MILNE'S PRACTICAL ARITHMETIC

INDUCTIVE SERIES

MILNE.

JONES BROTHERS & COMPANY,

Cincinnati, Philadelphia, Chicago.

Ranunculus bulbosus - Buttercup
...t Taraxacum - Dandelion
...folium pratense - Red Clover

...ous. Exogenous

Angiosperm. Polypetalous. Stam...
more than ten. Stamens on recep...
Pistils more than one + separa...
Leaves without Stipules. Crow fo...
family. Genus ranunculus Bu...
name - Ranunculus bulbosus

Dandelion = Dent-de-lion

IV

Eda Mills.

Moraga Valley.

Mch. 10. 1882.

Inductive Series.

THE

PRACTICAL ARITHMETIC

ON THE

INDUCTIVE PLAN,

INCLUDING

ORAL AND WRITTEN EXERCISES.

BY

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JONES BROTHERS & COMPANY:

CINCINNATI, CHICAGO,

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PREFACE

The design of the author in preparing this work has been to embrace within moderate compass all the essentials for A Practical Course in Arithmetic, and to present every subject in such a manner as to secure the highest mental development of the learner. To accomplish these results the author has spent much time in investigation, and in consultation with eminent educators and successful business men, and he believes that he has included in this volume all the subjects necessary for the arithmetical part of a business education.

The method of introducing each subject is such that the student is led to truth in the path of the original investigator—certainly the most natural and delightful road to the acquisition of knowledge. It is because of this special feature in connection with every subject that the series has been called The Inductive Series.

The work contains oral and written exercises sufficient in number to enable the student to master the principles underlying each subject and to give him facility in numerical processes.

In the problems given for solution it has been the aim of the author to use the language of trade, when no error is conveyed thereby, thus accustoming the student to the forms of expression needed in after life; and in general the author has striven after clearness of statement rather than technical accuracy of expression.

It would be pedantry to specify the departments in which excellence or originality may be found, but it is hoped that a careful examination will exhibit the logical sequence of the steps in all the processes, the perspicuity and accuracy of the analyses, and the brevity and correctness of the definitions, principles, and rules.

The author takes pleasure in acknowledging his indebtedness to Prof. J. B. De Motte, of Indiana Asbury University, and to several other teachers of ability and experience, for timely and valuable suggestions.

Trusting that the book will, in some measure, supply the popular demand for a brief and comprehensive treatise upon Arithmetic, the author presents his work to the public.

W. J. M.

State Normal School,
Geneseo, N. Y.,

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A Practical Arithmetic

NOTATION & NUMERATION

Article 1. A ***Unit*** is a single thing.

2. A ***Number*** is a unit or collection of units.

3. In counting a large number of objects it is natural to group them.

Thus, coins are put in piles and the piles counted, envelopes in packages and the packages counted, etc. These piles and packages may themselves be piled and put in larger packages, and the process continued indefinitely.

4. To express numbers so that all may understand what is meant by the characters which represent them, the system of grouping *by tens* has been adopted. There are, therefore, single things, or *units;* groups of ten units, or *tens;* groups containing ten tens, or *hundreds*, etc.

5. The method of grouping *by tens* is called the ***Decimal System.***

Decimal is derived from the Latin word *decem*, which means *ten*.

Numbers may be expressed by *words* or other *characters*, viz: *figures* and *letters*.

6. ***Notation*** is the method of *expressing* numbers by figures and letters.

The ***Arabic Notation*** is the method of expressing numbers by means of figures. Its name is derived from the Arabs, by whom it was introduced into Europe. The ***Roman Notation*** is the method of expressing numbers by means of letters. It is so called because it was used by the ancient Romans.

7. ***Numeration*** is the method of *reading* numbers expressed by figures or letters.

ARABIC SYSTEM.

8. In this system ten figures are employed to express numbers, viz:

Figures.	0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Names.	Naught,	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine.

Each of these, except naught, is called a *significant figure*.
Naught is also called *zero* and *cipher*.

9. By combining these figures according to certain principles, we can express any number.

10. PRINCIPLE.—*When figures are written side by side, the one at the right expresses units, the next tens, and the next hundreds.*

EXERCISES.

11. 1. In 79, what does the 7 express? What does the 9 express? Read the number, beginning at the left.

2. In 58, what does the 5 express? What does the 8 express? Read the number, beginning at the left.

3. In 740, what does the 7 express? What does the 4 express? What does the 0 express? Read the number.

4. Begin at the left and read 76, 176, 106, 360, 203.

12. Figures in *units'* place express *units of the first order;* those in *tens'* place, *units of the second order;* those in *hundreds'* place, *units of the third order*, etc.

13. Numbers between 1 ten and 2 tens are named thus:

1 ten and 1 unit or 11, eleven.
1 ten and 2 units or 12, twelve.
1 ten and 3 units or 13, thirteen.
1 ten and 4 units or 14, fourteen.
1 ten and 5 units or 15, fifteen.
1 ten and 6 units or 16, sixteen.
1 ten and 7 units or 17, seventeen.
1 ten and 8 units or 18, eighteen.
1 ten and 9 units or 19, nineteen.

The words thirteen, fourteen, fifteen, etc., mean *three* and ten, *four* and ten, *five* and ten, etc.

14. The *units of the second order* are named as follows:

2 tens or 20, twenty.	6 tens or 60, sixty.
3 tens or 30, thirty.	7 tens or 70, seventy.
4 tens or 40, forty.	8 tens or 80, eighty.
5 tens or 50, fifty.	9 tens or 90, ninety.

The suffix *ty* means *ten.* Thus, *forty* means *four tens,* etc.

The other numbers between 20 and 100 are read without the word *and* between the tens and units. Thus, 27 is read twenty-seven, instead of twenty *and* seven.

EXERCISES.

15. Read the following:

28	99	43	73	67	41
64	83	75	86	45	31
39	78	60	51	32	92
47	32	21	55	82	25

16. Express in figures the following:

Three tens and eight units.
Four tens and seven units.
Two tens and two units.
One ten and three units.
Six tens and nine units.
Five tens and seven units.
Eight tens and one unit.
Seven tens and three units.
One ten and eight units.
Four tens and two units.

Three units of the second order, six of the first order.
Two units of the second order, four of the first order.
Write all the numbers between 10 and 20. Between 30 and 50. Between 70 and 90.

17. In reading a number expressed by *three* figures, the tens are read after the hundreds without the word *and*. Thus, 235 is read two hundred thirty-five instead of two hundred *and* thirty-five.

18. Read the following:

746	932	786	849	534
678	453	777	391	585
963	378	243	873	412
531	217	918	855	248

19. Express in figures the following:

Two hundreds, three tens, five units.
Six hundreds, two tens, nine units.
Four hundreds, one ten, eight units.
Three units of the third order, six of the second order.

Three hundred eighteen.
Eight hundred thirty.
Four hundred four.
Six hundred eighty-one.
Seven hundred seventy.
Seven hundred.
Seventy.
Seven.
Seven hundred six.
Six hundred forty.
Two hundred six.
One hundred eleven.

Seven hundred seventy-seven.

From the previous examples we deduce the following general principles:

20. Principles.—1. *The representative value of a figure is increased tenfold by each removal one place to the left, and decreased tenfold by each removal one place to the right.*

2. *The figure* 0 *is used to give significant figures their positions.*

21. In reading numbers a new name is given the order of units *next higher than hundreds* of any denomination. Thus, the order next higher than hundreds is called thousands, that next higher than hundreds of thousands, millions, etc. Therefore each denomination can have but *three orders* of units.

22. A ***Period*** is a group of figures containing the *hundreds*, *tens*, and *units* of any denomination.

The present system of notation is illustrated by the following

TABLE.

Periods.	6*th.*	5*th.*	4*th.*	3*d.*	2*d.*	1*st.*
Names of Periods.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
Orders.	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units
	3 0,	2 3 0,	1 6 0,	7 0 0,	4 0 1,	6 9 0.

This number is read *thirty* quadrillion, *two hundred thirty* trillion, *one hundred sixty* billion, *seven hundred* million, *four hundred one* thousand, *six hundred ninety.*

1. In reading numbers, the *name* of units' period is omitted.
2. Each period, except the highest, must contain three figures.
3. The periods are separated from each other by *commas.*

23. The periods above Quadrillions, in their order, are *Quintillions*, *Sextillions*, *Septillions*, etc.

24. Give the number of each of the following periods:

Millions.	Trillions.	Billions.
Thousands.	Units.	Quadrillions.

25. Give the names of the following:

5th period.	2d period.	1st period.
3d period.	4th period.	6th period.

26. Repeat in order the names of the periods from:

Units to billions.	Billions to units.
Units to quadrillions.	Quadrillions to units.
Thousands to trillions.	Millions to thousands.

27. Copy and point off into periods:

1. 46825.	5. 38420058.	9. 5284325684.
2. 239746.	6. 33468204.	10. 7932468512.
3. 180040.	7. 8438206.	11. 83749275867.
4. 14168843.	8. 436784.	12. 1423789276586.

13. How many thousands are there in the first number?

14. How many thousands in the second number?

15. How many billions in the next to the last number?

16. How many trillions in the last number? How many billions? How many millions? How many thousands? How many units?

17. Point off into periods, and name in order, the billions, millions, thousands, and units of the next to the last number.

18. Point off into periods, and name in their order, the periods composing the 12th number.

19. In like manner point off and read each of the numbers.

28. Write in figures:

1. Thirty-four billion, eighteen thousand, forty.

PROCESS.

4th.	3d.	2d.	1st.
34	000	018	040

Or,

34,000,018,040

ANALYSIS.—Since the highest period is billions, which occupy the fourth period, we make four spaces for the periods. We write 34 in the fourth period, thus expressing the billions of the given number; 18 in the second period, thus expressing the thousands; and 40 in the first period, thus expressing the units. Since every period except the highest must contain three figures, we fill the vacant places with ciphers.

As soon as possible use commas instead of the lines, and cease to write both the number and name of the periods.

Write in figures, and read the number:

2. Thirty-six in the 3d period, two hundred eighteen in the 2d, eight hundred forty-six in the 1st.

3. Eighty-four in the 4th period, five hundred forty in the 3d, six hundred in the 2d, forty in the 1st.

4. Two hundred one in the 5th period, seventy-five in the 4th, five hundred sixty-two in the 3d, twelve in the 2d, one in the 1st.

5. Sixty in the 5th period, four hundred two in the 4th, three hundred thirty-three in the 3d, two hundred in the 2d, one hundred eleven in the 1st.

Write in figures:

6. Seventy-three million, two hundred fourteen thousand, seventy.

7. Eighty billion, forty million, six hundred twelve thousand, seven hundred eighty-eight.

8. Two hundred twenty-five million, six hundred forty-one thousand, three hundred fifty-one.

9. Three hundred fifty-four billion, six hundred four million, eight hundred ninety-two thousand, thirty-six.

29. Rule for Notation.—*Begin at the left and write the hundreds, tens, and units of each period in their proper order, putting ciphers in all vacant places and periods.*

While writing, separate each period from the next by a comma.

30. Rule for Numeration.—*Begin at the right and separate the numbers into periods of three figures each.*

Begin at the left hand and read each period as if it stood alone, adding its name.

EXERCISES.

31. Copy, point off, and read:

1. 116234.	8. 141120.	15. 7640.
2. 65231.	9. 101207.	16. 800900.
3. 20703.	10. 68978.	17. 2568242.
4. 71005.	11. 72020.	18. 1008003.
5. 3104.	12. 80001.	19. 212375647.
6. 48000.	13. 857000.	20. 609003588.
7. 60029.	14. 91029.	21. 897856846.

32. Write in figures, and read:

22. Two hundred in the 1st period.

23. Sixty in the 2d period, two in the 1st.

24. Seven hundred in the 3d period.

25. Two hundred thirty in the 3d period, sixty in the 1st.

26. Eighty-one in the 4th period, five hundred one in the 3d, seven in the 2d, twelve in the 1st.

27. Thirty in the 5th period, six hundred three in the 1st.

28. Seven hundred in the 5th period, eighty in the 4th.

29. Eight in the 4th period, seven in the 3d, fourteen in the 2d, and ten in the 1st.

30. Fifteen in the 6th period, eighteen in the 4th, two hundred seven in the 3d, and eighty-one in the 1st.

33. Copy, point off, and read:

1. 60701892.
2. 50607801.
3. 600000.
4. 49000000.
5. 593006070500.
6. 19019000190019019.
7. 163194568.
8. 3050050183.
9. 5000204.
10. 594900.
11. 12000012.
12. 200798013400019.
13. 212506067093012063067.

34. Write in figures:

14. Two in the 3d period, sixty in the 2d, one hundred fifty-three in the 1st.

15. Sixty in each of the 4th, 3d, 2d, and 1st periods.

16. 60 million, 200 thousand, 500.

17. 402 billion, 348 million, 213 thousand, 20.

18. 78 trillion, 640 billion, 9 million, 6 thousand, 16.

19. 6 billion, 542 million, 25.

20. Six billion, five hundred forty-two million, twenty-five.

21. Four hundred two billion, three hundred forty-eight million, two hundred thirteen thousand, twenty.

22. Five million, two hundred sixty-eight thousand, nine hundred forty-nine.

23. Two hundred million, three hundred thousand, eight hundred.

24. Twenty-nine billion, five hundred ninety-nine million, six hundred one.

25. Four trillion, five hundred fifty-eight million, two hundred forty-four thousand, seventy.

26. Thirty-two billion, sixty-one million, three hundred forty-three thousand, four hundred four.

27. Five hundred fifty-five million, seven hundred seventy-seven thousand, six hundred sixty-nine.

28. Eight hundred six billion, seventy million, three hundred eighty-five thousand, two hundred six.

29. Nine hundred forty-one trillion, one hundred sixteen thousand, twenty-two.

30. Twenty-three billion, twenty-three million, twenty-three thousand, twenty-three.

31. Six hundred thousand, seventy-five.

32. Twelve billion, eight million, nine hundred eighty-eight thousand, thirteen.

33. Twenty-nine quadrillion, seven hundred fifty-seven trillion, four hundred eighty million, thirteen thousand, five hundred sixty-five.

NOTATION AND NUMERATION OF U. S. MONEY.

35. The currency of the United States has a Decimal System of notation, thus:

10 mills make 1 cent.
10 cents make 1 dime.
10 dimes make 1 dollar.

36. The ***Sign of Dollars*** is $. It is written before the number.

Thus, $16 is read, sixteen dollars.

37. In writing decimal currency a mark called the decimal point is placed before cents and mills.

38. *Cents* occupy the *first two* places at the right of the decimal point, and *mills* the *third.*

Thus, $7.584 is read, seven dollars, fifty-eight cents, four mills; $.694 is read, sixty-nine cents, four mills.

39. If the number of cents is *less than ten,* write a cipher in the first place at the right of the decimal point.

Thus, five dollars, eight cents, is written, $5.08; three dollars, seven cents, $3.07.

40. Read the following:

$6.85	$7.843	$12.056
$31.095	$24.055	$20.20
$28.075	$40.04	$606.952
$500.50	$2103.094	$7000.16
$20000.	$6001.101	$300.416
$212012.12	$695.955	$200.204
$613.495	$211.12	$69.69
$203.033	$216.16	$75.25

41. Write the following:

1. Two dollars, twenty-three cents, five mills.
2. Two hundred two dollars, two cents, five mills.
3. One hundred twelve dollars, twenty-five cents.
4. Six hundred two dollars, nine cents.
5. Twenty thousand dollars, thirty-two cents.
6. Twelve million, seven hundred thousand dollars.
7. Six million dollars, eighty-eight cents.
8. Twelve thousand three hundred dollars, fifteen cents.

ROMAN SYSTEM.

42. In this system seven letters are used to express numbers, viz:

Letters.	I,	V,	X,	L,	C,	D,	M.
Values.	1,	5,	10,	50,	100,	500,	1000.

By combining these letters according to certain principles any number can be expressed.

PRINCIPLES.—1. *When a letter is repeated its value is repeated.*

Thus, I represents 1; II, two; III, three; X, ten; XX, twenty; XXX, thirty; C, one hundred; CCC, three hundred.

2. *When a letter is placed before another of greater value its value is to be taken from that of the greater.*

Thus, I represents one and V five, but IV represents four; IX, nine; XIX, nineteen; XL, forty; XC, ninety.

3. *When a letter is placed after another of greater value their values are to be united.*

Thus, XV represents fifteen; LX, sixty; LXXX, eighty; DC, six hundred; MD, fifteen hundred.

4. *A bar placed over a number increases its value a thousand-fold.*

Thus, V represents five; $\overline{V}$, five thousand; LX, sixty; $\overline{LX}$, sixty thousand; M, one thousand; $\overline{M}$, one million.

TABLE.

I	1	XIV	14	LX	60
II	2	XV	15	LXX	70
III	3	XVI	16	LXXX	80
IV	4	XVII	17	XC	90
V	5	XVIII	18	C	100
VI	6	XIX	19	CC	200
VII	7	XX	20	CCL	250
VIII	8	XXI	21	CCCC	400
IX	9	XXIX	29	D	500
X	10	XXX	30	DCC	700
XI	11	XXXIV	34	M	1000
XII	12	XL	40	MMM	3000
XIII	13	L	50	MDCCCLXXX	1880

Read the following numbers:

XV; XXIV; XXXIX; XL; XLIX; XCIX; LXXVII; CCCLXXXIX; DCCXXXVI; $\overline{V}$DLV; $\overline{DL}$DC; $\overline{CCX}$DVI; $\overline{LXX}$MMMDCCCXCIX; $\overline{MDXCV}$DCCCLXIV.

Express the following numbers by Roman Notation:

15, 18, 27, 81, 95, 86, 534, 684, 1050, 8004, 7000, 75869.

ADDITION

INDUCTIVE EXERCISES.

43. 1. How many are 2 pears and 1 pear? 2 pears and 2 pears?

2. How many are 3 leaves and 2 leaves? 3 leaves and 3 leaves? How many are 3 and 1? 3 and 2? 3 and 3?

3. Jane has 3 apples and Mary has 4 apples. How many apples have both? How many are 3 and 4? 4 and 3?

4. George gave me 2 apples and Mary gave me 4. How many apples did both give me? How many are 4 and 2? 2 and 4?

5. A farmer had 2 horses and bought 6 more. How many horses had he then? How many are 2 and 6? 6 and 2?

6. Henry paid 5 cents for a pencil and 7 cents for a writing-book. How many cents did he pay for both? How many are 5 and 7? 7 and 5?

7. If a barrel of flour is worth \$6, and a cord of wood \$4, how much are both worth? How many are 6 and 4?

8. A man plowed 8 acres of land in one week and 6 acres the next week. How many acres did he plow in both weeks?

9. On the Fourth of July, Ned spent 10 cents for fire-crackers and 6 cents for torpedoes. How many cents did he spend for both?

10. Harry is 6 years old and his sister is four years older. How old is his sister? How many are 6 and 4?

11. At Christmas, Horace received 9 gifts from his parents, and 4 from other friends. How many gifts did he receive?

12. A certain house has 5 windows in one side and 7 in another. How many windows in the two sides?

13. How many are 5 oranges and 4 oranges? 6 boys and 3 boys? 5 horses and 6 cents?

14. Why can you not tell how many 5 horses and 6 cents are?

15. Why can you tell how many 5 oranges and 4 oranges are?

Numbers that express things of the same name are called ***Like Numbers.***

16. What kind of numbers only can be united?

DEFINITIONS.

44. ***Addition*** is the process of finding a number which shall be equal to two or more given numbers.

45. The ***Sum*** or ***Amount*** is the result obtained by adding.

46. The ***Sign of Addition*** is an upright cross: $+$. It is called *plus*, and is placed between numbers to be added.

Thus, $3 + 4$ is read 3 plus 4, and means that 3 and 4 are to be added.

47. The ***Sign of Equality*** is two short horizontal lines: $=$. It is read *equals*, or *is equal to.*

Thus, $3 + 4 = 7$, is read 3 plus 4 equals 7.

The expression $3 + 4 = 7$, or any other expression of equality, is called an ***Equation.***

48. Principles.—1. *Only like numbers can be added.*

2. *The sum and numbers added must be like numbers.*

TABLE.

1+1= 2	1+2= 3	1+3= 4	1+4= 5	1+ 5= 6
2+1= 3	2+2= 4	2+3= 5	2+4= 6	2+ 5= 7
3+1= 4	3+2= 5	3+3= 6	3+4= 7	3+ 5= 8
4+1= 5	4+2= 6	4+3= 7	4+4= 8	4+ 5= 9
5+1= 6	5+2= 7	5+3= 8	5+4= 9	5+ 5=10
6+1= 7	6+2= 8	6+3= 9	6+4=10	6+ 5=11
7+1= 8	7+2= 9	7+3=10	7+4=11	7+ 5=12
8+1= 9	8+2=10	8+3=11	8+4=12	8+ 5=13
9+1=10	9+2=11	9+3=12	9+4=13	9+ 5=14
1+6= 7	1+7= 8	1+8= 9	1+9=10	1+10=11
2+6= 8	2+7= 9	2+8=10	2+9=11	2+10=12
3+6= 9	3+7=10	3+8=11	3+9=12	3+10=13
4+6=10	4+7=11	4+8=12	4+9=13	4+10=14
5+6=11	5+7=12	5+8=13	5+9=14	5+10=15
6+6=12	6+7=13	6+8=14	6+9=15	6+10=16
7+6=13	7+7=14	7+8=15	7+9=16	7+10=17
8+6=14	8+7=15	8+8=16	8+9=17	8+10=18
9+6=15	9+7=16	9+8=17	9+9=18	9+10=19

CASE I.

49. To add single columns.

1. A grocer sold 8 pounds of sugar to one man and 7 pounds to another. How many pounds did he sell to both?

Analysis.—Since he sold 8 pounds to one man and 7 pounds to another, to both he sold the sum of 8 pounds and 7 pounds, which is 15 pounds.

2. A man rode 7 miles the first hour and 6 miles the second hour. How far did he ride in the two hours?

3. On one tree are 8 birds, and on another 4 birds. How many birds are there on both?

4. Carl earned $2 in May, $5 in June, and $4 in July. How much did he earn in the three months?

5. I gave 6 nuts to one boy, 5 to another, and 3 to another. How many nuts did I give to all?

6. I paid 5 cents for paper, 3 cents for pens, and 5 cents for ink. How much did I pay for all?

7. A lemon cost 5 cents, an orange 6 cents, and a pineapple 8 cents. What did they all cost?

8. Esther gave her teacher 5 pinks, 7 roses, and 4 pansies. How many flowers did she give her?

9. James shot 9 birds, Henry shot 6, and William 5. How many did they all shoot?

10. A woman picked 9 quarts of blackberries one morning, while her son picked 3 quarts. How many quarts did both pick?

11. James solved 6 examples, John 5, William 8, and Henry 7. How many examples did they solve in all?

12. One boy picked 6 quarts of cherries, another 4 quarts, another 5 quarts. How many quarts did they all pick?

13. I gathered from one pear-tree, this year, 2 bushels of fruit, from another 4 bushels, from another 3 bushels, and from another 2 bushels. How many bushels did I gather from these four trees?

14. A merchant sold from a piece of cloth, 3 yards at one time, 6 yards at another, 8 yards at another, and 5 yards at another. How many yards did he sell in all?

15. A man picked 8 barrels of apples on Monday, 6 barrels on Tuesday, 4 barrels on Wednesday, and 5 barrels on Thursday. How many did he pick altogether?

16. Henry learned 7 verses of poetry on one day, 5 on another, 6 on another, and 8 on another. How many verses did he learn in the four days?

17. A man paid $9 for a coat, $4 for pants, and $2 for a hat. How much did he pay for all?

18. In a garden there are 8 apple-trees, 7 plum-trees, and 9 peach-trees. How many trees are there in the garden?

19. There are 4 boys and 7 girls in one class, and 6 boys and 8 girls in another. How many pupils in both classes?

20. Homer paid 8 dollars for a fur cap, and 5 dollars for a pair of skates. How much did both cost him?

21. A boy gathered nuts for three days. The first day he brought home 8 quarts, the next day 7 quarts, the next day 9 quarts. How many quarts did he bring home?

22. Repeat the addition table of ones. Of twos. Of threes. Of fours. Of fives. Of sixes. Of sevens. Of eights. Of nines. Of tens.

23. Count by 2's from 0 to 20; thus: 0, 2, 4, 6, 8, 10, 12, etc.

24. Count by 3's from 2 to 26. From 26 to 41.
25. Count by 4's from 0 to 36. From 5 to 53.
26. Count by 5's from 3 to 43. From 7 to 72.
27. Count by 6's from 0 to 42. From 4 to 46.
28. Count by 7's from 4 to 39. From 11 to 60.
29. Count by 8's from 2 to 58. From 7 to 63.
30. Count by 9's from 7 to 70. From 8 to 71.

WRITTEN EXERCISES.

50. 1. What is the sum of 5, 4, 7, and 6?

PROCESS.

$$\begin{array}{r} 5 \\ 4 \\ 7 \\ 6 \\ \hline 22 \end{array}\ \textit{Sum.}$$

ANALYSIS.—We write the numbers to be added in a column, and begin at the bottom to add; thus: 6, 13, 17, 22; and write the sum beneath. To prove the work we may begin at the top and add downwards. If the result agrees with the one formerly obtained the work is probably correct. In adding say, 6, 13, 17, etc., instead of 6 and 7 are 13 and 4 are 17, etc.

Copy, add, and prove:

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
5	6	5	6	7	8
3	7	6	4	3	9
4	8	2	3	8	8
2	1	3	4	5	7

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
8	6	5	8	9	2
7	4	4	7	7	5
6	3	3	9	8	7
3	0	2	8	9	9
5	3	1	8	8	3

51. Required the sum of the following:

14. 6, 7, 5, 3, 2, 4, and 5.
15. 8, 2, 0, 3, 3, 2, and 4.
16. 5, 6, 7, 6, 4, 2, and 8.
17. 3, 2, 6, 5, 8, 7, and 9.
18. 8, 3, 0, 5, 3, 8, and 2.
19. 7, 6, 6, 4, 3, 6, and 3.
20. 7, 8, 8, 9, 0, 3, and 3.
21. 8, 9, 7, 8, 5, 8, and 2.
22. 7, 6, 5, 4, 3, 2, and 1.
23. 5, 4, 4, 3, 2, 6, and 7.
24. 4, 3, 4, 5, 6, 8, and 8.
25. 3, 6, 8, 6, 7, 0, and 5.

26. There are 8 chickens in one coop, 9 in another, 7 in another, and 5 in another. How many chickens are there in all the coops?

27. My father has 5 horses, 9 cows, 7 sheep, and 3 pigs. How many animals has he in all?

28. A man walked from A to B in four hours. He went 4 miles the first hour, 3 miles the second hour, 5 miles the third hour, and 6 miles the fourth hour. What was the distance between the two places?

29. A house had 8 windows on the east side, 7 on the west, and 9 on the south. How many were there in all?

CASE II.

52. To add several columns.

1. Count by 10's from 7 to 107; thus, 7, 17, 27, 37, 47, etc.

2. Count by 10's from 5 to 95. From 9 to 79.

3. Count by 20's from 5 to 85. From 9 to 89.

4. Add 2 to each of the series of numbers 6, 16, 26, etc., to 76.

5. Add 3 to each of the series of numbers from 8, 18, etc., to 88.

6. A gentleman paid \$7 for a hat, \$8 for a vest, and \$13 for pantaloons. How much did he pay for all?

ANALYSIS.—Since he paid \$7 for a hat, \$8 for a vest, and \$13 for pantaloons, for all he paid the sum of \$7, \$8, and \$13, or \$28.

7. James gave 25 cents to his brother and 20 cents to his sister. How much did he give to both? 25 and 20 are how many?

8. Horace earned 35 cents on Monday, 20 cents on Tuesday, and 9 cents on Wednesday. How much did he earn during the three days? How many are 35 and 29?

9. William saw two flocks of wild geese; the first of 27 geese, the second of 23 $(20+3)$. How many geese did he see? How many are 27 and 23?

10. Paid 9 cents for raisins, 15 cents for plums, and 27 $(20+7)$ cents for currants. How much did all cost?

11. During a certain recitation 29 questions were answered correctly and 16 incorrectly. How many questions were asked? How many are 29 and 10? 39 and 6?

12. Add 2 to each of the numbers 2, 12, 22, 32, 42, etc., to 72.

13. Add 3 to each of the numbers 4, 14, 24, etc., to 94.

14. Add 4 to each of the numbers 9, 19, 29, 39, to 99.

15. Add each of the numbers 5, 6, 7, 8, and 9 to each of the numbers 6, 16, 26, etc., to 96.

16. A certain school had 40 girls and 30 boys in attendance. How many pupils were there in the school?

17. A music teacher paid $12 for a metronome and $15 for music. How much did she pay for both?

18. A boy bought a velocipede for $15 and a watch for $20. How much did both cost him?

19. Mary read 20 pages of history one day, 30 pages the next, and 25 the next. How many pages did she read in all?

20. In a certain book-case there were 18 books on the upper shelf, 20 on the next, 12 on the next, and 10 on the lowest. How many books in the case?

21. A merchant sold 15 yards of cloth to one woman, 25 to another, 30 to another, and 25 to another. How many yards did he sell to them all?

22. A postmaster sold on one day 50 three-cent stamps, 65 on another, and 55 on another. How many stamps did he sell in the three days?

23. James solved 31 oral problems and 24 written problems. Harry solved 35 oral problems and 25 written problems. How many problems did each solve? How many did both solve?

24. In an orchard there are 26 cherry-trees and 31 apple-trees. How many trees are there in the orchard?

25. Henry saw three flocks of wild ducks, the first containing 17 ducks, the second 25, and the third 30. How many ducks did he see?

26. James paid 28 cents for a slate, 20 cents for a writing-book, and 10 cents for ink. How much did he pay for all?

27. The month of January has 31 days, the month of February has 28 days, and the month of March has 31 days. How many days are there in these three months?

28. How many acres are there in three fields, containing respectively 22 acres, 33 acres, and 37 acres?

WRITTEN EXERCISES.

53. 1. What is the sum of \$535, \$213, and \$384?

PROCESS.
\$535
213
384
\$1132

ANALYSIS.—For convenience we arrange the numbers to be added so that units of the same order shall stand in the same column. Beginning with the lowest order of units we add each column separately. Thus, $4 + 3 + 5 = 12$, the sum of the units. 12 units are equal to 1 ten and 2 units. We write 2 under the column of units and reserve the 1 to add with the tens.

1 reserved $+ 8 + 1 + 3 = 13$, the sum of the tens. 13 tens are equal to 1 hundred and 3 tens. We write the 3 under the column of tens and reserve the 1 to add with the hundreds.

1 reserved $+ 3 + 2 + 5 = 11$, the sum of the hundreds. 11 hundreds are equal to 1 thousand and 1 hundred, which we write in thousands' and hundreds' places in the sum.

Hence the sum is \$1132.

1. In adding, name results only. Thus, instead of saying 4 and 3 are 7 and 5 are 12, say 4, 7, 12.

2. When the sum of any column is exactly 10, 20, or any number of *tens*, we write 0 under the column added and reserve the 1, 2, 3, etc., to add with the next column.

54. RULE.—*Arrange the numbers so that units of the same order shall stand in the same column.*

Begin at the right, add each column separately, and write the sum, if it is less than ten, under the column added.

If the sum of any column be ten or more, write the unit figure only under that column and add the ten or tens with the next column.

Write the entire sum of the last column.

PROOF.—*Add each column in the reverse order. If the results agree, the work is probably correct.*

EXAMPLES.

Copy, add, and prove:

(2.)	(3.)	(4.)	(5.)
310	512	\$24.15	\$12.25
114	415	10.21	9.08
523	371	8.34	7.15

(6.)	(7.)	(8.)	(9.)
POUNDS.	HORSES.	PLOWS.	RODS.
4134	8104	3910	45
2460	3673	418	3061
3782	1856	1916	418
469	7206	39	6

10. Add 4834, 3910, 4826, 8404.

11. Add 3159, 7816, 5459, 3568.

12. Add \$16.05, \$10.38, \$77.055.

13. Add \$317.50, \$600.10, \$514.085, \$6.16.

14. What is the sum of thirty-six thousand, three hundred five; eight hundred ninety-seven thousand, nineteen?

15. What is the sum of fifty-nine thousand; seven thousand, three hundred twelve; sixty-eight thousand, four hundred twenty-seven?

16. What is the sum of three hundred forty-four million, eight hundred ninety-six thousand, four hundred thirty-five; five million, six thousand, three; forty-eight thousand, two hundred?

17. What is the sum of eighteen dollars, five cents; fifty-one dollars, fifty-one cents; ten dollars, ten cents; eighteen dollars, twenty-four cents; thirty-five dollars, four cents?

18. A owns 345 sheep, B owns 295, C owns 436, D owns 524. How many sheep do all own?

19. A man sold his piano for $413, his collection of paintings for $536, his library for $719, his carpets for $728, other furniture for $1,736, his horses, carriage and two sets of harness for $1,324, and his house for $9,137. How much money did he obtain by the sale?

20. A fruit-dealer shipped for New York, 3,932 bushels of apples in one week, 2,436 in the next, 4,197 in the third, and, within the next month, 10,937 bushels. What was the entire number of bushels shipped by him during that time?

21. A man making his will, left $3,450 to his wife, $2,675 to his oldest son, $1,850 to his second son, and $1,290 to his youngest son. What amount of money was bequeathed in his will?

22. A man owns five horses. The first is worth $225, the second $325, the third $450, the fourth as much as the second and third, and the fifth as much as the first and fourth. What is the value of the five horses?

23. A and B were building a brick store. The first day A laid 2,136 bricks, and the second day he laid as many as the first day plus 207. B, on the first day, laid 1,936, and, on the second day, 341 more than on the first. How many bricks were laid by both in the two days?

24. The distance from Greening to Chatfield is 277 miles, from Chatfield to Glendale is 325 miles, from Glendale to Wyoming is 139 miles, from Wyoming to Dale is 193 miles. By this route what is the distance from Greening to Dale?

25. A man took 2,126 steps going from home to his place of business, 3,197 while in his store, 6,239 going from there to the park, 5,782 while in the park, 8,573 going from there home. What was the whole number of steps taken by him from the time he left until he re-entered his house?

26. In the first story of a house, the hall contained 117 square feet, the parlor 327, the sitting-room 296, the dining-room 257. How many square feet of carpeting would be required to cover the floors of these rooms?

27. In 1870, the population of Buffalo was 117,714; that of Rochester, 62,386; that of Albany, 69,422; that of Brooklyn, 396,099. How many inhabitants did these four cities contain?

28. The area of Spain is 195,773 square miles; that of France, 204,091; that of Switzerland, 15,922; that of Italy, 112,622. Over how many square miles do the four countries extend?

29. A speculator bought five lots for $1,375 each. He sold the first for $125 more than cost, the second for $319 more than cost, the third for $291 more than cost, the fourth for $739 more than cost, and the fifth for $135 more than cost. How much money did he receive for all?

30. The State of Alabama contains 1,430 libraries and 576,882 volumes; Mississippi, 2,788 libraries and 488,482 volumes; Louisiana, 2,332 libraries and 847,406 volumes; Texas, 455 libraries and 87,111 volumes. How many libraries and how many volumes do the four States contain?

31. The population of five of the principal cities of Ohio was in 1870 as follows: Cincinnati, 216,239; Cleveland, 92,829; Toledo, 31,584; Columbus, 31,274; Dayton, 30,473. What was the entire population of these cities in 1870?

32. The population of five of the principal cities of Illinois was in 1870 as follows: Chicago, 298,977; Quincy, 24,025; Peoria, 22,849; Springfield, 17,364; Bloomington, 14,590. What was the entire population of these cities at that time?

33. In 1870, the population of St. Louis, Mo., was 310,864; Memphis, Tenn., 40,226; Charleston, S. C., 48,956; Richmond, Va., 51,038; New Orleans, La., 191,418. What was the entire population of these cities at that time?

34. The Warsaw Manufacturing Company sawed 11,936 feet of pine on Monday, 12,117 feet of hemlock on Tuesday, 8,135 feet of maple on Wednesday, and 9,963 feet of ash on Thursday. How many feet of timber did they saw in the four days?

35. According to the census of 1870, the number of native Americans in Nebraska was 92,245; the number of Irish, 4,999; of Germans, 10,954; of English, 3,602; of Scotch, 792; of Canadians, 2,632; of French, 340; of Norwegians, 506; of Swedes, 2,352. What was the total population of the State in 1870?

36. In a certain State there were raised, last year, 7,771,009 bushels of potatoes, 278,798 bushels of wheat, 1,089,888 bushels of Indian corn, 2,351,354 bushels of oats, 658,816 bushels of barley. What was the entire number of bushels of farm products raised that year?

37. Mr. George Peabody gave to the poor of London \$2,250,000, to the town of Danvers \$60,000, to the Grinnell Arctic Expedition \$10,000, to the city of Baltimore \$1,000,000, to Phillips' Academy \$25,000, to the Massachusetts Historical Society \$20,000, to Harvard College \$150,000, to Yale College \$150,000, to the Southwest \$1,500,000. How much did this benevolent gentleman give away?

38. In 1870, there were, in the United States, 574 daily newspapers, with a circulation of 2,601,547; 107 tri-weeklies, with a circulation of 155,105; 115 semi-weeklies, with a circulation of 247,197; 4295 weeklies, with a circulation of 10,594,643; 96 semi-monthlies, with a circulation of 1,349,820; 622 monthlies, with a circulation of 5,650,843; 13 bi-monthlies, with a circulation of 31,650; 49 quarterlies, with a circulation of 211,670. How many periodicals were there in the United States during that year, and what was their entire circulation?

39. Mr. A. deposited in the First National Bank of Albany, N. Y., on July 3, 1877, $395.25; on July 5, $874.75; on July 8, $325.85. He also deposited in the National Park Bank of New York City, on July 12, 1877, $1,546.87; on July 16, $1,275; on July 20, $1,985.50. How much did he deposit in each of the banks? How much in both banks?

(40.)	(41.)	(42.)	(43.)
2134	6166	5873	46321
8060	5878	3858	69788
5032	9876	6430	76434
8797	7977	5082	68924
9888	6503	6353	96355
6432	4556	4202	88789
5421	6432	8792	93745

(44.)	(45.)	(46.)	(47.)
813	760	3945	5063
976	500	9204	2050
432	750	8769	3254
397	694	9876	4200
788	942	8020	6131
643	293	5612	5945
564	978	3424	2763
321	785	5861	4828
156	696	2188	7688
642	785	7654	3288
321	688	3210	5634
876	762	8765	6546
543	451	5849	3250
429	984	8574	7864
386	579	9836	9758
595	384	8759	8410

SUBTRACTION

INDUCTIVE EXERCISES.

55. 1. If I have 6 peaches and give away 3 of them, how many will be left?

2. If James has 4 bunches of grapes and eats 2 of them, how many will be left?

3. If I have 7 bunches of grapes and give away 4 of them, how many will be left?

4. If you find 8 acorns and lose 4 of them, how many will be left?

5. How many are left when 4 things are taken from 8 things? How many are 8 less 4? 7 less 4? 5 less 4? 9 less 4? 6 less 4?

6. A farmer who had 7 horses, sold 3 of them. How many had he left? How many are 7 less 3? 9 less 3?

7. James earned, during the summer, $9. He spent $5 of the money for a coat, and the rest for a pair of boots. How much did the boots cost him?

8. Nine is how many more than 5? Than 6? Than 4? Than 3?

9. A boy who had 9 chickens, sold 3 of them. How many had he left?

10. Lawrence had 10 pictures in his room. He gave his sister 3 of them. How many were left in his room?

11. A man earned $11 per week and spent $7. How much did he save weekly?

12. A hen had nine chickens, but 5 of them were killed. How many chickens were left? How many must be added to 5 to make 9?

13. When 5 is taken from 9, what number is left?

14. When 7 is taken from 10, what number remains? How many must be added to 7 to equal 10?

15. Howard is 10 years of age and Herbert is 8. What is the difference in their ages? What is the difference between 10 and 8?

16. If the difference between 10 and 8 be added to 8, what will the result be?

17. If the difference between any two numbers be added to the smaller number, to what will the result be equal?

18. What is the difference between 6 horses and 4 horses? Between 6 horses and 5 cents?

19. Why can you not find the difference between 6 horses and 5 cents?

20. Why can you find the difference between 6 horses and 4 horses?

21. Between what kinds of numbers only can the difference be found?

DEFINITIONS.

56. ***Subtraction*** is the process of taking one number from another.

57. The ***Minuend*** is the number from which another is to be subtracted.

58. The ***Subtrahend*** is the number to be subtracted.

59. The ***Remainder,*** or ***Difference,*** is the result obtained by subtracting.

60. The ***Sign of Subtraction*** is a short horizontal line: —. It is named *minus*.

When the sign *minus* is placed between two numbers it shows that the one after it is to be subtracted from the one before it.

Thus, 9 — 5 is read 9 minus 5, and means that 5 is to be subtracted from 9.

61. Principle.—1. *Only like numbers can be subtracted.*

2. *The sum of the subtrahend and remainder must be equal to the minuend.*

TABLE.

1—1= 0	2—2= 0	3—3= 0	4—4= 0	5— 5= 0
2—1= 1	3—2= 1	4—3= 1	5—4= 1	6— 5= 1
3—1= 2	4—2= 2	5—3= 2	6—4= 2	7— 5= 2
4—1= 3	5—2= 3	6—3= 3	7—4= 3	8— 5= 3
5—1= 4	6—2= 4	7—3= 4	8—4= 4	9— 5= 4
6—1= 5	7—2= 5	8—3= 5	9—4= 5	10— 5= 5
7—1= 6	8—2= 6	9—3= 6	10—4= 6	11— 5= 6
8—1= 7	9—2= 7	10—3= 7	11—4= 7	12— 5= 7
9—1= 8	10—2= 8	11—3= 8	12—4= 8	13— 5= 8
10—1= 9	11—2= 9	12—3= 9	13—4= 9	14— 5= 9
11—1=10	12—2=10	13—3=10	14—4=10	15— 5=10
6—6= 0	7—7= 0	8—8= 0	9—9= 0	10—10= 0
7—6= 1	8—7= 1	9—8= 1	10—9= 1	11—10= 1
8—6= 2	9—7= 2	10—8= 2	11—9= 2	12—10= 2
9—6= 3	10—7= 3	11—8= 3	12—9= 3	13—10= 3
10—6= 4	11—7= 4	12—8= 4	13—9= 4	14—10= 4
11—6= 5	12—7= 5	13—8= 5	14—9= 5	15—10= 5
12—6= 6	13—7= 6	14—8= 6	15—9= 6	16—10= 6
13—6= 7	14—7= 7	15—8= 7	16—9= 7	17—10= 7
14—6= 8	15—7= 8	16—8= 8	17—9= 8	18—10= 8
15—6= 9	16—7= 9	17—8= 9	18—9= 9	19—10= 9
16—6=10	17—7=10	18—8=10	19—9=10	20—10=10

CASE I.

62. When no figure of the subtrahend has a greater value than the corresponding figure of the minuend.

1. A merchant had 15 barrels of flour, and sold 4 of them. How many had he left?

ANALYSIS.—Since he had 15 barrels of flour and sold 4 of them, he had left the difference between 15 barrels and 4 barrels, which is 11 barrels.

2. Alice bought 18 cakes, and ate 6 of them. How many had she left?

3. James saw 17 birds on a tree, but 7 soon flew away. How many remained?

4. If a man earns $19 a week, and spends $9, how much will he save each week?

5. Lewis owed his brother $7, and paid him $3. How much did he still owe him?

6. Eliza had 16 plums, but gave 5 to her father. How many had she left?

7. James had $12, and lost $2. How many had he left?

8. If John is 19 years old, and Maggie 13, how much younger than John is Maggie?

9. In the same shop 6 boys and 17 men work. How many more men than boys are there in the shop?

10. There were 18 girls and 7 boys in a class. How many more girls than boys were there?

11. Laura had 14 cents, and lost 3 cents. How many had she then?

12. Henry solved 19 examples, and George solved 8. How many more did Henry solve than George?

13. William wrote 16 lines in his copy-book, and Peter wrote 5 lines less. How many did Peter write?

14. One piece of cloth contained 20 yards and another 10 yards. How many yards more were there in the larger piece?

15. A boy had 24 chickens, and 10 of them died. How many had he left?

16. Julia gave me 11 cents. If she had 16 cents at first, how many had she left?

17. A girl bought 18 eggs, and, on her way home, fell and broke 5 of them. How many had she left?

18. Subtract by 2's from 22 to 0; thus: 22, 20, 18, 16, 14, 12, etc.

19. Subtract by 3's from 35 to 2. From 45 to 0.

20. Subtract by 4's from 48 to 0. From 45 to 1.

21. Count back by 5's from 35 to 0. From 59 to 4.

WRITTEN EXERCISES.

63. 1. From 547 subtract 235.

PROCESS.

Minuend	547
Subtrahend	235
Remainder	312

ANALYSIS.—For convenience we write the less number under the greater, units under units, tens under tens, etc., and subtract each order of units separately from the same order of the minuend.

Thus, 7 units — 5 units = 2 units, which we write under the units.

4 tens — 3 tens = 1 ten, which we write under the tens.

5 hundreds — 2 hundreds = 3 hundreds, which we write under the hundreds.

Hence the remainder is 312.

PROOF.—312, the remainder, plus 235, the subtrahend, equals 547, the minuend. Hence the result is correct.

Copy, subtract, and prove:

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
713	458	986	854	795	7842
302	134	732	641	433	2310

(8.)	(9.)	(10.)	(11.)	(12.)
$48.25	$64.29	$45.78	$38.94	$41.89
23.13	30.29	34.65	27.83	20.45

13. A drover, having 1583 sheep, sold 1441 of them. How many had he left?

14. A speculator bought some land for $5849.75, and sold it for $6959.95. How much did he gain?

15. A cotton factory made 9875 yards of cloth in one week, and sold, during the same time, 7652 yards. How much more was made than sold?

16. A money-lender received for interest, during 1875, $1685.49, and during 1876, $2796.59. In which year did he receive the greater sum, and how much?

17. A man bought 7467 bricks, and carted away 3136. How many remained to be moved?

18. A has 736 sheep, and B has 213 less than A. How many sheep has B?

19. A man gave his note for $6792, without interest. In two years he had paid $3401. How much did he still owe on the note?

20. A man bought a house for $1765, and afterward sold it, thereby losing $504. For how much did he sell it?

21. A man bought a span of horses for $364, and a yoke of oxen for $120. How much more did he give for the horses than for the oxen?

22. A merchant having 6755 yards of cloth, sold 2532 yards. How many yards had he remaining?

23. A father having 3652 acres of land, gave his son 1230 acres. How many acres had he left?

24. A vintner had 38756 gallons of wine, and sold during the year, 34243 gallons. How much remained unsold?

25. The circulation of a newspaper in 1875 was 38293 copies, and in 1876, 37180. What was the decrease?

CASE II.

64. When any figure of the subtrahend has a greater value than the corresponding figure of the minuend.

1. A gentleman bought a coat at \$40, and a vest at \$9; he gave the merchant a hundred-dollar bill. How much change ought he to receive?

ANALYSIS.—Since he paid \$40 for a coat and \$9 for a vest, for both he paid the sum of \$40 and \$9, or \$49. And since he gave the merchant \$100, he should receive the difference between \$100 and \$49. \$100 — \$40 = \$60; \$60 — \$9 = \$51. Therefore he should receive \$51.

2. A boy saw 15 birds on a tree, and 9 of them flew away. How many remained?

3. John is 16 years old, and James is 8. How much older than James is John?

4. A jeweler bought a watch for \$75, and sold it for \$100. How much did he gain by the operation?

5. A grocer bought a quantity of sugar for \$36, and retailed the same for \$50. How much did he gain by the sale?

6. A boy had 34 marbles, and gave away 9 of them. How many had he left?

7. A lady bought a chair for \$3, and a table for \$5; she gave a twenty-dollar bill to the cabinet-maker. How much change ought she to receive?

8. A man set out to walk 50 miles; he walked 20 miles the first day, and 19 the second day. How many miles were left for him to walk?

9. A man bought a cow for \$35, and sold her for \$43, after keeping her 4 weeks at an expense of \$2 per week. How much did he gain?

10. A man who earned \$60 a month, paid \$25 a month for his board, and \$15 a month for other expenses. How much did he save per month?

11. Count back by 10's from 107 to 7; thus: 107, 97, etc.
12. Count back by 10's from 95 to 5. From 79 to 9.
13. Count back by 10's from 83 to 13. From 98 to 18.
14. Subtract by 20's from 106 to 26; thus: 106, 86, etc.

WRITTEN EXERCISES.

65. 1. From 643 subtract 456.

PROCESS.

643
456
———
187

ANALYSIS.—We write the numbers as in the previous case and begin at the right to subtract.

Since 6 units can not be subtracted from 3 units, we unite with the 3 units a unit of the next higher order, which is equal to 10 units, making 13 units: 6 units from 13 units leave 7 units, which we write under the units.

Since *one* of tens was united with the units, there can be but 3 tens left. Because 5 tens can not be subtracted from 3 tens, we unite with the 3 tens as before, a unit of the next higher order, which is equal to 10 tens, making 13 tens: 5 tens from 13 tens leave 8 tens, which we write under the tens.

Since *one* of the hundreds was united with the tens, there are but 5 hundreds left: 4 hundreds from 5 hundreds leave 1 hundred, which we write under the hundreds. Hence the result is 187.

PROOF.—187, the remainder, plus 456, the subtrahend, equals 643, the minuend. Hence the result is correct.

66. RULE.—*Write the subtrahend under the minuend, units under units, tens under tens, etc.*

Begin at the right and subtract each figure of the subtrahend from the corresponding figure of the minuend, writing the result beneath.

If a figure in the minuend has a less value than the corresponding figure in the subtrahend, increase the former by ten, and subtract; then diminish by one, the units of the next higher order in the minuend, and subtract as before.

PROOF.—*Add together the remainder and subtrahend. If the result be equal to the minuend the work is correct.*

EXAMPLES.

Copy, subtract, and prove:

(2.)	(3.)	(4.)	(5.)	(6.)
753	984	826	754	1426
448	756	534	482	547

(7.)	(8.)	(9.)	(10.)	(11.)
843	1846	1683	2897	3001
782	927	1395	1598	2851

(12.)	(13.)	(14.)	(15.)	(16.)
$24.45	$39.18	$63.25	$71.89	$42.34
21.38	27.92	47.18	47.93	18.67

Find the difference between

17. 583 and 294.
18. 690 and 508.
19. 763 and 574.
20. 966 and 599.
21. 982 and 796.
22. 891 and 798.
23. 5833 and 4968.
24. 7521 and 3635.
25. 7812 and 1984.
26. 8003 and 5872.
27. 63004 and 54872
28. 65432 and 54862.
29. 69721 and 49653.
30. 78303 and 49424.
31. 865932 and 785841.
32. 9050308 and 563420.

33. A man set out on a journey of 861 miles. During the first day he traveled 297 miles, and during the second day 308 miles. How many miles had he yet to travel?

34. A merchant deposited in a bank on Monday $584; on Tuesday, $759; on Wednesday, $463. He drew out $1298 during that time. How much did his deposits exceed what he drew out?

35. A grocer had 3715 pounds of sugar on hand. On one day he sold 1235 pounds, on the next 1317; the third day he sold to C all the sugar that remained. How many pounds did C buy?

36. I bought a horse for \$637, and a cow for \$317. I sold the horse for \$729, and the cow for \$356. How much did I gain by the sale?

37. In the first of three pavements there are 1317 bricks, in the second there are 2357, in the third there are 1719 less than in both the others. How many bricks in the third pavement?

38. In 1869 there were 264,146,900 bushels of wheat raised in the United States, and 874,120,005 bushels of corn. How much more corn than wheat was produced?

39. A bought 351 acres of land, and B bought 27 acres more than A; B sold his land to C, who then had 537 acres. How many acres did C have at first?

40. A grocer retailed a quantity of sugar for \$308.40, and so gained \$106.28. How much had he paid for it?

41. The year 1870 was just 378 years after the discovery of America by Columbus. In what year did that event take place?

42. On Monday morning a bank had on hand \$1826. During the day \$2191 were deposited and \$3412 drawn out; on Tuesday \$3256 were deposited and \$2164 drawn out. How many dollars were on hand Wednesday morning?

43. R. S. Hill is worth \$15795, of which \$2895 is invested in bank stock, \$3864 in mortgages and the rest in land. How much has he invested in land?

44. Of the two numbers 89346 and 56849, how much nearer is the one than the other to 68754?

45. The number of pupils who attended school in Boston in 1870 was 38944, and of this number 35442 attended the public schools. How many attended the other schools?

MULTIPLICATION

INDUCTIVE EXERCISES.

67. 1. How many books are there in 2 piles containing 3 books each?

2. If you place 4 apples in a group, how many apples are there in 3 such groups? In 4 groups?

3. When there are 3 roses in a cluster, how many are there in 3 clusters? In 4 clusters? In 5 clusters?

4. How many are $3+3+3+3$, or four 3's?

5. How many are $4+4+4$, or three 4's?

6. How many are four 4's? Four 5's? Four 6's?

7. James bought 5 pencils at 5 cents each. How much did they cost him? How many cents are 5 times 5 cents? How many are five 5's?

8. An orchard contains 5 rows of 6 trees each. How many trees are there in the orchard? How many trees are 5 times 6 trees? How many are 5 times 6?

9. James piled his blocks in 3 piles, each containing 5 blocks. How many blocks had he? How many are 3 times 5 blocks? How many are 3 times 5?

10. A boy earned $4 a week for 6 weeks. How much did he earn in all? How many dollars are 6 times $4? How many are 6 times 4?

11. Harry played 5 hours per day. How many hours did he play in 6 days? How many are 6 times 5 hours? How many are 6 times 5?

12. How does 5 times 4 compare with 4 times 5? 5 times 6 with 6 times 5?

13. When numbers are used without reference to any particular thing, they are called ***Abstract Numbers.***

DEFINITIONS.

68. ***Multiplication*** is a short process of finding the sum of equal numbers; or,

The process of repeating one number as many times as there are units in another.

69. The ***Multiplicand*** is the number to be repeated or multiplied.

70. The ***Multiplier*** is the number showing how many times the multiplicand is to be repeated.

71. The ***Product*** is the result obtained by multiplying.

72. The multiplicand and multiplier are called the *factors* of the *product.*

73. The ***Sign of Multiplication*** is an oblique cross: $\times$. It is read, *multiplied by*, or *times.* When placed between two numbers it shows that they are to be multiplied together.

Thus, 4×3 is read, 4 multiplied by 3, or 3 times 4.

74. PRINCIPLES.—1. *The multiplier must be regarded as an abstract number.*

2. *The multiplicand and product must be like numbers.*

3. *Either of the factors may be used as multiplicand or multiplier when both are abstract.*

In practice, for convenience, the smaller number is generally used as multiplier.

TABLE.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40	5 × 10 = 50
6 × 1 = 6	7 × 1 = 7	8 × 1 = 8	9 × 1 = 9	10 × 1 = 10
6 × 2 = 12	7 × 2 = 14	8 × 2 = 16	9 × 2 = 18	10 × 2 = 20
6 × 3 = 18	7 × 3 = 21	8 × 3 = 24	9 × 3 = 27	10 × 3 = 30
6 × 4 = 24	7 × 4 = 28	8 × 4 = 32	9 × 4 = 36	10 × 4 = 40
6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45	10 × 5 = 50
6 × 6 = 36	7 × 6 = 42	8 × 6 = 48	9 × 6 = 54	10 × 6 = 60
6 × 7 = 42	7 × 7 = 49	8 × 7 = 56	9 × 7 = 63	10 × 7 = 70
6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	9 × 8 = 72	10 × 8 = 80
6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81	10 × 9 = 90
6 × 10 = 60	7 × 10 = 70	8 × 10 = 80	9 × 10 = 90	10 × 10 = 100

CASE I.

75. When the multiplier is expressed by one figure.

1. What will 5 yards of ribbon cost at 6 cents a yard?

Analysis.—Since 1 yard of ribbon costs 6 cents, 5 yards will cost 5 times 6 cents, or 30 cents.

2. At 7 cents each, what will 8 pencils cost?
3. What will be the cost of 8 melons at 9 cents each?

4. What will 6 sheep cost at $6 a head?

5. When grapes are worth 7 cents a bunch, how much will 9 bunches cost?

6. James bought 5 rabbits for 7 dimes each, and sold them for 5 dimes each more than he paid for them. How much did he gain? How much did he get for them?

7. How much did he get for each rabbit? How does 5 times 12 compare with 5 times 7 *plus* 5 times 5?

8. How many are 5 times 15, or 5 times 10 *plus* 5 times 5?

9. How many are 3 times 18, or 3 times 10 *plus* 3 times 8?

10. James sold 7 doves for 19 cents each. How much did he get for them?

11. What will be the cost of 8 slates at 15 cents apiece?

12. If one inkstand costs 18 cents, how much will 4 cost?

13. A lady bought 7 yards of ribbon at 19 cents a yard. How much did she pay for it?

14. If a man walked 16 miles in one day, how far could he walk in 3 days?

15. How much can a boy earn in 8 hours if he earns 13 cents in an hour?

16. How many are 5 times 20? 5 times 7? 5 times 20 *plus* 5 times 7, or 5 times 27?

17. How many are 5 times 23, or 5 times 20 *plus* 5 times 3?

18. How many are 7 times 24, or 7 times 20 *plus* 7 times 4?

19. How many are 9 times 22, or 9 times 20 *plus* 9 times 2?

20. If there are 25 yards in one piece of cloth, how many yards are there in 8 pieces?

21. A boy bought 7 chickens at 27 cents apiece. How much did he pay for all?

22. If a steamship can sail 28 miles in one hour, how far can she sail in 8 hours?

23. If the cars run 26 miles an hour, how far will they run in 5 hours? How far in 6 hours? How far in 7 hours?

24. If 6 men can do a piece of work in 21 days, how long will it take one man to do the same work?

25. In a certain orchard there are 9 rows of trees and 27 trees in a row. How many trees are there in the orchard?

26. Count by 2's from 0 to 24; thus: 2, 4, 6, 8, 10, etc.

27. Count by 3's from 0 to 36. By 4's from 0 to 48.

28. Repeat all the numbers of times 5 from *once* 5 to 10 times 5. Thus, *once* 5 is 5, 2 times 5 are 10, 3 times 5 are 15, etc.

29. Repeat from *once* 6 to 10 times 6, and back from 10 times 6 to *once* 6.

30. Repeat from *once* 7 to 10 times 7, and reverse.

31. Repeat from *once* 8 to 10 times 8, and reverse.

32. Repeat from *once* 9 to 10 times 9, and reverse.

33. Repeat from *once* 10 to 10 times 10, and reverse.

34. At 25 cents a pound, how much will 6 pounds of raisins cost?

35. If a man can dig 28 bushels of potatoes in one day, how many can he dig in 4 days?

36. If a person spend 25 cents a day for cigars, how much will he spend in 7 days?

37. If a boy earns 33 cents a day, how much will he earn in 9 days?

38. When butter is selling at 37 cents a pound, what will 7 pounds cost me?

WRITTEN EXERCISES.

76. 1. How many are 3 times 275?

1ST PROCESS.

	275
	275
	275
Sum	825

ANALYSIS.—Since multiplication is a short process of adding equal numbers, it is evident that we can determine by addition how many 3 times 275, or three 275's, are. The sum is 825.

In practice, a shorter method is employed, which is given in the second process and analysis.

2D PROCESS.	
Multiplicand	275
Multiplier	3
Product	825

ANALYSIS.—For convenience we write the multiplier under the multiplicand, and begin at the right to multiply. Thus, 3 times 5 units are 15 units, or 1 ten and 5 units. We write the 5 units in units' place in the product and reserve the tens to add with the tens.

3 times 7 tens are 21 tens, plus 1 ten reserved are 22 tens, or 2 hundreds and 2 tens. We write the 2 tens in tens' place in the product and reserve the hundreds to add with the hundreds.

3 times 2 hundreds are 6 hundreds, plus 2 hundreds reserved are 8 hundreds, which we write in hundreds' place in the product. Hence the *product* is 825, the same as the *sum* in the first process.

PROOF.—If the results obtained by both processes agree, the work is probably correct.

In multiplying, pronounce the results only. Thus, in the example given above, instead of saying 3 times 5 are 15, 3 times 7 are 21, plus 1 reserved are 22; 3 times 2 are 6, plus 2 reserved are 8; say 15, 22, 8.

Solve and prove:

2. 3 times 314.	4. 4 times 987.	6. 5 times $819.
3. 4 times 568.	5. 5 times 345.	7. 3 times $769.

How many are

8. 5 times 314?	11. 3 times 830?	14. 8 times $42?
9. 4 times 655?	12. 6 times 734?	15. 6 times $32?
10. 7 times 764?	13. 9 times $48?	16. 7 times $57?

17. If a man earns $17.25 per week, how much can he earn in 8 weeks?

18. A benevolent man paid annually for the support of the poor $2365. How much did he pay in 7 years?

19. A shoe dealer sold 9 pairs of shoes at $3.75 a pair. How much did he receive for all?

20. A man bought 8 cows at an average price of $31.27. How much did they all cost him?

21. If a ship sail 425 miles in one week, how far will she sail in 9 weeks?

22. A barrel of flour weighs 196 pounds. How much will 8 barrels weigh?

23. When wheat is worth \$1.78 per bushel, how much can be realized from the sale of 9 bushels?

24. At \$6.25 a pair, what will be the cost of 7 pairs of boots?

25. There are 5280 feet in a mile. How many feet in 7 miles?

26. At \$37.50 an acre, what will be the cost of 8 acres of land?

27. What will be the cost of 7 thousand feet of lumber at \$18.25 per thousand?

28. When broom corn is selling at \$83.50 a ton what is the value of 8 tons?

CASE II.

77. When the multiplier is expressed by more than one figure.

1. There are 9 square feet in one square yard. How many are there in 10 square yards?

2. How many square feet in 6 square yards?

3. Since 10 square yards contain 90 square feet, and 6 square yards contain 54 square feet, how may the number of square feet in 10 + 6, or 16, square yards, be found? How, then, may you multiply by 16? By 18? By 13?

4. Find the cost of 17 yards of cloth at 18 cents a yard?

5. When eggs are 21 cents a dozen, what will 15 dozen cost?

6. Since 12 inches make one foot in length, how many inches are there in 18 feet?

7. A pound of sugar is equal to 16 ounces. How many ounces are there in a quantity of sugar weighing 16 pounds?

7. Find the cost of 17 yards of cloth at 8 cents a yard, by finding the cost of 9 yards, and then of 8 yards. Of 10 yards and 7 yards.

8. What will be the cost of 11 primers at 25 cents each?

9. Find the cost of 16 yards of cloth at 8 cents a yard, by finding the cost of 10 yards and 6 yards. 9 yards and 7 yards. 8 yards and 8 yards.

10. James is in school 5 hours a day. How many hours is he in school during three weeks, or 15 school-days?

11. A bought 4 sets of forks, each set containing 6 forks. How much did the forks cost him at $2 each?

12. Mary bought 15 pounds of sugar at 11 cents a pound, and 3 pounds of raisins at 15 cents a pound. After paying her bill she had 10 cents left. How much money had she at first?

13. A cooper can make 12 barrels a day. How many can he make in 12 days?

14. John bought 12 lead pencils at 8 cents each, and 2 erasers at 4 cents each. How much did all cost him?

15. The railroad fare from Rochester to New York is $7. How much will the tickets for a party of 9 cost?

16. If a cow give 9 quarts of milk a day, how much milk will she give in 9 days?

17. If a man put $8 in a savings-bank each month, how much will he deposit in a year?

18. At $4 a yard, what will 17 yards of broadcloth cost?

19. If a laborer can earn $2 a day, how much can he earn in 12 days?

20. What will 15 pairs of skates cost at $4 a pair?

21. At 20 cents a dozen, how much will 18 dozen eggs cost?

22. A coal dealer sold an average of 18 tons of coal per day for 12 days. How many tons did he sell in that time?

23. At 22 cents a pound, how much will 11 pounds of butter cost?

24. How far will a man travel in 15 days, if he travel 10 hours a day and 3 miles an hour?

25. A man bought 25 cows and 12 times as many sheep. How many sheep did he buy?

WRITTEN EXERCISES.

78. 1. Multiply 327 by 123.

	1ST PROCESS.
	327
	123
1st Partial Prod.	981
2d Partial Prod.	6540
3d Partial Prod.	32700
Entire Prod.	40221

ANALYSIS.—For convenience we write the numbers as in the preceding case. Since in multiplying we must multiply by the parts of the multiplier and add the partial products, to multiply by 123 we multiply by 3 units, 2 tens, and 1 hundred as partial multipliers.

3 times 327 are 981, the *first* partial product; 2 times 327 are 654 and 2 *tens* times 327 are 654 *tens*, or 6540, which we write for a second partial product.

1 time 327 equals 327, and 1 *hundred* times 327 are 327 hundreds, or 32700, which we write for a third partial product. The sum of the partial products will be the entire product.

	2D PROCESS.
	327
	123
1st Partial Prod.	981
2d Partial Prod.	654␣
3d Partial Prod.	327␣␣
Entire Prod.	40221

ANALYSIS.—In the second process the ciphers at the right of the partial products are omitted, the significant figures still occupying their proper places. Thus, in multiplying by 2 *tens* the product was 654 *tens*, or 6 thousand, 5 hundred, 4 tens, which we write in their places in the partial product.

In multiplying by hundreds, the lowest order of the product is hundreds, hence we write the first figure of the product under hundreds.

PROOF.—Multiply the multiplier by the multiplicand. (Prin. 3.) If the result agrees with that formerly obtained, the work is probably correct.

RULE.—*Write the multiplier under the multiplicand with units under units, tens under tens, etc.*

Multiply each figure of the multiplicand by each significant figure of the multiplier successively, beginning with units. Place the right hand figure of each product under the figure of the multiplier used to obtain it, and add the partial products.

PROOF.—*Review the work, or multiply the multiplier by the multiplicand. If the results agree the work is probably correct.*

EXAMPLES.

	(2.)	(3.)	(4.)	(5.)	(6.)
Multiply	325	219	384	$2.81	$3.18
By	42	54	46	23	36

Multiply:

7. 456 by 12.
8. 389 by 23.
9. 493 by 25.
10. 374 by 27.
11. 3625 by 28.
12. 2413 by 31.
13. 3681 by 63.
14. 67021 by 52.
15. 63583 by 62.
16. 84216 by 78.
17. 38413 by 35.
18. 29615 by 45.
19. 23423 by 25.
20. 24542 by 64.
21. 45684 by 73.
22. 41075 by 62.

Multiply:

23. 73982 by 321.
24. 42586 by 604.
25. 89258 by 703.
26. 84206 by 569.
27. 156783 by 423.
28. 248164 by 372.
29. 182642 by 419.
30. 192573 by 429.
31. 234567 by 612.
32. 467105 by 623.
33. 398120 by 706.
34. 683912 by 1684.
35. 312465 by 1827.
36. 468975 by 2946.
37. 416004 by 3009.
38. 329706 by 3802.

39. $ 18.61 by 73.	44. $ 18.37 by 127.
40. $115.81 by 45.	45. $113.41 by 613.
41. $164.32 by 81.	46. $281.69 by 247.
42. $123.45 by 804.	47. $312.09 by 684.
43. $415.05 by 367.	48. $425.27 by 618.

49. In a reaper factory an average of 2346 reapers is constructed annually. At this rate how many would be made in 25 years?

50. A farmer counted the trees in his orchard and found that he had 104 rows, each row containing 106 trees. How many trees were there in the orchard?

51. In a croquet factory a man makes 835 balls daily. How many balls can he make in 312 days?

52. The distance between Rochester and Syracuse is 81 miles. How many miles per month of 31 days, will a locomotive travel that goes from Rochester to Syracuse daily and returns?

53. Mr. Davis built 8 houses at a cost of $1925 each 6 at $2275 each, and 5 at $3897 each. What did they all cost him?

54. Sold my farm of 413 acres at $85 per acre. How much did I get for it?

55. How much will it cost to build 89 miles of railroad at an estimated expense of $57394 per mile?

CASE III.

79. Where there are ciphers on the right of either or both factors.

1. How many are are 10 times 2? 3? 4? 5? 6? 7? 8? 9?

2. Write the above multiplicands and products side by side and compare them.

3. How may the product be found from the multiplicand when the multiplier is 10 ?

4. How many are 100 times 2? 3? 4? 5? 6? 7? 8? 9?

5. How may any number be multiplied by 100? by 1000?

6. How may a number be multiplied by 1 with any number of ciphers affixed ?

80. PRINCIPLE.—*In multiplying by 10, 100, 1000, etc., as many ciphers must be annexed to the right of the multiplicand as there are ciphers in the multiplier.*

1. Multiply 36 by 1000.

PROCESS.

$$\begin{array}{r} 36 \\ 1000 \\ \hline 36000 \end{array}$$

ANALYSIS.—Since in multiplying by 1 with any number of ciphers annexed, we annex as many ciphers to the multiplicand as there are in the multiplier, to multiply by 1000 we annex *three* ciphers to the multiplicand, which gives the product 36000.

2. Multiply 2360 by 400.

PROCESS.

$$\begin{array}{r} 2360 \\ 400 \\ \hline 944000 \end{array}$$

ANALYSIS.—Since 2360 is equal to 236×10, and 400 is equal to 4×100, the product of 2360×400 may be obtained by multiplying 236 by 4, and this product by 10 times 100, or 1000. The product of 236×4 is 944, and this may be multiplied by 1000 by annexing *three ciphers* (Prin.), giving as a result 944000.

RULE.—*Multiply without regard to the ciphers on the right, and to the product annex as many ciphers as there are on the right of both multiplier and multiplicand.*

3. Multiply 375 by 10. By 100. By 40. By 300.
4. Multiply 845 by 30. By 70. By 600. By 900.
5. Multiply 176 by 500. By 700. By 400. By 1000.
6. Multiply 1385 by 200. By 2000. By 2200. By 3300.
7. Multiply 4860 by 250. By 3200. By 4200. By 6500.
8 Multiply 3120 by 210. By 3800. By 2700. By 4600.

9. In a mile there are 5280 feet. How many feet are there in 500 miles?

10. In an acre there are 160 square rods. How many square rods are there in a farm of 300 acres?

11. A farmer sold a flock of 260 sheep at $3.20 per head. How much did he get for them?

12. A drover sold 1120 hogs at an average price of $16.30 per head. How much did he receive for them?

EXAMPLES.

81. 1. What will be the cost of 896 chests of tea, each chest containing 58 pounds, at 63 cents a pound?

2. An agent sold 3923 Lyman's Historical Charts at $3.50 each. How much did he receive for them?

3. I have 6 bins that hold 119 bushels each. They are full of grain and I have already sold 515 bushels. It was all raised on my farm this year. How much grain was raised?

4. A. J. Newton & Co. bought 113 cases of calico, each case containing 64 pieces, and each piece 47 yards. How many yards did they buy?

5. A drover bought 25 oxen at $85 a head, 316 sheep at $4.50 a head, and 94 calves at $8 a head. What was the whole amount paid?

6. A man insured 2 houses valued at $3750 and $4650, respectively, at the rate of $2 per hundred dollars. How much did the insurance cost him?

7. If I have 219 acres of land, and each acre produces 47 bushels of corn, how many bushels do I receive?

8. How many quills can be obtained from 398 geese, if each wing furnishes 6 quills?

9. A grocer sold in one month 81 dozen eggs at 26 cents per dozen; in the next, 53 dozen at 28 cents per dozen. How much money did he receive for the eggs?

10. It requires 1716 pickets to fence one side of a square lot. How many pickets will be required to fence 13 lots of the same size and shape?

11. A sold 13 firkins of butter, each firkin containing 56 pounds, at $ .34 a pound. How much did he receive for it?

12. A coal dealer bought 13 car loads of coal, each load containing 10 tons, at $6.85 a ton. He retailed 48 tons of this at $7 per ton, 28 tons at $8.25 per ton, 27 tons at $8.75 per ton, and the remainder at $9.50 per ton. How much did he make by the transaction?

13. An army lost in battle 315 killed, 417 wounded; the enemy lost in killed and wounded, together, 13 times as many. How many soldiers were killed and wounded in this battle?

14. If two steamers should leave New York at the same time, and should sail in the same direction, the first at the rate of 18 miles an hour, the second at the rate of 15 miles an hour, how far apart would they be in 36 hours?

15. Mr. Hudson bought 350 bushels of corn at 65 cents a bushel, 215 bushels of wheat at $1.35 per bushel, and 273 bushels of oats at 43 cents a bushel. What did the whole cost him?

16. Mr. Henderson sold a farm of 325 acres at $65.50 per acre, and received in payment 345 sheep at $3.25 per head, a note for $2684.95 and the rest in cash. How much cash did he receive?

17. A cloth merchant sold two lots of cassimeres, the first containing 17 pieces of 28 yards each, at $1.75 per yard, the second containing 23 pieces averaging 29 yards each, at $1.85 per yard. What was the value of the whole?

18. An excursion train composed of 13 passenger coaches, each containing 37 persons, went from Syracuse to Niagara Falls and back. If the fare to Niagara Falls and return to Syracuse, was $3.25 per ticket, how much did the railroad company receive?

DIVISION

INDUCTIVE EXERCISES.

82. 1. How many groups of 2 birds each can be formed from 6 birds? How many 2's are there in 6?

2. How many groups of 3 sheep each can be formed from 9 sheep? How many 3's are there in 9?

3. How many groups of 2 chickens each can be formed from 10 chickens? How many 2's are there in 10?

4. At 5 cents apiece, how many pencils can be bought for 10 cents? How many 5's are there in 10?

5. When milk is worth 7 cents a quart, how many quarts can be bought for 28 cents? How many 7's are there in 28?

6. There are 20 panes of glass in the front of a block of stores. If each window contains 4 panes, how many windows are there? How many 4's are there in 20?

7. At 8 cents a dozen, how many peaches can be bought for 24 cents? How many times 8 cents are 24 cents? How many 8's are there in 24?

8. How many groups of 4 things each can be formed from 16 things? How many 4's are there in 16?

9. A merchant had 30 yards of calico which he cut into pieces 5 yards long. How many pieces did it make? How many 5's are there in 30? How many times is 5 contained in 30?

10. How many 9's are there in 18? How many times is 9 contained in 18?

11. How many 5's are there in 10? In 15? In 20? In 25? In 30?

12. If 15 cents are divided equally among 3 boys, how many cents will each receive? When 15 cents are divided into 3 equal parts, how many cents will each part contain?

13. If 12 peaches are arranged in 3 rows, how many will there be in each row?

14. What is one of the 4 equal parts of 8? Of 12? Of 16?

15. How many 3's are there in 30? How many are 10 threes, or 10 times 3?

16. How many 4's are there in 40? How many times is 4 contained in 40? How many are 10 fours?

DEFINITIONS.

83. ***Division*** is the process of finding how many times one number is contained in another; or,

The process of separating a number into equal parts.

84. The ***Dividend*** is the number to be divided.

85. The ***Divisor*** is the number by which we divide. It shows into how many equal parts the dividend is to be divided.

86. The ***Quotient*** is the result obtained by division. It shows how many times the divisor is contained in the dividend.

87. The part of the dividend remaining when the division is not exact is called the ***Remainder.***

88. The ***Sign of Division*** is $\div$. It is read *divided by.* When placed between two numbers it shows that the one at the left is to be divided by the one at the right.

Thus, $154 \div 7$, is read 154 divided by 7.

Division is also indicated by placing the dividend above the divisor with a line between them, and by writing the divisor at the left of the dividend with a curved line between them. Thus, 154 divided by 7, may also be written $\frac{154}{7}$, and 7)154.

89. PRINCIPLES.—1. *The dividend and divisor must be like numbers.*

2. *The quotient must be an abstract number.*

3. *The product of the divisor by the quotient, plus the remainder, is equal to the dividend.*

$$\begin{array}{r} 9 \\ 3 \\ \hline 6 \\ 3 \\ \hline 3 \\ 3 \\ \hline \end{array}$$

1. In problems where it is required to separate a number into equal parts, it is customary to regard the dividend and quotient as like numbers, and the divisor as an abstract number.

2. The example "How many 3's are there in 9" may be solved, as in the margin, by *subtraction*. All examples in division may be solved in the same manner. Hence, division may be regarded as a *short method of subtracting equal numbers.*

3. In multiplication two numbers are given to find their *product* In division the *product* is given and one of the factors to find the other. Hence, division is the *converse of multiplication.*

CASE I.

90. When the divisor is expressed by one figure.

1. At $8 each, how many plows can be bought for $24?

ANALYSIS.—Since each plow costs $8, as many plows can be bought for $24 as $8 is contained times in $24, which is 3 times. Therefore 3 plows can be bought for $24.

2. If a man can earn $7 in a day, how long will it take him to earn $28?

3. At $4 each, how many hats can be bought for $24?

4. When flour is selling at $6 a hundred-weight, how many hundred-weight can be bought for $36?

5. If a mason built 3 rods of walk per day, how long did it take him to build 21 rods?

6. B paid 96 cents for glass at 8 cents a pane. How many panes did he buy?

7. At $9 a cord, how many cords of wood can be bought for $45?

8. If a man earns $11 a week, how many weeks will he require to earn $66?

9. How many lots of 11 acres each can be made from a farm containing 132 acres?

10. If a farmer exchanges 6 firkins of butter worth $20 a firkin for cloth at $4 a yard, how many yards will he receive?

11. My coal cost me $35 at the rate of $7 a ton. How many tons did I purchase?

12. How many engravings must an artist sell for $12 apiece to realize $84?

13. When sugar is worth 9 cents a pound, how many pounds can be bought for 45 cents?

14. At the rate of $7 a rod, how many rods of fence can be built for $63?

15. I hired a man for $45 to do a piece of work at the rate of $5 a day. How many days did it take him?

16. A lady bought some silk worth $3 a yard, paying $36 for it. How many yards did she buy?

17. How many barrels of flour at $8 a barrel can be bought for $48?

18. How many pounds of nails can be bought for 75 cents at the rate of 4 pounds for 20 cents?

19. I bought 6 sheep for $30. How much did I pay per head?

20. At $5 per head, how many head of sheep can be bought for $37? *Ans.* 7 sheep and $2 left.

21. A man whose wages were $4 a day earned in a certain time $33. How many days did he work? *Ans.* $8\frac{1}{4}$ days.

91. From examples 20 and 21 it is apparent that the *remainder* may be written either *after* the quotient, as in the answer to the 20th, or as a *part* of it, as in the answer to the 21st.

When written as a *part* of the quotient, the *remainder* is expressed by placing the *divisor under it* with a line between them. Such an expression shows that each unit of the remainder is to be divided into as many equal parts as there are units in the divisor.

When any thing is divided into *two equal* parts, each of the parts is called *one half.*

When into *three* equal parts, each part is called *one third.*

When into *four* equal parts, each part is called *one fourth.*

When into five, six, seven, etc., equal parts, the parts are called fifths, sixths, sevenths, etc.

$\frac{1}{2}$ expresses *one half*, or one of two equal parts of any thing.

$\frac{1}{4}$ expresses *one fourth*, or one of four equal parts of any thing.

$\frac{2}{5}$ expresses *two fifths*, or two of five equal parts of any thing.

$\frac{5}{27}$ expresses *five twenty-sevenths*, or five of twenty-seven equal parts of any thing.

92. One or more of the equal parts of any thing is called a *Fraction.*

93. Read the following fractional expressions:

$\frac{2}{9}$	$\frac{8}{11}$	$\frac{5}{19}$	$\frac{21}{35}$	$\frac{19}{80}$	$\frac{18}{98}$
$\frac{11}{84}$	$\frac{10}{29}$	$\frac{18}{100}$	$\frac{25}{87}$	$\frac{8}{94}$	$\frac{24}{29}$

22. If James should divide 25 apples equally among 5 boys, what part of the whole would each receive? How many apples would each receive?

Analysis.—If he should divide 25 apples equally among 5 boys, each boy would receive *one-fifth* of 25 apples, which is 5 apples.

23. If flour is worth $8 a barrel, what will one-half barrel cost?

24. Mr Smith bought 8 bushels of chestnuts for $24. How much did he pay per bushel? How much is one-eighth of 24?

25. What is one-sixth of 36? Of 42? Of 48? Of 60?

26. What is one-tenth of 50? Of 60? Of 70? Of 80?

27. What is one-seventh of 14? Of 28? Of 42? Of 49?

WRITTEN EXERCISES.

94. 1. Divide 1396 by 4.

1st process.

```
Divisor. Dividend. Quotient.
   4)1396(300 times.
     1200   40 times.
     ----    9 times.
      196  ---
      160  349 times.
      ---
       36
       36
       --
```

2d process. hunds. tens. units.

```
4)1396(349
  12
  --
   19
   16
   --
    36
    36
    --
```

Analysis.—For convenience we write the divisor at the left, and the quotient at the right of the dividend, with curved lines between them, and begin at the left to divide.

4 is not contained in 1 thousand any thousand times, therefore the quotient can not contain units of any order higher than hundreds. Hence we find how many times 4 is contained in all the hundreds of the dividend. 1 thousand plus 3 hundreds equals 13 hundreds. 4 is contained in 13 hundreds 3 hundred times and a remainder. We write the 3 hundreds in the quotient and multiply the divisor by it, obtaining for a product 12 hundreds, or 1 thousand 2 hundred, which we write under units of the same order in the dividend. Subtracting this product from the partial dividend, there is a remainder of 1 hundred.

1 hundred plus 9 tens equals 19 tens. 4 is contained in 19 tens 4 tens times and a remainder. We write the 4 tens in the quotient and multiply the divisor by it, obtaining for a product 16 tens, or 1 hundred, and 6 tens, which we write under units of the same order in the partial dividend. Subtracting, there is a remainder of 3 tens and 6 units.

3 tens plus 6 units equals 36 units. 4 is contained in 36 units 9 times. We write the 9 units in the quotient and multiply the divisor by it, obtaining for a product 36 units, or 3 tens and 6 units, which we write under units of the same order in the partial dividend. Subtracting, there is no remainder. Hence the quotient is 349.

In the *second process* all ciphers are omitted from the right of the products and the significant figures are written under units of the same order. The quotient also is expressed by writing the different orders of units in proper succession.

PROOF.—349 the quotient, multiplied by 4 the divisor, is equal to 1396 the dividend.

Hence the work is correct. (Prin. 3.)

Solve in like manner and prove:

2. $738 \div 3$.	5. $4821 \div 3$.	8. $7848 \div 9$.
3. $845 \div 5$.	6. $3462 \div 6$.	9. $8432 \div 4$.
4. $385 \div 7$.	7. $3864 \div 8$.	10. $8308 \div 7$.

95. The solution of the preceding examples may be shortened by performing the multiplications and subtractions without writing the results. This process is called ***Short Division.***

The solution of Example 1 by *Short Division* is as follows:

PROCESS.

$$\begin{array}{r} 4\overline{)1396} \\ \hline \text{Quotient}\quad 349 \end{array}$$

ANALYSIS.—4 is contained in 13 hundred 3 hundred times and 1 hundred remainder. We write 3 hundreds in the quotient under units of the same order in the dividend. 1 hundred remainder united with 9 tens makes 19 tens. 4 is contained in 19 tens 4 tens times and 3 tens remainder. We write the 4 tens in the quotient under tens of the dividend. 3 tens remainder united with 6 units make 36 units. 4 is contained in 36 units 9 times. We write the 9 in the quotient. Hence the quotient is 349.

Solve by *short division:*

11. $4872 \div 4$.	17. $\frac{3248}{8}$.	23. $4567 \div 5$.
12. $6830 \div 5$.	18. $\frac{5256}{6}$.	24. $8932 \div 6$.
13. $2976 \div 6$.	19. $\frac{4986}{9}$.	25. $8174 \div 9$.
14. $2985 \div 5$.	20. $\frac{6765}{5}$.	26. $9185 \div 4$.
15. $4635 \div 3$.	21. $\frac{2836}{7}$.	27. $8436 \div 7$.
16. $3936 \div 4$.	22. $\frac{9872}{8}$.	28. $3885 \div 8$.

CASE II.

96. When the divisor is expressed by more than one figure.

1. How many barrels of flour at $10 a barrel can be bought for $80?

ANALYSIS.—Since 1 barrel costs $10, as many barrels can be bought for $80 as $10 are contained times in $80 which is 8 times. Therefore 8 barrels can be bought for $80.

2. How many pounds of mutton at 10 cents a pound can be bought for 50 cents? How many 10's in 50? In 60? In 70? In 80?

3. A man measured a stick and found it to be 60 inches long. There are 12 inches in a foot. How many feet long was it? How many 12's in 60? In 72? In 84? In 96?

4. At $13 a ton how much hay can be bought for $26?

5. At 15 cents each how many toys can be bought for 30 cents? For 45 cents? For 60 cents?

6. Mr. Henderson sold 20 lambs for $80. How much did he get apiece for them?

7. 25 cents make a quarter of a dollar. How many quarters of a dollar has a boy who has 50 cents?

8. Henry's father gave him a dollar. How many pineapples at 20 cents each can he buy with the money?

9. The railroad fare to a certain place is 35 cents. How many tickets can be bought with 70 cents?

10. If a boy earns 11 cents an hour, how long will it take him to earn 55 cents? 66 cents? 88 cents?

11. There are 20 hundred-weight in a ton. How many tons are there in 45 hundred-weight? How many in 55?

12. In one day there are 24 hours. How many days are there in 50 hours? In 60 hours? In 72 hours?

13. 12 articles make a dozen. How many dozen are there in 39 articles? In 48? In 51? In 60? In 65?

14. A farm of 60 acres was divided into 15 equal lots. How many acres were there in each lot?

15. At 18 cents a dozen, how many dozen of eggs can be bought for 36 cents? For 40 cents? For 54 cents?

16. When butter is 30 cents a pound, how many pounds can be bought for 90 cents? For $1.20? For $1.50?

17. How many 20's are there in 40? In 50? In 60?

18. How many 25's are there in 50? In 60? In 75?

19. By selling brooms at 25 cents each, I received $1.25. How many brooms did I sell?

20. In 100 how many 10's are there? How many 11's? 12's? 13's? 15's? 16's? 20's? 23's? 25's?

WRITTEN EXERCISES.

97. 1. Divide 7975 by 26.

PROCESS.

Divisor. Dividend. Quotient.

```
26)7975(306 19/26
   78
   ---
    175
    156
    ---
     19
```

ANALYSIS.—26 is not contained in 7 thousands any thousands times; hence we unite the thousands with the hundreds, making 79 hundreds. 26 is contained in 79 hundreds 3 hundred times and a remainder. We write the 3 hundreds in the quotient and multiply the divisor by it, obtaining for a product 78 hundreds, or 7 thousands and 8 hundreds, which we write under units of the same order in the dividend. Subtracting, there is a remainder of 1 hundred.

We unite the 1 hundred with the 7 tens, making 17 tens. 26 is not contained in 17 tens any tens times; therefore there are no tens in the quotient, and we write a cipher there.

We unite the 17 tens with the 5 units, making 175 units. 26 is contained in 175 units 6 times and a remainder. We write the 6 in units' place in the quotient and multiply the divisor by it, obtaining for a product 156 units, or 1 hundred, 5 tens, and 6 units, which we write under units of same order in the partial dividend. Subtracting, there is a remainder of 19. We write the remainder over the divisor as a part of the quotient.

Hence the quotient is $306\frac{19}{26}$.

PROOF.—$306 \times 26 + 19 = 7975$. Hence the work is correct. (Prin. 3.)

98. When the steps in the solution of an example in division are written, the process is called ***Long Division.***

RULE.—*Write the divisor at the left of the dividend with a curved line between them.*

Find how many times the divisor is contained in the fewest figures on the left hand of the dividend that will contain it, and write the quotient on the right.

Multiply the divisor by this quotient and place the product under the figures divided. Subtract the result from the partial dividend used, and to the remainder annex the next figure of the dividend.

Divide as before until all the figures of the dividend have been annexed to the remainder.

If any partial dividend will not contain the divisor, write a cipher in the quotient, then annex the next figure of the dividend and proceed as before.

If there is a remainder after the last division write it after the quotient, or with the divisor under it as part of the quotient.

PROOF.—*Multiply the divisor by the quotient, and to the product add the remainder, if any. If the work is correct, the result will equal the dividend.*

1. *To find the quotient figure,* see how many times the first *figure* of the divisor is contained in *first figures* of the partial dividend that will con-

tain it, making allowance for the addition of the tens from the product of the second figure of the divisor.

2. If the product of the divisor by the quotient figure be greater than the partial dividend from which it is to be subtracted, the quotient figure is *too large*.

3. Each remainder must be less than the divisor; otherwise the quotient figure is *too small*.

4. When there is no remainder the divisor is said to be *exact*.

EXAMPLES.

99. Divide:

2. 1240 by 10.
3. 3443 by 11.
4. 2592 by 12.
5. 3978 by 13.
6. 5684 by 14.
7. 6480 by 15.
8. 8736 by 21.
9. 1472 by 32.
10. 9672 by 31.
11. 9724 by 22.
12. 2952 by 72.
13. 1188 by 54.
14. 4235 by 55.
15. 5356 by 52.
16. 8733 by 41.
17. 9639 by 81.
18. 7991 by 61.
19. 2508 by 22.
20. 7332 by 52.
21. 4824 by 72.
22. 16665 by 33.
23. 13545 by 43.
24. 25578 by 63.

Divide:

25. 12456 by 24.
26. 28350 by 54.
27. 50854 by 94.
28. 58176 by 96.
29. 56394 by 78.
30. 54944 by 101.
31. 90992 by 121.
32. 199864 by 301.
33. 475524 by 612.
34. 1445204 by 802.
35. 1760225 by 905.
36. 3156584 by 722.
37. 5173302 by 834.
38. 5926431 by 643.
39. 3214664 by 566.
40. 6923471 by 555.
41. 14293624 by 675.
42. 56243121 by 686.
43. 692348726 by 897.
44. 496839715 by 1047.
45. 786935846 by 3118.
46. 1234589640 by 96813.
47. 31964875932 by 37425.

48. Into how many lots of 39 acres each, can a tract of land containing 6318 acres be divided?

49. Wm. Wallace has 17 horses, the aggregate value of which is $4386. What is the average worth of each horse?

50. A surveyor traveled 41600 rods in one week. How many miles did he travel, there being 320 rods in a mile?

51. How many eggs at $.38 per dozen can be bought for $6.84?

52. In 24 hours the earth moves 1575000 miles. How far does it move in one minute, 60 minutes making an hour?

53. Mount Everest in Asia is 29100 feet high. There are 5280 feet in a mile. How many miles high is it?

54. It required 4375480 bricks to build an orphan asylum. How many days did it require 5 teams to draw the bricks, if they drew 5 loads per day and 1250 bricks at a load?

55. The earth is 91500000 miles from the sun. How many seconds does it take light to come from the sun to the earth, if it travels 185000 miles per second?

56. A man bought a farm of 278 acres at $63 an acre. He paid $1275 down and agreed to pay the rest in 8 equal annual payments. How much was he required to pay yearly?

57. The earnings of a certain railroad were $3681452 during the year. The number of days in a year is 365. What was the average income per day?

58. How many feet are there in a mile, if 42 miles contain 221760 feet?

59. If the average wages of a laboring man are $500 per year, how many men will it require to earn $50000 per year, the salary of the President?

60. The area of the State of North Carolina is 50704 square miles, and the population, according to the census of 1870, was 1071400. How many persons, on an average, were there living on a square mile?

CASE III.

100. When the divisor has ciphers on the right.

1. How many 10's and what remainder in 46? 84? 97?

2. When a number is divided by 10, what part is remainder? What part is quotient?

3. How many hundreds, and what remainder in 434? 516? 639? 758?

4. When a number is divided by 100 what part of it is remainder? What part is quotient?

5. When a number is divided by 1000, what part is remainder? What part is quotient?

101. PRINCIPLE.—*In dividing by 10, 100, 1000, etc., the remainder will be as many of the figures at the right of the dividend as there are ciphers on the right of the divisor. The rest of the number is quotient.*

102. 1. Divide 6374 by 1000.

PROCESS.

$1|000)\underline{6|374}$

$6\frac{374}{1000}$

ANALYSIS.—Since the divisor contains no order of units lower than thousands, in dividing we may omit or cut off from the dividend for a remainder, all orders of units lower than thousands. (Prin.)

Dividing 6 thousands by 1 thousand we obtain 6 for the quotient and 374 for the remainder, or $6\frac{374}{1000}$.

2. Divide 39321 by 6000.

PROCESS.

$6|000)\underline{39|321}$

6 Rem. 3000

321

Entire remainder 3321

ANALYSIS.—Since the divisor contains no order of units lower than thousands, in dividing we may omit or cut off from the dividend all orders of units lower than thousands.

6 thousands are contained in 39 thousands 6 times and 3 thousand remainder. 3 thousand plus the other partial remainder equals the entire remainder. Hence the quotient is $6\frac{3321}{6000}$.

RULE.—*Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend.*

Divide the rest of the dividend by the rest of the divisor.

Annex to the remainder the figures cut off; the result will be the true remainder.

Divide the following:

3. Divide 1869 by 100.
4. Divide 12345 by 200.
5. Divide 89325 by 700.
6. Divide 35968 by 900.
7. Divide 2465 by 1000.
8. Divide 13692 by 4000.
9. Divide 83005 by 1100.
10. Divide 75684 by 1500.

11. How many miles of railroad at \$50000 a mile can be constructed for \$38968457?

12. How many schooners carrying 8300 bushels of wheat will it require to carry 984364 bushels?

13. The area of the State of New York is 47000 square miles, or 30080000 acres. How many acres in a square mile?

103. RELATION OF DIVIDEND, DIVISOR, AND QUOTIENT.

The value of the quotient depends upon that of the dividend and divisor. If one of these is changed, while the other remains the same, the quotient will be changed. If both are changed, the quotient may not be changed.

The changes may be illustrated as follows:

FUNDAMENTAL EQUATION.

$$64 \div 8 = 8.$$

CHANGED EQUATIONS.

1. *Dividend changed.*	1. $128 \div 8 = 16$	1. Multiplying the dividend by 2 multiplies the quotient by 2.
	2. $32 \div 8 = 4$	2. Dividing the dividend by 2 divides the quotient by 2.

2. *Divisor changed.*	1. $64 \div 16 = 4$	1. Multiplying the divisor by 2 divides the quotient by 2.
	2. $64 \div 4 = 16$	2. Dividing the divisor by 2 multiplies the quotient by 2.
3. *Both changed.*	1. $128 \div 16 = 8$	Multiplying or dividing both dividend and divisor by 2 does not change the quotient.
	2. $32 \div 4 = 8$	

From these illustrations the following principles are deduced:

104. Principles.—1. *Multiplying the dividend or dividing the divisor, multiplies the quotient.*

2. *Dividing the dividend or multiplying the divisor, divides the quotient.*

3. *Multiplying or dividing both dividend and divisor by the same number, does not change the quotient.*

ANALYSIS AND REVIEW.

105. ***Analysis*** is the process of solving problems by tracing the relation of the parts.

In analyzing we commonly reason from the *given number* to *one*, and then from *one* to the *required number.*

1. If 8 yards of cloth cost $16, what will 12 yards cost?

PROCESS.	
8 yards = $16.	Analysis.—Since 8 yards cost $16, 1 yard will cost one-eighth of $16, or $2; and since 1 yard costs $2, 12 yards will cost 12 times $2, or $24.
1 " = $ 2.	
12 " = $24.	

2. If 8 horses cost $2400, what will 6 horses cost?
3. If 8 lemons cost 40 cents, what will 11 lemons cost?
4. How much will 12 hats cost, if 8 hats cost $16?

5. If 25 pounds of sugar cost \$2.50, what will 36 pounds cost?

6. If 12 men can build a school-house in 25 days, how long will it take 25 men to build it?

7. If 12 barrels of flour are worth \$132, what are 22 barrels worth?

8. If it requires 576 feet of boards to build 18 rods of fence, how many feet will be required to build 13 rods?

9. If 6 men can do a piece of work in 10 days, how long will it take 5 men to do it?

10. If I exchanged 18 barrels of flour for 61 yards of cloth at \$4 a yard, how much did I get per barrel for the flour?

106. The ***Parenthesis,*** ( ), shows that the numbers included within it are to be subjected to the same operation.

Thus, $(5+6-2)\times 3$ shows that $5+6-2$, or 9, is to be multiplied by 3.

107. The ***Vinculum,*** $\overline{\quad\quad}$, may be used instead of the parenthesis.

Thus, instead of $(5+6-2)\times 3$, we may write $\overline{5+6-2}\times 3$.

Find the value of the following:

11. $(12+7-9)\times 5$.
12. $(13-6+8)\times 6$.
13. $(11-2+5)\times 8$.
14. $(3+4)\times 9-(3+6)\div 3$.
15. $(5+7-3)\times 3+(3+5-4)\div 4$.
16. $\overline{(36-7)\times 5}+\overline{(102+6)\div 9}$.
17. $\overline{(99-3)\div 8}-\overline{(86+10)\div 12}+\overline{(3+6)\div 3}$.
18. $\overline{(45+3)\div 6}+(10+15)\div\overline{(7-2)}+6$.

19. A man dying, left the following tracts of land to be divided equally among his five children. The first tract contained 1118 acres; the second, 3 times as much lacking 193

acres; the third, twice as much as the other two lacking 105 acres. What was each one's share?

20. A gentleman bought 1516 head of cattle at $39 per head. During the summer 97 died of disease, but he sold the remainder so as to gain on the whole number $1819. How much did he get for his cattle per head?

21. If a young man who has a salary of $30 per week, pays $7.25 for his board and $4.25 for other expenses, how long will it take him to save $1500?

22. A man bought a horse for $115 and after keeping him three months, sold him for $155. If he paid $30 for his keeping and received $50 for the use of him during that time, how much did he gain?

23. A speculator purchased a certain number of bushels of wheat for $8735. He sold it for $9215 and in so doing gained $ .25 per bushel. How many bushels did he buy?

24. I bought 25 barrels of flour for $200. For what must it be sold per barrel to gain $50? What will be the gain per barrel?

25. A tailor having $585 wished to purchase with this an equal number of yards of two kinds of broadcloth. One kind was worth $6 a yard, the other $7 a yard. How many yards of each kind could he buy?

26. Two men leave the same place at the same time and travel in opposite directions, one at the rate of 48 miles per day, the other at the rate of 52 miles per day. How far apart will they be at the end of 5 days?

27. If 20 men can do a piece of work in 31 days, how many days will be required to do an equal amount of work if 11 additional men are employed?

28. A farmer wished to obtain $120. He sold 16 barrels of apples at $3.50 per barrel, and enough barley at $ .80 a bushel to make up the sum required. How many bushels of barley did he sell?

29. Mr. B. bought 140 acres of land for $17500, and sold enough at $120 per acre to amount to $9600. The rest of the land he sold at cost. How many acres did he sell at cost, and what was the entire loss?

30. A man pays $628 a year for groceries, $350 for house rent, $262 for clothes, twice as much for traveling expenses as for house rent, $175 for annual premium for life insurance, and saves in 4 years enough money to purchase 130 acres of land at $53 an acre. What is his yearly income?

31. In October, 1871, the great fire in Chicago burned over an area of 2124 acres. The estimated loss occasioned by the fire was $196000000. What was the average loss per acre?

32. A boy has a velocipede which he can run at the rate of 140 rods in 4 minutes. How many minutes will it take him to run it 630 rods?

33. A farmer has 1000 head of cattle in 5 fields. In the first he has 315 head, in the second 175 head, in the third 300 head, and in the fourth the same number as in the fifth. How many has he in the fifth?

34. A man gave away $45000 in three equal amounts. One share he gave to his son, one share to his daughter, and the rest to his grandchildren, giving them $1500 apiece. How many grandchildren had he?

35. In the Centennial Exhibition, at Philadelphia, a section of a cable in process of construction for the new suspension bridge at New York was shown. It was composed of 6000 galvanized steel wires, and its ultimate strength was 22,300,000 pounds. What weight was each wire capable of sustaining?

36. The main building of the Centennial Exhibition at Philadelphia, the largest building in the world, contained on the ground floor an area of 872320 square feet, on the upper floors in projections 37344 square feet, in towers 26344 square feet. If there are 43560 square feet in an acre, how many acres did the floors of the building contain?

PROPERTIES OF NUMBERS

108. 1. What is the product of 4 times 5? What are 4 and 5 of their product?

2. What is 4 of 16? Of 24? What is 7 of 14? Of 28?

3. What numbers will exactly divide 18? 24? 36? 72?

4. Give the exact divisors of 42. 96. 108. 48. 32?.

5. What are the factors of 30? 24? 40? 56? 64?

6. What numbers between 0 and 10 *can not* be divided by any number except themselves and 1? Between 10 and 20?

7. What numbers between 0 and 10 *can* be divided by other numbers than themselves and 1? Between 10 and 20?

DEFINITIONS.

109. An ***Integer*** or ***Integral Number*** is one that expresses whole units.

Thus, 281, 36 houses, 46 men, are integral numbers.

110. An ***Exact Divisor*** of a number is an integer that will divide it without a remainder.

Thus, 2, 4, 6 and 12 are exact divisors of 24.

111. The ***Factors*** of a number are the integers which being multiplied together will produce the number.

Thus, 6 and 8 are factors of 48.

The *exact divisors* of a number are *factors* of it.

112. A ***Prime Number*** is one that has no exact divisors except itself and 1.

Thus, 1, 3, 5 and 7 are prime numbers.

113. A ***Composite Number*** is one that has exact divisors besides itself and 1.

Thus, 18 and 24 are composite numbers, for 18 is divisible by 6, and 24 by 8.

114. An ***Even Number*** is one that is exactly divisible by 2.

Thus, 2, 4, 6, 8, etc., are even numbers.

115. An ***Odd Number*** is one that is not exactly divisible by 2.

Thus, 1, 3, 5, 7, 9, etc., are odd numbers.

DIVISIBILITY OF NUMBERS.

116. In determining by inspection the divisibility of numbers, the following facts will be found valuable.

1. *Two* is an exact divisor of any even number.

Thus, 2 is an exact divisor of 12, 16, 30 and 44.

2. *Three* is an exact divisor of any number, the sum of whose digits is divisible by 3.

Thus, 3 is an exact divisor of 312, 135, 423, and 3816.

3. *Four* is an exact divisor of a number, if the number expressed by its two right hand figures is divisible by 4.

Thus, 4 is an exact divisor of 264, 1284, 1368, and 7932.

4. *Five* is an exact divisor of any number whose right hand figure is 0 or 5.

Thus, 5 is an exact divisor of 360, 1795, 3810, and 7895.

5. *Six* is an exact divisor of any *even* number, the sum of whose digits is divisible by 3.

Thus, 6 is an exact divisor of 732, 534, 798, and 8226.

6. *Eight* is an exact divisor of a number, if the number expressed by its three right hand figures is divisible by 8.

Thus, 8 is an exact divisor of 4328, 3856, 61360, and 5920.

7. *Nine* is an exact divisor of any number, the sum of whose digits is divisible by 9.

Thus, 9 is an exact divisor of 513, 1314, 252, 1341, and 312462.

8. 10, 100, 1000, etc., are exact divisors of any numbers that end respectively with one, two, three, etc., ciphers.

Thus, 10, 100, 1000, etc., are exact divisors respectively of 80, 800, 8000, etc.

9. If an even number is divisible by an odd number it is divisible by *twice* that number.

Thus, 72 is divisible by 9 and by twice 9 or 18. 312 by 3 and 6.

10. An exact divisor of a number is an exact divisor of any number of times that number.

Thus, 3 is an exact divisor of 12, and of any number of times 12, as 36.

11. An exact divisor of each of two numbers is an exact divisor of their *sum* and of their *difference*.

Thus, 3 is an exact divisor of 9 and 12 respectively, and therefore of $9 + 12$, or 21; of $12 - 9$, or 3.

117. Find by inspection some of the exact divisors of the following numbers:

1. 1524.	5. 2556.	9. 42840.	13. 376250.
2. 3432.	6. 7236.	10. 92475.	14. 428328.
3. 4264.	7. 27360.	11. 362088.	15. 4183200.
4. 9360.	8. 23661.	12. 438408.	16. 6853744.

FACTORING.

118. 1. What are the factors of 6? 8? 12? 16?

2. What factors of 18 are prime numbers or prime factors?

3. What are the prime factors of 30?

4. What are all the exact divisors of 30?

5. What numbers besides the prime factors of 30 are its exact divisors? How are they obtained from the prime factors?

6. Of what number are 2, 3, and 5, the prime factors?

7. How can a number be obtained from its prime factors?

8. The prime factors of a number are 2, 2, and 5. What is the number? Give all the exact divisors of this number.

9. What are the exact divisors of 60? 72? 96? 144?

DEFINITIONS.

119. ***Factoring*** is the process of separating a number into its factors.

120. ***Prime Factors*** are factors that are prime numbers.

121. The number of times a number is used as a factor is indicated by a small figure called an *exponent.* It is written above and at the right of the number.

Thus, $4 \times 4 \times 4 = 4^3$, and the 3 indicates that 4 is used as a factor three times.

122. Principles.—1. *Every prime factor of a number is an exact divisor of that number.*

2. *The only exact divisors of a number are its prime factors or the product of two or more of them.*

3. *Every number is equal to the product of its prime factors.*

1. What are the prime factors of 756?

PROCESS.
2)756
2)378
3)189
3)63
3)21
7

ANALYSIS.—Since every prime factor of a number is an exact divisor of the number, we may find the prime factors of 756 by finding all the prime numbers that are exact divisors of 756. Since the number is *even*, we divide by 2. Since the quotient obtained is an even number, we divide again by 2. Then we divide by the prime numbers 3, 3, 3, successively, and the last quotient is 7, which is a prime number.

Hence the prime factors are 2, 2, 3, 3, 3, 7, or 2^2, 3^3, 7.

RULE.—*Divide the given number by any prime number that will exactly divide it. Divide this quotient by another prime number, and so continue until the quotient is a prime number.*

The several divisors and last quotient will be the prime factors.

What are the prime factors

2. Of 35?	12. Of 484?	22. Of 3913?
3. Of 64?	13. Of 1280?	23. Of 3812?
4. Of 336?	14. Of 1008?	24. Of 7007?
5. Of 168?	15. Of 1140?	25. Of 3980?
6. Of 144?	16. Of 1184?	26. Of 26840?
7. Of 315?	17. Of 1872?	27. Of 38148?
8. Of 198?	18. Of 7644?	28. Of 11340?
9. Of 224?	19. Of 2310?	29. Of 24024?
10. Of 786?	20. Of 3204?	30. Of 18500?
11. Of 316?	21. Of 4725?	31. Of 124416?

MULTIPLICATION BY FACTORS.

123. 1. What are the factors of 12? 16? 18? 20?

2. What are the factors of 24? 42? 36? 30? 27?

3. What are the factors of 45? 48? 56? 63? 72?

4. When a number is multiplied by 4 and the product by 6, by what is the number multiplied?

5. What will 20 carriages cost at \$346 each?

PROCESS.	
\$346	cost of 1 carriage.
4	
\$1384	cost of 4 carriages.
5	
\$6920	cost of 20 carriages.

ANALYSIS.—Since 20 is 5 times 4, 20 carriages will cost 5 times as much as 4 carriages. 4 carriages will cost 4 times \$346, or \$1384, and 20 carriages will cost 5 times as much as 4 carriages, or 5 times \$1384, which is \$6920. Hence, 20 carriages will cost \$6920.

RULE.—*Multiply the multiplicand by one factor of the multiplier, the product thus obtained by another factor, and so continue until all the factors have been used successively as multipliers. The last product will be the product sought.*

Multiply in same manner, using the factors of the multiplier:

6. 425 by 32; by 36; by 48; by 72.
7. 1824 by 56; by 27; by 45; by 108.

8. What will be the cost of 35 cows at \$64 each?
9. What will 21 cords of wood cost at \$5.35 a cord?
10. What will 72 yoke of oxen cost at \$168 per yoke?
11. What will 36 boxes of lemons cost at \$6.25 per box?
12. What will 48 acres of land cost at \$46 per acre?
13. What will 24 paintings cost at \$55 each?
14. What will 45 cases of boots cost at \$36 a case?
15. What will 56 barrels of salt cost at \$2.35 a barrel?

DIVISION BY FACTORS.

124. 1. What are the factors of 32? 25? 64? 96?

2. If a number is divided by 8, by what must the quotient be divided that the number may be divided by 16?

3. If a number is divided by 8 and the quotient by 6, by what is the number divided?

4. What factors may be used to divide a number by 36?

5. What factors may be used to divide a number by 48?

6. A miller put up 500 pounds of hominy in packages containing 4 pounds each, and packed them in boxes containing 10 packages each. How many packages and how many boxes did he have?

7 Divide 888 by 24, using factors.

PROCESS.

$4\,)\,\underline{8\,8\,8}$
$6\,)\,\underline{2\,2\,2}$
$\quad\;\; 3\,7$

ANALYSIS.—24 is equal to 6 times 4. Hence to divide by 24 we may divide by 6 times 4. $888 \div 4 = 222$. But since we were to divide by 6 times 4, this quotient is 6 times too great, hence we must divide it by 6. $222 \div 6 = 37$ the true quotient.

8. Divide 5863 by 32, using factors.

PROCESS.

$4\,)\,\underline{5\,6\,8\,3}$
$2\,)\,\underline{1\,4\,2\,0}\ldots 3$
$\;4\,)\,\underline{7\,1\,0}$
$\quad 1\,7\,7\ldots 2$
$3 + (2 \times 8) = 19$ true Rem.
$177\frac{19}{32}$ Quotient.

ANALYSIS.—32 is equal to $4 \times 2 \times 4$. Dividing 5683 by 4 gives a quotient of 1420 *fours* and 3 units remaining.

Dividing 1420 *fours* by 2 gives a quotient of 710 *eights*. Dividing 710 *eights* by 4 gives a quotient of 177 *thirty-twos* and 2 *eights* remainder. The first partial remainder is 3 *units*, and the second, 2 *eights*, or 16; hence the entire remainder is $3 + 16$, or 19, and the quotient is $177\frac{19}{32}$.

RULE.—*Divide the dividend by one factor of the divisor, the quotient thus obtained by another factor, and so continue until all the factors have been used successively as divisors.*

If there be remainders, multiply each remainder by all the preceding divisors except the one that produced it. The sum of these products will be the true remainder.

Divide, using factors:

9. 1704 by 24.	13. 1288 by 56.	17. 3275 by 56.
10. 4725 by 15.	14. 3528 by 72.	18. 3276 by 27.
11. 5740 by 28.	15. 3824 by 32.	19. 4104 by 45.
12. 1428 by 42.	16. 2184 by 49.	20. 7304 by 24.

21. A wholesale grocer put up 1120 pounds of tea in 35-pound packages, containing 5-pound canisters. How many packages and canisters were there?

22. A paper manufacturer put up his paper so that each quire contained 4 packages of 6 sheets each. How many packages and quires were made up from 912 sheets?

CANCELLATION.

125. 1. How many times is 2 times 5 contained in 4 times 5? 2 times 7 in 4 times 7? 2 times 9 in 4 times 9? 2 times 24 in 4 times 24? 2 times any number in 4 times that number?

2. How many times is 4 times 8 contained in 12 times 8? 4 times 25 in 12 times 25? 4 times 75 in 12 times 75? 4 times any number in 12 times the same number?

3. How many times is 6×48 contained in 24×48? 6×144 in 24×144?

4. In determining the quotient, what numbers may be omitted from both dividend and divisor?

126. ***Cancellation*** is the process of shortening computations by rejecting equal factors from the dividend and divisor.

127. Principle.—*Rejecting equal factors from both dividend and divisor does not alter the value of the quotient.*

1. Divide 66 times 36 by 24 times 11.

PROCESS.

$$\frac{66 \times 36}{24 \times 11} = \frac{\cancel{11} \times \cancel{6} \times \cancel{4} \times 3 \times 3}{\cancel{6} \times \cancel{4} \times \cancel{11}} = 9$$

Analysis.—We write the numbers as in division, the dividend above, the divisor below a line. Instead of multiplying 66 by 36 we resolve 66 into its factors 11 and 6, and 36 into its factors 4, 3 and 3, and in the divisor resolve 24 into the factors 6 and 4.

Cancelling equal factors from both dividend and divisor, which is the same as dividing both by the same number, and does not alter the value of the quotient, we have remaining in the dividend the factors 3 and 3, or 9, which is the quotient.

2. Divide $72 \times 66 \times 49$ by $63 \times 40 \times 21$.

PROCESS.

$$\frac{\overset{\cancel{8}}{\cancel{72}} \times \overset{22}{\cancel{66}} \times \overset{\cancel{7}}{\cancel{49}}}{\underset{\cancel{7}}{\cancel{63}} \times \underset{5}{\cancel{40}} \times \underset{\cancel{3}}{\cancel{21}}} = \frac{22}{5} = 4\tfrac{2}{5}.$$

ANALYSIS.—We write the numbers as before. Since 9 is a factor of both 72 and 63 it may be rejected from both, leaving 8 instead of 72 in the dividend, and 7 instead of 63 in the divisor. We next cancel 8 from 8 and 40, leaving 5 instead of 40 in the divisor. We next cancel 7 from 7 and 49, leaving 7 instead of 49 in the dividend, and 7 again from 7 and 21, leaving 3 instead of 21. Rejecting the factor 3 from both 66 and 3, there is left for a dividend 22, and for a divisor 5, which gives a quotient of $4\frac{2}{5}$.

RULE.—*Reject from the dividend and divisor all factors common to both, and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

When all the factors of both dividend and divisor are cancelled, the quotient is 1, for the dividend will then exactly contain the divisor once.

EXAMPLES.

Divide, using cancellation:

3. $7 \times 5 \times 3 \times 11$ by $5 \times 11 \times 3$.
4. 12×14 by $6 \times 7 \times 2$.
5. $6 \times 3 \times 5 \times 2$ by $3 \times 5 \times 2 \times 2$.
6. $4 \times 2 \times 8 \times 24$ by $36 \times 8 \times 2$.
7. 24×32 by $8 \times 6 \times 4$.
8. $45 \times 60 \times 7$ by $49 \times 12 \times 9$.
9. $2 \times 3 \times 5 \times 8 \times 7$ by $6 \times 5 \times 2 \times 7$.
10. $5 \times 8 \times 12 \times 6$ by $20 \times 16 \times 2$.
11. $12 \times 60 \times 36 \times 35$ by $7 \times 30 \times 18 \times 24$.
12. $30 \times 49 \times 64 \times 25$ by $35 \times 15 \times 24$.

13. Divide the product of 26 times 18 times 35, by 78 times 30.

14. Find the quotient of 99 times 360 times 365, divided by 11 times 72.

15. Find the quotient of $175 \times 28 \times 72$ times 363, divided by $12 \times 11 \times 9$.

16. Four farms containing 80 acres each, worth \$65 per acre, were exchanged for 5 farms containing 95 acres each. What was the value per acre of the farms received in exchange?

17. A farmer buys 3 pieces of muslin each containing 44 yards at 11 cents a yard, and pays for it in wheat at \$2 per bushel. How many bushels are required?

18. A merchant bought 13 tubs of butter, each containing 39 pounds, at 32 cents a pound, paying for it in 4 patterns of silk of 13 yards each. How much was the silk a yard?

COMMON DIVISORS.

128. 1. What numbers will exactly divide 12? 15? 20?

2. What numbers will exactly divide both 12 and 15? 15 and 20? 24 and 48? 63 and 72?

3. What numbers will exactly divide both 12 and 24? What is the largest number that will exactly divide them?

4. What is the largest number that will exactly divide both 15 and 30? 16 and 32? 16 and 24? 24 and 32?

5. Name all the divisors common to 15 and 30.

6. Name all the prime divisors or factors common to 15 and 30?

7. How is the greatest divisor common to 15 and 30 found from the prime factors of those numbers?

8. What is the greatest divisor common to 24 and 30?

9. How is the greatest divisor common to 24 and 30 obtained from the prime factors of those numbers?

DEFINITIONS.

129. A ***Common Divisor*** of two or more numbers is an exact divisor of each of them.

Thus, 6 is a common divisor of 12, 24, 48; 8 of 16, 24 and 64.

130. ***The Greatest Common Divisor*** of two or more numbers is the greatest number that is an exact divisor of each of them.

Thus, 24 is the greatest common divisor of 24 and 48.

131. When numbers have no common divisor they are said to be ***Prime to each other.***

Thus, 7, 8 and 9 are prime to each other.

A common divisor is sometimes called a *common measure* and the greatest common divisor the *greatest common measure.*

132. PRINCIPLE.—*The greatest common divisor of two or more numbers is the product of all their common prime factors.*

1. What is the greatest common divisor of 45, 60, and 75?

1ST PROCESS.

$45 = 3 \times 3 \times 5$
$60 = 2 \times 2 \times 3 \times 5$
$75 = 3 \times 5 \times 5$
$3 \times 5 = 15$

ANALYSIS.—Since the greatest common divisor is equal to the product of all the prime factors common to the given numbers, we separate the numbers into their prime factors. The only prime factors common to all these numbers are 3 and 5. Hence their product, 15, is the greatest common divisor of the given numbers.

2D PROCESS.

$$\begin{array}{r|rrr} 3 & 45 & 60 & 75 \\ \hline 5 & 15 & 20 & 25 \\ \hline & 3 & 4 & 5 \end{array}$$

$3 \times 5 = 15$

ANALYSIS.—3 will divide each of the given numbers, and is therefore a factor of the greatest common divisor. 5 will divide each of the resulting quotients and is therefore a factor of the greatest common divisor. The quotients 3, 4, and 5, have no common divisor; therefore 3 and 5 are the only factors of the greatest common divisor, 15.

RULE.—*Separate the numbers into their prime factors and find the product of all the common factors.* Or,

Divide the numbers by any common divisor, the resulting quotients by another common divisor, and so continue to divide until quotients are obtained that have no common divisor.

The product of the divisors will be the greatest common divisor.

EXAMPLES.

What is the greatest common divisor of

2. 12, 16, 20, 24?
3. 18, 27, 36, 45?
4. 24, 48, 60, 72?
5. 36, 60, 72, 66?
6. 48, 72, 96, 84?
7. 18, 81, 72, 54?
8. 32, 48, 80, 96?
9. 45, 63, 99, 81?
10. 35, 56, 84, 63?
11. 16, 40, 72, 88?
12. 36, 60, 84, 96?
13. 30, 55, 85, 90?
14. 30, 54, 66, 78?
15. 14, 42, 63, 91?
16. 24, 28, 120, 144?
17. 33, 77, 143, 154?
18. 24, 72, 120, 168?
19. 42, 84, 252, 294?

133. When the numbers can not be factored readily, the following method is employed:

1. What is the greatest common divisor of 35 and 168?

PROCESS.

35)168(4
140
28)35(1
28
7)28(4
28

ANALYSIS.—The greatest common divisor can not be greater than the smaller number; therefore 35 will be the greatest common divisor if it is exactly contained in 168. By trial it is found that it is not an exact divisor of 168, since there is a remainder of 28. Therefore 35 is not the greatest common divisor.

Since 168 and 140, which is 4 times 35, are each divisible by the greatest common divisor, their difference, 28, must contain the greatest common divisor; therefore the greatest common divisor can not be greater than 28. 28 will be the greatest common divisor if it is exactly contained in 35; since if it be contained in 35, it will be contained in 140, and in 28 *plus* 140, or 168. By trial we find that it is not an exact divisor of 35, for there is a remainder of 7. Therefore 28 is not the greatest common divisor.

Since 28 and 35 are each divisible by the greatest common divisor, their difference, 7, must contain the greatest common divisor; therefore the greatest common divisor can not be greater than 7. 7 will be the greatest common divisor if it is exactly contained in 28; since if it be contained in itself and 28, it will be contained in their sum, 35, and also in 168, which is the sum of 28 and 4 times 35, or 140. By trial we find that it is an exact divisor of 28. Hence 7 is the greatest common divisor.

RULE.—*Divide the greater number by the less and if there be a remainder divide the less number by it, then the preceding divisor by the last remainder, and so on, till nothing remains. The last divisor will be the greatest common divisor.*

If more than two numbers are given, find the greatest common divisor of any two, then of this divisor and another of the given numbers, and so on. The last divisor will be the greatest common divisor.

Find the greatest common divisor of

2. 169 and 195.	8. 252 and 294.
3. 187 and 209.	9. 156 and 208.
4. 372 and 492.	10. 702 and 945.
5. 119 and 187.	11. 1029 and 1197.
6. 243 and 297.	12. 1666 and 1938.
7. 322 and 391.	13. 3596 and 3768.

What is the greatest common divisor of

14. 672, 352, 992?	17. 630, 1134, 1386?
15. 714, 867, 1088?	18. 462, 1764, 2562?
16. 462, 759, 1155?	19. 7955, 8769, 6401?

20. In a village some of the walks are 56 inches wide, some 70 inches, and others 84 inches. What is the width of the widest flagging that will suit all the walks?

21. A merchant has 60 pounds of tea of one kind, 75 pounds of another, and 100 pounds of another, which he wishes to put up in the largest possible equal packages without mixing the different kinds. How many pounds should be put in each package?

22. Mr. A. has 324 acres of land in one farm and 78 acres in another. He wishes to divide these into the largest possible fields of equal size. How many fields will there be, and how many acres in each field?

MULTIPLES.

134. 1. What numbers less than 25 will exactly contain 4? 5? 6?

2. What numbers less than 25 will exactly contain both 4 and 6?

3. Name some numbers that are exactly divisible by 5. By 4. By both 5 and 4.

4. Name some numbers that are exactly divisible by 2. By 3. By 4. By 2 and 4.

5. What is the smallest number that is exactly divisible by each of the numbers 2, 3, and 4?

6. What is the least number that will contain 10 and 15?

7. What common prime factors have 10 and 15? What factor occurs in 10 that does not in 15? What factor is found in 15 that is not found in 10?

8. What are all the different prime factors of 10 and 15?

9. How may the least number that will contain 10 and 15 be formed from their prime factors?

What is the least number that will exactly contain

10. 3, 6 and 9?	13. 2, 3, 5 and 6?
11. 3, 5 and 6?	14. 3, 4, 5 and 6?
12. 4, 8 and 12?	15. 3, 6, 8 and 12?

DEFINITIONS.

135. ***A Multiple*** of a number is a number that will exactly contain it.

A multiple of a number is obtained by *multiplying* the given number by some integer.

136. ***A Common Multiple*** of two or more numbers, is a number that will exactly contain each of them.

137. ***The Least Common Multiple*** of two or more numbers, is the least number that will exactly contain each of them.

138. PRINCIPLE.—*The least common multiple of two or more numbers is equal to the product of all the prime factors of the numbers, and no other factors.*

WRITTEN EXERCISES.

139. 1. Find the least common multiple of 30, 28 and 60?

1ST PROCESS.

$$30 = 2 \times 3 \times 5$$
$$28 = 2 \times 2 \times 7$$
$$60 = 2 \times 2 \times 3 \times 5$$
$$2 \times 2 \times 3 \times 5 \times 7 = 420$$

ANALYSIS.—Since the least common multiple is equal to the product of all the different prime factors of the numbers and no other factors, (Prin.) the numbers must be separated into their prime factors, and the product of all the different prime factors found. The prime factors of 60, the largest number, are 2, 2, 3 and 5. 28 contains a factor, 7, which is not found in 60. 60 contains all the factors of the other number, 30. Therefore all the different prime factors of the given numbers are 2, 2, 3, 5 and 7, and their product, 420, is the least common multiple.

RULE.—*Separate the given numbers into their prime factors.*

Find the product of all the different prime factors, using each factor the greatest number of times it occurs in any of the given numbers.

Find the least common multiple of

2. 28, 32 and 64.	5. 12, 16, 18 and 24.
3. 36, 72 and 144.	6. 15, 20, 25 and 30.
4. 45, 70 and 90.	7. 18, 54, 90 and 180.

What is the least common multiple of

8. 22, 55 and 77?	13. 12, 18, 24 and 96?
9. 48, 60 and 180?	14. 32, 56, 64 and 80?
10. 10, 64 and 96?	15. 14, 35, 50 and 28?
11. 81, 63 and 135?	16. 33, 99, 84 and 135?
12. 25, 70 and 95?	17. 17, 51, 65 and 121?

18. Find the least common multiple of 16, 20, and 30.

2D PROCESS.

$$\begin{array}{r|rrr} 2 & 16 & 20 & 30 \\ \hline 2 & 8 & 10 & 15 \\ \hline 5 & 4 & 5 & 15 \\ \hline & 4 & 1 & 3 \end{array}$$

$$2 \times 2 \times 5 \times 4 \times 3 = 240$$

ANALYSIS.—Since 2 is a prime factor of each of the numbers, it is also a factor of the least common multiple. (Prin.) Dividing, there remain as the other factors of the numbers, 8, 10, and 15. 2 is a prime factor of 8 and 10, and is therefore a factor of the least common multiple. Dividing, there remain 4, 5, and 15.

5 is a prime factor of 5 and 15, and is therefore another factor of the least common multiple. Dividing, there remain 4, 1, and 3, which are prime to each other. Therefore the product of the factors 2, 2, 5, 4, and 3, will be the least common multiple.

RULE.—*Write the given numbers in a horizontal line. Divide by any prime number that is an exact divisor of two or more of the given numbers, and write the quotients and undivided numbers in a line beneath.*

Thus continue to divide until the quotients and undivided numbers are prime to each other. The product of the divisors, and the numbers in the last horizontal line, will be the least common multiple.

In finding the least common multiple, all numbers that are factors of other given numbers may be disregarded. Thus, the multiples of 8, 16, 32, 64, 80, and 128 are the same as the multiples of 80 and 128.

EXAMPLES.

Find the least common multiple of

19. 60, 40, 120 and 72.
20. 81, 45, 108 and 135.
21. 40, 60, 80 and 120.
22. 32, 36, 72 and 80.
23. 30, 75, 60 and 90.
24. 24, 44, 65 and 100.

What is the least common multiple of

25. 8, 12, 16, 24 and 48?
26. 16, 20, 24, 32 and 40?
27. 25, 40, 75, 80 and 120?
28. 32, 45, 70, 64 and 90?

29. What is the smallest number that will exactly contain 16, 24, and 30?

30. How long must a box be that no room may be lost in packing in it books 6 inches, 8 inches, or 12 inches long?

31. A lady desires to purchase a piece of cloth that can be cut without waste, into parts 4, 5, or 6 yards long. How many yards must the piece contain?

32. I have a certain number of pennies which I can arrange in either 4, 6, 8, 10, or 12 equal piles. What number of pennies have I, if it is the least number that admits of such arrangement?

33. How many bushels will the smallest bin contain that can be emptied by taking out either 7 bushels, 10 bushels, or 30 bushels at a time?

34. Four agents start from New York at the same time. The first makes his trip in 8 weeks, the second in 9 weeks, the third in 15 weeks, and the fourth in 20 weeks. How many weeks will pass by before they will again start out from New York together?

35. Three men walk around a circular island, the circumference of which is 360 miles. A walks 15 miles a day, B 18 miles a day, and C 24 miles a day. If they start together and walk in the same direction, how many days will elapse before they will be together again?

36. Divide $5 \times 15 \times 80 \times 56 \times 81$ by $10 \times 5 \times 16 \times 78$.

37. If a man buys a lot whose sides measure respectively 48 feet, 60 feet, 96 feet and 108 feet, what will be the length of the longest boards which he can use to fence all the sides without cutting?

38. Find the greatest common divisor of 1744, 9564 and 8524.

39. What is the smallest number which can be divided by 250, 350, and 525 respectively, and leave a remainder of 25?

40. What is the greatest common divisor of 1728, 6912, and 8640?

41. A stock buyer wishes to invest the same amount of money in sheep at $3 each, hogs at $14 each, and cows at $21 each, as he does in beef cattle at $48 each. What is the smallest possible amount which he can invest in each?

42. Jones Brothers & Co., of Cincinnati, O., received an order for a number of Lyman's Historical Charts. It was found that if the charts were packed in boxes containing either 24, 28, 32, or 36 charts each, there was a remainder of 9 each time, but if packed in boxes containing 25 each, there was no remainder. How many charts were ordered?

43. A dealer in real estate purchased 3 lots of land whose width on the street were respectively 152 rods, 288 rods, and 184 rods. What is the width of the largest lots of equal size which can be formed from them?

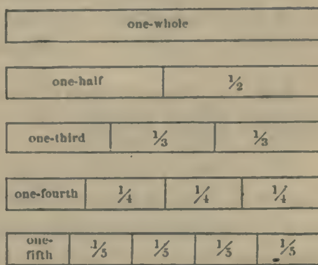
44. Divide $3 \times 5 \times 20 \times 10 \times 3 \times 13$ by $26 \times 9 \times 3 \times 4$.

45. Find the greatest common divisor of 2219, 4501, and 5964.

46. Divide $5 \times 8 \times 3 \times 7 \times 28 \times 99$ by $11 \times 4 \times 7 \times 5 \times 4$.

FRACTIONS

140. 1. When a line is divided into *two* equal parts, what is each part called?



2. When a line is divided into *three* equal parts, what is each part called? What are two of the parts called?

3. When a line is divided into *four* equals parts, what is each part called? What are two of the parts called? What are three of the parts called?

4. When any thing is divided into *five* equal parts, what is each part called? What are three parts called? What are four parts called?

5. When things are divided into 6, 7, 8, 9, 10, 15 equal parts respectively, what is each of the parts called? What are four of them called?

6. How many halves are there in any thing? How many thirds? Fourths? Fifths? Sixths? Tenths?

7. If 10 marbles are separated into 5 equal groups, what part of the marbles will be in each group?

8. How many are one-fifth of 10? Two-fifths? Three-fifths?

9. How many are one-sixth of 12? Two-sixths?

DEFINITIONS.

141. ***A Fraction*** is one or more of the equal parts of a unit.

142. The ***Unit of a Fraction*** is the unit which is divided into equal parts.

A fraction in which the unit has been divided into *any number* of equal parts is called a ***Common Fraction.***

A fraction in which the unit has been divided into *tenths, hundredths, thousandths,* etc., is called a ***Decimal Fraction.***

143. A ***Fractional Unit*** is one of the equal parts into which a unit is divided.

144. Since a fraction is one or more of the equal parts of any thing, to express a fraction two numbers are necessary, one to express the number of equal parts into which the unit has been divided, the other to express how many make the fraction. These numbers are written one above the other with a horizontal line between them.

145. The ***Denominator*** is the number which shows into how many equal parts the unit is divided.

It is written below the line.

Thus, in the fraction $\frac{5}{7}$, 7 is the denominator. It shows that the unit of the fraction has been divided into 7 equal parts.

146. The ***Numerator*** is the number which shows how many fractional units form the fraction.

It is written above the line.

Thus, in the fraction $\frac{5}{7}$, 5 is the numerator and shows how many fractional units form the fraction.

147. The numerator and denominator are called the ***Terms of a Fraction.***

148. Fractional units are named from the number of parts into which the unit is divided. Thus, $\frac{1}{6}$ is read one-*sixth*; $\frac{1}{7}$, one-*seventh*.

Fractions are read by naming the number and kind of fractional units. Thus, $\frac{5}{6}$ is read five-sixths; $\frac{5}{21}$, five twenty-firsts; $\frac{13}{35}$, thirteen thirty-fifths.

149. Read the following:

$\frac{5}{13}$	$\frac{11}{17}$	$\frac{24}{375}$	$\frac{19}{47}$	$\frac{18}{64}$	$\frac{24}{84}$	$\frac{595}{81}$
$\frac{23}{100}$	$\frac{85}{140}$	$\frac{29}{235}$	$\frac{35}{816}$	$\frac{814}{10606}$	$\frac{6845}{100000}$	$\frac{415684}{8070606}$
$\frac{825}{689}$	$\frac{829}{375}$	$\frac{469}{881}$	$\frac{385}{986}$	$\frac{5190}{1896}$	$\frac{18958}{38296}$	$\frac{51684}{789654}$

Express by figures:

1. Three elevenths. Five thirteenths. Eight twenty-firsts.
2. Forty-eight fiftieths. Twenty-seven eighty-fifths.
3. Sixty forty-eighths. Fifty-seven ninety-ninths.
4. Forty-two eighty-sevenths. Thirty-nine ninety-thirds.
5. Seventy-four one-hundredths. Ninety-seven one-hundred-fifths.
6. Fifty-two seventy-eighths. Thirty-six eighty-fourths.
7. Two hundred three-hundred-ninetieths.
8. Seven hundred seventy-one eight-hundred-sixtieths.
9. Two hundred forty-nine three-hundredths.
10. Five hundred sixty-six seven-hundred-fiftieths.
11. One hundred eleven two-hundredths.
12. Four thousand six hundred thirty five-thousandths.

Fractions are classified with reference to the relation of numerator and denominator thus:

150. A ***Proper Fraction*** is one in which the numerator is less than the denominator.

Thus, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{18}{35}$, etc., are proper fractions.

The value of a proper fraction is therefore less than 1.

151. An ***Improper Fraction*** is a fraction in which the numerator equals or exceeds the denominator.

Thus, $\frac{4}{4}$, $\frac{7}{3}$, $\frac{12}{7}$, are improper fractions.

The value of an improper fraction is therefore 1 or more than 1.

152. A ***Mixed Number*** is a number expressed by an integer and a fraction.

Thus, $2\frac{2}{3}$, $5\frac{1}{6}$, are mixed numbers.

Mixed numbers are read by naming the fraction after the whole number. Thus, $2\frac{3}{4}$ is read *two and three-fourths.*

Fractions may be regarded as expressing unexecuted *division.* Thus, $\frac{16}{8}$ is equal to $16 \div 8$; $\frac{15}{3}$ is read $15 \div 3$.

153. 1. Interpret the expression $\frac{5}{7}$.

Analysis.—$\frac{5}{7}$ represents 5 of 7 equal parts into which any thing is divided. It also represents one-seventh of five, and 5 divided by 7. It is read *five-sevenths.*

In like manner interpret:

2. $\frac{5}{9}$.	5. $\frac{13}{30}$.	8. $\frac{184}{196}$.	11. $\frac{81}{66}$.
3. $\frac{8}{11}$.	6. $\frac{24}{31}$.	9. $\frac{19}{134}$.	12. $\frac{99}{16}$.
4. $\frac{9}{13}$.	7. $\frac{45}{139}$.	10. $\frac{138}{130}$.	13. $\frac{368}{41}$.

REDUCTION.

CASE I.

154. To reduce fractions to larger, or higher terms.

1. In $\frac{1}{2}$ of an apple how many fourths are there? How many eighths?

2. How many sixths are there in $\frac{1}{3}$? How many ninths? How are the terms of the fraction $\frac{2}{6}$ obtained from those of $\frac{1}{3}$? $\frac{3}{9}$ from $\frac{1}{3}$?

3. How many eighths are there in $\frac{1}{4}$? How many twelfths?

4. How do the terms of the fraction $\frac{2}{8}$ compare with the terms of the fraction $\frac{1}{4}$?

5. In what equivalent fraction can $\frac{1}{8}$ be expressed ?

6. How do the terms of the fraction $\frac{1}{8}$ compare with those of $\frac{2}{16}$?

7. How are the terms of the fraction $\frac{2}{16}$ obtained from those of $\frac{1}{8}$?

8. How are the terms of the fraction $\frac{2}{8}$ obtained from $\frac{1}{4}$?

9. How are the terms of the fraction $\frac{4}{8}$ obtained from $\frac{1}{2}$?

10. What then may be done to the terms of a fraction without changing the value of the fraction ?

11. Change $\frac{1}{8}$ to 24ths.	15. Change $\frac{5}{12}$ to 36ths.
12. Change $\frac{3}{8}$ to 16ths.	16. Change $\frac{3}{5}$ to 20ths.
13. Change $\frac{5}{8}$ to 24ths.	17. Change $\frac{4}{7}$ to 14ths.
14. Change $\frac{3}{4}$ to 12ths.	18. Change $\frac{5}{9}$ to 18ths.

155. ***Reduction of Fractions*** is the process of changing their form without changing their value.

156. A fraction is expressed in ***Larger*** or ***Higher Terms*** when its numerator and denominator are expressed by *larger numbers*.

157. PRINCIPLE.—*Multiplying both terms of a fraction by the same number, does not change the value of the fraction.*

WRITTEN EXERCISES.

1. Change $\frac{7}{15}$ to forty-fifths.

PROCESS.

$45 \div 15 = 3$

$\frac{7 \times 3 = 21}{15 \times 3 = 45}$

ANALYSIS.—Since there are 45 forty-fifths in 1, in $\frac{1}{15}$ there are 3 forty-fifths; and in $\frac{7}{15}$ there are 7 times $\frac{3}{45}$, or $\frac{21}{45}$; or,

Since the denominator of the required fraction is 3 times that of the given fraction, we must multiply the terms of the fraction by 3.

RULE.—*Multiply the terms of the fraction by such a number as will change the given denominator to the required denominator.*

Reduce:

2. $\frac{13}{25}$ to 50ths.
3. $\frac{17}{30}$ to 60ths.
4. $\frac{25}{35}$ to 70ths.
5. $\frac{32}{42}$ to 84ths.
6. $\frac{18}{20}$ to 40ths.
7. $\frac{12}{27}$ to 54ths.
8. $\frac{24}{33}$ to 66ths.
9. $\frac{33}{27}$ to 54ths.
10. $\frac{15}{28}$ to 84ths.

Reduce:

11. $\frac{25}{40}$ to 120ths.
12. $\frac{31}{32}$ to 64ths.
13. $\frac{29}{37}$ to 74ths.
14. $\frac{36}{70}$ to 210ths.
15. $\frac{49}{80}$ to 240ths.
16. $\frac{30}{75}$ to 225ths.
17. $\frac{52}{87}$ to 348ths.
18. $\frac{71}{93}$ to 558ths.
19. $\frac{86}{94}$ to 235ths.

CASE II.

158. To reduce fractions to smaller, or lower terms.

1. How many *fourths* are there in $\frac{2}{8}$? How many in $\frac{4}{16}$?

2. How many *thirds* are there in $\frac{2}{6}$? How many in $\frac{8}{12}$?

3. How does the number of *eighths* of any thing compare with the *fourths?* Thirds with sixths? Halves with eights?

4. How do the terms of the fraction $\frac{1}{4}$ compare with those of $\frac{2}{8}$? How with those of $\frac{4}{16}$?

5. How do the terms of the fraction $\frac{1}{3}$ compare with those of $\frac{2}{6}$? How with those of $\frac{8}{12}$?

6. How are the terms of the fraction $\frac{1}{4}$ obtained from those of the fraction $\frac{2}{8}$? How from those of $\frac{4}{16}$?

7. How are the terms of the faction $\frac{1}{3}$ obtained from $\frac{2}{6}$?

8. What then may be done to the terms of a fraction without changing the value of the fraction?

9. Express $\frac{8}{12}$, $\frac{9}{12}$, $\frac{8}{16}$, in smaller or lower terms.

10. Express $\frac{12}{24}$, $\frac{16}{36}$, $\frac{18}{27}$, in smaller or lower terms.

11. Reduce $\frac{9}{45}$, $\frac{18}{54}$, $\frac{24}{48}$, to smaller or lower terms.

12. Reduce $\frac{30}{50}$, $\frac{60}{120}$, $\frac{45}{180}$, to smaller or lower terms.

159. A fraction is expressed in ***Smaller,*** or ***Lower Terms*** when its numerator and denominator are expressed in *smaller* numbers.

160. A fraction is expressed in the ***Smallest,*** or ***Lowest Terms*** when its numerator and denominator have *no common divisor.*

161. PRINCIPLE.—*Dividing both terms of a fraction by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

162. 1. Change $\frac{32}{48}$ to an equivalent fraction expressed in its smallest, or lowest terms.

PROCESS.

$$4\left|\frac{32}{48}\right. = 4\left|\frac{8}{12}\right. = \frac{2}{3}$$

Or,

$$\frac{32}{48} = \frac{32 \div 16}{48 \div 16} = \frac{2}{3}$$

ANALYSIS.—Since the denominator of the required fraction is to be smaller than that of the given fraction, we may obtain an equivalent fraction having smaller terms, by dividing the terms of the given fraction by any exact divisor, as 4 (Prin.), and the terms of the resulting fraction by 4. We thus obtain the fraction $\frac{2}{3}$, whose terms have no common divisor. The fraction is therefore in its *smallest* terms. Or,

Since fractions are in their smallest terms when their numerator and denominator have no common divisor, to reduce them to their smallest terms we may divide both terms by their greatest common divisor.

RULE.—*Divide the numerator and denominator by any common divisor, and continue to divide thus until the terms have no common divisor, Or,*

Divide both terms by their greatest common divisor.

2. Reduce $\frac{16}{36}$, $\frac{40}{60}$, $\frac{24}{30}$, $\frac{64}{80}$, to their smallest terms.
3. Reduce $\frac{32}{48}$, $\frac{96}{108}$, $\frac{21}{28}$, $\frac{120}{144}$, to their smallest terms.

Reduce to their smallest, or lowest terms:

4. $\frac{22}{77}$.	9. $\frac{182}{826}$.	14. $\frac{648}{810}$.	19. $\frac{3584}{3808}$.
5. $\frac{30}{42}$.	10. $\frac{276}{300}$.	15. $\frac{720}{1440}$.	20. $\frac{5643}{5940}$.
6. $\frac{305}{810}$.	11. $\frac{648}{810}$.	16. $\frac{1702}{1886}$.	21. $\frac{1710}{14304}$.
7. $\frac{280}{308}$.	12. $\frac{308}{815}$.	17. $\frac{2196}{3172}$.	22. $\frac{5040}{17160}$.
8. $\frac{480}{630}$.	13. $\frac{144}{1728}$.	18. $\frac{3432}{7804}$.	23. $\frac{4692}{161976}$.

CASE III.

163. To reduce integers or mixed numbers to improper fractions.

1. How many halves are there in 1 apple? In 4 apples? In 6 apples?
2. How many thirds are there in 1 orange? In 3 oranges? In 5 oranges?
3. How many fourths are there in 2? In 3? In 4?
4. How many fifths are there in 3? In 4? In 6?
5. How many fourths are there in $1\frac{1}{4}$? In $2\frac{3}{4}$?
6. How many thirds are there in $2\frac{2}{3}$? In $3\frac{1}{3}$? In $6\frac{2}{3}$?

WRITTEN EXERCISES.

164. 1. Reduce $8\frac{3}{7}$ to sevenths.

PROCESS.

$$8 = \frac{56}{7}$$

$$8\frac{3}{7} = \frac{56}{7} + \frac{3}{7} = \frac{59}{7}$$

ANALYSIS.—Since in 1 there are 7 sevenths, in 8 there are 8 times 7 sevenths, or $\frac{56}{7}$; and in $8 + \frac{3}{7}$ there are $\frac{56}{7} + \frac{3}{7}$, or $\frac{59}{7}$.

RULE.—*Multiply the integers by the given denominator, to this product add the numerator of the fractional part, if there be any, and write the result over the given denominator.*

2. Change $5\frac{1}{4}$ to fourths.
3. Change 15 to fifths.
4. Reduce $13\frac{1}{6}$ to sixths.
5. Reduce $18\frac{3}{11}$ to elevenths.

6. Change $5\frac{1}{3}$ to ninths. To eighteenths.
7. Change $6\frac{5}{12}$ to twelfths. To twenty-fourths.
8. Change $8\frac{3}{14}$ to fourteenths. To forty-seconds.
9. Reduce $9\frac{5}{20}$ to twentieths. To sixtieths.

Reduce to improper fractions:

10. $13\frac{5}{9}$.	14. $25\frac{6}{7}$.	18. $421\frac{12}{13}$.	22. $867\frac{5}{43}$.
11. $12\frac{3}{7}$.	15. $29\frac{6}{17}$.	19. $540\frac{17}{18}$.	23. $904\frac{73}{97}$.
12. $18\frac{6}{11}$.	16. $37\frac{13}{14}$.	20. $763\frac{19}{81}$.	24. $314\frac{16}{141}$.
13. $23\frac{5}{9}$.	17. $56\frac{23}{41}$.	21. $419\frac{3}{17}$.	25. $721\frac{12}{128}$.

CASE IV.

165. To reduce improper fractions to integers or mixed numbers.

1. How many days are there in 6 half-days? In 8 half-days? In 14 half-days?

2. How many yards are there in 9 thirds of a yard? In 15 thirds? In 18 thirds?

3. If a boy pick $\frac{1}{2}$ bushel of peaches per hour, how many bushels can he pick in 10 hours? How many are 10 halves?

4. If a man can earn $\frac{1}{4}$ of a dollar per hour, how much can he earn in 12 hours? How many are 12 fourths?

5. How many units are there in $\frac{15}{3}$? $\frac{36}{7}$? $\frac{32}{5}$? $\frac{61}{6}$? $\frac{81}{9}$?

6. How many dollars are there in $\$\frac{37}{4}$? $\$\frac{47}{5}$? $\$\frac{118}{40}$? $\$\frac{109}{9}$?

WRITTEN EXERCISES.

166. 1. Reduce $\frac{123}{7}$ to a mixed number.

PROCESS.

$\frac{123}{7} = 123 \div 7 = 17\frac{4}{7}$

ANALYSIS.—Since 7 sevenths equal 1 unit, 123 sevenths are equal to as many units as 7 sevenths are contained times in $\frac{123}{7}$, or $17\frac{4}{7}$ times. Therefore $\frac{123}{7} = 17\frac{4}{7}$.

RULE.—*Divide the numerator by the denominator.*

2. Change $\$\frac{125}{25}$ to dollars.
3. Change $\frac{134}{11}$ pounds to pounds.
4. Change $\frac{212}{20}$ ounces to ounces.
5. Change $\frac{135}{8}$ to a mixed number.
6. Change $\frac{146}{12}$ to a mixed number.
7. Change $\frac{212}{10}$ to a mixed number.

Reduce to integers or mixed numbers:

8. $\frac{136}{18}$.	12. $\frac{2915}{193}$.	16. $\frac{2728}{148}$.	20. $\frac{38495}{894}$.
9. $\frac{347}{15}$.	13. $\frac{1681}{258}$.	17. $\frac{31445}{242}$.	21. $\frac{51696}{3134}$.
10. $\frac{263}{19}$.	14. $\frac{3926}{208}$.	18. $\frac{27241}{324}$.	22. $\frac{21568}{3115}$.
11. $\frac{3846}{141}$.	15. $\frac{1866}{132}$.	19. $\frac{47246}{812}$.	23. $\frac{31246}{8103}$.

CASE V.

167. To reduce dissimilar fractions to similar fractions.

1. How many *fourths* are there in $\frac{1}{2}$ of an orange?
2. How many *sixths* of a field are there in $\frac{1}{3}$ of a field?
3. How many *eighths* in $\frac{3}{4}$? How many *ninths* in $\frac{2}{3}$?
4. Express each of the fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ as twelfths.
5. Express each of the fractions $\frac{4}{5}$ and $\frac{3}{4}$ as twentieths.
6. If $\frac{1}{2}$ is divided into 3 equal parts, how large is each part?
7. If $\frac{1}{3}$ is divided into 2 equal parts, how large is each part?
8. When $\frac{1}{2}$ and $\frac{1}{3}$ are divided into equal parts, what parts are common to both?
9. When $\frac{1}{3}$ and $\frac{1}{4}$ are divided into equal parts, what parts are common to both?
10. What equal parts are common to both $\frac{1}{4}$ and $\frac{1}{5}$?
11. When $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are divided into equal parts, what parts are common to all?
12. Change $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, to equivalent fractions having the same fractional unit. Express the resulting fractions in equivalent fractions having their least common denominator.

Reduce to fractions having the same fractional unit:

11. $\frac{3}{4}$ and $\frac{5}{8}$.	14. $\frac{3}{7}$ and $\frac{5}{14}$.	17. $\frac{5}{10}$ and $\frac{7}{15}$.
12. $\frac{2}{5}$ and $\frac{3}{15}$.	15. $\frac{5}{8}$ and $\frac{5}{12}$.	18. $\frac{3}{10}$ and $\frac{7}{12}$.
13. $\frac{2}{3}$ and $\frac{3}{5}$.	16. $\frac{6}{9}$ and $\frac{7}{12}$.	19. $\frac{5}{12}$ and $\frac{4}{18}$.

168. ***Similar Fractions*** are those that have the same fractional unit.

169. ***Dissimilar Fractions*** are those that have not the same fractional unit.

170. Similar fractions have a ***Common Denominator.***

171. When similar fractions are expressed in their *smallest terms* they have their ***Least Common Denominator.***

172. PRINCIPLES.—1. *A common denominator of two or more fractions is a common multiple of their denominators.*

2. *The least common denominator of two or more fractions is the least common multiple of their denominators.*

WRITTEN EXERCISES.

173. 1. Reduce $\frac{3}{4}$ and $\frac{5}{8}$ to similar fractions.

PROCESS.

$\frac{3}{4} = \frac{3 \times 8}{4 \times 8} = \frac{24}{32}$

$\frac{5}{8} = \frac{5 \times 4}{8 \times 4} = \frac{20}{32}$

ANALYSIS.—Since similar fractions have a common denominator, to make these fractions similar we must change them to equivalent fractions having a common denominator.

Since a common denominator of two or more fractions is a common multiple of their denominators (Prin.), we find a common multiple of the denominators 4 and 8, which is 32.

We then multiply the terms of each fraction by such a number as will change the fraction to thirty-seconds.

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to similar fractions having their least common denominator.

PROCESS.

$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$

ANALYSIS.—The least common denominator of several fractions is the least common multiple of their denominators (Prin. 2); therefore we find the least common multiple of 3, 4, and 6, which is 12.

We then multiply the terms of each fraction by such a number as will change it to *twelfths*, or to a fraction whose denominator is 12. Or,

Since 1 is equal to $\frac{12}{12}$, $\frac{1}{3}$ is equal to $\frac{1}{3}$ of $\frac{12}{12}$, or $\frac{4}{12}$, and $\frac{2}{3}$ are equal to 2 times $\frac{4}{12}$, or $\frac{8}{12}$, etc.

RULE.—*Find the common, or least common multiple of the denominators for a common, or least common denominator.*

Divide this denominator by the denominator of each fraction and multiply both terms of the fraction by the quotient.

Reduce all mixed numbers to improper fractions and all fractions to their smallest terms.

Change the following to similar fractions having their least common denominator:

3. $\frac{5}{6}$, $\frac{4}{9}$, $\frac{7}{12}$.
4. $\frac{1}{4}$, $\frac{11}{12}$, $\frac{13}{20}$.
5. $\frac{3}{5}$, $\frac{7}{20}$, $\frac{13}{30}$.
6. $\frac{5}{7}$, $\frac{3}{28}$, $\frac{15}{35}$.
7. $\frac{3}{7}$, $\frac{5}{8}$, $\frac{6}{12}$.
8. $\frac{2}{9}$, $\frac{3}{27}$, $\frac{5}{18}$.
9. $\frac{3}{5}$, $\frac{17}{20}$, $\frac{19}{24}$.
10. $\frac{5}{12}$, $\frac{13}{16}$, $\frac{25}{36}$.
11. $3\frac{5}{7}$, $1\frac{8}{9}$, $\frac{13}{27}$.
12. $\frac{13}{25}$, $1\frac{4}{5}$, $\frac{36}{90}$.
13. $\frac{12}{35}$, $\frac{13}{25}$, $\frac{15}{49}$.
14. $\frac{13}{36}$, $\frac{7}{45}$, $\frac{16}{75}$.

ADDITION.

174. 1. James has 2 fifths of a dollar, and his brother has 4 fifths of a dollar. How many fifths have both?

2. George spent $\$\frac{3}{7}$ on Monday, and $\$\frac{2}{7}$ on Tuesday. How much did he spend in both days? How many sevenths are $\frac{3}{7}$ and $\frac{2}{7}$?

3. Mr. A. sold $\frac{1}{4}$ of his farm at one time, and $\frac{2}{4}$ at another time. What part of it did he sell? What is the sum of $\frac{1}{4}$ and $\frac{2}{4}$?

4. James caught a fish in the morning that weighed $\frac{3}{8}$ of a pound, and another in the afternoon that weighed $\frac{7}{8}$ of a pound. What did both weigh? What is the sum of $\frac{3}{8}$ and $\frac{7}{8}$?

5. Marian gave $\$\frac{1}{2}$ for a book and $\$\frac{1}{4}$ for some writing paper. How much did she pay for both? What is the sum of $\frac{1}{2}$ and $\frac{1}{4}$?

6. Ella gave $\frac{3}{4}$ of her apple to a poor beggar and Julia gave him $\frac{1}{2}$ of hers. How many fourths did he receive? What is the sum of $\frac{3}{4}$ and $\frac{1}{2}$?

7. I bought $\frac{1}{2}$ of an acre of ground for a site for a house, and $\frac{1}{3}$ of an acre for a site for a barn. How much land did I buy? What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? Of $\frac{1}{3}$ and $\frac{1}{4}$? Of $\frac{3}{5}$ and $\frac{7}{10}$?

8. Mr. A. gave $\$\frac{3}{12}$ to one man and $\$\frac{7}{8}$ to another. How much did he give to both?

9. A merchant sold $\frac{3}{4}$ of a bushel of clover seed to one farmer, and $\frac{2}{3}$ of a bushel to another. How much did he sell to both?

10. Sarah paid $\$\frac{3}{4}$ for eggs and $\$\frac{7}{8}$ for butter. How much did both cost her?

11. I paid $\$\frac{3}{4}$ for turnips and $\$\frac{5}{8}$ for squashes. How much did I pay for both?

12. A merchant sold $\frac{3}{4}$ of a yard of silk to one lady and $\frac{3}{8}$ of a yard to another. How much did he sell to both?

13. A boy earned $\$\frac{2}{3}$ in the forenoon and $\$\frac{1}{6}$ in the afternoon. How much did he earn that day?

14. What is the sum of $\frac{5}{8}$ and $\frac{7}{12}$? $\frac{5}{6}$ and $\frac{7}{15}$? $\frac{8}{15}$ and $\frac{8}{10}$?

15. What kind of fractions can be added without changing their form?

16. What must be done to dissimilar fractions before they can be added? How are dissimilar fractions made similar?

175. Principles.—1. *Only similar fractions can be added.*

2. *Dissimilar fractions must be reduced to similar fractions before adding.*

WRITTEN EXERCISES.

176. 1. What is the sum of $\frac{5}{6}$, $\frac{3}{4}$ and $\frac{2}{9}$?

PROCESS.

$$\frac{5}{6}+\frac{3}{4}+\frac{2}{9}=\frac{30}{36}+\frac{27}{36}+\frac{8}{36}=\frac{65}{36}$$

Analysis.—Since the fractions are not similar, before adding we must change them to similar fractions, or equivalent fractions having a common denominator.

The least common denominator of the given fractions is 36; and $\frac{5}{6}=\frac{30}{36}$, $\frac{3}{4}=\frac{27}{36}$, and $\frac{2}{9}=\frac{8}{36}$. Hence the sum of the given fractions must be equal to the sum of $\frac{30}{36}$, $\frac{27}{36}$, and $\frac{8}{36}$, which is $\frac{65}{36}$, or $1\frac{29}{36}$.

2. What is the sum of $5\frac{1}{2}$, $6\frac{2}{3}$ and $2\frac{3}{4}$?

PROCESS.

$5\frac{1}{2}=5\frac{6}{12}$

$6\frac{2}{3}=6\frac{8}{12}$

$2\frac{3}{4}=2\frac{9}{12}$

Analysis.—Since the numbers are composed of both integers and fractions, we may add each separately and unite the sums. Thus, the sum of the fractional parts is $\frac{23}{12}$, or $1\frac{11}{12}$; the sum of the integers is 13; and the sum of both, $14\frac{11}{12}$.

Rule.—*Reduce the given fractions to similar fractions, add their numerators and write the sum over the common denominator.*

When there are mixed numbers, or integers, add the fractions and integers separately and then add the results.

If the sum be an improper fraction, reduce to an integral or mixed number.

Find the sum

3. Of $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$ and $\frac{11}{12}$.
4. Of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$.
5. Of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{7}{12}$.
6. Of $\frac{3}{5}$, $\frac{3}{8}$, $\frac{4}{7}$, $\frac{7}{8}$ and $\frac{9}{10}$.
7. Of $\frac{6}{9}$, $\frac{12}{14}$, $\frac{15}{18}$ and $\frac{21}{36}$.
8. Of $2\frac{1}{2}$, 4, $3\frac{1}{4}$ and $5\frac{5}{6}$.
9. Of $27\frac{4}{10}$, $8\frac{3}{4}$ and $40\frac{5}{8}$.
10. Of $13\frac{4}{7}$, $15\frac{3}{4}$ and $20\frac{17}{35}$.

Add the following:

11. $\frac{4}{7}$, $\frac{15}{21}$, $\frac{13}{14}$, $\frac{5}{28}$, $\frac{7}{35}$.
12. $4\frac{3}{7}$, $5\frac{2}{5}$, $8\frac{1}{9}$, $2\frac{3}{5}$, $7\frac{1}{3}$, $4\frac{3}{7}$.
13. $9\frac{1}{4}$, $7\frac{3}{5}$, $8\frac{2}{3}$, $7\frac{5}{12}$, $8\frac{5}{20}$.
14. $7\frac{4}{18}$, 8, $6\frac{5}{12}$, $3\frac{7}{60}$, $5\frac{4}{78}$.
15. $\frac{21}{23}$, $\frac{17}{46}$, $2\frac{3}{11}$, $3\frac{5}{44}$, $2\frac{13}{69}$.
16. $3\frac{5}{9}$, $4\frac{7}{18}$, 6, $9\frac{8}{19}$, $\frac{5}{27}$.

17. A farmer received \$$18\frac{3}{4}$ for hay, \$$65\frac{3}{5}$ for a cow, and \$$161\frac{3}{4}$ for a horse. How much did he receive for all?

18. A man earns \$$67\frac{5}{8}$ per month, and each of his two sons \$$23\frac{3}{4}$ per month. How much do all earn per month?

19. A pedestrian walked $45\frac{2}{3}$ miles on Monday, $47\frac{3}{5}$ on Tuesday, $50\frac{5}{7}$ on Wednesday. How far did he walk?

20. A has $5\frac{1}{4}$ acres of land, B has $10\frac{3}{5}$ acres more than A, C has as much as both A and B. How many acres have B and C together?

SUBTRACTION.

177. 1. Mary earned 5 ninths of a dollar and spent 2 ninths. How many ninths of a dollar had she left?

2. Mr. A. owning $\frac{5}{7}$ of a flouring mill, sold $\frac{3}{7}$ of it. How many sevenths did he then own?

3. From $\frac{5}{8}$ subtract $\frac{3}{8}$. From $\frac{7}{9}$ subtract $\frac{5}{9}$. From $\frac{8}{11}$ subtract $\frac{5}{11}$.

4. From $\frac{11}{13}$ subtract $\frac{9}{13}$. From $\frac{8}{15}$ subtract $\frac{4}{15}$.

5. Mr. B. owned a lot containing $\frac{7}{8}$ of an acre. How much had he left after selling $\frac{1}{4}$ of an acre?

6. A boy paid \$$\frac{3}{8}$ for a whip, but sold it after a time for \$$\frac{1}{4}$. How much did he lose?

7. Find the difference between $\frac{1}{2}$ and $\frac{1}{4}$. $\frac{1}{4}$ and $\frac{1}{8}$.

8. What kind of fractions can be subtracted without changing their form?

9. What must be done to dissimilar fractions before they can be subtracted? How are dissimilar fractions made similar?

178. Principles.—1. *Only similar fractions can be subtracted.*

2. *Dissimilar fractions must be reduced to similar fractions before subtracting.*

WRITTEN EXERCISES.

179. 1. What is the difference between $\frac{11}{12}$ and $\frac{2}{9}$?

PROCESS.

$\frac{11}{12} - \frac{2}{9} = \frac{33}{36} - \frac{8}{36}$

$\frac{33-8}{36} = \frac{25}{36}$

Analysis.—Since the fractions are not similar, before subtracting we must change them to similar fractions.

The common denominator of the given fractions is 36; and $\frac{11}{12} = \frac{33}{36}$, and $\frac{2}{9} = \frac{8}{36}$. Hence the difference between the given fractions is equal to the difference between $\frac{33}{36}$ and $\frac{8}{36}$, which is $\frac{25}{36}$.

2. What is the difference between $23\frac{1}{4}$ and $4\frac{5}{6}$.

PROCESS.

$$23\tfrac{1}{4} = 23\tfrac{3}{12} = 22\tfrac{15}{12}$$
$$4\tfrac{5}{6} = 4\tfrac{10}{12} = 4\tfrac{10}{12}$$
$$18\tfrac{5}{12}$$

Analysis.—Since the numbers are composed of both integers and fractions, we may subtract each separately.

We first reduce the given fractions to similar fractions. Since we can not take $\frac{10}{12}$ from $\frac{3}{12}$, we unite with the $\frac{3}{12}$ 1, or $\frac{12}{12}$, taken from 23, making $\frac{15}{12}$. Then $22\frac{15}{12} - 4\frac{10}{12} = 18\frac{5}{12}$, the remainder.

Rule.—*Reduce the fractions to similar fractions.*

Find the difference of the numerators and write it over the common denominator.

When there are mixed numbers or integers, subtract the fractions and integers separately.

Mixed numbers may be reduced to improper fractions and subtracted according to the first part of the rule.

	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
From	$\frac{7}{8}$	$\frac{2}{3}$	$\frac{6}{13}$	$\frac{13}{20}$	$\frac{17}{30}$	$\frac{14}{25}$
Take	$\frac{3}{5}$	$\frac{3}{11}$	$\frac{5}{17}$	$\frac{4}{15}$	$\frac{12}{25}$	$\frac{7}{32}$

9. From $\frac{3}{5}$ take $\frac{3}{10}$.
10. From $\frac{5}{8}$ take $\frac{3}{6}$.
11. From $\frac{6}{20}$ take $\frac{3}{15}$.
12. From $\frac{5}{24}$ take $\frac{5}{48}$.
13. From $\frac{17}{30}$ take $\frac{7}{24}$.
14. From $\frac{27}{35}$ take $\frac{15}{20}$.
15. From $10\frac{14}{30}$ take $\frac{13}{99}$.
16. From $66\frac{2}{3}$ take $33\frac{1}{3}$.
17. From $210\frac{1}{2}$ take $109\frac{2}{3}$.
18. From 112 take $75\frac{1}{2}$.
19. From $606\frac{3}{4}$ take $70\frac{1}{2}$.
20. From $589\frac{2}{3}$ take $67\frac{3}{4}$.

21. If from a bin containing $506\frac{2}{3}$ tons of coal, $418\frac{1}{2}$ tons are taken, how many tons still remain?

22. A lady having \$25, paid \$$2\frac{1}{2}$ for a pair of gloves, \$$15\frac{3}{4}$ for a bonnet, and \$$3\frac{3}{4}$ for some lace. How much money had she left?

23. A man owned a farm of 412 acres. He sold three parcels of land from it, the first containing $60\frac{2}{3}$ acres, the second $45\frac{1}{2}$ acres, and the third $116\frac{1}{3}$ acres. How many acres did he sell, and how many had he remaining?

24. A clerk earned \$$50\frac{1}{2}$ per month. He paid \$$20\frac{3}{4}$ for board, \$$5\frac{3}{4}$ for washing, and \$$4\frac{2}{5}$ for other expenses. How much did he save per month?

MULTIPLICATION.

CASE I.

180. To multiply a fraction by an integer.

1. At \$$\frac{1}{4}$ a yard what will 3 yards of cambric cost?

2. If a man can earn \$$\frac{3}{10}$ per hour, how much can he earn in 5 hours? How much can he earn in 8 hours?

3. James gave $\frac{2}{3}$ of an apple to each of 5 children. How many apples did he give to all? How much is 5 times $\frac{2}{3}$?

4. How many fifths are there in 6 times $\frac{4}{5}$? In 7 times $\frac{3}{5}$?

5. If Mr. A. spends \$$2\frac{1}{2}$ per day, how much will he spend in 5 days? How much in 10 days?

6. How much is 2 times $\frac{3}{8}$? How does the result compare with $\frac{3}{8}$? How is it obtained from $\frac{3}{8}$?

7. In multiplying a fraction what part of the fraction do we multiply?

8. Multiply $\frac{3}{5}$ by 2. $\frac{4}{7}$ by 3. $\frac{6}{11}$ by 7.

9. Express 2 times $\frac{3}{8}$ in smallest terms. How is *this* result obtained from the fraction $\frac{3}{8}$?

10. In what other way then may we multiply a fraction?

11. How much is 3 times $\frac{5}{6}$? 4 times $\frac{3}{8}$? 6 times $\frac{5}{12}$?

12. How much is 5 times $\frac{3}{7}$? 6 times $\frac{2}{5}$? 9 times $\frac{3}{8}$?

13. How much is 4 times $\frac{5}{8}$? 3 times $\frac{5}{12}$? 5 times $\frac{7}{8}$?

14. How much is 4 times $\frac{5}{9}$? 7 times $\frac{4}{21}$? 9 times $\frac{7}{27}$?

181. PRINCIPLE.—*Multiplying the numerator or dividing the denominator of a fraction by any number, multiplies the fraction by that number.*

WRITTEN EXERCISES.

1. Multiply $\frac{13}{24}$ by 6.

PROCESS.

$$\frac{13}{24} \times 6 = \frac{13 \times 6}{24} = \frac{78}{24} = 3\frac{1}{4}$$

Or,

$$\frac{13}{24} \times 6 = \frac{13}{24 \div 6} = \frac{13}{4} = 3\frac{1}{4}$$

ANALYSIS.—6 times 13 twenty-fourths are 78 twenty-fourths, or $3\frac{1}{4}$. Or,

Since dividing the denominator multiplies the fraction (Prin.), 6 times $\frac{13}{24}$ are $\frac{13}{4}$, or $3\frac{1}{4}$.

RULE.—*Multiply the numerator or divide the denominator by the integer.*

Multiply:

2. $\frac{3}{11}$ by 5.
3. $\frac{9}{14}$ by 7.
4. $\frac{7}{25}$ by 5.
5. $\frac{6}{26}$ by 3.
6. $\frac{14}{11}$ by 17.

Multiply:

7. $\frac{18}{21}$ by 7.
8. $\frac{7}{48}$ by 6.
9. $\frac{9}{56}$ by 8.
10. $\frac{17}{24}$ by 3.
11. $\frac{14}{28}$ by 7.

Multiply:

12. $\frac{12}{23}$ by 9.
13. $\frac{16}{39}$ by 13.
14. $\frac{15}{39}$ by 14.
15. $\frac{18}{37}$ by 18.
16. $\frac{16}{85}$ by 75.

17. What is the value of a load of 17 bushels of apples at $\$\frac{1}{3}$ a bushel?

18. If a boy earns $\$\frac{2}{3}$ per day, how much can he earn in 9 days?

19. At $\$7\frac{3}{4}$ a barrel what will 7 barrels of flour cost?

PROCESS.

$$\begin{array}{r} \$7\frac{3}{4} \\ 7 \\ \hline 5\frac{1}{4} \\ 49 \\ \hline 54\frac{1}{4} \end{array}$$

ANALYSIS.—In multiplying a mixed number, we multiply the fractional part and integer separately and add the results.

Thus, 7 times $\$\frac{3}{4} = \$\frac{21}{4} = \$5\frac{1}{4}$. 7 times \$7 are \$49, and the sum of \$49 and $\$5\frac{1}{4}$ is $\$54\frac{1}{4}$.

We may reduce the mixed number to an improper fraction before multiplying.

20. If a man travel $21\frac{2}{3}$ miles per day, how far can he travel in 4 days?

21. What will 13 yards of cloth cost at $\$6\frac{1}{2}$ a yard?

22. If a steamship sails $17\frac{1}{5}$ miles an hour, how far can she sail in 9 hours?

CASE II.

182. To multiply an integer by a fraction.

1. Henry had 6 rabbits and sold $\frac{1}{3}$ of them to James. How many did he sell?

2. Jane had 16 cherries and gave $\frac{1}{4}$ of them to her sister. How many did she give to her sister?

3. How much is $\frac{1}{6}$ of \$18? $\frac{3}{6}$ of \$18?

4. How much is $\frac{1}{5}$ of 7 apples? $\frac{1}{8}$ of 9 bushels? $\frac{1}{6}$ of 5 ounces? $\frac{1}{7}$ of 3 lemons?

5. How much is $\frac{1}{8}$ of 5? $\frac{3}{8}$ of 5? $\frac{7}{8}$ of 5?

6. How much is $\frac{3}{4}$ of 7? $\frac{5}{6}$ of 8? $\frac{3}{5}$ of 12?

7. What is $\frac{3}{4}$ of 36? $\frac{5}{8}$ of 32? $\frac{7}{9}$ of 54?

8. How much is $\frac{2}{5}$ of 35 tons? $\frac{3}{7}$ of 49 horses? $\frac{5}{8}$ of 80?

183. PRINCIPLE.—*Multiplying by a fraction is taking such a part of a number as is indicated by the fraction.*

WRITTEN EXERCISES.

1. Multiply 75 by $\frac{3}{5}$?

PROCESS.

$$75 \times \frac{3}{5} = \frac{\overset{15}{\cancel{75}} \times 3}{\cancel{5}} = 45$$

ANALYSIS.—To multiply 75 by $\frac{3}{5}$ is to find $\frac{3}{5}$ of 75. $\frac{3}{5} = 3$ times $\frac{1}{5}$. $\frac{1}{5}$ of 75 is 15, and $\frac{3}{5} = 3$ times 15, or 45. Or,

Since $\frac{3}{5} = \frac{1}{5}$ of 3, $\frac{3}{5}$ of $75 = \frac{1}{5}$ of 3 times 75, or $\frac{75 \times 3}{5} = 45$.

RULE.—*Multiply the integer by the numerator of the multiplier, and divide the product by the denominator.*

When possible use *cancellation.*

2. Multiply 9 by $\frac{7}{18}$.
3. Multiply 17 by $\frac{8}{51}$.
4. Multiply 12 by $\frac{18}{48}$.
5. Multiply 18 by $\frac{13}{36}$.
6. Multiply 100 by $\frac{3}{25}$.
7. Multiply 144 by $\frac{11}{16}$.
8. Multiply 51 by $\frac{5}{17}$.
9. Multiply 79 by $\frac{6}{19}$.
10. Multiply \$318 by $\frac{6}{25}$.
11. Multiply \$406 by $\frac{3}{14}$.
12. Multiply \$718 by $\frac{13}{17}$.
13. Multiply \$825 by $\frac{23}{45}$.

14. A man owned a mill worth \$7850. How much money should he receive for $\frac{3}{5}$ of it?

15. A sportsman shot 48 birds one day, and $\frac{5}{6}$ as many the next. How many did he shoot in both days?

16. Multiply 46 by $5\frac{3}{5}$.

PROCESS.

$$\begin{array}{r} 46 \\ 5\frac{3}{5} \\ \hline 27\frac{3}{5} \\ 230 \\ \hline 257\frac{3}{5} \end{array}$$

Or, $5\frac{3}{5} = \frac{28}{5}$

$46 \times \frac{28}{5} = \frac{1288}{5}$

$\frac{1288}{5} = 257\frac{3}{5}$

ANALYSIS.—In multiplying by a mixed number we multiply by the integer and fraction separately and add the products. Thus, $\frac{3}{5}$ of 46 is $\frac{138}{5}$, or $27\frac{3}{5}$. 5 times $46 = 230$, and $27\frac{3}{5} + 230 = 257\frac{3}{5}$, the product. Or, we may reduce the mixed number to an improper fraction and multiply.

17. Multiply 36 by $5\frac{1}{2}$; $6\frac{3}{5}$; $7\frac{5}{9}$; $8\frac{10}{11}$; $10\frac{5}{18}$.

CASE III.

184. To multiply a fraction by a fraction.

1. How much is $\frac{1}{2}$ of 4 fifths of a yard? How much is $\frac{1}{3}$ of 3 ninths of a yard? $\frac{1}{4}$ of 8 twelfths of a yard?

2. If James has $\frac{3}{8}$ of a dollar, and Anna has $\frac{1}{3}$ as much, how much has Anna? If Henry has $\frac{2}{3}$ as much as James, how much has he? How much is $\frac{1}{3}$ of $\frac{3}{8}$? $\frac{2}{3}$ of $\frac{3}{8}$?

3. A man who owned $\frac{5}{8}$ of a steamship sold $\frac{1}{5}$ of his share. What part of the vessel did he sell? How much is $\frac{1}{5}$ of $\frac{5}{8}$?

4. How much is $\frac{1}{7}$ of $\frac{14}{18}$? $\frac{2}{3}$ of $\frac{12}{17}$? $\frac{5}{6}$ of $\frac{12}{13}$? $\frac{7}{8}$ of $\frac{8}{13}$?

5. If $\frac{1}{2}$ of a yard be divided into 2 equal parts, what part of a yard will each part be? How much is $\frac{1}{2}$ of $\frac{1}{2}$ yard? $\frac{1}{2}$ of $\frac{1}{4}$ yard?

6. At $\$\frac{1}{2}$ a bushel what will $\frac{1}{4}$ of a bushel of oats cost? How much is $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{2}{3}$ of $\frac{1}{2}$? $\frac{3}{4}$ of $\frac{1}{2}$?

7. What is the value of $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{2}{3}$ of $\frac{1}{5}$? $\frac{3}{4}$ of $\frac{2}{5}$?

WRITTEN EXERCISES.

1. Multiply $\frac{4}{7}$ by $\frac{3}{5}$.

PROCESS.

$$\frac{4}{7} \times \frac{3}{5} = \frac{4 \times 3}{7 \times 5} = \frac{12}{35}$$

ANALYSIS.—To multiply $\frac{4}{7}$ by $\frac{3}{5}$ is to find $\frac{3}{5}$ of $\frac{4}{7}$, or 3 times $\frac{1}{5}$ of $\frac{4}{7}$. $\frac{1}{5}$ of $\frac{4}{7} = \frac{4}{7 \times 5}$, and $\frac{3}{5}$ are 3 times $\frac{4}{7 \times 5}$ or $\frac{4 \times 3}{7 \times 5} = \frac{12}{35}$.

RULE.—*Reduce all integers and mixed numbers to improper fractions.*

Multiply the numerators together for the numerator of the product, and the denominators together for its denominator.

1. When possible use cancellation.

2. The word *of* between fractions is equivalent to the *sign of multiplication*. Such expressions are sometimes called *compound fractions*. Thus, $\frac{3}{4}$ of $\frac{7}{5}$ is equal to $\frac{3}{4} \times \frac{7}{5}$.

3. Integers may be expressed in the form of fractions by writing 1 as a denominator. Thus, 4 may be written as $\frac{4}{1}$.

Multiply:

2. $\frac{7}{12}$ by $\frac{5}{6}$.
3. $\frac{5}{8}$ by $\frac{3}{7}$.
4. $\frac{6}{13}$ by $\frac{5}{9}$.

Multiply:

5. $\frac{5}{8}$ by $\frac{4}{7}$.
6. $\frac{7}{12}$ by $\frac{6}{11}$.
7. $\frac{8}{15}$ by $\frac{5}{8}$.

Multiply:

8. $\frac{11}{18}$ by $\frac{7}{11}$.
9. $\frac{24}{81}$ by $\frac{7}{12}$.
10. $\frac{15}{82}$ by $\frac{7}{15}$.

Find the value of

11. $\frac{15}{27} \times \frac{9}{20} \times \frac{4}{21} \times \frac{5}{7}$.
12. $\frac{45}{51} \times \frac{17}{40} \times \frac{9}{25} \times \frac{8}{11}$.
13. $\frac{24}{51} \times \frac{17}{48} \times \frac{12}{15} \times \frac{7}{16}$.
14. $\frac{31}{45} \times \frac{15}{62} \times \frac{5}{7} \times \frac{9}{10}$.
15. $\frac{17}{20} \times \frac{10}{11} \times \frac{3}{5} \times \frac{7}{12}$.
16. $\frac{24}{35} \times \frac{7}{15} \times \frac{15}{18} \times \frac{9}{13}$.
17. $\frac{25}{94} \times \frac{47}{86} \times \frac{32}{75}$.
18. $\frac{22}{120} \times \frac{60}{93} \times \frac{31}{114}$.
19. $\frac{3}{4} \times \frac{2}{3} \times \frac{6}{7} \times \frac{7}{8}$.
20. $\frac{3}{5} \times \frac{5}{7} \times \frac{6}{11} \times \frac{5}{18}$.
21. $\frac{5}{9} \times \frac{7}{8} \times \frac{5}{3} \times \frac{16}{21}$.
22. $\frac{9}{11} \times \frac{10}{13} \times \frac{26}{40} \times \frac{11}{27}$.

23. Multiply $\frac{3}{7}$ of $1\frac{2}{5}$ by $\frac{5}{6}$ of $\frac{9}{12}$ of 4.

PROCESS.

$\frac{3}{7} \times \frac{7}{5} \times \frac{5}{6} \times \frac{9}{12} \times \frac{4}{1} = \frac{3}{2}$, or $1\frac{1}{2}$

ANALYSIS.—All the mixed numbers must be changed to improper fractions, and all whole numbers expressed in the fractional form. Multiplying and *cancelling* we have $\frac{3}{2}$, or $1\frac{1}{2}$.

24. Multiply $\frac{5}{9}$ of $\frac{3}{7}$ of 5 by $\frac{3}{11}$ of $\frac{8}{5}$ of $3\frac{1}{2}$.
25. Multiply $\frac{3}{5}$ of $\frac{7}{12}$ of 8 by $\frac{5}{7}$ of $\frac{9}{10}$ of 15.
26. Multiply $3\frac{1}{2}$ times $\frac{3}{7}$ by 4 times $\frac{3}{4}$ of 7.
27. Multiply $5\frac{1}{4}$ times $\frac{7}{9}$ of 18 by $\frac{5}{6}$ of 3 times $\frac{1}{5}$ of 4.
28. What will be the cost of $\frac{9}{10}$ of a yard of cloth at \$$\frac{5}{6}$ a yard?
29. $16\frac{1}{2}$ feet make a rod. How many feet are there in $5\frac{1}{2}$ rods?
30. A man who owned $\frac{3}{5}$ of a mill, sold $\frac{5}{7}$ of his share. What part of the mill did he sell?
31. How many yards of cloth are there in $12\frac{1}{2}$ pieces of cloth, each piece containing $42\frac{3}{4}$ yards?
32. At \$$16\frac{7}{8}$ per ton how much can be realized from the sale of $4\frac{4}{5}$ tons of hay?

33. What is the value of $\frac{3}{5}$ of $\frac{9}{11}$ of $\frac{10}{21}$ of $\frac{8}{25}$ of 15?

34. What is the value of $\frac{8}{17}$ of $3\frac{4}{9}$ of $\frac{34}{49}$ of 29?

DIVISION.

CASE I.

185. To divide a fraction by an integer.

1. Mr. Allen divided 3 fourths of a dollar equally between 3 boys. How much did each receive?

2. A man divided $\frac{3}{5}$ of an acre into 3 equal lots. How large were they?

3. If 7 yards of cloth were bought for \$$\frac{7}{9}$, what was the cost per yard?

4. If 4 books cost \$$\frac{12}{13}$, what is the cost of each?

5. Mr. Hurd put $\frac{3}{5}$ of his crop of wheat on 3 wagons. What part of his crop was on each wagon? How much is $\frac{3}{5} \div 3$?

6. In dividing a fraction, what part of the fraction is divided?

7. A gentleman divided $\frac{1}{2}$ a barrel of flour equally between 2 people. What part of a barrel did each receive? How much is $\frac{1}{2} \div 2$?

8. 3 boys spent altogether \$$\frac{1}{4}$. If each spent the same amount, what part of a dollar did each spend? How much is $\frac{1}{4} \div 3$?

9. Mr. Smith divided $\frac{1}{6}$ of his farm into 3 equal fields. What part of his farm did each field contain? How much is $\frac{1}{6} \div 3$? How is the result obtained from the fraction $\frac{1}{6}$?

10. In what other way, then, besides dividing the numerator may a fraction be divided?

11. When a number is divided by 3 what part of it is found?

12. When a fraction is divided by 7 what part of it is found?

13. What is $\frac{1}{3}$ of $\frac{3}{7}$? $\frac{3}{7} \div 3$? $\frac{1}{5}$ of $\frac{5}{16}$? $\frac{5}{16} \div 5$?

14. What is the value of $\frac{3}{5} \div 2$? Of $\frac{5}{9} \div 3$? Of $\frac{6}{11} \div 9$?

186. Principle.—*Dividing the numerator or multiplying the denominator of a fraction by any number, divides the fraction by that number.*

WRITTEN EXERCISES.

1. Divide $\frac{12}{17}$ by 6.

PROCESS.

$$\frac{12}{17} \div 6 = \frac{12 \div 6}{17} = \frac{2}{17}$$

Or,

$$\frac{12}{17} \div 6 = \frac{12}{17 \times 6} = \frac{2}{17}$$

Analysis.—Since dividing the numerator of a fraction divides the fraction, the fraction $\frac{12}{17}$ may be divided by 6 by dividing the numerator by 6. Or,

Since multiplying the denominator divides the fraction, the fraction may be divided by 6 by multiplying the denominator by 6. The result by both processes is $\frac{2}{17}$.

Rule.—*Divide the numerator or multiply the denominator by the given integer.*

Divide:	Divide:	Divide:
2. $\frac{3}{8}$ by 4.	6. $\frac{24}{37}$ by 8.	10. $\frac{49}{24}$ by 21.
3. $\frac{8}{9}$ by 8.	7. $\frac{45}{73}$ by 15.	11. $\frac{25}{40}$ by 18.
4. $\frac{12}{29}$ by 3.	8. $\frac{63}{25}$ by 7.	12. $\frac{76}{84}$ by 38.
5. $\frac{17}{12}$ by 6.	9. $\frac{81}{43}$ by 18.	13. $\frac{63}{90}$ by 35.

14. Divide $16\frac{3}{4}$ by 5.

PROCESS.

$$16\tfrac{3}{4} = \frac{67}{4}$$

$$\frac{67}{4} \div 5 = \frac{67}{20} = 3\tfrac{7}{20}$$

Or, $5\,)\,16\frac{3}{4}$
$\quad 3\frac{7}{20}$

Analysis.—We may reduce $16\frac{3}{4}$ to an improper fraction, and divide as before. Or,

We may divide without reducing to an improper fraction. Thus, 5 is contained in $16\frac{3}{4}$, 3 times and a remainder of $1\frac{3}{4}$, or $\frac{7}{4}$, and $\frac{7}{4}$ divided by 5, equals $\frac{7}{20}$. Therefore the quotient is $3\frac{7}{20}$.

Divide:	Divide:	Divide:
15. $17\frac{3}{5}$ by 6.	17. $38\frac{5}{9}$ by 8.	19. $36\frac{1}{2}$ by 10.
16. $25\frac{3}{7}$ by 4.	18. $24\frac{5}{11}$ by 9.	20. $25\frac{3}{7}$ by 7.

21. A man gave each of his 5 sons an equal share of $\frac{5}{8}$ of his estate. What part of the whole did each receive?

22. 8 men built $\frac{7}{8}$ of a mile of wall in 10 days. What part of a mile did each man build daily?

23. Mr. B. earned $\$35\frac{1}{8}$ by working 8 days. How much did he earn per day?

24. I paid $\$10\frac{1}{8}$ for 27 pounds of butter. What did I pay a pound?

25. A farmer realized $\$233\frac{1}{2}$ for 21 bushels of clover seed. How much did he get per bushel?

26. Three men who have been partners in business gain $\$3216\frac{7}{9}$. If they share equally, what will be each one's part of the gain?

CASE II.

187. To divide an integer by a fraction.

1. How many fourths are there in an apple? In 2 apples?

2. How many apples, at $\frac{1}{2}$ cent each, can be bought for 2 cents? For 3 cents? For 10 cents?

3. At $\$\frac{1}{8}$ a yard, how many yards of cloth can be bought for \$1? For \$2? For \$3? For \$10?

4. At $\$\frac{1}{5}$ an ounce, how many ounces of nutmegs can be bought for \$2? How many at $\$\frac{2}{5}$ an ounce can be bought for \$2?

5. If a man can mow $\frac{1}{7}$ of a field per day, how long will it take him to mow the entire field? How long if he can mow $\frac{2}{7}$ per day?

6. At $\$\frac{1}{8}$ a bushel, how many bushels of lime can be bought for \$5? At $\$\frac{3}{8}$ a bushel how many bushels can be bought for \$5?

7. When apples are worth $\frac{3}{4}$ of a cent apiece, how many can be bought for 12 cents? How many times is $\frac{3}{4}$ contained in 12?

8. How many pieces of cloth $\frac{3}{4}$ of a yard long can be cut from a piece 9 yards long? How many times is $\frac{3}{4}$ contained in 9?

9. How many times is $\frac{5}{6}$ contained in 8? $\frac{3}{5}$ in 9?

10. What is the quotient when 8 is divided by $\frac{1}{7}$? 10 by $\frac{6}{7}$?

11. What is the value of $10 \div \frac{1}{8}$? $7 \div \frac{5}{8}$? $9 \div \frac{6}{7}$?

WRITTEN EXERCISES.

1. Divide 12 by $\frac{6}{7}$.

PROCESS.

$$12 \div \frac{6}{7} = \frac{\overset{2}{\cancel{12}} \times 7}{\cancel{6}} = 14$$

Or,

$$12 = \frac{84}{7};\ \frac{84}{7} \div \frac{6}{7} = 14$$

ANALYSIS.—$\frac{1}{7}$ is contained in 12 7 times 12, or 84 times; and $\frac{6}{7}$, one-sixth of 84 times, or 14 times. Or,

Reducing 12 to sevenths, we have 84 sevenths. 6 sevenths are contained in 84 sevenths 14 times.

RULE.—*Multiply the integer by the denominator of the fraction and divide the product by the numerator. Or,*

Reduce the dividend and the divisor to similar fractions, and divide the numerator of the dividend by the numerator of the divisor.

Divide:	Divide:	Divide:
2. 18 by $\frac{3}{7}$.	10. 72 by $\frac{8}{5}$.	18. 51 by $\frac{17}{20}$.
3. 64 by $\frac{8}{9}$.	11. 34 by $\frac{5}{6}$.	19. 69 by $\frac{23}{37}$.
4. 26 by $\frac{7}{3}$.	12. 15 by $\frac{10}{13}$.	20. 70 by $\frac{14}{61}$.
5. 48 by $\frac{6}{7}$.	13. 49 by $\frac{21}{40}$.	21. 65 by $\frac{13}{29}$.
6. 75 by $\frac{15}{16}$.	14. 91 by $\frac{13}{7}$.	22. 90 by $\frac{45}{64}$.
7. 31 by $\frac{9}{25}$.	15. 64 by $\frac{16}{17}$.	23. 36 by $\frac{13}{23}$.
8. 45 by $\frac{45}{60}$.	16. 54 by $\frac{18}{23}$.	24. 39 by $\frac{25}{31}$.
9. 39 by $\frac{3}{10}$.	17. 39 by $\frac{13}{41}$.	25. 24 by $\frac{17}{15}$.

26. Divide 24 by $3\frac{1}{2}$.

PROCESS.

$3\frac{1}{2} = \frac{7}{2}$

$24 \div \frac{7}{2} = \frac{24 \times 2}{7} = 6\frac{6}{7}$

Or,

$$\begin{array}{c|c} 3\frac{1}{2} & 24 \\ 2 & 2 \\ \hline 7 & 48 \\ & \overline{6\frac{6}{7}} \end{array}$$

ANALYSIS.—When the divisor is a mixed number we reduce it to an improper fraction and proceed according to the rule. Thus, $3\frac{1}{2} = \frac{7}{2}$; and 24 divided by $\frac{1}{2} = 48$, and by $\frac{7}{2} = \frac{1}{7}$ of 48, or $6\frac{6}{7}$. Or,

We may reduce $3\frac{1}{2}$ to halves, and 24 to halves, and divide the numerator of the dividend by the numerator of the divisor.

Divide the following by both processes:

27. 15 by $3\frac{1}{2}$.	29. 26 by $7\frac{3}{4}$.	31. 39 by $8\frac{3}{5}$.
28. 23 by $6\frac{2}{3}$.	30. 35 by $6\frac{1}{7}$.	32. 46 by $7\frac{5}{11}$.

33. At $\$\frac{3}{4}$ a yard, how many yards of cloth can be bought for \$15?

34. When corn is worth $\$\frac{5}{6}$ a bushel, how many bushels must a man sell to get money enough to pay \$18 taxes?

35. A man invested \$32 in peaches at $\$1\frac{3}{5}$ per basket. How many baskets did he purchase?

36. What is the price of hay, when $3\frac{4}{5}$ tons sell for \$37?

37. Mr. Shaw paid \$6 for $14\frac{2}{3}$ pounds of Java coffee. What did it cost him per pound?

38. It requires 656 pounds of meat to supply 34 soldiers $3\frac{1}{2}$ weeks. How much does each soldier eat daily?

CASE III.

188. To divide a fraction by a fraction.

1. How many fourths are there in 1? How many fifths? eighths? tenths? fifteenths? twentieths?

2. How many pieces of cloth $\frac{1}{8}$ of a yard long, can be cut from a yard?

3. If the pieces were $\frac{2}{8}$ of a yard long, how many would there be? How does the number compare with the number when the pieces are $\frac{1}{8}$ of a yard long?

4. If the pieces were $\frac{4}{8}$ of a yard long, how many would there be? How does the number of pieces compare with the number when the pieces are $\frac{1}{8}$ of a yard long?

5. How many times is $\frac{1}{8}$ contained in 1? $\frac{2}{8}$? $\frac{3}{8}$? $\frac{4}{8}$? $\frac{5}{8}$?

6. Since $\frac{1}{8}$ is contained in 1, eight times, how many times will it be contained in $\frac{1}{2}$? What part of 8 times will it be contained in $\frac{1}{2}$?

7. Since $\frac{3}{8}$ is contained in 1 $\frac{1}{3}$ of 8 times, or $\frac{8}{3}$ times, how many times will it be contained in $\frac{1}{2}$? How many times in $\frac{1}{3}$? $\frac{2}{3}$? $\frac{3}{4}$?

8. What is the value of $1 \div \frac{1}{7}$? Of $1 \div \frac{3}{7}$? $\frac{1}{2} \div \frac{3}{7}$? $\frac{3}{4} \div \frac{3}{7}$?

9. Into how many parts of 3 eighths of a dollar each, can 6 eighths of a dollar be divided?

10. How many sacks containing $\frac{3}{10}$ of a barrel each, can be filled from $\frac{9}{10}$ of a barrel of flour? How many times is $\frac{3}{10}$ contained in $\frac{9}{10}$? $\frac{3}{7}$ in $\frac{9}{7}$? $\frac{3}{20}$ in $\frac{9}{20}$? $\frac{3}{47}$ in $\frac{9}{47}$?

11. How many pine-apples at \$$\frac{1}{4}$ each, can be bought for \$$\frac{1}{2}$?

12. How many times is $\frac{1}{4}$ contained in $\frac{1}{2}$? $\frac{1}{8}$ in $\frac{1}{4}$? $\frac{3}{8}$ in $\frac{3}{4}$?

13. How many times is $\frac{1}{3}$ contained in 1? In $\frac{1}{2}$? In $\frac{1}{3}$?

14. How many times is $\frac{2}{3}$ contained in 1? In $\frac{1}{5}$? In $\frac{2}{3}$?

WRITTEN EXERCISES.

1. Divide $\frac{4}{7}$ by $\frac{3}{5}$.

PROCESS.

$$\frac{4}{7} \div \frac{3}{5} = \frac{4}{7} \times \frac{5}{3} = \frac{20}{21}$$

Or,

$$\frac{4}{7} \div \frac{3}{5} = \frac{20}{35} \div \frac{21}{35} = \frac{20}{21}$$

ANALYSIS.—$\frac{1}{5}$ is contained in 1, 5 times; and $\frac{3}{5}$ is contained in 1, one-third of 5 times, or $\frac{5}{3}$ times.

And since $\frac{3}{5}$ is contained in 1, $\frac{5}{3}$ times, in $\frac{4}{7}$ it will be contained $\frac{4}{7}$ of $\frac{5}{3} = \frac{20}{21}$ times. Or,

$\frac{4}{7}$ is equal to $\frac{20}{35}$, and $\frac{3}{5}$ is equal to $\frac{21}{35}$. 21 thirty-fifths are contained in 20 thirty-fifths $\frac{20}{21}$ times.

X RULE.—*Multiply the dividend by the divisor inverted.* Or, *Reduce the dividend and divisor to similar fractions and divide the numerator of the dividend by the numerator of the divisor.*

x When possible use cancellation.

Divide:	Divide:	Divide:
2. $\frac{13}{24}$ by $\frac{7}{8}$.	6. $\frac{13}{46}$ by $\frac{5}{13}$.	10. $\frac{19}{45}$ by $\frac{7}{15}$.
3. $\frac{23}{40}$ by $\frac{15}{23}$.	7. $\frac{18}{49}$ by $\frac{6}{7}$.	11. $\frac{26}{75}$ by $\frac{13}{25}$.
4. $\frac{24}{65}$ by $\frac{8}{13}$.	8. $\frac{25}{72}$ by $\frac{5}{24}$.	12. $\frac{25}{63}$ by $\frac{8}{21}$.
5. $\frac{34}{55}$ by $\frac{7}{11}$.	9. $\frac{36}{91}$ by $\frac{9}{13}$.	13. $\frac{31}{72}$ by $\frac{7}{24}$.

14. What is the quotient of $\frac{2}{3}$ of $\frac{3}{5}$ of $5\frac{2}{3}$ divided by $\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{7}{9}$?

PROCESS.

$$\frac{2}{3} \text{ of } \frac{3}{5} \text{ of } \frac{17}{3} \div \frac{3}{5} \text{ of } \frac{5}{7} \text{ of } \frac{7}{9} =$$

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{5}} \times \frac{17}{\cancel{3}} \times \frac{\cancel{5}}{\cancel{3}} \times \frac{\cancel{7}}{5} \times \frac{\overset{3}{\cancel{9}}}{\cancel{7}} = \frac{34}{5} = 6\frac{4}{5}$$

ANALYSIS.—In the solution of examples like this, all mixed numbers should be changed to improper fractions, and all fractions that are factors of the divisor, inverted, and the product found as in previous examples.

15. Divide $\frac{3}{7}$ of $\frac{5}{8}$ of 16 by $\frac{3}{5}$ of $\frac{6}{7}$ of $5\frac{1}{3}$.
16. Divide $\frac{5}{9}$ of $\frac{13}{15}$ of $5\frac{1}{3}$ by $4\frac{1}{3}$ times $\frac{1}{5}$ of 16.
17. Divide $\frac{7}{9}$ of $\frac{3}{4}$ of $\frac{3}{7}$ by $\frac{1}{7}$ of $\frac{3}{4}$.
18. Divide $\frac{1}{8}$ of $\frac{5}{3}$ of $\frac{9}{11}$ by 5 times $\frac{3}{8}$ of $\frac{6}{7}$.
19. Divide $\frac{3}{5}$ of $\frac{3}{7}$ of 15 by $\frac{2}{5}$ of $\frac{5}{9}$ of 6.
20. Divide $\frac{5}{8}$ of $\frac{6}{11}$ of 22 by $\frac{3}{10}$ of $\frac{5}{7}$ of 16.
21. Divide $\frac{3}{14}$ of $3\frac{2}{3}$ of 6 by $\frac{7}{9}$ of 6 times $1\frac{2}{7}$.
22. Divide $8\frac{1}{4}$ times $\frac{1}{8}$ of 7 by $\frac{3}{4}$ of $\frac{2}{3}$ of 5.

23. How many pieces of ribbon $\frac{2}{15}$ of a yard in length, can be made from $\frac{1}{2}$ of $\frac{9}{10}$ of a yard?

24. If a man spends \$$\frac{2}{3}$ per day for cigars, in how many days will he spend \$$17\frac{1}{2}$?

25. How many yards of cloth at \$$3\frac{2}{5}$ per yard can be bought for \$$317\frac{2}{3}$?

26. At $\$\frac{3}{8}$ per bushel, how many bushels of potatoes can be bought for $\$17\frac{1}{4}$?

27. If a family uses $\frac{2}{3}$ of a barrel of flour a week, how long will $5\frac{1}{4}$ barrels last?

28. If a boy earns $\$\frac{3}{7}$ daily, how long will it take him to earn $\$3\frac{5}{9}$?

29. A certain number multiplied by $\frac{3}{7}$ is equal to $\frac{21}{35}$. What is that number?

30. If a man can saw $1\frac{1}{4}$ cords of wood in one day, how long will he require to saw $17\frac{1}{5}$ cords?

31. If a horse eats $12\frac{1}{2}$ bushels of oats in 5 weeks, how much does he eat in a day?

32. When wheat is selling at $\$1\frac{7}{8}$ per bushel, how many bushels can be bought for \$3168?

FRACTIONAL FORMS.

189. Expressions of unexecuted division are often written in the *form of a fraction*.

190. A fractional form having an integral denominator and a fractional numerator is called a ***Complex Fraction.***

Thus, $\dfrac{\frac{3}{4}}{5}$ and $\dfrac{6\frac{1}{2}}{9}$ are complex fractions.

Expressions which have a fraction in the denominator can not properly be regarded as Complex Fractions, though they are commonly classified as such.

1. Find the value of the fractional form $\dfrac{\frac{5}{6}}{\frac{7}{8}}$.

PROCESS.

$$\dfrac{\frac{5}{6}}{\frac{7}{8}} = \frac{5}{6} \div \frac{7}{8} = \frac{5}{\underset{3}{\cancel{6}}} \times \frac{\overset{4}{\cancel{8}}}{7} = \frac{20}{21}$$

ANALYSIS.—$\dfrac{\frac{5}{6}}{\frac{7}{8}}$ is an expression of division, and is the same as $\frac{5}{6} \div \frac{7}{8}$, which is equal to $\frac{20}{21}$.

Reduce to simple fractions:

2. $\dfrac{\frac{5}{7}}{9}$.	6. $\dfrac{4}{6\frac{2}{7}}$.	10. $\dfrac{5\frac{1}{3}}{6\frac{1}{4}}$.	14. $\dfrac{\frac{2}{3} \text{ of } \frac{5}{6}}{\frac{2}{3} \text{ of } 9}$.
3. $\dfrac{\frac{3}{4}}{\frac{5}{6}}$.	7. $\dfrac{\frac{1}{3}}{4\frac{2}{3}}$.	11. $\dfrac{\frac{13}{24}}{\frac{5}{19}}$.	15. $\dfrac{\frac{2}{5} \text{ of } 3}{6}$.
4. $\dfrac{3\frac{1}{2}}{6}$.	8. $\dfrac{5\frac{1}{4}}{\frac{2}{3}}$.	12. $\dfrac{\frac{24}{19}}{\frac{12}{7}}$.	16. $\dfrac{3}{\frac{1}{5} \text{ of } \frac{7}{8}}$.
5. $\dfrac{\frac{23}{26}}{\frac{9}{13}}$	9. $\dfrac{6\frac{1}{7}}{5\frac{1}{3}}$.	13. $\dfrac{5\frac{1}{3}}{\frac{17}{36}}$.	17. $\dfrac{\frac{3}{5} \text{ of } \frac{5}{6}}{4\frac{1}{3} \times 3}$.

FRACTIONAL RELATION OF NUMBERS.

CASE I.

191. To find the relation of one number to another.

1. What part of 5 cents is 1 cent? 2 cents? 3 cents? 4 cents?

2. What part of 9 acres is 5 acres? 7 acres? 3 acres? 4 acres?

3. What part of 4 apples is 1 apple? $\frac{1}{2}$ of 1 apple? $\frac{3}{4}$ of 1 apple? $\frac{5}{6}$ of 1 apple?

4. What part of \$5 is \$2? \$$\frac{1}{2}$? \$$\frac{1}{3}$? \$$\frac{1}{5}$? \$$\frac{3}{7}$?

5. What part of \$6 is \$1? \$$\frac{1}{2}$? \$$\frac{1}{3}$? \$$\frac{2}{3}$?

6. Henry had \$5 and gave his brother \$$\frac{3}{8}$. What part of his money did he give his brother?

7. James earned \$7, and his brother \$2. What part of the whole did each earn?

PRINCIPLE.—*Only like numbers can have relation to each other.*

8. What is the relation of 5 to 9?

ANALYSIS.—1 is $\frac{1}{9}$ of 9, and 5 is 5 times $\frac{1}{9}$ of 9, or, $\frac{5}{9}$. Hence 5 is $\frac{5}{9}$ of 9.

What is the relation

9. Of 7 to 21?	12. Of 9 to 18?	15. Of 15 to 24?
10. Of 12 to 16?	13. Of 8 to 32?	16. Of 14 to 35?
11. Of 10 to 28?	14. Of 32 to 48?	17. Of 18 to 81?

18. What is the relation of $\frac{3}{5}$ to 2?

ANALYSIS.—1 is $\frac{1}{2}$ of 2, and $\frac{3}{5}$ of 1 is $\frac{3}{5}$ of $\frac{1}{2}$ of 2, or $\frac{3}{10}$ of 2.
Hence $\frac{3}{5}$ is $\frac{3}{10}$ of 2.

What is the relation

19. Of $\frac{3}{5}$ to 4?	22. Of $\frac{5}{6}$ to 8?	25. Of $\frac{2}{9}$ to 6?
20. Of $\frac{2}{7}$ to 9?	23. Of $\frac{2}{3}$ to 15?	26. Of $\frac{2}{3}$ to 18?
21. Of $\frac{3}{8}$ to 6?	24. Of $\frac{3}{7}$ to 25?	27. Of $\frac{5}{6}$ to 12?

28. What is the relation of $\frac{2}{3}$ to $\frac{5}{7}$?

ANALYSIS.—$\frac{1}{7}$ is $\frac{1}{5}$ of $\frac{5}{7}$, and 1 is 7 times $\frac{1}{5}$ of $\frac{5}{7}$, or $\frac{7}{5}$ of $\frac{5}{7}$; and since 1 is $\frac{7}{5}$ of $\frac{5}{7}$, $\frac{2}{3}$ of 1 is $\frac{2}{3}$ of $\frac{7}{5}$ of $\frac{5}{7}$, or $\frac{14}{15}$ of $\frac{5}{7}$.
Hence $\frac{2}{3}$ is $\frac{14}{15}$ of $\frac{5}{7}$.

What is the relation

29. Of $\frac{2}{3}$ to $\frac{2}{5}$?	32. Of $\frac{5}{7}$ to $\frac{6}{11}$?	35. Of $\frac{3}{8}$ to $\frac{6}{7}$?
30. Of $\frac{3}{7}$ to $\frac{2}{8}$?	33. Of $\frac{5}{8}$ to $\frac{2}{3}$?	36. Of $\frac{6}{7}$ to $\frac{4}{5}$?
31. Of $\frac{3}{11}$ to $\frac{4}{7}$?	34. Of $\frac{8}{2}$ to $\frac{6}{5}$?	37. Of $\frac{3}{10}$ to $\frac{5}{9}$?

CASE II.

192. A number and its relation to another number given, to find the other number.

1. 2 cents are $\frac{1}{2}$ of how many cents? $\frac{1}{3}$ of how many cents? $\frac{1}{4}$ of how many cents?

2. 3 is $\frac{1}{3}$ of what number? $\frac{1}{5}$ of what number? $\frac{1}{6}$ of what number?

3. 8 is $\frac{1}{5}$ of what number? $\frac{2}{5}$ of what number? $\frac{4}{5}$ of what number?

4. 12 is $\frac{1}{6}$ of what number? $\frac{3}{7}$ of what number? $\frac{3}{8}$ of what number?

5. $\frac{1}{2}$ is $\frac{1}{3}$ of what number? $\frac{1}{4}$ of what number?

6. $\frac{2}{3}$ is $\frac{1}{5}$ of what number? $\frac{2}{5}$ of what number?

7. 24 is $\frac{4}{5}$ of what number?

Analysis.—Since 24 is $\frac{4}{5}$ of a certain number, 1 fifth of the number is $\frac{1}{4}$ of 24, or 6; and since 6 is $\frac{1}{5}$ of the number, the number must be 5 times 6, or 30. Hence 24 is $\frac{4}{5}$ of 30.

8. 24 is $\frac{2}{3}$ of what number? 56 is $\frac{7}{8}$ of what number?
9. 18 is $\frac{3}{5}$ of what number? 49 is $\frac{7}{9}$ of what number?
10. 24 is $\frac{6}{7}$ of what number? 42 is $\frac{3}{5}$ of what number?
11. 45 is $\frac{5}{8}$ of what number? 96 is $\frac{12}{13}$ of what number?

12. $\frac{4}{7}$ is $\frac{2}{3}$ of what number?

Analysis.—Since $\frac{4}{7}$ is $\frac{2}{3}$ of a certain number, $\frac{1}{3}$ of the number is $\frac{1}{2}$ of $\frac{4}{7}$, or $\frac{2}{7}$; and since $\frac{1}{3}$ of the number is $\frac{2}{7}$, the number must be 3 times $\frac{2}{7}$, or $\frac{6}{7}$. Hence $\frac{4}{7}$ is $\frac{2}{3}$ of $\frac{6}{7}$.

13. $\frac{12}{17}$ is $\frac{4}{5}$ of what number? $\frac{28}{32}$ is $\frac{7}{11}$ of what number?
14. $\frac{18}{35}$ is $\frac{9}{7}$ of what number? $\frac{24}{19}$ is $\frac{12}{7}$ of what number?
15. $\frac{49}{65}$ is $\frac{7}{13}$ of what number? $\frac{35}{49}$ is $\frac{5}{7}$ of what number?
16. $\frac{18}{25}$ is $\frac{9}{15}$ of what number? $\frac{49}{50}$ is $\frac{7}{10}$ of what number?

REVIEW EXERCISES.

193. 1. Mr. B. bought a barrel of flour for $\$7\frac{3}{8}$, a cord of wood for $\$5\frac{7}{8}$, and gave the clerk a twenty-dollar bill. How much change should Mr. B. receive?

2. A merchant bought 360 pounds of sugar at $11\frac{1}{8}$ cents a pound, 50 pounds of tea at $62\frac{1}{2}$ cents a pound. How much did he pay for both?

3. If a man can cut in one day $\frac{1}{2}$ of a field containing 7 acres of wheat, how many acres can he cut in $\frac{5}{7}$ of a day?

4. What will be the cost of $3\frac{5}{12}$ dozen eggs at $18\frac{3}{4}$ cents a dozen?

5. If a man can hoe a field in $7\frac{1}{4}$ days, how long will it take 3 men to hoe a field $2\frac{2}{3}$ times as large?

6. From a barrel of kerosene containing $41\frac{1}{2}$ gallons it was estimated that $\frac{1}{7}$ leaked out. If I paid \$6.15 for it, at what price per gallon must I sell the remainder to balance the loss sustained by leakage?

7. James Henderson sold $\frac{3}{5}$ of his farm of 155 acres to Mr. Paine, and Mr. Paine soon sold $\frac{2}{5}$ of what he had bought, to Mr. Banker. How many acres did Mr. Banker buy?

8. Mr. A. built a block of stores which cost him \$3122$\frac{1}{4}$ for brick, \$1368$\frac{1}{5}$ for lumber, \$3258$\frac{3}{7}$ for labor, and \$1325$\frac{5}{12}$ for other expenses. He sold the block for \$10000. Did he gain or lose, and how much?

9. There are $272\frac{1}{4}$ square feet in a square rod. How many square feet are there in $\frac{3}{5}$ of a square rod?

10. I sold a house and lot for \$3215, which was $1\frac{5}{9}$ times what it cost me. How much did it cost?

11. How much will 8 carpenters earn in $6\frac{2}{3}$ days at \$2$\frac{7}{8}$ per day?

12. If a man walks $3\frac{1}{4}$ miles per hour, in how many hours can he walk $30\frac{1}{2}$ miles?

13. Mr. Jones left an estate valued at \$19000. $\frac{3}{5}$ of it was divided equally among 4 sons and the rest equally among 3 daughters. What was the share of each?

14. The price of maple sugar this year is only $\frac{7}{8}$ of what it was last year. How much more would I have received last year for 3140 pounds which I sold this year at \$.20 a pound?

15. Custom millers take $\frac{1}{8}$ of the quantity of grain as pay for grinding it. How many bushels must a man carry to the mill so that he may bring back 14 bushels of ground provender?

16. If \$45 is $\frac{5}{7}$ of my money, what part of it will that sum plus \$4$\frac{1}{2}$ be?

17. A farmer had two fields in which he kept his sheep. In one there was $\frac{1}{3}$ of the whole number of sheep, and in the other there were 148 sheep. How many sheep had he?

18. A merchant exchanged 21 barrels of flour worth $\$7\frac{3}{5}$ a barrel, for $24\frac{3}{7}$ cords of wood. What did the wood cost him per cord?

19. If I give A $\frac{1}{3}$ of my money, B $\frac{1}{4}$ of it, and C $\frac{1}{6}$ of it, what part of my money have I left?

20. Mr. A. owns $\frac{5}{7}$ of a vessel valued at \$18326. If he sells $\frac{2}{3}$ of his share to Mr. B., what part of the whole will he have left? What part will Mr. B. have? What is the value of Mr. A.'s share? Of Mr. B.'s share?

21. After buying a suit of clothes for \$60 I found I had $\frac{3}{7}$ of my money left. How much had I at first?

22. A man sold $\frac{1}{2}$ of $3\frac{1}{5}$ cords of wood for $\frac{5}{7}$ of $\$8\frac{1}{5}$. How much did he receive for it per cord?

23. How many tons of hay will be required to keep 7 horses for 6 months, if 9 horses eat $16\frac{1}{3}$ tons in that time?

24. If $\frac{3}{4}$ of a farm is sold for \$8516, what would be the worth of the whole at the same rate?

25. A gentleman spent $\frac{1}{3}$ of his annual income traveling, and $\frac{1}{5}$ of the remainder in the purchase of books. The rest, which was \$8526, he expended upon paintings and other works of art. What was his annual income?

26. Two men dug a ditch for \$53; one man worked $3\frac{1}{2}$ days and dug $14\frac{1}{2}$ rods; the other worked as many days as the first dug rods per day. How much did each receive, if they shared in proportion to the time they worked?

27. Two brothers together own $\frac{1}{2}$ of a flouring mill valued at \$13000. One owns $\frac{2}{3}$ as much as the other. What is the value of each one's share?

28. The loss caused by a fire was \$3865. The sum was paid by an insurance company which insured the stock for $\frac{2}{3}$ of its value. What was the entire value of the stock?

29. A can do a piece of work in 10 days. What part of it can he do in 1 day? If B can do the same piece of work in 8 days, what part of it can he do in 1 day? What part can both together do in 1 day? How many days would be required for both to do the work?

30. If A can do a piece of work in 5 days and B in 8 days, how long will it take both to do it?

31. If A and B can do a piece of work in 10 days, and A can do it alone in 15 days, how long will it take B to do it?

32. A tree 124 feet high was broken in two pieces by falling. $\frac{3}{5}$ of the length of the shorter piece, equaled $\frac{2}{7}$ of the length of the longer piece. What was the length of each piece?

33. A man who had spent $\frac{1}{2}$ his money and \$$\frac{1}{2}$ more, found that he had \$21 left. How much money had he at first?

34. A, B and C can do a piece of work in 9 days. A can do it in 25 days, and B in 30 days. In what time can C do it?

35. Two ladders will together just reach the top of a building 75 feet high. If the shorter ladder is $\frac{2}{3}$ the length of the longer one, what is the length of each?

36. There are two numbers whose sum is 140, one of which is $\frac{3}{4}$ the other. What are the numbers?

37. In 1875 a merchant's profits were $\frac{1}{5}$ of his receipts; in 1876 they decreased $\frac{1}{8}$, which diminished his profits $\frac{1}{5}$ of \$2756$\frac{1}{2}$. What were his receipts in 1875?

38. A and B together had \$5700. $\frac{2}{3}$ of A's money was equal to $\frac{3}{5}$ of B's. How much had each?

39. A man engaged to work a year for \$240 and a suit of clothes. At the end of 9 months an equitable settlement was made by giving him \$168 and the suit of clothes. What was the value of the clothes?

40. A and B can do a piece of work in 12 days. Assuming that A can do $\frac{3}{4}$ as much as B, how long will it take each to do it?

DECIMAL FRACTIONS

194. 1. If an apple be divided into ten equal parts, what part of the apple will one of these parts be? Two parts? Five parts?

2. If one-tenth of an apple be divided into ten equal parts, what part of the apple will one of these parts be? Two parts? What part of 1 is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{5}{10}$ of $\frac{1}{10}$? $\frac{8}{10}$ of $\frac{1}{10}$.

3. If one-hundredth of a dollar be divided into ten equal parts, what part of a dollar will one of these parts be? Eight parts? What part of 1 is $\frac{1}{10}$ of $\frac{1}{100}$?

4. What part of one-tenth is one-hundredth? Of 2 tenths is 2 hundredths? Of 3 tenths is 3 hundredths? Of 9 tenths is 9 hundredths?

5. What part of one-hundredth is one-thousandth? Of 2 hundredths is 2 thousandths? Of 8 hundredths is 8 thousandths?

6. What are the divisons of any thing into tenths, hundredths, thousandths, ten-thousandths, etc., called?

Ans. Decimal divisions.

DEFINITIONS

195. A ***Decimal Fraction*** is one or more of the decimal divisions of a unit.

The word decimal is derived from the Latin word *decem*, which signifies *ten*.

Decimal fractions, for the sake of brevity, are usually called *decimals.*

196. Since tenths are equal to ten times as many hundredths, and hundredths are equal to ten times as many thousandths, thousandths to ten times as many ten-thousandths, etc., it is evident that decimals have the same law of increase and decrease as integers, and that the *denominator* may therefore be indicated by the *position of the figures.*

According to the decimal system of notation, figures decrease in tenfold ratio in passing from left to right; therefore a figure *at the right of units* will express *tenths, at the right of tenths, hundredths, at the right of hundredths, thousandths,* etc., as is exhibited by the following expressions:

Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.			
2	3	4	5					$= 234\frac{5}{10}$
1	4	7	0	5				$= 147\frac{5}{100}$
3	1	3	0	0	5			$= 313\frac{5}{1000}$
1	6	4	0	3	4			$= 164\frac{34}{1000}$
1	6	4	0	3	4	0	0	$= 164\frac{3400}{100000}$
			0	1	6	4		$= \frac{164}{10000}$

From this mode of expressing decimal fractions the following principles are deduced:

197. PRINCIPLES.—1. *Decimals conform to the same principles of notation as integers.*

2. *Each decimal cipher prefixed to a decimal diminishes its value tenfold, since it removes each figure one place to the right.*

3. *Annexing ciphers to a decimal does not alter its value, since it does not change the place of any figure of the decimal.*

4. *The denominator of a decimal, when expressed, is 1 with as many ciphers annexed as there are orders in the decimal.*

The Decimal Point is a period placed before the decimal. Thus, .6 represents $\frac{6}{10}$; .54 represents $\frac{54}{100}$.

The decimal point is also called the *Separatrix*, since it is also used to separate integers from decimals.

198. A ***Pure Decimal Number*** is one which consists of decimals only; as .387.

199. A ***Mixed Decimal Number*** is one which consists of an integer and a decimal; as 46.3, which is equal to $46\frac{3}{10}$.

200. A ***Complex Decimal*** is one which has a common fraction at the right of the decimal; as $.3\frac{2}{3}$, which is equal to $\frac{3\frac{2}{3}}{10}$.

NUMERATION TABLE.

Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.	Ten-billionths
2	7	5	4	8	3	4	.	6	8	4	7	3	0	2	4	2	5
INTEGERS.								DECIMALS.									

By examining this table it will be seen that *tenths* occupy the *first* decimal place, *hundredths* the *second*, *thousandths* the *third*, *ten-thousandths* the *fourth*, *hundred-thousandths* the *fifth*, *millionths* the *sixth*, etc. Hence,

The place occupied by any order of decimals is one less than that occupied by the corresponding order of integers.

201. What order of decimals occupies

1st place?	5th place?	9th place?
4th place?	2d place?	8th place?
3d place?	6th place?	2d place?
7th place?	10th place?	3d place?

What decimal place is occupied by hundredths? Tenths? Hundred-millionths? Thousandths? Ten-thousandths? Ten-millionths? Millionths? Billionths? Hundred-thousandths?

EXAMPLES IN NUMERATION.

202. 1. Read the decimal 4.246.

Analysis.—The figures of the decimal express 2 tenths, 4 hundredths and 6 thousandths, which, reduced to equivalent fractions having a common denominator, become 200 thousandths, 40 thousandths and 6 thousandths, or 246 thousandths.

The whole expression is read 4 and 246 thousandths.

Rule.—*Read the decimal as an integral number and give it the denomination of the right-hand figure.*

Read the following:

2. .684.	14. .6231.	26. 4.16.
3. .084.	15. .4896.	27. 5.8406.
4. .004.	16. .3893.	28. .60000.
5. 6.839.	17. 18.468.	29. .00006.
6. 68.39.	18. 23.8009.	30. .40508.
7. 683.9.	19. 649.3804.	31. 40.0004.
8. .00450.	20. .0020064.	32. 4000.004.
9. 3.02304.	21. .4120465.	33. 518.6800.
10. .050600.	22. 6.932474.	34. 4000.129.
11. 4.00008.	23. 2.234006.	35. 80000.86.
12. .000000856.	24. 3.000600.	36. 8000.086.
13. 1.000003894.	25. 4.006006.	37. 800.0086.

EXAMPLES IN NOTATION.

203. 1. Express decimally forty-three thousandths.

ANALYSIS.—Since 43 thousandths are equal to 4 hundredths and 3 thousandths, we write 3 in thousandths' place and 4 in hundredths' place, and as there are no tenths, 0 in tenths' place. Hence, forty-three thousandths = .043.

RULE.—*Write the numerator of the decimal, prefix ciphers if necessary to indicate the denominator, and place the decimal point before tenths.*

Express decimally:

2. Eight tenths. Nine tenths. Five tenths.
3. Two hundredths. Eight hundredths. Six hundredths.
4. Six thousandths. Four-hundred-two thousandths.
5. Nine ten-thousandths. Eight hundred ten-thousandths.
6. Seventeen hundredths. Fifteen hundred-thousandths.
7. Forty-eight thousandths. Five hundred ten-millionths.
8. Ninety-three ten-thousandths. Ninety-three billionths.
9. Fifty-one hundred-thousandths. Fifty-one thousandths.
10. 107 millionths. 306 ten-millionths.
11. 3259 hundred-thousandths. 4268 hundred-millionths.
12. 429 ten-millionths. 3842 hundred-thousandths.
13. 4300 billionths. 38496 hundred-billionths.
14. 85 hundred-millionths. 85 hundred-billionths.
15. Six thousand ten-thousandths. Five ten-billionths.
16. Five and six-tenths. Eight and nine ten-thousandths.
17. Eighty ten-thousandths. Forty hundred-thousandths.

Express decimally:

18. $\frac{48}{100}$.
19. $\frac{13}{1000}$.
20. $\frac{714}{1000000}$.
21. $4\frac{9}{10}$.
22. $5\frac{84}{100}$.
23. $\frac{168}{1000000}$.
24. $\frac{135}{1000}$.
25. $4\frac{13}{1000}$.
26. $64\frac{5}{10}$.
27. $33\frac{418}{10000}$.
28. $\frac{9}{1000000}$.
29. $\frac{4000}{10000}$.

In reading expressions of United States currency, the cents, mills, etc., may be read as decimals of a dollar. Thus, \$4.7235 may be read 4 dollars $72\frac{35}{100}$ cents, or \$$4\frac{7235}{10000}$.

REDUCTION.

CASE I.

204. To reduce dissimilar decimals to similar decimals.

1. How many tenths of an apple are there in 1 apple? How many hundredths in 10 apples? How many thousandths?

2. How many hundredths are there in 6 tenths? How many thousandths? How many ten-thousandths?

3. Express 6 hundredths as thousandths. As ten-thousandths. As hundred-thousandths. As millionths.

4. Express 8 thousandths as ten-thousandths. As hundred-thousandths. As millionths.

205. PRINCIPLE.—*Annexing ciphers to a decimal does not alter its value.*

WRITTEN EXERCISES.

1. Reduce .5, .36, .406 and 3.3109 to similar fractions.

PROCESS.

$$\begin{aligned} .5 &= .5000 \\ .36 &= .3600 \\ .406 &= .4060 \\ 3.3109 &= 3.3109 \end{aligned}$$

ANALYSIS.—The lowest order of decimals in the given numbers is ten-thousandths, and to reduce the decimals to similar decimals, we must change them all to ten-thousandths, or to other decimals having an equal number of places.

Since annexing ciphers to a decimal does not alter its value, we give to each number four decimal places by annexing ciphers, and this renders them similar.

RULE.—*Give to all the given decimals the same number of decimal places by annexing ciphers.*

2. Reduce .6, .75, .089, to similar decimals.
3. Reduce .15, .0406, .0035, .051, to similar decimals.
4. Reduce .0045, .3846, .51, .51040, to similar decimals.
5. Reduce 3.35, .345, to similar decimals.

Reduce the following dissimilar decimals to similar decimals:

6. .0436, .04506, .82.	10. 5, .5, .005, 50.
7. .05, 4.825, 3.6046.	11. 3.5, .416, .34, 14.
8. .3854, .729, 8.053.	12. .214, 8.3, .8, 4.6.
9. .8104, .0008, 8000.4.	13. 8.1, 43, .68, 3.90.

CASE II.

206. To reduce a decimal to a common fraction.

1. If 5 tenths be written as a common fraction what will be the numerator? What will be the denominator?

2. What is the numerator and what the denominator of the decimal 18 hundredths, when expressed as a common fraction?

3. Express the value of the decimal 50 hundredths, by a common fraction in its smallest terms.

4. Express by a common fraction in its smallest terms, the following decimals: 20 hundredths. 30 hundredths. 50 hundredths. 250 thousandths. 375 thousandths.

WRITTEN EXERCISES.

1. Reduce .75 to its equivalent common fraction.

PROCESS.	
$.75 = \frac{75}{100} = \frac{3}{4}$	ANALYSIS.—.75 expressed as a common fraction is $\frac{75}{100}$, which, being reduced to its smallest terms, equals $\frac{3}{4}$.

RULE.—*Omit the decimal point, supply the denominator, and reduce the fraction to its lowest terms.*

Reduce the following decimals to equivalent common fractions in their smallest terms:

2. .054.	6. 4.0125.	10. .354.	14. .5675.
3. .03875.	7. .4355.	11. .00625.	15. 3.216.
4. .05625.	8. .0005.	12. .05375.	16. .4824.
5. .4375.	9. .5000.	13. .06506.	17. .005396.

18. Reduce $.15\frac{1}{7}$ to an equivalent common fraction.

PROCESS.

$$.15\tfrac{1}{7} = \frac{15\frac{1}{7}}{100} = \frac{\frac{106}{7}}{100} = \frac{106}{700} = \frac{53}{350}$$

ANALYSIS.—The expression $.15\frac{1}{7}$ is equal to $\frac{15\frac{1}{7}}{100}$, or $\frac{\frac{106}{7}}{100}$. Reducing the denominator also to sevenths, the expression becomes $\frac{106}{700}$, or $\frac{53}{350}$.

Change the following to equivalent common fractions, or to mixed numbers:

19. $.12\frac{1}{2}$.	22. $.87\frac{1}{2}$.	25. $.562\frac{1}{2}$.	28. $.0003\frac{1}{2}$.
20. $.33\frac{1}{3}$.	23. $.04\frac{2}{5}$.	26. $.003\frac{3}{4}$.	29. $2.756\frac{1}{5}$.
21. $.16\frac{2}{3}$.	24. $.037\frac{1}{2}$.	27. $.078\frac{3}{10}$.	30. $13.81\frac{4}{5}$.

CASE III.

207. To reduce a common fraction to a decimal.

1. One half of an apple is equal to how many tenths of an apple?

2. How many tenths are there in $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$?

3. How many hundredths are there in $\frac{1}{4}$? $\frac{2}{4}$? $\frac{3}{4}$? $\frac{3}{5}$? $\frac{4}{5}$?

4. How many hundredths are there in $\frac{1}{20}$? $\frac{3}{20}$? $\frac{7}{20}$? $\frac{1}{25}$?

5. How many hundredths are there in $\frac{1}{2}$, or 100 hundredths divided by 2? How many in $\frac{1}{4}$?

6. How many hundredths are there in $\frac{4}{5}$, or 400 hundredths divided by 5? How many in $\frac{3}{5}$?

7. How many thousandths are there in $\frac{5}{8}$, or 5000 thousandths divided by 8? How many in $\frac{3}{8}$? How many in $\frac{7}{8}$?

WRITTEN EXERCISES.

1. Reduce $\frac{5}{8}$ to an equivalent decimal.

PROCESS.

$$8\underline{)5.000}$$
$$.625 \quad \textit{Or,}$$

$$\tfrac{5}{8}=\tfrac{5000}{8000}=\tfrac{625}{1000}=.625$$

ANALYSIS.—$\frac{5}{8}$ is $\frac{1}{8}$ of 5, or 50 tenths; and $\frac{1}{8}$ of 50 tenths is 6 tenths and 2 tenths remaining. 2 tenths are equal to 20 hundredths, and $\frac{1}{8}$ of 20 hundredths is 2 hundredths and 4 hundredths remaining. 4 hundredths are equal to 40 thousandths, and $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Hence $\frac{5}{8}$ is equal to 6 tenths + 2 hundredths + 5 thousandths, or .625.

Or we may multiply both terms of the fraction by 1000 and divide the resulting terms by 8, and obtain the decimal $\frac{625}{1000}$, or .625.

RULE.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

In many cases the division is not exact. In such instances the remainder may be expressed as a common fraction, or the sign + may be employed after the decimal to show that the result is not complete; thus: $\frac{1}{6}=.166\frac{2}{3}$, or .166 +.

Reduce the following to equivalent decimals:

2. $\frac{1}{4}$.	8. $\frac{1}{16}$.	14. $\frac{5}{9}$.	20. $\frac{3}{16}$.
3. $\frac{3}{5}$.	9. $\frac{3}{20}$.	15. $\frac{5}{18}$.	21. $\frac{5}{128}$.
4. $\frac{5}{8}$.	10. $\frac{17}{20}$.	16. $\frac{7}{30}$.	22. $\frac{3}{7}$.
5. $\frac{3}{8}$.	11. $\frac{13}{25}$.	17. $\frac{9}{64}$.	23. $\frac{5}{11}$.
6. $\frac{4}{5}$.	12. $\frac{14}{40}$.	18. $\frac{63}{24}$.	24. $\frac{6}{19}$.
7. $\frac{3}{4}$.	13. $\frac{23}{60}$.	19. $\frac{3}{125}$.	25. $\frac{5}{21}$.

Change the following to the decimal form:

26. $15\frac{5}{8}$.	29. $3.4\frac{1}{4}$.	32. $.87\frac{1}{2}$.	35. $37.5\frac{4}{10}$.
27. $24\frac{3}{5}$.	30. $.23\frac{5}{8}$.	33. $.43\frac{3}{4}$.	36. $20.0\frac{3}{5}$.
28. $.82\frac{1}{2}$.	31. $.62\frac{1}{2}$.	34. $4.21\frac{5}{6}$.	37. $.000\frac{13}{80}$.

ADDITION.

208. 1. What is the sum of $\frac{5}{10}$ and $\frac{3}{10}$? $\frac{6}{10}$ and $\frac{5}{10}$? .3 and .7?

2. What is the sum of $\frac{40}{100}$ and $\frac{9}{100}$? $\frac{26}{100}$ and $\frac{30}{100}$? .12 and .20?

3. What is the sum of $\frac{3}{1000}$ and $\frac{7}{1000}$? $\frac{14}{1000}$ and $\frac{23}{1000}$? .005 and .043?

4. Find the sum of $\frac{2}{10}$ and $\frac{8}{100}$. Of .5 and .06. .7 and .19.

5. Find the sum of .6, .31, .004. Of .5, .08 and .006.

209. PRINCIPLES.—*The principles are the same as for addition of integers.*

WRITTEN EXERCISES.

1. What is the sum of .36, 2.136 and 4.5004?

PROCESS.

.36	=	.3600
2.136	=	2.1360
4.5004	=	4.5004
6.9964		6.9964

ANALYSIS.—We write the numbers so that units of the same order shall stand in the same column, and add as in integers, separating the decimal part of the sum from the integral part by the decimal point. The decimals are made similar by annexing ciphers until all the decimals have the same number of places.

It is not usual to make the decimals similar, for if they are written so that decimals of the same order stand in the same column it is unnecessary to supply the ciphers.

RULE.—*The rule is the same as for addition of integers.*

What is the sum of

2. 4.15, 3.86 and .487?
3. 3.9, 4.84 and .0507?
4. .004, 5.86 and 3.05?
5. 6.843, 48.25 and 17.286?
6. .35, .046 and .00435?
7. 106, .106, 1.06 and 10.6?

8. What is the sum of \$5.18, \$3.09, \$46. and \$54.185?

9. Find the sum of \$18.23, \$12.08, \$31.255 and \$6.625.

10. Add \$34.73, \$206.357, \$1200.18, \$3816 and \$137.

11. Express as decimals and add $6\frac{1}{4}$, $3\frac{2}{5}$, $5\frac{3}{8}$, $6\frac{1}{5}$ and $9\frac{3}{4}$.

12. A laborer earned \$7.25 in one week, \$7.12$\frac{1}{2}$ in another, \$9.18$\frac{3}{4}$ in another, and \$8$\frac{5}{8}$ in another. How much did he earn during that time?

13. What is the sum of 18 thousandths, 15 millionths, 81 hundredths, 146 ten-thousandths, 834 hundred-thousandths?

14. What is the sum of 8 dollars 5 cents, 13 dollars 19 cents, 18 dollars 3 cents 8 mills, 25 dollars 37 cents 5 mills, $12\frac{5}{8}$ dollars, and $\frac{7}{16}$ of a dollar?

15. Mr. A. paid the following bills for repairs upon his premises, viz: carpenter-work, \$381.45; plastering, \$215.385; plumbing, \$323.94; and other expenses, \$181.57. How much did he pay for repairs?

-16. A farmer purchased cloth for \$13$\frac{7}{8}$, boots for \$8$\frac{5}{16}$, crockery for \$10$\frac{11}{16}$, and groceries for \$15.49. How much did he pay for all his purchases?

SUBTRACTION.

210. 1. From $\frac{5}{10}$ take $\frac{3}{10}$. From .9 take .5.

2. Find the difference between $\frac{9}{100}$ and $\frac{6}{100}$; $\frac{13}{100}$ and $\frac{7}{100}$; .19 and .08.

3. Find the difference between $\frac{3}{1000}$ and $\frac{9}{1000}$; $\frac{7}{1000}$ and $\frac{2}{1000}$; .007 and .005.

4. What is the difference between $\frac{2}{10}$ and $\frac{8}{100}$? .5 and .06? .7 and .09?

5. What is the difference between .16 and .03? .15 and .08? .45 and .3?

211. Principles.—*The principles are the same as for the subtraction of integers.*

WRITTEN EXERCISES.

1. From 34.634 take 5.6857.

PROCESS.
34.6340
5.6857
28.9483
Or,
34.634
5.6857
28.9483

ANALYSIS.—We write the numbers so that units of the same order shall stand in the same column, and subtract as in integers, separating the decimal part of the remainder from the integral part by a decimal point.

In the first process the decimals are made similar by annexing a cipher to the minuend.

In the second process, which is the one commonly employed, the cipher is not written, but we suppose it to be annexed.

RULE.—*The rule is the same as for subtraction of integers.*

	(2.)	(3.)	(4.)	(5.)
From	48.356	39.82	\$43.25	\$118.375
Subtract	23.453	13.856	\$18.375	\$ 43.50

6. What is the difference between .7134 and .50645?
7. What is the difference between 8.34 and 6.3168?
8. What is the difference between 100 and .03846?
9. From 84 millionths take 84 ten-millionths.
10. From 80 thousand take 80 thousandths.
11. From 29 dollars 3 cents take 17 dollars 9 cents.
12. From 27 dollars 8 cents take 9 dollars 37 cents 5 mills.
13. If I spend \$45.89½ for merchandise, how much will I have left after paying for it with a fifty-dollar bill?
14. A gentleman's income was \$12384.16, and his expenses the same year were \$9864.18. How much of his income was left?
15. The receipts of a reaper manufactory for the year 1876 were \$1374837.64, and the expenditures \$1298369.58. What was the surplus?

MULTIPLICATION.

212. 1. What is the product of $\frac{3}{10} \times 2$? $\frac{3}{10} \times 3$? $.4 \times 2$?

2. How many decimal figures are there in the product of tenths by units?

3. What is the product of $\frac{2}{100} \times 4$? $\frac{3}{100} \times 4$? $.04 \times 2$?

4. How many decimal figures are there in the product of hundredths by units?

5. What is the product of $\frac{2}{10} \times \frac{3}{10}$? $\frac{3}{10} \times \frac{3}{10}$? $.4 \times .2$?

6. How many decimal places are required to express the product of tenths multiplied by tenths?

7. What is the product of $\frac{2}{10} \times \frac{3}{100}$? $\frac{3}{10} \times \frac{4}{100}$? $.4 \times .02$?

8. How many decimal places are required to express the product of tenths multiplied by hundredths? Tenths by thousandths? Hundredths by thousandths?

9. If the multiplicand has two decimal places, and the multiplier three, how many will there be in the product?

10. How does the number of places required to express the product of two decimals compare with the number of decimal places in the factors?

213. PRINCIPLE.—*The product of two decimals contains as many decimal places as there are decimal places in both factors.*

WRITTEN EXERCISES.

1. Multiply .312 by .24.

PROCESS.

$$\begin{array}{r} .312 \\ .24 \\ \hline 1248 \\ 624 \\ \hline .07488 \end{array}$$

ANALYSIS.—$.312 \times .24 = \frac{312}{1000} \times \frac{24}{100} = \frac{7488}{100000} = .07488$. Hence $.312 \times .24 = .07488$. Or,

We may multiply as if the numbers were integers; and since the multiplicand has three decimal places, and the multiplier two places, the product must have five places. (Prin.) Or, thousandths multiplied by hundredths give hundred-thousandths, the denomination of the product.

RULE.—*Multiply as if the numbers were integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

If the product does not contain as many figures as there are decimals in both factors the deficiency must be supplied by prefixing ciphers.

Multiply:

2. .65 by .34.
3. .45 by 4.5.
4. .436 by .46.
5. 348 by .44.
6. 3.48 by .64.
7. 34.8 by .74.
8. 8.75 by 8.5.
9. .579 by .035.
10. 486 by 3.75.
11. 2.48 by 2.37.
12. 3.94 by 3.84.
13. 5384 by .0064.

Multiply:

14. \$2.75 by $8\frac{1}{2}$.
15. \$31.16 by $5\frac{3}{8}$.
16. 34.165 by $3\frac{3}{5}$.
17. 3.845 by 7.3.
18. \$15.18 by $.666\frac{2}{3}$.
19. 500.15 by 5.36.
20. 37.856 by 30.04.
21. 70.05 by .0405.
22. 63.18 by 2.308.
23. 64.032 by .0634.
24. 51.27 by 5.321.
25. $\frac{3}{4}$ of .55 by $\frac{2}{5}$ of 6.5.

26. Multiply 4.639 by 100.

PROCESS.	
4.639	ANALYSIS.—Since each removal of a figure one place to the left increases its value tenfold, the removal of the decimal point one place to the right multiplies by 10, and the removal of the point two places to the right multiplies by 100. Hence,
100	
463.900	

RULE.—*To multiply by 1 with any number of ciphers annexed, remove the decimal point as many places to the right as there are ciphers annexed.*

27. Multiply 384.64 by 100. By 10. By 1000.
28. Multiply 1.8465 by 100. By 1000. By 10000.
29. What will be the cost of 34.5 yards of cloth at \$3.15 per yard?

30. When land is worth \$137.18 per acre, how much must be paid for a farm of 38 acres?

31. Since 16.5 feet make a rod, how many feet are there in 23.7 rods?

32. What will be the cost of 9 houses at \$3847.93 each?

33. When wheat is worth \$1.62½ per bushel, what will 37.3 bushels cost?

34. What is the value of 57 barrels of flour at \$8.37½ a barrel?

35. Mr. Orr sold 8 horses at an average price of \$213.27 each. How much did he receive for them?

36. A lady made the following purchases, viz: 37 yards of bleached sheeting at \$ .13½ per yard, 8 yards of velvet ribbon at \$ .37½ per yard, 27 yards of silk at \$2.35 per yard. What was the entire cost of her purchases?

DIVISION.

214. 1. What is the product of .6 by 8?

2. 4.8 is the product of two factors, one of which is 8: what is the other factor?

3. What is the product of .6 by .8?

4. .48 is the product of two factors, one of which is .6: what is the other factor?

5. What is the product of .06 by .8?

6. .048 is the product of two factors, one of which is .06· what is the other factor?

7. What is the product of .06 by .08?

8. .0048 is the product of two factors, one of which is .06: what is the other factor?

9. How many decimal places are there in the quotient when tenths are divided by units? Hundredths by tenths? Thousandths by hundredths? Ten-thousandths by hundredths?

10. How many decimal places are there in the product of any two factors?

11. If the product and one of two factors are given, how may the number of decimals in the other factor be found?

12. How may the factor be found?

13. Since the factor sought will be the quotient, how many decimal places will there be in the quotient?

215. PRINCIPLE.—*The quotient will contain as many decimal places as the number of decimal places in the dividend exceeds those in the divisor.*

WRITTEN EXERCISES.

1. Divide 8.88 by 2.4.

PROCESS.

```
2.4)8.88(3.7
    72
    ---
    168
    168
    ---
```

ANALYSIS.—$8.88 \div 2.4 = \frac{888}{100} \div \frac{24}{10} = \frac{888}{100} \times \frac{10}{24} = \frac{37}{10} = 3.7$. Or,

We divide as if the numbers were integers; and since the dividend has two decimal places, and the divisor one, the quotient will have one. (Prin.)

RULE.—*Divide as if the numbers were integers, and from the right of the quotient point off as many figures for decimals as the number of decimal places in the dividend exceeds the number of those in the divisor.*

1. If the quotient does not contain a sufficient number of decimal places the deficiency must be supplied by prefixing ciphers.

2. Before commencing the division, the number of decimal places in the dividend should be made at least equal to the number of decimal places in the divisor.

3. When there is a remainder after using all the figures of the dividend, annex decimal ciphers and continue the division.

4. For the ordinary purposes of business it is not necessary to carry the division further than to obtain four or five decimal figures in the quotient.

Divide:		Divide:
.25	2. 2.450 by 9.8.	9. .04905 by .327.
.005	3. .00335 by .67.	10. 135.05 by .037.
.6595	4. 6.2512 by 3.7.	11. 687.50 by .025.
	5. .05475 by 15.	12. 34.368 by .013.
	6. 18.312 by .24.	13. .014532 by .0692.
	7. 105.70 by 3.5.	14. 71.142 by .0071.
	8. .11928 by .056.	15. .027538 by .0326.

16. Divide 423.68 by 100.

PROCESS.

$$100\overline{)423.68}$$
$$4.2368$$

ANALYSIS.—Since each removal of a figure one place to the right decreases its value tenfold, the removal of the decimal point one place to the left divides by 10, and the removal of the decimal point two places to the left divides by 100. Hence,

RULE.—*To divide by 1 with any number of ciphers annexed, remove the decimal point as many places to the left as there are ciphers annexed.*

Divide:	Divide:
17. 48.26 by 100.	20. 4.897 by 100.
18. 382.457 by 1000.	21. .06045 by 1000.
19. 13.8542 by 1000.	22. 3845.63 by 10000.

23. At \$8.25 per ton, how much hay can be bought for \$29.35?

24. At \$.18 per dozen, how many dozen eggs can be bought for \$32.40?

25. If I pay \$106.40 for 35 hats, how much do they each cost me?

26. How many hogsheads of molasses, at \$57.38 each, can be purchased for \$1319.74?

27. How many stoves, at \$21.35 each, can be bought for \$789.95?

SHORT PROCESSES.

216. Many methods have been devised for abbreviating the processes of computation, among which the following are of much practical value:

CASE I.

217. To multiply by a number a little less than a unit of the next higher order.

1. How much less than 10 is 9? Than 100 is 99? Than 1000 is 999? Than 100 is 98? Than 100 is 97? Than 100 is 96?

2. How much less than 10 times a number is 9 times the number? Than 100 times is 99 times? Than 1000 times is 999 times? Than 100 times is 97 times? Than 1000 times is 997 times?

WRITTEN EXERCISES.

1. Multiply 4685 by 97.

PROCESS.

$$\begin{aligned} 468500 &= 100 \text{ times } 4685 \\ 14055 &= 3 \text{ times } 4685 \\ \hline 454445 &= 97 \text{ times } 4685 \end{aligned}$$

ANALYSIS.—Ninety-seven times a number is 100 times the number minus 3 times the number. We annex two ciphers to the multiplicand, thus multiplying by 100, and then subtract from the product three times the multiplicand, leaving 97 times the multiplicand.

Multiply:

2. 3856 by 99.
3. 4832 by 998.
4. 48567 by 999.
5. 89736 by 98.

Multiply:

6. 346725 by 997.
7. 486965 by 97.
8. 843256 by 96.
9. 586436 by 9996.

10. What will be the cost of 385 bushels of corn at $ .97 per bushel?

11. How much must be paid for 1373 pounds of tea at $ .96 per pound?

12. At $97 per acre, what will a farm of 139 acres cost?

CASE II.

218. To multiply when one part of the multiplier is a factor of another part.

1. Multiply 4 by 8; 4 by 8 tens; 4 by 16 tens.

2. Multiply 7 by 6; 7 by 6 tens; 7 by 18 tens.

3. When 7 times a number is known, how may 21 times a number be found? 21 tens times? 21 hundreds times?

WRITTEN EXERCISES.

1. Multiply 3684 by 124.

PROCESS.

```
  3684
   124
 -----
 14736
44208
------
456816
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ANALYSIS.—The multiplier may be regarded as composed of 12 tens and 4 units, or 3 times as many tens as units. We therefore first multiply by 4 units; and since there are 3 times as many tens as units, we multiply this product by 3, and write the result as tens by placing it one place to the left of units.

Multiply:

2. 3825 by 63.
3. 5973 by 93.
4. 8126 by 123.
5. 6924 by 213.
6. 14273 by 246.
7. 28653 by 328.
8. 68435 by 217.

Multiply:

9. 3692 by 357.
10. 6384 by 248.
11. 4239 by 369.
12. 12783 by 189.
13. 36412 by 279.
14. 36485 by 2408.
15. 29753 by 3609.

CASE III.

219. To multiply by a number which is an aliquot part of some higher unit.

1. What part of 10 is 2? Is 5? Is $2\frac{1}{2}$? Is $3\frac{1}{3}$?

2. What part of 100 is 10? Is 20? Is 25? Is 50? Is $12\frac{1}{2}$? Is $33\frac{1}{3}$?

3. What part of a dollar are 50 cents? 25 cents? 20 cents? 10 cents? $12\frac{1}{2}$ cents?

4. What part of a dollar are $33\frac{1}{3}$ cents? $16\frac{2}{3}$ cents?

220. An ***Aliquot Part*** of a number is such a part as will exactly divide the number.

Thus, 5, 10, $12\frac{1}{2}$, etc., are aliquot parts of 100.

The aliquot parts of 10, commonly used, are:

$5 = \frac{1}{2}$ of 10.	$3\frac{1}{3} = \frac{1}{3}$ of 10.	$2\frac{1}{2} = \frac{1}{4}$ of 10.

The aliquot parts of 100, commonly used, are:

$50 = \frac{1}{2}$ of 100.	$33\frac{1}{3} = \frac{1}{3}$ of 100.	$10 = \frac{1}{10}$ of 100.
$25 = \frac{1}{4}$ of 100.	$16\frac{2}{3} = \frac{1}{6}$ of 100.	$8\frac{1}{3} = \frac{1}{12}$ of 100.
$20 = \frac{1}{5}$ of 100.	$12\frac{1}{2} = \frac{1}{8}$ of 100.	$6\frac{1}{4} = \frac{1}{16}$ of 100.

Other parts of 100 are:

$40 = \frac{2}{5}$ of 100.	$37\frac{1}{2} = \frac{3}{8}$ of 100.	$66\frac{2}{3} = \frac{2}{3}$ of 100.
$60 = \frac{3}{5}$ of 100.	$62\frac{1}{2} = \frac{5}{8}$ of 100.	$75 = \frac{3}{4}$ of 100.
$80 = \frac{4}{5}$ of 100.	$87\frac{1}{2} = \frac{7}{8}$ of 100.	$43\frac{3}{4} = \frac{7}{16}$ of 100.

WRITTEN EXERCISES.

1. Multiply 434 by 25.

PROCESS.

$$\begin{array}{r} 4\underline{)43400} \\ 10850 \end{array}$$

ANALYSIS.—Since 25 is $\frac{1}{4}$ of 100, we may multiply by 25 by first multiplying by 100, and then taking $\frac{1}{4}$ of the product.

2. What will 85 yards of cloth cost at \$ .33$\frac{1}{3}$ per yard?

PROCESS.

$$\begin{array}{r} 3\underline{)85} \\ 28.33\frac{1}{3} \end{array}$$

ANALYSIS.—At \$1 per yard, the cloth would cost \$85; and at \$ .33$\frac{1}{3}$, or $\frac{1}{3}$ of a dollar, it will cost $\frac{1}{3}$ of \$85, or \$28.33$\frac{1}{3}$.

Multiply:

3. 688 by 12$\frac{1}{2}$.
4. 402 by 16$\frac{2}{3}$.
5. 5056 by 25.
6. 75630 by 33$\frac{1}{3}$.

Multiply:

7. 8404 by 50.
8. 2160 by 37$\frac{1}{2}$.
9. 4236 by 66$\frac{2}{3}$.
10. 7288 by 75.

11. What will be the cost of 27 yards of cloth at \$ .25 per yard?

12. When butter is worth 33$\frac{1}{3}$ cents a pound, what will 824 pounds be worth?

13. What will be the cost of 216 pounds of tea at \$ .75 per pound?

14. What will 287 bushels of oats cost at 37$\frac{1}{2}$ cents per bushel?

15. What will 394 bushels of potatoes cost at 62$\frac{1}{2}$ cents per bushel?

16. What is the value of 319 bushels of wheat at \$1.37$\frac{1}{2}$ per bushel?

CASE IV.

221. To find the cost when the quantity and the price of 100 or 1000 are given.

1. When the cost of 100 articles is known, how can the cost of 500 be found? 600? 800? 900?

2. When the cost of 100 articles is given, how can the cost of 250 be found? 350? 750? 850? 950?

3. How many times 100 is 250? 275? 280? 285?

4. How many times 1000 is 3000? 3500? 3750? 4585?

WRITTEN EXERCISES.

1. What will be the cost of 375 pounds of fish at \$6.75 per 100 pounds?

PROCESS.
\$6.75
3.75
\$25.3125

ANALYSIS.—Since 100 pounds cost \$6.75, 375 pounds, or 3.75 times 100 pounds, will cost 3.75 times \$6.75, or \$25.31¼. Or, the price may be multiplied by the quantity, and the decimal point removed two places to the left in the product.

The letters C and M are used instead of the words *hundred* and *thousand*, respectively.

2. How much will 6075 pounds of coal cost at \$.35 per hundred-weight?

3. When shingles cost \$4.75 per M, how much will 8609 shingles cost?

4. What is the price of a load of hay weighing 1925 pounds, at \$9.50 per ton (2000 pounds)?

5. What is the cost of 16795 pounds of plaster at \$4.50 per 100 pounds?

6. How much will 129765 laths cost at \$2.75 per M?

7. What is the cost of 6975 bricks @ \$3.25 per M?

8. What is the cost of 1825 pounds of iron @ \$45 per ton?

9. How much must be paid for 6780 envelopes @ \$2.75 per M?

10. What would be the cost of 550 pine-apples at \$13.25 per C?

11. What will be the cost of 1592 pounds of beef at \$4.50 per hundred pounds?

12. What will 15000 pounds of coal cost at \$7.50 a ton?

13. What will be the cost of 2294 pounds of broom-corn at \$55 per ton?

14. What will be the cost of 1964 pounds of maple-sugar at \$13.45 per hundred-weight?

ACCOUNTS AND BILLS.

222. A ***Debt*** is an amount which one person owes to another.

223. A ***Credit*** is an amount which is due to a person, or a sum paid towards discharging a debt.

224. A ***Debtor*** is a party owing a debt.

225. A ***Creditor*** is a party to whom a debt is due.

226. An ***Account*** is a record of debts and credits between parties doing business with each other.

227. The ***Balance of an Account*** is the difference between the amount of the debts and credits.

228. A ***Bill*** is a written statement given by the seller to the buyer, of the quantity and price of each article sold, and the amount of the whole.

229. The ***Footing of a Bill*** is the total cost of all the articles.

230. A bill is ***Receipted*** when the words *Received Payment* are written at the bottom, and the creditor's name is signed either by himself or some authorized person.

231. The following abbreviations are in common use:

@,	At.	Do.,	The same.	Mdse.,	Merchandise.
%,	Account.	Doz.,	Dozen.	No.,	Number.
Acc't,	Account.	Dr.,	Debtor.	Pay't,	Payment.
Bal.,	Balance.	Fr't,	Freight.	Pd.,	Paid.
Bbl.,	Barrel.	Hhd.,	Hogshead.	Per,	By.
Bo't,	Bought.	Inst.,	This month.	Rec'd,	Received.
Co.,	Company.	Int.,	Interest.	Yd.,	Yard.
Cr.,	Creditor.	Lb.,	Pound.	Yr.,	Year.

(1.)

Cincinnati, O., May 25, 1877.

Mr. Howard B. Judson,

Bought of Smith & Williams.

3 bbl. Coffee Sugar (sd)	647 lb., @ $ .11¼		$72.79
5 chests Oolong Tea	255 " "	.81	206.55
3 " Black "	167 " "	.67	111.89
7 boxes Cheese	301 " "	.13½	40.63
			$431.86

Received Payment,

Smith & Williams,

Per B. J.

Copy, fill out and find the footings of each of the following:

(2.)

ROCHESTER, *March* 1, 1877.

MR. J. B. ADAMS,

Bought of HOWE & ROGERS:

				$	
75½	yards of Carpeting	@	$2.12½		
37	yards of Drugget	"	1.20		
8	Rugs	"	4.16		
5	Mats	"	2.37½		
18	yards Oil-cloth	"	1.08		
9	yards Carpet Lining	"	.12½		
3	Carpet-sweepers	"	2.00		
2	doz. Stair-rods	"	8.25		
				$	

Received Payment,

HOWE & ROGERS.

(3.)

MEMPHIS, *May* 20, 1877.

MR. GEORGE B. SHERMAN,

To SAMUEL B. SMALLWOOD, *Dr.*

			$	
To 37 bbl. Pork	@	$24.35		
" 127 bbl. Flour	"	8.15		
" 3 hhd. Molasses—169 gal.	"	.43		
" 29 firkins Butter—2120 lb.	"	.31		
" 3 boxes Raisins	"	4.65		
" 5 bbl. Kerosene—207 gal.	"	.18¼		
" 25 doz. cans Fruit	"	2.40		
" 3 packages Tobacco—318 lb.	"	.45		
" 13 doz. Spices	"	1.10		
			$	

Received Payment by note at 60 days,

SAM'L B. SMALLWOOD.

(4.)

New York, *April* 1, 1877.

Mr. Erastus P. Gates,

To Sturdevant & Co., *Dr.*

1877.				
Jan.	9	To 3 Gold Watches—$124.50, $61.24, $57.18	$	
"	13	" 437 pwt. Gold Chains . . @ $1.15		
Feb.	3	" 35 sets Plated Tea-service. " 43.10		
"	15	" 7 " " " . " 51.		
Mar.	8	" 5 Silver Pie-knives . . " 12.		
"	12	" 12 Plated Ice-pitchers . . " 12.50		
			$	
		—*Cr.*—		
1877.				
Jan.	24	By Cash	$21	20
Feb.	10	" Draft	327	50
"	18	" Mdse. returned	78	67
			$	

How much is still due Sturdevant & Co.?

Make out in proper form and receipt the following:

5. Mrs. M. T. Dana bought of G. C. Smith & Co., 25 yd. of calico @ 10 cents, 37 yd. of sheeting @ $18\frac{1}{4}$ cents, 2 pairs of gloves @ $1.50, 1 sun-umbrella @ $6.75, 5 yd. of Hamburg edging @ 25 cents, 7 pairs of hose @ $ .85.

6. Mr. C. C. Lovell bought of R. P. Lawton 7568 feet of hemlock @ $12.75 per M, 8539 feet of pine flooring @ $23.50 per M, 5608 feet of clear pine @ $45 per M, 3815 feet of oak joists @ $32 per M, 7346 feet of ash flooring @ $34 per M.

7. Mr. George M. Line bought of Steele & Avery 15 reams of commercial note paper @ $1.25, 7500 envelopes @ $3.65 per M, 18 gross steel pens @ $ .75 per gross, 24 Ridpath's Histories @ $1.25, 9 Webster's Dictionaries @ $10.25.

REVIEW EXERCISES.

1. A farmer sold his butter at 34 cents a pound, and received for it $123.59. How many pounds did he sell?

2. A gallon of distilled water weighs 8.339 pounds. How much will $15\frac{1}{4}$ gallons weigh?

3. A square rod contains $272\frac{1}{4}$ square feet. How many square feet are there in $7\frac{5}{8}$ square rods?

4. The best anthracite coal is said to weigh 55.32 pounds per cubic foot. How many cubic feet will weigh a ton of 2000 pounds?

5. The number of cubic inches in a bushel is 2150.42. How many cubic inches are there in 1000 bushels?

6. What is the quotient when 3 is divided by 3 thousandths?

7. What is the quotient when 300 is divided by 3000 hundred-millionths?

8. A lumber merchant had 2182565 ft. of lumber. After selling .20, or 20 per cent., of it, he lost 15 per cent. of the remainder by fire. How many feet of lumber were burned?

9. What will 385 pounds of flour cost at $4.25 per hundred-weight?

10. At $ .11$\frac{1}{4}$ per pound, how many pounds of sugar can be bought for $31.25?

11. Bought 26 yards of broadcloth at 4.37\frac{1}{2}$ per yard, and paid for it in pork at $7.25 per hundred-weight. How much pork will it take to pay for the cloth?

12. If 15 tons of hay cost $125.25, what will 35 tons cost?

13. If Ridpath's histories retail at $1.25 each, what will be received for 350 sold at that rate?

14. When pork is selling at $6.25 per hundred-weight, how much can be bought for $325?

15. When 8000 is divided by .004, what is the quotient?

16. When .0008 is divided by 40000, what is the quotient?

17. How many days must a laborer work at \$1.37½ per day, to pay for 8 cords of wood at \$4.43¾ per cord?

18. A lady bought the following articles: 27 yards of silk at \$2.75 per yard, 11 yards of lace at \$6.37½ per yard, 9 pairs of gloves at \$2.15 per pair, 10 pairs of hose at \$1.10 per pair. What was the amount of the purchase?

19. If a man earns \$12½ per week, and spends \$7⅝ per week, in how many weeks can he save \$500?

20. What is the value of 95150 bricks at \$7.25 per M?

21. What is the value of a farm of 195 acres if 91 acres are worth \$6688.50, and the remainder \$1.12½ per acre more?

22. A drover bought 375 sheep at \$4.50 per head. He sold 200 of them at a loss of \$.20 per head, and gained enough on the rest to balance the loss. What did he get per head for the rest?

23. The expenses of conducting a business enterprise were .40 of the entire profits. If the profits were .15 of the value of the goods sold, how much was received from the sale of goods if the profits were \$9000 more than the expenses?

24. Express as a decimal $\dfrac{(\frac{2}{3}-\frac{1}{10})\times(3+\frac{2}{5})}{(1\frac{1}{2}+\frac{5}{7})+(3-1\frac{2}{3})\times 5}$.

25. A speculator bought 5000 bushels of corn at \$.65 per bushel. He sold .25 of it for \$.70 per bushel, and the remainder for such price that he realized a profit on the whole of \$447.50. How much did he get per bushel for the remainder?

26. The estimated value of Mr. A.'s farm was \$6500. If he sold a portion of it, at its estimated value per acre, for \$2275, what decimal part of the farm did he sell?

27. A, B and C divide 645⅕ bushels of wheat among themselves. A takes .37½, B $\frac{3}{16}$, and C the remainder. How many bushels had each?

DENOMINATE NUMBERS

DEFINITIONS.

232. A ***Concrete Number*** is a number used in connection with some specified thing.

Thus, 5 books, 7 trees, 8 horses, are concrete numbers.

233. An ***Abstract Number*** is a number that is not used in connection with any specified thing.

Thus, 5, 7, 8, are abstract numbers.

234. A ***Denominate Number*** is a concrete number in which the unit of measure is established by law or custom.

Thus, 5 yards, 3 feet, 7 pounds, 3 ounces, are denominate numbers.

235. A ***Simple Denominate Number*** is a denominate number composed of units of the same denominations.

Thus, 5 feet, 9 pounds, 3 miles, are simple denominate numbers.

236. A ***Compound Denominate Number*** is a denominate number composed of units of two or more denominations which are related to each other.

Thus, 6 feet and 4 inches, 8 hours and 32 minutes, are compound denominate numbers.

237. A ***Standard Unit*** is a unit of measure from which the other units of the same kind may be derived.

Thus, the yard is the standard unit from which all measures of length are formed; the Troy pound the standard unit of weight.

238. A ***Scale*** is the ratio by which numbers increase or decrease. Scales are either *uniform* or *varying.*

MEASURES OF VALUE.

239. ***Money*** is the measure of value.

It is also called *Currency*, and is of two kinds, viz: *coin* and *paper* money.

240. ***Coin*** or ***Specie*** is stamped pieces of metal having a value fixed by law.

241. ***Paper Money*** is notes and bills issued by the Government and banks, and authorized to be used as money.

UNITED STATES MONEY.

242. The *unit* of United States or Federal money is the *Dollar.*

TABLE.

10 Mills (m.)	=	1 Cent . . .	ct.
10 Cents	=	1 Dime . . .	d.
10 Dimes	=	1 Dollar. . .	$
10 Dollars	=	1 Eagle . . .	E.

$ *d.* *ct.* *m.*
1 = 10 = 100 = 1000

Scale—Decimal.

The *coins* of the United States are—

Gold: The double-eagle, eagle, half-eagle, quarter-eagle, three-dollar piece, one-dollar piece.

Silver: The dollar, half-dollar, quarter-dollar, the twenty-cent piece, the ten-cent piece.

Nickel: The five-cent piece and three-cent piece.

Bronze: The one-cent piece.

There are various other coins of the United States in circulation, but they are not coined now.

The denominations dimes and eagles are rarely used, the dimes being regarded as cents, and the eagles as dollars.

No examples in Reduction of U. S. Money are given, because the pupil has been familiarized with the process from the beginning.

CANADA MONEY.

243. The currency of Canada is decimal, and the *table* and *denominations* are the same as those of United States money. English money is still used to some extent.

The coins of Canada, are, for the most part, of the same denominations as those of the United States, except the gold coins, which are the sovereign and half-sovereign.

ENGLISH OR STERLING MONEY.

244. English money is the currency of Great Britain. The *unit* is the *Pound* or *Sovereign.*

TABLE.

4 Farthings (far.)	=	1 Penny. . .	d.
12 Pence	=	1 Shilling . .	s.
20 Shillings	=	1 Pound, or 1 Sovereign .	£

$$\begin{array}{cccc} \pounds & s. & d. & far. \\ 1 = & 20 = & 240 = & 960 \end{array}$$

Scale — 4, 12, 20.

1. Farthings are commonly written as fractions of a penny. Thus, 7 pence 3 farthings is written 7¾d.; 5 pence 1 farthing, 5¼d.
2. The value of £1 or sovereign is $4.8665 in American gold.

The coins of Great Britain in general use are—

Gold: Sovereign, half-sovereign, and guinea, which is equal to 21 shillings.

Silver: The crown (equal to 5 shillings), half-crown, florin (equal to 2 shillings), shilling, six-penny and three-penny pieces.

Copper: Penny, half-penny, and farthing.

REDUCTION DESCENDING.

245. 1. How many farthings are there in 2 pence? In 5 pence? In 7 pence? In 8 pence? In 6 pence?

2. How many pence are there in 2 shillings? In 5 shillings? In 7 shillings? In 8 shillings? In 6 shillings?

3. How many pence are there in 5s.? In 5s. and 3d.? In 7s. 4d.? In 4s. 5d.? In 6s. 8d.?

4. How many farthings are there in 5d.? In 6d. 3 far.? In $5\frac{1}{4}$d.? In $6\frac{1}{2}$d.? In $8\frac{3}{4}$d.? In $9\frac{1}{4}$d.? In $10\frac{3}{4}$d.?

5. How many shillings are there in £2 5s.? In £3 5s.?

246. ***Reduction*** of a denominate number is the process of changing it from one denomination to another without altering its value.

247. ***Reduction Descending*** is the process of changing a denominate number to an equivalent number of a *lower* denomination.

WRITTEN EXERCISES.

1. How many farthings are there in £3 5s. $6\frac{3}{4}$d.?

PROCESS.

£3 5s. $6\frac{3}{4}$d.
20
65s. = £3 5s.
12
786d. = £3 5s. 6d.
4
3147 far. = £3 5s. $6\frac{3}{4}$d.

ANALYSIS.—Since in 1 pound there are 20 shillings, in 3 pounds there are 3 times 20 shillings, or 60 shillings; and 60 shillings + 5 shillings = 65 shillings.

Since in 1 shilling there are 12 pence, in 65 shillings there are 65 times 12 pence, or 780 pence; and 780 pence + 6 pence = 786 pence.

Since in 1 penny there are 4 farthings, in 786 pence there are 3144 farthings; and 3144 farthings + 3 farthings = 3147 farthings.

Hence in £3 5s. $6\frac{3}{4}$d. there are 3147 farthings.

2. How many pence are there in £2 10s. 6d.?
3. How many shillings are there in £13 5s.?
4. How many farthings are there in £4 6s. 5d.?
5. How many pence are there in £$\frac{3}{7}$?

PROCESS.

£$\frac{3}{7} = \frac{3}{7}$ of 20 s. $= \frac{60}{7}$ s.

$\frac{60}{7}$ s. $= \frac{60}{7}$ of 12 d. $= \frac{720}{7}$ d.

$\frac{720}{7}$ d. $= 102\frac{6}{7}$ d.

ANALYSIS.—Since in 1 pound there are 20 shillings, in $\frac{3}{7}$ of a pound there are $\frac{3}{7}$ of 20 shillings, or $\frac{60}{7}$ of a shilling.

Since in 1 shilling there are 12 pence, in $\frac{60}{7}$ of a shilling there are $\frac{60}{7}$ of 12 pence, or $\frac{720}{7}$ pence; and $\frac{720}{7}$ pence $= 102\frac{6}{7}$d.

Therefore in £$\frac{3}{7}$ there are $102\frac{6}{7}$d.

RULE.—*Multiply the number of the highest denomination given, by the number of units of the next lower denomination which is equal to one of the next higher, and to the product add the number given of this lower denomination.*

Proceed in like manner with this and each successive result thus obtained, until the number is reduced to the denomination required.

6. How many pence are there in £$\frac{3}{8}$?
7. How many pence are there in £$\frac{7}{8}$?
8. How many farthings are there in $\frac{5}{9}$s.?
9. How many pence are there in £$\frac{5}{11}$?
10. How many shillings are there in £5 6s.? How many farthings?
11. Reduce 12s. 5d. 2 far. to farthings.
12. How many pence are there in £7 9s. 5d.?
13. Reduce 17s. $6\frac{3}{4}$d. to farthings.
14. What is the value of £$\frac{3}{8}$ in units of lower denominations?
15. Find the number of farthings in £5 13s. 3d.
16. Reduce £35 6s. 8d. to pence.
17. Reduce £45 3s. $9\frac{3}{4}$d. to farthings.
18. Reduce £29 18s. 5d. to farthings.

REDUCTION ASCENDING.

248. 1. How many pence are there in 12 farthings? In 16 farthings? In 20 farthings?

2. How many shillings are there in 24 pence? In 60 pence? 84 pence? 96 pence?

3. How many pounds are there in 40 shillings? In 60 shillings? In 120 shillings?

4. How many pounds sterling must be paid for 10 pairs of boots at 6 shillings a pair?

5. At 5 shillings each how many pounds sterling must be paid for 16 hats? For 20 hats?

6. Sold 8 pairs of skates at 5 shillings a pair. How many pounds sterling did I receive for them?

Reduction Ascending is the process of changing a denominate number to an equivalent number of a *higher* denomination.

WRITTEN EXERCISES.

1. How many pounds sterling are there in 7254 pence?

PROCESS.

$$\begin{array}{r l} 12 & \underline{)7254} \\ 20 & \underline{)604} \ldots 6 \\ & \;\; 30 \ldots 4 \end{array}$$

7254d. = £30 4s. 6d.

ANALYSIS.—Since 12 pence are equal to 1 shilling, there must be as many shillings in 7254 pence as 12 pence are contained times in that number. 12 pence are contained in 7254 pence 604 times with a remainder of 6 pence, therefore 7254 pence are equal to 604s. 6d.

Since 20 shillings are equal to 1 pound, there must be as many pounds in 604 shillings as 20 shillings are contained times in that number. 20 shillings are contained in 604 shillings 30 times and a remainder of 4 shillings.

Therefore 7254 pence are equal to £30 4s. 6d.

2. How many shillings are there in 345 farthings?
3. How many pounds are there in 456 shillings?
4. How many pounds are there in 1586 pence?
5. Reduce 3864 farthings to pounds.
6. Reduce $\frac{3}{7}$d. to a fraction of a pound.

PROCESS.

$\frac{3}{7}\text{d.} = \frac{3}{7}$ of $\frac{1}{12}\text{s.} = \frac{3}{84}\text{s.}$

$\frac{3}{84}\text{s.} = \frac{3}{84}$ of $£\frac{1}{20} = £\frac{3}{1680}$

ANALYSIS.—Since 1 penny is $\frac{1}{12}$ of a shilling, $\frac{3}{7}$ of a penny is equal to $\frac{3}{7}$ of $\frac{1}{12}$ of a shilling, or $\frac{3}{84}$ of a shilling.

Since 1 shilling is $\frac{1}{20}$ of a pound, $\frac{3}{84}$ of a shilling is equal to $\frac{3}{84}$ of $\frac{1}{20}$ of a pound, or $\frac{3}{1680}$ of a pound.

RULE.—*Divide the given number by the number of that denomination which is equal to a unit of the next higher denomination.*

Divide the quotient in like manner, and thus proceed until the required denomination is reached.

The last quotient and the several remainders will be the result sought.

7. Change $\frac{3}{5}$ of a shilling to a fraction of a pound.
8. Change $\frac{3}{7}$ of a farthing to a fraction of a shilling.
9. Change 384 pence to units of higher denominations.
10. Change 3146 shillings to pounds.

Reduce:

11. 3596d. to pounds.
12. 3846 far. to shillings.
13. 4856s. to pounds.
14. 5968 far. to pounds.
15. 3984d. to pounds.
16. 4685 far. to shillings.
17. 48567 far. to pounds.
18. £3 14s. 5d. to far.
19. 48596 far. to pounds.

Reduce:

20. £15 8s. to farthings.
21. £15 to dollars.
22. \$456 to pounds.
23. \$394.45 to pounds.
24. \$37.50 to pounds.
25. £25 to dollars.
26. £15 10s. to farthings.
27. \$973.30 to pounds.
28. \$1216.625 to pounds.

FRENCH MONEY.

249. In France the currency is *decimal.* The *unit* is the *Franc.*

TABLE.

10 Centimes (ct.)	[pronounced *son-teems*]	= 1	Decime . .	dc.
10 Decimes	[pronounced *des-seems*]	= 1	Franc . . .	fr.

Scale — Decimal.

The value of the franc, as determined by the Secretary of the Treasury, is \$ .193 in United States money.

1. How many centimes are there in 1 franc? In 5 francs?
2. How many decimes are there in 1 franc? In 7 francs?
3. How many centimes are there in 4 decimes?
4. How many dollars are there in 10 francs? In 20 francs?
5. In 3684 centimes how many francs are there?
6. How many francs are there in \$19.30? In \$9.65? In \$3.86?

MEASURES OF SPACE.

250. Space is extension in any direction. It has three dimensions or measurements—length, breadth and thickness.

251. A ***Line*** is that which has only length.

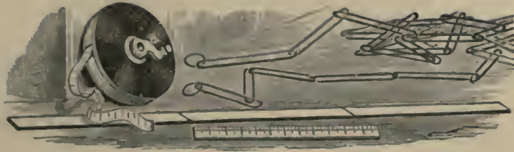
Thus, the edge of any thing, or the distance between any two objects or places, is a *line.*

252. A ***Surface*** is that which has only length and breadth.

Thus, the floor, this page, or the outside of any thing, is a *surface.*

253. A ***Solid*** is that which has length, breadth and thickness.

Thus, a stone, an apple, a block, a book, etc., are *solids.*



LINEAR MEASURES.

254. ***Linear Measures*** are used in measuring lengths and distances.

LINEAR MEASURE.			
12 Inches (in.)	= 1 Foot	.	ft.
3 Feet	= 1 Yard	.	yd.
$5\frac{1}{2}$ Yards, $16\frac{1}{2}$ Feet	= 1 Rod	.	rd.
320 Rods	= 1 Mile	.	mi.

SURVEYORS' LINEAR MEAS.			
7.92 Inches	= 1 Link	. .	l.
25 Links	= 1 Rod	. .	rd.
4 Rods, 100 Links	= 1 Chain	.	ch.
80 Chains	= 1 Mile	. .	mi.

$$\begin{array}{ccccc} mi. & rd. & yd. & ft. & in. \\ 1 = & 320 = & 1760 = & 5280 = & 63360 \end{array}$$

Scale—12, 3, $5\frac{1}{2}$, and 320.

The following are also used:

3 Barleycorns	= 1 Inch. Used by shoemakers.
4 Inches	= 1 Hand. Used to measure the height of horses.
6 Feet	= 1 Fathom. Used to measure depths at sea.
3 Feet	= 1 Pace. } Used in pacing distances.
5 Paces	= 1 Rod. } Used in pacing distances.
8 Furlongs	= 1 Mile.
1.15 Statute Miles	= 1 Geographical, or Nautical Mile.
3 Geographical Miles	= 1 League.
60 Geographic Miles, 69.16 Statute Miles	= 1 Degree { of Latitude on a Meridian, or of Longitude on the Equator.

1. For the purpose of measuring cloth and other goods sold by the yard, the yard is divided into halves, fourths, eighths, and sixteenths.

2. The length of a degree of latitude varies. 69.16 is the average length, and is that adopted by the United States Coast Survey.

1. How many inches are there in 4 feet? 6 feet? 8 feet? 10 feet? 12 feet?

2. How many feet are there in 2 rods? 3 rods? 4 rods?

3. How many inches are there in 2 yards? 4 yards? 5 yards?

4. How many inches are there in 2 yards and 2 inches? 3 yards and 4 inches?

5. How many rods are there in 2 miles? 3 miles?

6. How many feet are there in 1 rod and 2 yards? 2 rods and 3 yards?

7. How many feet are there in 45 inches? In 63 inches?

8. How many yards are in 22 feet? In 47 feet? In 34 feet?

9. How many miles in 640 rods? In 480 rods?

10. How many inches in 10 links? In 100 links?

11. How many links in 5 rods? In 3 rods? In 6 rods?

12. The length of a road was 400 links. What was its length in rods?

13. In 160 chains how many miles?

WRITTEN EXERCISES.

14. Reduce 5 mi. 18 rd. 4 yd. to yards.

15. Reduce 7 rd. 5 ft. 6 in. to inches.

16. How many inches are there in 7 miles? In 9 miles?

17. A building was 327 ft. long. How many rods was it in length?

18. A man sold a piece of wire 36828 in. long. How many rods was it in length?

19. In 3960 rods how many miles are there?

20. Reduce 15 mi. 8 rd. 5 yd. 3 ft. 4 in. to inches.

21. Reduce 8 mi. 14 rd. 5 ft. 4 in. to inches.

22. Reduce 66454 inches to miles, etc.

23. Reduce 158964 inches to miles, etc.

24. The diameter of the earth is 7912 miles. How many feet is it?

25. How high is a horse that measures 15 hands?

26. My farm is 67 ch. 83 l. long. How many rods long is it?

27. Reduce 59 ch. 75 l. to inches.

SURFACE MEASURES.

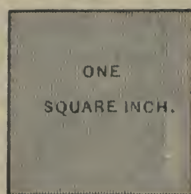
255. An ***Angle*** is the difference in the direction of two lines that meet.

ANGLE.

256. A ***Square*** is a figure that has four equal sides, and four equal angles.

A square inch is a square whose side is *one inch*. A square foot, a square whose side is *one foot*.

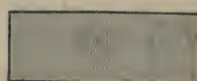
The angles of a square are called *right angles*.



SQUARE.

257. A ***Rectangle*** is a figure that has four straight sides and four equal angles.

The angles of a rectangle are all right angles.



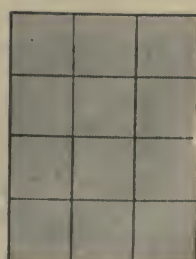
RECTANGLE.

258. The ***Area*** or extent of any surface is the number of square units it contains.

Thus, if a rectangle is 4 inches long and 3 inches wide the area will be 12 square inches.

For it may be divided into 4 rows, each containing 3 square *inches* or *units*, and the entire area will be 12 square inches.

The method of computing the area of figures that are not rectangular is given in MENSURATION.



259. *The area of a rectangle is equal to the product of the numbers that express its length and breadth.*

The length and breadth must be expressed in units of the same denomination.

1. How many square inches are there in a rectangle 6 inches long and 5 inches wide? 8 inches long and 3 inches wide? 7 inches long and 5 inches wide?
2. How many square feet are there in a rectangle 4 feet long and 3 feet wide? 7 feet long and 5 feet wide?
3. How many square inches are there in a square whose side is 2 inches? 5 inches? 8 inches? 12 inches?
4. How many square yards are there in a square whose side is 2 yards? 5 yards? 7 yards? 10 yards?
5. How many square feet are there in a square whose side is 1 yard long? 3 yards? 5 yards? 7 yards? 10 yards?
6. How many square rods are there in a lot 5 rods long and 4 rods broad? In a square whose side is 6 rods?
7. How many square feet in a square whose side is 3 yards? In a rectangle whose length is 4 yards and breadth 3 yards?
8. How many square inches are there in a square foot? Square feet in a square yard? Square yards in a square rod?

SQUARE MEASURE.

TABLE.

144 Square Inches (sq. in.)	= 1 Square Foot . . .	sq. ft.
9 Square Feet	= 1 Square Yard . . .	sq. yd.
$30\frac{1}{4}$ Square Yards	= 1 Square Rod . . .	sq. rd.
160 Square Rods	= 1 Acre	A.
640 Acres	= 1 Square Mile . . .	sq. mi.

sq. mi.	*A.*	*sq. rd.*	*sq. yd.*	*sq. ft.*	*sq. in.*
1 =	640 =	102400 =	3097600 =	27878400 =	4014489600

Scale—144, 9, $30\frac{1}{4}$, 160, 640.

1. Plastering, ceiling, etc., are commonly estimated by the *square yard;* paving, glazing, and stone-cutting, by the *square foot.*

2. Roofing, flooring and slating are commonly estimated by the *square* of 100 *feet.*

SURVEYORS' SQUARE MEASURE.

TABLE.

625 Square Links	= 1 sq. rd.	10 Square Chains	= 1 acre.
16 Square Rods	= 1 sq. chain.	640 Acres	= 1 sq. mi.

In some parts of the country a Township contains 36 square miles, or is 6 miles square.

1. How many square feet are there in 4 square yards? 7 square yards? 9 square yards?

2. How many square inches are there in 2 square feet? 3 square feet? 5 square feet?

3. How many square yards are there in 27 square feet? 36 square feet? 81 square feet?

4. How many square yards are there in 10 square rods?

5. How many square chains are there in 48 square rods? 64 square rods? 96 square rods?

6. How many square rods are there in 3 acres? In 5 acres?

7. How many acres are there in 480 square rods?

8. How many square feet are there in 288 square inches?

9. How many acres are there in 30 square chains? In 50?

WRITTEN EXERCISES.

10. Reduce 9 sq. yd. 3 sq. ft. 15 sq. in. to square inches.

11. Reduce 3 sq. mi. 15 sq. rd. to square inches.

12. Reduce 262685 sq. ft. to acres, etc.

13. Reduce 2 A. 37 sq. rd. 5 sq. yd. 7 sq. ft. to sq. in.

14. Reduce 184265 sq. in. to units of higher denominations.

15. Reduce $\frac{5}{7}$ of an acre to units of lower denominations.

PROCESS.

$$\tfrac{5}{7}\text{ A.} \times 160 = \tfrac{800}{7}\text{ sq. rd.} = 114\tfrac{2}{7}\text{ sq. rd.}$$
$$\tfrac{2}{7}\text{ sq. rd.} \times 30\tfrac{1}{4} = \tfrac{2}{7}\text{ sq. rd.} \times \tfrac{121}{4} = \tfrac{242}{28}\text{ sq. yd.} = 8\tfrac{9}{14}\text{ sq. yd.}$$
$$\tfrac{9}{14}\text{ sq. yd.} \times 9 = \tfrac{81}{14}\text{ sq. ft.} = 5\tfrac{11}{14}\text{ sq. ft.}$$
$$\tfrac{11}{14}\text{ sq. ft.} \times 144 = \tfrac{1584}{14}\text{ sq. in.} = 113\tfrac{1}{7}\text{ sq. in.}$$

Therefore $\frac{5}{7}$ A. = 114 sq. rd. 8 sq. yd. 5 sq. ft. $113\frac{1}{7}$ sq. in.

ANALYSIS.—We multiply by that number in the scale which will reduce the number to the next lower denomination, and so continue to multiply each fraction until the lowest denomination is reached.

16. Express $\frac{5}{9}$ of an acre in lower denominations.

17. What part of an acre are 100 sq. rd.? 80 sq. rd.? 120 sq. rd.?

18. Change $\frac{3}{8}$ of a sq. rd. to lower denominations.

19. How many sq. in. are there in a rectangle 7 inches wide by 11 inches long?

20. How many square feet are there in a floor 8 feet long by 15 feet wide?

21. How many square yards are there in a ceiling that is 18 feet wide by 21 feet long?

42 sq yd

22. What is the area of a square whose side is 5 feet?

23. How many square yards are there in a floor 18 feet wide by 24 feet long? How much would it cost to carpet it at \$1.15 per square yard?

24. How many yards of carpeting 1 yard wide will be required to cover a room 18 ft. long by 17 ft. wide?

25. What will it cost to carpet a room 18 ft. long by $15\frac{3}{4}$ ft. wide, with carpet $\frac{3}{4}$ of a yard wide, at \$1.90 per yard?

26. If the width of a lot is 66 feet, how long must it be to contain $\frac{1}{4}$ of an acre? What will be the cost of it at \$3.25 per square foot?

27. A pasture containing 10 acres had a width of 20 rods? How long was it? 4

28. Mr. A. sold a lot of land whose width was 20 rd. and whose length was 80 rd. at $47.25 per acre. How much did he get for it?

29. What is the difference between 10 square feet and 10 feet square? Illustrate this by drawings.

30. What will be the expense of painting a roof 48 feet long and 22 feet wide at $.30 a square yard? $35.20

31. What will be the cost of cementing the bottom of a cellar 45 feet by 32 feet at $.30 per square yard? $48

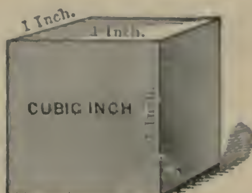
32. How many yards of plastering are there in the sides of a room 18 ft. long, 17 ft. wide, and 11 ft. high? How many in the ceiling? What will be the cost of plastering at $.37 a square yard?

33. What will be the cost of papering the side walls of the above room at $.25 per square yard?

MEASURES OF VOLUME.

260. A ***Solid*** has length, breadth, and thickness.

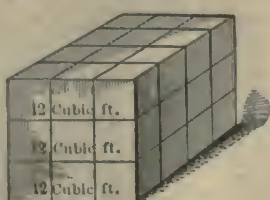
261. A ***Cube*** is a solid having six equal square sides called faces.



262. A ***Cubic Inch*** is a solid whose sides or faces are each a square inch.

263. A ***Cubic Foot*** is a solid whose sides are each a square foot.

264. The ***Volume***, or ***Solid Contents***, of any body is the number of solid units it contains.



Thus, if a solid is 4 ft. long, 3 ft. wide, and 3 ft. thick, its volume will be 36 cubic feet. For it may be divided into 3 blocks, each containing 12 cubic feet, making in all 36 cubic feet. That is, the number of cubic feet in each block will be equal to the product of the numbers expressing its length and breadth, and the number of blocks is equal to the number expressing the thickness. Therefore,

265. *The volume of any rectangular solid is equal to the product of the numbers expressing its length, breadth, and thickness.*

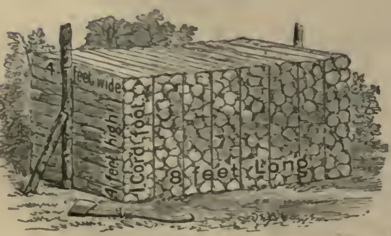
The length, breadth and thickness must be expressed in units of the same denomination.

1. How many cubic feet are there in a rectangular solid whose length is 3 ft., its breadth 2 ft., and its thickness 2 ft.?
2. How many cubic feet are there in a cube whose dimensions are each 3 feet; or, how many cubic feet are there in a cubic yard? In a cube whose sides are 5 ft. long?
3. How many cubic inches are there in a cube whose dimensions are each 12 inches; or, how many cubic inches are there in a cubic foot? In a cube whose sides are 10 in. long?
4. What is the volume of a cube whose sides are each 4 inches square? 9 inches square? 16 inches square?

CUBIC MEASURE.

TABLE.

1728 Cubic Inches (cu. in.)	= 1 Cubic Foot	. . .	cu. ft.
27 Cubic Feet	= 1 Cubic Yard	. . .	cu. yd.



A cord of wood or stone is a pile 8 feet long, 4 feet wide and 4 feet high.

A pile that is 1 foot long, 4 feet wide and 4 feet high, is a cord foot.

The following are the denominations:

16 Cubic Feet	= 1 Cord Foot	. . .	cd. ft.
8 Cord Feet } 128 Cubic Feet }	= 1 Cord	. . . . .	C.

1. A *perch* of stone or masonry is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. thick, and 1 foot high, and contains $24\frac{3}{4}$ cu. ft.

2. A *cubic yard* of earth is considered a *load*.

3. Brick-work is commonly estimated by the thousand bricks.

4. Brick-layers, masons and joiners commonly make a deduction of one-half the space occupied by windows and doors in the walls of buildings.

5. In computing the contents of walls, masons and brick-layers multiply the entire distance around on the outside of the wall by the height and thickness. The corners are thus measured twice.

WRITTEN EXERCISES.

1. How many cubic inches are there in 2 cubic feet? In 3 cu. ft.? In 15 cu. ft.? In 32 cu. ft.?

2. How many cubic feet are there in 2 cubic yards? In 3 cu. yd.? In 13 cu. yd.? In 25 cu. yd.?

3. How many cubic feet are there in 5 cords? In 8 cords?

4. How many perch of masonry are there in 418 cubic feet? What will be the cost of laying it at $1.75 per perch?

5. How many perch of masonry are there in a wall 38 feet long, 4 feet high, and $1\frac{1}{2}$ feet thick?

6. How many yards or *loads*, of earth, must be removed in digging a cellar 35 feet by 20, 8 feet deep?

7. Reduce 32 cu. ft. 114 cu. in. to cubic inches.

8. Reduce 13 cu. yd. 18 cu. ft. to cubic feet.

9. Reduce 15 perch $13\frac{1}{2}$ cu. ft. to cubic feet.

10. How many cubic blocks of one foot on a side can be cut from a cube that is 8 yards long on each edge?

11. How many cubic feet in a block of marble 9 feet long, 5 feet wide, and $3\frac{1}{2}$ feet thick? $157\frac{1}{2}$ cu ft

$20.62½ $10.31¼

2 12. A man sawed a pile of wood 40 ft. long, 4 ft. wide, and 5½ ft. high, for $1.50 per cord. How much did he earn? II

13. A bin is 8 ft. long, 7 ft. wide, and 5 ft. high. How
3 many cubic feet are there in it? How many cubic inches? How many bushels will it hold if a bushel contains 2150.4 cubic inches?

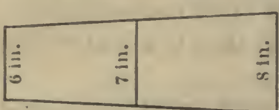
4 14. What will it cost to excavate a cellar 80 by 35 ft., and 8 ft. deep, at $.42 per yd.? What will be the expense of building a stone wall around it 1½ ft. thick, at $3.75 a perch?

15. How many bricks will it require to build a wall 35½
5 ft. long, 19 ft. high, and 3 ft. thick, allowing 22 bricks to the cubic foot when laid?

BOARD MEASURE.

266. In measuring lumber, when a board is one *inch thick*, the number of feet *board measure* is obtained by multiplying the length in feet by the breadth expressed in feet.

When the lumber is *more than one inch thick*, the number of feet, board measure, may be obtained by multiplying the length in feet by the breadth in feet, and this product by the number expressing the inches in thickness.



When a board tapers uniformly, the average or mean width is equal to half the sum of the two ends.

Board measure may also be computed by multiplying the number of feet in length by the number of inches in width, and then dividing the product by 12.

EXERCISES.

How many feet are there in the following boards:

1. 18 ft. by 16 in.?
2. 15 ft. by 11 in.?
3. 10 ft. by 13 in.?
4. 13 ft. by 15 in.?

5. How many feet of timber are there in a stick 40 feet long, 9 inches wide, and 6 inches thick?

6. Mr. B. bought 318 fence boards 16 feet long and 8 inches wide. What did they cost at \$11 per thousand feet?

7. A lumber dealer bought 35 three-inch planks, 22 feet long and 16 inches wide, at \$17.50 per M. How much did they cost?

8. What will it cost to floor a room 35 feet by 18, with $1\frac{1}{4}$ inch flooring, at \$30 per M, allowing $\frac{1}{6}$ for matching?

9. What will be the expense of flooring a room 20 feet by 25 with $1\frac{1}{2}$ inch flooring, at \$25 per M, allowing $\frac{1}{8}$ for matching?

MEASURES OF CAPACITY.

LIQUID MEASURE.

267. ***Liquid Measure*** is used in measuring liquids.

TABLE.

4 Gills (gi.)	= 1 Pint	. . .	pt.
2 Pints	= 1 Quart	. . .	qt.
4 Quarts	= 1 Gallon	. . .	gal.

gal.		*qt.*		*pt.*		*gi.*
1	=	4	=	8	=	32

Scale—4, 2, 4.

1. In determining the capacity of *cisterns, reservoirs*, etc., $31\frac{1}{2}$ gallons are considered a barrel (bbl.), and 2 barrels, or 63 gallons a hogshead (hhd.). In *commerce*, however, the barrel and hogshead are not fixed measures.

2. *Casks* of large size do not hold any fixed quantity. Their capacity is usually marked upon them.

3. The *standard gallon* of the United States contains 231 cubic inches.

4. The *beer gallon* is not now in use. It contained 282 cubic inches.

EXERCISES.

1. How many gills are there in 3 pints? 5 pints? 7 pints?

2. How many gills are there in 2 quarts? 3 quarts?

3. How many pints are there in 3 quarts? 8 quarts?

4. How many pints are there in a cask which contains 37 gallons?

5. A man sold 684 pints of milk at 20 cents a gallon. How much did he get for it? How many gallons were there?

6. Reduce 3846 gi. to gal. 4869 pt. to gal.

7. Reduce 3 gal. 4 qt. 1 pt. 3 gi. to gi.

8. Reduce 4 bbl. 6 gal. to gi. 484 pt. to gal.

9. Reduce 24 gal. to pt. 8459 gi. to bbl.

10. How many cubic inches are there in 7 gal.?

11. How many gallons will a vessel hold that contains 3846 cubic inches?

12. How many barrels of water will a cistern hold that is 15 feet long, 10 feet wide, and 8 feet deep?

APOTHECARIES' LIQUID MEASURE.

268. ***Apothecaries' Liquid Measure*** is used in compounding and measuring liquid medicines.

TABLE.

60 Drops (gtt.) or minims (♏)	= 1	Fluid drachm .	*f*ʒ.
8 Fluid drachms	= 1	Fluid ounce .	*f*℥.
16 Fluid ounces	= 1	Pint . . .	*O.*
8 Pints	= 1	Gallon . . .	*Cong.*

1. The abbreviation *Cong.* is from the Latin *congius*, a gallon. A pint being one-eighth of a gallon the abbreviation is *O.*, from the Latin *octavus*, one-eighth.

2. In writing prescriptions, physicians write the number after the symbol; thus: *O.* 5, *f*℥ 2, etc.

DRY MEASURE.

269. ***Dry Measure*** is used in measuring grain, roots, fruit, etc.

TABLE.

2 Pints (pt.)	= 1 Quart	. . .	qt.
8 Quarts	= 1 Peck	. . .	pk.
4 Pecks	= 1 Bushel	. . .	bu.

bu.	*pk.*	*qt.*	*pt.*
1 =	4 =	32 =	64

Scale—2, 8, 4.

1. In measuring grain, seeds, or small fruits, the measure must be *even full* or *stricken*. In measuring large fruits, coarse vegetables, corn in the ear, etc., the measure should be heaped at least six inches.

2. Five stricken bushels are considered equal to 4 heaped bushels.

3. A standard bushel contains 2150.4 cubic inches.

4. A pint, quart, or gallon, dry measure, is more than the same quantity liquid measure, for a quart is $\frac{1}{32}$ of a bushel, or $\frac{1}{32}$ of 2150.4 cubic inches, which is about $67\frac{1}{5}$ cubic inches, while a quart liquid measure is $\frac{1}{4}$ of 231 cubic inches, or $57\frac{3}{4}$ cubic inches.

	Cu. In. in One Gal.	Cu. In. in One Qt.	Cu. In. in One Pt.	Cu. In. in One Gi.
Liquid Meas.	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry Meas.	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

EXERCISES.

1. How many pints are there in 3 quarts? 7 quarts?

2. How many quarts are there in 2 pecks? 3 pecks? 5 pecks? 7 pecks?

3. How many pints are there in 1 bushel? 3 bushels? 5 bushels? 8 bushels?

4. How many pints are there in 3 bu. 3 pk. 5 qt. 1 pt.?

5. How many pints are there in 8 bu. 5 qt. 3 pt.?

6. Change 16845 qt. to units of higher denominations.

12

7. Change 13965 pt. to units of higher denominations.

8. Change 57364 qt. to units of higher denominations.

9. Change 35 bu. 3 pk. 6 qt. 1 pt. to pints.

10. How many cubic inches are there in 7 bu.? 8 bu.? 10 bu.? 20 bu.?

11. How many bushels are there in 13846 cu. in.? 35769 cu. in.? 48695 cu. in.

12. How many cubic inches are there in a bin 8 ft. long, 7 ft. wide, and 5 ft. high? How many bushels will it hold?

13. How many bushels will a bin hold that is 9 ft. long, 6 ft. wide, and 6 ft. high?

MEASURES OF WEIGHT.

270. ***Weight*** is the measure of the force that attracts bodies to the earth.

AVOIRDUPOIS WEIGHT.

271. ***Avoirdupois Weight*** is used in measuring all coarse and heavy articles, as hay, grain, groceries, coal, etc., and the metals, *except gold and silver.*

TABLE.

16 Ounces (oz.)	= 1 Pound	lb.
100 Pounds	= 1 Hundred-weight .	cwt.
20 Hundred-weight	= 1 Ton	T.

T.	*cwt.*	*lb.*	*oz.*
1 =	20 =	2000 =	32000

Scale — 16, 100, 20.

1. In weighing coal at the mines and in levying duties at the United States Custom House, the *long* ton of 2240 lb. is sometimes used.

2. The ounce is considered as 16 *drams.*

The following denominations are also used:

56 lb. Butter	= 1 Firkin.
100 lb. Grain or Flour	= 1 Cental.
100 lb. Dried Fish	= 1 Quintal.
100 lb. Nails	= 1 Keg.
196 lb. Flour	= 1 Barrel.
200 lb. Pork or Beef	= 1 Barrel.
280 lb. Salt at N. Y. Works	= 1 Barrel.

The following are the pounds in a bushel in the States named:

	Cal.	*Conn.*	*Del.*	*Ill.*	*Ind.*	*Iowa.*	*Ky.*	*La.*	*Me.*	*Mass.*	*Mich.*	*Minn.*	*Mo.*	*N. H.*	*N. J*	*N. Y.*	*Ohio.*	*Oregon.*	*Penna.*	*Vt.*	*Wash. T.*	*Wis.*	*N. C.*
Wheat	60	56	60	60	60	60	60	60		60	60	60	60		60	60	60	60	60	60	60	60	60
Indian Corn	52	56	56	52	56	56	56	56		56	56	56	52		56	58	56	56	56	56	56	56	54
Oats	32	28		32	32	32	$33\frac{1}{3}$	32	30	30	32	32	35	30	30	32	32	34	32	32	36	32	
Barley	50			48	48	48	48	32		46	48	48	48		48	48	48	46	47	46	45	48	48
Buckwheat	40	45		40	50	52	52			46	42	42	52		50	48		42	48	46	42	42	50
Rye	54	56		54	56	56	56	32		56	56	56	56		56	56	56	56	56	56	56	56	
Clover Seed				60	60	60	60				60	60	60		64	60	60	60			60	60	
Timothy Seed				45	45	45	45						45			44						46	

EXERCISES.

1. How many ounces are there in 5 lb.? In 3 lb. 5 oz.?
2. How many pounds are there in 5 cwt.? In 6 cwt.?
3. How many pounds are there in 1 ton? In 3 T.?
4. How many pounds are there in 3 T. 2 cwt. 5 lb.?
5. How many pounds are there in 5 T. 216 lb.?
6. How much will 5 lb. 7 oz. of indigo cost at \$ .12½ per oz.?
7. What will 3½ lb. of confectionery cost at \$ .04½ per oz.?
8. At 8 cents a pound, what must be paid for 5 cwt. 28 lb. of sugar?

9. How many pounds are there in $\frac{1}{2}$ barrel of pork? In $\frac{1}{4}$ barrel of salt? In $\frac{1}{2}$ barrel of flour? In $\frac{1}{4}$ keg of nails?

10. What will be the value of $\frac{1}{4}$ barrel of flour at $8.50 per cwt.?

11. What will $\frac{1}{2}$ quintal of codfish cost at $ .06$\frac{1}{2}$ per lb.?

12. What will be the cost of 13 cwt. 18 lb. of hay at $15 per ton?

13. When flour is $10 a barrel, how many pounds can I buy for $2.80?

14. A merchant sold 3 cwt. 19 lb. 9 oz. of cheese at $ .17 per lb. How much did he receive for it?

15. If a merchant buys flour at $9 per barrel and sells it at $5 per cental, how much will be his profit on the sale of 15 barrels?

16. How many barrels of salt are there in 275000 lb.?

17. If the weight of a bushel of wheat is 60 lb., how many bags that hold 2 bu. each will be required to carry away 3 T. 4 cwt. 20 lb. of wheat?

TROY WEIGHT.

272. ***Troy Weight*** is used in weighing gold, silver, and jewels.

TABLE.

24 Grains (gr.)	= 1 Pennyweight . . .	pwt.
20 Pennyweights	= 1 Ounce	oz.
12 ounces	= 1 Pound. . . . . :	lb.

lb. *oz.* *pwt.* *gr.*
1 = 12 = 240 = 5760

Scale — 24, 20, 12.

1. In weighing diamonds, pearls, and other jewels, the unit commonly employed is the *carat*, which is equal to 4 grains.

2. The term *carat* is also used to express the fineness of gold, and means $\frac{1}{24}$ part. Thus, gold that is 18 carats fine is $\frac{18}{24}$ gold and $\frac{6}{24}$ alloy.

APOTHECARIES' WEIGHT.

273. ***Apothecaries' Weight*** is used by apothecaries and physicians in weighing medicines.

TABLE.

20 Grains (gr.)	= 1 Scruple . . .	sc., or ℈
3 Scruples	= 1 Dram	dr., or ʒ
8 Drams	= 1 Ounce	oz., or ℥
12 Ounces	= 1 Pound	lb., or ℔

lb.	*oz.*	*dr.*	*sc.*	*gr.*
1 =	12 =	96 =	288 =	5760

Scale—20, 3, 8, 12.

1. In writing prescriptions, physicians express the number in Roman characters, using j instead of i final. They also write the symbol first; thus: ℥v, ʒvj, ℈ij.

2. Medicines are bought and sold in large quantities by Avoirdupois Weight.

1 lb. Avoirdupois = 7000 gr.	1 lb. {Troy and Apothecaries'} = 5760 gr.
1 oz. " = 437½ gr.	1 oz. " = 480 gr.

EXERCISES.

1. How many grains are there in 3 pwt.? In 5 pwt.?
2. How many pennyweights are there in 5 oz.? In 7 oz.?
3. How many grains are there in 7 oz. 5 pwt. 18 gr.?
4. Express 3456 grains Troy in higher units.
5. What will be the value of an ornament weighing 2 oz. 15 pwt., at $1.35 per pwt.?
6. How many spoons, weighing 5 ounces each, can be made from 3 lb. 5 oz. of silver?
7. How many powders, of 5 grains each, can be made from 5 oz. 7 dr. of quinine?

MEASURES OF TIME.

274. The following are the ordinary divisions of time:

TABLE.

60 Seconds (sec.)	= 1 Minute	. . .	min.
60 Minutes	= 1 Hour	. . . .	hr.
24 hours	= 1 Day	. . . .	da.
7 days	= 1 Week	. . . .	wk.
365 days	= 1 Year	. . . .	yr.
366 days	= 1 Leap Year	. .	yr.
100 years	= 1 Century	. . .	cen.

yr.	*mo.*	*da.*	*hr.*	*min.*	*sec.*
1 =	12 =	365 =	8760 =	525600 =	31536000

Scale—60, 60, 24, 365, 100.

1. In most business computations 30 days are considered a month, and 12 months a year. For many purposes 4 weeks constitute a month.

2. The common year contains 52 weeks and 1 day, the leap year 52 weeks and 2 days. Hence, commonly, each year begins one day later in the week, but the year succeeding leap year begins *two* days later.

3. The time required for the earth to revolve around the sun is one year, which is 365 da. 5 hr. 48 min. 49.7 sec., or very nearly 365¼ days. Instead of reckoning this part of a day each year, it is disregarded, and an addition made when this would amount to *one day*, which would be very nearly every fourth year. This addition of one day is made to the month of February. Since the part of a day that is disregarded when 365 days are considered as a year, is a *little less than one-quarter of a day*, the addition of one day every fourth year is a little too much, and, to correct this excess, addition is made to only every fourth centennial year. With this correction the error does not amount to much more than a day in 4000 years. Therefore,

Centennial years whose number is exactly divisible by 400, and other years whose number is exactly divisible by 4, are ***Leap Years.***

The year begins with the month of January, and ends with the month of December.

The months, their names and the number of days in each, are as follows:

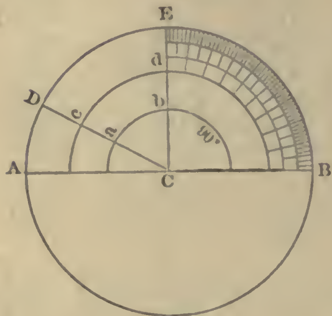
January,	31 da. . .	Jan.	July,	31 da. . .	July.
February,	28 or 29 da.	Feb.	August,	31 da. . .	Aug.
March,	31 da. . .	Mar.	September,	30 da. . .	Sept.
April,	30 da. . .	Apr.	October,	31 da. . .	Oct.
May,	31 da. . .	May.	November,	30 da. . .	Nov.
June,	30 da. . .	June.	December,	31 da. . .	Dec.

EXERCISES.

1. How many seconds are there in 5 minutes? In 6 min.?
2. How many minutes are there in $\frac{1}{2}$ hour? In $\frac{1}{4}$ hr.?
3. How many days are there in 4 weeks? In 5 wk.? In 8 wk.? In 10 wk.?
4. How many hours are there in $\frac{1}{4}$ day? In $\frac{1}{2}$ da.?
5. How many days are there in $\frac{1}{2}$ year? In $\frac{1}{2}$ month?
6. What part of an hour are 30 minutes? 15 min.?
7. How many hours are there in 90 minutes? In 120 min.? In 240 min.?
8. How many seconds are there in 5 hr. 15 min. 12 sec.?
9. How many seconds are there in 6 hr. 27 min. 38 sec.?
10. Express in units of higher orders 48695 sec.
11. Express in units of higher orders 38497 sec.
12. How many minutes are there in 5 yr. of 365 da. each?
13. How many days are there from Jan. 1st to May 1st?
14. How many days are there from April 1st to Oct. 15th?
15. Reduce 2 wk. 5 da. 13 hr. to hours.
16. Reduce 5 da. 10 hr. 15 min. to minutes.
17. Reduce 384600 sec. to higher denominations.
18. Reduce 15 hr. 12 min. 18 sec. to seconds.
19. Reduce 32965 min. to higher denominations.

CIRCULAR OR ANGULAR MEASURE.

275. A ***Circle*** is a plane surface, bounded by a curved line every point of which is equally distant from a point within called the ***Center.***



276. The ***Circumference*** is the line that bounds the circle.

277. An ***Arc*** of a circle is any part of the circumference.

278. A ***Degree*** is $\frac{1}{360}$ of the circumference of a circle.

279. The ***Measure of an Angle*** is that part of the circumference which is included between the lines which form the angle.

Each of the arcs of the circumferences a b, c d, D E, is a measure of the same angle, and therefore contains the same number of degrees; but since each degree is $\frac{1}{360}$ of the circumference, the *length* of a degree must vary.

280. ***Circular or Angular Measure*** is used to measure arcs of circles and angles, in determining latitude, longitude, direction, the position of vessels at sea, etc.

TABLE.

60 Seconds (″)	= 1 Minute . . .	′
60 Minutes	= 1 Degree . . .	°
360 Degrees	= 1 Circumference .	Cir.

$$\begin{array}{cccc} \textit{Cir.} & {}^{\circ} & {}' & {}'' \\ 1 = & 360 = & 21600 = & 1296000 \end{array}$$

Scale—60, 60, 360.

1. A ***Quadrant*** is $\frac{1}{4}$ of a circumference, or 90°; a ***Sextant*** is $\frac{1}{6}$ of a circumference, or 60°.

2. The length of a degree of longitude on the earth's surface at the Equator is 69.16 miles.

3. In astronomical calculations 30° are called a ***Sign,*** and there are therefore 12 signs in a circle.

EXERCISES.

1. How many minutes are there in 5°? 6°?

2. In 35 degrees how many seconds are there? In 27°? In 21° 12′ 18″?

3. How many seconds are there in 34° 12′ 43″?

4. In 468560 seconds how many minutes, etc., are there?

5. In 384500 seconds how many minutes, etc., are there?

6. How many seconds are there in $\frac{1}{2}$ Cir.? In $\frac{1}{4}$? In $\frac{1}{6}$?

7. How many minutes are there in 2 quadrants? In 2 sextants?

COUNTING.

281. The following denominations are used in counting some classes of articles:

12 Things	= 1 Dozen	doz.
12 Dozen	= 1 Gross	gr.
12 Gross	= 1 Great Gross	G. gr.

Two things are often called a *pair,* six things a *set,* and twenty things a *score;* as a *pair* of birds, a *set* of spoons, a *score* of years.

STATIONERS' TABLE.

282. The denominations used in the paper trade are:

24 Sheets	= 1 Quire.
20 Quires	= 1 Ream.
2 Reams	= 1 Bundle.
5 Bundles	= 1 Bale.

The terms *folio, quarto, octavo,* applied to books, indicate the number of leaves into which a sheet of paper is folded. Thus, when a sheet of paper is folded into 2, 4, 8, 12, 16, 18, or 24 leaves, the forms are called respectively, folio, 4to, or quarto, 8vo, or octavo, 12mo, 16mo, 18mo, and 24mo.

EXERCISES.

1. How many eggs are there in 5 dozen? 7 doz.? 10 doz.?
2. How many crayons are there in 2 gross? 3 gr. 5 doz.?
3. How many things are there in a great gross?
4. What will be the cost of 3 dozen brushes at $.45 each?
5. A man lived 3 score and 10 years. What was his age?
6. What will 3 reams of paper sell for at $.15 per quire?

REDUCTION OF DENOMINATE FRACTIONS.

283. The principles, processes and analyses are essentially the same as those of denominate integers.

CASE I.

284. To reduce denominate fractions to equivalent numbers of lower denominations.

EXERCISES.

1. How many hours are there in 1 day? In $\frac{1}{2}$ day? In $\frac{2}{3}$ of a day? In $\frac{3}{4}$ of a day? In $\frac{5}{6}$ of a day?
2. How many ounces are there in $\frac{1}{2}$ pound avoirdupois? In $\frac{1}{4}$ lb.? In $\frac{1}{8}$ lb.?
3. How many pints are there in $\frac{1}{4}$ of a peck? In $\frac{3}{4}$ pk.?
4. How many pecks and quarts are there in $\frac{3}{5}$ of a bushel?
5. How many pounds and ounces are there in $\frac{5}{8}$ cwt.?
6. How many inches are there in $\frac{5}{8}$ of a foot? $\frac{3}{4}$ ft.? $\frac{3}{5}$ ft.?

7. Change $\frac{5}{7}$ of a rd. to units of lower denominations.

PROCESS.

$\frac{5}{7}$ of $\frac{11}{2}$ yd. $= \frac{55}{14}$ yd. $= 3\frac{13}{14}$ yd.

$\frac{13}{14}$ of 3 ft. $= \frac{39}{14}$ ft. $= 2\frac{11}{14}$ ft.

$\frac{11}{14}$ of 12 in. $= \frac{132}{14}$ in. $= 9\frac{6}{14}$ in.

ANALYSIS.—Since in 1 rod there are $5\frac{1}{2}$ yards, in $\frac{5}{7}$ of a rod there will be $\frac{5}{7}$ of $5\frac{1}{2}$ yards, or $3\frac{13}{14}$ yards.

Since in 1 yard there are 3 feet, in $\frac{13}{14}$ of a yard there will be $\frac{13}{14}$ of 3 ft., or $2\frac{11}{14}$ ft.

Since in 1 foot there are 12 inches, in $\frac{11}{14}$ of a foot there will be $\frac{11}{14}$ of 12 inches, or $9\frac{6}{14}$ in.

Therefore $\frac{5}{7}$ of a rod is equal to 3 yd. 2 ft. $9\frac{6}{14}$ in.

Change the following to lower denominations:

8. $\frac{3}{5}$ of a pound Troy.
9. $\frac{3}{7}$ of a ton.
10. $\frac{2}{3}$ of a furlong.
11. $\frac{3}{11}$ of an acre.
12. $\frac{5}{6}$ of a peck.
13. $\frac{3}{10}$ of a day.
14. $\frac{5}{8}$ of a sq. rd.
15. $\frac{7}{20}$ of a cu. yd.

16. Express $\frac{1}{145}$ of a gallon as a fraction of a gill.

ANALYSIS.—Since in 1 gallon there are 32 gills, in $\frac{1}{145}$ of a gallon there are $\frac{1}{145}$ of 32 gi., or $\frac{32}{145}$ gi. Hence $\frac{1}{145}$ gal. $= \frac{32}{145}$ of a gill.

17. Express $\frac{11}{720}$ of a bushel as a fraction of a pint.
18. Express $\frac{3}{21120}$ of a mile as a fraction of a foot.
19. Express $\frac{5}{1944}$ of a pound as a fraction of a scruple.
20. Express .006 of a bushel as a decimal of a pint.
21. Express in lower denominations .685 of a pound Troy.

PROCESS.

```
 .685
   12
 ----
8.220 oz.
   20
 ----
4.400 pwt.
   24
 ----
9.600 gr.
```

ANALYSIS.—Since in 1 pound there are 12 ounces, in .685 of a pound there are .685 of 12 ounces, or 8.220 ounces.

Since there are 20 pennyweights in 1 ounce, in .220 of an ounce there are .220 of 20 pennyweights, or 4.400 pennyweights.

Since in 1 pennyweight there are 24 grains, in .400 of a pennyweight there are .400 of 24 grains, or 9.600 grains.

Therefore .685 lb. is equal to 8 oz. 4 pwt. 9.6 gr.

Express in units of lower denominations:

22. £.575.
23. .1935 of a pound Troy.
24. .436 of a ream.
25. .1845 of a gallon.
26. .135 of a rod.
27. .455 of a mile.
28. .4832 of a bushel.
29. .684 of a league.

CASE II.

285. To change denominate fractions to equivalent fractions of higher denominations.

EXERCISES.

1. What part of a pound Troy is 1 ounce? Is $\frac{1}{2}$ oz.? Is $\frac{1}{4}$ oz.?
2. What part of a ton is 1 pound? Is $\frac{1}{5}$ lb.? Is $\frac{1}{10}$ lb.?
3. What part of a mile is 1 rod? Is $\frac{1}{5}$ rd.? Is $\frac{3}{5}$ rd.?
4. What part of a league is 1 mile? Is 1 rd.? Is $\frac{1}{8}$ rd.?
5. What part of an hour is $\frac{7}{8}$ of a minute? Is $\frac{4}{5}$ of a min.?
6. What part of a week is $\frac{3}{7}$ of a day? $\frac{5}{6}$ of a day?
7. What part of a bushel is $\frac{5}{6}$ of a pint?

PROCESS.

$\frac{5}{6} \times \frac{1}{64} = \frac{5}{384}$ bu.

Or,

$\frac{5}{6} \div 2 = \frac{5}{12}$ qt.
$\frac{5}{12} \div 8 = \frac{5}{96}$ pk.
$\frac{5}{96} \div 4 = \frac{5}{384}$ bu.

ANALYSIS.—Since there are 64 pints in a bushel, 1 pint is $\frac{1}{64}$ of a bushel, and $\frac{5}{6}$ of a pint is $\frac{5}{6}$ of $\frac{1}{64}$ of a bushel, or $\frac{5}{384}$ of a bushel. Or,

Since we are required to change pints to bushels we have an example in reduction *ascending*, and hence we divide by 2, 8, and 4, respectively.

8. Reduce $\frac{5}{11}$ of an inch to the fraction of a yard.
9. Change $\frac{4}{9}$ of a second to the fraction of an hour.
10. Express .375 of a week as a fraction of a year.
11. Express .35 of a pound as a fraction of a ton.
12. Express $\frac{3}{7}$ of a cubic inch as a fraction of a cubic foot.

13. Change $\frac{8}{5}$ of a square yard to a fraction of an acre.
14. Reduce $\frac{5}{8}$ of a pint to a fraction of a barrel.

CASE III.

286. To express one denominate number as a fraction of another.

1. What part of a foot are 3 in.? 6 in.? 9 in.?

2. What part of an hour are 30 min.? 15 min.? 45 min.?

3. What part of a gallon is 1 pint? 2 pints? 1 quart?

4. What part of a gallon are 2 quarts? 2 qt. 1 pt.? 3 qt. 1 pt.?

5. What part of 3 ft. 6 in. are 2 ft. 3 in.?

ANALYSIS—Since 3 ft. 6 in. = 42 in., and 2 ft. 3 in. = 27 in., 27 in. = $\frac{27}{42}$ of 42 in.

6. What part of 3 yd. 2 ft. are 2 yd. 2 ft.?

7. What part of 5 gal. 3 qt. 1 pt. are 2 gal. 1 qt. 1 pt.?

8. What part of 2 pounds Troy are 3 oz. 10 pwt.?

9. What part of 3 pecks are 2 qt. 1 pt.?

10. What part of 3 barrels are 13 gal. 3 qt. 2 pt. 2 gi.?

11. Express 15s. 7d. in the decimal of a pound sterling:

1ST. PROCESS.

15s. 7d. = 187d.
£1 = 240d.
£$\frac{187}{240}$ = £.7791+

2D. PROCESS.

$$12\overline{)7\text{d.}}$$
$$.5833+\text{s.}$$

$$20\overline{)15.5833+\text{s.}}$$
$$£.7791+$$

ANALYSIS.—In order to find what part one number is of another, both must be reduced to the same denomination. 15s. 7d. = 187d. and £1 = 240d. Therefore 187d. = £$\frac{187}{240}$, which, reduced to a decimal, is equal to £.7791+. Or,

ANALYSIS.—Since 7d. is $\frac{7}{12}$ of a shilling, it may be reduced to a decimal by annexing ciphers to the numerator and dividing by 12, which gives .5833+s. Therefore 15s. 7d. = 15.5833+s.

Since 1 shilling is $\frac{1}{20}$ of a pound, 15.5833+s. = £$\frac{15.5833}{20}$, or £.7791+.

12. Reduce 4 hr. 15 min. to the decimal of a day.
13. Reduce 3 pk. 2 qt. to the decimal of a bushel.
14. Reduce 3 ft. 6 in. to the decimal of a rod.
15. Reduce 18s. $5\frac{3}{4}$d. to the fraction of a pound.
16. Reduce 18s. $5\frac{3}{4}$d. to the decimal of a pound.
17. Reduce 16 lb. 11 oz. to the fraction of a hundred-weight.
18. Reduce 37 rd. 14 ft. 3 in. to the decimal of a mile.
19. Reduce 3 da. 5 hr. 14 min. to the decimal of a week.
20. Reduce 8 quires, 15 sheets, to the decimal of a ream.
21. Change 3 cd. ft. 7 cu. ft. to the decimal of a cord.
22. Change 654 yd. 9 in. to the decimal of a mile.
23. Change 4 oz. 7 pwt. 13 gr. to the fraction of a pound Troy.
24. Write rules for each of the cases in denominate numbers.

REVIEW EXERCISES.

287. 1. What will be the cost of 15 lb. 8 oz. of butter at $.31 per pound?

2. What must be paid for 3 pk. 2 qt. of berries at 9 cents a quart?

3. Mr. A. sold 18 bu. 3 pk. of barley at $1.05 per bushel. How much did he get for it?

4. How much must be paid for making 42 rd. 7 ft. 8 in. of fence at $ .75 per foot?

5. How much butter at $ .30 a pound must be given for 12 gal. 3 qt. of molasses, at $ .50 per gallon?

6. Bought 15 bu. of oats at $ .37½ a bushel, and sold them at 15 cents a half-peck. How much did I gain?

7. How many cords of wood are there in a pile 4 ft. wide, 6 ft. high, 60 ft. long? What would it cost at $4.25 a cord?

8. A man built a cistern 10 ft. long and 6 ft. wide, that would hold 100 barrels. How high did he make it?

9. What is gained by selling 1 oz. Troy of opium for $1, which was purchased at the rate of $.75 per oz. Avoirdupois?

10. What are the contents of a field 15 rd. 8 ft. wide, 27 rd. 9 ft. long? What is its value at $150 per acre?

11. How many days of 10 hours each will it require to make a million marks if I make 2 per second?

12. What is the value of a plank 18 ft. long, 16 in. wide, and 4 in. thick, at $18 per M?

13. If at 10 cents a foot the Atlantic cable cost $1689600, what is its length?

14. A druggist put up 7℥ 3ʒ 4℈ in two-grain pills. How many pills did he put up?

15. Bought paper at $2.55 per ream and sold it at 20 cents per quire. How much did I gain?

16. How much sugar at 12 cents a pound can be obtained for 13 lb. 7 oz. butter at 27½ cents a pound?

17. A farmer sold 3 piles of wood at $4.60 per cord. The following are the dimensions of the piles: The first was 73 ft. 9 in. long, 6 ft. high, and 4 ft. wide; the second was 30 ft. long, 7 ft. 2 in. high, and 4 ft. wide; the third was 37 ft. long, 3 ft. 6 in. high, and 4 ft. wide. How much should he receive for his wood?

18. A printer used 4 reams 8 quires 12 sheets of paper for half-sheet posters. How many did he print? What did they cost at $6.50 per M?

19. Hay at $18 per ton is exchanged for flour at $6.85 per barrel. How many barrels are equal to a ton?

20. Two men who are equal partners, obtained from a field 327 bu. 3 pk. 5 qt. of oats. One of them claimed 167 bu. 3 pk. for his share. Did he claim too much or too little? How much?

21. A cubic foot of water weighs about 62 lb. 8 oz. What will be the weight or pressure on a square yard where the sea is 20 fathoms deep?

ADDITION.

288. The processes of adding, subtracting, multiplying, and dividing compound numbers are based upon the same principles as those governing similar operations in simple numbers.

The only difference between the processes is caused by compound numbers having a *varying scale*, while simple numbers have a *uniform decimal scale.*

EXERCISES.

1. What is the sum of 130 rd. 5 yd. 1 ft. 6 in., 215 rd. 2 ft. 8 in., 304 rd. 4 yd. 11 in.?

PROCESS.

	rd.	yd.	ft.	in.
	130	5	1	6
	215	0	2	8
	304	4	0	11
2 mi.	10	$4\frac{1}{2}$	2	1
Or,		$\frac{1}{2}$=	1	6
2 mi.	10	5	0	7

ANALYSIS.—The numbers should be written as in simple addition, so that units of the same denomination stand in the same column, and for convenience we begin at the right to add.

The sum of the inches is 25 in., which is equal to 2 ft. 1 in. We write the 1 under the inches and add the 2 ft. with the feet. The sum of the feet is 5 ft., or 1 yd. 2 ft. We write the 2 as feet in the sum and add the 1 yd. with the yards.

The sum of the yards is 10 yd., or 1 rd. $4\frac{1}{2}$ yd. We write the $4\frac{1}{2}$ yd. as yards of the sum, and add the 1 rd. with the rods. The sum of the rods is 650 rd., or 2 mi. 10 rd., which we write as miles and rods of the sum.

Therefore the sum is 2 mi. 10 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ yd. equals 1 ft. 6 in., the sum may be expressed as 2 mi. 10 rd. 5 yd. 7 in.

RULE.—*Change the rule for the addition of simple numbers so that it may be applicable to denominate numbers.*

2. What is the sum of 12 lb. 5 oz. 13 pwt., 21 lb. 8 oz. 15 pwt., 13 lb. 7 oz. 10 pwt., 51 lb. 3 oz. 17 pwt.?

3. What is the sum of £71 6s. $5\frac{3}{4}$d., £32 8s. $5\frac{1}{2}$d., £61 15s. $8\frac{1}{4}$d., £37 18s. $5\frac{3}{4}$d., £115 11s. 7d.?

4. Find the sum of 10 mi. 217 rd. 2 yd. 3 ft. 4 in., 7 mi. 185 rd. 3 yd. 9 in., 19 mi. 37 rd. 6 yd.

5. Find the sum of 3 T. 7 cwt. 39 lb. 8 oz., 8 T. 11 cwt. 48 lb., 11 oz., 13 T. 33 lb. 10 oz., 9 cwt. 18 lb. 9 oz.

6. Find the sum of 18 gal. 3 qt. 1 pt. 3 gi., 15 gal. 2 qt. 1 pt. 2 gi., 11 gal. 2 qt. 2 gi., 3 qt. 1 pt. 1 gi.

7. A miller bought four loads of grain containing, respectively, 25 bu. 3 pk., 28 bu. 2 pk., 32 bu. 3 pk. 5 qt., 28 bu. 2 pk. 7 qt. How much grain did he buy?

8. How much wood is there in 3 piles containing, respectively, 37 C. 21 cu. ft. 1140 cu. in., 29 C. 110 cu. ft. 708 cu. in., and 34 C. 121 cu. ft. 398 cu. in.?

9. Find the sum of $\frac{3}{7}$ mi., .35 rd. and $2\frac{3}{8}$ rd.

PROCESS.

	rd.	ft.	in.
$\frac{3}{7}$ mi. =	137	2	$4\frac{2}{7}$
.35 rd. =		5	$9\frac{3}{10}$
$2\frac{3}{8}$ rd. =	2	6	$2\frac{1}{4}$
	139	14	$3\frac{117}{140}$

ANALYSIS.—Each of the fractions is expressed in integers of lower denominations, and then they are added.

10. A merchant sold $12\frac{3}{4}$ yards of cloth to one person, $8\frac{7}{8}$ yards to another, $37\frac{1}{2}$ yards to another, $39\frac{3}{8}$ yards to another. How many yards, feet and inches did he sell?

11. What is the amount of land in the following lots, the first containing $\frac{1}{5}$ of an acre, the second $\frac{3}{8}$ of an acre, the third $129\frac{1}{4}$ sq. rd., and the fourth $118\frac{1}{4}$ sq. rd.?

12. A merchant sold the following quantities of molasses, viz: On June 15, 24 gal. 2 qt. 3 pt.; June 16, $45\frac{1}{8}$ gal.; June 17, $1\frac{1}{4}$ bbl. ($39\frac{3}{8}$ gal.) How much did he sell in that time?

13. James is 3 yr. 4 mo. 18 da. old, Henry is 2 yr. 8 mo. 6 da. older than James, William is 7 yr. 10 mo. 24 da. older than Henry, and Herbert is 20 mo. older than William. How old is Herbert?

14. Find the sum of $20\frac{1}{8}$ cwt., $16\frac{1}{7}$ T., $17\frac{1}{8}$ lb., 19 cwt. 18 lb. 7 oz., 15 lb. 8 oz., 2 T. 7 lb. 5 oz., $\frac{3}{7}$ lb., $\frac{5}{9}$ T., 2 T. 3 cwt. 57 lb. 4 oz.

SUBTRACTION.

289. 1. From 127 rd. 3 yd. 1 ft. 7 in., subtract 100 rd. 4 yd. 2 ft. 9 in.

PROCESS.

	rd.	yd.	ft.	in.
	127	3	1	7
	100	4	2	9
	26	$3\frac{1}{2}$	1	10
Or,		$\frac{1}{2}$	$=1$	6
	26	4	0	4

ANALYSIS.—The numbers should be written as in simple subtraction, so that units of the same order stand in the same column, and, for convenience, begin at the right to subtract.

Since we can not subtract 9 in. from 7 in., we unite with 7 in. a unit of the next higher order, making 1 ft. 7 in., or 19 in. Then 9 in. from 19 in. leaves 10 in., which we write as inches in the remainder. Inasmuch as 1 ft. was united with 7 in., there are no feet remaining in the minuend.

Since we can not subtract 2 ft. from 0 ft., we unite with 0 ft. a unit of the next higher order, making 3 ft. Then 2 ft. from 3 ft. leaves 1 ft., which we write as the feet of the remainder.

Since 4 yd. can not be subtracted from 2 yd., we unite with 2 yd. a unit of the next higher order and proceed as before. The remainder will be 26 rd. 4 yd. 0 ft. 4 in.

RULE.—*Change the rule for subtraction of simple numbers so that it may be applicable to denominate numbers.*

2. From 2 mi. 116 rd. 4 yd. 0 ft. 4 in., take 1 mi. 120 rd. 2 yd. 1 ft. 8 in.

3. From 15 cwt. 37 lb. 10 oz., take 8 cwt. 42 lb. 8 oz.

4. From 1 hhd. 38 gal. 3 qt. 2 pt., take 60 gal. 2 qt. 1 gi.

5. From 13 lb. 8 oz. 13 pwt. 15 gr., take 8 lb. 8 oz. 16 pwt. 15 gr.

6. From 18° 33′ 16″, take 9° 42′ 28″.

7. From 37 C. 7 cd. ft. 11 cu. ft., take 18 C. 7 cd. ft. 12 cu. ft.

8. From $\frac{3}{4}$ bbl. take $7\frac{3}{5}$ gal.

PROCESS.

	gal.	qt.	pt.	gi.
$\frac{3}{4}$ bbl. =	23	2	1	
$7\frac{3}{5}$ gal. =	7	2	0	$3\frac{1}{5}$
	16	0	0	$\frac{4}{5}$

ANALYSIS.—The fractions are first expressed in integers of lower denominations and then subtracted.

9. From $\frac{3}{4}$ of an acre of land a piece containing 72 sq. rd. 160 sq. ft. 39 sq. in. was sold. How much was left?

10. A merchant sold cloth for £384 6s. $5\frac{1}{2}$d. which cost him £297 9s. $8\frac{3}{4}$d. How much was his profit?

11. From a farm of 285 acres there were sold at one time $97\frac{5}{8}$ acres, and at another 38 A. $39\frac{1}{4}$ sq. rd. How much was left?

12. A merchant bought 9 reams 18 quires 15 sheets of paper, from which he sold $3\frac{5}{12}$ reams. How much remained unsold?

13. How long was it from Jan. 10, 1841, to May 7, 1853?

PROCESS.

1853	5	7
1841	1	10
12	3	27

ANALYSIS.—Since the later date expresses the greater period of time, we write it as the minuend, and the earlier date as the subtrahend, giving the month its number instead of the name. We then subtract as in denominate numbers, considering 30 days one month, and 12 months one year. The remainder will be the time as correct as it can be expressed in months and days.

14. How long was it from Jan. 3, 1843, to Mar. 15, 1851?

15. How old was a man who was born April 2, 1803, and who died Dec. 15, 1869?

16. A man bought a farm May 15, 1860, and paid for it Jan. 5, 1871. How long did it take to pay for it?

17. A legacy of $3000 was to be paid to a man 3 yr. 2 mo. 5 da. after Dec. 8, 1837. When was it to be paid?

18. How many years, months and days from the day of your birth? or, How old are you?

19. The American Civil War began April 11, 1861, and ended April 9, 1865. How long did it continue?

20. A note dated July 9, 1871, was paid October 10, 1876. How long did it run before it was paid?

MULTIPLICATION.

290. 1. How much is 5 times 147 rd. 4 yd. 2 ft. 8 in.?

PROCESS.

	rd.	yd.	ft.	in.
	147	4	2	8
				5
2 mi.	99	2	1	4

ANALYSIS.—We write the numbers as in simple numbers, and for convenience begin at the right to multiply.

5 times 8 in. are 40 in., or 3 ft. 4 in. We write the 4 in. as inches in the product, and reserve the 3 ft. to add with the product of feet.

5 times 2 ft. are 10 ft.; 10 ft. + 3 ft. reserved equal 13 ft., or 4 yd. 1 ft. We write the 1 ft. in the product and reserve the 4 yd. to add to the product of yards.

5 times 4 yd. equal 20 yd.; 20 yd. + 4 yd. reserved equal 24 yd., or 4 rd. 2 yd. We write the 2 yd. in the product and reserve the rods to add to the product of rods.

5 times 147 rd. are 735 rd.; 735 rd. + 4 rd. reserved equal 739 rd., or 2 mi. 99 rd., which we write in the product.

Therefore the product is 2 mi. 99 rd. 2 yd. 1 ft. 4 in.

RULE.—*Modify the rule for multiplication of simple numbers so that it may be applicable to denominate numbers.*

2. Multiply 9 gal. 3 qt. 1 pt. 3 gi. by 7.

3. Multiply 17 lb. 8 oz. 3 pwt. 15 gr. by 9.

4. Multiply 1 T. 4 cwt. 35 lb. 6 oz. by 10.

5. A farm consists of 7 fields each containing 18 A. 25 sq. rd. How much land does it comprise?

6. What is the length of a fence which encloses a square field each side of which is 28 rd. 5 yd. $2\frac{1}{2}$ ft. long?

7. How much wood is there in 7 piles, each containing 13 C. 7 cd. ft. 24 cu. ft.?

8. What will $14\frac{1}{2}$ yd. of lace cost at £2 5s. 6d. per yard?

9. What is the value of 4 loads of potatoes, each containing 27 bu. 3 pk., at \$.45 per bushel?

DIVISION.

291. 1. Divide 27 bu. 3 pk. 5 qt. 1 pt. into 6 equal parts.

PROCESS.

6)	27 bu.	3 pk.	5 qt.	1 pt.
	4	2	4	$1\frac{5}{6}$

ANALYSIS.—Since the quantity is to be divided into 6 equal parts, each part will contain *one-sixth* of the quantity.

One-sixth of 27 bu. is 4 bu. with a remainder of 3 bu. We write the 4 bu. in the quotient and unite the 3 bu. remaining with the number of the next lowest denomination, making 15 pk.

One-sixth of 15 pk. is 2 pk. and 3 pk. remaining. We write the 2 pk. in the quotient, and unite the 3 pk. remaining with the number of the next lower denomination, making 29 qt.

One-sixth of 29 qt. is 4 qt. and 5 qt. remaining. We write the 4 qt. in the quotient, and unite the 5 qt. with the number of the next lowest denomination, making 11 pt.

One-sixth of 11 pt. is $1\frac{5}{6}$ pt., which we write in the quotient.

Therefore the quotient is 4 bu. 2 pk. 4 qt. $1\frac{5}{6}$ pt.

RULE.—*Change the rule for the division of simple numbers so that it may be applicable to denominate numbers.*

2. In 8 bags there are 17 bu. 3 pk. 4 qt. How much does each bag contain?

3. A gentleman divided his farm of 427 A. 131 sq. rd. equally among his 5 sons. What was the share of each?

4. A brewer filled 4 casks of equal size from a vat containing 315 gal. 3 qt. How large was each cask?

5. 16 T. 1300 lb. of hay was drawn at 9 loads. What was the average weight per load?

6. If a pile of wood containing 8 cords 100 cu. ft. be equally divided among 3 persons, how much will each receive?

7. When £31 5s. 8d. is divided equally among 10 persons, how much does each receive?

8. If 31 cwt. 18 lb. of tea is put up in packages, each containing 3 lb. 8 oz., how many packages will there be?

PROCESS.

31 cwt. 18 lb. = 49888 oz.

3 lb. 8 oz. = 56 oz.

49888 oz. ÷ 56 oz. = $890\frac{6}{7}$

ANALYSIS.—Since the divisor and the dividend are similar denominate numbers, we may reduce them to the same denomination, and then proceed to divide as in simple numbers.

9. How many times must a man dip with a dipper holding 2 qt. 1 pt. so that he may empty a cask containing 31 gal.?

10. If a man walks at an average rate of 23 mi. 160 rd. 4 yd. 2 ft. per day, how long will it take him to walk 100 miles?

11. If a man can travel 300 mi. in 13 days, how far can he travel daily?

12. How many barrels of sugar, each containing 2 cwt. 35 lb., are there in 3 T. 4 cwt. 18 lb.?

13. How many spoons, each weighing 2 oz. 10 pwt., can be made from 13 lb. 7 oz. 15 pwt. of silver?

14. How many pickets 2 ft. 4 in. long and 2 in. wide, can be made out of 5 boards each 11 ft. 8 in. long and 8 in. wide?

LONGITUDE AND TIME.

292. 1. Where does the sun appear to rise?

2. How long will it be before it rises again?

3. Through how many degrees of space does it appear to pass in this daily motion? *Ans.* 360°.

4. Since it seems to travel 360° in one day, or 24 hours, how great will be its apparent motion in 1 hour?

5. If the earth moves 15° in 1 hour, how far will it move in 1 minute?

6. If it moves 15′ in one minute of time, how far will it move in 1 second?

7. How does the number of degrees passed over compare with the number of hours? The number of minutes of space with the number of minutes of time? The number of seconds of space with the number of seconds of time?

8. When it is sunrise at New York, how long will it be before it is sunrise at a place 15° west of New York? 30° west? 45° west? 60° west?

9. When it is sunrise at New York, how long before was it sunrise at a place 15° east? 30° east? 45° east?

10. When it is sunrise at any place, how long will it be before it is sunrise at a place 15° west? 15° east? 30° west? 30° east?

11. When it is noon at any place, what time is it at a place 15° west? 15° east? 30° west? 30° east?

12. If I travel eastward will my watch become too slow or too fast? If I travel westward what change will take place?

13. What places have sunrise at the same time? Noon at the same time? Midnight at the same time?

293. A ***Meridian*** is an imaginary line passing from the North Pole to the South Pole through any place.

294. ***Longitude*** is the distance east or west, from a given meridian.

RELATION BETWEEN LONGITUDE AND TIME.

15° of	Longitude	make 1 Hour	difference in time.
15′ of	"	make 1 Minute	difference in time.
15″ of	"	make 1 Second	difference in time.
1° of	"	makes 4 Minutes	difference in time.
1′ of	"	makes 4 Seconds	difference in time.

WRITTEN EXERCISES.

1. The longitude of Boston is 71° 3′ 30″ west; that of Cincinnati, 84° 29′ 31″ west. What is the difference in time?

PROCESS.

$$\begin{array}{rrrr} & 84° & 29' & 31'' \\ & 71 & 3 & 30 \\ \hline 15) & 13 & 26 & 1 \\ \hline & & 53 & 44 \end{array}$$

ANALYSIS.—We first find the difference in longitude of the two places, and since there are 15 times as many degrees, minutes and seconds as there are hours, minutes and seconds of time, we find $\frac{1}{15}$ of 13° 26′ 1″, which is 53 min. 44 sec., the difference in time.

2. When it is 12 o'clock M. at Philadelphia it is 5 o'clock, 10 min. P. M. at Paris. What is the longitude of Paris, the longitude of Philadelphia being 75° 10′ west?

PROCESS.

$$\begin{array}{rll} 5 \text{ hr.} & 10 & \text{min.} \\ & 15 & \\ \hline 77° & 30' & \text{difference in Long.} \\ 75 & 10 & \\ \hline 2° & 20' & \text{east, Long. of Paris.} \end{array}$$

ANALYSIS.—Since in 1 hour the earth moves 15° of distance, in 1 minute 15′ of distance, in 1 second 15″ of distance, the difference in longitude will be 15 times as many degrees, minutes and seconds of distance as there are hours, minutes and seconds of time. Since Philadelphia is 75° 10′ west, and the difference is 77° 30′, the longitude of Paris is 2° 20′ east.

To find the difference in time when the difference in longitude is given:

Divide the difference in longitude, expressed in degrees, etc., by 15; *the several quotients will be the difference in time in hours, minutes, and seconds.*

To find the difference in longitude when the difference in time is given:

Multiply the difference in time, expressed in hours, minutes and seconds, by 15; *the several products will be the difference in longitude, in degrees, minutes, and seconds.*

3. Two places are 32° 18′ 24″ apart. What is the difference in time between them?

4. When it is noon at San Francisco it is 3 hr. 9 min. 7 sec. P. M. at Philadelphia. What is the longitude of San Francisco if that of Philadelphia is 75° 10′ west?

5. New York is 74° 3′ west longitude, and Paris, France, is 2° 20′ east. How much earlier is it sunrise in Paris than in New York?

6. Washington is 77° west of Greenwich, England. What is their difference in time?

7. When it is noon at Washington, which is 77° west, what time is it at New York, which is 74° 3′ west?

8. The difference in time between Halifax, Nova Scotia, and Charleston, S. C., is 1 hr. 5 min. 8 sec. What is their difference in longitude?

9. Pekin, China, is 116° 27′ 30″ east longitude, and Washington is 77° west longitude. When it is noon on January 1st at Washington, what time is it at Pekin?

10. A gentleman traveling found, on arriving at his destination, that his watch, which kept correct time, was 1 hr. 11 min. slow. Which way was he traveling? How far had he traveled?

METRIC SYSTEM

295. The ***Metric System*** of weights and measures has been legalized by the United States, most of the countries of Europe. and several countries of Central and South America.

Although this system is extremely valuable on account of its simplicity, it is not in general use in this country, and hence is not treated as fully here as the other divisions of Denominate Numbers.

296. The ***Unit of Length,*** called the ***Metre*** (*meeter*), from which the system derives its name, is nearly one ten-millionth of a quadrant of the earth's circumference.

297. The ***Unit of Area,*** called the ***Are*** (*air*), is a square whose side is 10 metres. It contains 100 square metres.

298. The ***Unit of Solidity,*** called the ***Stere*** (*stair*), is a cube whose edge is one metre.

299. The ***Unit of Capacity,*** called the ***Litre*** (*leeter*), contains a volume equal to that of a cube whose edge is one-tenth of a metre.

300. The ***Unit of Weight,*** called the ***Gramme,*** is the weight of a cube of distilled water whose edge is one-hundredth of a metre.

It must be weighed in a vacuum and at the period of its greatest density, 39.2 Fahrenheit.

301. From these standard units are derived the multiples and sub-multiples which are named to express units of higher or lower orders in the decimal scale. Thus,

For multiples, Greek numerals are used:

Deka, 10; Hecto, 100; Kilo, 1000; Myria, 10000.

For sub-multiples the Latin ordinals are used:

Deci, 10th; Centi, 100th; Milli, 1000th.

Dekametre	means	10	Metres.
Dekagramme	"	10	Grammes.
Hectometre	"	100	Metres.
Kilolitre.	"	1000	Litres.
Myriagramme	"	10000	Grammes.
Centigramme	"	$\frac{1}{100}$	Gramme.
Milligramme	"	$\frac{1}{1000}$	Gramme.

MEASURES OF EXTENSION.

302. The ***Metre*** is the unit of length.

TABLE.

10 Millimetres	=	1 Centimetre	=	.3937079 in.
10 Centimetres	=	1 Decimetre	=	3.937079 in.
10 Decimetres	=	1 ***Metre***	=	39.37079 in.
10 Metres	=	1 Dekametre	=	32.808992 ft.
10 Dekametres	=	1 Hectometre	=	19.927817 rd.
10 Hectometres	=	1 Kilometre	=	.6213824 mi.
10 Kilometres	=	1 Myriametre	=	6.213824 mi.

303. The ***Are*** is the unit of land measure.

TABLE.

1 Centiare	=	1 Sq. Metre	=	1.196034 sq. yd.
100 Centiares	=	1 ***Are***	=	119.6034 sq. yd.
100 Ares	=	1 Hectare	=	2.47114 acres.

304. The ***Square Metre*** is the unit for measuring ordinary surfaces, as flooring, ceiling, etc.

TABLE.

100 Sq. Millimetres	= 1 Sq. Centimetre	=	.155 + sq. in.
100 Sq. Centimetres	= 1 Sq. Decimetre	=	15.5 + sq. in.
100 Sq. Decimetres	= 1 ***Sq. Metre***	=	1.196 + sq. yd.

305. The ***Stere*** is the unit of wood and solid measure.

TABLE.

	1 Decistere	=	3.531 + cu. ft.
10 Decisteres	= 1 ***Stere***	=	35.316 + cu. ft.
10 Steres	= 1 Dekastere	=	13.079 + cu. yd.

306. The ***Cubic Metre*** is the unit for measuring many ordinary solids; as excavations, embankments, etc.

TABLE.

1000 Cu. Millimetres	= 1 Cu. Centimetre	=	.061 + cu. in.
1000 Cu. Centimetres	= 1 Cu. Decimetre	=	61.026 cu. in.
1000 Cu. Decimetres	= 1 ***Cu. Metre.***	=	35.316 cu. ft.

MEASURES OF CAPACITY.

307. The ***Litre*** is the unit of capacity, both of liquid and dry measure. It contains about a quart, liquid measure.

TABLE.

10 Millilitres	= 1 Centilitre	= .6102 cu. in.	= .338 fluid oz.
10 Centilitres	= 1 Decilitre	= 6.1022 cu. in.	= .845 gills.
10 Decilitres	= 1 ***Litre***	= .908 quart	= 1.0567 qt.
10 Litres	= 1 Dekalitre	= 9.08 quarts	= 2.6417 gal.
10 Dekalitres	= 1 Hectolitre	= 2.8372 + bu.	= 26.417 gal.
10 Hectolitres	= 1 Kilolitre	= 28.372 + bu.	= 264.17 gal.
10 Kilolitres	= 1 Myrialitre	= 283.72 + bu.	= 2641.7 + gal.

MEASURES OF WEIGHT.

308. The ***Gramme*** is the unit of weight.

TABLE.

10 Milligrammes	= 1 Centigramme	=	.15432 + gr.
10 Centigrammes	= 1 Decigramme	=	1.54324 + "
10 Decigrammes	= 1 ***Gramme***	=	15.43248 + "
10 Grammes	= 1 Dekagramme	=	.35273 + oz. Av.
10 Dekagrammes	= 1 Hectogramme	=	3.52739 + " "
10 Hectogrammes	= 1 Kilogramme, or ***Kilo***	=	2.20462 + lb. "
10 Kilogrammes	= 1 Myriagramme	=	22.04621 + " "
10 Myriagrammes, or 100 Kilogrammes	= 1 Quintal	=	220.46212 + " "
10 Quintals, or 1000 Kilogrammes	= 1 Tonneau, or ***Ton***	=	2204.62125 + " "

The ***Kilogramme,*** or ***Kilo,*** is the unit of common weight in trade, and is a little less than $2\frac{1}{5}$ lb. Avoirdupois.

The ***Tonneau*** is used for weighing very heavy articles, and is about $204\frac{1}{2}$ lb. more than a ton.

EXERCISES.

1. What metric unit corresponds most nearly to our yard? How many metres are there in a rod?

2. What metric measure corresponds most nearly to our mile?

3. What unit expresses nearly one ton?

4. What unit is nearly equal to one quart?

5. Reduce 45 dekagrammes to grammes.

6. Express 7 dekametres, 25 centimetres, as metres.

7. How many litres are there in a kilolitre? In a dekalitre?

8. What is the value of an acre in metric units?

9. What metric measure corresponds most nearly to one bushel?

10. Reduce 586.431 metres to decimetres.

11. What will be the cost of 324.16 hectogrammes of sugar at 22 cents per hectogramme?

12. A merchant bought 38 gal. of wine at $2.15 per gal. Did he gain or lose, and how much, by selling it at $5 per dekalitre?

13. Which is cheaper, to buy cloth at $3 per metre, or at $2.90 per yard? How much cheaper?

14. How much carpet a metre in width, is required to carpet a room 4.2 metres long and 3.8 metres wide?

15. How long must a pile of wood be, so that it may contain 12 steres, if it is 3.5 metres wide and 3 metres high?

16. How much is gained by selling a piece of silk 100 metres in length, at $2.25 per yard, if it cost $2 per metre?

17. If a farm contains 1400 ares, what will be its value at $2.50 per are?

18. A barrel of flour contains 196 pounds. Express its weight in metric units.

19. How many hectares are there in a farm that is 1000 metres long and 180 metres broad? What is its value at $250 per hectare?

20. A bin is 5 metres square and 2.5 metres high. How many hectolitres of wheat will it hold?

21. A room is 5.2 metres long, 4.5 metres wide, and 3.2 metres high. What will be the cost of plastering it at 35 cents per square metre?

22. Which is more profitable, and how much per ton, to sell sugar at 11 cents per lb. or 23 cents per kilo?

23. A cask holding 2 hectolitres of molasses was sold at $18\frac{3}{4}$ cents per litre. How much more profitable would it be to sell the molasses at 90 cents per gallon?

2.206) 2.15 × 25 = $51.70
38 ÷ 2.6417 = 14.38 Defaulters
14.38 × 5 = $71.90
81.75 − 71.90 = $9.80 ÷ 38 = .257 ans

PERCENTAGE

309. 1. In a quantity of sugar, 4 lb. of every 100 lb. were wasted. What part of it was wasted?

2. A laborer digs potatoes for 10 bushels out of every 100. What part of the whole does he get?

3. A merchant lost $3 out of every $100 worth of goods sold, on account of bad debts. What part of his sales did he lose?

4. Millers take 1 bushel out of every 10 bushels which they grind for customers, as pay for grinding. How many hundredths do they take?

5. In a company of soldiers, 1 out of every 4 men was killed. How many was that *per hundred*, or *per cent.?*

6. In a school, 5 out of 20 pupils are more than 14 years old. How many is that per hundred? How many per cent.?

7. A man spent $3 out of every $4 earned. How many hundredths of his money did he spend? What per cent.?

8. A man whose income was $2500 annually, saved $\frac{10}{100}$, or 10 per cent. of it. How many dollars did he save?

9. What is $\frac{5}{100}$, or 5 per cent., of $800? 6 per cent. of $500?

10. What is 2 per cent. of $500? 4 per cent. of $900? 3 per cent. of $500?

11. What is 5 per cent. of $800? 6 per cent. of $900?

12. What is 8 per cent. of 500 bushels? 10 per cent. of 1000 pounds?

DEFINITIONS.

310. *Per Cent.* means *by the hundred.*

It is a contraction of the Latin *per centum*, by the hundred.

311. The ***Sign of Per Cent.*** is %. Thus, 8% is read 8 per cent.

312. *Percentage* treats of computations which involve per cent.

313. Since per cent. is a number of hundredths, it is usually expressed as a decimal. It may also be expressed as a common fraction. Thus,

2 per cent. is written 2%, .02, or $\frac{2}{100}$.
5 per cent. is written 5%, .05, or $\frac{5}{100}$.
47 per cent. is written 47%, .47, or $\frac{47}{100}$.
135 per cent. is written 135%, 1.35, or $\frac{135}{100}$.
$12\frac{1}{2}$ per cent. is written $12\frac{1}{2}$%, $.12\frac{1}{2}$, or $\frac{12\frac{1}{2}}{100}$.
$\frac{3}{4}$ per cent. is written $\frac{3}{4}$%, $.00\frac{3}{4}$, or $\frac{\frac{3}{4}}{100}$.
$31\frac{1}{4}$ per cent. is written $31\frac{1}{4}$%, $.31\frac{1}{4}$, or $\frac{31\frac{1}{4}}{100}$.

314. The expressions $.12\frac{1}{2}$, $.31\frac{1}{4}$, etc., may also be written .125, .3125, etc.; and the complex fractional forms $\frac{12\frac{1}{2}}{100}$, $\frac{\frac{3}{4}}{100}$, etc., may be expressed as simple fractions: as, $\frac{25}{200}$, $\frac{3}{400}$, etc.

Express decimally, and in the smallest terms of their equivalent common fractions, the following:

1. 10%.	9. $6\frac{1}{4}$%.	17. $18\frac{3}{4}$%.
2. $12\frac{1}{2}$%.	10. 125%.	18. $20\frac{5}{6}$%.
3. 20%.	11. $1\frac{1}{2}$%.	19. $\frac{3}{4}$%.
4. 25%.	12. $33\frac{1}{3}$%.	20. $\frac{1}{2}$%.
5. 30%.	13. $16\frac{2}{3}$%.	21. $4\frac{1}{2}$%.
6. 75%.	14. $212\frac{1}{2}$%.	22. $\frac{3}{10}$%.
7. $87\frac{1}{2}$%.	15. $31\frac{1}{4}$%.	23. $7\frac{3}{10}$%.
8. $3\frac{1}{8}$%.	16. $66\frac{2}{3}$%.	24. $37\frac{1}{2}$%.

Express in hundredths or per cent., $\frac{1}{2}$ of a number; $\frac{1}{4}$ of it; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$; $\frac{1}{25}$; $\frac{3}{4}$; $\frac{3}{10}$; $\frac{5}{20}$; $\frac{4}{25}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{16}$; $\frac{7}{30}$; $\frac{3}{5}$; $\frac{5}{16}$; $\frac{7}{16}$; $\frac{7}{50}$; $\frac{9}{75}$.

Problems in Percentage involve the following elements:

315. The ***Base*** is the number of which the per cent. is taken.

316. The ***Rate*** is the number of hundredths taken.

317. The ***Percentage*** is the number which is a certain number of hundredths of the base.

318. The ***Amount*** is the sum of the base and percentage.

319. The ***Difference*** is the base less the percentage.

In the formulas, *B.* represents base; *R.*, rate; *P.*, percentage; *A.*, amount; and *D.*, difference.

CASE I.

320. To find the Percentage when the Base and Rate are given.

EXERCISES.

1. What is 10 per cent., or $\frac{10}{100}$, of \$150?
2. What is 5 per cent., or $\frac{5}{100}$, of \$400?
3. What is 20 per cent., or $\frac{20}{100}$, of 300 bu.?
4. What is $12\frac{1}{2}$ per cent., or $\frac{12\frac{1}{2}}{100}$, of 800 gal.?
5. What is 15 per cent., or $\frac{15}{100}$, of \$400?
6. What is $33\frac{1}{3}$ per cent. of \$600?
7. What is $37\frac{1}{2}$ per cent. of 160 men?
8. What is 40 per cent. of 200 tons?
9. What is 4 per cent. of 800 horses?
10. What is 5 per cent. of 700 pupils?
11. What is 25 per cent. of 124 yards?

12. What is $6\frac{1}{4}$ per cent. of \$32.64?

PROCESS.

$\$32.64 \times \frac{1}{16} = \$2.04$

Or,

$\$32.64 \times .06\frac{1}{4} = \$2.04$

FORMULA.

$B \times R = P.$

ANALYSIS.—Since $6\frac{1}{4}\%$ of any number is $\frac{6\frac{1}{4}}{100}$, or $\frac{1}{16}$, of it, $6\frac{1}{4}\%$ of \$32.64 is $\frac{1}{16}$ of \$32.64, which is \$2.04. Or,

Since $6\frac{1}{4}\%$ of any number is $.06\frac{1}{4}$ of that number, $6\frac{1}{4}\%$ of \$32.64 is $.06\frac{1}{4}$ of \$32.64, which is \$2.04.

RULE.—*Multiply the base by the rate.*

13. Find 35% of \$21.75.
14. Find 48% of \$13.42.
15. Find $33\frac{1}{3}\%$ of 465 gal.
16. Find $37\frac{1}{2}\%$ of 816 mi.

17. A farmer who had a flock of 450 sheep, sold $33\frac{1}{3}\%$ of them. How many had he left?

18. A man whose salary was \$2000 per year, spent 85% of it. How much did he save annually?

19. A farmer sold $37\frac{1}{2}\%$ of his crop of 816 bu. of wheat at \$1.56 per bu., and the rest at \$1.60. How much did he realize from the sale of his wheat?

20. If a merchant makes a deduction of 5% from a bill of \$318.57, how much must be paid him?

21. A man bought a farm for \$30000, and sold it for a gain of 25%. How much did he get for it?

22. Mr. Seymour sold \$3000 worth of flour at a loss of $12\frac{1}{2}\%$. How much did he realize from the sale?

23. A man having \$40000, invested 15% in bank stock, 27% of it in bonds and mortgages, and the rest in a flouring mill. How much did the mill cost?

24. Two brothers each inherited \$18500. The elder increased his inheritance 8% per year for 3 years. The younger lost $33\frac{1}{3}\%$ of his in the same time. What was then the value of the inheritance of each?

CASE II.

321. To find the Rate when the Base and Percentage are given.

1. If a man earn \$100 and spend \$50 of it, what part of it does he spend? How many hundredths? How many per cent.?

2. In a piece of cloth containing 36 yards, 9 yards were damaged. What part of it was damaged? How many hundredths of it? How many per cent.?

3. When I spend $\frac{1}{4}$ of my money, how many hundredths of it do I spend? How many per cent.?

4. If a farmer loses $\frac{1}{2}$ of his crop by a flood, how many hundredths of it does he lose? How many per cent.?

5. If a merchant sells $\frac{1}{4}$ of his goods annually, how many hundredths does he sell? How many per cent.?

6. A farmer had 25 sheep, and 10 of them died. What part of his sheep died? What per cent. of them?

7. What part of \$15 are \$3? What per cent.?

8. What part of 12 bushels are 6 bushels? What per cent.?

9. What per cent. of 24 cows are 8 cows? What per cent. are 12 cows?

10. What per cent. of 200 students are 40 students? Are 60 students?

11. What per cent. of 150 acres are 30 acres? Are 50 acres? Are 75 acres?

12. What per cent. of 80 hours are 16 hours? Are 20 hours? Are 40 hours?

13. What per cent. of 90 gallons are 30 gallons? Are 60 gallons? Are 45 gallons?

14. If a man who earns \$60 per month, expends \$40 per month for necessary expenses, what per cent. of his earnings does he save?

15. A merchant having 375 yards of cloth, sold 150 yards of it. What per cent. did he sell?

PROCESS.

150 yd. $= \frac{150}{375}$ of 375 yd., or $\frac{2}{5}$ of 375 yd., or 40% of 375 yd.

Or,

150 yd. $\div$ 375 yd. $=$ .40, or 40%

FORMULA.

$$P \div B = R.$$

ANALYSIS.—150 yards are $\frac{150}{375}$, or $\frac{2}{5}$ of 375 yards. $\frac{2}{5}$ expressed in hundredths, equals 40 hundredths; therefore 150 yards are .40, or 40%, of 375 yards. Or,

Since the percentage is a product of the base by the rate, if we divide the percentage by the base we shall obtain the rate. Therefore we divide 150 by 375, and obtain for a quotient .40, or 40%.

RULE.—*Divide the percentage by the base.*

16. What per cent. of 360 men are 60 men?
17. What per cent. of 840 men are 360 men?
18. What per cent. of 380 pages are 120 pages?
19. What per cent. of 45 hours are 25 hours?
20. What per cent. of 50 yards are 27 yards?
21. What per cent. of 36 pounds are 24 pounds?
22. A farmer who had a farm of 540 acres, sold 210 acres of it. What per cent. of it did he sell?
23. A man whose annual income is $1800, spends $1600 of it. What per cent. of it does he spend? What per cent. of it has he left?
24. A grocer sold tea for $1 that cost him $.75. What per cent. of the cost did he gain?
25. What per cent. of 30000 bushels are 50 bushels?
26. What per cent. of the cost does a hatter gain by selling hats at $7 each, that cost $5.50?
27. A real-estate agent gets $60 for selling my house for $4000. What % of the sale does he receive for his services?

28. I paid \$120 for insuring a boat-load of wheat valued at \$10000. What % of the value of the cargo was received for insuring it?

29. A man who had 1000 acres of land, gave $\frac{1}{4}$ of it to his eldest son, $\frac{1}{5}$ of it to another, and the remainder he divided equally between his 3 daughters. What % of the whole did each receive?

CASE III.

322. To find the Base when the Percentage and Rate are given.

1. A man spent \$15, which was 10% or $\frac{10}{100}$ of all the money he had. How much money had he?

2. My net profit from an investment was \$800, which was 25% or $\frac{25}{100}$ of the amount invested. How much had I invested?

3. Of what sum is 18 dollars $33\frac{1}{3}\%$, or $\frac{33\frac{1}{3}}{100}$, or $\frac{1}{3}$?

4. Of how many days are 15 days 20%? 30 days $37\frac{1}{2}\%$?

5. Of what sum is 25 dollars $62\frac{1}{2}\%$, or $\frac{62\frac{1}{2}}{100}$, or $\frac{5}{8}$?

6. Of what number is 120 6%? 150, 30%? 180, 60%?

7. Of what number is 40 80%? 20, 60%? 30, 150%?

8. A drover lost 450 sheep, which was 75% of his flock. How many sheep had he?

PROCESS.

$\frac{75}{100}$ or $\frac{3}{4} = 450$

$\frac{1}{4} = 150$

Whole $= 600$

Or,

$450 \div .75 = 600$

FORMULA.

$P \div R = B.$

ANALYSIS.—Since 75% or $\frac{75}{100}$ or $\frac{3}{4}$ of the number is 450, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 450 or 150; and since 150 is $\frac{1}{4}$ of the number, the whole number of sheep will be 4 times 150, or 600. Therefore he had 600 sheep. Or,

Since the percentage is the product of the base by the rate, if the percentage is divided by the rate the quotient will be the base.

Therefore we divide 450 by .75.

RULE.—*Divide the percentage by the rate.*

Of what number is			Of what number is		
9.	385	$12\frac{1}{2}\%$?	13.	\$53.25	10%?
10.	245	10%?	14.	27.5 bu.	8%?
11.	125	15%?	15.	168 men	8%?
12.	7.15	$33\frac{1}{3}\%$?	16.	231 oxen	7%?

17. A farmer sold 275 barrels of apples, which were 75% of all he had. How many had he?

18. A man sold 25% of a mill for \$3750. At this rate what was the mill worth?

19. A man who owned 40% of a foundry sold 25% of his share for \$10000. What was the value of the foundry?

20. A farmer after selling 110 A. 43 sq. rd. of land had 90% of his land left. How much land had he at first?

21. A farm cost \$3000. One-third of this sum was $62\frac{1}{2}\%$ of what the house and barn on the farm cost. What was the cost of the house and barn?

22. A man indebted to me paid me \$80, which was $8\frac{1}{2}\%$ of $\frac{1}{2}$ the amount due. How much did he still owe?

23. A merchant sold 4500 bushels of wheat at \$1.60 per bushel. The amount received was 90% of the cost of the wheat. How much did it cost?

24. Mr. A. sold a lot for \$8000, which was only 40% of the amount he paid for it. How much did he pay for it?

25. A man pays \$600 a year rent; 75% of this sum is just $33\frac{1}{3}\%$ of $\frac{1}{2}$ his income. What is his income?

26. A man owning $\frac{1}{3}$ of a vessel sold 25% of his share for \$3350.50. At that rate what was the value of the vessel?

27. The amount paid by insurance companies to the people of St. John, New Brunswick, for losses caused by the great fire in 1877, was about \$7500000, which was $37\frac{1}{2}\%$ of the estimated loss. What was the estimated loss?

28. 25% of $\frac{1}{3}$ of 60 is 75% of $\frac{1}{3}$ of what number?

29. $\frac{2}{3}$ of 40% of 100 is 5% of 10 times $\frac{1}{4}$ of what number?

CASE IV.

323. To find the Base when the Amount and Rate are given.

1. A gentleman increased his collection of horses by an addition of $\frac{1}{4}$ of the number, and then he had 15. How many had he before he made the addition?

2. A coal dealer in selling coal at \$6 a ton received 20% or $\frac{1}{5}$ more than it cost him. What did it cost him?

3. A grocer in selling sugar at \$.11 a pound gains 10% or $\frac{1}{10}$ of the cost. What was the cost?

4. A merchant sold cloth at an advance of 25% on the cost, receiving \$1.25 per yard for it. What was the cost?

5. A man's monthly expenses were $33\frac{1}{3}$% more during 1876 than during 1875. During 1876 they were \$120. What were they in 1875?

6. A certain number increased by 20% of itself is 36. What is the number?

7. After adding to a number $37\frac{1}{2}$% of it, the sum is 33. What is the number?

8. What number increased by 35% of itself equals 540?

PROCESS.

$\frac{135}{100} = 540$

$\frac{1}{100} = 4$

The number $= 400$

Or,

$(1 + .35)$

$540 \div 1.35 = 400$

FORMULA.

$A \div (1 + R) = B.$

ANALYSIS.—Since the number is increased by 35%, or $\frac{35}{100}$ of itself, the amount will be $1\frac{35}{100}$ or $\frac{135}{100}$ times the number; and since $\frac{135}{100}$ of the number $= 540$, $\frac{1}{100}$ of it $= \frac{1}{135}$ of 540, which is 4; and since 4 is $\frac{1}{100}$ of the number, the number will be 100 times 4, which is 400. Or,

A number increased by 35% of itself equals 135% or 1.35 of itself. And since 1.35 times the number equals 540, the number may be found by dividing 540 by 1.35.

RULE.—*Divide the amount by* 1 + *the rate.*

9. What number increased by 27% of itself equals 508?

10. What number increased by $33\frac{1}{3}$% of itself equals 492?

11. What number increased by $16\frac{2}{3}$% of itself equals 329?

12. What number increased by $62\frac{1}{2}$% of itself equals 910?

13. A man owes $15400, which is 10% more than his property is worth. What is the value of his property?

14. A man sold a horse for $345, which was 15% more than it cost him. How much did it cost?

15. A clerk's salary was increased 30%, and now it is $1950. What was it before the increase?

16. A man expended $3750 in repairs upon his house. This sum was 25% more than $\frac{1}{2}$ the cost of the house. How much did it cost?

17. The number of pupils in a certain school during 1876 was 872, which was 9% more than the number in attendance during 1875. What was the attendance during 1875?

CASE V.

324. To find the Base when the Difference and Rate are given.

1. A gentleman sold 25% or $\frac{1}{4}$ of the number of his horses and had 15 left. How many had he?

2. By selling coal at $6 per ton a coal dealer lost 20% or $\frac{1}{5}$ of the cost. What was the cost?

3. A grocer sold sugar at 9c. per pound, and lost 10% or $\frac{1}{10}$ of the cost. What did it cost?

4. A merchant sold cloth at $.75 per yard, thereby losing 25% of the cost. What was the cost?

5. A man's monthly expenses are $33\frac{1}{3}$% less this year than last year. This year they are $120; what were they last year?

6. A certain number diminished by 20% of itself is 36. What is the number?

7. What number diminished by 10% of itself equals 45?

8. After subtracting from a number $37\frac{1}{2}\%$ of it, the remainder is 25. What is the number?

9. What number diminished by 27% of itself equals 401.5?

PROCESS.

$\frac{73}{100} = 401.5$

$\frac{1}{100} = \quad 5.5$

The number $= 550$

Or,

$(1 - .27)$

$401.5 \div .73 = 550$

FORMULA.

$D \div (1 - R) = B.$

ANALYSIS.—Since the number is decreased by 27%, or $\frac{27}{100}$ of itself, the remainder will be $\frac{73}{100}$ of the number, which equals 401.5; $\frac{1}{100}$ of it equals $\frac{1}{73}$ of 401.5, which is 5.5; and since 5.5 is $\frac{1}{100}$ of the number, the number will be 100 times 5.5, which is 550. Or,

A number diminished by 27% of itself, equals 73%, or .73, of itself; and since .73 of the number equals 401.5, the number will be equal to 401.5 ÷ .73, which is 550.

RULE.—*Divide the difference by 1 minus the rate.*

10. What number diminished by 36% of itself equals 336?

11. What number diminished by 40% of itself equals 432?

12. What number diminished by 55% of itself equals 285?

13. What number diminished by 28% of itself equals 307?

14. A clerk, after paying out 75% of his salary, had $450 left. What was his salary?

15. A farmer, after selling 30% of his wheat, found that he had 350 bushels left. How much had he at first?

16. A man sold some land for 30% less than he asked for it, getting $29.24 per acre. What was his asking price?

17. A regiment losing 15% of its men, had 527 left. How many had it at first?

18. A speculator lost 10% of his money during the year 1875 and 10% of the remainder during 1876. He then had $40500 left. How much had he at first?

19. A merchant's profit in 1876 was $10318, which was 23% less than in 1875. What was his profit in 1875?

INTEREST

325. 1. When a sum equal to 5% of the amount of money lent is paid for the use of it for one year, how much will be paid for the use of $100 for 1 year? For 2 years?

2. When the allowance for the use of money is 6% per year, what is the allowance for the use of $100 for 1 year? For 2 years? For 3 years? For $3\frac{1}{2}$ years?

3. When the sum paid for the use of money is 8% per year, what must be paid per year for $50? For $500?

4. When the sum paid for the use of money is 12% yearly, what must be paid for the use of $100 for 1 year?

5. When the allowance for the use of money is 8% per year, what must be paid for the use of $100 for 6 months? For 1 month? For $\frac{1}{2}$ month? For $\frac{1}{4}$ month? For $\frac{1}{5}$ month? For 10 days? For 20 days?

6. When 6% is paid per year for the use of money, how much will $500 amount to in 2 years? In 3 years?

7. When $500 is loaned for $1\frac{1}{2}$ years at 8% per year, what will be the amount?

DEFINITIONS.

326. ***Interest*** is the sum paid for the use of money.

327. The ***Principal*** is the sum for the use of which interest is paid.

328. The ***Amount*** is the sum of the principal and interest.

329. The ***Rate of Interest*** is the annual rate per cent.

330. ***Legal Interest*** is interest according to rate fixed by law.

331. ***Usury*** is interest computed at a higher rate than the law allows.

332. A ***Note,*** or ***Promissory Note,*** is a written promise to pay a sum of money at a given time.

333. PRINCIPLE.—*The interest is equal to the product of the principal, rate, and time expressed as years.*

334. When the rate per cent. is not specified in notes, accounts, etc., the *legal rate* is always understood.

On debts due the United States the rate is 6%.

The following table contains the rates of interest in the United States. The first column gives the legal rate, the second the rate that may be collected *if agreed to in writing.*

NAME OF STATE.	RATE PER CENT.		NAME OF STATE.	RATE PER CENT.	
Alabama	8	......	Mississippi	6	10
Arkansas	6	10	Missouri	6	10
Arizona	10	Any.	Montana	10	Any.
California	7	Any.	New Hampshire	6	......
Connecticut	6	Any.	New Jersey	6	......
Colorado	10	Any.	New Mexico	6	Any.
Dakota	7	18	New York	6	......
Delaware	6	......	North Carolina	6	8
District of Columbia	6	10	Nebraska	7	10
Florida	8	Any.	Nevada	10	Any.
Georgia	7	8	Ohio	6	8
Idaho	10	24	Oregon	10	12
Illinois	6	8	Pennsylvania	6	......
Indiana	6	8	Rhode Island	6	Any.
Indian Territory	......	......	South Carolina	7	Any.
Iowa	6	10	Tennessee	6	......
Kansas	7	12	Texas	8	12
Kentucky	6	......	Utah	10	Any.
Louisiana	5	8	Vermont	6	......
Maine	6	Any.	Virginia	6	......
Maryland	6	......	West Virginia	6	......
Massachusetts	6	Any	Washington Territory	10	Any.
Michigan	7	10	Wisconsin	7	10
Minnesota	7	10	Wyoming	10	Any.

TO COMPUTE INTEREST.

335. 1. What is the interest of \$200 for 1 year at 6%?

2. What is the interest of \$200 for 2 years at 7%?

3. What is the interest of \$300 for 3 years at 5%?

4. What is the interest of \$400 for $1\frac{1}{2}$ years at 8%?

5. What is the interest of \$400 for 3 months at 6%?

6. What is the interest of \$600 for 1 month at 6%?

7. What is the interest of \$600 for 10 days at 6%?

8. What is the interest of \$500 for 15 days at 8%?

9. What will be the amount of \$100 for 2 years at 6%?

10. What will be the amount of \$200 for 3 years at 4%?

11. What will be the amount of \$300 for $2\frac{1}{2}$ years at 6%?

12. What will be the amount of \$150 loaned for $1\frac{1}{2}$ years at 5%?

13. Find the interest of \$234.27 for 2 yr. 7 mo. 12 da. at 6%?

PROCESS.

\$234.27	
.06	
\$14.0562	Int. for 1 yr.
2	
\$28.1124	Int. for 2 yr.
8.6676	Int. for 7 2-5 mo.
\$36.78	Int. 2 yr. 7 mo. 12 da.

Or,

\$234.27	
.06	
12)\$14.0562	Int. for 1 yr.
\$1.1713	Int. for 1 mo.
31.4	
\$36.78	Int. 2 yr. 7 mo. 12 da.

ANALYSIS.—Since the interest for 1 year is 6% of the principal, we find .06 of \$234.27, which is \$14.0562; and since \$14.0562 is the interest for 1 year, the interest for 2 years will be twice that sum, which is \$28.1124. The interest for 1 month is one-twelfth of the interest for 1 year, or \$1.1713; and the interest for 7 mo. and 12 days, or $7\frac{2}{5}$ mo., is $7\frac{2}{5}$ times \$1.1713, or \$8.6676. This added to the interest for 2 years, gives the interest for 2 years, 7 months, 12 days. Or,

We may find the interest for 1 year as before, and then for 1 month. We then multiply the interest for 1 month by the number of months and fractions of a month. Thus, in 2 years, 7 months, there are 31 months, and in 12 days there are $\frac{12}{30}$ or $\frac{4}{10}$ of a month.

Therefore, the entire interest may be found by multiplying \$1.1713 by 31.4, which is \$36.78.

Since there are 30 days in a month, one-third of the number of days will be *tenths* of a month.

RULE.—I. *Find the interest for 1 year and multiply this by the time expressed as years and fractions of a year.* Or,

II. *Find the interest for 1 month and multiply this by the time expressed as months and fractions of a month.*

14. What is the interest of \$25.16 for 1 yr. 6 mo. at 6%?
15. What is the interest of \$36.24 for 2 yr. 4 mo. at 7%?
16. What is the interest of \$48.20 for 2 yr. 4 mo. at 8%?
17. What is the interest of \$2000 for 3 yr. 7 mo. at 9%?
18. Find the amount of \$590.50 for 3 yr. 6 mo. at 7%.
19. Find the amount of \$640.82 for 2 yr. 7 mo. at 8%.
20. Find the amount of \$725.83 for 3 yr. 6 mo. at 10%.
21. Find the amount of \$618.24 for 2 yr. 5 mo. at 8%.
22. Find the amount of \$312.29 for 3 yr. 5 mo. at 6%.
23. Find the interest of \$718.24 for 5 mo. 10 da. at 5%.
24. Find the interest of \$127.46 for 3 mo. 15 da. at 7%.
25. Find the interest of \$364.18 for 2 mo. 12 da. at $8\frac{1}{2}$%.
26. Find the interest of \$318.29 for 9 mo. 10 da. at $7\frac{1}{4}$%.
27. Find the interest of \$312.24 for 2 mo. 20 da. at 8%.
28. Find the interest of \$1614.25 for 20 da. at 7%.
29. Find the interest of \$1318.29 for 24 da. at 10%.
30. Find the interest of \$4684.68 for 11 da. at $12\frac{1}{2}$%.
31. If you lend \$500, how much will be due you in 3 yr. 6 mo. 21 da., interest at 7%?
32. What is the interest on \$784.25 from Aug. 7, 1874, to July 19, 1877, at 8%? What is the amount?
33. How much interest is due on \$500, that has been loaned at interest since Jan. 1, 1876?

OTHER METHODS.

ALIQUOT PARTS.

336. 1. What is the interest and amount of \$520.32 for 2 yr. 5 mo. 15 da. at 7%?

PROCESS.

\$520.32	
.07	
\$36.4224	Int. for 1 yr.
2	
\$72.8448	Int. for 2 yr.
12.1408	Int. for 4 mo.
3.0352	Int. for 1 mo.
1.5176	Int. for 15 da.
\$ 89.5384	Int. for 2 yr. 5 mo. 15 da.
\$520.32	Principal.
\$609.86	Amount.

ANALYSIS.—Since the interest is 7% of the principal, we find .07 of \$520.32, which is \$36.4224, the interest for 1 year. Twice \$36.4224 gives the interest for 2 years, which is \$72.8448. One-third of the interest for 1 year is \$12.1408. the interest for 4 months. One-fourth of the interest for 4 months is \$3.0352, the interest for 1 month. One-half the interest for 1 month is \$1.5176, the interest for 15 days. The sum of these amounts is the interest for 2 years 5 months 15 days, which is \$89.5384. This sum, added to the principal, gives the amount.

Solve the following by aliquot parts:

2. What is the interest of \$324.22 for 3 yr. 4 mo. at 6%?
3. What is the interest of \$218.90 for 2 yr. 7 mo. at 7%?
4. What is the interest of \$36.48 for 2 yr. 5 mo. 15 da. at 6%?
5. What is the interest of \$40.28 for 1 yr. 7 mo. 20 da. at 6%?
6. What is the interest of \$56.24 for 2 yr. 5 mo. 18 da. at 7%?
7. What is the interest of \$24 96 for 3 yr. 1 mo. 6 da. at 8%?

8. What is the interest of \$48.72 for 2 yr. 2 mo. 16 da. at 8%?

9. What is the interest of \$36.18 for 2 yr. 5 mo. 21 da. at 10%?

10. What is the interest of \$20.25 for 3 yr. 1 mo. 16 da. at $12\frac{1}{2}$%?

11. What is the interest of \$30.24 for 2 yr. 8 mo. 15 da. at 8%?

SIX PER CENT. METHOD.

337. The interest on \$1, at 6% per annum,

For 12 months, is	.06
For 2 months, is	.01
For 1 month (30 days), is . . .	.005
For 6 days ($\frac{1}{5}$ month), is . . .	.001
For 1 day, is	$.000\frac{1}{6}$

1. What is the interest of \$125 for 2 yr. 3 mo. 16 da. at 6%?

PROCESS.

\$125
$.137\frac{4}{6}$
\$17.208

ANALYSIS.—Int. of \$1 for 2 yr. at 6% is .12.
Int. of \$1 for 3 mo. at 6% is .015.
Int. of \$1 for 16 da. at 6% is $.002\frac{4}{6}$.
The sum of these, $.137\frac{4}{6}$, is the interest of \$1 for the given time at the given rate, and since the interest of \$1 is $.137\frac{4}{6}$, the interest of \$125 will be 125 times that sum, which is \$17.208.

1. When it is required to find the interest at any other rate than 6%, first find it at 6%, then increase or decrease this result by such a part of it as the given rate is greater or less than 6%. Thus, if the rate is 7%, increase the interest at 6% by $\frac{1}{6}$ of it; if the rate is 5%, decrease it by $\frac{1}{6}$ of it; if the rate is 8%, increase it by $\frac{2}{6}$, or $\frac{1}{3}$ of it; if the rate is 9%, increase it by $\frac{1}{2}$ of it, etc.

2. ***Exact*** or ***Accurate*** interest requires that the year should be considered 365 days, for a common year, and 366 days for a leap year, instead of the ordinary method of considering 12 months of 30 days each, or 360 days a year.

Find the interest on the following:

2. On a note for \$185.26 for 1 yr. 4 mo. 13 da. at 6%?
3. On a note for \$368.18 for 3 yr. 5 mo. 22 da. at 6%?
4. On a note for \$284.25 for 2 yr. 7 mo. 18 da. at 6%?
5. On a note for \$183.17 for 1 yr. 8 mo. 17 da. at 6%?
6. On a note for \$215.25 for 3 yr. 2 mo. 18 da. at 6%?
7. On a note for \$204.37 for 2 yr. 5 mo. 15 da. at 5%?
8. On a note for \$186.15 for 3 yr. 7 mo. 23 da. at 7%?
9. On a note for \$315.30 for 1 yr. 9 mo. 27 da. at 8%?
10. On a note for \$379.15 for 1 yr. 8 mo. 11 da. at 6%?
11. On a note for \$685.31 for 4 yr. 1 mo. 15 da. at 7%?
12. On a note for \$516.28 for 3 yr. 6 mo. 28 da. at 5%?
13. On a note for \$423.15 for 2 yr. 7 mo. 10 da. at 6%?
14. On a note for \$304.27 for 1 yr. 3 mo. 21 da. at 10%?
15. On a note for \$516.24 for 2 yr. 1 mo. 13 da. at 12%?

Find the amount of the following notes when due:

16. *\$150.15.* CINCINNATI, O., Jan. 31, 1877.

Three months after date, for value received, I promise to pay John T. Jones, or order, One Hundred Fifty $\frac{15}{100}$ Dollars, with interest at 6%.

CHARLES C. THOMSON.

17. *\$328.35.* ST. PAUL, MINN., Oct. 1, 1877.

On the 15th day of January, 1878, for value received, I promise to pay to S. E. Hoyt, or order, Three Hundred Twenty-Eight $\frac{35}{100}$ Dollars, with interest at 8%.

J. W. RAY.

18. *\$315.75.* LEAVENWORTH, KAN., May 3, 1876.

For value received, on demand I promise to pay to J. C. Coe, or order, Three Hundred Fifteen $\frac{75}{100}$ Dollars, with interest.

HENRY B. ROBESON.

Paid, June 5th, 1877. How much was due?

COMPOUND INTEREST.

338. ***Compound Interest*** is interest upon the principal and its unpaid interest, combined at regular intervals.

It is usually compounded annually, semi-annually, or quarterly. Unless some other condition is mentioned in the written obligation, the interest is understood to be compounded annually.

WRITTEN EXERCISES.

1. Find the compound interest of \$250 for 2 yr. 3 mo. at 6%.

PROCESS.

\$250	Prin. for 1st yr.
15	Int. for 1st yr.
\$265	Prin. for 2d yr.
15.90	Int. for 2d yr.
\$280.90	Prin. for 3d yr.
4.21	Int. for 3 mo.
\$285.11	Amount for 2 yr. 3 mo.
250.00	First Principal.
\$35.11	Comp. Int. for 2 yr. 3 mo.

ANALYSIS.—Since the interest is compounded annually, we first find the interest of \$250 for 1 yr. We add this interest to the principal, and compute the interest on this amount for another year. We add this interest to the principal as before, and compute interest on this amount for 3 months, which we add to the principal. From this amount we subtract the original principal, and obtain \$35.11, the compound interest required.

RULE.—*Find the interest of the principal for the first period of time at the end of which interest is due.*

Add this interest to the principal, and compute the interest upon this amount for the next period, and so continue.

Subtract the given principal from the last amount, and the remainder will be the compound interest.

1. If the interest is compounded semi-annually, the rate is considered as one-half the annual rate; if quarterly, one-fourth, etc.

2. When the time consists of years, months, and days, find the compound interest for the greatest number of entire periods, and to this add the simple interest upon the amount for the rest of the time.

2. Compute the compound interest on $315 for 2 yr. 6 mo. at 6%.

3. Find the amount of $324.18 for 3 yr. 5 mo. at 7% compound interest.

4. What is the compound interest on $525.75 for 3 yr. 4 mo. at 6%?

Computations in compound interest may be shortened very much by the use of the table on the following page.

5. What is the compound interest of $325.10 for 3 yr. 2 mo. at 6%?

ANALYSIS.—By referring to the table, the amount of $1 for 3 yr. is found to be $1.191016. Computing interest on this sum for the remaining 2 mo., the amount is $1.202926. Since the amount of $1 for the given time is $1.202926, the amount for $325.10 will be 325.10 times that sum. If from this product the principal is subtracted, the remainder is the compound interest.

6. Find the compound interest of $600.50 for 3 yr. 7 mo. at 6%.

7. Find the compound interest of $318.25 for 2 yr. 4 mo. at 7%.

8. Find the compound interest of $412.08 for 3 yr. 2 mo. 10 da. at 6%.

9. Find the compound interest of $310.24 for 2 yr. 5 mo. 15 da. at 8%.

10. What is the difference between the simple interest on $328 for 2 yr. 7 mo. at 7%, and the compound interest on same sum for the same time at 6%?

11. If I deposit $300 in a savings bank which compounds at 6% semi-annually, how much will be due me in 3½ years?

COMPOUND INTEREST TABLE,

Showing the amount of $1, at various rates, compound int. from 1 to 20 years.

Yrs.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yrs.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.269712	1.295029	1.331000	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469328	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.586874	1.677100	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.839420	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.310440	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6.727500	8.062309	9.646291

ANNUAL INTEREST.

339. ***Annual Interest*** is simple interest on the principal and upon any interest overdue, when the contract contains the words, "with annual interest," or, "with interest payable annually."

Annual interest is not considered legal in some States.

1. Find the amount of \$3500 for 4 yr. 6 mo., with interest payable annually at 6%.

PROCESS.

Int. of \$3500 for $4\frac{1}{2}$ yr. = \$945
Int. of \$210 for 8 yr. = \$100.80
Annual Interest = \$1045.80
\$3500 + \$1045.80 = \$4545.80, Amt.

ANALYSIS.—Since annual interest is simple interest on the principal and upon any over-due interest, we first find the interest upon the principal, which is \$945, and then upon the interest due. The interest for each year is \$210. The interest for the first year remained unpaid for $3\frac{1}{2}$ years; that for the second year, $2\frac{1}{2}$ years; that for the third year, $1\frac{1}{2}$ years; and that for the fourth year, for $\frac{1}{2}$ year; therefore the annual interest, \$210, drew interest for $3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2}$ years, or, 8 years, which is \$100.80. This sum, added to the simple interest, \$945 = \$1045.80, the annual interest. This sum added to the principal, \$3500 = \$4545.80, the amount due.

RULE.—*Compute the interest on the principal for the entire time, and on each year's interest from the time it was due up to the end of the period.*

The sum of these interests will be the annual interest.

2. How much is due upon a note of \$350 which has run 4 years, interest at 8%, payable annually?

3. How much was due April 15, 1877, on a note for \$750, dated Jan. 1, 1873, with interest at 6%, payable annually?

PARTIAL PAYMENTS.

340. A ***Partial Payment*** is a payment in part of a note or other obligation.

341. An ***Indorsement*** is the statement of the amount of a payment and the time when it was made. It is written on the back of the note or other written obligation.

342. Business men often settle notes and accounts running for *one year or less* by what is known as the ***Mercantile Rule.***

MERCANTILE RULE.—*Find the amount of the principal at the time of settlement.*

Find the amount of each payment from the time it was made until the time of settlement, and from the amount of the principal subtract the amounts of the payments.

1. A note for $850, on demand with interest at 7% dated Jan. 1, 1876, was indorsed as follows: April 10, 1876, $200; Sept. 15, 1876, $255. How much was due Nov. 15, 1876?

2. What is the balance due at the end of a year on a note for $1800, dated May 15, 1875, on which the following payments had been made: Sept. 20, 1875, $300; Jan. 18, 1876, $200; April 20, 1876, $1000; when the rate is 7%?

3. *$585.25.* CHARLESTON, S. C., March 3, 1876.

Eight months after date, for value received I promise to pay to the order of E. S. Farran, Five Hundred Eighty-five $\frac{25}{100}$ Dollars, with interest at 7%.

H. S. LAUPHIER.

This note was indorsed as follows: June 8, 1876, $325; Aug. 4, 1876, $84.30; Sept. 2, 1876, $100. What was due on the note at maturity?

343. Most of the States have adopted the ***United States Rule*** for computing the amount due upon any obligation where partial payments are made, based upon the following principle.

PRINCIPLE.—*The indebtedness should be computed whenever a payment is made, but the principal must not be increased by the addition of interest.*

WRITTEN EXERCISES.

1. A note was given, Jan. 1, 1870, for $700. The following payments were indorsed upon it: May 6, 1870, $85; July 1, 1871, $40; Aug. 20, 1871, $100; Jan. 10, 1873, $350. How much was due Sept. 30, 1874, interest at 6%?

PROCESS.

Principal		$700.00
Int. to May 6, 1870,—4 mo. 5 da.		14.58
Amount		714.58
First payment		85.00
New Principal		629.58
Int. from May 6, 1870, to July 1, 1871,—1 yr. 1 mo. 25 da.		43.55
Second payment, less than interest due	$40.00	
Int. on $629.58 from July 1, 1871, to Aug. 20, 1871,—1 mo. 19 da.		5.14
Amount		678.27
Third payment to be added to second	$100	140.00
New Principal		538.27
Int. from Aug. 20, 1871, to Jan. 10, 1873,—1 yr. 4 mo. 20 da.		44.85
Amount		583.12
Fourth payment		350.00
New Principal		233.12
Int. from Jan. 10, 1873, to Sept. 30, 1874,—1 yr. 8 mo. 20 da.		24.08
Amount due, Sept. 30, 1874		$257.20

UNITED STATES RULE.—*Find the amount of the principal to a time when a payment, or the sum of the payments, equals or exceeds the interest due, and from this amount subtract such payment or payments. With the remainder as a new principal, proceed as before.*

2. A note for \$2500, dated July 10, 1871, bore the following indorsements: Sept. 15, 1871, \$150; Nov. 12, 1871, \$300; Dec. 1, 1871, \$100; April 3, 1872, \$325; May 15, 1872, \$275; Sept. 20, 1872, \$1000. How much was due Jan. 1, 1873, the rate of interest being 6%?

3. How much was due at maturity on a note for \$2150, dated Sept. 20, 1873, to run 2 years 6 months, on which the following payments were indorsed: Dec. 15, 1873, \$75; Feb. 4, 1874, \$200; April 3, 1874, \$150; July 1, 1874, \$500; Dec. 16, 1874, \$1000, the rate of interest being 8%?

4. A note for \$6725, dated Feb. 10, 1875, had the following indorsements: May 5, 1875, \$275; Aug. 15, 1875, \$50; Nov. 12, 1875, \$1000; Jan. 3, 1876, \$184.25; Sept. 13, 1876, \$84.10; Dec. 23, 1876, \$1000. How much was due Feb. 10, 1877, interest at 6%?

5. A bond was given April 4, 1870, for \$5825, with interest at 8%. The following payments were indorsed upon it: May 15, 1871, \$728.50; April 8, 1872, \$1000; Dec. 12, 1872, \$125; July 9, 1873, \$980; June 12, 1874, \$1000. How much remained due upon the bond April 4, 1875?

6. Sept. 25, 1872, James Hanna gave his note for \$895.75 with interest at 10%. He paid on it as follows: Jan. 10, 1873, \$25; Oct. 12, 1873, \$200; Jan. 18, 1874, \$75; March 25, 1874, \$187.50; Jan. 1, 1875, \$375. How much was due when he paid the note, Nov. 15, 1875?

7. Required, the balance due on a note dated Jan. 1, 1875, for \$580 at 5%, to run 2 years, on which a payment of \$85 was made every 3 months.

8. A note for \$10000, with interest, dated Milwaukee, Wis., Dec. 12, 1875, was indorsed as follows: Feb. 23, 1876, \$750; July 17, 1876, \$108.25; Nov. 23, 1876, \$3000; Jan. 18, 1877, \$4000. How much was due May 12, 1877, interest at 8%?

9. Required, the balance due July 8, 1876, on a note for \$3124.75, at 8% interest, dated Feb. 15, 1874, on which a payment was made Dec. 23, 1874, of \$985; another of \$875.35, Feb. 15, 1875; another of \$1025, Feb. 20, 1876.

10. *\$1885.75.* DETROIT, MICH., Feb. 10, 1874.

Two years after date, for value received I promise to pay W. W. Heilbronner or order, One Thousand Eight Hundred Eighty-five $\frac{75}{100}$ Dollars, with interest. C. H. VIDRARD.

On this note were the following indorsements: June 30, 1874, \$50; Nov. 8, 1874, \$100; Feb. 5, 1875, \$125; April 17, 1875, \$500; Dec. 1, 1875, \$500. How much remained unpaid Mar. 1, 1876?

PROBLEMS IN INTEREST.

PROBLEM I.

344. Principal, rate, and time given, to find the interest.

This has already been solved. (See page 220.)

RULE.—*Multiply the interest of* \$1, *for the given rate and time, by the principal.*

PROBLEM II.

345. Principal, rate, and interest given, to find the time.

1. How much is the interest of \$100 for a year at 6%? For 2 years? For 3 years?

2. When \$100, loaned at 6%, brings an income of \$12, for how long a time was it loaned? How long when the interest was \$18? \$24? \$3? \$4? \$2? \$1.50?

3. When \$50, loaned at 10%, brings an income of \$10, for how long a time was it loaned?

RULE.—*Divide the given interest by the interest of the principal for one year.*

In what time will

4. \$250 produce \$30 interest at 6%?
5. \$600 produce \$24 interest at 8%?
6. \$115 produce \$13.80 interest at 6%?
7. \$12.60 produce \$4.15 interest at 7%?
8. \$35.25 produce \$13.25 interest at 7%?
9. \$25 produce \$25 interest at 6%?
10. \$150 double itself at 8%?
11. Any sum double itself at 5%? 6%? 7%?
12. Any sum triple itself at 5%? 6%? 7%?

PROBLEM III.

346. Principal, time, and interest given, to find the rate.

1. What is the interest of \$100 for 1 year at 1%? At 2%? At 3%?

2. When the interest of \$100 for 1 year was \$8, what was the rate?

3. When the interest of \$100 for 2 years was \$14, what was the rate?

4. When the interest of \$50 for 3 years was \$15, what was the rate?

RULE.—*Divide the given interest, by the interest of the principal for the given time, at 1 per cent.*

What is the rate per cent. when the interest

5. Of $125 for 2 years is $15?
6. Of $250 for 6 months is $8.75?
7. Of $415 for 2 years 6 months is $56.025?
8. Of $317 for 1 year 5 months is $31.44?
9. Of $215 for 2 years 7 months 10 days is $39.30?
10. Of $325.18 for 5 months 26 days is $11.13?
11. Of $30.18 for 2 months 3 days is $ .32?
12. Of $24.36 for 3 months 3 days is $ .44?
13. Of $25.40 for 1 month 15 days is $ .397?

PROBLEM IV.

347. Rate, time, and interest given, to find the principal.

1. At 6%, what sum will produce $6 yearly?

2. At 6%, what sum will produce $12 yearly? What $18 yearly? What $12 in two years? What $18 in 2 years?

RULE.—*Divide the given interest by the interest of* $1, *for the given time at the given rate.*

What sum of money will produce

3. $36.60 interest in 2 years at 6%?
4. $35.70 interest in 2 years 6 months at 8%?
5. $51.20 interest in 5 years 6 months at 5%?
6. $50.84 interest in 6 months 27 days at 6%?
7. $39.18 interest in 5 months 18 days at 6%?
8. $41.25 interest in 3 months 15 days at 9%?
9. $87.50 interest in 1 month 12 days at 7%?
10. $68.75 interest in 3 months 10 days at 6%?
11. $50.83 interest in 2 years 3 months at 6%?
12. $81.25 interest in 3 years 5 months at 8%?

NOTES.

348. A ***Note,*** or ***Promissory Note,*** is a written promise to pay a sum of money at a given time.

349. The ***Maker*** or ***Drawer*** is the person who signs the note.

350. The ***Payee*** is the person to whom it is made payable.

351. The ***Holder*** is the person who has the note in his possession.

352. The ***Indorser*** is the person who writes his name upon the back of the note to transfer it or guarantee its payment.

The payee may indorse by writing his name on the back of a note. It is then payable to the bearer. He may also indorse by writing, "Pay to A—— B——, or order." It then requires the signature of A—— B—— before it is payable.

353. The ***Face*** of a note is the sum named in it.

354. A ***Negotiable Note*** is a note payable to the order of the payee or bearer.

355. A ***Non-negotiable Note*** is a note payable to the ***payee only.***

Notes should contain: the date, the time when due, the amount of the note written in words, the words "for value received," and "with interest," if such is the contract.

356. ***Three Days of Grace*** are usually allowed after the specified time, before the note is said to *mature* or *be due*, except on notes payable *on demand*, and where the words "without grace" are written.

If, when a note is unpaid at its maturity, the holder fails to notify the indorsers of the fact, they are released from responsibility regarding its payment.

357. Notes without interest draw interest at the legal rate after they become due, but a note does not draw interest until after it is due, unless the words "with interest," or "with use," are written in it.

FORMS OF NEGOTIABLE NOTES.

$729.18. CINCINNATI, O., Oct. 5, 1876.

For value received, two months after date I promise to pay James J. Cone, or order, Seven Hundred Twenty-Nine $\frac{18}{100}$ Dollars, with interest.

H. B. BUCKHAM.

$600. DETROIT, Jan. 29, 1877.

For value received, three months after date I promise to pay H. G. Burlingame, or bearer, Six Hundred Dollars.

W. H. SARGENT.

WRITTEN EXERCISES.

1. Write a negotiable note for $500.25, making yourself the payee, and James J. Rogers the maker. Interest at the legal rate.

2. Write a non-negotiable note for $315.17, making W. R. Howard the payee, payable on demand without interest.

3. Write two forms of negotiable notes for $3184.25, due in three months to James P. Hermann, with interest.

4. Indorse them properly for transferring; one to bearer, and the other to H. H. Hurd, or order.

5. Write a note from the following data: Face, $5000; negotiable; maker, P. G. Sloane; payee, J. S. Orton; payable on demand; rate of interest, the legal rate.

COMMERCIAL DISCOUNT.

358. ***Discount*** is an allowance, or deduction, made from a sum of money to be paid.

Problems in discount may, or may not, have reference to time.

359. ***Commercial Discount*** is a deduction from the *price* of an article, or from a *bill*, without regard to time.

360. The ***Net Price*** is the selling price, minus the discount.

361. The ***Cash Value*** of a bill, is its face less the discount.

EXERCISES.

362. 1. If gloves marked at \$1.50 per pair, are sold at 10% discount, what is the discount? What is the net price?

2. If flour is offered at \$7.50 per barrel, with a discount of 5% for cash, what is the discount?

3. Find the value of a bill of goods, amounting to \$845 at 5% discount.

4. Find the value of a bill of goods amounting to \$680, when a discount of $2\frac{1}{2}$% is allowed for cash.

5. What is the cash value of a bill of goods, amounting to \$3215.45 at 20% discount, and 5% off for cash?

In problems like No. 5, it is always understood that the per cent. *off for cash* is to be reckoned upon the price after the previous discounts have been allowed.

6. What is the cash value of a bill amounting to \$3750 at 10% discount, and $2\frac{1}{2}$% off for cash?

7. What is the cash value of a bill of goods amounting to \$2157.25 at 15% discount, and 3% off for cash?

TRUE DISCOUNT.

363. 1. What will be the amount of $100 in 1 year, interest at 6%? In 2 years? In 3 years? In 4 years?

2. What is the value *now* of $106, to be paid in 1 yr., when money loans at 6%? Of $112, to be paid in 2 yr.?

3. What is the value *now* of $212, to be paid in 1 yr., money loaning at 6%? Of $224, to be paid in 2 yr.?

4. What is the worth *now* of a debt of $535, to be paid in 1 yr., when money can be loaned at 7%?

5. What is the present value of $672, due in $1\frac{1}{2}$ yr., when money is loaning at 8%? Of $316, due in 2 yr., money being worth 8%?

364. ***True Discount*** is a deduction made for the payment of a sum of money before it is due.

365. The ***Present Worth*** of a sum of money due at some future time, is a sum which, put at interest at the specified rate, will amount to the debt when it becomes due.

WRITTEN EXERCISES.

366. 1. What sum of ready money is equivalent to $784.25, payable in 2 years, when money is worth 7%?

PROCESS.

$1.14 = Amount of $1 for 2 yr.
$784.25 ÷ 1.14 = $687.94.
$687.94 = Present Worth.
$784.25 — $687.94 = $96.31 Discount.

ANALYSIS.—Since every dollar if put at interest now at 7% would amount to $1.14 in 2 years, it will require as many dollars now, to amount to $784.25 in 2 years, as $1.14 is contained times in $784.25, which is 687.94 times. Therefore, the present worth is $687.94.

The face of the debt, $784.25 — $687.94, leaves $96.31, the discount.

RULE.—*Divide the amount due by the amount of $1, for the given time and rate, and the quotient will be the present worth.*

Subtract the present worth from the amount due, and the remainder will be the true discount.

What are the present worth and discount of the following:

2. $975.50, payable in $1\frac{1}{2}$ years, when money is worth 6%?
3. $845.20, payable in $1\frac{1}{4}$ years, when money is worth 7%?
4. $958.75, payable in 3 years, when money is worth 7%?
5. $576.25, payable in 3 years, when money is worth 8%?
6. $8575, payable in 2 yr. 3 mo. 10 da., when money is worth 5%?
7. $4274, payable in 1 yr. 4 mo. 15 da., when money is worth 6%?
8. $2845, payable in 3 yr. 6 mo. 15 da., when money is worth 7%?
9. $1752.75, payable in 1 yr. 3 mo. 20 da., when money is worth 8%?
10. $5493.50, payable in 2 yr. 5 mo. 25 da., when money is worth 9%?
11. $3457.84, payable in 7 mo. 10 da., when money is worth $7\frac{1}{2}$%?

12. Bought a farm for $14500 cash, and sold it for $16000, payable one-half cash and the remainder in one year. To what amount of cash in hand was the selling price equal when money was loaning at 6%?

13. A merchant bought a bill of goods amounting to $5275, on 3 mo. credit, but was offered $2\frac{1}{2}$% discount for cash. How much would he gain by paying cash, money being worth 8%?

14. A merchant holds two notes, one for $187.25, due Feb. 15th, 1877, and the other for $382.75, due April 1st, 1877. What would be due him in cash, on both notes, Jan. 1st, 1877, money being worth 8%?

15. If I pay a debt of $4725 1 yr. 3 mo. 15 da. before it is due, what discount should be made from the face of the debt, money being worth 8%?

16. Bought 850 barrels of pork at $21.50 per barrel, on 3 mo. credit, and sold it the same day for $21.50 per barrel, cash. How much did I gain, money being worth 6½%?

17. How much will I gain if, instead of paying $5400, cash, for a piece of property, I pay $6000 in 1 yr. 4 mo., money being worth 9%?

18. How much should be discounted on a bill of $3725.87, due in 8 mo. 10 da., if it is paid immediately, money being worth 10%?

19. Bought 250 barrels of granulated sugar, weighing 282 lb. each, at 10⅝ cents per lb., on 1 mo. credit. How much cash would settle the account, money being worth 8%?

20. A crockery dealer bought $3725 worth of goods, for cash, at a discount of 37½% from the regular list prices, and sold them at the regular list prices on 4 months' time. How much did he gain, money being worth 8%?

BANK DISCOUNT.

367. A ***Bank*** is an institution established by law to receive money for safe-keeping, to loan money, or to issue notes or bills to circulate as money.

368. A ***Check*** is a written order upon a bank for money deposited with it.

369. ***Bank Discount*** is the simple interest on the amount due, paid in advance, for 3 days more than the specified time.

370. The ***Proceeds,*** or ***Avails,*** of a note are the face of the note, less the discount.

371. The ***Maturity*** of a note occurs on the last day of grace.

If the last day of grace falls on Sunday or a legal holiday the note matures one day earlier.

372. The ***Term of Discount*** is the number of days from the time of discounting to the maturity of the note.

1. Banks usually discount for short terms, not exceeding 3 mo. or 4 months.
2. Unless otherwise specified, 30 days are a month; 65 days, 2 mo. 5 da., etc.
3. Bank discount is undoubtedly wrong in principle, but has been sanctioned by custom.

CASE I.

373. To find the Bank Discount and Proceeds.

1. What is the bank discount of $385 for 3 mo. at 6%?

PROCESS.

$385, Face of Note.
.0155, Rate of Discount.
$5.9675, Discount.

$385 — $5.9675 = $379.0325, Proceeds.

ANALYSIS. — Since banks, in discounting, require the simple interest in advance, we find the interest of $385 for 3 mo. 3 da. at 6%, which is $5.9675. And since the present worth is the face of the debt minus the discount, we subtract $5.9675 from $385, which gives $379.0325, the proceeds or avails.

RULE.—*To find the bank discount: Compute the interest on the face of the note for 3 days more than the specified time at the given rate.*

To find the proceeds: Subtract the bank discount from the face of the note.

If the note bears interest, find the discount on the *amount* of the note at its maturity. This *amount*, less the discount, will be the proceeds.

Find the bank discount of:

2. \$ 318.25 for 2 mo. 15 da. at 6%.
3. \$3846.18 for 1 mo. 17 da. at 7%.
4. \$2184.39 for 3 mo. 10 da. at 7%.
5. \$ 824.17 for 3 mo. 28 da. at 8%.
6. \$4484.15 for 3 mo. 18 da. at 7%.
7. \$7318.69 for 2 mo. 15 da. at 6%.
8. \$8984.15 for 30 da. at 6%.
9. \$ 765.30 for 65 da. at 6%.

10. *\$375.* CHICAGO, ILL., Dec. 20, 1876.

Two months after date, for value received I promise to pay E. D. Bronson, or order, Three Hundred Seventy-Five Dollars, with interest at 10%, at the First National Bank, Chicago, Ill.

S. HOWARD BLACKWELL.

This note was discounted Jan. 23, 1877, at 10%. What was the bank discount? What were the proceeds?

11. What is the bank discount on a note for \$890.25 at 2 months, the rate being 7%? What are the proceeds?

12. What is the difference between the bank discount and the true discount, on a note for 3 months for \$3725.85, the rate being 8%?

13. *\$15,725.95.* ST. PAUL, MINN., May 15, 1876.

For value received, two months after date "The North River Sugar Refining Co." promises to pay Messrs. Smith & Haughton, or order, Fifteen Thousand Seven Hundred Twenty-five $\frac{95}{100}$ Dollars, at The Merchants National Bank, New Orleans, La.

NORTH RIVER SUGAR REFINING CO.,
BY C. MOLLER, *Sec'y.*

The above note was discounted May 25, 1876, at 6%. What was the discount? What were the proceeds?

14. Mr. A. sold goods amounting to \$3782.75, and received in payment a note at 3 mo., without interest, which he had discounted at a bank after holding it 1 mo. How much did he realize from the sale, the rate of discount being 8%?

15. A merchant sold 32 yards of silk velvet at \$8.75 per yard; 48 yards of silk at \$2.80 per yard; 25 yards of English broadcloth at \$6.25 per yard; receiving in payment a bank note payable in 1 mo. 15 da., without interest, which he had discounted on the same day at 9%. How much did he realize in cash from the sale?

CASE II.

374. To find the face of a note when the proceeds, time, and rate are given.

1. What is the bank discount of \$1 for 2 mo. 27 da. at 6%? What the proceeds?

2. Since \$.985 is the proceeds of \$1 when discounted at 2 mo. 27 da. at 6%, of what sum is \$1.97 the proceeds for the same time and rate? Of what sum is \$2.955 the proceeds for the same time and rate?

3. Of what sum is \$394 the proceeds, when discounted at a bank for 1 mo. 27 da. at 6%? Of what sum is \$197 the proceeds?

4. Of what sum is \$396 the proceeds, when discounted at a bank for 1 mo. 27 da. at 6%?

5. The proceeds of a note discounted at a bank for 2 mo. 12 da. at 7% were \$1182.50. What was the face of it?

PROCESS.

$\$1 - \$.0145\frac{5}{6} = \$.9854\frac{1}{6}$ The proceeds of \$1.

$\$1182.50 \div .9854\frac{1}{6} = \$1200$ The face of the note.

ANALYSIS.—Since the proceeds of \$1 are $\$.9854\frac{1}{6}$, \$1182.50 are the proceeds of as many dollars as $\$.9854\frac{1}{6}$ is contained times in \$1182.50, which is 1200 times. Therefore the face of the note was \$1200.

RULE.—*Divide the proceeds by the proceeds of $1, at the given rate for 3 days more than the specified time.*

6. The proceeds of a note discounted at a bank for 2 mo. at 6% are $989.50. What was its face?

7. What must be the face of a note so that when discounted at a bank for 3 mo. at 6%, the proceeds will be $1969?

8. A merchant wishes to use $975, which he can secure by giving a bank note payable in 2 mo., to be discounted at 7%. For what sum must the note be written?

9. For what sum must a note be drawn payable in 3 mo., so that when discounted at 6% the proceeds may be $1000?

10. A man owes a debt of $1375.38, which he can meet by giving his note payable in 3 mo., discounted at 9%. For what sum must the note be written so that the avails may be just large enough to discharge the debt?

11. For what sum must a note be drawn, payable in 15 days, so that when discounted at 8% the proceeds may be $1257.25?

12. A speculator wishes to raise $5250 on an indorsed note, payable in 1 mo. 15 da., discounted at 7%. For what amount must he draw the note so that the proceeds may be exactly that sum?

13. For what amount must a note be made payable in 3 mo., and discounted at 12½%, so that the proceeds may be $1875?

14. A merchant wishes to raise $500 at a bank by a note for 2 mo. For what sum must the note be drawn that he may receive $500 in cash from the banker after paying the discount at 8%?

15. For how large a sum must a note be drawn, payable in 3 mo., that the net proceeds may be $15000 after deducting the bank discount at 8%?

REVIEW EXERCISES.

375. 1. What sum must be invested at 8% to yield an annual income of $1400?

2. A merchant bought a bill of goods amounting to $7825 on 3 mo. credit, but was offered a discount for cash of 4%. What was the difference in the offers, money being worth 9%?

3. A man bought a farm of 135 A. 25 sq. rd. for $62.50 per acre. He paid one-third of the purchase money in cash; one-half the remainder in 6 mo., and the balance in 1 yr. 3 mo. Money being worth 6%, what would have been the cash price of the farm?

4. A will contained a bequest of $4500 to a charity hospital, to be paid in 1 yr. 3 mo. Money being worth 7%, what was the cash value of the bequest?

5. What is the amount of $3752 for 3 yr. 2 mo. 15 da., with compound interest at 7%?

6. $175. CINCINNATI, O., March 25, 1872.

For value received, on demand I promise to pay J. H. Sheppard, or bearer, One Hundred Seventy-five Dollars, with interest at 8%.

DAVID B. ASPELL.

This note was paid April 15, 1876. How much was due upon it?

7. What is the difference between the true discount and the bank discount of $5728 for 3 mo. at 8%?

8. On a note of $3729.75, dated Feb. 20, 1872, bearing 6% interest, were the following indorsements; July 15, 1872, $525; Dec. 15, 1872, $478; Feb. 20, 1873, $25; May 17, 1873, $75; Sept. 28, 1873, $1000. What was due Jan. 15, 1874?

9. A coal dealer bought 8790 T. of coal at $3.75 per T. He sold 15% of it at a gain of 10% on the cost, 40% of it at a gain of 5%, and the rest at a gain of 8%. He paid $3\frac{1}{2}$% of the cost for transportation. How much did he gain by the speculation?

10. A grain speculator bought 10000 bushels of barley, at 85 cents per bushel cash. He sold it the same day at an advance of 4%, receiving in payment a note due in 1 mo., which he had discounted at a bank at 9%. What was his gain in cash?

11. A man bought a house, paying $62\frac{1}{2}$% of the price in cash, and the rest in notes to the amount of $3300. What was the cost of the house?

12. A merchant sold 15% of his stock of dry goods the first month, 10% the second month and 25% of the remainder the third month, when he took an inventory of his stock on hand, and found that he had remaining $5300 worth of goods. What was the original value of the stock?

13. What number is that to which, if $\frac{3}{8}$ of 25% of $\frac{1}{5}$ of 480 be added, the sum will equal 25% of $\frac{8}{9}$ of 50% of 324?

14. A speculator bought 1000 bbl. of flour at a given price per bbl. paying $\frac{2}{3}$ of its value in cash and giving a bank-note for 2 mo. for the balance, which was discounted on the day it was given at 6%. The discount on the note was $31.50. How much did he pay for the flour per bbl.?

15. What is the difference between the simple and the compound interest of $4725.50 for 2 yr. 2 mo. 15 da. at 6%.

16. What is the difference between the amount of $3240 for 5 yr. 3 mo. 10 da. at 7% simple interest, and the amount of the same sum for the same time and rate, with interest payable annually?

17. A gentleman invested $\frac{1}{5}$ of his annual income in mortgages, paying 6% annual interest. In 6 mo. 12 da. his interest from them was $640. What was his annual income?

PROFIT AND LOSS.

376. 1. When 25% is gained what part is gained?

2. I sold a coat which cost me $12 for 25% more than it cost. How much did I gain? How much did I get for it?

3. Paid $15 for a ton of hay, and sold it at a loss of 20%. How much did I lose? How much did I get for the hay?

4. If I sell land that cost me $50 an acre, at an advance of 10%, how much do I get per acre?

5. If I paid $50 per acre for my land and sell it at $5 per acre less than I paid, what part of the cost do I lose? What % ?

6. If I sell flour for $8 per bbl. that cost me $6, what part of the cost do I gain? What per cent.?

7. Sold boots at $5 a pair that cost $4, what part of the cost was gained? What per cent.?

8. By selling flour at a profit of $2 per bbl., 20%, or $\frac{1}{5}$, of the cost was gained. What was the cost?

9. Sold wheat at a profit of $ .10 per bu. which was 5% of its cost. What was its cost?

10. If I sell a cow that cost me $50, at an advance of 20% on the cost, how much will my profit be? How much do I get for her?

11. By selling flour at $10 per bbl., a profit of 25% was made. What did it cost?

Analysis.—Since 25% or $\frac{1}{4}$ of the cost was gained, the selling price must have been $\frac{5}{4}$ of the cost; and since $\frac{5}{4}$ of the cost was $10, $\frac{1}{4}$ of the cost is $\frac{1}{5}$ of $10, which is $2; and since $\frac{1}{4}$ of the cost is $2, the cost is 4 times $2 or $8.

12. By selling dress goods at 66 cents per yd., a profit of 10% was made. What was the cost?

13. By selling tea at $ .80 a pound, a loss of 20% was incurred. What was the cost?

14. If I get \$60 for a cow, and thereby gain 20%, or $\frac{1}{5}$ of the cost, what did she cost me?

15. I bought a horse for \$150 and sold him at an advance of 20%. What did I get for him?

DEFINITIONS.

377. ***Profit*** and ***Loss*** are terms used to denote the gain or loss in business.

378. The processes in Profit and Loss involve the same elements as do the fundamental problems in Percentage. The corresponding terms are:

1. The *Cost*, or *Sum Invested*, is the ***Base.***
2. The *Rate Per Cent.* of profit or loss is the ***Rate.***
3. The *Gain* or *Loss* is the ***Percentage.***
4. The *Selling Price*, when more than the cost, is the ***Amount.***
5. The *Selling Price*, when less than the cost, is the ***Difference.***

PRINCIPLE.—*The gain or loss is reckoned at a certain rate per cent. on the cost or sum invested.*

WRITTEN EXERCISES.

379. 1. A paid \$3500 for a house, and sold it at 10% advance. How much did he gain? How much did he get for it?

PROCESS.

$\$3500 \times .10 = \$\ 350$, Gain.

$\$3500 + \$350 = \$3850$, Selling price.

Cost $\times$ Rate $=$ Gain.

ANALYSIS.—Since the house was sold at an advance of 10% on the cost, his gain was $\frac{10}{100}$ or $\frac{1}{10}$ of \$3500, which is \$350; and the selling price is equal to the sum of the cost and gain, or \$3850.

RULES.—*Since the same elements are involved as in the fundamental problems in Percentage the rules are the same.*

FORMULAS.

1. Gain or loss = Cost × Rate.
2. Rate = Gain or loss ÷ Cost.
3. Cost = Gain or loss ÷ Rate.
4. Cost = Selling price ÷ (1 + Rate).
5. Cost = Selling price ÷ (1 — Rate).

2. Mr. A. bought a piano for $275, and sold it at an advance of 25%. How much did he receive for it?

3. A bookseller bought $3584 worth of books, and sold them at a gain of 10%. How much did he gain?

4. A carriage maker sold a carriage at 25% advance on the cost. It cost him $318.25. How much did he get for it?

5. A harness maker sold a set of double harness at a profit of 15%. It cost him $45. What did he get for it?

6. A manufacturer of tools sold 5 dozen axes at a profit of 12%. They cost him $9 per dozen. What was his profit?

7. A merchant sold a bill of goods at a profit of 15%. The goods cost him $84.25. What did he receive for them?

8. A speculator bought 50000 pounds of sole leather, which he sold at a profit of 8%. If it cost him $6000, what did he get for it?

9. A man sold his house at a profit of 15%. If he paid $3000 for it, how much did he get for it?

10. A drover sold a flock of sheep at a profit of 7%. If they cost him $1500, what did he get for them?

11. A poultry dealer bought a quantity of poultry, which he sold at a gain of 9%. He paid $250 for it. How much did he get for it?

12. Mr. A bought cloth at $2.15 per yard. At what price must he sell it to gain 30%?

13. A farm which cost \$65 per acre was sold at a gain of 15%. For how much did it sell per acre?

14. A merchant bought 3950 yards of cotton at $9\frac{1}{2}$ cents a yard. How much will he get for it if he sells it at a gain of $12\frac{1}{2}$%?

15. A merchant desires to mark goods that cost him \$3.60 per yard so that he may gain $33\frac{1}{3}$%. At what price must the goods be marked?

16. A bankrupt stock was sold at 35% loss. What was the selling price of articles that cost 50c.? \$1? \$1.50? \$1.75?

17. What per cent. is lost by selling sugar at 10 cents per pound which cost 12 cents per pound?

PROCESS.

$\$.12 - \$.10 = \$.02$

$\$.02 \div \$.12 = 16\frac{2}{3}\%$

Loss ÷ Cost = Rate.

ANALYSIS.—Since sugar that cost 12 cents was sold for 10 cents, there was a loss of 2 cents per pound. And since the gain or loss is reckoned at a rate per cent. on the cost, we must find what per cent. 2 is of 12. 2 is $\frac{1}{6}$ of 12; or, expressed as hundredths, is $.16\frac{2}{3}$ of 12, or $16\frac{2}{3}$%.

18. What per cent. is gained by selling tea at \$1 which cost \$.75?

19. What per cent. is lost by selling tea at \$.75 which cost \$1?

20. What per cent. is lost by selling cloth at \$1.25 that cost \$1.75?

21. Bought goods at 50 cents a yard and sold them at 60 cents a yard. What per cent. did I gain?

22. Goods that are selling at $12\frac{1}{2}$ cents a yard cost 10 cents. What per cent. is gained by selling them at that rate?

23. A man bought a city lot for \$4500 and sold it for \$5000. What per cent. did he gain?

24. Sold a quantity of potatoes for \$850 which cost me \$970. What per cent. did I lose?

25. Bought a quantity of crude petroleum at 5 cents per gallon and sold it at $4\frac{7}{8}$ cents. What per cent. did I lose?

26. A fruiterer bought 10 boxes oranges at \$1.75 per box. Two of the boxes were worthless, but he sold the balance at such price that he gained 5% on the whole purchase. How much did he sell them for per box? How much did he gain on the purchase?

27. I bought books at 10% discount from the retail price, which was \$1.50 per volume, and sold them at the retail price. What was my gain per cent.?

28. An agent gets a discount of 40% from the retail price of articles and sells them at the retail price. What is his gain per cent.?

29. A merchant bought cloth at \$3.25 per yard, and after keeping it 6 months sold it at \$3.75 per yard. What was his gain per cent., reckoning 6% per annum for the use of money?

30. Which is more profitable, and how much per cent., to sell goods for cash, at once, at 25% advance, or in 1 year at 30% advance, money being worth 7%?

31. Mr. A. gets a discount of 30% from the retail or list price of goods. Mr. B. gets a discount of 30% also, and 5% off for cash. If both sell goods at the list price, what is each one's gain per cent.?

32. A merchant bought tea at 20% less than its market value, and received a discount of 5% for cash. He sold it at an advance of 15% above its market value. What was his gain per cent.?

33. By selling cloth at a gain of 12 cents a yard, I realized a gain of 8% on the cóst. What was the cost?

PROCESS.	
$\$.12 \div .08 = \$1.50.$	ANALYSIS.—Since the gain, 12 cents, is 8%, or $\frac{8}{100}$ of the cost, $\frac{1}{8}$ of 12 cents, or $1\frac{1}{2}$ cents, is $\frac{1}{100}$ of the cost, and the cost is 100 times $1\frac{1}{2}$ cents, or \$1.50.
Gain $\div$ Rate $=$ Cost	

34. I make 10% by selling tea at a profit of 10 cents a pound. What did it cost me? What do I sell it for?

35. Flour was sold at a profit of \$1.50 per bbl., which was $16\frac{2}{3}\%$ of the cost. What was the cost?

36. A merchant made 12% by selling cloth at an advance of 12 cents a yard. What did it cost?

37. By selling butter at 8 cents a pound more than cost, a grocer made 20%. What did he pay for it?

38. A merchant sold cloth which was damaged by fire, at a sacrifice of 22 cents per yard, which was 40% of the cost. What did the goods cost?

39. A farmer sold a yoke of cattle, to which he had fed \$10 worth of grain, at an advance of \$25, and still realized a profit of 15%. What did they cost?

40. A man sold a horse at an advance of \$75, which was a gain of 25%. What was the cost of the horse?

41. If I sell a quantity of apples at an advance of 25 cents a barrel, and thereby realize $12\frac{1}{2}\%$ profit, what was the cost?

42. By selling cloth at a gain of 23 cents per yard, I realize a profit of 20%. What did it cost?

43. A merchant asked 25% more for his goods than they cost him, but at last sold them at a reduction of 10% from his asking price, thus realizing from the sale \$4684 profit. What was the cost of the goods?

44. A gentleman sold a horse for \$180 and gained 20% on him. What did the horse cost?

PROCESS.

$$100\% + 20\% = 120\%$$

$$\$180 \div 1.20 = \$150$$

Selling Price $\div$ (1 + Rate) = Cost.

ANALYSIS.—Since 20% of the cost was gained, the selling price must have been 20% more than the cost, or 120% of the cost. And, since 120% of the cost is \$180, 1% of the cost is $\frac{1}{120}$ of \$180, or, \$1.50, and the whole cost is 100 times \$1.50, or \$150.

Therefore the horse cost the gentleman \$150.

45. A gentleman sold a carriage for $230, and thereby lost 8% of the cost. What was the cost?

PROCESS.

100% − 8% = 92%

$230 ÷ .92 = $250

Selling Price ÷ (1—Rate) = Cost.

ANALYSIS.—Since 8% of the cost was lost, the selling price must have been 8% less than the cost, or 92% of the cost. And, since 92% of the cost was $230, 1% of the cost was $\frac{1}{92}$ of $230, or $2.50, and the whole cost was 100 times $2.50, or $250.

46. By selling apples at $.50 per bushel a grocer gained 25% on the cost. What was the cost?

47. A farm was sold for $38000, which was a loss of 5% of the cost. What was the cost?

48. A block of stores was sold for $185000, which was a gain of 15%. What did they cost?

49. A merchant lost 5% by selling calico at 9½ cents a yard. What did it cost?

50. A bankrupt stock was sold for $3582, which was a loss of 33⅓%. What did it cost?

51. By selling molasses at 65 cents a gallon, a grocer gained 30%. What was its cost?

52. A man was compelled to sell his household furniture for $1250, which was a loss of 37½%. What did it cost?

53. A boot and shoe dealer lost 9% by selling boots at $3.75 a pair. What was the cost of the boots?

54. A stationer lost 26% by selling paper at $2.22 a ream. What did he pay for it?

55. A druggist gained 125% by selling alcohol for $3.50 per gallon. What did he pay for it?

56. Coal was sold at $4.56½ per ton, which was a loss of 17%. What was the cost?

57. When pork is selling at $4.50 per hundred-weight I lose 10%. What will be my gain per cent. if I sell at $6 per hundred-weight?

COMMISSION.

380. 1. If a man sells $500 worth of goods for me, how much will he receive if he gets 2% of the sales?

2. If I allow a man 2% for purchasing $3000 worth of silks for me, how much will he get for his services?

3. If I collect a debt of $350, and charge 4% of the amount for my services, how much will I receive?

4. Mr. B. paid his agent 5% for selling $3000 worth of cotton. How much did he pay him, or what was his *commission?*

5. At 3% commission, how much will A receive for selling $4200 worth of flour? How much will be left after paying the commission, or what will be the *net proceeds?*

6. If I pay 2% commission for buying goods, what is the cost of every dollar's worth of goods bought? Since every dollar's worth of goods bought costs the purchaser $1.02, how many dollars' worth can be bought for $102? For $204?

7. If I pay 3% commission for buying goods, how many dollars' worth of goods can be bought for $309, after paying the commission? For $515?

8. How many dollars' worth of goods can be purchased for $630, after making allowance for the agent's commission at 5% of the value of the goods purchased?

DEFINITIONS.

381. A ***Commission Merchant*** or ***Agent*** is a person who buys or sells goods, or transacts other business for another.

382. The ***Commission*** is the compensation or percentage allowed a commission merchant or agent.

383. A ***Consignment*** is a quantity of merchandise sent to a commission merchant or agent to be sold.

384. The ***Consignor*** is the person who sends the merchandise to be sold.

385. The ***Consignee*** is the person to whom the merchandise is sent.

386. The ***Net Proceeds*** of a sale is the sum left after the commission, expenses, etc., have been deducted.

387. The processes in Commission involve the same elements as do the fundamental problems in Percentage. The corresponding terms are:

1. The *Sales* or *Sum Invested* is the ***Base.***
2. The *Rate Per Cent.* is the ***Rate.***
3. The *Commission* is the ***Percentage.***
4. The *Purchase Price* plus the *Commission* is the ***Amount.***
5. The *Net Proceeds* is the ***Difference.***

388. PRINCIPLE.—*The commission is reckoned at a certain rate per cent. on the value of the sales and purchases.*

WRITTEN EXERCISES.

389. 1. What will be an agent's commission for selling \$385.15 worth of goods at $3\frac{1}{2}\%$?

PROCESS.

$\$385.15 \times .03\frac{1}{2} = \$13.48$

Sales $\times$ Rate = Commission.

ANALYSIS.—Since the rate of commission is $3\frac{1}{2}\%$, or $.03\frac{1}{2}$, the commission will be $.03\frac{1}{2}$ of \$385.15, which is \$13.48.

2. What is the commission for selling cattle to the value of \$3184 at $2\frac{3}{4}\%$?

3. What is the commission for selling cotton to the value of \$8000, at $2\frac{1}{4}$ per cent.?

4. Mr. B. sent his agent \$3468 to invest in goods, allowing him 2% commission. What sum did he invest in goods after deducting his commission? What was the agent's commission?

PROCESS.

\$3468 ÷ 1.02 = \$3400, Amount invested.
Remittance ÷ (1 + *Rate*) = *Purchase Price.*
\$3468 — \$3400 = \$68, Commission.

ANALYSIS.—Since the agent gets a commission of 2% for purchasing, it requires \$1.02 to purchase \$1 worth of goods; he can therefore purchase as many dollars' worth of goods for \$3468 as \$1.02 is contained times in \$3468, which is 3400 times. Therefore, he can purchase \$3400 worth of goods. The money sent minus the amount invested will be the commission.

RULES.—*Since the same elements are involved as in the fundamental problems in Percentage, the rules are the same.*

FORMULAS.

1. Commission = Sales or Purchase × Rate.
2. Rate = Commission ÷ Sales or Purchase.
3. Sales or Purchase = Commission ÷ Rate.
4. Purchase = Sum remitted ÷ (1 + Rate).
5. Sales = Net Proceeds ÷ (1 — Rate).

5. If I send my agent \$4050 to invest in goods, after deducting 3% commission, what sum will he invest?

6. If I send my agent \$875 to invest in calico, allowing him 2% commission, how many yards can he buy at 6 cents per yard?

7. How much is an agent's commission for selling 385 bbl. flour at \$6.50 per bbl., the rate of commission being 2%?

8. What is the commission for collecting bills to the amount of \$784.25 at 5%?

9. How much must an agent be paid for selling 25 bbl. pears at \$12.75 per bbl., his commission being 6%?

10. What is the commission at 3% for selling 125 bbl. of potatoes at \$2.37½ per bbl.

11. A commission merchant sold 20 firkins of butter, each containing 56 lbs., at 28 cents per lb. How much was his commission at 8%?

12. What is the commission at 7% on a sale of 20 boxes of eggs, each containing 22 doz., at 23 cents per doz.?

13. How much commission must be paid to a collector for collecting an account of \$928.75 at 3¾%?

14. I sent my agent \$1525 to be invested in goods after deducting his commission of 2%. How much did he invest?

15. A merchant sent his agent \$375.50 to invest in muslin at 8 cents per yard. After deducting 3% commission, how many yards of muslin did he purchase?

16. Mr. A. sent \$3320.10 to be invested in goods after paying his agent 2% commission. What sum was invested?

17. A man sent his agent \$3725.05 to invest in pork after deducting 1½% commission. How much did he invest?

18. A speculator sent his agent in Chicago \$8966.75 to invest in wheat. After deducting ¾% commission, how many bushels of wheat did he buy at \$1.11¼ per bushel? What was the commission?

19. An agent received \$24.52 for selling goods at a commission of 2%. How many dollars' worth of goods did he sell?

20. How much money must I send my agent, so that he may purchase for me 150 bbl. flour at \$8.25 per bbl. if I pay him 3% commission for his services?

21. A commission merchant received \$318.25 for selling \$12730 worth of bankrupt goods. What was the rate of commission?

22. A sale of real estate returned as net proceeds \$2396.49, after paying \$324.18 charges and a commission of 2%. For how much did it sell?

REVIEW EXERCISES.

390. 1. A man whose wages had been \$27 per week was obliged to take $33\frac{1}{3}\%$ less. How much was the reduction?

2. A man bought a horse for \$300 and sold him for \$375. What per cent. did he gain?

3. When sugars that cost 10 cents a pound are sold for 11 cents, what per cent. is gained?

4. When land is selling at an advance of \$40 an acre, what is the gain per cent. if it cost \$120 an acre?

5. A boy sold apples at the rate of 2 for 3 cents which he bought at the rate of 3 for 2 cents. What did he gain per cent.?

6. A news agent sold \$31 worth of goods in a day at a commission of 10%. How much was his commission?

7. A man lost \$80 which was just 20% of his money. How much money had he?

8. A man paid 2% for selling his wheat and realized \$1.47 per bushel. For how much did it sell per bushel?

9. The interest on \$240 for 2 years was \$28.80. At what per cent. was it loaned?

10. When money is loaned at 6% and the interest amounts to \$75 on \$1000, how long has it been loaned?

11. When a merchant buys goods at $\frac{2}{3}$ of their estimated value, and sells them at their estimated value, how much is his gain per cent.?

12. When a man sells goods at a price from which he received a discount of 30%, what is his gain per cent.?

13. When a man can borrow money at 8%, which is more profitable, and how much per cent., to buy goods at 3% off for cash or on 3 mo. credit?

14. If I sell a horse for \$125 which cost me \$175, what do I lose per cent.?

15. A man sold a cow at an advance of \$10, which was 25% of what she cost. How much did she cost?

16. A boy can pick $33\frac{1}{3}\%$ as many apples as his father. If his father can pick 18 barrels a day, how many can the boy pick?

17. When a merchant buys goods at a discount of 20% from the regular price, and sells them at 20% more than the regular price, what is his gain per cent.?

18. Which is more profitable, to sell goods *now*, that cost 18 cents a yard, for 20 cents a yard, or to keep them 1 year and sell them at 21 cents when money is worth 6%? How much more profitable on an investment of \$1000?

19. Mr. A. bought a horse and carriage, paying twice as much for the horse as the carriage. He afterward sold the horse for 25% more than he gave for it, and the carriage for 20% less than he gave for it, receiving for both \$577.50. What was the cost of each?

20. After getting a note, without interest, discounted at a bank for 3 months at 7%, I had \$468.39. What was the face of the note?

21. A man can borrow money at 6% and pay cash for goods, obtaining a discount of 2%, or he may pay for the goods in 2 mo. Which is the more advantageous, and how much, on an invoice of goods amounting to \$1500?

22. I had a note for \$1000 discounted at a bank for 3 months at 7%. The proceeds were invested in wheat at \$1.65 per bu. How many bushels did I buy?

23. Mr. A. sold a horse for \$198, which was 10% less than he asked for him, and his asking price was 10% more than the horse cost him. What was the cost of the horse?

24. I bought a horse of Mr. A. for 20% less than he cost him, and I immediately sold the horse for 25% more than I paid for him, gaining \$25. What did the horse cost Mr. A.? What did he cost me?

25. I directed my agent to purchase for me 30 village lots at $650 each, and to pay the expenses of examining the titles, which averaged $4.25 per lot. What did they all cost me if the agent's commission was 4% on the price of the lots?

26. For how much must the lots be sold to give me a net profit of 20%, besides allowing the agent 5% commission for selling?

27. After a certain time a sum of money which had been at interest, had increased $18\frac{1}{2}$% and amounted to $3896.74. What was the sum at interest?

28. A clothier sold two suits of clothes at $72 each. On one he gained 20%, and on the other he lost 20%. Did he gain or lose on the sale and how much? How much per cent.?

29. After marking goods at an advance of 25% over cost, a merchant made an abatement of 20% from the marked price. Did he gain or lose, and how much per cent.?

30. On $\frac{2}{3}$ of my property I gained $33\frac{1}{3}$% and sold the rest for $\frac{2}{3}$ of the cost of the whole, receiving in payment a note due in 3 months which I got discounted at a bank at 6%. What was my gain per cent. if my property was worth $10000?

31. I sent my agent $7000 to be invested in wheat at $1.15 per bushel, allowing him a commission of 3% on the purchase. I paid for storage 2 cents per bushel per month, and 1% per month for the use of the money. After 3 months my agent sold the wheat at $1.33 per bushel, charging 2% commission for making the sale, and took in payment a note for the amount for 1 mo., which I had discounted at a bank at 9%. Did I make or lose, and how much?

32. A man wishing to sell a horse and cow, asked three times as much for the horse as the cow, but, finding no purchaser, he reduced the price of the horse 20%, and the price of the cow 10%, and sold them both for $165. What did he get for each?

√33. I owe B $66\frac{2}{3}\%$ of the amount I owe A, and I owe C 40% of the amount I owe B. How much do I owe each if I owe B \$80 more than I do C?

√ 34. I bought a quantity of coffee at $28\frac{1}{2}$ cents per pound. Allowing the coffee to fall short 5% in weighing, and 10% of the sales to be lost through bad debts, for how much must I sell it per pound that I may make a clear gain of 20% on the cost?

35. A quantity of prints was sold at a commission of 2% and the proceeds invested in cambrics purchased at 3% commission. The commission for buying the cambrics was \$126.30. How much did the prints sell for?

36. A tailor sold a suit of clothes for \$46, thereby gaining 15%. He sold another for \$60, and lost the same amount of money which he gained upon the first suit. What per cent. did he lose upon the last suit sold?

37. A merchant having a quantity of pork asked $33\frac{1}{3}\%$ more than it cost him, but was obliged to sell it $12\frac{1}{2}$ per cent. less than his asking price. If he received \$7 per cwt., what was its cost?

38. What must be asked for apples which cost \$3 per bbl., that I may reduce my asking price 20%, and still gain 20% on the cost?

39. A merchant sold a consignment of blankets at 3% commission, receiving in payment a bank-note for 1 mo., which he had discounted at 6%. He then invested the proceeds in wool at 30 cents per lb., charging $\frac{3}{4}\%$ commission for buying it. His commission for purchasing the wool was \$45. What was the value of the consignment? How much wool did he buy?

40. A merchant sold a quantity of goods at a gain of 20%. If, however, he had purchased the goods for \$60 less than he did, his gain would have been 25%. What did the goods cost?

TAXES.

CASE I.

GENERAL TAXES.

391. 1. If a person has to pay annually, for public purposes, 2% of the value of his property, estimated at \$5000, how much will be his *tax?*

2. If I am taxed $1\frac{1}{2}$% on my land, houses, etc., or *real estate*, estimated at \$20000, how much will be my tax?

3. If I am taxed 1% on the value of my movable or *personal property*, what is the tax on \$6000?

4. My property is estimated by the *assessors* to be worth \$40000. What will be my tax at $1\frac{1}{4}$%?

5. I imported 500 yards of silk, invoiced at \$2 per yard. What will be the Government tax, or *duty*, upon it at 35%?

6. I imported 500 yards carpeting, invoiced at \$2 per yard. What will be the duty at 50% of the value, or 50% *ad valorem?*

7. Mr. A. imported 5000 yards sheeting. What will be the duty at $4\frac{1}{2}$ cents per yd., or the $4\frac{1}{2}$ cents *specific* duty?

DEFINITIONS.

392. ***Real Estate*** is fixed property; as, houses, lands, tenements, etc.

393. ***Personal Property*** is movable property; as money, stocks, mortgages, cattle, etc.

394. A ***Tax*** is a sum of money assessed upon the persons, property, income, or business of individuals for public purposes.

395. A ***Property Tax*** is a tax on property. It is reckoned at a certain rate per cent. on the estimated or assessed value of the property.

396. A ***Personal Tax*** is a tax assessed upon the person. It is called a *poll* or *capitation tax*.

397. An ***Assessor*** is an officer appointed to estimate the taxable value of property and prepare the assessment roll.

398. An ***Assessment Roll*** is a list of the names of taxable inhabitants, and the value of each person's property.

Before taxes are assessed a complete inventory of all the taxable property must be made. If the assessment includes a poll tax, then a complete list of taxable polls must be made out.

WRITTEN EXERCISES.

399. 1. A village is to be taxed \$3756 on property assessed at \$854315. The number of polls at \$1.50 is 350. A's property is assessed at \$5000, and he pays 5 polls; B's property, at \$7000, and he pays 5 polls; C's property, at \$3425, and he pays 3 polls. What will be the tax on \$1, and what will be the tax of each?

PROCESS.

\$1.50 × 350 = \$525, Amount of poll tax.
\$3756 — \$525 = \$3231, Amount to be levied on property.
\$3231 ÷ \$854315 = .00378, or $3\frac{78}{100}$ mills on \$1.
\$5000 × .00378 = \$18.90, A's property tax.
\$1.50 × 5 = \$ 7.50, A's poll tax.
\$26.40, A's entire tax.
\$7000 × .00378 = \$26.46, B's property tax.
\$1.50 × 5 = \$ 7.50, B's poll tax.
\$33.96, B's entire tax.

400. A table like the following is commonly made by those who compute the taxes.

ASSESSOR'S TABLE. (*Rate*, .00378.)

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	$.0037	$ 7	$ .026	$40	$ .151	$100	$ .378	$ 700	$ 2.646
2	.0074	8	.03	50	.19	200	.756	800	3.024
3	.011	9	.034	60	.23	300	1.124	900	3.402
4	.015	10	.038	70	.26	400	1.512	1000	3.78
5	.019	20	.076	80	.302	500	1.89	2000	7.56
6	.023	30	.111	90	.34	600	2.268	3000	11.34

2. Find C's tax in example No. 1, using the table.

PROCESS.

Tax by table on $3000 is $11.34
Tax by table on $ 400 is $ 1.512
Tax by table on $ 20 is $ .076
Tax by table on $ 5 is $ .019

$12.947, C's property tax.
$1.50 × 3 = ... $ 4.50 , C's poll tax.
$17.447, C's entire tax.

RULE.—*Divide the sum to be raised, after deducting the poll tax, by the whole amount of taxable property, and the quotient will be the rate.*

Multiply each man's property by the rate, and to the product add the poll tax, if any, and the sum will be the whole tax.

3. In a certain town the assessed value of property was $1250500, on which a tax was levied of $5008.125. The number of polls was 425 at $ .75 each. A's property is valued at $18500; B's, at $22180; C's, at 15200. A pays 5 polls; B, 8 polls; C, 9 polls. What is the tax of each?

4. At what rate must \$595000 worth of property be assessed to raise a tax of \$2587, if there are 300 polls at \$1.50 each? What will be the tax on property valued at \$3400?

5. In a town containing 275 polls, assessed at \$1 each, the assessment roll shows that the taxable property is valued at \$895970. The tax is \$2310.90. What is the rate of taxation?

6. A tax of \$35000 is assessed upon a certain town. The valuation of the taxable property amounts to \$4506000, and there are 650 taxable polls assessed at \$1.25 each. What will be the tax on property assessed at \$11000?

7. What sum must be assessed to raise \$3750, besides paying 2% for collection? What would be the taxable valuation of property to raise that sum if the rate were .003275?

CASE II.

DUTIES OR CUSTOMS.

401. ***Duties,*** or ***Customs,*** are taxes levied by government upon imported goods.

402. A ***Specific Duty*** is a fixed tax on certain articles without regard to their value.

403. An ***Ad Valorem Duty*** is a tax of a certain per cent. on the net value of the articles in the country from which they have been brought.

404. ***Tare*** is the allowance made for the weight of a box, bag, etc.

405. ***Leakage*** and ***Breakage*** are allowances made for leakage and breakage during transportation.

406. The *Duties*, or *Customs*, are collected by officers of the Government in the ***Custom-houses.***

WRITTEN EXERCISES.

407. 1. What is the duty on 20 hhd. molasses, containing 63 gal. each, at 9 cents per gal., allowing 5% for leakage?

PROCESS.

63 gal. × 20 = 1260 gal. gross.
1260 gal. × .05 = 63 gal. leakage.
1260 gal. — 63 gal. = 1197 gal. net.
$ .09 × 1197 = $107.73 duty.

ANALYSIS.—Since 5% is allowed for leakage, 5% of the gross amount, or 63 gal., are deducted for leakage; and since the duty on 1 gal. is 9 cents, on 1197 gal. it is 1197 times 9 cents, or $107.73.

2. What is the duty, at 5 cents a pound, on 3750 pounds of coffee, allowing 5% for tare?

3. What is the duty on 500 pounds of raisins, in boxes, valued at 10 cents a pound, allowing 15% for tare, when the rate of duty is 6% ad valorem?

4. What will be the duty on $3000 worth of merchandise if the rate of duty is 15% ad valorem?

5. What is the duty, at 20% ad valorem, on 7 tons of steel, of 2240 lb. each, invoiced at 17 cents per lb.?

6. H. K. Thurber & Co., of New York, imported from Havana 75 hhd. molasses, 63 gal. each, valued at 35 cents per gal.; 125 hhd. sugar, 500 lbs. each, valued at 6 cents per lb.; 800 boxes cigars, valued at $8 per box. 7% was allowed for leakage on the molasses; duty on same, 25%; tare on sugar, 45 lbs. per hhd.; duty on same, 30%; duty on cigars, 60%. What was the amount of duties paid?

7. A wine merchant imported 45 casks sherry wine, valued at $65 per cask; 56 casks Madeira wine, valued at $60 per cask; 38 casks German wines, valued at $37 per cask. If an allowance of 4% be made for leakage, what will the duty be at 45%?

STOCKS

408. 1. Into how many shares can $100000 capital stock of a company be divided, if the shares are $100 each?

Shares will be regarded as $100 each unless otherwise specified.

2. How much of the capital, or capital stock does a man own who has 30 shares? 25 shares?

3. How much stock is represented by a certificate entitling the holder to 40 shares?

4. What is the selling price or *market value* of 10 shares of railroad stock, when stock is selling at its original value or *at par?*

5. What is the market value of 10 shares of stock, if it is sold at 105% of the original value or 5% *above par?*

6. What is the market value of 10 shares of stock, if it is sold at 95% of the original value or 5% *below par?*

7. What will 10 shares of stock cost at 5% above par, if I pay a dealer in stocks or *stock-broker* ¼% of the par value of the stock for buying it?

8. What will 5 shares of stock cost at 5% below par, if I pay a broker ¼% for buying or for *brokerage?*

9. What is the value of 15 shares of bank stock at 90% of its par value?

10. What is the cost of 50 shares Chicago & Rock Island R. R. stock at 90% of its par value?

11. What is the cost of 100 shares Chicago & Alton R. R. stock at 95% of its par value?

12. What is the cost of 20 shares Pacific Mail stock at $26\frac{1}{2}\%$ of its par value?

13. A company whose capital stock was $50000, gained $5000 above expenses, which was divided among the stockholders. What per cent. of the capital stock was the amount divided, or what was the *dividend?*

14. What amount will a man receive who owns 20 shares of stock, if a dividend of 5% is declared?

15. What amount will a man receive who has 30 shares of stock, if a 6% dividend is declared?

16. A company whose capital stock was $50000, lacked $5000 of meeting its obligations. What per cent. of the stock was the deficiency?

17. If the deficiency is made up by the stockholders, how much will be the *assessment* on a stockholder who owns 20 shares of the above stock?

18. How much will be the assessment on 15 shares of the above stock?

19. What will be the annual income on a written obligation or *bond* for $5000 which yields 6% interest annually?

DEFINITIONS.

409. A ***Company*** is a number of persons associated together for the prosecution of some industrial pursuit.

410. A ***Corporate Company*** or ***Corporation*** is a company authorized by law to transact business as an individual.

411. A ***Charter*** is the legal document which defines the rights and obligations of a corporation.

412. ***Capital Stock*** is the property or money invested in the business of the company.

413. A ***Share*** is one of the equal divisions of the capital stock of a company.

The value of a share is different in different companies. Unless otherwise specified it will be regarded as $100.

414. A ***Certificate of Stock*** is a paper issued by a corporate company giving the number of shares to which the holder is entitled, and the original value of each.

415. ***Par Value*** is the value named in the certificate.

When shares sell for the value named on their face the stock is said to be *at par;* when for more than their face, above par, or at a *premium;* when for less than their face, below par, or at a *discount.*

416. The ***Market Value*** is the sum for which stocks sell.

417. An ***Installment*** is a portion of the capital stock paid by the stockholder.

418. A ***Dividend*** is a sum divided among the stockholders as the profits of the business.

419. An ***Assessment*** is a sum which the stockholders of a company are required to pay, to make up deficiencies or losses.

The Government and Corporations frequently issue *Bonds* for the purpose of raising money.

420. A ***Bond*** is a written obligation under seal, securing the payment of a sum of money on or before a specified time.

Interest is usually paid upon bonds at fixed dates, as annually or semi-annually.

421. ***Coupons*** are certificates of interest attached to bonds. They are cut off and presented for payment as often as the interest becomes due.

422. United States Government Securities are of two kinds, viz: bonds which are to be paid at a specified time, and bonds which are to be paid at a fixed date, or some earlier specified time at the option of the Government.

Bonds are sometimes designated by combining the rate of interest and the time of payment. Thus, bonds that pay 6% interest and are payable in 1881, are called 6's of '81.

When the rate is uniform for a class of bonds it is omitted, and the time of redemption and payment only are given. Thus, bonds that may be redeemed in five years, and are payable in twenty years, are called 5–20's, those redeemable in ten years and payable in forty years are called 10–40's.

The 4% *consols* are the bonds of the *consolidated* loan. They are redeemable after 30 years from July 1, 1877.

The bonds issued by States, counties, etc., are named from the rate of interest they bear. Thus, New Hampshire bonds that bear 6% interest are called New Hampshire 6's.

All bonds of the United States are payable in coin at maturity.

The various classes of United States bonds are:

1. 6's of '81. Interest payable semi-annually in coin.
2. 5–20's, issued in 1862, '64, '65, '67, '68. Interest payable semi-annually in coin at 6%.
3. 10–40's, issued in 1864. Interest payable in coin semi-annually, at 5% on $500 and $1000 bonds, and annually on $50 and $100 bonds.
4. 5's of '81. Interest payable quarterly in coin.
5. 4½'s of '86. Interest payable quarterly in coin.
6. 4's of 1901. Interest payable quarterly in coin.
7. 4% consols. Interest payable quarterly in coin.

423. ***Stocks*** is a term which includes the stock of a corporation, the various U. S., State, and county bonds, etc.

424. A ***Stock-broker*** is a person whose business it is to buy and sell stocks.

425. ***Brokerage*** is the compensation allowed a broker for buying and selling stocks.

426. The computations in stocks involve the same elements as the fundamental problems in Percentage. The corresponding terms are:

1. The *Par Value* of the stock is the ***Base.***
2. The *Rate of Premium*, or *Discount* is the ***Rate.***
3. The *Premium* or *Discount* is the ***Percentage.***
4. The *Market Value* is the ***Amount,*** or ***Difference.***

427. PRINCIPLE.—*Brokerage is computed on the par value of the stock.*

EXERCISES.

1. What is the cost of \$1000 Delaware, Lackawanna, and Western R. R. bonds at 108 when $\frac{1}{2}\%$ additional is paid for buying?

2. A broker received \$2510 to invest in first mortgage bonds at a *premium* of 25%. If he charged $\frac{1}{2}\%$ brokerage, how many bonds of \$100 each did he buy?

3. A broker invested \$26000 in quicksilver stocks at $12\frac{1}{2}$, charging $\frac{1}{2}\%$ brokerage. How many shares did he buy?

4. A man invested \$5125 in Union Pacific R. R. bonds at \$102, paying $\frac{1}{2}\%$ brokerage. How many bonds of \$1000 each did he buy?

5. A man bought a number of shares of bank-stock at 125 and sold it at 128, gaining \$300. How many shares did he buy?

6. Bought bonds at 115 and sold them at 110, losing \$300. How many bonds of \$1000 each did I buy?

7. How much must I invest in Missouri State bonds at par, which pay 6% interest to secure an income of \$1200 annually?

8. What sum must I invest in stock at par paying an annual dividend of 8% to realize an income of \$4800 yearly?

9. What per cent. on the investment does 6% stock yield if it is bought at half its par value? If at $\frac{1}{4}$ of its par value?

10. What per cent. does stock pay, if it yields an annual dividend of 4% and is bought at 50% of its par value?

11. How much must I pay a share for stock, which yields an annual dividend of 6%, so that I may realize 12% on my money annually? 18%? 24%? 30%?

12. For what price must bonds bearing 9% interest be bought, so that 12% may be realized annually? 6%?

13. How much currency can be obtained for \$100 in gold, when gold is at a premium of 4%? 9%? 12%?

14. How much currency can be obtained for \$100 in gold, when gold is at a premium of 10%? 100%? 150%? 5%?

15. How much gold at 6% premium can I buy with \$106 in currency? With \$318? With \$159?

16. When gold is at a premium of 20%, how much gold is \$1 in currency worth. How much when gold is at 50% premium?

17. When gold is selling at 125, what is the value in gold of a United States greenback for \$10?

WRITTEN EXERCISES.

428. 1. What is the cost of 500 shares Delaware and Hudson Canal Co. stock at $50\frac{1}{2}$, brokerage $\frac{1}{4}\%$?

PROCESS.

$50\frac{1}{2}\% + \frac{1}{4}\% = 50\frac{3}{4}\%$

$50\frac{3}{4}\%$ of \$100 = \$50.75, cost of 1 share.

\$50.75 × 500 = \$25375, the entire cost.

ANALYSIS.— Since $50\frac{1}{2}\%$ of the par value of the stock is the price paid for it, the entire cost of the stock, including the rate for brokerage, is $50\frac{3}{4}\%$ of the par value of the stock. And, since the par value of a share of the stock is \$100, the cost of a share will be $50\frac{3}{4}\%$ of \$100, or \$50.75, and the cost of 500 shares of the stock will therefore be 500 times \$50.75, or \$25375.

RULE.—*Since the same elements are involved as in the fundamental problems in Percentage, the rules are the same.*

FORMULAS.

1. *Premium* or *Discount* $=$ *Par Value* $\times$ *Rate*.
2. *Rate* $=$ *Premium* or *Discount* $\div$ *Par Value*.
3. *Par Value* $=$ *Premium* or *Discount* $\div$ *Rate*.
4. $\textit{Par Value} = \textit{Market Value} \div \begin{cases} (1 + \textit{Rate}). \\ (1 - \textit{Rate}). \end{cases}$
5. $\textit{Market Value} = \textit{Par Value} \begin{cases} + \textit{Premium}. \\ - \textit{Discount}. \end{cases}$

2. Find the cost of 125 shares Union Pacific R. R. stock, at $68\frac{1}{2}$, brokerage $\frac{1}{4}\%$?

3. What will \$8000 U. S. 5–20's, coupon bonds of '65, cost at $108\frac{1}{4}$, brokerage $\frac{1}{8}\%$?

4. How much will 55 shares C. C. C. & I. R. R. stock cost at $28\frac{3}{4}$, brokerage $\frac{1}{4}\%$?

5. What must be paid for \$5000 U. S. 10–40's. at $8\frac{1}{4}\%$ premium, brokerage $\frac{1}{8}\%$?

6. Bought 35 shares N. Y. C. & H. R. R. R. stock at $86\frac{1}{2}$, and sold them at $8\frac{1}{4}\%$ advance. How much did I gain?

7. Sold 135 shares railroad stock at a discount of $15\frac{1}{4}\%$, paying $\frac{1}{4}\%$ brokerage. How much did I receive for it?

8. How many shares of bank-stock at 5% discount can be purchased for \$3810, if $\frac{1}{4}\%$ is paid for brokerage?

PROCESS.

$100\% - 5\% = 95\% + \frac{1}{4}\% = 95\frac{1}{4}\%$

$\$3810 \div .95\frac{1}{4} = \$4000$, or 40 shares.

ANALYSIS.—Since the stock was bought at 5% discount it was bought at 95% of its par value, but the brokerage increased the cost $\frac{1}{4}\%$, so that each dollar's worth of stock cost $95\frac{1}{4}\%$ of its par value, or \$ $.95\frac{1}{4}$. Therefore, as many dollars' worth can be bought for \$3810 as \$ $.95\frac{1}{4}$ is contained times in \$3810, which is 4000 times, or 40 shares can be bought.

9. Find the number of shares of R. R. stock at $102\frac{5}{8}$, which can be bought for \$2575, brokerage $\frac{3}{8}\%$.

10. How many shares of N. Y. C. & H. R. R. R. stock at $98\frac{3}{4}$, can be bought for \$28710, brokerage $\frac{1}{4}\%$?

11. How many shares of C. B. & Q. R. R. stock at $109\frac{7}{8}$, can be bought for \$66075, brokerage $\frac{1}{4}\%$?

12. How many shares of Hartland Ferry stock at 4% discount, can be bought for \$3330.25, brokerage $\frac{1}{4}\%$?

13. How many shares of R. R. stock at 3% discount, can be bought for \$2150.50, brokerage $\frac{1}{4}\%$?

14. What income will be realized from investing \$4196.25 in 5% stock purchased at 93, allowing $\frac{1}{4}\%$ for brokerage?

PROCESS.

$\$4196.25 \div .93\frac{1}{4} = \$4500$, the par value.
$\$4500 \times .05 = \$225$, annual income.

ANALYSIS.—Since the stock cost $93\frac{1}{4}\%$ of its par value, every dollar's worth cost $\$.93\frac{1}{4}$; and as many dollars' worth can be bought for \$4196.25 as $\$.93\frac{1}{4}$ is contained times in that sum, which is 4500 times; and since the stock paid 5% income, the entire income from \$4500 is 5% of \$4500, which is \$225.

15. How much income will I receive annually by investing \$1299 in 6% stock purchased at 37%, allowing $\frac{1}{4}\%$ brokerage?

16. What will be the income from investing \$4696.25 in Crawford Co. 6's at 45, brokerage $\frac{1}{4}\%$.

17. Which is more profitable, and how much, to invest \$5000 in 6% stock purchased at 75%, or 5% stock purchased at 60%?

18. U. S. 5-20's pay 6% interest in gold. What will be my income in currency by investing \$11212.50 at $112\frac{1}{8}$, when gold is quoted at $6\frac{7}{8}\%$ premium?

19. Which is more profitable, to buy 6% bonds, purchased at 90, interest payable in currency; or 5% bonds, purchased

at 95, interest payable in gold, when gold is quoted at $6\frac{7}{8}\%$ premium? How much more profitable in currency is it on each $100 invested?

20. A. B. Howard sold a mill for $13850, which had been paying an annual profit of $5\frac{1}{2}\%$ of that sum, and invested the proceeds in U. S. 10-40's at $111\frac{3}{8}$, paying $\frac{1}{8}\%$ brokerage. Was his yearly income increased or diminished, and how much in currency, gold being at 5% premium?

21. How much must be invested in 6% stock, purchased at 90, to secure to the purchaser an income of $900 annually?

PROCESS.

$900÷.06=$15000, par value of stock.
$15000×.90=$13500, cost of stock.

ANALYSIS.—Since the income of $1 is $.06, it will require as many dollars to secure an income of $900 as $.06 is contained times in $900, which is 15000 times; and since the stock is selling at 90% of its par value, 90% of $15000, which is $13500, will be the cost.

22. I desire to invest in Ohio & Mississippi R. R. bonds which bear 6% interest, a sum of money sufficient to bring an income of $1000. If the bonds can be bought at 91%, how much money must I invest, brokerage $\frac{1}{4}\%$?

23. What sum must I invest in Louisiana 7's at $107\frac{1}{4}$ to secure an annual income of $1750?

24. What sum must I invest in U. S. 6's at $112\frac{3}{8}$ to secure an annual income of $1750?

25. What sum must be invested in 6% stock at $84.50 per share to yield an income of $900 annually?

26. What per cent. income on my investment will I receive if I buy 6% stock at 20% premium?

PROCESS.

$.06÷$1.20=.05, or 5%.

ANALYSIS.—Since $1 of the stock costs $1.20, and the income from it is $.06, the income is $\frac{6}{120}$, or $\frac{1}{20}$, or 5% of the amount of the investment.

27. What is the rate per cent. of income from bonds which pay 7% interest when they are bought at 105?

28. If stock which pays a semi-annual dividend of $5\frac{1}{2}$% be bought at 10% premium, what rate per cent. of income does it pay?

29. Which affords the greater per cent. of income, bonds bought at 125 which pay 8%, or bonds which pay 6% bought at a discount of 10%?

30. Which is more profitable, and how much per cent., to buy New York 7's at 105%, or Louisiana 6's at 98%?

31. What per cent. of income does stock paying 9% dividends afford if it is bought at 112?

32. How much must I pay for New York 6's so that I may realize an income of 9% on the investment?

PROCESS.

$\$.06 \div \$.09 = .66\frac{2}{3}$

ANALYSIS.—Since the income is 6% of every dollar of the par value of the stock, if an income of 9% on an investment be desired, then 6% of the par value of the stock must be 9% of the sum paid for \$1 of the stock, which is $\$.66\frac{2}{3}$, or, the stock must be bought for $66\frac{2}{3}$% of its par value.

33. How much must I pay for stock which pays a dividend of 15% so that I may realize 7% on my investment?

34. How much premium must I pay on stock which pays a 10% dividend so that I may realize 8% on my investment?

35. At what price must I buy 7% stocks so that they may yield an income equivalent to 10% stocks at par?

36. What must I pay for New York 6's so that my purchase may yield me 7%?

37. At what price must I purchase 15% stock that it may yield the same income as 6% stock purchased at 90?

38. What is the currency value of \$9280 in gold, when gold is selling at $107\frac{1}{8}$?

39. What is the currency value of \$7225 in gold, when gold is at a premium of $8\frac{1}{8}$%?

40. What is the value in gold of $5000 in currency, when the premium on gold is $6\frac{1}{2}\%$?

41. When gold is selling at $105\frac{1}{4}$, what is the value of $7250 in currency?

42. The net earnings of a company whose capital stock is $2000000 was $135000. If they reserve $5000 as a surplus fund, what per cent. dividend can they declare?

43. Mr. A. purchased 250 shares of stock at 75, in a company whose capital was $1500000. The gross earnings of the company for 1876 were $225000, the expenses were 40% of the gross earnings, and they reserved a surplus fund of $10000. What per cent. dividend did Mr. A. receive on his investment?

44. A capitalist owning 200 shares of stock of $150 per share, on which he was receiving a dividend of 3% semi-annually, exchanged them for 6% bonds purchased at 98. Did he gain or lose, and how much annually?

45. If a man who had $5000 of U. S. 6's of '81 should sell them at 115, and invest in U. S. 10-40's purchased at 105, would he gain or lose, and how much annually?

INSURANCE.

429. 1. How much must I pay to secure myself against loss by fire, or, *insure* my property for $5000, if an annual sum or *premium* of 1% is charged by those who take the risk?

2. What will be the annual premium for insuring property for $10000 at $\frac{1}{2}\%$?

3. What will be the annual premium for insuring property for $1500 at $1\frac{1}{2}\%$?

4. What is the premium at $\frac{3}{4}\%$ for insuring a vessel and cargo to the amount of $36000?

5. What will it cost to insure $1500 worth of tea at $\frac{5}{8}\%$?

6. What must be paid for insuring a building valued at \$3000, for $\frac{2}{3}$ of its value at $1\frac{1}{4}\%$?

7. If a merchant insured his goods for \$2000 at 2%, how much premium did he pay?

8. How much premium did a merchant pay who insured his stock of boots and shoes for \$6000 at $1\frac{1}{2}\%$?

9. A man paid \$25 premium for insuring his house and furniture against loss or damage by fire. For how much was he insured, if the rate of insurance was $\frac{1}{4}\%$?

10. For how much was a man insured who paid \$50 premium for insuring his barn and live stock at 1%?

11. How much was the amount of insurance when the premium paid for insuring a house and furniture at $\frac{3}{4}\%$ was \$75?

12. For how much is a farmer insured on his barns and the grain in them, who pays \$60 premium, when the rate is 2%?

DEFINITIONS.

430. ***Insurance*** is indemnity against loss or damage. It is of two kinds: *Property Insurance* and *Personal Insurance.*

CASE I.

PROPERTY INSURANCE.

431. ***Property Insurance*** is indemnity against loss or damage by fire, or *Fire Insurance;* against loss or damage by casualties at sea, or *Marine Insurance;* and against loss or damage by fire, lightning, etc., to cattle, horses, etc., or *Live Stock Insurance.*

432. The ***Policy*** is the contract or agreement between the insurance company and the person insured.

433. The ***Premium*** is the sum paid for insurance.

The insurance business is conducted mostly by corporate companies, which are either *Mutual* or *Stock companies.*

434. A ***Mutual Insurance Company*** is one in which the person insured participates in the profits and shares the losses of the company.

435. A ***Stock Insurance Company*** is one in which the capital is contributed by stockholders, who alone participate in the profits and share the losses.

A few companies combine the features of both stock and mutual companies. They are called *Mixed Companies.*

436. The computations in insurance involve the same elements as do the fundamental problems in Percentage. The corresponding terms are:

1. The *Amount Insured* is the ***Base.***
2. The *Rate of Premium* is the ***Rate.***
3. The *Premium* is the ***Percentage.***

WRITTEN EXERCISES.

1. How much is the premium on a policy of insurance on a dwelling for \$3000 if the rate of premium is $1\frac{1}{4}\%$?

PROCESS.

$\$3000 \times .01\frac{1}{4} = \$37.50$

ANALYSIS.—Since the premium is $1\frac{1}{4}\%$ of the amount insured, to find the premium, $1\frac{1}{4}\%$ of \$3000 must be found, which is \$37.50.

RULE.—*Since the same elements are involved as in the fundamental problems in Percentage the rules are the same.*

FORMULAS.

1. Sum insured $\times$ Rate $=$ Premium.
2. Premium $\div$ Sum insured $=$ Rate.
3. Premium $\div$ Rate $=$ Sum insured.

2. How much is the premium for insuring a stock of goods for $15000 at $1\frac{1}{4}\%$?

3. A man had his house insured for $5000 paying $\frac{3}{4}\%$, and his furniture for $3000 paying 1%. How much was the premium?

4. How much must be paid for insuring a flouring mill valued at $18000 for $\frac{2}{3}$ of its value at $2\frac{1}{4}\%$?

5. A vessel valued at $80000, with its cargo valued at $65000, was insured for $\frac{3}{4}$ of its value at $1\frac{1}{4}\%$. What was the premium? What would be the actual loss to the insurance company if the above vessel should be lost at sea?

6. Property was insured for $15850 at $3\frac{1}{4}\%$. What amount of premium was paid?

7. Paid $275 for insuring property at $\frac{1}{2}\%$. What was the amount insured?

8. Paid $325 for insuring property valued at $16250. What was the rate of premium?

9. A insured his buildings for $9500, paying a premium of $47.50. What rate did he pay?

10. Mr. Orcott paid $175 for insuring his block of mercantile buildings at $1\frac{1}{4}\%$. How much insurance did he procure?

11. Mr. James paid $652.50 for insuring property valued at $43500. What was the rate?

12. A man paid $180 for insuring his saw mill for $\frac{2}{3}$ its value at 3%. What was the value of the mill?

13. A merchant whose stock of goods was valued at $30000 insured it for three-fourths of its value at $\frac{3}{4}\%$. In a fire he saved $5000 of the goods. What was his loss? What was the loss of the insurance companies?

14. The price of a quantity of silks was discovered by knowing that they were insured at $4\frac{1}{2}\%$ for two-thirds of their value, and the premium paid was $400. What was the price of the silks?

15. A manufacturing company insured its works for $\frac{3}{4}$ of their value at $2\frac{1}{2}\%$, paying as premium \$1657.50. What was the value of the works?

16. A vessel and cargo were insured for $\frac{5}{8}$ of their value at $1\frac{1}{4}\%$. The premium amounted to \$2475. At what price were the vessel and cargo valued?

17. Paid \$225 for insuring a store and its contents for $\frac{3}{4}$ of their value at $1\frac{1}{2}\%$. The stock was worth half as much as the store. What was the value of each?

18. For how much must a block of stores worth \$20000 be insured, so that the insurance will cover $\frac{3}{4}$ of the value of the property and the amount of the premium at $1\frac{1}{2}\%$?

CASE II.

PERSONAL INSURANCE.

437. ***Personal Insurance*** is indemnity against loss of life, or *Life Insurance;* against loss occasioned by accidents, or *Accident Insurance;* and against loss occasioned by sickness, or *Health Insurance.*

The policies issued by Life Insurance Companies are of various kinds, the chief of which are the *Life Policy* and the *Endowment Policy.*

438. A ***Life Policy*** secures a sum of money at the death of the person insured.

439. An ***Endowment Policy*** secures a sum of money at a specified time, or at death, if it occur before the specified time.

440. ***Accident*** and ***Health Policies*** secure a stipulated sum for a certain time, in case of a disabling accident or sickness, and the face of the policy in case of death by accident.

441. The rates of premium are based upon the *expectation of life*, determined by observing the death rate per thousand inhabitants.

WRITTEN EXERCISES.

442. 1. What premium must be paid annually for a life policy of \$5000, at \$21.10 per \$1000?

PROCESS.

\$21.10 × 5 = \$105.50

ANALYSIS.—Since the rate of premium is \$21.10 annually for \$1000, the premium for \$5000 is 5 times \$21.10, which is \$105.50.

2. How much will be the annual premium on a life insurance policy for \$3000 at \$31.30 per \$1000?

3. What is the amount of annual premium on a life policy for \$5500 at \$26.38 per \$1000?

4. If a person who is insured for \$5000, at an annual premium of \$28.90 per \$1000, dies after 9 payments, how much more will his heirs get than has been paid in premiums?

5. If a man insures his life for \$5000, paying \$22.90 per \$1000, and dies immediately after paying his annual premium for 30 years, what is the result of the investment, reckoning simple interest at 6% on the premiums paid?

6. If Mr. Bowditch insures his life for \$5000 on the endowment plan when he is 40 years of age, the policy to be payable when he is 55 years of age, paying therefor \$54.90 for \$1000, will he gain or lose by insuring, reckoning simple interest at 7% on the premiums paid, if he lives till the policy is paid?

7. A traveling agent has an accident insurance policy for \$3000, for which he pays \$50 per year. His weekly compensation, in case of a disabling injury, is \$30. Immediately after he makes his fifteenth payment he is disabled by an injury for 20 weeks. Does he gain or lose by the insurance, reckoning simple interest at 6% on the premiums paid?

EXCHANGE

443. 1. When A owes B $500, and B owes A $500, how may the accounts be settled without any transfer of money taking place?

2. When A in Chicago owes B in New York $500, and C in New York owes A $1000, how can A pay his indebtedness to B without remitting the money?

3. What will be the indebtedness of A, B and C to each other after the transaction has taken place?

4. A and C live in the same city, and B in a distant city. A owes B $2000, and B owes C $1000. How may B pay his indebtedness to C without remitting the money?

5. What will be their indebtedness to each other after A has paid B's order, or *draft?*

6. What will a draft for $500 cost, payable when it is presented, or *at sight*, if ½% premium is charged for it?

7. How much should be deducted from the price of the above draft if it is not to be paid until two months, money being worth 6%?

8. What will be the cost of a draft for $50, payable at sight, if it is purchased at 1% discount?

9. What will be the cost of a sight draft for $300, purchased at ½% premium?

10. If A in Nashville owes B in New Orleans $1000, and C in New Orleans owes D in Nashville $1500, how may A pay his indebtedness without remitting the money?

11. If the premium is $\frac{1}{8}\%$, how much will it cost me to remit a draft for $800 from Cincinnati to Cleveland?

12. If a man sells a draft for $500, at a premium of $\frac{3}{4}\%$, how much does he receive for it?

13. A wishes to send to his agent in New Orleans a draft for $5000. If the premium on exchange is $\frac{3}{4}\%$, how much will the draft cost him?

14. When I pay $2025 for a sight draft on New York for $2000, what is the premium, or *rate* of exchange?

15. When I can buy a sight draft on Chicago for $2000, paying for it $1980, what is the rate of exchange?

16. If Mr. Burt pays $4975 for a sight draft on Cincinnati for $5000, at what rate is exchange?

DEFINITIONS.

444. ***Exchange*** is the method of making payments in distant places without transmitting money.

445. A ***Draft*** or ***Bill of Exchange*** is a written order by one person to another to pay a specified sum of money to the person named in the writing or his order.

FORM OF A DRAFT.

\$384$\frac{27}{100}$. CINCINNATI, O., July 20, 1877.

Twenty days after sight pay to the order of the First National Bank, Chicago, Ill., Three Hundred Eighty-four $\frac{27}{100}$ Dollars, value received, and charge to the account of

JONES BROS. & CO.

To JAMES H. HOOSE & CO.,
Chicago, Ill.

There are primarily *three* parties connected with a draft, viz: the person who signs it, the person who is ordered to pay the money, and the person to whom the money is to be paid.

446. The ***Drawer*** is the person who *signs* the draft.

447. The ***Drawee*** is the person who is ordered to pay the money.

448. The ***Payee*** is the person to whom it is ordered that the money be paid.

449. A ***Sight Draft*** is one which is to be paid when it is presented to the drawee.

450. A ***Time Draft*** is one payable at a specified time, after its presentation to the drawee, or after date.

On Time Drafts three days of grace are usually allowed.

451. ***Accepting*** a draft is agreeing to pay it when it is due. This is done by the *drawee* writing "Accepted" across the face of the draft, with his name and the date.

Exchange is of two kinds, viz: ***Domestic*** or ***Inland, and Foreign.***

CASE I.

DOMESTIC EXCHANGE.

452. ***Domestic Exchange*** treats of drafts payable in the country where they are made.

WRITTEN EXERCISES.

453. 1. What will be the cost of procuring a sight draft on New York for $5000 at $\frac{1}{2}\%$ premium?

PROCESS.

$$\$1 + \$.00\tfrac{1}{2} = 1.00\tfrac{1}{2}$$
$$\$1.00\tfrac{1}{2} \times 5000 = \$5025$$

ANALYSIS.—Since exchange on New York is at $\frac{1}{2}\%$ premium, every $1 of the draft will cost $1.00½, and a draft for $5000 will therefore cost the purchaser 5000 times $1.00½, which is $5025.

2. What will be the cost in New Orleans of a draft on New York, payable 60 days after sight, for \$5000, exchange being at $1\frac{1}{2}\%$ premium?

PROCESS.

$$\$1.+\$.015=\$1.015$$
$$\$1.015-\$.008\tfrac{3}{4}=\$1.006\tfrac{1}{4}$$
$$\$1.006\tfrac{1}{4}\times 5000=\$5031.25$$

ANALYSIS.—Since the exchange on New York is at $1\frac{1}{2}\%$ premium, every \$1 of the draft would cost \$1.015 if paid at sight. But since it is not to be paid in New York for 63 days, the banker in New Orleans who has the use of the money for that time, since he is not obliged to pay the money in New York for 63 days, allows the bank discount on the face of the draft for that time. The bank discount on \$1, at the legal rate in the State of Louisiana, for the given time, is \$.008$\frac{3}{4}$, which, subtracted from \$1.015, gives \$1.006$\frac{1}{4}$, the cost of \$1 of the draft. Since the cost of a draft for \$1 is \$1.006$\frac{1}{4}$, the cost of a draft of \$5000 will be 5000 times \$1.006$\frac{1}{4}$, or \$5031.25.

3. What will be the cost in Memphis, Tenn., of a sight draft on Cincinnati for \$1000, the rate of exchange being $\frac{1}{2}\%$ premium?

4. If exchange on Chicago is $1\frac{1}{4}\%$ premium, what will be the cost in Savannah, Ga., of a sight draft for \$3000?

5. A merchant in Chicago bought a draft on New York for \$5000, payable 1 mo. after sight. What did it cost if exchange was $\frac{1}{8}\%$ premium?

6. What will be the cost in Detroit, Mich., of a draft for \$1500 on Cleveland, O., payable 3 mo. after date, when exchange is $\frac{1}{8}\%$ discount?

7. How much will be realized from the sale of a draft for \$5000 at $\frac{1}{4}\%$ discount?

8. How much will be realized from the sale of a draft for \$3000 sold at $\frac{1}{8}\%$ premium?

9. When exchange is at $\frac{1}{8}\%$ premium, what will be the cost of a draft for \$5000, purchased in Chicago on Omaha, to be paid 3 mo. after date?

10. If exchange is at $\frac{1}{4}\%$ premium, what will a draft for \$1500 cost, purchased in St. Paul, Minn., on Dayton, O., payable in 2 mo., without grace?

11. If exchange is at $\frac{1}{10}\%$ premium, what will a draft for \$5000 on New York cost in Cincinnati, payable 2 mo. after date?

12. How large a draft on New Orleans can be purchased for \$5000, when exchange is at $1\frac{1}{2}\%$ premium?

PROCESS.

$$\$1+\$.01\tfrac{1}{2}=\$1.015$$
$$\$5000\div1.015=\$4926.11$$

ANALYSIS.—Since exchange is at $1\frac{1}{2}\%$ premium, it will cost \$1.015 to buy a draft for \$1, and \$5000 will buy a draft for as many dollars as \$1.015 is contained times in \$5000, or \$4926.11.

13. How large a draft on Washington, D. C., payable 60 days after sight, can be bought in Nashville, Tenn., for \$3000, when exchange is at 1% discount?

PROCESS.

$$\$1-\$.01=\$.99$$
$$\$.99-\$.0105=\$.9795$$
$$\$3000\div.9795=\$3062.78$$

ANALYSIS.—Since exchange is at 1% discount, it would cost \$.99 to buy a draft for \$1 if it were payable at sight; but since the draft is not to be paid until 63 days, the banker in Nashville who has the use of the money for 63 days, allows bank discount on the face of the draft for that time, or \$.0105 for every dollar. Therefore since it costs \$.9795 to purchase a draft for \$1, \$3000 will purchase a draft for as many dollars as \$.9795 is contained times in \$3000, or \$3062.78.

14. How large a sight draft can be purchased on Chicago for \$5725, when the rate of exchange is $\frac{1}{4}\%$ premium?

15. What will be the face of a 30-day draft purchased for \$1500, if the rate of exchange is $\frac{1}{8}\%$ premium and the rate of discount is 6%?

16. If I pay \$1200 for a draft payable in 2 mo., when

the premium on exchange is $\frac{1}{2}\%$, and the rate of discount is 9%, what will be the face of the draft?

17. How large a sight draft on New York can be purchased in Chicago for \$10000, if exchange is $\frac{1}{8}\%$ discount?

18. A commission merchant in St. Louis, Mo., sold goods amounting to \$3500 for a man in Denver, Col. He sent the amount due by a draft payable in 1 mo. after sight, exchange being $\frac{1}{8}\%$ premium. How large a draft did he purchase?

19. How large a draft at sight on San Francisco can I purchase for \$1750 if exchange is at $\frac{1}{8}\%$ premium?

CASE II.

FOREIGN EXCHANGE.

454. ***Foreign Exchange*** treats of drafts made in one country and payable in another.

Foreign bills of exchange in the United States are drawn on London, Paris, Berlin, Antwerp, Amsterdam, Hamburg, Bremen, and other commercial centers; but drafts on London and Paris are more common, inasmuch as they are paid anywhere on the continent of Europe.

455. A ***Set of Exchange*** consists of *three* drafts or bills of the same date and tenor, named, respectively, the *first*, *second*, and *third* of exchange. They are sent by different mails, so that if one is lost another may be presented. When one bill of the set is paid the others are void.

456. The value of a *pound sterling* previous to 1873 was fixed at \$4.44$\frac{4}{9}$. In 1873 Congress fixed the value of the sovereign in U. S. gold coin at \$4.8665, which is now the *par of exchange*.

The value of a *franc* is about \$.193, or about 5.18 francs to one dollar in gold. Exchange on Paris is quoted at a certain number of francs to a dollar in gold.

WRITTEN EXERCISES.

457. 1. What is the cost in currency, in New York, of a sight draft on London for £312 15s. 5d., when exchange is \$4.87 for a pound sterling and gold at 106.

ANALYSIS.

£312 15s. 5d. = £312.7708, value in pounds and decimals of a pound.
\$4.87 × 312.7708 = \$1523.193 +, the cost in U. S. gold.
\$1523.193 × 1.06 = \$1614.59, the cost in U. S. currency.

2. How large a bill of exchange at sight on London can be bought in New York for \$2984.38 in currency, exchange being at \$4.86 for a pound sterling and gold at 107½?

ANALYSIS.

\$2984.38 ÷ \$1.07½ = \$2776.167, value of currency in gold.
\$2776.167 ÷ \$4.86 = £571.2263 +, value in pounds sterling.
£571.2263 = £571, 4s. 6¼d., the face of the draft.

3. How large a bill of exchange at sight on London can be bought in New York for \$3762.50 in currency, when gold is at 105½ and exchange is at \$4.87?

4. What will be the face of a sight draft on London, which is purchased in Philadelphia for \$5928.75 in currency, when gold is at 106¼ and exchange is at \$4.85½?

5. What will be the face of a sight draft on London, which is purchased in Norfolk, Va., for \$5575.20 in currency, when exchange is at \$4.87½, and gold is selling at 107¾?

6. What must be paid in currency for a bill of exchange on Paris, at sight, for 3269 francs, exchange being at 5.15 francs to the dollar, and gold at 105⅜?

7. What must be paid in currency for a sight bill of exchange on Paris for 8950 francs, exchange being at 5.19 francs for one dollar, and gold at 106¼?

8. What will be the face of a sight draft on Paris which is bought in Baltimore for \$1575 in currency, when the rate of exchange is 5.19 francs for a dollar, and gold is at $107\frac{1}{4}$?

9. An American bought a sight draft on Paris for 5725 francs. What was the currency value of the draft when exchange was at 5.20 francs for a dollar and gold was $106\frac{1}{4}$?

10. What is the value in currency of a bill of exchange, at sight, on London, for £895 10s., when exchange is \$4.87 for a pound sterling and gold is quoted at $106\frac{5}{8}$?

11. An American in Philadelphia purchased a sight draft on London for £585 10s. 5d. What was the currency value of the draft, if exchange was at par and gold at $107\frac{1}{2}$?

12. What will be the cost, in currency, of a sight bill of exchange on London for £875 5s. 4d., when exchange is $\$4.88\frac{1}{2}$ for a pound sterling and gold is quoted at $104\frac{7}{8}$?

AVERAGE OF PAYMENTS.

458. 1. How long may \$1 be kept to balance the use of \$5 for 2 months?

2. How long may \$1 be kept to balance the use of \$7 for 3 months?

3. How long may \$10 be kept to balance the use of \$5 for 2 months?

4. How long may \$20 be kept to balance the use of \$10 for 4 months?

5. How long may \$50 be kept to balance the use of \$25 for 7 months?

6. How long may \$40 be kept to balance the use of \$80 for 3 months?

7. I owe B two debts of equal amount, one due in 3 months and the other in 6 months. When may I pay both at one payment?

8. I owe A two debts of $20 each, one due in 2 months and the other in 4 months. When may I pay both at one payment?

9. If I pay $20 three months before it is due, how long after it is due may I keep $30 to balance it?

10. If I owe $20, due in 4 months, and $40, due in 6 months, at what time can I equitably pay both debts at one payment?

11. If I owe $30 due in 3 months, and $10 due in 7 months, when may I equitably pay both debts by a single payment of $40?

DEFINITIONS.

459. ***Averaging Payments*** is finding the equitable time for discharging, by one payment, sums due at different times.

460. The ***Average Time*** is the date at which the debts may be equitably discharged by a single payment.

461. The ***Term of Credit*** is the time that must elapse before the debt becomes due.

462. The ***Average Term of Credit*** is the time that must elapse before the debts due at different times may be equitably discharged by a single payment.

CASE I.

463. When the terms of credit begin at the same date.

1. A. T. Stewart & Co. sold a bill of goods upon the following terms: $400 cash, $300 due in 2 months, and $400 due in 4 months. At what time might the whole indebtedness be equitably discharged by a cash payment?

PROCESS.

\$400 for	0 mo. = \$1 for	——.	
300 for	2 mo. = \$1 for	600 mo.	
400 for	4 mo. = \$1 for	1600 mo.	
\$1100		2200 mo.	

2200 mo. ÷ 1100 = 2 mo. Average term of credit.

ANALYSIS.—Since \$400 was to be paid in cash, there was no term of credit for that sum. Since \$300 was to be paid in 2 mo., the use of that sum for 2 mo. is equal to the use of \$1 for 600 mo., and the use of \$400 for 4 mo. is equal to the use of \$1 for 1600 months. Hence, the credit of the whole debt, \$1100, is equal to the credit of \$1 for 2200 mo., or \$1100 for $\frac{1}{1100}$ part of 2200 mo., which is 2 months, the average term of credit.

RULE.—*Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit.*

2. H. B. Claflin & Co. sold a bill of goods amounting to \$2300, on the following terms: \$300 cash, \$1200 due in 3 months, and the balance due in 4 months. What was the average term of credit.

3. Field, Leiter & Co. sold a bill of goods payable as follows: \$500 in 1 month, \$500 in 2 months, and \$800 in 4 months. What was the average term of credit?

4. Whitney & Co. sold a bill of lumber on the following terms: \$1500 cash, \$3000 payable in 30 days, and \$2000 payable in 90 days. At what time will the debt be payable in one cash payment?

5. H. K. Thurber & Co. sold to F. N. Burt a bill of goods amounting to \$2400, payable as follows: $\frac{1}{3}$ in 30 days, $\frac{1}{2}$ the remainder in 60 days, and the balance in 4 months. What was the average term of credit?

6. Mr. Birge bought a bill of goods amounting to \$3000, payable as follows: $\frac{1}{4}$ in 3 mo., $\frac{1}{4}$ in 2 mo., and the rest in 4 mo. What was the average term of credit?

CASE II.

464. When the terms of credit begin at different dates.

1. Find the average time of payment of the following bills: Feb. 10, 1877, \$400 due in 2 mo.; March 15, 1877, \$350 due in 3 mo.; and April 12, 1877, \$300 due in 3 mo.

PROCESS.

\$400 due April 10.	400	
350 due June 15.	$350 \times 66 = 23100$	
300 due July 12.	$300 \times 93 = 27900$	
	1050	51000

$51000 \div 1050 = 48\frac{1}{2}\frac{2}{1}$ days.

April $10 + 49$ days $=$ May 29, average term.

ANALYSIS.—Adding to the date of the purchase of each bill its term of credit, we obtain the time when it is due, and so we have \$400 due April 10, \$350 due June 15, \$300 due July 12. The average time when the bills will be due will be either after the earliest date, or before the latest date, and so we may select either of these dates from which to compute the average time. Selecting the earliest date, we find that \$350 was due 66 days after that time, and \$300 was due 93 days thereafter. Averaging, as in Case I, we find the term of credit to be $48\frac{1}{2}\frac{2}{1}$, or a fraction more than 48 days, which must be 49 days. This, added to April 10, gives May 29, the average time of payment.

RULE.—*Select the earliest date at which any debt becomes due for the standard date, and find how long after that date the other amounts become due.*

Find the average term of credit by multiplying each debt by the number of days from the standard date, and dividing the sum of the products by the sum of the debts.

Add the average term of credit to the standard date, and the result will be the average term of payment.

Instead of the *earliest* date, the *first of the month* may be used.

2. What is the average time at which the following bills become due: Feb. 1, 1877, \$200 on 1 mo. credit; March 10, 1877, \$500 on 3 mo. credit; April 12, 1877, \$275 on 2 mo. credit; and May 1, 1877, \$400 on 4 mo. credit?

3. A merchant owes bills dated as follows: Jan. 1, 1877, \$500 due in 2 mo.; Jan. 15, 1877, \$850 due in 3 mo.; Feb. 20, 1877, \$375 due in 3 mo.; and, Feb. 28, 1877, \$650 due in 4 mo. What will be the average time of payment?

4. A merchant purchased goods of Cragin Bros. & Co. as follows: Sept. 10, 1876, \$300 on 4 mo. credit; Oct. 15, 1876, \$400 on 6 mo. credit; Nov. 1, 1876, \$750 on 2 mo. credit; and, Nov. 15, 1876, \$300 on 1 mo. credit. What was the average time of payment?

5. Messrs. J. Rorbach & Son bought goods from George C. Buell & Co. as follows: Sept. 1, 1876, \$600 on 3 mo. credit; Oct. 3, 1876, \$400 on 4 mo. credit; Oct. 20, 1876, \$250 on 2 mo. credit; and, Nov. 10, 1876, \$375 on 1 mo. credit. What was the average time of payment?

6. Stevens & Shepard bought goods from the Russell Irwin Manufacturing Co. as follows: Dec. 10, 1876, a bill of \$460 on 4 mo. credit; Jan. 5, 1877, a bill of \$200 on 3 mo. credit; Jan. 30, 1877, a bill of \$200 on 4 mo. credit; and, Feb. 25, a bill of \$900 on 2 mo. credit. What was the average time of payment?

7. Bought goods of Carson, Pirie & Co. as follows: Jan. 25, 1877, \$850 on 4 mo. credit; Feb. 15, 1877, \$600 on 3 mo. credit; March 20, 1877, \$500 on 4 mo. credit; and, April 10, 1877, \$960 on 2 mo. credit. What was the average time of payment?

8. May 1, 1877, Mr. S. purchased goods to the amount of \$2400 on the following terms: $\frac{1}{4}$ payable in cash, $\frac{1}{4}$ payable in 2 months, and the balance in 6 months. When may the whole be equitably paid by one payment?

AVERAGE OF ACCOUNTS.

465. 1. What should be the date of a note given to settle the following account?

Dr. W. H. STEVENS. *Cr.*

1877.				1877.			
May	5	To Mdse.	50 00	May	15	By Cash	25 00
June	7	" " 2 mo.	140 00	June	10	" Draft, 10 da.	100 00
June	20	" " 1 "	150 00	June	30	" "	100 00

PROCESS. (*By Products.*)

Due.		*Amount.*	*Days.*	*Product.*	*Paid.*		*Amount.*	*Days.*	*Product.*
May	5	$ 50	94	4700	May	15	$ 25	84	2100
Aug.	7	140	0		June	23	100	45	4500
July	20	150	17	2550	June	30	100	38	3800
		340		7250			225		10400
		225							7250
		115			$3150 \div 115 = 27\frac{45}{115}$				3150

Aug. $7 + 28$ days $=$ Sept. 4, the average time.

ANALYSIS.—From the dates at which the various amounts become due, we select the latest, which is Aug. 7, for the assumed time of settlement, and multiply each amount by the number of days intervening between that date and the time when each item of the account becomes due. The debit side of the account shows there is due $340 and the use of $1 for 7250 days, and the credit side shows that $225 has been paid, and that the debtor is entitled to the use of $1 for 10400 days, if the time of settlement is Aug. 7. Subtracting the amounts, there is shown to be $115 due, and the debtor is entitled to the use of $1 for 3150 days. Therefore, he should not be required to pay the account until the time when the use of $115 is equal to the use of $1 for 3150 days, which is 28 days. 28 days after Aug. 7 is Sept. 4.

RULE.—*Multiply each amount due by the number of days intervening between the time it becomes due and the latest date at which any sum on either side of the account becomes due.*

Divide the difference between the sum of the products of the debit and credit side of the account, by the balance due on the account. The quotient will be the average term of credit.

1. When the balances are both on one side of the account, the term of credit is to be counted *backward* from the date at which the first amount becomes due, but *forward* from that date if the balances are on opposite sides.

2. The average term of credit may also be found by reckoning interest upon each sum due for the number of days intervening between the time it becomes due, and the earliest date at which any sum becomes due; then dividing the balance of the interest by the interest on the balance of the account for one day. This is called the *Method by Interest.* The result is the same whether the average term of credit is found by the method by products or by interest.

2. Find the average term of credit of the following account:

Dr. OLMSTEAD & BISHOP. *Cr.*

1877.				1877.			
Jan.	5	To Mdse., 2 mo.	$375	Jan.	30	By Cash	$200
Feb.	15	" " 1 "	200	Mar.	15	" "	600
Feb.	25	" " 4 "	800	Apr.	1	" "	200
Mar.	30	" " 3 "	450				

3. Find by both methods when the balance of the following account becomes due.

Dr. HAMILTON & MATTHEWS. *Cr.*

1876.				1877.			
Nov.	1	To Mdse., 4 mo.	$1600	Jan.	15	By Accept'ce, 2 mo.	$2000
Dec.	3	" " 3 "	3800	Mar.	20	" " 2 "	5000
1877.							
Jan.	15	" " 4 "	5500				
Mar.	1	" " 3 "	1500				

4. When should interest begin on the following account?

Dr. JAMES HOWARD, *in acc't with* HIRAM SIBLEY. *Cr.*

1877.				1877.			
Apr.	10	To Mdse.	$150	Apr.	12	By Cash	$250
Apr.	20	" "	400	May	1	" "	200
May	16	" "	100	June	7	" "	400
June	21	" "	500				

5. Find the average term of credit of the following account?

Dr. BREYFOGLE & CO. *Cr.*

1877.				1877.			
Jan.	1	To Mdse., 1 mo.	$ 500	Feb.	3	By Cash	$ 500
Jan.	20	" " 3 "	850	Feb.	28	" "	200
Feb.	15	" " 2 "	1500	May	15	" Draft, 1 mo.	1200
Apr.	3	" " 4 "	2500				

6. When should interest begin on the following account?

Dr. BURK, FITZSIMONS, HONE & CO. *Cr.*

1877.				1877.			
Feb.	1	To Mdse.	$1800	Feb.	20	By Cash	$3000
Mar.	15	" " 1 mo.	3000	May.	18	" Accept'ce, 2 mo.	8000
Mar.	20	" " 4 "	4800				
Apr.	3	" " 4 "	6000				

7. What will be the cash balance of the following account, Jan. 1, 1878, interest at 6%?

Dr. PRATT J. NELSON. *Cr.*

1877.				1877.			
July	10	To Mdse., 2 mo.	$500	July	20	By Cash	$ 400
Aug.	1	" " 3 "	700	Aug.	20	" "	1000
Sept.	8	" " 1 "	800			Jan 1	
Sept.	20	" " 2 "	600				

1200 × .061½ = 1400

1200 × 1.40 = 1198.60 ans

PARTNERSHIP

466. 1. If two men, who have equal sums invested in the same business, gain $100, what is each man's share of the gain?

2. If one man furnishes $\frac{2}{3}$ of the capital, and another $\frac{1}{3}$ of it, and the gain is $1200, what should be the gain of each?

3. Mr. A. furnishes $3000 of the capital, and Mr. B. furnishes the balance, which is $5000. What part of the profits should each receive?

4. Four partners furnish money in the proportion of $2000, $3000, $4000, and $5000 respectively. What part of the gain should each one receive?

5. Three men engage in business and furnish the following sums respectively: A, $5000; B, $4000; C, $3000. How much of the gain should each receive if $1200 was gained during the year?

6. The cost of a pasture was $27. A had in it 5 cows for 3 weeks, and B 3 cows for 4 weeks. What should each one pay?

7. The profits of a company were $800 for a certain time. What share of the profits did each partner receive, if the capital contributed by them was $900, $700, and $800 respectively?

8. A and B formed a partnership after A had been doing business alone for 6 months. A had $5000 invested during the year, and B had $10000 invested for 6 months. The gain was $5000. What was each one's share?

DEFINITIONS.

467. A ***Partnership*** is an association of two or more persons, for the purpose of conducting business.

468. ***Partners*** are the persons associated in business. They are called collectively *a company*, *a firm* or *house*.

469. The ***Capital*** is the money employed in business.

470. Principle.—*Partners share the gains and losses in proportion to the amount of the capital each invests, and the length of time it is employed.*

CASE I.

471. When the capital of each partner is employed for the same time.

WRITTEN EXERCISES.

1. A, B and C are partners, having furnished \$5000, \$6000 and \$8000 capital respectively. If during the year they gain \$2850, what is each partner's share of the gain?

PROCESS.

$\$5000 + \$6000 + \$8000 = \$19000$

$\frac{5000}{19000}$ or $\frac{5}{19}$ of $\$2850 = \$750$, A's share.

$\frac{6000}{19000}$ or $\frac{6}{19}$ of $\$2850 = \$900$, B's share.

$\frac{8000}{19000}$ or $\frac{8}{19}$ of $\$2850 = \$1200$, C's share.

\$2850, Whole gain.

Analysis.—Since each partner should share the gain in proportion to the amount of capital he contributed, we find what part of the whole capital each partner contributed. A furnished $\frac{5}{19}$ of the capital, and is therefore entitled to $\frac{5}{19}$ of the gain, or \$750. B furnished $\frac{6}{19}$ of the capital, and is entitled to $\frac{6}{19}$ of the gain, or \$900. C furnished $\frac{8}{19}$ of the capital, and is therefore entitled to $\frac{8}{19}$ of the gain, or \$1200.

RULE.—*Find such a part of the gain or loss as each partner's capital is of the whole capital.*

The result will be each partner's gain or loss.

2. A, B and C engaged in business, employing $20000 capital, of which A furnished $7000, B $7000, and C $6000. They gained in one year $6000. What was each partner's share?

3. Three men engage in business. A furnishes $3000 of the capital, B $6000 and C $4000. If they gain $2600, what is each partner's share?

4. Three men engaged in land speculation. A furnished $10000, B $8000 and C $12000. They lost in one year $6000. What was the loss of each partner?

5. A, B and C furnish capital to engage in business as follows: A $2500, B $2000 and C $3500. If the firm loses $640, what is the loss of each partner?

6. A, B, C and D engaged in buying produce. A contributed $8000, B $10000, C $9000 and D $13000. They gained $3000. What was each partner's share of the gain?

7. D and G furnish capital to engage in business and L does the work for $\frac{1}{3}$ of the profits; D contributes $8000 and G 10000 of the capital. They gain $5400. What is each one's share of the gain?

8. E, F and G bought a block of stores for $46000. E furnished $\frac{3}{8}$ of the money, F $11500 and G the rest. The property was sold for $48300. What was the gain of each?

9. A, B and C engage in business. A furnishes $6470, B $5420 and C $3410 capital. If they gain $6490.75, what is the gain of each?

10. Four persons rented conjointly a pasture containing 125 A. 60 sq. rd., for $3.75 an acre. A fed 125 sheep upon it, B 145 sheep, C 175 sheep, and D 340 sheep. How much rent should each one pay?

11. Three men engaged in business. A furnished \$6000 and B \$8000. They gained \$4200, of which C's share was \$1400. What was the gain of A and B and C's stock?

12. Five men trade in partnership. A furnishes \$500, B \$600, C \$800, D \$1000 and E \$1200 capital. They gain \$2750. What is the gain of each partner?

13. A, B and C bought a farm in partnership. A paid $\frac{1}{4}$ the purchase money, B $\frac{1}{3}$ and C the rest. They sold it at a gain of \$3000. What was each one's share of the profit?

CASE II.

472. When the capital of the partners is employed for different periods of time.

WRITTEN EXERCISES.

1. A began business with \$6000 capital. At the end of 6 months he took in B as a partner, who furnished \$5000 additional capital. If the gain, after 6 months more, was \$3400, what was each partner's share of the gain?

PROCESS.

$$\$6000 \times 12 = \$72000$$
$$\$5000 \times 6 = \$30000$$
$$\$102000$$

$\frac{72}{102}$ of \$3400 = \$2400, A's share.
$\frac{30}{102}$ of \$3400 = \$1000, B's share.

ANALYSIS.—A's capital of \$6000 was used for 12 months, and was therefore equal to the use of \$72000 for 1 month. B's capital of \$5000 was used for 6 months, which was equal to the use of \$30000 for 1 month. Both together had invested sums of money which were equal to the use of \$102000 for 1 month, of which A contributed a sum equal to \$72000 for 1 month, or $\frac{72}{102}$, and he was therefore entitled to $\frac{72}{102}$ of the gain, or \$2400. B contributed a sum equal to \$30000 for 1 month, or $\frac{30}{102}$, and was therefore entitled to $\frac{30}{102}$ of the gain, or \$1000.

RULE.—*Find such a part of the entire gain or loss, for each partner's share of the gain or loss, as the capital of each partner for a unit of time, is of the entire capital for a unit of time.*

2. A engaged in business with a capital of $4000. After 3 months he took in B with a capital of $6000, and in 6 more, C became a partner, with a capital of $8000. At the end of 18 months the profits were $9360. What was each partner's share of the gain?

3. A, B and C engage in business together. A puts in $4000 capital for 8 months, B $6000 for 7 months, and D $3500 for one year. If they gain $2320, what is each partner's share of the gain?

4. B, C and D entered into partnership, furnishing a joint capital of $5875, of which B furnished 20%, C 35%, and D the rest. B's capital was employed 15 months, C's 9 months, and D's 10 months. They lost $2502.75. What was each partner's loss?

5. A, B and C took a contract to build a block of stores. A furnished 20 men for 3 months, B 25 men for $3\frac{1}{2}$ months, and C 15 men for 4 months. After paying the expenses the profits were $1475. What was the share of each?

6. A, B and C lost $8500 by speculating in real estate. A furnished $5000 of the capital which was employed for 1 year, B $8000 for 10 months, and C $10000 for 6 months. What was each one's share of the loss?

7. A, B and C engaged in manufacturing rope and cordage. A invested $4500 for 6 months, B $5000 for 8 months, and C $6500 for 7 months. They gained $4500. What was the gain of each partner?

8. G, L and F entered into partnership. G furnished $1200, L $1500, and F $3000. After 6 months F withdrew $2000 of his capital. If at the end of a year the profits were $2200, what part of the profits belonged to each partner?

RATIO

473. 1. A was employed on a piece of work 6 days, and B 12 days on the same work. How does the number of days A worked compare with the number of days B was employed?

2. A laborer earned $12 a week, and spent $6. How does what he spent compare with what he earned?

3. How does $3 compare with $9? $4 with $12? $6 with $18?

4. How does 2 compare with 10? 3 with 18? 5 with 25?

5. What relation has 2 to 12? 3 to 21? 4 to 28?

6. What is the relation of 3 to 24? 6 to 30? 7 to 35?

7. How does 8 compare with 2? What is the relation of 8 to 2?

8. How does 9 compare with 3? What relation has 9 to 3?

9. What relation has 24 to 8? 30 to 6? 25 to 4?

10. What is the relation *between* 5 and 7? *Ans.* $\frac{5}{7}$ or $\frac{7}{5}$.

11. What is the relation between 6 and 9? Between 8 and 9?

12. What is the relation of 8 to 9? Between 8 and 9?

13. What is the relation of 12 to 4? Between 12 and 4?

14. What is the relation of 15 to 5? Between 15 and 5?

15. What is the relation of 16 to 8? Between 16 and 8?

16. What is the relation of 25 to 5? Between 25 and 5?

DEFINITIONS.

474. ***Ratio*** is the relation of one number to another of the same kind.

1. This relation is expressed either as quotient of one number divided by the other, and is called *Geometrical Ratio,* or simply *Ratio,* or, as the difference between two numbers, and is called *Arithmetical Ratio.*

2. When it is required to determine *what the relation of one number to another is,* it is evident that the *first* is the dividend, and the *second* the divisor.

3. When it is required to determine *the relation between two numbers,* either may be regarded as dividend or divisor.

4. The first number is commonly regarded as the dividend.

475. The ***Terms of a Ratio*** are the numbers compared.

476. The ***Antecedent*** is the first term.

Thus, in "What is the ratio of 6 to 8?" 6 is the antecedent.

477. The ***Consequent*** is the second term.

Thus, in "What is the ratio of 6 to 8?" 8 is the consequent.

478. The ***Sign*** of ratio is a colon (:).

Thus, the ratio of 12 to 6 is expressed, 12 : 6.

The colon (:) is sometimes regarded as the sign of division without the line. Thus, 12 : 8 is regarded as $12 \div 8$.

479. The antecedent and consequent together form a ***Couplet.***

480. PRINCIPLES.—1. *The terms of a ratio must be like numbers.*

2. *The ratio is an abstract number.*

3. *Multiplying or dividing both terms of a ratio by the same number does not change the ratio of the numbers.*

EXERCISES.

481. 1. What is the ratio of 3 to 6? 5 to 10? 7 to 21?

2. What is the ratio of \$3 to \$10? 12 lb. to 6 lb.? 27 bush. to 9 bush.?

3. What is the ratio of 7 to 35? 24 to 48? 13 to 39?

4. If the antecedent be 20, and the consequent 15, what is the ratio?

5. What is the ratio when the antecedent is 45, and the consequent 25?

6. What is the ratio of $\frac{2}{5}$ to $\frac{7}{5}$? $\frac{2}{3}$ to $\frac{3}{4}$? $\frac{5}{6}$ to $\frac{7}{11}$?

Fractions should be reduced to similar fractions. They will then have the ratio of their numerators.

7. What is the ratio of $5\frac{1}{4}$ to $3\frac{1}{3}$? $7\frac{1}{5}$ to $6\frac{1}{4}$? $9\frac{2}{3}$ to $5\frac{1}{8}$?

8. What ratio will the work of 12 men sustain to that of 8 men?

9. What will be the ratio of 8 yd. to 24 yd.? $6\frac{3}{4}$ yd. to 9 yd.?

10. When the antecedent is 3 and the ratio $\frac{2}{3}$, what is the consequent?

11. When the consequent is 8 and the ratio $\frac{2}{7}$, what is the antecedent?

12. When the antecedent is $\frac{1}{2}$ and the ratio $\frac{1}{4}$, what is the consequent?

13. What number has to 3 the ratio of 5 to 6?

14. What number has to 5 the ratio of 4 to 12?

15. What number has the ratio to 12 that 3 has to 1?

16. If two numbers have the relation of 6 to 8, and the first is 12, what is the other?

17. What number has to 12 the ratio of 8 to 9?

18. If two numbers have the relation of 10 to 15, and the antecedent is 40, what is the consequent?

PROPORTION

482. 1. What two numbers have the same relation to each other as 3 to 6? As 2 to 8? As 7 to 21?

2. What two numbers have the same ratio as 5 to 15? 6 to 30? 12 to 48? $12\frac{1}{2}$ to 25? $2\frac{1}{4}$ to $4\frac{1}{2}$? $12\frac{1}{2}$ to 50?

3. What number has the same relation to 6 that 3 has to 9?

4. What number has the same relation to 5 that 7 has to 14?

5. What number has the same relation to $\frac{2}{3}$ that 4 has to 8?

6. To what number has 5 the same relation that 3 has to 9?

7. To what number has $2\frac{1}{2}$ the same relation that 7 has to 21?

8. 24 is to 7 as 12 is to what number?

9. 12 is to 5 as what number is to 15?

10. If the cost of 9 yards of cloth is $5, how will the cost of 18 yards compare with that sum?

11. If 10 men can earn $30 per day, what ratio will the earnings of 15 men bear to that sum?

12. Write two equal ratios; multiply the first and last terms together; multiply the second and third terms together. How do the products compare?

13. Write two other equal ratios; multiply as before. How do the products compare?

DEFINITIONS.

483. A ***Proportion*** is an equality of ratios.

Thus, $9 : 18 = 6 : 12$ is a proportion.

484. The ***Sign*** of proportion is a double colon (: :).

The double colon (: :) may be regarded as the extremities of the sign of equality ($=$). It is written between the ratios.

A proportion must have four terms, viz: two antecedents, and two consequents.

Any four numbers that can be formed into a proportion are called *proportionals.*

485. The ***Antecedents*** of a proportion are the antecedents of the ratios, or the first and third terms.

Thus, in the proportion 5 : 10 : : 7 : 14, 5 and 7 are the *antecedents.*

486. The ***Consequents*** of a proportion are the consequents of the ratios, or the second and fourth terms.

Thus, in the proportion 5 : 10 : : 7 : 14, the *consequents* are 10 and 14.

487. The ***Extremes*** of a proportion are the first and fourth terms.

Thus, in the proportion 7 : 8 : : 14 : 16, 7 and 16 are the *extremes.*

488. The ***Means*** of a proportion are the second and third terms.

Thus, in the proportion 7 : 8 : : 14 : 16, 8 and 14 are the *means.*

489. PRINCIPLES.—1. *The product of the extremes is equal to the product of the means.*

2. *The product of the extremes divided by either mean gives the other mean.*

3. *The product of the means divided by either extreme gives the other extreme.*

EXERCISES.

Find the term that is wanting in the following:

1. 18 : 24 :: 6 : ( ).
2. 9 : 18 :: ( ) : 6.
3. 8 : 18 :: 7 : ( ).
4. ( ) : 18 :: 7 : 15.
5. 7 : ( ) :: 8 : 24.
6. 15 : 18 :: ( ) : 16.
7. 17 : 19 :: 15 : ( ).
8. 25 : ( ) :: 16 : 25.
9. ( ) : 14 :: 16 : 35.
10. 14 : 23 :: ( ): 69.
11. 13 : ( ) :: 15 : 65.
12. $\frac{1}{2}$: 5 :: ( ) : 6.
13. $\frac{2}{3}$: ( ) :: $\frac{1}{2}$: 10.
14. $\frac{2}{5}$: $\frac{1}{7}$ :: ( ) : 15.
15. 6 : $\frac{2}{9}$ :: 5 : ( ).
16. 13 : 7 :: ( ) : 8.
17. 5 men : 7 men :: $8\frac{1}{2}$: ( ).
18. $21.16 : $15.20 :: $\frac{3}{4}$: ( ).
19. 80.51 A. : 21.15 A. :: ( ) : 2.
20. 16 lb. 3 oz. : 18 lb. 2 oz. :: ( ) : 7.
21. 5 gal. 3 qt. : ( ) :: 5 : 9.
22. 14% : ( ) :: 6 : 15.

SIMPLE PROPORTION.

490. A ***Simple Ratio*** is a ratio between any two numbers.

Thus, 6:8, $10:$8, 5 lb. 6 oz.:7 lb. 3 oz., are simple ratios.

491. A ***Simple Proportion*** is an equality between two simple ratios.

492. A ***Direct Proportion*** is one in which each term *increases* or *diminishes*, as the one on which it depends *increases* or *diminishes*.

Thus, proportions involving *quantity* and *cost*, *men* and *work done*, etc., are direct proportions, for as the quantity *increases* or *diminishes*, the cost *increases* or *diminishes*, and as the number of men *increases* or *diminishes*, the amount of work done will *increase* or *diminish*.

493. An ***Inverse Proportion*** is one in which each term *increases* as the term upon which it depends *diminishes*, or *diminishes* as it *increases*.

Thus, in the problem, "If 6 men can mow a field of grass in 9 days, how long will it take 9 men to mow it," as the number of men *increases*, the number of days required to do the work *decreases*, and the proportion is an *inverse proportion*.

WRITTEN EXERCISES.

494. 1. If 8 yd. of silk cost $24, what will 15 yd. cost?

PROCESS.

(1) yd. 8 : yd. 15 :: $ 24 : $ ( )

(2) yd. 15 : yd. 8 :: $ ( ) : $ 24

The term wanting (1) $\frac{15 \times 24}{8} = \$45$

The term wanting (2) $\frac{15 \times 24}{8} = \$45$

ANALYSIS.—It is evident that 8 yd. have the same relation to 15 yd. that the cost of 8 yd. has to the cost of 15 yd. Hence we have the proportion, 8 yd. : 15 yd. :: $24, the cost of 8 yd. : the cost of 15 yd., or 15 yd. : 8 yd. :: the cost of 15 yd. : $24, the cost of 8 yd. To find the cost of 15 yards, the term wanting, we divide the product of the means by the extreme, as in (1); or the product of the extremes by the mean, as in (2).

2. If 5 men can cut a quantity of wood in 18 days, in how many days could 12 men do the same work?

PROCESS.

(1) men. 5 : men. 12 :: days. ( ) : days. 18

(2) men. 12 : men. 5 :: days. 18 : days. ( )

Term wanting $= \frac{18 \times 5}{12} = 7\frac{1}{2}$ da.

ANALYSIS.—It is evident that exactly in proportion as the number of men is *increased*, the number of days required to do the work is *diminished*, and therefore 5 men : 12 men :: the days it will require 12 men to do the work : 18 the number of days required for 5 men to do the work. Or,

12 men : 5 men :: 18 days, the number of days it requires 5 men to do the work : the number of days 12 men require to do the work.

RULE.—*Express the ratio between the two numbers that are like numbers. Consider, from the conditions of the problem, whether the proportion is direct or indirect, and arrange the other number and the term wanted so that the two ratios will be equal.*

Divide the product of the extremes or means by the single extreme or mean. The result will be the term wanted.

Problems in proportion are sometimes regarded as illustrations of cause and effect, in which two causes and their corresponding effects are compared, giving the following proportion:

1st cause : 2d cause : : 1st effect : 2d effect.

3. If 6 men earn $75 in one week, how much will 10 men earn in the same time?

4. If 16 yards of cloth cost $20, what will be the cost of 7 yards?

5. A man can buy 45 sheep for $112.50. How much will 18 sheep cost at the same rate?

6. If 8 horses consume 15 tons of hay in 6 months, how much hay will 14 horses consume in the same time?

7. If 6 men can do a piece of work in 45 days, how many days will it take 11 men to do the same work?

8. If 10 men can do a piece of work in 6 days, in how many days can 13 men do the same work?

9. How many men will it require to build 60 rods of wall in the same time that 8 men can build 40 rods?

10. If 8 men can dig a ditch in 15 days, how many days will it take 13 men to dig it?

11. If 6 bushels of wheat can be bought for $7.32, how many bushels can be bought for $45?

12. How many barrels of apples can be bought for $2250, if 15 barrels cost $33.75?

13. If it requires 13 men to lay a certain number of bricks in 28 days, how many days will it take 9 men to lay the same number?

14. If 165 bushels of potatoes can be raised on $1\frac{1}{32}$ acres of ground, how many bushels can be raised on $3\frac{1}{4}$ acres?

15. If it requires $1\frac{1}{8}$ acres of ground to raise 405 bu. of carrots, how many acres will it require to raise 975 bu.?

16. Five horses cost a man $626.25. What would be the cost of 13 horses at the same rate?

17. It required 26 men to build an embankment in 80 days. How long would it require 32 men to do the same work?

18. It took 9 horses to move a stick of timber weighing 12590 pounds. How many pounds would a stick weigh which could be moved by 7 horses?

19. If an ocean steamer sails 1775 miles in 5 days, how many miles will she sail in $6\frac{1}{2}$ days?

20. If a locomotive runs $96\frac{3}{4}$ miles in $3\frac{1}{2}$ hours, how many miles will it run in $5\frac{7}{8}$ hours?

21. A dog is chasing a rabbit, which has 45 rods the start of the dog. The dog runs 19 rods while the rabbit runs 17. How far must the dog run before he catches the rabbit?

22. A cistern has 3 pipes. The first will fill it in 12 hours, the second in 16 hours, and the third in 18 hours. If all run together how long will it take them to fill it?

23. If it requires 15 compositors 15 days to set up a book of 675 pages, how many days will they need to set up a book of 900 pages?

COMPOUND PROPORTION.

495. A ***Compound Ratio*** is the product of two or more simple ratios.

496. A ***Compound Proportion*** is a proportion in which either ratio is compound.

497. PRINCIPLE.—*The product of two or more proportions is a proportion.*

WRITTEN EXERCISES.

498. 1. If 6 men can mow 24 acres of grass in 2 days, by working 10 hours per day, how many days will it take 7 men to mow 56 acres, by working 12 hours per day?

PROCESS.

(1) $7 : 6 :: 2 : (1\frac{5}{7}$ days.)
(2) $24 : 56 :: 1\frac{5}{7} : (4$ days.)
(3) $12 : 10 :: 4 : (3\frac{1}{3}$ days.)

Or,

$$(4)\quad \left.\begin{cases} 7 : 6 \\ 24 : 56 \\ 12 : 10 \end{cases}\right\} :: 2 : 3\frac{1}{3}$$

$$\begin{array}{l} \textit{Means} \\ \textit{Extremes} \end{array} \quad \frac{6 \times 56 \times 10 \times 2}{7 \times 24 \times 12} = 3\frac{1}{3}$$

ANALYSIS.—A simple proportion is a proportion that has but one condition. A compound proportion has more than one condition. The conditions are introduced one at a time, therefore examples in compound proportion may be solved as several simple proportions. The first condition in this example is: If 6 men can mow 24 acres of grass in 2 days, how long will it take 7 men to do the same work? This, solved by simple proportion, (1), gives $1\frac{5}{7}$ days. The second condition is: If the men can mow 24 acres of grass in $1\frac{5}{7}$ days, how long will it take them to mow 56 acres? This, solved by simple proportion, (2), gives 4 days. The third condition is: If the work can be done by the men in 4 days, by working 10 hours per day, how many days will it take to do the work if they work 12 hours per day? This, solved by simple proportion, (3), gives $3\frac{1}{3}$ days, the time it will take 7 men to mow 56 acres of grass, by working 12 hours per day, if six men can mow 24 acres in 2 days by working 10 hours per day. Or,

Since every simple proportion is an equality of ratios, the product of the three proportions, (1), (2), (3), will be an equality of ratios; and, since $1\frac{5}{7}$ and 4 appear in both antecedent and consequent, they may be omitted, and the simple proportions will assume the form of the compound proportion, (4).

The problem may be stated, as in the second part of the process, by writing for the third term the term that is like the one sought, and by arranging the others in couplets, considering their relation to the ratio between the third term and the term sought.

RULE.—*Solve by successive simple proportions, introducing the conditions one at a time.* Or,

Use for the third term the number which is of the same kind as the term required.

Arrange the like numbers in couplets, as in simple proportion.

The product of the means divided by the product of the extremes will be the term required.

Problems in compound proportion are readily solved by *cause* and *effect*. Example 1, stated by cause and effect, is as follows:

$$\begin{array}{cccccccc} \textit{1st Cause.} & & \textit{2d Cause.} & & & \textit{1st Effect.} & & \textit{2d Effect.} \\ \left.\begin{array}{r} 6 \text{ men} \\ 2 \text{ days} \\ 10 \text{ hours} \end{array}\right\} & : & \left.\begin{array}{r} 7 \text{ men} \\ (\ ) \text{ days} \\ 12 \text{ hours} \end{array}\right\} & :: & \left\{\begin{array}{l} \\ \\ \\ \end{array}\right. & 24 \text{ acres} & : & \left\{\begin{array}{l} \\ \\ \\ \end{array}\right. 56 \text{ acres} \end{array}$$

$$\begin{array}{l} \textit{Means} \\ \textit{Extremes} \end{array} \quad \frac{6 \times 2 \times 10 \times 56}{7 \times 12 \times 24} = 3\tfrac{1}{3}$$

2. If 15 men can dig a ditch in 45 days by working 10 hours a day, how many days will it take 20 men to dig it, by working 12 hours a day?

3. If a block of granite 6 feet long, 3 feet wide, and 2 feet thick, weighs 5940 pounds, what will be the weight of a block of the same kind, which is 9 feet long, 4 feet wide and 3 feet thick?

4. If I place \$1500 at interest for 18 months and receive \$135 interest, what sum must I place at interest at the same rate, so that I may receive \$275 interest in 8 months?

5. If it cost \$180 to support 5 grown persons and 3 children for 3 weeks, what will it cost to support 8 grown persons and 6 children for 7 weeks, allowing that it costs $\frac{1}{2}$ as much to support a child as a grown person?

6. If 20 men working 8 hours a day, can dig a trench 65 feet long, 9 feet wide and 6 feet deep, in 25 days, how many days will it take 25 men, working 10 hours a day, to dig a trench 75 feet long, 8 feet wide, and 7 feet deep?

7. If it costs $240 to board 16 persons 5 weeks, how much will it cost to board 9 persons 22 weeks?

8. If $800 placed at interest, amounts to $880 in 15 months, what sum must be placed at interest at the same rate, to amount to $975 in one year?

9. If it requires 275 yards of cloth $\frac{3}{4}$ yd. wide to make 75 garments, how many yards of cloth $1\frac{1}{4}$ yd. wide, will it require to make 215 such garments?

10. If a bin which is 8 feet long, 6 feet wide and 8 feet deep, holds 309 bushels of wheat, how many bushels will a bin hold that is 14 feet long, 8 feet wide and 9 feet deep?

11. If 15 men, working 10 hours a day, can do a certain piece of work in 18 days, how many days will it require for 13 men to do the same work, by working 8 hours a day?

12. If 12 horses consume 40 bushels of oats in 8 days, how long will 140 bushels of oats last 16 horses?

13. If a regiment of 1025 soldiers consume 11500 pounds of bread in 15 days, how many pounds will 3 regiments of the same size, consume in 12 days?

14. If the water that fills a vat, which is 8 feet long, 4 feet wide and 5 feet deep, weighs 10000 pounds, what will be the weight of the water required to fill a vat, which is 10 feet long, 5 feet wide and 6 feet deep?

15. If 5 horses eat as much as 6 cattle, and 8 horses and 12 cattle eat 12 tons of hay in 40 days, how much hay will be needed to keep 7 horses and 15 cattle 65 days?

16. If 15 men working 6 hours a day, can dig a cellar 80 feet long, 60 feet wide and 10 feet deep in 25 days, how many days will it require 25 men working 8 hours a day, to dig a cellar 120 feet long, 70 feet wide and 8 feet deep?

17. If 52 men can dig a trench 355 feet long, 60 feet wide and 8 feet deep in 15 days, how long will a trench be, that is 45 feet wide and 10 feet deep, which 45 men can dig in 25 days?

INVOLUTION

499. 1. Of what number are 3 and 3 the factors? 4 and 4?

2. Of what number are 3 and 3 and 3 the factors? 4 and 4 and 4?

3. What is the product when 5 is used twice as a factor?

4. What is the product or *power*, when 6 is used twice as a factor? When 8 is used twice as a factor?

5. What is the product of $\frac{2}{3} \times \frac{2}{3}$? Of $\frac{3}{4} \times \frac{3}{4}$?

6. What is the product when $\frac{3}{5}$ is used twice as a factor? When $\frac{2}{3}$ is used three times as a factor?

7. What is the product of two 4's, or the *second* power of 4? What is the product of three 5's, or the *third power* of 5? What is the third power of 6?

8. What is the second power of $\frac{2}{3}$? Of $\frac{3}{4}$? Of $\frac{4}{5}$?

DEFINITIONS.

500. A ***Power*** of a number is the product arising from using the number a certain number of times as a factor.

501. The powers of a number are *named* from the number of times the number is used as a factor.

Thus, when 2 is used as a factor *twice*, the product, 4, is called the *second power* of 2. 9 is the *second power* of 3. 27 is the *third power* of 3.

The number itself is called the *first power*.

502. The number of times a number is used as a factor is indicated by a small figure called an *Exponent*, written a little above and at the right of the number.

Thus, 3^2 means the *second power* of 3; 5^4, the *fourth power* of 5, etc.

Inasmuch as the area of a square is the product of *two equal factors*, and the volume of a cube is the product of *three equal factors*, the second power of a number is also called the *square*, and the third power the *cube* of the number.

503. ***Involution*** is the process of finding the power of a number.

WRITTEN EXERCISES.

504. 1. Find the third power of 15.

PROCESS.

$15 \times 15 \times 15 = 3375$

ANALYSIS.—To find the third power of a number is to find the product, when the number is used 3 times as a factor. Therefore, the third power of 15 will be $15 \times 15 \times 15$, which is equal to 3375.

2. Find the third power of 12. 23. 39. 24.
3. Find the second power of 47. 51. 29. 34.
4. What is the square of 15? 33? 24? 36? 25?
5. What is the cube of 28? 45? 18? 21? 41?
6. What is the third power of $\frac{5}{7}$? Ans. $\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} = \frac{125}{343}$.
7. What is the cube of $\frac{2}{3}$? Of $\frac{3}{5}$? $\frac{5}{9}$? $\frac{6}{11}$? $\frac{7}{13}$?
8. What is the fourth power of $\frac{5}{7}$? Cube of $\frac{9}{17}$?

Find the value of the following:

9. 15^4.	12. $.05^4$.	15. $(\frac{12}{17})^2$.	18. $(25\frac{1}{3})^2$.
10. 25^3.	13. $.005^3$.	16. $(\frac{15}{23})^2$.	19. $(3.00\frac{1}{3})^2$.
11. 30^3.	14. 2.05^2.	17. $(4\frac{1}{2})^3$.	20. $(4.500\frac{1}{7})^2$.

21. Raise 10 to the fourth power; 8 to the third power; 3 to the 6th power.

505. To find the square of a number in terms of its parts.

1. Find the square of 35 in terms of its tens and units.

PROCESS.

$$\begin{array}{l} 35 \\ \underline{35} \\ 25 = u^2 \\ \left.\begin{array}{l}15\\15\end{array}\right\} = 2t \times u \\ \underline{9} \quad = t^2 \\ 1225 = t^2 + 2t \times u + u^2 \end{array}$$

ANALYSIS.—If we square 35 or multiply 35 by itself and write every step in the process, we shall have for the first product 25, or the square of the units, for the next two products 15 tens, or two times the product of the tens and units, and for the third product 9 hundreds or the square of the tens. Hence,

506. PRINCIPLE.—*The square of any number consisting of tens and units, is equal to the tens*^{2} + *2 times the tens* $\times$ *the units* + *the units*^{2}.

Thus, $25 = 20 + 5$, and $25^2 = 20^2 + 2(20 \times 5) + 5^2$.

The above principle is true into whatever two parts the number may be separated, and the principle stated in general terms would be, the square of any number consisting of two parts is equal to the first part^2 + 2 times the first part $\times$ the second + second part^2.

Thus, $14 = 8 + 6$, and $14^2 = 8^2 + 2\ (6 \times 8) + 6^2$.

Express in terms of their tens and units the square of the following numbers:

2. 54.	5. 47.	8. 74.	11. 39.
3. 71.	6. 89.	9. 95.	12. 44.
4. 68.	7. 26.	10. 82.	13. 67

14. Square 16 by squaring its parts 9 and 7.
15. Square 20 by squaring its parts 12 and 8.
16. Square 32 by squaring its parts 30 and 2.
17. Square 13 by squaring its parts 7 and 6.
18. Square 26 by squaring its parts 9 and 17.
19. Square 17 by squaring its parts 8 and 9.

507. To find the cube of a number in terms of its parts.

1. Find the cube of 35 in terms of its parts.

PROCESS.

$$35^3 = \left.\begin{array}{r} 25 \\ 15 \\ 15 \\ 9 \end{array}\right\} \times 35 = \begin{array}{rl} 125 & = u^3 \\ \left.\begin{array}{r} 75 \\ 75 \\ 75 \end{array}\right\} & = 3t \times u^2 \\ \left.\begin{array}{r} 45 \\ 45 \\ 45 \end{array}\right\} & = 3t^2 \times u \\ 27 & = t^3 \\ \hline 42875 & = t^3 + 3t^2 \times u + 3t \times u^2 + u^3 \end{array}$$

ANALYSIS.—By multiplying the second power expressed as in Art. *505*, by 35, and writing every step, we shall have the cube of the tens, plus the product of three times the square of the tens multiplied by the units, plus the product of three times the tens multiplied by the square of the units, plus the cube of the units. Hence,

508. PRINCIPLE.—*The cube of any number consisting of tens and units is equal to the tens* $^3 + 3$ *times the tens* $^2 \times$ *the units* $+ 3$ *times the tens* $\times$ *the units*$^2 +$ *the units*^{3}.

Thus, $25 = 20 + 5$, and $25^3 = 20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3$.

The above principle may be stated in general terms thus: The cube of any number when separated into two parts is equal to the first part ^{3} $+ 3$ times the first part $^2 \times$ second part $+ 3$ times the first part multiplied by the second part $^2 +$ the second part ^{3}.

Express in terms of their tens and units the cube of the following numbers:

2. 26.	5. 42.	8. 38.	11. 52.
3. 31.	6. 27.	9. 39.	12. 64.
4. 28.	7. 36.	10. 54.	13. 66.

EVOLUTION

509. 1. What are the factors of 36? What are the two equal factors of 36? Of 49? Of 81?

2. What number used three times as a factor will produce 27? 64? 125? 216?

DEFINITIONS.

510. A ***Root*** of a number is one of the equal factors of the number.

Thus, 4 is a root of 16, because it is one of *two* equal factors.

Roots are named in a manner similar to powers. Thus, one of *two* equal factors of a number is the *second*, or *square root;* one of *three* equal factors, the *third*, or *cube root;* one of *four* equal factors, the *fourth root*, etc.

511. ***Evolution*** is the process of finding roots of numbers.

512. The ***Radical,*** or ***Root Sign,*** is $\sqrt{\ }$. When placed before a number it shows that its root is to be found.

When no figure or *index* is written in the opening of the radical sign, the square root is indicated; if the figure 3 is placed there, as $\sqrt[3]{\ }$, the cube root is indicated; if 4, as $\sqrt[4]{\ }$, the fourth root; etc.

513. A ***Perfect Power*** is a number whose root can be found.

514. An ***Imperfect Power*** is a number whose root can not be found exactly.

EVOLUTION BY FACTORING.

515. 1. What is the square root of 1225?

PROCESS.

$$\begin{array}{r} 5)\underline{1225} \\ 5)\underline{245} \\ 7)\underline{49} \\ 7)7 \end{array} \quad \sqrt{1225} = 5 \times 7 = 35$$

ANALYSIS.—Since the square root of a number is one of its two equal factors, we may find the square root of 1225 by separating it into its prime factors, and finding the product of the numbers forming one of the two equal sets of factors. The prime factors are 5, 5, 7 and 7, and 5 and 7 form one of the two equal sets. Therefore their product, 35, is the square root of 1225.

RULE.—*Separate the numbers into their prime factors. Arrange these factors into two, three, four, or any number of sets containing the same factors, according as the second, third, fourth, or any root is to be found.*

The product of the factors which form a set will be the root.

This method is valuable only when the numbers whose roots are sought are perfect powers.

2. Find the square root of 144. 256. 324. 576.
3. Find the cube root of 64. 512. 4096. 13824.
4. Find the fourth root of 1296. The fifth root of 248832.

ORDERS OF UNITS IN POWERS AND ROOTS.

516. 1. How does the number of figures which express the square of units, compare with the number expressing units?

2. How does the number of figures required to express the cube of units compare with the number expressing units?

3. Write the numbers, 10, 99, 100, 999, 1000, and under them their second and third powers.

4. How does the number of figures required to express the second powers compare with the number of figures required to express the given numbers?

5. How does the number of figures required to express the third powers compare with the number of figures required to express the given numbers?

517. PRINCIPLES.—1. *The square of a number is expressed by twice as many figures as the number itself or one less.*

2. *The cube of a number is expressed by three times as many figures as the number itself or one or two less.*

EXERCISES.

518. Tell by referring to the principles, how many figures there are in the following:

1. In the square of	21.	Of 15.	Of 115.	Of 4156.
2. In the cube of	19.	Of 25.	Of 316.	Of 6184.
3. In the square of	35.	Of 29.	Of 584.	Of 8196.
4. In the cube of	59.	Of 67.	Of 999.	Of 9999.

5. How many figures or orders of units are there in a number if the second power of it is expressed by 4 figures? By 7 figures? By 9 figures?

6. How many figures or orders of units are there in a number if the third power of it is expressed by 6 figures? By 8 figures? By 12 figures? By 11 figures? By 21 figures? By 25 figures?

519. PRINCIPLES.—1. *The orders of units in the square root of a number correspond to the number of periods of two figures each into which the number can be separated, beginning at units.*

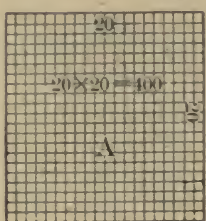
2. *The orders of units in the cube root of a number correspond to the number of periods of three figures each into which the number can be separated, beginning at units.*

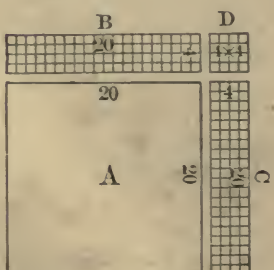
SQUARE ROOT.

520. 1. What is the square root of 576, or what is the side of a square whose area is 576 square units?

1st Process.

$$
\begin{array}{rrl}
 & 576 & (20 \\
20^2 = & \underline{400} & \quad \underline{4} \\
2 \times 20 = 40) & 176 & \quad 24 \\
(40 + 4) \times 4 = & \underline{176} &
\end{array}
$$





Analysis.—According to Prin. 1, Art. ***519,*** the orders of units in the square root of any number may be determined by separating the number into periods of two figures each, beginning at units. Separating 576 thus, there are found to be two orders of units in the root, or it is composed of tens and units. Since the square of tens is hundreds, 5 hundreds must be the square of at least 2 tens. 2 tens or 20 squared is 400, and 400 subtracted from 576 leaves 176, therefore the root 20 must be increased by such an amount as will exhaust the remainder.

The square (A) already formed from the 576 square units is one whose side is 20 units, but inasmuch as the number of units was not exhausted, such additions must be made to the square that they will exhaust the units and keep the figure a square. The necessary additions are two equal rectangles B and C, and a small square D.

Since the square D is small, the area of the rectangles B and C is nearly 176 units. The area, 176 units, divided by the length of the rectangles, will give the width, which is 4 units. The width of the additions is 4 units, and the entire length, including the small square, is 44 units; therefore the area of all the additions is 4 times 44 units, or 176 square units, which is equal to the entire number of units to be added. Therefore the side of the square is 24 units, or the square root of the number is 24.

2D PROCESS.

$$
\begin{array}{rl}
 & 5{\cdot}76\,(24 \\
t^2 = 2^2 = 4 & \\ \hline
2t = 40 & \big|\ 176 \\
u = 4 & \\
2t + u = 44 & \big|\ 176
\end{array}
$$

ANALYSIS.—In the same manner as before, the root of this number is shown to consist of tens and units. The tens can not be greater than 2; therefore we write 2 tens for a partial root. Squaring and subtracting, there is a remainder of 176, which is composed of 2 times the tens $\times$ units $+$ units^2, Art. ***506***. Since 2 times the tens multiplied by the units is much greater than the units squared, 176 is *nearly* two times the tens multiplied by the units. Therefore if 176 is divided by twice the tens, or 40, the quotient will be approximately the units of the root. Dividing, the units are found to be 4.

Since the tens are to be multiplied by the units, and the units are to be multiplied by the units or squared, and these results are to be added, to abridge the process the units are added to twice the tens and the sum multiplied by the units. Thus, $40 + 4$ is multiplied by 4, making 176. Therefore the square root of 576 is 24.

When the root consists of more than two orders of units, the process for solving is similar to that already given.

2. Find the square root of 137641.

1ST PROCESS.

$$
\begin{array}{rrr}
 & 13{\cdot}76{\cdot}41 & (300 \\
 & 9\ 00\ 00 & 70 \\
300\times2=600+70=670) & 4\ 76\ 41 & 1 \\
 & 4\ 69\ 00 & 371 \\
370\times2=740+1=741\ ) & 7\ 41 & \\
 & 7\ 41 &
\end{array}
$$

2D PROCESS.

$$
\begin{array}{rl}
 & 13{\cdot}76{\cdot}41(371 \\
 & 9 \\
67) & 476 \\
 & 469 \\
741) & 741 \\
 & 741
\end{array}
$$

In the first process the steps are given with considerable fullness, while in the second process all steps are abbreviated as much as possible.

RULE.—*Separate the number into periods of two figures each, beginning at units.*

Find the greatest square in the left-hand period, and write its root for the first figure of the required root.

Square this root and subtract the result from the left-hand period, and annex to the remainder the next period for a dividend.

Double the root already found for a trial divisor, and by it divide the dividend, disregarding the right-hand figure. The quotient or quotient diminished will be the second figure of the root.

Annex to the trial divisor for a complete divisor, the figure last found, multiply this divisor by the last figure of the root found, subtract the product from the dividend, and to the remainder annex the next period for the next dividend.

Proceed in this manner until all the periods have been used thus. The result will be the square root sought.

1. When the number is not a perfect square, annex periods of ciphers and continue the process.

2. Decimals are pointed off into periods of two figures each, by beginning at tenths and passing to the right.

3. The square root of a common fraction is found by extracting the square root of both numerator and denominator separately, or by reducing it to a decimal and then extracting its root.

Extract the square root of the following:

3. 2809.	7. 70756.	11. 938961.
4. 3969.	8. 118336.	12. 5875776.
5. 4356.	9. 674041.	13. 12574116.
6. 9216.	10. 784996.	14. 30858025.

15. Find the value of $\sqrt{222784}$; $\sqrt{11390625}$.

16. Find the value of $\sqrt{.763876}$; $\sqrt{.30858025}$.

17. What is the square root of .093636?
18. What is the square root of .099225?
19. What is the square root of $\frac{55225}{784996}$?
20. Extract the square root of $\frac{781456}{978121}$.
21. Extract the square root of $\frac{887364}{935089}$.
22. Extract the square root of $\frac{1}{2}$.
23. Extract the square root of $\frac{3}{4}$.
24. Extract the square root of $\frac{5}{8}$.
25. Extract the square root of .9.

APPLICATIONS OF SQUARE ROOT.

521. To find the side of a square when its area is given.

Since the area of a square is the product of two equal factors which represent its sides, the sides may be found by extracting the square root of the number expressing its area.

1. What is the side of a square whose area is 625 square feet?

2. What is the side of a square whose area is 2025 square rods?

3. A rectangle whose area is 5408 square feet is composed of two equal squares. What is the length of its sides?

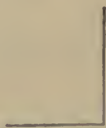
4. A man owns 50 acres of land in two square fields, one of which contains 4 times as much area as the other. How many rods of fence will be needed to fence both fields if they are not adjacent?

5. The length of a rectangular field containing 20 acres is twice its width. What is the distance around it?

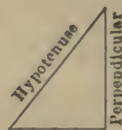
6. If it cost $572 to inclose with a fence a field that is 72 rods long and 32 rods wide, how much less will be the cost of inclosing a square field containing the same area?

522. To find any side of a right-angled triangle when the other sides are given.

Triangle



Right angle



Base

523. A ***Triangle*** is a figure which has three angles and three sides.

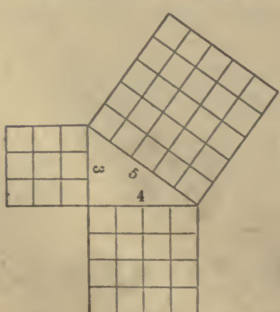
524. A ***Right Angle*** is the angle formed when one line is drawn perpendicular to another.

525. A ***Right-angled Triangle*** is a triangle which has a right angle.

526. The ***Hypotenuse*** of a right-angled triangle is the side opposite the right angle.

527. The ***Base*** of a triangle is the side on which it is assumed to stand.

528. The ***Perpendicular*** is the side which forms a right-angle with the base.



The relation of the squares described upon the sides of a right-angled triangle is expressed thus:

529. Principles.—1. *The square described upon the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.*

2. *The square on either of the other sides of a right-angled triangle is equal to the square on the hypotenuse diminished by the square on the other side.*

When the number of square units in the surface of any square figure is known, its side may be found by extracting the square root of the number according to the preceding case.

530. 1. The base of a right-angled triangle is 8 feet and the perpendicular 6 feet, what is the hypotenuse?

PROCESS.

$8^2 + 6^2 = 100.$

$\sqrt{100} = 10.$

ANALYSIS.—Before we can determine the length of the hypotenuse when the sides are given we must find the area of a square described upon it. The square described upon it is equal to the sum of the squares upon the other two sides, or the sum of $8^2 + 6^2$, which is 100. Since the area of the square described upon the hypotenuse contains 100 square units, the length of the side is the square root of 100 or 10.

To find the hypotenuse.

RULE.—*Extract the square root of the sum of the squares of the other two sides.*

To find the base or perpendicular.

RULE.—*Extract the square root of the difference of the squares on the hypotenuse and the other side.*

2. The base of a right-angled triangle is 15 feet and the perpendicular is 20 feet. What is the hypotenuse?

3. The base of a right-angled triangle is 40 feet and the hypotenuse is 120 feet. What is the perpendicular?

4. The perpendicular of a right-angled triangle is 30 feet and the hypotenuse is 50 feet. What is the base?

5. A tree 150 feet high, standing upon the bank of a stream, was broken off 125 feet from the top, and falling across the stream the top just reached the other shore. What was the width of the stream?

6. Two steamers start from the same point, one going due north at the rate of 15 miles an hour, and the other going due

west at the rate of 18 miles an hour. How far apart were they at the end of 6 hours?

7. A rectangular park, whose sides are respectively 45 rods and 60 rods in length has a walk crossing it from corner to corner. How long is the walk?

8. A certain assembly room is 100 feet in length, 60 feet in width, and 26 feet in height. What is the distance from a lower corner to the upper opposite corner?

9. Two buildings standing opposite each other are respectively 60 feet and 80 feet high. A ladder 125 feet long placed at a certain distance from the base of each just reaches the top of each. How far apart are the buildings?

10. The distance from the base of a building to a pole is 145 feet, and a string 225 feet long attached to the top of the pole just reaches the base of the building. What is the height of the pole?

11. A person who wished to ascertain the exact height of St. Paul's Cathedral in London, England, learned by inquiry that a rope extending from the top of the cross to a point 300 feet from the center of the circular pavement under the dome was 488 feet $10\frac{1}{2}$ inches long. If these data were correct, what is the height of St. Paul's?

SIMILAR FIGURES.

531. ***Similar Figures*** are such as are of the same form, and differ from each other only in size.

The truth of the following principles can be shown by geometry:

532. PRINCIPLES.—1. *Similar surfaces are to each other as the squares of their corresponding dimensions.* Hence,

2. *The corresponding dimensions of similar surfaces are to each other as the square roots of their areas.*

1. If the area of a triangle whose base is 16 rods, is 128 square rods, how many square rods are there in the area of a similar triangle whose base is 12 rods?

PROCESS.

$128 : x :: 16^2 : 12^2$ *or,*

$128 : x :: 256 : 144$

$x = 72$ sq. rd.

ANALYSIS.—Since the areas of similar figures are to each other as the squares of their like dimensions, the area of the first triangle (128 sq. rd.) will be to the area of the second triangle (x) as the square of the side of the first triangle (16^2) is to the square of the side of the second triangle (12^2). Solving the proportion, the area is 72 sq. rd.

2. If the area of a circle, whose diameter is 2 feet, is 6.2832 sq. ft. what will be the area of a circle whose diameter is 4 feet?

3. If the side of a rectangular field containing 25 acres is 40 rods, what is the side of a similar field containing 10 acres?

ANALYSIS.—Since the corresponding dimensions of similar surfaces are to each other as the square roots of their areas:

$$\sqrt{25} : \sqrt{10} :: 40 : x, \text{ or } 5 : \sqrt{10} :: 40 : x.$$

Extracting the square root of 10 and solving the proportion, x, or the corresponding side, is 25.296 rd.

4. If the side of a square field containing 40 acres is 80 rods, what will be the side of a similar field whose area is 25 acres?

5. If the area of a circle whose diameter is 20 feet is 314.16 square feet, what is the diameter of a circle whose area is 113.0976 square feet?

6. A farmer has two rectangular fields similar in form: one, whose length is 120 rods and whose breadth is 12 rods, contains 9 acres, the other contains $6\frac{1}{4}$ acres. What are its length and breadth?

7. A horse tied to a stake by a rope 7.13 rods long can graze upon just 1 acre of ground. How long must the rope be that he may graze upon 5 acres?

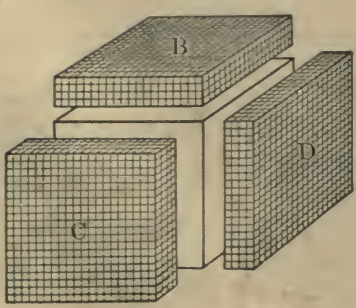
CUBE ROOT.

533. 1. What is the cube root of 13824, or what is the edge of a cube whose solid contents are 13824 units?

1ST PROCESS.

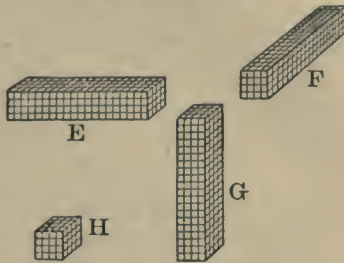
$$
\begin{array}{rr|l}
 & & 13\cdot824(20+4=24 \\
20^3 = & & 8\,000 \\
\cline{3-3}
3\times20^2 = & 1200 & 5\,824 \\
3\times4\times20 = & 240 & \\
4^2 = & 16 & \\
\cline{2-2}
 & 1456 & 5\,824
\end{array}
$$

ANALYSIS.—According to Pr. 2, Art. ***519,*** the orders of units in the cube root of any number may be determined from the number of periods obtained by separating the number into periods, containing three figures each, beginning at units. Separating the given number thus, there are two periods, or the root is composed of tens and units.

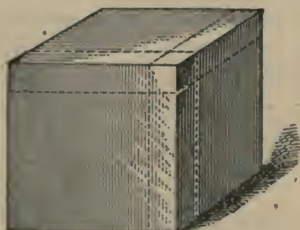


The tens in the cube root of the number can not be greater than 2, for the cube of 3 tens is 27000. 2 tens, or 20 cubed, are 8000, which, subtracted from 13824, leave 5824; therefore the root, 20, must be increased by a number such that the additions will exhaust the remainder.

The cube (A) already formed from the 13824 cubic units is one whose edge is 20 units. The additions to be made, keeping the figure formed a perfect cube, are 3 equal rectangular solids, B, C and D; 3 other equal rectangular solids, E, F and G; and a small cube, H. Inasmuch as the solids, B, C and D, comprise much the greatest part of the additions, their solid contents will be nearly 5824 cubic units, the contents of the addition.



Since the cubical contents of these three equal solids are nearly equal to 5824 units, and the superficial contents of a side of each of these solids are 20×20, or 400 square units, if we divide 5824 by 3 times 400, or 1200, since there are 3 equal solids, we shall obtain the thickness of the addition, which is 4 units.



Since all the additions have the same thickness, if their superficial contents, or area of each side, are multiplied by 4, the result will be the solid contents of these additions.

Besides the larger additions there are three others, E, F, and G, that are each 20 units long and 4 units wide, or whose sides have an area of 80 units each, and the area of all 240 units, and a small cube whose sides have an area of 16 units. The sum of these areas, 1456, multiplied by 4, the thickness of the additions, gives the solid contents of the additions, which are 5824 units.

Therefore the edge of the cube is 24 units in length, or the cube root of 13824 is 24.

2D PROCESS.

$$
\begin{array}{rr|l}
 & & 13.824\,(24 \\
t^3 = 20^3 = & & 8\,000 \\
3t^2 = 20^2 \times 3 = & 1200 & 5\,824 \\
3t \times u = (20 \times 4) \times 3 = & 240 & \\
u^2 = 4 \times 4 = & 16 & \\
3t^2 + 3tu + u^2 = & 1456 & \\
(3t^2 + 3tu + u^2) \times u = & & 5\,824
\end{array}
$$

ANALYSIS.—In the same manner as before, it may be shown that the root of the number contains only tens and units. The tens can not be greater than 2, for 3 tens cubed would equal 27000. Cubing and subtracting, there is left 5824, which is composed of three times the tens^2 $\times$ the units + 3 times the tens $\times$ the units^2 + the units^3, Art. **507.**

Since 3 times the tens^2 is much greater than 3 times the tens $\times$ the units^2 + the units^3, 5824 is a little more than 3 times the tens^2 $\times$ the units. If, then, 5824 is divided by 3 times the tens^2, or 1200, the trial divisor, the quotient 4, will be approximately the units of the root.

Since 5824 is equal to the sum of 3 times the tens^2 multiplied by the units, 3 times the tens multiplied by the units^2 and the units^3, the process may be shortened by adding together 3 times the tens^2, 3 times the tens $\times$ the units and the units^2, and multiplying this sum, 1456, by the units, 4. The product is 5824, which, subtracted from the number, leaves no remainder.

When the root consists of more than two orders of units the processes and analyses are similar to those already given.

2. What is the cube root of 48228544?

$$
\begin{array}{rr|r|r}
 & & 48\cdot228\cdot544 & 300 \\
300^3 = & & 27\ 000\ 000 & 60 \\
\cline{3-3}
3 \times 300^2 = & 270000 & 21\ 228\ 544 & \underline{\quad 4} \\
3 \times 300 \times 60 = & 54000 & & 364 \\
60^2 = & \underline{3600} & & \\
 & 327600 & \underline{19\ 656\ 000} & \\
3 \times 360^2 = & 388800 & 1\ 572\ 544 & \\
3 \times 360 \times 4 = & 4320 & & \\
4^2 = & \underline{16} & & \\
 & 393136 & 1\ 572\ 544 &
\end{array}
$$

RULE.—*Separate the number into periods of three figures each, beginning at units.*

Find the greatest cube in the left-hand period, and write its root for the first part of the required root.

Cube this root, subtract the result from the left-hand period, and annex to the remainder the next period for a dividend.

Take 3 times the square of the root already found for a trial *divisor, and by it divide the dividend. The quotient or the quotient diminished will be the second part of the root.*

To this trial divisor add 3 times the product of the first part of the root by the second part, and also the square of the second part. Their sum will be the entire divisor.

Multiply the entire divisor by the second part of the root and subtract the product from the dividend.

Continue thus until all the figures of the root have been found.

1. When there is a remainder, after subtracting the last product annex decimal ciphers, and continue the process. The figures of the root obtained will be decimals.

2. Decimals are pointed off into periods of three figures each, by beginning at tenths and passing to the right.

3. The cube root of a common fraction is found by extracting the cube root of both numerator and denominator separately, or by reducing it to a decimal and then extracting its root.

Extract the cube root of the following:

3. 74088.	6. 704969.	9. 5545233.
4. 262144.	7. 185193.	10. 2000376.
5. 166375.	8. 250047.	11. 153990656.

12. What is the cube root of 2 to 3 decimal places?

13. What is the cube root of 9 to 4 decimal places?

14. What is the cube root of .27? Of .64?

15. What is the cube root of $\frac{2}{3}$? $\frac{3}{5}$? $\frac{4913}{15252992}$?

APPLICATIONS OF CUBE ROOT.

534. 1. What is the length of the edge of a cubical box that contains 91125 cubic feet?

2. What are the dimensions of a cubical box that contains as much as a rectangular box, that is 2 feet 8 inches long, 2 feet 3 inches wide, and 1 foot 4 inches deep?

3. What is the depth of a cubical cistern whose contents are 2197 cubic feet?

4. What must be the depth of a cubical bin that will contain 1000 bushels?

5. What must be the depth of a cubical cistern that will hold 300 barrels of water?

6. A bin that contains 2000 bushels of grain, is just twice as long as it is wide or high. What is its length?

7. What is the depth of a cubical box that will hold a bushel?

8. What is the depth of a cubical box that will hold a barrel of water ($31\frac{1}{2}$ gal.)?

9. How much will it cost, at 30 cents per sq. yd., to plaster the bottom and sides of a cubical cistern that will hold 300 barrels?

10. A miller wishes to make a wagon-box large enough to hold 100 bushels, having the length 3 times the width and height. What will be its dimensions?

11. Which has the greater surface, a cube whose solid contents are 13824 cubic feet, or a rectangular solid having the same solidity, whose height is half its length and whose width is three-fourths its length? How much?

12. If a cubic metre contains 61026.048 cubic inches, what is the length of a linear metre?

SIMILAR FIGURES.

535. The truth of the following principles can be shown by geometry:

PRINCIPLES.—1. *Similar solids are to each other as the cubes of their like dimensions.* Hence,

2. *The corresponding dimensions of similar solids are to each other as the cube roots of their volumes.*

1. If a globe 4 inches in diameter weighs 8 lb., what will be the diameter of a similar one that weighs 125 lb.?

PROCESS.

$4 : x :: \sqrt[3]{8} : \sqrt[3]{125}$ (1)

$4 : x :: 2 : 5$ (2)

x or diameter is 10 in.

ANALYSIS.—Since the corresponding dimensions of similar solids are proportional to the cube roots of these volumes, we have the diameter of the smaller globe 4 inches : the diameter of the larger globe x : : the cube root of the weight of the smaller globe $\sqrt[3]{8}$: the cube root of the weight of the other globe $\sqrt[3]{125}$. (1). Extracting the cube root of 8 and 125, and we have Prop. (2). Whence solving, the diameter is 10 inches.

2. If a ball 5 ft. in diameter weighs 800 lb., what will be the diameter of a similar ball which weighs 3 T. 4 cwt.?

3. If a globe of gold 1 inch in diameter is worth $125, what will be the value of one 3 inches in diameter?

4. If a cubical bin 8 ft. long will hold 411.42 bu., what must be the dimensions of a similar bin, that will hold 1000 bushels?

5. A ball 3 feet in diameter weighed 2000 lb. What will be the diameter of one that weighs 1000 lb.?

6. The dimensions of a cubical bin were such that it would contain 1000 bushels of wheat. How would the dimensions of a similar bin that would hold 8000 bushels compare with the dimensions of such a bin?

7. The diameters of two spheres are respectively 4 and 12 inches. How many times the smaller sphere is the larger?

8. Three women own a ball of yarn 4 inches in diameter. How much of the diameter of the ball must each wind off, so that they may share equally?

9. A stack of hay in the form of a pyramid 12 ft. high, contained 8 tons. How high must a similar stack be, that it may contain 60 tons?

PROGRESSIONS

536. 1. How does each of the numbers 2, 4, 6, 8, 10, 12, compare with the number that follows it?

2. How may each of the numbers 4, 6, 8, etc., be obtained from the one that precedes it?

3. How does each of the numbers 2, 5, 8, 11, 14, 17, compare with the number that follows it? How with the one that precedes it?

4. Write in succession some numbers beginning with 3 having a *common difference* of 2.

5. Write a *series* of numbers beginning at 4, and having a common difference of 4.

6. Write a series of numbers beginning with 25, and decreasing by the common difference 4.

7. How does each of the numbers 2, 4, 8, 16, 32, etc., compare with the one that follows it? How may each be obtained from the one that precedes it?

8. Write a series of numbers beginning with 2 and increasing by a common multiplier 3.

9. Write a series of numbers beginning with 5, and increasing by a common multiplier 5.

DEFINITIONS.

537. A ***Series*** of numbers is numbers in succession, each derived from the preceding according to some fixed laws.

538. The first and last terms of a series are called the *extremes*, the intervening terms the *means*.

Thus, in the series 2, 4, 6, 8, 10, the numbers 2 and 10 are the extremes and the others are the means.

539. An ***Ascending Series*** is one in which the numbers increase regularly from the first term.

Thus, 2, 5, 8, 11, 14, 17, 20, etc., is an ascending series.

540. A ***Descending Series*** is one in which the numbers decrease regularly from the first term.

Thus, 48, 24, 12, 6, 3, is a descending series.

ARITHMETICAL PROGRESSION.

541. An ***Arithmetical Progression*** is a series of numbers which increase or decrease by a constant *common difference*.

Thus, 5, 9, 13, 17, 21, etc., is an arithmetical progression of which the common difference is 4.

542. 1. The first term of an arithmetical series is 3 and the common difference is 2. What is the 7th term?

PROCESS.

Com. diff., $2 \times 6 = 12$

First term, $3 + 12 = 15$, the 7th term.

ANALYSIS.—Since the common difference is 2, the second term is equal to the first plus *once* the common difference, the third term is equal to the first plus *twice* the common difference, the fourth term is equal to the first term plus *three times* the common difference. Hence, the seventh term will be equal to the first term plus *six times* the common difference, which is 15.

RULE.—*Any term of an arithmetical progression is equal to the first term, increased by the common difference multiplied by a number one less than the number of terms.*

22

2. The first term is 10 and the common difference 5. What is the 10th term? Prove it.

3. The first term is 6 and the common difference is 8. What is the 25th term?

4. A boy agreed to work for 50 days at 25 cents the first day, and an increase of 3 cents per day. What were his wages the last day?

5. A body falls $16\frac{1}{12}$ feet the first second, 3 times as far the second second, 5 times as far the third second. How far will it fall the seventh second?

6. An arithmetical series has 1000 terms, the first term of which is 75 and the common difference 5. What is the last term?

7. Find the sum of an arithmetical series of which the first term is 2, the common difference 3, and the number of terms 7.

PROCESS.

$$2 + (6 \times 3) = 20$$
$$2 + 20 = 22$$
$$22 \div 2 = 11$$
$$11 \times 7 = 77$$

ANALYSIS.—By examining the series 2, 5, 8, 11, 14, 17, 20, it is evident that the average term is 11, for if half the sum of any two terms equidistant from the extremes be found it will be 11, and in general in any arithmetical progression the average term is equal half the sum of the extremes or any two terms equidistant from the extremes. Since the first term is 2 and the common difference 3, the last term is found by the previous rule to be 20. The sum of the extremes is therefore 22, which, divided by 2, gives the average term. And since there are 7 terms, the sum will be 7 times the average term, or 77.

RULE.—*To find the sum of an arithmetical series: Multiply half the sum of the extremes by the number of terms.*

8. What is the sum of an arithmetical series composed of 50 terms, of which the first term is 2 and the common difference 3?

9. What is the sum of a series in which the first term is $\frac{1}{10}$ the common difference, $\frac{1}{10}$, and the number of terms 100?

10. A man walked 15 miles the first day, and increased his rate 3 miles per day for 10 days. How far did he walk in the eleven days?

11. How many strokes does a clock strike in 12 hours?

12. A person had a gift of $100 per year from his birth until he became 21 years old. These sums were deposited in a bank and drew simple interest at 6%. How much was due him when he became of age?

GEOMETRICAL PROGRESSION.

543. A ***Geometrical Progression*** is a series of numbers which increase or decrease by a constant *multiplier* or *ratio.*

Thus, 5, 10, 20, 40, 80, etc., is a geometrical progression, of which the multiplier or ratio is 2.

WRITTEN EXERCISES.

544. 1. The first term of a geometrical series is 3 and the multiplier or ratio is 2. What is the 5th term?

PROCESS.

$2^4 = 16$

$3 \times 16 = 48$

ANALYSIS.—Since the multiplier is 2, the second term will be 3×2, the third $3 \times 2 \times 2$ or 3×2^2, the fourth $3 \times 2^2 \times 2$ or 3×2^3 and the fifth $3 \times 2^3 \times 2$ or 3×2^4, that is, the *fifth* term is equal to the first term multiplied by the ratio raised to the *fourth* power.

RULE.—*Any term of a geometrical progression is equal to the first term, multiplied by the ratio raised to a power one less than the number of the term.*

2. The first term of a geometrical progression is 10, the ratio 3. What is the 6th term?

3. The first term of a geometrical progression is 10, the ratio 4, and the number of terms 6. What is the 6th term?

4. If a farmer should hire a man for 10 days, giving him 5 cents for the first day, 3 times that sum for the second day, and so on, what would be his wages for the last day?

5. If the first term is \$100 and the ratio 1.06, what is the 6th term? Or, what is the amount of \$100 at compound interest for 5 years at 6%?

6. What is the amount of \$520 for 6 years, at 5% compound interest?

7. What is the sum of a geometrical series, of which the first term is 5, the ratio 3, and the number of terms 5?

PROCESS.

$5 \times 81 = 405$, the 5th term.

$\frac{3 \times 405 - 5}{3 - 1} = 605$, the sum.

ANALYSIS.—Since in this series the first term is 5, the ratio 3, and the number of terms 5, their sum may be obtained by the following process, which illustrates the formation of the rule:

$$\begin{array}{rl} \text{Series} & 5 + 15 + 45 + 135 + 405 \\ \text{3 times Series} & \underline{\quad\;\; 15 + 45 + 135 + 405 + 1215} \\ \text{2 times Series} & = 1215 - 5 \\ \text{Series} & = \frac{1215-5}{2} \end{array}$$

RULE.—*The sum of a geometrical series is equal to the difference between the first term, and the product of the last term by the ratio, divided by the difference between the ratio and 1.*

8. The extremes of a geometrical progression are 4 and 1024, and the ratio 4. What is the sum of the series?

9. The extremes are $\frac{1}{5}$ and $\frac{346}{135}$ and the ratio $2\frac{1}{3}$. What is the sum of the series?

10. What is the sum of the series in which the first term is 2, the last term 0, and the ratio $\frac{1}{2}$; or what is the sum of the infinite series 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, etc?

MENSURATION

DEFINITIONS.

545. ***Mensuration*** treats of the measurement of lines, surfaces, and solids.

546. A ***Line*** is that which has length only.

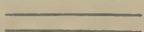
547. A ***Straight Line*** is a line that does not change its direction.

Straight Line.

548. A ***Curved Line*** is a line that changes its direction at every point.

Curved Lines.

549. ***Parallel Lines*** are such as are equidistant throughout their whole extent.



Parallel Lines.

550. A ***Plane Surface*** is a surface such that a straight line joining any two points of it is wholly in the surface.

551. A ***Curved Surface*** is a surface such that no part of it is a plane surface.

552. An ***Angle*** is the divergence of two lines that meet.

Angle.

553. A ***Right Angle*** is the angle formed when one straight line meets another making the adjacent angles equal.

The lines are *perpendicular* to each other when a right angle is formed.

Two Right Angles.


Acute Angle.

554. An ***Acute Angle*** is an angle which is less than a right angle.

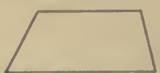

Obtuse Angle.

555. An ***Obtuse Angle*** is an angle which is greater than a right angle.

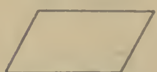
556. The ***Vertex*** of an angle is the point where the sides meet.


Triangle.

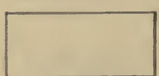
557. A ***Triangle*** is a figure which has three sides and three angles.


Quadrilateral.

558. A ***Quadrilateral*** is a figure bounded by four sides.


Parallelogram.

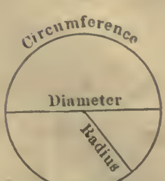
559. A ***Parallelogram*** is a quadrilateral whose opposite sides are parallel.


Rectangle.

560. A ***Rectangle*** is a parallelogram whose angles are right angles.


Polygon.

561. A ***Polygon*** is a plane figure bounded by straight lines.



Circle.

562. A ***Circle*** is a plane figure bounded by a curved line every point of which is equally distant from a point within called the *center*.

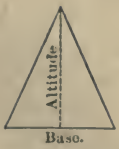
563. The ***Circumference*** is the line which bounds the circle.

564. A ***Radius*** of a circle is a straight line drawn from the center to the circumference.

565. A ***Diameter*** of a circle is a straight line drawn through the center and terminating at both ends in the circumference.

566. The ***Base*** of a figure is the side on which it is assumed to stand.

567. The ***Altitude*** of a figure is the perpendicular distance between the base and the highest point opposite it.



568. A ***Diagonal*** of a figure is a straight line joining the vertices of two angles not adjacent.

569. The ***Perimeter*** of a figure is the length of the lines that bound it.

570. The ***Area*** of a surface is the definite amount of surface it contains.

MEASUREMENT OF LINES.

571. It can be shown by geometry that the circumference of a circle is 3.1416 — times its diameter.

For ordinary measurements it is sufficiently accurate to consider the circumference $3\frac{1}{7}$ times the diameter.

Rule.—1. *The circumference is equal to the diameter multiplied by* 3.1416.

2. *The circumference divided by* 3.1416 *is equal to the diameter.*

WRITTEN EXERCISES.

572. 1. What is the circumference of a circle 10 feet in diameter?

2. What is the circumference of a circle 45 feet in diameter?

3. How far is it around a circular lake that is 300 rods in diameter?

4. What is the circumference of a circle whose radius is 20 rods?

5. What is the circumference of a circle whose radius is 5 feet 6 inches?

6. What is the diameter of a circle whose circumference is 318.5 rods?

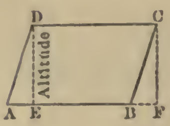
7. What is the radius of a circle whose circumference is 1284 rods?

MEASUREMENT OF SURFACES.

573. To compute the area of a parallelogram.

The truth of the following principle has been shown already:

PRINCIPLE.—*The area of any rectangular figure is equal to the product of its length by its breadth or altitude.*



By examining the figure A, B, C, D, it will be seen that it is equal to E, F, D, C, and that any *oblique* parallelogram is equal to a rectangular parallelogram of the same base and altitude. Therefore,

RULE.—*The area of any parallelogram is equal to the product of the base multiplied by the altitude.*

WRITTEN EXERCISES.

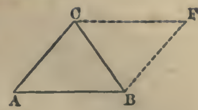
574. 1. How many square feet are there in a parallelogram, whose length is 40 feet and altitude 13 feet?

2. What is the area of a parallelogram whose base measures 7 feet and whose altitude is 3 feet 8 inches?

3. What is the area of a field in the form of a parallelogram, whose length is 30 rods and the perpendicular distance between the sides is 24 rods?

4. What is the area of a parallelogram whose length is 35 feet and whose altitude is 15 feet?

575. To compute the area of a triangle.



If C E be drawn parallel to the base of the triangle, and B E be drawn parallel to A C, the parallelogram A B E C will be formed, of which the original triangle is one-half. In the same manner it can be shown that every triangle is one-half of a parallelogram of the same base and altitude. Therefore,

RULE.—*The area of a triangle is equal to one-half the product of the base by the altitude.*

When the three sides are given, the following is the rule:

RULE.—*From half the sum of the three sides subtract each side separately. Multiply together the half sum and the three remainders, and extract the square root of the product. The result will be the area of the triangle.*

WRITTEN EXERCISES.

576. 1. What is the area of a triangle whose base is 24 feet, and whose altitude is 18 feet?

2. What is the area of a triangle whose base is 21 feet and whose altitude is 12 feet?

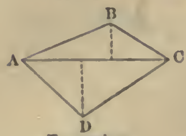
3. What is the cost, at $850 per acre, of a triangular piece of ground, the three sides of which are in a ratio of 5, 6 and 8, and whose shortest side is 120 feet?

4. What is the area of a triangle, the three sides of which are respectively 180 feet, 150 feet, and 80 feet?

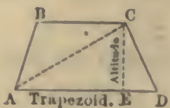
5. A house is 32 feet wide, and the rafters are 20 feet long on each side, exclusive of any projections. What will the lumber cost at $22.50 per M, which will inclose both gable ends of the house?

6. What is the area of a triangle whose base is 300 feet, and whose altitude is 100 feet?

577. To compute the area of a polygon.



Trapezium.



Trapezoid.

Regular Polygon.

Since any figure may be divided into triangles, its area will be the area of the triangles which compose it. Therefore,

RULES.—I. *The area of a trapezium is equal to the diagonal, multiplied by half the sum of the perpendiculars drawn from the vertices of the opposite angles to the diagonal.*

II. *The area of a trapezoid is equal to the sum of the parallel sides multiplied by half the altitude.*

III. *The area of a regular polygon is equal to the perimeter of the polygon multiplied by one-half the perpendicular distance from the center to one of the sides of the polygon.*

WRITTEN EXERCISES.

578. 1. What is the area of a trapezium, the diagonal of which is 110 feet, and the perpendiculars to the diagonal are 40 feet and 60 feet respectively?

2. The parallel sides of a trapezoid are respectively, 10 rods and 8 rods, and the altitude 6 rods. What is its area?

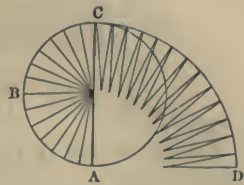
3. A figure is composed of 8 triangles, whose bases are each 12 ft. and their altitude 12 ft. What is the area?

4. What is the cost, at $125 per acre, of a piece of ground in the form of a trapezoid, whose parallel sides are respectively, 40 rods and 30 rods, and whose altitude is 20 rods?

5. I paid $110 per acre for a piece of ground in the form of a trapezium. A diagonal line crossing it was 120 rods long, and the perpendiculars drawn to the diagonal were, respectively, 30 rods and 20 rods. What did it cost me?

579. To compute the area of a circle.

From the accompanying figure it is evident that a circle may be regarded as composed of a large number of triangles, the sum of whose bases forms the circumference of the circle, and whose altitude is the radius of the circle. Hence,



RULE.—1. *The area of a circle is equal to the circumference, multiplied by half the radius;* or,

2.—*The square of the diameter multiplied by* .7854.

WRITTEN EXERCISES.

580. 1. What is the area of a circle whose diameter is 5 feet?

2. What is the area of a circle whose diameter is 8 feet?

3. What is the area of a circle whose circumference is 120 rods?

4. What is the area of a circle whose circumference is 100 feet?

5. A gentleman discovered that the distance around a circular pond was 320 rods. What was its area?

6. If a horse is tethered to a stake by a rope 15 rods long, over how much surface can he graze?

7. How long must a rope be that a horse can graze on just an acre?

8. The area of a circle is 113.0976 square rods. What is its diameter?

9. The round-house of the P. and S. Railroad is 350 feet in diameter. How much land does it cover?

10. What is the area of a railroad turn-table 35 feet in diameter?

MEASUREMENT OF SOLIDS.

DEFINITIONS.

581. A ***Solid*** or ***Body*** is that which has length, breadth and thickness.

The planes which bound a solid are called its *faces*, and their intersections its *edges*.

Triangular Prism. Quadrangular Prism. Parallelopipedon. Cylinder.

582. A ***Prism*** is a solid, having its two ends equal polygons, parallel to each other, and its sides parallelograms.

Prisms are named from the form of their bases *triangular*, *quadrangular*, *pentagonal*, etc.

583. A ***Parallelopipedon*** is a solid whose opposite faces are equal and parallel parallelograms.

584. A ***Cylinder*** is a regular solid bounded by a uniformly curved surface, and having for its ends two equal circles, parallel to each other.

The face of any section of a cylinder parallel to the base is a circle equal to the base.

585. A ***Pyramid*** is a solid whose base is a polygon and whose faces are triangles, meeting at a point called the vertex of the pyramid.

586. A ***Cone*** is a solid, whose base is a circle and whose surface tapers uniformly to a point called the vertex.

Pyramid.

Cone.

Frustum of Cone.

Frustum of Pyramid.

587. A ***Frustum*** of a pyramid or cone is the portion remaining, after the top has been cut off by a plane parallel to the base.

588. A ***Sphere*** is a solid, every point of whose surface is equally distant from a point within, called the center.

Sphere.

589. The ***Diameter*** of a sphere is a straight line passing through the center, and terminating in the surface at both ends.

590. The ***Radius*** of a sphere is one-half the diameter, or the distance from the center to the surface.

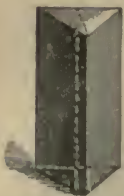
591. The ***Circumference*** of a sphere is the greatest distance around the sphere.

592. The ***Altitude*** of a solid is the perpendicular distance from its highest point to the plane of the base.

CONVEX SURFACE OF SOLIDS.

593. The ***Convex Surface*** of a solid, is all its surface except its base or bases. The *entire* convex surface includes the area of the bases also.

594. To find the convex surface of a prism or cylinder.



It is evident that if a prism or cylinder were 1 inch high, its convex surface would contain as many square units of surface as there were units in the perimeter of the base; and if it were 2 inches, 3 inches, or 4 inches high, the convex surface would contain 2, 3, or 4 times the number of units in the perimeter of the base. Hence the following

RULE.—*Multiply the perimeter of the base by the altitude.*

WRITTEN EXERCISES.

595. 1. What is the convex surface of a cylinder whose diameter is 2 feet and length 5 feet?

2. What is the convex surface of a quadrangular prism whose sides are each $2\frac{1}{2}$ feet and whose height is 4 feet?

3. What is the convex surface of a triangular prism whose sides are each 6 feet and whose altitude is 8 feet?

4. What is the entire surface of a cylinder which is 5 feet in length, and whose base is 2 feet in diameter?

5. What is the convex surface of a piece of timber in the form of a triangular prism, which is 18 feet long and the sides of whose base are 10 inches, 14 inches, and 18 inches?

596. To find the convex surface of a pyramid or cone.

It is evident that the convex surface of any pyramid is composed of triangles, and the convex surface of a cone may also be assumed to be made up of an infinite number of triangles. The bases of these triangles form the perimeter of the solid, and their height is the slant height of the solid. Therefore the following is the rule:

RULE.—*Multiply the perimeter of the base by one-half the slant height.*

WRITTEN EXERCISES.

597. 1. What is the convex surface of a quadrangular pyramid, whose base is 15 feet square and the slant height 18 feet?

2. What is the convex surface of a cone whose diameter at the base is 12 feet and whose slant height is 20 feet?

3. What is the convex surface of a cone whose base is 20 feet in diameter and whose slant height is 20 feet?

4. What is the cost of painting a church steeple, the base of which is an octagon 6 feet on each side, and whose slant height is 80 feet, at $.30 per square yard?

5. How many feet of convex surface are there on a cone, the base diameter of which is 6 feet and whose slant height is $9\frac{1}{2}$ feet?

6. How many feet of convex surface are there on a pyramid whose base is 10 feet square and whose slant height is 20 feet?

7. How many feet of convex surface are there on a cone whose base is 8 feet in diameter and whose slant height is 6 feet?

8. What is the convex surface of a cone whose base is 10 feet in diameter and whose slant height is 10 feet?

598. To find the convex surface of a frustum of a pyramid or cone.

It is evident that the convex surface of a frustum of a pyramid is composed of trapezoids, the sum of whose parallel sides forms the perimeter of the bases, and whose altitude is the slant height of the frustum; and the convex surface of a cone may be assumed to be made of an infinite number of trapezoids. Hence,

RULE.—*Multiply half the sum of the perimeter of the two bases by the slant height.*

WRITTEN EXERCISES.

599. 1. How many feet of convex surface are there in the frustum of a cone whose slant height is 8 feet, the diameter of whose lower base is 12 feet and upper base 8 feet?

2. What is the convex surface of the frustum of a pyramid the slant height of which is 25 feet, whose lower base is 40 feet square, and whose upper base is 20 feet square?

3. What did it cost, at $.15 per sq. yd., to paint the convex surface of a vat which was 10 feet in diameter at the bottom and 8 feet at the top, the slant height of which was 12 feet?

4. What is the convex surface of a vat, the base of which is 9 feet square whose top is 8 feet square and whose slant height is 10 feet?

5. What would the lumber cost at $40 per M, to build such a vat if the sides were of $1\frac{1}{2}$ inch plank, and the bottom was 2 inch plank?

600. To find the convex surface of a sphere.

The convex surface of a sphere is computed, according to geometrical principles, by the following rule:

RULE.—1. *Multiply the diameter by the circumference.*

2. *Multiply the square of the diameter by* 3.1416.

EXERCISES.

601. 1. What is the convex surface of a sphere whose diameter is 15 inches?

2. What is the convex surface of a spherical cannon-ball 8 inches in diameter?

3. What is the convex surface of a base-ball whose circumference is $9\frac{1}{8}$ inches?

4. What is the convex surface of a sphere whose circumference is 12 feet?

VOLUME OF SOLIDS.

602. The ***Volume*** of any body is the number of solid units it contains.

603. To find the volume of a prism or cylinder.

It is evident that if a prism or cylinder were 1 inch high, it would contain as many cubic inches as there were square inches in the area of the base; and if it were 2 inches, 3 inches, or 4 inches high, the volume would be 2 or 3 or 4 times as much. Hence the following is the rule:

RULE.—*Multiply the area of the base by the altitude.*

WRITTEN EXERCISES.

604. 1. What are the solid contents of a prism whose base is 12 inches square and whose height is 2 feet?

2. What is the volume of a cylinder whose diameter is $1\frac{1}{2}$ feet and whose length is 4 feet?

3. What would be the cost of a piece of timber 20 feet long, 18 inches wide and 12 inches thick at $.30 per cubic foot?

4. What will be the capacity in bushels of a square bin the base of which was 8 feet square, and the height of which was 9 feet on the inside?

5. How many gallons of water will a vat in the form of a cylinder hold, whose inside dimensions are—base 8 feet in diameter, height 7 feet?

6. How much would the wheat be worth at $1.85 per bushel, which would just fill a bin the base of which is 15 feet square, and the height of which is 12 feet?

605. To find the volume of a pyramid or cone.

It can be shown by geometry that a pyramid or cone is one-third of a prism or cylinder of the same base and altitude. Hence the following is the

RULE.—*Multiply the area of the base by one-third of the altitude.*

WRITTEN EXERCISES.

606. 1. What are the solid contents of a cone, the diameter of whose base is 6 feet and whose altitude is 9 feet?

2. What are the solid contents of a pyramid whose base is 30 feet square and whose altitude is 60 feet?

3. If a cubic foot of granite weighs 165 lb., what is the weight of a granite cone the diameter of whose base is 6 feet and whose altitude is 8 feet?

4. What is the weight of a marble pyramid whose base is 4 feet square and whose altitude is 8 feet, if a cubic foot of marble weighs 171 pounds?

607. To find the volume of a frustum of a pyramid or cone.

It can be shown by geometry that the frustum of a pyramid or cone is equal to three pyramids or cones, having for their bases, respectively, the upper base of the frustum, its lower base, and a mean proportional between the two bases. Hence the following is the

RULE.—*To the sum of the areas of the two ends add the square root of the product of these areas, and multiply the result by one-third of the altitude.*

EXERCISES.

608. 1. What is the volume of a frustum of a pyramid the lower base of which is 20 feet square, the upper base 10 feet square and the altitude 20 feet?

2. What are the solid contents of the frustum of a cone whose upper base is 5 feet in diameter, whose lower base is 8 feet in diameter, and whose altitude is 7 feet?

3. A tree was 3 feet in diameter at the butt and its diameter at a height of 40 feet was 1 foot. What were the cubical contents of that portion of the tree?

4. A vat whose inside measurements were as follows—diameter of the bottom 12 feet, diameter of the top 10 feet, height 9 feet—was filled with water. How many gallons did it contain?

609. To find the volume or contents of a sphere.

A sphere may be regarded as composed of pyramids whose bases form the surface of the sphere, and whose altitude is the radius of the sphere. Hence the following is the

RULE.—1. *Multiply the convex surface by one-third of the radius;* or,

2. *Multiply the cube of the diameter by* .5236.

EXERCISES.

610. 1. The diameter of a sphere is 5 feet. How many cubic feet does it contain?

2. Find the contents of a sphere whose diameter is 8 feet.

3. The circumference of a sphere is 9.4248. What are its cubical contents?

4. A cubic foot of cast-iron weighs about 450 pounds. What is the weight of a cannon-ball whose diameter is 18 inches?

5. What are the cubical contents of a spherical vessel the diameter of which is 2½ feet?

6. How many cubic feet are there in a spherical body whose diameter is 25 feet?

MISCELLANEOUS EXAMPLES.

611. 1. If 5 men can do a piece of work in 12 days, how long will it take 6 men to do the same work?

2. If 5 barrels of apples cost \$7.50, what will 8 barrels cost at the same rate?

3. It required 20 men to load a vessel in 6 days, how many men would it require to load it in $1\frac{1}{2}$ days?

4. A steamboat sailed $42\frac{1}{2}$ miles in $2\frac{1}{2}$ hours. How far did she sail in 20 minutes?

5. If 6 men can dig 28 rods of ditch in 1 day, how many men will it require to dig 56 rods in $\frac{3}{4}$ of a day?

6. If $\frac{2}{3}$ of a yard of broadcloth cost $\$3\frac{3}{5}$, what will $\frac{7}{8}$ of a yard cost?

7. If it costs \$50 to support a family of 8 persons for $2\frac{1}{2}$ weeks, what will it cost to support 10 persons 3 weeks?

8. If 3 pounds of tea are worth 14 pounds of coffee, and 5 pounds of coffee are worth 18 pounds of sugar, and 21 pounds of sugar are worth 60 pounds of flour, how many pounds of flour are equal in value to 7 pounds of tea?

9. A farmer sold 12 firkins of butter, each containing 56 pounds, for 23 cents a pound, and received in payment 5 pounds tea at 85 cents per pound, 60 pounds sugar at 13 cents per pound, 15 yards cloth at $\$1.12\frac{1}{2}$ per yard, and the balance in money. How much money did he receive?

10. A regiment of soldiers consisting of 1100 men, was furnished with bread sufficient to last it 8 weeks, allowing each man 15 oz. per day. If $\frac{1}{9}$ of it was found to be unfit for use, how many ounces per day shall each man receive so that the balance may last 8 weeks?

11. A man being asked how many sheep he had, replied, "If I had 3 times as many as I have and 5 sheep, I would have 185." How many had he?

12. A man paid $\frac{1}{2}$ of his money on a debt, $\frac{1}{2}$ of the remainder for a suit of clothes, $\frac{1}{2}$ of the remainder for provisions, and lost $\frac{1}{2}$ of the remainder, when he had \$5 left. How much had he at first?

13. Three men engage to reap a field of wheat. A can do it in 15 days, B in 18 days and C in 20 days. In what time can they do it together?

14. A farmer was offered \$1.45 per bu. for his wheat, but determined to have it ground and sell the flour. It cost to take it to the mill $2\frac{1}{4}$ cents per bu.; the miller took $\frac{1}{8}$ for grinding; it took $4\frac{40}{60}$ bu. to make a barrel of flour; he paid 45 cents apiece for barrels, and it cost 25 cents per barrel commission to sell it. 75 bbl. sold for \$550 and 25 bbl. for \$165. If the refuse was sold for \$100, did he make or lose, and how much per hundred barrels?

15. A farmer being asked how many apple-trees he had, replied, "If I had 3 times as many and 5 trees more, I would have 1358." How many had he?

16. $\frac{1}{2}$ of A's money is equal to $\frac{3}{5}$ of B's, and the difference is \$8. How much has each?

17. A, B and C hire a pasture for \$170. A puts in 70 sheep for $6\frac{1}{2}$ months, B 24 cattle for $4\frac{1}{6}$ months, C 10 cattle and 35 sheep for $5\frac{1}{2}$ months. If 2 cattle eat as much as 7 sheep, how much should each pay?

18. If a pole 10 feet long, casts a shadow 13 feet long, what is the length of a pole which will cast a shadow $62\frac{1}{2}$ feet long at the same time?

19. A's weight is $\frac{3}{4}$ that of B, and C's weight is as much as A's and B's together. The sum of their weights is 490 pounds. What is the weight of each?

20. $\frac{3}{8}$ of A's money is equal to $\frac{4}{9}$ of B's, and the difference is \$5. How much money has each?

21. The ages of A, B and C, are to each other as 3, 4 and 5, and their sum is 136 years. What is the age of each?

22. A boy bought a certain number of apples at the rate of 4 for 5 cents, and sold them at the rate of 3 for 4 cents. He gained 60 cents. How many did he buy?

23. A, B and C agree to build a house. A and B can do the work in 32 days, B and C in 28 days, and A and C in 26 days. How long will it take them to do it by working together? How long would it take each to do it alone?

24. A can build a wall in 10 days, by working 12 hours a day, B can build it in 9 days, by working 10 hours a day. In how many days can both build it, by working 8 hours a day?

25. A pair of horses is sold for $390. One of them is worth $\frac{5}{8}$ as much as the other. What is the value of each?

26. A hind wheel of a carriage 4 feet 6 inches high, revolved 720 times in going a certain journey. How many revolutions did the fore wheel make, which was 4 feet high?

27. The shadow of a pole 6 feet long is 9 inches, and the shadow of a steeple at the same time is 9 feet long. What is the height of the steeple?

28. What is the bank discount on a note for $245.30, due in 90 days, if discounted at 6%?

29. If a man takes 2 steps of 30 inches each in 3 seconds, how long will it take him to walk 10 miles?

30. It cost $150 to support 4 grown persons and 3 children 8 weeks. What will it cost to support 3 grown persons and 8 children for the same time, if 3 children cost as much as 2 grown persons?

31. A man bought 20 bushels of wheat and 15 bushels of corn for $36, and 15 bushels of wheat and 25 bushels of corn for $32.50? What did he pay per bu. for each?

32. A fox has 120 rods the start of a hound. If the hound runs 30 rods while the fox runs 26, how far will the hound run before he overtakes the fox?

33. A starts on a journey at the rate of 3 miles an hour. 6 hours afterward, B starts after him at the rate of 4 miles an hour. How far will B travel before he overtakes A?

34. One-fourth of a certain number is 10 more than $\frac{1}{5}$ of it. What is the number?

35. If to a certain number you add $\frac{1}{2}$ of itself and $\frac{1}{3}$ of itself, the sum will be 105. What is the number?

36. If to a certain number you add 15 more than $\frac{2}{3}$ of itself, the sum will be 40. What is the number?

37. How many days will it take 30 men to do a piece of work, which 20 men can do in 45 days?

38. If a man can earn $\frac{5}{8}$ of a dollar in $\frac{3}{4}$ of a day, how much can he earn in $\frac{8}{9}$ of a day?

39. How many yards of silk $\frac{3}{8}$ yard wide, will it take to line $4\frac{1}{4}$ yards of broadcloth $1\frac{3}{8}$ yards wide?

40. If 14 ounces of wool make $2\frac{1}{4}$ yards of cloth 1 yard wide, how much will it take to make $6\frac{1}{2}$ yards $1\frac{1}{4}$ yards wide?

41. How many tiles 14 inches long, will it take to make a drain which is $\frac{1}{8}$ of a mile long?

42. If $300 placed at interest yields an income of $18 in 9 months, how much must be placed at interest at the same rate to yield an income of $115 in 6 months?

43. If to a certain number you add $\frac{1}{4}$ of itself, the result will be 20 less than double the number. What is the number?

44. At what time between 5 and 6 o'clock will the hour and minute hands of a clock be exactly together?

45. Two soldiers start together for a certain fort. One, who travels 12 miles per day, after traveling 9 days, turns back as far as the other had traveled during those 9 days. He then turns and pursues his way toward the fort, where both arrive together 18 days from the time they set out. At what rate did the other travel?

46. A man agreed to execute a piece of work in 60 days,

and employed 30 men to perform the labor. At the end of 40 days it was only half finished. How many additional laborers was he obliged to employ to perform the work within the time agreed upon?

47. A person, being asked the time of day, replied that it was past noon, and that $\frac{3}{4}$ of the time past noon was equal to $\frac{3}{5}$ of the time to midnight. What was the time?

48. A gentleman wishes his son to have \$3000 when he is 21 years of age. What sum must be deposited at the son's birth, in a savings bank, which pays compound interest at the annual rate of 6%, so that the deposit shall amount to that sum when the boy becomes of age?

49. A note for \$100 was due on Sept. 1st, but on Aug. 11th the maker proposed to pay as much in advance as will allow him 2 mo. after Sept. 1st to pay the balance. How much must be paid Aug. 11th, money being worth 6%?

50. What sum must a person save annually, commencing at 21 years of age, so that he may be worth \$25000 when he is 40 years old, if he gets 6% compound interest on his money?

51. If a merchant sells $\frac{3}{4}$ of an article for what $\frac{7}{8}$ of it cost, what is his gain per cent.?

52. If goods are sold so that $\frac{5}{7}$ of the cost is received for half the quantity of goods, what is the gain per cent.?

53. A man sold a horse and carriage for \$597, gaining by the sale 25% on the cost of the horse and 10% on the cost of the carriage. If $\frac{3}{4}$ of the cost of the horse equaled $\frac{2}{3}$ of the cost of the carriage, what was the cost of each?

54. If 300 cats can kill 300 rats in 300 minutes, how many cats can kill 100 rats in 100 minutes?

55. A party of 8 hired a coach. If there had been 4 more the expense would have been reduced \$1 for each person. How much was paid for the coach?

56. I sold goods at a gain of 20%. If they had cost me

$250 more than they did, I would have lost 20% by the sale. How much did the goods cost me?

57. A laborer agreed to work for $1.25 per day and his board, paying $ .50 per day for his board when he was idle. At the end of 25 days he received $19. How many days was he idle?

58. A is 20 years of age; B's age is equal to A's and half of C's; and C's is equal to A's and B's together. What is the age of each?

59. A and B were partners in a profitable enterprise. A put in $4500 capital and received $\frac{3}{5}$ of the profits. What was B's capital?

60. A man spent $4 more than half his money traveling, one-half what he had left and $2 more for a coat, $6 more than half the remainder for other clothing, and had $2 left. How much money had he at first?

61. A boy bought at one time 5 apples and 6 pears for 28 cents, and at another time 6 apples and 3 pears for 21 cents. What was the cost of each kind of fruit?

62. A and B can do a piece of work in 20 days. If A does $\frac{3}{4}$ as much as B, in how many days can each do it?

63. A man bought a farm for $5000, agreeing to pay principal and interest in 5 equal annual installments. What will be the annual payment, including interest at 6%?

64. A carriage maker sold two carriages for $300 each. On one he gained 25%; on the other he lost 25%. Did he gain or lose by the sale? How much, and how much per cent.?

65. If a ladder placed 8 feet from the base of a building 40 feet high, just reached the top, how far must it be placed from the base of the building that it may reach a point 10 feet from the top?

66. Mr. A. is 35 years of age and his son is 10. How soon will the son be one-half the age of the father?

67. A person in purchasing sugar found that if he bought sugar at 11 cents he would lack 30 cents of having money enough to pay for it, so he bought sugar at $10\frac{1}{2}$ cents and had 15 cents left. How many pounds did he buy?

68. A farmer had his sheep in three fields. $\frac{2}{3}$ of the number in the first field was equal to $\frac{3}{4}$ of the number in the second field, and $\frac{2}{3}$ of the number in the second field was $\frac{3}{4}$ of the number in the third field. If the entire number was 434, how many were there in each field?

69. A and B can do a piece of work in 10 days, B and C can do it in 12 days, and A and C in 15 days. How long will it take each to do it?

70. A, B and C pasture an equal number of cattle upon a field of which A and B are the owners—A of 9 acres and B of 15 acres. If C pays $24 for his pasturage, how much should A and B each receive?

71. How many acres are there in a square tract of land containing as many acres as there are boards in the fence inclosing it, if the boards are 11 feet long and the fence is 4 boards high?

72. What is the greatest number which will divide 27, 48, 90, and 174, and leave the same remainder in each case?

73. A and B invested equal sums in business. A gained a sum equal to 25% of his stock, and B lost $225. A's money at this time was double that of B's. What amount did each invest?

74. A man at his marriage agreed that if at his death he should leave only a daughter, his wife should have $\frac{3}{4}$ of his estate; and if he should leave only a son, she should have $\frac{1}{4}$. He left a son and a daughter. What fractional part of the estate should each receive, and how much was each one's portion, if the estate was worth $6591?

TEST QUESTIONS.

612. Define a unit; a number. Explain the necessity for a uniform system of grouping objects. In how many ways may numbers be represented? Name them. Define numeration; notation; Arabic notation; Roman notation. Give the first principle of Arabic notation. Illustrate it. What is meant by "units of first order," etc.? Give the general principles of Arabic notation. What is meant by a period of figures? Give the names of the first seven periods. Give the rule for notation; for numeration. State how cents and mills are written in notation of U. S. money. What characters are employed in Roman notation? Give the principles of Roman notation.

Define addition; sum, or amount; equation; like numbers. Describe the sign of addition; the sign of equality. How many cases are there in addition? Show the truth of the principles of addition. Repeat the rule for addition. Why do we begin at the right to add? Why are the numbers of the same order written in the same column?

Define subtraction; minuend; subtrahend; remainder; difference. What is the sign of subtraction? What is it called? State the principles of subtraction. Show that they are true. Explain what is to be done when some figure of the subtrahend expresses more than the corresponding figure of the minuend.

Define multiplication; multiplicand; multiplier; product; factors of the multiplier; abstract number. Describe the sign of multiplication. Give the principles of multiplication. Show that they are true. Show that multiplication is a special case of addition. Repeat the rule for multiplication. What steps in the process are for convenience? How may you multiply when there are ciphers on the right of either or both factors?

Define division; dividend; divisor; quotient; remainder. What is the sign of division? In how many ways is division indicated? State the principles of division. Show that they are true. Show that division is a special case of subtraction. In how many ways may the remainder be expressed? Illustrate each way by an example. What is a fraction? What is meant by long division? What is meant by short division? Which should precede the other? Why? What steps in the process of division are for convenience? What are necessary? How may you proceed when there are ciphers on the right of either divisor or dividend? State the principles governing the relation of dividend, divisor, and quotient. Illustrate each by an example. Define analysis. Illustrate the process. Describe the parenthesis and vinculum, and show their uses.

Define and illustrate what is meant by an integer; exact divisor; factor; a prime number; a composite number; an even number; an odd number. Give eleven facts relating to exact divisibility of numbers. Illustrate each statement by an appropriate example. What is meant by factoring? Prime factors? What is an exponent? State the principles relating to the prime factors of numbers. Illustrate the truth of these principles by appropriate examples. Give the rule for finding the prime factors of a number. Explain the process of multiplying by factors. Show the use of this process. Show how to divide by factors. Explain how to find the true remainder in division by factors. Give the rule for dividing by the factors of a number.

What is meant by cancellation? Upon what principle is the process based? Illustrate the process.

Define what is meant by a common divisor; the greatest common divisor; numbers that are prime to each other. What is the principle underlying the greatest common divisor? Give the ordinary method of finding the greatest common divisor when the numbers are small. Solve an example, and give the analysis when the numbers can not be readily factored.

What is a multiple? Define what is meant by a common multiple; the least common multiple. State the principle upon which the processes in least common multiple are based. Solve an example showing the truth of the principle.

Define and illustrate what is meant by the terms fraction; unit of a fraction; fractional unit; the denominator; the numerator; the terms of a fraction; a proper fraction; an improper fraction; a mixed number; a common fraction; a decimal fraction. How are fractional expressions read? Interpret the expression $\frac{5}{7}$.

What is meant by reduction of fractions? What is Case I? When is a fraction reduced to larger or higher terms? Upon what principle does the process in Case I depend? What is Case II? What is meant by reducing a fraction to smaller or lower terms? To smallest or lowest terms? Upon what principle is the process in Case II based? What is Case III in reduction? Solve an example illustrating the process. What is Case IV? Solve an example illustrating the process. What is meant by similar fractions? Dissimilar fractions? When have similar fractions their least common denominator? Give the principles relating to the common and least common denominator of fractions. What is the rule for finding the least common denominator of several fractions?

What kind of fractions only can be added? Why? What must be done with dissimilar fractions before they can be added? How should mixed numbers be added? What kind of fractions only can be subtracted? What must be done to dissimilar fractions before they can be subtracted? How could mixed numbers be subtracted?

What is Case I in multiplication of fractions? What principle underlies the process? Demonstrate the truth of the principle. What is Case II? What is the principle? What is Case III? What is the general rule for multiplication of fractions? Solve and explain the following: Multiply $\frac{2}{3}$ by $\frac{5}{7}$.

What is Case I in division of fractions? What principle underlies the process? Show by an example that the principle is true. What is Case II? Give the rule for dividing an integer by a fraction. What is Case III? Solve the following: What is the value of $\frac{2}{5} \div \frac{3}{5}$? Give an analysis and explanation of the process. Give the general rule for division of fractions. Describe what are included among fractional forms How are they simplified? What is Case I in fractional relation of numbers? What is the principle upon which relation of numbers is based? What is Case II? Illustrate each case by an example.

What is a decimal fraction? From what is the word decimal derived? How are decimal fractions expressed? How are decimals distinguished from integers? State the principles of decimal fractions. Show each to be true. What is the decimal point? What other name has it? What is a pure decimal? What is a mixed decimal? What is a complex decimal? Name the orders of decimals as far as ten-millionths. How does the place occupied by any order of decimals compare with that occupied by integers of the corresponding name?

How are decimals reduced to a common denominator? Explain the process. How are common fractions reduced to decimals? Analyze the process. If a common fraction can not be exactly reduced to a decimal, what is done? How do addition and subtraction of decimals compare with the same processes in integers?

What is the principle upon which multiplication of decimals is based? Show that it is true. How may a decimal be multiplied by 1 with any number of ciphers annexed? What is the principle upon which the process of division of decimals is based? How may a decimal be divided by 1 with any number of ciphers annexed?

How may we multiply by a number that is a little less than a unit of the next higher order? How may we multiply when one part of the multiplier is a factor of another part? How may we multiply by a number that is a part of some higher unit? What is an aliquot part of a number? What are the common aliquot parts of 10? What of 100? How is the cost found when the quantity and price per 100 or 1000 are given?

What is a debt? Define what is meant by a credit; a debtor; a creditor; an account; the balance of an account; a bill; the footing of a bill. State some of the more common abbreviations used in business correspondence.

Tell what a concrete number is; an abstract number; a denominate number; a simple denominate number; a compound denominate number; a standard unit; a scale. Illustrate each of the preceding by an appropriate example. How many kinds of numerical scales are there?

What is money? Of how many kinds is it? What is coin, or specie? What is paper money? Give the table and denominations of the currency of the United States. What are the ordinary coins? What are the denominations and coins of Canada? Give the table of English money and the coins in common use. What are the currency and coins of France?

What is meant by reduction of denominate numbers? What is reduction descending? Give the rule. What is reduction ascending? Give the rule.

Define and illustrate what is meant by space, a line, a surface, a solid. For what are linear measures used? Repeat the table of Linear Measure, and of Surveyor's Linear Measure. What is an angle? A square? A square inch? A rectangle? What is the area of a surface? How is the area of a rectangular surface computed? Repeat the table of Square Measure, and of Surveyors' Square Measure. What is a solid? A cube? A cubic inch? A cubic foot? The volume or solid contents? How is the volume of a rectangular solid computed? Repeat the tables of Cubic Measure, and Wood and Stone Measure.

What are the measures of capacity? Recite the table of Liquid Measure. In estimating the capacity of cisterns, etc., how many gallons are considered a barrel? How many a hogshead? How many cubic inches are there in a gallon? Repeat the table of Apothecaries' Fluid Measure. For what is Dry Measure used? Repeat the table. How many cubic inches are there in a bushel?

What is weight? For what is Avoirdupois Weight used? Repeat the table. How many pounds are there in the long ton? How many grains are there in an avoirdupois pound? For what is Troy Weight used? Repeat the table. How many grains are there in a Troy pound? For what is Apothecaries' Weight used? Repeat the table. How many grains are there in a pound Apothecaries' Weight?

Repeat the table of Measures of Time. Explain how often leap year occurs. What is a circle? What is the circumference of a circle? An arc of a circle? A degree of the circumference? What is the measure of an angle? Repeat the table of Circular Measure. What a quadrant? A sextant? Give the Stationers' Table and the table of Counting. Give the cases in Reduction of Denominate Fractions. Solve an example illustrative of each case and explain the process.

How do the fundamental processes in Compound Denominate Numbers compare with the same processes in Simple Numbers?

How does the number of degrees apparently passed over by the sun compare with the number of hours occupied in passing that distance? The number of minutes of space with the number of minutes of time? The seconds of space with the seconds of time? Repeat the table showing the relation between longitude and time. What is a meridian? What is longitude? Give the rule for finding the difference in time when the difference in longitude of two places is given. Give the rule for finding the difference in longitude of two places when their difference in time is given.

What is the unit of length in the Metric System of measures? To what is it nearly equal? What is the metric unit of area? What the unit of solidity? What the unit of capacity? What the unit of weight?

Define per cent. What is the commercial sign of per cent.? Of what does Percentage treat? How may per cent. be expressed? What elements are involved in problems in Percentage? What is meant by the base? The rate? The percentage? The amount? The difference? What are the five fundamental problems or cases in

Percentage? Solve an example illustrating each, and give a rule for each case.

What is interest? Define the terms principal; amount; rate of interest; legal interest; usury; a note or promissory note. Give three methods for computing interest. What is compound interest? Give the rule for computing compound interest. How is the compound interest table formed? What is meant by annual interest? Give the rule for computing annual interest. In what respect does compound interest differ from annual interest?

What are partial payments? What is an indorsement? What is the Mercantile Rule for computing the amount due when partial payments have been made? When is the Mercantile Rule used? What is the principle upon which the United States Rule is based? Give the United States Rule. When the principal, rate, and interest are given, how is the time found? When the principal, time, and interest are given, how is the rate found? When the rate, time, and interest are given, how is the principal found?

What is a promissory note? Who is the maker or drawer? Who is the payee? Who is the holder? Who is the indorser? In how many ways may he indorse? What is the face of a note? When is a note negotiable? When is a note not negotiable? What are days of grace? Write a negotiable note and transfer it by indorsement.

What is discount? What is commercial discount? What is net price? What is the cash value of a bill? In cases where there is a discount of some per cent., as 20% off and 5% off for cash, upon what sum is the 5% computed? What is true discount? Define present worth. Give the rule for solving problems in true discount. What is a bank? A check? Bank discount? The proceeds or avails of a note? The maturity of a note? The term of discount? How is the bank discount computed? Is it right or wrong in principle? How can we find how large to make a note that we may have a certain sum left after paying the discount at a bank?

What elements in Profit and Loss correspond to the base, rate, percentage, amount, and difference? What is the principle upon which computations in Profit and Loss are based?

Define the terms commission merchant or agent; commission; a consignment; consignor; consignee; the net proceeds. What elements in commission correspond to base, rate, percentage, amount, and difference? Upon what principle is commission based?

What is a tax? What is real estate? What is personal property? What is a property tax? What is a personal tax? Who is an assessor? What is an assessment roll? Explain how taxes are levied and the individual taxes computed. Explain the formation of the assessor's table. What are duties or customs? What is meant by specific duty? Ad valorem duty? Tare? Leakage and breakage? Custom-houses?

What is meant by the terms, a company; a corporate company or corporation; a charter; capital stock; a share of stock; a certificate of stock; par value; above par; below par; market value? What is an installment? What is an assessment? What is a dividend? Describe a bond and coupons. How are Government securities designated? Name the various classes of Government securities, and state the rate of interest they bear. In what are all Government bonds payable? In what is the interest of all Government bonds payable? What are stocks? Who is a stock-broker? What is brokerage? What elements in the subject of stocks correspond to base, percentage, amount, and difference? What is meant by the expressions, stock is selling at $83\frac{1}{4}$, 112, etc.? If the market value and rate of premium or discount are given, how can the par value be found?

What is insurance? Of how many kinds is it? What is property insurance? What is a policy? What is the premium? Of how many kinds are insurance companies with regard to the parties who participate in the profits? What is a mutual insurance company? What is a stock company? What is a mixed company? What are the elements involved in insurance that correspond to base, rate, and percentage? What is personal insurance? What is a life policy? What an endowment policy? What an accident or health policy?

What is exchange? Explain the method. What is a draft or bill of exchange? Write a draft. How many parties are there connected with a draft primarily? Who is the drawer? The drawee? The payee? What is a sight draft? A time draft? What is meant by accepting a draft? Of how many kinds is exchange? What is domestic exchange? State and solve examples illustrating the rules. What is foreign exchange? What is a set of exchange? Upon what cities in Europe are drafts more commonly drawn? What is the value in United States gold coin of a sovereign? What is the value of a franc? Solve an example illustrating the principles of foreign exchange.

What is meant by averaging payments? The average time? The term of credit? The average term of credit? Solve an example illustrating the process of solution when the terms of credit begin at the same time. Solve an example when the terms of credit begin at different dates. Explain the process. Explain the process of averaging accounts, and give the rule.

What is partnership? Who are partners? What is the capital of a firm or company? Give the principle underlying partnership. What is Case I in partnership? What is Case II? Solve and analyze an example in Case II. Give the rule for partnership settlements.

What is ratio? Of how many kinds is it? What are the terms of a ratio? The antecedent? The consequent? What is the sign of ratio? From what may it be regarded as derived? What is a couplet? What are the principles of ratio? Illustrate each principle.

What is proportion? What is the sign of proportion? From what may it be regarded as derived? What are the antecedents of a proportion? The consequents? The extremes? The means? Give the principles of proportion regarding the relation of extremes and means. What is meant by a simple ratio? A simple proportion? A direct proportion? An inverse proportion? Give the rule for solving examples in simple proportion. What is a compound ratio? A compound proportion? The principle underlying compound proportion? Solve an example in compound proportion, by successive simple proportions and by cause and effect. Give the rule for compound proportion.

What is a power? How are powers named? What is an exponent? What is involution? How is the power of a number obtained? How is the square of a number expressed in terms of its tens and units? How in terms of any two parts? How is the cube of a number expressed in terms of its tens and units? How in terms of any two parts?

What is a root? How are roots named? What is evolution? What is the radical or root sign? Define a perfect power; an imperfect power. Give the rule for finding the root of a number by factoring. Give the principle relating to the number of figures required to express the square of a number; the cube of a number. Give the principle relating to the number of figures in the square root of a number; the cube root of a number. Solve and explain an example in square root. Repeat the rule. What is done when the number is not a perfect square? How are decimals pointed off? How is the square root of a common fraction found? How is the side of a square found when its area is given?

What relation do the squares described upon the sides of a right-angled triangle sustain to each other? How is the hypotenuse of a right-angled triangle found when the other sides are given? How is either side found when the hypotenuse and the other side are given? What are similar figures? What is the relation between similar surfaces?

Solve an example in cube root. Deduce from your solution a rule. What is done when there is a remainder after subtracting the last product? How are decimals pointed off? How is the cube root of a common fraction found? Give the principles relating to similar solids.

Define a series; an ascending series; a descending series; an arithmetical progression; a geometrical progression. How is the sum of an aritmetical series found? Illustrate by an example. How is the sum of a geometrical series found? Illustrate by an example.

Define mensuration; a line; a straight line; a curved line; parallel lines; a plane surface; a curved surface; an angle; a right angle; an acute angle; an obtuse angle; a vertex of an angle; a triangle; a quadrilateral; a parallelogram; a rectangle; a polygon; a circle; the circumference of a circle; a radius of a circle; a diameter of a circle; the base of a figure; the altitude of a figure; a diagonal of a figure; the perimeter of a figure; the area of a surface.

How is the circumference of a circle obtained from its diameter? The diameter from the circumference? How is the area of a parallelogram computed? How is the area of a triangle computed? Give the rule for computing the area of a trapezium; a trapezoid; a regular polygon; a circle.

What is a solid? A prism? A parallelopipedon? A cylinder? A pyramid? A cone? A frustum? A sphere? A diameter? A radius of a sphere? The circumference of a sphere? The altitude of a solid? The convex surface of a solid?

Give the rule for finding the convex surface of a prism or cylinder; a pyramid or cone; a frustum of a pyramid or cone; a sphere. Give the rule for finding the volume of a prism or cylinder; a pyramid or cone; a frustum of a pyramid or cone; a sphere.

ANSWERS

Page 28.

2. 947.
3. 1298.
4. $42.70.
5. $28.48.
6. 10845.
7. 20839.
8. 6283.
9. 3530.
10. 21974.
11. 20002.
12. $103.485.
13. $1437.845.
14. 933324.
15. 134739.
16. 349950638.
17. $132.94.
18. 1600.

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19. $14593.
20. 21502.
21. 9265.
22. 2775.
23. 8692.
24. 934.
25. 25917.

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26. 997.
27. 645621.
28. 528408.
29. $8484.
30. 7005 lib.; 1999881 vol.
31. 402399.
32. 377805.
33. 642502.

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34. 42151.
35. 118422.
36. 12149865.
37. $5165000.
38. 5871 period. 20842475 circ.

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39. $1595.85, Albany; $4807.37, N. Y.; $6403.22, both.
40. 45764.
41. 47388.
42. 40590.
43. 540356.
44. 8882.
45. 11031.
46. 109546.
47. 86672.

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2. 305.
3. 228.
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6. 879.
7. 61.
8. 919.
9. 288.
10. 1299.
11. 150.
12. $3.07.
13. $11.26.
14. $16.07.
15. $23.96.
16. $23.67.
17. 289.
18. 182.
19. 189.
20. 367.
21. 186.
22. 93.
23. 865.
24. 3886.
25. 5828.
26. 2131.
27. 8132.
28. 10570.
29. 20068.
30. 28879.
31. 80091.
32. 8486888.
33. 256 mi.
34. $508.

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35. 1163.
36. $131.
37. 1955.
38. 609973105.
39. 159 acres.
40. $202.12.
41. 1492.
42. $1697.
43. $9036.
44. 8687.
45. 3502.

Page 48.

2. 942.
3. 2272.
4. 3948.
5. 1725.
6. $4095.
7. $2307.

8. 1570.
9. 2620.
10. 5348.
11. 2490.
12. 4404.
13. $432.
14. $336.
15. $192.
16. $399.
17. $138.
18. $16555.
19. $33.75.
20. $250.16.

Page 49.

21. 3825.
22. 1568.
23. $16.02.
24. $43.75.
25. 36960.
26. $300.
27. $127.75.
28. $668.

Page 52.

2. 13650.
3. 11826.
4. 17664.
5. $64.63.
6. $114.48.
7. 5472.
8. 8947.
9. 12325.
10. 10098.
11. 101500.
12. 74803.
13. 231903.
14. 3485092.
15. 3942146.
16. 6568848.
17. 1344455.
18. 1332675.
19. 585575.
20. 1570688.
21. 3334932.
22. 2546650.
23. 23748222.
24. 25721944.
25. 62748374.
26. 47913214.
27. 66319209.
28. 92317008.
29. 76526998.
30. 82613817.
31. 143555004.
32. 291006415.
33. 281072720.
34. 1151707808.
35. 570873555.
36. 1381600350.
37. 1251756036.
38. 1253542212.

Page 53.

39. $1358.53.
40. $5211.45.
41. $13309.92.
42. $99253.80.
43. $152323.35.
44. $2332.99.
45. $69520.33.
46. $69577.43.
47. $213469.56.
48. $262816.86.
49. 58650.
50. 11024.
51. 260520.
52. 5022.
53. $48535.
54. $35105.
55. $5108066.

Page 54.

3. 3750;
37500;
15000;
112500.
4. 25350;
59150;
507000;
760500.
5. 88000;
123200;
70400;
176000.
6. 277000;
2770000;
3047000;
4570500.
7. 1215000;
15552000;
20412000;
31590000.
8. 655200;
11856000;
8424000;
14352000.

Page 55.

9. 2640000.
10. 48000.
11. $832.
12. $18256.

1. $32739.84.
2. $13730.50.
3. 1229.
4. 339904.
5. $4299.
6. $168.
7. 10293.
8. 4776.
9. $35.90.

Page 56.

10. 89232.
11. $247.52.
12. $169.25.
13. 10248.
14. 108 mi.
15. $635.14.
16. $17481.30.
17. $2066.95.
18. $1563.25.

Page 64.

11. 1218.
12. 1366.
13. 496.
14. 597.
15. 1545.
16. 984.
17. 406.
18. 876.
19. 554.
20. 1353.
21. 548.

22. 1234.
23. $913\frac{2}{3}$.
24. $1488\frac{1}{6}$.
25. $908\frac{3}{8}$.
26. $2296\frac{1}{7}$.
27. $1205\frac{1}{7}$.
28. $485\frac{5}{8}$.

Page 67.

2. 124.
3. 313.
4. 216.
5. 306.
6. 406.
7. 432.
8. 416.
9. 46.
10. 312.
11. 442.
12. 41.
13. 22.
14. 77.
15. 103.
16. 213.
17. 119.
18. 131.
19. 114.
20. 141.
21. 67.
22. 505.
23. 315.
24. 406.
25. 519.
26. 525.
27. 541.
28. 606.
29. 723.
30. 544.
31. 752.
32. 664.
33. 777.
34. 1802.
35. 1945.
36. 4372.
37. 6203.
38. $9216\frac{543}{643}$.
39. $5679\frac{350}{565}$.
40. $12474\frac{401}{555}$.
41. $21175\frac{499}{675}$.
42. $81987\frac{89}{856}$.
43. $771849\frac{173}{467}$.
44. $474536\frac{523}{1017}$.
45. $252384\frac{2544}{3118}$.
46. $12752\frac{30264}{50815}$.
47. $854104\frac{33783}{37123}$.

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48. 162.
49. $258.
50. 130 mi.
51. 18 doz.
52. $1093\frac{1018}{1116}$.
53. $5\frac{2700}{5280}$.
54. 140 da. 480 left.
55. $494\frac{119888}{185000}$.
56. \$$2029\frac{7}{8}$.
57. \$$10086\frac{62}{365}$.
58. 5280.
59. 100.
60. $21\frac{6616}{50701}$.

Page 70.

3. $18\frac{69}{100}$.
4. $61\frac{145}{200}$.
5. $127\frac{425}{700}$.
6. $39\frac{568}{900}$.
7. $2\frac{465}{1000}$.
8. $3\frac{1692}{4000}$.
9. $75\frac{505}{1100}$.
10. $50\frac{684}{1500}$.
11. $779\frac{18457}{50000}$.
12. 118; 4964 bu. rem.
13. 640.

Page 73.

19. $2546\frac{2}{5}$.
20. \$$42\frac{1345}{1419}$.
21. $81\frac{150}{1850}$ wk.
22. $60.
23. 1920 bu.
24. $2 per bbl.
25. 45 yd.
26. 500 mi.
27. 20 da.
28. 80 bu.
29 60 acres;
$400 loss.
30. $3837.50.
31. \$$92278\frac{1528}{2121}$.
32. 18 minutes.
33. 105 head.
34. 10 grandchild'n.
35. $3716\frac{4888}{6000}$ lb.
36. $21\frac{21248}{13500}$ acres.

Page 79.

2. 7, 5.
3. 2^6.
4. 2^4, 3, 7.
5. 2^3, 3, 7.
6. 2^4, 3^2.
7. 3^2, 5, 7.
8. 2, 3^2, 11.
9. 2^5, 7.
10. 2, 3, 131.
11. 2^2, 79.
12. 2^2, 11^2.
13. 2^3, 5.
14. 2^4, 3^2, 7.
15. 2^2, 3, 5, 19.
16. 2^5, 37.
17. 2^4, 3^2, 13.
18. 2^2, 3, 7^2, 13.
19. 2, 3, 5, 7, 11.
20. 2^2, 3^2, 89.
21. 3^3, 5^2, 7.
22. 7, 13, 43.
23. 2^2, 953.
24. 7^2, 11, 13.
25. 2^2, 5, 199.
26. 2^3, 5, 11, 61.
27. 2^2, 3, 11, 17^2.
28. 2^2, 3^4, 5, 7.
29. 2^3, 3, 7, 11, 13.
30. 2^2, 5^3, 37.
31. 2^9, 3^5.

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6. 13600;
15300;
20400;
30600.
7. 102144;
49248;
82080;
196992.

8. \$2240.
9. \$112.35.
10. \$12096.
11. \$225.
12. \$2208.
13. \$1320.
14. \$1620.
15. \$131.60.

Page 81.

9. 71.
10. 315.
11. 205.
12. 34.
13. 23.
14. 49.
15. $119\frac{16}{32}$.
16. $44\frac{28}{49}$.
17. $58\frac{27}{36}$.
18. $121\frac{9}{27}$.
19. $91\frac{9}{75}$.
20. $304\frac{8}{24}$.

Page 82.

21. 224 canisters; 32 packages.
22. 152 packages; 38 quires.

Page 83.

3. 7.
4. 2.
5. 3.
6. $2\frac{2}{3}$.
7. 4.
8. $3\frac{4}{7}$.
9. 4.
10. $4\frac{1}{2}$.
11. 10.
12. $186\frac{2}{3}$.

Page 84.

13. 7.
14. 16425.
15. 107800.
16. \$$43\frac{11}{18}$.
17. $7\frac{13}{36}$ bu.
18. \$3.12.

Page 86.

2. 4.
3. 9.
4. 12.
5. 6.
6. 12.
7. 9.
8. 16.
9. 9.
10. 7.
11. 8.
12. 12.
13. 5.
14. 6.
15. 7.
16. 4.
17. 11.
18. 24.
19. 42.

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2. 13.
3. 11.
4. 12.
5. 17.
6. 27.
7. 23.
8. 42.
9. 52.
10. 27.
11. 21.
12. 34.
13. 4.
14. 32.
15. 17.
16. 33.
17. 126.
18. 42.
19. 37.
20. 14 in.

Page 88.

21. 5 lbs.
22. 67 fields; 6 acres each.

Page 90.

2. 448.
3. 144.
4. 630.
5. 144.
6. 300.
7. 540.
8. 770.
9. 720.
10. 960.
11. 2835.
12. 6650.
13. 288.
14. 2240.
15. 700.
16. 41580.
17. 401115.

Page 91.

19. 360.
20. 1620.
21. 240.
22. 1440.
23. 900.
24. 85800.
25. 48.
26. 480.
27. 1200.
28. 20160.
29. 240.
30. 24 inches.
31. 60 yards.
32. 120.
33. 210 bu.
34. 360.

Page 92.

35. 120 days.
36. $436\frac{2}{15}$.
37. 12 feet.
38. 4.
39. 5275.
40. 1728.
41. \$336.
42. 2025.
43. 8 rods.
44. $41\frac{3}{5}$.
45. 7.
46. 378.

Page 98.

2. $\frac{33}{56}$.

3. $\frac{34}{60}$.
4. $\frac{60}{70}$.
5. $\frac{64}{84}$.
6. $\frac{32}{40}$.
7. $\frac{24}{54}$.
8. $\frac{48}{66}$.
9. $\frac{56}{84}$.
10. $\frac{45}{84}$.
11. $\frac{75}{120}$.
12. $\frac{32}{84}$.
13. $\frac{60}{77}$.
14. $\frac{108}{210}$.
15. $\frac{147}{210}$.
16. $\frac{90}{225}$.
17. $\frac{208}{318}$.
18. $\frac{426}{558}$.
19. $\frac{90}{235}$.

Page 100.

4. $\frac{2}{7}$.
5. $\frac{13}{14}$.
6. $\frac{61}{162}$.
7. $\frac{10}{11}$.
8. $\frac{7}{9}$.
9. $\frac{91}{180}$.
10. $\frac{23}{25}$.
11. $\frac{4}{5}$.
12. $\frac{44}{45}$.
13. $\frac{1}{12}$.
14. $\frac{4}{5}$.
15. $\frac{1}{2}$.
16. $\frac{37}{41}$.
17. $\frac{9}{13}$.
18. $\frac{39}{83}$.
19. $\frac{16}{17}$.
20. $\frac{19}{20}$.
21. $\frac{5}{42}$.
22. $\frac{42}{143}$.
23. $\frac{23}{791}$.

2. $\frac{21}{4}$.
3. $\frac{75}{5}$.
4. $\frac{79}{6}$.
5. $\frac{201}{11}$.

Page 101.

6. $\frac{46}{9}$; $\frac{92}{18}$.
7. $\frac{77}{12}$; $\frac{154}{24}$.
8. $\frac{115}{14}$; $\frac{345}{42}$.
9. $\frac{185}{20}$; $\frac{555}{60}$.
10. $\frac{142}{9}$.
11. $\frac{87}{7}$.
12. $\frac{204}{11}$.
13. $\frac{212}{9}$.
14. $\frac{181}{7}$.
15. $\frac{499}{17}$.
16. $\frac{531}{14}$.
17. $\frac{2312}{41}$.
18. $\frac{5455}{13}$.
19. $\frac{2737}{18}$.
20. $\frac{61822}{81}$.
21. $\frac{7136}{17}$.
22. $\frac{37286}{45}$.
23. $\frac{87701}{97}$.
24. $\frac{44290}{171}$.
25. $\frac{86695}{123}$.

Page 102.

2. \$5.
3. $12\frac{2}{11}$.
4. $10\frac{3}{5}$.
5. $16\frac{7}{8}$.
6. $12\frac{1}{6}$.
7. $21\frac{1}{5}$.
8. $7\frac{3}{8}$.
9. $23\frac{2}{15}$.
10. $13\frac{16}{19}$.
11. $27\frac{13}{47}$.
12. $15\frac{20}{193}$.
13. $61\frac{33}{258}$.
14. $18\frac{7}{8}$.
15. $14\frac{3}{22}$.
16. $18\frac{16}{37}$.
17. $129\frac{227}{242}$.
18. $84\frac{25}{324}$.
19. $58\frac{75}{406}$.
20. $97\frac{277}{394}$.
21. $16\frac{776}{1567}$.
22. $6\frac{2878}{3115}$.
23. $3\frac{6937}{8103}$.

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3. $\frac{30}{36}$; $\frac{16}{36}$; $\frac{21}{36}$.
4. $\frac{15}{60}$; $\frac{55}{60}$; $\frac{39}{60}$.
5. $\frac{36}{60}$; $\frac{21}{60}$; $\frac{26}{60}$.
6. $\frac{100}{140}$; $\frac{15}{140}$; $\frac{66}{140}$.
7. $\frac{72}{168}$; $\frac{105}{168}$; $\frac{84}{168}$.
8. $\frac{12}{54}$; $\frac{6}{54}$; $\frac{15}{54}$.
9. $\frac{72}{120}$; $\frac{102}{120}$; $\frac{95}{120}$.
10. $\frac{60}{144}$; $\frac{117}{144}$; $\frac{100}{144}$.
11. $\frac{102}{189}$; $\frac{357}{189}$; $\frac{91}{189}$.
12. $\frac{234}{450}$; $\frac{810}{450}$; $\frac{180}{450}$.
13. $\frac{420}{1225}$; $\frac{637}{1225}$; $\frac{875}{1225}$.
14. $\frac{325}{900}$; $\frac{140}{900}$; $\frac{182}{900}$.

Page 106.

3. $4\frac{187}{504}$.
4. $1\frac{9}{20}$.
5. $3\frac{91}{120}$.
6. $3\frac{9}{28}$.
7. $2\frac{79}{84}$.
8. $15\frac{7}{12}$.
9. $76\frac{31}{40}$.
10. $49\frac{113}{140}$.

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11. $2\frac{83}{140}$.
12. $32\frac{19}{63}$.
13. $41\frac{11}{60}$.
14. $29\frac{38}{85}$.
15. $8\frac{2603}{3036}$.
16. $23\frac{2215}{6669}$.
17. \$$246\frac{1}{10}$.
18. \$$115\frac{1}{8}$.
19. $143\frac{103}{105}$.
20. $36\frac{19}{20}$.

Page 108.

3. $\frac{11}{40}$.
4. $\frac{13}{83}$.
5. $\frac{87}{221}$.
6. $\frac{23}{60}$.
7. $\frac{13}{150}$.
8. $\frac{273}{800}$.

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9. $\frac{3}{10}$.
10. $\frac{1}{8}$.
11. $\frac{1}{10}$.
12. $\frac{5}{48}$.
13. $\frac{11}{40}$.
14. $\frac{3}{140}$.
15. $10\frac{17}{66}$.
16. $33\frac{1}{15}$.
17. $100\frac{17}{18}$.
18. $36\frac{1}{2}$.
19. $536\frac{1}{4}$.

20. $521\frac{11}{12}$.
21. $88\frac{1}{6}$.
22. \$3.
23. $222\frac{1}{2}$ sold;
$189\frac{1}{2}$ left.
24. \$$19\frac{3}{5}$.

Page 110.

2. $1\frac{4}{11}$.
3. $4\frac{1}{2}$.
4. $1\frac{3}{5}$.
5. $\frac{9}{13}$.
6. $21\frac{7}{11}$.
7. 6.
8. $\frac{7}{8}$.
9. $1\frac{3}{7}$.
10. $2\frac{1}{8}$.
11. $3\frac{1}{3}$.
12. $4\frac{12}{23}$.
13. $5\frac{1}{5}$.
14. $5\frac{3}{13}$.
15. $8\frac{28}{37}$.
16. $14\frac{2}{17}$.

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17. \$$5\frac{2}{3}$.
18. \$6.
20. $86\frac{2}{5}$.
21. \$$84\frac{1}{2}$.
22. $154\frac{1}{5}$.

Page 112.

2. $3\frac{1}{2}$.
3. $2\frac{2}{3}$.
4. $4\frac{3}{4}$.
5. $6\frac{1}{3}$.
6. 12.
7. 99.
8. 15.
9. $24\frac{15}{16}$.
10. \$$76\frac{8}{25}$.
11. \$87.
12. \$$549\frac{1}{17}$.
13. \$$421\frac{2}{3}$.
14. \$4710.
15. 88.
17. 198;
$237\frac{3}{5}$;
272;
$320\frac{8}{11}$;
370.

Page 114.

2. $\frac{35}{72}$.
3. $\frac{13}{56}$.
4. $\frac{19}{36}$.
5. $\frac{5}{14}$.
6. $\frac{7}{22}$.
7. $\frac{1}{3}$.
8. $\frac{7}{15}$.
9. $\frac{14}{31}$.
10. $\frac{7}{32}$.
11. $\frac{5}{117}$.
12. $\frac{81}{2200}$.
13. $\frac{7}{120}$.
14. $\frac{3}{28}$.
15. $\frac{119}{440}$.
16. $\frac{12}{65}$.
17. $\frac{4}{27}$.
18. $\frac{29}{684}$.
19. $\frac{3}{8}$.
20. $\frac{5}{77}$.
21. $\frac{20}{81}$.
22. $\frac{1}{6}$.
24. $3\frac{1}{35}$.
25. 27.
26. $31\frac{1}{2}$.
27. 147
28. \$$\frac{3}{4}$.
29. $90\frac{3}{4}$.
30. $\frac{3}{4}$.
31. $534\frac{3}{8}$.
32. \$81.

Page 115.

33. $1\frac{47}{385}$.
34. $32\frac{272}{441}$.

Page 116.

2. $\frac{3}{32}$.
3. $\frac{1}{9}$.
4. $\frac{4}{29}$.
5. $\frac{17}{72}$.
6. $\frac{3}{37}$.
7. $\frac{3}{75}$.
8. $\frac{9}{25}$.
9. $\frac{9}{86}$.
10. $\frac{7}{72}$.
11. $\frac{5}{144}$.
12. $\frac{1}{42}$.
13. $\frac{1}{50}$.

Page 117.

15. $2\frac{14}{15}$.
16. $6\frac{5}{17}$.
17. $4\frac{59}{72}$.
18. $2\frac{71}{99}$.
19. $3\frac{13}{20}$.
20. $3\frac{31}{49}$.
21. $\frac{1}{8}$.
22. $\frac{7}{640}$.
23. \$$4\frac{25}{64}$.
24. \$$\frac{3}{5}$.
25. \$$11\frac{5}{42}$.
26. \$$1072\frac{7}{27}$.

Page 118.

2. 42.
3. 72.
4. $11\frac{1}{7}$.
5. 56.
6. 80.
7. $86\frac{1}{9}$.
8. 60.
9. 130.
10. 120.
11. $40\frac{4}{5}$.
12. $19\frac{1}{2}$.
13. $93\frac{1}{3}$.
14. 49.
15. 68.
16. 69.
17. 123.
18. 60.
19. 111.
20. 305.
21. 145.
22. 128.
23. $63\frac{9}{13}$.
24. $48\frac{9}{25}$.
25. $21\frac{3}{17}$.

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27. $4\frac{2}{7}$.
28. $3\frac{9}{20}$.
29. $3\frac{11}{14}$.
30. $5\frac{39}{43}$.

31. $4\frac{23}{43}$.
32. $6\frac{7}{11}$.
33. 20 yd.
34. $21\frac{3}{5}$ bu.
35. 20.
36. $\$9\frac{14}{15}$.
37. $\$\frac{9}{22}$.
38. $\frac{656}{833}$ lb.

Page 121.

7. $\frac{3}{7}$
8. $1\frac{2}{5}$.
9. $\frac{4}{7}$.
10. $\frac{19}{21}$.
11. $\frac{2}{3}$.
12. $1\frac{1}{21}$.
13. $1\frac{10}{21}$.
15. $1\frac{9}{16}$.
16. $\frac{5}{27}$.
17. $2\frac{1}{3}$.
18. $\frac{7}{65}$.
19. $2\frac{23}{24}$.
20. $2\frac{3}{16}$.
21. $\frac{11}{14}$.
22. $7\frac{7}{10}$.
23. $3\frac{3}{4}$.
24. $26\frac{1}{4}$.
25. $93\frac{23}{31}$.

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26. 46.
27. $7\frac{7}{8}$.
28. $8\frac{8}{27}$.
29. $1\frac{2}{5}$.
30. $13\frac{19}{25}$.
31. $\frac{5}{14}$.
32. $1689\frac{8}{9}$ bu.

Page 123.

2. $\frac{5}{63}$.
3. $\frac{9}{10}$.
4. $\frac{7}{12}$.
5. $1\frac{5}{18}$.
6. $\frac{7}{11}$.
7. $\frac{1}{14}$.
8. $7\frac{7}{8}$.
9. $1\frac{17}{112}$.
10. $\frac{64}{75}$.
11. $2\frac{7}{120}$.
12. $\frac{14}{19}$.
13. $11\frac{5}{17}$.
14. $\frac{5}{34}$.
15. $\frac{1}{5}$.
16. $6\frac{3}{7}$.
17. $\frac{1}{26}$.

Page 124.

19. $\frac{3}{20}$.
20. $\frac{2}{65}$.
21. $\frac{1}{16}$.
22. $\frac{5}{48}$.
23. $\frac{2}{45}$.
24. $\frac{3}{175}$.
25. $\frac{1}{27}$.
26. $\frac{1}{27}$.
27. $\frac{5}{72}$.
29. $\frac{5}{8}$.
30. $\frac{24}{17}$.
31. $\frac{21}{44}$.
32. $\frac{35}{32}$.
33. $\frac{13}{16}$.
34. $\frac{15}{12}$.
35. $\frac{21}{45}$.
36. $\frac{30}{28}$.
37. $\frac{27}{56}$.

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13. $\frac{15}{17}$; $\frac{11}{8}$.
14. $\frac{2}{5}$; $\frac{14}{19}$.
15. $\frac{7}{5}$, 1.
16. $\frac{6}{5}$; $\frac{7}{5}$.

1. $\$6\frac{3}{4}$.
2. $71.30.
3. $2\frac{1}{2}$ acres.

Page 126.

4. $64\frac{1}{16}$ cents.
5. $6\frac{4}{9}$.
6. $\$.17\frac{24}{83}$.
7. $37\frac{1}{5}$.
8. $\$925\frac{74}{105}$ gain.
9. $163\frac{7}{20}$.
10. $2066\frac{11}{14}$.
11. $153\frac{1}{3}$.
12. $9\frac{5}{13}$.
13. $2850, son;
$\$2533\frac{1}{3}$, daughter.
14. $\$89\frac{3}{7}$.
15. 16 bu.
16. $\frac{11}{14}$.

Page 127.

17. 222.
18. $\$6\frac{153}{285}$.
19. $\frac{1}{4}$.
20. $\frac{5}{21}$, A's, or $\$4363\frac{1}{3}$;
$\frac{10}{21}$, B's, or $\$8726\frac{2}{3}$.
21. $105.
22. $\$3\frac{17}{56}$.
23. $12\frac{19}{27}$.
24. $\$11354\frac{2}{3}$.
25. $\$15986\frac{1}{4}$.
26. $\$24\frac{29}{107}$, first;
$\$28\frac{78}{107}$, second.
27. $2600, one's;
$3900, other's.
28. $\$5797\frac{1}{2}$.

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29. $\frac{1}{10}$, A;
$\frac{1}{8}$, B;
$\frac{9}{40}$, both;
$4\frac{4}{9}$ da.
30. $3\frac{1}{13}$ da.
31. 30 da.
32. 40, shorter;
84, longer.
33. $43.
34. $26\frac{8}{17}$ da.
35. 45 ft.; 30 ft.
36. 60 and 80.
37. $\$8269\frac{1}{2}$.
38. $2700, A;
$3000, B.
39. $48.
40. 28 da., A;
21 da., B.

Page 136.

2. $\frac{27}{500}$.
3. $\frac{31}{800}$.
4. $\frac{9}{160}$.
5. $\frac{7}{16}$.
6. $4\frac{1}{80}$.
7. $\frac{871}{2000}$.
8. $\frac{1}{2000}$.

9. $\frac{1}{2}$.
10. $\frac{177}{500}$.
11. $\frac{1}{100}$.
12. $\frac{43}{800}$.
13. $\frac{3253}{50000}$.
14. $\frac{227}{400}$.
15. $3\frac{27}{125}$.
16. $\frac{603}{1250}$.
17. $\frac{1849}{2500000}$.
19. $\frac{1}{8}$.
20. $\frac{1}{5}$.
21. $\frac{1}{6}$.
22. $\frac{7}{8}$.
23. $\frac{11}{250}$.
24. $\frac{3}{80}$.
25. $\frac{9}{16}$.
26. $\frac{3}{800}$.
27. $\frac{783}{10000}$.
28. $\frac{7}{20000}$.
29. $2\frac{3781}{5000}$.
30. $13\frac{193}{500}$.

Page 137.

2. .25.
3. .6.
4. .625.
5. .375.
6. .8.
7. .75.
8. .0625.
9. .15.
10. .85.
11. .52.
12. .35.
13. .3833 +.
14. .555 +.
15. .3846 +.
16. .233 +.
17. .1406 +.
18. 2.625.
19. .024.
20. .1875.
21. .03906 +.
22. .42857 +.
23. .4545 +.
24. .3157 +.
25. .23809 +.
26. 15.625.
27. 24.6.
28. .825.
29. 3.425.
30. .23625.
31. .625.
32. .875.
33. .4375.
34. 4.2155 +.
35. 37.54.
36. 20.06.
37. .0001625.

Page 138.

2. 8.497.
3. 8.7907.
4. 8.914.
5. 72.379.
6. .40035.
7. 117.766.

Page 139.

8. $108.455.
9. $68.19.
10. $5394.267.
11. 30.975.
12. $32.1875.
13. .850955.
14. $77.7155.
15. $1102.345.
16. $48.365.

Page 140.

2. 24.903.
3. 25.964.
4. $24.875.
5. $74.875.
6. .20695.
7. 2.0232.
8. 99.96154.
9. .0000756.
10. 79999.92.
11. $11.94.
12. $17.705.
13. $4.105
14. $2519.98.
15. $76468.06.

Page 142.

2. .2210.
3. 2.025.
4. .20056.
5. 153.12.
6. 2.2272.
7. 25.752.
8. 74.375.
9. .020265.
10. 1822.5.
11. 5.8776.
12. 15.1296.
13. 34.4576.
14. $23.375.
15. $167.485.
16. 122.994.
17. 28.0685.
18. $10.12.
19. 2680.804.
20. 1137.19424.
21. 2.837025.
22. 145.81944.
23. 4.0596288.
24. 272.80767.
25. 1.0725.
27. 38464; 3846.4; 384640.
28. 184.65; 1846.5; 18465.
29. $108.675.

Page 143.

30. $5212.84.
31. 391.05 rd.
32. $34631.37.
33. $60.6125.
34. $477.375.
35. $1706.16.
36. $71.445.

Page 145.

2. .25.
3. .005.
4. 1.6895 +.
5. .00365.
6. 76.3.
7. 30.2.
8. 2.13.
9. .15.
10. 3650.

11. 27500.
12. 2643.6923 +.
13. .21.
14. 10020.
15. .84472 +.
17. .4826.
18. .382457.
19. .0138542.
20. .04897.
21. .00006045.
22. .384563.
23. 3.5575 + T.
24. 180 doz.
25. $3.04.
26. 23 hhd.
27. 37 stoves.

Page 146.

2. 381744.
3. 4822336.
4. 48518433.
5. 8794128.
6. 345684825.
7. 47235605.
8. 80952576.
9. 5862014256.

Page 147.

10. $373.45.
11. $1318.08.
12. $13483.

2. 240975.
3. 555489.
4. 999498.
5. 1474812.
6. 3511158.
7. 9398184.
8. 14850395.
9. 1318044.
10. 1583232.
11. 1564191.
12. 2415987.
13. 10158948.
14. 87855880.
15. 107378577.

Page 149.

3. 8600.
4. 6700.
5. 126400.
6. 2521000.
7. 420200.
8. 81000.
9. 282400.
10. 546600.
11. $6.75.
12. 274.66\frac{2}{3}$.
13. $162.
14. $107.625.
15. $246.25.
16. $438.625.

Page 150.

2. 21.26\frac{1}{4}$.
3. $40.89 +.
4. $9.14 +.
5. 755.77\frac{1}{2}$.
6. $356.85 +.
7. $22.668 +.
8. 41.06\frac{1}{4}$.
9. $18.645.
10. $72.875.
11. $71.64.
12. $56.25.
13. $63.085.
14. $264.158.

Page 153.

2. $293.06.
3. $2935.00.

Page 154.

4. $2393.60.
5. $26.20.
6. $921.36.
7. $181.88.

Page 155.

1. 363$\frac{1}{2}$ lb.
2. 127.16975 lb.
3. 2075$\frac{29}{32}$ sq. ft.
4. 36.153 + cu. ft.
5. 2150420.
6. 1000.
7. 10000000.
8. 261907.8.
9. 16.36\frac{1}{4}$.
10. 277$\frac{7}{9}$ lb.
11. 1568.96 + lb.
12. $292.25.
13. $437.50.
14. 5200 lb.
15. 2000000.
16. .00000002.
17. 25.818 + da.
18. $174.725.
19. 102.56 + wk.
20. 689.83\frac{3}{4}$.
21. $14449.50.
22. $4.728.
23. $100000.
24. .2169 +.
25. $.75$\frac{4}{15}$.
26. .35.
27. 241.95, A;
    120.975, B;
    282.275, C.

Page 161.

2. 606d.
3. 265s.
4. 4148 far.
6. 90d.
7. 210d.
8. 26$\frac{2}{3}$ far.
9. 109$\frac{1}{11}$d.
10. 106s.;
    5088 far.
11. 598 far.
12. 1793d.
13. 843 far.
14. 7s. 6d.
15. 5436 far.
16. 8480d.
17. 43383 far.
18. 28724 far.

Page 163.

2. 7s. 2$\frac{1}{4}$d.
3. £22 16s.
4. £6 12s. 2d.
5. £4 0s. 6d.
7. £$\frac{3}{100}$.
8. $\frac{1}{112}$s.
9. £1 12s.
10. £157 6s.

11. £14 19s 8d.
12. 80s $1\frac{1}{2}$d.
13. £242 16s.
14. £6 4s. 4d .
15. £16 12s.
16. 97s. $7\frac{1}{4}$d.
17. £50 11s. $9\frac{3}{4}$d.
18. 3572 far.
19. £50 12s. 5d.
20. 14784 far.
21. $72.9975.
22. £93 14s. $\frac{1}{2}$d.
23. £81 1s. 1d.
24. £7 14s. $1\frac{1}{4}$d.
25. 121.66\frac{1}{4}$.
26. 14880 far.
27. £200.
28. £250.

Page 166.

13. 2 mi.
14. 8903 yd.
15. 1452 in.
16. 443520 in.; 570240 in.
17. $19\frac{9}{11}$ rd.
18. 186 rd.
19. $12\frac{3}{4}$ mi.
20. 952204 in.
21. 509716 in.
22. 1 mi. 15 rd. 3 yd. 1 ft. 4 in.
23. 2 mi. 162 rd. 4 yd. 2 ft.

Page 167.

24. 41775360 ft.
25. 5 ft.
26. 271 rd. 8 l.
27. 47322 in.

Page 169.

10. 12111 sq. in.
11. 12044056860 sq. in.
12. 6 A. 4 sq. rd. 26 sq. yd. 2 sq. ft.
13. 14003316 sq. in.
14. 4 sq. rd. 21 sq. yd. 1 sq. ft. 89 sq. in.

Page 170.

16. 88 sq.rd. 26 sq.yd. 8 sq. ft.
17. $\frac{5}{8}$; $\frac{1}{2}$; $\frac{3}{4}$.
18. 11 sq. yd. 3 sq. ft. $13\frac{1}{2}$ sq. in.
19. 77 sq. in.
20. 120 sq. ft.
21. 42 sq. yd.
22. 25 sq. ft.
23. 48 sq. yd.; $55.20, cost.
24. 34 yd.
25. $79.80.
26. 165 ft. long; $35392.50, cost.
27. 80 rd.

Page 171.

28 $472.50.
29. 90 sq. ft.
30. $35.20.
31. $48.
32. $85\frac{1}{3}$ sq. yd., sides; 34 sq. yd., ceiling; $44.23, cost.
33. $21.39.

Page 173.

1. 3456; 5184; 25920; 55296.
2. 54; 81; 351; 675.
3. 640; 1024.
4. $16\frac{5}{8}$ perch; $29.56, cost.
5. $9\frac{7}{33}$.
6. $207\frac{11}{27}$ yd.
7. 55410 cu. in.
8. 369 cu. ft.
9. $384\frac{3}{4}$ cu. ft.
10. 13824 blocks.
11. $157\frac{1}{2}$ cu. ft.

Page 174.

12. 10.31\frac{1}{4}$.
13. 225 bu.
14. $348.44, excav'g; $418.18, wall.
15. 44517.

1. 24 ft.
2. $13\frac{3}{4}$ ft.
3. $10\frac{5}{6}$ ft.
4. $16\frac{1}{4}$ ft.

Page 175.

5. 180 ft.
6. $37.312.
7. $53.90.
8. 27.56\frac{1}{4}$.
9. 21.09\frac{3}{8}$.

Page 176.

4. 296 pt.
5. $17.10; $85\frac{1}{2}$ gal.
6. 120 gal. 1 pt. 2 gi. 608 gal. 2 qt. 1 pt.
7. 135 gi.
8. 4224 gi.; 60 gal. 2 qt.
9. 192 pt.; 8 bbl. 12 gal. 1 qt. 3 gi.
10. 1617.
11. $16\frac{59}{77}$.
12. 284 bbl. 30.623 gal.

Page 177.

4. 251 pt.
5. 525 pt.
6. 526 bu. 1 pk. 5 qt.

Page 178.

7. 218 bu. 6 qt. 1 pt.
8. 1792 bu. 2 pk. 4 qt.
9. 2301 pt.
10. 15052.8; 17203.2; 21504; 43008.
11. 6.4388 +; 16.6336 +; 22.6446 +.
12. 483840 cu. in.; 225 bu.
13. 260.357 +.

Page 179.

4. 6205.
5. 10216.

6. \$10.875.
7. \$2.52.
8. \$42.24.

Page 180.

9. 100; 70; 98; 25.
10. \$4.165.
11. \$3.25.
12. \$9.885.
13. 54.88 lb.
14. \$54.32$\frac{9}{16}$.
15. \$12.
16. 982$\frac{1}{4}$.
17. 54 bags.

Page 181.

3. 3498.
4. 7 oz. 4 pwt.
5. \$74.25.
6. 8 spoons; 1 oz. left.
7. 564 powders.

Page 183.

8. 18912.
9. 23258.
10. 13 hr. 31 min. 35 sec.
11. 10 hr. 41 min. 37 sec.
12. 2628000.
13. 120 da.
14. 197 da.
15. 469 hr.
16. 7815 min.
17. 4 da. 10 hr. 50 min.
18. 54738 sec.
19. 3 wk. 1 da. 21 hr. 25 mins.

Page 185.

2. 126000 sec.; 97200 sec.; 76338 sec.
3. 123163 sec.
4. 130° 9′ 20″.
5. 106° 48′ 20″.
6. 648000; 324000; 216000.
7. 10800; 7200.

Page 186.

2. 288; 492.
3. 1728.
4. \$16.20.
5. 70 yr.
6. \$9.

Page 187.

8. 7 oz. 4 pwt.
9. 8 cwt. 57 lb. 2$\frac{2}{3}$ oz.
10. 26 rd. 3 yd. 2 ft.
11. 43 sq. rd. 19 sq. yd. 2 sq. ft. 36 sq. in.
12. 6 qt. 1 pt. 1$\frac{1}{3}$ gi.
13. 7 hr. 12 min.
14. 18 sq. yd. 8 sq. ft. 22$\frac{1}{4}$ sq. in.
15. 9 cu.ft. 777$\frac{3}{5}$ cu.in.
17. $\frac{4}{45}$ pt.
18. $\frac{3}{4}$ ft.
19. $\frac{20}{27}$ sc.
20. .384 pt.

Page 188.

22. 11s. 6d.
23. 2 oz. 6 pwt. 10.56 gr.
24. 8 qr. 17.28 sh.
25. 1 pt. 1.904 gi.
26. 2 ft. 2.73 in.
27. 145 rd. 9 ft. 10.8 in.
28. 1 pk. 7.4624 qt.
29. 2 mi. 16 rd. 10 ft. 6.72 in.

8. $\frac{5}{396}$ yd.
9. $\frac{1}{8100}$ hr.
10. $\frac{21}{2920}$ yr.
11. $\frac{7}{40000}$ T.
12. $\frac{1}{4032}$ cu. ft.

Page 189.

13. $\frac{3}{24200}$ A.
14. $\frac{5}{2268}$ bbl.

6. $\frac{8}{11}$.
7. $\frac{19}{47}$.
8. $\frac{7}{48}$.
9. $\frac{5}{48}$.
10. $\frac{225}{1512}$.

Page 190.

12. .17708 + da.
13. .8125 bu.
14. .2121 + rd.
15. £$\frac{887}{960}$.
16. .9239 + £.
17. $\frac{267}{1600}$ cwt.
18. .1183 + mi.
19. .4597 + wk.
20. .4312 + reams.
21. .4296 + C.
22. .3717 + mi.
23. $\frac{2101}{5760}$ lb.

1. \$4.805.
2. \$2.34.
3. \$19.68$\frac{3}{4}$.
4. \$525.50.
5. 21 lb. 4 oz.
6. \$12.375.
7. 11$\frac{1}{4}$ C.; \$47.81$\frac{1}{4}$.
8. 7.01822 ft.

Page 191.

9. \$.1772+ per oz. T.
10. 2.6658+ A.; \$399.87.
11. 13$\frac{8}{9}$ da.
12. \$1.728.
13. 3200 mi.
14. 1810.
15. \$1.45 per ream.
16. 30 lb. 12.7 + oz.
17. \$113.13.
18. 4248 posters; \$27.612, cost.
19. 2.6277 bbl.
20. 3 bu. 3 pk. 1 qt. 1 pt. too much.
21. 67500 lb.

Page 193.

2. 99 lb. 1 oz. 15 pwt.
3. £319 0s. 8$\frac{1}{4}$d.

4. 37 mi. 121 rd. 1 yd. 1 ft. 1 in.
5. 25 T. 8 cwt. 40 lb. 6 oz.
6. 47 gal. 1 pt.
7. 115 bu. 3 pk. 4 qt.
8. 101 C. 125 cu. ft. 518 cu. in.
10. 98 yd. 2 ft. $4\frac{1}{2}$ in.
11. 2 A. 19 sq. rd. 15 sq. yd. 1 sq. ft. 18 sq. in.
12. 109 gal. 1 qt. 1 pt.

Page 194.

13. 15 yr. 7 mo. 18 da.
14. 22 T. 17 cwt. 46 lb. $3\frac{25}{63}$ oz.

2. 316 rd. 1 yd. 1 ft. 8 in.

Page 195.

3. 6 cwt. 95 lb. 2 oz.
4. 41 gal. 1 qt. 1 pt. 3 gi.
5. 4 lb. 11 oz. 17 pwt.
6. 8° 50′ 48″.
7. 18 C. 7 cd. ft. 15 cu. ft.
9. 47 sq. rd. 12 sq. yd. 3 sq. ft. 141 sq. in.
10. £86 16s. $8\frac{3}{4}$d.
11. 148 A. 147 sq. rd. 12 sq. yd. 5 sq. ft. 63 sq. in.
12. 6 R. 10 qr. 7 sh.
14. 8 yr. 2 mo. 12 da.

Page 196.

15. 66 yr. 8 mo. 13 da.
16. 10 yr 7 mo. 20 da.
17. Feb. 13, 1841.
18.
19. 3 yr. 11 mo. 28 da.
20. 5 yr. 3 mo. 1 da.

Page 197.

2. 69 gal. 3 qt. 1 gi.
3. 159 lb. 1 oz. 12 pwt. 15 gr.
4. 12 T. 3 cwt. 53 lb. 12 oz.
5. 127 A. 15 sq. rd.
6. 116 rd. 1 yd. 1 ft.
7. 98 C. 3 cd. ft. 8 cu. ft.
8. £32 19s. 9d.
9. $49.95.

Page 198.

2. 2 bu. 7 qt. 1 pt.
3. 85 A. 90 sq. rd. 6 sq.yd. $64\frac{4}{5}$ sq.in.
4. 78 gal. 3 qt. 1 pt. 2 gi.
5. 1 T. 17 cwt.
6. 2 C. $118\frac{2}{3}$ cu. ft.
7. £3 2s. $6\frac{4}{5}$d.
9. 50 times.
10. 4 da. 6 hr. 6 min. 58 + sec.
11. 23 mi. 24 rd. 10 ft. $1\frac{1}{11}$ in.
12. $27\frac{73}{235}$ bbl.
13. 65 spoons; 1 oz. 5 pwt. left.
14. 100 pickets.

Page 201.

3. 2 hr. 9 min. $13\frac{9}{15}$ sec.
4. 122° 26′ 45″ W.
5. 5 hr. 5 min. 32 sec.
6. 5 hr. 8 min.
7. 11 min. 48 sec. past 12 o'clock noon.
8. 16° 17′.
9. 53 min. 50 sec. A. M. Jan. 2.
10. 17° 45′ east.

Page 206.

8. 40.4671 + A.
9. $\frac{1}{2}$ hectolitre.
10. 5864.31 dm.
11. $71.315.
12. $.257 + per. gal. loss.
13. At $3 per metre; $.156 + per yd.
14. 15.96 M.
15. $1\frac{1}{7}$ M.
16. $46.067 +.
17. $3500.
18. 88.904 + kilos.
19. 18 hectares; $4500.
20. 625 hectolitres.
21. $29.918.
22. 11 cents per lb.; $11.34.
23. $10.05.

Page 208.

1. $\frac{1}{10}$; .10.
2. $\frac{1}{8}$; .125.
3. $\frac{1}{5}$; .20.
4. $\frac{1}{4}$; .25.
5. $\frac{3}{10}$; .30.
6. $\frac{3}{4}$; .75.
7. $\frac{7}{8}$; .875.
8. $\frac{1}{32}$; .03125.
9. $\frac{1}{16}$; .0625.
10. $1\frac{1}{4}$; 1.25.
11. $\frac{3}{200}$; .015.
12. $\frac{1}{3}$; $.33\frac{1}{3}$.
13. $\frac{1}{6}$; $.16\frac{2}{3}$.
14. $2\frac{1}{8}$; 2.125.
15. $\frac{5}{16}$; .3125.
16. $\frac{2}{3}$; $.66\frac{2}{3}$.
17. $\frac{3}{16}$; .1875.
18. $\frac{5}{24}$; $.20\frac{5}{6}$.
19. $\frac{3}{400}$; .0075.
20. $\frac{1}{200}$; .005.
21. $\frac{9}{200}$; .045.
22. $\frac{3}{1000}$; .003.
23. $\frac{73}{1000}$; .073.
24. $\frac{3}{8}$; .375.

Page 210.

13. $7.61.
14. $6.44.
15. 155 gal.
16. 306 mi.
17. 300 sheep.

18. \$300.
19. \$1293.36.
20. \$302.64.
21. \$37500.
22. \$2625.
23. \$23200.
24. \$22940, elder; $12333.33\frac{1}{3}$, y'nger.

Page 212.

16. $16\frac{2}{3}\%$.
17. $42\frac{6}{7}\%$.
18. $31\frac{11}{19}\%$.
19. $55\frac{5}{9}\%$.
20. 54%.
21. $66\frac{2}{3}\%$.
22. $38\frac{3}{8}\%$.
23. $88\frac{8}{9}\%$ spent; $11\frac{1}{9}\%$ left.
24. $33\frac{1}{3}\%$.
25. $\frac{1}{8}\%$.
26. $27\frac{3}{11}\%$.
27. $1\frac{1}{2}\%$.

Page 213.

28. $1\frac{1}{5}\%$.
29. 25%; 20%; $18\frac{1}{3}\%$.

Page 214.

9. 3080.
10. 2450.
11. $833\frac{1}{3}$.
12. 21.45.
13. \$532.50.
14. 343.75 bu.
15. 2100 men.
16. 3300.
17. $366\frac{2}{3}$.
18. \$15000.
19. \$100000.
20. 1102 A. 110 sq. rd.
21. \$1600.
22. $\$1882\frac{6}{17}$.
23. \$8000.
24. \$20000.
25. \$2700.
26. \$40206.
27. \$20000000.
28. 20.
29. $213\frac{1}{3}$.

Page 216.

9. 400.
10. 369.
11. 282.
12. 560.
13. \$14000.
14. \$300.
15. \$1500.
16. \$6000.
17. 800.

Page 217.

10. 525.
11. 720.
12. $633\frac{1}{3}$.
13. $426\frac{7}{15}$.
14. \$1800.
15. 500 bu.
16. $\$41.77\frac{1}{2}$.
17. 620 soldiers.
18. \$50000.
19. \$13400.

Page 221.

14. \$2.26.
15. \$5.92.
16. \$8.99.
17. \$645.
18. \$735.17.
19. \$773.256.
20. \$979.87.
21. \$737.76.
22. \$376.309.
23. \$15.96.
24. \$2.60.
25. \$6.19.
26. \$17.948.
27. \$5.55.
28. \$6.28.
29. \$8.788.
30. \$17.89.
31. \$624.546.
32. \$185.08, interest. \$969.33, amount.

Page 222.

2. \$64.844.
3. \$39.584.
4. \$5.38.
5. \$3.96.
6. \$9.71.
7. \$6.19.

Page 223.

8. \$8.61.
9. \$8.95.
10. \$7.917.
11. \$6.55.

Page 224.

2. \$15.22.
3. \$76.826.
4. \$44.91.
5. \$18.835.
6. \$41.54.
7. \$25.12.
8. \$47.525.
9. \$46.03.
10. \$38.61.
11. \$197.88.
12. \$92.35.
13. \$66.29.
14. \$39.807.
15. \$131.29.
16. \$152.40.
17. \$335.938.
18. \$339.817.

Page 226.

2. \$49.55.
3. \$408.71.
4. \$112.95.
5. \$65.97.
6. \$139.73.
7. \$54.61.
8. \$84.43.
9. \$64.89.
10. \$5.87.
11. \$368.96.

Page 228.

2. \$475.44.
3. \$962.32.

Page 229.

1. \$435.55.

2. \$402.88.
3. \$91.45.

Page 231.

2. \$497.80.
3. \$439.277.
4. \$4841.52.
5. \$3986.35.
6. \$219.31.
7. \$15.086.

Page 232.

8. \$3022.56.
9. \$621.056.
10. \$833.818.

Page 233.

4. 2 yr.
5. 6 mo.
6. 2 yr.
7. 4 yr. 8 mo. 14 da.
8. 5 yr. 4 mo. 13 da.
9. $16\frac{2}{3}$ yr.
10. $12\frac{1}{2}$ yr.
11. 20 yr.; $16\frac{2}{3}$ yr.; $14\frac{2}{7}$ yr.
12. 40 yr.; $33\frac{1}{3}$ yr.; $28\frac{4}{7}$ yr.

Page 234.

5. 6%.
6. 7%.
7. $5\frac{5}{8}$%.
8. 7%.
9. 7%.
10. 7%.
11. 6%.
12. 7%.
13. $12\frac{1}{2}$%.

3. \$305.
4. \$178.50.
5. \$186.18.
6. \$1473.62.
7. \$1399.28.
8. \$1571.43.
9. \$10714.28.
10. \$4125.
11. \$376.518.

12. \$297.26.

Page 237.

3. \$802.75.
4. \$663.
5. \$2443.74.
6. \$3290.625.
7. \$1778.65.

Page 239.

2. \$894.95, present worth; \$80.54 discount.
3. \$777.195, present worth; \$68.005 discount.
4. \$792.355, present worth; \$166.395 disc't.
5. \$464.717, present worth; \$111.533 disc't.
6. \$7698.254, pres't worth; \$876.746 disc't.
7. \$3948.26, present worth; \$325.73 disc't.
8. \$2279.79, present worth; \$565.20 disc't.
9. \$1586.99, present worth; \$165.75 disc't.
10. \$4489.07, present worth; \$1004.43 disc't.
11. \$3306.30, present worth; \$151.54 disc't.
12. \$15547.169.
13. \$28.44 gain.
14. \$560.68.

Page 240.

15. \$142.52.
16. \$296.97.
17. \$42.85.
18. \$241.94.
19. \$7441.018.
20. \$1300.12 gain.

Page 242.

2. \$4.137.
3. \$37.393.
4. \$43.748.
5. \$22.16.
6. \$96.78.
7. \$95.14.
8. \$49.41.
9. \$8.67.
10. \$3.179 discount; \$378.383 proc'ds.
11. \$10.90 discount; \$879.34 proceeds.
12. \$3.94+.
13. \$138.91 discount; \$15587.04 proc'ds.

Page 243.

14. \$3729.79.
15. \$563.80.

Page 244.

6. \$1000.
7. \$2000.
8. \$987.09.
9. \$1015.74+.
10. \$1408.11+.
11. \$1262.29.
12. \$5299.46.
13. \$1937.56.
14. \$507.09.
15. \$15316.54.

Page 245.

1. \$17500.
2. \$140.81 in favor of 4% discount.
3. \$8168.80.
4. \$4137.93.
5. \$4663.39.
6. \$231.77.
7. \$6.06+.
8. \$1969.93.

Page 246.

9. \$1186.65.

10. \$267.07.
11. \$8800.
12. \$9422.22.
13. 27.
14. \$9 per bbl.
15. \$24.31.
16. \$180.81.
17. \$100000.

Page 248.

14. \$50.
15. \$180.

Page 249.

2. \$343.75.
3. \$358.40.
4. \$397.81$\frac{1}{4}$.
5. \$51.75.
6. \$5.40.
7. \$96.88$\frac{3}{4}$.
8. \$6480.
9. \$3450.
10. \$1605.
11. \$272.50.
12. \$2.795.

Page 250.

13. \$74.75.
14. \$422.15$\frac{5}{8}$.
15. \$4.80.
16. .325; .65; .975; 1.1375.
18. 33$\frac{1}{3}$%.
19. 25%.
20. 28$\frac{4}{7}$%.
21. 20%.
22. 25%.
23. 11$\frac{1}{9}$%.
24. 12$\frac{36}{97}$%.

Page 251.

25. 2$\frac{1}{2}$%.
26. \$2.296 per box;
    \$.875 gain.
27. 11$\frac{1}{9}$%.
28. 66$\frac{2}{3}$%.
29. 12+%.
30. For cash at once;
    by 3$\frac{54}{107}$%.
31. 42$\frac{6}{7}$%, A's gain;
    50$\frac{50}{133}$%, B's gain.
32. 51$\frac{6}{13}$%.

Page 252.

34. \$1, cost;
    \$1.10, sell'g price.
35. \$9.
36. \$1.
37. \$.40.
38. \$.55.
39. \$100.
40. \$300.
41. \$2.
42. \$1.15.
43. \$37472.

Page 253.

46. \$.40.
47. \$40000.
48. \$160869.56.
49. \$.10.
50. \$5373.
51. \$.50.
52. \$2000.
53. \$4.12$\frac{3}{51}$.
54. \$3.
55. \$1.55$\frac{5}{8}$.
56. \$5.50.
57. 20%.

Page 255.

2. \$87.56.
3. \$180.

Page 256.

5. \$3932.03.
6. 14297.3 + yd.
7. \$50.05.
8. \$39.21$\frac{1}{4}$.
9. \$19.125.

Page 257.

10. \$8.91.
11. \$25.088.
12. \$7.084.
13. \$34.828.
14. \$1495.09.
15. 4557.03 + yd.
16. \$3255.
17. \$3670.
18. 8000 bu.;
    \$66.75 commis'n.
19. \$1226.
20. \$1274.625.
21. 2$\frac{1}{2}$%.
22. \$2776.19.

Page 258.

1. \$9.
2. 25%.
3. 10%.
4. 33$\frac{1}{3}$%.
5. 125%.
6. \$3.10.
7. \$400.
8. \$1.50.
9. 6%.
10. 1$\frac{1}{4}$ yr.
11. 50%.
12. 42$\frac{6}{7}$%.
13. Pay cash; 1$\frac{2}{51}$%.
14. 28$\frac{4}{7}$%.

Page 259.

15. \$40.
16. 6 bbl.
17. 50%.
18. Sell now at 20c.;
    by \$10.48+.
19. \$350, cost of the horse;
    \$175, cost of the carriage.
20. \$477.01.
21. \$15.30 gain by borrow'g at 6%.
22. 595.1 + bu.
23. \$200.
24. \$125, A's cost;
    \$100, cost to me.

Page 260.

25. \$20407.50.
26. \$25777.89.
27. \$3288.38.
28. Lost \$6; 4%.
29. Neither.

30. 18.96 + %.
31. $73.229 gain.
32. $120, selling price of horse;
$45, selling price of cow.

Page 261.

33. A, $200;
B, 133\frac{1}{3}$;
C, 53\frac{1}{3}$.
34. 40 cents per lb.
35. $4424.79+
36. 9$\frac{1}{11}$%.
37. $6.
38. $4.50.
39. $6267.49;
20000 lb. wool.
40. $1500.

Page 264.

3. 73.12\frac{1}{2}$, A;
89.17\frac{1}{2}$, B;
$63.75, C.

Page 265.

4. $12.211 tax;
.003591 + rate.
5. .002272 + .
6. .007587 + rate;
$83.457 tax.
7. $3826.53, sum to be assessed;
$1168406+, value of property.

Page 266.

2. $178.125.
3. $2.55.
4. $450.
5. $533.12.
6. $5248.246.
7. $3322.512.

Page 273.

2. $8593.75.
3. $8670.
4. $1595.
5. $5418.75.
6. $288.75.
7. $11407.50.

Page 274.

9. 25 shares.
10. 290 shares.
11. 600 shares.
12. 34.6 shares.
13. 22.11 + shares.
15. $209.234.
16. $622.707.
17. 5% at 60;
by $16.666.
18. $641.25.
19. 6% at 90;
by $1.04.

Page 275.

20. $109.62 dimin'd.
21. $13500.
22. $15208.33.
23. $26812.50.
24. $32776.04.
25. $12675.

Page 276.

27. 6$\frac{3}{5}$%.
28. 10%.
29. 6% stock at 10% discount.
30. N. Y. 7's; $\frac{80}{147}$%.
31. 8$\frac{1}{17}$%.
33. 214$\frac{2}{7}$%.
34. 25%.
35. 70%.
36. 85$\frac{5}{7}$%.
37. 225%.
38. $9941.20.
39. $7812.03.

Page 277.

40. $4694.84.
41. $6888.36.
42. 6$\frac{1}{4}$%.
43. 11$\frac{1}{9}$%.
44. Gained $36.73.
45. Loss $26.19.

Page 280.

2. $187.50.
3. $67.50.
4. $270.
5. $1359.375, prem.;
$107390.625, loss.
6. $515.125.
7. $55000.
8. 2%.
9. $\frac{1}{2}$%.
10. $14000.
11. 1$\frac{1}{2}$%.
12. $9000.
13. $2668.75, loss of the merchant;
$22331.25, insurance co.'s loss.
14. 13333.33\frac{1}{3}$.

Page 281.

15. $88400.
16. $297000.
17. 6666.66\frac{2}{3}$, stock;
13333.33\frac{1}{3}$, store.
18. $15228.42.

Page 282.

2. $93.90.
3. $145.09.
4. $3699.50.
5. $1423.45 loss.
6. $1423.30 loss.
7. $465 loss.

Page 286.

3. $1005.
4. $3037.50.
5. $4978.75.
6. $1471.
7. $4987.50.
8. $3003.75.
9. $4928.75.

Page 287.

10. $1486.25.
11. $4952.50.
14. $5710.72.
15. $1506.40.

16. \$1213.04.

Page 288.

17. \$10012.51.
18. \$3514.93.
19. \$1747.81.

Page 289.

2. £571 4s. $6\frac{1}{4}$d.
3. £732 6s. $2\frac{3}{4}$d.
4. £1149 6s. $7\frac{1}{4}$d.
5. £1061 7s. $5\frac{3}{4}$d.
6. \$668.87.
7. \$1832.25.

Page 290.

8. 7621.67 fr.
9. \$1169.77.
10. \$4650.
11. \$3063.14.
12. \$4484.11.

Page 292.

2. 2 mo. 29 da.
3. 2 mo. 18 da.
4. 1 mo. 12 da.
5. 2 mo. 10 da.
6. $3\frac{1}{4}$ mo.

Page 294.

2. June 20, 1877.
3. May 2, 1877.
4. Jan. 24, 1877.
5. Dec. 22, 1876.
6. April 23, 1877.
7. June 7, 1877.
8. Aug. 16, 1877.

Page 296.

2. Aug. 22, 1877.
3. Mar. 21, 1877.

Page 297.

4. June 19, 1877.
5. June 15, 1877.
6. July 5, 1877.
7. \$1198.60.

Page 300.

2. A, \$2100; B, \$2100; C, \$1800.
3. A, \$600; B, \$1200; C, \$800.
4. A, \$2000; B, \$1600; C, \$2400.
5. A, \$200; B, \$160; C, \$280.
6. A, \$600; B, \$750; C, \$675; D, \$975.
7. D, \$1600; G, \$2000; L, \$1800.
8. E, \$862.50; F, \$575; G, \$862.50.
9. A, \$2744.78; B, \$2299.33; C, \$1446.63.
10. A, \$74.86; B, \$86.84; C, \$104.81; D, \$203.63.

Page 301.

11. A, \$1200 gain; B, \$1600 gain; C, \$7000 stock.
12. A, \$335.365; B, \$402.439; C, \$536.585; D, \$670.731; E, \$804.878.
13. A, \$750; B, \$1000; C, \$1250.

Page 302.

2. A, \$2880; B, \$3600; C, \$2880.
3. A, \$640; B, \$840; C, \$840.
4. B, \$705; C, \$740.25; D, \$1057.50.
5. A, \$426.505; B, \$621.987; C, \$426.505.
6. A, \$2550; B, \$3400; C, \$2550.
7. A, \$1080; B, \$1600; C, \$1820.
8. G, \$561.702; L, \$702.127; F, \$936.170.

Page 310.

3. \$125.
4. \$8.75.
5. \$45.
6. $26\frac{1}{4}$ T.
7. $24\frac{6}{11}$.
8. $4\frac{8}{15}$.
9. 12 men.
10. $9\frac{3}{15}$ da.
11. $36\frac{54}{81}$ bu.
12. 1000 bbl.
13. $40\frac{4}{9}$ da.

Page 311.

14. 520 bu.
15. $2\frac{17}{54}$ A.
16. \$1628.25.
17. 65 da.
18. $9792\frac{2}{9}$ lb.
19. $2307\frac{1}{2}$ mi.
20. $162\frac{45}{112}$ mi.
21. $427\frac{1}{2}$ rd.
22. $4\frac{28}{29}$ hr.
23. 20 da.

Page 313.

2. $28\frac{1}{8}$ da.
3. 17820 lb.
4. \$6875.

5. \$710.76 + .
6. $19\frac{17}{117}$ da.

Page 314.

7. \$594.
8. \$902.77.
9. 473 yd.
10. $811\frac{1}{8}$ bu.
11. $25\frac{2}{26}$.
12. 21 da.
13. 27600 lb.
14. 18750 lb.
15. $21\frac{1}{8}$ T.
16. $15\frac{3}{4}$ da.
17. $546\frac{2}{15}$ ft.

Page 316.

2. 1728; 12167; 59319; 13824.
3. 2209; 2601; 841; 1156.
4. 225; 1089; 576; 1296; 625.
5. 21952; 91125; 5832; 9261; 68921.
7. $\frac{8}{27}$; $\frac{27}{125}$; $\frac{125}{729}$; $\frac{216}{1331}$; $\frac{343}{2197}$.
8. $\frac{625}{2401}$; $\frac{729}{4913}$.
9. 50625.
10. 15625.
11. 27000.
12. .00000625.
13. .000000125.
14. 4.2025.
15. $\frac{144}{289}$.
16. $\frac{225}{529}$.
17. $91\frac{1}{8}$.
18. $641\frac{7}{9}$.
19. $9.0200\frac{1}{5}$.
20. $20.251285\frac{36}{49}$.
21. 10000; 512; 729.

Page 324.

3. 53.
4. 63.
5. 66.
6. 96.
7. 266.
8. 344.
9. 821.
10. 886.
11. 969.
12. 2424.
13. 3546.
14. 5555.
15. 472; 3375.
16. .874; .5555.

Page 325.

17. .306.
18. .315.
19. $\frac{235}{886}$.
20. $\frac{884}{989}$.
21. $\frac{942}{987}$.
22. .70710 + .
23. .86602 + .
24. .79056 + .
25. .9486 + .

1. 25 ft.
2. 45 rd.
3. 52 ft. wide; 104 ft. long.
4. 480 rd.
5. 240 rd.
6. \$44.

Page 327.

2. 25 ft.
3. 113.137 ft.
4. 40 ft.
5. 122.474 ft.
6. 140.584 mi.

Page 328.

7. 75 rd.
8. 119.482 ft.
9. 205.704 ft.
10. 172.046 ft.
11. 386.003 ft.

Page 329.

2. 25.1328.
4. 63.245.
5. 12 ft.
6. 100 rd. length; 10 rd. breadth.
7. 15.94268 rd.

Page 333.

3. 42.
4. 64.
5. 55.
6. 89.
7. 57.
8. 63.
9. 177.
10. 126.
11. 536.
12. 1.259+.
13. 2.0800+.
14 .6463+; .8617+.
15 .8735+; .8434+; $\frac{17}{248}$.

1. 45 ft.
2. 24 in.

Page 334.

3. 13 ft.
4. 10.75 ft.
5. 10.81 ft.
6. 21.50 ft.
7. 12.9 + in.
8. 19.37 in.
9. \$19.51.
10. 3.46 + ft., width; 10.38 + ft., length.
11. Rectangle; 143.55 + sq. ft.
12. 39.37 in.

Page 335.

2. 10 ft.
3. \$3375.
4. 10.75 ft.
5. 2.38.
6. Twice as great.
7. 27 times.
8. 1st, .506; 2d, .721; 3d. 2.773.
9. 23.48.

Page 338.

2. 55.
3. 198.
4. \$1.72.

5. $209\frac{1}{12}$ ft.
6. 5070.
8. 3775.
9. 505.
10. 330 mi.
11. 78.
12. $3360.

2. 2430.

Page 340.

3. 10240.
4. $984.15.
5. $133.82+.
6. $696.849.
8. 1364.
9. $4\frac{141}{516}$.
10. 4.

Page 343.

1. 31.416 ft.
2. 141.372 ft.
3. 2 mi. 302.48 rd.
4. 125.664 rd.

Page 344.

5. 34.557 ft.
6. 101.38 rd.
7. 204.354 rd.

1. 520 sq. ft.
2. $25\frac{3}{4}$ sq. ft.
3. 720 sq. rd.
4. 525 sq. ft.

Page 345.

1. 216 sq. ft.
2. 126 sq. ft.
3. $168.376.
4. 5935.85 sq. ft.
5. $8.64.
6. 15000 sq. ft.

Page 346.

1. 5500 sq. ft.
2. 54 sq. rd.
3. 576 sq. ft.
4. $546.875.
5. $2062.50.

Page 347.

1. 19.635 sq. ft.
2. 50.265 sq. ft.
3. 1145.9 sq. rd.
4. 795.77 sq. ft.
5. 50.929 A.
6. 706.86 sq. rd.
7. 7.136 rd.
8. 12 rd.
9. 2 acres 33.3939 sq. rd.
10. 962.115 sq. ft.

Page 350.

1. 31.416 sq. ft.
2. 40 sq. ft.
3. 144 sq. ft.
4. 37.6992 sq. ft.
5. 63 sq. ft.

Page 351.

1. 540 sq. ft.
2. 376.992 sq. ft.
3. 628.32 sq. ft.
4. $64.
5. 89.5356 sq. ft.
6. 400 sq. ft.
7. 75.3984 sq. ft.
8. 157.08 sq. ft.

Page 352.

1. 251.328 sq. ft.
2. 3000 sq. ft.
3. $5.654.
4. 340 sq. ft.
5. $26.88.

1. 4.908 sq. ft.
2. 1.396 sq. ft.
3. 26.50+ sq. in.

Page 353.

4. 45.836 sq. ft.

1. 2 cu. ft.
2. 7.0686 cu. ft.
3. $9.
4. 462.857+ bu.
5. 2632.089+ gal.
6. $4013.837.

Page 354.

1. 84.8232 cu. ft.
2. 18000 cu. ft.
3. 12440.736 lb.
4. 7296 lb.

1. $4666\frac{2}{3}$ cu. ft.

Page 355.

2. 236.405 cu. ft.
3. 136.136+ cu. ft
4. 6415.718 gal.

1. 65.45 cu. ft.
2. 268.0832 cu. ft.
3. 14.1372 cu. ft.
4. 795.217+ lb.
5. 8.181 cu. ft.
6. 8181.25 cu. ft.

Page 356.

1. 10 da.
2. $12.
3. 80 men.
4. $5\frac{2}{3}$ mi.
5. 16 men.
6. $$5\frac{1}{4}$.
7. $75.
8. 336 lb.
9. $125.635.
10. $13\frac{1}{3}$ oz.
11. 60 sheep.

Page 357.

12. $80.
13. $5\frac{25}{31}$.
14. $40.33 loss.
15. 451 trees.
16. $48, A's money; $40, B's money.
17. $65, A's; $50, B's; $55, C's.
18. $48\frac{1}{13}$ ft.
19. A's, 105 lb.; B's, 140 lb.; C's, 245 lb.

20. A's, \$32;
B's, \$27.
21. 34, A's age;
$45\frac{1}{3}$, B's age;
$56\frac{2}{3}$, C's age.

Page 358.

22. 720 apples.
23. $18\frac{298}{307}$ da., time in which all can do it;
$58\frac{82}{99}$ da., time in which A can do it;
$70\frac{14}{83}$ da., time in which B can do it;
$46\frac{74}{125}$ da., time in which C can do it.
24. $6\frac{3}{7}$ da.
25. \$240, one;
\$150, other.
26. 810 revolutions.
27. 72 ft.
28. \$3.80.
29. 8 hr. 48 min.
30. \$$208.33\frac{1}{3}$.
31. \$1.50, wheat;
\$ .40, corn.
32. 900 rd.

Page 359.

33. 72 mi.
34. 200.
35. $57\frac{3}{11}$.
36. 15.
37. 30 da.
38. \$$\frac{20}{27}$.
39. $15\frac{7}{12}$ yd.
40. $50\frac{5}{9}$ oz.
41. 566 tiles.
42. \$2875.
43. $26\frac{2}{3}$.
44. $27\frac{3}{11}$ min. past 5.
45. 6 mi. per day.
46. 30 men.

Page 360.

47. 20 min. past 5.
48. \$882.46.
49. \$74.07.
50. \$740.52 +.
51. $16\frac{2}{3}$% gain.
52. $42\frac{6}{7}$%.
53. \$270, carriage;
\$240, horse.
54. 300 cats.
55. \$24.
56. \$500, cost.

Page 361.

57. 7 da.
58. B's age, 60 yr.;
C's age, 80 yr.
59. \$3000.
60. \$80.
61. 2 cts., apples;
3 cts., pears.
62. A, $46\frac{2}{3}$ da.;
B, 35 da.
63. \$1186.98.
64. \$40 loss;
$6\frac{1}{4}$%.
65. 27.64 ft.
66. 15 yr.

Page 362.

67. 90 lb.
68. 162 in No. 1;
144 in No. 2;
128 in No. 3.
69. A, in 24 da.;
B, in $17\frac{1}{7}$ da.;
C, in 40 da.
70. A's share, \$3;
B's share, \$21.
71. 92160 A.
72. 21.
73. \$600.
74. \$4563, son's;
\$1521, widow's;
\$507, daughter's.

$\frac{480}{437\frac{1}{2}} = \frac{875}{2}$

9. - 191 -

$\frac{75}{100} \div \frac{875}{2} = \frac{75}{100} \times \frac{2}{875} = \frac{3}{1750}$

$\frac{1}{480} - \frac{3}{1750} = \frac{175}{84000} - \frac{144}{84000} = \frac{31}{84000}$

$\frac{31}{84000} \times \frac{480}{1} = \frac{1488}{8400} = .1772$

$\frac{31}{175} = .177142 +$

6. 212 - Ex 24 1700 - 550 = 150 ; 3750 = 24

210 24 = 15500 × 24 = 2 2946

213 27 - $\frac{250}{1000} - \frac{200}{1000}$ - 25 % 2

$25 + 20 = 45$; $100 - 45 = 55$ } $55 \times \frac{1}{3} = 18\frac{1}{3}\%$

4 - 206 = 3937.079 ÷ 36 = 109.3633 y

109.3633 × 2.25 = 246.067425

400 × 2 = 200

246.067 - 200 = 46.067 ans

38 Ex. - Q. 128

$3\frac{3}{3} = \frac{2}{3}$ of $

$\frac{1}{5} = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

$\frac{5}{5} = \frac{10}{9}$ A

$B = \frac{9}{9}$

A & B $= \frac{10}{9} + \frac{9}{9} = \frac{19}{9} =$ $57

$\frac{1}{9} = 300$ or $\frac{1}{19}$ of 5700

$\frac{10}{9} =$ $3000 $\frac{9}{9} =$ $2700

P = 15 % Ex ÷ 3

Ex = 40 % of 15 % = 6 % of sales

Prof = 15 % $1000 ÷ .09 = $11000 = sale of goods

Ex = 6 %

9 % = Excess of Prof over Ex

37 ex - P. 261

$\frac{7}{8} = \$7$ | $1\ \frac{4}{3} = \$8$

$\frac{1}{8} = 1$ | $1\ \frac{1}{3} = 2$

$\frac{8}{8} = \$8$ ask p. 1 $\frac{3}{3} = \$3$

38 ex P. 261 $\frac{10}{15} - \frac{4}{15} = \$80$

A $\frac{3}{3}$

B $= \frac{2}{3}$ | $\frac{6}{15} = 80$

C $= \frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$ | $1 = \$13\frac{1}{3}$

$\frac{15}{15} = \$200$ | $\frac{10}{15} = \$133\frac{1}{3}$ | $\frac{4}{15} = \$53$

34 - P. 261

.285
20
.05700
285
.342

95% = .342
1% = .0036
100% = .36

90% = .36
1 = .004
100 = .40

84 @ 36 2 "Saved $\frac{1}{2}$ on 1 lb = once 45
$11 - 30$ he would have to buy as many
$10\frac{1}{2} - 15$ lbs as $\frac{1}{2}$ is contained in 45
$\frac{1}{2} - 45$
$\frac{2}{2} - 90$ lbs - ans

What a sly bad boy to jump quick
over an old lazy dog.

MILNE'S PRACTICAL ARITHMETIC

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