THRESHOLDS OF SCIENCE

MECHANICS

by C. E. Guillaume

GENERAL FOREWORD

There are many men and women who, from lack of opportunity or some other reason, have grown up in ignorance of the elementary laws of science. They feel themselves continually handicapped by this ignorance. Their critical faculty is eager to submit, alike old established beliefs and revolutionary doctrines, to the test of science. But they lack the necessary knowledge.

Equally serious is the fact that another generation is at this moment growing up to a similar ignorance. The child, between the ages of six and twelve, lives in a wonderland of discovery; he is for ever asking questions, seeking explanations of natural phenomena. It is because many parents have resorted to sentimental evasion in their replies to these questionings, and because children are often allowed either to blunder on natural truths for themselves or to remain unenlightened, that there exists the body of men and women already described. On all sides intelligent people are demanding something more concrete than theory; on all sides they are turning to science for proof and guidance.

To meet this double need—the need of the man who would teach himself the elements of science, and the need of the child who shows himself every day eager to have them taught him—is the aim of the "Thresholds of Science" series.

This series consists of short, simply written monographs by competent authorities, dealing with every branch of science—mathematics, zoology, chemistry and the like. They are well illustrated, and issued at the cheapest possible price. When they were first published in France they met with immediate success, showing that science is not necessarily dull or fenced off by a barrier of technical jargon. Of course, specialisation in this as in other subjects is not for everyone, but the publication of this series of books enables any man or woman to learn, any child to be taught, to pass with understanding and safety the "Thresholds of Science."

THRESHOLDS OF SCIENCE

MECHANICS

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MECHANICS

BY

C. E. GUILLAUME

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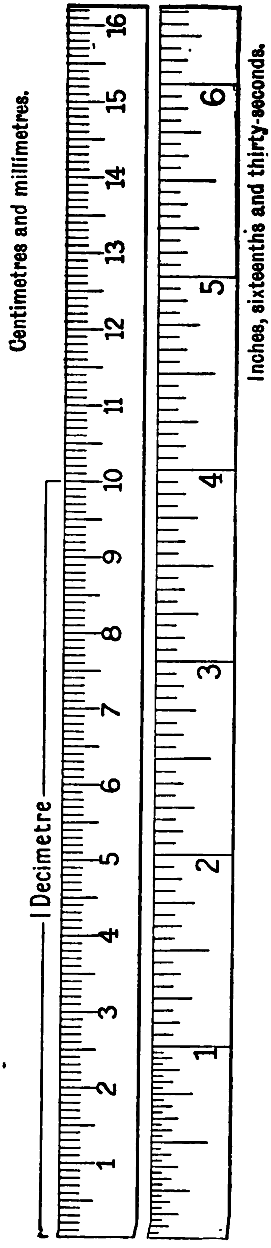

METRIC MEASURES

LENGTH

1000 metres	= 1 kilometre = 1093·6 yards = about $\frac{5}{8}$ mile.
100 m.	= 1 hectometre.
10 m.	= 1 decametre.
1 metre	= 39·37 ins. = 1·09 yard.
·1 m.	= 1 decimetre.
·01 m.	= 1 centimetre.
·001 m.	= 1 millimetre = about $\frac{1}{25}$ in.

MASS

1000 grammes	= 1 kilogramme = about $2\frac{1}{4}$ lbs.
100 g.	= 1 hectogramme.
10 g.	= 1 decagramme.
1 gramme	= weight of 1 c.c. of water at 4° C. = 15·43 grains = ·035 oz.
·1 g.	= 1 decigramme.
·01 g.	= 1 centigramme.
·001 g.	= 1 milligramme.

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MECHANICS

FIRST PART

Preparation for the Study of Mechanics

CHAPTER I

HOW TO STUDY NATURE

I.—Observation and Experiment.

If we wish to make any progress in the study of nature, we must, before everything, take a keen delight in contemplating its ever-changing aspects; we must allow them to become part of ourselves and we must be constantly under their charm; we must frequently ponder them, comparing them with each other in order to learn in what respect they are alike, and thus determine from similarity of results the existence of similar causes. The first step towards the discovery of their relations, that is, to the discovery of natural laws, is to classify these effects and ascertain their causes.

Watch a stone rolling down the side of a mountain. At first it rolls slowly, then its speed gradually increases; it meets a root, bounds into the air, falls, rolls and bounds again, until, arriving at the bottom of the valley, it makes a last leap and slowly rolls to a standstill.

Watch ten or a hundred stones; a resemblance can be recognised in their descent—they all travel faster and faster. There the resemblance stops; they all take different routes, because each one is acted upon by forces peculiar to itself.

If we were gifted with a penetration a hundred times keener than that of Sherlock Holmes, we might, by dint of perseverance, determine the complex forces that each stone obeys. But we are naturally dull and slow of apprehension; we must therefore simplify our task by persuading nature to disclose its secrets.

For this purpose we must arrange a place where these actions can happen in an unusually simple manner. We must replace direct *observation* by a kindred process—*experiment.*

An *experiment* is a question addressed to nature, which is always prepared to answer correctly, if the question is correctly put. There will be no false evidence to mislead us, no lawyers seeking to prejudice the inquiry. Nature discloses itself in all its sincerity, and our art will consist merely in asking it a simple question to which it can return only a simple answer.

Replace the mountain-side by a smooth plank, the stone by a ball made by a skilful turner, and let the ball run down the inclined plank. After our experiment, we shall be able to describe very closely not only the path that all balls will follow on planks having the same inclination, but other details of their descent; if our observation has been complete, we shall even know the moment at which the ball will pass any point on the plank, as well as the time occupied in travelling the whole length.

II.—Approximation and Simplification.

Can we describe *accurately* every feature of the descent of the ball? No; because the plank is not perfectly straight nor the ball perfectly round; in any other experiment both ball and plank would have been different in these respects. Yet the plank is much straighter than the mountain-side, and the ball much smoother than the stone. We have not attained complete simplicity, though we have very nearly reached it; the remaining complication is of small importance in comparison with all that we have suppressed.

Let us go into detail. A good method of determining the path of the ball would be to blacken the plank with smoke, on which the ball would trace its path. Let us examine this path; at first sight we should say that it was straight, and we should not be far wrong; but if we stretched a very fine thread between the extremities of the line we should discover the existence of small undulations.

To what are due these deviations from the straight line? If we worked in a large town where traffic shakes the houses, we should not have far to seek. We should know the traffic was responsible. But if we still have doubts, we can wait until everything is still and then begin again. The undulations of the line will have diminished greatly, and this will confirm our hypothesis; but some sign of it will remain, and we shall conclude that this hypothesis is insufficient. We shall then suspect that the plank is not smooth enough or the ball is not round enough for the path to be absolutely straight. We have not attained complete simplicity.

By carrying this experiment into detail we have gained many useful ideas; we have seen that a minute examination of what happened enabled us to discover peculiarities that previously escaped us; hence, even when we attempt to realise simplicity, we cannot completely succeed.

But we shall not confuse ourselves too much with these minute complications. We have attempted simple conditions and we have so nearly succeeded that our result was approximately simple; it would have been completely so if our experiment had been perfectly prepared.

What we have done is what all men of science do; they work as accurately as they can, and interpret the result by eliminating the minor interferences that attend it.

A well-conducted experiment will not then attempt an impossible perfection; it will approach as closely as possible; it will render a single action dominant in such a way that the others almost disappear, and subsequent reasoning will reject them entirely.

The details rejected in a first experiment can often be examined separately; thus, in the descent of the ball, the undulations are of great interest, for they reveal the vibration of the house shaken by movements from the street. If these movements of the ground have not, as is the case in towns, an artificial origin, they are called *seismic*, and are of extreme importance; but that is another problem, to be approached by other methods.

It has always been thus. The early naturalists described great animals in their general forms, then others examined their anatomy; next came histologists, who examined their tissues under the microscope; last of all, the study of microbes, of which they are the hosts, has concentrated upon the infinitely small the largest amount of interest. These are separate problems whose importance appears successively. The subject is first approached by simplifying, and retaining only the main facts; then little by little, the neglected detail is taken into account.

III.—The Need for Simplification.

Whether it is from natural indolence or from need of clearness, the mind prefers what is understood without much difficulty and, accordingly, it constantly resorts to simplification. When we say the number of inhabitants of France is 39 millions, we knowingly make a mis-statement. The truth is that we have not remembered the number of inhabitants. By saying 38,893,654 we should perhaps be nearer the truth, but that is not certain; and, given two numbers, one being so complicated that we are not sure of it or that rigid exactness is not of importance, the other being a simple number that we cannot vouch for further, but which is certainly approximate enough for our use, we always choose the simple number.

The need for simplification influences operations of all kinds. Why has the gun of 305 millimetres become the standard in the French navy? Simply because its calibre is 12 inches (304·8 mm.). There may be technical reasons for preferring a cannon of 12 inches to one of 11 or 13. But 304 and 306 are too near 305 for any inconvenience to be felt in adopting one or other of these dimensions instead of the one that has been chosen. In England, the advantages of a calibre of 12 inches having been recognised, this diameter has been adopted for simplicity; and other countries have followed because the English Admiralty is generally well advised in what it does.

IV.—Limits of Experiment.

The rougher our mode of investigation, the more perfect will our result appear; the line traced by the ball appeared straight until we compared it with a stretched thread, and only when we had resorted to this measure were the deviations obvious.

Similarly, a labourer touches a mirror and then a plank with his thick fingers; he thinks them equally smooth. A blind man, whose sense of touch is particularly refined, could arrange ten surfaces of decreasing roughness between the plank and the mirror.

On the other hand, a lacemaker will arrange by means of her delicate fingers a series of threads, the different thicknesses of which a smith could not detect at all if he had no other means than that of taking them separately between his finger and thumb.

These judgments do not depend entirely on the quality of our senses; they vary with the means adopted for examination.

An echo is an image of sound as a reflection in a mirror is the echo of a face. If the reflection is good it must be formed on a surface from which every trace of unevenness or irregularity has been removed. An echo, on the contrary, is returned by a stretched sheet, by a wall, and by a screen of trees—not very distinctly in the case of the last, but very clearly in the case of the house, which is therefore a very suitable mirror for sound, as it returns the image without appreciable distortion.

Why this difference? The structure of light is much finer than that of sound; the latter, like the labourer with the plank, declares the wall perfectly smooth, while the light penetrates and discloses all its irregularities, as the lacemaker separates and classifies threads.

This difference in the nature of light and sound is discernible in all their manifestations; it enables us to hear the words of a person through a wall, while, in order to see him, we must be able to look directly at him.

In studying nature, we must be perfectly aware of the degree of delicacy of our process. Sometimes it has to be exact, sometimes it is preferred less exact. Look at a good engraving through a lens; it is easy to distinguish, lighter and darker, and separated by more or less black, all the separate points that constitute the picture; but the picture, though now not so pleasant to look at, will not have completely disappeared. Let us now examine it under the microscope; only the points remain, and we cannot unite them. Just as trees prevent us from seeing the forest, so the points prevent us from seeing the picture. Too close an examination exaggerates the detail and leaves the whole indistinct.

The roughness of a process of examination brings about that simplification which otherwise our minds would be forced to effect. At first glance, the line on the plank appeared straight, but close examination proved it not so;

its deviations are interesting from one point of view, but for elementary study they are too subtle.

V.—Illusion.

Our senses are not only too dull or too refined; they can, in addition, mistake the true nature of what they perceive, and give us a false impression. They con-

FIG. 1.—Spiral or circle?

stantly mislead us, because they allow themselves to be deceived. They are like a judge who, listening to the statements of a person on trial, is incapable of separating what is true from what is added for the purpose of exciting pity.

To our eyes, the figure above represents a spiral. Follow this spiral with the point of a pencil. It resolves itself into circles, The inventor of this illusion desired to deceive our sight, and he has completely succeeded; our eyes in turn deceive us, without intending to do so.

Illusions are found everywhere. When, in the sixteenth century, the thermometer enabled temperature to be determined otherwise than by touching a body with the hand, men were astonished to discover that springs are colder in winter than in summer, since the evidence of the senses had always taught the contrary. The cause of this illusion we shall recognise if we plunge the right hand into cold water, the left into warm water, and then both into tepid water. The water will appear cold to the left and warm to the right.

Bodies of equal weight seem heavier to us in inverse proportion to their size. The following experiment is always successful. Put upon the hands of a friend a cardboard box and a lead ball of equal weight. Ask him the question, "How much heavier is the ball than the box?" The reply is generally, "Three or four times." We never trust, then, the evidence of our hands if we have to buy anything by weight.

Occupation enables us to forget the passing of time; but when a day has been very full of events, although it may have appeared short at the time, the morning seems very far off when evening comes. On a journey we often confuse yesterday with the day before.

Waiting makes time seem endless; the popular expression "I have waited an age for you," is a significant proof.

The loss of illusions is a cause for regret, and in ordinary life we are happier if we retain some of them. Painters, engravers, and all those who present images for our enjoyment appeal to our capacity for illusion; and the better they succeed, the more grateful we are.

When, instead of believing, we wish to know, we must rid ourselves of illusions, and try to see things as they really are, within the range of power and penetration of our senses.

Talleyrand advised mistrust of a first impulse, " because it is good " ; we shall learn to mistrust a first impression, but for the opposite reason—" because it is often bad."

VI.—Education of the Senses: Measurement.

Our first defence against the erroneous impressions given by our sense-organs is to educate them. Some people have good, others only moderately good, eyes ; but the education of the eye can be as much improved in the direction of accuracy of observation as the fingers of a pianist for playing or a goldsmith for fine work.

Some hands can carve while others can do no more than forge ; in the same way, the eye trained in close observation sees minute details that escape the ordinary eye. The eye of a painter or a dyer (for a profession, like art, refines the perception) distinguishes ten shades where the ordinary eye sees only one. The trained musician's ear distinguishes in the playing of an orchestra every note of flute or violin, while the amateur gains only a general impression.

Modern education makes much of the training of the senses ; but to equip the mind with the learning of others seems to be the sole aim of many teachers—if they deserve the name. Their pupils learn to *see by proxy*, when really in every-day life and in the performance of their profession they ought to be able to see with their own eyes.

The constant aim of a child who desires to be in a position to understand the environment in which his life is to be passed ought to be to compel himself to take full advantage of everything that the eye can see, the ear can hear, and the fingers can touch. Some, for special and often professional reasons, will train special senses ; and, if the sources of perception are sufficiently educated, understanding will soon follow.

The establishment of connections between the organs of sense is no less important and can be brought to astonishing precision. Although, for instance, a player at bowls may not be able to guess without difficulty the exact distance to the jack, he will almost certainly reach it in play; his skill is a sign of perfect co-ordination between his visual estimation and his muscular sense—a co-ordination that is partly unconscious, but is brought about by training and is applied almost automatically.

By means of extreme co-ordination modern life has created new men. How superior, in the rapidity of his perceptions, is the chauffeur to the countryman driving his horse at the trot! The chauffeur perceives a whole series of objects that in the tenth of a second may be dangerous obstacles; in a moment he gives to his muscles the orders they have to execute to avoid a catastrophe.

The modern aviator is a notable example of the result of training. He is a complete orchestra of perceptions and commands. If man has learned to fly in the air, it is not only because he has been able to construct flying machines, but quite as much because he has become acquainted with all their movements in that deceptive medium. Education of the senses has created this new man—a sort of superman—and has thus realised the ideal of many centuries of human hope.

Our second defence against illusions consists in replacing unaided observation by a more exact operation—*measurement.*

This is not necessarily a scientific operation; everyone measures more or less, but scientific measurement is generally more precise than measurement in ordinary life.

Perfection of measurement has accompanied, and often preceded, science, whose existence has frequently depended upon the existence of measurement. Yet rough measurements have led to interesting and even great discoveries. Later, Mechanics will teach us the laws of oscillation of a pendulum, and will give us the explanation. Galileo discovered them simply by feeling his pulse while watching a large chandelier swing slowly in the nave of the Cathedral at Pisa. His devotions were but half-hearted, it seems; later, he was made to realise that.

Afterwards, Galileo wanted to measure with greater accuracy the time occupied by a body in rolling down an inclined plane. For this purpose he placed above a balance a vessel filled with water and provided with an opening discharging into a glass fixed on the balance. He held his finger over the opening, opened it simultaneously with the release of the body, and closed it when the descent was finished. By weighing the water discharged into the glass he calculated the time he wished to know.

Galileo's measurements were still far from exact; but a beginning had to be made somewhere. His discovery of the first laws of Mechanics, and, in particular, of those governing the motion of the pendulum, had the precise result of making possible more accurate measurement of time. A century later Huyghens applied the pendulum to the regulation of the motion of clocks; still later he adapted an analogous principle to watches, by unceasingly restoring a balance-wheel to its position of equilibrium by means of a spring.

Measurement of time has progressed continuously towards perfection. After 120 years, the competition of the Observatory of Geneva is still instructive. In 1790, a prize of £20 was offered for the best watch, on condition that its error did not exceed one minute a day when at rest and two minutes when carried. The first year no watch satisfied the conditions; the next year three were satisfactory.

To-day very good watches vary by less than two-tenths of a second per day; for the watches that won the first prizes at the end of the eighteenth century we should not give ten shillings.

Certain delicate experiments necessitate the measurement of extremely short periods of time. Further on, we shall indicate a method of determining the infinitely short duration of an impact. The following is the means generally employed for measuring a very short interval of time.

When a tuning-fork is struck, its arms approach and separate alternately, in such a way that the successive

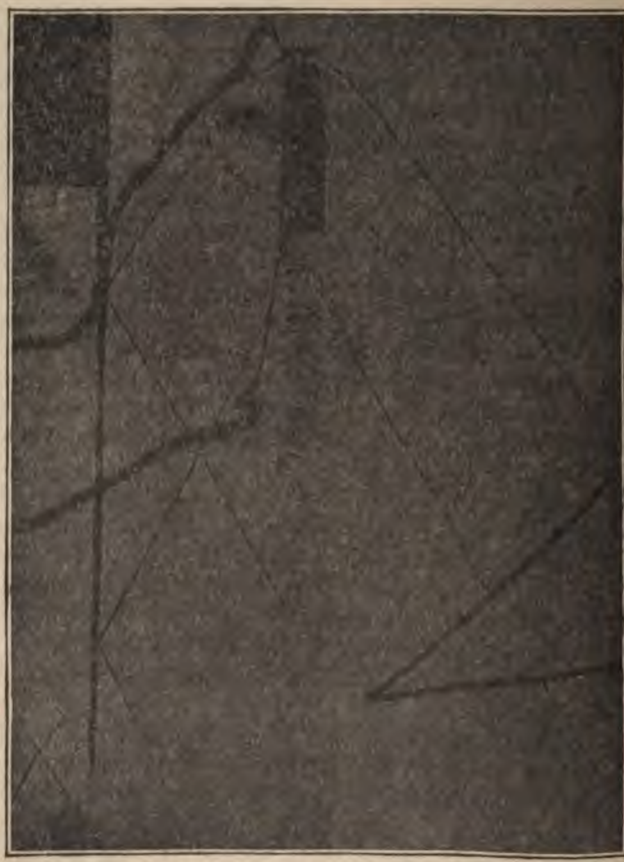


FIG. 2.—A rifle bullet in flight is accompanied by air waves and followed by a furrow.

movements are of exactly the same duration. A very fine point can be fixed to one of its arms so as to touch a piece of smoked paper turning rapidly on a drum. The point will trace on the paper a regular and sinuous line, and, if the tuning-fork is making 500 vibrations per second, each of the undulations corresponds to $\frac{1}{500}$th of a second. By marking the beginning and end of the interval by an electric spark, the duration can be found with a degree of approximation that will depend upon the measurement of a fraction of an undulation; but it is already apparent that there are no great difficulties in measuring an interval of time to the five thousandth of a second.

In experiments in which speed is essential, electricity inevitably plays an important part. We shall know better what can be done when we have examined the photograph reproduced in Fig. 2. This is a rifle bullet taken close to the gun, and travelling through the air with a velocity of nearly 600 metres per second.* The shot is perfectly clear at the line of its base—evidence that it has traversed hardly more than $\frac{1}{10}$th of a millimetre during the exposure. The exposure, then, has not lasted more than $\frac{1}{6,000,000}$ of a second, and if it were desired to expose a lens for so short a time the shutter would have to be given a velocity that would render it much more destructive than a rifle bullet.

The method is as follows :—

The rifle is so pointed that the shot cuts in its passage two wires forming part of a circuit, from which an extremely brief spark is given.

This throws on the plate the shadow of the bullet, which is photographed in outline. Apart from the difficulty, it is obviously not in the photograph of the shot that the interest lies; it is in what can be seen very clearly happening around the shot. From the point

* This photograph was taken by Mr. C. V. Boys, F.R.S.

and base of the bullet spring two oblique lines—projections of the cone formed by air waves, which, originating from the bullet, swell sharply into the air and drive it back. Then, immediately behind the shot, can be seen a disturbance similar to the back-wash set up in the wake of a boat.

The detailed study of this would take us too far and divert us from our purpose. It is important to note, however, how much better prepared we are to measure time or divide it into infinitely small parts than in the age when Galileo compared the time of oscillation of a pendulum with his pulse.

Our young readers, who probably have in their pockets watches showing seconds, will perhaps smile at such a pitiable outfit, but admiration can never be denied those wonderful men whose genius was equal to the discovery of such wonderful secrets with such meagre resources. If our age is gathering an abundant harvest, it is because former ages have tended and sown the ground from which to-day is springing such a profusion of fruit.

VII.—Induction and Deduction.

Another means of discovery is deduction, that is, the mental operation of tracing a cause to its effects, and determining from these data alone the course of a phenomenon.

Mathematicians always proceed thus, with the difference that the starting-point of their deduction is not a *cause*, properly speaking, but an axiom or a postulate. Men have not, however, always worked in this way; some centuries ago, when it was desired to know the area enclosed by a curve, the curve, after having been traced on cardboard, was cut out and weighed. Galileo, whose name occurs very often in this book (this is by no means the last time), discovered, by this experimental process, certain interesting relations. The mind must be disciplined, however, even in deduction, before it can continue its way unaided.

In our studies we can sometimes proceed like geometricians, and confine ourselves to reasoning, even when an experiment is easy.

Take, for example, an instance of experiment and deduction applied to the same physical problem.

Without going into details, we know it is possible to measure the illumination of a surface, that is, the amount of light that falls on a given area. Place a candle 1 yard distant from a wall, and then find how many candles must be placed at a distance of 2 yards to produce the same effect; it will be found that four are necessary; at 3 yards nine, and so on. If the experiment shows small differences, we shall apply the principle of simplification, and we shall say that the approximate figures are sufficient for the degree of accuracy to which we wish to carry our investigation.

Let us now set out the two series of numbers :—

Distances.	Number of candles.
1	1
2	4
3	9

We see that the last three numbers are the squares of the first three, and we conclude the illumination on the surface diminishes, when the distance from the light increases, as the square of this distance. This relation is one of the first of the laws of photometry. A moment's reflection would have enabled us to foretell it.

Represent the flame of the candle by a point, and imagine a square surface placed at a certain distance from the point, so that a perpendicular drawn from this point to the plane falls at its middle point. The light which illuminates the plane from the point O (Fig. 3) is contained within the pyramid OABCD. Now replace the plane by another, EFGH, twice as far from the point. The latter has a surface four times that of the former; since it does not receive more light, it is a quarter as well lighted. Thus the photometric law is evident.

Deduction, occupying one minute, has saved us from making an experiment that, to be conclusive, would have taken some hours. Deduction has thus one advantage; but there is another. Our reasoning has taught us that the law is true only if all the light received on the first surface falls on the second. Therefore, if the experiment had been carried out in a fog, the results would not have been true. They would have been very complicated and difficult to elucidate, though reasoning would at least have helped towards a solution.

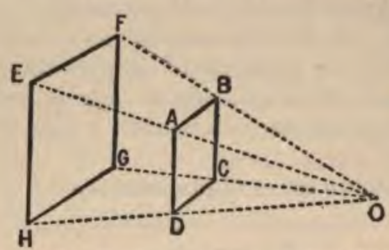


FIG. 3.—Evidence of the photometric law.

Since it is easy to discover laws by simple reasoning, why is this convenient and economical process not always resorted to? Simply because problems are not always free from vitiating influences, and our data are nearly always incomplete.

We already know how much we ought to distrust what we see; but how many things exist that we do not see? The man of science, perhaps more than any other, ought to bear the words of Hamlet in mind: "There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."

Though exercising great care, we may omit a factor, which may happen to be insignificant, but which may sometimes be essential.

Reason says that the surface of a liquid at rest is perfectly level, and, in general, the statement is justified. Yet, if mercury falls on a table, it divides into spherical drops; this simple example proves that our reasoning has not included everything. A force, generally negligible, but occasionally predominant, has been forgotten.

An unconscious belief that there exist unknown causes gives rise to superstition; the same belief, once conscious, prevents us from denying anything without due consideration. Yet it can be safely affirmed that there are limits to what is possible. In his famous "Physical and Mathematical Recreations" Ozanam gives a recipe for the preparation of an ointment by means of which a wound could be healed without contact—called by Paracelsus "warrior's ointment." According to Glocenius, when the weapon that had caused the wound was plunged into the curious mixture, of which he gives the formula, a cure was effected. This we deny, for the absence of any healing cause is so obvious that common sense refuses credence.

If scientific thought is, in general, more certain and more penetrating than ordinary thought, the man of science cannot, on the other hand, make a discovery without applying to the regulation of his thought that common sense that directs him in the ordinary acts of life. There is no strict division between scientific and every-day work.*

Science is not a kind of magic, but the cautious and logical extension of daily experience and reflection.

A thorough knowledge of causes enables us to complete phenomena mentally. We can guess the causes or we can infer them from their effects; this process is "induction." Here also caution must be exercised, for the

* "It is a great error to suppose that scientific truth differs from ordinary truth. The only difference is one of vastness and profundity. An eye at a microscope is still a human eye. It sees more than other eyes, but not differently."—ANATOLE FRANCE, "The Garden of Epicurus."

same effect may be produced by various causes; in the case of a crime, the whole skill of the judge is to disentangle them. Both expert and simple man are liable to msitake; the only difference is in degree.

When the railway first crossed the steppe, the peasants thought they saw the evil one *in propriâ personâ.* They knew, however, that holy images put evil spirits to flight, so, coming in procession, they knelt on the track, and, when the train came, elevated their protective images. The monster seemed to hesitate, slackened speed, and stopped just before reaching the poor people. They returned edified and convinced that the images had once more worked a miracle; they never imagined the driver did not want to run over his fellow-men.

Chemists no more see the devil in their retorts than surgeons seek him under the scalpel; a modern Faust would be far from calling Mephistopheles to his aid. One characteristic of the scientific spirit is that it progresses definitely by rejecting certain errors, after it has been deceived by them.

A citizen of Berlin, returning home one evening, ran into a burglar, who, surprised at his work, attempted to escape. He released himself from the hold of the citizen by dealing him a heavy blow. A suspected man was arrested and immediately recognised by the peaceful citizen. "How could you see in the dark?" questioned the magistrate. "His blow made me see so many stars that by their light I was able to see him with my other eye which was open."

The celebrated physiologist Müller, when consulted, declared this possible. "If light is seen," he said, "it must exist. Why, then, can it not be emitted from the eye, since it enters so freely?"

The man on trial was condemned; but Müller felt some doubt. As a result of later experiments he recognised his mistake, and formulated an important physiological law.

By using in turn each method of investigation, we are going to form an idea of the laws of Mechanics. The experiments we shall make, and the reasoning to which we shall submit the causes, would strictly lead to the discovery of the facts of the science, if they were not already known. Our sole advantage in working together will be like that of a party of tourists, exploring, by the aid of a map, a country covered with good roads and with a sign-post at every crossing. Having explored this, to us, new country, we shall feel a great admiration for the pioneers who opened it up and for the engineers who mapped it and made the roads. If, further, we succeed in knowing it well, we shall have gained, in addition to the enjoyment of the scenery, a facility in finding our way in future exploration in a wilder country, where the paths are aimless and the heights hidden in mist.

CHAPTER II

MOTION

VIII.—How Velocity is Defined.

As we shall see, motion holds a very important place in Mechanics; its nature must be perfectly understood, and we had better study it first alone. We shall thereby be diverted for a short time from the principal object of our study, but it will thereby be all the easier and will yield better results. Though we may lose a little time now, we shall gain much later on.

At first sight no difficulty is apparent in defining motion completely. Given the speed and direction of a moving body, we know where it has come from, where it is going, and in what time it will travel a certain distance. Its speed can, it is true, be given in miles per hour, in feet per second, in kilometres per hour, in metres per second, or in any other units; the transition from one system to another is simply a question of arithmetic, and will not detain us for a moment.

Yet, when we are told a train travels 60 miles an hour, are we sure that we fully understand?

Not if this is the only information, for it can have many separate meanings for a traveller. We can say, for instance, that the journey from New York to Albany is done at 60 miles per hour, or that the express from New York passing through Y nkers at a speed of 60 miles per hour collided with a freight train. In one case we mean that the total number of miles between New York and Albany, divided by the number of hours of the journey, gives 60 as the result; in the other case, that if the train had continued at a uniform speed, it would have gone 60 miles in an hour.

The first calculation gives what may be called *commercial velocity ;* this is composed of a combination of all the speeds from stoppage to the maximum speed on the straight parts of the track.

If there had been no stoppage, but only variation in the speed, the rate as just calculated is called the *average velocity.* Successive minutes will not be marked by successive milestones ; some minutes will show more than a mile, others less.

We can mark the position of the front of the train at the end of each minute, and then divide the intervals between these points into 60 equal parts ; each of these intervals will not be travelled in exactly a second ; but the differences will be relatively smaller than in the case of the minutes. If we subdivide the times again, the differences in the distances travelled in any two consecutive intervals of time will be negligible ; in order to detect a difference, two equal intervals of sufficient duration must be taken.

Let us now consider the distance travelled in a very short time, *e.g.*, one hundredth of a second, or, if the velocity is subject to sudden variations, even less, and divide the distance by this interval of time.

For a very short space we shall have a definite velocity that we shall call the *instantaneous velocity.* This is the velocity that interests us if we want to know the effects of a collision. If the moving body has gone 2 decimetres in one hundredth of a second, we can say its rate is 20 metres per second. When we express it in kilometres or miles per hour, we obviously sacrifice accuracy to custom.

It is not advisable, however, for the time in which a result is expressed to exceed the duration of the action by too much. For example, in experiments on the velocity of projectiles on emerging from a gun, a space of only 10 to 50 metres is involved, and extremely short intervals of time have to be measured. The velocities are then stated in metres per second, though the shot does not really travel in a second the distance indicated by this figure. Immediately it leaves the gun its speed begins to diminish, and the initial speed as expressed gives the distance it would travel if it moved with uniform velocity. We understand what is meant by the statement that a projectile has an initial speed of 600 metres per second; and as the second is the smallest unit of time, we can go no further. But it would be ridiculous to infer from the muzzle velocity that the shot travelled 36 kilometres per minute or 2,160 kilometres per hour.

This is the error into which automobilists fall when they speak of a speed of 100 miles per hour with a flying start. In reality, the driver travels a single mile in 36 seconds, and no one will deny it is an achievement; but that does not imply that car, tires, or driver himself could maintain that speed for an hour.

A man of limited means could say with as much reason, "I have lived at the rate of over $500,000 a year—for one minute." That would simply mean he had occupied a minute in making a purchase of $1.00.

In Mechanics we do not take commercial velocity into account, because we are not travelling or managing railways; even average velocity is of little interest. Our concern is with instantaneous velocity, and common sense will save us from the error of expressing it in inappropriate units.

The conception of velocity has still serious difficulties that must be encountered.

IX.—Relative and Absolute Velocity.

If, instead of travelling by train, we sailed in a fog, we should estimate our speed by the log or the number of revolutions of the screw, and we should rely on our calculation. If, however, we were sailing from the West

Indies to Europe, and if we had calculated the voyage by multiplying the average velocity by the time of the cruise, we should have the pleasant surprise of seeing the shores of Europe before the expected time. That would be (and we have no excuse for not knowing it) because, in addition to its own motion, the vessel would have had the Gulf Stream to assist it—the powerful, warm current that comes to our shores from the Gulf of Mexico.

Let us take another example. During the most violent storm, the sensations of absolute repose can be enjoyed, provided a person is in a balloon. When the ground is hidden from aeronauts by reason of mist or darkness, they are ignorant of their course, and may find themselves carried from England into France, still under the illusion that they are quietly soaring above the starting-place.

What does speed mean to a sailor? On rivers, those "moving roads," as Pascal called them, the speed calculated by reference to the water would be very different from that calculated by reference to the bank;* the idea of rest or motion would be reversed for an aeronaut, according as he referred to the earth or the air. With the ground in sight, he might have the impression of tremendous speed; with the ground hidden, he must laboriously determine his speed by an observation of the stars.

While on the earth, an aeronaut regards the ground as the fixed point: in the air, it is the balloon.

We shall agree, once for all, that the aeronaut is under an illusion, just as the sailor was, and that he is stationary if he is not moving in relation to the ground, and moving if he sees the ground moving.

This was the definition adopted in ancient times. For the ancients it was beyond dispute that the earth was a great fixed plain, constituting the world proper, to

* A boat can no more be steered when drifting with a current than it can be turned when at rest on a lake by merely turning the rudder.

which all the rest of the universe was but incidental. So long as the dimensions and distances of the stars were not known, it was naturally supposed that they revolved round the earth. But we know to-day that a shot travelling at a constant speed would reach the nearest fixed star only after thousands of years, while it would travel round the earth in less than a day.

We might, in the light of this fact, suppose that the stars revolve round the earth ; but it would be less absurd for the aeronaut, on seeing the earth moving under him, to suppose that it is moving while he is at rest.

Since Copernicus, Kepler, and Galileo elaborated a reasonable system of the universe, we have admitted that the earth rotates ; we believe, moreover, that it completes a revolution round the sun in a year ; it is not, then, at rest, and we can no longer say that a body is motionless because it undergoes no displacement in relation to the earth.

Where, then, can absolute rest be found ? Astronomers thought they knew when they placed the stars in the two classes of fixed stars and wandering stars—the sun and the planets with their satellites, and comets.

Science has deprived us of this last illusion. Exact observation has shown that the outlines of the constellations change. Seen from the earth, the stars have their own motions in various directions, but they have, in addition, a general motion which is so marked that we know the whole solar system is travelling with incredible speed towards a point in the constellation Hercules.

The idea of absolute rest becomes more and more difficult to grasp ; and, as we can never find absolute rest, we are released from what would prove a vain search. Accepting rest and motion as conventions, we shall choose as a standard anything we wish—the closed cabin of a ship, the air with the balloon, the river with the boat, the earth, the sun, a group of stars, in short, the point that is most convenient and leads to simple conceptions.

But it must never be forgotten that we are dealing with a pure convention.*

Nothing is easier than to be deceived with regard to rest and motion. Some years ago an ingenious inventor exhibited what he called a "devil's swing." Within a completely closed room was suspended a kind of a cradle, large enough for five or six people. When they were seated the whole room was caused to oscillate, the swing remaining still. It was amusing to see the people bend to right and left as if to maintain their balance on the swing, which they believed to be oscillating. After several increasing swings the room was given a complete turn and the illusion destroyed, though not before the people had suffered and exhibited considerable alarm.

X.—Illusion in the Observation of Motion.

The facts of the preceding paragraph will have brought us to the conclusion that the standard of rest or of motion is arbitrary. We always feel that it is so more or less, and that it would be absurd, when we see a fowl turning on a spit, to pretend that the fire, the house, the town, the earth, and the universe itself are revolving round it. Such an idea would be that of a fly alighting on the fowl, on the going down of the fire; certainly it is not ours.†

This question will be still clearer when we have taken into account the various facts that motion will bring to our notice. For the present, let us examine the action of the medium.

* Physicists have found a standard of rest that seems much more reliable than any yet suggested. This is the ether, which transmits light and electro-magnetic action, but it can be ignored at this stage of our subject.

† If, however, the spit lost its regulator, the fowl would spin round and throw the fly outwards; the centrifugal force would prove, without the possibility of doubt, the reality of rotation. We shall soon see that, though velocity is always relative, acceleration is not; and rotation implies change of the direction of motion, *i.e.*, acceleration.

Consider two dirigible balloons travelling at the same rate along parallel straight lines. If the ground is not visible to the aeronauts, the only guide each will have will be the other balloon; and as he will always see the other in the same direction and at the same distance, he will think he is at rest. The balloons are really in a state of relative rest, and each is for the other a standard of rest.

Suppose now the aeronauts travel side by side during the night and signal by means of whistles. We know, from frequent observation, that sound is not transmitted instantaneously; when a train whistles we see the steam rise from the engine without any sound, then the sound reaches our ears often after the steam has disappeared.

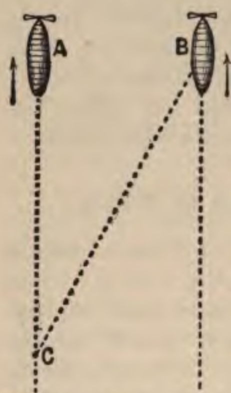


FIG. 4.—For two balloons travelling together, seeing is a more reliable guide to position than hearing.

In the same way, a sound from balloon A (Fig. 4) will reach balloon B at a point in its course in advance of its position when the sound was emitted. Balloon B will not hear the sound from the direction of balloon A, but from the point C where the balloon A was at the moment it made the sound. In other words, the aeronaut in balloon B will think balloon A has fallen behind. The greater the velocity of the balloons, the greater will be the interval; and if the balloons simultaneously increase their rate, each will think the other is left behind. As soon as they can see, they will discover that they are level, and will realise they have been under an illusion.

The curious result to which this experiment has led proves it is essential to take into account the velocity of transmission of the signals; if serious errors occur as the result of sound observations, it is due to the fact that the velocity of sound, though greater than that of most motions, is relatively small. The velocity of light, on the other hand, is immense in comparison with the velocity of any body on the earth, and even in comparison with celestial bodies. For this reason, when we observe a terrestrial body in motion, or when we are moving parallel to another body, we can affirm that it is, if not exactly at the point where we see it, at least so near that we are incapable of appreciating the difference between its true position and that in which we see it.

Again consider signals by sound. All who have been present at the firing of a cannon-ball or a shell must have noticed the strange whistling — not the same sound for the one firing as for the one fired at.

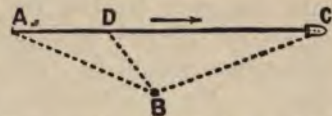


FIG. 5.—A projectile is heard far from the place where it really is.

Formerly, when balls were of great size, and had only a small velocity, they could be heard coming, preceded by the sound. Napoleon's soldiers used to greet the balls, as they passed over their heads, with the words, "That is for someone else, not for us."

With modern projectiles, the velocity of which is, at least for a part of the range, greater than that of sound, the shot is always in advance of the sound, and reaches the man aimed at before he can hear it.

Imagine a projectile passing over our heads with a velocity double that of sound, which is itself about 330 metres per second. During the time the sound occupies in travelling from the point A (Fig. 5), where the projectile is at a given moment, to the point B, where we are, the projectile has travelled AC. We therefore imagine we hear the shell at A when it is already at C.

This gives rise to a strange illusion. It is easy to see that, in the supposed conditions, the sum of the times required for the projectile to travel AD and for its sound to travel DB will be less than the time required for the propagation of sound from A to B. We should therefore hear the projectile at D before hearing it at A. It is also obvious that the sound from C would reach us after that from D. There is accordingly a certain point in the trajectory in respect of which sound is first heard; sounds proceeding from any other points of the flight are heard subsequent to this. In other words, the projectile seems to start its flight from a certain point and to be immediately divided into two projectiles, which move, one towards the gun, the other towards the target.*

This example is an indication of the caution with which the subject of motion must be approached. We certainly shall not err in trusting the evidence of our eyes so long as we are dealing with terrestrial objects, but a like affirmation cannot be ventured with regard to celestial objects. We believe that we see a star at a definite point in the sky; yet the light from some stars reaches us only after many years—hundreds or thousands of years. While the light has been travelling, the earth has rotated thousands of times and has travelled thousands of millions of miles. The star is then far from the point where we judged it to be.

In addition, a star sometimes appears, shines for some time, and then disappears. Thus, on the 21st of February, 1901, a splendid and hitherto unknown star was observed in the constellation Perseus. It rapidly increased in brilliance, then darkened and disappeared. The examination that could be made of it during the short time it existed was sufficient to allow its distance from the earth to be calculated and the time its light had required to reach us to be determined It was consequently found that, at an immense distance, a fearful conflagration had taken place in the middle of the sixteenth century.

* M. Durand-Greville has observed that the explosion frequently attributed to aerolites is simply due to an analogous illusion.

The time required for the transmission of light is not the only source of error in the subject of motion; the imperfection of our senses increases the chance of error considerably. The cinematograph is evidence of the degree to which an illusion of motion can be given by a succession of images at rest. A similar illusion leads us to think that waves advance to the shore, when each particle of water really describes a closed curve in its rise and fall. On the contrary, a polished disc, without any irregularity to arrest the eye, can turn at any speed without our suspecting it. Therefore rest, as well as motion, may be nothing more than an illusion.

We should not have imagined that the subject of motion concealed so many difficulties or was so extended in its application. We are now going to pursue the subject with the help of mathematics, but this will be neither difficult nor long.

XI.—Distance, Time, Velocity, and Acceleration.

The definition of velocity already given leads to its algebraical expression —

$$\text{Velocity} = \frac{\text{distance}}{\text{time}}, \text{ or } v = \frac{d}{t}.$$

If d is, for example, the distance from New York to Albany, t the duration of the journey, v will be the *commercial* velocity; between any two stations it will be the *average* velocity; finally, if a very short distance and time are taken, v will be the *instantaneous* velocity. Strictly, in order to obtain the instantaneous velocity, d and t must be infinitely short; v is then called the *derivative* of the distance in relation to the time. There is no need to be dismayed at the term "derivative" with its suggestion of higher mathematics, upon which we shall not enter. It is well, however, that the meaning of the word should be understood.

When it is desired to construct a reliable record of the journey of a train, a diagram containing complete information can be traced.* This diagram is obtained by carrying on the engine or in a carriage an apparatus of the following description.

A clockwork mechanism draws a roll of paper (Fig. 6) over a drum. Perpendicular to the paper is a pencil or pointer, connected with a band which is driven, through a reducing gear, by one of the wheels. When the train is stationary, the pointer traces a straight line parallel to the direction in which the paper moves; when the train is travelling very quickly, the pointer ascends rapidly and traces an oblique line. If the engine is reversed, the line descends; and it reaches its lowest position the moment the backward motion of the train ceases, and again ascends as soon as the motion is in the forward direction.

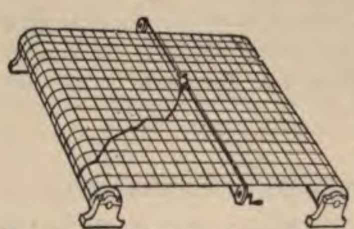


FIG. 6.—Tracing a diagram of a train's rate.

Suppose each square of the paper corresponds to 5 minutes from right to left, and to $2\frac{1}{2}$ miles in the perpendicular direction. Let us now analyse the diagram (Fig. 7). It shows first a stop (AB) of three minutes, then a start and an increase of speed until $\frac{1}{2}$ mile per minute is attained (CD); after another change of speed, the extreme limit of 60 miles an hour is attained (EF); after a decrease, there is a stop (GH), a short return is made for switching (HJ), the forward journey is resumed without a stop at J, a brief stoppage is made (KL), and the train finally departs from the station.

From B to G the rate is 16·5 miles in 25 minutes; the average rate in that time is therefore 39·6 miles per hour.

* In § LVI. and the following sections of the volume on Mathematics in this series will be found examples of calculations made with the help of diagrams.

If we wish to calculate the average rate between H and K, we shall find it to be zero, because the distance travelled is zero; yet the train was moving. There is no contradiction between the calculation and the actual fact, for it is understood that between H and J the velocity is negative; from that point it *ceased to count.* Obviously the notion of negative velocity has a particular meaning. If between H and J the train had collided with another, the passengers would have considered the velocity quite positive. We merely draw attention to this fact; it will be elaborated later. To know the instantaneous velocity, we take two points on the curve very close together, or even two points infinitely close together; the inclination of the tangent to the curve at the point under consideration gives the velocity.

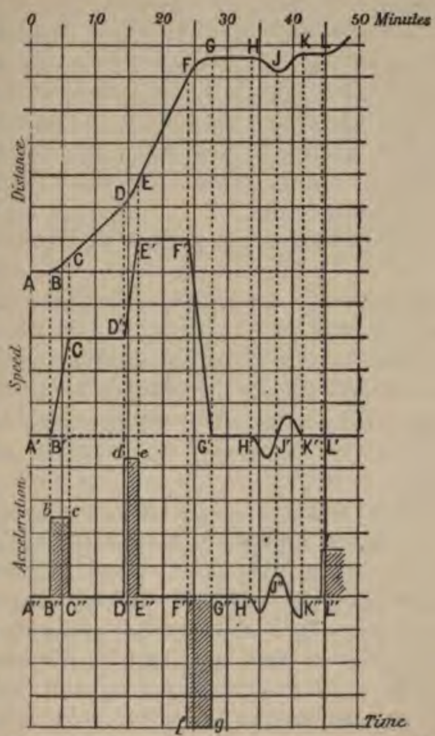


FIG. 7.—Diagram of distance, speed, and acceleration.

By examining the diagram we have been able to see not only the distance travelled by the train—the immediate record—but, in addition, the velocity attained in each section, by dividing the distance by the time required to travel it.

We can now construct another diagram such as is recorded by a certain apparatus of great value to motorists; this is a diagram of velocities.

Set out on a new sheet of squared paper, the times from left to right, and velocities up and down. During stops, the velocity is zero ; then it increases to a certain value, which is constant for the normal rate on the level. Before a stop at a station, the velocity diminishes and falls to zero. When the train travels backwards, it becomes negative.

We can construct a third and last diagram. This will show variations in the speed, or, as they are called, *accelerations*—a term that, in every-day language, means only increase of velocity, but, as we shall understand it, means any change, even negative. It is because acceleration is of extreme importance in Mechanics—more important, in fact, than velocity, as every observation shows—that we have troubled to carry the analysis of motion so far.

If, in a lift that is ascending slowly, we close our eyes and try to analyse our sensations, we shall, after some time, find it impossible to say in which direction we are moving. The moment the lift stops, we imagine it is suddenly descending. The explanation is that motion is relative. For the time, the closed lift is our little world, our standard of rest, and we can easily believe it to be really at rest. But when the lift stops, it can be said to descend in relation to the motion of a moment before, and the descent is what we feel.

Similarly, when the brakes are suddenly applied on a fast train, we continue our forward motion ; and if an interesting book has made us forget that the train is moving, we are suddenly brought back to reality. The velocity of the train is of the greatest importance to us, for we are using it in order to travel. But this velocity has, so to speak, only a commercial or ordinary importance. Its sudden change disagreeably emphasises the fact that variation plays the important part in Mechanics.

During an earthquake we may imagine the earth is acquiring a velocity; in reality, the earth moves in its orbit with a velocity of 19 miles per second; no one would suspect it if astronomers had not discovered it after centuries of investigation and thought; for an observer at rest, the cause of the terrible disasters at San Francisco and in Sicily was but an infinitely small local variation of the velocity.

Knowing the importance of acceleration, we can now trace the diagram.

Let us begin with the horizontal parts of the second curve, where a constant velocity is indicated. The acceleration is therefore zero, and we shall mark the sections A″B″, C″D″, E″F″, etc. When the velocity increases (when the motion is *accelerated*), the acceleration is positive (B″C″, D″E″). When it diminishes, the acceleration is negative (F″G″). There is this distinction between the two diagrams; the most interesting parts of the second (where the train travelled well) are the insignificant parts of the third.

We might, as in the case of the velocity, calculate an instantaneous acceleration, which is the derivative of the velocity in relation to the time; in this case the derivative varies suddenly.

Let us now consider an interesting peculiarity of the diagram; the lengths of the lines *b*B″ and *d*D″ indicate the acceleration at the instant under consideration; adding these accelerations, we have the total increase of velocity, which is represented by the areas of the quadrilaterals B″*bc*C″ and D″*de*E″. There have been two equal increases of velocity, after which the velocity fell to zero; the effect indicated by the first two quadrilaterals is therefore cancelled by F″*fg*G″, which corresponds to the stop; the area of the last is equal to the sum of the areas of the other two.

The third diagram, like the second, might have been traced automatically, as we might have inferred from the shaking the acceleration has given us. When we have studied acceleration, we shall be able to devise an apparatus to record it.

XII.—Direction of Velocity. Composition and Resolution of Motions.

We have hitherto confined ourselves to rectilinear motion; but we are not obliged to move in a straight line. In order to define motion completely, we must know not only the rate and the changes of rate, but changes of direction. The problem seems to become complicated, as indeed it is; but thanks to a common artifice, it can be greatly simplified.

Suppose a dirigible balloon, while travelling in one direction by means of its engine, is carried by the wind in another direction. Its motion is between that given by the engine and that given by the current of air. If the ground is hidden, the pilot is aware only of his motion in the air, and it is impossible for him to know any other. But if the ground is in sight, he can see that his motion is not that given by the engine alone. By directing his course towards various points, he can determine the direction and rate of the wind. We ourselves could do so as well, but for us it is easier to approach the problem in another way.

The wind is blowing, for example, from A to B (Fig. 8) with a speed of 20 miles per hour. The balloon, steered in the direction A to C, is travelling at 40 miles per hour. As it travels, the air in which it is travelling moves from AC to BD. The balloon therefore really reaches D. If we examine the figure, our conclusion is that a moving body acted upon simultaneously by two motions in different directions, follows the path indicated by the diagonal of the parallelogram representing these two motions.

The problem of the composition of motion being solved

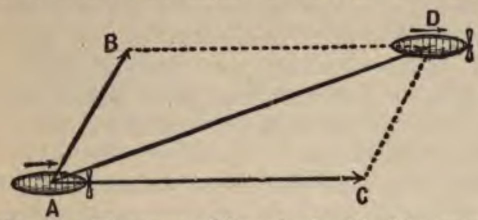


FIG. 8.—The composition of velocities illustrated by the deviation of a balloon in a cross wind.

we understand how the pilot knows the direction and rate of the wind. He knows he has to go from A to C. He arrives at D and knows at once that the wind has taken him from C to D.

The balloon could have been made to travel across the wind. It would then have taken the course A′D′ (Fig. 8A), which would have represented the resultant of its two motions.

Just as the motion of the balloon was a moment ago separated from the motion of the air, so it can be separated in the present case; A′C′ and A′B′ are the *rectangular components* of the motion of the balloon.

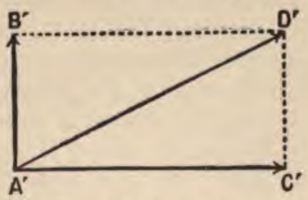


FIG. 8A.—Rectangular resolution is immediately suggested.

For purposes of the resultant, it is of little importance to know how the balloon is moved from A′ to D′; and, even if it had travelled in that direction by its own power, we could always suppose that it had been under the action of two distinct motions at right angles to each other.

This is, of course, a fiction, but a very useful fiction.

Suppose the balloon, instead of moving in a straight line, had described a curve. Each part of this line could be replaced by its two components, or by its two projections, as they are called; and the motion of the balloon would be known by describing the motion of two points simultaneously displaced along the straight lines A′C′ and A′B, so that by drawing from these points the perpendiculars to the two axes the position of the balloon would be at their intersection.

It is somewhat after this manner that the position of a warship forcing an entrance into a channel is determined, when a mine is to be fired beneath it. Two observers on the shore follow the course of the vessel with telescopes; each ascertains one of the projections of its movement, and, by prearranged signals, they communicate their observations. One of them, in charge of the mines, which are exactly marked on a map, traces on the map the course of the ship, and when it passes near a mine he causes an explosion by means of an electric current.

Let us watch, from a distance, a wheel turning in the horizontal plane and carrying a light. We can see only a motion from right to left, and from left to right, rapid in the middle of its course, but much slower at the ends; that is all our observation reveals, and we are reduced to conjecturing the real motion of the light. If another observer follows the motion from a point in the plane of the wheel, but at right angles to our line of sight, he will see exactly the same movement, except that, when the light is at the end of its course for us, it is at the middle for him. He will be as incapable of defining its motion as we are, for he will have observed only one of its elements. Yet, if we know his observation at every instant, or if he knows ours, the position of the luminous point can be determined, and hence the circumference that it is describing can be constructed.

We can also measure the distance travelled by the luminous point and calculate its velocity or acceleration.

In addition, we may consider the motion of the straight line joining it to the centre of the circle and describing angles. This motion will be known if we can indicate the *angular velocity*, expressed in revolutions per second, or, generally, by the quotient of any unit angle divided by any unit of time.

One peculiarity of the observation is noteworthy; while the light moves in a circle with a constant velocity, it appears to come and go, to stop and start. For each observer the apparent motion is subject to an acceleration, sometimes positive, sometimes negative. The uniform motion, of which the projection possesses variable velocity, is affected by an acceleration of direction, as it is called. The importance of this will soon be understood.

The analogy between an oscillation and a circular motion is not as distant as it appears; it is very close, and we shall see later (§ XXXVIII.) that the consideration of oscillation as the projection of a uniform circular motion is full of consequences.

SECOND PART

Exposition of Principles

CHAPTER III

FORCE, THE CAUSE OF ACCELERATION

XIII.—Velocity, Acceleration, Force, and Inertia.

Experts in the interesting game of bowls always choose with the very greatest care the piece of ground on which they are to play. They unhesitatingly reject ploughed land, and have no liking for gravel. They look for a place with a flat, smooth surface—either sand or rolled turf. Ice likewise would not suit them, for, if they disregarded the danger of slipping and falling, they would experience little pleasure in chasing bowls which a slight effort would send rolling undesirable distances over the smooth surface.

Let us follow the movements of the bowl from the moment it is taken in hand until it comes to a dead stop at the end of its course. The player takes hold of it, throws it forward with a swing of his arm, thereby imparting to it a certain velocity, and releases it; it does not fall right down at his feet, for it has acquired a new property by virtue of which it falls obliquely and then rolls away along the surface of the ground. Upon prepared ground it rolls a fairly moderate distance; on ploughed, soft. soil, however, it would very quickly come to rest, perhaps at its very first contact therewith, or perhaps after rebounding once or twice; on gravel, the motion would be quite indefinite—it might be in a straight line or it might not.

Thus, subject to the wish of the player and the object he had in view, the bowl has been raised from the ground, it has had velocity imparted to it, and this velocity, as well as its course, has been variously modified. There was a rapid decrease of velocity when the bowl was thrown on soft soil, a less rapid decrease on a pebbly surface, and still less on sand. If the bowl had been played on ice and equal velocity imparted, it would have travelled a very great distance before coming to rest.

These changes of speed from slow to rapid, or *vice versâ*, are found everywhere; thus a projectile fired from a heavy gun will travel as much as 6 miles, and even then cause very great destruction, but if it encounters a sheet of armour plate it may be able to penetrate scarcely more than 4 inches.

That which causes change of velocity in moving bodies is easily traced, and is called *force*. In the language of Mechanics, which we shall adopt, force is that which causes acceleration. The acceleration is considerable in the case of powerful forces, as, for example, when a projectile leaves a gun or collides with an armour plate; almost negligible when the operating forces are small, as when the projectile is passing through the air, or the bowl along smooth ice.

These causes of change in velocity are never completely absent or inoperative, at least in any experiment we can perform. Yet they differ so very greatly that we may frequently ignore the minor causes as having no effect when very potent causes are involved. In some instances, indeed, they can without any great effort be considered non-existent; thus the air may be imagined of such tenuity that it offers no resistance to the projectile, or the ice may be imagined so smooth that the velocity of a bowl would not be sensibly diminished. If the projectile and the bowl do not pursue their course indefinitely in a straight line, it is because there exist forces, extremely small it may be, which prevent; if these hindering forces could be eliminated, the moving bodies would continue moving for ever. Impossible as it may seem, we are encouraged to think that it may be accomplished from the fact that the stars maintain an eternal flight.

That property of matter by virtue of which a body persists in motion in a straight line with uniform velocity so long as nothing interferes to prevent it has been called *inertia*. It must be understood, however, that forces alone cause change of motion.

If we reflect for a moment, we shall see that in stating these two facts (they are generally separated in works on Mechanics) we have simply shown two aspects of one and the same thing; if for every change of motion there is of necessity a cause, then, in the absence of cause, there will be no change of motion. It is superfluous to assert that matter possesses a special property called inertia; it is sufficient to say that force alone can cause change of motion.

Rest, for instance, could be described as a special case of motion; it is really non-motion. If a body once in motion persists in moving, it equally persists in a state of rest in the entire absence of force.

A complete understanding of these principles is so important for the intelligent continuance of our subject that it ought to be thoroughly impressed upon our memories. This will present no great difficulty if we give careful attention to some of our every-day experiences.

When we mount a bicycle and are in a hurry, we press the pedals vigorously in order to get up speed; we perceive there is a resistance and that it would serve no purpose to exert all our strength with the object of starting at full speed; we should very likely do some damage. The velocity increases rapidly yet uniformly and not by sudden changes. Again, when coasting, we notice that the velocity decreases gradually—quickly if the road is rough or if the wind is opposing, and slowly if the road is a smooth, level asphalt, and if there is no wind. It does not decrease at all if the road descends or if the wind is behind. If we wish, on a level road, to return to our original speed, all that is required is a few vigorous turns of the pedals, after which moderate pressure suffices to maintain it.

Very little reflection is needed to convince us that the machine is subject to a great variety of forces; some of these are immediately disclosed by the sensation of muscular effort; we become aware of others by observation. The friction of the machine, friction with the road surface, the action of the wind and of gravity, may be cited as easily recognised instances. We reason quite correctly that increase of the driving force is followed by a corresponding increase in speed; that an increase of the retarding forces, either intentional or automatic, is followed by a decrease in speed; and, finally, that an increase of driving force, accompanied by a simultaneous and equivalent retarding force, leaves the speed unaltered. This is the condition when, by steady pedalling, we maintain a uniform speed.

In all other cases (*i.e.*, almost always, since we can never assert that our speed is absolutely uniform) force causes change of speed, and we feel that we are justified in saying that *forces are the causes of accelerations, be they positive or negative.*

True, this is not yet for us a mathematical certainty; the subject is so intricate that Aristotle and his followers ventured no further than to admit that force could, in certain circumstances, be employed to *maintain* velocity; this great philosopher did not carry his analysis of motion far enough, and failed to recognise the universal action of other causes such as friction. It is sufficient for us, for the moment, to know that we are on the right road; when we have more experience, belief will come to us, as it did to those who, before us, made exhaustive study of the causes of motion.

We observe that a great number of objects are in a state of rest, and, if our observation is correct, we ought to conclude that they are being subjected to no force. The actual conditions are somewhat more complicated. The bodies are, in fact, being acted upon by forces, but the effects of these forces are equal and opposite, and they balance each other; the result is equilibrium. Since bodies in a state of rest are very numerous, it is evident that equilibrium of forces is of frequent occurrence. Yet this equilibrium is the result of complexity of forces—usually acting automatically—the nature of which we shall better understand when we have examined them carefully. For the present, therefore, we shall put on one side the forces of equilibrium, in order that we may devote our attention to those that produce motion. The branch of Mechanics that treats of the latter is called *Dynamics*; that, on the contrary, that is concerned with forces in equilibrium is called *Statics*.

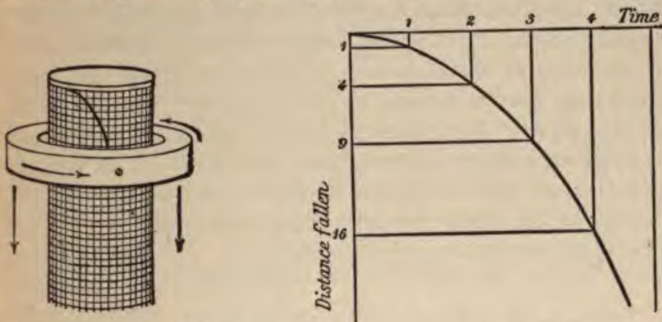
So far we have confined ourselves to the study of qualitative relations, which simply indicate the manner in which the phenomena are produced; in order to establish quantitative relations (*i.e.*, such as furnish the exact values of effects obtained from known values of causes), we must supplement simple observation by experiment. This we now proceed to do.

XIV.—Laws of Falling Bodies.

At the top of a cardboard cylinder (Fig. 9), covered with a sheet of white paper, let us fix to an apparatus, whose vertical axis is the same as that of the cylinder, a ring of which the inner surface is fitted with a brush moistened with colour and lightly touching the paper surface of the cylinder. By means of this apparatus we impart a smart rotatory motion to the ring, suddenly release it, and allow it to fall. As it falls, it continues to rotate round the cylinder, upon which the brush traces a spiral, the form and characteristics of which we shall now proceed to study.

First of all, we notice that the rotation has been accomplished with a uniform, or almost uniform, angular velocity, since the friction of the brush on the cylinder has had a retarding effect on the rotation of the ring. This effect is so minute that it can evidently be disregarded.

As a matter of fact, the ring, in its fall, was subjected to a known force—its own weight, *i.e.*, the force it exerted on the suspending apparatus before release.



FIGS. 9 AND 10.—Diagram of a fall under gravity, traced by a ring falling and rotating simultaneously.

Let us now unroll the paper, in order to examine the spiral traced by the brush. We see that it is in reality a curve (Fig. 10) which, beginning in a horizontal direction, is deflected in an increasing degree towards the vertical.

The horizontal velocity of the brush being uniform, equal distances in a horizontal direction are passed over in equal times, which result we may indicate by means of arbitrary units—1, 2, 3, 4. Let perpendiculars be drawn from the points thus marked. The lengths of these lines, measured from the level of the origin of the curve, are the respective distances fallen by the ring from the instant of its release.

The lengths of the successive lines are greater and greater, which shows that in equal times the vertical distances travelled by the ring in its fall have kept on increasing—in other words, its velocity has been increasing.

If we measure the distances fallen and divide them by the distance corresponding to the point 1, we shall find that they are represented very nearly by the figures 4, 9, 16, *i.e.*, the distances fallen are in the ratio of the squares of the natural numbers.

We shall hereafter see that certain causes have contributed to falsify slightly the result of our experiment. Let us disregard them for the moment, and assume that the differences between the numbers found and the squares of the natural numbers are due to the effect of these interfering forces which we have agreed to disregard. Let us apply the principle of simplification already established, and let us accept as a natural law that a body falling from rest traverses distances which are as the squares of the times the body has been falling.

Now set down these numbers.

Times.	Heights.	First differences (average velocities.)	Second differences (average accelerations.)
0	0		
1	1	1	2
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	

The differences between the distances traversed in successive equal periods of time are proportional to the average velocities during these periods. These velocities are represented by the series of odd numbers.

The differences between these average velocities are all represented by the number 2. Thus we can assert that the average acceleration is constant.

XV.—Deduction from the Laws of Falling. Symbols.

Since the average acceleration is constant, it can be confidently stated that the acceleration of a body falling freely is always the same. Taking this principle as the basis, we can, by intuitive methods, conclude that it is a necessary law for falling bodies.

Let us adopt a geometrical interpretation. Set out, on a horizontal line, certain distances (abscissæ) as measures of time, and, in a vertical direction, mark distances equivalent to accelerations. As the acceleration is the same at every instant, it will be indicated by the straight line AB parallel to the axis of time.

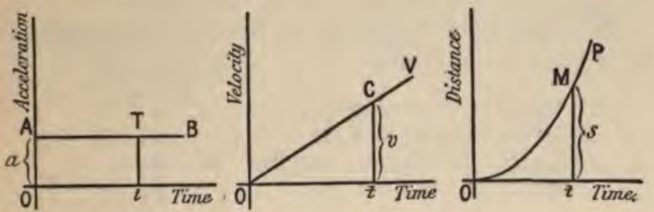


FIG. 11.—Acceleration, velocity and distance.

But acceleration is variation in velocity in a given time, or, otherwise, the quotient when the difference between the velocities at the beginning and end of the period is divided by the time in which the difference is produced; velocity, on the other hand, is acceleration multiplied by time. From the first moment until the instant t, the acceleration has produced a velocity which is equal to the product of the acceleration a by the time t. This is the area of the rectangle OATt.

This area is proportional to the distance from the origin; to express it numerically, it can be represented as in the second diagram, by the straight line OV, the time being still the abscissa and the velocity the ordinate.

As for the distance traversed, it is equal to the product of each of the velocities and the time during which it lasted. In the present instance, it cannot properly be said that any single velocity lasts any time, because it is continually changing. But the time can be conceived as divided infinitesimally, and then the velocities can be multiplied by the periods of time. The product will be the area of the triangle O C t, which is equal to $\frac{vt}{2}$, and as $v = at$, this area (*i.e.*, the distance traversed) is given by $\frac{at^2}{2}$. The course of the body can be represented by the curve OP, which is that of a parabola.

In the special case of falling bodies, it has been customary to denote acceleration by the letter g.

The three laws applicable to freely falling bodies can be written down thus :—

Constant acceleration	$= g$
Velocity varying as time	$v = gt$
Distance traversed varying as the square of the time	$s = \frac{gt^2}{2}$

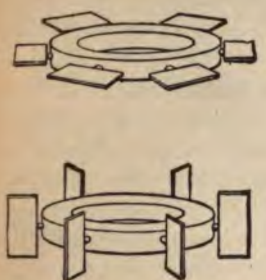
In order to discover the laws associated with falling bodies, we have had recourse to a series of symbols. We have represented time graphically in such a way as to convey a meaning to our mind. Such a mode of representation is not unfamiliar. The hand of a clock indicates time by the distance it has travelled ; the angle which it describes measures the time, the rotation of the earth furnishing the unit.

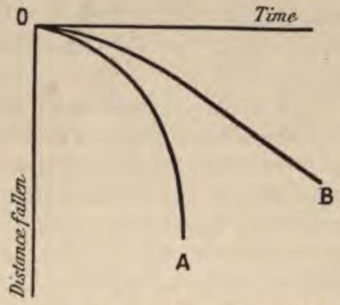
We can equally well employ a symbol to represent an acceleration, a velocity, or a distance. In fact, we find that in the third diagram a distance has been represented by another distance ; this is quite permissible. We have only represented a relatively long line by a short line. This is the method adopted in drawing maps.

We could have given a more striking example of the parabola ; it is met with continually in innumerable forms. A jet of water discharged in a horizontal direction falls in the form of a parabola. The distance travelled horizontally by the drops of water is proportional to the time; the vertical motion measures the distance fallen, and the combination of the two motions acting simultaneously results in a curve like that traced by our brush in the experiment.

XVI.—Exaggeration of the Disturbing Causes; their Study and Elimination.

It will be useful, after this first experiment, to take account of the nature and magnitude of the errors that contributed to render our result unreliable and necessi-





FIGS. 12 AND 13.—By exaggeration of the action of the air, the normal fall is changed.

tated a process of simplification. Let us substitute for our heavy ring a lighter one, say of wood, provided on its rim with a series of pegs having two slits cut across them at right angles, horizontally and vertically. Let us fix in the vertical slits a number of cards, so that the ring may present the appearance of a water-wheel, and let us allow it to fall after it has been set rotating. Similarly, let the same procedure be followed with the cards fixed in the horizontal slits. On unrolling the paper, we find that two distinct curves have been traced, and that each diverges from the original parabola in a different direction. In the first, OA, the curve falls much more rapidly, and at the finish is almost vertical; in the second curve, OB, the direction is obliquely towards the right, and the fall appears as proceeding with a uniform velocity.

Let us try to analyse our experiment. In the former of the two, the ring was so fitted that its rotatory motion was greatly retarded by the resistance of the air. Instead of proceeding, as formerly, with uniform velocity, this motion gradually slackened until it almost ceased entirely.

In the second experiment, the rotatory motion proceeded undisturbed, and, as the distances traversed vertically were equal in equal times, the ring, though at first accelerated, finally fell at a constant velocity. In accordance with principles already explained, the forces acting upon the ring during the last part of the fall were in equilibrium.

In a previous experiment we took precautions to ensure that the action of the air should be negligible in comparison with the other forces acting on the ring. In the two present experiments we designedly exaggerated this action, firstly as affecting the fall, secondly as affecting the rotatory motion; and the result has been to present the laws of falling bodies in a quite new aspect. In the first of our experiments, the rotation of the ring obviously cannot be taken as the measure of the time; in the second, the fall proceeds with uniform velocity, and not with uniform acceleration, as we found was the case in a previous experiment.

We can form further ideas of the two modes of falling just investigated by comparison with familiar objects. The first is a football, which, though sent horizontally, falls almost vertically; the second is an aeroplane, which, on its engine being stopped, alights by gliding at a constant angle.

In each instance the air, notwithstanding its tenuity, has so influenced the bodies as to change completely the aspect of the fall.

So complex are the laws governing the fall of bodies in air, and so difficult is it to eliminate the influence of the latter (whether, as in the majority of cases, by experiments purposely designed to that end, or by mathematical induction), that their discovery may be regarded as one of the achievements of modern times. Aristotle and his followers had always omitted to clear away the obscuring factors before attacking the problem. They discovered that, towards the termination of the fall, the downward motion of the body was accelerated, but they said that after a certain lapse of time the velocity became uniform, and deduced thence an erroneous generalisation, which persisted for more than a thousand years.

Galileo was the first to distinguish in the fall of bodies that which is essential (the weight of the body) from that which is accidental (the resistance of the air). By his insight the science of Dynamics was founded.

The increased use of firearms in modern times greatly promoted the discovery of the fundamental laws of Mechanics, by putting the problems of motion in an entirely new form; yet nearly two centuries elapsed before they were fully understood. It is true that Leonardo da Vinci (1452–1519), who was at once the greatest artist and the greatest mathematical genius of his age, had discovered the laws governing the fall of bodies, but had not published them. Santbach taught, in 1561, that a projectile travels in a straight line and falls vertically when its horizontal velocity is exhausted. His teaching was so far from seeming absurd to the uninitiated that less than a year ago it was revived by a journalist in order to explain how the celebrated Tom Canon was able to catch a shot from a cannon. The whole art of vigorous Tom was in knowing the exact point where the projectile stopped for an instant before falling!

Has it not been observed that the smoke from a cannon, emitted from the muzzle with the same velocity as the shot, stops in the air after a few yards? The difference between the particles of smoke and the actual projectile is that the latter is massive, while the former are tenuous, relative to the air. We shall see later what differentiates them so completely.

Progress in science is slow, and the most intelligent of us must avail himself of all the means at his command in order to eliminate mere appearances. The study of falling bodies presents a striking example of this. The laws were admitted to be incontestable only when the air-pump, invented by Otto von Guericke, proved that a feather and a piece of lead fall with the same acceleration when atmospheric action is eliminated, *i.e.*, *in vacuo*. In air, the effect of the weight is preponderant for the lead, whilst the least current of air is sufficient to cause the feather to ascend.

The dust ejected by Krakatoa or by Mount Pelée spread throughout the whole atmosphere, and for more than a year after the eruptions of these volcanoes an etxraordinary glow, especially noticeable at sunset, enabled its presence to be established; if the earth had been without atmosphere like the moon, a few minutes would have sufficed for all the dust to reach the ground.

The size remaining the same, dust falls much more slowly in water than in air. If a cloud of emery dust is produced in a flask, it will immediately settle at the bottom. But if the same dust is shaken up with water, it falls gradually in strata containing grains of decreasing size. This phenomenon is so constant that it affords the best means of separating polishing powders.

The deltas of rivers are formed by the direct deposit of the large grains of sand and mud. The finer particles remain longer in suspension; and, just in proportion as they diminish in size, they give to the water, by reflection of light, the successive tints of green, blue and violet.

Such examples prove that the accidental factors may, in combination, obscure the real nature of phenomena.

This helps us to understand how it is that long centuries of observation have not been rewarded by the discovery of the most elementary laws.

Natural law assumes certain aspects according to the environment of the observer. We have just seen what difficulties even our tenuous atmosphere introduces into the discovery of comparatively simple laws. It is obvious that birds would not have discovered them, nor should we if our relation to the air had not been such as it is. But let us return to simpler subjects.

XVII.—Generalisation of the Laws of Falling.

The results we have just arrived at would be interesting even if they were independent of all others. But we are able to give them an immensely wider application by generalising from them.

Forces produce accelerations; the acceleration produced by gravity is constant. These facts have already been ascertained by us; a further brief investigation will disclose their importance and value.

The effort required to raise a weight gives us a *qualitative* idea of weight; but we have seen how unreliable are our senses, how carefully their evidence has to be checked, and how frequently measurement has to be substituted for unaided observation.

Let us fix to a wooden cross-bar a spring balance with a scale-pan carrying a weight of 100 grammes; the spring will lengthen, and, if it is provided with a pointer, its extension can be read on a scale. Add 10 grammes and the pointer will descend further; remove them and it will return to its original position. We have no doubt as to the cause of the change in the spring, and we have an idea that its elongation *measures*, so to speak, the value of the weight supported.

Now let us place the apparatus on the floor, then on the table; further, let us carry it to an upper floor, or even into an attic; the pointer will remain at the same figure, though a very small addition to the weights in the pan had visibly stretched the spring. We conclude that, within the limits of our movements and measurements, which are at least as exact as those in the experiment on the fall of the ring, the force exerted by the weight on its support is the same; the ring therefore possessed the same weight throughout the whole extent of its fall, *i.e.*, it exerted upon itself a constant force. We can then give our result in the following more general form:—

A body acted upon by a constant force possesses a constant acceleration and a velocity proportional to the duration of application of the force; it describes a distance which, calculated from the position of rest, is proportional to the square of the time during which the force acts.

This law must be remembered, for it is fundamental and is encountered everywhere.

It was useful to make by means of the spring a preliminary survey of the *field of force* in which we operate. If the scale-pan had carried a piece of iron into the neighbourhood of a powerful magnet, we should have found that neither the force nor the acceleration was constant. The magnet would have been one of the accidental complications mentioned in our first experiment, and would have had to be eliminated. Fortunately for the discovery of mechanical laws, powerful magnets are rare and of modern invention; moreover, all bodies do not possess the properties of iron. If magnets had long been of common use, and if many substances were magnetic, the fundamental laws would perhaps be unknown even now.

CHAPTER IV

THE WORK OF FORCES

XVIII.—The Meaning of Work.

In the domain of manual labour, what is a worker? It is a man who makes a useful effort by moving his arms. If we saw a carpenter pressing his plane against a plank and remaining motionless in that position, we should not call him a worker; we do not expect him to press the plank against the stop of his bench, but to smooth it by passing his plane over its surface; it is by the intelligent application of this and similar movements that he makes his living. Whatever occupation the workman follows—be he mechanic, ploughman, or hod-carrier—we see that he exercises a force and continues to exercise it for a definite time with a definite purpose. Work may be said to be that which is paid for; we do not pay for mere unproductive expenditure of force, but for force that is active, and it is this which constitutes work.

The horse that stops on a slope is, for the moment, a useless animal; if it has to use force to keep the cart in place, the carter must have forgotten to put on the brake or put a stone behind the wheel; the horse will not do useful work until it again draws the cart.

The work of a goldsmith is more highly esteemed than that of a labourer, and a race-horse is admired without any thought of the patient work done by a cart-horse. Rightly or wrongly, we usually judge work by its *quality*, and we measure its value thereby. But Mechanics cannot enter into such fine distinctions; it takes account only of *quantity* of work, and for this purpose a precise definition of work must first of all be agreed upon.

It will be a good definition if it corresponds to the teaching of common sense. If we saw one horse dragging an equally heavy load twice as far up a slope as another horse, we should say that it was doing twice as much work; similarly, if it drew a cart twice as heavy over the same distance, its work would be doubled. It will be a good definition, therefore, if it predicates that work is proportional to the effort put forth and to the distance travelled. This definition is that adopted by Mechanics in the words:—*The work done by a force is equal to the product of this force and the distance through which its point of application is moved in the direction of the force.**

XIX.—Storage and Restitution of Work.

The application of a force has many different results. One, the raising of a weight, is of particularly easy calculation; others are more complex, such as the picking up of shavings, the making of iron filings, the breaking up and powdering of a substance, and the innumerable other cases around us. But the effect of an applied force may be to impart velocity to a body or projectile. The effort of the hand in throwing a stone, of the tense string upon the arrow, of the explosive force of gunpowder on a shot, results in imparting to these various bodies a velocity that enables them forthwith, by virtue of their inertia, to move of themselves.

A body once set in motion does not continue to move indefinitely; the forces acting upon it modify its direction, but the material through which these forces act is also affected by them. When a shell is stopped by an armour-plate, it strikes the plate with immense force and drives in the metal; thus, in an infinitely short time, it accomplishes work which would have occupied a machine for a considerable time.

* We shall return at the beginning of the Third Part to the true meaning of the product of a force and a distance, or, generally, the product or quotient of two concrete quantities.

Water which has acquired a velocity by flowing down the side of a mountain can be made to perform work by falling on the blades of a water-wheel: it leaves the wheel with less than its original velocity; the difference is the cost of the work.

A body charged with a store of work is a new object to us; the bullet, harmless at rest, becomes dangerous when in motion; its whole object is to utilise the power of doing work which it has acquired.

Frequently the capacity for work stored in a body is worth more than the body itself. Beauty aside, what we appreciate in a waterfall is its power of doing work, and not the water itself, for, although we collect it carefully as long as it retains velocity, we rid ourselves of it as waste as soon as it has lost that quality.

Accordingly, one of the essential properties of matter is its capacity for becoming a vehicle of work, whereby work is stored and given up in turn. The stone that is thrown, the shell discharged from a gun, the train that dashes against buffers, all produce in destructive effects the equivalent of the work they have absorbed. A heavy body which has acquired increased velocity in falling is capable of performing work—whether it is useful or destructive does not concern Mechanics. The market value of work has no meaning for us; nevertheless, opinions differ: a shell does useful work for the gunner, but harmful work for those against whom it is directed; its value is positive for the one, and negative for the other. But to the student of Mechanics, which is concerned with phenomena in themselves apart from their incidental effects, the value of work is always the same.

Amongst the numerous kinds of work that a body can effect in exchange for its velocity is one in which the relation of the velocity to the work stored is exceptionally simple; it is that of the ascent of a body that has acquired a velocity by falling.

A cyclist descending a hill without applying the brake can " coast " part way up the next hill. We know that he does not reach his original height exactly, and that, in order to do so, he has to pedal; but we know also that a part of the work of the descent has disappeared in friction the calculation of which (and we could conclude nothing from it) is difficult.

For the purpose of giving clearness to our ideas, we can arrange a much simpler experiment, in which we shall reduce the loss of work as much as possible.

Suspend a metal ball from a rigid support by a very fine thread. Draw it from its position of rest, then release it. So long as it falls its velocity continues to increase. At the moment it passes the position of rest, the velocity is a maximum, from which it gradually decreases so long as the ball continues to rise. When the ball has actually lost its velocity and is about to redescend, we can easily see that it has practically reached its original level.

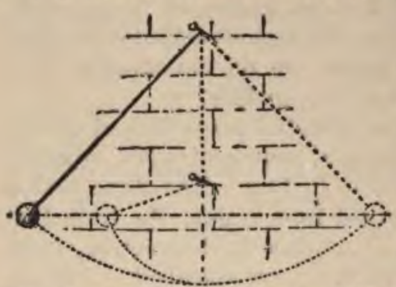


FIG. 14.—The ball of a pendulum ascends to the level of its starting-point.

We can profitably modify this experiment by fixing a nail in the wall exactly beneath the point from which the thread is suspended. As before, the ball will rise, but in an arc of a circle of less radius. We can still see that the ball almost reaches the level of its starting-point. If the nail is lowered sufficiently, a circle of so small a radius can be reached that the ball will make a complete revolution round the nail—it will, in fact, loop the loop.

The series of experiments just described is due to Galileo. Even in our days, they afford the simplest and most direct demonstration of the fact that a body that has accumulated a quantity of work by falling, and has thereby acquired a certain velocity, is capable of rising to the level of its starting-point, if no external cause disturbs the capacity for work imparted by the forces acting upon it.

In the oscillation of the ball, the force of gravity is applied so as to cause alternate acceleration and retardation. These two forms of work are periodically exchanged, and, at equal intervals, the work is transformed into velocity and the velocity into work. The equality of the height to which the ball practically ascends in successive oscillations shows that, when no portion of the capacity for work is dissipated, its whole value can be found in the velocity of the moving body which has absorbed it and is ready to restore it.

Before we can proceed further in the study of work, we must first learn what are the properties of matter by means of which capacity for work can be stored, or, being stored, can be made active.

XX.—Mass.

Bodies are, as we have just seen, vehicles for work; they store it by increasing their velocity, and restore it by losing velocity. Everything that can absorb and restore (a sponge, a bucket, a reservoir) is said to have *capacity*; we shall be intelligible, therefore, if we say that *bodies possess a capacity for absorbing work.*

It is essential to know what property gives matter this capacity. We already know that its state—solid, liquid, or gaseous—is not an essential factor of this capacity, because a waterfall and the wind itself produce work, just as a solid in motion does.

It is evident that the *quantity* of matter involved is a very important factor in this capacity for absorbing work; the larger a shot, the greater the damage it does, the velocity remaining the same, and the greater the volume of water passing through a turbine, the greater the work performed. The last instance is almost evidence for our surmise that the work given by a velocity is proportional to the quantity of the *same sort* of matter in action; and, so long as only one sort of matter is under consideration, *quantity* need not be further defined. But, if we pass to other substances, the idea of quantity has different meanings; and it is all the more necessary to be clear on this point, as this is one of the most important and elusive conceptions in Mechanics.

The bodies around us possess numerous properties, enabling them to be distinguished and assigned their utilities and conventional values.

A man's fortune is the quantity of valuables he has; these may be land, houses, industrial property, rights, royalties, or copyright. Numerically equal fortunes may be represented by very different objects, and a signature by Rockefeller may be worth an ocean liner or a thousand acres of forest.

To the financier, a paper, a ship, or diamonds have an equal value, and, in his domain, represent the same *quantity*.

But Mechanics does not recognise this meaning of the word "quantity." A piece of paper has a very small capacity for work, and the capacity is the same whether the piece of paper is a writ or one bearing the signature of Rothschild.

For a packer, quantity of matter is its volume; this is what decides the size of his cases. But this is not yet a characteristic of quantity fit for Mechanics. Take up a piece of pumice-stone and a flint of the same volume and throw them; the latter will offer the greater resistance to the motion of the hand and will break the window from which the pumice-stone would have simply rebounded.

Volume (capacity for occupying space) has then nothing in common with capacity for absorbing work.

We should be no more fortunate with the subject of capacity for absorbing heat. We might try all kinds of things with regard to capacity for different sorts of absorption, and we should find that they could be arranged in certain orders, without anything but complicated relations becoming apparent as the result of the comparison of the various series.

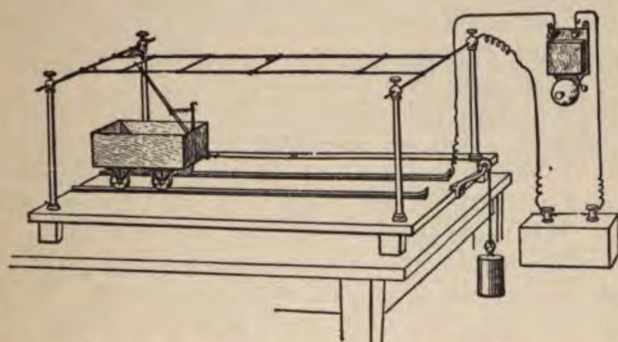


FIG. 15.—At the expense of work done upon it, matter acquires velocity.

To clear up this difficult question a direct experiment is necessary.

We have so far advanced in the elucidation of the idea of work as to know that by causing a body to fall we can obtain therefrom a quantity of work proportional to the height of its fall. This is our means of producing work; there remains the problem of utilising it.

Place on a level table a small car fastened to a string passing over a light pulley and attached to a weight. The car carries a flexible rod, like the pole of a trolley, in order to form in its passage a contact with cross wires over its track, and, at each contact, to close a circuit connected with a bell. The aim of the experiment is to cause the sounds of the bell to synchronise with the beats of a metronome, by a suitable arrangement of the cross wires.

We first start the metronome, and, at the moment a beat is struck, release the car. After some trials the wires can be placed; and, if we measure the distances they are apart, we shall discover them to be very nearly as 1 : 4 : 9, *i.e.*, as the squares of the natural numbers.

This result we have met before: it is similar to that of our first experiment on falling bodies; it corroborates, *a posteriori*, our enunciation of the law of motion of a body acted upon by a constant force.

We may now put a stone in the car and repeat the experiment. It is not surprising to hear the bell sound after the metronome, for now the car and its load have a greater capacity for absorbing work than the car alone had. It is necessary, however, that the sound of the bell and of the metronome should synchronise, and to accomplish this we raise the weight of the metronome. We can test what quantities of different substances—lead, sand, mercury, water—have the same capacity as the stone; this can be recognised by making the bell and metronome sound together.

The appearance of equal quantities of matter, as we now understand the term, will necessarily surprise us and will suggest a comparison.

In daily life, quantities of matter are easily determined by balancing them against known weights which serve as standards in the commercial or conventional sense. Our first task is to compare, by weighing in a balance, the quantities of matter that we have recognised as being equal. At last we find the coincidence sought for; the balance proves that these quantities—shown in our dynamic experiment to be equal, *i.e.*, to be capable of absorbing the same amount of work and of acquiring the same velocity—are also equal in weight.

Our experimental determination was none too exact; we must be satisfied with the assurance that measurements of extreme precision, made by delicate processes, the theory of which is not yet comprehensible to us, have led to exactly the same result:—*Capacity for absorbing work is, for all bodies, proportional to their weight.* This relation, to the verification of which Newton and Bessel devoted much patient research, is one of the most important and mysterious of those that govern the universe; it enables us to penetrate deeply into the secret of the constitution of matter, and furnishes us with the best argument in support of its unity.

For the sake of brevity, we shall replace the somewhat lengthy expression "capacity for absorbing work" by the word "mass," and we shall state the law that we have just established in the form that, for all bodies, *weight* and *mass* are proportional quantities.

In place of connecting the moving weight to a car, we might have utilised the work it produced for the purpose of imparting acceleration to itself and itself alone; we should have discovered, by similarly testing various materials, that all bodies fall to the ground *with the same acceleration.*

We have already learned why the discovery of this law was surrounded with so many difficulties; appearances are against it, and imagination has to be brought into play. The method by which we have just discovered it puts us in a position not only to understand its existence, but, what is of even greater value, to understand its significance.

The terms "weight" and "mass" are often confused in ordinary speech: to correct this carelessness in the use of words, *mass* is sometimes spoken of as the *quantity of matter* in a body; this is certainly an improvement in the use of the word, but we have just seen how vague is the word "quantity," and how its meaning escapes our grasp. We now have obtained so clear an idea of what is meant by "capacity for absorbing work" that we can safely dispense with it and substitute the word "mass," but the origin and exact meaning of the latter must never be forgotten.

XXI.—Kinetic Energy.

Examination of the motion of our car has already taught us much; but one fact is lacking. We do not know how velocity and mass interchange in the expression for a quantity of work. An experiment will make it clear.

Gradually load the car until it takes double the time to pass from one cross wire to the next. This will happen when the bell sounds once for every two beats of the metronome; the acceleration is half as much, while the work required is the same.

We have now to determine the relation between the masses employed in the two experiments. We know that it is sufficient to weigh them; and, as work is absorbed simultaneously by the moving weight, the car, and its load, we must weigh them all together.

The result is very simple; the balance shows that the mass employed in the second case is four times that employed in the first.

If we desire to return to our first acceleration, we must divide the load between the car and the descending weight so that the latter may be four times as great as in the first experiment; by making it four times as great again we obtain twice the acceleration, *i.e.*, the bell sounds twice for one beat of the metronome; and these combined results lead us to the following statements:—

1. Four times the mass gives half the velocity, the work remaining the same.
2. Four times the work gives the same velocity, the mass being four times as great.

3. Four times the work gives twice the velocity, the mass remaining the same.

All these relations are summarised in the generalisation:—The work absorbed by a mass m moving with velocity v is proportional to mv^2.

But we shall be satisfied with nothing less than an equality.

The preceding experiments have shown that, in a mass four times as great, acted upon by the same force, half the former velocity is attained in an interval twice as long; the acceleration is therefore only one-fourth as much; it follows that—

$$\text{Acceleration} = \frac{\text{force}}{\text{mass}}.$$

Now, when a body falls under gravity, the force acting is equal to its weight; therefore—

$$\text{Acceleration } g = \frac{\text{weight}}{\text{mass}} = \frac{w}{m}.$$

Whence $w = mg$.

Elsewhere, on the subject of bodies falling under gravity, we have found—

$$s = \frac{gt^2}{2}.$$

Multiplying the last two equations, we have—

$$ws \text{ (the work of the fall)} = m\frac{g^2t^2}{2}.$$

According to the first law of falling bodies, we can replace gt by v, which gives finally—

$$\text{Work} = \frac{mv^2}{2}.$$

This relation is fundamental in Mechanics. We shall soon learn to give a name to the expression on the right of the equation; but we shall first retrace our steps and review the path by which we have been led to this relation.

We caused work to be utilised by certain bodies, and difference is expressed by saying that the railway is more powerful than the mule and the mule more powerful than the ant; and the quality by virtue of which each one does more or less work in a given time is called *power* or *activity*. A powerful man can do much in a short time; so can an active man; the two words are equally appropriate, but the word "power" being good, there is no reason why we should not continue to use it.

In the case of the ant, the mule, and the railway, the power is liable to be confused with another quality—force. Such confusion must be carefully avoided. Although the mule was strong, it might have refused to move; it would not then have done any work, and consequently would not have developed any power whatever.

Power is the quotient when work is divided by the time required to perform the work; since work is *force* × *distance*,

$$\text{power} = \frac{\textit{force} \times \textit{distance}}{\textit{time}} = \textit{force} \times \textit{velocity}.$$

In order to effect work, the point of application of a force must move; to act with great power, the point of application of a moderate force may move rapidly, or that of a great force may move slowly.

A powerful machine may act with small force, provided it works at a high speed; and, conversely, a machine of small power may produce a great force, provided it works at a low speed.

The engine of a ship is of great power, and, acting at a high speed through the screw, maintains the progress of the hull, to which the water offers resistance. But a ship that is stranded cannot always be floated by its screw; a great force without rapid displacement is required, and a capstan, hauling on an anchor, must be used.

A punch which pierces a hole in a thick sheet of metal at each stroke is an extremely strong machine; but it works slowly and can be driven by an engine of small power, while the engine of a cruiser, a hundred times more powerful, would be incapable of giving nearly the same force with its screw. For the same reason, if an attempt is made to start a motor on a too high gear, the engine is stopped.

A strong machine must therefore be distinguished from a powerful machine; the two ideas are often found together, it is true, but in many instances they are in complete disagreement. Force in a machine is one desirable quality, and power is another. To confuse them would show the same disregard for accuracy as to mistake beauty for goodness; these are also desirable, and are often found together, but not always.

In calculations of work, rate and time can be interchanged. An engine of small power, working for a long time, may accumulate work in another machine which will utilise it in a very short time, and, consequently, with very great power. For example, a small engine is employed to raise a heavy weight, which, on falling, does work that the small engine could not do. For a similar purpose, an elevated reservoir may be filled with water, gas may be compressed in a receiver, as in an air-gun, etc. A large weight that is slowly raised in order that its rapid descent may be utilised is called an accumulator of work.

Conversely, work may be accumulated rapidly in order to be utilised slowly, as is the case with the weight of a clock or the spring of a watch.

Thus the possible combinations vary infinitely. Rapid or slow work, with corresponding small or great power, can be obtained; one thing alone can never be increased, that is, the total amount of work available. We may be careful or wasteful of our money, but the purse does not create more; when it is empty it must draw upon the source from which it has already been supplied.

XXIII.—Conservation of Work.

It will be of value to pursue further these ideas, the significance of which we are only now beginning to grasp.

We have realised that, in order to produce or utilise a given amount of work, we can select any one of the factors of the expressions (*force* × *distance*) or (*power* × *time*), and when that is done the other follows of necessity.

The first expression shows that a small force acting over a great distance may produce a great force for a short distance; the second, that a small power acting for a long time allows of the use of a great power for a short time.

All this would appear very complicated if we confined our attention to these statements; but we cannot look around without seeing some application of the principles.

Undoubtedly the packer would have trouble in stating them, although he continually applies them. It does not enter his head to drive a nail by pressing it with his hand or with the head of the hammer. He raises the hammer to enable it to accumulate work in its fall; he also adds that of his arm, and, the head of the hammer being arrested in a space many times shorter than that over which it has moved, the resulting force drives the nail into the wood.

Heavy pressure on the head of a hammer fails to make the least depression in a copper plate. But if the hammer is raised so as to strike even a light blow, a permanent mark will be left. It will be by no means shallow, and the inference is that the force that has produced it was considerable. For instance, if the blow of the hammer is equivalent to the work of 1 kilogramme displaced 2 decimetres it is capable of producing a force of 1,000 kilogrammes over two tenths of a millimetre. A child at play may make use of forces of this magnitude. From another point of view, he can develop the power of a large motor-car, but only for the ten thousandth of a second.

Formerly percussion was the chief means of working metals. The fixing of rivets was done by hand, with the help of very heavy hammers;* the force used by the workman had to be continuously increased, in order to obtain progressive change of shape in the metal. To-day the force is applied directly, without the intervention of a hammer; the place of the workman is taken by a powerful machine, capable of giving continuously the force formerly obtained by the impact of the hammer.

Galileo was acquainted with this relation of force, distance, and work. He explained the results just arrived at by saying that *live force* is infinitely greater than mere *force*, since it is the equivalent of the greatest force. A small stone, he said, dropped from a height upon a hewn stone on a stand will move it, and though the movement may be extremely small, there will nevertheless be some movement.

Galileo exaggerated a little, for the sake of emphasis, for he intentionally neglected certain properties of matter that interfere with the free action of this principle. But matter can be imagined with such properties as to make the statement of Galileo strictly true. With the force of both hands we might not be able to move the stone in question, but a blow would do so. True, the movement would be imperceptible, even under the microscope.

It is not for great forces alone that accumulated work can be utilised. Suppose an object has to be slightly moved. It resists, and if a person succeeds in moving it he may push it too far, because he cannot instantaneously relax his muscles. Yet there is always the resource of striking numerous slight blows; the work can be applied by striking with a piece of wood held in the hand, and this work is used up, over a short distance, by the forces that hold the object in place.

Having once understood this principle, we shall not try to answer the question so often put :—" How much does

* The French name (*masse*) given to these great hammers shows that mass is recognised as the principal quality in a hammer.

a person weigh when he jumps ?" "It depends," we shall reply. Yes, it depends on many causes. The work of his fall must be equalised by work done at the moment he touches the ground. If the ground is soft, he can alight as he pleases; the work will be met by the force between his feet and the ground; but, if he lands on hard ground, he must use up the work on his muscles by relaxing his knees and all other joints. If he omits to do so, he is subjected to forces that operate over such a very short distance, that they may be great enough to break his bones. This explains why a man on skis, relying too much on the elasticity of his fall, may be left on the frozen snow with a broken leg.

Here, again, the skill of the packer is great. Though he exerts a great force on the head of a nail, he must, on the other hand, protect fragile objects from the effects of force while on their journey, no great care being taken, at stations and wharves, even if the cases are marked "fragile." When a case falls heavily, its contents acquire work and velocity. They lose the latter by giving up their work; to avoid violent forces, the work must be done over a sufficiently great distance. Hence the need for the elasticity of the shavings, etc., used in packing.

In order to understand thoroughly what happens in consequence of a shock, certain properties of matter must be known. Before undertaking the study of this subject, we have first to work through a wide field.

XXIV.—Action and Reaction. Momentum.

Watch a bird fly from the end of a flexible twig; it bends its legs under it, extends them suddenly, and takes to the wing. At this instant pay attention to the twig; it will be seen to move quickly backwards, and, after some oscillations, to return to its normal position.

Why does this twig do this ? Obviously because the bird, in pushing itself forward, pushed the twig backward. So much for the qualitative statement.

We can go further: a horse drawing a carriage exerts a force on the traces. The carriage does not acquire an acceleration, on account of the resistance of the road; but the traces, being free along their length, would acquire an acceleration if they were subjected to different forces at their ends. For Mechanics, the traces may be regarded as part of the cart or part of the horse; a moment's thought will enable us to state the quantitative principle of action and reaction, which Newton first put into exact form. We can express it in this form:—*When one body acts on another, the force of the first on the second is equal to the force of the second on the first.*

Later, we shall consider some results of this principle, which is of the nature of an axiom; for the moment we shall confine ourselves to deducing, by a very simple process, another principle of great importance.

The forces begin and end at the same instant for the two bodies; therefore, for each, one quantity is the same —the force multiplied by its duration.

A careful distinction must be made between the product of force and time (*impulse*) and the product of force and distance (*work*).

In the firing of a cannon, the gases drive the shot from one end to the other; in propelling the shot forward, they press on the breech of the gun with a force at least as great as that on the shot—*at least*, for the breech does not yield under the pressure as the shot does; but we can neglect this action, provided we remember that when we say the forces are equal we are underestimating that on the gun. If the gases exert on the gun work equal to that discharging the shot, the former ought to become a dangerous projectile, since this work ought to start the gun moving.

But it is not the work of the gases that is the same for the two parts of the system, but the impulse.

These equal impulses can be compared with another product.

We have seen that the force f is the product of mass m and acceleration a, *i.e.*—

$$f = ma.$$

Elsewhere, we saw that velocity is the product of acceleration and time—

$$at = v.$$

Multiplying these two equations, we have

$$ft = mv.$$

The product of the mass and velocity of the shot ought then to be the same as for the gun, no other force having acted. The action of the gases preponderates to such a degree, during the very short time of the firing of the gun (about the one hundredth of a second), that the statement is very nearly correct for the instant the shot leaves the gun. We are not concerned with what happens after the projectile begins to travel in the air.

The velocities are in opposite directions; and, if we agree to give a positive sign to one direction, the other is consequently negative, and the algebraic sum of the products is zero.

Now imagine the gun placed in the forward turret of an ironclad, which is firing directly forward. To an observer in the turret, the conditions are the same as in the preceding experiment. But an observer of the motions from the shore would have to add the velocity of the vessel to each velocity measured by the observer in the turret. As the result of calculation, it would be found that after, as before firing, the sum of the products of the masses and their various velocities would be equal to the sum of the masses multiplied by the velocity of the vessel.

Descartes called the product of mass and its velocity *quantity of motion*, i.e., *momentum*. Our experiment, as well as any other we may do, proves that *the momentum of a number of masses acting upon each other, without inter-*

ference from any external force, is constant. This is the *principle of conservation of momentum.**

Thus a body cannot be displaced by internal force alone. A man seated in a boat or a carriage cannot put it in motion without an external point to which to apply his force. He can make a light boat move alternately backward and forward by moving to and fro. When he has a velocity, the boat has; when he stops, it stops; when he sits down, it resumes its original position. To displace it, some external force must act—pressure on the water, the pull of a rope, or the action of the wind. The same conditions apply to a leap from a boat to the bank. If any one tries it, he will see that the boat is indeed driven back, because, having lost its load, it is changed; so is he, for he will have fallen into the water.

We shall meet applications of the principle just explained; but we must now try to discover how a state of rest is brought about, since forces seem always to produce motion.

* "Although motion is only a condition of matter which is moved, there is nevertheless a fixed quantity of it that never increases or diminishes, even if there is sometimes more and sometimes less in certain parts."—DESCARTES: "Principles of Philosophy," 1644.

CHAPTER V

FORCES WITHOUT MOTION

XXV.—Reactions of Matter ; Elasticity and Friction.

FORCES do not always produce motion. Innumerable objects around us are under the action of forces and yet they do not move. Books on the shelf, an inkstand on the table, and the table resting on the floor are all subjected to a force—their weight; but at the point where each object rests is developed an antagonistic force (reaction) which exactly counteracts the *action* of the weight. The principle of equality of action and reaction, already recognised in the case of bodies in motion, is true for bodies at rest. We should have readily said that the principle would be applicable, *a fortiori*, if one of them could be more true than the other. In reality, they are equally true, but the reaction of a support on a body at rest is not so apparent as the reaction of an object free to move under a force exerted by another object.

Any one going out on a frosty day and observing the difficulty of keeping his feet must have been forcibly reminded of that property of the ground whereby he is enabled to maintain an upright position without great effort. The same idea may have come to him when his bicycle has slipped on wet paving or when he has seen a cab-horse fall on an asphalt road. He may have seen, too, some kind-hearted person throw a shovelful of cinders under the animal's feet, with the result that its hoofs gripped the ground and it could rise.

In consequence of these examples, it is easy to realise how much we owe to the beneficent action of friction.

Engineers avoid it as much as possible, and rightly so. In applied Mechanics friction is described as "the abomination of desolation," and here, too, rightly, but within very special limits. Elsewhere friction is to be blessed. It enables us to walk, to sit at a table and work without fear of the books and inkstand falling on the floor, the table sliding into a corner, or the very pen slipping from our fingers.

Friction is so all-pervading a phenomenon that, with rare exceptions, we are never under the necessity of applying it; it is always present as a matter of course. Sometimes it is disregarded with disastrous results. In consequence of coming into contact with Europeans, the Abyssinians have begun to wear shoes of which they are very proud. But are they justified? According to observant travellers, they are not. The earth of the mountains is often wet and slippery; the toes grip in such conditions, but shoes slip. The Abyssinians will become less active, less able to move freely, and finally less able to fight. For them, as for us, though for other reasons, shoes will be one of the penalties of civilisation.

Friction gives stability. A carpenter can level a floor without any difficulty so that tables and chairs will stay where they are put. Except at sea, plates and glasses when placed on a table give occasion for no uneasiness. But a wooden ball must be propped if it is to remain at rest on a smooth plank; and if a steel ball is to be kept in place on a sheet of glass, the glass must be on levelling screws. When the ball ceases rolling the glass is very nearly level, but not quite, for the level can be disturbed slightly without affecting the ball. It remains in place for a range of very small angles of inclination of the glass—much smaller angles than those possible in the case of the plank supporting the ball, or of the table supporting the book.

As we have hitherto done, we can say that we have experimentally reduced friction to a very small value; let us imagine it completely removed, in which case no two objects will any longer remain one upon the other—neither boulders nor grains of sand. Everything will slide or roll until a condition of uniform level is attained. In the absence of friction, the earth would present no irregularities and would be as smooth as the surface of a pond.

The cause of friction is often self-evident; when two rough planks are laid upon each other, the projections engage each other and offer direct obstacles to motion. Yet a steel ball resting on a sheet of glass is not held in place by any visible roughness. There are two causes; at the point of contact with the glass, the ball is slightly flattened (Fig. 16), whilst the glass is slightly hollowed. In order to move, the ball has to climb from the hollow; and it is only when, by inclination of the sheet of glass, the side of this minute hollow is made to slope slightly outwards that the ball rolls.

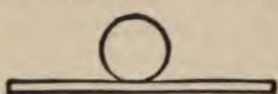


FIG. 16.—A ball depresses a surface, and is itself flattened at the point of contact.

Both in rolling and sliding, the cause of friction is frequently the one we have just examined. A cart sinks into soft ground, and the wheels have always a slope before them that they have to ascend.

The second reason why the ball stays in place is easy to discover. If it had been put in a basin, the cause of its remaining stationary would have been obvious. Now, if a piece of glass, the surface of which is a perfect plane, is rested on two supports, it bends into the form of an inverted arch, but an arch of such slight curvature that it can be detected only by the most delicate instruments.

If a sheet of glass is supported all round its edge a hollow is formed; it is not perceptible to the eye, because the deflection is infinitesimal; but it is perfectly well known that a full-grown man makes an appreciable depression in a rush chair, while a child causes very little; the weight being gradually diminished, a time will at last come when the depression will no longer be visible, but it is impossible to fix the precise moment when it ceases to exist, and the conclusion is that if we cannot see it our method is at fault.

A rail a dozen yards long, supported at its ends, will inevitably appear to be bent. A shorter rail also appears bent, but to a less degree; and, by means of very delicate experiments, the existence of a deflection could be proved in a rail only 1 yard long, to the middle of which a weight inadequate to break a hair was suspended.

Whether instruments can detect it or not, matter undergoes change of shape when in contact with matter; this change, either localised at the point of contact or affecting the whole area of support, is the cause of reaction that results in the equilibrium of the body supported.

We shall soon study the reactions of matter; it is enough to know that they, together with friction, are responsible for stability. We shall understand this better after considering a simple but generally neglected experiment.

Pass over a freely-running pulley a fine thread, provided at each end with a scale-pan containing sand. It will not be difficult to equalise the sand in each pan so that the system is in equilibrium. One grain of sand can be removed, then another, and so on, until the difference is enough to put the apparatus in motion. The experiment can be varied by adding grains of sand, when it will become manifest that there is a sort of region of rest within which the system has no tendency to move; yet one of the scale-pans is known to be heavier than the other.

The reason is that, in addition to the force acting on the string, several incidental forces are present—friction of the string, bending of the axle, and friction of the bearing—all of which are hindrances to movement and ensure a state of rest.

In the absence of these forces, absolute rest of the system could not be achieved; the difference might be no more than a single grain of sand, but that would be enough. This grain, a fraction of which constituted the excess, might be broken up and its microscopic particles put singly in the lighter scale-pan; a moment would come when one of the grains of dust would neutralise the upward tendency and cause its own scale-pan to descend.

Friction and the reactions of matter relieve us of this troublesome search for an equilibrium that is not attainable; they are the originating causes of those passive forces which offer resistance to active forces and thus render possible a state of rest.

We now know that there is such a condition as one of rest, and we know the causes that give rise to it. We can therefore study forces in equilibrium otherwise than by a sort of miraculous mathematical equality.

XXVI.—Forces in Equilibrium; Composition and Resolution of Forces.

When it rains in calm weather an umbrella is held upright; it is inclined towards the wind in stormy weather, if it is used effectively. But rain does not always fall vertically to the ground; carried by the wind, it falls at an angle and reaches the feet of the pedestrian, even if his head is protected.

We already know enough Mechanics to explain this. Each drop of rain is acted upon by two forces—one vertical, and the other horizontal; the former is the difference between its weight and the resistance of the air; the latter is the force of the wind—a force of no negligible power, for it may be sufficient to uproot trees and damage buildings. What direction will the drop take under the combined action of the two forces? In order to solve this, we shall make use of the hypothesis that the two forces act separately for a very short time; as soon as one has finished, time will be considered as set back, and the other will act for the same interval.

In consequence, each of the forces will impart to the drop of rain an acceleration proportional to its value. The velocities, which are the sums of the accelerations, will also be proportional to the forces, and the drop of rain will fall as in the diagonal of the parallelogram of velocities with which we have become familiar (§ XII.).

If the two lines OA and OB (Fig. 17) represent the distances the drop would travel in a given time if under the influence of one of the forces, the two together will cause it to travel OR.

Suppose now that the lines no longer represent distances, but *values*, *i.e.*, of forces. Then OR is the value of the force that drives the drop in the direction OR, with an acceleration represented by OR, in the same scale as OA and OB represent the separate accelerations. Therefore OR represents, in magnitude and direction, the force that would produce the same effect as OA and OB. This force is called a *resultant*, and the two others are its *components*; this theorem is known as the theorem of the parallelogram of forces. It can be stated in the following terms:—

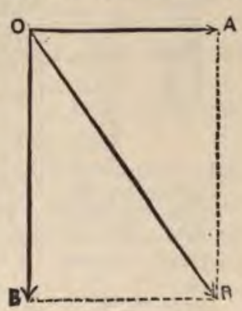


FIG. 17.—Composition of forces.

The resultant of two forces is represented, in magnitude and direction, by the diagonal of the parallelogram constructed on the straight lines representing these two forces.

We have found this principle simply by referring to what we have already learnt. We can, however, approach it in another way, by opposing the forces so that they balance.

Let us begin at the beginning. We can hang a weight to a string and hook a second weight to the first; equally well we can attach both weights directly to the string. There is no doubt that the string is stretched the same amount each time, and we are brought to the conclusion

FIG. 18.—Forces acting in the same direction are added. They can be balanced by a force equal to their sum.

that two forces (we might have said ten or twenty) can be represented by one force —their sum.*

The weights do not fall because the string supports them and opposes to the force that draws them to the earth a force exactly equal and opposite.

Forces can not only be added, but they can be subtracted and brought to equilibrium.

We shall use the same method of representation as before. Each of the forces will be represented by a line of a length proportional to the force. Place these lines so that they form one continuous straight line, and, in the opposite direction, draw a line whose length is equal to the sum of the other two.

This can be plainly demonstrated by a spring balance and weights which do not

* Observe what great caution is necessary; the fact stated above is evident, yet it is slightly erroneous in its present form. The fact is that the attaching of one weight to the other puts it nearer the centre of the earth than fastening it directly to the string. It is therefore heavier (§ XXVII.). If a very sensitive balance be substituted, a minute but still measurable difference will be detected. If a kilogramme be raised one decimetre, the force on its support is lessened by two to three hundredths of a milligramme.

It is necessary to mention this, in order to show (1) that evidence is sometimes misleading; (2) that highly accurate measurement teaches much more than a rough experiment; (3) how a perfectly true conclusion can apparently be falsified by a non-essential.

The best of experiments has to be interpreted with common sense. Would a very skilled physicist, who did not already know the result, and who was performing this experiment for the first time, doubt the possibility of adding forces without introducing a change? It is improbable; he would perhaps discover the explanation of the slight difference; still, it is not certain that he might not search a long time for it.

stretch the spring too far. Note the extension under two successive weights, and then attach both the weights together; the total extension will be the sum of the separate extensions.

Another definition:—The point at which a force is attached in order to act is its *point of application*.

A horse can be harnessed either to the shafts or to a rope attached to the cross-bar of the carriage, and this rope may be of any reasonable length. We know, therefore, that the point of application may be moved anywhere along its line of action without any alteration of the effect.

But a force cannot be displaced laterally without alteration. A horse harnessed to one end of the axle would simply turn the carriage round; we shall soon know the reason.

As with the drop of rain driven by the wind, forces have not always one and the same direction.

A horse dragging a railway waggon to form a train walks as near the rail as possible, and in a parallel direction, because oblique forces have less effect. Suppose (the extreme case) the horse pulls directly across the track; the waggon will be pressed against the rails, but will not move; the force exerted by the horse is not suddenly destroyed the moment it draws perpendicularly to the track; it diminishes gradually, as the line of application moves away from that of the rail.

We will examine the variation of forces that do not act directly.

Stevin, a Dutchman of great genius, who lived three centuries ago (1548-1620), thought out the following experiment, which was so evident that he had no need to perform it.

Over a triangle of wood ABC, with one side vertical and another horizontal (Fig. 19), is hung a string of balls united by frictionless bearings. This string hangs under the triangle in the form of a festoon which is necessarily symmetrical. Common sense says that the string will not run round the triangle of its own accord; therefore the forces acting on it are in equilibrium.

The string of balls can be divided into three parts—the festoon and the two sides of the triangle. The first being symmetrical, its halves balance each other; hence the two straight parts are in equilibrium. The part BC is shorter and therefore lighter than the part AB; the sloping side of the triangle must balance a part of the festoon, and it is the remainder that acts on the point B.

Fig. 19.—A string of balls resting on a triangle is in equilibrium.

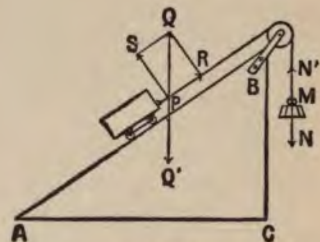


Fig. 20.—A weight can be held on an inclined plane by a force less than that of the weight.

Stevin's experiment can be developed into a form in which, though less evident, it is more easily carried out. This is the form in which Galileo thought it out.

Substitute (Fig. 20) for the parts of the chain AB and BC weights equal to them, and fasten them to a thread passing over a pulley fixed at B. One of the weights is suspended on the thread, whilst the other is represented by a car running on the sloping side of the triangle; the system will still be in equilibrium.

Only such a fraction of the weight of the car will act upon the thread as suffices to produce equilibrium; as the two weights are respectively proportional to the sides AB and BC of the triangle, it follows that the weight of the car is reduced in the proportion $\frac{BC}{AB}$.

We can now make use of our method of symbolic representation. Let MN be the value of the suspended weight. That of the car will be represented by PQ′. The fraction acting on the string will be PR (= MN). The triangle will support a force represented by PS. The triangle PQS being similar to the triangle ABC, the ratios of the forces are such as to agree with the results of Stevin's experiment. Here, again, we meet the parallelogram of forces.

If the string is cut, the car will run down the incline; its acceleration, being less than that of a body under gravity in the ratio $\frac{PR}{PQ}$, allows its motion to be easily

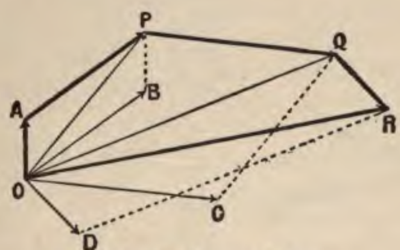


FIG. 21.—Polygon of forces.

followed. When Galileo (§ VI.) investigated the laws in accordance with which a body rolls down an inclined plane, his principal aim was to discover, by an easy but indirect method, the law of falling under gravity.

If, in our graphical method, it is desired to find the resultant of several forces, two are resolved, a third is introduced, and so on. OA and OB give the resultant OP (Fig. 21); this with OC gives OQ, and this, in turn, with OD gives the final resultant OR. We can remove the lines OP, BP, OQ, CQ, DR, and leave the polygon APQRO with the resultant OR. APQRO is called a *polygon of forces*.

Just as several forces can be replaced by one, so one force can be resolved into several others; and, as in the case of velocity, this apparent complication aids a final simplification. Project a force on two rectangular axes; its two components are found. Project on the same axes all the forces under consideration, and add the components falling on each axis; two partial resultants are found, from which a single resultant can be obtained.

Forces are not, however, always in the same plane; they may be in any direction in space. That is no obstacle, for two intercepting forces are always in the same plane, since they define the plane; they can be combined, as has already been done; with the first resultant and a third force a second resultant can be determined, and so on; a figure shows at once how the final resultant is discovered by means of a polygon of forces in space.

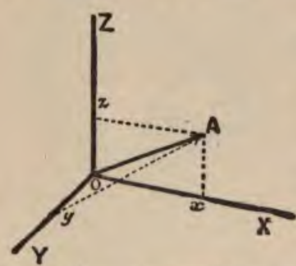


FIG. 22. — Composition of forces in space.

Similarly, a system of three axes can be established in place of two, so as to define any point in space, and, in any of the planes formed by one of the three axes and one force, this force can be projected so as to be replaced by its three components O*x*, O*y*, O*z*. Any number of concurrent forces can thus be resolved and reduced to forces lying in three rectangular axes; finally, the three rectangular forces have to be combined. This method is often very useful.

We are constantly utilising the principle of composition of forces, sometimes—as M. Jourdain spoke prose—without knowing it; sometimes we do it consciously, like the well-intentioned orator, who, deficient in Latin, concluded an appeal for unity of action with the words "*Sursum corda*—let us all pull on the same rope." It is, in fact, by pulling on the same rope that we add together our efforts and render them efficacious; it is surprising to see how mutual aid lightens the task of each person engaged. Without leaving the domain of Mechanics, our young friends can unite their efforts to raise one of their number, and he need not necessarily be the lightest. He must sit down, while each one must apply a finger wherever a point of support can be found—under his feet, his knees, his shoulders, and his chin. At the word "three" every one must lift, and the subject will appear to rise of his own accord, for each will experience only the slightest sensation of lifting.

A glance around shows how forces are associated; several horses are harnessed to a heavy load; though nothing is directly visible, we know that the equivalent of forty horses is utilised on a motor-car. A single grain of powder in a gun would produce no effect, but a quantity together would drive out a huge projectile.

In nature, it is nothing but the addition of forces on a grand scale that directs all the movements of the universe; the forces are those of the attraction of matter for matter. It is now time for us to become familiar with these, as we have already met them many times in the course of this book.

XXVII.—The Law of Attraction; Weight and Mass.

Our environment has many different aspects; it may be considered as happy or unhappy, as pleasant or unpleasant; but the fact that we live on the earth and not on the moon ought not to interfere in any way with our ideas of the principles of Mechanics. The man of science ought to be capable of such detachment from his surroundings that his judgment is free from their influence, as an impartial official ought to be able to repress his likes or dislikes when exercising his office. Before he has found the truth, the seeker cannot distinguish between the essential and the accidental. The immediate testimony of his senses is his first, and, for a long time, his only, guide. Hence, the development of Mechanics has been closely related to the facts of our surroundings. Have we not ourselves utilised weight (§ XIV.) to discover fundamental laws? We have thus recognised that weight is in close relation with mass; when we have considered weight itself, the importance of this relation will become apparent.

Among the ancients, motion "up and down" was a very definite idea, as it is for children and many men to-day. "Up" is the direction away from the earth, "down" that towards the earth. As the earth is immense in relation to its inhabitants, it seems, on the whole, to be a plane, and the verticals defining up and down seem, from immediate observation, to be everywhere parallel.

So long as this belief was held, the descent of bodies towards the earth was a necessity, as was the existence of "up and down." Aristotle and his school taught that every body seeks its level, and that for all bodies this level is as low as possible. If every body does not fall indefinitely, it is simply because the places are already filled.

The ancient belief was that the world was divided by the surface of the earth into two distinct regions—upper and lower. The latter was the abode of the wicked, whence the name *infernal* (L. *inferna*) that they had given it.

The shadow of the earth on the moon in eclipses, the gradual disappearance of the coast as a vessel leaves a port, astronomical observations during a voyage round the earth, and the mere fact that ships can sail round the earth, have convinced men that the earth is nearly spherical. Since, at every spot, the downward direction is towards the ground, every place on the earth has its own up and down direction, which is parallel to no other, except that diametrically opposite, and, in this case, "upwards" for one is "downwards" for the other.*

* The elaboration of this idea by M. Flammarion can be read with advantage in the volume on Astronomy in this series.

In modern scientific terms, the ancient idea of the fall of a body could be stated thus:—All bodies have a tendency to approach the earth.

As up and down directions are infinite, the modern idea is much more complex than the ancient, which was the simple observation of a single fact. For the sake of the simplicity that is always desirable, some facts about weight must first be experimentally determined.

All bodies show a tendency to fall, whether on a mountain, in a mine, or in a balloon. The action of the earth on matter is not limited to the surface of the globe, and it is obvious that it varies very little within the limits we can reach either upwards or downwards. Is there, then, a distance from the earth at which this action suddenly ceases? There is no reason for thinking so. Such a hypothesis would be bold to the point of absurdity. This was Newton's conclusion; he had no hesitation in admitting that the earth exercises its attraction as far as the moon —a feeble attraction, doubtless, but still well marked.

Why, therefore, does the moon not fall to the earth? For the reason that it has a motion that tends to carry it constantly away from the earth in the same measure as it is attracted.

We have seen that a projectile thrown horizontally describes a parabola in falling. The acceleration of the fall being constant, the parabola is the more extended as the speed is greater.

Suppose (these facts were available for Newton) a gun is trained horizontally outside the atmosphere (Fig. 23), so that the shot is free from the resistance of the air, and suppose the gun sends shots with increasing velocity. The projectiles will fall at increasing distances, until the velocity will cause the shot to fall beyond the limits of the earth; it will travel round the earth, because its acceleration at any instant will be enough to keep it in a circle round the globe. The curve of its flight will not be a parabola as before. Something is, in fact, changed in the conditions; so long as the trajectory was entirely within a space in which the verticals could be considered parallel, the curve was a parabola. Then its shape changed as the lines of action of its weight included greater and greater angles; finally, when the velocity was such that the variation in the direction of the trajectory was equivalent to the variation in the direction of the weight, the trajectory cut at right angles the perpendiculars from the places passed, *i.e.*, it was horizontal at every

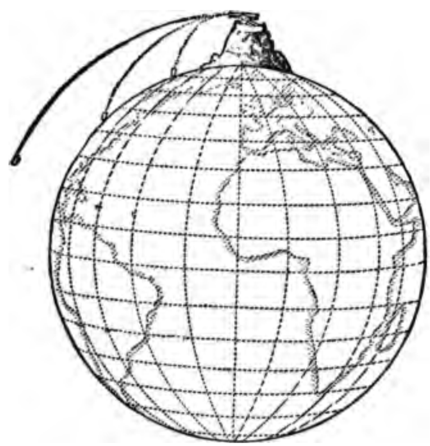


FIG. 23.—A shot sent with sufficient speed would travel round the earth.

point. By an exact adjustment of acceleration and tendency to travel away in a straight line, a shot sent horizontally could maintain a horizontal trajectory, and continue round the earth indefinitely. A speed of a little over 5 miles per second would be necessary—seven times greater than that of the fastest projectiles—but much less than that of certain meteorites that, crossing the orbit of the earth, deviate from their way and continue their course through space. Those of the meteorites that enter the atmosphere lose velocity, become heated, and form the shooting stars that cross the sky at certain times of the year.

It may be said that the moon has a speed of less than 5 miles per second; that is true, but it is far removed from the earth, and at that distance the earth's attraction on it is much reduced. The acceleration of its fall is then much less, and it requires less speed to escape. When calculating the attraction the earth exercises on each particle of the moon, Newton verified the law of attraction of masses, already formulated by Kepler, but not proved by him. This law is as follows:—*Matter attracts matter directly as the masses and indirectly as the square of the distance.*

Every-day facts do not seem to confirm this attraction. Houses do not attract one another; true, they are strongly fixed in the ground; but neither do billiard balls attract one another, and beginners think they have a curious tendency to repel one another. On smooth ice skaters are often seen to go towards one another, but this attraction can always be explained otherwise than by the action of masses.

In reality, the attraction of matter for matter is extraordinarily feeble, and its detection demands exceedingly delicate operations. It gives a sufficient idea to say that 2 kilogrammes, 1 metre apart, exercise a reciprocal attraction nearly equal to the force on a balance of six millionths of a milligram. If perfectly free to move, they would approach by one ten thousandth of a millimetre in the first minute.

To mutual attraction is due the motion of the stars; but every particle of one attracts every particle of the other, and thus the accumulated force becomes inconceivable. The earth contains 5,000 millions of millions of millions of tons of matter, producing the attraction that holds us to its surface. Moreover, attraction varies from star to star, according to the amount of matter; 1 kilogramme carried to the sun would extend a spring twenty-seven times as much as on the earth; and one sixth as much on the moon. If we could be transported there and could live, we should be crushed on the sun by a mass of 4 to 5 kilogrammes, while on the moon we could easily support from 400 to 500.

An experiment, sufficient to give a general idea of weight, showed that it is, as far as could be perceived, the same in all the rooms of a house: measurements of extreme delicacy, such as a scientist can make, reveal

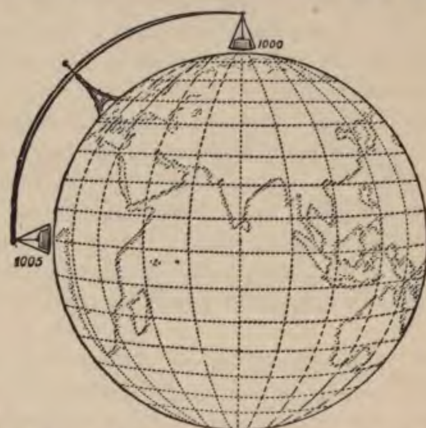


Fig. 24.—A ton weighs five kilogrammes more at the pole than at the equator.

small differences. They are not shown by a balance, for the weights in the scale-pans can be considered as being in the same place. But if there were a balance with a beam 10,000 kilometres long, supported by a tower at the 45th degree of latitude (Fig. 24), so that the pans were at the pole and at the equator respectively, 5 kilogrammes would have to be added to a ton in the equatorial pan in order to balance the ton in the polar.

On account of this difference in attraction, a clock adjusted at Paris loses four minutes a day at the equator.

Very far from any celestial body, weight is imperceptible. Between the earth and the moon, there is a point where the attractions of the two balance, and a body would not be attracted to either.

Weight is a property that bodies can lose, just as a shadow is lost in a uniformly dull light; weight is not in the objects themselves, but, like the shadow, depends on the surroundings. If a quantity of matter is defined in terms of its weight, nothing precise is said about it.

Further, although for convenience we have the goods we are buying weighed under our eyes, it is not (unless we want to break stones) on the weight that the value depends. Bread is bought for its nutritive value; but if it weighed nothing, there would be all the less trouble in carrying it home, and it would in consequence be worth more.

The fundamental difference between weight and mass must be insisted upon, for confusion is frequent, and is responsible for grave errors. Weight can increase, diminish, or disappear, while the mass remains of exactly the same value.

All this is perfectly plain, and the wonder is how confusion has arisen elsewhere than in the minds of those to whom everything is confused. The reason can soon be understood; the weight of a body is the product of its mass and the acceleration that its weight gives it. Now, as weight is the more conspicuous property, it is habitually spoken of rather than mass. It is considered as fundamental, and mass is accordingly spoken of as weight divided by the acceleration due to gravity.

Since the weight can disappear, the quotient is that of two non-existent quantities, *i.e.*, of an indeterminate quantity—an example of how the most stable property of a body can vanish.

If it had been possible for us to travel far from our planet, such a definition would never have come into existence; the weight of a body would be an accident like its colour; a definition of mass in terms of weight would seem as artificial as that of a crab in an old dictionary of the French Academy:—"Small red fish that walks backwards."

The idea that mass is the quotient of weight by acceleration leads to strange errors.

Some years ago an engineer, who had remembered only the formulæ of Mechanics, developed the following theory:—A balloon in equilibrium in the air weighs nothing; its weight being nothing, so is its mass; if it has no mass, $\frac{mv^2}{2} = 0$, whatever its speed; therefore a dirigible balloon can have no kinetic energy; it cannot maintain its motion, even for an infinitely short time, and, if it is to be kept moving, it must be constantly propelled.

True, it could be given a velocity without any effort at all. But the engineer had not drawn this last conclusion; he had gone no further than to condemn the balloon for the former reason. Apparently he did not know that, from time to time, a balloon knocks down a chimney or breaks a branch off a tree.

XXVIII.—Couples.

Have you ever wondered why the wheels of a carriage turn round? The horses draw on the traces (automobiles are quite different), and the force is transmitted to the axles, which draw the wheels forward; but these are on the ground and, as we have seen, are kept in place by friction. The two forces AB, CD (Fig. 25) are not directly opposite; one acts on the hub, the other on the rim. Whenever this is the case, the object on which the forces act begins to rotate.

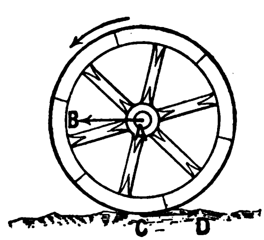


FIG. 25.—A carriage wheel turns because it is under the action of a couple.

The name *couple* is given to a combination of two forces that produce rotation, *i.e.*, two forces that are equal, parallel, and opposite in direction.

Often a couple is constituted of forces that act directly. Thus, a corkscrew is taken in the hand and turned, as is done when a door is opened by the knob.

At other times the couple comes into existence automatically, as a result of the actions that produce equilibrium of forces—the reaction of matter, and friction.

On a bicycle one pedal is pressed down and the other allowed to rise, rotation being produced. The bearing holds the axle of the crank in place, and gives the second force of the couple.

The necessity of the second force can easily be explained. Place the point of a joiner's bit on a hard plank, and try to bore a hole. So long as the point has not penetrated enough it has a tendency to escape, as generally happens with beginners ; the hole once begun, the bit will remain in position.

The reason of the efficiency of a couple is easy to discover. A door is always closed by the knob, not only to avoid finger-marking, but because it is known to be necessary to act as far from the hinges as possible. If unconsciously (*i.e.*, after forgotten experiments have left only a habit) we take hold of a door by the edge furthest from the hinges, we do so because we know that we economise our effort. The door turns without lateral displacement because its hinges complete the couple, and we now know that the further apart the forces, the more efficient they are ; it is not necessary to recall the many experiments proving that the value of a couple increases as the forces acting. A few measurements will show how force and distance determine the efficiency of a couple.

At the middle of a bar of wood (Fig. 26) screw a ring and, on the opposite side, eleven equidistant hooks, one of which is exactly under the ring. A string having been passed through the ring, weights can be hung from the hooks.

The ring will exercise a force upwards, the weights downwards. If one hook is loaded, the bar is pulled down on one side, because the weight forms a couple with the supporting force. But if two equal weights are attached to hooks 2 and 10, for example, it remains horizontal. This will be the case if equal weights are put on hooks 1, 2, 3, and 9, 10, 11. This effect can be interpreted in two different ways—first, that the six forces are equivalent in effect to one resultant passing through

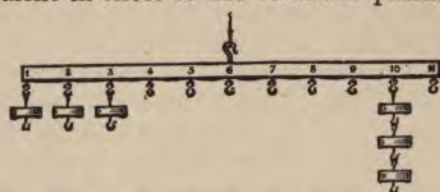


FIG. 26.—The equilibrium of the bar can be brought about in many ways; but always when equal couples are opposed at the point of suspension.

the point of suspension and balanced by the force exerted by the string; second, that one half of this force combines with the three forces on the right to form one couple, the other half with the three on the left, with the result that two equal and opposite couples are produced.

The first of these two interpretations leads to the conclusion that two equal and parallel forces can be composed into a resultant, and that the position of this resultant is midway between them. Without any change in the equilibrium, the three weights on the right can be hung together on hook 10 and three couples be balanced by one.

The experiment can be still further modified. Hang one of the equal weights on hook 6 and leave one on each of the hooks 2 and 10; equilibrium will be maintained if on the left two weights are treated as one and hung on hook 4. The weight from one point of the bar is double that hung from the other; but the latter is twice the distance of the former from the point of support, and equilibrium is still maintained. No matter how this experiment is varied, it is *always* found that a state of equilibrium is obtained when the weight on one side multiplied by its distance from the fulcrum is equal to the like product on the other side of the fulcrum; and, generally, the equality of the sum of the products on each side of the fulcrum is the necessary condition for equilibrium.

The principle of composition of parallel forces can be deduced immediately. Further, the efficiency of a couple depends on the product of one of the forces and the distance separating them; this is the *moment* of the couple, and experiments already made prove that a system of couples is in equilibrium if the sum of the clockwise is equal to the sum of the counter-clockwise moments.

XXIX.—The Lever.

Since very ancient times, it is probable that the principles just considered have been utilised to increase the force that a man can exert; for this purpose an instrument called the *lever* was and is still used. It is a strong bar, generally of iron or steel, which, inserted under the weight to be lifted and rested upon a support, is subjected to a force at the other end. If the distance from the weight to the support is small, while that between the hand and the support is great, the force is considerably increased; the increase is always in the ratio of these two distances, called the *arms of the lever*.

The principle of the lever, now established, can hereafter be stated as follows:—*A lever is in equilibrium when the sums of the products of the forces and their distances* *from the point of support are equal in magnitude but opposite in direction on each side of the point of support.*

It is to be remembered, when this fact is utilised, that a couple can be kept in equilibrium only by another couple, and not by a single force; for equilibrium to exist, the moments of the two couples must be equal, but these couples may be very different, either a large force acting on a short lever, or a small force on a long lever.

If our young pupils have understood, they will be able to puzzle their friends by lifting a hammer at the end, so that the handle rests on the back of the first finger and is held under the middle finger (Fig. 27). If the fingers are brought together, the hammer cannot be held horizontal without tremendous effort; the further the fingers are separated, the easier it is. This simply means that the two forces of the couple that prevent the hammer from turning have been separated. The finger and thumb holding the handle would not have kept the hammer horizontal; yet they could hold it vertical without any difficulty, for, in this case, they would be opposing a force to a force; in the former case, they would be trying to balance a couple by a force sufficient to support the weight of the hammer, but applied in a couple much too small to balance the couple produced by the hammer.

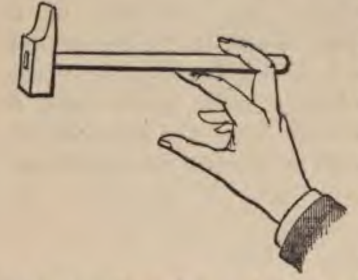


FIG. 27.—In order to balance a couple, a couple of the same moment is required.

The ordinary balance compares forces by equalising couples whose levers are as nearly as possible of the same length. If they are exactly equal, the equality of the couples implies equality of forces, *i.e.*, of weights; if they are only approximately equal, the equilibrium of the balance indicates that the weights are not quite equal.

But accurate weighing can be done with a false balance. Borda has shown that it is sufficient to put the two weights successively in the same pan, and to balance their moments by that of a weight (not necessarily of known value) in the other.

According to a method indicated by Gauss, the weights can be changed into the other pan; the two weighings give different results, but the mean is the result that an accurate balance would have given. This method is adopted by the International Bureau of Weights and Measures and similar establishments, in order to obtain very exact weights.

When rough weighing is satisfactory, as in the majority of commercial operations, the balances are considered as exact, and the small error is neglected. The authorities take precautions that these balances are within legal limits of error; they have false balances repaired or destroyed.

A balance can in all honesty be made false, but in a way to prevent misunderstanding. For example, one arm may be ten times as long as the other; the weights are put on the former, and the load on the latter. Thus, 100 pounds are weighed by means of 10 pounds—a much easier operation.

At railway stations the balances often have arms of different length, so that a constant weight (at least up to 2 hundredweights) can be made to slide along an arm.

These examples show the different combinations of which levers are capable, and each example might be taken as a verification of the principles.

The lever allows either the force to be increased or the distance it moves to be increased; this can be shown by placing four matches as in Fig. 28. Pressure at A

... motion of the

... force in

... that even

... may be

into rectangles, triangles, or circles. In every case a point, marking the limit of stable and unstable positions, will be found.

Another test can be made by suspending a triangle, for instance, by each angle in succession, and putting against it a string attached to a weight, and passing exactly in front of the point of suspension. The string will also pass through the point already marked. It seems as if all the weight of the card is centred in this point, since it hangs under that by which the triangle is supported. Therefore it is often called the *centre of gravity* of the triangle, as of any body whatever.

Every body has a point possessed of the same property; although it is often said of a person who has fallen that he has "lost his centre of gravity," that is no exception to the rule, but rather an incorrect expression. An experiment with a person, similar to that with the piece of cardboard, would soon show that his centre of gravity never leaves him.

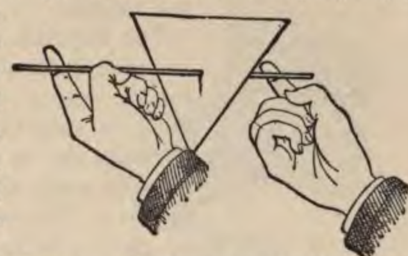


FIG. 29.—A body supported at its centre of gravity has no tendency to rotate.

We are, without suspecting it, constantly utilising the properties of our centre of gravity, as certain common experiences will show. If a person stands pressed against a wall, from shoulder to foot, and tries to lift the further foot, he will find he cannot do so without falling. Probably for the first time in his life he learns that, in order to raise the left foot, he must bend to the right.

He might also bend forward with his head resting against a wall and his feet set as far back as possible. In that position he could not rise without the help of his hands.

To return to the cardboard triangle, a knitting-needle could be inserted at the centre of gravity, and supported on the fingers (Fig. 29). The triangle, being supported at the very point where its weight acts, remains in any position indifferently. If, on the other hand, the needle is inserted elsewhere, the triangle turns round until the point marking the centre of gravity is directly below the place where the needle enters. The reason is that the supporting force of the needle and the weight of the triangle form a couple, and a couple always produces rotation. When the centre of gravity is below the needle, both are in the line of action of the force representing the weight.

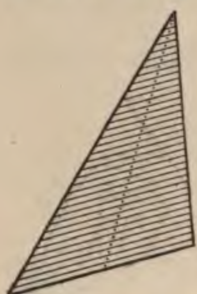


FIG. 30.—The centre of gravity can often be determined geometrically.

The experiments with the wooden bar give some idea of the part played by the centre of gravity. When a body is suspended at that point, its particles can be combined two by two, so as to make their couples equal and opposite. The resultant of all the forces of gravity acting on the body always passes through that point.

The following example is an instance of how the centre of gravity (*e.g.*, of a triangle) can be found without recourse to experimental methods; the triangle is imagined as cut into strips, parallel to the base (Fig. 30). The centre of gravity of each of these is its middle point. The figure, if placed with a median over the edge of a knife, will be in equilibrium; the same can be done with the other medians, and their point of intersection gives the centre of gravity.

The idea of centre of gravity is so instinctive that the term is often used in a figurative sense. The same problem is met in many other spheres; there is, for example, the case of a person who has to pay certain sums at different times; he wants to settle the debt by a single payment, without interest. He could set out the times on a rigid rod, hang at the points representing the different periods the sum due or an equivalent weight, and suspend the rod in order to find the point from which it hangs horizontal. This point marks the single payment, and is, so to speak, the centre of gravity of the various payments.

Let us conclude with some experiments of a less serious nature. Can the contents of a bottle be poured out while

FIG. 31.

FIG. 32.

The position of the centre of gravity is sometimes surprising.

the cork remains on the neck? Nothing easier; fasten into the cork two forks at an acute angle (Fig. 31). The cork being in position, its centre of gravity is always below the point of support; thus it has no tendency to fall over, and remains upright on the neck while the bottle is inclined.

Observe this cardboard lobster resting by one extremity on a needle point (Fig. 32). Our instinct for the centre of gravity tells us the position is contrary to all laws. But lift it up; the cunning maker has fastened two pieces of lead to its claws, which extend beyond the head; together they exercise a moment, produced by a relatively large force acting on a short lever, that balances a much smaller force acting on a long lever.

XXXI.—Pressure.

Our knowledge of forces in equilibrium enables us to undertake new investigations. We shall begin by what may be called the *distribution of forces.*

When snow lies thick, walking becomes troublesome and often dangerous. The foot sinks in at every step, and, instead of being carried forward, as in walking on firm ground, has to be lifted to the surface, only to sink in again at the next stride. But dwellers among snow know how to provide against this; they attach to their feet snow-shoes, resembling tennis rackets with short handles, and then they sink only slightly. Walking is no longer difficult, but merely tiring.

Perhaps these people have invented snow-shoes entirely; perhaps they have observed the mountain dog, an animal particularly well adapted for walking on snow, by reason of its webbed feet; in the same way, the camel, the classical example, has a large bearing surface, due to the form of its foot. In each case the force is distributed or separated, with certain advantages.

On the contrary, a force may be concentrated on one small spot and its effect immeasurably increased. The point of a nail has no other purpose; while foils are provided with caps, swords and knives are brought to an edge; penetration is prevented by spreading the force and assisted by concentrating it. In order to slide over snow, skis are used; for ice, on the contrary, skates with narrow, sharp blades are used.

This distribution of force demands, as is customary, an exact definition. We shall consider as representing this distribution, or rather concentration (for concentration is distribution carried no further than a certain degree), the force divided by the surface on which it acts, and this result will be called *pressure.* A force concentrated on a surface of small extent produces a great pressure and inversely.

Generally, the force acts uniformly throughout only in fluids—liquids or gases—which, on account of their mobility, can escape from regions of high pressure and adjust themselves so that the pressure is everywhere equal. In solids, on the other hand, pressure varies from part to part. A ball on a plane causes, as has been seen, a minute depression in which the reaction is proportional to the deformation of the material. The pressure is a maximum at the centre of the depression and diminishes towards the edge, where it is zero.

A uniform distribution of force can be instanced, even in solids. A wire subjected to a tractive force resists equally at every point of its section; the rod of a piston or a lift is also under a uniform pressure.

Pressure in solids permits of generalisation more readily than in the case of liquids. Exceptional conditions aside, a liquid cannot be subjected to tractive force. This, upon a solid, may be considered as negative pressure, tending outwards, while positive pressure tends inwards. Both are expressed by the same numbers.

We have gained a knowledge of pressure by considering the distribution of a force on a surface. But pressure can be deduced directly; the force is computed by combining the products of the pressure and the units of surface into one resultant. The wind driving a sail exerts pressure which generates force, and this is made as great as possible by increasing the sail-area as much as the stability of the boat allows. Cyclists, when racing, bend over the handle-bar to reduce the pressure of the air.

The force being constant, the pressure can be increased or diminished by increasing or diminishing the surface.

The weight on the bottom of a cylindrical vessel of liquid is obviously equal to the total weight of the liquid (Fig. 33). But at any level the vessel can be narrowed and continued as a thin tube. If the liquid rises to the same height as in the cylindrical vessel, that part of the bottom vertically below the tube will support the same force as before, for the tube can be supposed produced to the bottom so as to isolate a small column of liquid. In reality, the surrounding liquid forms the walls of this extension of the tube; and in the neighbourhood of the bottom, the liquid is pressed as if at the bottom. If it is not to escape, the surrounding liquid must support it with the same force, which is communicated to the sides and the whole of the bottom. The vessel might be made very flat and surmounted by a very high tube. The bottom would then be under a great force, produced by a small amount of liquid. This invariable condition is so incredible that it has been given the name of the *hydrostatic paradox*.

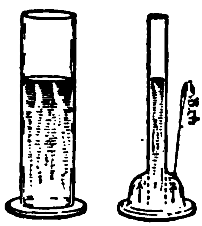


FIG. 33.—The pressure on the horizontal bottom of a vessel is independent of the shape of the vessel.

If the bottom of the vessel supports such a force, ought it not to be very heavy, and would not a balance show a weight greater than that of the vessel and liquid together? The difficulty is easily solved; the liquid, under pressure in all directions, is held with equal force against the dome of the vessel, and exerts as much pressure on it upwards as it exerts on the bottom downwards, the small surface of the tube excepted. If the dome is pierced, the water escapes in a strong jet.

This fact can be expressed by saying that the pressure at any point whatever in a liquid depends only on the height of the column above it, and not on the section.

However small the fissures in rock, water succeeds in penetrating, and the further it descends the more easily it makes a way, for it is under pressure from the whole column. This is why the cutting of tunnels such as the Simplon is so difficult. When the tunnel is under a high mountain, the fissures allow a powerful jet of water to escape. This is very difficult to stop, and has to be diverted into a side tunnel.

Since water penetrates to such a degree at great depths, ought it not to disappear entirely in the ground? The reply is surprising; under immense pressure bodies usually considered solid begin to flow like liquids. At a certain depth rocks form a compact obstacle to the water, exactly as if they were themselves water under pressure.

In the interior of stars the pressure is immense—thousands, or even millions, of times more than any pressure developed by powder in a gun. We have no conception of the effects these pressures can produce; but it is certain that they modify considerably the properties of matter. They may even set up such combinations as will produce new substances; probably high pressure in the interior of the earth is responsible for certain chemical elements, particularly uranium and radium.

Our existence is adapted to the pressure at the bottom of the atmospheric ocean, as it is called. When we ascend in the atmosphere and the pressure is greatly reduced, we experience discomfort which may lead to fainting, or even to death, if the pressure continues to be reduced. On the other hand, when we descend in a diving bell, the pressure of the water is added to that of the air, and at 44 yards below the surface it becomes almost unbearable.

The air-bladder of a fish brought up from a great depth forces its way through the fish's mouth, because the air it contains expands under the reduced pressure.

Men are obliged to take great precautions when under an increased pressure such as certain building operations require. When the foundations of a bridge are to be laid, or any other work is to be performed under water, a caisson—a large cylinder of sheet-iron—is first sunk, and is, of course, full of water. This is driven out by air, forced into the caisson by pumps. Then men enter by a series of air-tight doors isolating small spaces which open either to the interior or to the open air. On entering the caisson, the men experience a slight discomfort, though there is no danger. When coming out, on the contrary, if they are "decompressed" too quickly, the result may be fatal.

The reason is that under pressure the blood dissolves a great quantity of oxygen, which is given off as small bubbles in the blood-vessels if the pressure is suddenly diminished. These bubbles are carried by the blood into the capillaries, where they adhere and arrest the circulation. Nowadays the risks are well known, and such are the precautions taken that there is nothing to fear.

In a bottle of sparkling wine the gas is kept in the liquid by pressure; and as soon as the cork is drawn the gas comes off in bubbles.

A contractor once wished to celebrate the completion of an undertaking, so he sent some bottles of champagne to the workers in the caisson. The wine did not sparkle, and it was thought at the time that it had lost its quality. The error did not last long, for the wine recovered its virtue in the open air. Everything has its time and place. The champagne was out of place and could not sparkle; so the workers resolved that they would not drink again in a caisson.

When steam is introduced into the cylinder of an engine a pressure is caused and, by displacement of the piston, work is produced. The work is equal to the displacement of the piston multiplied by the force on it, this force being the product of the pressure and the area of the piston. In a calculation, the force can be dispensed with; only the pressure and expansion taking place concurrently with the movement of the piston need be considered. Work, in these circumstances, appears as the product of pressure and the increased volume in which the pressure exists.

This remark leads to the principle discovered by Pascal. If a reservoir, filled with a fluid, has two unequal openings fitted with pistons, when one piston is pushed in, the other will be forced out. If at the same level, they will be subjected to the same pressure; since they will displace the same volume, work done upon one can be recovered through the other, as in the case of the lever. As the distances are inversely proportional to their sections, they will exercise forces proportional to the latter.

Such is Pascal's principle, upon which he based a description of a machine that he could not make on account of constructional difficulties, but which was made a century later. This machine is the hydraulic press, consisting of two pistons closing unequal openings in the same reservoir. Pressure on the smaller produces a great force on the other. The hydraulic press is much employed in industries where materials have to be strongly compressed, either to press out liquid (brewery grains, etc.), or to reduce the volume for transport (hay, cotton, etc.).

Every time a bicycle tyre is pumped up, Pascal's principle is applied; by means of a moderate force the air is compressed, and supports the rider's weight by the surface of the tyre in contact with the ground.

CHAPTER VI

REST AND MOTION DUE TO FORCES

XXXII.—Virtual and Real Acceleration.

WE are now approaching a very difficult part of our subject, and we must bring our full attention to bear on it if we are to realise the meaning of the experiments we are going to make.

Our first laboratory will be an Elevated Railway train, at a time when it is not too crowded. Without paying attention to what other people may think, we shall walk from one end of one of the cars to the other, and try to analyse carefully our sensations. Between stations, we may hardly be conscious that the train is moving; but as soon as it commences to slow down, we shall observe the curious fact that the floor seems to tilt towards the front of the train, with the result that we imagine we are descending when walking towards the front and ascending towards the back. During a stop the floor apparently becomes horizontal, and, on starting, is inclined in the opposite direction. Thus, the horizontal (or vertical) is the cause of an illusion, and the question is whether the error can be explained by the facts of Mechanics.

An inanimate object, beyond the reach of any hallucination, will give a true answer.

At the risk of surprising the other passengers still more, we shall hang a small ball from the roof of the car, and then burn the thread which supports it. With the train at rest, the ball will mark a point on the floor directly underneath the point of support, and will still fall to this point when the train is travelling at a constant speed. But at the instant of starting the ball will fall behind this point, while at the moment the brakes are applied it will fall in front.

Recollecting that motions are relative, we can explain very simply the meaning of our experiment; if the thread is burnt at the instant the train is put into motion the ball falls vertically; but, as the car has its own apparent vertical, the ball falls, in appearance, in an inclined direction; and it is because the floor of the car is not at right angles to this direction of fall that it appears inclined to the horizontal plane.

The illusion that we have just experienced is of the same kind; but it has taught us a little more than the result already arrived at; our first observation showed that the two accelerations, that of gravity and that of the motion of the train, could combine without a normal fall resulting; even upon us, though we remained motionless in relation to the floor and did not yield to the attraction of the earth, the same combination of accelerations acted.*

We can otherwise demonstrate this deviation from the horizontal plane; it can be done by placing a flat vessel of viscous liquid (glycerine, for example) in the car. During periods of acceleration the surface has an inclination. We have all noticed a similar fact; when a train enters a station in wet weather, the rain-

* The cause of our perception of the vertical has for long been discussed, and it has been set down to the hydrostatic pressures at the extremities of the three semi-circular canals of the internal ear, which, being in three rectangular planes, define a complete system of co-ordinates, in which nature has preceded Descartes by many thousands of years. These pressures are altered according to the inclinations of the head; but the acceleration of the liquid in the canals produces dynamic pressure which may interfere with the sense of the vertical. Many close observations have been made, but the subject is difficult.

water from the roof is discharged forwards; at the departure, it escapes backwards; this is because the water tends to be heaped up in a direction opposite to the motion of the train. Thus, the deviation from the horizontal plane is associated in the simplest manner with the principle of inertia, which affords the explanation of our last observation.

For the sake of shortness, we shall adopt a particular term to indicate an acceleration that would be produced but is prevented; we shall call it *virtual acceleration.* The other is naturally real acceleration. We can now express the result by saying that *real acceleration and virtual acceleration acting simultaneously on a mass produce forces that combine into a single resultant.*

XXXIII.—The Price of Acceleration.

The experiments we have just done show that there is an element in force and acceleration of which we have not yet thought.

Bend a willow twig and fasten its ends to form a bow. Apparently nothing is happening in the bow; and if we did not know that the natural form of a willow twig is not an arc of a circle, we should never imagine that the twig and the string are exerting a force on each other. In reality, gravity being neglected, each end of the twig exercises a force on the string, which opposes an equal force; and, as the string is at rest, the conclusion is that the two forces acting on it neutralise each other, *i.e.*, the forces are equal and opposite. If the string is cut, the forces are no longer opposed; the twig suddenly recovers its shape under the action of internal forces. At the instant the string is cut, the force tending to straighten the twig is equal to the force that the string has just ceased to support. But the force is not destroyed the moment after; it is completely used in giving motion to the twig and restoring its shape. The mass of the twig absorbs the work of the force by acquiring an acceleration.

Similarly, a stone exercises a force on its support. Let the support be withdrawn, and the whole force is spent in giving the stone a constant acceleration (the resistance of the air neglected).

Suppose the support of the stone is a plank held by thread. The thread having been cut, the plank will fall in the same time as the stone; and, since the two bodies have the same acceleration, they will cease to act on each other. If the stone did press on the plank, the latter would be subject to a force greater than its weight, while the stone would be subject to the same force upwards, and would consequently be drawn to the earth with a force less than its weight. Thus the plank would acquire a speed greater than that natural to a body falling under gravity, while the stone would acquire a less speed. They would separate immediately, and the force we have imagined introduced would vanish.

Bodies falling under gravity do not, therefore, press upon each other; and if an arrow always falls point down, the reason is that, on account of the resistance of the air, the feathers do not fall freely.

What becomes of the weight of a falling body? It is not lost, but utilised in accelerating the motion; as each particle of the body requires the action of a force to maintain the constant acceleration of its fall, there is in this acceleration the exact equivalent of the weight of each particle. Here the acceleration is real, and the weight is utilised by the body in augmenting its speed. So long as the body rested on its support, the acceleration was virtual; it was not actually produced, but manifested itself as a *tendency to acceleration*, *i.e.*, as the weight of the body.

The analogy between a body on a support and the twig bent by a string is extremely close. In the latter case, the different parts of the twig and string were acting on each other, by pressure or traction, exactly as the book, the table, the floor, the house, and the ground acted on each other, in order to resist the attraction due to gravity. This action is comparable to the force that the twig exerts.

Rest and motion are two particular cases that occur among an infinite number.

Like the bird from the branch a person could jump from a weighing-machine; the pointer would indicate a weight much above the real, just as if he had fallen on the machine. He could slide the weight along the arm until he had found the place where the pointer just lifted when he gave himself his greatest acceleration. The difference between this instantaneous measurement and his usual weight would indicate the force he had used to raise himself. For a very short time, he would see that he weighed hundreds of pounds.

At the beginning of the descent of a mine lift, the fall is very rapid, and is greatly accelerated; the weight of the miners on the floor of the cage is less than their real weight, and the cage is far from exerting on the cable a force equal to its weight together with that of its load. At the moment of putting on the brakes, on the contrary, the motion decreases, or, what is the same thing, the positive direction of acceleration is upwards. Therefore the passengers exert on the floor a force greater than their weight; and if the stop were affected not by means of the brakes, but by the cable, such a strain would be put upon it that it would probably break.

It is easy to imagine mechanism that would give a cage, during part of its descent, a constant acceleration a, less than that of a fall under gravity. Let m be the mass of a passenger; the force devoted to giving him his acceleration is ma. The force he exerts on the floor is equal to his weight diminished by this accelerating force. It is therefore equal to $m(g-a)$. In this expression a is the real acceleration, $(g—a)$ the virtual acceleration. The sum of these two is always equal to g.

The real weight of a body—the force that it exerts on its support—is that which corresponds to the virtual part of its acceleration. The part of its weight producing an acceleration is not perceptible, and can be called the *virtual weight.* The sum of the real and virtual weight is constant;* its value is that of the real weight of the body on a support at rest or moving at a constant rate.

On the subject of the pendulum we shall meet an experiment that will fix these facts in our minds (§ XL.). It may be here remarked that they are part of a general theory due to d'Alembert, the great encyclopædist whose name is still associated with it.

XXXIV.—Centripetal Acceleration and Centrifugal Force.

We have already seen that a mass to which an acceleration is being imparted resists by exerting a force that is paid for by a loss of velocity. If we begin with a body at rest, the problem is a particular case; but if the body is already in motion, the force may act either along the line of this motion so as to produce an acceleration or a retardation, or it may act at an angle to the line of motion. In the latter case, which is the more general, the force may be regarded as the resultant of two other forces, one of which acts in the direction of the movement (tangential force), while the other is perpendicular to it (radial force). The former produces an *acceleration,* positive or negative, the second a *deviation,* which we have already recognised as an acceleration of direction.

If a ball is fastened to a string and revolved rapidly, after the manner of a sling, it will be forced into a constant deviation from its course; it will receive a *centripetal*

* This does not contradict the fact that the apparent weight of a body may be greater than its real weight; in which case its virtual weight is negative.

acceleration, and will exercise upon the string an outward force the stronger as the ball is turned the more rapidly. This is the force called *centrifugal force*.

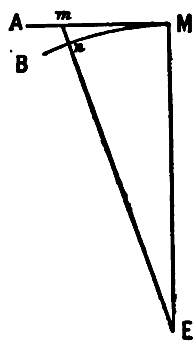


FIG. 34.—How centrifugal force is produced.

We have seen (§ XXVII.) a similar rotation, on a vast scale, in the movement of stars in a planetary system; by resuming this problem, we shall learn how to calculate the value of centrifugal force.

Consider, for example (Fig. 34), the rotation of the moon round the earth. If the moon were free, it would travel away in a straight line MA; but, in reality, its trajectory is inclined towards B. For a very short time, during which the straight lines joining the centre of the moon and the earth may be taken as parallel, the laws of falling may be applied to the motion of the moon; if its acceleration is a, the distance it will travel towards the earth will be $\frac{at^2}{2}$, after a time t. The mass of the moon being m, the force to produce the supposed acceleration is ma.

Now, the theorem of Pythagoras enables us to write

$$\overline{EM}^2 + \overline{Mm}^2 = \overline{Em}^2 = (\overline{EM} + \overline{nm})^2,$$

or, substituting r for EM and v for the velocity of the moon,

$$r^2 + (vt)^2 = \left(r + \frac{at^2}{2}\right)^2.$$

Suppressing r^2, this reduces to $v^2t^2 = rat^2 + \frac{a^2t^4}{2}$.

Since t is extremely short, t^4 is extremely small; it can therefore be neglected, giving

$$v^2t^2 = rat^2, \text{ or } v^2 = ra.$$

Multiplying by m and dividing by r, we have

$$\frac{mv^2}{r} = ma = \text{centrifugal force.*}$$

We have selected as one example from an unlimited number the case of the attraction between the sun and the moon; but we could just as well have taken the case of a string or any limiting influence whatever that gives a definite curve. The body describing the curve would always have exerted against the restraint a force measured by the expression that we have just found.

Obviously a body need not turn in a circle in order to produce centrifugal force; it need only be diverted from its course. Every part of the course can be represented as an arc of a circle, and the radius of this circle gives r of the formula.

Centripetal acceleration is capable of developing almost unlimited force. It is no rare occurrence to read in the newspapers that the *explosion* of a fly-wheel has caused serious damage and even loss of life. The very high speeds at which fly-wheels are frequently run produce centrifugal force sufficient to break the rim away from the spokes and to throw the pieces considerable distances.

Centrifugal force is really the cause of most of the accidents in automobile races. On turnings taken too sharply (with a too small value of r), the car tends to

* Those of our readers who are not familiar with the infinitesimal calculus will be surprised at our rejecting the last term of the equation; they may rest assured that the method is reliable, provided that infinitely small displacements are considered. Then the straight lines converging towards E may be considered as parallel, and, at the same time, terms containing t^2 or higher powers of t may be neglected. Those who have had doubts will be glad to know that the mathematicians of the seventeenth century had the same distrust of the infinitesimal method, and relied on it only when they had verified that it led to results already discovered by other methods.

persist on its way when an attempt is made to turn it; as the front wheels are turned at an angle to the direction in which the car was running up to that moment, the wheels slip sideways and the car is overturned. A safe radius with respect to a certain speed having been found, the corners must not be taken at a greater speed than that indicated by the fact that the radius increases as the square of the speed.

Centrifugal force obliges cyclists and circus horses to lean towards the inside of the track in such a way that the resultant of their weight and radial force may pass through the line joining their centre of gravity and their point of contact with the ground. If this line is too much inclined, the hold upon the ground is insufficient, and a fall results. For this reason, the track is raised at the turnings. Thus, for a certain average speed, cyclists remain upright; the faster riders lean more and the slower less; too slow a speed on the sloping parts of the track causes a fall.

Here again we meet, as a result of the combination of centrifugal force and weight, that deviation from the vertical already noticed in the train; here, however, the deviation is under a new and more intelligible form. While the time of stopping or starting of the train was of short duration, and was at the expense of velocity, centrifugal force can retain indefinitely the same value; it is part of a stable system; the illusion of the vertical that it gives can be made to last as long as may be necessary for full examination.

If we fix a glass half full of water on the end of a vertical rod, we can, by making the rod rotate, see a curious change in the surface of the water, which sinks in the centre and rises at the edges, the change becoming more pronounced as the speed increases. The surface of the water is necessarily perpendicular at every point to the resultant of the centrifugal force and the weight at that point. When the glass has turned for an instant, the angular velocity is the same throughout the liquid; then the surface is a paraboloid of revolution, which is the form generated by a parabola turning on its axis.

Melted wax (Fig. 35) can be used to continue the experiment, and, when it has hardened, a ball can be put upon it and will be under the influence of a force perpendicular to the surface of the wax. The conditions are the same as if the ball rested upon a motionless horizontal surface, for the weight OP and the centrifugal force OC give a resultant OR perpendicular to the surface of the paraboloid. If the speed of rotation of the paraboloid is now increased, the surface of the wax is too open in comparison with the surface required by the ball for equilibrium; the latter is hollow or rounded in comparison with the actual surface produced in the wax. As the speed increases, the value of OC increases, OR consequently tends towards the horizontal, and the ball of necessity moves upwards in order to maintain the direction of OR normal to the surface. As the speed diminishes the opposite is the case; the wax then presents too steep a curve, and the ball falls to the bottom, since the value of OC becomes zero.

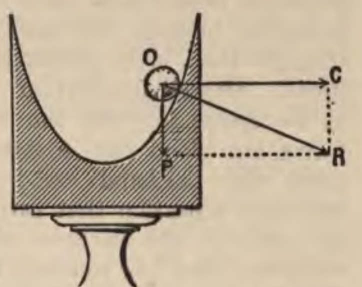


FIG. 35.—Equilibrium of a ball on the surface of a paraboloid turning at a suitable speed.

An insect on the wax could climb upon the sides exactly as if it were on the horizontal; it certainly would not suspect the tremendous significance of its action. We ourselves could do the same in suitable conditions, and we should experience very curious sensations. Some years ago an engineer proposed to construct a vast closed room with a rotating floor in the form of a paraboloid. The paraboloid constituting, for the time, the surface of equilibrium, people could walk anywhere, the walls not excepted, without suspecting their inclination to the vertical outside; but, on looking at each other's inclination, they might imagine themselves magically gifted with the climbing powers of flies.

One result of this experiment would not escape us, even if we walked with our eyes closed; we should feel very heavy; we should lift our feet with difficulty, and we should imagine we had weights in our hands. The reason is that the resultant of centrifugal force and weight is greater than weight alone; every particle of the body is driven towards the surface of the paraboloid more strongly than it is attracted to the earth at ordinary times. Afterwards, certain discomforts would be felt; as the frame supports the solid parts of the body but leaves the blood free, a lightness would be felt in the head, while the hands and feet would become congested, causing a sensation that can be experienced by turning the arms in the manner of a windmill. People who suffer from cold feet would have some compensation; but, if the experiment were prolonged, it would be at the risk of fainting.

Those who use skis are acquainted with this lightness of the head, which is felt when they describe a curve at full speed. Leaning inwards, they cut a deep track in the snow, every part of the body being at the same time pressed towards the feet. Generally a curve ends a run; otherwise, grave discomfort could not fail to be felt.

Centrifugal force is utilised in laboratories and factories to separate bodies of different densities, which are, when rotated, in the same state as if the weight of each particle had been considerably increased. The differences are increased in the same proportion, and the denser parts are rapidly drawn to the edge. In this way serum is separated from the other constituents of the blood, butter from milk, etc. The latter process is so common that advertisements can be seen in German and Swiss villages for "centrifugal butter."

Centrifugal force produces strange effects on bodies under its action. A sheet of paper or a piece of cloth can be rotated and will have a remarkable rigidity. The sheet of paper will serve to saw wood, and the cloth, if struck from the side, will vibrate like metal.

Centrifugal force manifests itself in nature on a grand scale. The Maelstrom, the ill-reputed whirlpool of the Lofoden Islands, is due to this cause. At the rise of the tide the sea rushes between two islands, the currents through the narrow passage meet at an angle and begin to turn round their surface of contact. The result is a depression into which boats may be drawn. When the tide falls the whirlpool subsides, only to begin when the next tide rises.

Further, it was centrifugal force that, after the formation of the solar system, detached the rings that later condensed into planets; to this force we therefore owe the earth, which itself bears the mark of its own rotation It is flattened at the poles and bulged at the equator, because the matter there has been forced outwards. The polar diameter of the earth is about 26 miles less than the equatorial (§ XXVII.).

To many people, centrifugal force seems a mystery, if not of effects, at least of causes; they think it is a special force, distinct from all others, and requiring special knowledge to be understood. We now know that it is nothing of the sort; centrifugal force is the necessary and immediate consequence of centripetal acceleration, or, more generally, of *acceleration of direction*; it is an equally necessary consequence of inertia—that property of matter that can be defined by saying that masses modify their motion only under the action of forces (§ XIII.).

XXXV.—Central or Eccentric Percussion; Reciprocal Centres.

A billiard ball struck at the centre rolls forward; at the top it moves rapidly; at the bottom it has a tendency to drag; if the movement of the ball is watched, it will be seen that the ball, when pushed forward, receives at the same time a rotation, as if the parts more remote from the point of contact had a tendency to keep the ball in place.

We now know enough to enable us to explain this fact; acceleration given to a mass develops a necessary and equal reaction; all these reactions, united in a body which is solid by reason of the cohesion of its particles, are composed either into a single force or a couple.

Suppose, first, that equal and parallel accelerations are communicated to all the particles; a series of parallel forces will result, each proportional to the mass of the particle, and similar to those produced by the *virtual* acceleration of gravity. They can be composed in a similar way, *i.e.*, into a resultant passing through the centre of gravity of the body.

Conversely, a blow at the centre of gravity of a body communicates accelerations of the same value to all the particles, on account of their solidity; the body travels backwards parallel to itself throughout, without any rotation. If it is struck eccentrically, it cannot move parallel to itself. Suppose it does; then its motion will develop a force passing through its centre of gravity. This, in conjunction with the force of the blow, will constitute a couple which will cause the body to rotate.

Thus arises a singular relation between a state of rest and of motion. The centre of gravity, which is already familiar as the centre of the forces of weight, is found to be identical with the centre of reaction in a body to which a real acceleration is being given; the explanation why these points coincide in a body is to be sought in the intimate relation existing between real and virtual acceleration. The identity of the two centres has a significance that is not enough insisted upon.

The further a body is struck from its centre of gravity, the more it rotates during its backward movement. In the immediate neighbourhood of the point struck the backward motion is considerable; in consequence of the rotation attending the motion of translation, parts can be found at some distance from the centre of gravity that have a greater motion backwards than that due to the motion of translation alone; but nearer the centre of gravity smaller displacements are to be found, and regions can be discovered where the motion of rotation neutralises or even exceeds that of translation; these parts, when the body is struck, will move forward.

All these movements could be easily studied if weight could be eliminated; but the experiment can be carried out by striking a floating stick horizontally at various points. A stick might also be suspended by a string and struck first at the lower end and then gradually higher. The stick would be seen to be driven back or to turn on itself.

This experiment, if systematically carried out, will disclose an interesting relation; the stick being hung by a string from any point A and the blow applied higher and higher, a point B will at length be found such that a blow there does not move the string either way—the point where the motion backwards is exactly compensated by rotation. The points A and B are reciprocal in the sense that if B is struck A remains motionless.

Some knowledge of the behaviour of bodies subjected to shock is useful to those whose work consists of striking—smiths and all other men that use the hammer, which receives a shock and rebounds the moment it strikes the metal. If the smith grasped the shaft near the head, his hand would receive a painful jar; but he grasps it near the end, where it remains almost still. Again, each smith between very narrow limits. It is the task of the fly-wheel to give the motor the necessary regularity; it absorbs the superfluous power by increasing its speed slightly, and restores it by losing speed.

On the other hand, fly-wheels are equally useful when the rate of consuming work is variable. A circular saw, for instance, consumes a large amount of work while it is working; the descending note that its teeth produce as they strike the plank shows that the impacts grow gradually less frequent; then, while the saw is running free, the motor restores the lost speed to the fly-wheel, to prepare it for the next plank.

Similarly, whenever an electric train or car starts, a heavy demand is made upon the current that drives the motors. Again it is the fly-wheel at the generating station that loses the energy necessary to accelerate the train; it afterwards recovers it during the interval between the stations.

Let us consider a fly-wheel formed by a ring of infinite thinness. When it rotates, its kinetic energy is $mv^2/2$; but its peripheral velocity is equal to the product of its radius and its angular velocity ($r\omega$). Its energy is therefore given by $\frac{mr^2\omega^2}{2}$. But mr^2 is its moment of inertia; we conclude, then, that the *kinetic energy of a fly-wheel is equal to half the product of its moment of inertia and the square of its angular velocity.*

The elementary ideas of Mechanics are so thoroughly a part of our mode of thought that we commonly think of a *regulator* in any connection whatever as a fly-wheel. In factories certain orders, whose delivery can be delayed, constitute a *fly-wheel of work*; they are resumed when work is scarce and left when urgent orders are numerous. Exactly like fly-wheels, they prevent either stoppage or overwork.

CHAPTER VII

OSCILLATORY MOTION

XXXVII.—Periodic Forces and Oscillations.

In nature, as in the mechanical world, motion backward and forward is of extreme importance; between the wing of a fly, with its distinctive note, and the rise and fall of the tide, are ranged an immense number of moving bodies that come and go—that *oscillate* between two limits and never exceed them. A tuning-fork and a vibrating string, like the air that transmits the sound, have a motion that is vibration; similarly, the pendulum of a clock, the balance of a watch, a ship swinging at anchor, the waves themselves, all move to right and left or up and down, though always tending to a mean position through which they continually pass.

This mean position is, in general, the position of stable equilibrium of the moving body—the position in which it would rest if an external cause did not force it to change; but as soon as it is moved, forces come into existence and increase in proportion as the body is still further moved from its position of equilibrium, towards which these forces tend to restore it. An acceleration results—the speed increases until the body passes its stable position; when it is once past, opposing forces retard its progress; and when all the kinetic energy is exhausted by the action of the opposing forces, the body stops and returns. The motion would continue indefinitely if the opposing forces were always of the same value at the same points of the path, for there would be a constant exchange of work and kinetic energy.

We know that in practice this is not the case; a tuning-fork gives a note that gradually sinks into silence; the pendulum of a clock that has not been wound up stops after minutes or hours, according to the perfection of its construction, as a result of the forces of friction that are always and everywhere consuming work.

XXXVIII.—Representation of Oscillatory Motion.

With a little attention, we can form an accurate idea of the conditions in which oscillatory motion takes place.

Let us first obtain a figure. In the chapter devoted to motion we have examined the apparent motion of a light turning in a circle (§ XII.). Let us continue this in greater detail.

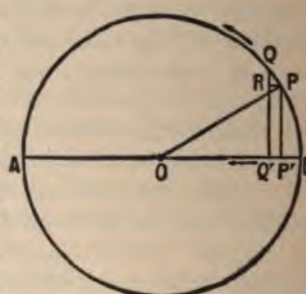


FIG. 36.—The projection of rotation gives the motion of oscillation.

Having laid a bicycle on a table, we can attach a piece of paper to one of the wheels; then, while it is rotating, we can sit some distance away, so as to see the wheel in elevation. Closing one eye in order to remove the sensation of relief, and concentrating our attention on the paper, we see it moving only from right to left and from left to right—rapidly in the middle, and slowly towards each end of its course. We can imagine that the paper is not turning in a circle, but that it is displaced along a straight line at right angles to our line of sight.

If we now set down what we have just seen (Fig. 36), we shall notice that when the paper is at P we imagine it is at P′, where our line of sight intersects the diameter perpendicular to it. When the paper goes from P to Q, we follow its motion from P′ to Q′. Its speed is reduced, for us, in the ratio $\frac{P'Q'}{PQ} = \frac{PR}{PQ}$.

Now, if the arc PQ is very small, it may be regarded as a straight line. The triangles PQR and POP′ being similar, the speed is reduced in the ratio of PP′ to the radius of the circle. Therefore, if the motion of the point is represented by the displacement of P′, its velocity is given in an appropriate scale, by its distance from the diameter AB.

But the apparent velocity constantly changes. From P to Q it varies by the quantity RQ. The similarity of the triangles already considered shows that this variation in velocity, which is the apparent acceleration of the point P, bears the ratio to the motion of $\frac{RQ}{PQ} = \frac{OP'}{OP}$.

If we take OB as the measure of the greatest acceleration, the distance OP′ always represents the value of the acceleration of the motion at the point P′.

We have to remark on a convention as to the signs of the quantities; we shall observe the general usage, and count as positive all distances to the right of O and negative all those to the left. Further, velocities and accelerations are positive when they go towards the right. In the particular case we are considering, the velocity and the acceleration are therefore negative at the place where P′ is, the direction of the motion of P being given. It is well that this should be thoroughly understood.

XXXIX.—Laws of Oscillatory Motion.

We have now studied the kinetic conditions of the problem. We are going to consider it in a dynamic sense.

Suppose a mass, which is under the action of no force when at the point O (Fig. 37), is, as soon as it is displaced, brought back to this point by a force proportional to its displacement. We can represent this force at any point E by the distance ED from the straight line OC to the straight line AB in which the point under consideration is moving.

Displace the mass as far as B and release it without initial velocity. It will be brought back towards O, and, at every point in its path, its acceleration will be proportional to the ordinate of the line OC; or, from the similarity of triangles, proportional to its distance from the point O.

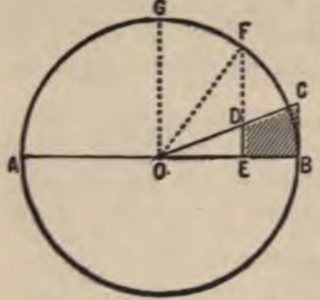


FIG. 37.—A mass under the action of a force restoring it to its position describes an oscillation.

It is easy to calculate the velocity of the mass at every point.

The work absorbed by the mass m being the product of the force and the distance traversed, the motion from B to E absorbs work equal in amount to the sum of the products of a series of small displacements and a series of forces represented by the ordinates of the trapezium BEDC, or, in total, by the surface s of this trapezium. Put OB $= r$, OE $= x$, BC $= k$. We have first, ED $= k\frac{x}{r}$, then

$$s = \text{OBC} - \text{OED} = \tfrac{1}{2}\,rk - \tfrac{1}{2}\,xk\frac{x}{r}.$$

If we put $\frac{k}{r} = a$ we have

$$s = \frac{a}{2}(r^2 - x^2).$$

This area s is equal (§ XXI.) to $\frac{mv^2}{2}$, that is, to the kinetic energy of mass m, whose velocity at E is v. Therefore we have the equation

$$\frac{mv^2}{2} = \frac{a}{2}(r^2 - x^2),$$

whence $$v = \sqrt{\frac{a}{m}(r^2 - x^2)}.$$

Describe a circle on AB as diameter. On a suitable scale of velocities, *e.g.*, if the greatest velocity of the oscillating point is represented by OG, the velocity at E is given by EF.

Thus we arrive at all the results of the preceding diagram, and the complete analogy enables us to state the following theorem, which is of great importance in the theory of oscillation :—*If a mass is brought back to its position of equilibrium by a force proportional to its distance from this position, the position of the mass is represented by the projection on the line of action of the force of a uniformly moving point which describes a circle on the greatest elongation of the motion of the mass ; the velocity is represented by the height of this point above the line of action of the force ; its acceleration is represented by the distance of its projection from the point of equilibrium.*

A fundamental relation remains.

The energy stored by the moving mass at the moment it passes its position of equilibrium is represented by the area of the triangle OBC ; it is therefore proportional to the square of OB. This energy being also proportional to the square of the velocity of the mass at O, this velocity is proportional to OB. If the mass is displaced only as far as E before release, it will oscillate in such a way that the uniformly moving point, whose projection it is, will describe, on the same scale as before, a circle of radius OE. The velocity being proportional to the radius, the period of revolution is always the same. This important fact is the result : *When a mass is drawn to its position of equilibrium by a force proportional to its distance from that position, the period of oscillation is independent of the amplitude of its motion.* This is the law of isochronism of oscillatory motion, discovered by Galileo from observation of the swinging of a candelabrum by Benvenuto Cellini in the Cathedral of Pisa (§ VI.).

We have yet to determine another relation, after which we shall abandon mathematics. The energy of oscillation is the energy of the mass at the instant of passing through its position of equilibrium. It is proportional to the square of the velocity; but we have seen that this velocity is itself proportional to the amplitude; obviously it is also inversely proportional to the period of oscillation. This quantity (the reciprocal of the period of oscillation) is called the *frequency;* it is equal to the number of complete oscillations (going and returning) in the unit of time. Our conclusion is that the *energy of oscillation is proportional to the square of the amplitude and, at the same time, to the square of the frequency.*

Let us examine the formula that expresses the velocity of the mass in motion. When this mass is at its position of equilibrium, its point of reference (P, Fig. 36) is at its highest position and is being displaced in a direction parallel to that of the mass; therefore both have the same velocity. Since, at this instant, $x = 0$, the velocity is given by

$$v = r\sqrt{\frac{a}{m}}.$$

This is the velocity of the point of reference at any instant. Now, the length of the circumference being $2\pi r$, the time required to describe it is given by

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{m}{a}}.$$

But $a = \frac{k}{r}$, being the rate of increase of the force when the point is displaced from O, or, numerically, the value of the force at a distance from O equal to unity.

This is a formula of general application; an identical formula would have been found for circular oscillation; m would then have been the moment of inertia of the fly-wheel (§ XXXVI.), a would have been the rate of variation of the couple or the moment of the couple for an angle equal to unity—a revolution divided by 2π.

In many ways we can imagine a mass restored to its position of rest by a force proportional to its distance from this position; for example, we can imagine it held between two springs which pull and push it alternately. In illustration of rotation, if we suspend a metal disc by a wire through its centre, and release it after having rotated it about the axis of the wire, it will describe isochronous oscillations.

Perhaps we shall be enabled to understand better the artifice of the moving point of reference projected upon the line of action of the force when we have applied it to an actual case, even though the performance of the experiment is beyond our power.

Newton demonstrated that the attraction of a homogeneous sphere on an external point acts as if the matter in the sphere were concentrated at its centre, while a hollow sphere has no action on an internal point. If we could make a shaft along a diameter of the earth, we should experience a decrease in weight proportional to the depth of the shaft, for attraction would be exerted by a proportionally smaller sphere concentric with the earth. Now, the mass of this portion of the earth would be proportional to the cube of its radius, while its attraction would be inversely proportional to the square of its radius; therefore the attraction would be proportional to the radius itself. A mass allowed to fall into the shaft would be under an attraction proportional to its distance from the centre; if the shaft were perfectly empty, so that the mass would not experience any resistance to its movement, it would make isochronous oscillations indefinitely. If, now, at the time the mass was released, a little moon were sent horizontally from the earth, with such a speed that its centrifugal force exactly compensated for its weight, it would revolve (in space, that is) in such a way as to be at any moment the point of reference of the motion of the mass falling in the shaft. Certain formulæ given above enable this curious relation to be demonstrated.

Oscillatory motion is not always so simple as we have assumed. The movement of a particle of air transmitting a sound, that of the membrane of a telephone, or that of the ear-drum, can seldom be represented by a point describing a circle at a uniform velocity. If it were so, we should hear sounds all of one kind, whereas they are all of different pitch. Without insistence on these facts (which belong rather to the domain of physics), it is sufficient to say that, however complicated the oscillatory motion, it can always be represented by the following arrangement.

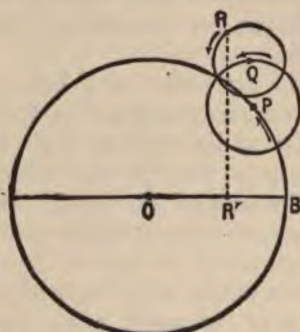


FIG. 38.—The most complicated oscillations can be represented by the motion of a tooth of a gear-wheel.

The moving point of reference P of the first motion (Fig. 38) is replaced by the centre of a circle that another point Q describes in half the time ; this point in turn becomes the centre of a third circle described in a third of the time of the first, and so on. As many circles are taken as are required to represent the motion to be examined. (The study of alternating currents necessitates as many as seventeen.) The projection R′ of the point R describing the last circle is the point of reference required.

These different circles can be compared with cog-wheels geared to each other. They are numbered from the second onwards and are called the harmonics of the motion of the first ; their radii are the amplitudes of the different harmonics.

This analysis of oscillatory motion is due to the great geometrician Fourier ; it is used in numberless scientific and technical problems. It furnished the solution to the problem of pitch, the theory of which, propounded as early as 1700 by Sauveur, remained incomplete until taken up by Helmholtz.

XL.—The Pendulum and Weight.

When the force is not proportional to the displacement, isochronism of small oscillations does not result; large oscillations are of shorter duration than small oscillations if the force increases more rapidly than the displacement, and of longer duration if the force increases more slowly. These facts are evident, for the straight line OC (Fig. 37) is replaced by a curve inclined from or towards the line OB according as the ordinates of OC (the forces) increase or decrease disproportionately to the displacement from O.

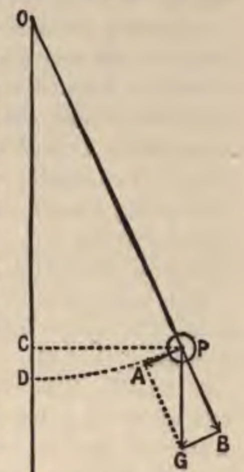


FIG. 39.—The force that brings the pendulum to its position of equilibrium increases more slowly than the distance moved.

In the case of the pendulum, the weight PG (Fig. 39) of the mass can be resolved into two forces, one of which (PB) is in the direction of the support, and does not act on the mass, while the other (PA) is perpendicular to it, and this alone acts on the mass. The triangles PAG and PCO being similar, the force is proportional to CP, *i.e.*, to the displacement of the mass P from the vertical. But the distance it travels, measured by the arc of a circle, increases more rapidly than this displacement; conversely the force increases less rapidly than the distance; the result is that large oscillations are slower than small. If the amplitude remains within narrow limits, the arc is approximately equal to its chord, and the oscillations are nearly of the same duration. This is the approximate principle of the *isochronism of small oscillations.*

By the above proceeding we have introduced into the explanation of the motion of the pendulum a simplification that is admissible if the pendulum consists of a concentrated mass suspended by a string of negligible mass. But the problem is really slightly different; as the mass travels in the arc of a circle whose centre is the point of suspension, its moment of inertia round this point is what ought to be considered; and the motion is produced by a couple formed by the weight of the oscillating mass and the equal reaction of the support. The problem necessarily appears under this form when, instead of a simple pendulum, we deal with a mass of any shape oscillating round an axis. For instance, the beam of a balance or the arm of a metronome are far from resembling a simple pendulum.

If a balance is to be sensitive, that is, if it is to show a great inclination for very small differences in weights, it is necessary that the couple acting on the beam should be very small. The moment of this couple is equal to the weight of the beam multiplied by the distance of its centre of gravity from the knife edge on which it rests. If this distance is small, the couple brings the balance very gently to its position of equilibrium, and the oscillation is of long duration.

Apart from its clock-work, a metronome consists of a mass whose weight, together with the spring, restores it to its position of equilibrium. The tendency in this direction is due to a piece of metal suspended under the axis of rotation. But another mass can be moved along an arm situated above the axis, in such a way that, when raised, it increases the moment of inertia of the system by bringing its centre of gravity nearer the axis. These two actions together increase the period of oscillation.

It is necessary to be perfectly clear as to the causes of the oscillation of a pendulum. Approaching the subject as we have done, we have been able to separate the two factors of oscillation: (1) the force or couple acting, and (2) the reacting force or couple of the matter in motion. When the force is provided by an alternately compressed and extended spring or when the couple is provided by a twisted thread, no confusion arises as to the work of the two factors. But when weight is involved, as in the pendulum, the action of the weight, which provides the moving force, and of the mass, which gives rise to the reaction, may easily be confused. As the mass is involved in two ways, first to furnish the weight, and then the reaction, it (m) occurs with the length (l) both in the numerator and denominator of the formula—

$$T = 2\pi\sqrt{\frac{ml^2}{mlg}},$$

in which the quantity under the radical contains the moment of inertia divided by the value of the couple; on simplifying, we are left with the length of the pendulum (the distance between the forces of the couple), and the weight. The period of oscillation of the simple pendulum is then given by

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

This relation is exact in consequence of the perfect proportion between weight and mass; but we have seen that this is purely experimental, and does not follow of necessity. We must never forget that the oscillation of the pendulum is regulated by the acting couple, equal to the product of its weight and the distance of its centre of gravity from the axis of rotation, and by the reacting couple, whose constant factor is its moment of inertia.

It is now evident that if the length and time of oscillation of a simple pendulum are known, a simple calculation gives the value of g. The pendulum is, in fact, used to determine weight over the surface of the globe. Formerly its value was found by means of an instrument closely resembling the simple pendulum; to-day a pendulum of more complicated appearance is used, but it makes easy and accurate operations possible.

When acceleration changes from virtual to real, the body has no longer any weight (§ XXXIII.); if, therefore, an oscillating pendulum is allowed to fall, g ceases to exist, and T becomes infinite—in other words, the pendulum turns indefinitely about its axis, unless it is let fall at the end of its swing; in which case it remains in this inclined position.*

XLI.—Resonance.

When we put a swing into motion, we avoid sending it as far as it will go in one push. We give the first push and wait for it to return; then a second push, which increases both its speed and amplitude, and so on; thus, avoiding great exertion, we take advantage of the motion itself to obtain as great an increase as we wish.

This is an instance of a common action to which the name *resonance* has been given. It is always produced when an oscillation is maintained by a force of the same period.

Resonance is of extreme importance; it enables us to see and hear, because vibrations are in accord with certain elements (partly molecular and partly mechanical) of our eyes and ears. As a result of resonance, the tide

* This question has troubled many who have never thought of the difference between the effects of real and virtual acceleration. They have not, of course, watched a clock fall from a height; but, on thinking of the moon, they must have often wondered whether it affects clocks. Obviously it would, if the axis of suspension of a pendulum occupied a fixed position in relation to the sun and the moon. But the earth is constantly falling towards these two stars, as towards every star of the universe, with an acceleration it would acquire if all the matter in it were collected at its centre. The action of the stars on the pendulum is therefore equal only to the difference between the action at the centre of the earth and at its surface; it is very small, and is within the limits of error of the best clocks.

attains the great amplitude known in certain oceans, the Atlantic in particular; 14 or 15 metres instead of the $\frac{1}{2}$ metre that it would have if the oscillation had to be generated anew each time; thus the ocean is like a swing that is rhythmically pushed. By means of resonance the heaviest bells are rung, and the greatest trees broken down after the bottom has been cut. Similarly, when a suspension bridge is caused to vibrate at its appropriate period, the amplitude of its motion may be so increased that the bridge may be broken. This was the case with the celebrated bridge at Angers in France, which gave way under the measured step of a regiment, and took with it all the soldiers, many of whom were drowned in the Loire.

Ships are so built as to avoid giving the engines the same period of oscillation as the hull, which would otherwise develop a dangerous vibration. The French cruiser *Jeanne d'Arc* vibrated to such a degree that the construction had to be altered.

In the same way, when a vessel cuts the waves so as to reach the successive crests at intervals equal to its period of pitching, the motion increases to an alarming extent. Many vessels have foundered in this way in a comparatively calm but very regular sea; danger could have been avoided by changing the speed of the vessel; after a short distance, the waves would have struck it at intervals that did not coincide with its pitching, and would have caused a gradual decrease and a short cessation of the motion, due to the fact that the waves would no longer have acted in a way to maintain the motion, but would have destroyed what they had generated.

This cross-beat is the result of imperfect resonance; it restricts the amplitude and gradually brings the vibrating body to rest. But, in order to avoid the effects of resonance completely, it is necessary to avoid even a cross-beat, especially if it is slow, since a slight change in the cause of vibration or in the vibrating body is sufficient to produce real resonance. It is wiser to break up the impulses whenever possible; for this reason the order is always given to troops passing over a suspension bridge to break step and to walk like a flock of sheep. It is a wise precaution, suggested by the catastrophe at Angers.

The effects of resonance would be disastrous more often, if a closely associated action did not always interfere and restrict it; this is the limiting action of friction and all the other causes that consume kinetic energy. The consumption increases with the amplitude of the motion, and finally equals the energy of the impulse; at this stage the oscillation has reached its maximum value. The difficulty is to determine whether this value is below that of the dangerous amplitude; if not, the limiting action must be increased, or the period of oscillation either of the moving body or of the exciting cause must be altered. This process is well known to engineers, who frequently make use of it.

The tenor Tamburini used to cause a cut glass vessel to break in pieces, by giving loudly the note corresponding with its natural vibration. The vibration, stimulated by the sound waves, gradually increased and exceeded the limit of deformation that cut glass can bear. It would have been enough to touch the rim with a feather to prevent the vibration from reaching the dangerous limit, and even to restrict it to a very small amplitude.

Thus, in turn, we utilise or avoid resonance. The commonest example of sustained resonance is furnished by the regulator of clocks and watches; the shock induced by the oscillating part itself restores to it the energy that it has expended in overcoming the resistance of the air and in moving the wheels.

THIRD PART

Development and Application

Although, in our rapid survey of the principles of Mechanics, we have always had every-day occurrences to impress upon us the reality of these principles, we have regarded them so far in a somewhat limited way, in consequence of our having selected for special attention only certain aspects of the facts under examination. We have now to undertake more complex problems in which various principles contribute to produce the effect we are studying. Thus we shall grasp more fully their co-ordination and relation, and after solving some problems we shall be better prepared to apply the knowledge we have gained. These problems will no longer be of a general kind; they will have reference to particular cases and will give numerical results. We must first, therefore, become acquainted with the units in which all the quantities that come within the domain of Mechanics are expressed.

CHAPTER VIII

CALCULATIONS IN MECHANICS

XLII.—Qualitative and Quantitative Relations.

BEFORE beginning the calculations involved in Mechanics, we must make a rapid summary, and review the various quantities concerned, so as to discern what may be called their family relations, and to establish their genealogy; first, however, we must notice an obvious principle.

If, after having put 100 apples into a basket, we found only 99, we should not be greatly surprised; we should assume some one had taken one out. But if we found 100 pears, we should want an explanation of this curious fact. We should not for a moment admit that the apples had changed into pears, but we should be certain the apples had been taken out and replaced by pears.

The first change would have been a question of quantity, the second of *quality*; the first a question of *number*, the second of *kind*. There is no need to emphasise the fact that the second change is more fundamental than the first. True, the contents of the basket have been modified in each case; but they have been modified little in the first, and much in the second case, although the number has remained the same.

If we substitute "equation" for "basket" and "forces" for "apples," we approach a mechanical problem. The basket contains apples or pears, just as the equation contains forces or pressures or any other magnitudes.

We now understand that if we put forces into an equation, we cannot obtain pressure as a result; further, that equality can exist only between quantities of the same kind, and before troubling about the numerical value of the quantities we must make sure of their nature. In other words, an equation, before being verified numerically (quantitatively), must be verified as to kind (qualitatively).

It would hardly be necessary to insist on such a simple precaution if it were not a fact that it is often ignored even by advanced students.

Yet the different units are not isolated magnitudes; they are all related, as their definitions show. Displacing a force in its own line of application, we have work; distributing work over time, we have power; spreading a force on a surface, we have pressure; and we are able to say that work is force multiplied by displacement, power is work divided by time, and pressure is force divided by surface.

There is a common belief that such relations are strictly numerical; in which case, to say that pressure is force divided by surface would simply signify this: the *numerical value of a pressure* is equal to the *numerical value of a force* divided by the *numerical value of a surface.* But this is a too restricted mode of considering the relations; the origin of pressure is actually the distribution of a force on a surface, that is, *the magnitude called force divided by the magnitude called surface;* and we shall soon see that equations established on this understanding lead to the discovery of similarities in the nature of the magnitudes.

If all the relations that we have established are utilised, all the magnitudes in Mechanics can be allocated to three categories—length, mass, and time, which are of a simple character, and, retained as fundamental magnitudes, enable us to derive all the others.

It has been agreed to indicate by means of the notation when the nature and not the numerical value of a quantity is concerned; for this purpose a notation consisting of capital letters in brackets has been created; the equations in which such symbols are found are therefore qualitative, not quantitative.

These equations show the origin of the various statements and how they are related. It is valuable to be able to recognise them, and therefore the relations between the magnitudes treated of in Mechanics are given here.

Fundamental Magnitudes.

Length [L]. Mass [M]. Time [T].

Derived Magnitudes.

Surface	= length	× length	= $[L^2]$
Volume	= surface	× length	= $[L^3]$
Velocity	= length	÷ time	= $[LT^{-1}]$
Acceleration	= velocity	÷ time	= $[LT^{-2}]$
Force	= mass	× acceleration	= $[MLT^{-2}]$
Pressure	= force	÷ surface	= $[ML^{-1}T^{-2}]$
Work	= force	× length	= $[ML^2T^{-2}]$
Kinetic energy	= mass	× velocity²	= $[ML^2T^{-2}]$
Power	= work	÷ time	= $[ML^2T^{-3}]$
Impulse	= force	× time	= $[MLT^{-1}]$
Momentum	= mass	× velocity	= $[MLT^{-1}]$

Reference to these symbols shows at once that work and kinetic energy, although of different origin, are quantities of the same kind; the same is true for impulse and momentum. Thus qualitative relations disclose the relations between the magnitudes, and prevent those that differ from being confused.*

* Persons who ignore the existence of qualitative equations naturally fall into confusion; they tolerate the term "live force," and they can often be heard to say "pressure" when *they mean* "force"; some of them even go so far as to assert

The symbols may be associated in other ways, thereby producing other relations. One example will suffice; let us multiply the symbol for pressure [$ML^{-1} T^{-2}$] by that for volume [L^3]. We shall have [$ML^2 T^{-2}$], which is the symbol for work; and this operation shows that when a volume under pressure is increased, work is performed. Elsewhere we have arrived at the same conclusion by careful reasoning.

XLIII.—Units in Mechanics.

For every magnitude in Mechanics we could choose any unit whatever; but we should then prevent ourselves from taking advantage of the simplifications that result from the inter-relations that we have just determined. Since only three fundamental magnitudes are required, their three units will enable any quantity in Mechanics to be measured.

For scientific purposes, the numerous units in use a century ago have given way to a single system, at once simple, rational, and possessed of such qualities as to render it superior to any other. Our heterogeneous English measures—best characterised as the absence of system—will doubtless in time be replaced by the metric system. We can therefore ignore them without loss.

The unit of length is the *metre* and the unit of mass the *kilogramme*. Their originals are deposited at the International Bureau of Weights and Measures in Paris. The unit of time is the fraction $1/24 \times 60 \times 60$, or 1/86,400 of the mean solar day; it is called *the second of mean time*. The standard is given by the rotation of the

that it is pedantic to seek rigid terminology in this domain; yet these same persons would be surprised if they were served with oranges when they ordered apples; the error would nevertheless be more pardonable than a similar confusion of scientific terms. Let our readers accustom themselves to accuracy of language; the effort will cost them but little trouble and will obviate many misunderstandings.

earth. Thus the standards of measurement are within the reach of every one.

In order to measure very great or very small quantities, multiples or sub-multiples of the fundamental units can be used. Thus physicists, who often have to measure exceedingly small quantities, have selected the centimetre as unit of length and the gramme as unit of mass, while retaining the second as unit of time. Their unit of velocity is therefore the velocity of a body that travels 1 centimetre per second. For calculations in Mechanics, metres per second are more convenient. When transportation is concerned, velocity is expressed in kilometres or miles per hour; but for the purposes of Mechanics the second is taken as the unit of time.

Change of velocity is acceleration. To establish a unit, we shall consider a train travelling on the level with a regularly increasing rate; we shall then suppose that, at the end of each of the first two seconds, the train loses its last car, and that these two cars continue to advance on their way. Now we can express the rate of these cars in metres per second. The difference of these rates will also be in metres per second; since this difference has been produced in a second, the acceleration is expressed in metres per second per second. It can, of course, be expressed in centimetres per second per second.

A force is always the product of a mass by an acceleration; if we work out the force that gives a mass of 1 gramme an acceleration of 1 centimetre per second per second, *i.e.*, if we multiply 1 gramme by the value of this acceleration, we obtain a unit of force that is called a *dyne*. Replacing this acceleration by one that has the metre and kilogramme as units, we have a force 100,000 times greater than the dyne. But this unit of force is seldom used; a unit ten times greater (a million dynes or a *megadyne*) is preferred; we shall soon see the reason.

We happen to live upon a planet where the acceleration *of a body falling* under gravity is about 9·81 metres per second per second.* The virtual acceleration of a body at rest generates a force equal to the mass multiplied by the value of this acceleration. The force of a mass of 1 kilogramme on its support is therefore equal to 1 kilogramme multiplied by 9·81 metres per second per second, or 9·81 m/sec^2.

The ease with which a standard of force of this value can be determined has led to its being frequently taken as a unit under the name of *kilogramme-force*. In the choice of a rational unit (a simple number) it is of importance to preserve the relations between the common units. The substitution of 10 for 9·81 in the expression for acceleration alters it only by 2 per cent. In many of its applications the megadyne can be substituted for the weight of 1 kilogramme. Similarly, the dyne can be used for the weight of 1 milligramme.

The displacement of a unit of force over a unit of length gives the unit of work. This is, for physicists, the dyne-centimetre, called the *erg*; it is, in Mechanics, the megadyne displaced 1 decimetre, and is called the *joule*; or the kilogramme-force displaced 1 metre, giving rise to the *kilogrammetre*.

Finally the unit of power is the joule per second, to which the name *watt* has been given, or a unit 1,000 times greater—the *kilowatt*.

Power multiplied by its duration gives a unit of work; thus the *kilowatt-hour* is the work done by 1 kilowatt in 1 hour.†

As for the unit of pressure, 1 megadyne on 1 square centimetre is a *megabar*; this is very nearly the pressure of a column of mercury 75 centimetres high at 0° Centigrade, and normal atmospheric pressure. More than a

* Acceleration varies, as we have seen (§ XXVII.), at different places. The value adopted is that at the sea-level at latitude 45°; it is taken as 9·80665 m/sec^2.

† In France the *poncelet* (100 kilogrammetres per second) is replacing the old unit "horse-power" (75 kilogrammetres per second).

century ago, Laplace proposed to use as the unit of pressure the mean pressure of the atmosphere at sea-level; this is called an *atmosphere*, and is defined by a column of mercury 76 centimetres high. The distribution of 1 kilogramme-force on 1 square centimetre gives a third unit of pressure which is much used in industry.

The numerical relations of the dynamic units are summarised below.

Force:	1 kilogramme-force	= 0·981 megadyne.
Work:	1 kilogrammetre	= 9·81 joules.
	1 joule	= 1 megadyne-decimetre.
Power:	1 poncelet	= 100 kilogrammetres per second.
		= 0·981 kilowatt.
Pressure:	1 kilogramme-force/cm^2	= 0·981 megabar.
	1 atmosphere	= 1·013 megabar.

CHAPTER IX

IMPACT

XLIV.—Definition of Impact.

: shall now consider a phenomenon that for a long remained very mysterious on account of its com-y and difficulty. Involving obscure properties of -r, the principle of impact naturally escaped the vation of early students of Mechanics, and its investi-n was possible only to workers equipped with the perfect apparatus. It may even be said that to-day ie features of impact are not known because of their me diversity; but they are well enough known to y the curiosity of our young friends.

ien two billiard balls are driven towards each other meet with a sharp sound, then separate and run in rent directions. A rubber ball thrown against a wall inds and falls into the hand of the thrower; on the r hand, a ball of clay is flattened and falls at the foot ie wall, or adheres to the wall as an ugly brown spot. hammer, on striking an anvil, rebounds; but if a of lead is interposed, the hammer emits a dull sound does not rebound. Smiths know well when to cease mering iron or steel. When the metal is at red heat, iammer sinks into it without any sign of a rebound; as the cooling proceeds, the sound of the blow becomes allic, and the hammer no longer falls so dully.

iere are therefore many kinds of impact that can be nged between two extremes called *elastic impact* and *istic impact.* In reality, all impacts are more or less tic, and have the common property of deforming ter and developing reactions in it.

XLV.—Representation of Elastic Impact.

If we desired to render the examination of impact easy, we might arrange two cars to run on rails. One car would have a buffer, but it would be a very peculiar sort of buffer. It would be composed of a long spiral spring wound on a rod (Fig. 40), and a tube, carried by the other car, to press against the spring. Each car would be provided with a pointer for the purpose of tracing its position on a sheet of paper moving perpendicularly to the rails; the records on the paper would show us in miniature what we could see on a large scale every time a railway car strikes another at rest and transfers

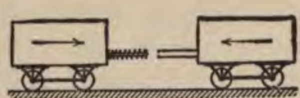


FIG. 40.

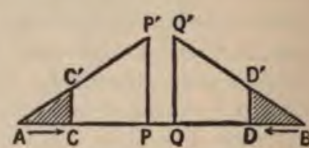


FIG. 41.

A spring forces apart two cars running towards each other. The diagram of the forces acting explains the variations of velocity.

to it part of its velocity. We already know enough Mechanics to be able almost to guess what would happen.

Let the diagram (Fig. 41) represent the value of the force acting on each car from the instant the tube comes into contact with the spring. This force gradually increases and does work in reducing the kinetic energy of the cars. The work done when the moving bodies have reached C and D is represented by the area of the triangles ACC′ and BDD′. A moment comes when the whole of the kinetic energy is consumed, *i.e.*, when the two cars are at rest. But the spring is then compressed and is forcing the cars apart; they separate, the spring expands, and contact ceases. All the forces that acted on the cars when they approached are reproduced in inverse order; the same work is effected by the spring, and the cars, separating with their original velocity, regain their kinetic energy.

The action is just as if the cars began at A and B to ascend slopes of gradually increasing gradient, after the manner of parabolic curves; a time would come when, having lost their velocity, the cars would come to a stop and then descend to their starting-points.

Considerations of symmetry alone, together with the knowledge that the whole of the kinetic energy in the system at the first contact of the spring must be reproduced at the instant the cars separate, have enabled us to treat the problem fully and in a way that does not admit of doubt.

We are now going to introduce complications and to suppose that one car (the right-hand one, for instance) is at rest while the other approaches it with a velocity of $2v$. This time we must imagine the railway car moving on rails parallel to a track n which we ourselves are travelling with a velocity v in the same direction as the left-hand car. Then, since we can imagine ourselves at rest, the two cars appear to approach with the same velocity, v; nothing will be altered in relation to the first case, and the c nditions will appear as below.

	Left.	Right.
Before the impact ..	$\longrightarrow v$	$\longleftarrow v$
Middle of the impact .	rest	rest
After the impact ..	$\longleftarrow v$	$\longrightarrow v$

A spectator standing at the side of the track can correct our results, and we shall know what he has seen by adding to each of the velocities noted that of our car, from left to right; the real conditions are as below.

	Left.	Right.
Before the impact ..	$\longrightarrow 2v$	rest
Middle of the impact	$\longrightarrow v$	$\longrightarrow v$
After the impact ..	rest	$\longrightarrow 2v$

The result, though surprising, is correct; *the velocities are exchanged.*

The theory of impact seemed so incredible to those who established it, that the celebrated Huyghens, in order to realise this exchange of velocities, utilised the very elementary experiment that we have just performed.

We can arrive at this result by considering the work done by the spring on the two cars.

The spring is compressed until the moment when the two velocities are equal, *i.e.*, until the cars are at relative rest. In accordance with the principle of conservation of momentum, the common velocity will be half the initial velocity of the left-hand car. The momentum before the impact was

$$m.2v + m.0 = 2mv.$$

At the middle of the impact it is

$$2m \times \text{common velocity} = 2mv.$$

Therefore the common velocity is $= v$.

The work of the spring having given the right-hand car a velocity v, the same amount of work done in the opposite direction (areas APP′ and BQQ′) will destroy the velocity of the left car.

We could have given the two cars any velocity whatever; the two methods we have used would have led to this final result: *When two equal masses meet in an elastic impact, they exchange their velocities.*

If the masses are unequal, it is evident that there can be no exchange. A shell striking an armour-plate does not, fortunately, communicate to it its own velocity. We could easily discover formulæ suitable for that case; but the time so spent could be more usefully employed; it will be enough to indicate the method. We should first say that the momentum of the two masses together remains constant; then we should imagine we were on one of the moving bodies against which the other is to strike, such as a rubber ball against a wall. The velocity of the ball for a person seeing it from a window simply changes in direction. It is sufficient to put these statements into algebraic language, and the result will be the values of the velocities.

Let us notice an interesting result of the method we have described; the rebound of two bodies from each other is equivalent to the change of sign of their relative velocities. Suppose a body A has a very great mass in relation to another B, and is approaching it. At the instant of the impact, A receives a negligible shock, and continues without apparent change of velocity; the whole change is in B, which recoils with double the velocity of A. Further, if B approaches A, its velocity changes in direction, so that after the impact its total velocity is equal to its own velocity increased by double that of A. In this way the great velocities that tennis and golf balls acquire are explained. We shall presently examine the latter game.

XLVI.—Real Impact.

If, in the above, we have studied something resembling impact, it is not impact proper that we have learned to recognise—a sudden impact accompanied with a sound so intimately associated with the phenomenon that we talk of *hearing* an impact. If impact had always been associated with the diagram we have just seen, it would sooner have been understood; but the figure seemed too remote to be of use.

Physicists have taught us to regard impact like the compressing of a spring; they have found that this property of acting like a spring is associated with matter itself, which is deformed at the point of contact, absorbs work, and then restores it by separating the bodies But, since the compression is very slight, the work is absorbed and restored in a very short distance, *i.e.*, a great force is called into play.

The compression of bodies in impact is so small that measurement is almost impossible; but there are two factors that can be easily and accurately determined in the case of two bodies that touch by curved surfaces, *e.g.*, two spheres. These factors are the extent of the surface that actually comes into contact (by the compression of a small spherical cap), and the time during which the bodies touch. In order to measure the surface, it is sufficient to clean one of the spheres perfectly and to rub the other with an oiled cloth; after the impact, the former has a small mark that is measured with the microscope.

The operation for measuring the duration is more complicated. The spheres are suspended by wires united to a circuit containing a battery and a galvanometer. When the spheres touch, the current flows through the galvanometer for an exceedingly short time, and gives it an impulse that ceases as soon as the bodies separate. The current can then be passed through the galvanometer for an easily measured time (one tenth of a second, for instance), and the duration of the impact is found by proportion.

In experiments thus carried out, Schneebeli found, amongst other results, that the duration of contact of two spheres 70 millimetres in diameter, meeting with a relative velocity of 776 millimetres per second, is about *two ten-thousandths of a second.*

In the same conditions, the diameter of the circle of contact is nearly 2 millimetres. The force when the spheres are in closest contact is nearly *one thousand megadynes*, or about 1,000 kilogrammes weight.

At the edges of the surface in contact, the deformation is very small, as is also the reaction of the matter; it is at the centre of this surface that the deformation, and therefore the pressure, is at a maximum. As a result of calculation, pressures of about 500 kilogrammes per square millimetre have been found. Such a pressure acting on a steel surface for a suitable period of time would crush the metal, just as the best steel wire (piano wire) would rupture under a stretching force of equal value. Yet, after pressures that necessarily produce equivalent deformations, the steel returns to its original shape, the pressures having been applied by impact. This is due to two causes—the first is the extremely short duration of the deformation; the second is the fact that the part that is most deformed reacts violently on the immediately surrounding material, which, instantly recovering, forces the central part back to the place it formerly occupied.

The most enthusiastic golfers are far from suspecting what an interesting instance of impact is to be found in the meeting of the club and the ball. The famous physicist P. G. Tait, whose son Freddy, the golf champion, was killed in the South African War, has devoted to the scientific treatment of the game a splendid work, some results from which we shall quote.

The ball is of gutta-percha; its mass is 42 grammes, while that of the club is 220 grammes. In the hands of a skilful and vigorous player, the ball may attain a velocity of 100 metres per second; a perfectly elastic impact with a ball at rest would give the latter a velocity not much less than 200 metres per second, since the mass of the club is much greater than that of the ball.

At the middle of the impact, the surface of contact is a circle of which the diameter sometimes reaches 20 millimetres; but the duration of contact is very short, probably less than one ten-thousandth of a second, so that the club does not travel with the ball for more than 1 centimetre. But how efficiently must this time be used, to assure the exchange of energy between club and ball! The mean force is 6,700 megadynes, 160,000 times greater than the weight of the ball. The acceleration is in the same proportion as the weight, or 1,600 kilometres per second per second. If the force of the club on the ball could be sustained for only 1 second, it would impart a velocity to the ball that, in the next second, would cause it to travel in space a distance equal to that between New York and St. Louis.

These figures are incomprehensible; they help us to understand how difficult the subject of impact must have appeared to an age that had not the means of measuring very short durations. Before experimental proofs were found, it must have been difficult to believe that such tremendous forces could be generated so simply, or that matter could withstand them.

XLVII.—Inelastic Impact.

If, at the meeting of our cars, the spring had broken when it was most compressed, the cars would not have separated; all the work they had used to compress it would have been dissipated, and they would have remained in contact. It would be as if the inclined planes of the figure had suddenly broken at the instant the cars reached their highest point. But we can also imagine other cases; we can suppose that the spring, having been compressed beyond the limit it can bear, does not expand; or that the slope is lowered before the return; or that the cars do not run on rails, but on a muddy surface. Therefore they will no longer be acted upon at each point by forces equal to those that retarded their ascent; they will not leave the slope with the same speed as they began, for something will have been lost.

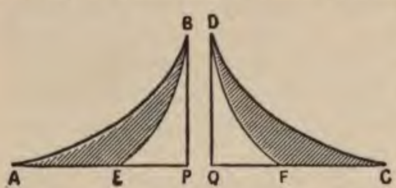


FIG. 42.—In an inelastic impact, the forces during separation are less than during the approach.

Let us first consider two similar cars; or, as we are more advanced, two spheres of equal mass meeting with equal and opposite velocities. The curves of the forces during the time of contact will be AB and CD (Fig. 42), but during the separation they will be represented by BE and DF. The work during contact will be given by the areas ABP and CDQ; during separation by BEP and DFQ. Some work, represented by areas ABE and CDF, will therefore be lost; the spheres will separate with equal velocities, but with less than the original velocity.

If one of the spheres is at rest, the work done by the other is the sum of the areas ABP and CDQ; but the work recovered is not represented by the areas BEP and DFQ.

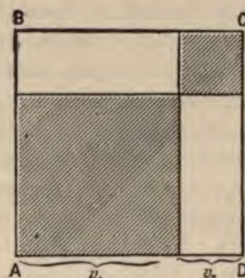


FIG. 43. -- The shaded areas represent the proportion of work recovered in an inelastic impact.

At the middle of the impact the spheres have no relative velocity, *i.e.*, both have a velocity equal to half that of the sphere A before the impact, the momentum being the same; but, as the available work is less than that already spent, sphere B will acquire a velocity less than that of sphere A before the impact. Now, the sum of the velocities is the same at every instant; therefore A will continue its motion, but with a diminished velocity.

Let $2v$ be the velocity of A before the impact. The velocities after, v_1 and v_2, will be such that $v_1 + v_2 = 2v$. But the kinetic energy, $\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$, will be less than $\frac{1}{2}m\,(2v^2)$, as we shall see immediately on looking at the square ABCD (Fig. 43), whose side is equal to $v_1 + v_2$, and whose area is $(v_1 + v_2)^2$ or $(2v)^2$, while $v_1^2 + v_2^2$ is given by the shaded areas.

We can now suppose the spheres to be quite soft; this is as if the spring were to break or the inclined plane to collapse; at the approach, the work consumed is ABP and CDQ; but then the force ceases suddenly, and the bodies do not rebound, but remain adhering. If one was originally at rest, both continue their way with half the velocity of the other, and they do not separate; as would be the case with two pieces of butter stuck together.

XLVIII.—Forms of Energy.

There was something in impact that appeared very mysterious to the ancient students of Mechanics, and that is still well calculated to give trouble. We know, as they did, at least from the time of Galileo, that the whole of the work can be recovered from the kinetic energy of bodies, and conversely. A mass descending a slope is capable of ascending to the same height again, or so nearly that the difference is immediately accounted for by the inevitable friction. But we have just verified that a part of the energy may disappear in an inelastic impact; and, in the impact of two soft bodies of equal mass, both continue with the average of their velocity before impact, in accordance with the principle of the conservation of energy. Therefore, if one of the bodies were at rest, they would acquire half the velocity of the other; the large square of Fig. 43 is then divided into four equal squares; and the energy remaining is half the original. Further, if the two meet with equal and opposite velocities, they adhere and remain where they are; their energy has therefore disappeared; it has vanished in some way without a sign.

Hence impact seems to be an exceptional action; at times (*e.g.*, in elastic impact) it manifests the conservation of energy; at other times it consumes energy more or less completely; in perfectly soft impact it leaves none at all.

Physicists, by distinguishing carefully different forms of energy, have solved this problem.

Work and kinetic energy are two forms we have learnt *to recognise*. But there is a third, directly manifested by elastic impact. When two perfectly elastic spheres of equal mass and equal and opposite velocities meet each other, an instant comes when they stop absolutely before rebounding. At this moment they have done work, and have no longer any kinetic energy; but the work is not lost, for it has generated a compression of the material around the surfaces of contact, and it is from the sudden distension resulting that the spheres derive their velocity and, consequently, their kinetic energy. The energy was not apparent, but it was ready to reappear; it had, as is said, assumed the *potential* form, a term indicating, in general, all the latent energy that can produce work or kinetic energy. If our cars, instead of compressing a spring, had ascended a slope, they would have stored potential energy that would have reached its maximum at the highest point of the slope. If they had been fastened together in this position, they would have preserved their potential energy until their release; at that instant it would have been transformed into kinetic energy.

Kinetic energy may be stored in many ways; so far, we have met only a compressed spring and a raised mass. But gas under pressure, gunpowder, dynamite, and all explosives, contain potential energy that can be liberated at a certain time, and can then produce work and kinetic energy.

A mass is ascending by virtue of its own velocity; it does work and accumulates potential energy; if it falls to the same level, the potential energy reproduces the whole of the kinetic energy used up in the ascent. Along with the ascending or descending motions there may be a simultaneous horizontal motion, but the vertical velocities will always be the same every time the body passes through the same plane—called a *level plane*—whether rising or falling.

The mass moves on a level plane without either gaining or losing work; as soon as it leaves that plane its velocity changes; if it absorbs work the velocity diminishes;

if it produces work the velocity increases. In the former case, the potential energy increases and the body rises; in the latter, it decreases and the body descends.

Whenever the motion of a mass is accomplished without friction, potential and kinetic energy are transformed without loss—what one gains the other loses; and the quantities gained or lost are equivalent to the quantity of work done by the mass as it loses velocity, or absorbed by it as it gains velocity.

But what happens in the case of the piece of butter? After the impact, the kinetic energy cannot be regained, and it is difficult to believe that some time the butter will expand and push apart the two adhering masses. In this instance Mechanics has evidently lost something; but it is found by Physics. This "something" is the heat that is generated, always in a quantity proportional to the two other forms of energy that have disappeared. There is therefore a relation between these forms of energy and the calorific form, which, when understood, renders impact quite intelligible.

In the practical application of Mechanics it is found that a small part of energy is transformed into heat; it is friction that effects the transformation; physicists say that it *degrades energy*—that it reduces it from a higher to a lower state. The fact is merely noticed here. It is fully dealt with in the volume on Physics in this series.

CHAPTER X

RESISTANCE OF MATERIALS

XLIX.—Different Properties of Matter.

THE singular results to which the subject of impact has led us give us a desire to know more of those properties of matter upon which depend the enormous and generally unsuspected forces that come into action at the point of contact of two hard bodies approaching with even a moderate speed.

The mechanical properties of bodies are infinitely diverse. If we violently throw a glass ball against a wall, it will certainly break; on the contrary, a rubber ball will recover its shape; and if we construct a star with six points from well-pressed new bread, we can throw it as strongly as we wish; it will rebound without losing its shape. Yet, taking it between our fingers, we can crush it easily; we can shape it again, for it will take any form we care to give it, and will preserve it when thrown against any object. The rubber ball can be pressed out of shape in the hand, but will become a sphere again as soon as released. The glass sphere can be pressed with any force at all, but will not be appreciably changed and cannot be crushed.

FIG. 44. — A star made of bread rebounds without being deformed.

What is responsible for the different behaviour of the rubber, the glass, and the bread? We might have added steel, lead, and cement, and we should have found another series of properties. In order to disentangle this terrible skein, we must proceed in an orderly manner; we shall therefore begin with some elementary experiments.

L.—Elastic Limit, Breaking Strain, and Modulus of Elasticity.

Our apparatus is very simple—a wooden frame, a balance-pan, and wires of different materials.

We fix a wire to the frame, and attach a balance-pan to it (Fig. 45); at the same time, we fasten a scale, divided into millimetres, to the frame. At the end of the wire we affix a needle* to serve as a pointer. For the sake of simplicity, we take a wire of about 1 metre.

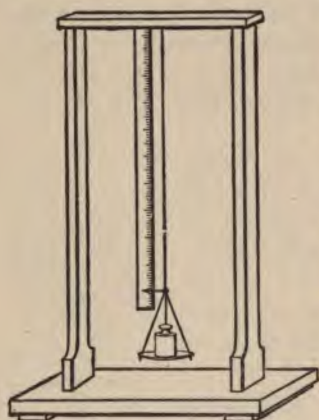


FIG. 45.—The mechanical properties of metals can be studied by measuring the elongation undergone by a wire loaded with a weight.

If we load the balance-pan, the needle descends; if we remove the weights, it ascends. If we have not subjected the wire to a too great strain, the needle will rise to its original mark; we say that the wire has undergone an *elastic elongation.* But if we use a greater load, the time will come when the needle will not return to its first position; the wire will be slightly deformed; in addition to the elastic elongation, it will have undergone what is called a *permanent elongation.* The precise moment when this elongation occurs is difficult to determine; it begins gradually, and exact experiments detect

* For shortness, we suppose the needle fastened directly to the wire; but, in order to render its movement more apparent, it ought to be arranged so as to have its motion amplified.

it much sooner than rough experiments. Yet it can be *approximately* fixed, and then it can be said the wire has reached its *elastic limit.*

We can then continue to load the wire; at a certain instant it breaks. The *breaking strain* has been reached.

Engineers calculate the elastic limit and the breaking strain in kilogramme-force applied to a wire of 1 square millimetre section. Eventually the megadyne will certainly be substituted for the kilogramme-force, but, for the present, we shall observe the general usage.

Within the limits of elastic deformation another property of matter can be defined—its *coefficient* or *modulus of elastic resistance.* Engineers define it numerically by loading a wire of 1 square millimetre section with 1 kilogramme-force; it increases by a certain fraction of its original length. The inverse of this fraction is the modulus of elastic resistance.

We can now attach to the frame a wire of steel, brass, or lead, or a band of elastic. Making accurate measurements, we find that the steel wire 1 square millimetre in section with a load of 1 kilogramme-force extends about $\frac{1}{20}$ millimetre, the brass wire $\frac{1}{10}$, and the lead wire $\frac{5}{10}$ to $\frac{6}{10}$; as for the elastic of the same section, we cannot load it with 1 kilogramme; 10 grammes elongate it by 10 centimetres.*

The modulus of elastic resistance is therefore 20,000 for steel, 10,000 for brass, 2,000 for lead, and ·1 for rubber.

There are, as we see, notable differences between the elastic elongation of various metals under the same stress; but their elastic limits vary still more widely. Thus, in order to produce an apparent permanent elongation in hard steel wire (such as good piano wire), it

* Until the moment when permanent deformation is manifested, wires lengthen almost proportionally to the traction (negative pressure) that they bear (Hooker's law); for an elastic band, the deformation varies at first more than the force, which is due to the rapid diminution of the section; then it tends to a limit.

must be loaded with 100 to 200 kilogrammes per square millimetre. With our restricted means we can determine this limit only by working with a finer wire. If our brass wire is moderately hardened, its elastic limit will be about 15 kilogrammes; the lead wire will be permanently deformed with a load of 500 grammes. The elastic, on the other hand, can be stretched to close upon its breaking point, without ceasing to return to its original length. The steel will break a little after its first permanent deformation; the brass will break at about 50 kilogrammes; the lead at a little more than 1 kilogramme.

For the same metal, the modulus of elastic resistance varies little, whatever its state; provided that it is subjected only to very small deformations, reactions are developed that are almost the same. But the elastic limit and breaking strain differ considerably, according to the state of hardness or temper. Brass, for example, can be reduced by heating to a state as soft as lead, while rolling, hammering, and drawing into wire raise its elastic limit to such a degree as to permit of its being used for springs of a good quality, which undergo great elastic deformations before showing permanent deformation.

A body with a high elastic limit gives a ringing sound when struck; on the contrary, if its elastic limit is reached by a small deformation, it gives a dull sound that does not last. The reason is almost evident; we shall see it in a moment.

In the scientific study of metals, great importance is attached to the modulus of elastic resistance; on the contrary, for building purposes, the elastic limit and the breaking stress are more important.

LI.—The Bending of Bars.

The examination just made of wires has one advantage —it is simple. Every particle of the metal is submitted to the same action, and its properties are immediately apparent. But metal is more often used in bars or girders, which bend under the action of forces. We are now going to discover how a bar behaves when made to bend, and how it resists force.

A sheet of paper bends without the least difficulty; similarly, we can roll a copy-book with very little effort; but a piece of cardboard of the same thickness resists much more. What is the difference? In each case we have the same quantity of the same kind of matter; but we have only to observe the way in which a copy-book or a sheet of cardboard bends to see that the matter is differently associated; the leaves of the book slide one over another, while the layers of cardboard cannot do so. We might have done the same with thin sheets of metal, and we should have come to the conclusion that, properly speaking, there is no such thing as resistance to flexion. *Matter does not resist bending;* it resists only when it is prevented from sliding.

FIG. 46.—Extension and compression.

Let us take two thin bars of metal that can be easily bent, and let us connect them by a series of wood blocks to which they are screwed (Fig. 46). If we try to bend this beam, we find it opposes a great resistance; and we can easily see that this resistance is due to the fact that we are trying to extend one of the bars and compress the other.

Our reasoning will still be applicable if we consider the two bars as forming one girder, connected by a middle part; actually, in a uniform bar that is bent, all the material in the concave half is compressed, and all that in the other is extended. The surface separating these regions is the *neutral fibre* of the girder, and we can, by working out the force required to deform two symmetric bars relative to their neutral fibre, ascertain the laws of flexion of beams.

Suppose that, beginning with a given beam, we double all the dimensions in the plane in which we are bending it. The two bars will become twice as thick; they will then resist twice as much for the same deformation; but, since they are now twice as far apart, they will undergo twice as much deformation for the same total flexion of the beam; for the same reason, the moment they exert in opposition to the couple that tends to turn them is twice as great. Thus this couple is doubled three times and is therefore eight times as strong as before. Let us remark that if we double the distance between the bars, without increasing their thickness, their moment of flexion will be four times as great. We conclude that a given quantity of material resists flexion better in the form of a tube than in the form of a solid rod or a beam. So much for thickness.

If we increase the breadth of a bar, the lateral layers that we add act exactly like those already forming part of the bar; they contribute to the resistance in the proportion of the added material, just as if a second bar were placed by the side of the first. So much for the breadth.

Let us now see how deflection varies with the length.

The bar being placed on two supports and bent by a weight at the middle, it will be seen that the deflection, *i.e.*, the difference in level between the supports and the lowest point of the bar, increases rapidly as the supports are moved apart.

If the bar preserved the same shape, the deflection would be proportional to the square of the distance between the two supports; but it is more bent as the supports are separated, for the weight acts on a lever the length of which depends on the distance between the points of support. The value of the weight being constant, the deflection is proportional to the cube of the distance between the supports.

Here, in a short form, is the law of deflection of bars :— *A bar placed on two supports and loaded in the middle bends proportionally to the cube of the distance between the supports, in inverse proportion to its breadth, and to the cube of its height.*

If it is true, as the great geometrician Laplace said, that nature makes light of difficulties of integration, nature is no less ready to apply the principles of resistance of materials, which man, as the result of many accidents due to his ignorance of these principles, has learned to know. We have just seen that a beam resists the more as its material is accumulated at its exterior; for the same amount of material, our bones support us better than they would if they were solid; and the very straw supports the heavy ear by a perfect marvel of mechanical construction.

LII.—Fragility, Plasticity, and Hardness.

We all have a clear idea of what these terms mean. But we must be able to associate them with the properties we have observed in the traction of wires.

The blow of a hammer does not leave an impression on hard steel, although it marks a piece of copper or lead, breaks a piece of glass into pieces, and reduces a piece of sandstone to powder. The steel is resistant, the copper and lead are plastic, the glass is fragile, and the sandstone friable.

Steel, as we have discovered, has a very high elastic limit; it can be deformed, but returns to its original shape; copper and lead have a low elastic limit; when they are deformed to a slight degree, they retain signs of the change.

On striking these objects, the hammer possessed a certain amount of energy; this energy could be used only by being transformed into its equivalent work. The hammer struck the steel and rebounded; the deformation of the metal, having for an instant absorbed the energy of the hammer by storing it under the potential form, reproduced it under the kinetic form in the hammer, which was driven violently backwards.

The limit of permanent deformation in copper and lead is reached before the hammer has exhausted its kinetic energy; it therefore penetrates and leaves a mark. The glass, which undergoes no permanent deformation, and has a relatively low breaking point, is deformed beyond that point before the force of the hammer is spent; nothing remains for the glass but to give way by dividing. The sandstone is composed of innumerable grains within which the breaking stress and the elastic limit are relatively high, but they are united by a cement whose breaking stress is relatively low; the grains are individually uninjured, but are separated from each other.

Which of all these properties can be called hardness? Steel is hard and not fragile; glass is fragile and hard, for steel can hardly mark it; brass and lead are certainly not hard, for they are easily deformed; sandstone, though friable, possesses a high degree of hardness, since, in the form of grindstones, it serves to grind steel tools and to sharpen knives, chisels, etc.

If a body is hard, its elastic limit must be high; that is evident from the above. If, at the same time, its breaking stress is high, it is hard and tenacious; if its breaking stress is of moderate value, it is hard and fragile.

The structure of sandstone gives it strange and paradoxical properties. Although it attacks tools in the process of grinding, it can, in the form of a grindstone, be turned with a mere file.

The reason is that the action of the metal on the stone is different in each case. Grinding subjects the cement to only a very small force, which it can resist; every grain attacks the steel by sliding over it, and forcing it away if it presses too strongly; on the other hand, a grindstone is made by fixing a file with one part against the trough and another against the stone, so as to exert a great force on the cement; the grains are not worn away, but broken off in the same way as the sandstone was broken into fragments with a hammer.

Grains of sand attack metal even when they are not united by cement; in the kitchen, pans are polished with sand on a piece of cloth; and pieces of metal are cleaned in foundries by means of a jet of sand carried by an air-blast.

Hard quartz sand, carried by the wind, attacks window panes and renders them dull. Thus, an American officer, observing the opaque condition of the windows of his hut during the War of Secession, was led to think of manufacturing articles of "frosted" glass, which were popular for many years.

The following appropriate experiment may appear paradoxical. If a piece of fine lace is laid on a sheet of glass, it protects the glass so effectively from the sand-blast that the surface is marked only in the spaces between the lace; the complete design of the lace then appears transparent on the opaque background.

The difficulty disappears if we think of the results of impact of grains of sand on lace and on glass. Every grain has, when it meets an obstacle, a certain amount of kinetic energy that it must transform into some other form of energy; the lace yields for an appreciable distance, and the force is very small. On striking the glass, the grain is stopped in a very short distance, and the force it exerts is consequently very great. Thus particles of glass are broken from the surface. The energy of the impact is too weak to break the glass; the total force at any instant is infinitely small, but the pressure, being exerted on a very restricted area, becomes enormous; the glass breaks, but the fracture is of microscopic dimensions.

LIII.—Gradual Rupture and Sudden Rupture.

The facts that we have studied are of a somewhat complex nature, and therefore it is possible that some of them are still obscure to our readers. Hence some further experiments may not be without use.

We first fix in a strong vice about 1 metre of piano wire 3 millimetres or less in diameter, and we fasten to its lower end a clamp with an old weight of 100 grammes. We then lift the weight and let it fall from a gradually increased height, until the wire breaks; it breaks when the fall is about 80 to 90 centimetres.

We next fasten the clamp with the weight to the end of the remaining piece of wire, after having put planks underneath to deaden the fall. The wire breaks when the clamp is loaded with about 15 kilogrammes; and if the wire is long enough, we can verify that, before breaking, it had increased by about one hundredth of its original length.

To take another extreme, we can repeat the experiment with an indiarubber band, but in the opposite order, *i.e.*, we first break the band by a static load (200 grammes, for example); we can verify that it becomes almost six times as long, and we can therefore decide, according to the space available, what length of indiarubber to use.

If we argued that the conditions would be analogous to those of the steel wire, we should wisely begin by letting a weight of 10 grammes fall. But we already know how well indiarubber resists; yet we shall be surprised to find that nearly 100 grammes, falling the whole length of the band, are required to break it.

Here, then, is a singular fact; in order to break the steel, we have had to take a static weight 150 times as great as the weight falling at least 1 metre, while, in order to break the rubber, the static weight was just double the falling weight.

Let us try to explain this paradoxical result.

The wire and the band broke when the force, instantaneous in one case, and lasting in the other, reached their breaking limits. The weight was 15 kilogrammes for the steel—that is to say, in the distance of 1 centimetre by which the wire was lengthened, the weight developed forces that increased from 0 to that developed by 15 kilogrammes. The work done on the wire was therefore represented by the area of the triangle ABC (Fig. 47), or 15/2 kilogrammes-force-centimetres, and it is when the energy of the shock exceeds this that the wire reaches an elongation sufficient to break it.

On the contrary, the rubber, before breaking, can withstand an elongation of five times its original length. Before a band of 1 metre broke with a static weight of 200 grammes, it had first supported weights of 0 to 200 grammes for a space of 5 metres. The work was then $\frac{\cdot 2 \times 500}{2} = 50$ kilogrammes-centimetres, and this was the work required to break the rubber.* The elongation of the elastic being such as it is, it was a matter of indifference whether we applied the weight gently to the end of the band, a very small part of which should be held in the clamp, or whether we allowed the weight to fall some decimetres; the kinetic energy gained by this initial fall would be small in comparison with the whole work done by the weight.

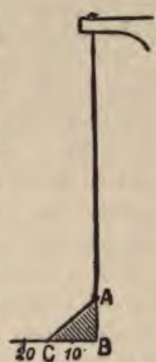


FIG. 47. — In rupture by a falling weight, the wire first absorbs the kinetic energy of the fall.

* For simplicity of calculation, we suppose the reaction of the indiarubber band is proportional to its elongation. But we have seen that experiments are exceptional with rubber. Our readers can find experimentally the factors of the problem; they can first draw a diagram of the elongations under a static weight, and then bring about rupture by a falling weight. The energy exactly required to break the rubber band will be represented by the area included by the elongations and the forces. This experiment, carefully executed, is very instructive.

It is interesting to notice that the static rupture of the rubber was brought about by one seventy-fifth of the weight required for the steel; but the energy necessary for sudden rupture was seven times as much; according to our method of estimating the relative merits of steel and rubber, we can express them by numbers whose ratios will vary from about 1 to 500. This example makes it plain that the method of testing materials ought to be perfectly definite, if any significance whatever is to be attached to the numbers that indicate their qualities.

We shall understand, too, without its being emphasised, that the steel would have resisted the shock much better if it had been fastened to its support by a piece of rubber. In order to withstand the static weight, this would have had to be of large section; but it would have served to nullify the energy of the falling weight in the second experiment, and would have prevented rupture.

For a static weight, the breaking stress is obviously almost independent of the length of the wire. It would be absolutely independent if the wire were weightless, and if it were quite homogeneous; the longer it is, the greater probability there is of a weak part whose breaking strain is a little less than that of the average of the other sections. The wire withstands a falling weight all the better if it is long, for the quantity of kinetic energy that it can reduce to the potential form is proportional to its length.

Let us now take an extreme case. A very short wire ought to be broken by the energy of a very slight shock. If we reduce the wire to a length of 1 millimetre, it ought to be enough to allow a weight of 100 grammes to fall from a height of about ·8 millimetre. But an experiment would not confirm this apparently accurate result. The reason is that the deformation of the clamps in contact with the wire could no longer be neglected; all these deformations transform kinetic energy into potential energy, and assist the wire to bear the stress.

Our young pupils will draw from these remarks all the conclusions implied; let them look around for every kind of shock, especially when considerable force is developed. If these forces are not destructive, the explanation is that all matter yields a little, consequently increasing the distance and decreasing the force of the shock. They will understand better the skill of the packer; they will grasp better the real significance of the saying of Galileo: "Live force is always infinitely greater than mere force, for force cannot always raise a body while live force always can." That would be true if bodies were infinitely hard, if they did not yield at the point struck, and if they did not suffer flexion. But flexion and local deformation minimise the force, and hence a blow cannot move a large stone, as it would necessarily do if there were no deformation.

These considerations are of great importance. If we do not master them, we are ill prepared to understand how mechanical principles are applied in the material world. For the purpose of rendering them more easily intelligible, we simplify Mechanics to an extreme degree; we create imaginary conditions—forces on the one hand, and material points on the other; then we associate them and obtain results which are perfectly exact when applied to these imaginary cases, but which are very often in complete disagreement with the actual facts. Between the two intervene deformations of the material, and these alter the phenomena to such a degree as to render them unrecognisable by the simplified science that constitutes the skeleton, so to speak, of real Mechanics.

LIV.—Conservation of Momentum, and Deadening of Shock.

There are in the world poor wretches who allow themselves to be struck with the full force of a heavy hammer, and, like Oliver Twist, want more. Let us explain that they do not receive the blows directly on their bodies, but through an anvil placed on the chest, an assistant meanwhile striking with all his strength. The mighty supporter of the anvil rests on the ground with his hands and feet, and raises his body as much as he can, thus forming a sort of elastic support that bends slightly at every blow, and absorbs the energy transmitted by the anvil. But we know this flexion is not enough to use up all the energy of the hammer, and that there must be something more.

It is not always easy to experiment on a man, and we can attain our end more directly by simplifying the problem and replacing the gymnast by the steel wire that has already taught us so much.

We shall cut a piece 1 metre long and load it with 5 kilogrammes; on this weight we shall let fall from a gradually increasing height the 100 grammes that previously broke the steel wire. This time, however, the wire will resist, although to the energy of the fall we have added a weight one-third of that required to break it.

We must analyse the action if we are to explain this paradox. We will take the shock first. We know a considerable force is produced; but it is of very short duration and can give to the suspended mass only a small impulse; this begins to move; but, before the wire has time to be deformed by an appreciable amount, the shock is already finished.

In shock or impact one thing is constant, the momentum. It is the same when the falling mass m touches the stationary mass M as it is immediately after, when the two have a common velocity. In this case, the velocity v' is equal to $\frac{M + m}{m} = \frac{1}{51}$ of the velocity v of the smaller mass; the energy is equal to half the total mass multiplied by the square of the new velocity $(M + m)\frac{v'^2}{2}$, and is $\frac{1}{51}$ of that before the impact; it corresponds to the energy of the small mass falling for $\frac{1}{51}$ metre or about 20 millimetres, and is therefore negligible, since it produces an insignificant elongation of the wire.

If it is a question not of measurement, but of a simple demonstration, we can clamp a piece of the same wire to a support, attach a heavy ball to it, and fasten another piece of wire to the ball (Fig. 48). If we pull gradually on the end, we shall break the upper piece, for to our force is added that of the ball; on the contrary, a quick jerk will break the lower wire.

FIG. 48.—A slow pull breaks the upper wire, a jerk the lower. In the first case, the ball acts by its weight; in the second, by its mass.

This experiment is interesting from another point of view. The ball produces different results because, in the first case, it acts by means of its weight, and in the second case by its mass. Thus weight and mass are not only separable; their effects can be opposite.

Unless our gymnast is an old engineer reduced to poverty, he has probably no idea of kinetic energy, of conservation of energy, of absorption of work, or of pressure; yet he doubtless has a clear intuition of what can be accomplished by a rational application of mechanical principles. The base of the anvil spreads the total force over a large area; the anvil reduces the kinetic energy to be supported to a very small fraction, and the remainder is used up in the spring that he forms with his body and limbs. There is a certain merit in holding an anvil in this manner; but as for the blows of the hammer, he feels them so little that they are not worth speaking about.

In practical construction, these principles are applied in a way that exceeds in utility the invention of this poor fellow to make a living. The resistance of material is combined with its capacity for absorbing energy in such a way that it may be able to resist the shocks it will suffer.

LV.—Speed of Transmission in Sudden Rupture.

When a rifle bullet penetrates a canvas target, the most attentive spectator cannot detect a quiver of the canvas; a stone, on the contrary, when penetrating the target, produces a movement that extends to the frame and may even knock that down. Similarly, a stone breaks a pane of glass into pieces, but a bullet makes a hole just sufficient for its passage.

We know that the great difference is due to the fact that the bullet is displaced much more quickly than the stone. The difference in speed produces a number of simultaneous actions; but, of these, one is predominant—the speed of transmission of forces and motions by the bodies struck, in comparison with the speed of the bodies striking them.

Recalling the wire, we shall understand that it must be so. When calculating the energy it could absorb in a longitudinal shock, we assumed the force was instantaneously transmitted throughout the length of the wire, so that every particle of it contributed equally to the transformation of the kinetic energy into potential energy. But that is approximately true only if the shock is not sudden and if the wire is not long. When a sudden elongation is given to a wire, the movement travels along the wire with a speed rather over 5 kilometres per second; if, therefore, the shock lasted only a thousandth of a second, only the first 5 metres of the wire were concerned in the absorption of the energy, and the remainder was not affected.

The difference between a sudden and a static force is still more evident from a modification of the experiment (Fig. 48). Instead of dividing the wire into two by a *ball,* we could work with a long wire; a gradual force would break it near the top, a sudden force near the bottom. The reason is to be sought in the slowness of transmission of the force; and analogy with the preceding experiment enables us to understand that if the transmission is not instantaneous, it is because each particle of the wire acts like a ball. It can propagate the effect of the force only by putting itself into motion, which requires time.

The results of rapidity of mechanical action are observable not only in solid bodies; liquids and gases give quite as curious results. A rifle bullet ricochets when fired at an angle to the surface of water, although it would have sunk if it had had time to settle. A strong jet of water behaves like a spring if it is struck with a stick; and, if the speed of the jet still increases, it can hardly be cut with a sword.

A thin ring of dynamite round the foot of a tree is enough to blow it up, though a large charge of gunpowder similarly placed would be without effect. The difference is due to the fact that the explosion of powder is relatively slow, while that of dynamite is extremely sudden. The surrounding air gives way to the former, which reacts on it without violence, but it forms a packing and acts almost like a solid towards the gases that result from the explosion of the dynamite.

These actions possess a great interest, but to study them would take us too far. It will be enough for us to have introduced our readers to the subject, and to have given them a desire to continue. If, further, this "Introduction" has prepared them to understand these subtle actions, the reason is that we have travelled far since the observation of falling bodies led us to discover the action of forces.

CHAPTER XI

ARTILLERY

FROM the beginning, Mechanics has profited from the questions raised by the motion of projectiles; the immense forces brought into play to give them their velocity, the equally immense forces to which they give rise on meeting an obstacle, the distance they travel—all must have impressed the ancient students of Mechanics. We who are deriving benefit from their work can still, in order to test the thoroughness of our knowledge of Dynamics, utilise the few instants during which a projectile fulfils its purpose.

LVI.—General Conditions in the Firing of a Projectile.

The explosion of the powder produces a large quantity of gas at a high temperature, which generates a very strong pressure. The sides of the gun, the breech, and the base of the shell are uniformly subjected to this pressure, except that the shell, in its forward movement before the gas, whose rate of expansion is not infinite, tends to create a vacuum behind it. The shell ought to receive most pressure, but it really has the least; the difference is not very great, and we need not take it into account. The sides generally withstand the strain; as for the breech, it drags back the gun, which travels on slides in modern guns, or recoiled together with its carriage in old guns.

The gases of the powder escape from the gun at the same time as the shell; their mass being very small, we can ignore it, for we are not making an exact calculation, but examining the main features of the action.

On this assumption, a sufficiently close approximation will be reached if the momentum of the gun be regarded as equal to that of the shell, for the force (*i.e.*, the pressure multiplied by the section of the loading chamber), as well as its duration, is the same for both. At each instant the acceleration is inversely proportional to the mass moved, and the momenta of the two moving bodies are always equal.

The work performed in putting the shell and the gun into motion, and absorbed by them, is inversely proportional to their respective masses, since the distance travelled by a body moving under the action of a force is inversely proportional to the mass. Hence the shell is very destructive, while the gun, in a sense, is never so.

All the parts of the gun-barrel ought to be solidly fitted together, so that they may acquire the necessary acceleration as a whole, and thus avoid strain. Those subject to the direct explosive action of the gunpowder ought all to be of exceptional strength, lest they themselves become dangerous projectiles.

The gun could be kept from recoiling, but it would necessitate the use of a brake capable of exerting a force equal to the maximum of that produced by the powder, which, as we shall soon see, is enormous. It is better to allow the gun alone to undergo the reaction—a reaction equal and opposite to that of the gas—that results from its having had an acceleration imparted to it. The effects of the kinetic energy of the gun are thus extended over as great a distance as may be necessary, and the energy is dissipated in work on the brake.

An actual example will make these facts much clearer.

LVII.—Actual Firing.

The old field-gun of 80 millimetres calibre will serve as a standard. In this gun the projectile travelled 171 centimetres in the barrel and attained a speed of 490 metres per second.

For the purpose of making the calculation clear, we shall not follow out the exact facts, but we shall introduce corrections later. Let us suppose the shell is acted on by a constant force the whole way down the barrel; neglecting friction, we have the simple though not quite accurate result that the shell has a constant acceleration. Putting a for this acceleration, and t for the time the projectile is travelling in the gun, we can write—

$$1{\cdot}71 = \frac{at^2}{2} \text{ and } 490 = at.$$

These two equations lead to the following results— $a = 72{,}000$ m/sec^2. and $t = {\cdot}007$ sec.

If our hypotheses were in accord with the facts, the shell would travel through the gun in seven thousandths of a second, and the force of it would be about 7,300 times its weight ($72{,}000 \times 9{\cdot}81$); that is, each gramme of matter in it reacts with a force of 7·3 kilogrammes-force.

In reality, the action is slightly different, in consequence of the fact that the powder requires time to explode and generate gas.

What is of primary importance is to give the shell a great velocity; at the same time, the gun must not be subjected to a force capable of bursting it. Therefore we are confined to these two limits, and the skill of the cannon-maker and that of the powder-maker consist in attaining conditions that, in practice, satisfy each as much as possible.

The greatest velocity will be given the shell when the greatest amount of work has been done on it; this work being represented by the area enclosed by the curve drawn with distances as abscissæ and forces (pressure × section) as ordinates, the problem is to make this area as large as possible.

We can now work out two special cases. The first is that in which the combustion of the powder takes place instantaneously. The pressure develops suddenly, and reaches its maximum value CA (Fig. 49) before the shell has moved in the least.

Then it begins to move and the space behind it increases, thus lowering the pressure continuously until the moment when the shell leaves the muzzle and the gases escape into the air. The pressures are represented by the curve AB. For the area enclosed by the curve to be sufficient,

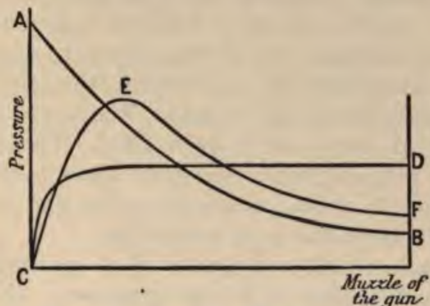


FIG. 49.—The throwing effect and shattering effect depend on the development of the pressure in a gun.

its first point A must be very high, *i.e.*, the initial pressure must be enormous. The gun is then subjected to such a great force that it can resist only if it is very thick. The powder produces immediate destructive effects, whence its description of *shattering*. On the other hand, its carrying power is only moderate, and it is not used for artillery.

Let us imagine a powder the deflagration of which takes place slowly, so as to sustain the pressure during the whole course of the projectile. After an initial increase of pressure, lasting for an appreciable time but represented in terms of the displacement of the projectile by a very small part of the curve CD (for the shell moves very slowly at first), the curve becomes almost horizontal, and the projectile is under a constant force. For the same total area enclosed by the curve CD, the maximum ordinate is as low as possible; in some ways, this is the ideal powder; yet it could not be recommended. At the moment the shell left the muzzle some gases with considerable energy would remain, and firing would be simply a waste of powder.

As very often happens, the solution is to be found in a middle course—a powder of gradual combustion, so that the pressure is maintained while the shell is moving in the gun; then, combustion being complete, the energy of the gases is utilised by allowing them to expand, and, when they have fully done their work, they are liberated when the shot leaves the muzzle. The normal curve of pressure is of the general form CEF. The maximum pressure is attained when the shell has travelled many times the length of the loading chamber, and its greatest value is reduced in the same proportion.

Let us again consider a cannon of 80 millimetres. Its projectile has a mass of 5·6 kilogrammes; the average force acting on it is $5{\cdot}6 \times 7{,}300 = 41{,}000$ megadynes (about), if the force is constant. But we have just seen that it is not; the maximum is very much above the average force, and in this case it is certainly greater than that exerted by a weight of 100 tons on its support.

The discharge of a projectile could be represented very simply by means of a lever with extremely unequal arms, the shorter of which would be horizontal, the other nearly vertical. From the horizontal arm would be suspended a very heavy weight, which would give it a corresponding acceleration, while the other would have a projectile placed in a cavity at the end, and would acquire velocity by the fall of the moving weight.

Before the invention of gunpowder, besiegers of towns proceeded in this fashion with the catapult. The lever was not on a pivot, as we have described it; it was held in a kind of band that, when twisted, added greatly to the effect of the weight. Such a system might be employed to-day, if the properties of matter had not to be taken into consideration—if, in particular, levers could be made absolutely rigid and devoid of mass. A lever sufficiently rigid to transmit to the projectile an acceleration to give it the speed desired to-day would have a mass out of all proportion to that of the projectile; the greater part of the work of the falling weight would be used in setting the lever in motion, and would afterwards have to be absorbed; therefore the day of catapults is past.

In proportion as the calibre of a gun is increased the length must be increased, because, the mass of the projectile increasing as the cube of its dimensions, while the area of the base increases only as the square, it is necessary, if the same pressure is to be utilised, to allow it to act for a longer time, *i.e.*, on a surface proportional to the mass. For the same final speed, and for similar conditions of discharge, the time the shell takes to pass through the barrel is, as is easily seen, proportional to the square of the length of the cannon. The duration of deflagration must therefore be proportional to the square of the calibre, if the same conditions are to be attained in large as in small guns. This is done by using, in large guns, powder of large grain, the surface of which is small relative to the volume. In their endeavour to make the grains in proportion to the size of the gun, makers of powder must not be suspected of having obeyed the rather rudimentary instinct of the famous colonel who wanted to give the large instruments to large men, and was annoyed to see a great fellow blowing a fife.

LVIII.—Around the Projectile.

The maximum force of the powder on the shell and the breech is, as we have seen, too great (100 tons in field-pieces) to be opposed. The gun is simply allowed to recoil, and is then checked within such limits of distance as conditions permit. The mass of the whole gun is practically 200 times more than that of the shell; its velocity is therefore 200 times less. When, after about one hundredth of a second (for field-pieces), the shell has gone the length of the barrel, the gun has recoiled at least 1 centimetre, and the work done during this displacement is dissipated in a distance 200 or 300 times greater. The force is reduced in the same proportion, and is therefore manageable.

Formerly the whole gun ran back on the ground. To-day all guns are formed of two independent parts—the gun proper and the carriage, united in such a way that the former can recoil on a slide and be brought back into position by springs; a viscid liquid, compressed in a cylinder by a piston attached to the gun, transforms the energy without changing it into the potential form. The function of the brake is to take up a certain portion of the kinetic energy of the gun. The part required to bring it back to the firing position is accumulated in springs; the remainder is degraded by the friction of the liquid. Thanks to this combination (the realisation of which presented great practical difficulty), quick firing has been made possible, because the aim is not destroyed as when the gun ran along the ground.

If the gun is firmly fixed, it may remain motionless. If on a ship, whatever be the method of recoil, the effect on the ship as a whole is the same; the whole ship recoils with a velocity which is that of the projectile in the inverse ratio of their masses. For example, a shell of 305 millimetres, weighing 440 kilogrammes and fired with a speed of 830 millimetres per second from the turret of a battleship of 20,000 tons, reacts on the ship with a velocity of about 2 centimetres per second.

If the shell, without loss of speed, reaches a ship *similar to* that it has left, it communicates to it a speed equal to that with which it started. But at the point it strikes it causes much more damage than the recoil of the gun; for, in addition to its momentum, it has acquired energy that passes from the kinetic form into work, and we have seen that the energy of the projectile is incomparably greater than that of the cannon.

The force of a shell striking a battleship is prodigious. The shell has, for instance, acquired its velocity in 12 metres, and gives it up in 2 or 3 decimetres. The force is therefore 40 or 60 times greater than that of the powder on the shell, and attains some 12,000 tons; but it lasts a very short time, so that the ship, though experiencing a severe local shock, suffers, as a whole, a much less motion than if it had been gently touched by another ship, which would have left no appreciable sign of its contact.

We have assumed without justification that the initial and final speeds of the projectile are equal. Tenuous though it is, the air absorbs and dissipates a large proportion of the energy of translation, partly by the definite waves which are generated from solid surfaces, as shown in Fig. 2 (page 12), and partly by the commotion in the wake of the shot. It is these regular waves that transmit to our ears the sound by which a shell or bullet passing above our heads announces its presence.

The local and general effects of the impact of a projectile are good examples of what is characteristic of kinetic energy and what of momentum in the meeting of two bodies. These questions arise everywhere; the lash of a whip on the skin gives severe pain; through a thick garment, it is hardly felt. A blow with a heavy stick is hardly more painful when direct than through the clothing, provided it does not meet a bone near the surface; in the latter case, impact proper reappears, with all the consequences of a mass being suddenly arrested.

The effects of kinetic energy and of momentum are often found together; but their chief characteristics can be separated by remarking that a small mass striking another at a great speed produces effects due to energy, while a large mass with a low speed acts through its momentum. Energy is preserved by being transformed; momentum is preserved without transformation; energy of impact produces a local result, and momentum a general result. All difficulties in application are removed by these tests.

A word more on a curious illusion. We judge a speed sometimes by the duration of the passage of a moving body, but more often by the damage it would do if it were suddenly stopped; the latter is proportional to the square of the speed; we are therefore unconsciously led to over-estimate high speeds, hence the conflicts of chauffeurs (whom we do not defend) with the police, who are very liable to see a speed of 100 miles an hour.

LIX.—Within the Projectile.

At the moment of firing, all kinds of interesting changes take place within the shell, as the result of the acceleration it receives. Gunners utilise the acceleration to set in action the mechanism intended to fire the charge in the shell. Now, if this mechanism were always set in readiness, handling of the shells would be extremely dangerous and would undoubtedly cause more deaths among the army using them than among the enemy.

The mechanism to effect the explosion is constructed of two hollow cylinders (Fig. 50) in such a way that one of them slides inside the other. Normally it is prevented from doing so by a small spring with projections, while a spiral spring holds the two so that they have no relative motion.

The instant the shell leaves the gun, the outer cylinder crushes the projections of the spring, and presses firmly against the other cylinder, which is now free to move. When the shell strikes, it suffers a sudden retardation.

The striker continues on its way and hits a percussion-cap, and the charge is fired.

The same effects of inertia have been utilised by certain gunners in order to record the acceleration of the shell within the barrel. For this purpose the shell is provided along its axis with a strong steel bar, upon which runs a slide with a sounding-fork attached. This fork, whose arms are held apart by a piece of metal, is attached to the front of the shell by a small pin, strong enough to support the weight on it, but no more. We have just seen that at the instant of firing the acceleration produces forces thousands of times greater than the weights of the objects. The pin can therefore break without appreciably moving the mass, which remains at rest. The shell proceeds while the sounding-fork (which is now vibrating, for its arms are free) traces with a pointer an undulating line on the bar over which it is sliding.

FIG. 50.—At the start of a shell the fuse is primed, and on its arrival the striker fires the charge.

The bar increases in speed, and the undulations extend; they record equal times but unequal distances, of which the curve can be drawn. This is all that is required to analyse the beginning of the shell's motion. A special and very ingenious mechanism then liberates a second fork, which, already having a rather greater speed, records a longer distance. When this, in turn, strikes the base of the shell, the interesting part of the trajectory is recorded.

The shell thus becomes a perfect laboratory, in which is written a page of its history—a short page, for it lasts only thousandths or, at most, hundredths of a second, but the most interesting page of its life, since it is that in which its purpose is attained.

LX.—The Journey from the Earth to the Moon.

There is another kind of laboratory for projectiles—a laboratory that Jules Verne invented for the delight of many generations of young readers, before Wells imagined even more fantastic journeys.

Let us recall the facts. After the War of Secession, the spirit of the Americans, so long devoted to the art of war and now without an object, continued with its recent impulse to devise machines of destruction. At this time, Barbicane, Director of Artillery during the war, made a sensational proposal to the Gun Club of Baltimore. He proposed to construct a gun of hitherto unknown power—so powerful as to send to the moon a shot large enough for astronomical instruments to enable it to be seen on its arrival on our satellite.

The project was far advanced when Michel Ardan of Paris offered to make the journey enclosed in the projectile. Two members of the Gun Club, naturally including Barbicane, insisted on accompanying him, and on sharing the dangers and sensations of this unprecedented voyage.

The cannon, consisting of a cast-iron tube 2·7 metres in internal diameter and 270 metres long, sunk vertically in the ground, was to send a projectile of about 8,000 kilogrammes with a speed sufficient to enable it to escape the earth's attraction—a speed which, according to the theorem of the level plane (§ XLVIII.), is equal to that with which a projectile falling from space without initial velocity would reach the earth, *i.e.*, rather more than 10 kilometres per second. The author, knowing, on the one hand, that he must take into account the resistance of the air, and, on the other, that it is sufficient to reach the point where the attraction of the moon just exceeds that of the earth, fixes the speed at 12,000 yards, or about 11,000 metres per second.

The explosive charge was considered with special care.

Gun-cotton was adopted, and of this 180,000 kilogrammes were used, occupying 60 metres of the barrel of the gun and leaving 210 metres for the shell to travel.

The passengers had to be protected from the shock, and it was thought that no spring would deaden it enough. Therefore a false bottom, enclosing a chamber full of water, was fixed 90 centimetres from the base of the shell. Its purpose was to be compressed during the firing, and to affect the travellers as gradually as possible. Thus everything was perfectly arranged for a successful experiment. We shall not do the author of so many works that have given men of to-day a genuine enthusiasm the injustice of admitting that he believed his experiment possible; he had certainly made an analysis, and had disguised the impossibility with the skill that creates illusion.

We shall not speak of the resistance of the gun, for the calculation of which the knowledge gained from this book would be enough, but shall begin with the question of the powder.

The speed of a projectile increases with the charge; when this is small, the speed is roughly proportional to the mass of the powder divided by that of the projectile. In long guns this quotient is about one quarter, and no great advantage is found in exceeding this limit. Moreover, after a certain value of the quotient, the gain is very slight; on the one hand, when the mass of the gases is comparable with that of the powder, a large part of the energy is used in driving them out; on the other hand, the final limit is the speed with which the gases would escape if they had nothing to force out. In Barbicane's cannon, the Columbiad, the quotient must have been equal to 23, and nine-tenths of the work must have been to no purpose.

Let us consider the shell in the barrel. In a distance of 210 metres the mean acceleration would communicate to the shell a speed of about 11,000 metres per second.

Assuming, as before (§ LVII.), a constant acceleration, we have :—

$$a = \frac{11{,}000^2}{420} = 288{,}000 \text{ m/sec.}^2$$

$$t = \frac{11{,}000}{288{,}000} = \cdot 04 \text{ sec.}$$

Hence the time in the gun would have been four-hundredths of a second, and the mean value of the acceleration 288,000 m/sec.^{2} Every object in the shell would have been at least 29,000 times its usual weight, and a hat of 100 grammes would have crushed its owner under a weight of nearly three tons. The passengers would certainly have been heavy-headed.

True, the water would have offered some protection. But a buffer of any kind acts by diminishing the acceleration; this is enormous, in the impact of solid bodies, because the change of velocity takes place in an extremely short space; to increase the space by some thousandths of a millimetre is often to diminish the force to one-half or one-third; but when the acceleration is to last over a distance of 210 metres, it is not altered greatly by another 90 centimetres; to protect the passengers effectively, the chamber of water would have had to be thousands of kilometres long.

On leaving the gun, the shell is submitted to the action of the air. The data are missing for the speed as calculated by Barbicane; at such high speeds the air would behave like a solid, and would offer a terrible resistance to the motion of the shell. The retardation would become enormous, and the shapeless remains of the unfortunate passengers to the moon, driven into the point of the shell, would have been reduced to pulp. The pressure of the air, together with its consequent rise of temperature, would have transformed the projectile into a shooting star like those that cross the high regions of our atmosphere, pleasing the sight for an instant, and leaving behind them a trail of cosmic dust that disappears the next moment.

CONCLUSION

At the end of our subject, an obstacle appeared before us that neither the power nor the knowledge of man can help us to surmount; the last problem, which opposed to a dream that was perhaps that of our childhood the *non possumus* of reality, showed us that thought, sometimes supreme, is often arrested in its free progress by a sound knowledge of the facts of existence on the earth.

Though our intelligence can reach out to the furthest star whose ray of light tells us its history, we are of necessity restricted to the planet on which we exist, a planet that was for philosophers of ancient times the centre and justification of the whole universe, but is for us a wandering and minute sphere, which is attached for ever to one of the many suns that fill the cosmos, and which is imperatively drawn by it through space.

Science has limited human ambition; it has humbled human pride; it has cast down man from the pedestal he had himself erected; yet it has not disheartened him; for, in place of the vain complacency of which it has deprived him, what understanding has it not offered his mind, what sublime consolation has it not offered his heart!

In the progress of thought towards the divination of natural forces, those facts falling within the domain of Mechanics have been the first, as we have seen, to present themselves as problems for urgent elucidation; by approaching nature in all humility, men of genius, restoring to humanity a little of that pride that it has voluntarily abandoned, have, by prodigous efforts, prepared the splendid recompense of invention and discovery that every day brings forth.

What Mechanics formerly did for the grown-up children it can still do for the children of to-day, who are the men of to-morrow. Therefore, if we are entrusted with the development of its intelligence, we must guide the young generation that is following us in the study of this science in such a way as to prepare it to seize with confidence the instruments that are soon to fall from our hands.

NOTES AND PROBLEMS

It has long been asked whether we ought to admit into Mechanics one or other of the following axioms: (1) The effect of a force on a material point is instantaneous; (2) a force requires a very short time to make its effects felt.

If we regard Mechanics as an artificial science whose only necessity is to be logical, we can add one or other indifferently to the collection of axioms on which it rests; we should thus establish two distinct sciences.

But Mechanics, treated as a natural science, knows no axioms; it is limited to drawing conclusions from experimental facts, simplified and reduced to their greatest probability.

Now, we know that in the ten-thousandth of a second an impact is often finished, and has produced all its local effects; a hundred-thousandth of a second after the surfaces have met, the forces have done their work. According as we consider as of finite or infinite duration a hundred-thousandth of a second or any other shorter period, at the end of which the force becomes just perceptible, we shall support the second or first axiom.

Let us add that what precedes has no meaning if we cannot define the first contact of the bodies independently of the impact. If the latter serves to establish it, the first contact and the evidence of the existence of a force will be, by definition, two simultaneous actions.

As for the transmission of a force within a body, we know it is relatively slow—some hectometres or kilometres per second.

It is necessary to know these and other facts, revealed by experiment, in order to give sound judgment in practical Mechanics, and to apply them in the world of realities. Thereafter, they become infinitely productive, for all visible phenomena bear on the science of forces.

A force applied to a point does not exist in nature; there exist only forces distributed on surfaces, *i.e.*, pressures. A force applied to an infinitely small area would produce an infinitely great pressure, which nothing could resist. The matter, by giving way, would enlarge the surface, and restore the pressure to a finite value.

Three points define a plane; similarly, three points can always be applied to a sufficiently large surface. A fourth point rests where it can. For this reason delicate instruments are supported on the points of three screws that define a triangle within which falls the perpendicular from the centre of gravity of the instrument.

Yet the four feet of a table or chair always rest on the floor. The reason is that the table or chair is deformed and bends until the fourth point of support is adjusted. For similar reasons, two material planes can in practice be applied to each other.

Imagine a tank waggon released on an inclined track, down which it can run freely. What form will the surface of the liquid assume? Reflection on this problem will make clear the difficulty encountered by inventors of instruments to show the gradient of roads to motorists, or that of aerial routes to aviators.

Every one has seen a plum tree shaken. This very efficient operation utilises many mechanical principles. To separate their effects would enable them to be better understood. One of them is analogous to that which results from the cracking of a whip. The energy, travelling along a lash of decreasing mass, produces increasing accelerations. The cord at the end acquires an enormous velocity, the sudden change of which breaks the threads at the extremity.

The great physiologist Marey proposed harness, at the time horse traction was general, to transmit the effort of the horse to the cart by means of the intervention of springs. Such harness would utilise the effort of the horse better, and save it from fatigue. It is interesting to think out the reasons.

When a stretching force is applied to a bar of metal the bar undergoes permanent deformations, which soon localise in a region where a contraction of the section is produced, and where a break finally occurs. If the bar is turned, to reduce it to a uniform section, it is discovered that the second constriction is not produced at the same place as the first. Continuation of the experiment proves that each constriction hardens the metal, which then becomes capable of greater resistance.

This action is particularly intense in certain alloys of steel and nickel, in which the constriction extends of its own accord until the whole of the metal is hardened. Though soft at first, it becomes a hard metal throughout its length, after a considerable elongation. It finally breaks after a tremendous resistance. This action suggests a kind of unity and internal activity of matter.

We are sometimes astonished that the jump of a flea is as high as that of a man. It is easy to show that no great merit is due to the disagreeable parasite. The reason is simple. A hundred million fleas jump as high as a single flea; collectively they have a mass about the same as a man, and muscles that are equivalent. Yet, in the speed of using them, there is a difference that is interesting to analyse.

This problem leads us to think of the similarities and differences between large and small animals. The problem of the fly-wheel (§ XXXVI.) suggests that we cannot argue directly from the small to the great. Recollecting that masses increase as the cubes of their dimensions, and sections as their squares, we realise that the same shapes cannot be retained. For a relatively equal solidity, a large animal will have a heavier appearance than a small animal, *i.e.*, its limbs will be relatively stronger.

In order to retain the advantages of the small animals, the large would have to live on a smaller earth, where the attraction would be less. They could then have smaller legs, or continue to grow while preserving the same forms.

We see, then, that environment exerts a decisive action on living beings. In this connection we may ask what would be the constitution of creatures adapted for life upon Jupiter.

First, they would have to be small, for the attraction is great ; yet they would walk quicker, for the motion of the legs is essentially the oscillation of a pendulum, the period of which is determined by its length. The atmosphere of Jupiter is dense, and, in order to move with ease and speed, the inhabitants would have to possess the shapes of fishes. Then they would not be well supported with two feet, and would require at least four. The hare seems to represent the type that Mechanics indicates as the best for the inhabitant of Jupiter, when life on that gigantic planet is possible.

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