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## Mechanisms with No Regret: Welfare Economics and Informaiton Reconsidered

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College of Commerce and Business Administration
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# Mechanisms with No Regret:
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# MECHANISMS WITH NO REGRET:

*Welfare Economics and Information Reconsidered*

by

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## Abstract

This paper achieves five objectives relating to the design of efficient mechanisms in "Postlewaite-Schmeidler" economies with asymmetric information. First, it is argued that the various notions of implementation used in the literature are either too strong or too weak; an appropriate notion is defined. Second, it is shown that it is impossible to design an individually rational and efficient mechanism using Bayesian equilibrium as the solution concept. Third, a complete characterization is given of "No regret-implementation" using mechanisms with communication and no binding commitments, introduced by Green and Laffont (1987). We propose a further refinement of their *posterior optimal* refinement of Bayesian equilibria. Fourth, it is shown that though such mechanisms cannot implement *interim* individually rational and efficient performance standards, they can implement *ex post* individually rational and efficient standards. Finally, it is shown that even under asymmetric information such mechanisms implement *any* performance standard that is Nash-implementable, such as the core or the Walrasian correspondence. Thus far, this is the only result that demonstrates the possibility of designing efficient mechanisms in general asymmetric information economies.

## 1. Introduction

Imagine an economy with asymmetric information. A *performance standard* is a non-empty set of socially "desirable" state-contingent allocations. Given the incompleteness of information about the realized state of the world, a *mechanism* is required to *implement* a given performance standard. An implementing mechanism is a game of incomplete information such that its set of equilibrium outcomes is non-empty and is contained within the standard. Most economies aspire towards the two basic objectives of individual rationality and efficiency. This paper reports a fundamental difficulty with any attempt to design mechanisms for implementing individually rational-efficient performance standards, and then proposes a solution to this crucial problem. In light of the difficulties that were just alluded to, this is the only result that demonstrates the possibility of designing individually rational-efficient mechanisms in general asymmetric information economies. Moreover, we show that even in asymmetric information environments it is possible to implement any standard that is Nash-implementable, such as the core or the Walrasian correspondence.

Broadly, our strategy will be to proceed in five steps. First, we shall argue that arriving at the appropriate definition of "implementation" is a rather subtle issue and that the bulk of the vast literature on the subject of mechanism design has appealed to definitions that are either too strong or too weak. Second, we show that we can never hope to design a mechanism such that all of its Bayesian equilibrium outcomes are individually rational and efficient (in either an *interim* or an *ex post* sense, as defined in Holmström and Myerson (1983)). In other words, there exist no individually rational-efficient performance standards that are also *Bayesian-implementable*. Third, we explore the implementation properties of "mechanisms with no regret": where the agents do not make binding commitments during the play of the game and an equilibrium is a voluntary signing of a contract representing a binding agreement. We further sharpen a criterion for refinement of Bayesian equilibria due to Green and Laffont (1987) which requires that the components of the equilibrium strategy profile retain their mutual best-response properties even if the players are permitted to revise their strategies after observing the actions of others. A complete characterization of *No Regret-implementability* (which is very different from Green and Laffont's characterization of *posterior implementability*) is provided using our refinement of Green and Laffont's equilibrium as the solution concept. Fourth, we show that there exists a mechanism with no regret that implements the ex post individually rational-efficient standard in the class of problems analyzed here. Interim individually rational-efficient standards still remain non-implementable. Finally, we show that such mechanisms implement *any* Nash-implementable standard.

Juxtaposed with recent literature on the subject, our findings have several interesting implications: (i) We prove some pessimistic conjectures made by Palfrey and Srivastava (1987a) regarding Bayesian-implementation. (ii) We show that the negative conclusions of Green and Laffont (1987) on implementation using mechanisms with no regret can be reversed for the class of problems that we study. (iii) The most widely studied concept of implementation is Nash-implementation whose origins lie in the classic work of Maskin (1977). Maskin's characterization of Nash-implementable standards (also see Saijo (1988)) has been criticized on the grounds that it applies only to complete information settings. Our results show that the class of standards that Maskin had identified as being implementable can be implemented (by mechanisms with no regret) even when information is asymmetric. (iv) Finally, we find that appropriate refinements of Bayesian equilibria broadens the scope mechanism design -- a fact discovered by Palfrey and Srivastava (1987b) for economies with private values and by Moore and Repullo (1988) for economies with complete information.

### *1.1 "Full", "Weak" and Just Plain Implementation*

At the heart of some of the differences between the findings of this paper and those of Palfrey and Srivastava (1987a) and Green and Laffont (1987) lies the definition of implementation itself. Our notion of implementation is as defined in the first paragraph of the paper. This is distinct from the stronger notion of *full implementation* (Maskin (1977), Dasgupta, Hammond and Maskin (1979), Postlewaite and Schmeidler (1986)), which requires that the set of equilibrium outcomes of a mechanism and the performance standard must <u>coincide</u>. On the other hand, there is a notion of *weak implementation*, which simply requires that the set of equilibrium outcomes should <u>contain</u> the standard. (A special case of weak implementation is *truthful implementation* (Myerson (1979), Harris and Townsend (1981)), which is widely used in theoretical and applied models that appeal to the Revelation Principle). As Postlewaite and Schmeidler (1986), Repullo (1986) and others have argued, rather convincingly, weak implementation ignores the possibility that the mechanism could have equilibrium outcomes that lie outside the standard which could, in addition, be more salient (Pareto-superior, for example) than those contained in the standard.

In other words, weak implementation is too weak. We argue that full implementation is, for some purposes, too strong. The latter may be an appropriate concept if one is interested in possibility results since a standard that is fully implementable is also implementable. However, if one is interested in impossibility results, implementation (as defined in the first paragraph of the paper) is the notion that one should focus on: a standard that is not fully implementable may still be implementable, whereas one that is not implementable can never be fully implementable. To summarize, given that both implementability and full implementability are satisfactory from the viewpoint of a mechanism designer, in any study of the limits and possibilities of mechanism design, both the notions deserve our attention.

## *1.2 Efficient Mechanisms*

An *efficient mechanism* is a game such that all of its equilibrium outcomes are efficient. Palfrey and Srivastava (1987a) have shown that neither the interim efficient set nor the ex post efficient set is fully Bayesian-implementable. Disappointing as this may be, this does not imply that we cannot hope to design an efficient mechanism since there still remains the possibility of Bayesian-implementing either the interim or ex post efficient set (and their subsets).

A crucial condition called "Bayesian Monotonicity" is necessary for full Bayesian-implementability. It can be checked that a standard that does not satisfy Bayesian monotonicity can still have subsets that do satisfy the condition. Consider a performance standard $\varphi$ defined such that $\varphi = \varphi' \cup \varphi''$ where $\varphi'$ is Bayesian monotonic and $\varphi''$ is not. By construction, $\varphi$ is not Bayesian monotonic. If $\varphi'$ is fully implementable, and under certain conditions it is, then $\varphi$ is Bayesian-implementable even though it is not fully Bayesian-implementable. To prove a stronger statement which says that neither one of the efficient sets can be Bayesian-implemented, it needs to shown that an even weaker condition, (given in this paper) which is necessary for Bayesian-implementation, is violated. In the paper, we prove such a proposition by imposing an additional restriction of individual rationality. The latter rules out uninteresting outcomes which assign all resources to one individual and yet are efficient and trivially implementable.

### *1.3 Efficient Mechanisms with No Regret*

The next step in the agenda is to address the problem of non-existence of individually rational-efficient mechanisms by exploring the possibilities with mechanisms where agents can communicate but make no binding agreements during the play of the game. To model such mechanisms, we follow the lead of Green and Laffont (1987). Their approach is to pose the following question: which one of the Bayesian equilibria of a normal form game will survive if the agents were not committed to their Bayesian equilibrium actions? An equilibrium that survives is interpreted as a voluntary signing of a contract representing a binding agreement.

Consider a normal form Bayesian game where every player takes into consideration the fact that the equilibrium actions of other players will be observable. Hence, a Bayesian equilibrium strategy profile will survive only if the mutual best response property of its components is not destroyed even after new information is acquired through the observation of the actions of others. Any Bayesian equilibrium that passes this test is a *posterior optimal* agreement in the sense of Green and Laffont.

In this paper, we shall be interested in those agreements that are robust in the following sense. We refine the set of equilibria even further by imposing the following consistency condition: two posterior optimal equilibria s and s′ must not destroy each other, i.e. the information revealed when s is played does not invalidate the optimality of s′ and vice versa. Any equilibrium that survives is a common knowledge potential endogenous source of information. A robust agreement/equilibrium is a list of strategies $s^*$ such that it is common knowledge that there is no other potential source of information that would destroy $s^*$. The fact that agents are not committed *a priori* to any agreement and that we do not expect agents to ignore common knowledge information sources motivates our consistency requirement. The set of posterior optimal equilibria that satisfies this consistency requirement is referred to as the set of *Bayesian equilibria with no regret* in the sequel. The argument behind this can be seen most easily by way of a simple example.

*Example 1:* Consider a game with two players and three states of the world: "Rain", "Shine" and "Cloudy". Player 1 chooses *T* or *B* and is completely uninformed. Her prior beliefs are that there is an equal chance of any one of the states occurring. Player 2 chooses *L*, *M* or *R* and is completely informed. Their payoffs are given in Figure 1. Four Bayesian equilibria of this game are given in Figure 2: $s = (s_1, s_2)$, $s' = (s'_1, s'_2)$, $s'' = (s''_1, s''_2)$ and $s^* = (s^*_1, s^*_2)$.

*[Insert Figures 1 and 2 here.]*

Each player can revise his/her strategy after observing the other player's action. A Bayesian equilibrium will survive Green and Laffont's test if and only if such observation does not lead to a revision of either player's strategy in equilibrium. Since an equilibrium is a pair of strategies that are common knowledge functions, Player 1 could conceivably acquire some information if Player 2's action were associated with a unique state. Observe that the equilibrium s is non-revealing. Given no revision of information, $s$ survives. In the equilibrium $s'$ Player 1 can distinguish "Shine" from the other states. However, $s'_1$ continues to remain a best response to $s'_2$ even though Player 1 can partition the state space in the following way: {{Rain, Cloudy}, {Shine}}. Thus, $s'$ also survives. Next, check that in the equilibrium $s''$ Player 1 can distinguish "Cloudy" from the other states. Given this new information, $s''_1$ is no longer a best response to $s''_2$ since in the "Cloudy" state, Player 1 would do better by switching to $T$. Thus, the Bayesian equilibrium $s''$ is not posterior optimal. Finally, check that $s^*$ is fully revealing and even under complete information $s^*_1$ is a best response to $s^*_2$. Therefore $s^*$ survives too.

Thus, we are left with three equilibria $s$, $s'$ and $s^*$ that are posterior optimal and are, therefore, three potential agreements according to Green and Laffont. We shall argue that $s$ and $s'$ may not be expected to survive. Suppose that $s$ is being considered as a potential agreement between the players. Check that $s^*$ cannot be destroyed by any new information and, therefore, represents an agreement that is a Bayesian equilibrium with no regret. Thus, it is common knowledge that there is an agreement $s^*$ which is fully revealing. Since there is no binding commitment, Player 1 could wait for $s^*$ to be played and then withdraw her commitment to $s^*$ and evaluate $s$ in light of the complete information revealed by the play of $s^*$. $s_1$ is no longer a best response to $s_2$ since in the "Cloudy" state Player 1 would do better by switching to $B$. $s'$ is also destroyed by the same argument. In a normal form game, the players make all these calculations before playing the game as if the sequence of play described above were possible, and in one shot arrive at an equilibrium. This formulation and the consistency condition appears to be a natural consequence of Green and Laffont's model of games with no binding commitments.

For arbitrary games, in general, the set of Bayesian equilibria with no regret may be empty. However, we shall prove existence of such a set for any mechanism that we use for our implementation results.

Green and Laffont's notion of posterior-implementability is a form of weak implementation with their refinement of equilibrium as the solution concept. They argue that mechanisms for posterior implementation have two limitations in general: (a) such mechanisms cannot posterior implement any performance standard that cannot be weakly Bayesian-implemented; and (b) in two-person economies, such mechanisms can posterior implement only those standards that can take on essentially two values throughout the range of observations of the players. We hope to show that neither (a) nor (b) is any cause for alarm. Green and Laffont's mechanisms can actually do better than Bayesian-implementation mechanisms. This apparent contradiction can be resolved by keeping in mind the fact that while Green and Laffont's analysis relates to weak implementation, we focus on implementability itself. As the Postlewaite and Schmeidler (1986), Repullo (1986) examples demonstrate, the latter approach is the more compelling one.

There are other differences between our framework and that of Green and Laffont: while they analyze a two-person abstract choice problem, we focus on a "Postlewaite-Schmeidler" economy (Postlewaite and Schmeidler (1986)), i.e. an $n$-person ($n > 2$) Arrow-Debreu-McKenzie pure exchange economy with asymmetric and non-exclusive information about individual preferences; they consider only singleton-valued performance standards while we analyze the more general case; in Green and Laffont's model, private information can take on uncountable values ranging from the "best news" to the "worst news", while we consider private information about a finite set of states of the world with no explicit ranking of the states.

As noted earlier, Palfrey and Srivastava's (1987b) results imply that, given non-exclusive information, it is possible to fully implement efficient performance in economies with private values by considering undominated Bayesian equilibrium as the solution concept. The more general situation, i.e. economies with common values still suffered from the non-existence of efficient mechanisms. Our findings offer a solution to this problem.

## 2. Preliminaries

An *asymmetric information economy*, $e$, is a triple $\{L, N, \Theta\}$. $L$ is a *set of goods*, $N$ is a *set of agents* and $\Theta$ is a *set of states of the world*. All of these sets are assumed to be non-empty and finite and the cardinalities of $L$ and $N$ are given by $\ell$ and $n$, respectively. $\mathcal{E}$ is the domain of all asymmetric information economies. In the definitions that follow, we focus on a given $e \in \mathcal{E}$. An explicit reference to $e$ is dropped to minimize notational burden.

Let $e = \{L, N, \Theta\}$ be given. Every agent $i \in N$ is completely characterized by a list $(u_i, \omega_i, \Pi_i, q_i^*)$, where $u_i\colon \mathbb{R}_+^\ell \times \Theta \to \mathbb{R}$ is *agent i's von Neumann-Morgenstern utility function*; $\omega_i$ $(\neq 0) \in \mathbb{R}_+^\ell$ is *agent i's initial endowment of goods*; $\Pi_i$ is *agent i's natural information partition of* $\Theta$ and $q_i^*\colon \Theta \to (0, 1)$ is *agent i's prior probability distribution on* $\Theta$. Each constituent of this list is assumed to be given exogenously, and is common knowledge in the sense of Aumann (1976). Let the function $I_i^o\colon \Theta \to \Pi_i$ be defined by $I_i^o(\theta) \equiv \{\theta' \in \Theta\colon$ there exists $\pi_i \in \Pi_i$ such that $\theta, \theta' \in \pi_i\}$. The latter is *agent i's natural information set in state* $\theta$. By "natural" information we refer to the information structure that the agent is exogenously endowed with. This distinguishes it from the information that can be acquired endogenously. In the sequel, let $\Pi \equiv \times_{i \in N} \Pi_i$ and $\Omega \equiv$

$\Sigma_{i \in N} \omega_i$. In addition, unless specified otherwise, $x \equiv (x_i)_{i \in N}$ and $x_{-i} \equiv (x_j)_{j \in N \setminus \{i\}}$.

Let $\mathbb{P}(X)$ denote the set of non-empty subsets of X. *Agent i's posterior probability distribution* is the function $q_i$: $\Theta \times \mathbb{P}(\Theta) \to [0, 1]$ defined by Bayes' Law, i.e. for all $\theta \in \Theta$ and for all $\mathcal{I} \in \mathbb{P}(\Theta)$,

$$q_i(\theta, \mathcal{I}) = \begin{cases} \dfrac{q_i^*(\theta)}{\Sigma_{\theta' \in \mathcal{I}} q_i^*(\theta')}, & \text{if } \theta \in \mathcal{I}; \\ 0, & \text{otherwise.} \end{cases}$$

*Agent i's expected utility from* $f \in F$, *given* $\mathcal{I} \in \mathbb{P}(I_i(\theta))$ is $\sum_{\theta' \in \mathcal{I}} q_i(\theta', \mathcal{I}) u_i(f_i(\theta'), \theta')$, and is written more compactly as $EU_i(f \mid \mathcal{I})$. *Agent i's* $\mathcal{I}$*-expected lower contour set at f* is given by $EL_i(f \mid \mathcal{I}) \equiv \{g \in F: EU_i(f \mid \mathcal{I}) \geq EU_i(g \mid \mathcal{I})\}$.

The domain under consideration, $\mathcal{E}$ is defined as the collection of all economies $e = \{L, N, \Theta\} \in \mathcal{E}$, that satisfy the following:

[A1] (*strict monotonicity of preferences*) $\forall i \in N$, $\forall \theta \in \Theta$, $u_i(.,\ .)$ is strictly increasing in $z_i \in \mathbb{R}_+^{\ell}$, and

[A2] (*non-exclusivity of information*) $\forall i \in N$, $\forall \theta \in \Theta$, $\bigcap_{j \in N \setminus \{i\}} I_j^o(\theta) = \{\theta\}$.

[A3] $|N| > 2$.

The assumption A2 implies that once $n - 1$ agents pool their private information, they can tell exactly what information the last agent has. Such an assumption is clearly restrictive. However, as discussed in Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1987a) and Blume and Easley (1987) some such restriction is required to analyze implementability using Bayesian equilibrium or one of its refinements. This is a consequence of the Revelation Principle (for a full discussion, see the references given). Presumably, the A2 restriction can be mildly weakened. However, given that it is still descriptive of a large class of interesting problems with asymmetric information, as in these earlier papers, our focus will not be on weakening this restriction.

$A \equiv \{z \in \mathbb{R}_{+}^{\ell n}: \sum_{i\in N} z_i \leq \Omega\}$ is the *set of feasible allocations*. A *state-contingent allocation* is a random variable $f: \Theta \to A$. $F$ is the domain of such functions. A *performance standard* $\varphi$ is a non-empty subset of $F$ such that for all $\theta \in \Theta$, $0 \notin \varphi(\theta)$. $\Phi$ is the class of all performance standards.

A *mechanism* is a game $\Gamma = \{N, M, \xi\}$, where, given that $M_i$ is agent $i$'s *message (or action) space*, $M \equiv \times_{i\in N} M_i$; and $\xi: M \to A$ *is an outcome function*. *Agent $i$'s strategy* is a random variable $s_i: \Theta \to M_i$ such that $s_i$ is $\Pi_i$-measurable. Let $S_i$ be the domain of such functions. Let $S \equiv \times_{i\in N} S_i$.

## 3. Bayesian Equilibria with No Regret

The fundamental solution concept for games with asymmetric information is that of Bayesian equilibrium due to Harsanyi (1967). This concept is defined as follows:

$s \in S$ is a *Bayesian equilibrium of* $\Gamma = \{N, M, \xi\}$ if $\forall i \in N$, $\forall \theta \in \Theta$, $\forall s_i' \in S_i$,

$$\xi\circ(s_i', s_{-i}) \in EL_i(\xi\circ s \mid I_i^o(\theta)).$$

Let $E^o(\Gamma)$ denote the *set of Bayesian equilibria of* $\Gamma$ and $E_F^o(\Gamma) \equiv \{\xi\circ s \in F: s \in E^o(\Gamma), \Gamma = \{N, M, \xi\}\}$.

An implicit assumption underlying the Bayesian equilibrium concept is that the agents are committed to their equilibrium strategies. In this paper, we shall drop this assumption and attempt to identify the set of Bayesian equilibria that survive when the agents are not committed to their messages and can revise their strategies after having observed the actions/messages of others.

Let $I_i(\theta, s) \equiv \{\theta' \in I_i^o(\theta): \forall j \in N\backslash\{i\},\ s_j(\theta) = s_j(\theta')\}$. Green and Laffont (1987) propose the following refinement of Bayesian equilibrium for games with no binding commitments:

$s \in S$ is a *posterior optimal Bayesian equilibrium of* $\Gamma = \{N, M, \xi\}$ if $\forall i \in N$, $\forall \theta \in \Theta$, $\forall s_i' \in S_i$,

$$\xi\circ(s_i', s_{-i}) \in EL_i(\xi\circ s \mid I_i^o(\theta)) \bigcap EL_i(\xi\circ s \mid I_i(\theta, s)).$$

Let $E^*(\Gamma)$ denote the *set of posterior optimal Bayesian equilibria of* $\Gamma$.

The *set of Bayesian equilibria with no regret*, denoted $E(\Gamma)$, is a subset of $E^*(\Gamma)$ that satisfies the following condition:

$\forall s, s' \in E(\Gamma)$, $\forall i \in N$, $\forall \theta \in \Theta$, $\forall s_i'' \in S_i$,

$$\xi\circ(s_i'', s_{-i}) \in EL_i(\xi\circ s \mid I_i(\theta, s')).$$

Let $E_F(\Gamma) \equiv \{\xi\circ s \in F: s \in E(\Gamma),\ \Gamma = \{N, M, \xi\}\}$. We interpret $E(\Gamma)$ as the set of binding agreements which each agent will voluntarily endorse. A game with no commitment which has a non-empty set of such agreements is a *mechanism with no regret*.

## 4. Bayesian-Implementation and Welfare Implications

The classical approach to welfare economics has been to identify the subset of allocations that are Pareto-efficient within the set of all physically and technologically feasible allocations. Among these efficient allocations, attention is generally focused on ones that are individually rational. However, in asymmetric information economies, these welfare evaluations must also take account of informational constraints -- an uninformed social planner or the group of agents as a whole cannot identify the set of individually rational-efficient allocations in the absence of complete information about the state of the world which affects the agents' preferences. The notion of an individually rational-efficient allocation would vary depending on the extent of insurance that the allocation provides each agent. Individual rationality and Pareto-efficiency are thus extended to take account of different levels of insurance and the appropriate notion depends on the timing of the welfare analysis (see Holmström and Myerson (1983) for a detailed discussion). The two primary concepts of efficiency are:

*Interim-efficiency*: A state-contingent allocation $f$ is *interim-efficient* if there is no $g \in F$ such that $\forall i \in N$, $\forall \theta \in \Theta$, $f \in EL_i(g \mid I_i^o(\theta))$ and $f \in \text{int}(EL_i(g \mid I_i^o(\theta))$ for some $i \in N$ and some $\theta \in \Theta$.

*Ex post-efficiency*: A state-contingent allocation $f$ is *ex post-efficient* if there is no $g \in F$ such that $\forall i \in N$, $\forall \theta \in \Theta$, $f \in EL_i(g \mid \{\theta\})$ and $f \in \text{int}(EL_i(g \mid \{\theta\})$ for some $i \in N$ and some $\theta \in \Theta$.

Given $w \in F$ defined by $w(\theta) = \omega$ for all $\theta \in \Theta$, let $\mathcal{P}^i \equiv \{f \in F\colon f$ is interim efficient and $\forall i \in N$, $\forall \theta \in \Theta$, $w \in EL_i(f \mid I_i^o(\theta))\}$ and $\mathcal{P}^e \equiv \{f \in F\colon$ $f$ is ex post efficient and $\forall i \in N$, $\forall \theta \in \Theta$, $w \in EL_i(f \mid \{\theta\}\}$ denote, respectively, the *sets of interim individually rational-efficient* and *ex post individually rational-efficient performance standards*.

Once a social planner decides on the appropriate notion of efficiency, the question of implementing an efficient performance standard arises. Once again, the informational asymmetry poses a constraint. A naive mechanism in which each agent is asked to report his/her private information to the planner will generally not ensure truth-telling as the unique equilibrium strategy profile. Thus, we need to be guaranteed the existence of a mechanism that implements the given standard. This is defined (for the case where Bayesian equilibrium is the solution concept) as follows:

A performance standard $\varphi$ is *Bayesian-implementable by* $\Gamma$ *in* $e = \{L, N, \Theta\}$ if $\emptyset \neq E^{o}_{F}(\Gamma) \subseteq \varphi$.

A performance standard $\varphi$ is *Bayesian implementable* (in a global sense) if $\forall e \in \mathcal{E}$, $\exists \Gamma$ such that $\varphi$ is Bayesian-implementable by $\Gamma$ in $e$.

Remark: *Full* Bayesian-implementation of $\varphi$ by $\Gamma$ requires that $E^{o}_{F}(\Gamma) = \varphi$, whereas *weak* Bayesian-implementation of $\varphi$ by $\Gamma$ requires that $\varphi \subseteq E^{o}_{F}(\Gamma)$.

Next, we present a condition that is both necessary and, in economies in $\mathcal{E}$, sufficient for a performance standard to be Bayesian-implementable. Before we present the condition, some additional definitions are needed. We shall use the approach of Postlewaite and Schmeidler (1986) and Palfrey and Srivastava (1987) to define a "collection of compatible manipulation operators". The intuition behind these operators is simple. Consider a mechanism in which each agent is asked to report his/her initial information set to the game designer as part of a message. If the realized state of the world is $\theta$, then agent $i$ observes $I^{o}_{i}(\theta)$. The game designer cannot observe this set. By the common knowledge assumption, agent $i$'s natural information partition $\Pi_i$ is observable. Second, the assumption A2 is common knowledge. These two facts constrain the extent to which agent $i$ can manipulate his/her report of his/her private information. An $n$-tuple of manipulated reports will fool the game designer only if the manipulation is "compatible" with the common knowledge structure. This is formalized in the definitions below.

A *collection of compatible manipulation operators for* $\Pi$ *(CCMO)*, denoted $\alpha = (\alpha_i)_{i \in N}$, is defined by

$$\text{(i) } \forall i \in N,\ \alpha_i\colon \Pi_i \to \Pi_i,$$

$$\text{(ii) } \forall \pi \in \Pi,\ \{\textstyle\bigcap_{i \in N} \pi_i \neq \emptyset\} \Rightarrow \{\textstyle\bigcap_{i \in N} \alpha_i(\pi_i) \neq \emptyset\}.$$

Let $\theta^{\alpha}\colon \Theta \to \Theta$ be the *deception induced by* $\alpha$ and defined by $\theta^{\alpha}(\theta) \equiv$

$\bigcap_{i \in N} \alpha_i(I_i^o(\theta))$. By assumption A2, $\theta^\alpha$ is a well-defined function.

A performance standard $\varphi$ satisfies *Property M* if the following is true:

$\exists \varphi' \in \Phi$ such that $\varphi' \subseteq \varphi$ and $\forall f \in F$, $\forall$CCMO's $\alpha$,

if (i) $f \in \varphi'$ and

(ii) $\forall i \in N$, $\forall\, g \in F$, $\forall \theta \in \Theta$, $\forall \theta' = \theta^\alpha(\theta)$, $\langle g \in EL_i(f \mid I_i^o(\theta')) \rangle \Rightarrow \langle g \circ \theta^\alpha \in EL_i(f \circ \theta^\alpha \mid I_i^o(\theta)) \rangle$,

then $f \circ \theta^\alpha \in \varphi'$.

**Remark:** $\varphi$ satisfies *Bayesian monotonicity* if the Property M is modified as follows: $\forall f \in F$, $\forall$CCMO's $\alpha$, if (i)′ and (ii)′ imply $f \circ \theta^\alpha \in \varphi$, where (i)′ $f \in \varphi$ and (ii)′ is the same as (ii) in the definition above.

**Fact 1:** *There exists $\varphi \in \Phi$ such that $\varphi$ satisfies Property M and violates Bayesian monotonicity.*

To check that this true, simply choose $\varphi$ such that it is the union of a Bayesian monotonic set $\varphi' \in \Phi$ and a set $\varphi'' \in \Phi$ which is not Bayesian monotonic. For examples of such sets, see Palfrey and Srivastava (1987a).

**Fact 2:** *Let $e = \langle L, N, \Theta \rangle$ be an economy in $\mathcal{E}$. A performance standard $\varphi$ is fully Bayesian-implementable by a game in e if and only if it satisfies Bayesian monotonicity.*

Proof: See Palfrey and Srivastava (1985). ■

The following proposition provides a parallel characterization of Bayesian-implementability.

**Proposition 1:** *Let $e = \langle L, N, \Theta \rangle$ be an economy in $\mathcal{E}$. A performance standard $\varphi$ is Bayesian-implementable by a game in e if and only if it satisfies Property M.*

Proof: $\varphi$ is Bayesian-implementable if and only if there exists $\varphi' \in \Phi$ such that $\varphi' \subseteq \varphi$ and $\varphi'$ is fully Bayesian-implementable. Given Fact 2, the proposition follows from the definitions. ∎

In conjunction with the facts given above, the proposition has an interesting implication: a performance standard may be Bayesian-implementable even though it violates Bayesian monotonicity.

Palfrey and Srivastava's (1987a) examples suggest that the $\mathcal{P}^i$ and $\mathcal{P}^e$ sets are not fully Bayesian-implementable. However, disappointing as this may be, we are interested in a more crucial question: can we ever hope to design a mechanism so that all of its Bayesian equilibrium outcomes are individually rational-efficient (in either an interim or an ex post sense)? In other words, is it possible to Bayesian-implement (i.e. to fully Bayesian-implement <u>some</u> subset of) $\mathcal{P}^i$ or $\mathcal{P}^e$ in the domain of economies that Palfrey and Srivastava have examined? It must be noted that their result does not answer this latter question. The following results confirm that the answer to the question is, indeed, negative.

The theorems are proved by way of counterexamples. We shall present simple ones using linear economies. The negative results do not depend on the assumption of linearity and hold for other utility specifications too.

Theorem 1: *If $\varphi \subseteq \mathcal{P}^i$, then $\varphi$ is not Bayesian-implementable.*

<u>Proof:</u> Consider the following example:

*Example 2:* We define $e = \{L, N, \Theta\}$ as follows. Let $L = \{X, Y\}$ with the quantities of the two goods being denoted by the corresponding lower case letters. Let $N = \{1, 2, 3, 4\}$ and let $\Theta = \{a, b, c\}$. $\Pi_1 = \Pi_2 = \{(a), (b), (c)\}$ and $\Pi_3 = \Pi_4 = \{(a), (b, c)\}$, i.e. agents 1 and 2 are fully informed and agents 3 and 4 cannot distinguish between states $b$ and $c$ when one of them occurs. An allocation $z$ is written as $(x_i, y_i)_{i\in N}$. $\omega = ((0, 1), (0, 1), (1, 0), (1, 0))$. Each state is equally likely, i.e. $q^*_i(a) = q^*_i(b) = q^*_i(c) = \frac{1}{3}$ for all $i \in N$. $u_i$: $\mathbb{R}^{\ell}_{+} \times \Theta \to \mathbb{R}$ is given as follows:

$\forall i \in \{1, 2\}, \forall\theta \in \Theta, \quad u_i(z_i, \theta) = \quad x_i + 1.1y_i,$

$$u_3(z_3, \theta) = \begin{cases} (x_3 + y_3), & \text{if } \theta = a \\ 0.25(x_3 + y_3) & \text{if } \theta = b \\ 0.75(x_3 + y_3) & \text{if } \theta = c. \end{cases}$$

$$u_4(z_4, \theta) = \begin{cases} (x_4 + y_4), & \text{if } \theta = a \\ 0.75(x_4 + y_4) & \text{if } \theta = b \\ 0.25(x_4 + y_4) & \text{if } \theta = c. \end{cases}$$

It can be checked that $\mathcal{P}^i \neq \emptyset$. Consider the following state-contingent allocation, denoted $f^*$:

| | $a$ | $b$ | $c$ |
|---|---|---|---|
| $z_1 =$ | (0, 1) | (0, 1) | (0, 1) |
| $z_2 =$ | (0, 1) | (0, 1) | (0, 1) |
| $z_3 =$ | (1, 0) | (0, 0) | $(\frac{5}{3}, 0)$ |
| $z_4 =$ | (1, 0) | (2, 0) | $(\frac{1}{3}, 0)$ |

It can be checked that $f^* \in \mathcal{P}^i$. Also check that assumptions A1 and A2 are satisfied.

Choose any $\varphi \in \Phi$ such that $\varphi \subseteq \mathcal{P}^i$. We shall show that the hypothesis of Property M is satisfied. Consider the function $\alpha_i\colon \Pi_i \to \Pi_i$ for all $i \in N$ defined by $\alpha_i(\pi_i) = \{a\}$ for all $i \in N$ and all $\pi_i \in \Pi_i$. Thus, $\alpha$ is a CCMO and for all $\theta \in \Theta$, $\theta^\alpha(\theta) = a$. Pick $f \in \varphi$ and $g \in F$. Write $f(a)$ as $(x, y)$ and $g(a)$ as $(x', y')$. By construction, the utility functions of agents 1 and 2 are state-independent. Therefore, given that they are completely informed, the relevant portion of the hypothesis of Property M is trivially satisfied for these two agents. The following relationships imply that the relevant portion of the hypothesis of Property M is met for the remaining agents:

$$x_3 + y_3 \geq x_3' + y_3' \Rightarrow 0.5[0.25(x_3 + y_3) + 0.75(x_3 + y_3)] \geq 0.5[0.25(x_3' + y_3') + 0.75(x_3' + y_3')].$$

$$x_4 + y_4 \geq x_4' + y_4' \Rightarrow 0.5[0.75(x_4 + y_4) + 0.25(x_4 + y_4)] \geq 0.5[0.75(x_4' + y_4') + 0.25(x_4' + y_4')].$$

For $\mathcal{P}^i$ to satisfy Property M, we must have $f \circ \theta^\alpha \in \varphi$. The following allocation is recommended by $f \circ \theta^\alpha$:

| | $b$ | $c$ |
|---|---|---|
| Agent 3 | $(x_3, y_3)$ | $(x_3, y_3)$ |
| Agent 4 | $(x_4, y_4)$ | $(x_4, y_4)$ |

By interim individual rationality of $f$, $(x_3, y_3) \neq (0, 0)$. By the construction of this example and given that $f \in \varphi \subseteq \mathcal{P}^i$, $x_3 > 0$ and $x_4 > 0$. Given $\min\{x_3, x_4\} > \varepsilon > 0$, consider an alternative rule $h \in F$ defined by

$h(a) = f(a)$,

$\forall i \in \{1, 2\}$, $\forall \theta \in \{b, c\}$, $h_i(\theta) = f_i(a)$,

$h_3(b) = (x_3 - \varepsilon, y_3)$, $\quad h_3(c) = (x_3 + \frac{\varepsilon}{3}, y_3)$,

$h_4(b) = (x_4 + \varepsilon, y_4)$, $\quad h_4(c) = (x_4 - \frac{\varepsilon}{3}, y_4)$.

Thus, $EU_3(h \mid \{b, c\}) = 0.5[0.25(x_3 - \varepsilon + y_3) + 0.75(x_3 + \frac{\varepsilon}{3} + y_3)] = 0.5[0.25(x_3 + y_3) + 0.75(x_3 + y_3)] = EU_3(f \circ \theta^\alpha \mid \{b, c\})$. Trivially, for $i \in \{1, 2\}$ and all $\theta \in \Theta$, $EU_i(h \mid \{\theta\}) = EU_i(f \circ \theta^\alpha \mid \{\theta\})$. Also, trivially, for $i \in \{3, 4\}$, $EU_i(h \mid \{a\}) = EU_i(f \circ \theta^\alpha \mid \{a\})$. However, $EU_4(h \mid \{b, c\}) = 0.5[0.75(x_4 + \varepsilon + y_4) + 0.25(x_4 - \frac{\varepsilon}{3} + y_4)] > 0.5[0.25(x_4 + y_4) + 0.75(x_4 + y_4)] = EU_4(f \circ \theta^\alpha \mid \{b, c\})$. Thus, $f \circ \theta^\alpha \notin \mathcal{P}^i$.

For any $\varphi \subseteq \mathcal{P}^i$ with $f \in \varphi$, we find that $f \circ \theta^\alpha \notin \varphi$. Thus, $\mathcal{P}^i$ violates Property M. By Proposition 1, it is not Bayesian-implementable in $e$. ■

Theorem 2: *If $\varphi \subseteq \mathcal{P}^e$, then $\varphi$ is not Bayesian-implementable.*

Proof: The proof is by a counter-example. Consider an economy $e'$ which retains all the specifications of $e$ from Example 2 above with one exception: agent 3's utility function is altered as follows:

$$u_3(z_3, \theta) = \begin{cases} (x_3 + y_3), & \text{if } \theta = a \\ (x_3 + 1.5y_3) & \text{if } \theta = b \\ (x_3 + 0.5y_3) & \text{if } \theta = c. \end{cases}$$

Given these specifications, $\mathcal{P}^e \neq \emptyset$. Choose $\alpha_i: \Pi_i \to \Pi_i$ as in the previous proof, i.e. for all $i \in N$, for all $\pi_i \in \Pi_i$, $\alpha_i(\pi_i) = \{a\}$. Thus, $\alpha$ is a CCMO and for all $\theta \in \Theta$, $\theta^\alpha(\theta) = a$. Choose any $\varphi \in \Phi$ such that $\varphi \subseteq \mathcal{P}^e$ and pick $f \in \varphi$. Next, we establish that the hypothesis of Property M is met. From the relationships derived in the proof of Theorem 1, we know that the relevant relationships for agents 1, 2 and 4 hold. To check that the relevant relationship holds for agent 3, consider the following observations:

Pick $f \in \varphi$ and $g \in F$. Write $f(a)$ as $(x, y)$, and $g(a)$ as $(x', y')$. Thus, we have

$$x_3 + y_3 \geq x'_3 + y'_3 \Rightarrow 0.5[(x_3 + 1.5y_3) + (x_3 + 0.5y_3)] \geq 0.5[(x'_3 + 1.5y'_3) + (x'_3 + 0.5y'_3)].$$

Thus, the hypothesis of Property M is satisfied. For $\mathcal{P}^e$ to satisfy Property M, we must have $f \circ \theta^\alpha \in \varphi$. The following allocation is recommended by $f \circ \theta^\alpha$:

| | $a$ | $b$ |
|---|---|---|
| Agent 1 | $(x_1, y_1)$ | $(x_1, y_1)$ |
| Agent 3 | $(x_3, y_3)$ | $(x_3, y_3)$ |

By ex post individual rationality of $f$, $(x_3, y_3) \neq (0, 0)$. By the construction of this example and given that $f \in \varphi \subseteq \mathcal{P}^e$, $x_3 > 0$ and $y_1 > 0$. Given $\min\{\frac{10}{11}x_3, y_1\} > \varepsilon > 0$, consider an alternative rule $h \in F$ defined by:

$h_1(b) = (x_1 + 1.1\varepsilon, y_1 - \varepsilon)$

$h_3(b) = (x_3 - 1.1\varepsilon,\ y_3 + \varepsilon)$

$\forall i \in \{1, 3\}, \forall \theta \in \{a, c\}, h_i(\theta) = f_i(a)$

$\forall i \in \{2, 4\}, h_i = f_i \circ \theta^\alpha$.

Thus, $EU_3(h \mid \{b\}) = x_3 - 1.1\varepsilon + 1.5\varepsilon + 1.5y_3 = x_3 + 0.4\varepsilon + 1.5y_3 > x_3 + 1.5y_3 = EU_3(f \circ \theta^\alpha \mid \{b\})$ and $EU_1(h \mid \{b\}) = x_1 + 1.1\varepsilon + 1.1y_1 - 1.1\varepsilon = x_1 + 1.1y_1 = EU_1(f \circ \theta^\alpha \mid \{b\})$. For all $i \in N$ and all $\theta \in \Theta$, $f \circ \theta^\alpha \in EL_i(h \mid \{\theta\})$. Thus, $f \circ \theta^\alpha \notin \varphi \subseteq \mathcal{P}^e$. Given that this holds for all $\varphi \subseteq \mathcal{P}^e$, $\mathcal{P}^e$ does not satisfy Property M. By Proposition 1, it is not Bayesian-implementable in $e'$. ■

## 5. No Regret Implementation

In this section, we discuss the implementability properties of mechanisms with no regret and provide necessary and sufficient conditions for this method of implementation.

A performance standard $\varphi$ is *No regret-implementable (NR-implementable) by* $\Gamma$ *in* $e = \{L, N, \Theta\}$ if $\varnothing \neq E_F(\Gamma) \subseteq \varphi$.

A performance standard $\varphi$ is *NR-implementable* (in a global sense) if $\forall e \in \mathcal{E}$, $\exists \Gamma$ such that $\varphi$ is NR-implementable by $\Gamma$ in $e$.

**Remark:** $\varphi$ is *fully NR-implementable* by $\Gamma$ if $E_F(\Gamma) = \varphi$.

Next, we shall define some properties that shall be used to characterize NR-implementable standards.

A performance standard $\varphi$ satisfies *Property M1 with respect to* $\Gamma = (N, M, \xi)$ if the following is true:

$\exists \varphi' \in \Phi$ such that $\varphi' \subseteq \varphi$ and $\forall f \in F$, $\forall$CCMO's $\alpha$,

<u>if</u> (i) $f \in \varphi'$

(ii) $\forall i \in N$, $\forall g \in F$, $\forall \theta \in \Theta$, $\forall \theta' = \theta^\alpha(\theta)$, $\{\forall s \in E(\Gamma)$, $g \in EL_i(f \mid$

$I_i^o(\theta')) \bigcap EL_i(f \mid I_i(\theta', s))\rangle \Rightarrow \langle \forall s' \in S,\ g\circ\theta^{\alpha} \in EL_i(f\circ\theta^{\alpha} \mid I_i^o(\theta)) \bigcap EL_i(f\circ\theta^{\alpha} \mid I_i(\theta, s'))\rangle$,

<u>then</u> $f\circ\theta^{\alpha} \in \varphi'$.

A performance standard $\varphi$ satisfies *Property M2 with respect to* $\Gamma$ if $\forall f \in \varphi$, $\forall$CCMO's $\alpha$, (i)' and (ii)' imply $f\circ\theta^{\alpha} \in \varphi$, where (i)' $f \in \varphi$ and (ii)' is the same as (ii) in the definition above.

Green and Laffont (1987) present a characterization of "posterior implementable" performance standards in two-person economies and show that a posterior implementable standard can take essentially only two values throughout the range of observations of the two agents. We shall attempt a parallel characterization of NR-implementable performance standards.

**Theorem 3:** *Let $e = \langle L, N, \Theta\rangle$ be an economy in $\mathcal{E}$, and let $\varphi$ be a performance standard. If $\varphi$ is NR-implementable by $\Gamma$ in e, then $\varphi$ satisfies Property M1 with respect to $\Gamma$.*

<u>Proof:</u> By definition of NR-implementation by a game $\Gamma = \{N, M, \xi\}$, there exists $\varphi' \subseteq \varphi$ such that $\varphi'$ is fully NR-implemented by $\Gamma$. Thus, for all $f \in \varphi'$ there exists $s \in E(\Gamma)$ that $f = \xi\circ s$. This means that the following holds for all $i \in N$, for all $\theta \in \Theta$, for all $s_i' \in S_i$, for all $s'' \in E(\Gamma)$:

$$\xi\circ(s_i', s_{-i}) \in EL_i(f \mid I_i^o(\theta)) \bigcap EL_i(f \mid I_i(\theta, s'')) \qquad (1)$$

Choose a CCMO $\alpha$. Next, suppose that for all $i \in N$, for all $\theta \in \Theta$, for all $\theta' = \theta^{\alpha}(\theta)$, for all $s'' \in E(\Gamma)$, for all $g \in EL_i(f \mid I_i^o(\theta')) \bigcap EL_i(f \mid I_i(\theta', s''))$, the following holds for all $s^* \in S$:

$$g\circ\theta^{\alpha} \in EL_i(f\circ\theta^{\alpha} \mid I_i^o(\theta)) \bigcap EL_i(f\circ\theta^{\alpha} \mid I_i(\theta, s^*)) \qquad (2)$$

Given (1) and (2), for all $i \in N$, for all $\theta \in \Theta$, for all $s_i' \in S_i$, for all $s^* \in S$, the following holds:

$$\xi\circ(s_i', s_{-i})\circ\theta^{\alpha} \in EL_i(f\circ\theta^{\alpha} \mid I_i^o(\theta)) \bigcap EL_i(f\circ\theta^{\alpha} \mid I_i(\theta, s^*)) \qquad (3)$$

By definition of $\alpha$, $s_i\circ\theta^{\alpha}$ is $\Pi_i$-measurable for all $i \in N$. By definition of

Bayesian equilibrium with no regret, (3) implies that $s \circ \theta^{\alpha} \in E(\Gamma)$ and $f \circ \theta^{\alpha} \in E_F(\Gamma)$. By definition of full NR-implementation of $\varphi'$, $E_F(\Gamma) \subseteq \varphi'$. Thus, $f \circ \theta^{\alpha} \in \varphi'$ and $\varphi$ satisfies Property M1. ■

**Theorem 4:** *Let $e = \langle L, N, \Theta \rangle$ be an economy in $\mathcal{E}$ and let $\varphi$ be a performance standard. There exists $\Gamma$ such that if $\varphi$ satisfies Property M2 with respect to $\Gamma$, then $\varphi$ is NR-implementable by $\Gamma$ in $e$.*

Proof: The proof of this statement derives from the following lemmata.

**Lemma 1:** *There exist $\Gamma$ and $s \in E(\Gamma)$ such that for all $i \in N$, for all $\theta \in \Theta$, $I_i(\theta, s) = \langle \theta \rangle$.*

**Lemma 2:** *There exists $\Gamma$ that satisfies Lemma 1 such that if $\varphi$ satisfies Property M2 with respect to $\Gamma$, then $E_F(\Gamma) \subseteq \varphi$.*

The proofs of these results are given in the appendix. ■

**Corollary to Theorem 4:** *Let $e = \langle L, N, \Theta \rangle$ be an economy in $\mathcal{E}$, and let $\varphi$ be a performance standard. If $\varphi$ is fully NR-implementable by $\Gamma$ in $e$, then $\varphi$ satisfies Property M2 with respect to $\Gamma$. Conversely, there exists $\Gamma$ such that if $\varphi$ satisfies Property M2 with respect to $\Gamma$, then $\varphi$ is fully NR-implementable by $\Gamma$ in $e$.*

## 6. No Regret Implementation and Welfare Implications

This section explores whether or not the individually rational-efficient performance standards are implementable by no regret mechanisms. We have bad news and good news. The bad news is that the interim standard is not NR-implementable. The good news is that the ex post standard is. Often we are not interested in implementing the entire ex post individually rational-efficient set. Instead, we would like to find out which subsets of this set are implementable. This brings us to the most significant result: *any* performance standard that is Nash-implementable is NR-implementable. Hence, extremely important sets like the *core* or the *Walrasian correspondence* are implementable by mechanisms with no regret even in asymmetric information economies.

**Theorem 5:** *$\mathcal{P}^i$ is not NR-implementable.*

Proof: Suppose that the theorem were not true, i.e. for all $e \in \mathcal{E}$, there exists a game $\Gamma$ such that $\mathcal{P}^i$ is NR-implementable by $\Gamma$ in $e$. Consider the economy $e$ defined in Example 2 (see proof of Theorem 1 above). By definition of NR-implementation by $\Gamma = \{N, M, \xi\}$ in $e$, there exists $\varphi \subseteq \mathcal{P}^i$ such that for all $f \in \varphi$, there exists $s \in E(\Gamma)$ with $\xi \circ s = f$. In addition, check that in the economy $e$ in Example 2, the hypotheses of Property M1 are satisfied with respect to $\Gamma$. Choose a CCMO $\alpha$ as in Example 2. Pick $f \in \varphi$ and $g \in F$. Write $f(a)$ as $(x, y)$ and $g(a)$ as $(x', y')$. By construction, the utility functions of agents 1 and 2 are state-independent. Therefore, given that they are completely informed, the relevant portion of the hypothesis of Property M1 is trivially satisfied for these two agents. The following relationships imply that the relevant portion of the hypothesis of Property M1 is met for the remaining agents:

$$x_3 + y_3 \geq x'_3 + y'_3 \Rightarrow 0.5[0.25(x_3 + y_3) + 0.75(x_3 + y_3)] \geq 0.5[0.25(x'_3 + y'_3) + 0.75(x'_3 + y'_3)].$$

$$x_4 + y_4 \geq x'_4 + y'_4 \Rightarrow 0.5[0.75(x_4 + y_4) + 0.25(x_4 + y_4)] \geq 0.5[0.75(x'_4 + y'_4) + 0.25(x'_4 + y'_4)]$$ and

$$x_3 + y_3 \geq x'_3 + y'_3 \Rightarrow 0.25(x_3 + y_3) \geq 0.25(x'_3 + y'_3)$$

$$x_3 + y_3 \geq x'_3 + y'_3 \Rightarrow 0.75(x_3 + y_3) \geq 0.75(x'_3 + y'_3)$$

$$x_4 + y_4 \geq x'_4 + y'_4 \Rightarrow 0.75(x_4 + y_4) \geq 0.75(x'_4 + y'_4)$$

$x_4 + y_4 \geq x'_4 + y'_4 \Rightarrow 0.25(x_4 + y_4) \geq 0.25(x'_4 + y'_4)$

Given these observations, the hypothesis of Property M1 is met for any game $\Gamma$. The rest of the proof of Theorem 5 follows the proof of Theorem 1 and it is established that $f \circ \theta^{\alpha} \notin \varphi$. Thus, $\mathcal{P}^{i}$ does not satisfy Property M1 with respect to $\Gamma$. Given Theorem 3 and the assumption that $\mathcal{P}^{i}$ is NR-implementable by $\Gamma$, we have a contradiction. ∎

**Theorem 6:** *$\mathcal{P}^{e}$ is NR-implementable.*

This result follows from Lemma 3 and a more general result given in Theorem 7. The lemma is proved in the appendix. We shall use the following definition:

A performance standard $\varphi$ satisfies *Property $M^{*}$* if the following is true:

$\exists \varphi' \in \Phi$ such that $\varphi' \subseteq \varphi$ and $\forall f \in F$, $\forall$CCMO's $\alpha$,

<u>if</u> (i) $f \in \varphi'$ and

(ii) $\forall i \in N$, $\forall\, g \in F$, $\forall \theta \in \Theta$, $\forall \theta' = \theta^{\alpha}(\theta)$, $(g \in EL_i(f \mid \{\theta'\})) \Rightarrow (g \circ \theta^{\alpha} \in EL_i(f \circ \theta^{\alpha} \mid \{\theta\}))$,

<u>then</u> $f \circ \theta^{\alpha} \in \varphi'$.

**Lemma 3:** *$\mathcal{P}^{e}$ satisfies Property $M^{*}$.*

**Theorem 7:** *Let $\varphi$ be a performance standard. If $\varphi$ is Bayesian-implementable (Nash-implementable) in economies with complete information, i.e. when for all $i \in N$ and all $\theta \in \Theta$, $I_i^{o}(\theta) = \langle\theta\rangle$, then $\varphi$ is NR-implementable.*

<u>Proof:</u> By Proposition 1, if $\varphi$ is Bayesian-implementable in economies with complete information, it must satisfy Property $M^{*}$. By definition, if $\varphi$ satisfies Property $M^{*}$, then there exists $\varphi' \subseteq \varphi$ which satisfies Property M2 with respect to any game $\Gamma$ such that for all $i \in N$ and all $\theta \in \Theta$, there

exists $s \in E(\Gamma)$ with $I_i(\theta, s) = \{\theta\}$. By Lemma 1 and Lemma 2 and the Corollary to Theorem 4, $\varphi'$ is fully NR-implementable. Thus, $\varphi$ is NR-implementable. ■

## 7. Extensions

In many economic problems of interest, initial endowments are not specified (for example, when resources are owned collectively) or a social planner may wish to efficiently allocate resources in an "equitable" manner. In such situations, we may want to replace the individual rationality condition with some equity requirement, such as *freedom from envy* (Foley (1967). The results reported in this paper, both and positive ones, hold for the corresponding problem of implementing the interim and ex post envy-free-efficient performance standards.

To summarize, we have shown that it is impossible to design "interesting" efficient mechanisms using Bayesian equilibrium as the solution concept. By employing mechanisms with no regret, introduced in the literature by Green and Laffont (1987), we have a solution to the problem. In fact, any success that has been achieved by Nash-implementation theory for complete information economies can be mimicked in situations where information is non-exclusive and incomplete.

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In this appendix, we shall prove the lemmata presented in the main body of the paper. For this purpose, we shall devise an algorithm which generates a game for every performance standard. Let $\mathcal{G}$ denote the algorithm and let $\mathcal{G}(\varphi)$ denote the game that is generated when a particular $\varphi$ is applied to the algorithm given below. For all $\varphi \in \Phi$, $\mathcal{G}(\varphi) = \{N, M, \xi\}$ is defined as follows:

(I) $\forall i \in N$, $M_i = \{m_i = (\pi_i(i), f(i), \delta(i)) \in \Pi_i \times F \times \mathbb{R}_+\}$.

*Definition:* $\forall i \in N$, $m_{-i}$ satisfies *Property* $\gamma|i$ if the following conditions hold:

(i) $\bigcap_{j\in N\setminus\{i\}}\pi_j(j) \neq \emptyset$.

(ii) $\exists f \in \varphi$ such that $\forall j \in N\setminus\{i\}$, $f(j) = f$.

(iii) $\forall j \in N\setminus\{i\}$, $\delta(j) = 0$.

*Definition:* $\forall m \in M$, $\theta^*(m)$ is defined such that $\bigcap_{i\in N}\pi_i(i) = \{\theta^*(m)\}$.

*Definition:* $\forall i \in N$, $\forall m_{-i} \in M_{-i}$, $\theta_i(m_{-i})$ is defined such that $\bigcap_{j\in N\setminus\{i\}}\pi_j(j) = \{\theta_i(m_{-i})\}$.

*Definition:* $\forall m \in M$, $K(m) \equiv \{i \in N: \forall j \in N\setminus\{i\}, \delta(i) \geq \delta(j)\}$.

(II) $\xi$: $M \rightarrow A$ is given by the schematic diagram in Figure 3.

Proof of Lemma 1: Choose $f \in \varphi$. We shall show that there exists $s \in E(\mathcal{G}(\varphi))$ such that $\xi \circ s = f$ and for all $i \in N$ and all $\theta \in \Theta$, $I_i(\theta, s) = \{\theta\}$. Construct a strategy list $s \in S$ as follows: for all $i \in N$, for all $\theta \in \Theta$, $s_i(\theta) = (I_i(\theta), f, 0)$. It may be checked that for all $i \in N$, for all $\theta \in \Theta$, $s_{-i}(\theta)$ satisfies Property $\gamma|i$. By construction, Case 1 applies and $\xi \circ s = f$.

Consider a unilateral deviation to $s'_i \in S_i$ by agent $i$. We write $s'_i = (s'_{i1}, s'_{i2}, s'_{i3})$. Note that by assumption A2, for all $\theta \in \Theta$, $\theta_i(s_{-i}(\theta)) = \theta$. For all $\theta \in \Theta$, there are two possibilities:

(a) $s'_{i2}(\theta) = f$, in which case either Case 1 or Case 2 applies and $\xi(s'_i(\theta), s_{-i}(\theta)) \in \{f(\theta), 0\}$.

(b) $s'_{i2}(\theta) \neq f$. Case 3 applies and $\xi(s'_i(\theta), s_{-i}(\theta)) \in \{f'(i)(\theta), 0\}$.

By strict monotonicity of preferences, $u_i(f, \theta) \geq u_i(0, \theta)$ for all $\theta \in \Theta$. By construction for all $j \in N$, $s_{j1}(\theta) = I^0_j(\theta)$. In conjunction with assumption A2, this implies that for all $\theta \in \Theta$, for all $\theta' \in I^0_i(\theta)\backslash\{\theta\}$, there exists $j \in N\backslash\{i\}$ such that $s_{j1}(\theta) \neq s_{j1}(\theta')$. Thus, for all $i \in N$, for all $\theta \in \Theta$, $I_i(\theta, s) = \{\theta\}$. If possibility (b) occurs, by Case 3, we have $\xi \circ (s'_i, s_{-i}) \in EL_i(f \mid \{\theta\})$ which implies that for all $s'' \in S$, $\xi \circ (s'_i, s_{-i}) \in EL_i(f \mid I^0_i(\theta)) \bigcap EL_i(f \mid I_i(\theta, s''))$. Thus, we conclude that $s \in E(\mathcal{G}(\varphi))$. Given $\xi \circ s = f \in \varphi$, we have $E(\mathcal{G}(\varphi)) \neq \emptyset$. ■

Proof of Lemma 2: By Lemma 1, $E(\mathcal{G}(\varphi)) \neq \emptyset$. We need to show that $E_F(\mathcal{G}(\varphi)) \subseteq \varphi$.

Step 1: The proof of this lemma makes use of the following result:

**Lemma 2.1:** *Let $s \in E(\mathcal{G}(\varphi))$. For all $\theta \in \Theta$, $s(\theta)$ satisfies Case 1.*

Proof of Lemma 2.1: We shall establish that there cannot be a state $\theta \in \Theta$ such that $s(\theta) = (\pi_i(i), f(i), \delta(i))_{i \in N}$ satisfies either of the cases 2, 3 or 4. We write $s_i$ as $(s_{i1}, s_{i2}, s_{i3})$ where the components of this list denote the functions induced by restricting the range of $s_i$ to $\Pi_i$, $F$ and $\mathbb{R}_+$ respectively.

Suppose that for some $\theta \in \Theta$, $s(\theta)$ satisfies one of the following: Case 2, Case 3 or Case 4. A contradiction will be established. Consider a unilateral deviation by agent $i \in N$ to $s'_i \in S_i$ such that $s'_i = (s_{i1}, s_{i2}, s'_{i3})$. $s'_{i3}$ is such that for all $j \in N \setminus \{i\}$, for all $\theta' \in \Theta$, $s_{j3}(\theta') < s'_{i3}(\theta')$. Let $m = s(\theta)$ and $m' = (s'_i(\theta), s_{-i}(\theta))$. We shall show that there exists $i \in N$ such that the following hold:

$$\xi_i(m') > \xi_i(m) \qquad (*)$$

$$\forall \theta' \in \Theta,\ \xi_i(s'_i(\theta'), s_{-i}(\theta')) \geq \xi_i(s(\theta')) \qquad (**)$$

There are two possibilities:

(i) *Possibility 1:* There exists $j \in N$ such that $K(m) = \{j\}$. Therefore, for all $i \in N \setminus \{j\}$, $m_{-i}$ does not satisfy Property $\gamma|i$. Choose $i \in N \setminus \{j\}$. By definition, Property $\gamma|i$ is not met even if $i$ deviates to $s'_i$. By the outcome rule associated with Case 4A, we have $\xi_i(m') = \Omega$. Since $|N \setminus \{j\}| > 1$, there exists $i \in N \setminus \{j\}$ such that $\xi_i(m) \neq \Omega$. Thus, (*) holds.

(ii) *Possibility 2:* There does not exist any $j \in N$ such that $K(m) = \{j\}$. In this case, by construction, $\xi(m) = 0$. By the outcome rules associated with Cases 2A, 3A and 4A, and given that for all $\theta' \in \Theta$, for all $f \in \varphi$, $f(\theta') \neq 0$, there exists $i \in N$ such that $\xi_i(m') > 0$. Thus, (*) holds.

To check that (**) is true for the agent $i$ for whom (*) holds, choose $\theta'' \in \Theta$ with $\theta'' \neq \theta$. There are, again, two possibilities:

(a) *Possibility 1:* There exists $j \in N$ such that $K(s(\theta'')) = \{j\}$. The arguments given in (i) above apply. Thus, for all $k \in N \setminus \{j\}$, $\xi_k(s'_k(\theta''), s_{-k}(\theta'')) = \Omega$. Also, $\xi(s'_j(\theta''), s_{-j}(\theta'')) = \xi(s(\theta''))$. Thus, (**) holds for $i$.

(b) *Possibility 2:* There does not exist $j \in N$ such that $K(s(\theta'')) = \{j\}$. By construction, for some $f \in \varphi$, $\xi(s(\theta'')) \in \{f, 0\}$. By the outcome rules associated with Cases 2A, 3A and 4A, we conclude that $\xi(s_i'(\theta''), s_{-i}(\theta'')) \in \{f, \Omega\}$. Thus, (**) holds for $i$.

Given that (*) and (**) are true, and given strict monotonicity of preferences, we conclude that if $s(\theta)$ does not satisfy Case 1, for some $\theta \in \Theta$, then $s \notin E(\mathcal{G}(\varphi))$. This contradicts the assumption that $s \in E(\mathcal{G}(\varphi))$. ■

<u>Step 2:</u> Choose $s \in E(\mathcal{G}(\varphi))$. By Lemma 2.1, for all $\theta \in \Theta$, $s(\theta)$ must satisfy Case 1. We write $s_i$ as $(s_{i1}, s_{i2}, s_{i3})$. Thus, for all $\theta \in \Theta$, $\bigcap_{i \in N} s_{i1}(\theta) \neq \emptyset$ and $\theta^*(s(\theta))$ is well-defined. By $\Pi_i$-measurability of $s_i$, $(s_{i1})_{i \in N}$ defines a CCMO and we shall write it as $\alpha$, where for all $i \in N$, for all $\theta \in \Theta$, $\alpha_i(I_i(\theta)) \equiv s_{i1}(\theta)$. Observe that for all $\theta \in \Theta$, $\theta^*(s(\theta)) = \theta^{\alpha}(\theta)$. By construction, for all $\theta \in \Theta$, there exists $f^* \in \varphi$ such that $s_{i2}(\theta) = f^*$. This implies that there exists $f \in \varphi$ such that for all $\theta \in \Theta$, $\xi(s(\theta)) = f(\theta^*(s(\theta))$, i.e. $\xi \circ s = f \circ \theta^{\alpha}$. We need to show that $f \circ \theta^{\alpha} \in \varphi$.

We shall first show that for all $i \in N$, for all $\theta \in \Theta$, for all $\theta' = \theta^{\alpha}(\theta)$, if $g \in EL_i(f \mid I_i(\theta', s^*))$ for all $s^* \in E(\mathcal{G}(\varphi))$, then $g \circ \theta^{\alpha} \in EL_i(f \circ \theta^{\alpha} \mid I_i^o(\theta)) \bigcap EL_i(f \circ \theta^{\alpha} \mid I_i(\theta, \hat{s}))$ for all $\hat{s} \in S$. In the case where $f = g$, this is trivially true. To show that this is true even when $f \neq g$, choose $i \in N$ and $g \in F$ such that $g \neq f$ and for all $\theta \in \Theta$ and for all $\theta' = \theta^{\alpha}(\theta)$, for all $s^* \in E(\mathcal{G}(\varphi))$, $g \in EL_i(f \mid I_i(\theta', s^*))$. By Lemma 1, there exists $s^* \in E(\mathcal{G}(\varphi))$ such that for all $\theta \in \Theta$, $I_i(\theta, s^*) = \{\theta\}$. Thus, $g \in EL_i(f \mid I_i(\theta', s^*))$ for all $s^* \in E(\mathcal{G}(\varphi))$ implies that $g \in EL_i(f \mid \{\theta'\})$. Suppose $i$ unilaterally deviates to $s_i' \in S_i$ where for all $\theta \in \Theta$, $s_i'(\theta) = (s_{i1}(\theta), g, \delta'(i))$. $\delta'(i)$ is such that for all $j \in N\setminus\{i\}$, for all $\theta \in \Theta$, $\delta'(i) > s_{j3}(\theta)$. By construction, for all $\theta \in \Theta$, $\theta_i(s_{-i}(\theta)) = \theta^*(s(\theta))$. Thus, for all $\theta \in \Theta$, $(s_i'(\theta), s_{-i}(\theta))$ satisfies Case 3A and $\xi((s_i'(\theta), s_{-i}(\theta)) = g(\theta_i(s_{-i}(\theta))) = g(\theta^*(s(\theta)) = g(\theta^{\alpha}(\theta))$. Observe that $s \in E(\mathcal{G}(\varphi))$ implies that for all $\theta \in \Theta$, for all $s^* \in E(\mathcal{G}(\varphi))$, $g \circ \theta^{\alpha} \in EL_i(f \circ \theta^{\alpha} \mid I_i^o(\theta)) \bigcap EL_i(f \circ \theta^{\alpha} \mid I_i(\theta, s^*))$. By Lemma 1, there exists $s^* \in E(\mathcal{G}(\varphi))$ such that

$I_i(\theta, s^*) = \{\theta\}$ for all $\theta \in \Theta$. Thus, $g \circ \theta^\alpha \in EL_i(f \circ \theta^\alpha \mid \{\theta\})$ for all $\theta \in \Theta$ and we conclude that for all $\hat{s} \in S$, $g \circ \theta^\alpha \in EL_i(f \circ \theta^\alpha \mid I_i^o(\theta)) \bigcap EL_i(f \circ \theta^\alpha \mid I_i(\theta, \hat{s}))$.

Given that the last conclusion holds for all $i \in N$, by the fact that $\varphi$ satisfies Property M2 with respect to $\mathcal{G}(\varphi)$, we have $f \circ \theta^\alpha \in \varphi$. ■

<u>Proof of Lemma 3</u>: Choose $f \in \mathcal{F}^e$ and a CCMO $\alpha$. Suppose that for all $i \in N$, for all $\theta \in \Theta$, for all $\theta' = \theta^\alpha(\theta)$, if $f' \in EL_i(f \mid \{\theta'\})$, then $f' \circ \theta^\alpha \in EL_i(f \circ \theta^\alpha \mid \{\theta\})$.

Next, suppose that $f \circ \theta^\alpha \notin \mathcal{F}^e$. We shall prove that this yields a contradiction. $f \circ \theta^\alpha \notin \mathcal{F}^e$ implies either one or both of the following possibilities:

*Possibility A:* $f \circ \theta^\alpha$ is not individually rational, i.e. given $w \in F$ defined by $w(\theta) = \omega$ for all $\theta \in \Theta$, there exists $j \in N$ and $\hat{\theta} \in \Theta$ such that $w \notin EL_j(f \circ \theta^\alpha \mid \{\hat{\theta}\})$.

*Possibility B:* $f \circ \theta^\alpha$ is not ex post efficient, i.e. there exists $g, h \in F$ such that $h \equiv g \circ \theta^\alpha$ with $h \notin EL_k(f \circ \theta^\alpha \mid \{\theta^*\})$ for some $k \in N$ and some $\theta^* \in \Theta$ and $f \circ \theta^\alpha \in EL_i(h \mid \{\theta'\})$ for all $i \in N$ and all $\theta' \in \Theta$.

By assumption, for all $i \in N$, for all $\theta \in \Theta$, for all $\theta'' = \theta^\alpha(\theta)$, for all $f' \in F$ if $f' \in EL_i(f \mid \{\theta''\})$, then $f' \circ \theta^\alpha \in EL_i(f \circ \theta^\alpha \mid \{\theta\})$. This implies that for $w$, $g$, $\hat{\theta}$, $\theta^*$ $j$ and $k$ given above, Possibility A implies (+) and Possibility B implies (++) and (+++), where

$$w \notin EL_j(f \mid \{\theta''\}) \text{ for } \theta'' = \theta^\alpha(\hat{\theta}) \quad (+)$$

$$g \notin EL_k(f \mid \{\theta''\}) \text{ for } \theta'' = \theta^\alpha(\theta^*) \quad (++)$$

and for all $i \in N$, for all $\theta' \in \Theta$,

$$f \in EL_i(g \mid \{\theta'\}) \quad (+++)$$

(+) contradicts the fact that $f \in \mathcal{F}^e$ implies ex post individual rationality of $f$. (++) and (+++) contradict the fact that $f \in \mathcal{F}^e$ implies ex post efficiency of $f$. Thus Possibilities A and B cannot occur and we conclude that $f \circ \theta^{\alpha} \in \mathcal{P}^{e}$. ■

Rain

| | L | M | R |
|---|---|---|---|
| T | 10, 10 | 5, 5 | 0, 0 |
| B | 5, 5 | 10, 10 | 0, 0 |

Shine

| | L | M | R |
|---|---|---|---|
| T | 10, 10 | 5, 5 | 10, 10 |
| B | 5, 5 | 10, 10 | 0, 0 |

Cloudy

| | L | M | R |
|---|---|---|---|
| T | 5, 10 | 5, 10 | 10, 0 |
| B | 10, 10 | 5, 5 | 5, 10 |

*Figure 1*

$s_1(\text{Rain}) = T;$

$s_1(\text{Shine}) = T;$

$s_1(\text{Cloudy}) = T;$

$s_2(\text{Rain}) = L$

$s_2(\text{Shine}) = L$

$s_2(\text{Cloudy}) = L$

$s'_1(\text{Rain}) = T:$

$s'_1(\text{Shine}) = T;$

$s'_1(\text{Cloudy}) = T;$

$s'_2(\text{Rain}) = L$

$s'_2(\text{Shine}) = R$

$s'_2(\text{Cloudy}) = L$

$s''_1(\text{Rain}) = B;$

$s''_1(\text{Shine}) = B;$

$s''_1(\text{Cloudy}) = B;$

$s''_2(\text{Rain}) = M$

$s''_2(\text{Shine}) = M$

$s''_2(\text{Cloudy}) = R$

$s^*_1(\text{Rain}) = T:$

$s^*_1(\text{Shine}) = T;$

$s^*_1(\text{Cloudy}) = T;$

$s^*_2(\text{Rain}) = L$

$s^*_2(\text{Shine}) = R$

$s^*_2(\text{Cloudy}) = M$

*Figure 2*

Let $m = (\pi_i(i),\ f(i)\ \delta(i))_{i \in N}$

**Case 1:**

$\forall i \in N$, (i) $\exists \theta \in \Theta$ such that $\theta^*(m) = \{\theta\}$, (ii) $\exists f \in \varphi$ such that $f(i) = f$, and (iii) $\delta(i) = 0$.

$\xi(m) = f(\theta)$

**Case 2:**

(i) $\exists f \in \varphi$ such that $\forall j \in N$, $f(j) = f$, (ii) $\exists i \in N$ such that $m_{-i}$ satisfies Property $\gamma | i$ and (iii) the conditions of Case 1 are not all met.

**Case 2A**

$K(m) = \{i\}$

$\xi(m) = f(\theta_i(m_{-i}))$

**Case 2B**

Otherwise

$\xi(m) = 0$

*Figure 3*

**Case 3:**

$\exists i \in N$ such that (i) $\forall j \in N \backslash \{i\}$, $f(i) \neq f(j)$ and (ii) $m_{-i}$ satisfies Property $\gamma | i$.

**Case 3A**

(i) $K(m) = \{i\}$

(ii) $f(i) \in EL_i(f(j) | \{\theta_i(m_{-i})\})$, $j \neq i$.

↓

$\xi(m) = f(i)(\theta_i(m_{-i}))$

**Case 3B**

Otherwise

↓

$\xi(m) = 0$

**Case 4:**

Otherwise

**Case 4A:**

$\exists i \in N$ with $K(m) = \{i\}$

↓

$(\xi_i(m), \xi_{-i}(m)) = (\Omega, 0)$

**Case 4B**

Otherwise

↓

$\xi(m) = 0$

*Figure 3 (Contd.)*

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