METHODS OF LOCATION

FOR

RAILWAY ENGINEERS,

BY

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CIVIL ENGINEER.

METHODS OF LOCATION

OR MODES OF

DESCRIBING AND ADJUSTING

RAILWAY CURVES AND TANGENTS,

AS PRACTISED BY THE

ENGINEERS OF PENNSYLVANIA,

REVISED AND EXTENDED

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PREFACE.

In submitting this work to the public, I do not wish to claim for myself its exclusive authorship. A few of the first solutions were the work of my comrades on the Pennsylvania Railway; these falling into my hands suggested the idea of a complete series of Geometric solutions applicable to all cases that might occur.

In such a work I considered it of importance to dispense with all difficult calculations, and even with tabular statements, which cannot be committed to memory. In this I have happily succeeded; there is nothing in the following pages which may not be remembered by an assistant after a short practice, and executed in the field *even if the book be left at home.*

There are a few instances, however, in which a table of chords may facilitate operations, and one has therefore been placed upon the last page for the use of those who prefer it.

EXPLANATIONS.

1. SINCE all the curves described in this work are circular, the words curve and circle will be used indiscriminately.
2. All measurements in this work are referred to some chord of convenient length as a unit, which may be either the common four pole chain of 100 links, or one of 100 feet, and for brevity sake the word chain will be used to designate such chord.
3. The angle subtended by the above chord at the centre of the circle is called the degree of curvature, or simply the curvature.
4. The letters m and n are used to express degrees of curvature, and when both are used m is the greatest, that is, it belongs to the smallest circle.
5. A central angle is that which a chord subtends at the centre of the circle.
6. A circumferential angle is that which a chord subtends at any point in the circumference.
7. A tangential angle is the smallest angle made by a chord at its extremity, with a tangent to the curve at that extremity.
8. A compound curve is composed of two curves of different radii turning in the same direction, having a common tangent at their point of meeting.
9. This point of meeting is called the point of compound curvature, or simply P. C. C.
10. A reversed curve is composed of two curves turning in opposite directions and having a common tangent at their point of meeting.

11. This point is called P. R. C., or point of reversed curvature.

12. A differential curve is one whose radius is equal to the difference between the radii of any two curves to which it is applied.

13. An integral curve is one whose radius equals the sum of the radii of two other curves.

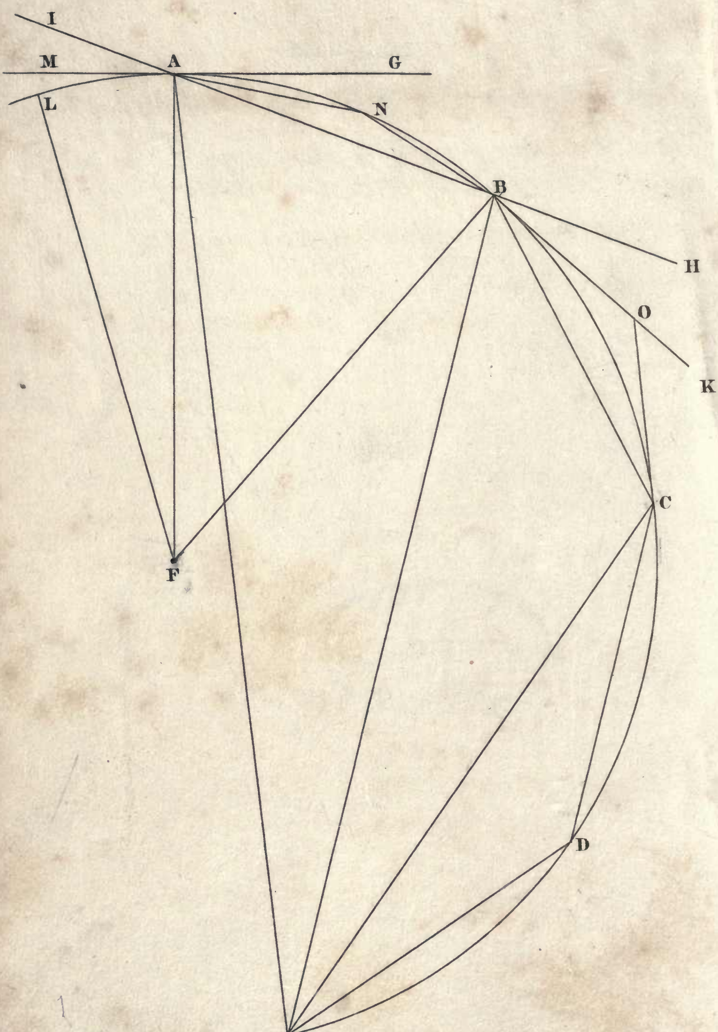
14. Equivalent arcs or curves are such as subtend equal central angles.

15. Corresponding points in different circles are any points, where the tangents and of course the radii are parallel.

16. The terms origin and termination are used in reference to the course of location. The termination of a tangent being the point where a curve is commenced, and the origin of the next tangent the point where the curve terminates.

17. The origin is also called the point of curve, or point of tangent, or simply P. C. or P. T.

Fig. 1



PRELIMINARY PROPOSITIONS.

Condensed from Euclid, Book Third.

1. The Angle AFB (Fig. 1st,) subtended by any chord AB at the centre, is double the angle AEB at any part of the circumference on the same side of the chord.

2. Equal chords AB, BC, CD, subtend equal angles whether at the centre or circumference.

3. The angle BAG formed by any chord AB with a tangent at either extremity, is equal to half the angle AFB at the centre, or to the angle AEB at the circumference.

4. The exterior angle HBC formed by two equal chords AB, BC, is equal to the central angle AFB, or CFB, or double the tangential angle GAB.

5. The exterior angle LAI of two unequal chords, LA, AB, is equal to half the sum of their central angles LFB, or to the sum of their tangential angles LAM + MAI or GAB.

6. The exterior angle of any two chords AN, NB, is equal to one half the central angle of AB, or its exterior angle with its equal BC.

7. The exterior angle KOC, of any two tangents CO, BK, is equal to the central angle BFC of the chord BC, which joins their point of contact.

The following Propositions, which are likewise used in this work, although not strictly correct, are sufficiently so for all purposes of Location.

8. The central, circumferential, and tangential angles of chords of unequal lengths are directly as the lengths.

9. The radii of circles are directly as their degrees of curvature.

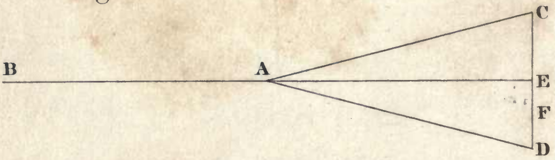
10. The radius of a circle is half the circumference divided by 3.1416.

11. If the chord of one degree be taken as a unit, the circumference may be considered equal to 360.

Hence, the radius is equal to $\frac{180}{3.1416} = 57.30$ and by proposition 9 the radius of any other circle is $\frac{57.3}{m}$

NOTE.—The 8th, 9th and 11th propositions are true to the second place of decimals, so long as m is not greater than 10°, which is double what is required in ordinary cases.

Fig. 2



PRELIMINARY EXERCISES.

In the Use of the Transit.

The Transit is an instrument invented and manufactured by W J. Young of Philadelphia. It is in many respects more convenient than the Goniometer or Theodolite, and being the instrument to which I have been most accustomed, I have adapted the phraseology of this treatise to its use. There is, however, no difficulty in solving all the propositions in this series with either of the other instruments above mentioned.

It is not my purpose to give a description of the Transit instrument, but, supposing the student to have one before him, and to be acquainted with the uses of its various parts, I shall proceed to describe some of its most common applications.

PROPOSITION I.

To adjust the vertical hair of the Telescope so that the Lines of sight forward and backwards shall be parts of the same Straight Line.

Choose a piece of perfectly level ground, from 500 to 800 feet long, and clear of all obstruction to the sight, set the instrument in the middle as at A, Fig. 2d, level and clamp it, and with the tangent screws bring the sight to bear upon a chain-pin or any other suitable object which an assistant must hold at B.

Then reverse the *Telescope* on its axis and set up another pin in the opposite direction, and at the same distance as B is from A: if the instrument be out of adjustment this will not fall in the line AB produced but on one side of it as at C.

Now unscrew the clamp and without touching the Telescope reverse the *Transit* on its axis, and fix the sight upon B as before, and screw up the clamp. Again reverse the *Telescope* and set up a third pin, which will now fall upon the point D precisely, as far to the right of AE, as C is to the left. Divide accurately the distance between C and D, and set up a fourth pin at E ; B, A, and E will then be in the same straight line. Now remove the pin from C, and set it up at F, precisely in the middle of E D, and with the adjusting pin remove the vertical hair until it coincides with F, then with the tangent screws fix the sight upon E and reverse the *Telescope.* If the operation has been carefully performed, the sight will strike the chain-pin at B and the adjustment is effected.

It generally happens, however, that a second slight movement of the hair is necessary to perfect the operation, which should be tested by several reversions on the axis of both Transit and Telescope, until the coincidence of the hair with B and E is fully established.

PROPOSITION II.

To discover whether the Telescope revolves truly in the Meridian.

After completing the adjustment by the last proposition, choose a steeple or any other lofty object upon whose top a steady and accurate sight can be obtained ; set the instrument as near to its base as possible, and after leveling and clamping it, fix the sight upon the top of the object and turn the head of the Telescope in the opposite direction, so as to bear upon the ground at some convenient distance from the instrument, and set up a pin.

Then reverse the *Transit* on its axis and take sight as before to the top of the steeple : again turn the head of the Telescope towards the pin just set up, and if the vertical hair coincides with it, the instrument is sound, but if not, half the distance between them will be the error. As this inaccuracy is always the result of accident, a blow, or a fall, there is no method of removing it in the field; when discovered it should be sent to the maker for repairs.

Fig. 3

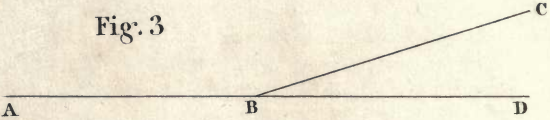


Fig. 4

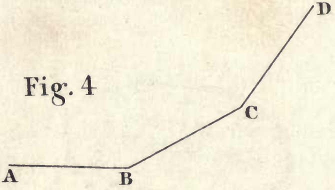
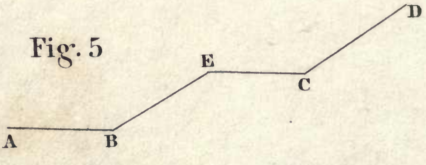


Fig. 5



PROPOSITION III.

To Measure the Angle between any two Lines or Deflection from one Line to another.

In performing this operation, it is necessary to bear in mind the course of survey : thus, if the survey proceeds from A to B, Fig. 3d, and afterwards from B to C, then the angle required will be CBD, but if the course of survey is from D to B, and afterwards from B to C, then the angle required is ABC. To measure the angle proceed as follows : place the instrument over B, and set the index to Zero, then take a back sight from B to A, and *reverse* the *Telescope:* turn the rack until the sight bears upon C, and the index will then show the angle DBC required.

Case 2d.—*When the intersection of two lines* AB *and* CD, *Fig.* 4*th, are inaccessible.* Place the instrument over B, set the index to Zero, and take a back sight to A, then *reverse the Telescope*, turn the rack and take sight upon C ; screw the rack fast, and the instrument to C ; take a back sight from C to B : again *reverse the Telescope* and turn the rack till the sight falls upon D. The index will then show the required angle.

Case 3d.—*When the ground is so encumbered that no part of* CD *can be seen from* AB. Take a point E, Fig. 5th, from which both lines may be seen, and deflect from AB to BE in the same manner, as from AB to BC in the last case, then remove to E, and continue the process by deflecting from BE to EC, and so on, until the instrument is brought into the position CD, the index will then show the angle as before.

PROPOSITION III.

To Measure the Angle between any two Lines or Deflection from one Line to another.

In performing this operation, it is necessary to bear in mind the course of survey: thus, if the survey proceeds from A to B, Fig. 45, and afterwards from B to C, then the angle required will be CBD; but if the course of survey is from D to B, and afterwards from B to C, then the angle required is ABC. To measure the angle proceed as follows: place the instrument over B, and set the index to Zero, then take a back sight from B to A, and reverse the *Telescope*; turn the rack until the sight bears upon C, and the index will then show the angle DBC required.

Case 2d.—*When the intersection of two lines AB and CD, Fig. 46, is inaccessible.* Place the instrument over B, set the index to Zero, and take a back sight to A, then reverse the *Telescope*, turn the rack and take sight upon C; screw the rack fast, and the instrument to C; take a back sight from C to B; again reverse the *Telescope* and turn the rack till the sight falls upon D. The index will then show the required angle.

Case 3d.—*When the ground is so encumbered that no part of CD can be seen from AB.* Take a point E, Fig. 46, from which both lines may be seen, and deflect from AB to BE in the same manner, as from AB to BC in the last case, then remove to E, and continue the process by deflecting from BE to EC, and so on, until the instrument is brought into the position CD, the index will then show the angle as before.

Fig: 6

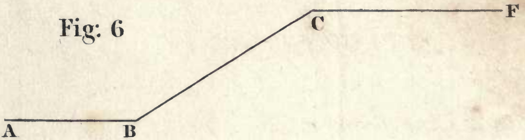
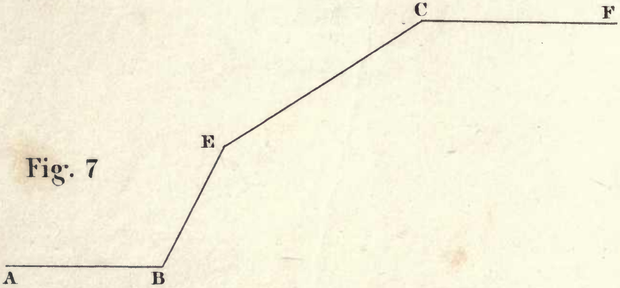


Fig. 7

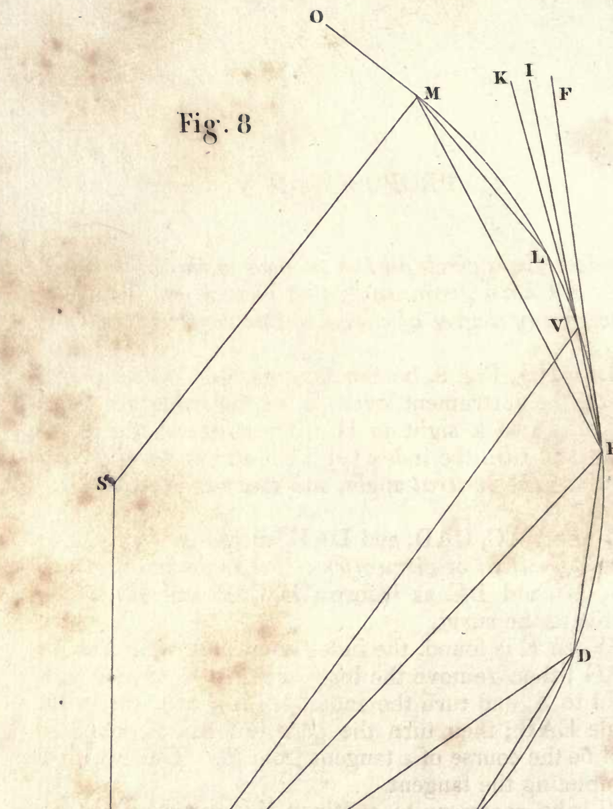


PROPOSITION IV.

To run a Line from a given Point, parallel to a given line.

LET C, Fig. 6th and 7th, be the point, and AB the given line, set the instrument at B and proceed as directed in the second and third cases of the last proposition until arriving at C, then turn the index back to Zero and the glass will then be in the line CF parallel to AB.

Fig. 8



PROPOSITION V.

To describe a circle on the ground with the Transit and Chain from any point in u given Tangent with any degree of curvature or central angle.

LET HG, Fig. 8, be the tangent, and A the point. Place the instrument over A, set the index to Zero, and take a back sight to H. Then reverse the Telescope and turn the index till it shows the *tangential*, i. e. *half the central* angle, and measure AB with the chain.

Make BAC, CAD, and DAE, successively, equal to the *tangential* or *circumferential* angle and measure BC, CD, and DE as before ; B, C, D and E will be points in the curve.

When E is found, the index shows the whole angle EAG ; then remove the instrument to E, take a back sight to A, and turn the index till it shows twice the angle EAG ; then turn the glass towards F, and E F will be the course of a tangent from E. This is called completing the tangent.

If it be necessary to continue the curve, repeat the above process by adding the tangential angle successively, and measuring EL, LM, &c. as before. To complete the tangent at M, after taking a back sight from M to E, it is necessary to add only the tangential angle FEM to what is already shown by the index, which will then show the whole central angle ASM, or exterior angle of the tangents HA and MO. But if the proper course of the tangents require it to be drawn from a point between E and L, ascertain by Proposition III, page 11, the angle it would make with EF, and make VEF equal to half that angle, and take EV : EL :: VEF : LEF, then make IVK equal to VEF and VK is the course of the tangent required.

If the angle VEF were known, that is, if the whole exterior angle of VK with HA were known, the point V may be fixed from A, by making VAE equal to VEF, i. e. VAG equal to half the exterior angle of VK with AG, and measuring EV as before.

Intermediate points *a*, *b*, *c*, must be found by calculating the ordinates by Proposition XV, page 39, or they may be found by making the angles proportionate to the distances measured on AB, and setting off *a*. *b*. *c*. &c. at right angles, but the ordinates are best suited to an unpractised hand.

The demonstration of the above method is evident from the preliminary propositions.

Note. Since the chain EL whose central angle is *m* contains 100 links, each link of EV will have a central angle of $\frac{m}{100}$.

The index should, therefore, be divided into hundredths instead of minutes. More perplexity will be avoided by this simple contrivance than can well be imagined by any one who has not made the experiment.

Fig. 9

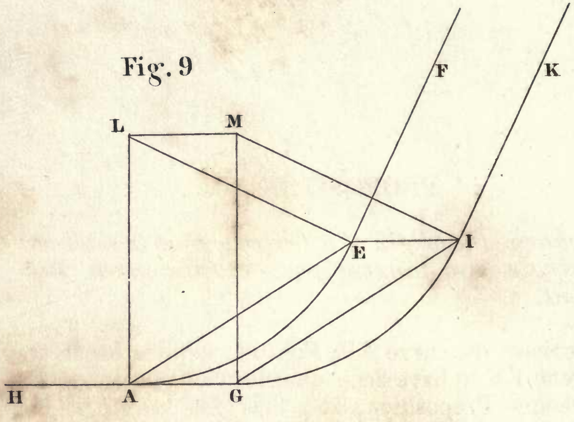
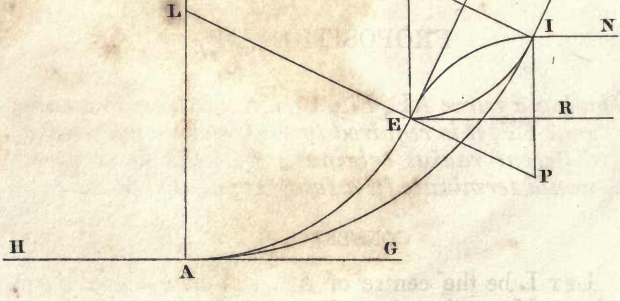


Fig. 10



PROPOSITION VI.

To change the origin of a Curve so that it shall terminate in a tangent, parallel to a given Tangent.

SUPPOSE the curve AE, Fig. 9th, terminating in the tangent EF to have been described as directed in the preceding Proposition, and that the nature of the ground requires that it should terminate in IK parallel to EF.

Measure the distance EI on the line parallel to AG, and make AG equal to EI, G will be the origin of a curve, similar to AE, that will terminate in IK at the point I.

The parallelism of all the lines drawn from A and E, with similar lines from G and I, sufficiently show the correctness of this method without a formal demonstration.

PROPOSITION VII.

Having a curve AE, *Fig.* 10, *terminating in a tangent* EF, *it is required to find where a curve of a different radius originating in the same point would terminate in a tangent parallel to* EF.

CONSTRUCTION.

LET L be the centre of AE, and on AL produced, take A M equal to the radius of the other circle, draw

ES parallel and equal to LM, and from S with the radius SE describe the differential circle EI, join MS and produce it till it cuts EI in I, then will I be the terminating point required.

For ES being equal and parallel to LM, MS must be equal and parallel to LE; hence, MI is equal to MA, and the angles LMS and ESI are equal to ALE, and therefore, the tangent EF is parallel to IK.

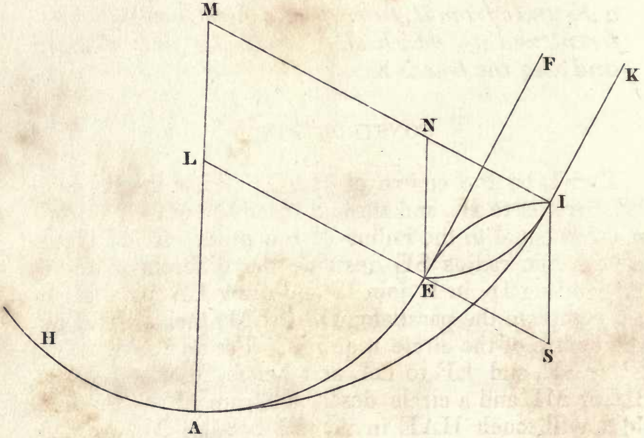
Hence, the field operation is evident. The instrument must be set over E, and the Telescope brought into the position ER parallel to AG. From ER as a tangent describe the differential curve EI, and make it equivalent to ALE, and I will be the point sought.

By completing the parallelogram SEPI, it will be perceived that EI may be described in the opposite direction, by starting off the tangent EF, instead of ER, and also if AI were given, and E required, IE might be described either from IK or IN the parallel to AG.

The degree of curvature for EI is found thus: $a = 57.30$ m and $n =$ curvatures of AE and AI; then $\frac{a}{m} =$ Radius of AE and $\frac{a}{n}$ Radius of AI, and their difference $= \frac{am - an}{mn}$ and a divided by this result $= \frac{mn}{m - n}$; that is, multiply the curvatures together, and divide by their difference.

Note. When the product of the two curvatures is very great and their difference small, a differential curvature will result which is too large to be used with the chord of 100 feet without occasioning a sensible error in the result the proper method then is, to take smaller chords of 10 or 20 feet; reducing the curvature in a like proportion. Thus, if m be 5 and n be 6 $\frac{mn}{m - n}$ will be 30°; then instead of a chord of 100 feet, take 20 feet and a central angle of 6°.

Fig. 11



PROPOSITION VIII.

Having a Curve HAE, (*Fig.* 11,) *and a line* NK *at a distance from it, to draw another Curve of a different radius, which shall touch the first Curve, and also the line* NK.

CONSTRUCTION.

Let L be the centre of HAE. Draw the tangent EF parallel to IK, and through L and E draw LS, and make it equal to the radius of the other circle: from S with the radius SE describe the differential circle EI, cutting IK in I: join IS and draw LA parallel to it; complete the parallelogram ISLM; then will M be the centre of the circle required. For MS is equal to SI or SE, and LE to LA, and hence, MA is equal to LS or MI, and a circle described from M with radius MA will touch HAE in A, and because MI and LS are parallel, and EF and IK are parallel, the angle MIK is equal to LEF, which is a right angle, and therefore the circle AI touches IK in I.

APPLICATION.

It is evident from the mathematical solution, that the curve HAE must be continued until its tangent becomes parallel to IK, then from EF as a tangent the differential curve EI must be traced as directed in the preceding proposition until it intersects IK in I, the central angle ESI must then be ascertained from the index, (after completing the tangent as in Proposition V,) and the curve A E made equivalent to it; A will then be the P. C. C. sought, from which if a curve be described with the proper curvature, it will touch IK in the point I.

It is also evident, that if AI were given, and AE required that the centre S would fall upon N, and that IE must be described from the tangent IK in the opposite direction till it intersects EF.

Note. The observations in the note attached to the preceding Proposition will apply to this, as well as several other succeeding Propositions.

It often happens, that the continuation of HAE to the tangent EF is rendered difficult or impracticable by the roughness of the ground; recourse must then be had to the succeeding Proposition.

Fig. 12

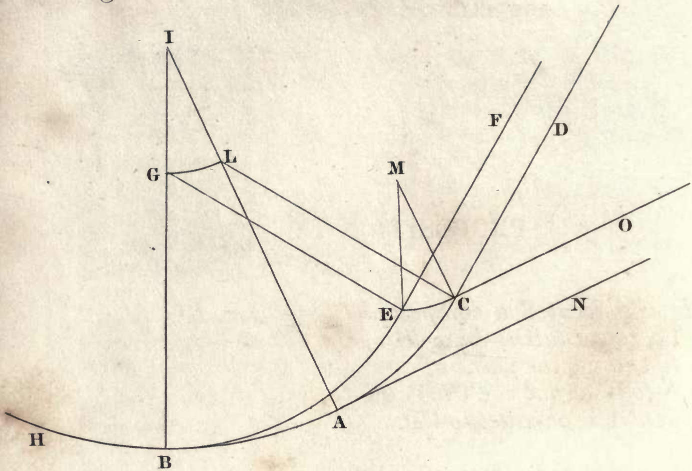
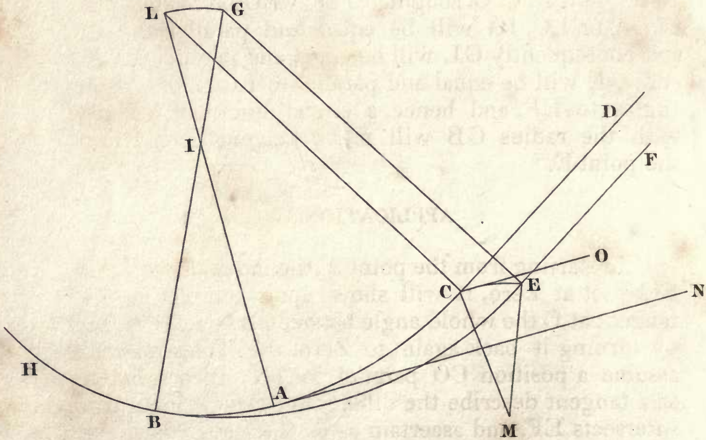


Fig 13



PROPOSITION IX.

Having located a compound Curve HA, AC, *Fig.* 12, *terminating in a Tangent* CD, *it is required to change the point of compound curvature from* A *to* B *so that the Curve will terminate in a Tangent* EF *parallel to* CD.

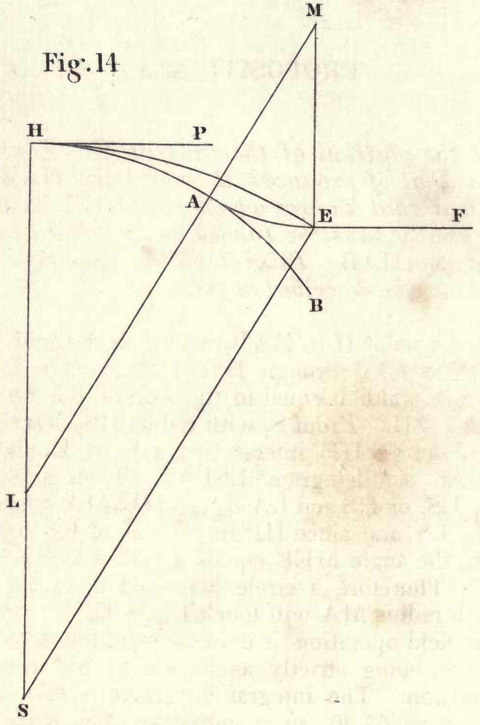
LET I be the centre of HBA, and L the centre of AC, and draw ILA, then IL is the difference of the radii of the two curves: make CM equal and parallel to IL, and describe the differential curve CE, cutting EF in E. Draw IB parallel to ME, then will B be the new P. C. C. sought. For if BG be made equal to LA or LC, IG will be equal and parallel to ME, and consequently GL will be equal and parallel to CE, and GE will be equal and parallel to LC, and at right angles to EF, and hence, a circle described from G with the radius GB will make tangent upon EF at the point E.

APPLICATION.

If, in starting from the point A, the index of the Transit be set at Zero, it will show upon completing the tangent at C the whole angle between AN and CD, and by turning it back again to Zero, the Telescope will assume a position CO parallel to AN: then from CO as a tangent describe the differential curve CE, until it intersects EF, and ascertain as in the last Proposition, the central angle of EC which is equivalent to AB, and consequently the point B may be readily obtained by setting the transit over A, and retracing a portion of the curve equivalent to EC.

A second case of this Proposition occurs, when the radius of AC, Fig. 13, is greater than that of HA, the effect of which is to throw the point M on the opposite side of EC, and consequently the curve must be turned in the opposite direction in passing from C to E.

Fig. 14



PROPOSITION X.

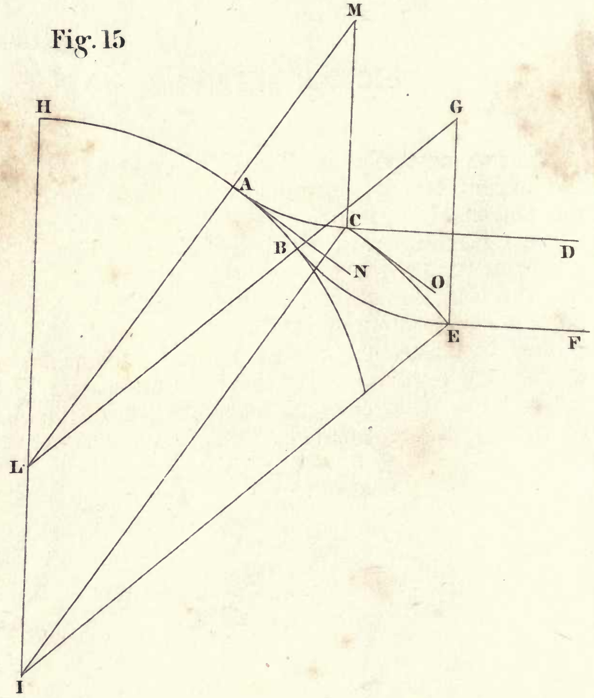
When the position of the tangent EF, *Fig.* 14, *is such that if produced it would cut* HAB, *it is evident that a curve which shall touch them both externally, must be turned in the opposite direction from* HAB. *This is termed a Reversed Curve, and may be described as follows.*

Take a point H in HAB, where the tangent HP is parallel to EF, through H and the centre L draw HLS and make it equal to the sum of the two radii HL and MI. From S, with radius HS, describe the integral curve HE intersecting EF in E, and complete the parallelogram LSEM. Then since ML equals ES, or HS and LA equals HL, MA must equal ME or LS, and since HP is parallel to EF, and ME to HL, the angle MEF equals LHP, which is a right angle. Therefore a circle described from the centre M with radius MA will touch EF in E.

The field operation is evident from the above construction, being strictly analogous to the preceding Proposition. The integral curvature is found as follows: $a = 57.30$, $m =$ curvature AE, $n =$ curvature HA, then radius AE, $= \frac{a}{m}$, and radius HA $\frac{a}{n}$ and their sum $= \frac{ma + na}{mn}$, and a divided by this result gives $\frac{mn}{m + n}$; i. e. multiply the curvatures together and divide by their sum.

When H is inaccessible, as it often will be, the Proposition must be solved by the method immediately following.

Fig. 15



SECOND METHOD.

Taking any point A, Fig. 15th, describe AC until the tangent becomes parallel to EF, then from O and the tangent CO parallel to AN describe the integral curve CE, intersecting EF in E; then ascertain the whole central angle of CE, and extend HA to B, so that AB shall be equivalent to CE, and B will be the point of reversed curvature required.

The demonstration of this method is so analogous to that of Proposition VIII, that it is unnecessary to go into it here; it depends on the parallelism of the lines connecting different parts of the figure.

Fig. 17

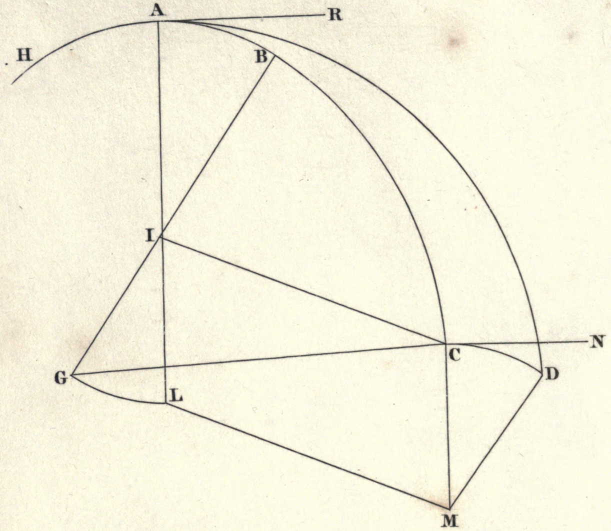


Fig. 18

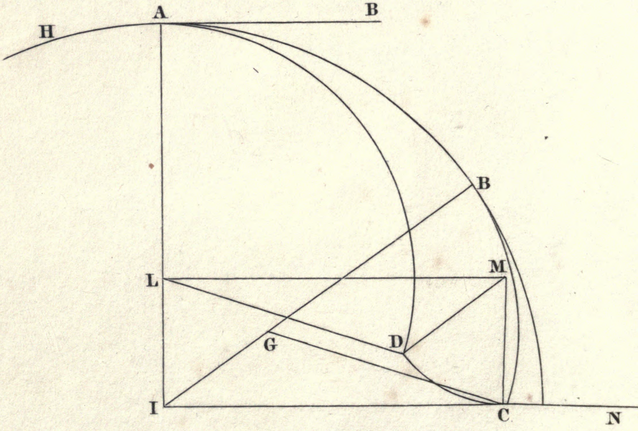
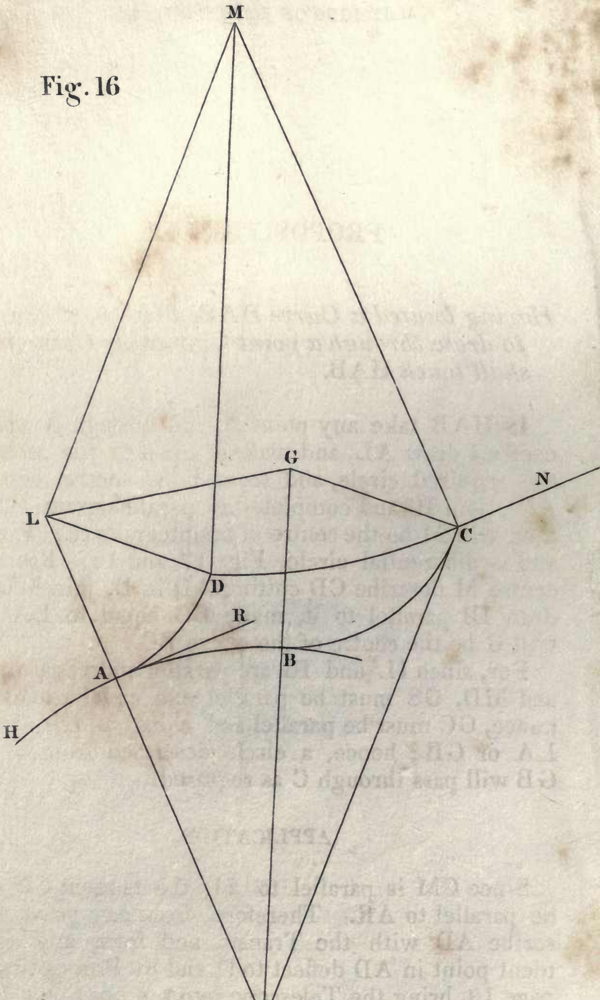


Fig. 16



PROPOSITION XI.

Having located a Curve HAB, *Fig.* 16, 17 *and* 18, *to draw through a point* C, *another Curve which shall touch* HAB.

In HAB take any point A, and through A and the centre I draw AL, and make it equal to the radius of the required circle, and from L as centre, describe AD ; join IC and complete the parallelogram ILMC, then will M be the centre of an integral circle, Fig. 16, and a differential circle, Fig. 17 and 18. From the centre M describe CD cutting AD in D, join MD and draw IB parallel to it, make BG equal to LA then will G be the centre of the curve BC.

For, since IL and IG are parallel and equal to MC and MD, GS must be parallel and equal to CD, and hence, GC must be parallel and equal to LD, i. e. to LA or GB ; hence, a circle described from G with GB will pass through C as required.

APPLICATION.

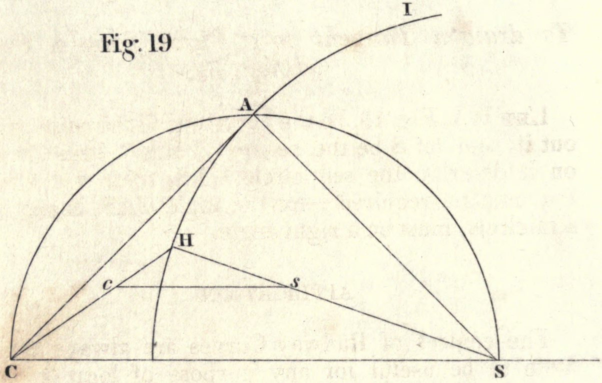
Since CM is parallel to AI, the tangent CN must be parallel to AR. Therefore, from any point A describe AD with the Transit, and from any convenient point in AD deflect to C, and by Proposition IV, page 13, bring the Telescope into the position CN, parallel to AR, then describe the differential or integral curve CD till it intersects AD ; complete the tangent AD, and ascertain the central angle of CD and make

AB equivalent: from a tangent at B describe BC, which must pass through the point C.

Observe that in Fig. 17, CD is turned towards the same hand as AB, and in Fig. 16 and 18 towards the opposite hand. The reason is sufficiently evident from the position of the parallelogram ILMC.

The distinction of the several cases must be carefully observed. In Fig. 16 ABC is a *reversed* curve. In Fig. 17 and 18 ABC is a compound curve. In Fig. 17 C is outside of AB and the radius of BC is of course the greatest. In Fig. 18 C is inside of AB, and of course the radiaus of BC must be least.

Fig. 19



PROPOSITION XII.

To draw a Tangent to a Curve from a Point without it.

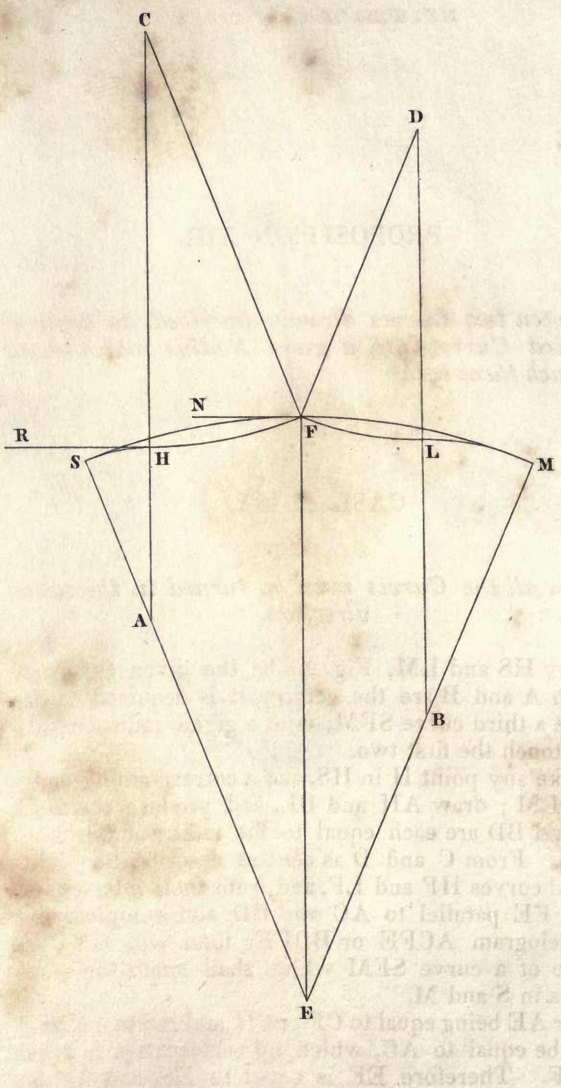
Let HA, Fig. 19, be the curve and C the point without it, and let S be the centre of HA. Join CS and on it describe the semicircle CAS, then will CS be the tangent required ; for the angle CAS, being in a semicircle, must be a right angle.

APPLICATION.

The centres of Railway Curves are always too remote to be useful for any purpose of location, and therefore, other methods must be resorted to in all cases, where the position of their centres with regard to objects beyond their circumferences are involved.

In the present case take any point H in the curve and measure CH ; from H towards the centre lay off HS, any convenient multiple of the radius, and make H*c* a similar multiple of HC, then will *cs* be a similar multiple of CS, and by the similarity of triangles is evidently parallel to it. Divide 57.30 by ½ of CS, and the quotient will be the curvature of CAS. With this curvature and from a tangent at right angles to C*s* or CS, describe CAS, and where it cuts HA will be the P.T. required.

Fig. 20



PROPOSITION XIII.

Between two Curves already described, to draw a third Curve with a given Radius which shall touch them both.

CASE FIRST.

When all the Curves must be turned in the same direction.

LET HS and LM, Fig. 20, be the given curves of which A and B are the centres, it is required to describe a third curve SFM, with a given radius which shall touch the first two.

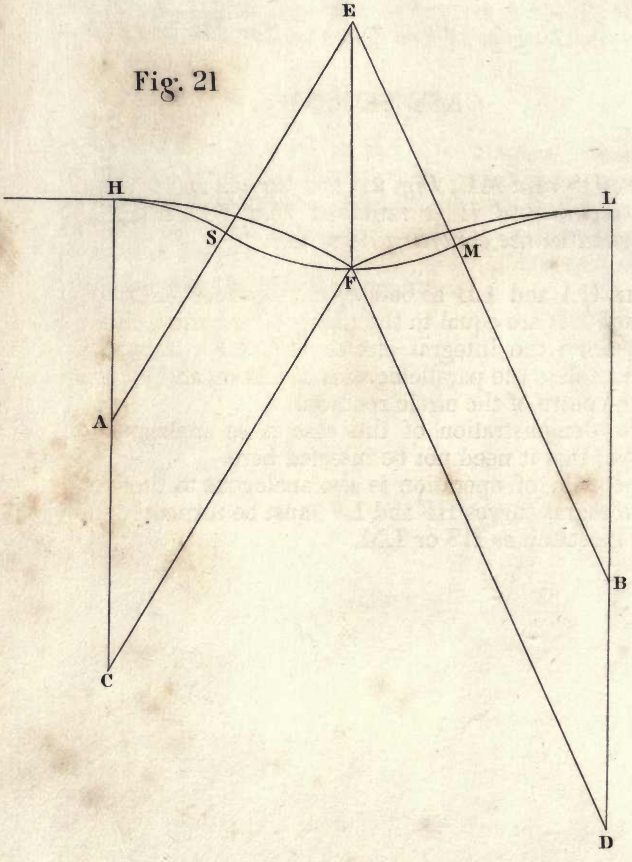
Take any point H in HS, and a corresponding point L in LM; draw AH and BL, and produce them till AC and BD are each equal to the radius of the third curve. From C and D as centres describe the differential curves HF and LF, and from their intersection draw FE parallel to AC and BD, and complete the parallelogram ACFE or BDFE, then will E be the centre of a curve SFM which shall touch the other curves in S and M.

For AE being equal to CF or CH, and AS to AH, ES must be equal to AC, which by construction is equal to EF. Therefore EF is equal to ES, and by the same reasoning EM may be proved to be equal to EF; therefore E is the centre of the curve SFM, and since SE and EM pass through the centres A and B, SFM must touch HS and LM in S and M respectively.

APPLICATION.

Since FE is parallel to HA, the tangent FN must be parallel to HR. Therefore from any two corresponding points, H and L, describe the differential curves HF and LF and from their intersection with a tangent parallel to HR describe the required curve each way till it touches the others as required.

Fig. 21



CASE SECOND.

When HS *and* ML, *Fig.* 21, *are turned in the same direction and it is required that* SM *shall be turned in the contrary direction.*

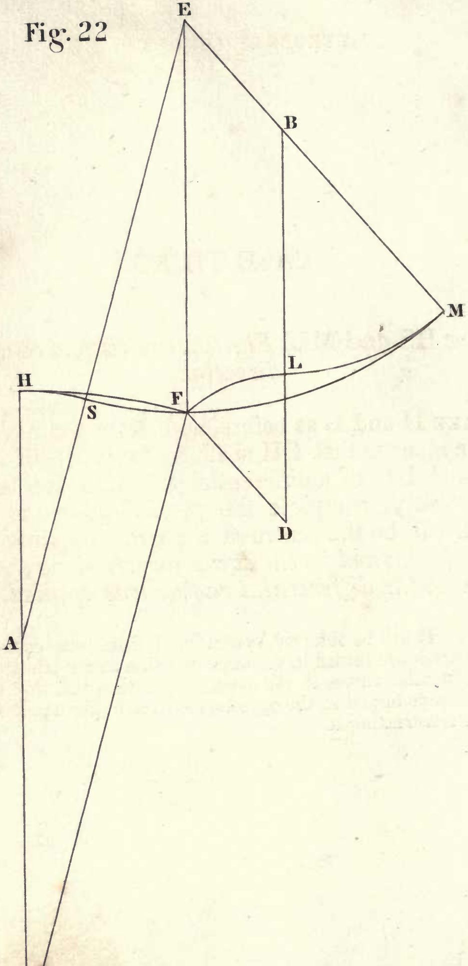
Join HA and LB as before, and produce them till AC and BD are equal to the radius of the third circle.

Describe the integral circles HF, LF ; draw FE and complete the parallelograms as before, and E will be the centre of the circle required.

The demonstration of this case is so analogous to the last, that it need not be inserted here.

The field of operation is also analogous to the last. The integral curves HF and LF must be turned in the same direction as HS or LM.

Fig. 22



CASE THIRD.

Where HS *and* ML, *Fig. 22, are turned in opposite directions.*

TAKE H and L as before, and draw CA and BD in such a manner that CH shall be the radius of an integral and DL of a differential curve, to HS and LM respectively, complete the parallelograms as before, and E will be the centre of the circle required, *which must be turned in the same direction as the curve to which the differential radius was applied.*

NOTE. It will be observed, generally, of these three cases, that integral curves are turned in the same direction as the primary curves, and differential curves in the opposite direction, and that the curve SM is always turned in the opposite direction to the auxiliary curves used in constructing it.

Fig. 23

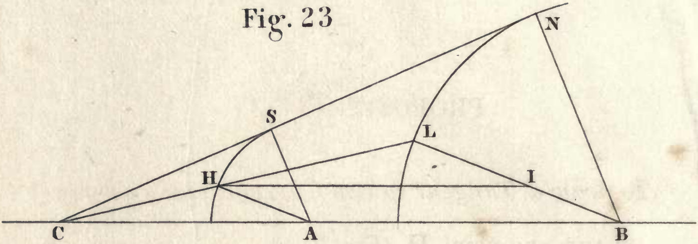
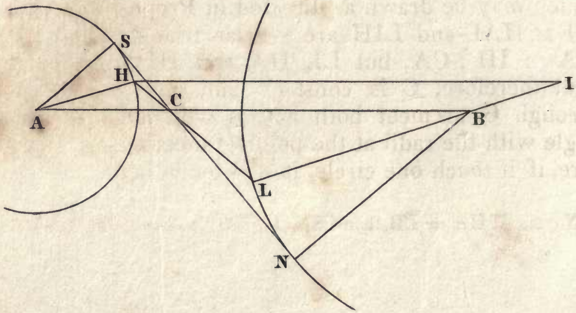


Fig. 24



PROPOSITION XIV.

To draw a Tangent to two Curves already located.

TAKE any point H, (Fig. 23 and 24,) in the circle HS, and take L a corresponding point in the other circle LN, and measure HL. Then suppose HI drawn parallel to AB and take CH : HL : : HA : LI,

That is in Fig. 23, $CH = HL \times \frac{n}{m-n}$.

And in Fig. 24, $CH = HL \times \frac{n}{m+n}$.

Then C will be a point in the common Tangent which may be drawn as directed in Proposition XII.

For HAC and LIH are similar triangles, and LI : HA : : HI : CA, but LI, HA and HI are constant and, therefore, C is constant, and any line drawn through C to meet both circles will make the same angle with the radii at the point of meeting, and therefore, if it touch one circle, it will touch both.

NOTE. If HA = LB, then CSN (Fig. 23,) will be parallel to HL.

Fig. 25

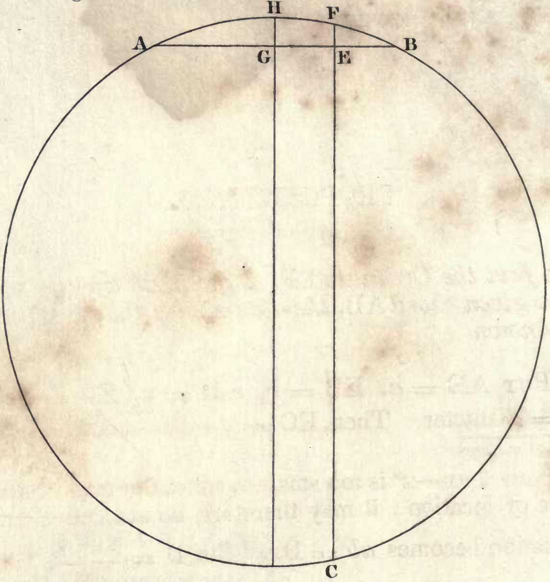
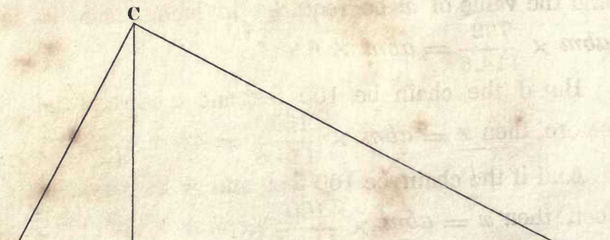


Fig. 26



PROPOSITION XV.

To find the Ordinate EF, *Fig.* 25, *at any point in a given chord* AB, *the diameter of the circle being known.*

PUT AE $= a$, EB $= b$, GH $= v$, EF $= x$ and D = diameter. Then, EC $= D - \overline{2v - x}$ and $ab = Dx - \overline{2vx - x^2}$.

Now $2vx - x^2$ is too small to affect the result in any case of location ; it may therefore be omitted and the equation becomes $ab = Dx$. But $D = \frac{114.6}{m}$; therefore $ab = \frac{114.6x}{m}$; and $x = ab \times \frac{m}{114.6} = abm \times \frac{1}{114.6}$.

Now if ab be given in parts of a chain of 66 feet and the value of x be required in inches, then $x = abm \times \frac{792}{114.6} = abm \times 6.9$.

But if the chain be 100 feet and x be required as before, then $x = abm \times \frac{1200}{114.6} = abm \times 10.5$.

And if the chain be 100 feet, and x be required in feet, then $x = abm \times \frac{100}{114.6} = abm \times .875$.

NOTE. It must be remembered (see Explanations, No. 2,) that the chord of 100 parts is considered as a *unit*, and therefore, the decimal point in a and b must be so placed as to diminish their value in like proportion.

PROPOSITION XVI.

To measure the Width of a River or distance to any inaccessible object in the line of survey.

LET AB, Fig. 26, be part of a line of survey in which A and B are on opposite sides of a river.

From A at right angles to AB lay off any convenient distance AC, so that B may be seen from C; remove the instrument to C, and lay off CD at right angles to CB, fixing the point D in AB produced, and measure DA.

Then AB is a third proportional to DA and AC and of course $\frac{AC^2}{DA} = AB$.

On the opposite page will be found the table of chords referred to in the preface. The numbers in the table are the ratios of the base to the side of an isosceles triangle for every degree of vertical angle. It would be difficult to give a specific account of its various uses, since it may be applied to every case which can be resolved into an isosceles triangle of which one side and angle are known.

TABLE OF CHORDS. RADIUS BEING UNITY.

Deg,	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170
0		.1743	.3473	.5176	.6840	.8452	1.000	1.1471	1.2856	1.4142	1.5321	1.6383	1.7320	1.8126	1.8794	1.9320	1.9696	1.9924
1	.0174	.1917	.3645	.5345	.7004	.8610	1.0151	1.1614	1.2989	1.4265	1.5433	1.6482	1.7407	1.8200	1.8853	1.9365	1.9726	1.9938
2	.0349	.2091	.3816	.5513	.7167	.8767	1.0301	1.1756	1.3121	1.4387	1.5542	1.6581	1.7492	1.8270	1.8910	1.9406	1.9754	1.9952
3	.0524	.2264	.3987	.5680	.7330	.8924	1.0450	1.1896	1.3252	1.4517	1.5652	1.6673	1.7576	1.8341	1.8967	1.9447	1.9780	1.9963
4	0698	.2437	.4158	.5848	.7492	.9080	1.0598	1.2036	1.3384	1.4627	1.5760	1.6774	1.7659	1.8410	1.9020	1.9487	1.9805	1.9973
5	.0872	.2611	.4329	.6014	.7654	.9236	1.0746	1.2176	1.3512	1.4746	1.5867	1.6868	1.7740	1.8478	1.9074	1.9526	1.9830	1.9981
6	.1047	.2784	.4499	.6180	.7815	.9389	1.0892	1.2313	1.3640	1.4863	1.5973	1.6961	1.7820	1.8544	1.9126	1.9563	1.9851	1.9989
7	.1221	.2956	.4669	.6346	.7975	.9543	1.1039	1.2450	1.3767	1.4980	1.6077	1.7053	1.7897	1.8609	1.9177	1.9600	1.9871	1.9994
8	.1395	.3128	.4838	.6512	.8135	.9696	1.1184	1.2586	1.3893	1.5094	1.6180	1.7146	1.7976	1.8672	1.9226	1.9632	1.9890	1.9997
9	.1569	.3301	.5008	.6676	.8294	.9848	1.1328	1.2722	1.4018	1.5208	1.6282	1.7232	1.8052	1.8734	1.9273	1.9665	1.9910	2.0000