269	.0253	.1898
+ 30	490.66	8556.6	59.41	536.92	4.763	.0254	.2100
+ 35	495.66	9493.9	65.93	533.78	4.313	.0256	.2319
+ 40	500.66	10512	73.00	530.63	3.914	.0257	.2555
+ 45	505.66	11616	80.66	527.47	3.559	.0259	.2809
+ 50	510.66	12811	88.96	524.30	3.242	.0261	.3085
+ 55	515.66	14102	97.93	521.12	2.958	.0263	.3381
+ 60	520.66	15494	107.60	517.93	2.704	.0265	.3698
+ 65	525.66	16998	118.03	514.73	2.476	.0266	.4039
+ 70	530.66	18605	129.21	511.52	2.271	.0268	.4403
+ 75	535.66	20336	141.25	508.29	2.087	.0270	.4793
+ 80	540.66	22192	154.11	505.05	1.920	.0272	.5208
+ 85	545.66	24178	167.86	501.81	1.770	.0273	.5650
+ 90	550.66	26300	182.8	498.11	1.632	.0274	.6128
+ 95	555.66	28565	198.37	495.29	1.510	.0277	.6623
+ 100	560.66	30980	215.14	492.01	1.398	.0279	.7153
+ 105	565.66	33550	232.98	488.73	1.296	.0281	.7716
+ 110	570.66	36284	251.97	485.42	1.203	.0283	.8312
+ 115	575.66	39188	272.14	482.41	1.119	.0285	.8937
+ 120	580.66	42267	293.49	478.79	1.045	.0287	.9560
+ 125	585.66	45528	316.16	475.45	0.970	.0289	1.0309
+ 130	590.66	48978	340.42	472.11	0.905	.0291	1.1049
+ 135	595.66	52626	365.16	468.75	0.845	.0293	1.1834
+ 140	600.66	56488	392.22	465.39	0.791	.0295	1.2642
+ 145	605.66	60550	420.49	462.01	0.741	.0297	1.3495
+ 150	610.66	64833	450.20	458.62	0.695	.0299	1.4388
+ 155	615.66	69341	481.54	455.22	0.652	.0302	1.5337
+ 160	620.66	74086	514.40	451.81	0.613	.0304	1.6343
+ 165	625.66	79071	549.04	448.39	0.577	.0306	1.7333

Specific Heat of Ammonia Vapor at the Saturation Point. (Wood, Trans. A. S. M. E., x. 644.)—For the range of temperatures ordinarily used in engineeering practice, the specific heat of saturated ammonia is negative, and the saturated vapor will condense with adiabatic expansion, and the liquid will evaporate with the compression of the vapor, and when all is vaporized will superheat.

Regnault (*Rel. des. Exp.*, ii. 162) gives for specific heat of ammonia-gas 0.50836. (Wood, Trans. A. S. M. E., xii. 133.)

Properties of Brine used to absorb Refrigerating Effect of Ammonia. (J. E. Denton, Trans. A. S. M. E., x. 799.)—A solution of Liverpool salt in well-water having a specific gravity of 1.17, or a weight per cubic foot of 73 lbs., will not sensibly thicken or congeal at 0° Fahrenheit.

The mean specific heat between 39° and 16° Fahr. was found by Denton to be 0.805. Brine of the same specific gravity has a specific heat of 0.805 at 65° Fahr., according to Naumann.

Naumann's values are as follows (*Lehr- und Handbuch der Thermochemie*, 1882):

Specific heat....	.791	.805*	.863	.895	.931	.962	.978
Specific gravity.	1.187	1.170	1.103	1.072	1.044	1.023	1.012

* Interpolated.

Chloride-of-calcium solution has been used instead of brine. According to Naumann, a solution of 1.0255 sp. gr. has a specific heat of .957. A solution of 1.163 sp. gr. in the test reported in *Eng'g*, July 22, 1887, gave a specific heat of .827.

ACTUAL PERFORMANCES OF ICE-MAKING MACHINES.

The table given on page 996 is abridged from Denton, Jacobus, and Riesenberger's translation of Ledoux on Ice-making Machines. The following shows the class and size of the machines tested, referred to by letters in the table, with the names of the authorities:

Class of Machines.	Authority.	Dimensions of Compression-cylinder in inches.	
		Bore.	Stroke.
A. Ammonia cold-compression..	Schröter.	9.9	16.5
B. Pictet fluid dry-compression.	"	11.3	24.4
C. Bell-Coleman air	"	28.0	23.8
D. Closed cycle air..............	Renwick & Jacobus.	10.	18.0
E. Ammonia dry-compression..	Denton.	12.0	30.0
F. Ammonia absorption........	"	....	....

Performance of a 75-ton Ammonia Compression-machine. (J. E. Denton, Trans. A. S. M. E., xii, 326.)—The machine had two single-acting compression cylinders 12″ × 30″, and one Corliss steam-cylinder, double-acting, 18″ × 36″. It was rated by the manufacturers as a 50-ton machine, but it showed 75 tons of ice-refrigerating effect per 24 hours during the test.

The most probable figures of performance in eight trials are as follows:

No. of Trial.	Ammonia Pressures, lbs. above Atmosphere.		Brine Temperatures, Degrees F.		Capacity Tons Refrigerating Effect per 24 hours.	Efficiency lbs. of Ice per lb. of Coal at 3 lbs. Coal per hour per H.P.	Water-consumption, gals. of Water per min. per ton of Capacity.	Ratio of Actual Weights of Ammonia circulated.	Ratio of Capacities.
	Condensing	Suction.	Inlet.	Outlet.					
1	151	28	36.76	28.86	70.3	22.60	0.80	1.0	1.0
8	161	27.5	36.36	28.45	70.1	22.27	1.09	1.0	1.0
7	147	13.0	14.29	2.29	42.0	16.27	0.83	1.70	1.66
4	152	8.2	6.27	2.03	36.43	14.10	1.1	1.93	1.92
6	105	7.6	6.40	−2.22	37.20	17.00	2.00	1.91	1.88
2	135	15.7	4.62	3.22	27.2	13.20	1.25	2.59	2.57

The principal results in four tests are given in the table on page 998. The fuel economy under different conditions of operation is shown in the following table:

Condensing Pressure, lbs.	Suction-pressure, lbs.	Pounds of Ice-melting Effect with Engines— Non-condensing. Per lb. Coal.	Per lb. Steam	Non-compound Condensing. Per lb. Coal.	Per lb. Steam.	Compound Condensing. Per lb. Coal.	Per lb. Steam.	B.T.U. per lb. of Steam with Engines— Non-condensing.	Condensing.	Compound Condensing.
150	28	24	2.90	30	3.61	37.5	4.51	393	518	640
150	7	14	1.69	17.5	2.11	21.5	2.58	240	300	366
105	28	34.5	4.16	43	5.18	54	6.50	591	725	923
105	7	22	2.65	27.5	3.31	34.5	4.16	376	470	591

The non-condensing engine is assumed to require 25 lbs. of steam per horse-power per hour, the non-compound condensing 20 lbs., and the compound condensing 16 lbs., and the boiler efficiency is assumed at 8.3 lbs. of water per lb. coal under working conditions. The following conclusions were derived from the investigation:

1. The capacity of the machine is proportional, almost entirely, to the weight of ammonia circulated. This weight depends on the suction-pressure and the displacement of the compressor-pumps. The practical suction-pressures range from 7 lbs. above the atmosphere, with which a temperature of 0° F. can be produced, to 28 lbs. above the atmosphere, with which the temperatures of refrigeration are confined to about 28° F. At the lower pressure only about one half as much weight of ammonia can be circulated as at the upper pressure, the proportion being about in accordance with the ratios of the absolute pressures, 22 and 42 lbs. respectively. For each cubic foot of piston-displacement per minute a capacity of about one sixth of a ton of "refrigerating effect" per 24 hours can be produced at the lower pressure, and of about one third of a ton at the upper pressure. No other elements practically affect the capacity of a machine, provided the cooling-surface in the brine-tank or other space to be cooled is equal to about 36 sq. ft. per ton of capacity at 28 lbs. back pressure. For example, a difference of 100% in the rate of circulation of brine, while producing a proportional difference in the range of temperature of the latter, made no practical difference in capacity.

The brine-tank was 10½ × 13 × 10⅝ ft., and contained 8000 lineal feet of 1-in. pipe as cooling-surface. The condensing-tank was 12 × 10 × 10 ft., and contained 5000 lineal feet of 1-in. pipe as cooling-surface.

2. The economy in coal-consumption depends mainly upon both the suction-pressures and condensing-pressures. Maximum economy, with a given type of engine, where water must be bought at average city prices, is obtained at 28 lbs. suction-pressure and about 150 lbs. condensing-pressure. Under these conditions, for a non-condensing steam-engine, consuming coal at the rate of 3 lbs. per hour per I.H.P. of steam-cylinders, 24 lbs. of ice-refrigerating effect are obtained per lb. of coal consumed. For the same condensing-pressure, and with 7 lbs. suction-pressure, which affords temperatures of 0° F., the possible economy falls to about 14 lbs. of "refrigerating effect" per lb. of coal consumed. The condensing-pressure is determined by the amount of condensing-water supplied to liquefy the ammonia in the condenser. If the latter is about 1 gallon per minute per ton of refrigerating effect per 24 hours, a condensing-pressure of 150 lbs. results, if the initial temperature of the water is about 56° F. Twenty-five per cent less water causes the condensing-pressure to increase to 190 lbs. The work of compression is thereby increased about 20%, and the resulting "economy" is reduced to about 18 lbs. of "ice effect" per lb. of coal at 28 lbs. suction-pressure and 11 5 at 7 lbs. If, on the other hand, the supply of water is made 3 gallons per minute, the condensing-pressure may be confined to about 105 lbs. The work of compression is thereby reduced about 25%, and a proportional increase of economy results. Minor alterations of economy depend on the initial temperature of the condensing-water and variations of latent heat, but these are confined within about 5% of the gross result, the main element of control being the work of compression, as affected by the back pressure and condensing-pressure, or both. If the steam-engine supplying the motive power may use a condenser to secure a vacuum, an increase of economy of 25% is available over the above figures, making the lbs. of "ice effect" per lb. of

r 150 lbs. condensing-pressure and 28 lbs. suction-pressure 30.0, and s. suction-pressure, 17.5. It is, however, impracticable to use a con- in cities where water is bought. The latter must be practically cost to be available for this purpose. In this case it may be assumed ater will also be available for condensing the ammonia to obtain as ondensing-pressure as about 100 lbs., and the economy of the refrig- -machine becomes, for 28 lbs. back-pressure, 48.0 lbs. of "ice effect" of coal, or for 7 lbs. back-pressure, 27.5 lbs. of ice effect per lb.

If a compound condensing-engine can be used with a steam-con- on per hour per horse-power of 16 lbs. of water, the economy of the rating-machine may be 25% higher than the figures last named, mak- 28 lbs. back pressure a refrigerating effect of 54.0 lbs. per lb. of coal, 7 lbs. back pressure a refrigerating effect of 34.0 lbs. per lb. of coal.

Actual Performance of Ice-making Machines.

Absolute Pressure, in lbs. per square inch. Condenser.	Absolute Pressure, in lbs. per square inch. Suction.	Temperature corresponding to Pressure, in degrees Fahr. Condenser.	Temperature corresponding to Pressure, in degrees Fahr. Suction.	Temperature of Brine, in degrees Fahr.* Inlet.	Temperature of Brine, in degrees Fahr.* Outlet.	Revolutions per minute.	Horse-power of Steam-cylinder.	Per cent of Indicated Power of Steam-cylinder lost in Friction.	Ice-melting Capacity, in tons per 24 hours.	Ice-melting Capacity in pounds per pound of Coal. Actual.†	Difference between theoretical Ice-melting Capacity, no Cylinder Heating or Friction, and actual. Per cent.‡	Heat losses. Per cent of Theoretical Amount with Friction.§	Mean Effective Pressure, in lbs. per square inch.‖
135	55	72	27	43	37	44.9	17.9	14.4	26.2	40.63	30.8	19.1	54.8
131	42	70	14	28	23	45.1	18.0	16.7	19.5	30.01	33.5	20.2	53.4
128	30	69	1	14	9	45.1	16.8	16.0	13.3	22.03	37.1	25.2	50.3
126	22	68	−12	0	−5	44.8	15.5	19.5	9.0	16.14	42.9	29.1	44.7
200	42	95	14	28	23	45.0	24.1	10.5	10.5	19.07	36.0	28.5	77.0
136	60	72	30	44	37	45.2	17.9	10.7	29.8	46.29	28.5	19.9	66.8
131	45	71	18	28	23	45.1	18.0	12.1	21.6	33.23	31.3	21.9	56.4
126	24	68	−9	0	−6	44.7	15.6	18.0	9.9	17.55	41.1	28.3	46.1
117	41	64	13	28	23	45.0	16.4	13.5	20.0	33.77	33.1	22.9	50.6
130	60	70	31	43	37	31.7	12.0	14.8	19.5	45.01	35.2	23.8	52.0
57	21	77	28	43	37	57.0	21.5	22.9	25.6	33.07	39.9	22.2	24.1
56	15	76	14	28	23	56.8	20.6	22.9	17.9	24.11	41.3	24.0	23.1
55	10	75	−2	14	9	57.1	18.5	24.0	11.6	17.47	42.2	25.2	20.4
60	7	81	−16	0	−6	57.6	15.7	25.7	5.7	10.14	54.5	38.5	16.8
91	15	104	14	28	23	59.8	27.2	16.9	15.7	16.05	36.2	23.1	31.5
61	22	81	31	44	37	57.3	21.6	14.0	28.1	36.19	33.4	22.5	26.8
59	16	80	16	28	23	57.5	20.5	12.8	19.3	26.24	34.6	25.0	25.6
59	7	79	−16	0	−6	57.8	15.9	21.1	6.8	11.93	47.5	33.4	18.0
54	22	75	31	43	37	35.3	12.4	22.3	17.0	33.04	39.5	22.6	29.6
89	16	103	16	28	23	42.9	19.9	14.7	11.9	16.68	37.7	27.0	32.7
62	6	82	−17	0	−5	34.8	9.9	24.3	3.5	9.86	54.2	30.5	17.7
59	15	65*	−58*	....	....	63.2	83.2	21.9	10.3	3.42	71.7	56.9	25.6
175	54	81*	−40*	....	....	93.4	38.1	32.1	4.9	3.0	80.	63.	39.2
166	43	84	15	37	28	58.1	85.0	22.7	73.9	24.16	32.8	11.7	65.9
167	23	85	−11	6	2	57.7	72.6	18.6	37.9	14.52	37.4	22.7	57.6
162	28	83	−8	14	2	57.9	73.6	19.3	46.5	17.55	34.9	18.6	59.9
176	42	88	14	36	28	58.9	88.6	19.7	74.4	23.31	30.5	13.5	70.5
152	40	79	13	21	16	....	....	....	42.2	20.1	47.8	....	..

perature of air at entrance and exit of expansion-cylinder.
basis of 8 lbs. of coal per hour per H.P. of steam-cylinder of com- -machine and an evaporation of 11.1 lbs. of water per pound of tible from and at 212° F. in the absorption-machine.
cent of theoretical with no friction.
due to heating during aspiration of gas in the compression-cylinder adiation and superheating at brine-tank.
al, including resistance due to inlet and exit valves.

In class A, a German machine, the ice-melting capacity ranges from 46.29 to 16.14 lbs. of ice per pound of coal, according as the suction pressure varies from about 45 to 8 lbs. above the atmosphere, this pressure being the condition which mainly controls the economy of compression-machines. These results are equivalent to realizing from 72% to 57% of theoretically perfect performances. The higher per cents appear to occur with the higher suction-pressures, indicating a greater loss from cylinder-heating (a phenomenon the reverse of cylinder condensation in steam-engines), as the range of the temperature of the gas in the compression-cylinder is greater.

In E, an American compression-machine, operating on the "dry system," the percentage of theoretical effect realized ranges from 69.5% to 62.6%. The friction losses are higher for the American machine. The latter's higher efficiency may be attributed, therefore, to more perfect displacement.

The largest "ice-melting capacity" in the American machine is 24.16 lbs. This corresponds to the highest suction-pressures used in American practice for such refrigeration as is required in beer-storage cellars using the direct-expansion system. The conditions most nearly corresponding to American brewery practice in the German tests are those in line 5, which give an "ice-melting capacity" of 19.07 lbs.

For the manufacture of artificial ice, the conditions of practice are those of lines 3 and 4, and lines 25 and 26. In the former the condensing pressure used requires more expense for cooling water than is common in American practice. The ice-melting capacity is therefore greater in the German machine, being 22.03 and 16.14 lbs. against 17.55 and 14.52 for the American apparatus.

Class B. Sulphur Dioxide or Pictet Machines.—No records are available for determination of the "ice-melting capacity" of machines using pure sulphur dioxide. This fluid is in use in American machines, but in Europe it has given way to the "Pictet fluid," a mixture of about 97% of sulphur dioxide and 3% of carbonic acid. The presence of the carbonic acid affords a temperature about 14 Fahr. degrees lower than is obtained with pure sulphur dioxide at atmospheric pressure. The latent heat of this mixture has never been determined, but is assumed to be equal to that of pure sulphur dioxide.

For brewery refrigerating conditions, line 17, we have 26.24 lbs. "ice-melting capacity," and for ice-making conditions, line 13, the "ice-melting capacity" is 17.47 lbs. These figures are practically as economical as those for ammonia, the per cent of theoretical effect realized ranging from 65.4 to 57.8. At extremely low temperatures, −15° Fahr., lines 14 and 18, the per cent realized is as low as 42.5.

Cylinder-heating.—In compression-machines employing volatile vapors the principal cause of the difference between the theoretical and the practical result is the heating of the ammonia, by the warm cylinder walls, during its entrance into the compressor, thereby expanding it, so that to compress a pound of ammonia a greater number of revolutions must be made by the compressing-pumps than corresponds to the density of the ammonia-gas as it issues from the brine-tank.

Tests of Ammonia Absorption-machine used in storage-warehouses under approaches to the New York and Brooklyn Bridge. (*Eng'g*, July 22, 1887.)—The circulated fluid consisted of a solution of chloride of calcium of 1.163 sp. gr. Its specific heat was found to be .827.

The efficiency of the apparatus for 24 hours was found by taking the product of the cubic feet of brine circulating through the pipes by the average difference in temperature in the ingoing and outgoing currents, as observed at frequent intervals by the specific heat of the brine (.827) and its weight per cubic foot (73.48). The final product, applying all allowances for corrections from various causes, amounted to 6,218,816 heat-units as the amount abstracted in 24 hours, equal to the melting of 43,565 lbs. of ice in the same time.

The theoretical heating-power of the coal used in 24 hours was 27,000,000 heat-units; hence the efficiency of the apparatus was 23%. This is equivalent to an ice-melting effect of 16.1 lbs. per lb. of coal having a heating value of 10,000 B.T.U. per lb.

A test of a 35-ton absorption-machine in New Haven, Conn., by Prof. Denton (Trans. A. S. M. E., x. 792), gave an ice-melting effect of 20.1 lbs. per lb. of coal on a basis of boiler economy equivalent to 3 lbs. of steam per I.H.P. in a good non-condensing steam-engine. The ammonia was worked between 138 and 23 lbs. pressure above the atmosphere.

Performance of a 75-ton Refrigerating-machine.

	Maximum Capacity and Economy at 28 lbs. Back Pressure.	Maximum Capacity and Economy at Zero, Brine, and 8 lbs. Back Pressure.	Maximum Capacity and Economy for Zero, Brine, 18 lbs. Back Pressure.	Maximum Capacity and Economy at 27.5 lbs. Back Pressure.
Av. high ammonia press. above atmos.	151 lbs.	152 lbs.	147 lbs.	161 lbs.
Av. back ammonia press. above atmos.	28 "	8.2 "	13 "	27.5 "
Av. temperature brine inlet	36.76°	6.27°	14.29°	......
Av. temperature brine outlet	28.86°	2.03°	2.29°	28.45°
Av. range of temperature	7.9°	4.24°	12.00°	7.91°
Lbs. of brine circulated per minute	2281	2173	943	2374
Av. temp. condensing-water at inlet	44.65°	56.65°	46.9°	54.00°
Av. temp. condensing-water at outlet	83.66°	85.4°	85.46°	82.86°
Av. range of temperature	39.01°	28.75°	38.56°	28.80°
Lbs. water circulated p. min. thro' cond'ser	442	315	257	601.5
Lbs. water per min. through jackets	25	44	40.	14
Range of temperature in jackets	24.0°	16.2°	16.4°	29.1°
Lbs. ammonia circulated per min	*28.17	14.66	16.67	28.32
Probable temperature of liquid ammonia, entrance to brine-tank	*71.8°	*68°	*63.7°	76.7°
Temp. of amm. corresp. to av. back press.	+14°	− 8°	− 5°	14°
Av. temperature of gas leaving brine-tanks	34.2°	14.7°	3.0°	29.2°
Temperature of gas entering compressor	*39°	25°	10.13°	34°
Av. temperature of gas leaving compressor	218°	263°	239°	221°
Av. temp. of gas entering condenser	200°	218°	209°	168°
Temperature due to condensing pressure	84.5°	84.0°	82.5°	88.0°
Heat given ammonia:				
By brine, B.T.U. per minute	14776	7186	8824	14647
By compressor, B.T.U. per minute	2786	2320	2518	3020
By atmosphere, B.T.U. per minute	140	147	167	141
Total heat rec. by amm., B.T.U. per min.	17702	9658	11409	17708
Heat taken from ammonia:				
By condenser, B.T.U. per min	17242	9056	9910	17359
By jackets, B.T.U. per min	608	712	656	406
By atmosphere, B.T.U. per min	182	338	250	252
Total heat rej. by amm., B.T.U. per min.	18032	10106	10816	18017
Dif. of heat rec'd and rej., B.T.U. per min.	330	453	407	309
% work of compression removed by jackets.	22%	31%	26%	13%
Av. revolutions per min	58.09	57.7	57.88	58.89
Mean eff. press. steam-cyl., lbs. per sq. in.	32.5	27.17	27.83	32.97
Mean eff. press. amm.-cyl., lbs. per sq. in.	65.9	58.3	59.86	70.54
Av. H.P. steam-cylinder	85.00	71.7	73.6	88.63
Av. H.P. ammonia-cylinder	65.7	54.7	59.37	71.20
Friction in per cent of steam H.P.	23.0	24.0	20.0	19.67
Total cooling water, gallons per min. per ton per 24 hours	0.75	1.185	0.797	0.990
Tons ice-melting capacity per 24 hours	74.8	36.48	44.64	74.56
Lbs. ice-refrigerating eff. per lb. coal at 3 lbs. per H.P. per hour	24.1	14.1	17.27	23.37
Cost coal per ton of ice-refrigerating effect at $4 per ton	$0.166	$0.283	$0.231	$0.170
Cost water per ton of ice-refrigerating effect at $1 per 1000 cu. ft.	$0.128	$0.200	$0.136	$0.169
Total cost of 1 ton of ice-refrigerating eff.	$0.294	$0.483	$0.467	$0.339

Figures marked thus (*) are obtained by calculation; all other figures are obtained from experimental data; temperatures are in Fahrenheit degrees.

Ammonia Compression-machine.

ACTUAL RESULTS OBTAINED AT THE MUNICH TESTS.

(Prof. Linde, Trans. A. S. M. E., xiv. 1419.)

No. of Test	1	2	3	4	5
Temp. of refrigerated brine, Inlet, deg. F.	43.194	28.344	13.952	−0.279	28.251
Temp. of refrigerated brine, Outlet, t deg. F.	37.054	22.885	8.771	−5.879	23.072
Specific heat of brine	0.861	0.851	0.843	0.837	0.851
Quantity of brine circ. per h., cu. ft.	1,039.38	908.84	633.89	414.98	800.93
Cold produced, B.T.U. per hour	342,909	263,950	172,776	121,474	220,284
Quant. of cooling water per h., c. ft.	338.76	260.83	187.506	139.99	97.76
I.H.P. in steam-engine cylinder (Le).	15.80	16.47	15.28	14.24	21.61
Cold produced per h., B.T.U., Per I.H.P. in comp.-cyl.	24,818	18,471	12,770	10,140	11,151
Cold produced per h., B.T.U., Per I.H.P. in steam-cyl.	21,703	16,026	11,307	8,530	10,194
Cold produced per h., B.T.U., Per lb. of steam	1,100.8	785.6	564.9	435.82	512.12

Means for Applying the Cold. (M. C. Bannister, Liverpool Eng'g Soc'y, 1890.)—The most useful means for applying the cold to various uses is a saturated solution of brine or chloride of magnesium, which remains liquid at 5° Fahr. The brine is first cooled by being circulated in contact with the refrigerator-tubes, and then distributed through coils of pipes, arranged either in the substances requiring a reduction of temperature, or in the cold stores or rooms prepared for them; the air coming in contact with the cold tubes is immediately chilled, and the moisture in the air deposited on the pipes. It then falls, making room for warmer air, and so circulates until the whole room is at the temperature of the brine in the pipes.

In a recent arrangement for refrigerating made by the Linde British Refrigeration Co., the cold brine is circulated through a shallow trough, in which revolve a number of shafts, each geared together, and driven by mechanical means. On the shafts are fixed a number of wrought-iron disks, partly immersed in the brine, which cool them down to the brine temperature as they revolve; over these disks a rapid circulation of air is passed by a fan, being cooled by contact with the plates; then it is led into the chambers requiring refrigeration, from which it is again drawn by the same fan; thus all moisture and impurities are removed from the chambers, and deposited in the brine, producing the most perfect antiseptic atmosphere yet invented for cold storing; while the maximum efficiency of the brine temperature was always available, the brine being periodically concentrated by suitable arrangements.

Air has also been used as the circulating medium. The ammonia-pipes refrigerate the air in a cooling-chamber, and large wooden conduits are used to convey it to and return it from the rooms to be cooled. An advantage of this system is that by it a room may be refrigerated more quickly than by brine-coils. The returning air deposits its moisture in the form of snow on the ammonia-pipes, which is removed by mechanical brushes.

ARTIFICIAL ICE-MANUFACTURE.

Under summer conditions, with condensing water at 70°, artificial ice-machines use ammonia at about 190 lbs. above the atmosphere condenser-pressure, and 15 lbs. suction-pressure.

In a compression type of machine the useful circulation of ammonia, allowing for the effect of cylinder-heating, is about 18 lbs. per hour per indicated horse-power of the steam-cylinder. This weight of ammonia produces about 32 lbs. of ice at 15° from water at 70°. If the ice is made from distilled water, as in the "can system," the amount of the latter supplied by the boilers is about 33% greater than the weight of ice obtained. This excess represents steam escaping to the atmosphere, from the re-boiler and steam-condenser, to purify the distilled water, or free it from air; also, the loss through leaks and drips, and loss by melting of the ice in extracting it from the cans. The total steam consumed per horse-power is, therefore, about $32 \times 1.33 = 43.0$ lbs. About 7.0 lbs. of this covers the steam-consumption of the steam-engines driving the brine circulating-pumps, the several cold-water pumps, and leakage, drips, etc. Consequently, the main steam-engine must consume 36 lbs. of steam per hour per I.H.P., or else live steam must be condensed to supply the required amount of distilled water. There is, therefore, nothing to be gained by using steam at high rates of expansion in the steam-engines, in making artificial ice from distilled water. If the cooling water for the ammonia-coils and steam-condenser is not too hard for use in the boilers, it may enter the latter at about 175° F., by restricting the quantity to 1⅛ gallons per minute per ton of ice. With good coal 8½ lbs. of feed-water may then be evaporated, on the average, per lb. of coal.

The ice made per pound of coal will then be 32 ÷ (43.0 ÷ 8.5) = 6.0 lbs. This corresponds with the results of average practice.

If ice is manufactured by the "plate system," no distilled water is used for freezing. Hence the water evaporated by the boilers may be reduced to the amount which will drive the steam-motors, and the latter may use steam expansively to any extent consistent with the power required to compress the ammonia, operate the feed and filter pumps, and the hoisting machinery. The latter may require about 15% of the power needed for compressing the ammonia.

If a compound condensing steam-engine is used for driving the compressors, the steam per indicated steam horse-power, or per 32 lbs. of net ice, may be 14 lbs. per hour. The other motors at 50 lbs. of steam per horse-power will use 7.5 lbs. per hour, making the total consumption per steam horse-power of the compressor 21.5 lbs. Taking the evaporation at 8 lbs., the feed-water temperature being limited to about 110°, the coal per horse-power is 2.7 lbs. per hour. The net ice per lb. of coal is then about 32 ÷ 2.7 = 11.8 lbs. The best results with "plate-system" plants, using a compound steam-engine, have thus far afforded about 10½ lbs. of ice per lb. of coal.

In the "plate system" the ice gradually forms, in from 8 to 10 days, to a thickness of about 14 inches, on the hollow plates, 10 × 14 feet in area, in which the cooling fluid circulates.

In the "can system" the water is frozen in blocks weighing about 300 lbs. each, and the freezing is completed in from 40 to 48 hours. The freezing-tank area occupied by the "plate system" is, therefore, about twelve times, and the cubic contents about four times as much as required in the "can system."

The investment for the "plate" is about one-third greater than for the "can" system. In the latter system ice is being drawn throughout the 24 hours, and the hoisting is done by hand tackle. Some "can" plants are equipped with pneumatic hoists and on large hoists electric cranes are used to advantage. In the "plate system" the entire daily product is drawn, cut, and stored in a few hours, the hoisting being performed by power. The distribution of cost is as follows for the two systems, taking the cost for the "can" or distilled-water system as 100, which represents an actual cost of about $1.25 per net ton:

	Can System.	Plate System.
Hoisting and storing ice	14.2	2.8
Engineers, firemen, and coal-passer	15.0	13.9
Coal at $3.50 per gross ton	42.2	20.0
Water pumped directly from a natural source at 5 cts. per 1000 cubic feet	1.3	2.6
Interest and depreciation at 10%	24.6	32.7
Repairs	2.7	3.4
	100.00	75.4

A compound condensing engine is assumed to be used by the "plate system."

Test of the New York Hygeia Ice-making Plant.—(By Messrs. Hupfel, Griswold, and Mackenzie; *Stevens Indicator*, Jan. 1894.)

The final results of the tests were as follows:

Net ice made per pound of coal, in pounds	7.12
Pounds of net ice per hour per horse-power	37.8
Net ice manufactured per day (12 hours) in tons	97
Av. pressure of ammonia-gas at condenser, lbs. per sq. in. ab. atmos.	135.2
Average back pressure of amm.-gas, lbs. per sq. in. above atmos.	15.8
Average temperature of brine in freezing-tanks, degrees F	19.7
Total number of cans filled per week	4389
Ratio of cooling-surface of coils in brine-tank to can-surface	7 to 10

Ratio of brine in tanks to water in cans	1 to 1.2
Ratio of circulating water at condensers to distilled water	26 to 1
Pounds of water evaporated at boilers per pound of coal	8.085
Total horse-power developed by compressor-engines	444
Percentage of ice lost in removing from cans	2.2

APPROXIMATE DIVISION OF STEAM IN PER CENTS OF TOTAL AMOUNT.

Compressor-engines	60.1
Live steam admitted directly to condensers	19.7
Steam for pumps, agitator, and elevator engines	7.6
Live steam for reboiling distilled water	6.5
Steam for blowers furnishing draught at boilers	5.6
Sprinklers for removing ice from cans	0.5

The precautions taken to insure the purity of the ice are thus described: The water which finally leaves the condenser is the accumulation of the exhausts from the various pumps and engines, together with an amount of live steam injected into it directly from the boilers. This last quantity is used to make up any deficit in the amount of water necessary to supply the ice-cans. This water on leaving the condensers is violently reboiled, and afterwards cooled by running through a coil surface-cooler. It then passes through an oil-separator, after which it runs through three charcoal-filters and deodorizers, placed in series and containing 28 feet of charcoal. It next passes into the supply-tank in which there is an electrical attachment for detecting salt. Nitrate-of-silver tests are also made for salt daily. From this tank it is fed to the ice-cans, which are carefully covered so that the water cannot possibly receive any impurities.

MARINE ENGINEERING.

Rules for Measuring Dimensions and Obtaining Tonnage of Vessels. (Record of American & Foreign Shipping. American Bureau of Shipping, N. Y. 1890.)—The dimensions to be measured as follows:

I. Length, L.—From the fore side of stem to the after side of stern-post measured at middle line on the upper deck of all vessels, except those having a continuous hurricane-deck extending right fore and aft, in which the length is to be measured on the range of deck immediately below the hurricane-deck.

Vessels having clipper heads, raking forward, or receding stems, or raking stern-posts, the length to be the distance of the fore side of stem from aft-side of stern-post at the deep-load water-line measured at middle line. (The inner or propeller-post to be taken as stern-post in screw-steamers.

II. Breadth, B.—To be measured over the widest frame at its widest part; in other words, the moulded breadth.

III. Depth, D.—To be measured at the dead-flat frame and at middle line of vessel. It shall be the distance from the top of floor-plate to the upper side of upper deck-beam in all vessels except those having a continuous hurricane-deck, extending right fore and aft, and not intended for the American coasting trade, in which the depth is to be the distance from top of floor-plate to midway between top of hurricane deck-beam and the top of deck-beam of the deck immediately below hurricane-deck.

In vessels fitted with a continuous hurricane deck, extending right fore and aft, and intended for the American coasting trade, the depth is to be the distance from top of floor-plate to top of deck-beam of deck immediately below hurricane-deck.

Rule for Obtaining Tonnage.—Multiply together the length, breadth, and depth, and their product by .75; divide the last product by 100; the quotient will be the tonnage. $\frac{L \times B \times D \times .75}{100} = \text{tonnage}.$

The U. S. Custom-house Tonnage Law, May 6, 1864, provides that "the register tonnage of a vessel shall be her entire internal cubic capacity in tons of 100 cubic feet each." This measurement includes all the space between upper decks, however many there may be. Explicit directions for making the measurements are given in the law.

The Displacement of a Vessel (measured in tons of 2240 lbs.) is the weight of the volume of water which it displaces. For sea-water it is equal to the volume of the vessel beneath the water-line, in cubic feet, divided by 35, which figure is the number of cubic feet of sea-water at 60°

F. in a ton of 2240 lbs. For fresh water the divisor is 35.93. The U. S. register tonnage will equal the displacement when the entire internal cubic capacity bears to the displacement the ratio of 100 to 35.

The displacement or gross tonnage is sometimes approximately estimated as follows: Let L denote the length in feet of the boat, B its extreme breadth in feet, and D the mean draught in feet; the product of these three dimensions will give the volume of a parallelopipedon in cubic feet. Putting V for this volume, we have $V = L \times B \times D$.

The volume of displacement may then be expressed as a *percentage* of the volume V, known as the "*block coefficient.*" This percentage varies for different classes of ships. In racing yachts with very deep keels it varies from 22 to 33; in modern merchantmen from 55 to 75; for ordinary small boats probably 50 will give a fair estimate. The volume of displacement in cubic feet divided by 35 gives the displacement in tons.

Coefficient of Fineness.—A term used to express the relation between the displacement of a ship and the volume of a rectangular prism or box whose lineal dimensions are the length, breadth, and draught of the ship.

Coefficient of fineness $= \dfrac{D \times 35}{L \times B \times W}$; D being the displacement in tons of 35 cubic feet of sea-water to the ton, L the length between perpendiculars, B the extreme breadth of beam, and W the mean draught of water, all in feet.

Coefficient of Water-lines.—An expression of the relation of the displacement to the volume of the prism whose section equals the midship section of the ship, and length equal to the length of the ship.

Coefficient of water-lines $= \dfrac{D \times 35}{\text{area of immersed water section} \times L}$. Seaton gives the following values:

	Coefficient of Fineness.	Coefficient of Water-lines.
Finely-shaped ships	0.55	0.63
Fairly-shaped ships	0.61	0.67
Ordinary merchant steamers for speeds of 10 to 11 knots	0.65	0.72
Cargo steamers, 9 to 10 knots	0.70	0.76
Modern cargo steamers of large size	0.78	0.83

Resistance of Ships.—The resistance of a ship passing through water may vary from a number of causes, as speed, form of body, displacement, midship dimensions, character of wetted surface, fineness of lines, etc. The resistance of the water is twofold: 1st. That due to the displacement of the water at the bow and its replacement at the stern, with the consequent formation of waves. 2d. The friction between the wetted surface of the ship and the water, known as skin resistance. A common approximate formula for resistance of vessels is

Resistance = speed^2 $\times \sqrt[3]{\text{displacement}^2} \times$ a constant, or $R = S^2D^{\frac{2}{3}} \times C$.

If D = displacement in pounds, S = speed in feet per minute, R = resistance in foot-pounds per minute, $R = CS^2D^{\frac{2}{3}}$. The work done in overcoming the resistance through a distance equal to S is $R \times S = CS^3D^{\frac{2}{3}}$; and if E is the efficiency of the propeller and machinery combined, the indicated horse-power I.H.P. $= \dfrac{CS^3D^{\frac{2}{3}}}{E \times 33{,}000}$.

If S = speed in knots, D = displacement in tons, and C a constant which includes all the constants for form of vessel, efficiency of mechanism, etc., I.H.P. $= \dfrac{S^3D^{\frac{2}{3}}}{C}$.

The wetted surface varies as the cube root of the square of the displacement; thus, let L be the length of edge of a cube just immersed, whose displacement is D and wetted surface W. Then $D = L^3$ or $L = \sqrt[3]{D}$, and $W = 5 \times L^2 = 5 \times (\sqrt[3]{D})^2$. That is, W varies as $D^{\frac{2}{3}}$.

Another approximate formula is

$$\text{I.H.P.} = \frac{\text{area of immersed midship section} \times S^3}{K}.$$

The usefulness of these two formulæ depends upon the accuracy of the so-called "constants" C and K, which vary with the size and form of the ship, and probably also with the speed. Seaton gives the following, which may be taken roughly as the values of C and K under the conditions expressed:

General Description of Ship.	Speed, knots.	Value of C.	Value of K.
Ships over 400 feet long, finely shaped	15 to 17	240	620
" 300 " "	15 " 17	190	500
" " "	13 " 15	240	650
" " "	11 " 13	260	700
Ships over 300 feet long, fairly shaped	11 " 13	240	650
" " "	9 " 11	260	700
Ships over 250 feet long, finely shaped	13 " 15	200	580
" " "	11 " 13	240	660
" " "	9 " 11	260	700
Ships over 250 feet long, fairly shaped	11 " 13	220	620
" " "	9 " 11	250	680
Ships over 200 feet long, finely shaped	11 " 12	220	600
" " "	9 " 11	240	640
Ships over 200 feet long, fairly shaped	9 " 11	220	620
Ships under 200 feet long, finely shaped	11 " 12	200	550
" " "	10 " 11	210	580
" " "	9 " 10	230	620
Ships under 200 feet long, fairly shaped	9 " 10	200	600

Coefficient of Performance of Vessels.—The quotient

$$\frac{\sqrt[3]{(\text{displacement})^2} \times (\text{speed in knots})^3}{\text{tons of coal in 24 hours}},$$

gives a quotient of performance which represents the comparative cost of propulsion in coal expended. Sixteen vessels with three-stage expansion-engines in 1890 gave an average coefficient of 14,810, the range being from 12,150 to 16,700.

In 1881 seventeen vessels with two-stage expansion-engines gave an average coefficient of 11,710. In 1881 the length of the vessels tested ranged from 260 to 320, and in 1890 from 295 to 400. The speed in knots divided by the square root of the length in feet in 1881 averaged 0.539; and in 1890, 0.579; ranging from 0.520 to 0.641. (Proc. Inst. M. E., July, 1891, p. 329.)

Defects of the Common Formula for Resistance.—Modern experiments throw doubt upon the truth of the statement that the resistance varies as the square of the speed. (See Robt. Mansel's letters in *Engineering*, 1891; also his paper on The Mechanical Theory of Steamship Propulsion, read before Section G of the Engineering Congress, Chicago, 1893.)

Seaton says: In small steamers the chief resistance is the skin resistance. In very fine steamers at high speeds the amount of power required seems excessive when compared with that of ordinary steamers at ordinary speeds.

In torpedo-launches at certain high speeds the resistance increases at a lower rate than the square of the speed.

In ordinary sea-going and river steamers the reverse seems to be the case.

Rankine's Formula for total resistance of vessels of the "wave-line" type is:

$$R = ALBV^2(1 + 4\sin^2\theta + \sin^4\theta),$$

in which equation θ is the mean angle of greatest obliquity of the stream lines, A is a constant multiplier, B the mean wetted girth of the surface exposed to friction, L the length in feet, and V the speed in knots. The *power* demanded to impel a ship is thus the product of a constant to be determined by experiment, the area of the wetted surface, the *cube* of the speed, and the quantity in the parenthesis, which is known as the "coefficient of augmentation." The last term of the coefficient may be neglected in calculating the resistance of ships as too small to be practically important. In applying the formula, the mean of the squares of the sines of the angles of maximum obliquity of the water-lines is to be taken for $\sin^2 \theta$, and the rule will then read thus:

To obtain the resistance of a ship of good form, in pounds, multiply the length in feet by the mean immersed girth and by the coefficient of augmentation, and then take the product of this "augmented surface," as Rankine termed it, by the square of the speed in knots, and by the proper constant coefficient selected from the following:

For clean painted vessels, iron hulls........ $A = .01$
For clean coppered vessels.................. $A = .009$ to $.008$
For moderately rough iron vessels......... $A = .011 +$

The net, or effective, horse-power demanded will be quite closely obtained by multiplying the resistance calculated, as above, by the speed in knots and dividing by 326. The gross, or indicated, power is obtained by multiplying the last quantity by the reciprocal of the efficiency of the machinery and propeller, which usually should be about 0.6. Rankine uses as a divisor in this case 200 to 260.

The form of the vessel, even when designed by skilful and experienced naval architects, will often vary to such an extent as to cause the above constant coefficients to vary somewhat; and the range of variation with good forms is found to be from 0.8 to 1.5 the figures given.

For well-shaped iron vessels, an approximate formula for the horse-power required is $\text{H.P.} = \frac{SV^3}{20,000}$, in which S is the "augmented surface." The expression $\frac{SV^3}{\text{H.P.}}$ has been called by Rankine the *coefficient of propulsion*. In the Hudson River steamer "Mary Powell," according to Thurston, this coefficient was as high as 23,500.

The expression $\frac{D^{\frac{2}{3}}V^3}{\text{H.P.}}$ has been called the *locomotive performance*. (See Rankine's Treatise on Shipbuilding, 1864; Thurston's Manual of the Steam-engine, part ii. p. 16; also paper by F. T. Bowles, U.S.N., Proc. U. S. Naval Institute, 1883.)

Rankine's method for calculating the resistance is said by Seaton to give more accurate and reliable results than those obtained by the older rules, but it is criticised as being difficult and inconvenient of application.

Dr. Kirk's Method.—This method is generally used on the Clyde.

The general idea proposed by Dr. Kirk is to reduce all ships to so definite and simple a form that they may be easily compared; and the magnitude of certain features of this form shall determine the suitability of the ship for speed, etc.

The form consists of a middle body, which is a rectangular parallelopiped, and fore body and after body, prisms having isosceles triangles for bases, as shown in Fig. 168.

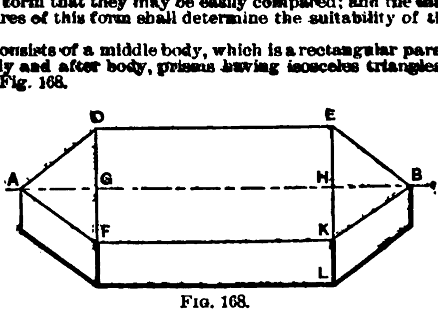


FIG. 168.

This is called a block model, and is such that its length is equal to that of the ship, the depth is equal to the mean draught, the capacity equal to the displacement volume, and its area of section equal to the area of immersed midship section. The dimensions of the block model may be obtained as follows:

Let $AG = HB$ = length of fore- or after-body $= F$;
GH = length of middle body $= M$;
KL = mean draught $= H$;
$$EK = \frac{\text{area of immersed midship section}}{KL} = B.$$

Volume of block $= (F + M) \times B \times H$;
Midship section $= B \times H$;
Displacement in tons = volume in cubic ft. ÷ 35.

$$AH = AG + GH = F + M = \text{displacement} \times 35 \div (B \times H).$$

The wetted surface of the block is nearly equal to that of the ship of the same length, beam and draught; usually 2% to 5% greater. In exceedingly fine hollow-line ships it may be 6% greater.

Area of bottom of block $= (F + M) \times B$;
Area of sides $= 2M \times H$.

$$\text{Area of sides of ends} = 4\sqrt{F^2 + \left(\frac{B}{2}\right)^2} \times H;$$

$$\text{Tangent of half angle of entrance} = \frac{\frac{1}{2}B}{F} = \frac{B}{2F}.$$

From this, by a table of natural tangents, the angle of entrance may be obtained:

	Angle of Entrance of the Block Model.	Fore-body in parts of length.
Ocean-going steamers, 14 knots and upward.	18° to 15°	.3 to .36
" " 12 to 14 knots.	21 to 18	.26 to .3
" cargo steamers, 10 to 12 knots.	30 to 22	.22 to .26

E. R. Mumford's Method of Calculating Wetted Surfaces is given in a paper by Archibald Denny, *Eng'g*, Sept. 21, 1894. The following is his formula, which gives closely accurate results for medium draughts, beams, and finenesses:

$$S = (L \times D \times 1.7) + (L \times B \times C),$$

in which S = wetted surface in square feet;
L = length between perpendiculars in feet;
D = middle draught in feet;
B = beam in feet;
C = block coefficient.

The formula may also be expressed in the form $S = L(1.7D + BC)$.

In the case of twin-screw ships having projecting shaft-casings, or in the case of a ship having a deep keel or bilge keels, an addition must be made for such projections. The formula gives results which are in general much more accurate than those obtained by Kirk's method. It underestimates the surface when the beam, draught, or block coefficients are excessive; but the error is small except in the case of abnormal forms, such as stern-wheel steamers having very excessive beams (nearly one fourth the length), and also very full block coefficients. The formula gives a surface about 6% too small for such forms.

To Find the Indicated Horse-power from the Wetted Surface. (Seaton.)—In ordinary cases the horse-power per 100 feet of wetted surface may be found by assuming that the rate for a speed of 10 knots is 5, and that the quantity varies as the cube of the speed. For example: To find the number of I.H.P. necessary to drive a ship at a speed of 15 knots, having a wetted skin of block model of 16,200 square feet:

The rate per 100 feet $= (15/10)^3 \times 5 = 16.875$.
Then I.H.P. required $= 16.875 \times 162 = 2734$.

When the ship is exceptionally well-proportioned, the bottom quite clean and the efficiency of the machinery high, as low a rate as 4 I.H.P. per 100 feet of wetted skin of block model may be allowed

The gross indicated horse-power includes the power necessary to overcome the friction and other resistance of the engine itself and the shafting, and also the power lost in the propellor. In other words, I.H.P. is no measure of the resistance of the ship, and can only be relied on as a means of deciding the size of engines for speed, so long as the efficiency of the engine and propellor is known definitely, or so long as similar engines and propellers are employed in ships to be compared. The former is difficult to obtain, and it is nearly impossible in practice to know how much of the power shown in the cylinders is employed usefully in overcoming the resistance of the ship. The following example is given to show the variation in the efficiency of propellers:

	Knots.		I.H.P.
H.M.S. "Amazon," with a 4-bladed screw, gave............	12.064	with	1940
H.M.S. "Amazon," with a 2-bladed screw, increased pitch, and less revolutions per minute..........................	12.396	"	1663
H.M.S. "Iris," with a 4-bladed screw......................	16.577	"	7503
H.M.S. "Iris," with 2-bladed screw, increased pitch, less revolutions per knot.....	18.587	"	7356

Relative Horse-power Required for Different Speeds of Vessels. (Horse-power for 10 knots = 1.)—The horse-power is taken usually to vary as the cube of the speed, but in different vessels and at different speeds it may vary from the 2.8 power to the 3.5 power, depending upon the lines of the vessel and upon the efficiency of the engines, the propeller, etc.

Speed, knots.	4	6	8	10	12	14	16	18	20	22	24	26	28	30
HP ∝														
$S^{2.8}$	.0769	.239	.535	1.	1.666	2.565	3.729	5.185	6.964	9.095	11.60	14.52	17.87	21.6
$S^{2.9}$	.0701	.227	.524	1.	1.697	2.653	3.908	5.499	7.464	9.841	12.67	15.97	19.80	24.1
S^{3}	.0640	.216	.512	1.	1.728	2.744	4.096	5.832	8.	10.65	13.82	17.58	21.95	27.
$S^{3.1}$	.0584	.205	.501	1.	1.760	2.838	4.293	6.185	8.574	11.52	15.09	19.34	24.33	30.1
$S^{3.2}$	.0533	.195	.490	1.	1.792	2.935	4.500	6.559	9.189	12.47	16.47	21.28	26.97	33.6
$S^{3.3}$	.0486	.185	.479	1.	1.825	3.036	4.716	6.957	9.849	13.49	17.98	23.41	29.90	37.5
$S^{3.4}$	.0444	.176	.468	1.	1.859	3.139	4.943	7.378	10.56	14.60	19.62	25.76	33.14	41.9
$S^{3.5}$	.0405	.167	.458	1.	1.893	3.247	5.181	7.824	11.31	15.79	21.42	28.34	36.73	46.7

EXAMPLE IN USE OF THE TABLE.—A certain vessel makes 14 knots speed with 587 I.H.P. and 16 knots with 900 I.H.P. What I.H.P. will be required at 18 knots, the rate of increase of horse-power with increase of speed remaining constant? The first step is to find the rate of increase, thus: $14^x : 16^x :: 587 : 900$.

$$x \log 16 - x \log 14 = \log 900 - \log 587;$$
$$x(0.204120 - 0.146128) = 2.954243 - 2.768638,$$

whence x (the exponent of S in formula H.P. $\propto S^{x}$) = 3.2.

From the table, for $S^{3.2}$ and 16 knots, the I.H.P. is 4.5 times the I.H.P. at 10 knots, ∴ H.P. at 10 knots = 900 ÷ 4.5 = 200.

From the table, for $S^{3.2}$ and 18 knots, the I.H.P. is 6.559 times the I.H.P. at 10 knots: ∴ H.P. at 18 knots = 200 × 6.559 = 1312 H.P.

Resistance per Horse-power for Different Speeds. (One horse-power = 33,000 lbs. resistance overcome through 1 ft. in 1 min.)—The resistances per horse-power for various speeds are as follows: For a speed of 1 knot, or 6080 feet per hour = 101⅓ ft. per min., 33,000 ÷ 101⅓ = 325.658 lbs. per horse-power; and for any other speed 325.658 lbs. divided by the speed in knots; or for

1 knot	325.66 lbs.	6 knots	54.28 lbs.	11 knots	29.61 lbs.	16 knots	20.35 lbs.
2 knots	162.83 "	7 "	46.52 "	12 "	27.14 "	17 "	19.16 "
3 "	108.55 "	8 "	40.71 "	13 "	25.05 "	18 "	18.09 "
4 "	81.41 "	9 "	36.18 "	14 "	23.26 "	19 "	17.14 "
5 "	65.13 "	10 "	32.57 "	15 "	21.71 "	20 "	16.28 "

Results of Trials of Steam-vessels of Various Sizes.
(From Seaton's Marine Engineering.)

	S.S. "Torpedo."	P.S. "John Penn."	S.S. "Africa."	P.S. "Mary Powell"	S.S. "Harrar."	R.M.P.S. "Connaught."
Length, perpendiculars	90′ 0″	171′ 9″	130′ 0″	286′ 0″	280′ 0″	327′ 0″
Breadth, extreme	10′ 6″	18′ 9″	21′ 0″	34′ 3″	29′ 0″	35′ 0″
Mean draught water	2′ 6″	6′ 9¼″	8′ 10″	6′ 0″	13′ 6″	13′ 0″
Displacement (tons)	29.78	280	370	800	1500	1900
Area Immersed mid. section	24?	99	148	200	340	336
By Kirk's System. Wetted skin	908	3798	3754	8222	10,075	15,782
By Kirk's System. Length, fore-body	45′ 0″	72′ 00″	42′ 6″	148′ 0″	79′ 6″	129′ 0″
By Kirk's System. Angle of entrance	12° 40′	11° 30′	23° 50′	13° 21′	17° 0′	11° 26′
$\frac{\text{Displacement} \times 35}{\text{Length} \times \text{Imm. mid area}}$	0.481	0.576	0.608	0.489	0.671	0.605
Speed (knots)	22 01	15.3	10.74	17.20	10.04	17.8
Indicated horse-power	460	798	371	1490	508	4751
I.H.P. per 100 ft. wetted skin	50.9	21.04	9.88	18.12	5.00	30.00
I.H.P. per 100 ft. wetted skin, reduced to 10 knots	4.78	5.87	7.97	3.56	4.90	5.32
$\frac{D^{\frac{2}{3}} \times S^3}{\text{I.H.P.}}$	223	192	172.8	293.7	266	182
$\frac{\text{Immersed mid area} \times S^3}{\text{I.H.P.}}$	556?	445	495	683	690	399

	H.M.S. "Active."	H.M.S. "Iris."	H.M.S. "Iris."	S.S. "Garonne."	H.M.S. "Hecla."	R.M.S.S. "Britannic."
Length, perpendiculars	270′ 0″	300′ 0″	300′ 0″	370′ 0″	392 0″	450′ 0″
Breadth, extreme	42′ 0″	46′ 0″	46′ 0″	41′ 0″	39 0″	45′ 2″
Mean draught water	18′ 10″	18′ 2″	18′ 2″	18′ 11″	21′ 4″	23′ 7″
Displacement (tons)	3057	3290	3290	4685	5767	8500
Area Imm. mid. section	682	700	700	656	738	926
By Kirk's System. Wetted skin	16,006	18,168	18,168	22,633	26,235	32,578
By Kirk's System. Length, fore-body	101′ 0″	135′ 6″	135′ 6″	128′ 0″	118′ 0″	129′ 0″
By Kirk's System. Angle of entrance	18° 44′	16° 16′	16° 16′	16° 4′	16° 30′	17° 16′
$\frac{\text{Displacement} \times 35}{\text{Length} \times \text{Imm. mid area}}$	0.629	0.548	0.548	0.668	0.698	0.714
Speed (knots)	14.966	18.573	15.746	13.80	12.054	15.045
Indicated horse-power	4015	7714	3958	2500	1758	4900
I.H.P. per 100 ft. wetted skin	25.08	42.46	21.78	11.04	6.7	15.04
I.H.P. per 100 ft. wetted skin, reduced to 10 knots	7.49	6.634	5.58	4.20	3.83	4.42
$\frac{D^{\frac{2}{3}} \times S^3}{\text{I.H.P.}}$	175.8	183.7	218.2	292	320	289.3
$\frac{\text{Immersed mid area} \times S^3}{\text{I.H.P.}}$	527.5	581.4	690.5	689	735	642.5

Results of Progressive Speed Trials in Typical Vessels.

(*Eng'g*, April 15, 1892, p. 463.)

	Torpedo-boat.	Torpedo-gunboat, "Sharpshooter" Class.	"Medusa," 3d-cl. Cruiser.	"Terpsichore," 2d-cl. Cruiser.	"Edgar," 1st-cl. Cruiser.	"Blenheim," 1st-cl. Cruiser.	Atlantic Passenger Steamer.
Length (in feet)	135	230	265	300	360	375	525
Breadth " "	14	27	41	43	60	65	63
Draught (mean) on trial	5' 1"	8' 3"	16' 6"	16' 2"	23' 9"	25' 9"	21' 3"
Displacement (tons)	108	735	2800	3330	7390	9100	11550
I.H.P.—10 knots	110	450	700	800	1000	1500	2000
" 14 "	260	1100	2100	2400	3000	4000	4600
" 18 "	870	2500	6400	6000	7500	9000	10000
" 20 "	1130	3500	10000	9000	11000	12500	14500

Speed	Ratio of speed³		Torpedo-boat	Torpedo-gunboat	"Medusa"	"Terpsichore"	"Edgar"	"Blenheim"	Atlantic Passenger Steamer
10	1.	Ratio of H.P. =	1	1	1	1	1	1	1
14	2.744	" " =	2.36	2.44	3	3	3	2.67	2.3
18	5.832	" " =	7.91	5.56	9.14	7.5	7.5	6.	5
20	8.	" " =	10.27	7.78	14.14	11.25	11	8.42	7.25

Admiralty coeff. $C = \frac{D^{\frac{2}{3}} \times S^3}{\text{I.H.P.}}$		Torpedo-boat	Torpedo-gunboat	"Medusa"	"Terpsichore"	"Edgar"	"Blenheim"	Atlantic Passenger Steamer
	10 knots.	200	181	284	279	380	290	255
	14 "	232	208	259	255	347	298	304
	18 "	147	190	181	217	295	282	297
	20 "	156	186	159	198	276	278	281

The figures for I.H.P. are "round." The "Medusa's" figures for 20 knots are from trial on Stokes Bay, and show the retarding effect of shallow water. The figures for the other ships for 20 knots are estimated for deep water.

More accurate methods than those above given for estimating the horse-power required for any proposed ship are: 1. Estimations calculated from the results of trials of "similar" vessels driven at "corresponding" speeds; "similar" vessels being those that have the same ratio of length to breadth and to draught, and the same coefficient of fineness, and "corresponding" speeds those which are proportional to the square roots of the lengths of the respective vessels. Froude found that the resistances of such vessels varied almost exactly as wetted surface × (speed)².

2. The method employed by the British Admiralty and by some Clyde shipbuilders, viz., ascertaining the resistance of a model of the vessel, 12 to 20 ft. long, in a tank, and calculating the power from the results obtained.

Speed on Canals.—A great loss of speed occurs when a steam-vessel passes from open water into a more or less restricted channel. The average speed of vessels in the Suez Canal in 1882 was only 5¼ statute miles per hour. (*Eng'g*. Feb. 15, 1884, p. 139.)

Estimated Displacement, Horse-power, etc.—The table on the next page, calculated by the author, will be found convenient for making approximate estimates.

The figures in 7th column are calculated by the formula H.P. = $S^3D^{\frac{2}{3}} \div c$, in which $c = 200$ for vessels under 200 ft. long when $C = .65$, and 210 when $C = .55$; $c = 200$ for vessels 200 to 400 ft. long when $C = .75$, 230 when $C = .65$, 240 when $C = .55$; $c = 230$ for vessels over 400 ft. long when $C = .75$, 250 when $C = .65$, 260 when $C = .55$.

The figures in the 8th column are based on 5 H.P. per 100 sq. ft. of wetted surface.

The diameters of screw in the 9th column are from formula $D = 3.31 \sqrt[5]{\text{I.H.P.}}$, and in the 10th column from formula $D = 2.71 \sqrt[5]{\text{I.H.P.}}$

To find the diameter of screw for any other speed than 10 knots, revolutions being 100 per minute, multiply the diameter given in the table by the 5th root of the cube of the given speed ÷ 10. For any other revolutions per minute than 100, divide by the revolutions and multiply by 100.

To find the approximate horse-power for any other speed than 10 knots, multiply the horse-power given in the table by the cube of the ratio of the given speed to 10, or by the relative figure from table on p. 1006.

Estimated Displacement, Horse-power, etc., of Ste vessels of Various Sizes.

Length, feet, L	Breadth, feet, B	Draught, feet, D	Coefficient of Fineness, C	Displacement. $\frac{LBD \times C}{35}$ tons.	Wetted Surface $L(1.7D + BC)$ sq. ft.	Estimated Horse-power at 10 knots. Calc. from Displacem't.	Estimated Horse-power at 10 knots. Calc. from Wetted Surface.	Diam. of Screw knots speed a revs. per min. If Pitch = Diam.	If P 1.4
12	3	1.5	.55	.85	48	4.3	2.4	4.4	[illegible]
16	3	1.5	.55	1.18	64	5.2	3.2	4.6	[illegible]
	4	2	.65	2.38	96	8.9	4.8	5.1	[illegible]
20	3	1.5	.55	1.41	80	6.0	4.0	4.7	[illegible]
	4	2	.65	2.97	120	10.8	6.0	5.8	[illegible]
24	3.5	1.5	.55	1.98	104	7.5	5.2	5	[illegible]
	4.5	2	.65	4.01	152	12.6	7.6	5.5	[illegible]
30	4	2	.55	3.77	168	11.5	8.4	5.4	[illegible]
	5	2.5	.65	6.96	224	18.2	11.2	5.9	[illegible]
40	4.5	2	.55	5.66	235	15.1	11.8	5.7	[illegible]
	6	2.5	.65	11.1	326	24.9	16.3	6.3	[illegible]
50	6	3	.55	14.1	420	27.8	21.0	6.4	[illegible]
	8	3.5	.65	26	558	43.9	27.9	7.1	[illegible]
60	8	3.5	.55	26.4	621	42.2	31.1	7.0	[illegible]
	10	4	.65	44.6	798	62.9	39.9	7.6	[illegible]
70	10	4	.55	44	861	59.4	43.1	7.5	[illegible]
	12	4.5	.65	70.2	1082	85.1	54.1	8.1	[illegible]
80	12	4.5	.55	67.9	1140	79.2	57.0	7.9	[illegible]
	14	5	.65	104.0	1408	111	70 4	8.5	[illegible]
90	13	5	.55	91.9	1408	97	70.4	8.3	[illegible]
	16	6	.65	160	1854	147	92.7	9	[illegible]
100	13	5	.55	102	1565	104	78.3	8.4	[illegible]
	15	5.5	.65	153	1910	148	95.5	8.9	[illegible]
	17	6	.75	219	2295	202	115	9.6	[illegible]
120	14	5.5	.55	145	2046	131	102	8.8	[illegible]
	16	6	.65	214	2472	179	124	9.4	[illegible]
	18	6.5	.75	301	2946	250	147	10	[illegible]
140	16	6	.55	211	2660	169	133	9.2	[illegible]
	18	6.5	.65	306	3185	227	159	9.8	[illegible]
	20	7	.75	420	3766	312	188	10.5	[illegible]
160	17	6.5	.55	278	3264	208	168	9.6	[illegible]
	19	7	.65	395	3880	269	194	10.1	[illegible]
	21	7.5	.75	540	4560	368	228	10.8	[illegible]
180	20	7	.55	396	4123	257	206	10.1	[illegible]
	22	7.5	.65	552	4869	337	248	10.6	[illegible]
	24	8	.75	741	5688	455	284	11.3	[illegible]
200	22	7	.55	484	4800	357	240	10.1	[illegible]
	25	8	.65	743	5970	373	299	10.8	[illegible]
	28	9	.75	1080	7260	526	363	11.6	[illegible]
250	28	8	.55	880	7250	383	363	10.9	[illegible]
	32	10	.65	1486	9450	592	473	11.9	[illegible]
	36	12	.75	2314	11850	875	593	12.8	[illegible]
300	32	10	.55	1509	10380	548	519	11.7	[illegible]
	36	12	.65	2407	13140	806	657	12.6	[illegible]
	40	14	.75	3600	17140	1175	857	13.6	[illegible]
350	38	12	.55	2508	14455	769	723	12.5	[illegible]
	42	14	.65	3822	17885	1111	894	13.5	[illegible]
	46	16	.75	5520	21595	1562	1080	14.4	[illegible]
400	44	14	.55	3872	19200	1028	960	13.3	[illegible]
	48	16	.65	5705	23360	1451	1168	14.2	[illegible]
	52	18	.75	8023	27840	2906	1392	15.2	[illegible]
450	50	16	.55	5657	24515	1221	1226	13.7	[illegible]
	54	18	.65	8128	29565	1616	1478	14.5	[illegible]
	58	20	.75	11157	34875	2171	1744	15.4	[illegible]
500	52	18	.55	7354	29600	1454	1480	14.2	[illegible]
	56	20	.65	10400	35200	1966	1760	15.1	[illegible]
	60	22	.75	14143	41200	2543	2060	15.9	[illegible]
550	56	20	.55	9680	38245	1747	1812	14.7	[illegible]
	60	22	.65	13483	42735	2266	2187	15 5	[illegible]
	64	24	.75	18103	49665	2998	2483	16.4	[illegible]
600	60	22	.55	12416	42900	2065	2145	15.2	[illegible]
	64	24	.65	17115	50220	2656	2511	15.4	[illegible]
	68	26	.75	22731	58020	3489	2901	16.9	[illegible]

THE SCREW-PROPELLER.

The "pitch" of a propeller is the distance which any point in a blade, describing a helix, will travel in the direction of the axis during one revolution, the point being assumed to move around the axis. The pitch of a propeller with a uniform pitch is equal to the distance a propeller will advance during one revolution, provided there is no slip. In a case of this kind, the term "pitch" is analogous to the term "pitch of the thread" of an ordinary single-threaded screw.

Let P = pitch of screw in feet, R = number of revolutions per second. V = velocity of stream from the propeller $= P \times R$, v = velocity of the ship in feet per second, $V - v$ = slip, A = area in square feet of section of stream from the screw, approximately the area of a circle of the same diameter, $A \times V$ = volume of water projected astern from the ship in cubic feet per second. Taking the weight of a cubic foot of sea-water at 64 lbs., and the force of gravity at 32, we have from the common formula for force of acceleration, viz.: $F = M\frac{v_1}{t} = \frac{W}{g}\frac{v_1}{t}$, or $F = \frac{W}{g}v_1$, when $t = 1$ second, v_1 being the acceleration.

$$\text{Thrust of screw in pounds} = \frac{64AV}{32}(V - v) = 2AV(V - v).$$

Rankine (Rules, Tables, and Data, p. 275) gives the following: To calculate the thrust of a propelling instrument (jet, paddle, or screw) in pounds, multiply together the transverse sectional area, in square feet, of the stream driven astern by the propeller; the speed of the stream relatively to the ship in knots; the real slip, or part of that speed which is impressed on that stream by the propeller, also in knots; and the constant 5.66 for sea-water, or 5.5 for fresh water. If S = speed of the screw in knots, s = speed of ship in knots, A = area of the stream in square feet (of sea-water),

$$\text{Thrust in pounds} = A \times S(S - s) \times 5.66.$$

The real slip is the velocity (relative to water at rest) of the water projected sternward; *the apparent slip* is the difference between the speed of the ship and the speed of the screw; i.e., the product of the pitch of the screw by the number of revolutions.

This apparent slip is sometimes negative, due to the working of the screw in disturbed water which has a forward velocity, following the ship. Negative apparent slip is an indication that the propeller is not suited to the ship.

The apparent slip should generally be about 8% to 10% at full speed in well-formed vessels with moderately fine lines; in bluff cargo boats it rarely exceeds 5%.

The effective area of a screw is the sectional area of the stream of water laid hold of by the propeller, and is generally, if not always, greater than the actual area, in a ratio which in good ordinary examples is 1.2 or thereabouts, and is sometimes as high as 1.4; a fact probably due to the stiffness of the water, which communicates motion laterally amongst its particles. (Rankine's Shipbuilding, p. 89.)

Prof. D. S. Jacobus, Trans. A. S. M. E., xi. 1028, found the ratio of the effective to the actual disk area of the screws of different vessels to be as follows:

Tug-boat, with ordinary true-pitch screw		1.42
" " screw having blades projecting backward		.57
Ferryboat "Bergen," with ordinary true-pitch screw	at speed of 12.09 stat. miles per hour.	1.53
	" " 13.4 " " " "	1.48
Steamer "Homer Ramsdell," with ordinary true-pitch screw		1.20

Size of Screw.—Seaton says: The size of a screw depends on so many things that it is very difficult to lay down any rule for guidance, and much must always be left to the experience of the designer, to allow for all the circumstances of each particular case. The following rules are given for ordinary cases. (Seaton and Rounthwaite's Pocket-book):

P = pitch of propeller in feet $= \frac{10133S}{R(100 - x)}$, in which S = speed in knots, R = revolutions per minute, and x = percentage of apparent slip.

For a slip of 10%, pitch $= \frac{112.6S}{R}$.

D = diameter of propeller $= K\sqrt{\frac{\text{I.H.P.}}{\left(\frac{P \times R}{100}\right)^3}}$, K being a coefficient given in the table below. If $K = 20$, $D = 20000\sqrt[5]{\frac{\text{I.H.P.}}{(P \times R)^3}}$.

Total developed area of blades $= C\sqrt{\frac{\text{I.H.P.}}{R}}$, in which C is a coefficient to be taken from the table.

Another formula for pitch, given in Seaton's Marine Engineering, is $P = \frac{C}{R}\sqrt[3]{\frac{\text{I.H.P.}}{D^2}}$, in which $C = 737$ for ordinary vessels, and 660 for slow-speed cargo vessels with full lines.

Thickness of blade at root $= \sqrt{\frac{d^3}{nb}} \times k$, in which d = diameter of tail-shaft in inches, n = number of blades, b = breadth of blade in inches where it joins the boss, measured parallel to the shaft axis; $k = 4$ for cast iron, 1.5 for cast steel, 2 for gun-metal, 1.5 for high-class bronze.

Thickness of blade at tip: Cast iron $.04D + .4$ in.; cast steel $.03D + .4$ in.; gun-metal $.03D + .2$ in.; high-class bronze $.02D + .3$ in., where D = diameter of propeller in feet.

Propeller Coefficients.

Description of Vessel.	Approximate Speed in Knots.	Number of Screws.	Number of Blades per Screw.	Values of K.	Values of C.	Usual Material of Blades.
Bluff cargo boats.......	8-10	One	4	17 -17.5	19 -17.5	Cast iron
Cargo, moderate lines..	10-13	"	4	18 -19	17 -15.5	" "
Pass. and mail, fine lines.	13-17	"	4	19.5-20.5	15 -13	C. I. or S.
" " " " "	13-17	Twin	4	20.5-21.5	14.5-12.5	" " "
" " " very fine.	17-22	One	4	21 -22	12.5-11	G. M. or B
" " " " "	17-22	Twin	3	22 -23	10.5- 9	" " "
Naval vessels, " "	16-22	"	4	21 -22.5	11.5-10.5	" " "
" " " "	16-22	"	3	22 -23.5	8.5- 7	" " "
Torpedo-boats, " "	20-26	One	3	25	7- 6	B. or F. S.

C. I., cast iron; G. M., gun-metal; B., bronze; S., steel; F. S., forged steel.

From the formulæ $D = 20000\sqrt[5]{\frac{\text{I.H.P.}}{(P \times R)^3}}$ and $P = \frac{737}{R}\sqrt[3]{\frac{\text{I.H.P.}}{D^2}}$, if $P = D$ and $R = 100$, we obtain $D = \sqrt[5]{400 \times \text{I.H.P.}} = 3.31\sqrt[5]{\text{I.H.P.}}$

If $P = 1.4D$ and $R = 100$, then $D = \sqrt[5]{145.8 \times \text{I.H.P.}} = 2.71\sqrt[5]{\text{I.H.P.}}$

From these two formulæ the figures for diameter of screw in the table on page 1009 have been calculated. They may be used as rough approximations to the correct diameter of screw for any given horse-power, for a speed of 10 knots and 100 revolutions per minute.

For any other number of revolutions per minute multiply the figures in the table by 100 and divide by the given number of revolutions. For any other speed than 10 knots, since the I.H.P. varies approximately as the cube of the speed, and the diameter of the screw as the 5th root of the I.H.P., multiply the diameter given for 10 knots by the 5th root of the cube of one tenth of the given speed. Or, multiply by the following factors:

For speed of knots:

	4	5	6	7	8	9	11	12	13	14	15	16
$\sqrt[5]{(S \div 10)^3}$ =	.577	.660	.736	.807	.875	.939	1.059	1.116	1.170	1.224	1.275	1.327

Speed:	17	18	19	20	21	22	23	24	25	26	27	28
$\sqrt[5]{(S+10)^2} =$	1.375	1.422	1.470	1.515	1.561	1.605	1.648	1.691	1.733	1.774	1.815	1.855

For more accurate determinations of diameter and pitch of screw, the formulæ and coefficients given by Seaton, quoted above, should be used.

Efficiency of the Propeller.—According to Rankine, if the slip of the water be s, its weight W, the resistance R, and the speed of the ship v,

$$R = \frac{Ws}{g}; \quad Rv = \frac{Wsv}{g}.$$

This impelling action must, to secure maximum efficiency of propeller, be effected by an instrument which takes hold of the fluid without shock or disturbance of the surrounding mass, and, by a steady acceleration, gives it the required final velocity of discharge. The velocity of the propeller overcoming the resistance R would then be

$$\frac{v + (v + s)}{2} = v + \frac{s}{2};$$

and the work performed would be

$$R\left(v + \frac{s}{2}\right) = \frac{Wvs}{g} + \frac{Ws^2}{2g},$$

the first of the last two terms being useful, the second the minimum lost work; the latter being the wasted energy of the water thrown backward. The efficiency is

$$E = v \div \left(v + \frac{s}{2}\right);$$

and this is the limit attainable with a perfect propelling instrument, which limit is approached the more nearly as the conditions above prescribed are the more nearly fulfilled. The efficiency of the propelling instrument is probably rarely much above 0.60, and never above 0.80.

In designing the screw-propeller, as was shown by Dr. Froude, the best angle for the surface is that of 45° with the plane of the disk; but as all parts of the blade cannot be given the same angle, it should, where practicable, be so proportioned that the "pitch-angle at the centre of effort" should be made 45°. The maximum possible efficiency is then, according to Froude, 77%.

In order that the water should be taken on without shock and discharged with maximum backward velocity, the screw must have an axially increasing pitch.

The true screw is by far the more usual form of propeller, in all steamers, both merchant and naval. (Thurston, Manual of the Steam-engine, part ii., p. 176.)

The combined efficiency of screw, shaft, engine, etc., is generally taken at 50%. In some cases it may reach 60% or 65%. Rankine takes the effective H.P. to equal the I.H.P. ÷ 1.63.

Pitch-ratio and Slip for Screws of Standard Form.

Pitch-ratio.	Real Slip of Screw.	Pitch-ratio.	Real Slip of Screw
.8	15.55	1.7	21.3
.9	16.22	1.8	21.8
1.0	16.88	1.9	22.4
1.1	17.55	2.0	22.9
1.2	18.2	2.1	23.5
1.3	18.8	2.2	24.0
1.4	19.5	2.3	24.5
1.5	20.1	2.4	25.0
1.6	20.7	2.5	25.4

Results of Recent Researches on the efficiency of screw-propellers are summarized by S. W. Barnaby, in a paper read before section G of the Engineering Congress, Chicago, 1893. He states that the following general principles have been established:

(*a*) There is a definite amount of real slip at which, and at which only, maximum efficiency can be obtained with a screw of any given type, and this amount varies with the pitch-ratio. The slip-ratio proper to a given ratio of pitch to diameter has been discovered and tabulated for a screw of a standard type, as below (see table on page 1012);

(*b*) Screws of large pitch-ratio, besides being less efficient in themselves, add to the resistance of the hull by an amount bearing some proportion to their distance from it, and to the amount of rotation left in the race.

(*c*) The best pitch-ratio lies probably between 1.1 and 1.5.

(*d*) The fuller the lines of the vessel, the less the pitch-ratio should be.

(*e*) Coarse-pitched screws should be placed further from the stern than fine-pitched ones.

(*f*) Apparent negative slip is a natural result of abnormal proportions of propellers.

(*g*) Three blades are to be preferred for high-speed vessels, but when the diameter is unduly restricted, four or even more may be advantageously employed.

(*h*) An efficient form of blade is an ellipse having a minor axis equal to four tenths the major axis.

(*i*) The pitch of wide-bladed screws should increase from forward to aft, but a uniform pitch gives satisfactory results when the blades are narrow, and the amount of the pitch variation should be a function of the width of the blade.

(*j*) A considerable inclination of screw-shaft produces vibration, and with right-handed twin-screws turning outwards, if the shafts are inclined at all, it should be upwards and outwards from the propellers.

For results of experiments with screw-propellers, see F. C. Marshall, Proc. Inst. M. E. 1881; R. E. Froude, Trans. Institution of Naval Architects, 1886; G. A. Calvert, Trans. Institution of Naval Architects 1887; and S. W. Barnaby, Proc. Inst. Civil Eng'rs 1890, vol. cii.

One of the most important results deduced from experiments on model screws is that they appear to have practically equal efficiencies throughout a wide range both in pitch-ratio and in surface-ratio; so that great latitude is left to the designer in regard to the form of the propeller. Another important feature is that, although these experiments are not a direct guide to the selection of the most efficient propeller for a particular ship, they supply the means of analyzing the performances of screws fitted to vessels, and of thus indirectly determining what are likely to be the best dimensions of screw for a vessel of a class whose results are known. Thus a great advance has been made on the old method of trial upon the ship itself, which was the origin of almost every conceivable erroneous view respecting the screw-propeller. (Proc. Inst. M. E., July, 1891.)

THE PADDLE-WHEEL.

Paddle-wheels with Radial Floats. (Seaton's Marine Engineering.)—The effective diameter of a radial wheel is usually taken from the centres of opposite floats; but it is difficult to say what is absolutely that diameter, as much depends on the form of float, the amount of dip, and the waves set in motion by the wheel. The slip of a radial wheel is from 15 to 20 per cent, depending on the size of float.

$$\text{Area of one float} = \frac{\text{I.H.P.}}{D} \times C.$$

D is the effective diameter in feet, and C is a multiplier, varying from 0.25 in tugs to 0.175 in fast-running light steamers.

The breadth of the float is usually about ¼ its length, and its thickness about ⅙ its breadth. The number of floats varies directly with the diameter, and there should be one float for every foot of diameter.

(For a discussion of the action of the radial wheel, see Thurston, Manual of the Steam-engine, part ii., p, 182.)

Feathering Paddle-wheels. (Seaton.)—The diameter of a feathering-wheel is found as follows: The amount of slip varies from 12 to 20 per cent, although when the floats are small or the resistance great it is as high as 25 per cent; a well-designed wheel on a well-formed ship should not exceed 15 per cent under ordinary circumstances.

If K is the speed of the ship in knots, S the percentage of slip, and R the revolutions per minute,

$$\text{Diameter of wheel at centres} = \frac{K(100 + S)}{3.14 \times R}.$$

The diameter, however, must be such as will suit the structure of the ship, so that a modification may be necessary on this account, and the revolutions altered to suit it.

The diameter will also depend on the amount of "dip" or immersion of float.

When a ship is working always in smooth water the immersion of the top edge should not exceed 1/8 the breadth of the float; and for general service at sea an immersion of 1/2 the breadth of the float is sufficient. If the ship is intended to carry cargo, the immersion when light need not be more than 2 or 3 inches, and should not be more than the breadth of float when at the deepest draught; indeed, the efficiency of the wheel falls off rapidly with the immersion of the wheel.

$$\text{Area of one float} = \frac{\text{I.H.P.}}{D} \times C.$$

C is a multiplier, varying from 0.3 to 0.35; D is the diameter of the wheel to the float centres, in feet.

The number of floats $= \frac{1}{2}(D + 2)$.
The breadth of the float $= 0.35 \times$ the length.
The thickness of floats $= 1/12$ the breadth.
Diameter of gudgeons $=$ thickness of float.

Seaton and Rounthwaite's Pocket-book gives:

$$\text{Number of floats} = \frac{60}{\sqrt[4]{R}},$$

where R is number of revolutions per minute.

$$\text{Area of one float (in square feet)} = \frac{\text{I.H.P.} \times 33000 \times K}{N \times (D \times R)^3},$$

where $N =$ number of floats in one wheel.

For vessels plying always in smooth water $K = 1200$. For sea-going steamers $K = 1400$. For tugs and such craft as require to stop and start frequently in a tide-way $K = 1600$.

It will be quite accurate enough if the last four figures of the cube $(D \times R)^3$ be taken as ciphers.

For illustrated description of the feathering paddle-wheel see Seaton's Marine Engineering, or Seaton and Rounthwaite's Pocket-book. The diameter of a feathering-wheel is about one half that of a radial wheel for equal efficiency. (Thurston.)

Efficiency of Paddle-wheels.—Computations by Prof. Thurston of the efficiency of propulsion by paddle-wheels give for light river steamers with ratio of velocity of the vessel, v, to velocity of the paddle-float at centre of pressure, V, or $\frac{v}{V}$, $= \frac{3}{4}$, with a dip $= 3/20$ radius of the wheel, and a slip of 25 per cent, an efficiency of .714; and for ocean steamers with the same slip and ratio of $\frac{v}{V}$, and a dip $= \frac{1}{2}$ radius, an efficiency of .685.

JET-PROPULSION.

Numerous experiments have been made in driving a vessel by the reaction of a jet of water pumped through an orifice in the stern, but they have all resulted in commercial failure. Two jet-propulsion steamers, the "Waterwitch," 1100 tons, and the "Squirt," a small torpedo-boat, were built by the British Government. The former was tried in 1867, and gave an efficiency of apparatus of only 18 per cent. The latter gave a speed of 12 knots, as against 17 knots attained by a sister-ship having a screw and equal steam-power. The mathematical theory of the efficiency of the jet was discussed by Rankine in *The Engineer*, Jan. 11, 1867, and he showed that the greater the quantity of water operated on by a jet-propeller, the greater as the efficiency. In defiance both of the theory and of the results of earlier experiments, and also of the opinions of many naval engineers, more than $200,000 were spent in 1888–90 in New York upon two experimental boats, the "Prima Vista" and the "Evolution," in which the jet was made of very small size, in the latter case only ⅝-inch diameter, and with a pressure of 2500 lbs. per square inch. As had been predicted, the vessel was a total failure. (See article by the author in *Mechanics*, March, 1891.)

The theory of the jet-propeller is similar to that of the screw-propeller. If A = the area of the jet in square feet, V its velocity with reference to the orifice, in feet per second, v = the velocity of the ship in reference to the earth, then the thrust of the jet (see Screw-propeller, *ante*) is $2AV(V-v)$. The work done on the vessel is $2AV(V-v)v$, and the work wasted on the rearward projection of the jet is $\frac{1}{2} \times 2AV(V-v)^2$. The efficiency is $\frac{2AV(V-v)v}{2AV(V-v)v + AV(V-v)^2} = \frac{2v}{V+v}$. This expression equals unity when $V = v$, that is, when the velocity of the jet with reference to the earth, or $V - v, = 0$; but then the thrust of the propeller is also 0. The greater the value of V as compared with v, the less the efficiency. For $V = 20v$, as was proposed in the "Evolution," the efficiency of the jet would be less than 10 per cent, and this would be further reduced by the friction of the pumping mechanism and of the water in pipes.

The whole theory of propulsion may be summed up in Rankine's words: "That propeller is the best, other things being equal, which drives astern the largest body of water at the lowest velocity."

It is practically impossible to devise any system of hydraulic or jet propulsion which can compare favorably, under these conditions, with the screw or the paddle-wheel.

Reaction of a Jet.—If a jet of water issues horizontally from a vessel, the reaction on the side of the vessel opposite the orifice is equal to the weight of a column of water the section of which is the area of the orifice, and the height is twice the head.

The propelling force in jet-propulsion is the reaction of the stream issuing from the orifice, and it is the same whether the jet is discharged under water, in the open air, or against a solid wall. For proof, see account of trials by C. J. Everett, Jr., given by Prof. J. Burkitt Webb, Trans. A. S. M. E., xii. 904.

RECENT PRACTICE IN MARINE ENGINES.

(From a paper by A. Blechynden on Marine Engineering during the past Decade, Proc. Inst. M. E., July, 1891.)

Since 1881 the three-stage-expansion engine has become the rule, and the boiler-pressure has been increased to 160 lbs. and even as high as 200 lbs. per square inch. Four-stage-expansion engines of various forms have also been adopted.

Forced Draught has become the rule in all vessels for naval service, and is comparatively common in both passenger and cargo vessels. By this means it is possible considerably to augment the power obtained from a given boiler; and so long as it is kept within certain limits it need result in no injury to the boiler, but when pushed too far the increase is sometimes purchased at considerable cost.

In regard to the economy of forced draught, an examination of the appended table (page 1018) will show that while the mean consumption of coal in those steamers working under natural draught is 1.578 lbs. per indicated horse-power per hour, it is only 1.336 lbs. in those fitted with forced draught. This is equivalent to an economy of 15%. Part of this economy, however, may be due to the other heat-saving appliances with which the latter steamers are fitted.

Boilers.—As a material for boilers, iron is now a thing of the past, though it seems probable that it will continue yet awhile to be the material for tubes. Steel plates can be procured at 132 square feet superficial area and 1¼ inches thick. For purely boiler work a punching-machine has become obsolete in marine-engine work.

The increased pressures of steam have also caused attention to be directed to the furnace, and have led to the adoption of various artifices in the shape of corrugated, ribbed, and spiral flues, with the object of giving increased strength against collapse without abnormally increasing the thickness of the plate. A thick furnace-plate is viewed by many engineers with great suspicion; and the advisers of the Board of Trade have fixed the limit of thickness for furnace-plates at ⅝ inch; but whether this limitation will stand in the light of prolonged experience remains to be seen. It is a fact generally accepted that the conditions of the surfaces of a plate are far greater factors in its resistance to the transmission of heat than either the material or the thickness. With a plate free from lamination, thickness being a mere secondary element, it would appear that a furnace-plate might be increased from ½ inch to ¾ inch thickness without increasing its resistance more than 1¼%. So convinced have some engineers become of the soundness of this view that they have adopted flues ¾ inch thick.

Piston-valves.—Since higher steam-pressures have become common, piston-valves have become the rule for the high-pressure cylinder, and are not unusual for the intermediate. When well designed they have the great advantage of being almost free from friction, so far as the valve itself is concerned. In the earlier piston-valves it was customary to fit spring rings, which were a frequent source of trouble and absorbed a large amount of power in friction; but in recent practice it has become usual to fit springless adjustable sleeves.

For low-pressure cylinders piston-valves are not in favor; if fitted with spring rings their friction is about as great as and occasionally greater than that of a well-balanced slide-valve; while if fitted with springless rings there is always some leakage, which is irrecoverable. But the large port-clearances inseparable from the use of piston-valves are most objectionable; and with triple engines this is especially so, because with the customary late cut-off it becomes difficult to compress sufficiently for insuring economy and smoothness of working when in "full gear," without some special device.

Steam-pipes.—The failures of copper steam-pipes on large vessels have drawn serious attention both to the material and the modes of construction of the pipes. As the brazed joint is liable to be imperfect, it is proposed to substitute solid drawn tubes, but as these are not made of large sizes two or more tubes may be needed to take the place of one brazed tube. Reinforcing the ordinary brazed tubes by serving them with steel or copper wire, or by hooping them at intervals with steel or iron bands, has been tried and found to answer perfectly.

Auxiliary Supply of Fresh Water—Evaporators.—To make up the losses of water due to escape of steam from safety-valves, leakage at glands, joints, etc., either a reserve supply of fresh water is carried in tanks, or the supplementary feed is distilled from sea-water by special apparatus provided for the purpose. In practice the distillation is effected by passing steam, say from the first receiver, through a nest of tubes inside a still or evaporator, of which the steam-space is connected either with the second receiver or with the condenser. The temperature of the steam inside the tubes being higher than that of the steam either in the second receiver or in the condenser, the result is that the water inside the still is evaporated, and passes with the rest of the steam into the condenser, where it is condensed and serves to make up the loss. This plan localizes the trouble of the deposit, and frees it from its dangerous character, because an evaporator cannot become overheated like a boiler, even though it be neglected until it salts up solid; and if the same precautions are taken in working the evaporator which used to be adopted with low-pressure boilers when they were fed with salt water, no serious trouble should result.

Weir's Feed-water Heater.—The principle of a method of heating feed-water introduced by Mr. James Weir and widely adopted in the marine service is founded on the fact that, if the feed-water as it is drawn from the hot-well be raised in temperature by the heat of a portion of steam introduced into it from one of the steam-receivers, the decrease of the coal necessary to generate steam from the water of the higher temperature bears a greater ratio to the coal required without feed-heating than the power which would be developed in the cylinder by that portion of steam would bear to the whole power developed when passing all the steam through all the cylinders. Suppose a triple-expansion engine were working under the following conditions without feed-heating: boiler-pressure 150 lbs.; I.H.P. in high-pressure cylinder 398, in intermediate and low-pressure cylinders together 790, total 1188. The temperature of hot-well 100° F. Then with feed-heating the same engine might work as follows: the feed might be heated to 220° F., and the percentage of steam from the first receiver required to heat it would be 10.9%; the I.H.P. in the h.p. cylinder would be as before 398, and in the three cylinders it would be 1103, or 93% of the power developed without feed-heating. Meanwhile the heat to be added to each pound of the feed-water at 220° F. for converting it into steam would be 1005 units against 1125 units with feed at 100° F., equivalent to an expenditure of only 89.4% of the heat required without feed-heating. Hence the expenditure of heat in relation to power would be 89.4 ÷ 93.0 = 96.4%, equivalent to a heat economy of 3.6%. If the steam for heating can be taken from the low-pressure receiver, the economy is about doubled.

Passenger Steamers fitted with Twin Screws.

Vessels.	Length between Perpendiculars.	Beam.	Cylinders, two sets in all. Diameters.	Stro.	Boiler-pressure per sq. in.	Indicated Horse-power
	Feet	Feet	Inches	In.	Lbs.	I.H.P.
City of New York / " " Paris	525	63¼	45, 71, 113	60	150	20,000
Majestic / Teutonic	565	58	43, 68, 110	60	180	18,000
Normannia	500	57½	40, 67, 106	66	160	11,500
Columbia	463½	55½	41, 66, 101	66	160	12,500
Empress of India / " " Japan / " " China	440	51	32, 51, 82	54	160	10,125
Orel	415	48	34, 54, 85	51	160	10,000
Scot	460	54½	34½, 57½, 92	60	170	11,656

Comparative Results of Working of Marine Engines, 1872, 1881, and 1891.

Boilers, Engines, and Coal.	1872.	1881.	1891.
Boiler-pressure, lbs. per sq. in	52.4	77.4	158.5
Heating-surface per horse-power, sq. ft.	4.410	3.917	3.275
Revolutions per minute, revs.	55.67	59.76	63.75
Piston-speed, feet per min	376	467	529
Coal per horse-power per hour, lbs	2.110	1.828	1.522

Weight of Three-stage-expansion Engines in Nine Steamers in Relation to Indicated Horse-power and to Cylinder-capacity.

No. of Steamer.	Weight of Machinery. Engine-room.	Boiler-room.	Total.	Relative Weight of Machinery. Per Indicated Horse-power. Engine-room.	Boiler-room.	Total	Engine-room per cu. ft. of Cylinder-capacity.	Boiler-room per 100 sq. ft. of Heating-surface.	Type of Machinery.
	tons.	tons.	tons.	lbs.	lbs.	lbs.	tons.	tons.	
1	681	662	1343	226	220	446	1.30	3.75	Mercantile
2	638	619	1257	259	251	510	1.46	4.10	"
3	134	128	262	207	198	405	1.23	3.23	"
4	38.8	46.2	85	170	203	373	1.29	3.30	"
5	719	695	1414	167	162	329	1.41	3.44	"
6	75.2	107.8	183	141	202	343	1.37	3.37	"
7	44	61	105	77	108	185	1.21	2.72	Naval horizontal
8	73.5	109	182.5	78	116	194	1.11	2.78	do.
9	262	429	691	62.5	102	165	0.82	2.70	Naval vertical

Particulars of Three-stage-expansion Engines in Twenty-eight Steamers. (A. Blechynden.)

No.	Cylinders. Diameter.			Cylinders. Stroke.	Condenser. Cooling Surface.	Propeller. Diameter.		Propeller. Pitch.		Boilers. Heating-surface.	Boilers. Fire-grate Area.	Boilers. Steam-pressure.	Results of Trials. Revolutions per minute.	Piston-speed, Ft. per min.	Indicated H.P.	Heating-surface. Per I.H.P.	Heating-surface. Per lb. of Coal per hour.	I.H.P. per sq. ft. of grate.	Coal burnt per sq. ft. of grate per hour.	Coal burnt per I.H.P. per hour.	Remarks.
	ins.			ins.	sq. ft.	ft.	in.	ft.	in.	sq. ft.	sq. ft.	lbs.	revs.	ft.	I.H.P.	sq. ft.	sq. ft.	I.H.P.	lbs.	lbs.	
1	40	66	100	72	11,586	22	0	28	6	17,640	626	155	52.2	627	4295	4.11	2.46	6.86	11.45	1.67	H
2	40	66	100	72	11,586	22	0	28	6	17,640	626	155	51.3	616	4402	4.04	2.55	7.03	11.14	1.584	H
3	39	61	97	66	11,000	20	10	26	0	15,107	540	155	57.3	630	3587	4.21	2.22	6.65	13.60	1.896	H
4	39	61	97	66	11,000	20	10	26	0	15,107	540	155	57.4	631	3822	3.95	2.14	7.08	13.02	1.841	H
5	23	38	61	42	2,008	16	0	17	6	3,972	133	160	61	427	1120	3.54	2.02	8.43	14.75	1.75	
6	25½	42	70	51	3,209	16	6	20	0	6,162	193	180	61.3	521	1700	3.62	2.40	8.82	13.25	1.505	
7	21	34	55½	36	1,447	14	0	17	6	3,350	99	160	64	384	900	3.72	2.31	9.09	14.67	1.612	H
8	22	35	59	39	1,430	15	6	15	6	3,324	102	160	70	455	1065	3.12	2.38	10.42	13.70	1.312	
9	29	45	74	54	3,900	19	6	20	0	6,875	240	160	56	504	2250	3.055	2.04	9.38	14.00	1.494	H
10	31	48	82	54	4,150	19	0	19	0	8,000	260	160	61.5	553	2600	3.075	2.04	10.00	15.10	1.505	H
11	25	41	67	48	2,800	.....	.....	.....	.....	4,645	142	160	58	464	1300	3.57	2.26	9.16	14.46	1.580	
12	21½	36	59	42	2,000	15	0	16	6	3,852	122	160	67	469	1100	3.50	2.29	9.02	13.79	1.529	
13	32	51	82	54	12,562	16	6	23	0	20,192	710	160	58.5	526	3670	5.50	3.64	5.17	7.80	1.510	H
14	27	44	71	48	2,800	17	9	17	6	6,164	220	150	63	504	1680	3.67	2.12	7.65	13.18	1.723	H
15	29	45	74	60	4,020	19	0	24	0	6,950	196	150	53.8	538	2360	2.94	1.78	12.03	19.85	1.650	
16	29	45	74	54	3,850	18	0	21	0	6,960	216	160	64	576	2550	2.73	1.82	11.80	17.70	1.500	
17	23	37	64	48	2,400	16	6	18	0	4,715	144	180	62	496	1500	3.14	2.00	10.40	16.31	1.568	
18	28	44	74	51	3,700	17	9	22	9	8,000	2 4	150	62	527	1727	4.63	2.85	10.53	17.06	1.620	
19	23	36½	58	36	2,218	15	0	15	6	3,271	126	160	76	456	1269	2.58	1.84	10.07	14.10	1.400	
20	17.17	38	60	42	2,900	15	6	15	6	4,400	168	150	75	525	1530	2.875	1.96	9.11	13.32	1.464	
21	25	39	62	36	2,700	14	0	16	3	4,000	150	160	73	[illegible]	1250	3.20	2.40	8.35	11.13	1.330	
22	31	46	72	51	3,713	16	3	22	6	5,076	110	150	72	612	2513	2.02	1.34	22.85	33.95	1.488	D H
23	22½	35½	58½	39	1,750	14	7	16	6	2,338	50	160	76	494	1350	1.73	1.28	27.00	36.42	1.350	D H
24	25	42	68½	48	2,763	16	10	17	9	4,346	84	160	65	520	1800	2.41	1.94	21.42	26.62	1.242	D H
25	22¼	35¾	58½	48	3,530	15	6	18	0	3,486	63	160	69.5	552	1360	2.56	1.91	21.59	28.90	1.338	D H
26	31	50	83	60	6,860	19	0	23	0	6,438	154	150	59	590	2600	2.435	1.78	16.88	23.05	1.365	D H
27	32	53	87½	60	7,500	19	0	23	9	8,751	210	160	66	660	3400	2.52	2.04	16.18	19.97	1.234	D H
28	23½	46	75	42	3,450	16	0	21	0	6,618	188	160	73	511	2058	3.215	2.05	17.10	10.92	1.565	

	Per I.H.P.	Per lb. of Coal per hour.	I.H.P. per sq. ft. of grate.	Coal burnt per sq. ft. of grate per hour.	Coal burnt per I.H.P. per hour.
Average of all twenty-eight	3.275	2.14	11.22	17.08	1.522
" " natural draught	3.560	2.25	8.91	13.92	1.573
" " forced draught	2.412	1.72	20.98	28.15	1.336

REMARKS.—D, forced draught; H, feed heater

Dimensions, Indicated Horse-power, and Cylinder-capacity of Three-stage-expansion Engines in Nine Steamers.

Number of Steamer.	Single or Twin Screws.	Cylinders. Diameters.			Cylinders. Stroke	Revolutions per minute.	Boiler-pressure per sq. in.	Indicated Horse-power.	Cylinder-capacity.	Heating-surface. Total.	Heating-surface. Per I.H.P.
			ins.		ins.	revs.	lbs.	I.H.P.	cu. ft.	sq. ft.	sq. ft.
1	Single	40	66	100	72	64.5	160	6751	522	17,640	2.62
2	"	39	61	97	66	67.8	160	5525	436	15,107	2.73
3	"	23	38	61	42	83	160	1450	109	3,973	2.73
4	"	17	26½	42	24	90	150	510	30	1,403	2.75
5	Twin	32	54	82	54	88	160	9625	508	20,193	2.10
6	"	15	24	38	27	113	150	1194	55	3,200	2.68
7	Single	20	30	45	24	191	145	1265	36.3	2,227	1.76
8	Twin	18½	29	43	24	182.5	140	2105	66.2	3,928	1.87
9	"	33½	49	74	39	145	150	9400	319	15,882	1.62

CONSTRUCTION OF BUILDINGS.*

(Extract from the Building Laws of the City of New York, 1893.)

Walls of Warehouses, Stores, Factories, and Stables.—25 feet or less in width between walls, not less than 12 in. to height of 40 ft.;

If 40 to 60 ft. in height, not less than 16 in. to 40 ft., and 12 in. thence to top;
60 to 80 " " " " " 20 " 25 " 16 " "
75 to 85 " " " " " 24 " 20 ft.; 20 in. to 60 ft., and 16 in. to top;
85 to 100 ft. in height, not less than 28 in. to 25 ft.; 24 in. to 50 ft.; 20 in' to 75 ft., and 16 in. to top;
Over 100 ft. in height, each additional 25 ft. in height, or part thereof, next above the curb, shall be increased 4 inches in thickness, the upper 100 feet remaining the same as specified for a wall of that weight.

If walls are over 25 feet apart, the bearing-walls shall be 4 inches thicker than above specified for every 12½ feet or fraction thereof that said walls are more than 25 feet apart.

Strength of Floors, Roofs, and Supports.

	Floors calculated to bear safely per sq. ft., in addition to their own weight.
Floors of dwelling, tenement, apartment-house or hotel, not less than	70 lbs.
Floors of office-building, not less than	100 "
" public-assembly building, not less than	120 "
" store, factory, warehouse, etc., not less than	150 "
Roofs of all buildings, not less than	50 "

Every floor shall be of sufficient strength to bear safely the weight to be imposed thereon, in addition to the weight of the materials of which the floor is composed.

Columns and Posts.—The strength of all columns and posts shall be computed according to Gordon's formulæ, and the crushing weights in pounds, to the square inch of section, for the following-named materials, shall be taken as the coefficients in said formulæ, namely: Cast iron, 80,000;

* The limitations of space forbid any extended treatment of this subject. Much valuable information upon it will be found in Trautwine's Civil Engineer's Pocket-book, and in Kidder's Architect's and Builder's Pocket-book. The latter in its preface mentions the following works of reference: "Notes on Building Construction," 3 vols., Rivingtons, publishers, Boston; "Building Superintendence," by T. M. Clark (J. R. Osgood & Co., Boston.); "The American House Carpenter," by R. G. Hatfield; "Graphical Analysis of Roof-trusses," by Prof. C. E. Greene; "The Fire Protection of Mills," by C. J. H. Woodbury; "House Drainage and Water Service," by James C. Bayles; "The Builder's Guide and Estimator's Price-book," and "Plastering Mortars and Cements," by Fred. T. Hodgson; "Foundations and Concrete Works," and "Art of Building," by E. Dobson. Weale's Series, London.
J. H. Woodbury; "House Drainage and Water Service," by James C, Bayles; "The Builder's Guide and Estimator's Price-book," and "Plastering Mortars and Cements," by Fred. T. Hodgson; "Foundations and Con-

wrought or rolled iron, 40,000; rolled steel, 48,000; white pine and spruce, 3500; pitch or Georgia pine, 5000; American oak, 6000. The breaking strength of wooden beams and girders shall be computed according to the formulæ in which the constants for transverse strains for central load shall be as follows, namely: Hemlock, 400; white pine, 450; spruce, 450; pitch or Georgia pine, 550; American oak, 550; and for wooden beams and girders carrying a uniformly distributed load the constants will be doubled. The factors of safety shall be as one to four for all beams, girders, and other pieces subject to a transverse strain; as one to four for all posts, columns, and other vertical supports when of wrought iron or rolled steel; as one to five for other materials, subject to a compressive strain; as one to six for tie-rods, tie-beams, and other pieces subject to a tensile strain. Good, solid, natural earth shall be deemed to safely sustain a load of four tons to the superficial foot, or as otherwise determined by the superintendent of buildings, and the width of footing-courses shall be at least sufficient to meet this requirement. In computing the width of walls, a cubic foot of brickwork shall be deemed to weigh 115 lbs. Sandstone, white marble, granite, and other kinds of building-stone shall deemed to weigh 160 lbs. per cubic foot. The safe-bearing load to apply to good brickwork shall be taken at 8 tons per superficial foot when good lime mortar is used, 11½ tons per superficial foot when good lime and cement mortar mixed is used, and 15 tons per superficial foot when good cement mortar is used.

Fire-proof Buildings—Iron and Steel Columns.—All cast-iron, wrought-iron, or rolled-steel columns shall be made true and smooth at both ends, and shall rest on iron or steel bed-plates, and have iron or steel cap-plates, which shall also be made true. All iron or steel trimmer-beams, headers, and tail-beams shall be suitably framed and connected together, and the iron girders, columns, beams, trusses, and all other ironwork of all floors and roofs shall be strapped, bolted, anchored, and connected together, and to the walls, in a strong and substantial manner. Where beams are framed into headers, the angle-irons, which are bolted to the tail-beams, shall have at least two bolts for all beams over 7 inches in depth, and three bolts for all beams 12 inches and over in depth, and these bolts shall not be less than ¾ inch in diameter. Each one of such angles or knees, when bolted to girders, shall have the same number of bolts as stated for the other leg. The angle-iron in no case shall be less in thickness than the header or trimmer to which it is bolted, and the width of angle in no case shall be less than one third the depth of beam, excepting that no angle-knee shall be less than 2½ inches wide, nor required to be more than 6 inches wide. All wrought-iron or rolled-steel beams 8 inches deep and under shall have bearings equal to their depth, if resting on a wall; 9 to 12 inch beams shall have a bearing of 10 inches, and all beams more than 12 inches in depth shall have bearings of not less than 12 inches if resting on a wall. Where beams rest on iron supports, and are properly tied to the same, no greater bearings shall be required than one third of the depth of the beams. Iron or steel floor-beams shall be so arranged as to spacing and length of beams that the load to be supported by them, together with the weights of the materials used in the construction of the said floors, shall not cause a deflection of the said beams of more than 1/30 of an inch per linear foot of span; and they shall be tied together at intervals of not more than eight times the depth of the beam.

Under the ends of all iron or steel beams, where they rest on the walls, a stone or cast iron template shall be built into the walls. Said template shall be 8 inches wide in 12-inch walls, and in all walls of greater thickness said template shall be 12 inches wide; and such templates, if of stone, shall not be in any case less than 2½ inches in thickness, and no template shall be less than 12 inches long.

No cast-iron post or column shall be used in any building of a less average thickness of shaft than three quarters of an inch, nor shall it have an unsupported length of more than twenty times its least lateral dimensions or diameter. No wrought-iron or rolled-steel column shall have an unsupported length of more than thirty times its least lateral dimension or diameter, nor shall its metal be less than one fourth of an inch in thickness.

Lintels, Bearings and Supports.—All iron or steel lintels shall have bearings proportionate to the weight to be imposed thereon, but no lintel used to span any opening more than 10 feet in width shall have a bearing less than 12 inches at each end, if resting on a wall; but if resting on an iron post, such lintel shall have a bearing of at least 6 inches at each end, by the thickness of the wall to be supported.

Strains on Girders and Rivets.—Rolled iron or steel beam girders, or riveted iron or steel plate girders used as lintels or as girders, carrying a wall or floor or both, shall be so proportioned that the loads which may come upon them shall not produce strains in tension or compression upon the flanges of more than 12,000 lbs. for iron, nor more than 15,000 lbs. for steel per square inch of the gross section of each of such flanges, nor a shearing strain upon the web-plate of more than 6000 lbs. per square inch of section of such web-plate, if of iron, nor more than 7000 pounds if of steel; but no web-plate shall be less than ¼ inch in thickness. Rivets in plate girders shall not be less than ⅝ inch in diameter, and shall not be spaced more than 6 inches apart in any case. They shall be so spaced that their shearing strains shall not exceed 9000 lbs. per square inch, on their diameter, multiplied by the thickness of the plates through which they pass. The riveted plate girders shall be proportioned upon the supposition that the bending or chord strains are resisted entirely by the upper and lower flanges, and that the shearing strains are resisted entirely by the web-plate. No part of the web shall be estimated as flange area, nor more than one half of that portion of the angle-iron which lies against the web. The distance between the centres of gravity of the flange areas will be considered as the effective depth of the girder.

The building laws of the City of New York contain a great amount of detail in addition to the extracts above, and penalties are provided for violation. See An Act creating a Department of Buildings, etc., Chapter 275, Laws of 1892. Pamphlet copy published by Baker, Voorhies & Co., New York.

MAXIMUM LOAD ON FLOORS.

(*Eng'g*, Nov. 18, 1892, p. 644.)—Maximum load per square foot of floor surface due to the weight of a dense crowd. Considerable variation is apparent in the figures given by many authorities, as the following table shows:

Authorities.	Weight of Crowd, lbs. per sq. ft.
French practice, quoted by Trautwine and Stoney	41
Hatfield ("Transverse Strains," p. 80)	70
Mr. Page, London, quoted by Trautwine........	84
Maximum load on American highway bridges according to Waddell's general specifications...	100
Mr. Nash, architect of Buckingham Palace..................	120
Experiments by Prof. W. N. Kernot, at Melbourne	126 / 143.1
Experiments by Mr. B. B. Stoney ("On Stresses," p. 617)....	147.4

The highest results were obtained by crowding a number of persons previously weighed into a small room, the men being tightly packed so as to resemble such a crowd as frequently occurs on the stairways and platforms of a theatre or other public building.

STRENGTH OF FLOORS.

(From circular of the Boston Manufacturers' Mutual Insurance Co.)

The following tables were prepared by C. J. H. Woodbury, for determining safe loads on floors. Care should be observed to select the figure giving the greatest possible amount and concentration of load as the one which may be put upon any beam or set of floor-beams; and in no case should beams be subjected to greater loads than those specified, unless a lower factor of safety is warranted under the advice of a competent engineer.

Whenever and wherever solid beams or heavy timbers are made use of in the construction of a factory or warehouse, they should not be painted, varnished or oiled, filled or encased in impervious concrete, air-proof plastering, or metal for at least three years, lest fermentation should destroy them by what is called "dry rot."

It is, on the whole, safer to make floor-beams in two parts, with a small open space between, so that proper ventilation may be secured, even if the outside should be inadvertently painted or filled.

These tables apply to distributed loads, but the first can be used in respect to floors which may carry concentrated loads by using half the figure given in the table, since a beam will bear twice as much load when evenly distributed over its length as it would if the load was concentrated in the centre of the span.

The weight of the floor should be deducted from the figure given in the table, in order to ascertain the net load which may be placed upon any floor. The weight of spruce may be taken at 36 lbs. per cubic foot, and that of southern pine at 48 lbs. per cubic foot.

Table I was computed upon a working modulus of rupture of Southern pine at 2160 lbs., using a factor of safety of six. It can also be applied to ascertaining the strength of spruce beams if the figures given in the table are multiplied by 0.78; or in designing a floor to be sustained by spruce beams, multiply the required load by 1.28, and use the dimensions as given by the table.

Theses tables are computed for beams one inch in width, because the strength of beams increases directly as the width when the beams are broad enough not to cripple.

EXAMPLE.—Required the safe load per square foot of floor, which may be safely sustained by a floor on Southern pine 10 × 14 inch beams, 8 feet on centres, and 20 feet span. In Table I a 1 × 14 inch beam, 20 feet span, will sustain 118 lbs. per foot of span; and for a beam 10 inches wide the load would be 1180 lbs. per foot of span, or 147½ lbs. per square foot of floor for Southern-pine beams. From this should be deducted the weight of the floor, which would amount to 17½ lbs. per square foot, leaving 130 lbs. per square foot as a safe load to be carried upon such a floor. If the beams are of spruce, the result of 147½ lbs. would be multiplied by 0.78, reducing the load to 115 lbs. The weight of the floor, in this instance amounting to 16 lbs., would leave the safe net load as 90 lbs. per square foot for spruce beams.

Table II applies to the design of floors whose strength must be in excess of that necessary to sustain the weight, in order to meet the conditions of delicate or rapidly moving machinery, to the end that the vibration or distortion of the floor may be reduced to the least practicable limit.

In the table the limit is that of load which would cause a bending of the beams to a curve of which the average radius would be 1250 feet.

This table is based upon a modulus of elasticity obtained from observations upon the deflection of loaded storehouse floors, and is taken at 2,000,000 lbs. for Southern pine; the same table can be applied to spruce, whose modulus of elasticity is taken as 1,200,000 lbs., if six tenths of the load for Southern pine is taken as the proper load for spruce; or, in the matter of designing, the load should be increased one and two thirds times, and the dimension of timbers for this increased load as found in the table should be used for spruce.

It can also be applied to beams and floor-timbers which are supported at each end and in the middle, remembering that the deflection of a beam supported in that manner is only four tenths that of a beam of equal span which rests at each end; that is to say, the floor-planks are two and one half times as stiff, cut two bays in length, as they would be if cut only one bay in length. When a floor-plank two bays in length is evenly loaded, three sixteenths of the load on the plank is sustained by the beam at each end of the plank, and ten sixteenths by the beam under the middle of the plank; so that for a completed floor three eighths of the load would be sustained by the beams under the joints of the plank, and five eighths of the load by the beams under the middle of the plank: this is the reason of the importance of breaking joints in a floor-plank every three feet in order that each beam shall receive an identical load. If it were not so, three eighths of the whole load upon the floor would be sustained by every other beam, and five eighths of the load by the corresponding alternate beams.

Repeating the former example for the load on a mill floor on Southern-pine beams 10 × 14 inches, and 20 feet span, laid 8 feet on centres: In Table II a 1 × 14 inch beam should receive 61 lbs. per foot of span, or 75 lbs. per sq. ft. of floor, for Southern-pine beams. Deducting the weight of the floor, 17½ lbs. per sq. ft., leaves 57 lbs. per sq. ft. as the advisable load.

If the beams are of spruce, the result of 75 lbs. should be multiplied by 0.6, reducing the load to 45 lbs. The weight of the floor, in this instance amounting to 16 lbs., would leave the net load as 29 lbs. for spruce beams.

If the beams were two spans in length, they could, under these conditions, support two and a half times as much load with an equal amount of deflection, unless such load should exceed the limit of safe load as found by Table I, as would be the case under the conditions of this problem.

Mill Columns.—Timber posts offer more resistance to fire than iron pillars, and have generally displaced them in millwork. Experiments made on the testing-machine at the U. S. Arsenal at Watertown, Mass., show that sound timber posts of the proportions customarily used in millwork yield by direct crushing, the strength being directly as the area at the smallest part. The columns yielded at about 4500 lbs. per square inch, confirming the general practice of allowing 600 lbs. per square inch, as a safe load. Square columns are one fourth stronger than round ones of the same diameter.

I. Safe Distributed Loads upon Southern-pine Beams One Inch in Width.

(C. J. H. Woodbury.)

(If the load is concentrated at the centre of the span, the beams will sustain half the amount as given in the table.)

Span, feet.	Depth of Beam in inches.														
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	Load in pounds per foot of Span.														
5	38	86	154	240	346	470	614	778	960						
6	27	60	107	167	240	327	427	540	667	807					
7	20	44	78	122	176	240	314	397	490	593	705	828			
8	15	34	60	94	135	184	240	304	375	454	540	634	735		
9	...	27	47	74	107	145	190	240	296	359	427	501	581	667	759
10	...	22	38	60	86	118	154	194	240	290	346	406	470	540	614
11	...	...	32	50	71	97	127	161	198	240	286	335	389	446	508
12	...	...	27	42	60	82	107	135	167	202	240	282	327	375	474
13	...	...	...	36	51	70	90	115	142	172	205	240	278	320	364
14	...	...	...	31	44	60	78	99	123	148	176	207	240	276	314
15	...	...	...	27	38	52	68	86	107	129	154	180	209	240	273
16	...	...	...	...	34	46	60	76	94	113	135	158	184	211	240
17	...	...	...	...	30	41	53	67	83	101	120	140	163	187	217
18	...	...	...	...	...	36	47	60	74	90	107	125	145	167	190
19	...	...	...	...	...	...	43	54	66	80	96	112	130	150	170
20	...	...	...	...	...	...	38	49	60	73	86	101	118	135	154
21	...	...	...	...	...	...	...	44	54	66	78	92	107	122	139
22	...	...	...	...	...	...	...	...	50	60	71	84	97	112	127
23	...	...	...	...	...	...	...	...	45	55	65	77	89	102	116
24	...	...	...	...	...	...	...	...	...	50	60	70	82	94	107
25	...	...	...	...	...	...	...	...	...	46	55	65	75	86	98

II. Distributed Loads upon Southern-pine Beams sufficient to produce Standard Limit of Deflection.

(C. J. H. Woodbury.)

Span, feet.	Depth of Beam in inches.															Deflection, inches.
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
	Load in pounds per foot of Span.															
5	3	10	23	44	77	122	182	259								.0300
6	2	7	16	31	53	85	126	180	247							.0432
7	...	5	12	23	39	62	93	132	181	241						.0588
8	...	4	9	17	30	48	71	101	139	185	240	305				.0768
9	...	...	7	14	24	38	56	80	110	146	190	241	301			.0972
10	...	...	6	11	19	30	46	65	89	118	154	195	244	300		.1200
11	...	...	...	9	16	25	38	54	73	98	127	161	202	248	301	.1452
12	...	...	...	...	13	21	32	45	62	82	107	136	169	208	253	.1728
13	...	...	...	...	11	18	27	38	53	70	91	116	144	178	215	.2028
14	...	...	...	...	...	16	23	33	45	60	78	100	124	153	186	.2352
15	...	...	...	...	...	14	20	29	40	53	68	87	108	133	162	.2700
16	...	...	...	...	...	...	18	25	35	46	60	76	95	117	147	.3072
17	...	...	...	...	...	...	16	22	31	41	53	68	84	104	126	.3468
18	...	...	...	...	...	...	...	20	27	37	47	60	75	93	112	.3888
19	...	...	...	...	...	...	...	18	25	33	43	54	68	83	101	.4332
20	...	...	...	...	...	...	...	...	22	30	38	49	61	75	91	.4800
21	...	...	...	...	...	...	...	...	20	27	35	44	55	68	83	.5292
22	...	...	...	...	...	...	...	...	...	24	32	40	50	62	75	.5808
23	...	...	...	...	...	...	...	...	...	22	29	37	46	57	69	.6348
24	...	...	...	...	...	...	...	...	...	...	27	34	42	52	63	.6912
25	...	...	...	...	...	...	...	...	...		25	31	39	48	58	.7500

ELECTRICAL ENGINEERING.

STANDARDS OF MEASUREMENT.

C.G.S. (Centimetre, Gramme, Second) or "Absolute" System of Physical Measurements:

Unit of space or distance = 1 centimetre, cm.;
Unit of mass = 1 gramme, gm.;
Unit of time = 1 second, s.;
Unit of velocity = space ÷ time = 1 centimetre in 1 second;
Unit of acceleration = change of 1 unit of velocity in 1 second;
Acceleration due to gravity, at Paris, = 981 centimetres in 1 second;

$$\text{Unit of force} = 1 \text{ dyne} = \frac{1}{981} \text{ gramme} = \frac{.0022046}{981} \text{ lb.} = .000002247 \text{ lb.}$$

A dyne is that force which, acting on a mass of one gramme during one second, will give it a velocity of one centimetre per second. The weight of one gramme in latitude 40° to 45° is about 980 dynes, at the equator 973 dynes, and at the poles nearly 984 dynes. Taking the value of g, the acceleration due to gravity, in British measures at 32.185 feet per second at Paris, and the metre = 39.37 inches, we have

$$1 \text{ gramme} = 32.185 \times 12 \div .3937 = 981.00 \text{ dynes.}$$

Unit of work = 1 erg = 1 dyne-centimetre = .00000007373 foot-pound;
Unit of power = 1 watt = 10 million ergs per second,
= .7373 foot-pound per second,

$$= \frac{.7373}{550} = \frac{1}{746} \text{ of 1 horse-power} = .00134 \text{ H.P.}$$

C.G.S. Unit of magnetism = the quantity which attracts or repels an equal quantity at a centimetre's distance with the force of 1 dyne.

C.G.S. Unit of electrical current = the current which, flowing through a length of 1 centimetre of wire, acts with a force of 1 dyne upon a unit of magnetism distant 1 centimetre from every point of the wire. The ampere, the commercial unit of current, is one tenth of the C.G.S. unit.

The Practical Units used in Electrical Calculations are:

Ampere, the unit of current strength, or rate of flow, represented by C.

Volt, the unit of electro-motive force, electrical pressure, or difference of potential, represented by E.

Ohm, the unit of resistance, represented by R.

Coulomb (or ampere-second), the unit of quantity, Q.

Ampere-hour = 3600 coulombs, Q'.

Watt (ampere-volt, or volt-ampere), the unit of power, P.

Joule (volt-coulomb), the unit of energy or work, W.

Farad, the unit of capacity, represented by K.

Henry, the unit of induction, represented by L.

Using letters to represent the units, the relations between them may be expressed by the following formulæ, in which t represents one second and T one hour:

$$C = \frac{E}{R}, \quad Q = Ct, \quad Q' = CT, \quad K = \frac{Q}{E}, \quad W = QE, \quad P = CE.$$

As these relations contain no coefficient other than unity, the letters may represent any quantities given in terms of those units. For example, if E represents the number of volts electro-motive force, and R the number of ohms resistance in a circuit, then their ratio $E \div R$ will give the number of amperes current strength in that circuit.

The above six formulæ can be combined by substitution or elimination, so as to give the relations between any of the quantities. The most important of these are the following:

$$Q = \frac{E}{R}t, \quad K = \frac{C}{E}t, \quad W = CEt = \frac{E^2}{R}t = C^2Rt = Pt,$$

$$P = \frac{E^2}{R} = C^2R = \frac{W}{t} = \frac{QE}{t}.$$

The definitions of these units as adopted at the International Electrical Congress at Chicago in 1893, and as established by Act of Congress of the United States, July 12, 1894, are as follows:

The *ohm* is substantially equal to 10^9 (or 1,000,000,000) units of resistance of the C.G.S. system, and is represented by the resistance offered to an unvarying electric current by a column of mercury at 32° F., 14.4521 grammes in mass, of a constant cross-sectional area, and of the length of 106.3 centimetres.

The *ampere* is 1/10 of the unit of current of the C.G.S. system, and is the practical equivalent of the unvarying current which when passed through a solution of nitrate of silver in water in accordance with standard specifications deposits silver at the rate of .001118 gramme per second.

The *volt* is the electro-motive force that, steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere, and is practically equivalent to 1000/1434 (or .6974) of the electro-motive force between the poles or electrodes of a Clark's cell at a temperature of 15° C., and prepared in the manner described in the standard specifications.

The *coulomb* is the quantity of electricity transferred by a current of one ampere in one second.

The *farad* is the capacity of a condenser charged to a potential of one volt by one coulomb of electricity.

The *joule* is equal to 10,000,000 units of work in the C.G.S. system, and is practically equivalent to the energy expended in one second by an ampere in an ohm.

The *watt* is equal to 10,000,000 units of power in the C.G.S. system, and is practically equivalent to the work done at the rate of one joule per second.

The *henry* is the induction in a circuit when the electro-motive force induced in this circuit is one volt, while the inducing current varies at the rate of one ampere per second.

The ohm, volt, etc., as above defined, are called the "international" ohm, volt, etc., to distinguish them from the "legal" ohm, B.A. unit, etc.

The value of the ohm, determined by a committee of the British Association in 1863, called the B.A. unit, was the resistance of a certain piece of copper wire preserved in London. The so-called "legal" ohm, as adopted at the International Congress of Electricians in Paris in 1884, was a correction of the B.A. unit, and was defined as the resistance of a column of mercury 1 square millimetre in section and 106 centimetres long, at a temperature of 32° F.

1 legal ohm	= 1.0112 B.A. units,	1 B.A. unit	= 0.9889 legal ohm;
1 international ohm	= 1.0136 " "	1 " "	= 0.9866 int. ohm;
1 " "	= 1.0023 legal ohm,	1 legal ohm	= 0.9977 " "

DERIVED UNITS.

1 megohm = 1 million ohms;
1 microhm = 1 millionth of an ohm;
1 milliampere = 1/1000 of an ampere;
1 micro-farad = 1 millionth of a farad.

RELATIONS OF VARIOUS UNITS.

1 ampere	= 1 coulomb per second;
1 volt-ampere	= 1 watt = 1 volt-coulomb per second;
1 watt	= .7378 foot-pound per second, = .0009477 heat-units per second (Fahr.), = 1/746 of one horse-power;
1 joule	= .7378 foot-pound, = work done by one watt in one second, = .0009477 heat-unit;
1 British thermal unit	= 1055.2 joules;
1 kilowatt, or 1000 watts	= 787.3 foot-pound per second, = .9477 heat-units per second, = 1000/746 or 1.3405 horse-powers;
1 kilowatt-hour, 1000 volt-ampere hours, 1 British Board of Trade unit,	= 1.3405 horse-power hours, = 2,654,200 foot-pounds, = 3412 heat-units;
1 horse-power	= 746 watts = 746 volt-amperes, = 33,000 foot-pounds per minute.

The ohm, ampere, and volt are defined in terms of one another as follows: Ohm, the resistance of a conductor through which a current of one ampere will pass when the electro-motive force is one volt. Ampere, the quantity

Equivalent Values of Electrical and Mechanical Units.

Unit.	Equivalent Value in Other Units.
1 K. W. Hour =	1,000 watt hours. 1.34 horse-power hours. 2,654,200 ft.-lbs. 3,600,000 joules. 3,412 heat-units. 367,000 kilogram metres. .235 lb. carbon oxidized with perfect efficiency. 3.53 lbs. water evap. from and at 212° F. 22.75 lbs. of water raised from 62° to 212° F.
1 H.P. Hour =	.746 K. W. hours. 1,980,000 ft.-lbs. 2,545 heat-units. 273,740 k. g. m. .175 lb. carbon oxidized with perfect efficiency. 2.64 lbs. water evaporated from and at 212° F. 17.0 lbs. water raised from 62° F. to 212° F.
1 Kilo-watt =	1,000 watts. 1.34 horse-power. 2,654,200 ft.-lbs. per hour. 44,240 ft.-lbs. per minute. 737.3 ft.-lbs. per second. 3,412 heat-units per hour. 56.9 heat-units per minute. .948 heat-unit per second. .2275 lb. carbon oxidized per hour. 3.53 lbs. water evap. per hour from and at 212° F.
1 H.P. =	746 watts. .746 K. W. 33,000 ft.-lbs. per minute. 550 ft.-lbs. per second. 2,545 heat-units per hour. 42.4 heat-units per minute. .707 heat-units per second. .175 lbs. carbon oxidized per hour. 2.64 lbs. water evap. per hour from and at 212° F.
1 Joule =	1 watt second. .000000278 K. W. hour. .102 k. g. m. .0009477 heat-units. .7373 ft.-lb.
1 Ft.-lb. =	1.356 joules. .1383 k. g. m. .000000377 K. W. hours. .001285 heat-units. .0000005 H.P. hour.
1 Watt =	1 joule per second. .00134 H.P. 3.412 heat-units per hour. .7373 ft.-lbs. per second. .0035 lbs. water evap. per hr. 44.24 ft.-lbs. per minute.
1 Watt per sq. in. =	8.19 heat-units per sq. ft. per minute. 6371 ft.-lbs. per sq. ft. per minute. .193 H.P. per sq. ft.
1 Heat-unit =	1,055 watt seconds. 778 ft.-lbs. 107.6 kilogram metres. .000293 K. W. hour. .000393 H.P. hour. .0000688 lbs. carbon oxidized. .001036 lbs. water evap. from and at 212° F.
1 Heat-unit per Sq. Ft. per min. =	.122 watts per square in. .0176 K. W. per sq. ft. .0236 H.P. per sq. ft.
1 Kilogram Metre =	7.233 ft.-lbs. .00000365 H.P. hour. .00000272 K. W. hour. .0093 heat-units.
1 lb. Carbon Oxidized with perfect Efficiency =	14,544 heat-units. 1.11 lb. Anth'cite coal ox. 2.5 lbs. dry wood oxidized. 21 cu. ft. illuminating-gas. 4.26 K. W. hours. 5.71 H.P. hours. 11,315,000 ft.-lbs. 15 lbs. of water evap. from and at 212° F.
1 lb. Water Evapor'ed from and at 212° F. =	.283 K. W. hour. .379 H.P. hour. 965.7 heat-units. 103,900 k. g. m. 1,019,000 joules. 751,300 ft.-lbs. .0664 lb. of carbon oxidized.

of current which will flow through a resistance of one ohm when the electro-motive force is one volt. Volt, the electro-motive force required to cause a current of one ampere to flow through a resistance of one ohm.

Units of the Magnetic Circuit.—(See Electro-magnets, page 1058.)

For Methods of making Electrical Measurements, Testing, etc., see Munroe & Jamieson's Pocket-Book of Electrical Rules, Tables, and Data; S. P. Thompson's Dynamo-Electric Machinery; and works on Electrical Engineering.

Equivalent Electrical and Mechanical Units.—H. Ward Leonard published in *The Electrical Engineer*, Feb. 25, 1895, a table of useful equivalents of electrical and mechanical units, from which the table on page 1026 is taken, with some modifications.

ANALOGIES BETWEEN THE FLOW OF WATER AND ELECTRICITY.

WATER.	ELECTRICITY.
Head, difference of level, in feet. Difference of pressure per sq. in., in lbs.	Volts; electro-motive force; difference of potential or of pressure; E. or E.M.F.
Resistance of pipes, apertures, etc., increases with length of pipe, with contractions, roughness, etc.; decreases with increase of sectional area. The law of increase and decrease is expressed by complex formulæ. See Flow of Water.	Ohms, resistance, R. The resistance increases directly as the length of the conductor or wire and inversely as its sectional area, $R \propto l \div s$. It varies with the nature or quality of the conductor. Conductivity is the reciprocal of specific resistance.
Rate of flow, as cubic ft. per second, gallons per minute, etc., or volume divided by the time. In the mining regions sometimes expressed in "miners' inches."	Amperes; current; current strength; intensity of current; rate of flow; 1 ampere = 1 coulomb per second. Amperes $= \frac{\text{volts}}{\text{ohms}}$; $C = \frac{E}{R}$; $E = CR$.
Quantity, usually measured in cubic feet or gallons, but is also equivalent to rate of flow × time, as cubic feet per second for so many hours.	Coulomb, unit of quantity, Q, = rate of flow × time, as ampere-seconds. 1 ampere-hour = 3600 coulombs.
Work, or energy, measured in foot-pounds; product of weight of falling water into height of fall; in pumping, product of quantity in cubic feet into the pressure in lbs. per square foot against which the water is pumped.	Joule, volt-coulomb, W, the unit of work, = product of quantity by the electro-motive force = volt-ampere-second. 1 joule = .7373 foot-pound. If C (amperes) = rate of flow, and E (volts) = difference of pressure between two points in a circuit, energy expended $= CEt$, $= C^2Rt$, since $E = CR$.
Power, rate of work. Horse-power, ft.-lbs. of work done in 1 min. ÷ 33,000. In falling water, pounds falling in one second ÷ 550. In water flowing in pipes, rate of flow in cubic feet per second × pressure resisting the flow in lbs. per sq. ft. ÷ 550.	Watt, unit of power, P, = volts × amperes, = current or rate of flow × difference of potential. 1 watt = .7373 foot-pound per second = 1/746 of a horse-power.

Analogy between the Ampere and the Miner's Inch. (T. O'Connor Sloane.)—The miner's inch is defined as the quantity of water which will flow through an aperture an inch square in a board two inches thick, under a head of water of six inches. Here, as in the case of the ampere, we have no reference to any abstract quantity, such as gallons or pounds. There is no reference to time. It is simply a rate of flow. We may consider the head of water, six inches, as the representative of electrical pressure; i.e., one volt. The aperture restricting the flow of water may be assumed to represent the resistance of one ohm; the flow through a resistance of one ohm under the pressure of one volt is one ampere; the flow through the resistance of a one-inch hole two inches long under the pressure of six inches to the upper edge of the opening is one miner's inch.

The miner's inch-second is the correct analogue of the ampere-second; the one denotes a specific quantity of water, 0.194 gallon; the other a specific quantity of electricity, a coulomb.

ELECTRICAL RESISTANCE.

Laws of Electrical Resistance.—The resistance, R, of any conductor varies directly as its length, l, and inversely as its sectional area, s, or $R \propto \frac{l}{s}$.

EXAMPLE.—If one foot of copper wire .01 in. diameter has a resistance of .10323 ohm, what will be the resistance of a mile of wire .3 in. diam. at the same temperature? The sectional areas being proportional to the squares of the diameters, the ratio of the areas is $.3^2 : .01^2 = 900$ to 1. The lengths are as 5280 to 1. The resistances being directly as the lengths and inversely as the sectional areas, the resistance of the second wire is $.10323 \times 5280 \div 900 = .6056$ ohm.

Conductance, c, is the inverse of resistance. $R = \frac{l}{sc}$, $c = \frac{l}{sR}$. If c and c_2 represent the conductances, and R and R_2 the respective resistance of two substances of the same length and section, then $c : c_2 : : R_2 : R$.

Equivalent Conductors.—With two conductors of length l, l_1, of conductances c, c_1, and sectional areas s, s_1, we have the same resistance, and one may be substituted for the other when $\frac{l}{cs} = \frac{l_1}{c_1 s_1}$.

The specific resistance, also called resistivity, a, of a material of unit length and section is its resistance as compared with the resistance of a standard conductor, such as pure copper. Conductivity, or specific conductance, is the reciprocal of resistivity.

$$R = \frac{l}{sc}, \quad R = \frac{al}{s}.$$

If two wires have lengths l, l_1, areas s, s_1, and specific resistances a, a_1, their actual resistances are $R = \frac{al}{s}$, $R_1 = \frac{a_1 l_1}{s_1}$, and $\frac{R}{R_1} = \frac{a l s_1}{a_1 l_1 s}$.

Electrical Conductivity of Different Metals and Alloys.—Lazare Weiler presented to the Société Internationale des Électriciens the results of his experiments upon the relative electrical conductivity of certain metals and alloys, as here appended:

1. Pure silver	100
2. Pure copper	100
3. Refined and crystallized copper	99.9
4. Telegraphic silicious bronze	98
5. Alloy of copper and silver (50%)	86.65
6. Pure gold	78
7. Silicide of copper, 4% Si	75
8. Silicide of copper, 12% Si	54.7
9. Pure aluminum	54.2*
10. Tin with 12% of sodium	46.9
11. Telephonic silicious bronze	35
12. Copper with 10% of lead	30
13. Pure zinc	29.9
14. Telephonic phosphor-bronze	29
15. Silicious brass, 25% zinc	26.49
16. Brass with 35% of zinc	21.5
17. Phosphor tin	17.7
18. Alloy of gold and silver (50%)	16.12
19. Swedish iron	16
20. Pure Banca tin	15.45
21. Antimonial copper	12.7
22. Aluminum bronze (10%)	12.6
23. Siemens steel	12
24. Pure platinum	10.6
25. Copper with 10% of nickel	10.6
26. Cadmium amalgam (15%)	10.2
27. Dronier mercurial bronze	10.14
28. Arsenical copper (10%)	9.1
29. Pure lead	8.88
30. Bronze with 20% of tin	8.4
31. Pure nickel	7.89
32. Phosphor-bronze, 10% tin	6.5
33. Phosphor-copper, 9% phos.	4.9
34. Antimony	3.8

The above comparative resistances may be reduced to ohms on the basis that a wire of soft copper one millimetre in diameter at a temperature of 0° C. has a resistance of .02029 international ohms per metre; or a wire .001 inch diam. has a resistance of 9.59 international ohms per foot.

* This figure is too low. J. W. Richards (*Jour. Frank. Inst.*, Mar. 1895) gives for hard-drawn aluminum of purity 98.5, 99.0, 99.5, and 99.75% respectively a conductivity of 55, 59, 61, and 63 to 64%, copper being 100%. The Pittsburg Reduction Co. claims that its purest aluminum has a conductivity of over 64.5%. (*Eng'g News*, Dec. 17, 1896.)

Relative Conductivities of Different Metals at 0° and 100° C. (Matthiessen.)

Metals.	Conductivities. At 0° C. " 32° F.	Conductivities. At 100° C. " 212° F.	Metals.	Conductivities. At 0° C. " 32° F.	Conductivities. At 100° C. " 212° F.
Silver, hard	100	71.56	Tin	12.36	8.67
Copper, hard	99.95	70.27	Lead	8.32	5.86
Gold, hard	77.96	55.90	Arsenic	4.76	3.33
Zinc, pressed	29.02	20.67	Antimony	4.62	3.26
Cadmium	23.72	16.77	Mercury, pure	1.60	....
Platinum, soft	18.00	....	Bismuth	1.245	0.878
Iron, soft	16.80	....			

Conductors and Insulators in Order of their Value.

Conductors.	Insulators (Non-conductors).	
All metals	Dry Air	Ebonite
Well-burned charcoal	Shellac	Gutta-percha
Plumbago	Paraffin	India-rubber
Acid solutions	Amber	Silk
Saline solutions	Resins	Dry Paper
Metallic ores	Sulphur	Parchment
Animal fluids	Wax	Dry Leather
Living vegetable substances	Jet	Porcelain
Moist earth	Glass	Oils
Water	Mica	

According to Culley, the resistance of distilled water is 6754 million times as great as that of copper.

Resistance Varies with Temperature.—For every degree Centigrade the resistance of copper increases about 0.4%, or for every degree F. 0.222%. Thus a piece of copper wire having a resistance of 10 ohms at 32° would have a resistance of 11.11 ohms at 82° F.

The following table shows the amount of resistance of a few substances used for various electrical purposes by which 1 ohm is increased by a rise of temperature 1° F., or 1° C.

Material.	Rise of R. of 1 Ohm when Heated— 1° F.	1° C.
Platinoid	.00013	.00021
Platinum-silver	.00018	.00031
German silver (see below)	.00024	.00044
Gold, silver	.00036	.00065
Cast iron	.00044	.00080
Copper	.00222	.00400

Annealing.—The degree of hardness or softness of a metal or alloy affects its resistance. Resistance is lessened by annealing. Matthiessen gives the following relative conductivities for copper and silver, the comparison being made with pure silver at 100° C.:

Metal.	Temp. C.	Hard.	Annealed.	Ratio.
Copper	11°	95.31	97.83	1 to 1.027
Silver	14.6°	95.36	103.33	1 to 1.084

Dr. Siemens compared the conductivities of copper, silver, and brass with pure mercury at 0° C., with the following results:

Metal.	Hard.	Annealed.	Ratio.
Copper	52.207	55.253	1 to 1.058
Silver	56.252	64.380	1 to 1.145
Brass	11.439	13.502	1 to 1.180

Edward Weston (Proc. Electrical Congress 1893, p. 179) says that the resistance of German silver depends on its composition. Matthiessen gives it as nearly 13 times that of copper, with a temperature coefficient of .0004433 per degree C. Weston, however, has found copper-nickel-zinc alloys (German silver) which had a resistance of nearly 28 times that of copper, and a temperature coefficient of about one half that given by Matthiessen. Kennelly and Fessenden (Proc. Elec. Cong., p. 186) find that copper has a uniform temperature coefficient of 0.406% per degree C., between the limits of 20° and 250° C.

Standard of Resistance of Copper Wire. (Trans. A. I. E. E., Sept. and Nov. 1890.)—Matthiessen's standard is: A hard-drawn copper wire 1 metre long, weighing 1 gramme has a resistance of 0.1469 B.A. unit at 0° C. (1 B.A. unit = 0.9889 legal ohm = 0.9866 international ohm.) Resistance of hard copper = 1.0226 times that of soft copper. Relative conducting power (Matthiessen): silver, 100; hard or unannealed copper, 99.95; soft or annealed copper, 102.21. Conductivity of copper at other temperatures than 0° C., $C_t = C_0(1 - .00387t + .000009009t^2)$.

The resistance is the reciprocal of the conductivity, and is

$$R_t = R_0(1 + .00387t + .00000597t^2).$$

The shorter formula $R_t = R_0(1 + .00406t)$ is commonly used.

A committee of the Am. Inst. Electrical Engineers recommend the following as the most correct form of the Matthiessen standard, taking 8.89 as the sp. gr. of pure copper:

A soft copper wire 1 metre long and 1 mm. diam. has an electrical resistance of .02057 B.A. unit at 0° C. From this the resistance of a soft copper wire 1 foot long and .001 in. diam. (mil-foot) is found to be 9.720 B.A. units at 0° C.

Standard Resistance at 0° C.	B.A. Units.	Legal Ohms.	Internat. Ohms.
Metre-millimetre, soft copper.........	.02057	.02034	.02029
Cubic centimetre " "	.000001616	.000001598	.000001593
Mil-foot " "	9.720	9.612	9.590
1 mil-foot of soft copper at 10°.22 C. or 50°.4 F...		10	9.977
" " " " " " 15°.5 " 59°.9 F...		10.20	10.175
" " " " " " 23°.9 " 75° F...		10.53	10.505

For tables of the resistance of copper wire, see pages 218 to 220, also pp. 1034, 1035.

Taking Matthiessen's standard of pure copper as 100%, some refined metal has exhibited an electrical conductivity equivalent to 103%.

Matthiessen found that impurities in copper sufficient to decrease its density from 8.94 to 8.90 produced a marked increase of electrical resistance.

DIRECT ELECTRIC CURRENTS.

Ohm's Law.—This law expresses the relation between the three fundamental units of resistance, electrical pressure, and current. It is:

$$\text{Current} = \frac{\text{electrical pressure}}{\text{resistance}}; \quad C = \frac{E}{R}; \quad \text{whence } E = CR, \text{ and } R = \frac{E}{C}.$$

In terms of the units of the three quantities,

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}}; \quad \text{volts} = \text{amperes} \times \text{ohms}; \quad \text{ohms} = \frac{\text{volts}}{\text{amperes}}.$$

EXAMPLES: *Simple Circuits.*—1. If the source has an effective electrical pressure of 100 volts, and the resistance is two ohms, what is the current?

$$C = \frac{E}{R} = \frac{100}{2} = 50 \text{ amperes.}$$

2. What pressure will give a current of 50 amperes through a resistance of 2 ohms? $E = CR = 50 \times 2 = 100$ volts.

3. What resistance is required to obtain a current of 50 amperes when the pressure is 100 volts? $R = \frac{E}{C} = \frac{100}{50} = 2$ ohms.

The following examples are from R. E. Day's "Electric Light Arithmetic:"

1. The internal resistance of a certain Brush dynamo-machine is 10.9 ohms, and the external resistance is 73 ohms; the electro-motive force of the machine being 839 volts. Find the strength of the current flowing in the circuit.

$$E = 839; \quad R = 73 + 10.9 = 83.9 \text{ ohms;}$$
$$C = E \div R = 839 \div 83.9 = 10 \text{ amperes.}$$

2. Three arc lamps in series have a combined resistance of 9.36 ohms, while the resistance of the leading wires is 1.1 ohm, and that of the dynamo is 2.8 ohms. Find what must be the electro-motive force of the machine when the strength of the current produced is 14.8 amperes.

$$R = 2.8 + 9.36 + 1.1 = 13.26 \text{ ohms}; \quad C = 14.8 \text{ amperes};$$
$$E = C \times R = 13.26 \times 14.8 = 196.3 \text{ volts}.$$

3. Calculate from the following data the average resistance of each of three arc lamps arranged in series. The electro-motive force of the machine is 244 volts and its resistance is 3.7 ohms, while that of the leading wires is 2 ohms, and the strength of current through each lamp is 21 amperes.

If x represent the average resistance in ohms of each lamp, then the total resistance of the circuit is $R = 3x + 2 + 3.7$.

But by Ohm's law $R = E \div C$, $\therefore 3x + 5.7 = 244/21 = 11.61$ ohms, whence $x = 1.97$ ohms, nearly.

4. Three Maxim incandescent lamps were placed in series. The average resistance, when hot, of each lamp was 39.3 ohms, and that of the dynamo and leading wires 11.2 ohms. What electro-motive force was required to maintain a current of 1.2 amperes through this circuit?

In this case we have

$$R = 3 \times 39.3 + 11.2 = 129.1 \text{ ohms, and}$$
$$C = 1.2 \text{ ampere};$$

and therefore, by Ohm's law,

$$E = C \times R = 1.2 \times 129.1 = 154.9 \text{ volts}.$$

5. The resistance of the arc of a certain Brush lamp was 3.8 ohms when a current of 10 amperes was flowing through it. What was the electro-motive force between the two terminals?

$$E = C \times R = 10 \times 3.8 = 38 \text{ volts}.$$

6. Twenty-five exactly similar galvanic cells, each of which had an average internal resistance of 15 ohms, were joined up in series to one incandescent lamp of 70 ohms resistance, and produced a current of 0.112 amperes. What would be the strength of current produced by a series of 30 such cells through 2 lamps, each of 30 ohms resistance?

The data of the first part of the problem enable us to determine the average electro-motive force of each cell of the battery. Let this be represented by E; then we have

$$25E = C \times R = .112 \times (25 \times 15 + 70) = .112 \times 445;$$

$$\therefore E = \frac{.112 \times 445}{25} = 2 \text{ volts, nearly}.$$

Then from the data in the second part of the problem, we have, by Ohm's law,

$$C = \frac{30 \times 2}{30 \times 15 + 2 \times 30} = \frac{60}{510} = 0.118 \text{ ampere}.$$

Divided Circuits.—If the circuit has two paths, the total current in both divides itself inversely as the resistances.

If R and R_1 are the resistances of the two branches, and C and C_1 the currents, $C \times R = C_1 \times R_1$, and $\frac{C}{C_1} = \frac{R_1}{R}$, whence

$$C = \frac{C_1 R_1}{R}; \quad C_1 = \frac{CR}{R_1}; \quad R = \frac{C_1 R_1}{C}; \quad R_1 = \frac{CR}{C_1}.$$

In the case of the double circuit, one circuit is said to be in *shunt* to the other, or the circuits are in multiple arc, in multiple, or in parallel.

Conductors in Series.—If conductors are arranged one after the other they are said to be in series, and the total resistance is the sum of their several resistances. $R = R_1 + R_2 + R_3$.

Internal Resistance.—In a simple circuit we have two resistances, that of the circuit R and that of the internal parts of the source of electro-motive force, called internal resistance, r. The formula of Ohm's law when the internal resistance is considered is $C = \frac{E}{R+r}$.

Total or Joint Resistance of Two Branches.—Let C be the total current, and C_1, C_2 the currents in branches whose resistances respectively are R_1, R_2. Then $C = C_1 + C_2$; $C = \frac{E}{R}$; $C_1 = \frac{E}{R_1}$; $C_2 = \frac{E}{R_2}$; or, if $E = 1$, $C = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, whence $R = \frac{R_1 R_2}{R_1 + R_2}$, which is the joint resistance of R_1 and R_2.

Similarly, the joint resistances of three branches have resistances respectively of R_1, R_2, R_3, is $R = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$.

When the branch resistances are equal, the formula becomes

$$\frac{R_1^{\,n}}{R_1^{\,n-1} \times n} = \frac{R_1}{n},$$

where R_1 = the resistance of one branch, and n = the number of branches.

Kirchhoff's Laws.—1. The sum of the currents in all the wires which meet in a point is nothing.

2. The sum of all the products of the currents and resistances in all the branches forming a closed circuit is equal to the sum of all the electrical pressures in the same circuit.

When $E = E_1 + E_2 + E_3$, etc., and $C = C_1 + C_2 + C_3$, etc., and R is the total resistance of $R_1 R_2 R_3$, etc., then

$$E_1 + E_2 + E_3, \text{ etc.} = C_1 R_1 + C_2 R_2 + C_3 R_3, \text{ etc.}$$

Power of the Circuit.—The power, or rate of work, in watts = current in amperes × electro-motive force in volts = $C \times E$. Since $C = E \div R$, watts $= \frac{E^2}{R}$ = electro-motive force^2 ÷ resistance.

EXAMPLE.—What H.P. is required to supply 100 lamps of 40 ohms resistance each, requiring an electro-motive force of 60 volts?

The number of volt-amperes for each lamp is $\frac{E^2}{R} = \frac{60^2}{40}$, 1 volt-ampere = .00134 H.P.; therefore $\frac{60^2}{40} \times 100 \times .00134 = 12$ H.P. (electrical) very nearly.

If the loss in the dynamo is 20 per cent, then 12 H.P. is 80 per cent of the actual H.P. required; which therefore is $\frac{12}{.80} = 15$ H.P.

Heat Generated by a Current.—Joule's law shows that the heat developed in a conductor is directly proportional, 1st, to its resistance; 2d, to the square of the current strength; and 3d, to the time during which the current flows, or $H = C^2Rt$. Since $C = E \div R$,

$$C^2Rt = \frac{E}{R} CRt = ECt = E\frac{E}{R}t = \frac{E^2t}{R}.$$

Or, heat = current^2 × resistance × time
= electro-motive force × current × time
= electro-motive force^2 × time ÷ resistance.

$$Q = \text{quantity of electricity flowing} = Ct = \frac{E}{R}t.$$

$$H = EQ; \text{ or heat} = \text{electro-motive force} \times \text{quantity.}$$

The electro-motive force here is that causing the flow, or the difference in potential between the ends of the conductor.

The electrical unit of heat, or "joule" = 10^7 ergs = heat generated in one second by a current of 1 ampere flowing through a resistance of one ohm = .239 gramme of water raised 1° C. $H = C^2Rt \times .239$ gramme calories = $C^2Rt \times .0009478$ British thermal units.

In electric lighting the energy of the current is converted into heat in the lamps. The resistance of the lamp is made great so that the required quantity of heat may be developed, while in the wire leading to and from the lamp the resistance is made as small as is commercially practicable, so that as little energy as possible may be wasted in heating the wire. The transformations of energy from the fuel burned in the boiler to the electric light are the following:

Heat energy is transformed into mechanical energy by means of the boiler and engine.

Mechanical energy is transformed into electrical energy in the dynamo.

Electrical energy is transformed into heat in the electric light.

The heat generated in a conductor is the equivalent of the energy causing the flow. Thus, rate of expenditure of energy in watts = electro-motive force in volts × current in amperes = EC, and the energy in joules = watts × time in seconds = ECt. Heat = $C^2Rt = ECt$.

Heating of Conductors. (From Kapp's Electrical Transmission of Energy.)—It becomes a matter of great importance to determine beforehand what rise in temperature is to be expected in each given case, and if that rise should be found to be greater than appears safe, provision must be made to increase the rate at which heat is carried off. This can generally be done by increasing the superficial area of the conductor. Say we have one circular conductor of 1 square inch area, and find that with 1000 amperes flowing it would become too hot. Now by splitting up this conductor into 10 separate wires each one tenth of a square inch cross-sectional area, we have not altered the total amount of energy transformed into heat, but we have increased the surface exposed to the cooling action of the surrounding air in the ratio of 1 : $\sqrt{10}$, and therefore the ten thin wires can dissipate more than three times the heat, as compared with the single thick wire.

Heating of Wires of Subaqueous and Aerial Cables (insulated with Gutta-percha). (Prof. Forbes.)

Diameter of cable ÷ Diameter of conductor = 4.
Temperature of air = 20° C. = 68° F.
t = excess of temperature of conductor over air.

Diameter in centimetres and mils.		Current in amperes.				
Cm.	Mils.	t = 1° C. = 1.8° F.	t = 9° C. = 16.2° F.	t = 25° C. = 45° F.	t = 49° C. = 92.2° F.	t = 81° C. = 145.8° F.
.1	40	3.7	11.0	17.8	24.0	29.5
.2	80	9.1	27.0	43.8	59.0	72.5
.3	120	15.0	44.4	72.1	97.3	119
.4	160	21.2	62.5	102	137	168
.5	200	27.4	81.0	131	177	218
.6	240	33.7	100	164	219	268
.7	280	40.1	119	192	259	319
.8	310	46.4	137	223	301	369
.9	350	52.9	157	253	342	420
1.0	390	59.3	175	285	384	472
2.0	780	124	367	595	803	988
3.0	1180	189	559	908	1225	1508
4.0	1570	254	753	1221	1646	2021
5.0	1970	319	945	1534	2068	2523
6.0	2360	385	1138	1846	2491	3058
7.0	2760	450	1330	2158	2846	3575
8.0	3150	514	1525	2472	3335	4094
9.0	3540	580	1716	2785	3755	4611
10.0	3940	645	1909	3097	4178	5130

Prof. Forbes states that an insulated wire carries a greater current without overheating than a bare wire if the diameter be not too great. Assuming the diameter of the cable to be twice the diam. of the conductor, a greater current can be carried in insulated wires than in bare wires up to 1.9 inch diam. of conductor. If diam. of cable = 4 times diam. of conductor, this is the case up to 1.1 inch diam. of conductor.

Copper-wire Table.—The table on pages 1034 and 1035 is abridged from one computed by the Committee on Units and Standards of the American Institute of Electrical Engineers (Trans. Oct. 1893).

Weights, Lengths, and Resistances of Cool, Warm, and Hot Copper Wires.

Gauges.		Diameter, inches.	Area, Circular mils.	Weight.		Length.		Resistance in International Ohms.			
A. W. G. B. & S.	B. W. G. Stubbs'.			Lbs. per Foot.	Lbs. per Ohm, at 20° C., 68° F.	Feet per Lb.	Ft. per Ohm, at 20° C., 68° F.	Ohms per Lb. at 20° C., 68° F.	O. per ft., at 20° C., 68° F.	O. per ft., at 50° C., 122° F.	O. per ft., at 80° C., 176° F.
0000		0.460	211,600	0.6405	13,090	1.561	20,440	0.00007639	0.00004893	0.00005467	0.00006058
	0000	0.454	206,100	0.6239	12,420	1.603	19,910	0.00008051	0.00005023	0.00005612	0.00006220
	000	0.425	180,600	0.5468	9,538	1.829	17,450	0.0001048	0.00005732	0.00006404	0.00007097
000		0.4096	167,800	0.5080	8,232	1.969	16,210	0.0001215	0.00006170	0.00006893	0.00007640
	00	0.380	144,400	0.4371	6,096	2.288	13,950	0.0001640	0.00007170	0.00008011	0.00008878
00		0.3648	133,100	0.4028	5,177	2.482	12,850	0.0001931	0.00007780	0.00008692	0.00009633
	0	0.340	115,600	0.3499	3,907	2.858	11,160	0.0002560	0.00008957	0.0001001	0.0001109
0		0.3249	105,500	0.3195	3,256	3.130	10,190	0.0003071	0.00009811	0.0001096	0.0001215
	1	0.3000	90,000	0.2724	2,368	3.671	8,692	0.0004223	0.0001150	0.0001285	0.0001424
1		0.2893	83,690	0.2533	2,048	3.947	8,083	0.0004883	0.0001237	0.0001382	0.0001532
	2	0.2840	80,660	0.2441	1,902	4.096	7,790	0.0005258	0.0001284	0.0001434	0.0001589
	3	0.2590	67,080	0.2031	1,316	4.925	6,479	0.0007601	0.0001543	0.0001724	0.0001911
2		0.2576	66,370	0.2009	1,288	4.977	6,410	0.0007765	0.0001560	0.0001743	0.0001932
	4	0.2380	56,640	0.1715	938.0	5.832	5,471	0.001066	0.0001828	0.0002042	0.0002263
3		0.2294	52,630	0.1593	810.0	6.276	5,084	0.001235	0.0001967	0.0002198	0.0002435
	5	0.2200	48,400	0.1465	684.9	6.826	4,675	0.001460	0.0002139	0.0002390	0.0002649
4		0.2043	41,740	0.1264	509.4	7.914	4,031	0.001963	0.0002480	0.0002771	0.0003071
	6	0.2030	41,210	0.1247	496.5	8.017	3,980	0.002014	0.0002513	0.0002807	0.0003111
5		0.1819	33,100	0.1002	320.4	9.980	3,197	0.003122	0.0003128	0.0003495	0.0003873
	7	0.1800	32,400	0.09808	306.9	10.20	3,129	0.003258	0.0003196	0.0003570	0.0003957
	8	0.1650	27,230	0.08241	216.7	12.13	2,629	0.004615	0.0003803	0.0004249	0.0004709
6		0.1620	26,250	0.07946	201.5	12.58	2,535	0.004963	0.0003944	0.0004406	0.0004883
	9	0.1480	21,900	0.06630	140.3	15.08	2,116	0.007129	0.0004727	0.0005281	0.0005853
7		0.1443	20,820	0.06302	126.7	15.87	2,011	0.007892	0.0004973	0.0005556	0.0006158
	10	0.1340	17,960	0.05435	94.26	18.40	1,734	0.01061	0.0005766	0.0006442	0.0007140
8		0.1285	16,510	0.04998	79.69	20.01	1,595	0.01255	0.0006271	0.0007007	0.0007765
	11	0.1200	14,400	0.04359	60.62	22.94	1,391	0.01650	0.0007190	0.0008033	0.0008903
9		0.1144	13,090	0.03963	50.12	25.23	1,265	0.01995	0.0007908	0.0008835	0.0009791
	12	0.1090	11,880	0.03596	41.27	27.81	1,147	0.02423	0.0008715	0.0009736	0.001079
10		0.1019	10,380	0.03143	31.52	31.82	1,003	0.03173	0.0009972	0.001114	0.001235
	13	0.0950	9,025	0.02732	23.81	36.60	871.7	0.04199	0.001147	0.001282	0.001420
11		0.09074	8,234	0.02493	19.82	40.12	795.3	0.05045	0.001257	0.001405	0.001557
	14	0.08300	6,889	0.02085	13.87	47.96	665.4	0.07207	0.001503	0.001679	0.001861
12		0.08081	6,530	0.01977	12.47	50.59	630.7	0.08022	0.001586	0.001771	0.001963
	15	0.07200	5,184	0.01569	7.857	63.73	500.7	0.1273	0.001997	0.002231	0.002473
13		0.07196	5,178	0.01568	7.840	63.79	500.1	0.1276	0.001999	0.002231	0.002476
	16	0.06500	4,225	0.01279	5.219	78.19	408.1	0.1916	0.002451	0.002738	0.003034
14		0.06408	4,107	0.01243	4.931	80.44	396.6	0.2028	0.002521	0.002817	0.003122
	17	0.0580	3,364	0.01018	3.308	98.23	324.9	0.3023	0.003078	0.003439	0.003811
15		0.05707	3,257	0.009858	3.101	101.4	314.5	0.3225	0.003179	0.003552	0.003936
16		0.05082	2,583	0.007818	1.950	127.9	249.4	0.5128	0.004009	0.004479	0.004964
	18	0.04900	2,401	0.007265	1.685	137.6	231.9	0.5963	0.004318	0.004818	0.005339

Gauges.		Diameter, inches.	Area, Circular mils.	Weight.		Length.		Resistance in International Ohms.			
A. W. G. B. & S.	B. W. G. Stubbs'.			Lbs. per Foot.	Lbs. per Ohm, at 20° C., 68° F.	Feet per Lb.	Ft. per Ohm, at 20° C., 68° F.	Ohms per Lb. at 20° C., 68° F.	Ohms per ft. at 20° C., 68° F.	Ohms per ft. at 50° C., 122° F.	Ohms per ft. at 80° C., 176° F.
17		0.04526	2,048	0.006200	1.226	161.3	197.8	0.8153	0.005055	0.005648	0.006259
	19	0.04200	1,764	0.005340	0.9097	187.3	170.4	1.099	0.005870	0.006558	0.007287
18		0.04030	1,624	0.004917	0.7713	203.4	156.9	1.296	0.006374	0.007122	0.007892
19		0.03589	1,288	0.003899	0.4851	256.5	124.4	2.061	0.008038	0.008980	0.009952
	20	0.03500	1,225	0.003708	0.4387	269.7	118.3	2.279	0.008452	0.009443	0.01047
	21	0.03200	1,024	0.003100	0.3066	322.6	98.90	3.262	0.01011	0.01130	0.01252
20		0.03196	1,022	0.003092	0.3051	323.4	98.66	3.278	0.01014	0.01132	0.01255
21		0.02846	810.1	0.002452	0.1919	407.8	78.24	5.212	0.01278	0.01428	0.01583
	22	0.02800	784.0	0.002373	0.1797	421.4	75.72	5.565	0.01321	0.01475	0.01635
22		0.02535	642.4	0.001945	0.1207	514.2	62.05	8.287	0.01612	0.01801	0.01996
	23	0.0250	625.0	0.001892	0.1142	528.6	60.36	8.756	0.01657	0.01851	0.02051
23		0.02257	509.5	0.001642	0.07589	648.4	49.21	13.18	0.02032	0.02271	0.02516
	24	0.0220	484.0	0.001465	0.06849	682.6	46.75	14.60	0.02139	0.02390	0.02649
24		0.02010	404.0	0.001223	0.04773	817.6	39.02	20.95	0.02563	0.02863	0.03173
	25	0.0200	400.0	0.001211	0.04678	825.9	38.63	21.38	0.02588	0.02892	0.03206
	26	0.0180	324.0	0.0009808	0.03069	1,020	31.29	32.58	0.03196	0.03570	0.03957
25		0.01790	320.4	0.0009699	0.03002	1,031	30.95	33.32	0.03231	0.03610	0.04001
	27	0.0160	256.0	0.0007749	0.01916	1,290	24.73	52.19	0.04045	0.04519	0.05008
26		0.01594	254.1	0.0007692	0.01888	1,300	24.54	52.97	0.04075	0.04552	0.05045
27		0.0142	201.5	0.0006100	0.01187	1,639	19.46	84.23	0.05138	0.05740	0.06362
	28	0.0140	196.0	0.0005933	0.01123	1,685	18.93	89.04	0.05283	0.05902	0.06541
	29	0.0130	169.0	0.0005116	0.008350	1,955	16.32	119.8	0.06127	0.06845	0.07586
28		0.01264	159.8	0.0004837	0.007466	2,067	15.43	133.9	0.06479	0.07239	0.08022
	30	0.0120	144.0	0.0004359	0.006062	2,294	13.91	165.0	0.07190	0.08033	0.08903
29		0.01126	126.7	0.0003836	0.004696	2,607	12.24	213.0	0.08170	0.09128	0.1012
30		0.01003	100.5	0.0003042	0.002953	3,287	9.707	338.6	0.1030	0.1151	0.1276
	31	0.0100	100.0	0.0003027	0.002924	3,304	9.658	342.0	0.1035	0.1157	0.1282
	32	0.0090	81.0	0.0002452	0.001918	4,078	7.823	521.3	0.1278	0.1428	0.1583
31		0.008928	79.70	0.0002413	0.001857	4,145	7.698	538.4	0.1299	0.1451	0.1608
	33	0.0080	64.0	0.0001937	0.001197	5,162	6.181	835.1	0.1618	0.1807	0.2003
32		0.007960	63.21	0.0001913	0.001168	5,227	6.105	856.2	0.1638	0.1830	0.2028
33		0.007080	50.13	0.0001517	0.0007346	6,591	4.841	1,361	0.2066	0.2308	0.2558
	34	0.0070	49.0	0.0001483	0.0007019	6,742	4.733	1,425	0.2113	0.2361	0.2616
34		0.006305	39.75	0.0001203	0.0004620	8,311	3.839	2,165	0.2605	0.2910	0.3225
35		0.005615	31.52	0.00009543	0.0002905	10,480	3.045	3,441	0.3284	0.3669	0.4067
36	35	0.0050	25.0	0.00007568	0.0001827	13,210	2.414	5,473	0.4142	0.4627	0.5129
37		0.004453	19.83	0.00006001	0.0001149	16,660	1.915	8,702	0.5222	0.5835	0.6466
	36	0.0040	16.	0.00004843	0.00007484	20,650	1.545	13,360	0.6471	0.7230	0.8011
38		0.003965	15.72	0.00004759	0.00007210	21,010	1.519	13,870	0.6585	0.7357	0.8154
39		0.003531	12.47	0.00003774	0.00004545	26,500	1.204	22,000	0.8304	0.9277	1.028
40		0.003145	9.888	0.00002993	0.00002858	33,410	0.9550	34,980	1.047	1.170	1.296

The data from which the foregoing table has been computed are as follows: Matthiessen's standard resistivity, Matthiessen's temperature coefficients, specific gravity of copper = 8.89. Resistance in terms of the international ohm.

Matthiessen's standard 1 metre-gramme of hard-drawn copper = 0.1469 B. A. U. @ 0° C. Ratio of resistivity hard to soft copper 1.0226.

Matthiessen's standard 1 metre-gramme of soft-drawn copper = 0.14365 B. A. U. @ 0° C. One B. A. U. = 0.9866 international ohm.

Matthiessen's standard 1 metre-gramme of soft-drawn copper = 0.141729 international ohm @ 0° C.

Temperature coefficients of resistance for 20° C., 50° C., and 80° C., 1.07968, 1.20625, and 1.33681 respectively. 1 foot = 0.3048028 metre, 1 pound = 453.59256 grammes.

Heating of Coils.—To calculate the heating of a coil, given the cooling surface and its resistance. (Forbes.)

Let ρ = the resistance of a coil in ohms at the permissible temperature (the resistance (cold) must be increased by 1/5 of its value to give ρ);

S = the surface exposed to the air measured in square centimetres (1 square cm. = .155 square inch; 1 sq. in. = 6.45 square cm.);

t = the rise in temperature, centigrade scale;

C = the current in amperes.

$$.24C^2\rho = \text{heat generated} = etS.$$

where e is McFarlane's constant, varying from .0002 to .0003. The latter value may be taken. If 50° C. be the permissible rise in temperature,

$$C = \sqrt{\frac{.0003 \times 50 \times S}{.24 \times \rho}} = .25\sqrt{\frac{S}{\rho}}.$$

EXAMPLE.—The resistance of the field-magnets of a dynamo is 1.5 ohms cold, and the surface exposed to the air is 1 square metre; find the current to heat it not more than 50° C.

$$\text{Here } S = 10{,}000;\ \rho = 1.8 \text{ ohms; and } C = .25\sqrt{\frac{10{,}000}{1.8}} = 18.6 \text{ amperes.}$$

For the heating of coils of field-magnets Carl Hering gives 1 watt of energy dissipated for every 223 square inches of cooling-surface for each degree F. of difference between the temperature of the coil and the surrounding air.

$W = CE = 1/223TS = 0.004476TS$, in which W = watts lost in coil, T = degrees Fahr., and S = square inches.

$C = \frac{TS}{223E}$ is the greatest current which can be used in the magnet coils of a shunt machine having a certain pressure in order that they do not heat above a certain temperature. Thus for a rise of temperature of 50° F. above the surrounding air,

$C = \frac{50S}{223E} = .224\frac{S}{E}$. Substituting for E its equivalent CR, we get

$$C = \sqrt{.224\frac{S}{R}}.$$

If 80° F. is the maximum difference of temperature,

$$C = \frac{80S}{223E} = .36\frac{S}{E} = .60\sqrt{\frac{S}{R}}.$$

The formula can be used for series machines when C is known, for writing

$$C^2R = 1/223TS, \quad \text{we get } R = \frac{TS}{223C^2}.$$

With a permissible rise of 50° F. or 80° F., we have respectively,

$$R = \frac{.224S}{C^2}; \text{ and } R = .36\frac{S}{C^2}.$$

The surface area of the coil in square inches may be found from

$$S = \frac{223W}{T} = \frac{223CE}{T} = \frac{223C^2R}{T}.$$

For a rise of temperature of 50° F. or 80° F., respectively, the surface will be

$$S = \frac{223W}{50} = 4.46W; \quad \text{and } S = \frac{223W}{80} = 2.8W.$$

Fusion of Wires.—W. H. Preece gives a formula for the current required to fuse wires of different metals, viz.: $C = ad^{\frac{3}{2}}$, in which d is the diameter in inches and a a coefficient whose value for different metals is as follows: Copper 10244; aluminum 7585; platinum 5172; German silver 5250; platinoid 4750; iron 3148; tin, 1642; lead, 1379; alloy of 2 lead and 1 tin, 1318.

Diameters of Various Wires which will be Fused by a given Current.

Formula, $d = \left(\frac{C}{a}\right)^{\frac{2}{3}}$; a = 1642 for tin = 1379 for lead = 10244 for copper = 3148 for iron.

Current, in amperes.	Tin Wire.		Lead Wire.		Copper Wire.		Iron Wire.	
	Diam. inches.	Approx. A. W. G.	Diam. inches.	Approx. A. W. G.	Diam. inches.	Approx. A. W. G.	Diam. inches.	Approx. A. W. G.
1	.0072	33	.0081	32	.0021	....	.0047	36
2	.0118	29	.0128	28	.0034	39	.0074	32.5
3	.0149	26.5	.0168	25.5	.0044	37	.0097	30
4	.0181	25	.0203	24	.0053	35.5	.0117	28.5
5	.0210	23.5	.0236	22.5	.0062	34	.0136	27.5
10	.0334	19.5	.0375	18.5	.0098	30	.0216	23.5
15	.0437	17	.0491	16	.0129	28	.0283	21
20	.0529	16	.0595	15	.0156	26	.0343	19
25	.0614	14.5	.0690	13	.0181	25	.0398	18
30	.0694	13	.0779	12	.0205	24	.0450	17
35	.0769	12.5	.0864	11.5	.0227	23	.0498	16
40	.0840	11.5	.0944	11	.0248	22	.0545	15.5
45	.0909	11	.1021	10	.0268	21.5	.0589	15
50	.0975	10.5	.1095	9.5	.0288	21	.0632	14
60	.1101	9	.1237	8.5	.0325	20	.0714	13
70	.1220	8.5	.1371	7.5	.0360	19	.0791	12
80	.1334	7.5	.1499	7	.0394	18	.0864	11.5
90	.1443	7	.1621	6	.0426	17.5	.0935	11
100	.1548	6.5	.1739	5.5	.0457	17	.1003	10
120	.1748	5.5	.1964	4.5	.0516	16	.1183	9
140	.1937	4.5	.2176	3.5	.0572	15	.1255	8
160	.2118	4	.2379	3	.0625	14	.1372	7.5
180	.2291	3	.2573	2	.0676	13.5	.1484	7
200	.2457	2	.2760	1.5	.0725	13	.1592	6
250	.2851	1	.3203	0	.0841	11.5	.1848	5
300	.3220	0	.3617	00	.0950	10.5	.2086	4

Current in Amperes Required to Fuse Wires According to the Formula $C = ad^{\frac{3}{2}}$.

Approx. A. W. G.	Diameter, inches.	$d^{\frac{3}{2}}$.	Tin. a = 1642.	Lead a = 1379.	Copper a = 10244	Iron. a = 3148.
12	.080	.022627	37.15	31.20	231.8	71.22
14	.064	.016191	26.58	22.32	165.8	50.96
16.5	.048	.010516	17.27	14.50	107.7	33.10
19	.036	.006831	11.22	9.419	69.97	21.50
21	.028	.004685	7.692	6.461	48.00	14.75
23	.022	.003263	5.357	4.499	33.43	10.27
25	.018	.002415	3.965	3.330	24.74	7.602
27	.0148	.001801	2.956	2.483	18.44	5.667
28	.0124	.001381	2.267	1.904	14.15	4.347
29	.0108	.001122	1.843	1.548	11.50	3.533

ELECTRIC TRANSMISSION, DIRECT CURRENTS.

Cross-section of Wire Required for a Given Current.—*Constant Current* (*Series*) *System*.—The cross-sectional area of copper necessary in any circuit for a given constant current depends on the difference between the pressure at the generating station and the maximum pressure required by all the apparatus on the circuit, and on the total length of the circuit. The following formulæ are given in "Practical Electrical Engineering:"

If V = pressure in volts at generators;
v = sum of all the pressures (in volts) required by apparatus supplied in the circuit;
n = total length (going and return) of circuit in miles;
C = current in amperes;
r = resistance of 1 mile of copper-conductor of 1 square inch sectional area in ohms;
a = required cross-sectional area of copper in square inches,—

$$a = \frac{nrC}{V - v}.$$

If we take the temperature of the conductor when the current has been flowing for some time through it, as 80° F.,

$$r = 0.0455 \text{ ohm, and } \quad a = \frac{0.0455nC}{V - v}.$$

It generally happens, however, that we are not tied down to a particular value of V, as the pressure at the generators can be varied by a few volts to suit requirements. In this case it is usual to fix upon a current density and determine the cross-sectional area of copper in accordance with it.

If D = current density in amperes per square inch determined upon,

$$a = \frac{C}{D}.$$

The current density is frequently taken at 1000 amperes to the square inch, but should in general be determined by economical considerations for every case in question.

Constant Pressure (Parallel System).—To determine the loss in pressure in a feeder of given size in the case of two-wire parallel distribution.

Let a = cross-sectional area of copper of one conductor of the feeder in square inches;
n = length of feeder (going and return) in miles;
C = current in amperes;
$V - v$ = loss of pressure in feeder in volts;
r = resistance of 1 mile of copper conductor of 1 square inch sectional area in ohms.

$$V - v = \frac{nrC}{a}.$$

If the temperature of the conductor with this current flowing in it is assumed to be 80° F.,

$$r = 0.0455 \text{ ohm, and } \quad V - v = \frac{0.0455nC}{a}.$$

Short-circuiting.—From the law $C = \frac{E}{R}$ it is seen that with any pressure E the current C will become very great if R is made very small. In short-circuiting the resistance becomes small and the current therefore great. Hence the dangers of short-circuiting a current.

Economy of Electric Transmission. (R. G. Blaine, *Eng'g*, June 5, 1891.)—Sir W. Thomson's rule for the most economical section of conductor is that for which the "annual interest on capital outlay is equal to the annual cost of energy wasted."

Tables have been compiled by Professor Forbes and others in accordance with modifications of Sir W. Thomson's rule. For a given entering horse-power the question is merely one as to what current density, or how many amperes per square inch of conductor, should be employed. Sir W. Thomson's rule gives about 393 amperes per square inch, and Professor Forbes's tables—for a medium cost of one electrical horse-power per hour—give a current density of about 380 amperes per square inch as most economical.

When a given horse-power is to be delivered at a given distance, the case is somewhat different, and Professors Ayrton and Perry (*Electrician*, March, 1886) have shown that in that case both the current and resistance are variables, and that their most economical values may be found from the following formulæ:

$$C = \frac{w}{P}(1 + \sin\phi), \quad \text{and} \quad r = \frac{P^2}{nw}\,\frac{\sin\phi}{(1+\sin\phi)^2},$$

in which C = the proper current in amperes; r = resistance in ohms per mile which should be given to the conductor; P = pressure at entrance in volts; n = number of miles of conductor; w = power delivered in watts; ϕ = such an angle that $\tan\phi = nt \div P$, t being a constant depending on the price of copper, the cost of one electrical horse-power, interest, etc.: it may be taken as about 17.

In this case the current density should not remain constant, but should diminish as the length increases, being in all cases less than that calculated by Sir W. Thomson's rule.

EXAMPLE.—If the current for an electric railway is sent in at 200 volts, 100 horse-power being delivered, find the waste of power in heating the conductor, the distance being 5 miles and there being a return conductor.

Here $n = 10$, $t = 17$, $P = 200$; $\tan\phi = 170 \div 200 = .85$, $\phi = 40°\ 22'$, $\sin\phi = .6477$.

Hence most economical resistance

$$r = \frac{200^2}{10 \times 74600} \times \frac{.6477}{1.6477^2} = .01279 \text{ ohm per mile},$$

or .1279 ohm in its total length.

The most economical current, $C = \frac{74600}{200} \times 1.6477 = 614.58$ amperes, and W, the power wasted in heat, $= \frac{C^2R}{746} = \frac{614.58^2 \times .1279}{746} = 64.75$ horse-power.

The following tables show the power wasted as heat in the conductor.

HORSE-POWER WASTED IN TRANSMITTING POWER ELECTRICALLY TO A GIVEN DISTANCE, THE ENTERING POWER BEING FIXED. PRESSURE AT ENTRANCE, 200 VOLTS. CURRENT DENSITY, 380 AMPERES PER SQUARE INCH.

Horse-power sent in.*	Horse-power Wasted, the Distance to which the Power is Transmitted being one Mile (there being a Return Conductor).	Horse-power Wasted. Distance Five Miles.
10	1.663	8.318
20	3.327	16.636
40	6.654	33.27
50	8.318	41.59
80	13.308	66.54
100	16.636	83.18
200	33.272	166.36

* That is, horse-power at the generator terminals.

PRESSURE AT ENTRANCE, 2000 VOLTS.

Horse-power sent in.	Horse-power Wasted. Distance One Mile (there being a Return Conductor).	Horse-power Wasted. Distance Five Miles.	Horse-power Wasted. Distance Ten Miles.	Horse-power Wasted. Distance Twenty Miles.
100	1.663	8.318	16.636	33.27
200	3.327	16.636	33.272	66.54
400	6.654	33.272	66.54	133.08
500	8.318	41.59	83.18	166.36
800	13.308	66.54	133.08	266.17
1000	16.636	83.18	166.36	332.72
2000	33.272	166.36	332.72	665.44

It will be seen from these numbers that when the current density is fixed the power wasted is proportional to the entering horse-power and the length of the conductor, and is inversely proportional to the potential. For a copper conductor the rule may be simply stated as

$$W = 16.6358 \frac{E}{P} \times l,$$

E being the horse-power and P the pressure at entrance, and l the length of the conductor in miles.

HORSE-POWER WASTED IN ELECTRIC TRANSMISSION TO A GIVEN DISTANCE, THE POWER TO BE DELIVERED AT THE DISTANT END BEING FIXED. PRESSURE AT ENTRANCE, 200 VOLTS. CURRENT AND RESISTANCE CALCULATED BY AYRTON AND PERRY'S RULES.

Horse-power Delivered.	Horse-power Wasted, the Distance to which the Power is Transmitted being One Mile (there being a Return Conductor).	Horse-power Wasted. Distance Five Miles.	Horse-power Wasted. Distance Ten Miles.
10	1.676	6.476	8.620
20	3.352	12.952	17.24
40	6.704	25.904	34.48
50	8.38	32.38	43.10
80	13.408	51.808	68.96
100	16.76	64.86	86.20
200	33.52	129.52	172.4

PRESSURE AT ENTRANCE, 2000 VOLTS.

Horse-power Delivered.	Horse-power Wasted. Distance One Mile.	Horse-power Wasted. Distance Five Miles.	Horse-power Wasted. Distance Ten Miles.
100	1.716	8.484	16.763
200	3.432	16.968	33.526
400	6.864	33.938	67.052
500	8.58	42.42	83.815
800	13.728	67.87	134.104
1000	17.16	84.84	167 63
2000	34.32	169.68	335.26

If H = horse-power sent in, w = power delivered in watts, C = current in amperes, r = resistance in ohms per mile, P = pressure at entrance in volts, and n = number of miles of conductor,

$$(w + C^2r) \div 746 = H; \quad w = 746H - C^2r;$$

id the formulæ for best current and resistance become

$$C = \frac{746H - C^2r}{P}(1 + \sin\phi); \quad r = \frac{P^2}{n(746H - C^2r} \times \frac{\sin\phi}{1 + \sin\phi}.$$

Energy wasted as heat in watts per mile $= C^2r = \dfrac{746H \sin\phi}{n + \sin\phi}$.

Horse-power wasted per mile $= W_1 = \dfrac{H \sin\phi}{n + \sin\phi}$.

(ϕ = angle whose tangent $= nt \div P$, and the value of t corresponding to a urrent density of 880 amperes per sq. in. is 16.636.)

TABLE OF ELECTRICAL HORSE-POWERS.

Formula: $\dfrac{\text{Volts} \times \text{Amperes}}{746} = \text{H.P.}$, or 1 volt-ampere = .0013405 H.P.

Read amperes at top and volts at side, or *vice versa.*

Volts or Amperes	Volts or Amperes.												
	1	10	20	30	40	50	60	70	80	90	100	110	120
1	.00134	.0134	.0268	.0402	.0536	.0670	.0804	.0938	.1072	.1206	.1341	.1475	.1609
2	.00268	.0268	.0536	.0804	.1072	.1341	.1609	.1877	.2145	.2413	.2681	.2949	.3217
3	.00402	.0402	.0804	.1206	.1609	.2011	.2413	.2815	.3217	.3619	.4022	.4424	.4826
4	.00536	.0536	.1072	.1609	.2145	.2681	.3217	.3753	.4290	.4826	.5362	.5898	.6434
5	.00670	.0670	.1341	.2011	.2681	.3351	.4022	.4692	.5362	.6032	.6703	.7373	.8043
6	.00804	.0804	.1609	.2413	.3217	.4022	.4826	.5630	.6434	.7239	.8043	.8847	.9652
7	.00938	.0938	.1877	.2815	.3753	.4692	.5630	.6568	.7507	.8445	.9384	1.032	1.126
8	.01072	.1072	.2145	.3217	.4290	.5362	.6434	.7507	.8579	.9652	1.072	1 180	1.287
9	.01206	.1206	.2413	.3619	.4826	.6032	.7239	.8445	.9652	1.086	1.206	1.327	1.448
10	.01341	.1341	.2681	.4022	.5362	.6703	.8043	.9383	1.072	1.206	1.341	1.475	1.609
11	.01475	.1475	.2949	.4424	.5898	.7373	.8847	1.032	1.180	1.327	1.475	1.622	1.769
12	.01609	.1609	.3217	.4826	.6434	.8043	.9652	1.126	1.287	1.448	1.609	1.769	1.930
13	.01743	.1743	.3485	.5228	.6970	.8713	1.046	1.220	1.394	1.568	1.743	1.917	2.091
14	.01877	.1877	.3753	.5630	.7507	.9384	1.126	1.314	1.501	1.689	1.877	2.064	2.252
15	.02011	.2011	.4022	.6032	.8043	1.005	1.206	1.408	1.609	1.810	2.011	2.212	2.413
16	.02145	.2145	.4290	.6434	.8579	1.072	1.287	1.501	1.716	1.930	2.145	2.359	2.574
17	.02279	.2279	.4558	.6837	.9115	1.139	1.367	1.595	1.823	2.051	2.279	2.507	2.735
18	.02413	.2413	.4826	.7239	.9652	1.206	1.448	1.689	1.930	2.172	2.413	2.654	2.895
19	.02547	.2547	.5094	.7641	1.019	1.273	1.528	1.783	2.037	2.292	2.547	2.801	3.056
20	.02681	.2681	.5362	.8043	1.072	1.340	1.609	1.877	2.145	2.413	2.681	2.949	3.217
21	.02815	.2815	.5630	.8445	1.126	1.408	1.689	1.971	2.252	2.533	2.815	3.097	3.378
22	.02949	.2949	.5898	.8847	1.180	1.475	1.769	2.064	2.359	2.654	2.949	3.244	3.539
23	.03083	.3083	.6166	.9249	1.233	1.542	1.850	2.158	2.467	2.775	3.083	3.391	3.700
24	.03217	.3217	.6434	.9652	1.287	1.609	1.930	2.252	2.574	2.895	3.217	3 539	3.861
25	.03351	.3351	.6703	1.005	1.341	1.676	2.011	2.346	2.681	3.016	3.351	3.686	4.022
26	.03485	.3485	.6971	1.046	1.394	1.743	2.091	2.440	2.788	3.137	3.485	3.834	4.182
27	.03619	.3619	.7239	1.086	1.448	1.810	2.172	2.534	2.895	3.257	3.619	3.981	4.343
28	.03753	.3753	.7507	1.126	1.501	1.877	2.252	2.627	3.003	3.378	3.753	4.129	4.504
29	.03887	.3887	.7775	1.166	1.555	1.944	2.332	2.721	3.110	3.499	3.887	4.276	4.665
30	.04022	.4022	.8043	1.206	1.609	2.011	2.413	2.815	3.217	3.619	4.022	4.424	4.826
31	.04156	.4156	.8311	1.247	1.662	2.078	2.493	2.909	3.324	3.740	4.156	4.571	4.987
32	.04290	.4290	.8579	1.287	1.716	2.145	2.574	3.003	3 432	3.861	4.290	4.719	5.148
33	.04424	.4424	.8847	1.327	1.769	2.212	2.654	3.097	3.539	3.981	4.424	4.866	5.308
34	.04558	.4558	.9115	1.367	1.823	2.279	2.735	3.190	3.646	4.102	4.558	5.013	5.469
35	.04692	.4692	.9384	1.408	1.877	2.346	2.815	3.284	3.753	4.223	4.692	5.161	5.630
36	.04826	.4826	.9652	1.448	1.930	2.413	2.895	3.378	3.861	4.343	4.826	5.308	5.791
37	.04960	.4960	.9920	1.488	1.984	2.480	2.976	3.472	3.968	4.464	4.960	5.456	5.952
38	.05094	.5094	1.019	1.528	2.038	2.547	3.056	3.566	4.075	4.585	5.094	5.603	6.113
39	.05228	.5228	1.046	1.568	2.091	2.614	3.137	3.660	4.182	4.705	5.228	5.751	6.274
40	.05362	.5362	1.072	1.609	2.145	2 681	3.217	3.753	4.290	4.826	5.362	5.898	6.434
41	.05496	.5496	1.099	1.649	2.198	2.748	3.298	3.847	4.397	4.946	5.496	6.046	6.595
42	.05630	.5630	1.126	1.689	2.252	2.815	3.378	3.941	4.504	5.067	5.630	6.193	6.756
43	.05764	.5764	1.153	1.729	2.306	2.882	3.458	4.035	4.611	5.187	5.764	6.341	6.917
44	.05898	.5898	1.180	1.769	2.359	2.949	3.539	4.129	4.719	5.308	5.898	6.488	7.078
45	.06032	.6032	1.206	1.810	2.413	3.016	3.619	4.223	4.826	5.429	6.032	6.635	7.239
46	.06166	.6166	1.233	1.850	2.467	3.083	3.700	4.316	4.933	5.550	6.166	6.783	7.400
47	.06300	.6300	1.260	1.890	2.520	3.150	3.780	4.410	5.040	5.670	6.300	6.930	7.560
48	.06434	.6434	1.287	1.930	2.574	3.217	3.861	4.504	5.148	5.791	6.434	7.078	7.721
49	.06568	.6568	1.314	1.970	2.627	3.284	3.941	4.598	5.255	5.912	6.568	7.225	7.882
50	.06703	.6703	1.341	2.011	2.681	3.351	4.022	4.692	5.362	6.032	6.703	7.373	8.043

TABLE OF ELECTRICAL HORSE-POWERS—(*Continued.*)

Amperes or Volts.	Volts or Amperes.												
	1	10	20	30	40	50	60	70	80	90	100	110	120
55	.07373	.7373	1.475	2.212	2.949	3.686	4.424	5.161	5.898	6.635	7.373	8.110	8.847
60	.08043	.8043	1.609	2.413	3.217	4.022	4.826	5.630	6.434	7.239	8.043	8.847	9.651
65	.08713	.8713	1.743	2.614	3.485	4.357	5.228	6.099	6.970	7.842	8.713	9.584	10.46
70	.09384	.9384	1.877	2.815	3.753	4.692	5.630	6.568	7.507	8.445	9.384	10.32	11.26
75	.10054	1.005	2.011	3.016	4.021	5.027	6.032	7.037	8.043	9.048	10.05	11.06	12.06
80	.10724	1.072	2.145	3.217	4.290	5.362	6.434	7.507	8.579	9.652	10.72	11.80	12.87
85	.11394	1.139	2.279	3.418	4.558	5.697	6.836	7.976	9.115	10.25	11.39	12.53	13.67
90	.12065	1.206	2.413	3.619	4.826	6.032	7.239	8.445	9.652	10.86	12.06	13.27	14.48
95	.12735	1.273	2.547	3.820	5.094	6.367	7.641	8.914	10.18	11.46	12.73	14.01	15.28
100	.13405	1.341	2.681	4.022	5.362	6.703	8.043	9.384	10.72	12.06	13.41	14.75	16.09
200	.26810	2.681	5.362	8.043	10.72	13.41	16.09	18.77	21.45	24.13	26.81	29.49	32.17
300	.40215	4.022	8.043	12.06	16.09	20.11	24.13	28.15	32.17	36.19	40.22	44.24	48.26
400	.53620	5.362	10.72	16.09	21.45	26.81	32.17	37.53	42.90	48.26	53.62	58.98	64.34
500	.67025	6.703	13.41	20.11	26.81	33.51	40.22	46.92	53.62	60.32	67.03	73.73	80.43
600	.80430	8.043	16.09	24.13	32.17	40.22	48.26	56.30	64.34	72.39	80.43	88.47	96.52
700	.93835	9.384	18.77	28.15	37.53	46.92	56.30	65.68	75.07	84.45	93.84	103.2	112.6
800	1.0724	10.72	21.45	32.17	42.90	53.62	64.34	75.07	85.79	96.52	107.2	118.0	128.7
900	1.2065	12.06	24.13	36.19	48.26	60.32	72.39	84.45	96.52	108.6	120.6	132.7	144.8
1,000	1.3405	13.41	26.81	40.22	53.62	67.03	80.43	93.84	107.2	120.6	134.1	147.5	160.9
2,000	2.6810	26.81	53.62	80.43	107.2	134.1	160.9	187.7	214.5	241.3	268.1	294.9	321.7
3,000	4.0215	40.22	80.43	120.6	160.9	201.1	241.3	281.5	321.7	361.9	402.2	442.4	482.6
4,000	5.3620	53.62	107.2	160.9	214.5	268.1	321.7	375.3	429.0	482.6	536.2	589.8	643.4
5,000	6.7025	67.03	134.1	201.1	268.1	335.1	402.2	469.2	536.2	603.2	670.3	737.3	804.3
6,000	8.0430	80.43	160.9	241.3	321.7	402.2	482.6	563.0	643.4	723.9	804.3	884.7	965.1
7,000	9.3835	93.84	187.7	281.5	375.3	469.2	563.0	656.8	750.7	844.5	938.4	1032	1126
8,000	10.724	107.2	214.5	321.7	429.0	536.2	643.4	750.7	857.9	965.2	1072	1180	1287
9,000	12.065	120.6	241.3	361.9	482.6	603.2	723.9	844.5	965.2	1086	1206	1327	1448
10,000	13.405	134.1	268.1	402.2	536.2	670.3	804.3	938.3	1072	1206	1341	1475	1609

Wire Table.—The wire table on the following page (from a circular of the Westinghouse El. & Mfg. Co.) shows at a glance the size of wire necessary for the transmission of any given current over a known distance with a given amount of drop, for 100-volt and 500-volt circuits, with varying losses. The formula by which this table has been calculated is

$$\frac{D \times 1000}{C \times 2L} = R,$$

in which D equals the volts drop in electro-motive force, C the current, L the distance from the dynamo to the point of distribution, and R the line resistance in ohms per thousand feet.

EXAMPLE 1.—Required the size of wire necessary to carry a current of 60 amperes a distance of 650 feet with a loss of 5% at 100 volts.

Referring to the table, under 60 amperes, we find the given distance, 650 feet. In the same horizontal line and under 5% drop at 100 volts, we find No. 000 wire, which is the size required.

EXAMPLE 2.—What size will be required for 10 amperes 2000 feet, with a drop of 10% at 500 volts.

Under 10 amperes find 1930—the nearest figure to 2000—and in the same horizontal line under 10% at 500 volts find No. 11, the size required.

Wiring Formulæ for Incandescent Lighting. (W. D. Weaver, *Elec. World*, Oct. 15, 1892.)—A formula for calculating wiring tables is

$$A = \frac{2150\,W}{aE^2} LN, \quad \text{or,} \quad A = \frac{2150LC}{aE},$$

where A = section in circular mils; W = watt rating of lamps; E = voltage; L = distance to centre of distribution, in feet; N = number of lamps; a = percentage of drop; C = current in amperes.

EXAMPLE.—Volts, 50; amperes, 100; feet to centre of distribution, 100; drop, 2%.

$$\frac{2150 \times 100 \times 100}{2 \times 50} = 215{,}000 \text{ circular mils},$$

or about 0000 B. & S. gauge.

Wire Table for 100 and 500 Volt Circuits.

Per cent Drop at 500 Volts.								Amperes to Centre of Distribution.																										
		1		2	5	10	12.8																											
Per cent Drop at 100 Volts.																																		
2	4	5	8	10				10	12	14	16	18	20	25	30	35	40	45	50	60	70	80	90	100	125	150	175	200	225	250	275	300	350	400
Brown & Sharpe Gauge								Distances in Feet.																										
				0000	1	4	5	9800	8200	7000	6100	5500	4900	3920	3270	2810	2450	2180	1960	1630	1400	1220	1090	980	785	654	560	490	435	392	356	327	281	245
			0000	000	2	5	6	7800	6500	5600	4900	4400	3910	3115	2610	2240	1950	1735	1560	1300	1110	980	870	780	625	520	447	391	347	312	284	261	224	195
			000	00	3	6	7	6200	5160	4400	3900	3400	3090	2470	2090	1770	1540	1370	1240	1050	880	770	690	620	494	410	354	309	274	247	225	209	177	154
		0000	00	0	4	7	8	4900	4100	3500	3050	2730	2450	1960	1640	1400	1230	1090	980	820	700	610	550	490	392	326	280	245	218	196	178	164	140	123
	0000	000	0	1	5	8	9	3900	3240	2800	2450	2175	1950	1550	1290	1110	970	870	770	650	560	490	430	390	310	258	222	194	172	155	141	129	110	97
	000	00	1	2	6	9	10	3100	2600	2200	1940	1700	1550	1228	1030	880	770	682	610	510	440	380	340	310	245	204	175	155	136	123	111	103	88	77
	00	0	2	3	7	10	11	2440	2033	1740	1525	1355	1220	976	813	700	610	543	485	407	346	305	271	244	195	162	140	122	108	98	89	81	70	61
0000	0	1	3	4	8	11	12	1930	1608	1378	1206	1072	965	774	643	553	483	430	386	322	276	241	214	193	154	128	110	96	86	77	70	64	55	48
000	1	2	4	5	9	12	13	1530	1278	1095	959	852	767	615	511	449	384	341	307	256	219	192	170	153	123	102	88	77	68	62	56	51	44	38
00	2	3	5	6	10	13	14	1216	1013	867	760	675	608	488	405	348	304	270	243	203	173	152	135	122	97	81	70	61	54	49	44	40	35	30
0	3	4	6	7	11	14	15	965	804	689	603	536	483	386	322	276	241	214	193	161	138	121	107	97	77	64								
1	4	5	7	8	12	15	16	835	695	596	521	464	418	334	278	239	208	176	167	129	119	104	93	84	67	51								
2	5	6	8	9	13	16	17	609	506	434	380	338	304	243	201	174	152	135	121	101	87	76	67	61	48	40								
3	6	7	9	10	14	17	18	481	400	344	300	267	240	192	160	137	120	107	96	80	69	60	52	48	38	32								
4	7	8	10	11	15	18	19	381	318	272	238	212	191	152	127	109	96	85	77	64	55	52	46	42	32	25								
5	8	9	11	12	16	19	20	302	252	216	189	168	151	121	101	87	76	67	61	50														
6	9	10	12	13	17	20		240	200	172	150	133	120	96	80	69	60	53	48	40														
7	10	11	13	14	18			189	157	135	118	105	95	76	62	54	48	42	38	32														
8	11	12	14	15	19			152	126	108	95	84	76	60	50	43	38	33	30	25														
9	12	13	15	16	20			120	100	85	75	66	60	48	40	35	30	26	24	20														
10	13	14	16	17				95	79	68	59	53	48	38	31	27																		
11	14	15	17	18				75	63	54	48	42	38	30	25	21																		
12	15	16	18	19				58	48	43	38	33	30	24	20	17																		
13	16	17	19	20				47	40	34	30	26	24	19	15	13																		
14	17	18	20					38	31	27	24	21	19	15	13	10																		
15	18	19						29	24	21	19																							
16	19	20						24	20	17	15																							
17	20							19	15	14	12																							
18								14	12	10	9																							
19								12	10	8	7																							

Safe Current in Amperes.

Size.	Bare Overhead.	Insulated House Wire.
0000	620	175
000	525	145
00	438	120
0	369	100
1	309	85
2	260	70
3	219	60
4	184	50
5	154	45
6	130	35
7	109	30
8	92	25
9	77	22
10	65	20
12	54	15
14	32	10
16	23	5
18	16	4
20	10	2

The horse-power and efficiency of a motor being given, the size of the conducting wire in circular mils can be found from the following formula:

$$A = \frac{160{,}400{,}000 \times \text{H.P.} \times L}{aE^2 \times \text{efficiency}}.$$

EXAMPLE.—Horse-power, 10; volts, 500; drop, 3%; feed to distributing point, 600: efficiency of motor, 75%.

$$A = \frac{160{,}400{,}000 \times 10 \times 600}{3 \times 500 \times 500 \times 75} = 17{,}109 \text{ circular mils, or about No. 8 B. \& S.}$$

Cost of Copper for Long-distance Transmission.

(Westinghouse El. & Mfg. Co.)

COST OF COPPER REQUIRED FOR THE DELIVERY OF ONE MECHANICAL HORSE-POWER AT MOTOR SHAFT WITH 1000, 2000, 3000, 4000, 5000, AND 10,000 VOLTS AT MOTOR TERMINALS, OR AT TERMINALS OF LOWERING TRANSFORMERS.

Loss of energy in conductors (drop), equals 20%.
Distances equal one to twenty miles.
Motor efficiency equals 90%.
Length of conductor per mile of single distance, 11,000 feet, to allow for sag.
Cost of copper taken at 16 cents per pound. (This figure is too high. An approximate figure now, 1897, is 12 cents per pound.)

Miles.	1000 v.	2000 v.	3000 v.	4000 v.	5000 v.	10,000 v.
1	$2.08	$0.52	$0.23	$0.13	$0.08	$0.02
2	8.33	2.08	0.93	0.52	0.33	0.08
3	18.70	4.68	2.08	1.17	0.75	0.19
4	33.30	8.32	3.70	2.08	1.33	0.33
5	52.05	13.00	5.78	3.25	2.08	0.52
6	74.90	18.70	8.32	4.68	3.00	0.75
7	102.00	25.50	11.30	6.37	4.08	1.02
8	133.25	33.30	14.80	8.32	5.33	1.33
9	168.60	42.20	18.70	10.50	6.74	1.69
10	208.19	52.05	23.14	13.01	8.33	2.08
11	251.90	63.00	28 00	15.75	10.08	2.52
12	299.80	75.00	33.30	18.70	12.00	3.00
13	352.00	88.00	39.00	23.00	14.08	3.52
14	408 00	102.00	45.30	25.50	16.32	4.08
15	468.00	117.00	52.00	29.25	18.72	4.68
16	533.00	133.00	59.00	33.30	21.32	5.33
17	600.00	150.00	67.00	37.60	24.00	6.00
18	675.00	169.00	75.00	42.20	27.00	6.75
19	750.00	188.00	83.50	47.00	30.00	7.50
20	833.00	208.00	92.60	52.00	33.32	8.33

Weight of Copper required for Long-distance Transmission.—W. F. C. Hasson (Trans. Tech. Socy. of the Pacific Coast, vol. x, No. 4) gives the following formula:

$$W = \frac{D^2}{E^2} \text{H.P.} \frac{(100 - L)}{L} 266.5,$$

where W is the weight of copper wire in pounds; D, the distance in miles; E, the E.M.F. at the motor in hundreds of volts; H.P., the horse-power delivered to the motor; L, the per cent of line loss.

Thus, to transmit 200 horse-power ten miles with 10 per cent loss, and have 3000 volts at the motor, we have

$$W = \frac{10 \times 10}{30 \times 30} \times 200 \times \frac{(100 - 10)}{10} \times 266.5 = 53{,}300 \text{ lbs.}$$

COST OF COPPER REQUIRED TO DELIVER ONE MECHANICAL HORSE-POWER AT MOTOR-SHAFT WITH VARYING PERCENTAGES OF LOSS IN CONDUCTORS, UPON THE ASSUMPTION THAT THE POTENTIAL AT MOTOR TERMINALS IS IN EACH CASE 3000 VOLTS.' (Westinghouse El. & Mfg. Co.)

Distances equal one to twenty miles.
Motor efficiency equals 90%.
Length of conductor per mile of single distance, 11,000 feet, to allow for sag.
Cost of copper equals 16 cents per pound.

Miles.	10%	15%	20%	25%	30%
1	$0.52	$0.33	$0.23	$0.17	$0.13
2	2.08	1.31	0.93	0.69	0.54
3	4.68	2.95	2.08	1.55	1.21
4	8.32	5.25	3.70	2.77	2.15
5	13.00	8.20	5.78	4.33	3.37
6	18.70	11.75	8.32	6.23	4.85
7	25.50	16.00	11.30	8.45	6.60
8	33.30	21.00	14.80	11.00	8.60
9	42.20	26.60	18.75	14.00	10.90
10	52.05	32.78	23.14	17.31	13.50
11	63.00	39.75	28.00	21.00	16.30
12	75.00	47.20	33.30	24.90	19.40
13	88.00	55.30	39.00	29.20	22.80
14	102.00	64.20	45.30	33.90	26.40
15	117.00	73.75	52.00	38.90	30.30
16	133.00	83.80	59.00	44.30	34.50
17	150.00	94.75	67.00	50.00	39.00
18	169.00	106.00	75.00	56.20	43.80
19	188.00	118.00	83.50	62.50	48.70
20	208.00	131.00	92.60	69.25	54.00

Efficiency of Long-distance Transmission. (F. R. Hart, *Power*, Feb. 1892.)—The mechanical efficiency of a system is the ratio of the power delivered to the dynamo-electric machines at one end of the line to the power delivered by the electric motors at the distant end. The commercial efficiency of a dynamo or motor varies with its load. Under the most favorable conditions we must expect a loss of say 9% in the dynamo and 9% in the motor. The loss in transmission, due to fall in electrical pressure or "drop" in the line, is governed by the size of the wires, the other conditions remaining the same. For a long-distance transmission plant this will vary from 5% upwards. With a loss of 5% in the line the total efficiency of transmission will be slightly under 79%. With a loss of 10% in the line it will be slightly under 75%. We may call 80% the practical limit of the efficiency with the apparatus of to-day. The methods for long-distance transmission may be divided into three general classes: (1) continuous current; (2) alternating current; and (3) regenerating or "motor-dynamo" systems.

There are many factors which govern the selection of a system. For each problem considered there will be found certain fixed and certain unfixed conditions. In general the fixed factors are: (1) capacity of source of power; (2) cost of power at source; (3) cost of power by other means at point of delivery; (4) danger considerations at motors; (5) operation conditions; (6) construction conditions (length of line, character of country, etc.). The partly fixed conditions are: (7) power which must be delivered, i.e., the efficiency of the system; (8) size and number of delivery units. The variable conditions are: (9) initial voltage; (10) pounds of copper on line; (11) original cost of all apparatus and construction; (12) expenses, operating (fixed charges, interest, depreciation, taxes, insurance, etc.); (13) liability of trouble and stoppages; (14) danger at station and on line; (15) convenience in operating, making changes, extensions, etc. Assuming that the cost of dynamos, motors, etc., will be approximately the same whatever the initial pressure, the great variation in the cost of wire at different pressures is shown by Mr. Hart in the following figures, giving the weights of copper required for transmitting 100 horse-power 5 miles:

Voltage.	Drop 10 per cent.	Drop 20 per cent.
2,000	16,800 lbs.	8,400 lbs.
3,000	7,400 "	3,700 "
10,000	620 "	310 "

The subdivisions of each of the general methods of transmission are tabulated as follows:

Continuous current	2-wire	Low voltage	One machine.
			Machines in parallel.
		High voltage	One machine.
			Machines in parallel.
			Machines in series.
	3-wire		2 machines in series.
			Machines in multiple series
	Multiple-wire		Machines in series.
Alternating current	Alternating single phase		Without conversions.
			With conversions.
	Alternating multiphase		Without conversions.
			With conversions.
Regenerating systems	Alternating continuous.		
	Alternating converter; line converter; alternating continuous.		
	Continuous-continuous.		
	Partial reconversion of any system.		

The relative advantages of these systems vary with each particular transmission problem, but in a general way may be tabulated as below.

	System.		Advantages.	Disadvantages.
Continuous.	2-wire	Low voltage.	Safety, simplicity.	Expense for copper.
		High voltage.	Economy, simplicity.	Danger, difficulty of building machines.
	3-wire.		Low voltage on machines and saving in copper.	Not saving enough in copper for long distances. Necessity for "balanced" system.
	Multiple-wire.		Low voltage at machines and saving in copper.	
Alternating.	Single phase.		Economy of copper.	Cannot start under load. Low efficiency.
	Multiphase.		Economy of copper, synchronous speed unnecessary; applicable to very long distances.	Requires more than two wires.
	Motor-dynamo.		High-voltage transmission. Low-voltage delivery.	Expensive. Low efficiency.

A Graphical Method of calculating leads for wiring for electric lighting is described by Carl Hering in Trans. A. I. E. E., 1891. He furnishes a chart containing three sets of diagonal straight-line diagrams so connected that the examples under the general formula for wiring may be solved without calculation by simply locating three points in succession on the chart.

Systems of Electrical Distribution in Common Use. (Chas. T. Scott, Proc. Engrs. Soc'y of Western Penna., 1895.)

I. CONTINUOUS OR DIRECT CURRENT.

A. Constant Potential.

110 Volts.—Distances less than, say, 1500 feet.
For incandescent lamps.
For arc-lamps, usually 2 in series,
For motors.

220 Volts.—Distances less than, say, 3000 feet.
For incandescent lamps, usually 2 in series.
For arc-lamps, usually 4 in series.
For motors.
220 Volts, 3-wire.—Distances less than, say, 3000 feet.
For incandescent lamps.
For arc-lamps, usually 2 in series on each branch.
For motors 110 or 220 volts, usually 220 volts.
500 Volts.—Distances less than, say, 8000 feet.
For incandescent lamps, usually 5 in series.
For arc-lamps, usually 10 in series.
For motors, stationary and street-car.

B. Constant Current.
Usually about 10 amperes, the volts increasing to several thousand, as demanded.
For arc-lamps.
For motors.

II. ALTERNATING CURRENT.
A. Constant Potential.
Ordinarily, about 16,000 or 7200 alternations per minute. Primary circuit, 1000 or 2000 volts; secondary circuit, 50 or 100 volts.
For incandescent lamps.
For arc-lamps.
For small motors.
Multiphase Systems.
For lighting.
For motors.
For rotary transformers for giving direct current.

B. Constant Current.
Usually 10 amperes.
For arc-lamps.

Efficiency of a Combined Engine and Dynamo.—A compound double-crank Willans engine mounted on a single base with a dynamo of the Edison-Hopkinson type was tested in 1890, with results as follows: The low-pressure cylinder is 14 in. diam., 16 in. stroke; steam-pressure 120 lbs. It is coupled to a dynamo constructed for an output of 475 amperes at 110 volts when driven at 430 revolutions per minute. The armature is of the bar construction, is plain shunt-wound, and is fitted with a commutator of hard-drawn copper with mica insulation. Four brushes are carried on each rocker-arm.

Resistance of magnets	16.	ohms
Resistance of armature	0.0055	"
I.H.P	83.3	
E.H.P	72.2	
Total efficiency	86.7	per cent
Consumption of water per I.H.P. hour	21.6	pounds
Consumption of water per E.H.P. hour	25	"

The engine and dynamo were worked above their full normal output, which fact would tend to slightly increase the efficiency.

The electrical losses were: Loss in magnet coils, 756 watts, equal to 1.4%; loss in armature coil, 1386 watts, equal to 2.6%; so that the electrical efficiency of the machine due to ohmic resistance alone was 96%. The remainder of the losses, a little over 8 horse-power, is due to friction of engine and dynamo, hysteresis, and the like.

Electrical Efficiency of a Generator and Motor.—A twelve-mile transmission of power at Bodie, Cal., is described by T. H. Leggett (Trans. A. I. M. E. 1894). A single-phase alternating current is used. The generator is a Westinghouse 120 K. W. constant-potential 12-pole machine, speed 860 to 870 revs. per min. The motor is a synchronous constant-potential machine of 120 horse-power. It is brought up to speed by a 10-H.P. Tesla starting motor. Tests of the electrical efficiency of the generator and motor gave the following results;

TEST ON GENERATOR.

	Amperes	Volts.	Watts.
Self-excited field	15.8	60	948
Separately-excited field	18.2	78	1419.6
Resistance of armature, 1.6618 ohms.			
C^2R, loss in armature			664.72
Total loss in machine			3032.32
Load	20	3414	68280

Apparent electrical efficiency of generator, 95.559%.

TEST ON MOTOR.

	Amperes	Volts.	Watts.
Self-excited field	52	62.4	3244.8
Resistance of armature, 1.4 ohms.			
C^2R, loss in armature			560.0
Total loss in machine			3804.08
Load	20	3110	62200

Apparent electrical efficiency of motor, 93.883%.

Efficiency of an Electrical Pumping-plant. (*Eng. & M. Jour.*, Feb. 7, 1891.)—A pumping-plant at a mine at Normanton, England, was tested, with results given below:

Above ground there is a pair of 20½ × 48-in. engines running at 20 revs. per min., driving two series dynamos giving 690 volts and 59 amperes. The current from each dynamo is carried into the mine by an insulated cable about 3000 feet long. There they are connected to two 50-h.p. motors which operate a pair of differential ram-pumps, with rams 6 in. and 4¼ in. diam. and 24 in. stroke. The total head against which the pumps operate is 890 feet. Connected to the same dynamos there is also a set of gearing for driving a hauling plant on a continuous-rope system, and a set of three-throw ram-pumps with 6-inch rams and 12-inch stroke can also be thrown into gear. The connections are so made that either motor can operate any or all three of the sets of machinery just described. Indicator-diagrams gave the following results:

Friction of engine	6.9 H.P.	9.4%
Belt and dynamo friction	4.8 "	6.5%
Leads and motor	6.7 "	9.4%
Motor belt, gearing and pumps empty	10.2 "	14.0%
Load of 117 gallons through 890 feet	31.5 "	43.1%
Water friction in pumps and rising main	12.9 "	17.6%
	73.0 H.P.	100.0%

At the time when these data were obtained the total efficiency of the plant was 43.1%, but in a later test it rose to 47%.

References on Power Distribution.—Kapp, Electric Transmission of Energy; Badt, Electric Transmission Handbook; Martin and Wetzler, The Electric Motor and its Applications; Hospitalier, Polyphased Electric Currents.

ELECTRIC RAILWAYS.

Space will not admit of a proper treatment of this subject in this work. Consult Crosby and Bell, The Electric Railway in Theory and Practice, price $2 50; Fairchild, Street Railways, price $4 00; Merrill, Reference Book of Tables and Formulæ for Street Railway Engineers, price $1.00.

Test of a Street Railway Plant.—A test of a small electric-railway plant is reported by Jesse M. Smith in Trans. A. S. M. E., vol. xv. The following are some of the results obtained:

Friction of engine, air-pump, and boiler feed-pump; main belt off	9.22 I.H.P.
Friction of engine, air and feed pumps, and dynamo, brushes off.	11.34 I.H.P.
Friction of dynamo and belt	2.12 I.H.P.
Power consumed by engine, air and feed pumps and dynamo, with brushes on and main circuit open	14.34 I.H.P.
Power required to charge fields of dynamo	3.00 I.H.P.
Rated capacity of engine and dynamo ....each	150 I.H.P.
Power developed by engine........min. 21.27; max. 141.4; mean,	70.1 I.H.P.
Volts developed by dynamo..............range, 480 to 520; average,	501 volts
Amperes developed by dynamo.....max. 200; min. 4.7; average,	67 amperes
Average watts delivered by dynamo	33,567 Watts
Average electrical horse-power delivered by dynamo	45 E.H.P.
Average I.H.P. del'd to pulley of dynamo, estimating friction of armature shaft to be the same as friction of belt	59.8 I.H.P.
Average commercial efficiency of dynamo............45 ÷ 59.8 =	75.25%
Average number of cars in use during test	2.89 cars.
Number of single trips of cars	64
Average number of passengers on cars per single trip	15.2
Weight of cars	14,500 lbs.
Est. total weight of cars and persons	15,900 lbs.
Average weight in motion	45,950 lbs.
Average electrical horse-power per 1000 lbs. of weight moved	0.98 E.H.P.
Average horse-power developed by engine per 1000 lbs. of weight moved	1.52 I.H.P.
Average watts required per car	11,615 watts
Average electrical horse-power per car	15.54 E.H.P.
Average horse-power developed in engine per car	24.25 I.H.P.
Length of road	10.5 miles.

Average speed, including all stops, 21 miles in 1.5 hours = 14 miles per hour.
Average speed between stops, 21 m. in 1.366 hours = 15.38 miles per hour.

ELECTRIC LIGHTING.

Life of Incandescent Lamps. (*Eng'g*, Sept. 1, 1893, p. 282.)—From experiments made by Messrs. Siemens and Halske, Berlin, it appears that the average life of incandescent lamps at different expenditure of watts per candle-power is as follows:

Watts per candle-power	1.5	2	2.5	3	3.5
Life of lamp, hours	45	200	450	1000	1000

Life and Efficiency Tests of Lamps. (P. G. Gossler, *Elec. World*, Sept. 17, 1892.)—Lamps burning at a voltage above that for which they are rated give a much greater illuminating power than 16 candles, but at the same time their life is very considerably shortened. It has been observed that lamps received from the factory do not average the same candle-power and efficiency for different invoices; that is, lamps which are received in one invoice are usually quite uniform throughout that lot, but they vary considerably from lamps made at other times.

The following figures show the different illuminating-powers of a 16-c.p., 50-volt, 52-watt lamp, for various voltages from 25 to 80 volts:

Volts:	25	34.8	40	48	50	52.5	55.6	59.5	62	68.2	72.5	80
Amperes:	.561	.774	.898	.968	1.055	1.097	1.161	1.226	1.29	1.419	1.484	1.58
Candles:	.4	2.47	5.1	12.6	15.8	20.5	28.4	39.3	50.7	74.5	103.2	141
Watts:	14.03	26.94	35.92	46.34	52.75	57.57	64.55	72.92	79.98	96.78	107.5	126.4
Watts per c.p.:	35.1	10.81	7.04	3.68	3.34	2.81	2.30	1.96	1.58	1.30	1.04	.90

Street-lighting. (H. Robinson, M.I.C.E., *Eng'g News*, Sept. 12, 1891.) —For street-lighting the arc-lamp is the most economical. The smallest size of arc-lamp at present manufactured requires a current of about 5 amperes; but for steadiness and efficiency it is desirable to use not less than 6 amperes. (Good 8-ampere lamps are now on the market. 1897.) The candle-power of arc-lamps varies considerably, according to the angle at which it is measured. The greatest intensity with continuous-current lamps is found at an angle of about 40° below the horizontal line. The following table gives the approximate candle-power at various angles. The height of the lamps should be arranged so as to give an angle of not less than 7° to the most distant point it is intended to serve.

Lighting-power of Arc-lamps.

Current in Amperes.	Candle-power. Horizontal	At Angle of 7°.	At Angle of 10°.	At Angle of 20°.	Maximum at Angle of 40°.
6	92	175	207	322	460
8	156	300	350	546	780
10	220	420	495	770	1100

The following data enable the coefficient of minimum lighting-power in streets to be determined:

Let P = candle-power of lamps;
L = maximum distance from lamp in feet;
H = height of lamp in feet;
X = a coefficient.

The light falling on the unit area of pavement varies inversely as the square of the distance from the lamp, and is directly proportional to the angle at which it falls. This angle is nearly proportional to the height of the lamp divided by the distance. Therefore

$$X = \frac{P}{L^2} \times \frac{H}{L} \quad \text{or} \quad X = \frac{PH}{L^3}.$$

The usual standard of gas-lighting is represented by the amount of light falling on the unit area of pavement 50 feet away from a 12-c.p. gas-lamp 9 feet high, which gives a coefficient as follows:

$$X = \frac{12 \times 9}{50^3} = 0.000864.$$

The minimum standard represents the amount of light on a unit area 50 feet away from a 24-c.p. lamp, 9 ft. high, and gives the coefficient .001728.

Adopting the first of the above coefficients, Mr. Robinson calculates that the before-mentioned sizes of arc-lights will give the same standard of light at the heights and distances stated in Table A. Table B gives the corresponding distances, assuming the minimum standard to be adopted.

TABLE A.

Hgt. of Lamps.	20 ft.	25 ft.	30 ft.	35 ft.
Current in Amperes.	Max. distances served from lamp, in ft.			
6	160	175	190	202
8	185	202	220	235
10	205	225	243	260

TABLE B.

Height.....	20 ft.	25 ft.	30 ft.	35 ft.
Amperes.	Max. distances served from Lamp.			
6	130	144	155	166
8	150	165	180	193
10	170	190	205	[illegible]

The distances the lamps are apart would, of course, be double the distances mentioned in Tables A and B. One arc-lamp will take the place of from 3 to 6 gas-lamps, according to the locality, arrangement, and standard of light adopted. A scheme of arc-lighting, based on the substitution of one arc-light on the average for 3½ to 4 gas-lamps, would double the minimum standard of light, while the average standard would be increased 10 or 12 times.

Candle-power of the Arc-light. (Elihu Thomson, *El. World*, Feb. 28, 1891.)—With the long arc the maximum intensity of the light is from 40° to 60° downward from the horizontal. The spherical candle-power is only a fraction of the rated c.p., which is generally taken at the maximum obtainable in the best direction. For this reason the term 2000 c.p. has little significance as indicating the illuminating-power of an arc. It is now generally taken to mean an arc with 10 amperes and not less than 45 volts between the carbons, or a 450-watt arc. The quality of the carbons will determine whether the 450 watts are expended in obtaining the most light or not, or whether that light will have a maximum intensity at one angle or another within certain limits. The larger the current passing in an arc, the less is its resistance. Well-developed arcs with 4 amperes will have about 11 ohms, with 10 amperes 4.5 ohms, and with 100 amperes .45 ohm.

It is not unusual to run from 50 to 60 lights in a series, each demanding from 45 to 50 volts, or a total of, say, 3000 volts. In going beyond this the difficulties of insulation are greatly increased.

Reference Books on Electric Lighting.—Noll, How to Wire Buildings, $1.00; Hedges, Continental Electric-light Central Stations, $6.00; Fleming, Alternating Current Transformers in Theory and Practice, 2 vols., $8.00; Atkinson, Elements of Electric Lighting, $1.50; Algave and Boulard, Electric Light: its History, Production, and Application, $5.00.

ELECTRIC WELDING.

The apparatus most generally used consists of an alternating-current dynamo, feeding a comparatively high-potential current to the primary coil of an induction-coil or transformer, the secondary of which is made so large in section and so short in length as to supply to the work currents not exceeding two or three volts, and of very large volume or rate of flow. The welding clamps are attached to the secondary terminals. Other forms of apparatus, such as dynamos constructed to yield alternating currents direct from the armature to the welding-clamps, are used to a limited extent.

The conductivity for heat of the metal to be welded has a decided influence on the heating, and in welding iron its comparatively low heat conduction assists the work materially. (See papers by Sir F. Bramwell, Proc. Inst. C. E., part iv., vol. cii. p. 1; and Elihu Thomson, Trans. A. I. M. E., xix. 877.)

Fred. P. Royce, *Iron Age*, Nov. 28, 1892, gives the following figures showing the amount of power required to weld axles and tires:

AXLE-WELDING.

	Seconds.
1-inch round axle requires 25 H.P. for	45
1-inch square axle requires 30 H.P. for	48
1¼-inch round axle requires 35 H.P. for	60
1¼-inch square axle requires 40 H.P. for	70
2-inch round axle requires 75 H.P. for	95
2-inch square axle requires 90 H.P. for	100

The slightly increased time and power required for welding the square axle is not only due to the extra metal in it, but in part to the care which it is best to use to secure a perfect alignment.

TIRE-WELDING.

	Seconds.
1 × 3/16-inch tire requires 11 H.P. for	15
1¼ × ⅜-inch tire requires 23 H.P. for	25
1½ × ⅜-inch tire requires 20 H.P. for	30
1½ × ½-inch tire requires 23 H.P. for	40
2 × ½-inch tire requires 29 H.P. for	55
2 × ¾-inch tire requires 42 H.P. for	62

The time above given for welding is of course that required for the actual application of the current only, and does not include that consumed by placing the axles or tires in the machine, the removal of the upset and other finishing processes. From the data thus submitted, the cost of welding can be readily figured for any locality where the price of fuel and cost of labor are known.

In almost all cases the cost of the fuel used under the boilers for producing power for electric welding is practically the same as the cost of fuel used in forges for the same amount of work, taking into consideration the difference in price of fuel used in either case.

Prof. A. B. W. Kennedy found that 2½-inch iron tubes ¼ inch thick were welded in 61 seconds, the net horse-power required at this speed being 23.4 (say 33 indicated horse-power) per square inch of section. Brass tubing required 21.2 net horse-power. About 60 total indicated horse-power would be required for the welding of angle-irons 3 × 3 × ½ inch in from two to three minutes. Copper requires about 80 horse-power per square inch of section, and an inch bar can be welded in 25 seconds. It takes about 90 seconds to weld a steel bar 2 inches in diameter.

ELECTRIC HEATERS.

Wherever a comparatively small amount of heat is desired to be automatically and uniformly maintained, and started or stopped on the instant without waste, there is the province of the electric heater.

The elementary form of heater is some form of resistance, such as coils of thin wire introduced into an electric circuit and surrounded with a substance, which will permit the conduction and radiation of heat, and at the same time serve to electrically insulate the resistance.

This resistance should be proportional to the electro-motive force of the current used and to the equation of Joule's law:

$$H = C^2Rt \times 0.24,$$

where C is the current in amperes; R, the resistance in ohms; t, the time in seconds; and h, the heat in gram-centigrade units.

Since the resistance of metals increases as their temperature increases, a thin wire heated by current passing through it will resist more, and grow hotter and hotter until its rate of loss of heat by conduction and radiation equals the rate at which heat is supplied by the current. In a short wire, before heat enough can be dispelled for commercial purposes, fusion will begin; and in electric heaters it is necessary to use either long lengths of thin wire, or carbon, which alone of all conductors resists fusion. In the majority of heaters, coils of thin wire are used, separately embedded in some substance of poor electrical but good thermal conductivity.

The Consolidated Car-heating Co.'s electric heater consists of a galvanized iron wire wound in a spiral groove upon a porcelain insulator. Each heater is 30⅝ in. long, 8⅞ in. high, and 6⅝ in. wide. Upon it is wound 392 ft. of wire. The weight of the whole is 23⅛ ibs.

Each heater is designed to absorb 1000 watts of a 500-volt current. Six heaters are the complement for an ordinary electric car. For ordinary weather the heaters may be combined by the switch in different ways, so that five different intensities of heating-surface are possible, besides the position in which no heat is generated, the current being turned entirely off.

For heating an ordinary electric car the Consolidated Co. states that from 2 to 12 amperes on a 500-volt circuit is sufficient. With the outside temperature at 20° to 30°, about 6 amperes will suffice. With zero or lower temperature, the full 12 amperes is required to heat a car effectively.

Compare these figures with the experience in steam-heating of railway-cars, as follows:

1 B.T.U. = 0.29084 watt-hours.

6 amperes on a 500-volt circuit = 3000 watts.

A current consumption of 6 amperes will generate 3000 ÷ 0.29084 = 10,315 B.T.U. per hour.

In steam-car heating, a passenger coach usually requires from 60 lbs. of steam in freezing weather to 100 lbs. in zero weather per hour. Supposing the steam to enter the pipes at 20 lbs. pressure, and to be discharged at 200° F., each pound of steam will give up 983 B.T.U. to the car. Then the equivalent of the thermal units delivered by the electrical-heating system in pounds of steam, is 10,315 ÷ 983 = 10½, nearly.

Thus the Consolidated Co.'s estimates for electric-heating provide the equivalent of 10½ lbs. of steam per car per hour in freezing weather and 21 lbs. in zero weather.

Suppose that by the use of good coal, careful firing, well-designed boilers, and triple-expansion engines we are able in daily practice to generate 1 H.P. delivered at the fly-wheel with an expenditure of 2½ lbs. of coal per hour.

We have then to convert this energy into electricity, transmit it by wire to the heater, and convert it into heat by passing it through a resistance-coil. We may set the combined efficiency of the dynamo and line circuit at 85%, and will suppose that all the electricity is converted into heat in the resistance-coils of the radiator. Then 1 brake H.P. at the engine = 0.85 electrical H.P. at the resistance-coil = 1,683,000 ft.-lbs. energy per hour = 2180 heat-units. But since it required 2½ lbs. of coal to develop 1 brake H.P., it follows that the heat given out at the radiator per pound of coal burned in the boiler furnace will be 2180 ÷ 2½ = 872 H.U. An ordinary steam-heating system utilizes 9652 H.U. per lb. of coal for heating; hence the efficiency of the electric system is to the efficiency of the steam-heating system as 872 to 9652, or about 1 to 11. (*Eng'y News*, Aug. 9, '90; Mar. 30, '92; May 15, '93.)

ELECTRICAL ACCUMULATORS OR STORAGE-BATTERIES.

Storage-batteries may be divided into two classes: viz., those in which the active material is formed from the substance of the element itself, either by direct chemical or electro-chemical action, and those in which the chemical formation is accelerated by the application of some easily reducible salt of lead. Elements of the former type are usually called Planté, and those of the latter "Faure," or "pasted."

Faraday when electrolyzing a solution of acetate of lead found that peroxide of lead was produced at the positive and metallic lead at the negative pole. The surfaces of the elements in a newly and fully charged Planté cell consists of nearly pure peroxide of lead, PbO_2, and spongy metallic lead, Pb, respectively on the positive and negative plates.

During the discharge, or if the cell be allowed to remain at rest, the sulphuric acid (H_2SO_4) in the solution enters into combination with the peroxide and spongy lead, and partially converts it into sulphate. The acid being continually abstracted from the electrolyte as the discharge proceeds, the density of the solution becomes less. In the charging operation this action is reversed, as the reducible sulphates of lead which have been formed are apparently decomposed, the acid being reinstated in the liquid and therefore causing an increase in its density.

The difference of potential developed by lead and lead peroxide immersed in dilute H_2SO_4 is, as nearly as may be, two volts.

A lead-peroxide plate gradually loses its electrical energy by local action, the rate of such loss varying according to the circumstances of its preparation and the condition of the cell. Various forms of both Planté and Faure batteries are illustrated in "Practical Electrical Engineering."

In the Faure or pasted cells lead plates are coated with minium or litharge made into a paste with acidulated water. When dry these plates are placed in a bath of dilute H_2SO_4 and subjected to the action of the current, by which the oxide on the positive plate is converted into peroxide of lead and that on the negative plate reduced to finely divided or porous lead.

Gladstone and Tribe found that the initial electro-motive force of the Faure cell averaged 2.25 volts, but after being allowed to rest some little time it was reduced to about 2.0 volts.

The following tables give the elements of several sizes of "chloride" accumulators made by the Electric Storage Battery Co., Philadelphia. Type G is furnished in cells containing from 11 to 125 plates, and type H from 21 plates to any greater number desired. The voltage of cells of all sizes is

	TYPE "B." Size of Plates, 3 × 3 in.	TYPE "C." Size of Plates, 4 × 4 in.			TYPE "D." Size of Plates, 6 × 6 in.					
Number of plates	3	3	5	7	3	5	7	9	11	13
Discharge in amperes: For 8 hours	⅝	1¼	2½	3¾	2½	5	7½	10	12½	15
Discharge in amperes: " 5 "	⅞	1¾	3½	5¼	3½	7	10½	14	17½	21
Discharge in amperes: " 3 "	1¼	2½	5	7½	5	10	15	20	25	30
Normal charge rate	⅝	1¼	2½	3¾	2½	5	7½	10	12½	15
Weight of each element, lbs	3	5	8	11	8	14	20	26	32	38
Outside measurement of rubber jar in inches: Width	1¾	2	3	4	2	3	4	5	6	7
Outside measurement of rubber jar in inches: Length	3⅝	4¾	4¾	4¾	6½	6½	6½	6½	6½	6½
Outside measurement of rubber jar in inches: Height	5	7	7	7	9	9	9	9	9	9
Outside measurement of glass jar in inches: Width	2¼	3	4	5¼	3¼	4¾	6½	7⅝	7⅝	7⅝
Outside measurement of glass jar in inches: Length	4	5¼	5¼	5¼	7⅞	7⅞	7⅞	8¾	8¾	8¾
Outside measurement of glass jar in inches: Height	4½	6½	6½	6½	9½	9½	8⅝	8⅝	8⅝	8⅝
Weight of acid in glass jars in lbs	1	3¼	4	5	6	9¼	13¾	16¼	14¼	18
Weight of acid in rubber jars in lbs	¾	1	2¼	2⅝	2¼	3½	5¼	7	8½	10
Weight of cell complete, with acid, in rubber jars in lbs.	4	6½	10½	14¼	11½	18½	26	35	42¼	50
Height of cell over all in inches.	8	10	10	10	12½	12½	12½	12½	12½	12½

	TYPE "E." Size of Plates, 7¾ × 7¾ in.						TYPE "F." Size of Plates, 10½ × 10½ in.					
Number of plates......	5	7	9	11	13	15	9	11	13	15	17	19
Discharge in amperes: For 8 hours	10	15	20	25	30	35	40	50	60	70	80	90
" 5 "	14	21	28	35	42	49	56	70	84	98	112	126
" 3 "	20	30	40	50	60	70	80	100	120	140	160	180
Normal charge rate....	10	15	20	25	30	35	40	50	60	70	80	90
Weight each element, lbs..................	23	33	43	52	62	71	86	106	125	145	165	184
Outside meas'm't: rubber jar. Width, in.,	3	4	5	6⅛	7¼	8½	....	lead tank	16¾	18⅝	20	21¾
Length, "	8½	8½	8½	8½	8½	8½	....	lead tank	15	15	15	15
Height, "	11	11	11	11	11	11	..	lead tank	17¾	17¾	17¾	17¾
glass jar. Width, "	5½	6¾	8	8⅝	11	11	9	10⅝	10⅝	12	....	....
Length, "	9⅛	9⅛	9⅛	9⅛	9⅛	9⅛	12½	12½	12½	12¾	....	....
Height, "	11¼	11¼	11¼	11¼	11¼	11¼	15½	15½	15½	15½	....	...
Weight of acid in glass jars in lbs..........	17	21	25	27	35	34	53	61	58	70	....	....
Weight of acid in rubber jars in lbs........	6½	9	11½	14½	17½	21	....	lead tank	94	104	114	124
Weight of cell complete, with acid, in rubber jar in lbs...	31	42	54	66	79	91	....		202	339	376	415
Height of cell over all, in inches......... ...	14½	14½	14½	14½	14½	14½	18	18	18	19	19	19

	TYPE "G." Size of Plates, 15½ × 15½ in.							TYPE "H." Size of Plates, 15½ × 32 in.				
Number of plates.	11	13	15	17	25	125	D*	21	23	25	125	D*
Discharge in amperes: For 8 hrs.	100	120	140	160	240	1240	10	400	440	480	2480	20
" 5 "	140	168	196	224	336	1736	14	560	616	672	3472	28
" 3 "	200	240	280	320	480	2480	20	800	880	960	4960	40
Normal charge rate....	100	120	140	160	240	1240	10	400	440	480	2480	20
Weight of each element, lbs....	219	260	300	341	503	2538	20.4	790	866	942	4741	38
Outside measurement of tank in inches. Width	15⅛	16¾	18⅝	20	27⅝	111¼	⅞	25⅛	26¾	28¾	111¼	⅞
Length	19¾	19¾	19¾	19¾	20¾	21½	...	21½	21½	21½	21½	...
Height	22⅞	22⅞	22⅞	22⅞	22⅞	24¾	....	42⅞	42⅞	42⅞	43⅞	...
Weight of acid in pounds	160	179	197	216	292	1242	9.5	515	552	590	2512	19.2
Weight of cell, complete, with acid in lead-lined tank in pounds:	482	552	621	689	992	4560	36	1635	1769	1904	8696	68
Height of cell over all, inches............	26	26	26	26	26	29	....	45	45	45	46	...

* D = addition per plate from 25 to 125 plates; approximate as to dimensions and weights.

slightly above two volts on open circuit, and during discharge varies from that point at the beginning to 1.75 at the end.

Accumulators are largely used in central lighting and power stations, in office buildings and other large isolated plants, for the purpose of absorbing the energy of the generating-plant during times of light load, and for giving it out during times of heavy load or when the generating-plant is idle. The advantages of their use for such purposes are thus enumerated:

1. Reduction in coal-consumption and general operating expenses, due to the generating machinery being run at the point of greatest economy while in service, and being shut down entirely during hours of light load, the battery supplying the whole of the current,

2. The possibility of obtaining good regulation in pressure during fluctuations in load, especially when the day load consists largely of elevators and similar disturbing elements.

3. To meet sudden demands which arise unexpectedly, as in the case of darkness caused by storm or thunder-showers; also in case of emergency due to accident or stoppage of generating-plant.

4. Smaller generating-plant required where the battery takes the peak of the load, which usually only lasts for a few hours, and yet where no battery is used necessitates sufficient generators, etc., being installed to provide for the maximum output, which in many cases is about double the normal output.

They are also in common use for furnishing current for electric motors for a great variety of purposes, and as a substitute for primary electric batteries.

For a very full description of various forms of storage-batteries, see "Practical Electrical Engineering," part xii. For theory of the battery and practice with the Julien battery, see paper on Electrical Accumulators by P. G. Salom. Trans. A. I. M. E., xviii. 348.

Use of Storage-batteries in Power and Light Stations. (*Iron Age*, Nov. 2, 1893.)—The storage-batteries in the Edison station, in Fifty-third Street, New York, relieve the other stations at the hours of heavy load, by delivering into the mains a certain amount of current that would otherwise have to come, and at greater loss or "drop," from one or another of the stations connecting with the network of mains. Hence the load may be varied more or less arbitrarily at these stations according to the proportion of load that the larger stations are desired or able to carry.

The battery consists of 140 cells each of about 1000 ampere-hour capacity, weighing some 750 lbs., and of about 48 inches in length, 21 inches in width, and 15 inches in depth. The battery has a normal discharge rate of about 200 amperes, but can be discharged, if necessary, at 500 amperes.

A test made when the station was running only 12 hours per day, from noon to midnight, showed that the battery furnished about 23.2% of the total energy delivered to the mains. The maximum rate of discharge attained by the battery was about 270 amperes. Thus, in this case, we have an example of a battery which is used for the purpose: 1. Of giving a load to station machinery that would otherwise be idle. 2. Utilizing the stored energy to increase the rate of output of the station at the time of heavy load, which would otherwise necessitate greater dynamo capacity.

The Working Current, or Energy Efficiency, of a storage-cell is the ratio between the value of the current or energy expended in the charging operation, and that obtained when the cell is discharged at any specified rate.

In a lead storage cell, if the surface and quantity of active material be accurately proportioned, and if the discharge be commenced immediately after the termination of the charge, then a current efficiency of as much as 98% may be obtained, provided the rate of discharge is low and well regulated. In practice it is found that low rates of discharge are not economical, and as the current efficiency always decreases as the discharge rate increases, it is found that the normal current efficiency seldom exceeds 90%, and averages about 85%.

As the normal discharging electro-motive force of a lead secondary cell never exceeds 2 volts, and as an electro-motive force of from 2.4 to 2.5 volts is required at its poles to overcome both its opposing electro-motive force and its internal resistance, there is an initial loss of 20% between the energy required to charge it and that given out during its discharge.

As the normal discharging potential is continually being reduced as the rate of discharge increases, it follows that an energy efficiency of 80% can never be realized. As a matter of fact, a maximum of 75% and a mean of 60% is the usual energy efficiency of lead-sulphuric-acid storage-cells.

ELECTRO-CHEMICAL EQUIVALENTS.

Elements.	Valency.*	Atomic Weight.†	Chemical Equivalent.	Electro-chemical Equivalent (milligrammes per coulomb).	Coulombs per gramme.	Grammes per ampere hour.
ELECTRO-POSITIVE.						
Hydrogen..............	H_1	1.00	1.00	.010384	96293.00	0.03738
Potassium.......... ...	K_1	39.04	39.04	.40539	2467.50	1.45950
Sodium	Na_1	22.99	22.99	.23873	4188.90	0.85942
Aluminum	Al_2	27.3	9.1	.09449	1058.30	0.34018
Magnesium.....	Mg_2	23.94	11.97	.12430	804.03	0.44747
Gold	Au_3	196.2	65.4	.67911	1473.50	2.44480
Silver............	Ag_1	107.66	107.66	1.11800	894.41	4.02500
Copper (cupric).........	Cu_2	63.00	31.5	.32709	3058.60	1.17700
" (cuprous)......	Cu_1	63.00	63.00	.65419	1525.30	2.35500
Mercury (mercuric)....	Hg_1	199.8	99.9	1.03740	963.99	3.73450
" (mercurous)..	Hg_1	199.8	199.8	2.07470	481.99	7.46900
Tin (stannic)...........	Sn_4	117.8	29.45	.30581	3270.00	1.10090
" (stannous)	Sn_2	117.8	58.9	.61162	1635.00	2.20180
Iron (ferric)..	Fe_4	55.9	18.64‡	.19356	5166.4	0.69081
" (ferrous)..........	Fe_2	55.9	27.95	.29035	3445.50	1.04480
Nickel.......	Ni_2	58.6	29.3	.30425	3286.80	1.09530
Zinc.....................	Zn_2	64.9	32.45	.33696	2967.10	1.21830
Lead.......	Pb_2	206.4	103.2	1.07160	933.26	3.85780
ELECTRO-NEGATIVE.						
Oxygen.................	O_2	15.96	7.98	.08286		
Chlorine.......	Cl_1	35.37	35.37	.36728		
Iodine..................	I_1	126.53	126.53	1.31390		
Bromine................	Br_1	79.75	79.75	.82812		
Nitrogen................	N_3	14.01	4.67	.04849		

* Valency is the atom-fixing or atom-replacing power of an element compared with hydrogen, whose valency is unity.

† Atomic weight is the weight of one atom of each element compared with hydrogen, whose atomic weight is unity.

‡ Becquerel's extension of Faraday's law showed that the electro-chemical equivalent of an element is proportional to its chemical equivalent. The latter is equal to its combining weight, and not to atomic weight ÷ valency, as defined by Thompson, Hospitalier, and others who have copied their tables. For example, the ferric salt is an exception to Thompson's rule, as are sesqui-salts in general.

ELECTROLYSIS.

The separation of a chemical compound into its constituents by means of an electric current. Faraday gave the nomenclature relating to electrolysis. He called the compound to be decomposed the Electrolyte, and the process Electrolysis. The plates or poles of the battery he called Electrodes. The plate where the greatest pressure exists he called the Anode, and the other pole the Cathode. The products of decomposition he called Ions.

Lord Rayleigh found that a current of one ampere will deposit 0.017253 grain, or 0.001118 gramme, of silver per second on one of the plates of a silver voltameter, the liquid employed being a solution of silver nitrate containing from 15% to 20% of the salt.

The weight of hydrogen similarly set free by a current of one ampere is .00001038 gramme per second.

Knowing the amount of hydrogen thus set free, and the chemical equivalents of the constituents of other substances, we can calculate what weight of their elements will be set free or deposited in a given time by a given current.

Thus the current that liberates 1 gramme of hydrogen will liberate 8 grammes of oxygen, or 107.7 grammes of silver, the numbers 8 and 107.7 being the chemical equivalents for oxygen and silver respectively.

To find the weight of metal deposited by a given current in a given time, find the weight of hydrogen liberated by the given current in the given time, and multiply by the chemical equivalent of the metal.

Thus: Weight of silver deposited in 10 seconds by a current of 10 amperes = weight of hydrogen liberated per second × number seconds × current strength × 107.7 = .00001038 × 10 × 10 × 107.7 = .11178 gramme.

Weight of copper deposited in 1 hour by a current of 10 amperes =

$$.00001038 \times 3600 \times 10 \times 31.5 = 11.77 \text{ grammes.}$$

Since 1 ampere per second liberates .00001038 gramme of hydrogen, strength of current in amperes

$$= \frac{\text{weight in grammes of H. liberated per second}}{.00001038}$$

$$= \frac{\text{weight of element liberated per second}}{.00001038 \times \text{chemical equivalent of element}}.$$

The table on page 1057 (from "Practical Electrical Engineering") is calculated upon Lord Rayleigh's determination of the electro-chemical equivalents and Roscoe's atomic weights.

ELECTRO-MAGNETS.

Units of Electro-magnetic Measurements.

C.G.S. unit of force = 1 dyne = 1.01936 milligrammes in localities in which the acceleration due to gravity is 981 centimetres, or 32.185 feet, per second.

C.C.S. unit of energy = 1 erg = energy required to overcome the resistance of 1 dyne at a speed of 1 centimetre per second. 1 watt = 10^7 ergs.

Unit magnetism = that amount of magnetic matter which, if concentrated in a point, will repel an equal amount of magnetic matter concentrated in another point one centimetre distant with the force of one dyne.

Unit strength of field = that flow of magnetic lines which will exert unit mechanical force upon unit pole, or a density of 1 line per square centimetre.

The following definitions of practical units of the magnetic circuit are given in Houston and Kennelly's "Electrical Engineering Leaflets."

Gilbert, the unit of magneto-motive force; such a M.M.F. as would be produced by $\frac{10}{4\pi}$ or 0.7958 ampere-turn.

If an air-core solenoid or hollow anchor-ring were wound with 100 turns of insulated wire carrying a current of 5 amperes, the M.M.F. exerted would be 500 ampere-turns = 628.5 gilberts.

Weber, the unit of magnetic flux; the flux due to unit M.M.F. when the reluctance is one oersted.

Gauss, the unit of magnetic flux-density, or one weber per normal square centimetre.

The flux-density of the earth's magnetic field in the neighborhood of New York is about 0.6 gauss, directed downwards at an inclination of about 72°.

Oersted, the unit of magnetic reluctance; the reluctance of a cubic centimetre of an air-pump vacuum.

Reluctance is that quantity in a magnetic circuit which limits the flux under a given M.M.F. It corresponds to the resistance in the electric circuit.

The reluctivity of any medium is its specific reluctance, and in the C.G.S. system is the reluctance offered by a cubic centimetre of the body between opposed parallel faces. The reluctivity of nearly all substances, other than the magnetic metals, is sensibly that of vacuum, is equal to unity, and is independent of the flux density.

Permeability is the reciprocal of magnetic reluctivity.

The fundamental equation of the magnetic circuit is

$$\text{Webers} = \frac{\text{gilberts}}{\text{oersteds}};$$

or, magnetic flux = magneto-motive force ÷ magnetic reluctance.

From this equation we have

$$\text{Gilberts} = \text{webers} \times \text{oersteds}; \quad \text{oersteds} = \text{gilberts} \div \text{webers}.$$

There are therefore two ways of increasing the magnetic flux: 1. by increasing the M.M.F.; 2. by decreasing the reluctance.

Lines and Loops of Force.—In discussing magnetic and electrical phenomena it is conventionally assumed that the attractions and repulsions as shown by the action of a magnet or of a conductor upon iron filings are due to "lines of force" surrounding the magnet or conductor. The "number of lines" indicates the magnitude of the forces acting. As the iron filings arrange themselves in concentric circles, we may assume that the forces may be represented by close curves or "loops of force." The following assumptions are made concerning the loops of force in a conductive circuit:

1. That the lines or loops of force in the conductor are parallel to the axis of the conductor.

2. That the loops of force external to the conductor are proportional in number to the current in the conductor, that is, a definite current generates a definite number of loops of force. These may be stated as the strength of field in proportion to the current.

3. That the radii of the loops of force are at right angles to the axis of the conductor.

The magnetic force proceeding from a point is equal at all points on the surface of an imaginary sphere described by a given radius about that point. A sphere of radius 1 cm. has a surface of 4π square centimetres. If F = total field strength, expressed as the number of lines of force emanating from a pole containing M units of magnetic matter,

$$F = 4\pi M; \quad M = F \div 4\pi.$$

Magnetic moment of a magnet = product of strength of pole M and its length, or distance between its poles L. Magnetic moment $= \frac{LF}{4\pi}$.

If B = number of lines flowing through each square centimetre of cross-section of a bar-magnet, or the "specific induction," and A = cross-section,

$$\text{Magnetic moment} = \frac{LAB}{4\pi}.$$

If the bar-magnet be suspended in a magnetic field whose induction is H, and so placed that the lines of the field are all horizontal and at right angles to the axis of the bar, the north pole will be pulled forward, that is, in the direction in which the lines flow, and the south pole will be pulled in the opposite direction, the two forces producing a torsional moment or torque,

$$\text{Torque} = MLH = LABH \div 4\pi, \text{ in dyne-centimetres.}$$

Magnetic attraction or repulsion emanating from a point varies inversely as the square of the distance from that point. The law of inverse squares, however, is not true when the magnetism proceeds from a surface of appreciable extent, and the distances are small, as in dynamo-electric machines. (For an analogy see "Radiation of Heat," page 467.)

Strength of an Electro-magnet.—In an electric magnet made by coiling a current-carrying conductor around a core of soft iron, the space in which the loops of force have influence is called the magnetic field, and it is convenient to assume that the strength of the field is proportional to the number of loops of magnetic force surrounding the magnet. Under this assumption, if we take a given current passing through a given number of conductor-turns, the number of magnetic loops will depend upon the resistance of the magnetic circuit, just as the current with a given pressure in the conductive circuit depends upon the resistance of the circuit.

The following laws express the most important principles concerning electro-magnets:

(1) The magnetic intensity (strength) of an electro-magnet is nearly proportional to the strength of the magnetizing current, provided the core is not saturated.

(2) The magnetic strength is proportional to the number of turns of wire in the magnetizing coil; that is, to the number of ampere turns.

(3) The magnetic strength is independent of the thickness or material of the conducting wires.

These laws may be embraced in the more general statement that the strength of an electro-magnet, the size of the magnet being the same, is proportional to the number of its ampere turns.

Force in the Gap between Two Poles of a Magnet.—If P = force exerted by one of the poles upon a unit pole in the gap, and m = density of lines in the field (that is, that there are m absolute or C.G.S. units on each square centimetre of the polar surface of the magnet), the polar surface being large relative to the breadth of the gap, $P = 2\pi m$. The total force exerted upon the unit pole by both north and south poles of the magnet is $2P = 4\pi m$, in dynes $= B$, or the induction in lines of force per square centimetre. If S = number of square centimetres in each polar surface, SB = total flow of force, or field strength $= F$; Sm = total pole strength $= M$, spread over each of the polar surfaces. We then have $F = 4\pi M$, as before; that is, the total field is 4π times the total pole strength.

Total attractive force between the two opposing poles of a magnet, when the distance apart is small, $= \frac{SB^2}{8\pi}$, in dynes.

This formula may be used to determine the lifting-power of an electro-magnet, thus:

A bent magnet provided with a keeper is 3 cm. square on each pole, and the induction B = 20,000 lines per square centimetre. The attractive force of each limb on the keeper in dynes $= \frac{9 \times 20000^2}{8 \times 3.14}$, or in kilogrammes for both limbs, $\frac{9 \times 400 \times 10^6}{25.12 \times 981000} \times 2 = 292$ kilogrammes.

The Magnetic Circuit.—In the conductive circuit we have $C = \frac{E}{R}$;

$$\text{Current} = \frac{\text{electro-motive force}}{\text{resistance}} = \frac{\text{volts}}{\text{ohms}}.$$

In the magnetic circuit we have
Number of lines, or loops, of force, or magnetism

$$= \frac{\text{Current} \times \text{conductor turns}}{\text{Resistance of magnetic circuit}} = \frac{\text{Ampere turns}}{\text{Resistance of magnetic circuit}}.$$

Or, in the new notation, webers $= \frac{\text{gilberts}}{\text{oersteds}}$.

Let N = No. of lines of force, Rm = total magnetic resistance, At = ampere turns, then $N = \frac{At}{Rm}$.

The magnetic pressure due to the ampere turns $= \frac{4}{10}\pi TC = 1.257TC$, where T = turns and C = amperes, whence $N = \frac{.4\pi TC}{Rm} = \frac{1.257TC}{Rm}$.

If Rm = total magnetic resistance, and Ra, R_A, R_F the magnetic resistances of the air-spaces, the armature, and the field-magnets, respectively,

$$Rm = Ra + R_A + R_F; \quad \text{and} \quad N = \frac{.4\pi TC}{Ra + R_A + R_F}.$$

Determining the Polarity of Electro-magnets.—If a wire is wound around a magnet in a right-handed helix, the end at which the current flows into the helix is the south pole. If a wire is wound around an ordinary wood screw, and the current flows around the helix in the direction from the head of the screw to the point, the head of the screw is the south pole. If a magnet is held so that the south pole is opposite the eye of the observer, the wire being wound as a right-handed helix around it, the current flows in a right-handed direction, with the hands of a clock.

DYNAMO-ELECTRIC MACHINES.

There are three classes of dynamo-electric machines, viz.:

1. Generators, for the conversion of mechanical into electrical energy.
2. Motors, for the conversion of electrical into mechanical energy.

Generators and motors are both subdivided into direct-current and alternating-current machines.

3. Transformers, for the conversion of one character or voltage of current into another, as direct into alternating or alternating into direct, or from one voltage into a higher or lower voltage.

Kinds of Dynamo-electric Machines as regards Manner of Winding. (Houston's Electrical Dictionary.)

1. *Dynamo-electric Machine.*—A machine for the conversion of mechanical energy into electrical energy by means of magneto-electric induction.

2. *Compound-wound Dynamo.*—The field-magnets are excited by more than one circuit of coils or by more than a single electric source.

3. *Closed-coil Dynamo.*—The armature-coils are grouped in sections communicating with successive bars of a collector, so as to be connected continuously together in a closed circuit.

4. *Open-coil Dynamo.*—The armature-coils, though connected to the successive bars of the commutator, are not connected continuously in a closed circuit.

5. *Separate-coil Dynamo.*—The field-magnets are excited by means of coils on the armature separate and distinct from those which furnish current to the external circuit.

6. *Separately-excited Dynamo.*—The field-magnet coils have no connection with the armature-coils, but receive their current from a separate machine or source.

7. *Series-wound Dynamo.*—The field-current and the external circuit are connected in series with the armature circuit, so that the entire armature current must pass through the field-coils.

Since in a series-wound dynamo the armature-coils, the field, and the external-series circuit are in series, any increase in the resistance of the external circuit will decrease the electro-motive force from the decrease in the magnetizing currents. A decrease in the resistance of the external circuit will, in a like manner, increase the electro-motive force from the increase in the magnetizing current. The use of a regulator avoids these changes in the electro-motive force.

8. *Series and Separately-excited Compound-wound Dynamo.*—There are two separate circuits in the field-magnet cores, one of which is connected in series with the field-magnets and the external circuit, and the other with some source by which it is separately excited.

9. *Shunt-wound Dynamo.*—The field-magnet coils are placed in a shunt to the armature circuit, so that only a portion of the current generated passes through the field-magnet coils, but all the difference of potential of the armature acts at the terminals of the field-circuit.

In a shunt-dynamo machine an increase in the resistance of the external circuit increases the electro-motive force, and a decrease in the resistance of the external circuit decreases the electro-motive force. This is just the reverse of the series-wound dynamo.

In a shunt-wound dynamo a continuous balancing of the current occurs. The current dividing at the brushes between the field and the external circuit in the inverse proportion to the resistance of these circuits, if the resistance of the external circuit becomes greater, a proportionately greater current passes through the field-magnets, and so causes the electro-motive force to become greater. If, on the contrary, the resistance of the external circuit decreases, less current passes through the field, and the electro-motive force is proportionately decreased.

10. *Series- and Shunt-wound Compound-wound Dynamo.*—The field-magnets are wound with two separate coils, one of which is in series with the armature and the external circuit, and the other in shunt with the armature. This is usually called a compound-wound machine.

11. *Shunt and Separately-excited Compound-wound Dynamo.*—The field is excited both by means of a shunt to the armature circuit and by a current produced by a separate source.

Current Generated by a Dynamo-electric Machine.—Unit current in the C.G.S. system is that current which, flowing in a thin wire forming a circle of one centimetre radius, acts upon a unit pole placed in the centre with a force of 2π dynes. One tenth of this unit is the unit of current used in practice, called the ampere.

A wire through which a current passes has, when placed in a magnetic field, a tendency to move perpendicular to itself and at right angles to the lines of the field. The force producing this tendency is $P = lcB$ dynes, in which l = length of the wire, c = the current in C.G.S. units, and B the induction in the field in lines per square centimetre.

If the current C is taken in amperes, $P = lCB10^{-1}$.

If P_k is taken in kilogrammes,

$$P_k = \frac{lCB}{9810000} = 10.1937lCB10^{-8} \text{ kilogrammes.}$$

EXAMPLE.—The mean strength of field, B, of a dynamo is 5000 C.G.S. lines; a current of 100 amperes flows through a wire; the force acts upon 10 centimetres of the wire $= 10.1937 \times 10 \times 100 \times 5000 \times 10^{-8} = .5097$ kilogrammes.

In the "English" or Kapp's system of measurement a total flow of 6000 C.G.S. lines is taken to equal one English line. Calling B_E the induction in English, or Kapp's, lines per square inch, and B the induction in C.G.S. lines per square centimetre, $B_E = B \div 930.04$; and taking l'' in inches and P_p in pounds, $P_p = 531Cl''B_E10^{-6}$ pounds.

Torque of an Armature.—P_p in the last formula, = the force tending to move one wire of length l'', which carries a current of C amperes through the field whose induction is B_E English lines per square inch. The current through a drum-armature splits at the commutator into two branches, each half going through half of the wires or bars. The force exerted upon one of the wires under the influence of a pole-piece $= \frac{1}{2}P_p$. If t = the number of wires under the pole-pieces, then the total force $= \frac{1}{2}P_pt$. If r = radius of the armature to the centre of the conductors, expressed in feet, then the torque $= \frac{1}{2}P_ptr, = \frac{1}{2} \times 531 \times Cl''B_E \times 10^{-6} \times tr$ foot-pounds of moment, or pounds acting at a radius of 1 foot.

EXAMPLE.—Let the length l of an armature = 20 in., the radius = 6 in. or .5 ft., number of conductors = 120, of which t = 80 are under the influence of the two pole-pieces at one time, the average induction or magnetic flux through the armature-field B_E = 5 English lines per square inch, and the current passing through the armature = 400 amperes; then

$$\text{Torque} = \tfrac{1}{2} \times 531 \times 400 \times 20 \times 5 \times 80 \times .5 \times 10^{-6} = 424.8.$$

The work done in one revolution = torque × circumference of a circle of 1 foot radius $= 424.8 \times 6.28 = 2670$ foot-pounds.

Let the revolutions per minute = 500, then the horse-power

$$= \frac{2670 \times 500}{33000} = 40.5 \text{ H.P.}$$

Electro-motive Force of the Armature Circuit.—From the horse-power, calculated as above, together with the amperes, we can obtain the E.M.F., for CE = H.P. × 746, whence E.M.F. or E = H.P. × 746 ÷ C.

If H.P., as above, = 40.5, and $C = 400$, $E = \frac{40.5 \times 746}{400} = 75.5$ volts.

The E.M.F. may also be calculated more directly by the following formulæ given by Gisbert Kapp:

C = Total current through armature; c, current through single armature conductor;
e_a = E.M.F. in armature in volts;
τ = Number of active conductors counted all around armature;
p = Number of pairs of poles (p = 1 in a two-pole machine);
n = Speed in revolutions per minute;
F = Total induction in C.G.S. lines;
Z = Total induction in English lines.

$$\text{Electro-motive force} \begin{cases} \left.\begin{array}{l} e_a = F\tau\frac{n}{60}10^{-8} \\ e_a = Z\tau n10^{-6} \end{array}\right\} \text{for two-pole machines.} \\ \left.\begin{array}{l} e_a = pF\tau\frac{n}{60}10^{-1} \\ e_a = pZ\tau n10^{-6} \end{array}\right\} \text{for multipolar machines with series-wound armature.} \end{cases}$$

$$\text{Torque}\begin{cases}\text{Kilogramme-metres} = 1.615 F r C\,10^{-10} \\ \text{Foot-pounds}\ldots\ldots\ldots = 7.05 Z r C\,10^{-6}\end{cases}\text{for two-pole machines.}$$

$$\begin{cases}\text{Kilogramme-metres} = 3.23 F r c p\,10^{-10} \\ \text{Foot-pounds}\ldots\ldots\ldots = 14.10 Z r c p 10^{-6}\end{cases}\text{for multipolar machines.}$$

EXAMPLE.—$r = 120$, $n = 500$, length of armature $l = 20$ in., diameter $d = 12$ in., cross-section $= 20 \times 12 = 240$ sq. in., induction per sq. in. $B_E = 5$ lines per sq. in., total induction $Z = 240 \times 5 = 1200$; then

$$E = Zrn10^{-6} = 1200 \times 120 \times 500 \times 10^{-6} = 72 \text{ volts.}$$

A formula for horse-power given by Kapp is

$$\text{H.P.} = 1/746\ ZNtn10^{-6}Ca$$
$$= 1/746\ 2abmNtn10^{-6}Ca.$$

Ca = current in amperes, n = revs. per min., $2ab$ = sectional area of armature-core, m = average density of lines per sq. in. of armature-core, Nt = total number of external wires counted all around the circumference, t = number of wires corresponding to one plate in the commutator, N = number of plates, $Z = 2abm$ = total number of English lines of force.

Kapp says that experience has shown that the density of lines m in the core cannot exceed a certain limit, which is reached when the core is saturated with magnetism. This value is reached when $m = 30$. A fair average value in modern dynamos and motors is $m = 20$, and the area ab must be taken as that actually filled by iron, and not the gross area of the core. 20 English lines per sq. in. = 18,600 C.G.S. lines per square centimetre. Silvanus P. Thompson says it is not advisable in continuous-current machines to push the magnetization further than $B = 17{,}000$ C.G.S. lines per square centimetre.

Thompson gives as a rough average for the magnetic field in the gap-space of a dynamo or motor 6300 lines per sq. cm., or 40,000 lines per sq. in., and the drag per inch of conductor .00354 lb. for each ampere of current carried.

Pounds average drag per conductor $= \dfrac{\text{H.P.} \times 33{,}000}{\text{ft. per min.} \times C}$, in which C is the number of conductors around the armature.

Strength of the Magnetic Field.—Kapp gives for the total number of lines of force (Kapp's lines = C.G.S. lines ÷ 6000) in the magnetic circuit. $Z = \dfrac{X}{Ra + RA + RF}$, in which Z = number of magnetic lines, X = the exciting pressure due to the ampere turns $= .4\pi TC$, Ra, RA, and RF, = respectively the resistances of the air-spaces, the armature, and the field-magnets.

Kapp gives the following empirical values of Ra, RA, and RF, for dynamos and motors made of well-annealed wrought iron, with a permeability of $\mu = 940$:

$$Ra = 1440\frac{2\delta}{\lambda b};\quad RA = \frac{l}{ab};\quad RF = 2\frac{L}{AB};$$

in which δ = distance across the span between armature-core and polar surface, b = breadth of armature measured parallel to axis, λ = length of arc embraced by polar surface, so that λb = the polar area out of which magnetic lines issue, a = radial depth of armature-core, so that ab = section of armature-core (space actually occupied by iron only being reckoned), AB = area of field-magnet core, l = length of magnetic circuit within armature, L = length of magnetic circuit in field magnet; all dimensions in inches or square inches.

For cast-iron magnets, $Z = \dfrac{0.8X}{1800\frac{2\delta}{\lambda b} + \frac{l}{ab} + \frac{8L}{AB}}$.

For double horse-shoe magnets of wrought iron,

$$\frac{Z}{2} = \frac{X}{1440\frac{2\delta}{\lambda b} + \frac{2l}{ab} + \frac{2L}{AB}};$$

and of cast iron,
$$\frac{Z}{2} = \frac{0.8X}{1800\frac{2\delta}{\lambda b} + \frac{2l}{ab} + \frac{3L}{AB}}.$$

These formulæ apply only to cases in which the intensity of magnetization is not too great—say up to 10 Kapp's lines per square inch.

Silvanus P. Thompson gives the following method of calculating the strength of the field, or the magnetic flux, M_F, or the whole number of magnetic lines flowing in the circuit in C.G.S. lines:

The magnetic resistance of any magnetic conductor is proportional directly to its length and inversely to its cross-section and its permeability.

Magnetic resistance $= \frac{L}{S\mu}$, in which L = length of the magnetic circuit passing through any piece of iron, S = section of the magnetic circuit passing through any piece of iron, μ = permeability of that piece of iron.

In a dynamo-machine in which the resistances are three, viz.: 1. The field-magnet cores; 2. The armature-core; 3. The gaps or air-spaces between them,—

let L_m, S_m, μ_m refer to the field-magnet part of the circuit;
L_{as}, S_{as}, μ_{as} refer to the air-space part of the circuit;
L_a, S_a, μ_a refer to the armature part of the circuit;

the lengths across each of the air-spaces being L_{as}, and the exposed area of polar surface at either pole being S_{as}.

Total magnetic resistance $= \frac{L_m}{S_m\mu_m} + \frac{L_{as}}{S_{as}\mu_{as}} + \frac{L_a}{S_a\mu_a}$.

Magnetic flux, or total number of magnetic lines, =

$$M_F = \frac{1.257 T_w C}{\frac{L_m}{S_m\mu_m} + \frac{L_{as}}{S_{as}\mu_{as}} + \frac{L_a}{S_a\mu_a}}.$$

T_w = turns of wires, or number of turns in the spiral;
C = current in amperes passing through spiral.

Application to Designing of Dynamos. (S. P. Thompson.)—Suppose in designing a dynamo it has been decided what will be a convenient speed, how many conductors shall be wound upon the armature, and what quantity of magnetic lines there must be in the field, it then becomes necessary to calculate the sizes of the iron parts and the quantity of excitation to be provided for by the field-magnet coils. It being known what M_F is to be, the problem is to design the machine so as to get the required value. Experience shows that in every type of dynamo there is magnetic leakage; also, that it is not wise to push the saturation of the armature-core to more than 16,000 lines to the square centimetre at the most highly saturated part, and that the induction in the field-magnet ought to be not greater than this, even allowing for leakage. Leakage may amount to ¼ of the whole: hence, if the magnet-cores are made of same quality of iron as the armature-cores, their cross-section ought to be at least 5/4 as great as that of the armature-core at its narrowest point. If the field-magnets are of cast iron, the section ought to be at least twice as great.

Now, B_a (the induction in the armature-core) $= M_a \div S_a$ (or magnetic flux through armature ÷ cross-sectional area of the armature; hence, if this is fixed at 16,000 lines per centimetre of cross-section, we at once get $S_a = M_a \div B_a$. This fixes the cross-section of the armature-core. (Example: If M_a = 4,000,000 of lines, then there must be a cross-section equal to 250 square centimetres for $\frac{4,000,000}{16,000} = 250$.)

Magnetic Length of Armature Circuit.—The size of wires on the armature is fixed by the number of amperes which it must carry without risk. Remembering that only half the current (in ring or drum armatures) passes through any one coil, and as the number is supposed to have been fixed beforehand, this practically settles the quantity of copper that must be put on the armature, and experience dictates that the core should be made so large that the thickness of the external winding does not exceed 1/6 of the radial depth of the iron core. This settles the size of the armature-core, from which an estimate of L_a, the average length of path of the magnetic lines in the core, can be made.

Length and Section or Surface Area of Air-space.—Experience further dictates the requisite clearance, and the advantage of making the pole-pieces subtend an arc (in two-pole machines) of at least 135° each, so as to gain a large polar area. This settles La_s and Sa_s.

Length of Field-magnet Iron Cores, etc.—As shown above, the minimum value of Sm is settled by leakage and materials; Lm therefore remains to be decided. It is clear that the magnet-cores must be long enough to allow of the requisite magnetizing coils, but should not be longer. As a rule, they are made so stout, especially in the yoke part, that they do not add much to the magnetic resistance of the circuit, then a little extra length assumed in the calculation does not matter much. It now only remains to calculate the number of ampere-turns of excitation for which it will be needful to provide.

It will now be more convenient to rewrite the formula of the magnetic circuit as follows:

$$A \times Tmw = Ma \frac{\left\{ \lambda \dfrac{Lm}{Sm\mu m} + 2\dfrac{Las}{Sas.\mu as} + \dfrac{La}{Sa.\mu a} \right\}}{1.257};$$

where A = amperes of current passing through the field-magnet coils;
Tmw = total turns of the magnet wire;
λ = leakage coefficient (say 5/4).

Or,

$$4 \times Tmw = Ma \frac{\lambda Rm + Ras + Ra}{1.257}.$$

Or, as before,

$$Ma = 1.257 \frac{A \times Tmw}{\lambda Rm + Ras + Ra},$$

where Rm, Ras, Ra stand for the magnetic resistance of magnets, air-space, and armature, respectively.

But we cannot use this formula yet, because the values of μ in it depend on the degree of saturation of the iron in the various parts. These have to be found from the Hopkinson tables, given below; and, indeed, it is preferable first to rearrange the formula once more, by dividing it into its separate members, ascertaining separately the ampere-turns requisite to force the required number of magnetic lines through the separate parts, and then add them together.

1. Ampere-turns required for magnet-cores $= \lambda \dfrac{Ma}{Sm} \times \dfrac{Lm}{\mu m} \div 1.257$.

2. Ampere-turns required for air-spaces $= \dfrac{Ma}{Sas} \times 2\dfrac{Las}{\mu as} \div 1.257$.

3. Ampere-turns required for armature-core $= \dfrac{Ma}{Sa} \times \dfrac{La}{\mu a} \div 1.257$.

Now $\lambda\dfrac{Ma}{Sm}$ is the value of B in the magnet-cores, and reference to the table of permeability will show what the corresdonding value of μm must be. Similarly, $\dfrac{Ma}{Sa}$ will afford a clue to μa. When the total number of ampere-turns to be allowed for is thus ascertained, the size and length of wire will be determined by the permissible rise of temperature, and the mode of exciting the field-magnets, whether in series, or as a shunt machine, or with a compound-winding.

Permeability.—Materials differ in regard to the resistance they offer to the passage of lines of force; thus iron is more permeable than air. The permeability of a substance is expressed by a coefficient μ, which denotes its relation to the permeability of air, which is taken as 1. If H = number of magnetic lines per square centimetre which will pass through an air-space between the poles of a magnet, and B the number of lines which will pass through a certain piece of iron in that space, then $\mu = B \div H$. The permeability varies with the quality of the iron, and the degree of saturation, reaching a practical limit for soft wrought iron when B = about 18,000 and for cast iron when B = about 10,000 C.G.S. lines per square centimetre.

The following values are given by Thompson as calculated from Hopkinson's experiments:

Annealed Wrought Iron.			Gray Cast Iron.		
B	H	μ	B	H	μ
5,000	2	2,500	4,000	5	800
9,000	4	2,250	5,000	10	500
10,000	5	2,000	6,000	21.5	279
11,000	6.5	1,692	7,000	42	133
12,000	8.5	1,412	8,000	80	100
13,000	12	1,083	9,000	127	71
14,000	17	823	10,000	188	53
15,000	28.5	526	11,000	292	37
16,000	52	308			
17,000	105	161			
18,000	200	90			
19,000	350	54			

Permissible Amperage and Permissible Depth of Winding for Magnets with Cotton-covered Wire.—Walter S. Dix (*El. Engineer*, Dec. 21, 1892) gives the following formula:

$$C = \sqrt{\frac{12 \times W}{\frac{\omega_m f}{M} \times T \times L}},$$

where C = current;
W = emissivity in watts per square inch;
$\omega_m f$ = ohms per mil-foot;
M = circular mils;
T = turns per linear inch;
L = number of layers in depth.

The emissivity is taken at .4 watt per sq. in. for stationary magnets for a rise of temperature of 35° C. (63° F.). For armatures, according to Esson's experiments, it is approximately correct to say that .9 watt per sq. in. will be dissipated for a rise of 35° C.

The insulation allowed is .007 inch on No. 0 to No. 11 B. & S.; .005 inch on No. 12 to No. 24; and .0045 inch on No. 25 to No. 31 single; twice these values for insulation of double-covered wires. Fifteen per cent is allowed for imbedding of the wires.

Formulæ of Efficiency of Dynamos.

(S. P. Thompson in "Munro and Jamieson's Pocket-Book.")

Total Electrical Energy (per second) of any dynamo (expressed in watts) is the product of the whole E.M.F. generated by armature-coils into the whole current which passes through the armature.

Useful Electrical Energy (per second), or useful output of the machine, is the product of the useful part of the E.M.F. (i.e., that part which is available at the terminals of the machine) into the useful part of the current (i.e., that part of the current which flows from the terminals into the external circuit).

Economic Coefficient or "electrical efficiency" of a dynamo is the ratio of the useful energy to the total energy.

Commercial Efficiency of a dynamo is the ratio of the useful energy or output to the power actually absorbed by the machine in being driven.

Let E_a = total E.M.F. generated in armature;
E_e = useful E.M.F. available at terminals;
C_a = total current generated in armature;
C_s = current sent round shunt-coils;
C_e = useful current supplied to external circuit;
R_a = resistance of armature-coils;
R_m = resistance of magnet-coils in main circuit (series);
R_s = resistance of magnet-coils in shunt;
R_e = resistance of external circuit (lamps, mains, etc.);

W_a = Watts lost in armature;
W_m = Watts lost in magnet-coils;
V_l = lost volts;
T_e = total electrical energy (per second);
U_e = useful electrical output;
c = economic coefficient;
p = commercial efficiency (percentage).

When only one circuit (series machine) $C_e = C_a$.

In shunt machines C_s should not be more than 5% of C_a. Also, $C_a = C_e + C_s$.

In *all* dynamos, R_a ought to be less than 1/40 as great as the working value of R_e.

In series (and compound) machines, R_m should be not greater than R_a, and preferably only ⅔ as great.

In shunt (and compound) machines, R_s should be not less than 300 times as great as R_a and preferably 1000 to 1200 times as great.

	Series Machine.	Shunt Machine.	Compound Machine (Short Shunt).
W_a	$C_a^2R_a$	$C_a^2R_a$	$C_a^2R_a$
W_m	$C_a^2R_m$	$C_s^2R_s = E_e^2 \div R_s$	$C_a^2R_m + C_s^2R_s$
V_l	C_aR_a	C_aR_a	$C_aR_a + C_eR_m$
T_e	$E_aC_a = C_a^2(R_a + R_m + R_e)$	$E_aC_a = C_a^2\left(R_a + \frac{R_sR_e}{R_s+R_e}\right)$	$E_aC_a = C_a^2\left(R_a + \frac{R_s(R_m+R_e)}{R_s+R_m+R_e}\right)$
U_e	$E_eC_a = C_a^2R_e$	$E_eC_e = C_e^2R_e$	$E_eC_e = C_e^2R_e$
c	$\frac{E}{E_a} = \frac{R_e}{R_a+R_m+R_e}$	$\frac{C_e^2R_e}{C_e^2R_e+C_a^2R_a+C_s^2R_s}$ *	$\frac{C_e^2R_e}{C_e^2R_e+C_a^2R_a+C_s^2R_s+C_e^2R_m}$
p	$100 \times E_eC_e \div$ (H.P. $\times$ 746)	$100 \times E_eC_e \div$ (H.P. $\times$ 746)	$100 \times E_eC_e \div$ (H.P. $\times$ 746)
	N.B. Horse-power is converted into watts (so as to compare with electric output of the machine) by multiplying by 746.	* This will be a maximum when R_e is a mean proportional between R_s and R_a.	In well-constructed compound machines the difference between "short shunt" and "long shunt" is very slight, as R_m is so small.

Alternating Currents, Multiphase Currents, Transformers, etc.—The proper discussion of these subjects would take more space than can be afforded in this work. Consult S. P. Thompson's "Dynamo-Electric Machinery," Bedell and Crehore on "Alternating Currents," Fleming on "Alternating Currents," and Kapp on "Dynamos, Alternators and Transformers."

The Electric Motor.—The electric motor is the same machine as the dynamo, but with the nature of its operation reversed. In the dynamo mechanical energy, such as from a belt, is converted into electric current; in the motor the current entering the machine is converted into mechanical energy, which may be taken off by a belt. The difference in the action of the machine as a dynamo and as a motor is thus explained by Prof. F. B Crocker, (*Cassier's Mag.*, March, 1895):

In the case of the dynamo there exists only one E.M.F., whereas in the motor there must always be two.

One kilowatt dynamo, $C = E \div R$; 10 amperes = 100 volts ÷ 10 ohms.

One kilowatt motor, $C = \frac{E - e}{R_1}$; 10 amperes $= \frac{100 \text{ volts} - 90 \text{ volts}}{1 \text{ ohm}}$.

C is the current; E, the direct E.M.F.; e, the counter E.M.F.; R, the total resistance of the circuit; R_1, the resistance of the armature. The current and direct E.M.F. are the same in the two cases, but the resistance is only one tenth as much in the case of the motor, the difference being replaced by the counter E.M.F., which acts like resistance to reduce the current. In the case of the motor the counter E.M.F. represents the amount of the electrical energy converted into mechanical energy. The so-called electrical efficiency or conversion factor = counter E.M.F. ÷ direct E.M.F. The actual or commercial efficiency is somewhat less than this, owing to friction, Foucault currents, and hysteresis.

For full discussions of the theory and practice of electric motors see S. P. Thompson's "Dynamo-Electric Machinery," Kapp's "Electric Transmission of Energy," Martin and Wetzler's "The Electric Motor and its Applications," Cox's "Continuous Current Dynamos and Motors," and Crocker and Wheeler's "Practical Management of Dynamos and Motors."

STANDARD BELTED MOTORS AND GENERATORS.

(Crocker-Wheeler Electric Co., 1898.)

Size.	No. Poles.	Output. Motor. H.P.	Output. Motor. Speed.	Output. Dynamo. K.W.	Output. Dynamo. Speed.	Efficiency. ½ Load.	Efficiency. Full Load.	Net Weight, pounds.	Outside Dimensions in inches. Net Over All. Length.	Height.	Width.	Size of Pulley. Diam.	Size of Pulley. Face.	Rise Temp. C.°
225	4	225	400	200	450	88	93	30000	133	73¾	67¼	38	29	45
150	6	150	400	130	450	85	92	11300	85½	65⅛	67	32	23	45
100	4	100	600	90	650	88	92	11000	78⅝	58¼	51¾	23	16	45
75	4	75	625	60	675	90	92	6500	69¾	52¼	46½	20	14	45
50	4	50	650	45	700	89	91½	4500	61½	46¼	42	17	12	45
35	4	35	700	31.5	750	88	91	3350	54⅞	40¼	37¼	15	11	45
25	4	25	750	22.5	825	86	88½	2400	46⅞	30⅝	33	13	9	45
15	4	15	800	13	900	82½	88	1510	41	31½	28¾	11	8	45
10	2	10	850	10	1000	83	87	920	36¼	25¾	23¼	9	7	45
7½	2	7½	900	7.5	1050	83	86	760	33	23½	21¼	8	6	45
5	2	5	950	5	1100	82	85	510	28¼	21⅝	19¼	7	5	45
3	2	3	975	3	1175	80	84½	410	26⅝	18⅝	16¼	6	4¼	45
2	2	2	1000	2	1200	75	82	288	22½	15¾	14¼	5	4	45
1	2	1	1000	1	1300	76	81	205	19¼	15	13¼	4	3½	45
½	2	½	1200	.5	1600	67	75	100	17¾	12¾	10	3	3	45
¼	2	¼	1375	.25	1800	55	73	70	15	10⅝	8⅝	3	2¼	45
1/6	2	1/6	1600	.11	2200	55	61	27	9⅞	8½	6½	1½	1	45

APPENDIX.

STRENGTH OF TIMBER.

Safe Loads in Tons, Uniformly Distributed, for White-oak Beams.

(In accordance with the Building Laws of Boston.)

Formula : $W = \frac{4PBD^2}{3L}$.

W, safe load in tons of 2000 lbs.; P, extreme fibre-stress = 1000 lbs. per sq. in. for white oak; B, breadth in inches; D, depth in inches; L, distance between supports in inches.

Size of Timber.	Distance between Supports in feet.														
	6	8	10	11	12	14	15	16	17	18	19	21	23	25	26
2 × 6	0.67	0.50	0.40	0.36	0.33	0.29	0.27	0.25	0.24	0.22					
2 × 8	1.19	0.89	0.71	0.65	0.59	0.51	0.47	0.44	0.42	0.40	0.37	0.34	0.31	0.28	
2 × 10	1.85	1.39	1.11	1.01	0.93	0.79	0.74	0.69	0.65	0.62	0.58	0.53	0.48	0.44	0.43
2 × 12	2.67	2.00	1.60	1.45	1.33	1.14	1.07	1.00	0.94	0.89	0.84	0.76	0.70	0.64	0.62
3 × 6	1.00	0.75	0.60	0.55	0.50	0.43	0.40	0.37	0.35	0.33	0.32	0.29	0.26		
3 × 8	1.78	1.33	1.07	0.97	0.89	0.76	0.71	0.67	0.63	0.59	0.56	0.51	0.46	0.43	0.41
3 × 10	2.78	2.08	1.67	1.52	1.39	1.19	1.11	1.04	0.98	0.93	0.88	0.79	0.72	0.67	0.64
3 × 12	4.00	3.00	2.40	2.18	2.00	1.71	1.60	1.50	1.41	1.33	1.26	1.14	1.04	0.96	0.92
3 × 14	5.45	4.08	3.27	2.97	2.72	2.37	2.18	2.04	1.92	1.82	1.72	1.56	1.42	1.31	1.25
3 × 16	7.11	5.33	4.27	3.88	3.56	3.05	2.84	2.67	2.51	2.37	2.25	2.03	1.86	1.71	1.64
4 × 10	3.70	2.78	2.22	2.02	1.85	1.59	1.48	1.39	1.31	1.23	1.17	1.06	0.97	0.89	0.85
4 × 12	5.33	4.00	3.20	2.91	2.67	2.29	2.13	2.00	1.88	1.78	1.68	1.52	1.39	1.28	1.23
4 × 14	7.26	5.44	4.36	3.96	3.63	3.11	2.90	2.72	2.56	2.42	2.29	2.07	1.90	1.74	1.68
4 × 16	9.48	7.11	5.69	5.17	4.74	4.06	3.79	3.56	3.35	3.16	3.00	2.71	2.47	2.28	2.19
4 × 18	12.00	9.00	7.20	6.55	6.00	5.14	4.80	4.50	4.24	4.00	3.79	3.43	3.13	2.88	2.77

For other kinds of wood than white oak multiply the figures in the table by a figure selected from those given below (which represent the safe stress per square inch on beams of different kinds of wood according to the building laws of the cities named) and divide by 1000.

	Hemlock.	Spruce.	White pine.	Oak.	Yellow Pine.
New York......	800	900	900	1100	1100*
Boston.	....	750	750	1000†	1250
Chicago.... ...	....	....	900	1080	1440

* Georgia pine. † White oak.

MATHEMATICS.

Formula for Interpolation.

$$a_n = a_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{1.2} d_2 + \frac{(n-1)(n-2)(n-3)}{1.2.3} d_3 + \ldots$$

a_1 = the first term of the series; n, number of the required term; a_n, the required term; d_1, d_2, d_3, first terms of successive orders of differences between a_1, a_2, a_3, a_4, successive terms.

EXAMPLE.—Required the log of 40.7, logs of 40, 41, 42, 43 being given as below.

Terms a_1, a_2, a_3, a_4:	1.6021	1.6128	1.6232	1.6335
1st differences:	.0107	.0104	.0103	
2d "	− .0003	− .0001		
3d "	+ .0002			

For log. 40 $n = 1$; log 41 $n = 2$; log 40.7 $n = 1.7$, $n - 1 = 0.7$, $n - 2 = -0.3$, $n - 3 = -.1.3$.

$$a_n = 1.6021 + 0.7(.0107) + \frac{(0.7)(-0.3)(-.0003)}{2} + \frac{(0.7)(-0.3)(-1.3)(.0002)}{6}$$

$$= 1.6021 + .00749 + .000031 + .000009 = 1.6096 +.$$

Maxima and Minima without the Calculus.—In the equation $y = a + bx + cx^2$, in which a, b, and c are constants, either positive or negative, if c be positive y is a minimum when $x = -b \div 2c$; if c be negative y is a maximum when $x = -b \div 2c$. In the equation $y = a + bx + c/x$, y is a minimum when $bx = c/x$.

APPLICATION.—The cost of electrical transmission is made up (1) of fixed charges, such as superintendence, repairs, cost of poles, etc., which may be represented by a; (2) of interest on cost of the wire, which varies with the sectional area, and may be represented by bx; and (3) of cost of the energy wasted in transmission, which varies inversely with the area of the wire, or c/x. The total cost, $y = a + bx + c/x$, is a minimum when item 2 = item 3, or $bx = c/x$.

RIVETED JOINTS.

Pressure Required to Drive Hot Rivets.—R. D. Wood & Co., Philadelphia, give the following table (1897):

POWER TO DRIVE RIVETS HOT.

Size.	Girder-work.	Tank-work.	Boiler-work.	Size.	Girder-work.	Tank-work.	Boiler-work.
in.	tons.	tons.	tons.	in.	tons.	tons.	tons.
½	9	15	20	1⅛	38	60	75
⅝	12	18	25	1¼	45	70	100
¾	15	22	33	1½	60	85	125
⅞	22	30	45	1¾	75	100	150
1	30	45	60				

The above is based on the rivet passing through only two thicknesses of plate which together exceed the diameter of the rivet but little, if any.

As the plate thickness increases the power required increases approximately in proportion to the square root of the increase of thickness. Thus, if the total thickness of plate is four times the diameter of the rivet, we should require twice the power given above in order to thoroughly fill the rivet-holes and do good work. Double the thickness of plate would increase the necessary power about 40%.

It takes about four or five times as much power to drive rivets cold as to drive them hot. Thus, a machine that will drive ¾-in. rivets hot will usually drive ⅜-in. rivets cold (steel). Baldwin Locomotive Works drive ½-in. soft-iron rivets cold with 15 tons.

HEATING AND VENTILATION.

Table of Capacities for Hot-blast or Plenum Heating with Fans or Blowers.

(Computed by F. R. Still, American Blower Co., Detroit, Mich.)

Size of Blower-housing.	Diam. of Fan-wheel.	Revolutions per minute.	H.P. required to drive Fan.	Cu. Ft. of Air Delivered per minute by Fan through Heater.	Cu. Ft. of Air per hour.	Heat Units required per hour to raise air from 0° to 120°.	Velocity of Air through Coils in ft. per minute.	Free Area between Pipes in sq. ft.	Heat Units given off per sq. ft. surface per hour.	Sq. Ft. Heating Surface required.
70	42	360	2½	6,900	415,200	1,021,000	900	7.7	1760	580
80	48	320	3	8,500	510,000	1,255,000	"	9.45	"	714
90	54	280	4	10,500	630,000	1,550,000	"	11.66	"	880
100	60	250	5	12,500	750,000	1,845,000	"	13.9	"	1050
110	66	230	6	15,800	948,000	2,335,000	"	17.55	"	1325
120	72	210	8	19,800	1,118,000	2,900,000	"	22.	"	1650
140	84	180	10	26,200	1,572,000	3,870,000	"	29.1	"	2200
160	96	160	12	33,000	1,980,000	4,870,000	"	36.7	"	2770
180	108	140	15	41,600	2,496,000	6,130,000	"	46.3	"	3490
200	120	125	18	50,000	3,000,000	7,375,000	"	55.5	"	4140

Size of Blower-housing.	Lineal Feet of One-inch Pipe required.	Pounds of Steam condensed per hour to 212°.	Size Steam-main required.	Size Return-main required.	Boiler Capacity required, H.P.; 30 lbs. steam per hour = 1 H.P.	Sq. Ft. Heating Surface in Boiler at 15 sq. ft. per H.P.	Sq. Ft. Grate-surface at 35 sq. ft. heating surface to sq. ft. grate.	Volume Air will expand to by heating from 0° to 120° Capacity per minute.	Area of Conduit in sq. ft. for 900 ft. velocity per minute.	Net Volume delivered, allowance being made for friction equal to 100 ft. of conduit.
70	1,740	1055	3½	2	35	525	15	8,700	9.67	8,200
80	2,142	1295	4	2	43	645	18	10,700	13.05	10,000
90	2,640	1600	4½	2½	53	795	23	13,200	14.72	12,500
100	3,150	1900	5	2½	63	945	27	15,800	17.55	15,000
110	3,975	2410	5½	3	80	1200	34	19,900	22.20	18,900
120	4,950	2990	6	3	100	1500	43	25,000	27.80	23,800
140	6,600	3990	7	3½	133	1995	57	33,100	36.80	31,400
160	8,310	5025	8	4	167	2505	72	41,700	46.30	39,600
180	10,470	6325	9	4½	211	3165	90	52,500	58.40	50,000
200	12,420	7560	10	5	252	3780	108	63,200	70.25	60,000

Temperature of fresh air, 0°; of air from coils, 120°; of steam, 227°. Pressure of steam, 5 lbs.

Peripheral velocity of fan-tips, 4000 ft.; number of pipes deep in coil, 24; depth of coil, 60 inches; area of coils approximately twice free area.

WATER-WHEELS.

Water-power Plants Operating under High Pressures.—The following notes are contributed by the Pelton Water Wheel Co.:

The Consolidated Virginia & Col. Mining Co., Virginia, Nev., has a 3-ft. steel-disk Pelton wheel operating under 2100 ft. fall, equal to 911 lbs. per sq. in. It runs at a peripheral velocity of 10,804 ft. per minute and has a capacity of over 100 H.P. The rigidity with which water under such a high pressure as this leaves the nozzle is shown in the fact that it is impossible to cut the stream with an axe, however heavy the blow, as it will rebound just as it would from a steel rod travelling at a high rate of speed.

The London Hydraulic Power Co. has a large number of Pelton wheels from 12 to 18 in. diameter running under pressure of about 1000 lbs. per. sq. in. from a system of pressure-mains. The 18-in. wheels weighing 30 lbs. have a capacity of over 20 H.P. (See Blaine's "Hydraulic Machinery.")

Hydraulic Power-hoist of Milwaukee Mining Co, Idaho.—One cage travels up as the other descends; the maximum load of 5500 lbs. at a speed of 400 ft. per min. is carried by one of a pair of Pelton wheels (one for each cage). Wheels are started and stopped by opening and closing a small hydraulic valve at the engineer's stand which operates the larger valves by hydraulic pressure. An air-chamber takes up the shock that would otherwise occur on the pipe line under the pressure due to 850 ft. fall.

The Mannesmann Cycle Tube Works, North Adams, Mass., are using four Pelton wheels, having a fly-wheel rim, under a pump pressure of 600 lbs. per sq. in. These wheels are direct-connected to the rolls through which the ingots are passed for drawing out seamless tubing.

The Alaska Gold Mining Co., Douglass Island, Alaska, has a 22-ft. Pelton wheel on the shaft of a Riedler duplex compressor. It is used as a fly-wheel as well, weighing 25,000 lbs.—and develops 500 H.P. at 75 revolutions. A valve connected to the pressure-chamber starts and stops the wheel automatically, thus maintaining the pressure in the air-receiver.

At Pachuca in Mexico five Pelton wheels having a capacity of 600 H.P. each under 800 ft. head are driving an electric transmission plant. These wheels weigh less than 500 lbs. each, showing over a horse-power per pound of metal.

Formulæ for Calculating the Power of Jet Water-wheels, such as the Pelton (F. K. Blue).—HP = horse-power delivered: δ = 62.36 lbs. per cu. ft.; E = efficiency of turbine; q = quantity of water, cubic feet per minute; h = feet effective head; d = inches diameter of jet; p = pounds per square inch effective head; c = coefficient of discharge from nozzle, which may be ordinarily taken at 0.9.

$$HP = \frac{\delta Eqh}{33000} = .00189Eqh = .00436Eqp = .00496Ecd^2\sqrt{h^3} = .0174Ecd^2\sqrt{p^3}.$$

$$q = 529.2\frac{HP}{Eh} = 229\frac{HP}{Ep} = 2.62cd^2\sqrt{h} = 3.99cd^2\sqrt{p}.$$

$$d^2 = 201.6\frac{HP}{Ec\sqrt{h^3}} = 57.4\frac{HP}{Ec\sqrt{p^3}} = .381\frac{q}{c\sqrt{h}} = .25\frac{q}{c\sqrt{p}}.$$

GAS FUEL.

Average Volumetric Composition, Energy, etc., of Various Gases. (Contributed by R. D. Wood & Co., Philadelphia, 1898.)

	Natural Gas.	Coal-gas.	Water-gas.	Producer-gas. Anthra.	Producer-gas. Bitum.	Air.
CO	0.50	6.0	45.0	27.0	27.0	....
H	2.18	46.0	45.0	12.0	12.0	....
CH_4	92.6	40.0	2.0	1.2	2.5	....
C_2H_4	0.31	4.0	....	....	0.4	....
CO_2	0.26	0.5	4.0	2.5	2.5	trace
N	3.61	1.5	2.0	57.0	55.3	79
O	0.34	0.5	0.5	0.3	0.3	21
Vapor	....	1.5	1.5	....	....	trace
Lbs. in 1000 cu. ft.	45.6	32.0	45.6	65.6	65.9	76.1
H. U. in 1000 cu. ft.	1,100,000	735,000	322,000	137,455	156,917*	....
Cu.ft. from each lb. of coal approx.	....	5	25	85	75	200†

* The real energy of bituminous producer-gas when used hot is far in excess of that indicated by the above table, on account of the hydrocarbons, which do not show, as they are condensed in the act of collecting the gas for analysis. In actual practice there is found to be about 50% more effective energy in bituminous gas than in anthracite gas when used hot enough to prevent condensation in the flues.

† Cubic feet of air required to burn 1 lb. of coal with blast.

STEAM-BOILERS.

Steam-boiler Construction. (Extract from the Rules and Specifications of the Hartford Steam Boiler Inspection & Insurance Co., 1898.)

Cylindrical boiler shells of fire box steel, and tube-heads of best flange steel. Limits of tensile strength between 55,000 and 62,000 lbs. per sq. in.

Iron rivets in steel plates, 38,000 lbs. shearing strength per sq. in. in single shear, and 85% more, or 70.300 lbs., in double shear.

Each shell-plate must bear a test-coupon which shall be sheared off and tested. Each coupon must fulfil the above requirements as to tensile strength, but must have a contraction of area of not less than 56% and an elongation of 25% in a length of 8 in. It must also stand bending 180° when cold, when red hot, and after being heated red hot and quenched in cold water, without fracture on outside of bent portion.

Crow-foot braces are required for boiler-heads without welds, and if of iron limit the strain to 7500 lbs. per sq. in., and stay-bolts must not be subjected to a greater strain than 6000 lbs. per sq. in.

The thickness of double butt-straps 8/10 the thickness of plates. In lap-joints the distance between the rows of rivets is ⅝ the pitch. In double-riveted lap-joints of plates up to ½ in. thick the efficiency is 70% and in triple-riveted lap-joints 75% of the solid plate.

In triple-riveted double-strapped butt-seams for plates from ¼ in. to ½ in. thick, the efficiency ranges from 88% to 86% of the solid plate.

In high-pressure boilers the holes are required to be drilled in place; that is, all holes may be punched ¼ in. less than full size, then the courses are rolled up, tube-heads and joint-covering plates bolted to courses, with all holes together perfectly fair. Then the rivet-holes are drilled to full size, and when completed the plates are taken apart and the burr removed.

The rule for the bursting-pressure of cylindrical boiler-shells is the following: Multiply the ultimate tensile strength of the weakest plate in the shell by its thickness in inches and by the efficiency of the joint, and divide result by the semi-diameter of shell; the quotient is the bursting-pressure per square inch. This pressure divided by the factor 5 gives the allowable working pressure.

BOILER FEEDING.

Gravity Boiler-feeders.—If a closed tank be placed above the level of the water in a boiler and the tank be filled or partly filled with water, then on shutting off the supply to the tank, admitting steam from the boiler to the upper part of the tank, so as to equalize the steam-pressure in the boiler and in the tank, and opening a valve in a pipe leading from the tank to the boiler the water will run into the boiler. An apparatus of this kind may be made to work with practically perfect efficiency as a boiler-feeder, as an injector does, when the feed-supply is at ordinary atmospheric temperature, since after the tank is emptied of water and the valves in the pipes connecting it with the boiler are closed the condensation of the steam remaining in the tank will create a vacuum which will lift a fresh supply of water into the tank. The only loss of energy in the cycle of operations is the radiation from the tank and pipes, which may be made very small by proper covering.

When the feed-water supply is hot, such as the return water from a heating system, the gravity apparatus may be made to work by having two receivers, one at a low level, which receives the returns or other feed-supply, and the other at a point above the boilers. A partial vacuum being created in the upper tank, steam-pressure is applied above the water in the lower tank by which it is elevated into the upper. The operation of such a machine may be made automatic by suitable arrangement of valves. (See circular of the Scott Boiler Feedor, made by the Q. & C. Co., Chicago.)

FEED-WATER HEATERS.

Capacity o Feed-water Heaters.—The following extract from a letter by W. R. Billings, treasurer of the Taunton Locomotive Manufacturing Co., builders of the Wainwright feed-water heater, to *Engineering Record*, February, 1898, is of interest in showing the relation of the heating surface of a heater to the work done by it:

"Closed feed-water heaters are seldom provided with sufficient surface to raise the feed temperature to more than 200°. The rate of heat transmission may be measured by the number of British thermal units which pass through a square foot of tubular surface in one hour for each degree of difference in temperature between the water and the steam. The difficulties which attend experiments in this direction can only be appreciated by those who have attempted to make such experiments. Certain results have been reached, however, which point to what appears to be a reasonable conclusion. One set of experiments made quite recently gave certain results which may be set forth in the table herewith.

Difference between final temperatures of water and steam	5° F.	67 B.T.U.	Transmitted in one hour by each sq. ft. of surface for each degree of average difference in temperatures.
	6° "	79 "	
	8° "	89 "	
	11° "	114 "	
	15° "	129 "	
	18° "	139 "	

"In other words, when the water was brought to within 5° of the temperature of the heating medium, heat was transmitted through the tubes at the rate of 67 B.T.U. per square foot for each degree of difference in temperature in one hour. When the amount of water flowing through the heater was so largely increased as to make it impossible to get the water any nearer than within 18° of the temperature of the steam, the heat was transmitted at the rate of 139 B.T.U. per sq. ft. of surface for each degree of difference in temperature in one hour. Note here that even with the rate of transmission as low as 67 B.T.U. the water was still 5° from the temperature of the steam. At what rate would the heat have been transmitted if the water could have been brought to within 2° of the temperature of the steam, or to 210° when the steam is at 212°?

"For commercial purposes feed-water heaters are given a H.P. rating which allows about one-third of a square foot of surface per H.P.—a boiler H.P. being 30 lbs. of water per hour. If the figures given in the table above are accepted as substantially correct, a heater which is to raise 3000 lbs. of water per hour from 60° to 207°, using exhaust steam at 212° as a heating medium, should have nearly 84 sq. ft. of heating surface—that is, a 100 H.P. feed-water heater which is to maintain a constant temperature of not less than 207°, with water flowing through it at the rate of 3000 lbs. per hour, should have nearly a square foot of surface per H.P. That feed-water heaters do not carry this amount of heating surface is well known."

THE STEAM-ENGINE.

Current Practice in Engine Proportions, 1897 (Compare pages 792 to 817.)—A paper with this title by Prof. John H. Barr, in Trans. A. S. M. E., xviii. 737, gives the results of an examination of the proportions of parts of a great number of single-cylinder engines made by different builders. The engines classed as low speed (L. S.) are Corliss or other long-stroke engines usually making not more than 100 or 125 revs. per min. Those classed as high speed (H. S.) have a stroke generally of 1 to 1½ diameters and a speed of 200 to 300 revs. per min. The results are expressed in formulas of rational form with empirical coefficients, and are here abridged as follows:

Thickness of Shell, L. S. only.—$t = CD + B$; $D =$ diam. of piston in in.; $B = 0.3$ in.; C varies from 0.4 to 0.6, mean $= 0.5$.

Flanges and Cylinder-heads.—1 to 1.5 times thickness of shell, mean 1.2.

Cylinder-head Studs.—No studs less than ¾ in. nor greater than 1⅜ in. diam. Least number, 8, for 10 in diam. Average number $= 0.7D$. Average diam. $= D/40 + ½$ in.

Ports and Pipes.—$a =$ area of port (or pipe) in sq. in.; $A =$ area of piston, sq. in.; $V =$ mean piston-speed, ft. per min.; $a = AV/C$, in which $C =$ mean velocity of steam through the port or pipe in ft. per min.

Ports, H. S. (same ports for steam as for exhaust).—$C = 4500$ to 6500, mean 5500. For ordinary piston-speed of 600 ft. per min. $a = KA$; $K = .09$ to $.13$, mean .11.

Steam-ports, L. S.—$C = 5000$ to 9000, mean 6800; $K = .08$ to $.10$, mean .09.

Exhaust-ports, L. S.—$C = 4000$ to 7000, mean 5500; $K = .10$ to $.125$, mean .11.

Steam-pipes, H. S.—$C = 5800$ to 7000, mean 6500. If $d =$ diam. of pipe and $D =$ diam. of piston, $d = .29D$ to $.32D$, mean $.30D$.

Steam-pipes, L. S.—$C = 5000$ to 8000, mean 6000; $d = .27$ to $.35D$, mean $.32D$.

Exhaust-pipes, H. S.—$C = 2500$ to 5500, mean 4400; $d = .33$ to $.50D$, mean $.37D$.

Exhaust-pipes, L. S.—$C = 2800$ to 4700, mean 3800; $d = .35$ to $.45D$, mean $.40D$.

Face of Pistons.—F = face; D = diameter. $F = CD$. H. S.: C = .30 to .60 mean .46. L. S.: C = .25 to .45, mean .32.

Piston-rods.—d = diam. of rod; D = diam. of piston; L = stroke, in.; $d = C\sqrt{DL}$. H. S.: C = .12 to .175, mean .145. L. S.: C = .10 to .13, mean .11.

Connecting-rods.—H. S. (generally 6 cranks long, rectangular section): b = breadth; h = height of section; L_1 = length of connecting-rod; D = diam. of piston; $b = C\sqrt{DL_1}$; C = .045 to .07, mean .057; $h = Kb$; K = 2.2 to 4, mean 2.7. L. S. (generally 5 cranks long, circular sections only): C = .082 to .105, mean .092.

Cross-head Slides.—Maximum pressure in lbs. per sq. in. of shoe, due to the vertical component of the force on the connecting-rod. H. S.: 10.5 to 38, mean 27. L. S: 29 to 33, mean 40.

Cross-head Pins.—l = length; d = diam.; projected area = $a = dl = CA$; A = area of piston; $l = Kd$. H. S.: C = .06 to .11, mean .08; K = 1 to 2, mean 1.25. L. S.: C = .054 to .10, mean .07; K = 1 to 1.5, mean 1.3.

Crank-pin.—HP = horse-power of engine; L = length of stroke; l = length of pin; $l = C \times HP/L + B$; d = diam. of pin; A = area of piston; $dl = KA$. H. S.: C = .13 to .46, mean .30; B = 2.5 in.; K = .17 to .44, mean .24. L. S.: C = .4 to .8, mean .6; B = 2 in.; K = .065 to .115, mean .09.

Crank-shaft Main Journal.—$d = C\sqrt[3]{HP \div N}$; d = diam.; l = length; N = revs. per min.; projected area = MA; A = area of piston. H.S.: C = 6.5 to 8.5, mean 7.3; K = 2 to 3, mean 2.2; M = .37 to .70, mean .46. L. S.: C = 6 to 8, mean 6.8; K = 1.7 to 2.1, mean 1.9; M = .46 to .64, mean .56.

Piston-speed.—H. S.: 530 to 660, mean 600; L. S.: 500 to 850, mean 600.

Weight of Reciprocating Parts (piston, piston-rod, cross-head, and one-half of connecting-rod).—$W = CD^2 \div LN^2$; D = diam. of piston; L = length of stroke, in.; N = revs per min. H. S. only: C = 1,200,000 to 2,300,000, mean 1,860,000.

Belt-surface per I.H.P.—$S = CHP + B$; S = product of width of belt in feet by velocity of belt in ft. per min. H. S.: C = 21 to 40 mean 28; B = 1800. L. S.: $S = C \times HP.$; C = 30 to 42, mean = 35.

Fly wheel (H. S. only).—Weight of rim in lbs.: $W = C \times HP \div D_1{}^2N^3$; D_1 = diam. of wheel in in.; $C = 65 \times 10^{10}$ to 2×10^{12} mean = 12×10^{11}, or 1,200,000,000,000.

Weight of Engine per I.H.P. in lbs., including fly-wheel.—$W = C \times H.P.$ H. S.: C = 100 to 135, mean 115. L. S.: C = 135 to 240, mean 175.

Work of Steam-turbines. (See p. 791.)—A 300-H.P. De Laval steam-turbine at the 12th Street station of the Edison Electric Illuminating Co. in New York City in April, 1896, showed on a test a steam-consumption of 19.275 lbs. of steam per electrical H.P. per hour, equivalent to 17.348 lbs. per brake H.P., assuming an efficiency of the dynamo of 90%. The steam-pressure was 145 lbs. gauge and the vacuum 26 in. It drove two 100-K.W. dynamos. The turbine-disk was 29.5 in. diameter and its speed 9000 revs. per min. The dynamos were geared down to 750 revs. The total equipment, including turbine, gearing, and dynamos, occupied a space 13 ft. 3 in. long, 6 ft. 5 in. wide, and 4 ft. 3 in. high.

The "Turbinia," a torpedo-boat 100 ft. long, 9 ft. beam, and 44⅛ tons displacement, was driven at 31 knots per hour by a Parsons steam-turbine in 1897, developing a calculated I.H.P. of 1576 and a thrust H.P. of 946, the steam-pressure at the engine being 130 lbs. and at the boilers 200 lbs. The vacuum was 13½ lbs. The revolutions averaged 2100 per minute. The calculated steam-consumption was 15.86 lbs. per I.H.P. per hour. On another trial the "Turbinia" developed a speed of 32¾ knots.

Relative Cost of Different Sizes of Steam-engines.

(From catalogue of the Buckeye Engine Co., Part III.)

Horse-power ..	50	75	100	125	150	200	250	300	350	400	500	600	700	800
Cost per H.P, $	20	17½	16	15	14½	13½	13	12¾	12.6	12.6	12.8	13¼	14	15

LOCOMOTIVES.

Resistance of Trains.—The Baldwin Locomotive Works contribute the following notes to the text on pages 852 to 862.

"*On page* 852, we think the resistances 'y' for increasing speeds were originally intended to be added to a coefficient for the total frictional resistance, for, if we assume a straight, level track and a speed of 5 miles per hour, then according to the formula the total resistance per ton would be 3.8 lbs. This is less than we are actually able to obtain under most favorable conditions, and we know that in some cases, for instance, in mine construction, the frictional resistance has been shown to be as much as 60 lbs. per ton at slow speed. This resistance should be approximate to suit the conditions of each individual case, and the increased resistances due to speed added thereto.

"*On page* 853, in the formula

$$uP - W(.0005c \pm .00019m) = Ll + .00025C \pm .00019m,$$

the journal and rolling resistances of engine and tender at different speeds are not accounted for, unless the author includes them in the coefficient 'u,' under the supposition that the tractive power will be in proportion to the total weight of engine and tender at different speeds. As the proportion of driver, or adhesive weight, to the total weight of engine and tender varies considerably in different classes, we think this rather indefinite. If the coefficient 'u' were made to embrace only the resistances of the working parts, and the coefficient 'l' (after the modification suggested above), were applied to the weight of engine and tender, we think the formula would be more generally applicable. For instance, in the formula assume, as before, a straight, level track: then $W(.005c \pm .00019m)$ would reduce to 0, and the total weight of engine and tender would disappear entirely, except in their indirect influence upon coefficient 'u.'

"*Approximate Formula for Train Resistance.* (See Holmes on the Steam Engine, pages 141 to 148.)

"*Page* 856, '*Exhaust Nozzles.*' Refer to the Annual Report of the American Railway Master Mechanics' Association for 1896, which gives some interesting data on the subject.

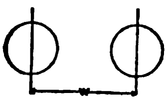
"*Page* 855, '*Boilers.*' Refer to Holmes on the Steam Engine, pages 371 to 377, and 383 to 389, and also to the Master Mechanics' Report for 1897, pages 218 to 232, for a very important list of data and formulæ.

"*Page* 864, '*Counterbalancing.*' Refer to the Master Mechanics' Report for 1896, pages 148 to 155, for some interesting formulæ.

"*Formulæ for Curves.*

Approximate Formula for Radius.

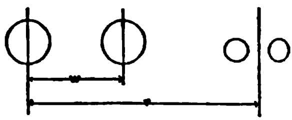
$$R = \frac{.7646W}{2P}.$$



R = radius of min. curve in feet.
P = play of driving-wheels in decimals of 1 ft.
W = rigid wheel-base in feet.

Approximate Formula for Swing.

$$\frac{WT}{2R} = S.$$



W = rigid wheel base.
T = total " "
R = radius of curve.
S = swing on each side of centre."

Performance of a High-speed Locomotive.—The Baldwin compound locomotive No. 1027, on the Phila. & Atlantic City Ry., in July and August, 1897, made a record of which the following is a summary:

On July 2d a train was placed in service scheduled to make the run between the terminal cities in 1 hour. Allowing 8 minutes for ferry from Philadelphia to Camden, the time for the 55½ miles from the latter point to Atlantic City was 52 minutes, or at the rate of 64 miles per hour. Owing to the inability of the ferry-boats to reach Camden on time, the train always left late, the average detention being upwards of 2 minutes. This loss was invariably made up, the train arriving at Atlantic City ahead of time 2 minutes on an average, every day. For the 52 days the train ran, from July 2d to August 31st, *the average time consumed on the run was 48 minutes*, equivalent to a uniform rate of speed from start to stop of 69 miles per hour. On July 14th the run from Camden to Atlantic City was made in 46½ min., *an average of 71.6 miles per hour for the total distance.* On 22 days the train consisted of 5 cars and on 30 days it was made up of 6, the weight of cars being as follows: combination car, 57,200 lbs.; coaches, each, 59,200 lbs.; Pullman car, 85,500 lbs.

The general dimensions of the locomotive are as follows: cylinders, 13 and 22 × 26 in.; height of drivers, 84¼ in.; total wheel-base, 26 ft. 7 in.; driving-wheel base, 7 ft. 3 in.; length of tubes, 13 ft.; diameter of boiler, 58¾ in.; diameter of tubes, 1¾ in.; number of tubes, 278; length of fire-box, 113⅞ in.; width of fire-box, 96 in.; heating-surface of fire-box, 186.4 sq. ft.; heating-surface of tubes, 1614.9 sq. ft.; total heating-surface, 1835.1 sq. ft.; tank capacity, 4000 gallons; boiler-pressure, 200 lbs. per sq. in.; total weight of engine and tender, 227,000 lbs.; weight on drivers (about), 78,600 lbs.

Locomotive Link Motion.—Mr. F. A. Halsey, in his work on "Locomotive Link Motion," 1898, shows that the location of the eccentric-rod pins back of the link-arc and the angular vibrations of the eccentric-rods introduce two errors in the motion which are corrected by the angular vibration of the connecting-rod and by locating the saddle-stud back of the link-arc. He holds that it is probable that the opinions of the critics of the locomotive link motion are mistaken ones, and that it comes little short of all that can be desired for a locomotive valve motion. The increase of lead from full to mid gear and the heavy compression at mid gear are both advantages and not defects. The cylinder problem of a locomotive is entirely different from that of a stationary engine. With the latter the problem is to determine the size of the cylinder and the distribution of steam to drive economically a given load at a given speed. With locomotives the cylinder is made of a size which will start the heaviest train which the adhesion of the locomotive will permit, and the problem then is to utilize that cylinder to the best advantage at a greatly increased speed, but under a greatly reduced mean effective pressure.

Negative lead at full gear has been used in the recent practice of some railroads. The advantages claimed are an increase in the power of the engine at full gear, since positive lead offers resistance to the motion of the piston; easier riding; reduced frequency of hot bearings; and a slight gain in fuel economy. Mr. Halsey gives the practice as to lead on several roads as follows, showing great diversity:

	Full Gear Forward, in.	Full Gear Back, in.	Reversing Gear, in.
New York, New Haven & Hartford	1/16 pos.	¼ neg.	¼ pos.
Maine Central	0	¼ neg.	....
Illinois Central	1/32 pos.	....	abt. 3/16
Lake Shore	1/16 neg.	9/64 neg.	5/16 pos.
Chicago Great Western	0	0	3/16 to 9/16
Chicago & Northwestern	3/16 neg.	....	¼ pos.

The link-chart of a locomotive built in 1897 by the Schenectady Locomotive Works for the Northern Pacific Ry. is as follows:

Lead.		Valve Open.		Cut-off.	
Forward Stroke, in.	Rearward Stroke, in.	Forward Stroke, in.	Rearward Stroke, in.	Forward Stroke, in.	Rearward Stroke, in.
− ⅛	− ⅛	1 ⅛	1 ⅛	22 9/16	22⅝
− 1/32	− 1/32	1 7/16	1 7/16	21	21
+ 1/32	+ 1/32	1 1/16	1 1/16	19	19
3/32	3/32	23/32	23/32	16	16
⅛	⅛	½	½	13	13⅛
9/64	9/64	⅜	⅜	10	10
5/32 s.	5/32 s.	5/16	5/16	8	8
5/32	5/32	¼	¼	6	6
5/32 f.	5/32 f.	7/32	7/32	4	4 1/16

Cylinders 20 × 26 in., driving-wheels 69 in., six coupled wheels, main rods 126¼ in., radius of link 40 in., lap 1⅛ in., travel 6 in., Allen valve.

GEARING.

Efficiency of Worm Gearing. (See also page 898.)—Worm gearing as a means of transmitting power, has until recently, generally been looked upon with suspicion, its efficiency being considered necessarily low and its life short. Recent experience, however, indicates that when properly proportioned it is both durable and reasonably efficient. Mr. F. A. Halsey discusses the subject in *Am. Machinist*, Jan. 13 and 20, 1898. He quotes two formulas for the efficiency of worm gearing due to Prof. John H. Barr:

$$E = \frac{\tan \alpha \, (1 - f \tan \alpha)}{\tan \alpha + f}, \quad . \; . \; . \quad (1) \qquad E = \frac{\tan \alpha \, (1 - f \tan \alpha)}{\tan \alpha + 2f} \text{ approx.}, \quad . \; . \; . \quad (2)$$

in which E = efficiency; α = angle of thread, being angle between thread and a line perpendicular to the axis of the worm; f = coefficient of friction.

Eq. (1) applies to the worm thread only, while (2) applies to the worm and step combined, on the assumption that the mean friction radius of the two is equal. Eq. (1) gives a maximum for E when $\tan \alpha = \sqrt{1 + f^2} - f$ . . . (3) and eq. (2) a maximum when $\tan \alpha = \sqrt{2 + 4f^2} - 2f$. . . . (4) Using a value .05 for f gives a value for α in (3) of 43° 34′ and in (4) a value of 52° 49′.

On plotting equations (1) and (2) the curves show the striking influence of the pitch-angle upon the efficiency, and since the lost work is expended in friction and wear, it is plain why worms of low angle should be short-lived and those of high angle long-lived. The following table is taken from Mr. Halsey's plotted curves:

RELATION BETWEEN THREAD-ANGLE SPEED AND EFFICIENCY OF WORM GEARS.

Velocity of Pitch-line, feet per minute.	Angle of Thread. 5	10	20	30	40	45
	Efficiency.					
3	35	52	66	73	76	77
5	40	56	69	76	79	80
10	47	62	74	79	82	82
20	52	67	78	83	85	86
40	60	74	83	87	88	88
100	70	82	88	91	91	91
200	76	85	91	92	92	92

The experiments of Mr. Wilfred Lewis on worms show a very satisfactory correspondence with the theory. Mr. Halsey gives a collection of data comprising 16 worms doing heavy duty and having pitch-angles ranging between 4° 30′ and 45°, which show that every worm having an angle above 12° 30′ was successful in regard to durability, and every worm below 9° was unsuccessful, the overlapping region being occupied by worms some of which were successful and some unsuccessful. In several cases worms of one pitch-angle had been replaced by worms of a different angle, an increase in the angle leading in every case to better results and a decrease to poorer results. He concludes with the following table from experiments by Mr. James Christie, of the Pencoyd Iron Works, and gives data connecting the load upon the teeth with the pitch-line velocity of the worm:

LIMITING SPEEDS AND PRESSURES OF WORM GEARING.

	Single-thread Worm 1″ Pitch, 2⅞ Pitch Diam.				Double-thread Worm 2″ Pitch, 2⅞ Pitch Diam.			Double-thread Worm 2¼″ Pitch, 4½ Pitch Diam.		
Revolutions per minute........	128	201	272	425	128	201	272	201	272	425
Velocity at pitch-line in feet per minute................	96	150	205	320	96	150	205	235	319	498
Limiting pressure in pounds...	1700	1300	1100	700	1100	1100	1100	1100	700	400

LIST OF AUTHORITIES QUOTED IN THIS BOOK.

When a name is quoted but once or a few times only, the page or pages are given. The names of leading writers of text-books, who are quoted frequently, have the word "various" affixed in place of the page-number. The list is somewhat incomplete both as to names and page numbers.

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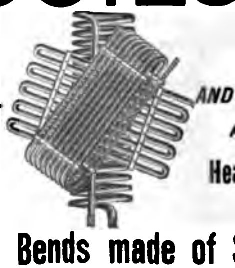

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