VENABLE'S

MENTAL ARITHMETIC.
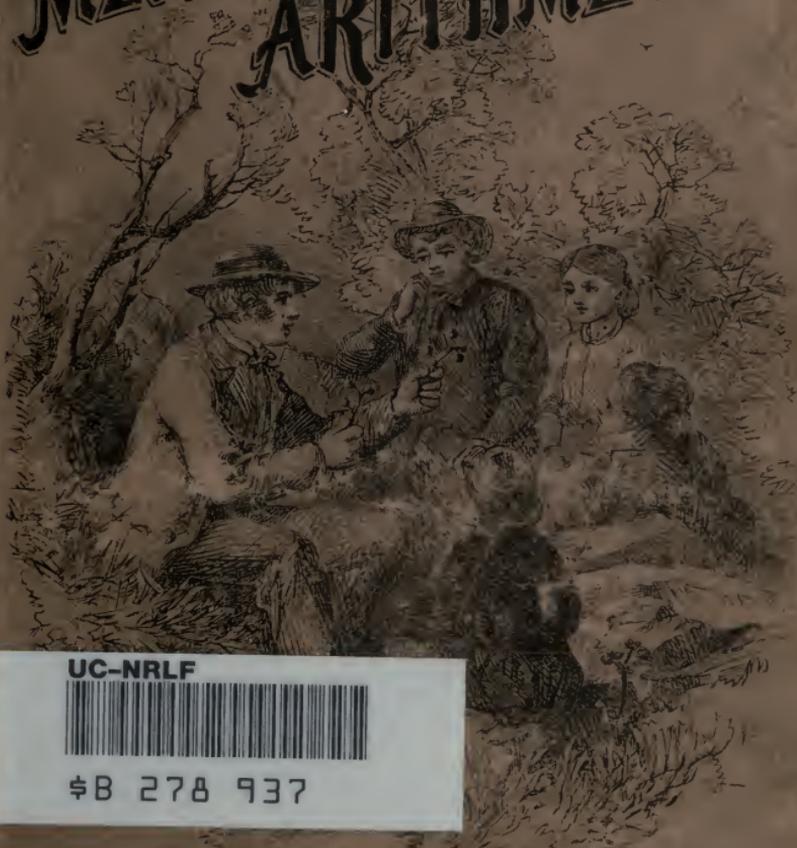
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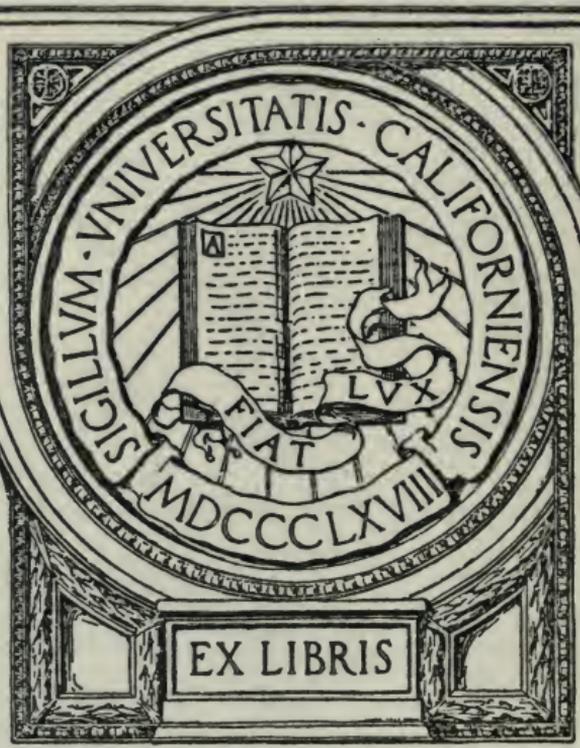
$$p = \frac{11}{3}$$

$$y^2 = \frac{12}{p-1}$$

$$y^2 = \frac{12}{\frac{11}{3} - \frac{3}{3}} = \frac{12}{\frac{8}{3}} = 12 \times \frac{3}{8}$$

$$= y^2 = \frac{36}{8}$$

$$y^2 = \frac{6^2}{4 \times 2}$$

$$y = \frac{3}{2}\sqrt{2}$$
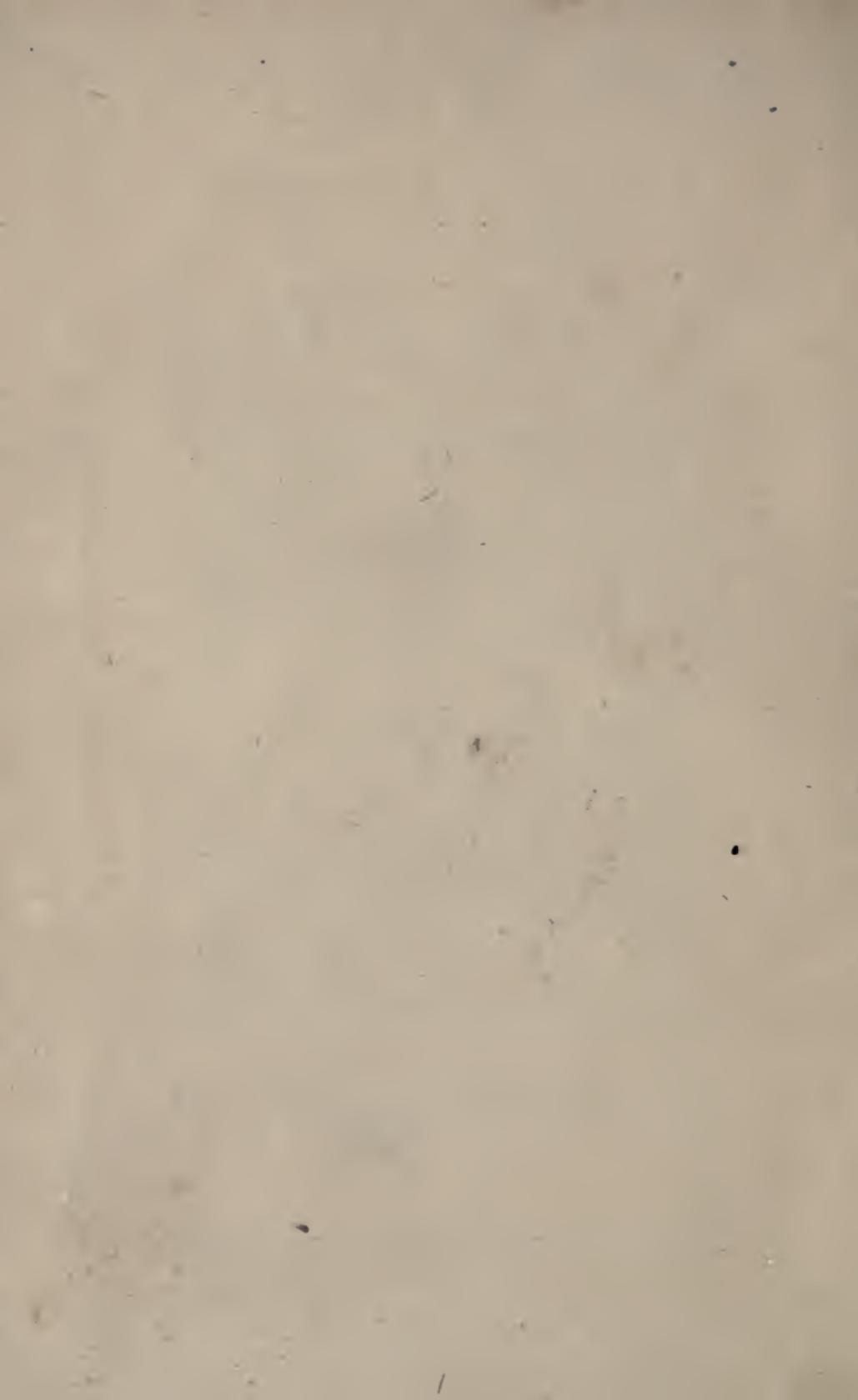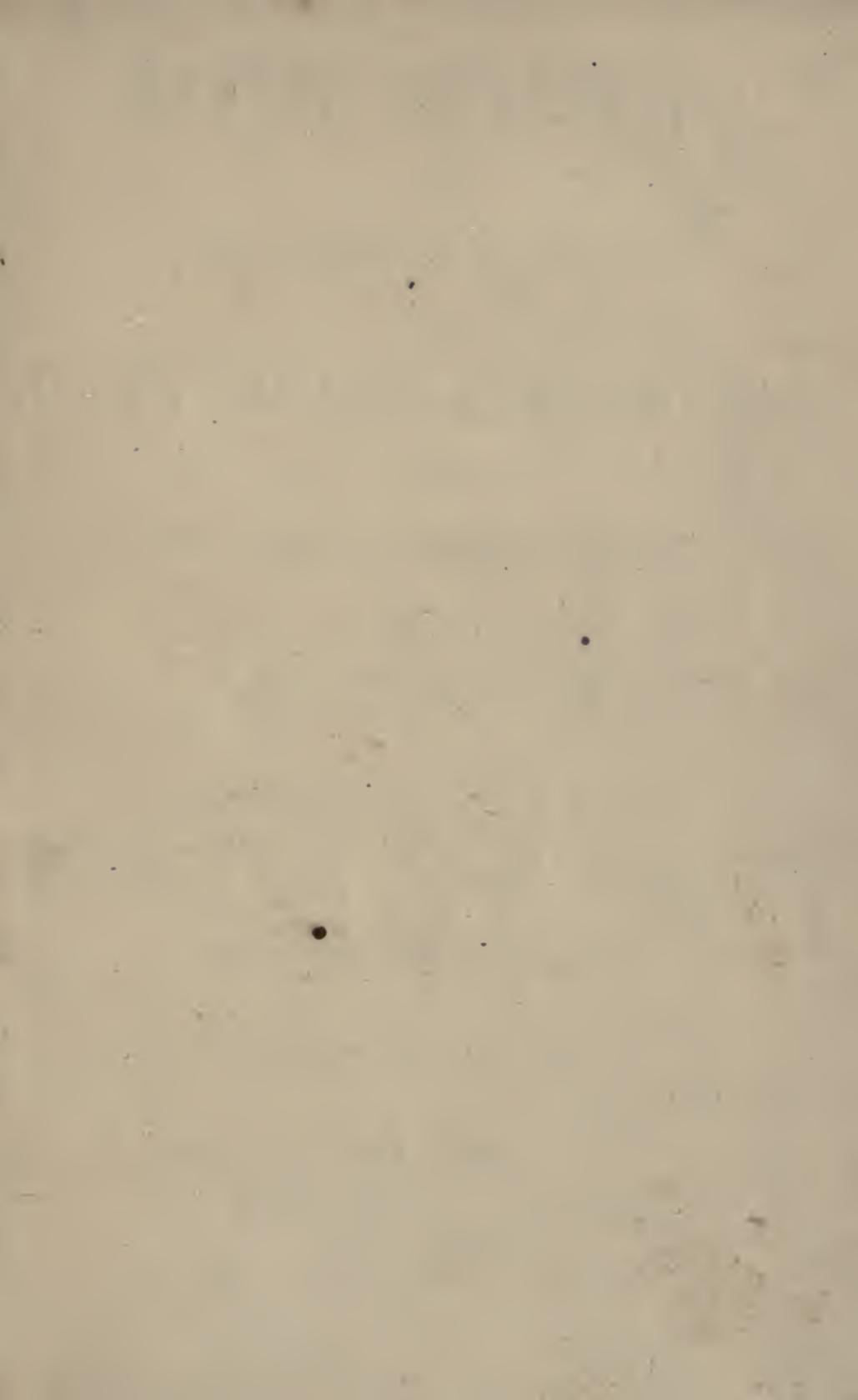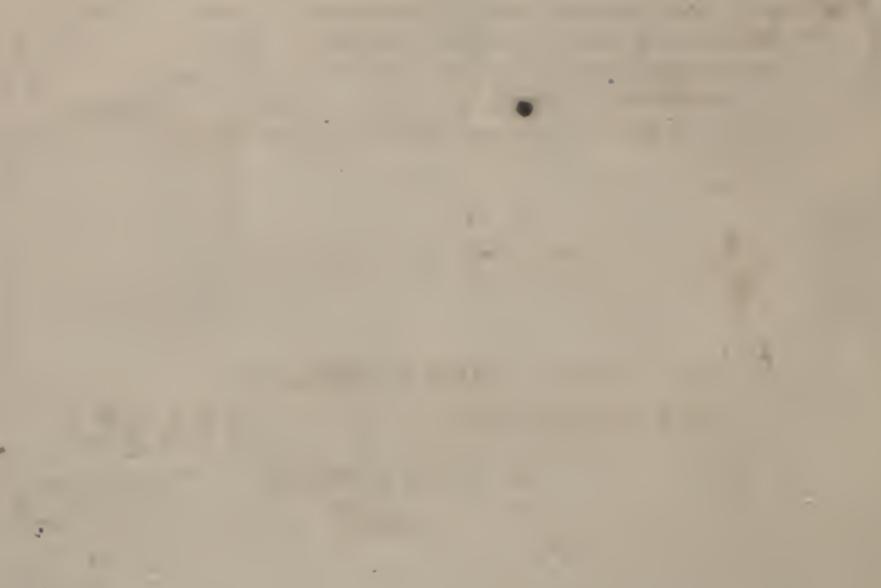
MENTAL ARITHMETIC;

OR,

ORAL EXERCISES IN ABSTRACT

AND

COMMERCIAL ARITHMETIC,

WITH

FIRST LESSONS IN WRITTEN ARITHMETIC.

FOR THE

USE OF SCHOOLS.

BY CHAS. S. VENABLE,

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OF MATHEMATICS AND ASTRONOMY, UNIVERSITY OF SOUTH CAROLINA; FOR-
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PREFACE.

THE advantages of the study of Mental, or Oral, Arithmetic are very generally acknowledged, and yet it is much neglected in the schools. The present work is prepared as a companion to those treatises on written Arithmetic which are not mere collections of examples, but whose aim is to place the study in its proper position among other branches of a rational education.

Great care has been taken to arrange the subjects and to select and frame the questions in such a manner that the pupil may not only acquire readiness and accuracy in the solution of numerical problems, but may gain a thorough knowledge of all the simpler relations among numbers, and learn the reason for each step taken in the various operations performed on them.

The method pursued is that identified with the name of Pestalozzi. The clearest exposition of this method in abstract numbers is to be found, perhaps, in Reiner's Lessons in a Pestalozzian school in Surrey, England. Use has been made of this, and of Menzel's valuable German collection of examples for "Head-reckoning."

The attention of teachers is called to the following points in this little work: Its order and method; the chapters on multiples, divisors, and prime factors; the treatment of vulgar fractions; the mode of teaching proportional numbers; the brief and direct analysis of the various solutions of questions given; the tables of French decimal weights and measures (now authorized in this country by the United States Government), with appropriate questions, illustrating the superiority of these tables over the ordinary ones for ready-reckoning.

A few pages of written Arithmetic are appended, for the convenience of those who may wish to acquire some familiarity with the operations on higher abstract numbers before entering upon the next book of the series. These pages may also be used to advantage concurrently with the rest of the book.

SUGGESTIONS TO TEACHERS.

1. Require the pupil to commit to memory the necessary tables of Addition, Subtraction, Multiplication, and Division.

2. Practice the pupil frequently in counting forwards and backwards by twos, threes, tens, &c., &c. This will cause him to understand thoroughly the basis of all the operations on whole numbers, and that Multiplication and Division are abridged forms of Addition and Subtraction.

3. Do not enter upon the subject of fractions until the pupil is very familiar with the questions on factors, multiples, and divisors.

4. Add examples and illustrations to those in the book, and require your pupils in each lesson to frame questions and give their solutions. This will be found a most useful exercise, especially in the miscellaneous questions on Proportion and its applications.

5. Pupils in written Arithmetic should be called on to solve, orally, questions similar to those in each set of examples which they work on the slate.

MENTAL ARITHMETIC.

CHAPTER I.

ADDITION.

LESSON I.

1. John has one knife; if his father gives him another, how many will he have?

2. Frank has two oranges, and Charles one; how many have they both together?

3. James is in school three hours every morning, and one hour each evening; how many hours is he in school each day?

4. A boy took four eggs from one bird's nest, and one from another; how many birds' eggs did he take?

5. Mary has five tulips in bloom on one bed in the garden, and one upon another; how many tulips bloom on both beds?

6. One and one are how many? Two and one? Three and one? Four and one? Five and one? Six and one? Seven and one? Eight and one? Nine and one? Ten and one?

7. Count from one to ten. Count the birds you know. Count the flowers you know. Count from ten to twenty. Count the number of words in the first two questions of this lesson. Count from twenty to thirty. Count from thirty to forty.

8. Begin at twenty and add successively the letters of the sentence, " Labor overcomes the greatest diffi-

culties." Count from fifty to sixty. From sixty to seventy, eighty, ninety, one hundred. Commence at the first word of this lesson, and count the words as far as one hundred.

9. How many tens have you counted in counting twenty? in counting thirty? forty? fifty? sixty? seventy? eighty? ninety? one hundred?

10. Count from one hundred to two hundred.

11. How would you count from two hundred to three hundred?

ANS.—*By counting from one to one hundred, and repeating two hundred before each number counted.*

12. How would you count from three hundred to four hundred? to five hundred? six hundred? seven hundred? eight hundred? nine hundred? ten hundred?

13. What do we call ten hundred?

ANS.—*One thousand.*

LESSON II.

1. A man has fifty-two white sheep and one black one; how many has he altogether?

2. One company of soldiers has sixty-five men in it, a second company has one man more; how many men in the second company?

3. Of a flock of blackbirds, forty-five are on a tree, and one on the fence; how many blackbirds in the flock?

4. Ten and one are how many? Twenty and one? Thirty and one? Forty and one? Fifty and one? Seventy and one? Ninety and one?

5. Eleven and one are how many? Twenty-one and one? Thirty-one and one? Fifty-one and one? Sixty-one and one? Eighty-one and one? One hundred and one and one?

6. Twelve and one are how many? Twenty-two

and one? Thirty-two and one? Forty-two and one? Fifty-two and one? Sixty-two and one? Seventy-two and one? Eighty-two and one? Ninety-two and one?

7. Fourteen and one? Twenty-four and one? Thirty-four and one? Forty-four and one? Fifty-four and one? Sixty-four and one? Seventy-four and one? Eighty-four and one? Ninety-four and one?

8. Fifteen and one are how many? Twenty-five and one? &c.

9. Nineteen and one are how many? Twenty-nine and one? &c.

10. One ten and one ten are how many? Two tens and one ten? Three tens and one ten? Seven tens and one ten? Eight tens and one ten? Ten tens and one ten?

11. One hundred and one hundred are how many? Two hundred and one hundred? Three hundred and one hundred? Four hundred and one hundred? Nine hundred and one hundred?

LESSON III.

1. Two oranges and two oranges are how many oranges?

2. A planter made three bales of cotton on one acre of land, and two on another acre; how many bales did he make on both?

3. Charles has three marbles, and James has three; how many have they together?

4. A traveller walked three miles the first hour, and four miles the second; how many miles did he walk in the two hours?

5. A planter makes five hogsheads of tobacco one year, and four the next; how many hogsheads in the two years?

1*

6. One and two are how many? One and three?
One and four?

7. Two and two? Two and three? Two and four?

8. Three and two? Three and three? Three and four?

9. Four and two? Four and three? Four and four?

10. Five and two? Five and three? Five and four?

11. Six and two? Six and three? Six and four?

12. Seven and two? Seven and three? Seven and four?

13. Eight and two? Eight and three? Eight and four?

14. Nine and two? Nine and three? Nine and four?

15. Ten and two? Ten and three? Ten and four?

16. Twenty and two are how many? Thirty and two? Forty and two? Fifty and two? Sixty and two? Seventy and two? Eighty and two? Ninety and two?

17. Twenty and three are how many? &c.

18. Twenty and four are how many? &c.

The teacher will cause the pupil to go through the same addition as above with three and four.

19. Eleven and two are how many? Twenty-one and two? Thirty-one and two? Forty-one and two? Fifty-one and two? Sixty-one and two? Seventy-one and two? Eighty-one and two? Ninety-one and two?

20. Eleven and three are how many? Twenty-one and three? &c.

21. Eleven and four are how many? Twenty-one and four? &c.

22. Twelve and two are how many? Twenty-two and two? &c.

23. Twelve and three are how many? Twenty-two and three? &c.

24. Twelve and four are how many? Twenty-two and four? &c.

25. Thirteen and two? Twenty-three and two? &c.

26. Thirteen and three? Twenty-three and three? &c.

27. Thirteen and four? Twenty-three and four? &c.

28. Fourteen and two? Twenty-four and two? &c.

29. Fifteen and two? Twenty-five and two? &c.

30. Sixteen and two? Twenty-six and two? &c.

31. Seventeen and two? Twenty-six and two? &c.

32. Eighteen and two? Twenty-eight and two? &c.

33. Nineteen and two? Twenty-nine and two? &c.

The teacher should extend these questions as far as ninety-three in, the same manner with those which are completed, and make the same series of additions with three and four.

34. Count by twos to one hundred and one, beginning with one.

35. Count by twos to one hundred, beginning with two.

36. Count by threes to one hundred, beginning with one.

37. Count by threes to one hundred and one, beginning with two.

38. Count by threes to ninety-nine, beginning with three.

39. Eighty-seven and one, and two, and three, are how many?

40. Ninety two and two, and three, and four, are how many?

41. Thirty-five and two, and three? Forty-three and four, and two? Fifty-four and four, and three?

LESSON IV.

We have thus far written out the names of the different numbers. For the sake of abbreviation in the succeeding lessons, we will use the figures of the Arabic notation.

These are—

one	two	three	four	five	six	seven	eight	nine	zero or naught
1	2	3	4	5	6	7	8	9	0

Note.—For a more detailed explanation of the use of figures, see first lessons in Numbers, page 31.

We have seen, in counting, there are two tens in twenty, three in thirty, four in forty, &c., and ten tens in one hundred. If now we wish to express numbers greater than nine by those figures, we may do so readily by the use of columns and the 0 (zero).

Hundreds.	Tens.	Units or ones.
	5	6
	4	0
1	2	3
1	0	0

Thus 5 in the column of tens and 6 in the column of ones or units, means five tens and six units, or fifty-six; 4 in the column of tens and 0 in the column of units indicates four tens and no units, or forty; 1 in the column of hundreds, and 2 in the column of tens, and 3 in that of units, indicates one hundred, two tens, and three units, or one hundred and twenty-three; 1 in the hundreds column, 0 in tens, and 0 in units, indicates one hundred. We leave out the columns in the actual writing of numbers and write them thus: 56, 40, 123, 100, and simply recollect the *local* values of the figures.

1. What is the local value of the figure on the right of a number?

Ans.— *Units.*

2. What is the local value of the figure next to the right-hand figure?

3. The third figure from the right? the fourth?

4. What is the meaning and use of the 0?

Note.—The pupil should now be required to read the following table, and be drilled in the use of figures, until he can readily write any number from one to one thousand in figures:

10	20	40	30	88
11	21	41	31	90
12	22	42	32	93
13	23	43	60	99
14	24	44	61	100
15	25	45	62	110
16	26	50	70	101
17	27	51	78	255
18	28	52	80	999
19	29	53	81	1000

LESSON V.

1. John bought a lemon for five cents, and an apple for five cents; how much did they both cost him?

2. One man rides six miles in one hour, and another rides three miles more than the first; how many miles does the latter ride in one hour?

3. Edward begins to work at six o'clock in the morning and works five hours; at what o'clock does he stop work?

4. Harry has seven peaches, and Frank has five; how many peaches have both?

5. A farmer has seven cows in one pasture, and nine in another; how many has he in both?

6. 1 and 5 are how many?   1 and 6?   1 and 7?
7. 2 and 5?   2 and 6?   2 and 7?
8. 3 and 5?   3 and 6?   3 and 7?
9. 4 and 5?   4 and 6?   4 and 7?
10. 5 and 5?   5 and 6?   5 and 7?
11. 6 and 5?   6 and 6?   6 and 7?
12. 7 and 5?   7 and 6?   7 and 7?
13. 8 and 5?   8 and 6?   8 and 7?
14. 9 and 5?   9 and 6?   9 and 7?

15. 10 and 5 are how many?   20 and 5?   30 and 5?   40 and 5?   50 and 5?   60 and 5?   70 and 5? 80 and 5?   90 and 5?

16. 10 and 6 are how many?   20 and 6? &c.
17. 10 and 7 are how many?   20 and 7? &c.
18. 11 and 5 are how many?   21 and 5? &c.
19. 11 and 6 are how many?   21 and 6? &c.
20. 11 and 7 are how many?   21 and 7? &c.
21. 12 and 5 are how many?   22 and 5? &c.
22. 12 and 6 are how many?   22 and 6? &c.
23. 12 and 7 are how many?   22 and 7? &c.
24. 13 and 5 are how many?   23 and 5? &c.
25. 13 and 6 are how many?   23 and 6? &c.

26. 13 and 7 are how many ?   23 and 7 ? &c.
27. 14 and 5 are how many ?   24 and 5 ? &c.
28. 14 and 6 are how many ?   24 and 6 ? &c.
29. 14 and 7 are how many ?   24 and 7 ? &c.
30. 15 and 5 are how many ?   25 and 5 ? &c.
31. 16 and 5 are how many ?   26 and 5 ? &c.
32. 17 and 5 are how many ?   27 and 5 ? &c.
33. 18 and 5 are how many ?   28 and 5 ? &c.
34. 19 and 5 are how many ?   29 and 5 ? &c.

To be carried out as far as 91, 92, 93, 94, &c., by the pupil, for 5, 6, and 7.

LESSON VI.

1. Count by 5's to 101, beginning with 1.   To 102, beginning with 2.   To 103, beginning with 3.   To 104, beginning with 4.   To 100, beginning with 5.

2. Count by 6's, beginning with 1, 2, 3, 4, 5, 6, respectively.   Thus, 1, 7, 13, &c.   2, 8, 14, &c., &c.

3. Count by 7's, beginning with 1, 2, 3, 4, 5, 6, respectively.   Thus, 1, 8, 15, &c.   2, 9, 16, &c., &c.

4. A man had sixty-five sheep in one fold, five in a second, six in a third, and seven in a fourth ; how many had he altogether ?

5. A farmer gathered fifty-six barrels of apples from his orchard, and seven barrels from two trees which stood in the corner of his yard ; how many barrels did he gather ?

6. There are three hundred soldiers in one regiment, and six hundred in another ; how many in the two regiments ?

7. 2 tens and 5 tens are how many ?   2 tens and 6 tens ?   2 tens and 7 tens ?

8. 9 tens and 5 tens are how many ?   9 tens and 6 tens ?   9 tens and 7 tens ?

9. 3 hundred and 5 hundred are how many ?   3 hundred and 6 hundred ?   3 hundred and 7 hundred ?

10. 4 hundred and 5 hundred are how many ?　4 hundred and 6 hundred ?　4 hundred and 7 hundred ?

11. Add fifty-seven, seven, six, and five.

12. Add forty-nine, seven, five, three, and four.

13. How many are ninety-seven, six, seven, four, seven, and three ?

14. How many are forty-five, six, three, five, six, four, three, and four ?

LESSON VII.

1. Frank is seven years old, his sister is eight years older; how old is his sister ?

2. Thomas killed six squirrels on Monday, and nine on Tuesday ; how many squirrels did he kill on both days ?

3. A gardener gathered five bushels of apples from one tree, and ten bushels from another; how many bushels from the two trees ?

4. 1 and 8 are how many ?　1 and 9 ?　1 and 10 ?

5. 2 and 8 ?　2 and 9 ?　2 and 10 ?

6. 3 and 8 ?　3 and 9 ?　3 and 10 ?

7. 4 and 8 ?　4 and 9 ?　4 and 10 ?

8. 5 and 8 ?　5 and 9 ?　5 and 10 ?

9. 6 and 8 ?　6 and 9 ?　6 and 10 ?

10. 7 and 8 ?　7 and 9 ?　7 and 10 ?

11. 8 and 8 ?　8 and 9 ?　8 and 10 ?

12. 9 and 8 ?　9 and 9 ?　9 and 10 ?

13. 10 and 8 are how many ?　20 and 8 ?　30 and 8 ? &c., to 98.

14. 11 and 9 are how many ?　21 and 9 ?　31 and 9 ? &c., to 10.

15. 12 and 10 are how many ?　22 and 10 ?　32 and 10 ? &c., to 102.

16. 13 and 8 are how many ?　23 and 8 ?　33 and 8 ? &c., to 101.

17. 14 and 9 are how many ?　24 and 9 ?　34 and 9 ? &c., to 103.

18. 15 and 10 are how many? 25 and 10? 35 and 10? &c., to 105.

19. 16 and 8 are how many? 26 and 8? 36 and 8? &c., to 104.

20. 17 and 9 are how many? 27 and 9? 37 and 9? &c., to 106.

21. 18 and 10 are how many? 28 and 10? 38 and 10? &c., to 108.

22. 19 and 8 are how many? 29 and 8? 39 and 8? &c., to 107.

23. 10 and 10 are how many? 20 and 10? 30 and 10? &c., to 100.

24. How much is forty-five and eight? Two and seven and five and eight and nine are how many?

25. Fifty-seven and eight are how many?

26. Ninety-three and nine? one hundred and sixteen and ten? 5, 9, 6, 9, 3, 8, 7, 2, and 1 are how many? Ninety-seven, eight, nine, and ten are how many?

LESSON VIII.

1. Count by 8's, beginning with 1, 2, 3, 4, 5, 6, 7, and 8 respectively. Thus, 1, 9, 17, &c. 2, 10, 18, &c., &c.

2. Count by 9's, beginning with 1, 2, 3, 4, 5, 6, 7, 8, and 9 respectively. Thus, 1, 10, 19, &c. 2, 11, 20, &c., &c.

3. Count by 10's, beginning with 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 respectively. Thus, 1, 11, 21, &c. 2, 12, 22, &c., &c.

4. A farmer sends sheep to market in two droves, one drove has twenty sheep in it and the other eighty; how many sheep in both droves?

5. A man makes five hundred bushels of wheat on one farm, and nine hundred on another; how many hundred bushels does he make on both farms?

6. A grazier has one hundred and seventeen head of

cattle in one field, and ten head in another; how many head in the two fields?

7. 3 tens and 8 tens are how many tens? 3 tens and 9 tens? 3 tens and 10 tens?

8. 8 tens and 8 tens are how many tens? 8 tens and 9 tens? 8 tens and 10 tens?

9. 2 hundred and 8 hundred are how many hundred? 2 hundred and 9 hundred? 2 hundred and 10 hundred?

10. 7 hundred and 8 hundred? 7 hundred and 9 hundred? 7 hundred and 10 hundred?

LESSON IX.

1. A father has three books, and wishes to give them to his three sons; how many would he give to each son?

2. If he wished to give them to two sons, how many would he give to one and how many to the other?

3. Three can be broken up how?

Ans.—*Into three ones, or into two and one.*

4. 4 can be separated into what numbers?

Ans.—*3 and 1, 2 and 2, 1 and 1 and 1 and 1.*

5. What different numbers added together make 5?

6. A boy has six marbles; into what different heaps can he distribute them?

7. 6 can be separated into what numbers?

8. 7 can be separated into what numbers?

9. What different numbers added together will make 8? 9? 10?

When a number is greater than ten, and less than one hundred, the most usual and useful separation in our system of numbers is into tens and units; or, if greater than one hundred, and less than a thousand, into hundreds, tens, and units. The notation or mode of writing numbers, which has been explained in a previous lesson, makes this separation very easy.

10. Separate 11, 12, 13, 14, 15, 27, 49, 58, and 97 into tens and units.

11. Separate 137, 142, 156, 207, 320, 197 into hundreds, tens, and units, and into tens and units.

12. 25 and 19 are how many?

PROCESS.—*Separate* 19, *and we have* 1 *ten and* 9 *units; * 25 *and* 10 *are* 35; 35 *and* 9 *are* 44. *Hence,* 25 *and* 19 *are* 44.

13. 34 and 14 are how many? 31 and 18? 32 and 17? 32 and 16?

14. 14 and 19? 95 more 13? 46 and 12?

15. 11 and 11 are how many? 22 and 11? 33 and 11?

16. Count by 11's to 99, beginning with 11.

17. 12 and 12 are how many? 24 and 12? 36 and 12?

18. Count by 12's to 96, beginning with 12.

19. 40 and 20 are how many? 50 and 30? 60 and 40? 30 and 60? 70 and 20? 90 and 50?

20. 40 and 27 are how many?

PROCESS.—27 *is* 20 *and* 7; 40 *and* 20 *are* 60, 60 *and* 7 *make* 67.

21. How much is 60 and 32? 70 and 29? 90 and 67? 50 and 48? 80 and 22?

22. 47 and 20 are how many? 58 and 30? 65 and 40? 33 and 30? 72 and 20?

23. 48 and 27 are how many?

PROCESS.—40 *and* 20 *are* 60; 8 *and* 7 *are* 15; 60 *and* 15 *are* 75. *Or thus,* 27 *is* 20 *and* 7, 48 *and* 20 *make* 68, 68 *and* 7 *make* 75.

24. 72 and 25 are how many? 64 and 56? 31 and 22? 48 and 63? 84 and 26?

CHAPTER II.

SUBTRACTION.

LESSON I.

1. A boy has one apple, and gives it away; how many has he left? Another has twenty apples and gives them away; how many has he left?

2. 1 from 1 leaves how many? 20 from 20 leaves how many?

3. Any number taken from itself leaves what?

4. A boy has five peaches, and eats one of them; how many has he left?

5. Mary had five books, but she has lost two of them; how many has she now?

6. William had ten canary birds in a cage, but allowed two of them to escape; how many has he left?

7. What do you call the result obtained by adding one number to another?

Ans.—*Their sum.*

8. What do you call the result obtained by taking one number from another?

Ans.—*Their difference.*

9. 1 from 1 leaves how much?   2 less 2 is?

10. 1 from 2 leaves?   3 less 2?

11. 1 from 3 leaves?   4 less 2?

12. 1 from 4 leaves?   5 less 2?

13. 1 from 5 leaves?   6 less 2?

14. 1 from 6 leaves?   7 less 2?

15. 1 from 7 leaves?   8 less 2?

16. 1 from 8 leaves?   .9 less 2?

17. 1 from 9 leaves?   10 less 2?

18. 1 from 10 leaves?

19. 11 less 1 is?   21 less 1?   31 less 1?   41 less 1?
51 less 1?   61 less 1?   71 less 1?   81 less 1?   91 less 1?

20. 11 less 2 ? 21 less 2 ? &c., to 101.

21. 22 less 1 ? 23 less 1 ? 24 less 1 ? 35 less 1 ? 46 less 1 ?

22. 57 less one ? 68 less 1 ? 79 less 1 ? 110 less 1 ?

23. 12 less 2 is how much ? 22 less 2 ? &c., to 102.

24. 13 less 2 ? 23 less 2 ? &c., to 103.

25. 14 less 2 ? 24 less 2 ? &c., to 94.

26. 15 less 2 ? 25 less 2 ? &c., to 95.

27. 16 less 2 ? 26 less 2 ? &c., to 96.

28. 18 less 2 ? 28 less 2 ? &c., to 98.

29. 19 less 2 ? 29 less 2 ? &c., to 99.

30. Begin at 100, and count backward down to 1.

31. Begin with 100, and count backwards by 2's down to 2.

32. Begin with 99, and count backwards by 2's down to 1.

33. 1 ten from 2 tens leaves how many? 1 ten from 3 tens?

34. 1 ten from 6 tens? 2 tens from 8 tens?

35. 1 hundred from 4 hundred leaves how many? 1 hundred from 8 hundred? 2 hundred from 7 hundred? 2 hundred from 9 hundred?

LESSON II.

1. George had ten chickens, but a weasel killed three of them; how many remained?

2. One farmer has seven horses and another has four; how many more has the first than the second?

3. A man planted twelve trees, but five of them died; how many lived?

4. A man owing thirteen dollars, paid three of them; how much did he still owe?

5. A farmer had fifteen cows in one pasture, and five in a second; how many more were in the first pasture than in the second?

6.	3 less 3		4 less 4 ? 5 less 5 ?
7.	4 less 3 ?		5 less 4 ? 6 less 5 ?
8.	5 less 3 ?		6 less 4 ? 7 less 5 ?
9.	6 less 3 ?		7 less 4 ? 8 less 5 ?
10.	7 less 3 ?		8 less 4 ? 9 less 5 ?
11.	8 less 3 ?		9 less 4 ? 10 less 5 ?
12.	9 less 3 ?		10 less 4 ? 11 less 5 ?
13.	10 less 3 ?		11 less 4 ? 12 less 5 ?
14.	11 less 3 ?		12 less 4 ? 13 less 5 ?
15.	12 less 3 ?		13 less 4 ? 14 less 5 ?
16.	13 less 3 ?		14 less 4 ? 15 less 5 ?
17.	10 less 3 ?	20 less 3 ?	30 less 3 ? &c., to 90.
18.	11 less 3 ?	21 less 3 ?	31 less 3 ? &c., to 91.
19.	12 less 3 ?	22 less 3 ?	32 less 3 ? &c., to 92.
20.	13 less 3 ?	23 less 3 ?	33 less 3 ? &c., to 93.
21.	14 less 3 ?	24 less 3 ?	34 less 3 ? &c., to 94.
22.	15 less 3 ?	25 less 3 ?	35 less 3 ? &c., to 95.
23.	16 less 3 ?	26 less 3 ?	36 less 3 ? &c., to 96.
24.	17 less 3 ?	27 less 3 ?	37 less 3 ? &c., to 97.
25.	18 less 3 ?	28 less 3 ?	38 less 3 ? &c., to 98.
26.	19 less 3 ?	29 less 3 ?	39 less 3 ? &c., to 99.

NOTE.—The teacher should cause the pupil to go through several of the above subtractions with 4 and 5 respectively.

27. Count by 3's backwards from 100 to 1.

28. Count by 4's backwards from 100 to 0.

29. Count by 5's backwards from 100 to 0.

30. 3 tens from 8 tens leaves how many? 4 tens from 7 tens? 5 tens from 9 tens?

31. One veteran regiment has 400 men in it, and another 500 men; how many more in the last than in the first?

32. 3 hundred from 6 hundred leaves how many? 7 hundred less 5 hundred is?

33. 4 hundred from 8 hundred leaves how many? 6 hundred from 9 hundred? 2 hundred from 7 hundred?

LESSON III.

1. A horse travelled six miles in one hour, and nine miles the next; how many more miles did he travel in the second hour than in the first?

2. A farmer paid twelve dollars to one laborer, and eight dollars to another; how much less did he pay the second laborer than the first?

3. Julian found thirteen eggs, and gave John seven; how many had he left?

4. A hunter killed ten bears in the canebrake one day, and six on the next day; how many more did he kill on the first day than the second?

5. 6 less 6	7 less 7?	8 less 8?
6. 7 less 6?	8 less 7?	9 less 8?
7. 8 less 6?	9 less 7?	10 less 8?
8. 9 less 6?	10 less 7?	11 less 8?
9. 10 less 6?	11 less 7?	12 less 8?
10. 11 less 6?	12 less 7?	13 less 8?
11. 12 less 6?	13 less 7?	14 less 8?
12. 13 less 6?	14 less 7?	15 less 8?
13. 14 less 6?	15 less 7?	16 less 8?
14. 15 less 6?	16 less 7?	17 less 8?
15. 16 less 6?	17 less 7?	18 less 8?

16. 20 less 7 is how much?   30 less 7? &c., to 90.
17. 21 less 7 is how much?   31 less 7? &c., to 91.
18. 22 less 7?   32 less 7? &c., to 92.
19. 23 less 7?   33 less 7? &c., to 93.
20. 24 less 7?   34 less 7? &c., to 94.
21. 25 less 7?   35 less 7? &c., to 95.
22. 26 less 7?   36 less 7? &c., to 96.
23. 27 less 7?   37 less 7? &c., to 97.
24. 28 less 7?   38 less 7? &c., to 98.
25. 29 less 7?   39 less 7? &c., to 99.

NOTE.—The pupil should be required to go through several of the above with 6 and 8 respectively.

26. Count by 6's backward from 100 to 4.

27. Count by 7's backwards from 100 to 2.

28. Count by 8's backwards from 100 to 4.

29. A man has 60 sheep in one fold, and 90 in a second fold; how many more in the second than in the first?

30. 6 tens from 9 tens leaves how many? 7 tens from 10 tens leaves? 8 tens from 12 tens?

31. There were 700 bison in one herd on the prairie, and 900 in another herd; how many more in the last herd than the first?

32. 600 from 900 leaves how many? 8 hundred from 11 hundred?

LESSON IV.

1. A man agreed to labor for a farmer 15 days, and left at the end of nine days; how many days' labor did he still owe the farmer?

2. A hunter on the Texas prairies killed 10 deer on one day, and 14 the next; how many more did he kill on the second day than on the first?

3. 9 less 9 are how many? 10 less 10 are how many?

4. 10 less 9? 11 less 10?

5. 11 less 9? 12 less 10?

6. 12 less 9? 13 less 10?

7. 13 less 9? 14 less 10?

8. 14 less 9? 15 less 10?

9. 15 less 9? 16 less 10?

10. 16 less 9? 17 less 10?

11. 17 less 9? 18 less 10?

12. 18 less 9? 19 less 10?

13. 19 less 9? 20 less 10?

14. 20 less 9 are how many? 30 less 9? &c., to 90.

15. 21 less 9? 31 less 9? &c., to 91.

16. 22 less 9? 32 less 9? &c., to 92.

17. 23 less 9? 33 less 9? &c., to 93.

18. 24 less 9 ?   34 less 9 ? &c., to 94.
19. 25 less 9 ?   35 less 9 ? &c., to 95.
20. 26 less 9 ?   36 less 9 ? &c., to 96.
21. 27 less 9 ?   37 less 9 ? &c., to 97.
22. 28 less 9 ?   38 less 9 ? &c., to 98.
23. 29 less 9 ?   39 less 9 ? &c., to 99.

NOTE.—The pupil should go through in the same manner with the subtraction of 10.

24. Count backwards by 9's from 100 to 1.

25. Count backwards by 10's from 100 to 0.

26. One squadron of cavalry contains 90 mounted men, and a second contains 110; how many more mounted men does the second contain than the first? 9 tens from 11 tens leaves how many?   10 tens from 15 tens leaves how many?

27. In one of the butcheries in Paris, 9 hundred sheep were slaughtered in one day, and in a second 12 hundred; how many more were slaughtered in the second than in the first?

28. 10 hundred or a thousand from 14 hundred leaves how many?

LESSON V.

1. How much is 20 less 11?

PROCESS.—11 *is* 10 *and* 1.   10 *from* 20 *leaves* 10, *and* 1 *from* 10 *leaves* 9.

2. How much is 45 less 11?   67 less 11?   99 less 11?   88 less 11?   66 less 11?

3. How much is 25 less 12?

PROCESS.—12 *is* 10 *and* 2.   10 *from* 25 *leaves* 15, *and* 2 *from* 15 *leaves* 13.

4. How much is 43 less 12?   54 less 12?   60 less 12?   72 less 12?   96 less 12?   84 less 12?

5. 26 less 13?   52 less 13?   39 less 13?   65 less 13?

2

6. 28 less 14? 42 less 14? 70 less 14?

7. 36 less 18? 54 less 18? 90 less 18?

8. Count backwards by 11's, beginning with 99.

9. Count backwards by 12's, beginning with 96.

10. How much is 30 less 20? 40 less 20? 50 less 30? 60 less 20? 70 less 50? 80 less 60? 90 less 30?

11. How much is 40 less 25?

PROCESS.—25 *is* 20 *and* 5. 40 *less* 20 *is* 20. 20 *less* 5 *is* 15.

12. How much is 30 less 22? 50 less 36? 60 less 37? 70 less 24? 80 less 48? 90 less 55?

13. How much is 55 less 20?

PROCESS.—50 *less* 20 *is* 30. *Hence* 55 *less* 20 *is* 35.

14. How much is 36 less 20? 48 less 20? 59 less 30? 67 less 40? 77 less 50? 87 less 30?

15. How much is 33 less 23? 66 less 46? 56 less 36? 87 less 67? 97 less 57? 79 less 49? 88 less 28?

16. How much is 55 less 22?

PROCESS.—55 *is* 52 *and* 3. 52 *less* 22 *is* 30. 30 *and* 3 *are* 33. *Or thus*, 22 *is* 20 *and* 2. 55 *less* 20 *is* 35. 35 *less* 2 *is* 33.

17. How much is 67 less 39? 78 less 23? 82 less 47? 96 less 51? 47 less 28?

LESSON VI.

1. A boy had 25 marbles, and gave 5 of them to one comrade, 6 to another, and 4 to another; how many did he have left?

2. A farmer raised 50 bushels of oats in one field, 20 in a second, and 30 bushels in a third, and sold 40 bushels; how many bushels did he raise on the three fields, and how many bushels had he after selling 40?

3. Charles' father gave him 50 cents to make some purchases. He bought an arithmetic for 15 cents, a grammar for 10 cents, and a writing-book for 8 cents; how much of the money had he left?

4. A traveller had 100 miles to go. On the first day he travelled 22 miles, on the second 31 miles, on the third 17 miles; how far did he go in the three days? How many miles had he yet to travel?

5. In a conservatory there were 20 oranges on one tree, 15 on another, 7 on a third, and 5 on a fourth. Of these, 10 on the first tree were ripe, 7 on the second, 2 on the third, and 3 on the fourth; how many oranges were on all the trees, how many were ripe, and how many unripe?

6. 36 bees were sucking honey from the flowers of a wild jessamine; 15 of them flew to the blossoms of a magnolia tree, 7 to the roses on a sweetbriar near by, while 6, loaded with honey, flew back to the hive; how many bees still lingered on the jessamine vine?

7. A squadron of cavalry went into battle one hundred strong, 20 of them were captured, 10 killed, and 22 wounded; how many men were left in the squadron?

8. 43 and 8 and 5, less 11, are how many?

9. Twenty-eight and four and nine, less seven, are how many?

10. Fifty-two and six and three and nine, less two and three and five, are how many?

11. Seventy-seven and eleven and three and four, less seven, less eight, are how many?

12. Ninety-eight and two and eight and three, less twenty and twelve, are how many?

13. Eighty-six less twenty, less seven, less nine, less twenty-two, is how much?

LESSON VII.

TABLES FOR PRACTICE IN ADDITION AND SUBTRACTION.

For the sake of brevity in these tables, we will use the sign of addition +, called *plus*, and the sign of subtraction −, called *minus*. Plus means *more*, minus means *less*. Thus, 7 + 6 is read 7 plus 6, which means 7 more 6, or 7 and 6. 7 − 6 is read 7 minus 6, and means 7 less 6, or 6 taken from 7.

I. ?

10 + 10	40 + 20	70 + 60	80 + 80
20 + 20	40 + 40	70 + 40	90 + 30
20 + 10	50 + 40	80 + 50	70 + 80
30 + 20	50 + 20	90 + 20	90 + 90
30 + 10	50 + 10	90 + 80	60 + 50
30 + 30	50 + 30	60 + 40	60 + 20
40 + 10	50 + 50	90 + 50	30 + 80
40 + 30	80 + 10		

II. ?

40 + 17	40 + 34	40 + 39	40 + 41
29	23	24	25
38	32	31	36
22	43	19	42
18	21	26	28
27	33	35	37

III. ?

47 + 30	67 + 30	51 + 30	54 + 30
55	49	62	64
69	66	44	58
59	57	52	63
48	61	68	53
56	46	65	45

IV. ?

17 + 27	17 + 56	47 + 19	49 + 28
29	29	55	66

38 + 27	38 + 56	69 + 19	57 + 28
22	22	59	61
18	19	48	46
27	32	56	51

V. ?

50 — 30	100 — 60	95 — 15	83 — 23
40 — 10	100 — 50	25	13
70 — 50	110 — 10	55	53
80 — 40	90 — 30	75	73
70 — 30	80 — 70	65	63
90 — 20	60 — 20	45	43

VI. ?

58 — 30	82 — 40	100 — 58	90 — 83
67	77	47	79
42	45	76	64
75	66	67	61
84	94	38	42
97	58	24	34

VII. ?

93 — 58	93 — 85	36 — 16	89 — 24
47	73	73	74
76	69	85	47
67	56	97	38
38	45	58	56
24	32	64	95

8. 44 less 27 is how much? 47 less 27? 44 less 30?

9. If we add 3 to the number from which we are to subtract another, what change does it make in the remainder?

10. If we add it to the number which we subtract from another number, what change do we make in the remainder?

11. 24 — 16 ?   24 — 23 ?   31 — 16 ?   31 — 23 ?

12. If we add seven to the number subtracted and also to the number from which it is taken, is there any change in the remainder?

13. 5 — 3 is how much ?   8 — 5 ?   12 — 9 ?

14. 40 — 20 is how much ?   41 — 21 ?   49 — 29 ?

15. One farmer had 32 sheep, and another 28 sheep; each lost 10 sheep; how many more sheep did the first have than the second before the loss? After the loss?

———————

CHAPTER III.

MULTIPLICATION.

LESSON I.

1. Five boys gave each a dime to a blind man; how many dimes did they all give?   1 and 1, and 1 and 1, and 1, are how many?   5 times 1 are how many?   one times 5 are how many ones?

2. Six boys are flying kites, and each boy has one kite; how many kites have they all?   1 and 1, and 1 and 1, and 1 and 1, are how many?   6 times 1?   Once 6?

3. Seven girls are coming from the garden, each with a bunch of roses; how many bunches of roses have they all together?   1 and 1, and 1 and 1, and 1 and 1, are how many?   7 times 1?   One time 7 is how many ones?

4. Eight boys are going out to gather blackberries, and each boy has one basket; how many baskets have they all?   8 times 1 are how many?

  5. 1 time 1?
  6. 2 times 1?   1 time 2?
  7. 3 times 1?   1 time 3?
  8. 4 times 1?   1 time 4?
  9. 5 times 1?   1 time 5?
 10. 6 times 1?   1 time 6?
 11. 7 times 1?   1 time 7?
 12. 8 times 1?   1 time 8?
 13. 9 times 1?   1 time 9?
 14. 10 times 1?   1 time 10?

15. 27 times 1 are how many? 36 times 1? 48 times 1? 87 times 1? 108 times 1?

16. Twice 1 ten is how many tens? 3 times 1 ten? 4 times 1 ten? 5 times 1 ten?

17. Twice 1 hundred is how much? 6 times 1 hundred? 8 times 1 hundred? 9 times 1 hundred?

18. Two row boats have three oars each? how many oars have both boats? 3 and 3 are how many? Twice 3?

19. Hayes saw two Esquimaux sledges drawn by 8 dogs each; how many dogs drew both sledges? 8 and 8 are how many? Twice 8?

20. There are two orange trees in a gentleman's garden, each of which has 20 oranges on it; how many oranges on both trees? 20 and 20 are how many? Twice 20?

21. A man bought two cloaks, and paid 9 dollars apiece for them; how much did he pay for both?

22. How much is twice 1?
                 twice 2?
                 twice 3?
                 twice 4?
                 twice 5?
                 twice 6?
                 twice 7?
                 twice 8?
                 twice 9?

23. How much is twice 10?
twice 11?

PROCESS.—11 *is* 10 *and* 1. *Twice* 11 *is twice* 10 *and twice* 1, or 20 *and* 2 (*i. e.*) 22.

24. How much is twice 12? Twice 16? Twice 17? Twice 18? Twice 19?

25. How much is twice 2 tens or 20? Twice 30? Twice 40? Twice 50? Twice 60? Twice 70? Twice 80? Twice 90?

26. How much is twice 1 hundred? Twice 2 hundred? Twice 3 hundred? Twice 4 hundred? Twice 5 hundred? Twice 8 hundred? Twice 9 hundred?

The sign of multiplication is ×, which we will read *times*. Thus, 2 × 12 or 3 × 6 means 2 times 12 and 3 times 6. The result obtained by the multiplication of numbers is called their *product*.

LESSON II.

1. What do 3 oranges at 2 cents apiece cost? 2 + 2 + 2?

2. What do 3 oranges at 3 cents apiece cost? 3 + 3 + 3?

3. Each wagon has four wheels; how many wheels have three wagons? How many wheels have four wagons? How many have five wagons? 4 + 4 + 4? 4 + 4 + 4 + 4? 4 + 4 + 4 + 4 + 4?

4. There are three rows of trees, and ten trees in each row; how many trees? How many trees in 4 rows of ten trees each? In 5 rows of ten trees each?

5. 3 times 10 are how many? 3 times 10 and once 10? 4 times 10 and once 10?

6. 3 times 1? 4 times 1? 5 times 1?
7. 3 times 2? 4 times 2? 5 times 2?
8. 3 times 4? 4 times 3? 5 times 3?
9. 3 times 5? 4 times 4? 5 times 4?

10. 3 times 6 ?   4 times 5 ?   5 times 5 ?
11. 3 times 7 ?   4 times 6 ?   5 times 6 ?
12. 3 times 8 ?   4 times 7 ?   5 times 7 ?
13. 3 times 9 ?   4 times 8 ?   5 times 8 ?
14. 3 times 10 ?   4 times 9 ?   5 times 9 ?

15. 3 times 2 are how many times 3 ?   4 times 2 are how many times 4 ?

16. 5 times 2 are how many times 5 ?   3 times 4 are how many times 3 ?

17. 3 times 5 are how many times 3 ?   4 times 5 are how many times 4 ?

18. How much is 3 times 11 ?

PROCESS.—11 *is* 10 *and* 1. 3 *times* 11 *is* 3 *times* 10 *and* 3 *times* 1 ; *i. e.*, 30 *and* 3, *or* 33.

19. 3 times 12 ?   3 times 13 ?   3 times 14 ?   3 times 15 ?   3 times 16 ?   3 times 17 ?   3 times 18 ?   3 times 19 ?

20. How much is 4 times 11 ?   4 times 12 ?   4 times 13 ?   4 times 14 ?   4 times 15 ?   4 times 16 ?   4 times 17 ?   4 times 18 ?   4 times 19 ?

21. How much is 5 times 11 ?   5 times 12 ?   5 times 13 ?   5 times 14 ?   5 times 15 ?   5 times 16 ?   5 times 17 ?   5 times 18 ?   5 times 19 ?

22. How much is four times 2 tens ?   4 times 3 tens ?   4 tens ?   8 tens ?   9 tens ?

23. How much is 3 times 2 tens ?   3 times 3 tens ?   3 times 4 tens ?   3 times 5 tens ?   3 times 6 tens ?   8 tens ?

24. How much is 5 times 2 tens?   3 tens ?   4 tens ?   5 tens ?   6 tens ?   7 tens ?   8 tens ?

25. How much is 4 times 2 hundred ?   3 hundred ?   6 hundred ?   5 times 3 hundred ?   5 hundred ?   7 hundred ?   3 times 2 hundred ?   9 hundred ?   4 hundred ?

LESSON III.

1. I planted 6 rows of trees with 6 trees in each row; how many trees did I plant? 5 times 6 and once 6 are how many? 7 rows of trees, 6 trees in a row, are how many trees? 8 rows of trees, 6 trees in a row, are how many trees? 6 × 6 and once 6? 7 × 6 and once 6?

2. What do 6 pounds of sugar cost at 7 cents a pound? at 8 cents? at 10 cents?

3. There are 7 days in one week; how many days in 7 weeks? in 8 weeks? 7 and 7 and 7 and 7 and 7 and 7 and 7 are how many? 7 times 7 and once 7 are how many?

4. If flour is 8 dollars a barrel, how much will 5 barrels cost? 8 barrels? 9 barrels? 10 barrels?

5. 6 times 2? 7 times 2? 8 times 2?
6. 6 times 3? 7 times 3? 8 times 3?
7. 6 times 4? 7 times 4? 8 times 4?
8. 6 times 5? 7 times 5? 8 times 5?
9. 6 times 6? 7 times 6? 8 times 6?
10. 6 times 7? 7 times 7? 8 times 7?
11. 6 times 8? 7 times 8? 8 times 8?
12. 6 times 9? 7 times 9? 8 times 9?
13. 6 times 10? 7 times 10? 8 times 10?

14. 6 times 5 are how many times 6? 7 times 5 are how many times 7? 8 times 5 are how many times 8?

15. 6 times 7 are how many times 6? 8 times 7 are how many times 8? 6 times 8 are how many times 6?

16. How much is 6 times 11? 12? 13? 14? 15? 16? 17? 18? 19?

17. How much is 7 times 11? 12? 13? 14? 15? 16? 17? 18? 19?

18. How much is 8 times 11? 12? 13? 14? 15? 16? 17? 18? 19?

19. How much is 6 times 2 tens? 3 tens? 5 tens? 7 tens? 8 tens? 9 tens?

20. How much is 7 times 2 tens? 3 tens? 4 tens? 6 tens? 8 tens? 9 tens?

21. How much is 8 times 3 tens? 5 tens? 7 tens? 8 tens? 9 tens? 11 tens?

22. How much is 6 times 2 hundred? 5 hundred? 8 hundred?

23. How much is 7 times 3 hundred? 4 hundred? 9 hundred?

24. How much is 8 times 2 hundred? 6 hundred? 7 hundred?

LESSON IV.

1. An elephant has two tusks; how many tusks have 9 elephants? How many tusks have 10 elephants?

2. In a certain village there are 9 cottages in a row, and each cottage has four windows in front; how many front windows have all the cottages?

3. Ten laborers received each 5 dollars for a week's labor; how much did they all receive together?

4. At 8 dollars a barrel, how much will 9 barrels of flour cost? Ten barrels?

5. 9 times 2? 10 times 2?
6. 9 times 3? 10 times 3?
7. 9 times 4? 10 times 4?
8. 9 times 5? 10 times 5?
9. 9 times 6? 10 times 6?
10. 9 times 7? 10 times 7?
11. 9 times 8? 10 times 8?
12. 9 times 9? 10 times 9?
13. 9 times 10? 10 times 10?

14. 9 times 8 are how many times 9? 10 times 8 are how many times 10?

15. 9 times 10 are how many times 9? 10 times 2 are how many times 10?

16. 9 times 2 tens are how many? 3 tens? 5 tens? 7 tens? 8 tens? 10 tens?

17. 10 times 3 tens? 4 tens? 5 tens? 6 tens? 8 tens? 9 tens?

18. How much is 9 times 4 hundred? 7 hundred? 8 hundred?

19. How much is 10 times 3 hundred? 6 hundred? 9 hundred?

20. How much is 9 times 11? 12? 13? 14? 15? 16? 17? 18? 19?

21. How much is 10 times 11? 12? 13? 14? 15? 16? 17? 18? 19?

22. How much is 11 times 11?

PROCESS.—10 *times* 11 *and once* 11 *gives* 121.

23. How much is 12 times 11? 12 times 12?

24. What will 11 dozen eggs cost, at 10 cents a dozen? At 11 cents? At 12 cents?

DEFINITION.—The number by which we multiply is called the *multiplier*, and the number multiplied is called the *multiplicand*. The two are called *factors*.

LESSON V.

1. How much is 5 times 26?

PROCESS.—5×26 *is* 5×2 *tens,* $+ 5 \times 6,$ *which is* 10 *tens, or* 100, + 30.

ANS.—130.

TABLES FOR PRACTICE.

I.?

2×40	5×90	2×13	3×13
9×30	6×80	2×23	3×21
4×70	3×50	2×14	3×25
9×60	4×60	2×42	3×31
8×80	7×30	2×35	3×28
7×50	2×90	2×49	3×32

II. ?

4 × 13	5 × 35	6 × 45	8 × 58	10 × 22
6 × 14	6 × 58	6 × 36	8 × 47	10 × 34
4 × 16	7 × 26	6 × 83	8 × 76	10 × 45
5 × 22	4 × 23	7 × 42	8 × 67	10 × 56
6 × 23	7 × 54	7 × 34	9 × 45	10 × 65
4 × 42	5 × 62	7 × 53	9 × 36	10 × 72
4 × 35	6 × 56	7 × 82	9 × 64	10 × 91

4. A man sold 57 barrels of apples at 5 dollars a barrel; how much did he get for all?

5. If he had sold them at 6 dollars a barrel, how much would he have received? at 7 dollars, how much?

6. What distance does a man travel in 9 days who goes 42 miles each day? what distance in ten days?

7. Out of a flock of sheep, 34 ewes had two lambs each; how many lambs did they all have?

8. In an orchard there are 8 rows of apple trees, and each row contains 22 trees; how many apple trees, in all?

LESSON VI.

1. How much is 2 × 2 × 2? 2 × 2 × 3? 2 × 2 × 4? 2 × 2 × 5? 2 × 2 × 6? 2 × 3 × 4? 2 × 5 × 6?

2. What are these products called?

ANS.—*Continued products.*

3. What is the continued product of 3, 4, and 5? 5, 4, and 3? 3, 5, and 6? 5, 6, and 3?

4. In a continued product, can we begin the multiplication with any two of the numbers, without changing the product?

5. What single number multiplied by 5 will give the same product as 2 × 3 multiplied by 5? as 2 × 2 × 5? as 2 × 4 × 5? as 3 × 4 × 5? as 2 × 2 × 2 × 5?

6. How much is $4 \times 5 \times 6$? $4 \times 6 \times 7$? $3 \times 6 \times 7$? $2 \times 2 \times 2 \times 3 \times 5$? $2 \times 2 \times 2 \times 4 \times 4$?

7. The square of any number is what we obtain by multiplying that number by itself. The square of 1 is 1×1; of 2, is 2×2, &c.

8. What is the square of 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

9. A man cuts two oranges into two parts each, and then cuts each of the parts into two parts; into how many parts were both oranges cut?

10. A miller wished to divide 3 bushels of meal among some beggars. He divided each bushel in four heaps, and then each heap in two parts. In how many parts were the three bushels divided?

11. If I cut a melon in 6 parts, and cut each of these parts in 5 pieces, and each of these pieces in 4 parts; in how many parts will the melon be divided?

12. A man had 10 orchards; each orchard contained 10 rows of trees, and each row of trees contained 10 trees; how many trees in all?

LESSON VII.

MISCELLANEOUS.

1. I subtracted 15 from a certain number, and obtained 5 for a remainder; what is that number?

2. 22 is 13 less than what number? 40 is 15 less than what number?

3. What number is composed of the two parts 34 and 26?

4. A farmer sold 5 barrels of flour at 5 dollars a barrel, and 10 bushels of wheat at 2 dollars a bushel; how much did he receive for the flour and wheat?

5. The difference between 46 and a greater number is 15; what is that number?

6. John has 6 marbles, and William has 3 times as many, less 10; what number has William?

7. A farmer sold 3 lambs at 2 dollars each; 5 pigs at 2 dollars each; 8 bushels of wheat at 3 dollars a bushel, and 5 barrels of corn at 8 dollars a barrel; what sum of money did he receive for all?

8. July, August, and October have each 31 days; September, March, and June have each 30 days; how many more days in the first three than the last three?

PROCESS.—*In the first three, there are* 3×31, *or* $3 \times 30 + 3 \times 1$; *and in the last three there are* 3×30; *hence, there are* 3×1, *or* 3 *more days in the first three than in the last.*

9. How much greater is 3×32 than 3×30? Ans.—3×2.

10. How much less is 5×10 than 5×15? Ans.—5×5.

11. How much less is 5×35 than 5×40? Ans.—5×5.

12. How much greater is 9×10 than 9×3? Than 9×4? Than 9×5?

13. How much more is 5×50 than 5×40? Than 5×41? Than 5×45? Than 5×49?

14. How much more is 8×8 than 7×7?

15. If a man buys 5 barrels of flour at 6 dollars a barrel, and sells them at 7 dollars, how much will he gain? How much if he sells at 8 dollars a barrel? 9 dollars a barrel? 10 dollars a barrel?

16. Frank has 30 chinquepins. William has 3 times as many, less 40, and Charles has twice as many, less 30; how many have they all together?

17. Multiply the sum of 8 and 9 by 9? Find the sum of the two products 8×9 and 9×9?

18. Two travellers set out from the same point at the same time, and travel in opposite directions. One travels 5 miles an hour, and the other 4 miles an hour; how far are they apart at the end of 5 hours? At the end of 6 hours? 7 hours? 8 hours?

19. $5 \times 5 + 5 \times 4$ is how much?

20. Two travellers set out from the same point at the same time, and travel in the same direction. One travels 6 miles an hour, and the other four miles an hour; how far will the slowest one be behind the other in 5 hours? 6 hours? 9 hours?

21. 5×6, less 5×4, is how much?

22. Two travellers set out at the same moment, one from Richmond and one from Washington, and travel toward each other. One travels 4 miles an hour, and the other 5 miles an hour. Putting the distance between Richmond and Washington at 100 miles; how far will the travellers be apart 3 hours after setting out? 4 hours? 5 hours?

23. 100 less $3 \times 4 + 3 \times 5$ is how much?

24. Ten times eight, plus eighteen, less twenty, less six, are how many?

25. Five times twenty, plus fifty, plus eight, are how many?

26. Six times twenty, less five times ten, are how many?

27. If a man earns 60 cents a day, and pays out 20 cents a day, how much will he have made at the end of 6 days?

CHAPTER IV.

DIVISION.

LESSON I.

We have seen how to separate large numbers into different sets of smaller ones. Thus, 5 is separated into $2 + 3$, $2 + 2 + 1$, $2 + 1 + 1 + 1$, $1 + 1 + 1 + 1 + 1$. 5 is said to *contain* these smaller numbers. A number can be separated into smaller

numbers, all of which are equal to each other. We say that the first number contains the other a certain number of times.

1. Into what equal numbers may every number be seperated?

Ans.—*Into ones.*

2. How many ones in 6? In 7? 8? 10? 105?

3. 1 is contained in 6 how many times? In 7? 8? 9? 110?

4. Distribute six cannon balls into heaps of two balls each; how many heaps will you have?

5. Distribute them into heaps of three balls each; how many heaps?

6. 2 is contained in 6 how many times?

7. 3 is contained in 6 how many times? 2 + 2 + 2 are how many?

8. How many times can you take 2 from 6 and 2 from the remainder until nothing remains? 3 + 3 is how much?

To divide one number by another is to find how often the latter is contained in the former.

9. How do we divide 12 by 2?

Ans.— *We distribute* 12 *into* 2's, *and then count the number of* 2's.

10. How is this done?

Ans.—*Either,* 1*ˢᵗ, By adding* 2's *thus:* 2 + 2 + 2 + 2 + 2 + 2, *until we get* 12; *or,* 2*ⁿᵈ, by subtracting* 2 *from* 12 *and* 2 *from the remainder, and so forth, until nothing remains; or,* 3*ʳᵈ, we may remember, from multiplication, how many times* 2 *make* 12, *or what number taken* 2 *times makes* 12.

11. Which of these is the shortest process?

The number which is divided by the other is called the *dividend.* The number by which we divide is called the *divisor.* The result obtained is called the *quotient.*

12. How many pair of spurs will make eight spurs?

13. How many pair of spectacles require ten glasses?

14. An ivory hunter gets two tusks from each elephant which he kills; how many elephants must he kill to get 12 tusks?

15. If Frank studies two hours a day, in how many days will he study 14 hours?

16. How many 2's in 4?   6?   8?   10?   12? 14?   16?   18?   20?

17. In what number is 2 contained exactly 2 times? 3 times?   4 times?   5 times?   6 times?   7 times?   8 times?   9 times?   10 times?

18. How much is 2 divided by 2?   12 divided by 2?
                            4 divided by 2?   14 divided by 2?
                            6 divided by 2?   16 divided by 2?
                            8 divided by 2?   18 divided by 2?
                          10 divided by 2?   20 divided by 2?

The numbers which contain 2 an exact number of times we say are *divisible* by 2.

19. How many pair of spurs will five spurs make?

20. If we endeavor to distribute 5 into 2's what do we find?   7 into 2's?   9 into 2's?

21. What is the 1 which is left in each case called?
Ans.—*The remainder.*

22. How much is 3 divided by 2?
Ans.—*1 and 1 remainder.*

23. How much is 5 divided by 2?   13 divided by 2?
                            7 divided by 2?   15 divided by 2?
                            9 divided by 2?   17 divided by 2?
                          11 divided by 2?   19 divided by 2?

24. What are the numbers called which are divisible by 2?
Ans.—*Even numbers.*

25. What are the numbers called which are not divisible by 2?
Ans.—*Odd numbers.*

26. What remainder results always when we divide the odd numbers by two?

27. Give the even numbers from 2 to 100.

28. Give the odd numbers from 1 to 99.

29. How much is 4 tens or 40 divided by 2 ?

Ans.—2 *tens or* 20.

30. 4 hundred divided by 2 ?

31. 6 tens or 60 divided by 2 ?

32. 6 hundred divided by 2 ?

33. 8 tens or 80 divided by two ?   8 hundred divided by 2 ?

34. 10 tens or 100 divided by 2 ?   10 hundred divided by 2 ?

35. 12 tens or 120 divided by 2 ?   14 tens divided by 2 ?   16 tens divided by 2 ?   18 tens divided by 2 ?

36. How much is 30 divided by 2 ?

Process.—30 *is* 20 + 10.   20 *divided by* 2 *is* 10, *and* 10 *divided by* 2 *is* 5.

Ans.—10 + 5 *or* 15.

37. How much is 50 divided by 2 ?   70 divided by 2 ?   90 divided by 2 ?

38. How much is 21 divided by 2 ?

Process.—21 *is* 20 + 1.   *Hence*, 21 *divided by* 2 *is* 10 *and* 1 *remainder.*

39. How much is 23 divided by 2 ?   24 ?   25 ?   36 ? 45 ?

40. If we multiply 5 by 2, and divide the product by 2, what will be the quotient ?

41. If we divide 8 by 2, and multiply the quotient by 2, what will be the product ?

42. If we divide any number by 2, and multiply the quotient by 2, what will be the result ?

LESSON II.

1. If one flour barrel holds 3 bushels, how many barrels will hold 9 bushels ?   12 bushels ?   15 bushels ?   30 bushels ?

2. Into how many ranks of four men each can you

divide a squad of 8 soldiers? a squad of 16 soldiers? a company of 36 soldiers? of 40 soldiers?

3. Into how many heaps of five balls each can you distribute 25 cannon balls? 30 cannon balls? 45 cannon balls? 50 cannon balls?

4. How many 3's in 6? 9? 12? 15? 18? 21? 24? 27? 30?

5. How many 4's in 8? 12? 16? 20? 24? 28? 32? 36? 40?

6. How many 5's in 10? 15? 25? 20? 30? 35? 45? 40? 50?

7. In what number is 3 contained exactly once? Twice? 6 times? 4 times? 7 times? 5 times? 9 times? 8 times? 10 times? 3 times?

8. In what number is 4 contained exactly twice? Once? 4 times? 8 times? 10 times? 6 times? 5 times? 7 times? 3 times? 9 times?

9. In what number is 5 contained exactly 4 times? Twice? 6 times? 5 times? 3 times? 7 times? 9 times? 8 times? 10 times?

10. How much is

3 divided by 3?	4 divided by 4?	5 divided by 5?
6 divided by 3?	8 divided by 4?	10 divided by 5?
9 divided by 3?	12 divided by 4?	15 divided by 5?
12 divided by 3?	16 divided by 4?	20 divided by 5?
15 divided by 3?	20 divided by 4?	25 divided by 5?
18 divided by 3?	24 divided by 4?	30 divided by 5?
21 divided by 3?	28 divided by 4?	35 divided by 5?
24 divided by 3?	32 divided by 4?	40 divided by 5?
27 divided by 3?	36 divided by 4?	45 divided by 5?
30 divided by 3?	40 divided by 4?	50 divided by 5?

11. How much is 3 × 11? 3 × 12? 3 in 33 how many times? 3 in 36?

12. How much is 4 × 11? 4 × 12? 4 in 44 how many times? 4 in 48?

13. How much is 5 × 11? 5 × 12? 5 in 55 how many times? 5 in 60?

14. 3 is contained in 4 how many times, and what remainder? in 5 how many times?

15. How many three bushel bags can you fill with 7 bushels of wheat, and what will remain? How many with 8 bushels?

16. What are the remainders of numbers divided by 3?

17. Give the numbers from 1 to 100 which are divisible by 3.

18. 4 in 5 how many times? 4 in 6? 4 in 7? 4 in 9? 4 in 10? 4 in 11?

What are the remainder of numbers divided by 4?

19. Give the numbers from 1 to 100 which are divisible by 4.

20. 5 in 6 how many times? 5 in 7? 5 in 8? 5 in 9? 5 in 11? 5 in 12? 5 in 13? 5 in 14?

What are the remainders of numbers divided by 5?

21. Give the numbers from 1 to 100 which are divisible by 5.

22. 3 in 3 tens how many times? in 6 tens? in 9 tens? in 12 tens? &c.

23. 3 in 3 hundred how many times? in 6 hundred? in 9 hundred? in 12 hundred?

24. 4 in 4 tens? in 8 tens? in 16 tens? in 24 tens?

25. 4 in 4 hundred? in 8 hundred? in 16 hundred?

26. 5 in 5 tens? 10 tens? 15 tens? in 5 hundred? 15 hundred?

27. Multiply 9 by 3, 4 and 5, and divide the products by 3, 4 and 5 respectively.

28. Divide 60 by 3, 4 and 5, and multiply the quotients by 3, 4 and 5 respectively.

29. A number multiplied by 3 and the product divided by 3 gives what result?

LESSON III.

1. If it requires 6 horses to draw each cannon on the field of battle, how many cannon will require 12 horses? 24 horses? 42 horses? 36 horses? 60 horses?

2. How many weeks do 14 days make? 28 days? 21 days? 35 days? 49 days? 56 days? 70 days?

3. If 8 leaves only are left to grow and ripen on each tobacco plant, how many plants will yield 16 leaves? 32 leaves? 48 leaves? 40 leaves? 80 leaves?

4. How many 6's in 12? 24? 18? 36? 48? 42? 54? 60? 30?

5. How many 7's in 14? 28? 21? 35? 49? 42? 56? 63? 70?

6. How many 8's in 16? 24? 40? 32? 48? 64? 56? 72? 80?

7. In what number is 6 contained exactly once? 3 times? Twice? 5 times? 7 times? 8 times? 9 times? 10 times?

8. In what number is 7 contained exactly once? 4 times? 6 times? 3 times? 5 times? 7 times? 9 times? 10 times? 8 times?

9. In what number is 8 contained exactly once? 5 times? 3 times? 4 times? 7 times? 9 times? 6 times? 8 times? 10 times?

10. How much is

6 divided by 6?	7 divided by 7?	8 divided by 8?
24 divided by 6?	14 divided by 7?	24 divided by 8?
36 divided by 6?	21 divided by 7?	40 divided by 8?
12 divided by 6?	28 divided by 7?	32 divided by 8?
18 divided by 6?	35 divided by 7?	16 divided by 8?
48 divided by 6?	42 divided by 7?	48 divided by 8?
42 divided by 6?	49 divided by 7?	56 divided by 8?
54 divided by 6?	56 divided by 7?	72 divided by 8?

60 divided by 6? 63 divided by 7? 64 divided by 8?
30 divided by 6? 70 divided by 7? 80 divided by 8?

11. How much is 6×11? 6×12? How much is 66 divided by 6? How much is 72 divided by 6?

12. How much is 11×7? 12×7? How much is 77 divided by 7? 84 divided by 7?

13. How much is 11×8? 12×8? How much is 88 divided by 8? 96 divided by 8?

14. 6 in 7 how many times, and what remainder? In 8? In 9? In 10? In 11? In 13? In 14? In 15? 16? 17?

What are the remainders of numbers divided by 6?

15. 7 in 15 how many times, and what remainder? In 16? 17? 18? 19? 20? 22? 23? 24? 25? 26? 27?

What are the remainders of numbers divided by 7?

16. 8 in 9 how many times, and what remainder? In 10? 11? 12? 13? 14? 19? 25? 26? 27? 28? 29? 30? 31?

What are the remainders of numbers divided by 8?

17. Give the numbers from 1 to 100 which are divisible by 6.

18. Give the numbers from 1 to 100 which are divisible by 7.

19. Give the numbers from 1 to 100 which are divisible by 8.

20. 6 in 6 tens how many times? In 12 tens? In 18 tens? In 24 tens? In 36 tens? In 60 tens?

21. 6 in 6 hundred how many times? In 12 hundred? In 18 hundred?

22. 7 in seven tens how many times? In 14 tens? In 21 tens? In 49 tens? In 56 tens? In 70 tens?

23. 7 in 7 hundred? In 14 hundred? In 21 hundred?

24. 8 in 8 tens? 16 tens? 24 tens? 32 tens? 56 tens? 80 tens?

25. 8 in 8 hundred? in 16 hundred? in 24 hundred?

26. How much is 6 × 30 divided by 6? How much is 56 divided by 8 and the quotient multiplied by 8?

27. 6 in 40 how many times?

PROCESS.—40 *is* 36, *and* 4, 6 *in* 36 6 *times.* *Hence* 6 *in* 40 6 *times and remainder* 4.

28. 6 in 50 how many times? 6 in 100? 8 in 50? 8 in 60? 8 in 90? 7 in 60? 7 in 80? 7 in 100? 8 in 100?

LESSON IV.

1. If I wish to plant 36 trees in parallel rows of nine trees each, how many rows must I plant? 27 trees will give how many rows of 9 trees each? 45 trees how many rows? 54 trees? 72 trees? 63 trees? 81 trees? 90 trees?

2. A vessel was wrecked on a rocky coast, and there were more than a hundred men clinging to the wreck. The life-boat from the shore could only save ten men on each trip to the wreck. How many trips did its bold rowers have to make in order to save 30 men? 40 men? 60 men? 100 men? 80 men?

3. How many 9's in 18? 27? 36? 45? 63? 54? 72? 81? 90?

4. How many 10's in 20? 80? 90? 70? 60? 50? 40? 30? 100?

5. What number contains 9 exactly once? 3 times? Twice? 5 times? 6 times? 8 times? 7 times? 4 times? 9 times? 10 times?

6. What number contains 10 exactly 10 times? 9 times? 8 times? 7 times? 6 times? 5 times? 4 times? 3 times? Twice?

7. How much is
9 divided by 9? 10 divided by 10?

18 divided by 9 ?   20 divided by 10 ?
27 divided by 9 ?   30 divided by 10 ?
36 divided by 9 ?   40 divided by 10 ?
45 divided by 9 ?   50 divided by 10 ?
54 divided by 9 ?   60 divided by 10 ?
63 divided by 9 ?   70 divided by 10 ?
72 divided by 9 ?   80 divided by 10 ?
81 divided by 9 ?   90 divided by 10 ?
90 divided by 9 ? 100 divided by 10 ?

8. How much is 11×9? 12×9? How many times does 99 contain 9? How many times does 108 contain 9?

9. How much is 11×10? 12×10? 10 in 110 how many times? 10 in 120 how many times?

10. 9 in 10 how many times, and what remainder? In 11? 12? 13? 14? 15? 16? 17? In 19? 20? 21? 22? 23? 24? 25? 26?

What are the remainders of numbers divided by 9?

11. 10 in 21 how many times, and what remainder? In 22? 23? 24? 25? 26? 27? 28? 29? In 41? 42? 43? 44? 45? 46? 47? 48? 49?

What are the remainders of numbers divided by 10?

12. Give the numbers, from 1 to 100, which are divisible by 9.

13. Give the numbers, from 1 to 200, which are divisible by 10.

14. What figure do all numbers divisible by 10 end in?

15. 9 in 9 tens how many times? In 18 tens? 27 tens? 36 tens? 45 tens? 90 tens?

16. 9 in 9 hundred how many times? 18 hundred? 27 hundred? 36 hundred? 45 hundred? 63 hundred?

17. 10 in 10 tens how many times? In 20 tens? 30 tens? 40 tens? 50 tens? 90 tens?

3

18. 10 in 20 hundred? In 3000? 4000? 60 hundred? In 7000? In 9000?

19. 9 in 86 how many times?

Process.—86 *is* 81 *and* 5. 9 *in* 81, 9 *times.* *Hence,* 9 *in* 86, 9 *times, and* 5 *remainder.*

20. 9 in 97 how many times? In 105? In 111? 113? In 960? 963? 999? In 456?

21. 10 in 770 how many times? 220? 440? 1010? 77? 78? 101? 105? 362? 456?

22. When a number is not divisible by 10, which figure of the number is always the remainder?

23. How much is $7 \times 8 \times 10$, divided by 10? How much is 60×9, divided by 9?

24. How much is $9 \times 8 + 3$? $9 \times 8 + 2$? $9 \times 7 + 5$? $9 \times 6 + 7$?

25. How much is $10 \times 3 + 4$? $10 \times 4 + 5$? $10 \times 7 + 6$? $10 \times 8 + 9$?

LESSON V.

1. A workman, desiring to finish a piece of work speedily, labors 11 hours a day. In how many days will he have worked 33 hours? 44 hours? 55 hours? 77 hours? 22 hours? 88 hours? 99 hours? 110 hours?

2. Twelve eggs make a dozen. How many dozen eggs are 24 eggs? 36 eggs? 72 eggs? 48 eggs? 60 eggs? 84 eggs? 96 eggs? 108 eggs? 120 eggs?

3. How many 11's in 22? 33? 44? 55? 66? 77? 88? 99? 110?

4. How many 12's in 24? 36? 48? 60? 72? 84? 96? 108? 120?

5. What number contains 11 exactly once? Twice? 4 times? 7 times? 3 times? 5 times? 9 times? 6 times? 8 times? 10 times?

6. What number contains 12 exactly twice? Once? 3 times? 5 times? 6 times? 4 times? 8 times? 7 times? 9 times? 10 times?

7. How much is

11 divided by 11?	12 divided by 12?
22 divided by 11?	24 divided by 12?
33 divided by 11?	36 divided by 12?
44 divided by 11?	48 divided by 12?
55 divided by 11?	60 divided by 12?
66 divided by 11?	72 divided by 12?
77 divided by 11?	84 divided by 12?
88 divided by 11?	96 divided by 12?
99 divided by 11?	108 divided by 12?
110 divided by 11?	120 divided by 12?

8. How much is 11×11? 12×11? 12×12? 11 in 121 how many times? 11 in 132? 12 in 132? 12 in 144?

9. 11 in 12 how many times, and what remainder? 11 in 13? in 14? in 15? in 16? in 17? in 18? in 19? in 20? in 21? in 23? &c.

What are the remainders of numbers divided by 11?

10. How much is $7 \times 11 + 3$? $5 \times 11 + 4$? $5 \times 11 + 5$? $6 \times 11 + 7$? $6 \times 11 + 10$?

11. 12 in 25 how many times, and what remainder? in 26? 27? 28? 29? 30? 31? 32? 33? 34? 35? 37?

12. How much is $2 \times 12 + 1$? $2 \times 12 + 2$? $2 \times 12 + 3$? $2 \times 12 + 4$? $2 \times 12 + 5$? $2 \times 12 + 10$? $2 \times 12 + 11$?

What are the remainders of numbers divided by 12?

13. Give the numbers, from 1 to 200, which are divisible by 11.

14. Give the numbers, from 1 to 200, which are divisible by 12.

15. 11 in 11 tens how many times? in 22 tens? 33 tens? 66 tens? 99 tens?

16. 11 in 11 hundred how many times? in 22 hundred? 44 hundred? in 88 hundred?

17. 12 in 12 tens how many times? in 24 tens?
in 36 tens? in 60 tens? in 84 tens? in 96 tens?

18. 12 in 12 hundred how many times? in 24
hundred? in 48 hundred? in 72 hundred? in 96
hundred?

19. 11 in 79 how many times? in 89? in 101?
772? 330? 334? 67? 667? 446?

20. 12 in 38 how many times? in 360? in 362?
in 484? in 75? in 721? 964?

21. How much is 55 divided by 11 and the quo-
tient multiplied by 11?

22. How much is 60 divided by 12 and the quotient
multiplied by 12? How much is 7×12, divided by
12?

LESSON VI.

1. Pythagoras, a Greek philosopher, united very

ingeniously in a square table all the products of the numbers from 1 to 12. This table can be used as a table of quotients.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	83
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

2. The first row at the top, moving from left to right, contains the numbers 1, 2, 3, &c., to 12. The first row on the left, moving from the top to the bottom of the table, contains also the numbers 1, 2, 3, &c., to 12.

The numbers in the second horizontal row, counting from the top, and those in the second vertical row, counting from the left, are formed by adding the

NOTE.—Horizontal means here across the page from left to right or right to left. Vertical means up and down the page. The horizontal we will call rows; the vertical, columns.

numbers in the first row to themselves. Those in the third rows are formed by adding those in the second rows to those in the first respectively. The numbers in the fourth rows by adding those in the third rows to those in the first respectively, &c.

3. The first row and the first column contain 1, 2, 3, &c.

The second row and second column contain 1, 2, 3, &c., multiplied by 2.

The third row and third column contain 1, 2, 3, &c., multiplied by 3, &c., &c.

4. If we wish to find the product of two numbers, as 5 multiplied by 6, we find 5 in the top row, and 6 in the first column on the left. We come down the fifth column until we get to the 6th row, and there find 30, the product.

5. If we wish to find the quotient of 108 divided by 9, we find 9 in the top row, come down the column to 108, and find 12, the first number on the left, in the same horizontal row with 108.

6. How is the table of Pythagoras formed?

7. What do the first row and first column contain?

8. The second row and the second column?

9. The fourth row and the fourth column?

10. Write the table on your slate.

11. Use the table to count by 2's from 2 to 24. Count by 3's from 3 to 36. Count by 5's from 5 to 60. Count by 9's from 9 to 108. Count by 11's from 11 to 132. Count by 12's from 12 to 144.

12. Begin with the numbers 24, 36, 48, &c., in the column on the right, and count backward by 2's, 3's, 4's, &c., respectively.

LESSON VII.

1. Seven are how many times 2? How many times 3? 4?

2. Ten are how many times 2? 3? 4? 5?

3. Nine are how many times 2? 3? 4? 5?

4. Eleven are how many times 3? 4? 6?

5. Twelve are how many times 4? 6? 3? 2? 5? 8?

6. Thirteen are how many times 2? 3? 4? 5? 6? 7? 8? 9?

7. Fourteen are how many times 2? 7? 5? 3? 4?

8. Fifteen are how many times 3? 5? 6? 4?

9. Sixteen are how many times 4? 5? 6? 3?

10. Seventeen are how many times 3? 4? 5? 2? 6? 9?

11. Eighteen are how many times 2? 3? 4? 6? 9?

12. Nineteen are how many times 6? 3? 10? 2? 5?

13. Twenty are how many times 2? 3? 4? 5? 6? 7? 8? 9? 10?

14. Twenty-one are how many times 7? 3? 8? 2? 4? 6? 10?

15. Twenty-two are how many times 2? 3? 4? 5? 6? 7? 8? 9? 10? 11?

16. Twenty-three are how many times 2? 3? 4? 6? 8? 10? 11?

17. Twenty-four are how many times 2? 3? 4? 5? 6? 6? 8? 9?

18. Twenty-five are how many times 2? 3? 4? 5? 6? 7? 8? 10?

19. Twenty-six are how many times 3? 4? 5? 6? 7? 8? 9? 10?

20. Twenty-seven are how many times 3? 4? 5? 6? 7? 8? 10? 11?

21. Twenty-eight are how many times 3? 4? 5? 6? 8? 10?

22. Twenty-nine are how many times 3? 4? 5? 6? 7? 8? 9? 10?

23. Thirty are how many times 3? 4? 5? 6? 7? 8? 9? 10?

24. Thirty-one are how many times 3?   4?   5?
6?   7?   8?   9?   11?

25. Thirty-two are how many times 9?   10?   8?
7?   6?   5?   4?   3?

26. Thirty-three are how many times 6?   8?   7?
4?   9?   5?   10?   3?

27. Thirty-four are how many times 7?   3?   9?
10?   6?   8?   4?   5?

28. Thirty-five are how many times 3?   4?   5?
6?   7?   8?   9?   10?

29. Thirty-six are how many times 3?   4?   5?   6?
7?   8?   9?   11?

30. Thirty-seven are how many times 3?   4?   5?
7?   8?   9?   10?

31. Thirty-eight are how many times 2?   4?   3?
5?   6?   7?   8?   9?

32. Thirty-nine are how many times 3?   4?   5?
6?   7?   8?   9?   10?   11?

33. Forty are how many times 8?   10?   6?   4?
3?   9?   5?   7?

34. Forty-one are how many times 4?   5?   6?   7?
8?   9?   10?

35. Forty-two are how many times 4?   5?   6?   7?
8?   9?   10?   11?

36. Forty-three are how many times 4?   5?   6?
7?   8?   9?

37. Forty-four are how many times 4?   5?   6?
7?   8?   9?   11?

38. Forty-five are how many times 4?   5?   6?   7?
8?   9?   10?

39. Forty-six are how many times 4?   5?   6?   7?
8?   9?   10?

40. Forty-seven are how many times 4?   5?   6?
7?   8?   9?   10?

41. Forty-eight are how many times 4?   5?   6?
7?   8?   9?   10?   12?

42. Forty-nine are how many times 4?   5?   6?   &c.

43. Fifty are how many times 4? 5? 6? 7? 8?
9? 10? 11?

LESSON VIII.

1. Fifty-one are how many times 10? 5? 3? 6?
9? 7? 8? 4?

2. Fifty-two are how many times 4? 8? 7? 6?
12? 10? 13?

3. Fifty-three are how many times 9? 10? 6?
7? 8? 12? 11?

4. Fifty-four are how many times 9? 10? 6? 7?
8? 12? 11?

5. Fifty-five are how many times 4? 8? 10? 12?
11? 2?

6. Fifty-eight are how many times 2? 3? 4? 5?
6? 7? 8? 9? 10?

7. Sixty are how many times 3? 4? 12? 15?
30?

8. Sixty-four are how many times 4? 8? 16? 12?
30?

9. Sixty-five are how many times 5? 6? 7? 8?
10? 12? 13?

10. Sixty-nine are how many times 6? 7? 10?
20?

11. Sixty-eight are how many times 11? 12? 8?
7? 10? 6?

12. Sixty-seven are how many times 8? 7? 11?
12? 5? 9?

13. Seventy are how many times 2? 6? 7? 8?
9? 11? 10?

14. Seventy-two are how many times 3? 5? 7?
6? 8? 9? 12?

15. Seventy-three are how many times 6? 7? 8?
9? 10?

16. Seventy-six are how many times 3?. 4? 5?
6? 7? 8? 9? 10? 11?

3*

17. Seventy-seven are how many times 6 ?   7 ?   8 ?
9 ?   10 ?   11 ?

18. Eighty are how many times 7 ?   8 ?   9 ?   10 ?
11 ?   20 ?   40 ?

19. Eighty-one are how many times   7 ?   8 ?   9 ?
10 ?   11 ?   12 ?   6 ?

20. Eighty-four are how many times 6 ?   7 ?   8 ?
9 ?   10 ?   11 ?   12 ?

21. Eighty-five are how many times 6 ?   7 ?   8 ?
9 ?   10 ?   11 ?   12 ?

22. Eighty-eight are how many times 7 ?   8 ?   9
10 ?   11 ?   12 ?

23. Eighty-nine are how many times 7 ?   11 ?   12 ?
8 ?   9 ?   10 ?

24. Ninety are how many times 10 ?   12 ?   7 ?   9 ?
8 ?   6 ?   30 ?

25. Ninety-one are how many times 7 ?   8 ?   9 ?
10 ?   11 ?   12 ?   30 ?

26. Ninety-four are how many times 12 ?   7 ?   10 ?
9 ?   8 ?   11 ?

27. Ninety-five are how many times 9 ?   10 ?   7 ?
8 ?   11 ?   12 ?

28. Ninety-six are how many times 9 ?   7 ?   8 ?
10 ?   11 ?   12 ?

29. Ninety-seven are how many times 8 ?   9 ?   10 ?
11 ?   12 ?

30. Ninety-eight are how many times 8 ?   9 ?   10 ?
11 ?   12 ?   30 ?

31. Ninety-nine are how many times 8 ?   9 ?   10 ?

32. One hundred are how many times 4 ?   5 ?   20 ?
50 ?   9 ?   10 ?   2 ?   8 ?   25 ?

33. One hundred and eight are how many times 9 ?
12 ?   3 ?   6 ?   25 ?   20 ?

34. One hundred and ten are how many times 10 ?
11 ?   5 ?   2 ?   20 ?

35. One hundred and twenty are how many times
10 ?   12 ?   6 ?   20 ?   5 ?   8 ?

36. One hundred and forty-four are how many times 12? 9? 6? 20? 50?

LESSON IX.

1. Divide 126, 180, 162, and 138 each by 2.

PROCESS.—*126 is 12 tens and 6 units. 12 tens contain 2 6 tens, or 60, times, and 6 contains 2 3 times. Hence, 126 contains 2 63 times.*

2. Divide 114, 150, 168, 243, and 408 each by 3.

PROCESS.—*114 is 11 tens and 4 units, or 9 tens and 2 tens and 4 units. 3 in 9 tens, 3 tens, or 30 times. 3 in 24, 8 times. Hence, 3 in 114, 38 times.*

3. Divide 140, 180, 408, 252, and 156 each by 4.

4. Divide 120, 240, 315, 225, and 550 each by 5.

5. Divide 96, 126, 162, 186, 198, and 216 each by 6.

6. Divide 91, 105, 154, 224, and 308 each by 7.

7. Divide 104, 176, 200, 216, and 464 each by 8.

8. Divide 126, 207, 333, 306, and 405 each by 9.

9. Divide 500, 690, 780, 320, and 450 each by 10.

10. Divide 154, 198, 253, 374, and 495 each by 11.

11. Divide 192, 300, 480, 492, and 708 each by 12.

CHAPTER V.

MISCELLANEOUS.

LESSON I.

For the sake of abbreviation, the sign ÷ is used in division to indicate "divided by." 30 ÷ 6 is read, 30 divided by 6. This sign, as well as the × for "times," or "multiplied by," will be used hereafter, when it is more convenient to do so.

1. Thomas divided 12 apples equally among his

three brothers; how many apples did he give each of them?

2. If 4 yards of cloth cost 20 dollars, how much does one yard cost? 2 yards?

3. Mary divided 10 roses equally among her 5 play-mates, how many did she give to each one?

4. If I pay 35 cents for 5 pounds of sugar, how much does one pound cost?

5. 42 bushels of corn meal were divided equally among 7 persons; how many bushels did each receive?

6. 60 dollars are paid for 12 cords of wood; how much does the wood cost per cord?

7. If 10 oranges are worth as much as 40 apples, how many apples are worth as much as one orange?

8. If 63 men reap a field in one day, how many men, laboring at the same rate, will reap it in 9 days?

9. How many barrels of flour can be bought for 84 dollars, at 12 dollars a barrel? at 7 dollars a barrel? at 14 dollars a barrel?

10. Two boats on the Mississippi are 100 miles apart. The hindmost boat gains on the other 5 miles an hour. In how many hours will it overtake the other?

11. A man, having 86 dollars, bought 12 cords of wood, and had 2 dollars left; how much did he give for each cord?

12. What is the price of a pound of beef, if 12 pounds can be bought for 60 cents? for 84 cents? 96 cents?

13. One man can do a piece of work in 25 days; how long will it take 5 men to do the same work?

14. 5 men can do a piece of work in 10 days; how long will it take one man to do the same work?

NOTE.—Each day's labor of each man is considered the same in these examples.

15. If one pipe discharges a cistern of water in 84 hours, how long will it take 7 pipes of the same size to discharge the cistern?

16. If 12 pipes discharge a cistern in 96 hours, how long will it take one pipe of the same dimensions to discharge it?

17. Two boats are 50 miles apart, approaching each other. One is moving at the rate of 6 miles an hour, and the other 4 miles an hour; how long before they will come together?

18. For 65 dollars, how many sheep can you buy, at 6 dollars apiece, and how much money would you have left?

19. If 51 dollars be divided among 3 men, how many dollars will each receive?

20. If 12 tons of coal cost 96 dollars, how much does one ton cost? 2 tons?

21. In how many days will 5 men reap a field of wheat, if 40 men reap it in one day?

22. A laborer engaged to work 4 months for 96 dollars; how much did he receive for each month? How much for each week, allowing 4 weeks to each month? How much each day, allowing 6 working days to each week?

23. Six whaling vessels brought from their voyages 1800 barrels of sperm oil, and each vessel brought the same number of barrels; how many barrels did each bring?

24. How much butter at 12 cents a pound can be bought with 6 dozen eggs at 10 cents a dozen? 6 times 10 are how many times 12?

25. 4 times 9 are how many times 3?

26. 4 times 10 are how many times 5? 8?

27. 5 times 12 are how many times 15? 6?

28. 6 times 7 are how many times 14?

29. How many times 12 are 9 times 4?

30. A man travels 8 miles an hour for 9 hours,

how many hours must a man travel 6 miles an hour in order to go the same distance?

31. How many times 7 are 3 times 14?

32. How many times 12 are 9 times 4 + 24?

33. 5 times 9 + 9 are how many times 6?

34. 7 times 12 — 14 are how many times 5?

35. How many yards of cloth at 12 dollars a yard can be bought for 360 dollars?

36. How many tons of coal at 11 dollars a ton can be bought for 136 dollars, and how many dollars will remain?

37. 8 times 8 are how many times 4? 32? 16?

38. 7 times 10 are how many times 5? 7? 14?

39. How long will it take a man to ride 72 miles, if he rides 8 miles an hour? 9 miles an hour? 3 miles an hour? 6 miles an hour? 4 miles an hour?

40. 10 times 6 are how many times 12? 20? 5? 4? 30?

41. How much is 67 less 9 times 3? less 4 times 9?

42. A boat sailed up stream 8 miles an hour for 12 hours, and came back over the same distance in 6 hours; how many miles an hour did she sail down stream?

43. How much is 25 less 45 ÷ 5?

44. How much is 27 ÷ 3 and the quotient multiplied by 4?

LESSON II.

1. If 3 oranges cost 9 cents, how much will 5 oranges cost?

PROCESS.—1. *If 3 oranges cost 9 cents, 1 orange costs 3 cents.* 2. *If 1 orange costs 3 cents, 5 oranges cost 15 cents.*

2. If 4 lemons cost 24 cents, how much will 7 lemons cost?

3. If 12 barrels of apples cost 108 dollars, how much will one barrel cost? If 1 barrel of apples costs 9 dollars, how much will 3 barrels cost? 4 barrels?

4. A man travels 16 miles in 4 hours; how far does he travel in 1 hour? How far in 6 hours? 7 hours? 9 hours?

5. How many workmen can do in 1 day what 4 workmen can do in 12 days?

Ans.—12 *times as many, or* 48 *workmen.*

6. How many workmen will require 12 days to do what 48 workmen can do in 1 day?

Ans.—48 *is* 12 *times as many as the number of workmen required.*

Hence 48 *divided by* 12, *or* 4 *workmen.*

7. 3 workmen in 15 days will do as much as 15 workmen in how many days?

Process.—1. *The work done by 3 workmen in 15 days can be done in one day by* 15 × 3, *or* 45 *workmen.* 2. *What* 45 *workmen can do in one day,* 15 *workmen can do in as many days as* 45 ÷ 15, *or* 3 *days.*

8. How many workmen will do in 4 days what 2 workmen require 28 days to accomplish?

Process.—1. *The work which 2 workmen can do in 28 days, can be done by one workman in* 2 × 28, *or* 56 *days.* 2. *What one workman can do in 56 days, can be done in 4 days by as many workmen as* 56 ÷ 4, *or* 14 *workmen.*

9. 15 laborers in 4 days do as much as 5 laborers in —— days?

10. 24 laborers in 2 days do as much as 16 laborers in —— days?

11. 12 laborers in 6 days do as much as 9 laborers in —— days?

12. 9 laborers in 10 days do as much as 30 laborers in —— days?

13. 10 laborers in 18 days do as much as 60 laborers in —— days?

14. 10 laborers in 12 days do as much as 20 laborers in —— days?

15. 12 laborers in 12 days do as much as 18 laborers in —— days?

16. 40 laborers in 3 days do as much as 6 laborers in —— days?

17. 15 days' work of 3 men is equal to 9 days' work of —— men?

18. 15 days' work of 4 men is equal to 5 days' work of —— men?

19. 3 days' work of 10 men is equal to 16 days' work of —— men?

20. 6 days' work of 6 men is equal to 12 days' work of —— men?

21. 5 days' work of 15 men is equal to 15 days' work of —— men?

22. 24 days' work of 5 men is equal to 12 days' work of —— men?

23. 24 days' work of 4 men is equal to 48 days' work of —— men?

24. How much will 30 pounds of sugar cost if 6 pounds cost 42 cents?

25. If 10 pencils cost 30 cents, what will be the cost of 7 pencils? of 9? of 15? of 12?

26. If 7 days' board cost 14 dollars, what will be the cost of 30 days' board?

27. If 8 quarts of milk cost 48 cents, how much will 30 quarts cost?

28. A merchant bought 20 yards of cloth for 100 dollars; he gave 10 yards of it for some flour which cost 10 dollars a barrel; how many barrels did he get?

29. A greyhound, chasing a rabbit, is 30 yards behind the rabbit, and gains on him 6 yards every 3 minutes; in how many minutes will he overtake him?

30. A man hired a laborer and agreed to give him 10 dollars for every 3 days' work; how much did he have to pay him each week of 6 working days, and how much for each month of 4 weeks?

31. If 6 oranges are worth 1 pine-apple, and 2 pine-apples are worth 1 melon, how many oranges are worth 4 melons?

32. If 9 bushels of apples cost 18 dollars, how much do 100 bushels cost?

33. If 60 pounds of pork cost 300 cents, how much will 9 pounds cost? 10 pounds? 3 pounds?

34. If a man travels 56 miles in 8 hours, how far does he travel in 3 hours? 4 hours? 5 hours?

35. On the 12th of November, 25 meteors appear and shoot across the sky every 2 hours; how many appear in 3 hours? In 5 hours?

36. A certain number of rations last 20 soldiers 5 days; how long would they last 2 soldiers? 50 soldiers? 25 soldiers?

37. 40 days' rations for 5 men are 8 days' rations for —— men?

38. 16 days' rations for 5 men are 10 days' rations for —— men?

39. 18 days' rations for 6 men are 12 days' rations for —— men?

40. 30 days' rations for 60 men are 18 days' rations for —— men?

41. 3 days' rations for 90 men are 10 days' rations for —— men?

42. 24 days' rations for 10 men are 3 days' rations for —— men?

43. 16 days' rations for 6 men are 12 days' rations for —— men?

44. 8 days' rations for 8 men are 2 days' rations for —— men?

45. 10 yards of cloth at 8 dollars a yard will pay for how many barrels of flour at 5 dollars a barrel?

46. 9 times 8 are how many times 24 ÷ 2 ?

47. 9 times 11 less 18 are how many times 36 ÷ 4 ?

48. 10 times 6 and 12 are how many times 40 ÷ 8 ?

49. 12 times 7 and 15 are how many times 44 ÷ 4 ?

50. 4 times 12 and 8 are how many times 49 ÷ 7 ?

51. Bought 20 barrels of flour, at 6 dollars a barrel, and 6 yards of cloth, at 5 dollars a yard. Paid for the whole with 15 loads of hay. How many dollars did I get for each load of hay ?

52. If 4 cords of wood are worth 24 dollars, and 10 cords of wood are given for 40 thousand feet of plank, how many dollars will each thousand feet of plank cost ?

53. 5 men bought a horse for 63 dollars, and paid two dollars a week for keeping him ; at the end of 8 weeks, they sold him for 54 dollars. How much did each man lose by the bargain ?

54. A man bought 12 bales of cotton at 5 hundred dollars for every two bales. He sold all the cotton at 6 hundred dollars for every three bales. How many hundred dollars did he lose ?

55. What is the rent of 12 acres of land, that of 16 acres being 20 dollars ?

56. How long will 10 months' rations for 300 soldiers, issued at the same rate, last a garrison of 500 soldiers ?

CHAPTER VI.

MULTIPLES AND DIVISORS.

LESSON I.

1. 12 contains 6 an exact number of times. By adding 6 to itself once, or multiplying it by 2, we obtain 12. Hence 12 is called a *multiple* of 6. Thus, 24 is a *multiple* of 6, and 50 is a *multiple* of 5.

2. When is one number said to be a *multiple* of another number?

Ans.— *When one number contains another number an exact number of times, it is said to be a multiple of that number.*

3. 6 divides 12 exactly, and hence is called a *divisor* of 12, or one of its *factors*. In the same manner, 4 is a *factor* or *divisor* of 12. 5 is one of the *factors* or *divisors* of 60.

4. What is meant by a factor or divisor of a given number?

Ans.—*Any number which is contained an exact number of times in the given number.*

5. Every number contains itself once. 5 contains 5 once, 100 contains 100 once. Hence, every number is a multiple and divisor of itself.

6. What number divides exactly all numbers? How often is 1 contained in 5? In 6? In 500?

7. What two divisors has every number, at least?

Ans.—*The number itself, and one.*

8. Give 4 numbers which are multiples of 9.

9. Give 5 numbers which are multiples of 10. Of 11. Of 12.

10. Give 3 multiples of each of the numbers, 20, 30, and 40.

11. Find some numbers which are factors or divisors of 15. Of 20. Of 40. Of 50.

Including one and the numbers themselves, respectively—

12. Find the 3 divisors of 4. Of 9. Of 25. Of 49.

13. What are the four divisors of 8? of 10? of 14? 15? 21? 22? 26? 33? 34? 35? 38? 39? 46? 51? 55? 57? 58? 62? 65? 69? 74? 77? 82? 85? 86? 87? 91? 93? 94? 95?

14. What are the five divisors of 16? of 81?

15. What are the six divisors of 12? 18? 20? 28? 32? 44? 45? 50? 52? 63? 68? 75? 76? 92? 98? 99?

16. What are the seven divisors of 64?

17. What are the eight divisors of 24? of 30? 40? 42? 54? 56? 66? 70? 78? 88?

18. What are the nine divisors of 36? of 100?

19. What are the ten divisors of 48? of 80?

20. What are the 12 divisors of 60? of 72? of 84? of 90? of 96?

LESSON II.

A number which has several divisors is called a *common multiple* of these divisors. Thus, 10 is a common multiple of 2 and 5, because 2 and 5 are both contained in 10 an exact number of times. 4 is a common multiple of 2 and 4. 12 is a common multiple of 1, 2, 3, 4, 6, and 12.

1. What is a common multiple of two or more numbers?

Ans.—*It is any number which contains each of the given numbers an exact number of times.*

2. Find some common multiples of 5 and 4. Of 6, 3, and 2. Of 3 and 4.

3. What is the least common multiple of two or more numbers?

ANS.—*It is the smallest number which these numbers will exactly divide.*

4. What is the least common multiple of 2 and 3? of 3 and 4? of 4 and 5? of 5 and 10?

5. of 2 and 4? of 3 and 5? of 5 and 6? of 5 and 100?

6. of 2 and 5? of 3 and 6? of 4 and 7? of 6 and 20?

7. of 2 and 6? of 3 and 7? of 4 and 8? of 7 and 21?

8. of 2 and 7? of 3 and 8? of 4 and 9? of 8 and 12?

9. of 2 and 8? of 3 and 9? of 4 and 10? of 9 and 30?

10. of 2 and 9? of 3 and 10? of 4 and 11? of 10 and 8?

11. What is the least common multiple of 2, 3, and 4?

PROCESS.—*The least common multiple of 2 and 3 is 6, of 6 and 4 is 12. Hence, 12 is the least common multiple of 2, 3, and 4.*

12. What is the least common multiple

of 2, 3, and 5? of 3, 4, and 5?
of 2, 3, and 6? of 3, 4, and 6?
of 2, 3, and 7? of 3, 4, and 7?

13. What is the least common multiple of 2, 3, 4, and 6?

PROCESS.—*The least common multiple of 2 and 3 is 6, of 6 and 4 is 24, of 24 and 6 is 24. Hence, the least common multiple of 2, 3, 4, 6 is 24.*

14. What is the least common multiple

of 2, 3, 4, 5? of 3, 4, 5, 6? of 4, 5, 6, 8?
of 2, 3, 4, 7? of 3, 4, 5, 10? of 4, 5, 6, 9?
of 2, 3, 4, 10? of 3, 4, 5, 8? of 4, 5, 6, 10?

15. What is the least common multiple of 2, 3, 4, 5, 6?

LESSON III.

1. Give two or more numbers of which 3 is a divisor. 3 is called a *common divisor* or *common factor* of these numbers.

2. What numbers are common divisors of 8 and 12 ?

Ans.—*2 and 4—because they divide 8 and 12 without a remainder.*

3. What numbers are common divisors of 6, 12, and 24 ?

Ans.—*2, 3, and 6.*

4. When is a number said to be a common divisor or common factor of two or more numbers ?

Ans.— *When it is contained an exact number of times in each of these numbers.*

5. What number is a common divisor of all numbers ?

6. Which is the greatest common divisor of 8 and 12 ?

Ans.—*4. Because it is the greatest number which divides 8 and 12 without a remainder.*

7. What is the greatest common divisor of 6, 12, and 24 ?

Ans.—*6.*

8. What is the greatest common divisor of two or more numbers ?

Ans.—*The greatest number which is contained exactly in these numbers.*

9. What is the greatest common divisor of 15 and 20 ? of 6 and 24 ? of 12 and 30 ? of 25 and 45 ? of 22 and 33 ?

10. What is the greatest common divisor of 4, 8, and 16 ?

Process.—*The greatest common divisor of 8 and 16 is 8; of 8 and 4 is 4. Hence, 4 is the greatest common divisor of 4, 8, and 16.*

11. What is the greatest common divisor of 5, 10, and 15? of 9, 12, and 15? of 6, 7, and 8?

12. What is the greatest common divisor of 18 and 48? of 24 and 36? of 60 and 84? of 24 and 100?

LESSON IV.

A number which has no other divisors except the number 1 and the number itself is called a *prime* number. Thus, 2 has no divisor except itself and 1; 3 has no divisor except 3 and 1; 5 has no divisor except 5 and 1, &c. Hence, 2, 3, and 5 are prime numbers. 1 is also a prime number.

1. Are any of the even numbers which are greater than 2, prime numbers? Why not?

2. Find the prime numbers from the number 1 to the number 25.

3. Find the prime numbers between 25 and 50.

4. Find the prime numbers between 50 and 75.

5. Find the prime numbers between 75 and 100.

The numbers which are not prime are the products of two or more numbers:—4 is 2 × 2. 6 is 2 × 3. 8 is 2 × 4, or 2 × 2 × 2. 24 is 8 × 3, or 2 × 4 × 3, or 2 × 2 × 2 × 3.

The divisors or factors 2, 2, 2, are called the *prime* factors of 8. 2, 2, 2, 3, the *prime* factors of 24.

6. 24 is the continued product of what 4 numbers? 9 is the product of what 2 prime numbers? 8 is the continued product of what three prime factors?

7. What are the prime factors of any number?

ANS.—*The smallest numbers whose continued product will make the given number.*

8. Find the 2 prime factors of each of the numbers, 4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, 38, 39, 46, 49, 51, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 91, 93, 94, 95. (Thirty-four numbers.)

9. Find 3 prime factors of each of the numbers, 8,

12, 18, 20, 27, 28, 30, 42, 44, 45, 50, 52, 63, 66, 68, 70, 75, 76, 78, 92, 98, 99. (Twenty-two numbers.)

10. What are the 4 prime factors of 16, 24, 36, 40, 54, 56, 60, 81, 84, 88, 90, 100? (Twelve numbers.)

11. What are the 5 prime factors of 32, 48, 72, 80? (Four numbers.)

12. Find the 6 prime factors of each of the numbers, 64 and 96.   (Two numbers.)

13. There are $32 + 22 + 12 + 4 + 2$ numbers, from 1 to 100, which have factors; how many are there?

14. How many prime numbers from 1 to 100?

15. What number multiplied by itself gives 4? 25? 9? 49? 16? 36? 64? 81? 100? 121? 144? 169? 196?

CHAPTER VII.

FRACTIONS.

LESSON I.

Halves, Thirds, and Fourths, or Quarters.

1. IF a line be divided into two equal parts, what is each one of these parts called?

Ans.—*One-half of the line.*

2. How do you get the half of an apple or orange, or any thing?

Ans.—*Divide it into two equal parts and take one of these parts.*

3. How many half cents in 1 cent? How many half cents in two cents? How many half apples in

three apples? How many half cents in 50 cents? How many half dollars in 8 dollars?

4. How many halves in 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 61? 40? 500?

5. How do you find the number of halves in any number?

Ans.—*Multiply the number by* 2.

6. How many half apples can you make of 2 half apples? How many cents are equal to 10 half cents? How many dollars do 12 half dollars make?

7. How many ones in 2 halves? 4 halves? 8 halves? 6 halves 10 halves? 12 halves? 24 halves? 48 halves? 100 halves?

8. How do you get the number of ones or units in any number of halves?

Ans.—*Divide the number of halves by* 2.

9. How many half apples in 3 apples and one half apple? How many dollars in 5 dollars and a half dollar? How many half yards of cloth in 7 yards and a half yard?

10. How many halves in 10 and one-half? 11 and one-half? 15 and one-half? 50 and one-half? 60 and one-half? 4 and one-half? 19 and one-half?

11. How many apples in 3 half apples? How many dollars in 7 half dollars? What remains over in each case?

12. How many ones in 7 halves? 9 halves? 11 halves? 13 halves? 27 halves? 99 halves? 57 halves? 45 halves? 23 halves? 31 halves?

13. 7 divided by 2 is how much?

Ans.—3 *and one-half*.

14. 2 in 27 how many times? 2 in 99? 2 in 31? 2 in 23? 2 in 81? 2 in 11?

15. 1 half of 2 apples is —— ? one-half of 3 dollars is how much?

16. How much is one-half of 2? of 3? of 5? of

6 ? of 7 ? of 8 ? of 9 ? of 10 ? of 11 ? of 12 ?
of 25 ? of 100 ? 50 ? 40 ? 32 ? 22 ? 400 ? 61 ?

17. How do we get the half of a number ?

ANS.—*Divide it by 2.*

18. 3 is the half of what number ? 6 is the half
of what number ? the half of what number is 7 ?
9 ? 12 ? 20 ? 30 ? 25 ? 60 ? 15 ? 19 ?

PROCESS.—*3 is the half of two 3's, or* 2 × 3. *7 is
the half of* 2 × 7, *&c., &c.*

19. How much is three halves of 12 ?

PROCESS.—*One-half of* 12 *is* 6 ; *hence, three halves
of* 12 *is three times* 6.

20. How much is three halves of 24 ? of 50 ? of
30 ? of 18 ? of 20 ?

21. How much is 7 halves of 20 ? 18 ? 30 ? 12 ?

22. How much is 9 halves of 18 ? 24 ? 10 ? 90 ?

23. How much is 11 halves of 12 ? 14 ? 30 ? 22 ?

_____

24. If a line be divided into three equal parts, what
is each part called ?

ANS.—*One-third.*

25. How do you get the third of an apple, orange,
or of any thing ?

26. How do you get two-thirds of any thing ?

27. How many thirds of an apple in one apple ?
in 2 apples ? How many thirds of a cent in 3 cents ?
How many thirds of a yard of cloth in 6 yards of
cloth ?

28. How many thirds in 1 ? 2 ? 3 ? 4 ? 5 ? 8 ?
7 ? 6 ? 9 ? 10 ? 30 ? 40 ? 12 ? 11 ? 300 ?

29. How do you find the number of thirds in any
number ?

30. How many apples do 3 thirds of an apple make ?
How many dollars are equal to 6 thirds of a dollar ?
How many oranges do 12 thirds of an orange make ?

31. How many ones in 3 thirds? 6 thirds? 9 thirds? 15 thirds? 12 thirds? 18 thirds? 24 thirds? 21 thirds? 27 thirds? 99 thirds? 60 thirds?

32. How do you find the number of ones or units in any number of thirds?

33. How many thirds of an orange in 3 oranges and 1 third of an orange? in 3 oranges and 2 thirds? How many thirds of a bushel of meal in 6 bushels and one-third of a bushel? How many thirds of a dollar in 8 dollars and 2 thirds of a dollar?

34. How many thirds in 3 and one-third? in 3 and 2 thirds? in 5 and 2 thirds? in 30 and a third? 21 and 2 thirds? in 40 and one-third? in 11 and 2 thirds? in 100 and one-third?

35. How many oranges can you make of 4 thirds of an orange? How many dollars do 8 thirds of a dollar make? How many thirds remain in each case? How many bushels of meal do 10 thirds of a bushel make? What remains?

36. How many ones in 4 thirds? 5 thirds? 7 thirds? 8 thirds? 11 thirds? 13 thirds? 16 thirds? 20 thirds? 61 thirds? 100 thirds?

37. 7 divided by 3 is how much?

Ans.—2 *and one-third*.

38. 3 in 13 how many times? in 14? 80? 62? 44? 22? 8?

39. How much is one-third of 2? Show by the picture above that one-third of two is 2 thirds of one. How much is one-third of 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 24? 27? 15? 18? 30? 60? 90? 600? 900? 99? 22? 17? 31?

40. How do you find one-third of any number?

How much is 2 thirds of 6?

Process.—*One-third of* 6 *is* 2. *Hence,* 2 *thirds of* 6 *is* 2 × 2, *or* 4.

41. How much is two-thirds of 21? 2 thirds of 9? 2 thirds of 5? of 7? of 100? of 60?

42. How much is 4 thirds of 6 ?

PROCESS.—*One-third of* 6 *is* 2.   *Hence,* 4 *thirds of* 6 *is* 4 × 2, *or* 8.

43. Which is the greater, 4 thirds of a number or the number itself?   How much is 3 thirds of 6 ?

44. How much is 4 thirds of 21 ?   24 ?   27 ?   60 ? 15 ? 10 ?   12 ?

45. How much is 5 thirds of 5 ?   12 ?   21 ?   24 ? 27 ?   60 ?   33 ?

46. How much is 6 thirds of 27 ?   9 ?   12 ?   18 ? 24 ?   30 ?   42 ?   36 ?

47. How much is 10 thirds of 15 ?   12 ?   21 ?   27 ? 20 ?   60 ?   90 ?   99 ?

48. 2 is one-third of what number ?   6 is one-third of what number ?

49. One-third of what number is 7 ?   9 ?   12 ?   8 ? 20 ?   30 ?   60 ?   19 ?   15 ?   100 ?   300 ?   600 ?

PROCESS.—2 *is one-third of three* 2's, *or* 3 *times* 2. 6 *is one-third of* 3 *times* 6, &c.

50. If a line be divided into 4 equal parts, what is each one of these parts called ?

ANS.—*One-fourth of the line, or sometimes one-quarter of the line.*

51. How do you get one-fourth or a quarter of an apple or orange, or of any thing?   How do you get 2 fourths or quarters ?   3 fourths or quarters ?

ANS.—*Divide it into four equal parts, and take one of the parts, two of the parts, and three of the parts, respectively.*

52. How many fourths of an apple in one apple ? How many fourths of a bushel of meal in 2 bushels of meal? How many fourths of a pint of wine in one pint of wine ?   How many quarter dollars in 5 dollars ?

53. How many fourths in 1 ?   2 ?   3 ?   4 ?   5 ?   6 ?

7 ?   8 ?   9 ?   10 ?   11 ?   12 ?   13 ?   15 ?   20 ?   30 ?
40 ?   100 ?   200 ?   202 ?

54. How do you find the number of fourths in any number?

55. How many fourths in 3 and one-fourth?

PROCESS.—*There are* 12 *fourths in* 3, *and these added to one-fourth give* 13 *fourths.*

56. How many fourths in 6 and 2 fourths?   7 and 3 fourths?   20 and one-fourth?   How many quarter dollars in 10 dollars and 3 quarter dollars?   How many fourths in 30 and 3 fourths?   in 100 and 2 fourths?   in 12 and 3 quarters?

57. How many oranges do 4 fourths of an orange make?   How many dollars do 12 quarter dollars make?   How many gallons of wine do eight quarter gallons make?

58. How many ones in 4 fourths?   8 fourths?   12 fourths?   24 fourths?   32 fourths?   28 fourths?   60 fourths?   40 fourths?   400 fourths?   88 fourths?

59. How do you find the number of ones in any number of fourths?

60. How many oranges do 5 quarter oranges make? How much over?

61. How many dollars do 6 quarter dollars make? How much over?

62. How many yards of cloth do 17 quarter yards make?   How much over?

63. How many ones in 5 fourths?   6 fourths?   7 fourths?   21 fourths?   22 fourths?   61 fourths?   49 fourths?   37 fourths?   39 fourths?

64. If we divide 9 apples among four boys, how many whole apples would each boy get?   In order to give each boy an equal share, what must we do with the remaining apple?   9 divided by 4 is how much?

ANS.—*2 and one-fourth.*

65. If we divide 10 apples equally among 4 boys, what would be the share of each one?

Ans.—2 *apples and* 2 *fourths of an apple.*

66. If we divide 11 apples equally among 4 boys, what would be the share of each?

67. 4 in 13 how many times? in 14? 15? 22? 21? 25? 27? 30? 62? 63? 43? 103? 101? 401?

68. How much is one-fourth of 2? of 3? Show by the picture of lines above that one-fourth of 2 is 2 fourths of 1. Show that one-fourth of 3 is 3 fourths of 1.

69. How much is one-fourth of 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 16? 18? 20? 44? 60? 64? 80? 400? 444?

70. How do you find one-fourth of any number?

71. How much is 2 fourths of 8? 3 fourths of 8? 4 fourths of 8? 5 fourths of 8? 10 fourths of 8?

72. 5 fourths of eight is how many times 8?

Ans.—*Once* 8 *and one-fourth of* 8.

73. How much is 3 fourths of 12? of 9? of 24? of 28? 32? 80? 100? 400? How much is 2 fourths of 8? 16? 12? 40? 100? 400?

74. How much is 5 fourths of 56? 60? 92? 80? 20? 28? 36? 40? 48?

75. How much is 7 fourths of 12? 96? 20? 16? 24? 52? 40? 44?

76. How much is 10 fourths of 8? 12? 24? 6? 16? 80? 400?

77. 2 is one-fourth of what number? 3 is one-fourth of what number? One-fourth of what number is 4? 5? 6? 7? 8? 9? 10? 11? 12? 60? 40? 100? 200? 15? 400? 900?

78. 8 is what part of 16? 24? 32? 6 is what part of 24? 12? 18? 7 is what part of 21? 28? 14? 10 is what part of 40? 20? 30? 60 is what part of 120? 240? 180? 9 is what part of 27? 36? 18? 12 is what part of 36? 15 is what part of 60? 14 is what part of 28? 100 is what part of 400?

79. 3 halves is $3 \div ?$ 5 halves is $5 \div ?$ 4 thirds is $4 \div ?$ 5 fourths is $5 \div ?$ 100 fourths is $100 \div ?$

We use figures to write briefly halves, thirds, and fourths, or in general, fractions. (The sign $=$ signifies equal to.) 3 halves is written $\frac{3}{2}$; 5 fourths is written $\frac{5}{4}$; 20 thirds is written $\frac{20}{3}$.

80. What does the number above the line stand for?

ANS.—*It gives the number of halves, fourths, or thirds.*

81. What does the number below the line show?

ANS.—*To write halves we put 2 below the line; to write thirds we put 3; to write fourths we put 4 below the line.*

82. Give the figures and their places which will represent 7 halves, 10 thirds, 14 fourths. Read the fractions $\frac{50}{2}$, $\frac{46}{3}$, $\frac{27}{4}$, $\frac{3}{4}$, $\frac{8}{3}$.

83. 5 and one-half, 6 and 2 thirds, 8 and 3 fourths, &c., are written without any sign between the fraction and the whole number, thus: $5\frac{1}{2}$, $6\frac{2}{3}$, $8\frac{3}{4}$, and these are called *mixed numbers.*

Write 100 and one-fourth, 20 and 2 fourths, 5 and 2 thirds, 7 and a half. Read $25\frac{3}{4}$, $42\frac{1}{2}$, $8\frac{1}{3}$.

NOTE.—In the chapters preceding Chapter VII. the word *number* means whole number. In this and succeeding chapters it is applied to fractions as well as whole numbers.

LESSON II.

1. If a line be divided in five equal parts, what do we call one part of it?

Ans.—*One-fifth of the line.*

2. If a line be divided into 6 equal parts? Seven equal parts?

Ans.—*One part is called one-sixth and one-seventh, respectively.*

3. How do you get one-seventh of any thing? one-sixth? one-fifth? 2 sevenths? 3 sevenths? 4 sevenths? 5 sevenths? 2 fifths? 3 fifths? 4 fifths? 3 sixths? 5 sixths?

4. If we conceive the number 1 divided into five equal parts, what will these parts be called? If it is divided into six equal parts, what do we call them? into 7 equal parts?

5. How many fifths in 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 20? 15? 40? 60? 24? 32? 100? 200?

6. How many fifths in 2? 3? 4? 5? 6? 7? 8? 9? 12? 10? 11? 20? 15? 40? 30? 31? 50? 100? 400?

7. How many sevenths in 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 75? 16? 19? 20? 30? 41? 100? 120?

8. How do you find the number of fifths in any number? the number of sixths? the number of sevenths?

9. How many apples do 5 fifths of an apple make? 6 fifths of an apple? How many dollars are equal to 12 sixths of a dollar? to 14 sevenths of a dollar?

10. How many ones in 5 fifths? 6 fifths? 7 fifths? 8 fifths? 9 fifths? 10 fifths? 15 fifths? 20 fifths?

25 fifths? 26 fifths? 27 fifths? 28 fifths? 29 fifths? 30 fifths? 60 fifths? 100 fifths? 200 fifths?

11. How many ones in 6 sixths? 7 sixths? 8 sixths? 9 sixths? 10 sixths? 11 sixths? 12 sixths? 24 sixths? 30 sixths? 36 sixths? 48 sixths? 60 sixths? 72 sixths? 100 sixths? 31 sixths? 32 sixths? 34 sixths? in 600 sixths?

12. How many ones in 7 sevenths? 8 sevenths? 9 sevenths? 10 sevenths? 11 sevenths? 12 sevenths? 13 sevenths? 14 sevenths? 15 sevenths? 21 sevenths? 42 sevenths? 35 sevenths? 70 sevenths? 84 sevenths? in 100 sevenths? 490 sevenths? 210 sevenths?

13. How do you find the number of ones in any number of fifths? in any number of sevenths? in any number of sixths?

14. 7 in 8 how many times? in 10? in 26? in 30?

15. 6 in 13 how many times? in 14? in 15? in 28? in 29?

16. 5 in 11 how many times? in 12? in 13? in 24? in 39?

17. If I wish to divide 13 oranges among 5 boys, how many oranges and what parts of an orange would the equal share of each boy be? What would be the equal share of each of 6 boys? of each of 7 boys?

18. How many fifths in 3 and one-fifth? 20 and 3 fifths? 60 and 4 fifths? 5 and one-sixth how many sixths? 4 and 3 sixths? 100 and 5 sixths? 8 and 2 sevenths how many sevenths? 10 and 3 sevenths? 15 and 5 sevenths? 40 and 6 sevenths?

19. How much is one-fifth of 2? Draw lines on your slate, and show that one-fifth of 2 is 2 fifths of one. Show that one-sixth of 5 is 5 sixths of 1. Show that one-seventh of 3 is equal to 3 sevenths of 1.

20. How much is one-fifth of 3? 4? 5? 10? 15? 20? 24? 25? 30? 31? 35? 37? 40? 46? 50? 100? 200? 500? 150? 250?

21. How much is one-sixth of 2? 3? 4? 5? 6? 12? 13? 14? 18? 24? 28? 29? 30? 36? 58? 72? 70? 100? 600? 120? 300?

22. How much is one-seventh of 2? 3? 4? 5? 6? 7? 11? 12? 13? 14? 28? 30? 35? 36? 42? 49? 70? 63? 100? 84? 490? 700?

23. How much is 2 fifths of 10? 3 fifths of 45? 4 fifths of 50? 6 fifths of 30? 4 fifths of 80? 5 fifths of 25?

24. How much is 3 sixths of 12? 4 sixths of 18? 5 sixths of 96? 6 sixths of 72? 7 sixths of 42.

25. How much is one-seventh of 84? 2 sevenths of 84? 3 sevenths of 84? 4 sevenths of 21? 5 sevenths of 28? 6 sevenths of 35? 7 sevenths of 98? 10 sevenths of 70?

26. 3 is one-fifth of what number? One-fifth of what number is 4? 5? 6? 7? 8? 9? 10? 12? 60? 100? 25? 33? 200?

27. 3 is one-sixth of what number? One-sixth of what number is 2? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15? 20? 22? 30? 40? 50? 100? 45? 200? 400?

28. 2 is one-seventh of what number? One-seventh of what number is 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 20? 21? 30? 33? 40? 100? 200? 300?

29. What part of 15 is 3? What part of 30 is 5? 6? What part of 42 is 6? 7? What part of 35 is 5? 7? What part of 72 is 12? What part of 63 is 9? of 80 is 16? 8 is what part of 16? 24? 32? 40? 48? 56? 1 is what part of 2? 3? 4? 5? 6? 7? 100 is what part of 200? 300? 400? 500? 600? 700?

30. 4 fifths of 1 is one-fifth of 4, or 4 divided by what? 6 sevenths is one-seventh of 6, or 6 divided by what? 5 sixths is one-sixth of 5, or 5 divided by what?

31. 4 fifths is written $\frac{4}{5}$; 6 sevenths is written $\frac{6}{7}$; 4 sixths is written $\frac{4}{6}$. What does the number under

the line show? What does the number over the line? What figures, and how placed, stand for 1 sixth? 2 fifths? 3 sevenths? 4 sevenths? 3 sixths? 1 fifth? 1 seventh? 4 and 2 sevenths? 5 and 5 sixths? 100 and 3 fifths?

32. Read the following fractions: $\frac{7}{5}$, $\frac{9}{6}$, $\frac{10}{7}$, $\frac{57}{5}$, $\frac{27}{3}$, $\frac{47}{2}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{7}{7}$, $\frac{100}{4}$, $47\frac{1}{5}$, $3\frac{4}{7}$, $5\frac{2}{5}$, $20\frac{1}{3}$.

LESSON III.

1. If a line be divided into 8 equal parts, what is each one of those parts called? Ans.—*One-eighth of the line.* If the line is divided into nine equal parts, what is each part? Ans.—*One-ninth.* If the line be divided into 10 equal parts, what is each part? Ans. —*One-tenth of the line.*

2. How do you find the eighth, ninth, and tenth of any thing? How do you get 4 eighths of a thing? 3 ninths? 7 tenths?

3. How many eighths of a dollar in one dollar? in 2 dollars? 4 dollars? How many ninths of a gallon of wine in one gallon? 3 gallons? How many tenths of a dollar in 5 dollars?

4. How many eighths in 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 20? 30? 40? 22? 35? 44? 100? 200?

5. How many ninths in 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15? 16? 20? 24? 30? 40? 100? 120? 300? 900?

6. How many tenths in 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15? 16? 20? 37? 49? 56? 100? 240? 500? 1000?

7. How many bushels do 16 eighths of a bushel make? How many dollars are equal to 21 dimes or tenths of a dollar? How many yards of cloth are equal to 27 ninths of a yard?

8. How many ones in 8 eighths? 10 eighths? 12

eighths? 16 eighths? 24 eighths? 32 eighths? 33 eighths? 35 eighths? 39 eighths? 40 eighths? 46 eighths? 56 eighths? 64 eighths? 71 eighths? 80 eighths? 100 eighths? 800 eighths?

9. How many ones in 9 ninths? 10 ninths? 17 ninths? 18 ninths? 25 ninths? 27 ninths? 30 ninths? 36 ninths? 38 ninths? 45 ninths? 50 ninths? 54 ninths? 58 ninths? 63 ninths? 69 ninths? 72 ninths? 81 ninths? 90 ninths? 100 ninths? 126 ninths?

10. How many ones in 10 tenths? 14 tenths? 20 tenths? 22 tenths? 30 tenths? 36 tenths? 40 tenths? 45 tenths? 50 tenths? 53 tenths? 60 tenths? 61 tenths? 70 tenths? 72 tenths? 80 tenths? 88 tenths? 90 tenths? 99 tenths? 100 tenths? 150 tenths? 200 tenths? 305 tenths? 1000 tenths?

11. If you wish to divide 17 pounds of candy equally among 8 boys, how many pounds and what parts of a pound would you give to each? If you divided 17 pounds of candy equally among 9 boys, what would be the share of each? If it was divided equally among 10 boys, what would be the share of each?

12. How many eighths in 5 and 2 eighths? in 20 and 5 eighths? in 3 and 7 eighths? 10 and 3 eighths? How many ninths in 5 and 2 ninths? 10 and 6 ninths? 12 and 8 ninths? How many tenths in 6 and 3 tenths? in 25 and 7 tenths? in 30 and 9 tenths?

13. How much is 1 eighth of 3? Show by means of lines that 1 eighth of 3 is 3 eighths of 1. Show that 1 ninth of 2 is 2 ninths of 1. Show that 1 tenth of 3 is 3 tenths of 1.

14. How much is 1 ninth of 2? 3? 4? 5? 6? 10? 15? 9? 20? 24? 18? 27? 36? 45? 54? 63? 72? 75? 81? 88? 90? 100? 900? 1800?

15. How much is 1 eighth of 2? 3? 4? 5? 10? 15? 16? 8? 21? 24? 32? 36? 40? 44? 48? 50? 56? 64? 72? 80? 96? 97? 100? 800? 1600?

16. How much is 1 tenth of 2? 3? 4? 5? 6? 10? 12? 14? 17? 19? 20? 30? 35? 48? 60? 67? 70? 80? 89? 90? 100? 111? 500? 120? 600? 1000?

17. How much is 1 eighth of 24? 2 eighths of 24? 3 eighths of 24? 4 eighths of 16? 6 eighths of 40? 7 eighths of 32? 2 eighths of 72? 10 eighths of 80?

18. How much is 1 ninth of 27? 2 ninths of 27? 5 ninths of 27? 10 ninths of 27? 4 ninths of 36? 6 ninths of 72? 8 ninths of 100?

19. How much is 1 tenth of 50? 5 tenths of 50? 6 tenths of 50? 11 tenths of 50? 3 tenths of 40? 4 tenths of 70? 7 tenths of 210? 12 tenths of 120? 9 tenths of 110?

20. 3 is one-eighth of what number? One-eighth of what number is 4? 2? 5? 7? 6? 8? 9? 10? 13? 14? 25? 100? 200? 80?

21. One-ninth of what number is 1? 2? 3? 4? 5? 6? 7? 8? 10? 12? 19? 22? 31? 100? 200? 90?

22. One-tenth of what number is 1? 2? 3? 4? 5? 6? 8? 9? 10? 11? 12? 17? 20? 36? 45? 100? 120? 200?

23. 2 is what part of 20? 18? 16? 14? 12? 10? 8? 6? 4? 2? 5 is what part of 10? 5? 20? 15? 30? 25? 40? 35? 45? 50? 1 is what part of 8? 9? 10? 10 is what part of 80? 100? 90? 100 is what part of 100? 900? 800? 1000?

24. How do you express in figures one-ninth? one-tenth? one-eighth? five-eighths? six-ninths? four-tenths? eight and three-tenths? nine and four-eighths? two hundred and seven-tenths?

25. Read the fractions $\frac{7}{9}$, $\frac{4}{10}$, $\frac{5}{8}$, $\frac{3}{10}$, $5\frac{3}{8}$, $7\frac{6}{9}$, $8\frac{9}{10}$, $200\frac{1}{9}$.

26. The numbers above the lines in a fraction are called what?

Ans. *The numerators.*

27. The numbers below the line are called what? ANS. *The denominators.*

28. The denominator shows in how many parts the unit is divided. It denominates or names the parts. And the numerator shows how many of these parts form the fraction. It numbers the parts taken. In how many parts is the unit divided, and how many of these parts are taken in $\frac{6}{9}$? $\frac{5}{4}$? $\frac{2}{3}$? $\frac{7}{10}$? $\frac{3}{8}$? $\frac{5}{7}$? $\frac{1}{2}$?

29. If we divide any thing into 11 equal parts, what is each one of these parts called? Into 12 equal parts? Into 20 equal parts? Into 35 equal parts? Into 60 equal parts? Into 100 equal parts?

30. How many twelfths in 3? 4? 5? 12? How many twentieths in 1? 2? 3? 20? How many hundredths in 5? 6? 7? 10?

31. How many ones in $\frac{24}{12}$, $\frac{120}{60}$, $\frac{300}{100}$, $\frac{70}{35}$?

32. How much is $\frac{1}{12}$ of 12? 24? 25? 60? How much is $\frac{7}{30}$ of 60? of 90? 120? How much is $\frac{1}{100}$ of 200? 300? 360?

33. We know now three sorts of fractions. 1st. Proper fractions, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{10}$, in which the numerator is less than the denominator. 2d. Improper fractions, as $\frac{4}{4}$, $\frac{5}{4}$, $\frac{20}{9}$, $\frac{12}{12}$, $\frac{200}{3}$, in which the numerator is equal to or is greater than the denominator. 3d. Mixed numbers, as $4\frac{2}{3}$, $5\frac{2}{10}$, $7\frac{3}{9}$, &c., made up of whole numbers and fractions.

REMARK.—An improper fraction always stands for either a whole or mixed number.

34. If you increase the denominator of a fraction, do you increase or diminish the fraction? If you increase the numerator of a fraction, do you increase or diminish the fraction?

LESSON IV.

1. If an orange costs 9 cents, what is the cost of $\frac{1}{3}$ of it? $\frac{2}{3}$ of it?

2. If a bushel of potatoes costs 60 cents, what is the cost of $\frac{1}{4}$ of a bushel? of $\frac{3}{4}$ of a bushel? of $\frac{1}{5}$ of a bushel? of $\frac{4}{5}$ of a bushel? of $\frac{3}{5}$ of a bushel? What is the cost of $\frac{1}{10}$ of a bushel? of $\frac{5}{10}$ of a bushel? of $\frac{1}{6}$ of a bushel? of $\frac{3}{6}$ of a bushel? of $\frac{1}{12}$ of a bushel? of $\frac{7}{12}$ of a bushel? of $\frac{3}{20}$ of a bushel? of $\frac{6}{30}$ of a bushel? of $\frac{10}{60}$ of a bushel?

3. If an acre of land costs 72 dollars, what is the cost of $\frac{1}{9}$ of the acre? of $\frac{6}{9}$ of it? of $\frac{1}{12}$ of it? of $\frac{4}{36}$ of it? of $\frac{1}{2}$ of it? of $\frac{1}{6}$ of it? of $\frac{5}{6}$ of it? of $\frac{7}{8}$ of it? of $\frac{4}{18}$ of it? of $\frac{3}{24}$ of it? of $\frac{2}{3}$ of it?

4. If a railway car travels 40 miles an hour, how many miles does it go in $\frac{3}{10}$ of an hour? in $\frac{4}{5}$ of an hour? in $1\frac{2}{10}$ of an hour? in $1\frac{0}{8}$ of an hour?

5. If a man earns 48 dollars a month, how much does he earn in $\frac{1}{4}$ of a month? in $\frac{3}{4}$ of a month? in $\frac{7}{12}$ of a month? in $\frac{3}{8}$ of a month? in $\frac{5}{6}$ of a month? in $\frac{1}{2}$ of a month? in $\frac{2}{3}$ of a month? in $\frac{5}{16}$ of a month? in $\frac{9}{24}$ of a month?

6. If a man can do a piece of work in 90 days, how long will it take him to do $\frac{1}{2}$ the work? $\frac{1}{3}$ of the work? $\frac{2}{3}$ of the work? $\frac{3}{10}$ of the work? How long will it take him to do $\frac{2}{15}$ of the work? $\frac{5}{6}$ of the work? $\frac{8}{9}$ of the work? $\frac{4}{5}$ of the work? $\frac{3}{18}$ of it? $\frac{4}{30}$ of it? $\frac{20}{45}$ of it?

7. 84 gallons of water flow from a spring in one hour, how many gallons flow out in $\frac{1}{2}$ an hour? in $\frac{2}{3}$ of an hour? in $\frac{3}{4}$ of an hour? in $\frac{5}{6}$ of an hour? in $\frac{3}{7}$ of an hour? in $\frac{10}{12}$ of an hour? in $1\frac{1}{14}$ of an hour? in $1\frac{2}{21}$ of an hour? in $1\frac{0}{28}$ of an hour? in $1\frac{5}{42}$ of an hour?

8. If $\frac{1}{10}$ of an acre of land costs 5 dollars, what is the whole acre worth? What is $\frac{5}{10}$ of the acre worth? $\frac{7}{10}$ of the acre? 5 is $\frac{1}{10}$ of what number?

9. If $\frac{1}{3}$ of a yard of cloth costs 4 dollars, what is the cost of the whole yard? the cost of $\frac{2}{3}$ of a yard?

10. If 20 gallons of water flow from a spring in $\frac{1}{7}$ of an hour, how many gallons flow in one hour? in $\frac{3}{7}$ of an hour? in $\frac{10}{7}$ of an hour?

11. If $\frac{1}{8}$ of an ounce of the otto of roses costs 5 guineas, what is the cost of 1 ounce? of $\frac{5}{8}$ of an ounce? of $\frac{10}{8}$ of an ounce?

12. If one pound of sugar costs 30 cents, what part of a pound can be bought for 2 cents? for 3 cents? 5 cents? 6 cents? 10 cents? 15 cents?

13. If one yard of ribbon costs 40 cents, what part of a yard costs 10 cents? 4 cents? 8 cents? 20 cents? 5 cents? 1 cent? 2 cents?

14. A minute contains 120 half seconds, what part of a minute do 12 half seconds make? 10 half seconds? 6 half seconds? 15 half seconds? 8 half seconds? 20 half seconds? 40 half seconds? 30 half seconds? 60 half seconds? 1 half second? 2 half seconds? 3 half seconds?

15. A whole hive of bees yield 36 pounds of honey in one year, what part of the hive yields 6 pounds? 9 pounds? 12 pounds? 4 pounds? 2 pounds? 18 pounds?

16. How long will it take a vessel which sails 10 miles an hour, to sail 67 miles? 45 miles? 102 miles? 507 miles? 422 miles? 53 miles?

17. At 9 dollars a barrel, how many barrels of flour can be bought for 28 dollars? 39 dollars? 69 dollars? 71 dollars?

18. If corn costs 5 dollars a barrel, how much can be bought for 47 dollars? 54 dollars? 61 dollars? 23 dollars?

LESSON V.

1. If 5 bushels of wheat cost 9 dollars, what is the cost of 10 bushels?

PROCESS.—*10 bushels is 2 times 5 bushels, hence costs twice as much, or 18 dollars.*

2. If 12 bushels of wheat cost 20 dollars, what is the cost of 3 bushels? 4 bushels? 6 bushels? 2 bushels? 1 bushel?

PROCESS.—*3 bushels is $\frac{1}{4}$ of 12 bushels, and costs therefore $\frac{1}{4}$ of 20 dollars, or 5 dollars.*

3. How many dozen in 107? 112? 145? 26?

4. 4 men can do a piece of work in 45 days, in how many days can 12 men do the same work? 20 men? 40 men? 36 men? 60 men? 32 men? 28 men?

5. If each yard of a certain cloth cost $2\frac{1}{5}$ dollars, what is the cost of 5 yards? Twice 5 and 1 fifth of 5 are how much?

6. If a man travels 7 miles an hour, how far does he travel in $1\frac{1}{7}$ hour? in 4 and $\frac{2}{7}$ hours? in $6\frac{6}{7}$ hours? $8\frac{5}{7}$ hours?

7. How much is 3 times 10 and $\frac{1}{10}$ of 10? 5 times 10 and $\frac{6}{10}$ of 10? 8 times 10 and $\frac{9}{10}$ of 10? 10 times 10 and $\frac{8}{10}$ of 10?

8. How much is 9 times 8 and $\frac{1}{8}$ of 8? 10 times 8 and $\frac{5}{8}$ of 8? 7 times 8 and $\frac{7}{8}$ of 8?

9. At 2 dollars a bushel, what will be the cost of $1\frac{1}{2}$ bushel of wheat? $5\frac{1}{2}$ bushels? $100\frac{1}{2}$ bushels? $25\frac{1}{2}$ bushels?

10. A boy gave away 7 apples, which was $\frac{1}{3}$ of the number of apples which he had; how many apples had he?

11. Frank is 10 years old and is $\frac{1}{4}$ as old as his father, how old is his father?

12. If $\frac{5}{8}$ of an acre of land costs 25 dollars, what is

the cost of $\frac{1}{6}$ of the acre ? Of the whole acre ? Of $\frac{2}{6}$ of the acre ?

13. If $\frac{9}{10}$ of a gallon of water leaks from a vessel in 27 hours, in how many hours will $\frac{1}{10}$ of a gallon leak out? In how many hours will one gallon leak out ? $\frac{7}{10}$ of a gallon ? $1\frac{2}{10}$ of a gallon ?

14. Two boats are moving in the same direction; one gains on the other $\frac{7}{8}$ of a mile in 35 minutes ; in how many minutes does it gain $\frac{1}{8}$ of a mile ? 1 mile ? $\frac{3}{8}$ of a mile ?

15. A merchant gains $\frac{4}{5}$ of a dollar in each sale of 8 dollars ; what sales must he make in order to gain $\frac{1}{5}$ of a dollar ? 1 dollar ? $\frac{6}{5}$ of a dollar ?

16. 8 is $\frac{4}{6}$ of what number ? Is $\frac{3}{6}$ of what number ? $\frac{10}{6}$ of what number ? $1\frac{5}{6}$ of what number ? $\frac{2}{6}$ of what number ? $\frac{6}{6}$ of what number ?

PROCESS.—*If 8 is $\frac{4}{6}$ of the number, then $\frac{1}{4}$ of 8, or 2, is $\frac{1}{6}$ of the number. If $\frac{1}{6}$ of the number is 2, $\frac{6}{6}$ of it, or the entire number, is 6 times 2, or 12.*

17. 3 is $\frac{2}{7}$ of what number ? Is $\frac{5}{7}$ of what number ? $\frac{10}{7}$ of what number ?

18. 8 is $\frac{3}{4}$ of what number ? $\frac{4}{5}$ of what number ? $\frac{9}{10}$ of what number ? $\frac{7}{12}$ of what number ? $\frac{3}{100}$ of what number ? $\frac{10}{3}$ of what number ? $\frac{8}{3}$ of what number ?

19. 12 is $\frac{3}{2}$ of what number ? $\frac{5}{3}$ of what number ? $\frac{10}{12}$ of what number ? $\frac{12}{15}$ of what number ? $\frac{6}{10}$ of what number ?

20. 9 is $\frac{3}{8}$ of what number ? $\frac{6}{8}$ of what number ? $\frac{24}{8}$ of what number ? $\frac{9}{10}$ of what number ? $\frac{5}{6}$ of what number ? $\frac{9}{100}$ of what number ?

LESSON VI.
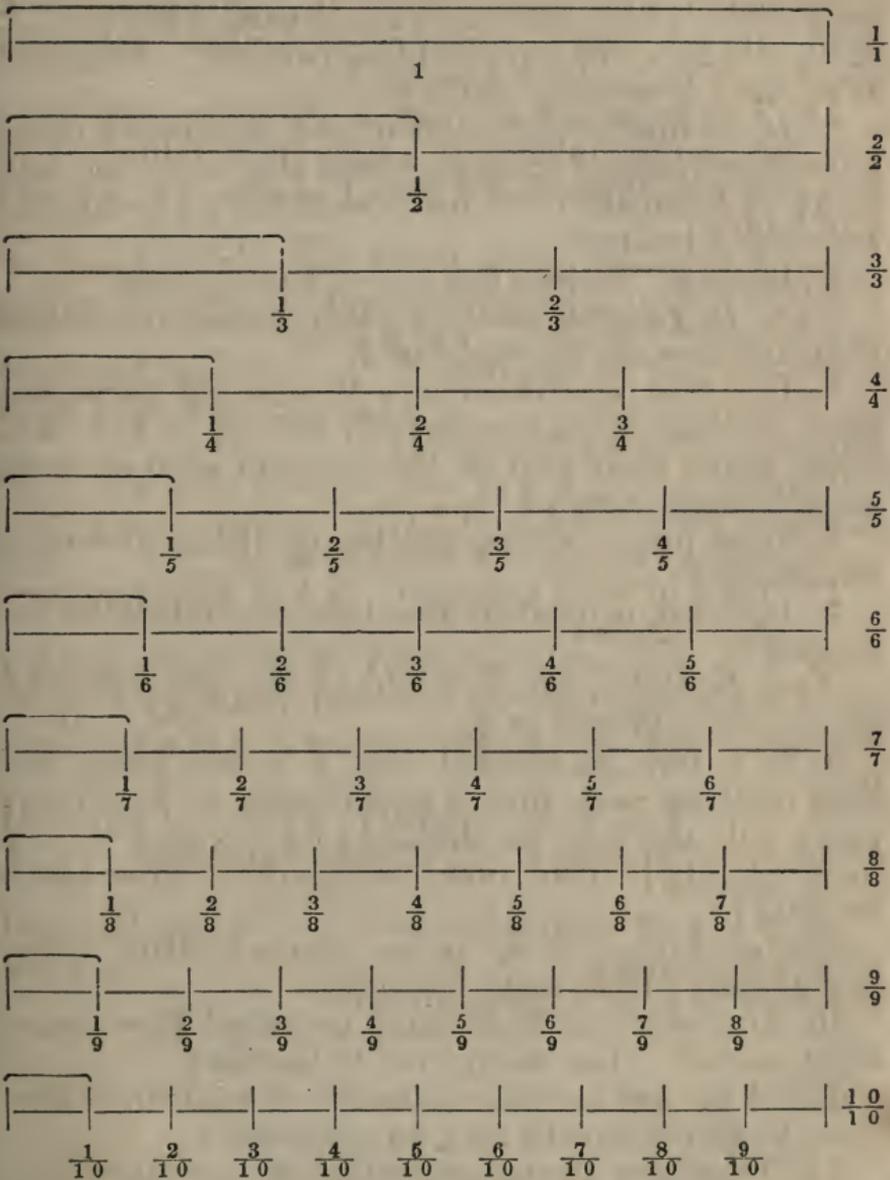
1. If a line be divided into two equal parts, and each of these parts be divided into two equal parts,

into how many equal parts will the line be divided?
One half is equal to how many fourths?

2. If a line be divided into two equal parts, and
each of these parts into three equal parts, what part of
the whole line will each of those parts be? One half
is equal to how many sixths?

3. If we divide each half of the line into 4 equal
parts, how many of these parts will the whole line con-
tain? $\frac{1}{2}$ is equal to how many eighths? $\frac{1}{2}$ is equal to
how many tenths?

4. In what fractions can halves be expressed?

Ans. *In fourths, sixths, eighths, tenths, &c., whose
denominators are multiples of 2.*

5. If a line be divided into three equal parts, and
each of these parts successively into 2, 3, 4, 5, &c.,
equal parts, what part of the line will each of these
parts be respectively?

6. How many sixths, ninths, twelfths, fifteenths,
twentieths?

7. In what equivalent fractions can thirds be ex-
pressed?

Ans. *In sixths, ninths, twelfths, &c., whose denomi-
nators are multiples of 3.*

8. If a line be divided into 4 equal parts, and
each of these parts into 2 equal parts, in how many
parts will the line be divided? $\frac{1}{4}$ is equal to how
many eighths? How many sixteenths? How many
twelfths?

9. One fifth is how many tenths? How many
twentieths? How many fifteenths.

10. One sixth is how many twelfths? How many
eighteenths? How many twenty-fourths?

11. What are the denominators of equivalent frac-
tions in which fourths may be expressed?

12. What are the denominators of equivalent frac-
tions in which fifths may be expressed?

13. What are the denominators of equivalent frac-
tions in which sixths may be expressed?

14. Express $\frac{1}{4}$ in parts of a unit 3 times as small. In parts 5 times as small. 10 times as small. In parts 100 times as small.

15. Express $\frac{3}{4}$ in parts 5 times as small. Find four fractions, each of which is equal to $\frac{3}{4}$. $\frac{3}{4}$ is equal to how many forty-fourths?

16. How many parts of a unit, 10 times as small, is $\frac{1}{5}$ equal to? How many parts of a unit 10 times as small is $\frac{4}{5}$ equal to? Find 5 fractions, each equal to $\frac{3}{5}$. $\frac{2}{5}$ is equal to how many sixtieths?

17. Find 10 fractions, each of which is equal to $\frac{3}{5}$. How many parts of a unit 15 times as small is $\frac{1}{6}$ equal to? Is $\frac{5}{6}$ equal to? Find 6 fractions, each of which is equal to $\frac{5}{6}$.

18. $1 = \frac{1}{2} = \frac{1}{3} = \frac{1}{4} = \frac{1}{5} = \frac{1}{6} = \frac{1}{7} = \frac{1}{12} = \frac{1}{20} = \frac{1}{100} = \frac{1}{588} = \frac{2}{7} = \frac{5}{7} = \frac{100}{7}$.

(Read 1 = how many halves, &c., &c.? $1 = 2$ what? 3 what? &c.)

19. $2 = \frac{1}{2} = \frac{1}{3} = \frac{1}{4} = \frac{1}{10} = \frac{1}{100} = \frac{12}{7} = \frac{16}{7} = \frac{18}{7} = \frac{66}{7}$.

20. $3 = \frac{1}{3} = \frac{1}{10} = \frac{1}{100} = \frac{1}{33} = \frac{6}{7} = \frac{12}{7} = \frac{45}{7}$.

21. What multiples must the denominators of fractions be in which sevenths can be expressed? In which eighths can be expressed? In which ninths can be expressed? In which tenths can be expressed?

22. Give 5 fractions, equal to $\frac{1}{7}$. Give 5 equal to $\frac{1}{8}$. Give 5 fractions equal to $\frac{1}{9}$. Give 5 equal to $\frac{1}{10}$.

23. Express $\frac{3}{7}$ in parts 4 times as small. Express $\frac{5}{9}$ in parts 6 times as small. $\frac{9}{10}$ is equal to how many twentieths? how many fiftieths? how many hundredths? how many two hundredths? $\frac{7}{8}$ is how many forty-eighths?

24. To change fifths into tenths, into how many equal parts must each fifth be divided? To change fifths into fifteenths, into how many? into twentieths, into how many? into thirtieths, into how many must each $\frac{1}{5}$ be divided?

25. $1\frac{5}{6}$ are how many forty-eighths? how many sixty-fourths? how many ninety-sixths?

26. Change $\frac{2}{5}$, $\frac{1}{10}$, $\frac{7}{10}$, and $\frac{11}{15}$, each to thirtieths.

27. Change $\frac{2}{5}$, $\frac{4}{9}$, $\frac{13}{15}$, and $\frac{7}{30}$ each to ninetieths.

28. Change $\frac{2}{5}$, $\frac{3}{5}$, $\frac{5}{5}$, and $\frac{9}{2}$ to tenths.

29. Change $\frac{7}{10}$ to three-hundredths. Change $\frac{5}{13}$ to thirty-ninths. Change $\frac{7}{8}$ to ninety-sixths.

30. How many 72ds in $\frac{8}{9}$? $\frac{3}{4}$? $\frac{4}{3}$? $\frac{5}{6}$? $\frac{7}{8}$? $\frac{11}{12}$? $\frac{13}{18}$? $\frac{21}{24}$? $\frac{31}{36}$?

31. How many 100ths in $\frac{1}{2}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{3}{5}$? $\frac{7}{10}$? $\frac{19}{20}$? $\frac{11}{25}$? $\frac{9}{50}$?

32. How many 100ths of a dollar make $\frac{1}{10}$ of a dollar? How many half a dollar?

33. How do you find a fraction of a greater denominator equal to a given fraction?

LESSON VII.

1. A line is divided into 10 equal parts; what part of the line will $\frac{5}{10}$ make? If a line is divided into 12 equal parts, what part will $\frac{6}{12}$ make?

2. Reduce $\frac{5}{10}$; $\frac{4}{8}$; $\frac{6}{12}$; $\frac{10}{20}$; $\frac{50}{100}$; $\frac{12}{24}$; to halves. What must be done in order to bring $\frac{6}{12}$ back to $\frac{1}{2}$?

3. Change $\frac{5}{25}$ to fifths. Change $\frac{10}{25}$ to fifths. Change $\frac{10}{4}$ to halves. Change $\frac{16}{24}$ to thirds; also to fourths.

4. $\frac{8}{12}$ of a bushel is how many sixths of a bushel? How many thirds of a bushel?

5. Express $\frac{10}{60}$ in parts of a unit twice as great. Express it in parts of a unit 5 times as great. Express it in parts of a unit 10 times as great.

6. $\frac{36}{72}$ is equal to what fraction with a denominator half as great? One-third as great? $\frac{1}{4}$ as great? $\frac{1}{6}$ as great? $\frac{1}{9}$ as great? $\frac{1}{12}$ as great? $\frac{1}{36}$ as great?

7. How do you change a fraction into an equal fraction of smaller denominator? In order to express a fraction in parts 3 times as great, the numerator and

denominator must be divisible by what? In parts 4 times as great, divisible by what? In parts 6 times as great?

8. Can $\frac{5}{7}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{6}$, $\frac{9}{10}$, be expressed exactly in greater parts of a unit? Can they be expressed in smaller parts of a unit?

9. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{100}{200}$, &c. $\frac{1}{2}$ is said to be in lower terms than any of its equivalent fractions, being expressed in smaller figures. In the same manner $\frac{2}{3}$ is a fraction at its lowest terms, or simplest form.

10. When is a fraction in its simplest form?

ANS.— *When its numerator and denominator have no common divisor greater than* 1.

11. Reduce $\frac{58}{80}$ to its simplest form.

12. Reduce $\frac{18}{54}$ to its simplest form.

13. Reduce $\frac{24}{32}$ and $\frac{25}{100}$ each to its simplest form.

14. Reduce $\frac{45}{60}$ to its simplest form.

15. Reduce $\frac{48}{64}$ to its simplest form.

16. Reduce $\frac{63}{105}$ to its simplest form.

17. How do you reduce a fraction to its simplest form?

18. If you multiply the denominator of the fraction $\frac{2}{3}$ by 4, what change do you make in the value of the fraction? If you multiply the numerator of the fraction $\frac{2}{3}$ by 4, what change do you make in the value?

19. If you multiply both the numerator and denominator of $\frac{2}{3}$ by 4, the fraction $\frac{2 \times 4}{3 \times 4} = \frac{2}{3}$; why?

ANS.—*Because we take* 4 *times as many parts, each of which is one fourth as great.*

20. If you divide the denominator of the fraction $\frac{8}{12}$ by 4, what change of value do you make in the fraction? If you divide the numerator by 4 what change do you make? If you divide both numerator and denominator by 4 you make no change; why?

ANS.—*Because you take one fourth as many parts, each of which is* 4 *times as great.*

NOTE.—The teacher should illustrate this with other examples until it is perfectly clear to each pupil.

21. How many 9ths in $\frac{56}{63}$? How many 7ths in $\frac{60}{70}$?

22. How many 8ths in $\frac{24}{64}$? In $\frac{27}{72}$?

23. Reduce to the lowest terms the following fractions:

$\frac{5}{30}, \frac{12}{30}, \frac{8}{18}, \frac{14}{24}, \frac{9}{18}, \frac{21}{30}, \frac{40}{60}, \frac{15}{25}, \frac{400}{600}, \frac{600}{1000}, \frac{18}{20}, \frac{8}{30}, \frac{10}{16},$
$\frac{15}{18}, \frac{22}{30}, \frac{18}{24}, \frac{20}{25}, \frac{22}{24}, \frac{300}{1000}, \frac{225}{900}.$

24. Give five fractions which cannot be reduced to lower terms.

LESSON VIII.

1. How do you express $\frac{1}{2}$ and $\frac{1}{3}$ by two other fractions having the same denominator—how do you proceed?

Ans.—$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12}$, &c. $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$, &c., *and we can express either* $\frac{1}{2}$ *by* $\frac{3}{6}$ *and* $\frac{1}{3}$ *by* $\frac{2}{6}$, *or* $\frac{1}{2}$ *by* $\frac{6}{12}$, *and* $\frac{1}{3}$ *by* $\frac{4}{12}$, *and go on higher and find others.*

2. What is the 6 or 12 called in this case?

Ans.—*The common denominator, and must be a common multiple of 2 and 3.*

3. What is the least common denominator which any two fractions equal to $\frac{1}{2}$ and $\frac{1}{3}$ can have?

Ans.—6, *the least common multiple of* 2 *and* 3.

4. What is the least common denominator of $\frac{1}{2}$ and $\frac{1}{4}$? Of $\frac{1}{2}$ and $\frac{1}{5}$? Of $\frac{1}{3}$ and $\frac{1}{4}$? Of $\frac{1}{3}$ and $\frac{5}{6}$?

5. What are 6 of the common denominators of $\frac{3}{4}$ and $\frac{2}{3}$?

Process.—6 *common multiples of* 3 *and* 4 *are* 12, 24, 36, 48, 60, 72. *Hence these are* 6 *of the common denominators of fractions equal to* $\frac{2}{3}$ *and* $\frac{3}{4}$ *respectively.*

6. What is the least common denominator of $\frac{2}{3}$ and $\frac{3}{4}$?

7. Reduce the fractions $\frac{3}{8}$ and $\frac{4}{5}$ to the least common denominator.

Process.—*The least common multiple of* 8 *and* 5 *is* 40, $\frac{3}{8} = \frac{15}{40}$; *and* $\frac{4}{5} = \frac{32}{40}$.

8. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to the least common denominator.

9. What is the least common denominator of $\frac{3}{7}$ and $\frac{4}{14}$?

10. What is the least common denominator of $\frac{1}{6}$ and $\frac{3}{20}$?

11. What is the least common denominator of $\frac{1}{9}$ and $\frac{9}{10}$?

12. What is the least common denominator of $\frac{4}{11}$ and $\frac{3}{22}$?

13. What is the least common denominator of $\frac{3}{8}$ and $\frac{5}{24}$?

14. What is the least common denominator of $\frac{1}{4}$, $\frac{5}{6}$, and $\frac{3}{8}$?

15. What is the least common denominator of $\frac{1}{4}$, $\frac{1}{10}$, and $\frac{1}{8}$?

16. What is the least common denominator of $\frac{1}{8}$, $\frac{1}{12}$, and $\frac{1}{24}$?

Reduce to common denominators—

17. $\frac{3}{4}$ and $\frac{5}{9}$.

18. $\frac{4}{5}$ and $\frac{7}{8}$.

19. $\frac{2}{7}$ and $\frac{3}{8}$.

20. $\frac{9}{10}$ and $\frac{7}{12}$.

21. $\frac{7}{9}$ and $\frac{5}{18}$.

22. $\frac{11}{12}$ and $\frac{9}{24}$.

23. Reduce to the least common denominator $\frac{1}{4}$ and $\frac{3}{10}$.

24. Reduce to the least common denominator $\frac{1}{15}$ and $\frac{5}{30}$.

25. Reduce to the least common denominator $\frac{1}{4}$, $\frac{3}{10}$, and $\frac{1}{8}$.

26. Reduce to the least common denominator $\frac{1}{6}$, $\frac{5}{12}$, and $\frac{7}{24}$.

27. Reduce $\frac{1}{8}$, $\frac{5}{12}$, and $\frac{7}{24}$ to the least common denominator.

28. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{10}$ and $\frac{11}{12}$ to the least common denominator.

29. How do you reduce fractions to common denominators?

30. How do you reduce fractions to the least com-

mon denominators, when the denominators have no common factor?

31. How do you find the least common denominator, when the denominators have a common factor?

LESSON IX.

1. A boy gave his brother $\frac{1}{4}$ of an apple, and his sister $\frac{3}{4}$ of an apple; how many fourths did he give to both?

2. A man sold a bushel of apples for $\frac{3}{5}$ of a dollar, a bushel of potatoes for $\frac{4}{5}$ of a dollar, and a bushel of meal for $\frac{2}{5}$ of a dollar. How many dollars did he receive for all?

3. A farmer divided his land into several lots. One lot contained $\frac{6}{7}$ of an acre, a second lot $\frac{4}{7}$ of an acre, a third lot $\frac{5}{7}$ of an acre, and a fourth lot $1\frac{1}{7}$ of an acre. How many acres in all the lots?

4. One man pays me $8\frac{1}{2}$ dollars, and another pays me $1\frac{1}{2}$ dollar; how much do both together pay me?

5. $\frac{5}{8}$ and $\frac{3}{8}$ are how much? $\frac{7}{9} + \frac{4}{9}$? $\frac{17}{8} + \frac{10}{8}$? $\frac{9}{10} + \frac{27}{10}$?

6. $3\frac{1}{2} + 1\frac{1}{2} = ?$ $4\frac{3}{5} + 6\frac{1}{5} = ?$ $7\frac{1}{8} + 4\frac{3}{8} = ?$ $16\frac{2}{7} + 3\frac{6}{7} = ?$ $15\frac{3}{11} + 4\frac{9}{11} = ?$ $1\frac{5}{7} + 3\frac{6}{7} = ?$ $\frac{5}{11} + \frac{7}{11} + \frac{9}{11} = ?$ $\frac{1}{10} + \frac{2}{10} + \frac{4}{10} + \frac{5}{10} + \frac{6}{10} + \frac{7}{10} + \frac{8}{10} = ?$

7. A workman receives from his employer at different times $1\frac{1}{12}$ of a dollar, $4\frac{5}{12}$ of a dollar, $3\frac{7}{12}$ of a dollar, $11\frac{5}{12}$ of a dollar, and $2\frac{9}{12}$ of a dollar. How many dollars did he receive altogether?

8. A vintner has in his cellar 4 casks; the first contains $5\frac{1}{10}$ gallons of wine, the second $7\frac{6}{10}$, the third $8\frac{4}{10}$, and the fourth $5\frac{9}{10}$ gallons; how many gallons of wine in the four casks?

9. $\frac{1}{2} + \frac{1}{3} = ?$ Can these be added without changing them first into the same denomination?

PROCESS.—$\frac{1}{2} = \frac{3}{6}$; $\frac{1}{3} = \frac{2}{6}$; $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

10. How do you add fractions having a common

demominator? How do you add fractions of different denominators?

11. $\frac{1}{5}+\frac{1}{6}=$? $\frac{2}{3}+\frac{1}{4}=$? $\frac{1}{6}+\frac{4}{5}=$? $\frac{3}{4}+\frac{5}{12}=$? $\frac{8}{9}+\frac{9}{10}=$? $\frac{6}{7}+\frac{7}{8}=$? $\frac{2}{3}+\frac{3}{11}=$? $\frac{5}{6}+\frac{7}{9}=$?

12. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=$? $\frac{1}{3}+\frac{1}{6}+\frac{1}{9}=$? $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=$? $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}=$? $\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}=$? $\frac{1}{6}+\frac{5}{12}+\frac{7}{24}=$?

13. $1\frac{1}{2}+\frac{1}{3}=$?

PROCESS.—$1\frac{1}{2}$ is $\frac{3}{2}$. *Hence we add $\frac{3}{2}$ and $\frac{1}{3}$, or* $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, 1 *and* $\frac{5}{6}=1\frac{5}{6}$.

14. $4\frac{1}{2}+\frac{5}{8}=$? $3\frac{2}{3}+\frac{6}{7}=$? $10+6\frac{1}{3}+5\frac{1}{5}=$? $3\frac{4}{9}+\frac{1}{3}+\frac{4}{6}=$? $11+\frac{1}{2}+\frac{7}{8}=$?

15. What is the sum of $14\frac{1}{2}$, $10\frac{5}{6}$, and $5\frac{1}{2}$?

16. What is the sum of $6\frac{4}{9}$, $8\frac{1}{7}$, and $5\frac{2}{3}$?

17. What is the sum of $\frac{9}{10}$, $12\frac{1}{2}$, $15\frac{1}{4}$, and $9\frac{1}{5}$?

18. Add $\frac{5}{6}$, $3\frac{1}{3}$, 4, and $5\frac{5}{8}$.

19. A man planted $2\frac{3}{10}$ acres in corn, sowed $10\frac{3}{4}$ acres in oats, and $5\frac{2}{3}$ acres in wheat. How many acres had he in corn, wheat, and oats?

20. I bought $2\frac{1}{2}$ bushels of potatoes of one man, $6\frac{2}{3}$ bushels of another, and $4\frac{1}{4}$ bushels of a third. How many bushels did I buy from the three men?

LESSON X.

1. I purchased a lot containing $\frac{3}{4}$ of an acre, and cut off from it a lot containing $\frac{1}{4}$ of an acre. How many fourths of an acre did the other part contain?

2. Mary had $\frac{6}{7}$ of an orange, and gave $\frac{2}{7}$ away. How much had she left?

3. Charles had a melon, and gave $\frac{3}{10}$ of it away. How much had he left?

4. Julian had 5 oranges, and gave away $\frac{2}{3}$ of one of them. How many had he left?

5. From $\frac{5}{10}$ take $\frac{3}{10}$. From $\frac{21}{10}$ take $\frac{11}{10}$. From $\frac{10}{12}$ take $\frac{4}{12}$. From $18\frac{5}{8}$ take $13\frac{1}{8}$. From $10\frac{3}{4}$ take $6\frac{2}{4}$.

6. How much is 1 less $\frac{1}{2}$? 2 less $\frac{1}{3}$? 3 less $\frac{3}{7}$? 15 less $\frac{4}{5}$? 16 less $2\frac{1}{3}$? 5 less $1\frac{3}{4}$? 10 less $\frac{9}{10}$? 5 less $4\frac{5}{6}$?

7. How much is $\frac{1}{2}$ less $\frac{1}{3}$?

SOLUTION.—$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$.

How much is $\frac{1}{2}$ less $\frac{1}{4}$? $\frac{2}{3}$ less $\frac{3}{4}$? $\frac{6}{7}$ less $\frac{5}{6}$? $\frac{8}{9}$ less $\frac{7}{8}$? $\frac{5}{7}$ less $\frac{3}{8}$?

8. What is the difference between $\frac{3}{5}$ and $\frac{7}{8}$? $\frac{4}{9}$ and $\frac{7}{11}$? $\frac{17}{18}$ and $\frac{11}{12}$? $\frac{7}{12}$ and $\frac{8}{11}$? $\frac{4}{5}$ and $\frac{9}{15}$?

9. From $3\frac{1}{4}$ take $2\frac{2}{3}$.

PROCESS.—$3\frac{1}{4} - 2\frac{2}{3} = 1\frac{3}{4} - \frac{8}{3} = \frac{39}{12} - \frac{32}{12} = \frac{7}{12}$.

From $3\frac{1}{5}$ take $2\frac{7}{8}$. How much is $3\frac{2}{3} - 1\frac{1}{2}$? What is the difference between $10\frac{5}{8}$ and $9\frac{1}{3}$?

How much is—

10.	$3\frac{3}{4} - 2\frac{7}{8}$?	13.	$7\frac{1}{4} - 4\frac{3}{5}$?
11.	$4\frac{1}{5} - 2\frac{6}{7}$?	14.	$9\frac{1}{2} - 8\frac{2}{3}$?
12.	$3\frac{3}{7} - 1\frac{3}{4}$?	15.	$12\frac{11}{12} - 5\frac{5}{6}$.

16. What is the difference between 19 and $\frac{9}{10}$? Between 12 and $\frac{10}{11}$? Between 16 and $1\frac{7}{8}$?

17. $\frac{1}{3} + \frac{1}{4}$ is how much less than 1 ?

18. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ is how much less than 2 ?

19. I sold $\frac{1}{3}$ of my land to one man, $\frac{1}{5}$ of it to another, and $\frac{1}{15}$ of it to a third. How much of it did I have left ?

20. I conceive two numbers, one is $\frac{3}{4}$ and the other is $\frac{2}{3}$ greater—what is the number ? What number is $\frac{2}{3}$ less than $\frac{3}{4}$?

21. Of two numbers $\frac{5}{6}$ is the greater and $\frac{1}{4}$ is the difference between them—what is the smaller ?

22. How much greater is $\frac{6}{7}$ than $\frac{5}{6}$?

23. How do you subtract one fraction from another ?

24. How do you subtract a fraction from a whole number ?

25. How do you subtract one mixed number from another ?

LESSON XI.

1. What is the cost of 5 pounds of tea at $\frac{3}{4}$ of a dollar a pound? 5 times three-fourths is how many fourths? $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = ?$

2. If $\frac{4}{5}$ of a barrel of flour last a family one month, how much flour will last the family 6 months? $\frac{4}{5} + \frac{4}{5} + \frac{4}{5} + \frac{4}{5} + \frac{4}{5} + \frac{4}{5} = ?$ 6 times $\frac{4}{5} = ?$

3. How much is 15 times $\frac{8}{9}$? 5 times $\frac{2}{3}$? 11 times $\frac{3}{4}$?

4. How much is 5 times $\frac{3}{4}$? 19 times $\frac{3}{4}$? 11 times $\frac{5}{6}$? 26 times $\frac{3}{4}$? 14 times $\frac{3}{8}$? 25 times $\frac{2}{3}$? 10 times $\frac{3}{8}$? 15 times $\frac{4}{5}$? 20 times $\frac{3}{5}$? $\frac{3}{10}$? $\frac{4}{5}$? $\frac{7}{10}$? $\frac{9}{10}$? 24 times $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{5}{12}$?

5. What is meant by multiplying 5 by $\frac{1}{2}$?

Ans.—*To take $\frac{1}{2}$ of 5.*

6. If a number be multiplied by a number less than unity, is the product greater or less than the first number? Multiply 5 by $\frac{3}{4}$.

7. How much is 10 times $\frac{2}{3}$? $\frac{2}{3} \times 10$? $\frac{2}{3}$ of 10? How much is 32 times $\frac{3}{4}$? $\frac{3}{4} \times 32$? $\frac{3}{4}$ of 32? How much is 23 times $\frac{2}{3}$? $\frac{2}{3} \times 23$? $\frac{2}{3}$ of 23?

8. How do you find a given fraction of any number?

9. A man sold 10 yards of cloth at $2\frac{1}{3}$ dollars a yard; what did he receive for the 10 yards?

10. At 2 dollars a yard, what will $3\frac{1}{3}$ yards of cloth cost?

11. At 5 dollars a barrel, what will $4\frac{3}{4}$ barrels of flour cost?

12. If a man walks 5 miles an hour, how far will he walk in $6\frac{4}{5}$ hours?

13. How much is $5 \times 6\frac{1}{4}$? $6 \times 7\frac{1}{8}$? $6\frac{2}{7} \times 6$? $9\frac{3}{7} \times 6$? $4\frac{5}{11} \times 3$? $1\frac{5}{6} \times 4$? $5 \times 4\frac{3}{13}$? $31\frac{1}{10} \times 10$? $12\frac{4}{9} \times 9$? $91\frac{3}{8} \times 8$? $20\frac{2}{7} \times 7$?

14. How much is 12 times 10 and $\frac{3}{10}$ of 10?

15. How much is 9 times 7 and $\frac{6}{7}$ of 7?

16. $10 \times \frac{5}{12} = ?$ $\frac{5}{12} \times 10 = ?$ $\frac{5}{12}$ of $10 = ?$ $11 \times \frac{3}{4} = ?$ $\frac{7}{15} \times 12$? $15 \times \frac{6}{23}$?

17. How much do you get when you multiply $\frac{1}{3}$ by 1 ?

18. How much do you get when you multiply $\frac{1}{3}$ by $\frac{1}{2}$? How much is $\frac{1}{3} \times \frac{1}{2}$?

Ans.—$\frac{1}{2}$ of $\frac{1}{3}$.

19. If you take a line and divide it into 3 equal parts, and take $\frac{1}{2}$ of one of these parts, what part of the whole line do you take ? If you divide the line into two equal parts, and take $\frac{1}{3}$ of one of these parts, what part of the whole line do you take ? $\frac{1}{3} \times \frac{1}{2} = \frac{1}{2}$ of $\frac{1}{3} = \frac{1}{3}$ of $\frac{1}{2}$. Show this by a line on your slate.

20. $\frac{1}{2} \times \frac{1}{4} = \frac{1}{2}$ of $\frac{1}{4} = \frac{1}{4}$ of $\frac{1}{2} = ?$ $\frac{1}{2} \times \frac{1}{5} = \frac{1}{2}$ of $\frac{1}{5}$ $= \frac{1}{5}$ of $\frac{1}{2} = ?$ $\frac{1}{3}$ of $\frac{1}{6} = \frac{1}{3} \times \frac{1}{6} = ?$ $\frac{1}{6}$ of $\frac{1}{3} = ?$ $\frac{1}{5}$ of $\frac{1}{7}$ $= \frac{1}{5} \times \frac{1}{7} = \frac{1}{7}$ of $\frac{1}{5} = ?$ $\frac{1}{5}$ of $\frac{1}{10} = \frac{1}{10} \times \frac{1}{5} = ?$ $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{10} \times \frac{1}{10} = ?$

21. $\frac{1}{4}$ multiplied by $\frac{2}{3}$ is 2 times $\frac{1}{3} \times \frac{1}{4} = ?$ How much is $\frac{1}{2}$ of $\frac{3}{7}$? Of $\frac{5}{8}$? Of $\frac{7}{12}$? Of $\frac{9}{10}$? Of $\frac{15}{12}$?

Process.—$\frac{1}{2} \times \frac{3}{7}$, or $\frac{1}{2}$ of $\frac{3}{7}$, is $3 \times \frac{1}{2}$ of $\frac{1}{7} = \frac{3}{14}$.

22. How much is $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{3}{5}$ of $\frac{1}{5}$? $\frac{5}{5}$ of $\frac{1}{5}$? $\frac{7}{5}$ of $\frac{1}{5}$? How much is $\frac{1}{5}$ of $\frac{1}{2}$? $\frac{1}{5}$ of $\frac{3}{2}$? $\frac{4}{5}$ of $\frac{3}{2}$? $\frac{1}{5}$ of $\frac{5}{2}$? $\frac{4}{5}$ of $\frac{5}{2}$?

23. How much is $\frac{2}{3} \times \frac{2}{4}$? $\frac{4}{5} \times \frac{2}{3} = \frac{4}{5}$ of $\frac{2}{3} = \frac{2}{3}$ of $\frac{4}{5}$? How much is $\frac{5}{8} \times \frac{3}{7}$? $\frac{9}{5} \times \frac{4}{10}$? $\frac{7}{10} \times \frac{9}{10}$? $\frac{6}{5} \times \frac{5}{6}$? $\frac{4}{3} \times \frac{3}{4}$? $\frac{2}{3} \times \frac{2}{2}$? $\frac{2}{3} \times \frac{3}{2}$? $\frac{7}{10} \times \frac{6}{7}$? $\frac{2}{7} \times \frac{7}{7}$?

24. How much is $\frac{6}{7} \times \frac{6}{7}$? $\frac{5}{12}$ of $\frac{3}{4}$? $\frac{6}{7}$ of $\frac{5}{8}$? $\frac{3}{20} \times \frac{7}{20}$? $\frac{3}{20} \times \frac{20}{8}$?

25. A man sold a bushel of wheat for $1\frac{1}{2}$ dollars. What is the cost of $\frac{1}{3}$ of the bushel ?

26. How much is $1\frac{1}{2}$ multiplied by $\frac{1}{3}$? What does it mean to multiply $1\frac{1}{2}$ by $\frac{1}{3}$?

Ans.—*To take $\frac{1}{3}$ of it.* Process.—$1\frac{1}{2}$ *is* $\frac{3}{2}$. $\frac{1}{3}$ *of* $\frac{3}{2} = \frac{3}{6} = \frac{1}{2}$; *or,* $\frac{1}{3}$ *of* $\frac{3}{2} = \frac{1}{2}$.

27. Sold a barrel of flour for $7\frac{3}{4}$ dollars. What was the cost of $\frac{1}{5}$ of the barrel ?

28. How much is $8\frac{1}{2} \times \frac{1}{5} = \frac{1}{5}$ of $8\frac{1}{2}$? How much is $2\frac{1}{3} \times \frac{4}{7}$? $9\frac{1}{3} \times \frac{2}{3}$? $3\frac{1}{2} \times \frac{4}{5}$? $4\frac{4}{5} \times \frac{4}{5}$? $1\frac{2}{3} \times 3\frac{1}{10}$? $\frac{2}{3} \times 3\frac{3}{4}$? $\frac{2}{3} \times 10\frac{1}{2}$?

29. What is the cost of $3\frac{1}{4}$ yards of cloth at $1\frac{1}{2}$ dollar a yard? How much is $3\frac{1}{4} \times 1\frac{1}{2}$?

SOLUTION.—$3\frac{1}{4} \times 1\frac{1}{2} = \frac{13}{4} \times \frac{3}{2} = \frac{3}{2}$ of $\frac{13}{4} = \frac{39}{8}$ or $4\frac{7}{8}$.

30. How much is $1\frac{3}{5} \times 1\frac{1}{2}$? $1\frac{1}{4} \times 2\frac{2}{5}$? $2\frac{2}{3} \times 7\frac{1}{2}$? $2\frac{2}{5} \times 1\frac{7}{8}$? $6\frac{2}{3} \times 2\frac{1}{10}$? $1\frac{1}{9} \times 6\frac{3}{4}$? $2\frac{1}{5} \times 3\frac{1}{6}$?

31. Find the sum, difference, and product of $\frac{3}{4}$ and $\frac{7}{8}$. Of $1\frac{1}{2}$ and $2\frac{1}{3}$. Of $2\frac{1}{4}$ and $3\frac{2}{3}$. Of $1\frac{1}{8}$ and $2\frac{1}{4}$. Of $7\frac{1}{2}$ and $3\frac{1}{4}$.

32. How do you multiply a fraction by a whole number? How do you multiply a fraction by a fraction, or take a part or parts of a fraction? How do you multiply a mixed number by a mixed number?

33. A man bought a ton of coal for $17\frac{3}{5}$ dollars a ton. What was the cost of $\frac{1}{6}$ of a ton?

34. A man bought 10 bushels of wheat for $27\frac{3}{8}$ dollars. What did he pay for each bushel?

35. 5 yards of cloth cost $18\frac{3}{4}$ dollars. What is the cost of one yard?

LESSON XII.

1. If an apple is divided into two equal parts, each part is $\frac{1}{2}$. 1 divided by 2 = ? If 2 apples each be divided into 3 equal parts and $\frac{1}{3}$ part of each be taken, we will have $\frac{2}{3}$ of an apple. 2 divided by 3 = what?

2. What do we mean by dividing 1 by 2? 2 by 3?

ANS.—*To divide* 1 *into two equal parts and take one of them, which gives* $\frac{1}{2}$. *In the same manner to divide* 2 *by* 3 *gives* $\frac{2}{3}$.

3. What fraction is equal to 1 divided by 5? $1 \div 6$? $1 \div 7$? $1 \div 8$? $1 \div 9$?

4. If you divide half an orange into two equal parts, what part of the orange do you get? $\frac{1}{2}$ divided by 2 is what?

5. How much is $\frac{1}{2}$ divided by 3? 4? 5? 6? 7? 8? 9? 10?

Ans.—$\frac{1}{2}$ *divided by* 3 *is* $\frac{1}{3}$ *of* $\frac{1}{2}$, *or* $\frac{1}{6}$, *&c.*

6. How much is $\frac{1}{3}$ divided by 3? 4? 5? How much is $\frac{2}{3}$ divided by 3? 4? 5? 6? 7? 8?

7. A man had $\frac{1}{4}$ of a barrel of flour, and divided it equally among 6 persons. How much did he give to each? How much is $\frac{1}{4} \div 6 = \frac{1}{6}$ of $\frac{1}{4}$?

8. A person wished to divide $\frac{5}{6}$ of an acre of land into 10 equal lots? How much land would each lot contain?

9. Divide $\frac{7}{8}$ by 9. By 2. By 5. By 7. By 6. Divide $\frac{8}{9}$ by 11. $\frac{19}{20}$ by 5. $\frac{30}{40}$ by 4. $\frac{50}{81}$ by 5.

10. How many halves make 1? How often is $\frac{1}{2}$ contained in one? How many halves in 2? How often is $\frac{1}{2}$ contained in 2?

11. A person wishes to distribute 4 barrels of flour so as to give $\frac{1}{3}$ of a barrel to each of a number of families. How many families could he supply?

12. How many thirds in 10? $\frac{1}{3}$ is contained in 10 how many times? $\frac{1}{4}$ is contained in 5, 6, 7, 8, 9, how many times?

13. How much is 1 divided by $\frac{1}{3}$? 2 divided by $\frac{1}{5}$? 5 divided by $\frac{1}{6}$? 9 divided by $\frac{1}{7}$? 11 divided by $\frac{1}{10}$? 12 divided by $\frac{1}{20}$?

14. How much is 10 divided by $\frac{1}{5}$? $\frac{2}{5}$ is contained in 10 how many times?

Solution.—$\frac{1}{5}$ *is contained in* 10, 50 *times; hence* $\frac{2}{5}$ *is contained half as many times, or* 25 *times.*

15. How much is 5 divided by $\frac{6}{9}$?

Solution.—5 *divided by* $\frac{1}{9}$ *gives* 45; *hence* 5 *divided by* $\frac{6}{9}$ *must give a quotient* 6 *times as small* = $\frac{45}{6}$ = $7\frac{3}{6}$.

16. How much is 6 divided by $\frac{3}{3}$? 6 is $\frac{3}{3}$ of what number? How much is 6 divided by 5? 6 is 5 times what number? How much is 10 divided by $\frac{4}{5}$? 10 is $\frac{4}{5}$ × by what number? 10 is $\frac{4}{5}$ of what number?

17. How much is $6 \div \frac{3}{4}$? 6 is $\frac{3}{4}$ of what number?

18. How much is $4 \div 1\frac{1}{3}$? 4 is $1\frac{1}{3}$ of what number?

19. How much is $16 \div \frac{5}{8}$? 16 is $\frac{5}{8}$ of what number?

20. How much is $12 \div 1\frac{2}{9}$? $10 \div \frac{10}{11}$? 6 divided by $\frac{6}{5}$? $9 \div \frac{9}{10}$? 5 divided by $\frac{5}{7}$? 11 divided by $1\frac{1}{12}$?

21. How much is 100 divided by $\frac{4}{5}$? $\frac{5}{8}$? $\frac{2}{3}$? $\frac{5}{6}$? $\frac{5}{12}$?

22. 2 is $\frac{5}{8}$ of what number? 5? 3? 7? 10? 11? 12? 15?

23. 60 is $\frac{4}{5}$ of what number? $\frac{5}{8}$ of what number? $\frac{2}{3}$ of what number? $\frac{5}{12}$ of what number?

24. How do you divide a fraction by a whole number? How do you divide a whole number by a fraction?

25. If one soldier's ration of flour is $1\frac{1}{3}$ pound, how many rations will 8 pounds of flour make? $1\frac{1}{3}$ is contained how often in 8?

SOLUTION.—$1\frac{1}{3}$ *is* $\frac{4}{3}$, *and* $8 \div \frac{4}{3}$ *is* 6.

26. How often is $6\frac{2}{3}$ contained in 9? How much is 60 divided by $3\frac{1}{3}$? By $1\frac{1}{9}$? $3\frac{3}{5}$? How much is 2 divided by $3\frac{1}{5}$? $4\frac{1}{3}$? How much is 5 divided by $4\frac{1}{2}$? $3\frac{1}{6}$?

27. How much is $2\frac{2}{5} \div 2$? $8\frac{2}{5} \div 7$? $14\frac{2}{5} \div 12$?

28. If $\frac{3}{4}$ of a ton of coal is burned in a stove in one month, how long will $\frac{9}{4}$ of a ton last for the same stove? $\frac{3}{4}$ is contained how often in $\frac{9}{4}$? 3 in 9 how many times? $\frac{3}{6}$ in $\frac{9}{6}$ how many times? $\frac{3}{10}$ in $\frac{9}{10}$? $\frac{3}{60}$ in $\frac{9}{60}$?

29. $\frac{4}{5}$ in $\frac{20}{5}$ how many times? $\frac{30}{6}$ divided by $\frac{5}{6}$ is how much? How much is $\frac{15}{4}$ divided by $\frac{3}{7}$? $\frac{100}{12}$ contains $\frac{10}{12}$ how many times? $\frac{100}{6} \div \frac{10}{6} = ?$ $\frac{200}{11} \div \frac{50}{11} = ?$ $\frac{300}{15} \div \frac{30}{15} = ?$ $\frac{15}{6} \div \frac{3}{6} = ?$ $\frac{6}{3}$ contains $\frac{5}{3}$ how many times? $\frac{6}{3}$ divided by $\frac{5}{3}$ is how much? 6 contains 5 how often? 5 divided by 6 is how much?

30. 1 divided by $\frac{1}{3}$ is how much? $\frac{1}{2}$ divided by $\frac{1}{3}$ is how much?

ANS.—*Since* $1 \div \frac{1}{3}$ *is* 3, $\frac{1}{2} \div \frac{1}{3}$ *must be* $\frac{1}{2}$ *of* 3, *or* $\frac{3}{2}$.

31. How much is $\frac{1}{5}$ divided by $\frac{1}{6}$?

Ans.—$\frac{1}{5} = \frac{6}{30}$ and $\frac{1}{6} = \frac{5}{30}$; $\frac{6}{30} \div \frac{5}{30} = 6$ divided by $5 = \frac{6}{5} = 1\frac{1}{5}$.

32. How much is $\frac{5}{2}$ divided by $\frac{1}{3}$?

Process.—$\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$. Hence, $\frac{5}{2} \div \frac{1}{3} = 5$ times $\frac{3}{2} = \frac{15}{2}$.

33. How much is $\frac{1}{2}$ divided by $\frac{5}{3}$?

Process.—$\frac{1}{2}$ divided by $\frac{1}{3}$ is $\frac{3}{2}$, and $\frac{1}{2}$ divided by $\frac{5}{3}$ must be $\frac{1}{5}$ of $\frac{3}{2} = \frac{3}{10}$.

34. How much is $\frac{2}{3}$ divided by $\frac{5}{6}$?

Ans.—$\frac{2}{3}$ divided by $\frac{1}{6}$ is $\frac{12}{3}$, and $\frac{2}{3}$ divided by $\frac{5}{6}$ is $\frac{1}{5}$ of $\frac{12}{3} = \frac{12}{15}$.

35. How much is $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5}$?

36. How much is $\frac{5}{14} \div \frac{9}{10} = \frac{5}{14} \times \frac{10}{9}$?

37. How much is $\frac{3}{13} \div \frac{7}{8} = \frac{3}{13}$ of $\frac{8}{7} = \frac{3}{13} \times \frac{8}{7}$?

38. How much is $\frac{1}{4}$ divided by $\frac{2}{3}$? by $\frac{3}{4}$? $\frac{2}{5}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{3}{10}$?

39. How much is $\frac{3}{4}$ divided by $\frac{7}{12}$? by $\frac{3}{5}$? $\frac{5}{8}$? $\frac{5}{12}$?

40. How much is $\frac{4}{5}$ divided by $\frac{9}{10}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{2}{3}$?

41. How do you divide one fraction by another?

42. A tailor having $8\frac{3}{4}$ yards of cloth, cut it into pieces, each $2\frac{1}{2}$ yards long—how many such pieces did he get? $2\frac{1}{2}$ is contained how often in $8\frac{3}{4}$?

Process.—$2\frac{1}{2}$ is $\frac{5}{2}$, $8\frac{3}{4}$ is $\frac{35}{4}$, $\frac{35}{4}$ divided by $\frac{5}{2} = \frac{35}{4} \times \frac{2}{5} = \frac{70}{20} = \frac{7}{2} = 3\frac{1}{2}$.

43. How much is $1\frac{1}{2}$ divided by $2\frac{1}{2}$? $2\frac{1}{2}$ divided by $1\frac{1}{2}$? $1\frac{2}{3}$ divided by $1\frac{3}{4}$? $1\frac{2}{7} \div 2\frac{3}{5}$? $6\frac{1}{3} \div 1\frac{1}{2}$.

44. How much is $2\frac{4}{9}$ divided by $6\frac{4}{5}$? $5\frac{2}{7}$ divided by $4\frac{2}{3}$? $5\frac{1}{2}$ divided by $2\frac{3}{4}$? $\frac{4}{9}$ divided by $3\frac{3}{4}$? $4\frac{7}{8}$ divided by $\frac{5}{11}$?

45. If one bushel of wheat cost $\frac{2}{3}$ of a dollar, what is the cost of $\frac{4}{5}$ of a bushel? $\frac{10}{9}$ of a bushel? $5\frac{1}{2}$ bushels? $10\frac{3}{4}$ bushels? $1\frac{2}{3}$ bushel?

46. If one bushel of wheat cost $\frac{2}{3}$ of a dollar, how much wheat can you buy for $\frac{4}{5}$ of a dollar? $\frac{10}{9}$ of a dollar? $5\frac{1}{2}$ dollars? $10\frac{3}{4}$ dollars? $1\frac{2}{3}$ dollar?

Note.—Let the pupil make 5 problems like the above, and solve them.

47. Add $\frac{1}{2}$ and $\frac{1}{3}$, and multiply the sum by $\frac{3}{4}$.

48. From $3\frac{1}{5}$ take $\frac{5}{6}$, and divide the remainder by $\frac{2}{3}$.

49. Multiply $\frac{3}{4}$ by $1\frac{1}{2}$, and divide the product by $\frac{7}{8}$.

50. Multiply $2\frac{1}{3}$ by $\frac{1}{5}$, and divide the product by $4\frac{1}{2}$.

51. Is the dividend always larger than the divisor? Is the quotient always smaller than the dividend? Give examples.

52. Does the quotient always give the number of times the divisor is contained in the dividend?

LESSON XIII.

1. 40 is $\frac{5}{8}$ of what number?

ANALYSIS.—40 *is* $\frac{5}{8}$ × *the number.* *Hence the number is* 40 *divided by* $\frac{5}{8}$ = 40 ÷ 5 *and quotient* × 8.

2. 27 is $\frac{9}{10}$ of what number? $\frac{3}{2}$ of what number?

3. 32 is $\frac{4}{5}$ of what number? $\frac{2}{3}$ of what number?

4. 60 is $\frac{5}{4}$ of what number? $\frac{12}{11}$ of what number?

5. 72 is $\frac{9}{8}$ of what number? $\frac{36}{5}$ of what number?

6. 48 is $\frac{6}{5}$ of what number? $\frac{3}{10}$ of what number?

7. 49 is $\frac{7}{10}$ of what number? $\frac{7}{2}$ of what number?

8. 54 is $\frac{9}{11}$ of what number? $\frac{6}{5}$ of what number?

9. 63 is $\frac{7}{8}$ of what number? $\frac{9}{4}$ of what number?

10. 20 is $\frac{4}{3}$ of what number? $\frac{10}{11}$ of what number?

11. 10 is $\frac{3}{2}$ of what number?

SOLUTION.—*If* 10 *is* $\frac{3}{2}$ *of a number, then* $\frac{1}{3}$ *of* 10, *or* $3\frac{1}{3}$, *is* $\frac{1}{2}$ *of the same number; and* $3\frac{1}{3}$ *is* $\frac{1}{2}$ *of* 2 × $3\frac{1}{3}$, *or* $6\frac{2}{3}$.

16 is $\frac{5}{7}$ of what number?

12. 15 is $\frac{2}{3}$ of what number? $\frac{4}{5}$ of what number?

13. 50 is $\frac{3}{2}$ of what number? $\frac{11}{1}$ of what number?

14. 9 is $\frac{5}{4}$ of what number? $\frac{10}{12}$ of what number?

15. 11 is $\frac{5}{6}$ of what number? $\frac{3}{4}$ of what number?

16. $5\frac{1}{2}$ is $\frac{1}{4}$ of what number? $\frac{3}{4}$ of what number?

17. $15\frac{3}{4}$ is $\frac{7}{3}$ of what number? $\frac{4}{7}$ of what number?

18. $7\frac{5}{6}$ is $\frac{2}{3}$ of what number? $\frac{9}{10}$ of what number?

19. $\frac{5}{6}$ is $\frac{3}{4}$ of what number? $\frac{7}{8}$ of what number?

20. $\frac{9}{10}$ is $\frac{2}{3}$ of what number? $\frac{9}{10}$ of what number?

21. $\frac{10}{11}$ is $\frac{5}{3}$ of what number? $\frac{2}{5}$ of what number?

22. A man paid 37 dollars for $\frac{2}{3}$ of an acre of land. What was the cost of the whole acre at this rate?

23. A man sold his horse for thirty dollars, which was $\frac{4}{5}$ of what it cost him. What was the cost of the horse?

NOTE.—The pupil will form four questions like the above, and give the solutions.

LESSON XIV.

1. 20 is $\frac{4}{10}$ of how many times 5?

SOLUTION.—20 *is* $\frac{4}{10}$ *of* 50, *and* 50 *is* 10 *times* 5.

2. 72 is $\frac{9}{10}$ of how many times 16?

3. 60 is $\frac{5}{12}$ of how many times 18?

4. 24 is $\frac{3}{4}$ of how many times 2?

5. 48 is $\frac{8}{9}$ of how many times 19?

6. 12 is $\frac{4}{5}$ of how many times 6?

7. 11 is $\frac{5}{6}$ of how many times 5?

8. 36 is $\frac{7}{10}$ of how many times 11?

9. 20 is $\frac{4}{10}$ of 5 times what number?

SOLUTION.—20 *is* $\frac{4}{10}$ *of* 50, *and* 50 *is* 5 *times* 10.

10. 16 is $\frac{2}{3}$ of 6 times what number?

11. 40 is $\frac{5}{12}$ of 16 times what number?

12. 18 is $\frac{6}{7}$ of 10 times what number?

13. 60 is $\frac{1}{4}$ of 12 times what number?

14. 20 is how many tenths of 10 times 5?

15. 20 is 4 times what part of 10 times 5?

16. What number is $\frac{4}{10}$ of 10 times 5?

17. 48 is $\frac{2}{20}$ of how many thirds of 36?

18. 54 is $\frac{6}{11}$ of how many times $\frac{1}{16}$ of 48?

19. 55 is $\frac{5}{8}$ of how many sixths of 24?

20.* 64 is $\frac{32}{33}$ of how many nineteenths of 38?

21. 9 is $\frac{3}{5}$ of how many sixths of 18?

22. $\frac{5}{6}$ is $\frac{3}{7}$ of how many times $\frac{1}{2}$ of 10?

23. $8\frac{2}{4}$ is $\frac{4}{5}$ of how many times $\frac{1}{4}$ of 24?

24. $7\frac{1}{2}$ is $\frac{5}{8}$ of how many times $\frac{1}{3}$ of 10 ?
25. $\frac{1}{2}$ of $\frac{5}{6}$ is $\frac{3}{4}$ of how many times $\frac{1}{20}$ of 100 ?
26. $\frac{2}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$ of how many times $\frac{1}{6}$ of $\frac{1}{5}$?
27. A man bought a horse for $60, which was $\frac{4}{5}$ of 10 times what it cost to keep the horse one month. How much did it cost to keep the horse a month ?

NOTE.—The pupil will frame three questions similar to question 27, and give the solutions. The teacher will vary some of the questions, beginning with question 17, after the manner in which question 1 is varied in 9, 14, 15, and 16.

LESSON XV.

1. $\frac{4}{3}$ of 15 is how many fifths of 100 ?
SOLUTION.—$\frac{4}{3}$ *of* 15 *is* 20. 20 *is* $\frac{1}{5}$ *of* 100.
2. $\frac{4}{3}$ of 15 is $\frac{1}{5}$ of what number ?
3. $\frac{4}{3}$ of what number is $\frac{1}{5}$ of 100 ?
4. $\frac{5}{8}$ of 48 is how many fourths of 12 ?
SOLUTION.—$\frac{5}{8}$ *of* 48 *is* 30. $\frac{1}{4}$ *of* 12 *is* 3. 30 *is* 10 *times* 3, *or* 10 *times* $\frac{1}{4}$ *of* 12, *or* 10 *fourths of* 12.
5. $\frac{3}{12}$ of 60 is how many tenths of 30 ?
6. $\frac{4}{7}$ of 49 is how many thirds of 42 ?
7. $\frac{2}{3}$ of 9 is how many fourths of 12 ?
8. $\frac{5}{7}$ of 21 is how many fourths of 60 ?
9. $\frac{3}{4}$ of 16 is how many sixths of 36 ?
10. $\frac{2}{3}$ of 12 is how many fifths of 20 ?
11. $\frac{5}{6}$ of 48 is how many tenths of 40 ?
12. $\frac{5}{11}$ of 77 is $\frac{7}{9}$ of what number ?
SOLUTION.—$\frac{5}{11}$ *of* 77 *is* 35. 35 *is* $\frac{7}{9}$ *of what number ?*
13. $\frac{5}{16}$ of 96 is $\frac{10}{11}$ of what number ?
14. $\frac{6}{4}$ of 44 is $\frac{6}{10}$ of what number ?
15. $\frac{5}{9}$ of 63 is $\frac{7}{2}$ of what number ?
16. $\frac{7}{18}$ of 72 is $\frac{4}{5}$ of what number ?
17. $\frac{4}{8}$ of 11 is $\frac{3}{4}$ of what number ?
18. $\frac{4}{9}$ of 36 is $\frac{8}{10}$ of how many times 5 ?
19. $\frac{4}{7}$ of 56 is $\frac{4}{5}$ of how many times 10 ?
20. $\frac{7}{6}$ of 54 is $\frac{9}{10}$ of how many times 7 ?
21. $\frac{8}{9}$ of 27 is $\frac{6}{7}$ of how many times 14 ?

22. $\frac{5}{6}$ of 36 is $\frac{3}{15}$ of how many times 10?

23. $\frac{4}{6}$ of 12 is $\frac{1}{3}$ of how many times 5?

24. $\frac{1}{2}$ of $\frac{1}{5}$ is $\frac{1}{6}$ of how many times $\frac{1}{10}$? How much is $\frac{3}{4}$ of $\frac{2}{3}$ of 36?

25. $\frac{2}{3}$ of $\frac{2}{3}$ of 60 is $\frac{5}{4}$ of what number?

SOLUTION.—$\frac{2}{3}$ *of* 60 *is* 40. $\frac{3}{8}$ *of* 40 *is* 15. 15 *is* $\frac{5}{4}$ *of what number?*

26. $\frac{4}{5}$ of $\frac{1}{3}$ of 45 is $\frac{10}{12}$ of what number?

27. $\frac{6}{5}$ of $\frac{4}{7}$ of 35 is $\frac{1}{2}$ of what number?

28. $\frac{5}{3}$ of $\frac{3}{10}$ of 100 is $\frac{4}{5}$ of what number?

29. $\frac{3}{7}$ of $\frac{1}{2}$ of 84 is $\frac{1}{3}$ of $\frac{6}{5}$ of what number?

30. $\frac{3}{2}$ of $\frac{5}{6}$ of 48 is $\frac{3}{5}$ of $\frac{2}{3}$ of what number?

31. $\frac{4}{9}$ of 36 is $\frac{8}{10}$ of 4 times what number?

32. $\frac{4}{9}$ of 36 is how many tenths of 4 times 5?

33. $\frac{3}{7}$ of 63 is $\frac{9}{10}$ of 5 times what number?

34. $\frac{4}{5}$ of 80 is $\frac{8}{9}$ of 12 times what number?

35. $\frac{8}{9}$ of 81 is $\frac{18}{10}$ of 8 times what number?

36. $\frac{6}{2}$ of 11 is $\frac{3}{5}$ of 10 times what number?

37. $\frac{3}{4}$ of $\frac{2}{3}$ of 24 is $\frac{6}{5}$ of 4 times what number?

38. $\frac{4}{5}$ of $\frac{9}{10}$ of 50 is $\frac{12}{10}$ of 5 times what number?

39. $\frac{4}{5}$ of $\frac{9}{10}$ of 50 is how many tenths of 5 times 6?

40. $\frac{4}{5}$ of $\frac{9}{10}$ of 50 is 12 times what part of 5 times 6?

NOTE.—The teacher will vary the above questions after the manner indicated in 32, 39, 40. The pupil will form four questions similar to the above, and give the solutions.

LESSON XVI.

1. $\frac{4}{5}$ of 15 is $\frac{6}{10}$ of how many thirds of 21?

SOLUTION.—$\frac{4}{5}$ *of* 15 *is* 12; 12 *is* $\frac{6}{10}$ *of* 20; $\frac{1}{3}$ *of* 21 *is* 7; 20÷7 *gives* $2\frac{6}{7}$. *Hence,* $\frac{4}{5}$ *of* 15 *is* $\frac{6}{10}$ *of* $2\frac{6}{7}$ *thirds of* 21.

2. $\frac{3}{4}$ of 40 is $\frac{5}{2}$ of how many ninths of 63?

3. $\frac{5}{11}$ of 77 is $\frac{7}{5}$ of how many tenths of 100?

4. $\frac{3}{4}$ of 24 is $\frac{9}{10}$ of how many twentieths of 80?

5. $\frac{10}{6}$ of 30 is $\frac{10}{12}$ of how many thirds of 36?

6. $\frac{5}{3}$ of 18 is $\frac{5}{9}$ of how many twelfths of 72?

7. $\frac{4}{5}$ of 9 is $\frac{3}{10}$ of how many thirds of 12?

8. $\frac{4}{5}$ of 80 is $\frac{8}{9}$ of how many fifths of 60?

9. $\frac{5}{13}$ of 39 is $\frac{3}{12}$ of how many ninths of 90?

10. $\frac{4}{5}$ of $\frac{3}{10}$ of 40 is 8 times what part of 36?

11. $\frac{3}{4}$ of $\frac{5}{6}$ of 48 is 10 times what part of 24?

12. $\frac{1}{2}$ of $\frac{1}{3}$ of 72 is 6 times what part of 50?

13. $\frac{1}{10}$ of $\frac{1}{10}$ of 90 is $\frac{1}{10}$ time what part of 20?

14. $\frac{4}{5}$ of $\frac{3}{7}$ of 70 is 3 times what part of 40?

15. $\frac{3}{8}$ of 48 is $\frac{6}{5}$ of how many times $\frac{8}{9}$ of 45?

16. $\frac{3}{7}$ of 56 is $\frac{4}{9}$ of how many times $\frac{2}{8}$ of 27? 54 is how many times $\frac{2}{3}$ of 27.

17. $\frac{3}{8}$ of 64 is $\frac{6}{10}$ of how many times $\frac{2}{7}$ of 35?

18. $\frac{4}{7}$ of 63 is $\frac{3}{8}$ of how many times $\frac{4}{5}$ of 40?

19. $\frac{5}{4}$ of 24 is $\frac{5}{7}$ of how many times $\frac{3}{4}$ of 20?

20. $\frac{3}{4}$ of 60 is $\frac{9}{10}$ of how many times $\frac{5}{6}$ of 24?

21. $\frac{3}{4}$ of $\frac{1}{2}$ of 96 is $\frac{9}{10}$ of how many times $\frac{2}{3}$ of 12?

22. $\frac{5}{9}$ of 81 is $\frac{3}{5}$ of how many times $\frac{1}{2}$ of $\frac{3}{4}$ of 40? How much is $\frac{1}{2}$ of $\frac{3}{4}$ of 40? $\frac{5}{9}$ of 81 is $\frac{3}{5}$ of what number?

23. $\frac{3}{8}$ of $\frac{1}{2}$ of 60 is $\frac{4}{9}$ of how many times $\frac{3}{4}$ of $\frac{5}{6}$ of 24?

24. $\frac{1}{5}$ of $\frac{6}{10}$ of 100 is $\frac{2}{3}$ of $\frac{3}{4}$ of how many times $\frac{2}{5}$ of $\frac{1}{2}$ of 30? $\frac{1}{5}$ of $\frac{6}{10}$ of 100 is how much? 12 is $\frac{2}{8}$ of $\frac{3}{4}$ of what number? How much is $\frac{2}{5}$ of $\frac{1}{2}$ of 30?

25. $\frac{5}{4}$ of $\frac{3}{8}$ of 72 is $\frac{2}{3}$ of $\frac{5}{4}$ of how many times $\frac{1}{2}$ of $\frac{1}{3}$ of 54?

NOTE.—The pupil will form three questions similar to the above, and give the answers.

LESSON XVII.

1. A farmer having 70 sheep sold $\frac{1}{5}$ of them at one time, and $\frac{1}{10}$ of them at another. How many of them had he left?

2. A woman is 50 years old, and her age is $\frac{5}{8}$ of the age of her husband. How old is he?

3. I bought a horse for $100; I paid $\frac{3}{4}$ as much for

an ox, and $\frac{3}{4}$ as much for a colt. How much did I pay for all three?

4. A man bought 40 dozen eggs for 400 cents. He sold $\frac{5}{8}$ of them for 15 cents a dozen, and the remainder at 10 cents a dozen. How much did he gain?

5. If $\frac{6}{5}$ of a barrel of flour cost 12 dollars, what will be the cost of a barrel?

6. A farm contains 60 acres of wood land, and $1\frac{9}{12}$ of the wood land is $\frac{5}{7}$ of the cleared land. How much cleared land in the farm?

7. A person owned in a town 17 lots, each lot containing $\frac{1}{5}$ of an acre. How many acres did he own?

8. From a lot of 5 acres of land, I sold $\frac{1}{2}$ an acre to one man, $\frac{4}{5}$ of an acre to another, and $\frac{7}{10}$ to a third. How many acres did I have left?

9. I paid out of a sum of $100, at different times, $20\frac{7}{12}$ dollars, $30\frac{5}{8}$ dollars, $40\frac{5}{6}$ dollars. How much did I have left?

10. Frank gave $\frac{3}{8}$ of an orange, $\frac{1}{4}$ of an orange, and $\frac{2}{3}$ of an orange to one boy; $\frac{1}{4}$ of an orange, $\frac{2}{3}$ of an orange, and $\frac{5}{12}$ of an orange to another boy. How much more did he give to the first boy than the second?

11. What is the cost of 4 yards of tape at $\frac{7}{8}$ of a dollar a yard?

12. What is the cost of 10 yards at $\frac{7}{10}$ of a dollar a yard?

13. What is the cost of 9 barrels of flour at $6\frac{2}{3}$ dollars a barrel?

14. A man paid 12 dollars for 3 yards of cloth, and sold it at $5\frac{2}{3}$ dollars a yard. How much did he gain?

15. What is the difference between $5\frac{1}{4}$ times 10 and 3 times $8\frac{5}{8}$?

16. 5 times 7 and $\frac{6}{7}$ of 7 is how many times 10?

17. If a bushel of potatoes is worth $\frac{1}{2}$ a dollar, how much is $\frac{1}{6}$ of a bushel worth? $\frac{5}{6}$ of a bushel? $\frac{2}{3}$ of a bushel? $\frac{3}{10}$ of a bushel?

18. If a yard of cloth is worth $\frac{9}{10}$ of a dollar, what is $\frac{3}{4}$ of a yard worth? $\frac{5}{8}$ of a yard? $\frac{1}{20}$ of a yard?

19. What is the cost of $2\frac{1}{2}$ yards of cloth at $\frac{3}{4}$ of a dollar a yard? At $1\frac{1}{2}$ dollar a yard? At $5\frac{2}{3}$ dollars a yard? At $6\frac{3}{4}$ dollars a yard? At $3\frac{3}{4}$ dollars a yard?

20. I bought a horse for $82\frac{1}{2}$ dollars, and sold him for $\frac{4}{5}$ of that sum. What did I lose on him?

21. If a man can mow $1\frac{1}{2}$ acre of grass in one day, how much can he mow in $5\frac{5}{6}$ days?

22. What will $\frac{1}{5}$ of $\frac{1}{7}$ of 70 gallons of wine cost at $1\frac{1}{5}$ dollar a gallon?

23. At $\frac{5}{8}$ of a dollar a bushel, how many bushels of meal can be bought for five dollars? for 6 dollars? 10 dollars? 20 dollars?

24. If a horse eats $\frac{3}{5}$ of a bushel of oats a day, how long will 15 bushels of oats last him? How long 5 bushels? 7 bushels?

25. If 12 pounds of sugar cost $\frac{6}{7}$ of a dollar, what is the cost of 1 pound?

26. At $\frac{2}{5}$ of a dollar a yard, how much cloth can be bought for $\frac{3}{4}$ of a dollar? for $\frac{5}{6}$ of a dollar?

27. If 7 bushels of wheat cost $8\frac{1}{2}$ dollars, what is the cost of one bushel?

28. In $\frac{9}{10}$ of an acre of land, how many building lots of $\frac{1}{5}$ of an acre each, and how much over?

29. If a horse eats $\frac{3}{4}$ of a bushel of oats a day, how long will $1\frac{1}{12}$ of a bushel last him?

30. If $\frac{3}{4}$ of a pound of tea costs 1 dollar, what is the cost of $\frac{4}{5}$ of a pound?

31. If $\frac{3}{5}$ of a bushel of barley be given for $\frac{3}{4}$ of a bushel of corn, and the corn costs $\frac{2}{3}$ of a dollar a bushel, what is the cost of a bushel of barley?

32. In an orchard $\frac{1}{2}$ of the trees are apple-trees, $\frac{1}{4}$ peach-trees, $\frac{1}{4}$ pear-trees, and the remaining 11 are plum-trees. How many trees in the orchard?

33. What number added to 4 times $\frac{3}{5}$ of 21 will make 52?

34. 24 is $\frac{2}{3}$ of some number. What is $\frac{4}{4}$ of the same number?

35. How long will it take two wood-choppers to cut 14 cords of wood, if one of them cuts $\frac{3}{4}$ of a cord a day, and the other cuts 1 cord?

36. Several workmen received together $6\frac{2}{3}$ dollars, and the wages of each one was $\frac{5}{6}$ of a dollar. How many workmen were there?

37. A man plants a bed of 36 square feet in cabbage, and gives to each cabbage $\frac{3}{4}$ of a square foot. How many cabbages can he plant in the bed?

38. How many steps in a staircase 39 feet high, when each step is $1\frac{1}{12}$ foot high?

39. A yard of a certain cloth, 4 quarters wide, is worth 3 dollars. What will 1 yard of the same cloth, 5 quarters wide, be worth?

NOTE.—The pupil will frame four questions similar to the above, and give the answers.

LESSON XVIII.

PROPORTION.

1. What is the difference between 6 and 2? Give two other numbers which have the same difference between them.

2. If we take 8 and 4 as the two numbers, we will have 6 less 2 equal to 8 less 4. Now if we add the greater of the first pair of numbers to the less of the second pair, and the less of the first to the greater of the first, what are the results? Try the same with any pair whose difference is 4.

3. Find 5 pairs of numbers, the difference between the two forming each pair being 5.

4. What number is as much greater than 10 as 12 is greater than 7? How is this number found?

Ans.—*By adding the difference between* 12 *and* 7 *to* 10.

5. What is the quotient of $6 \div 2$? Give two other numbers which have the same quotient. If we take the numbers $12 \div 4$, we will have $6 \div 2$ equal to $12 \div 4$. If, now, we multiply the greater of the first pair of numbers by the less of the second pair, and the less of the first pair by the greater of the second, what are the results? Try the same with another pair of numbers whose quotient is 3.

6. Find 5 pairs of numbers, the quotient of the numbers in each pair being 5.

7. What number is contained in 15 as often as 4 is contained in 20? or $20 \div 4$ is equal to $15 \div$ what number? How is this number found?

Ans.— *We divide 15 by the quotient of 20 divided by 4.*

8. $12 \div 3$ is equal to $16 \div$ what number? $12 \div 4$ $= 15 \div$ what number? $10 \div 2 = 20 \div$ what number?

9. $20 \div 4$ is called the *ratio* of 20 to 4. The line between the two dots is left out for convenience; thus, $20 : 4$, which is read 20 *to* 4.

10. The ratio of 20 to 4 is $\frac{20}{4}$, or 5.

So, $12 \div 3 = 12 : 3 = \frac{12}{3} = 4$; $3 : 12 = \frac{3}{12} = \frac{1}{4}$; $5 : 7 = \frac{5}{7}$; $1 : 3\frac{1}{2} = 1 \div \frac{7}{2} = 1 \times \frac{2}{7} = \frac{2}{7}$; $3\frac{1}{3} : 1 = \frac{10}{3} : 1 = \frac{10}{3}$.

What is the ratio of $10 : 4$? of $5 : 10$? of $3 : 9$? of $9 : 3$? of $5 : 11$? of $11 : 5$? of $8\frac{1}{3} : 1\frac{2}{3}$? of $\frac{3}{4} : \frac{4}{5}$? of $2\frac{1}{2} : 5$? of $5 : 2\frac{1}{3}$?

11. Find 6 pairs of numbers that have the same ratio as $10 : 5$.

12. $4 : 12 = 13 :$ what number?

Solution.—$4 : 12 = \frac{1}{3}$. *The required number is, then,* $13 \div \frac{1}{3} = 13 \times 3$, *or* 39. *Or, since* 12 *is* 3 *times* 4, *the required number must be* 3×13, *or* 39.

13. $1 : 5 = 7 : ?$ $9 : 7 = 14 : ?$ $6 : 9 = 8 : ?$ $2 : 5 = 7 : ?$ $3 : 7 = 6 : ?$—$3 : 7 = 6 : 14$ is called a PRO-PORTION.

14. What is the ratio of one number to another? (See Q. 9.)

When are .four numbers said to be in proportion? Ans. *When the ratio of the 1st and 2d = the ratio of the 3d and 4th.*

How do you find the fourth term of a proportion when three terms are known? (See Q. 12.)

15. $4 : 10 = 11 : ?$　$5 : \frac{1}{4} = 40 : ?$　$6 : 4 = 7\frac{1}{2} : ?$ $1 : 2\frac{2}{3} = 4\frac{1}{2} : ?$　$(1 : 2\frac{2}{3} = \frac{3}{8}.$ The required number is, then, $4\frac{1}{2} \div \frac{3}{8} = \frac{9}{2} \times \frac{8}{3} = \frac{72}{6} = 12).$　$\frac{4}{5} : \frac{2}{3} = \frac{5}{6} : ?$

16. 12 is $\frac{3}{4}$ of what number?　$\frac{3}{4} : 1 = 12 :$ what number?　18 is $\frac{6}{7}$ of what number?　$6 : 7 = 18 :$ what number?　$\frac{6}{7} : 1 = 18 :$ what number?

17. 6 workmen in $2\frac{1}{2}$ days will do as much as 10 workmen in how many days?

SOLUTION.—$6 : 10 = \frac{3}{5}.$ *The number of days required is, then,* $2\frac{1}{2} \div \frac{3}{5} = \frac{5}{2} \div \frac{3}{5} = \frac{5}{2} \times \frac{5}{3} = \frac{25}{6},$ *or* $4\frac{1}{6}.$

6 men in $12\frac{1}{2}$ days do as much as 15 men in　　days?

12 men in $6\frac{2}{3}$ days do as much as 8 men in　　days?

6 men in $7\frac{1}{2}$ days do as much as 4 men in　　days?

18. 20 days' work for six men is 10 days' work for how many?

$3\frac{3}{4}$ days' work for 14 men is $7\frac{1}{2}$ days' work for　men?

19. 7 days' rations for 6 men is how many days' rations for 4 men?

$6\frac{2}{3}$ days' rations for 5 men is how many days' rations for 7 men?

$3\frac{3}{4}$ days' rations for 14 men is how many days' rations for 7 men?

$2\frac{1}{2}$ days' rations for 8 men is how many days' rations for 6 men?

20. 3 men do a piece of work in 5 days; how long will it take 7 men to do the same work?

5 men in 6 days do as much as 11 men. in　　days?

21. 3 days' rations for 11 men are how many days' rations for 13 men?

22. 3 men can do a piece of work in 5 days; how long will it take 7 men to do twice the work? 3 times the work? 4 times the work? $\frac{1}{2}$ the work?

23. If 5 men build a wall 50 feet long in 6 days, in what time will 9 men build a similar wall 150 feet long?

24. 3 days' rations for 12 men are 16 days' half-rations for how many men?

25. If 3 horses eat 8 bushels of oats in 2 weeks, how long will 40 bushels last them? How long will 40 bushels last 5 horses at the same rate?

26. How many yards of cloth, 3 quarters wide, are equal to 7 yards of the same cloth 5 quarters wide?

27. How many yards of carpeting, 1 yard wide, are equal to 20 yards $\frac{3}{4}$ of a yard wide?

28. A workman laboring 11 hours a day does a certain piece of work in 5 days; in how many days will he do the same work, laboring 5 hours a day?

29. A traveller has enough money to last him 32 days, at $\frac{3}{4}$ of a guinea a day; how many days will it last him if he limits his expenses to $\frac{2}{3}$ of a guinea a day?

30. A man bought 6 barrels of flour at 7 dollars a barrel; at what price per barrel must he sell it in order to gain 11 dollars?

31. A man being asked how much he had given for each of two horses, stated that he gave $60 for the first, and $\frac{3}{4}$ of the price of the first was $\frac{5}{2}$ of $\frac{1}{3}$ of the price of the second. What was the price of the second?

32. A man can do a piece of work in $4\frac{3}{4}$ days, if he labor 10 hours a day; how many days will it take him to do the work, laboring 7 hours a day?

33. If 5 bushels of oats cost $2\frac{2}{5}$ dollars, what will 7 bushels cost?

34. If 10 yards of cloth cost 45 dollars, what will

be the cost of 6 yards of cloth worth twice as much per yard?

35. If 6 yards of cloth cost 2½ dollars, what cost 11 yards?

36. If 5 yards of cloth cost 7 dollars, what cost 13 yards?

37. If 8 bushels of meal cost 10 dollars, how much can be bought for 12 dollars?

38. If 9 barrels of flour cost 58½ dollars, how many can be bought for 75 dollars?

39. If 10 acres of land cost 300 dollars, what is the cost of 50 acres?

40. At the rate of 10 acres for 500 dollars, how much land can be bought for 3000 dollars?

41. A railway train travels at the rate of 90 miles in 3 hours; how many miles will it go in 10 hours?

42. If a railway train travels at the rate of 100 miles in 5 hours, how many hours will it require to go 320 miles?

NOTE.—In this and all the succeeding lessons the pupils should be required to form questions for themselves and each other.

CHAPTER VIII.

DENOMINATE NUMBERS.

LESSON I.

Table of Long Measure.

This table is used in measuring distances.
The smallest units are the inch and its subdivisions.

1 Inch.

12 inches (in.) make 1 foot—marked ft.
3 feet make one yard—marked yd.
5½ yards make 1 rod or pole—marked rd.
40 rods make 1 furlong—marked fur.
8 furlongs make 1 mile—marked m.
3 miles make 1 league—marked lea.
69¼ miles (nearly) } make 1 degree—marked deg.—
60 nautical miles } of the circumference of the earth.
360 degrees make 1 circumference—marked circ.

1. How many inches in 2 feet? 3 feet? 5 feet? in 10 feet? in ⅓ of a foot? ½ of a foot? ¼ of a foot? ⅜ of a foot? ⅚ of a foot? in ¾ of a foot?

2. How many feet in 24 inches? 36 inches? 30 inches? in 15 inches? in 17 inches? in 44 inches?

3. How many feet in 2 yards? 11 yards? 3½ yards? 4¾ yards? in ½ of a yard? in ⅔ of a yard? What part of a yard is 1 foot? What part of a yard is 1 inch?

4. How many yards in 6 feet? 8 feet? 9 feet? 12 feet? 13 feet? 25 feet?

5. How many yards in 3 rods? in 5 rods? in 10 rods? in 3½ rods? in 40 rods?

6. How many rods in 2 furlongs? 3 furlongs? in 6

furlongs? in one mile? What part of a mile is a furlong? What part of a mile is one rod? How many furlongs in 40 rods? in 50 rods? in 120 rods? in 150 rods?

7. How many furlongs in 2 miles? 3 miles? 4½ miles? ¾ of a mile how many furlongs? $\frac{1}{10}$ of a furlong is what part of a mile? How many miles in 16 furlongs? in 24 furlongs? in 25 furlongs? in 40 furlongs? in 45 furlongs? How many yards in ⅛ of a mile? in ¼ of a mile? in ½ of a mile?

8. How many miles in 3 leagues? in 4 leagues? in 10 leagues? A mile is what part of a league? ⅔ of a league are how many miles?

9. How many leagues in 9 miles? in 11 miles? in 24 miles?

10. If a man travels a mile in ten minutes, in how many minutes will he travel 1 league? 3 leagues? 4 leagues?

11. If a ship sails 20 leagues a day, how long will it take her to sail 600 miles? 300 miles? 3000 miles?

12. How many inches in 3 yds. 2 ft. 4 in.? in 2 yds. 1 ft. 5 in.?

Table of Cloth Measure.

This measure is used in measuring cloth and other articles sold by the yard. The yard and inch are the same in this as in Long Measure.

2¼ inches (in.) make 1 nail—marked na.
4 nails make 1 quarter of a yard—marked qr.
4 quarters make 1 yard—marked yd.
5 quarters make 1 ell English—marked E. E.

1. How many inches in ¼ of a yard? How many in ½ a yard? in ¾ of a yard? How many inches in an ell?

2. How many quarters in 12 nails? 13 nails? in 24 nails?

3. How many yards in 6 quarters? in 20 quarters? How many nails in 3 yds. 3 qrs. 3 nails? How many inches in 3 qrs. 2 na. 4 in.?

4. If 4 nails of cloth cost 20 cents, what is the cost of 5 yards? What part of a yard is one nail? What part of an ell is a quarter?

5. What is the cost of 12 yards of cloth at 5 dollars an ell? $\frac{5}{8}$ of a yard is what part of an ell? what part of a rod? $\frac{2}{8}$ of a rod is what part of a furlong? $\frac{1}{10}$ of a yard is what part of a foot? $\frac{1}{10}$ of a foot is how many inches?

LESSON II.

Square Measure.

Square Measure is used for measuring surfaces. The square inch is the smallest unit in the table.

Square Inch.

TABLE.

144 square inches (sq. in.) make one square foot—marked sq. ft.

9 square feet make 1 square yard—marked sq. yd.

$30\frac{1}{4}$ square yards make 1 square rod or perch—marked P.

40 square rods make 1 rood—marked R.

4 roods make 1 acre—marked A.

640 acres make 1 square mile—marked sq. m.

1. How many square feet in 2 square yards? in $2\frac{1}{2}$

square yards? in 5 square yards; in $3\frac{3}{4}$ square yards? How many square yards in 36 square feet? in 81 square feet? in 44 square feet? $\frac{1}{2}$ of a square foot is what part of a square yard? $\frac{1}{7}$ of a square yard is how many square feet?

2. How many square yards in 3 square rods or perches? in 4 perches? in 10 perches? How many square rods in 3 roods? in $\frac{1}{2}$ of a rood? in $\frac{5}{8}$ of a rood? in $3\frac{1}{2}$ roods? What part of a rood is a square rod?

3. How many acres in 10 roods? 16 roods? 40 roods? What part of an acre is 1 rood? $\frac{1}{2}$ of a rood? $\frac{1}{10}$ of a rood? How many roods in $\frac{5}{8}$ of an acre? 20 square rods is what part of an acre?

4. At 5 dollars a square rod, what is the cost of 2 roods and 5 sq. rods of land? 1 A. 2 R. 15 P. are how many perches?

5. At $2\frac{3}{4}$ dollars a rood, what is the cost of 2 acres of land?

Solid or Cubic Measure.

This measure is used in measuring timber and stone, and gives the unit for Liquid and Dry Measure, and measures weights and coins.
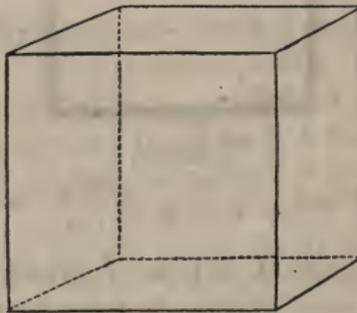
Cubic Inch.

TABLE.

1728 cubic inches (cu. in.) make 1 cubic foot—marked cu. ft.

27 cubic feet make 1 cubic yard—marked cu. yd.

16 cubic feet make 1 cord foot—marked co. ft.

8 cord feet, or 128 cubic feet, make 1 cord of wood —marked C.

1. How many cubic feet in 2 cubic yards? in 3 cubic yards? in $\frac{1}{3}$ of a cubic yard? in $\frac{1}{9}$ of a cubic yard? in $\frac{1}{10}$ of a cubic yard?

2. How many cubic yards in 27 cubic feet? in 36 cubic feet? in 54 cubic feet? in 57 cubic feet? What part of a cubic yard is 1 cubic foot? 3 cubic feet? 9 cubic feet? How many cubic feet in $\frac{2}{3}$ of a cubic yard?

3. How many cord feet in 2 cords of wood? in $2\frac{1}{2}$ cords of wood? in $1\frac{3}{4}$ cord of wood? $\frac{1}{2}$ of a cord is how many cord feet?

4. How many feet of plank, 1 inch thick, in 50 feet of plank 2 inches thick? $4\frac{1}{2}$ inches thick?

LESSON III.

Liquid or Wine Measure.

This measure is used in measuring all kinds of liquids. The wine pint-cup will hold $28\frac{8}{10}$ cubic inches.

TABLE.

4 gills (gi.) make 1 pint—marked pt.
2 pints make 1 quart—marked qt.
4 quarts make 1 gallon—marked gal.
63 gallons make 1 hogshead—marked hhd.
2 hogsheads make 1 pipe—marked pi.
2 pipes make 1 tun—marked tun.

1. How many gills in 2 pints? 3 pints? $\frac{1}{2}$ of a pint? $\frac{3}{4}$ of a pint? in $2\frac{1}{2}$ pints? in $6\frac{5}{8}$ pints? How many pints in 8 gills? 9 gills? 15 gills? 60 gills? What part of a pint is 1 gill? 2 gills? 3 gills?

2. How many pints in 5 quarts? 3 quarts? $10\frac{1}{2}$ quarts? $2\frac{1}{3}$ quarts? in 12 quarts? How many gills in 2 quarts? What part of a quart is 1 pint? How many quarts in 12 pints? in 13 pints? 50 pints?

3. How many quarts in 3 gallons? 2½ gallons? 5¾ gallons? What part of a gallon is a quart? How many gallons in 2 hogsheads? 9 gallons is what part of a hogshead? 21 gallons is what part of a hogshead?

4. If 3 quarts of oil cost 60 cents, what is the cost of 5 gallons and 2 quarts?

5. How many quart bottles can you fill from a hogshead of wine? How many gallons in a pipe?

Dry Measure.

This measure is used in measuring grain, fruit, coal, salt, &c. A pint-cup of this measure contains $33\frac{6}{10}$ cubic inches.

TABLE.

2 pints (pt.) make 1 quart—marked qt.
8 quarts make 1 peck—marked pk.
4 pecks make 1 bushel—marked bu.

1. How many cubic inches larger is the pint, Dry Measure, than the pint, Liquid Measure? How much larger is the quart, Dry Measure, than the quart, Liquid Measure?

2. How many quarts in 2 pecks? in 4½ pecks? in 3⅝ pecks? in ½ peck? ¾ of a peck? ⅘ of a peck? How many pecks in 16 quarts? 18 quarts? 21 quarts? 40 quarts? What part of a peck is a pint?

3. How many pecks in 4 bushels? in ⅓ of a bushel? in 5½ bushels? in 10 bushels? in 100 bushels? How many bushels in 12 pecks? in 15 pecks? in 32 pecks? in 45 pecks?

4. What is the cost of 1 bushel of corn at 5 cents a quart? What of 2 quarts of corn at 60 cents a bushel?

5. How many quarts in 4 bushels? in 3½ bushels? in 2 bu. 3 pk. 3 qt.?

6. If 2 pints of meal cost 5 cents, what is the cost of a bushel? What part of a bushel is a pint?

LESSON IV.
Troy Weight.

Troy Weight is used in weighing gold, silver, and jewels. The Troy pound is a piece of brass or other metal, which weighs as much as $22\frac{8}{10}$ cubic inches of water, at a certain temperature.

TABLE.

24 grains (gr.) make 1 pennyweight—marked pwt.
20 pennyweights make 1 ounce—marked oz.
12 ounces make 1 pound—marked lb.

1. In 1 pennyweight how many grains? in 2? in 3? in $4\frac{1}{2}$? in $5\frac{3}{4}$?
2. How many pennyweights in 48 grains? in 72? in 75?
3. In 1 pound how many ounces? in 2? in 4? in 5? in $\frac{1}{3}$? in $2\frac{3}{4}$?
4. In 24 oz. how many lbs.? in 36? 48? 28? 39?

Apothecaries' Weight.

The Troy pound is sometimes divided differently, to weigh medicines in mixing them. The pound, ounce, and grain are the same in Troy and Apothecaries' weight.

20 grains (gr.) make 1 scruple—marked sc. or ℈.
3 scruples make 1 dram—marked dr. or ℨ.
8 drams make 1 ounce—marked oz. or ℥.
12 ounces make 1 pound—marked lb. or ℔.

1. In 1 scruple how many grains? in 2? in 3? in $\frac{1}{5}$ of a scruple? in $2\frac{3}{4}$ scruples?
2. In 2 drams how many scruples? in 3? in 4? in $\frac{1}{3}$ of a dram? in $2\frac{2}{3}$ drams?

3. In 2 ounces how many drams? in 3? 4? 2¾?

4. In $\frac{1}{12}$ of a pound how many grains? How many grains in 4 drams, 3 scruples, 5 grains?

Avoirdupois Weight.

This weight is used in all ordinary weighing. The Avoirdupois pound is heavier than the Troy pound, 144 Avoirdupois pounds being equal to 175 Troy pounds.

TABLE.

16 drams make 1 ounce—marked oz.

16 ounces make 1 pound—marked lb.

25 pounds make 1 quarter—marked qr.

4 quarters, or 100 lbs., make 1 hundred weight—marked cwt.

20 hundred weight make 1 ton, marked T.

1. In 2 pounds how many ounces? in 3? in 4? in $\frac{1}{2}$ of a pound? in 2$\frac{5}{8}$?

2. In 32 ounces how many pounds? in 48? 64? 80? 82?

3. How many pounds in 2 quarters? in 3? in 4$\frac{1}{5}$?

4. How many quarters in 50 pounds? 100 pounds? 200 pounds? 76 pounds?

5. How many cwt. in 8 quarters? in 16? in 25? in 33?

6. How many cwt. in 2 tons? 4 tons? $\frac{1}{2}$ a ton? $\frac{1}{10}$ of a ton? in $\frac{3}{4}$ of a ton? in 5$\frac{1}{2}$ tons? What part of a ton is 1 cwt.?

7. How many tons in 40 cwt.? in 80? in 60? in 45?

8. What is the cost of 4$\frac{1}{2}$ cwt. of sugar, at 9 cents a pound? at 10 cents?

9. What is the cost of 5$\frac{1}{2}$ tons of hay, at 20 dollars a ton?

10. How much will 2 cwt. 2 qrs. of sugar cost at 10 cents a pound?

11. Bought 3 tons and 10 cwt. of hay at 12 dollars a ton? What was the cost of it?

LESSON V.

United States Money.

The units are coins of gold, silver, copper, and nickel, of fixed weight, size, and shape.

TABLE.

Gold.   Gold.   Gold.

Nickel.   Silver.   Silver.

10 mills make one cent.—marked c.
10 cents make 1 dime—marked d.
10 dimes make 1 dollar—marked $.
10 dollars make 1 eagle—marked E.

1. How many mills in 2 cents? 9 cents? 2½ cents? How many cents in 50 mills? 40 mills? 25 mills?

2. How many cents in 2 dimes? in 3? in 4½? in 5⅗? How many dimes in 20 cents? 25 cents? 50 cents? 75 cents?

3. How many dimes in 2 dollars? $4\frac{1}{2}$ dollars? $5\frac{1}{10}$ dollars? $6\frac{3}{10}$ dollars? How many dollars in 20 dimes? 40 dimes? 45 dimes?

4. How many dollars in 2 eagles? in a half-eagle? in $3\frac{1}{2}$ eagles? in $5\frac{1}{4}$ eagles? How many eagles in 20 dollars? 35 dollars? 42 dollars?

5. What part of a dollar is 1 cent? is 1 mill? is 50 cents? $33\frac{1}{3}$ cents? 25 cents? 20 cents? 1 dime? $16\frac{2}{3}$ cents? $12\frac{1}{2}$ cents?

6. What part of an eagle is 1 dime? 5 dimes?

7. How many cents in $\frac{1}{2}$ of a dollar? in $\frac{1}{4}$? in $\frac{1}{3}$? in $\frac{1}{5}$? in $\frac{1}{8}$? in $\frac{1}{6}$? $\frac{1}{16}$? $\frac{1}{20}$? $\frac{1}{50}$? $\frac{3}{10}$ of a dollar? $\frac{7}{8}$ of a dollar?

8. Bought six barrels of flour at 9.33\frac{1}{3}$ cents a barrel; what was the cost of the whole?

9. Paid $6\frac{1}{4}$ cents a pound for 48 pounds of nails; how many dollars did the whole cost?

10. At $37\frac{1}{2}$ cents a yard, what is the cost of 6 yards of ribbon?

11. If 6 pounds of sugar can be bought for 60 cents, how much can be bought for an eagle?

12. What is the cost of 100 articles at 1 cent each? 2 cents? 3 cents? 12 cents?

Table of English Money.

4 farthings (far.) make 1 penny—marked d.
12 pence make 1 shilling—marked s.
20 shillings make 1 pound—marked £.

The coins of this table are gold, silver, and copper. A sovereign is a gold coin = 1 pound or 20 shillings. The guinea is 21 shillings. A pound is 4 dollars and 84 cents.

1. How many farthings in 2 pence? in 3 pence? in $6\frac{1}{2}$ pence? $\frac{1}{2}$ a penny? $\frac{3}{4}$ of a penny? in $4\frac{3}{4}$ pence?

2. How many pence in 8 farthings? in 10? in 20? in 60?

3. How many pence in 2 shillings? in $\frac{1}{3}$ of a shilling? in $\frac{1}{4}$ of a shilling? in $\frac{1}{6}$ of a shilling? $\frac{5}{6}$ of a shilling? in 5s. 6d.? in 10s. 2d.? in 10s. 6d.? in 2s. 8d.?

4. How many shillings in 24 pence? 48 pence? 32 pence? 100 pence?

5. How many shillings in £2? in £$\frac{1}{2}$? in £$\frac{1}{5}$? in £$\frac{4}{5}$? in £$\frac{3}{10}$? in £2? £2$\frac{1}{2}$? £5? in £7? in £2 7s.? in £10 6s.?

6. How many pounds in 25 shillings? in 40 shillings? in 68 shillings? How many guineas in 63 shillings? How many dollars in £10.

7. What part of a shilling is 2 pence? 3 pence? 4 pence? 6 pence? 18 pence?

8. What part of a pound is 1 shilling? 2 shillings? 4 shillings? 2s. 6d.? 3s. 5d.?

9. What is the cost of 12 articles at 1 penny apiece? 2 pence apiece? at 4d.? at 3d.? at 10d.?

10. What is the cost of 24 or 2×12 articles, at 2d. apiece? at 3 pence? 4d.? 6d.? 10d.?

11. What is the cost of 20 articles at 1 shilling apiece? 3s. apiece? 4s.? 6 shillings? 10 shillings?

12. What is the cost of 40 articles, or 2×20 at 3 shillings apiece? at 4s.? 6s.? 10s.?

13. What is the cost of 8 bushels of corn at 2s. 8d. a bushel? What part of a pound is a penny?

LESSON VI.

Measure of Time.

The natural units in this table are the day and the year. The smallest unit, the second, is the interval between two ticks of a seconds clock, the pendulum of the clock being $39\frac{1}{10}$ inches long.

TABLE.

60 seconds (sec.) make 1 minute—marked min.
60 minutes make 1 hour—marked hr.
24 hours make 1 day—marked dy.
7 days make 1 week—marked wk.
365 days make 1 common year—marked yr.
366 days make 1 leap year.
100 years make 1 century—marked c.

The year is divided into 12 months.

TABLE.

1st month, January, has 31 days.
2d month, February, has 28 days—in leap year, 29.
3d month, March, has 31 days.
4th month, April, has 30 days.
5th month, May, has 31 days.
6th month, June, has 30 days.
7th month, July, has 31 days.
8th month, August, has 31 days.
9th month, September, has 30 days.
10th month, October, has 31 days.
11th month, November, has 30 days.
12th month, December, has 31 days.

This table is usually given in the following rhyme:

> Thirty days hath September,
> April, June, and November;
> All the rest have thirty-one,
> Except the second month alone,
> And this has twenty-eight, in fine,
> Till leap year gives it twenty-nine.

1. How many seconds in 2 minutes? 3 minutes? 4 minutes? 10 minutes? in $\frac{1}{2}$ minute? $\frac{1}{4}$ minute? $\frac{1}{12}$ of a minute? $\frac{3}{5}$ of a minute? in $\frac{7}{10}$ of a minute?

2. How many minutes in 120 seconds? in 180

seconds? in 2 hours? in $3\frac{1}{2}$ hours? in 5 hours? in $1\frac{3}{4}$ hour? $\frac{1}{2}$ an hour? $\frac{1}{4}$ of an hour?

3. How many hours in 2 days? 3 days? 4 days? in $2\frac{1}{2}$ days? in $1\frac{2}{3}$ day? How many days in 48 hours? 72 hours? 75 hours? 36 hours?

4. How many days in 4 weeks? in $\frac{5}{7}$ of a week? in $4\frac{1}{2}$ weeks? in 10 weeks? How many weeks in 28 days? 56 days? 31 days?

5. If October commences on Monday, on what day of the week will November begin?

6. How many months in 2 years? $2\frac{1}{2}$ years? 6 years?

7. If a man earns $2\frac{1}{2}$ cents a minute, how much will he earn in 5 hours? What part of an hour is 2 minutes? 3 minutes? 15 minutes?

MISCELLANEOUS TABLE.

> 12 units make 1 dozen.
> 12 dozen make 1 gross.
> 20 units make 1 score.
> 24 sheets of paper 1 quire.
> 20 quires make 1 ream.

1. How many units in 3 dozen? $\frac{1}{2}$ dozen? $5\frac{1}{2}$ dozen?

2. How many scores in 40? 50? 80? 60? 200? in 5 dozen? 10 dozen? in $4\frac{1}{3}$ dozen?

3. What is the cost of a ream of paper at 15 cents a quire? at 10 cents? $12\frac{1}{2}$ cents? at $5\frac{1}{10}$ cents? What part of a ream is half a quire?

CHAPTER IX.

METRIC SYSTEM OF WEIGHTS AND MEASURES.

The adoption of the French or metric system of weights and measures in this country, has been authorized by the Government. The great advantage of this system is the uniformity in the names and in the division of the units in the different tables. The division is the successive division into tenths, after the manner of the eagle, dollar, dime, cent, and mill of the United States money. In addition to these advantages, this system has *one* table of weights instead of three. One table for cubic, liquid, and dry measure. The names of the units indicate what multiple or part of the principal unit they are.

Long Measure.

The *Metre* is the principal unit or basis of the table of Long Measure.

The *Metre* is one yard 3 and $\frac{4}{10}$ inches, or $39\frac{37}{100}$ inches.

TABLE.

10 millimetres make 1 centimetre.
10 centimetres make 1 decimetre.
10 decimetres make 1 metre=$39\frac{37}{100}$ in.—marked M.
10 metres make 1 decametre—marked D .
10 decametres make 1 hectometre—marked H^m.
10 hectometres make 1 kilometre—marked K^m.
10 kilometres make 1 myriametre=$6\frac{21}{100}$ miles, marked M^m.

The prefixes, milli, centi, deci, mean $\frac{1}{1000}$, $\frac{1}{100}$, and $\frac{1}{10}$, respectively. The prefixes deca, hecto, kilo,

myria, mean 10, 100, 1000, 10,000, respectively. These are used in all the tables of this system. In commerce the expressions 10 metres, 100 metres, 1000 metres, 10,000 metres, are invariably used instead of decametre, hectometre, kilometre.

1. How many millimetres in 2 centimetres? 4 centimetres? in $3\frac{1}{2}$ centimetres? What part of a centimetre is 1 millimetre? 2 millimetres? How many centimetres in 50 millimetres? in 65 millimetres?

2. What part of a metre is a millimetre? a centimetre? What part of a myriametre is a metre? How many metres make a kilometre? how many centimetres? how many decimetres?

3. Bought 5 decimetres of ribbon at 1 dollar a metre, what was the cost?

4. What is the cost of 30 metres of cloth at half a dollar a decimetre? at $\frac{1}{4}$ of a dollar a decimetre? at 1 dollar a decimetre?

5. A tower is 55 metres high; how many centimetres?

6. How many miles in 100 myriametres? 40 myriametres?

Square Measure.

The natural unit is the centiare or square metre, equal to $1\frac{2}{10}$ square yard, nearly.

100 centiares make 1 are = 100 square metres.
10 ares make 1 decare.
10 decares make 1 hectare = $2\frac{47}{100}$ acres, $2\frac{1}{2}$ acres, nearly.

1. How many square metres in 15 centiares? how many square metres in 1 are? in 1 hectare? in 2 hectares? How many acres in 100 hectares? How many square yards in 100 square metres?

2. What is the cost of 15 hectares of land at 5 dol-

lars the decare? What is the cost of 15 acres of land, at 50 dollars the hectare?

NOTE.—To make the table of square measure complete, we should have the square decimetre, square centimetre, and square millimetre—but these are seldom used.

Cubic Measure.

The unit of this table is the *cubic metre* = 1.3 cub. yard. This is called a stere when used in measuring wood, or hectolitre when applied to liquid or dry measure, of which the principal unit is the litre or cubic decimetre = $\frac{9}{10}$ of a quart, dry measure, and $1\frac{5}{100}$ of a quart, liquid measure.

TABLE.

10 millilitres make 1 centilitre = 10 cubic centimetres.

10 centilitres make 1 decilitre.

10 decilitres make 1 litre = 1 cubic decimetre = $\frac{9}{10}$ of a dry quart, D. M., $1\frac{5}{100}$ of a quart, L. M.

10 litres make 1 decalitre.

10 decalitres make 1 hectolitre.

10 hectolitres make 1 kilolitre or stere = 1 cubic metre.

1. What part of a kilolitre is 1 litre? 2 litres? 1 decilitre? 1 decalitre? 5 hectolitres? 100 decalitres?

2. How many millilitres make one decalitre?

3. How many quarts D. M. in 10 litres? How many quarts, Liquid Measure, in 100 litres?

4. What is the cost of 20 hectolitres of wine at 10 cents a litre?

5. 1 hectolitre is what part of a cubic metre? 1 decilitre is what part of a cubic decimetre? is how many cubic centimetres?

Weights.

The principal unit is the gramme, or the weight of a millilitre of pure water when it is heaviest.

The gramme is $15\frac{43}{100}$ grains Avoirdupois.

TABLE.

10 milligrammes make 1 centigramme.

10 centigrammes make 1 decigramme.

10 decigrammes make 1 gramme = $15\frac{43}{100}$ grains Avoirdupois.

10 grammes make 1 decagramme.

10 decagrammes make 1 hectogramme.

10 hectogrammes make one kilogramme = $2\frac{2}{10}$ lbs. Avoirdupois.

10 kilogrammes make 1 myriagramme.

10 myriagrammes make 1 quintal.

10 quintals make 1 millier or tonneau = $1\frac{1}{10}$ ton, nearly.

1. How many kilogrammes in 1 quintal? in 5 quintals? in $4\frac{1}{2}$ quintals? How many grammes in 1 tonneau? How many myriagrammes in 3 tonneaux?

2. 1 milligramme is what part of two grammes? of 3 decagrammes? 1 centigramme is what part of a kilogramme? How many grammes in 5 kilogrammes?

3. What is the cost of two kilogrammes of hay at 1 dollar the quintal?

4. At 2 dollars the kilogramme, what is the cost of a hectogramme of tobacco? of a decagramme? of 15 grammes?

5. How many pounds Avoirdupois in 1 myriagramme?

6. One hundred pounds how many kilogrammes? One hundred kilogrammes how many pounds?

7. What are some of the advantages of the metric system of weights and measures?

TABLE OF FRENCH MONEY.

10 millimes make 1 centime.
100 centimes make 1 franc = 19 cents 3 mills, U. S. money.
20 francs make 1 louis.

1. 5 centimes make 1 sou. The franc is how many sous? 5 francs are nearly one dollar.

2. A franc in silver weighs 5 grammes. 40 five-franc pieces in silver weigh one kilogramme.

3. What is the weight of 1000 silver francs?

4. What is the value of a sum of money weighing two kilogrammes?

5. What is the weight of a litre of pure water when it is heaviest? of a decilitre? of a decalitre?

6. Bought 500 kilogrammes of hay at 1 dollar the quintal; how much did it cost?

7. A garden of 500 square metres is sold for $50; how much is that for each are?

8. A staircase of 12 steps descends into a cellar 2 metres and 76 centimetres deep; what is the height of each step?

CHAPTER X.

PERCENTAGE.

LESSON I.

Per cent. is a contraction for *per centum,* which means *by the hundred, for each hundred,* or *in a hundred.* Thus 5 per cent. of any number means 5 parts in every hundred parts of the number, or 5 hundredths of the number. To take 6 per cent. of a number is to take 6 hundredths of the number, or 6 times 1 per cent., or 6 times 1 hundredth of the number. The 5 and 6 in 5 per cent. and 6 per cent. are called the rates per cent. 5 per cent. and 6 per cent. are merely other forms for the fractions $\frac{5}{100}$ and $\frac{6}{100}$.

1. What is meant by per cent.? What is, then, 5 per cent. of any number? What is meant by the *rate* per cent.?

2. How much is 1 per cent. of 40? 50? 60? 4? 5? 6?

3. How much is 1 per cent. of 20? 2 per cent. of 20? 3 per cent.? 5 per cent.? 4 per cent.? $\frac{1}{2}$ of 1 per cent.? $\frac{1}{4}$ of 1 per cent.? $2\frac{1}{2}$ per cent.? 10 per cent.?

PROCESS.—1 *per cent. of* 20 *is* $\frac{1}{100}$ *of* 20 = $\frac{20}{100}$ = $\frac{1}{5}$; $\frac{1}{2}$ *of* 1 *per cent. is* $\frac{1}{2}$ *of* $\frac{1}{5}$, *or* $\frac{1}{10}$; 4 *per cent. of* 20 *is* 4 × $\frac{1}{5}$, *or* $\frac{4}{5}$; 10 *per cent. is* 10 × $\frac{1}{5}$ = 2.

4. How much is 10 per cent. of 540? 320? 250? 100? How much is 20 per cent. of the same numbers?

5. How much is 100 per cent. of 5? of 6? of any number? How much is 200 per cent. of 8? 40? of any number?

6. Every per cent. fraction has what for numerator and what for denominator?

ANS.—*The rate per cent. for the numerator, and one hundred for the denominator.*

7. Which per cent. fractions, or which hundredths, can be most readily reduced to simpler fractions?

ANS.—*Those whose numerators are divisors of* 100.

8. What fraction of a number having 1 for its numerator is 2 per cent. of the number?

PROCESS.—*2 per cent. of a number is* $\frac{2}{100}$ *of the number,* $\frac{2}{100}$ *in its simplest form* $= \frac{1}{50}$.

9. What is the simplest form of 4 per cent.? 5 per cent.? 10 per cent.? 20 per cent.? 25 per cent.? 50 per cent.? 75 per cent.?

10. What per cent. of a number is $\frac{1}{2}$ of the number? $\frac{1}{2}$ is how many hundredths?

11. What per cent. of a number is $\frac{1}{4}$ of it? $\frac{1}{5}$ of it? $\frac{1}{10}$ of it? $\frac{1}{20}$ of it? $\frac{1}{25}$ of it? $\frac{1}{50}$ of it? $\frac{1}{100}$ of it? $\frac{1}{200}$ of it? $\frac{3}{4}$ of it? $\frac{9}{10}$ of it? $\frac{3}{10}$ of it?

12. Which fractions can be extended most easily to hundredths?

ANS.—*Those of whose denominators* 100 *is a multiple.*

13. $\frac{1}{3}$ of a number is what per cent. of the number? Extend $\frac{1}{3}$ to hundredths.

PROCESS.—*100* $\div$ *3 is* $33\frac{1}{3}$, *hence* $\frac{1}{3}$ *is* $\frac{33\frac{1}{3}}{100}$; $\frac{1}{3}$ *of a number is* $\frac{33\frac{1}{3}}{100}$, *or* $33\frac{1}{3}$ *per cent. of it.*

14. What per cent. of a number is $\frac{1}{8}$ of it? $\frac{1}{6}$ of it? $\frac{1}{12}$ of it? $\frac{1}{16}$ of it? $\frac{2}{3}$ of it? $\frac{3}{8}$ of it? $\frac{7}{8}$ of it? $\frac{2}{5}$? $\frac{4}{5}$?

15. What fraction of a number is $33\frac{1}{3}$ per cent. of the number? $16\frac{2}{3}$ per cent. of it? $12\frac{1}{2}$ per cent. of it? $8\frac{1}{3}$ per cent. of it? $6\frac{1}{4}$ per cent. of it? $66\frac{2}{3}$ per cent.? $87\frac{1}{2}$ per cent.? 60 per cent.? 80 per cent.? 90 per cent.?

16. What part of 100 is $16\frac{2}{3}$? $12\frac{1}{2}$? $66\frac{2}{3}$? $6\frac{1}{4}$? 60? 80? $37\frac{1}{2}$? 90?

17. 5 is what per cent. of 60?

PROCESS.—*5 is* $\frac{1}{12}$ *of 60, and* $\frac{1}{12}$ *of a number is* $\frac{8\frac{1}{3}}{100}$ *of it, or* $8\frac{1}{3}$ *per cent. Hence 5 is* $8\frac{1}{3}$ *per cent. of 60.*

The question, What per cent? is simply, how many hundredths?

18. What per cent. of 30 is 5? 10? 20? 25? 30? 15? 2?

19. 8 is what per cent. of 16? 32? 36? $40\frac{1}{2}$? 60? 72?

20. 2 is what per cent. of 10? 8? 20? 40? 50? 12? 100? 2? 200?

21. $\frac{1}{2}$ is what per cent. of 5?

PROCESS.—$\frac{1}{2}$ *is what part of* 5? $\frac{1}{10}$ *of* 5 *is* 10 *per cent. of it; or,* 1 *is* $\frac{1}{5}$ *of* 5, *or* 20 *per cent., hence* $\frac{1}{2}$ *is* 10 *per cent.*

22. $\frac{3}{4}$ is what per cent. of 2? $\frac{2}{5}$ is what per cent. of 10?

23. $\frac{1}{2}$ is what per cent. of $\frac{3}{5}$?

PROCESS.—$\frac{1}{2}$ *is* $\frac{5}{6}$ *of* $\frac{3}{5}$; $\frac{1}{6}$ *of a number is* $16\frac{2}{3}$ *per cent., and* $\frac{5}{6}$ *is* $5 \times 16\frac{2}{3}$, *or* $83\frac{1}{3}$ *per cent.*

24. $\frac{4}{10}$ is what per cent. of $\frac{3}{4}$?

25. 10 is 5 per cent. of what number?

PROCESS.—5 *per cent. is* $\frac{1}{20}$, 10 *is* $\frac{1}{20}$ *of* 200, *or* 5 *per cent. of* 200.

26. 18 is 6 per cent. of what number?

PROCESS.—18 *is* $\frac{6}{100}$ *of what number?* 6 *in* 18, 3 *times,* $3 \times 100 = 300$; 18 *is* 6 *per cent. of* 300.

27. 20 is 5 per cent. of what number? 4 per cent.? 10 per cent.? $2\frac{1}{2}$ per cent.? 6 per cent. of what number?

28. $\frac{1}{2}$ is 6 per cent. of what number? 8 per cent.? 10 per cent.?

29. $2\frac{1}{2}$ is 5 per cent. of what number? 12 per cent.? $6\frac{1}{4}$ per cent.? $33\frac{1}{3}$ per cent.? 40 per cent.?

30. What is 2 per cent. of 5 per cent. of a number?

PROCESS.—2 *per cent. of* 5 *per cent.* = $\frac{1}{50}$ *of* 5 *per cent.* = $\frac{5}{50}$ *per cent.* = $\frac{1}{10}$ *per cent.*

31. How much is 50 per cent. of 25 per cent. of 80?

PROCESS.—*First take* 25 *per cent. of* 80 *and then* 50 *per cent. of the result.*

32. 25 per cent. of 40 per cent. is what per cent. of 20 per cent. of a number?

33. What fraction is 5 times 6 per cent.? 12 is how many times 5 per cent. of 40? 12 is 6 times what per cent. of 50?

LESSON II.

1. What is 2 per cent. of $10? $8? $20? $50? of 1 dime?

2. What is 5 per cent. of $100? $300? $600? $1000?

3. What is 6 per cent. of $1? $5? $50? $200? $150?

4. What is ½ per cent. of $5? $10? 100? $500? $1000?

5. What is 4½ per cent. of $100? $200? $500? $1000?

6. 6 cents is what per cent. of $1? 60 cents is what per cent. of $10? $6 is what per cent. of $600? $60 is what per cent. of $1000? $600 is what per cent. of $10,000?

7. 5 centimetres is what per cent. of a metre? 5 metres is what per cent. of a hectometre? 1 litre is what per cent. of a decalitre? 5 decilitres is what per cent. of a litre? A gramme is what per cent. of a kilogramme?

8. How much is 10 per cent. of 60 metres? 6 per cent. of 20 litres? 5 per cent. of 40 kilogrammes?

9. What is 5 per cent. of £50? £100? £600? £1000?

10. How much is ⅛ per cent. of £1200? ½ per cent. of £500?

11. 5 shillings is what per cent. of £1? 3d. is what per cent. of a shilling?

12. What per cent. of a yard is 1 foot? 6 inches?

13. The number of inhabitants of a village was last

year 650, this year it is 30 per cent. greater. What is the increase for the year?

14. A commission merchant sells $600 worth of wheat for a farmer, and deducts 4 per cent. commission; how much does he pay over to the farmer?

15. I bought 5 shares of railroad stock, which cost $100 a share; since the purchase each share has fallen 10 per cent.; what are the 5 shares worth now?

16. A house valued at $1200 is insured by a company at a cost of 2 per cent. on this sum. How much does the owner of the house pay the company?

17. How much is $6\frac{1}{4}$ per cent. of $32? 80 bushels of corn? 64 barrels of flour?

18. How much is $33\frac{1}{3}$ per cent. of 60 gallons of wine?

19. A man owes $6000 and is able to pay only $66\frac{2}{3}$ per cent. of it. How much do his creditors lose?

20. Of a pound of gunpowder 75 per cent. is saltpetre, $12\frac{1}{2}$ per cent. is sulphur, and $12\frac{1}{2}$ is charcoal. How many ounces of each form the pound?

21. Cannon metal consists of copper and tin; it contains 90 per cent. of copper. What per cent. of tin?

22. Bought merchandise for $44, sold the same for $22. What was the loss per cent.?

23. Bought for $66, sold for $88. What was the gain per cent.?

24. Bought for $96, sold for $100. Gained what per cent.?

25. Bought for $72, sold for $63. Lost what per cent.?

26. Bought for $23, sold for $69. Gained what per cent.?

27. Bought for $64, sold for $56. Lost what per cent.?

28. Bought for $48, sold for $44. Lost what per cent.?

29. Bought for $69, sold for $23. Lost what per cent.?

30. I bought merchandise for $75 and sold it at a loss of 6¼ per cent. For how much did I sell? For how much would I have to sell it to gain 6¼ per cent.?

31. Bought for $80, lost on sale 10 per cent. Sold for how much?

32. Bought for $72, gained on sale 33⅓ per cent. Sold for how much?

33. Bought for $88, gained 25 per cent. Sold for how much?

34. Bought for $57, gained 33⅓ per cent. Sold for how much?

35. Bought for $57, lost 33⅓ per cent. Sold for how much?

36. Bought for $112, lost 12½ per cent. Sold for how much?

37. A man has 200 bushels of wheat, he sells 75 per cent. of it at $2 a bushel and the remainder at $3 a bushel. How much does he get for the wheat?

38. The rent of a house was last year $75, but this year the owner increased the rent 20 per cent. What is the rent this year?

39. Gunpowder contains 75 per cent. of saltpetre. How much saltpetre in 120 lbs. of it?

40. It requires 16,000 bricks to build a certain house. If we allow 6¼ per cent. for loss of bricks by breakage, how many must be purchased for the house?

41. The rent of house was increased this year 10 per cent., and is now $99. What was it before the increase? 99 is 11/10 of what number?

42. A man gave $4000 for a house, and it costs him for repairs each year 2 per cent. on the purchase money. What is the yearly cost of repairs?

43. If 30 quarts of wine contain 6 quarts of water, what per cent. of water does the wine contain?

44. From $35.50 subtract 20 per cent.

45. A merchant bought cloth at $4½ a yard, and wishes to sell it at 25 per cent. profit. What must be the selling price per yard?

46. The *gross weight* of an article of merchandise is its weight including the box, barrel, or bag which contains it. The *net weight* is the weight after the weight of the box, &c., is deducted. The allowance for the weight of the box or barrel, &c., is called the tare.

47. The gross weight of some merchandise is 160 lbs., and the tare is 6¼ per cent. What is the net weight?

48. Gross weight 210 pounds, tare 10 per cent. What is the net weight?

49. Gross weight 250 pounds, net weight 230 pounds. What per cent. of tare?

50. Gross weight 220 pounds, tare 5 per cent. What is the net weight?

51. A man bought a horse for $100, and sold him for $300. What per cent. did he gain?

52. The person who bought the horse at $300 sold it for $100. What per cent did he lose?

53. $5 is what per cent. of $6? $6 is what per cent. of $5.

54. Bought an article for 2s. 6d., and sold it for 3s. 6d. What was the gain per cent.?

55. Bought an article at $4.30, sold it for $5.30. What was the gain per cent.?

56. Bought 5 pounds of sugar at 1s. 6d., and sold it at 5d. a pound. What per cent. did I gain?

57. Sold a horse for $63, and lost 12½ per cent. of what he cost me. What did he cost?

SOLUTION.—*Lost* ⅛. *Hence* $63 *is* ⅞ *of his cost.*

58. If $150 in currency are worth only $100 in gold, what percentage is lost in receiving in currency a debt payable in gold? $50 is what part of $150? ⅓ of a number is what per cent. of it?

59. If $100 in gold are worth $150 in currency, at what per cent. premium is gold?

NOTE.—Let the pupil frame questions similar to the above, and give the solutions.

LESSON III.

SIMPLE INTEREST.

Interest on money is the sum which the borrower pays the lender for the use of the borrowed money. The money loaned is called the *Principal*. The sum of the principal and interest is called the *Amount*.

Simple interest is computed at so much for a given time—usually a year—for every hundred dollars, pounds, or francs of the *loan;* in other words, at a given *rate per cent.* per year on the principal.

In computing for shorter periods than a year, thirty days are usually considered a month, and twelve months a year.

1. At 6 per cent., what is the interest on $1 for a year? $5 for a year? $10? $50? $500? $20?

2. At 7 per cent., what is the interest on $1 for one year? on $2 for one year? on $5? $10? $20? $100? $1000? $300?

3. At 8 per cent., what is the interest on $1 for one year? on $10 for one year? on $100? on $1000? on $5? $50? $500? $5000?

4. At 10 per cent., what is the interest on $1 for one year? on $10? on $5? $100? $1000? $500? $3000?

5. At 3½ per cent., what is the interest on $1 for one year? on $10? on $100? on $1000? on $60? on $600?

6. What is the interest on $1 for 2 years at 6 per cent.? 7 per cent.? 8 per cent.? 10 per cent.? 3½ per cent.? 4½ per cent.?

7. What is the interest on $20 for 3 years at 5 per cent.? 6 per cent.? 7 per cent.? 8 per cent.? 10 per cent.? $3\frac{1}{2}$ per cent.?

8. What is the interest on $200 for $3\frac{1}{2}$ years at 5 per cent.? 7 per cent.? 10 per cent.? $3\frac{1}{2}$ per cent.? 6 per cent.?

9. What is the interest on £25 for one year at 6 per cent.?

10. What is the interest on £20 for 10 years at 5 per cent.?

11. What is the interest on $1 for 1 month at 6 per cent.?

Ans.—$\frac{1}{12}$ of 6 cents, or $\frac{1}{2}$ cent = 5 mills.

12. $\frac{1}{12}$ of 6 per cent. of any number is what per cent. of that number? 6 per cent. per year on a sum of money is what per cent. per month on it?

13. What is the interest on $10 for 1 month at 6 per cent.? on $100? on $1000? on $3? $30? $25?

14. What is the interest on $1 at 6 per cent. per year for 2 months? 3 months? 5 months? 9 months? 7 months? On $10 for $2\frac{1}{2}$ months? on $50 for 3 months? on $100 for 6 months? 8 months? 10 months?

15. What is the interest on $1 for 1 month at 8 per cent.? 7 per cent.? 10 per cent.? 5 per cent.? 8 per cent. for 12 months is what per cent. for 1 month?

16. How much is $\frac{2}{3}$ of 1 per cent. of $1? $\frac{7}{12}$ of 1 per cent. of $1? $\frac{5}{6}$ of 1 per cent. of $1? $\frac{5}{12}$ of 1 per cent. of $1?

17. What is the interest on $100 for 7 months at 6 per cent.? 7 per cent.? 10 per cent.? 8 per cent.?

18. What is the interest on $84 at 7 per cent. for 2 months? 3 months? 6 months? 8 months? 10 months? 9 months? 2, 3, 6, 8, 9, and 10 form what parts, respectively, of 12?

19. At 6 per cent., what is the interest on $50 for 4 years and 3 months? for 3 years and 2 months? for 3 years and 6 months? 2 years and 4 months? 1 year and 7 months? 6 years and 9 months?

20. What is the interest on $200 for 3 years and 5 months at 7 per cent. ? 8 per cent. ? 10 per cent. ? 3½ per cent. ? 5 per cent. ?

21. The interest on any sum for a month is what part of the interest for one year?   The interest on any sum for 1 day is what part of the interest on it for 1 month ? on any sum for 2 days? 3 days? 5 days? 6 days? 15 days? 10 days? 9 days? 12 days? 18 days? 24 days?

22. At 6 per cent., what is the interest on $1 for 6 days ? on $5 for 6 days ? on $50 for 6 days ? on $500 for 6 days ? on $5000 for 6 days ? on $1 for 12 days ? 24 days? 60 days? 3 days? 5 days? 7 days?

23. What is the interest on $100 for 4 months and 15 days at 7 per cent. ? 8 per cent. ? 6 per cent. ? 5 per cent. ? 10 per cent. ? 3½ per cent. ?

24. What is the interest on $50 at 8 per cent. for 3 days? 5 days? 10 days? 12 days? 24 days? 8 days?

25. At 6 per cent., what is the interest on $1 for 60 days ? on $10 for 60 days ? on $50 ? on $250 ? on $144 ?

26. At 6 per cent., what is the interest on $120 for 3 years 2 months and 12 days ? for 4 years 5 months and 10 days?

27. What is the interest on $300 for 2 years 4 months and 15 days at 7 per cent. ? 8 per cent. ? 10 per cent. ? 5 per cent. ?

28. The interest on a certain sum for a given time at 6 per cent. is $18.   What is it at 5 per cent. ? at 1 per cent. ?   What part of 6 per cent. is 5 per cent. ?

29. The interest on any principal at 1 per cent. for one year is what part of the principal ? at 4 per cent., is what part of the principal ? at 12½ per cent. ?

LESSON IV.

1. What is the amount of $1 at 6 per cent. for 1 year? What is the amount of $10 at 5 per cent. for 1 year?

Ans.—1. $1.06 ; 2. $10.50.

2. What is the interest on $150 for 2 years at 6 per cent.? What is the amount?

3. At 6 per cent., what is the amount of $200 for 60 days? What is the amount of $80 for 90 days?

4. What is the amount of $50 for 2 years and 6 months at 4 per cent.? 7 per cent.? 8 per cent.? 10 per cent.? 5 per cent.?

5. What is the amount of $200 for 2 years 6 months and 15 days at 6 per cent.? at 10 per cent.? 5 per cent.?

6. What is the amount of $300 for 9 years at 8 per cent.? at 7 per cent.?

7. At what rate per cent. will $100 yield $5 interest in 1 year? At what rate per cent. will $20 yield $1 in one year? $5 is what per cent. of $100? $1 is what per cent. of $20?

8. The interest on $200 for 2 years is $14. What is the rate per cent.?

Solution.— *What is the interest for* 1 *year?* $7 *is what per cent. of* $200 ?

9. If the interest on $60 for 1 year and 4 months is $4, what is the rate per cent.?

10. If the interest on $90 for 3 years and 4 months is $30, what is the rate per cent.?

11. At what per cent. will $20 gain $6 in 1 year and 3 months?

12. At what per cent. will $100 gain $100 interest, or double itself, in 20 years? in 10 years? in 5 years? in 16⅔ years? in 12½ years? in 14⅞ years? At what

rate per cent. will any sum double itself in the times
mentioned?

SOLUTION.—*If* $100 *gain* $100 *in* 20 *years, how much
must it gain in one year?   If* $100 *gain* $100 *in* 20
years, how many dollars interest will $500 *gain?*

13. If the interest on $60 at 5 per cent. is $6, how
long has it been on interest?

SOLUTION.—*The interest on* $60 *for one year at* 5 *per
cent. is how much?   * $3 *is contained in* $6 *how many
times?*

14. In what time, at 6 per cent., will $10 gain $6
interest?   In what time, at 8 per cent., will $50 gain
$12 interest?

15. In what time, at 7 per cent., will $40 gain $6
interest?

16. In what time, at 10 per cent., will $50 gain
$12½?

17. In what time will $1 gain $1 interest, at 6 per
cent.?

SOLUTION.—*At* 1 *per cent.* $1 *will double itself, or
make* 100 *per cent., in* 100 *years. Hence, at* 6 *per cent.,
it will double itself in* $\frac{100}{6}$ *years, or* 16⅔ *years.*

18. In what time will a given principal double
itself at 5 per cent.? 7 per cent.? 8 per cent.? 10 per
cent.? 3½ per cent.? 25 per cent.?

19. In what time will $100 gain $200 interest, at 6
per cent.? at 7 per cent.? at 8 per cent.? at 10 per
cent.?

20. What principal, at 6 per cent., will, in 1 year,
gain $48 interest?   48 is $\frac{6}{100}$ of what number?

21. What principal, at 6 per cent., will, in 4 years,
yield $48 interest? $48 in 4 years is how much in
1 year?   12 is $\frac{6}{100}$ of what number?

22. What principal, in 1 year and 4 months, at 8
per cent., will yield $32 interest?

23. What principal, in 2 years and 3 months, at 7
per cent., will yield $21 interest?

24. What principal, in 7 months and 15 days, at 8 per cent., will yield $40 interest?

25. What principal, in 7 months and 15 days, at 4 per cent., will yield $40 interest?

26. What principal, at 10 per cent., will in 3 years and 6 months yield $60 interest?

27. What principal, at 10 per cent., will amount to $55 in one year?

SOLUTION.—*The interest is $\frac{1}{10}$ of the principal. The amount is therefore $\frac{11}{10}$ of the principal, and $55 is $\frac{11}{10}$ of $50.*

28. What principal will, in 12 years, at 5 per cent., amount to $120?

29. What principal will, in 5 years, at 7 per cent., amount to $85?

30. What principal, at 8 per cent., will amount to $116 in 4 years?

31. What principal, at 10 per cent., will amount to $44 in 2 years?

32. What principal, at 10 per cent., will amount to $200 in 10 years? What at 5 per cent. will amount to $200 in 20 years? What at 6 per cent. will amount to $200 in $16\frac{2}{3}$ years? What at 8 per cent. to $200 in $12\frac{1}{2}$ years?

33. The *present worth* of a debt due at some future time without interest is equal to a principal which put at interest will amount to the debt when it is due.

The discount on a debt is the difference between the debt and its present worth.

34. What is the present worth and the discount of $45 due five years hence without interest, money being at 10 per cent.?

SOLUTION.— *What principal at 10 per cent. will in 5 years amount to $45? What is the difference between $45 and $30?*

35. What is the present worth of $100 due in 90 days, money being at 6 per cent.? at 7 per cent.? at

8 per cent. ? at 10 per cent.? at 5 per cent.? What is the discount in each case?

36. What is the discount of $100 due in 10 years, money being at 10 per cent. ? due in 12 years at 8 per cent. ? in 14$\frac{2}{7}$ years at 7 per cent. ?

37. What is the discount of $600 dollars due in 5 years at 4 per cent. ? due in 4 years at 5 per cent. ? due in 2 years at 10 per cent. ? due in 1 year at 20 per cent. ? 10 years at 2 per cent. ?

38. What is the discount on $160, due in 4 months, at 6$\frac{1}{4}$ per cent. ?

39. What is the discount on $144, due in 15 days, at 8 per cent. ?

40. What sum placed at interest will amount to $960 in 6 years at 10 per cent. ? in 12 years at 5 per cent. ? in 10 years at 6 per cent. ?

41. When banks lend money, they usually discount or take off from the sum for which the borrower gives his note, the interest for the time, and lend the borrower the difference.

42. What is the bank discount on $100 for 6 months, at 6 per cent.

SOLUTION.—*The bank discount is the interest on $100 for 6 months at 6 per cent.*, $3. *The borrower gives his note for $100 without interest, and gets $97.*

43. What is the bank discount on $5,000 for 60 days, at 6 per cent. ? at 10 per cent ?

44. I wish to borrow $990 from a bank for 60 days, at 6 per cent. For what sum must I give my note to the bank?

45. What is the bank discount on $100 for 90 days, at 6 per cent. ? 7 per cent. ? 8 per cent. ? 10 per cent. ?

CHAPTER XI.

MISCELLANEOUS.

LESSON I.—*Proportional Parts.*

1. Divide 30 into two numbers proportional to the numbers 1 and 2; or parts which shall be to each other as 2 is to 1; or which divided one by the other shall give the same quotient as $2 \div 1$.

SOLUTION.—*2 and 1 make 3. Divide 30 into 3 equal parts, and 2 of these parts will make one of the numbers, and the remaining part will be the other. We thus get 20 and 10, and $20 \div 10 = 2 \div 1$.*

2. Divide 35 into two parts proportional to 3 and 4.

3. Divide 50 into two parts proportional to 4 and 5.

4. Divide 40 into two such parts that the first shall be 4 times the second.

5. I divided 60 oranges between Charles and Frank, giving Charles 7 for each 5 I gave to Frank; how many did each receive?

6. I wish to divide $190 between two men, giving $10 to one for each $9 which I give to the other; how much will each receive?

7. Divide 36 into 3 parts which shall be to each other as the numbers 2, 3, and 4.

8. Three men worked together for a certain time at different wages. The first at $3 a day, the second at $2 a day, and the third at $1 a day. At the end of the time they received $60 together. What is the share of each?

9. Divide 80 into two parts which shall be to each other as $\frac{1}{2}$ is to $\frac{1}{8}$.

SOLUTION.—$\frac{1}{2}$ *is to* $\frac{1}{3}$ *means* $\frac{1}{2} \div \frac{1}{3}$, *or* $3 \div 2$.   *Then divide* 80 *into* 2 *parts proportional to* 3 *and* 2 ; *or thus,* $\frac{1}{2}$ *is to* $\frac{1}{3}$ *as* $\frac{3}{6}$ *is to* $\frac{2}{6}$—*as* 3 *is to* 2.

10. Divide 34 into two parts proportional to $\frac{2}{3}$ and $\frac{3}{4}$.

11. Divide 51 into three parts proportional to $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$.

NOTE—Fractions reduced to a common denominator are to each other as their numerators.

12. In an orchard containing 96 trees there are 5 apple trees for every 4 peach trees, and 4 peach trees for every 3 pear trees ; how many trees of each kind in the orchard ?

13. Divide $200 among A, B, and C, so that B may get twice as much as A, and C three times as much as A.

14. Two men cut a certain amount of wood for $55 ; one cuts 6 cords while the other cuts 5 ; what should be the share of each one ?

15. Two men engage in trade together ; one puts in $3,000 for a year and the other $8,000 for a year.   At the end of the year the profits are $2,200 ; how much should each receive ?

16. Three men engage in trade ; A puts in $2,000 for one year, B $3,000 for the same time, and C $5,000 for the same time also.   They gain together $1,200 ; what is the share of each in the profits ?

17. Three men hire a pasture for a given time for $60.   One of them keeps 3 cows in it for the whole time, the second keeps 5 cows, and the third keeps 4 cows.   What part of the $60 should each one pay ?

REMARK.—When partners put sums of money in business together for different times, their respective shares of the profit or loss are sometimes by agreement proportional to the amounts put in multiplied by the times respectively.

18. Two men form a partnership ; A puts in $200

for 5 months, and B puts in $300 for 2 months. They gain $96; what is each man's share of the profits?

19. In trading together, A puts in $100 for 6 months, and B $50 for 4 months. They lose $64; what is each one's share of the loss?

20. Three men rent a pasture for $60. In it A pastures 5 cows for 2 weeks, B pastures two cows for 4 weeks, and C 4 cows for 3 weeks; how much must each one pay for the pasturage?

21. Reduce the ratio of 7,000 ÷ 5,000 to its simplest form.

22. Reduce the ratio of $\frac{5}{6} \div \frac{3}{4}$ to the ratio of two whole numbers.

23. Change the terms of the ratios 25 ÷ 5, 40 ÷ 8, 120 ÷ 12, 4 ÷ 3, 11 ÷ 5, in such manner that each divisor becomes 1 and each quotient remains unchanged.

24. What number is that to which if its half and fourth be added the sum will be 28?

25. A and B are 50 miles apart, traveling toward each other. When they meet it is found that $\frac{1}{3}$ of the distance traveled by A is $\frac{3}{5}$ of that traveled by B; what distance had each traveled?

26. $\frac{1}{3}$ of the time past noon = $\frac{1}{5}$ of the time before midnight; what o'clock is it?

SOLUTION.—*Divide* 12 *into two parts, which bear to each other the ratio of* $\frac{1}{5}$ *to* $\frac{1}{3}$, *i. e.*, 3 *to* 5.

27. What o'clock is it when the time past noon is $\frac{2}{5}$ of the time before midnight?

28. What number added to twice itself gives 24?

LESSON II.

1. The sum of two numbers is 25, and the greater exceeds the less by 11 what are the two numbers?

SOLUTION.—*Twice the less number added to* 11 *is equal to* 25. *Hence, twice the less number is equal to*

the difference between 25 *and* 11, *or the less number* = ½ *this difference* = 7. *The greater is then* 7 + 11 = 18.

2. Divide 40 into two such parts that the greater shall exceed the less by 22.

3. Of a pole 50 feet long, ½ as much is in the mud as in the water, and 3 times as much in the air as in the mud and water; how much in the mud, air, and water, respectively?

4. A boy being asked how many apples he had, replied, if besides what he then had, he also had ¼ and ⅓ as many more he would have 57. How many had he?

5. One man can do a piece of work in 5 days, and another in 20 days; in what time can both do the work?

SOLUTION.—*The first does* ⅕ *of the work in one day, and the second* 1/20. *Together they do* ⅕ + 1/20 = 5/20 = ¼ *of the work in one day; or the whole work in 4 days.*

6. A can reap a field of wheat in 10 days, B can reap the same field in 90 days; in what time can they both do the work together?

7. A can do a piece of work in 4 days, B can do it in 6 days, and C in 3 days; in what time can they all do it together?

8. A barrel of flour lasts a certain man and his wife 12 weeks, but when the man is absent it lasts his wife 20 weeks; how long would a barrel last the man alone?

9. A and B can do a piece of work in 12 days, but when B is absent A requires 20 days to do the same work; how long would it require B to do the work alone?

10. A and B can do a piece of work in 3 days, A and C in 4 days, B and C in 5 days; in what time can each one, working alone, do it? In what time can they all do the work together?

11. A cistern is discharged by three pipes; the first and second together will discharge it in one hour, the

first and third together in 2 hours, the second and third in 3 hours; in what time can they all three discharge it? In what time can each discharge it?

12. A, B, and C can perform a certain work in 4 days, C working alone can do the same work in 12 days, and B alone in 10 days; in what time can C do the work? In what time can B and C together?

13. One pipe leading into a cistern can fill it in 3 hours, and another pipe leading from a cistern can empty it in 5 hours. If the water is running through both pipes at the same time, in what time will the cistern be filled?

14. A man and his wife would eat one cwt. of flour in 18 days; after living together for 10 days, the woman could eat the remainder in 30 days; in how many days does the man eat one cwt. of flour?

15. A man wishes to give 10 cents apiece to some beggars, but finds he has not money enough by 9 cents: he gives them, however, 8 cents apiece, and has 5 cents remaining; how many beggars were there?

16. A can reap $\frac{4}{5}$ of a certain field in 2 days, and B can reap $\frac{9}{10}$ of the same field in 3 days; in what time can they both together reap the whole field?

17. A can reap 1 acre in 2 hours, B can reap 2 acres in 3 hours; in what time will they together reap 14 acres? And if the payment is half a dollar an acre, how much will each one receive?

18. A man hired a laborer, giving him one dollar a day for each day that he labored, and exacting 25 cents for each day that he was idle. At the end of 18 days he received 8 dollars; how many days did he labor and how many was he idle?

19. The sum of two numbers is 20, and three times the less is less by 15 than twice the greater part; what are the two parts?

20. The sum of three numbers is 20. The sum of the first and second is equal to the third, and half the

sum of the first and third is equal to twice the second.

21. If $\frac{3}{4}$ of a number is increased by 10 the sum is 40. What is the number?

22. If $\frac{5}{6}$ of a number be diminished 10 the remainder is 25. What is the number?

23. A can do a piece of work in $\frac{1}{3}$ of a day and B in $\frac{1}{4}$ of a day. How long will it take both to do it?

24. Three men, A, B, and C, can do a piece of work in 6 days; A can do it alone in 10 days, and B in 12 days; in how many days can C do the work?

25. A cistern of 200 gallons is emptied in 30 minutes by 3 pipes. The first pipe discharges $\frac{1}{2}$ gallon in a minute, the second $1\frac{1}{2}$ gallon; how much does the third pipe discharge per minute?

26. A can do a piece of work in 2 days, B in 3 days, C in 4 days, and D in 6 days. In what time can they all together do the work?

LESSON III.

1. Eggs are sold at the rate of 5 for 3 pence. At what rate must they have been bought in order to make this a gain of 20 per cent.?

2. A man mixed 20 gallons of ale worth 11 pence a gallon with 30 gallons of beer worth 8 pence a gallon; what is the price per gallon of the mixture? What must be the selling price per gallon in order to gain 20 per cent.?

3. A merchant sold his flour at various prices—one barrel for $9, a second for $10, a third barrel for $12, a fourth barrel for $11, a fifth barrel for $8. What was the average price per barrel?

4. What is the average of the numbers 7, 8, 12, $7\frac{1}{2}$ and $5\frac{1}{2}$?

5. If a pole 30 feet high at a given time of day casts a shadow 15 feet long, how long a shadow will be cast

at the same time by a pole 13 feet high? How high a pole will at the same time cast a shadow 20 feet long?

6. If ten oxen eat 2 tons of hay in 30 days, how many oxen will it take to eat 16 tons in 50 days? in 60 days? in 75 days?

7. If 6 men can dig a ditch of a certain depth and width 8 rods long in 4 days, how long will it take 12 men to dig a ditch of the same depth and width, 40 rods long? 72 rods long?

8. The posts of an electric telegraph by the side of a railway are placed at intervals of 60 yards; at what rate must a train move to pass 10 of these in 20 seconds?

9. The work of 54 men in 8 days is equal to that of 16 workmen in how many days?

10. If 15 workmen in 30 days finish 100 rods of a wall of certain thickness and height, how many rods can 6 workmen finish in 81 days? How many rods 60 workmen in 9 days? 9 workmen in 90 days? 12 workmen in 54 days?

11. If 12 workmen can dig 48 rods of a ditch of certain width and depth in 6 days, in how many days can 50 workmen dig 600 rods? 30 workmen 180 rods?

12. A small steam-engine of one horse-power burns in an hour 20 pounds of coal; how many hundred-weight will it require in 5 days?

13. When $100 in gold is worth $148 in currency, how much is $5 in gold worth?

14. What is the price of 240 articles at 3 pence apiece? at 8 pence apiece? What is the price of 300 articles at 5 shillings each? of 240 articles at 3 cents each? of 300 articles at $5 each?

15. A fox is 50 rods ahead of a hound, and is running 70 rods a minute; the hound is pursuing at the rate of 75 rods a minute; in how many minutes will the hound overtake the fox?

16. If an army marches 130 miles in $6\frac{1}{2}$ days, marching 9 hours a day, how far can it march in $4\frac{1}{4}$ days by marching 10 hours a day?

LESSON IV.

1. 5 is $\frac{10}{2}$. Hence to multiply a number by 5, annex 0 and divide by 2. $42 \times 5 = 420 \div 2 = 210$.

Multiply 94, 48, 63, 75, and 448 by 5.

2. $25 = \frac{100}{4}$. Hence to multiply a number by 25, annex 00 and divide by 4. $36 \times 25 = 3600 \div 4 = 900$.

Multiply 36, 72, 68, 84, and 79 by 25.

3. $125 = \frac{1000}{8}$. Hence to multiply a number by 125 we annex 000 and divide by 8.

4. Multiply 56, 72, 88, and 91 by 125.

5. 99 is $100 - 1$. Hence to multiply a number by 99 we multiply it by 100 and then subtract the number itself from the product. $67 \times 99 = 6700 - 67 = 6633$.

Multiply 34, 76, 29, and 34 by 99.

6. Multiply 45, 63, 72, and 51 by 999.

7. Multiply 32, 24, 48, and 52 by 98; also by 102.

8. What change is made in the product of 28 multiplied by 32 by adding 10 to the multiplier? What change by adding 10 to the multiplicand? What change by multiplying the multiplicand by 10? the multiplier by 10? by multiplying the multiplier and multiplicand both by 10? by multiplying the multiplicand by 10 and dividing the multiplier by 10 at the same time?

9. What change is made in the quotient of $80 \div 16$ by dividing the dividend by 5? by dividing the divisor by 4? by dividing both by 4? What change is made in the quotient by multiplying both the divisor and dividend by 4?

10. What is the square of 24?

Solution.—$24 = 20 + 4$; $(20 + 4) \times (20 + 4)$ *gives*

$20 \times 20 + 4 \times 20 + 4 \times 20 + 4 \times 4 = 28 \times 20 + 16 = 560 + 16 = 576$. *Hence, to square a number of two figures we add the number of units to the number, multiply this by the tens, and add to it the square of the units. Thus, to square 46, add 6 to 46, giving 52. Multiply 52 by 4 tens or 40, giving 2080, and to this add 36.*

11. What is the square of 13? of 14? of 15? 16? 17? 18? 19?

12. What is the square of 20? of 30? of 40? of 90?

13. Find the square of 31; of 37; of 52; of 56; 27.

In all the preceding chapters of miscellaneous questions the pupil should be required to frame several questions of each sort presented, and the teacher should add questions in each recitation.

CHAPTER XII.

ELEMENTARY LESSONS IN WRITTEN ARITHMETIC.

LESSON I.

Numeration and Notation.

1. In the preceding chapters the pupil has learned orally the numeration and notation of numbers as high as thousands. The numbers from one to one thousand form a class of three orders of numbers. The first order is units; second, tens of units; and third, hundreds of units. In the same manner there are three orders of the class *thousands:* first, *thousands;* second, *tens of thousands;* third, *hundreds of thousands.* As to *ten hundred* we gave a new name, *thousand;* so to *ten hundred thousand* it is convenient to give a new name, and thus form a third class. This we call a *million.*

2. We have also learned in a former chapter how to express numbers as high as thousands by means of the nine Arabic or Indian figures, together with the symbol of place, 0. These nine figures are called *significant figures*, and 0 is called a *cipher*, or *naught*, or *zero.* We have learned that a figure, as 7, standing by itself expresses 7 simple units; that a 0 or another figure placed to the right of it, thus, 70 or 72, makes it express seven tens, that is, seven units of the next higher order; that two 0's, or two figures, or a cipher and a significant figure, placed to the right of it, as 700, 708, 718, or 780, make it express seven units of

the next higher order, or seven hundreds; and that three significant figures or ciphers placed to the right of it cause the seven to express seven thousands.

3. *The law here is, that each additional figure to the right of a figure increases tenfold the value of the original figure.* We can now extend this law to all higher numbers. To make the figure 7 express seven tens of thousands, we annex four figures to it; to make it express seven hundreds of thousands, we annex five figures; to make it express seven millions, we annex six figures. If a number is expressed in seven figures, the first figure on the right expresses *units*, the next *tens*, the next *hundreds*, the next *thousands*, the next *tens of thousands*, and so on to *millions*. That is, moving a figure one place to the left, makes its value ten times as great as before.

Table.

Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	
			1,	3	7	4	(1)
		3	5,	6	4	1	(2)
	2	9	0,	1	6	2	(3)
4,	9	0	2,	6	0	0	(4)

(1) is read, One thousand three hundred and seventy-four.

(2) is read, Thirty-five thousand six hundred and forty-one.

(3) is read, Two hundred and ninety thousand one hundred and sixty-two.

(4) is read, Four million nine hundred and two thousand six hundred.

4. The 0 is put in the place of any order of units which is wanting in the number to be expressed. Thus five thousand and six has no tens nor hundreds. Hence it is written 5006, because in order that 5 may express five thousand, it must have three figures after it, and as we have only one significant figure to write, we therefore use two 0's.

NOTE.—When the number of tens is more than 1, we use the termination *-ty*. 74 is seven-ty-four, instead of seven-tens-and-four. Twenty, thirty, fifty, are slightly changed from twain-ty, three-ty, four-ty, five-ty. In 62,354, instead of saying six-tens-and-two thousand three hundred five-tens-and-four, we say sixty-two thousand three hundred and fifty-four.

5. EXERCISES.

Let the pupil read, or write in words, the following numbers :

1. 684, 596, 4000, 24000, 1000, 1001, 1021.
2. 50, 500, 5000, 50000, 50500, 50505.
3. 403, 430, 4308, 4764, 40764.
4. 10000, 10094, 10904, 10094.
5. 60, 604, 1604, 21604, 421640.
6. 123, 321, 312, 444, 4444, 44444, 444444.
7. 2000, 20000, 200200, 2200200, 2220202.

Express in figures the following numbers :

1. Six hundred ; six hundred and forty ; six hundred and forty-six.

2. Four hundred and four ; four hundred and fourteen ; four hundred and forty.

3. Three hundred and thirty ; three thousand ; three thousand and three hundred.

4. Six thousand; six thousand four hundred and forty; six thousand four hundred and four.

5. Nine hundred and one; one hundred and nine; one hundred and ninety.

6. Eight hundred; eight hundred thousand; eight hundred thousand four hundred and fifty; four hundred and fifty-five.

7. One thousand; one million; six million; six million six thousand and six hundred; six million six hundred and fifty-four thousand three hundred and twelve.

LESSON II.

Addition.

6. By ADDITION we unite two or more numbers in one number.

The result of the addition is called the SUM or AMOUNT.

In adding large numbers, we add the units, the tens, the hundreds, &c., separately. Two cases occur.

7. I. When the sum of each column is not more than 9.

EXAMPLE 1.—Add 641, 132, 213.

<div align="center">

Operation.

641
132
213
———
986

</div>

We write the numbers so that units are in the same column, tens in the same column, hundreds in the same column. Beginning with units, we add: 3 and 2 and 1 are 6.; 1 and 3 and 4 are 8; 2 and 1 and 6 are 9. The sum is 986.

Add the following numbers:

2. 234	2222	3333	2000	5004	4210
312	3132	2141	1423	2111	1434
421	2321	2203	1004	422	1244
22	2314	1221	4352	1241	3101
—	—	—	—	—	—

3. What is the sum of 3241, 1320, 2103, 1025?

4. Add together 2345, 1201, 4321, 1102.

5. Add together 1234, 4321, 2222, 2222.

6. A man bought a house for 2300 dollars, a horse for 250 dollars, and a wagon for 120 dollars; what did he pay for them all?

8. II. When the amount in any column is more than 9.

EXAMPLE.—If a merchant has 1276 dollars in bank bills, 479 dollars in gold, 784 dollars in silver, and 4 dollars in cents, how much money has he in all?

Operation.

$$\begin{array}{r} 1276 \\ 479 \\ 784 \\ 4 \\ \hline 2543 \end{array}$$

We write, as before, units under units, tens under tens, &c. 4, 4, 9 and 6 units are 23 units, or 2 tens and 3 units. We write 3 units under units, and add the 2 tens with the column of tens, thus: 2, 8, 7 and 7 are 24 tens, or 2 hundreds and 4 tens. Writing 4 tens under tens, we add 2 hundreds with the column of hundreds: 2, 7, 4 and 2 are 15 (hundred). We write 5 under hundreds, and add 1 with thousands: 1 and 1 are 2. The sum is 2543. The merchant has 2543 dollars.

EXAMPLES.

1.	2704	4623	66	704	358
	48502	598	749	8506	4854
	79508	1942	8515	9402	987
	706	749	15652	56708	74749

2.	490	777	333	888	888	123
	244	444	330	22	99	221
	130	999	444	3	11	456
	355	442	440	44	2	654
	240	111	505	555	33	789
	166	222	606	6	590	978

3.	8649	9876	5436	6	6789
	5842	5432	4563	66	9876
	3876	1234	3654	6836	7890
	9204	5678	5463	6956	9876
	7587	1875	6345	6796	1234

4. How many are 902010 + 178393 + 643421 + 366780 + 703020 + 683463 + 604050 + 221894?

5. What is the sum of forty-four thousand three hundred and twelve; six thousand eight hundred and twenty-seven; nine hundred and fifty-two; thirty thousand and thirty?

6. Find the sum of ninety; eighty thousand seven hundred and one; two thousand seven hundred and fifty-four; one thousand five hundred and seventy; forty-two thousand and four.

7. In a nursery of trees there are 1250 apple trees, 2050 pear trees, 3500 peach trees, 1750 plum trees, 850 apricot trees; how many trees in all?

LESSON III.

Subtraction.

9. By SUBTRACTION we find the difference of two numbers, by taking the less number from the greater.

The greater number is the MINUEND.

The less number is the SUBTRAHEND.

The difference is also called the REMAINDER.

To subtract, we place the subtrahend under the minuend; units under units, tens under tens, &c. Two cases may arise.

10. I. Each figure in the subtrahend may be less than the figure above it in the minuend.

EXAMPLE 1.—From 576 subtract 324.

Operation.

576
324
———
252

Subtracting each figure in the subtrahend from the figure above it, beginning with units, we have 4 from 6, 2; 2 from 7, 5; 3 from 5, 2. The remainder is 252.

PROOF.—Add the units, tens, &c. of the subtrahend and remainder—2 and 4 are 6; 5 and 2 are 7; 2 and 3 are 5—and we have the minuend, 576.

EXAMPLES.

2. From 540 take 200.   From 860 take 300.

3.	800	5000	60000	9000	790	80000
	200	2000	20000	3000	400	3000

4.	8760	9630	6320	5670	4810	5945
	3000	7000	2000	3000	1000	3000

| 5. | 937 | 877 | 697 | 867 | 633 | 857 | 6848 |
| | 122 | 352 | 245 | 423 | 121 | 307 | 3007 |

| 6. | 85674 | 4789 | 62476 | 34897 | 8975 |
| | 1352 | 3254 | 231 | 12244 | 75 |

11. II. One or more figures in the subtrahend may be greater than the figures above them in the minuend.

EXAMPLE.—From 468 take 349.

Operation.

$$\overset{10}{4}68$$
$$349$$
_____
$$119$$

Write the less number under the greater. As we cannot take 9 units from 8 units, we mentally add 10 to 8, and then say, 9 from 18 leaves 9, which we write down. Since we added 10 to the minuend, we add 1 ten,to the 4 tens of the subtrahend, and say 5 from 6 leaves 1, which we write down. Finally, 3 hundred from four hundred leaves 1 hundred. 119 is the difference between the numbers, because we have taken all the parts of the less from the greater, and we have balanced the ten added to the greater by adding ten to the less.

EXAMPLE.—From 9434 take 3537.

Operation.

$$\overset{10\ 10\ 10}{9}434$$
$$3537$$
_____
$$5897$$

We say, 7 from 14, 7; 4 from 13, 9; 6 from 14, 8; 4 from 9, 5. The remainder is 5897.

If there are fewer figures in the lower line than in the upper, proceed as if 0's were written on the left of the lower number until the number of figures is the same in both.

EXAMPLES.

1.
$$\overset{10}{6}70 \quad 860 \quad 490 \quad 540 \quad 650 \quad 880 \quad 732$$
$$\ \ 25 \quad \ \ 43 \quad \ \ 38 \quad \ \ 26 \quad \ \ 39 \quad \ \ 32 \quad \ \ 13$$

2.
$$\overset{10}{7}860 \quad 1780 \quad 2690 \quad \overset{\cdot 10}{8}06 \quad 709 \quad 807$$
$$\ \ \ 243 \quad \ \ 437 \quad \ \ 272 \quad \ \ 243 \quad 333 \quad 718$$

3.
$$\overset{10}{6}072 \quad 7043 \quad 4068 \quad 5037 \quad 8403$$
$$\ \ 2342 \quad 2520 \quad 3660 \quad 1717 \quad 2390$$

4.
$$\overset{10\ \ 10}{4}372 \quad 782 \quad 965 \quad 585 \quad 7184 \quad 4360 \quad \overset{10\ 10\ 10}{7}343$$
$$\ \ 2657 \quad 355 \quad 428 \quad 247 \quad 6526 \quad 3608 \quad 6740$$
$$\ \ 1715$$

5.
$$2814 \quad 375 \quad \overset{10\ 10\ 10}{5}006 \quad 4176 \quad 7583 \quad 8064$$
$$1033 \quad 185 \quad 2818 \quad 3648 \quad 3065 \quad 3975$$

6.
$$6004 \quad 60010 \quad 10000 \quad 8200102 \quad 815240$$
$$3716 \quad \ \ 528 \quad \ \ 3196 \quad 4866769 \quad 379468$$

7.
$$7802008 \quad 10000000 \quad 951315 \quad 9555555$$
$$3357564 \quad \quad \quad \ \ 1 \quad 947688 \quad 2888889$$

8. How much smaller is 38567 than 60600?

9. How much is 4009000 less 5678?

10. How much greater is 700000 than 599328?

11. Take from 43518 the number 7253, and from the remainder 7253, and so on, and give the remainder after the sixth subtraction.

12. How much is 3820 + 16488 + 347 + 5688 + 2433 − 31210?

13. A man owns a farm which cost him 5500 dollars; he has paid 2875 dollars; how much more has he to pay.

14. A merchant sold for 2825 dollars goods which cost 2138 dollars; how much did he gain?

15. A man bought a house and lot for 2750 dollars, and sold it for 3125 dollars; how much did he gain?

16. How old would a man be in 1868 who was born in 1839?

LESSON IV.

Multiplication.

12. If a man can earn 8 dollars in one week, in 4 weeks he can earn 4 times 8 dollars, or 32 dollars.

This is multiplication: 8, the number to be repeated, is called the MULTIPLICAND: 4, which shows how many times the multiplicand is to be taken, is called the MULTIPLIER, and 32, the result of the multiplication, is the PRODUCT.

In multiplying large numbers, there are two cases:

13. I. When the multiplier is a single figure.

EXAMPLE 1.—Multiply 467 by 5.

Operation.

$$467$$
$$5$$
$$\overline{}$$
$$2335$$

8

We write the multiplier under the multiplicand, units under units, and say, 5 times 7 units are 35 units, or three tens and 5 units. We write 5 units below, and reserve 3 tens. 5 times 6 tens are 30 tens, which with 3 tens make 33 tens, or 3 hundreds and 3 tens. Writing 3 tens in tens' place, we reserve 3 hundred. 5 times 4 hundred are 20 hundred, which with 3 hundred make 23 hundred. This we write down, and have for the product, 2335.

2.	356	825	6745	4567	Multiplicand.
	4	5	3	6	Multiplier.
	———	———	————	————	
	1424			27402	Product.

3.	8474	9876	5432	12345
	2	7	8	9
	————	————	————	—————

4. Multiply 4825 by 2; by 3; by 4; by 5; by 6.

5. Multiply 3775 by 5; by 6; by 7; by 8; by 9.

6. Multiply 5709 by 4; by 5; by 6; by 7; by 8; by 9.

7. If one horse costs 275 dollars, what is the cost of 8 horses?

8. If one house is worth 3275 dollars, what is the value of 7 similar houses?

9. If the multiplicand is 3917 and the multiplier is 6, what is the product?

10. Multiply the sum of 1847 and 2043 by 8.

11. Multiply the difference between 3194 and 1536 by 9.

14. II. When the multiplier contains more than one figure.

EXAMPLE 1.—Multiply 843 by 26.

Operation.

843
26
———
5058
1686
———
21918

Write the multiplier under the multiplicand, units under units, tens under tens, &c.

Multiply by 6, the units figure, as in Case I.   In multiplying by 2 tens, place the first figure of this product under 2, the multiplier.   For, 2 tens times 3 are 6 tens; therefore, the 6 should be in the second, or tens', place.

Add the two partial products, and find the entire product, 21918.

EXAMPLE 2. Multiply 4357 by 345.

Operation.

4357
345
———
21785
17428
13071
———
1503165

$5 \times 7 = 35$; write 5 and " carry " 3.
$5 \times 5 = 25, + 3 = 28$; write 8.
$5 \times 3 = 15, + 2 = 17$; write 7.
$5 \times 4 = 20, + 1 = 21$; write 21.

$4 \times 7 = 28$; write 8 under 4.
$4 \times 5 = 20, + 2 = 22$; write 2.
$4 \times 3 = 12, + 2 = 14$; write 4.
$4 \times 4 = 16, + 1 = 17$; write 17.

$3 \times 7 = 21$; write 1 under 3.
$3 \times 5 = 15, + 2 = 17$; write 7.
$3 \times 3 = 9, + 1 = 10$; write 0.
$3 \times 4 = 12, + 1 = 13$; write 13.

The sum of the partial products is the entire product, 1,503,165.

3. 3458	8431	1357	2465	5438
24	35	54	89	146

4. 5417	5104	3547	. 1596	2705
123	456 .	258	357	403

5. Multiply 1047 by 284; by 207; by 615.
6. Multiply 3579 by 123; by 612; by 246.
7. Multiply 4680 by 340; by 360; by 801.
8. Multiply 6143 by 200; by 4200.

```
      6143              6143
       200              4200
    --------         --------
    1228600          1228600
                       24572
                     --------
                     25800600
```

9. Multiply 3434 by 300; by 5300.
10. Multiply 1234 by 600; by 1054.
11. 1417×34; 6417×27; 6946×123.
12. $340 \times 20 \times 16$; $853 \times 300 \times 4$; 803×101.

NOTE.—The teacher will readily supply additional examples for practice under each rule. They can be written on the blackboard, and copied by the pupils.

LESSON V.

Division.

15. By DIVISION we find how many times one number is contained in another larger number.

The DIVIDEND is the number to be divided.

The DIVISOR is the number by which we divide.

The number of times the divisor is contained in the dividend is the QUOTIENT.

The REMAINDER is what is sometimes left after dividing.

16. There are two methods of performing division.

Ex. 1. How many times is 4 contained in 948?

1st Operation.

$$
\begin{array}{r}
4)948(237 \\
8 \\
\hline
14 \\
12 \\
\hline
28 \\
28 \\
\hline
0
\end{array}
$$

We begin at the left of the dividend to divide. 4 is contained in 9, 2 times. Place 2 at the right of the dividend, as the first figure of the quotient. 2 times 4 are 8, which we set under 9, and subtract. To the remainder, 1, we bring down 4, the next figure of the dividend, making 14. 4 in 14, 3 times. Place 3 in the quotient and multiply the divisor, and proceed as before, till all the figures are brought down and divided. The entire quotient is 237.

2d Operation.

4)948
———
237

In this method, we do in the mind a part of what we wrote out in the 1st operation, and write the quotient under the dividend.

4 is contained in 9, 2 times and 1 over. We place 2 under 9 as the first figure of the quotient, and *imagine* the remainder 1 placed before 4, the next figure of the dividend, making 14. 4 in 14, 3 times and 2 over. We place 3 under 4, and imagine 2 set before 8, making 28. 4 in 28, 7 times. The entire quotient is 237, as before.

17. This second method is sometimes called SHORT DIVISION, and is used when the divisor is not more than 12.

The first method is called LONG DIVISION, and is used when the divisor is more than 12.

2. 9)1251	9)1251(139	7)28345(4049
———	9	28
139	—	—
	35	34
	27	28
	—	—
	81	65
3. 7)28345	81	63
———	—	—
4049 2 Rem.	0	2 Rem.

Work the following examples by Short Division:

4. 4)57432   6)31740   8)17344   9)5562
   ———   ———   ———   ———
                                            618

5. 5)74035   4)67434   6)10746   8)84755

                                        10594—3 Rem.

Prove all the examples in Division, by multiplying the quotient and divisor together and adding the remainder, if there is any, to the product. The result must equal the dividend.

Work the following by both methods:

6. 6)74789   7)78743   8)97145   9)23474

EXAMPLE 7.—Divide 9274 by 35. Here, of course, we use Long Division.

Operation.   Proof.

35)9274(264   264
   70   35
   ___   ___
   227   1320
   210   792
   ___   34
   174   ___
   140   9274
   ___

34 Rem.

How many times is 35 contained in 92? 3 in 9, 3 times; but, multiplying 35 by 3, we have 105, which is more than 92; we therefore take a quotient figure one less, 2, and find 2 times 35=70, which is less than 92. So, to find the second quotient figure: 3 in 22, 7 times. Finding 7 too large, we try 6.

8. Divide 3754 by 24. Quo. 156. Rem. 10.
9. Divide 17473 by 56. Quo. 312. Rem. 1.

10. Divide 3746 by 36. How can you know whether your quotient is correct?

11. Divide 2756 by 48.   Operation.
The divisor, 48, is nearly 50. To   48)2756(57
find the quotient figures we say, 5 in   240
27, 5 times; multiplying the divisor   ———
by 5, we have 240, less than 275;   356
subtracting, we find 35 for remain-   336
der, a number less than the divisor.   ———
So, 5 in 35, 7 times, &c.   20 Rem.

12. Divide 9437 by 27; by 31; by 35; by 14.
13. Divide 10743 by 45; by 450; by 412.
14. Divide 12345 by 103; by 301; by 345.
15. Since there are 365 days in one year, how many years are there in 2000 days?
16. If I pay 9744 dollars for 112 acres of land, how much is it an acre?
17. How many cows can I buy for 581 dollars, if each cow is worth 83 dollars?
18. If the dividend is 7839, and the divisor is 214, what are the quotient and remainder?
19. Add together 359, 895, and 1054; subtract 1343 from the sum; multiply the remainder by 8, and divide the product by 4. What is the quotient?

Note.—The same example expressed by means of signs is $359 + 895 + 1054 - 1343 \times 8 \div 4 = ?$

20. $874 + 917 - 1056 \times 6 \div 5 = ?$ Quotient 882.
21. $3846 + 1904 + 3841 - 3489 \times 100 \div 75 = ?$
22. $804 \times 200 \div 25 + 3568 - 10000 = ?$

$4 \times 8 =$ $3 \times 8 = 24$

6×8 $5 \times 8 = 40$

8×8 $7 \times 8 = 56$

          $9 \times 8 =$

$$x + \frac{y}{2} - 3 = \frac{x}{2}$$

$$x + \frac{y}{2} = 20$$

$$x + u + y = 600$$

7 m.

7. 1 hour

$2 =$

$1\frac{3}{3}$
$\frac{1}{3}$

$\frac{9}{4} \times \frac{9}{3}$

$8 \div \frac{4}{5}$

$1\frac{2}{3}$

$\frac{8 \times 5}{4} = \frac{40}{4} =$

$1\frac{1}{2} = 1\frac{1}{2}$

$= 1$

$= 3$

$= 5.$

$= 4$

$9 \times 9 =$ $10 \times 9 = 9$

$11 \times 9 = 99$